DEVELOPING A COMPUTER PROGRAMME FOR SIMULATING THE REFLECTANCE AND THE TRANSMITTANCE OF N-LAYERED SYSTEMS

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ABSTRACT

DEVELOPING A COMPUTER PROGRAMME FOR SIMULATING THE REFLECTANCE AND THE TRANSMITTANCE OF N-LAYERED SYSTEMS

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This study is the computer simulation of the method developed by F. Abeles to represent the optical multilayers as 2X2 matrices to calculate the overall reflection and transmission characteristics of an n-layered optical system for a selected wavelength region. It is assumed that all layers are made of non-absorbing, plane-parallel sided dielectrics. The number of layers are unlimited. The programme is developed in Fortran 77 language and performed on an IBM compatible PC. The results are represented in graphical form showing the variation of reflectance with wavelength.

Key Words: Optical Coatings, Reflectance, Transmittance, Computer Programme

ÖZET

N TABAKALI BİR SİSTEMİN YANSIMA VE GEÇİRMESİNİ SİMULE EDEN BİR BİLGİSAYAR PROGRAMININ GELİŞTİRİLMESİ

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Bu çalışma, seçilmiş bir dalgaboyu aralığında n-tabakalı optik sistemlerin yansıma ve geçirme katsayılarını hesaplamak için yüzeylerin herbirini 2X2 matrislerle ifade ederek bütün sistemin yansıma ve geçirme katsayısını hesaplayan F.Abeles'in metodunun bilgisayar simulasyonudur. Bütün katmanların soğurmayan, düzlemsel paralel kenarlı dielektrik oldukları kabul edildi. Bilgisayar programı Fortran 77 dilinde yazıldı ve IBM uyumlu bir bilgisayarda gerçekleştirildi. Sonuçlar dalgaboyuna karşı yansıma yüzdesi şeklinde grafik olarak sunuldu.

Anahtar Kelimeler: Optiksel Kaplama, Yansıma, Geçirme, Bilgisayar Programı

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θ_i	Incident angle
θ_r	Reflection angle
n	Index of refraction
V	Potential
Φ	Flux
E	Electric field
B	Magnetic Field
A	Area
D	Electric flux density
Q	Total charge enclosed
ρ	Volume charge density
v	Volume
H	Magnetic field strength
I	Current
J	Current density
J_D	Displacement current
μ	Permeability
σ	Conductivity
ϵ	Dielectric constant (Permittivity)
ψ	Wave function
w	Frequency
S	Poynting vector
ϵ_0	Permittivity of free space
μ_0	Permeability of free space

$E_\parallel$ Parallel component of incoming electromagnetic wave

$E_\perp$ Perpendicular component of incoming electromagnetic wave

c Speed of light

T Transmission factor

R Reflection factor

J Radiant power per unit area of interface

z_j Thickness of the jth layer

n_0 Refractive index of the air

n_{subs} Refractive index of the substrate

N Number of layers

CHAPTER 1

INTRODUCTION

Advances in thin film technology and a continuing need for a better performance of optical systems have created a strong interest in reducing or increasing the reflection of optical systems that achievable by single layer.

During the twenty years after the Second World War, considerable development of the multilayer system of films took place, in a manner which had had a considerable impact on many areas of optics. Most attention has been confined to systems of plane-parallel-sided, transparent homogeneous layers with thicknesses generally of the order of the wavelength of the radiation used.

Multilayer films can be designed and synthesized by means of a numerous methods. Most of the synthesis methods are described in the first section of chapter 4. It is possible to evaporate multiple layers while maintaining control over both refractive index (choice of material) and individual layer thicknesses. Such techniques provide a great deal flexibility in designing interference filters with almost any specified wavelength dependent reflectance or transmittance characteristics. Useful applications of such coatings include antireflecting multilayers for use on the lenses of optical instruments and display windows, multipurpose broad and narrow band-pass filters, available from near ultraviolet to near infrared wavelengths; thermal reflectors and cold mirrors which reflect and transmit infrared, respectively, and are used in projectors; dichroic mirrors consisting of band-pass filters deposited on the faces of prismatic beam splutters to divide light into red, green, and blue channels in colour television cameras; highly reflecting dielectric mirrors for use in gas laser and in Fabry-Perot interferometer. Nonpolarizing dielectric beam splitters and edge filters, phase control coatings, low loss laser coatings,antireflecting coatings on plastics,antireflecting coatings for simultaneous use in the visual and infrared wavelength region can be added to the applications of thin films in optics. In addition to all, front surface mirrors, hot and cold mirrors, artificial jewellery, sun glass coatings, regenerative lamb, optical data storage, coatings for fiber communication and architectural coatings must be considered as the price sensitive products of the multilayer coatings. Under appropriate conditions a multilayer may also be employed as a filter which transmit (or reflects) only selected regions of the spectrum. The present study deals with specially on this property.

In 1917, Lord Rayleigh, gave an explanation of the spectral colours observed in the reflection of light on the colours of some coleopterous beetles, known to have laminar structures. We have essentially here a stop-band theory of dielectric multilayers, but no artificial stratifications of this kind could be produced at that time. An occasional observation of some atmospherically tarnished lenses led the optics manufacturer D. Taylor to the elaboration of artificial ageing of glass surfaces by etching with the effect of reducing unwanted reflection from boundaries.

The real birth of thin film applications had to wait until the middle 1930's, when it was derived from the advances of vacuum technology achieved within the framework of the electronics industry. A. Smakula and J. Strong are cited as independent fathers of the single antireflecting layer in Germany and in the U.S.A. Parallel to this A.M. Pfund discovered the reflection increasing potentialities of evaporated high-index layers.

It was subsequently realized that one may evaporated more different layers in succession. This led, in period 1937-1947, to the elaboration of the first multilayer theories for antireflecting and reflecting systems as well as for monochromatic interference filters. P. Rouard, A. Vasicek, and A.V. Crook may be regarded as the main contributors to the generalization by recurrence of the Airy formulae to multilayer systems, although a number of other authors have made various computations as well. Importing design work of deep foresight unfortunately confined to patent literature, is due to W. Geffcken, who also pioneered wet and gaseous-reaction deposition processes. A. G. Vlasov seemed to be the Russian classic of this era.

Exacting technological requirements for the production of optically effective and the physically stable multilayers have caused the practical developments to be mostly confined to the laboratory phase. The manufacture of The Fabry-Perot type filter, which spread after 1945, may be regarded as a real commercial hit. The Airy summation was also used by S. Tolansky in 1942 when analysing the Fizeau fringes in a silvered-wedge film.

Post-war development is characterized by increased confidence in the role of optical multilayers, with corresponding efforts in the experimental and theoretical spheres. Special evaporation plants were developed for optical multilayers. Large scale research was undertaken into materials suitable for various spectral regions and satisfying the requirements of economical manufacture and prolonged use of the films. Methods of the controlled deposition were developed, optical constants, porosity, structure, adhesion, etc. were studied and enormous amount of knowledge and skill was put into this new field.

The published thesis of F. Abeles [1] is one of the milestone papers of post-war developments. He treated the stratified media in terms of electromagnetic theory of light. Derivations of the expressions for the reflectance and transmittance of a stratified system is straightforward. The usual electromagnetic boundary conditions are applied at each interface so that the amplitude of the waves (both directions) in any layer may be readily written in terms of those of the adjacent layers. Thus by successive operations, the amplitude of the waves emerging from either side of the system may be expressed in terms of that of the incident wave. Each layer is expressed in terms of 2X2 matrices and each matrix is a function of film thickness, refractive index, and the angle of incidence. The evaluation of the matrix products and getting the percent reflectance and transmittance values will be described in Chapter 4.

The purpose of the present investigation is to computerize the method developed by F. Abeles. A computer programme in Fortran 77 language is developed for this reason. The computer programme, named REFLECT, takes necessary input to construct the 2X2 matrices, called characteristics matrix, for each medium. Then the necessary calculations are done to get the final reflectance and transmittance rates. Then the results are transferred to a drawing software, Harward Graphics 3, where the results are converted to graphical form.

In introduction section, the need for the multilayer coating is stated and the fields in which the multilayer coatings are applied are exampled with a brief historical development. In the second chapter, literature survey, the related literature is given in three main groups: Theoretical studies, computer aided designs (CAD's), and applications. Third chapter summarize the Maxwell's and Fresnel's equations. In the fourth chapter, the methods of synthesis for the multilayers are given and the characteristic matrix model which is used in the preparation of the computer programme is constructed. The description of the computer programme and the explanations of the parameters used in the preparation of the computer program are presented in Chapter 5. The results of the computer programme in graphical form and their discussions are given in Chapter 6.

CHAPTER 2

LITERATURE SURVEY

The use of reflection existed before written history, as it evidenced by the discovery of a mirror from the period around 1900 B.C. Some of the earliest written comments about reflection can be found in Exodus 38:8 and Job 37:18, Euclid in about 300 B.C. discussed the focus of a spherical mirror in Cotoprics. Cleomedes (50 A.D) discussed the reflection of light an air-water interface. He described an experiment whereby a coin at the bottom of a bowl and hidden by the bowl's sides could be made visible by pouring water into the bowl.

Claudius Ptolemy of Alexandria (139 A.D.) made table of the angles of incidence and reflection. His work is one of the few examples during that time. The concept of a sine of an angle was not developed yet so his tables were only approximately correct. One of the most interesting individuals active in optics during the middle ages was Alhazen (Abu Ali al Hasan ibn al Hasan ibn al Haitam) (965-1038) who developed optics during the Golden age of The Arabic Empire. He added to the law of reflection developed by Ptolemy in fact that the incident angle and the reflected angle lie in the same plane, called the plane of incidence. He also corrected Ptolemy's tables of incident and reflected angles. Alhazen failed, however, to discover the law of refraction. Alhazen was a successful optical scientist but some of his civil engineering projects got him into trouble with his caliph and Alhazen had to feign insanity and hide to escape the wrath of the caliph.

Vitello (in 1270) repeated the experiments of Ptolemy but also failed to discover the law of reflection. Johannes Kepler (1571–1630) gave a broad outline of the correct theory of the telescope and discussed total reflection without knowledge of the law of refraction. He used an empirical expression $\theta_i=n\theta_r$, where n=3/2. The law of refraction was discovered, evidently through experimentation, by Willebrord Snell Von Royen (1592–1626) a professor of mechanics at Leyden. He never published but Huygens and Isaac Voss claimed to have examined Snell's manuscripts. Descartes in 1637 deduced the law of refraction theoretically and expressed it in its present form. It is interesting that one of his assumption was wrong. He often confidently deduced theory without allowing himself to be disturbed by any possible discrepancy between his final conclusion and the actual facts. Piere de Fermat (1601–1665) deduced the law of refraction from the assumption that the light travels from a point in one medium to a point in another medium in the least time.

Interference between waves reflected from the interfaces of a dielectric film can be used to reduce the reflection from an optical surface. This concept can be extended to a multilayer dielectric coating to produce any desired reflection property. Frounhofer produced antireflection layers on glass surfaces by acid etching in 1817, but it was not until 1891 that Dennis Taylor associated the reduced reflectivity with an increased transparency. Full utilization of the concept of interference filters had to wait until the late 1940's and 1950's when techniques were developed that allowed the production of rugged dielectric films. Cold mirrors designed to reflect the visible wavelengths and transmit the infrared wavelengths were one of the first products of this technology and today are found in every dentists lamp.

The related literature survey may be grouped into three categories: Theoretical studies about the multilayers, related computer aided design (CAD) studies, and applications of multilayer thin films into different areas of technology . The cited literature in all groups will be summarized below.

F. Abeles [1] developed a matrix method to examine the multilayer filters. A quantitative approach to designing multilayer filters that allows the computer generation of the spectral characteristics of the filters is the use matrices.The approach is called the method of resultant waves or the E^+, E^- matrix method. The boundary conditions associated with Maxwell's equations, are placed into a matrix equation format. To accomplish the reformulation, the boundary conditions are manipulated so that the information about the angle of incidence and the polarization are placed into an effective index of refraction. The fields on each sides of the boundary can then be represented by plane waves incident normal to the interface.

A.J. FOX [2] presented a plane wave theory for the optical grating guide. His treatment uses the results of multilayer theory to obtain the reflectivity and phase of the grating guide walls.He discussed the characteristics of the grating guide in terms of reflectivity, phase attenuation, Q factor. Finally he proposed two method of realizing the grating guide: 1) Electro-optic guide 2) Cladding perturbation guide. He discussed the characteristics of both methods.

H. Uğur, R. Johanson and H. Fritzsche [3] used the matrix method with a difference of choosing the refractive indices of the layers. They had been used periodic layers of high and low refractive indices. After getting their results, they compared them with numerical calculations obtained by solving the boundary value problem for all layers. After establishing the correctness of the validity of the effective medium expression, they used it to analyze optical transmittance data of multilayer films consisting of hydrogenated amorphous silicon and amorphous silicone nitride.

T. Buffeteau and B. Desbat [4] studied on the determination of thin film optical constants from infrared reflectance and transmittance measurements. Their computer program consists of three main parts: 1) A matrix formalism, 2) An iterative Newton-Raphson method to estimate the optical constants, 3) A fast Kramers-Kronig transform to improve the accuracy of calculating the refractive index.

An analytical theory is proposed for a multilayer interference filter, with layer thicknesses close to the quarter wavelength by I.V. Bogatyreva [5]. Expansion of the elements of the full characteristics matrix of the filter in a series in terms of the fluctuations of the layer thicknesses gives the analytic dependence of these elements and the complex impedance of the filter as a function of the filter parameters.

A. Primal and M.L. Rastello [6] presented a new (stochastic) synthesis method which supplies the optimal design without requiring any sort of designer's experience which is essentially based on the mixed use of Montecarlo techniques and descent optimization algorithms.

W. P. Stricland [7] reviewed the evaluation of the vacuum coating industry. Vacuum science progressed slowly until the late nineteenth century due to an incomplete understanding of vacuum and lack of applications. Edison's invention of the light bulb launched the vacuum industry and increased development of improved vacuum systems. The thin film coating industry arose from the needs of the German and U.S.A. military efforts during the World War II.

T.W. Barbee [8] discussed the current status of the multilayer optics introducing significant instrumental applications for these reflecting optics such as linear plasma diagnostics, synchrotron radiation monochromators, solar astrophysics, x-ray microprobe

In the second group of the related literature the computer aided designs will be investigated. M.K. Rao, T.C. Chye, O.C. Seng [9] presented a versatile software package for computer aided design of thin film optical coatings. Both numerical and/or graphical output for transmittance and reflectance of a multilayer dielectric structure at various wavelengths are easily obtained. Several subprograms include the analytical program providing variation of transmittance, reflectance and optical density with wavelength, analytical program with normalized wavelength, display program for listing the optical coating materials, and etc.

An elaborate computer programme established by A.J. Aba El-Haija and H.Y. Omari [10] to calculate the optical properties and their first order derivatives for arbitrary multilayer systems employing Chebychev polynomials. The obtained reflectivity, transmissivity, and absorptivity and their derivatives are compared with their values calculated from direct multiplication of matrices using the characteristics transfer technique.

A number of programs for the design of the optical thin film systems are described and the advantages and disadvantages are discussed by J.A. Dobrowolski [11]. C. Gunkel [12] developed a program that allows to calculate reflectance, transmittance, and phase of randomly polarizes light which interacts with marginal surfaces with unlimited number of layers.

In the final section of this part, the related literature for the applications of the thin film optical coatings will be pointed out. F. Trier [13] made the application of the thin film optical multilayer coatings to solve the problem of decreasing the visible light transmittance with increasing the angle of inclination of the front windows of the modern automobiles. At an angle of 70° the transmittance through the windshield is only 50% from the driver's viewpoint. This study shows that this problem can be solved by special antireflecting coating.

A write-once optical data storage media has been developed by A.J.G. Strandjord, S.P. Webb, D.R. Beaman, S.L.B. Carroll [14] which is suitable for tape format. The recording layer is a metal alloy system which is sputter deposited onto polymeric substrates as a single layer thin film. S. Duan [16] studied the multilayer films for magnetic recording. Also J.G. Zhu [15] modelled a multilayer thin film recording media. Silver-based high-reflectance coatings that can withstand the humid and polluted conditions common in the open air have been developed by D.Y. Song and H.A. Macleod [17] for astronomical telescope optics. The successful designs incorporate a silver reflective layer with a copper underlayer and a stack of dielectric overlayers. The improved durability, which is due to the copper underlayer, has been investigated with analytical techniques, including Rutherford backscattering.

The optical coating requirements for high energy laser systems have advanced the state-of-the-art in coating design and manufacturing technology over the past decade. L.P. Mott and G.W. DeBell [18] are intended to provide guidance with respect to the spectral properties that can be achieved in practice with manufacturing technology. Realistic specifications are presented for several generic types of coating relevant to high energy laser optical system in the spectral region from 230 to 2000nm.

CHAPTER III

THEORY

In this chapter, the basic equations which will be used in this study will be derived. To do this, firstly, Maxwell's equations will be introduced and the solution of the wave equation obtained from the Maxwell's equations will be given. Then, the general properties of the plane wave obtained from this solution such as Poynting vector, polarization, and the reflection and transmission on a dielectric surface will be studied. Finally, applying the boundary conditions for a plane wave, Fresnel's equations will be obtained and also it will be shown that how reflection and transmission coefficients will be obtained from these equations.

3.1 Maxwell's Equations

3.1.1 Faraday's Law and Maxwell's First Equation

A time varying magnetic field which links a wire loop induces a voltage (emf) in the loop. The induced emf is proportional to the rate of the change of the magnetic flux passing through the loop. The polarity of the induced voltage is given by Lenz' law: Emf causes a current in the wire loop which produces a magnetic field opposing the change in flux. Combining Lenz' law, which determines the sign in Eqn. (3.1) with

Faraday's experimental result, Faraday's law can be written in the form:

$$V = -\frac{d\phi}{dt} \tag{3.1}$$

or

$$\int \vec{E}.d\vec{l} = -\frac{\partial}{\partial t}\iint \vec{B}.d\vec{A} \tag{3.2}$$

By applying Stoke's theorem to the left side, the above equation becomes:

$$\iint (\vec{\nabla} X \vec{E}).d\vec{A} = -\frac{\partial}{\partial t}\iint \vec{B}.d\vec{A} \tag{3.3}$$

Since this statement must be true for any surface, the integrands may be equated, giving

$$\vec{\nabla} X \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{3.4}$$

3.1.2 Gauss' Law and Maxwell's Second Equation

Gauss' law is given as

$$\int \vec{D}.d\vec{A} = Q \tag{3.5}$$

where D is the electric flux density. (Namely, the total flux out of a closed surface equals the charge enclosed by this surface.) If a charge Q is distributed continuously throughout a volume v, Gauss' law is

$$\int \vec{D}.d\vec{A} = \int_v \rho . dv \tag{3.6}$$

where ρ is the volume charge density (C/m^3). Applying the divergence theorem to the left side of the above equation, it can be written as

$$\int (\vec{\nabla}.\vec{D})\, dv = \int \rho . dv \qquad (3.7)$$

Since the above statement must be true for any volume, the integrands can be equated to give

$$\vec{\nabla}.\vec{D} = \rho \qquad (3.8)$$

3.1.3 Ampere's Law and Maxwell's Third Equation

Faraday's experimental law showed that a time varying magnetic field produces an electric field,was the basis for Maxwell's first equation. The converse of this, a time varying electric field gives rise to a magnetic field, will be expressed by Maxwell's third equation. Ampere's law can be stated like: a line integral of magnetic field taken about any given closed path must equal to the current enclosed by that path; that is

$$\oint \vec{H}.d\vec{l} = I \qquad (3.9)$$

where H is the magnetic field strength. For distributed currents, Ampere's circuital law can be written as

$$\oint \vec{H}.d\vec{l} = \int \vec{J}.d\vec{A} \qquad (3.10)$$

and, by use of Stoke's theorem, this equation can be converted to the differential form

$$\vec{\nabla} x \vec{H} = \vec{J} \qquad (3.11)$$

3.1.4 Displacement Current

The implication of Ampere's law is that a magnetic field can only be produced by a motion of charges. Maxwell showed that there is another current, the displacement current,which can produce a time-varying magnetic field (but not a steady one) just as effectively as a flow of charges can. The introduction of the new current allowed Maxwell to unify the separate laws of electricity and magnetism into an electromagnetic theory. It also let him to predict that electromagnetic waves which can propagate energy must exist and that light is an electromagnetic wave. Let's see how the need for a displacement current arises: The continuity equation

$$\nabla.\vec{J}+\frac{\partial\rho}{\partial t}=0 \tag{3.12}$$

and observed that it must be very general relation because it was derived from the principle of conservation of charge. We can now reasons as follows.

If the divergence of both sides of (3.11) is taken, it becomes as following

$$\begin{aligned}\nabla.\nabla\mathrm{x}\vec{H}&=\nabla.\vec{J}\\ 0&=\nabla.\vec{J}\end{aligned} \tag{3.13}$$

Since in the general case the divergence of J cannot be equal to zero, it follows that Ampere's law in the form (3.11) is not very general. This can be expected as (3.11) is a relationship valid for static fields. Can we make Ampere's law sufficiently general to include the time varying fields by adding a missing term? As the divergence operation in (3.12) removes the magnetic field part of Ampere's law, the missing term must be a current density, which we call J_D. Now Eqn.(3.13) must be written in new form as:

$$\vec{\nabla} \times \vec{H} = \vec{J} + \vec{J}_D$$
$$0 = \vec{\nabla} . \vec{J} + \vec{\nabla} . \vec{J}_D \qquad (3.14)$$

in place of Eqn. (3.13). Comparing this to the continuity equation, we obtain

$$0 = -\frac{\partial \rho}{\partial t} + \vec{\nabla} . \vec{J}_D \qquad (3.15)$$

as the relationship for the new term. Since the charge density ρ is related by Eqn.(3.8), we can substitute for ρ and identify J_D as $J_D = \partial D/\partial t$. Hence the generalized Ampere's law which now satisfies the continuity equation is given by:

$$\vec{\nabla} \times \vec{H} = \vec{J} + \vec{J}_D$$
$$= \vec{J} + \frac{\partial \vec{D}}{\partial t} \qquad (3.16)$$

and in integral form

$$\oint \vec{H} . d\vec{l} = \iint \vec{J} . d\vec{A} + \iint \frac{\partial \vec{D}}{\partial t} d\vec{A} \qquad (3.17)$$

3.1.5 Magnetic Flux and Maxwell's Fourth Equation

Since no magnetic charges exist, the magnetic field cannot terminate on charges and therefore must be solenoidal; i.e.magnetic flux lines must exist as closed lines. All lines entering a closed surface must also leave it. Mathematically, this can be expressed as;

$$\oint \vec{B}.d\vec{A} = 0$$
$$\nabla.\vec{B} = 0 \tag{3.18}$$

3.2 Wave Equation and Its Solution

To obtain the wave equations, let's write the Maxwell's equations

$$\nabla X \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu \frac{\partial \vec{H}}{\partial t} \tag{3.19}$$

$$\nabla X \vec{H} = \sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t} \tag{3.20}$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon} \tag{3.21}$$

$$\nabla \cdot \vec{B} = 0 \tag{3.22}$$

Taking the curl of both sides of Eqns (3.19) and (3.20) assuming a homogeneous medium, we obtain the following equations

$$\nabla X (\nabla X \vec{E}) = -\mu \nabla X \left(\frac{\partial \vec{H}}{\partial t}\right) = -\mu \frac{\partial}{\partial t} (\nabla X \vec{H}) \tag{3.23}$$

$$\nabla X (\nabla X \vec{H}) = \sigma \nabla X \vec{E} + \epsilon \nabla X \left(\frac{\partial \vec{E}}{\partial t}\right) = \sigma \nabla X \vec{E} + \epsilon \frac{\partial}{\partial t} (\nabla X \vec{E}) \tag{3.24}$$

Substituting Eqn.(3.20) into the right side of (3.23) and using the vector identity

$$\nabla X \nabla X \vec{F} = \nabla (\nabla \cdot \vec{F}) - \nabla^2 F \tag{3.25}$$

Eqn.(3.23) can be written as

$$\nabla(\nabla\cdot\vec{E}) - \nabla^2\vec{E} = -\mu\frac{\partial}{\partial t}(\sigma\vec{E} + \epsilon\frac{\partial\vec{E}}{\partial t}) = -\mu\sigma\frac{\partial\vec{E}}{\partial t} - \mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2} \qquad (3.26)$$

$$\nabla^2\vec{E} = \mu\sigma\frac{\partial\vec{E}}{\partial t} + \mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2} \qquad (3.27)$$

In a similar manner, by substituting Eqn.(3.19) into the right side of Eqn. (3.24) and using the vector identity given by Eqn. (3.25) in the left side of Eqn. (3.24) we obtain

$$\nabla(\nabla\cdot\vec{H}) - \nabla^2\vec{H} = -\sigma\mu\frac{\partial\vec{H}}{\partial t} - \mu\epsilon\frac{\partial}{\partial t}(\frac{\partial\vec{H}}{\partial t}) = -\sigma\mu\frac{\partial\vec{H}}{\partial t} - \mu\epsilon\frac{\partial^2\vec{H}}{\partial t^2} \qquad (3.28)$$

$$\nabla^2\vec{H} = \sigma\mu\frac{\partial\vec{H}}{\partial t} + \mu\epsilon\frac{\partial^2\vec{H}}{\partial t^2} \qquad (3.29)$$

Equations (3.27) and (3.29) are referred to as the vector wave equations for E and H.

For source free regions lossless media (σ=0) the wave equations simplify to

$$\nabla^2\vec{E} = \mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2} \qquad (3.30)$$

and

$$\nabla^2\vec{H} = \mu\epsilon\frac{\partial^2\vec{H}}{\partial t^2} \qquad (3.31)$$

Assuming the medium is nonconducting (σ=0) then the three dimensional wave equation becomes :

$$\nabla^2\psi - \mu\epsilon\frac{\partial^2\psi}{\partial t^2} = 0 \qquad (3.32)$$

For any t,ψ is independent of x and y so that for each value of z, ψ is constant on the corresponding infinite plane parallel to the xy-plane. Then Eqn. (3.32) becomes

$$\frac{\partial^2\psi}{\partial z^2} = \frac{1}{u^2}\frac{\partial^2\psi}{\partial t^2} \qquad (3.33)$$

$$\frac{\partial^2\psi}{\partial z^2} - \mu\epsilon\frac{\partial^2\psi}{\partial t^2} = 0 \qquad (3.34)$$

Using the method of separation of variables ψ can be written in the form $\psi(z,t) = Z(z)\ Y(t)$.

$$\frac{1}{Z}\frac{d^2Z}{dz^2} = \frac{1}{u^2Y}\frac{d^2Y}{dt^2} = -k^2 = \text{constant} \qquad (3.35)$$

so we have two differential equations :

$$\frac{d^2Z}{dz^2} + k^2Z = 0 \qquad \text{and} \qquad \frac{d^2Y}{dt^2} + k^2u^2Y = 0 \qquad (3.36)$$

$k^2u^2 = w^2$

General solution of Eqn. (3.36) can be written as

$$Z(z) = \alpha e^{ikz} + \beta e^{-ikz}$$
$$Y(t) = \gamma e^{i\omega t} + \delta e^{-i\omega t} \qquad (3.37)$$

where α, β, γ, δ are constants.

$$\psi(z,t) = Z(z) \, . \, Y(t) \qquad (3.38)$$

Since there may be many possible values of k and Eqn. (3.32) is a linear differential equation, the sum of solutions each of this form will also be a solution. Thus, if we sum the products Z.Y over all possible values of k we get the general solution of the wave equation and we get the form

$$\psi(z,t) = \sum_k \left(\alpha\delta e^{i(kz-wt)} + \beta\gamma e^{-i(kz-wt)}\right) + \sum_k \left(\alpha\gamma e^{i(kz+wt)} + \beta\delta e^{-i(kz+wt)}\right) \qquad (3.39)$$

Consider the term $e^{i(kz-wt)}$; if w and k are positive, this form represents a plane wave travelling in the direction of positive with speed v = w/k. If k is negative, we can write k = |k|, and this takes the form $e^{-i(|k|z+wt)}$, which is a plane wave propagating in the direction of negative z with speed v = w/|k|. Therefore, as far as the first term in each sum of Eqn. (3.39) is concerned, we need only concern ourselves with the single form $e^{i(kz-wt)}$ and the sign of k (positive or negative) will tell the sense of travel of the wave. The other exponential terms in Eqn. (3.39) are simply the complex conjugates of those we have just discussed still have the appropriate form. Then the plane wave takes the form

$$\psi(z,t) = \psi_0 e^{i(kz-wt)} \qquad (3.40)$$

where ψ_0 is a constant.

3.3 Wave Propagation

For a plane-polarized wave having its E in the x-direction as shown in Figure (3.1) below

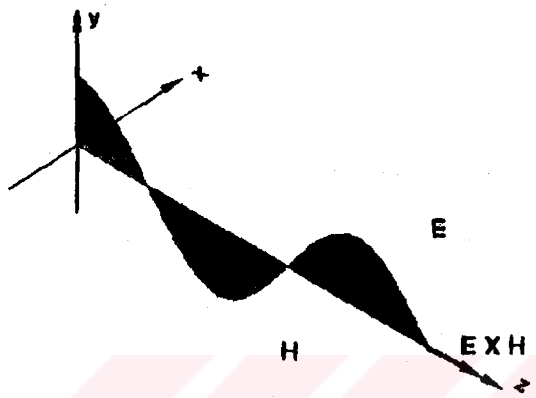

FIGURE 3.1 Graphical Representation of an Electromagnetic Plane Wave.

$$\vec{E} = E_0 \exp[j\omega(t - \frac{z}{c})]\,\hat{i} \qquad (3.41)$$

where w is the angular frequency, and magnetic field component H is

$$\vec{H} = \epsilon_0 c E_0 \exp jw(t - \frac{z}{c})\,\hat{j} \qquad (3.42)$$

The ratio of E to the H is called as impedance and it is found for vacuum as

$$\frac{\vec{E}}{\vec{H}} = \frac{1}{\epsilon_0 c} = \mu_0 c = \sqrt{(\frac{\mu_0}{\epsilon_0})} = 377\,\Omega \qquad (3.43)$$

and if B is substituted instead of H this ratio is

$$\frac{\vec{E}}{\vec{B}} = c = 3.00 X 10^8 \ m/s \quad (3.44)$$

The E and H vectors are perpendicular and oriented in such a way that their vector product EXH point in the direction of propagation. The E and H vectors are in phase, since E_x/H_y is real, and they have the same relative magnitudes at all points at all times.

3.4 The Poynting Vector

A plane electromagnetic wave in free space propagates in the direction of the vector EXH. The divergence of EXH in free space:

$$\begin{aligned} \vec{\nabla}\cdot(\vec{E}X\vec{H}) &= -\vec{E}\cdot(\vec{\nabla}X\vec{H}) + \vec{H}\cdot(\vec{\nabla}X\vec{E}) \\ &= -(\vec{E}\cdot\epsilon_0\frac{\partial\vec{E}}{\partial t}) - (\vec{H}\cdot\mu_0\frac{\partial H}{\partial t}) \\ &= -\frac{\partial}{\partial t}(\frac{\epsilon_0 E^2}{2} + \frac{\mu_0 H^2}{2}) \end{aligned} \quad (3.45)$$

Integrating over a volume τ bounded by a surface *S*, and using the divergence theorem

$$\int_s (\vec{E}X\vec{H}) \, . da = -\frac{\partial}{\partial t}\int_\tau (\frac{\epsilon_0 E^2}{2} + \frac{\mu_0 H^2}{2}) \, d\tau \quad (3.46)$$

The integral on the right side is the sum of electric and magnetic energies. The right-hand side is thus the energy lost per unit time by the volume τ and the left-hand side must be the total outward flux of energy through the surface *S* bounding τ. The quantity

$$\vec{S} = \vec{E} X \vec{H} \quad (3.47)$$

is called the POYNTING VECTOR. When integrated over a closed surface, it gives the total outward flow of energy per unit time. Vector S points in the direction of propagation of the wave.

3.5 Polarization

The polarization of a radiated wave is defined as "that property of a radiated electromagnetic wave describing the time varying direction and relative magnitude of the electric field vector;specifically,the figure traced as a function of time by the extremity of the vector at a fixed location in space, and the sense in which it is traced, as observed along the direction of the propagation.In other words, polarization is the curve traced out by the end point of the arrow representing the instantaneous electric field. The field must be observed along the direction of propagation.

Polarization may be classified into three categories: Linear, Circular, Elliptical. If the vector describes the electric field at a point in space as a function of time, it is always directed along a line, which is normal to the direction of propagation, the field is said to be LINEARLY POLARIZED. In general, however, the figure that the electric field traces is an ellipse, and the field is said to be ELLIPTICALLY POLARIZED. Linear and circular polarizations are special cases of elliptical, and they can be obtained when the ellipse becomes a straight line or circle,respectively.

3.5.1 Transverse Electromagnetic Modes

A mode is a particular field configuration. For a given electromagnetic boundary value problem, many field configurations that satisfy the wave equations, Maxwell's equations, and the boundary conditions usually exist.All these different field configurations (solutions) are usually referred to as *modes.*

A TEM mode is one whose field intensities, both E and H at every point in space are contained in a local plane, referred to as equiphase plane, that is independent of time. In general, the orientations of the local planes associated with the TEM wave are different at different points in space. In other words, at point (x_1,y_1,z_1) all the field components are contained in a plane. At another point (x_2,y_2,z_2) all field components are again contained in a plane; however, the two planes need not be parallel.

If the space orientations of the planes for a TEM mode is the same (equiphase planes are parallel) then the fields form *plane waves.* In other words, the equiphase surfaces are parallel planar surfaces. If in addition to having planar equiphases the field has equiamplitude planar surfaces (the amplitude is the same over each plane), then it is called a *uniform plane wave*; that is, the field is not a function of the coordinates that form the equiphase and equiamplitude planes.

3.6 Reflection and Refraction in Nondispersive Media

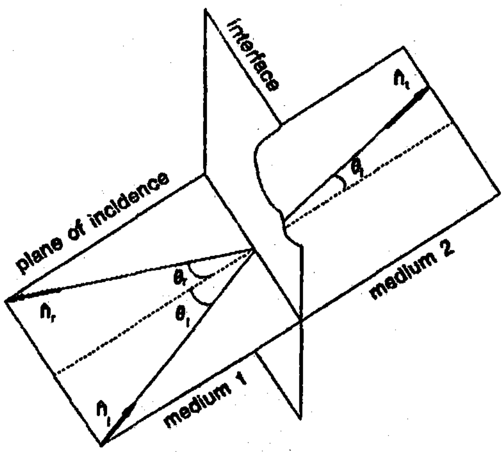

FIGURE 3.2 Coordinate System Used for the Study of the Reflection and Refraction.

An electromagnetic wave in medium 1 incident on the interface between media 1 and 2 and gives rise to both a reflected and a transmitted wave as seen in Fig. (3.2). The vectors n_i, n_r, and n_t are unit vectors normal to the respective wavefronts, and point in the direction of propagation. The angles θ_i, θ_r, and θ_t are the angles of incidence, reflection, and refraction, respectively. If we assume that the electromagnetic wave incident on the interface as in figure above, is both plane and plane polarized, than its electric field intensity E_i is of the form

$$\vec{E}_i = \vec{E}_0 \exp j\omega_i \left(t - \frac{\hat{n}_i \cdot \hat{r}}{u_1}\right) \tag{3.48}$$

where u_1 is the phase velocity of the wave medium 1. The time t=0 and the origin can be chosen arbitrarily. This equation defines a plane wave for all values of t and for all values of r, that is a wave that extends throughout all time and all space. However, it is used only in medium 1.

Both the reflected and refracted waves from a plane interface are then also plane and plane-polarized, since the laws of reflection and refraction for any given incident ray must be the same at all points on the interface. Then the reflected and the transmitted waves are of the form:

$$\vec{E}_r = \vec{E}_{0r} \exp j\omega_r \left(t - \frac{\hat{n}_r \cdot \hat{r}}{u_1}\right) \tag{3.49a}$$

$$\vec{E}_t = \vec{E}_{0t} \exp j\omega_t \left(t - \frac{\hat{n}_t \cdot \vec{r}}{u_2}\right) \tag{3.49b}$$

where u_2 is the phase velocity of the wave in medium 2. Note that we have made no assumption whatever as to the amplitudes, phases, frequencies or directions of the reflected and transmitted wave. The amplitudes can be complex to take phase differences into account.

The characteristics of the reflected and the transmitted waves can be determined from the fact that the tangential components of both E and H must be continuous across the interface. In other words, the sum of tangential components of E_i and E_r just above the interface must be equal to the tangential component of the E_t just below the interface; a similar situation holds for H. The reflected and the transmitted waves could also be obtained from the continuity of the normal components of the D and of B across the interface. To obtain continuity of the tangential components of E and H at the interface, some valid relations must exist between E_i, E_r, and E_t for all time for all points on the interface. Such a relation will be possible if :

a) All three vectors E_i,E_r, and E_t are identical function of time.

b) All three vectors are identical function of position r_I on the interface.

c) There exist certain relations between E_{0i}, E_{0r}, and E_{0t}.

From condition (a) we obtain a relationship between w_i, w_r, and w_t as

$$w_i = w_r = w_t \tag{3.50}$$

All three waves must be of the same frequency.

From condition (b), we must also have, at any point on the interface,

$$\frac{\hat{n}_i \cdot \vec{r}_I}{u_1} = \frac{\hat{n}_r \cdot \vec{r}_I}{u_1} = \frac{\hat{n}_t \cdot \vec{r}_I}{u_2} \tag{3.51}$$

Then from the first two of these equations;

$$(\hat{n}_i - \hat{n}_r) \cdot \vec{r}_I = 0 \tag{3.52}$$

Since the vector r_I lies in the interface, the vector $n_i - n_r$ must be normal to the interface and the tangential components of these two vectors must be equal and of the same sign as in Figure (3.3).

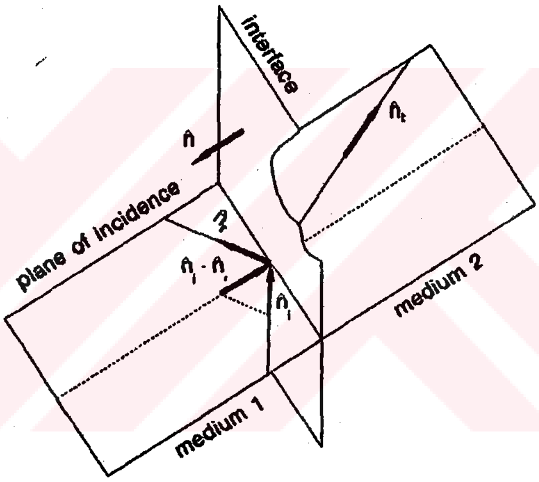

FIGURE 3.3 The Vectors n_i, n_r, n_t, n.

Then $\theta_i = \theta_r$, the angle of reflection is equal to the angle of incidence. Since $(n_i - n_r)$ is parallel to the normal **n**, the three vectors n_i, n_r, and n_t are coplanar. These are the laws of reflection. The plane of these three vectors is normal to the interface and is called the *plane of incidence.*

Now, consider Eqn.(3.51)

$$\left(\frac{\vec{n}_i}{u_1} - \frac{\vec{n}_t}{u_2}\right) \cdot r_I = 0 \tag{3.53}$$

Hence the vectors between parenthesis must also be normal to the interface, so that n_i, n_t, and **n** are coplanar and all four vectors n_i, n_t,

n_r, n are in the plane of incidence. Moreover, the tangential components of n_i / u_1 and of n_t / u_2 must be equal, and

$$\frac{\sin\theta_i}{u_1} = \frac{\sin\theta_t}{u_2} \qquad (3.54)$$

or since the wave number k is w/u

$$k_1 \sin\theta_i = k_2 \sin\theta_t \qquad (3.55)$$

The quantity k Sin θ is therefore conserved in crossing the interface. We can also write that

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{k_1}{k_2} = \frac{n_1}{n_2} \qquad (3.56)$$

since $k = n / \lambda_0$, where n is the index of refraction. This is Snell's law.

It is important to note that this law, as well as the laws of reflection, are general. They apply to any two media; they even hold true for total reflection.

3.7 Fresnel's Equations

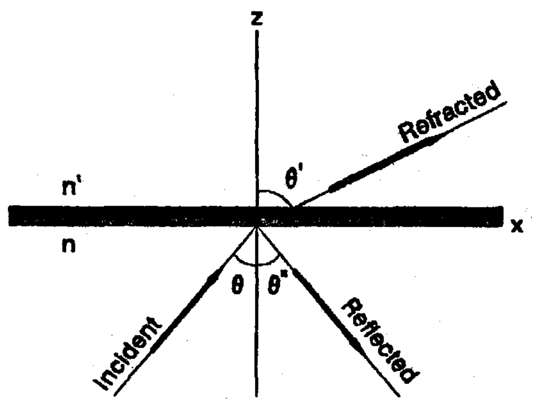

FIGURE 3.4 Wave Vectors of the Incident, Reflected, and Refracted Waves and Angle of Incidence, Refraction, and Reflection.

The electric and magnetic fields of incident, reflected and refracted waves are as follows

$$\text{Incident :} \qquad \vec{E} = \vec{E}_0 \exp -i(wt - \vec{k}.\vec{x}) \qquad\qquad (3.57)$$
$$\vec{B} = \frac{c}{w}\vec{k} \times \vec{E}_0 \exp -i(wt - \vec{k}.\vec{x})$$

$$\text{Refracted :} \qquad \vec{E}' = \vec{E}_0{}' \exp -i(wt - \vec{k}'.\vec{x}) \qquad\qquad (3.58)$$
$$\vec{B}' = \frac{c}{w}\vec{k}' \times \vec{E}_0 \exp -i(wt - \vec{k}'.\vec{x})$$

$$\text{Reflected :} \qquad \vec{E}'' = \vec{E}_0{}'' \exp -i(wt - \vec{k}''.\vec{x}) \qquad\qquad (3.59)$$
$$\vec{B}'' = \frac{c}{w}\vec{k}'' \times \vec{E}_0{}'' \exp -i(wt - \vec{k}''.\vec{x})$$

Since the magnitudes of the wave vectors are $k = nw/c$, $k' = n'w/c$, $k'' = n''w/c$, the magnitude of each magnetic field equals the index of refraction times the magnitude of the electric field, as it should.

At the interface z=0, the fields must satisfy the boundary conditions which express the continuity of the appropriate electric and magnetic fields components parallel or perpendicular to the interface:

$$\vec{E}_\parallel + \vec{E}_\parallel{}'' = \vec{E}_\parallel{}' \qquad (3.60)$$

$$\vec{D}_\perp + \vec{D}_\perp{}'' = \vec{D}_\perp{}' \qquad (3.61)$$

$$\vec{H}_\parallel + \vec{H}_\parallel{}'' = \vec{H}_\parallel{}' \qquad (3.62)$$

$$\vec{B}_\perp + \vec{B}_\perp{}'' = \vec{B}_\perp{}' \qquad (3.63)$$

To deal with these boundary conditions, it is convenient to assume that the incident wave has one or another of two linear polarizations: either the electric field is parallel to the plane of incidence (x–z plane) or else it is perpendicular to the plane of incidence.Figure (12.7) shows the directions of the fields for the former case; the electric field is parallel to the plane of incidence and the magnetic field is perpendicular.Here, $E_\parallel$ coincides with E_x, $D_\perp$ with D_z, $H_\parallel$ with H_y, and $B_\perp$ with B_z. At z=0, the boundary conditions given by Eqns (3.60) and (3.62) are then

$$-E_0 \cos\theta \, e^{-i(wt-kx\sin\theta)} + E_0'' \cos\theta'' \, e^{-i(wt-kx\sin\theta'')} = -E_0' \cos\theta' \, e^{-i(wt-k'x\sin\theta')} \tag{3.64}$$

$$\frac{n}{\mu} E_0 \, e^{-i(wt-kx\sin\theta)} + \frac{n}{\mu} E_0'' \, e^{-i(wt-kx\sin\theta'')} = \frac{n'}{\mu'} E_0' \, e^{-i(wt-k'x\sin\theta')} \tag{3.65}$$

Since the time dependence in these equations is contained in the overall factor e^{-iwt}, the equations will be satisfied at all times if they are satisfied at one initial time, say t=0. We can now ignore the time dependence and cancel the overall factor e^{-iwt}.

The boundary conditions must hold everywhere along the interface, that is, they must hold for all values of x. This requires that the factors $e^{ikx\sin\theta}$, $e^{ikx\sin\theta''}$, $e^{ik'x\sin\theta'}$ be equal. Accordingly, $k\sin\theta = k\sin\theta''$ and $k\sin\theta = k'\sin\theta'$. The first of these equations implies that the angles of incidence and of reflection are equal, $\theta=\theta''$. This is the law of reflection as mentioned before. With $k=nw/c$, $k'=nw/c$, the second equation implies that $n\sin\theta = n'\sin\theta'$. This is the law of refraction. Note that both of these laws are independent of the details of the boundary conditions; they only they only hinge on the existence of a boundary condition of some kind (3.64) leads to these laws, and so does (3.65), or any other equation expressing the equality of some wave fields at the interface.

We can now cancel all of the exponential factors in Eqn. (3.64) and Eqn.(3.65)

$$-E_0 \cos\theta + E_0'' \cos\theta = -E_0' \cos\theta' \qquad (3.66)$$

$$\frac{n}{\mu} E_0 + \frac{n}{\mu} E_0'' = \frac{n'}{\mu'} \qquad (3.67)$$

Solving these for E_0' and E_0'' in terms of E_0, we obtain the following:

For E_0 parallel to the plane of incidence:

$$E_0' = \frac{2\left(\frac{n}{\mu}\right)\cos\theta}{\frac{n'}{\mu'}\cos\theta + \left(\frac{n}{\mu}\right)\cos\theta'} E_0 \qquad (3.68)$$

For E_0 perpendicular to the plane of incidence:

$$E_0'' = \frac{\frac{n'}{\mu'}\cos\theta - \left(\frac{n}{\mu}\right)\cos\theta'}{\frac{n'}{\mu'}\cos\theta + \left(\frac{n}{\mu}\right)\cos\theta'} E_0 \qquad (3.69)$$

These are the Fresnel's equations.

3.8 Reflection and Transmission on a Dielectric Surface

Consider a plane surface separating two isotropic dielectric media with refractive indices n and n′ respectively.The surface normal is assumed to be parallel to the x-axis of a cartesian coordinate system. A plane quasi-monochromatic electromagnetic wave is incident on the surface from the left.The unit vector n normal to the incident wave is assumed to be parallel to the xy plane. Let n' and n'' denote the unit vectors normal to the refracted and reflected waves, respectively. By symmetry, n' and n'' are also parallel to the xy plane.The vectors n, n', n'' make angles θ, θ',θ'', respectively,with the x-axis.

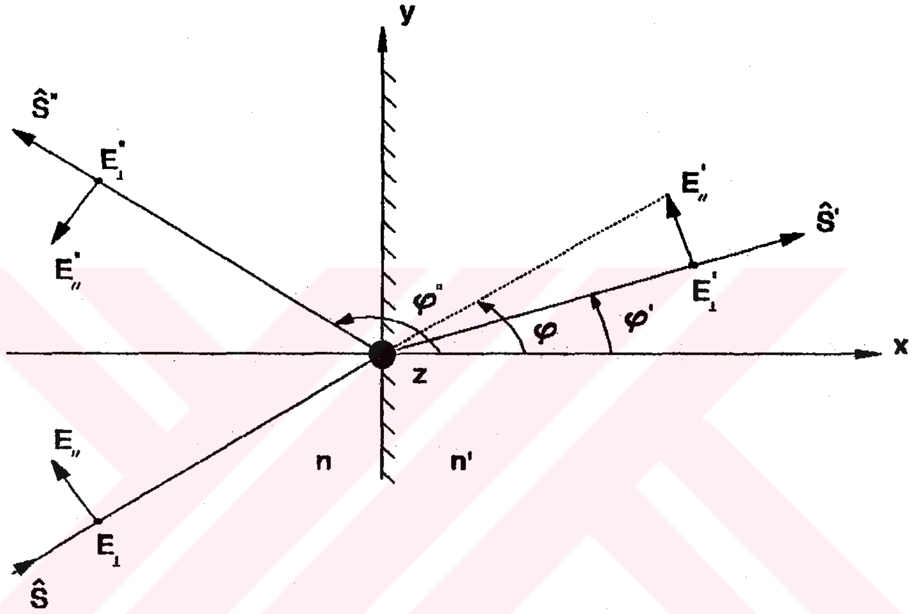

FIGURE 3.5 Reflection and Transmission at a Dielectric Interface. Vectors Parallel to z-axis are Indicated by the Symbol $\perp$.

By elementary electromagnetic theory, the electric and magnetic vectors E and H in each medium are perpendicular to the direction of propagation of the wave in that medium. In fact, in Heaviside-Lorentz Units (if the permeability μ=1)

$$\vec{H}=n\,\hat{s}\times\vec{E} \qquad (3.70)$$

Let $E_\perp$ and $E_\parallel$ denote the components of E perpendicular (TE wave) and parallel (TM wave) respectively to the xy plane. The same symbols with primes or double primes will refer to the refracted or reflected waves respectively. Let i, j, k represent the unit vectors parallel to the x,y,z axis, respectively. Then

$$\hat{e} = \hat{i}\,\mathrm{Cos}\theta + \hat{j}\,\mathrm{Sin}\theta$$

$$\vec{E} = -\hat{i}\,E_\parallel \mathrm{Sin}\theta + \hat{j}\,E_\parallel \mathrm{Cos}\theta + \hat{k}\,E_\perp \qquad (3.71)$$

$$\vec{E}' = -\hat{i}\,E'_\parallel \mathrm{Sin}\theta' + \hat{j}\,E'_\parallel \mathrm{Cos}\theta' + \hat{k}\,E'_\perp$$

$$\vec{E}'' = -\hat{i}\,E''_\parallel \mathrm{Sin}\theta'' + \hat{j}\,E''_\parallel \mathrm{Cos}\theta'' + \hat{k}\,E''_\perp$$

Using (3.17) and (3.18), the corresponding expressions for H become

$$\vec{H} = n\,(\hat{i}\,E_\perp \mathrm{Sin}\theta - \hat{j}\,E_\perp \mathrm{Cos}\theta + \hat{k}\,E_\parallel)$$

$$\vec{H}' = n'\,(\hat{i}\,E'_\perp \mathrm{Sin}\theta' - \hat{j}\,E'_\perp \mathrm{Cos}\theta' + \hat{k}\,E'_\parallel) \qquad (3.72)$$

$$\vec{H}'' = n\,(\hat{i}\,E''_\perp \mathrm{Sin}\theta'' - \hat{j}\,E''_\perp \mathrm{Cos}\theta'' + \hat{k}\,E''_\parallel)$$

The boundary conditions for B and H at the interface require that the total tangential components of E and H separately be continuous across the interface. Applying the boundary conditions in E, and equating components, one obtains

$$E_\parallel \mathrm{Cos}\theta + E_\parallel'' \mathrm{Cos}\theta'' = E_\parallel' \mathrm{Cos}\theta' \qquad (3.73)$$

$$E_\perp + E_\perp'' = E_\perp'$$

Similarly, applying the boundary condition in H to Eq.(3.47) yields

$$n\,(E_\perp \cos\theta + E_\perp'' \cos\theta'') = n' E_\perp' \cos\theta'$$
$$n\,(E_\parallel + E_\parallel'') = n' E_\parallel' \qquad (3.74)$$

By the law of reflection $\theta'' = \pi - \theta$, hence $\cos\theta'' = -\cos\theta$ so that Eq.(3.73) and Eq.(3.74) may be rearranged to give

$$E_\perp + E_\perp'' = E_\perp'$$
$$n\,(E_\perp - E_\perp'')\cos\theta = n' E_\perp' \cos\theta' \qquad (3.75)$$

and

$$(E_\parallel - E_\parallel'')\cos\theta = E_\parallel' \cos\theta'$$
$$n\,(E_\parallel + E_\parallel'') = n' E_\parallel' \qquad (3.76)$$

Equations (3.75) and (3.76) show the components of E parallel and perpendicular, respectively, to the plane of incidence are independent of each other. The two polarizations can therefore be treated separately. Eq. (3.75) may be solved for $E_\perp'$ and $E_\perp''$ to give

$$E_\perp' = \frac{2n\cos\theta}{n\cos\theta + n'\cos\theta'} E_\perp \quad ; \quad E_\perp'' = \frac{n\cos\theta - n'\cos\theta'}{n\cos\theta + n'\cos\theta'} E_\perp \qquad (3.77)$$

Similarly, solving for $E_\parallel'$ and $E_\parallel''$ from (3.76) yields

$$E_\parallel' = \frac{2n\cos\theta}{n\cos\theta + n'\cos\theta'} E_\parallel \quad ; \quad E_\parallel'' = \frac{n\cos\theta - n'\cos\theta'}{n\cos\theta + n'\cos\theta'} E_\parallel \qquad (3.78)$$

The radiant power propagated per unit area of the wavefront is equal to the Poynting Vector $S = c\, E \times H$. Using Eqn.(3.47) one obtains

$$\vec{S} = c\vec{E} \times (n\hat{s} \times \vec{E}) = cnE^2\hat{s} \qquad (3.79)$$

Usually, however, one is more interested in the radiant power per unit area of interface, which will be denoted by J. The quantity J is just the projection of S onto the surface normal

$$J = \vec{S}\,.\,\hat{1} = ncE^2 \mathrm{Cos}\theta \qquad (3.80)$$

The intensity transmission and reflection factors T and R, respectively, are defined by the relations

$$T = \frac{J'}{J} = \left(\frac{n'\mathrm{Cos}\theta'}{n\mathrm{Cos}\theta}\right)\left(\frac{E'}{E}\right)^2$$
$$R = \frac{J''}{J} = \left(\frac{E''}{E}\right)^2 \qquad (3.81)$$

where T + R = 1.

CHAPTER 4

METHODS OF SYNTHESIS AND MODELLING OF N-LAYERED SYSTEMS

In this chapter, first some commonly used method of synthesis will be discussed and then the characteristic matrix method will be outlined for a stratified medium using Maxwell's equations. Using the characteristics matrices, the reflection and transmission coefficients will be obtained at the and of this chapter.

4.1 Methods of Synthesis for Dielectric Multilayer Filters

The methods of synthesis have been subdivided into four classes:

1. Special Methods: This class includes methods which are either not true synthesis methods, or else not generally applicable to all types of filters. Most of these methods are well known and useful because of the insight that they provide to the designer.

i) Graphical methods

ii) Concept of equivalent layer

iii) Periodic multilayers

iv) Method of two effective interfaces

v) Method of hyperbolic functions

2. Approximate Methods: These methods are generally applicable to all types of problems, but are based on the assumption that the Fresnel reflectances are small.

i) Vector method

ii) Fourier Sampling method

3. Exact Methods: This class includes the methods for rigorous synthesis. Three of these methods are applicable in general. The fourth method is applicable to a broad subclass of filters.

i) Synthesis by continued fractions

ii) Synthesis when R/T is a perfect square

iii) Synthesis using radical factors

iv) Rational function synthesis

4. Methods of Differential Correction: This class includes iterative methods which make use of small changes (usually in layer thicknesses) to approach the desired optical performance in small steps. Although not a true synthesis process, differential correction is usually applied to the filter prototype which has been "synthesized" by any of the other methods.

i) General principles

ii) Design by evolution

iii) The orthonormal method

4.2 The Characteristic Matrix of a Stratified Medium

Consider a multilayer of n plane-parallel, homogeneous, isotropic dielectric layers with refractive indices n_j and geometrical thicknesses z_j where $j = 1,2,3,...,n$. The layers are assumed to be perpendicular to the z-axis and numbered from up to down, in the direction of propagation of light (see Fig. 4.1). The bounding media on the top and bottom of the multilayer are also assumed to be homogeneous, isotropic dielectrics,and have refractive indices n_0 and n_{subs} respectively. A plane electromagnetic wave is assumed to be incident from the top at an angle θ_0 with the z-axis. The corresponding angle θ_j within the jth layer and the angle θ_{subs} within the substrate, are related to θ_0 by Snell's law :

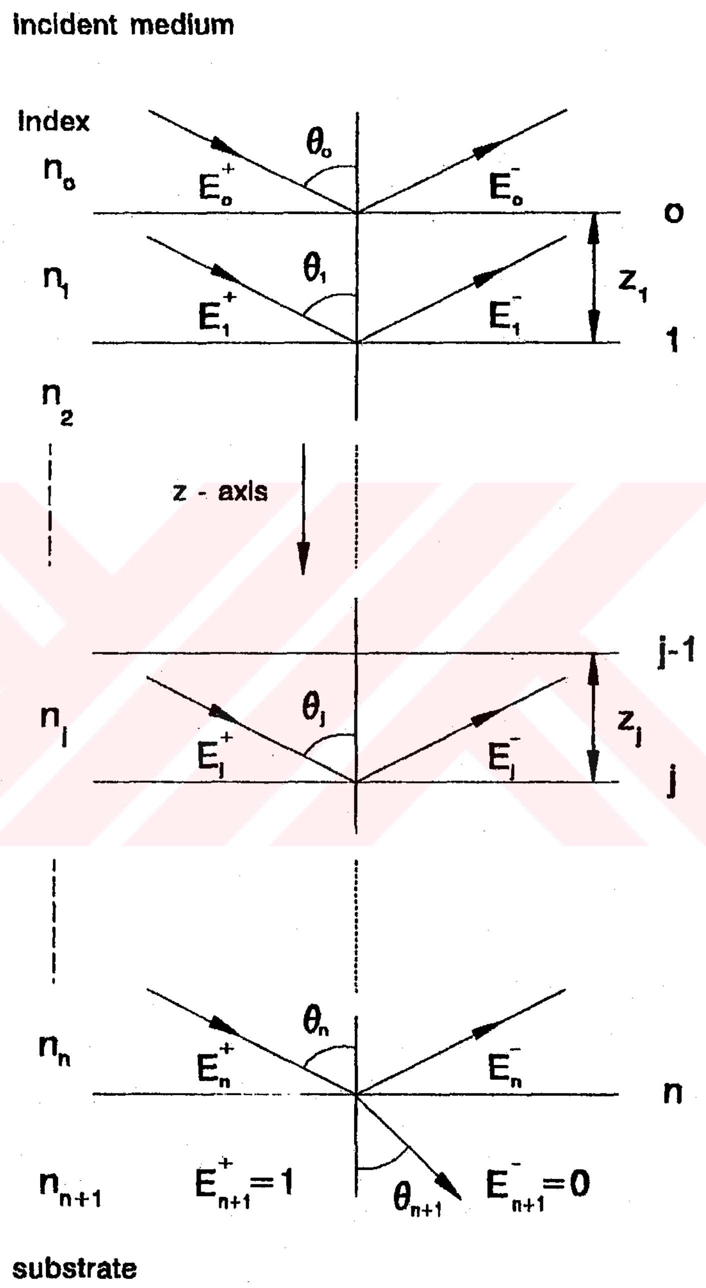

FIGURE 4.1 Reflection and Transmission for a Multilayer.

$$n_j \sin\theta_j = n_{j+1} \sin\theta_{j+1} \qquad j=1,2,3,\ldots,n,subs \qquad (4.1)$$

Consider a plane, time–harmonic electromagnetic wave propagated through a stratified medium. In the special case when the wave is linearly polarized with its electric vector perpendicular to the plane of incidence we shall speak of a transverse electric wave (denoted by TE); when its linearly polarized with its magnetic vector perpendicular to the plane of incidence we shall speak of a transverse magnetic wave (denoted by TM). Any arbitrarily polarized plane wave may be resolved into two waves, one of which is a TE wave and the other a TM wave. The boundary conditions at a discontinuity surface for the perpendicular and parallel components are independent of each other, these two waves will also be mutually independent. Moreover, Maxwell's equations remain unchanged when E and H and simultaneously ϵ and μ are interchanged. Thus any theorem relating to TM waves may immediately be deduced from the corresponding result for TE waves by making this change. It will, therefore, be sufficient to study in detail the TE waves only.

We take the plane of incidence to be y–z plane, z being the direction of stratification. For a TE wave, $E_y=E_z=0$ and Maxwell's equations reduce to the following six scalar equations (time dependence exp(–iwt) being assumed):

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} + \frac{i\varepsilon\omega}{c} E_x = 0 \qquad (4.2)$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = 0 \qquad (4.3)$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = 0 \tag{4.4}$$

$$\frac{i\omega\mu}{c} H_x = 0 \tag{4.5}$$

$$\frac{\partial E_x}{\partial z} - \frac{i\omega\mu}{c} H_y = 0 \tag{4.6}$$

$$\frac{\partial E_x}{\partial y} + \frac{i\omega\mu}{c} H_z = 0 \tag{4.7}$$

These equations show that H_y, H_z, and E_x are functions of y and z only. Eliminating H_y and H_z between Eqn. (4.2), (4.6)and (4.7) it follows that

$$\frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} + n^2 k_0^2 E_x = \frac{d(\log\mu)}{dz} \frac{\partial E_x}{\partial z} \tag{4.8}$$

where

$$n^2 = \varepsilon\mu, \qquad k_0 = \frac{\omega}{c} = 2\pi\lambda_0 \tag{4.9}$$

To solve Eqn. (4.8) we take, as a trial solution, a product of two functions, one involving y only and the other involving z only:

$$E_x(y, z) = Y(y)\, U(z) \tag{4.10}$$

Eqn. (4.8) then becomes

$$\frac{1}{Y}\frac{d^2Y}{dy^2} = -\frac{1}{U}\frac{d^2U}{dz^2} - n^2 k_0^2 + \frac{d(\log\mu)}{dz}\frac{1}{U}\frac{dU}{dz} \tag{4.11}$$

Now the term on the left is a function of y only while the terms on the right depend only on z. Hence Eqn. (4.11) can only hold if each side is equal to a constant ($-K^2$ say)

$$\frac{1}{Y}\frac{d^2Y}{dy^2} = -K^2 , \qquad (4.12)$$

$$\frac{d^2U}{dz^2} - \frac{d(\log\mu)}{dz}\frac{dU}{dz} + n^2 k_0^2 U = K^2 U \qquad (4.13)$$

It will be convenient to set

$$K^2 = k_0^2 \alpha^2 . \qquad (4.14)$$

Then Eqn. (4.12) gives

$$Y = \text{constant} . e^{ik_0\alpha y} \qquad (4.15)$$

and consequently E_x is of the form

$$E_x = U(z)\, e^{i(k_0\alpha y - \omega t)} \qquad (4.16)$$

where U(z) is a (possibly complex) function of z. From Eqn. (4.6) and Eqn. (4.7) it is seen that H_y and H_z are given by expressions of the same form:

$$H_y = V(z)\, e^{i(k_0\alpha y - \omega t)} \qquad (4.17)$$

$$H_z = W(z)\, e^{i(k_0\alpha y - \omega t)} \qquad (4.18)$$

On account of Eqn. (4.2),(4.6), and (4.7), the amplitude functions U, V, and

W are related by the following equations:

$$V' = i k_0 [\alpha W + \varepsilon U] \tag{4.19}$$

$$U' = i k_0 \mu V \tag{4.20}$$

$$\alpha U + \mu W = 0 \tag{4.21}$$

the primes denote differentiations of V and U functions with respect to z. Substituting the value of W obtained from Eqn. (4.21) into Eqn. (4.19), we have together with Eqn. (4.20), a pair of simultaneous first-order differential equations for U and V:

$$\left. \begin{aligned} U' &= i k_0 \mu V \\ V' &= i k_0 \left(\varepsilon - \frac{\alpha^2}{\mu} \right) U \end{aligned} \right\} \tag{4.22}$$

Elimination between these equations finally gives the following second-order linear differential equations for U and V:

$$\frac{d^2U}{dz^2} - \frac{d(\log\mu)}{dz} \frac{dU}{dz} + k_0^2 (n^2 - \alpha^2) U = 0 \tag{4.23}$$

$$\frac{d^2V}{dz^2} - \frac{d\left[\log\left(\varepsilon - \frac{\alpha^2}{\mu}\right)\right]}{dz} \frac{dV}{dz} + k_0^2 (n^2 - \alpha^2) V = 0 \tag{4.24}$$

According to the substitution rule which is a consequence of the symmetry of Maxwell's equations, it immediately follows that for the TM wave ($H_y = H_z = 0$), the non-vanishing components of the field vectors are of the form :

$$H_x = \overline{U}(z)\, e^{i(k_0 \alpha y - \omega t)} \tag{4.25}$$

$$E_y = -\overline{V}(z)\, e^{i(k_0 \alpha y - \omega t)} \tag{4.25}$$

$$E_z = -\overline{W}(z)\, e^{i(k_0 \alpha y - \omega t)} \tag{4.26}$$

where

$$\left.\begin{aligned} \overline{U}' &= i k_0 \varepsilon \overline{V} \\ \overline{V}' &= i k_0 \left(\mu - \frac{\alpha^2}{\varepsilon} \right) \overline{U} \end{aligned}\right\} \tag{4.27}$$

and W is related to U by means of the equation

$$\alpha \overline{U} + \varepsilon \overline{W} = 0 \tag{4.28}$$

U and V now satisfy the following second-order linear differential equations:

$$\frac{d^2\overline{U}}{dz^2} - \frac{d(\log\varepsilon)}{dz}\frac{d\overline{U}}{dz} + k_0^2(n^2 - \alpha^2)\overline{U} = 0 \tag{4.29}$$

$$\frac{d^2\overline{V}}{dz^2} - \frac{d\left[\log\left(\mu - \frac{\alpha^2}{\varepsilon}\right)\right]}{dz}\frac{d\overline{V}}{dz} + k_0^2(n^2 - \alpha^2)\overline{V} = 0 \tag{4.30}$$

U, V and W are in general complex functions of z. The surfaces of constant amplitude of E_x are given by

$$|\tilde{U}(z)| = \text{constant}$$

while the surfaces of constant phase have the equation

$$\Phi(z) + k_0\,\alpha\,y = \text{constant}$$

where $\Phi(z)$ is the phase of U. The two sets of surfaces do not in general coincide so that E_x (and similarly H_y and H_z) is an inhomogeneous wave. For a small displacement (dy, dz) along a co-phaseal surface, $\Phi'(z)dz + k_0\alpha dy = 0$; hence if θ denotes the angle which the normal to the co-phasal surface makes with OZ, then

$$\tan\theta = -\,dz/dy = k_0\alpha \,/\, \Phi'(z)$$

In the special case when the wave is a homogeneous plane wave,

$$\Phi(z) = k_0 n z \cos\theta, \qquad \alpha = n \sin\theta \tag{4.32}$$

Hence the relation

$$\alpha = \text{constant}$$

imposed by Eqn. (4.14) may be regarded as a generalization of Snell's law of refraction to stratified media.

Since the functions U(z) and V(z) each satisfy a second-order linear differential equation {Eqns. (4.23) and (4.24)}, it follows that U and V may each be expressed as a linear combination of two particular solutions, say U_1, U_2, and V_1, V_2. These particular solutions cannot be arbitrary; they must be coupled by the first order differential equations (Eqn. (4.22)):

$$\left.\begin{aligned} U_1' &= i k_0 \mu V_1 \\ V_1' &= i k_0 \left(\varepsilon - \frac{\alpha^2}{\mu}\right) U_1 \end{aligned}\right\} \qquad \left.\begin{aligned} U_2' &= i k_0 \mu V_2 \\ V_2' &= i k_0 \left(\varepsilon - \frac{\alpha^2}{\mu}\right) U_2 \end{aligned}\right\} \tag{4.33}$$

From these relations it follows that

$$V_1 U_2' - U_1' V_2 = 0 , \qquad U_1 V_2' - V_1' U_2 = 0 \tag{4.34}$$

so that

$$\frac{d}{dz} (U_1 V_2 - U_2 V_1) = 0 \tag{4.35}$$

This relation implies that the determinant

$$D = \begin{vmatrix} U_1 & V_1 \\ U_2 & V_2 \end{vmatrix} \tag{4.36}$$

associated with any two arbitrary solutions of Eqn. (4.22) is a constant, i.e., that D is an invariant of our system of equations.

For our purposes the most convenient choice of the particular solution is

$$\left.\begin{array}{l} U_1 = f(z) \\ V_1 = g(z) \end{array}\right\} \qquad \left.\begin{array}{l} U_2 = F(z) \\ V_2 = G(z) \end{array}\right\} \tag{4.37}$$

such that

$$f(0) = G(0) = 0 \quad \text{and} \quad F(0) = g(0) = 1 \tag{4.38}$$

Then the solutions with

$$U(0) = U_0 , \qquad V(0) = V_0 \tag{4.39}$$

may be expressed in the form

$$\left.\begin{aligned} U &= F\,U_0 + f\,V_0 \\ V &= G\,U_0 + g\,V_0 \end{aligned}\right\} \tag{4.40}$$

or, in matrix notation ,

$$Q = N\,Q_0 \tag{4.41}$$

where

$$Q = \begin{bmatrix} U(z) \\ V(z) \end{bmatrix}, \qquad Q_0 = \begin{bmatrix} U_0 \\ V_0 \end{bmatrix}, \qquad N = \begin{bmatrix} F(z) & f(z) \\ G(z) & g(z) \end{bmatrix} \tag{4.42}$$

On account of the relation D=constant, the determinant of the square matrix N is a constant. The value of this constant may immediately be found by taking z=0, giving

$$|N| = Fg - fG = 1 \tag{4.43}$$

It is usually more convenient to express U_0 and V_0 as functions of U(z) and V(z). Solving for U_0 and V_0, we obtain

$$Q_0 = M\,Q \tag{4.44}$$

where

$$M = \begin{bmatrix} g(z) & -f(z) \\ -G(z) & F(z) \end{bmatrix} \tag{4.45}$$

This matrix is unimodular,

$$|M| = 1 \tag{4.46}$$

The significance of M is clear: it relates the x and y components of the electric (or magnetic) vectors in the plane z=0, to the components in an arbitrary plane z=constant. Now we saw that the knowledge of U and V is sufficient for the complete specification of the field. Hence for the purposes of determining the propagation of a plane monochromatic wave through a stratified medium, the medium only need be specified by an appropriate two by two unimodular matrix M. For this reason we shall call M the characteristic matrix of the stratified medium.

In the case of a homogeneous dielectric film ϵ, μ and $n=\sqrt{\epsilon\mu}$ are constants. If θ denotes the angle which the normal to the wave makes with the z-axis, we have by Eqn. (4.32)

$$\alpha = n\sin\theta$$

For a TE wave, we have according to Eqn. (4.23) and Eqn. (4.24),

$$\left.\begin{aligned} &\frac{d^2U}{dz^2} + (k_0^2 n^2 \cos^2\theta)\,U = 0 \\ &\frac{d^2V}{dz^2} + (k_0^2 n^2 \cos^2\theta)\,V = 0 \end{aligned}\right\} \qquad (4.31)$$

The solutions of these equations, subject to the relations Eqn. (4.22), are easily seen to be

$$\left.\begin{aligned} U(z) &= A\cos(k_0 nz\cos\theta) + B\sin(k_0 nz\sin\theta) \\ V(z) &= \frac{1}{i}\sqrt{\frac{\epsilon}{\mu}}\cos\theta\,[B\cos(k_0 nz\cos\theta) \\ &\quad - A\sin(k_0 nz\cos\theta)] \end{aligned}\right\} \qquad (4.32)$$

Hence the particular solutions Eqn. (4.37) which satisfy the boundary conditions given by Eqn. (4.38) are

$$\left.\begin{aligned} U_1 &= f(z) = \frac{i}{\cos\theta}\sqrt{\frac{\mu}{\varepsilon}}\sin(k_0 n z \cos\theta) \\ V_1 &= g(z) = \cos(k_0 n z \cos\theta) \\ U_2 &= F(z) = \cos(k_0 n z \cos\theta) \\ V_2 &= G(z) = i\sqrt{\frac{\varepsilon}{\mu}}\cos\theta \sin(k_0 n z \cos\theta) \end{aligned}\right\} \tag{4.49}$$

If we set

$$p = \sqrt{\frac{\varepsilon}{\mu}}\cos\theta \tag{4.50}$$

the characteristics matrix is seen to be

$$M = \begin{bmatrix} \cos(k_0 n z \cos\theta) & -\frac{i}{p}\sin(k_0 n z \cos\theta) \\ -i p \sin(k_0 n z \cos\theta) & \cos(k_0 n z \cos\theta) \end{bmatrix} \tag{4.51}$$

For a TM wave, the same equations hold, with p replaced by

$$q = \sqrt{\frac{\mu}{\varepsilon}}\cos\theta \tag{4.52}$$

4.3 The Reflection and Transmission Coefficients

Consider a plane wave incident upon a stratified medium that extends from $z=0$ to $z=z_1$ and that is bounded on each side by a homogeneous, semi-infinite medium. We shall derive expressions for the amplitudes and intensities of the reflected and transmitted waves.

Let A, R, T denote the amplitudes of the electric vectors of the incident, reflected and transmitted waves. Further, let, ϵ_1, μ_1, and ϵ_l, μ_l be the dielectric constant and the magnetic permeability of the first and the last medium, and let θ_1 and θ_l be the angles which the normals to the incident and the transmitted waves make with the z-direction (direction of stratification). The boundary conditions demand that the tangential components of E and H shall be continuous across each of the two boundaries of the stratified medium. This gives, if the relation

$$H=\sqrt{(\epsilon/\mu)}.E$$

is also used, the following relations for a TE wave :

$$\left.\begin{aligned} U_0 &= A+R \\ V_0 &= p_1\,(A-R) \end{aligned}\right\} \qquad \left.\begin{aligned} U_{(z1)} &= T \\ V_{(z1)} &= p_1 T \end{aligned}\right\} \tag{4.53}$$

where

$$p_1=\sqrt{\frac{\varepsilon_1}{\mu_1}}\,\mathrm{Cos}\,\theta_1\,, \qquad p_1=\sqrt{\frac{\varepsilon_1}{\mu_1}}\,\mathrm{Cos}\,\theta_1 \tag{4.54}$$

The four quantities U_0, V_0, U and V given by Eqn. (4.53) are connected by the relation Eqn. (4.44) ; hence

$$\left.\begin{aligned} A+R &= (m_{11}' + m_{12}'\,p_1)\,T \\ p_1\,(A-R) &= (m_{21}' + m_{22}'\,p_1)\,T \end{aligned}\right\} \tag{4.55}$$

m_{ij}' being the elements of the characteristic matrix of the medium, evaluated for $z=z_l$. From Eqn. (4.55) we obtain the reflection and transmission coefficients of the film:

$$r = \frac{R}{A} = \frac{(m_{11}' + m_{12}' p_l)\, p_1 - (m_{21}' + m_{22}' p_l)}{(m_{11}' + m_{12}' p_l)\, p_1 + (m_{21}' + m_{22}' p_l)} \qquad (4.56)$$

$$t = \frac{T}{A} = \frac{2 p_1}{(m_{11}' + m_{12}' p_l)\, p_1 + (m_{21}' + m_{22}' p_l)} \qquad (4.57)$$

In terms of r and t, the reflectivity and transmissivity are

$$\text{Reflectivity} = |r|^2 \ , \quad \text{Transmissivity} = \frac{p_l}{p_1} |t|^2 \qquad (4.58)$$

The phase δ_r of r may be called the phase change on reflection and the phase δ_t of t the phase change on transmission. The phase change δ_r is referred to the first surface of discontinuity, while the phase change δ_t is referred to the plane boundary between the stratified medium and the last semi-infinite medium.

The corresponding formulae for a TM wave are immediately obtained from Eqn. (4.56)-(58) on replacing the quantities p_1 and p_l by

$$q_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}} \cos\theta_1 , \qquad q_l = \sqrt{\frac{\mu_l}{\varepsilon_l}} \cos\theta_l \qquad (4.59)$$

r and t then the ratios of the amplitudes of the magnetic and not the electric vectors.

CHAPTER 5

DESCRIPTION OF THE COMPUTER PROGRAMME

In this chapter the description of the prepared computer programme REFLECT will be done, then the explanations of the parameters used in the preparation of the programme will be given.

5.1 Description of the Computer Programme

A simulation programme, called REFLECT was written in Fortran 77 language, and developed to compute the percent reflectance of a multilayer dielectric coating. The computer programme was executed on both vax system and PC.

The program follows the theory developed by F. Abeles, [1]. Because of that, it firstly constructs the characteristics matrix of each layer. The necessary input elements to do this are given as follows:

1) Number of layers, NL
2) Refractive index of the incident medium, N_0
3) Refractive index of the substrate, N_{subs}
4) The angle at which the incident ray comes to the first layer, ACI
5) The wavelength at which the layer thicknesses are to be calculated, B

6) Refractive index of each medium, N_j

7) Thicknesses of each medium, Z_j. (If the layer thicknesses are to be calculated via any relation then, the computer programme must be modulated to be able to compute the layer thicknesses.)

Then using the input list given, the characteristics matrix

$$M_j = \begin{bmatrix} \cos\beta_j & -(\frac{i}{g_j})\sin(\beta_j) \\ -i\,g_j\sin(\beta_j) & \cos(\beta_j) \end{bmatrix} \quad (5.1)$$

is constructed for each layer.

After the construction process is completed, the multiplication process starts. In this stage, the matrices are multiplied one by one. For example, if there are N layer, it means that we have n matrices. Then the transfer matrix M for the multilayer is obtained as following

$$M = M_1 . M_2 . M_3 \ldots M_N \quad (5.2)$$

The resultant matrix can be written with its elements as

$$M_j = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \quad (5.3)$$

Then, by using the elements of the transfer matrix, Eqn. (5.3), the reflection and transmission factors are obtained (as previously described at Chapter 4).

Then, via the relations given at Chapter 4, the percentage reflection and transmittance values are computed through a wavelength region determined by the user. (The wavelength region is predetermined as $1.0X10^{-7}$m. and $10.0X10^{-7}$m. But if it is necessary to examine a different region, it is possible by changing that part of the programme.)

The results are stored in a data file for further usage. After getting the results of the REFLECT, it is time to plot the graphs. To do this, data is transferred to a drawing software, Harward Graphics 3. There, necessary operations take place and the graphs are ready to be discussed.

5.2 Description of the Parameters Used in the Computer Programme

The parameters used in the preparation in the computer program and their explanations are as follows:

N	Number of layers in the system
RI(J)	Refractive Index of the Jth layer
RIHAVA	Refractive Index of the Incoming Medium (Generally Air)
RISUBS	Refractive Index of the Substrate
THETA(J)	Incoming Angle to the Jth Layer
Z(J)	Thickness of the Jth Layer
BE(J)	Phase Difference for the Jth Layer
G(J)	Effective Refractive Index of the Jth Layer
GO	Effective Refractive Index of the Incoming Medium
GN	Effective Refractive Index of the Substrate
PI	Constant π
WL	Lower Value of the Wavelength Used
WU	Upper Value of the Wavelength Used
RTHF	First Value of the Incoming Angle to the First Medium in Radians

RTHS	Last Value of the Incoming Angle to the First Medium in Radians
RSTEP	Amount of Incrimentation Between RTHF and RTHS
WINC	Amount of Incrimentation in the Wavelength
R	Reflection Coefficient
T	Transmission Coefficient
REF	% Reflection
TRA	% Transmission

In addition to these parameters, some dummy variables such as SUM, MUL, X, Y, RAR, SON, TEMP used in different stages of the program to make the calculations easy to write and follow.

CHAPTER 6

RESULTS AND DISCUSSIONS

In this chapter, the graphs obtained from the execution of the computer program REFLECT will be discussed. All the curves have been calculated using the theory presented in Chapter 4. The overall transfer matrix elements are first determined by forming the product of the transfer matrices of the individual layers. In these elements, the phase difference is expressed as a function of wavelength. These matrix elements are then used for the reflection coefficient. When squared, the reflectance as a function of wavelength is obtained.

Firstly, the computer programme developed by us was used to calculate the reflectance of two-layered system according to the wavelength of the incident light. This system consists of two layers. The first layer is called as upper layer and the second which is under the first layer is called as lower layer. The refractive indexes of upper and lower layers are $n_1=1.4$ and $n_2=2.0$ and also the thicknesses of these two layers are $z_1=7.1X10^{-8}$m. and $z_2=1.0X10^{-7}$m. respectively. The thicknesses of these layers are calculated in terms of the wavelength whose value has been chosen arbitrarily. In these calculations the arbitrary value of the wavelength of the light is taken as $\lambda=4.0X10^{-7}$m. On the other hand, it was assumed that the incident light makes zero angle with the normal of the layers. Using these parameters, a curve which gives the reflectance of this two-layered system according to the wavelength of the incident light is obtained as seen in Fig. 6.1. As it is seen from the figure, this kind of two-layered system having the physical parameters given as above can be used as an antireflecting coating system in between the wavelength region which varies from λ_1=3.5X10^{-7}m. to λ_2=5.0X10^{-7}m. Because, the reflectance of this system is nearly zero in this wavelength region. The values of physical parameters used in these calculations are the same with the values used by Turner [10] who first developed a method to calculate the reflectance of this kind of two-layered system. Fig. 6.1 shows the comparison of two reflectance curves obtained from these two different calculations. As seen in this figure, these two curves are in good agreement with each other.

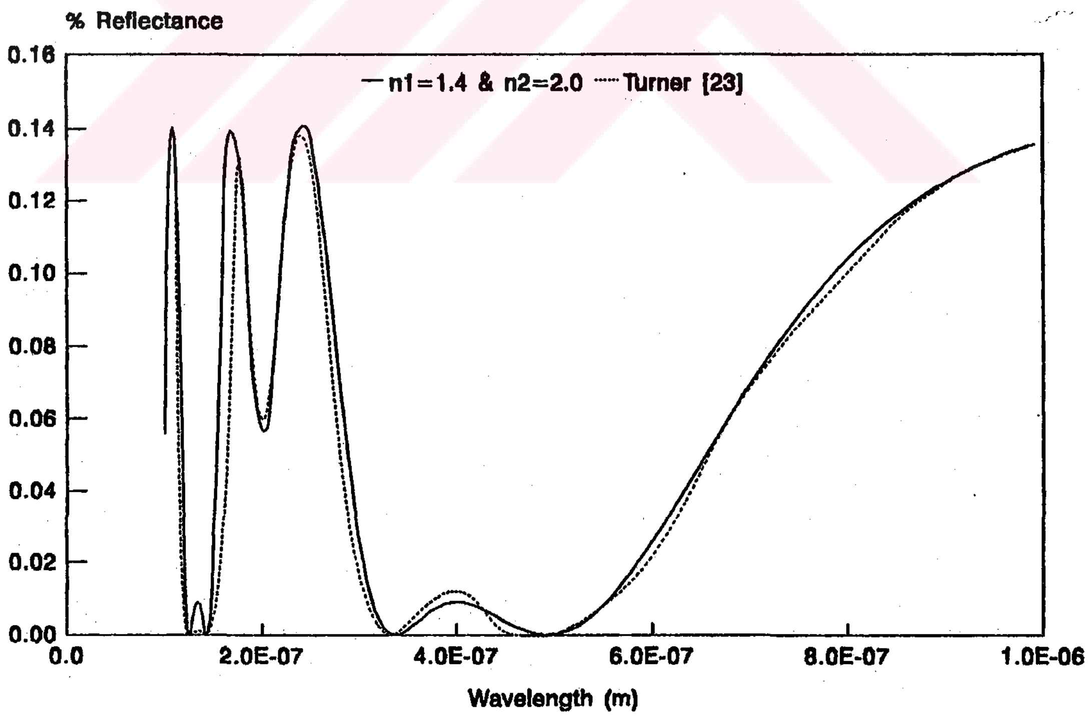

Fig. 6.1 Comparison of the Computed Reflectance Values
for a Two Layer System, n1=1.4, n2=2.0, z1=7.1X10^-8m and z2=1.0X10^-7 m
with the Model Given by Turner [23].

Secondly, the computer model has been used for the calculation of the reflectance of the same two-layered system keeping the value of every physical parameter as it is in the first calculation, only changing the value of the refractive index of the second layer to see the effect of this change. Reflectance calculations have been done for four different values of the refractive index of the second layer. Four reflectance curves obtained from computer model for four different values of the refractive index of the second layer are seen in Fig. 6.2. Thin solid, dotted, thick solid, and dashed curves represents the reflectances of the system corresponding to different values of the refractive index of the second layer for n_2=2.4, n_2=2.2, n_2=2.0, and n_2=1.8 respectively. As it is seen, all the minimum and maximum of percent reflectances are corresponding to the same wavelength. For example, in all of these four systems, reflectance has a local minimal value at $\lambda_{min}=2.0X10^{-7}$m. and has a maximal value at $\lambda_{max}=4.0X10^{-7}$m. On the other hand, the reflectance of the system increases with increasing the value of the refractive index of the second layer. There is another interesting result obtained from the comparison of these four curves. As seen in figure, the last minimum of the reflectance curve which falls between the wavelength region $2.0X10^{-7}$m. and $6.0X10^{-7}$m. shifts to the left with the increasing value of n_2. If the refractive index of the second layer is continued to be increased, it is estimated that there will be a narrow region about the wavelength $\lambda=5.0X10^{-7}$m. in which there is no reflection.

The third reflectance calculation has been done for a two-layered system again, using the same values of the thicknesses of the first and second layers as they were in previous two calculations, keeping the value of the refractive index of the second layer at a constant value which is equal to 2.0, and giving four different values to the value of the refractive index of the first layer. Four reflectance curves obtained from

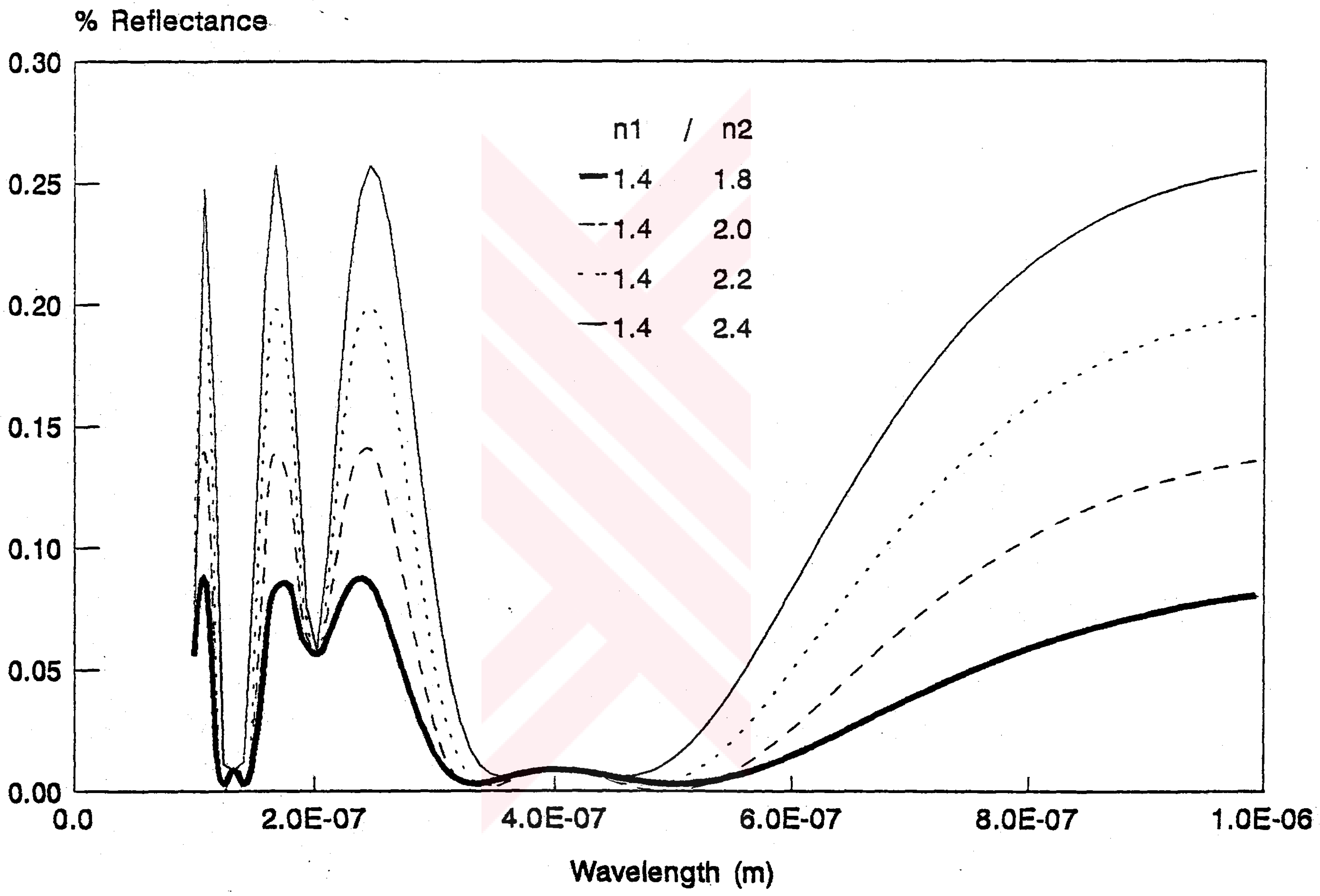

Fig. 6.2 The Effect of the Change in the Value of n2 on the Reflectance Values for a Two Layer System, Keeping n1 Constant, with $z1 = 7.1X10$^-8 m and $z2 = 1.0X10$^-7 m

computer model for four different values of the refractive index of the first layer are seen in Fig. 6.3. Here, thick solid, dashed, dotted, and thin solid curves represent the reflectance of the system corresponding to the different values of the refractive index of the first layer for n_1=1.4, n_1=1.7, n_1=2.0, n_1=2.3. As seen in Fig. 6.3, all minimum and maximum values of the reflectance curves correspond to the same wavelengths and also it is seen that the reflectance of the system increases with the increasing value of the refractive index of the first layer. Beyond $\lambda=6.0\times10^{-7}$m. region, four curves reach to the saturation and also they come close to each other at these saturation points. Another interesting point results from the comparison of the last minimum points of these curves. As seen in figure, the last minima of these curves shift to right with increasing value of the refractive index of the first layer.

To investigate the effect of the change in the thickness of the layers on the reflectance of a two layered system, a few calculations have been done. The refractive indices of the first and second layers of this system are n_1=1.4, and n_2=2.0, respectively. While the thickness of the first layer is being kept at a constant value z_1=0.071µm., the thickness of the second layer has been changed three times giving three different values. In Fig. 6.4, three curves corresponding to three different values of the thickness of the second layer are seen. In this figure, dashed, solid,and dotted curves represent the reflectance of the system corresponding to different values of the thickness of the second layer for z_2=0.1µm., z_2=0.2µm., and z_2=0.3µm. There are slight differences between these curves obtained for three cases. But this difference is not very important. The common property between these systems is that the reflectance of every system increases with increasing thickness of the second layer.

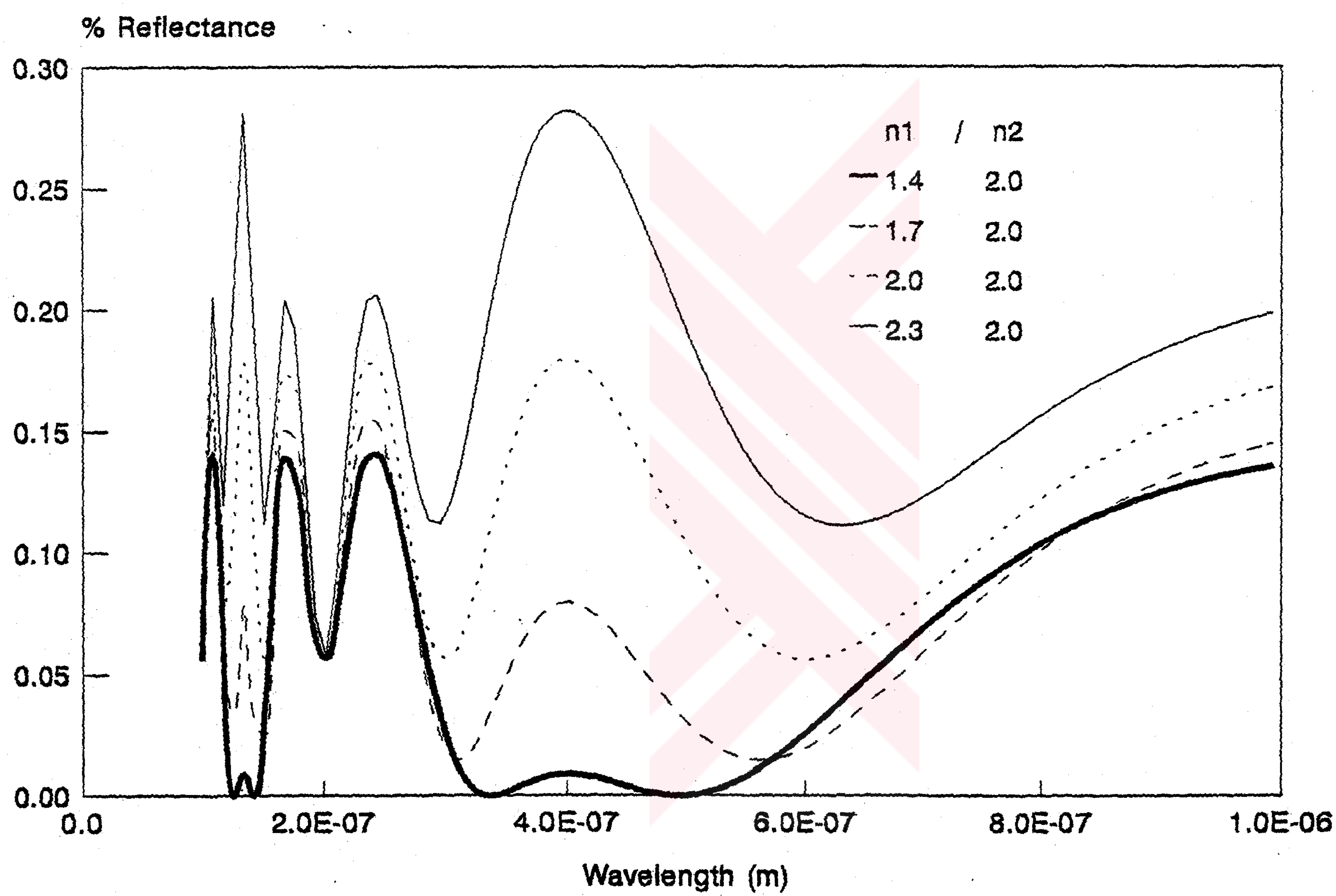

Fig. 6.3 The Effect of the Change in the Value of n1 on the Reflectance Values for a Two Layer System, Keeping n2 Constant, with z1=7.1X10^-8 m and z2=1.0X10^-7 m.

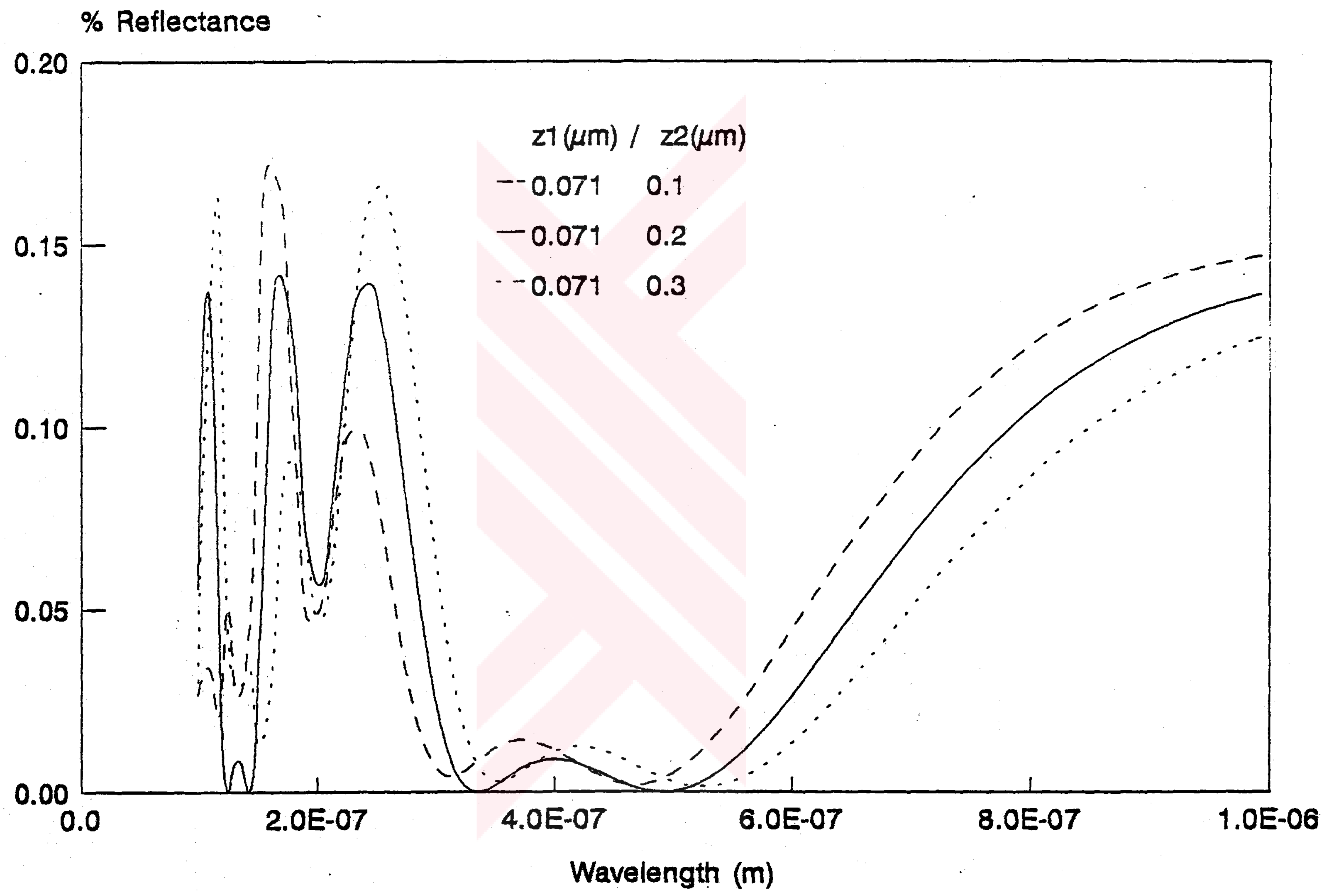

Fig. 6.4: The Effect of the Change in the Thickness of the Second Layer z2 on the the Reflectance Values for a Two Layer System with n1=1.4 and n2=2.0, Keeping the Thickness of the First Layer as z1=7.1X10^-8m.

To investigate the effect of the coating thickness of the first layer on the reflectance of the two-layered system, the values of the refractive indices of the first and second layers and the thickness of the second layer are kept at constant values as $n_1=1.4$, $n_2=2.0$, and $z_2=1.0X10^{-7}m.$ respectively, and only the value of the thickness of the first layer is changed. Fig. 6. 5 shows four curves obtained for four different values of the thickness of the first layer. In this figure, dashed, thin solid, dotted, and thick solid curves represent the reflectance of this system corresponding to the different values of the thickness of the first layer for $z_1=0.061\mu m.$, $z_1=0.071\mu m.$, $z_1=0.081\mu m.$, $z_1=0.091\mu m.$ If these curves are compared with the curve seen in Fig. 6.1 , it will be seen that these curves have main characteristic properties on it. The maximum and minimum points of these curves do not correspond to the same wavelengths, while maximum and minimum points of the curves obtained from the previous calculations are corresponding more or less to the same wavelengths in certain wavelength regions. This results from the variation in the thickness of the first layer.

Antireflecting and totally reflecting systems are especially used in industry and research laboratories as mentioned in the previous chapters. The aim in this field is to develop the most suitable and effective systems. In the previous sections, it was studied how the physical parameters effect on the reflectance of two-layered systems. Now, it will be seen how the increase in the number of layers effects on the reflectance of an n-layered system. To see this effect, a three-layered system was studied. The thicknesses of the first, second, and third layers have been chosen as $z_1=9.9X10^{-8}m.$, $z_2=1.2X10^{-7}m.$, and $z_3=2.3X10^{-7}m.$ and the refractive indexes of these layers are $n_1=1.38$, $n_2=2.30$, and $n_3=1.76$. The reflectance curves which were obtained from the computer calculation using the values of the parameters given as above is seen in Fig. 6.6.

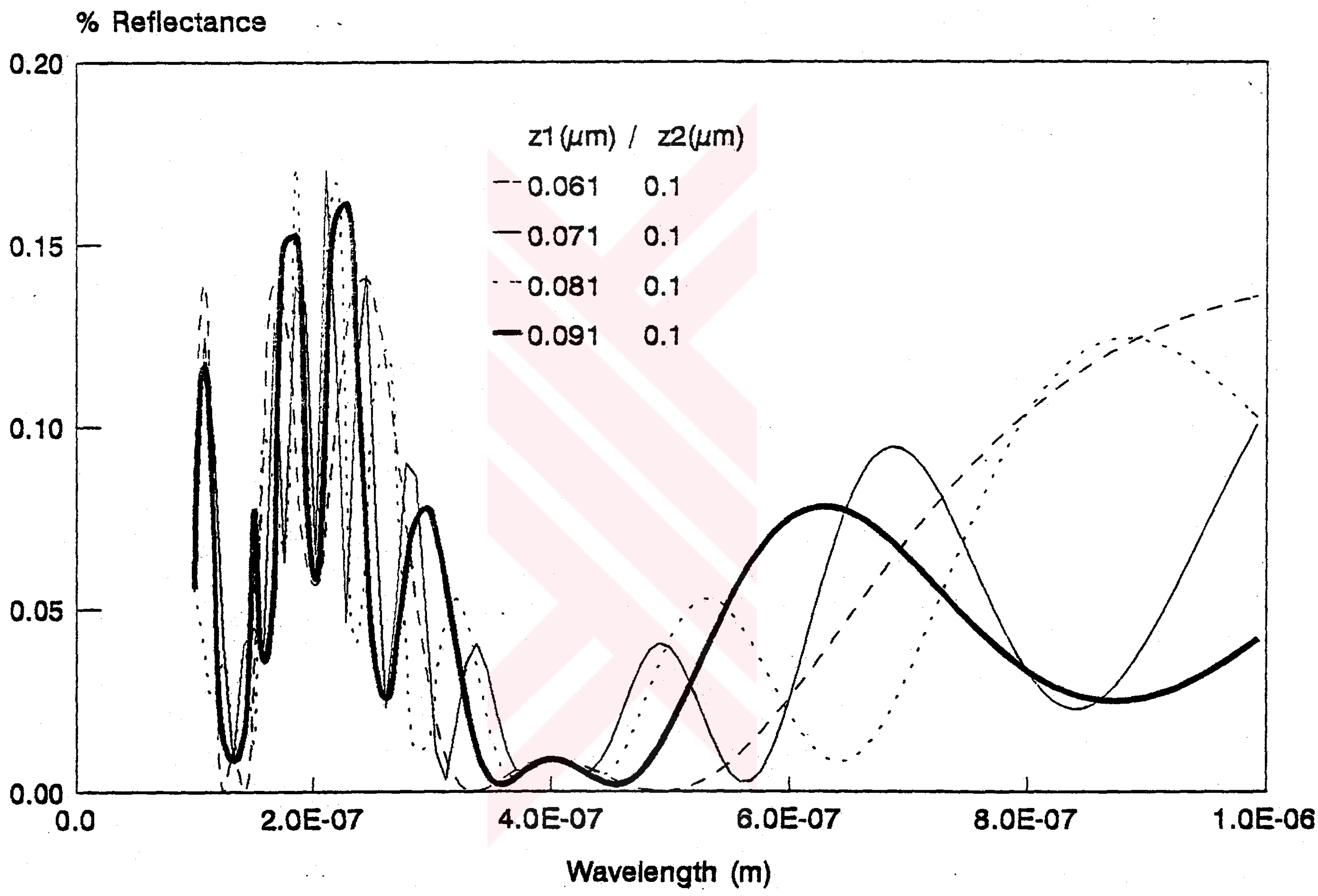

Fig. 6.5 The Effect of the Change in the Thickness of the First Layer z1 on the Reflectance Values for a Two Layer System with n1=1.4 and n2=2.0, Keeping the Thickness of the Second Layer Constant as z1=1.0X10^-7m.

Similar calculations have been done for the same system by Turner [23] too. The results obtained from these two calculations are the same. As seen from these reflectance curves, the reflectance of this system is exactly zero between the wavelengths λ_1=5.0X10^{-8}m. and λ_2=7.0X10^{-8}m.

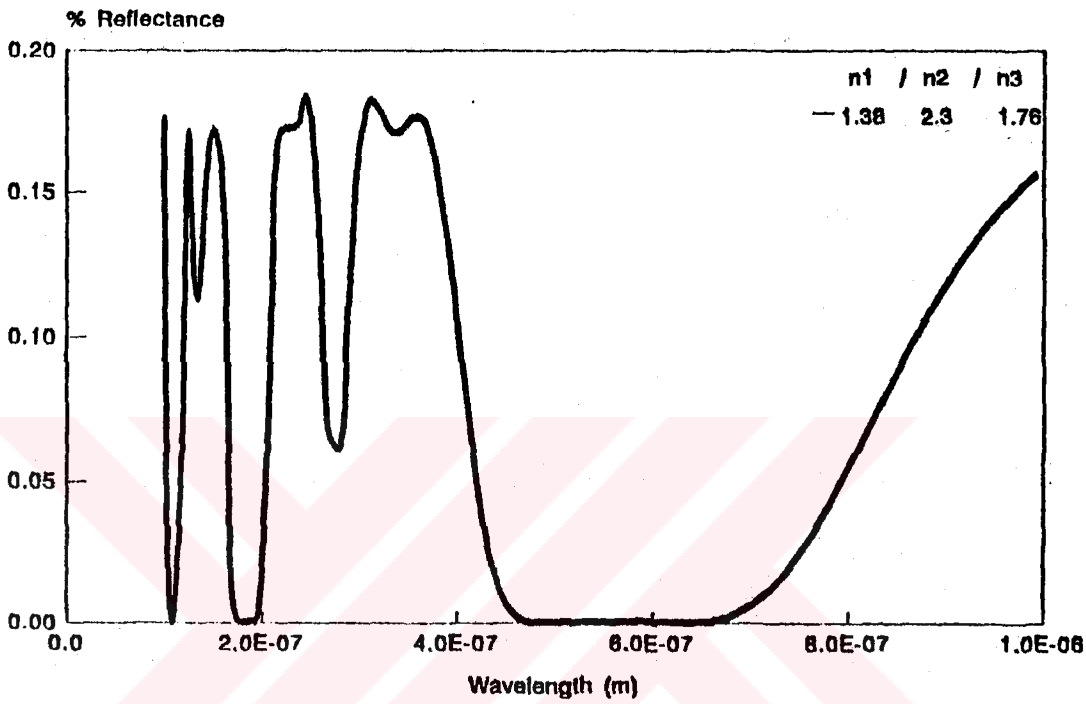

Fig. 6. 6. The Reflectance Values for a Three Layer System with Refractive Indices n1=1.38, n2=2.30, and n3=1.76 and with Thicknesses z1=9.96X10^-8 m. z2=1.195X10^-7m. z3=2.34X10^-7m.

Reflectance curves obtained for a three layer system with thicknesses z_1=7.2X10^{-8}m., z_2=8.7X10^{-8}m., and z_3=1.7X10^{-7}m., giving different values to the index of refraction of each layer, keeping two of them constant and changing the other alternately are given in Fig. 6.7. The thick solid curve represents the reflectance of the system for n_1=1.38, n_2=2.30, and n_3=2.00. The dotted curve represents the reflectance of the system for n_1=1.38, n_2=2.50, and n_3=1.76. The dashed curve represents the reflectance of the system for n_1=1.38, n_2=2.30, and n_3=1.76. Finally, the

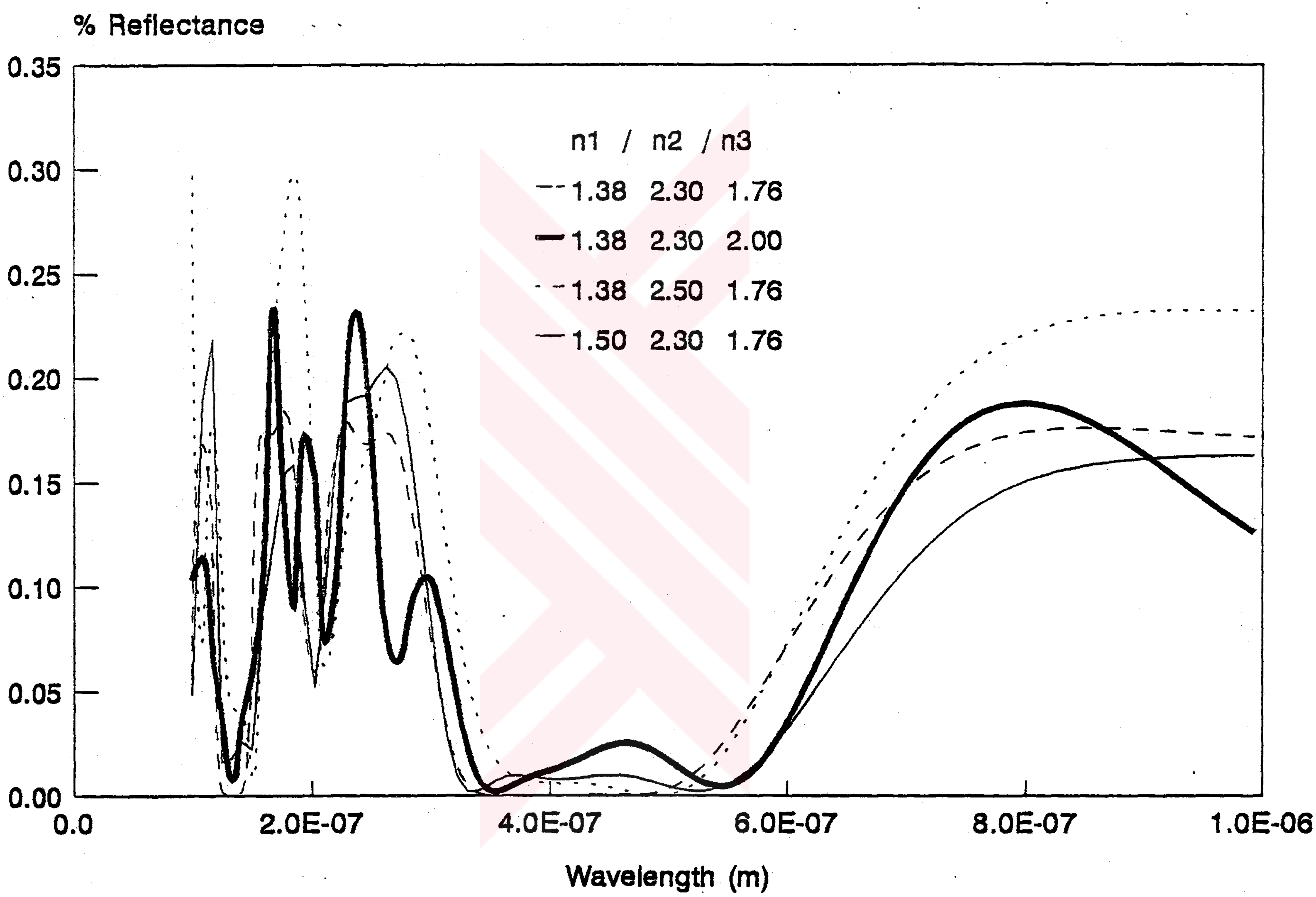

Fig. 6.7 Reflectance Curves Obtained for a Three Layer System with Thicknesses z1=7.2X10^-8m., z2=8.7X10^-8m., and z3=1.7X10^-7m., Giving Different Values to the Index of Refraction of Each Layer, Keeping Two of Them Constant and Changing the Other Alternately.

thin solid curve represents the reflectance of the system for n_1=1.50, n_2=2.30, and n_3=1.76. In all of four cases the general characteristic of the reflectance curves show similar properties. Although the magnitudes of the reflectances vary with changing the refractive indexes of each layer at a time, the corresponding wavelength values remain almost constant.

Till now, the effects of the physical parameters such as number of layers, refractive indexes of the layers, thicknesses of the layers on the reflectance values were studied. In addition to these parameters, angle of incidence is another parameter of the system effecting the value of the reflectance. To see the effect of the angle of incidence on the reflectance of a system a two-layered system is used. The refractive index of the first layer n_1=1.4 and the refractive index of the second layer n_2=2.0. The thickness of the first layer is equal to 7.1×10^{-8} m. and the thickness of the second layer is equal to 1.0×10^{-7} m. To see the effect of the angle of incidence on the reflectance, all these physical parameters are kept constant and four different values have been given for the angle of incidence as $\theta_1=0^0$, $\theta_2=30^0$, $\theta_3=45^0$, and $\theta_4=60^0$ for this calculations. The reflectance curves were obtained using the data calculated by computer model for four different values of the angle of incidence according to the wavelength of the light are seen in Fig. 6.8. Thick solid, thin solid, dashed, and dotted curves represent the reflectance values for the angles of incidence 0^0, 30^0, 45^0, and 60^0 respectively. It was seen from this figure that, if the angle of incidence is increased, then corresponding value of the reflectance will be increased too.

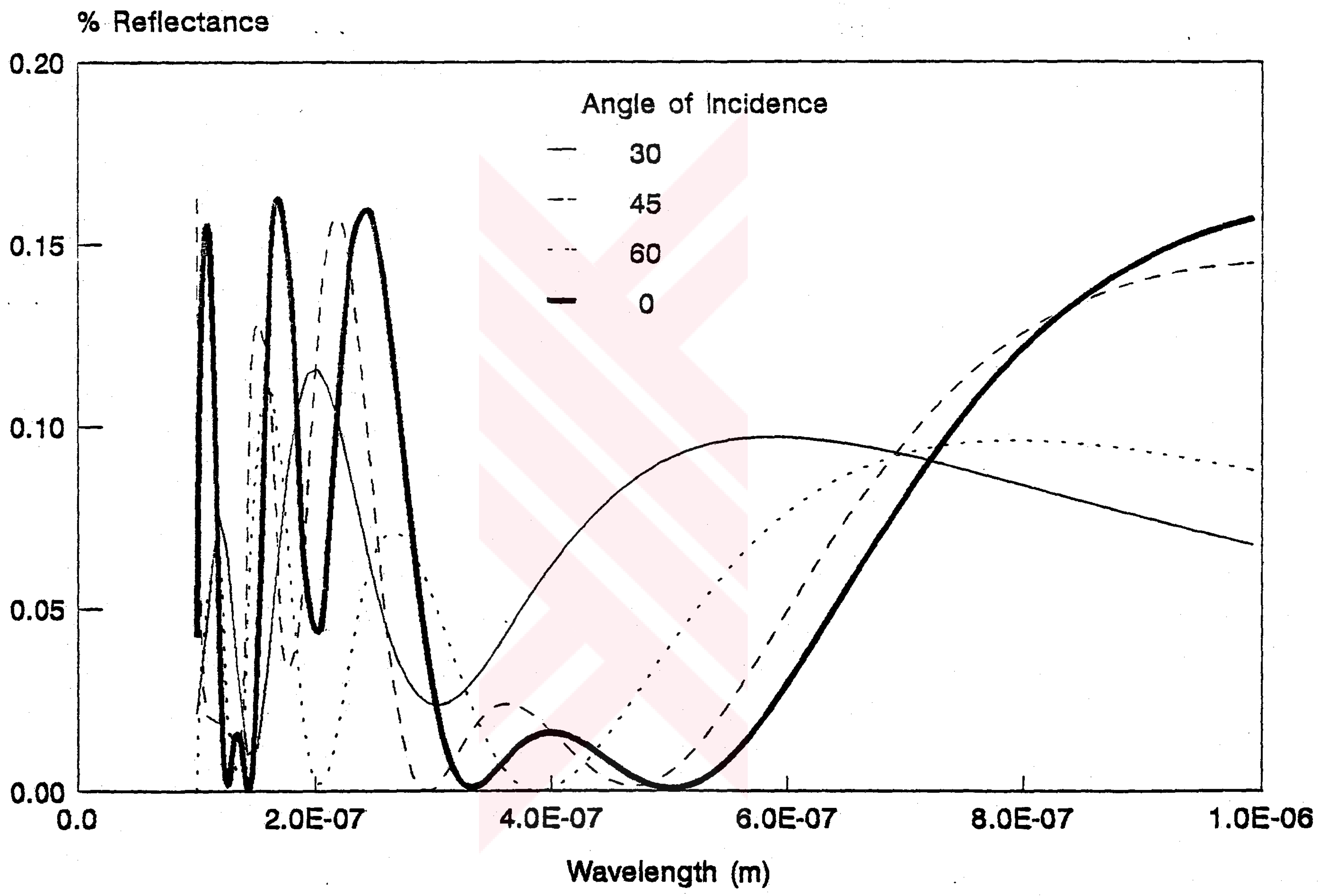

Fig. 6.8 The Effect of the Angle of Incidence for a Two Layer System with Refractive Indices n1=1.4 and n2=2.0 and with Thicknesses z1=7.1X10^-8m and z2=1.0X10^-7m.

CHAPTER 7

RECOMMENDATIONS FOR FURTHER STUDY

When preparing the computer programme the layers are characterized by 2X2 matrices and using those matrices the reflectance and transmittance characteristics were obtained. Instead of that, using only the Maxwell's equations and the Fresnel's equations the reflection and transmittance characteristics could be obtained without making any assumptions.

Also, in this study it was assumed that film layers are isotropic, homogeneous, and non-absorbing dielectric materials; incident light is unpolarized; and the refractive index values are real, and constant over the range of working wavelengths. This study may be reorganized for non-homogeneous, absorbing layers with complex and wavelength dependent refractive index. A look-up table of commonly used optical thin film materials may also be added to the programme.

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APPENDIX

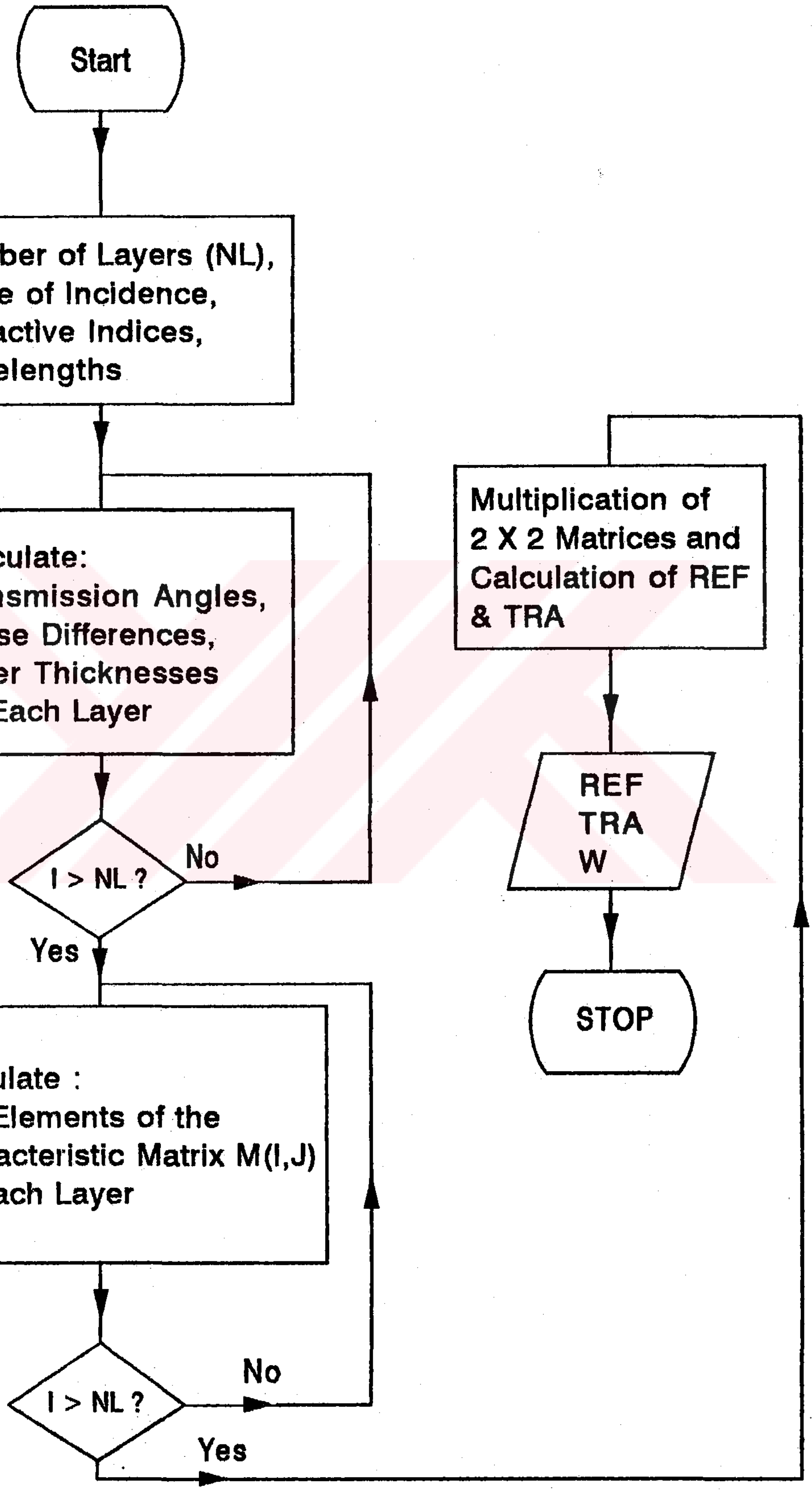

Fig. A1.1 Flow Chart of the REFLECT Programme

LISTING OF REFLECT PROGRAM

```
      COMPLEX*8 SUM,MUL,M,X,Y,R,T,TIP
      REAL RI(0:100),BE(100),G(100),Z(100),THETA(0:100)
      REAL RAR,SON,PI,WL,WU,RTHF,RTHS,RSTEP,WINC,ACI,A
      REAL TEMP,S,REF,TRA,TOP,B
      DIMENSION MUL(2,200),TIP(2,200),M(2,200)
      OPEN(UNIT=3,FILE=' T.DAT',STATUS='NEW')
      WRITE(3,*) '  WAVELENGTH          REFLECTANCE   '
C     ENTER THE NECASSARY PARAMETERS FOR THE SYSTEM
      PRINT*,'PLEASE ENTER THE NUMBER OF LAYERS'
      READ*,N
      THETA(0)=0.00
      RI(0)=1.00
      RISUBS=1.62
      RIHAVA=1.00
      DO 1 I=1,N,1
C     ENTER THE REFRACTIVE INDICES OF THE LAYERS
      PRINT*,'PLEASE ENTER THE REFRACTIVE INDEX OF  ',I,'
TH
      LAYER'
      READ*,RI(I)
      WRITE(3,*)RI(I)
    1 CONTINUE
C     B IS THE CENTRAL WAVELENGTH
      B=4.0E-7
      PI=4.00*ATAN(1.00)
```

```
      DO 1000 WL=1.0E-7,10.0E-7,85E-10
      DO 2 I= 1,N,1
C     THETA IS THE ANGLE OF TRANSMITTANCE
C     AND IS CALCULATED BY MEANS OF SNELL'S LAW
      THETA(I)= ASIN(RI(I-1)/(RI(I))*(SIN(THETA(I-1))))
C     Z IS THE THICKNESS OF I'TH LAYER
      Z(I)=I*B/(4.00*RI(I))
      BE(I)=2.00*PI*RI(I)*Z(I)*COS(THETA(I))/(WL)
      G(I)=RI(I)
    2 CONTINUE
      DO 200 I=1,N,1
      J=2*I
      S=BE(I)
C     M'S ARE THE ELEMEMNTS OF THE CHARAECTERISTIC MATRICES
      M(2,J)  =COS(S)
      M(1,J-1)=m(2,J)
      SON=-SIN(S)/G(I)
      M(1,J)=CMPLX(0.00,SON)
      RAR=-G(I)*SIN(S)
      M(2,J-1)=CMPLX(0.00,RAR)
  200 CONTINUE
 1001 FORMAT (2X,6(E10.5))
      J=2
C     MUL AND TIP ARE DUMMY VARIABLES
      MUL(1,1)=M(1,1)
      MUL(1,2)=M(1,2)
      MUL(2,1)=M(2,1)
      MUL(2,2)=M(2,2)
      TIP(1,1)=M(1,3)
```

```
      TIP(1,2)=M(1,4)
      TIP(2,1)=M(2,3)
      TIP(2,2)=M(2,4)
100   FINK=12
C     MULTIPLICATION OF CHARECTERISTICS MATRICES
      DO 3 I=1,2,1
      DO 3 L=1,2
      SUM=CMPLX(0.0,0.0)
      DO 4 K=1,2,1
      SUM=CMPLX(SUM+MUL(I,K)*TIP(K,L))
4     CONTINUE
      MUL(I,L)=CMPLX(SUM)
      WRITE(3,*)'THE RESULT OF THE MULTIPLICATION IS'
      WRITE(3,*)MUL(1,1),MUL(1,2)
      WRITE(3,*)MUL(2,1),MUL(2,2)
3     CONTINUE
      IF (J.EQ. N) THEN
      GO TO 150
      ELSE
      J=J+1
      TIP(1,1)=M(1,2*J-1)
      TIP(1,2)=M(1,2*J)
      TIP(2,1)=M(2,2*J-1)
      TIP(2,2)=M(2,2*J)
      ENDIF
      GO TO 100
1500  ALI=188
      PRINT*,MUL(1,1),MUL(1,2)
      PRINT*,MUL(2,1),MUL(2,2)
```

```
C          CALCULATION OF REFLECTION AND TRANSMITTANCE
     150   G0=RIHAVA*COS(THETA(0))
           THETA(N+1)=ASIN((RI(N)/(RISUBS))*(SIN(THETA(N))))
           GN=RISUBS*COS(THETA(N))
           X=CMPLX(G0*(MUL(1,1)+GN*MUL(1,2)))
           Y=CMPLX(MUL(2,1)+GN*MUL(2,2))
           R=CMPLX((X-Y)/(X+Y))
           T=CMPLX(2*G0/(X+Y))
           REF=R*CONJG(R)
           WRITE(3,*)'REF=',REF
           TRA=((RISUBS*COS(THETA(N+1)))/
                (RI(1)*COS(theta(0))))*(T*CONJG(T))
           WRITE(3,*)'TRA=',TRA
           TOP=TRA+REF
           ABSP=1-TOP
           PRINT*,X,Y,R,T,REF,TRA
           WRITE(3,*)A,TRA,REF,TOP
           FORMAT(2X,e14.7,2X,e14.7,2X,e14.7,2X,e14.7)
           WRITE (3,*)WL,TRA,REF
     1000  CONTINUE
           STOP
           END
```