THE REPUBLIC OF TURKEY
BAHÇEŞEHİR UNIVERSITY

ULTRAFLAT OPTICAL COMB GENERATION BY PHASE-ONLY MODULATION OF CONTINUOUS-WAVE LIGHT

Master's Thesis

MEHMET BAY

İSTANBUL, 2012

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Asst. Prof. Tunç BOZBURA
Director

This is to certify that we have read this thesis and that we find it fully adequate in scope, quality and content, as a thesis for the degree of Master of Science.

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ABSTRACT

ULTRAFLAT OPTICAL COMB GENERATION BY PHASE-ONLY MODULATION OF CONTINUOUS-WAVE LIGHT

Mehmet Bay

Electrical and Electronics Engineering

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Scientific studies about laser engineering specifically, generation and applications of stable optical frequency combs, have gained great interest of the scientists due to their intrinsic physical properties and very large area of application in technology such as, arbitrary waveform generation and spectral measurements. The main source of optical frequency combs are mode-locked laser. However, mode-locked lasers are not easy to build and have very high cost nad high maintenance compared to our method. Moreover our proposed approach can be replace mode locked lasers in some areas such as,communication and microwave photonics and network testing.

In this thesis, we are particularly focused on theoretical approach of the generation of stable optical frequency combs. Furthermore, Optical frequency combs are consist of equally spaced optical frequency with a fixed phase relation between themselves. Our method requires a single optical element and it is very practical and efficient in terms of both power budget and cost compared to modelocked lasers.

The optimization where the necessary modulation parameters are calculated, is the most important part of this thesis. In order to calculate them, an error function is defined. Moreover, the variance, the maximum and the mean value of the comb lines amplitudes are used in defining the error function. Using this approach we have generated an optical spectrum with 26 comb lines and 3 db flatness theoretically.

There are two kind of modulations that are explained in detail in their corresponding sections: one is amplitude modulation and the second is phase modulation which is used in the theoretical work.
Finally the thesis concludes with comments about the future work which includes the development of the following amendments of the results presented in this study.

Keywords: Generation Continuous-Wave (CW) Modulation, Variation, Optical Frequency Comb, Phase Modulation.

ÖZET

ULTRAFLAT OPTICAL COMB GENERATION BY PHASE-ONLY MODULATION OF CONTINUOUS-WAVE LIGHT

Mehmet Bay

Elektrik-Elektronik Mühendisliği
Tez Danışmanı: Yrd. Doç. Dr. SARPER ÖZHARAR

Haziran 2012, 47 Sayfa

Lazer mühendisliği alanında yapılan bilimsel çalışmalar, özellikle optik frekans tarak üretimi ve uygulamaları, fiziksel özellikleri ve teknoloji alanında bir çok uygulaması açısından bilim dünyasının ilgisini çekmiştir. Tezle bağlantılı olarak bazı örnekler verilebilir, arbitrary waveform generation and mode locked lazerler bu uygulamanın önemli alanlarıdır. Ayrıca lazerlerin kullanım alanında bazıları şu şekildedir, elektronik, haberleşme sektörü, nano boyuttaki malzemelerin karakter analizleri, optik, plazmonik nanofotonik ve son olarak nanoteknoloji.

Bu tezde özellikle, durağan frekansın tarağı üretilmesinin teorik amaçlanmıştır.Üretilen frekands tarağının her bir elemanı eşit faz farkı ile konumlandırılmıştır. Kullanılan method ise çok pratik olmakla birlikte maliyet ve bandgenişliği bakımından da etkili bir yöntemdir. Tezin en önemli kısımlarından biri de uygun parametrelerin hesaplandığı optimizasyon kısmıdır. Uygulanabilir eş kuvvetli optiksel frekans taraklarının elde edelebilmesi için, 3dB'lik bir salınım aralığını vardır ve bu aralıkta taraklanma elde etmek için hata fonksiyonuna ihtiyaç vardır. Buna ek olarak hata fonsiyonunun sıfıra yaklaşması için maksimum değer, ortalama değer, varyans terimleri de hata fonksiyonu içerisinde kullanılmıştır. Bu yaklaşım ile 3 dB salınımla birlikte 26 li optiksel çizgi elde ettik ve ayrıca teorik olarak kanıtlanmıştır. Ayrıntıları hali kendi kısımlarında anlatılmış olan modülasyon methodları, genlik modülasyonu ve faz modülasyonu olmak üzere iki çeşittir. Teorik çalışmada ise çoklu faz modülasyonu kullanılmıştır.

Tez, sonuçları hazırda bulunan bu çalışmayı, gelecekte takip edecek geliştirme ve iyileştirme çalışmalarından kısaca bahsederek tamamlanır.

Anahtar Kelimeler: Devamlı-Dalga (CW) Modülasyonu, Optik Frekans Tarağı, Varyans, Faz Modülasyonu.

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ABBREVIATIONS

CW	:	Continuous-Wave
PM	:	Phase Modulation
AM	:	Amplitude Modulation
RF	:	Radio Frequency
OAW	:	Optical Arbitrary Waveform Generation
AWG	:	Arbitrary Waveform Generation
CDMA	:	Code-Division Multiple Acces
HNLF	:	Highly Nonlinear Fiber
OFDM	:	Orthogonal Frequency-Division Multiplexing

SYMBOLS

Bessel function : $J(A),J(B),J(C)$

Input electromagnetic Wave : E_{in}

Radial Frequency : w

Frequency : v

Time : t

Phase of A : PhA

Phase of B : PhB

Phase of C : PhC

Phase multiplier between A and B : $m1$

Phase multiplier between A and C : $m2$

Amplitude of frequency component A: : A

Amplitude of frequency component B: : B

Amplitude of frequency component C: : C

The number of frequency comb line : n

1. INTRODUCTION

This chapter gives an introduction which shows us the big picture of the whole thesis report and gives intuitive motivation about this work.

1.1 SCOPE

Laser modulation is widely used in many application areas such as, telecommunication, and laser ranging on the other hand, equally spaced, and stable optical frequency combs are used in application such as, arbitrary waveform generation [1]. Mode locked lasers are the most common method to produce stable optical frequency combs. In addition, the mode locked lasers have an ability to produce optical spectrumwith a large bandwidths. However, these lasers are very demanding and expensive due to complexity of the requirements and maintaining problems. To mitigate these problems, it is obvious that an alternative method of generating optical frequency combs is needed. Due to its facilities like easyness and cheapness, by external modulation of continuous wave (CW) light can conribute profoundly new ideas for generating of optical frequency comb lines (Supradeepa, Weiner,2011).

In this thesis, it is specifically focused on a theoretical approach of the generation of stable optical frequency combs via phase only modulation of CW light. Moreover, optical frequency combs are descriptive of equally spaced optical frequencies with a fixed phase relation between them. Even though the optimization part is long at first implementation, this method requires a simple experimental set up and it is very practical and efficient in terms of both power budget comparing the modelocked lasers. The resultant optical spectrum has large bandwidth

The optimization section is the most important part of this thesis due to have decisively stable optical frequency comb generation. In mathematical terms the behavior of a power signal of a laser beam is exactly the sum of bessel functions from negative infinity and positive infinity.

$$E_{in}e^{iA\sin(wt)} = \sum_{-\infty}^{\infty} E_{in} J(A) e^{in(wt)} \qquad \textbf{(1.1)}$$

According to this equation, the laser beam behavior is working as bessel function therefore, all the basic bessel properties are exist in laser beam behavior such as, combinations of more than one bessel functions (Özharar, Quinlan, Gee, Delfyett, 2005).

These Bessel functions have different peaks for different values but, these peaks are not periodic they are irregularly shaped peaks. One bessel function can consist of irregular peaks however, if this function can combine with another appropriate irregular function which is unstable, the stable optical frequency combs could be existed.

Figure 1.1: Illustration of Bessel function

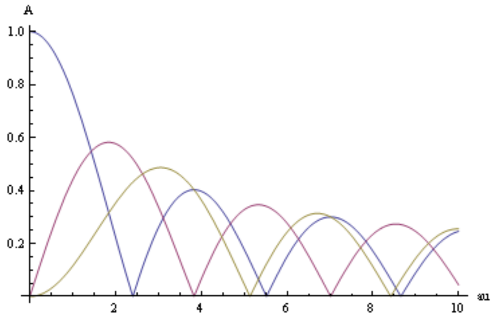
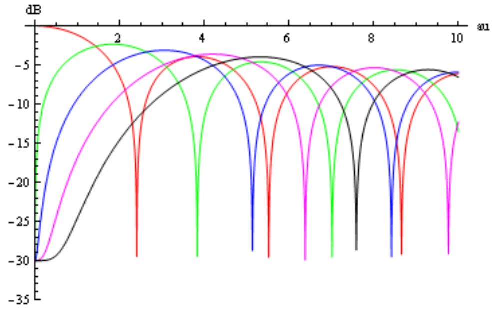


Figure 1.2: Illustration of Bessel function in dB scale



In other words, when these different bessel functions are combined themselves with some appropriate amplitude values, the stable optical frequency combs can be found. The figure 1.1 and the figure 1.2 illustrate the harmonics which are between zeroth hormoninc to infinitest harmonicss. Since the harmonics are symmecricly same negatif part of the figure 1.1 and 1.2 are same as the positive sides.

In order to have flat optical frequency combs, the values of the modulation parameters such as; amplitudes and relative phases need to be determined an error function mechanism. Making a good optimization is naturally a neccessity. Moreover, there is a target flatness of 3dB to have a practically useful optical comb. Therefore, an error function is needed which can find appropriate amplitude values, phase differences between them and the resulting peak has to be in the range of 3 dB. By minimizing the error function and approach it to zero. We can obtain perfectly flat optical frequency combs. The variance of the amlitudes of the optical comb lines, the maximum value and the mean value of these comblines are used to aproximate the error function a value as value of zero. Using this approach we have generated an optical spectrum with 25 comb lines and 3 dB flatness theoretically.

There are two kinds of modulations that are explained in detail in following sections one is the amplitude modulation, and the second is the phase modulation which is used in the theoretical work.

1.2 GOAL OF THE THESIS

The main purpose of this thesis is to produce stable optical frequency comb lines by phase modulation of continuous wave (CW). While considering for the number of comb lines, flatness is also considered as a purpose. We make a good approach with an efficient optimization and lowering the error value to zero.

In the mean time our method offers an alternative source to mode locked lasers based on reduced the complexity, and decreasing the cost. Last but not least, also the power efficiency of the output signal is considered as main part of theoretical approach of the study. It is intuitively expected that, generating the optical frequency comb lines with signal power which is centered around the center frequency.

1.3 OUTLINE OF THESIS REPORT

In the chapter 2, relevant previous works (literature survey) are described. In the chapter 3, the need for optical frequency combs is explained. In the chapter 4, amplitude modulation mentioned with usages and application, phase modulation is effectively detailed and some application examples are given besides mathematical explanation. In the chapter 5, multiple sine-phase modulation is explained by deriving formula. In the chapter 6, the optimization and error function are demonstrated with the convolution method. In the chapter 7 sythesis and results of the thesis are explained, In the chapter 8, conclusion and discussion about the thesis are given together with possible future research directions.

2. PREVIOUS WORK

In this chapter, a review of works in the literature review that have the same scope as this work is presented. Generally, at the previous work shows us there are 2 main modulation methods, the first one is the amplitude modulation and the second one is the phase modulation. There are some essentially individual published papers in the literature and it has to be mentioned that their crucial ideas gives us an understanding of flatness of comb lines.

S. Ozharar, F. Quinlan, I. Ozdur, S. Gee, and P. J. Delfyett, (2008) make the dual-sine-wave phase modulation approach to produce flat optical frequency combs. They claim that generation of stable and flat optical comb lines is a crutial issue for many applications such as, arbitrary waveform generation (AWG) (Özharar, Quinlan, Gee, and Delfyett 2005). spectral-phase-encoded optical code-division multiple acces (CDMA) (Delfyett, Gee, Choi, Izadpanah, Lee, Özharar, Quinlan, and Yilmaz, 2006), and mode locked lasers (Özharar, Quinlan, Gee, Delfyett, 2005).

In the theorical part of the study, they state that they need to produce an error function to obtain flat optical frequency comb lines. For the error function they used direct calculation of the flatness to make the error value minimized to the value of zero. In addition to this, in the experimental part of the study and they generated an optical spectrum of 11 comb lines with 1.9-dB flatness and 3-GHz spacing. Moreover, they generated another spectrum which has 9 comb lines with 0.8-dB flatness. It is quite good result and the results are match with the theorical study as we expected. They also clarified that this method is efficient economically and it has an advantage about power loss (Özharar, Quinlan, Gee, Delfyett, 2008).

On the other hand, Yuelan Lu, Yongwei Xing, and Yongkang Dong(2010) come up with a new method of generating an equal-amplitude optical comb by exploiting multi-frequency phase modulation. First of all, they pointed out that why they need a new method by comparing new method and mode locking method with its complicated design, starting and maintaing of mode locking problems (Ramond, Hollberg, Juodawlkis, Calawa, Calawa, 2007). They perfom that the results are theorically and experimentally matched. They also claim that, with an *n* frequency component and including odd harmonic frequencies, it is possible to have that 4*n-1* optical comb lines which are equally spaced and have same amplitude with the range of 0.3 dB flatness. Likely, for the even harmonic and *n* frequency componets this number shown as, 2*n+1* optical comb line which have equal amplitudes and same flatness range. In the similation part they used fourier transform of Bessel function which is characteristically and symetricly same. They used small amplitudes of Bessel function in case, for the higher ampllitudes, it is not possible and easy to have good result for the optical comb lines because, new side band amplitudes are produced that makes the flatness of produced optical comb lines poorer (Fujiwara, Teshima, Kani, Suzuki, Taka- chio, and Iwatsuki, 2003). Up to here one of the most important point that has to be mentioned, is that the flatness of the optical comb lines seem to be quite perfect but, we need more optical comblines which's range of flatness is around 2-3 dB. The power efficiency is also considered as an important issue (Yuelan, Yongwei, Yongkang, 2009).

There is another intersting study which is made by V.R. Supradeepa and Andrew M. Weiner. The study is about producing flat optical frequency combs generated by the phase modulation as well. They mentioned that comb generation is widely used in many areas such as optical communication(Ohara, Takara, Yamamoto, Masuda, Morioka, Abe, and Takahashi, 2007), Radio frequency (RF) photonics and optical arbitrary wave form generation (OAWG) (Jiang, Huang, Leaird, and Weiner, 2004). The methods of generating optical comb lines are mentioned such as, a short pulse and then spectral broadening in dispersion decreasing fiber or highly nonlinear fiber (HNLF). They found that, there are more than one phase modulators which are cascaded themselves due to weakness of flatness of optical comb lines of only one phase modulator which is not efficient. Moreover, they also clarified that the applications of flat comb lines need very smoth flatness which is not easy to obtain from behavior of laser beam same as Bessel function behavior. Finally, they demostrated that one phase modulator can generate more than 100 comb lines with 10-dB flatness range (Supradeepa, Weiner Andrew, 2011).

There are several methods for optical frequency comb generation, some of them are explained above. In addition to these studies, T. Sakamoto, T. Kawanishi and M. Izutsu came up with another method which is using a conventional Mach-Zehnd modulator that is asymmetrically dual driven large amplitude sinusoidal signals with different amplitudes (Kourogi, Enami, and Ohtsu, 1994), and (Morioka, Mori, and Saruwatari, 1993). They claim that, generation of an optical frequency comb is crucially important for generating multi-wavelength continuous-wave (CW) lights, ultra-short pulses and micro-millimetre-wave clocks (Izutsu, Yamane, and Sueta, 1977), (Sakamoto, Kawanishi, Izutsu, 2007) They mention that as it is difficult to have flat comb lines on account that the frequency components obey the Bessel function as an adversity of flatness of optical comb lines (Murphy, Udem, Holzwarth, Sizmann, Pasquini, Araujo-Hauck, Dekker, 2005). They further stated that odd harmonics can have optically flat comb lines and even harmonics can not have flat comb lines as much as odd harmonics. The light intensity graphs are shown in dB scale and each component of comb lines has the same phase difference. They clarified that, both experimentally and theorically, 11 comb line can be generated by this method with the flatness range of 1.1 dB. Moreover, 19 component can be generated with the range of 4.3 dB (Hung, Chung, Chao, 2001)

3. OPTICAL FREQUENCY COMBS

The optical frequency comb lines are widely used in many areas such as optical communication, arbitrary waveform generation, and even in calibration of astronomical telescopes. There are many published work by several research groups on finding the most practical method which is inexpensive and very power efficient (Özharar, Quinlan, Gee, Delfyett, 2005).

The optical frequency of a comb component with the index *n* can be defined as;

$$v_n = n.v_0 + v_{ceo} \ , \qquad \textbf{(3.1)}$$

Where, v_0 is the spacing between the sides frequency components, and v_{ceo} is the carrier envelop ofset frequency defined in range of 0 to v_0

The optical frequency comb lines are spreading around the carrier envelop offset frequency. In addition to this fact the phase difference of any arbitrary comb lines has a static value. The value of phase difference is changable. The outcome of this study is based on these parameters.

3.1 THE NEED FOR STABLE OPTICAL COMB LINES

Each of the optical frequency comb lines behaves as a continuous wave, in other words, each of these lines can be named as a single channel in communication. There are some applications about generating optical comb lines such as, calibration of astrophysical spectrographs (Ramond, Hollberg, Juodawlkis, Calawa, Calawa, 2007). Since each comb line has the same frequency difference, they can be used to replace multiple single wavelength lasers and data can be encoded on these individual comb lines for secure communication (Fujiwara, Teshima, Kani, Suzuki, Taka- chio, and Iwatsuki, 2003). However, most of the present methods have complicated design and high cost. In addition, they can be difficult to optimize and modify, which limits their use by non-field scientists. Therefore, there is a constant need for an efficient way to generate optical combs that has low loss of power and low cost.

Besides, the generation of frequency comb lines via side band generation based continuonus wave modulation (CW), there is another main method of generation of optical frequency comb lines which is through mode locked lasers. In other words, mode locked lasers can be the main example of generation of optical frequency combs. These lasers are common and well studied. Moreover, method of mode locked laser is very efficient in order to have large bandwidth of optical frequency comb.
Whereas its efficiency and large bandwidth, the complexity of the equipments that are included in the set up is difficult to maintain for the stablity of the optical frequency comb lines. On the contrary is the continuous wave (CW) modulation method is comparably cheaper and easier for the set up of the study.

An optical ring cavity lenght l, can conribute many optical modes separated by.

$$f_c = \frac{c}{nl} \qquad \textbf{(3.2)}$$

These modes are separeted at a frequency which is called the cavity fundamental frequency ‘f_c’ and c is the speed of light, n is the refractive index. The frequency of modes and the total number of modes which are allowed also depend on other parameters such as, gain bandwidth, spectral filtering, dispersion, etc. The mode locked lasers can be studied as an active mechanism which can be named as active mode locking, and passive mechanism can be named as passive mode locking or combination of both mechanism which is hybrit mode locking. As a consequence these are well defined phase relation and they are all coherent modes.

For active modelocking, there are some frequency types based on the relation among the cavity fundamental frequency (f_c), the pulse repetition rate (f_p), and the modulation frequency (f_{mod}). These are harmonic modelocking, fundamental modelocking, and rational harmonic modelocking (Shenping, L., Caiyun, L., Chan, K.T., 1998), and(Jeon, Kyu, Ahn, Lim, Kim, Kim, Lee, 1998).

Fundamental modelocking is where all three of the above mentioned parameters are equal to each other

$$f_p = f_c = f_{mod} \tag{3.3}$$

If the pulse repetition rate is equal to modulation frequency and an discrite number multiple of the cavity fundamental (fp), the resulting modelocking is called harmonic modelocking.

$$fp = n.fc = fmod \tag{3.4}$$

Rational harmonic modelocking is called when pulse repetition rate is an integer multiple of modulation frequency and modulation frequency is equal to an integer multiple of cavity fundamental plus an offset term

$$fp = (m.n \pm 1) .fc = m. Fmod \tag{3.5}$$

Passive mechanism or passive mode locking is usually fundamental modelocking, since the intracavity nonlinear effects such as saturable absorption, nonlinear polarization rotation, etc prefer a pulse repetition rate at the cavity fundamental frequency.

Mode-locked lasers referenced to an external or internal optical reference can generate optical frequency combs with large bandwidth and high stability.

4. METHODS OF MODULATION

There are two main modulation methods, the first one is amplitude modulation and the second one is the phase modulation. These methods are used in many areas such arbitrary waveform generation, communication (O’Reilly, Lane, Heidemann, and Hofstetter, 1992). In order to have flat optical comb lines modulation is inevitable due to its facilities such as side band generation.

The modulation method either phase modulation or amplitude modulation need to be proved by the mathematical expressions but, the optimization part is vary from fourier basis to convolution method. In case the result should be the same for all methods of optimization. Lithium niobite crystals also used in this type of lasers.

4.1 AMPLITUDE MODULATION

Amplitude modulation is theoretically basic but it is partially the base of modulation of light wave. In the amplitude modulation, the voltage is applied to the carrier waveform.

The advantage of amplitude modulation method is efficient for the loss of power which is quite important. There are two main methods for amplitude modulation, the first one is interferometric based amplitude modulation and the second one is absorbtion based amplitude modulation. The input signal of electromagnetic wave and amplitude modulated out put signal is shown in the figures 4.1 and 4.2

Figure 4.1 : Illustration of signal before amplitude modulation in time domain.

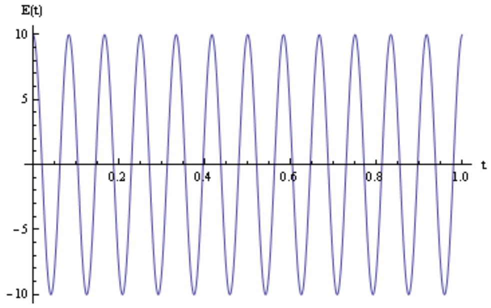
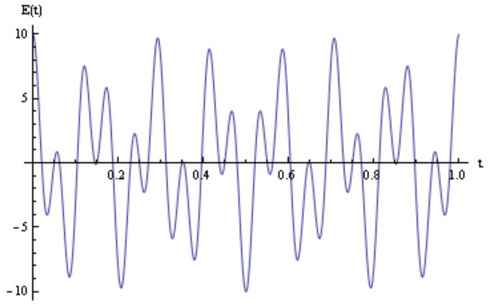


Figure 4.2 : Illustration of signal after amplitude modulation in time domain



For the mathematical model;

$$m(t) = A + m.\cos(w_m t + \phi) = A + \frac{e^{iwt} + e^{-iwt}}{2}.m \tag{4.1}$$

Where A is amplitude of frequency component, m is the constant, w_m is angular frequency, t is time, ϕ is phase difference.
For a sin wave the input electromagnetic wave can be shown as;

$$\begin{aligned} E_{in} &= E_0 \sin(w_c t + \phi_c) \\ E_{out} &= m(t) E_0 \sin(w_c t + \phi_c) \end{aligned} \tag{4.2-4.3}$$

Where, E_{in} is the input electromagtic signal, E_{out} is the output electromagtic signal, w_c is angular frequency.Now replace equivalent of *m(t)* in to the equation of 4.3

$$E_{out} = A + m.\cos(w_m t + \phi) E_0 \sin(w_c t + \phi_c) \tag{4.4}$$

After replacing some component the equation becomes as the following equation.

$$\begin{aligned} E_{out} &= E_0 A \sin(w_c t + \phi_c) + E_0 \frac{m}{2}.\sin(w_c + w_m)t \\ &+ (\phi_c + \phi) + E_0 \frac{m}{2}.\sin(w_c - w_m)t - (\phi_c - \phi) \end{aligned} \tag{4.5}$$

Intensity is proportional to the square of electric field. Therefore, it is possible to obtain the relation between input intensity and output intensity by taking square of the equation of 4.5 and put it in to the intensity the intensity formula which is written below in equation 4.6

$$I_0 = \frac{cn\varepsilon_0}{2}\left|E_0{}^2\right| \qquad \textbf{(4.6)}$$

Here I_0 is the intensity of the light and c is the speed of light. ε_0 is the permitivity of free space and n is the refractive index. Now we can plug the equation 4.5 into the equation 4.6 then new equation conributes us the relation the input intensity and the output intensity as follows, In figure 4.3,The spectrum of input wave has one peak however, in figure 4.2 there are more than one peak after modulation the side band generation occurs. The central angular frequency, $w_{c,}$ is adjustable and the the phase differences of right and left of the central freuency comb line are same.

$$I_{out} = I_0\left[A^2 + \frac{m^2}{2} + 2Am\cos(w_m t) + \frac{m^3}{2}.\cos(2w_m t)\right] \qquad \textbf{(4.7)}$$

Figure 4.3: Illustration of signal before amplitude modulation

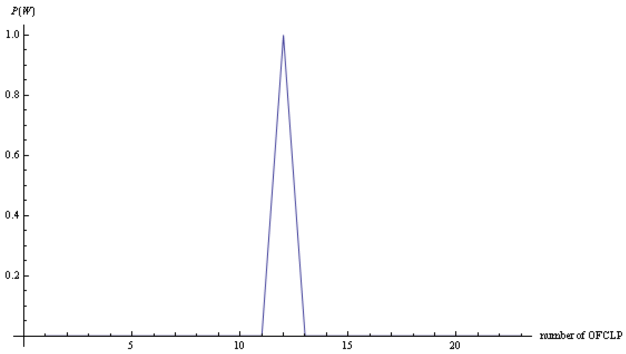
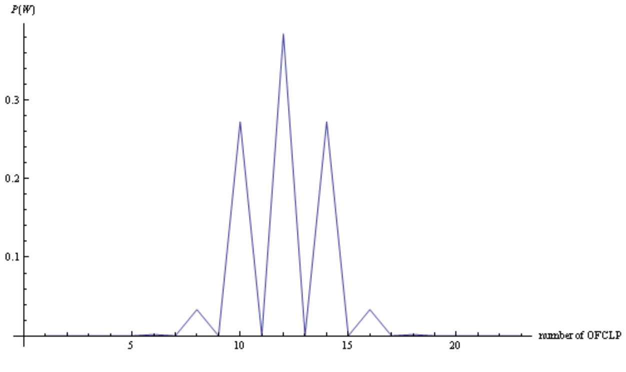


Figure 4.4: The spectrum of signal before amplitude modulation



Unlike the complexity of the mode locked laser for generating optical comb lines, interferometric based amplitude modulators are quite easy to set up. Moreover, these modulators are simply Mach-Zehnder interferometers which's arms are phase modulated. As a consequence the relative phase between two arms can be controlled same as the power output of the interferometer.

Absorption based modulators use direct absorption in the electro-absorption modulator case, or indirect absorption as amplitude modulation via polarization rotation and polarization filtering. Optical comb lines can be generated via amplitude modulation (AM) by using lithium niobate crystal (Supradeepa, Weiner Andrew, 2011). In the case of linearity of input optical field by linear polarization at a 45 degree angle with respect to principal axis of the lithium niobate crystal, its polarization can be rotated due to magnitude of the applied electric field by the electrically induced birefringence, the same principle can be applied to liquid crystal based amplitude modulators, which are very profoundly devices for spatial light modulators.

4.2 PHASE MODULATION

In phase modulation (PM) the phase of the frequency parameters are crutial. The phase of an optical signal is a relative parameter, and it is defined with respect to another optical signal and/or a reference point of time. Moreover, the phase has a specific value for adjusting. There is another main difference between the phase and amplitude of a signal. Since, the signal propagates the phase of a signal continually evolves, therefore phase modulation is simply based on modulation of the optical path length that the optical signal travels.

Lithyum niobate based modulators is the most widely used electro-optic phase modulators and amplitude modulators. The crystal that is made from Lithium niobate is an interesting crystal and it has many features for instance, ferroelectricity, piezoelectricity, and pyroelectricity and also Lithium niobate crystal has high electro-optic coefficients. Pockel‟s electro-optic effect basically states that crystals do not have inversion symmetry like lithium niobate will produce birefringence under an electric field. To put it another way, if there is an electric field which is applied, refractive index of the lithium niobate crystal a will change along x axis. Therefore, there is a change in optical path length and a change in the phase of optical signal traveling through the crystal of lithium niobate.

Besides of the crytal of lithyum niobate there is another family of modulators which is liquid crystal based modulators. (Yıldız, Polat, San, Kaya, 2011). Liquid crystals are simply semi-crystals, and these crystal's molecules are arranged in a crystal like structure however these are still the overall material is in liquid form. Same as the birefringe effect of lithyum niobate crysal, if there is an applied electric field, there can be change in the orientation of the liquid crystal molecules. As a concequence, the refractive index in a given axis, since the crystalline molecules are birefringent which depends on applied electric field, they change both the input light polarization and the input light phase according to their physical orientation. Liquid crystal based on the phase modulators which are generally used in *n*x*m* arrays, these modulators are very profoundly practical in pulse shaping

Similar to mathematical model the amplitude modulation, the mathematical model of phase modulation is exist, however for the phase modulation based on optical path length (Fujiwara, Teshima, Kani, Suzuki, Taka- chio, and Iwatsuki, 2003).

For the mathematical model;

$$m(t) = e^{iA.\sin(w_m.t+\phi_m)} \tag{4.7}$$

Here A can be any number, and w_m angular frequency, $Ø_m$ is phase difference between two optical comb lines.

$$E_{out} = E_0 e^{iw_c t} \otimes e^{i\phi_c} \tag{4.8}$$

For the equation 4.8, before the phase modulation of a signal the electromagnetic wave propagates the phase of a signal continually evolves as *Ø_c* or we can write the equation 4.8 as 4.9

$$E_{out} = E_0 \sin(w_c t + \phi_c) \tag{4.9}$$

After phase modulation of a electromagnetic signal, the equation becomes the following equation 4.10.

$$E_{out} = E_0 \sin(w_c t + \phi_c + A\sin(w_m t + \phi_m) \tag{4.10}$$

or we can write the equation propagation of a signal 4.10 as 4.11

$$E_{out} = E_0 e^{iw_c t} . e^{i\phi_c} . e^{iA\sin(w_m t + \phi_m)} \tag{4.11}$$

Since each frequency components obeys the behavior of bessel function, then it is possible to modify the equation 4.11 to the equation 4.12 which is modified version as the following.[4]

$$E_{out} = E_0 e^{iw_c t} . \sum_{n=-\infty}^{\infty} J_n(A) . e^{inw_m t} e^{in\phi_m} \tag{4.12}$$

The summation of the bessel function cope with a question that is why do not we take the integral of bessel function. The answer is quite simple, due to necesstiy of discreteness of the values of n, it can not be integrated. In other words, the summation of bessel function from negative infinity to positive infinity includes only the values which are discrite such as ±1, ±2, ±3, ±4.

Modified version of signal which is phase modulated can be shown as expanded to make it clear,

$$\begin{aligned}
E_{out} &= E_0 e^{iw_c t}.(J_{-\infty}(A)e^{-i\infty w_m t}.e^{-i\infty\phi_m} + ...\\
& J_{-2}(A)e^{-i2w_m t}.e^{-i2\phi_m} + J_{-1}(A)e^{-i1w_m t}.e^{-i1\phi_m} +\\
& J_0(A) + J_1(A)e^{i1w_m t}.e^{i1\phi_m} +\\
& J_2(A)e^{i2w_m t}.e^{i2\phi_m} + ...J_{+\infty}(A)e^{+i\infty w_m t}.e^{i\infty\phi_m} +)
\end{aligned} \quad \textbf{(4.13)}$$

As it can be seen basically, the summation discrete values of the bessel function is clear. more optical frequency comb but, by using one frequency component we can not have profoundly flat optical comb line as we expect.

This phenomenon come up with an idea about phase modulation of more than one frequency components. This idea will be explained in the multiple sine-phase modulation deeply. The input signal of electromagnetic wave and amplitude madulated out put signal is shown in the figures 4.5 and 4.6

Figure 4.5 : Illustration of signal after amplitude modulation in time domain

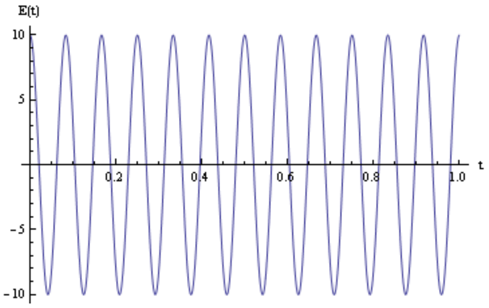
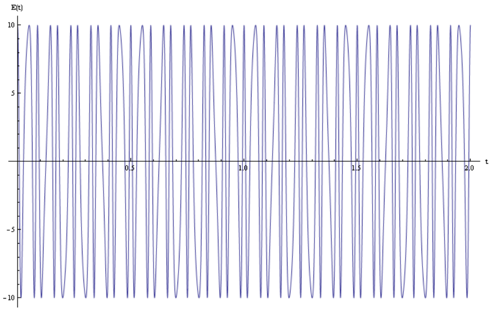


Figure 4.6 : Illustration of signal after amplitude modulation in time domain



The spectrum of one component is shown below, and the spectrum that is phase modulated is pictured in the following.

Figure 4.7: The spectrum of signal before phase modulation

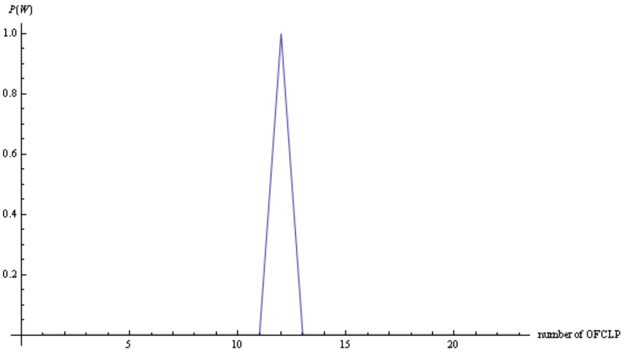
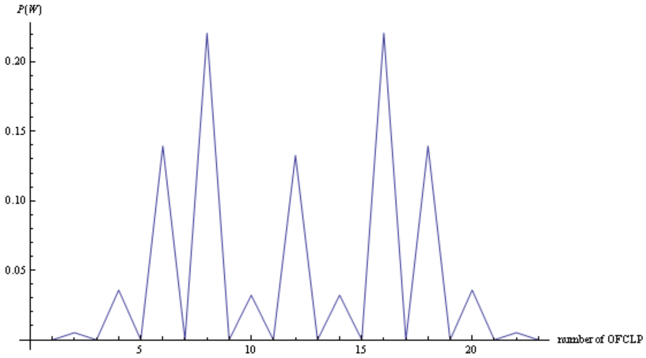


Figure 4.8 : The spectrum of signal after phase modulation



During the phase modulation of a signal, from one optical comb line conribute more than one optical frequency comb lines, however these comb lines are not flat the only positive advantage of this spectrum, the phase difference between two consecutive optical comb lines are almost constant. Also the phase difference of two comb lines is adjustable that can be change as how much is needed.The idea of phase modulation of multiple frequency component is tried in the next chapter. When there is no modulation, there is only one comb line which can be called as $J_0(A)$ on the other hand, the phase modulated signal have several comb lines after modulation, the center frequency comb lines again called as $J_0(A)$ and the comb line which is the right side of centered optical comb line is called as $J_1(A)$ first harmonic of the bessel function also the same as for the left side $J_{-1}(A)$ negative first harmonic of the bessel function. There is another interesting property of bessel function, every comb line is the same as the negative sign of combline $J_{+1}(A)= J_{-1}(A)$ which is symmetric, unless the multipliers of the frequency components are consist of odd harmonics. The last important thing but not least, the flatness range, the flatness or smothness is considered the proximity of frequency comb lines between the highest comb line. For example, there is a spectrum which has amplitude in dB scale by the following values, 4, 5, 6, 7, 6, 5, 4 which is symmetricly spread. Here the difference gives us the flatness or other terms smothness. The flatness range can be found as taking the maximum value and subtructing a specific value from it. Then the numbers of comb line in the range of flatness value give us the flatness of the given spectrum. In the example if the flatness is taken as 3 dB. In this range 7 flat optical frequency comb lines are obtained however, if the flatness of the spectrum is 2 dB, there will be 5 flat optical comblines in that range. The purpose of this study having more stable comb lines both numerically alot and smothly flat. Therefore given a spectrum the flatness range is important which make is it valuable for the aplications of the phase modulation such as communication systems.

Figure 4.9 : The spectrum of output signal after phase modulation power efficiency centered around enter frequency

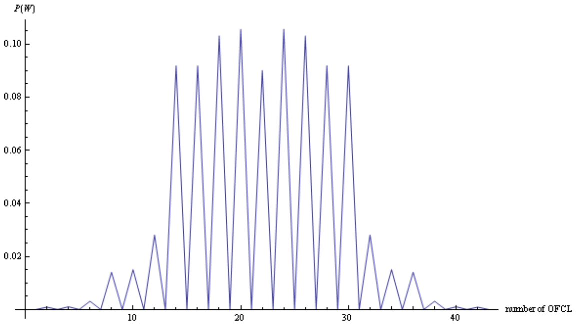
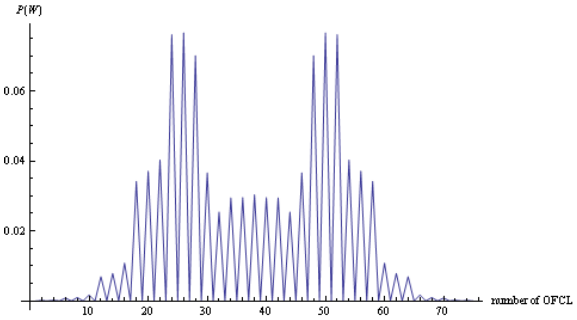


Figure 4.10 : The spectrum of output signal after phase modulation power efficiency spread sides of the center frequency



The last consideration is the efficiency of the signal power. After phase modulation the power spreads for the side frequency components to generate another comb lines by obeying the bessel funcion however, the values of bessel function send the power to the higher order bessel harmonics that makes the signal power spread to the sides instead of centers then the result comes very ineffectively the power accumulated at the sides. The main purpose is to use power efficiently by spreading the singal power around the central frequency. As one can the the difference between the figure 4.9 and the figure 4.10, it is obvous that in the figure 4.9, the power centered around the center frequency which is quite good for the efficiciency of the signal power but in the figure 4.10 the signal power is collected at the sides. Even in the middle part of the the spectrum of figure 4.10 there is a flatness which is quite acceptable but, it is not preferable as the figure 4.9 on account of the spreading the signal power through the side parts of the spectrum.

As a conclusion of this chapter, the appropirate flatness which is beter than 3 dB, centered singal power, and the quantity of flatt comb lines are considered. These can be obtained unless there will be multiple sine phase modulation of the frequency components with convenient values of phase difference, multiple number of phase, amplitudes of the frequency components. All of these can contribute a spectrum of flat optical frequency comblines.

5. MULTIPLE SINE –PHASE MODULATION

Multiple sine phase modulation is the combination of more than one modulationfrequency component. The modulation methods have some application such as, optical comb generation (Özharar, Quinlan, Gee, and Delfyett, 2005). Also, mode locked laser can be an example of generating optical frequency comb lines.

Since the spectrum due to each modulation frequency component obeys the Bessel function, it is not easy to obtain flat optical frequency comb line by one frequency component. More than one modulation frequency components are needed simultaneously to realize flat spectrum with multiple number of comb lines.

The resultant spectrum was given in the last chapter. Making the long story short, in order to have flat optical comb lines, multiple sine phase modulation is required. S. Ozharar, F. Quinlan, I. Ozdur, S. Gee, and P. J. Delfyett, (2008) studied the dual-sine-wave phase. They demand that the need for stable and flat optical comb lines is an essential topic. In the theorical part of the study, they reported that they need to produce an error function in order todetermine the necessary parameters to realize equal amplitude optical frequency comb lines. For the error function they used direct calculaltion of flatness and optimized the flatness as a function of modulation parameters. In addition to this, in the experimental part of the study, they realized an optical spectrum of 11 comb lines with 1.9-dB flatness which is quite practical and 3-GHz spacing. Moreover, they found a spectrum which has 9 comb lines with 0.8-dB flatness. It is quite good result and the results are in agreement with the theoretical study as we expected.

Remember the mathematical model for the phase modulation of one frequency element based on Bessel functions.

$$E_0 e^{iA\sin(wt)} = \sum_{n=-\infty}^{\infty} E_0 J_n(A) e^{in(wt)} \qquad \textbf{(5.1)}$$

If the frequency components are increased and the result of their combination is given as the following.

$$
\begin{aligned}
E_{out} &= E_0 e^{i(A\sin(w_m t+\Delta\phi)+B\sin(mw_m t+\Delta\phi))} \\
&= E_0 e^{iw_c t}.(\sum_{k=-\infty}^{\infty} J_k(A).e^{ikw_m t} e^{in\phi_m} \times \sum_{l=-\infty}^{\infty} J_l(B).e^{ilmw_m t} e^{in\phi_m})
\end{aligned}
\qquad \textbf{(5.2)}
$$

Then the equation 5.2 is expanded below in the equation 5.3

$$
\begin{aligned}
& E_{out} = E_0 e^{iw_c t}.[(J_{-\infty}(A)e^{-i\infty w_m t}.e^{-i\infty\phi_m} + \ldots\ldots \\
& J_{-2}(A)e^{-i2w_m t}.e^{-i2\phi_m} + J_{-1}(A)e^{-i1w_m t}.e^{-i1\phi_m} + \\
& J_0(A) + J_1(A)e^{i1w_m t}.e^{i1\phi_m} + \\
& J_2(A)e^{i2w_m t}.e^{i2\phi_m} + \ldots\ldots\ldots J_{+\infty}(A)e^{+i\infty w_m t}.e^{i\infty\phi_m} +)] \times \\
& [(J_{-\infty}(B)e^{-i\infty mw_m t}.e^{-i\infty\phi_m} + \ldots\ldots \\
& J_{-2}(B)e^{-i2mw_m t}.e^{-i2\phi_m} + J_{-1}(B)e^{-i1mw_m t}.e^{-i1\phi_m} + \\
& J_0(B) + J_1(B)e^{i1mw_m t}.e^{i1\phi_m} + \\
& J_2(B)e^{i2mw_m t}.e^{i2\phi_m} + \ldots\ldots\ldots J_{+\infty}(B)e^{+i\infty mw_m t}.e^{i\infty\phi_m} +)]
\end{aligned}
\qquad \textbf{(5.3)}
$$

After some cancellation and summation now we have the mathematical formula for the combination of 2 sine wave modulation as in the equation 5.4

$$
E_{out} = E_0 \sum_{n=-\infty}^{\infty}(\sum_{h=-\infty}^{\infty}(J_h(B).J_{n-mh}(A).e^{i(n-mh).\Delta\phi}).e^{inw_m t} \qquad \textbf{(5.4)}
$$

After this, *n*th order harmonic becomes an infinite sum of both sine waves as in combination of them. Yuelan *et al* states that ‘*n*’ number of modulation sine waves can result in *4n-1* number optical frequency comb lines in which *n* is odd. They also state that if ‘*n*’ is an even number, then one can generate an optical frequency comb with *2n+1* comb lines.

For 3 frequency components the result sholud be the following,

$$\begin{aligned}
E_{out} &= E_0 e^{i(A\sin(w_m t+\Delta\phi_1)+B\sin(m_1 w_m t+\Delta\phi_2)+C\sin(m_2 w_m t+\Delta\phi_3))} \\
&= E_0 e^{iw_c t}.(\sum_{j=-\infty}^{\infty} J_j(A).e^{ijw_m t} e^{ik\phi_a} \times \sum_{l=-\infty}^{\infty} J_l(B).e^{ilmw_m t} e^{il\phi_b}) \\
&\times \sum_{h=-\infty}^{\infty} J_h(C).e^{ihmw_m t} e^{ih\phi_c})
\end{aligned} \tag{5.5}$$

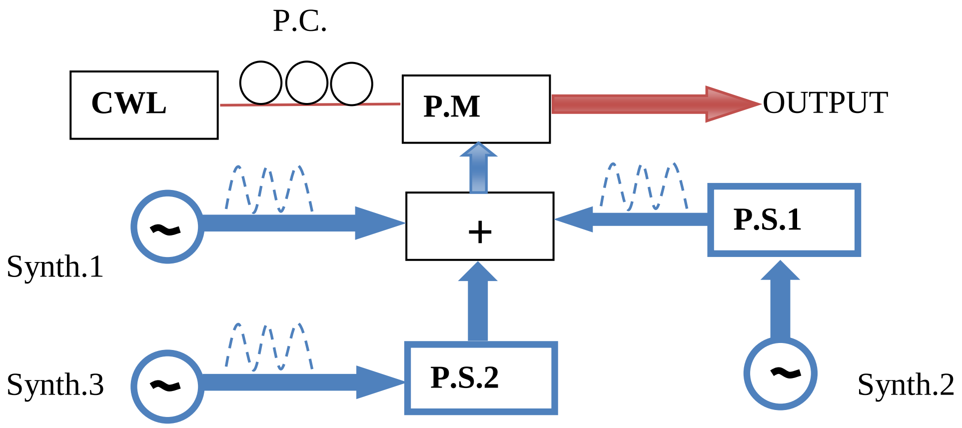
$$\begin{aligned}
&E_{out} = E_0 e^{iw_c t}.[(J_{-\infty}(A)e^{-i\infty w_m t}.e^{-i\infty\phi_m} + ... \\
&J_{-2}(A)e^{-i2w_m t}.e^{-i2\phi_m} + J_{-1}(A)e^{-i1w_m t}.e^{-i1\phi_m} + \\
&J_0(A) + J_1(A)e^{i1w_m t}.e^{i1\phi_m} + \\
&J_2(A)e^{i2w_m t}.e^{i2\phi_m} + ...J_{+\infty}(A)e^{+i\infty w_m t}.e^{i\infty\phi_m} +)]\times \\
&[(J_{-\infty}(B)e^{-i\infty m_1 w_m t}.e^{-i\infty\phi_m} + ... \\
&J_{-2}(B)e^{-i2m_1 w_m t}.e^{-i2\phi_m} + J_{-1}(B)e^{-i1m_1 w_m t}.e^{-i1\phi_m} + \\
&J_0(B) + J_1(B)e^{i1m_1 w_m t}.e^{i1\phi_m} + \\
&J_2(B)e^{i2m_1 w_m t}.e^{i2\phi_m} + ...J_{+\infty}(B)e^{+i\infty m_1 w_m t}.e^{i\infty\phi_m} +)]\times \\
&[(J_{-\infty}(C)e^{-i\infty m_2 w_m t}.e^{-i\infty\phi_m} + ... \\
&J_{-2}(C)e^{-i2m_2 w_m t}.e^{-i2\phi_m} + J_{-1}(C)e^{-i1m_2 w_m t}.e^{-i1\phi_m} + \\
&J_0(C) + J_1(C)e^{i1m_2 w_m t}.e^{i1\phi_m} + \\
&J_2(C)e^{i2m_2 w_m t}.e^{i2\phi_m} + ...J_{+\infty}(C)e^{+i\infty m_2 w_m t}.e^{i\infty\phi_m} +)]
\end{aligned} \tag{5.6}$$

For the 3 frequency component is quite difficult comparing with the dual sine phase modulation after bunch of mathematical calculation, the *k*th order component is given by the following equation. All of the parameter are explained before explanations.

$$\begin{aligned}
&E_{out} = E_0 \sum_{k=-\infty}^{\infty}(\sum_{n=-\infty}^{\infty}(\sum_{h=-\infty}^{\infty}(J_h(C)\times \\
&J_{n-hm_1}(B)\times J_{k+m_1 m_2 h-hm_1-nm_2}(A).e^{i(\Delta\phi_c+\Delta\phi_b(n-hm_1)+\Delta\phi_a(k+m_1 m_2 h-hm_1-nm_2))}))).e^{i(k)w_m t}
\end{aligned} \tag{5.7}$$

After all mathematical expressions, finally we can find the amplitude of nth order harmonic of the Bessel function. In other words if the amplitude of the comb lines can be controlled, with good approximation of a strong optimization several comb lines can be obtained in the available range of flatness. Therefore, a precise and sensitive error function is needed. Due to millions of possibilities of optical frequency comb lines spreading, there must be practical iteration of the error function. These will be explained in detail in the next chapter.

Figure 5.1: The experimental setup



The experimental setup is simply settled, these components are orderly, P.M. is the phase modulator, P.S. is the phase shifter to ensure that all sine waves are in phase, P.C. is the polarization controller, and CWL is the CW laser. A power signal divides into three signals then all signals are modified and multiplied by the phase multipliers. Finally the phase modulation progresses after summation of three signals.

6. OPTIMIZATION

In the optimization section, there is an approximation to generate flat optical comb lines. Since the function that finds the resultant spectrum of amplitudes of the combinations of frequency components. The function has some variables such as, frequency components, phase differences of frequency components, multipliers of phase differences. The function is made by convolution method which is explained in the section 6.1. After the optical frequency comb line function, it is possible to generate the spectrum of optical comb lines however, there are millions of possibilities of spectrum of comb lines due to boundrries of variables and numerically their precisions. Therefore, there is a constant need for an error function which is practically efficient, sensible and has profound accuracy.

6.1 CONVOLUTION METHOD

In the theoretical part of this study the convolution method is used due to its straightforwardness and facilities instead of classical mathematical methods.The calculation of the function is progressed in Fourier basis, The fourier transform of multiplication of functions is equal to the convolution of their Fourier transforms as shown in the equation below. Firstly, the frequency spectrum A and B are convolved the the result is reconvolved with the frequency spectrum C. The progresssors are commutative means that the order between A, B, and C is not important, it contributes the same result.

$$Fourier[f(t)\bullet g(t)] = Fourier[f(t)] * Fourier[g(t)] \qquad \textbf{(6.2)}$$

$$E_0 \cdot \left(\begin{array}{l} \sum_{j=-\infty}^{\infty} J_j(A)\cdot\delta(w-jw_m)e^{ij\phi_a} * \sum_{l=-\infty}^{\infty} J_l(B)\cdot\delta(w-lm_1w_m)e^{il\phi_b} * \\ \sum_{h=-\infty}^{\infty} J_h(C)\cdot\delta(w-hm_2w_m)e^{ih\phi_c} \end{array} \right) \qquad \textbf{(6.2)}$$

6.2 ERROR FUNCTION

In the error function, there are 9 variables with 18 boundaries of these variables these are amplitudes of frquency components which are *A, B, C* and their phase *PhA, PhB, PhC,* and their relative frequencies *m1* and *m2*. The final parameter is the number of comb line that is given by *2n+1* which is searched by the error function as *n*.
If the precision is taken as 6 digit accuracy then the boundaries of frequency components *A, B, C* are between 0 and 6 and *m1* and *m2* have to be integer numbers such as 1,2,3. Phase of them, *PhA, PhB, PhC,* are between 0 and 1. The values of *m1* and *m2* are between 1 and 9. Finally *n*, the number of comb line can be any number but to be practical, it is neccessary to narrow *n* between 1 and 30. Therefore, the multiplication of them gives the total possibility which is difficult to check one by one with mathematical calculation. The running process of the program could take along time that is not preferable and unwanted. To solve that, the iteration methods can be used such as Newton's method for minimizing the error function.

Up to here the function is constructed with boundaries and iteration methods however, still it is not completed. There has to be main function which consideres the flatness of the spectrum. As mentioned before the flatness range canbe defined as the distance from the maximum hight of comb line in dB scale. Unless, the average distance of the optical frequency comb lines to the maximum peak of the comb line is getting smaller and smaller, there will be more stable freqyency comblines.

For example,let us consider a symmetric specterum is spread as with the following 4, 5, 6, 7, 6, 5, 4 amplitudes. The maximum peak is 7 and the average distance to the maximum can be found as [(7-4)+ (7-5)+ (7-6)+ (7-7)+ (7-6)+ (7-5)+ (7-4)], then 12/7 is found as the average distance to the maximum peak.

By the approximation if it could be came closer to zero then the flatness of the spectrum can be increased and more frequency comb lines with closer amplitudes can be produced however, again giant number of possibilities do not let it be possile for computer programing. Very efficient way is needed, not structurely the same but consequently the same way is existed which is called as variance. Mathematical programs can find the variance value of the spectral amplitudes easily. Therefore, as the variance goes the zero value then the flatness goes the best smothness as expected.

Figure 6.1 : Variance example1

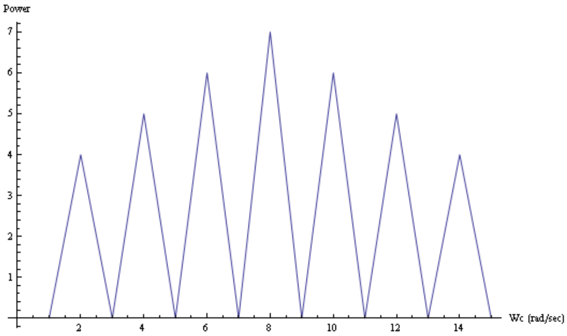
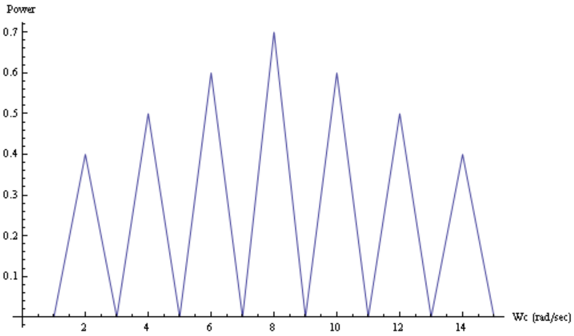


Figure 6.2: Variance example2



On the other hand the variance alone inefficient to determine the flatness. As the variance is the average distance to the mean, in the figure 6.1, the result is numerical but by using the same example 4, 5, 6, 7, 6, 5, 4 the variance is simply found as1.23 and int figure6.2, the variance of 0.4, 0.5, 0.6, 0.7, 0.6, 0.5, 0.4 is found as 0.123. As seen here two different spectra with different amplitudes, have the same flatness but different variance. Even both of them same in term of flatness the program will chose the smallest. Due to this reason the parameters of the maximum and the mean of the comb lines are needed to be considered in to the error function as well. In addition to this, the ratio consideration of the variance and the mean parameters that mitigates the problem on account of the variance which is mentioned above. It means that, only the variance can not give a good result due to dB scale format. Also the ratio consideration of the maximum and the mean parameters confine the resultant frequency comb lines in the flatness range of 3 dB. The equation 6.1 can simply contribute the values that are needed for generating optical frequency comb lines unless the error function value limits itself to aproach to zero value.

$$ERROR(A,B,C,PhA,PhB,PhC,m_1,m_2,n)=\frac{VARIANCE\,(dataset)}{MEAN\,(dataset)}\,\frac{MAXIMUM\,(dataset)}{MEAN\,(dataset)}$$

(6.2)

The last consideration is the efficiency of the signal power. The percentage of total power which is centered around the center frequency is very crucial. Accumulation of signal power which occurs at center is expected. In other words, the main purpose is to use power efficiently by spreading the signal power around the central frequency. Even the spectrum of a signal is flat, the power efficiency is still needed to be considered at the around of center frequency.

Finally, it can be obviously stated that, by using all boundaries, values, the variance, the mean, the maximum parameters, and the power efficiency of the signal, as a result of error function goes to zero numerically, the flatness of the spectrum can be diligently increased and the number of the flat frequency comb lines can be profoundly increased as well.

7. SYNTHESIS RESULTS

In this chapter, the experimental setup and some of theoretical spectrum results of flat optical frequency comb lines are demonstrated over a few millions of possibilities of parameters.

In the theoretical study of the thesis, convolution method, and some iteration methods are used to find the best fit spectrum which is profoundly flat and considering number of comblines in terms of quality and quantity. Moreover one thousand spectrums of flat optical frequency comb lines are searched out to find the best the spectrum. The most flat spectrum is came up with 25 optical frequency comb lines with better than 3 dB flatness. The output power signal has more than 85 percentage power efficiency.
Convolution of certain irregularly shaped spectrums of optical frequency comb lines can result in a flat optical frequency comb spectrum, when the appropirate values of parameters which are given by the error function. In the figure 7.1 only the frequency component A is existed and A is the amplitude of the voltage, and the others are not included. The value of A is taken as 3.1446 which is quiet practical value for the experimental set up due to range of the frequency component as being 5. For this value of A itself does not have enough comblines and the flatnessof the comb lines is quiet poor. In addition to this in the range of 3 dB flatness, there is only 2 comb lines and if the range of flatness of the optical frequency comb lines is taken as 10 dB, there are 9 optical frequency comblines which are in the range but still it is not applicable for any application. On the other hand, the power efficiency is not bad actually. The signal power is located around the center frequency and it does not accumulate far sides of the center frequency.

In the figure 7.2 only the modulation component B of frequency $3w$ present. B is the amplitude of the voltage and the others are not included. The value of B is taken as 2.4102 For this value of B itself does not have enough comblines and the flatnessof the comb lines is quiet poor. It is obvious that the optical frequency comblines has a Gaussian spread. In addition to this in the range of 3 dB flatness, there is only 4 comb lines and if the range of flatness of the optical frequency comb lines is taken as 10 dB, there are 6 optical frequency comblines which are in the range but still it is not applicable for any application. On the other hand, the power efficiency is not bad actually. Again the signal power is located around the center frequency and it does not accumulate at the right and left sides of the center frequency. The interesting thing that differenties the figure 7.2 from other figures is having zero amplitude at the center frequency. In other words, center frequency or zero order Bessel function has zero amplitude value for the value of *B* that is taken 2.4102.

Figure 7.1: The spectrum of frequency component A

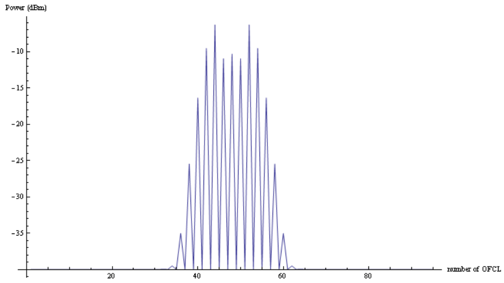


Figure 7.2: The spectrum of frequency component B

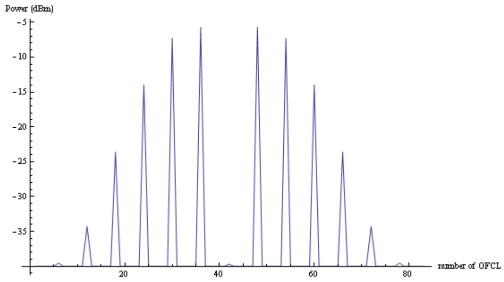
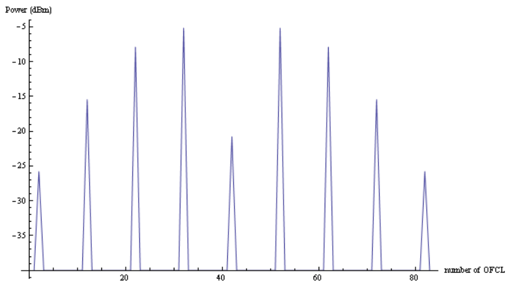


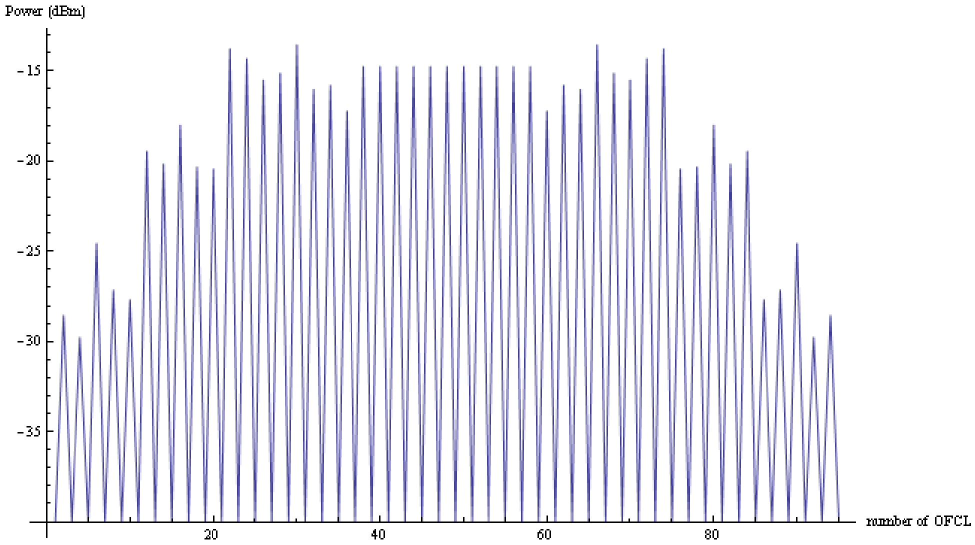
Figure 7.3 : The spectrum of frequency component C



In the figure 7.3 only the frequency component *C* of frequency 5*w* is present, *C* is the amplitude of the voltage and the others are not included. The value of *C* is taken as 2.2354 for this value of *C* itself does not have enough comblines and the flatnessof the comb lines is quiet poor. Moreover, the number of the optical comblines are very few. It is not difficult to see that the optical frequency comblines having Gaussian -like spread as well.

In addition to this in the range of 3 dB flatness, there are only 4 comb lines as in the figure of 7.2. Likely, if the range of flatness of the optical frequency comb lines is taken as 10 dB, there are 6 optical frequency comblines which are in the range but still it is not applicable for any application. On the other hand, the power efficiency is again acceptable. The signal power is located around the center frequency and it does not accumulate at the right and left sides of the center frequency. The intresting thing that differs the figure 7.3 that the center frequency has a 15 dB lower amplitude that corresponds to a power that is approximately 300 times lower. The value of *C* that is taken 2.2354.

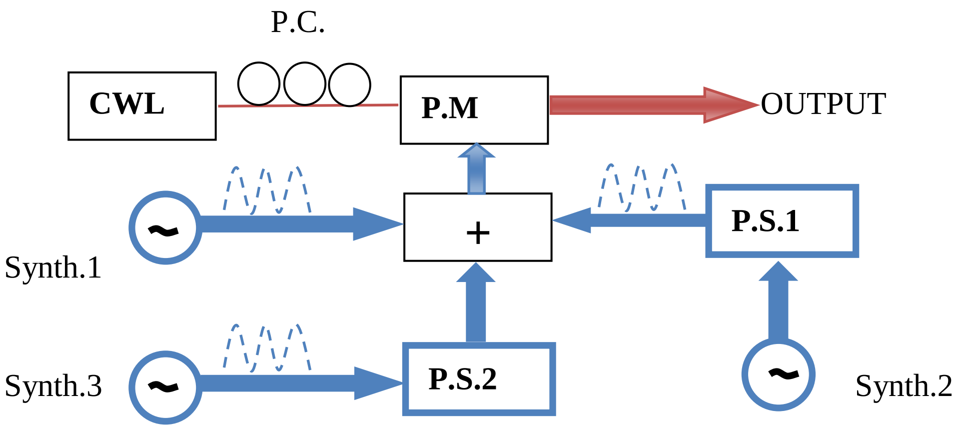
Figure 7.4: The spectrum of resultant A, B, C



In the figure 7.4 the combination of all of three the frequency components *A, B, C* are included. In this case all of the three modulations are applied simultaneously with the calculated relative phases. According the error function, the smallest error is contributed the best fit spectrum of the optical comb lines. The result are given by the error function, orderly the values of *A*, *B*, *C* are taken as 3.1446, 2.4102, 2.2354 for these values there are also phase differences of them *PhA* is taken as 0.4461. *PhB* is taken as 0.2600. *PhC* is taken as 0.7232.The frequency ratio between *A* and *B* as *m1* that is taken as 3. The frequency ratio between *A* and *C* defined as *m2* that is taken as 5. The last parameter, the number of optical frequency comb lines could be taken in the range between 1and 30.The resultant spectrum of the combination of frequency component *A*, *B*, *C* with their phase differences and phase multipliers is quite good. It has profoundly enough comblines and the flatness of the comb lines is practically applicable for many aplication such as optical communication.

It is not difficult to see that the optical frequency comblines have very flat shape. In addition, in the range of 3 dB flatness, there are 25 optical frequency comb lines. Likely, if the range of flatness of the optical frequency comb lines is taken as 10 dB, there are 38 optical frequency comblines which are in the range. Furthermore, the power efficiency is effectively good. Again the signal power is located around the center frequency and it does not accumulate at the right and left sides of the center frequency. The most important point that differs the figure 7.5 from other figures is having 25 frequency comb lines with similar amplitudes around the center frequency.

Figure 7.5: Illustration of the experimental setup



The experimental setup is basically shown in the figure above. These components are explained as follows:, P.M. is the phase modulator where all the modulating signals are sent, P.S1 and P.S.2 are the phase shifters to ensure that all sine waves have the right relative phase, P.C. is the polarization controller, and CWL is the CW laser. Three different modulation sine waves are amplified separately to ensure they have the correct amplitudes and are combined by an RF power combiner and sent to the phase modulator.

As a consequence, 25 flat optical frequency comb lines are succesfully generated with a flatness better than 3 db.

The power efficiency is more than 85 percentage of input signal power. The experimental part is quite simple and the cost is inexpensive. Three different signals can be also generated from a single RF source, by incorporating frequency multipliers as well. Each optical frequency combline can be considered a channel for data transmitting or data recieving. These comb lines can be used for decoding and coding in communication sector. Budget cost is very low on account of the simplicity of equipment of multiple sine phase modulation comparing the mode locked lasers. For the experimental part, the maintenance and starting problem due to cavity and isolation problems are negligible.

8. CONCLUSIONS AND FUTURE WORK

The multiple-sine-wave phase modulation approach is simple and robust to generate flat optical frequency comb lines with 85 percentage of power efficiency which is in the range of applicable flatness for many application such as arbitrary waveform generation, optical communication, optical network testing, filter testing, and microwave photonics. The experimental setup does not require a special modulator and can be incorporated with RF frequency multipliers and RF filters. The stability of the output combs is limited by the stability of the CW laser used in the setup. Using our multiple-sine-wave phase modulation approach, we have showed the possibility an optical spectrum of 27 comb lines with beter than 3-dB flatness.

REFERENCES

BOOKS

Yariv, A., 1997. *Optical Electronics in Modern Communications.* New York: Oxford University Press.

Yariv, A., 1998. *Quantum Electronics.* USA: John Wiley & Sons

PERIODICALS

Özharar, S., Quinlan, F. , Gee, S. , and Delfyett P. J. , 2005. Demonstration of endless phase modulation for arbitrary waveform generation," *IEEE Photon. Technol. Letters*, **17** (12), pp. 2739–2741.

Delfyett, P. J., Gee, S., Choi, M. T., Izadpanah , H., Lee , W., Özharar, S., Quinlan, F., and Yilmaz ,T., 2006 Optical frequency combs from semiconductor lasers and applications in ultrawideband signal processing and communications. *J. Lightw. Technol.*, **24** (7), pp. 2701–2719.

O'Reilly, J. J., Lane, P. M., Heidemann, R., and Hofstetter, R.,1992. Optical generation of very narrowlinewidth millimeterwave signals. *Electron. Letters*, **28** (12), pp. 2309–231.

Ozharar, S., Quinlan, F., Ozdur, I., Gee, S., and Delfyett , P. J., 2008. Ultraflat Optical Comb Generation by Phase-Only Modulation of Continuous-Wave Light, *IEEE Photonics Technology Letters* , **20** (4), pp. 36-38.

Ramond, T. M., Hollberg, L., P. Juodawlkis, Calawa, W., Calawa, S. D., 2007. Low-noise optical injection locking of a resonant tunneling diode to a stable optical frequency comb. *Appl. Phys. Lett*. **90** (17), pp. 171124-171127.

Fujiwara, M., Teshima, M., Kani, J., Suzuki, H., Taka- chio, N., and Iwatsuki, K.,2003. Optical carrier supply module using flattened optical multicarrier generation based on sinusoidal amplitude and phase hybrid modulation. *J. Lightwave Technol.* **24** (11), pp. 2705–2717.

Yuelan, L., Yongwei, X.,Yongkang, D., 2009. Equal-amplitude optical comb generation using multi-frequency phase modulation in optical fibers, *Chinese Optics Letters,* **8** (3), pp. 316-319.

Ohara, T., Takara, H., Yamamoto, H., Masuda, H., Morioka, T., Abe, M., and Takahashi, H., "Over-1000- Channel Ultradense WDM Transmission With Supercontinuum Multicarrier Source," *J. Lightwave Technol.* **24** (2311)- pp. 213-216.

Jiang, Z., Huang, C. B., Leaird, D. E., and Weiner, A. M., "Optical arbitrary waveform processing of more than 100 spectral comb lines,"
*Nature Photonics.***11** (67), pp. 463-467.

Supradeepa, V. R., Weiner Andrew, M.A , 2011. Broadband, Spectrally Flat, High Rep-rate Frequency Comb: Bandwidth Scaling and Flatness Enhancement of Phase Modulated CW through Cascaded Four-Wave Mixing, *Optical Society of America*, **5** (34), pp. 1-2.

Kourogi, M., Enami, T., and Ohtsu, M.,1994.A monolithic opticalfrequency comb generator, *IEEE Photonics Technol. Lettters*, **6** (2), pp. 214–217.

Morioka, T., Mori, K., and Saruwatari, M.,1993. More than 100-wavelengthchannel picosecond optical pulse generation from single laser source using supercontinuum in optical fibers. *Electron. Letters*, **29** (19), pp. 862–864.

Izutsu, M., Yamane, Y., and Sueta, T., 1977 Broad-band traveling –wave modulator using a LiNbO3 optical waveguide. *IEEE J. Quantum Electron.*, **13** (4), pp. 287–290.

Sakamoto T.; Kawanishi T.; Izutsu M. , 2007. Widely wavelength-tunable ultra-flat frequency comb generation using conventional dual-drive Mach Zehnder modulator, Electronic Letters. **43** (19), pp. 1039-1040.

Murphy ,M. T., Udem, T., Holzwarth, R., Sizmann ,A., Pasquini, L., Araujo-Hauck , C., Dekker , H., D‟Odorico , S., Fischer, M., Hansch, T. W., Manescau, A., 2004.

High-precision wavelength calibration of astronomical spectrographs with laser frequency combs. *Mon. Not. R. Astron. Soc.,* **45** (380), pp. 839-847.

Hung, F. C., Chung, C. D., Chao, Y. L., 2001 Coherent frequency/phase modulation scheme. *IEEE Proceedings Communications*, **23** (149), pp. 36-44.

Lee,W., Izadpanah, H., Menendez, R., Etemad, S., Delfyett, P.J., 1999. Synchronized Mode-Locked Semiconductor Lasers and Applications in Coherent Communications. *Journal of Lightwave Technology*, **34** (26), pp. 908-921.

Shenping, L., Caiyun, L., Chan, K.T., 1998. Rotional harmonic active and passive modelocking in a figure of eight fibre laser. *Electronics Letters*, **34** (4), pp.375-376.

Jeon, M.Y., Kyu, H. L., Ahn, J. T., Lim, D. S., Kim, H. Y., Kim, K. H., Lee, H.,1998. External fibre laser based pulse amplitude equalisation scheme for rational harmonicmodelocking in a ring-type fibre laser. *Electronic Letters,* **34** (2); pp. 182 – 184.

Yıldız, N., Polat, Ö., San, S.E. Kaya, N., 2011. Light-scattering determination of visco-elastic and electro-optic parameters of azo and anthraquinone dye-doped liquid crystal molecules and consistent neural network empirical physical formula construction for scattering intensities. *Journal of Molecular Structure,* **9** (91), :pp.127–135

OTHER SOURCES

Takita, Y., Futami, F., Doi, M., and Watanabe, S.,2004. ‘Highly stable ultrashort pulse generation by filtering out flat optical frequency components’. Conf. on Lasers and Electro-Optics (CLEO’04), CtuN1

Yamamoto, T., Takara, H., and Kawanishi, S.: 2002. Generation and transmission of tunable terahertz optical clock’. Int. Topical Meeting on Microwave Photonics (MWP’02), T2-2, Awaji, Japan

APPENDICES

Appendix A1 The algorithm written in Mathematica

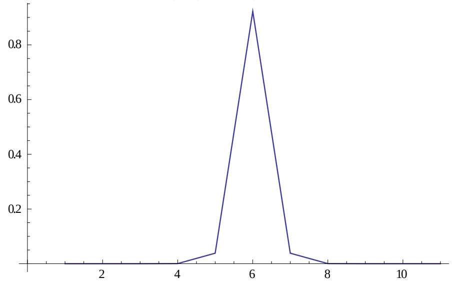
The algorithm written in Mathematica and some of the result are given below.

For 1 sine wave
(* This function returns the corresponding amplitude of n th harmonic of an optical signal that is phase modulated by 1 sine waves at frequency (f) and at a depth of modulation (A)
Also relative phase is considered and included to equation as the variable PH

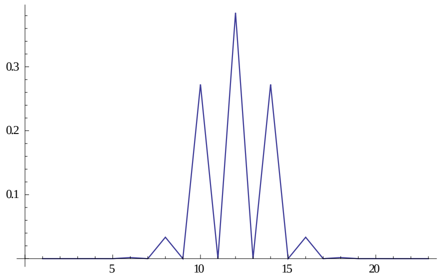
$$e^{i*A*\mathrm{Sin}[2\pi f t+2\pi \mathrm{PH}]} = \left(\sum_{k=-\infty}^{\infty} \mathrm{BesselJ}[k,A] * e^{i2\pi \mathrm{kft}} * e^{i2\pi k\mathrm{PH}}\right)$$

(* This function gives the real and imaginary amplitudes of the spectrum of a phase modulated signal;
A: the amplitude of modulation,
PhA: the phase of the modulation signal,
n: the number of harmonics to be calculated,

This spectrum is INDEPENDENT of the MODULATION FREQUENCY *)
S1[A_,PhA_,n_]:= Chop[Table [BesselJ[k,A] Exp[ⅈ k PhA 2π]//N, {k,-n,n}]]
ListPlot[Abs[S1[0.4,0,5]]^2,Joined→True]



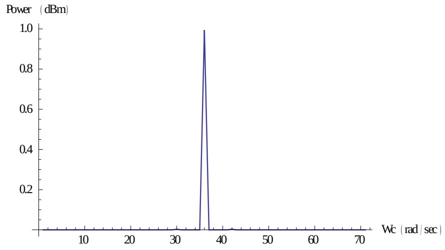
(* In order to do convolution of spectra, we need to introduce extra zeros between the harmonics according to the modulation frequencies *)
(* The function Riffle insert another element between each member of a list (in this case introduces 0 between the harmonics) *)
S1C[A_,PhA_,n_]:=Riffle[S1[A,PhA,n],0,{1,-1,2}]
S1C[0.4,0.0,5]
{0,-2.64894$\times10^{-6}$,0,0.0000661351,0,-0.00132005,0,0.0197347,0,-0.196027,0,0.960398,0,0.196027,0,0.0197347,0,0.00132005,0,0.0000661351,0,2.64894$\times10^{-6}$,0}
ListPlot[Abs[S1C[1.3,0.0,5]]^2,Joined→True, PlotRange→ All]



This function adds 2m-1 zeros between the data points
m is the ratio of the frequencies and it MUST be an integer
(* A is the modulation depth, PhA is the phase of the electrical signal, m is the ratio of the frequency w.r.t. w1(the smallest modulation freq.), and n is the highest harmonic number to be calculated *)

(* The number of zeros between harmonics should be proportional to the frequency of the sine waves *)

```
S1Cm[A_,PhA_,m_,n_]:=(
  DataLength=(2 n+1)+(2 m -1) (2n+2);
  Harmonics=Chop[Table [BesselJ[k,A] Exp[ⅈ k PhA 2π]//N, {k,-n,n}]];
  Table[If[Mod[k,2m]==0,Harmonics[[k/(2m)]],0],{k,DataLength}]
  )
ListPlot[Abs[S1Cm[0.11,0.5,3,5]]^2,Joined→True,Background→White,AxesLabel→{"Wc (rad/sec)","Power (dBm)"}]
```

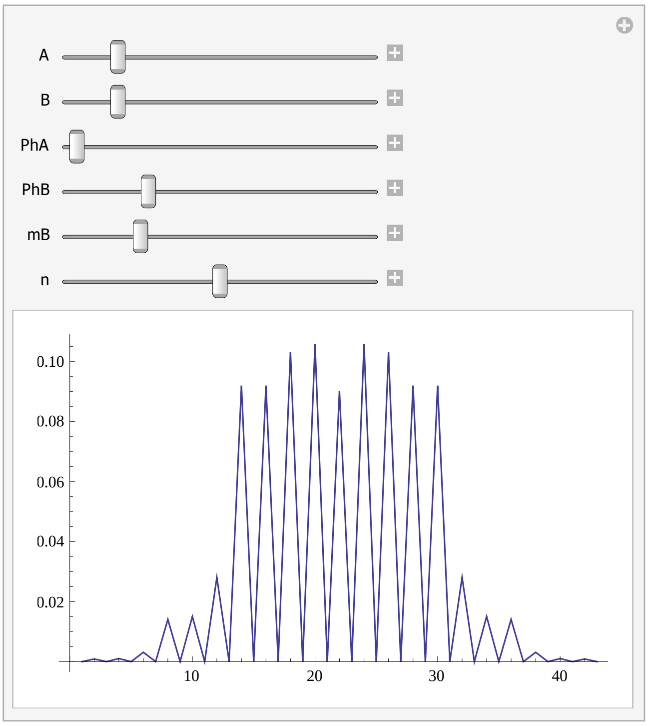


CONVOLUTION

By taking the convolution of the different spectra we can find the resulting spectrum
(* Flatten is introduced to add extra zeros to the second spectrum for a better convolution *)
(* Electric Field of spectrum after 2 sine wave *)
(* Upto 60th harmonic is considered but only upto nth harmonic is given in the output *)
(* the output of this function gives the harmonics of order -nth to +nth *)

```
Spectrum2Sine[A_,B_,PhA_,PhB_,mB_,n_]:=(
  (* Length of first data set *)
  DLength=4 60+3 ;
  (* Length of second sine wave data set *)
  D2Length=(2 IntegerPart[60/mB]+1) 2 mB+(2 mB-1) ;
```

```
Take[Chop[ListConvolve[S1Cm[A,PhA,1,60],Flatten[{Table[0,{i,1,DLength}],S1Cm[
B,PhB,mB,IntegerPart[60/mB]],Table[0,{i,1,DLength}]}]]],{(D2Length+DLength)/2-2
n,(D2Length+DLength)/2+2 n+2}]
  )
Spectrum2Sine[1,1,0.1,0,3,5]
{0,0.025512_+0.0777805 ⅈ,0,0.159214_-0.102379 ⅈ,0,-0.332802+0.0163807 ⅈ,0,-
0.129612-0.197275 ⅈ,0,-0.288952+0.149193 ⅈ,0,0.590844,0,0.288952_+0.149193 ⅈ,0,-
0.129612+0.197275 ⅈ,0,0.332802_+0.0163807 ⅈ,0,0.159214_+0.102379 ⅈ,0,-
0.025512+0.0777805 ⅈ,0}
Manipulate[ListPlot[Abs[Spectrum2Sine[A,B,PhA,PhB,mB,n]]^2,Joined→True,PlotRang
e→All],{{A,1.4347},0,10},{{B,1.4347},0,10},{PhA,0,1},{{PhB,0.25},0,1},{{mB,3},1,
10,1},{{n,10},0,20,1}]
```



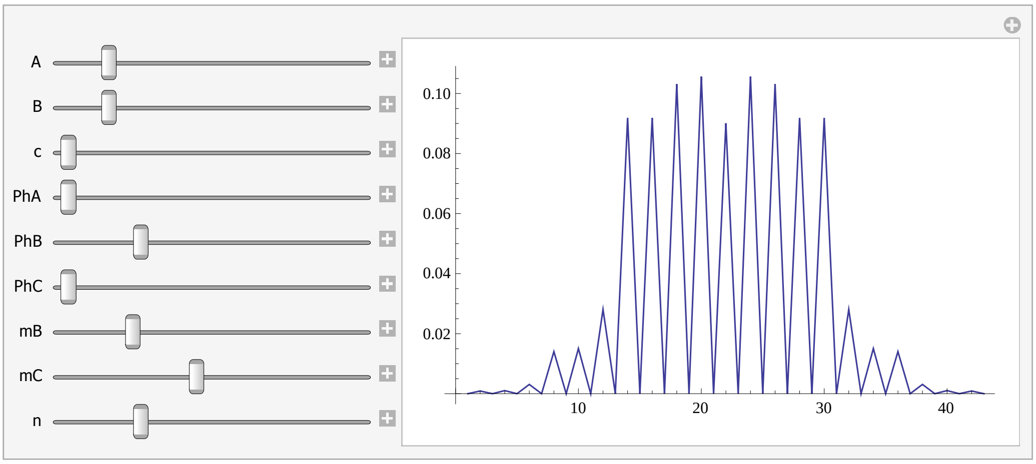
3 Sine Wave

```
(* Upto 60th harmonic is considered but only upto nth harmonic is given in the output
*)
(* the output of this function gives the harmonics of order -nth to +nth *)
Spectrum3Sine[A_,B_,c_,PhA_,PhB_,PhC_,mB_,mC_,n_]:=(
  (* Length of first data set *)
  DLength=4 60+3 ;
  (* Length of second sine wave data set *)
  D3Length=(2 IntegerPart[60/mC]+1) 2 mC+(2 mC-1) ;


Take[Chop[ListConvolve[Spectrum2Sine[A,B,PhA,PhB,mB,60],Flatten[{Table[0,{i,1,
```

```
DLength}],S1Cm[c,PhC,mC,IntegerPart[60/mC]],Table[0,{i,1,DLength}]}]]],{(D3Len
gth+DLength)/2-2 n,(D3Length+DLength)/2+2 n+2}]
 )
```

```
Manipulate[ListPlot[Abs[Spectrum3Sine[A,B,c,PhA,PhB,PhC,mB,mC,n]]^2,Joined→Tru
e,PlotRange→All],{{A,1.4347},0,10},{{B,1.4347},0,10},{{c,0},0,12},{PhA,0,1},{{Ph
B,0.25},0,1},{PhC,0,1},{{mB,3},1,10,1},{{mC,5},1,10,1},{{n,10},0,40,1}]
```



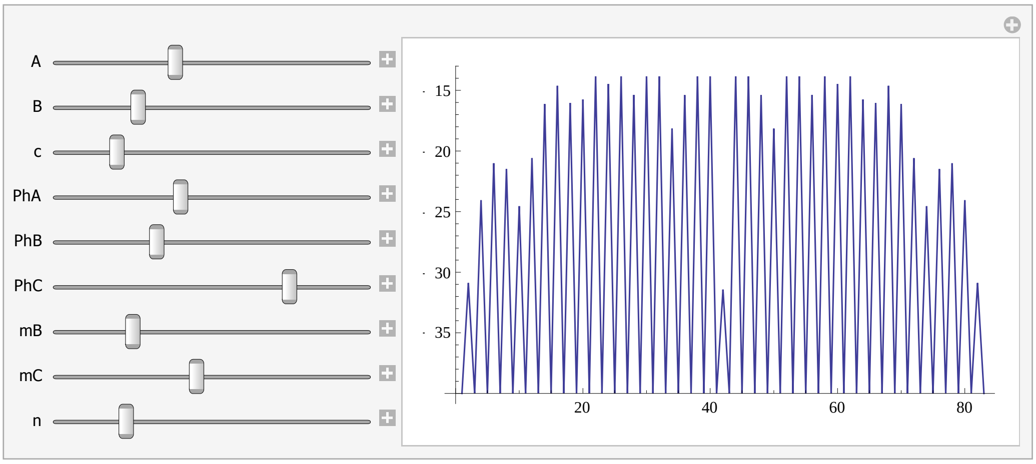
Optimization Part

```
DataSet[A_,B_,c_,PhA_,PhB_,PhC_,mB_,mC_,n_]:=Cases[Abs[Spectrum3Sine[A,B,c,
PhA,PhB,PhC,mB,mC,n]]^2,Except[0]]
DataSet[1,1,1,0,0,0,3,5,10]
{0.0052335,0.00389724,0.020646,0.00214014,0.0711079,0.0607181,0.0000451815,0.0
453078,0.066293,0.117785,0.190761,0.117785,0.066293,0.0453078,0.0000451815,0.0
607181,0.0711079,0.00214014,0.020646,0.00389724,0.0052335}
```

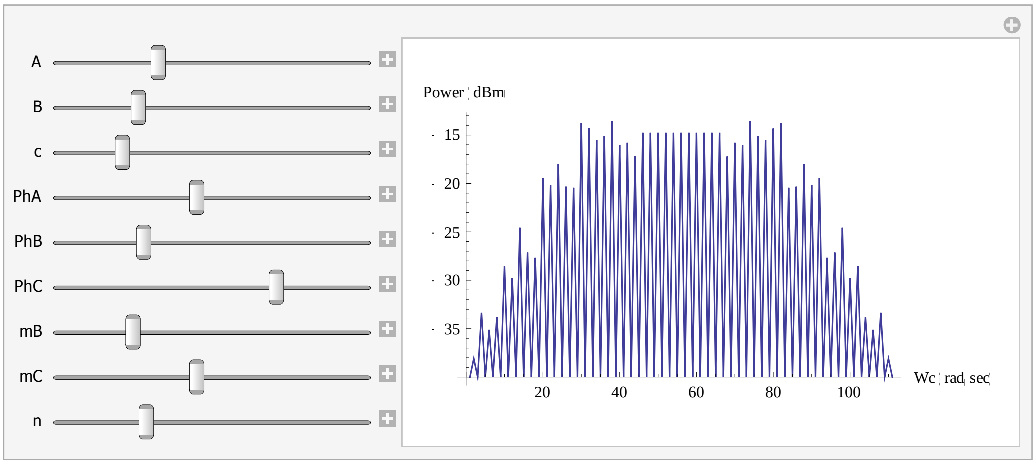
```
HAWK[A_,B_,c_,PhA_,PhB_,PhC_,mB_,mC_,n_]:=(Variance[DataSet[A,B,c,PhA,Ph
B,PhC,mB,mC,n]]/Mean[DataSet[A,B,c,PhA,PhB,PhC,mB,mC,n]])*(Max[DataSet[A,B
,c,PhA,PhB,PhC,mB,mC,n]]/Mean[DataSet[A,B,c,PhA,PhB,PhC,mB,mC,n]])
K=HAWK[1.2,1.3,1.2,0.75,0.5,0,3,5,2]
0.0171609
DataSet[9.947655015379084`,6.423864971202666`,6.746172090872834`,0.987640711
7475685`,0.3700033831648554`,0,3,5,10]
{0.00739249,0.0462265,0.00586013,0.000697488,0.00589494,0.00834053,0.00457054
,0.00645519,0.0101197,0.0375265,0.0137967,0.0375265,0.0101197,0.00645519,0.004
57054,0.00834053,0.00589494,0.000697488,0.00586013,0.0462265,0.00739249}
```

```
NMinimize[{HAWK[A,B,c,PhA,PhB,PhC,3,5,4],0.6<A<6&&0.6<B<6&&0.6<c<6&&
0<PhA<1&&0<PhB<1&&0<PhC<1},{A,B,c,PhA,PhB,PhC},MaxIterations→10000]
{3.25514×10^-
10,{A→4.60671,B→2.17739,c→3.4345,PhA→0.779788,PhB→0.214068,PhC→0.713977}
}
```

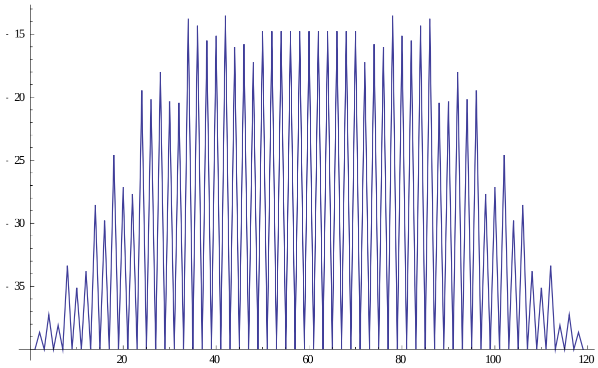
```
NMinimize[{HAWK[A,B,c,PhA,PhB,PhC,3,5,5],0.6<A<6&&0.6<B<6&&0.6<c<6&&
0<PhA<1&&0<PhB<1&&0<PhC<1},{A,B,c,PhA,PhB,PhC},MaxIterations→10000]
{4.54787×10^-
10,{A→3.14461,B→2.41018,c→2.23536,PhA→0.446124,PhB→0.259962,PhC→0.723227
}}
NMinimize[{HAWK[A,B,c,PhA,PhB,PhC,5,3,5],0.6<A<6&&0.6<B<6&&0.6<c<6&&
0<PhA<1&&0<PhB<1&&0<PhC<1},{A,B,c,PhA,PhB,PhC},MaxIterations→10000]
{1.74359×10^-
9,{A→2.24313,B→1.56779,c→4.3961,PhA→0.504278,PhB→0.114076,PhC→0.831985}}
Manipulate[ListPlot[10*Log[10,0.0001+Abs[Spectrum3Sine[A,B,c,PhA,PhB,PhC,mB,
mC,n]]^2],Joined→True,PlotRange→All],{{A,3.7057342448024784},0,10},{{B,2.42128
3790437267},0,10},{{c,2.018033680095632},0,12},{{PhA,0.38975631080835516},0,
1},{{PhB,0.3070076317707022},0,1},{{PhC,0.7692834244012707},0,1},{{mB,3},1,1
0,1},{{mC,5},1,10,1},{{n,20},0,100,1}]
```



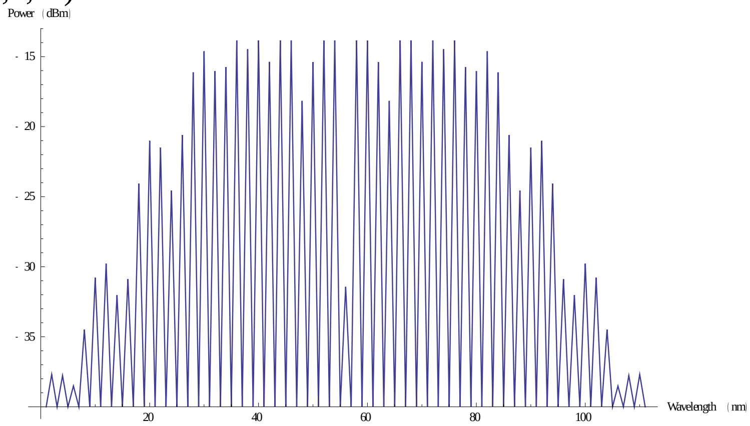
```
Manipulate[ListPlot[10*Log[10,0.0001+Abs[Spectrum3Sine[A,B,c,PhA,PhB,PhC,mB,
mC,n]]^2],Joined→True,PlotRange→All, Background→White,AxesLabel→{"Wc
(rad/sec)","Power
(dBm)"}],{{A,3.1446110010067327`},0,10},{{B,2.410177750522197`},0,10},{{c,2.23
5361960062508`},0,12},{{PhA,0.4461239596409412`},0,1},{{PhB,0.2599622032819
0645`},0,1},{{PhC,0.723227225937783`},0,1},{{mB,3},1,10,1},{{mC,5},1,10,1},{{n,
20},0,100,1}]
```



```
(*25  comb line are founded
NMinimize[{HAWK[A,B,c,PhA,PhB,PhC,3,5,5],0.6<A<6&&0.6<B<6&&0.6<c<6&&
0<PhA<1&&0<PhB<1&&0<PhC<1},{A,B,c,PhA,PhB,PhC},MaxIterations→10000]
{4.547868960856496`*^-
10,{A→3.1446110010067327`,B→2.410177750522197`,c→2.235361960062508`,PhA→
0.4461239596409412`,PhB→0.25996220328190645`,PhC→0.723227225937783`}}
3.1446110010067327`,B→2.410177750522197`,c→2.235361960062508`,PhA→0.44612
39596409412`,PhB→0.25996220328190645`,PhC→0.723227225937783`*)
```



(*a,b,c*)



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