THE REPUBLIC OF TURKEY
BAHÇEŞEHİR UNIVERSITY

CAPACITY BOUNDS AND CHANNEL ESTIMATION FOR MIMO RELAY CHANNELS WITH COVARIANCE FEEDBACK AT THE TRANSMITTERS

Master's Thesis

BENGİ AYGÜN

İSTANBUL, 2012

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BAHÇEŞEHİR UNIVERSITY

THE GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES
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Title of the Master's Thesis	:	Capacity Bounds and Channel Estimation for MIMO Relay Channels with Covariance Feedback at the Transmitters
Name/Last Name of the Student	:	Bengi AYGÜN
Date of Thesis Defense	:	10 September, 2012

The thesis has been approved by The Graduate School of Natural and Applied Sciences.

Assoc. Prof. Tunç BOZBURA
Acting Director

This is to certify that we have read this thesis and that we find it fully adequate in scope, quality and content, as a thesis for the degree of Master of Science.

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Prof. Hakan Deliç	: ....
Assoc. Prof. Ufuk Türeli	: ....

ACKNOWLEDGEMENTS

I would like to thank to;

My supervisor, Assist. Prof. Alkan Soysal, for giving me his advices and helping me to find the substantial opportunities in my carrier; Scientific and Technological Research Council of Turkey (TUBITAK) for the financial support to Project EEEAG 109E048.

My fellow researchers, Jbid Arsenyan, Erdem Erzurum, Ceyhun Can Ülker, Ertunç Erdil, Selçuk Keskin, Emin Akşehirli, and Güneş Akşehirli for their technical supports and making my days more pleasing so making me feel I can handle with all rubbish in the world.

My family for their support whole my life; my sister, Begüm Aygün, for dealing with my hysteria and sharing my dream with me without any hesitation; Zen tradition for making me much more patient and concentrating on my main purposes without being down because of the temporary complications.

Robert F. Winkelman for helping me to improve my English with pure love and for his very unique existence; Aslı Sakallı for being the unofficial editor of my academic carrier and the fellow traveler in my life; the Universe for presenting its perfect beauty and making the human think about what is more; all resistances against the injustices for reassuring me the world is going to be more brilliant in the next days.

10 September 2012 Bengi AYGÜN

ABSTRACT

CAPACITY BOUNDS AND CHANNEL ESTIMATION FOR MIMO RELAY CHANNELS WITH COVARIANCE FEEDBACK AT THE TRANSMITTERS

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Electrical and Electronics Engineering
Supervisor: Assist. Prof. Alkan SOYSAL

September 2012, 66 Pages

In this thesis, some resource allocation problems for fading multiple input multiple output (MIMO) relay channels are considered, where decode and forward (DF) relay strategy is used. In our model, the transmitters have partial channel state information (CSI). The receivers are assumed to have prefect channel state in some parts of the thesis, while channel estimation errors are taken into account in others parts of the thesis. The resource allocation problems are in terms of finding optimum transmission parameters (like average transmission powers, the source and relay transmit covariance matrices) and channel estimation parameters (like training duration, training sequence and training power).

In the first part of the thesis, the transmitters have partial CSI, while the receivers have perfect CSI. The capacity of this channel is not known, however, we obtain lower and upper bounds to the capacity for both half-duplex and full-duplex transmission. These bounds require a joint optimization over the source and relay transmit covariance matrices. The methods utilized in the previous literature cannot handle this joint optimization over the transmit covariance matrices for the system model considered in this thesis. First, we propose a sub-optimal solution by solving the source and relay transmit covariance matrices consecutively, i.e., not jointly. This suboptimal solution make use of the previous literature and finds the eigenvectors of the transmit covariance matrices before proposing the algorithm that finds the eigenvalues of the transmit covariance matrices. Then, in order to solve the joint optimization problem, we utilize matrix differential calculus and propose iterative algorithms that find the transmit covariance matrices of source and relay nodes. In this method, there is no need to specify the eigenvectors of the transmit covariance matrices first. The algorithm updates both the eigenvectors and the eigenvalues at each iteration. Through simulations, we observe that lower and upper bounds are close to each other. However, the distance between the lower and the upper bound depends on the channel conditions.

In the fifth part of the thesis, the transmitters have partial CSI, while the receivers experience channel estimation errors. The capacity of this channel is also not known, however, we obtain lower and upper bounds to the capacity. These bounds require joint optimization over not only the source and relay transmit covariance matrices, but also training sequence matrix. We deal with the trade-off between estimating the channel better and increasing the channel rate. We use minimum mean square error to minimize the estimation error.

Keywords: MIMO Relay Channels, Partial Channel State Information at the Transmitter, Channel Estimation, Full-duplex, Half-duplex, Decode-and-forward, Channel Estimation

ÖZET

VERİCİDE KOVARYANS BİLGİSİ MEVCUTKEN MIMO AKTARMA KANALLARINDA KAPASİTE SINIRLARI VE KANAL KESTİRİMİ

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Eylül 2012, 66 Sayfa

Bu tez çalışmasında, çok girişli çok çıkışlı (MIMO) çöz-ilet aktarma kanallarında, güç tahsis problemi incelenmiştir. Kanal modelinde, vericide kısmi kanal durum bilgisi (CSI) bulunmaktadır. Çalışmanın ilk bölümlerinde, alıcıda tam kanal bilgisinin mevcut olduğu durum incelenirken, son bölümde kanal tahmininde oluşan hatalar göz önünde bulundurulmuştur. Kaynak tahsis problemi, optimum iletim parametreleri (ortalama iletim gücü, kaynak ve aktarma iletim kovaryans matrisleri) ve kanal tahmin parametreleri (iletim süresi, iletim dizisi, iletim gücü) üzerinden çözümlenmiştir.

Tez çalışmasının ilk bölümünde, vericilerde kısmi CSI mevcut iken, alıcılarda tam CSI bulunmaktadır. Bu durumda, kanalın kapasite değeri tam olarak bulunamaz; ancak kapasite alt ve üst sınırları bulunabilmektedir. Bu tez çalışmasında, hem tam çift yönlü hem de yarı çift yönlü iletim için kapasite sınırları incelenmiştir. Bu kapasite sınırlarını elde ederken, kaynak ve aktarma iletim kovaryans matrisleri üzerinden birleşik optimizasyon yapılmıştır. Literatürde mevcut olan yöntemler, iletim kovaryans matrisleri üzerinden birleşik optimizasyon yapmak için yeterli olmamaktadırlar. Bu nedenle, ilk bölümde, kaynak ve aktarma iletim kovaryans matrisleri art arda çözümlenerek elde edilmiştir. Optimum iletim kovaryans matrisi birleşik optimizasyon ile elde edilebildiği için, art arda çözümleme yöntemi ile optimuma yakın kapasite sınırlarİelde edilmiştir. Bu yöntemde, iletim kovaryans matrislerinin özvektörleri (iletim yönleri) bulunmuştur; ardından iletim kovaryans matrislerinin özdeğerlerini (iletim yönlerinde kullanılan güç değerlerini) bulan algoritma önerilmiştir. Diğer bölümlerde, birleşik optimizasyon probleminin çözmek amacıyla, matrislerin diferansiyelini alma yöntemleri uygulanmıştır. Bu yöntemle, optimum kaynak ve aktarma iletim kovaryans matrislerini bulan iteratif algoritma elde edilmiştir. Bu teknikte, özvektörler ve özdeğerleri ayrıayrıbulmaya gerek yoktur. Algoritma, her ite-rasyonda hem özvektörleri hem de özdeğerleri güncellemektedir. Simülasyon

sonuçlarına göre, kapasite alt ve üst sınırları birbirine oldukça yakındır. Kapasite sınırlarının birbirine yakınlığı, kanal koşullarına göre değişmektedir.

Tezin son bölümünde, vericilerde kısmi CSI mevcutken, alıcılarda kanal tahmininde oluşan hatalar göz önünde bulundurulmuştur. Bu durum için de kapasite değeri tam olarak bulunamazken, kapasite alt ve üst sınırları elde edilmiştir. Bu kapasite sınırları, kaynak ve aktarma iletim kovaryans matrislerinin yanında, kanal kestirme matrislerinin üzerinden birleşik optimizasyon yapılarak bulunmuştur. Sistem gücünü kullanırken, kanal durumunu daha iyi kestirmek ve kanal kapasite değerlerini arttırmak arasında optimum nokta bulunmuştur. Kanal tahminindeki hatayı en aza indirgemek için en küçük ortalamalı kareler hatası (MMSE) tekniği kullanılmıştır.

Anahtar Kelimeler: MIMO Aktarma Kanalları, Vericide Kısmi Kanal Bilgisi, Kanal Tahmini, Tam Çift Yönlü İletim, Yarı Çift Yönlü iletim, Çöz-ilet, Kanal Tahmini

CONTENTS

FIGURES

ABBREVIATIONS

AF	:	Amplify-and-Forward
BC	:	Broadcast Channel
CSI	:	Channel State Information
CF	:	Compress-and-Forward
DF	:	Decode-and-Forward
FD	:	Full-Duplex
GCO	:	General Convex Optimization
HD	:	Half-Duplex
KKT	:	Karush-Kuhn-Tucker
LMMSE	:	Linear Minimum Mean Square Error
MAC	:	Multiple Access Channel
MIMO	:	Multiple Input Multiple Output
MMSE	:	Minimum Mean Square Error
SNR	:	Signal-to-Noise-Ratio

SYMBOLS

Random Channel Matrix	:	$\mathbf{H}_{xy}$
Received Signal at the relay	:	$\mathbf{r}$
Received Signal at the receiver	:	$\mathbf{y}$
Transmitted signal from the source	:	$\mathbf{x_s}$
Transmitted signal from the relay	:	$\mathbf{x_r}$
Source transmit covariance matrix	:	$\mathbf{Q_s}$
Relay transmit covariance matrix	:	$\mathbf{Q_r}$
The correlation matrix on the receiver	:	$\mathbf{\Phi}_{xy}$
Identity covariance random channel matrix	:	$\mathbf{Z}_{xy}$
The correlation matrix on the transmitter	:	$\mathbf{\Sigma}_{xy}$
Identity matrix	:	$\mathbf{I}$
Unitary matrix	:	$\mathbf{U}$
Eigenvalue	:	λ
Diagonal matrix of eigenvalue	:	$\mathbf{\Lambda}$
Power	:	P
Mutual information of MAC	:	$\mathbf{I}_{mac}$
Mutual information of source to relay link	:	$\mathbf{I}_{sr}$
Mutual information of BC	:	$\mathbf{I}_{bc}$
Lagrangian multiplier	:	μ
Noise vector	:	$\mathbf{n}$
Training signal	:	$\mathbf{S}$
Channel estimation error vector	:	$\tilde{\mathbf{h}}$
Channel estimated vector	:	$\hat{\mathbf{h}}$
Channel vector	:	$\mathbf{h}$

1. INTRODUCTION

Increasing user count and next generation technologies demand higher data rates in wireless communication. Wireless communication challenges by reason of its random fluctuations, which is called fading, in the channel and multi-user interference. Using multiple antennas both the receivers and the transmitters achieve higher data rates but it increases the challenge of wireless system. When fading is considered, the channel state estimation qualifies the achievable rates. The channel state should be estimated at the receiver since the channel fading and multi-user interference. In this thesis, we derive the capacity bounds and the channel estimation of MIMO relay channels with covariance feedback at the transmitters.

MIMO networks are capable of realizing higher throughput without increasing bandwidth and transmit power. MIMO systems gain ground based on two reasons: diversity gain and spatial multiplexing gain. Besides the advantages of spatial diversity in MIMO systems, they can also offer a remarkably gain in terms of information rate or capacity. In additive white Gaussian noise channels, the channel capacity is achieved with Gaussian input signalling when there is perfect CSI at the receiver. In single antenna systems, the variance of the Gaussian input signal is adapted to the realization of the channel. In Gaussian MIMO channels, the optimum covariance matrix of the transmit vector needs to be chosen.

In addition to multiple antennas, applying cooperative strategies such as adding a relay node to the system can further increase the capacity (Cover & El Gamal, 1979). The relay channel has a transmission topology where there is a third node between the source and destination (Van der Meulen, 1971). Generally, the relay both transmits its own information and helps forwarding information of other sources. The relay systems have high benefits such as coverage extension and network throughput enhancement which help to solve the problems of traditional base station about cost, flexibility and complexity.

In relay channels, several achievable schemes, such as Decode-and-Forward (DF), Amplify-and-Forward (AF), and Compress-and-Forward (CF), can be used as lower bounds to the capacity of MIMO relay channels. On the other hand, while the cut-set theorem provides a valid upper bound. In CF, after the relay node receives the signal from the source node, it compresses the signal with a quantization rate and forwards the compressed signal to the destination (Jiang et al., 2009). In AF, the relay node amplifies the received signal and transmits it to the destination node (Varanese et al., 2006). In DF, first, the relay node receives the signal from the source node and decodes it. Then, the relay node recodes the signal and sends it to the destination node. In this thesis, we study DF scheme, since DF allows the source and the relay to form a collaborative transmit antenna array when extended to wireless fading channels (Liu et al., 2007).

There are two relay transmission modes: full-duplex (FD) and half-duplex (HD). In FD, the source and the relay transmit their signals at the same time (Wang et al., 2005). In HD, they consecutively send their signals in two phases (Nabar et al., 2004). The mode which has more benefit depends on the system conditions. In practice, HD systems are modeled more accurately. In this thesis, we consider both phases to obtain fading channel capacity and channel estimation.

Process of estimation and feedback CSI uses up time, bandwidth and power. Theoretically, using perfect channel state information (CSI) both at the receiver and the transmitter, we can obtain the highest data rates. When the channel knowledge is not perfect, achievable rate decrease significantly. The channel knowledge is especially effective when there are multiple channels to estimate and feedback, as in the case with multiple antennas. Adding to that, measuring CSI at the receiver and feeding it back to the transmitter uses communication resources. One way of measuring the CSI is that the transmitters send known training sequences, and the receivers estimate the channel using these known training sequences and the received vector. Then, the receivers extract the statistical information (according to the feedback model) from the estimated channel, and feed the extracted information back to the transmitters. Because of those constraints, perfect CSI at the transmitters is not practical. Therefore, we work on partial CSI at the transmitters and noisy CSI at the receivers which is more practical.

In Chapter 3, we focus our attention on the achievable rate of FD mode MIMO relay channel which has DF scheme. We derive a lower bound to the ergodic capacity for this scenario in terms of a max-min problem and solve this problem. We propose an iterative algorithm that finds lower bound achieving transmit covariance matrices of the source and relay nodes. We describe lower bound in three cases. In the first case, lower bound on the capacity is equal to the capacity of the link from source to relay. In the second case, lower bound on the capacity is equal to the multi access channel capacity from source and relay to the destination. The optimization problems in the first and second cases can be solved by developing fast and efficient algorithms in order to solve for the transmit covariance matrices. In the last case, the lower bound to the capacity depends on both multi access channel and source to relay channel. Therefore, obtaining the lower bound for third case is not possible by using scalar derivation. However, by exploiting the nature of the relay channel and assuming that the source to destination channel is weaker than the source to relay channel, we are able to propose a fast and efficient algorithm that results in a rate which is very close to the lower bound. We propose an iterative algorithm which is faster and more efficient than classical convex optimization methods (Aygun & Soysal, 2011) .

In Chapter 4, we evaluate lower and upper bounds on the ergodic channel capacity for FD MIMO relay channel. In Chapter 5, we work on the capacity bounds on the ergodic channel capacity for HD MIMO relay channel (Aygun & Soysal, 2012a). For both FD and HD cases which derived in these two chapters, the capacity bounds require a joint optimization over the source and relay transmit covariance matrices. The methods utilized in the previous literature cannot handle this joint optimization over the transmit covariance matrices for the system model considered in this paper. Therefore, we utilize matrix differential calculus and propose iterative algorithms that find the transmit covariance matrices, in order to solve the joint optimization problem (Magnus & Neudecker, 1999). In this method, there is no need to specify the eigenvectors of the transmit covariance matrices first. The algorithm updates both the eigenvectors and the eigenvalues at each iteration. Through simulations, we observe that lower and upper bounds are close to each other. However, the distance between the lower and the upper bound depends on the channel conditions. If the mutual information of source to relay channel and the broadcast channel get closer to each other, the bounds on capacity also get closer. Adding to the advantages of the proposed iterative algorithm in Chapter 3, we obtain the exact lower and upper bounds on the capacity (Aygun & Soysal, 2012b).

In Chapter 6, the channel estimation is derived for full-duplex decode-and-forward MIMO relay channels. We divide the transmission into two parts: training sequence and data sequence. In training sequence, we send known training signals to the receiver. The receiver estimates the CSI using the output of the channel and the known training signals. In this process, a block fading scenario is assumed. The receiver performs a linear minimum mean square error (MMSE) estimation using training sequence (Soysal & Ulukus, 2010a,b). After the channel is estimated in the training phase, we send data in data transmission. We jointly optimize the achievable rate over source and relay transmit covariance matrices. In data transmission phase, the training signals and the transmit covariance matrices are optimized jointly.

2. LITERATURE REVIEW

Single antenna systems with perfect CSI available at both at the receiver and the transmitter are deeply studied. In Goldsmith & Varaiya (1992), it is proved that the CSI at the transmitter does not affect the achievable rate for single user system. In multiple access channel (MAC) which includes a transmitter and multiple receivers, the sum capacity is defined in Knopp & Humblet (1995) and the entire capacity region is defined in Tse & Hanly (1998).

Contrary to single antenna systems, the results in multi-input multi-output (MIMO) channels change with CSI at the transmitters. In Telatar (1999), a single-user MIMO system is considered when both the receiver and transmitter have perfect CSI and the channel is fixed. In this case, the optimum power allocation is to water-fill over the singular values of the deterministic channel matrix. For single user MIMO systems, the power is allocated equally into all channels when the receivers have perfect CSI and the transmitters have no CSI.

In Xie & Kumar (2004) and Reznik et al. (2002), degraded Gaussian channel with multiple relays is derived and capacity bounds are obtained. In Liang et al. (2007), a max-min type of problem is introduced for fading relay channels. Bounds on channel capacity are derived for synchronized and asynchronized cases. In Kramer et al. (2004); Gastpar & Vetterlj (2002), single user MIMO relay channels are presented when both the receivers and transmitters have perfect CSI. In Wang et al. (2005), a more realistic scenario is considered where only the receiver side knows the perfect CSI and transmitters do not know the channel. Moreover, it is found in Wang et al. (2005) that the channel inputs of the source and relay nodes are independent when the channel is fading. FD transmission is derived both for both CF and DF mode for different fading cases in Kramer et al. (2007). In Chen et al. (2011), the diversity-multiplexing tradeoff of FD single-user multihop relay channel is discussed for AF strategy. MIMO system with partial CSI is derived in Je et al. (2008). In this work, the system model includes source to relay and relay to destination links without broadcast and multi access channel.

HD transmission mode is discussed in Khojastepour et al. (2003); Xiao et al. (2009); Zhang & Duman (2007). The partial DF MIMO relay channels which means the relay decodes the received signal partially, is derived in Simoens et al. (2008). According to simulation results, partial DF strategy can achieve a rate very close to the capacity for realistic values of the source to relay signal-to-noise ratio. In Calvo et al. (2009), path loss is the only CSI for MIMO HD relay channels. In this work, the power is separately allocated for uplink and downlink. Both covariance and mean feedback in MIMO HD relay channels are discussed in Chen et al. (2010).

In Zhu et al. (2008); Host-Madsen & Zhang (2005) the ergodic rate and outage probability are studied for single antenna relay channels with DF operation. The results show that optimum relay channel signaling outperforms multihop protocols. DF case with finite feedback from the receiver to the transmitter is explained in Liu et al. (2007).

For MIMO relay channels with covariance and mean information at the transmitters, AF strategy is examined in Je et al. (2009). When there is perfect CSI at each nodes, achievable rate is maximized in Varanese et al. (2006). It proves that the optimal linear processing at the relay node is the outer product of the beamformers for the source-relay and relay-destination channels.

When there is instantaneous knowledge of perfect CSI at both the receiver and the transmitter, the optimum power allocation is obtained by using water-filling (Goldsmith & Varaiya, 1992; Telatar, 1999; Yu et al., 2004). There are some researches about the case which is more practical that there is perfect CSI at the receiver and partial CSI at the transmitter (Boche & Jorswieck, 2002; Jafar & Goldsmith, 2004; Visotsky & Madhow, 2001). Adding to that, there are some researches about the actual estimation of the channel at the receiver which is noisy. In Hassibi & Hochwald (2003), using more training symbols the number of transmit antennas is sub-optimal. In Soysal & Ulukus (2010a,b), the iterative algorithms which give optimum eigenvalues of the transmit covariance matrices are proposed when there is noisy CSI at the transmitter and partial CSI at the transmitters.

The channel estimation error at the receiver is discussed in Klein & Gallager (2001); Medard (2000); Yoo & Goldsmith (2006). In MIMO relay systems, the researches about the channel estimation are generally for AF transmission scheme (Gao & Nallanathan, 2008; Jiang et al., 2010; Behbahani & Eltawil, 2008). In (Yi et al., 2007), Voronoi cell boundary and Cramer-Rao bound are used for channel quantization and codebook design. In Behbahani & Eltawil (2008), LMMSE channel estimation is used for the case that relay estimates source to relay link.

There are some researches about the channel estimation for DF transmission scheme.

In Gao et al. (2008), maximum likelihood and LMMSE techniques are used for MIMO multi-relay systems. In Chen et al. (2010), the derivation is very close to our solution. However, it is assumed that the source to relay link is stronger than source to destination link. Therefore, the result is not valid for all conditions.

3. ACHIEVING A SUBOPTIMAL LOWER BOUND OF A MIMO RELAY CHANNEL

Wireless networking constitutes an important component of future information technology applications. Over the last decade, multi antenna systems became very popular since they increase the available spectral efficiency with the same transmit power. However, this increase crucially depends on the amount of channel knowledge at the transmitters and receivers. In addition to utilizing multiple antennas to increase the capacity of the system, adding a relay node and applying cooperative strategies can also increase capacity. In this chapter, we analyze the achievable rate on MIMO relay channels when the receivers have perfect CSI and the transmitters have partial CSI.

In Yu et al. (2004), a multi-user MIMO system is considered when all the transmitters and the receiver have perfect CSI and the channels are fixed. In this scenario, the optimum transmit directions and the power allocation policies are found using an iterative algorithm. However, the case that all the transmitters and the receiver have perfect CSI is not practical. In practice, the receiver feeds the information back to the transmitters and it is not possible to have an instantaneous feedback. Therefore, it is more realistic that the receivers have perfect CSI and the transmitters have only a statistical knowledge of the channel (Jafar & Goldsmith, 2004; Soysal & Ulukus, 2007, 2009). If the channel follows a Gaussian process, statistics of the channel are the mean and covariance information. Although the capacity is not known in general, upper and lower bounds can be derived. In decode and forward (DF) relay systems, the relay node demodulates and decodes the received signal from the source node, and retransmits it to the destination node.

In this chapter, we consider a MIMO relay channel in DF mode when the receivers have the perfect CSI and the transmitters have only the statistics of the channel. The capacity of the MIMO relay channel under such an assumption is not known in general, but lower and upper bounds to the capacity can be derived. For the model in this work, we derive a lower bound in terms of a max-min problem and solve this problem using similar techniques in Liang et al. (2007). In this technique, we describe lower bound in three cases. In the first case, lower bound on the capacity is equal to the capacity of the link from source to relay. In the second case, lower bound on the capacity is equal to the multi access channel capacity from source and relay to the destination. In the last case, lower bound on the capacity depends on both multi access channel and source to relay channel. The source to destination channel is assumed to be weaker than the source to relay channel. For all three cases, transmit covariance matrices of the source and relay nodes are determined in order to achieve the best lower bound.

3.1 SYSTEM MODEL

We consider a MIMO relay channel when the receivers have perfect CSI and the transmitters only have the transmit covariance information. The channel between a transmitter and a receiver is represented by a random matrix $\mathbf{H}_{xy}$ where $\mathbf{x}$ is the transmitter node and the $\mathbf{y}$ is the receiver node. The dimension of the channel matrix are the number of receive antennas times the number of transmitter antennas. The received signals at the relay and destination nodes for general MIMO relay channels are defined as

$$\mathbf{r} = \mathbf{H}_{sr}\mathbf{x}_s + \mathbf{n}_r \tag{3.1}$$

$$\mathbf{y} = \mathbf{H}_{sd}\mathbf{x}_s + \mathbf{H}_{rd}\mathbf{x}_r + \mathbf{n}_y \tag{3.2}$$

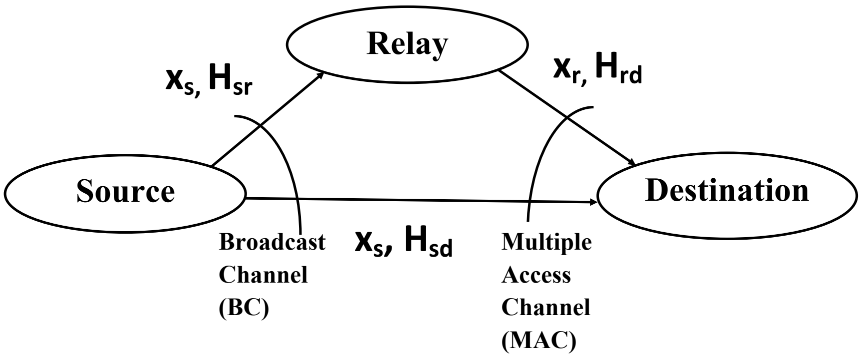
where $\mathbf{x}_s$ is an $M_s \times 1$ transmitted signal from the source node to the destination node and $\mathbf{x}_r$ is an $M_r \times 1$ transmitted signal from the relay node to the destination node. The covariance matrices of the transmitted signals are $\mathbf{Q}_s = E[\mathbf{x}_s\mathbf{x}_s^\dagger]$ and $\mathbf{Q}_r = E[\mathbf{x}_r\mathbf{x}_r^\dagger]$. The received signal at the destination node, $\mathbf{y}$, is $N_d \times 1$. The received signal at the relay node, $\mathbf{r}$, is $N_r \times 1$. The relay node is assumed to operate in full-duplex mode. As shown in Figure 3.1, $\mathbf{H}_{sr}$, $\mathbf{H}_{sd}$ and $\mathbf{H}_{rd}$ are $N_r \times M_s$, $N_d \times M_s$ and $N_d \times M_r$ dimensional channel matrices. Noise vectors at the relay, $\mathbf{n}_r$, and at the destination, $\mathbf{n}_d$ are zero-mean, identity covariance complex Gaussian random vectors. The part of the system which includes both the direct channel and the channel from source to relay is called as broadcast channel. Both the channel from relay to destination and the direct channel is defined as multiple access channel.

In the case that partial CSI with covariance information at the transmitters, there exists correlation between the signals transmitted by or received at different antenna elements. For each user, the channels is modeled as (Chuah et al., 2002)

$$\mathbf{H}_{xy} = \mathbf{\Phi}_{xy}^{1/2}\mathbf{Z}_{xy}\mathbf{\Sigma}_{xy}^{1/2} \tag{3.3}$$

where subscript xy refers to either sr (source to relay), sd (source to destination), or rd (relay to destination). The receive antenna correlation matrix, $\mathbf{\Phi}_{xy}$, is the correlation

Figure 3.1: MIMO relay channel



between the signals and the antennas on the receiver; $\mathbf{Z}_{xy}$ is an identity covariance random channel matrix, $\mathbf{\Sigma}_{xy}$ is the correlation matrix between the signals transmitted from the antennas on the transmitter. In this work, we assume that the receiver does not have physical restrictions. Therefore, there is adequate spacing between the antenna elements on the receiver. When the antenna spacing is sufficiently large, the correlation introduced by antenna element spacing is low enough that the fades associated with two different antenna elements can be considered independent (Jakes & Cox, 1974). Thus, the receive antenna correlation matrix becomes the identity matrix, $\mathbf{\Phi}_{xy} = \mathbf{I}$. As a result, the channel is written as

$$\mathbf{H}_{xy} = \mathbf{Z}_{xy}\mathbf{\Sigma}_{xy}^{1/2} \tag{3.4}$$

3.2 LOWER BOUND ON THE CAPACITY

When the receivers have perfect CSI and the transmitters have partial CSI, the channel capacity is not known in general, but lower and upper bounds on the capacity can be found. In this paper, we find a lower bound to the MIMO relay channel capacity in terms of the capacity of the link from the source to relay, capacity of the multi access channel (MAC) from source and relay to the destination and capacity of the link from the relay to destination. This lower bound involves a max-min type problem (Liang et al., 2007). This problem is solved by choosing the transmit covariance matrices of the source and the relay for each of the three cases.

Since our results will depend on single-user MIMO and MIMO-MAC mutual information expressions, here we state them for the sake of completeness. Single user MIMO channel capacity where the receiver has perfect CSI and the transmitter has only statistical knowledge of channel is known as

$$C = \max_{\text{tr}(\mathbf{Q}) \leq P} E\left[\log \left|\mathbf{I} + \mathbf{H}\mathbf{Q}\mathbf{H}^{\dagger}\right|\right] \tag{3.5}$$

where E[·] is the expectation operator with respect to the channel matrices $|\cdot|$ is the determinant operator, tr(·) is the trace of a matrix. Using eigenvalue decomposition, the channel covariance matrix and the transmit covariance matrix can be written, as $\mathbf{\Sigma} = \mathbf{U}_{\Sigma}\mathbf{\Lambda}_{\Sigma}\mathbf{U}_{\Sigma}^{\dagger}$ and $\mathbf{Q} = \mathbf{U}_{Q}\mathbf{\Lambda}_{Q}\mathbf{U}_{Q}^{\dagger}$ respectively where $\mathbf{U}_{Q}$ and $\mathbf{U}_{\Sigma}$ are unitary matrices, and $\mathbf{\Lambda}_{Q}$ and $\mathbf{\Lambda}_{\Sigma}$ are diagonal matrices that include ordered eigenvalues. Eigenvectors of transmit covariance matrix must be equal to the eigenvectors of the channel covariance matrix, $\mathbf{U}_{Q} = \mathbf{U}_{\Sigma}$ (Soysal & Ulukus, 2009). The eigenvalues of the covariance matrix are optimized using the algorithm in Soysal & Ulukus (2007).

The lower bound also depends on the two-user MAC channel, the capacity of which is defined as

$$C_{mac} = \max_{\substack{\text{tr}(\mathbf{Q}_s) \leq P_s \\ \text{tr}(\mathbf{Q}_r) \leq P_r}} E\left[\log \left|\mathbf{I} + \mathbf{H}_{sd}\mathbf{Q}_s\mathbf{H}_{sd}^{\dagger} + \mathbf{H}_{rd}\mathbf{Q}_r\mathbf{H}_{rd}^{\dagger}\right|\right] \tag{3.6}$$

where the receivers have perfect CSI and the transmitters have only transmit covariance feedback. The eigenvectors of the transmit covariance matrices of each user depend only on the eigenvectors of their own channel covariance matrices, i.e., $\mathbf{U}_{Q_s} = \mathbf{U}_{\Sigma_{sd}}$, and $\mathbf{U}_{Q_r} = \mathbf{U}_{\Sigma_{rd}}$ (Soysal & Ulukus, 2009). The eigenvalues of the covariance matrices of the source and relay are found using the iterative algorithm in Soysal & Ulukus (2007).

On the other hand, there are some results on MIMO relay channels, when the receivers have perfect CSI and the transmitters have no CSI. Lower bound of the ergodic capacity in this case is found in Wang et al. (2005). We also state this result by assuming that source to destination link is weaker than source to relay link.

$$C \geq C_{lower} = \max_{\substack{\text{tr}(\mathbf{Q}_s) \leq P_s \\ \text{tr}(\mathbf{Q}_r) \leq P_r}} (\min(C_{mac}, C_{sr})) \tag{3.7}$$

$$C_{mac} = E\left[\log \left|\mathbf{I} + \mathbf{H}_{sd}\mathbf{H}_{sd}^{\dagger} + \mathbf{H}_{rd}\mathbf{H}_{rd}^{\dagger}\right|\right] \tag{3.8}$$

$$C_{sr} = E\left[\log \left|\mathbf{I} + \mathbf{H}_{sr}\mathbf{H}_{sr}^{\dagger}\right|\right] \tag{3.9}$$

Since the transmitters have no CSI, the lower bound is maximized by choosing $\mathbf{x}_s$ and $\mathbf{x}_r$ to be independent circular-symmetric vectors with $\mathbf{Q}_s = \mathbf{I}$ and $\mathbf{Q}_r = \mathbf{I}$.

In this study, when the receivers know the channel perfectly, and the transmitters know the covariance information of the channel, we derive a lower bound on MIMO relay capacity. Theorem 3.1 gives this lower bound in terms of the capacity of the link from the source to relay, capacity of the MAC from source and relay to destination and the transmit covariance matrices at the source and relay.

Theorem 3.1. *When there is only transmit covariance information at the transmitters and perfect CSI at the receives, lower bound on ergodic capacity of a MIMO relay channel is given as*

$$C \geq C_{lower} = \max_{\substack{tr(\mathbf{Q}_s)\leq P_s \\ tr(\mathbf{Q}_r)\leq P_r}} \min(I_{mac}, I_{sr}) \tag{3.10}$$

$$I_{mac} = E\left[\log\left|\mathbf{I} + \mathbf{H}_{sd}\mathbf{Q}_s\mathbf{H}_{sd}^{\dagger} + \mathbf{H}_{rd}\mathbf{Q}_r\mathbf{H}_{rd}^{\dagger}\right|\right] \tag{3.11}$$

$$I_{sr} = E\left[\log\left|\mathbf{I} + \mathbf{H}_{sr}\mathbf{Q}_s\mathbf{H}_{sr}^{\dagger}\right|\right] \tag{3.12}$$

where $\mathbf{Q}_s = E[\mathbf{x}_s\mathbf{x}_s^{\dagger}]$ *and* $\mathbf{Q}_r = E[\mathbf{x}_r\mathbf{x}_r^{\dagger}]$ *are transmit covariance matrices with* $tr(\mathbf{Q}_s) \leq P_s$ *and* $tr(\mathbf{Q}_r) \leq P_r$.

Proof: Using block Markov coding technique, the achievable rate for decode and forward scenario is written below (Cover & El Gamal, 1979, Section VI).

$$R = \max_{p(\mathbf{x}_s,\mathbf{x}_r)} \min\left(I(\mathbf{x}_s;\mathbf{r}|\mathbf{x}_r), I(\mathbf{x}_s,\mathbf{x}_r;\mathbf{y})\right) \tag{3.13}$$

$$I(\mathbf{x}_s,\mathbf{x}_r;\mathbf{y}) = E\left[I(\mathbf{x}_s,\mathbf{x}_r;\mathbf{y}|\mathbf{H}_{sd},\mathbf{H}_{rd})\right] \tag{3.14}$$

$$I(\mathbf{x}_s;\mathbf{r}|\mathbf{x}_r) = E\left[I(\mathbf{x}_s;\mathbf{r}|\mathbf{x}_r,\mathbf{H}_{sr})\right] \tag{3.15}$$

where $\mathbf{x}_s$ and $\mathbf{x}_r$ are circularly-symmetric complex Gaussian random vectors. In order to prove the theorem, we have to calculate the mutual information expressions in (3.14) and (3.15), and then solve the max-min problem in (3.13). The expectation in (3.14) is calculated in Wang et al. (2005) as

$$E\left[I(\mathbf{x}_s,\mathbf{x}_r;\mathbf{y}|\mathbf{H}_{sd},\mathbf{H}_{rd})\right] = E\left[\log\left|\mathbf{I} + [\mathbf{H}_{sd} \quad \mathbf{H}_{rd}]\begin{bmatrix}\mathbf{Q}_s & \mathbf{Q}_{sr} \\ \mathbf{Q}_{rs} & \mathbf{Q}_r\end{bmatrix}\left[\mathbf{H}_{sd}^{\dagger} \quad \mathbf{H}_{rd}^{\dagger}\right]^{\dagger}\right|\right] \tag{3.16}$$

We can replace $\mathbf{H}_{sd}$ with $-\mathbf{H}_{sd}$ since the beginning phase is not important for any $\mathbf{H}_{sd}$.

That change is equal to using the same $\mathbf{H}_{sd}$, but replacing $\mathbf{x}_s$ with $-\mathbf{x}_s$. However, this makes cross-correlation matrices to change sign while the result is the same. Using the concavity of the logarithm, when we choose the cross-correlation matrices, $\mathbf{Q}_{sr} = E[\mathbf{x}_s\mathbf{x}_r{}^\dagger]$ and $\mathbf{Q}_{rs} = E[\mathbf{x}_r\mathbf{x}_s{}^\dagger]$ to be zero, the mutual information cannot decrease. Thus, the signals are independent (Kramer et al., 2004). After taking the cross correlations matrices to be zero matrices, (3.16) becomes (3.14).

The expression in (3.15) gives the single user capacity from the source to the relay node. This problem can be calculated as the following equation in Wang et al. (2005).

$$E\left[I(\mathbf{x}_s;\mathbf{r}|\mathbf{x}_r,\mathbf{H}_{sr})\right] \leq E\left[\log\left|\mathbf{I}+\mathbf{H}_{sr}(\mathbf{Q}_s-\mathbf{Q}_{sr}\mathbf{Q}_r^{-1}\mathbf{Q}_{rs}^\dagger)\mathbf{H}_{sr}^\dagger\right|\right] \tag{3.17}$$

$$= \log\left[\left|\mathbf{I}+\mathbf{H}_{sr}\mathbf{Q}_s\mathbf{H}_{sr}^\dagger\right|\right] \tag{3.18}$$

We used the fact that $\mathbf{Q}_{sr} = \mathbf{Q}_{rs} = 0$ (Kramer et al., 2004). Finally, we insert the mutual information expressions into (3.13) and obtain (3.10)-(3.12). □

3.3 THE ITERATIVE ALGORITHM GIVES OPTIMUM TRANSMIT COVARIANCE MATRICES

In this section, we will find optimum transmit covariance matrices which gives the maximum achievable rate. The eigenvalue decomposition of the channel covariance matrix, $\mathbf{\Sigma}_{xy}$, is $\mathbf{\Sigma} = \mathbf{U}_{\mathbf{\Sigma}}\Lambda_{\mathbf{\Sigma}}\mathbf{U}_{\mathbf{\Sigma}}^\dagger$, and the eigenvalue decomposition of the transmit covariance matrix, $\mathbf{Q}$, is $\mathbf{Q} = \mathbf{U}_{\mathbf{Q}}\Lambda_{\mathbf{Q}}\mathbf{U}_{\mathbf{Q}}^\dagger$. In these expressions, $\Lambda_{\mathbf{\Sigma}}$ and $\Lambda_{\mathbf{Q}}$ are diagonal matrices of ordered eigenvalues of $\mathbf{\Sigma}$ and $\mathbf{Q}$. The diagonal elements are the power values which are used in the channels. Adding to that, $\mathbf{U}_{\mathbf{\Sigma}}$ and $\mathbf{U}_{\mathbf{Q}}$ are unitary matrices which shows transmit directions.

It is important to note that the optimum $\mathbf{Q}_s$ maximizing I_{sr} and the optimum $\mathbf{Q}_s$ maximizing I_{mac} are different in (3.10). If we maximize I_{mac}, that choice of $\mathbf{Q}_s$ will result in a low I_{sr}. As a result, I_{sr} will come out of the minimization in (3.10), and the achievable rate will attain a lower value. As a solution to this, a max-min type of optimization is given in Liang et al. (2007). The following function R of α and $\mathbf{Q}$ is defined as

$$R(\alpha,\mathbf{Q}) = \alpha I_{mac}(\mathbf{Q}) + (1-\alpha)I_{sr}(\mathbf{Q}), \quad 0 \leq \alpha \leq 1 \tag{3.19}$$

where $\mathbf{Q} = [\mathbf{Q}_s \quad \mathbf{Q}_r]$. The maximization in (3.10) corresponds to maximizing the two end points of the line $R(\alpha, \mathbf{Q})$ over all values of $\mathbf{Q}$.

$$\mathbf{V}(\alpha) = \max_{\mathbf{Q}} R(\alpha, \mathbf{Q}) \tag{3.20}$$

where $\mathbf{Q}$ maximizes $R(\alpha, \mathbf{Q})$ for fixed α. We suppose that α^* provides the minimum value of $\mathbf{V}(\alpha)$,

$$\mathbf{V}(\alpha^*) = \min_{0 \leq \alpha \leq 1} \mathbf{V}(\alpha). \tag{3.21}$$

It is important to note that the optimum source and relay covariance matrices may be different in all three cases. After finding the optimum transmit covariance matrices for a given α, for all $0 \leq \alpha \leq 1$, we will find the optimum α^* that minimizes $\mathbf{V}(\alpha)$. In the following, we present the derivation for all three cases in detail.

Case 1: In the first case ($\alpha^* = 0$), $R(0, \mathbf{Q}) = I_{sr}(\mathbf{Q})$ and the condition $I_{mac}(\mathbf{Q}) \geq I_{sr}(\mathbf{Q})$ should be satisfied. Since the achievable rate is found by maximizing $I_{sr}(\mathbf{Q})$ only, we find the optimum source transmit covariance matrix, $\mathbf{Q}_s$, as a solution to a single-user problem from source to relay. Then we find the optimum relay transmit covariance matrix, $\mathbf{Q}_r$, by maximizing $I_{mac}(\mathbf{Q})$ with a fixed $\mathbf{Q}_s$ in order to satisfy $I_{mac}(\mathbf{Q}) \geq I_{sr}(\mathbf{Q})$.

In this chapter, we find the the transmit covariance matrices in two steps: finding the optimum transmit directions, which are the eigenvectors of the transmit covariance matrix and the optimum power allocation policies, which are the eigenvalues of the transmit covariance matrix. In the following theorem, we prove that all users should transmit along the eigenvectors of their own channel covariance matrices, regardless of the power allocation scheme.

Theorem 3.2. *Let us assume the channel covariance matrix from the relay to the destination, $\mathbf{\Sigma}_{sr}$, has the eigenvalue decomposition $\mathbf{\Sigma}_{sr} = \mathbf{U}_{\Sigma_{sr}} \mathbf{\Lambda}_{\Sigma_{sr}} \mathbf{U}_{\Sigma_{sr}}^{\dagger}$. Then, optimum relay transmit covariance matrix $\mathbf{Q_s}$ has the spectral decomposition $\mathbf{Q}_s = \mathbf{U}_{\Sigma_{sr}} \mathbf{\Lambda}_{Q_s} \mathbf{U}_{\Sigma_{sr}}^{\dagger}$ for any $\mathbf{Q}_r$.*

Proof: Using the channel model, $\mathbf{H}_{sr} = \mathbf{Z}_{sr} \mathbf{U}_{\Sigma_{sr}} \mathbf{\Lambda}_{\Sigma_{sr}}^{1/2} \mathbf{U}_{\Sigma_{sr}}^{\dagger}$ can be inserted into (3.12). Noting that $\mathbf{ZU}$ and $\mathbf{Z}$ have the same joint distribution for zero mean identity covariance Gaussian $\mathbf{Z}$ and unitary $\mathbf{U}$ (Soysal & Ulukus, 2007)

$$\max_{\mathbf{Q}_s} I_{sr} = \max_{\mathbf{Q}_s} E\left[\log\left|\mathbf{I} + \mathbf{Z}_{sr} \mathbf{\Lambda}_{\Sigma_{sr}}^{1/2} \mathbf{U}_{\Sigma_{sr}}^{\dagger} \mathbf{Q}_s \mathbf{U}_{\Sigma_{sr}} \mathbf{\Lambda}_{\Sigma_{sr}}^{1/2} \mathbf{Z}_{sr}^{\dagger}\right|\right] \tag{3.22}$$

The matrix between $\mathbf{Z}_{sr}$ and $\mathbf{Z}_{sr}{}^{\dagger}$ can be decomposed as $\mathbf{\Lambda}_{\Sigma_{sr}}^{1/2}\mathbf{U}_{\Sigma_{sr}}^{\dagger}\mathbf{Q}_s\mathbf{U}_{\Sigma_{sr}}\mathbf{\Lambda}_{\Sigma_{sr}}^{1/2} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^{\dagger}$. Inserting this into (3.22) and using the fact that $\mathbf{ZU}$ and $\mathbf{Z}$ have the same joint distribution one more time, we have

$$\max_{\mathbf{Q}_s} E\left[\log\left|\mathbf{I} + \mathbf{Z}_{sr}\mathbf{\Lambda}\mathbf{Z}_{sr}^{\dagger}\right|\right] \tag{3.23}$$

Since the optimization problem in (3.23) does not involve $\mathbf{U}$, and choosing $\mathbf{U} = \mathbf{I}$ does not violate the power constraint (Soysal & Ulukus, 2009), we have $\mathbf{Q}_s = \mathbf{U}_{\Sigma_{sr}}\mathbf{\Lambda}\mathbf{\Lambda}_{\Sigma_{sr}}\mathbf{U}_{\Sigma_{sr}}^{\dagger}$.
□

Using Theorem 3.2, (3.12) is written as

$$\mathbf{I}_{sr}(\mathbf{Q}) = \max_{\text{tr}(\mathbf{Q}_s)\le P_s} E\left[\log\left|\mathbf{I} + \sum_{i=1}^{M_s}\lambda_i^{Q_s}\lambda^{\Sigma_{sri}}z_{sri}z_{sri}^{\dagger}\right|\right] \tag{3.24}$$

The Lagrangian of the optimization problem is shown as

$$L = E\left[\log\left|\mathbf{I} + \sum_{i=1}^{M_s}\lambda^{\Sigma_{sri}}z_{sri}z_{sri}^{\dagger}\right|\right] - \mu_s\left(\sum_{i=1}^{M_s}\lambda_i^{Q_s}\lambda^{\Sigma_{sri}} - P_s\right) \tag{3.25}$$

In order to derive the Karush-Kuhn-Tucker (KKT) conditions, we use the following derivation rule (Jafar & Goldsmith, 2004)

$$\frac{\partial}{\partial x}\log|\mathbf{A} + x\mathbf{B}| = \text{tr}[(\mathbf{A} + xB)^{-1}\mathbf{B}] \tag{3.26}$$

Using this identity, the KKT condition is obtained as

$$\lambda_{sri}^{\Sigma}E\left[z_{sri}^{\dagger}\left(\mathbf{I} + \sum_{i=1}^{M_s}\lambda_i^{Q_s}\lambda_i^{\Sigma_{sr}}z_{sri}z_{sri}^{\dagger}\right)^{-1}z_{sri}\right] \le \mu_s, \quad i = 1,\ldots,M_s \tag{3.27}$$

For inversion of matrix, the lemma in Horn & Johnson (1985, page 19) is used.

$$E_i(\lambda^{Q_s}) \triangleq E\left[\frac{\lambda_i^{\Sigma sr}z_{sri}^{\dagger}A_i^{-1}z_{sri}}{1 + \lambda_i^{Q_s}\lambda_i^{\Sigma_{sr}}z_{sri}^{\dagger}A_i^{-1}z_{sri}}\right] \le \mu_s, \quad i = 1,\ldots,M_s \tag{3.28}$$

where the parameters are $\mathbf{A} = \mathbf{I} + \sum_{i=1}^{M_s}\lambda_i^{Q_s}\lambda_i^{\Sigma_{sr}}z_{sri}z_{sri}^{\dagger}$ and $\mathbf{A}_i = \mathbf{A} - \lambda_i^{Q_s}\lambda_i^{\Sigma_{sr}}z_{sri}z_{sri}^{\dagger}$. The left side is defined as $E_i(\lambda^{Q_s})$. According to Lagrangian rules (3.28) is equality when λ^{Q_s} is nonzero, but it is inequality when λ^{Q_s} is zero. To obtain the strict equality for all

λ^{Q_s}, both side of (3.28) is multiplied with λ^{Q_s}. Consequently, if λ^{Q_s} is zero, then both side of the equation is zero. Thus, the equality is achieved for all λ^{Q_s} (Soysal & Ulukus, 2009).

$$\lambda_i^{Q_s} E_i(\lambda^{Q_s}) = \mu_s \lambda^{Q_s}, \quad i = 1, \ldots, M_s \tag{3.29}$$

The sum operator is inserted to the both side of the equation and total power constraint is obtained at right side.

$$\sum_{j=1}^{M_s} \lambda_j E_j(\lambda^{Q_s}) = \mu_s P_s \qquad \longrightarrow \qquad \mu_s = \frac{\sum_{j=1}^{M_s} \lambda_j^{Q_s} E_j(\lambda^{Q_s})}{P_s} \tag{3.30}$$

The equivalent of the Lagrangian Multiplier μ_s is written in (3.29).

$$\lambda^{Q_s} = \frac{\lambda^{Q_s} E_i(\lambda^{Q_s})}{\sum_{j=1}^{M_s} \lambda_j^{Q_s} E_j(\lambda^{Q_s})} P_s, \quad i = 1, \ldots, M_s \tag{3.31}$$

To solve for optimum eigenvalues, (3.32) update the eigenvalues at step $n+1$ as a function of the eigenvalues at step n.

$$\lambda^{Q_s}(n+1) = \frac{\lambda^{Q_s}(n) E_i(\lambda^{Q_s}(n))}{\sum_{j=1}^{M_s} \lambda_j^{Q_s}(n) E_j(\lambda^{Q_s}(n))} P_s \tag{3.32}$$

The optimum eigenvalues of $\mathbf{Q}_s$ which are found above are used in I_{mac}. Then, I_{mac} is optimized over λ^{Q_r}, and the optimum eigenvalues of $\mathbf{Q}_r$ which satisfies $I_{mac}(\mathbf{Q}) \geq I_{sr}(\mathbf{Q})$ are found.

Case 2: In the second case, ($\alpha^* = 1$), $R(1, \mathbf{Q}) = I_{mac}(\mathbf{Q})$ and the condition $I_{mac}(\mathbf{Q}) \leq I_{sr}(\mathbf{Q})$ should be satisfied. In this case, the achievable rate is found by maximizing $I_{mac}(\mathbf{Q})$. Therefore, we find the optimum source transmit covariance matrix, $\mathbf{Q}_s$, and relay transmit covariance matrix $\mathbf{Q}_r$, as a solution to a MAC problem (Soysal & Ulukus, 2007).

Case 3: In the third case, ($0 < \alpha^* < 1$), $R(\alpha^*, \mathbf{Q}) = \alpha^* I_{mac}(\mathbf{Q}) + (1 - \alpha^*) I_{sr}(\mathbf{Q})$ and the condition $I_{mac}(\mathbf{Q}) = I_{sr}(\mathbf{Q})$ should be satisfied. In this case, we find the optimum transmit covariance matrices of the source and relay as functions of α^*. This case is the most interesting case as the solution is not trivial. When $0 < \alpha^* < 1$, (3.19) transforms to following optimization problem.

$$\mathbf{V}(\alpha^*) = \max_{\substack{\text{tr}(\mathbf{Q}_s)\leq P_s \\ \text{tr}(\mathbf{Q}_r)\leq P_r}} (\alpha^* I_{mac}(\mathbf{Q}) + (1-\alpha^*)I_{sr}(\mathbf{Q})) \tag{3.33}$$

In addition, the following condition has to be satisfied because of max-min rule (Liang et al., 2007)

$$I_{mac}(\mathbf{Q}) = I_{sr}(\mathbf{Q}) \tag{3.34}$$

We will solve (3.33) for a given α, and search over $0 < \alpha < 1$ to find $\mathbf{V}(\alpha^*)$. In order to find the transmit covariance matrices of the source and the relay nodes, we start by determining the eigenvectors (i.e., the transmit directions) of the transmit covariance matrices. In doing this, we assume that source to destination link is weaker that the source to relay link. Therefore, source node chooses to transmit along the eigenvectors of the covariance of the source to relay channel. Once this is given, transmit directions of the relay node can be found using Theorem 3.2, which shows that the eigenvectors of the transmit covariance matrix of each user are equal to the eigenvectors of its own channel covariance matrix. In MIMO relay channel as well, both the source and relay transmit along the eigenvectors of their own channel.

Having found the eigenvectors (i.e., transmit directions) of the source and relay transmit covariance matrices, next we find the optimum power allocated along these transmit directions. The amount of power allocated in each direction depends on both transmit directions and the power allocations on the system. The transmit directions are chosen the same directions with related channels. Next, we find the eigenvalues of the transmit covariance matrices. Re-writing (3.5) with transmit directions, we have

$$\mathbf{V}(\alpha^*) = \max_{\boldsymbol{\lambda}^{Q_s}, \boldsymbol{\lambda}^{Q_r}} (\alpha^* I_{mac}(\boldsymbol{\lambda}) + (1-\alpha^*)I_{sr}(\boldsymbol{\lambda})) \tag{3.35}$$

$$I_{mac}(\boldsymbol{\lambda}) = E\left[\log\left|\mathbf{I} + \sum_{i=1}^{M_s} \lambda_i^{Q_s} \boldsymbol{\Phi}_i \boldsymbol{\Phi}_i^\dagger + \sum_{i=1}^{M_r} \lambda_i^{Q_r} \lambda_i^{\Sigma_{rd}} \mathbf{z}_i^{rd} \mathbf{z}_i^{rd\dagger}\right|\right] \tag{3.36}$$

$$I_{sr}(\boldsymbol{\lambda}) = E\left[\log\left|\mathbf{I} + \sum_{i=1}^{M_s} \lambda_i^{Q_s} \lambda_i^{\Sigma_{sr}} \mathbf{z}_i^{sr} \mathbf{z}_i^{sr\dagger}\right|\right] \tag{3.37}$$

where $\boldsymbol{\lambda} = [\boldsymbol{\lambda}^{Q_s} \quad \boldsymbol{\lambda}^{Q_r}]$ and $\boldsymbol{\lambda}^{Q_s}$ and $\boldsymbol{\lambda}^{Q_r}$ are the eigenvalue vectors of the source and relay transmit covariance matrices respectively, and $\boldsymbol{\Phi}_i$ is the i^{th} column of the matrix $\boldsymbol{\Phi} = \mathbf{Z}^{sd} \boldsymbol{\Lambda}_{\boldsymbol{\Sigma}_{sd}}^{1/2} \mathbf{U}_{\boldsymbol{\Sigma}_{sd}}^\dagger \mathbf{U}_{\boldsymbol{\Sigma}_{sr}}$.

The Lagrangian of (3.35) is given as

$$R(\alpha^*, \boldsymbol{\lambda}) - \mu_s \left(\sum_{i=1}^{M_s} \lambda_i^{Q_s} - P_s \right) - \mu_r \left(\sum_{i=1}^{M_r} \lambda_i^{Q_r} - P_r \right) \tag{3.38}$$

By taking the derivative of (3.38) with respect to $\lambda_i^{Q_s}$ and $\lambda_i^{Q_r}$, we obtain the Karush-Kuhn-Tucker (KKT) conditions for all $0 \leq i \leq M_s$ and $0 \leq j \leq M_r$

$$\alpha^* E \left[\boldsymbol{\Phi}_i^\dagger \mathbf{A}^{-1} \boldsymbol{\Phi}_i \right] + (1 - \alpha^*) \lambda_i^{\Sigma_{sr}} E \left[\mathbf{z}_i^{sr\dagger} \mathbf{B}^{-1} \mathbf{z}_i^{sr} \right] \leq \mu_s \tag{3.39}$$

$$\alpha^* \lambda_j^{\Sigma_{rd}} E \left[\mathbf{z}_j^{rd\dagger} \mathbf{A}^{-1} \mathbf{z}_j^{rd} \right] \leq \mu_r \tag{3.40}$$

where μ_s and μ_r are the Lagrangian multipliers, $\mathbf{A}$ and $\mathbf{B}$ are the matrices inside the determinants of (3.36) and (3.37) respectively. Let us denote the left hand side of (3.39) as $E_i^s(\boldsymbol{\lambda})$ and the left hand side of (3.40) as $E_j^r(\boldsymbol{\lambda})$. By multiplying both sides of (3.39) with $\lambda_i^{Q_s}$ and (3.40) with $\lambda_j^{Q_r}$ and summing over i and j respectively, we have

$$\mu_s = \frac{\sum_{i=1}^{M_s} \lambda_i^{Q_s} E_i^s(\boldsymbol{\lambda})}{P_s}, \qquad \mu_r = \frac{\sum_{j=1}^{M_r} \lambda_j^{Q_r} E_j^r(\boldsymbol{\lambda})}{P_r} \tag{3.41}$$

Combining these with (3.39)-(3.40), we have the following fixed point equations for the eigenvalues of the source and relay transmit covariance matrices

$$\lambda_i^{Q_s} = \frac{\lambda_i^{Q_s} E_i^s(\boldsymbol{\lambda}) P_s}{\sum_{i=1}^{M_s} \lambda_i^{Q_s} E_i^s(\boldsymbol{\lambda})}, \quad \lambda_j^{Q_r} = \frac{\lambda_j^{Q_r} E_j^r(\boldsymbol{\lambda}) P_r}{\sum_{j=1}^{M_r} \lambda_j^{Q_r} E_j^r(\boldsymbol{\lambda})} \tag{3.42}$$

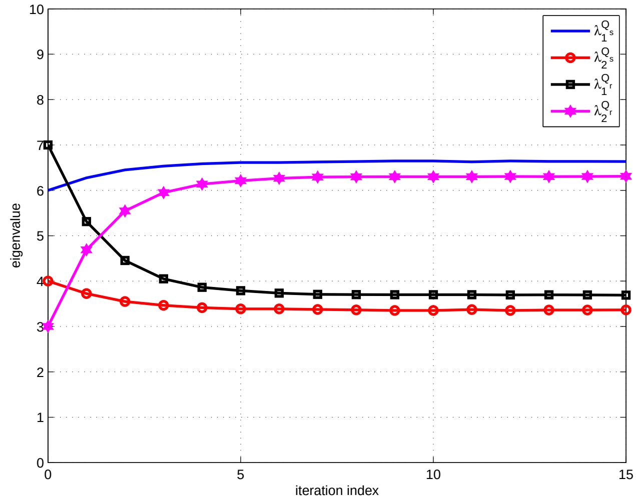
We propose the following iterative algorithm to solve for the above fixed point equations.

$$\lambda_i^{Q_s}(n+1) = \frac{\lambda_i^{Q_s(n)} E_i^s(\boldsymbol{\lambda}(n))}{\sum_{i=1}^{M_s} \lambda_i^{Q_s(n)} E_i^s(\boldsymbol{\lambda}(n))} P_s \tag{3.43}$$

$$\lambda_j^{Q_r}(n+1) = \frac{\lambda_j^{Q_r(n)} E_j^r(\boldsymbol{\lambda}(n))}{\sum_{j=1}^{M_r} \lambda_j^{Q_r(n)} E_j^r(\boldsymbol{\lambda}(n))} P_r \tag{3.44}$$

This iterative algorithm finds the optimum eigenvalues of the transmit covariance matrices of the source and relay nodes for Case 3. Finally, a minimization over α is performed in order to find which case results in the lower bound.

Figure 3.2: Eigenvalues of the capacity from source to relay



3.4 NUMERICAL RESULTS

In this section, we simulate our proposed solution. In Figure 3.2, Case 3 is analyzed and optimum eigenvalues of the transmit covariance matrix of the source and relay are plotted. Here, $\lambda_1^{Q_s}$ is the first eigenvalue and $\lambda_2^{Q_s}$ is the second eigenvalue of the source, $\lambda_1^{Q_r}$ is the first eigenvalue and $\lambda_2^{Q_r}$ is the second eigenvalue of the relay. The power constraints (P_s and P_r) are 10 dB. In figure, we observe that our algorithm converges to the optimum eigenvalues. When the source power is fixed at 10 dB, the relay power is increased in

Figure 3.3: Achievable rate to the capacity with increasing relay power

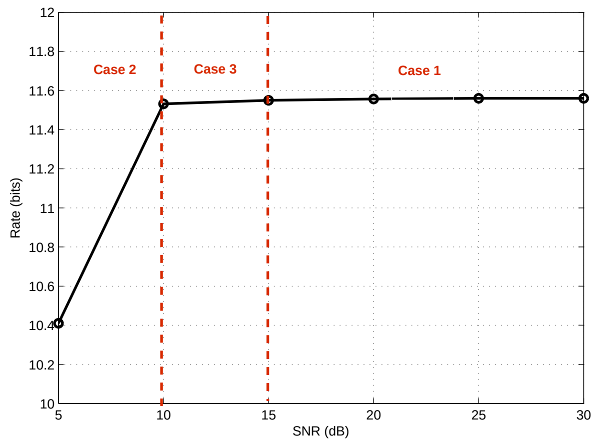
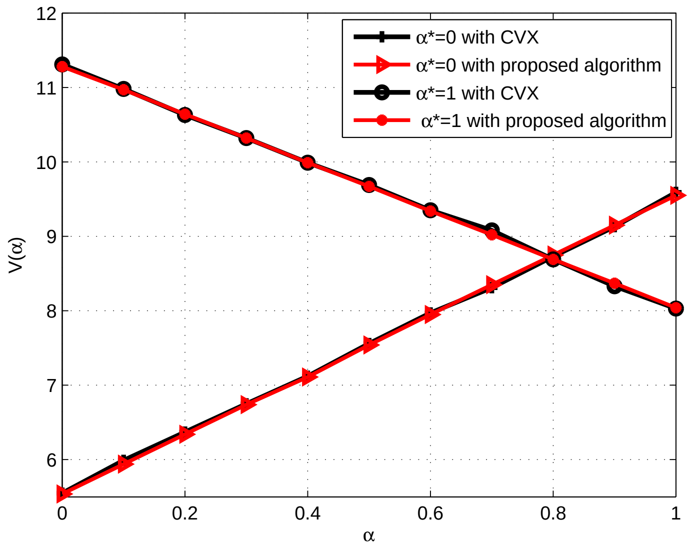


Figure 3.3. We observe that the channel is subject to Case 2 condition when the relay

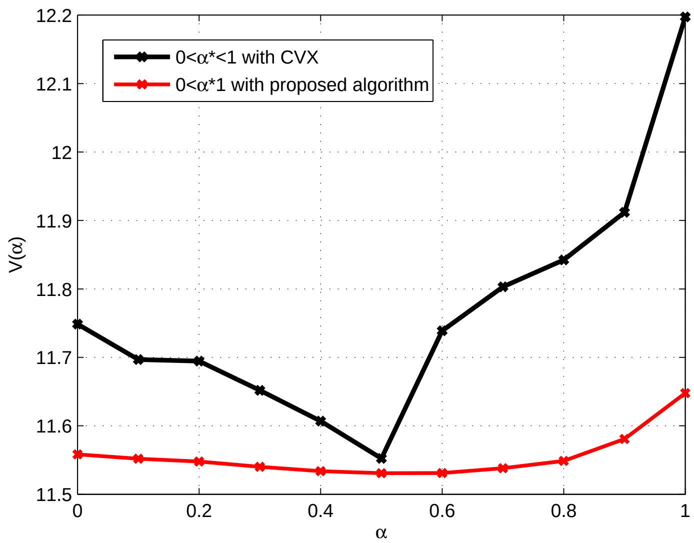
power is 5-10 dB. When the relay power is 10-15 dB, the channel is subject to Case 3 condition. Lastly, the channel is subject to Case 1 condition when the relay power is 15-30 dB. The channel saturates with relay power since in Case 1 the relay power is large enough to forward all the information decoded at the relay node to the destination node, and the achievable rate is limited by the capacity of the source to relay link (Liang et al., 2007).

Figure 3.4: Achievable rate as a function of α in case 1 and case 2



In this thesis work, the lower bound on relay channel capacity is determined in three cases. In Figure 3.4, the capacity variations with α are shown for Case 1 and Case 2. The power constraints (P_s and P_r) are 10 dB. In Figure 3.5, the minimum value of the convex is function about $\alpha* = 0.55$ in Case 3. We use CVX toolbox for general convex optimization (GCO). The results of iteration algorithms are so similar to the results of GCO for all three cases. Adding to that, the iteration algorithm analyzes the lower bound in a shorter time than GCO.

Figure 3.5: Achievable rate as a function of α in case 3



4. CAPACITY BOUNDS OF FULL DUPLEX MIMO RELAY CHANNEL

For single antenna fading relay channels, capacity bounds and power allocations are given in Host-Madsen & Zhang (2005) for both full-duplex and half-duplex transmissions, where perfect CSI is available everywhere. A similar setting with individual power constraints at the source and relay is considered in Liang et al. (2007), where a max-min type of solution is also introduced. In Kramer et al. (2004), MIMO relay channels with different fading assumptions are discussed, when only the receivers have the perfect CSI. For full-duplex, fading MIMO relay channels, capacity upper bound and DF achievable rate are found in Wang et al. (2005), where only the receivers know the perfect CSI and transmitters do not know the channel.

A more practical channel model, for which the receivers have the perfect CSI and the transmitters have partial CSI, was utilized for point-to-point MIMO and MIMO multiple access channels (MAC) in Jafar & Goldsmith (2004), Soysal & Ulukus (2009), Soysal & Ulukus (2007). In both of these channels, it is possible to find the eigenvectors of the transmit covariance matrices in closed form, and solve a reduced optimization problem over the eigenvalues of the transmit covariance matrices, using an iterative algorithm (Soysal & Ulukus, 2009)-(Soysal & Ulukus, 2007). However, in relay channels, it is not always possible to find a closed form expression for the eigenvectors of the transmit covariance matrices. One solution we offered to this problem in the previous chapter was to choose the eigenvectors of the transmit covariance matrices similar to point-to-point channels (Aygun & Soysal, 2011). However, this choice is clearly suboptimal. Therefore, in this chapter, we propose a new method for solving the transmit covariance matrices directly (i.e., without the need of finding the eigenvectors first). In this method, matrix differential calculus is extremely functional, since it offers a new way for optimizing rate expressions by taking derivatives of scalar functions with respect to matrix variables (transmit covariance matrices)(Magnus & Neudecker, 1999). This eliminates the need for calculating cumbersome partial differentials that need to be taken with respect to the eigenvalues of matrix variables. By using matrix differential calculus, the resulting iterative algorithm updates the entire matrix at once, at each iteration.

In this work, we consider both full-duplex and half-duplex MIMO relay channels where the transmitters have partial CSI in the form of covariance feedback. The source and relay terminals have individual power constraints. We evaluate DF lower bound and cut-set upper bound on the channel capacity that are given in terms of max-min type optimization problems over the source and relay transmit covariance matrices. The main contribution of this chapter is to find the transmit covariance matrices that satisfy the lower and upper bound optimization problems. We solve these joint optimization problems using techniques from Liang et al. (2007) and also using matrix differential calculus (Magnus & Neudecker, 1999). The solutions to the optimization problems are in terms of iterative algorithms that find the transmit covariance matrices directly (i.e., without first finding the eigenvectors and then calculating the eigenvalues). Through simulations, we show that the proposed algorithms converge. Moreover, we observe that, for certain channel covariance matrix settings, lower and upper bounds meet to give the exact capacity.

4.1 MATRIX DIFFERENTIAL CALCULUS

In this section, we introduce matrix differential calculus that will be useful in later sections. We start by defining the "differential" of a scalar function. Let $\phi : \Re \rightarrow \Re$ be a real-valued function. The differential is the linear part of the increment of the value of a function, $\phi(x+u) - \phi(x)$, at a fixed point x with an increment u. The derivative of the function ϕ at the point x is found by dividing the differential of the function with the increment u, and by taking the limit as u goes to 0.

$$\phi'(x) = \lim_{u \rightarrow 0} \frac{\phi(x+u) - \phi(x)}{u} \tag{4.1}$$

The differential is denoted by d(x; u) and it is equal to $\mathrm{d}\phi(x;u) = u\phi'(x)$. Similarly, let $f : \Re^n \rightarrow \Re^m$ be a vector valued function, and $\mathbf{x}, \mathbf{u} \epsilon \Re^n$. The differential of f is defined as $\mathrm{d}f(\mathbf{x};\mathbf{u}) = \mathbf{A}(\mathbf{x})\mathbf{u}$, where $m \times n$ dimensional matrix $\mathbf{A}(x)$ is called the first derivative of f at x. It is important to note here that while the differential of a vector valued function is a vector, derivative of a vector valued function is a matrix. Since dealing with a matrix is cumbersome, partial derivatives are often used in optimization problems involving vector valued functions. In fact, as the first identification theorem in Magnus & Neudecker (1999) states, the elements of $m \times n$ matrix $\mathbf{A}(x)$ are the partial derivatives of f evaluated at $\mathbf{x}$, and $\mathbf{A}(x)$ is called the Jacobian matrix of f, $\mathrm{D}f(\mathbf{x}) = \mathbf{A}(\mathbf{x})$. As a result of this, if f is differentiable at x and we have found a differential $\mathrm{d}f$ at $\mathbf{x}$, then the value of the partial derivatives at $\mathbf{x}$ can be immediately determined. Finally, the differential of a matrix valued function can be determined using the vector representation of matrices.

Let $F : \Re^{n\times q} \rightarrow \Re^{m\times p}$ be a matrix function, and differentiable at $\mathbf{X}\epsilon\Re^{n\times q}$. Then the differential can be written as vec $\mathrm{d}F(\mathbf{X};\mathbf{U}) = \mathbf{A}(\mathbf{X})\mathrm{vec}\mathbf{U}$, where the Jacobian is an $mp \times nq$ matrix $DF(\mathbf{X}) = \mathbf{A}(\mathbf{X})$.

Given a matrix function $F(\mathbf{X})$, determining the derivative of this function from its differential is carried out as follows: (i) compute the differential of $F(\mathbf{X})$, (ii) vectorize to obtain $\mathrm{d}vecF(\mathbf{X}) = \mathbf{A}(\mathbf{X})\mathrm{d}vec\mathbf{X}$, and (iii) conclude that $\mathrm{D}F(\mathbf{X}) = \mathbf{A}(\mathbf{X})$. In this paper, we mainly deal with scalar functions, $\phi : \Re^{n\times q} \rightarrow \Re$, of matrix variables. In this case, the differential can be written as

$$\mathrm{D}\phi(\mathbf{X}) = \frac{\partial\phi(\mathbf{X})}{\partial(vec\mathbf{X}^T)} \tag{4.2}$$

However, the idea of arranging the partial derivatives of $\phi(\mathbf{X})$ into a matrix (rather than a vector) is appealing and sometimes useful, so with a slight abuse of notation we will use

$$\mathrm{D}\phi(\mathbf{X}) = \frac{\partial\phi(\mathbf{X})}{\partial\mathbf{X}} \tag{4.3}$$

For scaler functions of matrix variable, the differential of $\phi(\mathbf{X})$ is given as $\mathrm{d}\phi = (vec\mathbf{A})^T$ $\mathrm{d}vec\mathbf{X}$ which is also equal to $\mathrm{d}\phi = \mathrm{tr}(\mathbf{A}^T\mathrm{d}\mathbf{X})$, where Jacobian matrix is obtained as $\mathrm{D}\phi(\mathbf{X}) = \frac{\partial\phi(\mathbf{X})}{\partial\mathbf{X}} = \mathbf{A}$.

Using this, we now give some important differentials that will be useful later. Differential of $\mathrm{tr}(\mathbf{X})$ with respect to $\mathbf{X}$ can be calculated as

$$\mathrm{dtr}(\mathbf{X}) = \mathrm{tr}(\mathrm{d}\mathbf{X}) \tag{4.4}$$

Therefore, the derivative of $\mathrm{tr}(\mathbf{X})$ is

$$\mathrm{Dtr}(\mathbf{X}) = \mathbf{I} \tag{4.5}$$

Given a matrix $\mathbf{H}$, the differential with respect to $\mathbf{X}$ of the expression $\mathrm{d}\log|\mathbf{I} + \mathbf{H}\mathbf{X}\mathbf{H}^\dagger|$ can be calculated as

$$\mathrm{d}\log|\mathbf{I} + \mathbf{H}\mathbf{X}\mathbf{H}^\dagger| = \mathrm{tr}(\mathbf{H}^\dagger(\mathbf{I} + \mathbf{H}\mathbf{X}\mathbf{H}^\dagger)^{-1}\mathbf{H}\mathrm{d}\mathbf{X}) \tag{4.6}$$

Therefore, the derivative of the expression is

$$\mathrm{D}\log|\mathbf{I} + \mathbf{H}\mathbf{X}\mathbf{H}^\dagger| = \mathbf{H}^\dagger(\mathbf{I} + \mathbf{H}\mathbf{X}\mathbf{H}^\dagger)^{-1}\mathbf{H} \tag{4.7}$$

4.2 LOWER BOUND ON THE CAPACITY

Theorem 3.1 gives the DF achievable rate in terms of a max-min type optimization problem that still needs to be solved. The solution to this problem requires a joint optimization over the source and relay transmit covariance matrices. Because, the optimum $\mathbf{Q}_s$ that maximizes I_{mac} in (3.11) and the optimum $\mathbf{Q}_s$ that maximizes I_{sr} in (3.12) are different. If we maximize I_{mac}, that choice of $\mathbf{Q}_s$ will result in a low I_{sr}. As a result, I_{sr} will come out of the minimization in (3.10), and the achievable rate will attain a lower value. In the same way if we maximize I_{sr}, that choice of $\mathbf{Q}_s$ will result in a low I_{mac}. In order to solve this trade-off, $\mathbf{Q}_s$ and $\mathbf{Q}_r$ should be found jointly.

We utilize a method that is proposed in Liang et al. (2007). In this method, the following function R_{fl} of α and $\mathbf{Q}$ is defined as

$$R_{fl}(\alpha, \mathbf{Q}) = \alpha I_{mac}(\mathbf{Q}) + (1-\alpha) I_{sr}(\mathbf{Q}), \quad 0 \leq \alpha \leq 1 \tag{4.8}$$

where $\mathbf{Q} = [\mathbf{Q}_s \quad \mathbf{Q}_r]$. The max-min problem in (3.10) corresponds to first maximizing $R_{fl}(\alpha, \mathbf{Q})$ over $\mathbf{Q}$ for a fixed α, and then taking the minimum over α (Liang et al., 2007). It is important to note that Liang et al. (2007) applied this method for a different channel assumption, in particular when both the transmitters and the receivers know the channel state information. Under this assumption, Liang et al. (2007) solved the max-min problem. In this chapter, we apply the same method but since our channel state information assumption is different, the solution of the max-min problem is completely different, more complex and results in an iterative algorithm.

Let us define $\mathbf{V}_{fl}(\alpha)$ as $\mathbf{V}_{fl}(\alpha) = \max R(\alpha, \mathbf{Q})$ and suppose that α^* provides the minimum value of $\mathbf{V}_{fl}(\alpha)$. Depending on the value of α, we have three cases. Optimum source and relay covariance matrices may be different in all three cases. In the first case ($\alpha = 0$), $R_{fl}(\alpha, \mathbf{Q}) = I_{sr}(\mathbf{Q})$ and the condition $I_{mac}(\mathbf{Q}) \geq I_{sr}(\mathbf{Q})$ should be satisfied (Liang et al., 2007). Since the achievable rate is found by maximizing $I_{sr}(\mathbf{Q})$ only, we find the source transmit covariance matrix, $\mathbf{Q}_s$, as a solution to the point-to-point problem from source to relay. When the receiver knows perfect CSI and the transmitter knows partial CSI, point-to-point problem is already solved in Soysal & Ulukus (2007). Then, we find the relay transmit covariance matrix, $\mathbf{Q}_r$, by maximizing $I_{mac}(\mathbf{Q})$ with a fixed $\mathbf{Q}_s$. This is also equivalent to a single user problem which is solved in Soysal & Ulukus (2007), and also solved in previous chapter. Therefore we omitted the Case 1 here.

In the second case, ($\alpha = 1$), $R_{fl}(1, \mathbf{Q}) = I_{mac}(\mathbf{Q})$ and the condition $I_{mac}(\mathbf{Q}) \leq I_{sr}(\mathbf{Q})$ should be satisfied. In this case, the achievable rate is found by maximizing $I_{mac}(\mathbf{Q})$, which is a MAC problem. When the receiver knows perfect CSI and the transmitters know partial CSI, MIMO-MAC system is already solved in (Soysal & Ulukus, 2007).

In the third case, ($0 < \alpha^* < 1$), $R_{fl}(\alpha^*, \mathbf{Q}) = \alpha^* I_{mac}(\mathbf{Q}) + (1 - \alpha^*) I_{sr}(\mathbf{Q})$ and the condition $I_{mac}(\mathbf{Q}) = I_{sr}(\mathbf{Q})$ should be satisfied. In this case, we find the transmit covariance matrices of the source and relay as functions of α. The third case is the most interesting case as the solution is not trivial. In that case, $\mathbf{Q}_s$ and $\mathbf{Q}_r$ must be optimized jointly since objective function $R_{fl}(\alpha^*, \mathbf{Q})$ includes both I_{sr} and I_{mac}. However, this joint optimization problem cannot be solved by using the methods from the previous literature. In studies like Jafar & Goldsmith (2004) and Soysal & Ulukus (2007), the transmit covariance matrices are always found by determining their eigenvectors first. This reduces the problem of finding the eigenvalues of the transmit covariance matrix, from a matrix variable to a vector (and sometimes scalar) variable problem. Since the eigenvectors cannot be determined in closed form in this joint optimization, one needs to come up with another solution. It is always possible to solve this joint optimization problem using classical convex optimization methods (Boyd & Vanderberghe, 2004). Disadvantage of classical convex optimization methods is that they are very slow, and therefore cannot be used in real-time communications in a fast fading wireless environment. However, under certain assumptions on the channel, it might be possible to choose eigenvectors of the transmit covariance matrices cleverly and propose fast and efficient algorithms to find the eigenvalues. One such assumption is that source to destination link is weaker than the source to relay link. Therefore, source node chooses to transmit along the eigenvectors of the covariance of the source to relay channel, instead of the jointly optimal directions.

Jointly optimal directions are possibly a combination of the eigenvectors of the covariance of the source to relay channel, and those of the source to destination channel. In vague terms, the source node chooses to transmit towards the relay. Once the transmit directions of the source node is given, the transmit directions of the relay node can be found as the eigenvectors of the relay to destination channel as explained in the previous chapter (Aygun & Soysal, 2011). Having chosen the eigenvectors (i.e., transmit directions) of the source and relay transmit covariance matrices, next one can find the jointly optimum power values allocated along these transmit directions by modifying the methods previously offered in the literature. Clearly, this solution is suboptimal. Although we omit the details of this derivation, we will compare the performance of this solution to the optimum solution in the Numerical Results section. The optimal solution uses matrix differential calculus.

First, (4.8) will be maximized over $\mathbf{Q}$ for a fixed α^*, $0 < \alpha^* < 1$. Note that, transmit covariance matrices that will result from this optimization will depend on α^*.

$$\mathbf{V}_{fl}(\alpha^*) = \max_{\text{tr}(\mathbf{Q}_s)\leq P_s, \text{tr}(\mathbf{Q}_r)\leq P_r} (\alpha^* I_{mac}(\mathbf{Q}) + (1-\alpha^*) I_{sr}(\mathbf{Q})) \tag{4.9}$$

The Lagrangian of (4.9) can be written as

$$L = R_{fl}(\alpha^*, \mathbf{Q}) - \mu_s(tr(\mathbf{Q}_s) - P_s) - \mu_r(tr(\mathbf{Q}_r) - P_r) \tag{4.10}$$

where μ_s and μ_r are Lagrange multipliers corresponding to source and relay power constraints, respectively. Here, we will take the derivative of (6.77) with respect to $\mathbf{Q}_s$ and $\mathbf{Q}_r$ directly. By using matrix differential calculus and referring to the examples in Matrix Differential Calculus Section, one can take the derivative of (6.77) with respect to $\mathbf{Q}_s$ and $\mathbf{Q}_r$ to obtain the following KKT conditions

$$E\Big[\alpha^* \mathbf{H}_{sd}^\dagger (\mathbf{D}_{mac})^{-1} \mathbf{H}_{sd} + (1-\alpha^*) \mathbf{H}_{sr}^\dagger (\mathbf{D}_{sr})^{-1} \mathbf{H}_{sr}\Big] \leq \mu_s \mathbf{I} \tag{4.11}$$

$$E\Big[\alpha^* \mathbf{H}_{rd}^\dagger (\mathbf{D}_{mac})^{-1} \mathbf{H}_{rd}\Big] \leq \mu_r \mathbf{I} \tag{4.12}$$

where $\mathbf{D}_{mac}$ is the expression inside the determinant in (3.11) and $\mathbf{D}_{sr}$ is the expression inside the determinant in (3.12). Note that we omitted the complementary slackness conditions while writing KKT conditions. The KKT conditions in (4.11) and (4.12) are satisfied with equality when matrices $\mathbf{Q}_s$ and $\mathbf{Q}_r$ are positive definite, respectively. Otherwise, KKT conditions are satisfied with strict inequalities. In order to solve for $\mathbf{Q}_r$ and $\mathbf{Q}_s$, we need equalities. Therefore, we utilize the reasoning that is first introduced in (Soysal & Ulukus, 2007). Let us denote the left hand side of (4.11) as E_1 and the left hand side of (4.12) as E_2. We multiply both sides of (4.11) with $\mathbf{Q}_s$ from the right hand side and both sides of (4.12) with $\mathbf{Q}_r$ from the right hand side, we have

$$E_1 \mathbf{Q}_s = \mu_s \mathbf{Q}_s \tag{4.13}$$

$$E_2 \mathbf{Q}_r = \mu_r \mathbf{Q}_r \tag{4.14}$$

We note that when $\mathbf{Q}_s = 0$ and $\mathbf{Q}_r = 0$, both sides of (4.13) and (4.14) are zero. Therefore, unlike (4.11)-(4.12), (4.13)-(4.14) are always satisfied with equality for optimum transmit covariance matrices. By applying the trace operator, Lagrange multipliers are calculated as

$$\mu_s = \frac{\text{tr}(E_1\mathbf{Q})}{P_s}, \qquad \mu_r = \frac{\text{tr}(E_2\mathbf{Q})}{P_r}. \tag{4.15}$$

By substituting these μ_s and μ_r into (4.13) and (4.14), we find the fixed point equations which have to be satisfied by the optimum transmit covariance matrices

$$\mathbf{Q}_s = \frac{E_1\mathbf{Q}_s}{\text{tr}(E_1\mathbf{Q}_s)}P_s, \qquad \mathbf{Q}_r = \frac{E_2\mathbf{Q}}{\text{tr}(E_2\mathbf{Q}_r)}P_r. \tag{4.16}$$

We propose the following iterative algorithm to solve for the fixed point equations that are obtained from (4.13)-(4.14)

$$\mathbf{Q}_s(n+1) = \frac{E_1(n)\mathbf{Q}_s(n)}{\text{tr}(E_1(n)\mathbf{Q}_s(n))}P_s, \qquad \mathbf{Q}_r(n+1) = \frac{E_2(n)\mathbf{Q}(n)}{\text{tr}(E_2(n)\mathbf{Q}_r(n))}P_r. \tag{4.17}$$

This iterative algorithm finds the optimum transmit covariance matrices of the source and relay for Case 3. After running this algorithm for different α values, a minimization over α is performed in order to find the lower bound. It is important to note that the algorithm in (4.17) updates every element of the transmit covariance matrices at once. As mentioned before, the eigenvectors of the transmit covariance matrices were not determined beforehand, they are found implicitly after the algorithm in (4.17) converges. The convergence of the algorithm in (4.17) is an important issue. Due to mathematical complexity, convergence analysis of the algorithm seems intractable. However, we observe through numerous simulations that the algorithm converges irregardless of the initial points.

4.3 UPPER BOUND ON THE CAPACITY

Having derived the DF achievable rate and jointly optimized the source and transmitter covariance matrices, in this section we consider the cut-set upper bound. This bound is introduced in Cover & El Gamal (1979) and evaluated for different channel model assumptions in the literature. For example, when the receivers have perfect CSI and the transmitters have no CSI, cut-set upper bound on MIMO relay channel capacity is found in Wang et al. (2005). In this paper, we consider a case where there is transmit covariance information at the transmitters. In this case, similar to the lower-bound development, we first evaluate the mutual information expressions in the cut-set bound, and then optimize the upper bound over $\mathbf{Q}_s$ and $\mathbf{Q}_r$.

Theorem 4.1. *When there is only channel covariance information at the transmitters and perfect CSI at the receivers, cut-set upper bound of a full-duplex MIMO relay channel is given as*

$$C_{fd} \leq \max_{tr(\mathbf{Q}_s)\leq P_s, tr(\mathbf{Q}_r)\leq P_r} \min(I_{mac}, I_{bc}) \tag{4.18}$$

where

$$I_{mac} = E\left[\log\left|\mathbf{I} + \mathbf{H}_{sd}\mathbf{Q}_s\mathbf{H}_{sd}^{\dagger} + \mathbf{H}_{rd}\mathbf{Q}_r\mathbf{H}_{rd}^{\dagger}\right|\right] \tag{4.19}$$

$$I_{bc} = E\left[\log\left|\mathbf{I} + \mathbf{H}_{bc}\mathbf{Q}_s\mathbf{H}_{bc}^{\dagger}\right|\right] \tag{4.20}$$

Proof:

$$C_{upper} = \max_{p(\mathbf{x}_s,\mathbf{x}_r)} \min(I(\mathbf{x}_s, \mathbf{x}_r; \mathbf{y}), I(\mathbf{x}_s; \mathbf{r}, \mathbf{y}|\mathbf{x}_r)) \tag{4.21}$$

In that equality, the mutual information expressions can be written as

$$I(\mathbf{x}_s, \mathbf{x}_r; \mathbf{y}) = E\left[I(\mathbf{x}_s, \mathbf{x}_r; \mathbf{y}|\mathbf{H}_{sd}, \mathbf{H}_{rd})\right] \tag{4.22}$$

$$I(\mathbf{x}_s; \mathbf{r}, \mathbf{y}|\mathbf{x}_r) = E\left[I(\mathbf{x}_s; \mathbf{r}, \mathbf{y}|\mathbf{x}_r, \mathbf{H}_{sr}, \mathbf{H}_{sd})\right] \tag{4.23}$$

where x_s and x_r are circularly-symmetric complex Gaussian random vectors. The expectation in (4.22) is calculated in our previous chapter. The expectation in (4.23) is calculated in Wang et al. (2005) as

$$I(\mathbf{x}_s; \mathbf{r}, \mathbf{y}|\mathbf{x}_r) \leq \log\left[|\mathbf{I} + \mathbf{H}_{bc}\mathbf{Q}_s\mathbf{H}_{bc}^{\dagger}|\right] \tag{4.24}$$

where

$$\mathbf{H}_{bc} = \begin{bmatrix} \mathbf{H}_{sd} \\ \mathbf{H}_{sr} \end{bmatrix} \tag{4.25}$$

Finally, we insert the MAC expression and (4.24) into (4.21) and obtain (4.18). □

The proof of Theorem 3 is very similar to the proof of Theorem 1 and it is omitted here due to space restrictions and due to the fact that the contribution of the paper is not the evaluation of the upper bound expression but providing its solution. The proof basically calculates the cut-set upper bound with zero cross-correlation matrices. Note that the

DF achievable rate and the cut-set upper bound expressions both involve the same I_{mac}. Therefore, lower and upper bounds meet and provide the capacity if I_{mac} comes out of the minimization in both cases. As in the case of the lower bound, we have a max-min problem to solve in the upper bound as well. The method for this solution is similar to the lower bound solution and utilizes matrix differential calculus. We skip some of the development where it can easily be obtained from lower-bound analysis. This time, we define R_{fu} as

$$R_{fu}(\alpha, \mathbf{Q}) = \alpha I_{mac}(\mathbf{Q}) + (1-\alpha) I_{bc}(\mathbf{Q}), \quad 0 \leq \alpha \leq 1 \tag{4.26}$$

Note that unlike the DF achievable rate, the upper bound, R_{fu} depends on I_{bc}, not on I_{sr}. Depending on the value of minimum α, the solution again has three cases. In the first case ($\alpha^* = 0$), $R(0, \mathbf{Q}) = I_{bc}(\mathbf{Q})$ and the condition $I_{mac}(\mathbf{Q}) \geq I_{bc}(\mathbf{Q})$ should be satisfied. For this case, the Lagrangian can be written as

$$L = I_{bc}(\mathbf{Q}) - \mu_s \left(tr(\mathbf{A}\mathbf{A}^\dagger) - P_s \right) \tag{4.27}$$

Using matrix differential calculus, and by taking the derivative of (4.27) with respect to $\mathbf{Q}_s$, we obtain the KKT conditions. Then, similar to the lower bound we derive the following algorithm

$$\mathbf{Q}_s(n+1) = \frac{E_3(n)\mathbf{Q}_s(n)}{\text{tr}(E_3(n)\mathbf{Q}_s(n))} P_s \tag{4.28}$$

where $E_3 = E\left[\mathbf{H}_{bc}^\dagger (\mathbf{D}_{bc})^{-1} \mathbf{H}_{bc}\mathbf{A}\right]$, and $\mathbf{D}_{bc}$ is the matrix inside the determinant of I_{bc}. Next, $\mathbf{Q}_r$ is found by maximizing I_{mac} using fixed $\mathbf{Q}_s$ found above. This is equivalent to a single user problem that is solved in Soysal & Ulukus (2007).

The second case is again a MIMO-MAC channel and is already known. In the third case ($0 < \alpha^* < 1$), $R_{ub-fd}(\alpha^*, \mathbf{Q}) = \alpha^* I_{mac}(\mathbf{Q}) + (1-\alpha^*) I_{bc}(\mathbf{Q})$ and the condition $I_{mac}(\mathbf{Q}) = I_{bc}(\mathbf{Q})$ should be satisfied. The Lagrangian for this case is given as

$$L = R_{fu}(\alpha^*, \mathbf{Q}) - \mu_s \left(tr(\mathbf{A}\mathbf{A}^\dagger) - P_s \right) - \mu_r \left(tr(\mathbf{B}\mathbf{B}^\dagger) - P_r \right) \tag{4.29}$$

Using matrix differential calculus and by taking the derivative of (4.29) with respect to $\mathbf{Q}_s$ and $\mathbf{Q}_r$, we obtain the KKT conditions. Then, using the similar method as in the lower bound we derive the algorithm below

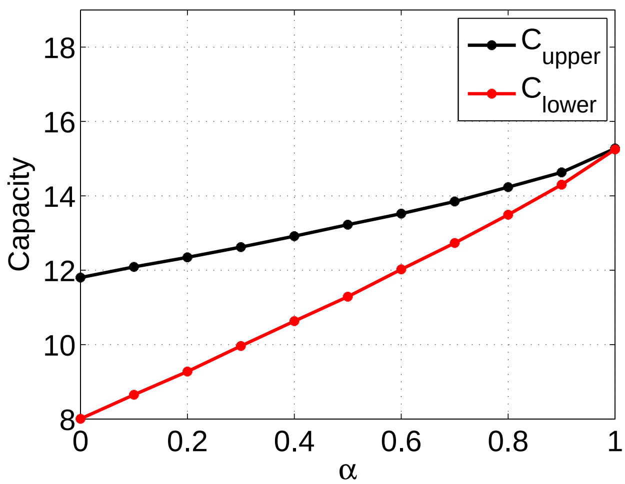
$$\mathbf{Q}_s(n+1) = \frac{E_4(n)\mathbf{Q}_s(n)}{\text{tr}(E_4(n)\mathbf{Q}_s(n))}P_s, \qquad \mathbf{Q}_r(n+1) = \frac{E_2(n)\mathbf{Q}_r(n)}{\text{tr}(E_2(n)\mathbf{Q}_r(n))}P_r \tag{4.30}$$

where $E_4 = E\left[\alpha^*\mathbf{H}_{sd}^\dagger(\mathbf{D}_{mac})^{-1}\mathbf{H}_{sd}\mathbf{A} + (1-\alpha^*)\mathbf{H}_{bc}^\dagger(\mathbf{D}_{bc})^{-1}\mathbf{H}_{bc}\mathbf{A}\right]$. This iterative algorithm finds the transmit covariance matrices of the source and relay nodes that solves the Case 3 of the optimization problem in the upper bound. Finally, a minimization over α is performed in order to find which case results in the upper bound.

4.4 NUMERICAL RESULTS

Capacity bounds on full-duplex MIMO relay channel are simulated using the proposed algorithms. Power constraints are chosen to be 10 dB for all cases. Figures 4.1, 4.2 and 4.3 give those bounds for different channel covariance matrices. In Figure 4.1, the covariance matrix provides Case 1 ($\alpha^* = 0$). It means the minimum points of capacity functions are at the point of $\alpha^* = 0$. For this channel matrix, the lower bound is 8 bit/sec and the upper bound is 14 bit/sec.

Figure 4.1: Full-duplex transmission capacity lower and upper bounds that result in $\alpha^* = 0$ at which point, both curves meet and give the capacity.



For the covariance matrix corresponding to Figure 4.2, lower and upper bounds are given by $\alpha^* = 1$ point (Case 2), which is the minimum value of the curves with respect to α. As expected, the lower bound is equal to upper bound at Case 2, and the capacity is in fact achieved for this covariance matrix setting.

Similarly, for the covariance matrix corresponding to Figure 4.3, lower and upper bounds are given by $\alpha^* = 0.9$ point (Case 3), which is the minimum value of the curves with respect to α. The difference between the lower and the upper bounds for this case is about 1%. Maximum difference between the bounds happens in Case 1, the point of $\alpha^* = 0$. At that point, the difference between the rates is 10%. Besides, we observe that (not shown in the figures) optimum transmit covariance matrices in the lower bound are almost the same as those in the upper bound for each case.

Figure 4.2: Full-duplex transmission capacity lower and upper bounds that result in $\alpha^* = 1$ at which point, both curves meet and give the capacity.

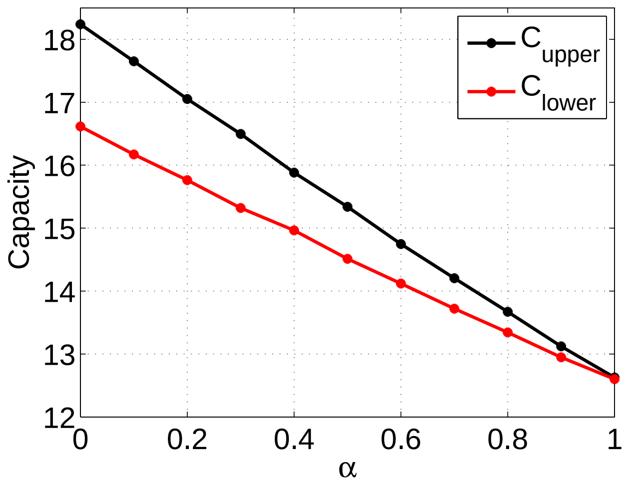
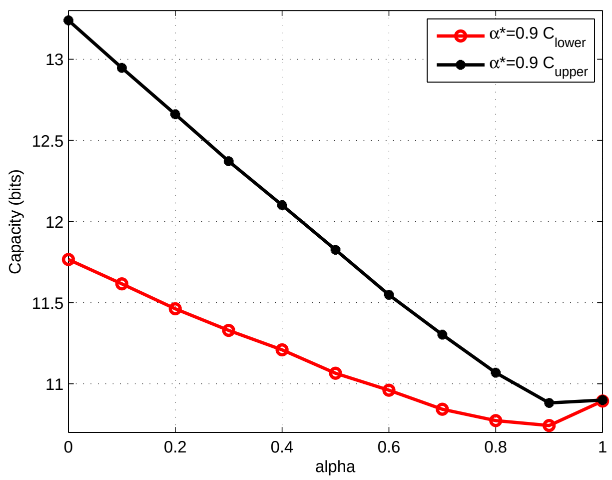


Figure 4.3: Full-duplex transmission capacity lower and upper bounds that result in $\alpha^* = 0.9$ at which point, both curves meet and give the capacity.



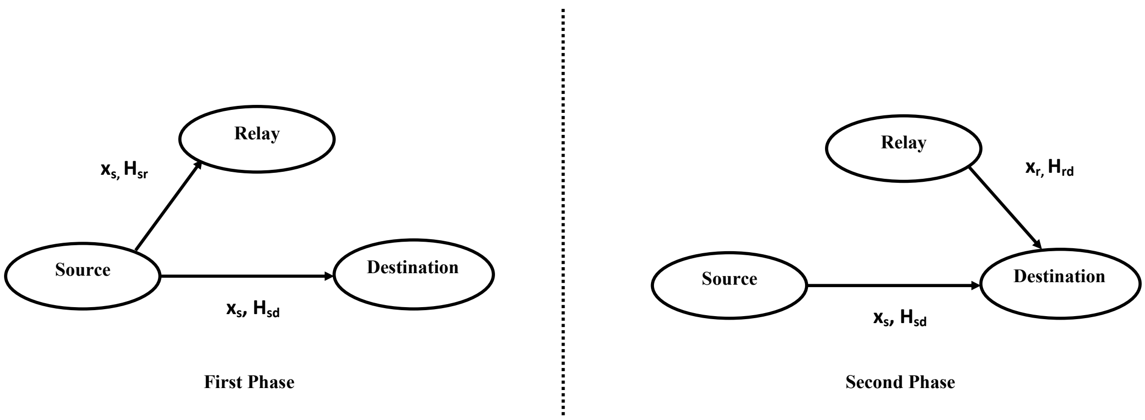
5. CAPACITY BOUNDS OF HALF DUPLEX MIMO RELAY CHANNEL

In Chapter 4, we considered full-duplex transmission where the relay was assumed to receive and transmit at the same time. However, it might be difficult to implement full-duplex transmission in practice. In this chapter, we consider a half-duplex transmission where the transmission block is divided into two phases. In the first phase, the relay receives the signal and in the second phase it transmits. The DF achievable rate and the cut-set upper bound are derived for half-duplex channels in Host-Madsen & Zhang (2005) and Liang et al. (2007) for single antenna systems and in Simoens et al. (2008) for MIMO systems. In this section, we generalize these bounds to the case where the transmitters have the covariance information of the channel. Then, we find the source and transmit covariance matrices that achieve those bounds.

5.1 SYSTEM MODEL

In half-duplex transmission, the relay cannot transmit and receive signals simultaneously. Therefore, one transmission frame is divided into two phases (Fig. 5.1).

Figure 5.1: Half-Duplex MIMO relay channel



Correspondingly, source input is also divided into two parts. In the first phase the relay behaves as a receiver only and the source transmits the first part of its input, ${\mathbf{x}_s}^{(1)}$. In this phase, the received signals at the relay and destination are

$$\mathbf{r} = \mathbf{H}_{sr}{\mathbf{x}_s}^{(1)} + \mathbf{n}_r, \qquad \mathbf{y}_1 = \mathbf{H}_{sd}{\mathbf{x}_s}^{(1)} + \mathbf{n}_y^{(1)} \tag{5.1}$$

where the covariance matrix of $\mathbf{x}_s^{(1)}$ is $\mathbf{Q}_s^{(1)} = E\left[\mathbf{x}_s^{(1)}{\mathbf{x}_s^{(1)}}^\dagger\right]$. In the second phase, the relay behaves as a transmitter. The source transmits the second part of its input, ${\mathbf{x}_s}^{(2)}$, and the relay transmits $\mathbf{x}_r$. In this phase, the received signal at the destination is

$$\mathbf{y}_2 = \mathbf{H}_{sd}\mathbf{x}_s^{(2)} + \mathbf{H}_{rd}\mathbf{x}_r + \mathbf{n}_y^{(2)} \tag{5.2}$$

where the covariance matrix of $\mathbf{x}_s^{(2)}$ is $\mathbf{Q}_s^{(2)} = E\left[\mathbf{x}_s^{(2)}{\mathbf{x}_s^{(2)}}^\dagger\right]$, and the noise vectors at the destination, $\mathbf{n}_y^{(1)}$ and $\mathbf{n}_y^{(2)}$ are zero-mean, identity covariance complex Gaussian random vectors.

5.2 LOWER BOUND ON THE CAPACITY

In DF half-duplex transmission, the relay listens to the source in the first phase, decodes the message and cooperates with the source in the second phase. Let us assume that the first phase has duration t, and the second phase has duration $1-t$, then we have the following theorem.

Theorem 5.1. *When there is only channel covariance information at the transmitters and perfect CSI at the receivers, DF achievable rate of a half-duplex MIMO relay channel is given as*

$$C_{hd} \geq \max_{\substack{t tr(\mathbf{Q}_s^{(1)})+(1-t)tr(\mathbf{Q}_s^{(2)})\leq P_s \\ (1-t)tr(\mathbf{Q}_r)\leq P_r \\ 1\geq t\geq 0}} \min(I_A, I_B) \tag{5.3}$$

$$I_A = tE\left[\log\left|\mathbf{I}+\mathbf{H}_{sr}\mathbf{Q}_s^{(1)}\mathbf{H}_{sr}^\dagger\right|\right] + (1-t)E\left[\log\left|\mathbf{I}+\mathbf{H}_{sd}\mathbf{Q}_s^{(2)}\mathbf{H}_{sd}^\dagger\right|\right] \tag{5.4}$$

$$I_B = tE\left[\log\left|\mathbf{I}+\mathbf{H}_{sd}\mathbf{Q}_s^{(1)}\mathbf{H}_{sd}^\dagger\right|\right] + (1-t)E\left[\log\left|\mathbf{I}+\mathbf{H}_{sd}\mathbf{Q}_s^{(2)}\mathbf{H}_{sd}^\dagger+\mathbf{H}_{rd}\mathbf{Q}_r\mathbf{H}_{rd}^\dagger\right|\right] \tag{5.5}$$

Proof: A general lower bound for half-duplex channels is given by Host-Madsen &

Zhang (2005)-Zhang & Duman (2007) as

$$C_{hd} \geq \min\left(I_A, I_B\right) \tag{5.6}$$

$$I_A = tE\left[I(\mathbf{x}_s^{(1)}; \mathbf{r}|\mathbf{x}_r = 0)\right] + (1-t)E\left[I(\mathbf{x}_s^{(2)}; \mathbf{y}^{(2)}|\mathbf{x}_r)\right] \tag{5.7}$$

$$I_B = tE\left[I(\mathbf{x}_s^{(1)}; \mathbf{y}|\mathbf{x}_r = 0)\right] + (1-t)E\left[I(\mathbf{x}_s^{(2)}, \mathbf{x}_r; \mathbf{y}^{(2)})\right] \tag{5.8}$$

Here, we will calculate the mutual information expressions for the system model in this paper. The first expression in I_A is the single user capacity from the source to the relay, while the second expression in I_A is the single user capacity from the source to the destination. The first expression in I_B is also the single user capacity from the source to the destination, while the second expression in I_B is the MAC capacity from source and relay to the destination. Since all these expressions are known, we can calculate them to get (5.4) and (5.5). Finally, the best lower bound is found by maximizing $\min(I_A, I_B)$ over power constraints and the time duration of the relay receive period. □

Theorem 4 defines the half-duplex DF achievable rate in terms of a max-min optimization problem. When the source to relay channel is better than the source to destination channel, half-duplex achievable rate is clearly less than the full-duplex achievable rate, as $I_A < I_{sr}$ and $I_B < I_{mac}$. Next, we will solve the optimization problem in (5.3) with the assumption that the relay transmit duration, t, is fixed. We analyze the effect of relay transmit duration in Numerical Results Section.

We use the same approach as in the full-duplex case. The following function R_{hl} of α and $\mathbf{Q}$ is defined as

$$R_{hl}(\alpha, \mathbf{Q}) = \alpha I_A(\mathbf{Q}) + (1-\alpha) I_B(\mathbf{Q}), \quad 0 \leq \alpha \leq 1 \tag{5.9}$$

Depending on the value of α^*, we have three cases. In the first case, When $\alpha^* = 1$, $R_{hl}(1, \mathbf{Q}) = I_A(\mathbf{Q})$ and $I_A(\mathbf{Q}) \leq I_B(\mathbf{Q})$ has to be satisfied (Liang et al., 2007). In that case, Lagrangian can be written as

$$L = I_A(\mathbf{Q}) - \mu_s\big(t\mathrm{tr}(\mathbf{Q}_s^{(1)}) + (1-t)\mathrm{tr}(\mathbf{Q}_s^{(2)}) - P_s\big) \tag{5.10}$$

Using matrix differential calculus and by taking the derivative of (5.10) with respect to $\mathbf{Q}_s^{(1)}$ and $\mathbf{Q}_s^{(2)}$, we obtain the following KKT conditions.

$$E_5 = E\left[\mathbf{H}_{sr}^{\dagger}(\mathbf{D}_k)^{-1}\mathbf{H}_{sr}\right] \leq \mu_s \mathbf{I}, \quad E_6 = E\left[\mathbf{H}_{sd}^{\dagger}(\mathbf{D}_l)^{-1}\mathbf{H}_{sd}\right] \leq \mu_s \mathbf{I} \tag{5.11}$$

where $\mathbf{D}_k$ is the inside of the determinant of the first expression in I_A and $\mathbf{D}_l$ is the inside of the determinant of the second expression in I_A. Then, using the same arguments as in the full-duplex mode, we derive the algorithm below

$$\mathbf{Q}_s^{(1)}(n+1) = \frac{E_5(n)\mathbf{Q}_s^{(1)}(n)^\dagger}{t\text{tr}(E_5(n)\mathbf{Q}_s^{(1)}(n)^\dagger) + (1-t)\text{tr}(E_6(n)\mathbf{Q}_s^{(2)}(n)^\dagger)} P_s \tag{5.12}$$

$$\mathbf{Q}_s^{(2)}(n+1) = \frac{E_6(n)\mathbf{Q}_s^{(2)}(n)^\dagger}{t\text{tr}(E_5(n)\mathbf{Q}_s^{(1)}(n)^\dagger) + (1-t)\text{tr}(E_6(n)\mathbf{Q}_s^{(2)}(n)^\dagger)} P_s. \tag{5.13}$$

After finding the source transmit covariance matrices, $\mathbf{Q}_r$ is calculated by maximizing I_B with source transmit covariance matrices fixed. This is equivalent to a single-user problem (Soysal & Ulukus, 2007).

In the second case, $\alpha^* = 0$, $R_{lb-hd}(0, \mathbf{Q}) = I_B(\mathbf{Q})$ and $I_A(\mathbf{Q}) \geq I_B(\mathbf{Q})$ has to be satisfied (Liang et al., 2007). In this case, Lagrangian can be written as

$$L = I_B(\mathbf{Q}) - \mu_s\big(t\text{tr}(\mathbf{Q}_s^{(1)}) + (1-t)\text{tr}(\mathbf{Q}_s^{(2)}) - P_s\big) - \mu_r\left((1-t)\text{tr}(\mathbf{Q}_r) - P_r\right) \tag{5.14}$$

Using matrix differential calculus and by taking the derivative of (5.14) with respect to $\mathbf{Q}_s^{(1)}$, $\mathbf{Q}_s^{(2)}$ and $\mathbf{Q}_r$, we obtain the KKT conditions.

$$E_7 = E\left[\mathbf{H}_{sd}^\dagger(\mathbf{D}_m)^{-1}\mathbf{H}_{sd}\right] \leq \mu_s \mathbf{I} \tag{5.15}$$

$$E_8 = E\left[\mathbf{H}_{sd}^\dagger(\mathbf{D}_n)^{-1}\mathbf{H}_{sd}\right] \leq \mu_s \mathbf{I} \tag{5.16}$$

$$E_9 = E\left[\mathbf{H}_{rd}^\dagger(\mathbf{D}_n)^{-1}\mathbf{H}_{rd}\right] \leq \mu_r \mathbf{I} \tag{5.17}$$

where $\mathbf{D}_m$ is the inside of the determinant of the first expression in I_B and $\mathbf{D}_n$ is the inside of the determinant of the second expression in I_B. Then, using the same arguments as before, we obtain the following algorithm

$$\mathbf{Q}_s^{(1)}(n+1) = \frac{E_7(n)\mathbf{Q}^{(1)}(n)}{t\text{tr}(E_7(n)\mathbf{Q}_s^{(1)}(n)) + (1-t)\text{tr}(E_8(n)\mathbf{Q}_s^{(2)}(n))} P_s \tag{5.18}$$

$$\mathbf{Q}_s^{(2)}(n+1) = \frac{E_8(n)\mathbf{Q}_s^{(2)}(n)}{t\text{tr}(E_7(n)\mathbf{Q}_s^{(1)}(n)) + (1-t)\text{tr}(E_8(n)\mathbf{Q}_s^{(2)}(n))} P_s \tag{5.19}$$

$$\mathbf{Q}_r(n+1) = \frac{E_9(n)\mathbf{Q}_r(n)^\dagger}{(1-t)\text{tr}(E_9(n)\mathbf{Q}_r(n)^\dagger)} P_r \tag{5.20}$$

In the third case, $0 < \alpha^* < 1$, $R_{hl}(\alpha, \mathbf{Q})$ is maximized with the condition that $I_A(\mathbf{Q}) =$

$I_B(\mathbf{Q})$ (Liang et al., 2007). The Lagrangian can be written as

$$L = R_{hl}(\alpha^*, \mathbf{Q}) - \mu_s\big(tr(\mathbf{Q}_s^{(1)}) + tr(\mathbf{Q}_s^{(2)}) - P_s\big) - \mu_r(tr(\mathbf{Q}_r) - P_r) \tag{5.21}$$

Using matrix differential calculus and by taking the derivative of (5.21) with respect to $\mathbf{Q}_s^{(1)}$, $\mathbf{Q}_s^{(2)}$ and $\mathbf{Q}_r$, we obtain the KKT conditions as

$$\alpha^* E_5 + (1-\alpha^*)E_7 \leq \mu_s \mathbf{Q}_s^{(1)}\mathbf{I} \tag{5.22}$$

$$\alpha^* E_6 + (1-\alpha^*)E_8 \leq \mu_s \mathbf{Q}_s^{(2)}\mathbf{I} \tag{5.23}$$

$$(1-\alpha^*)E_9 \leq \mu_r I_r \tag{5.24}$$

Using the same arguments as before, we obtain the following algorithm

$$\mathbf{Q}_s^{(1)}(n+1) = \frac{F_1(n)\mathbf{Q}_s^{(1)}(n)}{t\mathrm{tr}(F_1(n)\mathbf{Q}_s^{(1)}(n)) + (1-t)\mathrm{tr}(F_2(n)\mathbf{Q}_s^{(2)}(n))} P_s \tag{5.25}$$

$$\mathbf{Q}_s^{(2)}(n+1) = \frac{F_2(n)\mathbf{Q}_s^{(2)}(n)}{t\mathrm{tr}(F_1(n)\mathbf{Q}_s^{(1)}(n)) + (1-t)\mathrm{tr}(F_2(n)\mathbf{Q}_s^{(2)}(n))} P_s \tag{5.26}$$

$$\mathbf{Q}_r(n+1) = \frac{E_9(n)\mathbf{Q}_r(n)}{(1-t)\mathrm{tr}(E_9(n)\mathbf{Q}_r(n))} P_r \tag{5.27}$$

where $F_1 = \alpha^* E_5 + (1-\alpha^*)E_7$, and $F_2 = \alpha^* E_6 + (1-\alpha^*)E_8$.

Finally, after running these algorithms, we have to take a minimum over α and find the α^* that results in the minimum rate. As it can be seen, half-duplex algorithms are more complex than full-duplex algorithms, since they involve three transmit covariance matrices. None of the cases over α can be solved using previous point-to-point or MAC results.

5.3 UPPER BOUND ON THE CAPACITY

Having derived the DF achievable rate and jointly optimized the source and transmitter covariance matrices, in this section we consider the cut-set upper bound. This bound is introduced in Cover & El Gamal (1979) and evaluated for different channel model assumptions in the literature. For example, when the receivers have perfect CSI and the transmitters have no CSI, cut-set upper bound on MIMO relay channel capacity is found in Wang et al. (2005). In this chapter, we consider a case where there is transmit covariance information at the transmitters. In this case, similar to the lower-bound development, we first evaluate the mutual information expressions in the cut-set bound, and then optimize the upper bound over $\mathbf{Q}_s$ and $\mathbf{Q}_r$.

Theorem 5.2. *When there is only channel covariance information at the transmitters and perfect CSI at the receivers, cut-set upper bound of a half-duplex MIMO relay channel is given as*

$$C_{hu} \leq \max_{\substack{tr(\mathbf{Q}_s^{(1)})+(1-t)tr(\mathbf{Q}_s^{(2)})\leq P_s \\ (1-t)tr(\mathbf{Q}_r)\leq P_r \\ 1\geq t\geq 0}} \min(I_C, I_B) \tag{5.28}$$

where

$$I_C = tE\left[\log\left|\mathbf{I}+\mathbf{H}_{bc}\mathbf{Q}_s^{(1)}\mathbf{H}_{bc}^{\dagger}\right|\right] + (1-t)E\left[\log\left|\mathbf{I}+\mathbf{H}_{sd}\mathbf{Q}_s^{(2)}\mathbf{H}_{sd}^{\dagger}\right|\right] \tag{5.29}$$

and I_B is given in (5.5).

Proof: The cut-set upper bound is written below (Cover & El Gamal, 1979) (Host-Madsen & Zhang, 2005).

$$C \leq= \max_{p(\mathbf{x}_s,\mathbf{x}_r)} \min\left(I(\mathbf{x}_s;\mathbf{r},\mathbf{y}|\mathbf{x}_r), I(\mathbf{x}_s,\mathbf{x}_r;\mathbf{y})\right) \tag{5.30}$$

$$I(\mathbf{x}_s;\mathbf{r},\mathbf{y}|\mathbf{x}_r) = E\left[tI(\mathbf{x}_s^{(1)};\mathbf{r},\mathbf{y}|\mathbf{x}_r=0)\right] + E\left[(1-t)I(\mathbf{x}_s^{(2)};\mathbf{y}^{(2)}|\mathbf{x}_r)\right] \tag{5.31}$$

$$I(\mathbf{x}_s,\mathbf{x}_r;\mathbf{y}) = E\left[tI(\mathbf{x}_s^{(1)};\mathbf{y}|\mathbf{x}_r=0)\right] + E\left[(1-t)I(\mathbf{x}_s^{(2)},\mathbf{x}_r;\mathbf{y}^{(2)})\right] \tag{5.32}$$

First addend of the mutual information in (5.31) is calculated as

$$E\left[I(\mathbf{x}_s^{(1)};\mathbf{r},\mathbf{y}|\mathbf{x}_r=0)\right] \leq E\left[\log(\pi e)^{N_d+N_r}\left|Cov(\mathbf{H}_{bc}\mathbf{x}_s^{(1)})\right|\right] - \log(\pi e)^{N_d+N_r} \tag{5.33}$$

The second addend mutual information of (5.31) and both of addends mutual information of (5.32) are calculated at Theorem 3.1. Finally, we combine all four mutual information expression in (5.30)-(5.32) and obtain (5.28)-(5.29). □

5.4 NUMERICAL RESULTS

We start this section with a convergence analysis. This analysis is carried out for more complicated half-duplex case, similar results can be obtained for full-duplex case as well.

For all calculations, the power constraints (P_s and P_r) are fixed at 10 dB. At each iteration of the lower-bound and upper-bound algorithms, we calculate the matrix norms of transmit covariance matrices of the source and relay terminals. Then, in Figures 5.2 and 5.3, we plot the difference in matrix norms between successive iterations. We clearly see that as the iteration index increases, covariance matrices converge to their optimum values.

Figure 5.2: Convergence of the lower-bound algorithm for the half-duplex case.

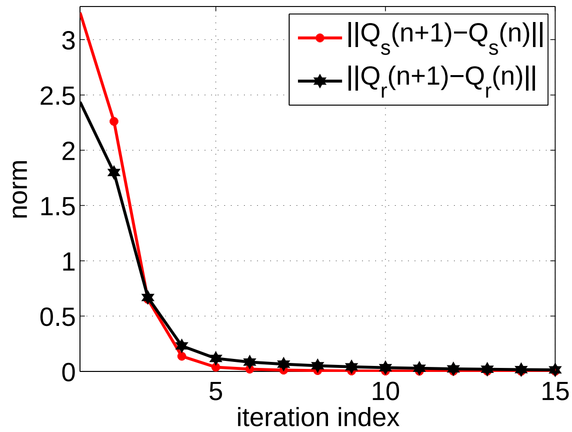
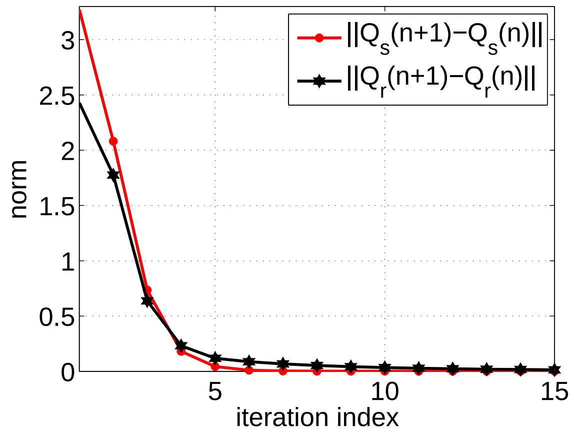


Figure 5.3: Convergence of the upper-bound algorithm for the half-duplex case.



In Figure 5.4, we obtain the plot for half-duplex scenario for which capacity is obtained when $\alpha = 0$. It means the minimum points of capacity functions are at the point of $\alpha = 0$. For Case 1, we can obtain exact capacity value since the lower and upper bounds are on the same point. The reason of it is the capacity expression is equal to $\mathbf{I}_{mac}$ for both bounds. For this channel parameters, the capacity is 7.5 bit/sec.

In Figure 5.5, we obtain the plot for $\alpha = 1$ since the minimum points of the capacity functions on the point of $\alpha = 1$. For this channel conditions, the lower bound is 7.1 bit/sec and the upper bound is 7.9 bit/sec.

The optimum α values for the lower and upper bounds turn out to be different in Figure 5.6 for Case 3. For the channel matrix, the lower bound on the capacity is the value at the point $\alpha = 0.7$. The upper bound on the capacity is at the point of $\alpha = 0.4$. Since the capacity values between those points are almost same, the capacity bounds can be obtained. For instance, the lower bound is 8.85 bit/sec and the upper bound is 8.98 bit/sec in Figure 5.6.

Figure 5.4: Half-duplex transmission capacity lower and upper bounds that result in $\alpha^* = 0$.

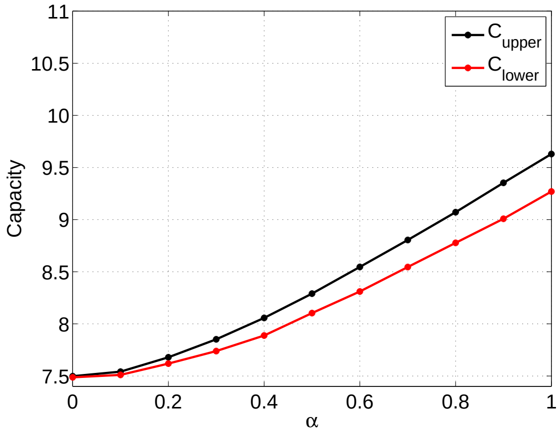
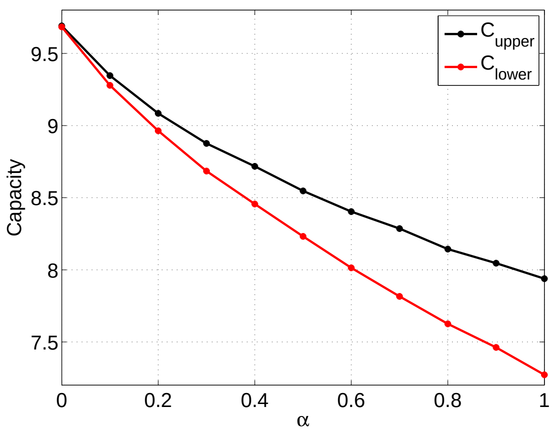


Figure 5.5: Half-duplex transmission capacity lower and upper bounds that result in $\alpha^* = 1$.



Finally, we compare the performance of the algorithm proposed in this paper that is based on matrix calculus to an algorithm that can be derived from previous chapter. In this second algorithm, transmit covariance matrices are decomposed into eigenvectors and eigenvalues. The eigenvectors are chosen cleverly but this choice is most probably not optimal. Then, only the eigenvalues are determined using an algorithm. In Figure 5.7, we clearly see that the algorithm proposed in this chapter out-performs the algorithm with fixed channel directions, especially at low SNR conditions. As SNR increases, we know from Figure 3.3 that Case 1 gives the lower bound. Since Case 1 results in a single-user solution, it is no surprise that two algorithms give the same lower-bound in high SNR scenario.

Figure 5.6: Half-duplex transmission capacity lower and upper bounds for Case 3.

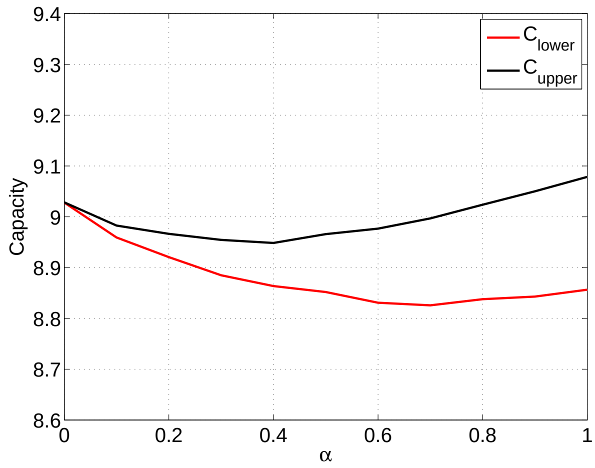
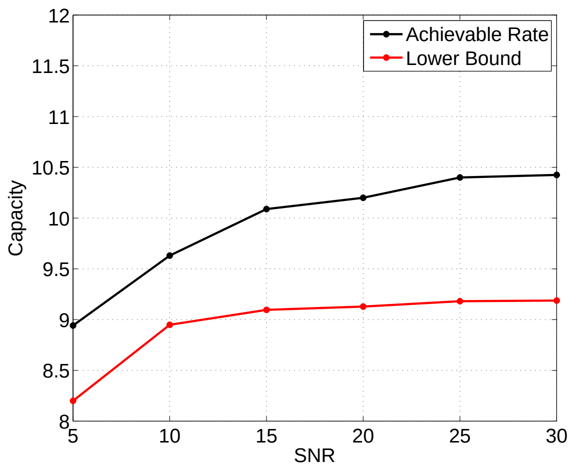


Figure 5.7: Comparison of the lower-bounds that are obtained by the algorithm with matrix calculus and by fixing the channel directions first and proposing an algorithm for the power values only.



6. CHANNEL ESTIMATION OF FULL DUPLEX MIMO RELAY CHANNELS

In MIMO transmission systems, the achievable rate of the system depends on the amount of CSI available at the receivers and transmitters. The CSI is estimated by the receiver and the receiver feeds the estimated CSI back to the transmitter. After the transmitter acquires the estimated CSI, it adapts its transmission scheme according to received CSI estimate. As a result, higher data rates can be obtained. In practice, the channel estimation is always noisy, and is not perfect due to round-trip delay, channel estimation error, codebook limitation, etc. Therefore, the feedback to the transmitter is limited.

Amount of the CSI can be figured out by sending a known training sequence from which the receiver measures the channel. The measured channel is used by the receiver in decoding the messages. The training process spends time and power.

In this chapter, we estimate the channel by sending a training sequence and optimize source and relay transmit covariance matrices jointly for decode-and-forward (DF) full duplex MIMO relay channels. The process includes two phases: the training phase and the data transmission phase.

The training phase have three parameters: the training signal, the training sequence, and the training sequence power. Similarly, the data transmission phase is characterized by the data carrying input signal, data transmission length and the data transmission power. The receiver uses linear MMSE detection to estimate the channel during the training phase. We first choose the training signal that minimizes the MMSE. This choice is increases the achievable rate of the data transmission phase (Hassibi & Hochwald, 2003). Then, the data transmission phase is started. We jointly optimize the achievable rate over the source and relay transmit covariance matrices. Finally, we provide simulation results.

6.1 SYSTEM MODEL

We consider a MIMO relay channel when the receivers have perfect CSI and the transmitters only have the transmit covariance information. The channel between a transmitter and a receiver is represented by a random matrix $\mathbf{H}_{xy}$ where x is the transmitter node and the y is the receiver node. The dimension of the channel matrix are the number of receive antennas times the number of transmitter antennas. The received signals at the relay and destination nodes for general MIMO relay channels are defined as

$$\mathbf{r} = \mathbf{H}_{sr}\mathbf{x}_s + \mathbf{n}_r \tag{6.1}$$

$$\mathbf{y} = \mathbf{H}_{sd}\mathbf{x}_s + \mathbf{H}_{rd}\mathbf{x}_r + \mathbf{n}_y \tag{6.2}$$

where $\mathbf{x}_s$ is an $M_s \times 1$ transmitted signal from the source node to the destination node and $\mathbf{x}_r$ is an $M_r \times 1$ transmitted signal from the relay node to the destination node. The covariance matrices of the transmitted signals are $\mathbf{Q}_s = E[\mathbf{x}_s\mathbf{x}_s^\dagger]$ and $\mathbf{Q}_r = E[\mathbf{x}_r\mathbf{x}_r^\dagger]$. The received signal at the destination node, $\mathbf{y}$, is $N_d \times 1$. The received signal at the relay node, $\mathbf{r}$, is $N_r \times 1$. The relay node is assumed to operate in full-duplex mode. The channel matrices, $\mathbf{H}_{sr}$, $\mathbf{H}_{sd}$ and $\mathbf{H}_{rd}$, are $N_r \times M_s$, $N_d \times M_s$ and $N_d \times M_r$ dimensional matrices. Noise vectors at the relay, $\mathbf{n}_r$, and at the destination, $\mathbf{n}_d$ are zero-mean, identity covariance complex Gaussian random vectors. The part of the system which includes both the direct channel and the channel from source to relay is called as broadcast channel. Both the channel from relay to destination and the direct channel is defined as multiple access channel.

The statistical model that we consider in this chapter, as in the previous chapters, is the partial CSI with covariance information at the transmitters. The channel is written as (Chuah et al., 2002)

$$\mathbf{H}_{xy} = \mathbf{Z}_{xy}\mathbf{\Sigma}_{xy}^{1/2} \tag{6.3}$$

where $\mathbf{Z}_{xy}$ is an identity covariance random channel matrix, $\mathbf{\Sigma}_{xy}$ is the correlation matrix between the signals transmitted from the antennas on the transmitter.

6.2 CHANNEL ESTIMATION AND LOWER BOUND ON THE CAPACITY

In the transmission model, a coherence interval, over which the channel is fixed, is divided into two phases: training phase and data transmission phase (Fig. 6.1). The transmitter uses P_t amount of power during the training phase, and P_d amount of power during the data transmission phase. We obtain $PT = P_tT_t + P_dT_d$ using the conservation of energy. In MIMO relay channel with partial CSI with only covariance feedback and channel esti-

Figure 6.1: A single coherence time, over which the channel is fixed

mation error at the receiver, the joint optimization problem is to maximize the achievable rate of the data transmission phase. The data rate depends on the channel estimation parameters: training signal, $\mathbf{S}_s$ for the source node and $\mathbf{S}_r$ for the relay node, training signal power, P_{ts} for the source node and P_{tr} for the relay node, and the training signal duration, T_t for the source node and T_t for the relay node. Therefore, we need to jointly optimize the achievable rate over these parameters and data transmission phase parameters. A longer training phase provide better channel estimation and larger achievable rate during the data transmission phase, since the channel estimation error contributes the effective noise. However, channel resources, time and power, are used during the training phase. Since the block length and total power are fixed by increased training phase duration, the duration and power of the data transmission phase becomes shorter. The result of this, the achievable rate decreases. In this thesis, we solve these trade-offs, and find the optimum training and data transmission parameters.

Firstly, the channel estimation process during the training phase is considered and were chosen the optimum training signals which minimize the channel estimation error, then, the data transmission phase is considered and the lower bound on the capacity is developed.

6.3 TRAINING AND CHANNEL ESTIMATION PHASE

In practice, the channel is estimated at the receiver. A way to channel estimation is to send the training symbols before the data transmission phase. The receiver estimates the channel by using the known training signals and the output of the channel. Since the channel is fixed during the entire block, the input-output relationship during the training phase in matrix form is written as

$$\mathbf{R} = \mathbf{H}_{sr}\mathbf{S}_s + \mathbf{N}_r \qquad \mathbf{Y} = \mathbf{H}_{sd}\mathbf{S}_s + \mathbf{H}_{rd}\mathbf{S}_r + \mathbf{N}_y \tag{6.4}$$

where $\mathbf{S}_s$ and $\mathbf{S}_r$ are $M_s \times T_t$ dimensional training signals that will be chosen and known at both the transmitter and the receiver, $\mathbf{R}$ and $\mathbf{N}_r$ are $N_r \times T_t$ dimensional received signal and noise matrices at the relay node, $\mathbf{Y}$ and $\mathbf{N}_y$ are $N_d \times T_t$ dimensional received signal and noise matrices at the destination node, respectively. The power constraint for the training input signal are $\frac{1}{T_t}\text{tr}(\mathbf{S}_s\mathbf{S}_s^\dagger) \leq P_{ts}$ for the relay node and $\frac{1}{T_t}\text{tr}(\mathbf{S}_r\mathbf{S}_r^\dagger) \leq P_{tr}$ for the destination node (Soysal & Ulukus, 2010a,b).

Due to the channel model in (6.3), the entries in a row of $\mathbf{H}_{xy}$ are correlated, and the entries in a column of $\mathbf{H}_{xy}$ are uncorrelated. Therefore, row i of $\mathbf{H}_{xy}$ is $\mathbf{h}^{xy\dagger}_{i}$, with $E[\mathbf{h}_i^{xy}\mathbf{h}^{xy\dagger}_{i}] = \mathbf{\Sigma}_{xy}$. The row vector of $\mathbf{H}_{sr}$ is $\mathbf{h}_i^{sr}$ and the row vector of $\mathbf{H}_{mac}$ is shown as

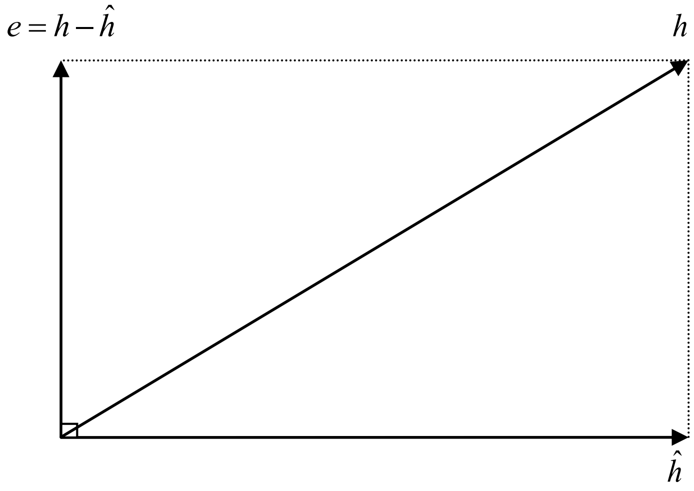
$$\bar{\mathbf{h}}_i^{mac} = \begin{bmatrix} \mathbf{h}_i^{sd} \\ \mathbf{h}_i^{rd} \end{bmatrix} \tag{6.5}$$

The channel covariance matrices are $\mathbf{\Sigma}_{sr} = E\left[\mathbf{h}_i^{sr}\mathbf{h}_i^{sr\dagger}\right]$ for source to relay link and $\bar{\mathbf{\Sigma}}_{mac} = E\left[\bar{\mathbf{h}}_i^{mac}\bar{\mathbf{h}}_i^{mac\dagger}\right] = diag\{\mathbf{\Sigma}_{sd}, \mathbf{\Sigma}_{rd}\}$ for multi access channel. Since the rows are independent identical distribution (i.i.d.), the receiver can estimate each of them independently using the same training signal. Re-writing (6.4), we get

$$\mathbf{r}_{ti} = \mathbf{S}_s^\dagger\mathbf{h}_i^{sr} + \mathbf{n}_{ti}^r \qquad \mathbf{y}_{ti} = \bar{\mathbf{S}}^\dagger\bar{\mathbf{h}}_i^{mac} + \mathbf{n}_{ti}^y \tag{6.6}$$

The receiver estimates the channel vector, $\mathbf{h}_i^{sr}$ and $\bar{\mathbf{h}}_i^{mac}$, using the received signal, $\mathbf{r}_{ti}$ and $\mathbf{y}_{ti}$, and the training signal, $\mathbf{S}_s$ and $\mathbf{S}_r$. In this thesis, the estimated channel vector, $\hat{\mathbf{h}}_i^{sr}$ and $\hat{\mathbf{h}}_i^{mac}$, is set to linear Minimum Mean Square Error (MMSE) estimation. The reason to use linear MMSE estimation is that it is optimal when the random variables involved in the estimation are Gaussian. To find the linear MMSE estimator, the following optimization problem is solved with $\hat{\mathbf{h}}_i^{sr} = \mathbf{M}_{sr}\mathbf{r}_{ti}$ and $\hat{\mathbf{h}}_i^{mac} = \bar{\mathbf{M}}^{mac}\mathbf{y}_{ti}$ as the estimates of $\mathbf{h}_i^{sr}$ and $\mathbf{h}_i^{mac}$, and $\tilde{\mathbf{h}}_i^{sr} = \mathbf{h}_i^{sr} - \hat{\mathbf{h}}_i^{sr}$ and $\tilde{\mathbf{h}}_i^{mac} = \bar{\mathbf{h}}_i^{mac} - \hat{\mathbf{h}}_i^{mac}$ as the channel estimation errors. As shown in Figure 6.2, the channel estimate vector, $\hat{\mathbf{h}}_i^{xy}$, is perpendicular to the channel estimation error vector, $\tilde{\mathbf{h}}_i^{xy}$. The optimization problem is written as

Figure 6.2: Spatial illustration of channel vectors



$$\min_{\mathbf{M}_{sr}} E\left[\tilde{\mathbf{h}}_i^{sr}\tilde{\mathbf{h}}_i^{sr\dagger}\right] = \min_{\mathbf{M}} E\left[\text{tr}\left(\tilde{\mathbf{h}}_i^{sr}\tilde{\mathbf{h}}_i^{sr\dagger}\right)\right] \quad (6.7)$$

$$= \min_{\bar{\mathbf{M}}_{sr}} E\left[\text{tr}\left(\left(\mathbf{h}_i^{sr} - \mathbf{M}_{sr}\mathbf{r}_{ti}\right)\left(\mathbf{h}_i^{sr} - \mathbf{M}_{sr}\mathbf{r}_{ti}\right)^\dagger\right)\right] \quad (6.8)$$

$$\min_{\mathbf{M}} E\left[\tilde{\mathbf{h}}_i^{mac}\tilde{\mathbf{h}}_i^{mac\dagger}\right] = \min_{\mathbf{M}} E\left[\text{tr}\left(\tilde{\mathbf{h}}_i^{mac}\tilde{\mathbf{h}}_i^{mac\dagger}\right)\right] \quad (6.9)$$

$$= \min_{\mathbf{M}} E\left[\text{tr}\left(\left(\mathbf{h}_i^{mac} - \bar{\mathbf{M}}_{mac}\mathbf{y}_{ti}\right)\left(\mathbf{h}_i^{mac} - \bar{\mathbf{M}}_{mac}\mathbf{y}_{ti}\right)^\dagger\right)\right] \quad (6.10)$$

Solving optimum transformation matrices $\mathbf{M}_{sr}^*$ and $\bar{\mathbf{M}}_{mac}^*$ from these equal to solve $\mathbf{M}_{sr}^*$ and $\bar{\mathbf{M}}_{mac}^*$ from the orthogonality principle for vector random variables, shown as (Kamen & Su, 1999).

$$E\left[(\tilde{\mathbf{h}}_i^{sr} - \mathbf{M}_{sr}^*\mathbf{r}_{ti})\mathbf{r}_{ti}^\dagger\right] = \mathbf{0} \quad (6.11)$$

$$E\left[(\tilde{\mathbf{h}}_i^{mac} - \bar{\mathbf{M}}_{mac}^*\mathbf{y}_{ti})\mathbf{y}_{ti}^\dagger\right] = \mathbf{0} \quad (6.12)$$

where $\mathbf{0}$ is the zero matrix. The optimum transformation matrices are solved as

$$\mathbf{M}_{sr}^* = E\left[\mathbf{h}_i^{sr}\mathbf{r}_{ti}^\dagger\right]\left(E\left[\mathbf{r}_{ti}\mathbf{r}_{ti}^\dagger\right]\right)^{-1} \quad (6.13)$$

$$\bar{\mathbf{M}}_{mac}^* = E\left[\bar{\mathbf{h}}_i^{mac}\mathbf{y}_{ti}^\dagger\right]\left(E\left[\mathbf{y}_{ti}\mathbf{y}_{ti}^\dagger\right]\right)^{-1} \quad (6.14)$$

Using the expressions in (6.6), we obtain $E\left[\mathbf{h}_i^{sr}\mathbf{r}_i^\dagger\right] = \mathbf{\Sigma}_{sr}\mathbf{S}_s$, $E\left[\mathbf{r}_i\mathbf{r}_i^\dagger\right] = \mathbf{S}_s\mathbf{\Sigma}_{sr}\mathbf{S}_s^\dagger + \mathbf{I}$, $E\left[\bar{\mathbf{h}}_i^{mac}\mathbf{y}_i^\dagger\right] = \bar{\mathbf{\Sigma}}_{mac}\bar{\mathbf{S}}$, $E\left[\mathbf{y}_i\mathbf{y}_i^\dagger\right] = \bar{\mathbf{S}}\bar{\mathbf{\Sigma}}_{mac}\bar{\mathbf{S}}^\dagger + \mathbf{I}$. Then, the transformation matrices,

$\mathbf{M}_{sr}^*$ and $\bar{\mathbf{M}}_{mac}^*$, are used in (6.7) and (6.9). We have,

$$\min_{\mathbf{M}} E\left[\tilde{\mathbf{h}}_i^{sr}\tilde{\mathbf{h}}_i^{sr\dagger}\right] = \text{tr}(\boldsymbol{\Sigma}_{sr} - \boldsymbol{\Sigma}_{sr}\mathbf{S}_s(\mathbf{S}_s\boldsymbol{\Sigma}_{sr}\mathbf{S}_s^\dagger + \mathbf{I})^{-1}\mathbf{S}_s\boldsymbol{\Sigma}_{sr}) \tag{6.15}$$

$$= \text{tr}((\boldsymbol{\Sigma}_{sr}^{-1} + \mathbf{S}_s\mathbf{S}_s^\dagger)^{-1}) \tag{6.16}$$

$$\min_{\mathbf{M}} E\left[\tilde{\mathbf{h}}_i^{mac}\tilde{\mathbf{h}}_i^{mac\dagger}\right] = \text{tr}(\bar{\boldsymbol{\Sigma}}_{mac} - \bar{\boldsymbol{\Sigma}}_{mac}\bar{\mathbf{S}}(\bar{\mathbf{S}}\bar{\boldsymbol{\Sigma}}_{mac}\bar{\mathbf{S}}^\dagger + \mathbf{I})^{-1}\bar{\mathbf{S}}\bar{\boldsymbol{\Sigma}}_{mac}) \tag{6.17}$$

$$= \text{tr}((\bar{\boldsymbol{\Sigma}}_{mac}^{-1} + \bar{\mathbf{S}}\bar{\mathbf{S}}^\dagger)^{-1}) \tag{6.18}$$

where the results follow from the matrix inversion lemma (Horn & Johnson, 1985). The mean square error of the channel estimation process can be further decreased by choosing the training signals $\mathbf{S}_s$ and $\mathbf{S}_r$ to minimize (6.16) and (6.18). Furthermore, the training signals affect the achievable rate (Hassibi & Hochwald, 2003).

6.4 DATA TRANSMISSION PHASE

When the CSI at the receiver noisy, the optimum input signalling that achieves the capacity is unknown. Using the channel estimation error, $\tilde{\mathbf{H}} = \mathbf{H} - \hat{\mathbf{H}}$, the channel models are written as

$$\mathbf{r} = \hat{\mathbf{H}}_{sr}\mathbf{x}_s + \tilde{\mathbf{H}}_{sr}\mathbf{x}_s + \mathbf{n}_r \tag{6.19}$$

$$\mathbf{y} = \hat{\mathbf{H}}_{sd}\mathbf{x}_s + \tilde{\mathbf{H}}_{sd}\mathbf{x}_s + \hat{\mathbf{H}}_{rd}\mathbf{x}_r + \tilde{\mathbf{H}}_{rd}\mathbf{x}_r + \mathbf{n}_y \tag{6.20}$$

where $\mathbf{x}_s$ and $\mathbf{x}_r$ are the information carrying input, $\mathbf{n}^r$ and $\mathbf{n}^y$ are zero-mean, identity covariance complex Gaussian vectors. The effective noise of source to relay and MAC links are

$$\mathbf{R}_{sr} = \mathbf{I} + E\left[\tilde{\mathbf{H}}_{sr}\mathbf{Q}_s\tilde{\mathbf{H}}_{sr}^\dagger\right] \tag{6.21}$$

$$\mathbf{R}_{mac} = \mathbf{I} + E\left[\tilde{\mathbf{H}}_{sd}\mathbf{Q}_s\tilde{\mathbf{H}}_{sd}^\dagger + \tilde{\mathbf{H}}_{rd}\mathbf{Q}_r\tilde{\mathbf{H}}_{rd}^\dagger\right] \tag{6.22}$$

By denoting each row of $\tilde{\mathbf{H}}_{xy}$ as $\tilde{\mathbf{h}}_i^{xy\dagger}$, we can write the $(i,j)^{th}$ entry of effective noises as

$$E\left[\tilde{\mathbf{h}}_i^{sr\dagger}\mathbf{Q}\tilde{\mathbf{h}}_j^{sr\dagger}\right] = \text{tr}\left(\mathbf{Q}E\left[\tilde{\mathbf{h}}_i^{sr}\tilde{\mathbf{h}}_j^{sr\dagger}\right]\right) \tag{6.23}$$

$$= \begin{cases} \text{tr}\left(\mathbf{Q}_s\tilde{\mathbf{\Sigma}}_{sr}\right) & , \quad \text{when} \quad i=j \\ 0 & , \quad \text{when} \quad i \neq j \end{cases} \tag{6.24}$$

$$E\left[\tilde{\mathbf{H}}_{sd}\mathbf{Q}_s\tilde{\mathbf{H}}_{sd}^\dagger + \tilde{\mathbf{H}}_{rd}\mathbf{Q}_r\tilde{\mathbf{H}}_{rd}^\dagger\right] = \text{tr}\left(\mathbf{Q}_sE\left[\tilde{\mathbf{h}}_i^{sr}\tilde{\mathbf{h}}_j^{sr\dagger}\right]\right) + \text{tr}\left(\mathbf{Q}_rE\left[\tilde{\mathbf{h}}_i^{rd}\tilde{\mathbf{h}}_j^{rd\dagger}\right]\right) \tag{6.25}$$

$$= \begin{cases} \text{tr}\left(\mathbf{Q}_s\tilde{\mathbf{\Sigma}}_{sd}\right) + \text{tr}\left(\mathbf{Q}_r\tilde{\mathbf{\Sigma}}_{rd}\right) & , \quad \text{when} \quad i=j \\ 0 & , \quad \text{when} \quad i \neq j \end{cases} \tag{6.26}$$

where the transmit covariance matrices are $\mathbf{Q}_s = E[\mathbf{x}_s\mathbf{x}_s^\dagger]$ and $\mathbf{Q}_r = E[\mathbf{x}_r\mathbf{x}_r^\dagger]$ and their average power constraints during data transmission phase are tr$(\mathbf{Q}_s) \leq P_{ds}$ and tr$(\mathbf{Q}_r) \leq P_{dr}$.The lower bound on the capacity with channel estimation error is (Yoo & Goldsmith, 2006; Wang et al., 2005)

$$C \geq I_{lower} = \max_{\substack{\text{tr}(\mathbf{Q}_s)\leq P_{ds} \\ \text{tr}(\mathbf{Q}_r)\leq P_{dr}}} \min(C_{sr-est}, C_{mac-est}) \tag{6.27}$$

$$C_{sr-est} = E\left[\log\left|\mathbf{I} + \frac{\hat{\mathbf{H}}_{sr}\mathbf{Q}_s\hat{\mathbf{H}}_{sr}^\dagger}{\mathbf{I} + \tilde{\mathbf{H}}_{sr}\mathbf{Q}_s\tilde{\mathbf{H}}_{sr}^\dagger}\right|\right] \tag{6.28}$$

$$C_{mac-est} = E\left[\log\left|\mathbf{I} + \frac{\hat{\mathbf{H}}_{sd}\mathbf{Q}_s\hat{\mathbf{H}}_{sd}^\dagger + \hat{\mathbf{H}}_{rd}\mathbf{Q}_r\hat{\mathbf{H}}_{rd}^\dagger}{\mathbf{I} + \tilde{\mathbf{H}}_{sd}\mathbf{Q}_s\tilde{\mathbf{H}}_{sd}^\dagger + \tilde{\mathbf{H}}_{rd}\mathbf{Q}_r\tilde{\mathbf{H}}_{rd}^\dagger}\right|\right] \tag{6.29}$$

Since our goal is to find the largest such achievable rate, the rate maximization problem over the entire block becomes

$$C \geq I_{lower} = \max_{\substack{(\mathbf{Q},P_t,T_t)\epsilon S \\ \text{tr}(\mathbf{Q}_s)\leq P_{ds},\text{tr}(\mathbf{Q}_r)\leq P_{dr}}} \min(I_{sr-est}, I_{mac-est}) \tag{6.30}$$

$$I_{sr-est} = \frac{T-T_t}{T}E\left[\log\left|\mathbf{I} + \frac{\hat{\mathbf{H}}_{sr}\mathbf{Q}_s\hat{\mathbf{H}}_{sr}^\dagger}{1 + \text{tr}\left(\mathbf{Q}_s\tilde{\mathbf{\Sigma}}_{sr}\right)}\right|\right] \tag{6.31}$$

$$I_{mac-est} = \frac{T-T_t}{T}E\left[\log\left|\mathbf{I} + \frac{\hat{\mathbf{H}}_{sd}\mathbf{Q}_s\hat{\mathbf{H}}_{sd}^\dagger + \hat{\mathbf{H}}_{rd}\mathbf{Q}_r\hat{\mathbf{H}}_{rd}^\dagger}{1 + \text{tr}\left(\mathbf{Q}_s\tilde{\mathbf{\Sigma}}_{sd}\right) + \text{tr}\left(\mathbf{Q}_r\tilde{\mathbf{\Sigma}}_{rd}\right)}\right|\right] \tag{6.32}$$

where $S = \{(\mathbf{Q}_s, P_{ts}, T_t), (\mathbf{Q}_r, P_{tr}, T_t)|\text{tr}(\mathbf{Q}_s) + \text{tr}(\mathbf{Q}_r) + P_{ts}T_t + P_{ts}T_t = PT\}$ and $\frac{T-T_t}{T}$ reflects the amount of time spent during the training phase. The maximization is over the training parameters and the data transmission parameters.

As in the previous chapters, a max-min type optimization problem needs to be solved. The solution to this problem requires a joint optimization over the source and relay transmit covariance matrices. We utilize a method that is proposed in Liang et al. (2007). In this method, the following function R of α and $\mathbf{Q}$ is defined as

$$R(\alpha, \mathbf{Q}) = \alpha I_{mac-est}(\mathbf{Q}) + (1-\alpha) I_{sr-est}(\mathbf{Q}), \quad 0 \leq \alpha \leq 1 \tag{6.33}$$

where $\mathbf{Q} = [\mathbf{Q}_s \quad \mathbf{Q}_r]$. The max-min problem in (6.30) corresponds to first maximizing $R(\alpha, \mathbf{Q})$ over $\mathbf{Q}$ for a fixed α, and then taking the minimum over α (Liang et al., 2007). Let us define $\mathbf{V}(\alpha)$ as $\mathbf{V}(\alpha) = \max R(\alpha, \mathbf{Q})$ and suppose that α^* provides the minimum value of $\mathbf{V}(\alpha)$. Depending on the value of α, we have three cases. Optimum source and relay covariance matrices may be different in all three cases.

Case 1: In the first case ($\alpha = 0$), $R(\alpha, \mathbf{Q}) = I_{sr-est}(\mathbf{Q})$ and the condition $I_{mac-est}(\mathbf{Q}) \geq I_{sr-est}(\mathbf{Q})$ should be satisfied (Liang et al., 2007). Since the achievable rate is found by maximizing $I_{sr-est}(\mathbf{Q})$ only, we find the source training signal, $\mathbf{S}_s$, the source transmit covariance matrix, $\mathbf{Q}_s$, as a solution to the point-to-point problem from source to relay. Then, we find the relay training signal, $\mathbf{S}_r$, and the relay transmit covariance matrix, $\mathbf{Q}_r$, by maximizing $I_{mac-est}(\mathbf{Q})$ with a fixed $\mathbf{Q}_s$ and $\mathbf{S}_s$. We have

$$I_{sr-est} = \max_{\{\mathbf{Q}_s, P_{ds}, T_d\}} \frac{T_d}{T} E\left[\log\left|\mathbf{I} + \frac{\hat{\mathbf{H}}_{sr}\mathbf{Q}_s\hat{\mathbf{H}}_{sr}^{\dagger}}{1+\text{tr}(\mathbf{Q}_s\tilde{\mathbf{\Sigma}}_{sr})}\right|\right] \tag{6.34}$$

where the power constraint is $P_{ds}T_d + P_{ts}T_t = PT$. The parameter of T_d is discrete valued, and P_{ds} and $\mathbf{Q}_s$ are continuous valued. Since for every value of T_d both the coefficient in front of the expectation and the number of terms in the sum in the numerator of (6.34) are different, the form of the objective function is different. Since T_d is discrete, and $1 \leq T_d \leq M_s$, we can perform an exhaustive search over T_d and solve M_s reduced optimization problems with fixed T_d in each one. Then, we take the solution that results in the maximum rate

$$I_{sr-est} = \max_{1\leq T_d \leq M_s} \max_{\{\mathbf{Q}_s, P_{ds}\}} \frac{T_d}{T} E\left[\log\left|\mathbf{I} + \frac{\hat{\mathbf{H}}_{sr}\mathbf{Q}_s\hat{\mathbf{H}}_{sr}^{\dagger}}{1+\text{tr}(\mathbf{Q}_s\tilde{\mathbf{\Sigma}}_{sr})}\right|\right] \tag{6.35}$$

While solving the inner maximization problem we define the expression for all T_d, $1 \leq$

$T_d \le M_s$. Therefore, the optimization problem is

$$\max_{\{\mathbf{Q}_s, P_{ds}\}} \frac{T_d}{T} E\left[\log\left|\mathbf{I} + \frac{\hat{\mathbf{H}}_{sr}\mathbf{Q}_s\hat{\mathbf{H}}_{sr}^{\dagger}}{1+\text{tr}(\mathbf{Q}_s\tilde{\boldsymbol{\Sigma}}_{sr})}\right|\right] \tag{6.36}$$

where the convergence of the energy provides $P_{ds}T_d = \text{tr}(\mathbf{Q}_s)T_d + \text{tr}(\mathbf{S}_s\mathbf{S}_s^{\dagger})$. The Lagrangian of the capacity expression is written as

$$L = \frac{T_d}{T} E\left[\log\left|\mathbf{I} + \frac{\hat{\mathbf{H}}_{sr}\mathbf{Q}_s\hat{\mathbf{H}}_{sr}^{\dagger}}{1+\text{tr}(\mathbf{Q}_s\tilde{\boldsymbol{\Sigma}}_{sr})}\right|\right] - \mu_s(\text{tr}(\mathbf{Q}_s)T_d + \text{tr}(\mathbf{S}\mathbf{S}^{\dagger}) - P_{ds}T_d) \tag{6.37}$$

where μ_s is Lagrangian multiplier. To differentiate the Lagrangian, the matrix differential rules which are indicated in previous chapters are applied. Adding to them, we utilize the partial differential rule (Magnus & Neudecker, 1999, pg167).

$$\text{d}\left(\frac{\mathbf{K}}{\mathbf{M}}\right) = \frac{\text{d}(\mathbf{K})\mathbf{M} - \mathbf{K}\text{d}(\mathbf{M})}{\mathbf{M}^2} \tag{6.38}$$

Using the matrix differential rules, the differential of the Lagrangian over $\mathbf{A}$ is written as

$$\text{d}L = \frac{T_d}{T} E\left[\text{tr}\left(\mathbf{D}_{sr}^{-1}\hat{\mathbf{H}}_{sr}\text{d}\left(\frac{\mathbf{Q}_s}{1+\text{tr}(\mathbf{Q}_s\tilde{\boldsymbol{\Sigma}}_{sr})}\right)\hat{\mathbf{H}}_{sr}^{\dagger}\right)\right] - 2\mu_s\text{tr}\left(\text{d}\mathbf{Q}_s\right)T_d \tag{6.39}$$

where $\mathbf{D}_{sr}$ is the inside of the determinant of (6.34). We use the partial differential rule and $\text{tr}(\mathbf{A}-\mathbf{B}) = \text{tr}(\mathbf{A}) - \text{tr}(\mathbf{B})$. After defining the denominator as $\mathbf{M}$, $\mathbf{M} = 1+\text{tr}(\mathbf{Q}_s\tilde{\boldsymbol{\Sigma}}_{sr})$, we get

$$\begin{aligned}\text{d}L = &\frac{T_d}{T} 2E\left[\frac{1}{\mathbf{M}^2}\left[\text{tr}\left(\mathbf{D}_{sr}^{-1}\hat{\mathbf{H}}_{sr}\text{d}\mathbf{Q}_s\mathbf{M}\hat{\mathbf{H}}_{sr}^{\dagger}\right) - \text{tr}\left(\mathbf{D}_{sr}^{-1}\hat{\mathbf{H}}_{sr}\left(\mathbf{Q}_s\text{tr}\left(\text{d}\mathbf{Q}_s\tilde{\boldsymbol{\Sigma}}_{sr}\right)\hat{\mathbf{H}}_{sr}^{\dagger}\right)\right)\right]\right] \\ &- 2\mu_s\text{tr}\left(\text{d}\mathbf{Q}_s\right)T_d\end{aligned} \tag{6.40}$$

We use the rules of trace operators as $\text{tr}(\mathbf{AB}) = \text{tr}(\mathbf{BA})$ and $\text{tr}(\mathbf{A}\text{tr}(\mathbf{B})) = \text{tr}(\mathbf{A})\text{tr}(\mathbf{B})$ (Magnus & Neudecker, 1999).

$$\begin{aligned}\text{d}L = &\frac{T_d}{T} 2E\left[\frac{1}{\mathbf{M}}\text{tr}\left(\hat{\mathbf{H}}_{sr}^{\dagger}\mathbf{D}_{sr}^{-1}\hat{\mathbf{H}}_{sr}\text{d}\mathbf{Q}_s\right) - \frac{1}{\mathbf{M}^2}\text{tr}\left(\hat{\mathbf{H}}_{sr}^{\dagger}\mathbf{D}_{sr}^{-1}\hat{\mathbf{H}}_{sr}\mathbf{Q}_s\right)\text{tr}\left(\text{d}\mathbf{Q}_s\tilde{\boldsymbol{\Sigma}}_{sr}\right)\right] \\ &- 2\mu_s\text{tr}\left(\text{d}(\mathbf{Q}_s)\right)T_d\end{aligned} \tag{6.41}$$

After finding the differential of Lagrangian, we derive the expression over $\mathbf{Q}_s$. Deriving the Lagrangian, we obtain Karush-Kuhn-Tucker(KKT) conditions.

$$\frac{T_d}{T}E\left[\frac{1}{\mathbf{M}}\hat{\mathbf{H}}_{sr}^{\dagger}\mathbf{D}_{sr}^{-1}\hat{\mathbf{H}}_{sr}-\frac{1}{\mathbf{M}^2}\text{tr}\left(\hat{\mathbf{H}}_{sr}^{\dagger}\mathbf{D}_{sr}^{-1}\hat{\mathbf{H}}_{sr}\mathbf{Q}_s\right)\tilde{\boldsymbol{\Sigma}}_{sr}\right] \leq \mu_s\mathbf{I}T_d \tag{6.42}$$

The KKT conditions in (6.42) is satisfied with equality when $\mathbf{Q}_s$ is different from zero, respectively. Otherwise, KKT conditions is satisfied with strict inequalities. In order to solve for $\mathbf{Q}_s$, we need equalities. Multiplying the both side of (6.42) with $\mathbf{Q}_s$, we obtain the equality for all $\mathbf{Q}_s$. We define the expectation expression at left hand side of (6.42) as E_1.

$$\frac{1}{T}\mathbf{E}_1\mathbf{Q}_s = \mu_s\mathbf{Q}_s \tag{6.43}$$

Applying trace operator, we get

$$\frac{1}{T}\text{tr}(\mathbf{E}_1\mathbf{Q}_s) = \mu_s\text{tr}(\mathbf{Q}_s) \tag{6.44}$$

Since the power is allocated to both the training phase and the data transmission phase, the joint optimization problem is about both of the phases. Therefore, we optimize the training signal and the transmit covariance matrix jointly. Differentiating the Lagrangian over the training signal, $\mathbf{S}_s$, we obtain

$$\mathrm{d}L = \frac{T_d}{T}E\left[\text{tr}\left(\mathbf{D}_{sr}^{-1}\mathrm{d}\mathbf{D}_{sr}\right)\right] - 2\mu_s\text{tr}\left(\mathrm{d}(\mathbf{S}_s)\mathbf{S}_s^{\dagger}\right) \tag{6.45}$$

To find $\mathrm{d}\mathbf{D}_{sr}$, we use the partial differential rule in (6.38). The identity matrix, which is added to the fraction expression in $\mathbf{D}_{sr}$, is removed by differentiating over $\mathbf{S}_s$. We define the numerator of the fraction expression in $\mathbf{D}_{sr}$ as $\mathbf{K}$, $\mathbf{K} = \hat{\mathbf{H}}_{sr}\mathbf{Q}_s\hat{\mathbf{H}}_{sr}^{\dagger}$.

$$\begin{aligned}\mathrm{d}\mathbf{K} =&\mathrm{d}\Big(\mathbf{H}_{sr}-\tilde{\mathbf{H}}_{sr}\Big)\mathbf{Q}_s(\mathbf{H}_{sr}-\tilde{\mathbf{H}}_{sr})^{\dagger}+(\mathbf{H}_{sr}-\tilde{\mathbf{H}}_{sr})\mathbf{Q}_s\mathrm{d}\Big(\mathbf{H}_{sr}-\tilde{\mathbf{H}}_{sr}\Big)^{\dagger} && (6.46)\\ =&-2\mathrm{d}\tilde{\mathbf{H}}_{sr}\mathbf{Q}_s\hat{\mathbf{H}}_{sr}^{\dagger} && (6.47)\end{aligned}$$

We use the training signal model which is explained in (6.16).

$$\begin{aligned}\mathrm{d}\mathbf{K} =&-2\mathrm{d}\tilde{\mathbf{Z}}_{sr}(\boldsymbol{\Sigma}_{sr}^{-1}+\mathbf{S}_s\mathbf{S}_s^{\dagger})^{-1/2}\mathbf{Q}_s\hat{\mathbf{H}}_{sr}^{\dagger} && (6.48)\\ =&2\tilde{\mathbf{Z}}_{sr}(\boldsymbol{\Sigma}_{sr}^{-1}+\mathbf{S}_s\mathbf{S}_s^{\dagger})^{-3/2}\mathrm{d}\mathbf{S}\mathbf{S}^{\dagger}\mathbf{Q}_s\hat{\mathbf{H}}_{sr}^{\dagger} && (6.49)\\ =&2\tilde{\mathbf{H}}_{sr}\tilde{\boldsymbol{\Sigma}}_{sr}^{2}\mathrm{d}\mathbf{S}_s\mathbf{S}_s^{\dagger}\mathbf{Q}_s\hat{\mathbf{H}}_{sr}^{\dagger} && (6.50)\end{aligned}$$

We defined the denominator of the fraction expression in $\mathbf{D}_{sr}$ as M, $\mathbf{M} = 1 + \text{tr}(\mathbf{Q}_s(\boldsymbol{\Sigma}_{sr}^{-1} + \mathbf{S}_s\mathbf{S}_s^\dagger)^{-1})$. Differential of the denominator is obtained as (Magnus & Neudecker, 1999, pg 207)

$$\begin{aligned} \mathrm{d}\mathbf{M} =& \mathrm{dtr}\left((\boldsymbol{\Sigma}_{sr}^{-1} + \mathbf{S}_s\mathbf{S}_s^\dagger)^{-1}\mathbf{Q}_s\right) && (6.51)\\ =& -\text{tr}\left((\boldsymbol{\Sigma}_{sr}^{-1} + \mathbf{S}_s\mathbf{S}_s^\dagger)^{-1}\mathrm{d}(\boldsymbol{\Sigma}_{sr}^{-1} + \mathbf{S}_s\mathbf{S}_s^\dagger)(\boldsymbol{\Sigma}_{sr}^{-1} + \mathbf{S}_s\mathbf{S}_s^\dagger)^{-1}\mathbf{Q}_s\right) && (6.52)\\ =& -2\text{tr}\left(\tilde{\boldsymbol{\Sigma}}_{sr}\mathrm{d}\mathbf{S}_s\mathbf{S}_s^\dagger\tilde{\boldsymbol{\Sigma}}_{sr}\mathbf{Q}_s\right) && (6.53) \end{aligned}$$

We use these differentials in (6.38).

$$\mathrm{d}L = 2\frac{T_d}{T}E\left[\text{tr}\left(\mathbf{D}_{sr}^{-1}\frac{\mathrm{d}\mathbf{K}\mathbf{M} - \mathbf{K}\mathrm{d}\mathbf{M}}{\mathbf{M}^2}\right)\right] - 2\mu_s\text{tr}\left(\mathrm{d}(\mathbf{S}_s)\mathbf{S}_s^\dagger\right) \tag{6.54}$$

where $\text{tr}\left(\mathbf{D}_{sr}^{-1}\mathrm{d}\mathbf{D}_{sr}\right)$ is

$$\frac{1}{\mathbf{M}}\text{tr}\left(\mathbf{S}_s^\dagger\mathbf{Q}_s\hat{\mathbf{H}}_{sr}^\dagger\mathbf{D}_{sr}^{-1}\tilde{\mathbf{H}}_{sr}\tilde{\boldsymbol{\Sigma}}_{sr}^2\mathrm{d}\mathbf{S}_s\right) + \frac{1}{\mathbf{M}^2}\text{tr}\left(\hat{\mathbf{H}}_{sr}^\dagger\mathbf{D}_{sr}^{-1}\hat{\mathbf{H}}_{sr}\mathbf{Q}_s\right)\text{tr}\left(\mathbf{S}_s^\dagger\tilde{\boldsymbol{\Sigma}}_{sr}\mathbf{Q}_s\tilde{\boldsymbol{\Sigma}}_{sr}\mathrm{d}\mathbf{S}_s\right) \tag{6.55}$$

The geometric interpretation of the the real channel vector, estimated channel vector and the estimation error is shown in Figure 6.2. The estimation error vector is perpendicular to the estimated channel vector. This information gives us $\hat{\mathbf{H}}_{sr}\tilde{\mathbf{H}}_{sr} = \mathbf{0}$ where $\mathbf{0}$ is the zero matrix. Eventually, the first trace expression in (6.55) is zero. Deriving the Lagrangian, we get KKT conditions.

$$\frac{T_d}{T}E\left[\frac{1}{\mathbf{M}^2}\text{tr}\left(\hat{\mathbf{H}}_{sr}^\dagger\mathbf{D}_{sr}^{-1}\hat{\mathbf{H}}_{sr}\mathbf{Q}_s\right)\tilde{\boldsymbol{\Sigma}}_{sr}\mathbf{Q}_s\tilde{\boldsymbol{\Sigma}}_{sr}\mathbf{S}_s\right] \leq \mu_s\mathbf{S}_s \tag{6.56}$$

We define the left side of (6.56) as E_2. We multiply the both side of the expression with $\mathbf{S}_s^\dagger$ and apply trace operator, we get

$$\frac{T_d}{T}\text{tr}(\mathbf{E}_2\mathbf{S}_s^\dagger) = \mu_s\text{tr}(\mathbf{S}_s\mathbf{S}_s^\dagger) \tag{6.57}$$

As mentioned before, the total power constraint is $P_{ds}T_d = \text{tr}(\mathbf{Q}_s)T_d + \text{tr}(\mathbf{S}_s\mathbf{S}_s^\dagger)$. We multiply (6.44) with T_d, and then add to (6.57), we have

$$\frac{T_d}{T}\text{tr}(\mathbf{E}_1\mathbf{Q}_s) + \frac{T_d}{T}\text{tr}(\mathbf{E}_2\mathbf{S}_s^\dagger) = \mu_s P_{ds}T_d \tag{6.58}$$

We use the Lagrangian multiplier and obtain optimum training signal and transmit covariance matrix as

$$\mathbf{Q}_s = \frac{1}{T_d}\frac{\mathbf{E}_1\mathbf{Q}_s}{\text{tr}(\mathbf{E}_1\mathbf{Q}_s) + \text{tr}(\mathbf{E}_2\mathbf{S}_s^\dagger)}P_{ds} \tag{6.59}$$

$$\mathbf{S}_s = \frac{\mathbf{E}_2}{\text{tr}(\mathbf{E}_1\mathbf{Q}_s) + \text{tr}(\mathbf{E}_2\mathbf{S}_s^\dagger)}P_{ds}T_d \tag{6.60}$$

Using these fixed point equations, we propose the following iteration algorithms.

$$\mathbf{Q}_s(n+1) = \frac{1}{T_d}\frac{\mathbf{E}_1(n)\mathbf{Q}_s(n)}{\text{tr}(\mathbf{E}_1(n)\mathbf{Q}_s(n)) + \text{tr}(\mathbf{E}_2(n)\mathbf{S}_s^\dagger(n))}P_{ds} \tag{6.61}$$

$$\mathbf{S}_s(n+1) = \frac{\mathbf{E}_2(n)}{\text{tr}(\mathbf{E}_1(n)\mathbf{Q}_s(n)) + \text{tr}(\mathbf{E}_2(n)\mathbf{S}_s^\dagger(n)}P_{ds}T_d \tag{6.62}$$

We find the relay transmit covariance matrix, $\mathbf{Q}_r$, and optimum training signal, $\mathbf{S}_r$, by deriving $\mathbf{I}_{mac}(\mathbf{Q})$ with a fixed $\mathbf{Q}_s$ and $\mathbf{S}_s$. This is also equivalent to a single user problem which is explained above.

Case 2: In the second case, ($\alpha = 1$), $\mathbf{R}(1, \mathbf{Q}) = \mathbf{I}_{mac-est}(\mathbf{Q})$ and the condition $\mathbf{I}_{mac-est}(\mathbf{Q}) \leq \mathbf{I}_{sr-est}(\mathbf{Q})$ should be satisfied. In this case, the achievable rate is found by maximizing $\mathbf{I}_{mac-est}(\mathbf{Q})$, which is a MAC problem.

$$I_{mac-est} = \max_{1\leq T_d \leq min(M_s,M_r)} \max_{\{\mathbf{Q},P_d\}} \frac{T_d}{T} E\left[\log\left|\mathbf{I} + \frac{\hat{\mathbf{H}}_{sd}\mathbf{Q}_s\hat{\mathbf{H}}_{sd}^\dagger + \hat{\mathbf{H}}_{rd}\mathbf{Q}_r\hat{\mathbf{H}}_{rd}^\dagger}{1 + \text{tr}(\mathbf{Q}_s\tilde{\mathbf{\Sigma}}_{sd}) + \text{tr}(\mathbf{Q}_r\tilde{\mathbf{\Sigma}}_{rd})}\right|\right] \tag{6.63}$$

where the coefficient $\frac{T_d}{T}$ reflects the amount of time that is spend during the training phase. The conservation of the energy provides $P_{ds}Td = \text{tr}(\mathbf{Q}_s)T_d + \text{tr}(\mathbf{S}_s\mathbf{S}_s^\dagger)$ and $P_{dr}Td = \text{tr}(\mathbf{Q}_r)T_d + \text{tr}(\mathbf{S}_r\mathbf{S}_r^\dagger)$. The joint optimization problem is about the training signals and the transmit covariance matrices for both the source and the relay. Deriving the Lagrangian, we obtain Karush-Kuhn-Tucker(KKT) conditions.

$$\frac{T_d}{T}E\left[\frac{1}{\mathbf{W}}\hat{\mathbf{H}}_{sd}^\dagger\mathbf{D}_{mac}^{-1}\hat{\mathbf{H}}_{sd} - \frac{1}{\mathbf{W}^2}\text{tr}\left(\hat{\mathbf{H}}_{sd}^\dagger\mathbf{D}_{mac}^{-1}\hat{\mathbf{H}}_{sd}\mathbf{Q}_s\right)\tilde{\mathbf{\Sigma}}_{sd}\right] \leq \mu_s\mathbf{I}T_d \tag{6.64}$$

$$\frac{T_d}{T}E\left[\frac{1}{\mathbf{W}}\hat{\mathbf{H}}_{rd}^\dagger\mathbf{D}_{mac}^{-1}\hat{\mathbf{H}}_{rd} - \frac{1}{\mathbf{W}^2}\text{tr}\left(\hat{\mathbf{H}}_{rd}^\dagger\mathbf{D}_{mac}^{-1}\hat{\mathbf{H}}_{rd}\mathbf{Q}_r\right)\tilde{\mathbf{\Sigma}}_{rd}\right] \leq \mu_r\mathbf{I}T_d \tag{6.65}$$

where $\mathbf{D}_{mac}$ is the inside of the determinant of (6.63). The denominator of the fraction expression is shown $\mathbf{D}_{mac}$ as $\mathbf{W}$, $\mathbf{W} = 1 + \text{tr}(\mathbf{Q}_s\tilde{\mathbf{\Sigma}}_{sd}) + \text{tr}(\mathbf{Q}_r\tilde{\mathbf{\Sigma}}_{rd})$. We define the expectation at the left hand side of (6.64) as E_3 and the expectation at left hand side of (6.65) as E_4 . To obtain equalities for all $\mathbf{Q}_s$ and $\mathbf{Q}_r$, we multiply the both side of (6.64) with $\mathbf{Q}_s$ and the both side of (6.65) with $\mathbf{Q}_r$. Finally, we apply trace operator.

$$\frac{1}{T}\text{tr}(\mathbf{E}_3\mathbf{Q}_s) = \mu_s\text{tr}(\mathbf{Q}_s) \tag{6.66}$$

$$\frac{1}{T}\text{tr}(\mathbf{E}_4\mathbf{Q}_r) = \mu_r\text{tr}(\mathbf{Q}_r) \tag{6.67}$$

Now, we optimize the training signals. By using matrix differential calculus and referring to the examples in Case 1, one can take the derivative of (6.63) with respect to $\mathbf{S}_s$ and $\mathbf{S}_r$ to obtain the following KKT conditions.

$$\frac{T_d}{T}E\left[\frac{1}{\mathbf{W}^2}\text{tr}\left(\hat{\mathbf{H}}_{sd}^{\dagger}\mathbf{D}_{mac}^{-1}\hat{\mathbf{H}}_{sd}\mathbf{Q}_s\right)\tilde{\mathbf{\Sigma}}_{sd}\mathbf{Q}_s\tilde{\mathbf{\Sigma}}_{sd}\mathbf{S}_s\right] \leq \mu_s\mathbf{S}_s \tag{6.68}$$

$$\frac{T_d}{T}E\left[\frac{1}{\mathbf{W}^2}\text{tr}\left(\hat{\mathbf{H}}_{rd}^{\dagger}\mathbf{D}_{mac}^{-1}\hat{\mathbf{H}}_{rd}\mathbf{Q}_r\right)\tilde{\mathbf{\Sigma}}_{rd}\mathbf{Q}_r\tilde{\mathbf{\Sigma}}_{rd}\mathbf{S}_r\right] \leq \mu_r\mathbf{S}_r \tag{6.69}$$

We define the expectation at the left hand side of the expressions as E_5 and E_6 respectively. To obtain equalities for all $\mathbf{S}_s$ and $\mathbf{S}_r$, we multiply the both side of the expressions above with $\mathbf{S}_s^{\dagger}$ and $\mathbf{S}_r^{\dagger}$ respectively. Finally, we apply trace operator.

$$\frac{T_d}{T}\text{tr}(\mathbf{E}_5\mathbf{S}_s^{\dagger}) = \mu_s\text{tr}(\mathbf{S}_s\mathbf{S}_s^{\dagger}) \tag{6.70}$$

$$\frac{T_d}{T}\text{tr}(\mathbf{E}_6\mathbf{S}_r^{\dagger}) = \mu_r\text{tr}(\mathbf{S}_r\mathbf{S}_r^{\dagger}) \tag{6.71}$$

Using power conservation, we obtain the Lagrange multipliers and obtain optimum training signals and transmit covariance matrices. Using the fixed point equations, we propose the following iteration algorithms as in the previous chapters. We define to write easier.

$$\mathbf{Q}_s(n+1) = \frac{1}{T_d}\frac{\mathbf{E}_3(n)\mathbf{Q}_s(n)}{\text{tr}(\mathbf{E}_3(n)\mathbf{Q}_s(n)) + \text{tr}(\mathbf{E}_5(n)\mathbf{S}_s^{\dagger}(n))}P_{ds} \tag{6.72}$$

$$\mathbf{Q}_r(n+1) = \frac{1}{T_d}\frac{\mathbf{E}_4(n)\mathbf{Q}_r(n}{\text{tr}(\mathbf{E}_4(n)\mathbf{Q}_r(n)) + \text{tr}(\mathbf{E}_6(n)\mathbf{S}_r^{\dagger}(n))}P_{dr} \tag{6.73}$$

$$\mathbf{S}_s(n+1) = \frac{\mathbf{E}_5(n)}{\text{tr}(\mathbf{E}_3(n)\mathbf{A}^{\dagger}(n)) + \text{tr}(\mathbf{E}_5(n)\mathbf{S}_s^{\dagger}(n))}P_{ds}T_d \tag{6.74}$$

$$\mathbf{S}_r(n+1) = \frac{\mathbf{E}_6(n)}{\text{tr}(\mathbf{E}_4(n)\mathbf{B}^{\dagger}(n)) + \text{tr}(\mathbf{E}_6(n)\mathbf{S}_r^{\dagger}(n))}P_{dr}T_d \tag{6.75}$$

We do the iteration for all T_d values. Finally, we determine the maximum achievable rate and the parameters that gives the maximum achievable rate.

Case 3: In the third case, $(0 < \alpha < 1)$, $R(\alpha, \mathbf{Q}) = \mathbf{I}_{mac-est}(\mathbf{Q}) + (1-\alpha)\mathbf{I}_{sr-est}(\mathbf{Q})$

and the condition $\mathbf{I}_{mac-est}(\mathbf{Q}) = \mathbf{I}_{sr-est}(\mathbf{Q})$ should be satisfied. In this case, we find the transmit covariance matrices of the source and relay as functions of α. As in the previous chapters, $R(\alpha, \mathbf{Q})$ will be maximized over $\mathbf{Q}$ and $\mathbf{S}$ for a fixed α, $0 < \alpha < 1$. Note that, transmit covariance matrices that will result from this optimization will depend on α.

$$\mathbf{V}(\alpha^*) = \max_{\text{tr}(\mathbf{Q}_s)\leq P_s, \text{tr}(\mathbf{Q}_r)\leq P_r} (\alpha^* I_{mac-est}(\mathbf{Q}) + (1-\alpha^*) I_{sr-est}(\mathbf{Q})) \tag{6.76}$$

The Lagrangian of (6.76) can be written as

$$L = R(\alpha^*, \mathbf{Q}) - \mu_s(tr(\mathbf{Q}_s) - P_{ds}T_d) - \mu_r(tr(\mathbf{Q}_r) - P_{dr}T_d) \tag{6.77}$$

As we know from Case 1 and Case 2, derivation of Lagrangian with respect to $\mathbf{Q}_s$ and $\mathbf{Q}_r$ gives KKT conditions. When we multiply both hand side with $\mathbf{Q}_s$ and $\mathbf{Q}_r$ respectively and apply trace operator, we get

$$\frac{1}{T}\text{tr}(\alpha \mathbf{E}_3 \mathbf{Q}_s + (1-\alpha)\mathbf{E}_1 \mathbf{Q}_s) = \mu_s \text{tr}(\mathbf{Q}_s) \tag{6.78}$$

$$\frac{1}{T}(1-\alpha)\text{tr}(\mathbf{E}_4 \mathbf{Q}_r) = \mu_r \text{tr}(\mathbf{Q}_r) \tag{6.79}$$

We derive Lagrangian with respect to $\mathbf{S}_s$ and $\mathbf{S}_r$ and then apply the same process.

$$\frac{T_d}{T}\text{tr}(\alpha \mathbf{E}_5 \mathbf{S}^\dagger + (1-\alpha)\mathbf{E}_2 \mathbf{S}^\dagger) = \mu_s \text{tr}(\mathbf{S}_s \mathbf{S}_s^\dagger) \tag{6.80}$$

$$\frac{T_d}{T}(1-\alpha)\text{tr}(\mathbf{E}_6 \mathbf{S}_r^\dagger) = \mu_r \text{tr}(\mathbf{S}_r \mathbf{S}_r^\dagger) \tag{6.81}$$

We define $X = \text{tr}(\alpha\mathbf{E}_3(n)\mathbf{Q}_s(n) + (1-\alpha)\mathbf{E}_1(n)\mathbf{Q}_s(n)) + \text{tr}(\alpha\mathbf{E}_5(n)\mathbf{S}_s^\dagger(n) + (1-\alpha)\mathbf{E}_2(n)\mathbf{S}_s^\dagger(n))$ and $Y = (1-\alpha)\text{tr}(\mathbf{E}_4(n)\mathbf{Q}_r(n)) + (1-\alpha)\text{tr}(\mathbf{E}_6(n)\mathbf{S}_r^\dagger(n))$.The iteration algorithm for the training signals and the transmit covariance matrices are shown below

$$\mathbf{Q}_s(n+1) = \frac{1}{T_d}\frac{\alpha\mathbf{E}_3(n)\mathbf{Q}_s(n) + (1-\alpha)\mathbf{E}_1(n)\mathbf{Q}_s(n)}{X}P_{ds} \tag{6.82}$$

$$\mathbf{Q}_r(n+1) = \frac{1}{T_d}\frac{(1-\alpha)\mathbf{E}_4(n)\mathbf{Q}_r(n)}{Y}P_{dr} \tag{6.83}$$

$$\mathbf{S}_s(n+1) = \frac{\alpha\mathbf{E}_5(n) + (1-\alpha)\mathbf{E}_2(n)}{X}P_{ds}T_d \tag{6.84}$$

$$\mathbf{S}_r(n+1) = \frac{(1-\alpha)\mathbf{E}_6(n)}{Y}P_{dr}T_d \tag{6.85}$$

These iteration algorithms are determined for all T_d values and chosen the maximum achievable rate. As a result we solved the joint optimization and resource allocation problem in full-duplex decode-and-forward MIMO relay channels when noisy channel estimation and partial CSI at the transmitters.

6.5 NUMERICAL RESULTS

In this chapter, channel estimation for MIMO relay channels are derived. We start this section with a convergence analysis. We derived the convergence analysis for Case 3. In Figure 6.3 and Figure 6.4, the source and relay transmit covariance matrices are shown for three antennas. In Figure 6.5, the training signals which are transmitted in the training phase are shown. For channel estimation, we need to iterate the algorithm more than the previous chapters to obtain convergence to optimum power values. For all calculations, the power constraints (P_s and P_r) are fixed at 10 dB. The results prove that as the iteration index increases, covariance matrices converge to their optimum values.

Figure 6.3: Convergence of the source transmit covariance matrix for channel estimation algorithm

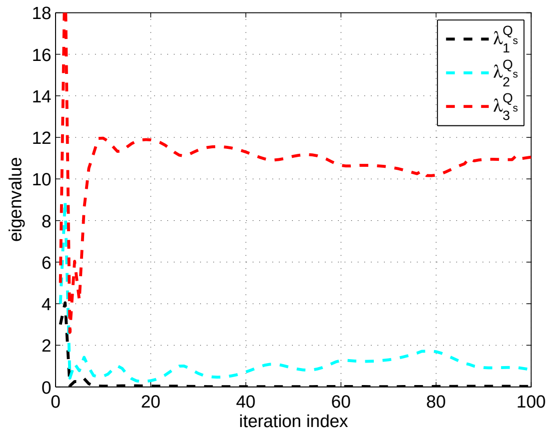
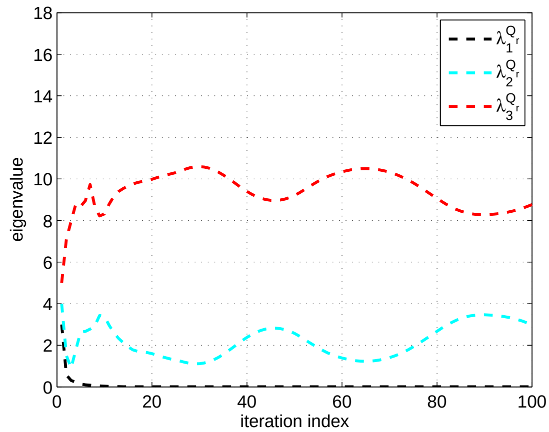
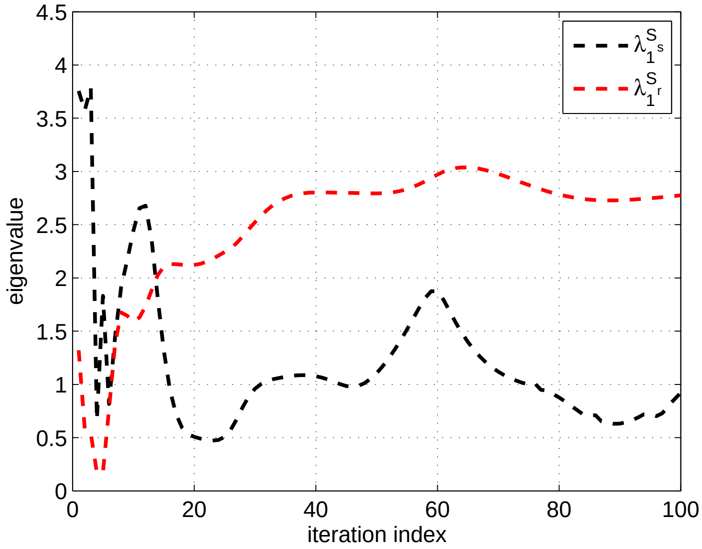


Figure 6.4: Convergence of the relay transmit covariance matrix for channel estimation algorithm



In Figure 6.6, we obtain the plot for channel estimation for which capacity is obtained when $\alpha = 0$. It means the minimum points of capacity function is at the point of $\alpha = 0$. For Case 1, the capacity expression is equal to $\mathbf{I}_{sr}$. For this channel parameters, the lower bound is 2.9 bit/sec.

Figure 6.5: Convergence of the training signals for channel estimation algorithm



In Figure 6.7, we obtain the plot for $\alpha = 1$ since the minimum point of the capacity function on the point of $\alpha = 1$. For this channel conditions, the lower bound is 2.58 bit/sec.

In Figure 6.8 for Case 3. For this channel matrix, the lower bound on the capacity is the value at the point $\alpha = 0.2$. The lower bound value is 2.7 bit/sec.

Figure 6.6: Capacity lower bound that result in $\alpha^* = 0$

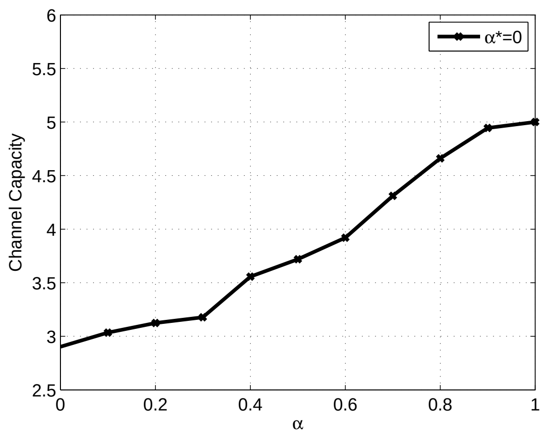


Figure 6.7: Capacity lower bound that result in $\alpha^* = 1$

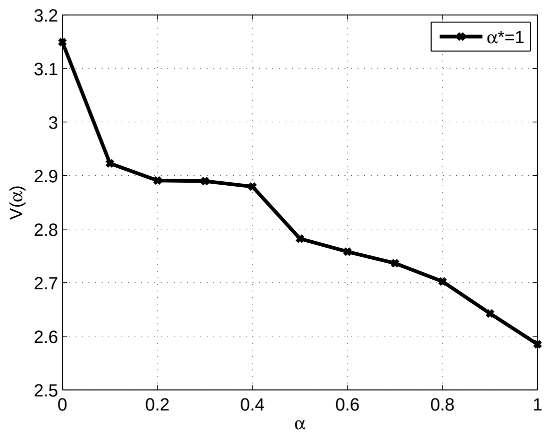
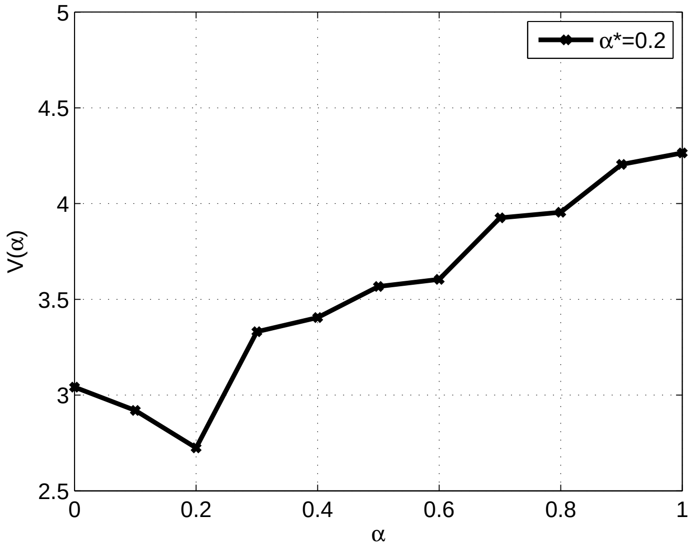


Figure 6.8: Capacity lower bound that result in $\alpha^* = 0.2$



7. CONCLUSIONS

In this thesis, we analyzed a MIMO relay channel when the transmitters have partial channel state information and the receivers have perfect channel state information. In Chapter 3, a suboptimal lower bound is derived. We proposed an iterative algorithm that gives the achievable rate on the channel capacity which was in terms of a max-min optimization problem for full-duplex decode-and-forward MIMO relay channels. To solve this problem, we combine our system model with the results in Liang et al. (2007). We found the transmit directions of the source and relay nodes, and in order to achieve best lower bound, we found the optimum power allocation policies (over the antennas) of the source and relay nodes. The solution of the max-min problem is given in three cases. For two of these cases, it is possible to propose efficient and fast algorithms that give the optimum source and relay transmit covariance matrices. For the third case, most general situation can only be solved with classical convex optimization methods. However, by making a reasonable assumption on the relay channel, we propose an efficient and fast algorithm for the third case as well. This assumption is validated through simulations by showing that the data rate obtained by our proposed algorithm is almost the same as the data rate obtained by the classical convex optimization methods.

In Chapter 4 and 5, we analyzed both full-duplex and half-duplex fading MIMO relay channels when the transmitters have partial CSI and the receivers have the perfect CSI. The channel capacity for such a system is not known in general. We derived decode-and-forward achievable rates and cut-set upper bounds on the channel capacity which were given in terms of max-min type optimization problems. When the transmitters know the channel covariance information, finding optimum source and relay transmit covariance matrices become important. Because, power allocation over the spatial dimension of the channel has a significant impact on the performance. We use matrix differential calculus to solve the source and relay transmit covariance matrices jointly. In our method, optimum transmit covariance matrices are found directly using a fast and efficient iterative algorithm.

In Chapter 6, we analyzed the noisy channel estimation and optimum achievable rate for full-duplex decode-and-forward MIMO relay channels. Firstly, the optimum training signal to minimize MMSE is determined. Then, the joint optimization problem is determined for both the training signal and the transmit covariance matrices of the source and the relay nodes. Iterative algorithms which give the optimum transmit covariance matrices and the training signals are obtained by using matrix differential calculus. The trade off between estimating the channel better and increasing the achievable rate is derived.

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