DOKUZ EYLÜL UNIVERSITY
GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

RANKING ERROR MODELS, COST AND OPTIMAL SET SIZE IN RANKED SET SAMPLING

by
Sami AKDENİZ

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İZMİR

RANKING ERROR MODELS, COST AND OPTIMAL SET SIZE IN RANKED SET SAMPLING

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Sami AKDENİZ

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M.Sc THESIS EXAMINATION RESULT FORM

We have read the thesis entitled "**RANKING ERROR MODELS, COST AND OPTIMAL SET SIZE IN RANKED SET SAMPLING**" completed by **SAMİ AKDENİZ** under supervision of **ASSOC. PROF. DR. TUĞBA YILDIZ** and we certify that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

....

Assoc. Prof. Dr. Tuğba YILDIZ

Supervisor

....

Dr. Öğr. Üyesi Özge Elmastaş Gültekin

Jury Member

....

Dr. Öğr. Üyesi Senem VAHAPLAR

Jury Member

Prof. Dr. Kadriye ERTEKİN
Director
Graduate School of Natural and Applied Sciences

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RANKING ERROR MODELS, COST AND OPTIMAL SET SIZE IN RANKED SET SAMPLING

ABSTRACT

Ranked Set Sampling (RSS) is a sampling method commonly used in recent years. RSS is developed as an alternative to Simple Random Sampling (SRS) in order to estimate population parameters more efficiently where the measurement of sampling units is difficult or costly but the units are easier to rank. There are several factors that make this method useful especially for studies in medicine, agriculture, forestry and ecology. The most important of these factors are the set size and the relative costs of some operations such as sampling, measurement and ranking. Ranking of the units in the set is made on the basis of the visual judgment of the researcher or a concomitant variable which has a strong correlation with the variable of interest. These ranking methods are defined as ranking error models. In this thesis, the widely used cost and ranking error models in RSS literature are investigated. Also, it is aimed to explore the effect of ranking error models on the mean estimator based on RSS and some of its modified methods for different distribution, set and cycle size in infinite population. Besides, it is aimed to examine whether RSS is cost effiective with respect to SRS in terms of mean squared error of the mean estimator considering ranking error models and the N-KPST cost model in infinite population and if so, to determine the optimal set size for RSS. Monte Carlo simulation studies are conducted for these purposes. Additionally, the study is supported by real life data.

Keywords: Ranked set sampling, ranking error models, cost, optimal set size, relative efficiency, relative cost efficiency

SIRALI KÜME ÖRNEKLEMESİNDE SIRALAMA HATA MODELLERİ, MALİYET VE EN UYGUN KÜME BÜYÜKLÜĞÜ

ÖZ

Sıralı Küme Örneklemesi (SKÖ) son yıllarda yaygın olarak kullanılan bir örnekleme yöntemidir. SKÖ örnekleme birimlerinin ölçümünün zor veya maliyetli olduğu, fakat bu değişkene ait birimleri sıralamanın daha kolay olduğu durumda kitle parametrelerini daha etkin bir şekilde tahmin etmek amacıyla Basit Rastgele Örnekleme'ye (BRÖ) alternatif olarak geliştirilmiş bir örnekleme yöntemidir. Bu yöntemi özellikle tıp, tarım, ormancılık ve ekoloji gibi çalışma alanlarında kullanışlı kılan çeşitli faktörler vardır. Bu faktörlerin en önemlileri küme büyüklüğü ve örnekleme, ölçüm ve sıralama gibi çeşitli işlemlerin göreceli maliyetidir. Kümedeki birimler ilgilenilen değişken ile arasında yüksek korelasyon bulunan bir yardımcı değişkene ya da araştırmacının görsel yargısına dayanarak sıralanır. Bu sıralama yöntemleri sıralama hata modelleri olarak tanımlanır. Bu tezde SKÖ literatüründe yaygın olarak kullanılan maliyet ve sıralama hata modelleri incelenmiştir. Ayrıca, sonsuz kitlede sıralama hata modellerinin, farklı dağılım, küme boyutu ve döngü sayıları için, SKÖ ve onun bazı modifiye edilmiş yöntemlerine ait ortalama tahmin edicilerine etkisinin belirlenmesi amaçlanmıştır. Ayrıca, sonsuz kitle için sıralama hata modelleri ve N-KPST maliyet modeli göz önünde bulundurularak ortalama kestircisinin hata kareler ortalaması bakımından SKÖ'nün BRÖ'ye göre maliyet etkin olup olmadığını araştırmak, eğer etkin ise belirlenen maliyet koşulları altında SKÖ için en uygun küme boyutunu belirlemek amaçlanmıştır. Bu amaçlar doğrultusunda Monte Carlo benzetim çalışmaları yapılmıştır. Ayrıca, bu tez gerçek hayat verisiyle desteklenmiştir.

Anahtar kelimeler: Sıralı küme örneklemesi, sıralama hata modelleri, maliyet, en uygun küme büyüklüğü, göreli etkinlik, göreli maliyet etkinliği

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CHAPTER ONE
INTRODUCTION

The most important objective of sampling is to obtain information from the population to make correct inference about population of interest. There are several factors that affect the amount of information obtained from sample. These factors include the sample size chosen from the population and the sampling method utilized in the sample selection process. In the sampling theory, sample selection procedure is defined as sampling survey design (Scheaffer et al. (2011)). Several sampling methods and procedures, as described in the literature, are used to obtain observations of a fixed size of n. The process including the sample selection from the population of interest and the acquisition of observations requires cost, time, etc. For this reason, several methods can be used to obtain an effective estimator of the parameter. Also these methods provide the researcher savings in time, cost, etc.

Simple Random Sampling (SRS) is the most commonly known and used method in sampling theory. It is a sampling method in which the probabilities of all units to be selected are identical within a sample size of n. The sample obtained by this method is called simple random sample from a population size of N. The probability of a unit to be selected is mathematically defined by $\frac{1}{N}$. Although it is the most widely used sampling method, SRS is affected by several factors such as time, cost and etc. due to large sample size. These factors limit the accuracy of inferences that represent the population characteristics obtained by SRS. For this reason, another sampling method, called Ranked Set Sampling (RSS) was proposed as an alternative to SRS

RSS is a sampling method which is commonly used in medicine, pharmacy, ecology, forestry and agriculture where the measurement of the sampling units is difficult or costly but these units are easier to rank. RSS requires shorter time, less cost and provides better results with the least possible sample size to represent the population. McIntyre (1952) is the first to use this method to estimate the mean amount of crops obtained in a pasture yield in Australia. Halls & Dell (1966) used RSS to estimate the mean weights of trees and plant leaves in pine forests located in the east of Texas. Evans (1967) carried out a study on long leaf pine trees in order to compare the variances of the means obtained from RSS and SRS methods. Takahasi & Wakimoto (1968) presented a detailed mathematical development on RSS. They also showed that the estimator of the population mean obtained by RSS is unbiased and its variance is smaller than SRS when the errors in the ranking are ignored. Dell & Clutter (1972) showed that these unbiased estimator with lower variance are independent from ranking errors for RSS. David & Levine (1972) conducted a study to determine the effects of the errors in the ranking in RSS. Stokes (1977) proposed the concept of concomitant variable for RSS which is an effective way to increase accuracy of ranking. This variable should be highly correlated with the variable of interest. Stokes (1980) proposed RSS based variance estimator which is asmptotically unbiased and more efficient compared to SRS based variance estimator. Martin et al. (1980) used RSS to estimate shrub phytomass in Appalachian Oak forests, in Virginia, USA. Cobby et al. (1985) used RSS to estimate the amount of greenery in the grass in Hurley region, England. Ridout & Cobby (1987) conducted a study focusing on the results of the errors in the ranking due to the non-random selection of units in the set. Bohn & Wolfe (1992) investigated the use of non-parametric tests such as Wilcoxon and Mann Whitney in RSS. Johnson et al. (1993) used RSS in the estimation of the mean forest and other vegetation supplies. Patil et al. (1994) applied RSS to determine the effect of soil pollution created by the gas lines of a company around the region. Kaur et al. (1995) proposed RSS version of the sign test. Stokes (1995) proposed Maximum Likelihood Estimator (MLE) to estimate population parameters with RSS. Barnett & Moore (1997) used RSS to estimate the mean root weight of the Arabidopsis Thaliana. Philip & Lam (1997) proposed an effective regression estimator to estimate the population mean when the mean of the concomitant variable is known. Patil et al. (1999) reviewed the basic studies on RSS. Detailed information regarding theoretical and applicational studies based on RSS can be found in the book,”Ranked Set Sampling: Theory and Appplication”, written by Chen et al. (2003).

Many modified RSS methods have been developed and introduced in the literature in order to reduce the errors in ranking and to estimate the population parameters more efficiently. First study on modified RSS was performed by Samawi et al. (1996) in which extreme RSS was proposed. Then, Muttlak (1996) carried out a series of studies and proposed different modified RSS methods such as paired and median RSS (1997), quantile RSS (2003a), percentile RSS (2003b). In addition, double RSS was proposed by Al-Saleh & Al-Kadiri (2000) which formed the basis for multi stage RSS proposed by Al-Saleh & Al-Omari (2002). Also, Al-Omari & Bouza (2014) presented a study that included RSS, some of its modified procedures and their applications.

On the other hand, there is a number of studies in the literature that are focused on the modelling of ranking errors. Primarily, Dell & Clutter (1972) developed a model including a term of random error for the observations. Then, Stokes (1977) developed the idea of concomitant variable based ranking method. In this model, it was shown that, if exists, concomitant variable can be used to determine relative efficiency in an imperfect ranking and under certain assumptions in RSS. Afterwards, Bohn & Wolfe (1994) proposed a ranking error model based on the expected value of the difference between two order statistics. Fligner & MacEachern (2006) used the principle of monotone likelihood ratio to model the ranking information in RSS. New class of models is presented for imperfect rankings, in a study carried out by Frey (2007). A calibration model is developed by Ozturk (2007) to reduce errors in the ranking for RSS. Also Ozturk (2010) studied obtaining non parametric maximum likelihood estimators by taking the ranking errors in RSS into account.

Several authors developed cost models and compared RSS to SRS based on cost efficiency. The first cost model in RSS was proposed by Dell & Clutter (1972) simply to find the relative cost efficiency of RSS with respect to SRS. Kaur et al. (1996) developed KPST cost model to compare RSS with stratified random sampling. This model includes more cost categories than Dell & Clutter (1972) cost model such as sampling, measuring and pairwise comparison costs. Also, Philip & Lam (1997) considered a cost model in RSS as a special case of KPST model. Mode et al. (1999) proposed a cost model with an assumption of linear relationship between the ranking cost and the set size in RSS. Nahhas et al. (2002) modified the KPST cost model to determine the difference between visual ranking and concomitant based ranking in

RSS. Wang et al. (2004) suggested General RSS (GRSS) method for such cases that have considerable amount of ranking costs. In this method, more than one observations can be selected from each ranked set. Buchanan et al. (2005) used a cost model for various distributions to determine the minimum ratio of measurement to ranking costs that will make RSS is cost effective than SRS.

In this thesis, the effect of different cost structures on the optimal set size in RSS are evaluated. For this purpose, the ranking error models and cost models presented in the literature are considered as the cost structures. Besides, to investigate the effects of ranking error models for various distributions, mean squared error (MSE) of the mean estimators based on RSS and some of its modified methods such as extreme RSS (ERSS) and percentile RSS (Per-RSS) are computed and compared with MSE of the mean estimator based on SRS. Chapter 2 contains detailed information about RSS, extreme RSS (ERSS) and percentile RSS (Per-RSS). Ranking error models in RSS literature such as visual ranked set sampling (VRSS) and concomitant ranked set sampling (CRSS) are examined in Chapter 3. In addition, a Monte Carlo simulation study is conducted to determine the effects of ranking error models on the MSE of the mean estimators based on RSS and some of its modified methods for different distributions, set sizes and number of cycles. The existing cost models in the RSS literature are examined in Chapter 4. In addition, a Monte Carlo simulation study is carried out to see whether RSS is cost effective with respect to SRS, considering NKPST cost model and ranking error models, in terms of MSE of the mean estimator in infinite population, and if so, to determine the optimal set size for RSS. This study is supported by real life data application in Chapter 5. The final chapter contains the conclusions.

CHAPTER TWO
RANKED SET SAMPLING AND SOME OF ITS MODIFIED METHODS

2.1 Ranked Set Sampling (RSS)

In recent years, RSS is a commonly used sampling method in literature. RSS was developed by McIntyre (1952) as an alternative sampling method to SRS in order to estimate the population parameters more efficiently. RSS is useful and preferable method because of several important factors. Set size and the relative costs of various operations such as sampling, ranking and measurement are the most important ones among these factors. Also RSS provides advantages due to its features such as the ability to work with finite or infinite populations and it does not require to measure all units in the selected sample.

There are two important parameters in RSS. These are the set size and the number of cycles which are denoted by n and m, respectively. The set size in RSS usually ranges from 2 to 5. Also, there are many studies available in the literature in which more sets are used. McIntyre (1952) stated that the set sizes larger than 5 would not increase RSS efficiency too much, because such large set sizes would cause too many errors in the ranking. But McIntyre (1952) did not provide enough evidence for this. On the other hand, there is no limit for the number of cycles.

Order statistics are commonly used especially in areas such as life analysis and reliability analysis. Order statistics, which are known as one of the important concepts in statistical theory, form the basis of RSS.

Let $X_1, X_2, ..., X_k$ be derived from a population which is absolutely continuous. Order statistics are formed in an increasing manner $X_{(1)}<X_{(2)}<...<X_{(k)}$ or decrasing manner $X_{(k)}>X_{(2)}>...>X_{(1)}$. Here, $X_{(i)}$ is the i^{th} order statistic where $i = 1, 2, ..., k$. The minimum of the sample, therefore, corresponds to the first order statistic and defined as $X_{(1)} = min(X_1, X_2, ..., X_k)$ while the maximum of the sample corresponds to the k^{th} order statistic which is $X_{(k)} = max(X_1, X_2, ..., X_k)$.

The probability density function and distribution function of $X_{(i)}$ can be determied using the following equations (Chen et al. (2003)).

$$f_i(x) = \frac{k!}{(i-1)!(k-i)!}[F(x)]^{i-1}[1-F(x)]^{k-i}f(x) \tag{2.1}$$

$$F_i(x) = P(X_{(i)} \leq x) = \sum_{i=r}^{k} \binom{k}{r} [F(x)]^r [1-F(x)]^{k-r} \tag{2.2}$$

Probability density functions specific to the smallest $(i = 1)$ and the largest $(i = k)$ order statistics are obtained as follows (Chen et al. (2003)).

$$f_1(x) = k[1-F(x)]^{k-1}f(x), \qquad -\infty < x < \infty \tag{2.3}$$

$$f_k(x) = k[F(x)]^{k-1}f(x) \qquad -\infty < x < \infty \tag{2.4}$$

In addition to these, RSS works with a mechanism based on ranking for which the ranked observations correspond to order statistics under certain assumptions defined below:

- The sets and judgment order statistics are independent.
- A consistent ranking mechanism is used.

RSS procedure is applied in 5 steps which are described as below:

Step 1: Select a sample of size n^2 from the population of interest using SRS.

Step 2: Divide this randomly chosen sample of size n^2 into n sets with size n.

Step 3: Units which are randomly distributed between sets are ranked increasingly via cost effective and straightforward measurement, rather than a precise measurement based on the population of interest. This ranking can be made by using visual ranking method which or a concomitant variable.

Step 4: Select the smallest ranked unit from the first set, the second smallest ranked unit from the second set and the n^{th} smallest ranked unit from the n^{th} set for actual measurement of units.

Step 5: This process is repeated m times, until maintaining the required sample size $(N = nm)$.

The following expression represents the RSS procedure for one cycle:

$$\begin{pmatrix} \boldsymbol{X_{1[1:n]}} \leq X_{1[2:n]} \leq X_{1[3:n]} \leq \ldots \leq X_{1[n:n]} \\ X_{2[1:n]} \leq \boldsymbol{X_{2[2:n]}} \leq X_{2[3:n]} \leq \ldots \leq X_{2[n:n]} \\ X_{3[1:n]} \leq X_{3[2:n]} \leq \boldsymbol{X_{3[3:n]}} \leq \ldots \leq X_{3[n:n]} \\ \vdots \\ X_{n[1:n]} \leq X_{n[2:n]} \leq X_{n[3:n]} \leq \ldots \leq \boldsymbol{X_{n[n:n]}} \end{pmatrix}$$

$X_{i[m:n]}$ is the m^{th} ranked unit in the i^{th} set where $i = 1, 2, 3 \ldots, n$ and $m = 1, 2, 3 \ldots, n$.

For m cycles, the RSS procedure is defined as :

1^{st} Cycle

$$\begin{pmatrix} \boldsymbol{X_{1[1:n]1}} \leq X_{1[2:n]1} \leq X_{1[3:n]1} \leq \ldots \leq X_{1[n:n]1} \\ X_{2[1:n]1} \leq \boldsymbol{X_{2[2:n]1}} \leq X_{2[3:n]1} \leq \ldots \leq X_{2[n:n]1} \\ X_{3[1:n]1} \leq X_{3[2:n]1} \leq \boldsymbol{X_{3[3:n]1}} \leq \ldots \leq X_{3[n:n]1} \\ \vdots \\ X_{n[1:n]1} \leq X_{n[2:n]1} \leq X_{n[3:n]1} \leq \ldots \leq \boldsymbol{X_{n[n:n]1}} \end{pmatrix}$$

2^{nd} Cycle

$$\begin{pmatrix} \boldsymbol{X_{1[1:n]2}} \leq X_{1[2:n]2} \leq X_{1[3:n]2} \leq \ldots \leq X_{1[n:n]2} \\ X_{2[1:n]2} \leq \boldsymbol{X_{2[2:n]2}} \leq X_{2[3:n]2} \leq \ldots \leq X_{2[n:n]2} \\ X_{3[1:n]2} \leq X_{3[2:n]2} \leq \boldsymbol{X_{3[3:n]2}} \leq \ldots \leq X_{3[n:n]2} \\ \vdots \\ X_{n[1:n]2} \leq X_{n[2:n]2} \leq X_{n[3:n]2} \leq \ldots \leq \boldsymbol{X_{n[n:n]2}} \end{pmatrix}$$

3^{rd} Cycle

$$\begin{pmatrix} \boldsymbol{X_{1[1:n]3}} \leq X_{1[2:n]3} \leq X_{1[3:n]3} \leq \ldots \leq X_{1[n:n]3} \\ X_{2[1:n]3} \leq \boldsymbol{X_{2[2:n]3}} \leq X_{2[3:n]3} \leq \ldots \leq X_{2[n:n]3} \\ X_{3[1:n]3} \leq X_{3[2:n]3} \leq \boldsymbol{X_{3[3:n]3}} \leq \ldots \leq X_{3[n:n]3} \\ \vdots \\ X_{n[1:n]3} \leq X_{n[2:n]3} \leq X_{n[3:n]3} \leq \ldots \leq \boldsymbol{X_{n[n:n]3}} \end{pmatrix}$$

$$\vdots$$

m^{th} Cycle

$$\begin{pmatrix} \boldsymbol{X_{1[1:n]m}} \leq X_{1[2:n]m} \leq X_{1[3:n]m} \leq \ldots \leq X_{1[n:n]m} \\ X_{2[1:n]m} \leq \boldsymbol{X_{2[2:n]m}} \leq X_{2[3:n]m} \leq \ldots \leq X_{2[n:n]m} \\ X_{3[1:n]m} \leq X_{3[2:n]m} \leq \boldsymbol{X_{3[3:n]m}} \leq \ldots \leq X_{3[n:n]m} \\ \vdots \\ X_{n[1:n]m} \leq X_{n[2:n]m} \leq X_{n[3:n]m} \leq \ldots \leq \boldsymbol{X_{n[n:n]m}} \end{pmatrix}$$

Here, $X_{i[m:n]j}$ represents the unit which has the rank of m in the i^{th} set and j^{th} cycle where $i = 1, 2, ..., n$, $m = 1, 2, ..., n$ and $j = 1, 2, ..., m$. The obtained ranked set sample for n set and m cycle can be defined as:

$$\begin{pmatrix} X_{[1]1} & X_{[2]1} & X_{[3]1} & \ldots & X_{[i]1} \\ X_{[1]2} & X_{[2]2} & X_{[3]2} & \ldots & X_{[i]2} \\ X_{[1]3} & X_{[2]3} & X_{[3]3} & \ldots & X_{[i]3} \\ \vdots & \vdots & & & \vdots \\ X_{[1]j} & X_{[2]j} & X_{[3]j} & \ldots & X_{[i]j} \end{pmatrix}$$

where, $X_{[i]j}$ denotes the i^{th} ranked observation in the j^{th} cycle for i and j changing from 1 to n and m, respectively. The sets in RSS are random samples that are elements of the i^{th} set $X_{[i]1}$, $X_{[i]2}$, ..., $X_{[i]j}$ and each set has the same distribution function $F(x;\theta)$ and same probability density function $f(x;\theta)$.

This procedure might seem to be somehow wasteful since there is no measurement made for $mn(n-1)$ of the sampled units. However, the motivation is the low cost for the sampling and ranking of units from the population.

The estimator of the population mean for RSS is defined as;

$$\bar{X}_{RSS} = \frac{1}{mn} \sum_{j=1}^{m} \sum_{i=1}^{n} X_{[i]j} \tag{2.5}$$

Also the variance of the mean estimator for RSS is defined as;

$$Var(\bar{X}_{RSS}) = \frac{\sigma_x^2}{mn} \left(1 - \sum_{i=1}^{n} \frac{(E(X_{[i]1}) - \mu_x)^2}{n\sigma_x^2} \right) \tag{2.6}$$

where, μ_x and σ_x^2 are mean and variance of the population of interest, respectively.

2.2 Extreme Ranked Set Sampling (ERSS)

Samawi et al. (1996) developed Extreme Ranked Set Sampling (ERSS) to estimate the population parameter more efficiently by using the minimum and maximum ranked units or median value in the set which resulted in reduced errors in ranking. In ERSS, the procedure is the same as in RSS for the first 3 steps and the last step, while the only difference is at step 4.

Step 1: Select a sample of size n^2 from the target population with SRS method.

Step 2: Divide this randomly chosen sample into n sets with size n.

Step 3: Rank the units in each set based on an interested variable by visual judgement or by using any concomitant variable.

Step 4: If the set size n is even, select the smallest ranked unit from the first $(n/2)$ sets and the largest ranked unit from the other $(n/2)$ sets for the actual measurement of units. Otherwise, select the smallest ranked unit from the first $(n - 1/2)$ sets, the largest ranked unit from the second $(n - 1/2)$ sets and the median value from the remaining set (last set).

Step 5: Repeat first 4 steps for m cycles until the required sample size is obtained. $(N = nm)$.

For example, when $n = 6$, extreme ranked set sample is described below:

$$\begin{pmatrix} \boldsymbol{X_{1[1:6]}} \leq X_{1[2:6]} \leq X_{1[3:6]} \leq X_{1[4:6]} \leq X_{1[5:6]} \leq X_{1[6:6]} \\ \boldsymbol{X_{2[1:6]}} \leq X_{2[2:6]} \leq X_{2[3:6]} \leq X_{2[4:6]} \leq X_{2[5:6]} \leq X_{2[6:6]} \\ \boldsymbol{X_{3[1:6]}} \leq X_{3[2:6]} \leq X_{3[3:6]} \leq X_{3[4:6]} \leq X_{3[5:6]} \leq X_{3[6:6]} \\ X_{4[1:6]} \leq X_{4[2:6]} \leq X_{4[3:6]} \leq X_{4[4:6]} \leq X_{4[5:6]} \leq \boldsymbol{X_{4[6:6]}} \\ X_{5[1:6]} \leq X_{5[2:6]} \leq X_{5[3:6]} \leq X_{5[4:6]} \leq X_{5[5:6]} \leq \boldsymbol{X_{5[6:6]}} \\ X_{6[1:6]} \leq X_{6[2:6]} \leq X_{6[3:6]} \leq X_{6[4:6]} \leq X_{6[5:6]} \leq \boldsymbol{X_{6[6:6]}} \end{pmatrix}$$

Since the set size $n = 6$ is even, the actual measurement of units is made over the smallest ranked units ($X_{1[1:6]}$, $X_{2[1:6]}$,$X_{3[1:6]}$) from the first three sets and the largest ranked units ($X_{4[6:6]}$, $X_{5[6:6]}$,$X_{6[6:6]}$) from the last three sets, where $X_{i[m:n]}$ represents the m^{th} ranked unit in the i^{th} set for $i = 1, 2, \ldots, n$, $m = 1, 2, \ldots, n$. On the other hand, an example for odd set size, $n = 7$, is given below:

$$\begin{pmatrix} \boldsymbol{X_{1[1:7]}} \leq X_{1[2:7]} \leq X_{1[3:7]} \leq X_{1[4:7]} \leq X_{1[5:7]} \leq X_{1[6:7]} \leq X_{1[7:7]} \\ \boldsymbol{X_{2[1:7]}} \leq X_{2[2:7]} \leq X_{2[3:7]} \leq X_{2[4:7]} \leq X_{2[5:7]} \leq X_{2[6:7]} \leq X_{2[7:7]} \\ \boldsymbol{X_{3[1:7]}} \leq X_{3[2:7]} \leq X_{3[3:7]} \leq X_{3[4:7]} \leq X_{3[5:7]} \leq X_{3[6:7]} \leq X_{3[7:7]} \\ X_{4[1:7]} \leq X_{4[2:7]} \leq X_{4[3:7]} \leq X_{4[4:7]} \leq X_{4[5:7]} \leq X_{4[6:7]} \leq \boldsymbol{X_{4[7:7]}} \\ X_{5[1:7]} \leq X_{5[2:7]} \leq X_{5[3:7]} \leq X_{5[4:7]} \leq X_{5[5:7]} \leq X_{5[6:7]} \leq \boldsymbol{X_{5[7:7]}} \\ X_{6[1:7]} \leq X_{6[2:7]} \leq X_{6[3:7]} \leq X_{6[4:7]} \leq X_{6[5:7]} \leq X_{6[6:7]} \leq \boldsymbol{X_{6[7:7]}} \\ X_{7[1:7]} \leq X_{7[2:7]} \leq X_{7[3:7]} \leq \boldsymbol{X_{7[4:7]}} \leq X_{7[5:7]} \leq X_{7[6:7]} \leq X_{7[7:7]} \end{pmatrix}$$

In this case, the actual measurement of units is made over the smallest ranked units ($X_{1[1:7]}$, $X_{2[1:7]}$, $X_{3[1:7]}$) from the first three sets and the largest ranked units ($X_{4[7:7]}$, $X_{5[7:7]}$,$X_{6[7:7]}$) from the following three sets. In addition to this, the fourth ranked unit ($X_{7[4:7]}$) is selected from the remaining set for the measurement where $X_{i[m:n]}$ represents the m^{th} ranked unit in the i^{th} set for $i = 1, 2, \ldots, n$, $m = 1, 2, \ldots, n$. For this case, the last unit corresponds to the median value of the last set in the sample.

For even set sizes, the mean estimator of ERSS is given as;

$$\bar{X}_{ERSS} = \frac{1}{n}\left(\sum_{i=1}^{n/2} X_{2i-1[1:n]} + \sum_{i=1}^{n/2} X_{2i[n:n]}\right) \tag{2.7}$$

Also the variance of the mean estimator based on ERSS is given as;

$$Var(\bar{X}_{ERSS}) = \frac{1}{n^2}\left(\sum_{i=1}^{n/2} Var(X_{2i-1[1:n]}) + \sum_{i=1}^{n/2} Var(X_{2i[n:n]})\right) \tag{2.8}$$

For odd set sizes, the mean estimator of ERSS is given as;

$$\bar{X}_{ERSS} = \frac{1}{n}\left(\sum_{i=1}^{n-1/2} X_{2i-1[1:n]} + \sum_{i=1}^{n-1/2} X_{2i[n:n]} + X_{n[(n-1/2):n]}\right) \tag{2.9}$$

Also the variance of the mean estimator based on ERSS is given as;

$$Var(\bar{X}_{ERSS}) = \frac{1}{n^2}\left(\sum_{i=1}^{n-1/2} Var(X_{2i-1[1:n]}) + \sum_{i=1}^{n-1/2} Var(X_{2i[n:n]}) + Var(X_{n[(n+1)/2:n]})\right) \tag{2.10}$$

2.3 Percentile Ranked Set Sampling (Per-RSS)

Muttlak (2003) developed Per-RSS to estimate the population parameter more efficiently with less errors in the ranking. In Per-RSS, the procedure is the same as in RSS for the first 3 steps and the last step, while the only difference is in step 4.

Step 1: Select a sample of size n^2 from the population via SRS method.

Step 2: Divide this randomly chosen sample into n sets of size n.

Step 3: Rank the units in each set based on a variable by visual judgement or by using any concomitant variable.

Step 4: If the set size n is even, select the $[p(n+1)]^{th}$ ranked unit from the first $(n/2)$ sets, $[q(n+1)]^{th}$ ranked unit from the second $(n/2)$ sets for actual measurement of units. Otherwise, select the $[p(n+1)]^{th}$ ranked unit from the first $(n-1/2)$ sets , the $[q(n+1)]^{th}$ ranked unit from the second $(n-1/2)$ sets and the median value from the remaining set (last set).

Step 5: Repeat first 4 steps for m cycles until the required sample size is obtained. $(N = nm)$.

In this sampling method, p is denoted as the percentile value and takes value between 0 and 1, $(0 < p < 1)$. On the other hand, $q = 1 - p$ and $[p(n+1)]$ and $[q(n+1)]$ are rounded to the nearest integer.

The working principles of Per-RSS procedure are presented in the following examples.

$$\begin{pmatrix} X_{1[1:6]} \leq \boldsymbol{X_{1[2:6]}} \leq X_{1[3:6]} \leq X_{1[4:6]} \leq X_{1[5:6]} \leq X_{1[6:6]} \\ X_{2[1:6]} \leq \boldsymbol{X_{2[2:6]}} \leq X_{2[3:6]} \leq X_{2[4:6]} \leq X_{2[5:6]} \leq X_{2[6:6]} \\ X_{3[1:6]} \leq \boldsymbol{X_{3[2:6]}} \leq X_{3[3:6]} \leq X_{3[4:6]} \leq X_{3[5:6]} \leq X_{3[6:6]} \\ X_{4[1:6]} \leq X_{4[2:6]} \leq X_{4[3:6]} \leq X_{4[4:6]} \leq \boldsymbol{X_{4[5:6]}} \leq X_{4[6:6]} \\ X_{5[1:6]} \leq X_{5[2:6]} \leq X_{5[3:6]} \leq X_{5[4:6]} \leq \boldsymbol{X_{5[5:6]}} \leq X_{5[6:6]} \\ X_{6[1:6]} \leq X_{6[2:6]} \leq X_{6[3:6]} \leq X_{6[4:6]} \leq \boldsymbol{X_{6[5:6]}} \leq X_{6[6:6]} \end{pmatrix}$$

Let the set size be $n = 6$, $p = 0.35$ and $q = 0.65$. Following the ranking process of units, the second ranked units from the first half of the sets ($X_{1[2:6]}$, $X_{2[2:6]}$, $X_{3[2:6]}$) and the fifth ranked units from the following three sets ($X_{4[5:6]}$, $X_{5[5:6]}$,$X_{6[5:6]}$) are selected.

$$\begin{pmatrix} X_{1[1:7]} \leq X_{1[2:7]} \leq \boldsymbol{X_{1[3:7]}} \leq X_{1[4:7]} \leq X_{1[5:7]} \leq X_{1[6:7]} \leq X_{1[7:7]} \\ X_{2[1:7]} \leq X_{2[2:7]} \leq \boldsymbol{X_{2[3:7]}} \leq X_{2[4:7]} \leq X_{2[5:7]} \leq X_{2[6:7]} \leq X_{2[7:7]} \\ X_{3[1:7]} \leq X_{3[2:7]} \leq \boldsymbol{X_{3[3:7]}} \leq X_{3[4:7]} \leq X_{3[5:7]} \leq X_{3[6:7]} \leq X_{3[7:7]} \\ X_{4[1:7]} \leq X_{4[2:7]} \leq X_{4[3:7]} \leq X_{4[4:7]} \leq \boldsymbol{X_{4[5:7]}} \leq X_{4[6:7]} \leq X_{4[7:7]} \\ X_{5[1:7]} \leq X_{5[2:7]} \leq X_{5[3:7]} \leq X_{5[4:7]} \leq \boldsymbol{X_{5[5:7]}} \leq X_{5[6:7]} \leq X_{5[7:7]} \\ X_{6[1:7]} \leq X_{6[2:7]} \leq X_{6[3:7]} \leq X_{6[4:7]} \leq \boldsymbol{X_{6[5:7]}} \leq X_{6[6:7]} \leq X_{6[7:7]} \\ X_{7[1:7]} \leq X_{7[2:7]} \leq X_{7[3:7]} \leq \boldsymbol{X_{7[4:7]}} \leq X_{7[5:7]} \leq X_{7[6:7]} \leq X_{7[7:7]} \end{pmatrix}$$

This time, let $n = 7$, $p = 0.4$ and $q = 0.6$. Following the ranking process of units, the third ranked units from the first three sets ($X_{1[3:7]}$, $X_{2[3:7]}$,$X_{3[3:7]}$), the fifth ranked units from the following three sets ($X_{4[5:7]}$, $X_{5[5:7]}$,$X_{6[5:7]}$) and the median unit ($X_{7[4:7]}$) of the last set are selected.

For even set sizes, the mean estimator of Per-RSS is determined by;

$$\bar{X}_{Per-RSS} = \frac{1}{n}\left(\sum_{i=1}^{n/2} X_{i[a:n]} + \sum_{i=(n/2)+1}^{n} X_{i[b:n]}\right) \tag{2.11}$$

Also the variance of the mean estimator based on Per-RSS is determined by;

$$Var(\bar{X}_{Per-RSS}) = \frac{1}{n^2}\left(\sum_{i=1}^{n/2} Var(X_{i[a:n]}) + \sum_{i=(n/2)+1}^{n} Var(X_{i[b:n]})\right) \tag{2.12}$$

For odd set size; the mean estimator of Per-RSS is determined by;

$$\bar{X}_{Per-RSS} = \frac{1}{n}\left(\sum_{i=1}^{(n-1/2)} X_{i[a:n]} + \sum_{i=(n-1/2)+1}^{n-1} X_{i[b:n]} + X_{i[(n+1/2):n]})\right) \tag{2.13}$$

Also the variance of the mean estimator based on Per-RSS is determined by;

$$Var(\bar{X}_{Per-RSS}) = \frac{1}{n^2}\left(\sum_{i=1}^{n-1/2} Var(X_{i[a:n]}) + \sum_{i=(n-1/2)+1}^{n-1} Var(X_{i[b:n]}) + Var(X_{i[(n+1/2):n]})\right) \tag{2.14}$$

where, $a = [p(n+1)]$ and $b = [q(n+1)]$.

CHAPTER THREE
RANKING ERROR MODELS IN RANKED SET SAMPLING

One of the most important challenges in RSS is the ranking error of the units in a set. The reason for this is that the ranking of units according to the variable of interest is done using low quality measurement techniques without any precise measurement of units. Ranking errors may occur depending on the ranking method used during the sample selection process in RSS. That is, if all the units in a set are ranked randomly, then the expected value of the i^{th} order statistic will correspond to the population mean. Under this condition, there is no difference between RSS and SRS methods. In RSS, there are several ranking methods that can be used. The most commonly ones are visual ranking and concomitant based ranking. In the case of visual ranking, variation in relative efficiency may vary depending on the researcher's knowledge and experience. On the other hand, in the case of ranking with respect to a concomitant variable, it may vary depending on the correlation between the variable of interest (X) and the concomitant variable (Y). Ranking errors would be reduced as the correlation coefficient between X and Y gets closer to 1 and, on the other hand, if a visual ranking method is administered by a well educated and experienced researcher. This chapter contains information about these two ranking error models. In addition, a simulation study is included that focused on the investigation of the effects of ranking error models on mean estimators based on RSS, ERSS and Per-RSS in infinite population. The bias and MSE values of the mean estimators are computed and compared with MSE of the mean estimator based on SRS.

3.1 Visual Ranked Set Sampling (VRSS)

McIntyre (1952) used visual judgement ranking for units in the set to estimate the mean amount of products. The visual judgement ranking method is a subjective ranking method since the ranking of units in the set is based on the personal judgement of the researcher. The reliability of visual ranking depends on the knowledge and experience of the researcher based on the subject of study and also on the materials used to rank the units. For example, an expert can estimate how much the soil is affected by toxic wastes by looking at the colour change on the soil and the effects seen on the vegetation. Dell & Clutter (1972) proposed modelling the visual score V_i belonging to the i^{th} observation in the set by;

$$V_i = X_i + \tau_i \tag{3.1}$$

where

V_i : i^{th} visual judgement order statistic

X_i : i^{th} true order statistic

τ_i : i^{th} random error term where $\tau_i \sim iid(0, \sigma_\tau^2)$ and X_i 's are mutually independent of τ_i 's .

In RSS for n^{th} set and m^{th} cycle, visual ranking process can be defined step by step as follows:

Step 1: Generate $V_i = X_i + \tau_i$ with $\tau_i \sim iid(0, \sigma_\tau^2)$ where X_i's and τ_i's are mutually independent.

Step 2: Rank the visual scores ($V_1, V_2, \ldots, V_n$) from the smallest to the largest.

Step 3: Select the sampling unit corresponding to the i^{th} visual score (V_i) and measure the $X_{[i]}$ value for this unit.

This method is called Visual Ranked Set Sampling (VRSS). Assume that, $X_i \sim N(0, \sigma_x^2)$ and $\tau_i \sim N(0, \sigma_\tau^2)$ for each $i = 1, 2, ..., n$. Then, the expected value of the r^{th} order statistic from $N(0, 1)$ with sample size n which is defined as α_r is calculated by;

$$\alpha_r = E(X_{[r]}) = \mu_x + \rho_{xv}^2 \frac{\sigma_x}{\sigma_v} E(V_{[r]} - \mu_v) \tag{3.2}$$

where

ρ_{xv}: The correlation between true order statistics and visual scores

μ_x : The mean of the true order statistic

σ_x : The standard deviation of true order statistic

μ_v : The mean of the visual score

σ_v : The standard deviation of visual score.

On the other hand, the correlation between X_i and V_i is calculated by;

$$\rho_{xv} = \frac{\sigma_x}{\sqrt{\sigma_x^2 + \sigma_\tau^2}} \tag{3.3}$$

When visual ranking is used, the variance of mean estimator of RSS is calculated by;

$$Var(\bar{X}_{VRSS}) = \frac{\sigma_x^2}{nm}\left(1 - \frac{\rho_{xv}^2}{n}\sum_{r=1}^{n}\left(\frac{E(V_{[r]} - \mu_v}{\sigma_v}\right)^2\right) \tag{3.4}$$

Then, the relative efficiency of variance of mean estimator based VRSS with respect to SRS is calculated by;

$$RE(\bar{X}_{VRSS}) = \frac{Var(\bar{X}_{SRS})}{Var(\bar{X}_{VRSS})} = \left(1 - \frac{\rho_{xv}^2}{n}\sum_{r=1}^{n}\left(\frac{E(V_{[r]} - \mu_v}{\sigma_v}\right)^2\right)^{-1} \tag{3.5}$$

It is shown by Nahhas et al. (2002) that the expected probability of ranking a pair (V_i,X_i) incorrectly is defined as;

$$p = \frac{1}{2} - \frac{tan^{-1}(\sigma_x/\sigma_\tau)}{\pi} \tag{3.6}$$

Table 3.1 σ_τ values corresponding to the p values for different distributions in VRSS

σ_τ											
p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
$N(0,1)$	0	0.158	0.325	0.509	0.727	1	1.376	1.963	3.077	6.314	318.3
$U(0,1)$	0	0.045	0.093	0.147	0.21	0.289	0.397	0.566	0.888	1.822	91.864
$Exp(1)$	0	0.158	0.325	0.509	0.727	1	1.376	1.963	3.077	6.314	318.3
$\Gamma(2,1)$	0	0.224	0.496	0.721	1.028	1.414	1.947	2.776	4.353	8.929	450.157

It can be clearly seen from Table 3.1 that when the standard deviation of random error term, the expected probability of ranking error increases as well.

3.2 Concomitant Ranked Set Sampling (CRSS)

In RSS, another method used to rank the units in the set, is concomitant variable based ranking which is suggested by Stokes (1977). The concomitant variable (Y) is a variable that has a high correlation with the variable of interest (X) and it provides the ranking of units in the set in less time and with less cost. The accuracy of the ranking is increased throught using this variable. Therefore, when a concomitant variable is used in RSS, the units are ranked according to the precise measurements taken with respect to the concomitant variable instead of the variable of interest. The probability of the error in the ranking will increase with the decreasing correlation. As an example, to estimate the mean weight of a certain number of fish belonging to a population, a researcher may use a concomitant variable, such as fish size, which has a high correlation with the fish weight. It allows the units to be ranked and measured at a lower cost and in a shorter time. Also, a more accurate ranking and correct estimates can be obtained with the help of the concomitant variable.

Concomitant variable is first studied byDavid & Levine (1972). Stokes (1977) studied concomitant variable in detail and developed some limiting assumptions in its use in order to determine its effects on RSS. These assumptions are given below:

- There is a linear relationship between concomitant variable and the variable of interest.
- Standardized concomitant variable and the variable of interest have identical distribution.

Concomitant based ranking can be modelled as follows;

$$X_i = \mu_x + \frac{\rho_{xy}\sigma_x}{\sigma_y}(Y_i - \mu_y) + \tau_i \tag{3.7}$$

where

μ_x : The mean of the interested variable

σ_x : The standard deviation of the interested variable

μ_y : The mean of the concomitant variable

σ_y : The standard deviation of the concomitant variable

ρ_{xy} : The correlation between interested variable and concomitant variable

X_i : The true order statistic of X

Y_i : The correctly ranked order statistic of Y

τ_i : i^{th} random error term. $\tau_i \sim iid(0, \sigma_\tau^2)$ and τ_i's and Y_i's are mutually independent.

The stepwise period of ranking the units in the set with respect to the concomitant variable for n^{th} set and m^{th} cycle is given below:

Step 1: Generate Equation (3.7) where τ_i's and $Y_{(i)}$'s are mutually independent.

Step 2: Rank the correctly ranked order statistics $Y_{(i)}$'s from the smallest to the largest.

Step 3: Select the r^{th} correctly ranked order statistic $Y_{(r)}$ and measure the r^{th} true order statistic $X_{[r]}$ from the sampling unit.

This method is defined as Concomitant Ranked Set Sampling (CRSS) method. Then the expected value of r^{th} order statistic of X is calculated by;

$$E(X_{[r]}) = \mu_x + \rho_{xy}\frac{\sigma_x}{\sigma_y}(E(Y_{(r)} - \mu_y) \tag{3.8}$$

When concomitant based ranking is used, the variance of mean estimator is calculated by;

$$Var(\bar{X}_{CRSS}) = \frac{\sigma_x^2}{nm}\left(1 - \frac{\rho_{xy}^2}{n}\sum_{r=1}^{n}\left(\frac{E(Y_r) - \mu_y}{\sigma_y}\right)^2\right) \tag{3.9}$$

Then, relative efficiency of variance of mean estimator based on CRSS with respect to

SRS is given as follows.

$$RE(\bar{X}_{CRSS}) = \frac{Var(\bar{X}_{SRS})}{Var(\bar{X}_{CRSS})} = \left(1 - \frac{\rho_{xy}^2}{n}\sum_{r=1}^{n}\left(\frac{E(Y_r) - \mu_y}{\sigma_y}\right)^2\right)^{-1} \tag{3.10}$$

In CRSS, ρ_{xy} is used to define the accuracy of ranking. Incorrect ranking of units in the set depends on the weak correlation between the variable of interest and the concomitant variable. p is defined as the probability of error in the ranking. Wang et al. (2004) suggested if the variable of interest and concomitant variable have a bivariate normal distribution, then the correlation between them is computed by;

$$\rho_{xy} = \sqrt{\left(1 + \left(tan^2\left(\frac{\pi(1-2p)}{2}\right)\right)^{-1}\right)^{-1}} \tag{3.11}$$

Table 3.2 Correlation values corresponding to the p values in CRSS

p	0	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
ρ	1	0.99	0.95	0.89	0.8	0.7	0.59	0.45	0.3	0.15	0

According to Table 3.2 obtained from Equation (3.11), when the correlation between the variable of interest and the concomitant variable decreases the expected probability of ranking error increases.

3.3 Simulation Results

Our basic goal in this simulation study is the investigation of the effects of ranking error models on the mean estimators based on RSS, ERSS and Per-RSS. For this reason, bias and MSE of the mean estimators are computed and compared with MSE of mean estimator based on SRS for different set and cycle sizes, distributions and ranking error models, VRSS and CRSS in infinite population. For this simulation study, R statistical programming language is used with 10000 repetitions. In this simulation study;

- The population of interest is generated from $N(0,1)$, $U(0,1)$, $Exp(1)$, and $\Gamma(2,1)$ distributions, of size 10000.

- Set sizes (n) are determined to be 2, 3, 4 and 5. Also cycle sizes (m) are determined to be 5 and 10.

- For VRSS, the variances of the random error term (σ_τ^2) are selected as 0.05, 0.15, 0.3 and 0.5.

- For CRSS, the correlation values between interested varible and concomitant variable are determined as 0.95, 0.75, 0.50 and 0.25.

- Four sampling methods are used. These sampling methods are SRS, RSS, ERSS and Per-RSS (p value for Per-RSS is determined as 0.4).

Let $\theta = \mu$ and $\hat{\theta}_1$, $\hat{\theta}_2$, $\hat{\theta}_3$, $\hat{\theta}_4$ represent $\bar{X}_{SRS}$, $\bar{X}_{RSS}$, $\bar{X}_{ERRS}$, $\bar{X}_{Per-RSS}$ respectively. Also the formulas given below are used:

$$Bias(\hat{\theta}_i, \theta) = \sum_{i=1}^{10000} \frac{(\hat{\theta}_i - \theta)}{10000} \tag{3.12}$$

$$MSE(\hat{\theta}_i, \theta) = \sum_{i=1}^{10000} \frac{(\hat{\theta}_i - \theta)^2}{10000} \tag{3.13}$$

$$RE_1(\hat{\theta}_2, \hat{\theta}_1) = \frac{MSE(\hat{\theta}_1)}{MSE(\hat{\theta}_2)} \tag{3.14}$$

$$RE_2(\hat{\theta}_3, \hat{\theta}_1) = \frac{MSE(\hat{\theta}_1)}{MSE(\hat{\theta}_3)} \tag{3.15}$$

$$RE_3(\hat{\theta}_4, \hat{\theta}_1) = \frac{MSE(\hat{\theta}_1)}{MSE(\hat{\theta}_4)} \tag{3.16}$$

RE values obtained from simulation study greater than 1 means that RSS, ERSS and

Per-RSS methods are more efficient than SRS method based on the MSE of the mean estimators. The results obtained from the simulation study are presented in tables.

Table 3.3 RE values for $N(0,1)$ in VRSS

$N(0,1)$		$\sigma_\tau^2 = 0.05$			$\sigma_\tau^2 = 0.15$			$\sigma_\tau^2 = 0.30$			$\sigma_\tau^2 = 0.50$		
m	n	RE_1	RE_2	RE_3	RE_1	RE_2	RE_3	RE_1	RE_2	RE_3	RE_1	RE_2	RE_3
5	2	1.430	1.435	1.440	1.376	1.373	1.362	1.357	1.319	1.389	1.305	1.256	1.247
	3	1.848	1.854	2.153	1.725	1.708	1.871	1.596	1.563	1.732	1.463	1.473	1.607
	4	2.238	1.908	2.529	1.954	1.823	2.254	1.726	1.697	1.933	1.602	1.539	1.722
	5	2.509	2.300	3.037	2.301	2.024	2.549	2.023	1.792	2.137	1.766	1.619	1.831
10	2	1.437	1.445	1.437	1.406	1.365	1.368	1.328	1.294	1.361	1.293	1.286	1.266
	3	1.845	1.849	2.132	1.734	1.691	1.912	1.612	1.579	1.716	1.474	1.443	1.565
	4	2.294	1.948	2.553	2.013	1.786	2.253	1.823	1.602	1.995	1.633	1.439	1.726
	5	2.485	2.288	2.985	2.264	2.119	2.442	2.025	1.847	2.115	1.826	1.700	1.837

Table 3.4 RE values for $U(0,1)$ in VRSS

$U(0,1)$		$\sigma_\tau^2 = 0.05$			$\sigma_\tau^2 = 0.15$			$\sigma_\tau^2 = 0.30$			$\sigma_\tau^2 = 0.50$		
m	n	RE_1	RE_2	RE_3	RE_1	RE_2	RE_3	RE_1	RE_2	RE_3	RE_1	RE_2	RE_3
5	2	1.299	1.288	1.257	1.155	1.130	1.141	1.115	1.038	1.105	1.046	1.056	1.030
	3	1.469	1.424	1.268	1.212	1.237	1.109	1.118	1.111	1.055	1.119	1.103	0.998
	4	1.635	2.002	1.369	1.291	1.424	1.150	1.146	1.241	1.057	1.100	1.165	1.099
	5	1.766	2.099	1.533	1.316	1.474	1.247	1.167	1.258	1.110	1.084	1.174	1.063
10	2	1.297	1.317	1.257	1.136	1.164	1.128	1.045	1.079	1.068	1.049	1.029	1.041
	3	1.457	1.482	1.237	1.184	1.255	1.059	1.101	1.116	1.057	1.081	1.072	1.040
	4	1.683	1.990	1.352	1.260	1.458	1.181	1.174	1.179	1.052	1.109	1.118	0.997
	5	1.742	2.130	1.557	1.331	1.471	1.180	1.148	1.218	1.108	1.121	1.153	1.081

Table 3.5 RE values for $Exp(1)$ in VRSS

$Exp(0,1)$		$\sigma_\tau^2 = 0.05$			$\sigma_\tau^2 = 0.15$			$\sigma_\tau^2 = 0.30$			$\sigma_\tau^2 = 0.50$		
m	n	RE_1	RE_2	RE_3	RE_1	RE_2	RE_3	RE_1	RE_2	RE_3	RE_1	RE_2	RE_3
5	2	1.294	1.285	1.263	1.269	1.250	1.205	1.232	1.269	1.309	1.181	1.210	1.208
	3	1.571	1.597	1.166	1.502	1.525	1.149	1.389	1.535	1.187	1.288	1.277	1.103
	4	1.839	0.741	1.063	1.694	0.668	1.026	1.562	0.477	1.006	1.485	0.686	0.982
	5	1.976	0.770	1.104	1.847	0.727	1.038	1.650	0.510	0.998	1.551	0.731	0.988
10	2	1.255	1.296	1.301	1.292	1.269	1.236	1.255	1.230	1.220	1.190	1.166	1.190
	3	1.550	1.593	0.758	1.479	1.535	0.735	1.400	1.409	0.736	1.397	1.315	0.750
	4	1.760	0.518	0.636	1.698	0.477	0.618	1.591	0.693	0.609	1.445	0.496	0.650
	5	2.066	0.509	0.656	1.831	0.510	0.630	1.676	0.727	0.627	1.563	0.495	0.654

Table 3.6 RE values for $\Gamma(2,1)$ in VRSS

$\Gamma(2,1)$		$\sigma_\tau^2 = 0.05$			$\sigma_\tau^2 = 0.15$			$\sigma_\tau^2 = 0.30$			$\sigma_\tau^2 = 0.50$		
m	n	RE_1	RE_2	RE_3	RE_1	RE_2	RE_3	RE_1	RE_2	RE_3	RE_1	RE_2	RE_3
5	2	1.322	1.359	1.375	1.313	1.334	1.371	1.244	1.281	1.350	1.289	1.241	1.239
	3	1.759	1.675	1.468	1.719	1.570	1.425	1.625	1.526	1.318	1.452	1.522	1.284
	4	2.065	1.031	1.531	1.926	0.989	1.330	1.756	0.940	1.292	1.716	0.910	1.270
	5	2.338	1.133	1.607	2.170	1.060	1.405	2.045	1.047	1.377	1.846	0.963	1.291
10	2	1.374	1.380	1.359	1.324	1.342	1.366	1.357	1.292	1.320	1.252	1.289	1.258
	3	1.642	1.726	1.092	1.644	1.705	1.013	1.558	1.579	0.990	1.517	1.487	0.926
	4	2.014	0.787	0.991	1.921	0.705	0.920	1.740	0.719	0.863	1.774	0.693	0.866
	5	2.251	0.831	1.025	2.109	0.775	0.957	1.949	0.715	0.877	1.792	0.696	0.860

Table 3.7 RE values for $N(0,1)$ in CRSS

$N(0,1)$		$\rho_{xy} = 0.95$			$\rho_{xy} = 0.75$			$\rho_{xy} = 0.50$			$\rho_{xy} = 0.25$		
m	n	RE_1	RE_2	RE_3	RE_1	RE_2	RE_3	RE_1	RE_2	RE_3	RE_1	RE_2	RE_3
5	2	1.351	1.398	1.361	1.188	1.236	1.180	1.115	1.060	1.072	1.024	1.020	0.993
	3	1.739	1.728	2.028	1.401	1.360	1.449	1.118	1.147	1.144	1.061	0.998	1.044
	4	2.050	1.890	2.302	1.452	1.388	1.511	1.166	1.185	1.220	1.048	1.043	1.041
	5	2.395	2.074	2.676	1.509	1.562	1.620	1.194	1.154	1.206	1.040	1.033	1.064
10	2	1.405	1.402	1.511	1.213	1.222	1.224	1.082	1.062	1.107	0.999	1.043	1.038
	3	1.740	1.726	1.962	1.345	1.382	1.430	1.147	1.149	1.152	1.043	1.019	1.061
	4	2.127	1.882	2.372	1.558	1.317	1.588	1.188	1.138	1.223	1.029	0.995	1.041
	5	2.349	2.062	2.735	1.559	1.529	1.686	1.182	1.137	1.216	1.065	1.068	1.041

Table 3.8 RE values for $U(0,1)$ in CRSS

$U(0,1)$		$\rho_{xy} = 0.95$			$\rho_{xy} = 0.75$			$\rho_{xy} = 0.50$			$\rho_{xy} = 0.25$		
m	n	RE_1	RE_2	RE_3	RE_1	RE_2	RE_3	RE_1	RE_2	RE_3	RE_1	RE_2	RE_3
5	2	1.474	1.408	1.442	1.193	1.251	1.208	1.079	1.106	1.093	1.022	1.000	1.047
	3	1.804	1.789	1.509	1.388	1.386	1.233	1.116	1.163	1.085	0.994	1.035	1.022
	4	2.202	2.670	1.844	1.497	1.648	1.340	1.186	1.206	1.101	1.043	1.025	1.029
	5	2.470	2.921	2.153	1.559	1.698	1.374	1.168	1.254	1.120	1.053	1.056	1.015
10	2	1.447	1.414	1.416	1.214	1.197	1.225	1.097	1.094	1.102	1.008	1.022	1.022
	3	1.752	1.866	1.545	1.393	1.369	1.275	1.108	1.092	1.104	1.055	1.021	0.992
	4	2.177	2.755	1.802	1.417	1.601	1.308	1.132	1.214	1.117	1.031	1.044	1.067
	5	2.513	2.923	2.182	1.569	1.693	1.415	1.208	1.199	1.158	1.041	1.022	0.995

Table 3.9 RE values for $Exp(1)$ in CRSS

$Exp(1)$		$\rho_{xy} = 0.95$			$\rho_{xy} = 0.75$			$\rho_{xy} = 0.50$			$\rho_{xy} = 0.25$		
m	n	RE_1	RE_2	RE_3	RE_1	RE_2	RE_3	RE_1	RE_2	RE_3	RE_1	RE_2	RE_3
5	2	1.323	1.284	1.297	1.167	1.186	1.182	1.099	1.077	1.102	1.017	1.010	1.040
	3	1.526	1.501	1.347	1.288	1.293	1.387	1.018	1.110	1.221	1.034	1.027	1.009
	4	1.705	0.802	1.286	1.362	0.944	1.312	1.101	1.009	1.197	1.028	1.011	1.017
	5	1.912	0.882	1.271	1.381	0.986	1.324	1.112	1.079	1.182	1.021	0.991	1.050
10	2	1.384	1.281	1.232	1.165	1.177	1.145	1.097	1.064	1.040	1.023	0.998	0.998
	3	1.539	1.551	0.912	1.321	1.224	1.077	1.097	1.102	1.138	0.997	1.070	1.042
	4	1.801	0.586	0.754	1.313	0.810	1.150	1.166	0.970	1.087	1.039	1.028	0.996
	5	1.939	0.614	0.818	1.480	0.826	1.077	1.143	0.979	1.162	1.054	0.961	1.023

Table 3.10 RE values for $\Gamma(2,1)$ in CRSS

$\Gamma(2,1)$		$\rho_{xy} = 0.95$			$\rho_{xy} = 0.75$			$\rho_{xy} = 0.50$			$\rho_{xy} = 0.25$		
m	n	RE_1	RE_2	RE_3	RE_1	RE_2	RE_3	RE_1	RE_2	RE_3	RE_1	RE_2	RE_3
5	2	1.260	1.344	1.381	1.176	1.180	1.221	1.065	1.064	1.120	1.021	0.988	1.003
	3	1.600	1.636	1.530	1.329	1.290	1.451	1.126	1.062	1.226	0.984	1.052	1.022
	4	1.903	1.090	1.570	1.402	1.051	1.420	1.132	1.062	1.187	1.030	0.990	1.035
	5	2.118	1.126	1.705	1.445	1.172	1.477	1.206	1.125	1.230	1.062	1.034	1.086
10	2	1.333	1.325	1.363	1.149	1.185	1.191	1.101	1.065	1.053	1.040	1.002	1.015
	3	1.601	1.626	1.192	1.374	1.324	1.299	1.152	1.118	1.180	1.030	0.988	1.032
	4	1.841	0.833	1.154	1.394	0.981	1.198	1.124	1.039	1.140	1.043	1.008	1.017
	5	2.191	0.880	1.178	1.422	1.030	1.257	1.156	0.993	1.186	1.075	1.012	1.054

Table 3.11 MSE values for $N(0,1)$ in VRSS

		$\sigma_\tau^2 = 0.05$			$\sigma_\tau^2 = 0.15$		
m	n	RSS	$ERSS$	$Per-RSS$	RSS	$ERSS$	$Per-RSS$
5	2	0.0697	0.0712	0.0687	0.0727	0.0728	0.0732
	3	0.0357	0.0362	0.0315	0.0386	0.0391	0.0358
	4	0.0224	0.0259	0.0196	0.0254	0.0279	0.0224
	5	0.0161	0.0174	0.0134	0.0174	0.0198	0.0160
10	2	0.0352	0.0346	0.0349	0.0365	0.0366	0.0368
	3	0.0179	0.0181	0.0157	0.0195	0.0196	0.0173
	4	0.0112	0.0129	0.0098	0.0124	0.0137	0.0110
	5	0.0080	0.0089	0.0066	0.0090	0.0096	0.0082
		$\sigma_\tau^2 = 0.30$			$\sigma_\tau^2 = 0.50$		
m	n	RSS	$ERSS$	$Per-RSS$	RSS	$ERSS$	$Per-RSS$
5	2	0.0736	0.0756	0.0719	0.0785	0.0804	0.0797
	3	0.0412	0.0430	0.0388	0.0452	0.0456	0.0416
	4	0.0287	0.0301	0.0254	0.0309	0.0329	0.0290
	5	0.0201	0.0218	0.0185	0.0225	0.0246	0.0218
10	2	0.0385	0.0383	0.0376	0.0384	0.0388	0.0389
	3	0.0209	0.0213	0.0195	0.0225	0.0228	0.0214
	4	0.0139	0.0155	0.0127	0.0149	0.0172	0.0143
	5	0.0101	0.0110	0.0094	0.0111	0.0120	0.0109

Table 3.12 MSE values for $U(0,1)$ in VRSS

		$\sigma_\tau^2 = 0.05$			$\sigma_\tau^2 = 0.15$		
m	n	RSS	$ERSS$	$Per-RSS$	RSS	$ERSS$	$Per-RSS$
5	2	0.0064	0.0066	0.0066	0.0073	0.0073	0.0074
	3	0.0038	0.0038	0.0044	0.0046	0.0045	0.0051
	4	0.0026	0.0021	0.0030	0.0033	0.0029	0.0036
	5	0.0019	0.0016	0.0022	0.0026	0.0023	0.0028
10	2	0.0032	0.0033	0.0033	0.0036	0.0036	0.0037
	3	0.0019	0.0019	0.0023	0.0023	0.0022	0.0026
	4	0.0013	0.0010	0.0016	0.0016	0.0014	0.0018
	5	0.0009	0.0008	0.0011	0.0013	0.0011	0.0014
		$\sigma_\tau^2 = 0.30$			$\sigma_\tau^2 = 0.50$		
m	n	RSS	$ERSS$	$Per-RSS$	RSS	$ERSS$	$Per-RSS$
5	2	0.0075	0.0079	0.0076	0.0078	0.0079	0.0079
	3	0.0049	0.0050	0.0053	0.0051	0.0051	0.0054
	4	0.0036	0.0034	0.0039	0.0038	0.0036	0.0038
	5	0.0028	0.0026	0.0030	0.0031	0.0029	0.0031
10	2	0.0039	0.0039	0.0040	0.0040	0.0041	0.0039
	3	0.0025	0.0025	0.0026	0.0026	0.0026	0.0027
	4	0.0018	0.0017	0.0020	0.0019	0.0019	0.0020
	5	0.0015	0.0014	0.0015	0.0015	0.0015	0.0015

Table 3.13 MSE values for $Exp(1)$ in VRSS

		$\sigma_\tau^2=0.05$			$\sigma_\tau^2=0.15$		
m	n	RSS	$ERSS$	$Per-RSS$	RSS	$ERSS$	$Per-RSS$
5	2	0.0758	0.0768	0.0780	0.0779	0.0795	0.0821
	3	0.0425	0.0424	0.0573	0.0445	0.0437	0.0583
	4	0.0271	0.0693	0.0478	0.0303	0.0727	0.0490
	5	0.0201	0.0511	0.0365	0.0218	0.0554	0.0398
10	2	0.0385	0.0381	0.0383	0.0395	0.0399	0.0399
	3	0.0215	0.0210	0.0429	0.0225	0.0217	0.0454
	4	0.0140	0.0500	0.0386	0.0148	0.0514	0.0409
	5	0.0097	0.0386	0.0306	0.0109	0.0398	0.0322
		$\sigma_\tau^2=0.30$			$\sigma_\tau^2=0.50$		
m	n	RSS	$ERSS$	$Per-RSS$	RSS	$ERSS$	$Per-RSS$
5	2	0.0815	0.0811	0.0797	0.0849	0.0843	0.0833
	3	0.0480	0.0466	0.0593	0.0505	0.0510	0.0591
	4	0.0324	0.0738	0.0504	0.0339	0.0745	0.0506
	5	0.0239	0.0549	0.0390	0.0258	0.0549	0.0404
10	2	0.0405	0.0412	0.0412	0.0428	0.0417	0.0428
	3	0.0238	0.0235	0.0454	0.0245	0.0250	0.0431
	4	0.0157	0.0515	0.0406	0.0173	0.0505	0.0383
	5	0.0119	0.0394	0.0321	0.0128	0.0395	0.0300

Table 3.14 MSE values for $\Gamma(2,1)$ in VRSS

		$\sigma_\tau^2=0.05$			$\sigma_\tau^2=0.15$		
m	n	RSS	$ERSS$	$Per-RSS$	RSS	$ERSS$	$Per-RSS$
5	2	0.1488	0.1431	0.1456	0.1515	0.1484	0.1474
	3	0.0776	0.0796	0.0890	0.0784	0.0841	0.0945
	4	0.0494	0.0968	0.0664	0.0510	0.1030	0.0725
	5	0.0345	0.0708	0.0506	0.0371	0.0756	0.0557
10	2	0.0726	0.0724	0.0725	0.0753	0.0735	0.0734
	3	0.0395	0.0392	0.0622	0.0413	0.0393	0.0643
	4	0.0249	0.0642	0.0500	0.0258	0.0702	0.0544
	5	0.0175	0.0478	0.0390	0.0187	0.0526	0.0424
		$\sigma_\tau^2=0.30$			$\sigma_\tau^2=0.50$		
m	n	RSS	$ERSS$	$Per-RSS$	RSS	$ERSS$	$Per-RSS$
5	2	0.1561	0.1557	0.1507	0.1608	0.1543	0.1566
	3	0.0821	0.0855	0.1007	0.0900	0.0875	0.1035
	4	0.0556	0.1075	0.0771	0.0589	0.1099	0.0799
	5	0.0392	0.0770	0.0593	0.0434	0.0816	0.0625
10	2	0.0740	0.0767	0.0760	0.0789	0.0768	0.0792
	3	0.0427	0.0415	0.0673	0.0442	0.0446	0.0718
	4	0.0286	0.0701	0.0572	0.0292	0.0726	0.0583
	5	0.0197	0.0552	0.0449	0.0224	0.0573	0.0460

Table 3.15 MSE values for $N(0,1)$ in CRSS

		$\rho_{xy} = 0.95$			$\rho_{xy} = 0.75$		
m	n	RSS	$ERSS$	$Per-RSS$	RSS	$ERSS$	$Per-RSS$
5	2	0.0733	0.0710	0.0728	0.0813	0.0827	0.0818
	3	0.0388	0.0391	0.0320	0.0477	0.0500	0.0456
	4	0.0245	0.0269	0.0215	0.0346	0.0361	0.0328
	5	0.0166	0.0200	0.0148	0.0263	0.0262	0.0244
10	2	0.0358	0.0362	0.0328	0.0406	0.0399	0.0403
	3	0.0186	0.0187	0.0166	0.0242	0.0237	0.0225
	4	0.0116	0.0132	0.0105	0.0166	0.0184	0.0158
	5	0.0085	0.0094	0.0073	0.0132	0.0131	0.0117
		$\rho_{xy} = 0.50$			$\rho_{xy} = 0.25$		
m	n	RSS	$ERSS$	$Per-RSS$	RSS	$ERSS$	$Per-RSS$
5	2	0.0917	0.0909	0.0914	0.1015	0.0963	0.0977
	3	0.0592	0.0591	0.0568	0.0646	0.0648	0.0630
	4	0.0434	0.0428	0.0413	0.0502	0.0488	0.0472
	5	0.0337	0.0333	0.0328	0.0386	0.0379	0.0372
10	2	0.0458	0.0462	0.0454	0.0501	0.0485	0.0495
	3	0.0296	0.0288	0.0287	0.0320	0.0322	0.0304
	4	0.0213	0.0222	0.0207	0.0245	0.0247	0.0239
	5	0.0166	0.0175	0.0163	0.0185	0.0186	0.0184

Table 3.16 MSE values for $U(0,1)$ in CRSS

		$\rho_{xy} = 0.95$			$\rho_{xy} = 0.75$		
m	n	RSS	$ERSS$	$Per-RSS$	RSS	$ERSS$	$Per-RSS$
5	2	0.0058	0.0060	0.0058	0.0070	0.0067	0.0069
	3	0.0031	0.0031	0.0036	0.0040	0.0040	0.0044
	4	0.0019	0.0016	0.0023	0.0028	0.0025	0.0030
	5	0.0013	0.0011	0.0015	0.0021	0.0019	0.0024
10	2	0.0029	0.0029	0.0029	0.0034	0.0034	0.0035
	3	0.0016	0.0015	0.0018	0.0020	0.0020	0.0022
	4	0.0010	0.0008	0.0011	0.0014	0.0013	0.0016
	5	0.0007	0.0006	0.0008	0.0011	0.0010	0.0012
		$\rho_{xy} = 0.50$			$\rho_{xy} = 0.25$		
m	n	RSS	$ERSS$	$Per-RSS$	RSS	$ERSS$	$Per-RSS$
5	2	0.0078	0.0074	0.0078	0.0083	0.0082	0.0079
	3	0.0049	0.0049	0.0051	0.0055	0.0054	0.0054
	4	0.0035	0.0034	0.0037	0.0040	0.0040	0.0039
	5	0.0029	0.0027	0.0029	0.0032	0.0032	0.0032
10	2	0.0038	0.0038	0.0038	0.0041	0.0040	0.0041
	3	0.0025	0.0025	0.0026	0.0027	0.0027	0.0027
	4	0.0018	0.0017	0.0018	0.0020	0.0020	0.0020
	5	0.0014	0.0014	0.0015	0.0016	0.0016	0.0017

Table 3.17 MSE values for $Exp(1)$ in CRSS

		$\rho_{xy}=0.95$			$\rho_{xy}=0.75$		
m	n	RSS	$ERSS$	$Per-RSS$	RSS	$ERSS$	$Per-RSS$
5	2	0.0748	0.0762	0.0754	0.0884	0.0852	0.0891
	3	0.0449	0.0412	0.0499	0.0521	0.0525	0.0510
	4	0.0280	0.0616	0.0402	0.0356	0.0540	0.0389
	5	0.0198	0.0450	0.0313	0.0276	0.0413	0.0293
10	2	0.0370	0.0385	0.0398	0.0425	0.0415	0.0423
	3	0.0211	0.0213	0.0359	0.0248	0.0257	0.0306
	4	0.0138	0.0421	0.0321	0.0180	0.0303	0.0219
	5	0.0101	0.0352	0.0245	0.0136	0.0255	0.0172
		$\rho_{xy}=0.50$			$\rho_{xy}=0.25$		
m	n	RSS	$ERSS$	$Per-RSS$	RSS	$ERSS$	$Per-RSS$
5	2	0.0921	0.0871	0.0885	0.0914	0.1017	0.0079
	3	0.0650	0.0614	0.0546	0.0656	0.0664	0.0054
	4	0.0442	0.0518	0.0429	0.0471	0.0487	0.0041
	5	0.0341	0.0364	0.0337	0.0413	0.0381	0.0032
10	2	0.0450	0.0480	0.0472	0.0472	0.0474	0.0040
	3	0.0302	0.0302	0.0288	0.0320	0.0330	0.0027
	4	0.0224	0.0246	0.0219	0.0248	0.0248	0.0020
	5	0.0180	0.0214	0.0173	0.0180	0.0206	0.0016

Table 3.18 MSE values for $\Gamma(2,1)$ in CRSS

		$\rho_{xy}=0.95$			$\rho_{xy}=0.75$		
m	n	RSS	$ERSS$	$Per-RSS$	RSS	$ERSS$	$Per-RSS$
5	2	0.1524	0.1483	0.1463	0.1625	0.1680	0.1628
	3	0.0842	0.0795	0.0868	0.0994	0.1033	0.0938
	4	0.0503	0.0886	0.0669	0.0663	0.0914	0.0711
	5	0.0373	0.0720	0.0495	0.0560	0.0680	0.0547
10	2	0.0728	0.0774	0.0736	0.0856	0.0834	0.0846
	3	0.0424	0.0413	0.0561	0.0484	0.0530	0.0513
	4	0.0282	0.0579	0.0415	0.0362	0.0508	0.0406
	5	0.0188	0.0452	0.0321	0.0270	0.0391	0.0340
		$\rho_{xy}=0.50$			$\rho_{xy}=0.25$		
m	n	RSS	$ERSS$	$Per-RSS$	RSS	$ERSS$	$Per-RSS$
5	2	0.1880	0.1814	0.1873	0.1981	0.1988	0.2012
	3	0.1141	0.1242	0.1104	0.1325	0.1242	0.1253
	4	0.0893	0.0916	0.0818	0.0964	0.0970	0.0943
	5	0.0667	0.0733	0.0645	0.0755	0.0776	0.0756
10	2	0.0903	0.0931	0.0971	0.0999	0.1010	0.0985
	3	0.0590	0.0599	0.0570	0.0711	0.0683	0.0654
	4	0.0438	0.0486	0.0420	0.0497	0.0480	0.0500
	5	0.0341	0.0387	0.0335	0.0377	0.0393	0.0381

Table 3.19 Bias values for $N(0,1)$ in CRSS

		$\rho_{xy}=0.95$				$\rho_{xy}=0.75$			
m	n	SRS	RSS	$ERSS$	$Per-RSS$	SRS	RSS	$ERSS$	$Per-RSS$
5	2	0.007	0.001	0.003	0.001	0.000	0.004	−0.004	0.000
	3	−0.001	0.000	0.001	−0.006	−0.005	0.001	−0.001	0.005
	4	0.001	0.002	−0.003	−0.003	−0.003	0.002	−0.001	0.000
	5	0.000	0.001	0.005	−0.002	−0.001	−0.002	0.004	0.000
10	2	0.001	0.000	−0.002	0.000	−0.004	0.003	0.001	−0.001
	3	−0.002	0.001	0.001	−0.003	0.001	0.003	−0.001	−0.002
	4	0.000	0.000	0.001	0.002	−0.003	0.000	−0.001	0.002
	5	0.000	0.001	0.003	−0.005	0.002	0.000	0.003	0.004
		$\rho_{xy}=0.50$				$\rho_{xy}=0.25$			
m	n	SRS	RSS	$ERSS$	$Per-RSS$	SRS	RSS	$ERSS$	$Per-RSS$
5	2	−0.003	−0.006	−0.005	0.003	−0.003	−0.001	0.001	0.002
	3	−0.002	−0.004	−0.003	−0.002	0.004	−0.002	0.002	−0.002
	4	−0.001	0.000	0.012	0.005	0.004	0.001	0.001	0.004
	5	−0.002	−0.001	0.000	−0.004	0.000	−0.002	0.005	−0.001
10	2	0.001	−0.003	−0.002	0.001	0.005	0.000	0.002	−0.007
	3	−0.001	0.000	0.001	−0.005	0.000	−0.003	0.000	−0.004
	4	0.001	0.001	0.003	0.001	0.003	0.000	0.000	0.001
	5	0.000	0.000	0.008	−0.002	−0.001	0.000	0.005	0.003

Table 3.20 Bias values for $U(0,1)$ in CRSS

		$\rho_{xy}=0.95$				$\rho_{xy}=0.75$			
m	n	SRS	RSS	$ERSS$	$Per-RSS$	SRS	RSS	$ERSS$	$Per-RSS$
5	2	0.002	−0.002	0.000	0.000	0.000	0.001	0.000	0.000
	3	0.001	0.000	−0.001	0.001	0.000	−0.001	−0.001	0.001
	4	0.000	0.000	0.001	−0.001	0.000	−0.001	−0.001	−0.001
	5	0.000	0.000	0.000	0.000	0.001	0.000	0.001	−0.001
10	2	−0.002	−0.001	0.000	0.000	0.000	0.000	0.000	0.000
	3	0.001	0.000	0.001	−0.001	0.000	0.000	0.000	0.001
	4	0.000	0.000	−0.001	0.000	0.000	0.000	0.000	0.002
	5	0.000	0.000	0.000	0.000	0.000	−0.001	0.000	0.000
		$\rho_{xy}=0.50$				$\rho_{xy}=0.25$			
m	n	SRS	RSS	$ERSS$	$Per-RSS$	SRS	RSS	$ERSS$	$Per-RSS$
5	2	0.001	−0.001	0.001	0.000	0.000	0.000	0.000	−0.001
	3	−0.001	0.001	0.000	0.000	0.000	0.000	0.000	−0.001
	4	0.001	0.000	−0.001	0.001	−0.001	−0.001	0.000	0.000
	5	0.000	0.001	0.000	−0.001	0.000	0.000	0.000	0.002
10	2	−0.001	0.000	0.000	−0.001	−0.001	0.000	−0.002	−0.001
	3	0.000	0.000	0.000	0.000	0.000	−0.001	0.000	−0.002
	4	0.001	0.000	−0.001	0.000	0.000	0.000	0.001	−0.001
	5	0.000	0.000	0.001	0.001	0.000	0.000	0.000	0.000

Table 3.21 Bias values for $Exp(1)$ in CRSS

		$\rho_{xy}=0.95$				$\rho_{xy}=0.75$			
m	n	SRS	RSS	$ERSS$	$Per-RSS$	SRS	RSS	$ERSS$	$Per-RSS$
5	2	−0.002	−0.002	0.001	0.001	0.001	0.001	0.001	−0.004
	3	−0.001	0.002	0.003	−0.152	0.007	0.002	0.002	−0.094
	4	−0.003	0.002	0.152	−0.149	−0.002	−0.003	0.094	−0.099
	5	0.000	0.000	0.134	−0.135	0.000	0.000	0.086	−0.081
10	2	−0.001	−0.003	−0.002	0.001	0.002	0.000	−0.001	0.003
	3	0.001	0.001	0.000	−0.148	−0.001	−0.002	0.002	−0.096
	4	0.001	0.000	0.151	−0.153	0.000	0.002	0.045	−0.088
	5	0.000	0.000	0.143	−0.133	0.001	0.000	0.029	−0.079
		$\rho_{xy}=0.50$				$\rho_{xy}=0.25$			
m	n	SRS	RSS	$ERSS$	$Per-RSS$	SRS	RSS	$ERSS$	$Per-RSS$
5	2	0.000	−0.001	−0.001	−0.002	0.000	−0.002	0.000	−0.001
	3	0.001	0.001	0.002	−0.046	0.003	−0.003	0.002	0.001
	4	−0.002	0.001	0.045	−0.039	0.002	−0.004	0.006	0.001
	5	−0.001	−0.001	0.029	−0.032	0.003	−0.001	0.012	0.002
10	2	0.003	−0.001	0.000	0.001	0.001	−0.002	0.003	0.000
	3	0.000	0.000	0.002	−0.045	−0.004	0.001	0.002	−0.002
	4	0.001	−0.001	0.040	−0.047	0.002	0.001	0.007	−0.002
	5	0.000	0.002	0.034	−0.038	0.003	−0.001	0.009	0.000

Table 3.22 Bias values for $\Gamma(2,1)$ in CRSS

		$\rho_{xy}=0.95$				$\rho_{xy}=0.75$			
m	n	SRS	RSS	$ERSS$	$Per-RSS$	SRS	RSS	$ERSS$	$Per-RSS$
5	2	0.000	0.001	0.001	0.005	−0.001	−0.003	0.001	−0.007
	3	−0.004	0.001	0.002	−0.163	−0.007	−0.001	0.000	−0.095
	4	−0.008	−0.002	0.156	−0.167	0.000	0.000	0.096	−0.093
	5	0.001	0.002	0.143	−0.145	−0.002	−0.003	0.087	−0.092
10	2	−0.005	0.001	0.001	0.002	0.001	0.001	−0.003	0.003
	3	−0.003	0.000	−0.004	−0.157	0.003	0.004	−0.003	−0.094
	4	0.000	−0.001	0.156	−0.151	0.004	−0.001	0.102	−0.101
	5	0.001	−0.001	0.146	−0.135	0.000	−0.001	0.090	−0.097
		$\rho_{xy}=0.50$				$\rho_{xy}=0.25$			
m	n	SRS	RSS	$ERSS$	$Per-RSS$	SRS	RSS	$ERSS$	$Per-RSS$
5	2	−0.008	0.000	0.012	0.002	0.006	0.202	0.010	0.005
	3	0.000	−0.005	0.004	−0.044	0.000	−0.001	−0.001	−0.015
	4	−0.002	0.003	0.043	−0.045	−0.004	−0.001	0.013	−0.013
	5	0.002	0.003	0.036	−0.032	0.000	0.003	0.007	−0.018
10	2	0.003	0.002	0.004	0.005	−0.002	−0.003	0.002	−0.001
	3	0.001	−0.002	0.000	−0.048	−0.001	0.000	−0.003	−0.013
	4	−0.001	0.000	0.043	−0.041	−0.002	−0.002	0.004	−0.016
	5	0.002	0.000	0.045	−0.032	0.000	0.003	0.014	−0.008

Table 3.23 Bias values for $N(0,1)$ in VRSS

		$\sigma_\tau^2 = 0.05$				$\sigma_\tau^2 = 0.15$			
m	n	SRS	RSS	$ERSS$	$Per-RSS$	SRS	RSS	$ERSS$	$Per-RSS$
5	2	0.000	0.000	0.004	0.002	−0.004	0.000	0.002	0.002
	3	0.000	0.003	0.000	0.001	0.000	0.001	0.001	0.001
	4	−0.001	0.001	−0.002	−0.001	0.002	−0.003	−0.001	0.000
	5	0.001	0.002	0.002	0.000	0.000	0.000	0.002	0.001
10	2	0.002	−0.001	0.002	−0.001	−0.003	−0.003	0.001	0.000
	3	−0.001	0.000	0.001	0.003	−0.002	0.000	0.000	0.001
	4	−0.003	−0.001	0.001	−0.001	0.000	0.000	0.001	0.002
	5	0.000	0.000	0.000	0.000	0.000	0.000	−0.001	0.002
		$\sigma_\tau^2 = 0.30$				$\sigma_\tau^2 = 0.50$			
m	n	SRS	RSS	$ERSS$	$Per-RSS$	SRS	RSS	$ERSS$	$Per-RSS$
5	2	0.003	0.004	0.003	−0.001	−0.002	0.000	−0.003	0.004
	3	0.001	−0.001	−0.002	0.003	−0.001	0.000	−0.001	−0.002
	4	−0.001	0.001	0.001	0.001	−0.001	0.002	−0.002	0.000
	5	−0.001	0.001	0.002	0.002	0.001	0.000	0.001	0.001
10	2	0.000	0.004	0.000	0.002	−0.001	−0.001	0.003	0.002
	3	0.000	−0.001	−0.001	0.001	0.000	−0.001	0.001	0.000
	4	−0.004	0.000	−0.001	0.003	0.000	0.000	−0.001	0.000
	5	−0.002	0.000	0.000	−0.001	−0.002	0.000	0.000	0.000

Table 3.24 Bias values for $U(0,1)$ in VRSS

		$\sigma_\tau^2 = 0.05$				$\sigma_\tau^2 = 0.15$			
m	n	SRS	RSS	$ERSS$	$Per-RSS$	SRS	RSS	$ERSS$	$Per-RSS$
5	2	0.002	−0.001	0.001	0.000	0.000	0.000	−0.002	0.000
	3	−0.002	0.000	0.000	−0.001	0.001	0.000	0.000	0.000
	4	−0.001	0.000	0.001	0.000	0.001	0.000	0.001	0.000
	5	−0.001	0.000	0.001	−0.001	−0.001	0.000	0.000	−0.001
10	2	0.000	0.000	0.000	−0.001	0.000	0.001	0.000	0.000
	3	0.001	0.002	0.000	−0.001	0.001	0.000	0.000	0.000
	4	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.000
	5	−0.001	0.000	0.000	0.000	−0.001	0.000	−0.001	0.000
		$\sigma_\tau^2 = 0.30$				$\sigma_\tau^2 = 0.50$			
m	n	SRS	RSS	$ERSS$	$Per-RSS$	SRS	RSS	$ERSS$	$Per-RSS$
5	2	0.000	0.000	0.001	−0.001	0.000	−0.001	0.000	−0.002
	3	−0.001	0.001	0.000	0.000	0.001	0.000	0.000	−0.001
	4	0.001	0.000	0.001	0.000	0.001	0.000	0.000	0.000
	5	0.000	0.000	−0.001	0.000	0.000	0.000	0.000	−0.001
10	2	0.000	0.000	0.001	0.000	0.001	0.001	0.001	0.000
	3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	4	0.000	−0.001	0.000	0.000	−0.001	0.000	−0.001	−0.001
	5	0.000	0.000	0.000	0.001	0.000	0.000	0.001	0.000

Table 3.25 Bias values for $Exp(1)$ in VRSS

		$\sigma_\tau^2 = 0.05$				$\sigma_\tau^2 = 0.15$			
m	n	SRS	RSS	$ERSS$	$Per-RSS$	SRS	RSS	$ERSS$	$Per-RSS$
5	2	0.004	−0.001	0.001	0.006	0.000	0.000	0.004	0.007
	3	−0.003	0.000	0.002	−0.178	−0.002	−0.001	−0.003	−0.178
	4	−0.002	0.004	0.176	−0.177	0.004	−0.003	0.179	−0.176
	5	0.000	0.000	0.157	−0.157	−0.001	−0.001	0.163	−0.161
10	2	0.001	−0.002	0.001	−0.003	0.001	−0.001	−0.002	−0.001
	3	−0.003	0.001	0.000	−0.173	−0.001	−0.001	−0.001	−0.179
	4	0.001	0.000	0.174	−0.174	0.002	0.000	0.176	−0.178
	5	−0.001	0.001	0.158	−0.157	0.000	0.000	0.159	−0.159
		$\sigma_\tau^2 = 0.30$				$\sigma_\tau^2 = 0.50$			
m	n	SRS	RSS	$ERSS$	$Per-RSS$	SRS	RSS	$ERSS$	$Per-RSS$
5	2	0.003	−0.002	−0.003	−0.006	0.002	0.004	0.001	−0.004
	3	0.003	0.001	−0.002	−0.171	0.002	0.002	0.003	−0.165
	4	0.005	−0.001	0.173	−0.172	−0.004	−0.002	0.166	−0.169
	5	0.001	0.002	0.155	−0.153	0.002	−0.001	0.146	−0.152
10	2	0.005	−0.001	−0.003	−0.001	−0.003	0.002	−0.003	0.001
	3	−0.001	−0.003	−0.001	−0.174	0.002	−0.001	0.001	−0.165
	4	0.000	0.002	0.173	−0.174	0.000	0.002	0.167	−0.165
	5	−0.001	0.000	0.155	−0.156	0.000	−0.001	0.150	−0.146

Table 3.26 Bias values for $\Gamma(2,1)$ in VRSS

		$\sigma_\tau^2 = 0.05$				$\sigma_\tau^2 = 0.15$			
m	n	SRS	RSS	$ERSS$	$Per-RSS$	SRS	RSS	$ERSS$	$Per-RSS$
5	2	−0.003	0.001	−0.008	−0.001	−0.006	−0.004	−0.001	0.008
	3	−0.004	0.002	0.004	−0.182	0.001	0.000	0.003	−0.189
	4	0.004	0.003	0.179	−0.177	0.001	0.000	0.191	−0.186
	5	0.003	−0.001	0.163	−0.162	0.002	0.002	0.171	−0.170
10	2	0.000	−0.001	0.000	−0.002	0.001	0.003	0.000	−0.005
	3	0.001	0.001	0.002	−0.184	0.000	0.003	−0.001	−0.187
	4	0.003	−0.001	0.179	−0.182	−0.001	0.002	0.191	−0.188
	5	0.002	−0.001	0.162	−0.164	−0.003	0.002	0.170	−0.170
		$\sigma_\tau^2 = 0.30$				$\sigma_\tau^2 = 0.50$			
m	n	SRS	RSS	$ERSS$	$Per-RSS$	SRS	RSS	$ERSS$	$Per-RSS$
5	2	0.003	−0.005	0.003	0.001	0.002	0.008	0.007	0.002
	3	0.000	−0.002	0.007	−0.191	0.005	−0.003	0.002	−0.195
	4	0.000	0.006	0.191	−0.193	−0.002	−0.003	0.189	−0.191
	5	0.000	−0.002	0.170	−0.171	0.004	0.003	0.172	−0.174
10	2	0.002	0.002	−0.002	−0.002	0.004	0.001	−0.001	0.002
	3	−0.003	0.000	−0.001	−0.190	0.000	0.001	0.002	−0.196
	3	0.001	0.001	0.002	−0.184	0.000	0.003	−0.001	−0.187
	4	0.001	0.000	0.189	−0.193	−0.003	−0.001	0.190	−0.191
	5	−0.002	0.002	0.174	−0.174	0.002	0.000	0.174	−0.172

The results of obtained from this simulation study are given below:

For VRSS;

- The relative efficiency increases with the increasing set size.
- The relative efficiency decreases with increasing variance of random error term (σ_τ^2).
- It is observed that RSS, ERSS and Per-RSS methods are more efficient than SRS for $N(0,1)$ and $U(0,1)$ based on MSE of mean estimators.
- For $Exp(1)$ and $\Gamma(2,1)$, RSS method is more efficient than SRS. On the other hand, SRS is more efficient than ERSS and Per-RSS methods for some set sizes (especially 4 and 5) based on MSE of mean estimators. As, Gamma and Exponential distributions are skewed distribution, bias values obtained them are greater than symmetric distributions.

For CRSS;

- The relative efficiency increases with increasing the set size.
- The relative efficiency increases with increasing correlation between the variable of interest and concomitant variable.
- It is observed that RSS, ERSS and Per-RSS methods are more efficient than SRS for $N(0,1)$ and $U(0,1)$ based on MSE of mean estimators.
- It is observed that RSS method is more efficient than SRS for $Exp(1)$ and $\Gamma(2,1)$. On the other hand, SRS is more efficient than ERSS and Per-RSS methods for some set sizes (especially 4 and 5) based on MSE of mean estimators for $Exp(1)$ and $\Gamma(2,1)$. Gamma and Exponential distributions are skewed distributions, bias values obtained from them are greater than symmetric distributions.

According to the distributions used in the simulation study and the results obtained from the sampling methods;

- In both visual and concomitant based ranking methods MSE decreases when set size and number of cycles increase.
- MSE increases as the variance of the error term increases in visual ranking and as the correlation between the concomitant variable and the variable of intereste decreases in concomitant based ranking.
- MSE obtained from skewed distributions such as $Exp(1)$ and $\Gamma(2,1)$ are larger than the MSE obtained from the symmetric distributions such as $N(0,1)$ and $U(0,1)$.
- When visual ranking is used, observations from $N(0,1)$, $U(0,1)$ and $Exp(1)$ give the best results of relative efficiency in Per-RSS, ERSS and RSS methods, respectively. Additionally, $\Gamma(2,1)$ yielded the best results in RSS.
- When concomitant based ranking is used, observations from $N(0,1)$, $U(0,1)$ and $Exp(1)$ give the best results of relative efficiency in Per-RSS, ERSS and RSS, respectively. Additionally, observations from $\Gamma(2,1)$ yielded the best results of relative efficiency in Per-RSS.
- Change in the number of cycles does not result in a consistent increase or decrease in relative efficiency for VRSS and CRSS. For this reason, exact comment can not be made about the effect of number of cycles on relative efficiency.

CHAPTER FOUR
COST MODELS AND OPTIMAL SET SIZE IN RSS

Many authors who are interested in RSS reported in their studies that the cost efficiency of RSS depends on the relative cost of sampling, ranking and measuring of the variable of interest. In general opinion, RSS method is very useful when the sampling of the units is easy while their measurement based on the variable of interest is difficult and expensive. On the other hand, terms of ”expensive” and ”difficult” can be expressed as general term, ”cost”. Therefore, as the difficulty in measuring units increases, the cost will also increase. In RSS, in a set of size n, only one of the n units which are sampled considering the variable of interest is measured. The efficiency of the estimator obtained by RSS depends on the information based on ranking of the remaining unmeasured $n-1$ units in the set. For visual ranking, this information depends on the cost for sampling of the n units and for obtaining the next ranking. For concomitant variable based ranking, it depends on the measurement of all units of X with respect to concomitant variable (Y). This situation causes the cost to increase even more. This chapter of the thesis includes an examination over the existing cost models in the RSS literature together with the factors affecting the optimal set size. Also, for infinite population, by using different ranking error models for the N-KPST cost model which proposed by Nahhas et al. (2002), in order to see whether RSS is cost effective with respect to SRS in terms of MSE of the mean estimator and if so, to determine the optimal set size for RSS.

4.1 Existing Cost Models in RSS

4.1.1 Dell and Clutter Cost Model

This cost model is proposed by Dell & Clutter (1972). It provides to determine the relative cost efficiency of RSS compared to SRS. This cost model is a basic cost model and includes two cost categories. These categories are defined as follows;

c_s : Cost of obtaining a unit to be measured in RSS (This includes sampling of units and ranking n units)
c_q : Cost of measurement for a X unit.

As a result, the relative cost efficiency of RSS according to SRS in terms of the MSE of the mean estimator is given as;

$$RCE(MSE_{\bar{X}_{RSS}}, MSE_{\bar{X}_{SRS}}) = \frac{c_q}{c_q + c_s}\left(\frac{MSE_{\bar{X}_{SRS}}}{MSE_{\bar{X}_{RSS}}}\right) \tag{4.1}$$

For RSS to be cost effective compared to SRS based on constant cost, the value of $RE(MSE_{\bar{X}_{RSS}}, MSE_{\bar{X}_{SRS}})$ must be greater than $1 + \frac{c_q}{c_s}$.

4.1.2 KPST Cost Model

This cost model was proposed by Kaur et al. (1996). Compared to Dell & Clutter (1972) cost model, this includes more cost categories. The purpose of developing this cost model is to compare the relative cost efficiency of RSS according to SSRS. Cost categories for the KPST cost model are defined as follows;
c_R : Total cost for RSS
c_S : Total cost for SSRS
c_{OR} : Setup cost for RSS
c_{OS} : Setup cost for SSRS
c_i : Cost of sampling for a unit
c_q : Cost of measurement of a unit
c_r : Cost of ranking units in a set.

Total cost for RSS and SSRS is given as follows;

$$c_R = c_{OR} + nm(nc_i + f(n)c_r + c_q) \tag{4.2}$$

$$c_s = c_{OS} + nm^{'}(c_i + c_q) \tag{4.3}$$

where

n : The set size for RSS and the strata size for SSRS

m : The cycle size for RSS

m' : The number of the units in each strata for SSRS

$f(n)$: The number of rankings required to completely rank the observations made in a set with size n.

Since, Kaur et al. (1996) assumed c_r to be approximately 0, the calculation of $f(n)$ is not necessary. Following this simplifying assumption, the relative cost efficiency of RSS compared to SSRS in terms of MSE of the mean estimator for a constant cost $(c = c_R = c_S)$ is given as;

$$RCE(MSE_{\bar{X}_{RSS}}, MSE_{\bar{X}_{SSRS}}) = \frac{(c - c_{OS})(nc_i + c_q)}{(c - c_{OR})(c_i + c_q)} \left(\frac{MSE_{\bar{X}_{SSRS}}}{MSE_{\bar{X}_{RSS}}} \right) \tag{4.4}$$

4.1.3 N-KPST Cost Model

In KPST model, the cost of ranking units in a set (c_r) is considered to be approximately zero. However, it may not be negligible. Also, for simplicity, it is preferred to combine the overall cost of sampling with other cost parameters in the model. This part contains information about the N-KPST cost model, which is a revised version of the KPST cost model. This cost model is very similar to the KPST cost model. This model was proposed by Nahhas et al. (2002).

Suppose that X is the variable of interest and Y is the concomitant variable. The cost categories for the N-KPST cost model are as follows:

c_o : General cost for SRS

c_i : Cost of sampling of a unit

c_{qx} : Cost of measurement of X for a unit

c^*_{qy} : Cost of measurement of Y for a unit

c^*_r : Cost of a single pairwise comparison

c_{OR} : General cost for RSS, in addition to c_o

c_{SRS} : Total cost of SRS

c_{CRSS} : Total cost of CRSS

c_{VRSS} : Total cost of VRSS

$f(n)$: The expected value of the paired comparisons required to do ranking in a set with size n.

In N-KPST cost model, the total cost of SRS includes sampling and measurement of N units of the variable of interest X. For CRSS, it includes the sampling of n units, measurement of each unit with respect to the concomitant variable, Y ranking the units in the set with size n based on these measurements and also the measurement of X for a single unit. On the other hand, for VRSS, it includes the sampling and ranking of n units, together with the measurement of X for a unit, for each set. For CRSS and VRSS, the number of sets and cycles does not need to be equal. Furthermore, the total number of measured units in CRSS and VRSS is not necessarily equal to the sample size in the SRS. This is due to the reason that the comparisons made here are based on a constant cost, rather than the measurement of constant X as the variable of interest. The expected costs for SRS, CRSS and VRSS in N-KPST cost model are calculated respectively as follows;

$$c_{SRS} = c_o + N(c_i + c_{qx}) \tag{4.5}$$

$$c_{VRSS} = c_{OR} + nm(nc_i + f(n)c_r^* + c_{qx}) \tag{4.6}$$

$$c_{CRSS} = c_{OR} + nm(nc_i + nc_{qy} + f(n)c_r^* + c_{qx}) \tag{4.7}$$

In N-KPST cost model, general cost for SRS is considered to be zero (i.e $c_o = 0$) without any loss of generality. Also, in RSS, when a concomitant variable is used for the ranking of units, general cost for RSS c_{OR} is assumed to be absorbed in c_{qy}^*. In other words, in this cost model, measurement cost for a unit of concomitant variable Y

is determined using the following equation:

$$c_{qy} = c^*_{qy} + \frac{c_{OR}}{n^2 m} \tag{4.8}$$

In N-KPST cost model, for visual ranking, general cost for SRS is assumed to be zero, similar to the concomitant based ranking model. Also, when the visual ranking method is used for the ranking of units in RSS, General cost for RSS c_{OR} is assumed to be absorbed in c^*_r. In other words, in this cost model, cost for a paired comparison carried out to rank the units using visual ranking method is determined using the following equation:

$$c_r = c^*_r + \frac{c_{OR}}{f(n)nm} \tag{4.9}$$

Besides, it must be noted that both the cost for measurement of Y in concomitant based ranking method and the cost for paired comparison in visual ranking method exceed c_o. Therefore, overall cost for the RSS which is well beyond the SRS includes all the additional steps in RSS such as the measurement of Y or visual ranking of X.

In KPST cost model, paired comparisons carried out for the rankings based on concomitant variable, include actual measurements of concomitant variable which are assumed to require no additional cost ($c^*_r = 0$). The general costs in the ranking process are included by c_{qy}. The cost of a paired comparison for visual ranking is not zero and it is defined as the cost per comparison rate determined by the researcher. Even if the researcher does not charge this cost, it can be determined in relation to the time spent to rank the units. In most of the studies focusing on medicine and biology that include human as object (unit), all units can not be determined at the same time. In this case, ranking cost may also include the cost necessary for the combination of a set of n units, for visual ranking. These costs will be defined under c_r.

Visual ranking may not be considered as a completely random ranking since the order of the selection of units depends on the knowledge and experience of researcher as well as the materials used in the ranking. But, this is against the ranking strategy mentioned before. This is because the fact that, when a researcher examines the units and selects the one to be ranked first, that means he already made paired comparison and therefore visual ranking. In the strategy of comparing a unit with the unit already ranked as the largest, it is assumed that the rank of unit selection has a uniform distribution over the set which contains all possible rankings. That is to say, the expected value of the paired comparisons needed to rank a set of size n is given as;

$$f(n) \approx \frac{(n-1) + \binom{n}{2}}{2} = \frac{(n+2)(n-1)}{4} \tag{4.10}$$

Actually, it must be noted that the determination of the number of paired comparisons this way is a conservative approach. Ranking algorithms working out with fewer paired comparisons are presented in the literature. However, when the capabilities of human researchers are considered rather than algorithms which work with ideal mechanisms of computation, this conservative approach seems to be more reasonable. Because, this approach determines a conservative upper limit for optimal set size and therefore provides prevention of the effect of imperfect rankings which can emanate from the less conservative set sizes. Suppose a fixed cost condition in order to compare the sampling methods ($c = c_{SRS} = c_{CRSS} = c_{VRSS}$). In this case, relative cost efficiency of CRSS with respect to SRS in terms of MSE of the mean estimator are given as ;

$$RCE(MSE_{\bar{X}_{CRSS}}, MSE_{\bar{X}_{SRS}}) = \frac{c_i + c_{qx}}{nc_i + nc_{qy} + c_{qx}} \left(\frac{MSE_{\bar{X}_{SRS}}}{MSE_{\bar{X}_{CRSS}}} \right) \tag{4.11}$$

On the other hand, relative cost efficiency of VRSS with respect to SRS in terms of MSE of the mean estimator is given as;

$$RCE(MSE_{\bar{X}_{VRSS}}, MSE_{\bar{X}_{SRS}}) = \frac{c_i + c_{qx}}{nc_i + f(n)c_r + c_{qx}} \left(\frac{MSE_{\bar{X}_{SRS}}}{MSE_{\bar{X}_{VRSS}}} \right) \tag{4.12}$$

For each of the relative efficiency in these equations, dependency on the related set size should not be ignored.

4.2 Optimal Set Size in RSS

In RSS, there are several factors influencing the determination of optimal set size under the preset cost conditions, also affecting the preferability of RSS against SRS. These factors may vary depending on the preset conditions of cost and the ranking method used to rank the units in the set. In RSS, when visual ranking is used to rank the units in the set, the accuracy of the visual ranking, that is, the probability of error in ranking, directly affects the optimal set size. Interactions between the sampling cost, measurement cost of the variable of interest X and the cost of ranking units in the set affect the optimal set size as well. On the other hand, when concomitant based ranking is used to rank the units in the set, the optimal set size is directly affected by the correlation between the variable of interest and the concomitant variable. In addition, interactions between the sampling cost, measurement cost of the variable of interest X and the concomitant variable Y affect optimal set size as well. For SRS, the optimal set size is the maximum size which is tolerated by the cost. In other words, the optimal set size for SRS is;

$$N = \frac{c}{c_i + c_q} \tag{4.13}$$

where c is a fixed cost. On the other hand, for VRSS, the optimal set size is the n value that maximizes $RCE(\bar{X}_{VRSS}, \bar{X}_{SRS})$, just like the way in CRSS. Namely for CRSS, the optimal set size is the n value that maximizes $RCE(\bar{X}_{CRSS}, \bar{X}_{SRS})$.

For RSS, increasing probability of ranking errors (p), when ranking units in a set using the visual ranking method will reduce the optimal set size and decrease relative cost efficiency. The accuracy of visual ranking used to rank the units in a set in RSS is based on the varaince of the random error term σ_τ^2. On the other hand, σ_τ^2 varies due to the researcher's knowledge and experience and materials which are used in study. In other words, the high accuracy of visual ranking will result in high optimal set size. For RSS, when concomitant variable is used to rank units in a set, the accuracy of the ranking depends on the correlation between the variable of interest and the concomitant variable. The p will reduce as the coefficient of this correlation approaches 1. This will result in an increase in the optimal set size and also in relative cost efficiency with respect to SRS.

Without the loss of generality, the changes in the cost ratios given below cause a change in the relative cost efficiency of RSS with respect to SRS. Owing to this change, there will be a change in the optimal set size for RSS as well (Nahhas et al. (2002)).

$$\alpha_0 = \frac{c_{qy}}{c_i} \tag{4.14}$$

$$\alpha_1 = \frac{c_{qx}}{c_i} \tag{4.15}$$

$$\alpha_2 = \frac{c_r}{c_i} \tag{4.16}$$

where c_r is obtained by dividing the cost of ranking unit in set by $f(n)$.

4.3 Simulation Results

The main purpose of the simulation study is carried out in this section is to use the N-KPST cost model and ranking error models (VRSS and CRSS) in order to see whether RSS is cost efficient with respect to SRS in terms of MSE of the mean estimator and if so, to determine the optimal set size for RSS for infinite population. For this simulation study, R statistical programming language is used with 10000 repetitions. Units belonging to all cost categories values are in dollar. In this simulation study;

- The population of interest is generated from $N(0,1)$, $U(0,1)$ and $Exp(1)$ and $\Gamma(2,1)$, with size 10000.
- Set sizes (n) are determined to be 2, 3, 4, 5, 10 ,15, 20 and 25. Also cycle size (m) is determined to be 1.
- For VRSS and CRSS, the p's are determined to be 0, 0.05, 0.10, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50. σ_τ term corresponding to the p are given in Table 3.1, where the visual ranking method is used to rank the units in the set, in RSS.

On the other hand, the ρ_{xy} corresponding to the p are shown in Table 3.2, where a concomitant variable is used to rank the units in the set, in RSS.

- For N-KPST cost model
  - Cost of sampling of a unit (c_i) is determined to be 2.5 and 5.
  - Cost of measurement of X for a unit (c_{qx}) is determined to be 50 and 100.
  - For CRSS, cost of measurement of Y for a unit (c_{qy}) is determined as 5, 10 and 20.
  - Based on these cost values, α_1 is determined as 10, 20, 40 respectively. Also α_0 is determined as 1, 2, 4 and 8 respectively.
  - For VRSS the cost of ranking of unit in the set is determined to be 1.25 and 2.5 respectively. In this simulation study, α_2 values which vary depending on the ranking cost of units in the set and the number of paired comparisons $(f(n))$ required to rank n units in a set are shown in tables from 4.1 to 4.4.

Table 4.1 α_2 values when cost of ranking a unit in set is 1.25 and c_i = 2.5

n	$f(n)$	$cost\ of\ ranking\ ranking\ a\ unit\ in\ set$	$c_r = \frac{cost\ of\ ranking\ ranking\ a\ unit\ in\ set}{f(n)}$	$\alpha_2 = \frac{c_r}{c_i}$
2	1	2.5	2.5	1
3	2.5	3.75	1.5	0.6
4	4.5	5	1.11	0.44
5	7	6.25	0.89	0.36
10	27	12.5	0.46	0.18
15	59.5	18.75	0.31	0.12
20	104.5	25	0.24	0.10
25	162	31.25	0.19	0.08

Table 4.2 α_2 values when cost of ranking a unit in set is 1.25 and c_i = 5

n	$f(n)$	$cost\ of\ ranking\ ranking\ a\ unit\ in\ set$	$c_r = \frac{cost\ of\ ranking\ ranking\ a\ unit\ in\ set}{f(n)}$	$\alpha_2 = \frac{c_r}{c_i}$
2	1	2.5	2.5	0.5
3	2.5	3.75	1.5	0.3
4	4.5	5	1.11	0.22
5	7	6.25	0.89	0.18
10	27	12.5	0.46	0.09
15	59.5	18.75	0.31	0.06
20	104.5	25	0.24	0.05
25	162	31.25	0.19	0.04

Table 4.3 α_2 values when cost of ranking a unit in set is 2.5 and c_i = 2.5

n	$f(n)$	$cost\ of\ ranking\ ranking\ a\ unit\ in\ set$	$c_r = \frac{cost\ of\ ranking\ ranking\ a\ unit\ in\ set}{f(n)}$	$\alpha_2 = \frac{c_r}{c_i}$
2	1	2.5	5	2
3	2.5	7.5	3	1.20
4	4.5	10	1.82	0.88
5	7	12.50	1.79	0.72
10	27	25	0.93	0.36
15	59.5	37.50	0.65	0.24
20	104.5	50	0.48	0.20
25	162	62.50	0.38	0.16

Table 4.4 α_2 values when cost of ranking a unit in set is 2.5 and c_i = 5

n	$f(n)$	$cost\ of\ ranking\ ranking\ a\ unit\ in\ set$	$c_r = \frac{cost\ of\ ranking\ ranking\ a\ unit\ in\ set}{f(n)}$	$\alpha_2 = \frac{c_r}{c_i}$
2	1	2.5	5	1
3	2.5	7.5	3	0.60
4	4.5	10	1.82	0.44
5	7	12.50	1.79	0.36
10	27	25	0.93	0.18
15	59.5	37.50	0.65	0.12
20	104.5	50	0.48	0.10
25	162	62.50	0.38	0.08

For CRSS and VRSS, RCE values of RSS with respect to SRS are computed by Equation (4.15) and (4.17) respectively. RCE values obtained from simulation study are greater than 1 means that RSS is cost efficient than SRS under the constant cost conditions. (In addition, the results obtained for different values of cost categories of N-KPST cost model are shown in the tables in the Appendix.)

Table 4.5 RCE values for $N(0,1)$ when c_{qx} = 100, c_i = 2.5, α_1 = 40 and cost of ranking a unit in set is 1.25 by using VRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.443	1.342	1.325	1.273	1.235	1.095	1.070	0.993	0.967	0.954	0.958
3	1.731	1.712	1.598	1.476	1.363	1.202	1.107	**1.030**	0.960	0.922	0.908
4	2.058	1.980	1.819	1.671	1.427	1.224	**1.108**	0.985	0.961	0.915	0.870
5	2.349	2.343	2.085	1.782	1.437	**1.290**	1.091	0.962	0.909	0.871	0.802
10	3.291	3.289	2.555	1.983	**1.545**	1.249	1.056	0.896	0.805	0.804	0.739
15	4.045	3.957	2.861	**2.056**	1.408	1.136	0.938	0.793	0.720	0.651	0.672
20	4.420	4.351	**3.042**	1.987	1.388	1.008	0.837	0.729	0.646	0.617	0.604
25	**4.735**	**4.403**	2.940	1.867	1.318	0.930	0.772	0.642	0.566	0.536	0.535

Table 4.6 RCE values for $U(0,1)$ when c_{qx} = 100, c_i = 2.5, α_1 = 40 and cost of ranking a unit in set is 1.25 by using VRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.411	1.364	1.382	1.298	1.226	1.156	1.075	**1.055**	0.963	0.995	0.959
3	1.840	1.789	1.689	1.508	1.386	1.250	**1.139**	1.002	0.936	0.959	0.898
4	2.250	2.128	1.901	1.727	1.513	1.294	1.127	1.006	0.970	0.891	0.897
5	2.551	2.472	2.238	1.863	1.533	**1.333**	1.151	1.032	0.910	0.888	0.880
10	4.193	3.786	2.997	2.194	**1.658**	1.279	1.050	0.929	0.807	0.740	0.742
15	5.192	4.521	3.308	**2.262**	1.560	1.159	0.961	0.800	0.725	0.673	0.641
20	6.146	5.103	3.322	2.195	1.513	1.105	0.867	0.698	0.639	0.614	0.562
25	**6.808**	**5.458**	**3.548**	2.006	1.404	1.022	0.775	0.650	0.591	0.535	0.521

Table 4.7 RCE values for $Exp(1)$ when c_{qx} = 100, c_i = 2.5, α_1 = 40 and cost of ranking a unit in set is 1.25 by using VRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.251	1.277	1.307	1.158	1.168	1.060	1.086	**1.065**	0.981	0.924	0.946
3	1.525	1.491	1.420	1.340	1.187	1.100	1.071	1.016	0.928	0.914	0.952
4	1.679	1.605	1.550	1.433	1.297	1.191	**1.133**	1.034	0.929	0.919	0.836
5	1.884	1.778	1.718	1.451	1.364	**1.275**	1.090	0.984	0.910	0.891	0.861
10	2.655	2.326	2.074	1.734	**1.417**	1.187	1.011	0.890	0.804	0.787	0.786
15	2.992	2.796	2.195	1.743	1.412	1.123	0.923	0.801	0.726	0.678	0.662
20	3.253	2.847	2.302	**1.750**	1.413	1.066	0.900	0.743	0.632	0.603	0.602
25	**3.512**	**2.984**	**2.343**	1.671	1.282	0.988	0.825	0.625	0.574	0.536	0.493

Table 4.8 RCE values for $\Gamma(2,1)$ when c_{qx} = 100, c_i = 2.5, α_1 = 40 and cost of ranking a unit in set is 1.25 by using VRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.328	1.313	1.258	1.202	1.158	1.174	1.020	1.004	0.981	0.953	0.944
3	1.588	1.571	1.445	1.353	1.311	1.175	**1.141**	1.028	0.977	0.936	0.912
4	1.911	1.842	1.677	1.504	1.346	1.183	1.109	**1.048**	0.974	0.899	0.876
5	2.096	2.039	1.774	1.625	1.344	**1.242**	1.084	0.993	0.926	0.869	0.862
10	2.883	2.691	2.304	1.800	**1.542**	1.231	1.030	0.911	0.803	0.754	0.733
15	3.603	3.043	2.418	**1.900**	1.446	1.105	0.954	0.805	0.738	0.667	0.659
20	3.970	3.338	**2.523**	1.821	1.357	1.032	0.841	0.705	0.628	0.627	0.575
25	**4.180**	**3.680**	2.494	1.766	1.327	0.991	0.780	0.652	0.588	0.535	0.510

Table 4.9 RCE values for $N(0,1)$ when c_{qx} = 100, c_{qy}= 5, c_i = 2.5, α_1 = 40 and α_0 = 2 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.311	1.301	1.213	1.207	1.160	1.061	**1.032**	0.964	0.909	0.898	0.890
3	1.590	1.568	1.475	1.346	1.217	1.097	0.999	0.892	0.914	0.860	0.833
4	1.809	1.826	1.592	1.455	1.270	**1.124**	0.992	0.891	0.856	0.838	0.753
5	2.104	1.977	1.777	1.534	**1.350**	1.071	0.970	0.872	0.793	0.750	0.739
10	2.742	2.565	2.061	**1.579**	1.261	1.006	0.806	0.734	0.639	0.587	0.592
15	3.364	2.853	**2.156**	1.471	1.086	0.810	0.683	0.607	0.518	0.512	0.476
20	3.439	3.078	2.052	1.371	0.982	0.747	0.579	0.486	0.441	0.422	0.401
25	**3.760**	**3.151**	1.989	1.278	0.876	0.636	0.527	0.429	0.392	0.368	0.341

Table 4.10 RCE values for $U(0,1)$ when c_{qx} = 100, c_{qy} = 5, c_i = 2.5, α_1 = 40 and α_0 = 2 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.336	1.323	1.275	1.213	1.140	1.050	**1.022**	0.978	0.940	0.878	0.902
3	1.675	1.629	1.503	1.351	1.201	1.102	0.983	0.922	0.869	0.831	0.826
4	1.980	1.908	1.723	1.494	**1.296**	**1.130**	1.000	0.910	0.859	0.791	0.797
5	2.165	2.059	1.811	1.534	1.283	1.104	0.941	0.837	0.791	0.772	0.757
10	3.237	2.907	2.241	**1.642**	1.264	0.959	0.782	0.698	0.641	0.613	0.589
15	3.832	3.207	**2.232**	1.532	1.087	0.824	0.665	0.580	0.530	0.492	0.464
20	4.379	**3.568**	2.174	1.391	0.976	0.728	0.597	0.504	0.458	0.416	0.406
25	**4.645**	3.545	2.015	1.234	0.840	0.646	0.507	0.422	0.385	0.371	0.354

Table 4.11 RCE values for $Exp(1)$ when c_{qx} = 100, c_{qy} = 5, c_i = 2.5, α_1 = 40 and α_0 = 2 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	0.971	1.027	0.961	0.932	0.795	0.776	0.689	0.641	0.637	0.615	0.609
3	1.328	1.330	1.301	1.169	1.136	0.984	0.979	0.956	0.844	0.814	0.836
4	1.497	1.533	1.377	1.318	**1.135**	**1.005**	0.960	0.865	0.825	0.803	0.783
5	1.686	1.582	1.446	1.305	1.131	0.965	0.896	0.800	0.739	0.753	0.732
10	2.006	1.907	**1.641**	**1.311**	1.035	0.857	0.767	0.679	0.635	0.610	0.579
15	2.128	2.083	1.587	1.290	0.945	0.764	0.634	0.540	0.531	0.503	0.500
20	**2.288**	2.015	1.554	1.091	0.846	0.672	0.562	0.476	0.427	0.426	0.417
25	2.247	**2.091**	1.506	1.045	0.750	0.568	0.488	0.405	0.374	0.384	0.351

Table 4.12 RCE values for $\Gamma(2,1)$ when c_{qx} = 100, c_{qy} = 5, c_i = 2.5, α_1 = 40 and α_0 = 2 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.259	1.298	1.171	1.120	1.104	1.050	**1.007**	0.920	0.898	0.895	0.875
3	1.487	1.460	1.353	1.278	1.137	0.981	0.940	0.911	0.897	0.849	0.845
4	1.656	1.579	1.475	1.369	**1.212**	1.049	0.951	0.877	0.851	0.828	0.769
5	1.748	1.716	1.623	**1.422**	1.181	**1.076**	0.950	0.833	0.763	0.761	0.714
10	2.333	2.168	1.753	1.366	1.143	0.865	0.782	0.686	0.616	0.594	0.577
15	2.468	2.390	**1.781**	1.342	0.965	0.772	0.649	0.575	0.509	0.487	0.480
20	2.784	**2.488**	1.716	1.198	0.842	0.678	0.558	0.487	0.432	0.428	0.403
25	**2.846**	2.458	1.631	1.106	0.803	0.624	0.498	0.422	0.382	0.361	0.353

Based on the results obtained from the simulation study, the following conclusions can be reached for the N-KPST cost model for RSS by Nahhas et al. (2002):

- When VRSS is used to rank the units in the set, larger σ_τ will lead to higher p. As a result, the relative cost efficiency of RSS will be lower according to SRS. For this reason, the optimal set size for RSS will also decrease.
- When CRSS is used to rank the units in a set, lower degrees of ρ_{xy} will lead to higher p. As a result, the relative cost efficiency of RSS will be lower according to SRS. For this reason, the optimal set size for RSS will also decrease.
- When concomitant variable is used to rank the units in a set, decrease in cost ratio which is defined as α_0, indicates RSS is cost efficient against SRS. This cost ratio increases with the decreasing cost of measurement for Y (c_{qy}) or sampling cost for a unit (c_i).
- When visual judgement is used to rank the units in a set, increase in cost ratio which is defined as α_1, indicates RSS is cost efficient against SRS. This cost ratio increases with the increasing cost of measurement for the variable of interest X (c_{qx}) or the decreasing cost of sampling of a unit (c_i).
- When visual judgement is used to rank the units in a set, decrease in cost ratio which is defined as α_2, indicates RSS is cost efficient against SRS. This cost ratio decreases with the increasing number of sets and sampling cost of a unit. It also decreases with the decreasing ranking cost of a unit in a set and cost of a single pairwise comparison (c_r).
- For $N(0,1)$, $U(0,1)$, $Exp(1)$ and $\Gamma(2,1)$ RSS is cost effective than SRS when c_{qx} = 100, c_i = 2.5 and cost of ranking a unit in the set is 1.25 even with $p = 0.35$ by using VRSS.
- For $N(0,1)$, $U(0,1)$ and $\Gamma(2,1)$ RSS is cost effective than SRS when c_{qx} = 100, c_i = 2.5 and c_{qy} = 5 even with $p = 0.30$ by using CRSS. Also for $Exp(1)$ RSS is cost effective than SRS when c_{qx} = 100, c_i = 2.5 and c_{qy} = 5 even with $p = 0.25$ by using CRSS.

CHAPTER FIVE
REAL LIFE DATA APPLICATION

5.1 About Real Life Data

Abalone is a common name for any of a group of small to very large sea snails, marine gastropod molluscs in the family Haliotidae. Age of an abalone can be determined by making some physical measurements which, in advance, include cutting and staining of the shell. After the staining process, the rings become clear and they are counted under a microscope to obtain age information. This process is tedious and time-consuming which pushes someone to prefer alternative measurement methods.

This data set (it can be found at https://archive.ics.uci.edu/ml/datasets/abalone) is taken from a machine learning repository that is generated by the University of California, in 2019 (Dua & Graff (2017)). It includes 4177 samples with 9 variables. Information about these variables are given in the table below:

Table 5.1 Descriptions of Abalone dataset

Variable	Data Type	Measurament Unit of Data	Description
Length	Continuous	mm	Longest shell measurement
Diameter	Continuous	mm	Perpendicular to length
Height	Continuous	mm	With meat in shell
Whole weight	Continuous	gr	Whole abalone
Shucked weight	Continuous	gr	Weight of meat
Viscera weight	Continuous	gr	Gut weight (after bleeding)
Shell weight	Continuous	gr	After being died
Rings	Integer	-	+1.5 gives the age in years
Sex	Nominal	-	Male,Female and Infant

Rings variable is selected as the interested variable. For concomitant based ranking, the linear relationships between the variable of interest and other variables are given in the table below together with the ρ corresponding to the p.

Table 5.2 ρ values between variable of interest and concomitant variables in Abalone dataset for CRSS

Variable of interest	Concomitant Variable(s)	ρ_{xy}	p
	Shell weight	0.628	0.284
	Diameter	0.575	0.305
	Height	0.559	0.311
Rings	Length	0.557	0.312
	Whole weight	0.536	0.320
	Viscera weight	0.504	0.332
	Shucked weight	0.420	0.362

Also, p values corresponding to the σ_τ are calculated for visual ranking by using Equation (3.6). The results are given in table below :

Table 5.3 σ_τ^2 and σ_τ corresponding to p in Ablone dataset for VRSS

p	0.284	0.305	0.311	0.312	0.32	0.332	0.362
σ_τ	3.998	4.587	4.776	4.809	5.080	5.531	6.965
σ_τ^2	15.984	21.040	22.810	23.126	25.806	30.591	48.511

5.2 Cost Categories Values for Real Life Data

Suppose, for rings variable, the cost of measurement cost a unit is determined as 100 dollars (i.e $c_{qx} = 100$). The cost of measuring is determined as 2 dollars for all concomitant variables (i.e $c_{qy} = 2$). Also the cost of sampling a unit (c_i) is determined as 1 dollar. For visual ranking, the cost of ranking a unit is determined as 1.25 dollars.

5.3 Real Life Data Results

Using the above mentioned information about the abalone data set, a simulation study with 10000 repetitions is performed using R statistical programming language. Set size (n) is determined as 2, 3, 4 , 5, 10, 15, 20 and 25. Also cycle size (m) is determined as 1. This data set is used to determine whether RSS is cost effective with respect to SRS in terms of MSE of the mean estimator and if so, to determine which optimal set size would be the most appropriate. In addition, MSE and bias values are calculated to see if RSS is more efficient than SRS. The results are shown in table below:

Table 5.4 RE values when VRSS is used

p	0.284	0.305	0.311	0.312	0.320	0.332	0.362
σ_τ^2	15.984	21.040	22.810	23.126	25.806	30.591	48.511
n	$Relative\ Efficiency$						
2	1.169	1.079	1.059	1.120	1.070	1.073	1.074
3	1.215	1.141	1.125	1.148	1.106	1.111	1.081
4	1.281	1.182	1.229	1.195	1.183	1.161	1.080
5	1.326	1.223	1.204	1.208	1.200	1.166	1.109
10	1.463	1.289	1.283	1.278	1.277	1.225	1.131
15	1.466	1.393	1.377	1.342	1.291	1.269	1.141
20	1.528	1.434	1.389	1.362	1.342	1.260	1.173
25	1.637	1.440	1.348	1.391	1.357	1.306	1.193

Table 5.5 RCE values when VRSS is used

p	0.284	0.305	0.311	0.312	0.320	0.332	0.362
σ_τ^2	15.984	21.040	22.810	23.126	25.806	30.591	48.511
n	$Relative\ Cost\ Efficiency$						
2	1.130	1.043	1.023	1.082	1.034	1.037	**1.038**
3	1.149	1.079	1.064	1.086	1.047	1.051	1.023
4	1.187	1.095	**1.139**	**1.107**	**1.096**	**1.076**	1.001
5	1.204	**1.111**	1.093	1.097	1.089	1.059	1.006
10	**1.206**	1.063	1.058	1.053	1.053	1.010	0.933
15	1.107	1.052	1.040	1.013	0.975	0.958	0.862
20	1.064	0.999	0.967	0.949	0.935	0.878	0.817
25	1.058	0.931	0.871	0.899	0.877	0.844	0.771

Table 5.6 MSE values for SRS when VRSS is used

p	0.284	0.305	0.311	0.312	0.320	0.332	0.362
σ_τ^2	15.984	21.040	22.810	23.126	25.806	30.591	48.511
n	$MSEs$						
2	5.166	5.197	5.075	5.230	5.161	5.108	5.413
3	3.456	3.423	3.407	3.401	3.408	3.343	3.425
4	2.542	2.532	2.649	2.609	2.562	2.542	2.580
5	2.096	2.049	2.019	2.129	2.066	2.030	2.037
10	1.038	1.006	1.024	1.030	1.036	1.046	1.038
15	0.698	0.694	0.693	0.693	0.667	0.686	0.677
20	0.516	0.516	0.516	0.506	0.513	0.511	0.514
25	0.418	0.408	0.400	0.410	0.410	0.407	0.412

Table 5.7 MSE values for RSS when VRSS is used

p	0.284	0.305	0.311	0.312	0.320	0.332	0.362
σ_τ^2	15.984	21.040	22.810	23.126	25.806	30.591	48.511
n	$MSEs$						
2	4.419	4.815	4.792	4.671	4.824	4.759	5.042
3	2.845	3.000	3.030	2.962	3.080	3.009	3.169
4	1.984	2.143	2.155	2.183	2.166	2.189	2.388
5	1.581	1.675	1.677	1.763	1.722	1.741	1.837
10	0.710	0.780	0.798	0.806	0.811	0.854	0.917
15	0.476	0.498	0.503	0.517	0.517	0.541	0.593
20	0.337	0.360	0.371	0.371	0.382	0.406	0.438
25	0.256	0.283	0.297	0.294	0.302	0.312	0.346

Table 5.8 Bias values for SRS when VRSS is used

p	0.284	0.305	0.311	0.312	0.320	0.332	0.362
σ_τ^2	15.984	21.040	22.810	23.126	25.806	30.591	48.511
n	$Biases$						
2	-0.034	-0.024	0.010	0.027	0.013	0.017	0.024
3	0.013	0.005	-0.021	-0.037	-0.015	-0.020	-0.009
4	0.009	-0.001	0.033	0.004	0.001	-0.027	0.017
5	0.008	-0.012	-0.005	-0.004	-0.009	-0.023	-0.010
10	-0.009	0.002	0.004	-0.005	0.006	0.002	-0.005
15	0.003	0.003	-0.002	-0.006	0.001	0.002	0.000
20	0.008	0.006	-0.006	0.004	0.000	0.016	-0.001
25	0.000	-0.007	0.007	0.001	0.009	-0.002	0.001

Table 5.9 Bias values for RSS when VRSS is used

p	0.284	0.305	0.311	0.312	0.320	0.332	0.362
σ_τ^2	15.984	21.040	22.810	23.126	25.806	30.591	48.511
n	$Biases$						
2	−0.009	0.020	−0.033	−0.022	−0.004	−0.003	−0.023
3	0.001	−0.031	0.012	−0.014	−0.009	−0.041	0.039
4	−0.012	0.009	0.013	0.009	−0.007	0.004	0.025
5	−0.001	−0.003	−0.006	−0.002	−0.009	−0.017	0.006
10	−0.005	0.002	0.011	0.009	0.011	0.008	0.005
15	0.006	−0.014	0.011	−0.006	−0.004	0.002	−0.004
20	−0.004	0.005	0.000	0.003	0.006	0.002	0.007
25	0.005	0.002	0.000	−0.004	−0.001	−0.003	0.000

Table 5.10 RE Values when CRSS is used

Variables	Shell weight	Diameter	Height	Length	Whole weight	Viscera weight	Shucked weight
p	0.284	0.305	0.311	0.312	0.320	0.332	0.362
ρ	0.628	0.575	0.559	0.557	0.536	0.504	0.420
n	$Relative\ Efficiency$						
2	1.174	1.081	1.162	1.140	1.086	1.100	1.089
3	1.279	1.161	1.262	1.158	1.161	1.172	1.111
4	1.358	1.261	1.285	1.209	1.200	1.202	1.126
5	1.418	1.267	1.318	1.237	1.266	1.225	1.167
10	1.464	1.329	1.433	1.339	1.337	1.349	1.284
15	1.552	1.342	1.468	1.391	1.410	1.415	1.327
20	1.565	1.420	1.492	1.438	1.461	1.447	1.276
25	1.605	1.443	1.502	1.341	1.448	1.425	1.323

Table 5.11 RCE Values when CRSS is used

Variables	Shell weight	Diameter	Height	Length	Whole weight	Viscera weight	Shucked weight
p	0.284	0.305	0.311	0.312	0.320	0.332	0.362
ρ	0.628	0.575	0.559	0.557	0.536	0.504	0.420
n	$Relative\ Cost\ Efficiency$						
2	1.118	1.030	1.107	1.087	1.034	1.048	**1.038**
3	1.185	1.076	**1.169**	1.073	1.076	**1.086**	1.029
4	1.225	**1.137**	1.159	**1.090**	1.082	1.084	1.016
5	**1.246**	1.113	1.158	1.087	**1.112**	1.076	1.025
10	1.137	1.032	1.114	1.040	1.039	1.048	0.998
15	1.081	0.935	1.023	0.969	0.982	0.986	0.924
20	0.988	0.896	0.942	0.908	0.922	0.913	0.806
25	0.926	0.833	0.867	0.774	0.836	0.822	0.763

Table 5.12 MSE values for RSS when CRSS is used

Variables	Shell weight	Diameter	Height	Length	Whole weight	Viscera weight	Shucked weight
p	0.284	0.305	0.311	0.312	0.320	0.332	0.362
ρ	0.628	0.575	0.559	0.557	0.536	0.504	0.420
n	$MSEs$						
2	4.485	4.750	4.477	4.542	4.742	4.648	4.871
3	2.751	2.971	2.840	2.918	2.945	2.956	3.080
4	1.941	2.097	1.565	2.177	2.109	2.153	2.241
5	1.473	1.655	0.729	1.665	1.632	1.667	1.750
10	0.685	0.771	0.485	0.773	0.759	0.774	0.819
15	0.436	0.504	0.472	0.519	0.487	0.488	0.528
20	0.333	0.373	0.350	0.369	0.357	0.361	0.408
25	0.260	0.290	0.272	0.301	0.286	0.290	0.316

Table 5.13 MSE values for SRS when CRSS is used

Variables	Shell weight	Diameter	Height	Length	Whole weight	Viscera weight	Shucked weight
p	0.284	0.305	0.311	0.312	0.320	0.332	0.362
ρ	0.628	0.575	0.559	0.557	0.536	0.504	0.420
n	$MSEs$						
2	5.264	5.133	5.200	5.180	5.148	5.114	5.305
3	3.519	3.450	3.583	3.378	3.420	3.464	3.421
4	2.636	2.644	2.064	2.633	2.530	2.587	2.524
5	2.089	2.098	1.045	2.060	2.066	2.043	2.042
10	1.003	1.024	0.678	1.035	1.014	1.044	1.052
15	0.677	0.676	0.693	0.721	0.687	0.691	0.701
20	0.521	0.530	0.523	0.530	0.522	0.523	0.520
25	0.417	0.418	0.409	0.404	0.414	0.413	0.418

Table 5.14 Bias values for SRS when CRSS is used

Variables	Shell weight	Diameter	Height	Length	Whole weight	Viscera weight	Shucked weight
p	0.284	0.305	0.311	0.312	0.320	0.332	0.362
ρ	0.628	0.575	0.559	0.557	0.536	0.504	0.420
n	$Biases$						
2	0.049	−0.001	−0.013	−0.014	−0.004	−0.039	0.034
3	−0.002	0.004	0.010	−0.060	−0.004	−0.015	0.007
4	0.007	−0.009	0.003	0.010	−0.006	−0.003	0.008
5	0.009	−0.001	−0.011	−0.014	−0.008	−0.007	−0.002
10	−0.007	−0.022	0.004	0.005	-0.002	0.019	−0.001
15	0.008	−0.007	0.003	0.000	0.004	−0.007	−0.003
20	−0.005	−0.014	−0.007	0.007	−0.007	0.010	0.007
25	−0.008	0.006	−0.008	−0.009	0.014	0.002	−0.006

Table 5.15 Bias values for RSS when CRSS is used

Variables	Shell weight	Diameter	Height	Length	Whole weight	Viscera weight	Shucked weight
p	0.284	0.305	0.311	0.312	0.320	0.332	0.362
ρ	0.628	0.575	0.559	0.557	0.536	0.504	0.420
n	$Biases$						
2	0.009	0.023	−0.016	−0.038	0.002	0.002	−0.016
3	0.018	0.006	0.014	−0.030	0.001	−0.001	−0.003
4	0.016	0.002	−0.005	−0.017	−0.014	−0.007	−0.004
5	0.009	0.010	0.006	−0.002	0.005	0.009	0.010
10	0.001	0.001	0.008	−0.008	−0.003	0.005	0.000
15	0.001	0.001	0.003	0.005	0.000	−0.009	0.002
20	-0.007	0.012	0.003	0.002	0.004	0.003	-0.006
25	0.006	0.000	-0.006	0.004	0.003	0.005	-0.002

The results from real data are given below;

- As the set size increases, the relative efficiency increases.
- When using visual and concomitant based ranking methods in the RSS method, MSE values of the mean estimator based on RSS are smaller than the MSE values of the mean estimator based on SRS. Therefore, we can say that RSS is more efficient than SRS.
- Generally, when using concomitant based ranking and visual ranking in RSS, the bias values obtained from RSS are smaller than the bias values obtained from SRS.
- When the concomitant based ranking is used in RSS, it is observed that p increases as ρ_{xy} decreases. Therefore, the relative efficiency and relative cost efficiency of RSS with respect to SRS decreases. In addition, the optimal set size, which maximizes relative cost efficiency of RSS with respect to SRS, also decreases.
- When visual ranking method is used in RSS, the p increases as σ_{τ}^2. Therefore, the relative efficiency and relative cost efficiency of RSS with respect to SRS

decreases. In addition, the optimal set size, which maximizes relative cost efficiency of RSS with respect to SRS, also decreases.

- For this data set, the best results in terms of relative cost efficiency of RSS with respect to SRS are obtained as 1.206 and 1.246 for visual and concomitant based ranking methods, respectively. The optimal set size from which these results are obtained is 10 for the visual ranking, while it is 5 for the concomitant based ranking.

CHAPTER SIX
CONCLUSIONS

In this thesis, we aimed to use N-KPST cost model (proposed by Nahhas et al. (2002) for RSS) and ranking error models (VRSS and CRSS) to see whether the RSS is cost effective with respect to SRS in terms of MSE of the mean estimator in infinite population and if so, to determine the optimal set size for RSS. Also bias and MSE of the mean estimators based on RSS and some of its modified methods such as Extreme RSS and Percentile RSS are computed and compared with MSE of the mean estimator based on SRS to investigate the effects of the ranking error models in infinite population.

In Chapter 3, to investigate the effects of ranking errors in RSS and in some of its modified methods are examined the simulation study. In this study, it is deduced that ranking errors may occur depending on the ranking method used. In VRSS, σ_τ^2 varies depending on the researcher's knowledge, experience and materials used in the study. The greater knowledge of researcher involved in the study and the use of more appropriate materials would yield lower σ_τ^2 with a higher accuracy in the ranking. On the other hand for CRSS, the accuracy of the ranking depends on the correlation between the variable of interest and the concomitant variable. Generally, when $\rho_{xy} \geq 0.5$, the error in the ranking decreases and the accuracy of the ranking increases. Thus, better results can be achieved by minimizing the error in the ranking.

The results obtained from the simulation study in Chapter 4, indicated that as p increases in both VRSS and CRSS, the relative cost efficiency and relative efficiency of RSS decreases with respect to SRS in terms of estimating population mean. On the other hand, it is observed that changes in the values of cost categories in the N-KPST cost model lead to changes in the optimal set size as well as the relative cost efficiency of RSS according to SRS. When using visual ranking method, in order for RSS to be cost effective according to SRS, c_{qx} should be high, c_i and ranking cost of a unit in the set should be low. When concomitant based ranking is used in RSS, c_{qx} should be high, while c_{qy} and c_i should be low. If these cost conditions are met and p is low, then the relative cost efficiency of RSS according to SRS will increase together with optimal set size.

For VRSS, according to the observations generated from $N(0,1)$, $U(0,1)$ $Exp(1)$ and $\Gamma(2,1)$, the best results of the cost efficiency of RSS with respect to SRS the N-KPST cost model are obtained when $c_{qx} = 100$, $c_i = 2.5$, and ranking cost of a unit in the set is equal to 1.25. These results are 4.735, 6.808, 3.512 and 4.180 respectively, for $p = 0$ and $n = 25$. RSS is observed to be cost effective than SRS even when $p = 0.35$ and $n = 3$ for $N(0,1)$, $p = 0.35$ and $n = 2$ for $U(0,1)$, $p = 0.35$ and $n = 2$ for $Exp(1)$ and $n = 2$, $p = 0.35$ and $n = 4$ for $\Gamma(2,1)$. These results are 1.030, 1.055 1.065, 1.048.

For CRSS, according to the observations generated from $N(0,1)$, $U(0,1)$, $Exp(1)$ and $\Gamma(2,1)$, the best results of the cost efficiency of RSS with respect to SRS the N-KPST cost model are obtained model are obtained when $c_{qx} = 100$, $c_{qy} = 5$ and $c_i = 2.5$. These results are 3.760, 4.645 and 2.288, respectively, for $p = 0$ and n of 25. In addition, RSS is observed to be cost efficient than SRS even when $p = 0.3$ and $n = 2$ for $N(0,1)$, $p = 0.3$ and $n = 2$ for $U(0,1)$, $p = 0.25$ and $n = 4$ for $Exp(1)$, $p = 0.3$ and $n = 2$ for $\Gamma(2,1)$. These results are 1.032, 1.022 1.005, 1.007.

In the chapter 5, a study is performed using the real data set in order to support the simulation studies performed in the chapter 3 and 4. It is observed that the results obtained from the real data and simulation studies are similar. For this data set, the best results in terms of relative cost efficiency of RSS with respect to SRS are obtained as 1.206 when $\sigma_\tau^2 = 15.984$ and $p = 0.284$ for visual ranking and 1.246 when $p = 0.284$ and $\rho = 0.628$ for concomitant based ranking. The optimal set size from which these results are obtained is 10 for the visual ranking, while it is 5 for the concomitant based ranking.

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APPENDICES

Table A.1 RCE values for $N(0,1)$ when c_{qx} = 50, c_i = 2.5, α_1 = 20 and cost of ranking a unit in set is 1.25 by using VRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.332	1.297	1.281	1.222	1.131	1.081	1.025	0.983	0.967	0.928	0.895
3	1.641	1.626	1.491	1.427	1.268	1.120	**1.037**	0.979	0.914	0.892	0.867
4	1.828	1.825	1.714	1.468	1.283	1.107	0.979	0.874	0.854	0.810	0.802
5	2.020	2.008	1.784	**1.571**	**1.329**	**1.140**	0.956	0.919	0.809	0.754	0.778
10	2.611	2.549	2.137	1.569	1.237	0.993	0.834	0.717	0.649	0.617	0.595
15	3.035	2.986	**2.150**	1.528	1.099	0.874	0.703	0.606	0.516	0.498	0.502
20	3.133	3.077	2.122	1.430	0.967	0.760	0.595	0.513	0.468	0.417	0.440
25	**3.295**	**3.166**	1.967	1.291	0.905	0.659	0.516	0.434	0.394	0.403	0.366

Table A.2 RCE values for $N(0,1)$ when c_{qx} = 50, c_i = 5, α_1 = 10 and cost of ranking a unit in set is 1.25 by using VRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.275	1.243	1.189	1.169	1.111	**1.053**	0.997	0.951	0.904	0.897	0.872
3	1.478	1.452	1.347	1.268	1.152	1.033	0.977	0.909	0.840	0.815	0.794
4	1.685	1.621	1.491	1.341	1.147	1.039	0.949	0.846	0.775	0.746	0.742
5	1.760	1.759	1.615	**1.410**	**1.192**	1.007	0.883	0.778	0.745	0.673	0.677
10	2.165	2.151	**1.776**	1.342	1.001	0.830	0.658	0.599	0.531	0.506	0.483
15	**2.392**	**2.228**	1.665	1.162	0.865	0.659	0.548	0.454	0.407	0.386	0.381
20	2.323	2.221	1.589	1.068	0.785	0.561	0.459	0.384	0.338	0.318	0.325
25	2.317	2.151	1.477	0.950	0.667	0.480	0.393	0.339	0.291	0.271	0.267

Table A.3 RCE values for $N(0,1)$ when c_{qx} = 100, c_i = 5, α_1 = 20 and cost of ranking a unit in set is 1.25 by using VRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.371	1.354	1.299	1.240	1.207	1.068	1.059	1.000	0.935	0.959	0.935
3	1.674	1.634	1.558	1.413	1.317	1.135	**1.069**	0.959	0.890	0.877	0.873
4	1.963	1.917	1.713	1.575	1.328	1.180	1.013	0.948	0.869	0.832	0.847
5	2.142	2.068	1.899	1.628	**1.393**	**1.198**	1.041	0.929	0.865	0.807	0.809
10	2.961	2.803	2.281	**1.800**	1.315	1.064	0.858	0.769	0.712	0.655	0.646
15	3.321	3.286	2.302	1.695	1.220	0.961	0.762	0.667	0.613	0.541	0.546
20	3.474	3.452	**2.344**	1.590	1.107	0.831	0.674	0.582	0.502	0.474	0.464
25	**3.613**	**3.489**	2.249	1.379	1.014	0.734	0.604	0.492	0.449	0.428	0.419

Table A.4 RCE values for $N(0,1)$ when c_{qx} = 50, c_i = 2.5, α_1 = 20 and cost of ranking a unit in set is 2.5 by using VRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.251	1.240	1.234	1.173	1.084	1.052	0.973	0.921	0.877	0.883	0.869
3	1.537	1.502	1.407	1.307	1.165	1.052	0.939	0.909	0.877	0.811	0.782
4	1.658	1.626	1.518	1.366	**1.219**	**1.055**	0.955	0.853	0.801	0.762	0.755
5	1.893	1.891	1.718	**1.425**	1.215	1.017	0.926	0.827	0.759	0.711	0.711
10	2.244	2.250	1.764	1.422	1.087	0.871	0.742	0.636	0.585	0.537	0.525
15	2.540	2.424	**1.833**	1.274	0.969	0.737	0.596	0.514	0.461	0.429	0.417
20	2.583	2.480	1.754	1.121	0.817	0.619	0.522	0.443	0.379	0.365	0.368
25	**2.611**	**2.564**	1.625	1.028	0.734	0.554	0.454	0.374	0.335	0.319	0.309

Table A.5 RCE values for $N(0,1)$ when c_{qx} = 100, c_i = 2.5, α_1 = 40 and cost of ranking a unit in set is 2.5 by using VRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.386	1.379	1.290	1.244	1.170	1.079	1.033	**1.017**	0.947	0.943	0.946
3	1.736	1.710	1.568	1.486	1.292	1.198	**1.056**	0.985	0.906	0.882	0.899
4	2.011	1.951	1.786	1.610	1.387	**1.241**	1.089	0.997	0.913	0.867	0.861
5	2.195	2.193	2.004	1.660	1.416	1.226	1.076	0.967	0.861	0.833	0.813
10	3.125	2.974	2.305	**1.823**	**1.460**	1.091	0.920	0.820	0.727	0.724	0.704
15	3.533	3.508	2.472	1.821	1.321	1.033	0.855	0.716	0.642	0.604	0.581
20	3.767	3.757	**2.590**	1.710	1.231	0.935	0.730	0.626	0.558	0.522	0.512
25	**4.047**	**4.011**	2.541	1.596	1.143	0.834	0.658	0.570	0.470	0.470	0.453

Table A.6 RCE values for $N(0,1)$ when c_{qx} = 50, c_i = 5, α_1 = 10 and cost of ranking a unit in set is 2.5 by using VRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.288	1.279	1.187	1.123	1.072	0.995	0.969	0.904	0.840	0.869	0.869
3	1.488	1.422	1.294	1.239	1.104	**1.007**	0.946	0.827	0.815	0.763	0.765
4	1.559	1.541	1.400	**1.284**	**1.112**	0.965	0.826	0.776	0.745	0.704	0.674
5	1.654	1.649	1.494	1.275	1.079	0.893	0.788	0.727	0.644	0.641	0.623
10	1.986	1.979	**1.554**	1.138	0.919	0.725	0.603	0.517	0.477	0.451	0.454
15	**2.074**	**1.991**	1.492	1.049	0.779	0.590	0.489	0.407	0.364	0.348	0.341
20	2.054	2.016	1.438	0.924	0.655	0.478	0.410	0.331	0.305	0.277	0.281
25	2.007	1.974	1.306	0.827	0.564	0.425	0.332	0.281	0.249	0.247	0.235

Table A.7 RCE values for $N(0,1)$ when c_{qx} = 100, c_i = 5, α_1 = 20 and cost of ranking a unit in set is 2.5 by using VRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.336	1.292	1.264	1.202	1.113	1.039	1.021	0.993	0.958	0.935	0.888
3	1.654	1.642	1.481	1.360	1.234	1.139	**1.052**	0.947	0.897	0.854	0.865
4	1.839	1.745	1.734	1.505	1.254	**1.141**	0.986	0.897	0.836	0.830	0.820
5	2.055	2.045	1.741	1.542	**1.319**	1.089	1.001	0.874	0.817	0.750	0.779
10	2.693	2.645	2.072	**1.614**	1.229	0.984	0.816	0.721	0.667	0.503	0.597
15	3.027	2.917	**2.178**	1.495	1.102	0.884	0.701	0.586	0.575	0.629	0.509
20	3.132	3.030	2.090	1.372	0.984	0.758	0.622	0.507	0.464	0.428	0.429
25	**3.202**	**3.128**	1.984	1.242	0.859	0.671	0.541	0.439	0.408	0.377	0.368

Table A.8 RCE values for $U(0,1)$ when c_{qx} = 50, c_i = 2.5, α_1 = 20 and cost of ranking a unit in set is 1.25 by using VRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.393	1.389	1.293	1.259	1.193	1.068	**1.042**	0.969	0.949	0.942	0.914
3	1.654	1.720	1.558	1.454	1.317	1.160	1.040	0.967	0.906	0.901	0.854
4	2.053	1.962	1.786	1.510	1.300	**1.184**	1.040	0.964	0.875	0.833	0.831
5	2.344	2.171	1.944	1.624	**1.356**	1.172	0.971	0.868	0.815	0.775	0.767
10	3.328	3.018	2.326	**1.794**	1.310	1.003	0.834	0.719	0.655	0.628	0.617
15	3.890	3.415	**2.458**	1.625	1.213	0.909	0.721	0.620	0.555	0.494	0.489
20	4.415	3.642	2.391	1.548	1.04	0.786	0.605	0.515	0.465	0.428	0.432
25	**4.738**	**3.761**	2.401	1.413	0.966	0.700	0.530	0.455	0.398	0.378	0.358

Table A.9 RCE values for $U(0,1)$ when c_{qx} = 50, c_i = 5, α_1 = 10 and cost of ranking a unit in set is 1.25 by using VRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.305	1.295	1.264	1.188	1.130	1.083	**1.008**	0.963	0.899	0.876	0.882
3	1.584	1.528	1.507	1.342	1.223	**1.097**	0.950	0.874	0.863	0.822	0.804
4	1.823	1.786	1.675	1.431	**1.238**	1.087	0.905	0.816	0.796	0.742	0.769
5	2.040	1.971	1.747	1.443	1.207	1.024	0.894	0.786	0.729	0.693	0.704
10	2.790	2.415	**1.996**	**1.475**	1.077	0.838	0.660	0.593	0.532	0.506	0.466
15	2.929	2.652	1.935	1.303	0.965	0.687	0.557	0.468	0.417	0.387	0.390
20	3.196	2.701	1.813	1.184	0.815	0.591	0.467	0.391	0.348	0.315	0.321
25	**3.533**	**2.733**	1.682	1.049	0.697	0.516	0.418	0.331	0.291	0.273	0.270

Table A.10 RCE values for $U(0,1)$ when c_{qx} = 100, c_i = 5, α_1 = 20 and cost of ranking a unit in set is 1.25 by using VRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.375	1.336	1.302	1.279	1.184	1.112	1.011	0.994	0.964	0.955	0.922
3	1.771	1.671	1.609	1.497	1.303	1.198	1.047	0.998	0.939	0.874	0.867
4	2.084	2.005	1.872	1.616	1.399	1.220	**1.064**	0.978	0.904	0.854	0.826
5	2.408	2.291	2.057	1.730	**1.512**	**1.250**	1.056	0.957	0.860	0.820	0.795
10	3.537	3.198	2.519	**1.951**	1.428	1.085	0.891	0.779	0.697	0.665	0.650
15	4.320	3.762	2.628	1.855	1.316	1.014	0.756	0.664	0.589	0.549	0.535
20	4.856	4.068	**2.708**	1.721	1.170	0.855	0.669	0.584	0.489	0.464	0.469
25	**5.495**	**4.084**	2.646	1.593	1.044	0.774	0.600	0.508	0.443	0.413	0.419

Table A.11 RCE values for $U(0,1)$ when c_{qx} = 50, c_i = 2.5, α_1 = 20 and cost of ranking a unit in set is 2.5 by using VRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.312	1.307	1.265	1.223	1.120	1.058	0.998	0.918	0.904	0.897	0.890
3	1.631	1.582	1.503	1.388	1.222	1.058	0.977	0.897	0.835	0.816	0.814
4	1.912	1.826	1.565	1.443	1.260	1.073	0.971	0.880	0.760	0.743	0.750
5	2.132	2.033	1.768	1.491	**1.280**	**1.082**	0.923	0.794	0.757	0.703	0.698
10	2.911	2.672	**2.066**	**1.562**	1.128	0.921	0.723	0.641	0.547	0.559	0.533
15	3.247	2.802	1.967	1.430	1.007	0.786	0.609	0.505	0.453	0.432	0.412
20	3.563	2.989	1.978	1.315	0.896	0.655	0.521	0.424	0.381	0.36	0.350
25	**3.911**	**3.052**	1.916	1.200	0.773	0.581	0.439	0.369	0.329	0.304	0.291

Table A.12 RCE values for $U(0,1)$ when c_{qx} = 100, c_i = 2.5, α_1 = 40 and cost of ranking a unit in set is 2.5 by using VRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.377	1.374	1.363	1.302	1.193	1.118	1.030	0.994	0.951	0.946	0.926
3	1.746	1.766	1.623	1.475	1.331	1.192	1.070	**1.011**	0.926	0.934	0.885
4	2.155	2.023	1.841	1.699	1.446	1.221	1.062	0.959	0.912	0.879	0.854
5	2.484	2.288	2.093	1.786	1.479	**1.256**	**1.081**	0.939	0.853	0.823	0.822
10	3.745	3.328	2.720	1.915	**1.506**	1.168	0.945	0.833	0.726	0.699	0.684
15	4.749	3.927	**2.915**	**2.044**	1.423	1.042	0.837	0.727	0.667	0.599	0.573
20	5.317	4.440	2.899	1.944	1.272	0.925	0.729	0.621	0.561	0.528	0.517
25	**5.847**	**4.675**	2.823	1.805	1.171	0.872	0.646	0.580	0.503	0.477	0.454

Table A.13 RCE values for $U(0,1)$ when c_{qx} = 50, c_i = 5, α_1 = 10 and cost of ranking a unit in set is 2.5 by using VRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.263	1.257	1.258	1.166	1.088	**1.025**	0.928	0.915	0.859	0.857	0.846
3	1.523	1.487	1.395	1.277	**1.180**	1.004	0.915	0.840	0.790	0.768	0.765
4	1.706	1.668	1.527	1.314	1.156	1.013	0.893	0.799	0.716	0.707	0.684
5	1.944	1.847	1.590	**1.358**	1.165	0.984	0.815	0.764	0.701	0.644	0.626
10	2.424	2.145	**1.712**	1.311	0.987	0.760	0.601	0.537	0.478	0.436	0.439
15	2.652	2.350	1.675	1.161	0.809	0.626	0.505	0.408	0.358	0.342	0.343
20	2.956	**2.430**	1.538	1.019	0.702	0.503	0.397	0.345	0.303	0.281	0.271
25	**2.964**	2.349	1.468	0.932	0.601	0.426	0.351	0.288	0.250	0.241	0.226

Table A.14 RCE values for $U(0,1)$ when c_{qx} = 100, c_i = 5, α_1 = 20 and cost of ranking a unit in set is 2.5 by using VRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.415	1.339	1.321	1.234	1.142	1.079	1.009	0.999	0.937	0.935	0.947
3	1.604	1.653	1.543	1.445	1.279	1.133	**1.032**	0.960	0.861	0.872	0.850
4	1.998	1.943	1.765	1.562	1.359	**1.180**	1.000	0.913	0.871	0.825	0.803
5	2.251	2.113	1.936	1.674	**1.369**	1.162	1.007	0.844	0.810	0.783	0.726
10	3.319	2.968	2.272	**1.748**	1.34	1.056	0.826	0.727	0.666	0.60	0.592
15	3.991	3.419	**2.455**	1.662	1.175	0.888	0.719	0.602	0.534	0.510	0.502
20	4.352	3.645	2.436	1.551	1.090	0.767	0.634	0.520	0.458	0.418	0.414
25	**4.787**	**3.718**	2.260	1.493	0.969	0.698	0.526	0.447	0.400	0.371	0.355

Table A.15 RCE values for $Exp(1)$ when c_{qx} = 50, c_i = 2.5, α_1 = 20 and cost of ranking a unit in set is 1.25 by using VRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.223	1.202	1.097	1.076	1.087	1.052	**1.030**	0.964	0.997	0.940	0.854
3	1.421	1.347	1.356	1.256	1.111	**1.077**	1.006	0.937	0.882	0.917	0.866
4	1.560	1.529	1.462	1.289	**1.168**	1.050	0.955	0.894	0.859	0.812	0.806
5	1.679	1.664	1.507	1.277	1.157	1.092	0.963	0.878	0.756	0.761	0.749
10	2.040	1.926	**1.675**	**1.396**	1.105	0.933	0.827	0.721	0.650	0.616	0.593
15	2.226	2.000	1.658	1.351	1.045	0.822	0.714	0.612	0.543	0.498	0.498
20	2.352	2.105	1.647	1.285	1.002	0.755	0.637	0.533	0.461	0.420	0.422
25	**2.446**	**2.128**	1.593	1.165	0.879	0.692	0.552	0.454	0.411	0.370	0.367

Table A.16 RCE values for $Exp(1)$ when c_{qx} = 50, c_i = 5, α_1 = 10 and cost of ranking a unit in set is 1.25 by using VRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.167	1.146	1.140	1.097	1.079	**1.061**	0.949	0.935	0.915	0.879	0.869
3	1.356	1.316	1.186	1.113	**1.134**	1.033	0.965	0.848	0.833	0.789	0.774
4	1.361	1.336	1.227	**1.152**	1.068	0.995	0.920	0.814	0.751	0.710	0.712
5	1.459	1.479	1.307	1.145	1.075	0.918	0.861	0.755	0.740	0.694	0.659
10	1.684	1.525	**1.369**	1.130	0.926	0.804	0.685	0.578	0.528	0.511	0.491
15	1.674	**1.585**	1.318	1.034	0.821	0.666	0.519	0.482	0.439	0.395	0.395
20	1.736	1.520	1.255	0.962	0.726	0.576	0.463	0.392	0.348	0.324	0.322
25	**1.739**	1.496	1.180	0.880	0.666	0.496	0.396	0.333	0.305	0.275	0.263

Table A.17 RCE values for $Exp(1)$ when c_{qx} = 100, c_i = 5, α_1 = 20 and cost of ranking a unit in set is 1.25 by using VRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.239	1.193	1.141	1.187	1.144	1.053	1.058	0.955	0.931	0.955	0.991
3	1.446	1.322	1.362	1.337	1.157	1.062	1.001	0.960	0.912	0.905	0.878
4	1.584	1.583	1.546	1.386	1.175	**1.117**	1.018	0.929	0.900	0.872	0.860
5	1.770	1.704	1.520	1.389	**1.254**	1.111	0.975	0.910	0.865	0.794	0.826
10	2.172	2.046	1.726	**1.494**	1.212	1.049	0.869	0.774	0.713	0.668	0.663
15	2.389	2.198	1.813	1.446	1.172	0.947	0.816	0.663	0.593	0.543	0.561
20	2.603	**2.350**	**1.854**	1.401	1.091	0.868	0.698	0.579	0.500	0.487	0.465
25	**2.716**	2.328	1.794	1.301	0.990	0.771	0.610	0.519	0.451	0.409	0.414

Table A.18 RCE values for $Exp(1)$ when c_{qx} = 50, c_i = 2.5, α_1 = 20 and cost of ranking a unit in set is 2.5 by using VRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.176	1.142	1.090	1.147	1.039	0.998	0.981	0.881	0.899	0.871	0.878
3	1.334	1.302	1.236	1.184	**1.093**	0.980	0.956	0.884	0.835	0.812	0.823
4	1.417	1.401	1.336	1.198	1.088	0.972	0.891	0.858	0.765	0.775	0.759
5	1.510	1.474	1.323	1.199	1.056	0.953	0.844	0.790	0.720	0.677	0.696
10	1.770	1.621	**1.475**	**1.205**	1.000	0.817	0.688	0.616	0.542	0.546	0.540
15	1.924	**1.806**	1.457	1.132	0.915	0.737	0.606	0.537	0.473	0.431	0.424
20	1.936	1.687	1.344	1.072	0.790	0.666	0.520	0.432	0.384	0.360	0.346
25	**1.996**	1.758	1.328	0.958	0.736	0.561	0.464	0.375	0.323	0.305	0.301

Table A.19 RCE values for $Exp(1)$ when c_{qx} = 100, c_i = 2.5, α_1 = 40 and cost of ranking a unit in set is 2.5 by using VRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.241	1.207	1.173	1.120	1.149	1.082	0.970	**1.008**	0.972	1.010	0.885
3	1.509	1.393	1.330	1.289	1.177	1.121	1.018	0.965	0.896	0.890	0.888
4	1.586	1.589	1.478	1.286	1.209	**1.134**	**1.052**	0.953	0.935	0.891	0.886
5	1.800	1.690	1.538	1.391	1.283	1.111	1.029	0.901	0.868	0.810	0.831
10	2.384	2.150	1.861	1.581	**1.304**	1.103	0.934	0.851	0.719	0.716	0.661
15	2.727	2.434	1.942	**1.593**	1.243	1.012	0.835	0.744	0.637	0.611	0.569
20	2.804	2.421	2.021	1.490	1.157	0.955	0.756	0.622	0.580	0.521	0.515
25	**3.005**	**2.701**	**2.047**	1.502	1.091	0.850	0.679	0.559	0.500	0.455	0.468

Table A.20 RCE values for $Exp(1)$ when c_{qx} = 50, c_i = 5, α_1 = 10 and cost of ranking a unit in set is 2.5 by using VRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.116	1.048	1.022	1.073	0.994	0.953	0.922	0.811	0.891	0.856	0.802
3	1.272	1.249	1.159	1.084	0.983	0.964	0.872	0.812	0.764	0.779	0.785
4	1.339	1.295	1.184	**1.118**	**1.022**	0.901	0.834	0.745	0.738	0.656	0.691
5	1.342	1.323	**1.245**	1.082	0.976	0.855	0.797	0.712	0.682	0.643	0.631
10	1.503	1.394	1.236	1.027	0.838	0.717	0.600	0.496	0.484	0.450	0.441
15	**1.521**	1.394	1.181	0.920	0.722	0.579	0.479	0.393	0.375	0.350	0.334
20	1.503	**1.408**	1.047	0.823	0.604	0.508	0.402	0.338	0.303	0.286	0.286
25	1.508	1.326	1.009	0.747	0.562	0.433	0.353	0.288	0.265	0.234	0.231

Table A.21 RCE values for $Exp(1)$ when c_{qx} = 100, c_i = 5, α_1 = 20 and cost of ranking a unit in set is 2.5 by using VRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.122	1.139	1.162	1.156	1.112	**1.109**	**1.011**	**1.016**	0.940	0.911	0.942
3	1.412	1.319	1.306	1.199	1.119	0.997	0.984	0.921	0.920	0.845	0.863
4	1.560	1.492	1.424	1.280	1.193	1.093	0.976	0.899	0.833	0.843	0.805
5	1.678	1.625	1.362	1.304	**1.199**	1.058	0.943	0.851	0.851	0.795	0.801
10	2.003	1.954	1.607	**1.380**	1.125	0.970	0.825	0.730	0.667	0.611	0.601
15	2.175	**2.065**	**1.680**	1.308	1.082	0.882	0.719	0.601	0.540	0.508	0.518
20	2.271	2.026	1.619	1.275	1.003	0.766	0.624	0.519	0.455	0.430	0.426
25	**2.318**	2.032	1.631	1.154	0.890	0.657	0.553	0.442	0.413	0.359	0.368

Table A.22 RCE values for $\Gamma(2,1)$ when c_{qx} = 50, c_i = 2.5, α_1 = 20 and cost of ranking a unit in set is 1.25 by using VRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.312	1.221	1.219	1.120	1.123	1.056	0.998	0.921	0.918	0.917	0.905
3	1.458	1.503	1.394	1.263	1.156	1.092	0.994	0.953	0.854	0.894	0.897
4	1.719	1.652	1.526	1.299	1.186	**1.103**	**1.019**	0.895	0.828	0.811	0.823
5	1.789	1.654	1.625	1.384	**1.238**	1.101	0.973	0.877	0.786	0.774	0.769
10	2.417	2.184	1.771	**1.477**	1.165	0.993	0.816	0.702	0.655	0.626	0.612
15	2.677	2.411	**1.852**	1.407	1.104	0.826	0.694	0.595	0.524	0.521	0.524
20	**2.846**	**2.465**	1.744	1.283	0.968	0.781	0.618	0.540	0.440	0.423	0.416
25	2.840	2.424	1.848	1.234	0.901	0.678	0.551	0.473	0.402	0.375	0.361

Table A.23 RCE values for $\Gamma(2,1)$ when c_{qx} = 50, c_i = 5, α_1 = 10 and cost of ranking a unit in set is 1.25 by using VRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.182	1.154	1.127	1.110	1.066	1.040	0.942	0.918	0.874	0.866	0.848
3	1.382	1.338	1.291	1.181	1.136	**1.052**	0.937	0.866	0.830	0.820	0.852
4	1.543	1.451	1.357	1.208	**1.146**	1.029	0.907	0.802	0.723	0.726	0.715
5	1.660	1.574	1.428	**1.266**	1.080	0.946	0.877	0.761	0.726	0.682	0.665
10	1.919	1.750	**1.504**	1.192	0.976	0.809	0.675	0.587	0.536	0.481	0.481
15	2.007	**1.828**	1.434	1.116	0.838	0.623	0.538	0.464	0.414	0.381	0.379
20	2.070	1.792	1.353	1.016	0.727	0.572	0.455	0.390	0.348	0.329	0.316
25	**2.116**	1.777	1.289	0.918	0.638	0.501	0.400	0.336	0.294	0.281	0.261

Table A.24 RCE values for $\Gamma(2,1)$ when c_{qx} = 100, c_i = 5, α_1 = 20 and cost of ranking a unit in set is 1.25 by using VRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.321	1.302	1.262	1.178	1.120	1.075	1.037	0.995	0.995	0.957	0.965
3	1.557	1.527	1.468	1.292	1.200	1.107	1.053	0.962	0.907	0.938	0.916
4	1.802	1.727	1.599	1.357	1.270	**1.153**	**1.063**	0.923	0.887	0.863	0.819
5	1.968	1.912	1.646	1.504	1.284	1.117	1.015	0.914	0.860	0.662	0.795
10	2.472	2.340	1.952	**1.599**	**1.323**	1.024	0.909	0.752	0.690	0.653	0.628
15	2.862	2.614	1.980	1.529	1.143	0.937	0.774	0.661	0.598	0.552	0.549
20	3.101	2.621	**2.063**	1.476	1.112	0.857	0.689	0.562	0.517	0.483	0.461
25	**3.314**	**2.742**	2.012	1.377	1.001	0.747	0.615	0.496	0.442	0.402	0.400

Table A.25 RCE values for $\Gamma(2,1)$ when c_{qx} = 50, c_i = 2.5, α_1 = 20 and cost of ranking a unit in set is 2.5 by using VRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.271	1.215	1.173	1.113	1.101	**1.079**	0.946	0.927	0.863	0.849	0.874
3	1.368	1.370	1.322	1.226	1.124	1.022	0.950	0.914	0.794	0.832	0.793
4	1.577	1.530	1.416	1.226	**1.135**	1.040	0.896	0.849	0.753	0.748	0.757
5	1.699	1.620	1.464	**1.281**	1.123	0.982	0.846	0.763	0.564	0.739	0.528
10	2.020	1.898	**1.610**	1.269	1.049	0.842	0.721	0.627	0.562	0.535	0.422
15	2.405	1.989	1.545	1.127	0.940	0.740	0.593	0.519	0.460	0.427	0.354
20	**2.424**	**2.031**	1.508	1.110	0.824	0.630	0.521	0.441	0.394	0.357	0.295
25	2.401	2.019	1.451	0.989	0.747	0.552	0.434	0.372	0.331	0.313	0.300

Table A.26 RCE values for $\Gamma(2,1)$ when c_{qx} = 100, c_i = 2.5, α_1 = 40 and cost of ranking a unit in set is 2.5 by using VRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.298	1.247	1.200	1.223	1.148	1.122	1.002	**1.032**	0.931	0.941	0.950
3	1.588	1.480	1.442	1.352	1.205	1.110	1.020	1.030	0.954	0.919	0.896
4	1.757	1.809	1.533	1.472	1.289	1.115	**1.054**	0.948	0.901	0.882	0.860
5	2.013	1.891	1.692	1.507	1.318	**1.155**	1.035	0.934	0.840	0.817	0.821
10	2.640	2.580	2.110	**1.724**	**1.359**	1.149	0.923	0.827	0.738	0.706	0.685
15	3.116	2.808	2.208	1.627	1.292	1.027	0.833	0.686	0.630	0.586	0.562
20	3.524	2.992	**2.250**	1.687	1.196	0.921	0.778	0.654	0.571	0.533	0.497
25	**3.585**	**3.140**	2.210	1.573	1.143	0.863	0.677	0.556	0.510	0.462	0.457

Table A.27 RCE values for $\Gamma(2,1)$ when c_{qx} = 50, c_i = 5, α_1 = 10 and cost of ranking a unit in set is 2.5 by using VRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.167	1.171	1.140	1.082	1.003	0.967	0.924	0.913	0.871	0.851	0.849
3	1.321	1.256	1.266	1.180	**1.051**	0.970	0.865	0.850	0.835	0.760	0.752
4	1.428	1.321	1.282	1.159	0.976	0.930	0.847	0.772	0.703	0.717	0.699
5	1.504	1.467	**1.343**	**1.210**	1.010	0.892	0.789	0.706	0.650	0.640	0.609
10	1.665	**1.664**	1.300	1.054	0.871	0.701	0.612	0.520	0.474	0.458	0.444
15	1.764	1.605	1.331	0.959	0.746	0.590	0.474	0.431	0.359	0.345	0.330
20	**1.892**	1.619	1.230	0.870	0.643	0.511	0.406	0.330	0.297	0.289	0.284
25	1.822	1.593	1.109	0.789	0.563	0.423	0.358	0.289	0.250	0.239	0.229

Table A.28 RCE values for $\Gamma(2,1)$ when c_{qx} = 100, c_i = 5, α_1 = 20 and cost of ranking a unit in set is 2.5 by using VRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.285	1.252	1.197	1.150	1.123	1.055	0.974	0.968	0.956	0.909	0.914
3	1.513	1.456	1.402	1.308	1.174	1.066	1.001	0.938	0.894	0.863	0.889
4	1.665	1.595	1.524	1.347	**1.237**	**1.125**	**1.017**	0.913	0.861	0.809	0.877
5	1.835	1.816	1.581	1.439	1.221	1.079	1.021	0.877	0.827	0.758	0.770
10	2.333	2.229	**1.925**	**1.509**	1.181	0.935	0.828	0.734	0.635	0.615	0.753
15	2.665	2.351	1.892	1.410	1.093	0.838	0.696	0.607	0.519	0.506	0.500
20	2.829	2.494	1.782	1.342	1.027	0.770	0.612	0.524	0.462	0.435	0.408
25	**2.843**	**2.537**	1.758	1.193	0.872	0.664	0.561	0.455	0.410	0.369	0.365

Table A.29 RCE values for $N(0,1)$ when c_{qx} = 50, c_{qy} = 5, c_i = 2.5, α_1 = 20 and α_0 = 2 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.237	1.148	1.112	1.096	1.015	0.951	0.909	0.882	0.837	0.844	0.802
3	1.397	1.403	1.304	1.166	1.042	0.997	0.874	0.793	0.754	0.733	0.732
4	1.541	1.508	1.419	1.228	**1.061**	0.924	0.836	0.741	0.686	0.659	0.642
5	1.664	1.600	1.435	**1.233**	1.031	0.871	0.768	0.714	0.655	0.628	0.601
10	2.087	1.798	**1.477**	1.123	0.913	0.671	0.592	0.486	0.439	0.425	0.416
15	2.104	1.870	1.394	1.024	0.747	0.572	0.471	0.393	0.338	0.332	0.326
20	2.162	**1.915**	1.304	0.888	0.637	0.470	0.383	0.326	0.284	0.270	0.264
25	**2.222**	1.900	1.164	0.765	0.543	0.409	0.311	0.266	0.241	0.233	0.224

Table A.30 RCE values for $N(0,1)$ when c_{qx} = 50, c_{qy} = 10, c_i = 2.5, α_1 = 20 and α_0 = 4 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.018	0.983	1.003	0.960	0.865	0.830	0.773	0.743	0.722	0.697	0.683
3	1.173	1.100	1.023	0.986	0.882	0.766	0.723	0.655	0.633	0.613	0.618
4	1.249	1.220	1.102	0.941	0.850	0.725	0.632	0.604	0.553	0.546	0.523
5	1.283	1.266	1.087	0.938	0.775	0.650	0.618	0.533	0.489	0.454	0.456
10	1.393	**1.356**	**1.105**	0.802	0.607	0.507	0.409	0.366	0.319	0.305	0.303
15	1.472	1.298	0.948	0.687	0.492	0.388	0.311	0.263	0.239	0.222	0.222
20	**1.500**	1.264	0.867	0.569	0.424	0.315	0.252	0.210	0.186	0.177	0.175
25	1.494	1.221	0.788	0.511	0.340	0.265	0.205	0.179	0.160	0.152	0.145

Table A.31 RCE values for $N(0,1)$ when c_{qx} = 100, c_{qy} = 10, c_i = 2.5 ,α_1 = 40 and α_0 = 4 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.198	1.238	1.134	1.101	1.029	0.973	0.897	0.907	0.820	0.826	0.812
3	1.416	1.398	1.337	1.193	**1.145**	**1.017**	0.883	0.809	0.795	0.768	0.742
4	1.610	1.536	1.466	1.260	1.102	0.934	0.854	0.779	0.726	0.693	0.665
5	1.685	1.641	1.452	**1.268**	1.092	0.908	0.791	0.755	0.674	0.659	0.637
10	2.183	1.991	**1.649**	1.201	0.947	0.730	0.628	0.551	0.493	0.467	0.482
15	2.452	2.136	1.505	1.112	0.799	0.623	0.499	0.423	0.408	0.364	0.357
20	2.532	**2.213**	1.479	0.986	0.684	0.531	0.421	0.345	0.322	0.298	0.297
25	**2.635**	2.162	1.335	0.844	0.625	0.448	0.365	0.299	0.280	0.252	0.251

Table A.32 RCE values for $N(0,1)$ when c_{qx} = 50, c_{qy} = 20, c_i = 2.5, α_1 = 20 and α_0 = 8 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	0.810	0.817	0.778	0.731	0.664	0.680	0.611	0.598	0.572	0.555	0.568
3	0.850	0.823	0.780	0.729	0.643	0.584	0.546	0.510	0.480	0.460	0.435
4	0.842	0.884	0.745	0.685	0.618	0.518	0.475	0.426	0.398	0.374	0.377
5	0.898	0.826	0.759	0.649	0.565	0.469	0.408	0.352	0.355	0.328	0.329
10	0.882	0.832	0.662	0.524	0.400	0.318	0.259	0.162	0.212	0.197	0.189
15	0.928	0.805	0.599	0.423	0.305	0.234	0.187	0.166	0.146	0.139	0.137
20	0.897	0.769	0.533	0.338	0.247	0.186	0.153	0.130	0.116	0.103	0.104
25	0.920	0.744	0.487	0.299	0.201	0.156	0.124	0.107	0.096	0.088	0.086

Table A.33 RCE values for $N(0,1)$ when c_{qx} = 100, c_{qy} = 20, c_i = 2.5, α_1 = 40 and α_0 = 8 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.053	0.991	0.973	0.933	0.881	0.845	0.796	0.730	0.732	0.717	0.683
3	1.194	1.165	1.077	0.991	0.930	0.792	0.715	0.718	0.679	0.618	0.606
4	1.272	1.238	**1.139**	0.944	0.878	0.751	0.694	0.603	0.566	0.559	0.544
5	1.325	1.291	1.126	0.968	0.871	0.740	0.614	0.546	0.521	0.492	0.474
10	1.532	1.389	1.111	0.875	0.856	0.642	0.522	0.430	0.376	0.332	0.319
15	1.562	1.366	1.014	0.730	0.535	0.424	0.336	0.289	0.259	0.248	0.232
20	1.663	**1.376**	0.904	0.631	0.443	0.334	0.285	0.216	0.202	0.190	0.189
25	**1.665**	1.345	0.874	0.550	0.376	0.293	0.232	0.193	0.167	0.155	0.152

Table A.34 RCE values for $N(0,1)$ when c_{qx} = 50, c_{qy} = 5, c_i = 5, α_1 = 10 and α_0 = 1 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.184	1.118	1.127	1.058	1.012	0.934	0.870	0.828	0.814	0.818	0.798
3	1.301	1.246	1.206	1.148	**1.051**	0.878	0.795	0.749	0.722	0.709	0.691
4	1.431	1.366	1.256	**1.181**	0.953	0.905	0.743	0.682	0.640	0.613	0.610
5	1.491	1.464	**1.343**	1.106	0.964	0.796	0.682	0.650	0.582	0.540	0.548
10	1.755	1.587	1.251	0.966	0.755	0.589	0.505	0.431	0.396	0.386	0.361
15	1.860	**1.671**	1.154	0.853	0.625	0.474	0.408	0.339	0.295	0.285	0.273
20	1.864	1.620	1.100	0.742	0.507	0.411	0.327	0.272	0.245	0.223	0.212
25	**1.907**	1.597	1.023	0.641	0.446	0.338	0.262	0.233	0.199	0.194	0.187

Table A.35 RCE values for $N(0,1)$ when c_{qx} = 100, c_{qy} = 5, c_i = 5,α_1 = 20 and α_0 = 1 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.317	1.298	1.257	1.150	1.129	**1.109**	**1.009**	0.927	0.896	0.861	0.868
3	1.519	1.514	1.383	1.275	1.175	1.012	0.945	0.851	0.835	0.799	0.803
4	1.761	1.685	1.574	1.370	1.149	1.060	0.967	0.823	0.795	0.736	0.753
5	2.025	1.871	1.612	1.414	**1.179**	1.005	0.894	0.839	0.758	0.700	0.705
10	2.580	2.319	1.811	**1.454**	1.072	0.852	0.726	0.637	0.567	0.536	0.537
15	2.949	2.507	**1.874**	1.275	0.958	0.764	0.604	0.503	0.471	0.441	0.416
20	2.884	**2.586**	1.673	1.121	0.805	0.657	0.492	0.435	0.372	0.351	0.360
25	**3.021**	2.500	1.663	1.067	0.715	0.530	0.439	0.368	0.330	0.310	0.295

Table A.36 RCE values for $N(0, 1)$ when c_{qx} = 50, c_{qy} = 10, c_i = 5, α_1 = 10 and α_0 = 2 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.029	0.970	0.988	0.926	0.886	0.805	0.771	0.751	0.712	0.693	0.688
3	1.121	1.073	1.002	0.931	0.822	0.742	0.692	0.626	0.608	0.594	0.588
4	1.131	1.129	**1.047**	0.941	0.802	0.715	0.620	0.548	0.549	0.500	0.500
5	1.217	1.159	1.044	0.894	0.755	0.621	0.562	0.503	0.467	0.453	0.432
10	1.328	**1.229**	0.977	0.714	0.566	0.437	0.368	0.326	0.292	0.270	0.267
15	1.336	1.186	0.902	0.591	0.450	0.359	0.276	0.248	0.213	0.198	0.200
20	**1.354**	1.138	0.792	0.520	0.378	0.279	0.224	0.187	0.171	0.161	0.159
25	1.320	1.129	0.718	0.470	0.320	0.231	0.193	0.158	0.145	0.135	0.131

Table A.37 RCE values for $N(0, 1)$ when c_{qx} = 100, c_{qy} = 10, c_i = 5, α_1 = 20 and α_0 = 2 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.189	1.137	1.151	1.076	0.986	0.961	0.903	0.851	0.839	0.844	0.840
3	1.395	1.338	1.291	1.200	**1.058**	0.993	0.851	0.793	0.772	0.744	0.717
4	1.576	1.489	1.363	1.200	1.053	0.934	0.844	0.725	0.674	0.687	0.659
5	1.712	1.590	1.402	**1.232**	1.040	0.887	0.781	0.699	0.655	0.627	0.585
10	2.031	1.793	1.428	1.156	0.885	0.657	0.583	0.495	0.464	0.431	0.423
15	2.248	1.915	**1.435**	1.057	0.755	0.541	0.474	0.381	0.357	1.017	0.322
20	2.343	**1.928**	1.306	0.871	0.611	0.468	0.371	0.319	0.285	0.265	0.257
25	**2.424**	1.835	1.235	0.824	0.527	0.401	0.333	0.270	0.252	0.219	0.222

Table A.38 RCE values for $N(0, 1)$ when c_{qx} = 50, c_{qy} = 20, c_i = 5, α_1 = 10 and α_0 = 4 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	0.822	0.808	0.791	0.739	0.699	0.656	0.640	0.602	0.569	0.568	0.553
3	0.862	0.800	0.757	0.693	0.645	0.581	0.523	0.490	0.472	0.449	0.433
4	0.866	0.805	0.783	0.670	0.571	0.529	0.453	0.430	0.385	0.356	0.372
5	0.892	0.835	0.732	0.636	0.545	0.476	0.403	0.360	0.338	0.313	0.306
10	0.889	0.817	0.660	0.491	0.382	0.303	0.259	0.224	0.199	0.188	0.188
15	0.858	0.771	0.562	0.401	0.291	0.220	0.193	0.156	0.141	0.138	0.132
20	0.882	0.691	0.498	0.337	0.197	0.180	0.150	0.119	0.109	0.105	0.102
25	0.850	0.706	0.442	0.291	0.200	0.148	0.120	0.100	0.089	0.083	0.081

Table A.39 RCE values for $N(0,1)$ when c_{qx} = 100, c_{qy} = 20, c_i = 5, α_1 = 20 and α_0 = 4 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.018	1.022	1.028	0.953	0.881	0.838	0.803	0.752	0.702	0.689	0.693
3	1.158	1.155	1.060	0.948	0.902	0.777	0.722	0.658	0.608	0.613	0.598
4	1.286	1.200	**1.061**	0.943	0.882	0.762	0.650	0.591	0.549	0.512	0.532
10	1.446	1.276	1.044	0.824	0.625	0.482	0.402	0.366	0.325	0.305	0.312
15	1.487	**1.307**	0.985	0.683	0.499	0.379	0.315	0.274	0.235	0.222	0.218
20	1.480	1.294	0.873	0.589	0.420	0.312	0.265	0.218	0.187	0.178	0.176
25	**1.539**	1.186	0.816	0.510	0.348	0.257	0.209	0.172	0.155	0.148	0.146

Table A.40 RCE values for $U(0,1)$ when c_{qx} = 50, c_{qy}= 5, c_i = 2.5, α_1 = 20 and α_0 = 2 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.207	1.194	1.155	1.092	1.023	0.969	0.912	0.876	0.845	0.802	0.796
3	1.413	1.375	1.275	1.164	1.046	0.929	0.853	0.788	0.742	0.704	0.719
4	1.644	1.586	1.419	1.230	**1.055**	0.914	0.840	0.756	0.705	0.651	0.651
5	1.773	1.688	1.470	**1.248**	1.032	0.885	0.750	0.681	0.629	0.587	0.572
10	2.323	2.062	**1.573**	1.139	0.868	0.696	0.572	0.499	0.434	0.427	0.413
15	2.605	2.183	1.495	1.004	0.722	0.532	0.439	0.379	0.341	0.322	0.323
20	2.764	**2.194**	1.378	0.862	0.602	0.454	0.377	0.318	0.281	0.270	0.261
25	**2.801**	2.152	1.277	0.779	0.532	0.416	0.324	0.267	0.246	0.228	0.226

Table A.41 RCE values for $U(0,1)$ when c_{qx} = 50, c_{qy} = 10, c_i = 2.5, α_1 = 20 and α_0 = 4 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.045	1.034	1.025	0.947	0.878	0.807	0.808	0.768	0.733	0.712	0.693
3	1.192	1.114	1.066	0.990	0.897	0.781	0.695	0.661	0.615	0.619	0.597
4	1.327	1.266	**1.168**	0.991	0.823	0.739	0.634	0.606	0.564	0.545	0.522
5	1.407	1.335	1.098	0.960	0.807	0.689	0.576	0.524	0.489	0.466	0.475
10	1.677	**1.549**	1.014	0.961	0.655	0.497	0.423	0.357	0.310	0.306	0.296
15	1.719	1.487	0.877	0.692	0.478	0.380	0.316	0.266	0.233	0.223	0.224
20	1.806	1.499	0.877	0.626	0.419	0.312	0.245	0.215	0.187	0.180	0.179
25	**1.930**	1.439	0.852	0.538	0.358	0.253	0.210	0.171	0.161	0.142	0.143

Table A.42 RCE values for $U(0,1)$ when c_{qx} = 100, c_{qy} = 10, c_i = 2.5, α_1 = 40 and α_0 = 4 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.248	1.253	1.168	1.135	1.028	0.971	0.927	0.862	0.873	0.819	0.817
3	1.460	1.414	1.359	1.191	**1.098**	0.997	0.868	0.815	0.788	0.752	0.749
4	1.672	1.644	1.470	1.248	1.096	0.959	0.844	0.755	0.725	0.691	0.682
5	1.966	1.824	1.560	**1.276**	1.096	0.925	0.802	0.724	0.687	0.614	0.647
10	2.482	2.295	**1.683**	1.257	0.949	0.767	0.614	0.543	0.467	0.467	0.436
15	2.842	2.371	1.618	1.174	0.797	0.641	0.521	0.431	0.401	0.359	0.360
20	3.135	**2.480**	1.531	0.984	0.698	0.505	0.429	0.369	0.328	0.288	0.291
25	**3.237**	2.415	1.446	0.864	0.596	0.454	0.345	0.300	0.269	0.247	0.252

Table A.43 RCE values for $U(0,1)$ when c_{qx} = 50, c_{qy} = 20, c_i = 2.5, α_1 = 20 and α_0 = 8 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	0.831	0.822	0.798	0.726	0.686	0.654	0.615	0.606	0.588	0.567	0.547
3	0.900	0.836	0.797	0.714	0.644	0.599	0.549	0.496	0.467	0.433	0.434
4	0.935	0.922	0.793	0.709	0.619	0.515	0.479	0.430	0.399	0.378	0.371
5	0.972	0.924	0.802	0.642	0.578	0.465	0.414	0.362	0.345	0.324	0.336
10	1.060	0.952	0.699	0.527	0.401	0.320	0.259	0.231	0.208	0.200	0.194
15	1.071	0.902	0.626	0.414	0.307	0.244	0.195	0.161	0.148	0.142	0.133
20	1.091	0.876	0.549	0.357	0.250	0.186	0.150	0.129	0.114	0.106	0.104
25	**1.094**	0.856	0.492	0.311	0.193	0.150	0.122	0.103	0.096	0.087	0.086

Table A.44 RCE values for $U(0,1)$ when c_{qx} = 100, c_{qy} = 20, c_i = 2.5, α_1 = 40 and α_0 = 8 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.053	1.054	1.012	0.940	0.920	0.841	0.791	0.753	0.736	0.728	0.711
3	1.206	1.185	1.102	1.008	0.920	0.802	0.743	0.672	0.646	0.617	0.596
4	1.302	1.323	1.161	**1.034**	0.884	0.783	0.681	0.610	0.591	0.537	0.548
5	1.433	1.358	**1.183**	1.010	0.838	0.702	0.608	0.548	0.521	0.481	0.485
10	1.714	1.566	1.182	0.850	0.645	0.507	0.415	0.381	0.341	0.326	0.307
15	1.913	**1.591**	1.116	0.719	0.513	0.414	0.311	0.282	0.249	0.237	0.230
20	1.932	1.569	0.967	0.625	0.422	0.330	0.256	0.222	0.206	0.189	0.189
25	**2.016**	1.537	0.904	0.539	0.380	0.283	0.223	0.177	0.166	0.156	0.158

Table A.45 RCE values for $U(0,1)$ when c_{qx} = 50, c_{qy} = 5, c_i = 5, α_1 = 10 and α_0 = 1 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.202	1.161	1.142	1.090	**1.005**	0.939	0.871	0.844	0.813	0.791	0.770
3	1.421	1.355	1.232	1.124	0.993	0.902	0.811	0.765	0.716	0.682	0.680
4	1.550	1.430	1.363	**1.168**	0.997	0.771	0.755	0.673	0.640	0.631	0.608
5	1.693	1.566	1.386	1.154	0.943	0.808	0.695	0.622	0.599	0.563	0.563
10	2.013	1.805	**1.390**	0.994	0.772	0.612	0.496	0.444	0.402	0.376	0.374
15	2.147	**1.892**	1.327	0.859	0.625	0.472	0.377	0.342	0.296	0.279	0.274
20	2.298	1.848	1.141	0.742	0.518	0.388	0.311	0.265	0.237	0.217	0.219
25	**2.361**	1.765	1.054	0.671	0.442	0.338	0.252	0.219	0.192	0.192	0.183

Table A.46 RCE values for $U(0,1)$ when c_{qx} = 100, c_{qy} = 5, c_i = 5, α_1 = 20 and α_0 = 1 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.323	1.302	1.253	1.187	1.117	1.043	0.988	0.937	0.857	0.890	0.901
3	1.605	1.568	1.465	1.331	1.204	1.073	0.989	0.917	0.859	0.833	0.815
4	1.827	1.756	1.602	1.400	1.203	**1.083**	0.926	0.851	0.794	0.778	0.763
5	2.151	2.036	1.750	**1.459**	**1.222**	1.029	0.885	0.810	0.737	0.700	0.694
10	2.892	2.600	**2.012**	1.427	1.063	0.835	0.710	0.611	0.546	0.536	0.515
15	3.264	2.798	1.923	1.291	0.924	0.732	0.576	0.507	0.434	0.424	0.416
20	3.659	**2.947**	1.866	1.189	0.817	0.606	0.494	0.418	0.370	0.360	0.345
25	**3.767**	2.878	1.728	1.020	0.703	0.537	0.413	0.363	0.328	0.295	0.294

Table A.47 RCE values for $U(0,1)$ when c_{qx} = 50, c_{qy} = 10, c_i = 5, α_1 = 10 and α_0 = 2 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.108	1.017	0.983	0.932	0.877	0.823	0.752	0.742	0.711	0.699	0.708
3	1.223	1.100	1.019	0.954	0.863	0.780	0.692	0.654	0.614	0.597	0.585
4	1.393	1.209	**1.081**	0.933	0.802	0.691	0.624	0.561	0.525	0.519	0.509
5	1.408	1.238	1.078	0.917	0.736	0.646	0.562	0.498	0.457	0.440	0.436
10	1.653	1.350	1.030	0.747	0.572	0.440	0.369	0.329	0.295	0.281	0.270
15	1.731	**1.351**	0.926	0.615	0.440	0.340	0.273	0.235	0.215	0.202	0.198
20	1.837	1.313	0.825	0.534	0.367	0.269	0.229	0.184	0.170	0.162	0.155
25	**1.861**	1.260	0.748	0.440	0.303	0.222	0.184	0.152	0.135	0.127	0.127

Table A.48 RCE values for $U(0,1)$ when c_{qx} = 100, c_{qy} = 10, c_i = 5, α_1 = 20 and α_0 = 2 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.257	1.199	1.156	1.092	1.023	0.961	0.910	0.876	0.845	0.827	0.796
3	1.546	1.410	1.300	1.164	1.046	0.935	0.878	0.788	0.742	0.722	0.719
4	1.708	1.588	1.434	1.230	**1.055**	0.932	0.839	0.756	0.705	0.673	0.651
5	1.952	1.657	1.457	**1.248**	1.032	0.867	0.770	0.681	0.629	0.610	0.572
10	2.390	2.084	**1.607**	1.139	0.868	0.687	0.581	0.499	0.434	0.410	0.413
15	2.608	2.148	1.495	1.004	0.722	0.546	0.466	0.379	0.341	0.323	0.323
20	2.865	**2.284**	1.392	0.862	0.602	0.455	0.378	0.318	0.281	0.271	0.261
25	**3.138**	2.198	1.249	0.779	0.532	0.396	0.305	0.267	0.246	0.228	0.226

Table A.49 RCE values for $U(0,1)$ when c_{qx} = 50, c_{qy} = 20, c_i = 5, α_1 = 10 and α_0 = 4 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	0.832	0.829	0.801	0.748	0.704	0.668	0.633	0.603	0.580	0.567	0.557
3	0.878	0.845	0.785	0.710	0.632	0.566	0.526	0.485	0.457	0.441	0.435
4	0.913	0.866	0.779	0.695	0.603	0.522	0.463	0.423	0.400	0.379	0.371
5	0.956	0.869	0.754	0.647	0.541	0.457	0.404	0.353	0.334	0.315	0.319
10	1.003	0.928	0.697	0.514	0.396	0.303	0.252	0.218	0.201	0.187	0.184
15	**1.068**	0.886	0.616	0.411	0.292	0.224	0.177	0.156	0.142	0.130	0.125
20	1.051	0.826	0.510	0.339	0.238	0.181	0.143	0.123	0.112	0.103	0.099
25	1.039	0.792	0.478	0.282	0.192	0.145	0.114	0.096	0.088	0.082	0.081

Table A.50 RCE values for $U(0,1)$ when c_{qx} = 100, c_{qy} = 20, c_i = 5, α_1 = 40 and α_0 = 4 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.052	0.985	0.953	0.959	0.901	0.852	0.768	0.772	0.744	0.726	0.693
3	1.166	1.153	1.069	0.968	0.858	0.772	0.716	0.640	0.608	0.594	0.597
4	1.313	1.265	1.136	0.980	0.838	0.726	0.649	0.584	0.548	0.531	0.522
5	1.436	1.315	**1.147**	0.936	0.653	0.655	0.585	0.513	0.484	0.469	0.475
10	1.667	1.490	1.139	0.849	0.501	0.517	0.416	0.368	0.336	0.312	0.296
15	1.809	**1.499**	1.027	0.702	0.501	0.390	0.323	0.273	0.246	0.234	0.224
20	**1.884**	1.468	0.933	0.564	0.394	0.298	0.241	0.204	0.191	0.179	0.179
25	1.850	1.426	0.822	0.514	0.349	0.257	0.210	0.175	0.159	0.151	0.143

Table A.51 RCE values for $Exp(1)$ when c_{qx} = 50, c_{qy} = 5, c_i = 2.5, α_1 = 20 and α_0 = 2 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	$Relative\ Cost\ Efficiency$										
2	1.140	1.193	1.014	1.034	0.986	1.011	0.887	0.832	0.832	0.789	0.819
3	1.148	1.166	1.126	1.037	0.958	0.900	0.818	0.784	0.740	0.719	0.742
4	1.228	1.205	**1.216**	**1.084**	0.934	0.837	0.814	0.728	0.701	0.663	0.669
5	1.452	1.255	1.139	1.076	0.928	0.814	0.718	0.669	0.605	0.630	0.587
10	1.430	**1.311**	1.127	0.962	0.769	0.636	0.525	0.489	0.449	0.442	0.419
15	1.387	1.304	1.020	0.754	0.624	0.489	0.436	0.370	0.347	0.318	0.321
20	**1.515**	1.264	0.977	0.723	0.540	0.417	0.363	0.310	0.287	0.261	0.254
25	1.421	1.193	0.928	0.640	0.467	0.349	0.291	0.259	0.239	0.221	0.213

Table A.52 RCE values for $Exp(1)$ when c_{qx} = 50, c_{qy} = 10, c_i = 2.5, α_1 = 20 and α_0 = 4 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	$Relative\ Cost\ Efficiency$										
2	0.971	0.949	0.915	0.880	0.839	0.757	0.766	0.735	0.710	0.734	0.701
3	1.041	0.916	0.947	0.839	0.788	0.748	0.692	0.641	0.631	0.607	0.608
4	0.995	0.980	0.924	0.863	0.729	0.653	0.638	0.572	0.564	0.526	0.536
5	1.014	0.973	0.914	0.827	0.741	0.626	0.558	0.524	0.502	0.475	0.455
10	**1.027**	**1.033**	0.792	0.671	0.544	0.454	0.409	0.350	0.320	0.302	0.293
15	1.017	0.877	0.735	0.548	0.430	0.349	0.293	0.246	0.240	0.232	0.228
20	0.972	0.825	0.662	0.478	0.350	0.281	0.237	0.207	0.187	0.179	0.175
25	0.978	0.771	0.586	0.416	0.300	0.247	0.206	0.169	0.151	0.148	0.145

Table A.53 RCE values for $Exp(1)$ when c_{qx} = 100, c_{qy} = 10, c_i = 2.5, α_1 = 40 and α_0 = 4 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	$Relative\ Cost\ Efficiency$										
2	1.152	1.084	1.060	1.051	0.932	0.903	0.907	0.884	0.824	0.783	0.792
3	1.202	1.217	1.203	1.045	**1.007**	0.919	0.859	0.752	0.775	0.742	0.750
4	1.262	1.279	1.217	1.082	0.993	0.920	0.829	0.771	0.715	0.682	0.689
5	1.368	1.385	1.191	**1.128**	0.934	0.849	0.736	0.707	0.657	0.651	0.614
10	1.490	**1.455**	**1.250**	1.038	0.841	0.682	0.611	0.515	0.483	0.455	0.463
15	**1.782**	1.439	1.191	0.868	0.699	0.560	0.482	0.424	0.363	0.361	0.359
20	1.779	1.376	1.100	0.766	0.617	0.479	0.407	0.340	0.318	0.311	0.283
25	1.646	1.386	1.024	0.729	0.511	0.396	0.339	0.299	0.270	0.254	0.244

Table A.54 RCE values for $Exp(1)$ when c_{qx} = 50, c_{qy} = 20, c_i = 2.5, α_1 = 20 and α_0 = 4 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	$Relative\ Cost\ Efficiency$										
2	0.766	0.749	0.722	0.694	0.662	0.598	0.605	0.580	0.561	0.580	0.553
3	0.746	0.760	0.675	0.641	0.579	0.548	0.528	0.490	0.462	0.440	0.447
4	0.738	0.709	0.671	0.627	0.541	0.486	0.441	0.416	0.402	0.387	0.365
5	0.686	0.681	0.636	0.560	0.491	0.447	0.408	0.356	0.332	0.313	0.323
10	0.653	0.657	0.504	0.427	0.346	0.289	0.260	0.223	0.204	0.192	0.187
15	0.597	0.538	0.451	0.340	0.264	0.209	0.182	0.163	0.144	0.137	0.133
20	0.583	0.495	0.397	0.287	0.210	0.169	0.142	0.124	0.112	0.107	0.105
25	0.548	0.485	0.358	0.244	0.186	0.144	0.119	0.103	0.094	0.086	0.084

Table A.55 RCE values for $Exp(1)$ when c_{qx} = 100, c_{qy} = 20, c_i = 2.5, α_1 = 40 and α_0 = 8 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	$Relative\ Cost\ Efficiency$										
2	0.980	0.958	0.924	0.888	0.847	0.765	0.774	0.742	0.717	0.742	0.707
3	0.971	1.027	0.961	0.932	0.795	0.776	0.689	0.641	0.637	0.615	0.609
4	1.085	1.050	0.945	0.879	0.825	0.759	0.654	0.606	0.553	0.536	0.556
5	1.043	**1.065**	0.954	0.858	0.783	0.638	0.578	0.545	0.510	0.479	0.475
10	1.063	0.989	0.860	0.703	0.568	0.483	0.408	0.368	0.339	0.318	0.323
15	1.075	0.943	0.780	0.630	0.449	0.358	0.317	0.278	0.252	0.236	0.235
20	1.014	0.851	0.689	0.501	0.393	0.310	0.256	0.217	0.204	0.185	0.184
25	**1.144**	0.836	0.700	0.441	0.315	0.251	0.211	0.185	0.164	0.155	0.154

Table A.56 RCE values for $Exp(1)$ when c_{qx} = 50, c_{qy} = 5, c_i = 5, α_1 = 10 and α_0 = 1 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	$Relative\ Cost\ Efficiency$										
2	1.048	1.049	1.034	0.951	0.975	0.909	0.828	0.813	0.819	0.788	0.763
3	1.097	1.107	1.027	**1.061**	0.950	0.847	0.794	0.771	0.731	0.695	0.642
4	1.242	1.186	**1.102**	0.98	0.909	0.774	0.702	0.673	0.623	0.644	0.601
5	1.204	1.120	1.082	0.998	0.797	0.765	0.666	0.620	0.600	0.575	0.537
10	1.268	**1.146**	0.987	0.837	0.668	0.563	0.479	0.429	0.391	0.377	0.369
15	**1.316**	1.116	0.959	0.705	0.535	0.431	0.360	0.328	0.299	0.279	0.282
20	1.240	1.090	0.827	0.588	0.453	0.340	0.302	0.253	0.233	0.222	0.212
25	1.197	1.088	0.755	0.519	0.403	0.303	0.254	0.217	0.203	0.187	0.180

Table A.57 RCE values for $Exp(1)$ when c_{qx} = 100, c_{qy} = 5, c_i = 5, α_1 = 20 and α_0 = 1 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.222	1.176	1.131	1.102	1.024	**1.061**	**1.001**	0.909	0.894	0.884	0.893
3	1.366	1.319	1.263	1.122	**1.101**	1.012	0.949	0.865	0.866	0.807	0.799
4	1.443	1.441	1.304	1.153	1.045	1.007	0.873	0.837	0.758	0.714	0.795
5	1.497	1.444	1.352	1.160	1.096	0.929	0.824	0.778	0.750	0.719	0.691
10	1.759	1.726	**1.458**	**1.183**	0.991	0.790	0.677	0.636	0.561	0.521	0.531
15	1.833	1.670	1.386	1.037	0.839	0.660	0.543	0.483	0.443	0.422	0.419
20	**1.941**	**1.728**	1.336	0.953	0.744	0.555	0.463	0.417	0.364	0.357	0.336
25	1.932	1.694	1.230	0.851	0.620	0.518	0.395	0.367	0.331	0.296	0.299

Table A.58 RCE values for $Exp(1)$ when c_{qx} = 50, c_{qy} = 10, c_i = 5, α_1 = 10 and α_0 = 2 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	0.943	0.928	0.871	0.878	0.884	0.793	0.753	0.719	0.705	0.674	0.693
3	0.942	0.913	0.871	0.841	0.759	0.731	0.682	0.639	0.573	0.573	0.568
4	0.958	0.918	0.889	0.779	0.707	0.649	0.591	0.539	0.518	0.514	0.508
5	0.997	0.938	0.880	0.880	0.665	0.580	0.533	0.484	0.454	0.444	0.440
10	0.966	0.911	0.763	0.621	0.502	0.398	0.369	0.320	0.302	0.281	0.271
15	0.875	0.887	0.666	0.505	0.382	0.319	0.269	0.233	0.218	0.202	0.200
20	0.952	0.724	0.598	0.404	0.323	0.258	0.208	0.183	0.169	0.157	0.165
25	0.874	0.728	0.535	0.372	0.280	0.221	0.179	0.157	0.139	0.135	0.134

Table A.59 RCE values for $Exp(1)$ when c_{qx} = 100, c_{qy} = 10, c_i = 5, α_1 = 20 and α_0 = 2 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.108	1.090	1.023	1.031	1.038	0.932	0.885	0.845	0.829	0.792	0.815
3	1.159	1.138	1.155	1.024	**1.004**	0.914	0.842	0.785	0.748	0.759	0.712
4	1.215	1.213	1.061	0.949	0.857	0.739	0.703	0.703	0.678	0.674	0.667
5	1.294	1.264	**1.180**	**1.042**	0.924	0.859	0.755	0.645	0.642	0.632	0.575
10	**1.476**	1.392	1.165	0.949	0.767	0.608	0.563	0.488	0.461	0.429	0.414
15	1.414	**1.432**	1.077	0.817	0.617	0.515	0.435	0.376	0.352	0.326	0.323
20	1.459	1.237	0.993	0.717	0.525	0.422	0.355	0.310	0.280	0.268	0.262
25	1.404	1.312	0.905	0.653	0.466	0.369	0.302	0.263	0.244	0.227	0.222

Table A.60 RCE values for $Exp(1)$ when c_{qx} = 50, c_{qy} = 20, c_i = 5, α_1 = 10 and α_0 = 4 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	0.727	0.765	0.731	0.673	0.650	0.627	0.567	0.586	0.582	0.568	0.568
3	0.716	0.694	0.662	0.639	0.577	0.556	0.518	0.486	0.436	0.435	0.432
4	0.684	0.679	0.677	0.593	0.530	0.479	0.413	0.393	0.379	0.377	0.373
5	0.688	0.704	0.609	0.557	0.495	0.435	0.371	0.353	0.337	0.311	0.310
10	0.634	0.590	0.507	0.413	0.341	0.283	0.235	0.212	0.201	0.184	0.186
15	0.565	0.539	0.436	0.318	0.258	0.205	0.170	0.151	0.139	0.133	0.123
20	0.556	0.471	0.378	0.273	0.200	0.161	0.135	0.118	0.107	0.102	0.100
25	0.525	0.473	0.321	0.236	0.171	0.128	0.114	0.096	0.086	0.086	0.077

Table A.61 RCE values for $Exp(1)$ when c_{qx} = 100, c_{qy} = 20, c_i = 5, α_1 = 20 and α_0 = 4 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	0.934	0.921	0.873	0.885	0.857	0.798	0.758	0.750	0.694	0.705	0.669
3	0.976	0.947	0.902	0.872	0.786	0.758	0.707	0.663	0.594	0.594	0.589
4	1.010	0.973	0.920	0.844	0.786	0.670	0.594	0.570	0.528	0.535	0.525
5	**1.062**	**1.006**	0.869	0.851	0.743	0.634	0.577	0.498	0.509	0.475	0.459
10	1.037	0.965	0.830	0.677	0.559	0.456	0.380	0.360	0.315	0.315	0.323
15	1.003	0.964	0.713	0.555	0.410	0.367	0.290	0.261	0.239	0.225	0.231
20	0.932	0.864	0.644	0.507	0.349	0.285	0.240	0.211	0.183	0.171	0.175
25	0.933	0.840	0.570	0.420	0.304	0.230	0.202	0.171	0.153	0.152	0.137

Table A.62 RCE values for $\Gamma(2,1)$ when c_{qx} = 50, c_{qy} = 5, c_i = 2.5, α_1 = 10 and α_0 = 2 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.102	1.113	1.071	1.082	0.946	0.961	0.888	0.883	0.844	0.833	0.749
3	1.299	1.306	1.153	1.100	1.007	0.918	0.843	0.784	0.762	0.721	0.730
4	1.321	1.302	1.250	**1.109**	**1.035**	0.885	0.789	0.734	0.696	0.634	0.674
5	1.453	1.441	**1.355**	1.107	0.989	0.883	0.737	0.669	0.620	0.589	0.614
10	1.616	1.529	1.268	1.007	0.771	0.660	0.536	0.506	0.461	0.418	0.418
15	1.679	**1.600**	1.176	0.870	0.644	0.534	0.435	0.375	0.362	0.326	0.323
20	1.728	1.542	1.145	0.775	0.548	0.449	0.364	0.311	0.287	0.262	0.261
25	**1.749**	1.541	1.012	0.667	0.489	0.382	0.300	0.267	0.239	0.228	0.225

Table A.63 RCE values for $\Gamma(2,1)$ when c_{qx} = 50, c_{qy} = 10, c_i = 2.5, α_1 = 20 and α_0 = 4 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	0.951	1.011	0.978	0.938	0.891	0.839	0.801	0.724	0.701	0.687	0.711
3	1.058	1.044	0.992	0.930	0.847	0.784	0.733	0.686	0.642	0.619	0.596
4	1.118	1.050	0.966	0.873	0.764	0.681	0.631	0.574	0.534	0.530	0.510
5	1.118	1.065	0.967	0.846	0.725	0.628	0.570	0.546	0.505	0.484	0.462
10	1.160	**1.119**	0.918	0.723	0.560	0.465	0.402	0.355	0.327	0.311	0.314
15	**1.191**	1.070	0.810	0.607	0.459	0.367	0.302	0.262	0.235	0.219	0.227
20	1.096	0.988	0.712	0.507	0.309	0.292	0.243	0.209	0.189	0.180	0.182
25	1.174	0.982	0.664	0.455	0.284	0.238	0.200	0.176	0.160	0.150	0.145

Table A.64 RCE values for $\Gamma(2,1)$ when c_{qx} = 100, c_{qy} = 10, c_i = 2.5, α_1 = 40 and α_0 = 4 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.131	1.145	1.070	1.065	0.982	0.951	0.925	0.819	0.870	0.843	0.808
3	1.365	1.310	1.140	1.155	1.007	0.908	0.861	0.833	0.780	0.751	0.751
4	1.462	1.376	1.282	1.196	0.997	0.932	0.807	0.772	0.748	0.713	0.672
5	1.518	1.523	1.374	**1.213**	**1.039**	0.883	0.776	0.728	0.661	0.632	0.622
10	1.819	1.653	**1.420**	1.088	0.845	0.668	0.606	0.530	0.487	0.451	0.445
15	2.021	**1.732**	1.290	0.981	0.726	0.588	0.479	0.431	0.387	0.364	0.357
20	2.014	1.644	1.227	0.879	0.614	0.482	0.395	0.345	0.321	0.303	0.294
25	**2.023**	1.646	1.145	0.770	0.554	0.398	0.341	0.303	0.271	0.251	0.246

Table A.65 RCE values for $\Gamma(2,1)$ when c_{qx} = 50, c_{qy} = 20, c_i = 2.5, α_1 = 20 and α_0 = 8 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	0.788	0.798	0.741	0.707	0.670	0.694	0.614	0.582	0.550	0.579	0.524
3	0.809	0.778	0.760	0.678	0.625	0.578	0.518	0.464	0.459	0.461	0.452
4	0.780	0.750	0.718	0.661	0.559	0.491	0.454	0.422	0.392	0.375	0.375
5	0.806	0.737	0.682	0.596	0.508	0.446	0.402	0.367	0.334	0.329	0.317
10	0.781	0.712	0.577	0.464	0.358	0.299	0.250	0.221	0.206	0.205	0.189
15	0.707	0.656	0.488	0.379	0.270	0.218	0.182	0.165	0.142	0.133	0.133
20	0.737	0.593	0.428	0.318	0.229	0.171	0.142	0.123	0.113	0.110	0.101
25	0.673	0.581	0.415	0.269	0.189	0.143	0.119	0.103	0.094	0.087	0.085

Table A.66 RCE values for $\Gamma(2,1)$ when c_{qx} = 100, c_{qy} = 20, c_i = 2.5, α_1 = 40 and α_0 = 8 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.018	0.949	0.982	0.874	0.863	0.832	0.801	0.774	0.715	0.721	0.715
3	1.089	1.045	1.015	0.923	0.834	0.746	0.694	0.670	0.644	0.637	0.607
4	1.133	1.080	1.017	0.927	0.812	0.729	0.664	0.596	0.604	0.541	0.533
5	1.189	1.124	**1.028**	0.918	0.754	0.664	0.597	0.556	0.514	0.486	0.479
10	1.236	1.136	1.003	0.770	0.597	0.479	0.398	0.371	0.334	0.328	0.325
15	**1.242**	**1.159**	0.842	0.632	0.499	0.367	0.303	0.278	0.254	0.238	0.230
20	1.223	1.125	0.785	0.563	0.397	0.313	0.263	0.219	0.208	0.190	0.181
25	1.231	1.083	0.708	0.477	0.321	0.263	0.202	0.192	0.169	0.158	0.149

Table A.67 RCE values for $\Gamma(2,1)$ when c_{qx} = 50, c_{qy} = 5, c_i = 5, α_1 = 10 and α_0 = 1 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.120	1.107	1.072	1.028	0.965	0.907	0.867	0.860	0.797	0.823	0.790
3	1.245	1.225	1.154	**1.070**	0.976	0.894	0.801	0.741	0.712	0.689	0.700
4	1.272	1.242	1.140	1.032	0.906	0.805	0.714	0.686	0.645	0.629	0.589
5	1.372	1.334	**1.196**	1.055	0.901	0.796	0.702	0.625	0.545	0.539	0.553
10	1.501	**1.416**	1.148	0.907	0.695	0.571	0.501	0.501	0.384	0.369	0.352
15	1.436	1.313	0.968	0.729	0.539	0.443	0.373	0.373	0.313	0.292	0.276
20	**1.545**	1.383	0.958	0.662	0.467	0.380	0.317	0.264	0.235	0.224	0.214
25	1.440	1.255	0.845	0.577	0.407	0.306	0.250	0.226	0.197	0.188	0.182

Table A.68 RCE values for $\Gamma(2,1)$ when c_{qx} = 100, c_{qy} = 5, c_i = 5, α_1 = 20 and α_0 = 1 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.222	1.167	1.143	1.138	1.018	1.012	0.914	0.945	0.877	0.853	0.900
3	1.403	1.445	1.308	1.229	**1.148**	**1.035**	0.881	0.918	0.820	0.813	0.837
4	1.619	1.492	1.424	1.254	1.117	0.986	0.904	0.835	0.782	0.726	0.775
5	1.654	1.693	1.558	**1.327**	1.082	0.998	0.850	0.801	0.759	0.718	0.715
10	2.033	1.940	**1.642**	1.295	0.984	0.841	0.678	0.625	0.563	0.536	0.510
15	2.185	1.979	1.612	1.187	0.847	0.666	0.570	0.511	0.448	0.420	0.423
20	2.273	**2.133**	1.475	0.978	0.762	0.585	0.475	0.409	0.365	0.349	0.347
25	**2.287**	2.002	1.409	0.946	0.691	0.514	0.405	0.371	0.321	0.313	0.301

Table A.69 RCE values for $\Gamma(2,1)$ when c_{qx} = 50, c_{qy} = 10, c_i = 5, α_1 = 10 and α_0 = 2 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	0.928	0.952	0.922	0.887	0.839	0.798	0.764	0.755	0.726	0.708	0.701
3	1.027	1.037	0.985	0.919	0.827	0.760	0.706	0.662	0.625	0.612	0.583
4	1.049	1.028	0.958	0.862	0.758	0.677	0.601	0.560	0.513	0.501	0.495
5	1.066	**1.040**	0.929	0.811	0.695	0.597	0.524	0.493	0.460	0.444	0.429
10	1.061	1.014	0.831	0.664	0.510	0.426	0.360	0.327	0.295	0.282	0.279
15	1.020	0.972	0.720	0.537	0.411	0.328	0.271	0.239	0.213	0.205	0.199
20	**1.108**	0.903	0.640	0.459	0.329	0.258	0.216	0.192	0.171	0.161	0.165
25	1.057	0.962	0.627	0.417	0.287	0.224	0.184	0.158	0.139	0.130	0.128

Table A.70 RCE values for $\Gamma(2,1)$ when c_{qx} = 100, c_{qy} = 10, c_i = 5, α_1 = 20 and α_0 = 2 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	1.133	1.150	1.088	1.060	0.983	0.924	0.896	0.829	0.853	0.829	0.768
3	1.301	1.249	1.158	1.094	**1.031**	0.900	0.842	0.834	0.751	0.711	0.713
4	1.365	1.309	1.248	**1.145**	0.986	0.897	0.807	0.724	0.680	0.642	0.654
5	1.447	1.438	1.262	1.105	0.966	0.896	0.734	0.662	0.630	0.599	0.600
10	1.658	**1.593**	**1.269**	1.015	0.810	0.843	0.568	0.496	0.437	0.408	0.406
15	1.741	1.539	1.227	0.897	0.672	0.671	0.442	0.387	0.347	0.321	0.322
20	**1.862**	1.516	1.128	0.775	0.549	0.534	0.362	0.308	0.275	0.265	0.258
25	1.714	1.544	1.016	0.699	0.491	0.445	0.310	0.265	0.240	0.221	0.214

Table A.71 RCE values for $\Gamma(2,1)$ when c_{qx} = 50, c_{qy} = 20, c_i = 5, α_1 = 10 and α_0 = 4 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	0.788	0.775	0.750	0.719	0.676	0.635	0.580	0.609	0.580	0.524	0.543
3	0.809	0.784	0.738	0.685	0.625	0.572	0.518	0.462	0.456	0.463	0.425
4	0.780	0.745	0.684	0.619	0.544	0.483	0.448	0.408	0.380	0.382	0.359
5	0.806	0.762	0.684	0.603	0.515	0.455	0.387	0.362	0.323	0.318	0.312
10	0.781	0.708	0.574	0.453	0.348	0.286	0.240	0.208	0.193	0.185	0.183
15	0.707	0.618	0.456	0.343	0.253	0.209	0.169	0.154	0.141	0.130	0.130
20	0.737	0.629	0.435	0.301	0.212	0.173	0.143	0.118	0.108	0.101	0.098
25	0.673	0.558	0.376	0.256	0.181	0.136	0.109	0.099	0.085	0.081	0.080

Table A.72 RCE values for $\Gamma(2,1)$ when c_{qx} = 100, c_{qy} = 20, c_i = 5, α_1 = 20 and α_0 = 4 by using CRSS

p	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
n	*Relative Cost Efficiency*										
2	0.975	0.964	0.924	0.865	0.848	0.811	0.804	0.728	0.698	0.742	0.716
3	1.058	1.049	0.951	0.913	0.806	0.783	0.671	0.668	0.650	0.625	0.599
4	1.127	1.046	**1.011**	0.932	0.791	0.704	0.625	0.589	0.555	0.531	0.504
5	1.152	1.126	1.010	0.883	0.748	0.621	0.542	0.533	0.501	0.476	0.471
10	**1.222**	**1.137**	0.946	0.734	0.577	0.473	0.403	0.353	0.329	0.299	0.315
15	1.201	1.077	0.838	0.604	0.457	0.360	0.311	0.256	0.238	0.231	0.225
20	1.157	1.023	0.721	0.520	0.378	0.297	0.237	0.207	0.182	0.177	0.178
25	1.139	1.064	0.686	0.452	0.325	0.247	0.199	0.172	0.155	0.151	0.153