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COMPARING THE FORECASTING PERFORMANCE OF GARCH-BASED VOLATILITY MODELS FOR SOME TL-DENOMINATED FINANCIAL ASSETS

A MASTER'S THESIS

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Finally, I dedicated the dissertation to my son, Oğuz Kerem.

Istanbul, June 2015 Yavuz YUMRUKUZ

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ABBREVIATIONS

Autocorrelation Function:	ACF
Autoregressive:	AR
Autoregressive Conditional Heteroscedasticity:	ARCH
Autoregressive Integrated Moving Average:	ARIMA
Autoregressive Moving Average:	ARMA
Borsa Istanbul:	BIST
Brownian Motion:	BM
Central Bank of Turkey:	CBT
Central Limit Theorem:	CLT
Coefficient of Variation:	CV
Data Generating Process:	DGP
Dickey Fuller Test:	DF test
Efficient Method of Moment	EMM
Expected Weighted Moving Average:	EWMA
Full Information Maximum Likelihood:	FIML
Generalized Autoregressive Conditional Heteroscedasticity:	GARCH
Independent Normal:	IN
Kwiatkowski Phillips Schmidt Shin Test:	KPSS test
Logarithmic Error Statistics:	LL
Maximum Likelihood Estimation:	MLE
Mean Absolute Error:	MAE

Mean Mixed Error:	MME
Mean Squared Error:	MSE
Monte Carlo Likelihood:	MCL
Moving Average:	MA
Ornstein-Uhlenberg Process:	OU process
Over The Counter:	OTC
Partial Autocorrelation Function:	PACF
Phillips Peron Test:	PP test
Phillips Ouliaris Test:	PO test
Probability Density Function:	pdf
Quasi-Maximum Likelihood:	QME
Stochastic Differential Equation:	SDE
Stochastic Volatility:	SV
Value at Risk:	VaR
Vector Autoregression:	VAR
White Noise:	WN

SYMBOLS

α: scale or shape parameter

β: scale or shape parameter

σ: standard deviation or scale parameter

μ: mean

η: IID innovation term

δ: a parameter in some GARCH specifications

θ: parameter of moving average

ε: error term

ψ: unit root parameter

χ: chi-square distribution

ϕ: parameter of autoregression

ζ: partial correlation

ρ: correlation

ω: an event in probability space

γ: location parameter

a: autoregression parameter

ℓ: time lag

υ: error term of a Markov proces

Φ: parameter of AR process

λ: decay factor

$\mathfrak{L}$: log-likelihood

$\mathcal{F}$:subset of probability space

RÉSUMÉ

La volatilité est l'un des indicateurs les plus importants reflétant les fluctuations économiques et financières. La littérature portant sur le sujet traite principalement des différents types du modèle GARCH. Lorsqu'il est question de trouver la spécification GARCH la plus appropriée au série de donnée, en utilisant les statistiques *a priori*, il faut faire un choix dans trois domaines différents; à savoir le type de distribution qui représente au mieux le modèle variant, le modèle moyen et les termes d'erreurs. Autrement dit, la mesure GARCH comprend de façon simultanée et ensemble, l'estimation de l'égalité moyenne, de l'égalité de variance et de la distribution. Dans les études menées, de façon générale, l'objectif est, sans modifier deux de ces trois domaines et en testant les alternatives concernant ces deux domaines, d'obtenir la spécification de modèle qui a la capacité la plus efficace d'interprétation et/ou d'estimation. Par exemple, dans certains travaux, pour une série temporelle à variable unique, l'objectif est d'obtenir la répartition qui reflète au mieux les termes de l'erreur et dont la capacité d'estimation a été évaluée à partir des différents types d'alternatives de distribution du modèle GARCH(1,1). Ou encore l'objectif est d'appliquer différents modèles de volatilité dans différents pays et marchés, à une série de variation de prix d'instruments financiers comportant des caractéristiques similaires et d'obtenir ainsi le modèle de volatilité fournissant l'estimation la plus efficace. Dans cette thèse, il est question de présenter de façon structurée les recherches existantes sur le sujet et de ;

- Développer une fonction qui teste de façon simultanée, les sous-éléments du modèle GARCH que sont le modèle variance, le modèle moyenne et le type de distribution GARCH et de permettre son développement sous le logiciel R,

- En utilisant la fonction en question, de trouver la spécification GARCH qui offre l'estimation la plus efficace concernant les fluctuations de prix des instruments financiers en TL existant sur les marchés financiers turcs,
- L'interprétation de la spécification GARCH offrant l'estimation la plus efficace et d'évaluer la concordance entre les résultats concernant les séries temporelles et obtenu à travers les statistiques *a priori* des instruments financiers en TL.

Suivant cet objectif, dans un premier temps, les notions statistiques et mathématiques relatives à la notion de volatilité ont été définies. Dans ce cadre, des informations ont été données sur les processus stochastiques (les processus de Markov et Wiener et le "Brownian Motion") et les processus stationnaires (le bruit blanc et l'essai racine unité) Le bruit blanc, est le processus stationnaire le plus simple. La racine unité de son coté constitue l'une des hypothèses alternatives de ce processus. De fait, ces deux notions constituent les pierres angulaires du développement de modèle.

Dans un deuxième temps, afin de constituer l'infrastructure nécessaire pour l'étude des modèles de volatilité, nous avons étudié les différents types de distribution utilisés de façon courante. Les différents types et notions de volatilité ont fait l'objet de nombreuses recherches. De ce fait, avant de procéder à l'étude de cette notion, il paraît nécessaire de classifier ses différents types et acceptions. La correction, qui apporte une certaine perspective au sujet, a fait l'objet d'une attention particulière.

Après correction, des informations ont été données sur les modèles de volatilité (sous deux titres principaux, à savoir GARCH et la Volatilité Scholastique) et les modèles moyennes (les dérivés d'ARFIMA).

Dans la dernière partie, qui, par ailleurs, constitue une partie qui a pour objectif de préparer le lecteur à l'analyse de la volatilité, les différentes méthodes de comparaison hors échantillon ont été présentés et la phase d'analyse a été initiée.

Dans la phase d'analyse, afin de comprendre les fluctuations des prix, des informations sur les instruments financiers en TL et les marchés financiers turcs ont été fournies. Pour l'analyse, six instruments, à savoir l'or, l'opération de contrat d'échange sur risque de crédit, l'opération de swap à monnaie croisée, le swap de taux d'intérêt, la parité USD-TL et la valeur d'indice de la bourse XBN10 ont été pris en considération. Les instruments financiers présents sur les marchés financiers turcs ont été traités sous cinq titres. Ces titres sont ; les biens, les actions, les taux d'intérêts concernant les instruments d'endettement, le taux de change et les opérations dérivées. De plus, des marchés hors cote tels que BIST, Reuters et Bloomberg, qui constituent par ailleurs, des bourses fortement structurées, ont été présentés.

La période prise en considération, hormis le cas de l'indice XBN10, est celle entre 2007 et le milieu de l'année 2012. L'année 2007 est importante puisque il s'agit là de l'année où la crise financière mondiale, dont nous ressentons les effets encore aujourd'hui, a débuté. Cette année mérite l'attention puisqu'elle a fait augmenté les fluctuations dans les séries financières temporelles. Pour l'indice XBN10, c'est la période post 2010 qui a été pris en considération. Ainsi, XBN10 a joué le rôle de groupe témoin.

Parmi les séries en question, le swap à monnaie croisée constitue l'instrument financier dont la tarification comporte le plus grand nombre de facteur. Il en est ainsi car dans l'échantillon, c'est cet instrument financier qui comporte les taux d'intérêt fixes et variables à la fois en USD-TRY et en USD et en TRY. Tandis que le swap de taux d'intérêt se détermine par rapport au taux d'inflation et/ou au taux d'intérêt des prises en pension, le prix du contrat d'échange sur risque de crédit est déterminé par la probabilité de risque du Trésor de la Turquie suivant le marché.

Tout en insistant sur un ensemble de volatilité plus large, nous nous sommes centrés sur les modèles de volatilité GARCH. GARCH peut être définit comme un calcul de variance sous condition où la moyenne conditionnée est modélisée suivant ARMA. Parmi les modèles de type GARCH, seulement six d'entre eux ont fait l'objet d'analyse. Ces modèles sont ; les modèles EGARCH, IGARCH, APARCH, GARCH, GJR-GARCH et CGARCH.

En outre, tout en traitant d'un ensemble de distribution plus large, seule 9 d'entre elles ont fait l'objet d'analyse. Les types de distribution en question sont ; la distribution normale, la distribution normale biaisée, la distribution student t biaisée, la distribution student t, la distribution gaussienne inversée, la distribution d'erreur généralisée, la distribution d'erreur généralisée biaisée, SU de Johnson, les distributions hypothétiques généralisées.

Pour le modèle variance, des essais ont été réalisés pour les valeurs d'AR et MA de (1) et (2), pour le modèle moyen les valeurs d'AR et MA de (0) et (1). Par conséquent, 864 spécifications GARCH alternatives incluant 6 modèles variance, 9 types de distribution et 4 degré de moyenne ont été réalisée.

Les fluctuations des prix de ces 6 instruments financiers ont été comparées entre elles en appliquant le critère de la Moyenne des Carrés des Termes d'Erreurs (MSE) et à partir de 864 modèles GARCH alternatifs et une durée d'estimation de 10 jours. Afin de procéder à un essai consécutif de tous les alternatifs et de pouvoir faire une comparaison automatique, en utilisant "rugarch" du logiciel R, une fonction a été créée. Ainsi la fonction a permis d'identifier la spécification GARCH donnant la valeur MSE la plus faible. Le "Rugarch" offre aux utilisateurs la flexibilité nécessaire pour essayer les différents modèles de volatilités et ce grâce à son contenu de commandes incluant "ugarchspec", "ugarchfit" et "ugarchforecast". De ce fait, "Rugarch" est jugé supérieur aux autres logiciels proposés. L'intérêt de cette fonction est qu'elle est différentielle et elle permet de travailler sur les différentes spécifications non testées dans le cadre de la thèse. Par exemple, en tant que degré du modèle variance degré (1) et degré (2) ont fait l'objet d'essai mais un utilisateur qui dispose d'un ordinateur performant peut augmenter le nombre de différents degrés d'essai autant qu'il le souhaite.

Finalement, mais sans etre de maniere dominante, avec EGARCH, APARCH et la distribution normale de GARCH, il a ete vu que ceux-ci donnent la meilleure estimation de la spécification. EGARCH et APARCH sont des fonctionnalités d'asymétrie du modele GARCH. Bien que sur des séries chronologiques de la valeur élevée de la pression des valeurs trouvees de la proéminence des options, la distribution régulière de ces valeurs, se montre comme un résultat inattendu de l'étude. D'autre part, les resultats des series de l'index XBN10 et le prix de l'or apportent les changements dus au prix. Rappelons-nous que, sous controle, XBN10 suivait une courbe moins volatile. Toujours dans cette meme voie, CGARCH, se distinctant de l'autre serie a produit conformement a la valeur minimum de MSE.

Suite aux changements du prix de l'or, les valeurs ont montre que İGARCH produit la valeur minimum de la MSE. Bien plus que le prix de l'or sur les marches financiers Turcs, celui-ci est plutôt determine par les marches financiers d'or a l'echelle mondiale comme London Metal Exchange, NYMEX et TOCOM.

Cependant, dans la période après la crise, l'or a évolue comme un projet d'investissement fiable et le prix de l'or a poursuivi sa tendance à la hausse d'une manière stable pour une assez longue duree. Le taux d'intérêt, Swap, la devise Swap, les transactions par défaut de crédit Swap et le cours de la parite USD/TL manifestent de grands sauts au courant de la crise. IGARCH et CGARCH sont les series qui ont reflete avec choc les effets de la crise pendant une longue duree, lors de la période de stabilisation. D'apres le point de vue de la gestion des risques, les modeles comme APARCH et EGARCH sont ceux le mieux adaptes comme un deuxième résultat.

Du côté de la distribution, la vague émerge un peu des résultats ambigus. Le changement de la valeur d'echange de l'indice XBN10, se distincte des autres series du fait d'une erreur generale due a la distribution. Pour les autres series, trois d'entre elles sont asymetriques et normales, et en distribution t nous donne la valeur minimum de la MSE. Cependant, aucune valeur de la série en question n'est pas conforme à la distribution normale, comme il en ressort de l'alinéa précédent.

D'autre part, d'apres les resultats, Tsay (2010), fait valoir que lorsqu'il se prononce sur le modèle de volatilité (autocorrélation), il est nécessaire de déterminer l'équation qui permettra d'éliminer en moyenne (modèle ARMA) afin d'eviter cette autocorrelation.

Puis suite aux resultats des donnees de l'equation, il faudra tester les effets de la valeur ARCH. Troisièmement, un des effets ARCH sera déterminé à veiller à ce que le modèle statistique est déterminée à une volatilité importante et sont fabriqués avec des estimations de la volatilité sur les capitaux propres moyens. Enfin, les résultats du modèle devraient être évalués et, si nécessaire, des ajustements doivent être faits. Le but de la these vise à estimer au mieux le modèle convenant au projet, et également une fonction de facteur R offre la possibilité d'une comparaison automatique, tel une feuille de route à suivre.

Cependant, la valeur minimum de la MSE de la spécification GARCH du DAC et MAE valeurs indiquées pour réfléchir davantage le pouvoir explicatif du modèle Akaike Information Criterion (AIC), critère d'information bayésien (BIC) en tant que valeurs de test ARCH LM pour montrer les effets de ARCH et les statistiques sont également fournis. Thèse alternative juste 864 GARCH. Bien que la comparaison d'un cours par MSE MAE, critère AIC DAC, le critère BIC, ARCH test LM, "rugarch" ou un autre paquet R dans plus de tests peut écrire d'autres statistiques existantes ou nous en définir, et pour R une comparaison peut être faite egalement.

En plus de ce qui précède, en particulier les résultats de l'auto-corrélation, stationnaire, racine unitaire et menant par des statistiques sur les instruments financiers, y compris les changements de prix et leur normalité donnent une évaluation qui est faite pour TL. Par ACF et PACF diagrammes pour l'auto-corrélation des statistiques du test de Ljung-Box pour la stabilité et l'unité les tests racine KPSS, ADF et PP essais de statistiques, les diagrammes Cullen et Frey pour les types de distribution, l'asymétrie et la pression, Comme les indicateurs statistiques de test utilisés couramment, ainsi que Les statistiques descriptives qui ont été données. Mais encore, tous les changements de prix sont stables mais juges non conformes à distribution normale .

En conséquence, la thèse de la comparaison avec d'autres études dans la littérature que vous avez besoin de trier certaines fonctionnalités. İl faut souligner les situations différentes et de proposer une fonction de la flexibilité qui peut être adapté à l'objet, il est de plus en plus utilisé au cours des dernières années et l'écriture sur R, qui est un logiciel prestigieux, de code ouvert, utilise par les marchés financiers et sur les marchés financiers turcs egalement, instrument financier négocié et décrivant la manière brève mais concrete des marches financiers Turcs-TL, de les soumettre à l'analyse, de fournir la documentation la plus récente et juste de la distribution, expliquant avec un résumé en utilisant la litterature la plus appropriee et la plus reelle, la distribution des especes de la GARCH.

ABSTRACT

Volatility is one of the most significant indicators showing economic and financial fluctuations. It could be seen that GARCH based models are mostly dealt with in academic studies. While trying to find the GARCH specification best fitting a data set, three elements, including mean equation, variance equation and distribution type, should be considered by making use of some applicable individual statistics. In other words, GARCH measurement, simultenaously and jointly, involves estimation of mean equation, variance equation and the distribution. Academic studies have generally purposed looking for the best GARCH specification in terms of forecasting performance or explanatory power by trying the alternatives in one element given the others. For example, forecasting performance of GARCH(1,1) with different distribution alternatives in some emerging markets was studied and the best distributional characteristics were tried to be obtained in a study. Or, some volatility models are implemented to different countries, markets, or financial assets for reaching the best forecasting volatility model. The dissertation aims to give comprehensive summary of this extensive literature and also;

- Developing a function that can fulfil rolling with trying simultenaously the alternatives of GARCH elements including mean equation, variance model, distribution type, and writing it in R script,
- Finding the best forecasting GARCH specification for some TL financial instruments exchanged in Turkish financial markets by means of this script,
- Analysing and commenting the results of the script, evaluating whether there is a consistency between individual statistics of TL denominated financial assets and the results.

At the first stage within this scope, statistical and mathematical terms and concepts required for the "volatility" term are explained. To be more specific,

stochastic processes (Markov-Wiener processes and Brownian Motion) and, as a subset of it, stationary processes (unit root and Wiener process) are informed. While we can define white noise as the most simple stationary process, unit root means one of the keystones for model development due to the fact that it is one of the alternative hypothesis against stationarity.

As a second phase, the most widespread and well known distribution types are explained to obtain necessary fundemental before passing the analysis of the volatility models. Besides, there has been a requirement of classification to examine the volatility term and types that have been subject to many analysis before and now. It is emphasized because it also leads to gain a standpoint for road map and analysing approach.

Following the classification, volatility models (two titles as GARCH and stochastic volatility) and mean models (ARFIMA and its sub-models such as AR, MA, ARIMA) are explained.

In the last phase of getting preliminary knowledge for analysing volatility, out of sample criteria or measures are given in the text. Then, ultimately analysis phase begin after completing all of these previous required articulation.

In the analysis part, primarily it was presented knowledge about TL financial instruments and Turkish financial markets to evaluate the time path and fluctuations of price changes or returns, and six financial instruments including cross currency swap (CCS) rate, interest rate swap (IRS) rate, credit default swap (CDS) rate, USD-TL parity, XBN10 banking sector equity price index and gold are selected for analysis. Financial instruments in Turkish financial markets are classified into five categories. These are commodities consisting of agricultural products and mining product / energy, foreign exchange, equities (stocks), interest bearing debt agreements, derivatives including main options, swaps and forwards/ futures. In addition, the most significant organized market in Turkey, BIST, and also over the counter (OTC) markets whose deals are made in Reuters and Bloomberg screens are described briefly.

Analysing period is time interval between 2007 and mid-2012 except XBN10 series. The year of 2007 is crucial because of being the starting year of global financial crisis, and is worth analysing because of strong rise in fluctuations in time series. On the other hand, The period of 2010 and subsequent years is considered for XBN10 index. Thus, XBN10 index undertakes the duty of controlling sample.

CCS transactions in these series have the characteristic of the one whose valuation requires the most of factors that drive both USD-TL exchange rate and floating and fixed interest rates or spreads for USD and TL. Interest rate swaps involve inflation rates and/or repo rates, and TR credit defaults swaps reflect the probability of default of Turkish treasury.

Although a greater volatility groups and distribution types are touched upon in the dissertation mainly the GARCH models are focuses on. GARCH can be defines as a conditional variance calculation in which conditional mean is modelled by ARMA. Besides, only the six of GARCH models like EGARCH, IGARCH, APARCH, GARCH, GJR-GARCH and CGARCH are included in the analysis throughout the dissertation.

Similarly, even though a wide range of distribution groups are dwelled on in the dissertation, merely 9 of them are included in the analysis process. These distributional types are; normal, skewed, student t, skewed student t, normal inverted gaussian, generalized error, skewed generalized error, Johnson's SU and skewed hyperbolic distributions.

AR and MA values of (1) and (2) for the variance equation and, of (0) and (1) for the mean equation are multiplied with each other and then the matcing results are tested. Therefore, we get 864 alternative GARCH specifications jointly estimating 6 variance model, 9 distribution types, 4 variance degrees and 4 mean degrees.

The price changes of 6 financial assets are compared by Mean Squares Error (MSE) with 864 GARCH models and 10-day prediction period. To try all the alternatives subsequently and compare automatically "Rugarch" package of R was used to develop a function which provided a GARCH specification that gives the lowest MSE value. As “Rugarch” package has “ugarchspec”, “ugarchfit” and “ugarchforecast” instructions that present users flexibility trying the different volatility models commands and therefore is superior to other packets. The other advantage of the function is to possess the feature of being derivative and multiplicative and allowing various GARCH specifications. For instance, while (1) and (2) as the degree of variance model are tried in the thesis, the number of degree alternatives can be extended by a user who relies on his/her computer performance.

EGARCH, APARCH and GARCH involving a normal distribution were observed to yield the best result. EGARCH and APARCH are two of the GARCH models that have asymetrical characteristics. Desire the fact that there are the highest kurtosis values in time series, the options that include normal distribution is the unexpected result of the analysis. One of the two distinctive results is XBN10 index and the other is gold's return series. As known, XNB10 included less floating period. With respect to it, it was differentiated from the other series and produced CGARCH model as the minumum MSE value. As for the gold price changes, it showed the IGARCH as a minumum MSE value. Gold's price is determined by the global markets such as London Metal Exchange (LME), NYMEX and TOCOM rather than Turkish financial markets. Gold has emerged as a thrustworty investment and gold price has been on the increase for a long period of time. In the interest rate swap, currency swap transactions, cedit default swap transactions and USD-TL exchange large scaled jump was observed. As IGARCH and CGARCH are the series that maintain a shock effect, it will not be wrong to assert the idea that they reflect the stability periods better. Another deduction is that in terms of risk management, EGARCH and APARCH models are more appropriate.

When considered the distribution, there emerges a more ambigous result. First to begin with the explicit part, XBN10 index value diversed from the other series and reached the minumum MSE value by way of generalized error distribution. The other series give the minumum MSE values, three of them skewed and the other being normal and student’s t distribution. Nevertheless, as can be understood from the following statements, none of the series are in harmony with the normal distribution.

According to Tsay (2010) while deciding on a volatility model, the first step should be to determine a mean equity that will miss autocorrelation. Then the ARCH effects the residual values of equity should be tested. The next step should be selection of a volatility model that would provide a logical statistical result of ARCH effects and then make predictions between volatility equities and mean. Lastly, model's results should be evaluated. If needed, editions should be made. As the aim of the thesis is to reach the model that gives the best result / predictions and R function provides a-single-factor automatic comparison such a course of action was not preferred. Despite this, GARCH specification that gives the minumum MSE value, DAC and MAE values were given and Akaike Information Criterion (AIC) reflecting model's explanatory ability and set up, Bayes Information Criterion (BIC) are given. ARCH LM test values to show the ARCH effect are shown. Even though 864 criteria were tested within MSE criteria, it is possible to make comparisons using other tests in “rugarch” package and statistics in “rugarch” package available in other R packages such as MAE, DAC, AIC criteria, ARCH LM test. In addition to the results mentioned above, statistics autocorrelation, stability, unit root and normality about were given in relation to price changes of TL financial instruments. For autocorrelation, ACF and PACF diagrams and Ljung- Box test statistics are used, for stationarity and testing the unit root, KPSS is used. As for ADF and PP test statistics, for the distribution types such indicators as Cullen and Frey diagrams, skewness and kurtosis coefficients, and Jarque-Bera test statistics were used and descriptive statistics are given. All the price changes are observed to be stationary but not suitable for the normal distribution.

In conclusion, if the dissertation was compared with the other studies in the literature, the most remarkable distinctions could be given as follows: designing a function that is flexible to adapt it into different circumstances and purposes, performing this duty by developing a script in open-source R software that has been increasingly used in academic studies in last years and has increased its prestige in this way, expressing and analysing TL denominated assets and Turkish financial markets briefly, presenting the review and summary of the most recent and comprehensive literature about GARCH models and distribution types.

ÖZET

Volatilite, iktisadi ve finansal dalgalanmaların boyutunu yansıtan en önemli göstergelerden birisidir. Literatürde ağırlıklı olarak GARCH modelinin farklı türevlerinin işlendiği görülmektedir. Bir verisetine en uygun GARCH spesifikasyonu araştırılırken öncül istatistiklerden de yararlanılarak varyans modeli, ortalama modeli ve hata terimlerini en iyi temsil eden dağılım türü olmak üzere üç alanda tercih yapılması gerekir. Bir başka ifadeyle, GARCH ölçümü, eşzamanlı olarak ve birarada ortalama eşitliği, varyans eşitliği ve dağılımın tahmin edilmesini içerir. Yapılan çalışmalarda, genel itibarıyla bu üç alandan ikisi değiştirilmeden üçüncüsüne ilişkin alternatifler denenmek suretiyle açıklama ve/veya tahmin gücü en yüksek model spesifikasyonuna ulaşmanın hedeflendiği görülmektedir. Örneğin bazı çalışmalarda tek değişkenli bir zaman serisi için GARCH(1,1) modelinin farklı dağılım türü alternatifleri üzerinden tahmin gücü ölçülmüş ve hata terimlerini en iyi yansıtan dağılımın elde edilmesi hedeflenmiştir. Ya da belirli sayıda volatilite modelinin farklı ülke ve piyasalardaki benzer nitelikte finansal araç getiri oranları serisine uygulanması ve tahmin gücü en yüksek volatilite modeline ulaşılması hedeflenmiştir.

Tezde söz konusu geniş literatürün derli toplu bir özetinin verilmesi ve bunun yanısıra;

- GARCH modelinin alt unsurları olan varyans modeli, ortalama modeli ve dağılım türü alternatiflerini eş zamanlı deneyen bir fonksiyon geliştirip bunun R üzerinde yazılması,
- Söz konusu fonksiyondan yararlanarak Türk finans piyasalarındaki TL finansal araçların fiyat hareketlerine ilişkin en iyi tahmin gücünü veren GARCH spesifikasyonlarının bulunması,

- Elde edilen en iyi tahmin gücüne sahip GARCH spesifikasyonunun yorumlanması ve TL finansal araçların zaman serilerine ilişkin öncü istatistiklerle bulunan sonucun tutarlılığının değerlendirilmesi

amaçlanmıştır.

Bu amaca özgü olarak ilk aşamada, volatilite kavramı için gerekli istatistiki ve matematiksel kavramlar açıklanmıştır. Bu kapsamda stokastik süreçler (Markov ve Wiener süreçleri ile "Brownian Motion") ve bunun bir alt kümesi olan durağan süreçler (beyaz gürültü ve birim kök sınaması) hakkında bilgi verilmiştir. Beyaz gürültü, en basit durağan süreç iken birim kök ise durağanlığın alternatif hipotezlerinden birisini oluşturması bakımından model geliştirmenin temel taşlarındandır.

İkinci aşamada volatilite modellerine girmeden önce gerekli altyapıyı sağlayabilmek için yaygın olarak kullanılan dağılım tipleri anlatılmıştır. Ayrıca üzerine çok sayıda çalışma yapılan volatilite kavram ve tiplerini inceleyebilmek için ise öncelikle bunları sınıflandırma ihtiyacı ortaya çıkmıştır. Tasnife, aynı zamanda konuya bir bakış açısı kazandırdığı için önem atfedilmiştir.

Tasnif yapıldıktan sonra volatilite modelleri (GARCH ve Stokastik Volatilite olmak üzere iki ana başlıkta) ve ortalama modelleri (ARFIMA'nın türevleri) hakkında bilgi verilmiştir.

Volatilite analizine ön hazırlık mahiyetindeki son aşamada örneklem dışı karşılaştırma yöntemleri kısaca sıralanmış ve analiz aşamasına geçilmiştir.

Analiz aşamasında öncelikle fiyat hareketlerini anlamlandırabilmek için TL finansal araçlar ve Türk finans piyasaları hakkında bilgi verilmiş ve analiz için bunlardan altın, kredi temerrüt swap işlemi, çapraz para swap işlemi, faiz oranı swap işlemi, USD-TL paritesi ve XBN10 borsa endeks değeri olmak üzere altı tanesi seçilmiştir. Türk finans piyasalarında işlem gören finansal araçlar; beş ana başlık altında ele alınmıştır. Bu başlıklar; emtia, hisse senedi, borçlanma araçlarına ilişkin faiz oranları, döviz kuru ve türev işlemler olarak verilmiştir. Ayrıca en güçlü teşkilatlanmış borsa olan BIST ile Reuters ve Bloomberg gibi tezgahüstü piyasalar tanıtılmıştır.

Dikkate alınan periyot, XBN10 endeksi hariç, 2007 ve 2012 ortası arasındaki dönemdir. 2007 yılı etkileri hala hissedilen küresel finansal krizin başladığı tarih olması açısından önemlidir ve finansal zaman serilerinde dalgalanmaları oldukça artırması açısından analize değerdir. XBN10 endeksi için ise 2010 sonrası dönem göz önünde bulundurulmuştur. Böylece XBN10 bir kontrol grubu gibi fonksiyon üstlenmiştir.

Söz konusu serilerden çapraz para swapı hem USD-TRY hem de her iki para birimi cinsinden değişken ve sabit faiz oranlarını içerdiği için örneklem içerisinde fiyatlamasına en çok faktörün dahil edildiği finansal araç olma özelliği göstermektedir. Faiz oranı swapının temelinde enflasyon oranları ve/veya repo faiz oranları bulunurken kredi temerrüt swapının fiyatını piyasa gözüyle Türkiye hazinesinin temerrüt olasılığı belirlemektedir.

Daha geniş bir volatilite kümesi ve dağılım türünden bahsedilmiş olmakla birlikte sadece GARCH tipi volatilite modellerine odaklanılmıştır. GARCH; koşullu ortalamanın ARMA tarafından modellendiği bir koşullu varyans hesaplaması olarak tanımlamabilir. GARCH tipi modellerden de sadece altısı analize konu edilmiştir. Bunlar; EGARCH, IGARCH, APARCH, GARCH, GJR-GARCH ve CGARCH modellerinden oluşmaktadır.

Buna ilaveten benzer şekilde daha geniş bir dağılım türü kümesinden bahsedilmiş olmakla birlikte bunlardan 9 tanesi analize dahil edilmiştir. Söz konusu dağılım türleri; normal, eğik normal, eğik student t, student t, normal ters gaus, genelleştirilmiş hata, eğik genelleştirilmiş hata, Johnson’un SU, genelleştirilmiş hiperbolik dağılımlardır.

Varyans modeli için AR ve MA değerleri (1) ve (2), ortalama modeli için AR ve MA değerleri (0) ve (1) seçenekleri için denemeye tabi tutulmuştur. Dolayısıyla 6 varyans modeli, 9 dağılım türü, 4 varyans derecesi ve 4 ortalama derecesi olmak üzere 864 alternatif GARCH spesifikasyonu ortaya çıkmaktadır.

6 finansal aracın fiyat değişimleri; 864 alternatif GARCH modeli ve 10 günlük tahmin süresi üzerinden Hata Terimleri Karesinin Ortalaması (MSE) kriteri kullanılarak karşılaştırılmıştır. Tüm alternatifleri birbiri peşisıra denemek ve otomatik olarak karşılaştırabilmek amacıyla R yazılımının “rugarch” paketini kullanarak bir fonksiyon geliştirilmiş ve fonksiyonun en düşük MSE değerini veren GARCH spesifikasyonunu göstermesi sağlanmıştır. “Rugarch” paketi; içerdiği “ugarchspec”, “ugarchfit” ve “ugarchforecast” gibi komutlarla farklı volatilite

modellerini deneyebilmek için gerekli esnekliği kullanıcılara sunduğu için diğer yazılım ve paketlere üstünlük kurmaktadır. Fonksiyonun sağladığı önemli bir fayda türetken olması ve tezde denenmemiş farklı GARCH spesifikasyonlarına müsaade edebilmesidir. Örneğin, tezde varyans modelinin derecesi olarak (1) ve (2) denenmiş olmakla birlikte bu sayı bilgisayar performansına güvenen bir kullanıcı için istenildiği kadar artırılabilir.

Sonuçta, baskın olmamakla birlikte EGARCH, APARCH ve normal dağılımı içeren GARCH spesifikasyonlarının en iyi tahmini verdikleri görülmüştür. EGARCH ve APARCH asimetri özelliğine sahip GARCH modellerinden birisidir. Zaman serilerinde yüksek kurtosis değerleri bulunmasına rağmen normal dağılımı içeren seçeneklerin ağırlıkta olması çalışmanın beklenmeyen bir sonucu olarak kendisini göstermektedir. Diğer taraftan iki ayrık sonuçtan birisi XBN10 endeksi ve diğer altın fiyat değişimi serisidir. Hatırlanacağı üzere XBN10 kontrol amacıyla daha az dalgalı bir periyodu içermekteydi. Bu amaca uygun olarak diğer serilerden ayrılarak CGARCH modelini asgari MSE değeri olarak üretmiştir. Altın fiyat değişimleri ise IGARCH'ı asgari MSE değeri olarak vermiştir. Altının fiyatı Türk finans piyasalarının ötesinde daha ziyade Londra Metal Borsası, NYMEX ve TOCOM gibi küresel ölçekte piyasalarda belirlenmektedir. Altın, kriz ve sonrası dönemde güvenilir bir yatırım aracı olarak öne çıkmış ve altın fiyatı uzun süre istikrarlı bir şekilde yükseliş trendini devam ettirmiştir. Faiz oranı swap, çapraz para swap, kredi temerrüt swap işlemleri ve USD-TL kurunda ise krizle birlikte büyük ölçekli sıçramalar kendisini göstermektedir. IGARCH ve CGARCH bir şokun etkilerini uzun süre üzerinde taşıyan seriler oldukları için istikrar dönemlerini daha iyi yansıttıkları söylenebilir. Risk yönetimi bakış açısına EGARCH ve APARCH gibi modellerin daha uygun düştüğü de ikinci bir sonuç olarak ön plana çıkmaktadır.

Dağılım tarafına bakıldığında biraz daha muğlak bir sonuç ortaya çıkmaktadır. Önce benzer taraftan başlamak gerekirse XBN10 endeks değişim değeri yine diğer serilerden ayrışarak minimum MSE değerine genelleştirilmiş hata dağılımıyla ulaşmıştır. Diğer seriler ise üçü eğik olmak üzere normal ve t dağılımıyla minimum MSE değerini vermektedir. Ancak sonraki paragraftan da anlaşılacağı üzere söz konusu serilerden hiçbirisi normal dağılıma uymamaktadır.

Diğer taraftan Tsay (2010)'a göre bir volatilite modeline karar verirken öncelikle doğrusal bağımlılığı (otokorelasyonu) ortadan kaldıracak bir ortalama eşitliğini belirlemek gerekir (ARMA Modeli). Sonra bu eşitliğin artık değerlerinin ARCH etkileri test edilir. Üçüncü olarak belirlenecek ARCH etkilerinin istatistiken anlamlı olmalarını sağlayacak şekilde bir volatilite modeli belirlenir ve ortalama ile volatilite eşitliklerinin birlikte tahminleri yapılır. Son olarak model sonuçları değerlendirilmeli ve gerekiyorsa düzeltme yapılmalıdır. Tez'de amaçlanan en yüksek tahmini veren modele ulaşmak olduğu için ve ayrıca R fonksiyonu tek faktörlü otomatik bir karşılaştırma imkanı sunduğundan böyle bir yol haritası takip edilmemiştir. Ancak en düşük MSE değerini veren GARCH spesifikasyonununa ilişkin DAC ve MAE değerleri verilmiş, ayrıca modelin açıklama gücünü yansıtan Akaike Bilgi Kriteri (AIC), Bayes Bilgi Kriteri (BIC) gibi istatistikler ile ARCH etkisini göstermek üzere ARCH LM testi değerleri de verilmiştir. Tez'de sadece 864 GARCH alternatifinin MSE kriteri üzerinden bir karşılaştırılması yapılmış olmasına rağmen MAE, DAC, AIC kriteri, BIC kriteri, ARCH LM testi, "rugarch" veya diğer R paketlerinde mevcut olan diğer istatistikler veya kendi tanımlayacağımız ve R'da yazabileceğimiz testler üzerinden bir karşılaştırma yapılabilmesi mümkündür.

Yukarıda ulaşılan sonuçlara ilaveten başta otokorelasyon, durağanlık, birim kök ve normallik olmak üzere TL finansal araçların fiyat değişimlerine ilişkin öncü istatistikler verilmiş ve bunların bir değerlendirmesi yapılmıştır. Otokorelasyon için ACF ve PACF diyagramları ile Ljung-Box test istatistiğinden, durağanlık ve birim kök sınaması için KPSS, ADF ve PP test istatistiklerinden, dağılım türleri için Cullen ve Frey diyagramları, eğiklik ve basıklık katsayıları, Jarque – Bera test istatistiği gibi göstergelerden yararlanılmış, ayrıca tanımlayıcı istatistikler verilmiştir. Fiyat değişimlerinin tamamının durağan oldukları ama "normal dağılıma uygun" olmadıkları görülmüştür.

Sonuç olarak tez çalışmasının literatürde yer alan diğer çalışmalarla karşılaştırıldığında öne çıkan bazı özelliklerini sıralamak gerekirse farklı durum ve amaçlara uyarlanabilecek esneklikte bir fonksiyon ortaya koyması, bunu son yıllarda kullanımı artan ve prestijli açık kodlu bir yazılım olan R üzerinden yazması, TL finans piyasaları ve Türk finans piyasalarında işlem gören finansal araçları kısa ama özlü bir biçimde anlatması, analize bunları konu etmesi, GARCH ve dağılım türlerine ilişkin en güncel literatürün derli toplu bir özetinin sunulması bunlar arasında öne çıkanlardır.

1 INTRODUCTION

Volatility is a significant analysis tool and/or input directly or indirectly used in nearly all of the finance and risk governance literature ranging from value-at-risk (VaR) calculations to valuation of a fixed-income interest bearing asset, or a complicated option.

Although actual meaning of it corresponds to the standard deviation, when we talk about the times series, panel data, or would like to obtain volatility of volatility, the situation is likely to become complicated and involves the need to benefit from the facilities that stochactic calculus presents.

Volatility measurement models have been developed on three mainstreams including ARIMA- and GARCH-based models, stochastic volatility models and the others such as multivariate models, simulation techniques and other similar approaches such as neural networks, bootstrapping realized volatility etc.

In accordance with this context, in the first section of this dissertation, stochastic processes and concepts of time series related to the volatility measurement methods are summarized. Then probability distribution/density functions and related literature are given in the second section. Third and one of the main section of the dissertation include both the classification and the articulation of volatility methods. As for the fourth part, tools needed to compare forecasting performance of volatility methods are explained.

The fifth part contains the articulation of Turkish financial markets, TL-denominated financial instruments and data set used in the study. In the seventh part, the results derived from R implementation on volatility methods with respect to the data set have been analysed. To be more specific, only GARCH-based models are used.

Thus, this study aims to give the best volatility forecasting methods in terms of the intersection of TL-denominated assets and Turkish financial markets.

Therefore, a time series corresponds to the realisation of the underlying stochastic process. Also the stochastic process could be explained as the whole picture of possible realisations.

It can be considered that probability theory is regarded as the keystone of time series analysis. Kirchgässner and Wolters (2007) acknowledge that "while T-dimensional vector of random variables x_1, x_2, ..., x_T is given with multivariate distribution, this formation means a series of random variables$\{xt\}_{t=0}^{T}$ as stochastic process or as DGP. Consequently, the real numbers $\{x_1^{(1)}, x_2^{(1)} \dots x_T^{(1)}\}$ would be just one realisation of this stochastic process. On the other hand, although there is not just one realisation of such a process, all the realisations which all have the same statistical properties, since derived from the same DGP."

Even if T-dimensional stochastic process can be characterized with a T-dimensional distribution function, in many cases, to obtain the distribution becomes inapplicable. Rather, the first and second order moments of the distribution may present the adequate information about the process.

One way to measure the dependence between past and current realizations of a process is provided by the autocorrelation function (ACF).

$$\rho(h)=\mathrm{Corr}(y_t,y_{t-h})=\frac{\mathrm{Cov}(y_t,y_{t-h})}{\mathrm{Var}(y_t)}$$

The properties of the process generating the data is likely to be obtained from the ACF. In case of an AR(1) process, the ACF has the form of

$$\rho(h) = a_1\rho(h\text{-}1) = a_1^h,$$

where the last equality follows from $\rho(0) = 1$. It implies progressive decrease of any observation as the time lag increases. AR process is described with the exponential decay of the ACF.

Another tool to characterize the properties of autoregressive processes and to find the order of it is PACF. PACF coefficient measures the correlation between two random variables of different lags with removing the effects of other variables concerning other lags.

$$\zeta(h) = Corr(y_t, y_{t-h} | y_{t-1}, \ldots y_{t-h-1})$$

1.1. Stochastic Processes

The basic definitions and theorems concerning the stochastic calculus are briefly explained to the extent of which they are related to and underpins the volatility measurement techniques.

Dividing the time, progressing from the past and the present to the future via an operator, and designing a DGP requires primarily to characterize stochastic processes, and to identify their properties. Stochastic processes such as Levy, Markov, Gaussian, Poisson, Brownian, OU provide a basis for time series econometrics, and also volatility forecasts. Hence, starting point to examine the volatility measurement techniques must be to obtain knowledge about them.

1.1.1 Markov Processes

The properties of conditional expectations with respect to filtrations define various types of stochastic processes, the most important of which for us will be Markov processes.

"A stochastic process X is said to be a Markov process if for any $0 \leq s \leq t$ and any Borel measurable function f: $\mathbb{R} \to \mathbb{R}$ such that $f(X_t)$ has finite expectation, we have

$$E[f(X_t)|\mathcal{F}s] = E[f(X_t)|X_s]$$

where $\mathcal{F}s$ is filtration. This property means, roughly, that "the future is independent of the past given the present (Hunter, 2009)". In other words, a Markov process has no memory about where it goes, and only cares about its present state.

Definition with one another notation could be given as follows. It means that conditional density functions of (x,t) pairs coincide with conditional density of the last time.

$$p(x_{n+1},t_{n+1}| x_n, t_n;\ldots; x_2,t_2; x_1,t_1) = p\,(x_{n+1},t_{n+1}| x_n, t_n)$$

$$p(x_n,t_n;\ldots; x_2,t_2| x_1,t_1) = p(x_n,t_n| x_{n-1}, t_{n-1}) \ldots p(x_2,t_2| x_1,t_1)$$

Therefore, while transition density p(x,t |y,s) and the probability density of initial value X_0 are obtained, the density function of Markov process could be reached. Going in the right direction, we could get the following equation:

$$p(x,t \mid y,s) = \int_{-\infty}^{\infty} p(x,t \mid z,r)\, p(z,r \mid y,s)dz \text{ for any } s < r < t,$$

meaning that in going from y at time s to x at time t, the process must go though some point z at any intermediate time “r”. A continuous Markov process is time-homogeneous if

$$p(x,t \mid y,s) = p(x; t-s \mid y,0)$$

meaning that its stochastic properties are invariant under translations in time.

1.1.2 Wiener Process and Brownian Motion

Standard (one-dimensional) BM starting at 0 or the Wiener process is a stochastic process B(t,ω) with the following properties:

1) $B(0,\omega) = 0$ for every $\omega \in \Omega$;
2) for every $0 \leq t_1 < t_2 < t_3 < \ldots < t_n$, the increments $B_{t2} - B_{t1}$; $B_{t3} - B_{t2}$;…;$B_{tn} - B_{tn-1}$ are independent random variables;
3) for each $0 \leq s < t < \infty$, the increment Bt - Bs is a Gaussian random variable with mean 0 and variance t - s;
4) the sample paths $B^{\omega}: [0,\infty) \rightarrow \mathbb{R}$ are continuous functions for every $\omega \in \Omega$[1].

The existence of BM is a crucial point. Also the other one that must be shown is that normal distribution is consistent with continuity (Hunter, 2009). CLT supports the view that normality assumption could be obtained by independent increments and continuity.

The Gaussian assumption must, in fact, be satisfied by any process with independent increments and continuous sample paths due to the CLT. On the other hand, even though variance t of B_t is not constant and grows linearly in time, Gaussian Markov process does not become stationary (Hunter, 2009). However, OU process is a stationary, Gaussian, Markov process.

[1] This definition could be found in many textbooks concerning stochastic calculus and time series econometrics. Ruppert (2010) is utilized for the definition in this text.

“According to Donsker's theorem, the random walk approaches a BM in distribution as $\Delta t \rightarrow 0$. A key point is that although the total distance moved by the particle after time t goes to infinity as $\Delta t \rightarrow 0$, since it takes roughly on the order of $1/\Delta t$ steps of size $\sqrt{\Delta t}$, the net distance traveled remains finite almost surely because of the cancellation between forward and backward steps, which have mean zero. … BM could be considered as a basic building block for the construction of a large class of Markov processes with continuous sample paths, called diffusion processes (Hunter, 2009).”

It could be utilized from the time derivative of Brownian paths in a distributional sense to obtain a generalized stochastic process called WN. By using Ito’s Formula and Fokker-Planck Equations (see in Hunter (2009)), we could obtain the following equations:

- $dX=b(X,t)dt + \sigma(X,t)dB$
- $\dot{X}=b(X,t) + \sigma(X,t)\xi(t)$ (BM with drift)
- $X(t) = X(0) + \int_0^t b(X(s), s)ds + \int_0^t \sigma(X(s), s)dB(s)$ (drift and diffusion shown respectively)
- $dX=b(X,t)dt + \sigma(X,t)\, \partial B$
- In the absense of noise, we can get $S(t)=S_0e^{\mu t}$
- Due to the dependence of the noise in two previous equation, we can get **geometric BM** as $S(t)=S_0 \exp[(\mu - \frac{1}{2}\sigma^2)t + \sigma B(t)]$ where St is lognormal.

1.1. Stationary Process and Its Testing

1.2.1. Stationary Process

Stationary process requires the condition that A process X_t and X_{t+c} has the same distribution for all $-\infty < c < \infty$, meaning that all of the finite-dimensional distributions are effected by merely time differences.

“Strict stationarity is a very strong assumption, because it requires that all aspects of behavior be constant in time. The joint and conditional distribution of the process are unchanged if displaced in time. That is, the PDF of y_{t1}, y_{t2}, .. is identical to that of y_{t1+h}, y_{t2+h}, .. y_{tk+h}, for any positive h and t. Often, we can get by assuming less, namely, weak stationarity. A process is weakly stationary if its mean, variance, and covariance are unchanged by time shifts. Weakly stationary process:

- $E(Y_i) = \mu$(a constant) for all i;
- $Var(Y_i) = \sigma^2$ (a constant) for all i; and
- $Corr(Y_i,Y_j) = \rho(|i - j|)$ for all i and j for some function $\rho(h)$

(Ruppert, 2010)."

Therefore, it can be inferred that only some distributional characteristics including mean, variance and covariance excluding quantiles, skewness and kurtosis meet the requirements of weak stationarity.

The time-invariant feature of the time series has three types: trend, seasonal variation and change over time in the size of the seasonal oscillations. As it will be discussed throughout the following part, That a WN process is independent from the past and the present, the difference between the mean and the actual value couldnot be estimated.

1.2.2. White Noise Process

WN is simple form of a stationary process. "The sequence $Y_1,Y_2,...$ is a weak white noise process with mean μ and variance σ^2, which will be shortened to weak $WN(\mu,\sigma^2)$ if $E(Y_i) =\mu$ for all i; $Var(Y_i) = \sigma^2$ (a constant) for all i; and $Corr(Y_i, Y_j) = 0$ (Ruppert, 2010)". That $Y_1, Y_2,...$ is an IID process leads to IID $WN(\mu,\sigma^2)$.

The building block of all time series models is the strict WN process (Gaussian WN has), given by a sequence of IID random variables,

$$\epsilon_t \sim IID(0,\sigma^2).$$

While the independence assumption is superseded with uncorrelatedness of the series ϵ_t, ϵ_t is converted into the well-known white noise,

$$\epsilon_t \sim WN(0,\sigma^2)$$

$$y_t = a_0 + a_1 y_{t-1} + \epsilon_t$$

That "a" is equal to ∓1 is RW, but that "a" is less than ∓1, means a nonstationary process in terms of the equation above. These two results are too different with each other.

Another concept we have to mention within this context is "mean reversion", a feature of a time series showing oscillation around some fixed level. It is a necessary condition for stationary process.

1.2.3. Unit Root Tests

ADF and PP tests are used to respond the question whether the coefficients of autoregressors has the value of 1. For ADF test, while H_0 is that there is a unit root, H_1 may have two results including stationarity or explosive behavior. PP test has the similar characteristic, but some minor differences.

A third test is the KPSS test. The null hypothesis for the KPSS test is stationarity and the alternative is a unit root, just the opposite of the hypotheses for the DF and PP tests.

To start with describing ADF test,

$$y_t=\beta y_{t-1} + \epsilon_t \text{ where } \varepsilon t \sim IN[0,\sigma_\epsilon^2]$$

The 't-statistic' for testing H_0: $\beta = 1$, but does not have a standard t-distribution. It has a skewed distribution with a long left tail, making it hard to discriminate the null of a unit root from alternatives close to unity (Henry and Juselius, 1999).

When data are non-stationary purely due to unit roots, they can be brought back to stationarity by linear transformations.

When $\hat{u}_t=\hat{y}_t-\beta_1 x_t-\beta_0$ where β is the OLS estimate of the long-run parameter vector β, then the null hypothesis is H_0: $\rho = 1$, or equivalently, H_0: $1 - \rho = 0$ in:

$$\hat{u}_t = \rho\ \hat{u}_{t-1} + \varepsilon_t \quad \text{or} \qquad \Delta\hat{u}_t = (1\text{-}\rho)\ \hat{u}_{t-1} + \varepsilon_t$$

The test is based on the assumption that ε_t is WN, and if it is not, it has to be augmented by lagged differences of residuals ($\Delta\hat{u}_t = (1\text{-}\rho)\ \hat{u}_{t-1} + \psi_1\Delta\hat{u}_{t-1} + \cdots \psi_m\Delta\hat{u}_{t-m} + \varepsilon_t$). The test of H_0: $(1\text{-}\ \rho) = 0$ is called the ADF test.

It could be observed that large autocorrelations of the price levels at long lags, suggesting nonstationarity, and the lack of such autocorrelations for the differenced prices, suggesting stationarity.

Modeling non-stationary data by focusing on autoregressive processes with unit roots could be transformed back into stationarity by differencing and cointegration transformations. But "other sources of non-stationarity may remain, such as changes in parameters (particularly shifts in the means of equilibrium errors and growth rates) or data distributions, so careful empirical evaluation of fitted equations remains essential. We reiterate the importance of having WN residuals, preferably homoscedastic, to avoid mis-leading inferences (Henry-Juselius, 1999)."

2 RELEVANT PROBABILITY DISTRIBUTIONS

2.1 Probability Distribution as an Input

Probability distribution functions could be esteemed as one of the most significant drivers/inputs of volatility forecasting.

Chuang et al. (2007) asserted that suitable distribution function should have the following properties: primarily it must be flexible to comply with a extensive range of shapes. Second, the shape parameter reflects the skewness and kurtosis of the distribution. Also, in order to be functional, the parameters should be reached by means of statistical tools.

Volatility forecasting relates to the various models generally being separable with respect to "conditional heteroskedasticity specifications" and "error distributional assumptions". Generally, it was supported by many researches that "the financial time series exhibits leptokurtosis and the volatility of asset prices reveals a clustering effect. In a study of the time-varying conditional variances of economic variables, the (G)ARCH model based on normal conditional distribution has been shown not only to capture the volatility-clustering effect but also to accommodate some of the leptokurtosis (Chuang et al.,2007)".

Chuang et al. (2007) tested the predictive accuracy of volatility forecasts generated by the GARCH model with various distributional assumptions. Thirteen distribution functions were employed to examine stock price indices and exchange rates of seven countries. Within this perspective, several distribution functions combined with the GARCH model were investigated, including logarithmic, the exponential power distribution, the mixed diffusion jump, the Johnson's SU distribution, the student's t distribution, the scaled student's t distribution, the skewed generalized t distribution, the discrete mixture of two/three normal distributions, the exponential generalized beta type 2, the two-piece mixture normal distributions and normal distribution. “The performance of volatility forecasts is considered over alternative loss function measures in order to provide a complete performance assessment of the use of both symmetric and asymmetric loss functions. Therefore, the result of forecasting performance indicated that, using data from 1996 to 2003 and the comparable forecast error criteria, logarithmic, the skewed student’s t distributions and the Riskmetrics model are favoured in both markets. On the other hand, the exponential power distribution and a mixture of two normal distributions were less recommended (Chuang et al., 2007).”

In short, the actual volatility does not necessarily reflect the characteristics of a standard distribution. Previous works have found out that time series, specifically the returns of financial assets, comply with considerable range of distributions including Student’s t-distribution, the generalized error distribution etc. Studies have generally come to conclusion that distributions of logarithmic price changes are “timevarying, asymmetric and fat-tailed”. Therefore, assuming stationarity may cause a misunderstanding about the real level of uncertainty.

2.1. Probability Distributions Used in Model Specification

We could analyse “pdf” by focusing on three parameters: location, scale and shape. The location parameter measures where the range of values is. The scale parameter reflects the variability of a distribution function and the shape parameter controls how the variation is distributed around the location. The expressions in this section are mainly obtained by summing up Chuang et al. (2007), but also some contributions of several sources are added to the text. The following expressions are merely related to the distributions used in specifications of volatility.

2.2.1. Logistic distribution

The logistic distribution or the LOG proposed by Smith in 1981 is very similar to normal distribution, but ensures thick tails. “Thus, it is potentially capable of providing a better fit to empirical return distributions than the N distribution. The “pdf” of the LOG is defined as:

$$f(x|\mu,\alpha)=\frac{e^{(x-\mu)/\alpha}}{\alpha[1+e^{\frac{x-\mu}{\alpha}}]^2}$$

where μ is a location parameter, and α is a scale parameter (α>0) (Chuang et al., 2007)."

2.2.2. Exponential power (generalized error) distribution

The generalized error distribution was suggested by Box and Tiao in 1973 and has the following density function:

$$f(x|\mu,\alpha,\beta)=\frac{\exp[-1/2|(x-\mu)/\alpha|^{2/(1+\beta)\beta}]}{2^{\frac{3+\beta}{2}}\alpha\,\Gamma(\frac{3+\beta}{2})}$$

where Γ(.) denotes the gamma function; and μ and σ are location and scale parameters respectively, β is a parameter controlling the shape of the distribution with the range of (-1,1) and enables us to get excess kurtosis more flexible than the LOG and normal distributions.

- β>0 means the distribution displays leptokurtosis,
- β<0, it becomes less peaked and has thinner tails than the N distribution,
- β=0, the distribution resembles the N distribution;
- β=1, it means the Laplace distribution.
- If β goes to 1, it leads to the uniform distribution.

2.2.3. SU-normal distribution

The SUN proposed by Johnson in 1949 as a transformation of the N distribution enables us to get various levels of skewness and kurtosis. Then the density function of the SUN is defined as:

$$f(x\mid\mu,\alpha_1,\alpha_2,\gamma,\beta)=\frac{\alpha_2}{\sqrt{2\pi}\sqrt{(x-y)^2+\beta^2}}\,e^{\frac{-1}{2}\left\{\alpha_1+\alpha_2\ln\left[\left(\frac{x-y}{\beta}\right)+\sqrt{\left(\frac{x-y}{\beta}\right)^2+1}\right]\right\}^2}$$

where α_1 and α_2 are shape parameters, and γ and β are location and scale parameters, respectively. The distribution allows both platykurtosis and asymmetry.

2.2.4. Scaled student's t distribution

The well-known student's t distribution is traditionally used to test the hypothesis that the difference between the means is statistically different from zero.

$$f(x|\nu)=\frac{\Gamma(\nu+\frac{1}{2})}{\Gamma\left(\frac{\nu}{2}\sqrt{\nu-2}\right)}\left[1+\frac{x^2}{\nu-2}\right]^{-(\nu+1)/2}$$

where v is the degree of freedom ($2<v<\infty$) and $\Gamma(.)$ is the gamma function. When v approaches infinity, we have a N distribution, hence the lower the v the fatter the tail.

The scaled student's t distribution (SST) can also be represented as a mixture of N and inverted Gamma distributions. The density function of the SST distribution is as follows:

$$f(x|\nu,\mu,H)=\frac{\Gamma((\nu+1)/2)\nu^{\nu/2}\sqrt{H}}{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{\nu}{2}\right)}\left[\nu+H(x-\mu)^2\right]^{-(\nu+1)/2}$$

where μ is a location parameter and H (H>0) is a scale parameter. v (v>0), the degree of freedom, measures the extent of departure from normality. When the value of v is low, the distribution exhibits excess kurtosis and has a fatter tail compared to the N distribution. As v increases, the distribution becomes less leptokurtic with a descending rate, and when v approaches infinity the SST distribution converts to the N distribution.

2.2.5. Skewed generalized t distribution

"Besides, fat tails, empirical distributions of asset returns may also be skewed. To handle this additional characteristic of asset returns, the Student-t distribution has been modified to become a skew-Student-t distribution. There are multiple versions of skew-Student-t distribution (Tsay, 2010)."

As an alternative, Karian and Dudevicz (2011) gave the "pdf" of Hansen, McDonals and Turley's article in 2006. However, it was first proposed by Theodossiou in 1998 as a skewed extension of the generalized t distribution or the SGT. It has complicated "pdf" as follows:

$$f(x|m,k,n,\lambda,\sigma) = C\left[1+\frac{k}{(n-2)}\frac{|(x-m)/\sigma|^k}{\theta^k(1+\operatorname{sign}(x-m)\lambda)^k}\right]^{-(n+1)/k}$$

with:

$$C = \frac{kS}{2\sigma}B\left(\frac{1}{k},\frac{n}{k}\right)^{-1.5}B\left(\frac{3}{k},\frac{n-2}{k}\right)^{0.5}$$

$$\theta = \frac{1}{S}\left(\frac{k}{n-2}\right)^{1/k}B\left(\frac{1}{k},\frac{n}{k}\right)^{0.5}B\left(\frac{3}{k},\frac{n-2}{k}\right)^{-0.5}$$

$$S = \left[1+3\lambda^2-4\lambda^2 B\left(\frac{2}{k},\frac{n-1}{k}\right)^2 B\left(\frac{1}{k},\frac{n}{k}\right)^{-1} \times B\left(\frac{3}{k},\frac{n-2}{k}\right)^{-1}\right]^{0.5}$$

where $-1<\lambda<1$, $n>2$, $\sigma>0$ and $k>0$ and B(.) is the Beta function.

- m and σ are location and scale parameters,
- k and n are shape parameters that control the height and tail of the distribution (When they are small, there could be excess kurtosis)
- The λ parameter controls the skewness of the data (positive λ value means right-skewed distribution)

The SGT presents flexibility by allowing wide range of skewness and kurtosis. By restricting the parameters of the SGT distribution, other distributions could be obtained (i.e. the generalized-t distribution (λ=0), the skewed-t distribution (k=2), the ST distribution (λ=0, k=2), normal distribution (λ=0, k=2, n→∞), the Cauchy distribution (λ=0, k=2, n=1), the EXP (λ=0, n→∞) etc.

Chuang et al. (2007) examined the skewness in the conditional distribution of six equity index returns using the conditional SGT distribution. They found out that "the conditional SGT distribution offers a substantial improvement in the fit of both GARCH and EGARCH models. Specifically, the study strongly rejected the parameter restrictions on the SGT that are implied by the three most commonly used distributions, which are N, student-t and EXP distribution. Moreover, another study carried out by Degiannakis applied the conditional skewed-t distribution to predict the one-step-ahead volatility of three stock indice and found that the use of asymmetric ARCH-Skewed-t model generates more accurate volatility forecasts of stock returns (Chuang et al., 2007)."

2.2.6. Generalized Hyperbolic Distribution

Generalized hyperbolic distributions were introduced by Barndorff-Nielsen in 1977. “The one-dimensional generalized hyperbolic (GH) distribution is defined by the following Lebesgue density as

$$\begin{aligned} \mathrm{gh}(x;\lambda,\alpha,\beta,\delta,\mu) &= a(\lambda,\alpha,\beta,\delta)\left(\delta^2+(x-\mu)^2\right)^{(\lambda-\frac{1}{2})/2} \\ &\quad \times \mathrm{K}_{\lambda-1/2}\left(\alpha\sqrt{\delta^2+(x-\mu)^2}\right)\exp(\beta(x-\mu)) \\ a(\lambda,\alpha,\beta,\delta) &= \frac{(\alpha^2-\beta^2)^{\lambda/2}}{\sqrt{2\pi}\,\alpha^{\lambda-1/2}\,\delta^\lambda\,\mathrm{K}_\lambda\left(\delta\sqrt{\alpha^2-\beta^2}\right)} \end{aligned}$$

where K_λ is a modified Bessel function and x $\in \mathbb{R}$ (Aout,M,. The domain of variation of the parameters is $\mu \in \mathbb{R}$ and

$$\delta \geq 0,\ |\beta| < \alpha \quad \text{if } \lambda > 0$$

$$\delta > 0,\ |\beta| < \alpha \quad \text{if } \lambda = 0$$

$$\delta > 0,\ |\beta| \leq \alpha \quad \text{if } \lambda < 0$$

(Prause, 1999).”

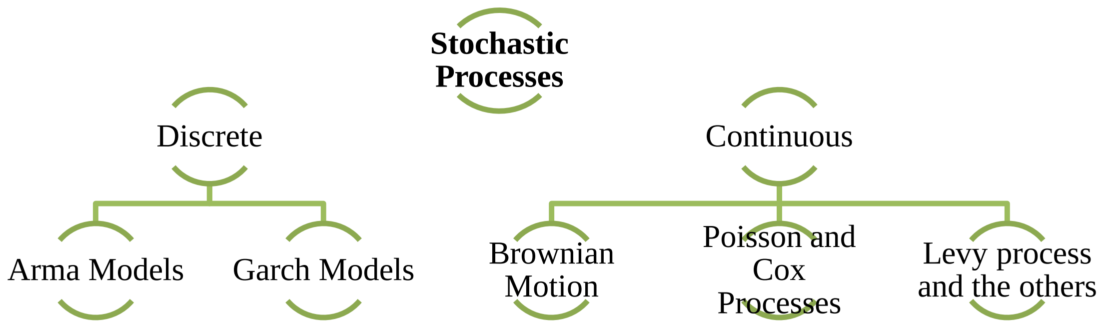
3 VOLATILITY MODELS

3.1. Classification and Definition

As mentioned before, there are various studies and as an outcome of them, various methods for volatility forecasting have emerged. In this respect, it is possible to categorize these methods with regard to the identical characteristics of varied techniques as done by subsequent articles compiling this broad and abundant literature. Since the field of volatility measurement produce many techniques and literature, analysing all the aspects of volatility without grouping them is impossible. Additionally, classification presents strong support to understand the logic behind each volatility model by making distinction with one another.

"Stochastic Processes" lecture notes of Rachev [2] is the first we scan within this scope. The text was organized by dividing the models of discrete time series into two groups in order to provide explanations about DGP's. According to him, there are two types of discrete-time models: static and dynamic.

[2] http://www.ams.sunysb.edu/QF/ExecEdNotes/ExecCourse3%209-20.pdf (16.06.2015)



Graphic 3.1: Classification of Rachev

Static models involve multivariate analysis of the same time, exemplified by capital asset pricing model. However, dynamic models assess one or more variables at two or more moments. In a dynamic model, the mathematical relationship between variables at different times is called the DGP. If we know the DGP, we can then simulate the process recursively, starting from the initial conditions.

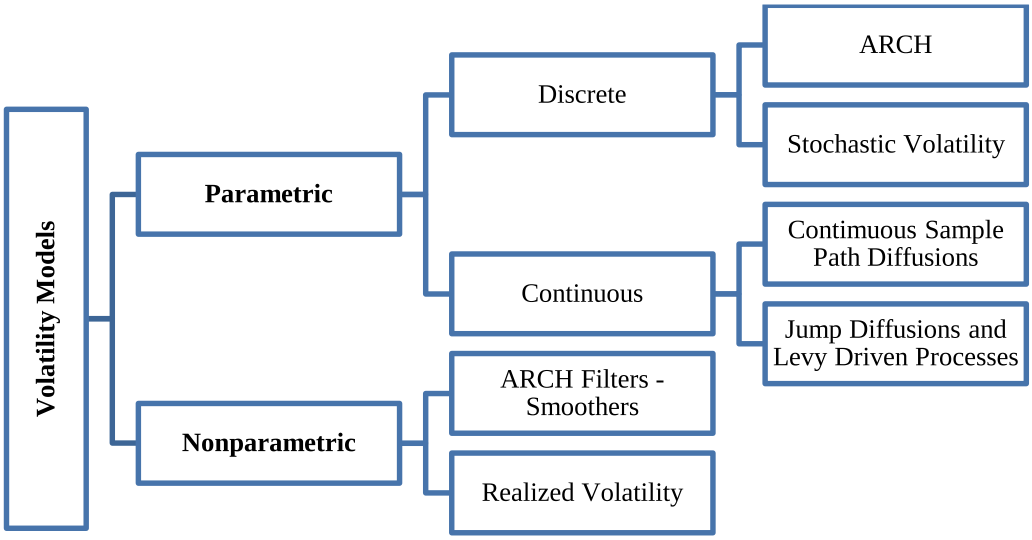
According to Andersen et al. (2002), volatility could be defined in three different concepts:

i) The notional volatility corresponding to the ex-post sample-path return variability over a fixed time interval,

ii) The ex ante expected volatility over a fixed time interval, and

iii) The instantaneous volatility corresponding to the strength of the volatility process at a point in time.

They made a little bit different classification and added nonparametric approaches to the coverage.



Graphic 3.2: Classification of Andersen et al. (2002) for Volatility Techniques

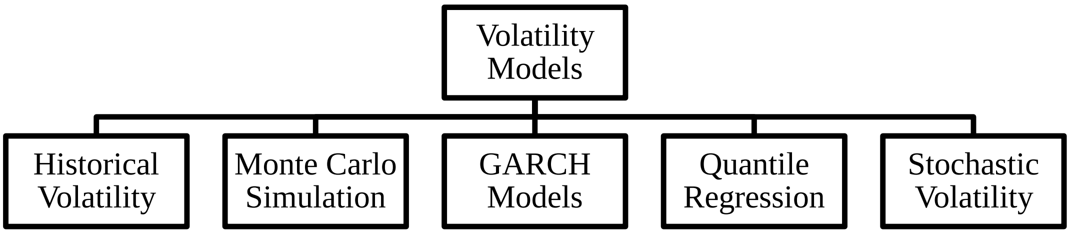
Parametric approaches are based on explicit functional form assumptions regarding the expected and/or instantaneous volatility. In the discrete-time ARCH class of models, the expectations are formulated in terms of directly observable variables, while the discrete- and continuous-time stochastic volatility models both involve latent state variable(s).

Nonparametric approaches does not have any similar functional form and produce the volatility estimation by flexible but consistent way. The nonparametric approaches are designed to measure the intraday volatility as well as realized volatility measures.

Poon (2005) could be regarded as another contribution how to approach volatility techniques. According to him, the simplest historical price model is the RW model, where σ_{t-1} is used as a forecast for σ_t. Extending this idea, we have historical average, MA, exponential smoothing method, and EWMA. All together, the four methods reflect a tradeoff between increasing the number of observations and sampling nearer to time t.

Simple regression method expressing volatility as a function of its past values and an error term is principally autoregressive. If past volatilities are also included, one gets the ARMA. Introducing a differencing order I(d), we get ARIMA when d=1 and ARFIMA when d<1. Also, we have the TAR model, where the thresholds separate volatility into states with independent simple regression models and noise processes for volatility in each state. It generally involves minimizing in-sample volatility forecast errors. More sophisticated models involve constant updating of parameter estimates when new information is observed and absorbed into the estimated period (Poon, 2005).

In Huang's article (2011) on stochastic volatility forecasting for emerging markets, 12 models are categorized under 5 segments:



Graphic 3.3: Classification of Huang (2011) for Volatility Models

Many of the classes in Huang's articles will be explained later. That the classes will not be mentioned later are Monte Carlo simulation and quantile regression models.

Monte Carlo simulation method is that the volatility forecast is estimated from a set of simulated random return paths with many iterations. In addition Quantile regression is summarized in the following chart. Conditional VaR quantile regression model of Taylor in 2005 should be mentioned in this title to comprise, and not to miss all the principal methods.

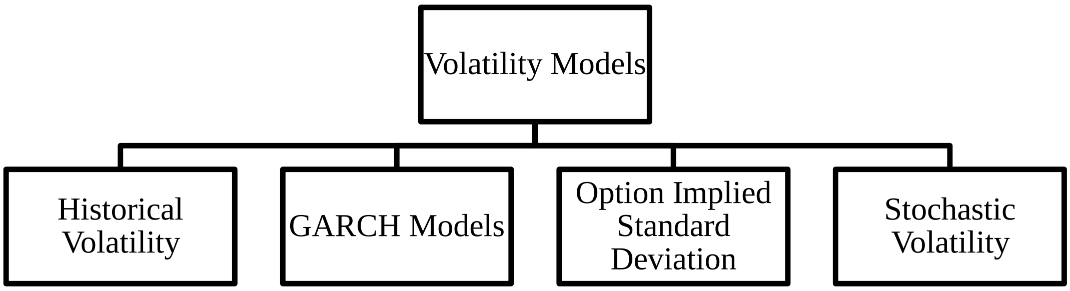
Conditional quantiles with parameter set of θ are estimated from the past.

Symmetric absolute value $Q_t(\theta)=\beta_1+\beta_2 Q_{t-1}(\theta)+\beta_3|r_{t-1}|$

Asymmetric slope $Q_t(\theta)=\beta_1+\beta_2 Q_{t-1}(\theta)+\beta_3 \max|r_{t-1},0|-\beta_4 \min|r_{t-1},0|$

The β's satisfy the following condition: min(β) $\{\sum_{r_t \geq Q t(\theta)} \theta \mid r_t - Q(\theta) \mid + \sum_{r_t < Q t(\theta)} (1 - \theta) \mid r_t - Q(\theta) \mid\}$

Another comprehensive classification was made by Poon ve Granger (2003). They studied on the forecasting performance of 93 papers developing/ related to a model by dividing them into 4 categories:



Graphic 3.4: Classification of Poon and Granger (2003) for Volatility Models

Each segment in the scheme is explained by summarizing and paraphrasing the work of Poon and Granger (2003).

1) Historical volatility: This category includes random walk, historical averages of squared returns, or absolute returns methods. Also included in this category are time series models based on historical volatility using moving averages, exponential weights, autoregressive models, or even fractionally integrated autoregressive absolute returns. The multivariate VAR realized volatility model of Andersen et al. (2002) is classified in this group.

All models in this group model volatility directly omit the goodness of fit of the returns distribution or any other variables such as option prices.

1. Random walk (RW): $\hat{\sigma}_t = \sigma_{t-1}$
2. Historical average (HA): $\hat{\sigma}_t = (\sigma_{t-1} + \sigma_{t-2} + \ldots \sigma_1) / (t-1)$
3. Moving average (MA): $\hat{\sigma}_t = (\sigma_{t-1} + \sigma_{t-2} + \ldots \sigma_{t-\tau}) / \tau$
4. Exponential smoothing (ES): $\hat{\sigma}_t = (1-\beta)\ \sigma_t\text{-}1 + \beta\ \hat{\sigma}_{t-1}$ and $0 \leq \beta \leq 1$
5. Exponentially weighted moving average (EWMA):

$$\hat{\sigma}_t = \frac{\sum_{i=1}^{\tau} \beta^i \sigma_{t-i}}{\sum_{i=1}^{\tau} \beta^i}$$

6. Smooth transition exponential smoothing (STES):

$$\hat{\sigma}_t = \alpha_{t-1}\epsilon_{t-1}^2 + (1-\alpha_{t-1})\hat{\sigma}_{t-1}^2$$

$$\alpha_{t-1} = 1/\ (1+\exp(\beta+\gamma V_{t-1})$$

where V_{t-1} is the transition variable; $V_{t-1} = \varepsilon_{t-1}$ for STES-E, $V_{t-1} = |\varepsilon_{t-1}|$ for STES-AE and V_{t-1} is a function of both for STES-EAE. It resembles IGARCH with restriction of 0 constant parameter.

7. Simple regression (SR): $\widehat{\sigma}_t = \gamma_{1,t-1}\sigma_{t-1} + \gamma_{2,t-2}\sigma_{t-2}$ ...

8. Threshold autoregressive (TAR): it was developed by Tsay and Cao in 1992 asserting that it generates better forecasting performance as compared with EGARCH and GARCH and should be explained briefly as:

$$\sigma_t=\phi_0^{(i)}+\phi_1^{(i)}\sigma_{t-1}\ldots\phi_p^{(i)}\sigma_{t-p}+\upsilon_t,$$

$$\widehat{\sigma}_t=\phi_0^{(i)}+\phi_1^{(i)}\sigma_t\ldots\phi_p^{(i)}\sigma_{t+1-p}$$

"where the thresholds separate volatility into states with independent simple regression models and noise processes in each state. The prediction σ_{t+1} could be based solely on current state information (i) assuming the future will remain on current state (Poon, 2005)."

2) GARCH: GARCH-based volatilities are included in this group from simple ARCH model to more complicated multivariate GARCH models. For all GARCH family models, returns "r_t" has the following process: $r_t = \mu + \epsilon_t$ where $\epsilon_t=\sqrt{h_t}\ z_t$ and h_t follows one of the following ARCH class models.

ARCH (q): $h_t = \omega + \sum_{k=1}^{q} \alpha_k \epsilon_{t-k}^2$ where $\omega>0$ and $\alpha\geq0$.

GARCH(p,q): $h_t = \omega + \sum_{k=1}^{q} \alpha_k \epsilon_{t-k}^2 + \sum_{j=1}^{p} \beta_{kj} h_{t-j}^2$

where $\omega>0$. For finite variance $\sum \alpha_k + \sum \beta_j < 1$.

3) Option implied standard deviation: This model is based on the Black-Scholes option pricing model model and various generalizations. Black Scholes model function is as follows:

$$c=g(S,X,\sigma,R,T)$$

where S = price of the underlying asset, X = exercise price, σ= volatility, R = risk-free interest rate, T = time to maturity.

4) Stochastic volatility: A stylized fact of time series of returns on financial assets is the clustering behaviour of volatility. Mainly two modelling approaches have been used to capture this behaviour. The GARCH model represents conditional variance as a function of lagged squared residuals and lagged conditional variance (Ding- Meade, 2010), and the other is stochastic volatility model (as implied by its name) or SV model based on the assumption that the variance follows a stochastic process. SV model of Taylor in 1982 includes the following equations:

$$r_t = \sigma_t \epsilon_t$$

$$\sigma_t = \exp(\frac{h_t}{2})$$

$$h_t = \mu + \phi(h_i - \mu) + \eta_i$$

where h is a non-zero mean Gaussian linear process (Shephard, 2005).

In short, several different classifications about the volatility forecasting are given above. On of them should be selected to analyse and explain the volatility and its measurement models. For this aim, the classification of Poon and Granger (2003) is referred in this dissertation, since the coverage and borders of my analysis are likely to be more efficiently represented in this manner.

3.2. Historical Volatility

3.2.1. AR process

As expressed before, the simplest correlated stationary processes are autoregressive (AR) processes, where Y_t is modeled as a weighted average of past observations plus a WN error which is also called the "noise" or "disturbance. AR(1) process is

$$Y - \mu = \Phi(Y_{t-1} - \mu) + \varepsilon_t$$

where if for some constant parameters μ and Φ.

The term $\Phi(Y_{t-1} - \mu)$ means the effect of "memory" on the present value of the process. The process Y_t is correlated because the deviation of Y_{t-1} from its mean is fed back into Y_t.

The first and second order moments of AR(1) are calculated as

$$E(y_t) = a_0 + a_1 E(y_{t-1}) + E(\epsilon_t) \Longrightarrow E(y_t) = \frac{a_0}{1 - a_1}$$

$$Var(y_t) = a_1^2 \, var(y_{t-1}) + \sigma^2 \Longrightarrow var(y_t) = \frac{\sigma^2}{1 - a_1^2}$$

where we implicitly use the fact that y_t is stationary. The mean and the variance above are called unconditional or long-term because no assumptions are made about possible additional knowledge about the process.

When a similar approach is used for y_{t+1} given today's information set $\mathfrak{F}_t$. Then,

$$E(y_{t+1}| \mathfrak{F}_t) = a_0+a_1y_t$$

$$Var(y_t| \mathfrak{F}_t)=\sigma^2$$

As seen above, the conditional mean changes through time, but the conditional variance remains constant.

3.2.2. MA process

Process Y_t is a MA process if Y_t can be expressed as a weighted average (moving average) of the past values of the WN process ε_t. The MA(1) process is

$$Y_t-\mu= \varepsilon_t+\Theta\varepsilon_t$$

where as before the σ_t are WN. Unlike AR(p) models, the mean μ in an MA(q) model is the same as β_0, the "constant" in the model. Thus, if a MA model is used, then only two or three MA parameters are needed. This is a strong contrast with AR models, which requires far more parameters, perhaps up to the six.

$$yt = \theta_0 + \sum_{i=1}^{p} \theta_i\epsilon_{t-i}+\epsilon_t$$

The unconditional mean and variance aren't based on "t":

$$E(y_t)= \theta_0 \text{ and } Var(y_t)=\sigma^2(1+ \sum_{i=1}^{q} \theta_i^2)$$

However, the conditional mean changes through time, but the conditional variance remains constant, as with an AR process:

$$E(y_{t+1}| \mathfrak{F}_t) = \theta_0+\sum_{i=1}^{q} \theta_i\epsilon_{t-i} \text{ and } Var(y_{t+1}| \mathfrak{F}_t)=\sigma^2$$

It can be shown that the ACF of an MA(q) process vanishes for lags greater than q. For example, for an MA(1) process, we have:

$$\rho(h)=\begin{cases} \frac{\theta_1}{1+\theta_1^2}, & h = \pm 1 \\ 0, & |h| > 1 \end{cases}$$

3.2.3. ARMA and ARIMA processes

An ARMA model combines both AR and MA terms and is defined by the equation

$$(Y_t-\mu)=\phi(Y_{t-1}-\mu) + \ldots + \phi_p(Y_{t-p}-\mu)+\epsilon_t+\theta_1\epsilon_{t-1}+\ldots+\theta_q\epsilon_{t-q}$$

which shows how Y_t depends on lagged values of itself and lagged values of the WN process. Equation can be written with the backwards operator as

$$(1-\phi_1B-\ldots-\phi_pB^p)(Y_t-\mu)=(1+\theta_1B+\ldots+\theta_qB^q)\epsilon_t$$

A WN process is ARMA(0,0) since if $p = q = 0$, $(Y_t-\mu)=\epsilon_t$. The differencing operator is another useful notation and is defined as $\Delta = 1-B$, where B is the backwards operator, so that ($\Delta Y_t=Y_t - BY_t=Y_t- Y_{t-1}$) where Δ^k can be derived from a binomial expansion:

$$\Delta^k Y_t=(1-B)^k Y_t=\sum_{\ell}^{k}\binom{k}{\ell}(-1)^{\ell}\,Y_{t-\ell}$$

The inverse of differencing is integrating. The integral of a process Y_t is the process ω_t, $\omega_t=\omega_{t_0}+Y_{t_0}+Y_{t_0+1}+\ldots+Y_t$, it is easy to check that $\Delta\omega_t=Y_t$.

Due to the fact that the first differences of a nonstationary process having constant mean have mean zero, differenced process is likely to be zero mean.

Although linear ARMA processes assume homoskedastic characteristic of conditional variance, time series generally reflects heteroskedastic behavior: the variances of the error terms are not equal, names as volatility clustering. It means the tendency of large changes to be followed by large changes and small changes to be followed by small changes. Hence, there is a non-linear temporal dependence in returns.

3.2.4. ARMAX (ARMA with Exogenous Variables)

ARMA processes can also include current and/or lagged, exogenously determined variables. Such processes are denoted by ARMAX processes. An ARMAX process has the form

$$y_t=a_0+\sum_{i=1}^{p} a_i y_{t-i}+\sum_{i=1}^{q}\theta_i\epsilon_{t-i}+\sum_{i=1}^{d} x_{t-i}\epsilon_t$$

These ideas can be extended to higher-degree polynomial trends and higher-order differencing.

3.2.5. FARIMA (Fractional ARIMA) or ARFIMA

The concept of fractional integration is often referred to as defining a time series with long-range dependence or long memory. Any pure ARIMA stationary time series can be considered as a short memory series. An AR(p) model has infinite memory, as all past values of 't' are embedded in Y_t, but the effect of past values of the disturbance process follows a geometric lag, damping off to near-zero values quickly. A MA(q) model has a memory of exactly q periods, so that the effect of the moving average component quickly dies off.

As unit root tests often lack the power to distinguish between a truly nonstationary I(1) series and a stationary series embodying a structural break or shift, time series are often first-differenced if they do not receive a clean bill of health from unit root testing[3]. Many time series exhibit too much long-range dependence to be classified as I(0) but are not I(1). The ARFIMA model is designed to represent these series.

3.3. (G)ARCH Based Models

The starting point for ARCH-based models is the article of Engle (1982). After dissemination of this article, hundreds of models or methods have been improved, and thousands of studies concerning volatility measurement and forecasting, or comparison of them has been done via using the improved models. Meanwhile, need for further analysis has occurred due to the improvements in financial engineering, and after a while, gathering and compiling them has required to accumulate them. One of such studies belongs to Bollerslev (2009).

Andersen et al. (2002) noted that since Engle's seminal paper on ARCH models in 1982, the econometrics literature has focused considerable attention on time-varying volatility and the development of new tools for volatility measurement, modeling and forecasting.

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[3] This expression is cited from the lecture notes of Baum, C. İn the following link: http://fmwww.bc.edu/ec-c/s2013/327/EC327.S2013.nn5.slides.pdf (12.05.2015)

As mentioned before, “ARMA models are used to model the conditional expectation of a process given the past, but in an ARMA model the conditional variance given the past is constant. However, an ARMA model cannot capture this type of behavior because its conditional variance is constant. So we need better time series models if we want to model the nonconstant volatility. We look at GARCH time series models that are becoming widely used in econometrics and finance because they have randomly varying volatility (Ruppert, 2011).”

Conditional variance of an ARCH model resembles conditional expectation (mean) of AR models, and this kind of relationship is similar for ARMA and GARCH models respectively (i.e. AR→ARCH and ARMA→GARCH).

Consider a GARCH(1,1) model for y_t:

$$y_t=\epsilon_t$$

$$\epsilon_t=\sigma_t\eta_t, \qquad \eta_t \sim N(0,1) \text{ where } \eta_t \text{ is IID}$$

$$h_t=\alpha_0+\alpha_1\epsilon_{t-1}^2+\beta_1\sigma_{t-1}^2$$

The first equation can be converted to an ARMA(1,1) model as follows:

$$\mu_t=a_0+a_1y_{t-1}+\theta_1\epsilon_{t-1}$$

The random variable, η_t, is an innovation term which is “assumed” to be IID with mean zero and unit variance. The random variable ϵ_t is conditionally normal when η_t is normally distributed.

The GARCH(1,1), a model for conditional variance σ_t^2, can be formulated as a linear function of last period's “squared error” and “conditional variance”. Making the proces more well-defined, the parameters should be restricted as follows: $\alpha_0>0$, $\alpha_1\geq0$, $\beta_1\geq0$, $\alpha_1+\beta_1<1$.

The term “persistence” to shocks or “long memory” in GARCH specifications could be measured with sum of ($\alpha_1+\beta_1$). That the sum goes to 1 makes the model more persistent. Empirical estimations find that nonnormally distributed return distribution generates persistence.

Two classes of stationary time series processes are based on their dependence structure: short-range dependence (or short-memory) and long-range dependent (or long-memory). The behavior of the autocorrelation function, ρ, is the differentiating factor. Short-range dependence means fast (i.e. exponential) decay of the autocorrelations. Long-range dependence means slow decay of the autocorrelations; past observations have a persistent impact on future realizations. A weakly stationary process has long memory if $\sum_{k=0}^{\infty} | \rho(k) |$ does not converge. The autocorrelation function has a hyperbolic instead of exponential decay. In practice, it is hard to distinguish between a long-memory stationary process and a nonstationary process. However, if the sample ACF is not large in magnitude but decays slowly, the series may have long memory.

Another characteristic of the GARCH specification is time varying nature of the model, since the conditional variance of ϵ_t is equal to $\alpha_0+\alpha_1\epsilon_{t-1}^2+\beta_1\sigma_{t-1}^2$ unlike the unconditional variance that does not depend on time: $\alpha_0/(1-\alpha_1-\beta_1)$.

The main obstacle for estimating GARCH models is that the conditional variance σ_t is an unobserved variable, which must explicitly be estimated, along with the parameters of the model. The lack of independence in the time series means that the joint density function is expressed as the product of conditional PDF's.

Numerical maximization yields the MLE of the parameters. The conditional mean of the data needs to be modeled well, so that the residuals obtained from it satisfy the assumption for the white-noise sequence $\{\epsilon_t\}$ which enters the conditional variance. A joint ARMA(1,1)-GARCH(1,1) model is written as

$$y_t = a_0 + a_1 y_{t-1} + b_1\epsilon_{t-1} + \epsilon_t$$

$$\epsilon_t = \sigma\eta_t \text{ where } \sigma_t^2 = \alpha_0 + \alpha_1\epsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2$$

where η_t is IID with zero mean and variance one. The parameters of the model can be jointly estimated via ML estimation. Alternatively, a two-step procedure can be preferred: The first is the estimation of the parameters of the conditional mean (first equation). Then from the residuals of the first model, the parameters of the GARCH model can be estimated. This two-step procedure is called as GARCH estimation after ARMA filtering.

A GARCH process implies (via the volatility clustering) that ϵ_t has heavy tails. Still, numerous empirical studies suggest that the GARCH(1,1) with normal distribution cannot match the large kurtosis found in return data. That is, for a GARCH model with Gaussian innovations, the assumption of conditional normality of $\{\epsilon_t\}$ usually does not hold. The error term ϵ_t is conditionally normal if the standardized residual

$$\hat{\eta}_t = \frac{\hat{\epsilon}_t}{\hat{\sigma}_t}$$

is normally distributed ($\hat{\sigma}_t$ is the fitted volatility at time t). Typically, the standard normality tests applied to $\hat{\eta}_t$ indicate that $\hat{\eta}_t$ are not normal. To reject the non-normality in conditional returns, Student's t distribution may be employed. The GARCH model with the assumption $\hat{\eta}_t \sim t(\upsilon)$ is denoted as GARCH-t model. Estimation is performed by numerical maximization of the likelihood function.

3.3.1. ARCH process (Engle, 1982)

In ARCH(q), h_t is a function of q past squared returns. "The basic idea of ARCH models is that (a) the shock a_t of an asset return is serially uncorrelated, but dependent, and (b) the dependence of a_t can be described by a simple quadratic function of its lagged values (Tsay, 2010)." Specifically, an ARCH(m) model assumes that

$$a_t = \sigma_t \epsilon_t, \quad \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \ldots \alpha_m a_{t-m}^2$$

where ϵ_t is the sequence of IID random variables with mean zero and variance 1, $\alpha_i \geq 0$.

For a given sample, a_t^2 is an unbiased estimate of σ_t^2. Therefore, it is expected that a_t^2 is linearly related to $a_{t-1}^2 \ldots a_{t-m}^2$ in a manner similar to that of an autoregressive model of order m. Note that a single a_t^2 is generally not an efficient estimate of σ_t^2, but it can serve as an approximation that could be informative in specifying the order m. Alternatively, define $\eta_t = a_t^2 - \sigma_t^2$. It can be shown that η_t is an autocorrelated series with mean 0.

$$\sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \ldots \alpha_m a_{t-m}^2 + \eta_t$$

PACF of a_t^2 is a useful tool to determine the order m. Because η_t are not identically distributed, OLS estimates of the prior model are consistent, but not efficient (thus, PACF may not be effective when the sample size is small).

Equation above gives some clues about the dynamics of (G)ARCH process. That a_{t-1} has an unusually large absolute value relatively increases the value of σ_t, and also a_t. Large deviation in a_t leads to crucial change in σ_{t+1}^2 and so on or vice versa. Equation implies persistence. The conditional variance tends to revert to the unconditional variance provided that $\alpha_1 < 1$, so that the process is stationary with a finite variance. Since independence implies a “0” correlation but not vice versa, a process, such as the GARCH processes, where the conditional mean is constant but the conditional variance is nonconstant is an example of an uncorrelated but dependent process. “An AR(1) process has a nonconstant conditional mean but a constant conditional variance, while an ARCH(1) process is just the opposite. If both the conditional mean and variance of the data depend on the past, then we can combine the two models (Ruppert, 2010).”

The q^{th}-order linear ARCH(q) model provides a particularly convenient and natural parameterizarion for capturing the tendency for large (small) variances to be followed by other large (small) variances,

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2$$

where for the conditional variance to non-negative and the model well-defined has to be positive and all of the i’s non-negative. Most of the early empirical applications of ARCH models were based on the linear ARCH(q) model with the additional constraint that the α_i’s decline linearly with the lag,

$$\sigma_t^2 = \omega + \alpha \sum_{i=1}^{q} (q + 1 - i) \varepsilon_{t-i}^2$$

in turn requiring the estimation of only a single parameter irrespective of the value of q. More generally, any non-trivial measurable function of the time (t-1) information set, such that

$$\varepsilon_t = \sigma_t z_t$$

where is a sequence of independent random variables with mean zero and unit variance, is now commonly referred as an ARCH model.

A deficiency of ARCH(q) models is that high volatility leads to high-frequency oscillations in the conditional variance process in a short fluctuation. It may be caused from the non-constant conditional variance that produces outliers when it is large. On the other hand, GARCH models enable us to reflect other alternative behaviors.

In addition other weaknesses could be added to the negative side of the model:

1. "The model assumes that positive and negative shocks have the same effects on volatility because it depends on the square of the previous shocks. In practice, it is well known that the price of a financial asset responds differently to positive and negative shocks.
2. The ARCH model is rather restrictive. For instance, α_1^2 of an ARCH(1) model must be in the interval [0,13] if the series has a finite fourth moment. The constraint becomes complicated for higher order ARCH models. In practice, it limits the ability of ARCH models with Gaussian innovations to capture excess kurtosis.
3. The ARCH model does not provide any new insight for understanding the source of variations of a financial time series. It merely provides a mechanical way to describe the behavior of the conditional variance. It gives no indication about what causes such behavior to occur.
4. ARCH models are likely to overpredict the volatility because they respond slowly to large isolated shocks to the return series (Tsay, 2010)."

3.3.2. GARCH (Bollerslev, 1986) and EWMA

In order to notice the difference, both the ARCH(q) and GARCH(p,q) models are given together below.

ARCH (q): $h_t = \omega + \sum_{k=1}^{q} \alpha_k \epsilon_{t-k}^2$ where ω>0 and α≥0.

GARCH(p,q): $h_t = \omega + \sum_{k=1}^{q} \alpha_k \epsilon_{t-k}^2 + \sum_{j=1}^{p} \beta_{kj} h_{t-j}^2$

where ω>0. For finite variance (stationarity) $\sum \alpha_k + \sum \beta_j < 1$.

The GARCH(p,q) model is also converted into the following form:

$$\hat{h}_{t+h} = \hat{\omega} \sum_{i=0}^{h-2} (\hat{\alpha} + \hat{\beta})^i + (\hat{\alpha} + \hat{\beta})^{h-1} \hat{h}_{t+1}$$

It can be seen from this formulation that as "(α+β) approaches unity, the long-run variance approaches nonstationarity (Ding and Meade, 2010)." For this reason we use (α+β) as a measure of the persistence of the GARCH volatility process.

Additionally, exponentially weighted moving average (EWMA) method of RiskMetrics in 2001 could be mentioned in the title of "GARCH". It is likely to be defined as the simplified form of persistent GARCH model by fixing the parameters of GARCH model, rather than optimizing them. EWMA implies that volatility follows a RW process. Therefore, it is assumed that any shock in return leads to persistent effect on volatility to the extent that volatility does not have a long-run mean level to revert to. EWMA model is given below.

$$\hat{\sigma}^2_{t+h} = \hat{\omega} \frac{1-\lambda}{1-\lambda^{n+1}} \sum_{i=0}^{n} \lambda^i r^2_{t-i}$$

and $0 \leq \lambda \leq 1$ is the decay factor, which typically takes a value between 0.94 and 0.97. Note that a single volatility prediction applies to all future time horizons.

In GARCH(p,q), additional dependencies are permitted on p lags of past h_t. EWMA is nonstationary version of GARCH(1,1) where the persistence parameters sum to 1 and there is no finite fourth moment. I(d) process has a positive drift term or a time trend in volatility level which is not observed in practice. There are many DGP's, other than an I(d) process, that also exhibit long memory in covariances.

3.3.3. IGARCH (Engle and Bollerslev, 1986)

"If the AR polynomial of the GARCH has a unit root, then we have an integrated GARCH or IGARCH model. Thus, IGARCH models are unit-root GARCH models. Similar to ARIMA models, a key feature of IGARCH models is that the impact of past squared shocks $\eta_{t-i} = a^2_{t-i} - \sigma^2_{t-i}$ for $i > 0$ on a^2_t is persistent (Tsay, 2010)." An IGARCH(1,1) model can be written as

$$a_t = \sigma_t \epsilon_t \qquad \sigma^2_t = \alpha_0 + \beta_1 \sigma^2_{t-1} + (1-\beta_1)\, a^2_{t-1}$$

where $\beta_1 \in (0,1)$.

IGARCH model formalizes this, so that for an IGARCH(1,1) model we have the restriction that $a_1 + b_1 = 1$. It implies that shocks to the conditional variance never die out. More formally, the simple GARCH(1,1) model can be rewritten in a form that resembles an ARMA process, making it easier to analyze certain of its properties:

$$(1-\alpha_1 L-\beta_1 L)\epsilon^2_t = \alpha_0 + (1-\beta_1 L)(\epsilon^2_t - \sigma^2_t)$$

where L denotes the lag operator ($L\epsilon^2_t = \epsilon^2_{t-1}$). The IGARCH model corresponds to the case when the polynomial $(1-\alpha_1 L-\beta_1 L)$ contains unit root.

Starica et al. (2006) had shown that the IGARCH model outperforms GARCH(1,1) in long-horizon forecasting, and documented a possible strong discrepancy between measures of unconditional variance provided by GARCH and by actual sample data. However, unlike IGARCH, the GARCH models do not satisfy weakly stationary conditions in the presence of autocorrelation. IGARCH is used as an auxiliary model in the EMMs procedure, which eases computation of the SVM estimates. The EMMs aims to combine the efficiency of MLE with the flexibility of the GMMs. The choice of IGARCH matches certain characteristics of long-horizon data, and ensures that the SVM model is feasible in this setting.

3.3.4. FIGARCH (Baillie, Bollerslev and Mikkelsen, 1996)

A richer class of models allowing for intermediate degrees of persistence (between short memory and infinite memory) is the fractionally-integrated GARCH (FIGARCH) model. Fractional orders of integration are as follows

$$(1-\alpha_1 L-\beta_1 L)\epsilon_t^2=\phi(L)(1-L)^d \epsilon_t^2 \ .$$

The FIGARCH model is between the IGARCH model (d=1) and the GARCH model (d=0). For values of $0<d<0.5$, the FIGARCH model implies eventual slow hyperbolic decay of the autocorrelations of ϵ_t^2, $\rho(h)\sim h^{2d-1}$. That is, it incorporates the long memory in volatility.

3.3.5. APARCH models (Ding, Granger and Engle, 1993)

"In some financial time series, large negative returns appear to increase volatility more than do positive returns of the same magnitude. This is called the leverage effect. Standard GARCH models cannot model the leverage effect because they model σ_t as a function of past values of a_t^2 -whether the past values of a_t are positive or negative is not taken into account. The problem here is that the square function x^2 is symmetric in x. The solution is to replace the square function with a flexible class of nonnegative functions that include asymmetric functions. The APARCH (asymmetric power ARCH) models do this (Ruppert, 2010)."

Its variance equation is defined as:

$$\varepsilon_t=\eta_t h_t, \qquad h_t^{\delta} =\alpha_0+\sum_{i=1}^{q} \alpha_i(|\varepsilon_{t-i}| - \gamma_i\varepsilon_{t-i})^{\delta} +\sum_{j=1}^{p} \beta_j h_{t-j}^{\delta}$$

where $\eta_t\sim N(0,1)$ and with the restrictions of $\alpha_0>0$, $\delta\geq 0$, $\alpha_i\geq 0$, $\gamma_i \in (0,1)$, and $\beta_i>0$. (Ruppert, 2010). The effect of ε_{t-i} upon σ_t is through the function $g_{\gamma i}$, where $g_\gamma(x) =|x|-\gamma x$. When $\gamma>0$, $g_\gamma(-x)> g_\gamma(x)$ for any $x>0$, so there is a leverage effect. If $\gamma<0$, then there is a leverage effect in the opposite direction – positive past values of ε_t increase the volatility.

“The APARCH model includes the following special cases:

- ✓ ARCH model, if $\delta = 2$, $\gamma_i = 0$ and $\beta_j = 0$;
- ✓ GARCH model, if $\delta = 2$ and $\gamma_i = 0$;
- ✓ TS-GARCH (Schwert in 1989; Taylor in 1986), if $\delta = 1$ and $\gamma_i = 0$;
- ✓ T-ARCH (Zakoian in 1994), if $\delta = 1$;
- ✓ N-ARCH (Higgins and Bera in 1992), if $\gamma_i = 0$ and $\beta_j = 0$;
- ✓ log-ARCH (Geweke in 1986; Pentula in 1986), if $\delta \to 0$;
- ✓ GJR-GARCH (Glosten, Jaganathan and Runkle in 1993), if $\delta = 2$ (Pfaff, 2013).”

3.3.6. GJR-GARCH (Glosten- Jaganathan- Runkle, 1993)

GJR-GARCH assumes the form of:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{s}(\alpha_i + \gamma_i I_{t-i})a_{t-i}^2 + \sum_{j=1}^{m}\beta_j\sigma_{t-j}^2$$

where I_{t-I} (indicator function) is 1 if $a_{t-i}<0$, and 0 if $a_{t-i}\geq 0$. To make a distinction, “0” is used as a threshold whether there is an impact of the past shocks. GJR-GARCH(1,1) is as follows:

$$\sigma_t^2 = \alpha_0 + (\alpha_1 + \gamma_i I_{t-1})a_{t-1}^2 + \beta\sigma_{t-1}^2$$

“GJR-GARCH allows the conditional variance to respond differently to the past negative and positive innovations (Bollerslev, 2009).”

It is closely similar to Threshold GARCH or TGARCH of Zakoian in 1994.

3.3.7. EGARCH (Nelson, 1991)

Exponential GARCH or EGARCH, as a further step to standard GARCH, can reflect asymmetric effects between positive and negative movements by weighting innovations. While the GARCH model imposes the nonnegative constraints on the parameters, EGARCH models the log of the conditional variance so that there are no restrictions on these parameters:

$$\log(\sigma_t^2) = \omega + \beta \log(\sigma_{t-1}^2) + \alpha\left(\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right| - E\left(\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right|\right)\right) + \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}}$$

Exponential, rather than quadratic, leverage effect could be seen in the left hand side. Logarithmic modeling leads to nonnegativity in volatility forecasts.

"The presence of leverage effects can be tested by the hypothesis that $\gamma < 0$. If $\gamma \neq 0$, then the impact is asymmetric. E-GARCH basically models the log of the variance (or standard deviation) as a function of the lagged logarithm of the variance and the lagged absolute error from the regression model. It also allows the response to the lagged error to be asymmetric, so that positive regression residuals can have a different effect on variance than an equivalent negative residual (Miron and Tudor, 2010)."

3.3.8. GARCH-Regime Switching (Hamilton, 1989)

Most generalizing form of GARCH regime switching or GARCH-RS model is as follows:

$$h_{t,St-1} = \omega_{St-1} + \alpha_{St-1}\epsilon_{t-1}^2 + \beta_{St-1} h_{t-1,St-1}$$

where S_t indicates the state of regime at time t.

It has long been argued that the financial market reacts to large and small shocks differently and the rate of mean reversion is faster for large shocks. "The earlier RS applications assume that conditional variance is state-dependent but not time-dependent. In these studies, only ARCH class conditional variance is entertained. Recent extensions allow GARCH-type heteroscedasticity in each state and the probability of switching between states to be time-dependent. More recent advancement is to allow more flexible switching probability (Poon,2005)."

Peria (2001) enabled the transition probabilities to vary according to economic conditions with the RS-GARCH model below:

$$r_t|\phi_{t-1}\ N(\mu_i, h_{it})\ with\ probability\ (p_{it})$$

$$h_{it} = \omega_i + \alpha_i\epsilon_{t-1}^2 + \beta_i h_{t-1}$$

where (i) represents a particular regime.

3.3.9. Smooth Transaction GARCH (Taylor, 2004)

GARCH-RS model has been extended to the ST-GARCH (Smooth transaction GARCH) model

$$ht = \omega + [1-F(\epsilon_{t-1})]\alpha\epsilon_{t-1}^2 + F(\epsilon_{t-1})\delta\epsilon_{t-1}^2 + \beta h_{t-1}$$

where $F(\epsilon_{t-1})=(1+\exp(-\theta\epsilon_{t-1}))^{-1}$ for logistic STGARCH, $1+\exp(-\theta\epsilon_{t-1})$ for exponential STGARCH. It belongs to the class of long memory GARCH models.

GARCH-RS could generate better forecasting performance for interest bearing assets, since “interest rates are different to the other assets in that interest rates exhibit ‘level’ effect, i.e. volatility depends on the level of the interest rate. … There is no such level effect in exchange rates Poon (2005).”

There has been many studies regarding forecasting performance of GARCH-RS empirically, one of which is Hamilton and Susmel (1994) finding that ARCH-RS with leverage effect that produces better volatility forecast than the asymmetric version of GARCH. “Gray in 1996 fits a GARCH-RS (1,1) model to US one-month T-Bill rates, where the rate of mean level reversion is permitted to differ under different regimes, and finds substantial improvement in forecasting performance. Klaassen in 1998 also applies RSGARCH (1,1) to the foreign exchange market and finds a superior, though less dramatic performance (Poon, 2005).”

3.3.10. CGARCH (Engle and Lee, 1999)

Component GARCH or C-GARCH’s volatility function is as follows:

$$(\sigma_t^2 - \sigma^2) = \alpha\,(\varepsilon_{t-1}^2 - \sigma^2) + \beta(\sigma_{t-1}^2 - \sigma^2) \text{ where } \sigma^2 = h = \omega / (1-\alpha-\beta)$$

$$(h_t - m_t) = \alpha(\varepsilon_{t-1}^2 - m_{t-1}) + \beta(h_{t-1} - m_{t-1}) \equiv u_t$$

$$m_t = \omega + \rho m_{t-1} + \varphi(\epsilon_{t-1}^2 - h_{t-1})$$

σ^2 means unconditional variance, but m_t is equal to long run variance. CGARCH accounts for long run volatility dependencies. “Volatility process is modelled as the sum of a time varying trend or permanent process, m_t, that has memory close to a unit root, and a (short-run) transitory mean reverting process, u_t, that has a more rapid time decay. The model can be seen as an extension of the GARCH(1,1) model with the conditional variance mean-revert to a long term trend level, m_t, instead of a fixed position at σ. m_t is permitted to evolve slowly in an autoregressive manner (Poon, 2005)”

“This model has various interesting properties: (i) both m_t and u_t are driven by (ϵ_{t-1}^2 -h_{t-1}), (ii) the short-run volatility component mean reverts to zero at a geometric rate of (α + β) if 0 <(α + β)< 1, (iii) the long-run volatility component evolves over time following an AR process and converge to a constant level defined by ω/ (1 − ρ) if 0 <ρ < 1, (iv) it is assumed that 0 < (α + β) < ρ< 1 so that the long-run component is more persistent than the short-run component (Poon, 2005).”

CGARCH model can reflect the dynamics behind pricing and valuation of financial instruments. For example, generally large jumps' persistence is not as long as mild shocks' persistence owing to the ordinary events. It enables large shocks to be transitory.

3.3.11. Model specification

To estimate the ACF, we use the sample ACF $\gamma(h)$. To estimate $\rho(.)$, we use the sample autocorrelation function (sample ACF) defined as $\hat{\rho}(h)=\frac{\hat{\gamma}(h)}{\hat{\gamma}(0)}$. The sample ACF decays to zero quickly, indicating clearly that the differenced series is stationary.

Many statistical software packages have functions to automate the search for the AR model that optimizes AIC or other criteria. On the other hand, The R function auto.arima can select all three parameters, p, d and q, for an ARIMA model. The differencing parameter d is selected using the KPSS test. If the null hypothesis of stationarity is accepted when the KPSS is applied to the original time series, then d = 0. Otherwise, the series is differenced until the KPSS accepts the null hypothesis. After that, p and q are selected using either AIC or BIC.

$$AIC(p,q) = \ln(\hat{\sigma}^2) + \frac{2(p+q)}{T}$$

$$BIC(p,q) = \ln(\hat{\sigma}^2) + \frac{\ln(T)(p+q)}{T}$$

$$HQ(p,q) = \ln(\hat{\sigma}^2) + \frac{\ln(\ln(T))(p+q)}{T}$$

where $\hat{\sigma}^2$ denotes the estimated variance of an ARMA(p, q) process (Pfaff, 2010)." The lag order (p, q) that minimizes the information criteria is then selected.

As an alternative, a likelihood-ratio test can be computed for an unrestricted and a restricted model. The test statistic is defined as:

$$2[\mathfrak{L}(\hat{\theta}) - \mathfrak{L}(\tilde{\theta})] \sim \chi^2(m)$$

where $\mathfrak{L}(\hat{\theta})$ denotes the estimate of the unrestricted log-likelihood and $\mathfrak{L}(\tilde{\theta})$ that of the restricted log-likelihood, m is the number of restrictions. Next, one should check the model's stability as well as the significance of its parameters.

3.4. Stochastic Volatility

One way of doing so is to model the evolution of volatility deterministically, i.e. through the (G)ARCH class of models. As an alternative, Taylor developed a model the volatility probabilistically, i.e. through a state-space model where the logarithm of the squared volatilities the latent states follow an autoregressive process of order one. Over time, this specification became known as the stochastic volatility (SV) model.

“GARCH models can easily be adjusted to incorporate multiple characteristics of volatility, but often require specific assumptions regarding the distribution of returns. These assumptions may not be consistent with the actual tail behaviours of targeted financial assets. SV utilizes latent factors that imitate specifications of asset returns and volatility behaviours to generate time-varying volatility in diffusion. Such approach provides the only modelling that incorporates dynamics of both the asset prices and volatilities together in volatility forecasting. However, the structure of this model is rather complicated and its estimation process often proves less efficient than other methods (Huang,.

Unlike the GARCH model, where conditional volatility is determined by lagged residuals and lagged conditional volatility, the SV model, introduced by Taylor in 1982 considers volatility as a stochastic process (Ding and Meade, 2010).

$$r_t = \bar{\sigma} e^{0,5 h_t} \xi_t \qquad where\, \xi_t \sim N(0,1)$$

$$h_t = \phi h_{t-1} + \eta_t \qquad where\, \eta_t \sim N(0, \sigma_\eta^2)$$

$$E(\xi_t \eta_t) = \rho \sigma_\eta$$

After the log transformation, the information regarding the correlation coefficient is lost.

$$ln\, r_t^2 = ln \bar{\sigma}^2 + h_t + \varepsilon_t \qquad where\, \varepsilon_t\ (= ln \xi_t^2) \sim \ln(\chi_1^2)$$

$$h_t = \phi h_{t-1} + \eta_t \qquad where\, \eta_t \sim N(0, \sigma_n^2) \quad E(\varepsilon_t \eta_t) = 0$$

The objective of Ding and Meade’s study (2010) was to identify the volatility scenarios that favour either GARCH or SV models, based on the persistence of volatility (its robustness to shocks) and the volatility of volatility. According to the study, SV model forecasts are only noticeably more accurate than GARCH in scenarios with very high volatility of volatility and a stochastic volatility generating process.

Volatility is subject to a source of innovations that may (not) be related to those that drive returns. Modelling volatility as a stochastic variable immediately leads to fat tail distributions for returns. Autoregressive term in the volatility process introduces persistence, and correlation between the two innovative terms in the volatility process and the return process produces volatility asymmetry. Long memory SV models have also been proposed by allowing volatility to have a fractional integrated order. The volatility noise term makes the SV model a lot more flexible, but as a result the SV model has no closed form, and hence cannot be estimated directly by MLE. QMLE approach is inefficient if volatility proxies are non-Gaussian. The alternatives are GMM approach through simulations by Duffie and Singleton in 2001, and likelihood approach through numerical integration by Fridman and Harris in 1988 or Monte Carlo integration using either importance sampling (Poon, 2005).

Huang (2011) explained the SV model being able to study the time-varying structure of an asset's random behaviour: the mechanisms driving volatility, jumping patterns and characteristics of formation.

$$r_t = \sigma_t Z_t$$

$$ln\,\sigma_t^2 = w + \beta\, ln\sigma_{t-1}^2 + \sigma_u u_t$$

$$(Z_t, u_t) \sim IID\ N(0, I_2)$$

The equations have only three free parameters: w, β and σ_u. Z_t and u_t are error terms for the asset return and log-normal volatility, respectively. Many financial assets present clustering, time-varying, diffusion and jumping behaviours or they are generated with a noise term that depends on current returns. SVM accounts for such variations by explicitly allowing for both persistent and time-dependent volatility terms in its diffusion equation while specifying a random walk behaviour for the return itself. In addition, the model generates a leptokurtic distribution, which is consistent with the observed outliers of many financial assets. The main drawback of the SVM is that estimation and empirical applications are very difficult. However, in recent years, SVM has gone through several significant refinements including a series of increasingly efficient estimation procedures and applications of the Levy process. Furthermore, for certain financial assets the SVM may have no closed form solution; in such cases one must rely on complex calibration procedures or simulation techniques for reliable estimates. These shortcomings have made SVMs less popular than GARCH models in the financial industry.

SVM estimation procedures fall into three classes: the method of moments, maximum likelihood and simulations such as Monte Carlo. Knowing that the method of moments is generally feasible, but may be less efficient than the other two approaches according to the prior research, he followed Andersen et al. (1999) based on the formation of simulated moments in employing the Efficient Method of Moments (EMMs) to estimate the SVM with one modification.

According to Wu (2005), “Levy processes can capture the behaviors of return innovations on a full range of financial securities. Applying stochastic time changes to the Levy processes randomizes the clock on which the processes run, thus generating stochastic volatilities and stochastic higher return moments.”

Therefore, with appropriate specification of Levy processes and stochastic time changes, the return dynamics of virtually all financial securities could be captured, and economic meanings of all components and its time change in return dynamics could explicitly be assigned in contrast to the hidden factor approach.

While the BM component in a Levy process generates a normal distribution, any non-normal distribution can be generated via the appropriate specification of the Levy density for a Levy jump process, which determines the arrival rate of jumps of all possible sizes. Accordingly, we can model the return innovation using several Levy processes as building blocks matching the distributional behavior of shocks from different economic forces. Furthermore, applying stochastic time change to each Levy component randomizes the clock on which the process runs, thus capturing the stochastically varying impacts from different economic forces. Statistically, applying stochastic time changes on different Levy components can generate both stochastic volatility and stochastic higher return moments, both of which are well-documented features for financial securities.

In designing models for a financial security return, the literature often starts by specifying a very general process with a set of hidden factors and then testing different restrictions on this general process.

A Levy process is a continuous time stochastic process with stationary independent increments, analogous to IID innovations in a discrete-time setting. "Until very recently, the finance literature narrowly focuses on two examples of Levy processes: the BM underlying the Black and Scholes (1973) model and the compound Poisson process with normal jump sizes underlying the jump diffusion model of Merton (1976) (Wu, 2005)." A BM generates normal innovations. The compound Poisson process in the Merton model generates return non-normality through a mixture of normal distributions with Poisson probability weightings. A general Levy process can generate a much wider range of distributional behaviors through different types of jump specifications. The compound Poisson process used in the Merton model generates a finite number of jumps within a finite time interval. Such a jump process is suitable to capture rare and large events such as market crashes and corporate defaults. Nevertheless, many observe that asset prices can also display many small jumps on a fine time scale. A general Levy process can not only generate continuous movements via a BM and rare and large events via a compound Poisson process, but it can also generate frequent jumps of different sizes.

4 PERFORMANCE OF VOLATILITY MEASURES

Analysis of forecasting ability of models by comparing simulated (or expected) and out-of-sample realized values could be made with some tools, having relatively certain pros and cons. These tools include mean absolute error (MAE), mean squared error (MSE), root mean squared error (RMSE), coefficient of variation (CV), Logaritmic error statistics (LL statistics), Mincer–Zarnowitz (MZ) regression etc. In this section, they are briefly explained in terms of which tools were used in previous analysis before and why MSE is selected in the thesis.

4.1. The Tools Used in Previous Analysis

Comparing forecasting performance of volatility models, Poon ve Granger (2005) compiled and reviewed 93 studies, 63 of which include pairwise comparisons between the groups of volatility methods. For those involving both historical volatility and GARCH models, 22 found historical volatility better at forecasting than GARCH (56% of the total). The ranking is that the first is implied volatility, the second and the third are historical volatility and GARCH respectively although historical volatility and GARCH roughly have identical performances. The result was not considered as surprising, since implied volatility uses option prices, and therefore utilize from a wide and more relevant range of data. The others are also less practical, not being available for all assets.

Another study, Hansen and Lunde (2005), sought an answer to the question of whether complicated volatility models provide a better description of returns as compared with less sophisticated models. They compared 330 GARCH-based models to 6 different loss functions with respect to their ability to forecast the 'one-day-ahead' conditional variance by substituting the realized variance for the latent σ_t, using the series of DM-USD parity and IBM stock returns. The realized variance for a particular day came from intraday returns, $r_{i,m}$ in this study. Superior Predictive Ability (SPA) of Hansen in 2001 and the reality check for data snooping (RC) by White in 2000 were used as a benchmark (testing vehicle), and six measures are calculated to make comparison, including MSE1, MSE2, QLIKE, R2LOG, MAE1 and MAE2. Main finding in this study is that “there is no evidence that GARCH(1,1) model is outperformed by other models.”

Another article related to the comparison of volatility forecasting methods belongs to Ding and Meade (2010), using exchange rates, equity indices, and commodities as time series; GARCH, SV and EWMA models as methods, CV and CV^2 as measures for volatility of volatility, RMSE and MME as measures of forecasting accuracy. They characterize a volatility method in terms of its persistence (its robustness to shocks) and the volatility of volatility. As mentioned before, CV and CV^2 measure the volatility of volatility, widely used as statistical measures of dispersion of a variable to its mean. The other dimension is persistence of volatility; the higher the measure is, the longer the effect of a shock persists. For a GARCH model, persistence is measured by $(\alpha+\beta)$, and for an SV model, it is measured by (Φ). These measures behave similarly in that$(\alpha+\beta)=\Phi= 0$ implies a constant variance and no persistence, whereas $(\alpha+\beta)=\Phi=1$ implies the variance process is integrated and has high persistence.”

The last but not the least article that we summarize in this context is Chuang et al. (2007) that aims at comparing the effects of alternative distributional assumptions on predictive accuracy of volatility forecasting. Some probability distributions are expected to reflect both the volatility clustering effect and also accomodation of leptocurtosis more than GARCH based models of normal conditional distribution. In this article, GARCH(1,1) volatilities of stock market indices and exchange rate returns belonging to seven countries were measured under the assumptions of 13 different probability distributions, then the results are compared via the tools of MAE, RMSE and LL. The properties of a suitable distribution function was qualified that, primarily, the distribution must be flexible to ensure considerable range of shapes, then, the skewness and kurtosis of a return series are reflected in shape parameteres of disribution, and ultimately, location, scale and shape parameters are estimable.

Other aspects that must be indicated to analyze the volatility are "fat tail distributions of asset returns, volatility clustering, asymmetry and mean reversion, and comovements of volatilities across assets and financial markets (Poon, 2005)". Also, forecast estimates will differ depending on the current level of volatility, volatility structure (e.g. the degree of persistence and mean reversion, etc.) and the forecast horizon. Poon (2005) pointed out that the prominent evaluation measures in the literature are ME, MSE, RMSE, MAE, MAPE, HMSE and LINEX, and stresses that these error statistics "are themselves subject to error and noise". Therefore, to reach a conclusion that the one is better than the others, test of significance should be made. The testing procedures were studied by West in 1996, West and Cho in 1995 and West and McCracken in 1998 to show how standard errors for ME, MSE, MAE and RMSE can be derived taking into account serial correlation in the forecast errors and uncertainty inherent in volatility model parameter estimates (Poon, 2005).

In other words, in case that returns are IID or strict WN, then variance of returns over a long time distance can be derived as a simple multiple of single period variance. However, this assumption is not valid for many return series due to the aforementioned reasons above. "While a point forecast of $\widehat{\sigma}_{T-1,T|t-1}$ becomes very noisy as $T \rightarrow \infty$, a cumulative forecast $\widehat{\sigma}_{t,T|t-1}$, becomes more accurate because of errors cancellation and volatility mean reversion unless there is a fundemental change in the volatility level or structure (Poon, 2005)."

Another decision whether to use intra-daily, daily, weekly or monthly data is significantly related to the intention of forecast horizon partly due to mean reversion. It is well known that volatility accuracy generally improves when data sampling frequency increases relative to forecast horizon. Figlewski (1997) finds that forecast error doubled in size when daily data, instead of monthly data, is used to forecast volatility over two years. "In some cases, e.g. when the forecast horizon exceeds ten years, a volatility estimate calculated using weekly or monthly data is better as volatility mean reversion is difficult to adjust using high frequency data." Volatility structure is not effected from the data frequency, but it is not valid for practice due to the volatility persistence, which is highly significant in the daily data, but weakens as the frequency of data decreases.

Another discussion point is whether there is direct relationship between predictive power and explanation capability of a model. Financial returns display pronounced volatility clustering, in other words, intertemporal volatility persistence. In spite of highly significant in-sample parameter estimates, some articles show that standard volatility models explain little of the variability in ex-post squared returns. In contrast, well-specified volatility models provide strikingly accurate volatility forecasts. Andersen and Bollerslev (1998) said that "there is, in fact, no contradiction between good volatility forecasts and poor predictive power for daily squared returns. Apparent poor predictive power of well-specified volatility models is explicit. Let the return innovation be written as $r_t=\sigma_t z_t$, where $z_t \sim N(0,1)$ while the latent volatility σ_t, evolves in accordance with the particular model entertained. A common approach for judging the practical relevance of any model is to compare the implied predictions with the subsequent realizations. However, volatility is not directly observed so this approach is not immediately applicable for volatility forecast evaluation. Still if the model for σ_t^2 is correctly specified, then $E_{t-1}(r_t^2) = E_{t-1}(\sigma_t^2 . z_t^2) = \sigma_t^2$, which appears to justify the use of the squared return innovation over the relevant horizon as a proxy for the ex post volatility. Unfortunately, while the squared innovation provides an unbiased estimate for the latent volatility factor, it may yield very noisy measurements due to the idiosyncratic error term, z_t^2. This component displays a large degree of observation-by-observation variation relative to σ_t^2. Thus, the poor predictive power of volatility models, when judged by standard forecast criteria using r_t^2 as a measure for ex-post volatility, is an inevitable consequence of the inherent noise in the return generating process."

4.2. Criteria for Comparison

If ARCH process could be properly designed, the standardized residuals $\tilde{a}_t=a_t/\sigma_t$ form a sequence of IID random variables. Hence, the adequacy of a fitted ARCH model can be clarified with examining the series $\{\tilde{a}_t\}$ (Tsay, 2010). Which ways to obtain information about the adequacy are descriptive statistics, including, first of all, the Ljung–Box statistics of $\tilde{a}_t$ can be used to check the adequacy of the mean equation and that of $\tilde{a}_t^2$ can be used to test the validity of the volatility equation. Also, skewness, kurtosis, and QQ plot of $\{\tilde{a}_t\}$ can be used to check the validity of the distribution assumption.

4.2.1. MAE, MSE and RMSE

Their formulations are as follows:

$$\text{MAE}=\frac{1}{m}\sum_{t=T+1}^{T+m}\left|\hat{h}_t-\sigma_t^2\right|$$

$$\text{MSE}=\frac{1}{m}\sum_{t=T+1}^{T+m}\left(\hat{h}_t-\sigma_t^2\right)^2$$

$$\text{RMSE}=\frac{1}{m}\sqrt{\sum_{t=T+1}^{T+m}\left(\hat{h}_t-\sigma_t^2\right)^2}$$

where m is the number of out-of-sample data and the true volatility is approximated by the squared returns. “The MAE is an orthodox forecast criterion which does not permit the offsetting effects of overand under-prediction, and weighs all forecast errors equally, while the RMSE is a conventional standard criterion and places a heavier penalty on outliers (Chuang er al., 2007)”.

4.2.2. LL statistics

MAE and RMSE statistics make some assumptions, one of which is that the loss function be symmetric, and the other is that the same absolute value of under- and over-predictions of volatility have both equal weights. However, prices of financial instruments do not be cared about up- and down-side movements equally. For example, a risk manager that concentrates more on the under-prediction of price volatility leads to a downward biased estimate of the option price. Logaritmic error statistics, or LL statistics eliminates the symmetry by summing up asymmetric bias:

$$\text{LL}=\frac{1}{m}\sqrt{\sum_{t=T+1}^{T+m}\left(\ln(\hat{h}_t)-\ln(\sigma_t^2)\right)^2}$$

4.2.3. CV, CV^2 and MME

CV and CV^2 were used in Ding and Meade (2010) in which the formulations were given as follows:

$$\text{CV}=\frac{\text{SD}}{\text{mean}}$$

$$\text{CV}_{\text{GARCH}}^2=\frac{2\alpha^2}{1-2\alpha^2-(\alpha+\beta)^2}$$

$$CV_{SV}^2=\exp(\frac{\sigma_\eta^2}{1-\phi^2}) - 1$$

Once the expected variance of the process is fixed to some arbitrary constant, the position of a GARCH model in the volatility scenario space is determined by the triplet of coefficients (α,β,ω). Similarly, the position of an SV model is determined by the triplet of coefficients (γ,Φ,σ). Since empirical studies shows that values of persistence range between 0.9 and 0.995, so that CV^2 takes the value of 10, 1, 0.1, the values of α are chosen given the values of β.

Ding and Meade (2010) designed and implemented an experiment involving the estimation of EWMA, GARCH and SV models using simulated data sets representing different scenarios, defined by the persistence of volatility and the volatility of volatility; and using volatility generating processes of GARCH and SV. "The first phase is to investigate how accurately the empirical CV^2 is calculated from the simulated series reflects the CV^2 used in the simulation. The range of parameter values (ω+α+β) for the GARCH model is chosen by first setting the persistence to realistic values, (α+β) are set to be 0.90, 0.95 and 0.98. Then for each value of α+β, the values of α are chosen such that the CV^2 takes the value of 10, 1 and 0.1. A high value of CV^2 indicates a high volatility of volatility. Lastly, values of ω are selected such that the expected variance ω/(1-α-β) equals (1%), taking the simulated data as daily returns, this corresponds to 16% annualized volatility. The values of these nine parameter triplets (ω,α,β) of GARCH and triplets of SV (σ_η, ϕ, γ) can be found in Ding and Meade (2010)."

RMSE and mean mixed error (MME) criteria measure the differences between the observed and simulated values. Since formula of RMSE is given before, only MME formulas are given below:

$$MME(U)= \frac{1}{T} \sum_{i\in U} \sqrt{|\sigma_i - \hat{\sigma}_i|} + \sum_{j\in O} \sqrt{|\sigma_j - \hat{\sigma}_j|}$$

$$MME(O)= \frac{1}{T} \sum_{i\in U} |\sigma_i - \hat{\sigma}_i| + \sum_{j\in O} |\sigma_j - \hat{\sigma}_j|$$

where $i\in U$ if $\sigma_i>\hat{\sigma}_i$ and $j\in O$ if $\sigma_i<\hat{\sigma}_i$. MME(U) penalizes underestimations more heavily than overestimations, whereas RMSE applies equal penalty regardless of the direction of errors.

They concluded that “the results for the SV model follow a similar pattern although the sample CV^2’s converge more quickly than the GARCH model. For a typical length of series, say 1000 observations, the sample CV^2 underestimates the theoretical CV^2. It is more noticeable for high values of CV2 than low values, and it is more pronounced for GARCH models than for SV models (Ding and Meade, 2010).”

5 TURKISH FINANCIAL MARKETS, ASSETS AND RETURNS

5.1 TL-denominated Financial Instruments

It is useful to separate OTC markets from exchange or other organized markets while beginning to evaluate markets for TL-denominated financial instruments. Major organized markets are BIST, Takasbank money market, and CBT Interbank Money Market; and major OTC markets are Reuters, Bloomberg, and FOREX.

Another aspect for describing the markets of TL-denominated assets is the regulation side. Main regulation and rule book for the markets is Capital Market Law with number of 6362 and date of 06.12.2012. In addition tens of communiques based on this law related to the financial products and markets were published in the years of 2013 and 2014. Hence, it could be asserted that regulation on financial markets have been renewed to a large extent. Besides that, regulation/rules of Ministry of Finance on taxation of incomes derived from financial assets, rules of Undersecreteriat of Treasury on market making and rules of CBT on banking, foreign exchange regime and markets are other remarkable legal factors needed to describe the framework for Turkish financial markets.

Describing aforesaid market and regulation framework above could enhance our ability to identify dynamics behind the price formation and the changes in markets.

In terms of transaction volume and its central position, the greatest organized market in Turkey is BIST, defined as a conglomerate of markets concerning debt securities, equities, some commodities including gold, platinium and silver, and derivatives.

BIST was established in 2012 with Capital Markets Law and by bringing Istanbul Gold Exchange (IAB), Izmir Turkish Derivatives Exchange (VOB or TURKDEX) and Istanbul Stock Exchange (ISE) all together. It can be characterized as a primary and secondary market of too many financial transactions of equities, debt securities, repo, commodities and derivatives. Thereby, as we talk about the BIST, a structure should come into mind that consists of many sub-markets, and a lot of financial products transacted in these sub-markets, and basket indices composed of the prices of these products.

Takasbank Money Market (TMM) and Securities Lending Market (SLM) were established to ensure that banks and other financial intermediaries make transactions of collateralized lending and/or borrowing with each other. Also, Takasbank has gained the title of central counterparty (CCP) since 14.08.2013, commiting to complete the clearing and settlement for markets and capital market instruments deemed to appropriate through open offer, novation or another legally binding method by acting as buyer against seller and seller against buyer.

Another platform which gives an opportunity to make depo and repo transactions for banks is CBT Interbank Money Market. Depo means the borrowing and lending transaction of banks with each other over the price/rate formerly declared by CBT. Depo transactions may be over-night (O/N), weekly, montly, and the term cannot exceed 91 days. Another facility enabled by CBT to the banks is late over night liquidity window (LON). Counterparties can use this facility at an interest rate set currently 300 bp above the central bank's O/N lending rate between 3 p.m. and 4. p.m..

The term of OTC markets corresponds to the platforms provided by the softwares of some private companies such as Thomson Reuters, Bloomberg, Forex, Telekurs, Money Line Telerate. These platforms has been used multifunctionally for trading financial assets, communicating and dealing with other agents, bidding, monitoring and actively involving in quotation process, obtaining and analyzing financial data including the behavior of market participants to improve strategy. Transactions and dialogs between the institutional agents are subsequently confirmed between the agents via messages on the SWIFT system[4]. However, these

[4] It stands for "Society for Worldwide Interbank Financial Telecommunication".

kinds of markets does not include the collateral system resembling the organized markets. This situation renders the credibility of counterparty (counterparty credit risk or CCR) significant for risk taking side. Over time, further steps have been taken by financial institutions not willing to undertake or willing to diminish their CCR. One example to these steps is to form central counterparties with the contribution of public authorities, which make cliring and settlement phases of transactions in centrally one hand. Besides, multilateral netting agreements has been performed, reducing the CCR by sharing the loss derived from the default of one side like an insurance system. Another institutional reaction to raise reliability and efficiency of OTC markets was to establish an agency named as International Swaps and Derivatives Association (ISDA) in 1985. ISDA has been improving the master agreements and wide range of related documentation materials, and ensuring the enforceability of their netting and collateral provisions. ISDA conditions turn into the international standards for netting and collateralization of the derivative transactions.

After summarizing the distinction between OTC and organized markets, and the knowledge about the organized markets, a crucial point, the possibility of leveraged finance, must be added to our information set in order to analyse effectively the formation of prices in these markets. Especially the instruments including forward settlement of cash flows and requirements of low level or unqualified collateral give a chance to create relatively high position as compared with the capital or investor’s own resources.

Collateral level given by the position holder does also determine the level of leverage. For instance an organized market stipulating 20 percent initial and maintenance margin requirements, one can make the position of 5 (=1/0.2) times higher than the level of collateral. Therefore, severe and rigorous volatility movements are expected in the markets ensuring high leveraged transactions.

On the other hand, the volatility of spot price is likely to be lower in the markets ensuring both spot and forward transactions, since the existence of prices for different maturities of the same asset could make the spot price be more stable and lead to the inertia.

Ultimately the last factor mentioned in this text which have negative effects on the volatility is deepness and size of the market. Notional amount of financial assets in a market is a stock value. The more this stock size is, the more quotations are needed to change the price level that must be both the level and the volume.

After describing the the framework of Turkish financial markets, then to complement the knowledge above, prominent features of some TL-denominated assets ought to be indicated there. By this way, representative power and coverage of our sample could be evaluated more influentially.

Table 5.1: List of TL-denominated financial instruments

1. Commodities (BIST)
   - Gold
   - Silver
   - Platinum and Palladium
2. Stocks (BIST)
   - Equity prices
   - Values of indices
   - Warrants
3. Interest rates (BIST, Bloomberg, Reuters)
   - Treasury and private bond prices
   - Investment bonds (A-B type)
   - Repo rate (O/N and one week)
   - Libor (3 months and 1 year)
   - Depo interest rate (O/N and one week)
   - Ijara bonds
4. Exchange rates (Bloomberg, Reuters, CBT Interbank)
   - USD-TRY
   - EUR-TRY
5. Derivatives (Bloomberg, Reuters, BIST)
   - Swap rates/ spreads (i.e.IRS, CCS, CS)
   - Option values (Vanilla and exotic) (OTC and VOB)
   - Credit derivatives and spreads (i.e.CDS rate)

5.1.1. TL interest bearing assets

Major markets for debt securities consist of BIST, TMM, SLM and CBT Interbank Money Market. Also bilateral transactions are performed via Reuters, Bloomberg and other platforms. Basic bond price can be calculated as the following formula:

$$P=\left(\sum_{n=1}^{N}\frac{C}{(1+i)^n}\right)+\frac{M}{(1+i)^n}=C\left(\frac{1-(1+i)^{-N}}{i}\right)+M(1+i)^{-N}$$

where C is coupon payments, i is interest rate, M is the final payment (nominal amount), N is the number of payments. While pricing or evaluating the fair value, “i” comes from the zero coupon rates obtained from the yield curve.

Specifically there are two points when calculating the bond price: daily basis (n/360 or n/365) and choice of suitable type of interest rate including simple, compound and continous compound. All interest bearing assets have their own rules and methods with respect to these choices created by market participants over time.

The main variable effecting the bond price, par value or market interest rate, comes from the equations below both of which decompose the interest rates into the components in terms of different aspects. The first explains that market interest rate comes from the risk free rate and risk premium, and the other indicates mark-up pricing (funding costs). These decompositions could be beneficial while analyzing the causes of price movements of mean or volatility.

Interest rate= risk free interest rate + risk premium

Interest rate= Funding costs (swap rate if the source of funding is denominated as foreign currency + LIBOR) + risk premium (i.e. CDS rate) + mark-up rate

In Turkish financial markets, types of assets and debt finance products have begun to have broad range of diversity for about ten years. These types vary in terms of term structures, payment schedules including fixed and flexible legs, and issuer types such as public or private. Major debt instruments are bonds and notes, profit and loss share certificates, ijara certificates, asset or mortgage backed securities, covered bonds, depo and repo financing etc. Taking into consideration this variety, volatility forecasts are performed by using the closing prices of the following interest bearing assets.

i) Benchmark three month - bond prices: debt instruments issued by the treasury, quoted and traded in BIST, and marked as benchmark or three months remained are selected. Logarithmic difference of clear price (obtained by excluding accrued interest) are considered to make calculations.

ii) Repo rates (O/N and weekly): Bloomberg and CBT Interbank Money Market data are used.

iii) Deposit rate of CBT (O/N): Bloomberg and CBT Interbank Money Market data are used.

5.1.2. Equities and their warrants

The sole equity market of Turkey is BIST. It ensures the primary (initial public offering) and secondary (trading) market operations.

Stock exchange give publicly available abundant series of stock prices, returns, indices, warrants[5] etc.

5.1.3. Foreign exchange

Turkish financial sector, reel sector firms and individuals hold extensively large amount of positions in foreign exchange (specifically in USD and EUR) from time to time, and also make foreign exchange trade. Since above-cited currencies are highly likely move in the same direction as compared with TL, only the parity of USD-TL is involved, and Bloomberg data is used.

5.1.4. Commodity

Commodities could be classified into the metal/energy and the agriculture, since they have different dynamics behind the price formation.

Agricultural products are traded in tens of regional mercantile exchanges organized under the body of The Union of Chambers and Commodity Exchanges of Turkey (TOBB). In addition, forward transactions are performed in Futures and Options Market organized under the BIST.

Mining (metal, mineral and energy) products are traded on spot price in Precious Metals and Diamond Markets organized under the BIST, and traded on forward price in Futures and Options Market organized under the BIST.

Precious Metals and Diamond Market has deemed as one of some markets for metals all over the world. Due to the fact that gold, platinium, silver and paladium must be imported on this market, trade volume seems to be relatively high. However, metal prices are determined globally in organized markets of NYMEX/COMEX settled in New York and TOCOM settled in Tokio, and OTC market of London Metal Exchange (LME). Therefore, spill-over effect occurred during the price formation in BIST for these products.

Because commodity prices are highly effected by the global markets aforesaid above, only spot price of gold is involved in this study.

[5] Warrants are capital markets instruments that give the holder the right, but not the obligation, to buy ('call' warrant) or to sell ('put' warrant) an underlying asset at a specified price (the 'strike' price or 'exercise' price) on or before a predetermined date where such right is exercised by registered delivery or cash settlement.

5.1.5. Derivatives

Derivative is a financial instrument having the following qualifications:

- Its valuation strictly depends on the change in an underlying (reference) asset. Reference asset is likely to be a financial or non-financial variable. Remarkable examples are specified interest rates, asset prices, commodity prices, exchange rates, index of prices or rates in a specified market, external credit ratings or any other indicator that could not be specified to any individual or contract.
- It includes forward cash flows.
- It requires less capital than notional amount of contract, hence includes leverage.

There are different types of derivatives in terms of counterparty type (central counterparty, multilateral or bilateral), markets in which contracts are made (OTC, or organized), reciprocity of cash flows (netting or change of gross amounts), type of cash flows (only forward cash flows or both forward and spot cash flows), complexity (vanilla -standard- or exotic –nonstandard-).

The basic derivative types are forward, futures, swap and option, and any instrument including an option has non-linear cash flows.

A sample being able to represent this broad variety mentioned above is selected and described in the following:

i) TL interest rate swap (IRS) rates: it means the change of floating rate with fixed rate or vice versa, based on a specified notional amount and for a terms structure. Receiver of floating rate based cash flow transfers its interest rate risk to the payer. Reference of the floating rate, if notional amount is TL-denominated, is likely to become Consumer Price Index or repo rates. In the mean time, diminishing notional amounts and/or different frequencies and terms for floating and fixed legs are possible options to change the standardized vanilla structure.

   When both legs are in the same currency, this notional amount is typically not exchanged between counterparties, but is used only for calculating the fair or present value of cash flows to be exchanged. When the legs are in different currencies (i.e. USD and TRY), the respective notional amount are typically exchanged at the start and the end of the swap.

ii) TL-USD cross currency swap (CCS) rates is an agreement between two parties to exchange interest payments and principals denominated in TL and USD. It includes both interes rate and foreign exchange risk factors, because its value will depend on not only USD and TRY interest rates but also forward exchange rate of USD-TRY. Therefore, it could be regarded as the combination of currency swap and IRS.

iii) TR CDS rate: “A credit default swap is a financial swap agreement that the seller of the CDS will compensate the buyer in the event of a loan default or other credit event. The buyer of the CDS makes a series of payments to the seller and, in exchange, it receives a payoff if the loan defaults. It was invented by Blythe Masters from JP Morgan in 1994”[6], and the most widespread one of credit derivatives protecting the buyer from default loss event of the counterparties. It is also deemed as a significant market signal indicating the default rate of Turkish treasury (sovereign). CDS types and definitions are generally referred to standardized ISDA contracts.

5.2 Data and Pre-assessment

Within each market, a small number of the examples are chosen for the analysis, including one FX series (USD-TL parity), one equity index (based on equity prices of 10 Turkish banks - XBN10), one CDS rate (Turkish sovereign), one commodity (gold), two derivatives (interest rate swaps and cross currency swaps), as given below.

Table 5.2: Characteristics of Data Set

No	Financial instrument / indices	Abbr.	# of data	In sample	Out of sample	Start date	Final date
1	USD-TRY exchange rate of Turkish Central Bank	USDTRY	1637	10	1627	04.01.2007	28.06.2013
2	IRS rate - 1 year	IRS1Y	1963	10	1953	03.01.2006	23.07.2013
3	Cross currency swap rate - USD/TRY - 1 y	CCS1Y	1920	10	1910	03.01.2006	23.07.2013
4	BIST commodity exchange spot gold price	GOLD	1899	10	1889	02.01.2006	19.07.2013
5	CDS rate (Turkish sovereign) - 1 year	CDS1Y	2013	10	2003	02.01.2006	28.06.2013
6	BIST stock exchange banking index	XBN10	1215	10	1205	04.01.2010	19.11.2014

[6] https://en.wikipedia.org/wiki/Credit_default_swap (26.06.2015)

All series are derived from daily closing price data, and calculated by taking logarithmic differences of consecutive closing prices.

To make an analysis of volatility and price movements between 2007 and 2010, it is indispensable to explain why the global economy experienced a crisis. There is concencus about the trigger of the crisis: problems in subprime mortgage markets of USA. Increase in probability of default (PD) rates of mortgages stemmed from some factors, one of which was that the individuals having low credibility were lended despite too high DTI[7] ve LTV[8] ratios. What is more, ARM[9] type of loans were mostly lended to them. By this way, as long as price level of real estate increased, borrowers could produce net asset and have oppotunity to refinance their loans. On the supply side of the mortgage market, investors demanded high revenue during the time of low interest rate level. ODM[10] type of financing was utilized to respond to these demands. According to the model, financial instruments having high leverages such as CDS, MBS, MBS based CDO's[11] and/or CDO based CDO^2 promised higher income to the investors having high risk appetite even if increasing the risk profile. Then, fall in credit standards, bubble in real estate prices and huge increase in leverages of financial institutions' balance sheets combined together, and led to huge increase in credit risk level. However, increase in credit risk could not be evaluated properly without delay by the credit rating agencies. Structured finance products mentioned above whose structures include hybrid, varied and complicated cash flows were subject to standard credit evaluation methodology by these agencies. Issuing MBS or CDO requires detailed and effective due diligence made by them to the extent of whether loan or CDS portfolio as a collateral of these securities are well-qualified and –rated. However, rating methodologies did not consider the changes in economic conjuncture, and were not flexible enough to respond quickly while the crisis evolved. Crisis was generally defined in the context

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[7] DTI: Debt to income ratio

[8] LTV: Loan to value rati

[9]ARM: Adjustable rate mortgage having cash flows that were structured as relatively low payments at an initial phase.

[10] ODM: Originate to distribute model. According to the model, loans are originated by financial institutions including banks, then accumulated to the buckets or tranches. After a while, credit risk of these buckets are transferred to the investors by securitizing these buckets.

[11] Mortgage backed securities and collateralized debt obligations

of Minsky moment - the point where credit supply starts to dry up, systemic risk emerges and the central bank is obliged to intervene – has duly arrived (Magnus, 2007).

Deterioration in the credit quality of collateral issuers and market illiquidity triggered the need of funding liquidity, especially for hedge funds and the similar business models having long term assets and short term liabilities. In order to compensate their increasing margin call requirements derived from the decrease in mark-to-market price of collaterals because of deteriorating credit quality, they had to sell more assets. Radical shifts in volatility and correlations could not be seen in time by financial actors, and the circumstances of one way markets, deep out of money positions having nonlinear cash flows and gapping of prices were experienced in exchange and interbank lending markets during the crisis.

Transmission mechanisms of spillover effect led to transfer the problems in US's mortgage market into the other markets no matter what the currency or the country was. Volatility and the level of TL-denominated assets' prices are substantially effected from global crisis and its transmission mechanisms after 2007. Therefore, volatility analysis of the period including the 2007 and 2008 could be beneficial for responding to the question which model is able to capture the behaviors (i.e. jumps) of financial markets and financial instruments under stressed conditions.

5.3 Analysis

Using a volatility model requires implementing four steps respectively:

1- "Specify a mean equation by testing for serial dependence in the data and, if necessary, building an econometric model (e.g., an ARMA model) for the return series to remove any linear dependence.

2- Use the residuals of the mean equation to test for ARCH effects.

3- Specify a volatility model if ARCH effects are statistically significant, and perform a joint estimation of the mean and volatility equations.

4- Check the fitted model carefully and refine it if necessary (Tsay, 2010)."

To implement the first step, there are two alternatives, one of which is to apply the usual Ljung–Box statistics Q(m) to the a_t^2 series. The null hypothesis is that the first m lags of ACF of the a_t^2 series are zero. “auto.arima” function of “forecast” package of R that utilizes AIC and BIC values to reach the best fitting mean equation may help us to set up suitable ARIMA mean equation.

The second test for conditional heteroscedasticity is the Lagrange multiplier test of Engle. Residuals of ARIMA model given by “auto.arima” function is likely to be used for ARCH test and best fitting distribution. To implement second step, GARCH-based models are used for different orders. It has to be taken into consideration that generally daily return series has the characteristics of time-varying volatility and heavy tails. GARCH models enable us to reflect both volatility clustering and unconditional heavy tails.

Since what we need to compare the forecasting performance of alternative GARCH specifications are elaborated before, the analysis phase can be started. Basic analysis tool of comparison in the thesis is the script that is designed with the help of “rugarch” package in R. The script used for CDS premium is given representatively below. Identical scripts were used to analyse forecasting performance of GARCH based volatilities for other five TL-denominated financial instruments.

Table 5.3: R script of Functional Form Implemented to Price Changes

```
dagilim<-list("norm","snorm","std","sstd","ged","sged","nig","ghyp","jsu")
varyans<-
list("sGARCH","eGARCH","apARCH","iGARCH","csGARCH","gjrGARCH")
speci<-expand.grid(ar=1:2,ma=1:2,arm=0:1,mam=0:1,varyans,dagilim)
kiyas<-function(fdata) {
  tekrar<-rep(0,nrow(speci))
  options(warn=-1)
  for(i in seq(along=tekrar)) {
    show(as.character(speci[i,5]))
    show(as.character(speci[i,6]))
sp<-
ugarchspec(variance.model=list(model=as.character(speci[i,5]),garchOrder=c(unli
st(speci[i,1:2]))),mean.model=list(armaOrder=c(unlist(speci[i,3:4])),include.mean
=TRUE),distribution.model=as.character(speci[i,6]))
    fit<-ugarchfit(spec=sp,data=fdata,out.sample=10)
```

```
    sp2=sp
    try(setfixed(sp2)<-as.list(coef(fit)))
    show(coef(fit))
    fit2<-ugarchfit(spec=sp2,data=fdata,out.sample=10,fit.control=list(fixed.se=1))
    pred<-ugarchforecast(fit2, n.ahead = 1,n.roll=10)
    tekrar<-fpm(pred)[,1,]
    show(tekrar)
  }
  speci[which.min(tekrar),]
}
kiyas(dlcds1)
```

Explanation about what the aim and coverage of this script should start by introducing some crucial instructions of rugarch package. Within this scope, “ugarchspec” undertakes introducing specification alternatives of GARCH alternatives. In the script designed for the analysis, both AR and MA degrees of variance equation have the value of (1) and (2). AR and MA degrees of mean equation have the value of (0) and (1). While variance equation alternatives are sGARCH, eGARCH, apARCH, iGARCH, csGARCH and gjrGARCH, distribution alternatives consist of normal distribution “norm”, skewed normal distribution “snorm”, skewed student t distribution “sstd”, student t distribution “std”, normal inverse gaussian distribution “nig”, generalized error distribution “ged”, skewed generalized error distribution “gged”, Johnson’s SU distribution “jsu”, generalized hyperbolic distribution “ghyp”. Thus, 864 (=2 x 2 x 2 x 2 x 6 x 9) alternative GARCH specifications of returns that six TL-denominated financial instruments had are compared in terms of their volatility forecasting performances in the dissertation. The one who comes into his/her mind why, for example, lag alternatives of variance are only (1) and (2) could be responded in such a way that main focus in the dissertation is designing a tool to find the GARCH specification having best-forecasting performance, rather than attempting to attain the one having the best value. The tool could be tried and developed extensively with other alternative specifications.

The last point we must stress is that outsample comparison criterion for finding best forecasting GARCH model is mean squared error (MSE) in this study. However, MAE and DAC indices are given for the best choice presented by MSE.

The results will be given below consecutively. Subtitles comprise descriptive statistics, testing of autocorrelation, stationary and unit root testing, distributional characteristics and the results of the R script explained above.

After reaching the best forecasting GARCH specification, the following script exemplifying the IRS rates representatively is likely to be used to obtain the properties of this specification.

Table 5.4: R script of Getting Results of the Best Forecasting GARCH Specification

```
spirs<-ugarchspec(variance.model=list(model="apARCH",garchOrder=c(2,1)),mean.model=list(armaOrder=c(1,1)),distribution.model="snorm")

fitirs<-ugarchfit(spec=spirs,data=dlirs1,out.sample=10)

spirs2<-spirs

setfixed(spirs2)<-as.list(coef(fitirs))

fitirs2<-ugarchfit(spec=spirs2,data=dlirs1,out.sample=10,fit.control=list(fixed.se=1))

predirs<-ugarchforecast(fitirs2, n.ahead = 1,n.roll=10)

show(predirs)

fpm(predirs)

plot(predirs)
```

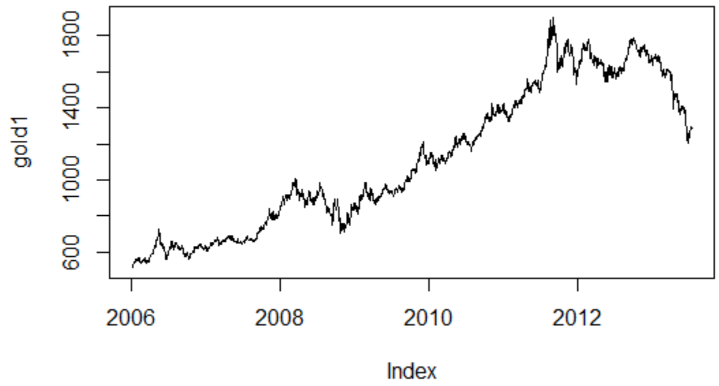
5.3.1. Gold price series

(i) Descriptive statistics

Descriptive statistics and plot of gold price series in BIST are given below.

```
> summary(gold1)
Index                    gold1
Min.   :2006-01-02       Min.   : 514.8
1st Qu.:2007-11-21       1st Qu.: 760.8
Median :2009-10-15       Median :1050.0
```

Mean :2009-10-13	Mean :1118.5
3rd Qu.:2011-09-05	3rd Qu.:1518.0
Max. :2013-07-19	Max. :1902.0



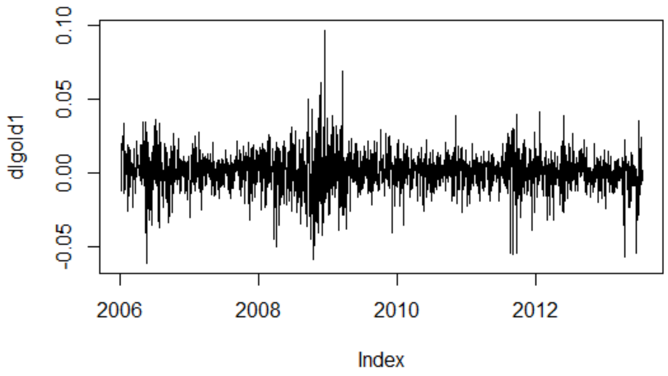
Graphic 5.1: Gold Price Series

The period between 2006 and 2012 had witnessed that gold prices jump from about 500 USD to 1900 USD level, and this was the most volatilite period within the last decades. Since global financial crisis derived from credit crunch in mortgage markets of developed countries led to diminish investors' risk appetite, but to increase liquidity due to monetary expansion, raising the pursuit of holding more reliable assets, i.e. gold instead of currencies.

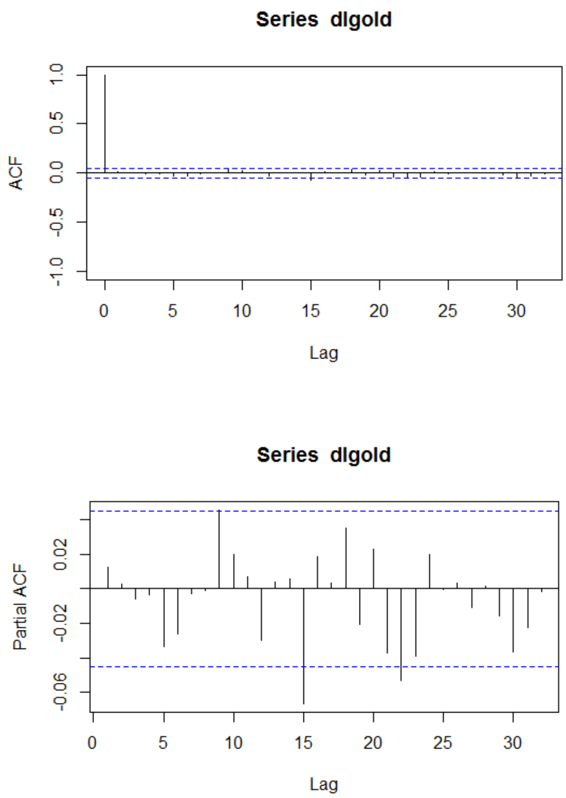
As could be seen directly from the graph, gold price series is non-stationary to the extent that logarithmic difference is required to get stationarity condition.

(ii) Autocorrelation

However, differential equation of gold price series has the following graphs, showing no significant autocorrelation.



Graphic 5.2: Logarithmic Difference of Gold Price Series



Graphic 5.3: ACF and PACF of Logarithmic Price Change of Gold

(iii) Unit root and stationary testing

Phillips-Perron and KPSS test results are presented in the following, indicating stationarity.

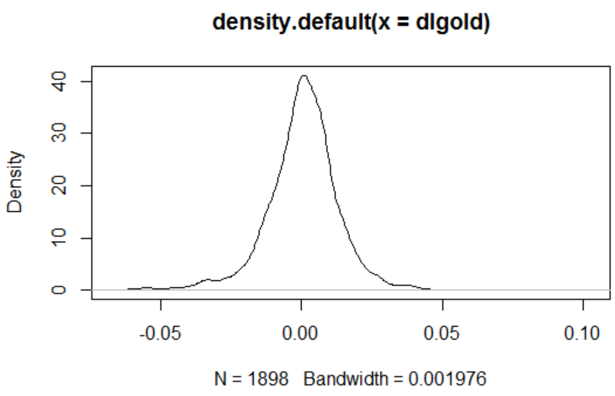
```
> pp.test(dlgold)
        Phillips-Perron Unit Root Test
data:  dlgold
Dickey-Fuller Z(alpha) = -1814.216, Truncation lag parameter = 8, p-value = 0.01
```

```
alternative hypothesis: stationary

> kpss.test(dlaltin1)
            KPSS Test for Level Stationarity
data:  dlaltin1
KPSS Level = 0.2599, Truncation lag parameter = 10, p-value = 0.1
```

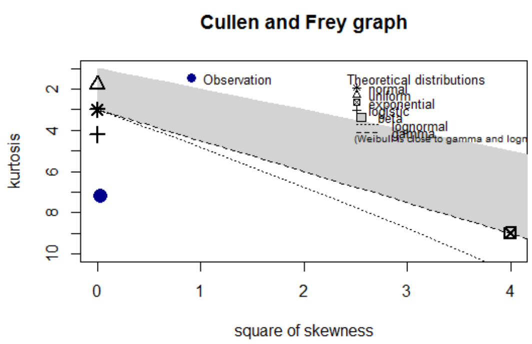
(iv) Distributional characteristics

Another element we have to analyse is which distributional characteristics the series has. Cullen-Frey graph, JB test results are given below. The results show that the series has leptokurtic, but slightly skewed distribution, thus is not normal.

```
> jarque.bera.test(dlgold1)
            Jarque Bera Test
data:  dlgold1
X-squared = 1386.238, df = 2, p-value < 2.2e-16
```



Graphic 5.4: pdf of Price Changes of Gold



Graphic 5.5: Cullen and Frey Graph for Price Changes of Gold

```
> descdist(dlgold)
summary statistics
------
min:  -0.06172011   max:  0.09689102
median:  0.0008573099
mean:  0.0004840443
estimated sd:  0.01306783
estimated skewness:  -0.1871604
estimated kurtosis:  7.184177
```

Therefore, the distribution reflects the characteristics of leptokurtic and slightly left skewed.

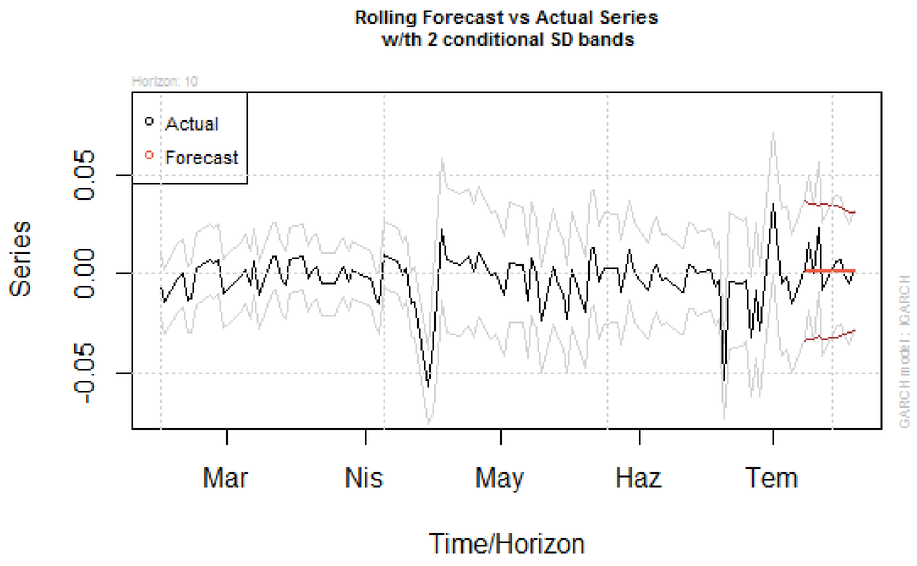
(v) Result of the script

However, the script operating the function that enables us to compare out sample measures of GARCH specifications has the following result.

	ar	ma	arm	mam	Var5	Var6
63	**1**	**2**	**1**	**1**	**iGARCH**	**norm**

The combination of IGARCH and normal distribution produce the minimum MSE with (1,2) degrees of variance equation and (1,1) degrees of mean equation. MSE and graph of forecasted and actual values with the interval of 3 standard deviation are given in the following.

```
> fpm(predgold)
         MSE         MAE DAC
1 9.064499e-05 0.006771845 0.7
> show(predgold)

0-roll forecast:
             sigma   series
2013-07-08 0.02206 0.001554
```



Graphic 5.6: Actual & Forecasted Series of Gold Returns

5.3.2. IRS rate

(i) Descriptive statistics

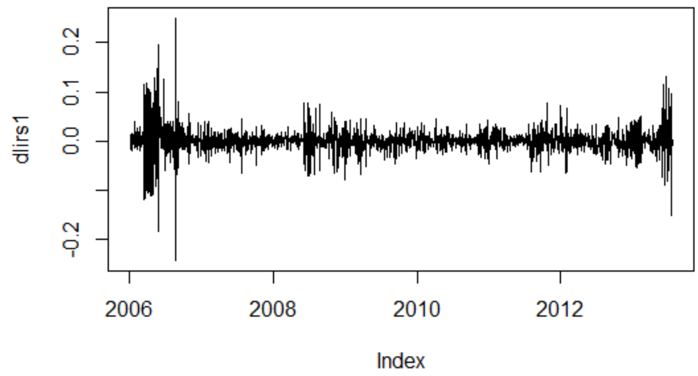
Descriptive statistics and graph of 1 year-IRS premium of TL fixed-floating rates are given below.

```
> summary(irs1)
    Index                irs1
Min.   :2006-01-03   Min.   : 5.13
1st Qu.:2007-11-30   1st Qu.: 8.45
Median :2009-10-19   Median :10.39
Mean   :2009-10-16   Mean   :12.80
3rd Qu.:2011-09-05   3rd Qu.:18.36
Max.   :2013-07-23   Max.   :23.45
```

Graphic 5.7: IRS Rate Series

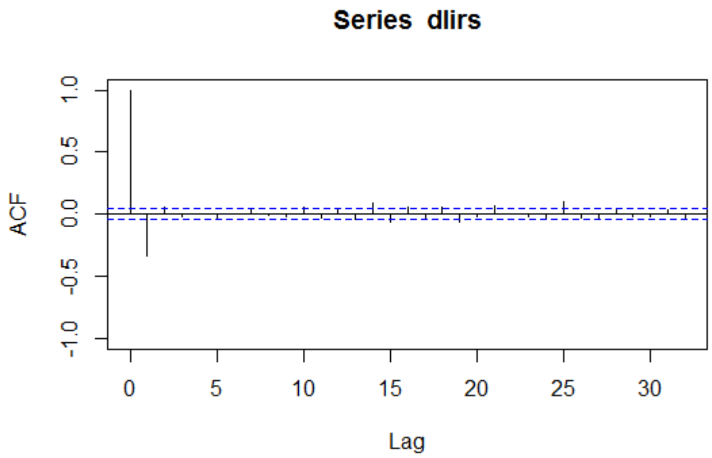
IRS rates had ranged from 5,13% to 23,45% between 2006 and the first half of 2013. IRS rates are highly correlated with interbank repo and deposit rates. In years from the middle of 2006 to 2008, IRS rounded around 20 percent, then has settled the neighborhood of 10 percent or below.

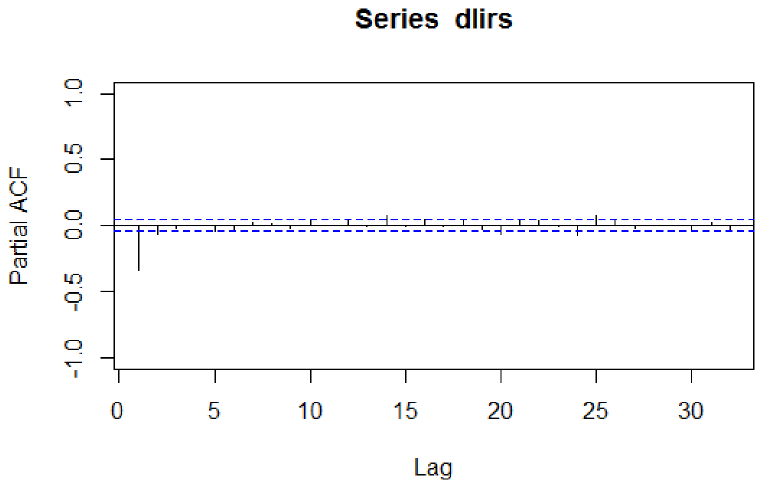
(ii) Autocorrelation



Graphic 5.8: Logarithmic Difference of IRS Rates

While observing the graph of logarithmic difference of IRS rates, volatility clustering explicitly be shown in the period of second half of 2006 and 2012. ACF and PACF diagrams state the following result.





Graphic 5.9: ACF and PACF of IRS Rate Changes

```
> Box.test(dlirs1,lag=1,type='Ljung')
  Box-Ljung test
data:  dlirs1
X-squared = 241.683, df = 1, p-value < 2.2e-16
> auto.arima(dlgold1)
Series: dlgold1
ARIMA(1,0,0) with zero mean
Coefficients:
         ar1
      -0.0044
s.e.   0.0273
sigma^2 estimated as 0.0001709:  log likelihood=5538.83
AIC=-11073.67   AICc=-11073.66   BIC=-11061.83
```

We concluded that IRS rate series is correlated with (t-1).

(iii) Unit root and stationary testing

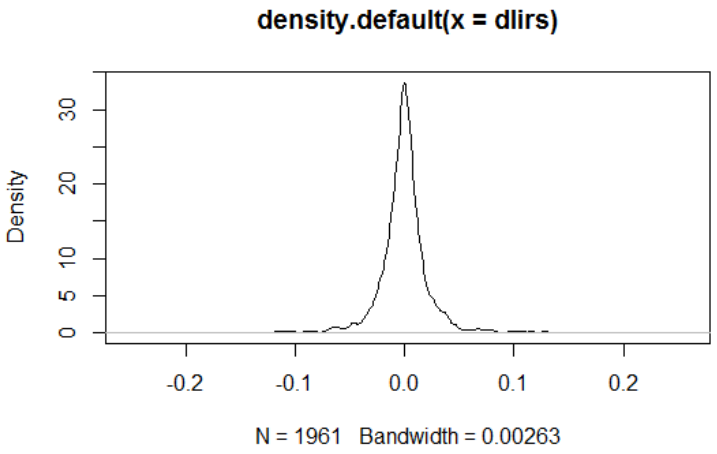
To reach the conclusion whether IRS rate time series is stationary, ADF and PP tests would be seen below. P-values show that there is no unit root problem in the series. KPSS test result means that null hypothesis of stationarity could not be rejected.

```
> adf.test(dlirs)
          Augmented Dickey-Fuller Test
data:  dlirs
Dickey-Fuller = -11.6255, Lag order = 12, p-value = 0.01
alternative hypothesis: stationary

> pp.test(dlirs)
          Phillips-Perron Unit Root Test
data:  dlirs
```

```
Dickey-Fuller Z(alpha) = -2475.999, Truncation lag parameter = 8,
p-value = 0.01
alternative hypothesis: stationary

> kpss.test(dlirs1)
        KPSS Test for Level Stationarity
data:  dlirs1
KPSS Level = 0.1406, Truncation lag parameter = 10, p-value = 0.1
```

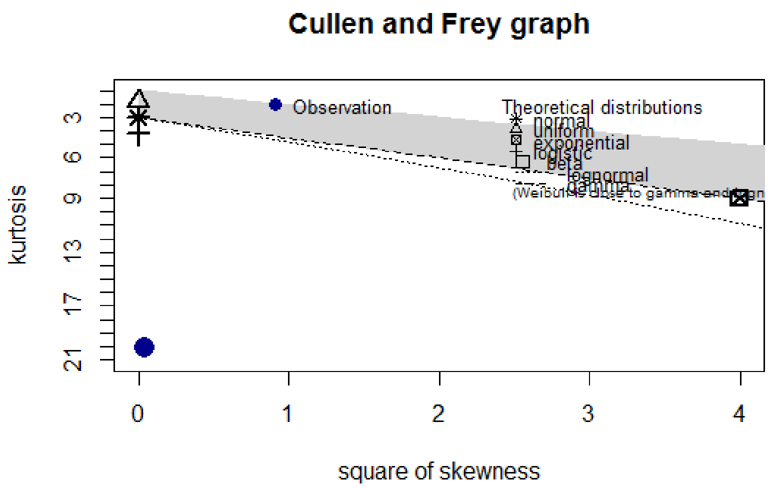
(iv) Distributional characteristics

J-B test statistic below enables us to know that error terms in IRS rate series could not be compatible with normal distribution. Leptokurtic and skewed distributional characteristics pertain to the series.



Graphic 5.10: Normal Q-Q Plot and pdf of IRS Rate Changes

```
> jarque.bera.test(dlirs1)
        Jarque Bera Test
data:  dlirs1
X-squared = 23736.41, df = 2, p-value < 2.2e-16

> descdist(dlirs)
        summary statistics
------
min:  -0.2443768   max:  0.2508205
median:  0
mean:  -0.0002998804
estimated sd:  0.0255065
estimated skewness:  0.2097878
estimated kurtosis:  20.08554
```



Graphic 5.11: Cullen and Frey Graph for IRS Rate Changes

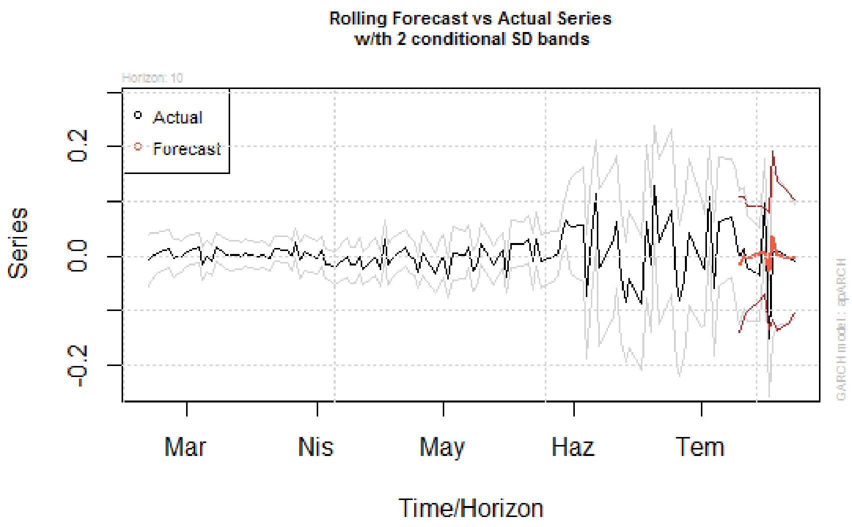
(v) Result of the script

After all the partial analysis were made before, the holistic approach would be passed with trying the best forecasting choice within 864 alternatives.

	ar	ma	arm	mam	Var5	Var6
142	**2**	**1**	**1**	**1**	**apARCH**	**snorm**

The function gives the choice of apARCH as variance model, skewed normal distribution, AR and MA degrees of (2) and (1) for variance equation, and respectively (1) and (1) for mean equation. MSE and the graph of actual and forecasted values with 2 standard deviation intervals are given below.

```
> show(predirs)
0-roll forecast:
           sigma   series
2013-07-10 0.06154 -0.01551
```



Graphic 5.12: Actual & Forecasted Series of IRS rates

```
> fpm(predirs)
         MSE        MAE DAC
1 0.002682677 0.03510588 0.8

> show(fitirs)

LogLikelihood : 5073.704

Information Criteria
------------------------------------

Akaike       -5.1899
Bayes        -5.1584
Shibata      -5.1899
Hannan-Quinn -5.1783

Q-Statistics on Standardized Residuals
------------------------------------
              statistic p-value
Lag[1]         0.003845 0.95055
Lag[p+q+1][3]  3.590509 0.05811
Lag[p+q+5][7]  9.704285 0.08406
d.o.f=2
H0 : No serial correlation

ARCH LM Tests
------------------------------------
             Statistic DoF P-Value
ARCH Lag[2]     0.9343   2  0.6268
ARCH Lag[5]     3.5642   5  0.6137
ARCH Lag[10]    4.1464  10  0.9405
```

ARCH LM test and the others point out that the model removes ARCH effect, and it could be seen that forecasted series has an ability to reflect actual series.

5.3.3. USD-TRY exchange rate

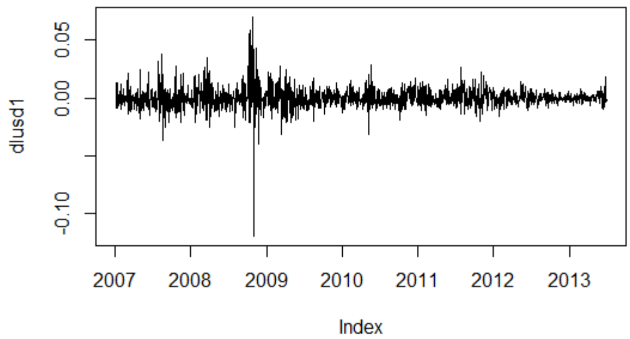
(i) Descriptive statistics

Descriptive statistics and graph of USD –TRY parity is given below.

```
> summary(usd1)
    Index                 usd1
 Min.   :2007-01-04   Min.   :1.145
 1st Qu.:2008-08-12   1st Qu.:1.392
 Median :2010-04-01   Median :1.532
 Mean   :2010-04-01   Mean   :1.540
 3rd Qu.:2011-11-18   3rd Qu.:1.764
 Max.   :2013-06-28   Max.   :1.941
```

Graphic 5.13: USD-TRY Parity Series

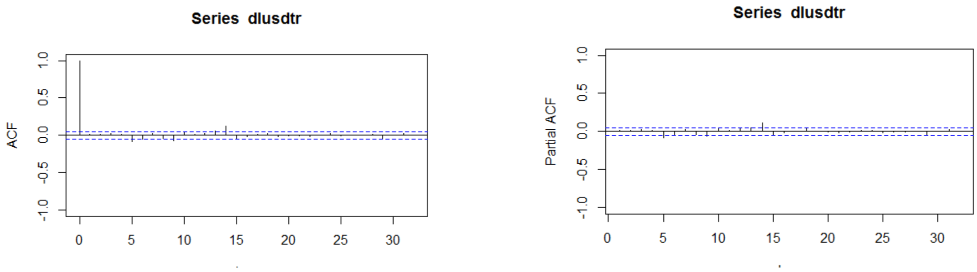
The period between 2007 and the first half of the 2013 witnessed that there was two jumps, one of which is from 1,2 to 1,7 in 2008, and the other is from about 1,6 to 1,8 in 2012. High volatility and also trend non-stationarity could explicitly be seen in the graph. Thus, we try to reach non-stationarity by logarithmic differencing of the series.



Graphic 5.14: Logarithmic Difference of USD-TRY Parity

(ii) Autocorrelation

The basic indicators of autocorrelation are presented in the following:



Graphic 5.15: ACF and PACF of USD-TRY Parity Change

```
> Box.test(dlusd1,lag=1,type='Ljung')
Box-Ljung test
data:  dlusd1
X-squared = 4.0122, df = 1, p-value = 0.04517

> auto.arima(dlusd1)
Series: dlusd1
ARIMA(0,0,0) with zero mean
sigma^2 estimated as 8.164e-05:  log likelihood=5378.58
AIC=-10755.16   AICc=-10755.15   BIC=-10749.39
```

According to the results, there is no autocorrelation in the series.

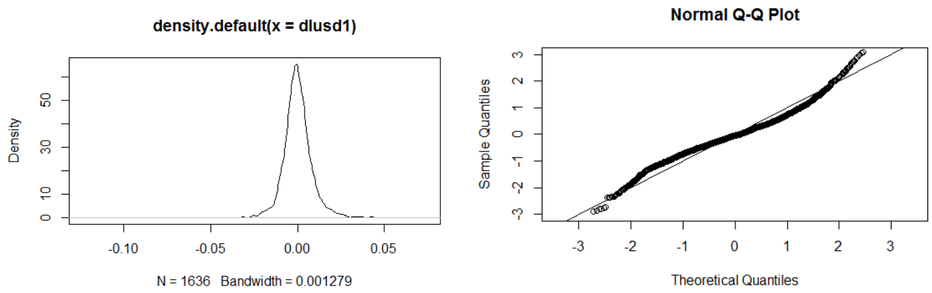
(iii) Unit root and stationary testing

PP test, KPSS test and ADF test results support the claim of stationarity.

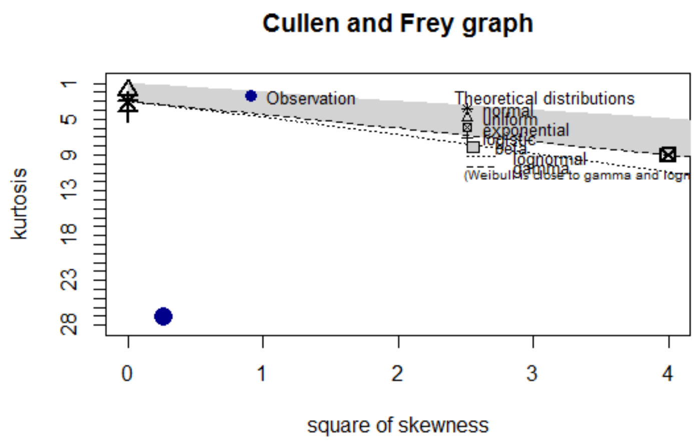
```
> pp.test(dlusdtr)
            Phillips-Perron Unit Root Test
data:  dlusdtr
Dickey-Fuller Z(alpha) = -1574.487, Truncation lag parameter = 8, p-value = 0.01
alternative hypothesis: stationary

> kpss.test(dlusd1)
            KPSS Test for Level Stationarity
data:  dlusd1
KPSS Level = 0.0919, Truncation lag parameter = 9, p-value = 0.1

> adf.test(dlusdtr)
            Augmented Dickey-Fuller Test
data:  dlusdtr
Dickey-Fuller = -11.8251, Lag order = 11, p-value = 0.01
alternative hypothesis: stationary
```

(iv) Distributional characteristics

Graphic 5.16: Normal Q-Q Plot and pdf of USD-TRY Parity




```
> jarque.bera.test(dlusd1)

        Jarque Bera Test

data:  dlusd1
X-squared = 39127.51, df = 2, p-value < 2.2e-16

> descdist(dlusdtr)
summary statistics
------
min:  -0.1193559   max:  0.07038781
median:  -0.0002281177
mean:  0.0001929119
estimated sd:  0.00903635
estimated skewness:  -0.5123117
estimated kurtosis:  27.01336
```



Graphic 5.17: Cullen and Frey Graph of USD-TRY Parity

(v) Results of the script

	ar	ma	arm	mam	Var5	Var6
21	**1**	**1**	**1**	**0**	**eGARCH**	**norm**

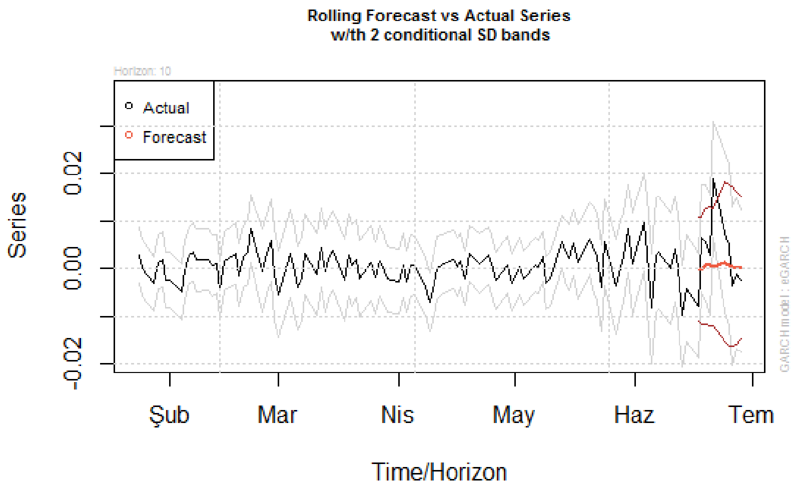
That the GARCH specification produces the best forecasting performance is that variance equation of eGARCH, normal distribution, AR degree of (1) and MA degree of (1) for variance equation, and AR degree of (1) and MA degree of (0) for mean equation.

Out-sample forecasting performance indicators are about 5,5 for MSE, 0,0058 for MAE and 0,7 for DAC.

MSE	MAE	DAC
5.499425e-05	**0.005798483**	**0.7**

The graph of forecasted and actual values with 2 standard deviation bands are given below.

```
> show(predusd)
0-roll forecast:
             sigma     series
2013-06-17 0.005449 -6.527e-05
```



Graphic 5.18: Actual & Forecasted Series of USD-TRY parity

5.3.4. CCS Rate

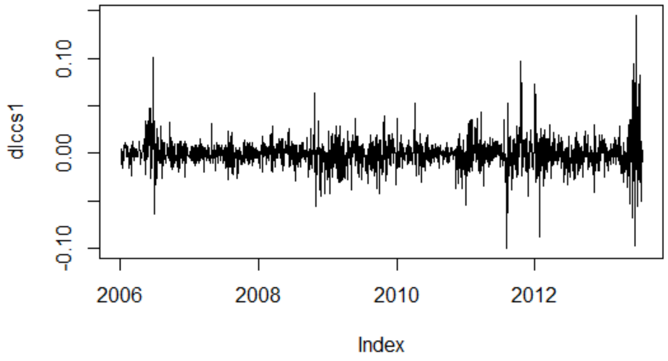
(i) Descriptive Statistics

Descriptive statistics and graph of CCS rates are given below.

```
> summary(ccs1)
    Index                 ccs1
 Min.    :2006-01-03   Min.    : 3.88
 1st Qu.:2008-01-15   1st Qu.: 7.07
 Median :2009-11-17   Median : 8.45
 Mean    :2009-11-12   Mean    :11.30
 3rd Qu.:2011-09-20   3rd Qu.:16.97
 Max.    :2013-07-23   Max.    :22.75
```

Graphic 5.19: CCS Rate Series

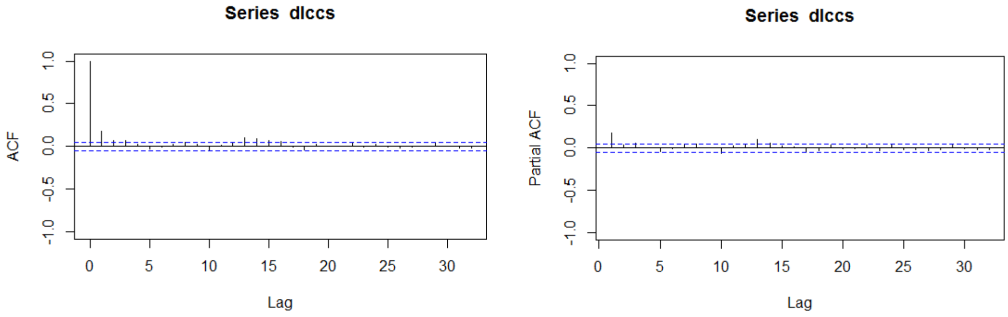
Sharp rise in 2006 and fall in 2009 had been witnessed, as seen in above. The main drivers of CCS rates are expected USD-TR parity, the difference between TL interest rate and USD interest rates and credit risk spread.



Graphic 5.20: Logarithmic Difference of CCS Rate Series

(ii) Autocorrelation

ACF and PACF of the series are given below, indicating the lag (1).



Graphic 5.21: ACF and PACF of CCS Rate Change

```
> Box.test(dlccs1,lag=1,type='Ljung')
            Box-Ljung test

data:  dlccs1
X-squared = 40.9522, df = 1, p-value = 1.56e-10
> auto.arima(dlccs1)
Series: dlccs1
ARIMA(0,0,0) with zero mean

sigma^2 estimated as 0.0002558:  log likelihood=5213.18
AIC=-10424.35   AICc=-10424.35   BIC=-10418.43
```

(iii) Unit root and stationary testing

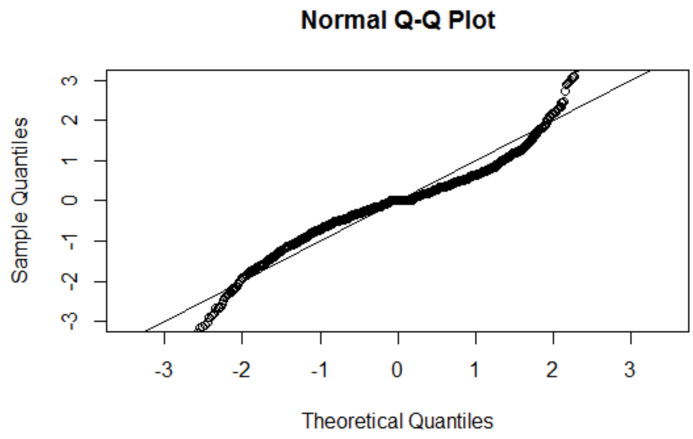
ADF, PP and KPSS test results means that there is no unit root.

```
> adf.test(dlccs)
            Augmented Dickey-Fuller Test
data:  dlccs
Dickey-Fuller = -9.9122, Lag order = 12, p-value = 0.01
alternative hypothesis: stationary

> pp.test(dlccs)
            Phillips-Perron Unit Root Test
data:  dlccs
Dickey-Fuller Z(alpha) = -1676.482, Truncation lag parameter = 8, p-value = 0.01
alternative hypothesis: stationary

> kpss.test(dlccs1)
            KPSS Test for Level Stationarity
data:  dlccs1
KPSS Level = 0.1501, Truncation lag parameter = 10, p-value = 0.1
```

(iv) Distributional characteristics

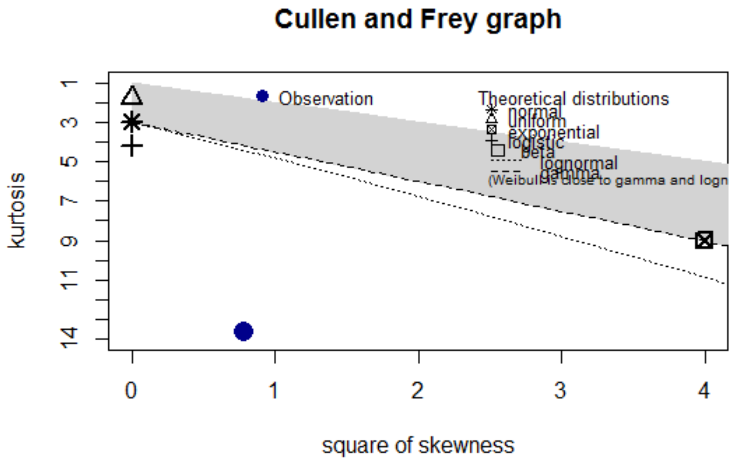
Some statistics about the distribution of the series strongly rejects normality.

```
> descdist(dlccs)
summary statistics
------
min:  -0.1003977   max:  0.1455076
median:  0
mean:  -0.0002899306
estimated sd:  0.01599542
estimated skewness:  0.8826388
estimated kurtosis:  13.66396
```



Graphic 5.22: Normal Q-Q Plot of CCS Rate Changes

```
> jarque.bera.test(dlccs1)
                Jarque Bera Test
data:  dlccs1
X-squared = 9289.039, df = 2, p-value < 2.2e-16
```



Graphic 5.23: Cullen and Frey Graph of CCS Rate Change

(v) Result of the script

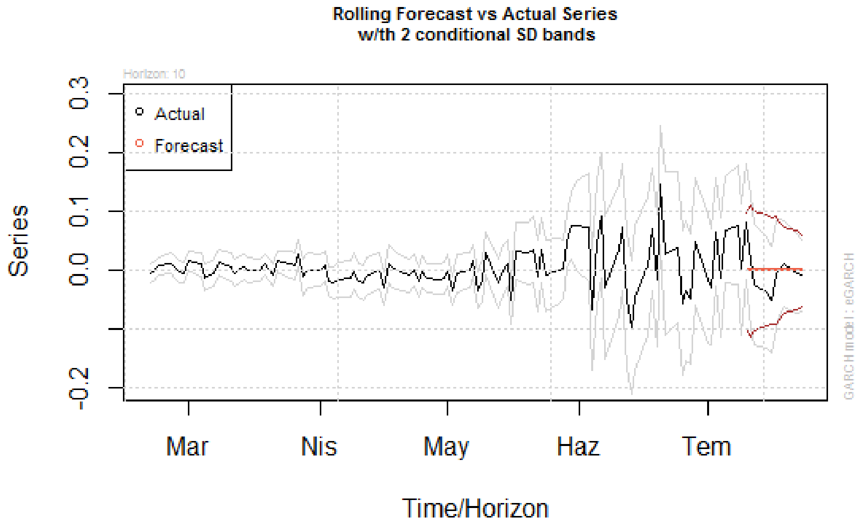
The function gives the choice of eGARCH as variance model, skewed student's t distribution, AR and MA degrees of (1) and (1) for variance equation, and respectively (1) and (1) for mean equation.

	ar	ma	arm	mam	Var5	Var6
317	1	1	1	1	eGARCH	sstd

Out of sample indicators and model results are given below.

```
> fpm(predccs)
        MSE        MAE       DAC
1 0.001003039 0.02173458 0.6363636

> show(fitccs)
Information Criteria
------------------------------------

Akaike       -6.0160
Bayes        -5.9898
Shibata      -6.0161
Hannan-Quinn -6.0064

Q-Statistics on Standardized Residuals
------------------------------------
              statistic p-value
Lag[1]            2.022 0.15502
Lag[p+q+1][3]     4.553 0.03287
Lag[p+q+5][7]    10.093 0.07265
d.o.f=2
H0 : No serial correlation

ARCH LM Tests
------------------------------------
             Statistic DoF P-Value
ARCH Lag[2]    0.07582   2  0.9628
ARCH Lag[5]    1.92916   5  0.8589
ARCH Lag[10]   8.64416  10  0.5662
```

```
> show(predccs)

0-roll forecast:
             sigma     series
2013-07-10 0.04933 -0.0006212
```



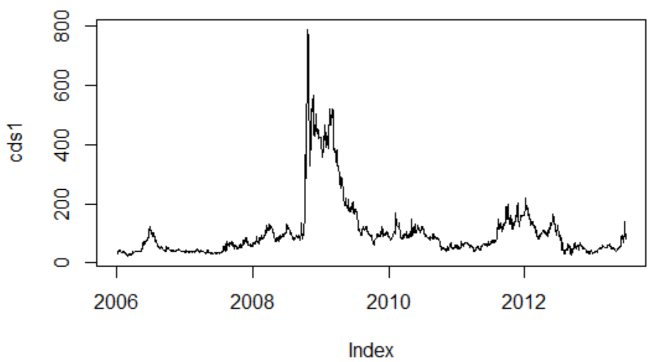
Graphic 5.24: Actual & Forecasted Series of CCS rate

5.3.5. CDS Rate

(i) Descriptive statistics

Turkish sovereign CDS spreads have the following values between 2006 and the first half of 2013.

```
> summary(cds1)
    Index                cds1
 Min.   :2006-01-03   Min.   : 21.80
 1st Qu.:2007-11-12   1st Qu.: 49.55
 Median :2009-09-29   Median : 77.52
 Mean   :2009-09-23   Mean   :104.81
 3rd Qu.:2011-08-04   3rd Qu.:112.41
 Max.   :2013-06-28   Max.   :790.05
```



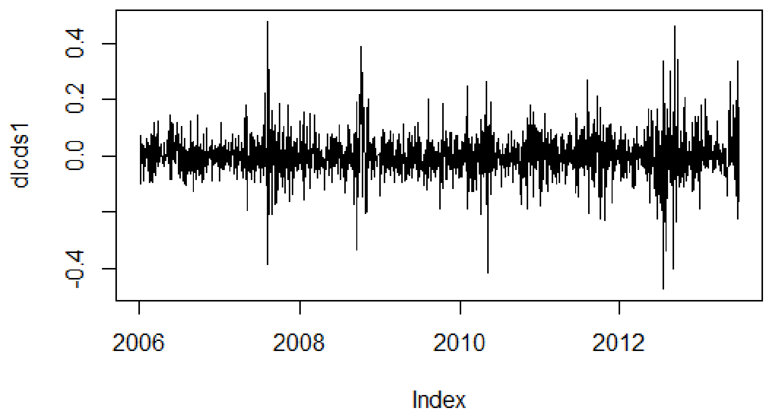
Graphic 5.25: CDS Rate Series

While range have become about 770 bps, there has been sharp rise in 2008 due to the financial crisis triggered by failures, defaults and downgrades in credit markets. CDS is one of the most reliable indicators of credit risk. Since the global crisis of 2008-2009 diminishes the credibility and relative accuracy of credit rating agencies, as an alternative, risk and valuation models based on CDS rates have increased its popularity and have superiority due to the fact that it becomes the product of real market transactions and directly reflect the views and behaviors of active market participants.

As seen in the graph above, stationarity condition is not likely be satisfied due to the fluctuations of years of 2008-2009 and the subsequent period. In order to ensure, ADF testing rejects null hypothesis of non-stationarity.

```
> adf.test(cds)
            Augmented Dickey-Fuller Test

data:  cds
Dickey-Fuller = -2.6464, Lag order = 12, p-value = 0.3047
alternative hypothesis: stationary
```

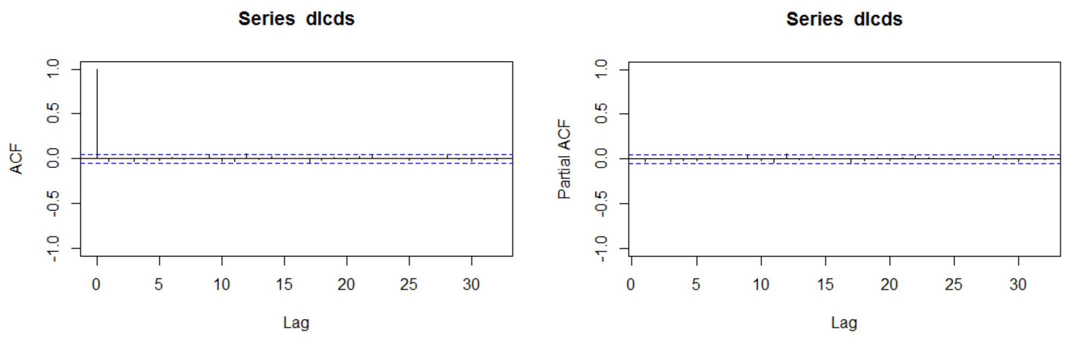
Following this result, logarithmic difference of the series is shown below to indicate stationarity.



Graphic 5.26: Logarithmic Difference of CDS Rates

(ii) Autocorrelation

The indicators for autocorrelation are given below.




```
> Box.test(dlcds1,lag=1,type='Ljung')
            Box-Ljung test
data:  dlcds1
X-squared = 0.8136, df = 1, p-value = 0.3671

> auto.arima(dlcds1)
Series: dlcds1
ARIMA(0,0,0) with zero mean
sigma^2 estimated as 0.005316:  log likelihood=2298.47
AIC=-4594.93   AICc=-4594.93   BIC=-4589.02
```

(iii) Unit root and stationarity testing

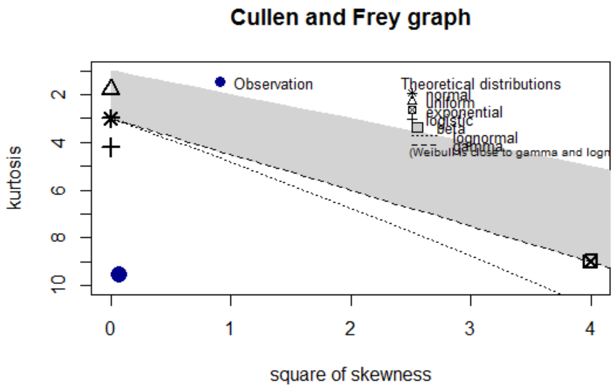
ADF, PP and KPSS testing results reject the claim of unit root, and support stationarity.

```
> adf.test(dlcds)
            Augmented Dickey-Fuller Test
data:  dlcds
Dickey-Fuller = -11.8466, Lag order = 12, p-value = 0.01
alternative hypothesis: stationary
> pp.test(dlcds)
            Phillips-Perron Unit Root Test
data:  dlcds
Dickey-Fuller Z(alpha) = -1902.335, Truncation lag parameter = 8, p-value = 0.01
alternative hypothesis: stationary
> kpss.test(dlcds1)
            KPSS Test for Level Stationarity
data:  dlcds1
KPSS Level = 0.0595, Truncation lag parameter = 10, p-value = 0.1
```

(iv) Distributional characteristics

JB testing rejects normality because of excess kurtosis.

```
(v)     > jarque.bera.test(dlcds1)
(vi)       Jarque Bera Test
(vii)   data:  dlcds1
(viii)  X-squared = 3444.889, df = 2, p-value < 2.2e-16
```



Graphic 5.28: Cullen and Frey Graph of CDS Rate Changes

```
> descdist(dlcds)
summary statistics
------
min:  -0.4760459   max:  0.4800663
median:  -0.0008058087
mean:  0.0005148577
estimated sd:  0.07292534
estimated skewness:  0.2600738
estimated kurtosis:  9.568609
```

(v) Result of the script

The function gives the choice of eGARCH as variance model, skewed normal distribution, AR and MA degrees of (1) and (1) for variance equation, and respectively (1) and (1) for mean equation.

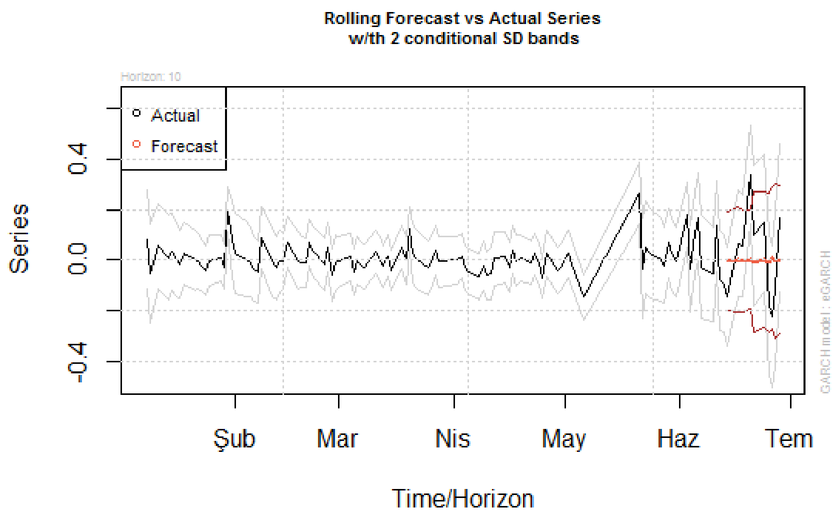
	ar	ma	arm	mam	Var5	Var6
125	**1**	**1**	**1**	**1**	**eGARCH**	**snorm**

Out of sample forecast error measures and test results of the model are given in the following:

```
> fpm(predcds)
        MSE       MAE DAC
1 0.03006966 0.1549128 0.3

> show(fitcds)
Information Criteria
------------------------------------

Akaike       -2.7205
Bayes        -2.6943
Shibata      -2.7206
Hannan-Quinn -2.7109
```

```
ARCH LM Tests
------------------------------------
           Statistic DoF P-Value
ARCH Lag[2]      4.169   2  0.1244
ARCH Lag[5]      6.189   5  0.2883
ARCH Lag[10]    13.912  10  0.1770
```

```
> show(predcds)
0-roll forecast:
          sigma    series
2013-06-14 0.0979 -0.001856
```



Graphic 5.29: Actual & Forecasted Series of CDS Rate

5.3.6. XBN10 Index

(i) Descriptive statistics

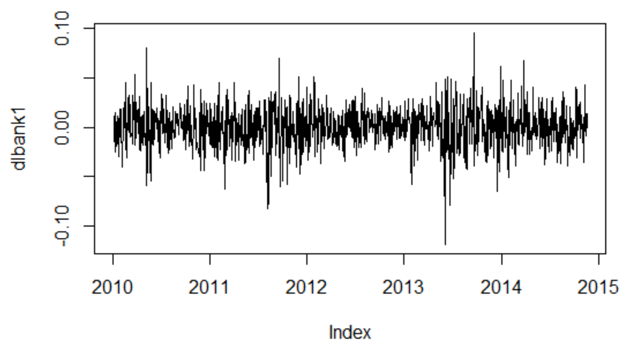
The assets of banking sector could be considered as the mix of all the sectors, financing the most profitable ones that is why the banking sector can represent all the economy and reflect the dynamics behind it. BIST stock exchange index data of 10 major Turkish banks share prices have the following values.

```
> summary(bank1)
    Index               bank1
 Min.   :2010-01-04   Min.   : 93557
 1st Qu.:2011-03-22   1st Qu.:121396
 Median :2012-06-06   Median :135477
 Mean   :2012-06-07   Mean   :135896
 3rd Qu.:2013-08-22   3rd Qu.:148012
 Max.   :2014-11-19   Max.   :199985
```

Graphic 5.30: XBN10 Index Series

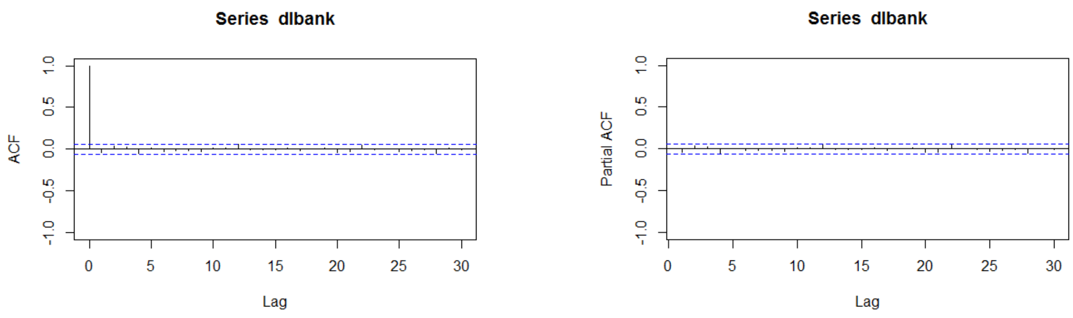
The series has a different characteristics as compared with the others. It includes the period between 2010 and mid-2015. This period is selected because of reflecting different economic story. While the period before 2010 includes the crisis environment, the last two or three years have been associated with low economic growth.

Graphic 5.31: Logarithmic Difference of XBN10 Index Series



(ii) Autocorrelation

ACF and PACF of the series are given below.



Graphic 5.32: ACF and PACF of XBN10 Series

```
> Box.test(dlbank1,lag=1,type='Ljung')
                Box-Ljung test
data:  dlbank1
X-squared = 4.0068, df = 1, p-value = 0.04532

> auto.arima(dlbank1)
Series: dlbank1
ARIMA(1,0,0) with zero mean
Coefficients:
         ar1
     -0.0574
s.e.   0.0324
sigma^2 estimated as 0.0004117:  log likelihood=3008.62
AIC=-6013.24   AICc=-6013.24   BIC=-6002.28
```

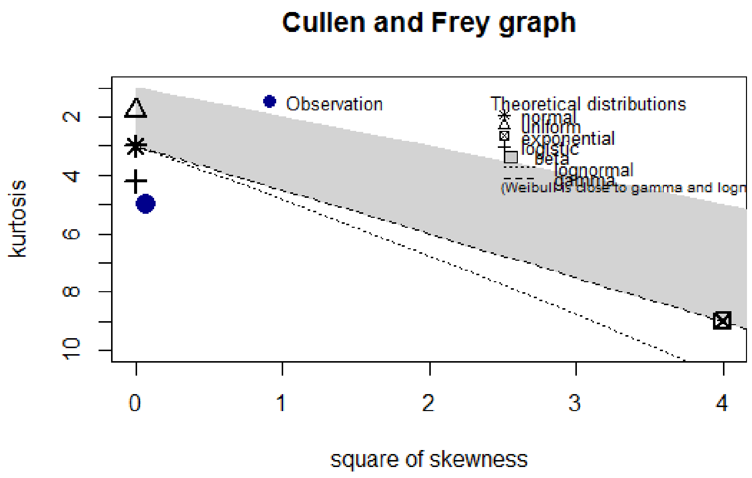
(iii) Unit root and stationary testing

ADF, PP and KPSS testing results indicate that there is no unit root.

```
> adf.test(dlbank)
                Augmented Dickey-Fuller Test
data:  dlbank
Dickey-Fuller = -10.8252, Lag order = 10, p-value = 0.01
alternative hypothesis: stationary

> pp.test(dlbank)
                Phillips-Perron Unit Root Test
data:  dlbank
Dickey-Fuller Z(alpha) = -1270.78, Truncation lag parameter = 7, p-value = 0.01
alternative hypothesis: stationary

> kpss.test(dlbank1)
                KPSS Test for Level Stationarity
data:  dlbank1
KPSS Level = 0.0418, Truncation lag parameter = 8, p-value = 0.1
```

(iv) Distributional characteristics

```
> jarque.bera.test(dlbank1)
                Jarque Bera Test
data:  dlbank1
X-squared = 213.6642, df = 2, p-value < 2.2e-16
```



Graphic 5.33: Cullen and Frey Graph of XBN10 Price Changes

```
> descdist(dlbank)
summary statistics
------
min:  -0.1183616   max:  0.09533358
median:  0.0006297902
mean:  0.0001878006
estimated sd:  0.02033216
estimated skewness:  -0.2627217
estimated kurtosis:  5.000275
```

(v) Result of the script

The best forecasting GARCH specification for XBN10 index after the period of 2010 is given below. CGARCH-generalized error distribution matching without the mean equation produces the least value of MSE.

	ar	ma	arm	mam	Var5	Var6
449	**1**	**1**	**0**	**0**	**csGARCH**	**ged**

MSE, MAE and DAC out-of-sample criteria, information criteria and ARCH LM test results have the following properties.

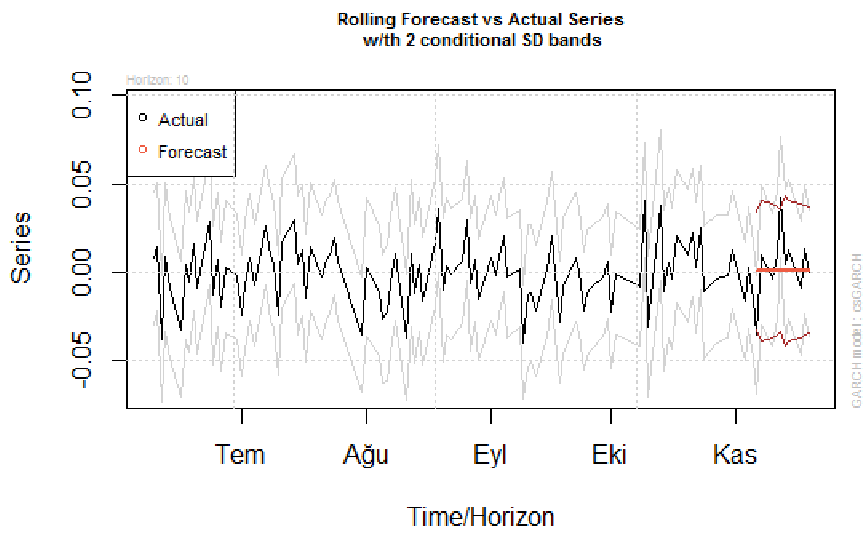
	MSE	MAE	DAC
1	**0.0003738979**	**0.01394597**	**0.7**

```
> show(fitbank)
Information Criteria
------------------------------------

Akaike       -5.0358
Bayes        -5.0062
Shibata      -5.0359
Hannan-Quinn -5.0247
```

```
ARCH LM Tests
------------------------------------
           Statistic DoF P-Value
ARCH Lag[2]     0.6884   2  0.7088
ARCH Lag[5]     3.4977   5  0.6237
ARCH Lag[10]   11.0739  10  0.3518
```

Forecasted-actual values and their plot with 2 standard deviation bands are presented in the following table.

```
> show(predbank)
0-roll forecast:
          sigma    series         q
2014-11-06 0.0168 0.0009341 0.0003217
```



Graphic 5.34: Actual & Forecasted Series of XBN10 Index

6 CONCLUSION

The aim of the thesis is to find the best forecasting volatility methods based on a range of GARCH-based volatility scenarios defined by persistence and volatility of volatility. To achieve this goal, a functional form was developed to roll alternatives recursively on rugarch package of R software, and 864 probable specifications were tried to obtain the best result. MSE was used as the major indicator of 10 days out-of-sample forecasting performance. 10-days forecasting horizon is selected because of that Basel II rules on risk measurement and capital adequacy assumes 10 days holding period for value at risk (VaR) calculations of market risk. Although there could not be unique rule to determine forecasting horizon, I suppose that Basel Committee backed and explained the meaning of the 10 days assumption thanks to the empirical analysis.

Univariate GARCH specifications involve the simultaneous estimation of mean equation, variance equation and distributional form. In the dissertation, as a variance equation, 6 alternatives including GARCH, EGARCH, IGARCH, APARCH, CGARCH and GJR-GARCH are tried. Likewise, as a distribution function, 9 alternatives including normal, skewed normal, skewed student t, student t, normal inverse gaussian, generalized error distribution, skewed generalized error, Johnson's SU, generalized hyperbolic are used.

To a more broad extent, EGARCH, APARCH and GJR-GARCH put forward asymmetric behavior of the return, while CGARCH and IGARCH represent long memory and persistence. On a distributional side, the distributions rather than normal distribution are generally designed to cover excess kurtosis and skewness values of financial data sets.

Before summing up the results, it should be noted that the data interval being subject to the thesis includes the global financial crisis of 2007-2008 and its effects on Turkish economy. Since the results indicate the reflections of the shock, rather than the features of more stable periods, they should be used as a benchmark for more volatile periods including jumps. However, as a benchmarking series, XBN10 index values are selected to reflect the post-crisis period.

All these these points considered, the results are summarized in the following table:

Table 6.1: Main Results of Dissertation

		Variance model	Variance order	Mean order	Distributional form
1	**CDS rate**	eGARCH	(1,1)	(1,1)	Skewed normal
2	**Gold prices**	iGARCH	(1,2)	(1,1)	Normal
3	**IRS rate**	apARCH	(2,1)	(1,1)	Skewed normal
4	**CCS rate**	eGARCH	(1,1)	(1,1)	Skewed student's t
5	**USD-TRY**	eGARCH	(1,1)	(1,0)	Normal
6	**XBN10**	csGARCH	(1,1)	(0,0)	Generalised error

As seen above, none of the specifications outperform the others. The result of MSE criterion indicates that 3 of 6 financial instruments' series including CDS rate, CCS rate and USD-TRY exchange rate are forecasted best by the identical variance equations (eGARCH with the order of (1,1)), but different mean equations and distributional forms.

In eGARCH, the conditional variance has an exponential specification, and unlike sGARCH, it does not produce negative values for conditional variance. Also, volatility can produce asymmetric behaviors for the positive or negative movements. In terms of asymmetric behavior, apARCH of IRS series could be regarded as similar with eGARCH and classified in the same group with eGARCH. It could be concluded from these results that the asymmetric GARCH models present a better explanation of volatility than the standard.

On the other hand, CGARCH and IGARCH models are designed to reflect slowly decaying stochastic long run volatility dependencies. IGARCH reflects persistence of volatility. CGARCH has observed and proposed that the volatility persistence of large jumps is shorter than shocks due to ordinary news events. The component model allows large shocks to be transitory.

If we mention that XBN10 index values are considered as a benchmark and reflects more stable period of time that coincides with post-crisis era, only Gold price change series reflects relatively different characteristics, and produces the result of IGARCH. It is meaningful if we notice the differences of price dynamics between gold and the others for the crisis period. During and following the crisis, gold prices had consistently increase while the others displayed instability and large jumps.

In short, within the GARCH specifications that have long memory and/or asymmetric characteristics achieve more robust forecasting performance. Stable periods generally lead to prioritize persistence, but the crisis environment produces leverage effect.

On the distributional form side, skewed normal distribution was selected by MSE for IRS and CDS rates. Gold price and USD-TRY exchange rate series produced minimum MSE with normal distribution, CCS and XBN10 did it with skewed t distribution and generalised error distribution respectively. Therefore, 3 of them including IRS rates, CDS rates and CCS rates reflect skewed location, and 2 of them reflects leptokurtic shape. Therefore, unlike pre-supposed, our data sets did not support leptokurtic distributions rather than normal distribution predominantly.

The best forecasting GARCH specifications that was selected by MSE criterion has the following properties:

Table 6.2: Results of Individual Statistics

	Return (log. diff.)	**Actual**	**Forecast**	**MSE**
1	CDS rate	0.16824	-0.00186	0.00009
2	Gold prices	0.00194	0.00155	0.00268
3	IRS rate	-0.00874	-0.01551	0.00005
4	CCS rate	-0.00928	-0.00062	0.00100
5	USD-TRY	-0.00249	-0.00007	0.03007
6	XBN10	-0.00033	0.00032	0.00037

The table shows that MSE criterion works relatively well for XBN10 and gold return series, but MSE values of all the series are less enough to present sound results.

Another aim of the thesis is to answer the question if there is a reconciliation between the results of the functional form trying to find the best forecasting GARCH specification within 864 alternatives and preliminary individual indicators about the distributional form, stationarity and unit root testing. The situation which we don't have a chance to employ this function is likely required either to try all the alternatives manually or to attempt to eliminate most of the choices by using that sort of indicators.

Individual results are given in the table below.

Table 6.3: Results of Individual Statistics

	CDS	**IRS**	**CCS**	**Gold**	**XBN10**	**USDTRY**
LB statistics (p-value)[12]	0.26	0.96	near 0	0.99	0.96	0.045
Auto-arima	(0,0,0)	(2,0,0)	(0,0,0)	(1,0,0)	(1,0,0)	(0,0,0)
ARCH LM test[13]	0	0	0	0	0	0
ADF statistics (p-value)	0.01	0.01	0.01	0.01	0.01	0.01
PP testing (p-value)	0.01	0.01	0.01	0.01	0.01	0.01
KPSS testing (p-value)	0.1	0.1	0.1	0.1	0.1	0.1
Kurtosis[14]	6.54	20.08	10.62	4.17	2.02	27.01
Skewness	0.259	0.209	0.882	-0.175	-0.285	-0.512

LB statistics show no autocorrelation for the series that auto.arima gives 1 or more degree for AR. Variance and mean orders generally does not coincided with AIC based auto-arima results. Also, ARCH LM tests for "auto.arima" results show no significant ARCH effect. ADF, PP and KPSS testing processes indicated stationarity for all series. Also, JB testing and kurtosis values pointed out that there is no evidence that the series have characteristics of normal distribution. However, R script chooses normal distribution skewed or not for 4 series despite high excess

[12] LB statistic values belong to residual distribution of auto.arima results.
[13] ARCH test statistic values belong to residual distribution of auto.arima results.
[14] Kurtosis and skewness values belong to residual distribution of auto.arima results.

kurtosis. Therefore, we can not obtain a result that distributional form of return series gives an opinion about the GARCH model with different distributional assumptions.

7 BIBLIOGRAPHY

Alberg, D., Shalit, H., Yosef, R. (2008), “Estimating stock market volatility using asymmetric GARCH models”, Applied Financial Economics, 18, p.1201-1208

Alexander, C., Market Risk Analysis - Practical Financial Econometrics, Volume II, 2008, John Wiley&Sons Ltd.

Andersen T.G. and T. Bollerslev, (1998), “Answering the Skeptics: Yes, ARCH Models Do Provide Good Volatility Forecasts”, International Economic Review, 39 (4): 885-905

Andersen T.G., T. Bollerslev, and S. Lange, (1999), “Forecasting Financial Market Volatility: Sampling Frequency vis-a-vis Forecast Horizon”, Journal of Empirical Finance, 6 (5): 457-77

Andersen T.G., T. Bollerslev, P. Christoffersen, and F.X. Diebold, (2005), “Volatility Forecasting”, NBER Working Paper No. 11188

Applebaum, D. (2009), Levy Processes and Stochastic Calculus, Cambridge University Press

Bauwens, L., Hafner, C. and Laurent S. (eds.) (2012), Handbood of Volatility Models and Their Applications, Wiley

Bedowska-Sojka, B. and Kliber, A. (2010), “Realized Volatility Versus GARCH and Stochastic Volatility Models, The Evidence From The WIG20 index and the EUR/PLN foreign exchange market”, Przegląd Statystyczny, R.LVII, 2010

Bollen, B., Skotnicki, A. and Veeraraghavan, M., “Idiosyncratic volatility and security returns: Australian evidence”

Bollerslev, T., (1986) “Generalized Autoregressive Conditional Heteroskedasticity”, Journal of Econometrics, 31(3), p.307-327

Bollerslev, T. (2009), “Glossary to ARCH (GARCH)” in Volatility and Time Series Econometrics: Essays in Honor of Robert F. Engle (2010) edited by Bollerslev, T., Russel, J.R. and Watson, M.J. (2010)

Bos, C.S. (2012), “Relating stochastic volatility estimation methods”, in L. Bauwens, C. Hafner, S. Laurent (eds.), Handbook of volatility models and their applications, pp:147-174, John Wiley&Sons Inc., chp. 6

Box, G. E. P. and Tiao, G. C. (1973) “Bayesian Inference in Statistical Analysis”, Addison-Wesley Press, Reading, MA, USA.

Brownlees, C.T. and Gallo G.M. (2010), “Comparison of Volatility Measures: a Risk Management Perspective”, Journal of Financial Econometrics, Vol. 8, No. 1, 29–56

Carr, P., Geman, H., Madan, D.B., Yor, M. (2003), “Stochastic Volatility for Levy Processes”, Mathematical Finance, Vol.13, No.3 (july 2003), 345-382

Christensen J. and Nielsen M. O. (2007), “The Effect of Long Memory in Volatility on Stock Market Fluctuations”, The Review of Economics and Statistics, 89(4), pp.684-700

Chuang, I.-Y., Lu, J. and Lee, P., “Forecasting accuracy in the financial markets: a comparison of alternative distributional assumptions”, Applied Financial Economics, 2007, 17, p:1051-1060.

Croissant, Y. and Millo, G., Panel data econometrics in R: The plm package

Ding, J. and Meade N., “Forecasting accuracy of stochastic volatility, GARCH and EWMA models under different volatility scenarios”, Applied Financial Economics, 2010, 20, p:771-783.

Dowd, K. (2005), Measuring market risk, John Wiley&Sons Ltd

Ederington, L.H. and Guan W. (2010), “Long term time-series volatility forecasts”, Journal of Financial and Quantitative Analysis, Vol.45, No.4, p:1055-1076

Engle, R. F., (1982), “Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of the United Kingdom Inflation”, Econometrica, 50(4) p.987-1008

Figlewski Stephen, (2004), “Forecasting Volatility”, New York University Stern School of Business Research Paper

Franke, R. and Westerhoff F. (2010), “Estimation of a structural stochastic volatility model of asset pricing”, Springer Science+Business Media, LLC

Ghalanos, A. (2013), “Package ‘rugarch’”, R CRAN document, http://cran.r-project.org/web/packages/rugarch/index.html (23.06.2015)

Ghalanos, A. (2014), “Introduction to rugarch package (version 1.3-0)”, R CRAN document, http://cran.rproject.org/web/packages/rugarch/vignettes/Introduction_to_the_rugarch_package.pdf (23.06.2015)

Halbleib, R., Voew, V. (2011), “Forecasting multivariete volatility using the VARFIMA model on realized covariance Cholesky factors”, Jahrbücher f. Nationalökonomie u Statistik (Lucius & Lucious, Stuttgart) Vol. 231/1

Hamilton, J.D. and Susmel R. (1994), Autoregressive conditional heteroskedasticity and changes in regime, Journal of Econometrics, Vol.64, pp:307-333

Hansen, P.R. (2001), “A test for superior predictive ability”

Hansen, P. and Lunde, A. (2005), “A forecast comparison of volatility models: does anything beat a GARCH(1,1)?”, Journal of Applied Econometrics, 20, p:873-889

Hansen, P. and Lunde, A. (2011), “Forecasting Volatility using High Frequency Data”, Michael P. CLEMENTS and David F. HENDRY (eds), The Oxford Handbook of Economic Forecasting, Oxford, Oxford University Press, 2011, Oxford Handbooks, 525-556

Harris, R.D.F., Kucukozmen, C. and Yilmaz, F. (2006), “Skewness in the conditional distribution of daily equity returns”, Applied Financial Economics, Volume 14, Issue 3, 2004, pp:195-202.

Hendry, D.F. and Juselius, K. (1999), Explaining Cointegration Analysis: Part I-II

Hentschel, L. (1995), “All in the family nesting symmetric and asymmetric GARCH models”, Journal of Financial Economics 39(1): 71-104.

Huang, A.Y. (2011), “Volatility forecasting in emerging markets with application of stochastic volatility model”, Applied Financial Economics, Vol.21, Issue 9, 2011.

Hunter, J.K. (2009), Lecture Notes on Applied Mathematics Methods and Models, from https://www.math.ucdavis.edu/~hunter/m280_09/ch.pdf (17.05.2014)

Jiang, Z. and Zhou, W., “Scale invariant distribution and multifractality of volatility multipliers in stock markets”, Physica A 381, 2007, 343-350

Johnson, N. L. (1949) “Systems of frequency curves generated by method of translation”, Biometrika, 36, 149–76

Karian, A.K. and Dudewicz, E.J. (2011), Handbook of Fitting Statistical Distributions With R, CRC Press A Chapman & Hall Book

Kastner, G., “Dealing wity stochastic volatility in time series using the R package stochvol”

Khalifa, A.A.A., Miao, H, and Ramchander, S., “Return distributions and volatility forecasting in metal futures markets: evidence from gold, silver, and copper”, The Journal of Futures Markets, 2011, Vol:31, No:1, p:55-80

Kleiber, C. and Zeileis, A. (2008), Applied econometrics with R, Springer

Knill, O. (2011), Probability and Stochastic Processes with Applications, from http://www.math.harvard.edu/~knill/teaching/math144_1994/probability.pdf (17.05.2014)

Magnus, George, (2007) “The Credit Cycle and Liquidity: Have We Arrived at a Minsky Moment?” Economic Insights UBS Investment Research.

Mills, T.C. and Markellos, R.N. (2008), The Econometric Modelling of Financial Time Series, Third Edition, Cambridge University Press

Miron, D. and Tudor, C. (2010), “Asymmetric Conditional Volatility Models: Empirical Estimation and Comparison of Forecasting Accuracy”, Romanian Journal of Economic Forecasting, 3/2010, pp.74-92

Mixon, S., “Option markets and implied volatility: past versus present”, Journal of Financial Economics, 2009, 94, 171-191

Pfaff, B. (2013), Financial Risk Modelling and Portfolio Optimization with R, John Wiley & Sons, Ltd

Poon, S. (2005), A Practical Guide to Forecasting Financial Market Volatility, John Wiley & Sons, Ltd

Poon, S. and Granger, C.W.J. (2003), "Forecasting volatility in financial markets: A review", Journal of Economic Literature, Vol: XLI, p:478-539

Poon, S. and Granger, C.W.J. (2005), "Practical Issues in Forecasting Volatility", Financial Analysts Journal, Vol: 61, No:1, p:45-56

Prause, K. (1999), The Generalized Hyperbolic Model: Estimation, Financial Derivatives, and Risk Measures, PhD Dissertation Thesis, Institut f¨ur Mathematische Stochastik Albert-Ludwigs-Universit¨at Freiburg

Ruppert, D. (2010), Statistics and Data Analysis for Financial Engineering, Springer

Shephard, N. (2005), "General Introduction" in Stochastic Volatility Selected Writings edited by Neil Shephard, Oxford Press

Smith, J. B. (1981) "The probability distribution of market returns: a logistic hypothesis", PhD Dissertation, University of Utah

Taylor, J.W. (2005) "Generating volatility forecasts from value at risk estimates", http://dx.doi.org/10.1287/mnsc.1040.0355

Theodossiou, P. (1998) "Financial data and the skewed generalized T distribution", Management Science, 44, 1650–61

Torben, G.A., Bollerslev, T. and Diebold, F.X., (2002), "Parametric and nonparametric volatility measurement", in L.P. Hansen and Y. Ait-Sahalia (eds.), Handbook of Financial Econometrics. Amsterdam: North-Holland

Tsay, R.S., (2010), Analysis of Financial Time Series, Wiley

Whalen, Charles, (Jan/Feb 2008), "Understanding The Credit Crunch As A Minsky Moment", Challenge Vol.51, 91-109

Wu, L. (2005), Modeling Financial Security Returns Using Levy Processes, Zicklin School of Business, Baruch College, City University of New York

Zuccini, W. and Nenadic, O., Time Series Analysis with R – Part I-II

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WORKING EXPERIENCE

2006-Present, Banking Regulation and Supervision Agency (BRSA) of Turkey

- 2014-Present, Bank examiner in an on-site examination team
- 2012-2014, Bank examiner in Risk Management Department, specialised in Basel 2 Pillar 2 implementation, macro- and micro-prudential stress testing, ICAAP and SREP processes
- 2013-2015, representative member of BRSA in Corporate Governance Task Force of Basel Committee on Banking Supervision (BCBS)
- 2006-2012, Bank examiner in on site examination teams

2005-2006, Ministry of Labor and Social Security of Turkey

- Assistant specialist team member of large scaled social security reform project of Turkey (backed by Worldbank)

ACTIVITIES AND LICENSES

- **Lecturer** of seminar programs organized by the Association of Turkish Capital Markets (TSPB) about the topics including hedge accounting, international financial reporting standards (IFRS), banking law, Basel III.
- **Board Member** of Association of Bank Examiners (2013-Present)

- **Team member of** the Oral History Project about Galata and Pera -historical districts of Istanbul- which were once the centre of commerce and diplomacy of Ottoman Empire at the time of Genoa colony, and which now are defined as the gentrification area (2003-2004).
- **Participants** of training courses or seminars below:
  - Advanced credit risk measurement, organized by Federal Reserve Bank (FED) between 03-07.08.2008 in İstanbul, Turkey
  - Risk management and risk focused supervision, organized by Financial Stability Institute (FSI) between 24-28.09.2012 in Beatenberg, Switzerland
  - Advanced course on financial stability stress testing for banking systems, organized by Joint Vienna Institute (JVI) under the sponsorship of IMF, 20-23.01.2014 in Wien, Austria
  - Quantitative risk measurement and its audit, organized by RiskActive, twice a week between October and December 2011 (throughout about ten weeks), Istanbul, Turkey
  - Risk assessment techniques and pricing of financial instruments, lectured by Prof.Burak SALTOĞLU, 5 Days Throughout December 2013, Istanbul, Turkey
  - Actuary and finance, organized by Hacettepe University Life Long Learning Centre (HUSEM), 05-29.09.2005, Ankara, Turkey
  - The Relationship between IFRS and Basel II, organized by The Banks Association of Turkey (TBB), 31.10-02.11.2007, Istanbul, Turkey
  - Comprehensive IFRS, organized by Deloitte Academy, August 2008, Istanbul, Turkey
  - Certificates on Microsoft Axcess, Excel, Excel-VBA
  - Observation program of French social security system, organized by Ecole nationale Supérieure de Sécurité Sociale (en3s), between 16.10-30.11-2005, St. Etienne and Lyon, France
- Certified Public Accountant (CPA) and Independent Auditor licenses

PUBLICATIONS

Dissertation for BRSA, Titled “Research on 2008 Global Financial Crisis by Examining Financial Statements of Some Crucial American Investment and Commercial Banks” Dec. 2009 (in Turkish).

“Implementation of Fair Value Option under the rules of IAS 39”, Active - Journal of Banking and Finance, Vol. 61, Nov. 2011 (in Turkish)

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