Hacettepe University Graduate School of Social Sciences

Department of Economics

PRIVATE-PUBLIC CAPITAL, ELASTICITY OF SUBSTITUTION AND ECONOMIC GROWTH

Abdulmecit YILDIRIM

Ph. D. Dissertation

Ankara, 2018

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ACCEPTANCE AND APPROVAL

The jury finds that Abdulmecit YILDIRIM has on the date of 05.06.2018 successfully passed the defense examination and approves his Ph.D. thesis titled "Private-Public Capital, Elasticity of Substitution and Economic Growth".

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Abdulmecit YILDIRIM

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In this thesis study, I declare that all the information and documents have been obtained in the base of the academic rules and all audio-visual and written information and results have been presented according to the rules of scientific ethics. I did not do any distortion in data set. In case of using other works, related studies have been fully cited in accordance with the scientific standards. I also declare that my thesis study is original except cited references. It was produced by myself in consultation with supervisor Doç. Dr. Bahar Sağlam, and written according to the rules of thesis writing of Hacettepe University Institute of Social Sciences.

Abdulmecit YILDIRIM

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ABSTRACT

YILDIRIM, Abdulmecit. *Private-Public Capital, Elasticity of Substitution and Economic Growth*, Ph. D. Dissertation, Ankara, 2018.

In the growth literature, Cobb-Douglas (CD), Constant Elasticity of Substitution (CES), and Variable Elasticity of Substitution (VES) production functions are frequently used in analyzing the elasticity of substitution between production factors. Most of the studies in the literature focus on the elasticity of substitution between capital and labor. However, studies on the sub-components of the capital stock are very scarce. In this thesis, the aggregate capital stock is analyzed by splitting it into two subcomponents, namely, public capital stock and private capital stock. The elasticity of substitution between public capital stock and private capital stock has been examined for 91 countries using the data for the period 1980-2011 in the framework of a nested VES type production function. The main advantage of the VES production function over other functions is that it provides more flexibility in the parameters. The parameters related to the VES production function are estimated by the help of nonlinear least squares (NLLS) method.

Findings of the thesis revealed that the elasticity of substitution between public capital stock and private capital stock is found to be negative in most models. Further, public capital stock and private capital stock are complements in the production of final goods and services. Therefore, public investments that are directed at improving infrastructure can positively affect economic growth by increasing the efficiency of private investments.

Key Words

Public Capital Stock, Private Capital Stock, Variable Elasticity of Substitution, Nonlinear Least Square (NLLS), Economic Growth

ÖZET

YILDIRIM, Abdulmecit. *Özel-Kamu Sermaye Stoku, Değişen İkame Esnekliği ve Ekonomik Büyüme*, Doktora Tezi, Ankara, 2018.

Büyüme literatüründe üretim faktörleri arasındaki ikame esnekliğinin hesaplanmasında Cobb-Douglas (CD), Sabit İkame Esneklikli (CES) ve Değişen İkame Esneklikli (VES) üretim fonksiyonlarından sıklıkla yararlanılmaktadır. Çalışmaların çoğu sermaye ile emek arasındaki ikame esnekliğine odaklanmaktadır. Toplam sermayeyi oluşturuan alt bileşenlere ilişkin yapılan çalışamalar sayıca azdır. Bu tezde toplam sermaye, kamu sermaye stoku ve özel kesim sermaye stoku olarak iki alt bileşene ayrılarak incelenmektedir. Kamu sermaye stoku ile özel kesim sermaye stoku arasındaki ikame esnekliği iç içe yuvalanmış VES tipi bir üretim fonksiyonu çerçevesinde 91 ülke için 1980-2011 dönemi baz alınarak incelenmiştir.VES üretim fonksiyonunun diğer fonksiyonlara göre temel avantajı parameterlerde daha fazla esneklik sağlamasıdır. VES üretim fonksiyonuna ilişkin parametreler lineer olmayan en küçük karaler (NLLS) yöntemiyle tahmin edilmiştir.

Kamu sermaye stoku ile özel kesim sermaye stoku arasındaki ikame esnekliğinin tahmin edildiği modellere ilişkin sonuçlara baktığımızda genel itibariyle bir çok modelde kamu sermaye stoku ile özel kesim sermaye stoku arasındaki ikame esnekliğini belirleyen parametre negative bulunmuştur. Bunun yanısıra, kamu sermaye stoku ile özel kesim sermaye stoku arasındaki ikame esnekliği seçilen örneklem için birden küçük bulunmuştur. Buna göre, kamu sermaye stoku ile özel kesim sermaye stoku nihai mal ve hizmet üretiminde birbirini tamamlayıcı girdiler konumundadır. Dolayısıyla, özellikle alt yapıyı iyileştirmeye dönük kamu yatırımları özel yatırımaların verimliliğini artırarak ekonomik büyümeyi olumlu etkileyebilir.

Anahtar Sözcükler

Kamu Sermaye Stoku, Özel Kesim Sermaye Stoku, Değişen İkame Esnekliği, Lineer Olmayan Enküçük Kareler, Ekonomik Büyüme

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ABBREVIATIONS

CD	: Cobb-Douglas
CES	: Constant Elasticity of Substitution
VES	: Variable Elasticity of Substitution
NLLS	: Nonlinear Least Square
OLS	: Ordinary Least Square
HDI	: Human Development Index
IMF	: International Monetary Fund

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CHAPTER 1: INTRODUCTION

1.1 CAPITAL STOCK AND ECONOMIC GROWTH

In recent years, there has been a growing debate on the effects of public capital on economic growth. It is generally considered that resource allocation in the public sector is less effective. This is in part due to the fact that, excessive weight of the public sector in the economy may disrupt the efficient distribution of the resources, thus impedes economic growth. On the other hand, many developing countries in the 1980s which were in debt, intended to balance public and private investments with structural adjustment programs (SAPs). This situation requires that the total capital should be divided between public capital and private capital and these two inputs should be treated as two different production factors.

Although the effect of public spending on private capital accumulation and output has been discussed for a long time, it is hard to say there is a common consensus about the relationship between public spending, private capital, and output. Some studies argue that public capital has a positive influence on private capital accumulation and output (see for example Aschauer 1989a, and 1989b; Lynde and Richmund 1992; Otto and Voss 1996; Nazmi and Ramirez 1997; Sanches-Robles 1998; Agenor and Blanca 2006). Some other studies (see for example, Tatom 1991; Holtz-Eakin 1994) could not find a statistically significant relationship between public spending, private capital and output. Moreover, there are studies such as Voss 2002 that finds the crowding-out effect of public capital on private capital accumulation and output.

According to Ashauer (1989b), public capital affects private capital accumulation from two different channels. First, high public investments lead to an increase in public capital accumulation, which then leads to an equivalent reduction in private capital. Second, public investments generate crowding-in effects on private capital accumulation by increasing the marginal productivity of private capital stock. Aschauer (1998b) advocates that the second effect will dominate the first, and therefore an increase in public investments will also increase private capital accumulation.

Further, public capital -particularly infrastructure investments- can be seen as an important determinant of long-run growth as it acts as an important complement to private sector capital (Dessus and Herrera, 2000). Mukuyana and Odhiambo (2016) states that public investments in form of infrastructure (education, airports, highways, roads, power generation and distribution facilities, etc.) often increase marginal productivity of private capital. The existence of this basic infrastructure in an economy can reduces the costs of firms operating in the private sector, leading to new private capital accumulation and economic growth. For this reason, the optimal allocation of capital between the public sector and the private sector is fundamental. The degree of elasticity of substitution between public capital and private capital plays an important role in this optimal allocation. For this reason, it is important for policymakers to know whether public capital and private capital are substitutes or complements for the effectiveness of policies. Therefore, before specifying economic policies, the degree of elasticity of substitution between factors of production should be determined.

Even though, there have been many studies to explore the growth-enhancing role of public and private capital accumulations (see, for example, Atukeren, 2005; Bucci and Del Bo, 2012), empirical studies suffer from controversial and conflicting results. In other words, the interaction between public and private capital stock depends on the period, sample selected, and technology used for final output production.

1.2. PRODUCTION FUNCTION AND ELASTICITY OF SUBSTITUTION

In growth models, production functions such as constant elasticity of substitution (CES) and variable elasticity of substitution (VES) are used to express the production process in the economy. Arrow et al. (1961) emphasize that it is important to know the elasticity of substitution between the factors of production, indicating that the growth process depends on the value of elasticity of substitution. The concept of elasticity of substitution used in these models refers to the ease of substitution of inputs used in production. Most of the studies investigating the dynamics of economic growth are based on the Cobb-Douglas type of production functions (see, for example, Solow 1956; Aschauer 1997; Dessus and Herrera 2000). Such production functions represent a process in which the elasticity of substitution between factors of production is equal to one. Recently, CES type production functions which assumed that the elasticity of substitution between production factors is a constant value, which is less than or greater than one, has been extensively used in many theoretical or empirical models (See, for example, Young 2013; Arrow et. al. 1961; Miyagiwa and Papageorgiou 2003).

It is necessary for a country to be able to substitute public capital with other production factors in order to achieve sustainable growth and development. In addition, the elasticity of substitution between public capital, private capital and other inputs of production is generally considered to be either one (Cobb-Douglas production function) or constant (CES production function) in the theory of economic growth. However, with the economic liberalization, the growing weight of the private sector in the economy does not make it possible to have a constant elasticity of substitution between public and private capital. Therefore, it is better to express the interaction among public capital, private capital and other production factors with a production function that allows for non-constant elasticity of substitution.

Standard growth models such as Solow 1956 can explain how different combinations of inputs affect economic growth under the assumption of constant elasticity of substitution (CD or CES). Yet, they are insufficient to explain a change in elasticity of substitution that increases or decreases economic growth. If we drop the assumption of traditional constant elasticity of substitution (CES), a change in the degree of substitutability between public capital and private capital can affect economic growth.

In this study, variable elasticity of substitution (VES) type production function is used to examine elasticity of substitution relationship between production factors. This production function assumes that, unlike the standard Cobb-

Douglas or CES functions, the elasticity of substitution between the factors is not constant. The VES type production function is more advantageous than other production functions as it provides more flexibility in parameters. The standard Cobb-Douglas production function used in many studies lacks this flexibility, since, the elasticity of substitution in Cobb-Douglas production function is equal to one. Therefore, the relative shares of production factors are constant. On the other hand, the production function with constant elasticity of substitution (CES) is also frequently used in the growth literature. Although comparing with Cobb-Douglas functions, CES functions place relatively few constraints on the parameters but it is not as flexible as the VES functions.

The theoretical model used in this study is based on the Solow-Swan growth model. Unlike the standard Solow-Swan growth model, in which a single capital stock variable is used, the distinction between public capital and private capital has been made in this thesis. In other words, public capital and the private capital are exist as two distinct inputs of production in our model. In addition, a production function of the VES type which allows variable elasticity of substitution between inputs is defined. Within this model, the degree of substitution of public capital and private capital can directly affect long-run economic growth.

The production function, which shows the variable elasticity of substitution property, was first used by Sato and Hoffman (1968) and Revenkar (1971). Karagiannis et. al. (2005) examined the elasticity of substitution between capital and labor in the framework of VES production function. Lazkano and Pham (2016) developed the model used by Karagiannis et. al. (2005) in the context of substitution elasticity between capital and energy. Our study in this direction contributes to the relevant literature.

1.3. PRODUCTION INPUTS AND ELASTICITY OF SUBSTITUTION

The most fundamental factor determining the direction and degree of the relationship between public investments, private capital accumulation, and output is the elasticity of substitution. Public investment expenditures can facilitate private capital accumulation and thus affect economic growth positively. For example, if the production factors are gross complement, a higher stock of public capital in form of infrastructure may positively affect the productivity inputs such as labor, private capital, and energy. Thereby reducing unit cost of production.

All of the models that investigate the growth process, even being implicit, takes elasticity of substitution into consideration. This situation indicates that elasticity of substitution is important in the theory of economic growth. First of all, elasticity of substitution is one of the determinant of the level of development. Klump and de la Grandville (2000) states that a higher elasticity of substation leads to not only a higher growth rate of income per capita, but also a higher steady state value of income per capita. Moreover, they conclude that the success of the growth miracles in East Asia countries was not necessarily due to high saving rates and technological progress, but to a higher elasticity of factor substitution. Second of all, elasticity of substitution affects the speed of convergence between countries. Klump and Preissler (2000) argue that if capital stock is relatively scarcer than labor input higher elasticity of substitution may rise the speed of convergence. Third of all, the elasticity of substitution may change the saving behavior of households during the transition period, which in turn affects capital accumulation. Smettters (2003) investigate this fact within the framework of Ramsey-Cass-Koopmans growth model. The results shows that if the elasticity of substitution between capital and labor is below unity the saving rates decreases monotonically. This situation is reversed when the elasticity of substitution is greater than unity. Finally, elasticity of substitution can influence the income distribution in an economy. Hicks (1932) argues that elasticity of substitution among production factors is the only determinant of changes in the relative share of production factors. According to Getachew (2009), this argument of Hicks shows a solution to the problem of income distribution. In other words, whether the public sector has an influence on the income distribution dynamics depends on the elasticity of substitution between public and private capital.

The balanced use of the public and private component of the total capital stock allows less developed countries to reduce inequality in income distribution (World Bank, 1994; Getachew, 2009). The interaction between these two components of the capital is important, since regardless of the technology used for good production, the degree of complementarity/substitutability between private and public capital is the main determinant of optimal growth (Bucci and Del Bo, 2012).

Empirical studies reveal that the relationship between public capital and private capital vary over time and across countries. In some sectors and countries, the public capital and private capital are complements, while in other sectors and regions these two factors of production are substitutes. One reason for these conflicting empirical results can be explained by the reduction of the state's weight in the economy and by the private sector filling this vacated field. However, most of the previous studies have focused mainly on the assumption of the existence of a constant elasticity of substitution between production inputs. To the best of our knowledge, this is the first study to examine the existence of a non-constant elasticity of substitution between the public capital and the private capital stocks.

While Barro (1990), Glomm and Ravikumar (1997) and Aschauer (2000) have shown that there is a non-linear relationship between public capital and economic growth, according to Aschauer (1998), much of the literature is based on assumption that there is a linear relationship between public capital and economic growth. Within this context, the elasticity of substitution between public and private capital is studied with the help of non-linear estimation method under the framework of VES production function. Moreover, the contribution of elasticity of substitution to economic growth is also investigated in this thesis.

To empirically examine the association between public and private capital stocks, a VES type general production function for 91 countries is estimated over the period 1980-2011. This function expresses a non-linear relationship between the factors of production and indicates a constant or variable elasticity of substitution between inputs. The validity of the hypothesis has also been tested for different income groups, considering the country classification according to the World Bank income groups and regions.

The empirical findings provide evidence that there is complementarity between public capital and private capital in developed and developing countries. The complementarity property between public and private capital stock is unchanged when we consider different country classifications. In general, in case of global data set a one-unit increase in public-private capital stock ratio reduces the elasticity of substitution approximately by 0.15 unit.

The rest of the thesis is organized as follows. The next chapter gives brief information about factor substitution and growth nexus. Chapter 3 reviews the theoretical and empirical studies about the role of public capital and private capital in economic growth. Methodological issues, and data set and variables are presented in chapter 4 and chapter 5, respectively. Chapter 6 presents the empirical results of VES production function. The last chapter concludes the study with some suggestions for development policies.

CHAPTER 2: PRODUCTION INPUTS, FACTOR SUBSTITUTION, AND GROWTH NEXUS

The relationship between input and output in the production process is represented by the production functions. Whether a production process is optimal or not is closely related to how much output is obtained from inputs used. The main objective here is to specify a feasible input composition that maximizes the output in final production. The elasticity of substitution plays a significant role in determining the optimal input composition.

In the growth models that make it easier to understand the dynamics of the growth process, capital and labor are used as fundamental inputs. Besides these two basic inputs, other factors (such as public capital, human capital, and energy) that affect growth process have become the center of interest for many researchers. Various nested CES and VES production functions have been developed to investigate the relationship between inputs and production (see, for example, Zha and Zhou, 2014; Shen and Whalley 1992; Lazkano and Pham 2016).

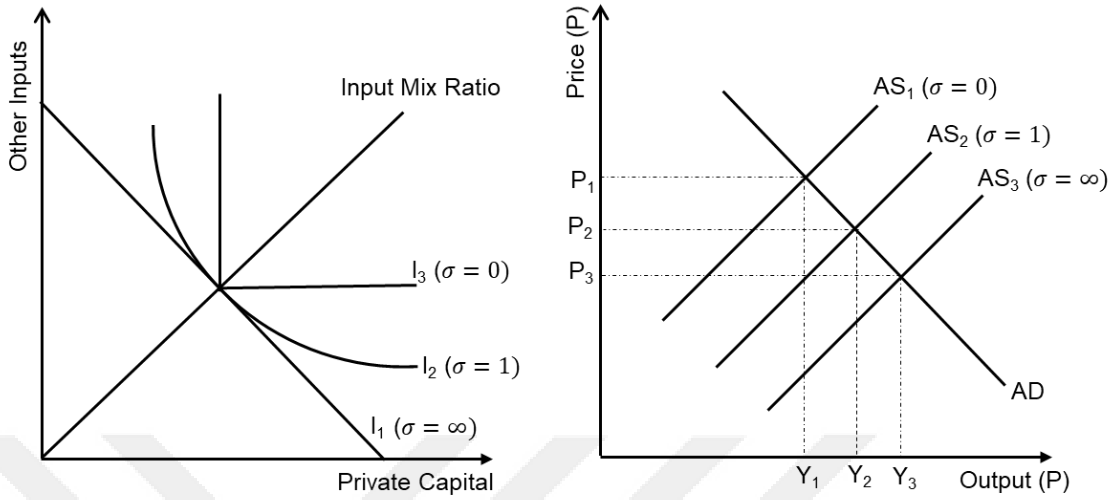
Nelson (1995, p. 6) states that when the growth rate of capital stock does not substantially differ from the growth rate of the labor force, the effects of the selection of production function in the short run analysis of growth process will be minimal. Starting from this assertion, the elasticity of substitution can play an important role in long-run total output, or in a situation where the growth rate of capital stock substantially differs from growth rate of the labor force. Therefore, in such cases the selection of production function for the analysis of the growth process becomes important.

The elasticity of substitution plays a significant role between production inputs and growth. The degree of elasticity of substitution between production factors gives information about whether inputs are substitutes or complements. Therefore, having knowledge about elasticity of substitution is important for policymakers. For example, the knowledge about the elasticity of substitution may be crucial for the determination of tax policy in manufacturing industries.

It is important to identify the elasticity of substitution among the factors of production and to take this into consideration in the socio-economic policy to be implemented by policymakers. The elasticity of substitution among the factors directly affects the results and the effects of policies. For instance, in an economy where labor and capital are used as inputs, assuming that capital stock is increasing, the marginal productivity of the capital will decrease faster if the substitution elasticity between these two inputs is low. Investment incomes will be directly affected by this situation. For this reason, the elasticity of substitution is so important that it cannot be neglected in economic growth. As Getachew (2009) points out, if the elasticity of public capital to substitute private capital is greater than one, an investment in public capital will increase the relative share of public capital in production and reduce the relative share of private capital. This will have a corrective effect on income distribution if the aim is to improve the living standards of the individuals in the low-income groups. However, if the elasticity of substitution is lower than one, an increase in public capital will increase the share of private capital, which may have a detrimental effect on income distribution.

Although capital stock plays a key role in economic growth, it may not be sufficient to explain the growth process alone. The sub-components of the capital stock need to be examined. In this context, examining public and private capital as two distinct inputs of production may shed light on better understanding the growth process. At this point, it is quite natural to think that whether public and private capital are substitutes or complement. The knowledge about the elasticity of substitution among production factors may give an answer to this issue.

Figure 1: Elasticity of Substitution and the Aggregate Output



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Hicks (1932) argues that elasticity of substitution among production factors is the only determinant of changes in the relative share of production factors. According to Getachew (2009), this argument of Hicks shows a solution to the problem of income distribution. In other words, whether the public sector has an influence on the income distribution dynamics depends on the elasticity of substitution between public and private capital.

In sum, it is important to determine the optimal factor composition in final output production. If it is correctly specified, the elasticity of substitution may help to choose the optimal composition of inputs. Therefore, the selection of production function is important. A production function that put less restrictions on the parameters may be an important step to investigate the production process. VES type production functions can be a good candidate to analyze the relationship between production inputs and elasticity of substitution.

CHAPTER 3: LITERATURE REVIEW

3.1. INTRODUCTION

In general, studies on the elasticity of substitution focus on Cobb-Douglas or CES production functions in which the elasticity of substitution is constant (see for example Arrow et. Al, 1961; Klump and de la Grandville, 2000). However, the studies that investigate the role of elasticity of substitution in the non-constant framework is relatively scarce. This chapter presents a literature review on the elasticity of substitution and economic growth far from being conclusive. The summary of the most cited studies in the context of substitutability and complementarity among production factors is presented in Appendix 4.

3.2. THE REVIEW OF THE STUDIES ON CONSTANT ELASTICITY OF SUBSTITUTION PRODUCTION FUNCTION (CD or CES)

Arrow et. al. (1961) investigate capital-labor substitution and economic efficiency under CD and CES framework. The data set contains three-digit industries of 19 countries belongs to different years of the period 1949-1955. The elasticity of substitution between capital and labor is less than unity for the manufacturing industries of 19 selected countries.

Zarembka (1970) empirically tests the suitability of the CES production function for 2-digit 13 the US manufacturing industry for the years 1957 and 1958. The results show that there is no evidence that the elasticity of substitution between capital and labor is different from unity for the selected sample of industries. Therefore, CD production function is a more appropriate specification than CES specification for the period specified.

Klump and de la Grandville (2000) examine CES production function in the context of neoclassical Solow model. Ceteris paribus, starting from the same initial conditions, a country with high elasticity of substitution between capital and labor will always have a higher per capita income. Hence, the study states that elasticity of substitution plays a significant role in economic growth.

Similarly, Klump and Preissler (2000) examined the CES production function in the context of the neoclassical Solow model. According to the results of the study, the higher elasticity of substitution between capital and labor leads to a higher steady state outcome, which confirms Klump and de la Grandville (2000).

Irmen (2010) construct a theoretical model to consider Klump and de la Grandville hypothesis. The author concludes that an economy with a higher elasticity of substitution between capital and labor has a higher steady-state growth rate of output per worker. This result supports Klump and de la Grandville (2000) assessment that the elasticity of substitution is a powerful engine of economic growth.

Duffy and Papageorgiou (2000) state that CD production function may not be empirically valid for aggregate production, and provide evidence that CES is a better specification. Moreover, the results show that the elasticity of substitution between capital and labor may vary with the level of development of countries. In rich countries, the elasticity of substitution is greater than one, whereas it is less than one in poor countries. Masanjala and Papageorgiou (2004) examine the Solow model in the CES framework for 98 countries. The results of this study show that elasticity of substitution between capital and labor is greater unity. CES specification better fits cross-country variation than CD specification. These results confirm Duffy and Papageorgiou (2000), which reject CD specification in favor CES.

In their theoretical study, Miyagiwa and Papageorgiou (2003) state that the relationship between factor substitutability and economic growth, as stated by Klump and de la Grandville (2000), may not be valid for Diamond overlapping generation model. The higher elasticity of substitution between capital and labor leads to lower per capita output growth in both transition and steady state. However, Miyagiwaa and Papageorgiou (2006) examine the endogenous aggregate elasticity of substitution for both CD and CES production functions in a theoretical framework. This study aims to test the hypothesis that aggregate elasticity of substitution evolve over time and varies with the level of economic development. The results show that aggregate elasticity of substitution between capital and labor is positively related to the level of economic development.

Antras (2004) and Young (2013) reach to similar results regarding the elasticity of substitution between capital and labor in the US economy. Antras (2004) investigates whether the private sector of the US economy over the period 1948-1998 is Cobb-Douglas or not. The results of the study show that the elasticity of substitution is well below unity, which suggests the rejection of Cobb-Douglas specification for United States economy during the sample period. Young (2013) examine the elasticity of substitution between capital and labor for 2-digit 35 US industry over the period 1960-2005. The results of the study show that the aggregate elasticity of substitution is less than unity. Moreover, when examining the elasticity of substitution for each individual industry, the elasticity of substitution is less than unity most of the industries.

Mallick (2010) explores how different values of elasticity of substitution between capital and labor affect a country’s growth performance in exogenous growth models. The study covers a sample of 112 countries over the period 1970-2000. The author uses both Solow-CES and Ramsey-CES framework in his analysis. The results show that the existence of the steady state and balanced growth path depend on the value of elasticity of substitution. After investigating that elasticity of substitution can affect growth performance of a country, Mallick (2012) tries to calculate this effect for 90 countries over the period 1950-200. CES framework is used to investigate the role of elasticity of substitution in economic growth. The results show that elasticity of substitution depart from unity, and it can explain the growth rate differential between East Asia and Sub-Saharan Africa about 20 to 25 percent. Moreover, the author suggests that the researchers should pay attention to the role of elasticity of substitution between capital and labor in both theoretical and empirical research, and reconsider the usefulness of CD production function in growth theory.

Daniels and Kakar (2017) examine the relationship between economic growth and human capital in CD and CES framework for non-oil producing countries over the period 1960-1985. Daniels and Kakar (2017) investigate this relationship for 98 countries in CD case, 84 countries for CES case. Moreover, the authors use a subset of countries whose population is less than 1 million in 1960 in their analysis. The results show that elasticity of substitution can lead to higher income per effective labor as well as a higher speed of convergence. Hence, the findings emphasize that elasticity of substitution is a significant determinant of economic growth. Moreover, the elasticity of substitution is significantly below unity for normalized aggregate CES production function.

Goldar et. al. (2013) examines the elasticity of substitutional between capital and labor in the context of CES production function for 22 Indian manufacturing industry over the period 1980-2008. The results of the study indicate that in the majority of Indian manufacturing industry the elasticity of substitution between capital and labor is less than one. This result implies that the substitution opportunities is relatively low in these industries over the sample period. Moreover, the elasticity of substitution between capital and labor varies among industries; indicating that different tax policies can be applied to different manufacturing industries.

Lukáš Rečka (2013) use CES framework to examines the elasticity substitution between production factors (capital, labor, and energy) for 13 Central and Eastern European countries over the period 1995-2009. According to the results, sector-specific estimates fit the data much more than the industry as a whole. Therefore, it seems to be desirable to estimate sector specific elasticities of substitution.

Leung (2003) examines elasticity of substitution in an endogenous growth model in the context of CES production function. The author shows that elasticity of substitution between capital and labor does not cause a change in growth rate, but it has a positive level effect on income. That is, a high elasticity of substitution will slow down the fall of the marginal product of the capital, which stimulates an increase in capital accumulation and saving. The author emphases that the increase in elasticity of substitution leads to a higher capital stock, indicating that it will only affect the level of income, not the balanced growth path.

Bucci and Bo (2011) theoretically examine the interaction between public-private capital in the context of CD and CES production functions. The share of public capital in total output can be exogenous or endogenous. In case of exogenous allocation of public capital in final output production, the degree of complementarity/substitutability between private and public capital investment is the main determinant behind the optimal growth. A higher degree of complementarity between private and public capital investment leads to higher growth rate. However, in case of endogenous allocation, an increase in the elasticity of substitution between private and public capital leads to a higher optimal growth rate.

3.3. THE REVIEW OF THE STUDIES ON NON-CONSTANT ELASTICITY OF SUBSTITUTION PRODUCTION FUNCTION (VES)

Revenkar (1971a) examine 12 two-digit United States manufacturing industries for the year 1957. The results show that in 5 out of 12 two-digit United States manufacturing industries VES specification is more appropriate than Cobb-Douglas. In another study, Revenkar (1971b) investigate capital-labor substitution for private non-Farm Sectors of United States over the period 1929-1953. According to the results, the elasticity of substitution varies both with capital-labor ratio and over time. This conclusion supports that VES specification is a correct functional form for the private nonfarm sector of the United States over the sample period.

Yuhn (1991) examines the size of elasticity of substitution by comparing the United States and South Korean manufacturing industries using translog cost function over the period 1962-1981. The result support de la Grandville hypothesis that states elasticity of substitution is an important source of economic growth. In addition, the result shows that the elasticity of substitution between capital and labor is higher for South Korean Economy.

Bairam (1989) found that the VES specification for the prewar Japanese economy generates better results than the CD type production function. That is, CD production function is rejected in favor of VES over the period 1978-1938 for the Japanese economy. The elasticity of substitution is greater than one and has an increasing trend. Hence, VES is a suitable specification for production function for Japanese economy during this period. This implies that for the period 1878-1938, capital deepening positively affects the elasticity of substitution and the performance of the Japanese economy. In another study, Bairam (1990) investigate the Soviet economy under the same theoretical framework over the period 1950-1975. The estimated VES production function shows that the elasticity of substitution is less than unity with a strong negative trend in the Soviet economy during this period. The low and declining elasticity of substitution between labor and capital is the main reason behind the slowdown in Soviet growth rates. These results demonstrate that VES production function is the most appropriate production function for the Soviet economy and industry as a whole. VES specification can explain the slowdown in the Soviet economy during the sample period.

Karagiannis et. al. (2004) investigate VES and economic growth for 82 countries over the period 1960-1987. They consider a one-sector Solow-Swan model in the context of VES production function. The empirical results support a VES between capital and labor, which is in general greater than one.

Henderson and Kumbhakar (2006) examine the productivity of public and private capital stock for 48 US states over the period 1970-1986 using CD and translog production function framework. Li-Racine (2004) generalized kernel estimation is used to avoid the model misspecification that stem from CD production function, which ignores nonlinearities inherent in functional relationship of production technology. This weakness of CD production function leads to incorrect estimates of input elasticities. The author shows that return to public capital is positive and significantly different from zero.

Ling (2010) examine elasticity of substitution and economic growth nexus using transcendental Logarithm Production Function for Malaysia manufacturing sector over the period 1970-2005. The results show that variable elasticity of substitution in manufacturing sector positively influences economic growth in Malaysia. The high factor substitution in the manufacturing sector could improve economic growth. This conclusions support de la Grandville hypothesis for Malaysia manufacturing industry.

Lazkano and Pham (2016) examine the relationship between energy and capital stock for 108 countries over the period 1971-2011 within the framework of the VES production function. The results show that the elasticity of substitution increases over time with the energy-capital ratio. The author's emphasis that the policies that increase the speed of capital-energy substitution can foster long-run economic growth.

There are several studies that consider VES production function in a theoretical model (see, for example, Brianzoni et al. (2012), Gamlath and Lahiri (2015) and Grassetti et. al. (2015)). Brianzoni et al. (2012) consider a one sector discrete time Solow-Swan growth model with differential savings in the context of VES production function. They demonstrate that, if the elasticity of substitution parameter is low enough (i.e. for a small value of elasticity of substitution) complex dynamics may occur. On the other hand, if the variable elasticity of substitution is greater than one, unbounded endogenous growth may occur.

Gamlath and Lahiri (2015) consider VES production function in standard Diamond Model. The result shows that if the elasticity of substitution between capital and labor is greater than 1, the economy reaches a unique and stable steady state. On the other hand, if the elasticity of substitution between capital and labor is less than 1, but take a positive value, the economy either fall into poverty trap or there could two steady states of which one is stable.

Grassetti et. al. (2015) consider VES production function in the form given by Revenkar et. al. (1971) for the Diamond overlapping generation model to examine the existence and stability conditions for steady state. According to the results, if the elasticity of substitution between production factors is low enough cycles or complex dynamics can occur.

3.4. THE STUDIES THAT COMPARE PRODUCTION FUNCTIONS

Sirmans, Kau and Lee (1977) investigate the elasticity of substitution in urban housing production using CES and VES production functions for single-family housing data for Santa Clara County, California. The results show that VES is a more correct specification than CES in analyzing the intensity of land use and relative factor prices. Fare and Yoon (1981) examine the same issue with emphasizing the weakness of the VES production function in urban housing production. Fare and Yoon (1981) emphasize that although VES is a better specification than CES, WDI is a more appropriate specification than VES in urban housing production. The reason behind this assumption is that in urban housing production model VES restrict elasticity of substitution to be equal or less than unity. WDI removes this restriction since it contains VES, CES, and CD as special cases. The results show that WDI exhibits more variability in the elasticity of substitution than VES.

Lovell (1968, 1973a, 1973b) examine US manufacturing industries for CD, CES or VES specifications in several studies for different periods. Lovell (1968) examine 16 two-digit US manufacturing Industries over the period 1949-1963. The results show that the elasticity of substitution varies over time. This conclusion suggests that CD or CES may not be a correct specification form for production function for the 16 two-digit US manufacturing industries during the postwar period. Lovell (1973a) consider CES and VES production functions in a cross-section context for 17 two-digit United States manufacturing industries for the year 1958. According to the results of the analysis, in seven industries CD hypothesis cannot be rejected in favor of CES or VES specifications. In five industries, CES hypothesis cannot be rejected in favor of CD or VES specifications. In only one industry, VES hypothesis is accepted in favor of CES. Hence, the author concludes that "…different industries have different technological characteristics and require different production functions to portray their behavior accurately". Lovell (1973b) consider CES and VES specifications for the US manufacturing sector over the period 1947-1963. Although elasticity of substitution varies significantly with the capital-labor ratio in VES functional forms, there is no evidence to reject CES specification in favor of VES for the US manufacturing service. Considering all these conclusions, we can say that the appropriateness of the production function depends on the time and sample analyzed.

Sato and Hoffman (1968) and Kazi (1980) are among the others that find the evidence for the appropriateness of the VES production function over CES or CD. Sato and Hoffman (1968) examine private non-farm sectors for the United States and Japan economies over the period 1909-1960. They conclude that VES specification for production function is more realistic than CES specification. Kazi (1980) consider variable elasticity of substitution for Indian 2 and 3- digit industries over the period 1973-1975. The results show that CES specification leads to an upward bias of the estimate of the elasticity of substitution. The estimation of elasticity of substitution from VES specification indicates that the elasticity of substitution varies among Indian industries. Therefore, VES specification is a correct functional form for Indian manufacturing industries.

Lu and Fletcher (1968) examine CES and VES specifications for 17 two-digit manufacturing industries of the United States for the year 1957. CES specification is rejected in favor of VES for 7 to 9 (depending on the various definition of capital and labor inputs) out of 17 two-digit industries.

Roskamp (1977) examines the elasticity of substitution between capital and labor in 38 West German industries over the period 1950-1960 using CES and VES specifications. The author does not state any information regarding which specification is more appropriate for West German industries. In case of VES, the author found a wide range of elasticity of substitution including positive and negative values. For positive values, the elasticity of substitution ranges from near 0 to 2.82. For 16 industries the elasticity of substitution is less than unity whereas in 7 industries it above unity. In CES, no negative values were found. The positive values range from 0.29 to 2.47. For 31 industries, the value of elasticity of substitution is below unity and for 7 it is above unity.

Lu (1967) gives detail information about derivation and estimation of CES and VES production functions. The results of the empirical estimates of CES and VES for 17 two-digit manufacturing industries of United States for 1957 shows that the elasticity of substitution estimated by the CES function and by the VES function are not the same.

This literature review shows that the elasticity of substitution among production factors can vary with the scope of the study, the type of industry, and the time spanned. Therefore, there is no consensus about whether CD, CES or VES production functions are more suitable to define the relationship between production inputs. However, since VES production function includes CD and CES as a special type; it is a more inclusive production function. If the elasticity of substitution between production inputs indicates CD or CES characteristic, this property will be captured by the VES production function. However, if the relationship between the production inputs indicates the VES characteristic, using CD or CES production functions may lead to biased results.

The investigation of elasticity of substitution at the aggregate and industry level is another reason behind the diverse results. Although industry-based elasticity of substitution gives more realistic results, the absence of industry-level data pushes researchers to examine the elasticity of substitution at the aggregate level. The fact that the data on public and private capital stocks are not available at the industry level is the main limitation of this study. Therefore, we investigate the elasticity of substitution between production inputs at the aggregate level.

CHAPTER 4: THEORETICAL FRAMEWORK

4.1. INTRODUCTION

In this paper a standard Solow-Swan growth model in which the production technology exhibit variable elasticity of substitution (VES) in production inputs is analyzed. VES type of production functions are more general than the constant elasticity of substitution (CES) production functions, which are commonly used in the growth literature. The main difference between these production technologies is about the assumption of elasticity of substitution between production inputs. CES type production functions restrict the elasticity of substitution to be constant along the isoquants. Nevertheless, VES technology allows the elasticity of substitution to vary between production inputs over time.

VES type production function first introduced by Revenkar (1971). Karagiannis, Palivos, and Papageorgiou (2004) used Revenkar's production function to analyze the role of variable elasticity of substitution between capital and labor within a standard Solow-Swan growth model.

4.2. THE VES PRODUCTION FUNCTION

The following VES production function uses capital and labor as production inputs. The variables *Y*, *K,* and *L* denote total output, capital stock, and labor respectively. *Y, K,* and *L* are all time-variant variables, but for simplicity, we omit the time subscripts.

$$Y = AK^{\alpha_1 v_1}(L + b_1\alpha_1 K)^{(1-\alpha_1)v_1} \tag{4.1}$$

α_1 indicates the role of capital in the production process, v_1 is the return to scale parameter, for example, if $v_1 = 1$ then the production function exhibit constant return to scale. b_1 is a parameter which affects the aggregate degree of substitutability between capital and labor. We assume that, capital in this model accumulates according to following VES production function:

$$K = K_g^{\alpha_2 v_2}(K_p + b_2\alpha_2 K_g)^{(1-\alpha_2)v_2} \tag{4.2}$$

Where K_g and K_p in the equation (4.1) indicate public and private capital respectively. For simplicity, the capital production function is abstracted from technological progress. As in equation (4.1), α_2 reflects the importance of public capital relative to private capital in production of total capital, v_2 is the return to scale parameter as before, and b_2 is a parameter that affects the aggregate degree of substitutability between public capital and private capital. Thus, substituting equation (4.2) into equation (4.1) we obtain the following general equation

$$Y = A[K_g^{\alpha_2 v_2}(K_p + b_2\alpha_2 K_g)^{(1-\alpha_2)v_2}]^{\alpha_1 v_1}[L + b_1\alpha_1 K_g^{\alpha_2 v_2}(K_p + b_2\alpha_2 K_g)^{(1-\alpha_2)v_2}]^{(1-\alpha_1)v_1} \quad (4.3)$$

b_1 and b_2 characterize whether the production technology is CES or VES. For example, in the equation (4.3), if $b_1 = 0$ the substitution between capital and labor is constant over time. That is the production technology return to standard Cobb-Douglas form, which is a special form of CES technology. Similarly, $b_2 = 0$ implies constant elasticity of substitution between public capital and private capital, while it is variable in the case $b_2 \neq 0$.

In order to simplify the theoretical model, we assume that $b_1 = 0$. This is a reasonable assumption, because we are interested in the elasticity of substitution between public capital and private capital. Moreover, we impose the constant return to scale assumption, i.e., $v_1 = v_2 = 1$. After the imposition of these simplifying assumptions, the production function in equation (4.3) reduced to

$$Y = A[K_g^{\alpha_2}(K_p + b_2\alpha_2 K_g)^{(1-\alpha_2)}]^{\alpha_1} L^{(1-\alpha_1)} \quad (4.4)$$

Table 1 summarizes the reduced form of our general production function for some parameter restrictions. As shows in table 1, after the imposition of the restriction the general VES production function reduced to some well-known production functions.

Table 1: Parameter Restrictions and the Properties VES Production Function

Parameter Restriction	First Level Production Function	Second Level Production Function	Reduced Form	Property
No Restriction	$Y = AK^{\alpha_1 v_1}(L + b_1\alpha_1 K)^{(1-\alpha_1)v_1}$	$K = K_g^{\alpha_2 v_2}(K_p + b_2\alpha_2 K_g)^{(1-\alpha_2)v_2}$	$Y = A[K_g^{\alpha_2 v_2}(K_p + b_2\alpha_2 K_g)^{(1-\alpha_2)v_2}]^{\alpha_1 v_1}[L + b_1\alpha_1 K_g^{\alpha_2 v_2}(K_p + b_2\alpha_2 K_g)^{(1-\alpha_2)v_2}]^{(1-\alpha_1)v_1}$	VES production function with public capital, private capital and Labor
$v_1 = v_2 = 1$	$Y = AK^{\alpha_1}(L + b_1\alpha_1 K)^{(1-\alpha_1)}$	$K = K_g^{\alpha_2}(K_p + b_2\alpha_2 K_g)^{(1-\alpha_2)}$	$Y = A[K_g^{\alpha_2}(K_p + b_2\alpha_2 K_g)^{(1-\alpha_2)}]^{\alpha_1}[L + b_1\alpha_1 K_g^{\alpha_2}(K_p + b_2\alpha_2 K_g)^{(1-\alpha_2)}]^{(1-\alpha_1)}$	CRTS property
$\alpha_1 = \alpha_2 = 1$ and $v_1 = v_2 = 1$	$Y = AK$	$K = K_g$	$Y = AK_g$	AK production function with public capital
$b_1 = b_2 = 0$ and $v_1 = v_2 = 1$	$Y = AK^{\alpha_1}L^{1-\alpha_1}$	$K = K_g^{\alpha_2}{K_p}^{1-\alpha_2}$	$Y = A(K_g^{\alpha_2}{K_p}^{1-\alpha_2})^{\alpha_1}L^{1-\alpha_1}$	Cobb-Douglas production function with public capital, private capital and labor
$b_1 = 0, \alpha_2 = 1$ and $v_1 = v_2 = 1$	$Y = AK^{\alpha_1}L^{1-\alpha_1}$	$K = K_g$	$Y = AK_g^{\alpha_1}L^{1-\alpha_1}$	Cobb-Douglas production function with public capital, and labor
$\alpha_2 = 1$ and $v_1 = v_2 = 1$	$Y = AK^{\alpha_1}(L + b_1\alpha_1 K)^{(1-\alpha_1)}$	$K = K_g$	$Y = AK_g^{\alpha_1}(L + b_1\alpha_1 K_g^{\alpha_1})^{1-\alpha_1}$	VES production function with public capital and Labor
$\alpha_1 = 1$ and $v_1 = v_2 = 1$	$Y = AK$	$K = K_g^{\alpha_2}(K_p + b_2\alpha_2 K_g)^{1-\alpha_2}$	$Y = AK_g^{\alpha_2}(K_p + b_2\alpha_2 K_g)^{1-\alpha_2}$	VES production function with public capital, and Private Capital

The marginal product of public capital and private capital are

$$MP_{K_g} = \frac{AL^{1-\alpha_1}\alpha_1\alpha_2(K_p+b_2K_g)(K_g^{\alpha_2}(K_p+b_2\alpha_2K_g)^{1-\alpha_2})^{\alpha_1}}{K_g(K_p+b_2\alpha_2K_g)} \quad (4.5)$$

$$MP_{K_p} = \frac{AL^{1-\alpha_1}\alpha_1(1-\alpha_2)(K_g^{\alpha_2}(K_p+b_2\alpha_2K_g)^{1-\alpha_2})^{\alpha_1}}{K_p+b_2\alpha_2K_g} \quad (4.6)$$

Hence, the above function satisfies the standard properties of a production function, namely

$Y > 0, \mathrm{MP}_{K_g} > 0, \mathrm{MP}_{K_p} > 0$ and diminishing marginal returns as long as,
$A > 0, 0 < \alpha_1, \alpha_2 \leq 1, b_2 > -1$ and $K_g^{-1} \geq -b_2$

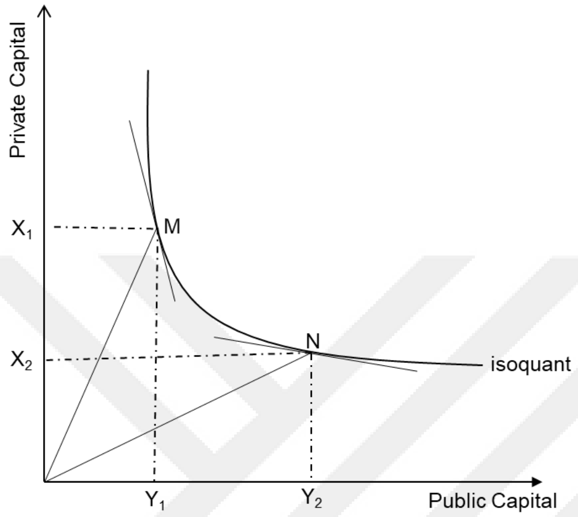
4.2.1 Elasticity of Substitution between Public and Private Capital

It is important to use the optimal input composition to maximize profit in the production process. When the use of an input in the production process is not profitable, or one of the inputs is more abundant and cheaper, the input combination in the production process may vary. Changing the price of one of the inputs causes the equilibrium conditions and the rate of marginal technical substitution to change. If the marginal rate of technical substitution decreases slowly, the elasticity of between the production factors will be high. On the other hand, if it falls quickly, the elasticity of substitution will be low.

The elasticity of substitution is introduced by Hicks (1932). It measures the percentage change in factor proportions depending on a change in the marginal rate of the technical substitution. It is is a measure of the degree of ease of substitution between production inputs.

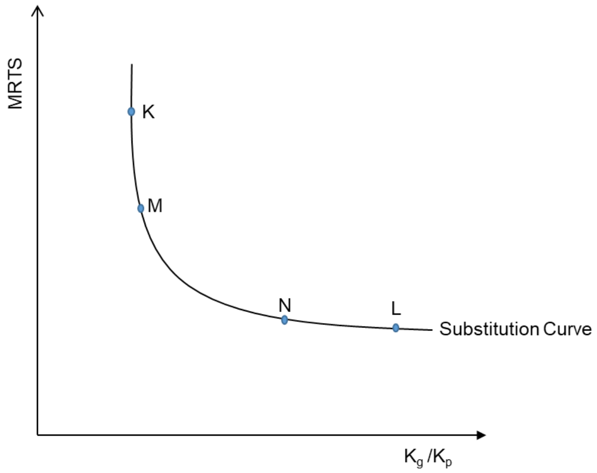
The shape of the isoquant curves is closely related to the elasticity of substitution. By looking at the slope of the isoquant curve, one can get an idea of about the elasticity of substitution between the production inputs. The more convex isoquants are associated with the lower elasticity of substitution. For example, consider the isoquant in the figure 2. The elasticity of substitution between public capital and private capital stocks is the ratio of proportionate change in the slopes of two rays from the origin to M and N points on the isoquant to the proportionate change in the slopes of isoquants at these points.

Figure 2: Elasticity of Substitution and Isoquants



The elasticity of substitution between public capital and private capital stocks is high in the flat portion of substitution curve (points N and L on the substitution curve), and low at the steep portion of the substitution curve (points K and M on the substitution curve).

Figure 3: Elasticity of Substitution Curve



In this section, we will derive the elasticity of substitution between public capital and private capital. Recall that K_g and K_p are public capital and private capital respectively. In this context, the elasticity of substitution (σ) is defined as the proportionate change in the ratio $\frac{K_g}{K_p}$ relative to the proportionate change in the marginal rate of technical substitution (MRTS). The elasticity of substitution between K_g and K_p can be calculated by the following formula:

$$\sigma(K_g, K_p) = \frac{\%\Delta(K_p/K_g)}{\%\Delta MRTS} = \frac{\partial \ln\left({}^{K_p}\!/_{K_g}\right)}{\partial \ln \mathrm{MRTS}} \tag{4.7}$$

The marginal rate of technical substitution is the ratio of the marginal product of the two inputs. That is, $MRTS = MP_{K_g} \big/ MP_{K_p}$.

Using equation (4.1) and (4.2) and assuming constant returns to scale in production, i.e. $v_1 = v_2 = 1$, we derive the marginal product of private capital and public capital to be

$$\mathrm{MP}_{K_g} = \frac{\partial Y}{\partial K_g} = \frac{\partial Y}{\partial K}\frac{\partial K}{\partial K_g} = \frac{\partial Y}{\partial K}\alpha_2 {K_g}^{\alpha_2 - 1}(K_p + b_2\alpha_2 K_g)^{-\alpha_2}(K_p + b_2 K_g) \tag{4.8}$$

$$\mathrm{MP}_{K_p} = \frac{\partial Y}{\partial K_p} = \frac{\partial Y}{\partial K}\frac{\partial K}{\partial K_p} = \frac{\partial Y}{\partial K}(1 - \alpha_2){K_g}^{\alpha_2}(K_p + b_2\alpha_2 K_g)^{-\alpha_2} \tag{4.9}$$

Where, $\frac{\partial Y}{\partial K} = A\alpha_1 K^{\alpha_1 - 1}(L + b_1\alpha_1 K)^{-\alpha_1}(\mathrm{L} + b_1\alpha_1 K)$ (4.10)

Therefore, MRTS is obtained by combining equation (4.8) and (4.9).

$$\mathrm{MRTS}_{K_g,K_p} = \frac{\mathrm{MP}_{K_g}}{\mathrm{MP}_{K_p}} = \frac{\alpha_2(K_p + b_2 K_g)}{(1-\alpha_2)K_g} = \frac{\alpha_2(b_2 + \frac{K_p}{K_g})}{1-\alpha_2} \tag{4.11}$$

Using chain rule, we have;

$$\frac{\partial \ln(\mathrm{MRTS})}{\partial(\frac{K_p}{K_g})} = \frac{1}{\mathrm{MRTS}} * \frac{\partial \mathrm{MRTS}}{\partial(\frac{K_p}{K_g})} \tag{4.12}$$

Equation (4.12) implies that,

$$\partial \ln(\text{MRTS}) = \frac{1}{\text{MRTS}} * \frac{\partial \text{MRTS}}{\partial\left(\frac{K_p}{K_g}\right)} * \partial(\frac{K_p}{K_g}) \tag{4.13}$$

Moreover, we have

$$\frac{\partial \text{MRTS}}{\partial\left(\frac{K_p}{K_g}\right)} = \frac{\alpha_2}{1-\alpha_2} \tag{4.14}$$

Substituting equation (4.11) and (4.14) into equation (4.13) we obtain

$$\partial \ln(\text{MRTS}) = \frac{1}{b_2+\frac{K_p}{K_g}} * \partial(\frac{K_p}{K_g}) \tag{4.15}$$

In addition,

$$\frac{\partial \ln(\frac{K_p}{K_g})}{\partial(\frac{K_p}{K_g})} = \frac{1}{\frac{K_p}{K_g}} \tag{4.16}$$

Which implies that,

$$\partial \ln\left(\frac{K_p}{K_g}\right) = \frac{1}{\frac{K_p}{K_g}} * \partial(\frac{K_p}{K_g}) \tag{4.17}$$

After substituting equation (4.15) and (4.17) into the formula stated in the equation (4.7), we obtain the elasticity of substitution between public capital and private capital as;

$$\sigma(K_g, K_p) = 1 + b_2(\frac{K_g}{K_p}) \tag{4.18}$$

Equation (4.18) shows that the elasticity of substitution between public and private capital stocks depends on the public capital and private capital ratio.

4.2.2. VES Production Function in Solow-Swan Growth Model

Following Mankiw, Romer, and Weill (1992), the accumulation equations for private capital and public capital are given in equations (4.19) and (4.20).

$$\dot{K_g} = s_g Y - (n + g + \delta) K_g \tag{4.19}$$

$$\dot{K_p} = s_p Y - (n + g + \delta) K_p \tag{4.20}$$

For analytical simplicity, we assume that there is no population growth, no depreciation, and no technological progress[1]. Moreover, a constant exogenous fraction of final output is saved for the accumulation of public capital and private capital. Imposing all these restrictions into the equations (4.19) and (4.20), and using the production function in equation (4.4) we obtain capital accumulation equations as;

$$\dot{K_g} = s_g A[K_g^{\alpha_2}(K_p + b_2\alpha_2 K_g)^{(1-\alpha_2)}]^{\alpha_1} L^{1-\alpha_1} \tag{4.21}$$

$$\dot{K_p} = s_p A[K_g^{\alpha_2}(K_p + b_2\alpha_2 K_g)^{(1-\alpha_2)}]^{\alpha_1} L^{1-\alpha_1} \tag{4.22}$$

$$\frac{\partial \dot{K_g}}{\partial b_2} = s_g \mathrm{AK}_g^{1+\alpha_1\alpha_2} L^{1-\alpha_1} \alpha_1\alpha_2(1-\alpha_2) K_p^{\alpha_1(1-\alpha_2)-1} (1 + b_2\alpha_2 \frac{1}{K_p/K_g})^{\alpha_1(1-\alpha_2)-1} \tag{4.23}$$

$$\frac{\partial \dot{K_p}}{\partial b_2} = s_p \mathrm{AK}_g^{1+\alpha_1\alpha_2} L^{1-\alpha_1} \alpha_1\alpha_2(1-\alpha_2) K_p^{\alpha_1(1-\alpha_2)-1} (1 + b_2\alpha_2 \frac{1}{K_p/K_g})^{\alpha_1(1-\alpha_2)-1} \tag{4.24}$$

When public capital and private capital are substitute, i.e., $b_2 > 0$, an increase in public capital leads to faster capital accumulation. However, when public capital and private capital are complements, i.e., $b_2 < 0$, equation (4.23) and (4.24) can be negative if private-public capital ratio is significantly low. That is, if the availability of private capital for production is limited relative to available public capital, ceteris paribus, a high level of complementarity between private capital and public capital can slow down the rate of public capital and private capital accumulation, thereby reducing long run economic growth.

4.2.3. The Share of Inputs in Production Function

In this part, we will derive the share of public capital, private capital and labor in production process. Let $\omega_l, \omega_g, \omega_p$ denote the share of labor, public capital and private capital in final output, respectively. Using equation (4.1) and (4.2) and

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[1] The steady state solution of the capital accumulation equation is given in Appendix 2.

assuming constant return to scale (i.e. $v_1 = v_2 = 1$) and perfect competition in the factor markets, the factor share of production inputs can be defined as;

$$\omega_l = \frac{MP_L * L}{Y} \tag{4.25}$$

$$\omega_g = \frac{MP_{K_g} * K_g}{Y} \tag{4.26}$$

$$\omega_p = \frac{MP_{K_p} * K_p}{Y} \tag{4.27}$$

From equation (4.1) we have,

$$\mathrm{MP}_L = \frac{\partial Y}{\partial L} = A(1-\alpha_1)K^{\alpha_1}(L + b_1\alpha_1 K)^{-\alpha_1} \tag{4.28}$$

Thus,

$$\omega_l = \frac{MP_L * L}{Y} = \frac{(1-\alpha_1)L}{L + b_1\alpha_1 K} = \frac{1-\alpha_1}{1 + b_1\alpha_1(K/L)} \tag{4.29}$$

Let define ω_C be the combined shares of public capital and private capital. Thus, we have

$$\omega_C = 1 - \omega_l = \frac{\alpha_1 + b_1\alpha_1(K/L)}{1 + b_1\alpha_1(K/L)} \tag{4.30}$$

Next, let derive the share of public capital (ω_g) in the final output.

$$\omega_g = \frac{\mathrm{MP}_{K_g} * K_g}{Y} = \frac{\frac{\partial Y}{\partial K_g}K_g}{Y} = \frac{\frac{\partial Y}{\partial K}\frac{\partial K}{\partial K_g}K_g}{Y} = \frac{\frac{\partial Y}{\partial K}K}{Y}\frac{\frac{\partial K}{\partial K_g}K_g}{K} \tag{4.31}$$

From equation (4.1) and (4.2) we have

$$\frac{\partial Y}{\partial K} = A\alpha_1 K^{\alpha_1 - 1}(L + b_1\alpha_1 K)^{1-\alpha_1} + A\alpha_1(1-\alpha_1)b_1 K^{\alpha_1}(L + b_1\alpha_1 K)^{-\alpha_1} \tag{4.32}$$

$$\frac{\partial K}{\partial K_g} = \alpha_2 {K_g}^{\alpha_2 - 1}(K_p + b_2\alpha_2 K_g)^{(1-\alpha_2)} + (1-\alpha_2)b_2\alpha_2 K_g^{\alpha_2}(K_p + b_2\alpha_2 K_g)^{-\alpha_2} \tag{4.33}$$

Plugging equations (4.1), (4.2), (4.32) and (4.33) into the expression in equation (4.31), and simplifying gives the share of public capital in the final output as

$$\omega_g = \frac{\alpha_1+b_1\alpha_1(K/L)}{1+b_1\alpha_1(K/L)}\ \frac{\alpha_2+b_2\alpha_2(K_g/K_p)}{1+b_2\alpha_2(K_g/K_p)} = \omega_C\ \frac{\alpha_2+b_2\alpha_2(K_g/K_p)}{1+b_2\alpha_2(K_g/K_p)} \tag{4.34}$$

Finally, let's derive the share of public capital in the final output. Since our production function, includes three inputs, and the sum of factor shares equals to one, the share of public capital in final output can be derived by subtracting the shares of labor and public capital from one.

$$\omega_p = 1 - \omega_l - \omega_g \tag{4.35}$$

We know that $1 - \omega_l$ is the combined shares (ω_C) of public capital and private capital. Plugging equation (4.34) into equation (4.35) and simplifying gives the share of private capital in the final output as

$$\omega_p = \omega_C\ \frac{1-\alpha_2}{1+b_2\alpha_2(K_g/K_p)} \tag{4.36}$$

In the case of $b_1 = 0$, i.e. the elasticity of substitution between labor and capital is constant, the share of labor, public capital and private capital reduced to

$$\omega_l = 1 - \alpha_1 \tag{4.37}$$

$$\omega_g = \alpha_1\ \frac{\alpha_2+b_2\alpha_2(K_g/K_p)}{1+b_2\alpha_2(K_g/K_p)} \tag{4.38}$$

$$\omega_p = \alpha_1\ \frac{1-\alpha_2}{1+b_2\alpha_2(K_g/K_p)} \tag{4.39}$$

4.2.4. Balanced Growth Path

A balanced growth path is a path $(K_g, K_p)_{t=0}^{t=\infty}$ along which the quantities K_g, K_p are positive and grow at constant rate.

$$K_g = e^{gt} \tag{4.40}$$

$$K_p = e^{pt} \tag{4.41}$$

The time path of public and private capital stock are;

$$\dot{K_g} = ge^{gt} \tag{4.42}$$

$$\dot{K}_p = pe^{pt} \tag{4.43}$$

Hence; for a balanced growth path, the left and right-hand side of the following equations should be growing at the same rate.

$$\dot{K}_g = s_g A[K_g^{\alpha_2}(K_p + b_2\alpha_2 K_g)^{(1-\alpha_2)}]^{\alpha_1} L^{1-\alpha_1} \tag{4.44}$$

$$\dot{K}_p = s_p A[K_g^{\alpha_2}(K_p + b_2\alpha_2 K_g)^{(1-\alpha_2)}]^{\alpha_1} L^{1-\alpha_1} \tag{4.45}$$

Replacing equations (4.42) and (4.43) into the equations (4.44) and (4.45) we get;

$$ge^{gt} = s_g A[e^{\alpha_2 gt}(e^{pt} + b_2\alpha_2 e^{gt})^{(1-\alpha_2)}]^{\alpha_1} L^{1-\alpha_1} \tag{4.46}$$

$$pe^{pt} = s_p A[e^{\alpha_2 gt}(e^{pt} + b_2\alpha_2 e^{gt})^{(1-\alpha_2)}]^{\alpha_1} L^{1-\alpha_1} \tag{4.47}$$

For balanced growth path, it is required that $p = g$

$$ge^{gt} = s_g A[e^{\alpha_2 gt}(1 + b_2\alpha_2)^{1-\alpha_2} e^{(1-\alpha_2)gt}]^{\alpha_1} L^{1-\alpha_1} \tag{4.48}$$

$$ge^{gt} = s_g A L^{1-\alpha_1}[(1 + b_2\alpha_2)^{\alpha_1(1-\alpha_2)} e^{\alpha_1 gt}] \tag{4.49}$$

Here, for the equality of the equation, we should have $\alpha_1 = 1$

$$ge^{gt} = s_g A(1 + b_2\alpha_2)^{(1-\alpha_2)} e^{gt} \tag{4.50}$$

After some simplification we have,

$$g = s_g A(1 + b_2\alpha_2)^{(1-\alpha_2)} \tag{4.51}$$

Similarly, for balanced growth path of equation (4.47), we require that $p = g$

$$pe^{pt} = s_p A[e^{\alpha_2 gt} e^{(1-\alpha_2)gt}(1 + b_2\alpha_2)^{(1-\alpha_2)}]^{\alpha_1} L^{1-\alpha_1} \tag{4.52}$$

Here, for the equality of the equation, we should have $\alpha_1 = 1$

$$pe^{gt} = s_p A e^{\alpha_2 gt} e^{(1-\alpha_2)gt}(1 + b_2\alpha_2)^{(1-\alpha_2)} \tag{4.53}$$

$$pe^{gt} = s_p A e^{gt}(1 + b_2\alpha_2)^{(1-\alpha_2)} \tag{4.54}$$

$$p = s_p A(1 + b_2\alpha_2)^{(1-\alpha_2)} \tag{4.55}$$

Using equations (4.51), (4.55), and $p = g$ equality; we have

$$s_p = s_g = s$$

Hence, for balanced growth path, public capital and private capital stocks are growing at the same constant rate.

4.3. NONLINEAR LEAST SQUARE ESTIMATION

Recall that the functions in linear regression models are linear in the unknown parameters. However, in case of nonlinear least square estimation the regression functions are nonlinear in parameters. Nonlinear least square regression method enable researcher to estimate the unknown parameters of more general and complicated functions. Unlike linear regression, there are very few limitations on the parameters in the functional part of the nonlinear regression model.

Suppose the nonlinear regression model to be estimated is of the following form:

$$Y = f(\beta_0, \beta_1, \beta_2, \dots; X_1, X_2, X_3, \dots) + \varepsilon$$

Where f is a function of unkonown parameters and the explanatory variables, and the error term ε is assumed to be linearly, independently and identically distributed. If the derivative of the function with respect to unknown parameters is a function of unknown parameters, then the function is called nonlinear in parameters. In some cases, it may be possible to transform a nonlinear function into a linear form by using an appropriate transformation. If the transformed model satisfies the properties of the Classical Linear Regression Models, then OLS technique can be used to estimate the unknown parameters. In order to use NLLS technique, the non-stochastic part of f should a nonlinear function of the parameters, $\beta_0, \beta_1, \beta_2, \dots$, and no transformation can renders the function linear in the parameters. As in OLS repression, the method of least squares is used to estimate the value of unknown parameters.

An example of a nonlinear regression model is the Variable Elasticity of Substitution (VES) production function

$$logY_i = logA_i + \alpha v logK_i + (1-\alpha)v \log(L_i + b\alpha K_i) + \varepsilon_i$$

where i refers to the individual, lnY_i is the logarithm of output; K_i and L_i are indivdual's i capital and labor usage, respectively; and ε_i is the regression error term.

Nonlinear least square estimation method uses iterative techniques to minimize the sum of squared residuals. Hence, for independent observations, the NLLS estimator $\hat{\beta}$ minimizes

$$h(\beta) = \sum_{i=1}^{N} (Y_i - f(X_i, \beta))$$

where $f(X_i, \beta)$ is the specified functional form for the conditional mean of Y given X (i.e. $E(Y|X)$). If the conditional mean function is correctly specified, then the NLLS estimator is consistent and asymptotically normally distributed.

NLLS has some advantages and disadvantages that can be shared with OLS. First, NLLS has a great advantage that allows researchers to fit a wide range of functions. Second, NLLS regression can provide fairly good estimates for a small sample of data. The main disadvantage of using NLLS is the need to use iterative optimization methods to estimate the parameters. The weakness of the iterative optimization method is the user-specified initial values. If the appropriate initial values are not chosen, then the estimations may converge to a local minimum rather than a global minimum.

Stata fits nonlinear regression models using the Gauss-Newton regression. This method requires specifying initial values for the estimates. The initial values are either specified by users or left as default. The nonlinear least square procedure finds the direction toward lower sum of square residuals to reach the second set of parameter values, and then repeats the same process until the gain in the sum of squared residuals is negligible. The set of parameter values in the final stage are provided as the nonlinear least square estimates along with the relevant test statistics.

4.4. PERPETUAL INVENTORY METHOD

OECD (2009) defines the perpetual inventory method based on the idea that "stocks constitute cumulated flow of investment, corrected for retirement and efficiency loss". The methodology applied here for perpetual inventory method is drawn from Kamps (2006). The capital stock at the beginning of the next period is defined as;

$$K_{t+1} = K_t + I_t - D_t \qquad (4.56)$$

Where K_t is the capital stock at the beginning of the current period, I_t is gross investment in the current period, and D_t is the depreciation in the current period. If one further assumes that capital stock depreciate at a constant rate δ, then equation (4.56) can be written as;

$$K_{t+1} = (1 - \delta)K_t + I_t \qquad (4.57)$$

The name perpetual come from the idea that all assets are forever part of the inventory of capital stocks. One can obtain the formula for the capital stock of the following period as the sum of the past investments by repeatedly substitute equation (4.57) for the capitals tock at the beginning of period t;

$$K_{t+1} = \sum_{i=0}^{\infty} (1 - \delta)^i I_{t-i} \qquad (4.58)$$

Equation (4.58) shows that capital stock at the beginning of the period t+1 is a weighted sum of the past investment. The weight in equation (4.58) is a decreasing function of the distance between current period and the investment period. To estimate capital stock by using equation (4.58), we need an infinite number of past investment data. Since an infinite number of past investment series is not available, equation (4.58) cannot be used in practice. Hence equation (4.58) is modified by using an initial capital stock, K_1. The initial capital stock is not available for many countries. It can be constructed from an artificial investment series by assuming that investment increased by a constant rate.

$$K_{t+1} = (1 - \delta)^t K_1 + \sum_{i=0}^{t-1} (1 - \delta)^i I_{t-i} \qquad (4.59)$$

Equation (4.59) states that the application of perpetual inventory method requires an initial capital stock, a time series of gross investment, and a depreciation method[2].

[2] For more information on perpetual inventory method see OECD Manual 2009, Kamps 2006

CHAPTER 5: DATA AND VARIABLES

5.1. INTRODUCTION

In the chapter, the abbreviation used for variables and source of the data will be introduced. The dataset will be examined with the aid of descriptive statistics and graphs.

5.1.1. Variables, Indicators, and Sources of Data

The estimation of the VES production function in equation 4.3 requires data on total output (Y), total factor productivity (TFP), public capital stock (K_g), private capital stock (K_p), labor force (L) and human capital (H). The dataset spans 32 years (1980-2011) and includes 91 countries. The data on public capital stock and private capital stock have been derived from the IMF investment and capital stock data set. Total output, total factor productivity, labor force and human capital data are obtained from Penn World Table (Feenstra et. al., 2015) data set. Countries and period under consideration were chosen to maximize the number of observation for a balanced panel of data. Table 1 below shows the variables and the source of data. Descriptive statistics are given in table 2.

Table 2: Variables and Sources of Data

Variable	Indicator	Unit of Measurement	Source
Public capital	K_g	PPP, Constant 2005 dollars	IMF
Private capital	K_p	PPP, Constant 2005 dollars	IMF
Real GDP	RGDP	Constant 2005 national prices (in millions of 2005 US Dollars)	Penn World Table (version PWT 8.1)
Real total factor productivity	RTFP	Constant 2005 national prices (TFP for 2005 = 1)	Penn World Table (version PWT 8.1)
Population	POP	Millions	Penn World Table

Variable	Indicator	Unit of Measurement	Source
			(version PWT 8.1)
Human capital index	HC	Greater than 1	Penn World Table (version PWT 8.1)

Note: The human capital index is in terms of average years of schooling and the return to education per person. PPP stands for purchasing power parity. TFP stands for total factor productivity. Penn World Table data is from Feenstra, Inklaar and Timmer 2015.

Aggregate output is measured by annual level of real gross domestic product at 2005 constant national prices for each individual countries. Penn World Tables construct the human capital index based on Barro and Lee average year of schooling and rate of return to education based on Mincer equation. The index for real total factor productivity computed with data on real GDP, capital stock, labor input, and share of labor compensation in GDP. This index evaluates the countries based on their 2005 real total factor productivity value. Therefore, it is not very helpful for cross-countries evaluation.

Public and private capital stocks data are constructed by IMF. The investment data is transformed into the net real cost stocks (constant 2011 US Dollars) by making an assumption on depreciation rate and initial capital stock. The depreciation rates depend on the countries income level. The countries are divided into three income groups; low-income, middle-income, and high-income countries. The depreciation for each income group varies over time. An artificial time series is constructed to derive the initial public capital and private capital stocks.

5.1.2. Descriptive Statistics

Table 3: Descriptive Statistics by Income Groups

Variables	Unit of Measurement	Statistics	(1) Low Income	(2) Lower Middle Income	(3) Upper Middle Income	(4) High Income OECD	(5) High Income nonOECD
Real GDP	Millions of 2005 US Dollar	Mean	12730.17	194836.4	411771.9	964151.7	157066
			(14148.56)	(531493.3)	(1169999)	(1929573)	(202287.4)
		Max	61249.59	4692698	1.40e+07	1.32e+07	918141.9
		Min	2222.9	1627.311	2026.299	4527.142	4639..98
Public Capital	Millions of 2005 US Dollar	Mean	9687.19	113037.5	420148.7	602218.9	109900
			(10992.88)	(350058)	(1558845)	(1388854)	(152509.6)
		Max	50363.59	2759030	1.71e+07	8915540	648413
		Min	252.677	1731.095	1813.058	3372.816	2138.176
Private Capital	Millions of 2005 US Dollar	Mean	21806.63	187034.7	367230.7	1488865	144292.5
			(40286.28)	(493688)	(707891.2)	(2967555)	(182836.3)
		Max	182644	4750460	8890140	2.08e+07	775684
		Min	1217.263	385.9934	2365.234	11198.26	1558.738
Real TFP	Constant 2005 National Prices	Mean	1.084	1.0155	0.9855	0.9428	0.9667
			(0.2935)	(0.1414)	(0.1599)	(0.0942)	(0.1932)
		Max	2.0387	1.6562	1.8494	1.1861	2.2112
		Min	0.5733	0.6916	0.2378	0.5953	0.5617
Population	Millions of People	Mean	9.8253	80.6014	75.0829	34.5454	5.7571
			(7.9962)	(224.6207)	(228.8028)	(54.6819)	(7.5099)
		Max	44.9240	1241.492	1324.353	313.0854	28.0825
		Min	2.2736	0.6034	0.6353	0.22816	0.2216
Human Capital Index	Greater Than 1	Mean	1.5343	2.0232	2.3147	2.8903	2.4685
			(0.3171)	(0.4114)	(0.3453)	(0.3224)	(0.3316)
		Max	2.4823	3.1615	3.0013	3.6187	3.2129
		Min	1.0905	1.2420	1.4136	1.9970	1.7674
Human Capital Adjusted Labor*		Mean	15.7465	145.2536	167.6553	104.4396	14.8327
			(15.1719)	(389.4655)	(517.1332)	(186.1086)	(20.3267)
		Max	91.9221	2395.579	3415.731	1132.977	74.5195
		Min	2.8342	1.0780	1.0920	0.5757	0.4107
Number of Observations (N)			352	576	800	864	320
Number of			11	18	25	27	10

Note: GDP and TFP stand for gross domestic product and total factor productivity, respectively. Numbers in parenthesis show the standard deviation. *see chapter 6 for derivation of human capital adjusted labor.

Table 4: Public and Private Capital Growth Rates 1980-2011, by income Groups

Income Groups	Public Capital Growth Rate (%)	Private Capital Growth Rate (%)
Low Income	1.724	1.338
Lower Middle Income	4.729	5.320
Upper Middle Income	5.332	4.011
High Income	2.154	2.263
Global	3.422	2.796

Note: Growth rates are calculated by $(1/T)\ ln\ (Y_t\ /Y_0)*100$ formula where T, $Y_{t,}$ and Y_0 denote period, last value of the variable, and the initial value of the variable, respectively.

The descriptive statistics for each income group during the period 1980-2011 are given in Table 3. In all income groups except upper-middle income, private capital stock is greater than public capital stock. Table 4 shows the growth rate of the mean public capital stock, and mean private capital stock between 1980 and 2011. On average, public and private capital stocks grow faster in lower-middle-income and upper-middle-income groups. Low-income economies experienced little change in their public and private capital accumulation. This is reasonable since low-income group includes many less developed countries.

The correlation coefficients for the main variables are shown in Table 5. The correlation between public and private capital is 0.76. This relationship also can be seen in Figure 5, which shows an upward trend between public and private capital stocks. In additions, there is a high correlation between real GDP and the two-sub component of capital stock. On the other hand, the correlation coefficient between real GDP, population, and the human capital index is not very high. However, when we consider real GDP and human capital adjusted labor the correlation coefficient increase to 0.78.

Table 5: Pairwise correlations of Variables (1980-2011)

	GDP	TFP	Public Capital	Private Capital	Population	Human Capital Index	Human Capital Adjusted Labor
GDP	1						
Total Factor Productivity		1					
Public Capital	0.927*	-0.052*	1				
Private Capital	0.937*		0.765*	1			
Population	0.530*	-0.129*	0.657*	0.301*	1		
Human Capital Index	0.281*	-0.213*	0.203*	0.321*	-0.056*	1	
Human Capital Adjusted Labor	0.659*	-0.119*	0.783*	0.424*	0.979*		1
Mean	456529	0.989	3.30e+11	5.98e+11	48.640	2.350	109.312
(S.D.)	(1289998)	(0.189)	(1.15e+12)	(1.77e+12)	(161.472)	(0.559)	(341.970)

* indicates 5 percent level of significance.

Figure 4: The Mean Ratio of Public and Private Capital to GDP (in logarithm)

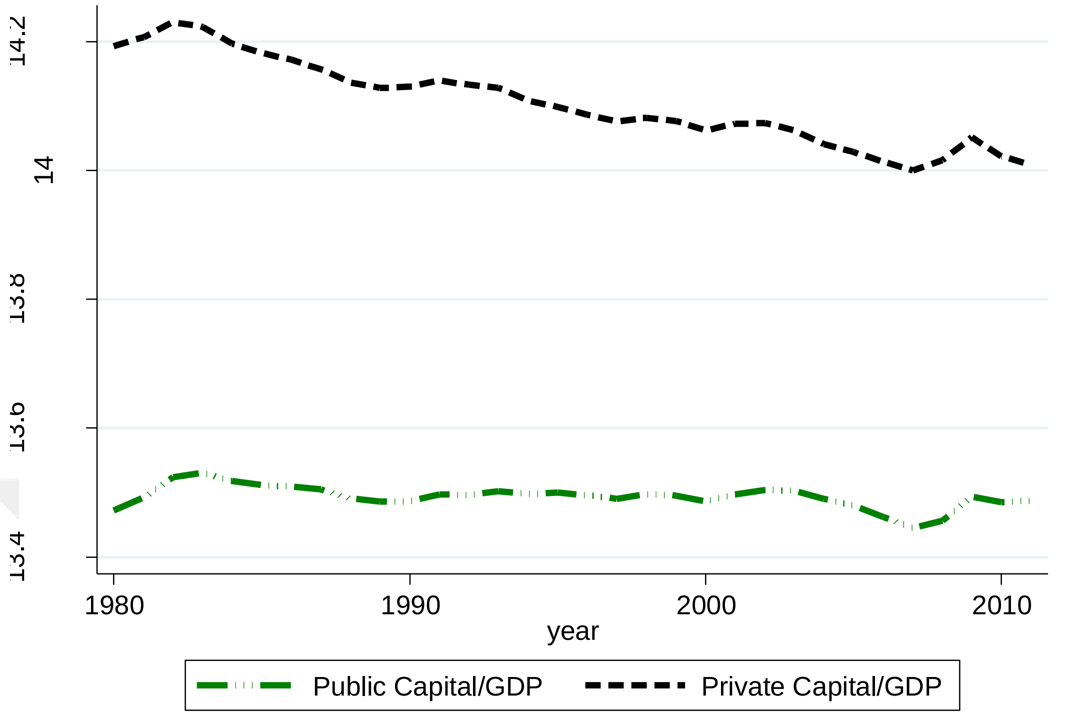
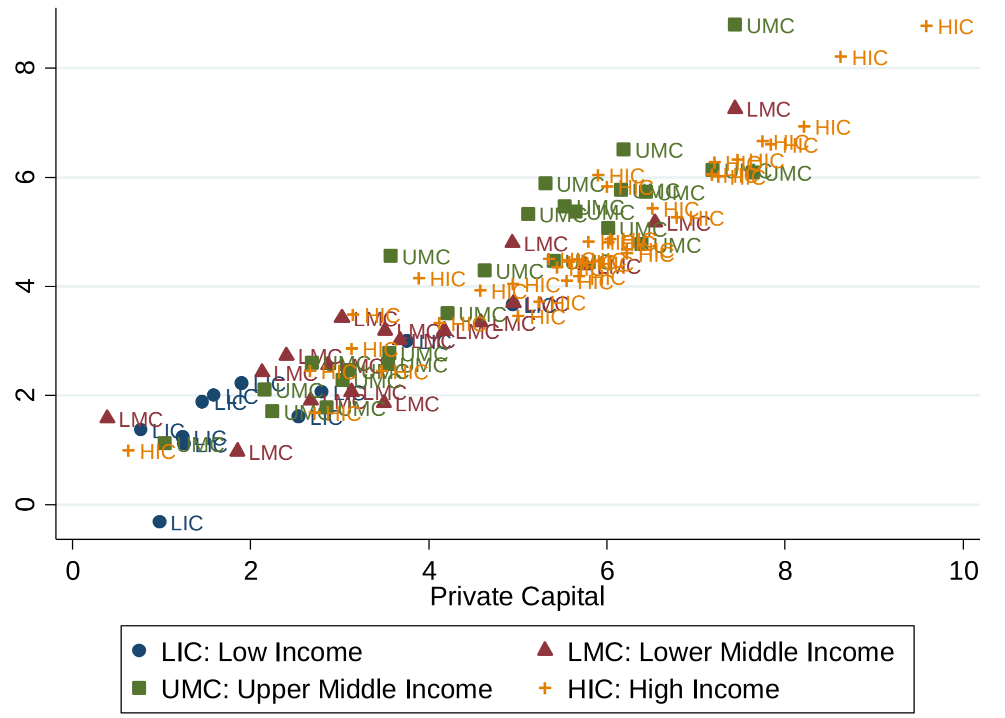


Figure 5: Public and Private Capital for Income Groups (Billion U.S. Dollars)



Total output (Y) is measured in real GDP at 2005 national prices for each country. In order to capture the vast income disparities across countries, countries are divided into four income groups based on World Bank country classification by income. These income groups are Low-income economies, Lower-middle-income economies, upper-middle-income economies and high-income economies. Moreover, we divide the high-income group into two subgroups. These two subgroups are high-income OECD and high-income non-OECD. The Country classification by income is shown in Table 6.

Table 6: World Bank Country Classification by Income (2018)

Low-Income Economies	Lower Middle Income Economies	Upper Middle Income Economies	High Income Economies (OECD)	High Income Economies (Non-OECD)
Benin	Bolivia	Botswana	Australia	Bahrain
Brundi	Cameroon	Brazil	Austria	Barbados
Central African Republic	Côte d'Ivoire	Bulgaria	Belgium	Hong Kong
Mozambique	Egypt	China	Canada	Kuwait
Niger	Guatemala	Colombia	Chile	Qatar
Rawanda	Honduras	Costa Rica	Denmark	Saudi Arabia
Senegal	India	Dominican Republic	Finland	Singapore
Sierra Leone	Indonesia	Ecuador	France	Taiwan
Togo	Jordan	Fiji	Germany	Trinidad & Tobago
Tanzania	Kenya	Gabon	Greece	Uruguay
Zimbabwe	Lesotho	Iran	Iceland	
	Mauritania	Iraq	Ireland	
	Mongolia	Malaysia	Israel	
	Morocco	Mauritius	Italy	
	Philippines	Mexico	Japan	
	Sri Lanka	Namibia	Korea	
	Swaziland	Panama	Luxembourg	
	Tunisia	Paraguay	Netherlands	
		Peru	New Zealand	
		Romania	Norway	
		South Africa	Poland	
		Thailand	Portugal	
		Turkey	Spain	
		Venezuela	Sweden	
			Switzerland	
			United Kingdom	
			United States	

In this thesis, we use public capital stock and private capital stock, which obtained from the IMF database. The use of this dataset has two main advantages:

1. This data set provides a long time series for a large panel of countries.

2. This dataset provides more consistent and comparable data than the data obtained from national or pre-1997 OECD databases.

Since the capital stock data generated by the IMF is calculated by the same methodology (perpetual inventory method), it provides a comparable homogeneous data set across countries.

Some studies (Ford and Paret 1991, Evans and Karras 1994) use the data on capital stock obtained from the OECD (1997) database. However, a heterogeneous set of data arises because different methods are used in the calculation of capital stock data in the OECD (1997) database. This situation makes it impossible to make comparable analyzes across countries.

In terms of the data set used, it is possible to distinguish two groups of studies that investigate the relationship between capital and economic growth. While some of the studies use the capital as a flow variable, others use the capital as a stock variable. Table 6 shows the studies that use capital as a flow or stock variable.

Table 7: Studies That Use Capital as Stock or Flow Variable

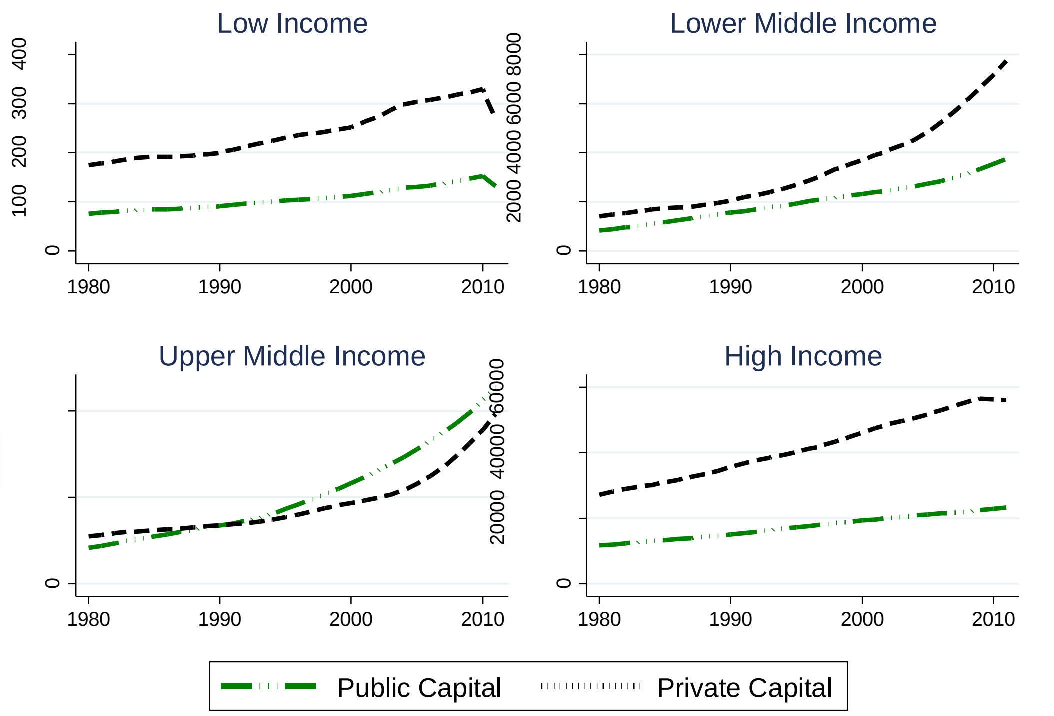
Studies that model capital as a stock variable	**Studies that model capital as a flow variable**
Aschauer (1989;2000a)	Easterly and Rebelo (1993)
Futagami et all. (1993)	Barro (1990)
Cassou and Lansing (1998)	Turnovsky and Fisher (1995)
Rioja (1999)	Aschauer (2000a)
Ziesemer (1990;1995)	Turnovsky (2000)
Turnovsky (1997; 2004)	Agênor (2008)

It is quite easier to obtain a consistent and comparable investment data for many countries. However, obtaining a consistent and comparable capital stock data across the country is not so easy. Therefore, many studies prefer investment data instead of capital stock data. For this reason, the results of the studies between capital and economic growth differ from each other. Moreover, some results might be the opposite of each other. Arslanalp et al. (2010) emphasize three main reasons for these contradictory results.

1. Most of the empirical studies use the public investment ratio instead of the rate of change in the capital stock while explaining the growth differences among countries. However, these two variables can vary greatly between countries. Moreover, these two variables can grow at different rates depending on the initial level of capital stock.
2. Since public investment and growth are flow variables, there is an endogenous link between public investment and economic growth that can complicate the econometric identification. The capital stock is calculated at the beginning of the period. For this reason, if the country faces a shortage in its resources public investment may fall, in contrast, public capital stock does not affect from this shortage.
3. If the public capital stock is above a certain threshold, a negative relationship between public investments and growth may occur. In other words, the productivity of public investments is closely related to the initial level of public capital stock.

Aschauer (1998a) examines the productivity of public spending for the US economy over the period 1949 -1985. The results show that during the period examined, non-military public capital is the main determinant of productivity growth relative to military and non-military flow spending. Main infrastructures such as highways, airports, public transport, sewerage have the most explanatory power for productivity. For these reasons, it would be more appropriate to use capital stock data instead of investment data.

Figure 6: Public and Private Capital for Countries in the Four Income Groups (1980-2011, in Billions of US Dollars)



While figure 4 shows the evolution of global public and private capital stocks over time, figure 6 displays the evolution of public and private capital stock for the countries in five income groups over the period1980-2011. In general, public capital and private capital stocks have increased over the period under consideration. A notable exception to the general pattern is upper-middle-income economies in which public capital stock exceed private capital stock after 1990. The year 1990 is a turning point for upper-income economies. Before 1990s private capital stock is greater than public capital stock, but the case has reversed after that year. This situation can be explained by the fact that the governments of the countries in this income group put more emphasis on development policies after the 1990s. Moreover, there is a huge discrepancy between these five income groups. Despite the increase in the capital stock for low-income economies, this increase is very small compared to other income groups.

CHAPTER 6: MODEL SPECIFICATIONS AND EMPIRICAL FINDINGS

6.1. INTRODUCTION

6.1.1. Presentation and Estimation of the Model

This section focuses on the estimation of VES. We obtain our baseline estimation equation by log-linearizing the production function stated in equation (4.3).

$$logY_{it} = logA_{it} + \alpha_1 v_1 \alpha_2 v_2 logK_{g,it} + (1-\alpha_2)v_2\alpha_1 v_1 \log\left(K_{p,it} + b_2\alpha_2 K_{g,it}\right) + (1-\alpha_1)v_1 \log\left(L_{it} + b_1\alpha_1 K_{g,it}^{\alpha_2 v_2}\left(K_{p,it} + b_2\alpha_2 K_{g,it}\right)^{(1-\alpha_2)v_2}\right) + \theta_{it} + \delta_{it} + \epsilon_{it} \quad (6.1)$$

i, t, θ, δ and ϵ are country index, time index, time fixed effect, region fixed effect and the error term, respectively. Following Tallman and Wang (1994), and Duffy and Papageorgiou (2000) we construct series for human capital adjusted labor input. We define the stock of human capital in country i, at time t, H_{it} as

$$H_{it} = E_{it}$$

Where E_{it} denotes average years of schooling[3]. Given the definition above, the human capital adjusted labor supply is defined as;

$$HL_{it} = H_{it} * L_{it} = E_{it} * L_{it}$$

In estimating the VES specification for aggregate production, we will use both raw labor (L) and human capital adjusted labor (HL) as measures of labor input[4].

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[3] we define E_{it} to denote Penn World Table human capital index. Penn World Table (version 9.0) define human capital index based on years of schooling and returns to education.

[4] For detail information on construction of human capital adjusted labor input, see Tallman and Wang (1994), and Duffy and Papageorgiou (2000).

We estimated equation (6.1) by nonlinear least square (NLLS) using data on total factor productivity, public capital, private capital, real GDP, and either raw labor supply (L) or human capital adjusted labor supply (HL) in place of labor input. In addition, we use total factor productivity as a proxy for technological improvement, to account for the differences in technological improvement across time and countries.

For the initial level of parameters in NLLS estimation, we estimated an ordinary least square (OLS) regression based on the $logY_{it}$, a constant, $logK_{g,it}$, $logK_{p,it}$, $logTFP$, and either $logL_{it}$ or $logHL_{it}$. We also consider other initial values for the parameters and obtain similar results.

In our baseline estimation equation, we are interested in the parameters b_1, b_2 and α_2. The first parameter (i.e. b_1) affects the aggregate degree of substitutability between capital and labor. The second parameter (i.e. b_2) affects the aggregate degree of substitutability between public capital and private capital. Hence, if these two parameters are different from zero (i.e. $b_1 \neq 0, b_2 \neq 0$), then we can state that the elasticity of substitution between production inputs is variable. This is our first testable hypothesis. Second testable hypothesis is related to the third parameter (i.e. α_2) which reflects the importance of public capital relative to private capital in production of total capital. Our second testable hypothesis is that private capital is significant in production (i.e. $\alpha_2 \neq 0$).

As we emphasized above b_1, b_2 reflect the aggregate degree of substitutability between inputs. These parameters will be estimated for aggregate sample of data, for each income groups, and for each region[5]. If $b_1 = 0$ and $b_2 = 0$ then CES specification is supported in favor of VES. Conversely, If $b_1 \neq 0$ and $b_2 \neq 0$ then we can say that VES specification better reflects the production process. While positive values of b_1, b_2 (i.e. $b_1 > 0$, $b_2 > 0$) implies substitutability between production factor, negative values of the parameters (i.e. $b_1 < 0$, $b_2 < 0$) implies complementarity between production inputs.

[5] Country classification by income and region are based on the World Bank country classification

6.1.2. Empirical Results

In this subsection, we estimate our main estimation equation presented in previous section. The main estimation specifications (6.1) includes public and private capital as production inputs. In the theory section, we assume constant returns to scale technology (i.e.$v_1 = v_2 = 1$). We follow Lazkano and Pham (2016) to test the validity of these restrictions. To test the validity of the constraints, the main equation is estimated separately for $v_1 = 1$ and $v_2 = 1$, respectively, and the results are reported in Table 8.

While column 1 of the Table 8 shows the value of the parameters obtained from the full specification by imposing the constraint only on v_1 (i.e. $v_1 = 1$), column 2 shows the same results by imposing the constrain only on v_2 (i.e. $v_2 = 1$). The post estimation tests after imposing the constraint on one parameter provides support to impose the same restriction on the other return to scale parameter. Therefore, we can conclude that, the production function in equation (3) exhibit constant return to scale property. Column (3) of Table 8 shows the results of the main specification in which both of the return to scale parameters are restricted to be 1 (i.e. $v_1 = v_2 = 1$). These restrictions do not fundamentally alter the result of the estimations. However, Insertion of these constraints will make the model clearer and easier to understand.

Table 8: Global Estimation of the Full Equation (All Countries)

Parameters	[1] $v_1 = 1$	[2] $v_2 = 1$	[3] $v_1 = v_2 = 1$
α_1	0.3033237*** (0.0027557)	0.302043*** (0.0028058)	0.3039112*** (0.0028455)
α_2	0.1461094*** (0.0544578)	0.1549679*** (0.0535331)	0.1326585** (0.0546316)
b_1	3.52e-09*** (5.80e-10)	4.00e-09*** (4.63e-10)	4.16e-09*** (5.04e-10)
b_2	4.854371* (2.767537)	4.461795* (2.429638)	5.510148* (3.336378)
v_1	-	1.008853*** (0.0037873)	-
v_2	1.005775*** (0.0040182)	-	-

Note: The terms in parentheses are standard errors. ***, **, and * display the level of significance at 1%, 5% and 10% respectively.

After the imposition of these restrictions into the equation (6.1), we obtain the following estimation equation:

$$logY_{it} = logA_{it} + \alpha_1\alpha_2 logK_{g,it} + (1-\alpha_2)\alpha_1 \log\left(K_{p,it} + b_2\alpha_2 K_{g,it}\right) + (1-\alpha_1)\log\left(L_{it} + b_1\alpha_1 K_{g,it}^{\alpha_2}(K_{p,it} + b_2\alpha_2 K_{g,it})^{1-\alpha_2}\right) + N + TX + \theta_{it} + \delta_{it} + \epsilon_{it} \quad (6.2)$$

In addition to the main variables, we include two other variables, N and TX, to control the countries with high natural resource rent, and high technology export, respectively. The dummy variable N equal to one if a country's average natural resource rent (% of GDP) is larger than 12 percent and TX equals to one if country's average high technology exports (% of manufactured export) are greater than 10 percent.[6] The nonlinear regression results of the restricted equation (6.2) are reported in table 9. Columns (1) and (2) of table 9 shows the results for the global dataset. While the top panel of table 9 reports the results for raw labor, the bottom panel reports the results for skilled labor. Moreover, the odd columns do not include any fixed effects, on the other hand, the even columns include time and regional fixed effects.

[6] The data on total natural resource rent (% GDP) and high technology export (% of manufactured exports) are obtained from World Bank database. We create the dummy variables (N and TX) based on the average natural resource rent over the period 1970-2015 and average high technology export over the period 2000-2015, respectively.

Table 9: Nonlinear Regression Estimates of VES, (Income Classification)

	Country Classification											
Parameters	Global		Low-Income Economies		Lower-Middle-Income Economies		Upper-Middle-Income Economies		High-Income OECD Economies		High-Income Non-OECD Economies	
Unadjusted Labor (L)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]
α_1	0.3431*** (0.0012)	0.2524*** (0.0020)	0.3333*** (0.0011)	0.2403*** (0.0030)	0.3506*** (0.0030)	0.2696*** (0.0035)	0.3645*** (0.0029)	0.2497*** (0.0087)	0.3839*** (0.0027)	0.2859*** (0.0061)	0.3920*** (0.0043)	0.3087*** (0.0075)
α_2	0.6892*** (0.0140)	0.6675*** (0.0200)	0.6408*** (0.0431)	0.6705*** (0679)	0.4350*** (0.0844)	0.4441*** (0.0710)	0.5688*** (0.0276)	0.5983*** (0.0335)	0.8028*** (0.0261)	0.7599*** (0.0455)	0.8584*** (0.0475)	0.7262*** (0.0863)
b_1	1.17e-9*** (7.49e-11)	2.12e-9*** (1.87e-10)	7.43e-10*** (1.16e-10)	1.92e-9*** (5.17e-10)	4.59e-10*** (9.76e-11)	8.49e-10*** (2.40e-10)	4.81e-10*** (9.94e-11)	3.05e-9*** (1.04e-9)	1.79e-10*** (3.66e-11)	6.50e-10*** (1.84e-10)	1.14e-10*** (4.02e-11)	2.25e-10** (1.05e-10)
b_2	-0.1454*** (0.0032)	-0.1506*** (0.0045)	-0.5709*** (0.0410)	-0.5971*** (0.0479)	-0.0128 (0.2822)	-0.2710*** (0.0485)	-0.1764*** (0.0083)	-0.1677*** (0.0094)	-1.7760*** (0.0541)	-1.8589*** (0.0861)	-0.3532*** (0.0164)	-0.4104*** (0.0385)
Adjusted Labor (HL)												
α_1	0.3291*** (0.0011)	0.2398*** (0.0017)	0.3232*** (0.0011)	0.2320*** (0.0025)	0.3291*** (0.0034)	0.2528*** (0.0028)	0.3429*** (0.0027)	0.2407*** (0.0057)	0.3568*** (0.0027)	0.2600*** (0.0055)	0.3674*** (0.0037)	0.2848*** (0.0059)
α_2	0.6861** (0.0145)	0.6555*** (0.218)	0.6049*** (0.0626)	0.6349*** (0.1067)	0.3872*** (0.1060)	0.4026*** (0.0852)	0.5815*** (0.0282)	0.5879*** (0.0373)	0.7936*** (0.0275)	0.7574*** (0.0486)	0.8718*** (0.0477)	0.7045*** (0.0995)
b_1	1.58e-9*** (9.95e-11)	2.56e-9*** (2.18e-10)	4.77e-10*** (1.34e-10)	1.19e-9** (5.07e-10)	8.34e-10*** (1.97e-10)	8.12e-10*** (2.89e-10)	9.18e-10*** (1.83e-10)	3.20e-9*** (8.50e-10)	4.39e-10*** (9.23e-11)	1.34e-9*** (3.60e-10)	2.89e-10*** (8.34e-11)	4.44e-10*** (1.72e-10)
b_2	-0.1464*** (0.0031)	-0.1535*** (0.0051)	-0.5358 (0.0839)	-0.6085*** (0.0764)	0.1479 (0.4910)	-0.3020*** (0.0601)	-0.1727*** (0.0082)	-0.1711*** (0.0108)	-1.7968*** (0.0589)	-1.8676*** (0.0953)	-0.3460*** (0.0184)	-0.4096*** (0.0511)
θ		✓		✓		✓		✓		✓		✓
δ		✓		✓		✓		✓		✓		✓
R^2 (for L)	0.9984	0.9967	0.9995	0.9970	0.9984	0.9960	0.9986	0.9970	0.9991	0.9979	0.9983	0.9963
R^2 (for HL)	0.9986	0.9969	0.9995	0.9974	0.9984	0.9964	0.9986	0.9971	0.9991	0.9979	0.9984	0.9965
N	2912	2912	352	352	576	576	800	800	864	864	320	320

Note: The terms in parentheses are standard errors. ***, **, and * display the level of significance at 1%, 5% and 10% respectively. ✓ shows that time and regional fixed effects dummies are included in the regression equations.

We see that the coefficient estimates of b_2 for the global data set is negative and statistically significant. This result provide evidence for non-constant elasticity of substitution between private and public capital. Hence, we can reject the Cobb-Douglas specification in favor of VES specification for our 32-year and 91-country sample. The parameter b_2 remains negative and significant when we used adjusted labor (HL) with and without time and regional fixed effects. Although b_2 is negative and statistically significant for all income groups, it takes its smallest value for High-income OECD countries. However, b_2 takes a larger value for lower-middle income and upper-middle-income countries. In addition, since the elasticity of substitution parameter is negative the elasticity of substitution between private and public capital is smaller than one[7]. The smaller b_2 value indicates the stronger complementarity between public and private capital. Hence, private and public capital are complements in the final production of output. Particularly, a one-unit increasing in public-private capital ratio reduces elasticity of substitution approximately by 0.15 unit. This improvement does not fundamentally change when we consider raw labor, skill labor, or time and regional fixed effects.

The situation is similar when we consider the estimations for different income levels. As in global estimates, the substitution parameter b_2 is highly significant and negative in most cases[8]. The parameter b_2 is insignificant in three cases. We found it to be insignificant for lower-middle-income economies when we used both raw and skilled labor, but not control for time and regional fixed effects. In addition, it is insignificant for low-income economies when skilled labor is used in place of labor input. However, these three cases turn to be significant when we include time and regional fixed effects to the estimation equation. The highly significant VES parameter b_2 support the existence of non-constant elasticity of substitution between private and public capital for five income groups. Moreover, a negative b_2 leads to an elasticity of substitution to

[7] Recall that $\sigma(K_g, K_p) = 1 + b_2(\frac{K_g}{K_p})$

[8] The parameter b_2 is positive but insignificant for lower-middle-income economies for the case adjusted labor and without time and regional fixed effects.

be smaller than one, which implies complementarity between private and public capital. Moreover, the existence of positive effects of public capital on private capital indicates a complementarity between the two inputs in the final output production. Therefore, an increase in public capital leads to an increase in marginal productivity of private capital which results in a greater use of private capital.

Table 10: Average Elasticity of Substitution by Income Groups (1980-2011)

		(1)	(2)	(3)	(4)	(5)	(6)
		Low Income	**Lower Middle Income**	**Upper Middle Income**	**High Income OECD**	**High Income nonOECD**	**Global**
Raw Labor (L)	Mean	0.4422 (0.3676)	0.7557 (0.3001)	0.8273 (0.2470)	0.3896 (0.2024)	0.5831 (0.2644)	0.8820 (0.1523)
	Max	0.8916	0.9555	0.9838	0.7021	0.8922	0.9855
	Min	-0.4480	-1.1517	-0.6661	-0.3016	-0.3439	-0.4965
Adjusted Labor (HL)	Mean	0.4315 (0.3746)	0.7277 (0.3344)	0.8238 (0.2520)	0.3867 (0.2034)	0.5839 (0.2639)	0.8798 (0.1552)
	Max	0.8895	0.9504	0.9835	0.7007	0.8925	0.9852
	Min	-0.4758	-1.3980	-0.6999	-0.3077	-0.3410	-0.5252
	N	352	576	800	864	320	2912

Note: Numbers in the parentheses are standard errors.

From table 10, it is observed that as the level of income increases, the elasticity of substitution between public capital and private capital stock increases up to a specific point and then decreases. In other words, we can say that there is an inverted U-shaped relationship between income level and substitution elasticity for the selected sample of data. In general, substitution elasticity is found to be less than one for all income groups[9].

The results of the analysis for income groups show that some countries have the negative elasticity of substitution for some years. But on average, the elasticity of substitution in positive and smaller than one. For example, Lesotho, which is a lower-middle income country, has the smallest substitution elasticity with -1.15. Many other countries have a moderate elasticity of substitution ranging from 0.5 to 1. It is emphasized that negative elasticity of substitution

[9] The time graphs of the elasticity of substitution for individual countries are given in Appendix 3.

implies a strong complementarity between public capital stock and private capital stock. An elasticity of substitution greater than 1 implies that public capital stock and private capital stock are strong substitutes for each other.

If the elasticity of substitution is greater than one, a fall in the public capital stock, along with a financial contraction, will make public goods more expensive. This may lead to a significant expansion in the private capital stock and remove the negative effect of fiscal contraction on aggregate demand. Moreover, this process may result in an increase in total demand. On the other hand, if the elasticity of substitution is smaller than one, public capital stock is a strong complement to private capital stock. In this case, public goods may become more expensive because of the financial contraction. Hence, this process may lead to a negative income and substitution effect. This situation may lead to a contraction in private sector investment expenditures, which constitute private sector capital stock. The total demand may be negatively affected in this case.

The same analysis is carried out for the country classification by region. Table 11 shows the estimation results of the elasticity of substitution between public capital stock and private capital stock for the groups of countries classified by region. The substitution elasticity parameter (b_2) is negative and statistically significant in many cases. The substitution elasticity effect is negative and statistically significant in 7 out of 10 models in which the time fixed effect is included. The elasticity of substitution parameter (b_2) is negative but statistically insignificant for the Middle East and North Africa countries in all of four models (see table 11 column 7 and 8). The b_2 coefficient is positive only in three cases (column 3 bottom panel, column 5 bottom panel, and column 9 top and bottom panels). In these cases, the b_2 coefficient is statistically insignificant. The negative coefficient of substitution elasticity indicates that the elasticity of substitution between public capital stock and private capital stock is smaller than 1. Hence, public capital stock and private capital stock act as complement in the final production of output. In sum, the results show that the elasticity of substitution is quite similar for country classification by income and by region.

The elasticity of substitution parameter (b_2) takes its smallest value in Europe and Central Asia countries. Contrary to other regions, b_2 is larger in East Asia and Pacific, and Sub-Saharan Africa countries. Considering the existence of more developed countries in Europe and Central Asia region, it is observed that the complementarity between public and private capital stock increase with the level of development. This result is consistent with the previous findings.

Our third classification is Human Development Index. The estimation results of the model examining the substitution effect between public and private capital stocks are shown in table 12. When the time and regional fixed effects are included in the models, the substitution parameter is negative in all groups. Therefore, even when the countries are classified according to human development index (HDI), there is a complementary feature between public capital stock and private capital stock. However, the elasticity of substitution is statistically significant only in the very high HDI and high HDI groups.

Table 11: Nonlinear Regression Estimates of VES, (Regional Classification)

	Country Classification									
Parameters	Latin America & the Caribbean		East Asia and Pacific		Europe and Central Asia		Middle East and North Africa		Sub-Saharan Africa	
Unadjusted Labor (L)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
α_1	0.3774***	0.3322***	0.3017***	0.2314***	0.3818***	0.3324***	0.3727***	0.3341***	0.3213***	0.2815***
	(0.0025)	(0.0045)	(0.0085)	(0.0223)	(0.0030)	(0.0071)	(0.0026)	(0.0049)	(0.0023)	(0.0036)
α_2	0.7980***	0.7533***	0.6967***	0.6924***	0.9964***	0.9612***	0.6690***	0.5504***	0.4592***	0.5416***
	(0.0161)	(0.0420)	(0.0403)	(0.0482)	(0.0185)	(0.0337)	(0.0662)	(0.1182)	(0.0565)	(0.0588)
b_1	2.91e-10***	6.88e-10***	3.95e-9***	1.32e-8	2.82e-10***	6.64e-10***	3.02e-10***	4.03e-10***	2.79e-9***	3.93e-9***
	(7.46e-11)	(2.06e-10)	(1.17e-9)	(9.03e-9)	(5.87e-11)	(2.36e-10)	(3.80e-11)	(9.34e-11)	(3.14e-10)	(6.53e-10)
b_2	-0.3955***	-0.4190***	-0.1038**	-0.1145***	-0.6533***	-0.6771***	-0.1503	-0.2114	0.1245	-0.1339
	(0.0161)	(0.0231)	(0.0410)	(0.0369)	(0.0396)	(0.0227)	(0.2177)	(0.3228)	(0.2373)	(0.1141)
Adjusted Labor (HL)										
α_1	0.3594***	0.3197***	0.2858***	0.2229***	0.3561***	0.3106***	0.3529***	0.3157***	0.3125***	0.2746***
	(0.0020)	(0.0031)	(0.0070)	(0.0151)	(0.0063)	(0.0062)	(0.0028)	(0.0049)	(0.0020)	(0.0030)
α_2	0.8613***	0.8360***	0.6952***	0.6952***	1.0333***	0.9559***	0.6884***	0.5673***	0.4777***	0.5434***
	(0.0369)	(0.0402)	(0.0396)	(0.0464)	(0.1572)	(0.0372)	(0.0747)	(0.1263)	(0.0498)	(0.0545)
b_1	2.67e-10***	4.66e-10**	6.08e-9***	1.62e-8**	7.23e-10***	1.20e-9***	6.02e-10	6.98e-10***	3.33e-9***	4.02e-9***
	(1.02e-10)	(1.89e-10)	(1.52e-9)	(7.71e-9)	(2.24e-10)	(4.08e-10)	(7.95e-11)	(1.62e-10)	(3.51e-10)	(6.45e-10)
b_2	-0.3666***	-0.3776***	0.1164***	-0.1271***	0.0970	-0.6809***	-0.1336	-0.2151	0.0029	-0.1834**
	(0.0166)	(0.0220)	(0.0311)	(0.0247)	(8.9676)	(0.0260)	(0.2553)	(0.3319)	(0.1693)	(0.0748)
θ		✓		✓		✓		✓		✓
R^2 (for L)	0.9984	0.9964	0.9986	0.9977	0.9992	0.9978	0.9993	0.9974	0.9987	0.9963
R^2 (for HL)	0.9985	0.9967	0.9987	0.9977	0.9992	0.9979	0.9993	0.9975	0.9988	0.9967
N	576	576	448	448	704	704	352	352	704	704

Note: The terms in parentheses are standard errors. ***, **, and * display the level of significance at 1%, 5% and 10% respectively. ✓ shows that time and regional fixed effects dummies are included in the regression equations.

The sign of the elasticity of substitution parameter is heterogeneous among the groups when the time and regional fixed effects are included in the model. The substitution parameter (b_2) in the very high HDI group is negative while it is positive in all the other groups. However, when the elasticity of substitution is positive, b_2 coefficient is statistically insignificant. In summary, as we classify the countries according to their human development index level, the negative sign of substitution parameter in 10 out of 15 cases shows that public capital and private capital are complements in the final output.

Similar to the previous results, it is observed that the complementarity property between public capital and private capital stock increase with the level of development. As seen in table 12, while b_2 takes its smallest value for very high HDI group, it takes its largest value in low HDI group. That is, the elasticity of substitution is larger in low-HDI countries. Hence, for the countries with the very high HDI public capital and private capital stocks are the complement in the final production of output. In contrast, for the countries with low-HDI, the two sub-components of capital stock are substitutes in the final production of output.

Table 12: Nonlinear Regression Estimates of VES, (HDI Classification)

	Country Classification							
Parameters	Very High HDI		High HDI		Medium HDI		Low HDI	
Unadjusted Labor (L)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]
α_1	0.3904*** (0.0020)	0.2890*** (0.0067)	0.3707*** (0.0042)	0.2888*** (0.0064)	0.3433*** (0.00800)	0.2756*** (0.0044)	0.3245*** (0.0066)	0.2433*** (0.0029)
α_2	0.1049*** (0.0340)	0.1150* (0.0609)	0.3141*** (0.0875)	0.3364*** (0.0799)	0.0937 (0.2199)	0.6498*** (0.0981)	0.0890 (0.2077)	0.5172*** (0.1127)
b_1	1.55e-10*** (1.47e-11)	6.32e-10*** (1.61e-10)	3.63e-10*** (6.46e-11)	6.93e-10*** (2.05e-10)	5.15e-10** (2.12e-10)	1.58e-9*** (3.65e-10)	3.79e-10*** (1.38e-10)	1.52e-9*** (3.54e-10)
b_2	-0.5378 (0.3522)	-0.9478** (0.3888)	0.6220 (0.6582)	-0.0776 (0.2255)	13.0500 (37.8492)	-0.2915 (0.2011)	9.2127 (27.2994)	-0.0199 (0.3393)
Adjusted Labor (HL)								
α_1	0.3655*** (0.0018)	0.2676*** (0.0053)	0.3444*** (0.0042)	0.2723*** (0.0042)	na	0.2558*** (0.0041)	0.3172*** (0.0044)	0.2342*** (0.0022)
α_2	0.0978*** (0.0359)	0.1088* (0.06637)	0.2363** (0.1049)	0.3760*** (0.0661)	na	0.6613*** (0.1056)	0.1815 (0.1521)	0.5073*** (0.0938)
b_1	3.66e-10*** (3.23e-11)	1.20e-9*** (2.53e-10)	7.62e-10*** (1.48e-10)	8.66e-10*** (2.55e-10)	na	2.57e-9*** (6.02e-10)	4.29e-10*** (1.11e-10)	1.23e-9*** (3.13e-10)
b_2	-0.1121 (0.5437)	-0.9781** (0.4032)	1.4371 (1.4995)	-0.2032*** (0.0679)	na	-0.2865 (0.2211)	2.6678 (4.0409)	-0.1654 (0.1513)
θ		✓		✓		✓		✓
δ		✓		✓		✓		✓
R^2 (for L)	0.9997	0.9985	0.9990	0.9971	0.9996	0.9975	0.9991	0.9965
R^2 (for HL)	0.9998	0.9986	0.9991	0.9973	na	0.9975	0.9993	0.9971
N	1120	1120	800	800	480	480	512	512

Note: The terms in parentheses are standard errors. ***, **, and * display the level of significance at 1%, 5% and 10% respectively. ✓ shows that time and regional fixed effects dummies are included in the regression equations.

6.2. THE CD-VES PRODUCTION FUNCTIONS

6.2.1. Introduction

In this chapter, we consider a CES production function (namely Cobb-Douglas) between aggregate capital and labor. We carried out the same analysis to see whether the elasticity of substitution between public capital and private capital is consistent with the previous results.

6.2.2. The Model

Consider the following CD production function with capital and labor as production inputs. As we illustrated in chapter 4, b_1 capture the VES between aggregate capital and labor. Although the post estimation tests of the estimations in section 6.1 shows that b_1 is statistically significant, it may be useful to consider the CES between capital and labor due to the very small value of b_1. In the case when b_1 is zero, the production function in equation (4.1) reduces to Cobb-Douglas Production function. Since our main purpose is to investigate the VES between public capital and private capital, this simplification makes the model more comprehensible. After imposition of this assumption into equation (4.1) we obtain the standard CD production function with capital and labor as in equation (6.4).

$$Y = AK^{\alpha_1 v_1} L^{(1-\alpha_1)v_1} \tag{6.4}$$

The capital production function remains the same as in equation (4.2) in chapter 4. Hence the nested production function with three inputs is:

$$Y = A[K_g^{\alpha_2 v_2}(K_p + b_2 \alpha_2 K_g)^{(1-\alpha_2)v_2}]^{\alpha_1 v_1} L^{(1-\alpha_1)v_1} \tag{6.5}$$

The above function satisfies the standard properties of a production function, namely,

$Y > 0, \mathrm{MP}_{K_g} > 0, \mathrm{MP}_{K_p} > 0$ and diminishing marginal returns as long as $A > 0, 0 < \alpha_1, \alpha_2 \leq 1, b_2 > -1 \text{ and } K_g^{-1} \geq -b_2$

Since the capital accumulation equation is still VES, the elasticity of substitution between public capital and private capital will be the same as in chapter 4. That is the elasticity of substitution between public capital and private capital is;

$$\sigma(K_g, K_p) = 1 + b_2(\frac{K_g}{K_p})$$

Log-linearizing equation (32), we obtain our main estimation equation:

$$logY = logA + \alpha_1 v_1 \alpha_2 v_2 logK_g + \alpha_1 v_1 (1-\alpha_2) v_2 \log\left(K_p + b_2 \alpha_2 K_g\right) + (1-\alpha_1) v_1 logL \quad (6.6)$$

6.2.3. Empirical Results

This section examines how the elasticity of substitution between the public capital stock and the private capital stock varies with the given production function. The main motivation here is how the assumption of constant elasticity of substitution between aggregate capital and labor affects the elasticity of substitution between public capital stock and private capital stock. The empirical results for equation (6.6), which is a Cobb-Douglas type of production function with a constant elasticity of substitution between K and L, are shown in Table 13. Table 13 contains the empirical results for groups of countries classified by their level of income.

Table 13 shows that the findings are largely parallel to the results in section 6.1.2. In general, we can say that the elasticity of substitution parameter is negative and statistically significant. Hence, under the assumption of constant elasticity of substitution between capital and labor, we can say that public capital and private capital stock show a complementary property in all income groups.

When the coefficients are examined in terms of significance levels, the only difference from the previous findings is that the elasticity of substitution coefficient for low-income economies become significant in this case (see Table 13 column 3 bottom panel). The coefficient is insignificant when we consider VES between capital and labor.

The most significant difference regarding the empirical results of the two models is the change in the magnitude of the elasticity of substitution parameter. In case of Cobb-Douglas production function, which assumes the constant elasticity of substitution between capital and labor, it can be said that the absolute value of the elasticity of substitution parameter significantly increases. There is a significant increase in absolute value of elasticity of substitution parameter when the Cobb-Douglas production function is used. A reverse situation occurred only

Table 13: Nonlinear Regression Estimates of CD-VES, (Income Classification)

	Country Classification											
Parameters	Global		Low-Income Economies		Lower-Middle-Income Economies		Upper-Middle-Income Economies		High-Income OECD Economies		High-Income Non-OECD Economies	
Unadjusted Labor (L)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]
α_1	0.3802***	0.3011***	0.3403***	0.2540***	0.3629***	0.2850***	0.3844***	0.3046***	0.4021***	0.3255***	0.4086***	0.3341***
	(0.0006)	(0.0001)	(0.0013)	(0.0018)	(0.0014)	(0.0015)	(0.0008)	(0.0011)	(0.0007)	(0.0011)	(0.0012)	(0.0019)
α_2	0.3409***	0.1102*	0.4844***	0.4532***	0.4687***	0.3494***	0.4610***	0.3118***	0.8614***	0.8589***	0.7518***	0.3983**
	(0.0356)	(0.0592)	(0.0772)	(0.1525)	(0.0811)	(0.1113)	(0.0412)	(0.0752)	(0.0379)	(0.0845)	(0.0895)	(0.1910)
b_2	-0.2926***	-0.9090*	-0.6782***	-0.8466***	-0.1970**	-0.3522***	-0.2177***	-0.3224***	-1.6578***	-1.6617***	-0.3954***	-0.7122***
	(0.0288)	(0.4845)	(0.0762)	(0.1819)	(0.0915)	(0.1012)	(0.0190)	(0.0773)	(0.0731)	(0.1578)	(0.0365)	(0.2506)
Adjusted Labor (HL)												
α_1	0.3573***	0.2752***	0.3262***	0.2381***	0.3416***	0.2613***	0.3608***	0.2780***	0.3734***	0.2931***	0.3880***	0.3075***
	(0.0005)	(0.0007)	(0.0013)	(0.0018)	(0.0016)	(0.0015)	(0.0008)	(0.0011)	(0.0007)	(0.0011)	(0.0013)	(0.0020)
α_2	0.4348**	0.2199***	0.5272***	0.5221***	0.4478***	0.3283***	0.4735***	0.3142***	0.8284***	0.8115***	0.8021***	0.4338**
	(0.0322)	(0.0567)	(0.0781)	(0.1585)	(0.0935)	(0.1149)	(0.0425)	(0.0782)	(0.0430)	(0.0934)	(0.1014)	(0.2205)
b_2	-0.2307***	-0.4569***	-0.5632***	-0.7032***	-0.1679	-0.3732***	-0.2121***	-0.3200***	-1.7236***	-1.7579***	-0.3648***	-0.6333***
	(0.0166)	(0.1172)	(0.1144)	(0.1262)	(0.1423)	(0.1166)	(0.0187)	(0.0794)	(0.0887)	(0.1940)	(0.0577)	(0.2060)
θ		✓		✓		✓		✓		✓		✓
δ		✓		✓		✓		✓		✓		✓
R^2 (for L)	0.9971	0.9947	0.9993	0.9965	0.9982	0.9957	0.9984	0.9964	0.9990	0.9975	0.9980	0.9959
R^2 (for HL)	0.9977	0.9957	0.9995	0.9973	0.9983	0.9963	0.9984	0.9967	0.9990	0.9976	0.9981	0.9960
N	2912	2912	352	352	576	576	800	800	864	864	320	320

Note: The terms in parentheses are standard errors. ***, **, and * display the level of significance at 1%, 5% and 10% respectively. ✓ shows that time and regional fixed effects dummies are included in the regression equations.

in the high-income OECD countries. That is, when the constant elasticity of substitution is assumed between capital and labor, there is a decline in the absolute magnitude of the elasticity of substitution parameter in high-income OECD countries (see Table 13 column 9).

Table 14 summarizes the regression results of equation (6.6) when the countries are classified by geographical region[10]. In general, the results found for country classification by geographical regions are largely similar to the results found for income groups. The substitution elasticity parameter is negative and statistically significant. Similar to the results for classification by income, there is also a significant increase in the absolute magnitude of the elasticity of substitution parameter when we used country classification by geographical regions. This situation does not change when adjusted labor is used for labor input. Under the constant elasticity of substitution between capital and labor assumption, the regressions converge to a maximum likelihood only if raw labor is used for labor input and no time fixed effect is included in the model (Table 14 column 1). The other regressions for this group of countries do not converge to a maximum likelihood. As a result, the existing complementarity between public capital and private capital stock remain valid for this classification.

When we consider human development index (HDI) as an indicator of economic progress the elasticity of substitution parameter becomes significant in most cases. In the case of nonconstant elasticity of substitution between capital and labor, the elasticity of substitution parameter is negative in most cases. However, it is statistically significant only for very high HDI countries (Table 12 column 2 both panels) and high HDI countries (Table 12 column 4 bottom panel). On the other hand, when we consider constant elasticity of substitution between capital and labor, the elasticity of substitution parameters becomes statistically significant in all cases except low HDI countries (see Table 15). One distinction of this classification from the two other classifications is that the magnitude of elasticity of substitution parameter is decreased in absolute terms.

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[10] See World Bank Country and Lending Groups classificaitons

However, the negative sign indicates that public capital and private capital are still complement in the final production of output.

Table 14: Nonlinear Regression Estimates of CD-VES, (Regional Classification)

	Country Classification									
Parameters	Latin America & the Caribbean		East Asia and Pacific		Europe and Central Asia		Middle East and North Africa		Sub-Saharan Africa	
Unadjusted Labor (L)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
α_1	0.3891*** (0.0009)	na	0.3756*** (0.0015)	0.3411*** (0.0019)	0.4030*** (0.0009)	0.3680*** (0.0016)	0.3981*** (0.0011)	0.3640*** (0.0020)	0.3574*** (0.0019)	0.3216*** (0.0019)
α_2	0.8622*** (0.0411)	na	0.4685*** (0.0717)	0.3477*** (0.0969)	0.9393*** (0.0547)	0.8229*** (0.0950)	0.7808*** (0.0635)	0.6202*** (0.1410)	0.1308 (0.1166)	0.1261 (0.1403)
b_2	-0.3662*** (0.183)	na	-0.2139*** (0.0313)	-0.2886*** (0.0789)	-0.6892*** (0.0400)	-0.7868*** (0.0734)	-0.3814*** (0.0230)	-0.4395*** (0.0784)	-0.4718 (0.3259)	-0.7914 (0.6710)
Adjusted Labor (HL)	na	na								
α_1	na	na	0.3481*** (0.0014)	0.3120*** (0.0018)	0.3762*** (0.0008)	0.3400*** (0.0014)	0.3776*** (0.0011)	0.3424*** (0.0019)	0.3403*** (0.0017)	0.3034*** (0016)
α_2	na	na	0.4881*** (0.0701)	0.3592*** (0.0974)	0.9641*** (0.0415)	0.8414*** (0.0848)	0.8467*** (0.0541)	0.7066)*** (0.1287)	0.1657 (0.1030)	0.1678 (0.1245)
b_2	na	na	-0.2054*** (0.0281)	-0.2794*** (0.0744)	-0.6751*** (0.0300)	-0.7728*** (0.0731)	-0.3586*** (0.0180)	-0.4039*** (0.0618)	-0.3721 (0.2476)	-0.6233* (0.3481)
θ				✓		✓		✓		✓
R^2 (for L)	0.9983	na	0.9969	0.9952	0.9990	0.9975	0.9984	0.9962	0.9964	0.9935
R^2 (for HL)	na	na	0.9975	0.9959	0.9990	0.9976	0.9984	0.9966	0.9974	0.9951
N	576	576	448	448	704	704	352	352	704	704

Note: The terms in parentheses are standard errors. ***, **, and * display the level of significance at 1%, 5% and 10% respectively. ✓ shows that time and regional fixed effects dummies are included in the regression equations.

Table 15: Nonlinear Regression Estimates of CD-VES, (HDI Classification)

	Country Classification							
Parameters	Very High HDI		High HDI		Medium HDI		Low HDI	
Unadjusted Labor (L)	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]
α_1	0.4024*** (0.0009)	0.3257*** (0.0012)	0.3845*** (0.0009)	0.3064*** (0.0012)	0.3748*** (0.0013)	0.2953*** (0.0016)	na	0.2590*** (0.0029)
α_2	0.8710*** (0.0461)	0.8131*** (0.0790)	0.5784*** (0.0477)	0.3551*** (0.0856)	0.9262*** (0.0762)	0.9693*** (0.0825)	na	0.2148 (0.2064)
b_2	-0.3465*** (0.0254)	-0.3749*** (0.0339)	-0.1739*** (0.0142)	-0.2831*** (0.0680)	-0.3262*** (0.0541)	-0.3157*** (0.0348)	na	-0.1215 (0.7080)
Adjusted Labor (HL)								
α_1	na	0.2978*** (0.0012)	0.3510*** (0.0009)	0.2787*** (0.0012)	0.3537*** (0.0013)	na	0.3255*** (0.0032)	0.2427*** (0.0018)
α_2	na	0.9965*** (0.0800)	0.5722*** (0.0493)	0.3230*** (0.0888)	0.9602*** (0.0746)	na	0.1466 (0.1364)	0.2950* (0.1524)
b_2	na	-0.3065*** (0.0354)	-0.1757*** (0.0150)	-0.3113*** (0.0853)	-0.3160*** (0.0786)	na	1.4993 (2.7221)	-0.2472 (0.2488)
θ		✓		✓		✓		✓
δ		✓		✓		✓		✓
R^2 (for L)	0.9989	0.9973	0.9978	0.9955	0.9973	0.9956	na	0.9957
R^2 (for HL)	na	0.9974	0.9979	0.9960	0.9974	na	0.9992	0.9968
N	1120	1120	800	800	480	480	512	512

Note: The terms in parentheses are standard errors. ***, **, and * display the level of significance at 1%, 5% and 10% respectively. ✓ shows that time and regional fixed effects dummies are included in the regression equations.

CHAPTER 8: CONCLUSION

This thesis studied the relationship between public capital stock, private capital stock, and economic growth by the help of a VES type nested production function. Actually, the interaction between public and private capital stocks is examined under two different models. First, we consider a three-factor two-level aggregate production function with inputs public capital (K_g), private capital (K_p) and labor (L). In this model, the first and the second level of two-level aggregate production function is a VES type production function. In other words, we consider variable elasticity of substitution between production inputs in the first and the second level. Second, the same relationship is examined by modifying the first level of the aggregate production function to have a constant elasticity of substitution between capital and labor.

We report the estimates of the parameters of nested VES aggregate production function for 91 countries with input factors real GDP, public capital, private capital, total factor productivity, and labor (with raw or unadjusted labor, and human capital adjusted labor). Moreover, in order to make a comparison with the existing literature, we estimate the model for aggregate capital stock. We used a panel of data set for the period 1980-2011 to estimate the parameters. Our dataset comes from two main sources: public and private capital stocks data comes from IMF database, and the data on the rest of the variables come from Penn World Tables version 8.1. We classify the countries by income level, by geographical regions, and by human development index. We adopt non-linear estimation technique using Stata12 for estimations.

The thesis examines how the parameter *b*, which is the determinant of the elasticity of substitutional between public capital and private capital stock, affects economic growth dynamics. The elasticity of substitution between these two production factors was estimated at the global level including all countries in the dataset, and for each group of countries subject to different classifications.

For the sensitivity of the findings, the countries are classified into three categories according to their income level, geographical region, and human development index. The parameters for each category are estimated for both raw labor (unadjusted), and human capital adjusted labor. Moreover, we estimate the elasticity of substitution for each country in each classification[11].

The results of the study show that the parameter of elasticity of substitution for the two nested structures of the two-level VES production function has the satisfactory goodness of fit and relatively high R^2 values. The goodness of fit and R^2 values does not substantially change when we use raw labor or human capital adjusted labor. The estimates for the elasticity of substitution parameter have comparatively low standard errors, which means there is not high variation in estimates of substitution parameter.

There is no consensus in the literature regarding the substitutability and complementarity between public capital stock and private capital stock. Studies regarding substitution elasticity show that the results changes from country to country. More importantly, even the studies on the same country can reach to different results. The main reason for these diverse results is the theoretical and econometric models used, and the chosen time-period.

In general, when we consider the elasticity of substitution between aggregate capital and labor, the results of the first model shows that the substitution parameter (b1) is positive and very small in magnitude but highly significant for all country classifications. These results indicate a nearly constant elasticity of substitution between aggregate capital and labor. More specifically, regardless of the country classification, the very low value of b1 signs a Cobb-Douglas case between aggregate capital and labor. Similarly when we consider elasticity of substitution between public capital stock and private capital stock the substitution parameter (b2) is negative and significant in most cases. The

[11] We only report estimates for income level. See the appendix for time graph of elasticity of substitution for individual countries.

results do not fundamentally change when we consider country classification by income, region or human development index (HDI).

In the second model (i.e. in case of Cobb-Douglas production function in the first stage) in general, the elasticity of substitution parameter increases in absolute magnitude for the country classifications by income and region. However, the negative value of the elasticity of substitution parameter shows the complementarity between public capital and private capital in the final output production.

The elasticity of substitution between public capital and private capital is about 0.8820 at the global level. The substitution elasticity takes it highest value in upper-middle-income countries and lowest value in high-income-OECD countries. The average value of substitution elasticity does not significantly change when we consider adjusted labor as a measure of labor input.

In the literature, it is emphasized that public investment, especially investments related to infrastructures, may complement private investments. A public investment that targeted to improve infrastructure facilities may increase the efficiency of private capital, and thereby improve private investment opportunities. In this case, an increase in the public capital stock leads to an increase in private capital investments. Therefore, the positive contribution of private capital stock to economic growth and its sustainability will be possible through properly directed public policies. The public sector may have a positive impact on economic growth by giving importance and priority to investments that boost private sector productivity. In short, it is important to know the behavior of the substitution elasticity in every stage of economic development to determine effective public policies.

Since the industrial level data on public and private capital stock is not available for many countries, we used to aggregate data at the country level to analyze the behavior of elasticity of substitution. Calculation of elasticity of substitution for more disaggregated data (data at the industry level, for example) will yield more appropriate results. For this reason, the calculated substitution elasticity coefficient needs to be evaluated cautiously. Although the aggregated data used in this thesis gives meaningful inference about substitution elasticity, it is possible to calculate a more exact substitution elasticity at the industry level by using a more disaggregated data set.

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APPENDIX 1. Estimated Elasticity of Substitution

The table below shows the elasticity of substitution calculated for countries classified by income level. We use the formula $\sigma(K_g, K_p) = 1 + b_2(\frac{K_g}{K_p})$ to calculate the value of elasticity of substitution between public capital and private capital stocks. The b_2 coefficient is obtained from the estimation that includes time and regional fixed effects, and human capital adjusted labor (HL) in place of labor input.

Country Classification by Income: Low Income Economies					
Country	**Year**	**σ**	**Country**	**Year**	**σ**
Benin	1980	-0,47575748	Rwanda	1996	0,320462584
Benin	1981	-0,473047376	Rwanda	1997	0,295080334
Benin	1982	-0,46584639	Rwanda	1998	0,274988502
Benin	1983	-0,392717123	Rwanda	1999	0,277065247
Benin	1984	-0,343497157	Rwanda	2000	0,297830373
Benin	1985	-0,327904791	Rwanda	2001	0,322740048
Benin	1986	-0,293479174	Rwanda	2002	0,343006581
Benin	1987	-0,27196306	Rwanda	2003	0,373063147
Benin	1988	-0,255846441	Rwanda	2004	0,401491314
Benin	1989	-0,245543703	Rwanda	2005	0,414533347
Benin	1990	-0,239408344	Rwanda	2006	0,422052085
Benin	1991	-0,184260935	Rwanda	2007	0,440755486
Benin	1992	-0,125031844	Rwanda	2008	0,454139948
Benin	1993	-0,054982405	Rwanda	2009	0,449816644
Benin	1994	0,004761112	Rwanda	2010	0,448938578
Benin	1995	0,041160312	Rwanda	2011	0,432216227
Benin	1996	0,075746365	Senegal	1980	0,711745203
Benin	1997	0,112071984	Senegal	1981	0,701421499
Benin	1998	0,151055768	Senegal	1982	0,702186584
Benin	1999	0,195164174	Senegal	1983	0,708239436
Benin	2000	0,232528329	Senegal	1984	0,712908149
Benin	2001	0,25475508	Senegal	1985	0,716345906
Benin	2002	0,274778634	Senegal	1986	0,718173504
Benin	2003	0,303145498	Senegal	1987	0,712739527
Benin	2004	0,326599985	Senegal	1988	0,709470689
Benin	2005	0,352983683	Senegal	1989	0,708343029
Benin	2006	0,371543676	Senegal	1990	0,706053197
Benin	2007	0,40100342	Senegal	1991	0,704731822
Benin	2008	0,409748435	Senegal	1992	0,703113258
Benin	2009	0,429113328	Senegal	1993	0,699108958
Benin	2010	0,421189338	Senegal	1994	0,699792027
Benin	2011	0,437560111	Senegal	1995	0,705637097
Burundi	1980	0,873687625	Senegal	1996	0,707615197
Burundi	1981	0,872565627	Senegal	1997	0,704622865
Burundi	1982	0,873450875	Senegal	1998	0,70261389
Burundi	1983	0,87656647	Senegal	1999	0,70486784
Burundi	1984	0,877046227	Senegal	2000	0,701760054
Burundi	1985	0,877510369	Senegal	2001	0,707334697
Burundi	1986	0,878208041	Senegal	2002	0,709663868
Burundi	1987	0,878889382	Senegal	2003	0,710677147
Burundi	1988	0,8754251	Senegal	2004	0,711426616
Burundi	1989	0,876546085	Senegal	2005	0,711027861

Burundi	1990	0,87445128	Senegal	2006	0,708683252
Burundi	1991	0,872096658	Senegal	2007	0,70836091
Burundi	1992	0,869657815	Senegal	2008	0,707674384
Burundi	1993	0,866343915	Senegal	2009	0,709383547
Burundi	1994	0,863459527	Senegal	2010	0,703029156
Burundi	1995	0,86029458	Senegal	2011	0,694494486
Burundi	1996	0,856795371	Sierra Leone	1980	0,549368799
Burundi	1997	0,853477776	Sierra Leone	1981	0,551900387
Burundi	1998	0,852732182	Sierra Leone	1982	0,553015292
Burundi	1999	0,853113651	Sierra Leone	1983	0,543037772
Burundi	2000	0,850489199	Sierra Leone	1984	0,542945027
Burundi	2001	0,849288106	Sierra Leone	1985	0,546760082
Burundi	2002	0,848596513	Sierra Leone	1986	0,548919678
Burundi	2003	0,848877013	Sierra Leone	1987	0,544987202
Burundi	2004	0,833042562	Sierra Leone	1988	0,540480733
Burundi	2005	0,80802983	Sierra Leone	1989	0,545978665
Burundi	2006	0,805346787	Sierra Leone	1990	0,548397422
Burundi	2007	0,796702385	Sierra Leone	1991	0,542164028
Burundi	2008	0,775210142	Sierra Leone	1992	0,524900913
Burundi	2009	0,752006531	Sierra Leone	1993	0,497046411
Burundi	2010	0,734397531	Sierra Leone	1994	0,475533724
Burundi	2011	0,720671356	Sierra Leone	1995	0,459882915
Central African Republic	1980	0,011073072	Sierra Leone	1996	0,450490743
Central African Republic	1981	-0,001204752	Sierra Leone	1997	0,453653336
Central African Republic	1982	-0,011575001	Sierra Leone	1998	0,439159036
Central African Republic	1983	-0,024912192	Sierra Leone	1999	0,410340548
Central African Republic	1984	-0,034200247	Sierra Leone	2000	0,398275018
Central African Republic	1985	-0,040310789	Sierra Leone	2001	0,369645357
Central African Republic	1986	-0,043775819	Sierra Leone	2002	0,371688485
Central African Republic	1987	-0,059246335	Sierra Leone	2003	0,377844244
Central African Republic	1988	-0,065210462	Sierra Leone	2004	0,386902004
Central African Republic	1989	-0,08405818	Sierra Leone	2005	0,393255532
Central African Republic	1990	-0,096353896	Sierra Leone	2006	0,398503333
Central African Republic	1991	-0,091699332	Sierra Leone	2007	0,401918054
Central African Republic	1992	-0,104846627	Sierra Leone	2008	0,417365968
Central African Republic	1993	-0,119538963	Sierra Leone	2009	0,40575242
Central African Republic	1994	-0,139864549	Sierra Leone	2010	0,381767273
Central African Republic	1995	-0,154827923	Sierra Leone	2011	0,429051369
Central African Republic	1996	-0,153451636	Togo	1980	-0,030951332
Central African Republic	1997	-0,151856795	Togo	1981	-0,091067091
Central African Republic	1998	-0,16315119	Togo	1982	-0,134358332
Central African	1999	-0,184465364	Togo	1983	-0,174813375

Republic					
Central African Republic	2000	-0,197515845	Togo	1984	-0,211577103
Central African Republic	2001	-0,191198602	Togo	1985	-0,240753189
Central African Republic	2002	-0,175641119	Togo	1986	-0,277251095
Central African Republic	2003	-0,178258315	Togo	1987	-0,318379104
Central African Republic	2004	-0,171978876	Togo	1988	-0,302336007
Central African Republic	2005	-0,169402376	Togo	1989	-0,340131491
Central African Republic	2006	-0,173175544	Togo	1990	-0,22815837
Central African Republic	2007	-0,16749543	Togo	1991	-0,182600379
Central African Republic	2008	-0,142129928	Togo	1992	-0,142986953
Central African Republic	2009	-0,107506	Togo	1993	-0,110248998
Central African Republic	2010	-0,082481124	Togo	1994	-0,111117877
Central African Republic	2011	-0,063460045	Togo	1995	-0,084546
Mozambique	1980	0,389893681	Togo	1996	-0,048713662
Mozambique	1981	0,38951245	Togo	1997	-0,009810043
Mozambique	1982	0,388790816	Togo	1998	0,025394557
Mozambique	1983	0,387783438	Togo	1999	0,066000804
Mozambique	1984	0,381413728	Togo	2000	0,081522152
Mozambique	1985	0,411116779	Togo	2001	0,121441878
Mozambique	1986	0,39027673	Togo	2002	0,165067434
Mozambique	1987	0,37554577	Togo	2003	0,216042891
Mozambique	1988	0,365702242	Togo	2004	0,269182771
Mozambique	1989	0,361270338	Togo	2005	0,311187476
Mozambique	1990	0,361768514	Togo	2006	0,343027204
Mozambique	1991	0,385593653	Togo	2007	0,370870262
Mozambique	1992	0,304979086	Togo	2008	0,399173409
Mozambique	1993	0,234405696	Togo	2009	0,419146061
Mozambique	1994	0,163981006	Togo	2010	0,42812717
Mozambique	1995	0,063079514	Togo	2011	0,428084493
Mozambique	1996	0,029377809	United Republic of Tanzania: Mainland	1980	0,690223992
Mozambique	1997	0,003655625	United Republic of Tanzania: Mainland	1981	0,688319266
Mozambique	1998	-0,037866376	United Republic of Tanzania: Mainland	1982	0,685787201
Mozambique	1999	-0,056373227	United Republic of Tanzania: Mainland	1983	0,682518065
Mozambique	2000	-0,089647442	United Republic of Tanzania: Mainland	1984	0,678778708
Mozambique	2001	0,024384821	United Republic of Tanzania: Mainland	1985	0,674980998
Mozambique	2002	-0,077635258	United Republic of Tanzania: Mainland	1986	0,671696424
Mozambique	2003	-0,003834054	United Republic of Tanzania:	1987	0,669784725

			Mainland		
Mozambique	2004	0,002842031	United Republic of Tanzania: Mainland	1988	0,667919636
Mozambique	2005	0,004031378	United Republic of Tanzania: Mainland	1989	0,666677773
Mozambique	2006	0,035664774	United Republic of Tanzania: Mainland	1990	0,667939425
Mozambique	2007	-0,020807771	United Republic of Tanzania: Mainland	1991	0,674663842
Mozambique	2008	-0,104051925	United Republic of Tanzania: Mainland	1992	0,680099189
Mozambique	2009	-0,166832134	United Republic of Tanzania: Mainland	1993	0,686035573
Mozambique	2010	-0,304027557	United Republic of Tanzania: Mainland	1994	0,694737673
Mozambique	2011	-0,360688448	United Republic of Tanzania: Mainland	1995	0,704082072
Niger	1980	0,889548123	United Republic of Tanzania: Mainland	1996	0,711652756
Niger	1981	0,883277237	United Republic of Tanzania: Mainland	1997	0,718156576
Niger	1982	0,878096759	United Republic of Tanzania: Mainland	1998	0,725197256
Niger	1983	0,874188066	United Republic of Tanzania: Mainland	1999	0,732563853
Niger	1984	0,870670021	United Republic of Tanzania: Mainland	2000	0,732492328
Niger	1985	0,865853429	United Republic of Tanzania: Mainland	2001	0,737295628
Niger	1986	0,862840533	United Republic of Tanzania: Mainland	2002	0,738939881
Niger	1987	0,86302352	United Republic of Tanzania: Mainland	2003	0,738840818
Niger	1988	0,864479542	United Republic of Tanzania: Mainland	2004	0,738447428
Niger	1989	0,856790304	United Republic of Tanzania: Mainland	2005	0,740306258
Niger	1990	0,850969195	United Republic of Tanzania: Mainland	2006	0,745308399
Niger	1991	0,834174097	United Republic of Tanzania: Mainland	2007	0,747148514
Niger	1992	0,823782206	United Republic of Tanzania: Mainland	2008	0,744717717
Niger	1993	0,806048691	United Republic of Tanzania: Mainland	2009	0,739522815
Niger	1994	0,786782801	United Republic of Tanzania: Mainland	2010	0,739884675
Niger	1995	0,76258117	United Republic	2011	0,691683888

			of Tanzania: Mainland		
Niger	1996	0,73927927	Zimbabwe	1980	0,836537838
Niger	1997	0,725854635	Zimbabwe	1981	0,835592508
Niger	1998	0,709407032	Zimbabwe	1982	0,835368633
Niger	1999	0,689536035	Zimbabwe	1983	0,835315645
Niger	2000	0,666772962	Zimbabwe	1984	0,835101664
Niger	2001	0,647045493	Zimbabwe	1985	0,834329784
Niger	2002	0,623380363	Zimbabwe	1986	0,833164871
Niger	2003	0,595817804	Zimbabwe	1987	0,832146943
Niger	2004	0,598093987	Zimbabwe	1988	0,83126843
Niger	2005	0,597304285	Zimbabwe	1989	0,830642164
Niger	2006	0,598415136	Zimbabwe	1990	0,829972863
Niger	2007	0,602402508	Zimbabwe	1991	0,830089092
Niger	2008	0,608438432	Zimbabwe	1992	0,830775082
Niger	2009	0,623663127	Zimbabwe	1993	0,83099097
Niger	2010	0,635573804	Zimbabwe	1994	0,83130759
Niger	2011	0,655089557	Zimbabwe	1995	0,831602156
Rwanda	1980	0,416782439	Zimbabwe	1996	0,831852853
Rwanda	1981	0,441668719	Zimbabwe	1997	0,831082046
Rwanda	1982	0,465752363	Zimbabwe	1998	0,830250978
Rwanda	1983	0,446971178	Zimbabwe	1999	0,830086887
Rwanda	1984	0,429468364	Zimbabwe	2000	0,829519808
Rwanda	1985	0,400048137	Zimbabwe	2001	0,830360591
Rwanda	1986	0,382563889	Zimbabwe	2002	0,831091404
Rwanda	1987	0,373389572	Zimbabwe	2003	0,832264066
Rwanda	1988	0,36568138	Zimbabwe	2004	0,83279109
Rwanda	1989	0,369319439	Zimbabwe	2005	0,831372321
Rwanda	1990	0,378689677	Zimbabwe	2006	0,829502583
Rwanda	1991	0,390578032	Zimbabwe	2007	0,82759434
Rwanda	1992	0,375246972	Zimbabwe	2008	0,825873673
Rwanda	1993	0,338395476	Zimbabwe	2009	0,823967636
Rwanda	1994	0,339409769	Zimbabwe	2010	0,82450211
Rwanda	1995	0,338955581	Zimbabwe	2011	0,822056949

Country Classification by Income: Lower-Middle-Income Economies					
Country	**Year**	**σ**	**Country**	**Year**	**σ**
Bolivia	1980	0,83103013	Kenya	1980	0,838197231
Bolivia	1981	0,829453945	Kenya	1981	0,83607322
Bolivia	1982	0,826996624	Kenya	1982	0,834406614
Bolivia	1983	0,823662281	Kenya	1983	0,833067596
Bolivia	1984	0,8214674	Kenya	1984	0,83320564
Bolivia	1985	0,819086611	Kenya	1985	0,833321691
Bolivia	1986	0,81682241	Kenya	1986	0,833426297
Bolivia	1987	0,816314876	Kenya	1987	0,833059669
Bolivia	1988	0,81838572	Kenya	1988	0,834172368
Bolivia	1989	0,817416072	Kenya	1989	0,834043562
Bolivia	1990	0,813920856	Kenya	1990	0,835218906
Bolivia	1991	0,812008202	Kenya	1991	0,833278418
Bolivia	1992	0,809838057	Kenya	1992	0,831993341
Bolivia	1993	0,806689262	Kenya	1993	0,830853343
Bolivia	1994	0,803625822	Kenya	1994	0,829909742
Bolivia	1995	0,795783281	Kenya	1995	0,827970982
Bolivia	1996	0,789669454	Kenya	1996	0,829725385
Bolivia	1997	0,78334862	Kenya	1997	0,83097434
Bolivia	1998	0,787019491	Kenya	1998	0,832389355
Bolivia	1999	0,798691988	Kenya	1999	0,83530736
Bolivia	2000	0,799792767	Kenya	2000	0,838536501
Bolivia	2001	0,800596535	Kenya	2001	0,842449009
Bolivia	2002	0,793506145	Kenya	2002	0,846716106
Bolivia	2003	0,788953543	Kenya	2003	0,84990567
Bolivia	2004	0,780619919	Kenya	2004	0,851955175
Bolivia	2005	0,762657046	Kenya	2005	0,854216456

Bolivia	2006	0,746525705	Kenya	2006	0,85798347
Bolivia	2007	0,726816714	Kenya	2007	0,86283648
Bolivia	2008	0,70474118	Kenya	2008	0,866408706
Bolivia	2009	0,683383107	Kenya	2009	0,867216945
Bolivia	2010	0,66135335	Kenya	2010	0,864347756
Bolivia	2011	0,64046067	Kenya	2011	0,858788788
Cameroon	1980	0,943412483	Lesotho	1980	-0,286669165
Cameroon	1981	0,94323951	Lesotho	1981	-0,330169708
Cameroon	1982	0,943799853	Lesotho	1982	-0,567634821
Cameroon	1983	0,944409907	Lesotho	1983	-0,826827765
Cameroon	1984	0,945170939	Lesotho	1984	-0,924791217
Cameroon	1985	0,946262538	Lesotho	1985	-1,102894783
Cameroon	1986	0,947393656	Lesotho	1986	-1,397979259
Cameroon	1987	0,948564053	Lesotho	1987	-1,268794298
Cameroon	1988	0,949512601	Lesotho	1988	-1,021383524
Cameroon	1989	0,950213194	Lesotho	1989	-0,988291621
Cameroon	1990	0,950422823	Lesotho	1990	-1,015289307
Cameroon	1991	0,950410187	Lesotho	1991	-1,186758399
Cameroon	1992	0,950004458	Lesotho	1992	-0,869666874
Cameroon	1993	0,949241042	Lesotho	1993	-0,488937438
Cameroon	1994	0,948376596	Lesotho	1994	-0,336371899
Cameroon	1995	0,947363853	Lesotho	1995	-0,184155613
Cameroon	1996	0,946345627	Lesotho	1996	-0,071386032
Cameroon	1997	0,945209086	Lesotho	1997	-0,070005655
Cameroon	1998	0,944249868	Lesotho	1998	-0,146332994
Cameroon	1999	0,943565905	Lesotho	1999	-0,102830604
Cameroon	2000	0,942974448	Lesotho	2000	-0,028250068
Cameroon	2001	0,942305624	Lesotho	2001	0,035198908
Cameroon	2002	0,939511657	Lesotho	2002	0,075746536
Cameroon	2003	0,939618707	Lesotho	2003	0,105553262
Cameroon	2004	0,939446926	Lesotho	2004	0,159113526
Cameroon	2005	0,939862669	Lesotho	2005	0,212314665
Cameroon	2006	0,937983632	Lesotho	2006	0,255026221
Cameroon	2007	0,934963882	Lesotho	2007	0,300838113
Cameroon	2008	0,929879904	Lesotho	2008	0,346573591
Cameroon	2009	0,922664464	Lesotho	2009	0,393960863
Cameroon	2010	0,917017817	Lesotho	2010	0,416192055
Cameroon	2011	0,912684679	Lesotho	2011	0,429455966
Côte d'Ivoire	1980	0,717806101	Mauritania	1980	0,892972887
Côte d'Ivoire	1981	0,694863737	Mauritania	1981	0,891503036
Côte d'Ivoire	1982	0,673260987	Mauritania	1982	0,890388489
Côte d'Ivoire	1983	0,653546512	Mauritania	1983	0,889342666
Côte d'Ivoire	1984	0,632257402	Mauritania	1984	0,888302982
Côte d'Ivoire	1985	0,599037766	Mauritania	1985	0,88773489
Côte d'Ivoire	1986	0,56605351	Mauritania	1986	0,886907935
Côte d'Ivoire	1987	0,564193964	Mauritania	1987	0,885641634
Côte d'Ivoire	1988	0,551610827	Mauritania	1988	0,884209871
Côte d'Ivoire	1989	0,540716708	Mauritania	1989	0,882353485
Côte d'Ivoire	1990	0,52835089	Mauritania	1990	0,880237818
Côte d'Ivoire	1991	0,515428245	Mauritania	1991	0,8731938
Côte d'Ivoire	1992	0,503958642	Mauritania	1992	0,872232258
Côte d'Ivoire	1993	0,488318384	Mauritania	1993	0,873409212
Côte d'Ivoire	1994	0,451491535	Mauritania	1994	0,871084273
Côte d'Ivoire	1995	0,449250668	Mauritania	1995	0,874284387
Côte d'Ivoire	1996	0,45834586	Mauritania	1996	0,875058651
Côte d'Ivoire	1997	0,477483094	Mauritania	1997	0,87352699
Côte d'Ivoire	1998	0,489841282	Mauritania	1998	0,871290565
Côte d'Ivoire	1999	0,501911402	Mauritania	1999	0,870211363
Côte d'Ivoire	2000	0,524676979	Mauritania	2000	0,866054714
Côte d'Ivoire	2001	0,528837264	Mauritania	2001	0,863511443
Côte d'Ivoire	2002	0,534162104	Mauritania	2002	0,857776642
Côte d'Ivoire	2003	0,533995867	Mauritania	2003	0,851733506
Côte d'Ivoire	2004	0,533034146	Mauritania	2004	0,84870255
Côte d'Ivoire	2005	0,532409728	Mauritania	2005	0,8633219
Côte d'Ivoire	2006	0,530988514	Mauritania	2006	0,887863457
Côte d'Ivoire	2007	0,526080191	Mauritania	2007	0,89020586
Côte d'Ivoire	2008	0,523588121	Mauritania	2008	0,875764549
Côte d'Ivoire	2009	0,523901045	Mauritania	2009	0,870002806
Côte d'Ivoire	2010	0,522189617	Mauritania	2010	0,855764747

Côte d'Ivoire	2011	0,521316588	Mauritania	2011	0,850298285
Egypt	1980	0,563366532	Mongolia	1980	0,357975543
Egypt	1981	0,569433391	Mongolia	1981	0,354182363
Egypt	1982	0,57366991	Mongolia	1982	0,347546488
Egypt	1983	0,577582359	Mongolia	1983	0,350612044
Egypt	1984	0,578335762	Mongolia	1984	0,35533902
Egypt	1985	0,577144444	Mongolia	1985	0,352712154
Egypt	1986	0,57495892	Mongolia	1986	0,338087887
Egypt	1987	0,571788371	Mongolia	1987	0,338360667
Egypt	1988	0,59323132	Mongolia	1988	0,333528161
Egypt	1989	0,6019876	Mongolia	1989	0,340806961
Egypt	1990	0,623137832	Mongolia	1990	0,366441339
Egypt	1991	0,633267224	Mongolia	1991	0,347150475
Egypt	1992	0,64365834	Mongolia	1992	0,432789862
Egypt	1993	0,647695005	Mongolia	1993	0,467159599
Egypt	1994	0,648883581	Mongolia	1994	0,484249562
Egypt	1995	0,659647465	Mongolia	1995	0,510132849
Egypt	1996	0,671593547	Mongolia	1996	0,508632302
Egypt	1997	0,682476342	Mongolia	1997	0,523449183
Egypt	1998	0,698930502	Mongolia	1998	0,565997958
Egypt	1999	0,719709158	Mongolia	1999	0,599430978
Egypt	2000	0,732688427	Mongolia	2000	0,623538375
Egypt	2001	0,746225834	Mongolia	2001	0,641843855
Egypt	2002	0,758380473	Mongolia	2002	0,655222178
Egypt	2003	0,7636168	Mongolia	2003	0,663441062
Egypt	2004	0,766443074	Mongolia	2004	0,678079307
Egypt	2005	0,770051062	Mongolia	2005	0,692375898
Egypt	2006	0,778470814	Mongolia	2006	0,709599674
Egypt	2007	0,792399108	Mongolia	2007	0,731504917
Egypt	2008	0,809159458	Mongolia	2008	0,742341101
Egypt	2009	0,823943794	Mongolia	2009	0,754943848
Egypt	2010	0,829025805	Mongolia	2010	0,76016438
Egypt	2011	0,834542632	Mongolia	2011	0,771802843
Guatemala	1980	0,908938408	Morocco	1980	0,911825657
Guatemala	1981	0,908504188	Morocco	1981	0,909090817
Guatemala	1982	0,908123493	Morocco	1982	0,904636621
Guatemala	1983	0,907429218	Morocco	1983	0,900902152
Guatemala	1984	0,906173706	Morocco	1984	0,898912907
Guatemala	1985	0,90477705	Morocco	1985	0,898315489
Guatemala	1986	0,903283775	Morocco	1986	0,897689521
Guatemala	1987	0,901833296	Morocco	1987	0,897116423
Guatemala	1988	0,900618553	Morocco	1988	0,897669196
Guatemala	1989	0,899609149	Morocco	1989	0,898118377
Guatemala	1990	0,898694336	Morocco	1990	0,900336027
Guatemala	1991	0,897505462	Morocco	1991	0,902956545
Guatemala	1992	0,896367431	Morocco	1992	0,905546486
Guatemala	1993	0,895846486	Morocco	1993	0,907738626
Guatemala	1994	0,895418644	Morocco	1994	0,908142626
Guatemala	1995	0,894801915	Morocco	1995	0,908448756
Guatemala	1996	0,894339204	Morocco	1996	0,908175111
Guatemala	1997	0,893699229	Morocco	1997	0,909023762
Guatemala	1998	0,893625498	Morocco	1998	0,909972429
Guatemala	1999	0,893952429	Morocco	1999	0,911921203
Guatemala	2000	0,894218385	Morocco	2000	0,914016783
Guatemala	2001	0,893879592	Morocco	2001	0,915746868
Guatemala	2002	0,892071903	Morocco	2002	0,917164207
Guatemala	2003	0,889792025	Morocco	2003	0,91760689
Guatemala	2004	0,888782978	Morocco	2004	0,918962181
Guatemala	2005	0,890374482	Morocco	2005	0,920764267
Guatemala	2006	0,89157176	Morocco	2006	0,922596157
Guatemala	2007	0,893081009	Morocco	2007	0,924802005
Guatemala	2008	0,892863631	Morocco	2008	0,927345753
Guatemala	2009	0,892569125	Morocco	2009	0,928777874
Guatemala	2010	0,88795656	Morocco	2010	0,928435385
Guatemala	2011	0,886707842	Morocco	2011	0,928780496
Honduras	1980	0,088839009	Philippines	1980	0,938415945
Honduras	1981	0,045262232	Philippines	1981	0,938565195
Honduras	1982	0,043446682	Philippines	1982	0,93873316
Honduras	1983	0,053634599	Philippines	1983	0,938805819

Honduras	1984	0,06841965	Philippines	1984	0,938850105
Honduras	1985	0,065790839	Philippines	1985	0,938300014
Honduras	1986	0,081141755	Philippines	1986	0,937338352
Honduras	1987	0,112434402	Philippines	1987	0,935278833
Honduras	1988	0,156089708	Philippines	1988	0,93530488
Honduras	1989	0,218166664	Philippines	1989	0,935537696
Honduras	1990	0,286115468	Philippines	1990	0,935408354
Honduras	1991	0,332033038	Philippines	1991	0,93371588
Honduras	1992	0,362582922	Philippines	1992	0,931147277
Honduras	1993	0,3876293	Philippines	1993	0,928225994
Honduras	1994	0,413853109	Philippines	1994	0,925806463
Honduras	1995	0,450199097	Philippines	1995	0,924311519
Honduras	1996	0,465470999	Philippines	1996	0,918010473
Honduras	1997	0,500389338	Philippines	1997	0,91785115
Honduras	1998	0,542981207	Philippines	1998	0,91708988
Honduras	1999	0,582272947	Philippines	1999	0,915736616
Honduras	2000	0,613737345	Philippines	2000	0,913852513
Honduras	2001	0,643206358	Philippines	2001	0,91336745
Honduras	2002	0,663982451	Philippines	2002	0,913745403
Honduras	2003	0,685410619	Philippines	2003	0,913592696
Honduras	2004	0,700080752	Philippines	2004	0,914970815
Honduras	2005	0,716804266	Philippines	2005	0,9164536
Honduras	2006	0,733595371	Philippines	2006	0,917683363
Honduras	2007	0,753449202	Philippines	2007	0,919247389
Honduras	2008	0,77676481	Philippines	2008	0,919691503
Honduras	2009	0,796057761	Philippines	2009	0,919053793
Honduras	2010	0,800202787	Philippines	2010	0,916865289
Honduras	2011	0,806227148	Philippines	2011	0,91595912
India	1980	0,757363915	Sri Lanka	1980	0,833302617
India	1981	0,746895611	Sri Lanka	1981	0,839472175
India	1982	0,735333085	Sri Lanka	1982	0,848333418
India	1983	0,719493508	Sri Lanka	1983	0,856555521
India	1984	0,700417817	Sri Lanka	1984	0,862446487
India	1985	0,679789245	Sri Lanka	1985	0,865949988
India	1986	0,658851326	Sri Lanka	1986	0,867594361
India	1987	0,632044673	Sri Lanka	1987	0,867497742
India	1988	0,624485135	Sri Lanka	1988	0,866597593
India	1989	0,61808002	Sri Lanka	1989	0,86484313
India	1990	0,620090723	Sri Lanka	1990	0,862663507
India	1991	0,630035639	Sri Lanka	1991	0,864500701
India	1992	0,627960443	Sri Lanka	1992	0,866013765
India	1993	0,633790076	Sri Lanka	1993	0,869938374
India	1994	0,641977131	Sri Lanka	1994	0,872863352
India	1995	0,647166073	Sri Lanka	1995	0,878230691
India	1996	0,66090709	Sri Lanka	1996	0,881507218
India	1997	0,679546535	Sri Lanka	1997	0,885135531
India	1998	0,696503162	Sri Lanka	1998	0,888017297
India	1999	0,711228728	Sri Lanka	1999	0,891314149
India	2000	0,724894404	Sri Lanka	2000	0,89491123
India	2001	0,734786153	Sri Lanka	2001	0,898359001
India	2002	0,744191408	Sri Lanka	2002	0,899496198
India	2003	0,752877951	Sri Lanka	2003	0,901382387
India	2004	0,763389945	Sri Lanka	2004	0,903339803
India	2005	0,777600765	Sri Lanka	2005	0,906803727
India	2006	0,790504515	Sri Lanka	2006	0,907043993
India	2007	0,800295949	Sri Lanka	2007	0,90865612
India	2008	0,810457766	Sri Lanka	2008	0,907258213
India	2009	0,814865351	Sri Lanka	2009	0,903248191
India	2010	0,819621027	Sri Lanka	2010	0,897372246
India	2011	0,824612319	Sri Lanka	2011	0,894807935
Indonesia	1980	0,926438928	Swaziland	1980	0,901026249
Indonesia	1981	0,926794529	Swaziland	1981	0,900074542
Indonesia	1982	0,927101672	Swaziland	1982	0,896877408
Indonesia	1983	0,927395761	Swaziland	1983	0,893227339
Indonesia	1984	0,927505016	Swaziland	1984	0,893895566
Indonesia	1985	0,927296758	Swaziland	1985	0,894172728
Indonesia	1986	0,927073181	Swaziland	1986	0,889922917
Indonesia	1987	0,926868081	Swaziland	1987	0,884280384
Indonesia	1988	0,926615238	Swaziland	1988	0,88215369

Indonesia	1989	0,926424325	Swaziland	1989	0,886316836
Indonesia	1990	0,926344037	Swaziland	1990	0,889060855
Indonesia	1991	0,926375747	Swaziland	1991	0,887816131
Indonesia	1992	0,926434875	Swaziland	1992	0,884360611
Indonesia	1993	0,926325262	Swaziland	1993	0,879860699
Indonesia	1994	0,926145375	Swaziland	1994	0,877099454
Indonesia	1995	0,926046133	Swaziland	1995	0,87314111
Indonesia	1996	0,926014721	Swaziland	1996	0,872822225
Indonesia	1997	0,926043928	Swaziland	1997	0,873229325
Indonesia	1998	0,925989866	Swaziland	1998	0,873878062
Indonesia	1999	0,924934387	Swaziland	1999	0,873854876
Indonesia	2000	0,923271358	Swaziland	2000	0,870239437
Indonesia	2001	0,924360216	Swaziland	2001	0,867152512
Indonesia	2002	0,922992647	Swaziland	2002	0,864921391
Indonesia	2003	0,922124743	Swaziland	2003	0,860726714
Indonesia	2004	0,918849885	Swaziland	2004	0,856747091
Indonesia	2005	0,91816926	Swaziland	2005	0,818118751
Indonesia	2006	0,918834865	Swaziland	2006	0,810733318
Indonesia	2007	0,91717422	Swaziland	2007	0,803178966
Indonesia	2008	0,917090952	Swaziland	2008	0,793378651
Indonesia	2009	0,918701053	Swaziland	2009	0,775189757
Indonesia	2010	0,921200454	Swaziland	2010	0,760352314
Indonesia	2011	0,92459029	Swaziland	2011	0,747118771
Jordan	1980	0,841241956	Tunisia	1980	0,929456234
Jordan	1981	0,819334567	Tunisia	1981	0,931480885
Jordan	1982	0,792762756	Tunisia	1982	0,929517806
Jordan	1983	0,770780444	Tunisia	1983	0,927031934
Jordan	1984	0,764365554	Tunisia	1984	0,927841663
Jordan	1985	0,763010323	Tunisia	1985	0,927669764
Jordan	1986	0,754029334	Tunisia	1986	0,927051008
Jordan	1987	0,741915345	Tunisia	1987	0,92402035
Jordan	1988	0,726265848	Tunisia	1988	0,922204554
Jordan	1989	0,720372915	Tunisia	1989	0,918494761
Jordan	1990	0,720046043	Tunisia	1990	0,916746497
Jordan	1991	0,730898023	Tunisia	1991	0,915709257
Jordan	1992	0,736204088	Tunisia	1992	0,913974941
Jordan	1993	0,750923753	Tunisia	1993	0,913630724
Jordan	1994	0,767178357	Tunisia	1994	0,913137674
Jordan	1995	0,780321121	Tunisia	1995	0,912968636
Jordan	1996	0,786499321	Tunisia	1996	0,911561549
Jordan	1997	0,79050535	Tunisia	1997	0,909294307
Jordan	1998	0,794867575	Tunisia	1998	0,908301353
Jordan	1999	0,793366015	Tunisia	1999	0,904816985
Jordan	2000	0,795143604	Tunisia	2000	0,904801965
Jordan	2001	0,796312571	Tunisia	2001	0,904909849
Jordan	2002	0,795450509	Tunisia	2002	0,904761195
Jordan	2003	0,788957	Tunisia	2003	0,904550731
Jordan	2004	0,778764546	Tunisia	2004	0,903959453
Jordan	2005	0,773550212	Tunisia	2005	0,903704822
Jordan	2006	0,785179436	Tunisia	2006	0,90418756
Jordan	2007	0,792163253	Tunisia	2007	0,90502274
Jordan	2008	0,799367249	Tunisia	2008	0,906553626
Jordan	2009	0,805293143	Tunisia	2009	0,90824151
Jordan	2010	0,799688518	Tunisia	2010	0,910238326
Jordan	2011	0,80141294	Tunisia	2011	0,911851943

Country Classification by Income: Upper-Middle-Income Economies					
Country	**Year**	**σ**	**Country**	**Year**	**σ**
Argentina	1980	0,971416	Iraq	1996	0,52272
Argentina	1981	0,971288	Iraq	1997	0,506041
Argentina	1982	0,971045	Iraq	1998	0,490418
Argentina	1983	0,970684	Iraq	1999	0,476096
Argentina	1984	0,970319	Iraq	2000	0,46155
Argentina	1985	0,96993	Iraq	2001	0,449377
Argentina	1986	0,969468	Iraq	2002	0,447201
Argentina	1987	0,969051	Iraq	2003	0,440985
Argentina	1988	0,968697	Iraq	2004	0,443757

Argentina	1989	0,968314	Iraq	2005	0,475629
Argentina	1990	0,967795	Iraq	2006	0,5723
Argentina	1991	0,967193	Iraq	2007	0,496437
Argentina	1992	0,966737	Iraq	2008	0,465992
Argentina	1993	0,966487	Iraq	2009	0,443255
Argentina	1994	0,966797	Iraq	2010	0,455664
Argentina	1995	0,966748	Iraq	2011	0,484259
Argentina	1996	0,966224	Malaysia	1980	0,865034
Argentina	1997	0,965963	Malaysia	1981	0,859583
Argentina	1998	0,965883	Malaysia	1982	0,849091
Argentina	1999	0,965926	Malaysia	1983	0,839193
Argentina	2000	0,965732	Malaysia	1984	0,831269
Argentina	2001	0,965759	Malaysia	1985	0,825966
Argentina	2002	0,9654	Malaysia	1986	0,823649
Argentina	2003	0,964908	Malaysia	1987	0,821443
Argentina	2004	0,964483	Malaysia	1988	0,822333
Argentina	2005	0,963827	Malaysia	1989	0,825792
Argentina	2006	0,962922	Malaysia	1990	0,830322
Argentina	2007	0,961775	Malaysia	1991	0,838246
Argentina	2008	0,960524	Malaysia	1992	0,845951
Argentina	2009	0,959505	Malaysia	1993	0,848147
Argentina	2010	0,957521	Malaysia	1994	0,85295
Argentina	2011	0,955872	Malaysia	1995	0,861188
Botswana	1980	0,867437	Malaysia	1996	0,87127
Botswana	1981	0,861184	Malaysia	1997	0,880316
Botswana	1982	0,866553	Malaysia	1998	0,887374
Botswana	1983	0,868688	Malaysia	1999	0,882298
Botswana	1984	0,868703	Malaysia	2000	0,873361
Botswana	1985	0,867834	Malaysia	2001	0,866372
Botswana	1986	0,866087	Malaysia	2002	0,856037
Botswana	1987	0,862375	Malaysia	2003	0,839884
Botswana	1988	0,855405	Malaysia	2004	0,824461
Botswana	1989	0,852595	Malaysia	2005	0,818289
Botswana	1990	0,861704	Malaysia	2006	0,814705
Botswana	1991	0,873298	Malaysia	2007	0,811013
Botswana	1992	0,877633	Malaysia	2008	0,808992
Botswana	1993	0,875363	Malaysia	2009	0,805805
Botswana	1994	0,869868	Malaysia	2010	0,800561
Botswana	1995	0,862671	Malaysia	2011	0,798062
Botswana	1996	0,857581	Mauritius	1980	0,921314
Botswana	1997	0,845844	Mauritius	1981	0,919596
Botswana	1998	0,835712	Mauritius	1982	0,916902
Botswana	1999	0,828573	Mauritius	1983	0,914341
Botswana	2000	0,823204	Mauritius	1984	0,912206
Botswana	2001	0,823119	Mauritius	1985	0,910658
Botswana	2002	0,823125	Mauritius	1986	0,9096
Botswana	2003	0,822668	Mauritius	1987	0,90823
Botswana	2004	0,823734	Mauritius	1988	0,906954
Botswana	2005	0,829322	Mauritius	1989	0,904046
Botswana	2006	0,835857	Mauritius	1990	0,903303
Botswana	2007	0,842893	Mauritius	1991	0,903658
Botswana	2008	0,847138	Mauritius	1992	0,903625
Botswana	2009	0,841698	Mauritius	1993	0,903984
Botswana	2010	0,833925	Mauritius	1994	0,904593
Botswana	2011	0,83375	Mauritius	1995	0,90365
Brazil	1980	0,970004	Mauritius	1996	0,901302
Brazil	1981	0,970409	Mauritius	1997	0,899879
Brazil	1982	0,970362	Mauritius	1998	0,897755
Brazil	1983	0,970306	Mauritius	1999	0,897477
Brazil	1984	0,970237	Mauritius	2000	0,898466
Brazil	1985	0,970118	Mauritius	2001	0,898686
Brazil	1986	0,96963	Mauritius	2002	0,897476
Brazil	1987	0,968671	Mauritius	2003	0,896024
Brazil	1988	0,968052	Mauritius	2004	0,891665
Brazil	1989	0,967616	Mauritius	2005	0,89131
Brazil	1990	0,967526	Mauritius	2006	0,891096
Brazil	1991	0,966142	Mauritius	2007	0,89036
Brazil	1992	0,96514	Mauritius	2008	0,8942
Brazil	1993	0,963703	Mauritius	2009	0,899278

Brazil	1994	0,962614	Mauritius	2010	0,900364
Brazil	1995	0,961434	Mauritius	2011	0,901225
Brazil	1996	0,961215	Mexico	1980	0,983488
Brazil	1997	0,96099	Mexico	1981	0,976061
Brazil	1998	0,96126	Mexico	1982	0,967354
Brazil	1999	0,960462	Mexico	1983	0,960491
Brazil	2000	0,960594	Mexico	1984	0,956123
Brazil	2001	0,960674	Mexico	1985	0,95173
Brazil	2002	0,960107	Mexico	1986	0,94768
Brazil	2003	0,959605	Mexico	1987	0,94417
Brazil	2004	0,96005	Mexico	1988	0,941481
Brazil	2005	0,960437	Mexico	1989	0,939334
Brazil	2006	0,960456	Mexico	1990	0,937307
Brazil	2007	0,96035	Mexico	1991	0,935347
Brazil	2008	0,960707	Mexico	1992	0,934403
Brazil	2009	0,960886	Mexico	1993	0,93447
Brazil	2010	0,960727	Mexico	1994	0,934438
Brazil	2011	0,960836	Mexico	1995	0,933057
Bulgaria	1980	0,907943	Mexico	1996	0,931559
Bulgaria	1981	0,907916	Mexico	1997	0,931674
Bulgaria	1982	0,907784	Mexico	1998	0,932635
Bulgaria	1983	0,907542	Mexico	1999	0,934463
Bulgaria	1984	0,907286	Mexico	2000	0,936166
Bulgaria	1985	0,906886	Mexico	2001	0,937441
Bulgaria	1986	0,906508	Mexico	2002	0,938198
Bulgaria	1987	0,903507	Mexico	2003	0,937999
Bulgaria	1988	0,905554	Mexico	2004	0,937324
Bulgaria	1989	0,908183	Mexico	2005	0,936511
Bulgaria	1990	0,910215	Mexico	2006	0,935647
Bulgaria	1991	0,912364	Mexico	2007	0,935586
Bulgaria	1992	0,91389	Mexico	2008	0,935292
Bulgaria	1993	0,913959	Mexico	2009	0,934105
Bulgaria	1994	0,913835	Mexico	2010	0,931447
Bulgaria	1995	0,914257	Mexico	2011	0,928933
Bulgaria	1996	0,91583	Namibia	1980	0,840504
Bulgaria	1997	0,916683	Namibia	1981	0,84007
Bulgaria	1998	0,916316	Namibia	1982	0,839127
Bulgaria	1999	0,915179	Namibia	1983	0,837493
Bulgaria	2000	0,912752	Namibia	1984	0,835117
Bulgaria	2001	0,911649	Namibia	1985	0,832389
Bulgaria	2002	0,911826	Namibia	1986	0,829658
Bulgaria	2003	0,913192	Namibia	1987	0,826751
Bulgaria	2004	0,914821	Namibia	1988	0,823935
Bulgaria	2005	0,91654	Namibia	1989	0,821434
Bulgaria	2006	0,919554	Namibia	1990	0,819206
Bulgaria	2007	0,923038	Namibia	1991	0,818575
Bulgaria	2008	0,92446	Namibia	1992	0,81402
Bulgaria	2009	0,927442	Namibia	1993	0,809657
Bulgaria	2010	0,928847	Namibia	1994	0,811675
Bulgaria	2011	0,928775	Namibia	1995	0,813244
China, People's Republic of	1980	-0,59757	Namibia	1996	0,817689
China, People's Republic of	1981	-0,59701	Namibia	1997	0,823394
China, People's Republic of	1982	-0,59993	Namibia	1998	0,820771
China, People's Republic of	1983	-0,6019	Namibia	1999	0,824778
China, People's Republic of	1984	-0,60159	Namibia	2000	0,820011
China, People's Republic of	1985	-0,60766	Namibia	2001	0,82335
China, People's Republic of	1986	-0,60877	Namibia	2002	0,825426
China, People's Republic of	1987	-0,6097	Namibia	2003	0,832166
China, People's Republic of	1988	-0,60806	Namibia	2004	0,834544
China, People's	1989	-0,60667	Namibia	2005	0,837885

Republic of					
China, People's Republic of	1990	-0,61548	Namibia	2006	0,841232
China, People's Republic of	1991	-0,62709	Namibia	2007	0,849623
China, People's Republic of	1992	-0,64882	Namibia	2008	0,853392
China, People's Republic of	1993	-0,69994	Namibia	2009	0,860607
China, People's Republic of	1994	-0,6624	Namibia	2010	0,866916
China, People's Republic of	1995	-0,56685	Namibia	2011	0,868325
China, People's Republic of	1996	-0,45125	Panama	1980	0,915037
China, People's Republic of	1997	-0,33037	Panama	1981	0,912554
China, People's Republic of	1998	-0,22418	Panama	1982	0,912187
China, People's Republic of	1999	-0,13507	Panama	1983	0,910362
China, People's Republic of	2000	-0,07432	Panama	1984	0,907411
China, People's Republic of	2001	-0,01364	Panama	1985	0,90386
China, People's Republic of	2002	0,061088	Panama	1986	0,902942
China, People's Republic of	2003	0,151186	Panama	1987	0,903674
China, People's Republic of	2004	0,247317	Panama	1988	0,906121
China, People's Republic of	2005	0,335081	Panama	1989	0,905382
China, People's Republic of	2006	0,411204	Panama	1990	0,903667
China, People's Republic of	2007	0,481929	Panama	1991	0,903835
China, People's Republic of	2008	0,544659	Panama	1992	0,905816
China, People's Republic of	2009	0,594056	Panama	1993	0,908438
China, People's Republic of	2010	0,634756	Panama	1994	0,91279
China, People's Republic of	2011	0,671202	Panama	1995	0,917556
Colombia	1980	0,923061	Panama	1996	0,922005
Colombia	1981	0,918192	Panama	1997	0,924434
Colombia	1982	0,913396	Panama	1998	0,926092
Colombia	1983	0,908096	Panama	1999	0,92587
Colombia	1984	0,897696	Panama	2000	0,927262
Colombia	1985	0,890235	Panama	2001	0,929825
Colombia	1986	0,88472	Panama	2002	0,929281
Colombia	1987	0,883056	Panama	2003	0,927744
Colombia	1988	0,881845	Panama	2004	0,926476
Colombia	1989	0,881366	Panama	2005	0,926201
Colombia	1990	0,879278	Panama	2006	0,927065
Colombia	1991	0,877157	Panama	2007	0,928615
Colombia	1992	0,873135	Panama	2008	0,929205
Colombia	1993	0,869051	Panama	2009	0,926276
Colombia	1994	0,865698	Panama	2010	0,922093
Colombia	1995	0,868421	Panama	2011	0,909445
Colombia	1996	0,869694	Paraguay	1980	0,938175
Colombia	1997	0,868753	Paraguay	1981	0,94057
Colombia	1998	0,865964	Paraguay	1982	0,942805
Colombia	1999	0,864495	Paraguay	1983	0,943337
Colombia	2000	0,855956	Paraguay	1984	0,942802
Colombia	2001	0,85024	Paraguay	1985	0,940766
Colombia	2002	0,845195	Paraguay	1986	0,937451
Colombia	2003	0,843113	Paraguay	1987	0,934529
Colombia	2004	0,841122	Paraguay	1988	0,931448

Colombia	2005	0,845367	Paraguay	1989	0,92897
Colombia	2006	0,850112	Paraguay	1990	0,928905
Colombia	2007	0,855437	Paraguay	1991	0,932027
Colombia	2008	0,859429	Paraguay	1992	0,931704
Colombia	2009	0,867004	Paraguay	1993	0,929527
Colombia	2010	0,870209	Paraguay	1994	0,930454
Colombia	2011	0,870834	Paraguay	1995	0,931627
Costa Rica	1980	0,948285	Paraguay	1996	0,923923
Costa Rica	1981	0,948195	Paraguay	1997	0,921533
Costa Rica	1982	0,947733	Paraguay	1998	0,912925
Costa Rica	1983	0,946984	Paraguay	1999	0,90782
Costa Rica	1984	0,946291	Paraguay	2000	0,896093
Costa Rica	1985	0,945802	Paraguay	2001	0,890173
Costa Rica	1986	0,945345	Paraguay	2002	0,88798
Costa Rica	1987	0,944989	Paraguay	2003	0,884106
Costa Rica	1988	0,944715	Paraguay	2004	0,883503
Costa Rica	1989	0,944338	Paraguay	2005	0,883496
Costa Rica	1990	0,944112	Paraguay	2006	0,884054
Costa Rica	1991	0,944022	Paraguay	2007	0,884059
Costa Rica	1992	0,944593	Paraguay	2008	0,884446
Costa Rica	1993	0,944421	Paraguay	2009	0,888358
Costa Rica	1994	0,944115	Paraguay	2010	0,885087
Costa Rica	1995	0,942688	Paraguay	2011	0,885239
Costa Rica	1996	0,94295	Peru	1980	0,952246
Costa Rica	1997	0,94209	Peru	1981	0,950399
Costa Rica	1998	0,941131	Peru	1982	0,948313
Costa Rica	1999	0,941352	Peru	1983	0,94554
Costa Rica	2000	0,94039	Peru	1984	0,941651
Costa Rica	2001	0,939498	Peru	1985	0,937568
Costa Rica	2002	0,937874	Peru	1986	0,934518
Costa Rica	2003	0,93699	Peru	1987	0,932924
Costa Rica	2004	0,934314	Peru	1988	0,932276
Costa Rica	2005	0,934597	Peru	1989	0,932051
Costa Rica	2006	0,935575	Peru	1990	0,931013
Costa Rica	2007	0,936892	Peru	1991	0,930257
Costa Rica	2008	0,938785	Peru	1992	0,929338
Costa Rica	2009	0,939864	Peru	1993	0,927738
Costa Rica	2010	0,940362	Peru	1994	0,926157
Costa Rica	2011	0,940395	Peru	1995	0,925675
Dominican Republic	1980	0,920131	Peru	1996	0,92654
Dominican Republic	1981	0,923604	Peru	1997	0,927225
Dominican Republic	1982	0,926487	Peru	1998	0,928375
Dominican Republic	1983	0,92809	Peru	1999	0,928966
Dominican Republic	1984	0,929952	Peru	2000	0,928176
Dominican Republic	1985	0,93228	Peru	2001	0,927929
Dominican Republic	1986	0,932462	Peru	2002	0,928094
Dominican Republic	1987	0,933976	Peru	2003	0,928308
Dominican Republic	1988	0,933094	Peru	2004	0,92869
Dominican Republic	1989	0,932279	Peru	2005	0,929276
Dominican Republic	1990	0,931055	Peru	2006	0,93003
Dominican Republic	1991	0,930842	Peru	2007	0,931221
Dominican Republic	1992	0,92936	Peru	2008	0,933031
Dominican Republic	1993	0,927772	Peru	2009	0,935173
Dominican Republic	1994	0,925422	Peru	2010	0,934133

Dominican Republic	1995	0,923522	Peru	2011	0,933132
Dominican Republic	1996	0,921566	Romania	1980	0,773296
Dominican Republic	1997	0,919502	Romania	1981	0,77384
Dominican Republic	1998	0,920144	Romania	1982	0,773359
Dominican Republic	1999	0,924646	Romania	1983	0,772298
Dominican Republic	2000	0,924573	Romania	1984	0,771067
Dominican Republic	2001	0,927649	Romania	1985	0,769864
Dominican Republic	2002	0,926075	Romania	1986	0,768472
Dominican Republic	2003	0,922836	Romania	1987	0,766888
Dominican Republic	2004	0,916805	Romania	1988	0,76501
Dominican Republic	2005	0,914516	Romania	1989	0,762842
Dominican Republic	2006	0,913752	Romania	1990	0,760444
Dominican Republic	2007	0,913226	Romania	1991	0,748797
Dominican Republic	2008	0,913379	Romania	1992	0,737208
Dominican Republic	2009	0,912556	Romania	1993	0,738285
Dominican Republic	2010	0,912268	Romania	1994	0,738538
Dominican Republic	2011	0,912845	Romania	1995	0,737537
Ecuador	1980	0,869814	Romania	1996	0,739093
Ecuador	1981	0,869868	Romania	1997	0,742703
Ecuador	1982	0,869429	Romania	1998	0,745948
Ecuador	1983	0,86878	Romania	1999	0,75047
Ecuador	1984	0,867262	Romania	2000	0,757786
Ecuador	1985	0,865617	Romania	2001	0,764791
Ecuador	1986	0,86406	Romania	2002	0,776088
Ecuador	1987	0,86268	Romania	2003	0,786509
Ecuador	1988	0,864258	Romania	2004	0,796662
Ecuador	1989	0,865514	Romania	2005	0,808714
Ecuador	1990	0,864422	Romania	2006	0,821925
Ecuador	1991	0,860927	Romania	2007	0,83377
Ecuador	1992	0,858444	Romania	2008	0,847507
Ecuador	1993	0,862559	Romania	2009	0,860633
Ecuador	1994	0,867094	Romania	2010	0,86556
Ecuador	1995	0,871595	Romania	2011	0,866015
Ecuador	1996	0,875434	South Africa	1980	0,94212
Ecuador	1997	0,878378	South Africa	1981	0,940771
Ecuador	1998	0,881666	South Africa	1982	0,939241
Ecuador	1999	0,884543	South Africa	1983	0,937589
Ecuador	2000	0,884632	South Africa	1984	0,93645
Ecuador	2001	0,886856	South Africa	1985	0,935574
Ecuador	2002	0,889632	South Africa	1986	0,934318
Ecuador	2003	0,890819	South Africa	1987	0,932697
Ecuador	2004	0,891939	South Africa	1988	0,931191
Ecuador	2005	0,893309	South Africa	1989	0,93013
Ecuador	2006	0,895224	South Africa	1990	0,929241
Ecuador	2007	0,897698	South Africa	1991	0,928789
Ecuador	2008	0,896196	South Africa	1992	0,928206
Ecuador	2009	0,886314	South Africa	1993	0,927854
Ecuador	2010	0,879407	South Africa	1994	0,92767
Ecuador	2011	0,87543	South Africa	1995	0,927928
Fiji	1980	0,868875	South Africa	1996	0,928508
Fiji	1981	0,858205	South Africa	1997	0,929041
Fiji	1982	0,846938	South Africa	1998	0,929544
Fiji	1983	0,839278	South Africa	1999	0,93018

Fiji	1984	0,833845	South Africa	2000	0,930466
Fiji	1985	0,833439	South Africa	2001	0,930624
Fiji	1986	0,835765	South Africa	2002	0,931084
Fiji	1987	0,837114	South Africa	2003	0,931384
Fiji	1988	0,833873	South Africa	2004	0,931822
Fiji	1989	0,829015	South Africa	2005	0,932655
Fiji	1990	0,824264	South Africa	2006	0,934052
Fiji	1991	0,826415	South Africa	2007	0,935385
Fiji	1992	0,820453	South Africa	2008	0,936493
Fiji	1993	0,811423	South Africa	2009	0,937707
Fiji	1994	0,800782	South Africa	2010	0,938682
Fiji	1995	0,791794	South Africa	2011	0,939714
Fiji	1996	0,780809	Thailand	1980	0,914086
Fiji	1997	0,775348	Thailand	1981	0,911125
Fiji	1998	0,765824	Thailand	1982	0,908023
Fiji	1999	0,750429	Thailand	1983	0,905693
Fiji	2000	0,737286	Thailand	1984	0,903852
Fiji	2001	0,753664	Thailand	1985	0,901823
Fiji	2002	0,762955	Thailand	1986	0,900752
Fiji	2003	0,774323	Thailand	1987	0,90089
Fiji	2004	0,786141	Thailand	1988	0,904233
Fiji	2005	0,80662	Thailand	1989	0,910255
Fiji	2006	0,818954	Thailand	1990	0,91727
Fiji	2007	0,827315	Thailand	1991	0,924184
Fiji	2008	0,83288	Thailand	1992	0,928743
Fiji	2009	0,839747	Thailand	1993	0,93077
Fiji	2010	0,837511	Thailand	1994	0,932803
Fiji	2011	0,840401	Thailand	1995	0,933658
Gabon	1980	0,948165	Thailand	1996	0,93451
Gabon	1981	0,947533	Thailand	1997	0,933604
Gabon	1982	0,94717	Thailand	1998	0,928941
Gabon	1983	0,946706	Thailand	1999	0,923434
Gabon	1984	0,946274	Thailand	2000	0,917696
Gabon	1985	0,94577	Thailand	2001	0,913479
Gabon	1986	0,94536	Thailand	2002	0,9101
Gabon	1987	0,944744	Thailand	2003	0,907564
Gabon	1988	0,943582	Thailand	2004	0,906096
Gabon	1989	0,942914	Thailand	2005	0,905575
Gabon	1990	0,942123	Thailand	2006	0,905652
Gabon	1991	0,941326	Thailand	2007	0,905605
Gabon	1992	0,940501	Thailand	2008	0,90503
Gabon	1993	0,938901	Thailand	2009	0,905104
Gabon	1994	0,937533	Thailand	2010	0,903633
Gabon	1995	0,935459	Thailand	2011	0,903563
Gabon	1996	0,93385	Turkey	1980	0,859381
Gabon	1997	0,93156	Turkey	1981	0,850723
Gabon	1998	0,927428	Turkey	1982	0,844696
Gabon	1999	0,923313	Turkey	1983	0,837283
Gabon	2000	0,92314	Turkey	1984	0,830042
Gabon	2001	0,923216	Turkey	1985	0,824104
Gabon	2002	0,923043	Turkey	1986	0,815732
Gabon	2003	0,924688	Turkey	1987	0,807566
Gabon	2004	0,925295	Turkey	1988	0,810962
Gabon	2005	0,925617	Turkey	1989	0,818494
Gabon	2006	0,926346	Turkey	1990	0,824574
Gabon	2007	0,926627	Turkey	1991	0,8322
Gabon	2008	0,926669	Turkey	1992	0,838739
Gabon	2009	0,926157	Turkey	1993	0,844528
Gabon	2010	0,924834	Turkey	1994	0,854348
Gabon	2011	0,922254	Turkey	1995	0,862289
Iran (Islamic Republic of)	1980	0,699459	Turkey	1996	0,872256
Iran (Islamic Republic of)	1981	0,704835	Turkey	1997	0,881091
Iran (Islamic Republic of)	1982	0,707752	Turkey	1998	0,888524
Iran (Islamic Republic of)	1983	0,703917	Turkey	1999	0,892394
Iran (Islamic	1984	0,716736	Turkey	2000	0,893385

Republic of)					
Iran (Islamic Republic of)	1985	0,729807	Turkey	2001	0,894876
Iran (Islamic Republic of)	1986	0,739366	Turkey	2002	0,893551
Iran (Islamic Republic of)	1987	0,742618	Turkey	2003	0,893091
Iran (Islamic Republic of)	1988	0,744251	Turkey	2004	0,895376
Iran (Islamic Republic of)	1989	0,732012	Turkey	2005	0,901038
Iran (Islamic Republic of)	1990	0,718076	Turkey	2006	0,906553
Iran (Islamic Republic of)	1991	0,724329	Turkey	2007	0,912496
Iran (Islamic Republic of)	1992	0,739818	Turkey	2008	0,917163
Iran (Islamic Republic of)	1993	0,751202	Turkey	2009	0,919358
Iran (Islamic Republic of)	1994	0,746612	Turkey	2010	0,918986
Iran (Islamic Republic of)	1995	0,741506	Turkey	2011	0,920194
Iran (Islamic Republic of)	1996	0,737225	Venezuela	1980	0,683074
Iran (Islamic Republic of)	1997	0,738176	Venezuela	1981	0,680677
Iran (Islamic Republic of)	1998	0,74406	Venezuela	1982	0,678253
Iran (Islamic Republic of)	1999	0,750749	Venezuela	1983	0,675398
Iran (Islamic Republic of)	2000	0,755863	Venezuela	1984	0,6708
Iran (Islamic Republic of)	2001	0,762509	Venezuela	1985	0,681187
Iran (Islamic Republic of)	2002	0,771516	Venezuela	1986	0,689268
Iran (Islamic Republic of)	2003	0,777095	Venezuela	1987	0,693941
Iran (Islamic Republic of)	2004	0,78363	Venezuela	1988	0,700119
Iran (Islamic Republic of)	2005	0,792482	Venezuela	1989	0,705098
Iran (Islamic Republic of)	2006	0,799542	Venezuela	1990	0,703074
Iran (Islamic Republic of)	2007	0,800543	Venezuela	1991	0,695614
Iran (Islamic Republic of)	2008	0,805993	Venezuela	1992	0,685549
Iran (Islamic Republic of)	2009	0,812902	Venezuela	1993	0,685361
Iran (Islamic Republic of)	2010	0,821154	Venezuela	1994	0,68784
Iran (Islamic Republic of)	2011	0,828726	Venezuela	1995	0,685154
Iraq	1980	0,618426	Venezuela	1996	0,678925
Iraq	1981	0,619566	Venezuela	1997	0,669391
Iraq	1982	0,626043	Venezuela	1998	0,67606
Iraq	1983	0,62864	Venezuela	1999	0,684786
Iraq	1984	0,624712	Venezuela	2000	0,693407
Iraq	1985	0,618902	Venezuela	2001	0,701628
Iraq	1986	0,612649	Venezuela	2002	0,713391
Iraq	1987	0,606443	Venezuela	2003	0,712022
Iraq	1988	0,599996	Venezuela	2004	0,705349
Iraq	1989	0,594079	Venezuela	2005	0,703431
Iraq	1990	0,589788	Venezuela	2006	0,708093
Iraq	1991	0,584427	Venezuela	2007	0,707147
Iraq	1992	0,572524	Venezuela	2008	0,710979
Iraq	1993	0,561293	Venezuela	2009	0,706369
Iraq	1994	0,551547	Venezuela	2010	0,703788

Iraq	1995	0,537829	Venezuela	2011	0,692912

Country Classification by Income: High-Income Economies (OECD Countries)					
Country	**Year**	**σ**	**Country**	**Year**	**σ**
Australia	1980	0,536726534	Italy	1996	0,441977024
Australia	1981	0,545142949	Italy	1997	0,436623812
Australia	1982	0,558125913	Italy	1998	0,430885673
Australia	1983	0,568191469	Italy	1999	0,423147917
Australia	1984	0,571230173	Italy	2000	0,414787322
Australia	1985	0,573168159	Italy	2001	0,408779889
Australia	1986	0,576947808	Italy	2002	0,401284695
Australia	1987	0,576694489	Italy	2003	0,395047933
Australia	1988	0,579149961	Italy	2004	0,385882199
Australia	1989	0,587329328	Italy	2005	0,388506621
Australia	1990	0,594879448	Italy	2006	0,391459048
Australia	1991	0,597649872	Italy	2007	0,395142376
Australia	1992	0,593150973	Italy	2008	0,398416996
Australia	1993	0,588866711	Italy	2009	0,399653673
Australia	1994	0,585501909	Italy	2010	0,388583243
Australia	1995	0,584363878	Italy	2011	0,383664072
Australia	1996	0,584515631	Japan	1980	-0,248687461
Australia	1997	0,585334718	Japan	1981	-0,261490047
Australia	1998	0,588744044	Japan	1982	-0,276885688
Australia	1999	0,592205465	Japan	1983	-0,288568169
Australia	2000	0,596577704	Japan	1984	-0,301898301
Australia	2001	0,595711052	Japan	1985	-0,307707369
Australia	2002	0,591454744	Japan	1986	-0,292036682
Australia	2003	0,591836393	Japan	1987	-0,275850445
Australia	2004	0,596059382	Japan	1988	-0,25673303
Australia	2005	0,599835575	Japan	1989	-0,22656244
Australia	2006	0,603260398	Japan	1990	-0,187043682
Australia	2007	0,607355297	Japan	1991	-0,149429858
Australia	2008	0,612416327	Japan	1992	-0,118921168
Australia	2009	0,612199724	Japan	1993	-0,11599265
Australia	2010	0,606784761	Japan	1994	-0,133807465
Australia	2011	0,593122661	Japan	1995	-0,156557858
Austria	1980	0,307898819	Japan	1996	-0,178425342
Austria	1981	0,310554773	Japan	1997	-0,200564057
Austria	1982	0,313180476	Japan	1998	-0,213124618
Austria	1983	0,312904924	Japan	1999	-0,232096344
Austria	1984	0,312609941	Japan	2000	-0,257915854
Austria	1985	0,311338365	Japan	2001	-0,271066189
Austria	1986	0,31369406	Japan	2002	-0,284129977
Austria	1987	0,313504219	Japan	2003	-0,300159544
Austria	1988	0,318570912	Japan	2004	-0,307352096
Austria	1989	0,328416497	Japan	2005	-0,307670623
Austria	1990	0,338156134	Japan	2006	-0,298939556
Austria	1991	0,349184752	Japan	2007	-0,285790086
Austria	1992	0,362121522	Japan	2008	-0,269917786
Austria	1993	0,372074753	Japan	2009	-0,257349998
Austria	1994	0,377871633	Japan	2010	-0,267063975
Austria	1995	0,383885473	Japan	2011	-0,277864158
Austria	1996	0,390681833	Luxembourg	1980	0,38200143
Austria	1997	0,401541829	Luxembourg	1981	0,377783716
Austria	1998	0,420766652	Luxembourg	1982	0,372097045
Austria	1999	0,439517796	Luxembourg	1983	0,366005987
Austria	2000	0,457340986	Luxembourg	1984	0,358047187
Austria	2001	0,477271736	Luxembourg	1985	0,349892735
Austria	2002	0,497548044	Luxembourg	1986	0,340309113
Austria	2003	0,51147151	Luxembourg	1987	0,335034162
Austria	2004	0,526339889	Luxembourg	1988	0,332161963
Austria	2005	0,540674806	Luxembourg	1989	0,330616057
Austria	2006	0,55327177	Luxembourg	1990	0,329405606
Austria	2007	0,565138042	Luxembourg	1991	0,327748954
Austria	2008	0,57745868	Luxembourg	1992	0,328253448
Austria	2009	0,587694466	Luxembourg	1993	0,323659152
Austria	2010	0,593725562	Luxembourg	1994	0,322411716

Austria	2011	0,600572884	Luxembourg	1995	0,319839627
Belgium	1980	0,652289271	Luxembourg	1996	0,314472586
Belgium	1981	0,632214904	Luxembourg	1997	0,295022905
Belgium	1982	0,608380914	Luxembourg	1998	0,311408132
Belgium	1983	0,587662816	Luxembourg	1999	0,304909438
Belgium	1984	0,569173932	Luxembourg	2000	0,311920226
Belgium	1985	0,557413101	Luxembourg	2001	0,316581339
Belgium	1986	0,548365712	Luxembourg	2002	0,348741144
Belgium	1987	0,542005539	Luxembourg	2003	0,332042217
Belgium	1988	0,539694309	Luxembourg	2004	0,322798342
Belgium	1989	0,541244924	Luxembourg	2005	0,316590667
Belgium	1990	0,553007066	Luxembourg	2006	0,304402679
Belgium	1991	0,568035841	Luxembourg	2007	0,293950468
Belgium	1992	0,577465415	Luxembourg	2008	0,309844315
Belgium	1993	0,584715247	Luxembourg	2009	0,321607053
Belgium	1994	0,587376416	Luxembourg	2010	0,305357814
Belgium	1995	0,588903964	Luxembourg	2011	0,27742961
Belgium	1996	0,591747999	Netherlands	1980	0,319365531
Belgium	1997	0,595954418	Netherlands	1981	0,323180437
Belgium	1998	0,601050973	Netherlands	1982	0,321808457
Belgium	1999	0,606705546	Netherlands	1983	0,319633067
Belgium	2000	0,608656824	Netherlands	1984	0,319747925
Belgium	2001	0,611369967	Netherlands	1985	0,319728255
Belgium	2002	0,616217852	Netherlands	1986	0,324482828
Belgium	2003	0,618122399	Netherlands	1987	0,333726078
Belgium	2004	0,619179726	Netherlands	1988	0,341044664
Belgium	2005	0,622828364	Netherlands	1989	0,349132389
Belgium	2006	0,62647891	Netherlands	1990	0,357534349
Belgium	2007	0,631519556	Netherlands	1991	0,364197165
Belgium	2008	0,63773185	Netherlands	1992	0,369420797
Belgium	2009	0,64347738	Netherlands	1993	0,37228772
Belgium	2010	0,642685711	Netherlands	1994	0,373276234
Belgium	2011	0,641450346	Netherlands	1995	0,373354137
Canada	1980	0,312408835	Netherlands	1996	0,37483269
Canada	1981	0,333901495	Netherlands	1997	0,377077878
Canada	1982	0,358205378	Netherlands	1998	0,386084586
Canada	1983	0,367224544	Netherlands	1999	0,395130783
Canada	1984	0,373493224	Netherlands	2000	0,405097544
Canada	1985	0,376776874	Netherlands	2001	0,40896219
Canada	1986	0,379626334	Netherlands	2002	0,408126354
Canada	1987	0,384652555	Netherlands	2003	0,398197323
Canada	1988	0,393665165	Netherlands	2004	0,386617988
Canada	1989	0,40440461	Netherlands	2005	0,377527535
Canada	1990	0,41322735	Netherlands	2006	0,367790401
Canada	1991	0,414586216	Netherlands	2007	0,361080974
Canada	1992	0,409840345	Netherlands	2008	0,355822772
Canada	1993	0,403255075	Netherlands	2009	0,350471735
Canada	1994	0,394815415	Netherlands	2010	0,33140856
Canada	1995	0,38860622	Netherlands	2011	0,307445824
Canada	1996	0,38122347	New Zealand	1980	-0,051309928
Canada	1997	0,37851131	New Zealand	1981	-0,059017498
Canada	1998	0,385724276	New Zealand	1982	-0,065075025
Canada	1999	0,392940879	New Zealand	1983	-0,076761447
Canada	2000	0,396973699	New Zealand	1984	-0,100104399
Canada	2001	0,401265681	New Zealand	1985	-0,095085815
Canada	2002	0,402872622	New Zealand	1986	-0,096248239
Canada	2003	0,401816994	New Zealand	1987	-0,106227487
Canada	2004	0,401210517	New Zealand	1988	-0,100490183
Canada	2005	0,402624816	New Zealand	1989	-0,097873725
Canada	2006	0,404628456	New Zealand	1990	-0,086909629
Canada	2007	0,40761137	New Zealand	1991	-0,081822224
Canada	2008	0,40777421	New Zealand	1992	-0,087727644
Canada	2009	0,404826432	New Zealand	1993	-0,07935711
Canada	2010	0,385148048	New Zealand	1994	-0,052595146
Canada	2011	0,367843539	New Zealand	1995	-0,018214783
Chile	1980	0,56126368	New Zealand	1996	0,017036045
Chile	1981	0,560256898	New Zealand	1997	0,048171252
Chile	1982	0,560443997	New Zealand	1998	0,065774918
Chile	1983	0,555064619	New Zealand	1999	0,072347663

Chile	1984	0,54828614	New Zealand	2000	0,082978427
Chile	1985	0,542815983	New Zealand	2001	0,107951395
Chile	1986	0,53624928	New Zealand	2002	0,115746886
Chile	1987	0,521791279	New Zealand	2003	0,108176515
Chile	1988	0,523523867	New Zealand	2004	0,119225174
Chile	1989	0,522984505	New Zealand	2005	0,127532557
Chile	1990	0,539398015	New Zealand	2006	0,127069652
Chile	1991	0,556349933	New Zealand	2007	0,127182499
Chile	1992	0,562541127	New Zealand	2008	0,131315306
Chile	1993	0,573155582	New Zealand	2009	0,119535998
Chile	1994	0,583680868	New Zealand	2010	0,080943868
Chile	1995	0,589695275	New Zealand	2011	0,046163008
Chile	1996	0,603146374	Norway	1980	0,700727582
Chile	1997	0,610273838	Norway	1981	0,696346879
Chile	1998	0,617836416	Norway	1982	0,694107473
Chile	1999	0,618957937	Norway	1983	0,693387091
Chile	2000	0,607618868	Norway	1984	0,692885101
Chile	2001	0,603971004	Norway	1985	0,692420602
Chile	2002	0,603221655	Norway	1986	0,689932466
Chile	2003	0,601126313	Norway	1987	0,686628044
Chile	2004	0,602891684	Norway	1988	0,68051523
Chile	2005	0,606927752	Norway	1989	0,672426343
Chile	2006	0,618367314	Norway	1990	0,660978794
Chile	2007	0,62516135	Norway	1991	0,647315383
Chile	2008	0,626934588	Norway	1992	0,629983842
Chile	2009	0,636033416	Norway	1993	0,610472798
Chile	2010	0,627797902	Norway	1994	0,596008301
Chile	2011	0,627729237	Norway	1995	0,583363652
Denmark	1980	0,061916888	Norway	1996	0,570613027
Denmark	1981	0,067657351	Norway	1997	0,560153723
Denmark	1982	0,067405403	Norway	1998	0,550276935
Denmark	1983	0,079929002	Norway	1999	0,54362148
Denmark	1984	0,098478928	Norway	2000	0,533471346
Denmark	1985	0,122742459	Norway	2001	0,526038945
Denmark	1986	0,151231781	Norway	2002	0,516764939
Denmark	1987	0,191434711	Norway	2003	0,506113589
Denmark	1988	0,226696715	Norway	2004	0,490846932
Denmark	1989	0,249104172	Norway	2005	0,48000899
Denmark	1990	0,27322492	Norway	2006	0,476220578
Denmark	1991	0,296700925	Norway	2007	0,473793209
Denmark	1992	0,316687316	Norway	2008	0,474458456
Denmark	1993	0,331570625	Norway	2009	0,471440494
Denmark	1994	0,341391236	Norway	2010	0,457866043
Denmark	1995	0,35297069	Norway	2011	0,442024052
Denmark	1996	0,369092703	Poland	1980	0,492886335
Denmark	1997	0,383826464	Poland	1981	0,491093934
Denmark	1998	0,402686268	Poland	1982	0,486293435
Denmark	1999	0,425215602	Poland	1983	0,479779065
Denmark	2000	0,443699807	Poland	1984	0,473911017
Denmark	2001	0,462169707	Poland	1985	0,468825489
Denmark	2002	0,473814905	Poland	1986	0,463917315
Denmark	2003	0,485184252	Poland	1987	0,459199488
Denmark	2004	0,496527374	Poland	1988	0,454613954
Denmark	2005	0,505342484	Poland	1989	0,450424522
Denmark	2006	0,514676034	Poland	1990	0,445423782
Denmark	2007	0,526301503	Poland	1991	0,438459486
Denmark	2008	0,537127972	Poland	1992	0,430628359
Denmark	2009	0,54344517	Poland	1993	0,422921598
Denmark	2010	0,538403511	Poland	1994	0,415402323
Denmark	2011	0,529649973	Poland	1995	0,409026533
Finland	1980	0,670411706	Poland	1996	0,400447518
Finland	1981	0,662783682	Poland	1997	0,395823359
Finland	1982	0,655856311	Poland	1998	0,397718579
Finland	1983	0,64857167	Poland	1999	0,406071901
Finland	1984	0,641440213	Poland	2000	0,419923127
Finland	1985	0,634792387	Poland	2001	0,447067916
Finland	1986	0,628130376	Poland	2002	0,443107635
Finland	1987	0,621683776	Poland	2003	0,43077752
Finland	1988	0,61411494	Poland	2004	0,418641299

Finland	1989	0,60876447	Poland	2005	0,406378746
Finland	1990	0,611803234	Poland	2006	0,394519955
Finland	1991	0,608822107	Poland	2007	0,380563498
Finland	1992	0,598190725	Poland	2008	0,371889383
Finland	1993	0,583183706	Poland	2009	0,358461052
Finland	1994	0,569011927	Poland	2010	0,326959103
Finland	1995	0,552702188	Poland	2011	0,280865997
Finland	1996	0,54183495	Portugal	1980	0,581644952
Finland	1997	0,531617403	Portugal	1981	0,579652607
Finland	1998	0,520021379	Portugal	1982	0,573824525
Finland	1999	0,513572633	Portugal	1983	0,57210964
Finland	2000	0,508889198	Portugal	1984	0,570641518
Finland	2001	0,508879006	Portugal	1985	0,56536752
Finland	2002	0,508305848	Portugal	1986	0,558998883
Finland	2003	0,501593411	Portugal	1987	0,554642975
Finland	2004	0,493639946	Portugal	1988	0,553288341
Finland	2005	0,486948282	Portugal	1989	0,553163826
Finland	2006	0,486420959	Portugal	1990	0,554498792
Finland	2007	0,48842594	Portugal	1991	0,555403352
Finland	2008	0,49317646	Portugal	1992	0,552454948
Finland	2009	0,495513618	Portugal	1993	0,543611765
Finland	2010	0,486624271	Portugal	1994	0,529183745
Finland	2011	0,481363773	Portugal	1995	0,519335032
France	1980	0,418288589	Portugal	1996	0,510550678
France	1981	0,422401458	Portugal	1997	0,49840793
France	1982	0,42416209	Portugal	1998	0,488238066
France	1983	0,423468083	Portugal	1999	0,487813771
France	1984	0,422003716	Portugal	2000	0,487513006
France	1985	0,419366151	Portugal	2001	0,491766453
France	1986	0,414655864	Portugal	2002	0,491051972
France	1987	0,410654694	Portugal	2003	0,490117073
France	1988	0,407452792	Portugal	2004	0,48699078
France	1989	0,403518379	Portugal	2005	0,481723011
France	1990	0,400914699	Portugal	2006	0,478719026
France	1991	0,3984029	Portugal	2007	0,480785251
France	1992	0,392082632	Portugal	2008	0,479188472
France	1993	0,382813245	Portugal	2009	0,474550098
France	1994	0,37118417	Portugal	2010	0,462249875
France	1995	0,360213637	Portugal	2011	0,439661205
France	1996	0,352062911	Republic of Korea	1980	0,405533522
France	1997	0,343953609	Republic of Korea	1981	0,415290684
France	1998	0,338629127	Republic of Korea	1982	0,41581434
France	1999	0,336837441	Republic of Korea	1983	0,412682623
France	2000	0,337314427	Republic of Korea	1984	0,414295793
France	2001	0,336516321	Republic of Korea	1985	0,415024698
France	2002	0,33665821	Republic of Korea	1986	0,409852773
France	2003	0,335478395	Republic of Korea	1987	0,416414142
France	2004	0,332580537	Republic of Korea	1988	0,431633741
France	2005	0,32953307	Republic of Korea	1989	0,448162496
France	2006	0,325581372	Republic of Korea	1990	0,46792832
France	2007	0,325353533	Republic of Korea	1991	0,493524045
France	2008	0,3276636	Republic of Korea	1992	0,511333823
France	2009	0,330152482	Republic of Korea	1993	0,516055048
France	2010	0,320754081	Republic of Korea	1994	0,521195173

France	2011	0,316170692	Republic of Korea	1995	0,528323829
Germany	1980	0,42336759	Republic of Korea	1996	0,541164398
Germany	1981	0,423040539	Republic of Korea	1997	0,545606434
Germany	1982	0,422176391	Republic of Korea	1998	0,540106833
Germany	1983	0,421473175	Republic of Korea	1999	0,517418444
Germany	1984	0,423168153	Republic of Korea	2000	0,498713166
Germany	1985	0,424902707	Republic of Korea	2001	0,484197497
Germany	1986	0,426189154	Republic of Korea	2002	0,466880411
Germany	1987	0,426098704	Republic of Korea	2003	0,456209719
Germany	1988	0,4263978	Republic of Korea	2004	0,441667944
Germany	1989	0,428322166	Republic of Korea	2005	0,428212881
Germany	1990	0,432317019	Republic of Korea	2006	0,418123603
Germany	1991	0,439314038	Republic of Korea	2007	0,41462639
Germany	1992	0,448543489	Republic of Korea	2008	0,413646728
Germany	1993	0,455833167	Republic of Korea	2009	0,412016869
Germany	1994	0,460725993	Republic of Korea	2010	0,395485282
Germany	1995	0,467292607	Republic of Korea	2011	0,392664969
Germany	1996	0,475472182	Spain	1980	0,30048421
Germany	1997	0,482578784	Spain	1981	0,318834186
Germany	1998	0,490532517	Spain	1982	0,326672196
Germany	1999	0,49849996	Spain	1983	0,317207277
Germany	2000	0,505602479	Spain	1984	0,311089575
Germany	2001	0,513420582	Spain	1985	0,295871764
Germany	2002	0,518455327	Spain	1986	0,266314864
Germany	2003	0,520088613	Spain	1987	0,244529709
Germany	2004	0,521960437	Spain	1988	0,23708345
Germany	2005	0,524336338	Spain	1989	0,230404004
Germany	2006	0,527380168	Spain	1990	0,216267869
Germany	2007	0,532852888	Spain	1991	0,193477497
Germany	2008	0,539501309	Spain	1992	0,173040628
Germany	2009	0,544452012	Spain	1993	0,16355297
Germany	2010	0,54159224	Spain	1994	0,140541688
Germany	2011	0,54106617	Spain	1995	0,121331543
Greece	1980	0,313298792	Spain	1996	0,117053352
Greece	1981	0,327142507	Spain	1997	0,124050409
Greece	1982	0,335511595	Spain	1998	0,133168578
Greece	1983	0,3446109	Spain	1999	0,1436373
Greece	1984	0,345916927	Spain	2000	0,160021469
Greece	1985	0,339781195	Spain	2001	0,197970927
Greece	1986	0,332718253	Spain	2002	0,230567649
Greece	1987	0,333545923	Spain	2003	0,255687356
Greece	1988	0,343914598	Spain	2004	0,280086786
Greece	1989	0,35104683	Spain	2005	0,307333797
Greece	1990	0,358625472	Spain	2006	0,33286801
Greece	1991	0,368558019	Spain	2007	0,35648936
Greece	1992	0,374655932	Spain	2008	0,373225361
Greece	1993	0,378753036	Spain	2009	0,380844861
Greece	1994	0,381943405	Spain	2010	0,36585328
Greece	1995	0,381973714	Spain	2011	0,354476303
Greece	1996	0,379374087	Sweden	1980	0,320292622
Greece	1997	0,380136937	Sweden	1981	0,311889648
Greece	1998	0,378983527	Sweden	1982	0,303168774
Greece	1999	0,380182713	Sweden	1983	0,297127753

Greece	2000	0,387530833	Sweden	1984	0,293379426
Greece	2001	0,389062166	Sweden	1985	0,295413941
Greece	2002	0,391259402	Sweden	1986	0,304259956
Greece	2003	0,399834245	Sweden	1987	0,313330531
Greece	2004	0,410188138	Sweden	1988	0,324548751
Greece	2005	0,413582146	Sweden	1989	0,335427731
Greece	2006	0,419439584	Sweden	1990	0,348207265
Greece	2007	0,433763683	Sweden	1991	0,359581143
Greece	2008	0,447689563	Sweden	1992	0,359338909
Greece	2009	0,452422321	Sweden	1993	0,351071149
Greece	2010	0,437407821	Sweden	1994	0,329174578
Greece	2011	0,428309143	Sweden	1995	0,302911729
Iceland	1980	0,43749103	Sweden	1996	0,28227818
Iceland	1981	0,442863584	Sweden	1997	0,269167721
Iceland	1982	0,441289932	Sweden	1998	0,261915326
Iceland	1983	0,437708169	Sweden	1999	0,257813096
Iceland	1984	0,430571049	Sweden	2000	0,257189006
Iceland	1985	0,427536994	Sweden	2001	0,264069736
Iceland	1986	0,421673179	Sweden	2002	0,267983973
Iceland	1987	0,416062087	Sweden	2003	0,265093446
Iceland	1988	0,415541112	Sweden	2004	0,264085889
Iceland	1989	0,404581994	Sweden	2005	0,265790462
Iceland	1990	0,389153689	Sweden	2006	0,271754712
Iceland	1991	0,376274854	Sweden	2007	0,281064421
Iceland	1992	0,361171246	Sweden	2008	0,294222444
Iceland	1993	0,34124577	Sweden	2009	0,303417832
Iceland	1994	0,307219863	Sweden	2010	0,29439801
Iceland	1995	0,276119411	Sweden	2011	0,287987918
Iceland	1996	0,251877785	Switzerland	1980	0,698646188
Iceland	1997	0,245581552	Switzerland	1981	0,695989013
Iceland	1998	0,242302701	Switzerland	1982	0,69286561
Iceland	1999	0,249187529	Switzerland	1983	0,687808573
Iceland	2000	0,246134296	Switzerland	1984	0,683008969
Iceland	2001	0,243954659	Switzerland	1985	0,679138184
Iceland	2002	0,232297152	Switzerland	1986	0,676432252
Iceland	2003	0,221532837	Switzerland	1987	0,674306154
Iceland	2004	0,223789662	Switzerland	1988	0,67233187
Iceland	2005	0,234523952	Switzerland	1989	0,671143711
Iceland	2006	0,290073663	Switzerland	1990	0,669542015
Iceland	2007	0,348609895	Switzerland	1991	0,667773545
Iceland	2008	0,365665197	Switzerland	1992	0,663057566
Iceland	2009	0,359684706	Switzerland	1993	0,654544115
Iceland	2010	0,324745059	Switzerland	1994	0,645090282
Iceland	2011	0,294785321	Switzerland	1995	0,63628304
Ireland	1980	0,131086007	Switzerland	1996	0,629264057
Ireland	1981	0,139104396	Switzerland	1997	0,621887267
Ireland	1982	0,150297612	Switzerland	1998	0,614714265
Ireland	1983	0,145549536	Switzerland	1999	0,607918501
Ireland	1984	0,1315649	Switzerland	2000	0,602436066
Ireland	1985	0,117958695	Switzerland	2001	0,600582659
Ireland	1986	0,093338318	Switzerland	2002	0,596604586
Ireland	1987	0,073791504	Switzerland	2003	0,590919852
Ireland	1988	0,070016667	Switzerland	2004	0,583572686
Ireland	1989	0,087448403	Switzerland	2005	0,579200685
Ireland	1990	0,10847193	Switzerland	2006	0,577167451
Ireland	1991	0,132577732	Switzerland	2007	0,57772243
Ireland	1992	0,144613698	Switzerland	2008	0,580480278
Ireland	1993	0,155831128	Switzerland	2009	0,581116974
Ireland	1994	0,155673698	Switzerland	2010	0,575813293
Ireland	1995	0,160365358	Switzerland	2011	0,570439279
Ireland	1996	0,172862485	United Kingdom	1980	0,165873796
Ireland	1997	0,193130702	United Kingdom	1981	0,184942588
Ireland	1998	0,220276326	United Kingdom	1982	0,205132052
Ireland	1999	0,25238356	United Kingdom	1983	0,2288775
Ireland	2000	0,281403631	United Kingdom	1984	0,24464418
Ireland	2001	0,299670488	United Kingdom	1985	0,260139942
Ireland	2002	0,295782954	United Kingdom	1986	0,275726944
Ireland	2003	0,290684074	United Kingdom	1987	0,28769356
Ireland	2004	0,302641034	United Kingdom	1988	0,304072708

Ireland	2005	0,324690461	United Kingdom	1989	0,330834806
Ireland	2006	0,353091747	United Kingdom	1990	0,352923751
Ireland	2007	0,37358433	United Kingdom	1991	0,364661127
Ireland	2008	0,371073961	United Kingdom	1992	0,370287478
Ireland	2009	0,336553693	United Kingdom	1993	0,371461689
Ireland	2010	0,294203639	United Kingdom	1994	0,369906843
Ireland	2011	0,232925579	United Kingdom	1995	0,368201196
Israel	1980	0,605517805	United Kingdom	1996	0,370835572
Israel	1981	0,60160774	United Kingdom	1997	0,383417398
Israel	1982	0,597803116	United Kingdom	1998	0,403837025
Israel	1983	0,594192386	United Kingdom	1999	0,425970435
Israel	1984	0,59139365	United Kingdom	2000	0,443844169
Israel	1985	0,584690213	United Kingdom	2001	0,462175727
Israel	1986	0,58076793	United Kingdom	2002	0,470519185
Israel	1987	0,574206293	United Kingdom	2003	0,477463156
Israel	1988	0,573160946	United Kingdom	2004	0,481733412
Israel	1989	0,568563163	United Kingdom	2005	0,483121902
Israel	1990	0,562488735	United Kingdom	2006	0,511561334
Israel	1991	0,558172703	United Kingdom	2007	0,516994715
Israel	1992	0,559417903	United Kingdom	2008	0,522815287
Israel	1993	0,560237586	United Kingdom	2009	0,515211701
Israel	1994	0,560796499	United Kingdom	2010	0,490522832
Israel	1995	0,567197442	United Kingdom	2011	0,468098044
Israel	1996	0,575010598	United States	1980	0,06220917
Israel	1997	0,581884325	United States	1981	0,07741987
Israel	1998	0,586306453	United States	1982	0,092894435
Israel	1999	0,586544633	United States	1983	0,100752942
Israel	2000	0,591774702	United States	1984	0,109128781
Israel	2001	0,596242607	United States	1985	0,1229214
Israel	2002	0,596258521	United States	1986	0,132279769
Israel	2003	0,591452956	United States	1987	0,135852262
Israel	2004	0,590428114	United States	1988	0,135234877
Israel	2005	0,588538349	United States	1989	0,13609159
Israel	2006	0,590717733	United States	1990	0,135819137
Israel	2007	0,597061276	United States	1991	0,129203394
Israel	2008	0,605632246	United States	1992	0,116445437
Israel	2009	0,612057865	United States	1993	0,107151054
Israel	2010	0,613797486	United States	1994	0,10571941
Israel	2011	0,618864477	United States	1995	0,111091629
Italy	1980	0,607583582	United States	1996	0,118815124
Italy	1981	0,600550294	United States	1997	0,130057469
Italy	1982	0,588649631	United States	1998	0,145480707
Italy	1983	0,574819207	United States	1999	0,164971769
Italy	1984	0,561270595	United States	2000	0,184468448
Italy	1985	0,549869955	United States	2001	0,204408735
Italy	1986	0,535403192	United States	2002	0,216042072
Italy	1987	0,521371305	United States	2003	0,218360141
Italy	1988	0,508020639	United States	2004	0,220371246
Italy	1989	0,497421443	United States	2005	0,225287944
Italy	1990	0,486953318	United States	2006	0,233747318
Italy	1991	0,478144974	United States	2007	0,239618734
Italy	1992	0,468786567	United States	2008	0,240182251
Italy	1993	0,458812356	United States	2009	0,231443986
Italy	1994	0,448790908	United States	2010	0,207631484
Italy	1995	0,443579733	United States	2011	0,184971094

Country Classification by Income: High-Income Economies (nonOECD Countries)					
Country	**Year**	**σ**	**Country**	**Year**	**σ**
Bahrain	1980	0,629285	Saudi Arabia	1980	0,381112
Bahrain	1981	0,627754	Saudi Arabia	1981	0,378533
Bahrain	1982	0,625793	Saudi Arabia	1982	0,34713
Bahrain	1983	0,625637	Saudi Arabia	1983	0,321827
Bahrain	1984	0,628407	Saudi Arabia	1984	0,327379
Bahrain	1985	0,63052	Saudi Arabia	1985	0,345038
Bahrain	1986	0,625229	Saudi Arabia	1986	0,351109
Bahrain	1987	0,619457	Saudi Arabia	1987	0,361113
Bahrain	1988	0,613368	Saudi Arabia	1988	0,364897

Bahrain	1989	0,601799	Saudi Arabia	1989	0,373258
Bahrain	1990	0,594204	Saudi Arabia	1990	0,380016
Bahrain	1991	0,580438	Saudi Arabia	1991	0,36551
Bahrain	1992	0,599615	Saudi Arabia	1992	0,367164
Bahrain	1993	0,621485	Saudi Arabia	1993	0,392863
Bahrain	1994	0,63938	Saudi Arabia	1994	0,422216
Bahrain	1995	0,654106	Saudi Arabia	1995	0,439161
Bahrain	1996	0,664887	Saudi Arabia	1996	0,449025
Bahrain	1997	0,671857	Saudi Arabia	1997	0,482286
Bahrain	1998	0,677834	Saudi Arabia	1998	0,511769
Bahrain	1999	0,686085	Saudi Arabia	1999	0,542205
Bahrain	2000	0,683301	Saudi Arabia	2000	0,569611
Bahrain	2001	0,685383	Saudi Arabia	2001	0,590299
Bahrain	2002	0,68162	Saudi Arabia	2002	0,606982
Bahrain	2003	0,678191	Saudi Arabia	2003	0,622971
Bahrain	2004	0,687719	Saudi Arabia	2004	0,632677
Bahrain	2005	0,705442	Saudi Arabia	2005	0,644401
Bahrain	2006	0,72699	Saudi Arabia	2006	0,647859
Bahrain	2007	0,742651	Saudi Arabia	2007	0,647505
Bahrain	2008	0,763973	Saudi Arabia	2008	0,640349
Bahrain	2009	0,784072	Saudi Arabia	2009	0,635538
Bahrain	2010	0,786734	Saudi Arabia	2010	0,627946
Bahrain	2011	0,778699	Saudi Arabia	2011	0,611185
Barbados	1980	0,438191	Singapore	1980	0,802552
Barbados	1981	0,437948	Singapore	1981	0,805331
Barbados	1982	0,448677	Singapore	1982	0,808298
Barbados	1983	0,462598	Singapore	1983	0,806392
Barbados	1984	0,471574	Singapore	1984	0,799313
Barbados	1985	0,470972	Singapore	1985	0,792973
Barbados	1986	0,466257	Singapore	1986	0,783668
Barbados	1987	0,463899	Singapore	1987	0,768897
Barbados	1988	0,450482	Singapore	1988	0,760466
Barbados	1989	0,455532	Singapore	1989	0,764041
Barbados	1990	0,460605	Singapore	1990	0,773372
Barbados	1991	0,430828	Singapore	1991	0,782014
Barbados	1992	0,435691	Singapore	1992	0,787972
Barbados	1993	0,401296	Singapore	1993	0,795808
Barbados	1994	0,360498	Singapore	1994	0,803557
Barbados	1995	0,345176	Singapore	1995	0,808248
Barbados	1996	0,3695	Singapore	1996	0,815218
Barbados	1997	0,334994	Singapore	1997	0,825416
Barbados	1998	0,348005	Singapore	1998	0,831129
Barbados	1999	0,39419	Singapore	1999	0,831395
Barbados	2000	0,442706	Singapore	2000	0,829111
Barbados	2001	0,444404	Singapore	2001	0,830279
Barbados	2002	0,419978	Singapore	2002	0,829389
Barbados	2003	0,3748	Singapore	2003	0,827954
Barbados	2004	0,348973	Singapore	2004	0,825886
Barbados	2005	0,37374	Singapore	2005	0,827189
Barbados	2006	0,369325	Singapore	2006	0,827909
Barbados	2007	0,368303	Singapore	2007	0,832934
Barbados	2008	0,357303	Singapore	2008	0,84112
Barbados	2009	0,37837	Singapore	2009	0,847837
Barbados	2010	0,403889	Singapore	2010	0,849512
Barbados	2011	0,421551	Singapore	2011	0,850544
China: Hong Kong SAR	1980	0,866874	Taiwan	1980	0,53775
China: Hong Kong SAR	1981	0,871078	Taiwan	1981	0,53535
China: Hong Kong SAR	1982	0,87463	Taiwan	1982	0,533052
China: Hong Kong SAR	1983	0,872192	Taiwan	1983	0,526638
China: Hong Kong SAR	1984	0,866744	Taiwan	1984	0,528846
China: Hong Kong SAR	1985	0,864787	Taiwan	1985	0,542972
China: Hong Kong SAR	1986	0,866411	Taiwan	1986	0,552152

China: Hong Kong SAR	1987	0,869821	Taiwan	1987	0,561942
China: Hong Kong SAR	1988	0,874171	Taiwan	1988	0,577761
China: Hong Kong SAR	1989	0,878459	Taiwan	1989	0,595234
China: Hong Kong SAR	1990	0,880957	Taiwan	1990	0,606283
China: Hong Kong SAR	1991	0,883298	Taiwan	1991	0,598894
China: Hong Kong SAR	1992	0,886641	Taiwan	1992	0,586777
China: Hong Kong SAR	1993	0,890646	Taiwan	1993	0,584408
China: Hong Kong SAR	1994	0,891387	Taiwan	1994	0,586599
China: Hong Kong SAR	1995	0,89246	Taiwan	1995	0,591864
China: Hong Kong SAR	1996	0,890873	Taiwan	1996	0,600375
China: Hong Kong SAR	1997	0,888344	Taiwan	1997	0,607961
China: Hong Kong SAR	1998	0,889134	Taiwan	1998	0,621819
China: Hong Kong SAR	1999	0,887667	Taiwan	1999	0,637607
China: Hong Kong SAR	2000	0,882266	Taiwan	2000	0,647399
China: Hong Kong SAR	2001	0,879238	Taiwan	2001	0,661966
China: Hong Kong SAR	2002	0,876311	Taiwan	2002	0,662134
China: Hong Kong SAR	2003	0,872241	Taiwan	2003	0,665798
China: Hong Kong SAR	2004	0,867619	Taiwan	2004	0,669653
China: Hong Kong SAR	2005	0,864278	Taiwan	2005	0,683442
China: Hong Kong SAR	2006	0,863375	Taiwan	2006	0,69374
China: Hong Kong SAR	2007	0,865529	Taiwan	2007	0,704305
China: Hong Kong SAR	2008	0,867899	Taiwan	2008	0,71299
China: Hong Kong SAR	2009	0,868986	Taiwan	2009	0,713857
China: Hong Kong SAR	2010	0,867885	Taiwan	2010	0,707323
China: Hong Kong SAR	2011	0,86462	Taiwan	2011	0,708343
Kuwait	1980	-0,15221	Trinidad and Tobago	1980	0,730364
Kuwait	1981	-0,26991	Trinidad and Tobago	1981	0,731535
Kuwait	1982	-0,32717	Trinidad and Tobago	1982	0,716251
Kuwait	1983	-0,28775	Trinidad and Tobago	1983	0,704522
Kuwait	1984	-0,10213	Trinidad and Tobago	1984	0,706865
Kuwait	1985	-0,09588	Trinidad and Tobago	1985	0,712234
Kuwait	1986	-0,22328	Trinidad and Tobago	1986	0,709481
Kuwait	1987	-0,28887	Trinidad and Tobago	1987	0,716768
Kuwait	1988	-0,29172	Trinidad and Tobago	1988	0,721635
Kuwait	1989	-0,32463	Trinidad and Tobago	1989	0,711079
Kuwait	1990	-0,34104	Trinidad and	1990	0,705092

			Tobago		
Kuwait	1991	-0,20408	Trinidad and Tobago	1991	0,695009
Kuwait	1992	-0,04561	Trinidad and Tobago	1992	0,680587
Kuwait	1993	0,044281	Trinidad and Tobago	1993	0,66826
Kuwait	1994	0,118726	Trinidad and Tobago	1994	0,657995
Kuwait	1995	0,190491	Trinidad and Tobago	1995	0,65434
Kuwait	1996	0,243771	Trinidad and Tobago	1996	0,65854
Kuwait	1997	0,310758	Trinidad and Tobago	1997	0,661411
Kuwait	1998	0,342086	Trinidad and Tobago	1998	0,666484
Kuwait	1999	0,402451	Trinidad and Tobago	1999	0,673777
Kuwait	2000	0,435389	Trinidad and Tobago	2000	0,666549
Kuwait	2001	0,458353	Trinidad and Tobago	2001	0,653739
Kuwait	2002	0,481557	Trinidad and Tobago	2002	0,659573
Kuwait	2003	0,511749	Trinidad and Tobago	2003	0,666005
Kuwait	2004	0,552533	Trinidad and Tobago	2004	0,684248
Kuwait	2005	0,587335	Trinidad and Tobago	2005	0,680896
Kuwait	2006	0,62962	Trinidad and Tobago	2006	0,690369
Kuwait	2007	0,675331	Trinidad and Tobago	2007	0,644391
Kuwait	2008	0,71447	Trinidad and Tobago	2008	0,576524
Kuwait	2009	0,742634	Trinidad and Tobago	2009	0,493548
Kuwait	2010	0,739045	Trinidad and Tobago	2010	0,418114
Kuwait	2011	0,741157	Trinidad and Tobago	2011	0,356095
Qatar	1980	0,785696	Uruguay	1980	0,102665
Qatar	1981	0,783295	Uruguay	1981	0,026286
Qatar	1982	0,78124	Uruguay	1982	-0,02668
Qatar	1983	0,78012	Uruguay	1983	-0,06493
Qatar	1984	0,776733	Uruguay	1984	-0,0145
Qatar	1985	0,773103	Uruguay	1985	0,003636
Qatar	1986	0,769598	Uruguay	1986	0,019011
Qatar	1987	0,765088	Uruguay	1987	0,037185
Qatar	1988	0,760367	Uruguay	1988	0,070528
Qatar	1989	0,756319	Uruguay	1989	0,104923
Qatar	1990	0,75272	Uruguay	1990	0,126564
Qatar	1991	0,756929	Uruguay	1991	0,171594
Qatar	1992	0,751514	Uruguay	1992	0,210353
Qatar	1993	0,749502	Uruguay	1993	0,278431
Qatar	1994	0,740424	Uruguay	1994	0,329629
Qatar	1995	0,738455	Uruguay	1995	0,371378
Qatar	1996	0,750026	Uruguay	1996	0,412986
Qatar	1997	0,762236	Uruguay	1997	0,456899
Qatar	1998	0,782147	Uruguay	1998	0,500731
Qatar	1999	0,791146	Uruguay	1999	0,538021
Qatar	2000	0,789023	Uruguay	2000	0,560138
Qatar	2001	0,798432	Uruguay	2001	0,572766
Qatar	2002	0,808445	Uruguay	2002	0,581716
Qatar	2003	0,814137	Uruguay	2003	0,585057
Qatar	2004	0,830511	Uruguay	2004	0,585654
Qatar	2005	0,837899	Uruguay	2005	0,5931
Qatar	2006	0,830882	Uruguay	2006	0,607049

Qatar	2007	0,845445	Uruguay	2007	0,623408
Qatar	2008	0,84936	Uruguay	2008	0,636793
Qatar	2009	0,856875	Uruguay	2009	0,651618
Qatar	2010	0,851322	Uruguay	2010	0,656183
Qatar	2011	0,837875	Uruguay	2011	0,667259

APPENDIX 2. Steady State Solution

The steady state solution of VES production function in Solow-Swan growth model. (A.1) and (A.2) are the accumulation equations for public capital and private capital stocks, respectively.

$$\dot{K}_g = s_g Y - (n+g+\delta)K_g \tag{A.1}$$

$$\dot{K}_p = s_p Y - (n+g+\delta)K_p \tag{A.2}$$

$$\dot{K}_g = s_g A[K_g^{\alpha_2}(K_p + b_2\alpha_2 K_g)^{(1-\alpha_2)}]^{\alpha_1} L^{1-\alpha_1} - (n+g+\delta)K_g \tag{A.3}$$

$$\dot{K}_p = s_p A[K_g^{\alpha_2}(K_p + b_2\alpha_2 K_g)^{(1-\alpha_2)}]^{\alpha_1} L^{1-\alpha_1} - (n+g+\delta)K_p \tag{A.4}$$

At the steady state $\dot{K}_g = \dot{K}_p = 0$

$$s_g A[K_g^{\alpha_2}(K_p + b_2\alpha_2 K_g)^{(1-\alpha_2)}]^{\alpha_1} L^{1-\alpha_1} = (n+g+\delta)K_g \tag{A.5}$$

$$s_p A[K_g^{\alpha_2}(K_p + b_2\alpha_2 K_g)^{(1-\alpha_2)}]^{\alpha_1} L^{1-\alpha_1} = (n+g+\delta)K_p \tag{A.6}$$

Dividing the equations (A.5) and (A.6)

$$K_g = \frac{s_g}{s_p} K_p \tag{A.7}$$

Equation (A.7) shows the relationship between public and private capital stocks. Substituting equation (A.7) into equation (A.5) and simplifying we obtain the steady state value for private capital stock.

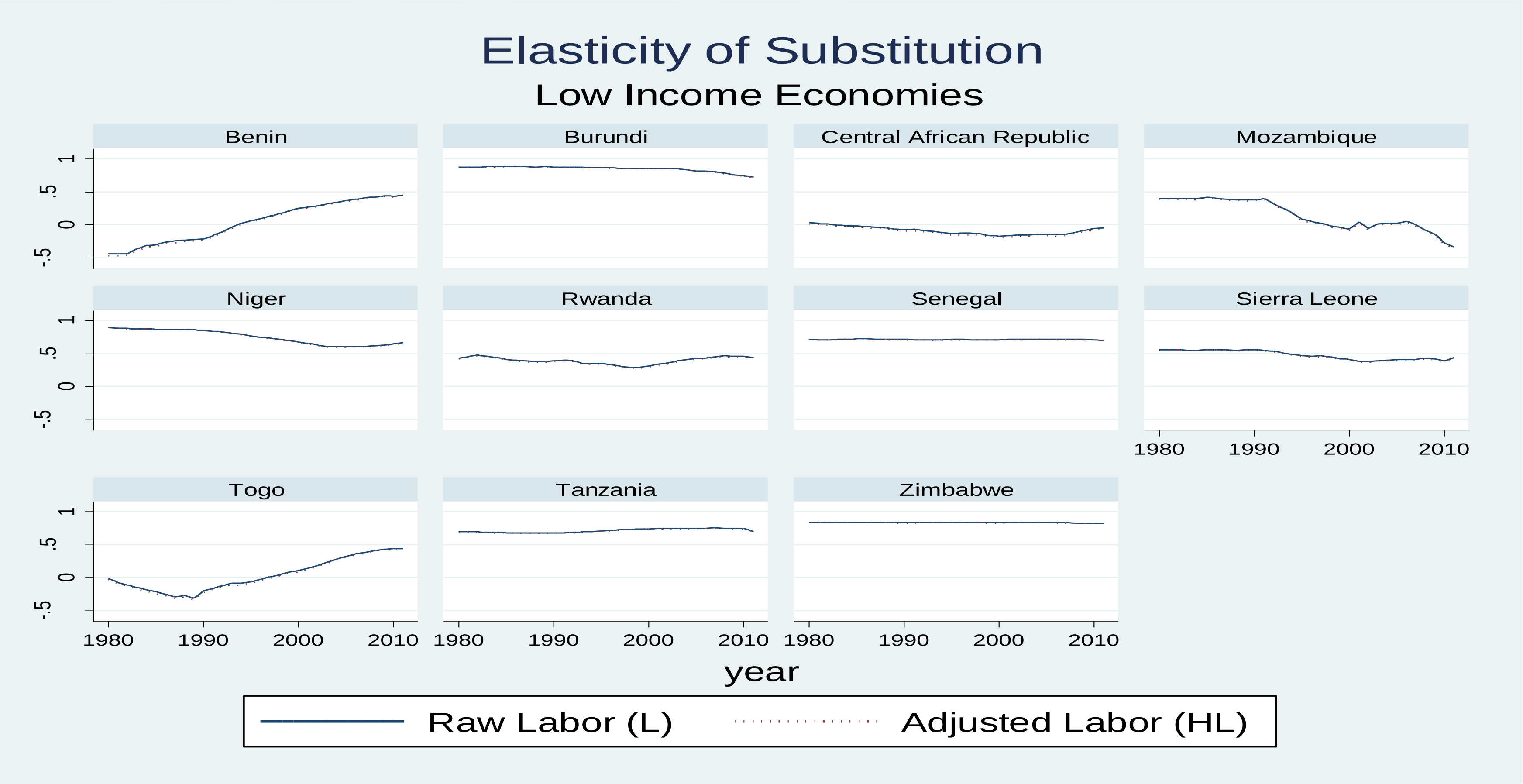
$$K_p = \left(\frac{A s_p \left(\frac{s_g}{s_p}\right)^{\alpha_1\alpha_2} \left(1 + b_2\alpha_2 \frac{s_g}{s_p}\right)^{\alpha_1(1-\alpha_2)}}{n+g+\delta} \right)^{1/1-\alpha_2} L \tag{A.8}$$

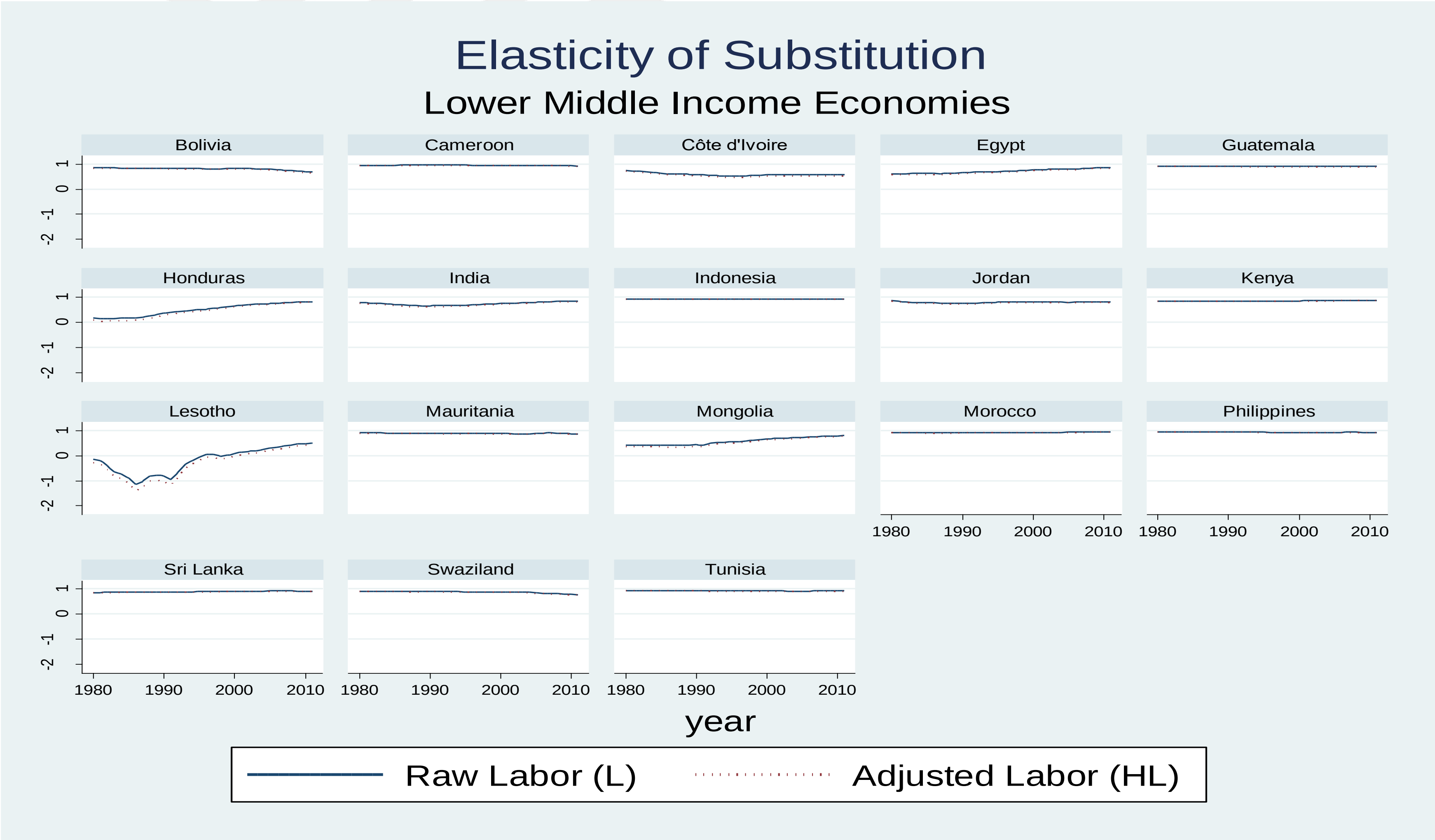
Similarly, we obtain the steady state for public capital stock as,

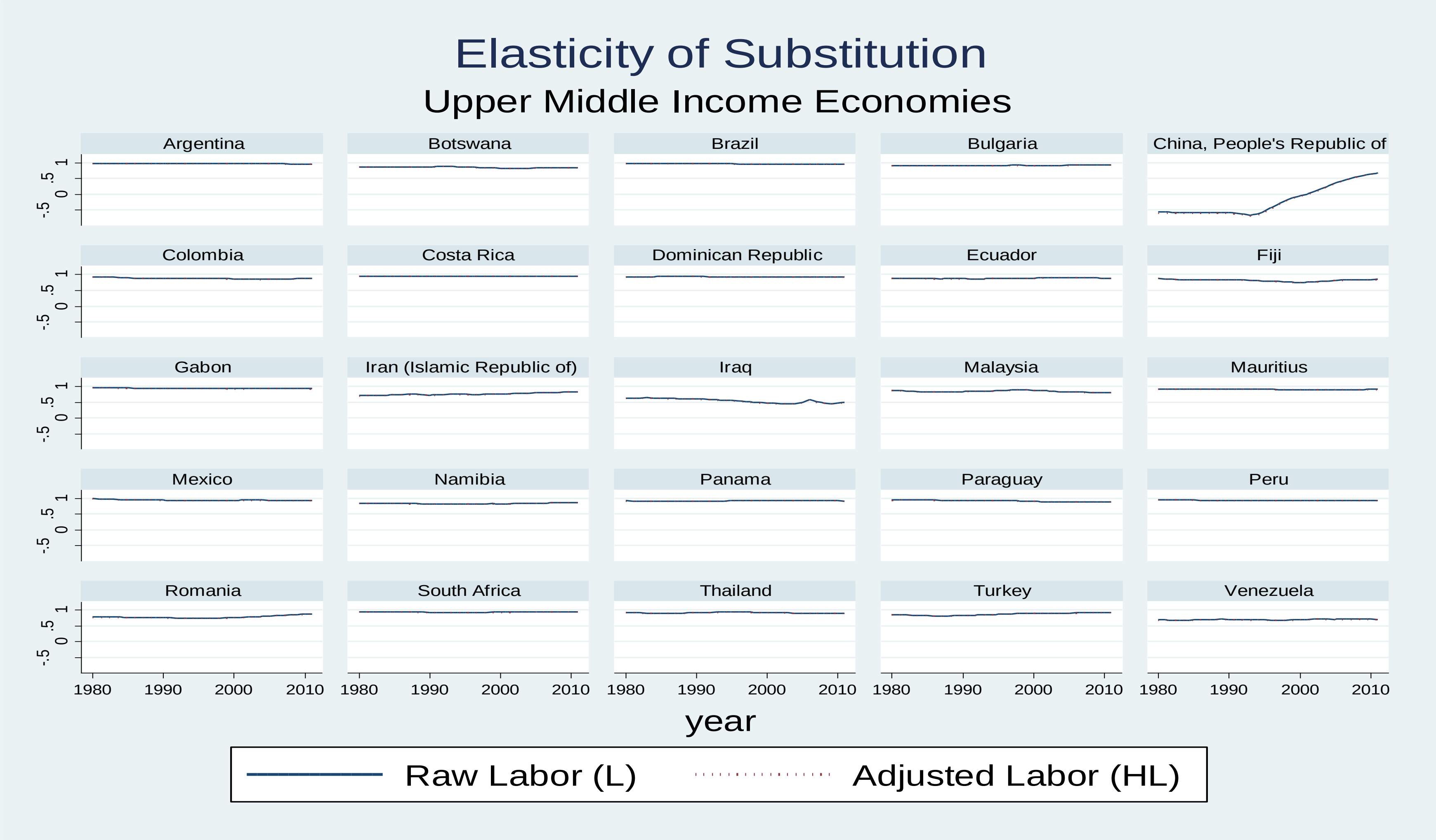
$$K_g = \frac{s_g}{s_p} \left(\frac{A s_p \left(\frac{s_g}{s_p}\right)^{\alpha_1\alpha_2} \left(1 + b_2\alpha_2 \frac{s_g}{s_p}\right)^{\alpha_1(1-\alpha_2)}}{n+g+\delta} \right)^{1/1-\alpha_2} L \tag{A.9}$$

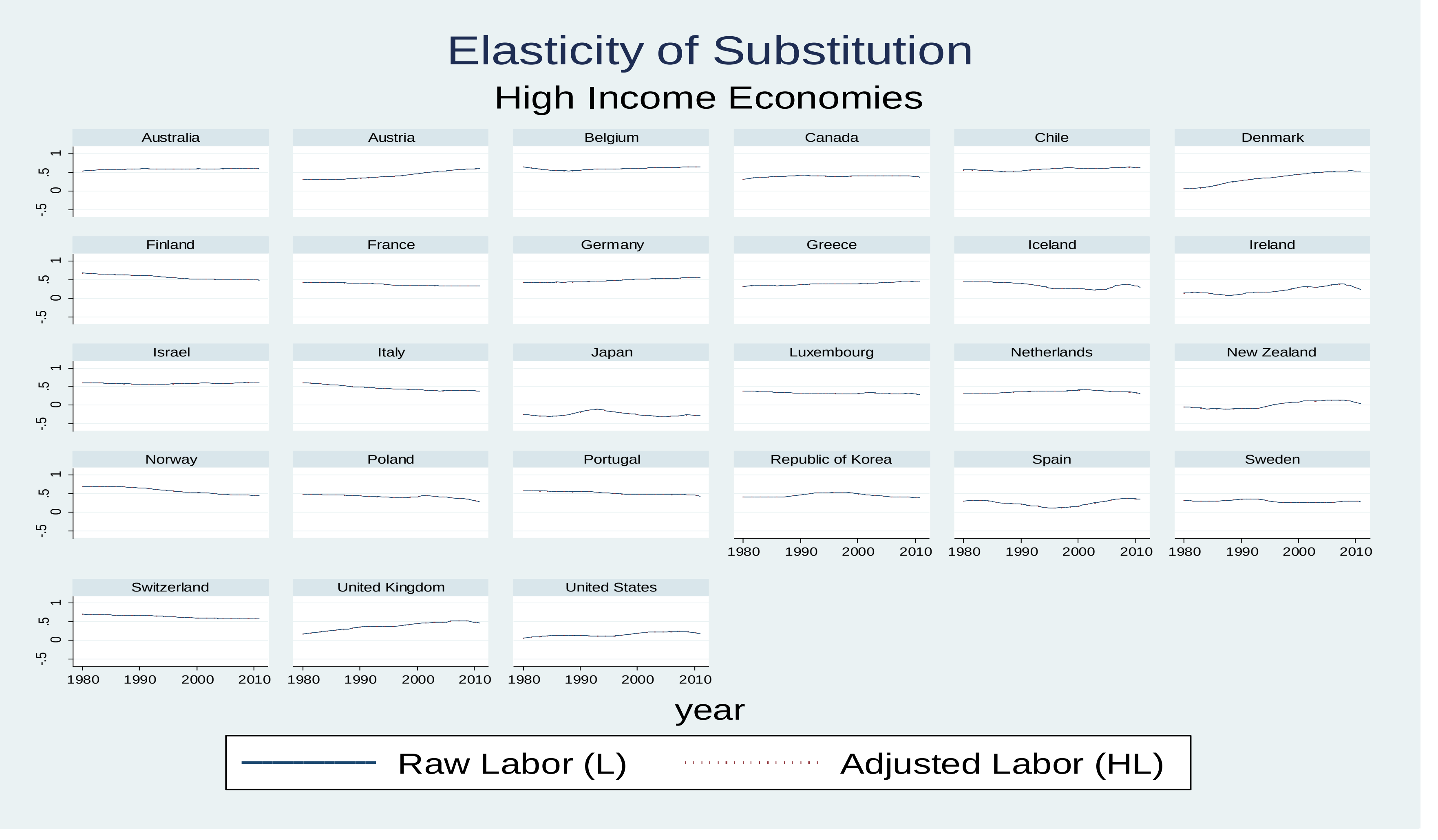
APPENDIX 3. The Change of Elasticity of Substitution over Time

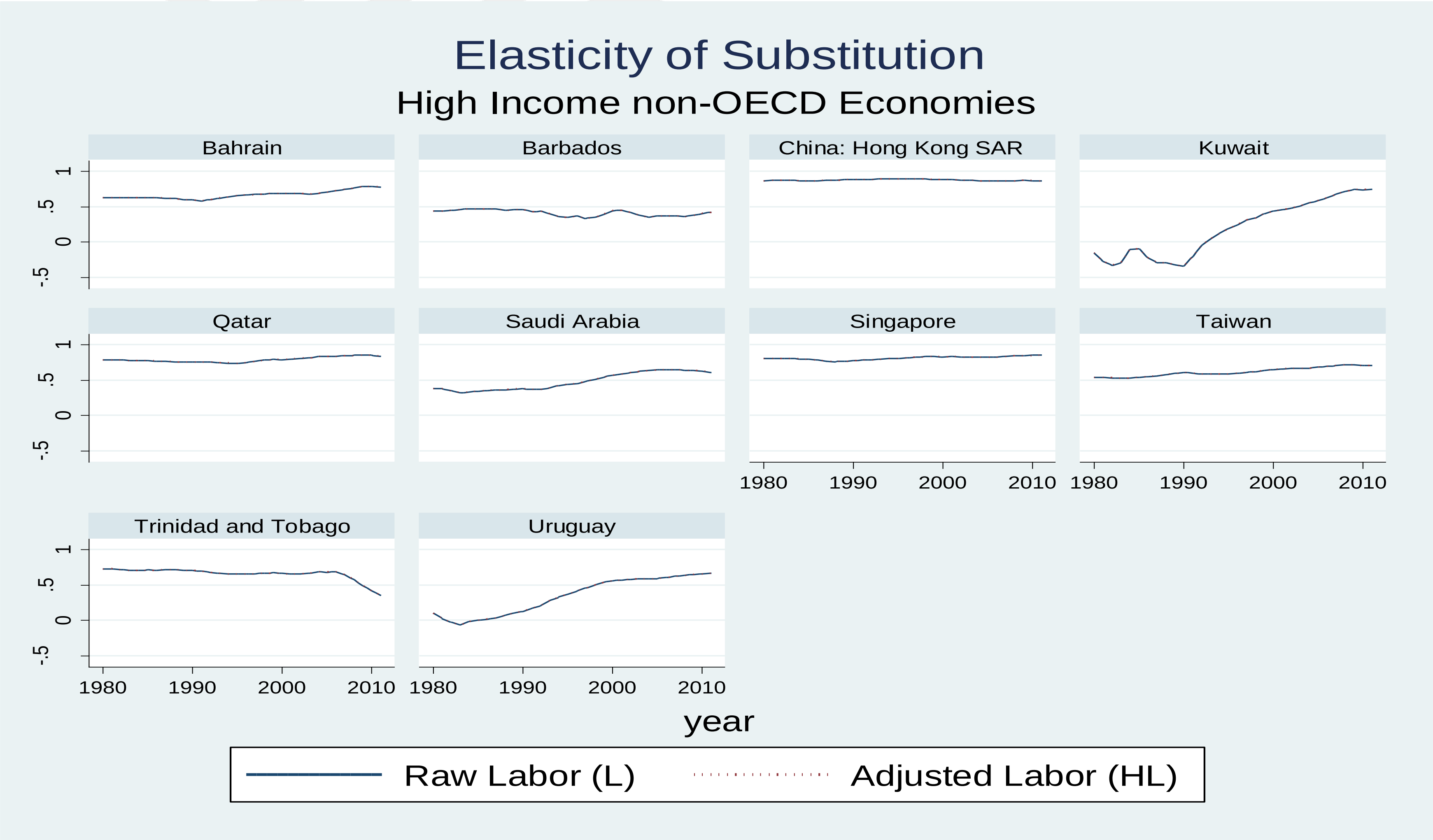
The evolution of the elasticity of substation for the countries classified by income level (low-income, lower-middle-income, upper-middle-income, high-income OECD, and high-income non OECD) is illustrated in the following figures.




Elasticity of Substitution
Lower Middle Income Economies
Bolivia
Cameroon
Côte d'Ivoire
Egypt
Guatemala
Honduras
India
Indonesia
Jordan
Kenya
Lesotho
Mauritania
Mongolia
Morocco
Philippines
Sri Lanka
Swaziland
Tunisia
-2 -1 0 1
1980 1990 2000 2010
year
Raw Labor (L)
Adjusted Labor (HL)


Elasticity of Substitution
Upper Middle Income Economies
Argentina
Botswana
Brazil
Bulgaria
China, People's Republic of
Colombia
Costa Rica
Dominican Republic
Ecuador
Fiji
Gabon
Iran (Islamic Republic of)
Iraq
Malaysia
Mauritius
Mexico
Namibia
Panama
Paraguay
Peru
Romania
South Africa
Thailand
Turkey
Venezuela
-.5
0
.5
1
1980
1990
2000
2010
year
Raw Labor (L)
Adjusted Labor (HL)


Elasticity of Substitution
High Income Economies
Australia
Austria
Belgium
Canada
Chile
Denmark
Finland
France
Germany
Greece
Iceland
Ireland
Israel
Italy
Japan
Luxembourg
Netherlands
New Zealand
Norway
Poland
Portugal
Republic of Korea
Spain
Sweden
Switzerland
United Kingdom
United States
-.5
0
.5
1
1980
1990
2000
2010
year
Raw Labor (L)
Adjusted Labor (HL)



Elasticity of Substitution
High Income non-OECD Economies
Bahrain
Barbados
China: Hong Kong SAR
Kuwait
Qatar
Saudi Arabia
Singapore
Taiwan
Trinidad and Tobago
Uruguay
-.5
0
.5
1
1980
1990
2000
2010
year
Raw Labor (L)
Adjusted Labor (HL)

APPENDIX 4. Selected Studies

Study	Specific ation	Period	Sample/Sector/Model
Duffy and Papageorgiou (2000): A Cross-Country Empirical Investigation of the Aggregate Production Function Specification	CD CES	1960-1987	82 Countries
Fare and Yoon (1981): Variable Elasticity of Substitution in Urban Housing Production	CD, CES, VES, WDI	-	Single-family housing data for Santa Clara County, California
Sirmans, Kau, and Lee (1977): The Elasticity of Substitution in Urban Housing Production: A VES Approach	CES, VES	-	Single-family housing data for Santa Clara County, California
Bairam (1989): Learning-by-doing, the variable elasticity of substitution and economic growth in Japan	CD, VES	1878-1939	Japanese economy
Bairam (1990): Capital-Labor Substitution and Slowdown in Soviet Economic Growth a Re-Examination	VES	1950-1975	Soviet economy
Kazi (1980): The Variable Elasticity of Substitution Production Function: A Case Study for Indian Manufacturing Industries	CD, CES, VES	1973-1975	Indian two digit and three digit industries
Brianzoni, Mammana, and Michetti (2012): Variable elasticity of substituition in a discrete time Solow–Swan growth model with differential saving	VES	-	One Sector Solow –Swan Growth Model
Karagiannis et. Al. (2004): Variable Elasticity of Substitution and Economic Growth	VES	1960-1987	82 Countries
Yuhn (1991): Economic Growth, Technical Change Biases, and the Elasticity of Substitution: A Test of the De La Grandville Hypothesis	Translog Cost function	1962-1981	The United States and Korean manufacturing industry
Miyagiwa and Papageorgiou (2003): Elasticity of substitution and growth:	CES	-	Diamond Overlapping Generation Model

Study	Specification	Period	Sample/Sector/Model
normalized CES in the Diamond model			
Irmen (2010): Steady-State Growth and the Elasticity of Substitution	CES	-	-
Gamlath and Lahiri (-): Technical Change, Variable Elasticity of Substitution and Economic Growth	VES	-	-
Sato and Hoffman (1968): Production Functions with Variable Elasticity of Factor Substitution: Some Analysis and Testing	CES, VES	1909-1960	Private non-Farm Sectors for the United States and Japan
Revenkar (1971b): Capital-Labor Substitution, Technological Change And Economic Growth: The U.S. Experience, 1929-1953	CD, VES	1929-1953	Private non-Farm Sectors for the United States
Lovell (1973b): Estimation and Prediction with CES and VES Production Functions	CES, VES	1947-1963	U.S. Manufacturing Sector
Lovell (1968): Capacity Utilization and Production Function Estimation in Postwar American Manufacturing	CD, CES, VES	1949-1963	16 two-digit US manufacturing Industries
Roskamp (1977): Labor Productivity and the Elasticity of Factor Substitution in West German Industries1950-1960	CES, VES	1950-1960	West German Industries
Lu and Fletcher (1968): A Generalization of the CES Production Function	CES, VES	1957	17 two-digit United States manufacturing industries
Revenkar (1971a): A Class of Variable Elasticity of Substitution Production Function	CD, CES, VES	1957	12 two-digit United States manufacturing industries
Lovell (1973a): CES and VES Production Functions in a Cross-Section Context	CD, CES, VES	1958	17 two-digit United States manufacturing industries
Klump and de la Grandville (2000): Economic Growth and the Elasticity of Substitution: Two Theorems and Some Suggestions	CES	-	Neoclassical growth model in the tradition of Solow

Study	Specific ation	Period	Sample/Sector/Model
Klump and Preissler (2000): CES Production Functions and Economic Growth	CES	-	Neoclassical growth model in the tradition of Solow
Masanjala and Papageorgiou (2004): The Solow Model With CES Technology: Nonlinearities and Parameter Heterogeneity	CES	-	98 countries
Bucci and Bo (2011): On the interaction between public and private capital in economic growth	CD, CES	-	-
Antras (2004): Is the U.S. Aggregate Production Function Cobb-Douglas? New Estimates of the Elasticity of Substitution	CD, CES	1948-1998	Private sector of the United States Economy
Arrow et. al. (1961): Capital-Labor Substitution and Economic Efficiency	CD, CES	The data pertain to different years between 1949-1955	Three-digit industries for 19 Countries
Dessus and Herrera (2000): Public Capital and Growth Revisited: A Panel Data Assessment	CD	1981-1991	28 developing countries
Grassetti et. Al. (2015): Variable Elasticity of Substitution in the Diamond Model: Dynamics and Comparisons	VES	-	-
Young (2013): U.S. Elasticities of Substitution and Factor Augmentation at the Industry Level	CES	1960-2005	2-digit 35 industries of United States
Aschauer (1997): Do States Optimize? Public Capital and Economic Growth	CD	1970-1990	US (48 contiguous)
Aschauer (1998a): Is Public Expenditure Productive?	CD	1949-1985	US
Aschauer (1998b): Public Capital	CD	1970-	46 low and middle-income

Study	Specification	Period	Sample/Sector/Model
and Economic Growth: Issues on Quantity, Finance, and Efficiency		1990	countries
Lazkano and Pham (2016): Can Capital-Energy Substitution Foster Economic Growth?	VES	1971-2011	108 countries
Mallick (2010): Capital-labor substitution and balanced growth	CES	1970-2000	112 countries
Singh (2012) Does public capital crowd-out or crowd-in private capital in India?	Granger, VAR	1950-2001	India
Daniels and Kakar (2017): Economic Growth and the CES Production Function with Human Capital	CD, CES	1960-1985	Non-oil producing countries (98 countries for CD case, 84 countries for CES case), and a subset of previous countries whose population is less than 1 million in 1960
Ling (2010): Elasticity Of Substitution And Economic Growth	Transcendental Logarithm Production Function	1970-2005	Malaysia manufacturing sector
Goldar et. al. (2013): Elasticity of substitution between Capital and Labor Inputs in Manufacturing Industries of Indian Economy	CES	1980-2008	22 Indian manufacturing industries
Miyagiwaa and Papageorgiou (2006): Endogenous Aggregate Elasticity Of Substitution	CD, CES	-	-
Lukáš Rečka (2013): Estimation of the elasticity of substitution of production factors in CEE economies	CES	1995-2009	13 Industry sector for CEE (Central and East Europe) countries
Zarembka (1970): On the Empirical Relevance of the CES Production Function	CES	1957,1958	13 two-digit United States Manufacturing industries
Lu (1967): Variable elasticity of substitution production functions, technical change, and factor shares	CD, CES, VES	1957	17 two-digit manufacturing industries of United States
Kadiyala (1972): Production Functions and Elasticity of	CD, CES, VES	-	-

Study	Specification	Period	Sample/Sector/Model
Substitution			
Mallick (2012): The role of the elasticity of substitution in economic growth: A cross-country investigation	CES	1950-2000	90 countries
Leung (2003): The Elasticity of Substitution and Endogenous Growth	CES	-	-
Henderson and Kumbhakar (2006): Public and Private Capital Productivity Puzzle: A Nonparametric Approach	CD, Translog Production Function	1970-1986	48 contiguous state of US

APPENDIX 5. Ethics Commission Form

HACETTEPE UNIVERSITY
GRADUATE SCHOOL OF SOCIAL SCIENCES
ETHICS COMMISSION FORM FOR THESIS

HACETTEPE UNIVERSITY
GRADUATE SCHOOL OF SOCIAL SCIENCES
DEPARTMENT OF ECONOMICS

Date: 08/07/2018

Thesis Title: Private-Public Capital, Elasticity of Substitution and Economic Growth

My thesis work related to the title above:

1. Does not perform experimentation on animals or people.
2. Does not necessitate the use of biological material (blood, urine, biological fluids and samples, etc.).
3. Does not involve any interference of the body's integrity.
4. Is not based on observational and descriptive research (survey, interview, measures/scales, data scanning, system-model development).

I declare, I have carefully read Hacettepe University's Ethics Regulations and the Commission's Guidelines, and in order to proceed with my thesis according to these regulations I do not have to get permission from the Ethics Board/Commission for anything; in any infringement of the regulations I accept all legal responsibility and I declare that all the information I have provided is true.

I respectfully submit this for approval.

08/07/2018

Date and Signature

Name Surname:	Abdulmecit YILDIRIM
Student No:	N12247960
Department:	Economics
Program:	Ph.D. in Economics
Status:	☐ MA ☒ Ph.D. ☐ Combined MA/ Ph.D.

ADVISER COMMENTS AND APPROVAL

Doç. Dr. Bahar SAĞLAM

APPENDIX 6. Originality Report

HACETTEPE UNIVERSITY
GRADUATE SCHOOL OF SOCIAL SCIENCES
Ph.D. DISSERTATION ORIGINALITY REPORT

HACETTEPE UNIVERSITY
GRADUATE SCHOOL OF SOCIAL SCIENCES
DEPARTMENT OF ECONOMICS

Date: 03/07/2018

Thesis Title: Private-Public Capital, Elasticity of Substitution and Economic Growth

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Program: Ph. D. in Economics

Status: ☒ Ph.D. ☐ Combined MA/ Ph.D.

ADVISOR APPROVAL

APPROVED.

Doç. Dr. Bahar SAĞLAM