SLIDING MODE CONTROL AND ITS APPLICATIONS

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KAYAN KİP KONUM KONTROLÜ VE UYGULAMALARI

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ABSTRACT

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Sliding Mode Control (SMC) method is one of the robust control methods to handle systems with model uncertainties, parameter variations and input and output disturbances. In this study, a sliding mode control system with an integral (SMC+I) operation is adopted to control speed of an electromechanical system. The proposed sliding mode controller is chosen to ensure the stability of overall dynamics during the reaching phase and sliding phase. The stability of the system is guaranteed in the sense of the Lyapunov stability theorem. Chattering problem is overcome using some continuous functions such as saturation function, and hyperbolic function. Experimental results verify that the proposed SMC+I controller can achieve favorable tracking performance and is robust with regard to parameter variations and disturbances compared with the conventional sliding mode controller and PID controller .

Keywords: sliding mode control, integral control, experimental application, electrical drive system, conventional control

ÖZET

KAYAN KİP KONUM KONTROL VE UYGULAMALARI

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Kayan kip konum kontrol metodu doğrusal olmayan sistemlere uygulanan bir metot olup, parametre değişimlerini ve gürültüleri de ele alır. Bu çalışmada kayan kip konum kontrolüne integral eklenerek, elektromekanik sistemin hızı kontrol edilmiştir. Kayan kip konum kontrol sistemi, kayan fazda ve uygulanan darbe yüzeyine ulaşımda sistem kararlılığı sağlandığı için seçilmiştir. Lyapunov karalılık teoremi ile sistem kararlılığı güvenceye alınmıştır. Çıkış sinyalindeki çatırdama problem hyperbolic fonksiyon kullanılarak üstesinden gelinmiştir. Deney sonuçları, SMC+I kontrolün geleneksel PID ve SMC ile karşılaştırıldığında, gürültü ve sistem parametre değişimlerine karşın, kararlılığı sonuç performansı ile popüler çalşmalar arasındadır. Kontrol metodunun verimliliği ve esnekliği deneysel olarak gösterilmiş ve kapalı devre sürücü sistemlerin dinamik tepkilerinde sonuçlar, geleneksel PID ile karşılaştırılmıştır.

Keywords: Kayan kip konum kontrolü, integral kontrol, deneysel uygulama, elektrik sürücü sistem, geleneksel kontrol.

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CHAPTER 1

INTRODUCTION

1.1 Overview of Control Theory

Automatic control Systems were first developed over two thousand years ago [1]. The first feedback control device on record is thought to be the ancient water clock of Ktesibios in Alexandria Egypt around the third century B.C. It kept time by regulating the water level in a vessel and, therefore, the water flow from that vessel. This certainly was a successful device as water clocks of similar design were still being made in ~Baghdad when the Mongols captured the city in 1258 A.D. A variety of automatic devices have been used over the centuries to accomplish useful tasks or simply to just entertain. The latter includes the automata, popular in Europe in the 17th and 18th centuries, featuring dancing figures that would repeat the same task over and over again; these automata are examples of open-loop control. Milestones among feedback, or "closed-loop" automatic control devices, include the temperature regulator of a furnace attributed to Drebbel, circa 1620, and the centrifugal flyball governor used for regulating the speed of steam engines by James Watt in 1978.

In his 1868 paper "On Governors", J. C. Maxwell (who discovered the Maxwell electromagnetic field equations) was able to explain instabilities exhibited by the flyball governor using differential equations to describe the control system. This demonstrated the importance and usefulness of mathematical models and methods in understanding complex phenomena, and signaled the beginning of mathematical control and systems theory. Elements of control theory had appeared earlier but not as dramatically and convincingly as in Maxwell's analysis [2].

Control theory made significant strides in the next 100 years. New mathematical techniques made it possible to control, more accurately, significantly more complex dynamical systems than the original flyball governor. These techniques include developments in optimal control in the 1950's and 1960's, followed by progress in stochastic, robust, adaptive and optimal control methods in the 1970's and 1980's. Applications of control methodology have helped make possible space travel and communication satellites, safer and more efficient aircraft, cleaner auto engines, cleaner and more efficient chemical processes, to mention but a few. There are two kind of control techniques. These are 'linear' and 'nonlinear' control system. Sliding mode control is a kind of nonlinear control system[3].

1.2 Introduction to Linear Control Systems

If you study about a system, which may be linear or nonlinear you should examine first mathematical description [4]. It is necessary therefore to analyse the relationship between the system variables and to obtain a mathematical model. The system considerations are dynamic in nature, the descriptive equations are usually differential equations.

If the mathematical equations can be linearised, then the Laplace transform can be appliable to simplify the method of solution. The transfer function of a linear system is defined as the ratio of the Laplace transform of the system output variable to the Laplace transform of the system input variable. But all initial values must be taken to be zero [2]. Linear control theory has been concerned with the study of linear time invariant (LTI) control systems. In a linear system, model simulation results are additive in their effects and satisfy the principle of superposition. In other words, the output is directly proportional to the input and two times of input leads to two times the output. In a non-linear model, the principle of superposition is not satisfied.

Time invariance is a second important property that can provide a basis for classification of models of systems. A time invariant system is one in which observed performance of the system is independent of the times at which the observations are made.

1.3 Non-Linear Control System

Non-linear systems can have more complex behavior than linear systems, therefore their analysis is much more difficult [4]. Mathematically, this is reflected in two aspects. First, non-linear equations, which is not like linear system, cannot be solved analytically and therefore a understanding of the behavior of non-linear system is very difficult. Second, powerful mathematical tools like Laplace and Fourier transforms do not apply to non-linear systems. As a result, there are no systematic tools for predicting the behavior of non-linear systems, nor are there systematic procedures for designing non-linear control systems.

No universal technique has been designed for the analysis of all non-linear control systems. In linear control, one can analysis a system in the time domain or in the frequency domain. In non-linear control systems, none of these standard approaches can be used. Direct solution of non-linear differential equations is generally impossible and frequency domain transformations do not apply. While the analysis of non-linear control systems is difficult. Many methods of non-linear control systems analysis have been proposed [6,7].

Traditionally, a nonlinear process has to be linearized first before an automatic controller can be effectively applied. This is typically achieved by adding a reverse nonlinear function to compensate for the nonlinear behavior, so that input-output relationship becomes linear. It is difficult application to match the nonlinear curve; and process uncertainties can easily disturb the effort.

The following trends show the nonlinear controller is able to control a severely nonlinear process with the entire operating range (linear and nonlinear). PID is appliable control solution for linear range but fails to control in the nonlinear range [8].

1.4 Historical Development of Sliding Mode Control

A several number of reserchers study about Variable Structure Systems (VSS) based on SlidingMode Control (SMC) strategy [7,9,10,11,12]. Typically, SMC is used to compansate against the modeling uncertainties and the unknown disturbances. These control strategies are shown to be detailed stable and robust despite the presence of the difficult result provided their upper bounds are known a priori to the controls. However, it may be difficult or sometimes impossible to obtain these upper bounds.

In SMC, the control task is to guide and keep the system state toward a hyperplane in the state space. This hyperplane, also referred to as sliding surface, represents a reduced order stable dynamics. Therefore, once the nonlinear system starts “sliding”, it assumes the behavior of that stable dynamics. One problem associated with SMC is the well known control chatter, which consists of infinite frequency of actuation. This phenomenon occurs as soon as the sliding motion starts. In order to keep the ideal sliding motion it is necessary to actuate with infinite bandwidth due to the destabilizing properties of the perturbations. Therefore ideal sliding mode an impractical achievement. A remedy to this problem was proposed by [7].

The concept of boundary layer thickness was introduced as a way to eliminate control chatter. This way, instead of ideal sliding mode, a quasi-sliding mode was achieved. A target various is defined in the state space. In this case the objective various surrounds the origin, which is the excellent target state to reach. In order to achieve the desired tracking, the state is first driven towards a sliding hyper surface. Next the control maintains the states within an acceptable proximity of the sliding surfaces (so called sliding manifold) despite the present perturbations. It is shown that the state, when within the sliding manifold, perfectly reaches the target manifold. It is a basic theory of the robust control application [14].

1.5 Advantages of Sliding Mode Control

The major advantage of sliding mode is low sensitivity to plant parameter variations and disturbances which eliminates the necessity of exact modeling. Sliding mode control enables the decoupling of the overall system motion into independent partial components of lower dimension. So that, it is reduced the complexity of feedback design. Sliding mode control implies that control actions are discontinuous state functions which may easily be implemented by conventional power converters with “on-off” as the only admissible operation mode. Due to these properties the intensity of the research at many scientific centers of industry and universities is maintained.

1.6 Industrial Application of Sliding Mode Control

Sliding mode control has been widely studied in recent years and has started to play an important role in the application of control theory to practical problems [16]. It has been successfully applied to underwater vehicles [17], automotive transmissions and engines, and power systems [7], induction motor [18], robots [19,20], electric drives [12,21], human neuromuscular process [22], electrical servo drive [43] and elevator velocity [23]. Sliding mode control has also been applied to dc motors using simulations [12,23,24] and experimental applications [8,Eker, 2005]. Firstly, input-output relation with pid surface is applied motor speed system in this study. Also PID+P sliding surface had applied motor position system. [43]

1.7 Structure of This Thesis

In this study, a theoratical study and application are performed such that a SMC+I controller is designed for the speed control of an electromechanical plant, a dc motor connected to a load using a belt mechanism via a shaft. The feasibility and effectiveness of the proposed sliding mode controller is experimentally demonstrated and the system is controlled using a computer. The results obtained from the present study are compared with the traditional PID control in dynamic responses of the closed-loop control system.

Chapter 2 is included to define the DC motor system, plant description and mathematical model.

Chapter 3, the structure of SMC controller is presented and introduced details. This chapter also includes the SMC and SMC+I controller mathematical model and presents the advantages these controllers.

In chapter 4, the responses of speed control of electrical drive system for, P, PD, PI, PID and SMC,SMC+I controllers and related actuated signals are shown. Additionally we compare the conventioal controllers P, PD, PI, PID with the SMC and SMC+I.

Conclusions are given in Chapter 5 to summarize the work achieved.

CHAPTER 2
PLANT DESCRIPTION

2.1 DC Motor Principles

In the late 1800s, several inventors built the first working motors, which used direct current (DC) power. After the invention of the induction motor, alternating current (AC) machines are used some application. However, DC motors still have many use DC motors consist of rotor-mounted windings (armature) and stationary windings (field poles). In all DC motors, except permanent magnet motors, current must be conducted to the armature windings by passing current through carbon brushes that slide over a set of copper surfaces called a commutator, which is mounted on the rotor. The commutator bars are soldered to armature coils. The brush/commutator combination makes a sliding switch that energizes particular portions of the armature, based on the position of the rotor. This process creates north and south magnetic poles on the rotor that are attracted to or repelled by north and south poles on the stator, which are formed by passing direct current through the field windings. It's this magnetic attraction and repulsion that causes the rotor to rotate[3,15].

The greatest advantage of DC motors may be speed control. Since speed is directly proportional to armature voltage and inversely proportional to the magnetic flux produced by the poles, adjusting the armature voltage and/or the field current will change the rotor speed. Today, adjustable frequency drives can provide precise speed control for AC motors, but they do so at the expense of power quality, as the solid-state switching devices in the drives produce a rich harmonic spectrum. The DC motor has no advers effects on power quality [24,25].

2.1.1 Permanent magnet motors

Here, permanent magnets instead of armature windings are mounted on the rotor [26]. Since the magnetic field produced on the rotor is limited in strength and isn't controllable, permanent magnet motors are typically small and produce little horsepower.

2.1.2 Series motors

Series motors connect the field windings in series with the armature [24]. Series motors lack good speed regulation, but are well-suited for high-torque loads like power tools and automobile starters because of their high torque production and compact size.

2.2 Model Identification

Dynamic model identification has been a major topic of interest in control engineering, motivated by the new achievements in control systems theory and requirements of new industrial and military applications. System identification can be performed in open loop or closed loop. However, open-loop identification is not applicable or is difficult to apply to plants which are open-loop unstable, have integrator behavior, or subject to significant drift in open-loop operation [27]. Some systems may contain inherent feedback mechanisms, which make open-loop methods inapplicable [28]. The fundamental problem in closed-loop identification is that the input signal and the immeasurable noise are correlated for any nonzero feedback mechanism [28]. Therefore, the subspace and nonparametric methods fail in closed loop unless special measures are taken. However, the prediction error approach to identification of plants operating in closed loop has proven to be safely applicable [4,28].

There are three different approaches to identification of systems working in closed loop: (1) Direct identification neglects the existence of feedback and uses the input–output data to identify the open-loop system. The feedback law, whether inherently present in the system or intentionally introduced for control purposes, is disregarded. Consequently, this approach is especially suitable for systems with nonlinear or unknown feedback mechanisms. This method which is called reaction curve method is applied in this thesis. Ziegler and Nichols have developed PID tuning methods back in the early forties based on open loop tests (less known than for example the Cohen-Coon formulas) and also based on a closed loop test, which is may be their most widely known achievement [8]. (2) Indirect identification is based on the assumption that the feedback law is known. The closed-loop system is identified and the open-loop system is determined using the identified system and the known feedback law [29]. (3) Joint input–output identification is carried out regarding the recorded input and output as outputs of a multivariable system in response to an external signal, for example noise. Open-loop parameters are obtained using the identified multivariable system [30].

2.3 Electromechanical System Model

The problem of controlling electromechanical plant is very important in many industrial applications [31]. DC motors have a wide range of profile in their motions that is required to follow a predetermined speed or position under load [32,33]. The main advantages of these motors are of easy speed or position control and wide adjustable range [32,33]. These motors have been extensively used in several industrial applications and control systems such as position control of robotic manipulators [33], disk motion control [34], liquid pumping [35]. Controller parameters of a dc motor have usually been calculated using linear fixed motor parameters at an operating point. However, fixed parameter controllers may not give desired performance under different operating conditions. There have been considerable developments in nonlinear control schemes for the dc motors [35,33,36]. This has attracted extensive researches in the field of control engineering.

The rough diagram of the system studied is shown in Fig. 1.

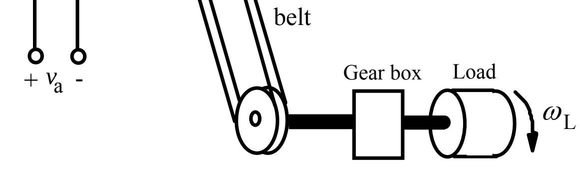


Figure 2.1 Diagram of the DC motor system

The electrical and mechanical equations representing a dc motor can be given as:

$$v_a(t) = L_a \frac{d}{dt} i_a + R_a i_a + K_m \omega + \omega_L \qquad (2.1)$$

$$K_m i_a(t) = J_m \frac{d}{dt} \omega_m(t) + R_m \omega_m(t) + d_1 \qquad (2.2)$$

$$\omega_m(t) - \omega_L(t) = sC\ T_{belt}(t) \qquad (2.3)$$

$$T_{belt}(t) - d_2 = J_L \frac{d}{dt} \omega_L + R_L \omega_L \qquad (2.4)$$

$$\omega_L(t) = r(\omega_{Lo} + d_3) \qquad (2.5)$$

Block diagram of dc motor; R_a and L_a are the armature coil resistance and inductance, i_a armature current K_m is torque coefficient, J_m is moment of inertia, R_m is coefficient of viscous friction, J_L and R_L load inertia and coefficient of viscous friction T_{belt} represents belt coefficient, d_1, d_2 and d_3 relates to system friction, disturbance, external load disturbance, nonlinear friction, and unmodelled dynamics, backlash friction e.t.c. The system equation is obtained from Eqs. (2.1)-(2.5)

The plant of block diagram;

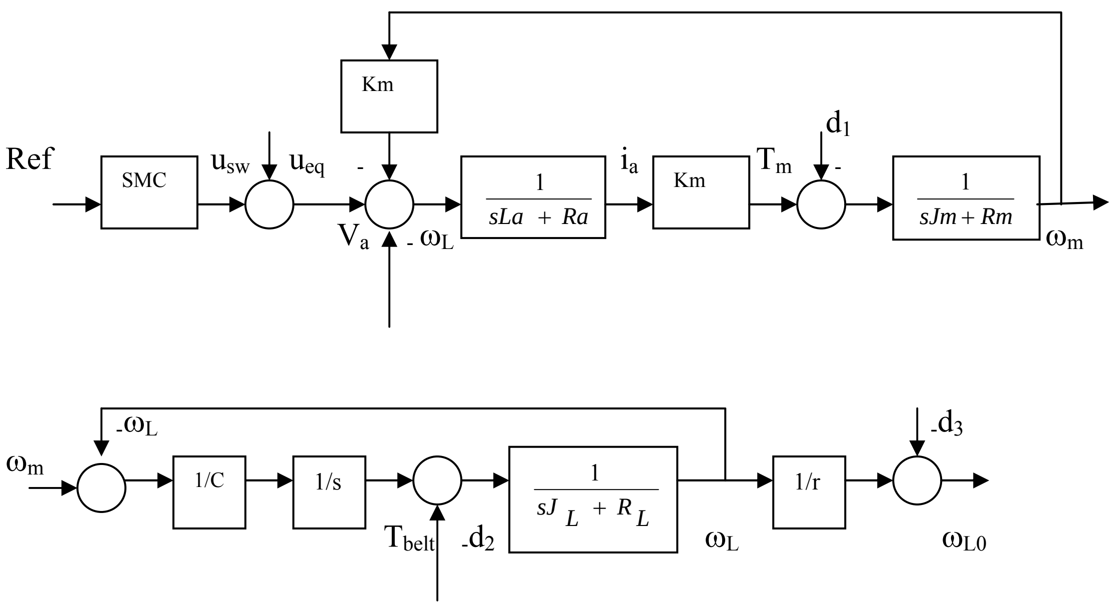


Figure 2.2 Plant Block Diagram

$$\ddddot{\omega}_L(t) = -A\dddot{\omega}_L(t) - B\ddot{\omega}_L(t) - C\dot{\omega}_L(t) - D\omega_L(t) \quad + V_a(t) + f(t) \tag{2.6}$$

where *A*, *B*, *C*, *D*, *E* are constants associated with the system parameters, $f(t)$ is the function that presents external load disturbance, nonlinear friction, and unmodelled dynamics, and unknown but bounded $|f(t)| \le f_{\max}$. The modelling procedure is given in Appendix A.

The system model is fourth order. The problem with this system model is that the system parameters are needed. It is difficult to define all these parameters. To simplify the modeling process, the model of the system can be approximated using the direct identification approach with Process Reaction Curve Method based on the experimental test. Step input signal change is applied to the system and the output is recorded. Based on the output data and using the output plot, the system model can be approximated as [37].

$$G(s) \cong K\frac{e^{-T_d s}}{1+Ts} \cong \frac{K}{(1+T_d s)(1+Ts)} \tag{2.7}$$

where T_d is the time delay, T is the time constant, *K* is the steady-state gain. Then, the output can be given as:

$$\ddot{\omega}_L(t) = -A_r\dot{\omega}_L(t) - B_r\omega_L(t) + C_r u(t) + d(t) \tag{2.8}$$

where the constants are $A_r = (T_d + T)/(T_d T)$, $B_r = 1/(T_d T)$, $C_r = K/(T_d T)$, and $d(t)$ presents external load disturbance, nonlinear friction, and unmodelled dynamics, and other uncertainties. It is is bounded but unknown, $|d(t)| \le d_{\max}$.

CHAPTER 3

SLIDING MODE CONTROL

3.1 Introduction

Sliding Mode Control is a robust control scheme based on the concept of changing the structure of the controller in response to the changing state of the system in order to obtain a desired response [10,39,40,41]. A high speed switching control action is used to switch between different structures and the trajectory of the system is forced to move along a chosen switching manifold in the state space. The behavior of the closed loop system is thus determined by the sliding surface. The biggest advantage of SMC is its insensitivity to variation in system parameters, external disturbances and modeling errors.

Sliding mode control enables separation of overall system motion into independent partial components of lower dimensions and low sensitivity to plant parameter variations and disturbances [42]. These properties make sliding mode an efficient tool to control high order dynamic plants operating under uncertainty conditions which is common for control in a wide range of modern technology processes. For example, the gains in each feedback path switch between two values according to a rule that depends on the value of the state at each instant. The purpose of the switching control law is to drive the nonlinear plants state trajectory onto a prespecified (user-chosen) surface in the state space and to maintain the plants state trajectory on this surface for subsequent time. This surface is called the switching surface. When the plant state trajectory is above the surface, a feedback path has one gain and a different gain if the trajectory drops below the surface. This surface defines the rule for proper switching. This surface is also called a sliding surface (sliding manifold). Ideally, once intercepted, the switched control maintains the plants state trajectory on the surface for all subsequent time and the plants state trajectory slides along this surface Fig. [3.1].

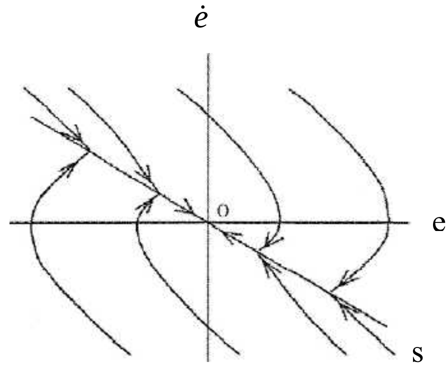


Figure 3-1: State Trajectories in Sliding Mode Control

SMC is the dynamic behavior of the system may be tailored by the particular choice of switching function, and insensitivity to variation in system parameters, external disturbances and modeling errors. Also, the ability to specify performance directly makes sliding mode control attractive from the design perspective. The motion of a SMC includes the reaching phase and the sliding phase. During the reaching phase, the system state is pushed towards the switching surface. During this period, however, the tracking error cannot be controlled directly and the system response is sensitive to parameter uncertainties and noise. Thus, one ideally would like to shorten the duration or even eliminate the reaching phase. One way to minimize the reaching phase and hence the reaching time is to employ larger control input. This however may cause extreme system sensitivity to unmodelled dynamics, actuator saturation etc [3].

Thus, the transient motion of a Sliding Mode Control System (SMCS) consists of two independent stage a (preferably rapid) motion bringing the state of the system to the manifold in which sliding occurs; and a slower sliding motion during which the state slides towards the origin of the state space while remaining in the sliding sub space. Once the state trajectory intercepts the surface, it remains on the surface for all subsequent time [10].

3.2 Sliding Surfaces

This section investigates Sliding Mode Control System (SMCS) as a high-speed switched feedback control resulting in sliding mode. The purpose of the switching control law is to drive the nonlinear plant's reference trajectory onto a prespecified (user-chosen) surface and to maintain the plants state trajectory on this surface for subsequent time. The surface is called a switching surface. When the plant trajectory is "above" the surface, a feedback path has one gain and a different gain if the trajectory drops "below" the surface. This surface defines the rule for proper switching. This surface is also called a sliding surface (sliding manifold). Ideally, once intercepted, the switched control maintains the plant's trajectory on the surface for all sebsequent time and the plant's trajectory slides along this surface. The most important task is to design a switched control that will drive the plant to the switching surface and maintain it on the surface upon interception.

A Lyapunov approach is uesd to characterize this task. Lyapunov method is usually used to determine the stability properties of an equlibrium point without solving V(0)=0 and V(t)>0 for t. It is said to be negative definite if V(0)=0 and V(t)>0 for t. Lyapunov method is to assure that the function ispositive definite when it is negative and function is negative definite if it is positive. In that way the stability is the equation. Let V(t) be a continuously differentiable scalar function defined in a domain D that contains the origin. A function V(t) is said to be positive definite if assured .

A generalized Lyapunov function, that characterizes the motion of the trajectory to the sliding surface, is defined in terms of the surface. For each chosen switched control structure, one chooses the "gains" so that the derivative of this Lyapunov function is negative definite, thus guaranteeing motion of the trajectory to the surface. After proper design of the surface, a switched controller is constructed so that the tangent vectors of the output trajectory point towards the surface such that the output is driven to and maintained on the sliding surface. Such controllers result in discontinuous closed-loop systems [28].

3.3 Principle and Properties of Sliding Mode Control

3.3.1 Conventional SMC

Sliding surface, $s(t)$ in the conventional SMC depends on the the error signal, $e(t)$ and derivative(s) of the error signal as [3.1]:

$$s(t) = \left(\lambda + \frac{d}{dt}\right)^{n-1} e(t) \tag{3.1}$$

where n is the order of uncontrolled system, λ is a positive constant, $\lambda \in R^{+}$, R^{+} is the space of positive real constants.

The sliding surface given in (3.1), however, causes steady-state error at the controlled output if the uncontrolled system does not include inherent integral action. The zero steady-state error is important in control system such as servo and regulation problems [3].

3.3.2SMC with Integral Action *(SMC+I)*

The sliding surface can be improved to overcome steady-state error problem by introducing integral action into the sliding surface for the present problem as:

$$s(t) = \left(\lambda + \frac{d}{dt}\right)^{n-1} e(t) + k_i \int_0^{\infty} e(t) \tag{3.2}$$

where k_i is the integral gain, $k_i \in R^{+}$

3.4 Stability

The control objective is to determine a control law, $u(t)$ such that the tracking error, $e(t)$ should converge to zero. The process of sliding mode control can be divided into two phases, that is, the sliding phase with $s(t) = 0$, $\dot{s}(t) = 0$, and the reaching phase with $s(t) \neq 0$. Corresponding to two phases, two types of control law can be derived separately [1,13]. Conceptually, in sliding mode the equivalent control is described when the trajectory is near $s(t)=0$, while the switching control is determined in the case of $s(t)\neq 0$ [10].

Derivative of the sliding surface in (3.2) can be given as (n=2):

$$\dot{s}(t)=\ddot{e}(t)+\lambda\dot{e}(t)+k_i e(t) \tag{3.3}$$

A necessary condition for the output trajectory to remain on the sliding surface $s(t)$ is $\dot{s}(t)=0$ [1,3,13]:

$$\ddot{e}(t)+\lambda e(t)+k_i e(t)=0 \tag{3.4}$$

If the control gains λ and k_i are properly chosen such that the characteristic polynomial in (3.4) is strictly stable, that is, roots of the polynomial are in the open left half of the complex plane, it implies that $\lim_{t\to\infty} e(t)=0$ meaning that the closed-loop system is globally asymptotically stable [21].

If the system parameters are inserted into (3.3), it becomes

$$\dot{s}(t)=\ddot{\omega}_r(t)+A_r\dot{\omega}_L(t)+B_r\omega_L(t)-C_r u(t) \quad -d(t)+\lambda\dot{e}(t)+k_i e(t) \tag{3.5}$$

where $\omega_r(t)$ is the set-point speed. A necessary condition for the output to remain on the sliding surface $s(t)$ is $\dot{s}(t)=0$ [10,13,24]. The equivalent control law is obtained when $\dot{s}(t)=0$ in the case of unknown $d(t)$:

$$u_{eq}(t)=\frac{1}{C_r}\left(\ddot{\omega}_r(t)+A_r\dot{\omega}_L(t)+B_r\omega_L(t) \ +\lambda\dot{e}(t)+k_i e(t)\right) \tag{3.6}$$

If the initial output trajectory is not on the sliding surface $s(t)$, or there is a deviation of the representative point from $s(t)$ due to parameter variations and/or disturbances, the controller must be designed such that it can drive the output trajectory to the sliding mode $s(t)=0$. The output trajectory under the condition that will move toward and reach the sliding surface is said to be on the reaching phase. For this purpose, the Lyapunov function can be chosen as

$$V(t) = \frac{1}{2} s^2(t) \qquad (3.7)$$

with $V(0) = 0$ and $V(t) > 0$ for $s(t) \neq 0$. A sufficient condition to guarantee that the trajectory of the error will translate from reaching phase to sliding phase is to select the control strategy, also known as reaching condition [10]:

$$\dot{V}(t) = s(t)\dot{s}(t) < 0\,,\ s(t) \neq 0. \qquad (3.8)$$

To satisfy the reaching condition, the equivalent control $u_{eq}(t)$ given in (13) is augmented by a hitting control term, $u_{sw}(t)$, to be determined. Consider the system given in (7) with uncertainties and disturbances and the sliding mode controller is designed such that;

$$u(t) = u_{eq}(t) + u_{sw}(t) \qquad (3.9)$$

If (14) is rewritten, it is obtained:

$$s(t)\dot{s}(t) = s\left[\ddot{\omega}_r(t) + A_r\dot{\omega}_L(t) + B_r\omega_L(t) - C_r(u_{eq}(t) + u_{sw}(t)) - d(t) + \lambda\dot{e}(t) + k_i e(t)\right] \qquad (3.10)$$

The switching control is chosen as

$$u_{sw}(t) = k_{sw}\,\mathrm{sgn}(s) \qquad (3.11)$$

where k_{sw} is a positive constant, $k_{sw} \in R^+$, sgn(.) denotes signum function defined as [10] .

$$\text{sgn}(s(t)) = \begin{cases} +1, & \text{if } s(t) > 0 \\ 0, & \text{if } s(t) = 0 \\ -1 & \text{if } s(t) < 0 \end{cases} \tag{3.11}$$

Derivative of the Lyapunov function is

$$\dot{V}(s(t)) = s(s)\dot{s}(t)$$
$$= -s(t)k_s C_r \text{ sgn}(s(t))$$
$$= -k_s C_r |s(t)| \leq 0 \tag{3.12}$$

This implies that $\dot{V}(s(t))$ is a negative semi-definite function. Define the following term:

$$X(t) = k_s C_r |s(t)| \tag{3.13}$$

therefore

$$X(t) \leq -\dot{V}(s(t)) \tag{3.14}$$

then

$$\int_0^t X(\tau)d\tau \leq V(s(0)) - V(s(t)) \tag{3.15}$$

Since $V(s(0))$ is bounded and $V(s(t))$ is non-increasing and bounded, the following result can be concluded

$$\lim_{t\to\infty} \int_0^t X(\tau)d\tau < \infty \tag{3.16}$$

Also, $\dot{X}(t)$ is bounded, and it can be shown that $\lim_{t\to\infty} X(t) = 0$ by the Barbalat's Lemma [1]. That is, $s(t) \to 0$ as $t \to \infty$. By applying this switching control law, the sliding mode control system can be guaranteed to be stable in the Lyapunov sense.

3.5 Sliding Signal

The continuous sliding mode control has been selected commonly in SMC problems to avoid chattering of the control force and to achieve the exponential stability [13]. Instead of signum function given in Eq. (18), a saturation function has been used via introducing a thin boundary layer around the sliding surface to avoid chattering [1,32]. For a more smooth change of the switching signal, a hyperbolic tangent function has also been used to improve the switching control effort and it is given as [10]

$$u_{sw}(t) = k_{sw} \tanh\left(\frac{s(t)}{\Omega}\right) \tag{3.17}$$

where Ω are positive constants, $\Omega \in R^{+}$, and it defines the thickness of the boundary layer.

3.6 Design of Sliding Mode Controller

The second stage of the design procedure involves the selection of the control which will ensure that the chosen sliding mode is attained. For this reason, the problem of determining a control structure and associated gains, which ensure the reaching or hitting of the sliding mode, is called the reachability problem-.

The solution of the reachability problem is dependent on the switching hyperplanes, and so cannot be achieved until the existence problem has been solved. The condition under which the state will move towards and reach a sliding surface is called a reaching condition. The system trajectory under the reaching condition is said to be in the reaching mode, or reaching phase. Three approaches for specifying the reaching condition are available :-

3.6.1 The Direct Switching Function Approach

The earliest condition proposed was:

$$s_i . \dot{s}_i < 0 \quad i = 1,2,3 \ldots m \tag{3.18}$$

where m is number of inputs.

This reaching condition is global but does not guarantee a finite reaching time. Also, it is very difficult to use for multiple input VSCS. Even with a simplifying assumption, such as adopting fixed order switching, the approach remains difficult.

3.6.2 The Reaching Law Approach

The crux of the reaching law approach is a new method called the reaching law method. It directly specifies the dynamics of the switching function. Let the dynamics of the switching function be specified by the differential equation :

$$s = Q\,\mathrm{sgn}(s) - K\,\mathrm{f}(s) \qquad (3.19)$$

where gains Q and K are diagonal matrices with positive elements.Equation (3.6) is called the 'reaching law'. Various choices of *Q* and *K* specify different rates for *s* and yield different structures in the reaching law. Three examples are :

a. The constant rate reaching law

$$s = -Qsgn\{s\} \qquad (3.20)$$

b. The constant plus proportional rate reaching law

$$s = -Qsgn\{s\}- Ks \qquad (3.21)$$

c. The power rate reaching law

$$\dot{s}_s = -K_i |s_i|^{\alpha}\,\mathrm{sgn}\, s_i \qquad (3.22)$$

where, $0 < a < 1y$. and $i = 1 \text{ to } m$ (3.23)

3.7 Basic Theorem of Lyapunov

Let V (x,t) be a non-negative function with derivative $\dot{V}$ along the trajectories of the system.

1. If $V(x,t)$ is locally positive definite and $V(x,t) \leq 0$ locally in x and for all t, the the origin of the system is locally stable (in the sense of Lyapunov).

2. If $V(x,t)$ is locally positive definite and decrescent, and $\dot{V}$ (x, t) ≤ 0 locally in x and for all t, then the origin of the system is uniformly locally stable (in the sense Lyapunov).

3. If $V(x,t)$ is locally positive definite and decrescent, and $\dot{V}$ (x, t) is locally positive definite, then the origin of the system is uniformly locally asymptotically stable.

4. If $V(x,t)$ is positive definite and decrescent, and $\dot{V}$ (x, t) is positive definite, then the origin of the system is globally uniformly asymptotically stable.

This situation gives sufficient conditions for the stability of the origin of a system. It does not, however, give a prescription for determining the Lyapunov function *V* (*x, t*). Since the theorem only gives sufficient conditions, the search for a Lyapunov function establishing stability of an equilibrium point could be arduous. However, it is a remarkable fact that the converse of (4) also exists: if an equilibrium point is stable, then there exists a function *V* (*x,t*) satisfying the conditions of the theorem. However, the utility of this and other converse theorems is limited by the lack of a computable technique for generating Lyapunov functions.

Asymptotically stable situation also stops short of giving explicit rates of convergence of solutions to the equilibrium. It may be modified to do so in the case of exponentially stable equilibria.

3.8 Barbarat's Lemma

A function tends toward a finite limit. Barbarat's lemma indicates that the derivative itself should have some smoothnes. To apply barbarat's lemma to the analysis of dynamics systems, one typically uses the following immediate corollary, which looks very much like an invariant set theorem in Lyapunov analysis:
Lemma:If a scalar $V(x,t)$ satisfies the following conditions,

1. $V(x,t)$ is lower bounded
2. $\dot{V}$ *(x,t)* is negative semi-define
3. $\dot{V}$ *(x,t)* is uniformly continous in time.

Then $\dot{V}$ (x,t)$\rightarrow 0$ a t$\rightarrow\infty$ indeed V then approaches a finite limiting value V∞ such that V$\infty \leq$V (x(0),0. (this does not require uniform continuity).

3.9 Properties of Sliding Mode Control

The design of VSC can proceed with the structure of the control u(x) free or preassigned at the outset. In either case, the objective is to satisfy a reaching condition. In the free structure approach, generally, the control u(x) can be solved by constraining the switching function to any one of the following conditions.

1. $s\dot{s}$ <0 (Direct switching approach)
2. $\dot{V} = \frac{d}{dt}(s\dot{s}) < 0$ (Lyapunov function approach)
3. $s_i = -q_i \operatorname{sgn}(s_i) - k_i f_i(s_i)$ (Reaching law approach).

Additional forms of constraint exist, including

4. $\dot{V}$ = $-q - kV$ $\quad V = s^T s$
5. $\dot{s}$ = -F(s) [16]

The latter forms are seldom used in practice. In some cases, it is convenient to preassign the structure of the VSC and then determine the values of the controller gains such that the desired reaching law is satisfied. Three popular types of preassigned structures are given here.

3. 10 Chattering Reduction

The term “chattering” describes the phenomenon of finite frequency, finite-amplitude oscillations appearing in many sliding mode implementations. These oscillations are caused by the high-frequency switching of a sliding mode controller exciting unmodeled dynamics in the closed loop. ‘Unmodeled dynamics’ may be those of sensors and actuators neglected in the principal modeling process since they are generally significantly faster than the main system dynamics.

However, since ideal sliding mode systems are infinitely fast, all system dynamics should be considered in the control design. Fortunately, preventing chattering usually does not require a detailed model of all system components. Rather, a sliding mode controller may be first designed under idealized assumptions of no unmodeled dynamics. The solution of the chattering problem is of great importance when exploiting the benefits of a sliding mode controller in a real life system. To some extend, chattering, without proper treatment in the control design, has been a major obstacle for implementation of sliding mode to a wide range of applications. It should be noted that the switching action itself as the core of a continuous-time sliding mode system is not referred to as chattering since in the ideal case, the switching is intended and its frequency tends to infinity; chattering, in the terminology used here, describes undesired system oscillations with finite frequency caused by system imperfections.

An ideal sliding mode exists only when the state trajectory x(t) of the controlled plant agrees with the desired trajectory at every $t \geq t_1$ for some t_1 . This may require infinitely fast switching. In real systes, a switched controller has imperfections which limit switching to a finite frequency. The representative point then oscillates within a neighborhood of the switching surface. This oscillation, called chattering, is illustrated on Figure 3.2

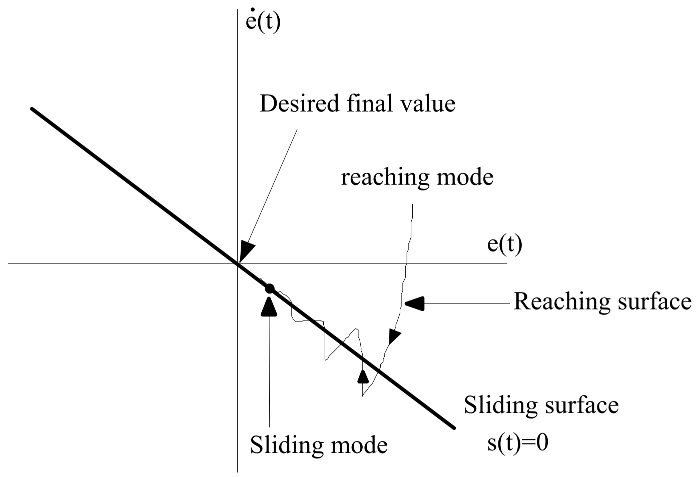


Figure 3.2: Chattering as a result of imperfect control switchings

Control laws which satisfying sliding condition (3.4) and lead to "perfect" tracking in the face of model uncertainty, are discontinuous across the surface S(t), thus causing

control chattering. Chattering is undesirable, since it involves extremely high control activity, and furthermore may excite high-frequency dynamics neglected in the course of modeling. Chattering must be reduced (eliminated) for the controller to perform properly. This can be achieved by smoothing out the control discontinuity in a thin boundary layer neighboring the switching surface

$$B(t) = \{x, |s(x,t)| \leq \phi\} \qquad \phi > 0 \tag{3.24}$$

where f is the boundary layer thickness, and $\varepsilon = \phi / \lambda^{n-1}$ is the boundary layer width. In other words, outside of B(t), we choose control law as before which guarantees that the boundary layer is attractive, hence invariant; all trajectories starting inside B(t=0) remain inside B(t) for all $t > 0$; and then u is interpolated inside B(t). For example, sgn(s) in (3.18) can be replaced by $\frac{s}{\phi}$ inside B(t).

This approach leads to tracking to within a guaranteed precisione (rather than perfect tracking), and more generally guarantees that for all trajectories starting inside

B(t=0)

$$\forall t \geq 0, \left|\tilde{x}^{(i)}(t)\right| \leq (2\lambda)^{i}\varepsilon \qquad i = 0,.........n-1 \qquad (3.25)$$

Due to its robustness properties, sliding mode controller can solve two major design difficulties involved in the design of a braking control algorithm:

1. The vehicle system is highly nonlinear with time-varying parameters and uncertainties;
2. The performance of the system depends strongly on the knowledge of the tire/roadsurface condition;

For the class of systems to which it applies, sliding controller design provides a systematic approach to the problem of maintaining stability and consistent performance in the face of modeling imprecision.
For wheel slip control system, the vehicle and brake system are highly nonlinear and time-varying systems. That makes sliding mode controller ideal candidate for the application.

3.11 Chattering Suppression Methods

1. The boundary layer solution: a continuous approximation of the discontinuity.
2. The observer-based solution: generating sliding mode in a observer loop without unmodeled dynamics.
3. The regular form solution: limiting sliding mode to an inner control loop of a cascaded control structure.
4. The disturbance rejection solution: generating integral sliding mode in an auxiliary control loop.

3.12 Boundary layer solution

The boundary layer solution, proposed e.g. by [Slotine and Sastry 1983] and [Slotine 1984], seeks to avoid control discontinuities and switching action in the control loop. The discontinous control law is replaced by a saturation function which approximates the sgn(s) term in a boundary layer of the sliding manifold s(t) = 0. Numerous types of saturation functions sat(s) have been proposed in the literature.

“In the large”, i.e. for s (t) $> \varepsilon$, sat(s)=sgn(s). However, in a small e -vicinity of the origin, the so-called boundary layer, sat(s) sgn(s) is continuous. As an illustrative example, consider a simple linear saturation function.

$$u(t) = \begin{cases} M \ \text{sgn (s(t))} & \text{for} \quad \text{s(t)} > \varepsilon \\ \dfrac{M}{\varepsilon} s(t) & \text{for} \quad \text{s(t)} < \varepsilon \end{cases} \tag{3.26}$$

Control loop with actuator dynamics neglected in ideal control design. Sliding mode does not occur since the actuator dynamics are excited by the fast switching of the discontinuous controller, leading to chattering in the loop with linear proportional feedback gain $\frac{M}{\varepsilon}$ within the boundary layer in the vicinity of the origin, s(t) $\leq \varepsilon$, and symmetrically saturated by M for s(t) $> \varepsilon$ outside the boundary layer. One of the benefits of the boundary layer approach is that sliding mode control design methodologies can be exploited to derive a *continuous* controller. The invariance property of sliding mode control is partially preserved in the sense that the system trajectories are confined to a s(ε)vicinity of the sliding manifold s (t) = 0, instead of exactly to s(t) = 0 as in ideal sliding mode. Within the s(ε) vicinity, however, the system behavior is not determined, i.e. further convergence to zero is not guaranteed. This type of control design is part of a class of robust controllers which satisfy the “globally uniform ultimate boundedness” condition proposed by [Leitmann 1981]. Note that no *real* sliding mode takes place since the switching action is replaced by a continuous approximation.

Instead of achieving ideal sliding mode, the system trajectories are confined to a boundary layer of the manifold s (t) = 0.

3.13 PID Based Sliding Mode Controller

We propose to overcome the problems associated with SMC and SMC with boundary layer in process control settings by introducing a sliding mode controller based on a PID design. (PIDSMC). The original SMC structure is retained in the proposed a sliding mode controller based on a PID design, except that the discontinuous switching control input is replaced with a continuous input determined by a PID algorithm. The PID controller in the sliding mode controller based on a PID design, takes the sliding surface function *s* as the input, not the controlled variable as in a conventional PID control-loop. The resulting over-all control input of the proposed sliding mode controller based on a PID design.is:

$$u_{pid} = k_p(\lambda + \frac{d}{dt})^{n-1} e(t) + k_i \int_0^t e(t) \ \mathrm{d(t)}) \qquad (3.27)$$

where k_p ,k_i and λ are the PID proportional gain, integral time constant and derivative time constant, respectively, and *s* is the sliding surface function. The order of the system *n=2* so results in a PID controller in terms of *s* :

$$u_{pid} = \lambda e(t) + k_i \int \mathrm{e(t)\ dt} + k_p \frac{de(t)}{dt} \qquad (3.28)$$

The sliding surface choosen like this, the $\dot{s} = 0$,[11]

$$\dot{s}(t) = \ddot{\omega}_r(t) + A_r \dot{\omega}_L(t) + B_r \omega_L(t) - C_r u(t) - d(t) + \lambda \dot{e}(t) + k_i e(t) \qquad (3.29)$$

The PID proportional term drives the states to the neighbourhood of the sliding surface. The PID integral action forces the states onto the sliding surface irrespective of the bounds of the uncertainties and disturbances, while the PID derivative action provides a stabilizing effect to counter the possible excessive control produced by the integral action. The integral term and the derivative term play important roles in ensuring that the states move onto the sliding surface.

$$u_{eq}(t) = \frac{1}{C_r}\left(\ddot{\omega}_r(t) + A_r \dot{\omega}_L(t) + B_r \omega_L(t) + \lambda \dot{e}(t) + k_i e(t)\right) \qquad (3.30)$$

The working principle of the PID SMC may be briefly explained as follows. Assume that the system is initially in the region of s > 0 and that the PID proportional action term is not sufficient to drive the states toward the sliding surface. This will result in an increasing *s* and the process states will move away from the surface. The integral action will increase the control action accordingly and become sufficient after a period of time to force the states to move toward the sliding surface, satisfying the reachability condition $\dot{s}s < 0$. As s approaches the sliding surface, the control action will automatically be reduced because that $\dot{s}$ is negative and s is decreasing. The reachablity condition can be ensured by a proper selection of the integral and derivative gains. An on-line adaptive strategy is proposed in the next section to tune automatically these two parameters to ensure the reachability condition $\dot{s}s < 0$

$$s(t)\dot{s}(t) = s\left[\ddot{\omega}_r(t) + A_r\dot{\omega}_L(t) + B_r\omega_L(t) - C_r(u_{eq}(t) + u_{sw}(t)) - d(t) + \lambda\dot{e}(t) + k_i e(t)\right] \tag{3.31}$$

The proposed PID SMC control as shown in consists of three major components: a nonlinear state feedback control that transforms the process into a simpler equivalent form, a linear state feedback control that completes the design of the set-point tracking, and a PID feedback control of the sliding surface function that makes the system robust and eliminates the disturbances. In terms of the control system structure, the PID SMC may be viewed a special case of the generalized feedback linearizing controller. The particularities of the PID SMC are in the use of the set-point and the controller tuning. The outer-most loop of the PID SMC system onto the sliding surface.

For a more smooth change of the switching signal, a hyperbolic tangent function has also been used to improve the switching control effort and it is given as [10]

$$u_{sw}(t) = k_{sw}\tanh\left(\frac{s(t)}{\Omega}\right) \tag{3.32}$$

where Ω are positive constants, $\Omega \in R^+$, and it defines the thickness of the boundary layer.

CHAPTER 4
EXPERIMENTAL SET-UP AND LABORATORY RESULTS

4.1 Experımental Set-Up and Test Results

Diagram of the experimental set-up is shown in Fig. 1. A computer (Pentiun II MMX, 300 MHz, 256 MB RAM) is used to implement proposed adaptive sliding mode controller. The output shaft speed is measured from an optical sensor (as rev/s) and a tachogenerator (as volts) connected to the motor shaft. The slotted opto sensor consists of a gallium-arsenide infra-red L.E.D. and silicon phototransistor mounted in a special plastic case which is transparent to light of the wavelenght. A series of pulses is generated when the slotted disk, that is mounted on the motor shaft, is rotated. When the shaft of the dc motor is turned, a voltage is induced at the tachogenerator terminals which is directly proportional to shaft speed. The motor to be studied operates with a maximum output shaft speed of 1500 rev/min. The motor drives a shaft that carries disks which operate various transducers, and a tachogenerator. Motor speed is reduced by 9:1, that means r=9. A low pass filter is used to filter the output speed signal from high frequency noise components. The motor speeds at different input armature voltages are measured to obtain the tachogenerator characteristics. The tachogenerator has a linear characteristics with a calculated gain of 2.5 volt/rad/sec. The measured output data are transferred to the computer by a Data Acquisition card (Advantech, Model: PCL-1800, 330 kHz in speed, a conversion time of 2.5 μsec., and 0.01% accuracy). The sampling period is taken to be 3 msecs for all cases. All control solutions and calculations are performed in matlab environment, in Simulink of Matlab. The measured signal is transferred to the simulink by the Data acquisition card. The measured output signal is compared with the reference signal and the other calculations are performed to produce the hitting signal and the equivalent signal. The overall control signal (switching signal + equivalent signal) is produced and sent to the power amplifier by the Data acquisition card to control the real plant.

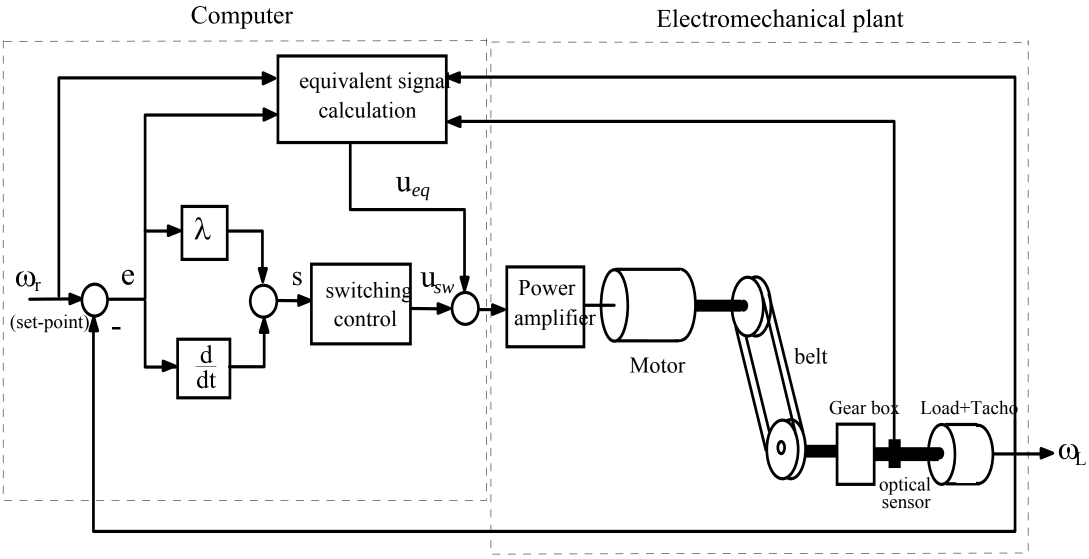


Figure 4.1 Diagram of the experimental set-up (SMC)

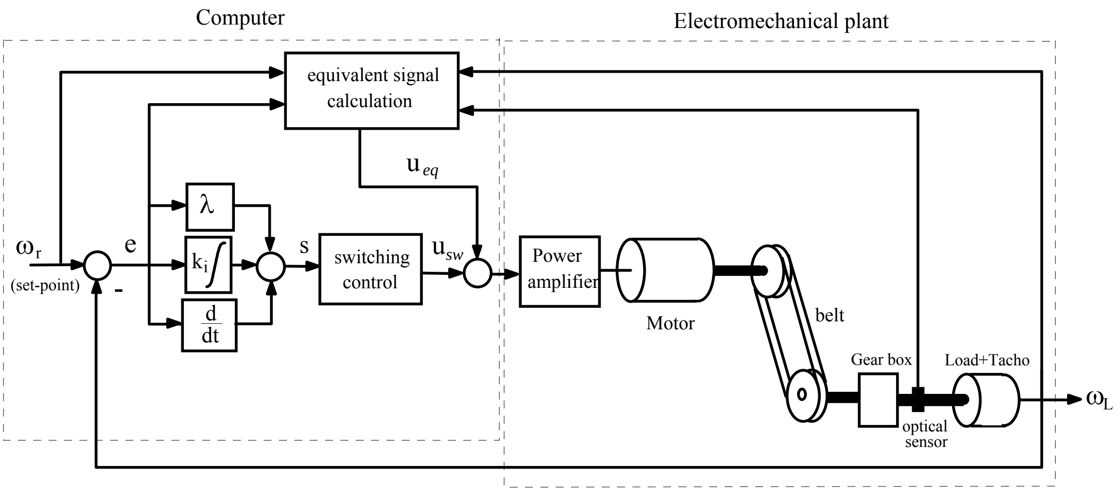


Figure 4.2. Diagram of the experimental set-up (SMC+I)

As a preliminary work, the plant should be tested to define the plant gain coefficients, A_r, B_r and C_r. This process is needed to obtain the equivalent control signal, $u_{eq}(t)$ and should be performed before the closed-loop operation is allowed. In open-loop conditions, a step input signal, amplitude of 2.5 volts, that corresponds to

1500 rpm is applied to the electromechanical plant from the computer. The approximate plant model that is based on the process reaction curve method is obtained as [36]:

$$G(s) \cong \frac{0.8}{(1+0.02s)(1+0.26s)} \tag{4.1}$$

4.1.1 Model Validation

The graph for the model validation is illustrated in Fig. 4.3 such that solid line presents the approximated model response and the dotted line is the real system output, and the speed error is illustrated in Fig. 4.4. The output speed settles down after 0.6 sec and the error is about ±6 rpm at the steady-state. Mean error in Fig. 4.4 is zero, that is, steady-state modeling error is -0.5. Using the approximate plant model, the plant gain coefficients are calculated to be $A_r = 53.84$, $B_r = 192.3$ and $C_r = 153.84$. Conventional PID controller parameters are determined using Process Reaction Curve Method and the gain coefficients are calculated to be $K_p = 7.5$, $T_i = 0.533$ and $T_D = 0.0115$. Heuristic method is used in general to choose the sliding mode controller parameters. There are certain guidelies here. First of all, the sliding mode controller parameters must be all positive real. Second, the polynomial, given in Eq. (3.3), must be stable. Third, the switching signal should be minimized not to hurt the actuator. The overshoot is not desired. From all these, to satisfy the requirements the parameters are selected to be $\lambda = 120$, $k_i = 1$, $\Omega = 30$ and $k_{sw} = 6$.

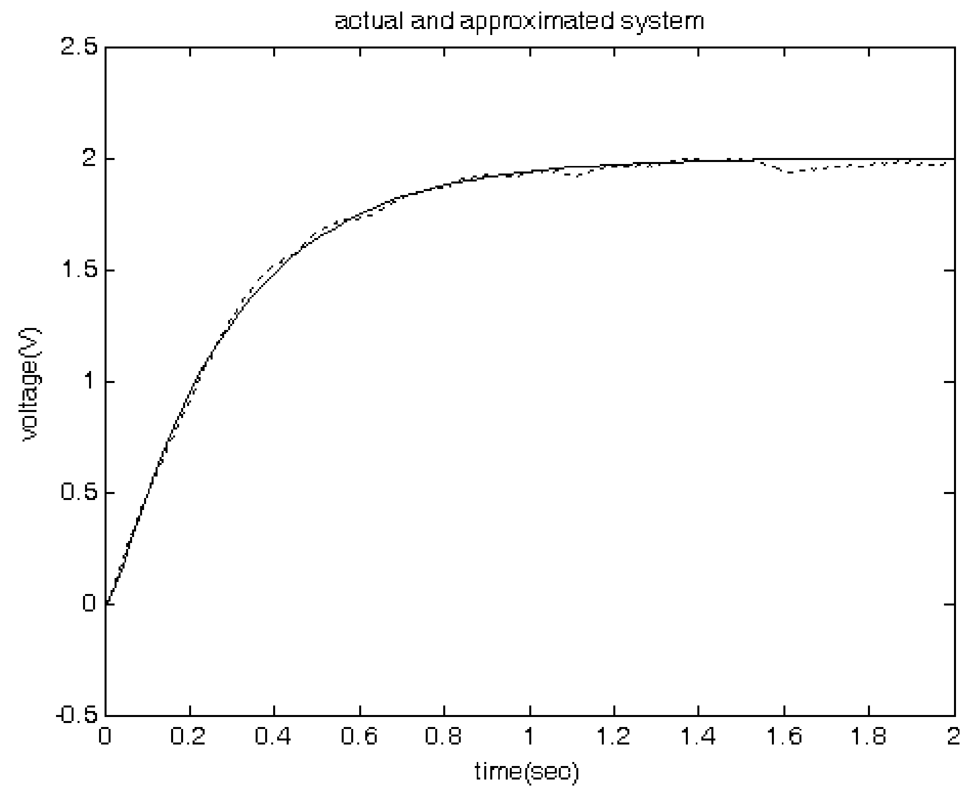


Fig. 4.3. Response of the actaul system and approximated system.

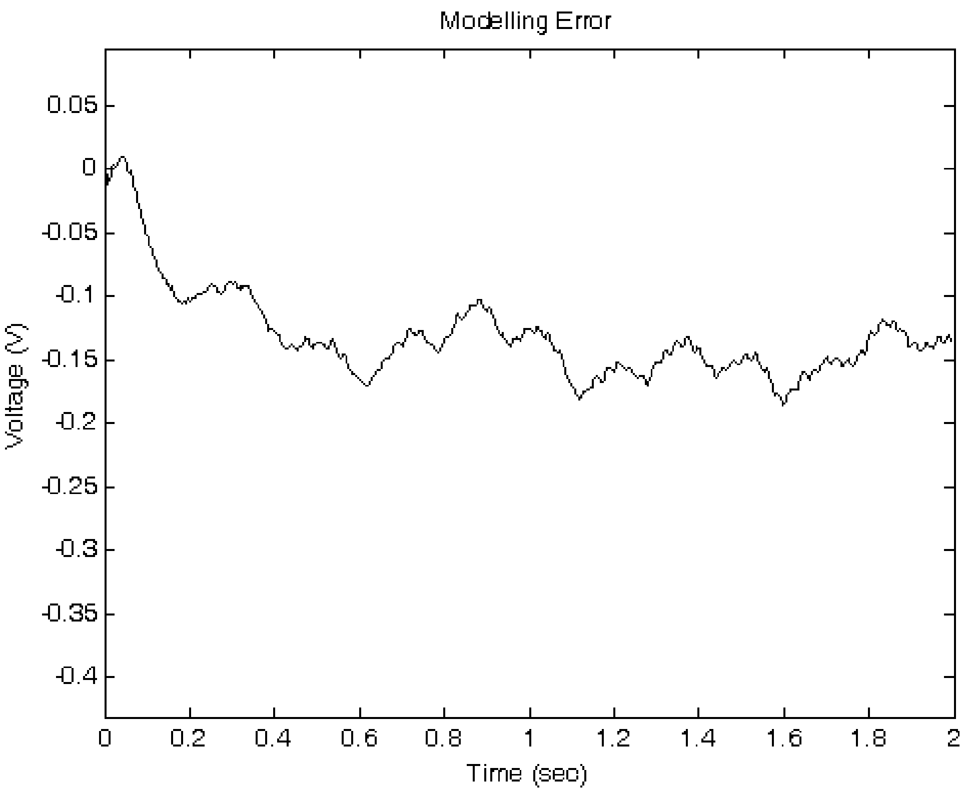


Fig. 4.4. Modeling error

4.5 Tracking Response for SMC and SMC+I

Tracking performance of conventional sliding mode controller (SMC) and proposed controller (SMC+I) are shown in Fig.4.11. The related control signals are given in Fig. 4.12. Square wave speed set-point signal change, 0 rpm to 1500 rpm and 1500 rpm to 0 rpm, is applied to the system. The performance of the system with the proposed sliding mode controller (SMC+I) is much beter than the system with the SMC controller.

The switching signals are shown in Fig.4.13 such that it is clearly seen that magnitudes of the variations are smaller in SMC+I control.

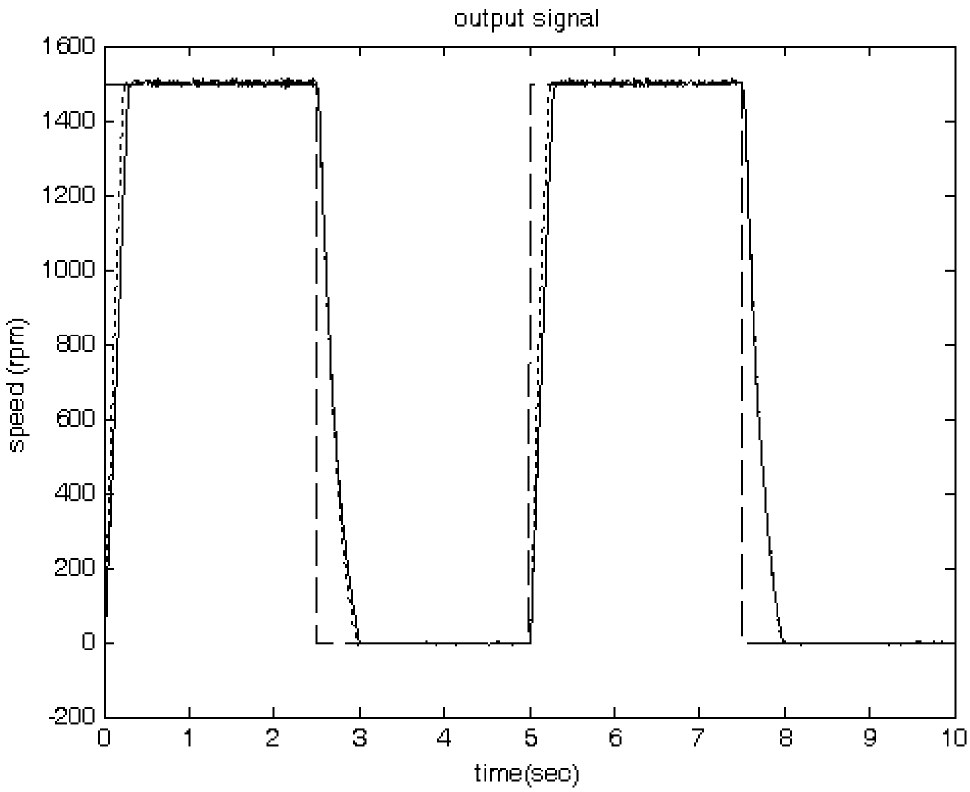


Figure 4.11 Tracking response of SMC and SMC+I control system.

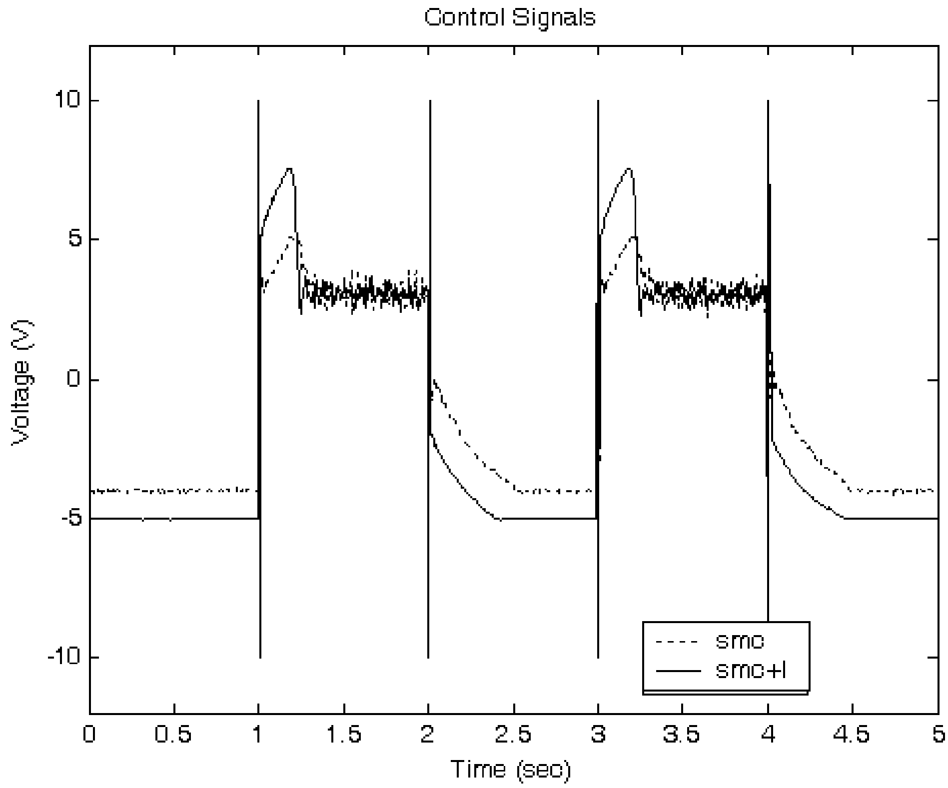
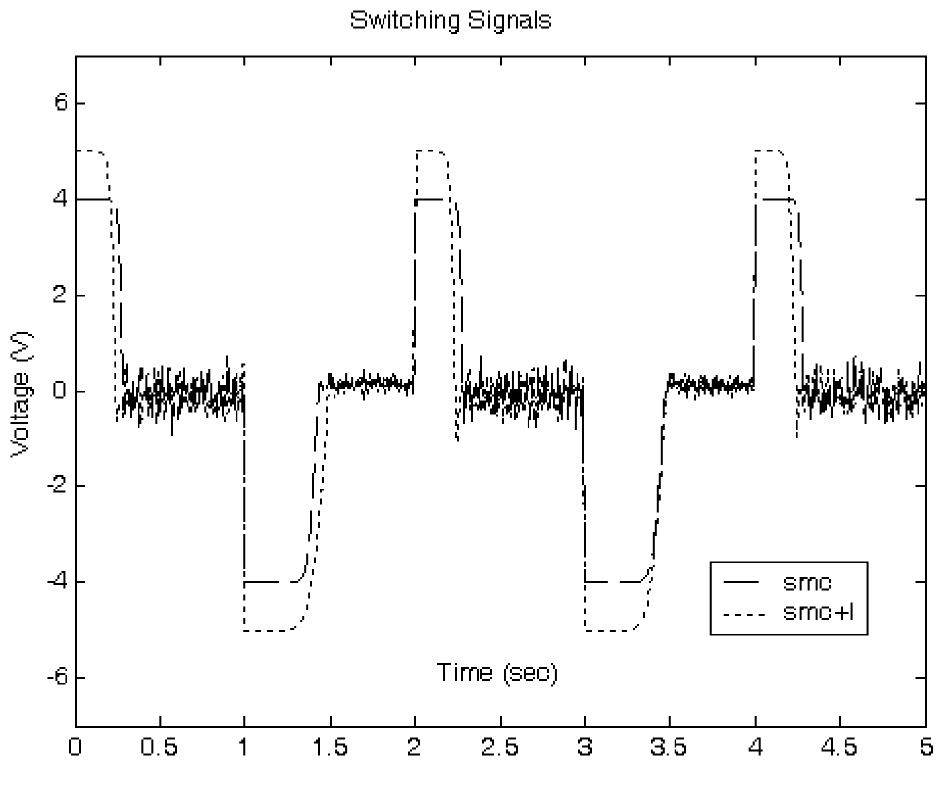


Fig. 4.12. Control signal response for SMC and SMC+I control system



4.13 Switching Signal Response for SMC and SMC+I control system.

4.1.2 Step test for P,PI,PD,PID

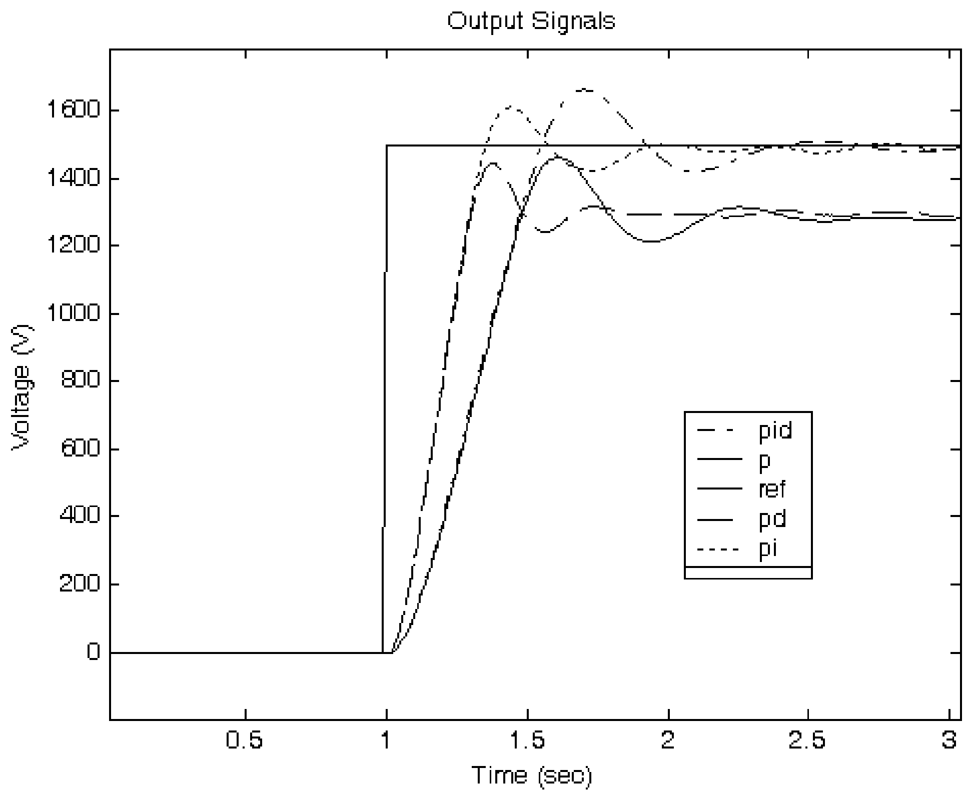


Figure 4.5 P, PD, PI, PID output signals

Figure 4.5 step set point test for P,PI,PD and PID . P and PD system have steady state. PID and PI sytem have no steady-state error. PID system gives so smooth response than others.

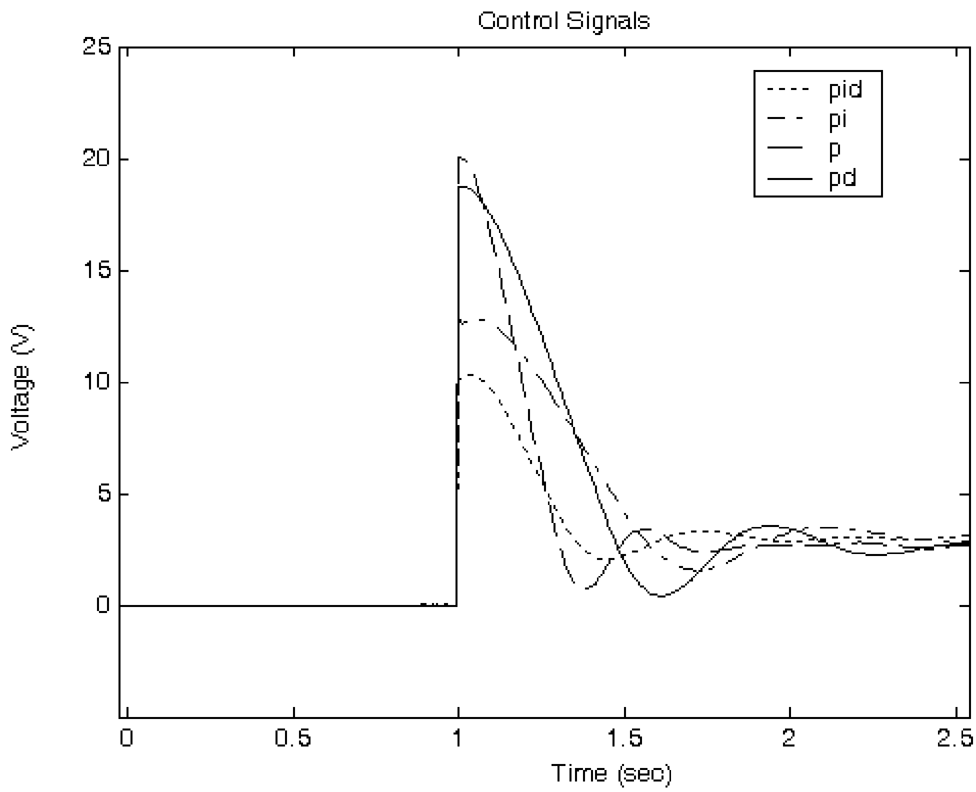


Figure 4.6 P, PD, PI, PID control

Related control signals are given in Figure 4.6. Response of the system to step set-point speed change (0-1500 rpm) are illustrated in Figure 4.5 Figure 4.5 for conventional P, PD, PI, PID controllers. PID controller is given smaller response than P, PD, PI.

4.5.3 Step test for SMC, SMC+I

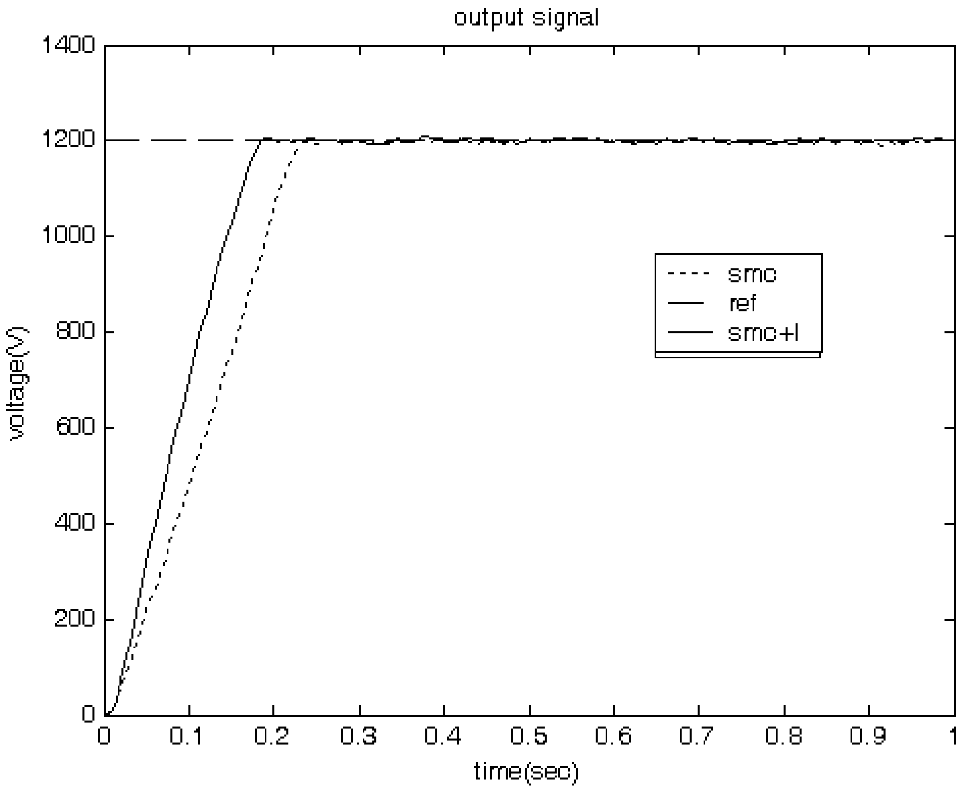


Fig 4.6 Step test for SMC and SMC+I response

The performance of the system with the proposed sliding mode controller (SMC+I) is much better than the system with the SMC controller and conventional sliding mode controller such that no overshoot, smaller rise time, and smaller settling time in magnitude were obtained from the proposed controller.

Table 1

Controller type	Rise time (sec)	Settling time (sec) (5%)	Overshoot (%)	Output deviations (rpm)
PID	0.3	2.5	50.9	±5.3
SMC	0.15	0.2	29.0	±3.8
SMC+I	0.18	0.18	0.0	±4.0

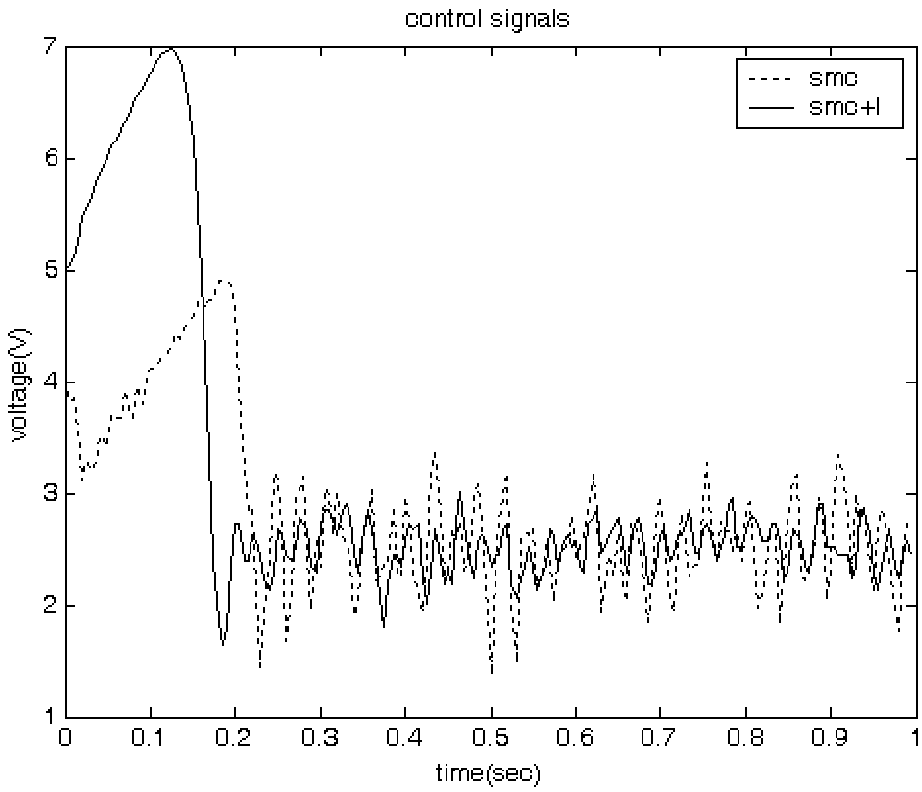


Fig. 4.7. Control signal response for SMC and SMC+I control system

Control signal response for SMC and SMC+I control system is given in Figure 4.7. SMC+I system gives higher response than the SMC system, but the settling time is changed with $\pm$ 0.5 in SMC+I.

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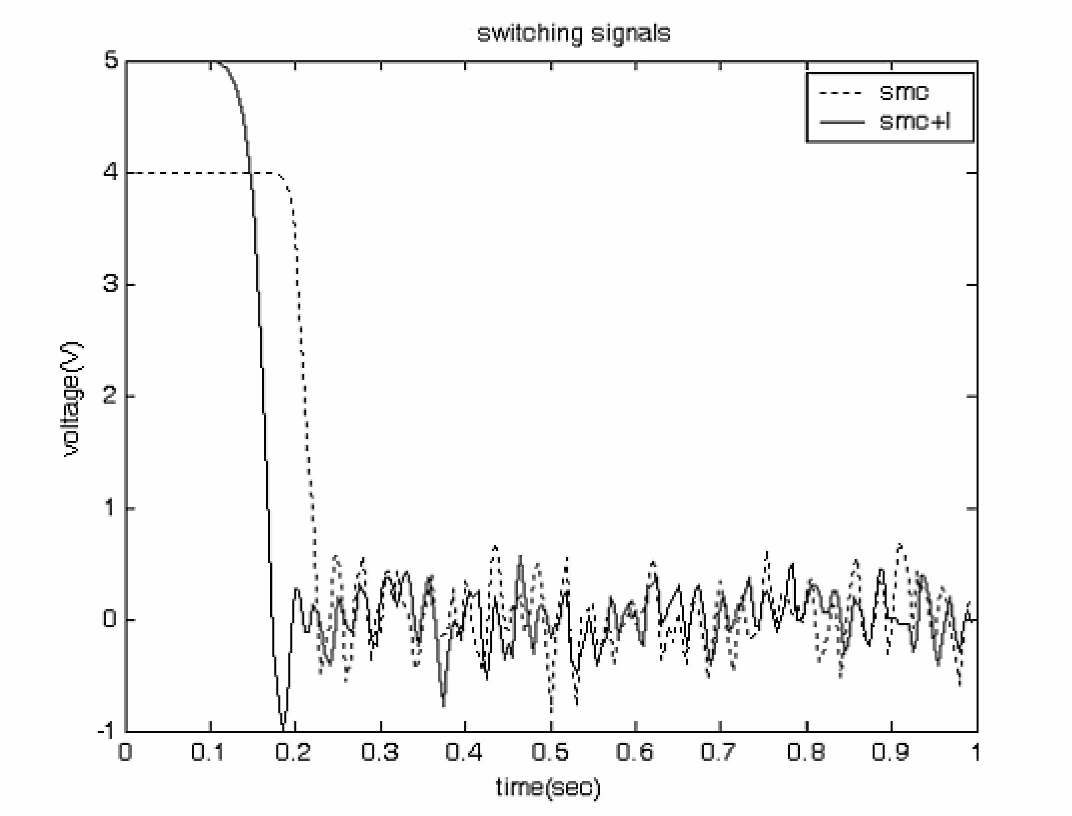

Fig.4.8 Switching signal response for SMC and SMC+I control system

Switching signal response for SMC and SMC+I control system is given in Figure 4.8. SMC+I system gives higher response than the SMC system, but the settling time is changed with ± 0.4 in SMC+I.

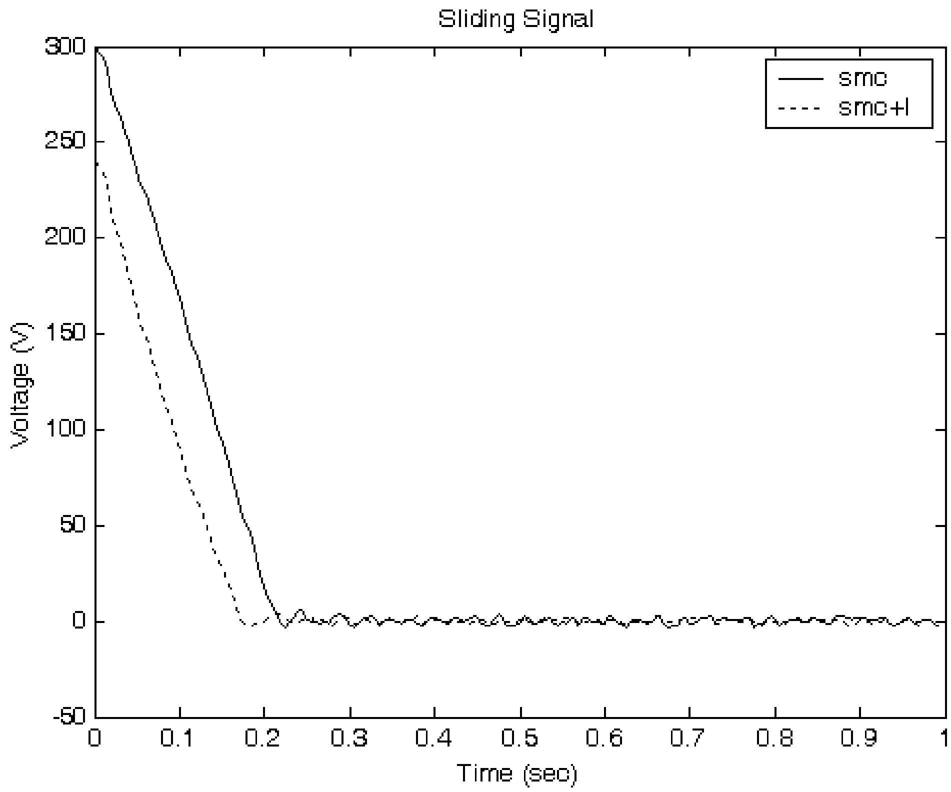

Figure 4.9 Sliding surface s(t)

Variation of the sliding surface $s(t)$ during the control is illustrated in Fig. 4.9. It can be noted that the sliding function is $s(t) \neq 0$ when the error signal is not zero. This means that the sliding mode is in the reaching surface up to about 0.18 sec and then arrives sliding surface. When it reaches to sliding surface, theoretically it is expected the sliding function to be zero, $s(t) = 0$. In practical applications, there are always some small deviations and fluctuations at the output measured variable because of uncertainties and disturbances. Here, the average value of the sliding function is zero, $s(t) = 0$. Variations in the sliding function occur due to errors or the disturbances and uncertainties.

Pulse generator is connected the system and SMC and SMC+I response is taken to system.

4.1.4 Tracking Response for SMC, SMC+I

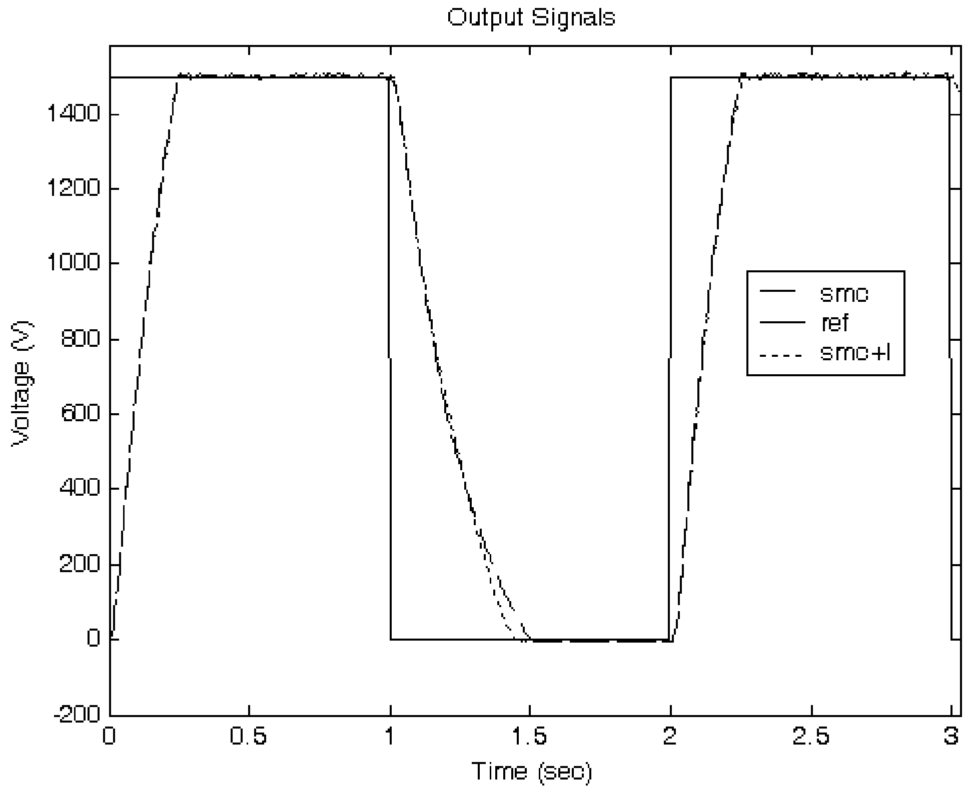

Fig. 4.10. Output signal response of SMC and SMC+I control system

The performance of the system with the proposed sliding mode controller (SMC+I) is much better than the system with the SMC controller. First pulse is given to system then SMC+I response is reach the 0 values rapidly. So, SMC+I system response is faster than SMC response.

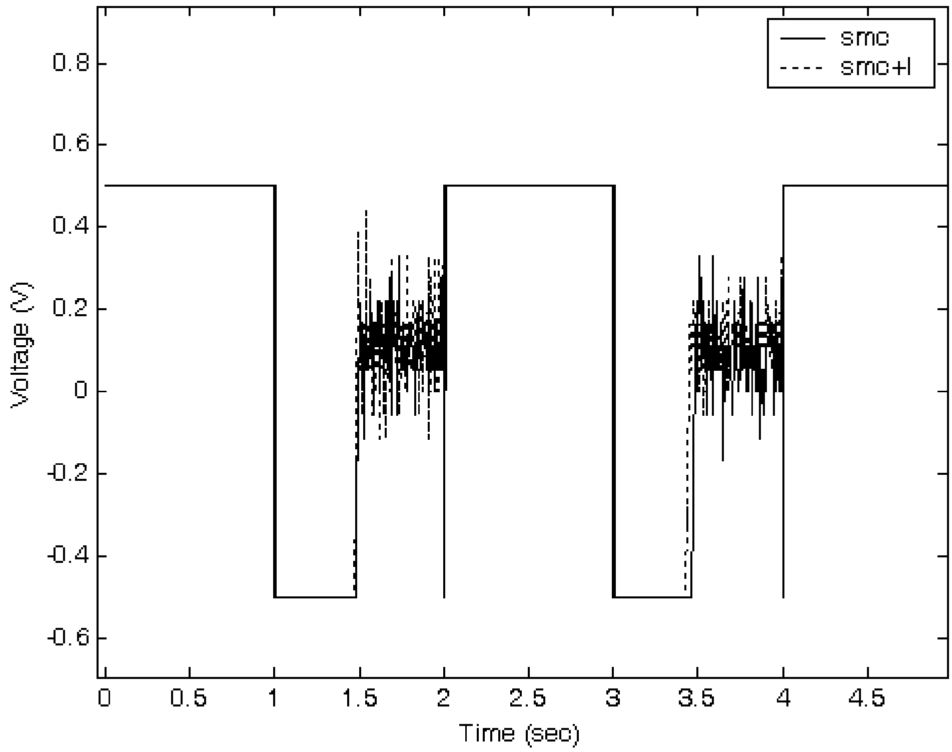

Fig. 4.11. Control signal response for SMC and SMC+I control system

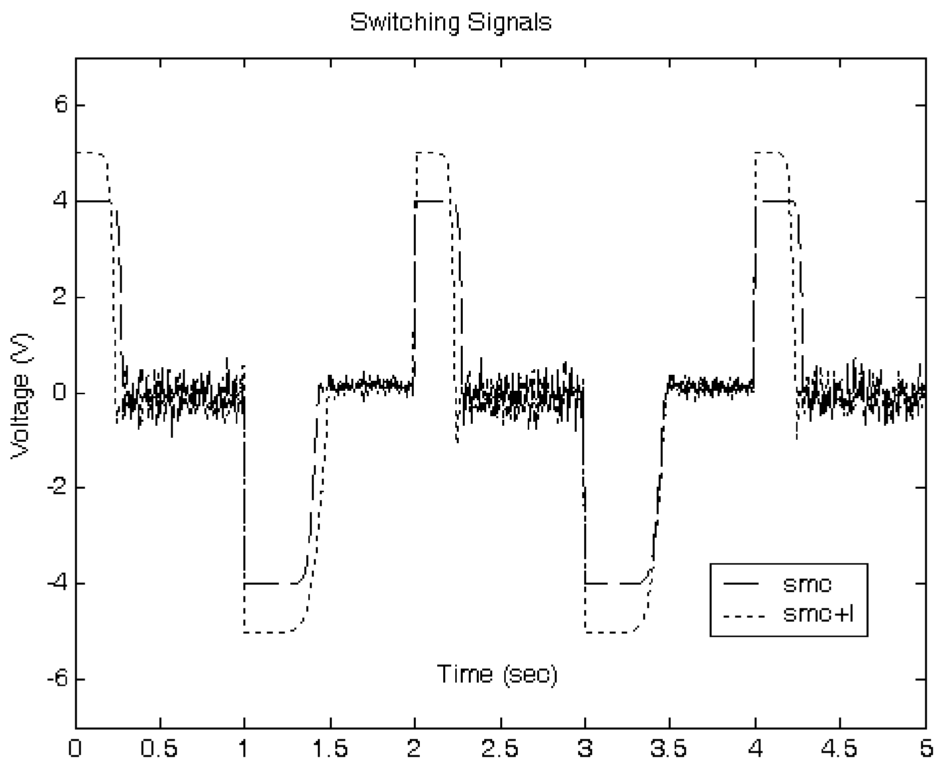

Figure 4.12 Switching Signal Response for SMC and SMC+I control system

It is clearly seen that magnitudes of the variations are smaller in SMC+I control. (Figure 4.11-12)

4.1.5 Load test

The load is connected the system at 1.5 sec. Then the output response is given in Figure 4.11-4.12.

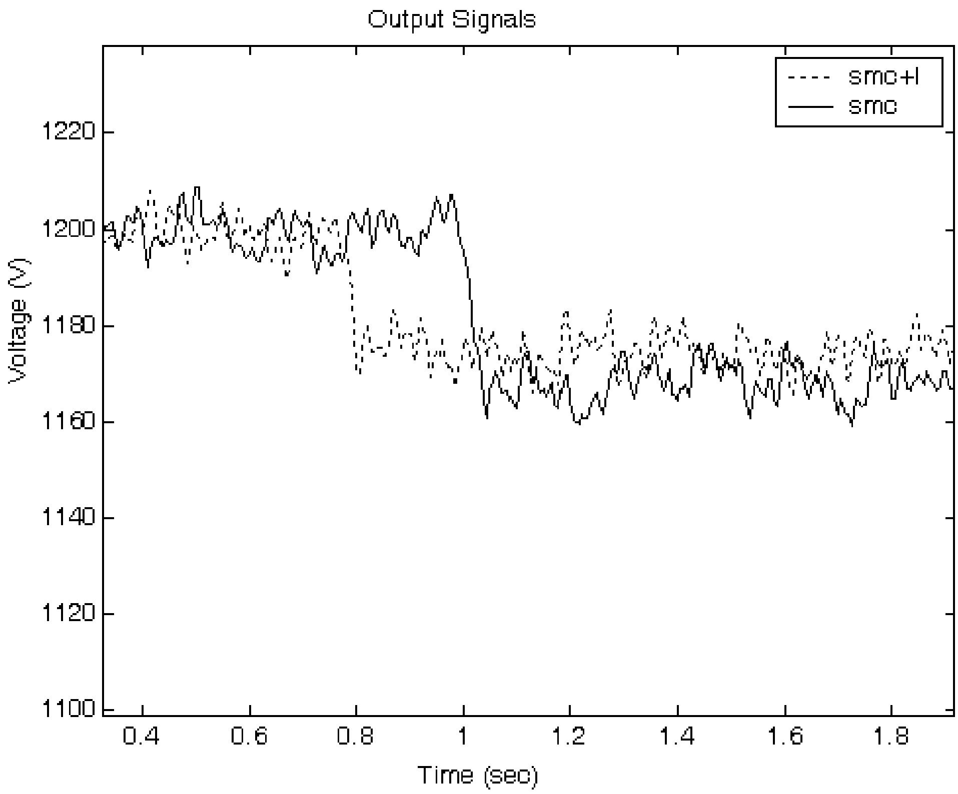

Fig. 4.13. Output response of SMC system with load.

Load test properly using the manually operated load called break system. When the system is running 1200 rpm, it is loaded using the manually brake system and the response are shown Fig.4.12.

The figure confirms the fact that, the magnitudes of the applied loads for SMC and SMC+I are same.

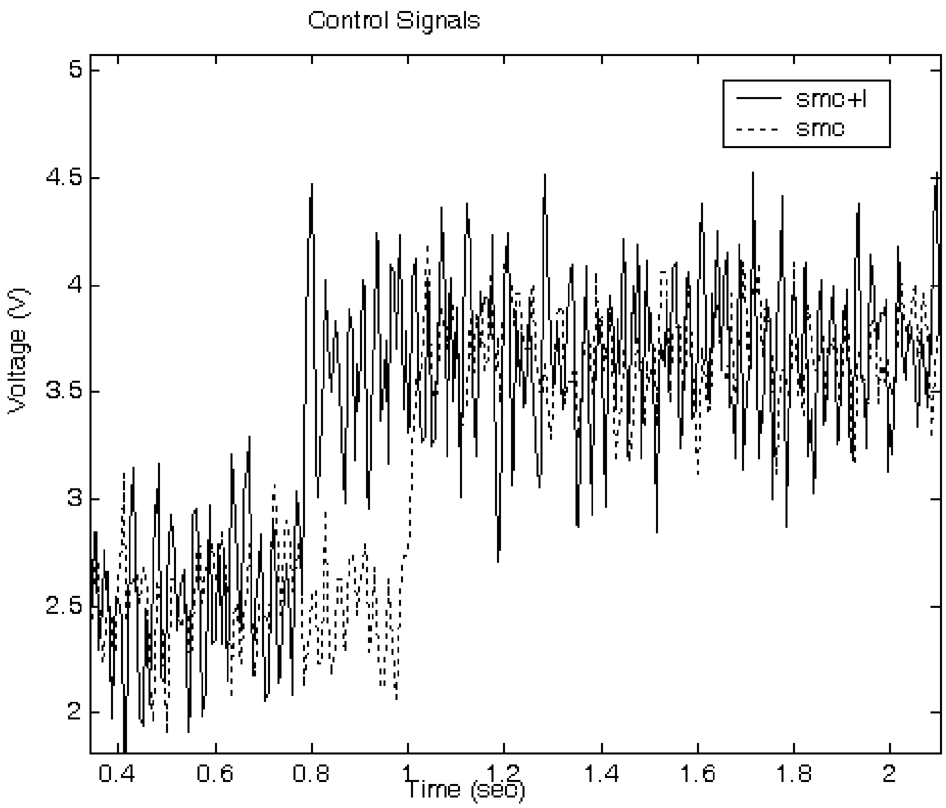

Fig. 4.14. Control response of SMC+I system with load

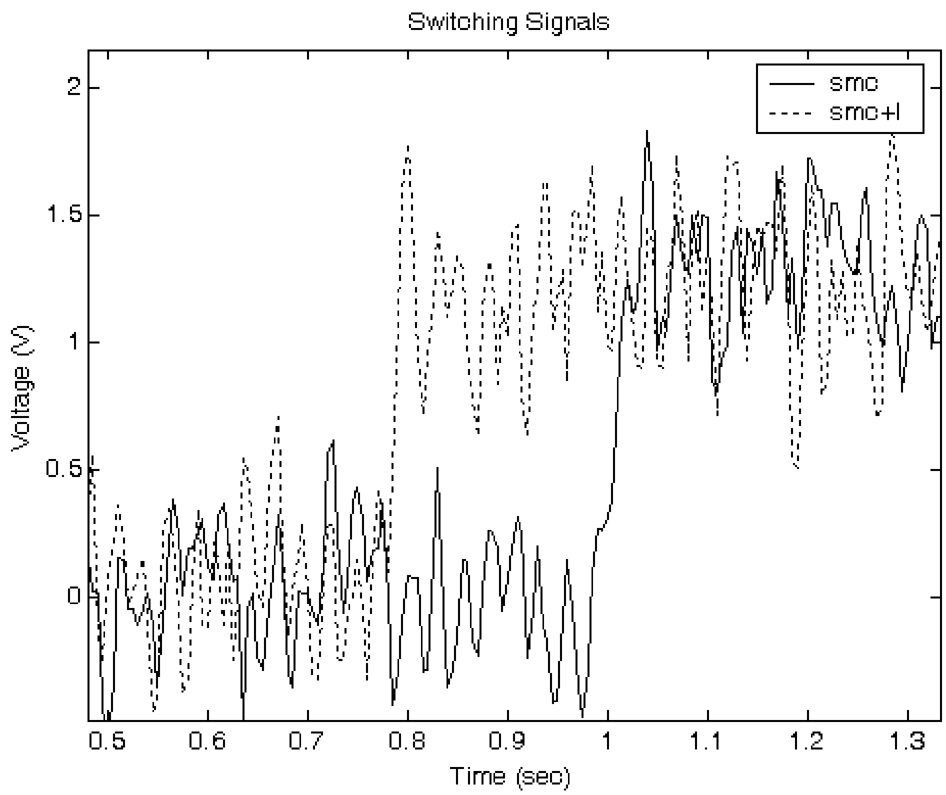

Fig. 4.15. Switching signal response of SMC+I system with load

It is clearly seen that magnitudes of the variations are smaller in SMC control (Fig. 4.14,4.15)but the system of SMC+I is reach steady state values rapidly .

CHAPTER 5
CONCLUSIONS

In this study, a sliding mode control with integral action has been adopted to control the speed of a computer-controlled electromechanical system while the nominal system is assumed to be known. Approximated second order system model is used in the present design, since many of the industrial plants can be modeled using a second order model. The practical application is associated with the sliding mode controller as a computational-intelligence approach to the engineering problems. Experimental application was carried out to test the effectiveness of the present sliding mode controller (SMC+I). From the experimental results the proposed SMC+I controller is more suitable to be applied for the speed control of the dc motor due to the uncertainty handling capabilities and disturbance rejection of the SMC design method. In order to avoid the chattering phenomena, a hyperbolic function has been used. The proposed controller ensures the invariance property against parameter uncertainties, compared with traditional PID controller and conventional SMC controller, without sacrificing the tracking accuracy. The closed-loop system is in the sliding mode at all times and the tracking error converges to zero exponentially under the existence of parameter uncertainties and disturbances. The closed-loop system has been shown to be globally exponentially stable in the sense of Lyapunov theorem.

Based on the experimental results and the time domain specification presented in Table 1, it can be concluded that the control performance of the electromechanical system was significantly improved with SMC+I controller compared with the conventional PID and SMC controllers. Experimental results also confirm the fact that the sliding mode controllers are reasonable condidates to use in industrial applications and these can be considered to be an alternative to usual PID controllers, since it is simple to use and easy to understand with its straightforward solution algorithm, and the computational task is not a problem any more because of high-speed computers and application tools to use in industrial applications.

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APPENDIX A

Mathematical solution of this plant diagram;

$$((\omega_m - \omega_L)\frac{1}{sC} - d2)\frac{1}{sJ_L + R_L} = \omega_L \tag{A.1}$$

$$\omega_m = (sJ_L + R_L)sC + 1)\omega_L + \frac{d_2}{sJ_L + R_L} \tag{A.2}$$

$$(T_{belt} - d_2)\frac{1}{sJ_L + R_L} = \omega_L \tag{A.3}$$

$$T_{belt} = \omega_L(sJ_L + R_L) + d_2 \tag{A.4}$$

$$(((([V_a - K_m[(sJ_L + R_L)sC + 1)\omega_L + \frac{d_2}{sJ_L + R_L})]\frac{K_m}{sL_a + R_a} - d_1)\frac{1}{sJ_m + R_m} - \omega_L)\frac{1}{sT_{belt}} - d_2)\frac{1}{sJ_L + R_L} = \omega_L$$

(A.5)

assume that; $sL_a + R_a = X$, $sJ_m + R_m = Y$ *and* $sJ_L + R_L = Z$

(A.6)

$$\frac{K_m V_a}{XYZsT_{belt}} - \frac{K_m{}^2 Z\omega_L sT_{belt}}{XYZsT_{belt}} - \frac{K_m{}^2 \omega_L}{XYZsT_{belt}} - \frac{K_m{}^2 d_2}{XYZ^2 sT_{belt}}$$

$$- \frac{X\omega_L Z}{XYZsT_{belt}} - \frac{(XYsT_{belt})d_1}{XYZsT_{belt}} - \frac{XY\omega_L}{XYZsT_{belt}} - \frac{XYd_2}{XYZsT_{belt}} = \omega_L$$

(A.7)

Denominator is shown that D_n function which is expanded;

$$D_{n1}(s) = s^4(L_a T_{belt} J_m J_L) + s^3(L_a T_{belt} R_m J_L + R_a T_{belt} J_m J_L + L_a T_{belt} J_m R_L)$$
$$+ s^2(R_a T_{belt} R_m J_L + L_a T_{belt} R_m R_L + T_{belt} R_L R_a J_m + K_m{}^2 T_{belt} J_L + L_a J_m + L_a J_L)$$
$$+ s(R_m L_a + R_a J_m + K_m{}^2 T_{belt} R_L + R_a T_{belt} R_L R_m + R_a J_L + L_a R_L) + (R_a R_L + R_m R_a + K_m{}^2) \quad \text{(A.8)}$$

$$\frac{D_{n1}(s)}{L_a T_{belt} J_m J_L} = D_n(s) = s^4 + As^3 + Bs^2 + Cs + D \quad \text{(A.9)}$$

Where;

$$A = \frac{L_a T_{belt} R_m J_L + R_a T_{belt} J_m J_L + L_a T_{belt} J_m R_L}{L_a T_{belt} J_m J_L} \quad \text{(A.10)}$$

$$B = \frac{(R_a T_{belt} R_m J_L + L_a T_{belt} R_m R_L + T_{belt} R_L R_a J_m + K_m{}^2 T_{belt} J_L + L_a J_m + L_a J_L)}{L_a T_{belt} J_m J_L} \quad \text{(A.11)}$$

$$C = \frac{(R_m L_a + R_a J_m + K_m{}^2 T_{belt} R_L + R_a T_{belt} R_L R_m + R_a J_L + L_a R_L)}{L_a T_{belt} J_m J_L} \quad \text{(A.12)}$$

$$D = \frac{(R_a R_L + R_m R_a + K_m{}^2)}{L_a T_{belt} J_m J_L} \quad \text{(A.13)}$$

$$\omega_L = \frac{K_m V_a}{s^4 + As^3 + Bs^2 + Cs + D} - d(s) \quad \text{(A.14)}$$

where Assume that d is not known exactly but can be written as $d = d' + \Delta d$, where d' is the nominal part and Δd is the uncertain part, which is bounded by a known function d, [38]

$d(s) \rightarrow |d(s)| \leq d_{\max}$

$$K_{m1} = K_m / L_a T_{belt} J_m J_L \tag{A.15}$$

$$\omega_L = \frac{K_{m1} V_a}{s^4 + As^3 + Bs^2 + Cs + D} - d(s) \tag{A.16}$$

$$\ddot{\ddot{\omega}}_L + A\dddot{\omega}_L + B\ddot{\omega}_L + C\dot{\omega}_L + D = K_{m1} V_a - d(t) * D_n(t) \tag{A.17}$$

$$\ddot{\ddot{\omega}}_L = -A\dddot{\omega}_L - B\ddot{\omega}_L - C\dot{\omega}_{-} - D + K_{m1} V_a - d(t) * D_n(t) \tag{A.18}$$