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**SUPPLIER SELECTION USING TOPSIS AND VIKOR
UNDER FUZZY ENVIRONMENT**

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ABSTRACT

SUPPLIER SELECTION USING TOPSIS AND VIKOR UNDER FUZZY ENVIRONMENT

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Supplier selection is a very important multi-criteria decision making problem. In real world, after determining the conflicting criteria, alternatives and opinions from different experts, a group of decision-makers rate criteria and alternatives with respect to criteria with crisp values or linguistic variables. In this study, supplier selection of a private hospital is thoroughly analyzed. To solve the problem, first a group of decision-makers are determined, the criteria and alternatives are chosen and each decision-maker rates the alternatives with respect to criteria using linguistic variables. These linguistic variables are shown with trapezoidal fuzzy numbers and decision matrices are formed. Then, problem is solved by fuzzy VIKOR and fuzzy TOPSIS methods, and the results are analyzed in a comparative way.

Keywords: supplier selection, multi-criteria decision making, group decision making, fuzzy TOPSIS, fuzzy VIKOR.

ÖZET

BULANIK VIKOR VE BULANIK TOPSIS YÖNTEMLERİ İLE TEDARİKÇİ SEÇİMİ

ZINGİL, Tuğba

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Tedarikçi seçimi önemli bir çok ölçütlü karar verme problemidir. Gerçek hayatta, çelişen ölçütler, alternatifler ve farklı görüşlere sahip uzmanlar belirlendikten sonra karar vericiler ölçütleri ve alternatifleri ölçütlere göre sayısal değerler veya sözel değişkenler ile derecelendirirler. Bu çalışmada, İstanbul'da bir özel hastanenin tedarikçi seçim problemi kapsamlı bir şekilde irdelenmektedir. Problemi çözmek için ilk olarak karar verici grup belirlenir, ölçütler ile alternatifler seçilir ve her bir karar verici sözel değişkenler ile alternatifleri ölçütlere göre derecelendirir. Bu çalışmada sözel değişkenler yamuk bulanık sayılar ile gösterilmekte ve karar matrisi oluşturulmaktadır. Problem bulanık VIKOR ve bulanık TOPSIS yöntemleriyle çözülmüş ve sonuçlar karşılaştırılmalı olarak irdelenmiştir.

Anahtar Kelimeler: tedarikçi seçimi, çok ölçütlü karar verme, grup karar verme, bulanık VIKOR, bulanık TOPSIS

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1. INTRODUCTION

A supplier manufactures or distributes or sells items to a customer and provides goods or services to a company. Suppliers may function as distributors of goods, or they function as manufacturers of goods. If they manufacture goods, they build to stock rather than build to order. Selecting an appropriate supplier is a serious and multi-dimensional process. Supplier selection is not only related with price; other factors affect the opinions of managements, especially the purchasing departments.

In today's conditions for selecting a right supplier, it is necessary to take right steps. As well as the purchasing department's supplier searching, the good conclusion of the research is also very important. If the intended efficiency is not achieved with the suppliers, it needed to look for another supplier. If the firms make a mistake while choosing the right supplier, then unsuccessful relation is guaranteed. This situation means losing time, losing market and extra cost for the management.

Choosing the right supplier involves much more than scanning a series of price lists. The choice depends on a wide range of factors such as value for money, quality and service. The importance of these factors depends on the business priorities and strategies. There are number of key factors that should be looked for when identifying and short listing possible suppliers. Good suppliers should be able to demonstrate that they can offer the benefits such as value for money, reliability, quality, strong service and clear communication, financial security, and partnership approach.

The lowest price does not always mean the best value for money. If the reliability and the quality from the supplier are expected, it has to be decided how much money is willing to be paid for the supplier, and the balance that is wanted to strike between cost, reliability, quality, and service. If the supplied product lets the firm down, the firm may have problems with their customers as a chain reaction. Thus, the products that are supplied should be reliable. The quality of the supplies needs to be consistent. The customers wait high quality from the firms. The supplies must be delivered on time. If the delivered period gets longer then, the firms need to give the suppliers plenty of

warning. The best supplier wants to talk regularly to find out what needs the firm have and how they can serve the firm better in the future.

It is always worth to making sure that the supplier has sufficiently strong cash flow to deliver what is wanted, when it is needed. A credit check will help reassure the firm that the supplier won't go out of business when the firm needs them most. A strong relationship benefits both sides. The suppliers have to know how important of the firm's custom. Thus, they make every effort to provide the best service possible. A strategic approach to choose suppliers can also help understanding how the potential customers weigh up their purchasing decision. The most effective suppliers offer product or services that match or exceed the needs of the business. While searching for suppliers, the needs of business have to be taken into consideration. For example, if the firm wants to cut down the time it takes to serve its customers, suppliers that offer the firm faster delivery rate higher than those that compete on price alone. It has to be examined how many suppliers that are really needed. Buying from a carefully targeted group could have number of benefits;

- it will be easier to control the suppliers,
- the business will become more important to suppliers,
- the company may be able to make deals that give the company an extra competitive advantage.

It is very important to have a choice of sources. Buying from only one supplier can be dangerous. If the supplier let the firm down, where can the firms go? Equally, while exclusivity may spur some suppliers to offer the firm better service, other may simply become complacent and drop their standards.

If the firm has got a clear idea of what it needs to buy and identifies some potential suppliers, the firm can build a shortlist of sources that meet the needs by asking these questions:

- Can these suppliers deliver what is wanted, when it is wanted?
- Are they financially secured?
- How long they have been established?

- Has anyone used the items before and recommended them?
- Are they on any approved supplier lists from trade associations, local or central government?

Once a manageable shortlist is done, the potential customer can approach by asking for a written quotation. It is best to provide them with a clear brief summarizing what is required, how frequently it is required and what level of business is hoped to place. It is worth asking potential suppliers to give a firm price in writing for, say, two months. The discounts can be asked for long term or high volume contacts. When a quotation is got, the management has to compare the potential suppliers in terms of what matters most. For example, the quality and the reliability of the product may be most important for a firm, while the location of supplier may not matter. Price is important, but it shouldn't be the only reason to choose a supplier. Lower price may reflect poorer quality goods and services which, in long term, may not be the most cost effective option. The firm has to be confident that its supplier can make a sufficient margin at the price quoted for the business to be commercially viable.

Wherever possible it is always a good idea to meet a potential supplier face to face and see how their business operates. The firm may see how the supplier's machines are, if they are old or not, they can check out the stock area and control if the conditions are healthy, or not. It makes good business sense to consider the ethical and environmental dimensions of the supply chain. The suppliers have to obey the environmental rules that the government or international foundations are forced. After setting on the suppliers, a contract that includes negotiating terms and condition has to be drawn up.

Supplier evaluation and selection problem is a serious problem and it has been studied extensively. Various decision-making approaches have been proposed to tackle the problem. In today's supply chain management, the performance of potential suppliers is evaluated against multiple criteria rather than considering a single factor. The contemporary supply management is to maintain long term partnership with suppliers, and use fewer but reliable suppliers. Therefore, to choose the right suppliers, it is needed to scan a series of price list and also the choices have to depend on wide range

of factors, which involve both quantitative and qualitative data. Extensive multi-criteria decision-making approaches have been proposed for supplier selection, such as analytic hierarchy process (AHP), analytic network process (ANP), case-based reasoning (CBR), data envelopment analysis (DEA), fuzzy set theory, genetic algorithm (GA), mathematical programming, and simple multi-attribute rating technique (SMART).

Normally, the supplier selection problem in supply chain is a group decision-making problem, under multiple criteria. The degree of uncertainty, the number of decision-makers and the nature of the criteria have to be taken into account while solving this problem. The decision-makers always express their preferences on alternatives of suppliers, which can be used to rank the suppliers or in selecting the most preferable ones. The preferences on different suppliers are decision-maker's subjective judgments. Generally, decision maker's judgments are not certain and cannot be estimated by exact numerical values. Under many conditions, crisp data are inadequate to model real-life situations; human judgments, including preferences, are often vague and preferences cannot be estimated in exact numerical values. In recent years, fuzzy set approaches have been proposed to deal with the supplier selection problem under uncertainty [8]. A more realistic approach may be to use linguistic assessments, instead of numerical values. In other words, ratings and weights of the criteria in the problem are assessed by means of linguistic variables. In reality, there is no avoidance of the coexistence of qualitative and quantitative data. In order to mediate the conflicts and contradictions in the reconciliatory process and act in response to the lack of flexibility while adopting traditional multi-criteria methods to solve the problems with inherent fuzziness, this study intends to use fuzzy VIKOR and fuzzy TOPSIS methods to solve the fuzzy multi-criteria decision-making problem. Fuzzy VIKOR method provides measurements of determining the aggregate distance to the ideal point and aims to find the decision-maker's preferable compromise that suits human objective cognition. On the other hand, fuzzy TOPSIS is employed to rank the alternatives considering distances to ideal and negative ideal solutions under a fuzzy environment. In this thesis, the comparison of fuzzy TOPSIS and fuzzy VIKOR is aimed via a real-world supplier selection study.

2. SUPPLY CHAIN MANAGEMENT

Supply chain management (SCM) is the oversight of materials, information and finances as they move in a process from supplier to manufacturer to wholesaler to retailer to customer. [28]

Table 2.1: History of Supply Chain Management. [36]

	1970's	1980's	1990's-Now
The Markets	<ul style="list-style-type: none"> • Focus on customer loyalty, • Quality is king, • Product engineering is competitive advantage. 	<ul style="list-style-type: none"> • Market demands variety, • Cost is king-technology drives manufacturing efficiencies, • Global market developing. 	<ul style="list-style-type: none"> • Throw away consumerism-product life measured of blink speed, • Cost is still king, but manufacturing has nothing left to give, • Global competition, • Global markets.
The Supply Chain	<ul style="list-style-type: none"> • Vertically integrated enterprises, • Primarily domestic, • Highly regulated, • Not managed beyond the extended enterprise, • Rigid, stable, slow but predictable, • Managed by function. 	<ul style="list-style-type: none"> • Deregulation, • Learning to manage global supply demand beginning of horizontal management craze, • Managed through functional collaboration (ERP hysteria), • Fragmented and unpredictable 	<ul style="list-style-type: none"> • Technologically enabled, • Service explosion, • The network is the enterprise, • Dynamic, agile and reconfigurable, • Supply chain as a strategic imperative.

In a supply chain system, all partners should be informed simultaneously, and the information they receive should be sufficient for making their own decision. The objective of a supply chain is customer satisfaction.

In table 2.1, supply chain management is analyzed according to market situation and supply chain events that have been used for the concept of supply chain management over the last 40 years. In 1970's, the essence of SCM was understood, this first phase is characterized as an inventory push era that focused primarily on physical distribution of finished goods. In 1980's, productivity could be increased significantly by managing relationships, information and material flow across enterprise borders. And in 1990's, computers change the way business is done, internal revolutionized the information pathway and the distribution system of the business, e-commerce has changed the definition of business itself.

Structuring the supply chain requires an understanding of the demand patterns, service requirements, distance considerations, cost elements and other related factors. It is easy to see that these factors are highly variable in nature and this variability needs to be considered during the supply chain analysis process [46]. In addition, the interplay of these complex considerations could have a significant bearing on the outcome of the supply chain analysis process.

A simple supply chain is made up of several elements that are linked by the movement of products along it. The supply chain elements are;

- **Location:** It's needed to be known where production facilities, stocking points and sourcing points are located; this information determines the paths along which goods will flow.
- **Production:** An organization must decide what products will be created at which plants, which suppliers will service those plants, which plants will supply specific

distribution centers, and, sometimes, how goods will get to the final customer. These decisions have a big impact on revenue, costs and customer service.

- **Inventory:** Each link in the supply chain has to keep a certain inventory of raw materials, parts, subassemblies and other goods on hand as a buffer against uncertainties. Shutting down an assembly plant because an expected part shipment didn't arrive is expensive. But inventory costs money too, so it's important to manage deployment strategies, determine efficient order quantities and reorder points, and set safety stock levels.

- **Transportation:** How do materials, parts and products get from one link in the supply chain to the next? Choosing the best way to transport goods often involves trading off the shipping cost against the indirect cost of inventory. For example, shipping by air is generally fast and reliable. Shipping by sea will likely be cheaper, especially for bulky goods and large quantities, but slower and less reliable. So if you ship by sea, you have to plan further in advance and keep larger inventories than you do if you ship by air.

Supply chain management includes coordinating and integrating the flows both within and among companies. It is said that the main aim of any effective supply chain management system is to reduce inventory. Supply chain management flows can be divided into three main flows:

- The product flow,
- The information flow,
- The finances flow.

The product flow involves the movement of goods from a supplier to a customer, as well as any customer returns or service needs. The information flow includes transmitting orders and updating the status of delivery. The financial flow consists of credit terms, payment schedules, consignment and title ownership arrangements.

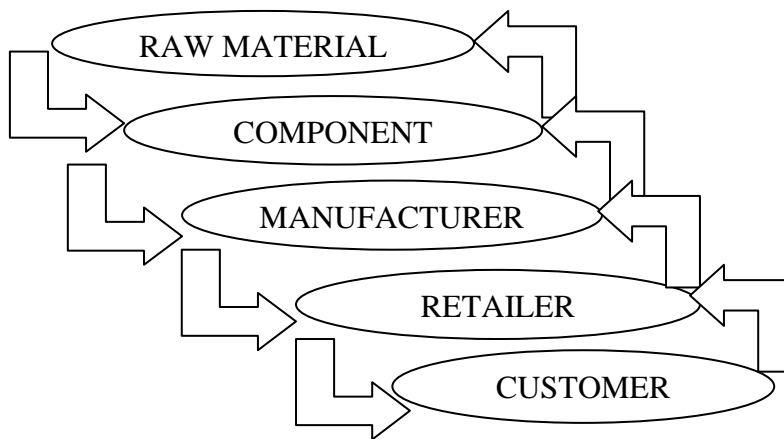


Figure 2.1: Key Supply Chain Management Concepts [70]

In the figure 1.1 each interface in the supply chain represents movement of goods, information flows, transfer of title, purchase and sale. Strategic supply chain management consists of developing smarter ways to choose, to buy from and to sell to your business partners.

Global economies have marked that significant challenges to companies wanting to fulfill the continuously changing requirements of the cost reduction, speedy time to-market and customization; they place increasing emphasis on SCM and establish a sounder strategic alliance against competitors. The major aims of SCM are to reduce production costs, maximize revenue, improve customer service, optimize inventory levels, business processes, and cycle times, and resulting in increased competitiveness, customer satisfaction and profitability.

SCM has recently received considerable attention in both academia and industry. If supply chain management is compared with materials management and logistics, materials management describes the material handling part of the movement of the material and components within the factory or firm and logistics describes the entire process of material and products moving into, through, and out of a firm however supply chain management is conceptualized as something even larger than logistics that links logistics more directly with the user's total communications network and with the firm's engineering staff.

3. LITERATURE REVIEW

Nowadays, many firms give extra importance on supplier selection for maximizing their revenue, reducing their total costs and improving situations. Thus, purchasing department plays important roles on firm's profit gaining. In such situation academic researches help industry and in academia supplier selection problems are studied in many different ways. Gaballa [47] is the first person who applied mathematical programming to a supplier selection in a real case. He used a mixed integer programming model to formulate this decision making problem. The objective of this programming is to minimize the total discounted price of allocated items to the suppliers, under constraints of suppliers' capacity and demand satisfaction. Sharma et al. [48] proposed a non-linear, mixed integer, goal programming model for supplier selection. They considered price, quality, delivery and service in their model, in which all criteria are considered as goals. The cost goal is decreased in relation to the increase in purchased quantity and is raised in relation to the increase in quality level. Chaudhry et al. [49] developed linear and mixed integer programming for supplier selection. In their model price, delivery, quality and quantity discount are included. The objective of the model is to minimize aggregate price by considering both cumulative and incremental discounts. Quality and delivery are included as constraints.

In spite of the importance of supplier selection problems only a few articles have addressed the decision making. Weber and Current [50, 51] stated that only a few articles analyzed the problem up to the time of their review. A comprehensive review of the articles which have addressed the problem can be found in Ghodsypour and O'Brien [7]. The most important articles are as follows: Weber and Current [50] used multi-objective linear programming for supplier selection to systematically analyze the trade-off between conflicting factors. In this model aggregate price, quality and late delivery are considered as goals, and two sets of constraints are taken into account: (1) systems' constraints, which are defined as the constraints which are not directly under the control of the purchasing managers such as vendor capacities, demand satisfaction, minimum order quantities established by the vendors and the total purchasing budget; and (2) policy constraints, including maximum and/or minimum order quantities purchased

from a particular supplier, and the maximum and/or minimum number of vendors to be employed. Current and Weber [51] proposed that mathematical constructs of facility location modeling can be applied to supplier selection. They did not solve any special supplier problem but they showed the similarities between the supplier selection problem and facility layout models. The complexity of both location models and supplier selection problems indicates that fitting these two methods together cannot be easy. Weber [20] developed a data envelopment analysis formulation for measuring vendor efficiency and showed how a baby food manufacturer applied DEA technique in a just-in-time environment. Ghodsypour and O'Brien [52] developed a decision support system for reducing the number of suppliers and managing the supplier's partnership. They used integrated analytical hierarchy process (AHP) with mixed integer programming and considered suppliers' capacity constraint and the buyers' limitations on budget and quality etc. Ghodsypour and O'Brien [53] proposed a model to deal with supplier selection, multiple sourcing, multiple criteria and discounted price. They considered the effects of limitations on budget, quality and suppliers' capacity. Ghodsypour and O'Brien [54] developed an integrated AHP and linear programming model to help managers consider both qualitative and quantitative factors in their purchasing activity in a systematic approach. They proposed an algorithm for sensitivity analysis to consider different scenarios in this decision making. Ghodsypour and O'Brien [22] introduced a mixed integer non-linear programming model to solve the multiple sourcing problems, which takes into account the total cost of logistics including net price, storage, and transportation and ordering costs.

Actually, the supplier selection problem is a group decision making problem, under multiple criteria. The degree of uncertainty, the number of decision-makers (DMs) and the nature of the criteria have to be taken into account while solving this problem. The decision-makers always express their preferences on alternatives or on attributes of suppliers, which can be used to rank the suppliers or in selecting the most desirable ones. The preferences on different suppliers and on attributes are DMs' subjective judgments. In conventional multi-criteria decision making (MCDM) methods, ratings and weights of the attributes are known precisely [30, 57]. Barbarasoğlu and Yazgaç [21] used an analytic hierarchy process model to solve supplier selection problem in

Turkish Electric Industry Inc. Shyur and Shih [23] formulated a vendor evaluation problem with the combined use of the multi-criteria decision making approach and proposed five-step hybrid process which incorporates the analytic network process (ANP). Then the modified TOPSIS was adopted to rank competing products in their overall performances. Bottani and Rizzi [24] focused on a subject that a wide number of vendors and purchased items exist, and these alternatives are needed to be reduced. Their approach integrated cluster analysis and multi-criteria decision making techniques.

Generally, DMs' judgments are uncertain and cannot be estimated by exact numerical values. Under many conditions, crisp data are inadequate to model real-life situations; human judgments, including preferences, are often vague and preferences cannot be estimated in exact numerical values. Decision-making in supplier selection problem includes a high degree fuzziness and uncertainty. Fuzzy set theory is one of the effective tools to handle uncertainty and vagueness. A more realistic approach may be to use linguistic assessments, instead of numerical values. In other words, ratings and weights of the criteria in the problem are assessed by means of linguistic variables [58, 27]. The fuzzy set theory offers a possibility of handling data and information involving subjective characteristics of human nature in the decision-making process. Zimmermann [38] illustrated a fuzzy set approach to multi-objective decision-making. He has compared some approaches to solve multi-attribute decision-making problems based on the fuzzy set theory. Yager [59, 60] presented a fuzzy multi-attribute decision-making method that uses crisp weights, and he introduced an ordered weighted aggregation operator and investigated its properties. To improve the fuzzy set, Gau and Buehrer [61] proposed the vague set theory. Then, based on the vague set theory, Chen and Tan [62] presented some new techniques for handling multi-criteria fuzzy decision-making problems. Chen et al. [33] presented a hierarchy model based on fuzzy sets theory to deal with the supplier selection problem. The linguistic values were used to assess the ratings and weights for the supplier evaluating factors. These linguistic ratings could be expressed by trapezoidal or triangular fuzzy numbers. The proposed model was capable of dealing with both qualitative and quantitative criteria. Kumar et al. [64] developed a fuzzy multi-objective integer programming vendor selection problem model and in the

proposed model, various input parameters have been treated as vague with linear membership function of fuzzy type. Jun [63] extended the study on vague sets-based MCDM method.

The foundation for compromise solution was established by Yu and Zeleny [65, 41]. The compromise solution is a feasible solution, which is the closest to the ideal, and a compromise means an agreement established by mutual concessions. TOPSIS (technique for order preference by similarity to ideal solution) generates the best compromise alternatives as the solution nearest to the positive ideal solution [30]. The TOPSIS method determines the solution with the shortest distance to the ideal solution and the greatest distance from the negative-ideal solution, but it does not consider the relative importance of these distances. A multi-criteria intuitionistic fuzzy group decision-making for supplier selection with TOPSIS method was studied by Boran et al. [25], they used the TOPSIS method combined with intuitionistic fuzzy set to select appropriate supplier in group decision-making environment. The VIKOR method was introduced as one applicable technique to implement within MCDM [32]. The VIKOR method compromise ranking determines a compromise solution, providing a maximum “group utility” for the “majority” and a minimum of an individual regret for the “opponent”. Sanayei [8] used linguistic values to assess the ratings and weights for the factors. These linguistic ratings can be expressed by trapezoidal or triangular fuzzy numbers. Then a hierarchy MCDM model based on fuzzy sets theory and VIKOR method was proposed to deal with the supplier selection problems in the supply chain system. Opricovic and Tzeng [5] made a comparative analysis of TOPSIS method and VIKOR method with a numerical example, showing their similarity and some differences.

4. MULTI-CRITERIA DECISION MAKING TECHNIQUES

Multiple criteria decision making (MCDM) was presented as an important field of study in the early 1970's. Since then the number of contributions to theories and models, which could be used as a basis for more systematic and rational decision making with multiple criteria, has continued to grow at a steady rate. A number of surveys, e.g. Bana e Costa [37], show the vitality of the field and the multitude of methods which have been developed. When Bellman and Zadeh [26], and a few years later Zimmermann [38], introduced fuzzy sets into the field, they cleared the way for a new family of methods to deal with problems which had been inaccessible to and unsolvable with standard MCDM techniques [2].

MCDM is a discipline aimed at supporting decision makers who are faced with making numerous and conflicting evaluations. MCDM aims at highlighting these conflicts and deriving a way to come to a compromise in a transparent process [1].

Unlike methods that assume the availability of measurements, measurements in MCDM are derived or interpreted subjectively as indicators of the strength of various preferences. Preferences differ from decision-maker to decision-maker, so the outcome depends on who is making the decision and what their goal and preferences are [1].

MCDM is one of the fastest growing fields of operational research because many concrete problems can only be solved by considering several conflicting criteria. It was described as the most well known branch of decision making. The decision-making process of selecting an appropriate alternative usually has to take many factors into consideration, for instance, organizational needs and goals, risks, benefits, limited resources, etc. The selection process gets complex and challenging if several qualitative and quantitative criteria affect each other mutually while evaluating alternatives.

Decision matrix or decision table is used for collecting criteria outcomes of decision alternatives and it comprises a set of columns and rows. The table rows represent decision alternatives, with table columns representing criteria. A value found at the

intersection of row and column in the table represents a criterion outcome - a measured or predicted performance of a decision alternative on a criterion. The decision matrix is a central structure of the MCDM because it contains the data for comparison of decision alternatives. A multi-criteria decision matrix can be expressed as follows;

$$D = \begin{matrix} & & x_1 & x_2 & \dots & x_n \\ & A_1 & & & & \\ & A_2 & & & & \\ & \vdots & & & & \\ & A_m & & & & \end{matrix} \left(\begin{array}{cccc} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \vdots & \vdots & & \vdots \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{array} \right)$$

where A_i represent the i^{th} alternative, $i = 1, 2, \dots, m$; x_j represent the j^{th} criterion, $j=1, 2, \dots, n$, and x_{ij} is the performance of alternative A_i with respect to the j^{th} criterion. The procedure for determining the best solution to an MCDM problem includes computing the utilities of alternatives and ranking these utilities. The alternative solution with the greatest utility is considered to be the optimal solution.

There are four major families of methods in MCDM:

- i. the outranking approach based on the pioneering work by Bernard Roy [56], and implemented in the ELECTRE and PROMETHEE methods;
- ii. the value and utility theory approaches mainly started by Keeney and Raiffa [55], and then implemented in a number of methods; a special method in this family is the Analytic Hierarchy Process (AHP) developed by Thomas L. Saaty [39] and then implemented in the Expert Choice software package;
- iii. the largest group is the interactive multiple objective programming approach with pioneering work done by P.L. Yu [40], Milan Zeleny [41], and a number of others; the MOLP family has been built around utility theory-based tradeoffs among objectives, with reference point techniques, ideal points, etc and the models have had a number of features including stochastic and integer variables; one of the best interactive methods available is the VIG software package developed by Pekka Korhonen [42];

- iv. group decision and negotiation theory introduced new ways to work explicitly with group dynamics and with differences in knowledge, value systems and objectives among group members.

In many cases, the decision-maker does not have the right information about the alternatives with respect to an attribute. The classical MCDM methods cannot effectively handle problems with such imprecise information. This has led to the development of fuzzy set theory by Zadeh, who proposed that the key elements in human thinking are not numbers but labels of fuzzy sets. Fuzzy set theory is a powerful tool to handle imprecise data and fuzzy expressions that are more natural for humans than rigid mathematical rules and equations. It is obvious that much knowledge in the real world is fuzzy rather than precise [3].

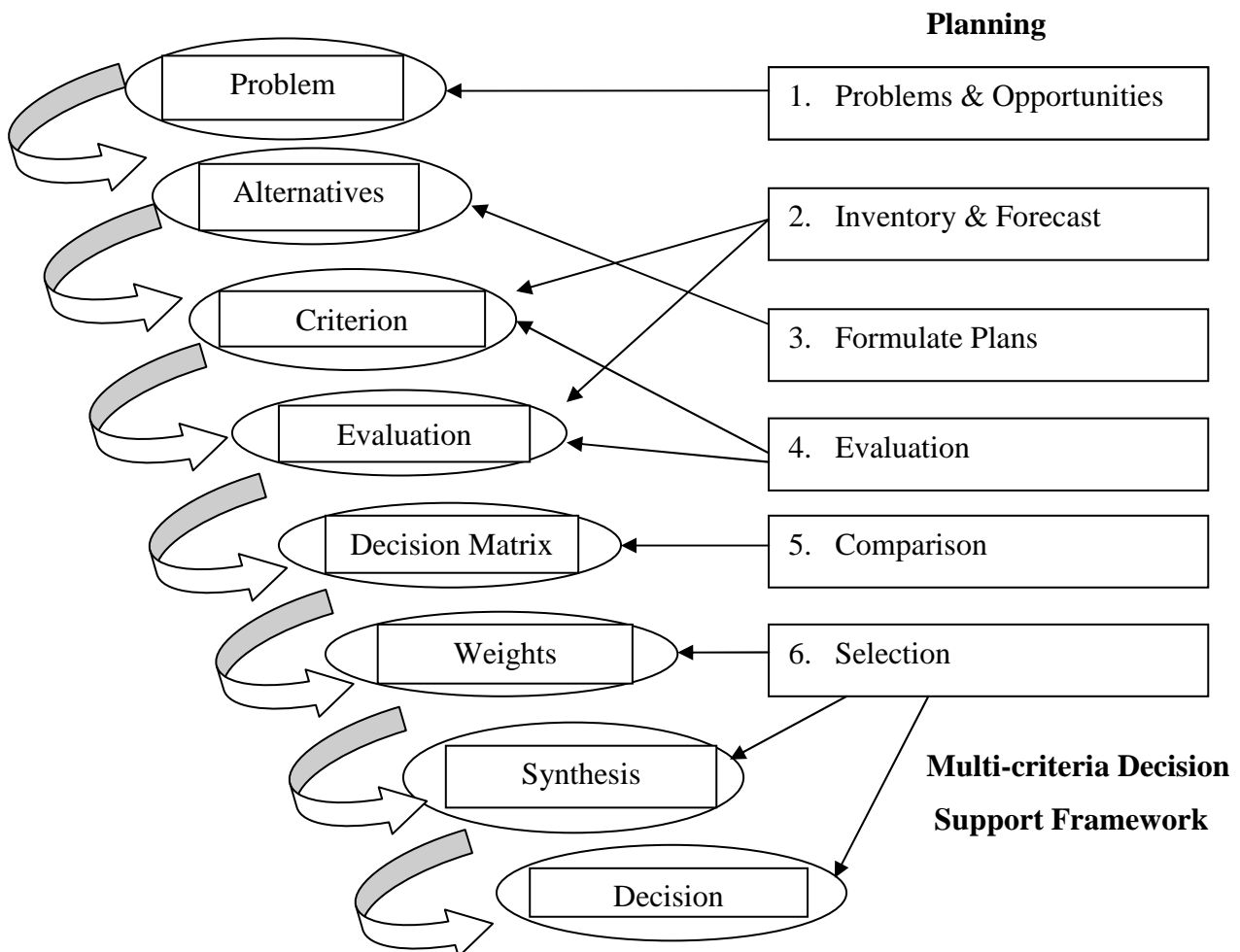


Figure 4.1: Relation of planning process to multi criteria decision support framework [34]

5. FUZZY SET THEORY

5.1 CRISPNESS, VAGUENESS, FUZZINESS, UNCERTAINTY

Most of the traditional tools for formal modeling, reasoning and computing are crisp, deterministic and precise in character. By crisp it is measured dichotomous, that is, yes-or-no type rather than more-or-less type. In conventional dual logic, for instance, a statement can be true or false and nothing in between. In set theory, an element can either belong to a set or not, and in optimization, a solution is either feasible or not.

Real situations are often uncertain or vague in a number of ways. This type of uncertainty or vagueness, stochastic uncertainty in contrast to the vagueness concerning the description of the semantic meaning of events, phenomena or statements themselves, which can be called fuzziness. Fuzziness can be found in many areas of daily life such as in engineering, medicine, meteorology, manufacturing, etc. It is particularly frequent, however, in all areas in which human judgment, evaluation and decision are important. These are the areas of decision-making, reasoning, learning and so on.

5.2 PRELIMINARIES

A fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership function which assigns to each objects a grade of membership ranking between zero and one [26]. Zadeh introduced fuzzy sets as an extension of the classical notion of set [26]. In classical set theory, the membership of elements in a set is assessed in binary terms according to a bivalent condition that means an element either belongs or does not belong to the set. However, fuzzy set theory allows the gradual assessment of the membership of elements in a set; this is shown with the membership function valued in the real unit interval $[0,1]$. Fuzzy set approaches are suitable to use when the modeling of human knowledge is necessary and human evaluations are needed.

While making a satisfactory decision if there exists imprecise and multi-criteria situations a decision-maker has to use fuzzy multi criteria decision making method. Fuzzy MCDM presents fuzzy multi attributes and multi objective decision making methodologies differentiated MCDM researchers. Fuzzy set theory found a large application area in MCDM. Most popular fuzzy multi-criteria methods are fuzzy AHP, fuzzy TOPSIS, fuzzy VIKOR, interactive fuzzy multi-objective stochastic linear programming, fuzzy multi-objective dynamic programming, grey fuzzy multi-objective optimization, etc. [4].

If X is a collection of objects then a fuzzy set \tilde{A} in X is a set of ordered pairs:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X\}$$

The range of the membership function is a subset of the nonnegative real numbers whose supremum is finite. If $\sup_{x \in X} \mu_{\tilde{A}}(x) = 1$, then the fuzzy set \tilde{A} is called normal. The height of a fuzzy set is the largest membership grade obtained by any element in that set. A fuzzy set \tilde{A} in the universe discourse X is called normalized when the height of \tilde{A} is equal to 1. A fuzzy number is a fuzzy subset in the universe of discourse X that is both convex and normal [8].

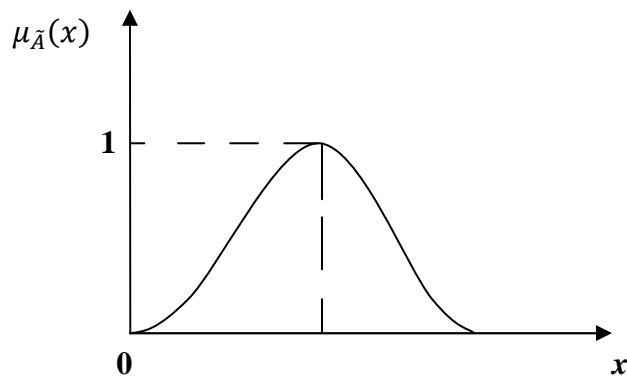


Figure 5.1 A Fuzzy number \tilde{A}

Some other definitions are presented as follows [68]:

- When X is continuous rather than a countable or finite set, the fuzzy set \tilde{A} is denoted as:

$$\tilde{A} = \int \frac{\mu_{\tilde{A}}(x_i)}{x} \text{ where, } x \in X.$$

- When X is countable rather than a continuous or finite set, the fuzzy set \tilde{A} is denoted as:

$$\tilde{A} = \sum \frac{\mu_{\tilde{A}}(x_i)}{x} \text{ where, } x \in X.$$

- The α -cut A_α and strong α -cut $A_{\alpha+}$ of the fuzzy set \tilde{A} in the universe of discourse X is defined by:

$$A_\alpha = \{x_i: \mu_{\tilde{A}}(x_i) \geq \alpha, x_i \in X\}, \text{ where } \alpha \in [0,1]$$

$$A_{\alpha+} = \{x_i: \mu_{\tilde{A}}(x_i) > \alpha, x_i \in X\}, \text{ where } \alpha \in [0,1]$$

A fuzzy number \tilde{A} is a fuzzy set whose membership function is $\mu_{\tilde{A}}(x)$ is called the membership value of $x \in X$ and it represent the degree of certainty that x belongs to fuzzy set [43]. The type of representation of the membership function depends on the base set. If this set consists of many values, or is the base set a continuum, then a parametric representation is appropriate. These functions are adapted for the changing of the parameters. Piecewise linear membership functions are preferred, because of their simplicity and efficiency with respect to computability. Mostly these are trapezoidal or triangular functions, which are defined by four and three parameters. The triangular fuzzy number is denoted as $\tilde{A} = (a, b, c)$ and its membership function is shown as,

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a}, & x \in [a, b] \\ \frac{c-x}{c-b}, & x \in [b, c] \\ 0, & \text{otherwise} \end{cases}$$

Where a and c stand for lower and upper bounds of the fuzzy number \tilde{A} , respectively, b for the modal value. A positive triangular fuzzy number can be defined as (a, b, c) , shown in **Fig 5.2**.

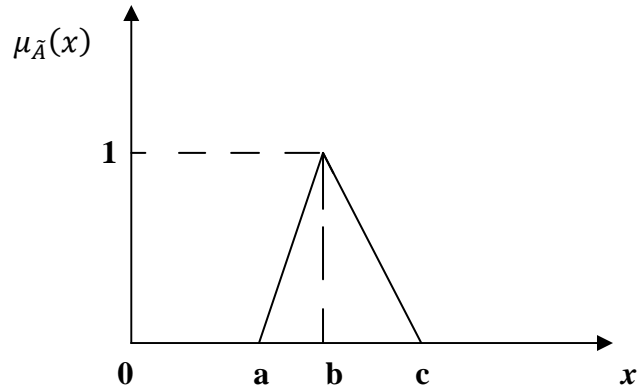


Figure 5.2 Triangular Fuzzy Number \tilde{A}

The trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$ is denoted with the membership function as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a \text{ or } x > d \\ \frac{x-a}{b-a}, & a \leq x \leq b \\ 1, & b < x < c \\ \frac{d-x}{d-c}, & c \leq x \leq d \end{cases}$$

A positive trapezoidal fuzzy number (PTFN)_n can be defined as (a, b, c, d) shown in **Fig 5.3**.

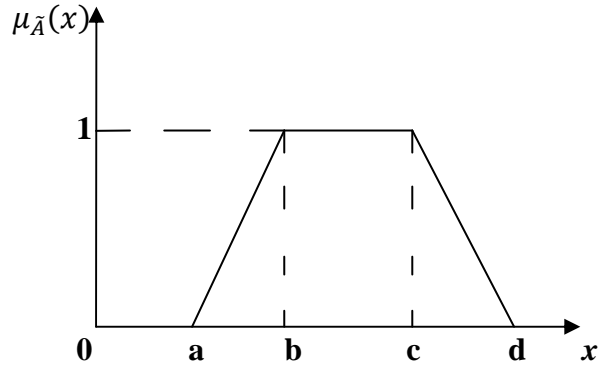


Figure 5.3 Trapezoidal Fuzzy Number \tilde{A}

For a trapezoidal fuzzy number if $b=c$, then the number is called a triangular fuzzy number.

Trapezoidal fuzzy number shows different patterns under different conditions shown in **Table 5.1**,

Table 5.1: Trapezoidal Fuzzy Number under Different Patterns

Different patterns	Trapezoidal fuzzy Numbers
$a=b<c=d$	interval (crisp interval)
$a=b=c=d$	number (crisp number, scalar)
$a<b<c$ or $b<c<d$	fuzzy interval
$a<b=c$ or $b=c<d$	fuzzy number

Let $\tilde{X}_1 = (a_1, b_1, c_1, d_1)$, $\tilde{X}_2 = (a_2, b_2, c_2, d_2)$ are two fuzzy numbers then, some basic arithmetic operators on fuzzy intervals are as follows:

- $\tilde{X}_1 + \tilde{X}_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2)$ (5.1)

- $\tilde{X}_1 - \tilde{X}_2 = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2)$ (5.2)

- $\widetilde{X}_1 * \widetilde{X}_2 = (\min(a_1a_2, a_1d_2, d_1a_2, d_1d_2), \min(b_1b_2, b_1c_2, c_1b_2, c_1c_2), \max(b_1b_2, b_1c_2, c_1b_2, c_1c_2), \max(a_1a_2, a_1d_2, d_1a_2, d_1d_2))$ (5.3)

- $\widetilde{X}_1 / \widetilde{X}_2 = (\min(a_1/a_2, a_1/d_2, d_1/a_2, d_1/d_2), \min(b_1/b_2, b_1/c_2, c_1/b_2, c_1/c_2), \max(b_1/b_2, b_1/c_2, c_1/b_2, c_1/c_2), \max(a_1/a_2, a_1/d_2, d_1/a_2, d_1/d_2))$ (5.4)
 $0 \notin [a_2, d_2]$ (i.e., defined if the support of \widetilde{X}_2 does not contain 0)

Special case: (a, b, c, d) , $b = c$ (triangular fuzzy number), in this case (since $b_1 = c_1$, $b_2 = c_2$), $\widetilde{X}_1 * \widetilde{X}_2$ reduces to;

- $\widetilde{X}_1 * \widetilde{X}_2 = (\min(a_1a_2, a_1d_2, d_1a_2, d_1d_2), b_1b_2, b_1b_2, \max(a_1a_2, a_1d_2, d_1a_2, d_1d_2))$

Since $b = c$, the fuzzy number can be represented as a triple (a, b, d) . By this notation, $\widetilde{X}_1 * \widetilde{X}_2 = (\min(a_1a_2, a_1d_2, d_1a_2, d_1d_2), b_1b_2, \max(a_1a_2, a_1d_2, d_1a_2, d_1d_2))$. In this way all operators $(+, -, *, /)$ are reduced to triple.

Let $\widetilde{X}_1 = (a_1, b_1, c_1, d_1)$, $\widetilde{X}_2 = (a_2, b_2, c_2, d_2)$ are two trapezoidal fuzzy numbers. Then the distance between them can be calculated by using the vertex method as: [45]

$$d_v(\widetilde{X}_1, \widetilde{X}_2) = \sqrt{\frac{1}{4}[(a_1 - a_2)^2 + (b_1 - b_2)^2 + (c_1 - c_2)^2 + (d_1 - d_2)^2]} \quad (5.5)$$

Also the crisp value of the fuzzy number \widetilde{A} based on Center of Area method can be expressed as:

$$\begin{aligned} \text{defuzz}(\widetilde{A}) &= \frac{\int x \cdot \mu(x) dx}{\int \mu(x) dx} \\ &= \frac{-a_1a_2 + a_3a_4 + \frac{1}{3}(a_4 - a_3)^2 - \frac{1}{3}(a_2 - a_1)^2}{-a_1 - a_2 + a_3 + a_4} \end{aligned} \quad (5.6)$$

6. DECISION MAKING TECHNIQUES

6.1 FUZZY VIKOR

The VIKOR method was developed for multi-criteria optimization for complex systems, to find a compromise priority ranking of alternatives according to the selected criteria [31]. A compromise solution for a problem with conflicting criteria can help decision-makers identify an acceptable answer [32].

The VIKOR method solves multiple criteria decision making (MCDM) problems with conflicting or non-commensurable criteria. This method assumes that compromising is acceptable for conflicting resolution. Although the VIKOR method is a popular method applied in multi-criteria analysis, it has some problems when solving MCDM problems.

According to Opricovic [5][29] the multi-criteria measure for compromise ranking is developed from the L_p -metric used as an aggregating function in a compromise programming method. The various m alternatives are denoted as x_1, x_2, \dots, x_m . For alternative x_i , the rating of the j th aspect is denoted by x_{ij} , i.e. x_{ij} is the value of j th criterion function for the alternative x_i ; n is the number of criteria. Development of the VIKOR method started with the following form of L_p -metric:

$$L_{pi} = \left\{ \sum_{j=1}^n [w_j (x_j^* - x_{ij}) / (x_j^* - x_j^-)]^p \right\}^{\frac{1}{p}}$$

$$1 \leq p \leq \infty; \tag{6.1}$$

The number of alternatives and the number of criteria are respectively denoted as m and n . The compromise-ranking algorithm has the following steps:

1. Calculate x_{ij} and determine the maximum x_j^* and the minimum x_j^- values of all criterion functions, $j=1,2,\dots,n$. x_{ij} is the value of j^{th} criterion function for the alternative x_i .

$$x_j^* = \max [(x_{ij}) \mid j=1,2,\dots,n]$$

$$x_j^- = \min [(x_{ij}) \mid j=1,2,\dots,n]$$

2. Compute the values S_i and R_i , $i = 1 \dots m$.

$$S_i = \sum w_j (x_j^* - x_{ij}) / (x_j^* - x_j^-)$$

$$R_i = \max [w_j (x_j^* - x_{ij}) / (x_j^* - x_j^-) \mid j=1,2,\dots,n] \quad (6.2)$$

where S_i and R_i respectively presents the utility measure and regret measure for the alternative x_i . w_j is the weights for the criteria.

3. Compute the values Q_i , $i= 1 \dots m$.

$$Q_i = v (S_j - S^*) / (S^- - S^*) + (1-v) (R_i - R^*) / (R^- - R^*)$$

$$S^* = \min [(S_i) \mid i=1,2,\dots,m]$$

$$S^- = \max [(S_i) \mid i=1,2,\dots,m]$$

$$R^* = \min [(R_i) \mid i=1,2,\dots,m]$$

$$R^- = \max [(R_i) \mid i=1,2,\dots,m] \quad (6.3)$$

4. Rank the alternatives by Q index. The smaller the value Q where Q represents the VIKOR value is the better decision of the alternative is.

In fuzzy VIKOR method;

Step 1: In a problem the inputs are expressed in matrix form as;

$$D = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{bmatrix}$$

$$W = [w_1, w_2, \dots, w_n]$$

where x_{ij} is the rating of alternative, A_i , with respect to C_j and w_j represents the importance weight of the j^{th} criterion holds, $x_{ij} = (x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4})$ and $w_j = (w_{j1}, w_{j2}, w_{j3}, w_{j4})$ for $i = 1, 2, \dots, m, j = 1, 2, \dots, n$, are linguistic variables can be approximated by positive trapezoidal fuzzy numbers.

Step 2: Calculate the aggregated fuzzy ratings (x_{ij}) of alternatives with respect to each criterion;

Let the fuzzy rating and importance weight of the k^{th} decision maker be $x_{ij} = (x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4})$ and $w_j = (w_{j1}, w_{j2}, w_{j3}, w_{j4})$ for $i = 1, 2, \dots, m, j = 1, 2, \dots, n$ respectively. Aggregated fuzzy ratings of alternatives (x_{ij}) are;

$x_{ij} = (x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4})$ where,

$$x_{ij1} = \min_k \{x_{ijk1}\}, x_{ij2} = \frac{1}{K} \sum_{k=1}^K x_{ijk2}, x_{ij3} = \frac{1}{K} \sum_{k=1}^K x_{ijk3}, x_{ij4} = \max_k \{x_{ijk4}\} \quad (6.4)$$

The aggregated fuzzy weights (w_j) of each criterion can be calculated as:

$w_j = (w_{j1}, w_{j2}, w_{j3}, w_{j4})$, where

$$w_{ij1} = \min_k \{w_{ijk1}\}, w_{ij2} = \frac{1}{K} \sum_{k=1}^K w_{ijk2}, w_{ij3} = \frac{1}{K} \sum_{k=1}^K w_{ijk3}, w_{ij4} = \max_k \{w_{ijk4}\} \quad (6.5)$$

Step 3: Defuzzification of the fuzzy decision matrix and fuzzy weight of each criterion into crisp values as follows:

$$\text{defuzz}(\tilde{A}) = \frac{\int x \cdot \mu(x) dx}{\int \mu(x) dx}$$

$$= \frac{-a_1 a_2 + a_3 a_4 + \frac{1}{3} (a_4 - a_3)^2 - \frac{1}{3} (a_2 - a_1)^2}{-a_1 - a_2 + a_3 + a_4} \quad (6.6)$$

Step 4: Determine the best x_j^* and the worst x_j^- vales of all criterion ratings, $j = 1, 2, \dots, n$

$$\begin{aligned} x_j^* &= \max_i x_{ij} ; \\ x_j^- &= \min_i x_{ij} \end{aligned} \quad (6.7)$$

Step 5: Calculate the values S_i and R_i by the relations

$$\begin{aligned} S_i &= \sum w_j (x_i^* - x_{ij}) / (x_j^* - x_j^-) \\ R_i &= \max [w_j (x_j^* - x_{ij}) / (x_j^* - x_j^-) \mid j=1, 2, \dots, n] \end{aligned} \quad (6.8)$$

Step 6: Compute the values Q_i by the relations

$$Q_i = v (S_j - S^*) / (S^- - S^*) + (1-v) (R_i - R^*) / (R^- - R^*) \quad (6.9)$$

Step 7: Rank the alternatives, sorting by the values S , R and Q in ascending order.

Step 8: Propose as a compromise solution the alternative $(A^{(1)})$ which is the best ranked by the measure Q (minimum) if the following two conditions are satisfied

C1. Acceptable advantage:

$$Q(A^{(2)}) - Q(A^{(1)}) \geq DQ \quad (6.10)$$

where $A^{(2)}$ is the alternative with second position in the ranking list by Q ; $DQ=1/(J-1)$

C2. Acceptable stability in decision making:

The alternative $A^{(1)}$ must also be best ranked by S or/and R . This compromise solution is stable within a decision making process, which could be the strategy of maximum group utility (when $v > 0.5$ is needed), or “by consensus” $v \approx 0.5$, or “with veto” ($v < 0.5$). Here v is the weight of decision making strategy of maximum group utility.

If one of the conditions is not satisfied, then a set of compromise solutions is proposed, which includes;

- Alternatives $A^{(1)}$ and $A^{(2)}$ if only the condition C2 is not satisfied, or
- Alternatives $A^{(1)}, A^{(2)} \dots A^{(M)}$ if the condition C1 is not satisfied $A^{(M)}$ is determined by the relation $Q(A^{(M)}) - Q(A^{(1)}) < DQ$ for maximum M [8].

As many multi-criteria decision making tools, The VIKOR method has the following characteristics,

1. The best alternative determined by the VIKOR method is the closeness to the ideal solution.
2. The best alternative according to the VIKOR method has the maximum group utility for decision makers and ensures the least regret.
3. The VIKOR method considers two distance measurements, L_{ij} and $L_{\infty j}$, based on the L_p metric in the compromising programming method to provide information about the utility and regret.
4. The VIKOR method considers two weights in decision-making. One is that of the criteria, the other that of the maximum group utility.

6.2 FUZZY TOPSIS

TOPSIS, the shortened name of the Technique for Order Preference by Similarity to Ideal Solution, is developed by Hwang and Yoon[30]. TOPSIS is used for ranking the preference order of alternatives and determining the optimal choice. TOPSIS is a useful method for multi-attribute decision making and it is simply deal with the chosen alternative's having of the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution. The ideal solution is formed as a composite of best performed values exhibited by any alternative for each attribute. The negative ideal solution is the composite of the worst performance values. Proximity to

each of these performance poles measured in the Euclidean sense with options weighting of each attribute.

Table 6.1: Some applications of TOPSIS

	Application Areas	Number of attributes	Number of alternatives	Published year	Authors
1	Manufacturing plant location analysis	5 attributes,	5 alternatives	1985	Yoon and Hwang[9]
2	Robot selection	4 attributes	27 alternatives	1999	Parkan and Wu [10]
3	Company financial ratios comparison	4 attributes	7 alternatives	2000	Deng et al. [11]
4	Facility location selection	5 attributes	4 alternatives	2002	Chu [12]
5	Solid waste management	12 attributes	11 alternatives	2002	Cheng et al. [13]
6	High-speed transport system selection	15 attributes	3 alternatives	2003	Janic [14]
7	Expatriate host country selection	6 attributes	10 alternatives	2004	Chen and Tzeng [15]
8	Gear material selection	5 attributes	9 alternatives	2005	Milani et al. [16]
9	Multiple response selection	2 attributes	18 alternatives	2005	Yang and Chou [17]
10	Rapid prototyping process selection	6 attributes	6 alternatives	2005	Byun and Lee [18]

There are so many articles related with TOPSIS method. For example, Opricovic and Tzeng conducted a comparative analysis VIKOR and TOPSIS [5], Abo Sinna and Amer

extended TOPSIS methods for solving multi-objective large scale nonlinear programming problems [6]. Jahashahloo developed an algorithmic method to extend TOPSIS for decision making problems with interval data [7].

Fuzzy TOPSIS is an extension of conventional TOPSIS. Fuzzy TOPSIS assigns the importance of attributes and the performance of alternatives with respect to various attributes by using fuzzy numbers instead of precise numbers. TOPSIS needs some steps for involving numerical measures of the relative importance of attributes and the performance of each alternative on these attributes. In real life conditions, that's why exact data may be difficult to be precisely determined. Thus, TOPSIS is extended naturally to fuzzy environment.

TOPSIS includes the following steps:

1. Firstly, calculate the normalized decision matrix. The normalized value r_{ij} is calculated as;

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^m x_{ij}^2}}, i = 1, \dots, m; j = 1, \dots, n. \quad (6.11)$$

Some normalization methods for TOPSIS are as follows:

- i. Vector normalization

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^m x_{ij}^2}}, i = 1, \dots, m; j = 1, \dots, n.$$

- ii. Linear normalization (1)

$$r_{ij} = \frac{x_{ij}}{x_j^*}, i = 1, \dots, m; j = 1, \dots, n; x_j^* = \max_i \{x_{ij}\} \text{ for benefit attributes}$$

$$r_{ij} = 1 - \frac{x_{ij}}{x_j^*}, i = 1, \dots, m; j = 1, \dots, n; x_j^* = \max_i \{x_{ij}\} \text{ for cost}$$

attributes.

iii. Linear normalization (2)

$$r_{ij} = \frac{x_{ij} - x_j^-}{x_j^* - x_j^-}, \text{ for benefit attributes}$$

$$r_{ij} = \frac{x_j^* - x_{ij}}{x_j^* - x_j^-}, \text{ for cost attributes}$$

2. Calculate the weighted normalized decision matrix. The weighted normalized value v_{ij} is calculated as;

$$v_{ij} = w_j r_{ij}, \quad i = 1, \dots, m; j = 1, \dots, n, \quad (6.12)$$

where, w_j is the weight of j^{th} criterion and $\sum_{j=1}^n w_j = 1$.

3. Determine the ideal and negative-ideal solutions as:

$$\begin{aligned} A^* &= \{v_1^*, \dots, v_n^*\} \\ &= \{(\max_i v_{ij} \mid j \in \check{I}), (\min_i v_{ij} \mid j \in \check{I}^c)\}, \\ A^- &= \{v_1^-, \dots, v_n^-\} \\ &= \{(\min_i v_{ij} \mid j \in \check{I}), (\max_j v_{ij} \mid j \in \check{I}^c)\}, \end{aligned} \quad (6.13)$$

Where, \check{I} is related with benefit criteria, and \check{I}^c is related with cost criteria.

4. Calculate the separation measures, using the n -dimensional Euclidean distance. The separation of each alternative from the ideal solution is given as;

$$D_i^* = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^*)^2}, \quad i = 1, \dots, m.$$

Similarly, the separation from the negative ideal solution is given as;

$$D_i^- = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^-)^2}, \quad i = 1, \dots, m. \quad (6.14)$$

5. Calculate of the relative closeness to the ideal solution. The relative closeness of the alternative a_j with respect to A^* is defined as;

$$C_i^* = \frac{D_i^-}{D_i^* + D_i^-}, \quad i = 1, \dots, m. \quad (6.15)$$

6. Rank the preference order according to decreasing values of C_i^* .

In the fuzzy TOPSIS procedure, the criteria weights and characteristic values of criteria ($x_{ij}; i = 1, 2, \dots, m, j = 1, 2, \dots, n$) are inputs and placed in matrix form, $x_{ij}^{\sim k}$ be the score which k th expert have allocated to i th alternative with respect to j th attribute, as shown in Step 1[35][27].

Step 1: Inputs are expressed in matrix form as;

$$D^{\sim k} = \begin{bmatrix} x_{11}^{\sim k} & x_{12}^{\sim k} & \dots & x_{1j}^{\sim k} \\ x_{21}^{\sim k} & x_{22}^{\sim k} & \dots & x_{2j}^{\sim k} \\ \vdots & \vdots & \ddots & \vdots \\ x_{i1}^{\sim k} & x_{i2}^{\sim k} & \dots & x_{ij}^{\sim k} \end{bmatrix}$$

$$W^{\sim k} = [w_1^{\sim k} w_2^{\sim k} \dots w_j^{\sim k}]$$

Step 2: Aggregate the results of decision makers' ideas.

Aggregated fuzzy ratings of alternatives (x_{ij}) are;

$$x_{ij} = (x_{ij1}, x_{ij2}, x_{ij3}, x_{ij4}) \text{ where,}$$

$$x_{ij1} = \min_k \{x_{ijk1}\}, x_{ij2} = \frac{1}{K} \sum_{k=1}^K x_{ijk2}, x_{ij3} = \frac{1}{K} \sum_{k=1}^K x_{ijk3}, x_{ij4} = \max_k \{x_{ijk4}\} \quad (6.16)$$

For the importance weight of criteria the aggregation matrix is:

$W = [w_1 w_2 \dots w_j]$ where,

$$w_{ij1} = \min_k \{w_{ijk1}\}, w_{ij2} = \frac{1}{K} \sum_{k=1}^K w_{ijk2}, w_{ij3} = \frac{1}{K} \sum_{k=1}^K w_{ijk3}, w_{ij4} = \max_k \{w_{ijk4}\} \quad (6.17)$$

Step 3: The normalized decision matrix is constructed using; [35][18].

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^m x_{ij}^2}} \quad (6.18)$$

Assume that the trapezoidal fuzzy number is $r_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij})$ and use linear scales transform normalization function which preserves the property that ranges of normalized TrFNs to be included in $[0,1]$ interval. The normalized decision matrix N is;

$$N = \begin{bmatrix} r_{11} & r_{12} & \dots & r_{1n} \\ r_{21} & r_{22} & \dots & r_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ r_{m1} & r_{m2} & \dots & r_{mn} \end{bmatrix}$$

Where,

$$r_{ij} = \begin{cases} \left(\frac{a_{ij}}{d_j^+}, \frac{b_{ij}}{d_j^+}, \frac{c_{ij}}{d_j^+}, \frac{d_{ij}}{d_j^+} \right) & \text{if } j \text{ is a benefit attribute} \\ \left(\frac{a_j^-}{d_{ij}^+}, \frac{a_j^-}{c_{ij}^+}, \frac{a_j^-}{b_{ij}^+}, \frac{a_j^-}{a_{ij}^+} \right) & \text{if } j \text{ is a cost attribute and } d_j^+ \text{ is not zero} \\ \left(1 - \frac{a_{ij}}{d_j^+}, 1 - \frac{b_{ij}}{d_j^+}, 1 - \frac{c_{ij}}{d_j^+}, 1 - \frac{d_{ij}}{d_j^+} \right) & \text{if } j \text{ is a cost attribute and } d_j^+ \text{ is zero} \end{cases}$$

$$\text{Where, } d_j^+ = \max (d_{ij}), \quad a_j^- = \min (d_{ij}), \quad i=1,2,\dots,m \quad (6.19)$$

Step 4: The weighted normalized decision matrix is,

$$v_{ij} = r_{ij} \otimes w_j \quad (6.20)$$

Note that x_{ij} ; is the performance rating of the i^{th} alternative, A_i , with respect to the j^{th} criterion, C_j and w_j represents the weight of the j^{th} criterion, C_j . The normalized fuzzy decision matrix denoted by R is shown;

$$R = [r_{ij}]_{m \times n}$$

The weighted fuzzy normalized decision matrix is shown;

$$V = \begin{bmatrix} v_{11} & v_{12} & \dots & v_{1n} \\ v_{21} & v_{22} & \dots & v_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ v_{m1} & v_{m2} & \dots & v_{mn} \end{bmatrix}$$

$$= \begin{bmatrix} w_1 r_{11} & w_2 r_{12} & \dots & w_n r_{1n} \\ w_1 r_{21} & w_2 r_{22} & \dots & w_n r_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ w_1 r_{m1} & w_2 r_{m2} & \dots & w_n r_{mn} \end{bmatrix}$$

Step 5: Determine the fuzzy positive-ideal solution (FPIS) and fuzzy negative ideal solution (FNIS). According to the weighted normalized fuzzy decision matrix, it is known that the elements v_{ij} are normalized positive fuzzy numbers and their ranges belong to the closed interval $[0, 1]$. Then, it is defined the FPIS A^* and FNIS A^- as following formula:

$$A^* = (v_1^*, v_2^*, \dots, v_n^*) \quad (6.21)$$

$$A^- = (v_1^-, v_2^-, \dots, v_n^-) \quad (6.22)$$

Where $v_j^* = (1, 1, 1, 1)$ and $v_j^- = (0, 0, 0, 0), j=1, 2, \dots, n$.

Step 6: Calculate the distance of each alternative from FPIS and FNIS

The distances (d_i^* and d_i^-) of each alternative A^* from and A^- can be currently calculated by the area compensation method.

$$d_i^* = \sum_{j=1}^n d(v_{ij}, v_j^*), \quad i = 1, 2, \dots, m; \quad j = 1, 2, \dots, n \quad (6.23)$$

$$d_i^- = \sum_{j=1}^n d(v_{ij}, v_j^-), \quad i = 1, 2, \dots, m; \quad j = 1, 2, \dots, n \quad (6.24)$$

Where,

Let $\widetilde{X}_1 = (a_1, b_1, c_1, d_1)$, $\widetilde{X}_2 = (a_2, b_2, c_2, d_2)$ are two trapezoidal fuzzy numbers. Then the distance between them can be calculated by using the vertex method as [45];

$$d_v(\widetilde{X}_1, \widetilde{X}_2) = \sqrt{\frac{1}{4} [(a_1 - a_2)^2 + (b_1 - b_2)^2 + (c_1 - c_2)^2 + (d_1 - d_2)^2]}$$

Step 7: Obtain the closeness coefficient and rank the order of alternatives. The CC_i is defined to determine the ranking order of all alternatives once the d_i^* and d_i^- of each alternative have been calculated. Calculate similarities to ideal solution. This step solves the similarities to an ideal solution. The CC_i is calculated using the equation below:

$$CC_i = \frac{d_i^-}{d_i^* + d_i^-} \quad i = 1, \dots, m. \quad (6.25)$$

According to the , the ranking order of all alternatives can be determined and the best one from among asset of feasible alternatives.

6.3 COMPARING VIKOR AND TOPSIS

The decision-makers may reach a final decision with the help of a compromise solution for a problem with conflicting criteria. The compromise solution is a feasible solution which is closest to the ideal solution. The VIKOR method introduces the multi-criteria ranking index based on the particular measure of closeness to the ideal solution [32]. The TOPSIS method determines a solution with the shortest distance from the ideal solution and the farthest distance from the negative ideal solution [30].

Multi-criteria decision making methods VIKOR and TOPSIS involves other differences. The VIKOR method is based on aggregating function, L_p -metric as follows:

$$L_{pi} = \left\{ \sum_{i=1}^n [w_j (x_j^* - x_{ij}) / (x_j^* - x_j^-)]^p \right\}^{\frac{1}{p}}$$

$1 \leq p \leq \infty$. The measure L_{pi} represents the distance of alternative a_j to the ideal solution [66]. However, TOPSIS method introduces an aggregating function for ranking as:

$$C_i^* = \frac{d_i^-}{d_i^* + d_i^-}, \quad i = 1, \dots, m.$$

The VIKOR method and the TOPSIS method use different normalization techniques. The VIKOR method uses a normalized value as:

$$d_{ij}(x) = (x_j^* - x_{ij}) / (x_j^* - x_j^-).$$

On the other hand, the normalized value r_{ij} in the TOPSIS method is calculated as:

$$r_{ij} = x_{ij} / \sqrt{\sum_{j=1}^J x_{ij}^2}$$

Linear normalization was subsequently introduced into the TOPSIS method by Lai and Hwang [67];

$$r_{ij}(x) = x_{ij} / (x_j^* - x_j^-), j \in I' \text{ (benefits)},$$

and

$$r_{ij}(x) = x_{ij} / (x_j^- - x_j^*), j \in I'' \text{ (costs)}.$$

Lastly, the differences between VIKOR and TOPSIS are summarized in Table 6.2.

Table 6.2: Comparison of VIKOR and TOPSIS

	VIKOR	TOPSIS
Aggregating function	Q_j function of L_1 and L_∞	C_j function of L_2
Normalization	Linear normalization	Vector normalization
Solution	Closeness to the ideal solution	Shortest distance from the ideal solution and farthest distance from the negative ideal solution

7. PROBLEM DEFINITION

For the case study, supplier selection process for a private hospital is analyzed. Supplier selection process is a very important issue for hospitals since they procure many types of equipment from the outsourcing firms. The medicaments, the medical equipments, high technology medical machines, foods and chemical cleaning products are all supplied from different suppliers. The hospitals have to be careful while choosing their suppliers. The products that have low quality may threat the people's life. For example, medical equipments that are exported from China are very cheap but they are not reliable, the hospitals should prefer European or American medical equipments. The purchasing department plays a big role for selecting the suitable supplier.

In Turkey, the hospitals have to use National Data Bank to buy the certificated equipments supplied from the officially allowed supplier. National Data Bank was developed in 2006, and the purpose of the data bank is to put a standard in health sector. In the beginning, into the National Data Bank database, the suppliers enter the codes of the products that they sell, their own addresses, and the names of the retailers that sell the same product. The purchasing manager of a hospital enter an appropriate code of a product and list the name of the all suppliers that sell this product, in addition, a purchasing manager have chance to check out the substitutes of the products and decide between each product. So, from the National Data Bank, the supplier alternatives and substitute products can be listed and a decision between the each supplier can be made by the hospital management.

Hospital purchasing decisions are very complex, multifaceted and involve many different decision maker's priorities or objectives. Most of the hospital management, when faced with such problems, will attempt to use intuitive and heuristic approaches to simplify the complexity until the problems seems more manageable. In such situation, important information may be lost, opposing points of view may be discarded and some elements may be ignored. In this case multi criteria supplier selection methods step in and help the hospital management to choose the most effective supplier according to hospital management's criterion. For applying a MCDM process in a supplier selection

problem, firstly decision-makers are identified, criterion are selected, alternatives are defined, criterion are weighted, performance of alternatives against the criterion are assessed, if required, the criteria performance values to commensurable units are transformed, and lastly final decision is made.

The hospital supplier selection problem is a multi-criteria decision making problem that deals with subjectiveness and vagueness. In such situation, the multi-criteria fuzzy decision making model is a useful technique. Fuzzy numbers are very useful to represent evaluating values of criteria to deal with fuzzy multi-criteria decision making problems. For the supplier selection problem of the hospital, the fuzzy multi criteria decision making techniques fuzzy VIKOR and fuzzy TOPSIS are proposed and these techniques are involved to the problem.

VIKOR is a helpful multi-criteria decision making technique where the group of decision-makers have difficulties at the beginning of the system design. The obtained compromise solution could be accepted by the group of decision-makers because it provides a maximum “group utility” of the “majority” and a minimum of the individual regret of the “opponent” [5].

TOPSIS is a useful technique in dealing with multi-criteria decision making problems in real world. It helps decision-makers organize the problems to be solved, and carry out analysis, comparisons and ranking alternatives. In this case, TOPSIS is extended into a group decision environment for fitting the real world.

7.1 MODEL DESCRIPTION

After many interviews with the hospital management and many investigations about the supplier selection problem, the model of the problem is structured. Firstly the opinions of the hospital management about the supplier selection problem are received. According to these opinions the frameworks of the decision matrices are formed. For ratings of decision matrices appropriate linguistic variables are defined and these linguistic terms are converted to positive trapezoidal fuzzy numbers.

Supplier selection problem is not only related with purchasing department, the other people who use or responsible from the supplied product in the hospital should have a decision for choosing the right supplier. Therefore, homogenous group of decision-makers are defined. The alternatives and the criteria are determined and the decision-maker group rates the decision matrix by answering with linguistic variables. Hence, the aggregated fuzzy weights of individual attributes are constructed and the aggregate fuzzy ratings are calculated. Congruently with the weight of individual attribute, fuzzy ratings of alternatives with respect to individual objective are computed and fuzzy rating matrix is constructed. With regarding the type of the attribute, (if it is a cost attribute or benefit attribute) total fuzzy scores of individual alternatives computed. Lastly, for the selection state, the fuzzy decision matrix and fuzzy weights of attributes are defuzzified and crisp values are determined. For ranking phase of the problem, fuzzy VIKOR and fuzzy TOPSIS methods are proposed and the best alternatives are found.

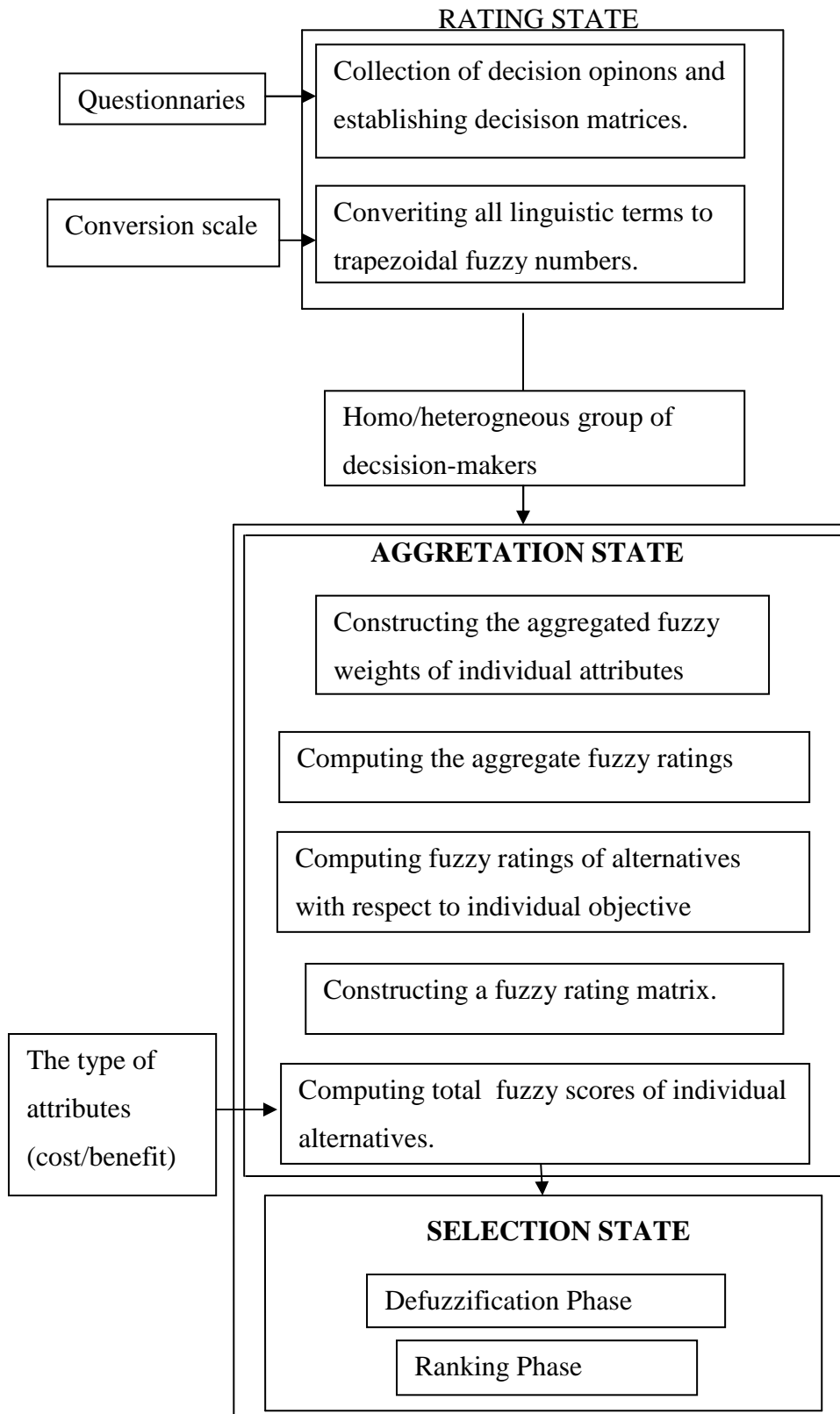


Figure 7.1: The conceptual model of the proposed approach

7.1.1 Supplier's Selection Criteria

The criteria must be defined according to the corporate strategies, company's competitive situation, and the level of buyer supplier integration. In 1966, Dickson made a survey with buyers to identify the factors they considered in awarding business to competing suppliers [43]. Although some supplier selection criteria were found to vary in different situations, three common criteria emerged as important regardless of the situation. These are quality, on time delivery and supplier performance history. In 1982, Lehmann and O'Shaughnessy made an another study and found that, in a fundamental sense, the key factors generally thought to affect supplier selection decisions were price, quality, delivery and service [44].

In the presents study similar to the two studies that are mentioned, the basic decision criteria in purchasing situations are chosen. These are on time delivery, quality, and price. The other criteria are chosen according to the necessities of the hospital management. These criteria are paying condition, product life and certificates.

1. On Time Delivery

Hospitals race with time to save the people's life. Stocking so many equipments is costly and need extra area. In addition to these, some of the equipments have to consume before the last consumption date. For that reason, the suppliers have to reach the products that are sold on time. (On time delivery is the 1st criteria. (C1))

2. Product Quality

In hospitals main important subject is life so every hospital has to take care about the quality of the products that they use. The products must not be damaged or broken. If we talk about the medicine, the medicine must not be spoilt. The hospitals have to supply their necessities from the suppliers that they trust their quality. (Product quality is the 2nd criteria. (C2))

3. Price

Lower price is directly related with cost saving, and profitability. The suppliers have to determine the price discounts. For competing with the other private hospitals, the hospitals prefer low cost and high technology. (Price is the 3rd criteria. (C3))

4. Paying Condition

Today in every sector paying conditions are various. Instead of paying cash the customer may choose paying on credit. The suitable credit conditions help the financial situation of the customer. (Paying Condition is the 4th criteria. (C4))

5. Product Life

The equipments have life span and the equipments that last long times are preferable by the customers. The time of usability turns as an economic benefit to the customer. If it is talked about the medicines, the fresher medicine is preferable and the products that are bought have not to be waited long time at the shelves. (Product life is the 5th criteria. (C5))

6. Certificates

For the wanted service performance, the experience is an important tool, the supplier have to meet this expectation. The candidate suppliers might have national and international certificates for better interpreting. (Certificate is the 6th criteria. (C6))

7.1.2 Decision-Makers and List of Suppliers

1. Decision-Makers

Purchasing department in a hospital management is responsible from the supplying process but in reality, they don't use or test the products. Therefore only one department's decision is not enough to choose a supplier. That's why in this case, there is not only a single decision-maker, decision making is done by group of decision-makers and number of decision-makers are arranged decision as four people that decide the purchasing item. After interview, a group of decision-makers is settled on, in daily

life these decision-makers use the product, check the products and have chance to be the witness of the patient's satisfaction. These decision-makers are:

D1: Doctor,

D2: Head Nurse,

D3: Head Doctor,

D4: Purchasing Department

2. List Of Suppliers

Suppliers are listed from the National Medicine and Medical Equipment Data Bank [19]. All the name of the suppliers that supplied the product appears on the list and the names of the suppliers are found from that list. Every supplier on that list has official authorization from the Ministry of Health. These suppliers are;

S1: Bıçakçılar,

S2: Telefex,

S3: Ad Tech,

S4: Calmed,

S5: Busse,

S6: Inhealth,

S7: Serres.

7.1.3 Appropriate Linguistic Variables

The appropriate linguistic variables are defined for fuzzy importance weight of criteria and the fuzzy rating for alternative for each criterion. In this case the linguistic variables are expressed with positive trapezoidal fuzzy numbers. The decision makers use linguistic variables for the importance weight of criterion and ratings of each alternative. For example, "Medium Low" for the importance weight of criteria can be expressed as (0.2, 0.3, 0.4, 0.5), the membership function of "Medium Low" is:

$$\mu_{Medium\ Low}(x) = \begin{cases} 0, & x < 0.2, \\ \frac{x - 0.2}{0.3 - 0.2}, & 0.2 \leq x \leq 0.3, \\ 1, & 0.3 \leq x \leq 0.4 \\ \frac{x - 0.5}{0.4 - 0.5}, & 0.4 \leq x \leq 0.5, \\ 0, & x > 0.5. \end{cases}$$

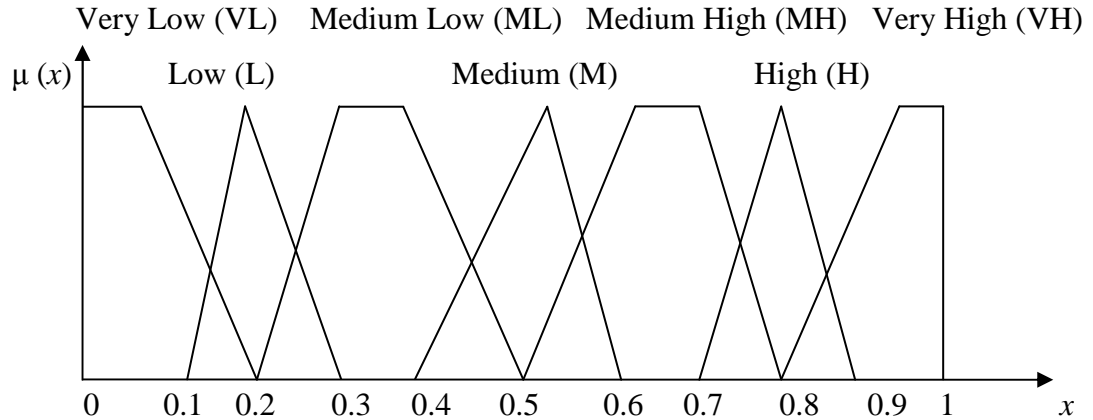


Figure 7.2 Linguistic variables for importance weight of criteria.

“Good” for ratings of alternatives can be expressed as (0.7, 0.8, 0.8, 0.9), the membership function of “Good” is:

$$\mu_{Good}(x) = \begin{cases} \frac{x - 0.7}{0.8 - 0.7} & 0.7 \leq x \leq 0.8, \\ \frac{0.9 - x}{0.9 - 0.8} & 0.8 \leq x \leq 0.9, \\ 0, & otherwise. \end{cases}$$

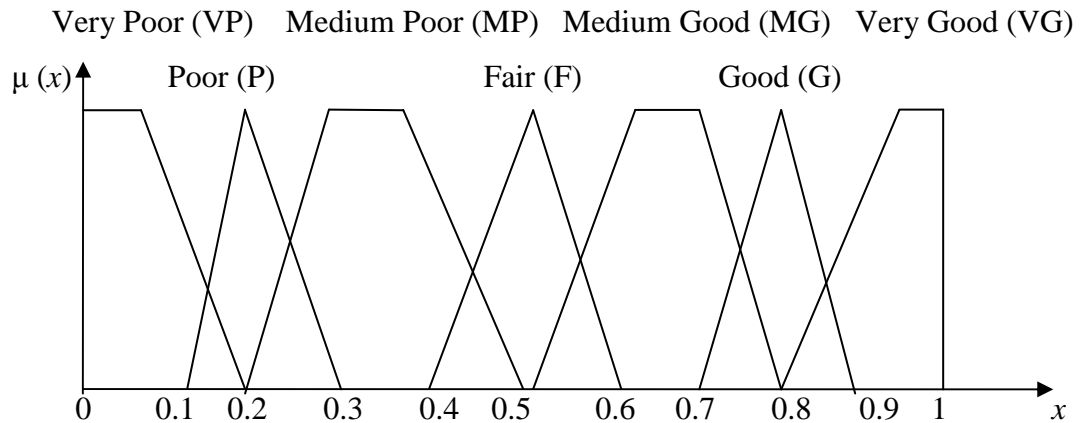


Figure 7.3 Linguistic variables for ratings

7.1.4 Fuzzy Decision Matrices

Four decision makers use the linguistic variables above and determine the importance weight of criteria matrix and ratings of the seven suppliers' matrix under the various criteria. For example, in Table 7.1, decision maker "doctor" is shown with "D1" and the criteria "on time delivery" is shown with "C1", according to the doctor's decision on time delivery is very highly important because the patients may not have chance to wait for a medical equipment.

Table 7.1: Importance weight of criteria from decision-makers.

Criteria	Decision Makers			
	D1	D2	D3	D4
C1	VH	VH	H	H
C2	H	H	H	H
C3	M	M	H	VH
C4	M	M	M	VH
C5	H	H	H	H
C6	H	H	H	VH

As it was mentioned before every linguistic variable is represented by fuzzy numbers. For example, in Table 7.2, doctor's decision about on time delivery is very highly important. Very high linguistic variable is represented by fuzzy number (0.8, 0.9, 1, 1)

Table 7.2: Importance fuzzy weight of criteria from decision-makers.

Criteria	Decision Makers			
	D1	D2	D3	D4
C1	(0.8, 0.9, 1, 1)	(0.8, 0.9, 1, 1)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)
C2	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)
C3	(0.4, 0.5, 0.5, 0.6)	(0.4, 0.5, 0.5, 0.6)	(0.7, 0.8, 0.8, 0.9)	(0.8, 0.9, 1, 1)
C4	(0.4, 0.5, 0.5, 0.6)	(0.4, 0.5, 0.5, 0.6)	(0.4, 0.5, 0.5, 0.6)	(0.8, 0.9, 1, 1)
C5	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)
C6	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.8, 0.9, 1, 1)

After determining the importance weight of criteria every decision-maker rates the seven suppliers under different criteria. Every decision-maker knows the qualifications of each supplier. Thus, the decision-makers rate the suppliers with regard to their past experience. Firstly, they rate each supplier with linguistic variables as it is shown in Table 7.3, then every linguistic variable that are given by four decision-makers are transformed into fuzzy numbers and fuzzy ratings are shown in Table 7.4, Table 7.5, Table 7.6 and Table 7.7.

Table 7.3: Ratings of the seven suppliers by four decision-makers under various criteria.

Decision Maker	Suppliers	Criteria					
		C1	C2	C3	C4	C5	C6
D1	S1	MG	G	G	MG	G	VG
	S2	G	MG	VG	VG	MG	MG
	S3	G	G	MG	MG	G	G
	S4	F	MG	G	G	MG	MG
	S5	VG	G	F	F	VG	VG
	S6	G	MG	G	G	G	MG
	S7	MG	MG	G	G	MG	MG
D2	S1	MG	G	G	G	G	G
	S2	MG	F	G	G	MG	MG
	S3	G	VG	G	G	G	G
	S4	MG	MG	MG	MG	MG	MG
	S5	G	G	MG	MG	G	G
	S6	G	MG	G	G	MG	MG
	S7	MG	MG	MG	MG	MG	MG
D3	S1	G	G	G	G	G	VG
	S2	MG	MG	G	MG	MG	MG
	S3	G	G	G	G	G	G
	S4	MG	MG	G	MG	MG	MG
	S5	G	G	MG	G	G	VG
	S6	G	G	G	G	G	MG
	S7	MG	MG	MG	MG	MG	MG
D4	S1	G	G	MG	G	G	VG
	S2	G	MG	G	MG	MG	MG
	S3	G	G	MG	G	G	G
	S4	F	MG	MG	F	F	MG
	S5	VG	G	MG	VG	VG	VG
	S6	G	MG	MG	MG	MG	MG
	S7	MG	G	MG	MG	MG	MG

Table 7.4: Fuzzy ratings of the seven suppliers by the first decision-maker (D1) under various criteria.

Supp.	Criteria					
	C1	C2	C3	C4	C5	C6
S1	(0.5, 0.6, 0.7, 0.8)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.5, 0.6, 0.7, 0.8)	(0.7, 0.8, 0.8, 0.9)	(0.8, 0.9, 1, 1)
S2	(0.7, 0.8, 0.8, 0.9)	(0.5, 0.6, 0.7, 0.8)	(0.8, 0.9, 1, 1)	(0.8, 0.9, 1, 1)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)
S3	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)
S4	(0.4, 0.5, 0.5, 0.6)	(0.5, 0.6, 0.7, 0.8)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)
S5	(0.8, 0.9, 1, 1)	(0.7, 0.8, 0.8, 0.9)	(0.4, 0.5, 0.5, 0.6)	(0.4, 0.5, 0.5, 0.6)	(0.8, 0.9, 1, 1)	(0.8, 0.9, 1, 1)
S6	(0.7, 0.8, 0.8, 0.9)	(0.5, 0.6, 0.7, 0.8)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.5, 0.6, 0.7, 0.8)
S7	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)

Table 7.5: Fuzzy ratings of the seven suppliers by the second decision-maker (D2) under various criteria.

Supp.	Criteria					
	C1	C2	C3	C4	C5	C6
S1	(0.5, 0.6, 0.7, 0.8)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)
S2	(0.5, 0.6, 0.7, 0.8)	(0.4, 0.5, 0.5, 0.6)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)
S3	(0.7, 0.8, 0.8, 0.9)	(0.8, 0.9, 1, 1)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)
S4	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)
S5	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)
S6	(0.7, 0.8, 0.8, 0.9)	(0.5, 0.6, 0.7, 0.8)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)
S7	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)

Table 7.6: Fuzzy ratings of the seven suppliers by the third decision-maker (D3) under various criteria.

Supp.	Criteria					
	C1	C2	C3	C4	C5	C6
S1	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.8, 0.9, 1, 1)
S2	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)	(0.7, 0.8, 0.8, 0.9)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)
S3	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)
S4	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)	(0.7, 0.8, 0.8, 0.9)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)
S5	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.5, 0.6, 0.7, 0.8)	(0.7, 0.8, 0.8, 0.9)	(0.5, 0.6, 0.7, 0.8)	(0.8, 0.9, 1, 1)
S6	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.5, 0.6, 0.7, 0.8)
S7	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)

Table 7.7: Fuzzy ratings of the seven suppliers by the fourth decision-maker (D4) under various criteria.

Supp.	Criteria					
	C1	C2	C3	C4	C5	C6
S1	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)	(0.7, 0.8, 0.8, 0.9)	(0.8, 0.9, 1, 1)
S2	(0.7, 0.8, 0.8, 0.9)	(0.5, 0.6, 0.7, 0.8)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)
S3	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)
S4	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)	(0.4, 0.5, 0.5, 0.6)	(0.5, 0.6, 0.7, 0.8)
S5	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)	(0.8, 0.9, 1, 1)	(0.8, 0.9, 1, 1)
S6	(0.7, 0.8, 0.8, 0.9)	(0.5, 0.6, 0.7, 0.8)	(0.7, 0.8, 0.8, 0.9)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)
S7	(0.5, 0.6, 0.7, 0.8)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)

7.2 PROPOSED METHOD

The fuzzy VIKOR and fuzzy TOPSIS methods are proposed to solve this supplier selection problem.

In reality; supplier selection in supply chain system is a group multiple criteria decision making (GMCDM) problem, which may be described by means of the following sets:

1. a set of K decision makers called $E = \{D_1, D_2, \dots, D_K\}$;
2. a set of m possible suppliers called $A = \{S_1, S_2, \dots, S_m\}$;
3. a set of n criteria, $C = \{C_1, C_2, \dots, C_n\}$; with which supplier performance are measured;
4. a set of performance ratings of A_i ($i = 1, 2, \dots, m$) with respect to criteria C_j ($j = 1, 2, \dots, n$), called $X = \{x_{ij}, i = 1, 2, \dots, m, j = 1, 2, \dots, n\}$

7.2.1 VIKOR under Fuzzy Environment

Step 1: Calculate the aggregate fuzzy weight of criteria and aggregate fuzzy ratings of alternatives with the help of the equation 6.4, 6.5. The aggregating process represents the distance from the ideal solution:

Table 7.8: Aggregated fuzzy weight of criteria and aggregated fuzzy rating of alternatives

		Criteria					
		C1	C2	C3	C4	C5	C6
Weight		(0.7, 0.85, 0.9, 1)	(0.7, 0.8, 0.8, 0.9)	(0.4, 0.68, 0.7, 1)	(0.4, 0.6, 0.63, 1)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.83, 0.85, 1)
S1		(0.5, 0.7, 0.75, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.5, 0.75, 0.78, 0.9)	(0.5, 0.7, 0.75, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.88, 0.95, 1)
S2		(0.5, 0.7, 0.75, 0.9)	(0.4, 0.58, 0.65, 0.8)	(0.7, 0.83, 0.85, 1)	(0.7, 0.83, 0.85, 1)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)
S3		(0.7, 0.8, 0.8, 0.9)	(0.7, 0.83, 0.85, 1)	(0.5, 0.7, 0.75, 0.9)	(0.5, 0.7, 0.75, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)
S4		(0.4, 0.55, 0.58, 0.8)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.7, 0.75, 0.9)	(0.5, 0.7, 0.75, 0.9)	(0.4, 0.58, 0.65, 0.8)	(0.5, 0.6, 0.7, 0.8)
S5		(0.7, 0.83, 0.9, 1)	(0.7, 0.8, 0.8, 0.9)	(0.4, 0.58, 0.65, 0.8)	(0.4, 0.58, 0.65, 0.8)	(0.7, 0.85, 0.9, 1)	(0.7, 0.88, 0.95, 1)
S6		(0.7, 0.8, 0.8, 0.9)	(0.5, 0.65, 0.73, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.75, 0.78, 0.9)	(0.5, 0.7, 0.75, 0.8)	(0.5, 0.6, 0.7, 0.8)
S7		(0.5, 0.6, 0.7, 0.8)	(0.5, 0.65, 0.73, 0.9)	(0.5, 0.75, 0.78, 0.9)	(0.5, 0.65, 0.73, 0.9)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)

Step 2: Calculate the crisp value of decision matrix and weight of criterion with the equation 5.6.

Table 7.9: Crisp values for decision matrix and weights of each criterion

		Criteria					
		C1	C2	C3	C4	C5	C6
Weight		0.86	0.80	0.70	0.67	0.80	0.85
S1		0.71	0.80	0.72	0.71	0.80	0.88
S2		0.71	0.61	0.85	0.85	0.65	0.65
S3		0.80	0.85	0.71	0.71	0.80	0.80
S4		0.59	0.65	0.71	0.71	0.61	0.65
S5		0.86	0.80	0.61	0.61	0.86	0.88
S6		0.80	0.70	0.80	0.79	0.68	0.65
S7		0.65	0.70	0.72	0.70	0.65	0.65

Step 3: Determine the maximum x_j^* and the minimum x_j^- values of criterion functions, $j=1, \dots, n$. Thus, the best and the worst values of all criterion ratings are:

Table 7.10: The best and the worst values of all criteria ratings

$x_1^* = 0.86$	$x_2^* = 0.85$	$x_3^* = 0.85$	$x_4^* = 0.85$	$x_5^* = 0.86$	$x_6^* = 0.88$
$x_1^- = 0.59$	$x_2^- = 0.61$	$x_3^- = 0.61$	$x_4^- = 0.61$	$x_5^- = 0.61$	$x_6^- = 0.65$

Step 4: Compute the values of S, R, and Q with the equation: 6.8, 6.9, where S_i , R_i represent the utility measure and regret measure respectively for the alternative x_i and v is the weight for the strategy of maximum group utility and $1-v$ is the weight of the individual regret.

Table 7.11: The values of S, R and Q for seven suppliers

Suppliers							
	S1	S2	S3	S4	S5	S6	S7
S	1.62	2.83	1.49	4.00	1.59	2.45	3.50
R	0.48	0.85	0.41	0.87	0.71	0.85	0.85
Q	0.90	0.22	1.00	0.00	0.67	0.33	0.12
$v \approx 0.5$							

Step 5= Rank the suppliers by S, R and Q decreasing order. The less the values Q_i is the better decision of the alternative is. Since it provides a maximum group utility of the majority and minimum individual regret of the opponent, the compromise solution is acceptable by decision-makers.

Table 7.12: The ranking of the suppliers by S, R and Q decreasing order.

RankingSuppliers							
	1	2	3	4	5	6	7
S	S3	S5	S1	S6	S2	S7	S4
R	S3	S1	S5	S2	S6	S7	S4
Q	S4	S7	S2	S6	S5	S1	S3

According to the equation 5.10 $Q_{S7} - Q_{S4}$ is not greater or equal to $DQ = 1/(1-J)$ where J is the number of alternatives. ($DQ = 1/(J-1)$) Thus the conditions C1 and C2 are not satisfied $Q(A^{(M)}) - Q(A^{(1)}) < DQ$ for maximum M is satisfied by S_7 . So S_7 is the best choice.

7.2.2 TOPSIS under Fuzzy Environment

Step 1: Calculate the aggregate fuzzy weight of criteria and aggregate fuzzy ratings of alternatives with the help of the equation 6.16, 6.17:

Table 7.13: Aggregated fuzzy weight of criteria and aggregated fuzzy rating of alternatives

	Criteria					
	C1	C2	C3	C4	C5	C6
Weight	(0.7, 0.85, 0.9, 1)	(0.7, 0.8, 0.8, 0.9)	(0.4, 0.68, 0.7, 1)	(0.4, 0.6, 0.63, 1)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.83, 0.85, 1)
S1	(0.5, 0.7, 0.75, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.5, 0.75, 0.78, 0.9)	(0.5, 0.7, 0.75, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.88, 0.95, 1)
S2	(0.5, 0.7, 0.75, 0.9)	(0.4, 0.58, 0.65, 0.8)	(0.7, 0.83, 0.85, 1)	(0.7, 0.83, 0.85, 1)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)
S3	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.83, 0.85, 1)	(0.5, 0.7, 0.75, 0.9)	(0.5, 0.7, 0.75, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)
S4	(0.4, 0.55, 0.58, 0.8)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.7, 0.75, 0.9)	(0.5, 0.7, 0.75, 0.9)	(0.4, 0.58, 0.65, 0.8)	(0.5, 0.6, 0.7, 0.8)
S5	(0.7, 0.83, 0.9, 1)	(0.7, 0.8, 0.8, 0.9)	(0.4, 0.58, 0.65, 0.8)	(0.4, 0.58, 0.65, 0.8)	(0.7, 0.85, 0.9, 1)	(0.7, 0.88, 0.95, 1)
S6	(0.7, 0.8, 0.8, 0.9)	(0.5, 0.65, 0.73, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.75, 0.78, 0.9)	(0.5, 0.7, 0.75, 0.8)	(0.5, 0.6, 0.7, 0.8)
S7	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.65, 0.73, 0.9)	(0.5, 0.75, 0.78, 0.9)	(0.5, 0.65, 0.73, 0.9)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)

Normalization eliminates anomalies with different measurement units and scales of the raw data in several MCDM problems.

Step 2: Normalize the aggregated fuzzy ratings according to the equation 6.19: (Note: C1, C2, C4, C5 and C6 are benefit attribute, C3 is a cost attribute)($d_1^+ = 1, d_2^+ = 1, d_4^+ = 1, d_5^+ = 1, d_6^+ = 1$ and $a_j^- = 0.8$, where $d_j^+ = \max (d_{ij})$, $a_j^- = \min (d_{ij})$)

Table 7.14: Normalized fuzzy rating of alternative

	Criteria					
	C1	C2	C3	C4	C5	C6
Weight	(0.7, 0.85, 0.9, 1)	(0.7, 0.8, 0.8, 0.9)	(0.4, 0.68, 0.7, 1)	(0.4, 0.6, 0.63, 1)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.83, 0.85, 1)
S1	(0.5, 0.7, 0.75, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.44, 0.51, 0.53, 0.8)	(0.5, 0.7, 0.75, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.88, 0.95, 1)
S2	(0.5, 0.7, 0.75, 0.9)	(0.4, 0.58, 0.65, 0.8)	(0.4, 0.47, 0.48, 0.57)	(0.7, 0.83, 0.85, 1)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)
S3	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.83, 0.85, 1)	(0.44, 0.53, 0.57, 0.8)	(0.5, 0.7, 0.75, 0.9)	(0.7, 0.8, 0.8, 0.9)	(0.7, 0.8, 0.8, 0.9)
S4	(0.4, 0.55, 0.58, 0.8)	(0.5, 0.6, 0.7, 0.8)	(0.44, 0.53, 0.57, 0.8)	(0.5, 0.7, 0.75, 0.9)	(0.4, 0.58, 0.65, 0.8)	(0.5, 0.6, 0.7, 0.8)
S5	(0.7, 0.83, 0.9, 1)	(0.7, 0.8, 0.8, 0.9)	(0.5, 0.62, 0.69, 1)	(0.4, 0.58, 0.65, 0.8)	(0.7, 0.85, 0.9, 1)	(0.7, 0.88, 0.95, 1)

S6	(0.7, 0.8, 0.8, 0.9)	(0.5, 0.65, 0.73, 0.9)	(0.44, 0.5, 0.5, 0.57)	(0.7, 0.75, 0.78, 0.9)	(0.5, 0.7, 0.75, 0.8)	(0.5, 0.6, 0.7, 0.8)
S7	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.65, 0.73, 0.9)	(0.44, 0.51, 0.53, 0.8)	(0.5, 0.65, 0.73, 0.9)	(0.5, 0.6, 0.7, 0.8)	(0.5, 0.6, 0.7, 0.8)

Considering the different weight of each criterion, the weighted normalized decision matrix can be computed by multiplying the importance weights of evaluation criteria and the values in the normalized fuzzy decision matrix.

Step 3: Construct the fuzzy weighted matrix according to equation 6.20:

Table 7.15: Fuzzy weighted matrix

	Criteria					
	C1	C2	C3	C4	C5	C6
S1	(0.35, 0.6, 0.68, 0.9)	(0.49, 0.64, 0.64, 0.81)	(0.18, 0.35, 0.37, 0.8)	(0.2, 0.42, 0.47, 0.9)	(0.49, 0.64, 0.64, 0.81)	(0.49, 0.73, 0.81, 1)
S2	(0.35, 0.6, 0.68, 0.9)	(0.28, 0.46, 0.52, 0.72)	(0.16, 0.32, 0.34, 0.57)	(0.28, 0.5, 0.54, 1)	(0.35, 0.48, 0.56, 0.72)	(0.35, 0.5, 0.6, 1)
S3	(0.49, 0.68, 0.72, 0.9)	(0.49, 0.66, 0.68, 0.9)	(0.18, 0.36, 0.4, 0.8)	(0.2, 0.42, 0.47, 0.9)	(0.49, 0.64, 0.64, 0.81)	(0.49, 0.66, 0.68, 0.9)
S4	(0.28, 0.47, 0.52, 0.8)	(0.35, 0.48, 0.56, 0.72)	(0.18, 0.36, 0.4, 0.8)	(0.2, 0.42, 0.47, 0.9)	(0.38, 0.46, 0.52, 0.72)	(0.35, 0.50, 0.6, 0.8)
S5	(0.49, 0.71, 0.81, 1)	(0.49, 0.64, 0.4, 0.81)	(0.3, 0.43, 0.48, 1)	(0.16, 0.35, 0.41, 0.8)	(0.49, 0.68, 0.72, 0.9)	(0.49, 0.73, 0.81, 1)
S6	(0.49, 0.68, 0.72, 0.9)	(0.35, 0.52, 0.58, 0.81)	(0.18, 0.34, 0.35, 0.57)	(0.28, 0.45, 0.49, 0.9)	(0.35, 0.56, 0.6, 0.72)	(0.35, 0.5, 0.6, 0.8)
S7	(0.35, 0.51, 0.63, 0.8)	(0.35, 0.52, 0.58, 0.81)	(0.18, 0.35, 0.37, 0.8)	(0.2, 0.39, 0.46, 0.9)	(0.35, 0.48, 0.56, 0.72)	(0.35, 0.5, 0.6, 0.8)

Step 4: Calculate the distance of each alternative from FPIS and FNIS by the equations: 6.23 and 6.24. The distances (d_i^* and d_i^-) of each alternative A^* from and A^- can be currently calculated by the area compensation method. Where $v_i^* = (1, 1, 1, 1)$ and $v_i^- = (0, 0, 0, 0)$, $i=1, 2, \dots, m$.

Table 7.16: Separation Measures

	d_i^*	d_i^-
S1	2.65	3.79
S2	3.07	3.32
S3	2.58	3.81
S4	3.18	3.25
S5	2.50	4.01
S6	2.91	3.42

S7	3.07	3.35
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An alternative with index CC_i indicates that the alternative is close to the positive ideal reference point and far from the negative ideal reference point. A large value of index CC_i indicates a good performance of the alternative A_i .

Step 5: Calculate the similarity to the ideal solution using equation 6.25:

Table 7.17: Similarity to the Ideal Solution

Supp.	CC_i
S1	0.59
S2	0.52
S3	0.60
S4	0.51
S5	0.62
S6	0.54
S7	0.52

Step 6: Rank order the solutions as shown in **Table 7.18**:

Table 7.18: Rank order of the suppliers

Supp.	CC_i
S5	0.62
S3	0.60
S1	0.59
S6	0.54
S2	0.52
S7	0.52
S4	0.51

S5 is the best choice according to the fuzzy TOPSIS method.

7.3 RESULTS AND DISCUSSION

The MCDM methods VIKOR and TOPSIS are applied to the supplier selection problem of a hospital. Firstly, for two methods a performance matrix is obtained by the evaluation of all alternatives in terms of each criterion. For eliminating the units of criterion values, normalization is used and for both of the methods aggregating function is applied. The difference between two methods occurs in the normalization and aggregation process.

VIKOR method uses linear normalization and the normalized value does not depend on the evaluation unit of criterion. TOPSIS method introduces vector normalization and the normalized value could be different for different evaluation unit of a particular criterion [5].

For aggregation process VIKOR method introduces the aggregating function representing the distance from the ideal solution. This ranking index is an aggregation of all criteria, the relative importance of the criteria and the balance between total and individual satisfaction. TOPSIS method introduces ranking index including the distance from the ideal and the negative ideal point.

The two of the methods provide a ranking list. The highest ranked alternative by VIKOR, **S7**, is the closest alternative to the ideal solution of the problem. The highest ranked alternative by TOPSIS, which considers both the shortest distance from the ideal solution and the farthest distance from the negative ideal solution, is **S5**. The alternative **S7** is a real compromise, as something between extremes. The TOPSIS method with vector normalization selects **S5** as a solution. However the highest ranked alternative by TOPSIS is the best in terms of the ranking index, which does not mean that it is always the closest to the ideal solution. In addition to ranking, the VIKOR method proposes a compromise solution with an advantage rate.

8. CONCLUSION

Today, while healthcare expenditures are rising up, the profit margins are continuously declining. This trend forces the health system managers to think operational based and to increase the system performance. For gaining competitive advantage, it is necessary to give importance on supply chain management. Supply chain management for health sector means satisfaction of every person that benefit from health care services. The constituent of supply chain management in hospitals are customer relation management, in sourcing supply chain management and supplier relation management.

Purchasing is essential constituent of supply chain management. In the hospitals that have only doctor oriented purchasing decision, it is very difficult to say the purchasing department works well and instituonization of these hospitals are not completed. The importance that is given to the purchasing activities directly affects the cost, service quality and profit.

In hospitals the services that are presented to the patients and healthy persons are gradually changed. In addition to recumbent patient or standing treatment, for continuous healthy situation, the number of people that benefit from the tests or protective services increase. Together with the legal legislation, the hospitals must focus on the purchasing. The right choice at the right time and the right service diminish the supply chain costs.

The hospitals are places where continuous expenditures of medicaments, medical products and other managerial issues are occurred. Additionally, the medical devices get more complex day by day and the sum paid for these high technology devices highly augment. Choosing the right device, buying it, setting and repairing necessities become more complex. For that reason it is very necessary that the suppliers and hospital managements need an arrangement. Applying the basic principles should be necessary for progressing of professional and ethical supplier-hospital relations. In this step, academic studies are very important. Traditional material management philosophy only focuses on product choosing, purchasing and distributing. However in real conditions

the cost of stocking, the price, quality of products and paying conditions should be evaluated. Hospital supply chain management should include purchasing, inventory control and stocking. With the help of the academic studies the purchasing strategies, supplier selection, price, cost and quality analysis can be examined totally.

In practice, supplier selection problem for a hospital is a decision making problem and many quantitative and qualitative factors with imprecision are considered in this type of problems. This makes the decision process very complicated and unstructured. In this thesis, the process of decision making is the selection of a supplier among different alternatives and the selected supplier will produce optimal result under some criteria of optimization. Thus the decision making process relies on information about the alternatives. The goal of the supplier selection problem of the hospital is to select a good choice from a number of available choices and this goal is put forth by a group of people.

In the multi-criteria decision-making (MCDM) context, the selection is facilitated by evaluating each choice on the set of criteria. The criteria must be measurable, even if the measurement is performed only at the nominal scale (for example: yes/no) and their outcomes must be measured for every decision alternative. Criterion outcomes provide the basis for comparison of choices and consequently facilitate the selection of one, satisfactory choice. Criterion outcomes of decision alternatives are collected in a table called decision matrix comprised of a set of columns and rows. In this thesis, the table rows represent decision alternatives of different suppliers, with table columns representing criteria. A value found at the intersection of row and column in the table represents a criterion outcome, a measured performance of a decision alternative on a criterion. The decision matrix is a central structure of this thesis since it contains the data for comparison of decision alternatives.

At a practical level, under many situations, the values for the qualitative criteria are often imprecisely defined for the decision-makers. It is not easy to precisely quantify the rating of each alternative supplier and the precision based methods are not adequate to deal with the supplier selection problem. Since human judgments including preference

are often vague and cannot estimate his preference with an exact numerical value. A more realistic way such as; linguistic terms are used for describing the desired value and important weight of criteria, e.g. “very low”, “medium”, “high”, “fair”, “very high”, etc. Due to this type of existing fuzziness in the supplier selection process, fuzzy set theory is used as an appropriate method for deal with uncertainty and the subjective evaluation data can be more adequately expressed in fuzzy linguistic variables. VIKOR method under fuzzy environment and TOPSIS method under fuzzy environment are the methods have emerged as powerful tools to assist in the process of searching for best supplier alternatives of the problem.

A comparative analysis of the VIKOR and TOPSIS methods are adapted to the supplier selection problem of the hospital and the results are compared. Mainly there are differences between VIKOR and TOPSIS, for example, the VIKOR method is based on the ranking index of “closeness” to the ideal solution; however the result of TOPSIS method introduces the chosen alternative have the “shortest distance” from the ideal solution and the “farthest” distance from the negative ideal solution. These two MCDM methods, VIKOR and TOPSIS are based on aggregating function representing closeness to the reference points. The VIKOR introduces aggregating function (L_p -metric) for ranking that is different compared to the TOPSIS; VIKOR introduces Q_j function of L_1 and L_∞ . On other hand TOPSIS introduces C_j^* function of L_2 . VIKOR and TOPSIS use different kinds of normalization, whereas The VIKOR method uses linear normalization and the TOPSIS method uses vector normalization.

As a result, the solution of fuzzy VIKOR and fuzzy TOPSIS methods are highlighted the management of the hospital. By working with a group of decision-makers and using fuzzy set theory, the solutions are able to reflect the reality. At the end of this study, it is shown that applying MCDM methods for supplier selection problem of a hospital is useful.

In the future, the globalization and competitiveness will make supply chain more efficient compared to today. The appropriate choice of the supplier will be relevant for a

product of good quality and low cost. Inadequate selection of suppliers will bring dissatisfaction to customer and prejudice to company as well.

Multi-criteria decision making methods are applied for the supplier selection problem. In this thesis, the supplier selection problem is solved by the two multi-criteria methods, namely fuzzy TOPSIS and fuzzy VIKOR. In the hospital supplier selection problem the importance weight of criteria are defined by the linguistic variables that the decision-makers decide. In the future research works, the importance weight of criteria may be determined by the AHP method. In addition, the number of decision-makers and the criteria may be increased. The selected problem solving method is only applied for one hospital; however, it may be applied to a group of hospitals.

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