DOKUZ EYLÜL UNIVERSITY
GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

POPULATION PROPORTION ESTIMATOR IN MODIFIED RANKED SET SAMPLING METHODS

by
Aylin GÖÇOĞLU

July, 2018
İZMİR

POPULATION PROPORTION ESTIMATOR IN MODIFIED RANKED SET SAMPLING METHODS

A Thesis Submitted to the
Graduate School of Natural And Applied Sciences of Dokuz Eylül University
In Partial Fulfillment of the Requirements for the Master of
Science in Statistics, Statistics Program

by
Aylin GÖÇOĞLU

July, 2018
İZMİR

M.Sc THESIS EXAMINATION RESULT FORM

We have read the thesis entitled "**POPULATION PROPORTION ESTIMATOR IN MODIFIED RANKED SET SAMPLING METHODS**" completed by **AYLİN GÖÇOĞLU** under supervision of **ASSOC. PROF. DR. NESLİHAN DEMİREL** and we certify that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

Assoc. Prof. Dr. Neslihan DEMİREL

Supervisor

Prof. Dr. Selma GÜRLER

Jury Member

Assoc. Prof. Dr. Ali Mert

Jury Member

Prof. Dr. Latif SALUM
Director
Graduate School of Natural and Applied Sciences

ACKNOWLEDGEMENTS

I would like to express my sincere thanks to my supervisor Assoc. Prof. Dr. Neslihan Demirel for her support, patience, guidance, advice, encouragements throughout the research. It was a great privilege and honor to work and study under her guidance. I am extremely grateful for what she has offered me.

I owe a special thanks to Ayça ÖLMEZ, sleepless nights we were working together before deadline, for all the fun we have had and your priceless support in the last two years. I would like to thanks to Yusuf Can SEVİL for our fruitful discussions and essential contributions and I also wish to thanks to Tolga YAMUT and Sami AKDENİZ for support.

I would like to thank my family who always believed in me and supported me to chase my goals.

Aylin GÖÇOĞLU

POPULATION PROPORTION ESTIMATOR IN MODIFIED RANKED SET SAMPLING METHODS

ABSTRACT

Ranked set sampling (RSS) is becoming an alternative sampling method in recent years. RSS method is used where sample units can be easily ranked, but where the exact measurement of sample units is time consuming, difficult or expensive. It is proved that the sample mean of RSS is always more efficient than the sample mean of SRS of the same sample size. Modified ranked set sampling methods have been developed for mean estimator to reduce the ranking errors of the units selected from the target population in RSS method. There are many studies for mean estimators based on modified RSS methods, but few contributions for proportion estimators to RSS research. In the literature, to estimate the population proportion and the variance of the proportion estimator were used only PRSS from the modified ranked set sampling methods. In this thesis, the proportion estimators and associated variance estimators are obtained for binary variable with a concomitant variable based on modified ranked set sampling methods which are Extreme Ranked Set Sampling (ERSS), Median Ranked Sampling (MRSS), Percentile Ranked Set Sampling (Per-RSS) and L Ranked Set Sampling (LRSS) methods. The Monte Carlo simulation study is performed to compare the performance of estimators based on bias, mean squared error and relative efficiency for different levels of correlation coefficient, set and cycle size under Normal and Log-Normal distributions.

Keywords: Ranked set sampling, modified ranked set sampling methods, proportion estimator, relative efficiency

MODİFİYE SIRALI KÜME ÖRNEKLEMESİ YÖNTEMLERİNDE ORAN KESTİRİCİSİ

ÖZ

Sıralı Küme Örneklemesi (SKÖ) son yıllarda oldukça tercih edilen bir örnekleme yöntemidir. SKÖ, örneklem birimlerinin kolaylıkla sıralanabileceği, ancak örneklem gözlemlerinin kesin ölçümlerinin zaman alıcı, zor ve maliyetli olduğu durumda kullanılır. SKÖ yönteminin çok yaygın olarak kullanılan Basit Rasgele Örnekleme (BRÖ) yöntemi göre etkinliği kanıtlanmıştır. SKÖ yöntemine göre ilgilenilen kitleden seçilen birimlerin sıralamasında yapılan hataların azaltılması amacı ile ortalama kestiricisi için birçok modifiye edilmiş SKÖ yöntemi geliştirilmiştir. Modifiye SKÖ yöntemlerinde ortalama kestiricisi ile ilgili birçok çalışma mevcutken, kitle oranı için yapılan çalışmalar çok daha azdır. Ayrıca literatürde, oran kestiricisi ve oranın varyans kestiricisi için sadece modifiye SKÖ yöntemlerinden eşleştirilmiş sıralı küme örneklemesi yöntemi kullanılmıştır. Bu çalışmada, uç sıralı küme örneklemesi, medyan sıralı küme örneklemesi, yüzde sıralı küme örneklemesi ve L sıralı küme örneklemesi gibi modifiye edilmiş SKÖ yöntemlerinde, oran kestiricileri ve oran için varyans kestiricileri elde edilmiştir. Kestiricilerin etkinlikleri Normal ve Log-Normal dağılımlar için farklı korelasyon katsayısı, set ve döngü boyutlarında yanlılık, ortalama hata kareler ve göreli etkinliğe göre karşılaştırılması için bir Monte Carlo simülasyon çalışması yapılmıştır.

Anahtar kelimeler: Sıralı küme örneklemesi, modifiye sıralı küme örneklemesi yöntemleri, oran kestiricisi, göreli etkinlik

CONTENTS

LIST OF TABLES

Page

CHAPTER ONE
INTRODUCTION

The most accurate, valid and usable result in a research is obtained by making use of the total population holding the desired information. However, a population comprises the full set of all possible units. Therefore, the best efficient approach would be having the researchers obtain a representative sample that is the subset of the population. It is crucial that the sample represents the entire population. Several sampling methods have been implemented in the literature to select the best sample.

The most commonly used sampling method is the simple random sampling (SRS) method. SRS is defined as selecting n units from population of N objects in which all possible samples has an equal probability of being chosen. Then measurements are made for each of the selected units. However, this method is not truly representative of the entire population which is very important flow since it can reduce the accuracy of population estimator. Additionally, while performing research through the environment, ecology, sociology, agriculture and medicine, the exact measurements of each of the subunits can be very troublesome in terms of time, cost or effort using simple random sampling method. Alternatively, a better sampling method can be applied in such situation by using ranked set sampling (RSS) method.

RSS is used in cases for which the exact measurement of sample units is time consuming, difficult or expensive whereas relative ranking of sample units is easy and inexpensive with visual inspection or concomitant variable. RSS was firstly introduced by McIntyre (1952) for estimation population mean of pasture yields in Australia. Halls & Dell (1966) introduced the name of RSS and examined for estimation of the forage yields. Evans (1967) applied the RSS on surveys in areas direct-seeded to longleaf pine. The mean seedling count and the variance between the means were calculated for RSS and SRS method. It was found that while there was no significant difference between the means of RSS and SRS methods, the variance of the means using SRS was four times greater than the variance of means using RSS. Takahasi & Wakimoto (1968) developed the first mathematical theory of RSS. They proved that the mean of the RSS has a smaller variance than the mean of the SRS under perfect ranking. Dell & Clutter (1972) investigated errors of ranking and they proved that estimator of RSS is unbiased when ranking is imperfect. David & Levine (1972) also investigated the performance of RSS with errors in ranking. They provided an analytical method for obtaining the relative precision. Stokes (1976) studied the estimation of parameters, variance, interval and a correlation coefficient. Stokes (1977) used a concomitant variable for ranking the sample, instead of visual inspection or personal examination when the measurement of the variable is difficult. Martin et al. (1980) applied the RSS procedure for estimating shrub phytomass in Appalachian Oak forests. Ridout & Cobby (1987) carried out an investigation to estimate spray deposits on the leaves of fruit trees for non-random selection of sets based on the concomitant variable in RSS. Later, Stokes & Sager (1988) used RSS to estimate distribution functions in an application of consumer spending. Patil et al. (1993) introduced that estimator of RSS is more efficient than regression estimator even if the correlation between the concomitant and interested variable is small. Followed by Fei et al. (1994) who showed the performance of the parameters of Weibull distribution in RSS.

In the following years, RSS was appreciated to be very useful in terms of efficiency. Different modified RSS methods have been developed to reduce ranking error as distinct from the first method used by McIntyre. Firstly, Samawi et al. (1996) suggested extreme ranked set sampling (ERSS) which was proposed to reduce the error of ranking and based on the only minimum and maximum units for estimating the population mean. Secondly, Muttlak (1996) proposed pair ranked set sampling (PRSS) to increase the efficiency of the mean estimator. Muttlak (1997) also investigated median ranked sampling (MRSS) which measured only median units of the sets to reduce the ranking errors. Al-Saleh & Al-Kadiri (2000) proposed double RSS procedure (DRSS) for estimating the population mean. Al-Saleh & Al-Omari (2002) studied a multistage ranked set sampling (MSRSS) as a generalization of the DRSS to increase the efficiency of the population mean estimator. For asymmetric distribution, Muttlak (2003a) suggested quartile ranked set sampling (QRSS) to estimate the population mean by using the quartiles of sets. Muttlak (2003b) also introduced percentile ranked set sampling (Per - RSS) which uses percentiles of the sets for estimating the population mean. Jemain & Al-Omari (2006) contributed many modified RSS methods for estimating the population mean. Jemain & Al-Omari (2007), and Jemain et al. (2008) investigated variously RSS methods for both mean and median estimators of population. Al-Nasser (2007) offered LRSS which is a generalization for some types of RSS.

While there are a lot of studies associated with the population mean estimator in RSS, only a few are about population proportion estimator. Lacayo & Sinha (2002) investigated the population proportion estimator using RSS method in the case where the binary variable of research is selected from a continuous variable. Terpstra (2004) investigated the sample proportion and maximum likelihood estimator in RSS and implemented this method to estimate population proportion estimator. Chen et al. (2005) proposed logistic regression to rank a binary variable of interest. They used RSS method to estimate the population proportion by using real data set about National Health and Nutrition Examination Survey III. Chen et al. (2006), Chen et al. (2009) first suggested unbalanced ranked set sampling to estimate a population proportion under perfect and imperfect ranking. Terpstra & Wang (2008) investigated the confidence interval for proportion estimator using RSS method. They examined coverage probability properties and expected width for the confidence interval. Kohlschmidt et al. (2012) applied RSS for a population proportion using unbalanced ranked set sampling as opposed to balanced RSS. They showed the application of unbalanced RSS even if the proportion p is uncertain. Zamanzade & Mahdizadeh (2018) used PRSS to estimate the population proportion estimator on an application to air quality monitoring.

The aim of this thesis is to estimate the population proportion and the variance of proportion estimator for some different RSS methods. The estimators are investigated for a binary variable with a concomitant variable for different ranking information. This study outlined as follows: Simple random sampling and ranked set sampling methods are described and the proportion estimator and associated variance estimator of methods are given in Chapter 2. Chapter 3 examines process of selection and proportion estimators that are based upon some RSS methods (Extreme RSS, Median RSS, Pair RSS, Percentile RSS and LRSS). In Chapter 4, the performance of estimators is investigated with the simulation study based on the symmetric and asymmetric distributions under different levels of correlation coefficient, set and cycle sizes. Chapter 5 includes concluding remarks. The remain results of simulation study are given in the appendix.

CHAPTER TWO
SIMPLE RANDOM SAMPLING AND RANKED SET SAMPLING

2.1 Simple Random Sampling

Simple random sampling is the most popular method for choosing a sample among the population. This sampling technique is a set of n objects in the population of N objects so that each of the $\binom{N}{n}$ certain sample have the same probability of being selected.

2.1.1 The Estimators of Proportion for SRS

Let X be a random variable that follows a Bernoulli distribution with the success probability p. Let $X_1, X_2, ..., X_n$ be a simple random sample of size n from the target population. The proportion estimator and associated variance estimator of SRS are defined as;

$$\hat{p}_{SRS} = \frac{1}{n}\sum_{i=1}^{n} X_i \tag{2.1}$$

$$Var(\hat{p}_{SRS}) = \frac{\hat{p}(1-\hat{p})}{n} \tag{2.2}$$

2.2 Ranked Set Sampling

In RSS method, order statistic is one of the most important concepts. $X_1, X_2, ..., X_k$ is as random sample from an obsulutely contunuous population. The order statistics of a random sample $X_1, X_2, ..., X_k$ are the sample values placed in ascending order. $X_{(1)} \leq X_{(2)} \leq ... \leq X_{(k)}$ denote the order statistics. Then $X_{(i)}$ $i = 1, 2, ..., k$ is called the i^{th} order statistics. The first order statistic is minimum of the sample, that is, $X_{(1)} = min(X_1, X_2, ..., X_k)$ and k^{th} order statistics is maximum, that is, $X_{(k)} = max(X_1, X_2, ..., X_k)$.

If $X_1, X_2, ..., X_k$ has probability density function $f_i(x)$ and distribution function $F_i(x)$, then the probability function of $X_{(i)}$ is given as Equation 2.3.(Chen et al., 2003).

$$f_i(x) = \frac{k!}{(i-1)!(k-i)!}[F(x)]^{i-1}[1-F(x)]^{k-i}f(x), \tag{2.3}$$

In this case, the probability density functions of the smallest $(i = 1)$ and the largest $(i = k)$ order statistics are given by

$$f_1(x) = k[1-F(x)]^{k-1}f(x), \qquad -\infty < x < \infty \tag{2.4}$$

and

$$f_k(x) = k[F(x)]^{k-1}f(x), \qquad -\infty < x < \infty \tag{2.5}$$

respectively.

The cumulative distribution function of order statistics for $X_1, X_2, ..., X_k$ is defined by

$$F_i(x) = P(X_{[i]} \leq x) = \sum_{l=i}^{k} \binom{k}{r} [F(x)]^l [1-F(x)]^{k-l} \tag{2.6}$$

There are two types of RSS method which is balanced or unbalanced. While unbalanced RSS allows unequal allocations, balanced RSS provide an equal allocation of measured sample units. The most fundamental version of RSS is balanced RSS. The steps of RSS method can be described as follows: The most fundamental version of RSS is balanced RSS. The steps of RSS method can be described as follows:

Step 1: Select randomly k^2 units by simple random sampling from the population of interest.

Step 2: Allocate the k^2 units randomly into k sets, each of size k.

Step 3: Rank the units within each set based on variable of interest by using any cost free method, visual inspection, expert opinion or concomitant variables that is highly correlated with variable of interest.

Step 4: Select i^{th} the smallest ranking unit from i^{th} set to measure unit of a ranked set sample, where $i = 1, 2, \ldots k$.

Step 5: The entire process can be repeated independently n cycles to obtain a sample size nk.

The RSS procedure where one cycle size can be illustrated as below:

$$\begin{bmatrix} \boxed{X_{1[1:k]}} \leq X_{1[2:k]} \leq \ldots \leq X_{1[k:k]} \\ X_{2[1:k]} \leq \boxed{X_{2[2:k]}} \leq \ldots \leq X_{2[k:k]} \\ \vdots \\ X_{k[1:k]} \leq X_{k[2:k]} \leq \ldots \leq \boxed{X_{k[k:k]}} \end{bmatrix}$$

$X_{i[m:k]}$ denote m^{th} ranked unit in the i^{th} set where $i = 1, 2 \ldots, k$ and $m = 1, 2, \ldots, k$.

The procedure of RSS for n cycle and the case of can be shown as below:

Cycle 1

$$\begin{bmatrix} \boxed{X_{1[1:k]1}} \leq X_{1[2:k]1} \leq \ldots \leq X_{1[k:k]1} \\ X_{2[1:k]1} \leq \boxed{X_{2[2:k]1}} \leq \ldots \leq X_{2[k:k]1} \\ \vdots \\ X_{k[1:k]1} \leq X_{k[2:k]1} \leq \ldots \leq \boxed{X_{k[k:k]1}} \end{bmatrix}$$

Cycle 2

$$\begin{bmatrix} \boxed{X_{1[1:k]2}} \leq X_{1[2:k]2} \leq \ldots \leq X_{1[k:k]2} \\ X_{2[1:k]2} \leq \boxed{X_{2[2:k]2}} \leq \ldots \leq X_{2[k:k]2} \\ \vdots \\ X_{k[1:k]2} \leq X_{k[2:k]2} \leq \ldots \leq \boxed{X_{k[k:k]2}} \end{bmatrix}$$

$$\vdots$$

Cycle n

$$\begin{bmatrix} \boxed{X_{1[1:k]n}} \leq X_{1[2:k]n} \leq \ldots \leq X_{1[k:k]n} \\ X_{2[1:k]n} \leq \boxed{X_{2[2:k]n}} \leq \ldots \leq X_{2[k:k]n} \\ \vdots \\ X_{k[1:k]n} \leq X_{k[2:k]n} \leq \ldots \leq \boxed{X_{k[k:k]n}} \end{bmatrix}$$

$X_{(i[m:k]j)}$ represents the m^{th} ranked unit in the i^{th} set and j^{th} cycle size for $i = 1, 2, .., k$, $m = 1, 2, .., k$, $j = 1, 2, .., n$.

The presentation of obtained ranked set sample for set size k and cycle size n can be shown as follows:

$$\begin{bmatrix} X_{[1]1} & X_{[2]1} & \ldots & X_{[i]1} \\ X_{[1]2} & X_{[2]1} & \ldots & X_{[i]2} \\ \vdots & \vdots & & \vdots \\ X_{[1]j} & X_{[2]j} & \ldots & X_{[i]j} \end{bmatrix}$$

Here $X_{[i]j}$ present the i^{th} ranked observation in the n^{th} cycle for $i = 1, ..., k$, $j = 1, 2, .., n$.

RSS method is based on obtaining without the exact measurement of the all sample units as opposed to the more traditional SRS. Although RSS require identification of

nk^2 units, only one nk of them are quantified. In this way, there is a significant saving on the measurement costs, time and effort in RSS.

Ranking process of a variable of interest in RSS method can be based on expert opinion, visual inspections. In these cases, ranking errors may occur. Firstly, Dell and Clutter (1972) discussed the fact that the efficiency of the RSS method depends on properties of population and errors of the ranking process. Also, David and Levine (1972) investigated the effects and consequences of error ranking error. Stokes (1977) studied the importance of error arising from ranking in RSS. Moreover, she suggested to use a concomitant variable instead of visual or judgment of the expert.

The units are ranked by Y which is a concomitant variable that represents the judgment status corresponding to the variable of interest X in RSS. This process yields n independent and identically distributed (iid) bivariate random variables,

$$(Y_{(i)1}, X_{[i]1}),(Y_{(i)2}, X_{[i]2}),\ ...,\ (Y_{(i)n}, X_{[i]n}),$$

where $X_{[i]}$: the i^{th} order statistic associated with SRS of size k from $F_x(X)$,

$Y_{(i)}$: the induced order statistic.

2.2.1 The Estimators of Proportion for RSS

Let $\{X_{[i]j} : i = 1, ..., k; j = 1, ..., n\}$ be a ranked set sample of size $N = nk$ which is the total sample size from the population of the interest. The proportion estimator and associated variance estimator for RSS design are given by Equation 2.7 and 2.8 respectively.

$$\hat{p}_{RSS} = \frac{1}{nk}\sum_{i=1}^{k}\sum_{j=1}^{n} X_{[i]j} \tag{2.7}$$

$X_{[i]j}$ present the i^{th} ranked observation in the n^{th} cycle for $i = 1, ..., k, j = 1, 2, .., n.$

$$
\begin{aligned}
Var(\hat{p}_{RSS}) &= \frac{1}{n^2k^2}\sum_{i=1}^{k}\sum_{j=1}^{n} Var(X_{[i]j}) \\
&= \frac{1}{nk^2}\sum_{i=1}^{k} p_{[i]}(1-p_{[i]}),
\end{aligned} \tag{2.8}
$$

where $Var(X_{[i]j}) = p_{[i]}(1 - p_{[i]})$ and $p_{[i]}$, is the i^{th} subsample mean and denotes the estimated probability of success for the i the judgment order statistic from a Bernoulli(p) distribution, for $i = 1, \ldots, k$. It is assumed that $p_{[i]} = E(X_{[i]j})$.

The proportion estimator for RSS is unbiased. It is shown as following:

$$
\begin{aligned}
E(\hat{p}_{RSS}) &= \frac{1}{nk}\sum_{i=1}^{k}\sum_{j=1}^{n} E(X_{[i]j}) \\
&= \frac{1}{k}\sum_{i=1}^{k} p_{[i]} \\
&= p
\end{aligned} \tag{2.9}
$$

CHAPTER THREE
MODIFIED RANKED SET SAMPLING

In this chapter, some of the modified RSS methods which are Extreme RSS, Median RSS, Pair RSS, Percentile RSS and LRSS are introduced. Moreover, the population proportion estimators and the variance of the estimators are given based on modified RSS methods.

3.1 Extreme Ranked Set Sampling

Samawi et al. (1996) proposed extreme ranked set sampling (ERSS) which is the first modified ranked set sampling method for estimating the population mean. The procedure of ERSS is described as follows:

Step 1: Select k random samples each of size k units from the population of interest,

Step 2: Rank the units within each set by cost-free methods, as visual inspection, personal professional judgment or concomitant variable correlated with the variable of interest.

Step 3: For actual measurement, if the set size k is even, from the first $k/2$ sets select the smallest ranked unit and from the other $k/2$ sets the largest ranked unit. If k is odd; from the first $(k-1)/2$ sets select the smallest ranked unit and from the second $(k-1)/2$ sets select the largest ranked unit, and from the remaining set median is selected.

Step 4: The process can be repeated n times to obtain nk units.

ERSS needs to identify only minimum and maximum ordered units in case of even set size or median unit for odd set size. Thus, it is a very practical method for an increase in set size without too many ranking errors. The procedure of ERSS method for one cycle and case of set size $k = 4$ can be illustrated as below:

$$\begin{bmatrix} \boxed{X_{1[1:4]}} \leq X_{1[2:4]} \leq X_{1[3:4]} \leq X_{1[4:4]} \\ \boxed{X_{2[1:4]}} \leq X_{2[2:4]} \leq X_{2[3:4]} \leq X_{2[4:4]} \\ X_{3[1:4]} \leq X_{3[2:4]} \leq X_{3[3:4]} \leq \boxed{X_{3[4:4]}} \\ X_{4[1:4]} \leq X_{4[2:4]} \leq X_{4[3:4]} \leq \boxed{X_{4[4:4]}} \end{bmatrix}$$

When the set size k is even number, the smallest ranked units ($X_{1[1:4]}$, $X_{2[1:4]}$) from the first two sets, the largest ranked units ($X_{3[4:4]}$, $X_{4[4:4]}$) from the last two sets are selected for measurement. ($X_{i[m:k]}$ represent the m^{th} ranked unit in the i^{th} set for $i = 1, 2, \ldots, k$, $m = 1, 2, \ldots, k$) and the procedure of ERSS method for set size $k = 5$ can be shown as below:

$$\begin{bmatrix} \boxed{X_{1[1:5]}} \leq X_{1[2:5]} \leq X_{1[3:5]} \leq X_{1[4:5]} \leq X_{1[5:5]} \\ \boxed{X_{2[1:5]}} \leq X_{2[2:5]} \leq X_{2[3:5]} \leq X_{2[4:5]} \leq X_{2[5:5]} \\ X_{3[1:5]} \leq X_{3[2:5]} \leq X_{3[3:5]} \leq X_{3[4:5]} \leq \boxed{X_{3[5:5]}} \\ X_{4[1:5]} \leq X_{4[2:5]} \leq X_{4[3:5]} \leq X_{4[4:5]} \leq \boxed{X_{4[5:5]}} \\ X_{5[1:5]} \leq X_{5[2:5]} \leq \boxed{X_{5[3:5]}} \leq X_{5[4:5]} \leq X_{5[5:5]} \end{bmatrix}$$

When the set size k is odd number, the smallest ranked units ($X_{1[1:5]}$, $X_{2[1:5]}$) from the first two sets, the largest ranked units ($X_{3[5:5]}$, $X_{4[5:5]}$) from other two sets and the third ranked unit ($X_{5[3:5]}$) from remain set are selected for measurement. ($X_{i[m:k]}$ represent the m^{th} ranked unit in the i^{th} set for $i = 1, 2, \ldots, k$, $m = 1, 2, \ldots, k$)

3.1.1 The Estimators of Proportion for ERSS

The proportion estimator of ERSS for even set size k is proposed as;

$$\hat{p}_{ERSS} = \frac{1}{nk} \left[\sum_{i=1}^{k/2} \sum_{j=1}^{n} X_{[2i-1](1:k)j} + \sum_{i=1}^{k/2} \sum_{j=1}^{n} X_{[2i](k:k)j} \right], \tag{3.1}$$

when set size k is even, $X_{[2i-1](1:k)j}$ represent the 1^{th} smallest ranked units in the $(2i-1)^{th}$ sets and $X_{[2i](k:k)j}$ represent the k^{th} smallest units in the $(2i)^{th}$ sets for $i = 1, 2, \ldots, l$, $j = 1, 2, \ldots, n$, where $l = k/2$. The variance of the population proportion estimator is given by Equation 3.2;

$$
\begin{aligned}
Var(\hat{p}_{ERSS}) &= \frac{1}{n^2k^2}\left[\sum_{i=1}^{k/2}\sum_{j=1}^{n} Var(X_{[2i-1](1:k)j}) + \sum_{i=1}^{k/2}\sum_{j=1}^{n} Var(X_{[2i](k:k)j})\right] \\
&= \frac{1}{n^2k^2}\left[\sum_{i=1}^{k/2} np_{[2i-1]}(1-p_{[2i-1]}) + \sum_{i=1}^{k/2} np_{[2i]}(1-p_{[2i]})\right] \\
&= \frac{1}{nk^2}\left[\sum_{i=1}^{k/2} p_{[2i-1]}(1-p_{[2i-1]}) + \sum_{i=1}^{k/2} p_{[2i]}(1-p_{[2i]})\right], \qquad (3.2)
\end{aligned}
$$

where $Var(X_{[2i-1](1:k)j}) = p_{[2i-1]}(1-p_{[2i-1]})$, $Var(X_{[2i](k:k)j}) = p_{[2i]}(1-p_{[2i]})$ for $i = 1, 2, \ldots, l$ $j = 1, 2, \ldots, n$, where $l = k/2$.

If the set size k is odd, the proportion estimator and the variance of ERSS are given in Equation 3.3 and Equation 3.4;

$$
\hat{p}_{ERSS} = \frac{1}{nk}\left[\sum_{i=1}^{(k-1)/2}\sum_{j=1}^{n} X_{[2i-1](1:k)j} + \sum_{i=1}^{(k-1)/2}\sum_{j=1}^{n} X_{[2i](k:k)j} + \sum_{j=1}^{n} X_{[k]((k+1)/2):k)j}\right], \qquad (3.3)
$$

where $X_{[2i-1](1:k)j}$ represent the 1^{th} smallest ranked units in the $(2i-1)^{th}$ sets, $X_{[2i](k:k)j}$ represent the k^{th} smallest units in the $(2i)^{th}$ sets and $X_{[k]((k+1)/2):k)j}$ represent the $((k+1)/2)^{th}$ smallest ranked units in the k^{th} sets, for $i = 1, 2, \ldots, l$, $j = 1, 2, \ldots, n$, where $l = (k-1)/2$.

The variance of the population proportion estimator of ERSS

$$
\begin{aligned}
Var(\hat{p}_{ERSS}) &= \frac{1}{nk}\Bigg[\sum_{i=1}^{(k-1)/2}\sum_{j=1}^{n} Var(X_{[2i-1](1:k)j}) + \sum_{i=1}^{(k-1)/2}\sum_{j=1}^{n} Var(X_{[2i](k:k)j}) \\
&+ \sum_{j=1}^{n} Var(X_{[k]((k+1)/2):k)j})\Bigg] \\
&= \frac{1}{n^2k^2}\Bigg[\sum_{i=1}^{(k-1)/2} np_{[2i-1]}(1-p_{[2i-1]}) + \sum_{i=1}^{(k-1)/2} np_{[2i]}(1-p_{[2i]}) \\
&+ np_{[k]}(1-p_{[k]})\Bigg] \\
&= \frac{1}{nk^2}\Bigg[\sum_{i=1}^{(k-1)/2} p_{[2i-1]}(1-p_{[2i-1]}) + \sum_{i=1}^{(k-1)/2} np_{[2i]}(1-p_{[2i]}) \\
&+ p_{[k]}(1-p_{[k]})\Bigg], \qquad (3.4)
\end{aligned}
$$

where $Var(X_{[2i-1](1:k)j}) = p_{[2i-1]}(1-p_{[2i-1]})$, $Var(X_{[2i](k:k)j}) = p_{[2i]}(1-p_{[2i]})$ and $Var(X_{[2i](k:k)j}) = p_{[k]}(1-p_{[k]})$ for $i = 1, 2, \ldots, l$ $j = 1, 2, \ldots, n$, where $l = (k-1)/2$.

The proportion estimator for ERSS method is biased for even and odd set sizes. In the case of the set size k is even, the expected value of the estimator can be shown as below:

$$
\begin{aligned}
E(\hat{p}_{ERSS}) &= \frac{1}{nk}\Bigg[\sum_{i=1}^{k/2}\sum_{j=1}^{n} E(X_{[2i-1](1:k)j}) + \sum_{i=1}^{k/2}\sum_{j=1}^{n} E(X_{[2i](k:k)j})\Bigg] \\
&= \frac{1}{k}\Bigg[\sum_{i=1}^{k/2} p_{[1]} + \sum_{i=1}^{k/2} p_{[k]}\Bigg] \qquad (3.5) \\
&= \frac{1}{2}[p_{[1]} + p_{[k]}],
\end{aligned}
$$

where $E(X_{[2i-1](1:k)j}) = p_{[1]}$ and $E(X_{[2i](k:k)j}) = p_{[k]}$. $(i = 1, 2, \ldots, l, j = 1, 2, \ldots, n,$ where $l = k/2)$.

For the set size k is odd, the expected value of $\hat{p}$ is

$$
\begin{aligned}
E(\hat{p}_{ERSS}) &= \frac{1}{nk}\Bigg[\sum_{i=1}^{(k-1)/2}\sum_{j=1}^{n} E(X_{[2i-1](1:k)j}) + \sum_{i=1}^{(k-1)/2}\sum_{j=1}^{n} E(X_{[2i](k:k)j}) \\
&\quad + \sum_{j=1}^{n} E(X_{[k]((k+1)/2):k)j})\Bigg] \\
&= \frac{1}{k}\Bigg[\sum_{i=1}^{(k-1)/2} p_{[1]} + \sum_{i=1}^{(k-1)/2} p_{[k]} + p_{[(k+1)/2]}\Bigg] \\
&= \frac{1}{k}\left[\frac{k-1}{2}(p_{[1]} + p_{[k]}) + p_{[(k+1)/2]}\right],
\end{aligned}
\tag{3.6}
$$

where $E(X_{[2i-1](1:k)j}) = p_{[1]}$, $E(X_{[2i](k:k)j}) = p_{[k]}$ and $E(X_{[k]((k+1)/2):k)j}) = p_{[(k+1)/2]}$ for $i = 1, 2, \ldots, l\ j = 1, 2, \ldots, n$, where $l = (k-1)/2$.

3.2 Pair Ranked Set Sampling

The procedure of pair ranked set sampling (PRSS) was revealed by Muttlak (1996). This method is developed to estimate the population mean. The steps of PRSS method are given below:

Step 1: For even set size k, identify $k/2$ sets each of set size k from the target population. For odd set size k, identify $(k + 1/2)$ sets each size k.

Step 2: Rank the units within each set by judgement or any inexpensive method.

Step 3: Select the smallest and largest ranked units from the first set, select the second smallest and second largest ranked units from the second set.

Step 4: If k is even, $(k/2)^{th}$ and $(k + 2)/2^{th}$ ranked unit are selected from the last set, and if k is odd, $(k + 1)/2^{th}$ ranked unit is selected from the last set.

Step 5: The process can be repeated n cycles to get a sample of size nk.

The procedure of PRSS method for one cycle and case of set size $k = 4$ can be illustrated as below:

$$\begin{bmatrix} \boxed{X_{1[1:4]}} \leq X_{1[2:4]} \leq X_{1[3:4]} \leq \boxed{X_{1[4:4]}} \\ X_{2[1:4]} \leq \boxed{X_{2[2:4]}} \leq \boxed{X_{2[3:4]}} \leq X_{2[4:4]} \end{bmatrix}$$

For even set size $k = 4$, identify $k/2$ sets each of set size $k = 4$. The ranked units ($X_{1[1:4]}$, $X_{1[4:4]}$) from the first set, the ranked units ($X_{2[2:4]}$, $X_{2[3:4]}$) from the last set are selected. ($X_{i[m:k]}$ represent the m^{th} ranked unit in the i^{th} set for $i = 1, 2, \ldots, k$, $m = 1, 2, \ldots, k$) and the procedure of PRSS method for set size $k = 5$ can be shown as below:

$$\begin{bmatrix} \boxed{X_{1[1:5]}} \leq X_{1[2:5]} \leq X_{1[3:5]} \leq X_{1[4:5]} \leq \boxed{X_{1[5:5]}} \\ X_{2[1:5]} \leq \boxed{X_{2[2:5]}} \leq X_{2[3:5]} \leq \boxed{X_{2[4:5]}} \leq X_{2[5:5]} \\ X_{3[1:5]} \leq X_{3[2:5]} \leq \boxed{X_{3[3:5]}} \leq X_{3[4:5]} \leq X_{3[5:5]} \end{bmatrix}$$

For odd set size $k = 5$, identify $(k + 1/2)$ sets each size $k = 5$. The ranked units ($X_{1[1:5]}$, $X_{1[5:5]}$) from the first set, the ranked units ($X_{2[2:5]}$, $X_{2[4:5]}$) from the second set and the third ranked unit ($X_{3[3:5]}$) from last set are selected. ($X_{i[m:k]}$ represent the m^{th} ranked unit in the i^{th} set for $i = 1, 2, \ldots, k$, $m = 1, 2, \ldots, k$)

3.2.1 The Estimators of Proportion for PRSS

The population proportion based on PRSS for even set size k can be defined,

$$\hat{p}_{PRSS} = \frac{1}{nk} \sum_{i=1}^{k/2} \sum_{j=1}^{n} \left[X_{[i](i:k)j} + X_{[i](k+1-i:k)j} \right], \tag{3.7}$$

where $X_{[i](i:k)j}$ represent the i^{th} smallest ranked units in thef i^{th} sets and $X_{[i](k+1-i:k)j}$ represent the $(k+1-i)^{th}$ smallest units in the i^{th} sets for $i = 1, 2, \ldots, l$ $j = 1, 2, \ldots, n$, where $l = k/2$.

Thus, the corresponding variance of the population propotion estimator can be obtained as

$$
\begin{aligned}
Var(\hat{p}_{PRSS}) &= \frac{1}{n^2k^2}\sum_{i=1}^{k/2}\sum_{j=1}^{n} Var\left[X_{[i](i:k)j} + X_{[i](k+1-i:k)j}\right] \\
&= \frac{1}{n^2k^2}\left[\sum_{i=1}^{k/2}\sum_{j=1}^{n} Var(X_{[i](i:k)j}) + \sum_{i=1}^{k/2}\sum_{j=1}^{n} Var(X_{[i](k+1-i:k)j})\right. \\
&+ \left.\sum_{i=1}^{k/2}\sum_{j=1}^{n} 2Cov(X_{[i](i:k)j}, X_{[i](k+1-i:k)j})\right] \\
&= \frac{1}{n^2k^2}\left[\sum_{i=1}^{k} np_{[i]}(1-p_{[i]}) + 2\sum_{i=1}^{k/2} np[i](1-p_{[k+1-i]})\right] \\
&= \frac{1}{nk^2}\left[\sum_{i=1}^{k} p_{[i]}(1-p_{[i]}) + 2\sum_{i=1}^{k/2} p_{[i]}(1-p_{[k+1-i]})\right]. \qquad (3.8)
\end{aligned}
$$

If set size k is odd, the proportion estimator for PRSS can be written as below:

$$
\hat{p}_{PRSS} = \frac{1}{nk}\left[\sum_{i=1}^{(k-1)/2}\sum_{j=1}^{n}(X_{[i](i:k)j} + X_{[i](k+1-i:k)j}) + \sum_{j=1}^{n} X_{[(k+2)/2]((k+2)/2:k)j}\right], \quad (3.9)
$$

Where $X_{[i](i:k)j}$ represent the i^{th} smallest ranked units from i^{th} sets, $X_{[i](k+1-i:k)j}$ represent the $(k+1-i)^{th}$ smallest units from i^{th} sets and $X_{[(k+2)/2]((k+2)/2:k)j}$ represent the $((k+2)/2)^{th}$ smallest ranked unit from $((k+2)/2)^{th}$ set for $i = 1, 2, \ldots, l$ $j = 1, 2, \ldots, n$, where $l = (k-1)/2$.

The associatted variance is given by

$$\begin{aligned}
Var(\hat{p}_{PRSS}) &= \frac{1}{n^2k^2}\Bigg[\sum_{i=1}^{(k-1)/2}\sum_{j=1}^{n} Var(X_{[i](i:k)j} + X_{[i](k+1-i:k)j})\\
&+ \sum_{j=1}^{n} Var(X_{[(k+1)/2]((k+1)/2:k)j})\Bigg]\\
&= \frac{1}{n^2k^2}\Bigg[\sum_{i=1}^{(k-1)/2}\sum_{j=1}^{n} Var(X_{[i](i:k)j}) + \sum_{i=1}^{(k-1)/2}\sum_{j=1}^{n} Var(X_{[i](k+1-i:k)j})\\
&+ \sum_{i=1}^{(k-1)/2}\sum_{j=1}^{n} 2Cov(X_{[i](i:k)j}, X_{[i](k+1-i:k)j})\\
&+ \sum_{j=1}^{n} Var(X_{[(k+1)/2]((k+1)/2:k)j})\Bigg]\\
&= \frac{1}{n^2k^2}\Bigg[\sum_{i=1}^{k} np_{[i]}(1-p_{[i]}) + 2\sum_{i=1}^{(k-1)/2} np_{[i]}(1-p_{[k+1-i]})\Bigg]\\
&= \frac{1}{nk^2}\Bigg[\sum_{i=1}^{k} p_{[i]}(1-p_{[i]}) + 2\sum_{i=1}^{k/2} p_{[i]}(1-p_{[k+1-i]})\Bigg], \qquad (3.10)
\end{aligned}$$

where $Var[X_{[i](i:k)j} + X_{[i](k+1-i:k)j}] \geq 0$ represent the covariance between $X_{i[i:k]}$ and $X_{i[k+1-i:k]}$. Thus, the sample units are not independent of each other completely.

$$\begin{aligned}
Cov(X_{[i](i:k)j}, X_{[i](k+1-i:k)j}) &= E(X_{[i](i:k)j}, X_{[i](k+1-i:k)j}) - E(X_{[i](i:k)j})E(X_{[i](k+1-i:k)j})\\
&= p_{[i]} - p_{[i]}p_{[k+1-i]}\\
&= p_{[i]}(1-p_{[k+1-i]}),\\
E(X_{[i](i:k)j}, X_{[i](k+1-i:k)j}) &= E(X_{[i](k+1-i:k)j} | X_{[i](i:k)j} = 1)P(X_{[i](i:k)j} = 1)\\
&= P(X_{[i](i:k)j} = 1)\\
&= p_{[i]},
\end{aligned}$$

where $E(X_{[i](i:k)j}) = p_{[i]}$, $E(X_{[i](k+1-i:k)j}) = (1 - p_{[k+1-i]})$.

Therefore, PRSS method reduces the number of sample units by almost half other modified RSS.

The proportion estimator for PRSS method is unbiased for set size even and odd. In case of set size k is even, the expected value of the estimator can be shown as below:

$$
\begin{aligned}
E(\hat{p}_{PRSS}) &= \frac{1}{nk}\left[\sum_{i=1}^{k/2}\sum_{j=1}^{n} E(X_{[i](i:k)j}) + \sum_{i=1}^{k/2}\sum_{j=1}^{n} E(X_{[i](k+1-i:k)j})\right] \\
&= \frac{1}{k}\left[\sum_{i=1}^{k/2} p_{[i]} + \sum_{i=1}^{k/2} p_{[k+1-i]}\right] \\
&= \frac{1}{k}\sum_{i=1}^{k} p_{[i]} \\
&= p
\end{aligned} \tag{3.11}
$$

In case of set size k is odd, the expected value of $\hat{p}$ can be shown as below:

$$
\begin{aligned}
E(\hat{p}_{PRSS}) &= \frac{1}{nk}\left[\sum_{i=1}^{(k-1)/2}\sum_{j=1}^{n} E(X_{[i](i:k)j}) + \sum_{i=1}^{(k-1)/2}\sum_{j=1}^{n} E(X_{[i](k+1-i:k)j})\right. \\
&\quad \left. + \sum_{j=1}^{n} E(X_{[(k+2)/2]((k+2)/2:k)j})\right] \\
&= \frac{1}{k}\left[\sum_{i=1}^{(k-1)/2} p_{[i]} + \sum_{i=1}^{(k-1)/2} p_{[k+1-i]} + p_{[(k+2)/2]}\right] \\
&= \frac{1}{k}\sum_{i=1}^{k} p_{[i]} \\
&= p
\end{aligned} \tag{3.12}
$$

3.3 Median Ranked Set Sampling

Muttlak (1997) suggested the median ranked set sampling (MRSS) method to estimate the population mean for reducing errors in ranking. MRSS method can be summarized as follow:

Step 1: Select k random sets each of size k from the target population.

Step 2: The each set unit are ranked by a concomitant variable, visual inspection.

Step 3: If the set size k is even, select the $(k/2)^{th}$ smallest ranked units for measurement from the $k/2$ sets and $((k+2)/2)^{th}$ smallest ranked units from the other $k/2$ sets. If k is odd; select for measurement the $((k+1)/2)^{th}$ smallest rank from each set.

Step 4: The whole process can be repeated n times to get nk units.

The procedure of MRSS method for one cycle and case of set size $k = 4$ can be illustrated as below:

$$\begin{bmatrix} X_{1[1:4]} \leq \boxed{X_{1[2:4]}} \leq X_{1[3:4]} \leq X_{1[4:4]} \\ X_{2[1:4]} \leq \boxed{X_{2[2:4]}} \leq X_{2[3:4]} \leq X_{2[4:4]} \\ X_{3[1:4]} \leq X_{3[2:4]} \leq \boxed{X_{3[3:4]}} \leq X_{3[4:4]} \\ X_{4[1:4]} \leq X_{4[2:4]} \leq \boxed{X_{4[3:4]}} \leq X_{4[4:4]} \end{bmatrix}$$

When the set size k is even number, the second smallest ranked units ($X_{1[2:4]}$, $X_{2[2:4]}$) from the first two sets, the third ranked units ($X_{3[3:4]}$, $X_{4[3:4]}$) from the last two sets are selected for measurement. ($X_{i[m:k]}$ represent the m^{th} ranked unit in the i^{th} set for $i = 1, 2, \ldots, k$, $m = 1, 2, \ldots, k$) and the procedure of MRSS method for set size $k = 5$ can be shown as below:

$$\begin{bmatrix} X_{1[1:5]} \leq X_{1[2:5]} \leq \boxed{X_{1[3:5]}} \leq X_{1[4:5]} \leq X_{1[5:5]} \\ X_{2[1:5]} \leq X_{2[2:5]} \leq \boxed{X_{2[3:5]}} \leq X_{2[4:5]} \leq X_{2[5:5]} \\ X_{3[1:5]} \leq X_{3[2:5]} \leq \boxed{X_{3[3:5]}} \leq X_{3[4:5]} \leq X_{3[5:5]} \\ X_{4[1:5]} \leq X_{4[2:5]} \leq \boxed{X_{4[3:5]}} \leq X_{4[4:5]} \leq X_{4[5:5]} \\ X_{5[1:5]} \leq X_{5[2:5]} \leq \boxed{X_{5[3:5]}} \leq X_{5[4:5]} \leq X_{5[5:5]} \end{bmatrix}$$

Where the set size k is odd number, the third smallest ranked $(X_{i(3:5)})$ units of the sets are selected. ($X_{i[m:k]}$ represent the m^{th} ranked unit in the i^{th} set for $i = 1, 2, \ldots, k$, $m = 1, 2, \ldots, k$)

3.3.1 The Estimators of Proportion for MRSS

The proportion estimator of MRSS for even set size case is defined as

$$\hat{p}_{MRSS} = \frac{1}{nk}\left[\sum_{i=1}^{k/2}\sum_{j=1}^{n} X_{[i](k/2:k)j} + \sum_{i=k/2+1}^{k}\sum_{j=1}^{n} X_{[i]((k+2)/2:k)j}\right], \tag{3.13}$$

where $X_{[i](k/2:k)j}$ represent the $(k/2)^{th}$ ranked units in the i^{th} sets for $i = 1, 2, \ldots, l$ and $X_{[i]((k+2)/2:k)j}$ represent the $((k+2)/2)^{th}$ ranked units in the i^{th} sets for $i = l+1, \ldots, k$, where $l = k/2$, $j = 1, 2, \ldots, n$. Its variance estimator can be obtained as

$$\begin{aligned} Var(\hat{p}_{MRSS}) &= \frac{1}{n^2k^2}\left[\sum_{i=1}^{k/2}\sum_{j=1}^{n} Var(X_{[i](k/2:k)j}) + \sum_{i=k/2+1}^{k}\sum_{j=1}^{n} Var(X_{[i]((k+2)/2:k)j})\right] \\ &= \frac{1}{n^2k^2}\left[\sum_{i=1}^{k/2} np_{[i]}(1-p_{[i]}) + \sum_{i=k/2+1}^{k} np_{[i]}(1-p_{[i]})\right] \\ &= \frac{1}{nk^2}\left[\sum_{i=1}^{k/2} p_{[i]}(1-p_{[i]}) + \sum_{i=k/2+1}^{k} p_{[i]}(1-p_{[i]})\right], \end{aligned} \tag{3.14}$$

where $Var(X_{[i](k/2:k)j}) = p_{[i]}(1 - p_{[i]})$ for $i = 1, 2, \ldots, l$, $Var(X_{[i]((k+2)/2:k)j}) = p_{[i]}(1 - p_{[i]})$ for $i = l + 1, \ldots, k$, where $l = k/2$, $j = 1, 2, \ldots, n$.

For odd set size k case is defined as

$$\hat{p}_{MRSS} = \frac{1}{nk}\left[\sum_{i=1}^{k}\sum_{j=1}^{n} X_{[i]((k+1)/2:k)j}\right], \tag{3.15}$$

where $X_{[i]((k+1)/2:k)j}$ represent the $((k + 1)/2)^{th}$ ranked units in the i^{th} sets for $i = 1, 2, \ldots, k$ $j = 1, 2, \ldots, n$.

The associated variance estimator can be obtained as below:

$$\begin{aligned} Var(\hat{p}_{MRSS}) &= \frac{1}{n^2k^2}\left[\sum_{i=1}^{k}\sum_{j=1}^{n} Var(X_{[i]((k+1)/2:k)j})\right] \\ &= \frac{1}{n^2k^2}\left[\sum_{i=1}^{k} np_{[i]}(1-p_{[i]})\right] \\ &= \frac{1}{nk^2}\left[\sum_{i=1}^{k} p_{[i]}(1-p_{[i]})\right], \end{aligned} \tag{3.16}$$

where $Var(X_{[i](k/2:k)j}) = p_{[i]}(1-p_{[i]})$ for $i = 1, 2, \ldots, k$ $j = 1, 2, \ldots, n$.

The proportion estimator for MRSS method is biased for set size even and odd. In case of set size k is even, it can be shown as below:

$$\begin{aligned} E(\hat{p}_{MRSS}) &= \frac{1}{nk}\left[\sum_{i=1}^{k/2}\sum_{j=1}^{n} E(X_{[i](k/2:k)j}) + \sum_{i=k/2+1}^{k}\sum_{j=1}^{n} E(X_{[i](k/2+1:k)j})\right] \\ &= \frac{1}{k}\left[\sum_{i=1}^{k/2} p_{[k/2]} + \sum_{i=k/2+1}^{k} p_{[k/2+1]}\right] \\ &= \frac{1}{2}p_{[k/2]} + \frac{k}{2}p_{[k/2+1]}, \end{aligned} \tag{3.17}$$

where $E(X_{[i](k/2:k)j}) = p_{[k/2]}$ for $i = 1, 2, \ldots, l$ and $E(X_{[i](k/2+1:k)j}) = p_{[k/2+1]}$ for $i = l+1, \ldots, k$, where $l = k/2$, $j = 1, 2, \ldots, n$.

When set size k is odd, it can be shown as below:

$$\begin{aligned} E(\hat{p}_{MRSS}) &= \frac{1}{nk}\left[\sum_{i=1}^{k}\sum_{j=1}^{n} E(X_{[i]((k+1)/2:k)j})\right] \\ &= \frac{1}{k}\left[\sum_{i=1}^{k} p_{[(k+1)/2]}\right] \\ &= p_{[(k+1)/2]}, \end{aligned} \tag{3.18}$$

where $E(X_{[i]((k+1)/2:k)j}) = p_{[(k+1)/2]}$ for $i = 1, 2, \ldots, k$, $j = 1, 2, \ldots, n$

3.4 Percentile Ranked Set Sampling

Muttlak (2003b) supposed the Per-RSS method. The procedure of Per-RSS method can be described as follows:

Step 1: The selected k random sets each of size k from the target population.

Step 2: Rank the units in sets within each sample with respect to a variable of interest by cost- free method.

Step 3: If set size k is even; select the $a = p(k+1)^{th}$ smallest ranked unit from the first $k/2$ sets, select the $b = q(k+1)^{th}$ smallest ranked unit from the second $k/2$ sets. If set size k is odd; select the $a = p(k+1)^{th}$ smallest ranked unit for measurement from the first $(k-1)/2$ sets and select the $b = q(k+1))^{th}$ smallest rank from the second $(k-1)/2$ sets, and select the median for actual measurement from remain set,where $0 \leq p \leq 1$, $q = 1-p$, a and b are the nearest integer value to x.

Step 4: Repeat the process n times to obtain a sample of size nk units.

In case of set size is $k = 4$, $p = 0.25$, the procedure of Per-RSS method can be represented for one cycle as below:

$$\begin{bmatrix} \boxed{X_{1[1:4]}} \leq X_{1[2:4]} \leq X_{1[3:4]} \leq X_{1[4:4]} \\ \boxed{X_{2[1:4]}} \leq X_{2[2:4]} \leq X_{2[3:4]} \leq X_{2[4:4]} \\ X_{3[1:4]} \leq X_{3[2:4]} \leq X_{3[3:4]} \leq \boxed{X_{3[4:4]}} \\ X_{4[1:4]} \leq X_{4[2:4]} \leq X_{4[3:4]} \leq \boxed{X_{4[4:4]}} \end{bmatrix}$$

The ranked units ($X_{1[1:4]}$, $X_{2[4:4]}$) from the first two sets, the ranked units ($X_{3[4:4]}$, $X_{4[4:4]}$) from the last two sets are selected for measurement. ($X_{i[m:k]}$ represent the m^{th} ranked unit in the i^{th} set for $i = 1, 2, \ldots, k$, $m = 1, 2, \ldots, k$) and the procedure of Per-RSS method for set size $k = 5$, $p = 0.3$ can be shown as below:

$$\begin{bmatrix} X_{1[1:5]} \leq \boxed{X_{1[2:5]}} \leq X_{1[3:5]} \leq X_{1[4:5]} \leq X_{1[5:5]} \\ X_{2[1:5]} \leq \boxed{X_{2[2:5]}} \leq X_{2[3:5]} \leq X_{2[4:5]} \leq X_{2[5:5]} \\ X_{3[1:5]} \leq X_{3[2:5]} \leq X_{3[3:5]} \leq \boxed{X_{3[4:5]}} \leq X_{3[5:5]} \\ X_{4[1:5]} \leq X_{4[2:5]} \leq X_{4[3:5]} \leq \boxed{X_{4[4:5]}} \leq X_{4[5:5]} \\ X_{5[1:5]} \leq X_{5[2:5]} \leq \boxed{X_{5[3:5]}} \leq X_{5[4:5]} \leq X_{5[5:5]} \end{bmatrix}$$

The ranked units ($X_{1[2:5]}$, $X_{2[2:5]}$) from the first two sets, the ranked units ($X_{3[4:5]}$, $X_{4[4:5]}$) from the other two sets and the ranked unit ($X_{5[3:5]}$) from the last set are selected for measurement. ($X_{i[m:k]}$ represent the m^{th} ranked unit in the i^{th} set for $i = 1, 2, \ldots, k$, $m = 1, 2, \ldots, k$).

3.4.1 The Estimators of Proportion for Per-RSS

The proportion estimator for Per-RSS for even set size k is given by

$$\hat{p}_{Per-RSS} = \frac{1}{nk}\left[\sum_{i=1}^{k/2}\sum_{j=1}^{n} X_{[i](a:k)j} + \sum_{i=(k/2)+1}^{k}\sum_{j=1}^{n} X_{[i](b:k)j}\right], \qquad (3.19)$$

where $X_{[i](a:k)j}$ represent the $(a = p(k+1))^{th}$ ranked units in the i^{th} sets for $i = 1, 2, \ldots, l$ and $X_{[i](b:k)j}$ represent the $(b = q(k+1))^{th}$ ranked units in the i^{th} sets for $i = l+1, \ldots, k$, where $l = k/2$, $j = 1, 2, \ldots, n$.

The corresponding variance is given by

$$\begin{aligned} Var(\hat{p}_{Per-RSS}) &= \frac{1}{n^2k^2}\left[\sum_{i=1}^{k/2}\sum_{j=1}^{n} Var(X_{[i](a:k)j}) + \sum_{i=(k/2)+1}^{k}\sum_{j=1}^{n} Var(X_{[i](b:k)j})\right] \\ &= \frac{1}{n^2k^2}\left[\sum_{i=1}^{k/2} np_{[i]}(1-p_{[i]}) + \sum_{i=(k/2)+1}^{k} np_{[i]}(1-p_{[i]})\right] \\ &= \frac{1}{nk^2}\left[\sum_{i=1}^{k/2} p_{[i]}(1-p_{[i]}) + \sum_{i=(k/2)+1}^{k} p_{[i]}(1-p_{[i]})\right], \qquad (3.20) \end{aligned}$$

where $Var(X_{[i](a:k)j}) = p_{[i]}(1-p_{[i]})$ for $i = 1, 2, \ldots, l$, $Var(X_{[i](b:k)j}) = p_{[i]}(1-p_{[i]})$ for $i = l+1, \ldots, k$, where $l = k/2$ $j = 1, 2, \ldots, n$.

If set size k is odd, the proportion estimator of Per-RSS are proposed as Equation 3.21;

$$\hat{p}_{Per-RSS} = \frac{1}{nk}\left[\sum_{i=1}^{(k-1)/2}\sum_{j=1}^{n} X_{[i](a:k)j} + \sum_{i=(k-1)/2}^{k}\sum_{j=1}^{n} X_{[i](b:k)j} + \sum_{j=1}^{n} X_{[k]((k+1)/2:k)j}\right], \tag{3.21}$$

where $X_{[i](a:k)j}$ represent the $(a = p(k+1))^{th}$ ranked units in the i^{th} sets for $i = 1, 2, \ldots, l$, $X_{[i](b:k)j}$ represent the $(b = q(k+1))^{th}$ ranked units in the i^{th} sets for $i = l, \ldots, k$ and $X_{[k]((k+1)/2:k)j}$ represent the $((k+1)/2)^{th}$ ranked units in the k^{th} sets where $l = (k-1)/2$, $j = 1, 2, \ldots, n$.

The corresponding variance is given by

$$\begin{aligned}
Var(\hat{p}_{Per-RSS}) &= \frac{1}{n^2k^2}\left[\sum_{i=1}^{(k-1)/2}\sum_{j=1}^{n} Var(X_{[i](a:k)j}) + \sum_{i=(k-1)/2}^{k}\sum_{j=1}^{n} Var(X_{[i](b:k)j})\right. \\
&+ \left.\sum_{j=1}^{n} Var(X_{k((k+1)/2:k)})\right] \\
&= \frac{1}{n^2k^2}\left[\sum_{i=1}^{(k-1)/2} np_{[i]}(1-p_{[i]}) + \sum_{i=(k-1)+1}^{k} np_{[i]}(1-p_{[i]})\right. \\
&+ \left. np_{[k]}(1-p_{[k]})\right] \\
&= \frac{1}{nk^2}\left[\sum_{i=1}^{k/2} p_{[i]}(1-p_{[i]}) + \sum_{i=(k-2)+1}^{k-1} p_{[i]}(1-p_{[i]})\right. \\
&+ \left. p_{[k]}(1-p_{[k]})\right],
\end{aligned} \tag{3.22}$$

where $Var(X_{[i](a:k)j}) = p_{[i]}(1-p_{[i]})$ for $i = 1, 2, \ldots, l$, $Var(X_{[i](b:k)j}) = p_{[i]}(1-p_{[i]})$ for $i = l, 2, \ldots, k$ and $Var(X_{k((k+1)/2:k)}) = p_{[k]}(1-p_{[k]})$ where $l = (k-1)/2$, $j = 1, 2, \ldots, n$.

The proportion estimator for Per-RSS method is biased for set size even and odd. In case of set size k is even, it can be shown as below:

$$
\begin{aligned}
E(\hat{p}_{Per-RSS}) &= \frac{1}{nk}\left[\sum_{i=1}^{k/2}\sum_{j=1}^{n} E(X_{[i](a:k)j}) + \sum_{i=(k/2)+1}^{k}\sum_{j=1}^{n} E(X_{[i](b:k)j})\right] \\
&= \frac{1}{k}\left[\sum_{i=1}^{k/2} p_{[a]} + \sum_{i=k/2+1}^{k} p_{[b]}\right] \\
&= \frac{1}{2}p_{[a]} + \frac{k}{2}p_{[b]},
\end{aligned} \tag{3.23}
$$

where $(a = p(k+1))$, $(b = q(k+1))$, $E(X_{[i](a:k)j}) = p_{[a]}$ for $i = 1, 2, \ldots, l$ and $E(X_{[i](b:k)j}) = p_{[b]}$ for $i = l+1, 2, \ldots, k$ where $l = k/2$, $j = 1, 2, \ldots, n$.

When set size k is odd, it can be shown as below:

$$
\begin{aligned}
E(\hat{p}_{Per-RSS}) &= \frac{1}{nk}\left[\sum_{i=1}^{(k-1)/2}\sum_{j=1}^{n} E(X_{[i](a:k)j}) + \sum_{i=(k-1)/2}^{k}\sum_{j=1}^{n} E(X_{[i](b:k)j})\right. \\
&+ \left.\sum_{j=1}^{n} E(X_{[k]((k+1)/2:k)j})\right] \\
&= \frac{1}{k}\left[\sum_{i=1}^{k/2} p_{[a]} + \sum_{i=k/2+1}^{k} p_{[b]} + p_{[(k+1)/2]}\right] \\
&= \frac{1}{k}\left[\frac{k-1}{2}p_{[a]} + \frac{k+3}{2}p_{[b]} + p_{[(k+1)/2]}\right],
\end{aligned} \tag{3.24}
$$

where $E(X_{[k]((k+1)/2:k)j}) = p_{[(k+1)/2]}$ for $j = 1, 2, \ldots, n$.

3.5 L Ranked Set Sampling

Al-Naseer (2007) developed LRSS method which is a generalization of RSS methods for estimating the population mean. The LRSS procedure is as follows:

Step 1: Randomly select k random samples of size k units from the population.

Step 2: Rank the units in each set by judgment or any inexpensive method.

Step 3: The LRSS coefficient is defined, $m = \lfloor k\alpha \rfloor$ where $0 \leq \alpha < 0.5$, such that $\lfloor x \rfloor$ is the largest integer part less than or equal to x.

Step 4: Select the unit with rank $m+1$ from the first m ranked samples, select the unit with rank $(k-m)$ for actual measurement from the last m ranked samples and for $j = k+1, \ldots, m-k-1$, the unit with rank j in the j^{th} ranked sample is selected for actual measurement.

The prodecure of L-RSS for $k = 4$, $m = 1$ is shown below:

$$\begin{bmatrix} X_{1[1:4]} \leq \boxed{X_{1[2:4]}} \leq X_{1[3:4]} \leq X_{1[4:4]} \\ X_{2[1:4]} \leq \boxed{X_{2[2:4]}} \leq X_{2[3:4]} \leq X_{2[4:4]} \\ X_{3[1:4]} \leq X_{3[2:4]} \leq \boxed{X_{3[3:4]}} \leq X_{3[4:4]} \\ X_{4[1:4]} \leq X_{4[2:4]} \leq \boxed{X_{4[3:4]}} \leq X_{4[4:4]} \end{bmatrix}$$

The second smallest ranked units ($X_{1[2:4]}$, $X_{2[2:4]}$) from the first $m+1$ sets, the third smallest ranked units ($X_{3[3:4]}$, $X_{4[3:4]}$) from the last $m+1$ sets are selected. ($X_{i[m:k]}$ represent the m^{th} ranked unit in the i^{th} set for $i = 1, 2, \ldots, k$, $m = 1, 2, \ldots, k$) and the another example for set size $k = 5$ and $m = 1$ can be shown as below:

$$\begin{bmatrix} X_{1[1:5]} \leq \boxed{X_{1[2:5]}} \leq X_{1[3:5]} \leq X_{1[4:5]} \leq X_{1[5:5]} \\ X_{2[1:5]} \leq \boxed{X_{2[2:5]}} \leq X_{2[3:5]} \leq X_{2[4:5]} \leq X_{2[5:5]} \\ X_{3[1:5]} \leq X_{3[2:5]} \leq \boxed{X_{3[3:5]}} \leq X_{3[4:5]} \leq X_{3[5:5]} \\ X_{4[1:5]} \leq X_{4[2:5]} \leq X_{4[3:5]} \leq \boxed{X_{4[4:5]}} \leq X_{4[5:5]} \\ X_{5[1:5]} \leq X_{5[2:5]} \leq X_{5[3:5]} \leq \boxed{X_{5[4:5]}} \leq X_{5[5:5]} \end{bmatrix}$$

The second smallest ranked units ($X_{1[2:5]}$, $X_{2[2:5]}$) from the first $m+1$ sets, the third smallest ranked unit ($X_{3[3:5]}$) from the third set and the fourth smallest ranked units ($X_{4[4:5]}$, $X_{5[4:5]}$) from the last $m+1$ sets are selected for measurement. ($X_{i[m:k]}$ represent the m^{th} ranked unit in the i^{th} set for $i = 1, 2, \ldots, k$, $m = 1, 2, \ldots, k$).

3.5.1 The Estimators of Proportion for LRSS

The proportion estimator of LRSS for set size of k is defined by

$$\hat{p}_{LRSS} = \frac{1}{nk} \left[\sum_{i=1}^{m} \sum_{j=1}^{n} X_{[i]((m+1):k)j} + \sum_{i=m+1}^{k-m} \sum_{j=1}^{n} X_{[i](i:k)j} + \sum_{i=k-m+1}^{k} \sum_{j=1}^{n} X_{[i]((k-m):k)j} \right], \tag{3.25}$$

where $X_{[i]((m+1):k)j}$ represent the $(m+1)^{th}$ ranked units in the i^{th} sets for $i = 1, 2, \ldots, m$, $X_{[i](i:k)j}$ represent the i^{th} ranked units in the i^{th} sets for $i = m+1, \ldots, k-m$ and $X_{[i]((k-m):k)j}$ represent the $(k-m)^{th}$ ranked units in the i^{th} sets , for $i = k-m+1, \ldots, k$, where $j = 1, 2, \ldots, n$.

The variance of the proportion estimator is defined by

$$
\begin{aligned}
Var(\hat{p}_{LRSS}) &= \hat{p}_{LRSS} = \frac{1}{nk}\left[\sum_{i=1}^{m}\sum_{j=1}^{n} Var(X_{[i]((m+1):k)j}) + \sum_{i=m+1}^{k-m}\sum_{j=1}^{n} Var(X_{[i](i:k)j})\right. \\
&+ \left.\sum_{i=k-m+1}^{k}\sum_{j=1}^{n} Var(X_{[i]((k-m):k)j})\right] \\
&= \frac{1}{n^2k^2}\left[\sum_{i=1}^{m} np_{[i]}(1-p_{[i]}) + \sum_{i=m+1}^{k-m} np_{[i]}(1-p_{[i]})\right. \\
&+ \left.\sum_{i=k-m+1}^{k} np_{[i]}(1-p_{[i]})\right] \\
&= \frac{1}{nk^2}\left[\sum_{i=1}^{m} p_{[i]}(1-p_{[i]}) + \sum_{i=m+1}^{k-m} p_{[i]}(1-p_{[i]}) + \sum_{i=k-m+1}^{k} p_{[i]}(1-p_{[i]})\right],
\end{aligned}
\tag{3.26}
$$

where $Var(X_{[i]((m+1):k)j}) = p_{[i]}(1-p_{[i]})$ for $i = 1,\ldots,m$, $Var(X_{[i](i:k)j}) = p_{[i]}(1-p_{[i]})$ for $i = m+1,\ldots,k-m$ and $Var(X_{[i]((k-m):k)j}) = p_{[i]}(1-p_{[i]})$ for $i = k-m+1,\ldots,k$.

It is important to note that, the proportion estimator for LRSS method is biased for $m \geq 1$ for any sample size. When $m = 0$, LRSS procedure leads to the novel RSS that is unbiased.

$$
\begin{aligned}
E(\hat{p}_{LRSS}) &= \frac{1}{nk}\left[\sum_{i=1}^{m}\sum_{j=1}^{n} E(X_{[i]((m+1):k)j}) + \sum_{i=m+1}^{k-m}\sum_{j=1}^{n} E(X_{[i](i:k)j})\right. \\
&+ \left.\sum_{i=k-m+1}^{k}\sum_{j=1}^{n} E(X_{[i]((k-m):k)j})\right] \\
&= \frac{1}{nk}\left[\sum_{i=1}^{m} np_{[m+1]} + \sum_{i=m+1}^{k-m} np_{[i]} + \sum_{i=k-m+1}^{k} np_{[k-m]}\right] \\
&= \frac{1}{k}\left[\sum_{i=1}^{m} p_{[m+1]} + \sum_{i=m+1}^{k-m} p_{[i]} + \sum_{i=k-m+1}^{k} p_{[k-m]}\right],
\end{aligned}
\tag{3.27}
$$

where $E(X_{[i]((m+1):k)j}) = p_{[m+1]}$, $E(X_{[i](i:k)j}) = p_{[i]}$, $E(X_{[i]((k-m):k)j}) = p_{[k-m]}$.

CHAPTER FOUR
SIMULATION STUDY

In this chapter, we investigate the performance of the proposed estimators of population proportion based on some modified ranked set sampling methods (Median RSS, Extreme RSS, Pair RSS, Percentile RSS and LRSS) by Monte Carlo simulation study. The interested data set is simulated from Normal and LogNormal distributions. The simulation study is performed via R Project with 10,000 repetitions. The comparisons of estimators are constructed in terms of bias, mean squared error and relative efficiency for different correlation levels, set and cycle sizes.

The process of simulation studies is given below:

Step 1: The populations are generated from Normal $(0, 1)$ and LogNormal $(0, 1)$ with size 1000.

Step 2: The quality of ranking information is controlled by the correlation coefficient ρ between a concomitant variable (Y) and the variable of interest (X). In this study, the values of the correlation coefficient ρ are taken to be 0.25 for poor ranking, 0.5 for imperfect ranking which is not strong, 0.75 for fair ranking and 0.95 for nearly perfect ranking.

Step 3: The estimators of population proportion are computed for each combination, set size $k = 3, 4, 5, 6$, cycle size $n = 2, 3, 4$ and true proportion $p = 0.1, 0.2, 0.3, \ldots, 0.9$.

Step 4: The simulation is run 10,000 times for simple random sampling, ranked set sampling, extreme ranked set sampling, median ranked set sampling, pair ranked set sampling, percentile ranked set sampling and L ranked set sampling.

Step 5: The bias of estimators is computed by $Bias = \hat{p} - p$ where p is true proportion and $\hat{p}$ is the proportion estimator.

Step 6: The mean square error is obtained by Equation (4.1)

$$MSE = bias^2 + variance \tag{4.1}$$

Step 7: To compare the performance of the proportion estimators is calculated in terms of relative efficiency; that is

$$RE_1(\hat{p}_{RSS}, \hat{p}_{SRS}) = \frac{MSE_{\hat{p}_{SRS}}}{MSE_{\hat{p}_{RSS}}} \tag{4.2}$$

$$RE_2(\hat{p}_{ERSS}, \hat{p}_{SRS}) = \frac{MSE_{\hat{p}_{SRS}}}{MSE_{\hat{p}_{ERSS}}} \tag{4.3}$$

$$RE_3(\hat{p}_{MRSS}, \hat{p}_{SRS}) = \frac{MSE_{\hat{p}_{SRS}}}{MSE_{\hat{p}_{MRSS}}} \tag{4.4}$$

$$RE_4(\hat{p}_{PRSS}, \hat{p}_{SRS}) = \frac{MSE_{\hat{p}_{SRS}}}{MSE_{\hat{p}_{PRSS}}} \tag{4.5}$$

$$RE_5(\hat{p}_{Per-RSS}, \hat{p}_{SRS}) = \frac{MSE_{\hat{p}_{SRS}}}{MSE_{\hat{p}_{Per-RSS}}} \tag{4.6}$$

$$RE_6(\hat{p}_{LRSS}, \hat{p}_{SRS}) = \frac{MSE_{\hat{p}_{SRS}}}{MSE_{\hat{p}_{LRSS}}} \tag{4.7}$$

The relative efficiency values greater than one indicate that the proportion estimator of related sampling method is more efficient than the proportion estimator of SRS method.

The Monte Carlo simulation results of relative efficiencies and bias values of the proportion estimators are given by Table 4.1- 4.20 for Normal and Log-Normal distributions. The results in these tables are obtained for different true proportions,

$p = 0.1, 0.2, 0.3, 0.4, 0.5$.

Bias values and relative efficiencies of proportion estimators based on sampling methods for Normal and Log-Normal distributions are given in Table 4.1, 4.2, 4.3 and 4.4 when true proportion p is 0.1, respectively. According to bias values in Table 4.1, the proportion estimators based on SRS, RSS, PRSS are unbiased. However, the proportion estimators based on Per- RSS, MRSS, LRSS and ERSS are biased. The bias values of these estimators increase, while the value of correlation coefficient increase.

According to Table 4.2, every values of relative efficiencies are greater than 1 and it indicates that the proportion estimators for RSS methods are more efficient than the proportion estimator for SRS methods. In more details, almost all efficiencies of proportion estimator for RSS method increase when the set size and the correlation level increase. Additionally, the results indicate that the proportion estimators for ERSS, MRSS, Per-RSS and LRSS methods are more efficient than the proportion estimator for SRS method for all case when the correlation coefficient ρ is around 0.25. However, the opposite is true for the efficiency of PRSS method. When the ranking information is nearly perfect, the proportion estimator for PRSS method is more efficient than the proportion estimator for SRS method. Similar results are obtained for the biased values and relative efficiecies for Log-Normal distribution in Table 4.3 and Table 4.4.

Table 4.1 Biases of the proportion estimator for Normal distribution when $p = 0.1$

		$Rho = 0.25$							$Rho = 0.5$						
n	k	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$
2	3	0.001	−0.002	0.001	0.009	−0.001	0.001	0.009	0.001	0.001	0.002	0.016	−0.003	0.000	0.020
	4	0.001	−0.002	−0.009	0.009	0.000	−0.008	0.008	0.001	−0.002	−0.018	0.018	−0.001	−0.019	0.018
	5	0.001	0.001	−0.009	0.011	−0.001	0.008	0.008	0.000	0.000	−0.018	0.023	0.000	0.016	0.017
	6	0.001	0.000	−0.015	0.012	0.001	0.004	0.006	−0.001	0.000	−0.033	0.023	0.000	0.009	0.016
3	3	0.000	0.000	0.001	0.007	0.000	0.000	0.008	−0.001	0.001	0.000	0.019	−0.001	0.001	0.018
	4	0.000	−0.001	−0.007	0.009	0.000	−0.007	0.008	0.000	0.001	−0.019	0.017	−0.001	−0.017	0.019
	5	−0.002	0.000	−0.008	0.011	−0.001	0.008	0.008	−0.001	0.001	−0.016	0.022	0.000	0.016	0.015
	6	0.000	0.000	−0.016	0.010	0.000	0.004	0.006	0.001	0.000	−0.035	0.022	0.000	0.011	0.015
4	3	0.001	0.000	−0.002	0.009	0.000	0.002	0.008	0.000	0.001	0.001	0.017	0.001	0.000	0.017
	4	0.000	0.002	−0.008	0.007	0.001	−0.008	0.008	0.001	0.001	−0.018	0.018	−0.001	−0.015	0.018
	5	0.002	0.000	−0.008	0.011	0.000	0.007	0.007	0.000	0.000	−0.016	0.022	0.001	0.016	0.016
	6	0.000	−0.001	−0.016	0.012	0.000	0.005	0.007	0.000	0.000	−0.033	0.022	0.000	0.011	0.016
		$Rho = 0.75$							$Rho = 0.95$						
n	k	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$
2	3	−0.001	0.001	−0.001	0.041	0.002	0.001	0.040	0.000	0.001	0.000	0.065	0.001	−0.001	0.064
	4	−0.003	−0.001	−0.039	0.041	0.000	−0.040	0.040	0.002	0.000	−0.067	0.065	0.001	−0.064	0.065
	5	0.000	−0.001	−0.034	0.051	0.000	0.037	0.036	0.000	0.002	−0.059	0.084	0.000	0.059	0.059
	6	0.001	0.001	−0.071	0.052	−0.001	0.021	0.031	0.002	0.001	−0.121	0.084	0.001	0.036	0.052
3	3	−0.001	0.000	−0.001	0.040	0.000	0.001	0.039	0.000	0.000	0.000	0.065	0.000	0.000	0.066
	4	0.001	−0.001	−0.041	0.040	0.001	−0.040	0.040	−0.001	0.001	−0.064	0.065	0.000	−0.064	0.065
	5	0.000	0.001	−0.037	0.053	−0.001	0.035	0.035	0.000	−0.001	−0.058	0.084	−0.001	0.058	0.059
	6	0.000	−0.001	−0.073	0.052	0.000	0.020	0.032	0.000	0.001	−0.120	0.084	0.000	0.037	0.052
4	3	−0.001	−0.001	0.000	0.039	0.000	0.001	0.040	0.000	0.000	0.000	0.065	0.000	0.000	0.065
	4	0.000	0.001	−0.039	0.039	0.000	−0.040	0.039	0.000	0.001	−0.066	0.066	0.000	−0.065	0.066
	5	0.000	0.000	−0.036	0.053	0.000	0.036	0.035	0.000	0.000	−0.058	0.084	0.000	0.059	0.059
	6	0.000	0.001	−0.073	0.053	0.000	0.021	0.032	−0.001	0.000	−0.120	0.084	−0.001	0.036	0.052

Table 4.2 Relative efficiencies of the proportion estimator for Normal distribution when $p = 0.1$

		$Rho = 0.25$						$Rho = 0.5$					
n	k	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6
2	3	1.639	1.698	1.799	0.976	1.719	1.770	1.737	1.733	1.853	1.177	1.729	1.887
	4	1.736	1.662	1.854	0.779	1.650	1.851	1.799	1.538	1.952	0.956	1.534	1.946
	5	1.838	1.694	1.928	0.956	1.903	1.910	1.919	1.611	1.989	1.216	2.004	2.017
	6	1.874	1.603	1.964	0.835	1.888	1.911	1.973	1.281	1.977	1.055	2.010	2.037
3	3	1.356	1.378	1.415	0.878	1.360	1.410	1.398	1.399	1.526	1.028	1.408	1.516
	4	1.399	1.322	1.464	0.721	1.326	1.470	1.460	1.193	1.533	0.851	1.230	1.532
	5	1.475	1.371	1.536	0.867	1.517	1.525	1.499	1.270	1.531	1.042	1.546	1.544
	6	1.455	1.224	1.513	0.751	1.471	1.489	1.506	0.937	1.498	0.906	1.539	1.530
4	3	1.241	1.215	1.292	0.828	1.251	1.285	1.282	1.293	1.364	0.979	1.285	1.370
	4	1.294	1.182	1.315	0.702	1.188	1.327	1.318	1.086	1.372	0.817	1.124	1.365
	5	1.279	1.199	1.333	0.803	1.317	1.318	1.334	1.138	1.343	0.967	1.373	1.373
	6	1.299	1.099	1.357	0.721	1.344	1.348	1.363	0.846	1.324	0.861	1.393	1.376
		$Rho = 0.75$						$Rho = 0.95$					
n	k	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6
2	3	1.856	1.815	1.990	1.675	1.880	1.985	1.937	1.948	1.759	1.914	1.920	1.773
	4	2.047	1.300	1.989	1.526	1.251	1.996	2.156	0.842	1.545	2.017	0.876	1.550
	5	2.070	1.360	1.635	1.812	1.918	1.928	2.410	0.979	1.040	2.354	1.524	1.521
	6	2.188	0.701	1.481	1.673	2.083	1.961	2.501	0.343	0.878	2.433	1.866	1.513
3	3	1.494	1.470	1.545	1.380	1.495	1.556	1.553	1.580	1.318	1.532	1.573	1.309
	4	1.537	0.932	1.466	1.242	0.936	1.459	1.761	0.676	1.145	1.660	0.685	1.149
	5	1.651	0.992	1.165	1.460	1.465	1.461	1.857	0.720	0.723	1.799	1.101	1.092
	6	1.675	0.496	1.069	1.369	1.604	1.470	1.986	0.249	0.619	1.920	1.394	1.110
4	3	1.348	1.371	1.363	1.254	1.383	1.358	1.443	1.450	1.098	1.419	1.448	1.091
	4	1.439	0.840	1.272	1.149	0.838	1.273	1.591	0.553	0.909	1.487	0.562	0.907
	5	1.484	0.879	0.961	1.347	1.247	1.253	1.688	0.608	0.558	1.674	0.882	0.881
	6	1.521	0.402	0.851	1.263	1.402	1.243	1.792	0.200	0.479	1.720	1.191	0.895

Table 4.3 Biases of the proportion estimator for Log-Normal distribution when $p = 0.1$

		$Rho = 0.25$							$Rho = 0.5$						
n	k	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$
2	3	0.001	0.000	0.002	0.007	0.000	0.000	0.005	0.000	−0.001	0.000	0.027	0.000	0.000	0.025
	4	0.002	−0.002	−0.007	0.005	−0.001	−0.007	0.005	0.001	0.000	−0.024	0.025	0.002	−0.024	0.026
	5	0.000	0.000	−0.007	0.011	0.001	0.005	0.006	−0.002	0.001	−0.024	0.034	0.000	0.021	0.023
	6	−0.002	0.002	−0.013	0.008	0.000	0.001	0.006	−0.001	−0.001	−0.047	0.032	0.001	0.014	0.020
3	3	−0.001	−0.001	0.001	0.007	0.000	−0.002	0.008	0.001	0.000	0.002	0.024	0.000	0.001	0.024
	4	0.001	0.001	−0.006	0.008	0.000	−0.008	0.007	0.000	0.000	−0.026	0.025	−0.001	−0.024	0.025
	5	0.000	−0.001	−0.006	0.010	0.001	0.007	0.005	−0.001	0.001	−0.022	0.034	0.001	0.023	0.023
	6	0.000	−0.001	−0.012	0.009	−0.001	0.003	0.005	0.000	0.000	−0.046	0.033	0.000	0.014	0.019
4	3	0.000	−0.001	0.001	0.007	0.002	0.000	0.007	0.001	−0.001	0.001	0.026	0.001	0.000	0.027
	4	0.000	0.001	−0.007	0.006	0.001	−0.007	0.006	0.000	0.000	−0.026	0.024	0.000	−0.026	0.025
	5	0.000	0.002	−0.005	0.009	0.000	0.005	0.005	−0.001	0.001	−0.022	0.032	0.000	0.024	0.023
	6	0.001	0.001	−0.012	0.008	0.000	0.002	0.005	−0.001	0.000	−0.048	0.032	0.000	0.014	0.020
		$Rho = 0.75$							$Rho = 0.95$						
n	k	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$
2	3	0.000	−0.001	0.001	0.049	0.000	−0.001	0.048	0.000	0.000	−0.001	0.062	0.001	−0.002	0.062
	4	0.002	−0.001	−0.050	0.050	0.000	−0.049	0.050	0.001	0.001	−0.062	0.063	0.000	−0.063	0.062
	5	0.000	−0.001	−0.044	0.066	−0.001	0.044	0.043	0.001	0.001	−0.055	0.081	0.001	0.056	0.056
	6	−0.001	0.000	−0.088	0.066	0.000	0.025	0.037	−0.001	0.001	−0.114	0.082	−0.001	0.034	0.049
3	3	−0.003	0.002	0.001	0.049	0.000	−0.001	0.049	0.000	0.001	0.001	0.062	−0.001	0.000	0.061
	4	−0.002	−0.001	−0.051	0.050	0.000	−0.050	0.049	−0.002	0.001	−0.063	0.062	0.001	−0.062	0.062
	5	0.001	0.000	−0.044	0.065	−0.002	0.044	0.043	−0.001	0.001	−0.056	0.082	0.000	0.056	0.056
	6	−0.001	0.000	−0.089	0.066	0.000	0.023	0.039	0.000	0.000	−0.117	0.082	0.000	0.033	0.050
4	3	0.000	0.000	−0.001	0.048	−0.001	0.000	0.049	0.000	0.000	−0.002	0.062	0.000	0.002	0.062
	4	0.001	0.001	−0.051	0.049	0.000	−0.050	0.049	0.000	−0.002	−0.063	0.062	0.000	−0.062	0.062
	5	0.000	0.000	−0.044	0.066	0.000	0.043	0.045	0.000	0.001	−0.056	0.082	0.000	0.057	0.056
	6	0.000	0.000	−0.090	0.066	−0.001	0.024	0.039	0.000	0.002	−0.115	0.082	0.000	0.033	0.050

Table 4.4 Relative efficiencies of the proportion estimator for Log-Normal distribution when $p = 0.1$

		$Rho = 0.25$						$Rho = 0.5$					
n	k	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6
2	3	1.666	1.699	1.740	0.974	1.655	1.723	1.765	1.765	1.976	1.367	1.772	1.949
	4	1.727	1.654	1.793	0.751	1.642	1.800	1.881	1.465	1.986	1.124	1.455	1.985
	5	1.839	1.750	1.942	0.976	1.882	1.910	2.006	1.564	1.950	1.469	2.069	2.064
	6	1.956	1.683	1.975	0.846	1.908	1.987	2.036	1.040	1.903	1.291	2.097	2.065
3	3	1.347	1.371	1.434	0.857	1.349	1.427	1.388	1.427	1.512	1.161	1.410	1.515
	4	1.408	1.330	1.461	0.710	1.300	1.460	1.497	1.105	1.548	0.970	1.148	1.541
	5	1.425	1.364	1.497	0.846	1.490	1.471	1.551	1.192	1.445	1.201	1.568	1.559
	6	1.431	1.280	1.498	0.730	1.470	1.484	1.564	0.780	1.390	1.083	1.590	1.567
4	3	1.233	1.253	1.298	0.849	1.245	1.292	1.289	1.307	1.379	1.074	1.301	1.378
	4	1.281	1.192	1.312	0.691	1.196	1.308	1.364	1.002	1.379	0.945	1.005	1.377
	5	1.310	1.242	1.345	0.797	1.327	1.322	1.405	1.075	1.268	1.122	1.377	1.382
	6	1.309	1.144	1.341	0.712	1.318	1.330	1.424	0.639	1.210	1.008	1.426	1.385
		$Rho = 0.75$						$Rho = 0.95$					
n	k	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6
2	3	1.887	1.903	1.928	1.779	1.836	1.951	1.930	1.906	1.806	1.912	1.902	1.809
	4	2.018	1.072	1.809	1.621	1.081	1.808	2.169	0.912	1.576	1.992	0.898	1.590
	5	2.212	1.194	1.344	1.979	1.815	1.823	2.349	1.019	1.078	2.308	1.580	1.558
	6	2.350	0.548	1.227	1.969	2.099	1.893	2.551	0.381	0.930	2.343	1.959	1.615
3	3	1.575	1.570	1.520	1.444	1.534	1.523	1.570	1.556	1.360	1.508	1.555	1.361
	4	1.618	0.822	1.367	1.390	0.827	1.388	1.751	0.697	1.192	1.632	0.699	1.194
	5	1.679	0.888	0.973	1.562	1.334	1.346	1.831	0.744	0.751	1.807	1.145	1.150
	6	1.789	0.388	0.862	1.574	1.606	1.365	1.939	0.261	0.645	1.852	1.457	1.149
4	3	1.392	1.379	1.288	1.302	1.385	1.277	1.423	1.406	1.124	1.408	1.438	1.125
	4	1.467	0.692	1.137	1.256	0.700	1.126	1.531	0.575	0.954	1.427	0.585	0.952
	5	1.550	0.773	0.763	1.440	1.124	1.108	1.650	0.629	0.579	1.607	0.910	0.915
	6	1.591	0.304	0.668	1.408	1.368	1.131	1.747	0.212	0.494	1.681	1.237	0.935

Table 4.5-4.8 are presented bias values and relative efficiencies of the proportion estimator for Normal and Log-Normal distributions when true proportion is 0.2.

In Table 4.5 and Table 4.7, the results are showed the proportion estimator for SRS, RSS, PRSS are unbiased but the estimator of proportion for ERSS, MRSS, Per-RSS and LRSS are biased for all values of correlation coefficient. The bias values increase for these modified RSS methods, while the correlation coefficients increase.

Table 4.6 and 4.8 present the relative efficiencies of proportion estimator for based on SRS, RSS, ERSS, MRSS, PRSS, Per-RSS under symmetric and asymmetric distributions, respectively. According to tables, the proportion estimator for RSS method is more efficient than the proportion estimator based on SRS method for all cases. Additionally, the efficiencies are increasing with the set size and the correlation coefficient between concomitant variable and variable of interest. The proportion estimators for ERSS, MRSS, Per-RSS and LRSS are more efficient than the proportion estimator for SRS under poor ranking.

Table 4.5 Biases of the proportion estimator for Normal distribution when $p = 0.2$

		$Rho = 0.25$							$Rho = 0.5$						
n	k	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$
2	3	0.000	0.002	0.000	0.010	−0.001	0.002	0.010	0.001	−0.001	−0.001	0.011	0.001	−0.001	0.011
	4	−0.002	0.000	−0.009	0.009	0.001	−0.007	0.010	0.003	0.001	−0.012	0.014	0.001	−0.014	0.015
	5	0.001	−0.002	−0.008	0.011	−0.001	0.008	0.009	0.000	0.000	−0.013	0.019	0.000	0.015	0.012
	6	−0.002	0.001	−0.016	0.013	0.000	0.004	0.008	0.001	−0.001	−0.026	0.016	−0.002	0.010	0.011
3	3	0.000	0.000	0.001	0.008	−0.002	−0.002	0.008	0.002	−0.001	0.001	0.016	−0.001	0.000	0.016
	4	−0.001	0.000	−0.007	0.010	−0.001	−0.009	0.009	0.000	−0.003	−0.013	0.013	0.001	−0.014	0.015
	5	−0.001	0.000	−0.007	0.011	0.000	0.006	0.004	0.000	−0.001	−0.015	0.016	0.000	0.014	0.013
	6	0.000	0.000	−0.013	0.011	0.000	0.003	0.005	−0.001	−0.003	−0.029	0.015	0.001	0.009	0.013
4	3	0.000	0.001	0.001	0.007	0.000	0.001	0.008	0.002	−0.002	−0.001	0.012	0.001	−0.001	0.014
	4	0.000	0.000	−0.009	0.006	0.000	−0.007	0.009	0.001	0.000	−0.014	0.015	0.000	−0.016	0.015
	5	−0.001	−0.001	−0.007	0.011	0.000	0.006	0.008	0.000	0.000	−0.014	0.017	−0.004	0.014	0.011
	6	−0.001	0.001	−0.015	0.013	0.000	0.003	0.005	0.000	0.000	−0.027	0.017	0.001	0.011	0.012
		$Rho = 0.75$							$Rho = 0.95$						
n	k	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$
2	3	0.000	0.000	0.000	0.046	0.000	0.000	0.044	0.000	0.001	−0.001	0.085	0.001	−0.002	0.085
	4	0.000	0.001	−0.044	0.046	0.000	−0.044	0.043	0.001	0.000	−0.084	0.085	0.002	−0.086	0.084
	5	0.000	0.001	−0.037	0.061	0.001	0.038	0.038	0.001	−0.001	−0.073	0.124	0.000	0.071	0.070
	6	0.000	0.001	−0.079	0.062	0.000	0.018	0.031	0.000	0.000	−0.148	0.124	0.000	0.024	0.060
3	3	0.001	0.000	0.000	0.046	0.002	0.002	0.045	0.000	0.000	0.001	0.085	−0.001	0.000	0.085
	4	0.001	0.000	−0.046	0.045	−0.001	−0.046	0.042	0.001	0.000	−0.085	0.084	0.000	−0.084	0.084
	5	0.002	0.002	−0.038	0.062	0.001	0.039	0.039	0.000	0.000	−0.071	0.124	0.001	0.071	0.072
	6	0.001	−0.001	−0.079	0.063	0.001	0.017	0.033	0.001	0.000	−0.150	0.124	−0.001	0.024	0.058
4	3	0.000	0.000	−0.002	0.043	0.001	−0.001	0.046	−0.001	0.001	−0.001	0.085	0.001	0.001	0.084
	4	0.000	−0.003	−0.045	0.042	−0.001	−0.046	0.044	−0.002	0.000	−0.086	0.085	0.000	−0.084	0.084
	5	0.001	−0.002	−0.039	0.062	−0.001	0.037	0.040	0.000	0.000	−0.073	0.124	−0.001	0.071	0.071
	6	0.001	0.001	−0.080	0.063	0.000	0.017	0.032	0.001	0.000	−0.150	0.123	0.000	0.026	0.058

Table 4.6 Relative efficiencies of the proportion estimator for Normal distribution when $p = 0.2$

		$Rho = 0.25$						$Rho = 0.5$					
n	k	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6
2	3	1.709	1.685	1.722	0.920	1.727	1.701	1.755	1.745	1.712	1.176	1.766	1.725
	4	1.785	1.773	1.803	0.740	1.773	1.802	1.870	1.880	1.820	0.929	1.841	1.809
	5	1.829	1.780	1.846	0.906	1.856	1.864	1.951	1.929	1.859	1.171	1.904	1.899
	6	1.920	1.788	1.899	0.797	1.923	1.909	2.017	1.835	1.884	0.992	1.987	1.953
3	3	1.368	1.370	1.373	0.824	1.352	1.367	1.399	1.413	1.379	1.006	1.404	1.386
	4	1.418	1.399	1.430	0.683	1.380	1.421	1.471	1.473	1.440	0.835	1.455	1.434
	5	1.447	1.423	1.444	0.811	1.442	1.440	1.508	1.487	1.442	1.017	1.469	1.478
	6	1.465	1.402	1.459	0.718	1.460	1.457	1.544	1.370	1.460	0.886	1.537	1.507
4	3	1.251	1.252	1.252	0.799	1.256	1.254	1.287	1.292	1.257	0.965	1.287	1.257
	4	1.279	1.249	1.276	0.657	1.267	1.280	1.346	1.318	1.298	0.793	1.298	1.298
	5	1.297	1.283	1.296	0.771	1.301	1.309	1.375	1.334	1.294	0.941	1.323	1.325
	6	1.328	1.241	1.309	0.685	1.322	1.318	1.402	1.228	1.297	0.832	1.369	1.343
		$Rho = 0.75$						$Rho = 0.95$					
n	k	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6
2	3	1.965	1.946	1.729	1.620	1.969	1.717	2.191	2.159	1.415	2.113	2.195	1.421
	4	2.117	1.658	1.740	1.324	1.671	1.764	2.535	1.174	1.330	2.121	1.161	1.342
	5	2.232	1.840	1.484	1.754	1.854	1.864	2.800	1.361	0.766	2.639	1.441	1.466
	6	2.350	1.048	1.402	1.511	2.149	1.974	3.067	0.465	0.675	2.647	2.286	1.587
3	3	1.559	1.564	1.345	1.321	1.556	1.354	1.779	1.762	1.076	1.676	1.749	1.071
	4	1.676	1.256	1.338	1.132	1.263	1.366	1.978	0.864	0.981	1.678	0.870	0.981
	5	1.738	1.360	1.082	1.436	1.390	1.392	2.187	1.020	0.543	2.092	1.057	1.052
	6	1.797	0.761	1.018	1.280	1.651	1.474	2.342	0.318	0.466	2.079	1.720	1.186
4	3	1.420	1.419	1.206	1.244	1.421	1.197	1.612	1.619	0.906	1.561	1.630	0.910
	4	1.495	1.109	1.199	1.071	1.089	1.192	1.826	0.719	0.810	1.571	0.734	0.817
	5	1.566	1.187	0.909	1.296	1.238	1.209	1.976	0.813	0.422	1.889	0.864	0.875
	6	1.631	0.613	0.837	1.177	1.472	1.287	2.114	0.247	0.368	1.874	1.501	0.985

Table 4.7 Biases of the proportion estimator for Log-Normal distribution when $p = 0.2$

		$Rho = 0.25$							$Rho = 0.5$						
n	k	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$
2	3	0.003	0.001	0.002	0.016	−0.001	−0.001	0.015	0.001	0.001	0.000	0.035	0.001	0.000	0.036
	4	0.002	0.002	−0.015	0.014	−0.002	−0.015	0.016	−0.001	−0.002	−0.036	0.035	0.002	−0.035	0.037
	5	−0.001	0.000	−0.016	0.020	0.001	0.013	0.013	0.000	−0.001	−0.031	0.047	0.002	0.029	0.029
	6	0.000	−0.002	−0.029	0.020	−0.001	0.007	0.013	0.003	0.001	−0.061	0.050	0.001	0.012	0.025
3	3	0.002	0.000	0.001	0.015	−0.002	−0.001	0.017	0.000	−0.001	−0.003	0.034	−0.001	0.000	0.036
	4	0.001	−0.001	−0.016	0.016	0.000	−0.017	0.014	0.001	0.002	−0.033	0.035	−0.001	−0.035	0.034
	5	−0.002	−0.001	−0.014	0.022	0.000	0.014	0.012	0.001	0.001	−0.029	0.047	0.001	0.029	0.029
	6	0.000	0.000	−0.030	0.021	−0.001	0.008	0.013	0.001	−0.001	−0.061	0.049	0.000	0.011	0.024
4	3	0.000	0.000	−0.001	0.016	0.002	0.003	0.016	0.002	0.002	0.000	0.035	0.003	0.002	0.034
	4	0.001	0.000	−0.015	0.015	0.001	−0.015	0.015	0.001	0.000	−0.035	0.034	−0.001	−0.033	0.036
	5	0.001	−0.001	−0.014	0.021	0.000	0.015	0.013	0.001	−0.001	−0.030	0.048	0.000	0.028	0.030
	6	0.001	0.000	−0.029	0.021	0.000	0.008	0.011	0.000	0.001	−0.062	0.050	−0.001	0.014	0.025
		$Rho = 0.75$							$Rho = 0.95$						
n	k	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$
2	3	−0.002	0.001	−0.006	0.055	0.000	0.001	0.057	0.001	0.001	−0.002	0.084	0.001	−0.001	0.083
	4	−0.002	0.001	−0.057	0.057	0.001	−0.055	0.057	0.003	0.000	−0.085	0.086	−0.001	−0.082	0.083
	5	−0.001	0.000	−0.047	0.080	−0.002	0.048	0.050	−0.001	0.001	−0.070	0.123	0.001	0.070	0.070
	6	−0.002	−0.001	−0.102	0.080	0.001	0.021	0.042	0.000	−0.001	−0.148	0.122	−0.001	0.025	0.056
3	3	0.001	0.001	0.000	0.056	0.000	0.000	0.057	0.000	0.000	0.004	0.083	0.001	−0.001	0.083
	4	0.000	0.001	−0.058	0.056	0.001	−0.055	0.056	−0.001	−0.001	−0.084	0.083	0.001	−0.083	0.083
	5	0.000	0.001	−0.049	0.079	−0.002	0.049	0.050	−0.002	0.001	−0.071	0.122	−0.001	0.072	0.070
	6	−0.001	0.000	−0.100	0.079	0.000	0.024	0.039	−0.001	0.000	−0.146	0.122	0.000	0.023	0.057
4	3	0.000	−0.001	0.000	0.057	−0.001	0.001	0.057	0.000	0.002	−0.002	0.085	−0.001	0.001	0.084
	4	0.001	0.000	−0.058	0.056	0.000	−0.056	0.055	0.000	0.000	−0.083	0.084	0.000	−0.082	0.083
	5	0.001	0.000	−0.049	0.080	0.002	0.049	0.048	0.001	0.001	−0.070	0.123	0.001	0.070	0.071
	6	0.000	0.001	−0.101	0.078	−0.001	0.023	0.040	0.001	0.000	−0.147	0.122	−0.001	0.024	0.055

Table 4.8 Relative efficiencies of the proportion estimator for Log-Normal distribution when $p = 0.2$

		$Rho = 0.25$						$Rho = 0.5$					
n	k	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6
2	3	1.698	1.696	1.720	0.942	1.681	1.726	1.846	1.848	1.742	1.380	1.850	1.721
	4	1.800	1.679	1.817	0.734	1.690	1.811	2.032	1.711	1.831	1.139	1.716	1.810
	5	1.869	1.746	1.851	0.918	1.889	1.880	2.067	1.829	1.657	1.432	1.899	1.904
	6	1.881	1.599	1.871	0.793	1.893	1.892	2.159	1.262	1.560	1.239	2.046	1.947
3	3	1.358	1.356	1.378	0.820	1.344	1.393	1.480	1.479	1.382	1.179	1.479	1.383
	4	1.398	1.311	1.425	0.682	1.305	1.419	1.575	1.339	1.407	0.975	1.324	1.400
	5	1.442	1.375	1.432	0.809	1.459	1.457	1.636	1.411	1.243	1.211	1.449	1.447
	6	1.460	1.196	1.433	0.710	1.461	1.465	1.659	0.936	1.173	1.066	1.586	1.500
4	3	1.246	1.246	1.271	0.804	1.264	1.268	1.354	1.357	1.233	1.103	1.368	1.242
	4	1.286	1.200	1.290	0.662	1.196	1.292	1.430	1.166	1.243	0.924	1.186	1.237
	5	1.293	1.216	1.274	0.762	1.295	1.297	1.478	1.227	1.060	1.132	1.290	1.278
	6	1.315	1.062	1.265	0.683	1.309	1.307	1.518	0.774	0.992	1.000	1.416	1.328
		$Rho = 0.75$						$Rho = 0.95$					
n	k	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6
2	3	2.016	1.953	1.673	1.661	2.009	1.664	2.198	2.178	1.415	2.107	2.180	1.437
	4	2.207	1.496	1.666	1.462	1.542	1.653	2.452	1.152	1.297	2.084	1.185	1.331
	5	2.321	1.696	1.236	1.794	1.756	1.709	2.814	1.405	0.778	2.676	1.467	1.464
	6	2.451	0.785	1.149	1.642	2.122	1.830	3.033	0.463	0.687	2.617	2.239	1.675
3	3	1.583	1.585	1.286	1.365	1.586	1.280	1.753	1.765	1.090	1.695	1.769	1.089
	4	1.701	1.105	1.240	1.219	1.145	1.244	1.974	0.880	0.994	1.705	0.886	0.998
	5	1.807	1.241	0.912	1.491	1.295	1.282	2.196	1.019	0.555	2.101	1.055	1.079
	6	1.898	0.569	0.835	1.372	1.609	1.399	2.329	0.334	0.480	2.073	1.747	1.214
4	3	1.455	1.465	1.120	1.285	1.462	1.123	1.628	1.598	0.904	1.551	1.610	0.909
	4	1.550	0.958	1.066	1.138	0.980	1.078	1.795	0.733	0.808	1.524	0.744	0.818
	5	1.633	1.065	0.730	1.375	1.102	1.116	1.977	0.858	0.430	1.876	0.879	0.876
	6	1.716	0.452	0.683	1.254	1.426	1.203	2.094	0.256	0.374	1.869	1.525	1.026

The result of bias values of the proportion estimators are given by Table 4.9 and Table 4.11 for the true proportion $p = 0.3$ under symmetric and asymmetric distribution. Based on the bias tables, the proportion estimators for SRS, RSS and PRSS methods are unbiased. However, the proportion estimators for ERSS, MRSS, Per-RSS and LRSS methods are biased under symmetric and asymmetric distribution for all ρ values.

The results of relative efficiencies of proportion estimators are given by Table 4.10 and 4.12 under symmetric and asymmetric distribution. According to the simulation results for $p = 0.3$ under distributions, the proportion estimators for RSS, Per-RS are more efficient than the proportion estimator for SRS method for all ρ values. The proportion estimators for ERSS, MRSS and LRSS are more efficient than SRS method for all combination, except for high values of ρ. However, when ρ close to perfect ranking, the proportion estimator for PRSS is more efficient than SRS.

Table 4.9 Biases of the proportion estimator for Normal distribution when $p = 0.3$

		$Rho = 0.25$							$Rho = 0.5$						
n	k	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$
2	3	0.001	−0.001	−0.002	0.009	0.002	0.001	0.006	0.001	0.001	0.001	0.005	−0.003	−0.001	0.012
	4	−0.003	0.000	−0.009	0.006	0.001	−0.006	0.005	0.000	0.002	−0.010	0.009	0.000	−0.009	0.006
	5	−0.003	0.001	−0.005	0.008	−0.001	0.007	0.005	0.002	0.002	−0.008	0.012	−0.001	0.006	0.008
	6	0.002	0.000	−0.010	0.011	0.000	0.003	0.006	0.000	−0.002	−0.014	0.006	0.003	0.007	0.007
3	3	0.000	0.001	−0.001	0.007	−0.002	0.002	0.006	0.001	−0.002	0.000	0.011	0.000	−0.002	0.007
	4	−0.001	0.000	−0.006	0.008	0.000	−0.006	0.005	0.001	0.001	−0.008	0.008	0.001	−0.008	0.009
	5	−0.001	−0.002	−0.006	0.008	0.001	0.005	0.006	−0.002	−0.001	−0.008	0.007	−0.001	0.010	0.007
	6	0.000	0.000	−0.012	0.008	0.002	0.003	0.004	−0.001	0.002	−0.015	0.006	0.000	0.008	0.006
4	3	−0.001	0.001	0.001	0.006	0.002	0.004	0.006	0.001	−0.001	0.000	0.007	0.002	0.002	0.008
	4	−0.001	0.000	−0.005	0.004	0.000	−0.006	0.007	0.000	0.001	−0.008	0.009	0.000	−0.009	0.009
	5	0.001	0.000	−0.006	0.009	0.000	0.006	0.005	0.000	0.000	−0.009	0.009	0.001	0.008	0.008
	6	0.000	0.000	−0.011	0.009	0.001	0.002	0.005	0.001	−0.001	−0.017	0.010	0.000	0.007	0.008
		$Rho = 0.75$							$Rho = 0.95$						
n	k	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$
2	3	−0.003	0.000	−0.002	0.038	0.001	0.002	0.035	−0.003	0.000	−0.004	0.070	−0.001	−0.001	0.070
	4	−0.004	−0.003	−0.037	0.037	0.002	−0.036	0.036	0.000	−0.001	−0.073	0.071	0.002	−0.074	0.070
	5	0.000	−0.002	−0.029	0.050	0.000	0.028	0.030	0.002	−0.001	−0.058	0.112	0.002	0.060	0.058
	6	−0.001	0.000	−0.065	0.051	0.000	0.012	0.025	0.003	−0.001	−0.123	0.112	0.001	0.010	0.048
3	3	−0.001	0.001	0.001	0.037	0.000	0.001	0.036	−0.002	−0.001	0.000	0.071	−0.001	−0.001	0.073
	4	−0.002	0.000	−0.035	0.038	0.001	−0.035	0.037	0.001	0.000	−0.073	0.071	0.000	−0.073	0.073
	5	0.000	−0.001	−0.030	0.053	0.000	0.031	0.031	0.000	−0.001	−0.059	0.112	−0.001	0.058	0.058
	6	−0.001	0.000	−0.064	0.051	−0.001	0.012	0.026	−0.001	0.000	−0.125	0.114	−0.001	0.010	0.046
4	3	−0.001	0.002	0.000	0.034	−0.003	−0.003	0.037	−0.002	−0.001	0.001	0.073	−0.001	0.002	0.074
	4	0.000	0.001	−0.036	0.033	0.001	−0.037	0.036	−0.001	0.000	−0.072	0.073	0.000	−0.071	0.071
	5	0.002	0.000	−0.029	0.051	0.001	0.029	0.031	0.000	0.000	−0.059	0.113	0.000	0.060	0.059
	6	−0.001	0.000	−0.063	0.052	0.000	0.012	0.024	−0.002	0.001	−0.124	0.112	−0.002	0.011	0.046

Table 4.10 Relative efficiencies of the proportion estimator for Normal distribution when $p = 0.3$

		$Rho = 0.25$						$Rho = 0.5$					
n	k	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6
2	3	1.697	1.691	1.681	0.894	1.685	1.667	1.806	1.774	1.675	1.131	1.779	1.689
	4	1.794	1.778	1.789	0.719	1.806	1.779	1.906	2.015	1.791	0.890	2.033	1.804
	5	1.869	1.878	1.840	0.877	1.845	1.844	1.986	2.078	1.813	1.140	1.856	1.872
	6	1.885	1.872	1.838	0.756	1.873	1.880	2.043	2.259	1.843	0.955	1.970	1.914
3	3	1.356	1.361	1.351	0.800	1.357	1.347	1.429	1.435	1.347	0.993	1.424	1.357
	4	1.403	1.416	1.398	0.655	1.419	1.407	1.501	1.600	1.403	0.808	1.577	1.409
	5	1.433	1.440	1.416	0.783	1.424	1.427	1.553	1.635	1.426	1.000	1.470	1.459
	6	1.446	1.441	1.432	0.691	1.448	1.448	1.586	1.727	1.436	0.856	1.529	1.493
4	3	1.247	1.244	1.235	0.765	1.252	1.236	1.307	1.315	1.235	0.949	1.317	1.242
	4	1.285	1.286	1.265	0.638	1.288	1.265	1.375	1.441	1.280	0.777	1.436	1.285
	5	1.290	1.291	1.274	0.745	1.284	1.284	1.393	1.453	1.275	0.934	1.315	1.315
	6	1.314	1.299	1.285	0.663	1.304	1.302	1.419	1.521	1.290	0.812	1.367	1.343
		$Rho = 0.75$						$Rho = 0.95$					
n	k	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6
2	3	2.031	2.025	1.681	1.580	2.033	1.701	2.418	2.364	1.483	2.206	2.396	1.487
	4	2.208	2.191	1.779	1.272	2.203	1.799	2.753	1.909	1.520	2.011	1.905	1.527
	5	2.344	2.367	1.585	1.710	1.935	1.919	3.093	2.264	0.933	2.722	1.708	1.733
	6	2.452	1.743	1.571	1.424	2.239	2.067	3.262	0.888	0.857	2.481	2.766	1.985
3	3	1.621	1.611	1.323	1.340	1.604	1.328	1.916	1.919	1.147	1.768	1.913	1.138
	4	1.735	1.700	1.361	1.106	1.699	1.380	2.155	1.435	1.147	1.649	1.427	1.115
	5	1.827	1.794	1.182	1.393	1.467	1.469	2.384	1.677	0.675	2.165	1.320	1.305
	6	1.889	1.277	1.171	1.203	1.718	1.574	2.565	0.612	0.605	2.001	2.152	1.521
4	3	1.486	1.473	1.201	1.235	1.476	1.189	1.746	1.743	0.986	1.644	1.758	0.979
	4	1.575	1.495	1.247	1.041	1.494	1.231	1.973	1.220	0.963	1.527	1.221	0.980
	5	1.645	1.592	1.035	1.283	1.312	1.303	2.156	1.392	0.542	1.977	1.089	1.108
	6	1.714	1.076	0.999	1.145	1.555	1.399	2.344	0.485	0.491	1.827	1.933	1.320

Table 4.11 Biases of the proportion estimator for Log-Normal distribution when $p = 0.3$

		$Rho = 0.25$							$Rho = 0.5$						
n	k	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$
2	3	−0.001	−0.001	0.001	0.017	−0.001	−0.003	0.019	−0.001	0.000	−0.001	0.029	0.000	0.001	0.029
	4	−0.002	−0.002	−0.017	0.017	−0.001	−0.021	0.016	−0.004	0.002	−0.028	0.030	0.003	−0.028	0.030
	5	−0.001	−0.004	−0.017	0.030	−0.001	0.016	0.014	−0.002	0.001	−0.023	0.042	0.001	0.026	0.023
	6	0.004	0.001	−0.033	0.027	0.000	0.006	0.013	0.002	0.001	−0.049	0.041	0.001	0.013	0.020
3	3	−0.002	−0.002	0.002	0.022	−0.001	−0.003	0.019	−0.001	−0.001	0.000	0.028	−0.001	−0.001	0.029
	4	−0.001	0.001	−0.021	0.022	0.000	−0.019	0.020	−0.002	0.000	−0.029	0.030	−0.001	−0.026	0.027
	5	−0.001	−0.001	−0.018	0.026	0.001	0.015	0.018	0.002	−0.001	−0.025	0.041	0.000	0.026	0.025
	6	0.000	−0.001	−0.033	0.029	−0.001	0.007	0.014	0.001	0.001	−0.050	0.039	−0.001	0.011	0.023
4	3	0.001	−0.001	−0.001	0.019	0.001	0.000	0.018	0.000	−0.002	0.000	0.027	0.003	−0.001	0.028
	4	−0.001	−0.002	−0.019	0.019	−0.001	−0.019	0.020	0.000	−0.001	−0.028	0.030	−0.001	−0.028	0.027
	5	0.001	0.001	−0.016	0.027	0.000	0.015	0.018	0.001	0.000	−0.025	0.040	0.000	0.025	0.025
	6	−0.001	−0.001	−0.034	0.026	0.000	0.007	0.014	−0.001	−0.001	−0.052	0.038	0.000	0.011	0.021
		$Rho = 0.75$							$Rho = 0.95$						
n	k	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$
2	3	0.000	0.004	0.002	0.046	0.002	0.001	0.045	−0.001	0.000	−0.002	0.074	0.000	−0.001	0.072
	4	−0.002	−0.001	−0.045	0.045	0.001	−0.044	0.041	0.000	0.001	−0.072	0.072	−0.001	−0.071	0.072
	5	0.005	0.000	−0.034	0.064	−0.003	0.039	0.038	0.001	−0.002	−0.057	0.112	0.000	0.059	0.059
	6	0.001	−0.001	−0.079	0.065	0.000	0.012	0.033	0.001	0.001	−0.123	0.110	−0.001	0.008	0.045
3	3	0.003	0.000	−0.001	0.045	−0.001	0.000	0.047	−0.001	0.000	0.001	0.072	0.001	−0.001	0.070
	4	0.002	0.001	−0.045	0.043	0.000	−0.045	0.043	−0.001	−0.002	−0.071	0.071	0.002	−0.070	0.070
	5	0.002	0.000	−0.038	0.064	−0.001	0.038	0.037	0.000	0.000	−0.056	0.111	0.000	0.058	0.059
	6	0.000	0.000	−0.079	0.066	0.001	0.011	0.031	−0.002	0.001	−0.122	0.111	0.000	0.011	0.045
4	3	0.000	0.001	−0.001	0.045	0.000	0.000	0.044	0.001	−0.002	0.001	0.073	−0.001	0.001	0.071
	4	−0.001	0.000	−0.044	0.046	0.000	−0.047	0.047	0.001	−0.001	−0.071	0.071	0.000	−0.071	0.072
	5	0.000	0.001	−0.036	0.066	0.001	0.037	0.038	−0.001	0.000	−0.059	0.112	0.001	0.058	0.057
	6	−0.001	0.000	−0.079	0.066	−0.001	0.011	0.032	−0.002	0.000	−0.122	0.112	0.000	0.012	0.045

Table 4.12 Relative efficiencies of the proportion estimator for Log-Normal distribution when $p = 0.3$

		$Rho = 0.25$						$Rho = 0.5$					
n	k	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6
2	3	1.715	1.699	1.702	0.967	1.717	1.690	1.886	1.886	1.683	1.297	1.870	1.700
	4	1.814	1.781	1.811	0.756	1.763	1.800	2.057	2.022	1.777	1.038	2.038	1.780
	5	1.879	1.829	1.770	0.923	1.823	1.831	2.130	2.138	1.675	1.321	1.863	1.893
	6	1.906	1.693	1.774	0.800	1.868	1.869	2.177	1.829	1.657	1.104	2.025	1.943
3	3	1.373	1.382	1.358	0.851	1.368	1.355	1.512	1.504	1.350	1.133	1.497	1.343
	4	1.431	1.358	1.384	0.695	1.372	1.391	1.597	1.570	1.382	0.912	1.593	1.397
	5	1.461	1.418	1.372	0.828	1.424	1.417	1.645	1.628	1.278	1.124	1.441	1.435
	6	1.480	1.302	1.349	0.723	1.455	1.432	1.696	1.352	1.279	0.973	1.581	1.484
4	3	1.259	1.256	1.238	0.812	1.258	1.234	1.383	1.374	1.226	1.061	1.376	1.218
	4	1.298	1.244	1.257	0.670	1.243	1.255	1.441	1.406	1.242	0.873	1.410	1.255
	5	1.324	1.280	1.224	0.780	1.281	1.270	1.493	1.450	1.130	1.067	1.289	1.295
	6	1.342	1.139	1.219	0.697	1.311	1.290	1.533	1.126	1.129	0.920	1.420	1.334
		$Rho = 0.75$						$Rho = 0.95$					
n	k	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6
2	3	2.064	2.035	1.629	1.606	2.039	1.643	2.328	2.380	1.458	2.190	2.388	1.465
	4	2.275	2.078	1.739	1.335	2.101	1.776	2.730	1.941	1.498	2.013	1.959	1.506
	5	2.394	2.308	1.424	1.727	1.834	1.862	3.033	2.290	0.932	2.717	1.730	1.718
	6	2.507	1.453	1.384	1.477	2.271	1.997	3.257	0.892	0.878	2.412	2.778	2.035
3	3	1.635	1.623	1.284	1.343	1.635	1.274	1.883	1.887	1.146	1.764	1.879	1.152
	4	1.781	1.601	1.335	1.145	1.599	1.343	2.157	1.467	1.146	1.632	1.467	1.153
	5	1.873	1.699	1.072	1.448	1.423	1.433	2.371	1.729	0.681	2.127	1.310	1.297
	6	1.948	1.046	1.018	1.266	1.760	1.530	2.581	0.637	0.625	1.954	2.140	1.546
4	3	1.494	1.497	1.147	1.274	1.491	1.158	1.705	1.730	0.982	1.614	1.737	1.001
	4	1.613	1.418	1.163	1.084	1.369	1.162	1.952	1.231	0.973	1.465	1.225	0.969
	5	1.695	1.530	0.903	1.343	1.259	1.254	2.155	1.381	0.545	1.950	1.118	1.132
	6	1.766	0.851	0.850	1.176	1.583	1.335	2.326	0.494	0.494	1.789	1.898	1.326

In case of true proportion p=0.4, the results of bias values and relative efficiencies of the proportion estimator based on SRS, RSS, ERSS, MRSS, PRSS, Per-RSS and LRSS methods are presented in Table 4.13-4.16 for different values of correlation coefficient ρ under Normal and Log-Normal distributions.

In Table 4.13 and 4.15, the proportion estimators for SRS, RSS, PRSS methods are unbiased but the estimators of proportion for ERSS, MRSS, Per-RSS and LRSS methods are biased. In more details for biased estimators, while the correlation coefficients increase, the bias values increase for these modified RSS methods.

According to Table 4.14 and 4.16, the proportion estimators under RSS, ERSS, Per-RSS and LRSS, MRSS methods are more efficient than the proportion estimator for SRS method except for MRSS method when sample size and correlation coefficient is high. The proportion estimator based on PRSS method is more efficient the proportion estimator based on SRS method for high level of correlation coefficient. In more details, the relative efficiencies of sampling methods with respect to SRS method increase as the coefficient correlation increase, except in same cases for MRSS method.

Table 4.13 Biases of the proportion estimator for Normal distribution when $p = 0.4$

		$Rho = 0.25$							$Rho = 0.5$						
n	k	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$
2	3	−0.001	0.000	0.002	0.000	0.001	0.003	0.004	0.001	0.001	0.004	−0.001	−0.001	−0.001	−0.003
	4	0.002	−0.004	−0.006	−0.002	0.000	−0.003	−0.001	0.000	0.000	0.004	−0.003	0.001	−0.001	−0.003
	5	−0.002	0.002	−0.002	0.003	−0.002	0.003	0.001	0.000	0.003	0.000	−0.006	0.001	−0.003	−0.001
	6	0.001	0.001	−0.002	0.002	0.001	0.005	0.004	−0.001	0.001	0.002	−0.004	−0.001	0.002	−0.001
3	3	0.001	0.000	−0.001	0.003	0.001	0.003	0.005	0.001	0.002	0.001	−0.001	−0.002	−0.001	−0.004
	4	0.000	−0.002	0.001	0.002	0.002	−0.001	0.003	−0.001	−0.001	0.004	−0.001	0.000	0.002	0.001
	5	0.001	−0.001	−0.002	0.001	0.000	0.001	0.004	0.000	0.002	0.002	−0.004	−0.002	−0.003	−0.003
	6	0.001	0.000	−0.004	0.004	0.001	0.000	0.003	−0.002	0.001	0.002	−0.006	0.000	0.002	−0.003
4	3	−0.001	0.001	−0.003	0.002	0.002	−0.001	0.001	0.002	0.000	0.000	−0.002	0.001	0.000	−0.003
	4	−0.001	0.002	−0.002	0.003	−0.001	−0.002	0.001	−0.001	−0.001	0.003	−0.003	0.000	0.003	−0.003
	5	−0.001	0.000	−0.001	0.000	0.003	0.001	0.001	−0.002	0.000	0.002	−0.005	−0.001	0.001	−0.002
	6	0.000	0.000	−0.005	0.003	0.001	0.001	0.003	0.001	0.000	0.004	−0.004	−0.001	0.003	−0.002
		$Rho = 0.75$							$Rho = 0.95$						
n	k	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$
2	3	0.005	0.000	−0.001	0.024	−0.002	0.000	0.022	0.001	−0.002	0.002	0.042	0.001	−0.001	0.043
	4	0.004	0.003	−0.025	0.023	0.001	−0.022	0.025	−0.001	0.001	−0.042	0.042	0.001	−0.040	0.041
	5	−0.001	0.001	−0.018	0.035	−0.002	0.021	0.020	0.001	0.000	−0.033	0.063	0.001	0.033	0.032
	6	0.001	0.000	−0.041	0.036	0.000	0.006	0.017	−0.003	0.001	−0.068	0.067	−0.001	0.002	0.025
3	3	−0.001	0.000	0.000	0.025	−0.001	−0.001	0.025	0.002	−0.002	−0.001	0.037	−0.001	−0.001	0.040
	4	0.000	0.000	−0.025	0.024	0.000	−0.025	0.024	0.000	−0.002	−0.040	0.039	0.000	−0.040	0.040
	5	0.000	0.000	−0.020	0.031	0.002	0.021	0.019	0.001	0.001	−0.033	0.066	0.000	0.031	0.031
	6	−0.002	0.001	−0.042	0.033	−0.002	0.009	0.016	−0.001	0.000	−0.069	0.067	0.000	0.006	0.024
4	3	−0.001	−0.001	0.000	0.023	0.003	0.002	0.025	0.001	−0.001	0.001	0.040	−0.002	−0.001	0.041
	4	0.001	0.000	−0.022	0.023	0.001	−0.023	0.025	−0.001	0.001	−0.040	0.040	0.000	−0.041	0.041
	5	0.000	0.000	−0.020	0.033	0.001	0.020	0.020	0.001	−0.001	−0.032	0.067	−0.001	0.032	0.032
	6	0.000	0.000	−0.042	0.035	0.000	0.007	0.015	0.001	0.000	−0.068	0.064	0.000	0.003	0.024

Table 4.14 Relative efficiencies of the proportion estimator for Normal distribution when $p = 0.4$

		$Rho = 0.25$						$Rho = 0.5$					
n	k	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6
2	3	1.709	1.698	1.656	0.892	1.712	1.679	1.800	1.797	1.664	1.112	1.788	1.648
	4	1.796	1.827	1.766	0.709	1.841	1.752	1.905	2.072	1.781	0.867	2.080	1.773
	5	1.874	1.905	1.807	0.883	1.841	1.843	1.982	2.122	1.776	1.124	1.870	1.858
	6	1.897	1.949	1.837	0.763	1.873	1.869	2.058	2.393	1.841	0.944	1.976	1.942
3	3	1.365	1.359	1.338	0.813	1.360	1.329	1.440	1.442	1.329	0.980	1.440	1.335
	4	1.419	1.449	1.390	0.661	1.446	1.394	1.509	1.645	1.398	0.789	1.631	1.398
	5	1.440	1.469	1.398	0.794	1.423	1.424	1.544	1.678	1.390	0.970	1.441	1.444
	6	1.464	1.520	1.424	0.696	1.448	1.437	1.588	1.859	1.421	0.839	1.531	1.497
4	3	1.256	1.244	1.229	0.784	1.252	1.232	1.314	1.314	1.215	0.929	1.319	1.211
	4	1.292	1.309	1.266	0.641	1.316	1.258	1.367	1.495	1.268	0.758	1.490	1.270
	5	1.309	1.332	1.267	0.757	1.283	1.283	1.405	1.506	1.262	0.915	1.314	1.309
	6	1.320	1.364	1.280	0.669	1.309	1.298	1.426	1.671	1.284	0.793	1.369	1.340
		$Rho = 0.75$						$Rho = 0.95$					
n	k	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6
2	3	2.070	2.057	1.645	1.636	2.076	1.659	2.437	2.453	1.579	2.200	2.464	1.578
	4	2.303	2.733	1.807	1.265	2.791	1.788	2.884	3.678	1.794	1.926	3.720	1.813
	5	2.466	2.916	1.683	1.719	1.980	1.982	3.138	3.799	1.422	2.731	2.175	2.163
	6	2.572	3.069	1.712	1.395	2.359	2.142	3.456	2.919	1.401	2.311	3.321	2.570
3	3	1.668	1.663	1.324	1.360	1.656	1.325	1.965	1.955	1.285	1.799	1.938	1.278
	4	1.801	2.156	1.410	1.111	2.140	1.413	2.245	2.843	1.406	1.571	2.867	1.414
	5	1.902	2.236	1.320	1.410	1.542	1.535	2.475	2.872	1.047	2.141	1.668	1.667
	6	1.988	2.269	1.331	1.190	1.809	1.650	2.691	2.035	1.034	1.878	2.553	1.963
4	3	1.514	1.528	1.211	1.272	1.529	1.205	1.796	1.801	1.147	1.660	1.810	1.141
	4	1.642	1.973	1.280	1.039	1.957	1.270	2.056	2.483	1.249	1.471	2.451	1.237
	5	1.716	1.987	1.165	1.318	1.377	1.384	2.243	2.556	0.889	1.947	1.474	1.482
	6	1.787	1.939	1.156	1.129	1.643	1.483	2.401	1.612	0.904	1.693	2.308	1.737

Table 4.15 Biases of the proportion estimator for Log-Normal distribution when $p = 0.4$

		$Rho = 0.25$							$Rho = 0.5$						
n	k	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$
2	3	0.003	0.000	0.000	0.014	−0.001	0.001	0.014	0.002	−0.003	0.001	0.013	0.001	0.001	0.014
	4	−0.001	0.003	−0.010	0.014	0.000	−0.014	0.010	0.000	0.000	−0.016	0.014	0.000	−0.014	0.012
	5	0.001	0.000	−0.010	0.016	−0.001	0.012	0.013	0.001	0.000	−0.013	0.023	0.001	0.015	0.012
	6	0.000	0.000	−0.022	0.017	−0.002	0.004	0.009	−0.003	0.000	−0.025	0.021	0.001	0.009	0.011
3	3	−0.002	0.000	−0.002	0.011	0.002	0.002	0.011	−0.001	−0.001	−0.002	0.018	0.002	0.001	0.018
	4	0.002	0.001	−0.012	0.013	0.001	−0.014	0.010	−0.002	0.000	−0.017	0.015	0.001	−0.016	0.014
	5	0.000	0.001	−0.011	0.018	−0.001	0.011	0.011	0.001	0.000	−0.012	0.020	−0.002	0.012	0.011
	6	−0.001	0.000	−0.021	0.017	−0.001	0.007	0.009	0.001	0.000	−0.028	0.021	0.002	0.006	0.013
4	3	−0.002	0.000	0.001	0.012	0.000	0.000	0.012	−0.001	−0.004	0.000	0.018	0.000	−0.001	0.012
	4	−0.002	−0.002	−0.010	0.012	0.001	−0.013	0.012	0.000	0.002	−0.015	0.015	0.000	−0.017	0.016
	5	−0.002	−0.002	−0.011	0.020	0.001	0.011	0.011	−0.002	0.000	−0.014	0.021	0.000	0.013	0.014
	6	−0.001	0.000	−0.022	0.017	0.000	0.004	0.009	0.001	0.001	−0.029	0.021	0.000	0.007	0.010
		$Rho = 0.75$							$Rho = 0.95$						
n	k	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$
2	3	−0.001	−0.002	0.002	0.034	0.001	−0.001	0.030	0.001	0.002	−0.002	0.039	−0.001	−0.003	0.039
	4	0.000	−0.003	−0.034	0.033	0.001	−0.031	0.030	−0.004	−0.001	−0.039	0.039	0.000	−0.041	0.039
	5	−0.001	−0.002	−0.026	0.046	−0.003	0.027	0.024	−0.001	0.000	−0.031	0.063	0.002	0.032	0.029
	6	0.000	−0.002	−0.056	0.045	0.000	0.005	0.020	0.001	0.000	−0.066	0.063	−0.002	0.004	0.022
3	3	0.001	0.000	0.001	0.032	0.000	−0.001	0.029	−0.001	0.002	−0.001	0.040	−0.001	0.001	0.040
	4	−0.001	0.002	−0.030	0.032	0.000	−0.030	0.029	0.001	0.000	−0.039	0.037	0.000	−0.037	0.040
	5	0.002	0.000	−0.024	0.048	−0.001	0.024	0.026	0.000	0.002	−0.030	0.062	0.001	0.031	0.031
	6	0.002	0.002	−0.053	0.049	−0.001	0.006	0.020	−0.003	0.001	−0.067	0.062	0.001	0.004	0.023
4	3	0.001	0.001	0.002	0.032	−0.002	0.002	0.031	0.000	0.000	0.001	0.036	0.001	0.000	0.038
	4	−0.001	0.000	−0.031	0.031	−0.002	−0.030	0.031	0.001	−0.001	−0.039	0.037	0.000	−0.039	0.039
	5	0.000	−0.001	−0.025	0.047	0.001	0.026	0.028	0.000	0.000	−0.031	0.060	0.000	0.031	0.031
	6	0.000	0.001	−0.053	0.046	0.001	0.007	0.020	0.001	0.000	−0.066	0.064	0.000	0.003	0.022

Table 4.16 Relative efficiencies of the proportion estimator for Log-Normal distribution when $p = 0.4$

		$Rho = 0.25$						$Rho = 0.5$					
n	k	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6
2	3	1.698	1.697	1.664	0.889	1.712	1.659	1.871	1.888	1.666	1.265	1.854	1.664
	4	1.831	1.843	1.760	0.729	1.819	1.763	2.027	2.235	1.795	0.992	2.252	1.813
	5	1.859	1.872	1.775	0.878	1.809	1.809	2.120	2.309	1.762	1.280	1.899	1.895
	6	1.897	1.901	1.830	0.760	1.852	1.848	2.198	2.556	1.809	1.072	2.061	1.980
3	3	1.368	1.364	1.343	0.828	1.370	1.335	1.503	1.500	1.327	1.095	1.514	1.333
	4	1.414	1.435	1.381	0.667	1.416	1.376	1.597	1.762	1.402	0.889	1.756	1.410
	5	1.450	1.459	1.380	0.789	1.404	1.415	1.652	1.804	1.373	1.095	1.467	1.472
	6	1.466	1.453	1.393	0.691	1.445	1.430	1.698	1.906	1.388	0.944	1.594	1.521
4	3	1.258	1.255	1.224	0.787	1.257	1.225	1.379	1.387	1.223	1.038	1.384	1.220
	4	1.288	1.306	1.258	0.646	1.289	1.259	1.455	1.586	1.271	0.843	1.587	1.277
	5	1.316	1.313	1.237	0.749	1.276	1.271	1.503	1.626	1.239	1.043	1.332	1.331
	6	1.326	1.293	1.255	0.667	1.294	1.291	1.526	1.671	1.242	0.892	1.436	1.382
		$Rho = 0.75$						$Rho = 0.95$					
n	k	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6
2	3	2.152	2.130	1.643	1.707	2.143	1.634	2.480	2.447	1.618	2.185	2.445	1.621
	4	2.364	2.730	1.757	1.342	2.792	1.785	2.869	3.836	1.834	1.927	3.646	1.827
	5	2.531	2.956	1.601	1.816	1.965	1.990	3.166	3.819	1.419	2.709	2.169	2.198
	6	2.676	2.681	1.616	1.480	2.435	2.157	3.432	2.999	1.450	2.268	3.281	2.594
3	3	1.700	1.702	1.294	1.423	1.691	1.310	1.949	1.947	1.273	1.785	1.954	1.266
	4	1.861	2.183	1.388	1.161	2.178	1.407	2.228	2.864	1.430	1.565	2.906	1.398
	5	1.963	2.280	1.199	1.498	1.531	1.516	2.462	2.975	1.082	2.144	1.663	1.673
	6	2.071	2.009	1.186	1.269	1.862	1.650	2.646	2.117	1.094	1.857	2.577	1.975
4	3	1.558	1.547	1.180	1.311	1.561	1.181	1.793	1.790	1.163	1.658	1.780	1.155
	4	1.689	1.946	1.250	1.100	1.950	1.242	2.040	2.482	1.270	1.445	2.482	1.255
	5	1.787	2.021	1.061	1.394	1.359	1.344	2.243	2.576	0.948	1.940	1.483	1.477
	6	1.871	1.682	1.054	1.198	1.684	1.482	2.400	1.689	0.901	1.689	2.316	1.765

In Table 4.17-4.20 are presented bias values and relative efficiencies of the proportion estimators based on SRS, RSS, ERSS, MRSS, PRSS, Per-RSS and LRSS methods for Normal and Log-Normal distributions with different values of correlation coefficient when the true proportion is 0.5.

Table 4.17 and 4.19 present the bias values for the proportion estimators based on sampling methods. The results indicate that the bias is almost zero, when perfect ranking and the proportion estimator for SRS, RSS, ERSS, MRSS, PRSS, Per-RSS and LRSS are unbiased estimators.

According to Table 4.18 and 4.20, the results clearly shows that almost every value of relative efficiency is over than 1 except for PRSS has poor correlation coefficient $\rho \leq 0.5$ and it indicates that the proportion estimators for RSS, ERSS, MRSS, PRSS, Per-RSS and LRSS methods are more efficient than the proportion estimator for SRS method. In this situation the proportion estimator for PRSS is valid when nearly perfect ranking.

Table 4.17 Biases of the proportion estimator for Normal distribution when $p = 0.5$

		$Rho = 0.25$							$Rho = 0.5$						
n	k	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$
2	3	−0.001	−0.002	0.000	−0.002	0.000	−0.001	0.002	0.003	0.000	−0.003	−0.017	−0.002	0.003	−0.014
	4	0.000	−0.005	0.003	0.003	0.001	0.000	0.001	0.002	−0.001	0.014	−0.014	0.000	0.014	−0.012
	5	0.001	0.002	−0.003	−0.001	0.001	0.000	0.002	0.001	−0.001	0.010	−0.019	0.003	−0.009	−0.012
	6	0.001	0.003	−0.003	−0.001	0.000	0.003	0.003	−0.001	0.000	0.023	−0.018	−0.003	−0.006	−0.009
3	3	0.000	0.002	−0.001	0.002	0.001	0.000	0.002	0.002	−0.002	0.000	−0.014	0.001	−0.002	−0.013
	4	−0.001	−0.003	−0.001	0.000	−0.002	−0.003	−0.001	0.000	0.003	0.014	−0.013	0.001	0.013	−0.014
	5	0.000	−0.001	0.000	−0.001	−0.001	0.000	0.001	−0.001	0.002	0.012	−0.019	−0.001	−0.009	−0.011
	6	0.000	0.000	−0.003	−0.002	0.000	0.003	0.003	0.001	−0.001	0.022	−0.020	0.000	−0.006	−0.009
4	3	−0.001	0.001	0.001	0.001	0.002	0.000	0.002	0.001	0.001	0.000	−0.015	−0.005	0.001	−0.012
	4	0.001	−0.001	0.001	0.001	−0.003	0.000	0.002	0.000	0.000	0.012	−0.014	−0.002	0.014	−0.013
	5	0.002	−0.001	−0.001	−0.001	0.000	−0.001	0.003	0.001	−0.002	0.011	−0.021	0.002	−0.011	−0.011
	6	−0.001	−0.001	−0.003	0.000	−0.001	0.006	0.002	0.000	−0.001	0.022	−0.020	0.001	−0.003	−0.009
		$Rho = 0.75$							$Rho = 0.95$						
n	k	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$
2	3	−0.001	0.002	0.002	0.004	0.000	0.002	0.002	−0.004	0.004	−0.001	0.000	−0.001	0.001	0.004
	4	0.003	0.000	−0.003	0.006	−0.001	−0.003	0.005	−0.001	0.001	0.000	0.003	0.000	−0.001	−0.003
	5	−0.001	0.001	−0.001	0.007	−0.002	0.003	0.001	−0.003	−0.002	−0.001	−0.001	0.002	0.002	0.001
	6	0.001	−0.001	−0.004	0.003	0.001	0.001	0.002	−0.002	0.001	0.000	0.000	0.001	−0.001	−0.002
3	3	0.000	0.002	0.001	0.003	0.000	0.000	0.004	0.001	−0.001	0.002	0.003	−0.001	0.001	0.002
	4	0.000	−0.001	−0.001	0.002	0.000	−0.001	0.001	0.000	0.000	0.000	0.002	0.000	−0.001	0.001
	5	0.002	−0.001	−0.005	0.005	−0.003	0.005	0.002	0.001	0.001	0.001	0.002	0.000	0.001	0.001
	6	0.001	−0.001	−0.008	0.007	0.001	0.001	0.002	0.002	0.000	−0.001	0.002	0.001	−0.001	0.001
4	3	−0.002	−0.002	0.002	0.004	0.001	0.000	0.002	0.002	−0.001	0.000	−0.001	0.000	0.001	0.002
	4	0.000	−0.002	−0.004	0.002	0.001	−0.004	0.006	−0.001	−0.001	−0.001	−0.001	0.000	0.000	0.000
	5	0.001	0.000	−0.003	0.005	0.000	0.002	0.003	0.000	0.000	0.000	0.002	−0.001	0.002	−0.001
	6	0.002	0.000	−0.007	0.006	−0.001	0.001	0.004	0.001	−0.001	−0.001	0.000	0.000	0.000	0.001

Table 4.18 Relative efficiencies of the proportion estimator for Normal distribution when $p = 0.5$

		$Rho = 0.25$						$Rho = 0.5$					
n	k	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6
2	3	1.703	1.716	1.672	0.877	1.710	1.672	1.791	1.806	1.655	1.094	1.806	1.649
	4	1.803	1.830	1.750	0.710	1.823	1.762	1.920	2.060	1.761	0.861	2.081	1.752
	5	1.851	1.891	1.805	0.866	1.817	1.829	1.999	2.124	1.743	1.088	1.850	1.833
	6	1.877	1.942	1.859	0.752	1.876	1.857	2.042	2.281	1.794	0.929	1.947	1.900
3	3	1.361	1.355	1.322	0.793	1.357	1.328	1.433	1.447	1.320	0.976	1.437	1.336
	4	1.414	1.441	1.378	0.654	1.430	1.389	1.507	1.621	1.380	0.778	1.617	1.383
	5	1.438	1.458	1.404	0.777	1.415	1.417	1.560	1.655	1.357	0.960	1.440	1.431
	6	1.455	1.500	1.416	0.681	1.446	1.438	1.578	1.743	1.368	0.823	1.502	1.483
4	3	1.247	1.253	1.220	0.770	1.249	1.220	1.318	1.311	1.205	0.925	1.322	1.208
	4	1.281	1.304	1.256	0.631	1.308	1.259	1.369	1.471	1.246	0.754	1.462	1.253
	5	1.298	1.320	1.269	0.738	1.283	1.282	1.403	1.498	1.215	0.906	1.297	1.289
	6	1.311	1.360	1.283	0.656	1.298	1.299	1.434	1.553	1.226	0.785	1.365	1.330
		$Rho = 0.75$						$Rho = 0.95$					
n	k	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6
2	3	2.090	2.079	1.657	1.634	2.098	1.666	2.474	2.463	1.657	2.195	2.473	1.651
	4	2.292	3.110	1.834	1.256	3.108	1.841	2.873	5.471	1.942	1.874	5.421	1.947
	5	2.472	3.118	1.785	1.704	2.036	2.020	3.194	4.872	1.806	2.658	2.363	2.378
	6	2.590	4.643	1.880	1.380	2.415	2.197	3.456	13.622	1.967	2.177	3.508	2.773
3	3	1.673	1.666	1.327	1.372	1.676	1.337	1.979	1.954	1.344	1.794	1.989	1.337
	4	1.818	2.424	1.452	1.093	2.424	1.454	2.253	4.309	1.554	1.528	4.266	1.540
	5	1.926	2.434	1.402	1.415	1.573	1.584	2.499	3.852	1.401	2.100	1.855	1.831
	6	1.985	3.516	1.451	1.196	1.855	1.699	2.684	10.433	1.518	1.813	2.715	2.146
4	3	1.534	1.530	1.214	1.272	1.528	1.222	1.823	1.811	1.222	1.649	1.814	1.228
	4	1.649	2.224	1.321	1.039	2.206	1.319	2.047	3.858	1.399	1.414	3.879	1.403
	5	1.736	2.217	1.266	1.318	1.426	1.428	2.261	3.490	1.266	1.920	1.661	1.672
	6	1.801	3.185	1.306	1.124	1.683	1.522	2.399	9.584	1.373	1.645	2.452	1.937

Table 4.19 Biases of the proportion estimator for Log-Normal distribution when $p = 0.5$

		$Rho = 0.25$							$Rho = 0.5$						
n	k	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$
2	3	−0.001	0.004	0.001	0.008	−0.001	−0.003	0.013	0.003	−0.001	0.000	0.002	−0.001	0.000	0.002
	4	−0.001	−0.002	−0.006	0.010	0.001	−0.008	0.011	0.000	0.001	−0.005	0.001	−0.001	−0.004	0.000
	5	0.004	−0.001	−0.009	0.013	0.000	0.008	0.009	0.001	−0.001	−0.002	0.005	0.001	−0.001	0.007
	6	−0.001	0.001	−0.016	0.012	−0.001	0.002	0.005	−0.002	0.001	−0.003	0.002	0.000	0.001	0.002
3	3	0.001	−0.003	−0.001	0.008	0.001	0.000	0.010	−0.001	−0.002	−0.001	0.000	−0.001	0.001	0.002
	4	0.000	−0.001	−0.010	0.008	−0.001	−0.008	0.009	0.000	−0.001	−0.002	0.003	−0.001	−0.003	0.004
	5	0.001	0.000	−0.009	0.013	0.000	0.008	0.007	−0.002	0.000	−0.002	0.005	0.001	0.004	0.000
	6	0.001	−0.001	−0.016	0.014	0.002	0.004	0.006	0.002	0.002	−0.006	0.002	0.000	0.004	0.003
4	3	−0.001	0.000	0.001	0.011	0.002	−0.001	0.008	0.001	0.001	−0.001	0.004	0.001	−0.001	0.003
	4	0.001	0.000	−0.007	0.008	0.001	−0.008	0.010	0.003	−0.001	−0.003	0.005	0.000	−0.003	0.002
	5	0.000	0.001	−0.008	0.014	0.000	0.008	0.008	0.000	0.002	−0.003	0.004	0.000	0.001	0.005
	6	0.001	−0.001	−0.018	0.013	−0.001	0.001	0.007	0.000	0.000	−0.008	0.001	0.000	0.001	0.001
		$Rho = 0.75$							$Rho = 0.95$						
n	k	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$
2	3	0.002	0.000	0.002	0.006	0.001	−0.001	0.004	0.001	0.000	−0.002	0.000	−0.001	0.000	0.000
	4	0.000	0.003	−0.006	0.004	0.002	−0.006	0.005	−0.002	−0.001	0.001	0.002	0.000	0.000	0.000
	5	−0.001	0.000	−0.005	0.012	0.000	0.005	0.005	−0.002	0.003	−0.002	0.000	0.001	−0.001	−0.001
	6	0.001	−0.001	−0.009	0.009	−0.002	−0.004	0.001	−0.002	0.002	−0.001	0.001	0.002	−0.001	−0.002
3	3	−0.004	−0.002	0.000	0.007	−0.002	0.001	0.006	−0.001	0.002	−0.005	0.003	0.001	0.002	0.000
	4	−0.002	−0.001	−0.005	0.005	−0.001	−0.003	0.003	0.001	−0.001	0.000	0.001	0.000	0.000	0.002
	5	0.002	0.001	−0.003	0.009	0.000	0.006	0.004	0.000	0.000	0.001	0.003	0.000	0.000	−0.001
	6	0.002	0.000	−0.008	0.007	−0.001	−0.002	0.002	−0.001	0.000	−0.001	0.001	−0.001	−0.001	0.000
4	3	−0.002	0.000	0.001	0.004	0.001	0.001	0.006	0.000	0.003	0.001	0.000	−0.001	0.001	0.001
	4	0.000	0.001	−0.005	0.003	0.000	−0.005	0.004	0.001	−0.001	−0.002	0.002	−0.001	−0.001	0.000
	5	0.002	0.000	−0.004	0.011	0.001	0.002	0.005	−0.001	0.002	−0.001	0.000	0.001	0.000	0.001
	6	−0.001	0.001	−0.008	0.010	0.000	−0.003	0.003	−0.001	0.001	−0.001	0.000	−0.002	0.000	0.000

Table 4.20 Relative efficiencies of the proportion estimator for Log-Normal distribution when $p = 0.5$

		$Rho = 0.25$						$Rho = 0.5$					
n	k	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6
2	3	1.741	1.727	1.662	0.904	1.704	1.682	1.879	1.879	1.670	1.235	1.872	1.683
	4	1.812	1.854	1.729	0.715	1.823	1.740	2.055	2.302	1.803	0.950	2.313	1.783
	5	1.854	1.893	1.765	0.878	1.804	1.808	2.090	2.354	1.788	1.239	1.886	1.878
	6	1.897	1.964	1.809	0.759	1.869	1.848	2.169	2.830	1.842	1.027	2.050	1.988
3	3	1.370	1.366	1.326	0.812	1.367	1.328	1.501	1.500	1.340	1.084	1.505	1.337
	4	1.419	1.453	1.375	0.668	1.454	1.377	1.577	1.805	1.404	0.862	1.815	1.408
	5	1.445	1.476	1.383	0.779	1.401	1.405	1.647	1.833	1.404	1.073	1.472	1.469
	6	1.469	1.504	1.394	0.688	1.440	1.426	1.671	2.182	1.426	0.909	1.574	1.517
4	3	1.250	1.253	1.217	0.777	1.255	1.218	1.368	1.372	1.224	1.008	1.371	1.227
	4	1.290	1.317	1.245	0.642	1.319	1.245	1.444	1.630	1.281	0.825	1.640	1.281
	5	1.312	1.334	1.247	0.750	1.270	1.276	1.476	1.663	1.262	0.999	1.332	1.323
	6	1.330	1.338	1.251	0.657	1.304	1.286	1.511	1.972	1.289	0.863	1.417	1.371
		$Rho = 0.75$						$Rho = 0.95$					
n	k	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6
2	3	2.116	2.129	1.673	1.658	2.124	1.662	2.567	2.553	1.672	2.308	2.531	1.672
	4	2.366	3.186	1.849	1.293	3.306	1.858	2.991	6.200	2.002	2.033	6.029	1.997
	5	2.488	3.301	1.778	1.785	2.040	2.056	3.360	5.352	1.805	2.919	2.456	2.477
	6	2.648	5.150	1.878	1.415	2.410	2.227	3.700	17.721	1.998	2.429	3.914	2.960
3	3	1.712	1.706	1.337	1.398	1.707	1.338	2.045	2.027	1.332	1.885	2.043	1.346
	4	1.850	2.543	1.446	1.137	2.567	1.459	2.350	4.851	1.573	1.660	4.722	1.562
	5	1.982	2.571	1.390	1.474	1.594	1.600	2.623	4.168	1.407	2.284	1.913	1.910
	6	2.038	3.932	1.451	1.232	1.900	1.722	2.858	13.904	1.542	1.931	3.033	2.291
4	3	1.552	1.546	1.220	1.304	1.564	1.226	1.861	1.876	1.232	1.727	1.853	1.226
	4	1.692	2.314	1.319	1.070	2.322	1.326	2.138	4.326	1.427	1.513	4.394	1.428
	5	1.786	2.320	1.256	1.378	1.440	1.441	2.376	3.753	1.267	2.067	1.735	1.737
	6	1.845	3.533	1.302	1.156	1.700	1.554	2.580	12.516	1.392	1.789	2.734	2.051

CHAPTER FIVE
CONCLUSIONS

Ranked set sampling method can be used where the exact measurement is expensive, true proportion consuming on difficult, but ranking observations are easy and expensive in ecological, medicine and environmental studies. The ranking procedure can be performed with a concomitant variable or an expert judgment which does not include exact measurement. The ranking error may arise from in these cases. In literature, modified ranked set sampling methods have been investigated to reduce ranking error. In addition, a substantial majority of literature involved the mean estimator using RSS methods. However, this study concerns about the population proportion estimator. In more details, the proportion estimator and associated variance estimator are obtained for ERSS, MRSS, Per-RSS, LRSS methods. By the help of the Monte Carlo simulation using R project, the performance of the estimators is evaluated in terms of bias, mean squared error and relative efficiency for different coefficient correlations, true proportion, set and cycle sizes under Normal and Log-Normal distributions.

Based on the results of simulations under symmetric and asymmetric distributions it can be summarized that the proportion estimator of RSS is more efficient than the proportion estimator for SRS for all cases. The proportion estimator for ERSS, MRSS, Per-RSS and LRSS are biased estimators. However, the bias amount is decreasing when true proportion $p = 0.5$. The relative efficiencies of the proportion estimators for RSS methods are higher than the proportion estimator for SRS method when the ranking quality is poor ($\rho < 0.5$). The proportion estimator for PRSS is unbiased estimator. The proportion estimator based on PRSS is more efficient than the estimator based on SRS under perfect ranking assumption. In addition, PRSS is nearly as efficient as RSS, while reducing the number of sample units needed for judgment ranking by almost half. Furthermore, all the patterns of relative efficiencies with respect to true proportion p are symmetric around $p = 0.5$ as expected and the results are given in the appendix in this thesis.

REFERENCES

Al-Nasser, A. D. (2007). L ranked set sampling: A generalization procedure for robust visual sampling. *Communications in Statistics—Simulation and Computation®*, *36*(1), 33–43.

Al-Saleh, M. F., & Al-Kadiri, M. A. (2000). Double-ranked set sampling. *Statistics & Probability Letters*, *48*(2), 205–212.

Al-Saleh, M. F., & Al-Omari, A. I. (2002). Multistage ranked set sampling. *Journal of Statistical planning and Inference*, *102*(2), 273–286.

Chen, H., Stasny, E. A., & Wolfe, D. A. (2005). Ranked set sampling for efficient estimation of a population proportion. *Statistics in Medicine*, *24*(21), 3319–3329.

Chen, H., Stasny, E. A., & Wolfe, D. A. (2006). Unbalanced ranked set sampling for estimating a population proportion. *Biometrics*, *62*(1), 150–158.

Chen, H., Stasny, E. A., Wolfe, D. A., & MacEachern, S. N. (2009). Unbalanced ranked set sampling for estimating a population proportion under imperfect rankings. *Communications in Statistics—Theory and Methods*, *38*(12), 2116–2125.

David, H., & Levine, D. (1972). Ranked set sampling in the presence of judgment error. *Biometrics*, *28*, 553–555.

Dell, T., & Clutter, J. (1972). Ranked set sampling theory with order statistics background. *Biometrics*, 545–555.

Evans, M. (1967). Application of ranked set sampling to regeneration surveys in areas direct-seeded to longleaf pine. *Master of Forestry Dissertation, Louisiana State University, Baton Rouge, LA*.

Fei, H., Sinha, B., & Wu, Z. (1994). Estimation of parameters in two-parameter weibull and extreme-value distributions using ranked set sampling. *Journal of Statistical Research*, *28*, 149–161.

Halls, L. K., & Dell, T. R. (1966). Trial of ranked-set sampling for forage yields. *Forest Science*, *12*(1), 22–26.

Jemain, A., & Al-Omari, A. (2006). Double percentile ranked set samples for estimating the population mean. *Advances and Applications in Statistics*, *6*(3), 261–276.

Jemain, A., & Al-Omari, A. (2007). Multistage percentile ranked set samples. *Advances and Applications in Statistics*, *7*(1), 127–139.

Jemain, A. A., Al-Omari, A., & Ibrahim, K. (2008). Some variations of ranked set sampling. *Electronic Journal of Applied Statistical Analysis*, *1*(1), 1–15.

Kohlschmidt, J. K., Stasny, E. A., & Wolfe, D. A. (2012). Ranked set sampling for a population proportion&58; allocation of sample units to each judgment order statistic. *Pakistan Journal of Statistics and Operation Research*, *8*(3), 511–530.

Lacayo, Herber, N. N. K., & Sinha, B. K. (2002). Ranked set sampling from a dichotomous population. *Journal of Applied Statistical Science*, *11*(1), 83–90.

Martin, W. L., Sharik, T. L., Oderwald, R. G., Smith, D. W. et al. (1980). Evaluation of ranked set sampling for estimating shrub phytomass in appalachian oak forests. *Publication, School of Forestry & Wildlife Resources, Virginia Polytechnic Institute and State University*, (FWS-4-80).

McIntyre, G. (1952). A method for unbiased selective sampling, using ranked sets. *Australian Journal of Agricultural Research*, *3*(4), 385–390.

Muttlak, H. A. (1996). Pair rank set sampling. *Biometrical Journal*, *38*(7), 879–885.

Muttlak, H. A. (1997). Median ranked set sampling. *Journal of Applied Statistical Sciences*, *38*(6), 245-255.

Muttlak, H. A. (2003a). Investigating the use of quartile ranked set samples for estimating the population mean. *Applied Mathematics and Computation*, *146*(2-3), 437–443.

Muttlak, H. A. (2003b). Modified ranked set sampling methods. *Pakistan Journal of Statistics-All Series-*, *19*(3), 315–324.

Patil, G., Sinha, A., & Taille, C. (1993). Relative precision of ranked set sampling: a comparison with the regression estimator. *Environmetrics*, *4*(4), 399–412.

Ridout, M., & Cobby, J. (1987). Ranked set sampling with non-random selection of sets and errors in ranking. *Applied Statistics*, 145–152.

Samawi, H. M., Ahmed, M. S., & Abu-Dayyeh, W. (1996). Estimating the population mean using extreme ranked set sampling. *Biometrical Journal*, *38*(5), 577–586.

Stokes, S. L. (1976). An investigation of the consequences of ranked set sampling. Technical report, North Carolina State University. Dept. of Statistics.

Stokes, S. L. (1977). Ranked set sampling with concomitant variables. *Communications in Statistics-Theory and Methods*, *6*(12), 1207–1211.

Stokes, S. L., & Sager, T. W. (1988). Characterization of a ranked-set sample with application to estimating distribution functions. *Journal of the American Statistical Association*, *83*(402), 374-381.

Takahasi, K., & Wakimoto, K. (1968). On unbiased estimates of the population mean based on the sample stratified by means of ordering. *Annals of the Institute of Statistical Mathematics*, *20*(1), 1–31.

Terpstra, J. (2004). On estimating a population proportion via ranked set sampling. *Biometrical Journal*, *46*(2), 264–272.

Terpstra, J. T., & Wang, P. (2008). Confidence intervals for a population proportion based on a ranked set sample. *Journal of Statistical Computation and Simulation*, *78*(4), 351–366.

Zamanzade, E., & Mahdizadeh, M. (2018). Estimating the population proportion in pair ranked set sampling with application to air quality monitoring. *Journal of Applied Statistics*, *45*(3), 426–437.

APPENDICES

Table A.1 Biases of the proportion estimator for Normal distribution when $p = 0.6$

		Rho = 0.25							Rho = 0.5						
n	k	SRS	RSS	ERSS	MRSS	PRSS	PerRSS	LRSS	SRS	RSS	ERSS	MRSS	PRSS	PerRSS	LRSS
2	3	−0.001	0.002	0.002	0.002	−0.002	0.001	0.001	0.001	−0.003	0.004	−0.020	0.002	−0.001	−0.019
	4	0.002	0.001	−0.001	0.002	0.000	0.000	0.001	0.000	−0.002	0.020	−0.021	−0.001	0.021	−0.019
	5	−0.002	0.001	−0.003	−0.003	0.001	0.002	0.001	0.000	0.000	0.017	−0.028	0.003	−0.014	−0.015
	6	0.001	−0.002	−0.001	−0.001	−0.002	0.002	0.003	−0.001	−0.002	0.032	−0.025	−0.003	−0.007	−0.013
3	3	0.001	−0.002	−0.002	0.002	−0.001	−0.003	0.001	0.001	0.000	0.001	−0.016	0.000	−0.002	−0.019
	4	0.000	0.001	−0.001	0.001	0.002	0.000	0.001	−0.001	0.001	0.017	−0.020	−0.001	0.018	−0.018
	5	0.001	0.000	0.001	0.000	0.000	0.001	0.000	0.000	0.001	0.016	−0.026	0.002	−0.015	−0.017
	6	0.001	0.000	−0.001	−0.001	0.000	0.001	0.001	−0.002	0.001	0.034	−0.028	−0.001	−0.005	−0.012
4	3	−0.002	0.000	0.000	−0.001	−0.001	0.000	0.000	0.002	−0.001	−0.001	−0.020	−0.001	0.000	−0.015
	4	−0.001	0.000	0.000	0.000	−0.001	0.001	0.000	−0.001	−0.001	0.019	−0.019	−0.001	0.019	−0.019
	5	−0.001	0.000	−0.002	0.000	−0.001	0.001	0.001	−0.002	−0.002	0.014	−0.026	0.002	−0.015	−0.015
	6	0.000	0.000	−0.003	0.001	0.001	0.004	0.002	0.001	0.000	0.032	−0.027	−0.001	−0.005	−0.012
		Rho = 0.75							Rho = 0.95						
n	k	SRS	RSS	ERSS	MRSS	PRSS	PerRSS	LRSS	SRS	RSS	ERSS	MRSS	PRSS	PerRSS	LRSS
2	3	0.005	0.002	−0.001	−0.015	0.004	0.001	−0.016	0.001	−0.003	0.001	−0.037	0.002	−0.001	−0.040
	4	0.004	−0.001	0.018	−0.016	0.001	0.018	−0.014	−0.001	−0.002	0.039	−0.037	−0.001	0.039	−0.041
	5	−0.001	0.000	0.014	−0.023	0.000	−0.016	−0.015	0.001	0.000	0.030	−0.060	0.001	−0.031	−0.030
	6	0.001	−0.001	0.032	−0.023	0.000	−0.007	−0.013	−0.002	0.001	0.066	−0.063	−0.001	−0.006	−0.024
3	3	−0.002	0.000	0.000	−0.018	−0.001	−0.001	−0.019	0.002	0.001	−0.001	−0.039	−0.002	0.000	−0.040
	4	0.000	−0.001	0.019	−0.017	0.002	0.015	−0.019	0.000	−0.002	0.039	−0.041	0.001	0.038	−0.039
	5	0.000	0.002	0.017	−0.022	0.000	−0.014	−0.016	0.001	0.001	0.030	−0.062	0.001	−0.032	−0.033
	6	−0.002	0.001	0.032	−0.021	−0.002	−0.008	−0.014	−0.001	0.000	0.065	−0.062	−0.001	−0.003	−0.024
4	3	−0.001	0.002	−0.001	−0.016	0.000	−0.001	−0.018	0.001	−0.001	0.000	−0.038	−0.001	0.000	−0.038
	4	0.001	0.000	0.017	−0.017	0.000	0.017	−0.018	−0.001	0.000	0.038	−0.040	0.000	0.039	−0.039
	5	0.000	−0.001	0.015	−0.023	0.000	−0.014	−0.015	0.001	0.001	0.032	−0.062	0.001	−0.030	−0.032
	6	0.000	−0.001	0.030	−0.022	0.000	−0.007	−0.013	0.001	0.000	0.066	−0.062	0.000	−0.005	−0.024

Table A.2 Relative efficiencies of the proportion estimator for Normal distribution when $p = 0.6$

		Rho = 0.25						Rho = 0.5					
n	k	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6
2	3	1.707	1.712	1.671	0.878	1.702	1.661	1.809	1.801	1.667	1.107	1.800	1.664
	4	1.785	1.834	1.769	0.714	1.840	1.775	1.919	1.969	1.772	0.863	1.940	1.762
	5	1.828	1.872	1.790	0.864	1.805	1.823	1.980	2.033	1.737	1.096	1.855	1.844
	6	1.895	1.954	1.831	0.748	1.865	1.847	2.043	2.037	1.782	0.912	1.957	1.900
3	3	1.370	1.370	1.337	0.797	1.366	1.336	1.444	1.435	1.333	0.956	1.431	1.330
	4	1.406	1.437	1.381	0.654	1.432	1.381	1.500	1.554	1.381	0.788	1.538	1.382
	5	1.442	1.460	1.399	0.781	1.417	1.423	1.535	1.588	1.353	0.946	1.434	1.428
	6	1.451	1.511	1.426	0.683	1.440	1.430	1.574	1.538	1.348	0.830	1.509	1.476
4	3	1.253	1.247	1.227	0.768	1.247	1.229	1.309	1.315	1.211	0.914	1.309	1.224
	4	1.284	1.307	1.258	0.636	1.305	1.260	1.365	1.386	1.246	0.749	1.396	1.252
	5	1.294	1.323	1.264	0.734	1.277	1.281	1.397	1.442	1.204	0.893	1.291	1.290
	6	1.313	1.358	1.283	0.656	1.301	1.292	1.420	1.363	1.197	0.788	1.362	1.328
		Rho = 0.75						Rho = 0.95					
n	k	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6
2	3	2.063	2.087	1.676	1.599	2.063	1.695	2.415	2.453	1.636	2.207	2.424	1.596
	4	2.270	2.765	1.842	1.230	2.780	1.833	2.860	3.704	1.828	1.885	3.751	1.807
	5	2.410	2.881	1.753	1.672	2.004	2.003	3.124	3.866	1.471	2.680	2.192	2.166
	6	2.507	3.354	1.827	1.364	2.319	2.140	3.383	2.962	1.433	2.259	3.251	2.546
3	3	1.653	1.632	1.322	1.343	1.653	1.326	1.960	1.944	1.274	1.776	1.949	1.264
	4	1.783	2.180	1.433	1.080	2.218	1.425	2.234	2.860	1.404	1.548	2.837	1.415
	5	1.875	2.193	1.372	1.391	1.547	1.544	2.451	2.950	1.085	2.101	1.654	1.641
	6	1.942	2.502	1.405	1.170	1.793	1.651	2.663	2.142	1.087	1.821	2.493	1.938
4	3	1.513	1.515	1.223	1.245	1.515	1.216	1.758	1.766	1.149	1.629	1.776	1.149
	4	1.613	1.990	1.293	1.026	1.969	1.292	2.026	2.505	1.243	1.432	2.483	1.261
	5	1.699	2.010	1.225	1.297	1.398	1.399	2.218	2.521	0.932	1.920	1.473	1.454
	6	1.751	2.236	1.250	1.108	1.620	1.480	2.364	1.663	0.924	1.659	2.259	1.719

Table A.3 Biases of the proportion estimator for Log-Normal distribution when $p = 0.6$

		$Rho = 0.25$							$Rho = 0.5$						
n	k	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$
2	3	0.000	−0.002	0.000	0.002	−0.001	0.001	−0.002	0.003	−0.002	0.001	−0.010	−0.001	0.002	−0.009
	4	0.001	0.000	0.002	−0.004	−0.001	−0.001	−0.002	−0.003	0.000	0.008	−0.011	0.001	0.011	−0.013
	5	0.003	0.001	0.001	0.001	0.001	0.000	−0.003	−0.001	0.001	0.010	−0.014	0.002	−0.007	−0.009
	6	0.004	0.004	0.001	0.001	−0.001	−0.001	−0.003	0.002	−0.001	0.019	−0.015	0.001	−0.006	−0.008
3	3	0.001	0.002	0.000	0.002	0.000	0.001	0.001	0.003	−0.001	0.001	−0.013	−0.002	−0.003	−0.012
	4	0.000	0.001	0.002	0.000	−0.001	0.000	−0.001	0.000	0.000	0.010	−0.011	0.000	0.010	−0.012
	5	−0.001	0.002	−0.001	0.004	0.001	−0.002	0.002	0.002	−0.001	0.011	−0.014	−0.001	−0.009	−0.012
	6	0.002	0.000	0.001	0.000	0.002	−0.003	−0.002	0.000	−0.001	0.019	−0.016	−0.001	−0.005	−0.008
4	3	−0.002	−0.001	0.001	0.000	−0.003	−0.001	0.002	0.000	0.000	−0.004	−0.013	0.000	0.000	−0.008
	4	0.000	0.003	0.000	0.000	0.000	0.000	−0.001	0.000	0.002	0.012	−0.011	0.002	0.011	−0.011
	5	0.000	−0.002	0.000	0.001	0.001	−0.002	0.000	0.002	−0.001	0.011	−0.015	0.001	−0.010	−0.009
	6	−0.002	−0.001	0.002	0.000	−0.001	0.000	−0.002	0.000	−0.001	0.019	−0.015	−0.001	−0.003	−0.008
		$Rho = 0.75$							$Rho = 0.95$						
n	k	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$
2	3	0.002	0.002	−0.001	−0.021	0.006	0.000	−0.022	−0.002	−0.001	0.000	−0.042	−0.002	0.000	−0.038
	4	0.002	−0.002	0.021	−0.022	−0.002	0.018	−0.019	0.000	−0.001	0.040	−0.039	0.000	0.040	−0.040
	5	−0.002	0.001	0.017	−0.027	−0.003	−0.018	−0.018	0.001	−0.001	0.034	−0.064	0.000	−0.032	−0.032
	6	0.003	0.001	0.038	−0.028	0.001	−0.009	−0.017	0.002	−0.001	0.067	−0.065	0.000	−0.003	−0.024
3	3	0.001	0.002	0.001	−0.020	−0.001	0.000	−0.019	−0.001	−0.003	−0.002	−0.039	0.001	−0.002	−0.041
	4	0.003	0.001	0.022	−0.021	0.004	0.021	−0.023	0.001	0.001	0.039	−0.041	0.000	0.040	−0.040
	5	0.001	−0.001	0.019	−0.029	−0.002	−0.017	−0.019	0.000	−0.001	0.032	−0.063	−0.001	−0.031	−0.030
	6	−0.001	−0.001	0.038	−0.028	0.001	−0.010	−0.017	−0.003	−0.001	0.068	−0.066	0.000	−0.003	−0.025
4	3	0.000	0.003	−0.001	−0.020	0.000	0.002	−0.019	0.000	−0.001	0.000	−0.040	0.001	−0.002	−0.039
	4	−0.001	0.001	0.020	−0.020	0.000	0.020	−0.019	−0.001	−0.002	0.040	−0.038	0.000	0.039	−0.039
	5	0.001	0.000	0.018	−0.028	−0.001	−0.017	−0.018	0.001	−0.001	0.031	−0.063	0.000	−0.032	−0.032
	6	−0.001	0.001	0.038	−0.028	0.001	−0.009	−0.015	−0.001	0.000	0.068	−0.065	0.000	−0.004	−0.023

Table A.4 Relative efficiencies of the proportion estimator for Log-Normal distribution when $p = 0.6$

		$Rho = 0.25$						$Rho = 0.5$					
n	k	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6
2	3	1.712	1.717	1.672	0.932	1.684	1.659	1.887	1.858	1.683	1.266	1.866	1.674
	4	1.808	1.881	1.762	0.737	1.877	1.764	2.035	2.312	1.816	0.981	2.290	1.781
	5	1.883	1.931	1.789	0.905	1.815	1.830	2.127	2.340	1.781	1.281	1.915	1.903
	6	1.912	2.052	1.839	0.782	1.877	1.868	2.208	2.664	1.846	1.074	2.076	1.990
3	3	1.384	1.381	1.337	0.843	1.382	1.345	1.512	1.512	1.338	1.105	1.509	1.339
	4	1.424	1.469	1.380	0.684	1.476	1.378	1.586	1.777	1.405	0.881	1.780	1.408
	5	1.459	1.499	1.394	0.810	1.424	1.410	1.655	1.806	1.398	1.100	1.486	1.472
	6	1.483	1.572	1.423	0.709	1.450	1.443	1.696	2.039	1.413	0.939	1.593	1.544
4	3	1.257	1.253	1.218	0.806	1.255	1.220	1.380	1.383	1.227	1.045	1.383	1.228
	4	1.292	1.335	1.256	0.658	1.337	1.259	1.446	1.619	1.280	0.848	1.629	1.281
	5	1.325	1.360	1.266	0.774	1.277	1.282	1.501	1.633	1.257	1.032	1.338	1.340
	6	1.335	1.423	1.281	0.676	1.309	1.301	1.528	1.828	1.270	0.895	1.436	1.383
		$Rho = 0.75$						$Rho = 0.95$					
n	k	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6
2	3	2.099	2.087	1.662	1.654	2.097	1.665	2.474	2.453	1.604	2.217	2.456	1.594
	4	2.335	2.896	1.830	1.298	2.902	1.822	2.892	3.725	1.816	1.961	3.747	1.810
	5	2.477	2.989	1.737	1.732	1.965	2.020	3.210	3.821	1.412	2.764	2.194	2.185
	6	2.605	3.309	1.809	1.460	2.403	2.161	3.494	2.992	1.424	2.368	3.337	2.560
3	3	1.672	1.682	1.329	1.393	1.691	1.336	1.968	1.962	1.274	1.791	1.985	1.269
	4	1.823	2.248	1.424	1.117	2.267	1.426	2.255	2.892	1.396	1.590	2.859	1.402
	5	1.917	2.262	1.335	1.459	1.560	1.545	2.524	2.960	1.078	2.172	1.689	1.685
	6	2.006	2.465	1.372	1.230	1.836	1.661	2.690	2.059	1.047	1.871	2.584	1.969
4	3	1.530	1.542	1.210	1.296	1.532	1.215	1.824	1.814	1.143	1.673	1.813	1.153
	4	1.656	2.027	1.294	1.067	2.044	1.299	2.045	2.496	1.266	1.472	2.516	1.257
	5	1.741	2.057	1.194	1.355	1.399	1.395	2.261	2.604	0.928	1.964	1.473	1.480
	6	1.800	2.116	1.217	1.153	1.658	1.494	2.414	1.633	0.892	1.727	2.331	1.764

Table A.5 Biases of the proportion estimator for Normal distribution when $p = 0.7$

		$Rho = 0.25$							$Rho = 0.5$						
n	k	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$
2	3	0.002	−0.002	0.001	0.006	0.001	−0.003	0.006	−0.001	0.001	0.001	−0.019	0.003	0.001	−0.020
	4	0.000	0.000	−0.007	0.005	0.001	−0.003	0.007	−0.001	−0.002	0.019	−0.021	0.001	0.024	−0.022
	5	0.001	0.001	−0.004	0.009	0.002	0.006	0.007	0.000	−0.001	0.020	−0.030	0.000	−0.019	−0.019
	6	0.001	0.001	−0.015	0.010	−0.001	0.004	0.005	0.001	0.000	0.039	−0.029	0.001	−0.009	−0.016
3	3	−0.002	0.000	−0.002	0.009	0.001	0.001	0.005	0.001	0.001	−0.003	−0.020	0.002	0.000	−0.021
	4	0.000	−0.001	−0.008	0.007	0.000	−0.008	0.007	0.001	−0.001	0.024	−0.021	0.002	0.023	−0.021
	5	0.001	0.001	−0.004	0.010	−0.001	0.007	0.006	0.001	−0.001	0.020	−0.028	−0.001	−0.019	−0.021
	6	0.000	0.001	−0.013	0.011	0.000	0.002	0.004	0.000	−0.001	0.042	−0.030	−0.001	−0.014	−0.018
4	3	0.000	0.001	0.001	0.007	−0.001	−0.001	0.009	0.000	0.002	0.001	−0.021	0.001	0.002	−0.021
	4	0.000	0.000	−0.006	0.007	−0.002	−0.008	0.009	0.001	0.000	0.022	−0.023	0.001	0.021	−0.022
	5	0.000	0.001	−0.006	0.009	0.001	0.007	0.005	−0.001	0.001	0.019	−0.028	0.000	−0.021	−0.019
	6	0.000	0.000	−0.013	0.010	−0.001	0.002	0.005	−0.001	0.001	0.041	−0.030	0.001	−0.013	−0.016
		$Rho = 0.75$							$Rho = 0.95$						
n	k	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$
2	3	0.001	−0.002	0.001	−0.031	0.001	−0.001	−0.032	−0.001	−0.001	0.002	−0.069	−0.002	−0.002	−0.070
	4	−0.001	0.000	0.031	−0.033	0.000	0.033	−0.032	−0.001	0.001	0.069	−0.069	−0.002	0.068	−0.071
	5	0.003	0.002	0.028	−0.044	0.000	−0.026	−0.029	0.001	−0.001	0.057	−0.107	0.003	−0.057	−0.056
	6	−0.001	0.001	0.059	−0.045	0.001	−0.014	−0.025	−0.001	0.000	0.121	−0.106	0.002	−0.013	−0.044
3	3	0.000	−0.001	0.001	−0.032	−0.001	0.001	−0.033	0.003	0.001	0.000	−0.069	−0.001	0.000	−0.069
	4	0.001	−0.001	0.032	−0.033	0.000	0.032	−0.030	−0.001	0.000	0.069	−0.070	0.000	0.070	−0.070
	5	0.000	0.000	0.029	−0.045	0.000	−0.028	−0.028	−0.001	0.000	0.058	−0.106	−0.001	−0.055	−0.058
	6	0.000	0.000	0.059	−0.046	0.001	−0.014	−0.024	0.000	0.002	0.120	−0.107	0.000	−0.014	−0.044
4	3	0.001	−0.001	0.001	−0.032	0.001	0.001	−0.033	−0.001	0.001	0.001	−0.068	0.001	0.001	−0.070
	4	0.001	0.000	0.033	−0.033	−0.001	0.031	−0.032	0.003	0.001	0.068	−0.071	0.000	0.067	−0.069
	5	−0.001	0.002	0.027	−0.046	0.000	−0.029	−0.029	0.000	0.000	0.056	−0.106	−0.001	−0.056	−0.056
	6	−0.002	−0.001	0.056	−0.044	0.000	−0.014	−0.024	0.000	0.000	0.118	−0.108	0.001	−0.012	−0.043

Table A.6 Relative efficiencies of the proportion estimator for Normal distribution when $p = 0.7$

		$Rho = 0.25$						$Rho = 0.5$					
n	k	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6
2	3	1.698	1.688	1.643	0.883	1.713	1.655	1.829	1.803	1.689	1.142	1.797	1.674
	4	1.761	1.825	1.738	0.695	1.812	1.728	1.939	1.949	1.790	0.905	1.915	1.798
	5	1.843	1.893	1.753	0.856	1.787	1.790	2.030	1.992	1.766	1.160	1.857	1.866
	6	1.871	1.941	1.790	0.748	1.854	1.828	2.062	1.871	1.792	0.991	1.988	1.937
3	3	1.344	1.358	1.301	0.784	1.355	1.312	1.439	1.471	1.354	1.016	1.450	1.360
	4	1.405	1.439	1.354	0.650	1.444	1.353	1.524	1.494	1.397	0.827	1.502	1.399
	5	1.435	1.462	1.361	0.781	1.387	1.398	1.566	1.548	1.362	1.016	1.445	1.445
	6	1.456	1.503	1.374	0.675	1.432	1.425	1.611	1.374	1.354	0.880	1.521	1.472
4	3	1.244	1.242	1.206	0.774	1.247	1.198	1.324	1.333	1.234	0.962	1.325	1.235
	4	1.279	1.309	1.233	0.629	1.307	1.232	1.387	1.371	1.263	0.794	1.373	1.267
	5	1.294	1.321	1.235	0.736	1.252	1.261	1.412	1.402	1.209	0.942	1.277	1.290
	6	1.312	1.348	1.240	0.648	1.290	1.280	1.446	1.202	1.197	0.827	1.368	1.328
		$Rho = 0.75$						$Rho = 0.95$					
n	k	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6
2	3	1.997	1.989	1.685	1.557	1.989	1.680	2.347	2.374	1.482	2.162	2.354	1.496
	4	2.180	2.260	1.799	1.236	2.229	1.772	2.711	1.999	1.536	1.952	2.002	1.518
	5	2.307	2.338	1.669	1.634	1.953	1.929	3.026	2.303	0.987	2.701	1.770	1.783
	6	2.396	1.851	1.641	1.360	2.203	2.029	3.248	0.919	0.917	2.387	2.742	2.058
3	3	1.593	1.592	1.333	1.306	1.594	1.331	1.881	1.896	1.165	1.758	1.891	1.169
	4	1.704	1.737	1.392	1.090	1.723	1.408	2.126	1.478	1.153	1.603	1.464	1.158
	5	1.803	1.796	1.251	1.378	1.490	1.483	2.338	1.670	0.716	2.102	1.337	1.308
	6	1.853	1.355	1.231	1.179	1.715	1.562	2.504	0.647	0.648	1.931	2.093	1.553
4	3	1.460	1.460	1.216	1.219	1.463	1.209	1.721	1.722	1.015	1.596	1.711	1.000
	4	1.562	1.535	1.247	1.026	1.555	1.250	1.931	1.265	0.979	1.484	1.292	0.997
	5	1.626	1.593	1.085	1.266	1.311	1.311	2.113	1.447	0.583	1.919	1.139	1.136
	6	1.670	1.182	1.072	1.099	1.519	1.378	2.274	0.516	0.516	1.760	1.905	1.338

Table A.7 Biases of the proportion estimator for Log-Normal distribution when $p = 0.7$

		$Rho = 0.25$							$Rho = 0.5$						
n	k	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$
2	3	0.002	0.002	−0.002	−0.006	−0.003	0.003	−0.010	0.000	−0.002	0.001	−0.015	−0.001	0.003	−0.014
	4	−0.001	0.002	0.007	−0.010	−0.001	0.009	−0.006	0.003	0.001	0.016	−0.016	0.000	0.015	−0.015
	5	0.002	0.000	0.005	−0.011	0.001	−0.004	−0.006	0.000	0.001	0.015	−0.020	0.001	−0.016	−0.016
	6	0.002	−0.002	0.011	−0.012	−0.001	−0.004	−0.006	0.000	0.000	0.030	−0.020	0.001	−0.009	−0.014
3	3	−0.002	0.000	0.001	−0.009	0.001	0.000	−0.004	−0.001	−0.001	0.000	−0.016	−0.002	0.002	−0.016
	4	−0.001	−0.001	0.007	−0.005	−0.001	0.006	−0.007	−0.001	0.000	0.015	−0.016	0.000	0.016	−0.017
	5	−0.001	0.001	0.007	−0.008	0.001	−0.006	−0.007	0.000	−0.001	0.015	−0.020	−0.001	−0.014	−0.013
	6	0.000	0.000	0.012	−0.009	0.002	−0.003	−0.005	−0.001	0.000	0.030	−0.019	0.000	−0.012	−0.015
4	3	0.002	0.000	0.000	−0.007	−0.002	0.000	−0.008	−0.002	0.000	0.000	−0.017	0.000	−0.001	−0.018
	4	−0.001	−0.001	0.007	−0.008	0.000	0.006	−0.007	−0.001	0.000	0.017	−0.015	0.001	0.016	−0.017
	5	−0.002	0.000	0.006	−0.009	0.001	−0.006	−0.006	0.000	0.000	0.014	−0.021	0.000	−0.014	−0.015
	6	0.001	−0.001	0.013	−0.009	−0.001	−0.004	−0.006	−0.002	0.000	0.029	−0.018	−0.001	−0.010	−0.013
		$Rho = 0.75$							$Rho = 0.95$						
n	k	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$
2	3	0.001	−0.003	−0.003	−0.044	0.004	−0.002	−0.044	−0.001	−0.004	−0.002	−0.070	−0.001	0.000	−0.076
	4	−0.001	0.000	0.040	−0.043	0.000	0.041	−0.042	0.000	0.002	0.074	−0.074	0.000	0.073	−0.073
	5	−0.001	0.001	0.036	−0.059	−0.001	−0.036	−0.034	−0.001	0.000	0.058	−0.115	0.000	−0.058	−0.059
	6	−0.004	0.000	0.075	−0.058	0.002	−0.017	−0.032	0.001	0.000	0.124	−0.115	−0.001	−0.011	−0.045
3	3	0.002	0.000	0.001	−0.043	−0.001	0.000	−0.042	−0.002	0.001	0.002	−0.073	0.000	−0.002	−0.074
	4	−0.002	0.000	0.041	−0.043	0.003	0.043	−0.041	0.000	0.000	0.073	−0.073	0.000	0.072	−0.072
	5	0.001	0.002	0.038	−0.059	−0.001	−0.034	−0.037	0.002	0.000	0.059	−0.114	−0.001	−0.059	−0.060
	6	0.001	−0.001	0.074	−0.056	0.000	−0.019	−0.030	0.002	−0.001	0.123	−0.114	0.000	−0.008	−0.045
4	3	0.001	−0.001	0.000	−0.044	0.000	0.001	−0.041	−0.002	0.002	0.002	−0.073	0.002	−0.001	−0.072
	4	0.000	0.001	0.042	−0.041	0.000	0.042	−0.042	0.001	−0.001	0.071	−0.069	−0.001	0.072	−0.072
	5	−0.001	0.001	0.036	−0.058	−0.001	−0.037	−0.036	0.000	0.001	0.060	−0.114	0.000	−0.059	−0.059
	6	0.000	−0.001	0.075	−0.058	0.000	−0.016	−0.032	−0.001	0.001	0.124	−0.114	−0.001	−0.010	−0.045

Table A.8 Relative efficiencies of the proportion estimator for Log-Normal distribution when $p = 0.7$

		$Rho = 0.25$						$Rho = 0.5$					
n	k	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6
2	3	1.727	1.743	1.690	0.977	1.733	1.699	1.846	1.822	1.707	1.279	1.833	1.692
	4	1.817	1.860	1.789	0.768	1.840	1.779	2.008	2.137	1.832	0.996	2.150	1.813
	5	1.887	1.933	1.806	0.937	1.825	1.840	2.054	2.191	1.802	1.289	1.897	1.906
	6	1.939	2.002	1.851	0.809	1.915	1.883	2.163	2.203	1.846	1.094	2.033	1.969
3	3	1.382	1.380	1.350	0.867	1.370	1.342	1.492	1.495	1.353	1.117	1.483	1.358
	4	1.421	1.446	1.394	0.700	1.456	1.390	1.563	1.658	1.410	0.890	1.643	1.418
	5	1.457	1.483	1.410	0.830	1.427	1.430	1.619	1.688	1.394	1.111	1.478	1.482
	6	1.485	1.531	1.431	0.724	1.451	1.446	1.659	1.665	1.416	0.949	1.570	1.516
4	3	1.274	1.276	1.245	0.837	1.271	1.241	1.350	1.348	1.226	1.036	1.353	1.225
	4	1.302	1.318	1.267	0.672	1.324	1.266	1.421	1.490	1.292	0.857	1.498	1.282
	5	1.328	1.343	1.275	0.789	1.285	1.289	1.474	1.529	1.254	1.036	1.340	1.335
	6	1.351	1.373	1.294	0.698	1.318	1.308	1.497	1.492	1.274	0.905	1.413	1.370
		$Rho = 0.75$						$Rho = 0.95$					
n	k	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6
2	3	2.084	2.077	1.654	1.671	2.077	1.658	2.383	2.367	1.488	2.183	2.359	1.435
	4	2.248	2.182	1.716	1.336	2.171	1.737	2.733	1.878	1.492	2.017	1.917	1.494
	5	2.432	2.325	1.485	1.791	1.870	1.908	3.072	2.306	0.910	2.742	1.752	1.753
	6	2.523	1.548	1.472	1.497	2.241	1.981	3.341	0.882	0.843	2.542	2.801	2.072
3	3	1.655	1.648	1.300	1.405	1.657	1.305	1.887	1.886	1.132	1.749	1.880	1.116
	4	1.78	1.696	1.340	1.151	1.668	1.356	2.178	1.429	1.130	1.650	1.436	1.136
	5	1.882	1.749	1.127	1.498	1.454	1.438	2.414	1.689	0.666	2.164	1.299	1.299
	6	1.973	1.146	1.127	1.276	1.732	1.547	2.584	0.628	0.607	2.015	2.173	1.551
4	3	1.516	1.512	1.166	1.311	1.513	1.179	1.734	1.742	0.986	1.628	1.736	0.990
	4	1.634	1.479	1.198	1.093	1.476	1.189	1.979	1.226	0.999	1.514	1.214	0.974
	5	1.71	1.567	0.978	1.364	1.265	1.272	2.161	1.374	0.533	1.963	1.098	1.104
	6	1.774	0.916	0.930	1.194	1.569	1.344	2.318	0.479	0.480	1.826	1.930	1.322

Table A.9 Biases of the proportion estimator for Normal distribution when $p = 0.8$

		$Rho = 0.25$							$Rho = 0.5$						
n	k	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$
2	3	0.001	−0.001	0.002	−0.001	0.000	0.000	−0.002	0.001	0.000	0.000	−0.024	−0.003	0.002	−0.022
	4	0.001	0.000	0.000	−0.001	−0.001	−0.002	−0.003	−0.001	−0.003	0.023	−0.024	0.000	0.024	−0.025
	5	0.000	−0.001	0.000	0.000	0.001	−0.001	−0.002	−0.002	0.000	0.021	−0.032	−0.001	−0.022	−0.021
	6	0.000	0.001	0.000	0.002	0.002	−0.001	−0.003	0.001	0.000	0.042	−0.031	0.001	−0.010	−0.018
3	3	−0.001	−0.003	0.001	0.000	0.000	0.003	0.000	−0.001	0.001	0.000	−0.023	0.000	−0.001	−0.023
	4	0.001	−0.001	−0.002	0.000	−0.001	0.001	−0.003	0.001	−0.002	0.021	−0.024	0.002	0.023	−0.023
	5	0.001	−0.001	0.002	0.002	0.002	0.001	0.000	0.001	0.001	0.021	−0.032	−0.001	−0.021	−0.021
	6	−0.001	0.001	0.002	−0.002	0.000	0.000	0.000	−0.001	0.001	0.043	−0.033	0.001	−0.011	−0.018
4	3	0.000	0.001	−0.001	−0.001	0.001	0.000	−0.001	0.000	0.000	−0.001	−0.025	0.000	0.001	−0.025
	4	0.000	0.000	0.000	−0.001	−0.001	0.000	0.000	0.000	0.000	0.024	−0.024	0.000	0.023	−0.023
	5	0.000	0.000	0.001	−0.001	0.000	−0.002	−0.001	0.000	0.000	0.022	−0.034	−0.001	−0.021	−0.022
	6	0.001	0.000	0.001	0.000	0.000	−0.001	−0.001	0.000	0.001	0.042	−0.031	0.000	−0.009	−0.017
		$Rho = 0.75$							$Rho = 0.95$						
n	k	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$
2	3	0.001	0.001	−0.002	−0.048	0.002	−0.003	−0.045	−0.001	0.001	−0.001	−0.083	0.000	−0.001	−0.086
	4	0.000	0.001	0.045	−0.048	−0.001	0.048	−0.044	0.001	0.000	0.087	−0.084	0.000	0.087	−0.086
	5	0.002	0.000	0.042	−0.063	−0.001	−0.038	−0.041	−0.001	0.001	0.071	−0.123	0.002	−0.073	−0.072
	6	0.001	0.001	0.084	−0.067	0.002	−0.019	−0.034	−0.001	0.001	0.150	−0.123	0.000	−0.026	−0.060
3	3	−0.001	−0.001	0.000	−0.047	−0.004	−0.001	−0.048	−0.001	0.001	0.002	−0.084	−0.001	−0.001	−0.086
	4	−0.002	0.000	0.047	−0.045	0.002	0.047	−0.046	−0.001	−0.001	0.085	−0.084	0.002	0.085	−0.085
	5	0.000	0.002	0.039	−0.065	0.000	−0.040	−0.039	0.001	−0.001	0.073	−0.123	−0.001	−0.072	−0.072
	6	0.000	0.002	0.084	−0.065	0.000	−0.020	−0.035	−0.002	0.002	0.151	−0.124	0.000	−0.025	−0.058
4	3	0.001	−0.001	−0.001	−0.046	0.000	0.000	−0.047	0.002	0.002	−0.001	−0.085	0.000	0.002	−0.085
	4	0.001	0.000	0.046	−0.048	0.000	0.047	−0.046	−0.002	0.000	0.085	−0.087	0.001	0.085	−0.085
	5	−0.001	0.000	0.040	−0.065	0.000	−0.041	−0.041	0.000	0.002	0.071	−0.123	0.000	−0.072	−0.073
	6	0.001	−0.001	0.085	−0.065	0.001	−0.020	−0.034	0.001	0.000	0.151	−0.124	0.000	−0.027	−0.060

Table A.10 Relative efficiencies of the proportion estimator for Normal distribution when $p = 0.8$

		$Rho = 0.25$						$Rho = 0.5$					
n	k	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6
2	3	1.685	1.676	1.678	0.837	1.682	1.664	1.778	1.826	1.783	1.249	1.803	1.750
	4	1.767	1.779	1.769	0.677	1.786	1.777	1.934	1.795	1.839	0.983	1.752	1.832
	5	1.828	1.831	1.818	0.806	1.828	1.815	1.986	1.836	1.789	1.192	1.887	1.886
	6	1.839	1.892	1.840	0.706	1.856	1.862	2.055	1.582	1.798	1.052	1.998	1.963
3	3	1.352	1.341	1.337	0.754	1.322	1.334	1.430	1.426	1.389	1.058	1.436	1.391
	4	1.395	1.401	1.372	0.629	1.401	1.395	1.521	1.407	1.434	0.854	1.392	1.443
	5	1.436	1.413	1.399	0.725	1.405	1.410	1.546	1.435	1.381	1.045	1.469	1.463
	6	1.421	1.446	1.422	0.642	1.420	1.414	1.568	1.161	1.349	0.907	1.526	1.495
4	3	1.219	1.234	1.218	0.726	1.226	1.230	1.315	1.321	1.261	0.993	1.308	1.269
	4	1.264	1.270	1.257	0.609	1.271	1.248	1.358	1.232	1.289	0.817	1.248	1.288
	5	1.279	1.282	1.270	0.701	1.278	1.274	1.402	1.270	1.195	0.984	1.314	1.304
	6	1.296	1.313	1.282	0.620	1.297	1.294	1.422	1.011	1.196	0.860	1.376	1.343
		$Rho = 0.75$						$Rho = 0.95$					
n	k	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6
2	3	1.967	1.992	1.737	1.619	1.995	1.733	2.181	2.192	1.430	2.096	2.191	1.408
	4	2.148	1.661	1.744	1.369	1.617	1.766	2.542	1.156	1.344	2.057	1.139	1.316
	5	2.278	1.773	1.475	1.764	1.862	1.833	2.792	1.392	0.770	2.618	1.400	1.416
	6	2.342	0.981	1.332	1.510	2.146	1.929	3.022	0.453	0.675	2.563	2.204	1.565
3	3	1.550	1.556	1.340	1.351	1.561	1.335	1.753	1.745	1.071	1.677	1.735	1.058
	4	1.673	1.236	1.336	1.149	1.244	1.343	1.981	0.858	0.982	1.661	0.861	0.968
	5	1.741	1.356	1.058	1.436	1.388	1.401	2.203	0.997	0.544	2.120	1.041	1.041
	6	1.798	0.708	0.993	1.278	1.623	1.446	2.333	0.311	0.467	2.031	1.689	1.171
4	3	1.431	1.430	1.195	1.247	1.429	1.190	1.624	1.624	0.907	1.570	1.626	0.913
	4	1.535	1.109	1.162	1.081	1.090	1.175	1.802	0.714	0.781	1.537	0.713	0.797
	5	1.587	1.176	0.884	1.319	1.202	1.192	1.968	0.841	0.426	1.880	0.856	0.853
	6	1.659	0.569	0.822	1.190	1.466	1.277	2.118	0.245	0.368	1.876	1.471	0.950

Table A.11 Biases of the proportion estimator for Log-Normal distribution when $p = 0.8$

		$Rho = 0.25$							$Rho = 0.5$						
n	k	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$
2	3	−0.001	−0.001	−0.002	−0.007	−0.002	0.001	−0.008	−0.001	0.000	−0.002	−0.013	0.000	0.001	−0.021
	4	−0.001	0.000	0.009	−0.012	−0.001	0.010	−0.012	−0.001	0.002	0.018	−0.018	−0.001	0.018	−0.017
	5	0.000	0.000	0.006	−0.011	0.000	−0.009	−0.011	−0.001	−0.001	0.015	−0.023	−0.001	−0.013	−0.015
	6	0.001	0.000	0.018	−0.011	0.000	−0.004	−0.007	0.000	0.000	0.034	−0.023	0.001	−0.006	−0.015
3	3	0.000	−0.003	0.001	−0.008	0.001	−0.001	−0.013	0.000	−0.002	0.001	−0.020	−0.001	0.000	−0.017
	4	0.000	0.000	0.009	−0.010	−0.001	0.011	−0.010	0.000	0.002	0.017	−0.018	0.001	0.017	−0.017
	5	0.001	0.000	0.009	−0.011	0.002	−0.007	−0.008	0.001	−0.002	0.015	−0.021	0.001	−0.016	−0.015
	6	0.002	0.002	0.018	−0.012	0.001	−0.006	−0.010	0.000	−0.001	0.033	−0.022	−0.001	−0.011	−0.015
4	3	−0.002	0.000	−0.001	−0.010	−0.001	0.000	−0.011	−0.002	0.000	0.001	−0.015	0.001	0.000	−0.015
	4	−0.001	0.000	0.008	−0.010	0.000	0.010	−0.012	0.000	0.001	0.017	−0.017	0.001	0.017	−0.019
	5	0.000	0.001	0.009	−0.012	0.000	−0.007	−0.009	−0.001	0.000	0.016	−0.023	−0.001	−0.017	−0.015
	6	0.001	0.000	0.017	−0.010	−0.001	−0.006	−0.009	−0.001	−0.001	0.033	−0.022	0.000	−0.013	−0.013
		$Rho = 0.75$							$Rho = 0.95$						
n	k	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$
2	3	−0.001	0.000	0.001	−0.051	0.004	0.001	−0.049	0.001	−0.001	−0.001	−0.086	0.000	−0.001	−0.084
	4	0.001	−0.001	0.050	−0.050	0.001	0.048	−0.050	0.003	0.002	0.085	−0.086	−0.001	0.083	−0.085
	5	0.003	0.002	0.043	−0.070	−0.001	−0.043	−0.045	−0.001	0.001	0.072	−0.123	0.000	−0.071	−0.072
	6	0.000	0.000	0.088	−0.068	0.002	−0.021	−0.038	0.001	0.000	0.149	−0.123	0.000	−0.026	−0.058
3	3	0.001	0.000	0.001	−0.050	0.000	0.000	−0.050	−0.001	0.000	−0.001	−0.086	−0.001	0.000	−0.084
	4	0.002	0.000	0.049	−0.050	0.002	0.050	−0.050	0.000	0.000	0.083	−0.085	0.000	0.084	−0.082
	5	0.000	0.000	0.043	−0.068	0.000	−0.045	−0.045	0.001	0.001	0.070	−0.122	−0.001	−0.071	−0.072
	6	0.001	0.000	0.091	−0.070	−0.001	−0.022	−0.038	0.000	0.000	0.149	−0.124	0.000	−0.027	−0.059
4	3	0.000	−0.001	−0.001	−0.051	0.001	0.000	−0.051	0.001	0.000	−0.001	−0.084	0.000	0.000	−0.085
	4	0.000	0.001	0.051	−0.050	0.001	0.050	−0.051	−0.001	0.000	0.082	−0.083	0.000	0.084	−0.083
	5	0.000	0.002	0.044	−0.068	−0.001	−0.043	−0.044	0.002	0.000	0.071	−0.122	−0.001	−0.071	−0.071
	6	0.001	0.000	0.089	−0.070	0.000	−0.021	−0.037	0.000	0.000	0.149	−0.123	0.000	−0.026	−0.059

Table A.12 Relative efficiencies of the proportion estimator for Log-Normal distribution when $p = 0.8$

		$Rho = 0.25$						$Rho = 0.5$					
n	k	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6
2	3	1.714	1.744	1.694	0.990	1.709	1.715	1.788	1.786	1.722	1.267	1.782	1.733
	4	1.797	1.759	1.807	0.777	1.771	1.815	1.915	1.871	1.849	1.016	1.887	1.854
	5	1.872	1.864	1.861	0.958	1.858	1.868	2.007	1.966	1.842	1.299	1.931	1.921
	6	1.900	1.822	1.880	0.827	1.902	1.916	2.065	1.777	1.860	1.119	2.007	1.972
3	3	1.384	1.367	1.362	0.873	1.378	1.380	1.453	1.437	1.396	1.121	1.449	1.394
	4	1.426	1.407	1.421	0.721	1.385	1.423	1.506	1.476	1.442	0.899	1.487	1.447
	5	1.455	1.432	1.442	0.847	1.448	1.448	1.581	1.538	1.433	1.117	1.496	1.495
	6	1.477	1.398	1.463	0.741	1.483	1.477	1.600	1.337	1.434	0.972	1.562	1.524
4	3	1.249	1.250	1.251	0.829	1.247	1.245	1.315	1.313	1.255	1.019	1.316	1.255
	4	1.286	1.274	1.282	0.677	1.262	1.287	1.381	1.339	1.312	0.861	1.331	1.309
	5	1.312	1.292	1.296	0.802	1.307	1.307	1.408	1.361	1.261	1.034	1.331	1.335
	6	1.340	1.251	1.308	0.717	1.328	1.325	1.443	1.168	1.272	0.920	1.397	1.363
		$Rho = 0.75$						$Rho = 0.95$					
n	k	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6
2	3	1.956	1.979	1.692	1.600	1.946	1.682	2.215	2.228	1.428	2.127	2.221	1.439
	4	2.173	1.581	1.740	1.381	1.623	1.727	2.546	1.174	1.334	2.099	1.194	1.331
	5	2.302	1.763	1.385	1.822	1.830	1.798	2.810	1.372	0.769	2.643	1.439	1.428
	6	2.381	0.935	1.312	1.549	2.133	1.870	3.078	0.463	0.687	2.635	2.257	1.636
3	3	1.577	1.574	1.334	1.353	1.589	1.327	1.742	1.765	1.057	1.690	1.752	1.073
	4	1.706	1.223	1.318	1.169	1.222	1.314	1.993	0.880	0.973	1.693	0.882	1.010
	5	1.785	1.318	1.023	1.468	1.340	1.341	2.205	1.038	0.555	2.085	1.068	1.055
	6	1.838	0.646	0.935	1.318	1.633	1.416	2.346	0.324	0.471	2.055	1.694	1.175
4	3	1.449	1.450	1.163	1.259	1.439	1.164	1.615	1.624	0.906	1.548	1.607	0.904
	4	1.531	1.030	1.132	1.101	1.047	1.127	1.803	0.738	0.813	1.540	0.722	0.814
	5	1.598	1.120	0.854	1.351	1.180	1.165	1.987	0.844	0.434	1.914	0.880	0.882
	6	1.673	0.536	0.761	1.227	1.451	1.236	2.106	0.252	0.369	1.896	1.503	0.974

Table A.13 Biases of the proportion estimator for Normal distribution when $p = 0.9$

		$Rho = 0.25$							$Rho = 0.5$						
n	k	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$
2	3	0.000	0.000	0.000	0.006	0.001	0.001	0.004	−0.001	−0.002	−0.002	−0.021	0.001	−0.001	−0.020
	4	0.000	0.001	−0.004	0.006	0.000	−0.003	0.004	0.002	−0.003	0.021	−0.022	−0.001	0.020	−0.022
	5	0.000	0.000	−0.002	0.006	0.001	0.003	0.005	−0.001	−0.001	0.018	−0.030	0.000	−0.020	−0.017
	6	0.002	0.000	−0.007	0.004	0.000	0.002	0.003	0.001	0.000	0.037	−0.028	0.001	−0.010	−0.016
3	3	−0.001	0.000	0.000	0.005	−0.001	0.001	0.003	0.000	0.001	0.001	−0.022	0.000	0.001	−0.021
	4	0.001	0.001	−0.003	0.004	−0.001	−0.004	0.003	0.000	0.000	0.020	−0.021	−0.001	0.021	−0.021
	5	0.000	0.000	−0.002	0.005	0.000	0.002	0.004	0.000	0.001	0.018	−0.028	0.000	−0.019	−0.019
	6	−0.001	0.000	−0.006	0.006	0.000	0.001	0.002	0.000	0.001	0.038	−0.027	0.000	−0.010	−0.016
4	3	0.000	0.000	0.001	0.005	−0.001	0.000	0.004	0.000	0.002	−0.001	−0.022	0.000	0.000	−0.020
	4	0.000	0.000	−0.004	0.004	0.000	−0.003	0.001	0.002	0.000	0.020	−0.021	0.001	0.020	−0.020
	5	0.001	−0.001	−0.003	0.006	0.000	0.003	0.004	0.000	0.000	0.018	−0.028	−0.001	−0.019	−0.019
	6	0.000	0.000	−0.007	0.004	0.001	0.002	0.003	−0.001	0.000	0.038	−0.029	0.000	−0.011	−0.017
		$Rho = 0.75$							$Rho = 0.95$						
n	k	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$
2	3	0.000	−0.001	0.001	−0.039	0.001	0.000	−0.039	−0.002	−0.001	0.000	−0.066	−0.001	−0.002	−0.066
	4	0.000	0.001	0.040	−0.038	0.000	0.039	−0.039	0.001	0.000	0.066	−0.065	−0.001	0.067	−0.066
	5	−0.001	−0.001	0.037	−0.053	−0.001	−0.035	−0.035	0.001	0.000	0.059	−0.086	0.000	−0.060	−0.060
	6	−0.001	−0.001	0.072	−0.052	−0.001	−0.021	−0.030	−0.001	0.000	0.123	−0.086	0.000	−0.037	−0.053
3	3	0.001	−0.001	−0.001	−0.038	0.000	0.000	−0.039	0.000	0.001	−0.001	−0.065	0.000	0.000	−0.065
	4	0.001	−0.001	0.041	−0.039	−0.001	0.039	−0.039	0.000	0.001	0.067	−0.066	−0.001	0.065	−0.066
	5	0.000	0.000	0.036	−0.053	0.001	−0.036	−0.035	0.001	0.001	0.059	−0.086	0.000	−0.060	−0.059
	6	0.000	0.000	0.073	−0.052	0.000	−0.022	−0.031	0.000	0.000	0.122	−0.085	0.000	−0.037	−0.053
4	3	0.001	0.000	0.002	−0.040	0.001	0.000	−0.039	−0.001	0.001	0.000	−0.066	−0.001	−0.001	−0.066
	4	0.000	0.000	0.040	−0.039	−0.001	0.039	−0.039	0.001	−0.001	0.065	−0.066	0.001	0.066	−0.066
	5	0.000	0.000	0.036	−0.052	0.000	−0.036	−0.036	−0.001	−0.001	0.059	−0.086	0.000	−0.060	−0.059
	6	0.000	0.000	0.072	−0.052	0.000	−0.021	−0.031	0.000	0.000	0.121	−0.086	0.001	−0.037	−0.054

Table A.14 Relative efficiencies of the proportion estimator for Normal distribution when $p = 0.9$

		$Rho = 0.25$						$Rho = 0.5$					
n	k	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6
2	3	1.664	1.670	1.574	0.899	1.644	1.596	1.761	1.758	1.928	1.208	1.755	1.894
	4	1.750	1.858	1.658	0.720	1.843	1.688	1.932	1.511	2.015	1.028	1.536	2.008
	5	1.829	1.869	1.710	0.887	1.760	1.722	1.909	1.602	1.958	1.230	2.014	1.994
	6	1.897	2.017	1.789	0.782	1.846	1.822	1.980	1.205	1.937	1.086	2.021	2.040
3	3	1.331	1.321	1.262	0.824	1.324	1.275	1.400	1.402	1.537	1.067	1.381	1.533
	4	1.388	1.446	1.330	0.683	1.466	1.339	1.462	1.182	1.550	0.902	1.172	1.538
	5	1.402	1.445	1.321	0.802	1.383	1.337	1.489	1.248	1.489	1.076	1.547	1.540
	6	1.423	1.512	1.316	0.689	1.405	1.383	1.513	0.891	1.456	0.947	1.553	1.553
4	3	1.233	1.228	1.171	0.808	1.229	1.178	1.265	1.294	1.378	0.979	1.280	1.376
	4	1.269	1.317	1.217	0.643	1.308	1.239	1.354	1.085	1.400	0.870	1.084	1.399
	5	1.304	1.328	1.203	0.769	1.244	1.234	1.349	1.105	1.297	0.998	1.368	1.368
	6	1.291	1.383	1.232	0.665	1.260	1.256	1.371	0.759	1.237	0.880	1.384	1.365
		$Rho = 0.75$						$Rho = 0.95$					
n	k	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6
2	3	1.875	1.862	1.987	1.654	1.876	1.997	1.979	1.937	1.740	1.930	1.987	1.750
	4	1.959	1.222	1.969	1.496	1.245	1.953	2.222	0.873	1.584	2.116	0.868	1.570
	5	2.122	1.310	1.603	1.836	1.942	1.947	2.435	0.971	1.013	2.390	1.507	1.506
	6	2.198	0.686	1.487	1.702	2.113	1.979	2.522	0.336	0.861	2.474	1.860	1.508
3	3	1.517	1.501	1.569	1.377	1.500	1.563	1.570	1.584	1.321	1.544	1.589	1.324
	4	1.584	0.936	1.492	1.264	0.961	1.488	1.733	0.652	1.131	1.689	0.673	1.120
	5	1.635	1.006	1.171	1.471	1.454	1.464	1.881	0.720	0.706	1.869	1.087	1.102
	6	1.685	0.499	1.071	1.390	1.597	1.482	1.989	0.245	0.606	1.959	1.402	1.098
4	3	1.367	1.356	1.347	1.249	1.361	1.355	1.431	1.426	1.063	1.416	1.433	1.062
	4	1.430	0.821	1.264	1.172	0.844	1.268	1.589	0.564	0.907	1.504	0.554	0.902
	5	1.495	0.885	0.965	1.354	1.246	1.238	1.682	0.590	0.535	1.664	0.860	0.867
	6	1.520	0.411	0.872	1.271	1.403	1.261	1.782	0.194	0.462	1.748	1.163	0.867

Table A.15 Biases of the proportion estimator for Log-Normal distribution when $p = 0.9$

		$Rho = 0.25$							$Rho = 0.5$						
n	k	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$
2	3	0.000	0.002	0.001	−0.011	−0.001	−0.002	−0.010	0.000	0.001	−0.001	−0.018	0.002	0.000	−0.018
	4	0.001	−0.002	0.012	−0.011	−0.002	0.010	−0.010	0.000	0.000	0.018	−0.017	−0.001	0.019	−0.017
	5	0.000	−0.001	0.011	−0.017	0.000	−0.010	−0.010	−0.001	0.001	0.015	−0.024	−0.001	−0.015	−0.016
	6	0.000	−0.001	0.021	−0.015	−0.002	−0.006	−0.011	0.000	0.000	0.034	−0.022	−0.001	−0.010	−0.014
3	3	0.003	0.002	0.000	−0.011	−0.001	0.000	−0.012	−0.001	0.000	0.000	−0.018	−0.001	0.000	−0.019
	4	0.000	0.000	0.012	−0.012	0.001	0.012	−0.012	−0.001	0.000	0.018	−0.018	0.000	0.018	−0.018
	5	0.000	0.001	0.011	−0.016	0.001	−0.009	−0.012	0.001	0.000	0.017	−0.024	0.001	−0.016	−0.016
	6	0.001	−0.001	0.022	−0.016	−0.001	−0.007	−0.009	0.000	0.001	0.034	−0.024	0.000	−0.011	−0.015
4	3	0.001	0.000	0.001	−0.010	0.001	0.001	−0.011	−0.001	0.001	0.001	−0.018	0.001	0.001	−0.019
	4	−0.001	0.000	0.012	−0.011	0.000	0.012	−0.011	0.001	−0.001	0.018	−0.018	0.000	0.019	−0.018
	5	0.000	0.001	0.010	−0.014	0.001	−0.010	−0.011	0.000	0.000	0.018	−0.024	0.000	−0.017	−0.016
	6	0.000	0.000	0.020	−0.015	0.000	−0.006	−0.008	−0.001	0.000	0.034	−0.024	−0.001	−0.009	−0.013
		$Rho = 0.75$							$Rho = 0.95$						
n	k	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$	SRS	RSS	$ERSS$	$MRSS$	$PRSS$	$PerRSS$	$LRSS$
2	3	0.000	−0.001	0.000	−0.046	0.000	0.000	−0.045	−0.001	−0.001	−0.002	−0.065	0.000	−0.001	−0.064
	4	−0.001	0.001	0.042	−0.045	−0.002	0.044	−0.044	0.001	−0.001	0.067	−0.064	−0.002	0.064	−0.066
	5	0.001	0.000	0.040	−0.057	0.000	−0.041	−0.040	0.001	−0.001	0.057	−0.084	−0.001	−0.059	−0.058
	6	0.001	−0.001	0.083	−0.058	0.001	−0.024	−0.035	0.001	0.000	0.119	−0.084	0.000	−0.036	−0.053
3	3	0.000	0.001	0.000	−0.045	0.002	0.000	−0.044	−0.001	0.000	0.000	−0.065	−0.001	0.000	−0.065
	4	0.000	0.001	0.045	−0.045	0.001	0.046	−0.044	0.002	0.001	0.066	−0.065	−0.001	0.064	−0.065
	5	0.000	0.000	0.039	−0.058	−0.001	−0.040	−0.039	−0.001	0.001	0.060	−0.085	0.000	−0.059	−0.059
	6	0.000	0.000	0.082	−0.058	−0.001	−0.024	−0.036	0.000	0.001	0.120	−0.085	0.001	−0.036	−0.052
4	3	0.000	0.000	0.000	−0.045	0.000	0.000	−0.044	0.001	0.001	0.000	−0.065	−0.001	0.001	−0.065
	4	0.002	−0.001	0.045	−0.043	0.000	0.045	−0.044	0.001	0.000	0.064	−0.066	0.001	0.066	−0.065
	5	0.000	0.000	0.041	−0.058	0.000	−0.039	−0.039	−0.001	0.000	0.058	−0.085	0.000	−0.058	−0.059
	6	0.001	0.001	0.082	−0.057	−0.001	−0.024	−0.035	−0.001	0.000	0.121	−0.084	0.000	−0.036	−0.053

Table A.16 Relative efficiencies of the proportion estimator for Log-Normal distribution when $p = 0.9$

		$Rho = 0.25$						$Rho = 0.5$					
n	k	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6
2	3	1.653	1.701	1.814	1.059	1.720	1.814	1.779	1.779	1.899	1.348	1.754	1.898
	4	1.836	1.650	1.910	0.844	1.668	1.901	1.878	1.597	1.968	1.099	1.597	1.985
	5	1.860	1.683	1.973	1.011	1.952	1.948	1.930	1.687	1.952	1.422	2.009	2.008
	6	1.912	1.486	1.973	0.888	1.922	1.971	1.984	1.284	1.964	1.236	2.073	2.054
3	3	1.390	1.394	1.492	0.938	1.406	1.500	1.401	1.405	1.503	1.150	1.408	1.503
	4	1.414	1.260	1.500	0.735	1.253	1.492	1.445	1.242	1.525	0.963	1.233	1.535
	5	1.428	1.299	1.524	0.880	1.501	1.526	1.531	1.302	1.524	1.188	1.577	1.573
	6	1.492	1.146	1.532	0.802	1.521	1.527	1.521	0.969	1.486	1.044	1.581	1.575
4	3	1.262	1.247	1.324	0.866	1.258	1.340	1.280	1.280	1.362	1.059	1.269	1.367
	4	1.271	1.142	1.341	0.712	1.137	1.337	1.360	1.127	1.401	0.928	1.131	1.401
	5	1.309	1.195	1.363	0.831	1.360	1.359	1.380	1.143	1.339	1.096	1.399	1.402
	6	1.324	1.027	1.356	0.755	1.354	1.360	1.382	0.831	1.293	0.971	1.400	1.390
		$Rho = 0.75$						$Rho = 0.95$					
n	k	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6	RE_1	RE_2	RE_3	RE_4	RE_5	RE_6
2	3	1.871	1.851	1.950	1.715	1.864	1.972	1.979	1.954	1.780	1.901	1.964	1.789
	4	2.006	1.192	1.871	1.591	1.139	1.883	2.208	0.868	1.605	2.070	0.904	1.563
	5	2.202	1.261	1.536	1.908	1.869	1.893	2.419	1.005	1.043	2.409	1.542	1.548
	6	2.238	0.587	1.374	1.781	2.088	1.925	2.588	0.356	0.891	2.488	1.903	1.530
3	3	1.474	1.489	1.507	1.361	1.486	1.514	1.569	1.548	1.307	1.559	1.571	1.315
	4	1.562	0.885	1.419	1.270	0.875	1.426	1.750	0.667	1.156	1.673	0.682	1.154
	5	1.672	0.966	1.091	1.537	1.404	1.417	1.835	0.697	0.714	1.829	1.076	1.088
	6	1.730	0.428	0.976	1.466	1.580	1.400	1.963	0.249	0.611	1.910	1.405	1.108
4	3	1.360	1.372	1.304	1.270	1.375	1.306	1.435	1.445	1.091	1.443	1.448	1.097
	4	1.490	0.787	1.245	1.233	0.775	1.233	1.571	0.575	0.901	1.456	0.552	0.924
	5	1.523	0.803	0.878	1.410	1.194	1.197	1.665	0.600	0.549	1.655	0.877	0.869
	6	1.563	0.350	0.796	1.353	1.383	1.198	1.769	0.194	0.472	1.724	1.177	0.877