STEADY-STATE AND TRANSIENT ANALYSIS OF TRANSMISSION LINES BY USING STATE-SPACE TECHNIQUES

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ABSTRACT

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In this study, the steady-state and transient analysis of transmission lines using state-space technique is investigated.

Transmission lines are considered as the interconnection of many lumped parameter sections. By this approach state-space equations are formulated for the system choosing the capacitor voltages and inductor currents as the state variables. These equations are solved using state-space techniques to compute steady-state and transient analysis of transmission lines for different source and load terminations. Both the formulation and solutions steps are programmed in Fortran language to be handled by a digital computer.

To illustrate the prepared program and to sight different aspects of the lumped parameter approach several examples are carried out. When it is compared with other methods, it is shown that this method has some superior properties cited in the conclusion section of the thesis.

Keywords: State Equations, Transmission Lines, Steady-state Response, Transient Response

ÖZET

İLETİM HATLARINDA MEYDANA GELEN AŞIRI GERİLİMLERİN UZAY-DURUM DENKLEMLERİ KULLANILARAK ÇÖZÜMÜ

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Bu çalişmada, durum uzayı tekniği kullanılarak iletim hatlarının kalıcı ve geçiçi rejim analizi incelenmiştir.

Enerji hatları cok sayıda toplu parametreli devrelerin birbirine bağlantısı olarak düşünülmüştür. Bu yaklaşimla kapasitans gerilimleri ve indüktans akımları durum değişkeni seçilerek sistemin durum uzayı denklemleri formüle edilmiştir. Bu denklemler iletim hatlarının çesitli yük ve kaynak sonlandırmaları altında kalıcı ve geçiçi çözümlerini bulmak için durum uzayı teknikleri kullanılarak çözülmüştür. Gerek formülasyon, gerekse çözüm kısımları Fortran programları dilini kullanarak sayısal bir bilgisayarda yapılmak üzere programlanmıştır.

Hazırlanan programı tanıtmak ve toplu-parametreler yaklaşımının farklı yönlerini göstermek için birkaç örnek üzerinde çalışılmıştır. Diğer bazı metodlarla yapılan karşılaştırmada, bu metodun tezin sonuçlar kısmında değinilmiş olan bazı üstün özelliklerinin olduğu gösterilmiştir.

Anahtar kelimeler: Durum Denklemleri, İletim Hatları, Kalıcı Çözüm, Geçiçi Çözüm

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LIST OF SYMBOLS

The following nomenclature defines the principal symbols used in the thesis.

a	Radius of conductor
a	Velocity
A, B, C, D	Coefficient matrices
c	Capacitance
C	Capacitance per section
C'	Capacitance per unit length
d	Spacing
D	Geometric mean distance
ε	Permitivity
g	Conductance
G	Conductance per section
G'	Conductance per unit length
i	Current
l	Inductance
L	Inductance per section
L'	Inductance per unit length
L_s	Source inductance
l	Length of line
λ	Eigenvalue
μ_o	Permeability of free space
N	Number of sections
Ψ	Phase
r	Resistance

R	Resistance per section
R'	Resistance per unit length
R_s	Source resistance
R_i	Residue matrices
σ	Number of distinct eigenvalues
t	Time
t_o	Initial time
u	Input vector
v	voltage
V_s	Source voltage
V_L	Load voltage
ω	Angular frequency
x	State vector
x_o	Initial state vector
x_c	Complete response
x_t	Transient response
$Z_{k\ell}$	Constituent matrices

CHAPTER 1

INTRODUCTION

Transmission line is one of the most important part of the electric power system. The function of the electric power transmission line is to transmit power to load centers and large industrial users beyond the primary distribution lines.

The steady-state analysis of the power transmission lines is an important subject in power system analysis. Steady-state operating conditions of an electric power transmission line are generally performed for transmission system planning and operational planning in connection with system operation and control. Many books such as [1-4] use very simple equivalent circuits for short and medium length transmission lines, which does not give accurate results. In the case of long transmission lines distributed parameter representation is used for more accurate results.

Transmission faults and switching operations on any power transmission system cause sudden changes in voltage and current. The troubles are usually in the form of broken conductors or circumstances in which conductors are temporarily connected to each other. The transients due to such faults need to be accurately predicted for the design of circuit breakers.

Switching transients generated by closing or opening of circuit breakers are important for insulation coordination and for the protection of system components. Although the system insulation level must be sufficiently high in order not to hazard the reliability of the system, at the same time there are economic reasons for keeping it as low as possible.

Temporary over voltages and currents as a result of faults and switching operations studies come under the general umbrella of electromagnetic transient analysis. The degree of the representation of plant components depend on the type of study. In the past Transient Analyzer was used for the prediction of transient voltages. Later by use of digital computer which is more general than Transient Analyzer, various methods have been developed, some of which are capable of high accuracy.

Lattice diagram technique [5,6] may be used in calculation of transients in power systems. This technique is an approximation of travelling wave equation. The reflection coefficients are calculated and by using space time diagrams the transient voltage or current at any point on the line can be obtained. In this method it is very difficult to incorporate the resistive effects in addition to switches and nonlinear elements in the network.

Most existing general purpose programs perform transient simulation in the time domain based on Bergeron's method [7]. This method uses linear relationships (characteristics) between current and voltage which are invariant from the point of view of an observer travelling with the wave. The discrete steps (or time intervals) of the digital solution cause truncating errors that often leads to numerical instability. The use of the trapezoidal rule for the integration of the ordinary differential equations has proved [8] invaluable in this respect.

In 1960's, Professor Dommel [9] improved a method which has become a general tool for electromagnetic transient simulation. In this method simple equivalent networks are derived for all components in the system to formulate nodal equations. By repeated solutions of these equations the transients are calculated. The program based on this method is called Electromagnetic Transient Program (EMTP). Although this method permits an accurate simulation of transients in networks involving distributed as well as lumped parameters, the disadvantage of this method is that past history of the network is needed. Also discrete time steps are important in the evaluation of trapezoidal integration.

Fourier transform technique [10-13] uses frequency domain for analysis of power system transients. For wide range of frequencies the computation is done and inverse Fourier transform is used for frequency to time domain transformation. Frequency dependent parameters can be included to the system and accurate results of transients on distributed parameter lines can be obtained. These programs are not general. Because it is difficult to implement a network containing nonlinear elements or multi-switching operations.

State-variable technique [14] uses Lumped-parameter representation for transmission line model and a state model is formulated. State equations are solved by using trapezoidal rule of integration. This technique is easy to implement on a digital computer and it can be extended to include nonlinear elements such as surge arresters [14]. The disadvantage of this method is that to deduce the state of the system at any time the previous state must be known. For this reason the computation must be carried out step by step from switching instant. This may accumulate errors in the analysis and spend the computer time.

The appearance of large computers paved the way for developments in power system analysis. In the early years of this development the mismatch between the size of the problems to be analyzed and the limited capability of the computer technology encouraged research into algorithmic efficiency. Such efforts made difficult to maintain the high levels of reliability.

The cost of processing information and computer memory is declining rapidly and the speed of processing is increasing. Under these conditions state-space technique can be used by an approximation; although the state dimension increases, this makes it possible to use explicit results valid for time-invariant linear lumped systems.

This thesis uses state-space technique for the solution of steady-state and transient voltages and currents in power lines. The state space equations are obtained by using conventional state variable technique [14]. Instead of trapezoidal rule of integration; explicite formulas of state space technique are used to solve these equations. The advantage over the numerical integration technique is that the state of the system at any time can be calculated without calculating the previous states.

After this introductory chapter, in the second chapter a general description of power transmission systems is given. Lumped parameter transmission line equivalent circuits and different exciting and load conditions are presented.

In Chapter 3, state-space equations for lumped parameter models introduced in the previous chapter are formulated. Also the state-space equations for switching operations and short and open circuit faults are given.

Analytical methods of steady-state and transient solutions of state space equations are examined as a subject of Chapter 4. Particular attention is focused on the computation of function of a matrix, which takes a particular attention rate on the solution of state-space equations.

On the base of the developed theory in Chapters 3, and

4, a general computer program (SSTAP: Steady-State and Transient Analysis Program) for the steady state and transient analysis of power transmission systems is written; and the description of the program including its usage, memory requirements and execution time, is presented in Chapter 5.

In Chapter 6 some examples are worked out by using SSTAP, The results are compared with data obtained by different methods, and a general critics on the performance of the prepared program is summarized. Problems associated in programming and execution stages are discussed.

CHAPTER 2

DESCRIPTION OF TRANSMISSION SYSTEMS

2.1 INTRODUCTION

The electrical energy is transferred from generating stations to consumers through overhead lines and underground cables.

Overhead lines are ideally suited for energy transfer in open country and rural areas, whereas underground cables are ideal for built-up areas. For the same power rating the cost ratio between underground cables and overhead lines is in the region of 10 to 15. Therefore there is a strong incentive to use in a particular system as much overhead line as practicable. It is interesting to note that the fundamental reason behind this is that overhead lines are immersed in a reasonably good insulation material, i.e., air, while underground cables are imbedded in a good conductor-wet earth. Of course, we capitalize on the latter when we use earth return systems. The high cost of underground cables is not only that of digging trenches, but the cost of very substantial insulating material.

Overhead line practice encompasses voltages between 120 V and 765 kV, while lines up to 1.5 MV are being considered for the future. For convenience we can label line voltages as follows:

Low voltage distribution	110 V to 415 V
High voltage distribution and subtransmission	6.6 kV to 70 kV
Extra high voltage transmission	110 kV to 500 kV
Ultra high voltage transmission	735 kV to 1500 kV

The design of lines at the lower end of the voltage scale has become standardized and there are very few basic problems to be resolved. In the u.h.v. range, however, a considerable amount of research and developments is being carried out.

A transmission system can not be thought independent from its terminations, namely the exciting system and the load. Transmission line, and including these terminations, are described in the following sections.

2.2 ELECTRICAL CHARACTERISTICS OF POWER TRANSMISSION LINES

Transmission lines have resistance R' due to the resistivity of the conductor, shunt conductance G' due to leakage currents in the insulation, inductance L' due to the magnetic field between conductors, and capacitance C' due to the electric field between the conductors; all expressed in per unit length.

These line parameters determine line performance, and equivalent circuits can be set up that enable us to represent the line as lumped components in a power system network. This is of great importance since such power system networks are used to study the flow of active and reactive power, the stability of the system and its performance under fault conditions, and the way the system should be operated to make the cost of generating a minimum.

It is the usual practice for books on power systems to derive the line parameters on the first. The topic is invariably cowered in elementary courses on fields, network theory, or transmission line theory. It seems superfluous to repeat here this material. Instead, a short summary of the salient points relating to line parameters is given.

2.2.1 Resistance and Conductance

These are the least important of line parameters as they effect the transmission line performance to a small degree. In power lines the the effect of shunt conductance is small and is usually neglected. However, for short lines, for which under emergency conditions the loading may be limited by conductor temperature rise, the series resistance plays an important part in defining the line active power loss and therefore its value should be known. If a calculation involving loss minimization or optimum economic operation is to be undertaken the line resistance should be known.

The effective a.c. resistance of small diameter conductors at power frequencies is very nearly equal to the d.c resistance. However, as the conductor cross section increases, the distribution of current becomes non-uniform. This phenomenon is called 'skin effect'. It is caused by the fact that portions of the conductor near the periphery are linked with fewer flux lines then portions near the conductor center. Since the inductance of a conductor element is proportional to the flux linkages per ampere, the inner areas of the conductor possess higher inductance than the outer areas and the current tends to congregate in the region of the conductor skin. This reduces the effective resistance of the conductor.

The resistance of lines is determined from manufacturer's tables where allowance is made for stranding, composite conductors, and skin effect. Resistance of transmission lines ranges from 0.5 to 0.015 Ω/km, the lower resistance being that of e.h.v. overhead and underground lines.

The shunt conductance represent loss due to leakage current along insulator strings and due to corona. There are no reliable data on shunt conductance of overhead lines as this is heavily dependent on atmospheric conditions and pollution. In the case of underground cables, data are given in manufacturer's tables and represent the loss of the dielectric material.

2.2.2 Inductance

Detailed derivations for the inductance of transmission lines can be found in references [2-4]. In such derivations both the partial flux linkages within the conductor cross section and the external flux linkages are taken into account. The inductance of a single-phase transmission line consisting of two conductors of radius a and spacing d is given by

$$L' = \frac{\mu_o}{\pi} \ln \frac{d}{\bar{r}} \text{ H/m} \tag{2.1}$$

where $\bar{r} = e^{-0.25} \times a = 0.779a$.

$\bar{r}$ is known as the geometric mean radius (g.m.r) and represents the radius of the hollow conductor of thickness small enough for no internal flux linkages to be present. Eq. 2.1 is valid where the relative permeability of the conductor is unity.

For a three-phase overhead line, the inductance of each phase is different unless the three conductor, occupy the vertices of an equilateral triangles, a geometry not usually adopted in practice. To equalize the inductance of the three phases with non-equilateral spacing, the lines are transposed in such a way that each phase occupies successively all three possible locations.

For a transposed three-phase line the inductance per phase is

$$L' = \frac{\mu_o}{2\pi} \ln \frac{D}{\bar{r}} \text{ H/m} \tag{2.2}$$

where $\bar{r}$ is again the g.m.r., which, is supplied by the manufacturers, takes into account not only the internal inductance but also the composition and stranding effect of the conductor, and D is the geometric mean distance (g.m.d.) This is a function of the distance d between the conductors of the three phases a, b, and c given by

$$D=\sqrt[3]{(d_{ab} \times d_{bc} \times d_{ca})} \tag{2.3}$$

The reactance per phase of overhead lines at 50 Hz ranges in practice between 0.2 and 0.5 Ω/km, which corresponds approximately to an inductance range of 0.637 and 1.6 mH/km.

The inductance of cables is complicated by the magnetic interaction between the conductors and sheaths. The reactance of e.h.v cables at 50 Hz ranges between 0.13 and 0.22 Ω/km, which corresponds 0.413 and 0.7 mH/km respectively.

2.2.3 Capacitance

As for the inductance the derivations of capacitance of transmission lines of different geometry can be found in references [2-4]. On the assumption that the radius of the line conductor is considerably smaller than the distance d between the two conductors (with radius r) of a single-phase line, the capacitance is given by

$$C' = \frac{\pi\varepsilon}{ln\ D/\bar{r}}\ \text{F/m} \tag{2.4}$$

where D and $\bar{r}$ are as defined before and ε is the electric permeability of the medium between conductors of each phase is again different unless the spacing is equilateral or the line is transposed. For a transposed line the capacitance of each phase to neutral is given to a reasonable approximation by

$$C'_n = \frac{2\pi\varepsilon}{ln\ D/\bar{r}}\ \text{F/m} \tag{2.5}$$

where D, $\bar{r}$, ε are defined before.

The capacitance of e.h.v. lines at 50 Hz is of the order of 0.2 MΩ/km which corresponds to approximately 0.0159 μF/km. In the case of cables, the capacitive reactance of e.h.v. cables at 50 Hz is of the order of 4 kΩ/km which corresponds to 0.318 μF/km.

The presence of the earth's surface, which is virtually an equipotential, will influence the electric flux lines between conductors and therefore the capacitance per phase. The problem of calculating the capacitance of a line in the presence of the earth can be solved very neatly using the idea of 'images', again the analysis can be found in books on transmission lines or power systems [2-4].

2.3 TRANSMISSION LINE MODEL

Whether the transmission line is overhead or underground its four electrical characteristics r,l,g and c are distributed along the line. The relationship between the terminal voltages and currents of a perfectly distributed transmission line can be arrived at through differential calculus.

Let us consider the differential length of line shown in Fig. 2.1 The voltage and current wave propagation along the lossless line (at a point x) are related to the line's distributed inductance L′and capacitance C′, by the equations

$$-\frac{\partial v}{\partial x} = L' \frac{\partial i}{\partial t} \tag{2.6}$$

$$-\frac{\partial i}{\partial x} = C' \frac{\partial v}{\partial t} \tag{2.7}$$

The general solutions of equations 1 and 2 are [12]

$$i(x,t)=f_1(x-at)+f_2(x+at) \tag{2.8}$$

$$v(x,t)=Z f_1(x-at)-Z f_2(x+at) \tag{2.9}$$

where f_1 and f_2 are arbitrary functions of the variables *(x-at) and (x+at)* to be determined from problem boundary and initial conditions. The physical interpretation of $f_1(x-at)$ is a wave travelling at velocity *a* in the forward direction, and of $f_2(x+at)$ is a wave travelling at velocity *a* in the backward direction.

Z and *a* are surge impedance and velocity of propagation, respectively, and for lossless line their values are

$$Z=\sqrt{L'/C'} \tag{2.10}$$

$$a=1/\sqrt{L'C'} \tag{2.11}$$

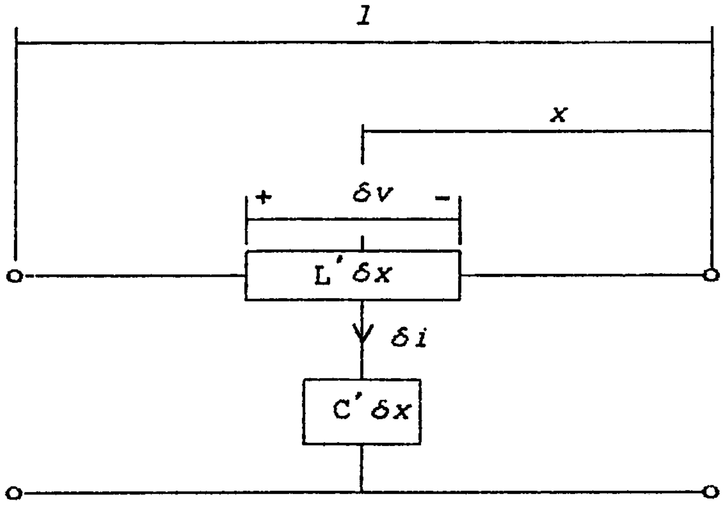


Figure 2.1 Differential length of line

An approximation to this distribution nature is to represent the transmission line as an interconnection of many lumped parameter identical sections. Each section is in the form of T, Π, ⅂, or Γ and contains a series resistance and inductance, and a shunt conductance and capacitance as seen in Fig. 2.2. R, L, G, and C are the resistance, inductance, conductance and capacitance of a section of the transmission line, respectively. The resistance and inductance for each network (section) are determined by dividing the total resistance and inductance for the line by the number of networks N. The shunt capacitance and conductance for each network can be determined in the same manner.

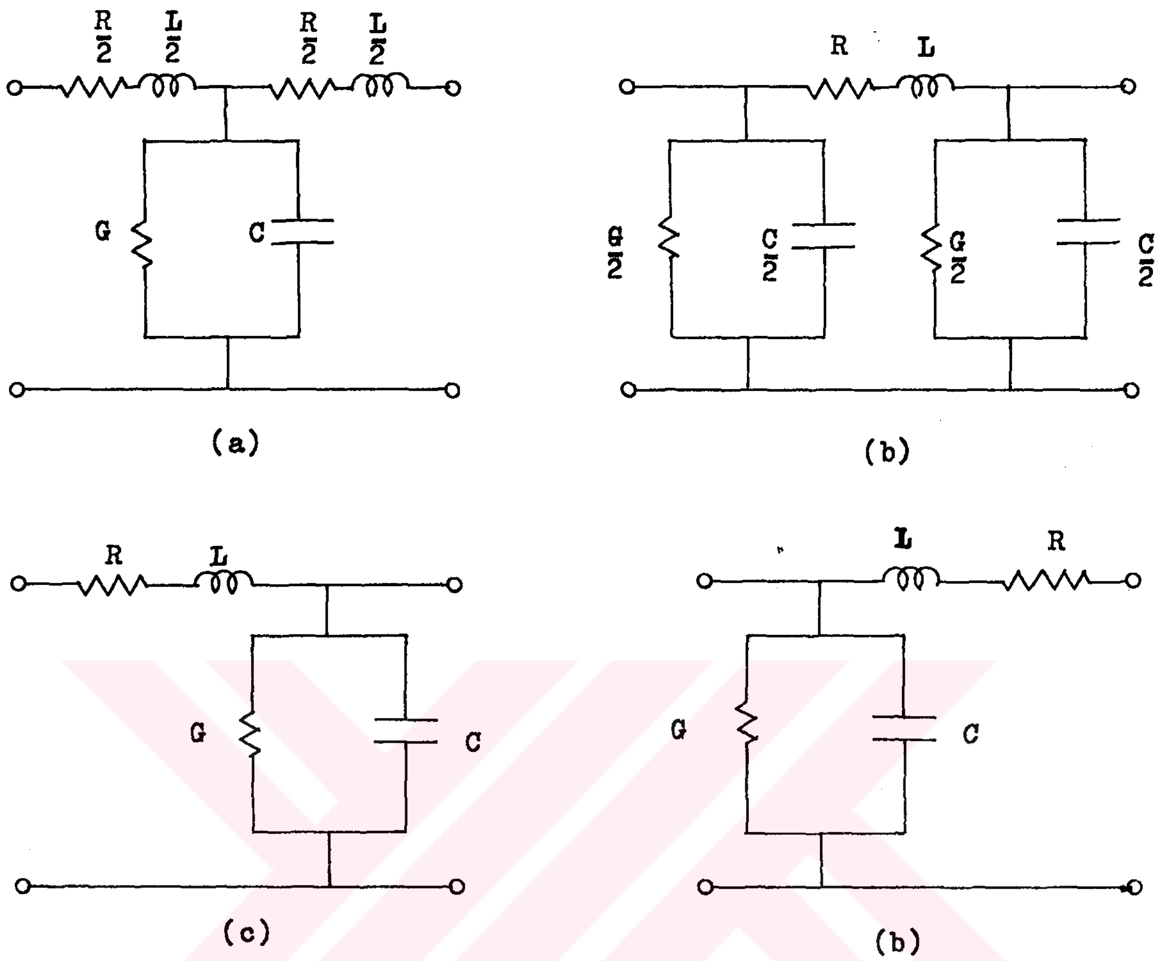


Figure 2.2 Different lumped parameter network models of each section of a transmission line; a) T-network, b) Π-network, c) ⅂-network, d) Γ-network.

When N sections are connected in cascade and some series elements are combined, the transmission line models shown in Fig. 2.3 are obtained. These models will be the fundamental basis for the state-space analysis of the transmission line and they are reconsidered in Chapter 3.

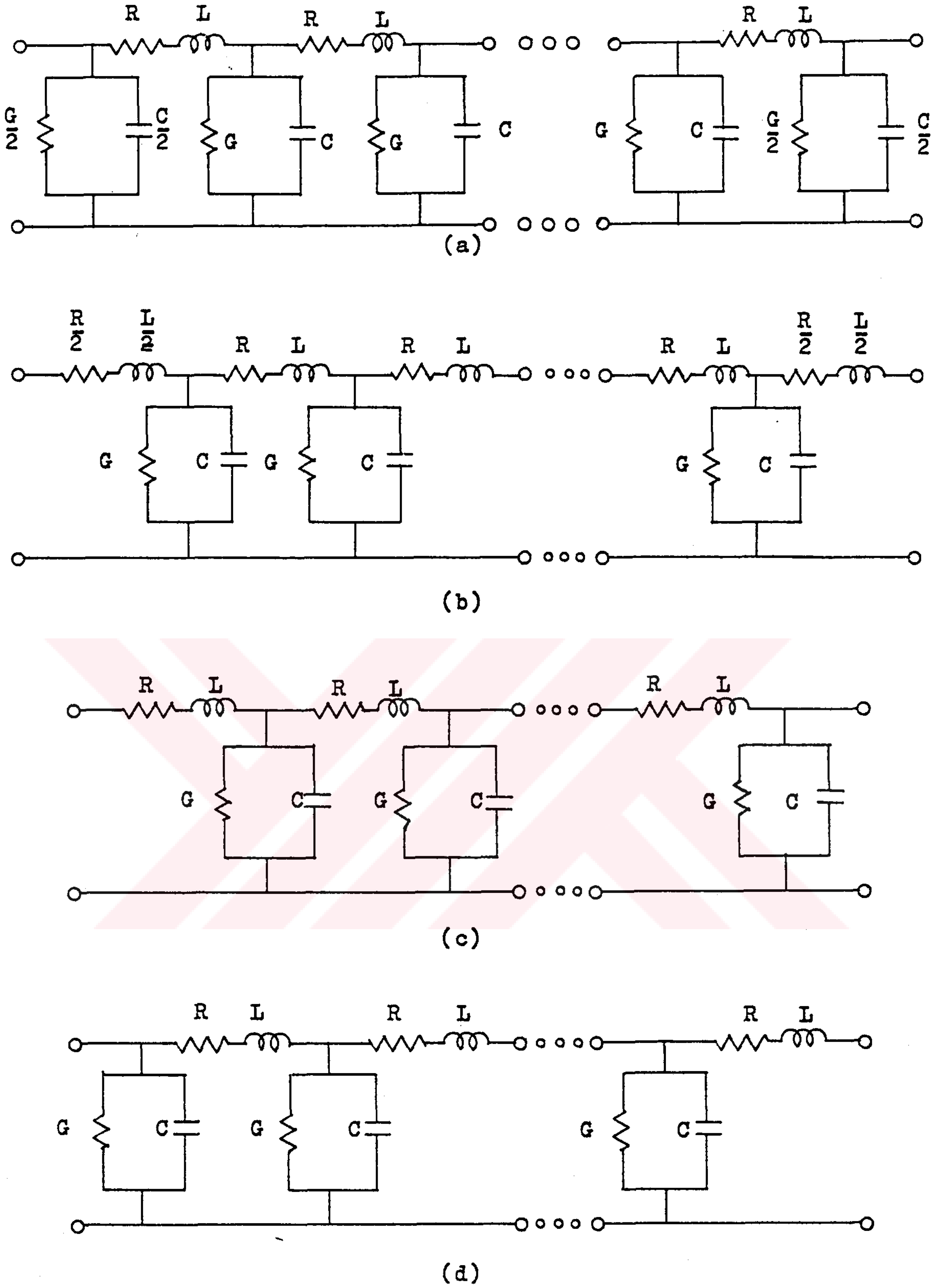


Figure 2.3 Transmission line models obtained by using a) Π-section, b) T-section c) T-section d) Γ-section.

2.4 REPRESENTATION OF THE SOURCE

Transmission systems are usually energized from bus-bars which are fed either by generating sources, i.e., generators or transformers or exclusively by other transmission lines and cables in the system. It is not always necessary to represent an overall source configuration of a network. the form of source representation should be chosen depending on the objectives of the particular study carried out. In view of this, for the studies carried out here, a simplified representation of the source side network, i.e., lumped parameters would be quite approximate and have been adopted.

Infinite bus-bar source has been simulated by a voltage source behind very small resistance of the order of $10^{-6}\Omega$. For practical purposes, this value is considered to be low enough as compared to the surge impedances of the line. A generating source is simulated by a voltage source behind an inductance, however, the source inductance may appear in series with a resistance. An illustration of the single line diagrams of an infinite bus-bar, and an inductive source being in series with the source resistance, is shown in Fig. 2.4.

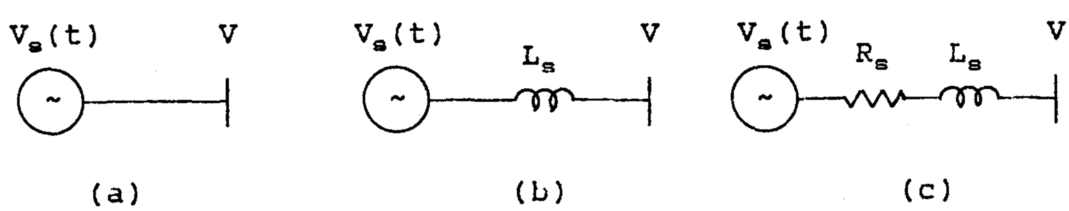


Figure 2.4 Source representations; a) Infinite bus-bar, b) Purely inductive finite source, c) Composite source.

2.5 LOAD REPRESENTATION

In general, the type and quantity of the load of a power transmission system varies by the hour, day and season. Different types of loads such as residential, commercial, industrial may accumulate to result with a load model which can be represented by one of the following equivalent circuits.

The analysis of a transmission line terminated by each of these loads is considered separately in Chapter 3.

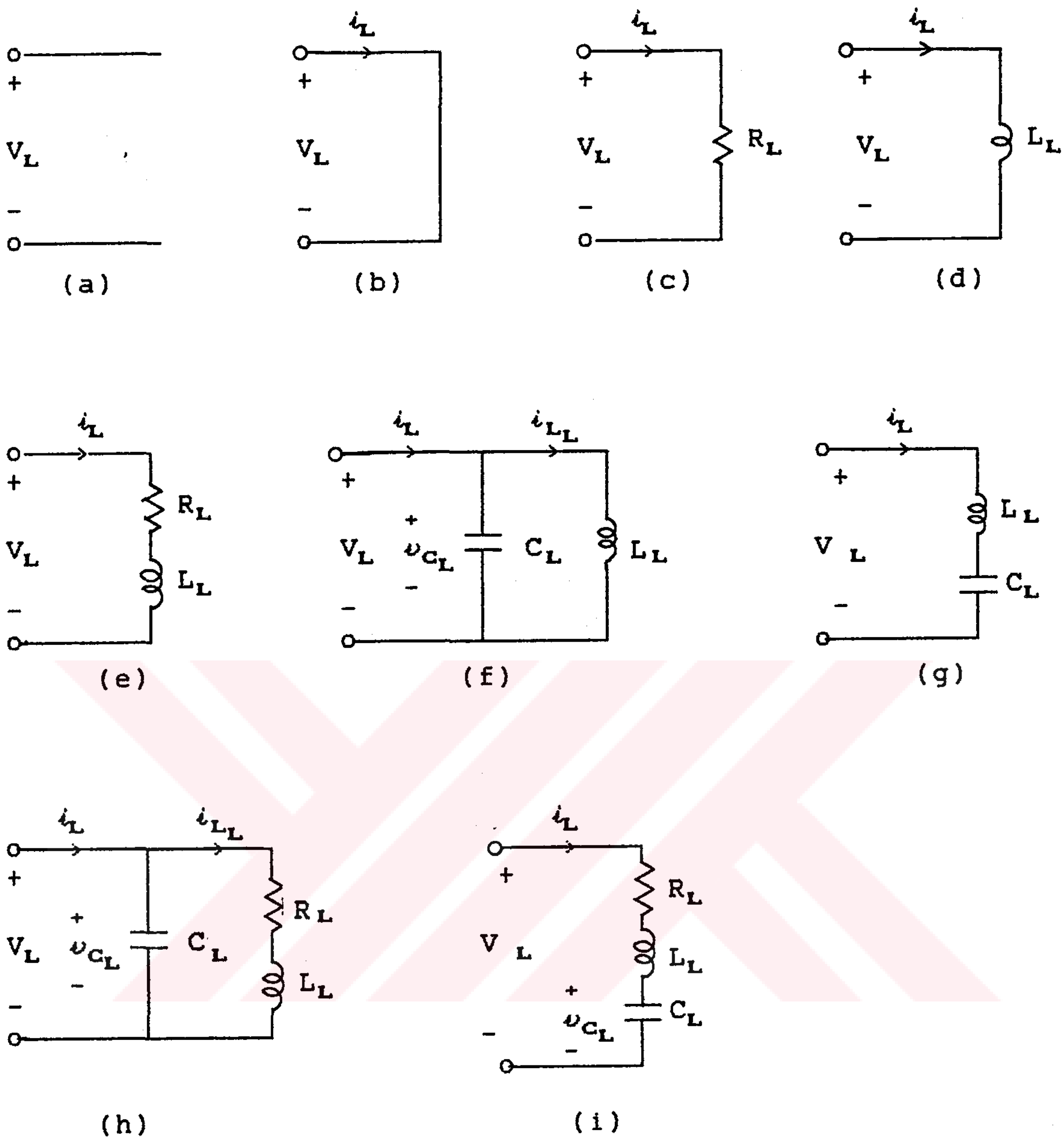


Figure 2.5 Load types for the termination of a transmission line: a) Open circuit, b) Short circuit, c) Resistive load, d) Inductive load, e) Resistive+inductive load, f) Tank circuit, g) Resonator h) Lossy tank circuit, i) Lossy resonator.

CHAPTER 3

STATE SPACE REPRESENTATION OF TRANSMISSION SYSTEMS

3.1. INTRODUCTION

Linear, lumped-parameter networks containing resistors, capacitors, inductors and independent voltage and current sources can be represented by the so called state-space equations written in the form [15]

$$\mathbf{x}(t)=\mathbf{A}\mathbf{x}(t)+\mathbf{B}\mathbf{u}(t), \qquad \mathbf{x}(o)=\mathbf{x}_o \qquad 3.1.a$$

$$\mathbf{y}(t)=\mathbf{C}\mathbf{x}(t)+\mathbf{D}\mathbf{u}(t) \qquad 3.1.b$$

In this equation the state vector **x** contains some of the capacitor voltages and inductor currents, the exciting vector u represents the input, the response vector **y** represents the output; **A, B, C, D** are the constant matrices which depend on the lumped parameter values of the network, t is the time parameter, $\mathbf{x}_o$ is the initial value of the state vector. Eq. 3.1.a is known to be a linear state equation and Eq. 3.1.b is the output equation.

In this chapter Eqs. 3.1 a and b are formulated for single phase transmission lines terminated in different ways on the source and load sides. The solutions of these state equations are then discussed in Chapter 4.

3.2 STATE-VARIABLE REPRESENTATION OF TRANSMISSION LINE

To derive the state-space representation of a transmission line the lumped parameter representation considered in Chapter 2 should be used. First an open ended transmission line with a nonideal sinusoidal excitation is considered. For such a line Ꞁ-section appearing in Fig. 2.3 is used as the lumped parameter representation for the reason to be explained at the end of this section; the total number of Ꞁ-sections is assumed to be N. With this lumped representation the line is approximated by the network shown in Fig. 3.1, which also includes the terminations.

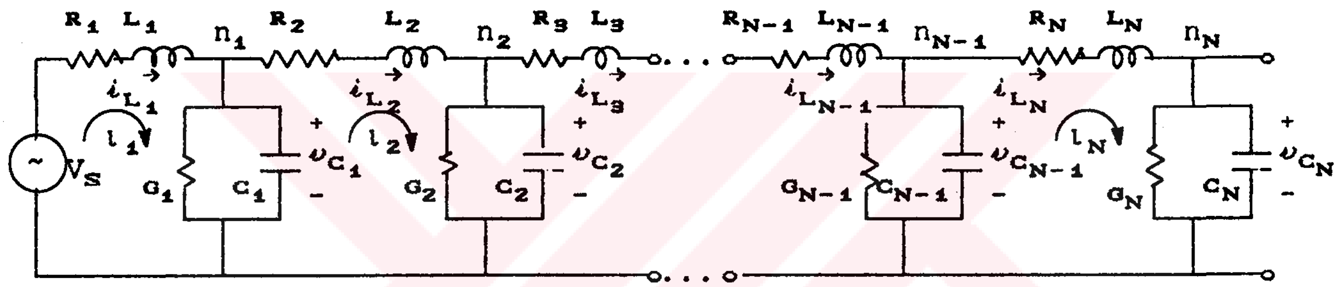


Figure 3.1 Open circuit transmission line excited by a voltage source

This circuit is of the type mentioned at the beginning of this chapter, and hence its state space equations can be obtained in the form (3.1). For this purpose, The state variables for the circuit are chosen to be

$$\begin{aligned} x_1(t) &= i_{L_1}(t) \\ x_2(t) &= v_{C_1}(t) \\ x_3(t) &= i_{L_2}(t) \\ x_4(t) &= v_{C_2}(t) \\ &\;\;\cdot \qquad \cdot \end{aligned} \tag{3.2}$$

$$\vdots \qquad \vdots$$

$$x_{K-1}(t)=i_{L_N}(t)$$

$$x_K(t)=v_{C_N}(t)$$

where current (i) and voltage (v) as are defined in the figure. Writing Kirchoff's Voltage Equations (KVE) and Kirchoff's Current Equations (KCE) for the loops l_j and nodes $n_j (j=1,2,....N)$ indicated in Fig. 3.1 we obtain

KVE:

$$
\begin{aligned}
l_1: &\quad -v_S+v_{R_1}+v_{L_1}+v_{C_1}=0 \\
l_2: &\quad -v_{C_1}+v_{R_2}+v_{L_2}+v_{C_2}=0 \\
l_3: &\quad -v_{C_2}+v_{R_3}+v_{L_3}+v_{C_3}=0 \\
&\quad \vdots \\
l_K: &\quad -v_{K-1}+v_{R_K}+v_{L_K}+v_{C_K}=0 \\
&\quad \vdots \\
l_N: &\quad -v_{C_{N-1}}+v_{R_N}+v_{L_N}+v_{C_N}=0
\end{aligned}
\tag{3.3}
$$

KCE:

$$
\begin{aligned}
n_1: &\quad -i_{L_1}+i_{C_1}+i_{G_1}+i_{L_2}=0 \\
n_2: &\quad -i_{L_2}+i_{C_2}+i_{G_2}+i_{L_3}=0 \\
n_3: &\quad -i_{L_3}+i_{C_3}+i_{G_3}+i_{L_4}=0 \\
&\quad \vdots \\
n_K: &\quad -i_{L_K}+i_{C_K}+i_{G_K}+i_{L_{K+1}}=0 \\
&\quad \vdots \\
n_N: &\quad -i_{L_N}+i_{C_N}+i_{G_N}=0
\end{aligned}
\tag{3.4}
$$

On the other hand element behavior equations (EBH) can be written as

EBH: $$v_{L_j} = L_j \frac{d i_{L_j}}{dt} = L_j \dot{i}_{L_j}$$

$$i_{L_j} = C_j \frac{d v_{C_j}}{dt} = C_j \dot{v}_{C_j}$$

$$v_{R_j} = R_j i_{R_j} = R_j i_{L_j} \qquad (3.5)$$

$$i_{G_j} = G_j v_{G_j} = G_j v_{C_j}$$

Replacing these equations into KVL and KCE in Eqs. 3.3 and 3.4 and rearranging the following equations are obtained.

$$
\begin{aligned}
&L_1 \dot{i}_{L_1} = v_S - R_1 i_{L_1} - v_{C_1} \\
&C_1 \dot{v}_{C_1} = i_{L_1} - G_1 v_{C_1} - i_{L_2} \\
&L_2 \dot{i}_{L_2} = v_{C_1} - R_2 i_{L_2} - v_{C_2} \\
&C_2 \dot{v}_{C_2} = i_{L_2} - G_2 v_{C_2} - i_{L_3} \\
&\vdots \qquad\qquad \vdots \\
&L_j \dot{i}_{L_j} = v_{C_{j-1}} - R_j i_{L_j} - v_{C_j} \qquad\qquad (3.6) \\
&C_j \dot{v}_{C_j} = i_{L_j} - G_j v_{C_j} - i_{L_{j+1}} \\
&\vdots \qquad\qquad \vdots \\
&L_N \dot{i}_{L_N} = v_{C_{N-1}} - R_N i_{L_N} - v_{C_N} \\
&C_N \dot{v}_{C_N} = i_{L_N} - G_N v_{C_N}
\end{aligned}
$$

Putting these results in matrix form , and omitting the subscripts C and L, we finally obtain the following state equations which is in the form of Eq. 3.1.a, i.e., $\dot{\mathbf{x}}(t)=\mathbf{A}\mathbf{x}(t)+\mathbf{B}\mathbf{u}(t)$.

$$
\begin{bmatrix} \dot{i}_1 \\ \dot{v}_1 \\ \dot{i}_2 \\ \vdots \\ \dot{v}_{K-1} \\ \dot{i}_K \\ \dot{i}_{K+1} \\ \dot{v}_{K+1} \\ \vdots \\ \dot{i}_N \\ \dot{v}_N \end{bmatrix}
=
\begin{bmatrix}
-R_1/L_1 & -1/L_1 & 0 & & & & & & \cdots & 0 \\
1/C & -G/C & -1/C & 0 & & & & & \cdots & . \\
0 & 1/L & -R/L & -1/L & 0 & & & & \cdots & . \\
\vdots & & \ddots & \ddots & \ddots & & & & & \vdots \\
0 & \cdots & 0 & 1/L & -R/L & -1/L & 0 & & \cdots & 0 \\
0 & \cdots & & 0 & 1/C & -G/C & -1/C & 0 & \cdots & 0 \\
\vdots & & & & & \ddots & \ddots & \ddots & & \vdots \\
\vdots & & & & & & & \ddots & \ddots & 0 \\
0 & \cdots & & & & & 0 & -1/L & -R/L & -1/L \\
0 & \cdots & & & & & & 0 & 1/C_N & -G/C_N
\end{bmatrix}
\begin{bmatrix} i_1 \\ v_1 \\ i_2 \\ \vdots \\ v_{K-1} \\ i_K \\ i_{K+1} \\ v_{K+1} \\ \vdots \\ i_N \\ v_N \end{bmatrix}
+
\begin{bmatrix} 1/L_1 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} u(t)
\qquad (3.7)
$$

Here x is a column vector whose elements are inductor currents (i_K, k=1,2,...N) and capacitor voltages (v_K, k=1,2,...N), **A** is a tridiagonal matrix of dimension 2Nx2N.

If a T or Γ-section were used for the open ended transmission line, the last resistance and inductance would not be included in the state-space representation; hence some part of the line would be ignored and this would effect the accuracy of the lumped parameter representation in a negative manner. On the other hand for a Π section, this problem would not occur, but instead the source parameters (R_S, L_S) would not be combined with the line parameter (R, L) and this would increase the total number of state variables from 2N to 2N+2.

3.3 STATE-VARIABLE REPRESENTATION OF TRANSMISSION LINE WITH A SHORT CIRCUIT TERMINATION

As it is mentioned at the end of the previous section in some cases for a specific load termination it will be convenient to use a specific lumped parameter section for the line model. Consider a line with a short circuit at the receiving end. As in Fig. 3.1 if ⅂ sections are used in the lumped parameter model this short circuit at the end of the line removes the effect of the shunt capacitance and conductance at node n_N; A similar situation arises if Π sections are used. This corresponds the chopping off some part of the line at the end. For this reason ⅂ or Π sections are not preferred for the representation of a short circuited transmission line.

Γ sections or T sections can be used without confronting with the above mentioned problem. However, T section is preferred in the following discussion since a more realistic approximation is achieved in the lumped parameterization of the line.

By using T sections in the lumped parameter model of the transmission line, the circuit shown in Fig. 3.2 is

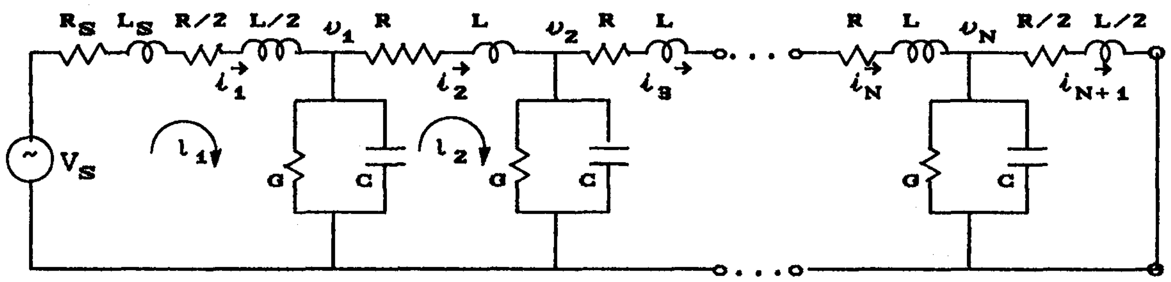


Figure 3.2 A transmission line load side is short circuit.

obtained for the short circuited transmission line. For this circuit following a similar procedure described in the previous section, the state equations can be obtained as

$$\begin{bmatrix} \dot{i}_1 \\ \dot{v}_1 \\ \dot{i}_2 \\ \vdots \\ \dot{v}_N \\ \dot{i}_{N+1} \end{bmatrix} = \begin{bmatrix} -R_1/L_1 & -1/L_1 & & & & \\ 1/C & -G/C & -1/C & & 0 & \\ & 1/L & -R/L & -1/C & & \\ & & \ddots & \ddots & \ddots & \\ & 0 & & 1/C & -G/C & -1/C \\ & & & & 2/L & -R/L \end{bmatrix} \begin{bmatrix} i_1 \\ v_1 \\ i_2 \\ \vdots \\ v_N \\ i_{N+1} \end{bmatrix} + \begin{bmatrix} 1/L_1 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} u(t) \tag{3.8}$$

where $L_1 = L/2 + L_s$ and $R_1 = R/2 + R_S$.

Taking the receiving end current as the desired output, the output equation becomes

$$i_L = \begin{bmatrix} 0 & \vdots & 1 \end{bmatrix} \begin{bmatrix} \bar{X} \\ \cdots \\ i_{N+1} \end{bmatrix} \tag{3.9}$$

Although the state equation in (3.8) is in the normal form given in Eq. 3.1a, it is written as in the following form for future use.

$$\begin{bmatrix} \dot{X} \\ \cdots \\ \dot{i}_{N+1} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} & \bar{A} & \end{bmatrix} \\ 0 \; . \; . \; . 0 \quad 2/L \;\; -R/L \end{bmatrix} \begin{bmatrix} \bar{X} \\ \cdots \\ i_{N+1} \end{bmatrix} + \begin{bmatrix} \bar{B} \\ \cdots \\ 0 \end{bmatrix} u(t) \tag{3.10}$$

where

$$\bar{X}=\begin{bmatrix} i_1 \\ v_1 \\ i_2 \\ \cdot \\ \cdot \\ \cdot \\ i_N \\ v_N \end{bmatrix}, \quad \bar{A}=\begin{bmatrix} -R/L_1 & -1/L_1 & 0 & & & \cdots & 0 \\ 1/C & -G/C & -1/C & 0 & & \cdots & 0 \\ 0 & 1/L & -R/L & -1/L & 0 & \cdots & 0 \\ \cdot & & \cdot & \cdot & \cdot & & \cdot \\ \cdot & & & \cdot & \cdot & \cdot & \cdot \\ \cdot & & & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdots & 0 & 1/L & -R/L & -1/L & 0 \\ 0 & \cdots & & 0 & 1/C & -G/C & -1/C \end{bmatrix}, \quad \bar{B}=\begin{bmatrix} 1/L_1 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 0 \end{bmatrix}$$

3.4 TRANSMISSION LINE WITH DIFFERENT TERMINATIONS

The state equations (including the output equation) of the transmission line for terminations other than open circuit and short circuit are listed in the sequel by considering each of the loading conditions given in Section 2.5. In each case transmission line is modeled by T sections as in Fig. 3.2; hence the results obtained in the previous section for the short circuited case can easily be adopted for different terminations as follows:

1) Resistive load (Fig. 2.5.c):

$$\begin{bmatrix} \dot{\bar{X}} \\ \cdots \\ \dot{i}_L \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} & \bar{A} & \end{bmatrix} & \\ 0 \; . \; . \; . \; 0 & 2/L \quad -2R_T/L \end{bmatrix} \begin{bmatrix} \bar{X} \\ \cdots \\ i_L \end{bmatrix} + \begin{bmatrix} \bar{B} \\ \cdots \\ 0 \end{bmatrix} u(t) \qquad (3.11)$$

where $R_T=R_L+R/2$. $\bar{X}$, $\bar{A}$, $\bar{B}$ in this equation and in the following equations are all the same as they are defined in Eq. 3.10.

The load voltage can be expressed as

$$V_L=\begin{bmatrix}0 & \vdots & R_L\end{bmatrix}\begin{bmatrix}\overline{X}\\ \cdots\\ i_L\end{bmatrix} \qquad (3.12)$$

2)Inductive load (Fig. 2.5.d):

$$\begin{bmatrix}\dot{\overline{X}}\\ \cdots\\ \dot{i}_L\end{bmatrix}=\begin{bmatrix}\begin{bmatrix} & \overline{A} & \end{bmatrix}\\ 0 \; . \; . \; . \; 0 \quad 1/L_T \quad -R/2L_T\end{bmatrix}\begin{bmatrix}\overline{X}\\ \cdots\\ i_L\end{bmatrix}+\begin{bmatrix}\overline{B}\\ \cdots\\ 0\end{bmatrix}u(t) \qquad (3.13)$$

where $L_T=L_L+L/2$,

$$V_L=\begin{bmatrix}0\ldots & 0 & L_L/L_T & -RL_L/2L_T\end{bmatrix}\begin{bmatrix}\overline{X}\\ \cdots\\ i_L\end{bmatrix} \qquad (3.14)$$

3) Resistive+inductive load (Fig. 2.5.e):

$$\begin{bmatrix}\dot{\overline{X}}\\ \cdots\\ \dot{i}_L\end{bmatrix}=\begin{bmatrix}\begin{bmatrix} & \overline{A} & \end{bmatrix}\\ 0 \; . \; . \; .0 \quad 1/L_T \quad -R_T/L_T\end{bmatrix}\begin{bmatrix}\overline{X}\\ \cdots\\ i_L\end{bmatrix}+\begin{bmatrix}\overline{B}\\ \cdots\\ 0\end{bmatrix}u(t) \qquad (3.15)$$

where $R_T=R_L+R/2$, $L_T=L_L+L/2$,

$$V_L = \left[0 \ \vdots \ L_L/L_T \quad (-R_T L_L/L_T)+R_L \right] \begin{bmatrix} \overline{X} \\ \cdots \\ i_L \end{bmatrix} \tag{3.16}$$

4) Termination by a tank circuit (Fig. 2.5.f):

$$\begin{bmatrix} \dot{\overline{X}} \\ \cdots \\ \dot{i}_L \\ \dot{v}_{CL} \\ \dot{i}_{L_L} \end{bmatrix} = \begin{bmatrix} \left[\quad \overline{A} \quad \right] & \begin{matrix} 0 \\ \vdots \\ 0 \end{matrix} & \begin{matrix} 0 \\ \vdots \\ 0 \end{matrix} \\ 0 \ . \ . \ . \ 0 \quad 2/L \quad -R/L & -2/L & 0 \\ 0 \ . \ . \ . \ 0 \quad 0 \quad 1/C_L & 0 & -1/C_L \\ 0 \ . \ . \ . \ 0 \quad 0 \quad 0 & 1/L_L & 0 \end{bmatrix} \begin{bmatrix} \overline{X} \\ \cdots \\ i_L \\ v_{C_L} \\ i_{L_L} \end{bmatrix} + \begin{bmatrix} \overline{B} \\ \cdots \\ 0 \\ 0 \\ 0 \end{bmatrix} u(t) \tag{3.17}$$

$$V_L = \left[0 \ \vdots \ 0 \ 1 \ 0 \right] \begin{bmatrix} \overline{X} \\ \cdots \\ i_L \\ v_{C_L} \\ i_{L_L} \end{bmatrix} \tag{3.18}$$

5) Termination by a resonator (Fig 2.5.g):

$$\begin{bmatrix} \dot{\overline{X}} \\ \cdots \\ \dot{i}_L \\ \dot{v}_{C_L} \end{bmatrix} = \begin{bmatrix} \left[\quad \overline{A} \quad \right] & \begin{matrix} 0 \\ \vdots \\ 0 \end{matrix} \\ 0 \ . \ . \ . 0 \quad 1/L_T \quad -R/2L_T & -1/L_T \\ 0 \ . \ . \ . \ 0 \quad 1/C_L & 0 \end{bmatrix} \begin{bmatrix} \overline{X} \\ \cdots \\ i_L \\ v_{C_L} \end{bmatrix} + \begin{bmatrix} \overline{B} \\ \cdots \\ 0 \\ 0 \end{bmatrix} u(t) \tag{3.19}$$

where $L_T = L_L + (L/2)$,

$$V_L = \begin{bmatrix} 0 & \vdots & L_L/L_T & -RL_L/2L_T & (L_T-L_L)/L_T \end{bmatrix} \begin{bmatrix} \overline{X} \\ \cdots \\ i_L \\ v_{L_L} \end{bmatrix} \qquad (3.20)$$

6) Termination by a lossy tank circuit (Fig. 2.5.h)

$$\begin{bmatrix} \dot{\overline{X}} \\ \cdots \\ \dot{i}_L \\ \dot{v}_{C_L} \\ \dot{i}_{L_L} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} & & \\ & \overline{A} & \\ & & \end{bmatrix} & \begin{matrix} 0 \\ \vdots \\ 0 \end{matrix} & \begin{matrix} 0 \\ \vdots \\ 0 \end{matrix} \\ 0 \;\; \cdots \;\; 2/L \;\; -R/L & -2/L & 0 \\ 0 \;\; \cdots \;\; 0 \;\; 1/C_L & 0 & -1/C_L \\ 0 \;\; \cdots \;\; 0 \;\; 0 & 1/L_L & -R_L/L_L \end{bmatrix} \begin{bmatrix} \overline{X} \\ \cdots \\ i_L \\ v_{C_L} \\ i_{L_L} \end{bmatrix} + \begin{bmatrix} \overline{B} \\ \cdots \\ 0 \\ 0 \\ 0 \end{bmatrix} u(t)$$

(3.21)

$$V_L = \begin{bmatrix} 0 & \vdots & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \overline{X} \\ \cdots \\ i_L \\ v_{C_L} \\ i_{L_L} \end{bmatrix} \qquad (3.22)$$

7) Termination by a lossy resonator (Fig. 2.5.i):

$$\begin{bmatrix} \dot{\bar{X}} \\ \cdots \\ \dot{i}_L \\ \dot{v}_{C_L} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} & & \bar{A} & & \end{bmatrix} & \begin{matrix} 0 \\ \vdots \\ 0 \end{matrix} \\ 0 \; . \; . \; . \; 0 \quad 1/L_T & -R_T/L_T \quad -1/L_T \\ 0 \; . \; . \; . \; 0 \quad 0 & 1/C_L \quad 0 \end{bmatrix} \begin{bmatrix} \bar{X} \\ \cdots \\ i_L \\ v_{C_L} \end{bmatrix} + \begin{bmatrix} \bar{B} \\ \cdots \\ 0 \\ 0 \end{bmatrix} u(t) \quad (3.23)$$

where $R_T=R_L+R/2$, $L_T=L_L+L/2$,

$$V_L = \begin{bmatrix} 0 & \vdots & L_L/L_T & (-R_T L_L/L_T)+R_L & (L_T-L_L)/L_T \end{bmatrix} \begin{bmatrix} \bar{X} \\ \cdots \\ i_L \\ v_{C_L} \end{bmatrix} \quad (3.24)$$

3.5 STATE-SPACE REPRESENTATION OF SHORT AND OPEN CIRCUIT FAULTS

In the previous section state equations for the transmission line with different terminations are formulated. In this section, these equations will be modified by considering short and open circuit faults on the transmission line.

For both kinds of faults T-model is used for the lumped parameter representation of the line; therefore the notation appearing in Fig. 3.2 and the definitions of the state vector $\bar{X}$ as well as the coefficient matrices $\bar{A}$ and $\bar{B}$ given in Eq. 3.10 are taken as reference in the following discussion.

3.5.1 Short Circuit Faults

Consider a short circuit at node n_K, k=1,2,..N of an N section line as shown in Fig. 3.3.

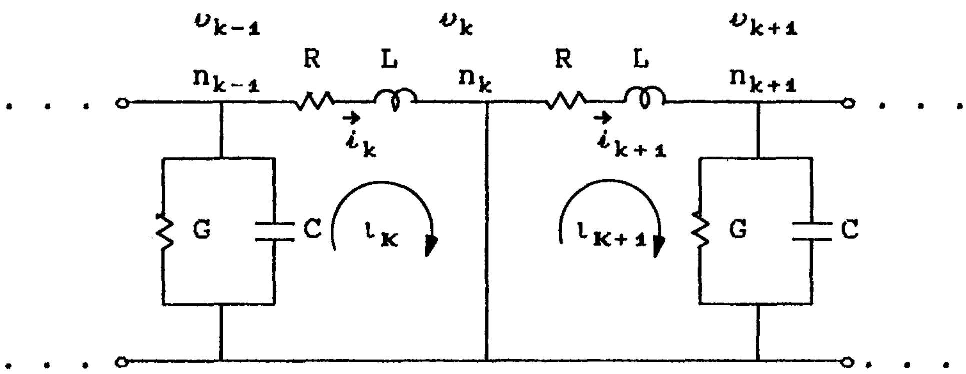


Figure 3.3 Transmission line model with short circuit at node n_K

Short circuit at this node removes the capacitance and conductance at this node and $\upsilon_K=0$ and it is not a state variable at all; hence, the number of state variables is reduced by 1. Writing KVE for the loops adjacent to this short circuit we obtain

$$\iota_K: \quad -\upsilon_{K-1}+Ri_K+L\dot{i}_K=0$$

$$\iota_{K+1}: \quad Ri_{K+1}+L\dot{i}_{K+1}+\upsilon_{K+1}=0 \qquad (3.25)$$

Rearranging these equations, the state equations expressing the derivatives of i_K and i_{K+1} become

$$\dot{i}_K=1/L\upsilon_{K-1}-R/Li_K$$

$$\dot{i}_{K+1}=-R/Li_{K+1}-1/L\upsilon_{K+1} \qquad (3.26)$$

Then the matrices $\bar{X}$, $\bar{A}$, and $\bar{B}$ can be written as follows:

$$\bar{X}=\left[i_1 \; \upsilon_1 \; i_2 \; \cdots \; \upsilon_{k-1} \; i_k \; i_{k+1} \; \upsilon_{K+1} \; \cdots \; i_N \; \upsilon_N \right]^T$$

$$\bar{A}=\begin{bmatrix}
-R_1/L_1 & -1/L_1 & & & & & & & & & \\
1/C & -G/C & -1/C & & & & & & \mathbf{O} & & \\
 & 1/L & -R/L & -1/L & & & & & & & \\
 & & \ddots & \ddots & \ddots & & & & & & \\
 & & & 1/C & -G/C & -1/C & & & & & \\
 & & & & 1/L & -R/L & 0 & & & & \\
 & & & & & 0 & -R/L & -1/L & & & \\
 & & & & & & 1/C & -G/C & -1/C & & \\
 & & \mathbf{O} & & & & & \ddots & \ddots & \ddots & \\
 & & & & & & & & -1/L & -R/L & -1/L \\
 & & & & & & & & & 1/C & -G/C
\end{bmatrix}$$

$$\bar{B}=\left[1/L_1 \; 0 \; 0 \; \cdots \cdots \; 0 \right]^T \tag{3.27}$$

Obviously υ_K is removed from $\bar{X}$, the corresponding rows from $\bar{A}$ and $\bar{B}$ are also removed, and the rows of $\bar{A}$ corresponding to the previous and the next state variables (i_K, i_{K+1}) have been changed.

3.5.2 Open Circuit Faults

Consider an open circuit between the two subsequent nodes n_{K-1} and n_K of the transmission line as shown in Fig. 3.4. As in the short circuit fault the number of state variables is reduced by 1.

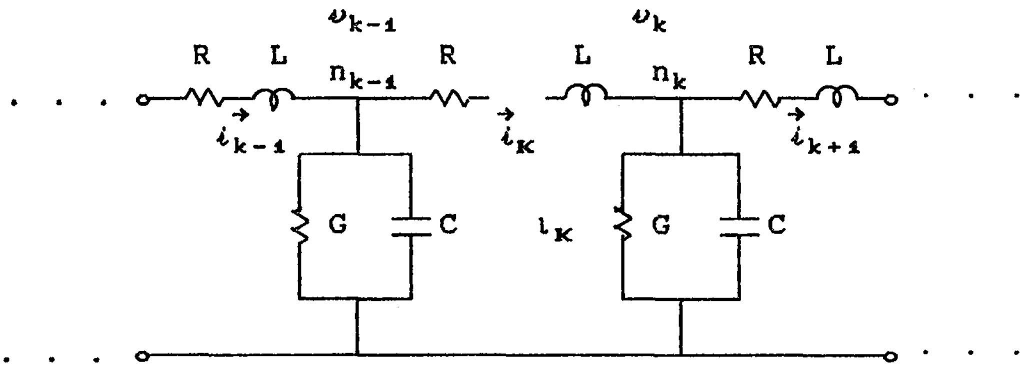


Figure 3.4 Open circuit between two subsequent nodes of transmission line model

Writing KCE for the nodes n_{K-1} and n_K , we obtain

$$n_{K-1}: \quad i_{K-1}+C\dot{v}_{k-1}+Gv_{K-1}=0$$
$$n_K: \quad i_{K+1}+C\dot{v}_k+Gv_K=0 \qquad (3.28)$$

Rearranging these equations, we found

$$n_{K-1}: \quad \dot{v}_{k-1}=(1/C)i_{K-1}-(G/C)v_{K-1}$$
$$n_{K-1}: \quad \dot{v}_k=-(G/C)v_K+(1/C)i_{K+1} \qquad (3.29)$$

With these modifications, the new form of the matrices $\overline{X}$, $\overline{A}$, and $\overline{B}$ in Eq. 3.10 become

$$\overline{X}=\left[i_1 \; v_1 \; i_2 \; . \; . \; . \; i_{K-1} \; v_{K-1} \; v_K \; i_{K+1} \; . \; . \; . \; i_N \; v_N\right]^T$$

$$\bar{A}=\begin{bmatrix} -R_1/L_1 & -1/L_1 & & & & & & & & & \\ 1/C & -G/C & -1/C & & & & & & & \mathbf{0} & \\ & 1/L & -R/L & -1/L & & & & & & & \\ & & \ddots & \ddots & \ddots & & & & & & \\ & & & 1/L & -R/L & -1/L & & & & & \\ & & & & 1/C & -G/C & 0 & & & & \\ & & & & & 0 & -G/C & -1/C & & & \\ & & & & & & 1/L & -R/L & -1/L & & \\ & \mathbf{0} & & & & & & \ddots & \ddots & \ddots & \\ & & & & & & & & 1/L & -R/L & -1/L \\ & & & & & & & & & 1/C_N & -G/C_N \end{bmatrix}$$

$$\bar{B}=\begin{bmatrix} 1/L_1 & 0 & 0 & . & . & . & . & . & . & . & . & . & . & . & . & 0 \end{bmatrix}^T \tag{3.30}$$

for the open circuit fault.

The state vector $\bar{X}$ and the coefficient matrices $\bar{A}$, $\bar{B}$ obtained in this section can be used in the previous sections to represent the line with different terminations under short and open circuit fault conditions.

CHAPTER-4

SOLUTION OF STATE-SPACE EQUATIONS

4.1 INTRODUCTION

In the previous chapter the state-space representations of the transmission systems under different load and fault conditions are derived as the state and output equations. Although the general form of these equations are given in Section 3.1, they are repeated here for convenience.

$$\dot{\mathbf{x}}(t)=\mathbf{A}\mathbf{x}(t)+\mathbf{B}\mathbf{u}(t), \quad \mathbf{x}(to)=\mathbf{x}_o \qquad (4.1.a)$$

$$\mathbf{y}(t)=\mathbf{C}\mathbf{x}(t)+\mathbf{D}\mathbf{u}(t) \qquad (4.1.b)$$

The solution of a linear time-invariant system described by these equations can be evaluated in terms of the initial state vector $\mathbf{x}_o$ and the excitation vector $\mathbf{u}(t)$ is given [15] by the expressions:

$$\mathbf{x}(t)=e^{\mathbf{A}(t-t_o)}\ \mathbf{x}_o + \int_{t_o}^{t} e^{\mathbf{A}(t-t')}\mathbf{B}\mathbf{u}(t')dt' \qquad (4.2)$$

$$\mathbf{y}(t)=\mathbf{C}e^{\mathbf{A}(t-t_o)}\ \mathbf{x}_o + \mathbf{C}\int_{t_o}^{t} e^{\mathbf{A}(t-t')}\mathbf{B}\mathbf{u}(t')dt'+\mathbf{D}\mathbf{u}(t') \qquad (4.3)$$

where t_o is the initial time. In this chapter by using these expressions the responses of a transmission line are derived under sinusoidal and step type excitations.

4.2 RESPONSES UNDER SINUSOIDAL EXCITATION

Consider that a transmission system is excited by a sinusoidal voltage source which can be expressed as

$$u(t)=V_s(t)= |V| \cos(\omega t+\phi) \tag{4.4}$$

where $|V|$, ω, and ϕ are real constant numbers representing the amplitude, angular frequency, and the phase of the sinusoidal source $V_s(t)$. To simplify the manipulations, we use phasor notation; more specifically, the phasor V is defined as a complex number given by

$$V= |V|e^{j\phi} \tag{4.5}$$

Then the input $V_s(t)$ can be written in terms of phasor V;

$$V_s(t)= \mathrm{Re}\left\{Ve^{j\omega t}\right\} \tag{4.6}$$

In other words, $V_s(t)$ is equal to the real part of $Ve^{j\omega t}$. Note that the phase of the source, ϕ, is included in the phasor V. Replacing $j\omega$ in the above equation by p where $p=\sigma+j\omega$, we write

$$V_s(t)=\mathrm{Re}\left\{Ve^{pt}\right\} \tag{4.7}$$

Inserting Eq. 4.7 into Eq. 4.2 we obtain

$$x(t)=e^{A(t-t_o)}x_o + \int_{t_o}^{t} e^{A(t-t')} B \; Re\left\{Ve^{pt'}\right\} dt' \qquad (4.8)$$

Since all the expressions are real in the integration except Ve^{pt} we can write **x**(t) as

$$x(t)=e^{A(t-t_o)}x_o + Re\left\{e^{At}\int_{t_o}^{t} e^{(pI-A)t'} B\, Vdt'\right\} \qquad (4.9)$$

By integration the following equation is derived

$$x(t)=e^{A(t-t_o)}x_o+Re\left\{(pI-A)^{-1}(e^{pt}BV-e^{A(t-t_o)}BVe^{pt_o})\right\} \qquad 4.10)$$

In this solution the matrix (pI-**A**) is assumed to be nonsingular; which is a valid assumption since the eigenvalues of **A** are different from the excitation frequency $p=j\omega$ in all practical applications of transmission systems.

i) Complete response

The response **x**(t) in Eq. 4.10 will be referred to as complete response of the system in the interval $0\le t\le\infty$, and will be denoted by $x_c(t)$.

ii) Zero-state response

The response of the system to an input applied at time t=0, subjected to the condition that the system is in the zero-state, i.e., $x_o=0$, just prior to the application of the input, is called the zero-state response. Thus the zero-state response of the system can be obtained from Eq. (4.10) and can be written as

$$x(t)=Re\left\{(pI-A)^{-1}(e^{pt}BV-e^{A(t-t_o)}BVe^{pt_o})\right\} \quad (4.11)$$

iii) Steady-state response

The steady-state response denoted by x_{ss} is defined as

$$x_{ss}(t)=\lim_{t\to\infty} x_C(t) \quad (4.12)$$

This definition is valid only for stable systems, that is when the eigenvalues of the **A** matrix in Eq. 4.1 lie in the open left half of the complex-plane. Then, using Eq. 4.10 the steady state response of the system can be written as:

$$x(t)=Re\left\{(pI-A)^{-1}Be^{pt}V\right\} \quad (4.13)$$

vi) Transient response

The difference between the complete response and the steady-state response is defined as the transient response $x_t(t)$ of the system. That is

$$x_t(t)=x_c(t)-x_{ss}(t) \quad (4.14)$$

or

$$x_t(t)=e^{A(t-t_o)}\left(x_o-Re\left\{BVe^{pt_o}\right\}\right) \quad (4.15)$$

Obviously $x_t(t)$ satisfies the equation

$$\lim_{t\to\infty} x_t(t)=0 \quad (4.16)$$

if all the eigenvalues of **A** are in the open left hand half plane (asymptotically stable system).

4.3 RESPONSES UNDER STEP EXCITATION

i) Complete response

If the excitation is a step voltage then the complete response of the system can be obtained directly from Eq. 4.10 by assuming that p=0, and V is a real number representing the value of step voltage. Then the complete response of a system under step excitation can easily be written as

$$\mathbf{x}_c(t)=e^{\mathbf{A}(t-t_o)}\,(\mathbf{x}_o-\mathbf{B}\mathbf{V})\;+(-\mathbf{A})^{-1}\mathbf{B}\mathbf{V} \qquad (4.17)$$

The other types of responses can be obtained from total response of the system as in the previous section.

ii) Zero-state response is

$$\mathbf{x}_c(t)=(-\mathbf{A})^{-1}\mathbf{B}\mathbf{V}-e^{\mathbf{A}(t-t_o)}\;\mathbf{B}\mathbf{V} \qquad (4.18)$$

iii) Steady-state response is

$$\mathbf{x}_{ss}(t)=(-\mathbf{A})^{-1}\;\mathbf{B}\mathbf{V} \qquad (4.19)$$

iv) Transient response is

$$\mathbf{x}_t(t)=e^{\mathbf{A}(t-t_o)}\;(\mathbf{x}_o-\mathbf{B}\mathbf{V}) \qquad (4.20)$$

4.4 COMPUTATION OF exp(At)

It is clear that computing the response of a linear time-invariant network involves computing the state transition matrix exp(**A**t) and simple matrix operations such as addition, subtraction, multiplication, inversion etc. Typically the computation exp(**A**t) is the major task in obtaining the response of linear time invariant networks.

Power-series method [16] can be used to compute exp(**A**t). The major advantage of this method is the simplicity of programming , and its main disadvantages are the computation need for each t and the large computation time is required [17]; also numerical errors may accumulate for large dimensions of matrix **A**. The method involves the computation of the series

$$e^{\mathbf{A}t} \cong I + \mathbf{A}t + \frac{(\mathbf{A}t)^2}{2!} + \cdots + \frac{(\mathbf{A}t)^N}{N!} \qquad (4.21)$$

until the last term added is considerably small with respect to the sum of the previous terms.

Although **A**t can be divided by an integer constant until its norm is smaller than 1 [18] so that its powers decay to zero quickly and the number of terms N taken in the above expression is not very large, this does not overcome the disadvantages above.

We can derive a closed-form expression for exp(**A**t) either as a special case of the functions of a matrix or by a purely algebraic method based on Laplace transform [19].

The second method, purely algebraic in its spirit. This computation needs, special background, however, when the size n of matrix **A** is large, it is important to be able to organize the computation more systematically and to be able to check the results. From a practical computational point of view either method is effective in general.

Now consider exp(**A**t) as a particular case of function of a matrix. The exponential function e^z is analytic everywhere in the finite complex-plane; therefore, the fundamental formula for the function of a matrix is directly applicable and yields[15]

$$e^{At}=\sum_{k=1}^{\sigma}\sum_{\ell=0}^{M_k-1} t^{\ell}e^{\lambda_k t}Z_{k\ell} \tag{4.22}$$

where $\lambda_1, \lambda_2, \ldots, \lambda_\sigma$ are the distinct eigenvalues of **A**, M_k is the multiplicity of the eigenvalue λ_k as a zero of the minimal polynomial of **A**, and the matrices $Z_{k\ell}$ have constant elements and depend exclusively on **A**.

For an nxn matrix **A** whose elements are real or complex numbers there is a convenient way for computing any function f(**A**) of **A**.

Let $M_1, M_2, \ldots, M_\sigma$ be the multiplicity of the eigenvalues; that is, λ_i is a zero of order M_i of the minimal polynomial of **A**. The interpolation method for the computation of function of a matrix is based on directly on the definition of f(**A**). If $p(\lambda)=\sum_{k=0}^{N} a_k\lambda^k$ is a polynomial, we define p(**A**) as the matrix $\sum_{k=0}^{N} a_k\mathbf{A}^k$ with $\mathbf{A}^0=I$. Now by by definition

$$f(\mathbf{A})=p(\mathbf{A}) \tag{4.23}$$

provided the interpolating polynomial p satisfies the following conditions [16].

$$\begin{aligned} &(1)\quad f(\lambda_k)=p(\lambda_k) && k=1,2,\ldots,\sigma \\ &(2)\quad f^{(\ell)}(\lambda_k)=p^{(\ell)}(\lambda_k) && k=1,2,\ldots,\sigma \\ & && \ell=1,2,\ldots,M_k-1 \end{aligned} \tag{4.24}$$

Thus the computation of f(**A**) contains two steps: (i) the determination of an interpolating polynomial p satisfying conditions 1 and 2, and (ii) the evaluation of p(**A**).

If the minimal polynomial $\psi(\lambda)$ has distinct roots, the interpolating polynomial is the well-known Lagrange interpolating polynomial

$$p(\lambda)=\sum_{k=1}^{\sigma} \frac{(\lambda-\lambda_1)(\lambda-\lambda_2)\ \dots\ (\lambda-\lambda_{k-1})(\lambda-\lambda_{k+1})\ \dots\ (\lambda-\lambda_\sigma)}{(\lambda_k-\lambda_1)(\lambda_k-\lambda_2)\ \dots\ (\lambda_k-\lambda_{k-1})(\lambda_k-\lambda_{k+1})\ \dots\ (\lambda_k-\lambda_\sigma)} f(\lambda_k) \tag{4.25}$$

Consequently with $f(\lambda)=e^{\lambda t}$, $f(\mathbf{A}t)=e^{\mathbf{A}t}$ becomes

$$e^{\mathbf{A}t}=\sum_{k=1}^{\sigma}\left[\frac{e^{\lambda_k t}}{\prod_{\substack{i=1\\ i\neq k}}^{\sigma}(\lambda_k-\lambda_i)}\prod_{\substack{i=1\\ i\neq k}}^{\sigma}(\mathbf{A}-\lambda_i\mathbf{I})\right] \tag{4.26}$$

Eq. 4.26, together with Eq. 4.22 ($M_k=1$) reveals the

$$Z_{k\ell}=\prod_{\substack{i=1\\ i\neq k}}^{\sigma}\frac{(\mathbf{A}-\lambda_i\mathbf{I})}{(\lambda_k-\lambda_i)} \tag{4.27}$$

and Eq. 4.26 is in a special form of Eq. 4.22 for distinct eigenvalues.

In the power system problems the eigenvalues of the matrix **A** are distinct, thus Eq. 4.26 or Eq. 4.22 ($\ell=0$) can be used to compute $e^{\mathbf{A}t}$.

4.5 COMPUTATIONAL DIFFICULTIES AND COMPUTATION ACCURACY

When Π representation of line is used to derive state equations and when the termination is resistive, one of the elements of matrix **A** becomes very large ($a_{2N+1,2N+1}=(2+R_LG)/R_LC$). This causes computational errors occurs in evaluating $e^{\mathbf{A}t}$ by the method described in the previous section (Eqs. 4.22, 4.26). For this reason the algebraic method should be used to calculate $e^{\mathbf{A}t}$.

To illustrate this method consider the Laplace transform of $e^{\mathbf{A}t}$;

$$\mathcal{L}(e^{\mathbf{A}t})=(s\mathbf{I}-\mathbf{A})^{-1} \qquad (4.28)$$

Performing a partial fraction expansion of left hand side, we obtain

$$(s\mathbf{I}-\mathbf{A})^{-1}=\sum_{i=1}^{n}\frac{1}{s-\lambda_i}\mathbf{R}_i \qquad (4.29)$$

where

$$\mathbf{R}_i=\lim_{s\to\lambda_i}(s-\lambda_i)(s\mathbf{I}-\mathbf{A})^{-1} \qquad (4.30)$$

For the computation purposes the limit is taken by setting

$$s=\lambda_i+|\lambda_i|\mathrm{x}10^{-\alpha} \qquad (4.31)$$

then,

$$\mathbf{R}_i=|\lambda_i|\mathrm{x}10^{-\alpha}(\lambda_i(1+10^{-\alpha})\mathbf{I}-\mathbf{A})^{-1} \qquad (4.32)$$

where α is a constant chosen such that $|\lambda_i| x 10^{-\alpha}$ is smaller than the difference of two closest eigenvalues.

Taking the inverse Laplace transform of Eq. 4.29

$$e^{\mathbf{A}t} = \sum_{i=1}^{\sigma} e^{\lambda_i t} \mathbf{R}_i \tag{4.33}$$

is obtained, which is again special form of Eq. 4.22 in the case of distinct eigenvalues.

Eq. 4.32 with Eq.4. 33 can be used to compute $e^{\mathbf{A}t}$. Although the computation time is reduced, for large dimension of **A** again errors occur in the computation. For this reason the problem discussed at the beginning of this section is tried to be removed and by using T-model to represent line, and in this way the accurate solution is obtained for large dimensions as well.

CHAPTER 5

DESCRIPTION OF THE PROGRAM

In this chapter, the prepared program SSTAP (Steady-State and Transient Analysis Program of power transmission lines), on the base of developed theory in Chapters 3 and 4, is outlined and various tasks necessary for accomplishing the steady-state and transient analysis of the transmission line including various terminations and switching operations are assembled.

5.1 DESCRIPTION OF THE MAIN PROGRAM SSTAP

At first, SSTAP reads input data which contains the information about the system such as the parameters of overhead line, source parameters, load type and load parameters, type of analysis etc.

Then, the coefficient matrices **A** and **B** in the state equations (Eq. 3.1) are built depending on the identifications given as input data. If a switch position changes or a fault occurs in the system, following such events, these matrices are altered as described in Chapter 3.

At the third stage, the steady state and/or transient analysis of the system are carried out and capacitor voltages and inductor currents in the lumped parameter representation of the system are computed as the values of the state variables.

5.1.1 Data input

An example of SSTAP input data file is illustrated as follows:

```
NUMBER   OF   SECTIONS=   25
RESISTANCE   PER UNIT LENGTH=   0.1500000  Ohms/km
INDUCTANCE   PER UNIT LENGTH=      1.910   mH/km
CONDUCTANCE  PER UNIT LENGTH=     0.00000  10E-6MHOSkm
CAPACITANCE  PER UNIT LENGTH=     0.03183  10E-6F/km

LENGTH OF LINE=    150.0  Km

SOURCE TYPE:   1 (INFINITE BUS-BAR SOURCE)
SOURCE RESISTANCE=   0.0000000  Ohms
SOURCE INDUCTANCE=       0.000  mH
SOURCE  FREQUENCY=       50   Hz
Vs=     79.6  kV

LOAD TYPE:    5 (RESONATOR)
LOAD RESISTANCE=    193.0000  Ohms
LOAD INDUCTANCE=     461.000  mH
LOAD CAPACITANCE=   0.00000  10E-6F

TYPE OF ANALYSIS:  1 (STEADY-STATE ANALYSIS)


to=        0.000000  mSEC
TIME LIMIT=       20.000000  mSEC
NUMBER   OF DATA: 400
```

At the start the program asks to the user whether new data will be used or not. If new data will be used the program calls the SUBROUTINE INPUT to read the input data. This subroutine reads input data in free format and is transferred to the main program through a data file called INPUT.DAT. This data can be modified to use in solving another problem without changing all of the information about system.

5.1.2 Construction of the Coefficient Matrices

The coefficient matrices **A** and **B** in the state equations (Eq. 3.1) are easily obtained by the main program by using the developed theory in Chapter 3. Since $\bar{\mathbf{A}}$ in Eq. 3.10 is constant for all type of terminations shown in Fig. 2.5, first, the elements of this matrix are built. Then the rest of the matrix **A** is replaced depending on the type of load termination.

5.1.3 Steady-State and Transient Analysis

The steady-state analysis of a given network is required in three cases.

1) Steady-state analysis
2) Loading
3) Fault analysis

The steady-state analysis is done by two subprograms.

SUBROUTINE HMAT(A,B,MO,N,U,H,P) computes the matrix $(pI-A)^{-1}BV$ and stores it to the array variable H. A and B are coefficient matrices in the Eq. 3.1, MO is the maximum dimension and N is the dimension of these matrices which are used in other subroutines discussed in this section also, U is the amplitude of the voltage source and P is the angular frequency of the source.

SUBROUTINE STEADS(H,MO,N,F,T,XT) computes steady-state response for each time T. The computed values are the elements of array variable XT. Argument F is frequency of the voltage source, H is the matrix calculated in the subroutine HMAT.

The control variable TO in the program is used to stop the steady state analysis. If the analysis type is steady-state then TO is the maximum time for execution. But for other cases the program continues by calculating transient analysis. Before doing this, the initial state vector X_0 is calculated to be used in the transient analysis and the coefficient matrices are modified as mentioned before.

In the case of loading, the steady-state analysis for the open circuit line is done by the program up to the switching time TO. At this time (TIME=TO) the coefficient matrices are altered depending on the connected load to the end of the transmission line. The new **A** matrix will be in the form of one of the matrices derived in Section 3.4. For short and open circuit faults a similar procedure is done by the program.

The transient analysis is done starting from the initial time to the time limit represented by the variables TO and TMAX in the program respectively. The subprograms used in the calculation of the total response of the system are given below with a short descriptions.

SUBROUTINE EV(A,MO,N,EGV) applies Rutishouser's LR transformation [20] to the matrix **A** and finds the eigenvalues of this matrix. This subroutine incorporates the following features: a) economy of storage, b) special handling of tridiagonal matrices, taking the advantage of high portions of zeros., c) double precision arithmetic.

The calculated eigenvalues of the matrix **A** with dimension N are the elements of array variable EIG.

SUBROUTINE RES(A,SMI,D,C,MN,M,EIG,LKA) computes the constituent matrices using Eq. 4.27 for calculation of $e^{\mathbf{A}t}$. MN is the same as MO which is defined previously, and array variable EIG contains the eigenvalues of matrix **A**. SMI, D, and C are work matrices.

SUBROUTINE EXPAT(LKA,R,EAT,EF,MO,N,EIG,T,EATF) calculates $e^{\mathbf{A}t}$ using Eq. 4.26 for every time instant t=T.

Subprograms HMAT and STEADS which are used in the steady-state analysis are also used in the transient analysis; and these subprograms have already been discussed at the beginning of this section.

The following subprograms perform general purpose matrix operations, and they are called by different subroutines described in the previous paragraphs. Due to their simplicity and generality, it is satisfied by stating their functions, names and argument lists shortly.

Matrix addition:
CMA(A,B,C,MO,NO,M,N)
A(M,N)=B(M,N)+C(M,N)

Matrix subtraction:
CMS(A,B,C,NO,MO,N,M)
A(M,N)=B(M,N)-C(M,N)

Matrix multiplication:
CMM(A,B,C,NO,NOMO,MO,N,NM,M)
A(M,N)=B(M,N)*C(M,N)

Matrix inversion:
CMI(QI,Q,MO,N)
$Q(M,N)=Q(N,N)^{-1}$

Scalar multiplication:
SCAMAT(S,A,B,NO,MO,N,M)
B(N,M)=S*A(N,M)

Identity matrix:
IMAT(MO,N,I)

These subprograms are combined in the file CMOP. Same subroutines in the case of double precision complex operations are combined in the file DMOP.

5.1.4 Data output

Although the data output are designed to be the load voltage and the source voltage, since all the state variables (inductor currents and capacitor voltages) are calculated for each time, they can be taken as output data if they are needed.

The output data are prepared by the SUBROUTINE OUTPUT. Further details about this subroutine and others can be found in the program list presented as Appendix B.

5.1.5 Variable names

The various arguments used in SSTAP are defined as follows:

Variable name and Dimension	Type	Notation in Theory
MO	INTEGER	-
P	REAL	p
U	REAL	U
A(MO,MO)	REAL*16	**A** matrix
B(MO,1)	COMPLEX*8	**B** matrix
XO(MO,1)	COMPLEX*8	$\mathbf{x}_o$
NTS		-
TO	REAL	t_o
TMAX	REAL	-
H(MO,1)	COMPLEX*8	$(p\mathbf{I}-\mathbf{A})^{-1}\,\mathbf{B}V$
HTO(MO,1)	COMPLEX*8	$(p\mathbf{I}-\mathbf{A})^{-1}\,\mathbf{B}Ve^{pt_o}$

Table 5.1 Variable names, types and their notations in the theory for the program SSTAP.

Variable name and Dimension	Type	Notation in Theory
EIGEN(MO)	COMPLEX*16	λ_k
EAT(MO,MO)	COPMLEX*8	e^{At}
LKA(MO,MO,MO)	COMPLEX*16	$Z_{k\ell}$
TRX(MO,1)	COMPLEX*8	$e^{At}[x_o-(pI-A)^{-1} BVe^{pt_o}]$
SSX(MO,1)	COMPLEX*8	$(pI-A)^{-1} BVe^{pt_o}$
XT(MO,1)	COMPLEX*8	$x(t)$

Table 5.1 (continued)

5.2 PROGRAM SIZE

The program sizes and number of statements for the main program and subprograms are given in Table 5.2.

Program	Size (block)	Number of statements
SSTAP	15	446
INPUT	12	168
HMAT	2	28
STEADS	1	16
EV	8	154
RES	4	44
EXPAT	2	35
CMOP	9	74
DMOP	8	64
OUTPUT	3	30
Total	64	1059

Table 5.2 Program size

5.3 MEMORY REQUIREMENTS

The memory requirement of the program is primarily depends on the state dimension of the system. The maximum state dimension is assumed to be 56. The parameter MO in the program represents this dimension. Considering the dimension of arrays listed in Table 5.1 and knowing that one REAL (REAL*8, REAL*16, COMPLEX*8, COMPLEX*16) number is stored in a 4 (8, 16, 8, 16) byte [21], the total memory requirement for the maximum state dimension is 2.842388 MB. Most of the memory is spent by the variable LKA, which is the set of constituent matrices used in calculation of the state transition matrix e^{At}.

5.4 COMPUTER TIME CONSIDERATIONS

Central Processor Unit (CPU) time of the program SSTAP depends on the state dimension. Most of the time is spent in the calculation of eigenvalues of the matrix **A** and in the calculation of constituent matrices. CPU time in VAX-VMS computer for both steady-state and transient analysis including the effect of choice of the number of sections N to represent the transmission line is shown in Table 5.3. Note that since steady-state analysis does not require the computation of the eigenvalues and the constituent matrices of **A**, the CPU time is almost independent of N in the first column of the table.

N	CPU time	
	Steady-state analysis	Transient analysis
5	2.90 sec.	10.37 sec.
10	3.00 sec.	1.02 min.
15	3.05 sec.	3.37 min.
20	3.10 sec.	17.00 min.

Table 5.3 Program CPU time.

5.5 STRUCTURE OF THE PROGRAM

An overview of the structure of the program SSTAP is described by a flowchart given in Fig. 5.1. Only the main parts of the program have been included in the flowchart.

While the flowchart is intended to bè self-evident several variables, need to be explained.

TIME: Time variable.

TO: Starting time to transient analysis.

TMAX: The predefined maximum time for the study.

ANLTYP: Type of analysis

1. Steady-state analysis
2. Energization
3. Loading
4. Fault analysis
5. Opening of circuit breaker

FTYP: Type of fault

1. Open circuit fault
2. Short circuit fault

LTYP: Load type (see Fig. 2.5)

AL: Logical variable used to represent the load switch.

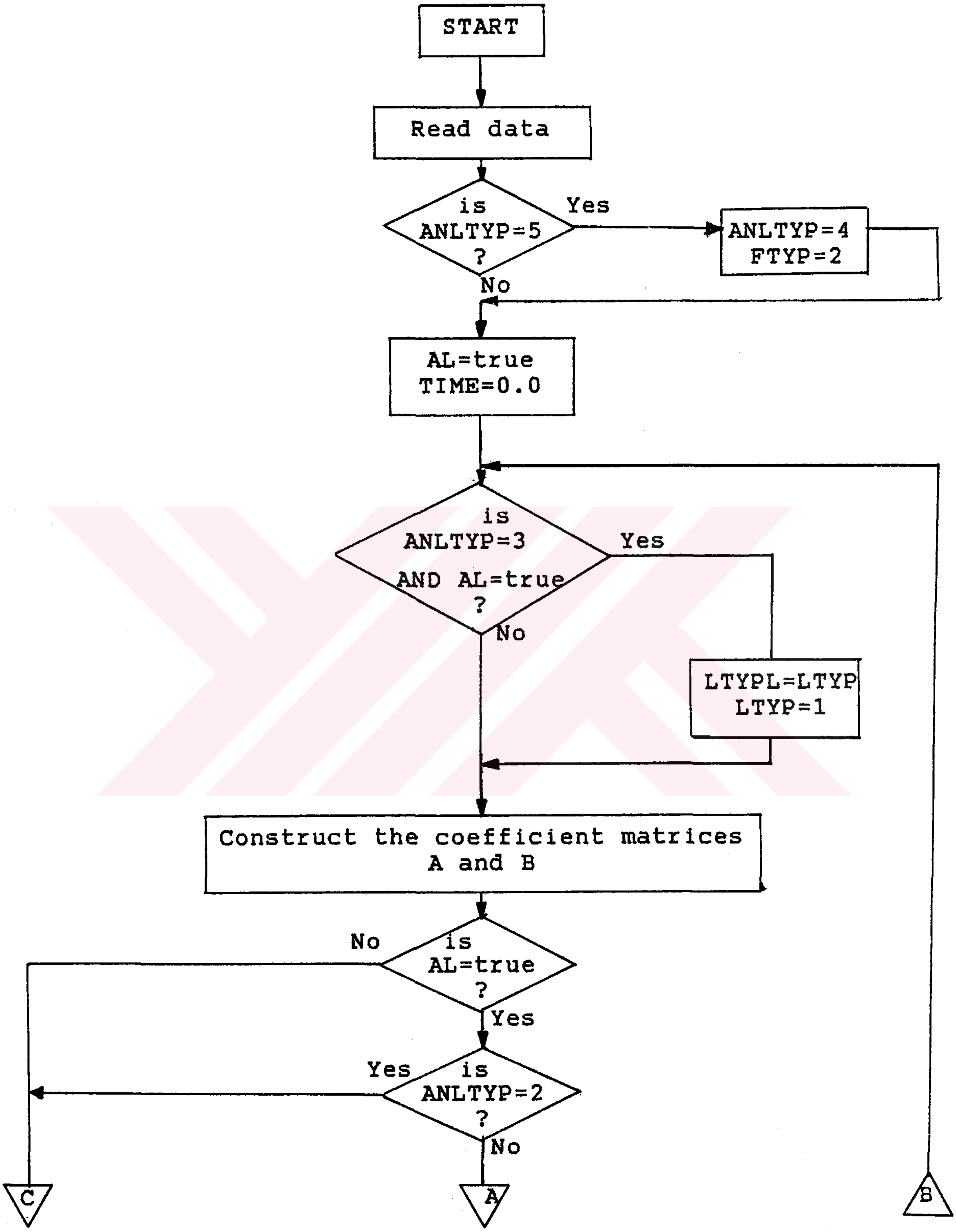


Figure 5.1 Flowchart for the program SSTAP

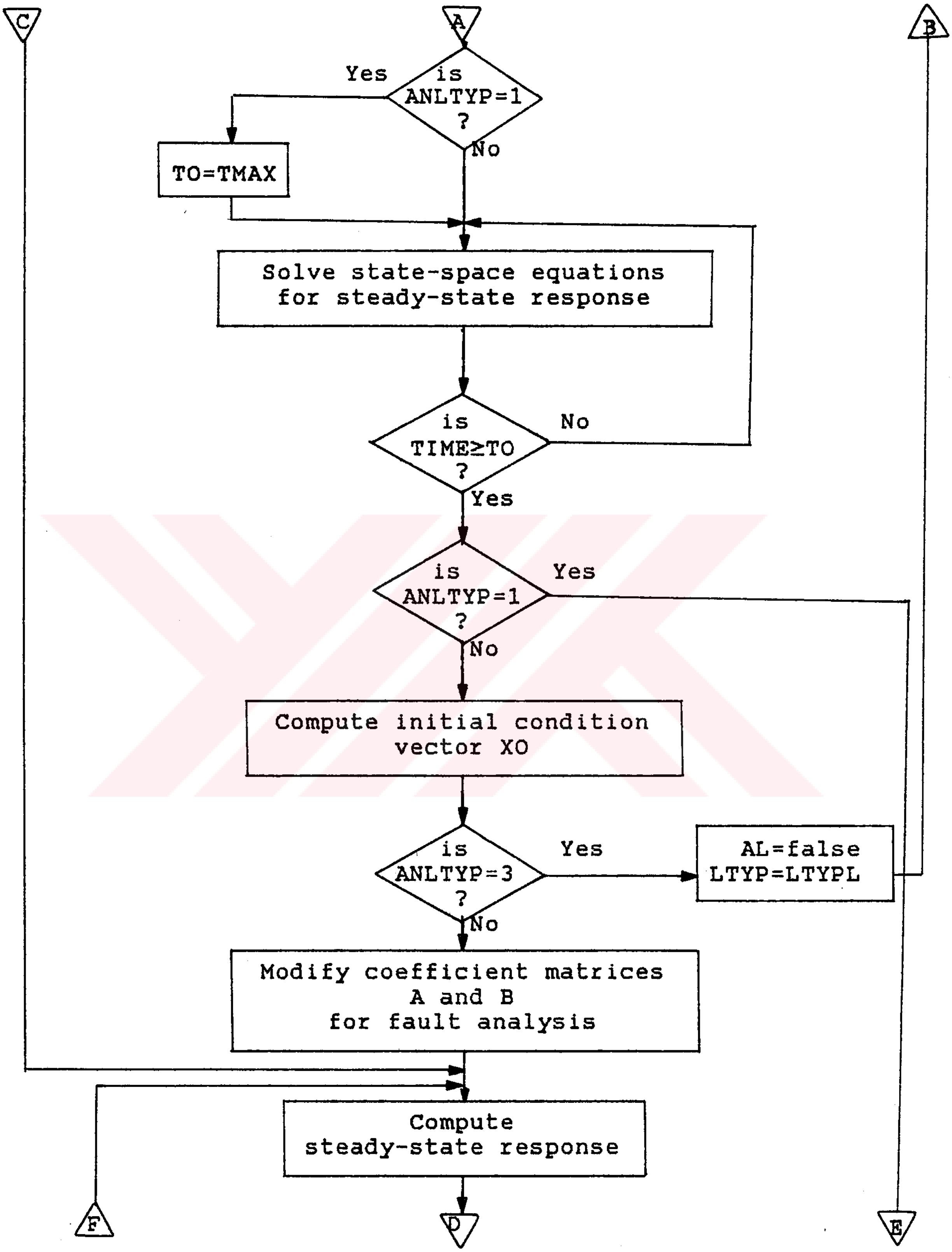


Figure 5.1 (continued)

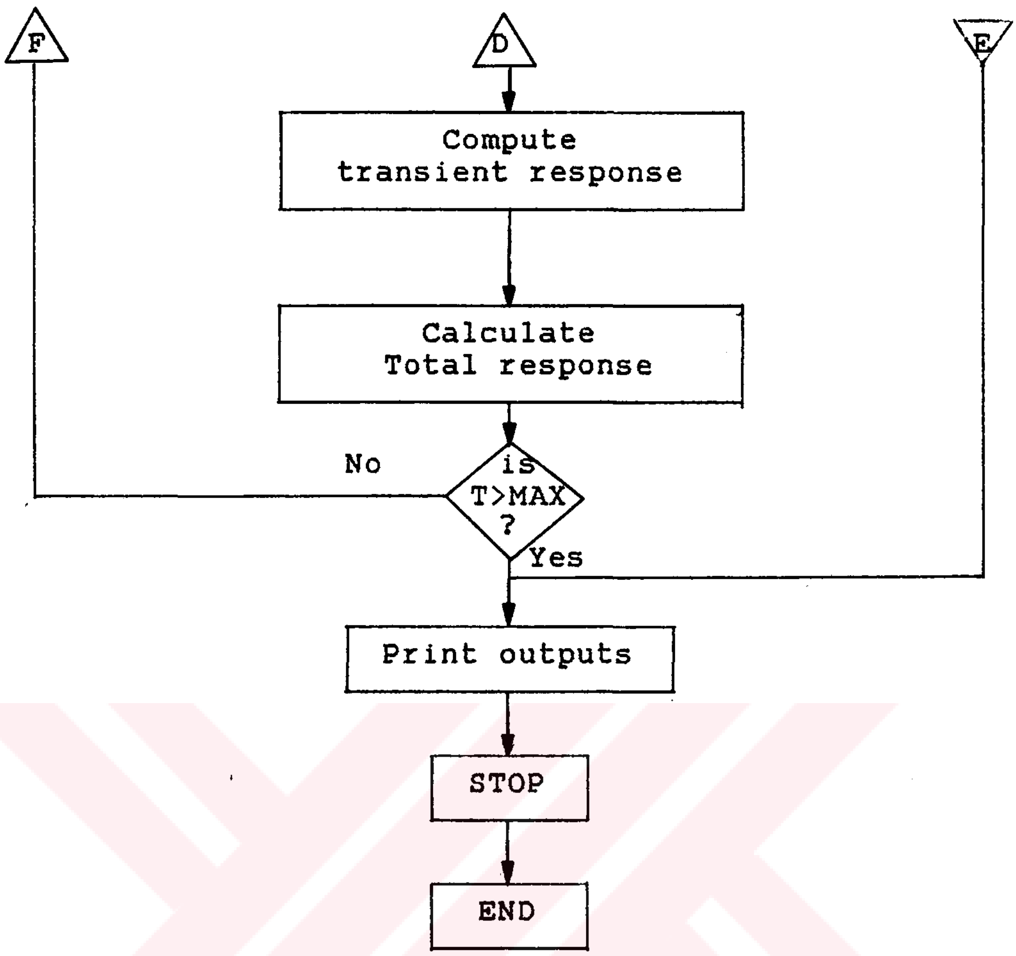


Figure 5.1 (continued)

CHAPTER 6

RESULTS, DISCUSSIONS AND CONCLUSIONS

To illustrate the method used in this thesis, the prepared program is applied to several examples and the obtained results are compared with the results of other methods. Different results for several examples are presented in the following sections.

6.1 STEADY-STATE ANALYSIS

The following example is studied to illustrate the steady-state performance of the program. A 150 km, 79.6 kV, 50 Hz line with resistance 0.15 Ω/km, inductance 1.91 mH/km and capacitance 0.03183 μF/km is terminated by a load type of shown in Fig. 2.5.e with R_L=193 Ω and L_L=0.466 H. When the line is connected to an infinite bus-bar and approximated by 20 T sections, the receiving end voltage and current is shown in Fig. 6.1.a and b, respectively. The magnitudes and phases of the load voltage and load current are computed to be:

$$V_L=63.701\underline{/-11.8^\circ}\ \text{kV}$$

$$I_L=264\underline{/-48.7^\circ}\ \text{A}$$

The phases are calculated taking input voltage as the reference which is assumed to be a cosine wave.

Transmission line voltage, when it is compared by using different number of sections (N) in the lumped parameter representation is recorded and shown in Table 6.1. As it is seen in the table, as the number of sections increases the load voltage approaches to a constant value. In fact for N≥10, the load voltage remains constant within 5 significant digits. Hence the result computed for 20 sections and appearing in the previous page can be assumed to be exact up to 5 decimals.

In general, to satisfy a given precision, it is not needed to increase the number of sections above a certain value, since this increases the computer time and sometimes it may cause the numerical instability.

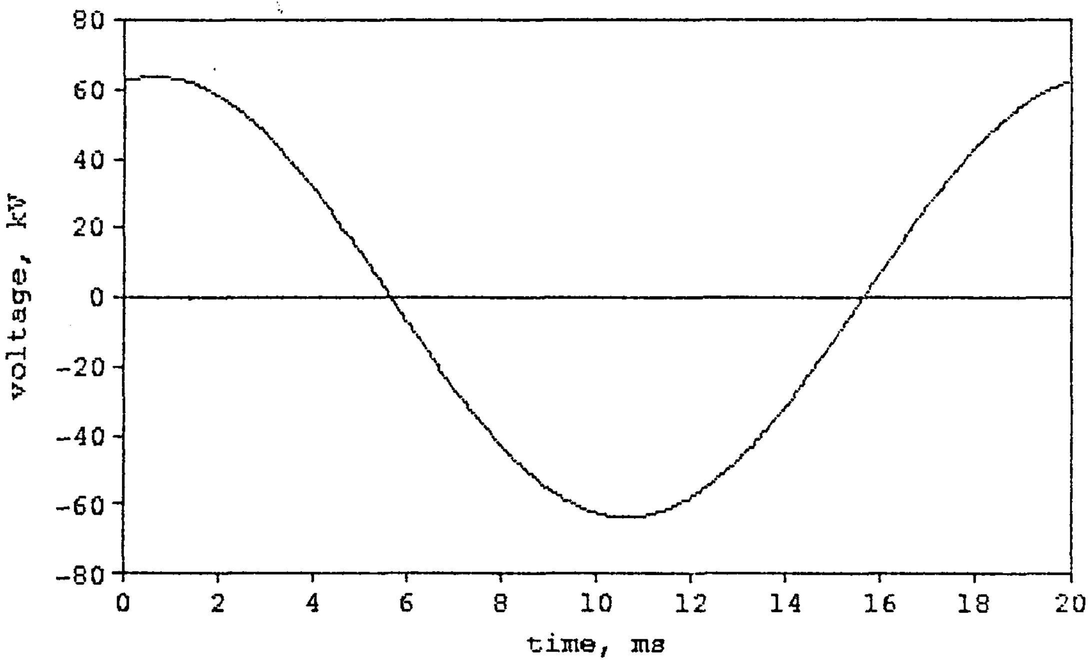
N	Time (ms)	Load voltage (kV)
1	0.656	63.956
2	↓	63.763
3		63.728
4		63.716
5		63.710
10		63.702
15		63.701
20		63.701
25		63.700

Table 6.1 Change in load voltage with respect to number of sections used to represent transmission line.

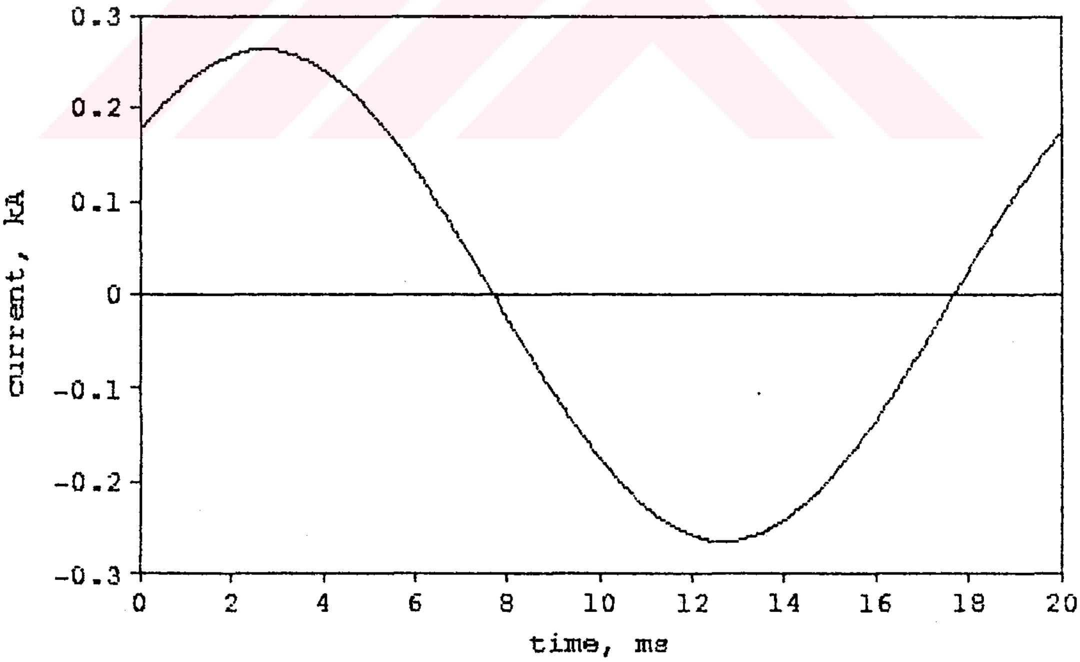
Same example is studied in reference [3] by the approximation of travelling wave method, and the phases are found to be the same but there is 3% difference between the magnitudes of load voltage and 3.8% difference between the magnitudes of load current calculated by two methods.

Considering the shortness of CPU time in Table 5.3

(approximately 3.05 sec.) and the accuracy for a single section (in load voltage: 0.402%), the state-space technique proves its superiority in being alternatives to the existing methods such as travelling wave and Fourier transform. In fact the above results show that using a single T section as the model for the line gives equally correct result with the travelling wave method by taking 3 terms.



(a)



(b)

Figure 6.1 Steady-state response of a transmission line; a) Receiving-end voltage, b) Load current

6.2 LINE ENERGISATION

In this section transients produced as a result of line energisation will be considered. The circuit breaker is represented by an ideal switch (R=0 when closing, R=∞ when opening).

6.2.1 Step Energisation

6.2.1.1 Example1: Open Circuit Line Response

A simple example is studied to compare the result with the exact analytical solution. A line with parameters $R'=2.75\times10^{-2}$/km $L'=1.386$ mH/km, $C'=0.0209$ μF/km and 10 Km long is excited by a step voltage function at t=0 and the receiving end voltage is calculated.

In the case of state-space method the line is represented by different numbers of sections to see the effect of line representation also. The receiving-end voltage for 5 and 10 sections is shown in Fig. 6.2.

In the application of the traveling wave method it's known that the wave travels along the line with velocity $v=1/\sqrt{LC}$. After reaching the receiving end; it is doubled due to the positive reflection from load side, and remains constant up to the source side reflection arrives to this point. This event occurs continuously. Fig. 6.3 shows the lossless line response and state-space solution when line is represented by 15 π-sections.

Fig. 6.2 and 6.3 reveal that high frequency oscillations are present in the solution of state-space method and the frequency of these oscillations increases by number of sections used in the representation of the line. At the same time when representing line by higher number of sections, the rise time of the output voltage decreases and it approaches to the actual result which is approximately zero in the case of low-loss line. Thus it is obvious that higher number of sections is giving more accurate result.

Although the travelling wave method gives the exact analytical solution and naturally preferable for this simple configuration, it can not be used with the same ease and advantage when terminations are realistic. Hence this example should not be taken as a base to prove the insufficiency of the proposed method. However it is included to show the validity of the lumped parameter representation as the number of sections is increased.

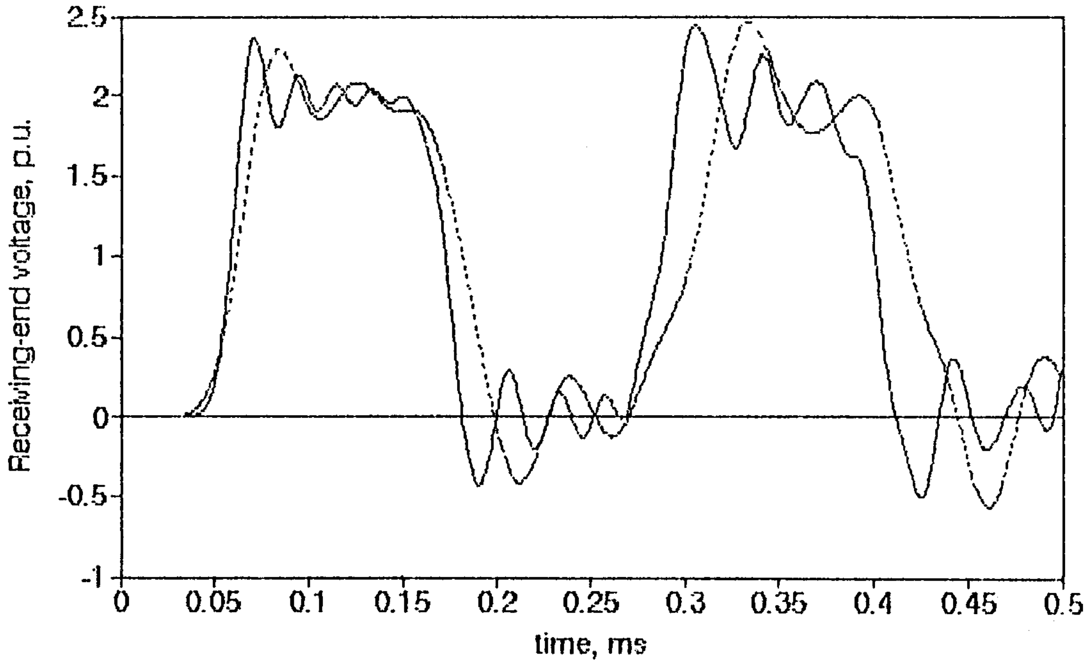


Figure 6.2 Effect of line representation

------- Line is represented by 5 T-sections

——— Line is represented by 10 T-sections

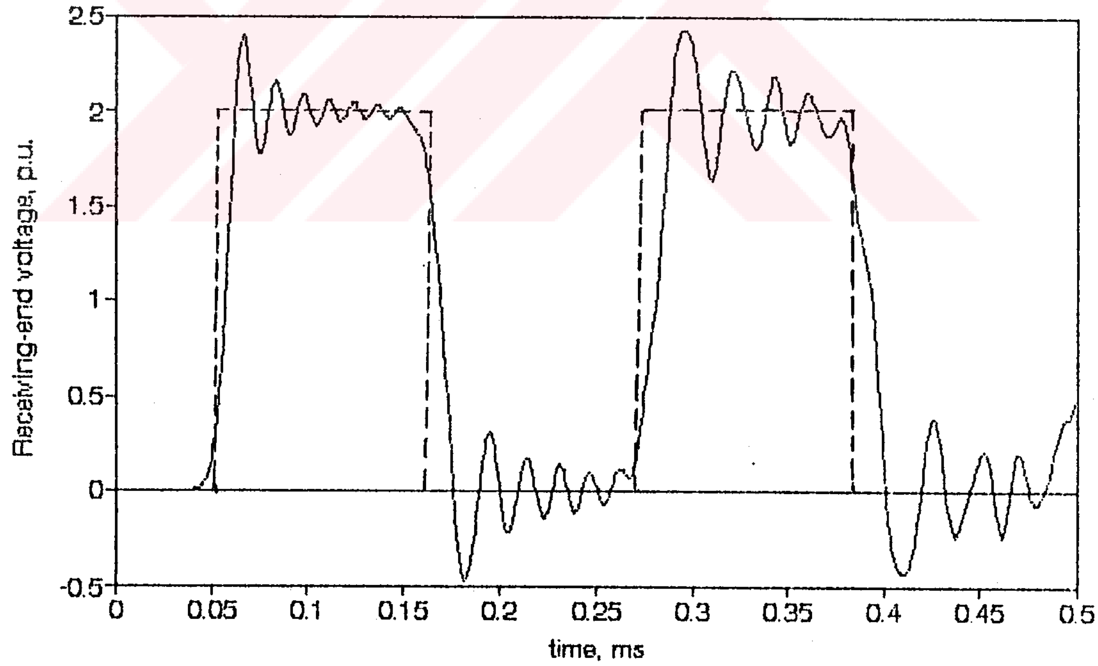


Figure 6.3 Open circuit line response

——— State-space method (line is represented by 15 T-sections)

------- Travelling-wave method

6.2.1.2 Example2: Loaded Line Response

This example is taken from [9] and it is used to compare the state-space technique used in this thesis and the trapezoidal rule of integration for the solution of state equations. The voltage at the receiving end of a single-phase line (320 km long, R'=0.0376 Ω/km, L'=1.52 mH/km, C'=0.0143 μF/km), that is terminated by an inductance of 0.1 H and excited with a step function $V_S(t)$=10 V, is calculated by using trapezoidal integration for the solution, by representing the line by 10 lumped-parameter T-sections and the results are plotted as shown in Fig 6.4. It is seen that the choice of time steps Δt influences primarily the phase position of the oscillations and effects the amplitudes also.

The receiving end voltage of the line is also plotted by using 15 lumped parameter T-sections and the results are shown in Fig. 6.4.b. Comparing Fig 6.4.a and 6.4.b, it is seen that the phase shift and the difference between the amplitudes increases by the increasing of number of sections (N) used to represent the transmission line.

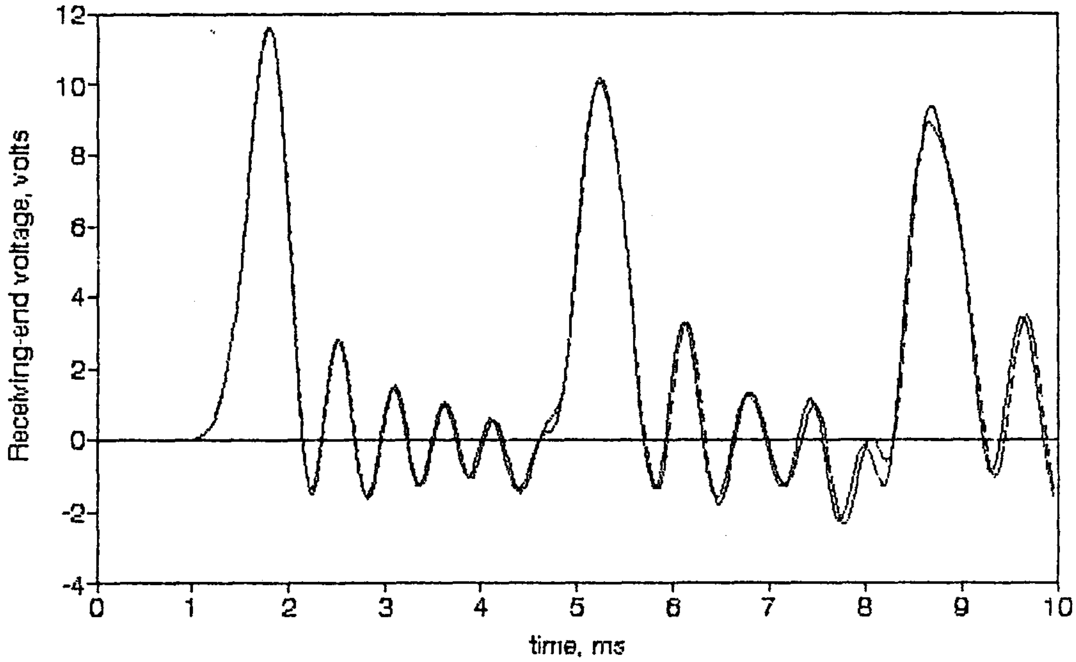
A comparison in terms of maximum error (based on results using smallest step length Δt) is given in Tables 6.1 and 6.2. The error is calculated in the case when it is maximum (t=8.2 msec when N=10, t=8.025 msec when N=15). From these tables Δt=0.25 μsec curve can be taken as the reference output for the trapezoidal integration in our study. This reference is superimposed with the result obtained by program SSTAP and they are shown in Fig. 6.5. As it is seen in this figure both results are almost coincident. However, the advantages of the state-space technique both from the computer time and accuracy point of views is well apparent from Tables 6.2 and 6.3.

	state-space technique	trapezoidal int. $\Delta t=0.2\mu sec.$	trapezoidal int. $\Delta t=2.5\mu sec.$	trapezoidal int. $\Delta t=25\mu sec.$
CPU time (min.)	1.02	80	6.30	0.39
Maximum error	-	-	0.00541	0.72254

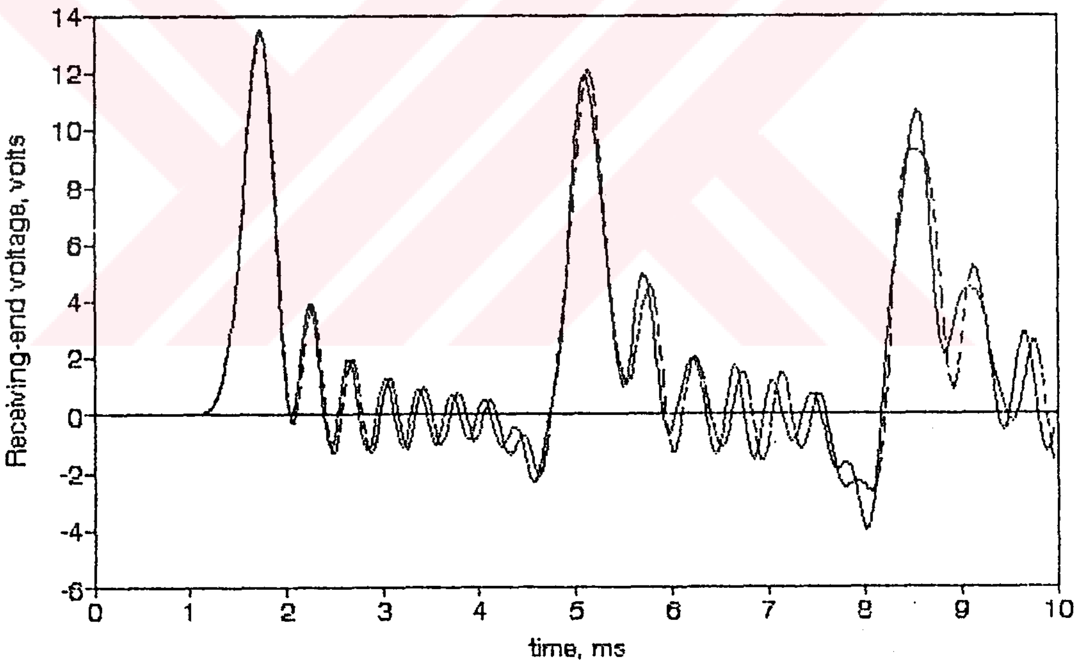
Table 6.2 CPU time and maximum error based on very small step length (Number of sections is 10)

	state-space technique	trapezoidal int. $\Delta t=0.2\mu sec.$	trapezoidal int. $\Delta t=2.5\mu sec.$	trapezoidal int. $\Delta t=5\mu sec.$
CPU time (min.)	3.37	80	13	1.27
Maximum error	0.00147	-	0.0123	1.595

Table 6.3 CPU time and maximum error based on very small step length (Number of sections is 15)



(a)



(b)

Figure 6.4 Effect of time steps in trapezoidal rule of integration; a) Line is represented by 10 T-sections, b) Line is represented by 15 T-sections

------- Δt=25 μsec.

——— Δt=2.5 μsec.

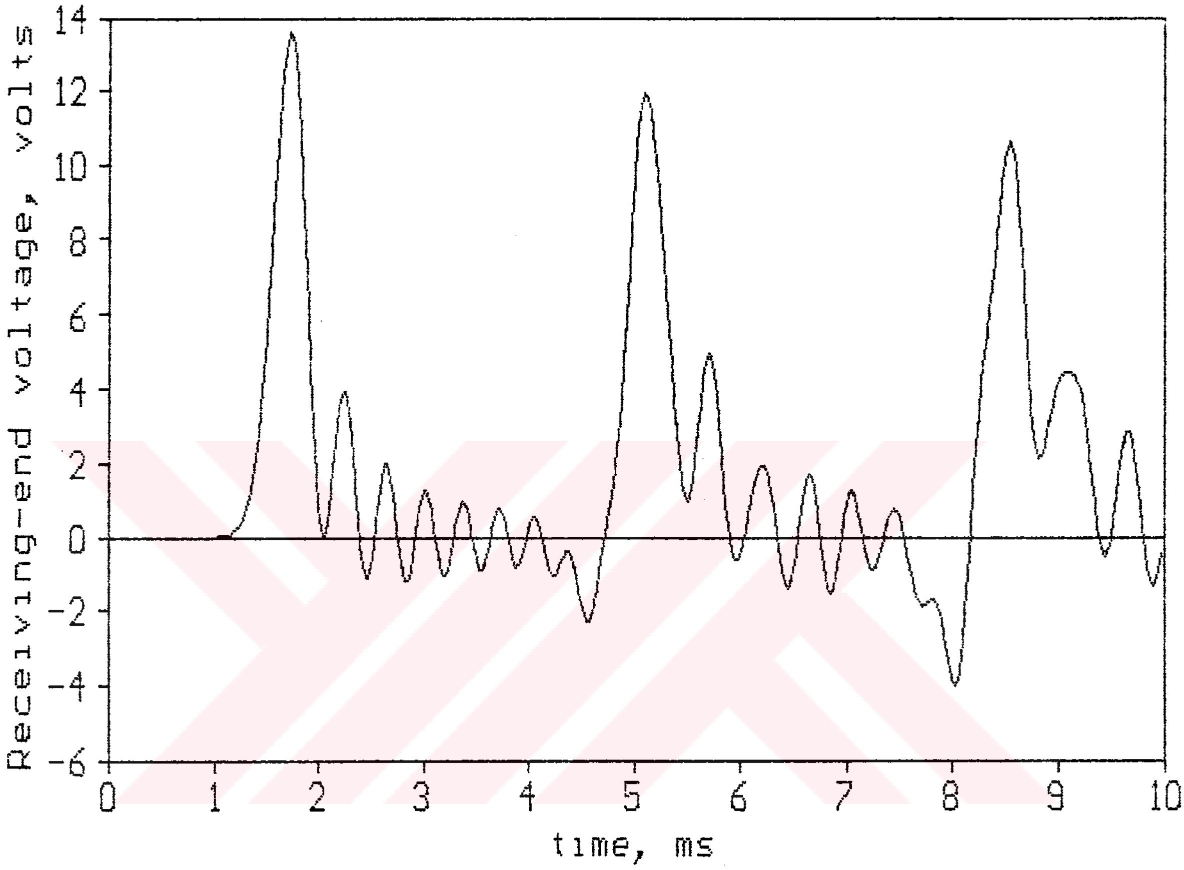


Figure 6.5 Comparison with the method of trapezoidal integration.

——— State-space technique

...... Trapezoidal rule of integration

6.2.2 Example-3: Sinusoidal Energisation

The sinusoidal response obtained by SSTAP of an open ended single-phase line (l=100 mile long, $R'=3.2X10^{-2}$ Ω/mile, $L'=1.42x10^{-3}$ H/mile, $C'=2.09X10^{-8}$ F/mile) in the case of infinite busbar source is shown in Fig. 6.6. The receiving and sending end voltages are plotted when the source voltage applied is at its positive peak at its instant of application. 15 π-sections are used to represent the transmission line. The voltage at the sending end changes as a step voltage at the initial time hence the response in this figure is similar to the square-wave shape response appearing in Fig. 6.3. However the sinusoidal variation of the input is obviously apparent in Fig. 6.6.

In the case of composite source (R_s=0.384 Ω, $L_s=4.88X10^{-2}$ H) the voltage changes exponentially due to the source parameters and "hump" in each curve is due to the reflected wave arriving back from the source, and owing to the inductance, not being immediately reflected as a negative wave. High frequency oscillations and their associated overshoots are not present as seen in Fig. 6.7. This shows that the overshoot and high-frequency oscillations do not occur as the voltage at the sending end to the line departs from the step waveform and becomes more continuous. This is due to the energisation of the line from a nonideal source, which is general in practice.

Comparing with [10], where the same example is studied by using Fourier transform method, the results obtained by SSTAP in the case of composite source is approximately the same. But in the case of infinite source, high frequency oscillations and overshoot exist in the case of state-space method, which is due to the representation of line by lumped parameters.

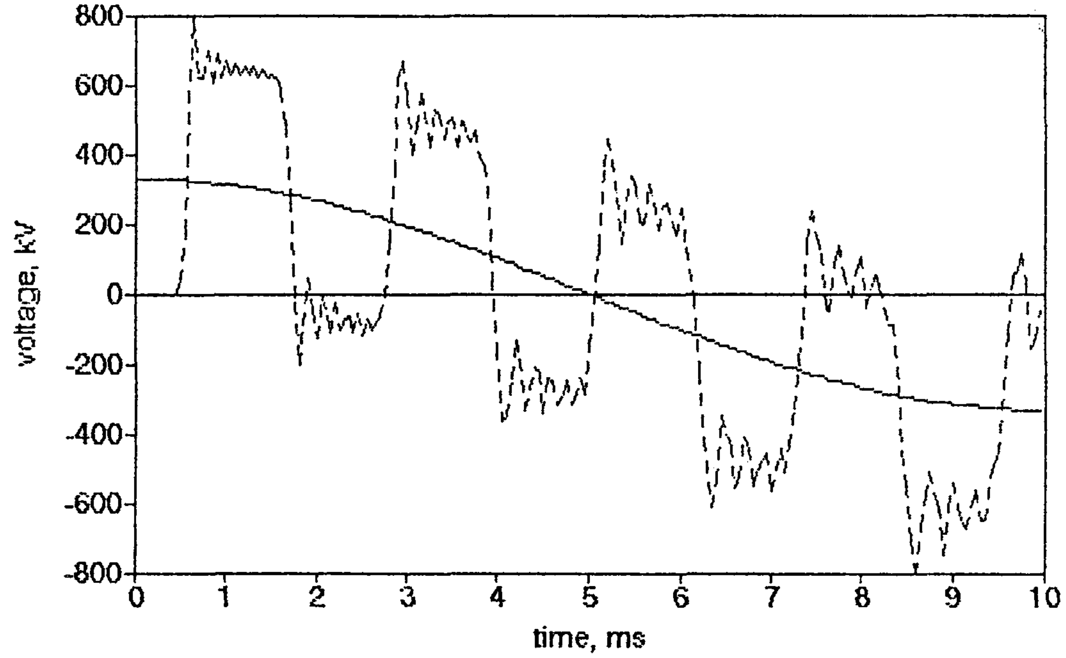


Figure 6.6 Line energization with infinite source
——— Source voltage

------ Receiving-end voltage

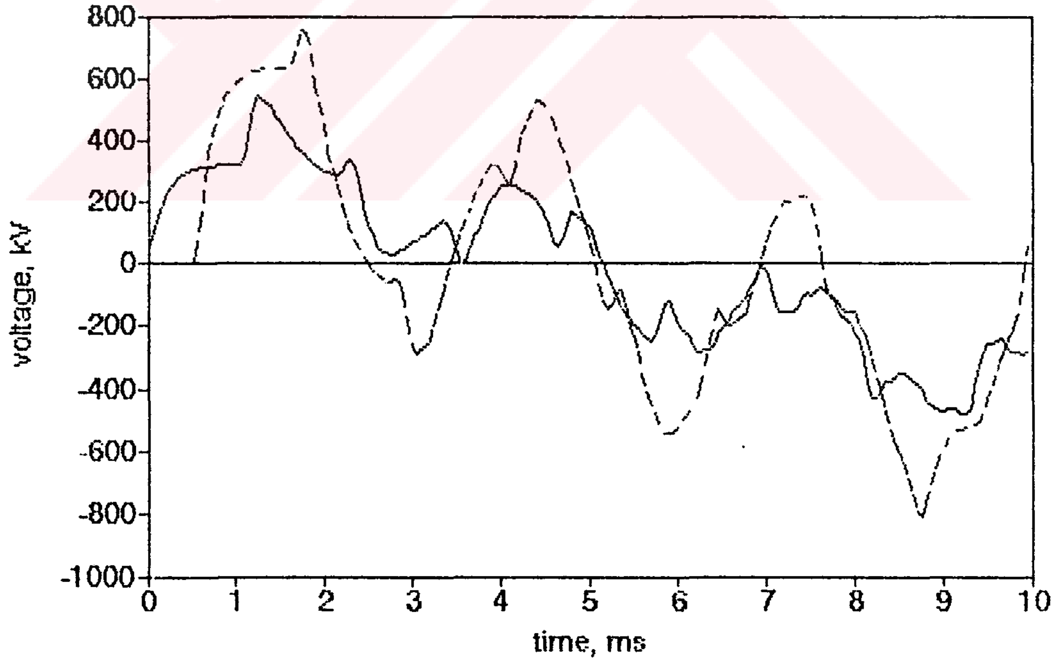


Figure 6.7. Line energization with impedance source
——— Sending-end voltage

------ Receiving-end voltage

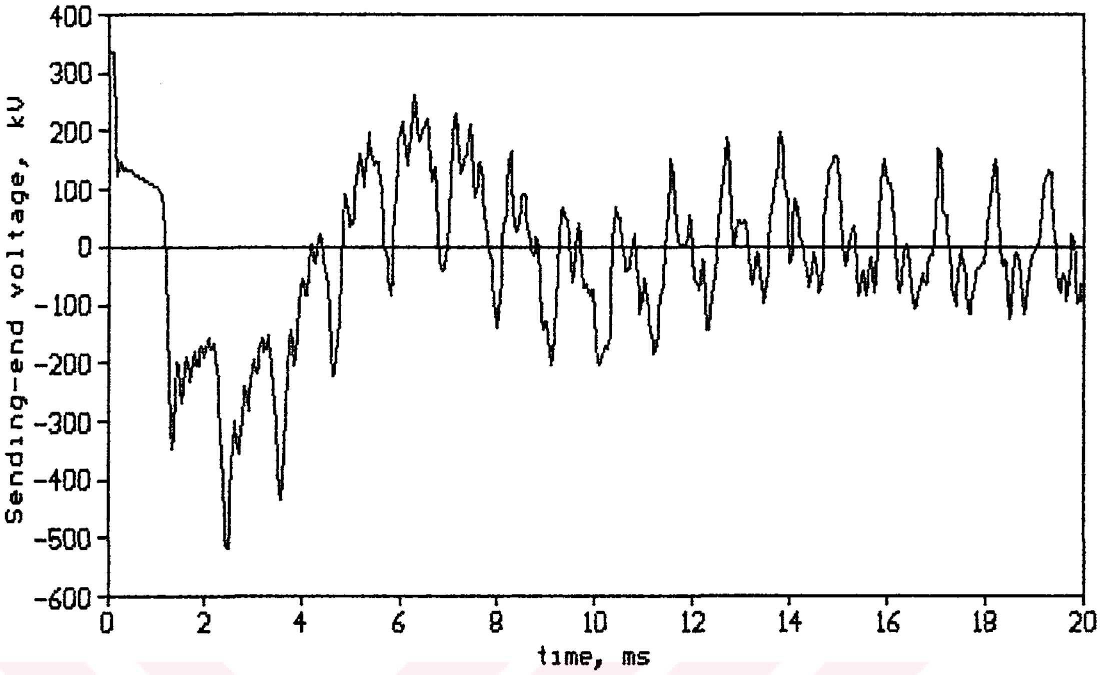
6.3 CIRCUIT BREAKER OPENING

The study of the transient behaviors of the transmission lines under different transient operations such as circuit breaker opening, fault transients, and loading, by using the program SSTAP has the same advantages (less computer time, high accuracy) listed in the previous section (6.2). Therefore in this section and in the following ones, it is satisfied by the problem identification and by presenting the results only.

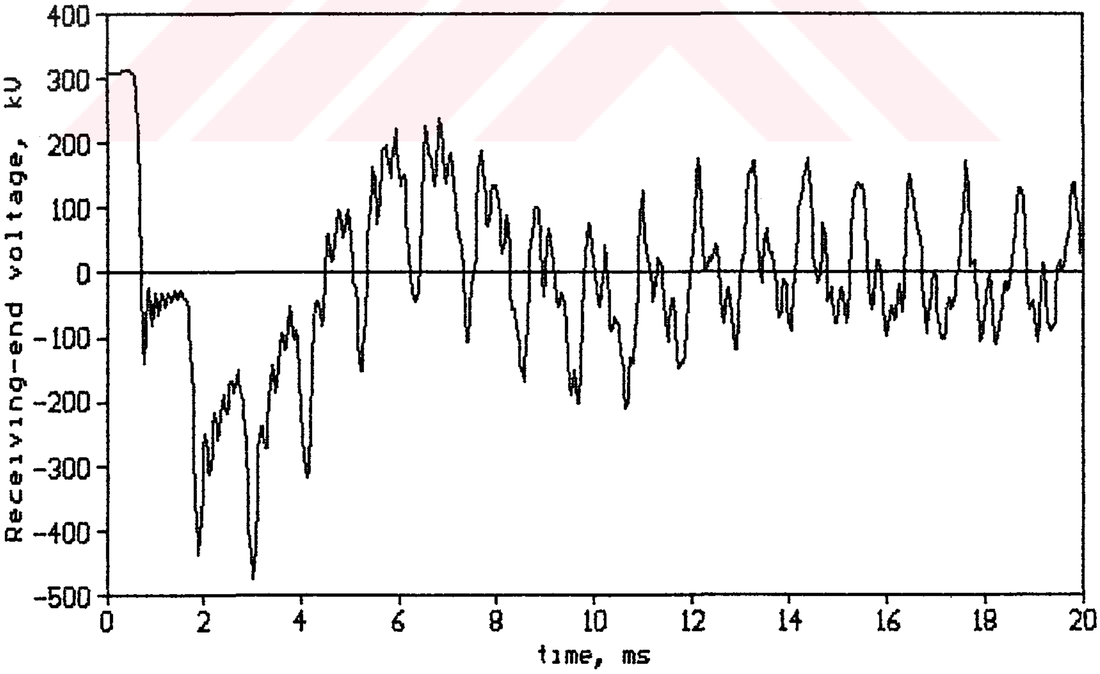
Consider the line of length l=100 mile, with parameters $R'=3.2X10^{-2}$ Ω/mile, $L'=1.42$ mH/mile, $G'=1.0X10^{-8}$ ℧/mile and $C'=2.09X10^{-8}$ F/mile. The line is under energisation by a composite source with parameters $R_s=0.384$ Ω, $L_s=0.0488$ H and $V_s=345$ kV, and it is terminated by a load type shown in Fig. 2.5.h ($R_L=250$ Ω, $L_L=0.6$ H, $C_L=2.2X10^{-8}$ μF). The receiving and sending-end voltages of the line resulting with the opening of circuit breaker at different time instants are shown in Figs. 6.8-6.11. The values of the source and load currents and voltages just before the circuit breaker opens are also shown in Table 6.4. The figures together with this table show that the peak of transient voltage depends on the magnitude of currents in the system. As the magnitude of current increases the peak value of the transient occurring in the system increases.

Switching time (msec.)	Source current (A)	Source voltage (kV)	Load current (A)	Load voltage (kV)	Receiving-end max. transient voltage (kV)
0.1	741	337	730	310	-474
2.5	855	246	994	247	-591
5.0	484	10	698	38	-412
7.0	-35	-189	149	-148	-93

Table 6.4 Maximum transient voltage at receiving-end, depending on the current and voltages in the system.

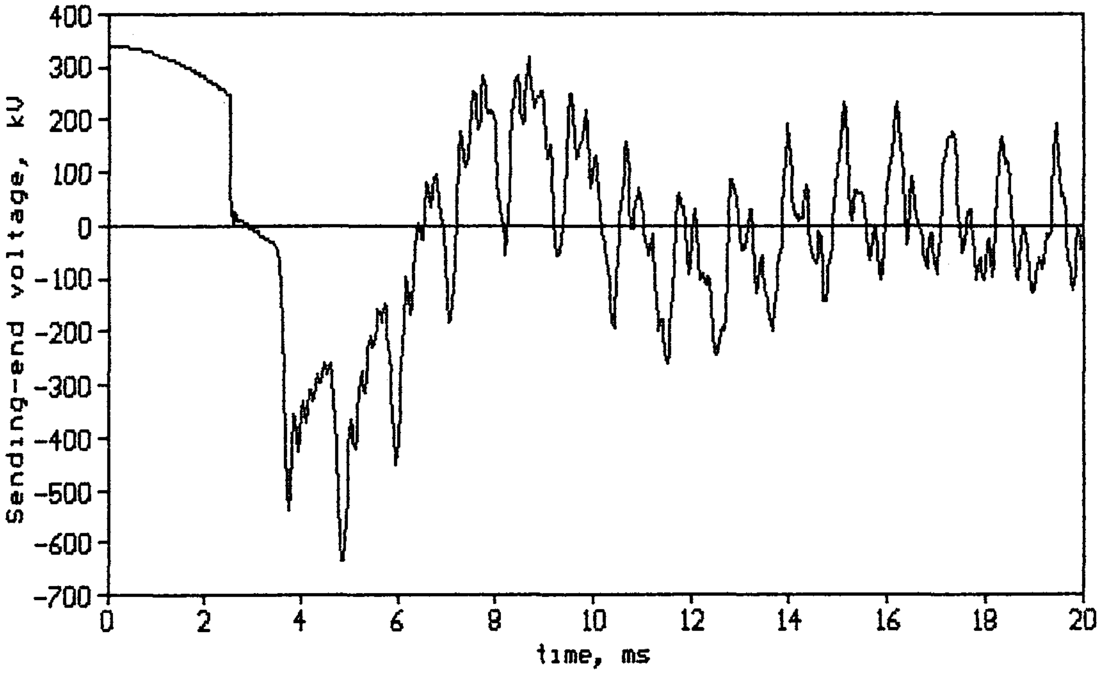


(a)

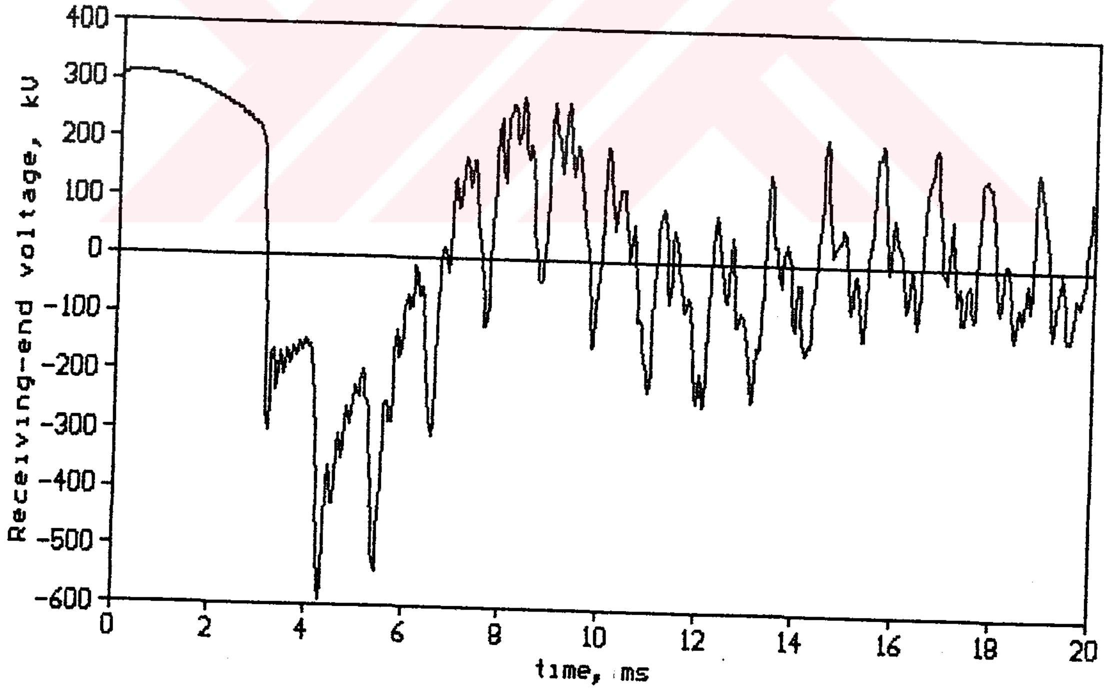


(b)

Figure 6.8 Circuit breaker opens at time t=0.1 msec.; a) sending-end voltage, b) receiving-end voltage.

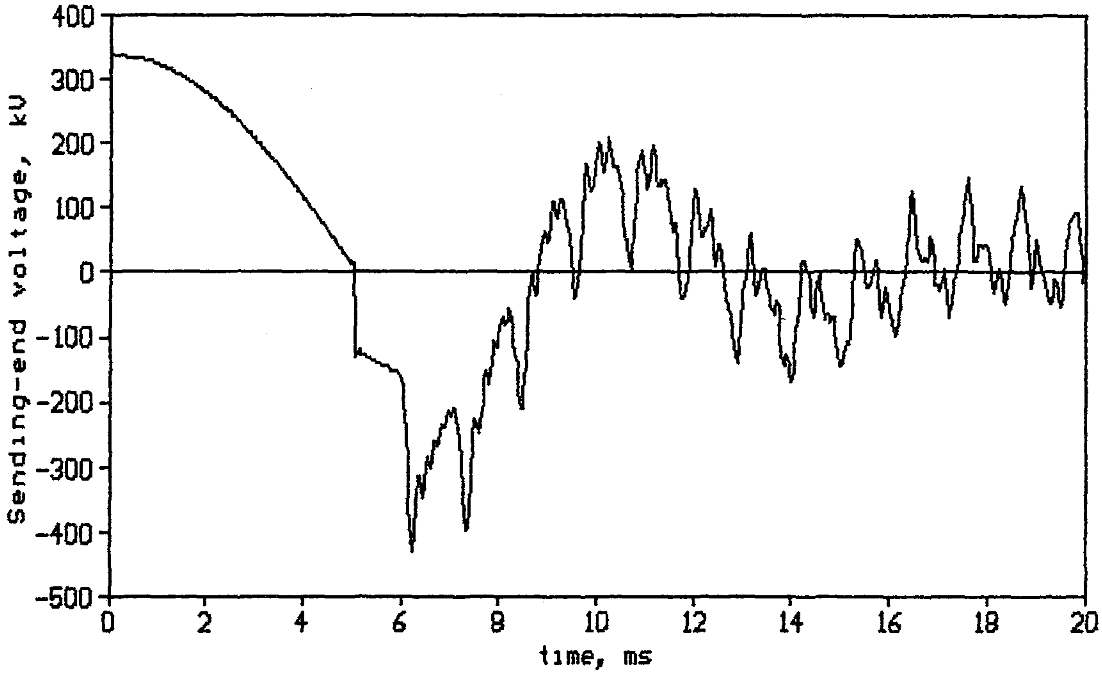


(a)

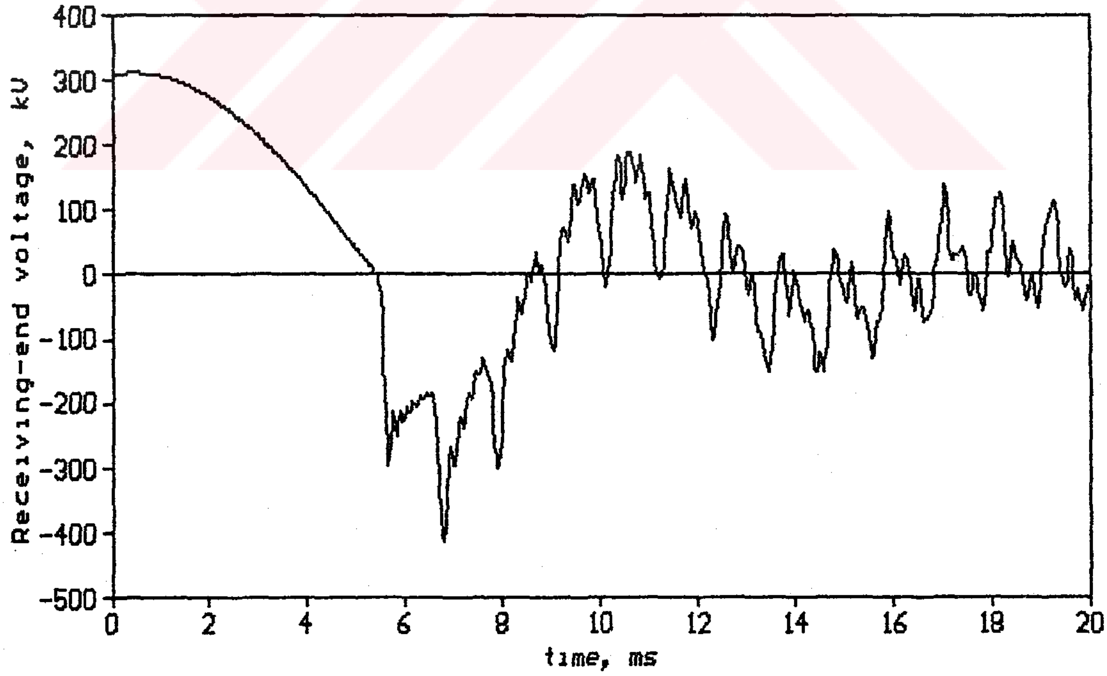


(b)

Figure 6.9 Circuit breaker opens at time t=2.5 msec.; a) sending-end voltage, b) receiving-end voltage.

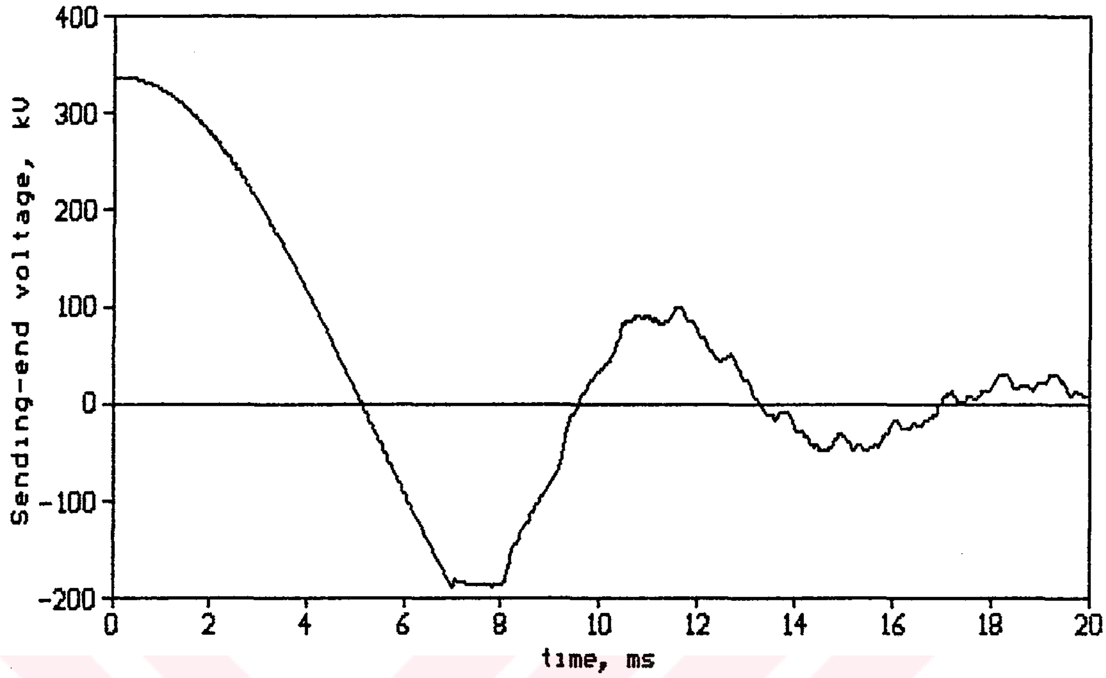


(a)

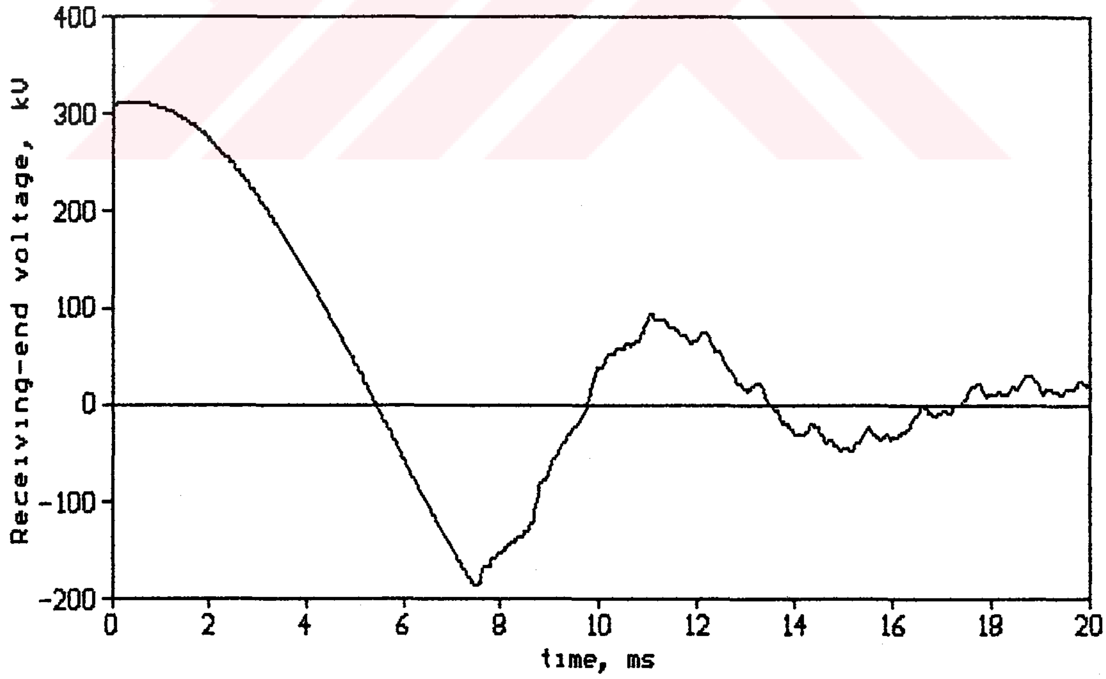


(b)

Figure 6.10 Circuit breaker opens at time t= 5.0 msec.; a) sending-end voltage, b) receiving-end voltage.



(a)



(b)

Figure 6.11 Circuit breaker opens at time t= 7.0 msec.; a) sending-end voltage, b) receiving-end voltage.

6.4 LOADING

Consider the line of length l=100 mile, with parameters $R'=3.2X10^{-2}$ Ω/mile, $L'=1.42$ mH/mile, $G'=1.0X10^{-8}$ ℧/mile and $C'=2.09X10^{-8}$ F/mile. While the line is under energisation by a composite source with parameters $R_s=0.384$ Ω, $L_s=0.0488$ H and $V_s=345$ kV, a load type shown in Fig. 2.5.e ($R_L=150$ Ω, $L_L=0.9$ H) is suddenly connected at the receiving end of the line at time t=3 msec. The transient obtained by SSTAP is shown in Fig. 6.12.

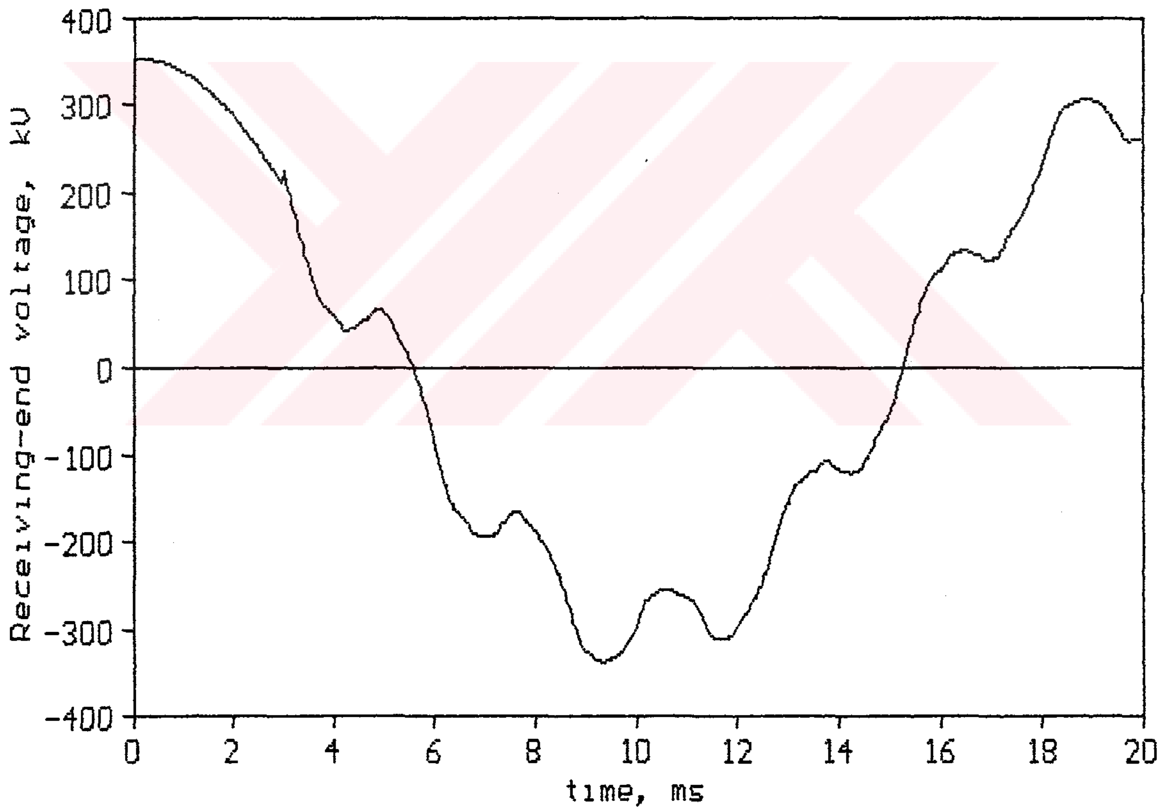


Figure 6.12 Loading transient

6.5 FAULT TRANSIENTS

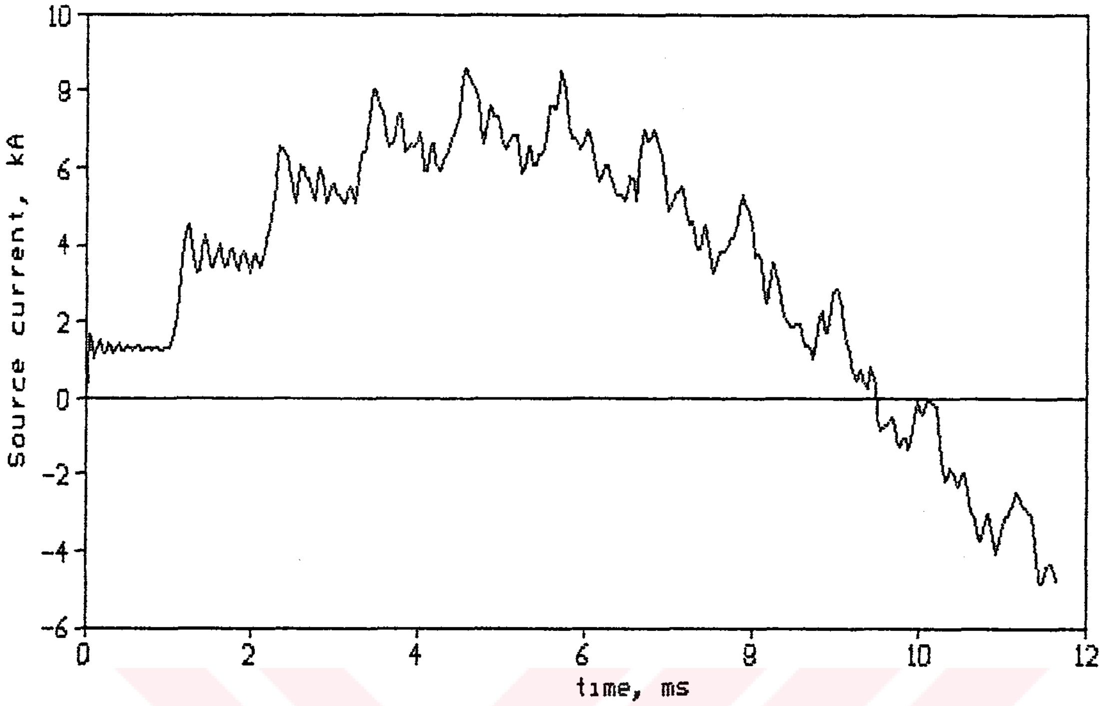
6.5.1 Short Circuit Faults

6.5.1.1 Example-1

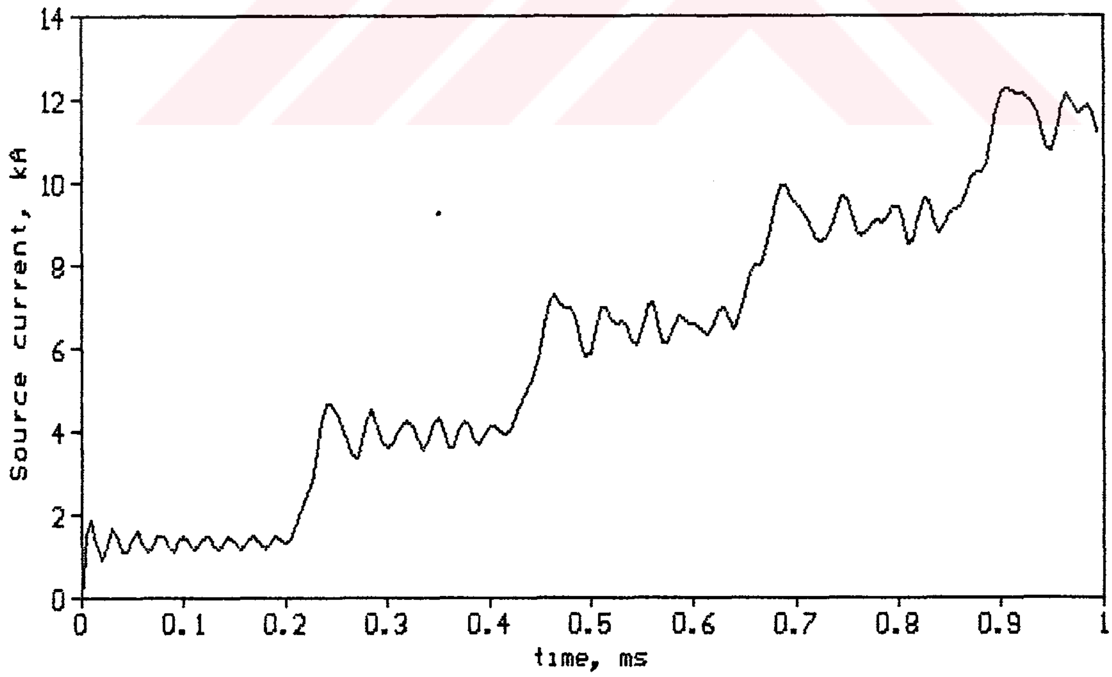
The system chosen for study contains a 400 kV single-phase line with parameters $R'=3.2X10^{-2}$ Ω/mile, $L'=1.42$ mH/mile, $G'=1.0X10^{-8}$ ℧/mile, and $C'=0.0209$ μF/mile, and the type of source used is an infinite bus-bar. It is assumed that the line is initially faulted by phase to earth short circuit at the load side remote from the source, and that the transient voltages and currents are initiated by closing of the circuit breaker at the source end of the line. Two line length are considered one 100 mile long, and the second 20 mile long.

The results are presented in Figs. 6.13.a and b and shows the currents at the source side when a prefaulted line is energized. The current waveforms at the sending end of the line are shown in Figs. 6.13.a and b for 100 mile and 20 mile lengths of line, respectively, when energisation takes place from an infinite bus-bar source at the instant when the voltage applied to the line is at positive peak. The results are for an observation time of 1 ms in the case of 20 mile line, and 12 ms in the case of longer line.

The current waveforms are characterized by steps, at intervals equal to twice the transit time of the line, which are caused by current waves reflected from the faulted end of the line arriving at the source side which also appears as a short circuit. The consecutive steps decrease in magnitude because of the attenuation as they travel along the line.



(a)



(b)

Figure 6.13 Source current in a prefaulted system for different lengths of line; a) line length is 100 mile, b) line length is 20 mile.

6.5.1.2 Example-2

The transmission line with same parameters in the previous example with length of 100 km and connected to the load (Fig. 2.5.h) with parameters R_L=150 Ω, L_L=0.9 H, and C_L=0.15X10^{-7} F is studied to illustrate the short circuit faults. In this case short circuit fault occurs suddenly at midpoint of the line when the line operates under steady-state conditions. The source current is shown in Fig. 6.14 and sending-end and receiving-end voltages are given in Fig. 6.15.a and b respectively.

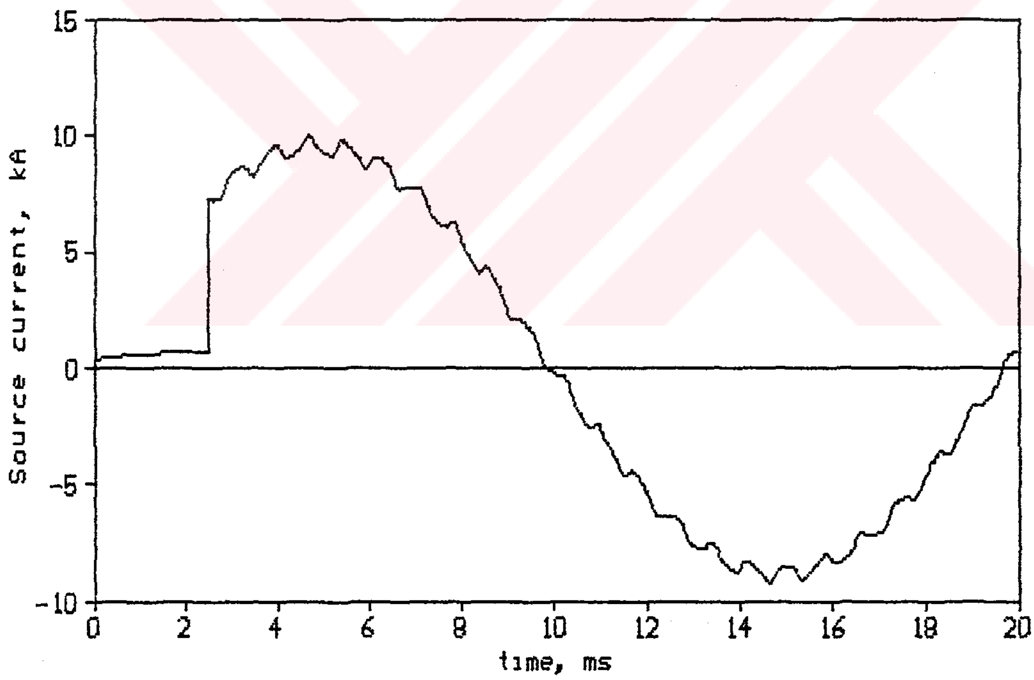


Figure 6.14 Short circuit transients; source current.

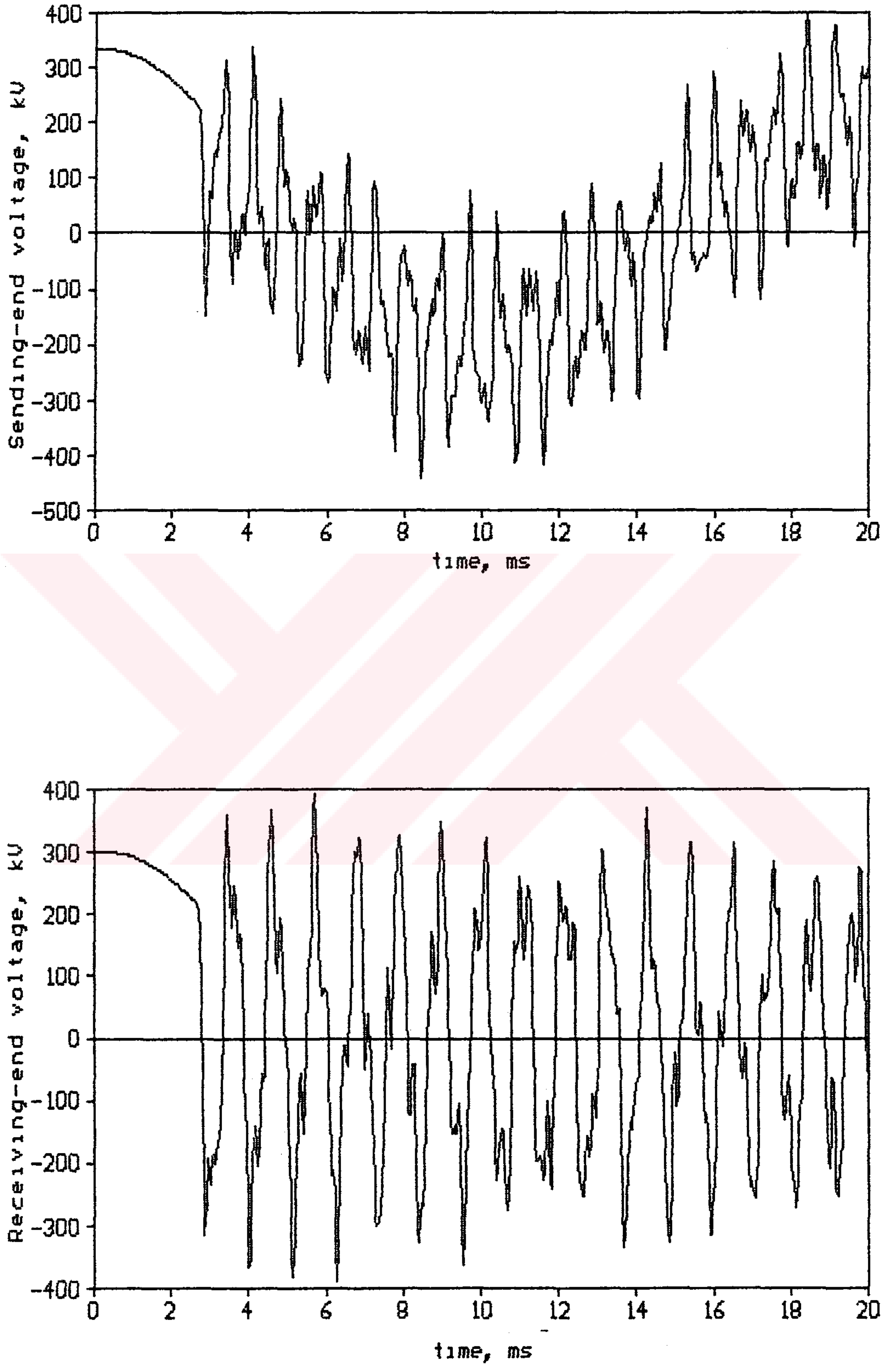


Figure 6.15 Short circuit transients; a) Sending-end voltage, b) receiving-end voltage

6.5.2 Open Circuit Faults

Open circuit faults in the system is in the form of broken conductors. Sudden changes in the network topology may change the state of the system and causes transients. the type of transients in this case is in the form of discharges. An example to open circuit faults is given in this section. Transmission line in Example 2 of the previous section with same the terminations is studied. The results are shown in Figs. 6.16-6.17.

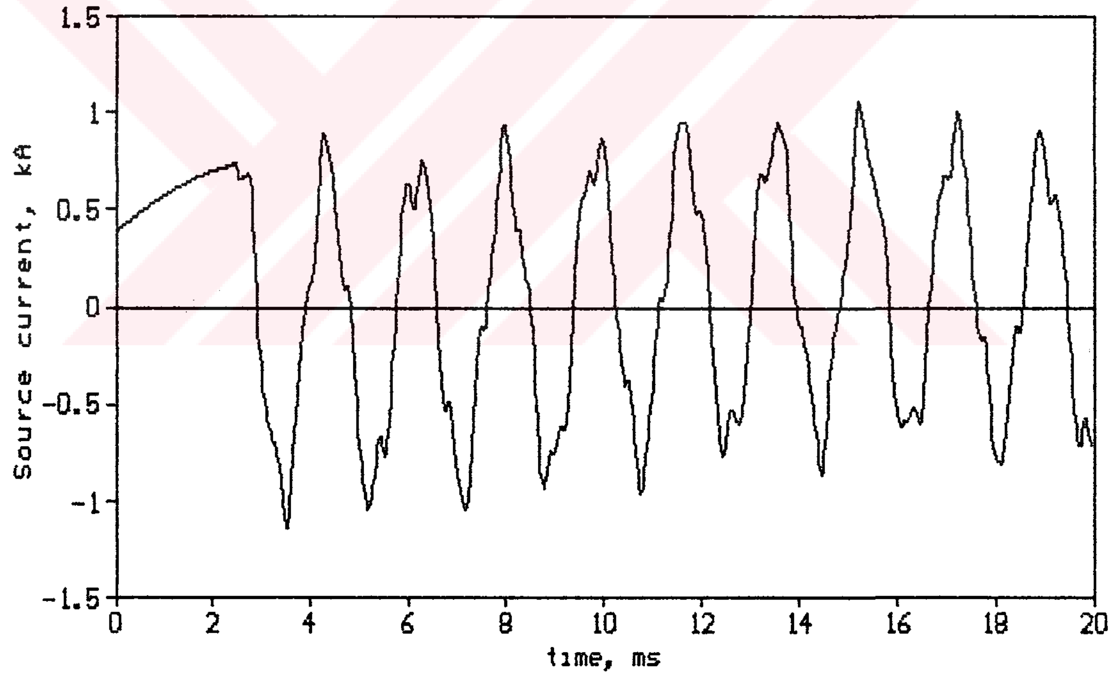
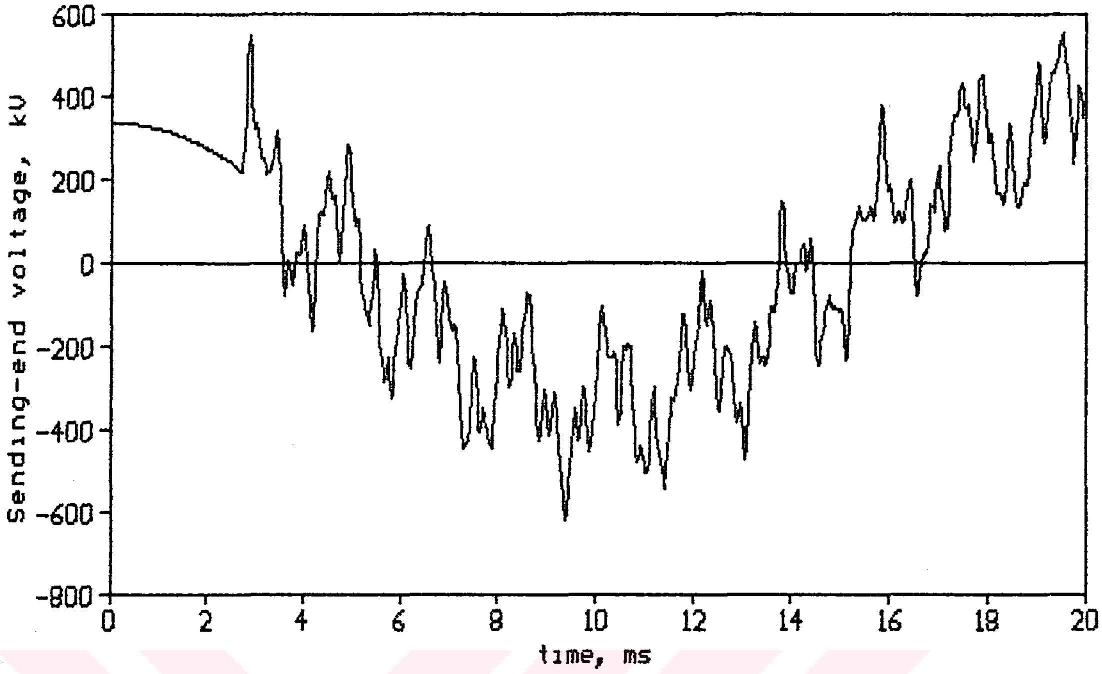
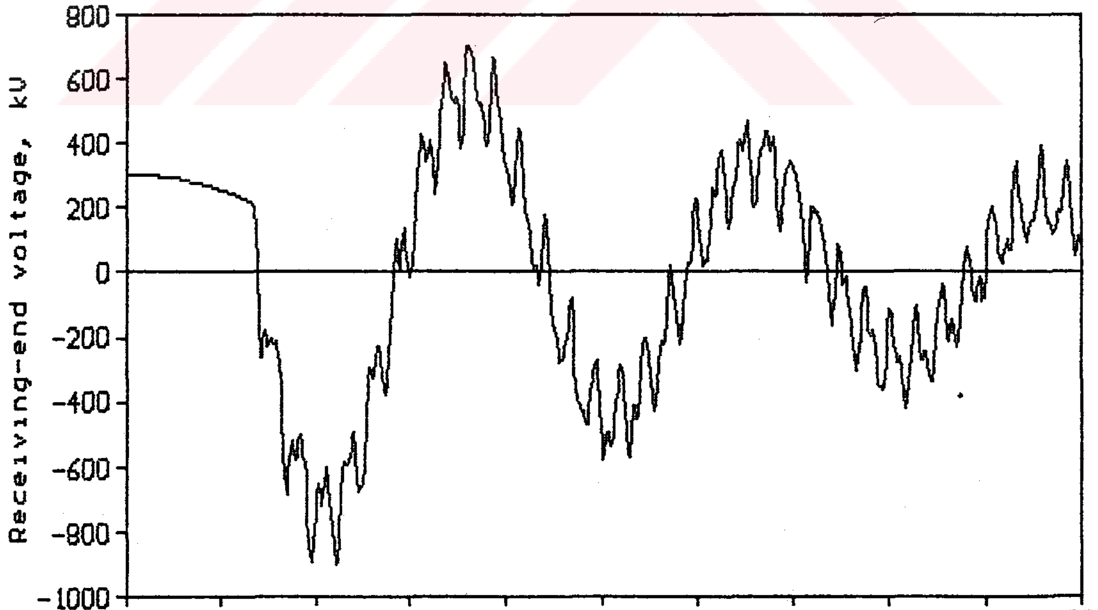


Figure 6.16 Open circuit transients; source current.



a)



b)

Figure 6.17 Open circuit transients; a) Sending-end voltage, b) receiving-end voltage

6.6 CONCLUSION

A digital computer program based on state-space technique, for solving steady-state and transient behaviors of single-phase power systems has been described.

Transmission line is represented by an interconnection of many lumped parameter sections. State-space equations are formulated by taking capacitor voltages and inductor currents as the state variables. Then the responses of the system are obtained by solving these equations, using state-space techniques. Both currents and voltages on the line can be obtained.

The prepared program is capable of solving various types of transients occurring in the power systems due to faults and switching operations and it is so flexible that line with several types terminations can be studied.

After several examples are studied, the followings are concluded:

1) The state of the system can be calculated without calculating the previous states. Therefore the method is past independent and this rejects the possibility of the accumulations of errors caused by repeated solutions, iterative or numerical solutions and saves the computer time.

2) Since lumped parameter representation is used for the line, the series and shunt resistive effects of the line and lumped parameters in the system can easily be implemented, which is not easy in travelling wave method.

3) The technique can be extended to include switching operations and nonlinear elements [14] without any difficulty; which is not the case in the method of Fourier transform and the methods based on travelling wave method such as lattice diagram technique.

4) The method described is easy to implement on a

digital computer.

Future proposals:

The following subjects stand up to be solved as future work.

1) The method can be extended to solve power transients in multi-phase systems.

2) The method can be developed to take the frequency dependency of the system parameters and earth resistance into account.

3) The method can be improved to handle the fault analysis more realistically when a better representation of the open circuit and short circuit faults are used by using nonlinear and/or time-dependent components.

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APPENDIX-A

LISTING OF COMPUTER PROGRAM

```
C*****************************************************************
C*******************  PROGRAM SSTAP ****************************
C*****************************************************************
C                       MAIN PROGRAM
C*****************************************************************
      PARAMETER(MO=56)
      INTEGER N,T,STYP,LTYP,BGROW,LROW,ANLTYP,M,F,MN,FTYP,MOC,MS
      REAL*16 LS,L,R,RS,RL,LL,G,C,CT,CL,A(MO,MO)
      REAL*8 RXT(MO,1),B11
      REAL LENGTH,P,U,TIME,TMAX,DT,TO,TMTO
      COMPLEX*8 B(MO,1),H(MO,1),XO(MO,1),TRX(MO,1),SSX(MO,1)
     1,XT(MO,1),CU,WM11(MO,1),EAT(MO,MO),D(MO,1),HTO(MO,1)
      COMPLEX*16 EIGEN(MO),LKA(MO,MO,MO),WM1(MO,MO),WM2(MO,MO),
     / WM3(MO,MO)
      CHARACTER*1  ST
      LOGICAL AL
      AL=.TRUE.
      OPEN (20,FILE='DISK$AKAD:[MAMIS]INPUT.DAT',STATUS='OLD')
      OPEN (50,FILE='DISK$AKAD:[MAMIS]OUT.DAT',STATUS='OLD')
      PRINT*,'NEW DATA'
      PRINT*,'Y/N'
      READ (6,30)(ST)
30    FORMAT(A1)
      IF (ST .EQ. 'Y') THEN
      CALL INPUT
      ENDIF
      READ(20,41)(N,R,L,G,C)
41    FORMAT(24X,I2,/,31X,F9.7,/,31X,F9.3,/,31X,F9.5,/,31X,F9.5)
      READ(20,42)(LENGTH)
42    FORMAT(/,18X,F6.1)
      READ(20,43)(STYP,RS,LS,F,U)
43    FORMAT(/,15X,I1,/,21X,F9.7,/,21X,F9.3,/,25X,I2,/,6X,F6.1)
      READ(20,44)(LTYP,RL,LL,CL)
44    FORMAT(/,13X,I1,/,19X,F9.7,/,19X,F9.3,/,18X,F9.5)
      READ(20,45)(ANLTYP)
45    FORMAT(/,19X,I1)
      IF (ANLTYP .EQ. 4) THEN
      READ(20,46)(FTYP,FAULTD)
      ELSE
      READ(20,47)
      ENDIF
46    FORMAT(I1,/,15X,F6.2)
```

```
47      FORMAT(/)
        READ(20,48)(TO,TMAX)
48      FORMAT(/,5X,F9.6,/,13X,F9.6)
        READ(20,49)(NTS)
49      FORMAT(15X,I4)
        IF (ANLTYP .EQ. 5) THEN
        ANLTYP=4
        FTYP=2
        FAULTD=0.0
        ENDIF
        C=C/1.0E+6
        G=G/1.0E+6
        CL=CL/1.0E+6
        L=L/1.0E+3
        LL=LL/1.0E+3
        LS=LS/1.0E+3
        TO=TO/1000
        TMAX=TMAX/1000
        MN=56
        R=(R*LENGTH)/N
        L=(L*LENGTH)/N
        G=(G*LENGTH)/N
        C=(C*LENGTH)/N
        P=2*3.14159*REAL(F)
        DT=TMAX/REAL(NTS)
        CU=CMPLX(U,0.0)
        TIME=0.0
100     CONTINUE
        IF ((ANLTYP .EQ. 3) .AND. AL) THEN
        LTYPL=LTYP
        LTYP=1
        ENDIF
C***************   COMPUTATION OF MATRIX A  *******************
        K=2*N
        T=K+3
        DO 110 I=1,MN
        DO 110 J=1,MN
        A(I,J)=0.0
110     CONTINUE
        DO 120 I=2,T
        IF (MOD(I,2) .EQ. 1) THEN
        A(I,I)= -R/L
        A(I,I-1)= 1./L
        A(I,I+1)= -1./L
        ELSE
        A(I,I)= -G/C
        A(I,I-1)= 1./C
        A(I,I+1)= -1./C
        ENDIF
120     CONTINUE
        BGROW=0
        A(1,1)=-(R/2+RS)/(L/2+LS)
        A(1,2)=-1/(L/2+LS)
        IF (LTYP .EQ. 1) THEN
```

```
 A(1,1)=-(R+RS)/(L+LS)
 A(1,2)=-1/(L+LS)
 LROW=K
ELSE IF (LTYP .EQ. 2) THEN
  LROW=K+1
  A(K+1,K)=2/L
ELSE IF (LTYP .EQ. 3) THEN
  A(K+1,K) = 2/L
  A(K+1,K+1)=-(R/2+RL)/(L/2)
  LROW=K+1
ELSE IF (LTYP .EQ. 4) THEN
  A(K+1,K)=1/(L/2+LL)
  A(K+1,K+1)=-(R/2)/(L/2+LL)
  LROW=K+1
ELSE IF (LTYP .EQ. 5) THEN
  A(K+1,K)=1/(LL+L/2)
  A(K+1,K+1)=-(RL+R/2)/(LL+L/2)
  LROW=K+1
ELSE IF (LTYP .EQ. 6) THEN
  A(K+1,K+1)=-R/L
  A(K+1,K)=2/L
  A(K+1,K+2)=-2/L
  A(K+2,K+2)=0.0
  A(K+2,K+1)=1/CL
  A(K+2,K+3)=-1/CL
  A(K+3,K+3)=0.0
  A(K+3,K+2)=1/LL
  LROW=K+3
ELSE IF (LTYP .EQ. 7) THEN
  A(K+1,K+1)=-(R/2)/(L/2+LL)
  A(K+1,K+2)=-1/(L/2+LL)
  A(K+1,K)=+1/(LL+L/2)
  LROW=K+2
ELSE IF (LTYP .EQ. 8) THEN
  A(K+1,K+1)=-R/L
  A(K+1,K)=2/L
  A(K+1,K+2)=-2/L
  A(K+2,K+2)=0.0
  A(K+2,K+1)=1/CL
  A(K+2,K+3)=-1/CL
  A(K+3,K+3)=-RL/LL
  A(K+3,K+2)=1/LL
  LROW=K+3
ELSE IF (LTYP .EQ. 9) THEN
  A(K+1,K+1)=-(R/2+RL)/(L/2+LL)
  A(K+1,K)=1/(L/2+LL)
  A(K+1,K+2)=-1/(L/2+LL)
  A(K+2,K+2)=0.0
  A(K+2,K+1)=1/CL
  LROW=K+3
ENDIF
M=LROW
DO 130 J=1,K
B(J,1)=(0.0,0.0)
```

```
 130   CONTINUE
       IF (LTYP .EQ. 1) THEN
       B(1,1)=CMPLX((1/(LS+L)))
       ELSE
       B(1,1)=CMPLX(1/(LS+L/2))
       ENDIF
       B11=REAL(B(1,1))
       IF (.NOT.(AL)) GOTO  240
        IF (ANLTYP .EQ. 2) THEN
        DO 140 I=1,M
        DO 140 J=1,M
        XO(I,J)=CMPLX(0.0,0.0)
140     CONTINUE
        TIME=TO
        GOTO 240
        ENDIF
       IF (ANLTYP .EQ. 1) THEN
       TO=TMAX
       ENDIF
C*********************** STEADY STATE ANALYSIS ***************
           CALL HMAT(A,B,MN,M,U,H,P)
150        CALL STEADS(H,MN,M,F,TIME,XT)
           DO 160 I=1,M
160        RXT(I,1)=REAL(XT(I,1))
           CALL OUTPUT(A,RXT,MN,M,TIME,U,F,RS,LS,LTYP,RL,LL,B11)
           TIME=TIME+DT
           IF (TIME .LT. TO) GO TO 150
           IF (ANLTYP .EQ. 1)  GO TO 290
           CALL STEADS(H,MN,M,F,TO,XO)
           IF (ANLTYP .EQ. 3) THEN
            AL=.FALSE.
            LTYP=LTYPL
            GOTO 100
           ENDIF
              ONESL=LENGTH/N
              RNF=FAULTD/ONESL
              MS=(NINT(RNF))*2
              MOC=(INT(FAULTD*(N+1)/LENGTH))*2+1
            IF (FTYP .EQ. 1)THEN
            DO 180 I=MOC+1,M
            DO 180 J=2,M
180         A(I-1,J-1)=A(I,J)
            IF (MOC .NE. 1) THEN
            A(MOC-1,MOC)=0.0
            A(MOC,MOC-1)=0.0
            ENDIF
            ELSE IF (FTYP .EQ. 2) THEN
            DO 183 I=MS+1,M
            DO 183 J=2,M
183         A(I-1,J-1)=A(I,J)
            A(MS-1,MS)=0.0
            A(MS,MS-1)=0.0
            ENDIF
              M=M-1
```

```
      DO 184 I=1,M
184   WRITE(10,190)(A(I,J),J=1,M)
190   FORMAT(1X,10F12.1)
C ****************** TRANSIENT ANALYSIS **********************
          IF (ANLTYP .EQ. 4) THEN
          IF (FTYP .EQ. 2) THEN
          DO 230 I=MS+1,M+1
230       XO(I-1,1)=XO(I,1)
          ELSE
          DO 231 I=MOC+1,M+1
231       XO(I-1,1)=XO(I,1)
          ENDIF
          ENDIF
240       CALL HMAT(A,B,MN,M,U,H,P)
          UTO=CEXP(CMPLX(0.0,2*3.14*REAL(F)*TO))
          CALL SCAMAT(UTO,H,HTO,MN,1,M,1)
          CALL EV(A,MN,M,EIGEN)
          CALL RES(A,WM1,WM2,WM3,MN,M,EIGEN,LKA)
          J=0
C          TIME=TIME+0.4
C          TMAX=TMAX+0.4
250       TMTO=TIME-TO
          J=J+1
          CALL EXPAT(LKA,WM1,WM2,WM3,MN,M,EIGEN,TMTO,EAT)
          CALL STEADS(H,MN,M,F,TIME,SSX)
          CALL CMS(D,XO,HTO,MN,1,M,1)
          CALL CMM(TRX,EAT,D,MN,MN,1,M,M,1)
          CALL CMA(XT,TRX,SSX,MN,1,M,1)
          DO 271 I=1,M
271       RXT(I,1)=REAL(XT(I,1))
          CALL OUTPUT(A,RXT,MN,M,TIME,U,F,RS,LS,LTYP,RL,LL,B11)
          TIME=TO+DT*J
          IF (TIME .LE. TMAX) GO TO 250
290       STOP
          END


      SUBROUTINE INPUT
C     SUBROUTINE INPUT READS INPUT DATA...
      INTEGER N,T,STYP,LTYP,BGROW,LROW,ANLTYP,DIM,M,F,MN,FTYP
     1 ,MOC,MS
      REAL LS,L,R,RS,RL,LL,U,LENGTH,P,NCYCLE,G,C,CT,CL,ALOAD,
     1TIME,TLIM,DELTAT,TO,TMTO
      LOGICAL AL
      OPEN (20,FILE='DISK$AKAD:[MAMIS]INPUT.DAT',STATUS='OLD')
      PRINT*,CHAR(27),'[H',CHAR(27),'[J'
      WRITE(6,85)
      PRINT*,CHAR(27),'#6      LINE PARAMETERS'
      WRITE(6,85)
      PRINT*,'NUMBER OF SECTIONS (N) ='
      READ*,N
      PRINT*,'RESISTANCE PER UNIT LENGTH (OHMS/KM)'
      PRINT*,'R='
      READ*,R
```

```
      PRINT*,'INDUCTANCE PER UNIT LENGTH (MH/KM)'
      PRINT*,'L='
      READ*,L
      PRINT*,'CONDUCTANCE PER UNIT LENGTH (10E-6MHOS/KM)'
      PRINT*,'G='
      READ*,G
      PRINT*,'CAPACITANCE PER UNIT LENGTH (10E-6F/KM)'
      PRINT*,'C='
      READ*,C
      PRINT*,'LENGH OF LINE (KM)'
      PRINT*,'L='
      READ*,LENGTH
      PRINT*,CHAR(27),'[H',CHAR(27),'[J'
      WRITE(6,85)
      PRINT*,CHAR(27),'#6SOURCE TYPE AND PARAMETERS'
      WRITE(6,85)
      PRINT*,CHAR(27),'[7;7MTYPE 1.',CHAR(27),'[0M',
     1 '     INFINITE SOURCE (SOURCE RESISTANCE=0,
     2 SOURCE INDUCTANCE =0)'
      PRINT*,CHAR(27),'[7;7MTYPE 2.',CHAR(27),'[0M',
     1 '     FINITE SOURCE   (SOURCE INDUCTANCE=0)'
      PRINT*,CHAR(27),'[7;7MTYPE 3.',CHAR(27),'[0M',
     2 '     COMPOSITE SOURCE'
      WRITE(6,85)
      PRINT*,CHAR(27),'[7;5MENTER SOURCE TYPE',CHAR(27),'[0M'
      WRITE(6,85)
      READ*,STYP
      IF (STYP.EQ.2) THEN
      PRINT*,'RS='
      READ*,RS
      ELSE IF (STYP .EQ. 3) THEN
      PRINT*,'RS= ------- OHMS,   LS= -------- MH'
      READ*,RS,LS
      ENDIF
      PRINT*,'SOURCE FREQUENCY (HZ)='
      READ*,F
      PRINT*,'LINE VOLTAGE (KV)='
      READ*,U
52    CONTINUE
      PRINT*,CHAR(27),'[H',CHAR(27),'[J'
      WRITE(6,85)
      PRINT*,CHAR(27),'#6LOAD TYPE AND PARAMETERS'
      WRITE(6,85)
      PRINT*,'TYPE 1.     OPEN CIRCUIT LOAD'
      PRINT*,'TYPE 2.     SHORT CIRCUIT LOAD'
      PRINT*,'TYPE 3.     RESISTANCE  '
      PRINT*,'TYPE 4.     INDUCTANCE  '
      PRINT*,'TYPE 5.     RESISTANCE + INDUCTANCE '
      PRINT*,'TYPE 6.     TANK CIRCUIT'
      PRINT*,'TYPE 7.     RESONATOR'
      PRINT*,'TYPE 8.     LOSSY TANK CIRCUIT'
      PRINT*,'TYPE 9.     LOSSY RESONATOR'
      WRITE(6,85)
      PRINT*,CHAR(27),'[7;5MENTER LOAD TYPE',CHAR(27),'[0M'
```

```
      READ*,LTYP
      PRINT*,CHAR(27),'[H',CHAR(27),'[J'
C*****************************************************************
      IF ((LTYP .EQ. 3) .OR. (LTYP .EQ. 5) .OR. (LTYP .EQ. 8)
     / .OR. (LTYP .EQ. 9))  THEN
      PRINT*,'LOAD RESISTANCE='
      READ*,RL
      ENDIF
      IF ((LTYP.EQ. 4).OR.(LTYP .EQ. 5).OR.(LTYP .EQ. 8).OR.
     1(LTYP .EQ.9).OR.(LTYP .EQ.7)) THEN
      PRINT*,'LOAD INDUCTANCE (MH)='
      READ*,LL
      ENDIF
      IF ((LTYP.EQ. 6).OR.(LTYP .EQ. 7).OR.(LTYP .EQ. 8).OR.
     / (LTYP .EQ. 9)) THEN
      PRINT*,'LOAD CAPACITANCE (10E-6F)='
      READ*,CL
      ENDIF
      PRINT*,CHAR(27),'[H',CHAR(27),'[J'
      WRITE(6,85)
      PRINT*,CHAR(27),'#6 TYPE OF ANALYSIS'
      WRITE(6,85)
      PRINT*,'TYPE 1. STEADY-STATE ANALYSIS'
      PRINT*,'TYPE 2. ENERGIZATION'
      PRINT*,'TYPE 3. LOADING'
      PRINT*,'TYPE 4. FAULT TRANSIENTS'
      PRINT*,'TYPE 5. OPENING OF CIRCUIT BREAKER'
      WRITE(6,85)
      PRINT*,CHAR(27),'[7;5MENTER THE TYPE OF ANALYSE',CHAR(27)
     1,'[0M'
      READ*,ANLTYP
      IF (ANLTYP .EQ. 4) THEN
      PRINT*,CHAR(27),'[H',CHAR(27),'[J'
      PRINT*,'FAULT DISTANCE (KM)='
      READ*,FAULTD
      PRINT*,CHAR(27),'[H',CHAR(27),'[J'
      WRITE(6,85)
      PRINT*,CHAR(27),'#6      FAULT TYPE'
      WRITE(6,85)
      PRINT*,'1. OPEN CIRCUIT FAULT'
      PRINT*,'2. SHORT CIRCUIT FAULT'
      WRITE(6,85)
      PRINT*,CHAR(27),'[7;5MENTER FAULT TYPE',CHAR(27),'[0M'
      READ*,FTYP
      ENDIF
      PRINT*,CHAR(27),'[H',CHAR(27),'[J'
      PRINT*,'TO='
      READ*,TO
      PRINT*,'TIME LIMIT'
      READ*,TLIM
      PRINT*,'NUMBER OF DATA ='
      READ*,NDATA
      PRINT*,CHAR(27),'[H',CHAR(27),'[J'
C_________________________________________________________
```

```
      WRITE(20,60)(N,R,L,G,C)
60    FORMAT('NUMBER  OF  SECTIONS=',3X,I2,/,
     1 'RESISTANCE  PER UNIT LENGTH=',3X,F9.7,2X,'OHMS/KM',/,
     2 'INDUCTANCE  PER UNIT LENGTH=',3X,F9.3,2X,'MH/KM',/,
     3 'CONDUCTANCE PER UNIT LENGTH=',3X,F9.5,2X,'10E-6MHOSKM',/,
     4 'CAPACITANCE PER UNIT LENGTH=',3X,F9.5,2X,'10E-6F/KM')
      WRITE(20,61)(LENGTH)
61    FORMAT(/,'LENGTH OF LINE=',3X,F6.1,2X,'KM')
      WRITE(20,62)(STYP,RS,LS,F,U)
62    FORMAT(/,'SOURCE TYPE:',3X,I1,/,
     1 'SOURCE RESISTANCE=',3X,F9.7,2X, 'OHMS',/,
     2 'SOURCE INDUCTANCE=',3X,F9.3,2X, 'MH',/,
     3 'SOURCE  FREQUENCY=',6X,I3,2X,'HZ',/,
     4 'VS=',3X,F6.1,2X,'KV')
      WRITE(20,63)(LTYP,RL,LL,CL)
63    FORMAT(/,'LOAD TYPE:',3X,I1,/,
     1 'LOAD RESISTANCE=',3X,F9.4,2X,'OHMS',/,
     2 'LOAD INDUCTANCE=',3X,F9.3,2X,'MH',/,
     3 'LOAD CAPACITANCE=',1X,F9.5,2X,'10E-6F',/)
      WRITE(20,64)(ANLTYP)
64    FORMAT('TYPE OF ANALYSIS:',2X,I1)
      IF (ANLTYP .EQ. 4) THEN
      IF (FTYP .EQ. 1) THEN
      WRITE(20,65)(FTYP)
      ELSE
      WRITE(20,65)(FTYP)
      ENDIF
      WRITE(20, 66)(FAULTD)
      ELSE
      WRITE(20,84)
      ENDIF
65     FORMAT(I1)
66    FORMAT('FAULT DISTANCE=',F6.2)
      WRITE(20,67)(TO*1000,TLIM*1000)
67    FORMAT(/,'to=',2X,F13.6,2X,'MSEC',/,'TIME LIMIT=',2X,
     1 F13.6,2X,'MSEC')
      WRITE(20,68)(NDATA)
68    FORMAT('NUMBER OF DATA:',I4)
84    FORMAT(/)
85    FORMAT(72('_'))
      RETURN
      END


       SUBROUTINE HMAT(A,B,MO,N,U,H,P)
       REAL U,P
       PARAMETER(M=56)
       COMPLEX*8  H(M,1),PMI(M,M),PMISA(M,M),C(M,M),
     / B(MO,1),CMB(M,1),ICOMP(M,M),ACOMP(M,M),CP
       INTEGER ID(M,M),N,MO
       REAL*16 A(MO,MO)
       COMPLEX CU
       MO=N
       CALL IMAT(MO,N,ID)
```

```
      DO 1 I=1,N
      DO 1 J=1,N
      ACOMP(I,J)=CMPLX(A(I,J))
      ICOMP(I,J)=CMPLX(ID(I,J))
1     CONTINUE
      CP=CMPLX(0.0,P)
      CALL SCAMAT(CP,ICOMP,PMI,MO,MO,N,N)
      CALL CMS( PMISA,PMI,ACOMP,MO,MO,N,N)
      CALL CMI(C,PMISA,MO,N)
      CALL CMM(CMB,C,B,MO,MO,1,N,N,1)
      CU=CMPLX(U,0.0)
      CALL  SCAMAT(CU,CMB,H,MO,1,N,1)
      RETURN
      END


      SUBROUTINE STEADS(H,MO,N,F,T,XT)
C     STEADY-STATE ANALYSIS IS DONE IN THIS SUBPROGRAM...
      REAL TKU ,T
      COMPLEX*8  XT(MO,1),H(MO,1)
      INTEGER N,F
      COMPLEX CU
      IF (F .GT. 0) THEN
      P=2*(REAL(F))*3.14159
      CU=CEXP(CMPLX(0.0,P*T))
      ELSE
      CU=CMPLX(1.0,0.0)
      ENDIF
      CALL  SCAMAT(CU,H,XT,MO,1,N,1)
      RETURN
      END


      SUBROUTINE EV(A,MO,N,EGV)
C     SUBROUTINE EV EVALUATES THE EIGENVALUES OF MATRIX A
      PARAMETER(M=56)
      IMPLICIT REAL*16 (A-H,O-Z)
      REAL*16 A(MO,MO), B(M,M),EPS1,EPS2,EPS4,V,X,SUM,SUBSUM
      DIMENSION V(M),X(M,M)
      INTEGER BEGIN(M),FINISH(M),FREQ
      LOGICAL SWEEP,TAG1,TAG2
      COMPLEX*16 EGV(MO)
      EPS1=1.0E-15
      EPS2=1.0E-12
      EPS4=1.0E-9
      FREQ=5
      SWEEP=.FALSE.
      ITMAX=10*N
      NM1=N-1
      DO 1 I=1,N
      DO 1 J=1,N
      X(I,J)=0.
 1    B(I,J)=0.
```

```
      L=0
      TAG1=.FALSE.
      TAG2=.FALSE.
      BEGIN(1)=1
      BEGIN(N)=NM1
      FINISH(1)=2
      FINISH(N)=N
      IF (N .LE. 2) GO TO 5
      DO 2 J=2,NM1
      BEGIN(J)=J-1
 2    FINISH(J)=J+1
      GO TO 5
 5    CONTINUE
      DO 6 I=1,N
      JLOW=BEGIN(I)
      JHIGH=FINISH(I)
      DO 6 J= JLOW, JHIGH
6     B(I,J)=A(I,J)
      DO 51 ITER=1,ITMAX
      DO 10 J=1,N
      ILOW=BEGIN(J)
      DO 8 I=ILOW,J
      SUM=0.
      IM1=I-1
      KLOW=BEGIN(I)
      IF (KLOW .GT.IM1) GO TO 8
          DO 7 K=KLOW,IM1
7         SUM=SUM+B(I,K)*B(K,J)
8     B(I,J)=B(I,J)-SUM
      JP1=J+1
      IHIGH=FINISH(J)
      IF (JP1 .GT. IHIGH) GO TO 18
      DO 10 I=JP1,IHIGH
      SUM=0.
      KLOW=BEGIN(I)
      JM1=J-1
      IF (KLOW .GT. JM1) GO TO 10
         DO 9 K=KLOW,JM1
9        SUM=SUM + B(I,K)*B(K,J)
10    B(I,J) = (B(I,J)-SUM)/B(J,J)
18    DO 24 I=1,N
      JLOW=BEGIN(I)
      IM1=I-1
      IF (JLOW .GT. IM1) GO TO 21
          DO 20 J=JLOW,IM1
          B(I,J)=B(I,I)*B(I,J)
          IP1=I+1
          KHIGH=FINISH(I)
          IF (IP1 .GT. KHIGH) GO TO 20
             DO 19 K=IP1,KHIGH
19           B(I,J)=B(I,J)+B(I,K)*B(K,J)
20    CONTINUE
21    JHIGH=FINISH(I)
      DO 23 J=I,JHIGH
```

```
      JP1=J+1
      KHIGH=FINISH(J)
      IF (JP1 .GT. KHIGH) GO TO 23
           DO 22 K=JP1,KHIGH
22         B(I,J)=B(I,J)+B(I,K)*B(K,J)
23    CONTINUE
24    CONTINUE
      DO 25 I=1,N
      JLOW=BEGIN(I)
      JHIGH=FINISH(I)
      DO 25 J= JLOW,JHIGH
25    IF(QABS(B(I,J)) .LT. 1.0D-18) B(I,J)=0.0
      L=L+1
      SUBSUM=0.
      DO 26 I=2,N
26    SUBSUM=SUBSUM+QABS(B(I,I-1))
      IF (.NOT.(L .EQ. FREQ  .AND.SUBSUM .LT. EPS4 .AND. SWEEP))
     1 GO TO 42
        DO 37 J=1,NM1
       DO 30 I=1,N
30     IF (QABS(B(J,J)-B(I,I)).LT.EPS2 .AND. J.NE.I) GO TO 37
      JP1=J+1
      DO 32 IT=JP1,N
       I=N+JP1-IT
      V(I)=B(I,J)
      IP1=I+1
      IF (I .EQ.N ) GO TO 32
      DO 31 K=IP1,N
31    V(I)=V(I)+B(I,K)*V(K)
32    V(I)=V(I)/(B(J,J)-B(I,I))
      DO 34 IT=JP1,N
      I=N+JP1-IT
      X(I,J)=X(I,J)+V(I)
      IM1=I-1
      IF (JP1 .GT.IM1) GO TO 34
          DO 33 K=JP1,IM1
33        X(I,J)=X(I,J)+X(I,K)*V(K)
34    CONTINUE
      DO 35 I=1,N
      DO 35 K=JP1,N
35    B(I,J)=B(I,J)+B(I,K)*V(K)
      DO 36 I=JP1,N
      DO 36 K=1,N
36    B(I,K)=B(I,K)-V(I)*B(J,K)
37    CONTINUE
         DO 40 J=1,N
         BEGIN(J)=1
40       FINISH(J)=N
41    CONTINUE
42    CONTINUE
       IF (.NOT.(L.EQ.FREQ .OR. ITER .EQ. ITMAX .OR.SUBSUM
     1           .LT.EPS1))                           GO TO 50
      L=0
50    CONTINUE
```

```
51    CONTINUE
52    DO 53 I=1,N
53    X(I,I)=0.
72    CONTINUE
      I=1
      K=1
100     IF (I .LE. N)  THEN
          J=I+1
          IF (B(J,I) .EQ. 0.0) THEN
          EGV(K)=B(I,I)
          I=I+1
          K=K+1
          ELSE
      EGV(K)=DCMPLX((B(I,I)+B(J,J))/2,-(SQRT(4*(B(I,I)*B(J,J)
     1              -B(I,J)*B(J,I)) -(B(I,I)+B(J,J))**2))/2)
      EGV(K+1)=DCMPLX((B(I,I)+B(J,J))/2,(SQRT(4*(B(I,I)*B(J,J)
     1                        -B(I,J)*B(J,I)) -(B(I,I)+B(J,J))**2))/2)
      I=I+2
      K=K+2
      ENDIF
      GOTO 100
      ENDIF
      RETURN
      END


      SUBROUTINE RES(A,SMI,D,C,MN,M,EIG,LKA)
C     SUBROUTINE COMPUTES THE CONSTITUENT MATRICES
      PARAMETER (MO=56)
      DIMENSION LKA(MN,MN,MN)
      INTEGER ID(MO,MO)
      COMPLEX*16  SMI(MN,MN),D(MN,MN),C(MN,MN),LK(MO,MO),
     1 ACOMP(MO,MO),EIG(MN),S,ICOMP(MO,MO),R(MO,MO),DIF,LKA
      REAL*16 A(MN,MN)
      CALL IMAT(MN,M,ID)
      DO 10 I=1,M
      DO 10 J=1,M
            ACOMP(I,J)=DCMPLX(A(I,J))
            ICOMP(I,J)=DCMPLX(ID(I,J))
10     CONTINUE
      DO 40 K=1,M
            DO 20 I=1,M
            DO 20 J=1,M
            LK(I,J)=ICOMP(I,J)
20          CONTINUE
         DO 30 J=1,M
            IF (K .NE. J) THEN
            S=EIG(J)
            CALL DCSM(S,ICOMP,SMI,MN,MN,M,M)
            CALL DCMS(D,ACOMP,SMI,MN,MN,M,M)
            DIF=EIG(K)-EIG(J)
            S=1/DIF
            CALL DCSM(S,D,C,MN,MN,M,M)
            CALL DCMM(R,LK,C,MN,MN,MN,M,M,M)
```

```
                DO 25 IA=1,M
                DO 25 JA=1,M
                LK(IA,JA)=R(IA,JA)
 25             CONTINUE
             ENDIF
 30          CONTINUE
         DO 35 IA=1,M
         DO 35 JA=1,M
         LKA(K,IA,JA)=LK(IA,JA)
 35      CONTINUE
 40    CONTINUE
       RETURN
       END


       SUBROUTINE EXPAT(LKA,R,EAT,EF,MO,N,EIG,T,EATF)
C      SUBROUTINE EXPAT CALCULATES EXP(A*T)
       PARAMETER(M=56)
       DIMENSION LKA(MO,MO,MO)
       INTEGER SIZE,N
       REAL T
       COMPLEX*8 EATF(MO,MO)
       COMPLEX*16  R(MO,MO),EAT(MO,MO),EF(MO,MO),S,EIG(MO)
      / ,DCT,LKA
       DCT=DCMPLX(T,0.0)
       DO 10 I=1,N
       DO 10 J=1,N
       EAT(I,J)=(0.0,0.0)
10     CONTINUE
       DO 40 K=1,N
       DO 30 I=1,N
       DO 30 J=1,N
       R(I,J)=LKA(K,I,J)
30     CONTINUE
       S=CDEXP(EIG(K)*DCT)
       CALL DCSM(S,R,EF,MO,MO,N,N)
       CALL DCMA(EAT,EAT,EF,MO,MO,N,N)
 40    CONTINUE
       DO 50 I=1,N
       DO 50 J=1,N
       EATF(I,J)=CMPLX(EAT(I,J))
50     CONTINUE
       RETURN
       END


       SUBROUTINE CMA(A,B,C,MO,NO,M,N)
C      SUBROUTINE CMA EVALUATES ADDITION OF TWO COMPLEX MATRICES.
       COMPLEX*8 A(MO,NO),B(MO,NO),C(MO,NO)
       DO 100 I=1,M
       DO 100 J=1,N
       A(I,J)=B(I,J)+C(I,J)
  100  CONTINUE
```

```
      RETURN
      END


      SUBROUTINE CMS(A,B,C,NO,MO,N,M)
C     SUBROUTINE EVALUATES SUBTRACTION OF TWO COMPLEX MATRICES .
      COMPLEX*8 A(NO,MO),B(NO,MO),C(NO,MO)
      DO 100 I=1,N
      DO 100 J=1,M
      A(I,J)=B(I,J)-C(I,J)
  100 CONTINUE
      RETURN
      END


      SUBROUTINE CMM(A,B,C,NO,NOMO,MO,N,NM,M)
C     SUBROUTINE EVALUATES MULTIPLICATION OF TWO COMPLEX MATRICES
      COMPLEX*8 A(NO,MO),B(NO,NOMO),C(NOMO,MO)
      DO 301 I=1,N
      DO 301 J=1,M
      A(I,J)=CMPLX(0.0,0.0)
      DO 301 K=1,NM
  301 A(I,J)=A(I,J)+B(I,K)*C(K,J)
      RETURN
      END


      SUBROUTINE CMI(QI,Q,MO,N)
C     SUBROUTINE INVERTS A COMPLEX SQUARE MATRIX
      COMPLEX*8  QI(MO,MO),Q(MO,MO)
      DO 140 I=1,N
      DO 140 J=1,N
  140 QI(I,J)=Q(I,J)
      DO 150 M=1,N
      QI(M,M)=1./QI(M,M)
      DO 151 KK=1,N
      IF(KK.NE.M) QI(M,KK)=-QI(M,M)*QI(M,KK)
  151 CONTINUE
      DO 152 KP=1,N
      IF(KP.NE.M) QI(KP,M)=QI(KP,M)*QI(M,M)
  152 CONTINUE
      DO 150 J=1,N
      DO 150 KP=1,N
      IF((J.NE.M).AND.(KP.NE.M)) QI(KP,J)=QI(KP,J)+
     1 QI(KP,M)*QI(M,J)/QI(M,M)
  150 CONTINUE
      RETURN
      END


      SUBROUTINE SCAMAT(S,A,B,NO,MO,N,M)
      COMPLEX*8 A(NO,MO),B(NO,MO),S
      DO 8 I=1,N
      DO 8 J=1,M
```

```
      B(I,J)=S*A(I,J)
8     CONTINUE
      RETURN
      END


      SUBROUTINE IMAT(MO,N,I)
      INTEGER I(MO,MO)
      DO 10 KI=1,N
      DO 10 J=1,N
      IF (KI .EQ. J) THEN
      I(KI,J)=1
      ELSE
      I(KI,J)=0
      ENDIF
10    CONTINUE
      RETURN
      END


      SUBROUTINE DCMA(A,B,C,NO,MO,M,N)
C     SUBROUTINE EVALUATES ADDITION OF TWO
C     DOUBLE COMPLEX MATRICES.....
      COMPLEX*16 A(NO,MO),B(NO,MO),C(NO,MO)
      DO 100 I=1,M
      DO 100 J=1,N
      A(I,J)=B(I,J)+C(I,J)
  100 CONTINUE
      RETURN
      END


      SUBROUTINE DCMS(A,B,C,NO,MO,M,N)
C     SUBROUTINE EVALUATES SUBTRACTION OF TWO
C     DOUBLE COMPLEX MATRICES ...
      COMPLEX*16 A(NO,MO),B(NO,MO),C(NO,MO)
      DO 100 I=1,M
      DO 100 J=1,N
      A(I,J)=B(I,J)-C(I,J)
  100 CONTINUE
      RETURN
      END



      SUBROUTINE DCMM(A,B,C,NO,NMO,MO,N,NM,M)
C
C     SUBROUTINE EVALUATES MULTIPLICATION OF TWO
C     DOUBLE COMPLEX MATRICES.
      COMPLEX*16 A(NO,MO),B(NO,NMO),C(NMO,MO)
      DO 301 I=1,N
      DO 301 J=1,M
      A(I,J)=DCMPLX(0.00,0.00)
      DO 301 K=1,NM
```

```
  301 A(I,J)=A(I,J)+B(I,K)*C(K,J)
      RETURN
      END


      SUBROUTINE DCSM(S,A,B,NO,MO,M,N)
      COMPLEX*16 A(NO,MO),B(NO,MO),S
      DO 8 I=1,M
      DO 8 J=1,N
      B(I,J)=S*A(I,J)
8     CONTINUE
      RETURN
      END


      SUBROUTINE DCMI(QI,Q,MO,N)
C     SUBROUTINE INVERTS A DOUBLE COMPLEX SQUARE MATRIX.
C
      DIMENSION  QI(MO,MO),Q(MO,MO)
      COMPLEX*16 Q,QI
C     INTEGER MO,N
      DO 140 I=1,N
      DO 140 J=1,N
  140 QI(I,J)=Q(I,J)
      DO 150 M=1,N
      QI(M,M)=1./QI(M,M)
      DO 151 KK=1,N
      IF(KK.NE.M) QI(M,KK)=-QI(M,M)*QI(M,KK)
  151 CONTINUE
      DO 152 KP=1,N
      IF(KP.NE.M) QI(KP,M)=QI(KP,M)*QI(M,M)
  152 CONTINUE
      DO 150 J=1,N
      DO 150 KP=1,N
      IF((J.NE.M).AND.(KP.NE.M)) QI(KP,J)=QI(KP,J)+QI(KP,M)*
     1 QI(M,J)/QI(M,M)
  150 CONTINUE
      RETURN
      END


      SUBROUTINE OUTT(A,XT,MO,M,TIME,U,F,RS,LS,LTYP,RL,LL,B11)
      INTEGER T,STYP,LTYP,F
      REAL*16 A(MO,MO),LS,L,R,RS,RL,LL,G,C,CT,CL
      REAL*8 XT(MO,1),B11
      REAL U,TIME,P,TO,TMTO
      OPEN (50,FILE='DISK$AKAD:[MAMIS]OUT.DAT',STATUS='OLD')
      VS=(1-LS*B11)*(U*COS(2*3.1415926*TIME*F))+XT(1,1)*
     1 (-RS-LS*A(1,1))-LS*A(1,2)*XT(2,1)
      IF (LTYP .EQ. 1) THEN
      VL=XT(M,1)
      ELSE IF(LTYP .EQ. 2)  THEN
      ELSE IF(LTYP .EQ. 3)  THEN
      VL=XT(M,1)*RL
```

```
      ELSE IF(LTYP .EQ. 4)  THEN
      VL=LL*(A(M,M-1)*XT(M-1,1)+A(M,M)*XT(M,1))
      ELSE IF(LTYP .EQ. 5)  THEN
      VL=LL*(A(M,M-1)*XT(M-1,1)+A(M,M)*XT(M,1))+RL*XT(M,1)
      ELSE IF(LTYP .EQ. 6)  THEN
      VL=XT(M,1)
      ELSE IF(LTYP .EQ. 7)  THEN
      VL=XT(M,1)+LL*(A(M-1,M-2)*XT(M-2,1)+A(M-1,M-1)*XT(M-1,1)+
     / A(M-1,M)*XT(M,1))
      ELSE IF(LTYP .EQ. 8)  THEN
      VL=XT(M-1,1)
      ELSE IF(LTYP .EQ. 9)  THEN
      ENDIF
      WRITE(50,280)((TIME*1000.),XT(M,1),VL)
  280 FORMAT(F7.3,2X,F9.3,F9.3)
      RETURN
      END
```