Calculation of Alpha Half-Lives of Radioisotopes by Using Different Techniques

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Abstract

Calculation of Alpha Half-Lives of Radioisotopes by Using Different Techniques

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The estimations and comparisions of alpha half-lives of alpha emitting isotopes of odd-odd, even-even, odd-even and even-odd nuclei are made by using different methods. Tunnelling probabilities are determined by the breaking the Coulomb barrier into segments. The obtained succesive tunnelling probabilities are multiplied to obtain tunnelling probability utilized to determine alpha half-lives. The advantages and disadvantages of the model are discussed. It is observed that an application of the simple model based on the tunnelling probability provides quite satisfactory results compared to some other techniques.

Key words: Alpha Decay, Tunnelling Probability, Half-life.

Öz

Radioizotopların Alfa Yarı Ömürlerinin Değişik Teknikler Kullanarak Hesaplanması

SÜR, Serap
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Alfa yayan çift-çift, tek-tek, tek-çift ve çift-tek izotopların alfa yarı ömürleri hesaplandı ve değişik metodlarla karşılaştırılması yapıldı. Tünelleme olasılığı Coulomb engelini parçalara bölerek tanımlandı. Elde edilen ardışık tünelleme olasılıkları alfa yarı ömrünü tanımlamaya yardımcı olan tünelleme olasılığını elde etmek için birbirleriyle çarpıldı. Modelimizin avantajları ve dezavantajları değerlendirildi. Tünelleme olasılığına dayanan basit modelimizin diğer tekniklerle karşılaştırıldığında oldukça uygun sonuçlar sağladığı gözlemlendi.

Anahtar kelimeler: Alfa Bozunması, Tünelleme olasılığı, Yarı ömür.

Acknowledgements

I am indebted to several people for the preparation of this thesis work. Most importantly, I would like to express my gratitude to my supervisor Prof. Dr. Zihni ÖZTÜRK for the many helpful comments and suggestions during his nearly three years of supervision, and especially for his commitment to guiding me through my research, as well as for the time he has spent reading the various drafts of this thesis. His critical commentary on my work has played a major role in both the content and presentation of my discussion and arguments.

Finally, I would like to give my sincere thankfulness to my family for their interest, patient and support and to my roommate Ress. Assist. Eser OLĞAR for friendship and his help through the work presented in this thesis.

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Chapter 1

INTRODUCTION

Alpha particle was played an important role for development of nuclear physics. Much empirical information on alpha decay was accumulated in early decades of radioactivity research. In 1896, Henri Becquerel studied the radiation emitted by phosphorescent materials. He was intrigued by Roentgen's recent discovery of X-rays in 1885 and looked for X-rays in Uranium salts. But the unexpected happened, as it has on numerous other occasions in physics: Becquerel discovered that these salts emit a new form of radiation, different from both phosphorescent light and X-rays, which he called Uranic rays. This marked the beginning of the field of nuclear physics.

While Becquerel went on to do research in atomic physics, Marie Sklodowska Curie was interested in his discovery of the Uranic rays and began to investigate them systematically. Soon afterwards her husband, Pierre Curie, joined her in this research. Their studies led them to propose that the radiation was emitted from single atoms. These ideas, based on the not yet fully confirmed theory of atomic structure of the elements, led them to the discovery of new elements Polonium and Radium. They showed that other elements besides Uranium emitted such rays, and coined the term Radioactivity by which the phenomenon of this sort of spontaneously emitted radiation has been known ever since.

A few years later, in 1898, Ernest Rutherford and Frederik Soddy found that substances like Uranium and Thorium radioactively transmute naturally into other elements by means of some decay processes which alpha and beta decays. In 1900, Soddy observed spontaneous disintegration of radioactive elements into variants he called "isotopes" or totally new elements, discovered "half-life", made initial calculations on energy released during decay.

Rutherford found that a certain fraction of a radioactive substance decays in a given time interval. This means that the original amount decays exponentially with time; the time it takes for half the material to decay is known as the half-life. For each radioactive decay, there is a characteristic half-life, which

Fanny Gates along with others showed was quite independent of chemical and thermal properties of the radioactive substance. It was eventually learned that alpha-rays are just helium atoms without electrons, carrying two units of positive charges each. This means that when an alpha ray is emitted, the atomic number Z of the atom decreases by two units.

In 1906, Hans Geiger developed an electrical device to "click" when hit with alpha particles. Before the development of this device, determinations of the ratio of the charge Q to the mass M of the alpha rays was made by Rutherford and Robinson. To determine the value of Q/M of the alpha particles it is necessary to perform another experiment to determine the velocity v of the alpha particle. The determination of the velocity and energy of the alpha particles will be discussed in some detail for several reasons. First, the accurate measurement of alpha particle energies made it possible to determine energies which differ only by small amounts, and this led to the discovery that some radionuclides actually emit a spectrum of the alpha particles. Second, knowledge of the energies of the components of the alpha spectra makes it possible to assign certain nuclear energy levels with confidence. Third, the methods for determining the energies of alpha particles are also used for protons and deuterons. These three charged particles are involved in many artificial disintegrations, and the accurate measurement of their energies yields accurate Q-values by which it is possible to determine nuclear masses and nuclear energy levels. Fourth, accurate values of alpha particle energies are needed in the development and use of the theory of the alpha decay.

Rutherford had continued his research over the those years. In 1908, he identified alpha particles as atomic bullets, probed the atoms in a piece of thin gold foil. He established that the nucleus was: very dense,very small and positively charged. He also assumed that the electrons were located outside the nucleus.

Rutherford's nuclear model pointed the way to the new world of modern physics, but it was Niels Bohr who opened its door. In 1913, he constructed a dynamical model of the hydrogen atom with an electron circulating a hydrogen nucleus, (which later acquired the name proton) in stable orbits called stationary states. By allowing the electron to emit light only when it jumps between these stationary states, Bohr was able to explain the known energies of light emitted by excited hydrogen atoms. Bohr's model was soon developed by others in a mathematical formulation called Quantum Mechanics. Quantum Mechanics and Albert Einstein's Theory of Relativity provide the conceptual basis for the theoretical description of all physical phenomena known to us today.

One of the early successes of quantum mechanics was its explanation of alpha decay. It had been known for some time that alpha decay half-lives depend very sensitively on the decay energy. Doubling the decay energy from 4 to 8 MeV causes the typical half-life to decrease from 10^{10} years to 10^{-2} seconds, a change by a factor of 10^{-19}! a qualitative inverse correlation between energy release. Half life was recognized by Rutherford in 1906, and in 1911 Geiger and Nuttall formulated a quantitative relation between decay constant λ and range in air. This extreme energy dependence was finally explained in 1928 by Gamow, and independently by Gurney and Condon, as a Quantum Mechanical phenomenon. The alpha-particle, held inside the nucleus by a potential barrier caused by the positive nuclear charges, cannot escape from it, according to classical physics. However, Quantum Mechanics does allow the alpha-particle to escape by "tunneling" through the barrier, with an energy-dependent half-life consistent with experiment.

In 1923, de Broglie discovered that electrons had a dual nature-similar to both particles and waves. Particle/wave duality is supported by Einstein. In 1929, Cockroft and Watson built an early linear accelerator and bombarded lithium with protons to produce alpha particles. In 1930, Schröndinger viewed electrons as continuous clouds and introduced "wave mechanics" as a mathematical model of the atom. Then in 1932 James Chadwick using alpha particles discovered a neutral atomic particle with a mass close to a proton. Thus was discovered the neutron [1, 2, 3].

With all these developments the theory of alpha decay can be easily understood and during these researches the four radioactive series was defined.

This thesis is organized as follows: the theory of alpha decay is given in details in chapter 2. The tunneling probability is considered with some approximation methods such as Wentzel-Kramers-Brillouin approximation and Bohr-Sommerfeld quantization condition. Chapter 3 deals with the comparisons of methods utilized to calculate alpha half-life. Then we formulate a simple method to estimate the alpha half-life. Our results are compared with the results of other methods and discussed in next chapter. Concluding remarks and future plans are given in last chapter.

Chapter 2

ALPHA DECAY

In a decay, the original particle disappears and two or more less massive particles are produced. Particle decays are like radioactive decays of atomic nuclei. When a nucleus decays radioactively, some of the decay products are constituents that were present before the decay, but others, such as photons or electrons, are entirely new objects produced by the decay process.

When a fundamental particle decays, all the produced particles are new objects that were not present before the decay. A single type of fundamental particle can have many possible sets of decay products.

Electrons, protons, photons, and neutrinos are the only fundamental particles that never decay. All other isolated particles are unstable and decay with a definite half-life decay distribution. There are three types of nuclear radioactive decay; these are alpha, beta and gamma emission. The decay of radioactive elements is independent of chemical and physical conditions imposed on them. It is dependent on the stability of nuclei.

2.1 STABILITY OF NUCLEI

The nucleus of an atom contains neutrons and protons. The protons are positively charged particles and, because like charges repel each other, one would expect the closely packed protons in the nucleus to fly apart. The reason that they do not is due to the nuclear force of attraction which at short distances is much stronger than the electric force of repulsion. This repulsion becomes so great in nuclei with more than 10 protons or there is an excess of neutrons leads to attractive forces, which is required for stability. The total binding energies of nuclei are very nearly proportional to the numbers of nucleons present. Both protons and neutrons are subject to this nuclear force, so the presence of neutrons acts to stabilize the nucleus. The neutrons, which are uncharged particles, add attractive force to the nucleus without adding to the electrical force of repulsion.

When the nuclear force overcomes the electrical force of repulsion a nucleus will remain intact indefinitely, and is said to be stable [4]. Most of the naturally occurring elements have stable atoms with the exception of the very heavy ones. Altogether some 287 stable nuclides have been identified as occurring naturally.

It might seem that the more neutrons the nucleus has the more stable it would become, but this is not the case. In general a nucleus is stable only for certain ratios between the number of neutrons and the number of protons. In the case of light nuclei the ratio is one, whereas for heavier nuclei the ratio rises to about one and a half.

A plot of the numbers of neutrons versus the number of protons for the stable nuclides is shown in figure (2.1). Notice that for $Z < 20$, the stability line is a straight line with $Z = N$. For heavier nuclides $Z > 20$, $N > 20$, the stability curve bends in the direction of $N > Z$. In general, light nuclei ($A < 20$) contain approximately equal numbers of neutrons and protons, while in heavier nuclei the proportion of neutrons become progressively greater [5]. In other words the total stability of nucleus is measured in terms of the energy of the incident particle and binding energy of the particles making up the nucleus. A nucleus undergoing radioactive decay spontaneously emits a helium nucleus, an electron, or a photon, thereby either ridding itself of nuclear excitation energy or achieving a configuration that is or will lead to one greater stability. In addition,

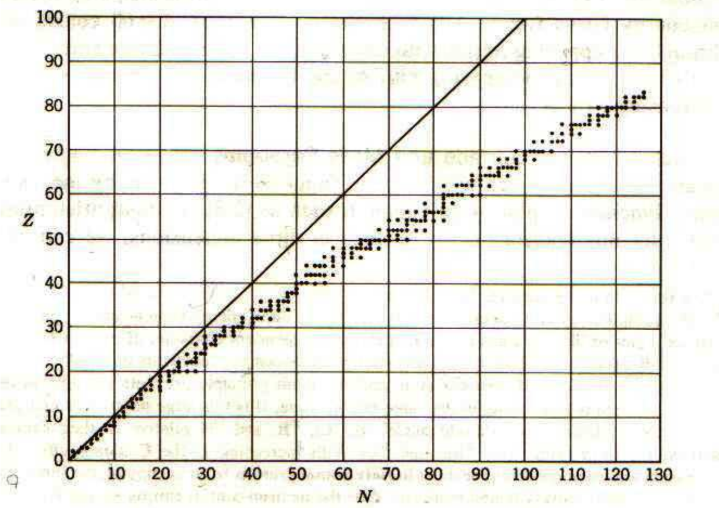

Figure 2.1: Stability curve for nuclides. The neutron number of each nuclide is plotted versus its proton number.

all nuclei with mass numbers greater than $A \gg 150$ are thermodynamically unstable against alpha emission($Q\alpha$), which is dominant decay process only for the heaviest nuclei, $A \approx 210$. As a result of physical and chemical research on the naturally occurring radioactive elements, it was proved that each radioactive nuclide is a member of one of three long chains, or radioactive series. These series are named the uranium, actinium, and thorium series respectively, after elements at, or near, the head of the series.

2.2 RADIOACTIVE SERIES

Nearly all the naturally occurring radioactive elements lie in the range of the atomic numbers, $Z = 81$ to $Z = 92$. These elements have been grouped into three genetically related series: the uranium series , the actinium series, and the thorium series [6]. The following three figures are used to explain the decay series for the actinium, the thorium and the uranium. The number located at the bottom of each box indicates the length of time—in years, days, minutes, or seconds—that a particular element (or isotope) in the decay series takes to lose half of its activity. At this point, it either transforms into a lower energy state of the same element, or transforms into a different element. Alpha and beta radiation given off during the decay series is indicated by the alpha and beta symbols appearing next to the lines between boxes. Many of these elements have two or more isotopes in these series. In the early work in radioactivity many of the isotopes were given names indicative of the manner of their discovery or their formation rather than those of the appropriate elements; for example, radium is the daughter formed in the alpha decay of radon, but it is not an isotope of radium. It is an isotope of the element polonium with $Z = 84$ and $A = 218$. Similarly thorium is the daughter of the uranium-238, but it is actually an isotope of thorium with $Z = 90$ and $A = 234$. Since these names appear in the extensive literature of the radioactivity, they are shown in figures (2.2, 2.3, 2.4); the appropriate elements are shown in top of the each figure. Wherever it is more convenient to retain the older name, the appropriate isotopic identification will be made. A long-lived isotope is at the head of each series and some stable isotope of lead ends each one. The uranium series originates with the uranium isotope $A = 238$ with half life of 4.51 billion years, as shown in Figure (2.3), and goes through a series of transformations that involves the emission of the alpha and beta particles, giving rise successively to radioactive isotopes of thorium, protoactinium, uranium, thorium again radium,.... down to lead ($A = 206, Z = 82$). In figure (2.3) the mass number A is plotted against the atomic number Z as abscissae. An emission of an alpha particle is indicated by a displacement down by four units and to the left two units; an emission of a beta particle is indicated by a displacement to the right by one unit. The thorium series starts with a long-lived isotope of the thorium ($A = 232$) with a half-life of 14.1 billion

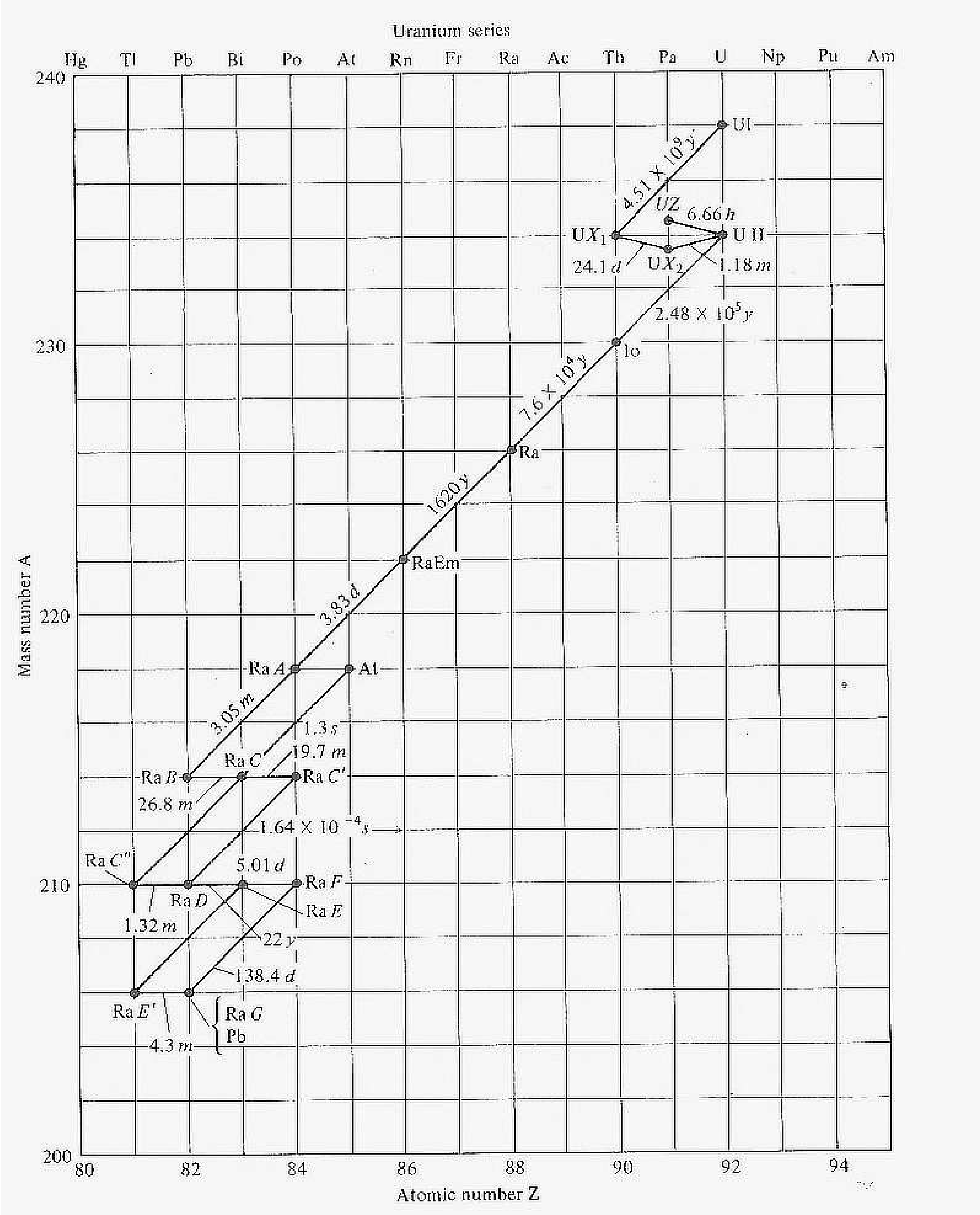

Figure 2.2: The uranium series.

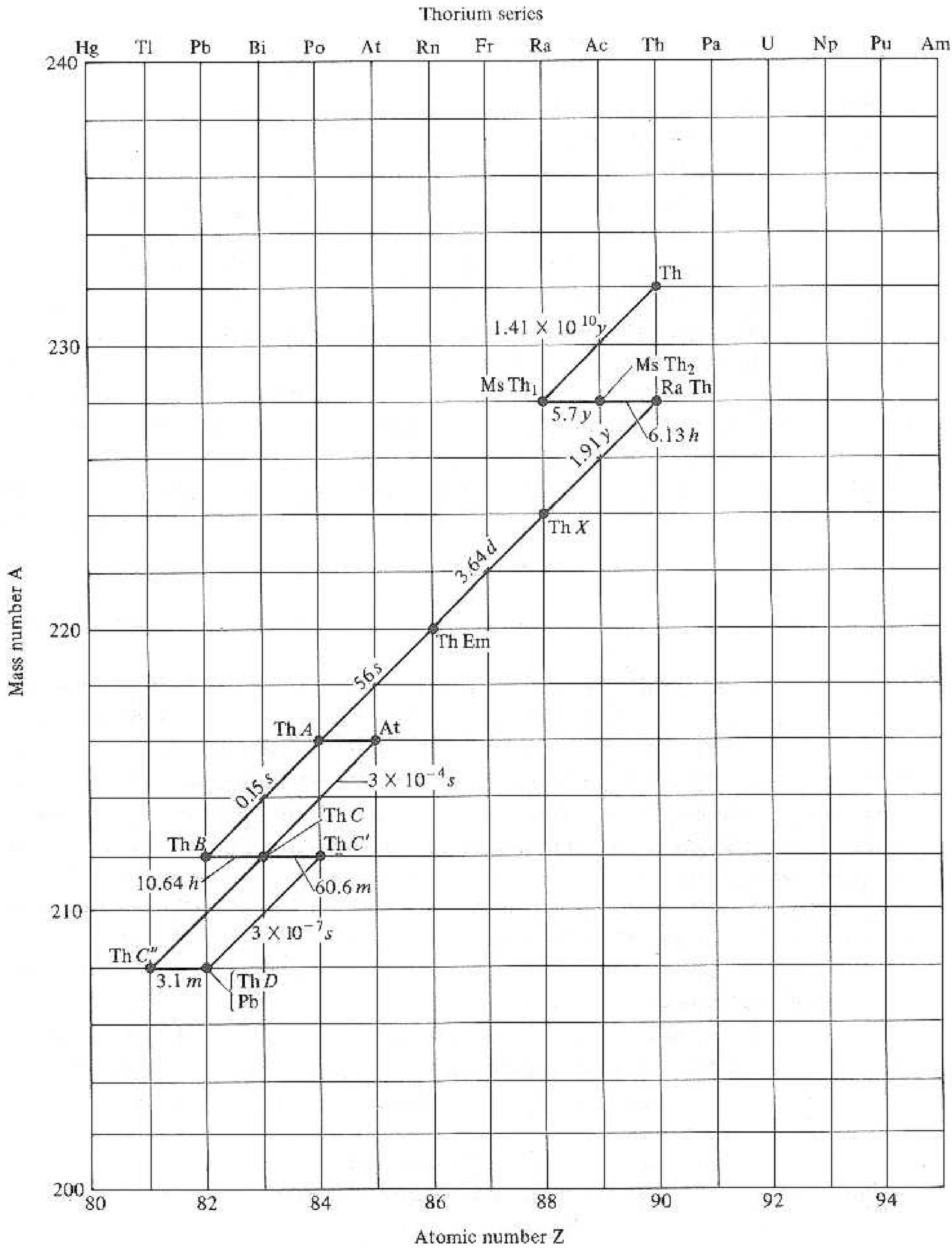

Figure 2.3: The thorium series.

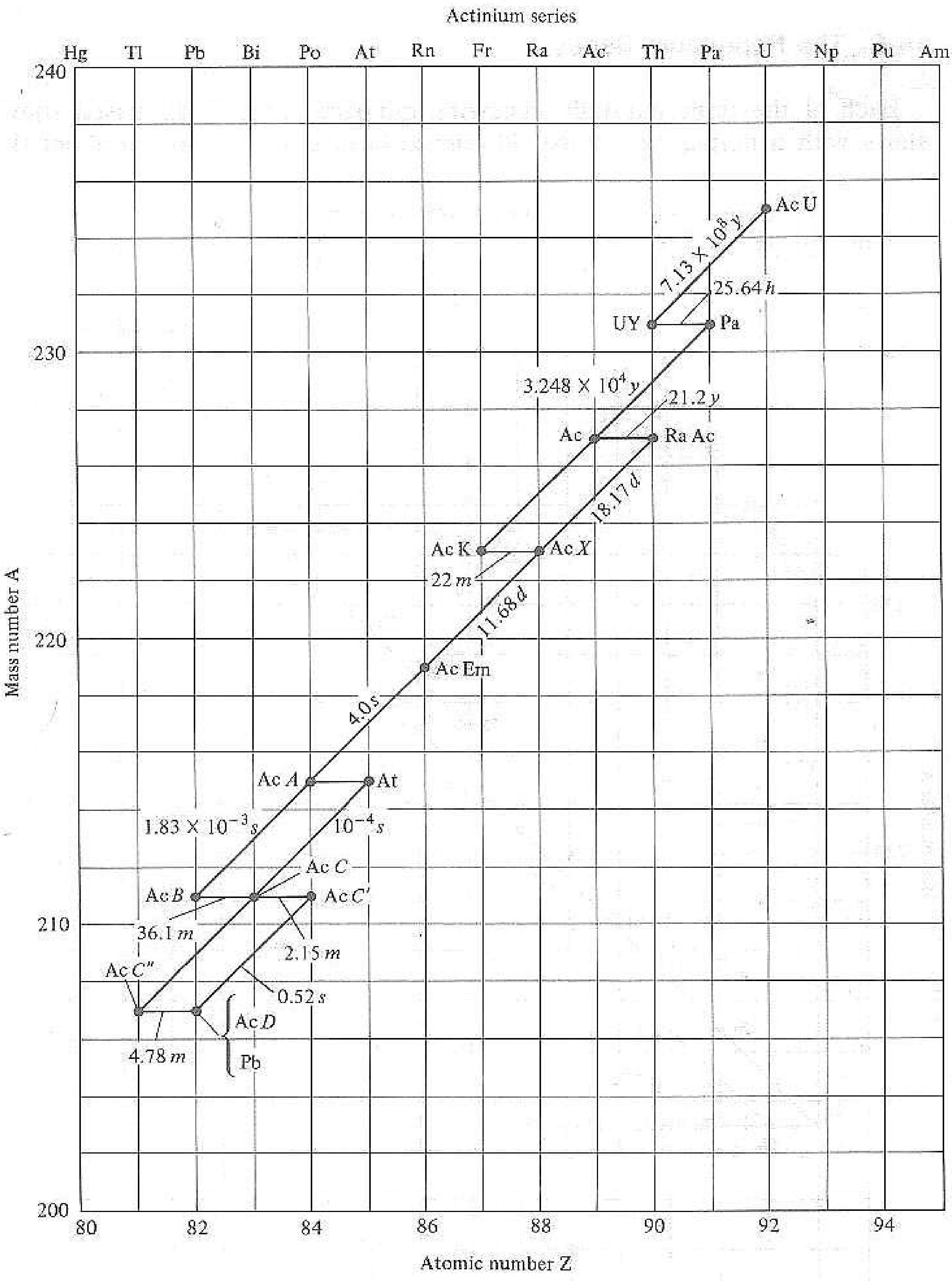

Figure 2.4: The actinium series.

years and goes through a series of alpha particle and beta particle decays similar in many respects to those of the uranium series and terminates with the isotope of lead of mass number $A = 208$.

The actinium series figure (2.4) was at one time believed to be an independent series, but its origin has been traced to the rarer isotope of uranium of mass number $A = 235$ and is sometimes called actino-uranium (AcU). The end product of the actinium series is an odd-numbered isotope of lead, $A = 207$.

2.2.1 The Neptunium Series

Each of the three naturally occurring radioactive series discussed above starts with a long-lived isotope. Physicists have often speculated about the possibility of the occurrence of other radioactive series, the isotopes of which may have disappeared or may be in such extremely small concentrations that they are undetectable by common methods. One type of speculation revolved around the fact that the mass numbers of the heads of the three known series could be represented by the following set of numbers: $4n$ (for the thorium series), $4n + 2$ (for the uranium series), and $4n + 3$ (for the actinium series), where n is an integer. It was felt that there might have been a $4n + 1$ series and that perhaps traces of it still exist.

With the transuranic elements -that is, elements of atomic number greater than 92 and with the ability to produce many different isotopes of both old and new elements - it was possible to trace a fourth radioactive series, a $4n + 1$ series. This series is called neptunium series, after the longest-lived isotope, neptunium, $Z = 93$, $A = 237$, of this series. It will be noted that the origin of this series can be traced back to americium and plutonium have been separated from pitchblende, a uranium-bearing more.

2.3 ALPHA DECAY

In series seminal experiments Ernest Rutherford and his collaborators established the important features of alpha decay [1]. The behavior of the radiations from natural sources of uranium and thorium and their daughters was studied in magnetic and electric fields. The least penetrating particles, labeled "$\alpha - rays$" because they were first absorbed, were found to be positively charged and quite massive in comparison to the more penetrating negatively charged "$\beta - rays$" and the most penetrating neutral "$\gamma - rays$". In subsequent experiment the alpha rays from a needle -like source were collected in a very small concentric discharged tube and the emission spectrum of helium was observed in the trapped volume. Thus, alpha rays were proven to be energetic helium nuclei. The alpha particles are the most ionizing radiation emitted by natural sources and are stopped by as a sheet of paper or a few centimeters of air. The particles are quite energetics.

Understanding these features of alpha decay allowed early researchers to use the emitted alpha particles to probe the structure of nuclei in scattering experiments and later, by reaction with beryllium to produce neutrons.

Alpha particles played an important role in nuclear physics before the invention of charged particle accelerators and were extensively used in research.

2.3.1 Theory of Alpha Decay

Alpha decay results for proton rich nuclei where a net reduction in mass energy occurs by the spontaneous emission of ^4He nucleus from the parent nucleus. This can be represented by the following process:

$$^{A}_{Z}P_N \longrightarrow ^{A-4}_{Z-4} D_{N-2} +^{4}_{2} He \tag{2.1}$$

where P stands for parent, D stands for daughter.

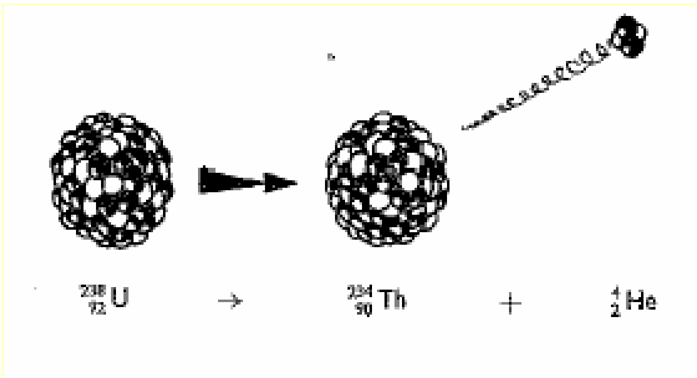

Figure 2.5: Decay scheme for uranium with alpha emitting.

The alpha particles, as was shown by Rutherford, is nucleus of ^4He, consisting of two neutrons and two protons.

Consider a nucleus of rest mass m_p which is at rest and undergoes alpha decay. Before the decay energy of the system is just the rest energy of the parent atom:

$$E_i = m_p c^2 \tag{2.2}$$

The final energy after the alpha particle is far from the daughter nucleus is just the sum of the rest energies and kinetic energies of both particles:

$$E_f = T_D + m_D c^2 + T_\alpha + m_\alpha c^2 \tag{2.3}$$

The disintegration occurs spontaneously, without any external forces and so the initial and final energies must be equal:

$$E_i = E_f \tag{2.4}$$

or

$$T_D + T_\alpha = (m_p + m_D - m_\alpha)\,c^2 \tag{2.5}$$

Kinetic energy can never be negative; hence, alpha decay cannot occur unless the mass of the parent nucleus is greater than the sum of masses of the two product nuclei; $m_p > m_D + m_\alpha$ [7]. In other words, alpha decay cannot occur unless the total rest mass decreases.

The energy converted from the mass energy to the kinetic energy is called the Q_{value} , also this energy is called disintegration energy.

$$Q = T_D + T_\alpha = (m_p + m_D - m_\alpha)\,c^2 \tag{2.6}$$

Q_{value} is equal to the decrease in rest energy or the increase in kinetic energy.

If the original nucleus P is at rest, then its linear momentum is zero, and conservation of linear momentum then requires that D and α move with equal and opposite momenta in order that the final momentum also be zero:

$$P_\alpha = P_D \tag{2.7}$$

Alpha decays typically release about 5 MeV of energy. Thus for both daughter and alpha , $T \ll mc^2$ and non-relativistic kinematics can be used. Writing $T = P^2/\ 2m$ and using (2.6) and (2.7) gives the kinetic energy of the alpha particle in terms of the Q_{value}:

$$T_\alpha = Q\left(\frac{A-4}{A}\right) \tag{2.8}$$

Typically, the alpha particle carries about 98% of the Q_{value}, with the much heavier nuclear fragment D carrying only about 2%. The kinetic energy T_α of the emitted alpha particle is never quite equal to the disintegration energy Q_{value} that recoils with a small amount of kinetic energy when the alpha particle emerges.

Such this alpha particle is in constant motion and is contained in the nucleus by surrounding potential well formed by nuclear and coulomb forces. Alpha decay can only occur if an a particle is permitted to penetrate the Coulomb barrier [8] around the outside of the nucleus. This barrier is a result of the electric energy of two charged bodies and outside the nucleus has the value

$$V = \frac{2(Z)e^2}{4\pi\varepsilon_0 r} \tag{2.9}$$

where Z is the atomic number of the daughter and r is the distance between the alpha and the center of the daughter nucleus [9]. The penetration of this barrier causes the exponentially decreasing half life as Q increases.

However Q alone does not determine whether or not we will observe an a decay. Since the decaying nucleus has a large Z, the coulomb barrier acts to suppress decay. The probability of the alpha getting out depends very critically on the energy of the alpha. This was found out by Geiger and Nuttall [10] in 1911 (the Geiger Nuttall law).

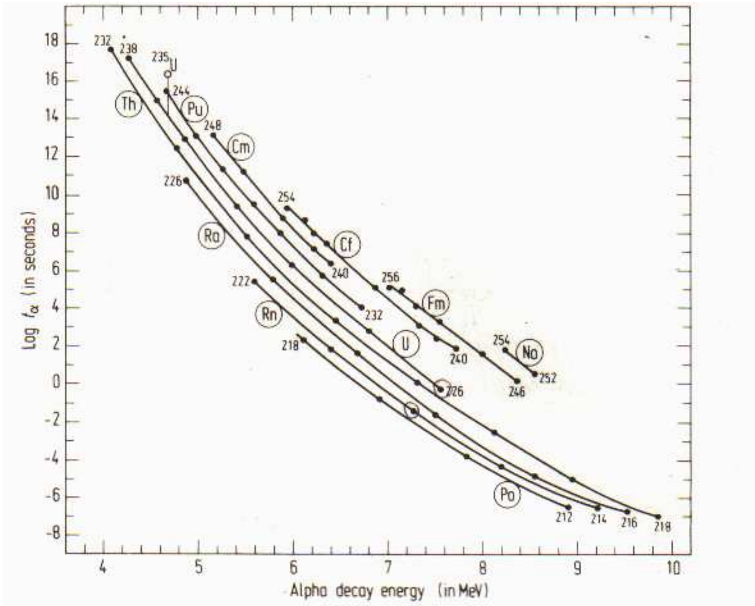

Figure 2.6: The inversely relationship between alpha decay half-life and decay energy, called the Geiger-Nuttall rule. Only even-Z, even-N nuclei are shown. Solid lines connect the data points.Note the Vertical scale is logarithmic, and ranges over some 25 orders of magnitude. Nuclei with the same Z are joined.

The energies of the emitted alpha particles can range from 1.8 MeV (^{144}Nd)to 11.6 MeV (^{212}Po), and most of them lie between 4 and 8 MeV. This relatively small range in energies is associated with an enormous range of half lives, from about $10^{-7}s$ to nearly $10^{16}y$, a factor of over 10^{30}. A quantitative relation between energy release and half life was recognized by Rutherford in 1906, and in 1911 Geiger and J. M. Nuttall formulated a quantitative relation between decay constant λ and range in air r:

$$\log \lambda = a + b \log r \tag{2.10}$$

A theoretical basis for understanding alpha decay was lacking until the advent of quantum mechanics. It was all the more gratifying that the basic quantum -mechanical theory, developed in 1928 independently by G. Gamow [11] and by

R. W. Gurney and E. U. Condon [12, 13], was successful in accounting for the relationship between half lives and energies.

While a heavy nucleus can, in principle, spontaneously reduce its bulk by alpha decay, there remains the problem of how an alpha particle can actually escape from the nucleus. Figure (2.7) is a plot of the potential energy V of an alpha particle as a function of its distance r from the center of a certain heavy nucleus. The height of the potential barrier is about 25 MeV, which is equal to the work that must be done against the repulsive electrostatic force to bring an alpha particle from infinity to a position adjacent to the nucleus but just outside the range of its attractive forces. Therefore, it may be regarded an alpha particle in such a nucleus as being inside a box whose walls require an energy of 25 MeV to be surmounted. However; alpha particles have energies that range from 4 to 9 MeV, depending upon the particular nuclide involved 16 to 21 MeV short of the energy needed for escape.

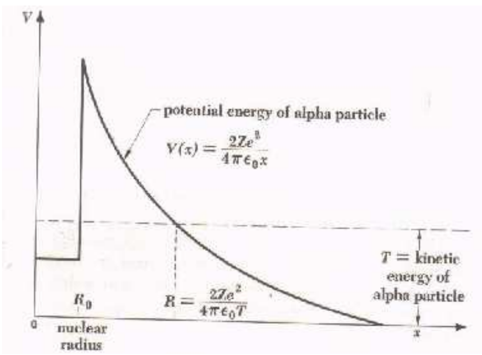

Figure 2.7: The potential energy of an alpha particle as a function of its distance from the center of a nucleus.

While alpha decay is inexplicable on the basis of the classical arguments, quantum mechanics provides a straightforward explanation. The Gamow-Gurney theory explains the alpha decay: the basic notations of this theory are:

(1) An alpha particle may exist as an entity within a heavy nucleus.

(2) Such a particle is in constant motion and is contained in the nucleus by the surrounding potential barrier.

(3) There is a small -but definite- likelihood that the particle may pass through the barrier (despite its height) each time a collision with it occurs.

Thus the decay probability per unit time λ can be expressed as:

$$\lambda = f * P \tag{2.11}$$

where f is the number of times per second an alpha particle within a nucleus strikes the potential barrier around it and P is the probability that the particle will be transmitted through the barrier. The frequency of alpha particle can be found by examining its movement in a nucleus also it is supposed that at any moment only one alpha particle exists as such in a nucleus and that it moves back and forth along a nuclear diameter.

$$f = \frac{v}{2R} \tag{2.12}$$

where v is the alpha-particle velocity when it eventually leaves the nucleus and R is the nuclear radius.

Since $V > E$, classical physics predicts a transmission probability P is zero. In quantum mechanics a moving alpha particle is regarded as a wave, and the results is a small but definite value for P. The probability density that a particle will be found is equal to the square of the absolute value of the wave function.

$$P = \Psi^2 \tag{2.13}$$

Hence the half life [14] of any alpha emitting isotopes can be easily evaluated by:

$$T_{\frac{1}{2}} = \frac{\ln 2}{\lambda} \tag{2.14}$$

2.3.2 Tunnelling Probability

In this section the tunnelling probability of an alpha particle will be discussed because the alpha decay of atomic nuclei could be explained by a strange phenomenon called tunnel effect. It will be discussed before that in classical mechanics, in regions where the total energy less than the potential energy are inaccessible: Therefore, a particle would need to have negative kinetic energy, hence a purely imaginary momentum, which does not make any sense. In wave mechanics [15], it amounts to replacing a purely oscillating wave function. Therefore, in quantum mechanics the wave function will be different from zero even

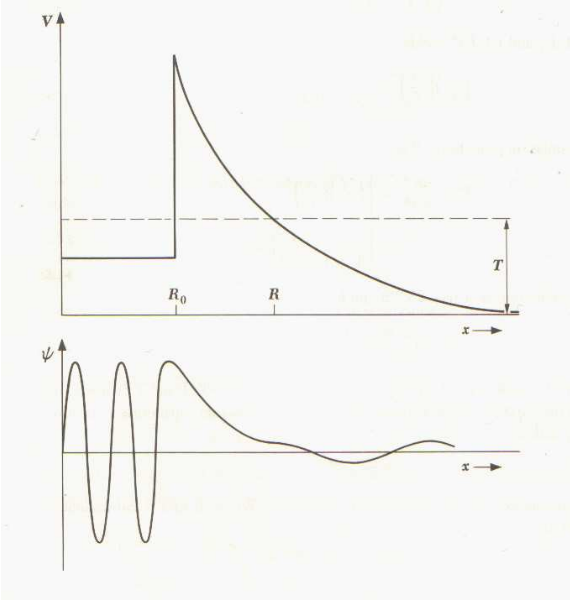

Figure 2.8: Alpha decay from point view of wave mechanics.

in classically forbidden regions. This leads to the possibility that an alpha may "tunnel" through a potential barrier which classically is completely insurmountable. In figure (2.8) the wave function behavior of an alpha decay is shown. Hence, the wave function of an alpha particle which penetrates into the barrier and escapes from the parent nucleus, can determined by the solution of the Schrödinger equation.

2.3.3 Schrödinger equation

The Schrödinger equation is nowadays the fundamental equation of quantum mechanics which governs the microscopic world. It is just contrasted with Newton's equation of motion which is the fundamental equation in the classical mechanics. By solving the Schrödinger equation, all the results from Bohr's quantum theory have completely been derived, and the mysteries in the microscopic world have been resolved one after another. We treat the motion of a free particle on which no force is exerted at all. The free-particle motion is the simplest one moving with a constant velocity. The substance particles are in the microscopic world combine the particle nature and the wave nature. Then, how can this double nature of the free particles be described (or formulated)? The momentum of a particle of mass m moving at a speed v is $p = mv$. The energy of this particle is:

$$E = \frac{1}{2}mv^2 \tag{2.15}$$

The relation between E and p is, of course,

$$E = \frac{1}{2m}p^2 \tag{2.16}$$

On one hand, these quantities, the momentum p and the energy E, representing the motion as a particle characterize the particle nature. On the other hand, the wavelength and the frequency of the de Broglie wave characterize the wave nature of the particle motion. The relations between these quantities characterizing the double nature are just Einstein-de Broglie's relations:

$$p = \frac{h}{\lambda} \tag{2.17}$$

and

$$E = hf \tag{2.18}$$

The function that represents the de Broglie wave is expressed as $\Psi(x,t)$ which is called the wave function. Taking into an account of the double nature mentioned above, Schrödinger proposed a wave equation which the wave function $\Psi(x,t)$ for a free particle must obey; i.e. The Schroedinger equation for a free particle in the one-dimensional space is:

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} \tag{2.19}$$

The simplest solution of the Schrödinger equation (2.19) is given by

$$\Psi(x,t) = \cos\left(\frac{p}{\hbar}x - \frac{E}{\hbar}t\right) + i\sin\left(\frac{p}{\hbar}x - \frac{E}{\hbar}t\right) \tag{2.20}$$

$$\Psi(x,t) = e^{i(px-Et)/\hbar} \tag{2.21}$$

This is the wave function of a free particle. A solution of the Schrödinger equation, i.e. a wave function, is in general a complex function. The wave function shown in (2.21) is a wave going to the positive direction on x axis with energy E. Let us consider a potential barrier as shown in figure (2.9).

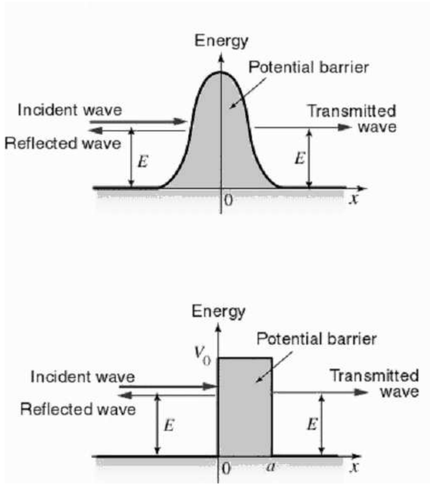

Figure 2.9: Potential barrier.

Suppose that an incident particle with energy E coming from the left collides the potential barrier. The energy E is assumed to be lower than the top of the barrier. In the classical mechanics, the particle is completely reflected by the potential barrier. In quantum mechanics, some will be reflected, but the other penetrates the barrier and pass through into the right-hand side region to proceed to the far right. This is the tunnel effect.

Tunnel Effect

Let us calculate an example of tunnel effect. Suppose a potential barrier with a width sign as a and a height V_0 as shown in figure (2.9). Let the region on the left -hand side of the barrier be A, the right-hand side be C, and the intermediate region be B. If the **incident wave** with an energy $E(< V_0)$ comes from the distant left and collides the barrier in the region A, a part of the wave is reflected to be the **reflected wave**, and another part penetrates the barrier to be the **transmitted wave** in the region C.

Since the waves treated here are of the constant E, the state is a **stationary state**. Hence the wave function is generally written:

$$\Psi(x,t) = e^{-iEt/\hbar} \tag{2.22}$$

The Schrödinger equation which the **spatial** wave function $\Psi(x)$ obeys is:

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V(x)\Psi = E\Psi \tag{2.23}$$

In the regions A and C, motions is of a free particle, because there is no potential, i.e. $V(x) = 0$. In these regions, the Schrödinger equation becomes

$$\frac{d^2\Psi}{dx^2} + k^2\Psi = 0 \tag{2.24}$$

where

$$k = \frac{\sqrt{2mE}}{\hbar} \tag{2.25}$$

It is easily seen that $\Psi(x) = e^{ikx}$ and $\Psi(x) = e^{-ikx}$ are solutions of the Schrödinger equation which is shown in (2.24). Writing $p = \hbar k$, we have the full wave function as $\Psi_1(x,t) = e^{i(px-Et)/\hbar}$ for the case of $\Psi(x) = e^{ikx}$, and $\Psi_2(x,t) = e^{i(-px-Et)/\hbar}$ for the case $\Psi(x) = e^{-ikx}$. The wave function $\Psi_1(x,t)$ denotes a right-going wave and $\Psi_2(x,t)$ a left-going wave.

In the region A, the total wave function must be a superposition of a right-going incident wave and a left-going reflected wave, i.e.,

$$\Psi_A(x) = Ae^{ikx} + Be^{-ikx} \tag{2.26}$$

where A and B are appropriate constants and k is shown in equation (2.25).

In the region C, there is only the transmitted wave going to the right, so that the wave function in this region is

$$\Psi_C(x) = Ce^{ikx} \tag{2.27}$$

where C is an appreciate constant and k is as shown as in equation (2.25).

In region B where there is potential barrier $V(x) = V_0\,(> E)$, the Schrödinger equation (2.23) becomes

$$\frac{d^2\Psi}{dx^2} + K^2\Psi = 0 \tag{2.28}$$

where

$$K = \frac{\sqrt{2m(V_0 - E)}}{\hbar} \tag{2.29}$$

It is easily understood that $\Psi(x) = e^{Kx}$ and $\Psi(x) = e^{-Kx}$ are special solutions of equation (2.28). Therefore, the general solution of equation (2.28) is a superposition of these functions as

$$\Psi_B(x) = Fe^{Kx} + Ge^{-Kx} \tag{2.30}$$

where F and G are appreciate constants.

2.3.4 The Approximation Methods

A fair amount of work was done to calculate the probability of tunnelling through a simple square barrier-unfortunately, reality is seldom so obliging and are faced with solving problems with a not so simple potential barrier. One way of approaching such problems is to adopt an approximation. There are many approximation methods for calculating the probability of tunnelling through a simple square barrier. The most used ones are Bohr-Sommerfeld quantization condition and Wentzel-Kramers-Brillouin (WKB) approximation.

Wentzel-Kramers-Brillouin Approximation

If we look at the probability of transmission through a square barrier we can see that it is:

$$\begin{aligned}\left|\frac{T}{I}\right|^2 &= \left[1+\frac{1}{4}\frac{V^2}{E\left(V-E\right)}\sinh^2\left(\frac{a\sqrt{2m\left(V-E\right)}}{\hbar}\right)\right]^{-1} \\ &\approx \frac{16E\left(V-E\right)}{V^2}\exp\left(-\frac{2a\sqrt{2m\left(V-E\right)}}{\hbar}\right) \qquad (2.31)\end{aligned}$$

Provided that

$$\frac{a\sqrt{2m\left(V-E\right)}}{\hbar} \gg 1 \qquad (2.32)$$

Since $\sinh x \approx \frac{e^x}{2}$ for $x \gg 1$. For many problems , the expression is dominated by the exponential term since the $2E(V-E)/V^2$ "is of the order of 1" so that the probability of transmission "is of the order of $P \approx \exp\left(-\frac{2a\sqrt{2m(V-E)}}{\hbar}\right)$".

Now we have a simple approximate expression for the probability of transmission across a square barrier. Any barrier of any shape may be approximated by a sequence of simple square barriers just illustrated in figure (2.10). In this

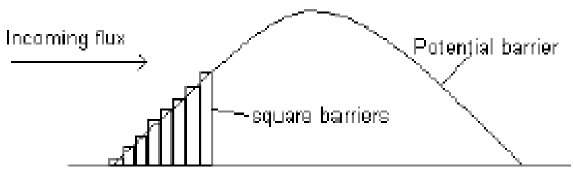

Figure 2.10: Breaking the potential barrier into the smaller square barriers.

way we can treat the probability of a particle passing over the barrier as the product of all the probabilities of passing over each of the smaller square barriers:

$$\begin{aligned}P &\approx \exp\left(-\frac{2\triangle x\sqrt{2m\left(V_1-E\right)}}{\hbar}\right)\exp\left(-\frac{2\triangle x\sqrt{2m\left(V_1-E\right)}}{\hbar}\right)\ldots\ldots\ldots \\ &= \exp\left(-2\triangle x\sum\frac{\sqrt{2m\left(V_i-E\right)}}{\hbar}\right) \qquad (2.33)\end{aligned}$$

Or, going to the limit of a continuous integration:

$$P \approx \exp\left(-2\int\frac{\sqrt{2m\left(V_{(x)}-E\right)}}{\hbar}dx\right) \qquad (2.34)$$

which is the WKB approximation [16].

Alpha Particle Decay

One of the most famous illustrations of the use of the WKB approximation is of the prediction of alpha particle decay lifetimes of atoms. Alpha particles can be thought of as "rattling" around inside the nucleus of an atom, held in by the nuclear force. If only they could overcome this potential barrier for a short while, the atom would decay [16]. Below is an illustration of the form of potential the alpha particle sees - the almost constant attractive potential well associated with the strong nuclear force up to a radius R, and then the long range electrostatic force out to infinity.

The probability that the alpha particle will tunnel its way through this barrier each time it bumps against it will therefore be:

$$P = \exp\left(-2\left(\frac{2m}{\hbar^2}\right)^{1/2}\int_R^b dr\left[\frac{Z_1 Z_\alpha e^2}{4\pi\varepsilon_0 r} - E\right]^{1/2}\right) \tag{2.35}$$

By noting that E, in terms of b is $E = \frac{Z_1 Z_\alpha e^2}{4\pi\varepsilon_0 b}$ the integrand can be rewritten as:

$$-2\left(\frac{2m}{\hbar^2}\frac{Z_1 Z_\alpha e^2}{4\pi\varepsilon_0}\right)^{1/2}\int_R^b dr\left[\frac{1}{r} - \frac{1}{b}\right]^{1/2} \tag{2.36}$$

The integral is difficult to do but can be found in a table of standard forms and is:

$$\int_R^b dr\left[\frac{1}{r} - \frac{1}{b}\right]^{1/2} = \sqrt{b}\left[\arccos\left(\frac{R}{b}\right)^{1/2} - \left(\frac{R}{b} - \frac{R^2}{b^2}\right)^{1/2}\right] \approx \frac{\pi}{2} \tag{2.37}$$

for $R \ll b$

So that the probability of escape will be (using $E = \frac{mV^2}{2}$):

$$p = \exp\left(-\frac{2\pi\pi}{\hbar v}\frac{Z_1 Z_\alpha e^2}{4\pi\varepsilon_0}\right) \tag{2.38}$$

The time between encounters with the barrier will be about 2R/v, so there will be v/2R encounters each second - giving a probability of decay each second of:

$$\lambda \approx \frac{v}{2R}\exp\left(-\frac{2\pi\pi}{\hbar v}\frac{Z_1 Z_\alpha e^2}{4\pi\varepsilon_0}\right) \tag{2.39}$$

Now the half-life of any nuclide which decays alpha can be found. In this study WKB approximation is used for predicting alpha half-life with a small change. The integration part in the equation (2.36) is calculated analytically by a computing program.

The Bohr Sommerfeld Quantization Condition

During the years 1913-1918 Bohr developed a quantum mechanical model for the electronic states in a hydrogen atom. This model supposes that the electron is described by Newton's laws of classical mechanics and a quantum condition. Bohr specifically postulated that an atomic system can only exist in a certain series of electronic states corresponding to a series of discrete values for its energy, and that consequently any change in energy of system, including the emission and absorption of photons of photons, must take place by a complete transition of the electron between two such states. These states are called as the stationary electron states of the system. Bohr further postulated that the radiation absorbed or emitted during a transition between two states possesses an angular frequency ω, given by the relation

$$E_m - E_n = \hbar\omega \tag{2.40}$$

where $\hbar = h/2\pi$ is reduced Planck constant and E_m and E_n are the energies of the two states (the mth and the nth state) under consideration. The quantum

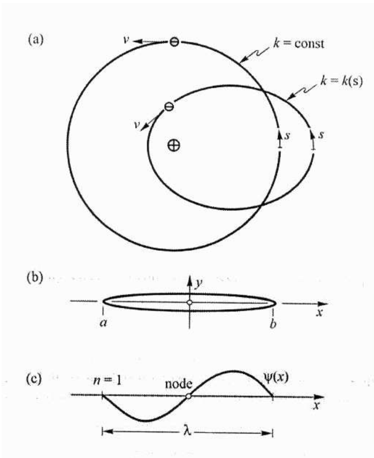

Figure 2.11: Orbiting of electrons around the positively charged nucleus on a circular or elliptical curve. (a) The motion of electrons in Bohr's atom model is fully described by (i) classical laws and (ii) the Bohr-Sommerfeld quantum condition. (b) The one-dimensional Bohr-Sommerfeld quantum condition can be obtained from an ellipsoid compressed onto the x axis. (c) Illustration of quantum state n=1, where the number n is the number of nodes of the wave function.

condition of Bohr can be visualized most easily in terms of the electron de Broglie wave orbiting the nucleus. The electron is moving in a circular orbit of radius r. The electrostatic potential of the nucleus has symmetry and the electron is consequently moving with a constant velocity about the nucleus. Electronic orbitals are allowed, only if the circumference is an integer multiple of the electron de Broglie wavelength

$$S = (n+1)\lambda \tag{2.41}$$

where n is integer ($n = 0, 1, 2, ...$) and S is the circumference of the electron orbit.

Only circular orbits have been considered in the equation (2.41) because the electron is assumed to move in a constant potential with constant momentum $p = h/\lambda$. However, the laws of classical mechanics also allow elliptical orbits. The nucleus is in one of the focal points of the ellipse as shown in figure (2.11). In such elliptical orbits the momentum is a function of the position. It is therefore necessary to generalize the quantum condition of equation (2.41) in order to make it applicable to orbits other than circular orbits.

A generalization of the quantum condition is obtained by first rearranging equation (2.41) and employing the wavenumber $k = 2\pi/\lambda$ according to

$$\frac{1}{2\pi} kS = n + 1 \tag{2.42}$$

where n=0,1,2,3,...

Because k depends on the position for the elliptical orbits an integration rather than a product must be employed

$${}_S k(s)ds = 2\pi(n+1) \tag{2.43}$$

where n=0,1,2,3,...

The integral is a closed line integral along the electron orbit S. Using the de Broglie relation $p = \hbar k$ one obtains

$${}_S p(s)ds = 2\pi\hbar(n+1) \tag{2.44}$$

which is known as the **Bohr-Sommerfeld quantization condition.**

Most wave function are oscillating functions. Oscillating functions have locations of zero amplitude, i.e. *nodes.* Assume, for example, $n = 1$. The corresponding wave function has one node. Thus the wave function with the quantum number n has n nodes. The quantum number is identical with the number of nodes of that wave function.

In the following chapter the simple cluster model for alpha decay is identified by using Bohr-Sommerfeld quantization condition. The parent nuclei is described as a core of nucleus and the daughter nuclei (alpha particle) is also assumed to move in an orbit around the daughter nuclei. The number nodes are identified then the orbit is specified by the large value of the global quantum number. The quantum number is used to calculation of the radius of the nucleus.

The wave numbers are explained by using this approximation method. Hence all of these parameters are also used to calculation of alpha half lives of radioisotopes and the details of this method is explained in the following chapter.

Chapter 3

COMPARISONS OF METHODS

In this chapter the different estimation methods of alpha half-life is discussed and the classification of these methods is done. The systematic variation of alpha decay half-lives with decay energy can be expressed in variety of ways. In this thesis three methods of estimation of alpha half-life are examined. These methods are discussed briefly in the following section.

3.1 FIRST METHOD

The relationship between energies and half-lives is recognized by Gamow [11], Gurney [12], and Condon [13] and the details of this theory was explained in previous chapter. In outline the theory takes the following form. The Schrödinger wave equation for an alpha particle of energy E inside the nuclear potential well [21] is set up and solved. The wave function representing the alpha particle does not go abruptly to zero at the wall of the potential barrier R_1 which is shown in figure (3.1) and has finite, although small, values outside the radial distance R_1.

$$R_1 = \left(1.3A^{1/3} + 1.2\right) \tag{3.1}$$

By applying the boundary condition that the wave function and its derivative must be continuous at R_1 and R_2, the wave equation can be solved for the region between R_1 and R_2, that is, inside the barrier where the potential $U(r)$ is greater than the total kinetic energy T (sum of kinetic energies of alpha particle and recoil nucleus).

$$T = \frac{2\,(Z-2)\,ke^2}{R_2} \tag{3.2}$$

The probability P for the alpha particle of mass M_α to penetrate that region, the so-called barrier penetrability factor, is given by the square of the wave function and turns out to be

$$P = \exp\left(-\frac{4\pi}{h}\sqrt{2\mu}\int_{R_1}^{R_2}\sqrt{U(r)-T dr}\right) \tag{3.3}$$

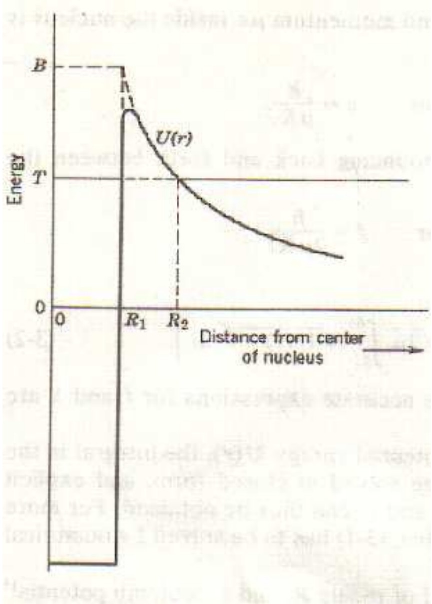

Figure 3.1: Potential energy for a nucleus-alpha particle system.

where

$$\mu = \frac{m_\alpha * M_R}{m_\alpha + M_R} \tag{3.4}$$

is the reduced mass of alpha particle and recoil nucleus. It is clear that the probability for the barrier penetration decreases with increasing the value of the integral in the exponent, that is, with increasing barrier height and width.

The decay constant λ may be considered as the product of P and the frequency f with which an α particle strikes the potential barrier; the order of magnitude of f may be estimated as follows. The de Broglie wavelength $h/\mu v$ of the particle of the velocity v and the momentum μv inside the nucleus is taken comparable to R_1, thus

$$\frac{h}{\mu v} \approx R_1 \tag{3.5}$$

or

$$v \approx \frac{h}{\mu R_1} \tag{3.6}$$

If the α particle is considered as bouncing back and forth between the potential walls,

$$f = \frac{v}{2R_1}$$

or

$$f \approx \frac{h}{2\mu R_1^2} \tag{3.7}$$

Therefore the decay constant is

$$\lambda \approx \frac{h}{2\mu R_1^2} \exp\left(-\frac{4\pi}{h}\sqrt{2\mu}\int_{R_1}^{R_2} \sqrt{U(r) - T} dr\right) \tag{3.8}$$

For a square-well nuclear potential of radius R_1 and Coulomb potential $U(r) = Zze^2/r$ for $r > R_1$ (the heavy dashed line in figure (3.1), the integral in equations (3.3) and (3.8) becomes

$$Int. = \int_{R_1}^{R_2} (Zze^2 - Tr)^{1/2} \frac{dr}{r^{1/2}} \tag{3.9}$$

which by the substitutions $x = r^{1/2}$ and $a^2 = Zze^2/T$, turns into the readily integrable form $2\sqrt{T}\int_{R_1}^{R_2} \sqrt{a^2 - x^2}dx$, with the solution

$$Int. = \sqrt{T}\left[x\left(a^2 - x^2\right)^{1/2} + a^2 \arcsin\frac{x}{a}\right]_{\sqrt{R_1}}^{\sqrt{R_2}} \tag{3.10}$$

Values of the radii R_1 and R_2 are obtained from the expressions for the total kinetic energy T and the barrier height B (see in figure (3.1)):

$$T = \frac{Zze^2}{R_2} \tag{3.11}$$

and

$$B = \frac{Zze^2}{R_1} \tag{3.12}$$

After substitution of the integration limits and some algebraic manipulations equation (3.10) becomes

$$Int = \frac{Zze^2}{\sqrt{T}}\left[\arccos\left(\frac{T}{B}\right)^{1/2} - \left(\frac{T}{B}\right)^{1/2}\left(1 - \frac{T}{B}\right)^{1/2}\right] \tag{3.13}$$

Finally, remembering that $T = \frac{1}{2}\mu v^2$, substitution of equation (3.13) in equation (3.8) gives

$$\lambda \approx \frac{h}{2\mu R_1^2}\exp\left\{-\frac{8\pi Zze^2}{hv}\left[\arccos\left(\frac{T}{B}\right)^{1/2} - \left(\frac{T}{B}\right)^{1/2}\left(1 - \frac{T}{B}\right)^{1/2}\right]\right\} \tag{3.14}$$

In this method, the WKB approximation discussed in the previous chapter is used.

3.2 SECOND METHOD

Figure (3.2) shows a plot of the potential energy between the alpha particle and the residual nucleus for various distances between their centers. The horizontal line Q is the disintegration energy. The Coulomb potential [21] is extended inward to a radius a and then arbitrarily cut off. The radius a can be taken as the sum of the radius of residual nucleus and of the alpha particle. There are three regions of interest. In the spherical region $r < a$ we are inside the nucleus and speak of the potential well depth $-V_0$, where V_0 is taken as a positive number. Classically the alpha particle can move in this region, with a kinetic energy $Q+V_0$ but it can escape from it. The annular-shell region $a < r < b$ forms a potential barrier because here the potential energy is more than the total available energy Q. Classically the alpha particle cannot enter this region from either direction; in each case the kinetic energy would have to be negative. The region $r > b$ is a classically permitted region outside the barrier. From the classical point of view,

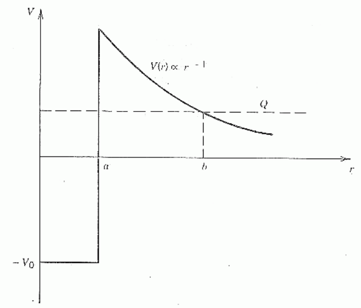

Figure 3.2: Relative potential energy of alpha particle, daughter-nucleus system as a function of their separation. Inside the nuclear surface at r=a, the potential is represented as a square well; beyond the surface, only coulomb repulsion operates. The alpha particle tunnels through the coulomb barrier from a to b.

an α particle in the spherical potential well would sharply reverse its motion every time it tried to pass beyond r=a. Quantum mechanically, however, there is a chance of leakage or tunnelling through such a barrier. This barrier accounts for the fact that α-unstable nuclei do not decay immediately. The alpha particle within the nucleus present itself again and again at the barrier surface finally penetrates.

The barrier also operates in reverse, in the case of the alpha particle scattering by nuclei. Alpha particles incident on the barrier from the barrier from outside the nucleus usually scatter in the Coulomb field if the incident energy is well below the barrier height. Tunnelling through the barrier, so that the nuclear force between the particle and target can cause nuclear reactions, is a relatively improbable process at low energy. The theoretical analysis of the nuclear reactions induced by charged particle uses a formalism similar to that of the alpha decay to calculate the barrier penetration probability.

The disintegration constant of an alpha emitter is given by

$$\lambda = fP \tag{3.15}$$

where f is the frequency with which the alpha particle presents itself at the barrier and P is the probability of transmission through the barrier.

The Coulomb barrier of figure (3.2) has height B at r=a, where

$$B = \frac{1}{4\pi\epsilon_0}\frac{zZ'e^2}{a} \tag{3.16}$$

In this expression the alpha particle has charge ze and the daughter nucleus, which provides the Coulomb repulsion, has charge $Z'e = (Z - z)\,e$.

The radius b at which the alpha particle 'leaves' the barrier is found from the equality of the particle's energy and the potential energy:

$$b = \frac{1}{4\pi\epsilon_0}\frac{zZ'e^2}{Q} \tag{3.17}$$

The Coulomb barrier can be thought as made up of sequence of infinitesimal rectangular barriers of height $V(r) = zZ'e^2/4\pi\epsilon_0 r$ and width dr. The probability to penetrate each infinitesimal barrier, which extends from r to $r + dr$, is

$$dP = \exp\left\{-2dr\sqrt{(2m/h^2)\,[V\,(r) - Q]}\right\} \tag{3.18}$$

The probability to penetrate the complete barrier is

$$P = e^{-2G} \tag{3.19}$$

where the Gamov factor G is

$$G = \sqrt{\frac{2m}{h^2}}\int_b^a [V\,(r) - Q]^{1/2}\,dr \tag{3.20}$$

which can be evaluated as

$$G = \sqrt{\frac{2m}{h^2}}\frac{zZ'e^2}{4\pi\epsilon_0}\left[\arccos\sqrt{x} - \sqrt{x\,(x-1)}\right] \tag{3.21}$$

where

$$x = \frac{a}{b} = \frac{Q}{B} \tag{3.22}$$

The quantity in brackets in equation (3.21) is approximately $\pi/2 - 2x^{1/2}$ when $x << 1$, as is the case for most decays of interest.

Thus the result of the quantum mechanical calculation for the half-life of alpha decay is

$$t_{1/2} = \frac{\ln 2}{\lambda} \tag{3.23}$$

3.3 SIMPLE CLUSTER MODEL FOR ALPHA DECAY

This method [17, 22, 23] is used for examining on partial half lives for alpha decay of heavy even-even nuclei. In this method Bohr Sommerfeld Quantization approximation is used. The traditional form of Geiger-Nuttall plots for various isotopic sequences is presented. A phenomenological connection between the half lives and Q-values of the alpha decays of radioactive series was noted by Geiger and Nuttall who obtained linear correlations of the form

$$\log_{10} T_{1/2} = aQ^{-1/2} + b \tag{3.24}$$

These relationships have often proved more effective than more microscopically based calculations in the prediction of alpha decay half lives. Their application to the decays of isotopic sequences of the heaviest elements with neutron number $N > 126$ has long been known to yield spectacular straight line plots.

The recent accumulation of accurate decay data has allowed the linear relation to be tested among the isotopes of lighter elements. It is found to be accurately followed in the lightest alpha emitters, such as the series of platinum isotopes, which all have $N < 126$. Here the data separate into two distinct groups with $N > 126$ and $N < 126$, each of which obeys but with different values for the coefficients.

Buck et al. proposed in a simple cluster model [22, 23] which would give rise to a natural explanation of these phenomena. The essence of the model is that it describes the parent nucleus as a core with a preformed alpha particle in orbit around it. The orbit is specified by a large value of the global quantum number $G = 2n + L$, where n is the number of nodes in the wave function and L is the orbital angular momentum (necessarily zero for transitions between the ground states of even-even nuclei). The value of G should be chosen so that the constituent nucleons of the alpha particle occupy states immediately above the Fermi surface of the daughter (core) nucleus. In this way G remains constant while a major neutron shell is being filled, (i.e. N = 52-82, 84-126 and 128-184 in the parent nuclei), but is forced to increase abruptly as the shell closure is crossed. Their model thus generates two linear relations for isotopic sequences which straddle $N = 126$, and predicts that the half lives for nuclei having $N < 126$ should be better reproduced using a smaller value of G than that, appropriate for nuclei having $N > 126$.

Buck et al. want to identify the minimum physical requirements essential for a good description of the large body of data pertaining to the alpha decays of heavy even-even nuclei. To this end they analyze these data with a very simple

form of the cluster model containing only three adjustable parameters. They treat the alpha cluster and core as distinct entities whose interaction may be modeled by a potential of the square-well+(surface-charge) Coulomb form:
when $(r < R)$

$$V = -V_N + \frac{C}{R} \tag{3.25}$$

and when $(r > R)$

$$V = \frac{C}{r} \tag{3.26}$$

where V_N is the depth of the nuclear potential, which acts out to some distance R (to be determined later) and $C = 2(Z-2)e^2$ is the product of charges.

This very simple parametrization has been chosen because it introduces the smallest number of free parameters. It is not suggesting that the alpha-core potential actually has the form of (3.25) and (3.26) and, ideally, it would be preferred to use a series of unique optical potentials derived from alpha-nucleus scattering experiments. However, no such optical potentials are presently available. Indeed, alpha-nucleus scattering experiments are essentially impossible for many of the nuclei of interest here because of the short lifetimes and/or extremely low isotopic abundances of the proposed targets. To use extrapolations of those alpha-nucleus optical potentials which are currently available would only obscure the physics. With typical realistic potentials, employing complicated radial geometries, surface and volume imaginary terms and energy dependent strengths it would become hopelessly enmeshed in a morass of parameter ambiguities. Even with only three adjustable parameters it is observed strong correlations in their values and are unable to identify a unique set of best values.

The surface charge Coulomb potential is also a simplification. However, it is not exceptionally unphysical, and it helps to lay bare the minimum physics really relevant to the calculation of alpha decay half lives. It is therefore proceeded with (3.25) and (3.26), and note that the considerable success achieved acts as a useful yardstick against which to judge more realistic potentials.

It has traditionally been found convenient to use some recipe to fix the radius parameter (and diffuseness when appropriate) of the nuclear potential and to vary the well depth so as to produce a quasi-bound state at the energy of the Q-value of each decay. It has found more profitable to fix V, and to vary R for each individual decay so as to produce the quasi-bound state (with n nodes) at energy Q. It is therefore related the radius R to the potential depth V, for an L = 0 alpha particle having global quantum number G through the Bohr-Sommerfeld condition.

$$\int_0^R dr \sqrt{\frac{2\mu}{\hbar^2}(Q + V - \frac{C}{R})} = (G+1)\frac{\pi}{2} \tag{3.27}$$

where μ is the reduced mass of the alpha-core system, in order to obtain a quadratic equation for R:

$$R = \frac{\pi}{2}(G+1)\left[\frac{2\mu}{\hbar^2}(Q+V-\frac{C}{R})\right]^{-1/2} \tag{3.28}$$

This approach strongly indicates that it is a better approximation to hold V fixed and let the $Q-value$ determine each individual value of N, then to assume that one knows R and to let the data determine V, separately for each decay. Small variations in R can lead to very large changes in the calculated decay widths Γ. This is apparent from the expression for Γ, given in semiclassical approximation by

$$\Gamma = \frac{P\hbar^2 K}{2\mu R}\exp\left[-2\int_R^{C/Q} drk(r)\right] \tag{3.29}$$

where P is the alpha particle formation probability, and K and $k(r)$ are the wave numbers in the internal and barrier regions, respectively:

$$K = \{\frac{2\mu}{\hbar^2}(Q+V-\frac{C}{R})\}^{-1/2} \tag{3.30}$$

$$k(r) = \{\frac{2\mu}{\hbar^2}(\frac{C}{r}-Q)\}^{-1/2} \tag{3.31}$$

The decay half-life is thus given by

$$T_{1/2} = \frac{\hbar\ln 2}{\Gamma} = P^{-1}2\ln 2\left(\frac{\mu R}{\hbar K}\right)\exp\left[2\int_R^{C/Q} drk(r)\right] \tag{3.32}$$

Explicitly, the square-well radius R is obtained from the quadratic equation (3.27) as

$$R = \frac{C+\sqrt{C^2+4(Q+V_N)\frac{\hbar^2}{2\mu}[(G+1)\frac{\pi}{2}]^2}}{2(Q+V_N)} \tag{3.33}$$

and the integral in the Gamow factor of equation (3.32) can be evaluated analytically as

$$2\int_R^{C/Q} drk(r) = 2\sqrt{\frac{2\mu}{\hbar^2}}\frac{C}{\sqrt{Q}}\{\frac{\pi}{2}-\sin^{-1}x - x\sqrt{1-x^2}\} \tag{3.34}$$

where $x = \sqrt{RQ/C}$

The parameters are thus the formation probability P , the potential depth V, and the global quantum number G. Once these have been fixed the corresponding radii R and half-lives $T_{1/2}$ are determined from the separation energies Q and the charge products C. A set of parameter values:

$$P = 1$$

$$V_N = 135.6 Mev$$
$$G_1 = 22 \tag{3.35}$$
$$G_2 = 24$$

where G_1 refers to nuclei having $N \leq 126$ and G_2 to those with $N > 126$.

3.4 A MODIFIED SIMPLE MODEL

The half-lives of alpha emitting elements greatly vary from about a nanosecond to billion years. and strongly depends on the alpha kinetic energy ranging only a factor of two, from about 4 to 10 MeV. This extraordinary dependence suggests an exponential process which can be modeled in the framework of effective quantum mechanical tunnelling through the Coulomb barrier. The tunnelling probability depends very strongly on the nature of the barrier. In this work a better estimate of the half-lives of some elements was obtained by evaluating the tunnelling probability based on breaking the barrier into segments and multiplying the successive tunnelling probabilities. We utilized and a simple model to obtain more accurate results for alpha half-lives than the others. Firstly, we increase the segment numbers. Then the formulas are rearranged for new segment numbers. And the obtained new formulas are used for calculating alpha half-lives of some isotopes.

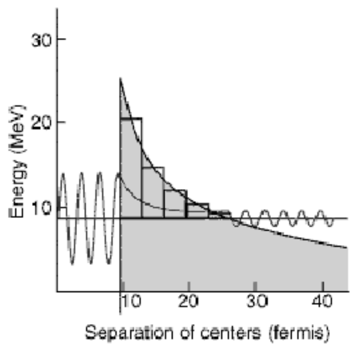

Figure 3.3: Breaking the Coulomb barrier into segments.

A number of parameters must be defined to model alpha decay and to estimate half lives of alpha emitting isotopes. These parameters are:

Nuclear separation distance: This parameter is calculated from the results of the experiments. These results can be expressed in the following empirical equation for the radius R of the nucleus of an isotope of mass number A:

$$R = R_0 A^{1/3} \tag{3.36}$$

where R_0 is the radius parameter. Numerical values of R_0 vary from about $1.2{*}10^{-15}$ to $1.5{*}10^{-15}$meter. The latest data favor then value $R_0 = 1.2{*}10^{-15} m$.

$$R = 1.2 \left[4^{1/3} + (A-4)^{1/3} \right] \tag{3.37}$$

Height of the Coulomb Barrier: Alpha decay can only occur if an alpha particle is permitted to penetrate the Coulomb barrier around the outside of the nucleus. This barrier is a result of the electric energy of two charged bodies and outside the nucleus has the value:

$$H = \frac{2\,(z-2) * ke^2}{R} \tag{3.38}$$

where $ke^2 = 1.44\ Mev\ fm$.

The distance at which the alpha particle leaves the barrier:

$$r = \frac{2(Z-2)ke^2}{E_\alpha} \tag{3.39}$$

where E_α is the kinetic energy of the alpha particle. It is necessary that E_α be positive in order to have alpha decay occur. In fact, it turns out that the E_α value must be range of 3-10 MeV in order to effectively compete with other decay modes. The half-life for alpha decay is strongly dependent on the E_α value. There is a rapid variation of half-life with Q value, this rapid variation is easily explained in terms of barrier penetration probabilities.

Width of the barrier:

$$W = r - R \tag{3.40}$$

Velocity of the alpha particle:

$$v = \left(\frac{2 * E_\alpha}{m_\alpha} \right)^{1/2} \tag{3.41}$$

Frequency of alpha particle hitting the walls of the barrier:

$$f = \frac{v}{2R} \tag{3.42}$$

Tunnelling probability: The square of the absolute value of the wave function of a particle gives the tunnelling probability:

$$P = \psi^2 \tag{3.43}$$

where ψ is the solution of the free particle Schrödinger equation and is given by:

$$\psi = \exp\left[\sqrt{\frac{2m_\alpha\left(H - E_\alpha\right)}{\hbar^2}} * W\right] \tag{3.44}$$

As shown in figure (3.3), we break the Coulomb barrier into segments in this simple model and the tunnelling probability for each segment is separately determined as:

$$\begin{aligned} P_1 &= \psi_1^2, \\ P_2 &= \psi_2^2, \\ P_3 &= \psi_3^2, \end{aligned} \tag{3.45}$$

and so on.

Then the product of probabilities is determined as:

$$P = P_1 * P_2 * P_3 *, ..., P_N \tag{3.46}$$

where N is the segment number.

Chapter 4

RESULTS AND DISCUSSIONS

It is known that in classical physics, a particle of energy E less than the height U of a barrier could not penetrate the barrier- the region inside the barrier is classically forbidden. But according to quantum physics the wavefunction associated with a free particle must be continuous at the barrier and will show an exponential decay inside the barrier as shown in figure (3.3). The wavefunction must also be continuous on the far side of the barrier, so there is a finite probability that the particle will eventually tunnel through the barrier. The shape of the barrier must obviously be taken into account since it drops rapidly. But it is instructive to calculate the half-life for a rectangular barrier of that height and width. The tunnelling probability depends very strongly on the nature of the barrier. The probability depends exponentially on the height and width, and in this case the height is dropping like $1/r$ with distance from the nucleus.

A better tunnelling probability can be obtained by breaking the barrier into segments and multiplying the successive tunnelling probabilities according to the basic rules for combining probabilities. Then using the product of tunnelling probabilities for the segments the alpha half lives of some heavy isotopes have been evaluated utilizing the relationship between the tunnelling probability P and half life $T_{1/2}$ described previously.

A computer program which enclosed in appendix for the modified simple model of alpha half-life is written and with aid of this computer program the alpha half-lives of nuclides are calculated. We choose the nuclides which have atomic number bigger than 80. Also these selected nuclides can become stable with only alpha decay. We did not deal with the other decays such as beta decay or gamma decay. The isotopes that make decay for that ground state to ground state are considered in this work.

The comparisons of the calculated half lives of some even-even, odd-odd, odd-even, and even-odd nuclei with the experimental values are made in Table (4.1-4.4). As seen from these tables, it can be easily concluded that much better results were obtained by breaking the barrier into segments. The results from these tables are analyzed for each table and it was found that similar results were obtained from all the alpha emitting nuclides considered in this thesis. Let us examine $^{203}_{86}Rn$ nuclei from table 4.1 it is seen that good results are provided. If the barrier is not breaking into segment (segment # 1), the calculation gives the alpha half life of 1.99 seconds where as the presented model provides a value of 27.33 seconds if the barrier were broken into 5 segments, which is much closer to the experimental value of 28 seconds as shown in table (4.1). Similarly, for $^{193}_{83}Bi$ isotope in table (4.2), for $^{204}_{86}Rn$ isotope in table (4.3), for $^{216}_{85}At$ in table (4.4), the method provides more accurate results when the barrier is broken into segments.

However our model does not provide good results for all nuclei investigated, for some nuclides, it works badly. But we can say that this modified simple model usually provides satisfactory results, for many isotopes.

An interesting point was observed when we investigated the alpha energy of nuclides versus half-life. It is that the model gives more suitable results for nuclides with high alpha energies over 8 MeV than others. For example; $^{256}_{102}No$ is an even-even isotope and with an alpha energy of 8.42 MeV and the calculated half-life is 2.3 seconds very near to the experimental value of 2.005 seconds.

An exciting situation is that alpha half-lives as a function of segment number show a similar trend for all nuclides. Figures (4.1-4.4) show the effect of segment number on the half-lives of different isotopes. Such as $^{190}_{83}Bi$ from even-odd isotopes, $^{193}_{83}At$ from an odd-odd isotopes, $^{200}_{86}Rn$ from an even-even isotope and $^{197}_{84}Po$ from an odd-even isotope. As seen from these four figures the half-lives first show an exponential increase with increasing segment number up to segment number 14 and then seem to level off.

The data on the four graphs are tabulated in table (4.5) for a clear demonstration of the effect of the segment number on evaluated half-lives of four isotopes described previously.

For some isotopes the alpha half-lives are calculated by using different methods explained in previous chapter and the results are shown in table 4.6. The obtained values by the presented method seem more closer to experimental values than the others. Generally, similar results can be obtained for other isotopes. Finally, we can say that the presented simple model provides fairly good results compared to other methods some of which are rather complex. Hence we conclude that this modified model generally gives quite satisfactory results although it is rather simple

ODD-EVEN			The calculated alpha half lives as function of segments				
Nuclie	$E_\alpha MeV$	$T_{1/2}$	1	5	10	50	100
$^{177}_{80}Hg$	6.58	0.17s	0.011s	0.133s	0.140s	0.139s	0.139s
$^{179}_{80}Hg$	6.29	1.09s	0.099s	1.603s	1.723s	1.718s	1.713s
$^{193}_{84}Po$	7.00	0.42sec	0.0100s	0.1005s	0.1053s	0.103s	0.103s
$^{195}_{84}Po$	6.70	2.0s	0.087s	1.178s	1.254s	1.241s	1.238s
$^{195}_{84}Po$	6.61	4.5s	0.175s	2.625s	2.808s	2.786s	2.777s
$^{197}_{84}Po$	6.385	26s	0.994s	19.17s	20.80s	20.72s	20.65s
$^{197}_{84}Po$	6.282	56s	2.341s	51.60s	56.44s	56.32s	56.15s
$^{199}_{86}Po$	7.06	0.29s	0.031s	0.330s	0.347s	0.342s	0.341s
$^{201}_{86}Rn$	6.77	3.8s	0.248s	3.65s	3.89s	3.86s	3.84s
$^{203}_{86}Rn$	6.548	28s	1.345s	25,25s	27.33s	27.18s	27.10s
$^{203}_{86}Rn$	6.5	45s	1.99s	39.68s	43.09s	42.89s	42.76s
$^{215}_{86}Rn$	8.67	2.3μs	0.686μs	1.983μs	1.961μs	1.908μs	1.902μs
$^{217}_{86}Rn$	7.740	0.54ms	0.153ms	0.801ms	0.811ms	0.793ms	0.791ms
$^{207}_{88}Ra$	7.13	1,3s	0.078s	0.906s	0.954s	0.941sec	0.938s
$^{209}_{88}Ra$	7.01	4.6s	0.177s	2.303s	2.439s	2.410sec	2.402s
$^{211}_{88}Ra$	6.91	13s	0.353s	5.045s	5.369s	5.313sec	5.296s
$^{213}_{88}Ra$	6.62	2.7min	0.054min	1.,082min	1.173min	1.166min	1.162min
$^{215}_{88}Ra$	8.99	1.6μs	0.52μs	1.142μs	1.140μs	1.362μs	1.358μs
$^{219}_{88}Ra$	7.68	10ms	1.101ms	7.034ms	7.187ms	7.037ms	7.014ms
$^{221}_{90}Th$	8.146	1.68ms	0.264ms	1.360ms	1.375ms	1.342ms	1.338ms
$^{223}_{90}Th$	7.29	0.66s	0.077s	0.828s	0.867s	0.853s	0.850s
$^{227}_{92}U$	6.87	1.1min	0.151min	3.333min	3.600min	3.290min	3.566min
$^{239}_{98}Cf$	7.63	39s	3.387s	48.95s	51.83s	51.11s	50.93s
$^{241}_{98}Cf$	7.33	3.8min	0.519min	10.28min	11.08min	10.98min	10.93min
$^{245}_{98}Cf$	7.137	44min	2.116min	51.50min	56.08min	55.71min	55.51min
$^{243}_{100}Fm$	8.55	0.18s	0.026s	0.191s	0.195s	0.191s	0.190s
$^{245}_{100}Fm$	8.15	4s	0.353s	3.53s	3.67s	3.60s	3.59s
$^{247}_{100}Fm$	8.18	9s	0.269s	2.585s	2.678s	2.625s	2.616s

Table 4.1: The Alpha half-lives of some odd-even isotopes

ODD-ODD			The calculated alpha half lives as function of segments				
Nuclie	$E_\alpha MeV$	$T_{1/2}$	1	5	10	50	100
$^{189}_{83}Bi$	6.67	¡1.5s	0.055s	0.732sec	1.034sec	0.771sec	0.769sec
$^{191}_{83}Bi$	6.32	13s	0.830s	16.33sec	17.76sec	17.70sec	17.64sec
$^{193}_{83}Bi$	6.48	3.5s	0.211s	3.344sec	3.59sec	3.91sec	3.55sec
$^{193}_{83}Bi$	5.91	64s	26.29s	91.54s	102.5s	103.9s	102.8s
$^{195}_{83}Bi$	6.11	1,.5min	0.07min	1.767min	1.949min	1.950min	1.944min
$^{197}_{85}At$	6.96	0.4s	0.028s	0.312s	0.328s	0.324s	0.323s
$^{199}_{85}At$	6.64	7.0s	0.298s	4.64s	4.97s	4.93s	4.92s
$^{201}_{85}At$	6.345	89s	3.05s	67.29s	73.55s	73.37s	73.14s
$^{213}_{85}At$	9.08	0.11μs	0.041μs	0.092μs	0.092μs	0.088μs	0.087μs
$^{213}_{89}Ac$	7.36	0.8s	0.028s	0.275s	0.286s	0.282s	0.281s
$^{217}_{89}Ac$	9.65	0.11μs	0.039μs	0.083μs	0.081μs	0.079μs	0.078μs
$^{219}_{89}Ac$	8.66	7μs	6.215μs	21.44μs	23.2μs	20.75μs	20.69μs
$^{221}_{89}Ac$	7.65	52ms	2.88ms	20.20ms	20.80ms	20.39ms	20.32ms
$^{215}_{91}Pa$	8.09	14ms	1.036ms	6.26ms	6.38ms	6.24ms	6.22ms
$^{217}_{91}Pa$	8.33	4.9ms	0.217ms	1.085ms	1.096ms	1.069ms	1.066ms
$^{223}_{91}Pa$	8.01	6.5ms	1.301ms	7.94ms	8.09ms	7.91ms	7.88ms
$^{223}_{91}Pa$	8.2	6.5ms	3.912ms	2.066ms	2.090ms	2.041ms	2.034ms
$^{225}_{91}Pa$	7.25	1.8s	0.231s	2.78s	2.92s	2.88s	2.87s
$^{229}_{93}Np$	6.89	4min	0.331min	6.95min	7.53min	7.49min	7.47min
$^{243}_{99}Es$	7.89	21s	1.031s	12.22s	12.80s	12.58s	12.54s
$^{245}_{99}Es$	7.73	1.3min	0.051min	0.701min	0.740min	0.729min	0.726min
$^{247}_{99}Es$	7.32	4.8min	1.093min	23.25min	25.12min	24.90min	24.81min
$^{249}_{101}Md$	8.43	˜3s	0.014s	0.937s	0.965s	0.870s	0.867s
$^{251}_{101}Md$	7.55	4min	0.881min	17.09min	18.35min	18.15min	18.08min
$^{255}_{103}Lr$	8.37	22s	0.627s	6.229s	6.454s	6.858s	6.300s
$^{257}_{103}Lr$	8.86	0.65s	0.023s	0.156s	0.158s	0.154s	0.154s
$^{257}_{105}X$	9.16	˜1s	0.017s	0.104s	0.105s	0.102s	0.102s
$^{261}_{105}X$	8.93	˜1.8s	0.062s	0.439s	0.448s	0.437s	0.435s

Table 4.2: The Alpha half-lives of some odd-odd isotopes

EVEN-EVEN			The calculated alpha half lives as function of segments				
Nuclie	$E_\alpha MeV$	$T_{1/2}$	1	5	10	50	100
$^{186}_{82}Pb$	6.32	8s	0.391s	7.17s	7.775s	7.774s	7.720s
$^{200}_{86}Rn$	6.91	1s	0.088s	1.112s	1.189s	1.176s	1.172s
$^{202}_{86}Rn$	6.636	9.9s	0.687s	11.692s	12.585s	12.497s	12.457s
$^{204}_{86}Rn$	6.417	1.24min	1.286min	1.398min	1.520min	1.522min	1.535min
$^{212}_{86}Rn$	6.264	24min	0.176min	4.47min	4.88min	4.87min	4.86min
$^{214}_{86}Rn$	9.04	0.27s	0.101s	0.244s	0.240s	0.233s	0.227s
$^{216}_{86}Rn$	8.05	˜45μs	0.23μs	96μs	97μs	94.88μs	94.58μs
$^{208}_{88}Ra$	7.13	1.4s	0.075s	0.867s	0.913s	0.901s	0.898s
$^{210}_{88}Ra$	7.02	3.7s	0.158s	2.02s	2.14s	2.11s	2.10s
$^{212}_{88}Ra$	6.901	13s	0.366s	5.23s	5.56s	5.50s	5.49s
$^{214}_{88}Ra$	7.14	2.46s	0.056s	0.615s	0.645s	0.636s	0.634s
$^{216}_{88}Ra$	9.35	0.18μs	0.085μs	0.197μs	0.193μs	0.188μs	0.187μs
$^{218}_{88}Ra$	8.39	14μs	0.139μs	53μs	53.2μs	51.87μs	51.70μs
$^{212}_{90}Th$	7.8	30ms	3.336ms	23.90ms	24.63ms	24.14ms	24.06ms
$^{214}_{90}Th$	7.68	0.09s	0.006s	0.054s	0.056s	0.055s	0.054s
$^{216}_{90}Th$	7.92	28ms	1.319ms	8.35ms	8.53ms	8.34ms	8.32ms
$^{218}_{90}Th$	9.67	0.11μs	0.069μs	0.154μs	0.150μs	0.146μs	0.145μs
$^{220}_{90}Th$	8.76	10μs	6.21μs	21.09μs	20.90μs	20.39μs	20.33μs
$^{222}_{90}Th$	7.98	2.8ms	0.726ms	4.21ms	4.28ms	4.18ms	4.17ms
$^{226}_{92}U$	7.43	0.5s	0.141s	1.544s	1.616s	1.591s	1.586s
$^{240}_{98}Cm$	6.291	27day	0.483day	32.16day	37.27day	37.72day	37.60day
$^{240}_{98}Cf$	7.59	1.1min	0.073min	1.099min	1.160min	1.150min	1.146min
$^{242}_{98}Cf$	7.385	3.5min	0.327min	6.03min	6.47min	6.40min	6.38min
$^{244}_{98}Cf$	7.21	20min	25.10min	27.19min	29.19min	29.23min	29.13min
$^{246}_{100}Fm$	8.24	1.2s	0.185s	1.696s	1.750s	1.718s	1.712s
$^{248}_{100}Fm$	7.87	36s	2.302s	29.46s	30.90s	30.45s	30.34s
$^{250}_{102}No$	8.76	0.25ms	0.021ms	0.178ms	0.180ms	0.177ms	0.177ms
$^{252}_{102}No$	8.42	2.3sec	0.223s	1.987s	2.004s	2.005s	1.998s

Table 4.3: The Alpha half-lives of some even-even isotopes

EVEN-ODD			The calculated alpha half lives as function of segments				
Nuclie	$E_\alpha MeV$	$T_{1/2}$	1	5	10	50	100
$^{190}_{83}Bi$	6.45	5.4s	0.299s	5.05s	5.44s	5.41s	5.40s
$^{196}_{85}At$	7.06	0.3s	0.014s	0.142s	0.149s	0.147s	0.146s
$^{198}_{85}At$	6.75	4.9s	0.131s	1.819s	1.937s	1.918s	1.912s
$^{198}_{85}At$	6.85	1.5s	0.061s	0.761s	0.805s	0.796s	0.796s
$^{200}_{85}At$	6.536	4.3s	0.658s	11.50s	12.40s	12.32s	12.328s
$^{212}_{85}At$	7.681	0.314ms	0.016ms	0.608ms	0.616ms	0.603ms	0.601ms
$^{214}_{85}At$	8.82	2μs	0.150μs	0.380μs	0.374μs	0.364μs	0.362μs
$^{216}_{85}At$	7.8	0.3ms	0.047ms	0.222ms	0.224ms	0.218ms	0.218ms
$^{202}_{87}Fr$	7.25	0.34s	0.016s	0.160s	0.167s	0.164s	0.164s
$^{204}_{87}Fr$	7.03	2.1s	0.075s	0.905s	0.955s	0.943s	0.940s
$^{206}_{87}Fr$	6.93	0.7s	0.149s	1.95s	2.07s	2.04s	2.04s
$^{208}_{87}Fr$	7.13	1.4s	0.031s	0.328s	0.344s	0.339s	0.338s
$^{214}_{87}Fr$	8.48	3.4ms	0.147ms	1.58ms	1.51ms	1.47ms	1.46ms
$^{216}_{87}Fr$	9.01	0.7ms	0.231ms	0.592ms	0.583ms	0.566ms	0.564ms
$^{218}_{87}Fr$	7.867	0.7ms	0.149ms	0.764ms	0.770ms	0.755ms	0.753ms
$^{210}_{89}Ac$	7.46	0.35s	0.015s	0.140s	0.146s	0.143s	0.143s
$^{212}_{89}Ac$	7.38	0.93s	0.025s	0.244s	0.254s	0.250s	0.249s
$^{214}_{89}Ac$	7.21	8.2s	0.080s	0.902s	0.947s	0.934s	0.931s
$^{218}_{89}Ac$	9.2	0.27μs	0.353μs	0.914μs	0.900μs	0.874μs	0.871μs
$^{222}_{89}Ac$	6.81	65s	1.290s	21.01s	22.50s	22.29s	22.22s
$^{222}_{89}Ac$	7.013	5s	0.266s	3.444s	3.643s	3.597s	3.585s
$^{216}_{91}Pa$	7.87	0.2s	0.004s	0.029s	0.030s	0.029s	0.029s
$^{218}_{91}Pa$	9.61	0.12ms	0.019ms	0.455ms	0.446ms	0.433ms	0.432ms
$^{222}_{91}Pa$	8.21	4.3ms	0.380ms	2.008ms	2.032ms	1.984ms	1.977ms
$^{224}_{91}Pa$	7.49	0.95s	0.042s	0.400s	0.417s	0.409s	0.408s
$^{226}_{91}Pa$	6.86	1.8min	0.075min	1.396min	1.504min	1.492min	1.487min
$^{252}_{101}Md$	7.73	2.3min	0.216min	3.410min	3.630min	3.583min	3.570min
$^{258}_{103}Md$	8.6	4.3s	0.122s	0.976s	1.001s	0.978s	0.974s

Table 4.4: The Alpha half-lives of some even-odd isotopes

	$T_{1/2}$ in seconds			
Segment number	$^{190}_{83}Bi$	$^{200}_{86}Rn$	$^{197}_{84}Po$	$^{193}_{83}At$
1	0.212	0.089	2.3620	0.302
2	1.659	0.601	23.047	2.456
3	2.641	0.913	39.153	3.960
4	3.115	1.050	47.436	4.660
5	3.344	1.122	51.600	5.050
6	3.461	1.155	53.800	5.239
7	3.525	1.172	55.037	5.340
8	3.560	1.182	55.753	5.397
9	3.580	1.186	56.181	5.430
10	3.592	1.189	56.440	5.440
11	3.599	1.190	56.599	5.460
12	3.602	1.190	56.694	5.466
13	3.603	1.190	56.748	5.469
14	3.604	1.190	56.776	5.470
15	3.603	1.189	56.787	5.469
16	3.602	1.188	56.786	5.468
17	3.601	1.187	56.778	5.466
18	3.599	1.187	56.765	5.464
19	3.597	1.187	56.748	5.462
20	3.596	1.186	56.730	5.460
25	3.588	1.183	56.632	5.448
30	3.582	1.181	56.544	5.440
40	3.573	1.177	56.414	5.426
45	3.570	1.176	56.367	5.422
50	3.568	1.176	56.328	5.418
60	3.564	1.174	56.269	5.412
70	3.561	1.173	56.227	5.408
80	3.559	1.173	56.195	5.405

Table 4.5: Half-lives of some isotopes as a function of segment numbers

	Half-lives of four radioisotopes				
Nuclides	Experimental	1^{st} method	2^{nd} method	Simple cluster	Our simple model
$^{190}_{83}Bi$	5.4 sec	11.68 sec	0,1994 sec	22.982 sec	5.430 sec
$^{193}_{83}At$	3.5 sec	9.201 sec	0,1046sec	17.936 sec	3.525 sec
$^{200}_{86}Rn$	1 sec	7.805 sec	9.852 sec	8.3133 sec	1.172 sec
$^{197}_{84}Po$	56 sec	89.65 sec	6470.2 sec	318.94 sec	56.181 sec

Table 4.6: Comparison of methods

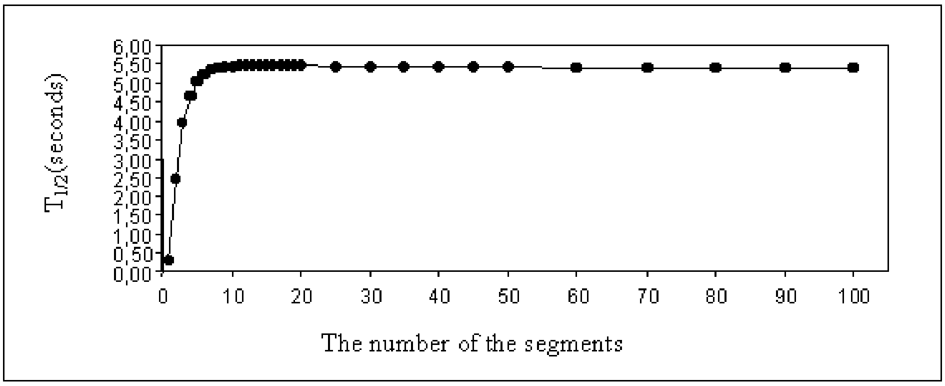

Figure 4.1: The calculated half-life of $^{190}_{83}Bi$ even-odd isotope as function of the segments.

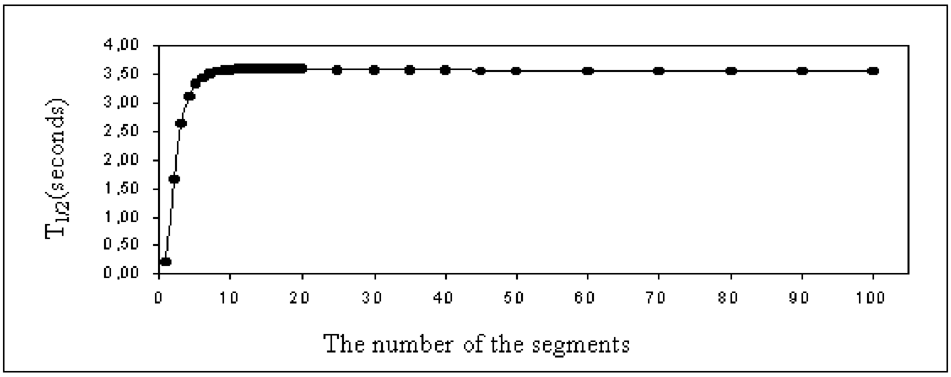

Figure 4.2: The calculated half life of $^{193}_{83}At$ odd-odd isotope as function of the segments.

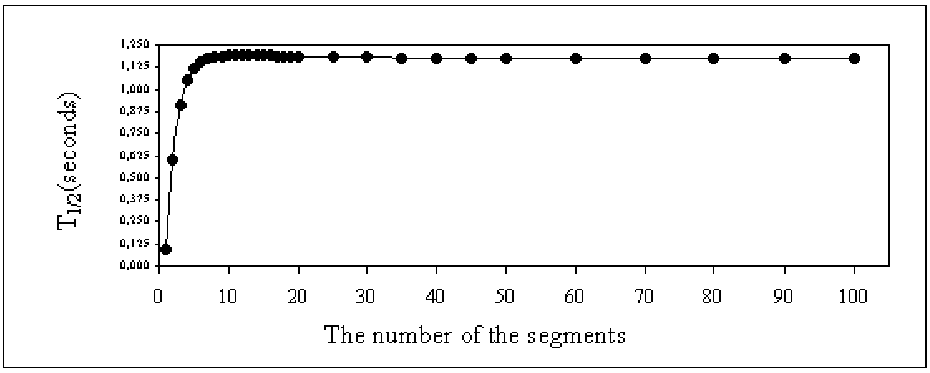

Figure 4.3: The calculated half-life $^{200}_{86}Rn$ of even-even isotope as function of the segments.

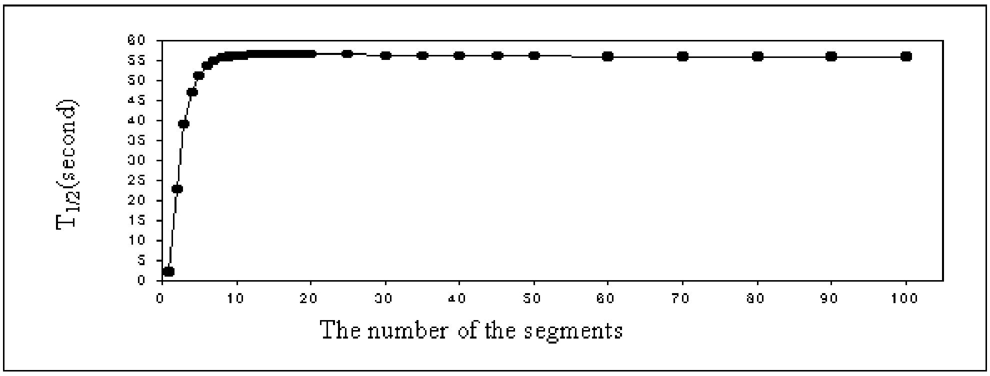

Figure 4.4: The calculated half-life $^{197}_{84}Po$ of odd-even isotope as function of the segments.

Chapter 5

CONCLUSION

The simple model presented in this work provides good reasonable results in compared to the experimental results. The model usually gives a better results for isotope with high alpha decay energy over 8 MeV. Another interesting point is that the half-life shows similar trend as a function of the segment number nearly for all isotopes considered in this work. It is that it first makes an exponential rise up to the segment number seven and then seems to level off with a slight decrease as the segment number increase. Hence it is better to use the segment number five for the calculation in order to save computer time since the results do not vary much as the segment number increases.

In the literature there are a lot of models that give more accurate results related to this problem. In contrast some of these models have disadvantages based on complexity but the presented model is very simple for calculation of the alpha half-life.

The next step on the future work about this study should be the utilization of different suitable potentials in place of Coulomb potential. It is expected that utilizing different potentials may result in much better results for the half-lives of alpha emitting nuclides.

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Appendix A

A computer Program for The Modified Simple Model

```
REAL AT,Z
INTEGER i,N
DOUBLE PRECISION R1,EK,EH,R2,R3,fR
DOUBLE PRECISION A1,A2,V,D1,D2,S1,S2,H1,H2,F1,F2,P1,P2
DOUBLE PRECISION PT,L,YO,P(100),F(100),W(100),H(100),S(100)
PRINT*,'ENTER AT=MASS NUMBER'
READ*,AT
PRINT*,'ENTER Z=ATOMIC NUMBER'
READ*,Z
PRINT*,'ENTER EK=KINETIC ENERGY OF ALPHA PARTICLE'
READ*,EK
PRINT*,'ENTER N'
READ*,N
R1=1.2*(4.**(1./3.)+(AT-4.)**(1./3.))
EH=2.88*(Z-2.) /R1
R2=(2.*(Z-2.)*1.44)/EK
R3=R2-R1
V=SQRT(4.82911E+13*EK)
fR=V/(2.*R1*(1E-15))
D1=R3/N
D2=D1/2.0
S1=R1+D2
H1=(R1*EH)/S1
A1=SQRT(1.909E+29*(H1-EK))
F1=EXP((-A1*D1)/1E+15)
P1=(F1)**2
S2=S1+D1
```

```
H2=(R1*EH)/S2
A2=SQRT(1.909E+29*(H2-EK))
F2=EXP((-A2*D1)/1E+15)
P2=(F2)**2
PT=P1*P2
PRINT*,R1,EH,R2,R3,V,fR,D1,D2,S1,H1,A1,F1,P1,S2,H2,A2,F2,P2
L=fR*PT
YO=(0.693)/L
PRINT*,'R12=',R1,R2,'EH=',EH,'L=',L,'PT=',PT
IF(N.LT.3) goto 10
S(2)=S2
DO i=3,N
    S(i)=S(i-1)+D1
    H(i)=(R1*EH)/S(i)
    W(i)=SQRT(1.909E+29*(H(i)-EK))
    F(i)=EXP((-W(i)*D1)/1E+15)
    P(i)=F(i)*F(i)
    PT=PT*P(i)
PRINT*,'P=',P(i),i
END DO
10 L=fR*PT
YO=(0.693)/L
PRINT*,R1
PRINT*,YO
END
```