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MODELLING OF FOUR-LINK GRÜBLER MECHANISMS BY "IMAGINARY JOINTS" METHOD

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Modelling of Four-Link Grübler Mechanisms By “Imaginary Joints” Method

M.Sc. Thesis
in
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Supervisor
Prof. Dr. Sedat BAYSEÇ

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ABSTRACT

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Machine designers require to see the dynamic behaviour of the mechanism they design before including them into their machines, and verify that the kinematics of the mechanisms are conformable and kinetics, realisable. For this purpose, motion equations of the mechanisms are required together with the actuation motors and control strategies used. Various general purpose simulation programs have been made and put forth to the use of design engineers. These programs separate the mechanism into free bodies and define the interaction between them. Responses of the free bodies are determined by Newton's second law in accelerations. This thesis presents a method called "Imaginary Joints", which is originally designed to model open chain planar robotic systems, and then describes how the tip of a robotic chain is fixed to the ground by appropriate constraint forces which can be generated by either a revolute or a prismatic joint and convert the open chain into a closed link loop. Motion equations are generated by Lagrange formulations. Two joints, a revolute and a prismatic are assumed to exist between each pair of link. Based on the type of mechanism being simulated, one of the two joints is allowed to function while other is constrained by appropriately shaped force profiles. By this method, a single system can be converted into anyone of the 4-link Grübler type mechanisms and motion equation is properly shaped up to model this particular mechanism. Compared to that of Newton's Second Law, Lagrangian method allows fewer motion equations and hence numerical integration is simpler, with less tendency to numerical errors to build up.

Keywords: Four-link Grübler mechanisms, imaginary joint, open chain planar, revolute joint, prismatic joint.

ÖZET

DÖRT KOLLU GRÜBLER MEKANİZMALARININ "SANAL EKLEMLER" METODU İLE MODELLENMESİ

NACARKAHYA, Ekrem
Yüksek Lisans Tezi, Makine Mühendisliği
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Makine tasarımcıları tasarladıkları mekanizmaların dinamik davranışlarının, yani hareket profillerinin uygun olup olmadığını görmek isterler. Bunun için çeşitli genel amaçlı simülasyon programları hazırlanarak tasarımcıların kullanımına sunulmuştur. Bu programlar mekanizmayı oluşturan elemanların her birisini bir serbest cisim olarak kabul eder. Bu serbest cisimlerin aralarındaki etkileşim tanımlandıktan sonra dinamik davranışları, ivmeleri bulmak üzere Newton'un 2. kanunu ile modellenir. İvmeler daha sonra nümerik integrasyondan geçilir ve hız ve konum profilleri elde edilir.

Bu tez çalışmasında önce robotik sistemleri oluşturan düzlemsel açık zincirleri modellemek için tasarlanmış "Sanal Eklemler" metodu tanıtılmış ve daha sonra bir açık zincirin uç noktasına toprağa bağlı bir döner yada kayar eklemin uygulayacağı kuvvet profili etki ettirilerek mekanizmaları modelleyerek bir kapalı zincir haline getirilmesi anlatılmıştır. Hareket denklemleri Lagrange formülasyonu ile oluşturulmuştur. Her peş peşe gelen eleman çifti arasında birisi döner, diğeri kayar olmak üzere iki eklem olduğu varsayılmış ve modellenmek istenen mekanizmanın tümüne göre bu eklemlerden birisi sabitlenip diğeri serbest bırakılmıştır. Bu metotla tek bir sistem her türlü 4 elemanlı Grübler mekanizmasını modelleye bilmekte ve hareket denklemlerinde gerekli değişiklikleri yapabilmektedir. Hareket denklemleri daha azdır, nümerik integrasyon daha kolaydır ve hesaplama hatalarının birikimi daha yavaştır.

Anahtar kelimeler: Dört kollu Grübler mekanizmaları, sanal eklem, düzlemsel açık zincir, döner eklem, kayar eklem.

ACKNOWLEDGEMENTS

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CHAPTER 1

DYNAMICS OF MECHANICAL SYSTEMS

The word “dynamic”, an adjective, often preceding nouns like “people”, “ideas” etc. means that they are in a state of change, hence embedding power. In mechanical systems, changing quantities are positions and variations of positions in time in various orders. These systems have masses and mass moments of inertia, and hence resist motion, so, to set a mechanical system into motion, there must be power and forces and torques must be acting for finite time durations.

The “dynamics” of a system, this time a noun, is the relationship between various variations in its kinematics and also between the kinematic variables and the forces acting on it. Motion of celestial bodies has always fascinated human beings at all times. They have sought and some still are seeking hints about the future events trough their current positions, and vast amounts of time and position data had been gathered from observations of the sky, for making cross-match between them and the important events occurring. Contrary to what clergy had been claiming from Ptolemaic times, Nikolaus Capernicus had found out that the observable celestial bodies are not revolving about the earth but about the sun and put forth his famous heliocentric system (1514) which was later verified by the works of Galileo Galilei, and had been the initiation of the end of religious dogmas in science, and lead to the reformations, the renaissance, the scientific revolution and the industrial revolution successively. Tycho Brahe (1546-1601) the imperial astronomer of Emperor Rudolph II of Austria compiled a catalogue with positions for 1004 celestial bodies.

The positions data recorded showed a cyclic variation in time, but the actual function had not been determined until Johannes Kepler (1571-1630), the apprentice astronomer-mathematician of Tycho, has noticed the error imposed on the data by the refraction of the light beams coming from planets by the atmosphere. Johannes

Kepler, the gifted polish mathematician have than clarified the laws governing the planetary motion, as; The first correct definition of the principles governing rigid body dynamics was published by Newton and comprises the basis of classical mechanics. His deductions known as " Newton's laws" are completely based on experimental observations and have no mathematical proof, but there has been no incident reported that does not follow these laws. Therefore they can be regarded as axioms.

1.1 Newton's Laws:

Written in three statements, they describe the dynamics of particle motion. The first indication that these laws could be used to define angular motion is due to Euler and hence the approach as a whole is known as the Newton-Euler formula.

First Law:

Newton's first law states that every material body remains in its state of rest or uniform rectilinear motion unless a net non-zero force acts on it. In conjunction with the first law, Newton defines the quantity of motion as the product of two factors, the velocity and the quantity of matter, that is the linear momentum as:

$$\vec{P} = \overrightarrow{mv} \tag{1.1}$$

Where m is the mass of the body, v is its rectilinear velocity and P is the linear momentum. For a body having constant mass and velocity, momentum remains unchanged; hence the first law is generally known as the "law of the conservation of momentum".

The first law can be applied to the angular motion of a rigid body as:

$$\vec{K} = \overrightarrow{I\,\dot{w}} \tag{1.2}$$

Where I is the mass moment of inertia of the body, ω is its angular velocity and K is the angular momentum. On a constant inertia body, if no external moments act, it conserves its angular velocity and hence angular momentum.

For static compatibility, i.e. for defining the state of a static body, its physical model and the accompanying mathematics must obey the first law. Application of the first law to a system of bodies require the setting up of some constraint equation, which describe the link dimensions and types of joints and the connectivities they provide. Static compatibility is based on kinematic compatibility described by the constraint equations. Equations 1.1 and 2.1 are vector equations, meaning that the expressions on both sides of the equations are equal in magnitude and direction.

Second Law:

Newton's second law describes the general motion of rigid bodies, stating that the change in the quantity of motion is equal to the net force acting on it and takes place in the direction of the straight line along which the force acts. Change is meant with respect to time and so:

$$\frac{d(P)}{dt} = \frac{d(mv)}{dt} = m\frac{dv}{dt} = ma = F \tag{1.3}$$

where F is the resultant vector of all external forces acting. The idea can be extended for rotating bodies so that the rate of change of angular momentum is equal to the net moment acting as:

$$T = \frac{d(k)}{dt} = \frac{d(Iw)}{dt} = I\frac{dw}{dt} = I\alpha \tag{1.4}$$

where T (τ) is the acting resultant moment or torque.

The second law defines how the motion of a body proceeds in time under the effects of a given forcing system. For dynamic compatibility, a model must obey the second law.

Third Law:

Newton's third law states that action always equals reaction. The forces that two bodies exert on each other are equal in magnitude and opposite in direction. The third law defining the nature of the force interaction between different bodies makes possible the transition from the mechanics of single bodies into compound systems. The third law incorporates both the kinematic and static compatibilities.

Newton's laws completely define the motion of bodies or systems of bodies as a function of time. Application of the laws for planar linkages leads to the systematic free-body definition of system subsets. Each free body moving in a plane has 3 degrees of freedom and 3 equations can be derived for each movability. Between the 3N coordinates defined for the system having N bodies and n degrees of freedom. (3N – n) many are related with each other leaving only n independent generalised coordinates. Existence of constraints create two problems in application: holonomic constraints between the coordinates defined and time in some cases add that many algebraic equations to be solved with the differential equations of motion simultaneously. This difficulty can be overcome by deriving the motion equations using the generalised coordinates, resulting in fewer but more complicated equations. Secondly, the force and motion constraint at the joints appear in the form of non-holonomic constraints, which can not be integrated alone, and thus, must be solved with the rest of the motion equations simultaneously. This process yields the forces interacting at the joints, which may or may not be required.

Starting with these fundamental laws, alternative techniques were developed to eliminate the necessity of obtaining explicit expressions for the constraint forces, such as Lagrange's and Hamilton's equations.

1.2 Lagrange's Equation

Lagrange's equation for holonomic systems:

Equations of motion for dynamic systems can be specified in the form of Lagrange's equation as:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = Q_j \tag{1.5.}$$

$j = 1$, ... n

where L is the Lagrangian , q_j are the generalised coordinates Q_j is the generalised force acting on the j'th generalised coordinate. By definition the Lagrangian of the system is:

$$L = T - V \tag{1.6}$$

Where T is the total kinetic energy and V is the total potential energy of the system. Lagrange equation describes the dynamics of the associated coordinate only. Therefore for a system of n degrees of freedom, n equations are derivable which simultaneously define the dynamics of the whole system.

There is complete freedom in the choice of generalised coordinates as long as they are independent of the each other, so that the kinematics of the system are uniquely defined.

The Lagrangian of the system is the difference between the kinetic and potential energies, both of which are scalar quantities. Therefore the Lagrangian of a system will have the same value for a given condition as long as the same definition is used in each case, because the Lagrangian of a system is not unique. If L (q, $\dot{q}$, t) is an appropriate Lagrangian and F (q,t) is any differentiable function of the generalised coordinates and time , then

$$L'\left(q,\dot{q},t\right) = L\left(q,\dot{q},t\right) + dF/dt \tag{1.7}$$

is also a Lagrangian of the system. For example the gravitational potential energy is defined with respect to a reference datum. If the datum is changed for the same system, both the analytical form and the numerical value of the Lagrangian changes.

The generalised force Q_j is the net effect of all the external forces on the j'th generalised coordinate, thus,

$$Q_j = \sum_k F_k \frac{\partial x_k}{\partial q_j} \tag{1.8}$$

where x are coordinates defining the position of the system in real and virtual displacements and F_k is the net external force applied on coordinate x_k . Generalised forces are composed of all the forces external to the system. These external forces can be arbitrary functions of the generalised coordinates and time. Physically, they may be forces involving an energy injection into the system such as actuator forces, or involving energy dissipation from the system such as the velocity dependent damping forces of viscous dampers and position, velocity and acceleration dependent forces due to Coulomb friction. They can further include forces exerted by potential fields not included in the Lagrangian such as weights and forces coming from energy storage devices like mechanical springs, air cylinders etc.

The concept of kinematic compatibility of mechanical networks is included in the Lagrangial and in the generalised forces. As Newton's equations, Lagrange's equations can be derived for any number of coordinates resulting in that many differential equations of motion. In conjunction with the constraint equations, which are equal to the number of equations less the number of degrees of freedom of the system, the dynamics of the systems is fully definitive.

Lagrange's Equation for non-holonomic systems:

Non-holonomic systems have constraints described by differential equations which can not be integrated independent of the system dynamics. Non-holonomic constraints apply constraint forces on the system to make it obey the constraints. With the inclusion of the constraint forces, Lagrange's application becomes:

$$\frac{d}{dt}\left(\frac{\partial \dot{L}}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = Q_j + \sum_{i=l}^{(m-n)} \lambda_i \frac{\partial f_i}{\partial q_j} \tag{1.9}$$

j = 1,2,.....m

where f_i are constraint equations , m is the number of coordinates and λ_i are Lagrange's undetermined multipliers. The constraints equations can be explicit function of the time , that is , rheonomous or may not be explicitly dependent on time, that is, scleronomous.

1.3 Hamilton's Equations

Equation of motion for dynamic systems can be specified in the form of the canonical equations of Hamilton as:

$$\dot{q}_j = \frac{\partial H}{\partial p_j} \tag{1.10}$$

$$\dot{p}_j - Q_j = -\frac{\partial H}{\partial q_j} \tag{1.11}$$

$$-\frac{\partial L}{\partial t} = \frac{\partial H}{\partial t} \tag{1.12}$$

where Q_j are the generalised forces , H is the Hamiltonian , L is the Lagrangian and p_j are the conjugate momenta , functions of generalised coordinates q_j , generalised velocities $\dot{q}_j$ and time t as :

$$\frac{\partial L}{\partial q_j} = \dot{p}_j \tag{1.13}$$

Hamiltonian H is defined as:

$$\mathrm{H}(\mathrm{q}, \mathrm{p}, \mathrm{t}) = \sum_j \dot{q}_j p_j - \mathrm{L}(\mathrm{q}, \dot{q}, \mathrm{t}) \tag{1.14}$$

If Lagrangian is independent of time and the potential energy independent of velocities, the Hamiltonian becomes:

$$H = T + V \tag{1.15}$$

To apply the canonical equations of Hamilton to mechanical networks, first a set of generalised coordinates are defined and the Lagrangian is formulated. Then conjugate momenta are derived using equation 13. Once momenta and Lagrangian are known, the Hamiltonian is formulated and substituted into the canonical equations.

Hamilton's method produces 2n first order differential equations, n being the degrees of freedom. The first canonical equation is used to obtain velocities from momenta. The second equation incorporates the principles of dynamics. Hamiltonian formulations in the form of equations 1.10-1.12 can describe the conservative and holonomic systems. For non-conservative and non-holonomic systems, the constraint equations must also be included in the set of equations of motion.

In the forthcoming chapters, the mechanical systems whose dynamic behaviour is sought for and explanations are given how their motion equations are generated by the abovementioned methods. Also a comparison is given to display the relative adventages and disadvantages of each over the others.

CHAPTER 2

FOUR LINK GRÜBLER MECHANISMS

2.1 A Brief History

Mechanisms are multi-body systems where each body is connected to some others by joints and developed for changing the type and extent of motion and the incorporated forces. Some mechanisms are very old. When man inverted the wheel is unknown for example. Levers have been used to magnify force or motion. Various mechanisms have been developed to get water from a deep well or be carry heavy objects up and down or in horizontal direction. As mechanisms are passive devices from power point of view, a human or an animal had been an integral part of any mechanism to provide power. Later we see means of getting power from flowing water and from wind. Very elaborate models of wind and water mills are still operational today. After the invention of the steam engine by James Watt in 1765, great amounts of cheap mechanical power became available and consequently building of machinery accelerated. Machines are composed of mechanisms. Each simple motion, comprising the more completed series of motions of a machine is generated by a single mechanism property designed. Techniques in mechanism design hence were developed starting from the beginning of 19. th century. Each new mechanism came up as a brilliant invention to solve a certain motion problem. As mechanisms pile up, some engineers tried to compile and classify them such that repetitive inventions or design work should be avoided. Roleaux and Chebyshev (1821 – 1894) have been the pioneers of mechanisms design.

Simplest mechanisms are composed of four links. One of them is the ground and remaining three are moving. One very important problem of technology was the generation of exact straightline profiles, be it a plinth surface like a machine table or a rail like the slideways of a machine tool. Making geometrically good cylinders on a lathe had been the most important problem to solve to make better steam engines and internal combustion engines later. Roleaux tried to find an exact straightline generation mechanism but could not succeed. It was French engineer Peaucellier and Chebychevs student Lipkin who separately found 8 link exact straight line generators. Though complicated mechanisms were required for more intricate motions, basic four-link systems have been used much extensively in building machines, as still the case is today.

Mechanisms are multi-body systems and hence each moving link does its own predetermined motion which is unique for itself, hence observing a mechanism in motion is surely fascinating. Motion of every link is under control and beach link is powered by a single crank , which also turns the cranks of all the mechanisms comprising the machine. This we call now as “single degree of freedom” mechanisms, which are driven by a single “prime mover”. Greater movabilities require more drives, which is not desirable.

2.2 Grübler’s Equation

Movabilities of mechanisms were discussed by Grübler and Kutzbach in detail first, though Roleaux, Kennedy and Chebshev [1] have considered movability in classifying mechanisms. Artobolevsky’s compilation [2] is now an important reference for designers of machinery. Grübler has considered the mechanisms which h are in practical usage in making machines. They were all planar and single degree of freedom mechanisms composed of revolute and prismatic joints only. Grübler has found that all such mechanism satisfy the following criterion[3]:

$$3L - 2J - 4 = 0 \qquad (2.1)$$

where L and J are the number of links and joints in the mechanism respectively. This equation consider only single degree of freedom joints only. Later, Kutcbach put forth the following formula:

$$F = 3(L-1) - 2J_1 - J_2 \tag{2.2}$$

where F is the total movability or degree of freedom of the mechanism, J1 and J2 are the number of one and two degree of freedom joints respectively. With his formula, Kutcbach says that a mechanism having L links have (L-1) many movingh links, each having 3 degrees of freedom without joints. A single degree of freedom joints constrains 2 and a two degree of freedom joint constrains 1 degree of freedom, remaining F movabilities free. A 3 degree of freedom joints is no joint at all. With this formula, Kutcbach puts forth the idea of multi-degree-of freedom mechanisms.

Spatial mechanisms, that is, mechanism working in three dimensional space are also possible and they provide more intricate input-output functions. In 3 dimensional space, degree of freedom is 6 and hence joints up to 5 degree of freedom are allowable. A list of possible joints will degrees of freedom ranging between 1 and 5 can be found is the "Mechanism" book of Eres Söylemez [4]. Kutcbach criterion for spatial mechanisms can be represented as follows:

$$F = 6(L-1) - 5J_1 - 4J_2 - 3J_3 - 2J_4 - J_5 \text{-} \tag{2.3}$$

and if the mechanism is of Grübler type, the composed of only revolute and prismatic pairs and having only 1 degree of freedom, Grübler equation becomes:

$$6L - 5J - 7 = 0 \tag{2.4}$$

The Kutcbach criterion is later put into a more suitable form as:

$$F = \lambda(L - J - 1) + \sum_{i=1}^{J} f_i$$

where λ is the degree of freedom space, i.e. λ = 3 for planar and λ = 6 for spatial mechanisms, f_i is the degree of freedom of joints i.

Some Further Characteristic of The Grübler Mechanisms:

In Grübler equation for planar mechanisms,

$$3L - 2J - 4 = 0$$
$$3L = 4 + 2J \tag{2.5}$$

J, the number of joints can be even or odd. Whether it is even or odd, twice that number ie 2J is even. 4 is also even and sun of two even numbers make an even number. So 3L is even. As 3 is odd, l must be even. From here, one can conclude that all Grübler Mechanisms have even number of links. With 2 links, a movable link loop is not possible. So, simplest Grübler mechanism contains 4,6,8 or higher even number of links.

If a number of links contain k kinematic elements, let us denote this number of links by l_k. We can not have l_1 (since there can be no link with one kinematic element). The total number of links, l l, in the mechanism will then be equal to:

$$l = l_2 + l_3 + l_4 + l_5 + ...l_n \tag{2.6}$$

or

$$3l = 3l_2 + 3l_3 + 3l_4 + 3l_5 + ...3l_n \tag{2.7}$$

(Where l_2 is the number of binary links, l_3 is the number of ternary links, etc. in the mechanisms)

The number of kinamatic elements in the mechanism will be equal to:

$$2l_2 + 3l_3 + 4l_4 + 5l_5 + ...nl_n = \text{\# of kinematic elements.} \tag{2.8}$$

Since two kinematic elements, when paired, will form a joint: The number of kinematic elements is equal to twice the number of joints in the mechanism:

$$2j = 2l_2 + 3l_3 + 4l_4 + 5l_5 + ...nl_n \tag{2.9}$$

Substituting equations 2.3. and 2.4. in to Grübler's equation, we obtain:

$$l_2 - (l_4 + 2l_5 + 3l_6.......(n-3)l_n = 4$$

or

$$l_2 = 4 + P$$

where $P = l_4 + 2l_5 + 3l_6.......(n-3)l_n$

P is always a positive quantity. It can at must be zero, if all the links in the mechanism are binary or ternary links. Hence, the number of binary links can at least be 4 if P=0, otherwise it is greater than 4.

Consider of link (a) with I kinematic elements and let this number of kinematic elements be the maximum that a link can have. A kinematic chain using this link can be formed if we attach links of type (b), and if we join these links with links of type (c). In this case the number of the kinematic elements on link (a) will be a maximum. The number of link will be:

$$l = 1 + i + (i-1)$$

or

$$i = l / 2$$

Simplest Grübler Mechanism has four links. Machine must be simple. So four links Grübler Mechanisms are the most widely used mechanism.

Cases Where Grübler Equation does not Hold True:

Grübler equation does not take link dimensions and combinations of joints types into consideration. There are cases where mechanism produces more movabilities than the Grübler equation suggests. These cases are as follows:

i) If a link has two prismatic elements only, their axes should not be parallel. If so, another degree of freedom comes up, which is the sliding of the link along the prismatic axes. This movability does not interfere with the rest of the mechanism motion and hence is redundant. It must be avoided.

ii) Binary links of a kinematic chain having only prismatic elements can not be connected to each other. If so, an extra movability comes up, redundant.

iii) No closed link loop can have less than 2 revolute joints.

So, 4 links Grübler mechanisms can be in R-R-R-R, RRRP, RRPP and RPRP types only, R standing for Revolute and P for Prismatic pairs.

2.3 Isomers of 4-Link Grübler Mechanisms

Isomer of a mechanism is synonymous to an isomer molecule. Two isomer mechanism have topologically the same structure, that is same number of links and joints, same joints types and sequencing, but look completely different, to different motions and hence used for different purposes. Releasing the fixed link, such that it becomes a movable link and fixing one of the originally moving links into a stationary state can obtain isomers of a link loop. This process is called “inversion”.

2.3.1 Isomers of The 4R Chain

4R or RRRR chain is the “four bar mechanism”. Movability of the four bar mechanism was first examined by Grashoff. (See fig. 2.1 and 2.3).

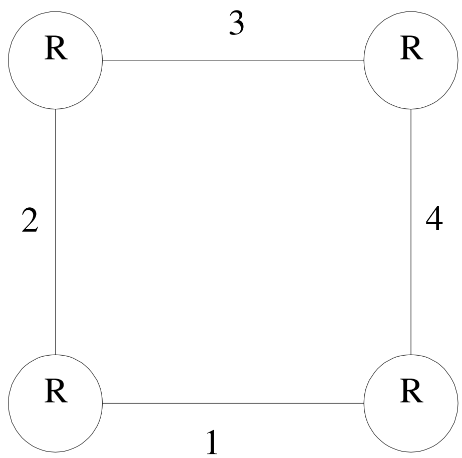


Figure 2.1 Topological map of the 4R chain.

If the length of the longest link is denoted as l, the shortest as s and two intermediary length links as p and q. As seen in figure (2.2) and Grashoff 's theorem says that four different 4-bar mechanisms exist if
L+S <p+q, as:

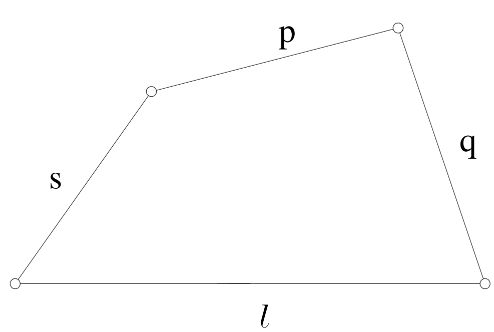


Figure 2.2 A Four-bar mechanism.

i) If one link adjacent to S is made the ground, a crank-rocker type mechanism is formed. S is a crank, which does full rotation. The link opposite to S is a rocker, which does a limited rotation in a to and manner. The link adjacent to the moving end of S is the coupler. This mechanism converts a full rotation into a limited rotation and is a useful mechanism, widely used in machines.

ii) If the other link adjacent to S is made the ground, another crank rocker mechanism is formed.

iii) If S is made the ground, the mechanism formed becomes a “double crank”. As its name implies, both links connected to the ground can do full rotation. Both cranks complete a full rotation with in the same time duration and hence their arrange velocities within a cycle are equal, but their instantaneous speeds are not equal. Output crank sometimes run faster and sometimes slower than the input crank, which is supposedly running at constant speed by the actuation of a drive motor. It is also a useful mechanism which can be directly coupled to a motor.

iv) If the link opposite to S is made3 the ground, the mechanism formed is a “double rocker”. Name of the links connected to the ground can do full rotation, but instead to a limited rotation or rocking. This mechanism can not be directly coupled to a rotating shaft and so is generally not a preferred mechanism. If the motion is essential the input rocker is made the output rocker of a crank rocker mechanism with the inclusion of two more links, in form of a 6 links Watt type Linkage.

According to Grashoff theorem if $r + L = p + q$, the same mechanisms as above come up but mechanisms will all have a cross over problem when the links are all aligned. When the mechanism is passing through the cross over position, it can change closure as governed by the instantaneous velocities and accelerations of the links at that moment. This is something not preferred and mechanisms of this sort are never operated in full cycle. If $r + L > p + q$, only double rocker mechanisms come up, in four different types.

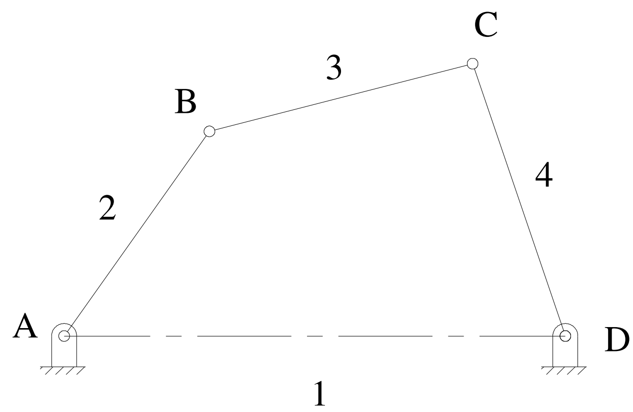


Figure 2.3: A Four-bar mechanism.

2.3.2 Isomers of The RRRP Chain

Topological map of the RRRP chain is seen figure (2.4) and its isomers can be obtained by fixing one of the links at a time and releasing all the others.

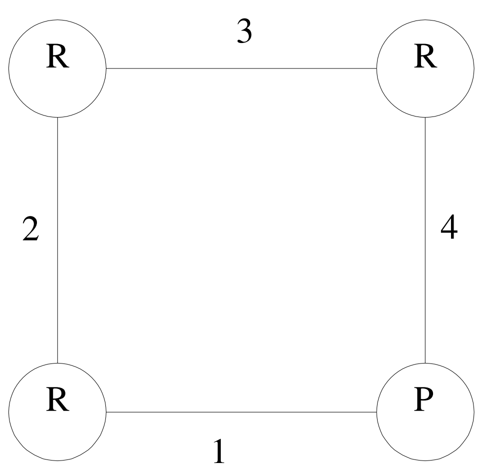


Figure 2.4: Topological map of the RRRP chain.

i) If link number 1 of the chain in figure 2.4 is fixed, the mechanism obtained is a “slider crank” mechanism. A crank, that is link number 2 is connected to the ground via a revolute joints. A P-R link, that is link number 4 is connected to the ground via a prismatic pair. Other kinematic pairs of links 2 and 4, both revolute, are coupled to each other by a coupler link, that is link 3.

This mechanism can be driven from either the crank or the slider end. Output motion then is obtained from anyone of the remaining two moving links. Links proportions can be made such that the crank is capable of doing full rotation. Then the mechanism can coupled directly to the rotating shaft of a motor. Output motion then, that of the slider, becomes a complicated, multi harmonic motion of the crank angle θ, or seen in figure (2.5).

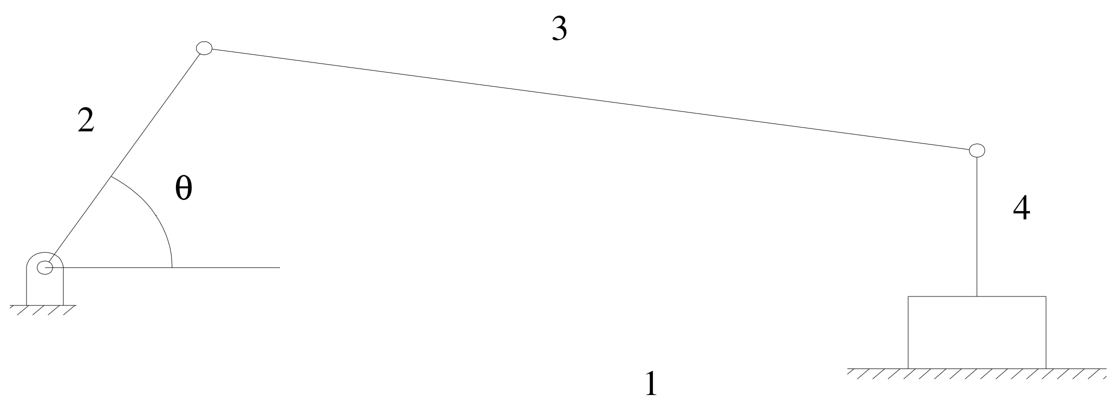


Figure 2.5 An off-set Slider-crank mechanism.

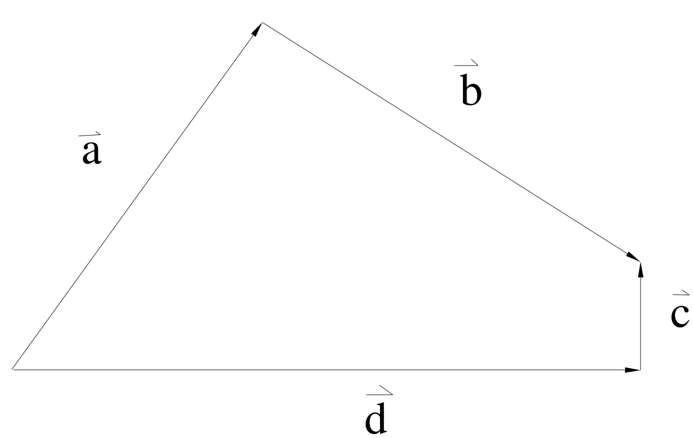


Figure 2.6 vector loop.

Obtain able by solving the vector loop equation.

$$\vec{a} + \vec{b} = \vec{c} + \vec{d} \tag{2.10}$$

vector as defined in figure 2.6. The length of link 4 can be made zero without altering the motion characteristics of the mechanism, thus making d = 0 in equation (2.10). This not only simplifies equation (2.10) and its derivatives, but also makes the mechanism is called as an “In line slider crank mechanism” contrary to the “Off-set mechanism” of figure (2.5). Slider crank mechanism is very useful as it can convert a continuous rotation into a reciprocation, in the design of machinery.

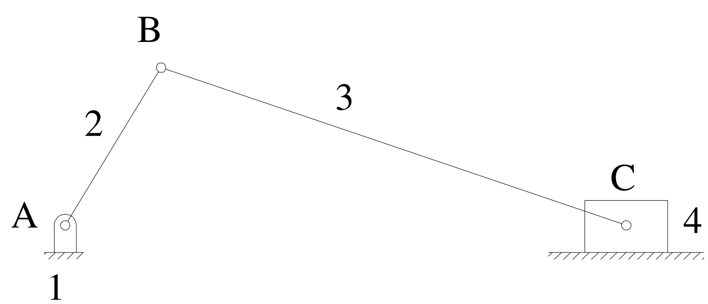


Fig 2.7: An in-line slider crank mechanism

ii) If the number 2 in Fig (2.4) is fixed, the mechanism obtained is a “Whitworth Quick Return Mechanism”. Link number 1 and 3 are connected to the ground via revolute joints and link number 4 coupler the two with a revolute joints at one end and a sliding joint at the other. Mechanism can be driven by link 3 or link 1. The output motion can be obtained from anyone of the remaining two moving links. Mechanism dimensions are so made that generally link 3 is rotated continuously by a motor shaft and output motion is from 1. when link 3 rotates with a constant speed, motion of link 1 becomes a to and pro swinging, stopping at two points and speed harmonically varying. Such a mechanism can be made to do work with its output link 1 while it is moving slowly and no work, but return fast in the completing motion as its name implies. A vector loop equation similar to that of Equation (2.10), which defines the position of the links. This equation can be differentiated with respect to time once and twice to obtain equations describing the velocities and accelerations of the mechanism.

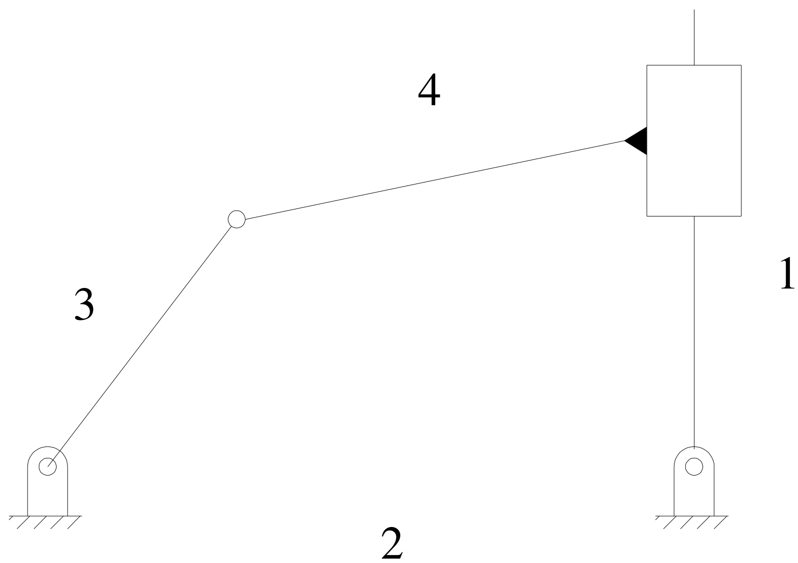


Figure 2.8 A Wtihworth Quick-Return Mechanism.

Length of link 4 can be made zero without altering the motion characteristics of the mechanism, hence a more familiar and simpler form of the mechanism is obtained. Due to the "Quick Return" action, this mechanism is also one of the very familiar mechanisms in machine design. Quick-return mechanisms are used in machine tools such as shapers and power-driven saws for the purpose of giving the reciprocating cutting tool a slow cutting stroke and a quick-return stroke with a constant angular velocity of the driving crank.

iii) If link number 3 in figure (2.4) is fixed, the mechanism obtained is a " Swinging Block". As seen in figure (2.9), the links connected to the ground are 2 and

4. Link number 2 can be made the input link, and then the multi harmonic swinging of link 4 becomes the output link. Revere order operation is not feasible.

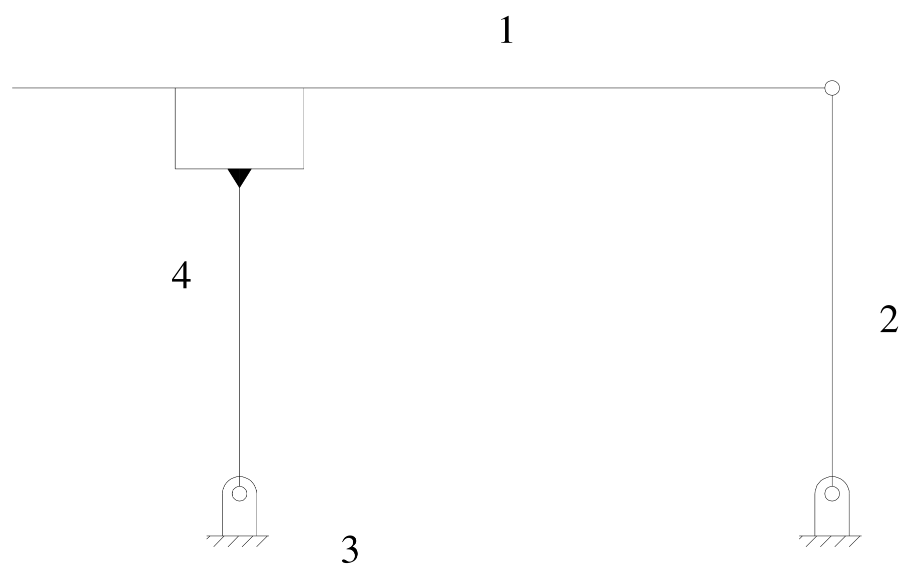


Figure 2.9 A "Swinging-Block" mechanism.

The relative motion between links 1 and 4 at the sliding joint can also be used to actuate the mechanism as an alternative, but still is not a very common mechanism in machine design. Length of link 4 can be made zero without altering the motion characteristics of the system and puts the mechanism into a simpler form physically and mathematically.

iv) If link number 4 in figure (2.4) is fixed, the mechanism obtained is called the "Slider- Crank Mechanism of the Second Type". As seen in figure (2.10), it is a peculiar mechanism not used very widely. Generally link 1 is made the input link and driven by a linear actuator, hydraulic, pneumatic or solenoid. Output motion can be obtained from either of link 2 or 3.

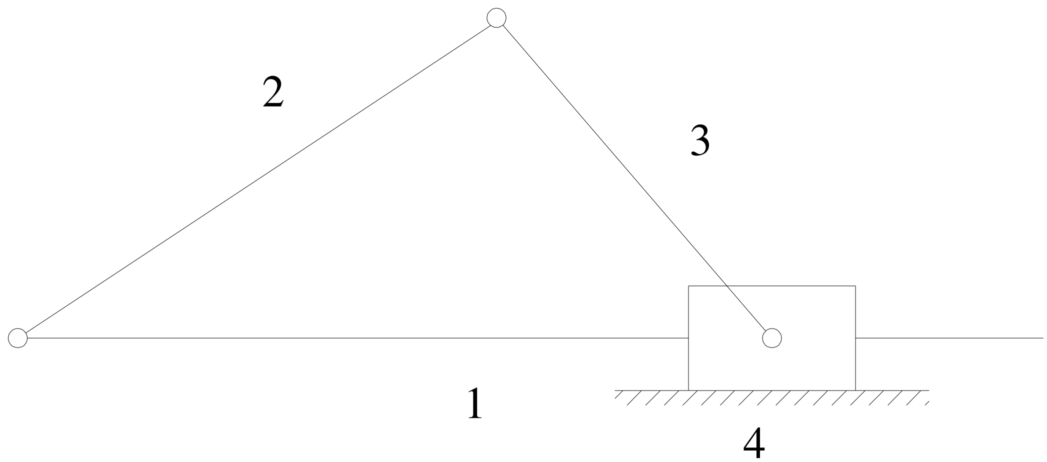


Figure 2.10 A Slider-Crank mechanism of the second type.

Similarly, length of link 4 can be made zero without altering the motion characteristics of the mechanism.

2.3.3 Isomers of the RRPP Chain

Figure (2.11) shows the topological map of the RRPP closed link loop, and its isomers can be obtained by fixing one of the links at a time and releasing all the others.

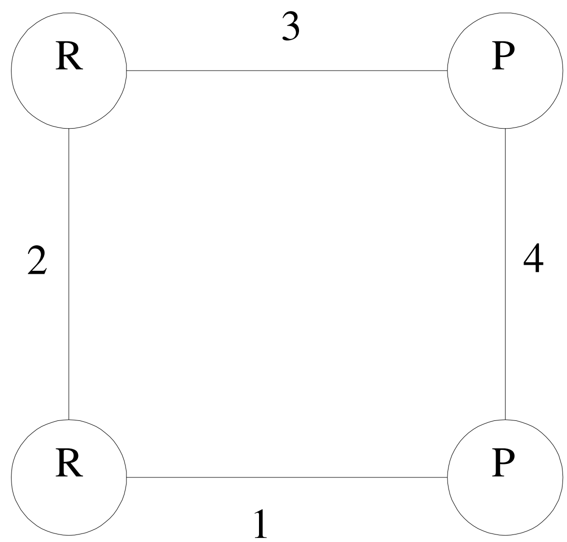


Figure 2.11 Topological map of the RRPP closed link loop.

i) If link number 1 of the chain in figure (2.11) is fixed, the mechanism obtained is a “Scotch Yoke” mechanism. Link number 2 is connected to the ground via a revolute joint and 4 by a prismatic, and these two links are coupled to each other by link 3 from their other ends. Link 4 is a PP link and hence axes of the prismatic pairs should hot be parallel.

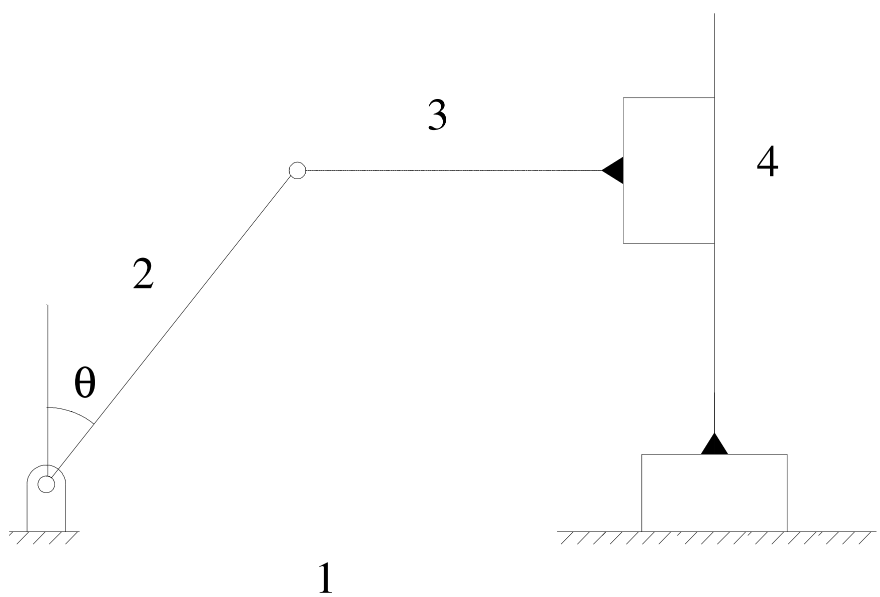


Figure 2.12 Basic topology of a “Scotch Yoke” mechanism.

Generally this mechanism is driven from the crank end, and the output motion is obtained from link number 4 as the projection of the motion of the crank tip in the horizontal plane if the prismatic pair axes are displaced by 90 degrees. So, output motion becomes a single harmonic pure sine of the crank angle θ. Length of link number 3 can be made zero without altering the motion characteristics. A more familiar form of the mechanism is shown in figure (2.13).

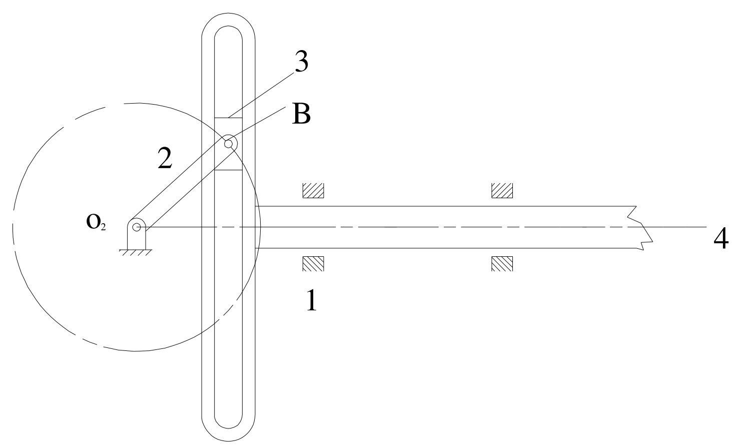


Figure2.13 Familiar form of the "Scotch Yoke" mechanism.

ii) If link number 2 of the chain in figure (2.11) is fixed, the mechanism obtained is an "Oldham Coupling". As seen in figure 2.14, links 3 and 1 are connected to the ground via revolute pairs. The prismatic pairs at the other end are coupled by the PP link number 4.

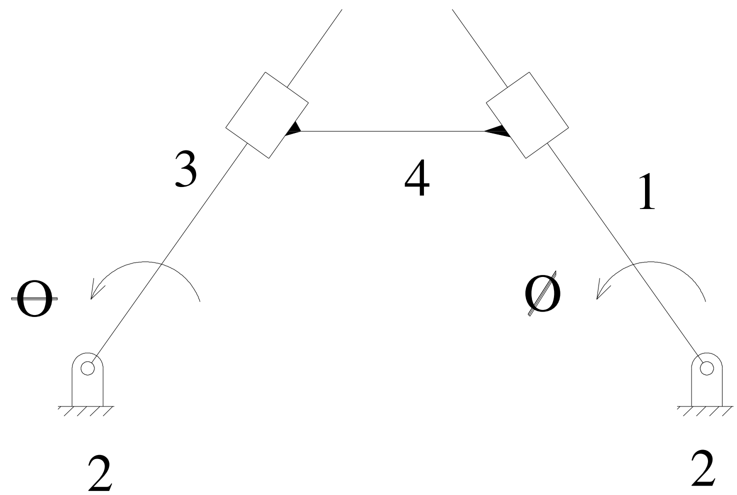


Figure 2.14 An Oldham Coupling.

In usage either of 1 or 3 is the input and the other the output link. As the joints in between are all prismatic, the motion of the input and output links are exactly the same, that is with the same velocity and acceleration. The lengths of the PR links of 1 and 3 can be made zero without altering the motion characteristic of the system and hence the mechanism reduces into a simpler and more familiar form seen in figure (2.15). This from of the mechanism is the familiar coupling to couple parallel –but- offset shafts, an essential system used in every motor coupling.

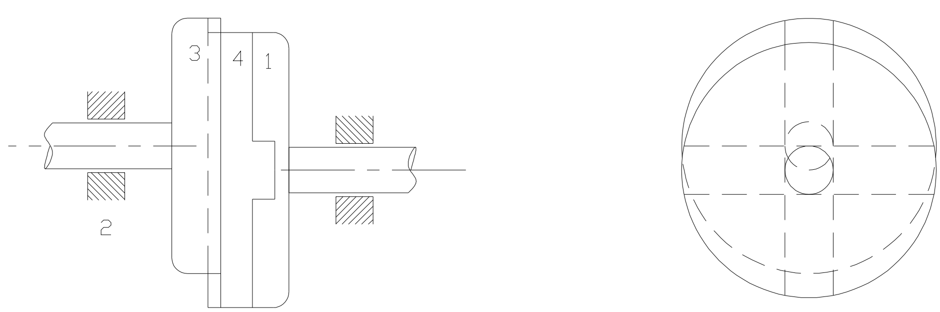


Figure 2.15 Familiar form of the "Oldham Coupling", which is used to couple parallel-offset shafts.

iii) If link number 3 of the chain in figure (2.11) is fixed, the mechanism is a "Double Slider". It converts a translation in same other direction. The prismatic pair axes on link 3 should not be parallel. The lengths of the PR links 2 and 4 can be made zero without altering the motion characteristic of the mechanism.

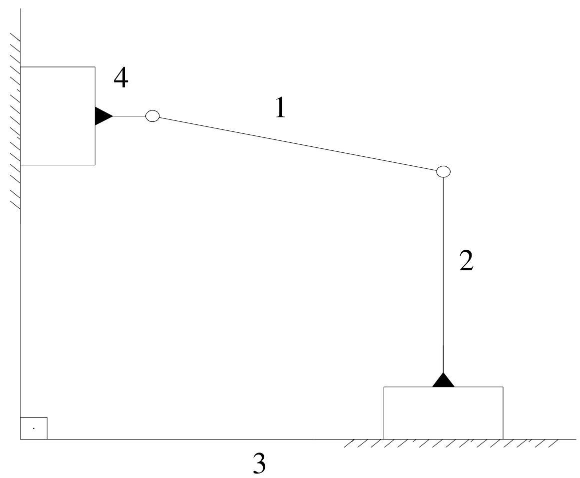


Figure 2.15 A Double Slider Mechanism.

If the prismatic axes are made perpendicular, the mechanism takes a special name, the "Elliptic Trammel". Each point on the coupler link traces exact ellipses, as shown in figure 2.16. It has been used in various calculating mechanisms, still usable whenever an elliptic motion is required.

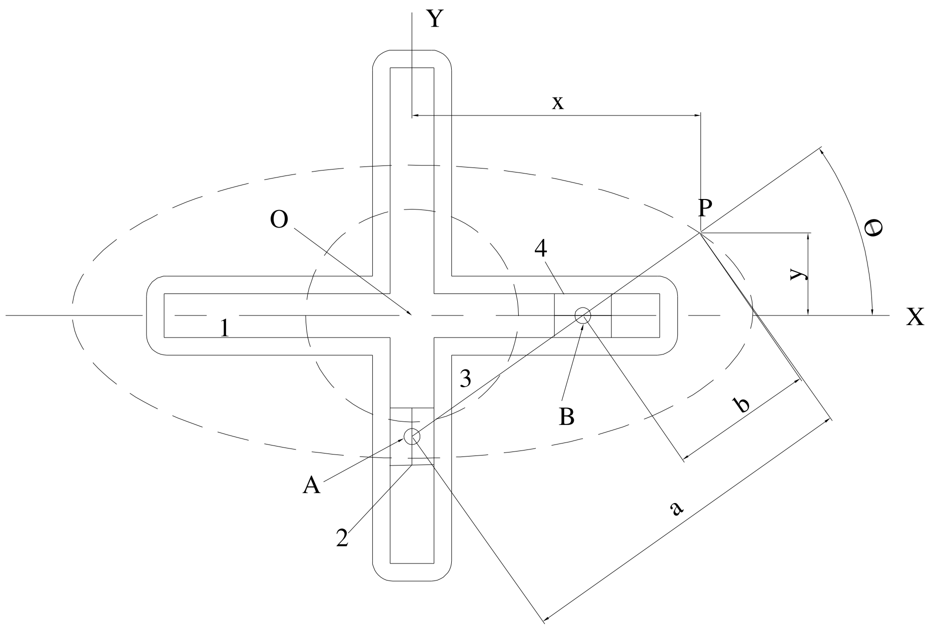


Figure 2.16 An Elliptic Trammel. Every paint on the coupler link AB of this mechanism traces an exact ellipse.

The elliptic trammel is an instrument for drawing ellipses. Link 3 is pivoted to sliders 2 and 4, which slide in link 1 and point P describes an ellipse. From the figure (2.16)

$x = a \cos \theta$

$y = b \sin \theta$

Then

$$\cos^2 \theta + \sin^2 \theta = \frac{x^2}{y^2} + \frac{y^2}{b^2} = 1 \tag{2.11}$$

which is the equation of an ellipse with center at the origin. Length a is half the major axis and b is minor axis. When the device is used as a drawing instrument, a pen or pencil carried at P and both lengths a and b are adjustable. If P is placed at point C, which is midway between A and B, then a and b are equal and Equation (2.11) becomes,

$x^2 + y^2 = a^2$

which is the equation of a circle of radius a.

iv) If link number 4 of the chain in figure 2.11 is fixed, the mechanism obtained is again a “Scotch Yoke”.

2.3.4 Isomers of the RPRP Chain

Figure 2.17 shows the topological map of the RPRP closed link loop and its isomers can be obtained by fixing one of the links at a time and releasing all the others.

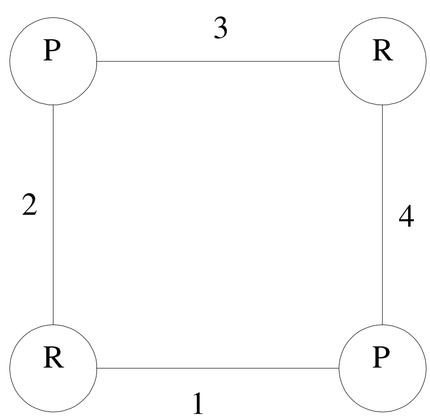


Figure 2.17: Topological map of the RPRP closed link loop.

Every link is RP and mechanism is symmetrical. So every isomer obtainable is the same. The resulting mechanism is called the conchoidal motion mechanism. An example of a mechanism based on the crossed double-slider linkage is as seen in figure (2.15) used to provide, conchoidal motion (fig. 2.18), occasionally used in the steering of ships. Frame 1 is a guide fixed to ship’s deck, link 2 is the tiller, and link 3 and link 4 are sliding blocks, pin-connected at A.

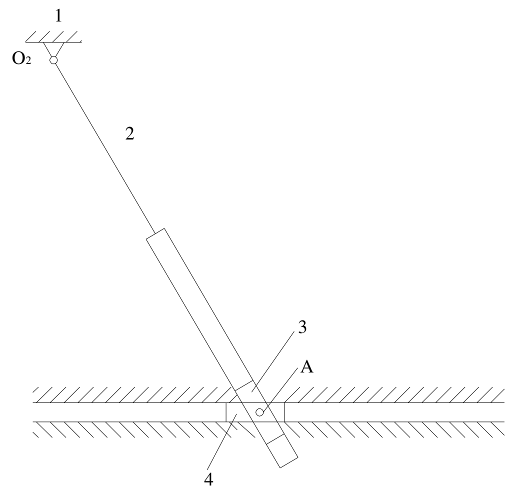


Figure 2.18: Conchoidal Motion mechanism.

CHAPTER 3

MECHANISM DYNAMICS

3.1 The Two Basic Problems of Dynamics

Dynamics of moving bodies are defined by Newton-Euler equations. For a single body of mass m and centroidal mass moment of inertia I_G,

$$\sum \overline{F} = m\bar{a}$$
$$\sum \overline{M} = I_G \overline{\alpha} \tag{3.1}$$

where a is the acceleration of the mass center and a is the angular acceleration of the body. Mechanisms are composed of several moving and fixed bodies which transfer force and motion to each other via joints in between. The interacting forces at the joints are clarified by Newton's third law, or namely the "Law of Action-Reaction". The inertial properties in equation (3.1) are mechanism parameters. Mass can be measured by a balance and inertia by experimental techniques know as the "Compound Pendulum", "Torsional Pendulum" and "Filar Pendulum". In each, the mechanism component is made a pendulum and set into free vibrations by an intial displacement. Period of natural oscillations, measured by a stop-watch, is definitive of the mass moment of inertia.

Knowing the inertial properties, equation (3.1) can be used to solve only two types of problems:

i) The forward dynamics problem: Accelerations are given and forces to generate these accelerations are required. This problem is simply solved by multiplying the masses with accelerations, or rather, in more technical terms, by pre-multiplying the acceleration vector with mass matrix. Mathematics involved is simple.

ii) The inverse dynamic problem: Forces are given and resulting motion are divided into masses or rather, in more technical terms, pre-multiplying both sides of equation (3.1) by the inverse of the mass matrix. This problem requires the inversion of the mass matrix, hence more elaborate than the first problem. Further, resulting accelerations are not too much meaningful for designers and velocities and positions must be known in time. To obtain velocities and positions, accelerations found must be integrated along time once and twice respectively. Accelerations and controlled forces of actuation are often too complicated and hence no closed form solutions for the differential motion equations are obtainable. In such cases, solution is obtained by numerical integration.

3.2 Mechanism Kinematics

The "forward dynamics problem" is the basic problem encountered in machine design, while the "inverse dynamics problem" provides an analysis. Actually at the end of each synthesis problem, an analysis comes to see if the synthesis done is correct or not. Also as analysis is simpler to understand than synthesis, in formal education, it is thought before synthesis. In general, it is not possible to do mechanisms design by using only the forward or inverse dynamics solutions, and solutions are often repeated in a cyclic manner.

Whether the dynamics problem is forward or inverse, the kinematics or motion geometry of the mechanism must be known. Kinematics or kinematic equations describe the relationship between the positions of the links comprising the mechanism. Once position equations are described, they can be differentiated with respect to time once and twice to obtain equations describing the relationship between the velocities and accelerations of the links.

Mechanisms are highly non-linear systems, hence their kinematic equations are also complicated and non-linear equations. Kinematic position equation for each of the mechanisms described in Chapter 2 is a problem unique in itself. A revolute joint providing a rotational motion always impose a harmonic component, comprising of the sine and cosine of the joint angle. A translational motion at a prismatic joint is linear on the contrary. With this simple definition in mind, the position equation for a four-bar mechanism is the most difficult, and the mechanisms containing two prismatic pairs is the simplest.

In kinematic analysis, each link is represented by a vector ranging between the points or axes representing the joints. As joints never disengage, these vectors form a closed link loop. For example, in case of a four-bar mechanism as shown in figure (3.1),

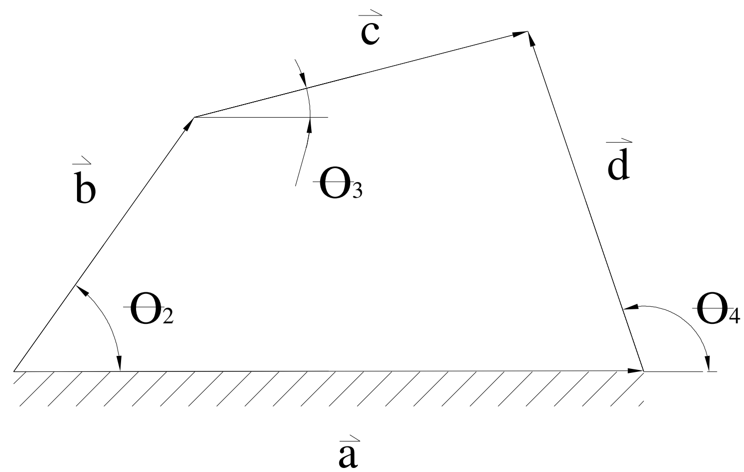


Figure 3.1 A four-bar mechanism and its vector representation.

The vector loop equation is as:

$$\vec{b}+\vec{c}=\vec{a}+\vec{d} \tag{3.2}$$

and is always valid. This equation has 4 vectors and each vector has two arguments, namely a magnitude and a direction. A vector has one or both of its arguments varying or fixed. In case of a four –bar mechanism, the magnitudes of the vectors in equation (3.2) are all constant. Angle of vector $\vec{a}$ is constant. Angle of vector $\vec{b}$, that is θ_2 is an independent variable, angles of $\vec{c}$ and $\vec{d}$ are dependent variables. So, equation (3.2) normally has two result calculable, θ_3 and θ_4. A vector equation is solvable for up to a maximum of two unknowns and hence, when θ_2 is given, equation (3.2) is solvable.

To solve a vector equation, various methods exist. A purely graphical technique is merely a scaled drawing of the mechanism. Once the mechanism is drawn to scale, angles θ_3 and θ_4 are measured directly from the figure by using a protractor. An analytical solution is difficult as simultaneous equations describing the x and y components of equation (3.2) are highly non-linear. A closed form solution has been obtained by Ferdinand Freundenstein and is known as Freundenstein's equation. In the first form θ_3 is eliminated and a closed form solution for θ_4 is obtained as:

$$A \;\; Tan^2 \frac{\theta_4}{2} + B \;\; Tan \frac{\theta_4}{2} + C = 0 \tag{3.3}$$

where

$$A = C\,\text{os}\,\theta_2\left(1 - K_2\right) + K_3 - K_1$$
$$B = -2Sin\theta_2$$
$$C = Co\,\text{s}\,\theta_2\left(1 + K_2\right) + K_3 + K_1 \tag{3.4}$$

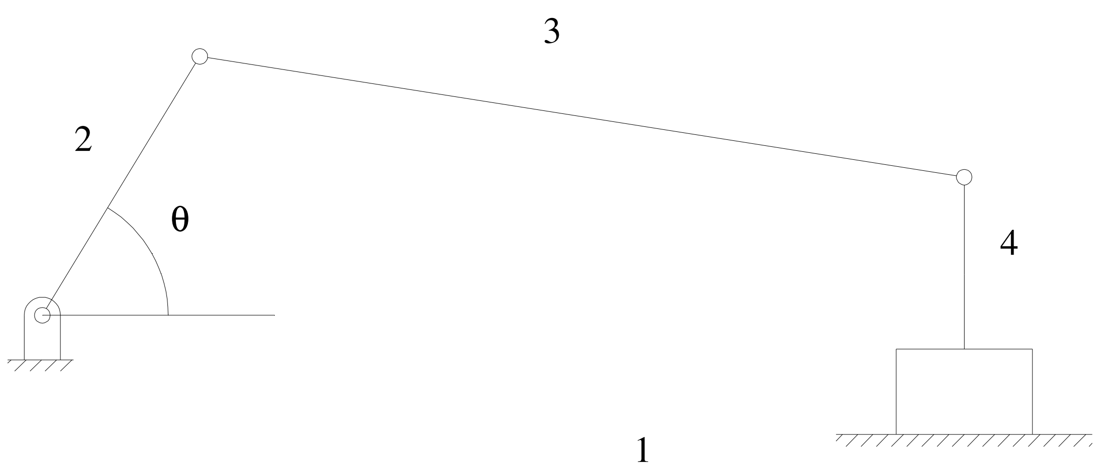
Coefficients A,B and C are variables. K's in them are parameter and hence are constant as:

$$K_1 = \frac{a}{b}$$
$$K_2 = \frac{a}{d}$$
$$K_2 = \frac{a^2 + b^2 - c^2 + d^2}{2ad} \tag{3.5}$$

A very similar set of equations can be derived for θ_3, the coupler link angle when θ_4 is eliminated from the companent equations of equation (3.2). resulting equation is known as "Freundenstein's equation of the second Type".

All isomers of the RRRP closed link loop has one prismatic joint in them and hence their position equations are simpler than that of the four-bar equation. For example, the slider-crank mechanism shown figure (3.2) has the following loop equation.

$$\vec{a} + \vec{b} = \vec{c} + \vec{d} \tag{3.6}$$



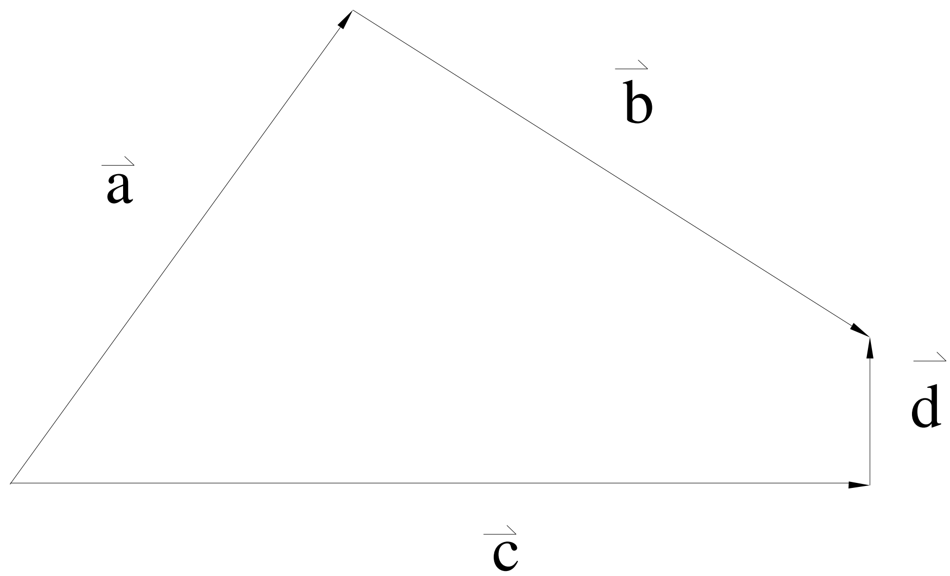


Figure 3.2 An off-set slider mechanism and its vector representation.

$\vec{a}$, $\vec{b}$, $\vec{d}$ has constant magnitudes, magnitude of $\vec{c}$ is variable. Angles of $\vec{a}$ and $\vec{b}$ are variables and angles of $\vec{c}$ and $\vec{d}$ are constant. This equation hence has three variables in all, namely magnitude of $\vec{c}$ and angles of $\vec{a}$ and $\vec{b}$. A decent input / output function will relate crank angle to slider position, eliminating the coupler link angle. The resulting position equation comes up as:

$$C = aCos\theta + \sqrt{b^2 - (aSin - d)^2} \tag{3.7}$$

which is much simpler to derive and use than Freundenstein's equation.

All isomers of 2R-2P closed link loop has two prismatic joints in them and hence their position equations are even simpler than equation (3.7). For example, the elliptic trammel shown in figure (3.3) has the following loop equation:

$$\vec{y}+\vec{l}=\vec{x} \tag{3.8}$$

where only variables are magnitudes of x and y vectors. Solution reduces to Pythogora's theorem as:

$$x^2+y^2=l^2 \tag{3.9}$$

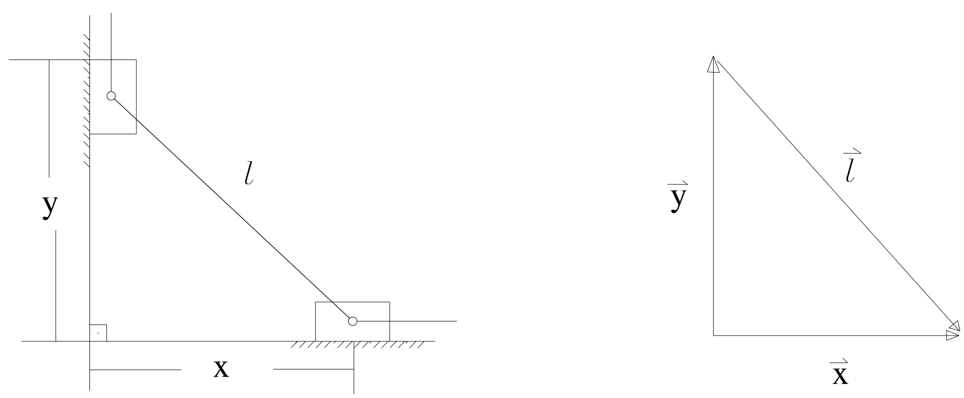


Figure 3.3 An elliptic trammel and its vector representation.

3.3 Mechanism Kinetics

3.3.1 Motion Equations by Newton-Euler Formulation

Mechanisms are composed of several rigid bodies and Newton-Euler laws given by equation (3.1) are valid for each moving body. Each moving body is considered by itself with all the external motor and loud forces and the constraint forces coming through the joints should be taken into consideration. A diagram of a single link with all the acting forces shown is called a "Free Body Diagram" and is very helpful in generating the differential motion equations. External actuation and load forces are arbitrarily definable and hence require their own individual definition. Joint forces obey Newton's third law, such that the force exerted by one link onto another is equal in magnitude and opposite in direction to the force that particular link exerts onto the former. Within this framework, differential motion equations can be derived. In case of the four-bar mechanism shown in figure (3.4) for example, active forcing can well be an arbitrary profile load torque on output crank CD and on accordingly calculated motor torque on input link AB. Motor torque must be at a level at least to overcome the load and friction forces and gravity forces if any and bring the mechanism to static equilibrium. A marginally greater motor torque will generate a marginal acceleration on all the moving links.

Types of joint forces depend on joint types. Söylemez has generated all the possible joints with degrees of freedom between 1 and 5 systemmatically in his book [4]. Shigley and Vicker have examined these joints from a force transmission point of view in their books.

This work takes only the 4-link Grübler mechanisms into consideration and hence only the revolute and prismatic joints will be considered. Degrees of freedom in planar motion is 3, two of which are translations along two mutually perpendicular directions and the remaining a rotation about an axis perpendicular to the plane of motion. A revolute joint can transmit forces in any direction, but it can not transmit any moments. That is why a revolute joint does not allow any relative translation, but allows a rotation only. Forces transferable at a revolute joint hence can be separated into two components, presumably x and y, and specified like that in free body diagrams and motion equations. A prismatic joint on the other hand allows translation along the joints axis. This is because it can not transmit any forces along the joint axis, but it can transmit a force in a direction perpendicular to the joint axis. Also it can transmit a moment and so presents any relative rotation of the links it is joining.

With this simplistic definition of joint forces, the free body diagram of the moving links are shown in figures (3.5), (3.6), and (3.7) showing characteristic external forcing, an arbitrary load torque on input link CD and an accordingly calculated motor torque must be at least at a level enough to bring the mechanism to static equilibrium.

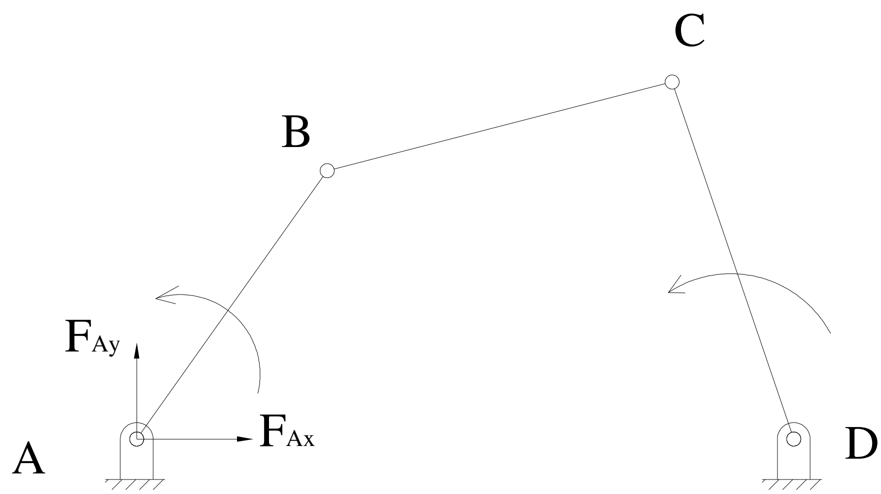


Figure 3.4 A four bar mechanism

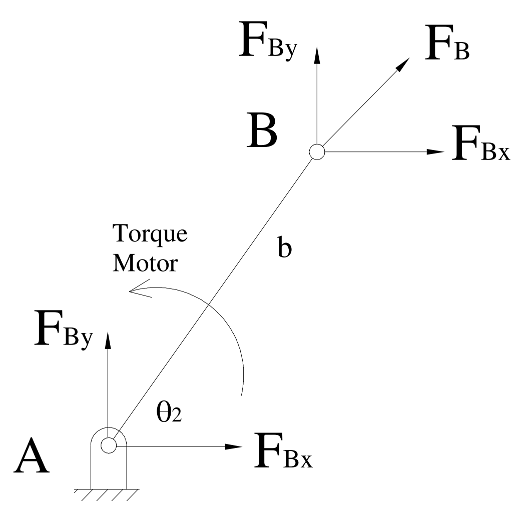


Figure 3.5 Free body diagram of the input crank of the mechanism of figigure (3.4)

Motion equations for the input crank AB hence become:

$$\sum F_x = F_{cx} - F_{Bx} = m_2 \ddot{x}_2 \tag{3.10}$$

$$\sum F_y = F_{cy} - F_{By} = m_2 \ddot{y}_2 \tag{3.11}$$

$$\sum \tau_A = F_{By} bCos\theta_2 - F_{Bx} bSin\theta_2 = I_{2A} \ddot{\theta}_2 \tag{3.12}$$

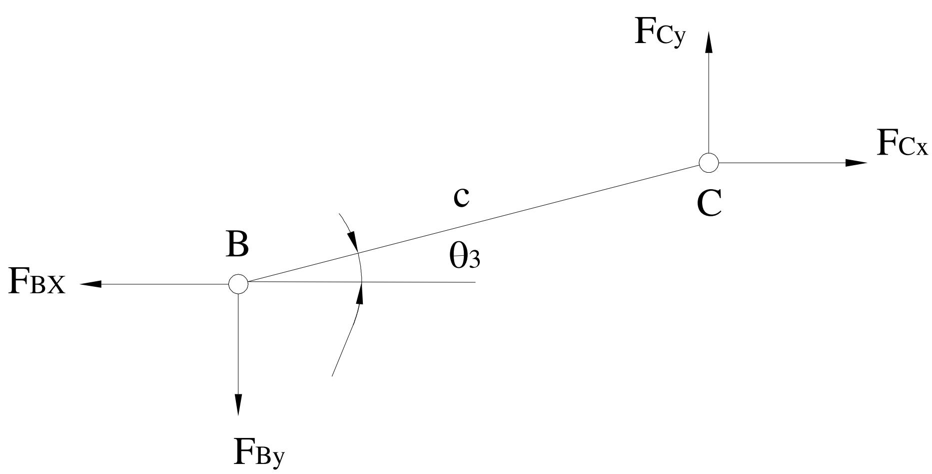


Figure 3.6 Free body diagram of the coupler link of the mechanism of figure (3.4)

Note the opposed directions of forces at joint B:

$$\sum F_x = F_{Cx} - F_{Bx} = m_3 \ddot{x}_3 \tag{3.13}$$

$$\sum F_y = F_{Cy} F_{By} = m_3 \ddot{y}_3 \tag{3.14}$$

$$\sum \tau_c = F_{cy} dSin\theta_3 + F_{cy} dCos\theta_3 = I_{3B}\, \ddot{\theta}_3 \tag{3.15}$$

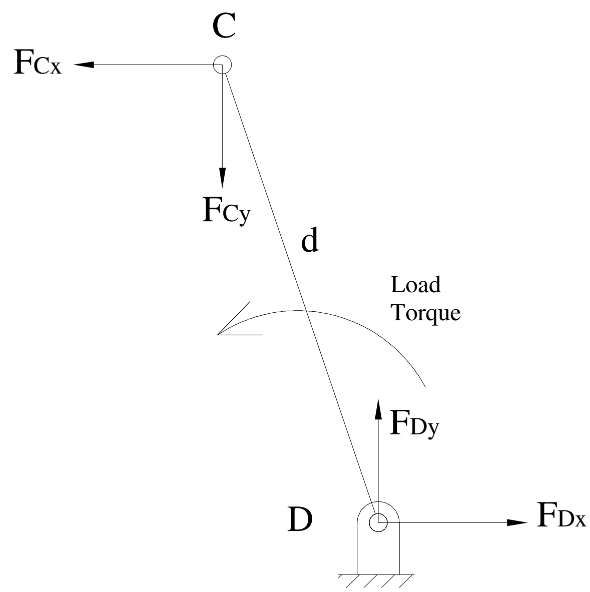


Figure 3.7 Freebody diagram of the output crank of the mechanism of the figure(3.4).

Note the opposed directions of forces at joint C and for output crank CD :

$$\sum F_x = F_{Dx} - F_{Cx} = m_4\, \ddot{x}_4 \tag{3.16}$$

$$\sum F_y = F_{Dy} - F_{Cy} = m_4\, \ddot{y}_4 \tag{3.17}$$

$$\sum \tau_D = F_{Cx} dSin\theta_4 - F_{Cy} dCos\theta_4 = I_{4D}\, \ddot{\theta}_4 \tag{3.18}$$

Resulting 9 differential equations in all. System has only one degree of freedom, so there must be 8 constraint equations in algebraic nature.

3.3.2 Motion Equation by Lagrange Formulation without Multipliers

Generating differential motion equations there times as many as the number of link is one extreme of the kinetics problem. Another extreme is generating a single equation describing the motion in the single degree of freedom, θ_2. This is done by Lagrange Formulation without multipliers. Lagrangian by definition, is the difference between the kinetic and potential energies of the system. Lagrange equation, re-written again as:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i$$

For i = 1,2,….,n (3.19)

where

L the Lagrangian,

q_i the i^{th} generalised coordinate,

Q_i the i^{th} generalised torque or force directly acting on coordinate q_i

The Lagrangian of the system is defined as:

$$L = T - V \tag{3.20}$$

where

T the kinetic energy of the system,

V the potential energy of the system, The crank-rocker mechanism shown in Figure (3.1) has a single degree of freedom resulting in one generalised coordinate as:

$$q_1 = \theta_2 \tag{3.21}$$

$$L = \sum T - \sum V$$
$$L = \sum_{i=1}^{3} \frac{1}{2} m_i v^2{}_{Gi} + \frac{1}{2} I_{Gi} \dot{\theta}^2{}_i - \sum_{i=1}^{3} m_i g y_{Gi} \tag{3.22}$$

$$v^2{}_{Gi} = \left(v_{ix}^2 + v_{iy}^2\right) \tag{3.23}$$

where $\dot{\theta}_i$ is the angular velocity of link i, V_{ix} and V_{iy} are the velocity components of its centre of mass.

The positions in Lagrange formulation should be found by using Freudenstein's equation of the first and second type, coordinate velocities are found from the time derivative of Freudenstein's equation. Using Lagrange formulation in deriving the motion equation of a four-bar mechanism is a laborious task and possibility of making mistakes is great. Resulting equation is large. A complete derivation of the motion equation of a four-bar mechanism has been done in [5].

3.3.3 Motion Equation by Hamilton Formulation

Canonical formulations of Hamilton generate two first order equations for each assumed movability shown by equations (1.10) and (1.11). The former of these describe time rate of change of the generalised momenta. A description of the Hamiltonian function is given by equations (1.12) and (1.15). Number of generalised coordinates defined need not be equal to the degrees of freedom, but can be greater. If no of generalised coordinates is equal to degrees of freedom, then Hamilton formulation ends up with first order differential equations twice in number of the degrees of freedom. If m many coordinates are defined for an n degrees of freedom systems, formulation by equation (1.9) generates m second order differential equations, which in turn require (m,n) many constraint equations, generally algebraic in nature.

3.4 Comparison of the Motion Equation Generation Methods

Motion equations are differential in nature and equations for linkage mechanisms are highly non-linear. Their closed form solutions are never available. They can be integrated by numerical integration routines. Assuming movabilities greater than the degree of freedom simplifies the appearance of the degree of freedom simplifies the appearance of the equations and reduces the labour in their generation, but as number of equations increase, their simultaneous integration becomes more difficult and rounding off errors build up faster, hence results of numerical integration becomes unreliable as time goes on.

Also there is the problem of solving algebraic constraint equations together with the differential equations. This is a difficult task in itself. There are no known software to the writer of this dissertation other than the software developed by L. Changgao of Beijing institute of Post and Telecommunications [6] named “difalg”. One shortcut method is to take the derivatives of the algebraic constraint equations and make them differential equations too. This method increaser the number of differential equations to integrate, hence worsens the accuracy problem.

3.5 Automatic Simulation Programs

As generation of motion equations for mechanisms is a difficult process, some researchers have generated computer programs, which upon description of the mechanism, form the motion equations automatically and once the load and actuation forces and initial conditions are described, integrates them accordingly.

From 1968, a general purpose program called DAMN (Dynamic Analysis of Mechanical Networks) was developed to simulate planar linkages by D.A. Smith and M.A. Chace at the University of Michigan. This program can handle dynamic or kinematic, constrained or unconstrained systems undergoing finite or infinitesimal displacements. System up to 30 links connected to each other by lower kinematic pairs can be modelled. Chonggao indicates a similarity between the way DAMN defines the linkage topology and Branin's technique for the automatic modelling of electrical networks in his 1981 study. Equations of motion are derived using Lagrange's formulation with multipliers. The number of equations developed is equal to the number of joints in the system. The constraint forces are calculated by an iterative determination of lagrange multipliers. The program has a facility to accommodate user-defined forcing functions. Results of integration can be printed out or presented in form of graphs or simple mechanism stick diagrams drawn at a certain position.

DRAM (Dynamical Response of Articulated Machinery) is the second generation of DAMN. It uses D'alambert's principle to define the equations of motion. DAMN and DRAM are powerful dynamic program which are commercially available. At the Central Electricity Generating Board of Britain. DRAM has been further modified and divided into 2 sections called AMP2D and AMP3D which can simulate planar and spatial mechanisms respectively. These programs can accommodate user-defined forcing functions and impact type forces, which are very difficult to integrate digitally. They both can solve forward and inverse dynamics problems.

Another commercially available program originating from the University Michigan is ADAMS: Almost during the same period, between 1968 and 1971 another general purpose program to simulate mechanical networks named IMP (Integrated Mechanisms Program) was developed by J.J. Uicker Jr., D.F. Livermore and P.N. sheth at the University of Wisconsin. IMP is based on the earlier work of Livermore, concisely described in his paper published in 1967. It was further developed by Sheth as a Ph. D. thesis and finally put into a commercially available multi-purpose program as reported by Sheth and Uicker in their paper published in 1972. IMP can handle planar or spatial, multi-degrees of freedom , multi-loop chains.

The program uses the concepts of freedom, multi-loop chains. The program uses the concepts of Graph Theory to define the mechanical network and formulates the constraint equations from the network topology. Constraint equations are used to generate a stiffness matrix, which is substituted into the Hamilton's equations. The program can work in kinematic, static and dynamic modes, and hence can do static force analysis of structures as well. It can calculate joint forces using virtual work. Springs and dampers can easily be included into the simulation. Any other forcing functions can be defined externally. IMP has a powerful graphics package, which can draw graphs or pictures of the network simulated and is commercially available.

Many other general purpose computer programs exist such as MEDUSA prepared by T.J. Lehman at the Illinois Institute of technology, VECNET developed by G.C. Andrews and H.K. Kesavan at the University of Waterloo, SKINAL developed by Paul and Hud at the University of Pennsylvania, KIDYAN developed by Brat at the Czech Technical University, DAPL developed by G.T. Rooney and J.S. Rai at Liverpool Polytechnic and CADOM developed by H. Rankers at Delft University of Technology.

CATIA is an intricate and voluminous general purpose mechanisms simulation software prepared by a team of researchers in 1983, at Dassault Systems, France. It can handle both open and closed loop linkages. It has a graphics package which can present mechanisms in wire frame or detailed polyhedra representation and is graphically interactive. Basic robot tasks and related operations can also be implemented onto the simulation. While in motion, continuous checking of geometric incompatibilities and collisions are carried out and avoided. It can accommodate revolute and prismatic pairs. Any other joint types required are represented by a combination of these. Robot simulations allow up to 20 joints and robot systems up to 20 robots.

Another general purpose dynamic software prepared by Görür as an M.Sc. dissertation in 1989 [7], initially aiming the simulation of internationally accepted commissioning tests of electric towers, can simulate the small and large scale displacements of structures due to external loads and material failure. In this software the topology is described by the initial positions of the joints and stiffness between them. System equations are automatically generated using Newton's laws. Moment equations for links are eliminated by converting the system into equivalent masses concentrated at the joints. This layout enables the simulation of mechanisms and open chain linkages composed of all revolute binary links also.

Most of those programs are either more complicated for 4-link Grübler mechanisms or already outdated now. Among newer software, "Simulink" and "Camlinks" can be referred to.

Simulink is a platform for multi-domain simulation and Model-based design for dynamic systems. It provides an interactive graphical communication environment and a customisable set of libraries by which users can modify them for their on application problems, including the dynamics of the mechanical system and the systems. Produced by "The Mathworks" it is commercially available in market. Components of the simulated system are embedded in blocks, which are shaped by the user. Interaction between the blocks are again defined by the user. The software can be coupled to any other software, such as Matlab for the generation of filters, windows etc. and for required more extensive arithmetic.

Camlinks is an interactive software, developed by the Camlinks company. It is an effective way of designing high-speed machines. Concentrating on the dynamic aspects of the machine, mechanisms and motions can be designed and analysed in detailed. It can design cam and linkage mechanisms, gives an animated motion and motion profiles. After design is completed, produces CAD drawing outlines. Both of these final two softwares act in form of interacting system blocks are similar to the free-bodies of the Newton method, and interaction of the blocks are similar to what Newton's third law defines. So, when one such program is used to simulate a mechanism, it spontaneously utilises Newton's laws to model. Equation numbers are spontaneously elevated, solution becomes time consuming and costly, accuracy lacking in time.

This dissertation presents the use of Lagrange formulation in a systematic definition of the system and generation of its motion equations in the forthcoming chapters.

CHAPTER 4

A GENERALISED APPROACH FOR THE MODELLING OF ARTICULATED OPEN CHAIN PLANAR LINKAGES; FICTITIOUS DEGREES OF FREEDOM

4.1 Introduction

This chapter explains a method developed by Bayseç and Jones [7] which they call "The Method of Fictitious Degrees of Freedom". This method was intended to model 4 link, planar open chain linkages composed of revolute and prismatic pairs only. These systems are parts of industrial robots. First link is the ground and the three links connected to it in articulation are moving and hence comprise a 3 degree of freedom motion. The aim of this dissertation has been to apply constraint forces and or moments to the tip of the last moving link either to keep that point fixed, as if there is a revolute joint there or to make it possible to move along a certain prismatic axis with fixed angular orientation, as if there is a prismatic joint there, to connect the last moving link to the ground and hence convert the open chain robotic system into a closed chain mechanism.

4.2 Definition of the 3-Degrees of Freedom Articulated Open Chain Planar Robotic System

This generalised chain is used to model any configuration of three degrees of freedom planar robot manipulators. Two of the movabilities locate the tip link to the required point in plane, third movability puts it to the correct angular orientation. Four links connected in an articulated manner by three joints form the generalised chain.

Ground is stationary, which is the first link. Main links are powered or actuated by intermediary links, which stretch between main link pairs.

In simulation of the mechanical systems, simulation must be compatible with the real system simulated. The compatibility must hold kinematically, statically dynamically. Kinematic compatibility requires that the motion of all bodies in the simulation must be same as the real system. The joint types, their degrees of freedom and ranges of these degrees must be defined and dimensions of the moving links must be specified in accordance to the real system. For static compatibility, all external, bearing and actuation forces appearing in the simulation must be the same as the real forces occurring in the real system, defining a static entity, when the system is static. All subsets of the static entity must also be static. Dynamic compatibility requires that the inertial properties of the simulation such as masses, mass moments of inertia, must be the same as that of the real system simulated, hence the dynamic forces such as actuator, friction, damping forces and inertia forces appearing in the simulation becomes the same as the real forces occurring in the system. Static compatibility is a subset of dynamic compatibility.

To provide static and dynamic compatibility between the model and the system, first a suitable Cartesian frame is selected, y axis upward, opposite to the g vector. Then the open chain is defined on this Cartesian frame by placing the joints first placement also describes the initial position of the chain. A joint can be either a revolute or a prismatic. Location of a revolute joint is described by the Cartesian coordinates of the point where revolute axis pierces the manipulator plane. A prismatic joint on the contrary, is described by the prismatic axis. Prismatic axis is a straight line its slope and y intercept should be known. Three such joints must be described in consecutive order, starting from the ground, ending at the tip point. Ground link or link number 1 starts from the origin of the coordinate frame and ends at the first joint. First moving link starts from the first joint and extends up to the second joint. Second moving link starts from the second joint and extends up to the third joint. Third moving link or link number 4 starts from the third joint and ands at the end point.

Vectors are used to model the three moving links and the ground link. There are possible 4 different links according to the type of joint.

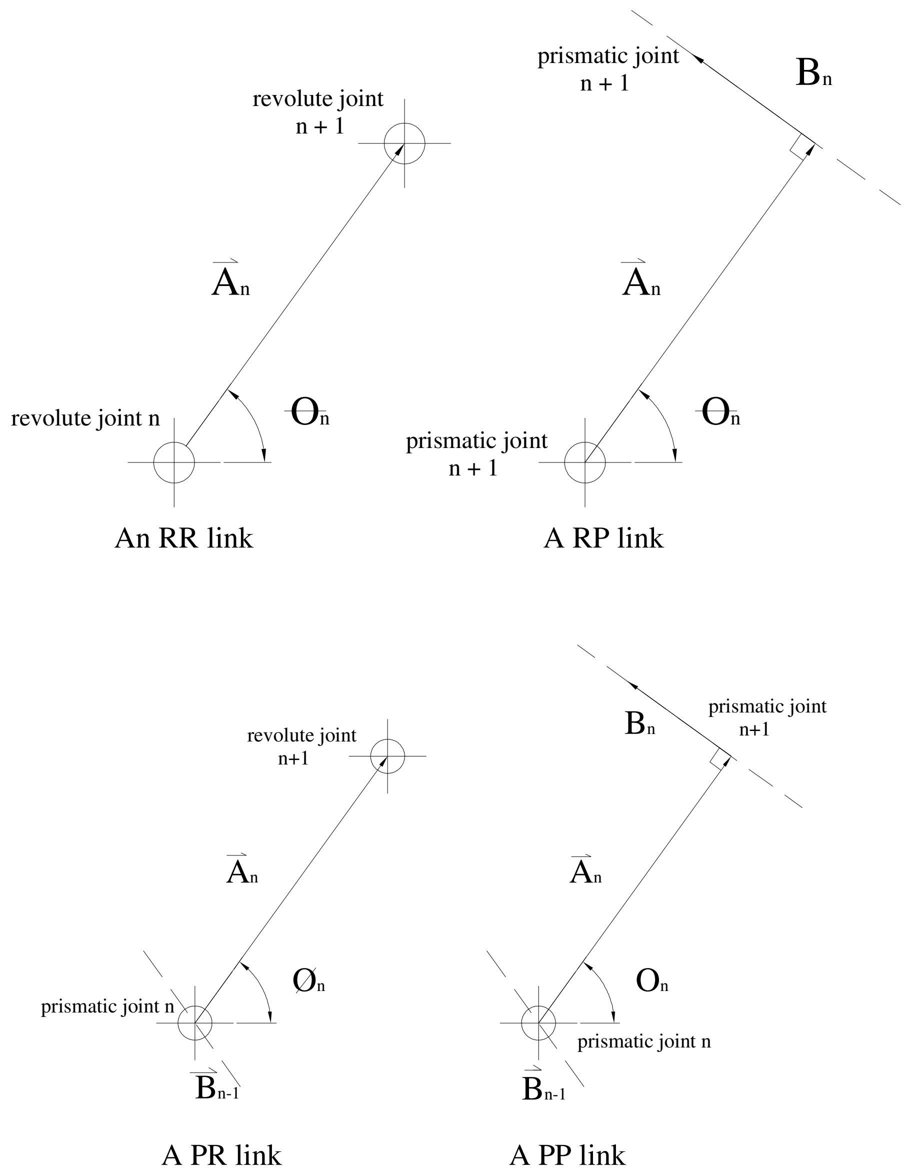


Figure 4.1 Four possible different links with revolute and / or prismatic elements on.

An R-R link is represented by vector $\overline{A}$, which star from the centre of the revolute joint nearer to the ground and ends at the next one. It has a changing argument but a constant modulus. An R-P link is shown by two or orthogonal vectors $\overline{A}$ and $\overline{B}$. A stars from the centre of the revolute joint and extends up to the prismatic axis perpendicular to it. Its argument is variable, but its modulus is constant. The prismatic joint is represented by a $\overline{B}$ vector. Its minimum magnitude is zero, maximum value is the range of sliding motion. Angle of $\overline{B}$ is a right angle ahead of the preceding $\overline{A}$ vector. A P-P link has a prismatic joint at the beginning. Its starts form the tip of the $\overline{B}_{n-1}$, the $\overline{B}$ vector of the preceding link and ends at the centre of the revolute joint. Its modulus is constant. It has the same angular velocity and acceleration as the preceding link. A P-P link starts from the tip of the $\overline{B}_{n-1}$ vector and extends up the succeeding prismatic pair, perpendicular to it. Modulus of A is constant, but its angular velocity and acceleration are the same as that of preceding link. Its B vector starts at the tip of A and represents the succeeding prismatic joint. These links are shown in figure (4.1).

This definition of links leads to an assumption that, each link is starting at a revolute joint and ending at a prismatic. Therefore each link has 2 degrees of freedom to form the generalized chain. Three of these freedoms are in excess for plane positioning and are constrained. Totally 8 different combinations, which are R-R-R, R-R-P, R-P-P, R-P-R, P-R-R, P-R-P, P-P-R, P-P-P are obtainable. But P-P-P combination does not provide a rotational degree, hence is not used for robotics applications.

This model satisfies kinematic compatibility and dynamic compatibility unless friction forces are included. The formulation friction forces are included in following chapters. The generalised chain is shown in figure (4.2).

4.3 Equations of Motion for the Generalised 6-Degrees of Freedom Model

The structure of the generalised 6 degrees of freedom manipulator model is shown in Figure (4.2)

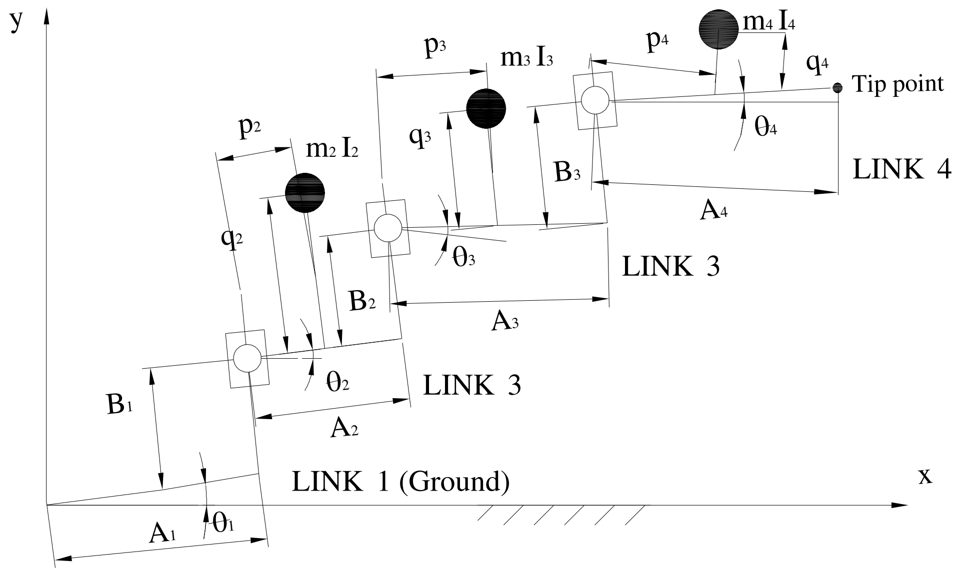


Figure 4.2 The generalised 6 degrees of freedom planar system.

The equations of motion for the generalised chain are derived using Lagrange's formulation that results six second order ordinary differential equations. Generalised coordinates used are B_1, θ_2, B_2, θ_3, B_3 and θ_4.

The coordinates of the mass centres of the moving links are:

$$x_2 = A_1 Cos\theta_1 - B_1 Sin\theta_1 + p_2 Cos\theta_2 - q_2 Sin\theta_2$$

$$y_2 = A_1 Sin\theta_1 + B_1 Cos\theta_1 + p_2 Sin\theta_2 + q_2 Cos\theta_2 \tag{4.1}$$

$$x_3 = A_1 Cos\theta_1 - B_1 Sin\theta_1 + A_2 Cos\theta_2 - B_2 Sin\theta_2 + p_3 Cos\theta_3 - q_3 Sin\theta_3$$

$$y_3 = A_1 Sin\theta_1 + B_1 Cos\theta_1 + A_2 Sin\theta_2 + B_2 Cos\theta_2 + p_3 Sin\theta_3 + q_3 Cos\theta_3 \tag{4.2}$$

$$x_4 = A_1 Cos\theta_1 - B_1 Sin\theta_1 + A_2 Cos\theta_2 - B_2 Sin\theta_2 + A_3 Cos\theta_3 - B_3 Sin\theta_3$$
$$+ p_4 Sin\theta_4 + q_4 Cos\theta_4$$

$$y_4 = A_1 Sin\theta_1 + B_1 Cos\theta_1 + A_2 Sin\theta_2 + B_2 Cos\theta_2 + A_3 Sin\theta_3 + B_3 Cos\theta_3$$
$$+ p_4 Cos_4 - q_4 Sin\theta_4 \qquad (4.3)$$

The expressions for the components of the velocity of mass centers are found by differentiating equations (4.1), (4.2) and (4.3) with respect to time. Substituting them into the Lagrangian and doing the necessary arithmetic operations of the Lagrange formulation. We get the final form of the six equations of motion as:

Equation of motion for the coordinate B_1 is:

$$\ddot{B}_1\left[m_2 + m_3 + m_4\right] + \ddot{\theta}_2\left[Sin(\theta_1 - \theta_2)\left\{m_2 q_2 + B_2(m_3 + m_4)\right\} + Cos(\theta_1 - \theta_2)\left\{m_2 p_2 + A_2(m_3 + m_4)\right\}\right] +$$

$$\ddot{B}_2\left[(m_3 + m_4)Cos(\theta_1 - \theta_2)\right] + \ddot{\theta}_3\left[Sin(\theta_1 - \theta_3)\left\{m_3 q_3 + m_4 B_3\right\} + Cos(\theta_1 - \theta_3)\left\{m_3 p_3 + m_4 A_3\right\}\right] +$$

$$\ddot{B}_3\left[m_4 C\text{os}(\theta_1 - \theta_3)\right] + \ddot{\theta}_4\left[m_4\left\{q_4 Sin(\theta_1 - \theta_4) + p_4 Cos(\theta_1 - \theta_4)\right\}\right] =$$
$$F_{B1} - gCos\theta_1(m_2 + m_3 + m_4) -$$
$$Sin(\theta_1 - \theta_2)\left[m_2 p_2 \dot{\theta}_2^2 + \dot{\theta}_2\left(A_2 \dot{\theta}_2 + 2\dot{B}_2\right)(m_3 + m_4)\right] + Cos(\theta_1 - \theta_2)\left[m_2 q_2 \dot{\theta}_2^2 + B_2 \dot{\theta}_2^2 (m_3 + m_4)\right] -$$

$$Sin(\theta_1 - \theta_3)\left[m_3 p_3 \dot{\theta}_3^2 + m_4 \dot{\theta}_3\left(A_3 \dot{\theta}_3 + 2\dot{B}_3\right)\right] + Cos(\theta_1 - \theta_3)\left[(m_3 q_3 + m_4 B_3)\dot{\theta}_3^2\right] -$$

$$Sin(\theta_1 - \theta_4)\left[m_4 p_4 \dot{\theta}_4^2\right] + Cos(\theta_1 - \theta_4)\left[m_4 q_4 \dot{\theta}_4^2\right] \qquad (4.4)$$

Equation of motion for the coordinate θ_2 is:

$$\ddot{B}_1\left[Sin(\theta_1-\theta_2)\{m_2q_2+B_2(m_3+m_4)\}+Cos(\theta_1-\theta_2)\{m_2p_2+A_2(m_3+m_4)\}\right]+$$

$$\ddot{\theta}_2\left[I_2+m_2\left(p_2^2+q_2^2\right)+(m_3+m_4)\left(A_2^2+B_2^2\right)\right]+$$

$$\ddot{B}_2\left[A_2(m_3+m_4)\right]+\ddot{\theta}_3\,[\,Sin(\theta_2-\theta_3)\,\{m_3(A_2q_3-B_2p_3)+m_4(A_2B_3-B_2A_3)\}+$$

$$Cos(\theta_2-\theta_3)\{m_3(A_2p_3-B_2q_3)+m_4(A_2A_3+B_2B_3)\}]+$$

$$\ddot{B}_3\left[m_4\{A_2Cos(\theta_2-\theta_3)-B_2Sin(\theta_2-\theta_3)\}\right]+$$

$$\ddot{\theta}_4\left[m_4\{(A_2q_4-B_2p_4)Sin(\theta_2-\theta_4)+(A_2p_4-B_2q_4)Cos(\theta_2-\theta_4)\}\right]=$$

$$\tau_{\theta 2}-(m_3+m_4)\left[2B_2\,\dot{B}_2\,\dot{\theta}_2+g\,(A_2Cos\theta_2-B_2Sin\theta_2)\right]-m_2\left[g\,(p_2Cos\theta_2-q_2Sin\theta_2)\right]-$$

$$Sin(\theta_2-\theta_3)\left[m_3\,\dot{\theta}_3^2\,(A_2p_3-B_2q_3)+m_4\left\{2A_2\,\dot{B}_3\,\dot{\theta}_3+\dot{\theta}_3^2\,(A_2A_3+B_2B_3)\right\}\right]-$$

$$Cos(\theta_2-\theta_3)\left[-m_3\,\dot{\theta}_3^2\,(A_2q_3-B_2p_3)+m_4\left\{2B_2\,\dot{B}_3\,\dot{\theta}_3-\dot{\theta}_3^2\,(A_2B_3-B_2A_3)\right\}\right]-$$

$$Sin(\theta_2-\theta_4)\left[m_4\,\dot{\theta}_4^2\,(A_2p_4-B_2q_4)\right]-Cos(\theta_2-\theta_4)\left[m_4\,\dot{\theta}_4^2\,(A_2q_4-B_2p_4)\right] \tag{4.5}$$

Equation of motion for the coordinate B_2 is:

$$\ddot{B}_1\left[(m_3+m_4)Cos(\theta_1-\theta_2)\right]+\ddot{\theta}_2\left[A_2(m_3+m_4)\right]+\ddot{B}_2\,[m_3+m_4]+$$

$$\ddot{\theta}_3\left[Sin(\theta_2-\theta_3)\{m_3q_3+m_4B_3\}+Cos(\theta_2-\theta_3)\{m_3p_3+m_4A_3\}\right]+$$

$$\ddot{B}_3\left[m_4Cos(\theta_2-\theta_3)\right]+\ddot{\theta}_4\left[Sin(\theta_2-\theta_4)\{m_4q_4\}+Cos(\theta_2-\theta_4)\{m_4p_4\}\right]=$$

$$F_{B2}+(m_3+m_4)\left[2B_2\dot{\theta}_2^2-gCos\theta_2\right]+Sin(\theta_2-\theta_3)\left[-m_3p_3\dot{\theta}_3^2-m_4\dot{\theta}_3^2\left(A_3\dot{\theta}_3+2\dot{B}_3\right)\right]+$$

$$Cos(\theta_2-\theta_3)\left[\dot{\theta}_3^2(m_3q_3+m_4B_3)\right]+Sin(\theta_2-\theta_4)\left[-m_4p_4\dot{\theta}_4^2\right]+Cos(\theta_2-\theta_4)\left[m_4p_4\dot{\theta}_4^2\right]$$

(4.6)

Equation of motion for the coordinate θ_3 is:

$$\ddot{B}_1\left[Sin(\theta_1-\theta_3)\{m_3q_3+m_4B_3\}+Cos(\theta_1-\theta_3)\{m_3p_3+m_4A_3\}\right]+$$

$$\ddot{\theta}_2\left[Sin(\theta_2-\theta_3)\{m_3(A_2q_3-B_2p_3)+m_4(A_2B_3-B_2A_3)\}+Cos(\theta_2-\theta_3)\{m_3(A_2p_3+B_2q_3)+m_4(A_2A_3+B_2B_3)\}\right]+$$

$$\ddot{B}_2\left[Sin(\theta_2-\theta_3)\{m_3q_3+m_4B_3\}+Cos(\theta_2-\theta_3)\{m_3p_3+m_4A_3\}\right]+$$

$$\ddot{\theta}_3\left[I_3+m_3(p_3^2+q_3^2)+m_4(A_3^2+B_3^2)\right]+\ddot{B}_3[m_4A_3]+$$

$$\ddot{\theta}_4\left[m_4\{(A_3q_4-B_3p_4)Sin(\theta_3-\theta_4)+(A_3p_4+B_3q_4)Cos(\theta_3-\theta_4)\}\right]=$$

$$\tau_{\theta 3}-m_3g(p_3Cos\theta_3-q_3Sin\theta_3)-m_4\left[2B_3\dot{B}_3\dot{\theta}_3+g(A_3Cos\theta_3-B_3Sin\theta_3)\right]+$$

$$Sin(\theta_2-\theta_3)\left[m_3\left\{2p_3\dot{B}_2\dot{\theta}_2+\dot{\theta}_2^2(A_2p_3+B_2q_3)\right\}+m_4\left\{2A_3\dot{B}_2\dot{\theta}_2+\dot{\theta}_2^2(A_2A_3+B_2B_3)\right\}\right]-$$

$$Cos(\theta_2-\theta_3)\left[m_3\left\{2q_3\dot{B}_2\dot{\theta}_2-\dot{\theta}_3^2(A_2q_3-B_2p_3)\right\}+m_4\left\{2B_3\dot{B}_2\dot{\theta}_2+\dot{\theta}_2^2(A_2B_3-B_2A_3)\right\}\right]-$$

$$Sin(\theta_3-\theta_4)\left[m_4\dot{\theta}_4^2(A_3p_4-B_3q_4)\right]+Cos(\theta_3-\theta_4)\left[m_4\dot{\theta}_4^2(A_3q_4-B_3p_4)\right] \qquad (4.7)$$

Equation of motion for the coordinate B_3 is:

$$\ddot{B}_1\left[m_4Cos(\theta_1-\theta_3)\right]+\ddot{\theta}_2\left[m_4\{A_2Cos(\theta_2-\theta_3)-B_2Sin(\theta_2-\theta_3)\}\right]+\ddot{B}_2\left[m_4Cos(\theta_2-\theta_3)\right]+$$

$$\ddot{\theta}_3\left[m_4A_3\right]+\ddot{B}_3\left[m_4\right]+\ddot{\theta}_4\left[m_4\left\{q_4Sin\left(\theta_3-\theta_4\right)+p_4Cos\left(\theta_3-\theta_4\right)\right\}\right]=$$

$$F_{B3}+m_4\left[B_3\dot{\theta}_3^2-gCos\theta_3\right]+Sin\left(\theta_2-\theta_3\right)\left[m_4\dot{\theta}_2^2\left(A_2\dot{\theta}_2+2\dot{B}_2\right)\right]+$$

$$Cos\left(\theta_2-\theta_3\right)\left[m_4B_2\dot{\theta}_2^2\right]+Sin\left(\theta_3-\theta_4\right)\left[-m_4p_4\dot{\theta}_4^2\right]+Cos\left(\theta_3-\theta_4\right)\left[m_4q_4\dot{\theta}_4^2\right] \qquad (4.8)$$

Equation of motion for the coordinate θ_4 is:

$$\ddot{B}_1\left[m_4\left\{q_4Sin\left(\theta_1-\theta_4\right)+p_4Cos\left(\theta_1-\theta_4\right)\right\}\right]+\ddot{\theta}_2\left[m_4\left\{\left(A_2q_4-B_2p_4\right)Sin\left(\theta_2-\theta_4\right)+\left(A_2p_4+B_2q_4\right)Cos\left(\theta_2-\theta_4\right)\right\}\right]+$$

$$\ddot{B}_2\left[m_4\left\{q_4Sin\left(\theta_2-\theta_4\right)+p_4Cos\left(\theta_2-\theta_4\right)\right\}\right]+\ddot{\theta}_3\left[m_4\left\{\left(A_3q_4-B_3p_4\right)Sin\left(\theta_3-\theta_4\right)+\left(A_3p_4+B_3q_4\right)Cos\left(\theta_3-\theta_4\right)\right\}\right]+$$

$$\ddot{B}_3\left[m_4\left\{q_4Sin\left(\theta_3-\theta_4\right)+p_4Cos\left(\theta_3-\theta_4\right)\right\}\right]+\ddot{\theta}_4\left[I_4+m_4\left(p_4^2+q_4^2\right)\right]=$$

$$\tau_{\theta4}-m_4g\left(p_4Cos\theta_4-q_4Sin\theta_4\right)+m_4Sin\left(\theta_2-\theta_4\right)\left\{2p_4\dot{B}_2\dot{\theta}_2+\theta_2^2\left(A_2\dot{p}_4+B_2q_4\right)\right\}-$$

$$m_4Cos\left(\theta_2-\theta_4\right)\left\{2q_4\dot{B}_2\dot{\theta}_2+\dot{\theta}_2^2\left(A_2q_4-B_2p_4\right)\right\}+m_4Sin\left(\theta_2-\theta_3\right)\left\{2p_4\dot{B}_3\dot{\theta}_3+\dot{\theta}_3^2\left(A_3p_4+B_3q_4\right)\right\}-$$

$$m_4Cos\left(\theta_3-\theta_4\right)\left\{2q_4\dot{B}_3\dot{\theta}_3+\dot{\theta}_3^2\left(A_3q_4-B_3p_4\right)\right\} \qquad (4.9)$$

Equation of motions of [(Equation (2.4) – (4.9)] are arranged in a matrix from that is given as:

$$\begin{bmatrix} M(1,1) & M(1,2) & M(1,3) & M(1,4) & M(1,5) & M(1,6) \\ M(2,1) & M(2,2) & M(2,3) & M(2,4) & M(2,5) & M(2,6) \\ M(3,1) & M(3,2) & M(3,3) & M(3,4) & M(3,5) & M(3,6) \\ M(4,1) & M(4,2) & M(4,3) & M(4,4) & M(4,5) & M(4,6) \\ M(5,1) & M(5,2) & M(5,3) & M(5,4) & M(5,5) & M(5,6) \\ M(6,1) & M(6,2) & M(6,3) & M(6,4) & M(6,5) & M(6,6) \end{bmatrix} * \begin{bmatrix} \ddot{B}_1 \\ \ddot{\theta}_2 \\ \ddot{B}_2 \\ \ddot{\theta}_3 \\ \ddot{B}_3 \\ \ddot{\theta}_4 \end{bmatrix} = \begin{bmatrix} F_{B1} + \Phi_{B1} \\ \tau_{\theta 2} + \Phi_{\theta 2} \\ F_{B2} + \Phi_{B2} \\ \tau_{\theta 3} + \Phi_{\theta 3} \\ F_{B3} + \Phi_{B3} \\ \tau_{\theta 4} + \Phi_{\theta 4} \end{bmatrix} \quad (4.10)$$

Where $[M]$ is the symmetric mass matrix, F and τ's are the generalised external forces and torques acting arbitrarily. Φ's are the generalised velocity dependent forces.

4.4 The Generalised Constraint Forces and Torques

The generalised constraint forces and torques, which are acted on the system like external forcing functions are added on each element of the force vectors. Generalised constraint force of each degree of freedom is equal in magnitude opposite in direction to the sum of all external and velocity dependent forces. Each generalised constraint force is a activated or kept inactive by an Existence Factor, that is a binary information bit defining the actual type of joint. If degree is real its EF is 1, if not its EF is 0, EF's 1 to 6 denote the existence of coordinates $B_1, \theta_2, B_2, \theta_3, B_3, \theta_4$ respectively and

$$EF1 = \overline{EF2} \text{ . OR . } EF2 = \overline{EF1}$$

$$EF3 = \overline{EF4} \text{ . OR . } EF4 = \overline{EF3}$$

$$EF5 = \overline{EF6} \text{ . OR . } EF6 = \overline{EF5} \quad (4.11)$$

are always true.

Angular degrees of freedom are defined as counter-clockwise from the positive x axis, and reactions of rotary actuators are acted on the preceding link. That is, the inclusion of reaction torques and generalised constraint forces convent equation (4.10) into the following form:

$$\begin{bmatrix} \\ M \\ \\ \end{bmatrix} * \begin{bmatrix} \ddot{B}_1 \\ \ddot{\theta}_2 \\ \ddot{B}_2 \\ \ddot{\theta}_3 \\ \ddot{B}_3 \\ \ddot{\theta}_4 \end{bmatrix} = \begin{bmatrix} F_{B1} + \phi_{B1} + GCF_{B1}.EF2 \\ \tau_{\theta 2} + \phi_{\theta 2} + GCF_{\theta 2}.EF1 - \phi_{\theta 3} - GCF_{\theta 3}.EF3 \\ F_{B2} + \phi_{B2} + GCF_{B2}.EF4 \\ \tau_{\theta 3} + \phi_{\theta 3} + GCF_{\theta 3}.EF3 - \phi_{\theta 4} - GCF_{\theta 4}.EF5 \\ F_{B3} + \phi_{B3} + GCF_{B3}.EF6 \\ \tau_{\theta 4} + \phi_{\theta 4} + GCF_{\theta 4}.EF5 \end{bmatrix} \tag{4.12}$$

where GCF's are the Generalised Constraint Force. Inclusion of existence factors and starting from the outermost coordinate yield equations that are given below:

θ_4 equation is:

$$\ddot{B}_1\left[M(6,1).EF6\right] + \ddot{\theta}_2\left[M(6,2).EF6\right] + \ddot{B}_2\left[M(6,3).EF6\right] + \ddot{\theta}_3\left[M(6,4).EF6 - M(6,6).EF5\right] +$$

$$\ddot{B}_3\left[M(6,1).EF6\right] + \ddot{\theta}_4\left[M(6,2)\right] = \left[\tau_{\theta 4} + \phi_{\theta 4}\right].EF6 \tag{4.13}$$

B_3 equation is:

$$\ddot{B}_1\left[M(5,1).EF5\right] + \ddot{\theta}_2\left[M(5,2).EF5\right] + \ddot{B}_2\left[M(5,3).EF5\right] + \ddot{\theta}_3\left[M(5,4).EF5\right] +$$

$$\ddot{B}_3\left[M(5,5)\right] + \ddot{\theta}_4\left[M(5,6)\right] = \left[F_{B3} + \phi_{B3}\right].EF5 \tag{4.14}$$

θ_3 equation is:

$$\ddot{B}_1\left[\left\{M(4,1)+M(6,1).EF5\right\}.EF4\right]+$$

$$\ddot{\theta}_2\left[\left\{M(4,2)+M(6,2).EF5\right\}.EF4-M(4,4).EF3-\left\{M(4,2)+M(6,2)\right\}.EF5EF3\right]+$$

$$\ddot{B}_2\left[M(6,3).EF6\right]+\ddot{\theta}_3\left[M(6,4).EF6-M(6,6).EF5\right]+$$

$$\ddot{B}_3\left[\left\{M(4,5)+M(6,5).EF5\right\}.EF4\right]+\ddot{\theta}_4\left[M(4,6).EF4\right]=$$

$$\left[\tau_{\theta 3}+\phi_{\theta 3}-\tau_{\theta 3}.EF6+\phi_{\theta 4}.EF5\right].EF4 \quad (4.15)$$

B_2 equation is:

$$\ddot{B}_1\left[M(3,1).EF3\right]+\ddot{\theta}_2\left[M(3,2).EF3\right]+\ddot{B}_2\left[M(3,3)\right]+\ddot{\theta}_3\left[M(3,4).EF3\right]+$$

$$\ddot{B}_3\left[M(3,5).EF3\right]+\ddot{\theta}_4\left[M(3,6).EF3\right]=\left[F_{B2}+\phi_{B2}\right].EF3 \quad (4.16)$$

θ_2 equation is:

$$\ddot{B}_1\left[\left\{M(2,1)+M(4,1)+M(6,1).EF5.EF3\right\}.EF2\right]+$$

$$\ddot{\theta}_2\left[M(2,2)\left\{M(4,2)+M(4,4)+M(6,2)+M(6,4)+M(6,6)\right).EF5\right\}.EF3\right]+$$

$$\ddot{B}_2\left[\left\{M(2,3)+\left(M(4,3)+M(6,3).EF5\right).EF3\right\}.EF2\right]+\ddot{\theta}_3\left[M(2,4).EF2\right]+$$

$$\ddot{B}_3\left[\left\{M(2,5)+M(4,5)+M(6,5).EF5).EF3\right\}.EF2\right]+\ddot{\theta}_4\left[\left\{M(2,6)+M(4,6).EF3\right\}.EF2\right]=$$

$$\left[\tau_{\theta 2}-\tau_{\theta 3}.EF4-\tau_{\theta 4}.EF3.EF6+\phi_{\theta 2}+\phi_{\theta 3}.EF3+\phi_{\theta 3}.EF3.EF5\right].EF2 \quad (4.17)$$

B_1 equation is:

$$\ddot{B}_1\left[M(1,1)\right]+\ddot{\theta}_2\left[M(1,2).EF1\right]+\ddot{B}_2\left[M(1,3).EF1\right]+\ddot{\theta}_3\left[M(1,4).EF1\right]+$$

$$\ddot{B}_3\left[M\left(1,5\right).EF1\right]+\ddot{\theta}_4\left[M\left(1,6\right).EF1\right]=\left[F_{B1}+\phi_{B1}\right].EF1 \tag{4.18}$$

If equations (4.13), (4.14), (4.15), (4.16), (4.17), and (4.18), are used whith proper existence factors, the motion of any 3 degrees of freedom open loop chain is defined excepting friction. The portion of the original model, which is presented here does not contain the dynamics of intermediary links such as actuators, balance springs, dashpots etc. The model in full can be found in [8].

4.5 Checking for Correctness

The derivations of motion equations are generally laborious and the possibility of making mistakes in the mathematics and computation is great. Once the computer implementation is done, results should be checked for correctness and accuracy. Checking should never be done by using the same arithmetic approach used in the derivation of the motion equation to prevent any possible repetition of mistakes. A kineto- static solution based on the D'alambert's principle with graphical solution of position, velocity, acceleration and force equations is probably the best means of checking. This method is easy to understand and apply, and as in the well-illustrated books by J. Shgley, fist published in 1961 and by R. Norton, published in 1992 are textbook materials now. A scaled stick-diagram of the mechanism comprises the position analysis. Velocity and acceleration polygons enable to grasp and get an insight into how the coordinated motion of each link is developing at that instant. As mathematics involved is minimum, the possibility of making human mistakes is negligible. Sample solutions must be carried out as many times as possible, enough to prove that the outputs of the computer simulation are correct.

Another way of checking for correctness is giving the conditions, which will produce a known and expected motion profile. Examples to this are numerous. Motion along a vertical slideway for example is a free fall. Free motion of a link about a revolute joint is harmonic, with the natural frequency of the link. To hinder a dynamic link from moving in a multi degree of freedom system, it can be brought to its minimum potential energy state and assigned a very large mass, which is initially at rest. This converts that particular link to a virtual ground. A conservative system keeps the level of its total energy, that is its Hamiltonian, constant. Kinetic and potential energy of individual links come up with complicated profiles, but the total kinetic and potential energies will vary in equal amounts but of opposite polarity, such that the total energy is conserved. If there are not any prismatic movements to infinite displacements, variation in total kinetic and potential energies will be periodic at a frequency equal to the system fundamental frequency. Energy injection or dissipation complicates the problem and therefore should be avoided in the first stages of the tests for correctness.

CHAPTER 5

CONVERSION OF THE 4 LINK OPEN CHAIN INTO A 4 LINK CLOSED LOOP AND EMULATION OF 4 LINK GRÜBLER MECHANISMS

5.1 Conversion Of The 4 Link Open Chain Into A 4 Link Closed Loop

5.1.1 Nature Of The Constraint Forces And Moments

The first link of the open chain is the ground and is not movable. Remaining three links are connected to one another in an articulated manner to form an open chain, one end of which is connected to the ground by a prismatic and a revolute joint in succession. When the tip point of the last moving link is connected to the first link, that is the ground, a closed loop is farmed. If the joint in between is a revolute, it transmits a force in appropriate direction and magnitude to keep the tip point fixed at a certain location, but capable of rotating about it. So, emulate a revolute joint there, a constraint force must be applied in the required direction and magnitude. As neither of these variable are known, it has to be under an appropriate control strategy. For simplicity, the constraint force can be examined in Cartesian components. Control strategy must definitely be aiming position control. If the location of the revolute joint needs be x_r and y_r defined with respect to the primary coordinate frame, which are constant numbers and x_T and y_T are the coordinates of the tip point of the open chain, which are variables, the position dependent constraint forces will be:

$$F_{Tx} = G_1 \left(x_r - x_T \right)$$
$$F_{Ty} = G_2 \left(y_r - y_T \right) \tag{5.1}$$

where the terms in parentheses are errors in x and y directions, G_1 and G_2 are gains, most simply constant numbers. They are large numbers in N/m, Newtons of force-per metre of positional error.

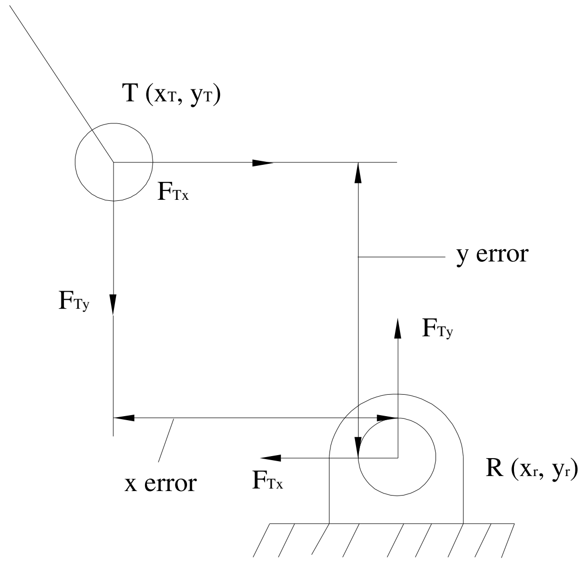


Figure 5.1 Tip point T of the open chain and constraining forces acting on it to more it towards R, where the revolute joint is located.

Position of the tip point can be calculated from:

$$x_T = A_1 Cos\theta_1 - B_1 Sin\theta_1 + A_2 Cos\theta_2 - B_2 Sin\theta_2 + A_3 Cos\theta_3 - B_3 Sin\theta_3 + A_4 Cos\theta_4$$
$$y_T = A_1 Sin\theta_1 + B_1 Cos\theta_1 + A_2 Sin\theta_2 + B_2 Cos\theta_2 + A_3 Sin\theta_3 + B_3 Cos\theta_3 + A_4 Sin\theta_4 \qquad (5.2)$$

For positional accuracy gains G_1 and G_2 must be large numbers but stiff positional control often goes out of control and starts to generate non-decaying limit cycles, which can only be eliminated by velocity dependent damping forces. So, a better constraint force should include damping terms on top of what is shown equation (5.1), such as:

$$F_{Tx} = G_1\left(x_r - x_T\right) - C_1\, \dot{x}_T$$
$$F_{Ty} = G_2\left(y_r - y_T\right) - C_2\, \dot{y}_T \qquad (5.3)$$

$\dot{x}_T$ and $\dot{y}_T$ are the components of the velocity of the tip point, calculable and derivable from equation (5.2), by faking their time derivatives. C_1 and C_2 are constant damping coefficients with unit N/m/sec.

A prismatic joint allows only a translation along the sliding axis and a constraint moment in the mechanism plane, in proper magnitudes and polarities.

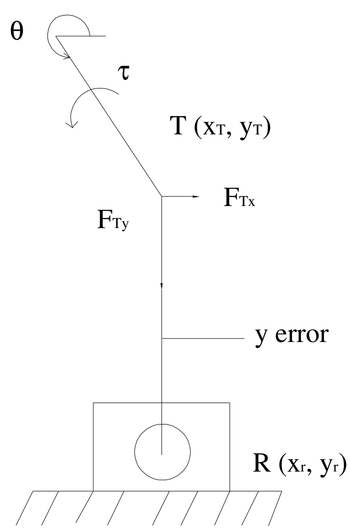


Figure 5.2 Tip point T of the open chain, dislocated from the prismatic details at P, being driven to correct location and orientation by the constraint force F_{Ty} and τ

A simplified problem is shown figure (5.2), where the tip point T is dislocated from the prismatic joint at P by a distance $(x_P - x_T)$ and twisted from its proper angular orientation by $(\psi - \theta_4)$. As a prismatic pairs can not transmit any forces along the slideway, there are no position errors in horizontal direction and hence:

$$F_{Ty} = G_2(y_P - y_T) - C_2\dot{y}_T$$

$$\tau = G_3(\psi - \theta_4) - C_3\dot{\theta}_4 \quad (5.5)$$

ψ, the required angular orientation of the tip most link is a constant number, G_3 is a positional gain in N-m/rad, a high number and C_3 again a damping coefficient in N-m/rad/sec. The prismatic axis can be in any angular position, so, the simple problem illustrated in figure (5.2) and resulting constraint forces of equation (5.5) should be written to emulate any direction for the prismatic axis.

5.1.2 Effect of Constraint Forces and Moments on the Generalized open Chain

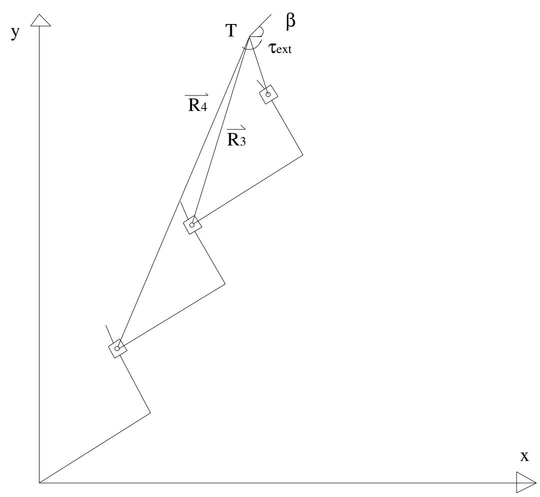


Figure 5.3 Effect of an arbitrary external force and arbitrary external torque on the generalized coordinates of the open chain.

Any external force F_{ext} and torque τ_{ext} applied onto the tip point T has effects on all the six joints on the generalized chain, and hence, must be added onto the generalized forcing functions.

Generalized force components to be added onto the prismatic joint forces are:

$$F_{B1ext} = \vec{b}_1 \overrightarrow{F}_{ext}$$

$$F_{B2ext} = \vec{b}_2 \overrightarrow{F}_{ext}$$

$$F_{B3ext} = \vec{b}_3 \overrightarrow{F}_{ext} \tag{5.4}$$

where $\vec{b}_n$ is a unit vector along the n th prismatic axis, or simply, projection of the external force along the prismatic axes. Note that "dot product" of two vectors is a scalar and arithmetic of equation (5.4). Describe the magnitudes only. Directions of these forces are the same as the generalized coordinates themselves. Similarly, the generalized torque components to be added onto the revolute joint generalized torques are:

$$\tau_{\theta 2ext} = \overline{R}_2 \overline{F}_{ext} + \vec{\tau}_{ext}$$
$$\tau_{\theta 3ext} = \overline{R}_3 \overline{F}_{ext} + \vec{\tau}_{ext}$$
$$\tau_{\theta 4ext} = \overline{R}_4 \overline{F}_{ext} + \vec{\tau}_{ext} \qquad (5.5)$$

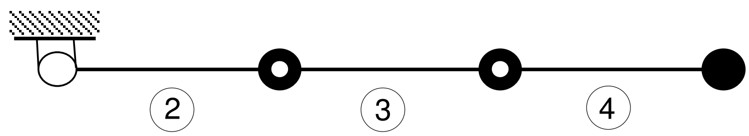
where $\overline{R}_n$ is the vector connecting the $\vec{\theta}_n$ revolute joint to the tip point. Transfer of these components from link to link is then done by the existence factors and generalized force arrangements in the force vector of equation (4.12).

5.2 Examples

To demonstrate the versatility of the method, several examples are presented in this section.

5.2.1 The Triple Pendulum

In this example, the generalized chain is converted into a triple pendulum. Shown in figure (5.4). Vector of Existence Factors is: $[101010]^T$. Link lengths are 1 m. each, masses are 1 kg. each, with zero centroidal mass moments of inertia. There are no external forces and moments, no motor forces and system released from a horizontal position at rest. Motion develop, as shown in figure (5.4), with a complicated nature as expected. Figure (5.5) shows the kinetic and potential energy profiles of each link, which are also complicated, but when total kinetic and total potential energies are calculated, but when total kinetic and total potential energies are calculated, they show equal profiles of opposite polarity, as seen in figure (5.6), hence total energy being zero all the time, as system is conservative. This is a very common approach in verifying the equations of motion of a system are derived correctly or not.



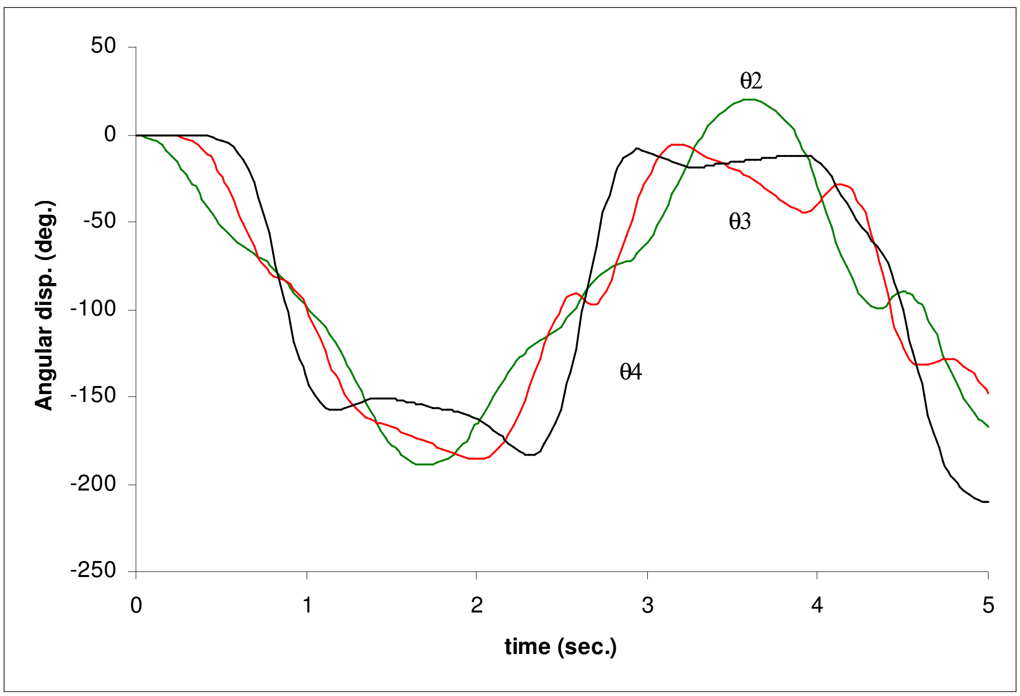
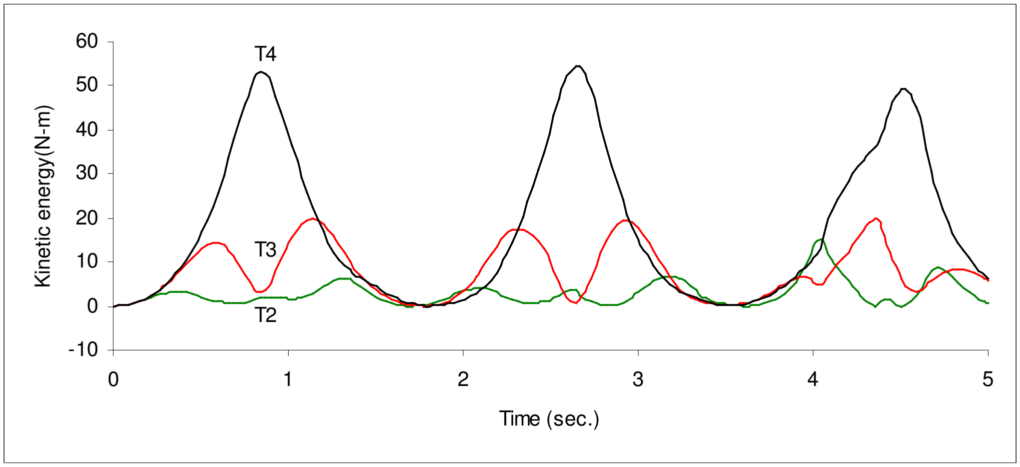


Figure 5.4 The triple pendulum of example 5.2.1 at initial conditions and its motion profile. Link lengths are 1 metre and masses 1 kg. all.



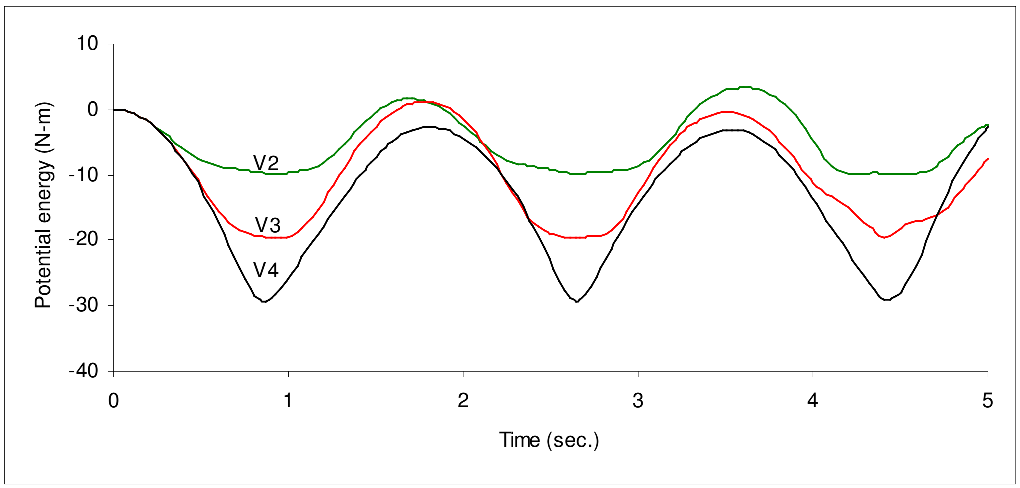


Figure 5.5 Kinetic and potential energies of the links of the triple pendulum whose motion profile is given in figure (5.4)

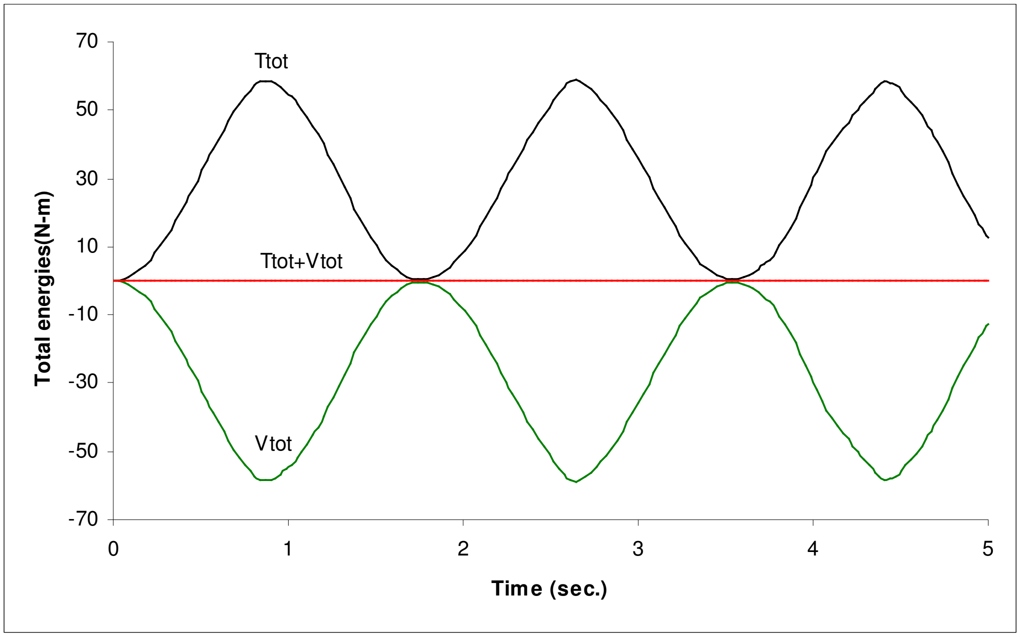


Figure 5.6 Total of kinetic and potential energies of the moving links of the triple pendulum whose motion profile is given in figure (5.4), and their sum. Note that the total energy is zero always as the system is conservative.

5.2.2 The 4-Bar Mechanism

When the tip of a triple pendulum is constrained not to more by two constraint forces, the system reduces into a 4-bar mechanism. In the first example, a triple pendulum whose first link is 0.3 meters long, second 0.8 meters and third 0.6 meters, with 1 kg. masses each, mass centers located at the joints is constrained to act like a 4-Bar mechanism whose second fixed pivot is displaced from origin by 1 meter. Crank is brought to zero degrees position and released from rest. As there are no external motor action or friction, system is conservative and displays non-ending oscillations.

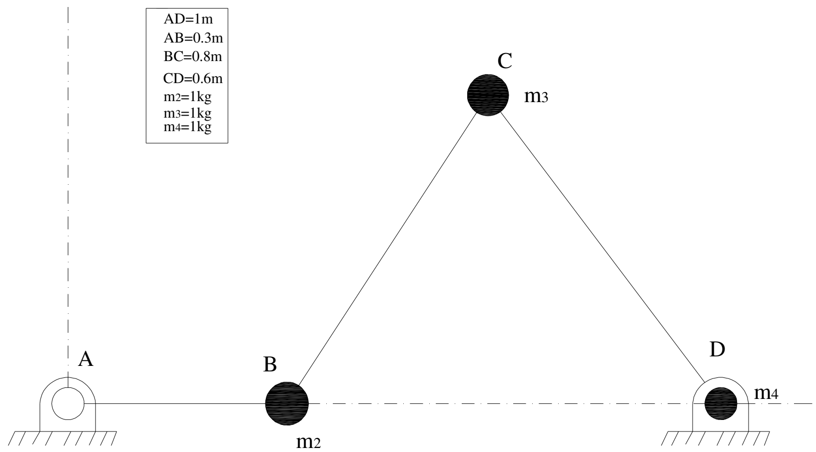


Figure 5.7 The tip of a triple pendulum is constrained not to move, converting it into a 4-Bar mechanism.

To bring the system to rest, a small amount of damping is provided onto the first revolute joint, that is the crank pivot with damping coefficient 0.25 N-m/rad/sec. Resulting motion profile is shown in figure (5.8). Due to damping, motion settles down after some time and system comes to rest at the position where $\theta_2 = -11.98569$ degrees, $\theta_3 = 43.62593$ degrees and $\theta_4 = 332.7809$ degrees. If the dynamic motion equations are derived correctly, the 4-Bar must be in balance of the static forces acting at this position as "static equilibrium" is a subset of "dynamic equilibrium".

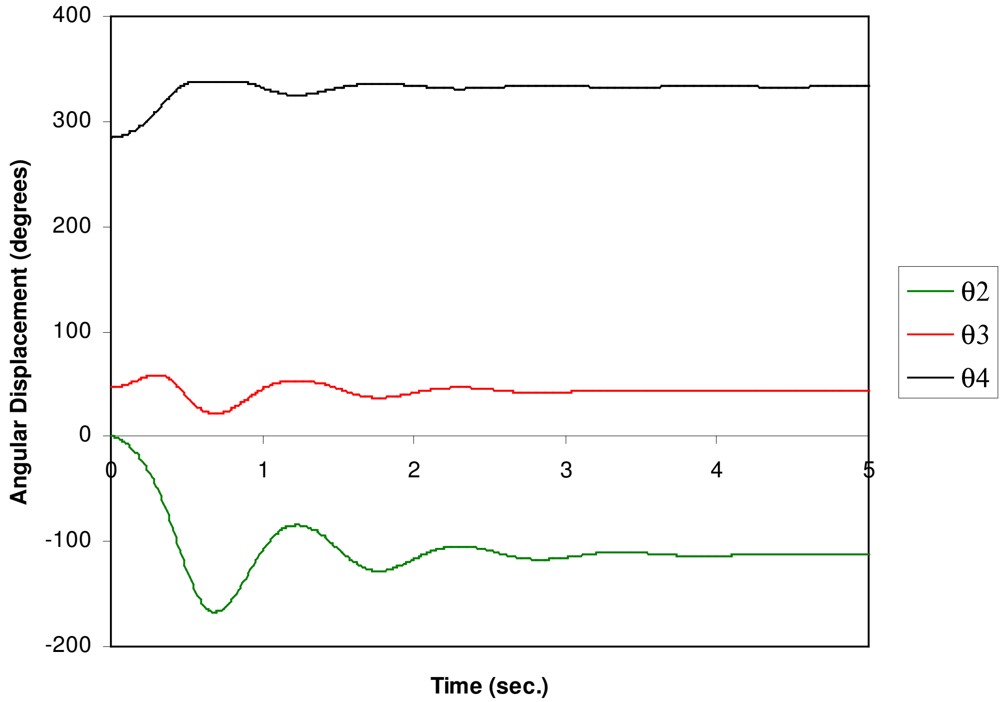


Figure 5.8 Motion profile of the 4-Bar mechanism whose initial position, dimensions and inertial properties are shown in figure (5.7).

There are two masses under the action of gravity only. Figure (5.9) shows the free body diagrams of the 4-Bar mechanism. Coupler link BC is a two forces member and hence is under a compressive load of 9.384 N.

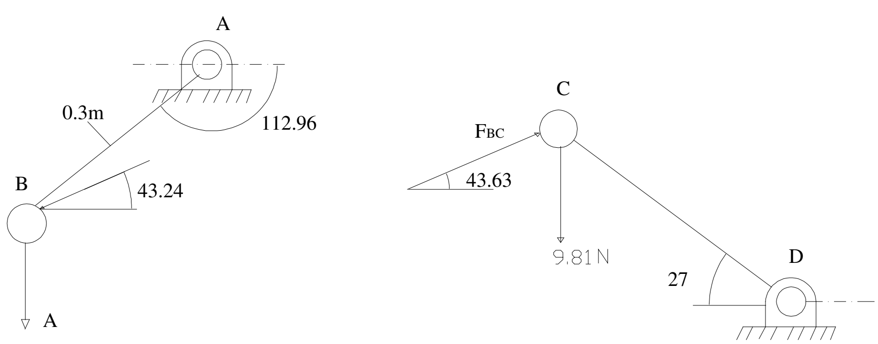


Figure 5.9 Free body diagrams of cranks of the 4-Bar mechanism at the rest position, acted upon by gravity only.

The coupler force F_{BC} required to keep the output crank CD in static equilibrium is also 9.384 N. constraint force components at D, as F_{Dx}=-6.8365 N and F_{Dy} N. The 4-Bar mechanism under consideration can be driven by an arbitrary constant or variable motor torque applied onto crank AB. In the second example a constant motor torque of 3 N-m is applied and mechanism is released from the rest position it attained in the previous example. Motor powers the mechanism into a continuous motion. Crank angle increases with increasing velocity, coupler link BC and output crank CD oscillates within their motion ranges as seen in figure (5.10).

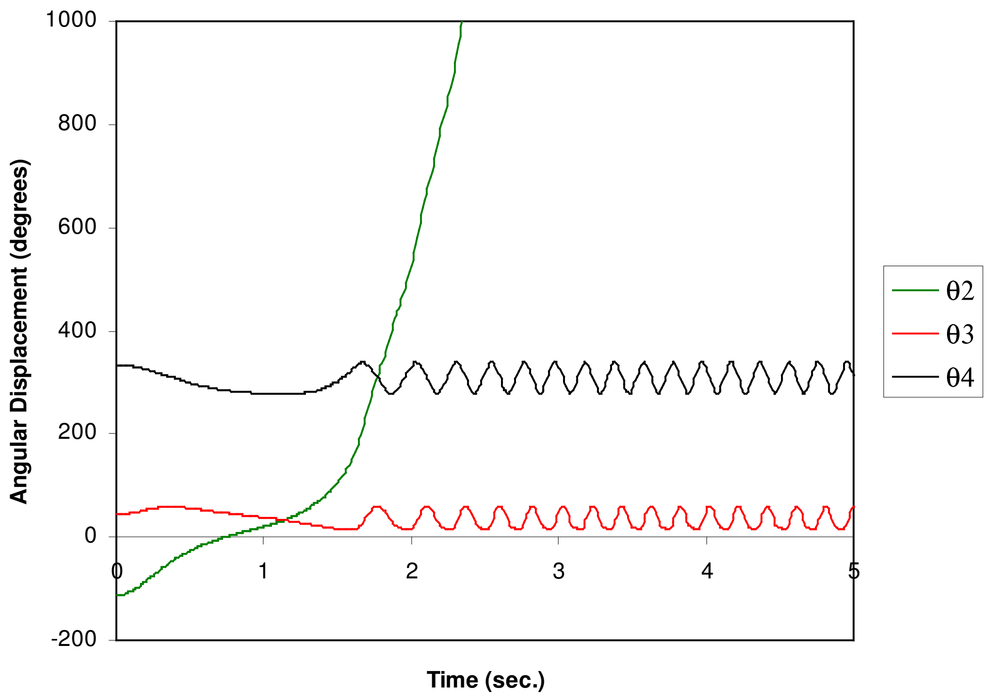


Figure 5.10 The input crank AB of the 4-Bar mechanism described in figure (5.7) and at initial conditions described in figure (5.9) is acted upon by a constant motor torque of 3 N-m. Resulting motion profile is as seen here.

In the third example, again a 4-Bar mechanism is considered in shape of a parallelogram, as shown in figure (5.11) each link is of a 1 meter long and have a mass of 1 kg., concentrated at the link tips. The system is released from rest, while the cranks are 30 degrees from the vertical.

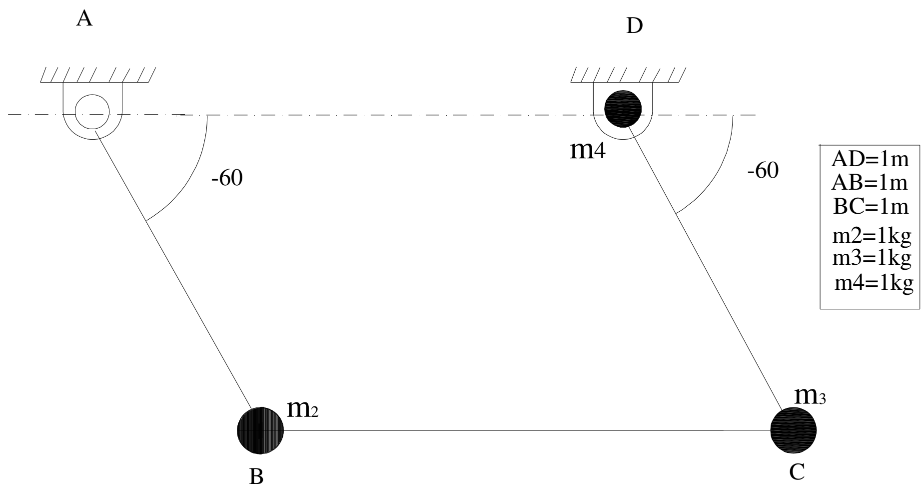


Figure 5.11 A parallelogram mechanism. When this mechanism is released from the initial condition shown here while at rest, generates the motion under the action of gravity only, shown in figure (5.12).

The resulting motion will be that of two identical simple pendulums of natural frequency 3.132 rad/sec or period 2.006 seconds as seen in figure (5.12). Coupler link BC of course does only circular translation and its angle never changes.

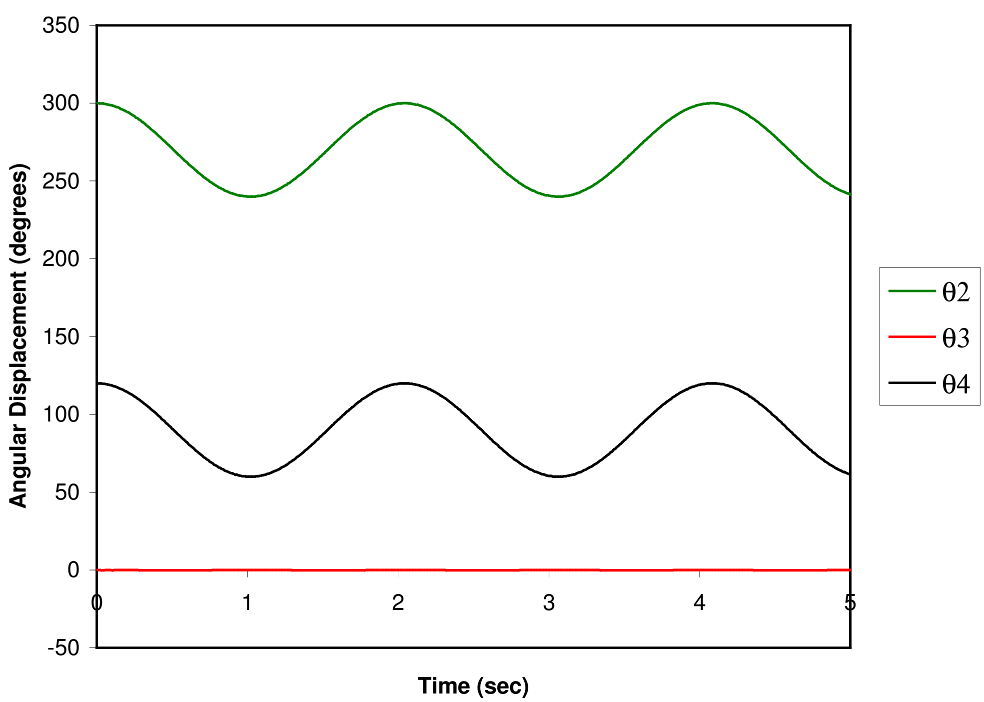


Figure 5.12 Motion profile of the parallelogram mechanism whose initial position and mechanism parameters are shown in figure (5.11).

In the fourth example, to be able to see how the program behaves in case of a dead center position,

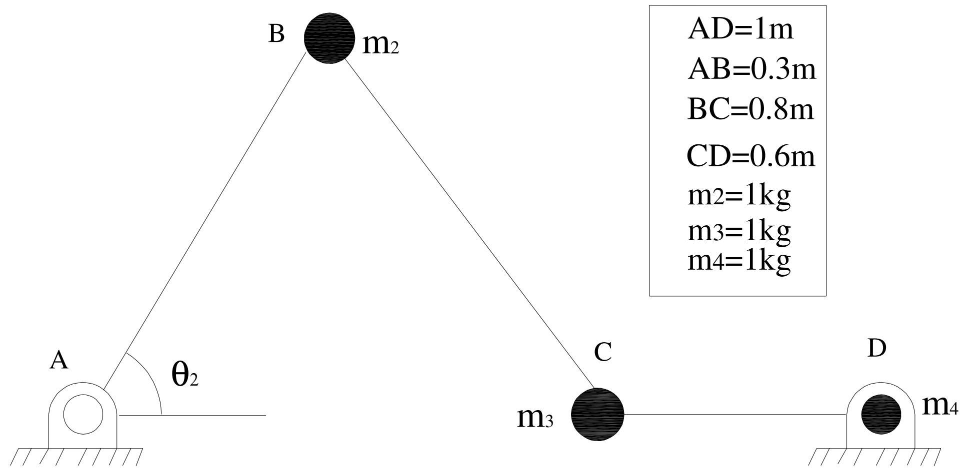


Figure 5.13 A 4-Bar mechanism, at rest at the position shown, where output crank CD is at horizontal position.

Mechanism is assigned the dimensions, inertial properties and initial conditions shown in figure (5.13) and allowed to more under the action of gravity and a 50 N-m motor torque on crank AB in counter-clockwise direction. Profile of the motion is as shown in figure (5.14).

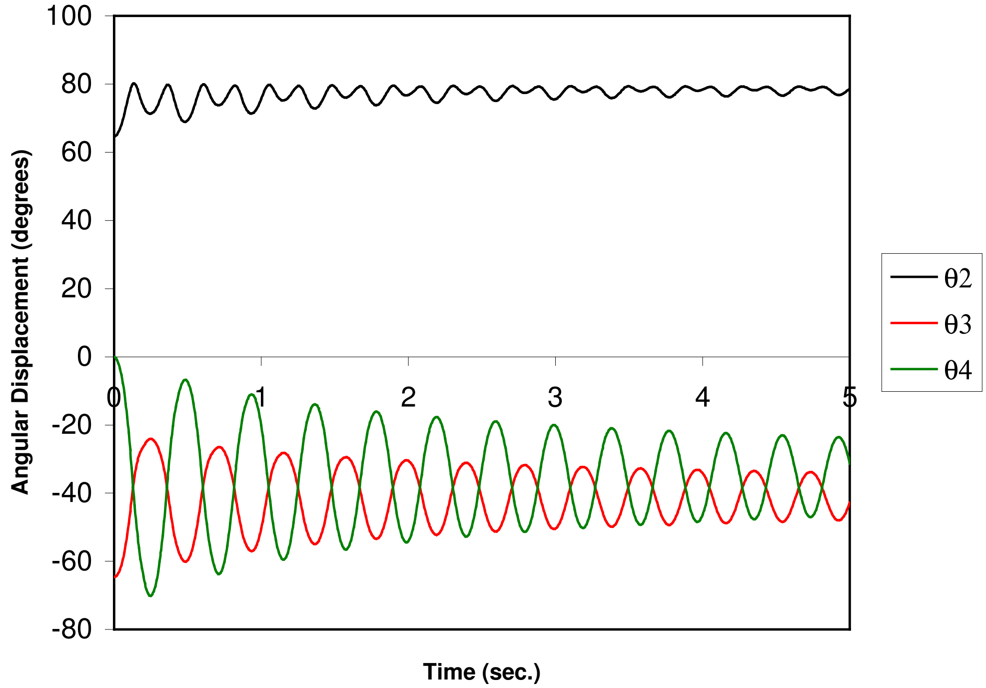


Figure 5.14 Motion of the 4-Bar mechanism whose parameters and initial conditions are shown in figure (5.13), under the action of a 50 N-m motor torque on crank AB and gravity.

In the fifth example, for the motion to settle down, a damping torque on crank AB of the previous example is applied. Damping coefficient is 10 N-m/rad/sec. Resulting motion profile is shown in figure (5.15). As seen there, all the links come to a stop at the extended dead center position. Angular positions when settled down are: θ_2 = 77.7 degrees, θ_3 = -44.74 degrees and θ_4 = -28.51 degrees. At this position, a force analysis shows that the mechanism is in static equilibrium under the action of gravity and the 20 N-m motor torques on crank AB. 20 N crank torque stretches the dyad of the coupler link and the output crank, but the two never attain the same angle due to gravity.

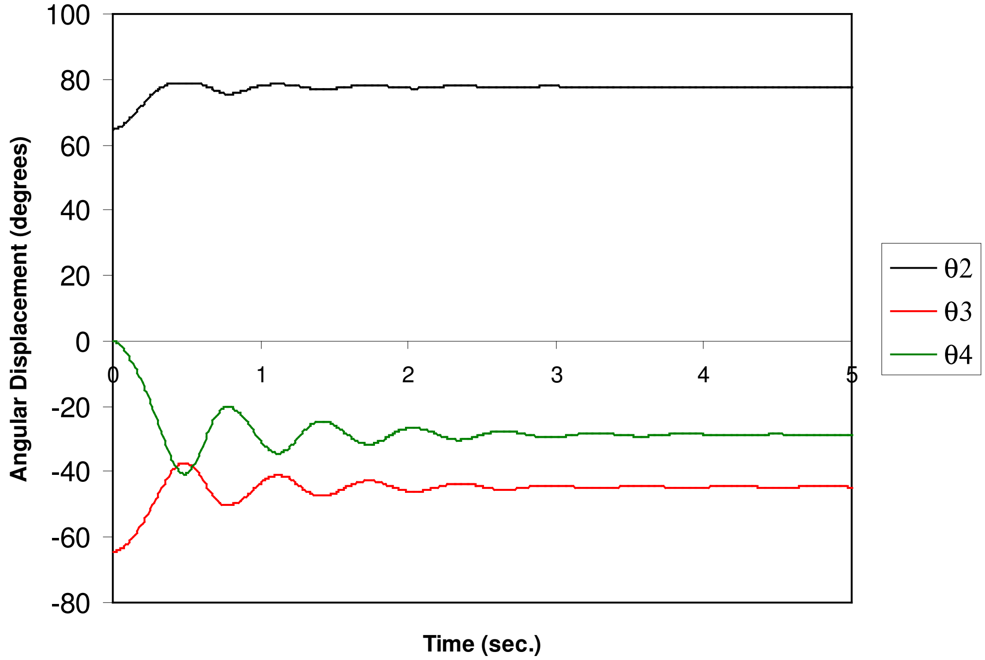


Figure 5.15 Motion of the 4-Bar mechanism shown in figure 5.13 under the action of gravity, 20 N-m of constant motor torque and damping of coefficient 10 N-m/rad/sec on crank AB. Links come to rest at the extended dead center position.

5.2.3 The Slider –Crank Mechanism

The next set of examples are on the slider-crank mechanism, well known member of the RRRP closed link loop. This mechanism requires the vector of existence factors as $[101010]^T$ if the sliding R-P link is of finite dimensions. If the sliding link of zero length, it can still be considered a double pendulum, a special version of the triple pendulum, a special version of the triple pendulum used to model the 4-Bar mechanisms described in the previous five examples, with the length of the sliding link is zero, it will not require any external torque for constraining any rotation.

In the sixth example, the mechanism considered is an in-line slider-crank, whose the dimensions, inertial properties and the initial condition is shown in figure (5.16). No motor torques are applied and the mechanism is released from rest, while the crank angle is at zero degrees. The system moves under the action of gravity only and to settle down, a damping is applied onto the crank with coefficient 9N-m/rad/sec.

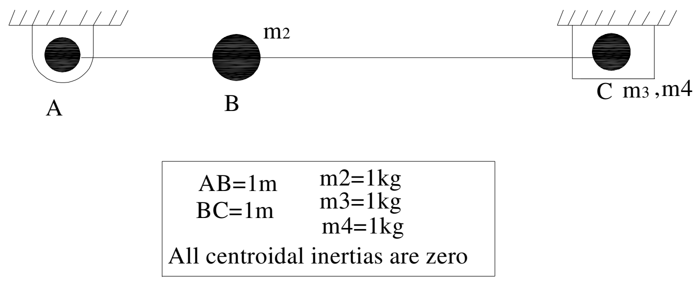
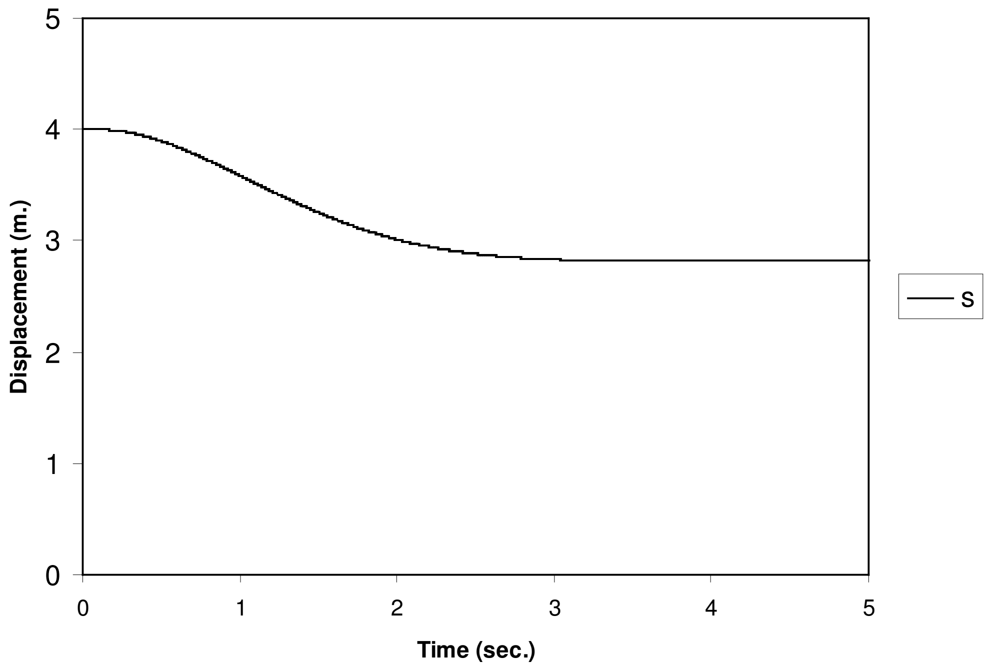


Figure 5.16 An in line slider-crank mechanism, initially at its extended dead center position. Mechanism parameters are shown aside.

Resulting motion profile is shown in figure (5.17), motion actuated by gravity only settles down at its minimum potential energy position where crank angle becomes -90º, coupler angle becomes 19.471º and slider position from crank pivot 3.1622 m.



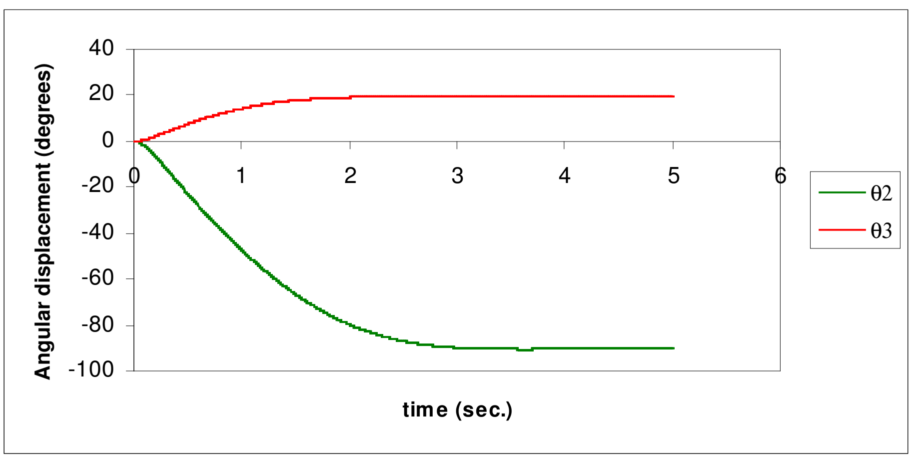
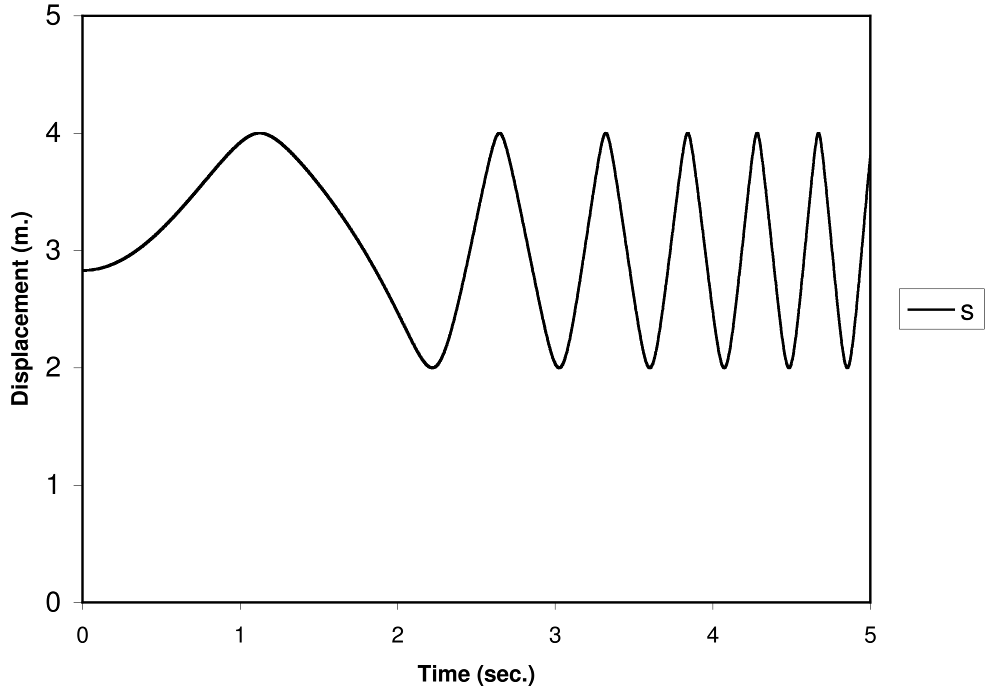
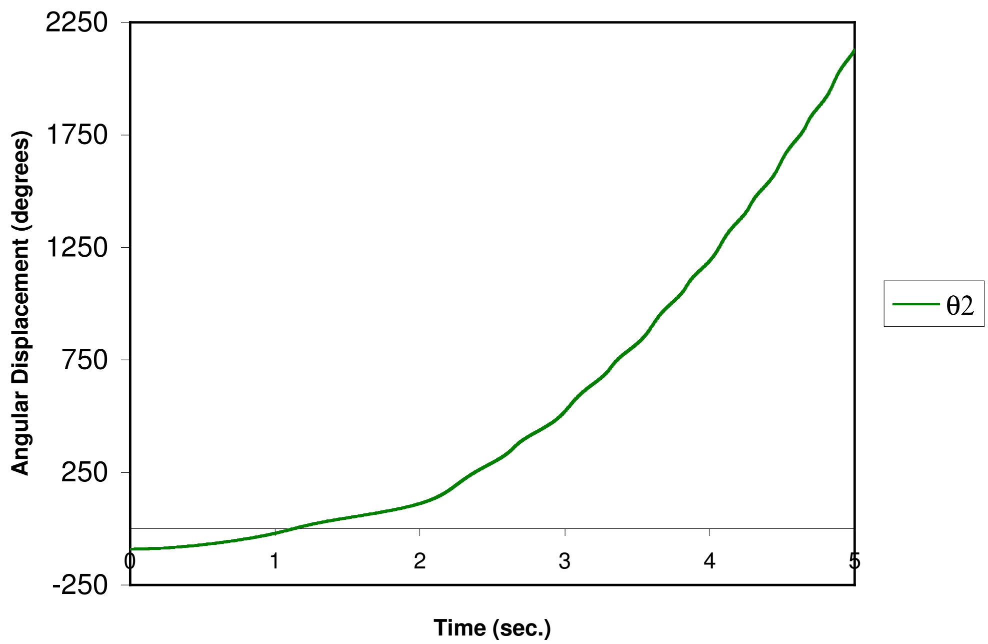


Figure 5.17 Motion profile of the slider-crank mechanism shown in figure (5.16). There is damping but no motor torque on the crank, hence system moves under the action of gravity only and settles down at minimum potential energy position.

Next example, the seventh one shows how can the slider-crank of the previous example be driven by a motor connected to point A in figure (5.16). An arbitrary torque of 9 N-m is applied, without any damping. Resulting motion profile is shown in figure 5.18, starting from the minimum potential energy position described in figure (5.17).


5
4
3
2
1
0
Displacement (m.)
0
1
2
3
4
5
Time (sec.)
s


2250
1750
1250
750
250
-250
Angular Displacement (degrees)
0
1
2
3
4
5
Time (sec.)
θ2

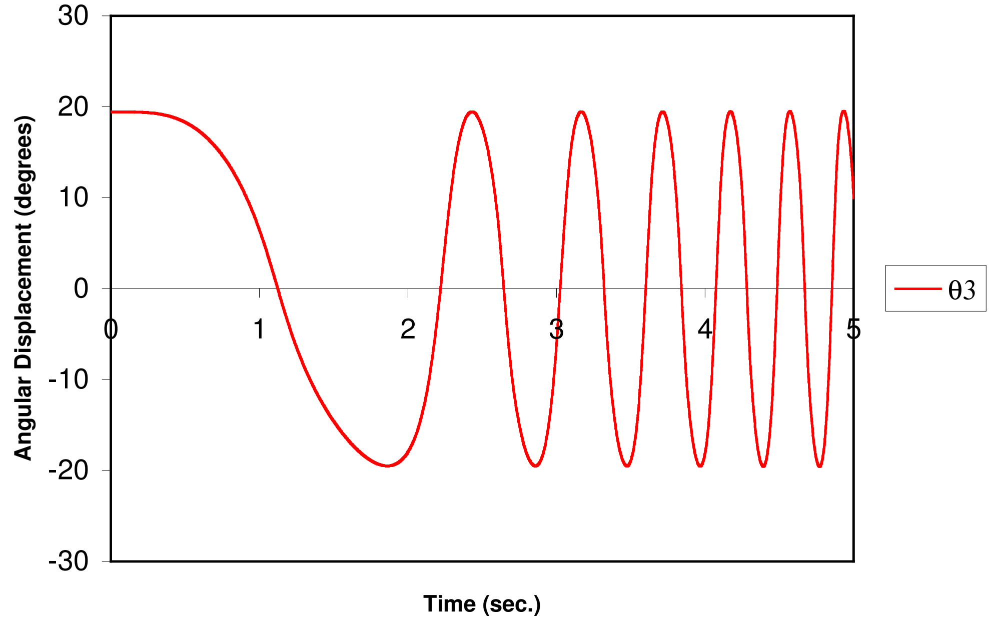


Figure 5.18 Motion profile of the slider-crank mechanism described in figure (5.16), generated by a 9 N-m crank torque. Motion starts from the settled down conditions shown in figure (5.17).

5.2.4 The Elliptic Trammel

To show how the method deals with two sliding joints, the elliptic trammel is emulated in two examples here. In the eight example of the chapter, the mechanism shown in figure (5.19) used. Vector of existence factors is $[100101]^T$, indicating that joint between the ground and first moving link is a prismatic, joint between the second moving link and third is a revolute too. The third is constrained to move on a vertical slideway by proffer constraint forces. Mechanism is released from the position shown in figure (5.19), while at rest. Normally it has to display non-ending oscillations under the action of gravity if no friction or damping acts.

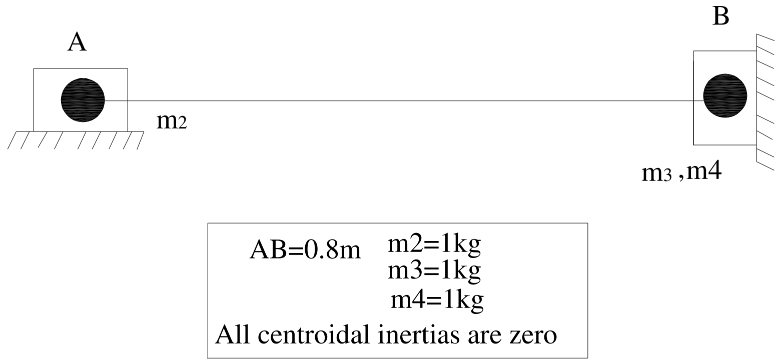


Figure 5.19 The Elliptic Trammel referred to in example 8 of section 5.2.4 in its initial condition, together with system parameters.

For the motion to settle, a damping force is applied onto the first prismatic joint with coefficient 10 N-m/rad/sec. Resulting motion profile is shown in figure (5.20). S_1 in the legend stands for coordinate B_1, that is position of point A, initially at zero, moves right as the mass at B moves down under the action of gravity, displays a slight overshoot to the other side of the vertical slideway and then settles at 0.8 meters.

Similarly, slider at point B is initially at origin moves down under the action of gravity. Displacement of slider B from origin is indicated by S_2 in the legend of figure (5.20). At max, it attains a 0.8 meter displacement, equal to the length of the coupler link, displays a rebounce and then settles down at o.8 m. displacement. As motion is in direction of gravity, mathematically it comes up with negative valves of displacement. Angle of the coupler link BC, as measured counterclockwise from the positive x axis, origin being at A, starts from zero degrees and end, its motion at –90 degrees. Motion profile is correct dynamically and the static position attained in the end is also static position attained in the end is also statically correct.

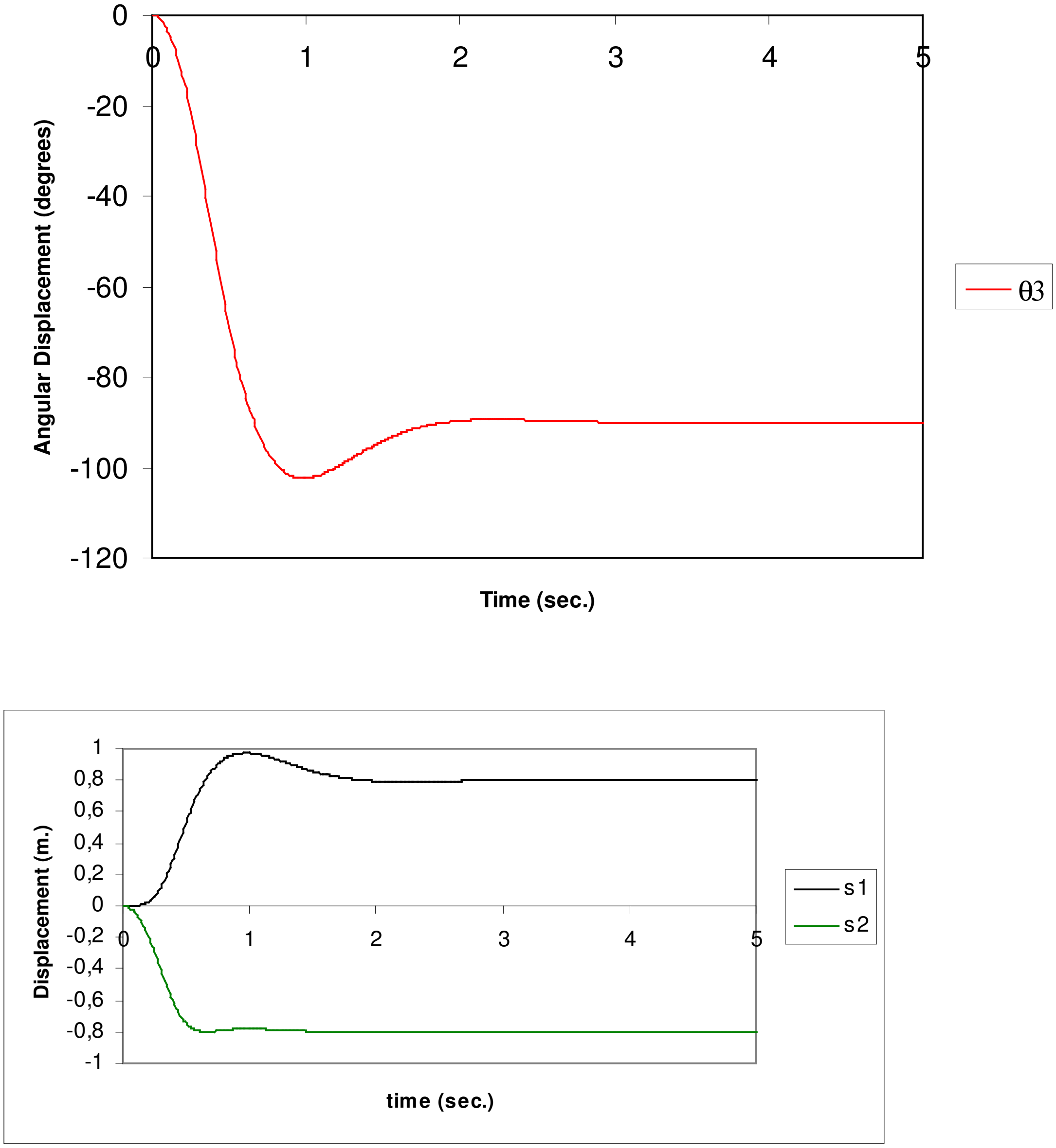
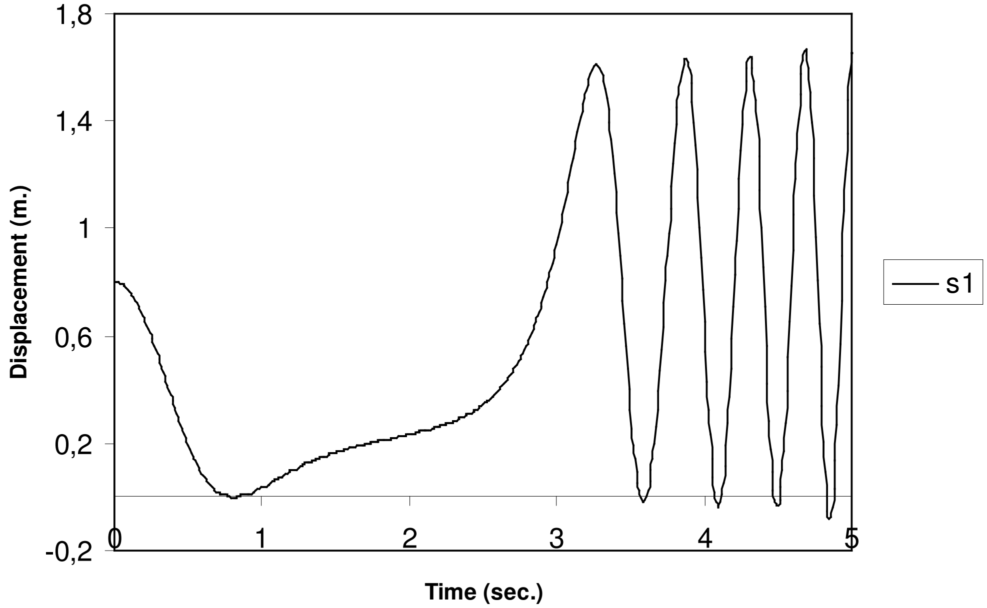
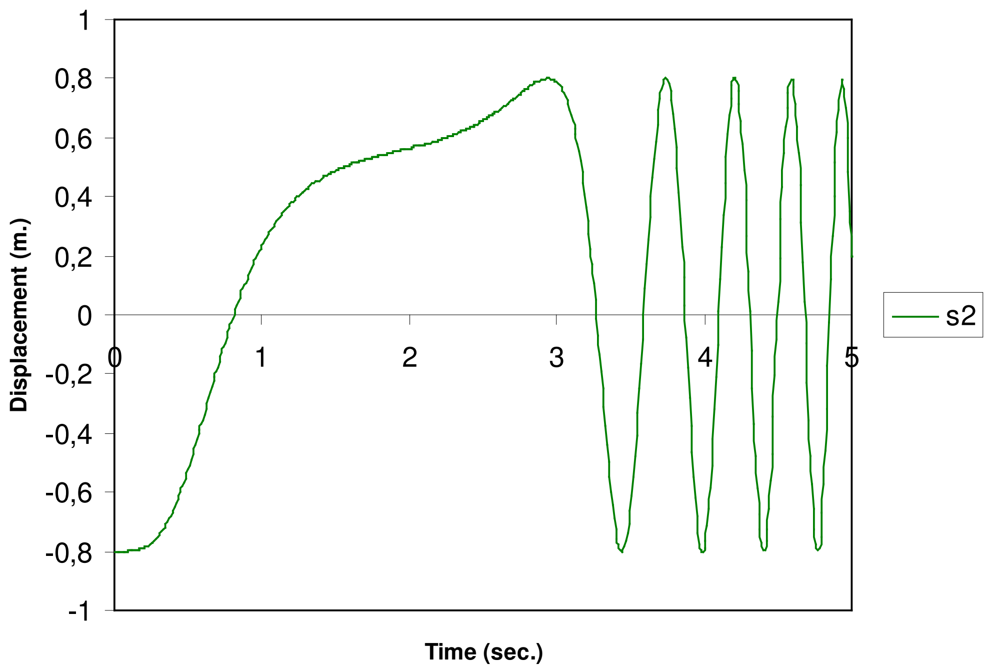


Figure 5.20 Motion profile of the elliptic trammel shown in figure 5.19 under the action of gravity only. Due to damping, motion settles down at the minimum potential energy position.

An elliptic trammel is a special form of a “double-slider” mechanism, where the angle between the sliding axes is 90 degrees. A double-slider is used to convent a rectilinear motion in one direction into another rectilinear motion in some other direction. As points on the coupler link of an elliptic trammel traces exact ellipses, this mechanism has been the part of mechanical calculating machines for solving second order equations. Normally, it is operated only in the first quadrant due to space limitations, by moving one of the sliders, manually in case of a computing machine or by a hydraulic actuator. Output is the motion of either the other slider or a coupler point. This mechanism often is not operated in all four quadrants. For that, the sliding axes should be crossing each other and so are made in different planes. Prismatic axes are in parallel but offset planes, and the coupler link operates in between. In such a construction, the system can be driven by a continuously rotating motor placed an one of the sliders and coupler connected to the motor shaft. In the ninth example of the chapter, a continuous operation of the mechanism is observed in all four quadrants. Motion starts from the final conditions of the motion of previous example, where mechanism has attained its minimum potential energy and staying still in stable static equilibrium. As seen in figure (5.21), motion needs some time to build up, and then sliders show to and fro oscillations in their slideways within a range of 1.6 meters, twice the length of the coupler link and coupler link, starting its motion from –90 degrees, displays an ever increasing angular position, rotating counterclockwise. Driving torque of the motor is constant and arbitrarily chosen to be 11.38 N-m.


1,8
1,4
1
0,6
0,2
-0,2
0
1
2
3
4
5
Displacement (m.)
Time (sec.)
s1


1
0,8
0,6
0,4
0,2
0
-0,2
-0,4
-0,6
-0,8
-1
0
1
2
3
4
5
Displacement (m.)
Time (sec.)
s2

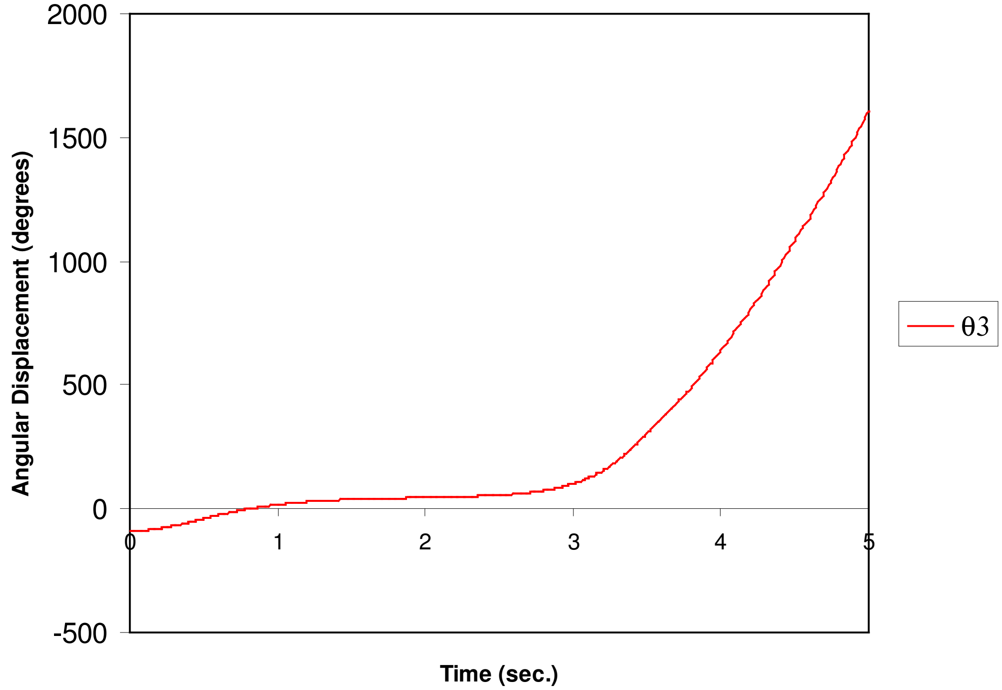


Figure 5.21 Motion profile of an elliptic trammel working in all 4-quadrants Actuation is done by a motor on the first sliding link, rotating counterclockwise and the coupler link connected to its shaft. After the motion builds up both sliders oscillate within a range of 1.6 meters each, twice the length of the coupler and the coupler displays an ever increasing angular position in counterclockwise or positive sense.

5.3 Tuning Up Of The Motion Equations

To tune up the motion equations to model a particular mechanisms, existence factors, mechanism dimensions and location of mass centers, the masses and centroidal mass moments of inertias must be declared. Tuning may seem difficult at the beginning and therefore, for the completeness of the dissertation, sample tunings for all possible 4-link Grübler mechanisms are given in this section. Nomenclature is as follows:

Often symbols come up with subscripts. Subscript is a number and indicates a particular link. There are 4 links in total, enumerated starting from the ground. Link 1 is the ground, immovable with infinite dimensions and inertial properties. Links 2,3 and 4 are the first, second and third moving links in succession, hence:

m_n , n = 2,3,4 : mass of nth link.

mi_n , n = 2,3,4 : mass moment of inertia of nth link.

p_n , n = 2,3,4 : relative coordinate of mass nth link along vector $\overline{A}_n$.

q_n , n = 2,3,4 : relative coordinate of mass center of nth link perpendicular to vector $\overline{A}_n$.

a_n , n = 1,2,3,4 : magnitude of vector $\overline{A}_n$ of nth link, all constants.

Theta 1 : Angle of vector $\overline{A}_1$, a constant value.

flx : x coordinate of the point where tipmost point of the open chain is to be fixed at, a constant number.

Fly : y coordinate of the point where tipmost point of the open chain is to be fixed at, a constant number.

gain : position gain of the constraint force fixing the tip of open chain.

cgain : velocity gain of the constraint force fixing the tip of open chain.

y functions are as follows :

y(1) : B_1 (Prismatic joint between the ground and link 2)

y(2) : $\dot{B}_1$ Joint velocity

y(3) : θ_2 (Revolute joint between the ground and link 2)

y(4) : $\dot{\theta}_2$ Joint velocity

y(5) : B_2 (Prismatic joint between links 2 and 3)

y(6) : $\dot{B}_2$ Joint velocity

y(7) : θ_3 (Revolute joint between links 2 and 3)

y(8) : $\dot{\theta}_3$ Joint velocity

y(9) : B_3 (Prismatic joint between links 3 and 4)

y(10) : $\dot{B}_3$ Joint velocity

y(11) : θ_4 (Revolute joint between links 3 and 4) y(12) : $\dot{\theta}_4$ Joint velocity

ef_n , n = 1….6 : existence factors

fx : x component of the constraint force closing the open chain

fy : y component of the constraint force closing the open chain

tt : constraint torque closing the open chain

Sample values for different 4-link Grübler mechanisms are as follows:

5.3.1 The Four Bar Mechanism

Masses;
m2 = 1 kg. (for link2)
m3 = 2 kg. (for link 3)
m4 = 3 kg. (for link 4)

Mass moment of inertias;
mi2 =0. 1 (for link2)
mi3 = 0.2 (for link 3)
mi4 = 0.3 (for link 4)

P;
p2 =0. 1 m. (for link2)
p3 = 0.4 m. (for link 3)
p4 = 0.4 m. (for link 4)

q ;
q2 =0 (for link2)
q3 = 0 (for link 3)
q4 = 0 (for link 4)

length of links;
a2 = 0.2 m.(for link2)
a3 = 0.8 m. (for link 3)

a4 = 0.8 m. (for link 4)

Theta 1 = 0

flx = 1 m.(position of end of link 4)

fly = 0 (position of end of link 4)

gain = 10000

cgain = 100

Initial valves for displacements and velocities;

y(1) = 0

y(2) = 0

y(3) = 0

y(4) = 0

y(5) = 0

y(6) = 0

y(7) = pi / 3

y(8) = 0

y(9) = 0

y(10) = 0

y(11) = - pi / 3

y(12) = 0

Joints ;

$$\left.\begin{aligned} ef\,1 &= 0 \\ ef\,2 &= 1 \end{aligned}\right\} \text{Revolute joint}$$

$$\left.\begin{aligned} ef\,3 &= 0 \\ ef\,4 &= 1 \end{aligned}\right\} \text{Revolute joint}$$

$$\left.\begin{aligned} ef\,5 &= 0 \\ ef\,6 &= 1 \end{aligned}\right\} \text{Revolute joint}$$

forces for end of link 4 (joint 4);

fx = - gain * (dx – flx) – cgain * vx (for end of link 4)

fy = - gain * (dy – fly) – cgain * vy (for end of link 4)

Torque;

tt = 0

joint 4 is revolute joint.

5.3.2 The Slider-crank mechanism

Masses;

m2 = 1 kg. (for link2)

m3 = 2 kg. (for link 3)

m4 = 3 kg. (for link 4)

Mass moment of inertias;

mi2 = 0 (for link2)

mi3 = 0 (for link 3)

mi4 = 0 (for link 4)

P;

p2 = 0.25 m. (for link2)

p3 = 1 m. (for link 3)

p4 = 0 (for link 4)

q ;

q2 =0 (for link2)

q3 = 0 (for link 3)

q4 = 0 (for link 4)

length of links;

a2 = 0.25 m. (for link2)

a3 = 1 m. (for link 3)

a4 = 0 (for link 4)

Theta 1 = 0

flx = 2.35 m. (position of end of link 4)

fly = 0.2 m. (position of end of link 4)

gain = 10000

cgain = 100

Initial valves for displacements and velocities;

y(1) = 0

y(2) = 0

y(3) = 0

y(4) = 0

y(5) = 0

y(6) = 0

y(7) = 0

y(8) = 0

y(9) = 0

y(10) = 0

y(11) = 0

y(12) = 0

Joints ;

$$\left.\begin{array}{l} ef\ 1 = 0 \\ ef\ 2 = 1 \end{array}\right\} \text{Revolute joint}$$

$$\left.\begin{array}{l} ef\ 3 = 0 \\ ef\ 4 = 1 \end{array}\right\} \text{Revolute joint}$$

$$\left.\begin{array}{l} ef\ 5 = 0 \\ ef\ 6 = 1 \end{array}\right\} \text{Revolute joint}$$

Forces for end of link 4 (joint 4);

fx = 0

fy = - gain * (dy – fly) – cgain * vy (for end of link 4)

Torque;

tt = - gain * (y(11) - pi) –cgain * y(12),

joint 4 is prismatic joint.

5.3.3 The Quick-return mechanism

Masses;

m2 = 1 kg. (for link2)

m3 = 2 kg. (for link 3)

m4 = 3 kg. (for link 4)

Mass moment of inertias;

mi2 = 0 (for link2)

mi3 = 0 (for link 3)

mi4 = 0 (for link 4)

P;

p2 = 0.1 m. (for link2)

p3 = 0.4 m. (for link 3)

p4 = 0.4 (for link 4)

q ;

q2 =0 (for link2)

q3 = 0 (for link 3)

q4 = 0 (for link 4)

length of links;

a2 = 0.2 m. (for link2)

a3 = 0.8 m. (for link 3)

a4 = 0.8 m. (for link 4)

Theta 1 = 0

flx = 0.6 m. (position of end of link 4)

fly = 0.7 m. (position of end of link 4)

gain = 20000

cgain = 100

Initial valves for displacements and velocities;

y(1) = 0

y(2) = 0

y(3) = 0

y(4) = 0

y(5) = 0

y(6) = 0

y(7) = pi / 3

y(8) = 0

y(9) = 0

y(10) = 0

y(11) = - pi / 3

y(12) = 0

Joints ;

$$\left.\begin{aligned} ef\,1 &= 0 \\ ef\,2 &= 1 \end{aligned}\right\} \text{Revolute joint}$$

$$\left.\begin{aligned} ef\,3 &= 0 \\ ef\,4 &= 1 \end{aligned}\right\} \text{Revolute joint}$$

$$\left.\begin{aligned} ef\,5 &= 1 \\ ef\,6 &= 0 \end{aligned}\right\} \text{Prismatic joint}$$

Forces for end of link 4 (joint 4);

fx = - gain * (dx – flx) – cgain * vx (for end of link 4)

fy = - gain * (dy – fly) – cgain * vy (for end of link 4)

Torque;

tt = 0

joint 4 is revolute joint.

5.3.4 The Swinging-block mechanism

Masses;

m2 = 1 kg. (for link2)

m3 = 1 kg. (for link 3)

m4 = 1 kg. (for link 4)

Mass moment of inertias;

mi2 = 0 (for link2)

mi3 = 0 (for link 3)

mi4 = 0 (for link 4)

P;

p2 = 0.1 m. (for link2)

p3 = 0.4 m. (for link 3)

p4 = 0.4 (for link 4)

q ;

q2 =0 (for link2)

q3 = 0 (for link 3)

q4 = 0 (for link 4)

length of links;

a2 = 0.2 m. (for link2)

a3 = 0.6 m. (for link 3)

a4 = 0 (for link 4)

Theta 1 = 0

flx = 1 m. (position of end of link 4)

fly = 0 (position of end of link 4)

gain = 15000

cgain = 100

Initial valves for displacements and velocities;

y(1) = 0

y(2) = 0

y(3) = 0

y(4) = 0

y(5) = 0

y(6) = 0

y(7) = - pi / 3

y(8) = 0

y(9) = 0

y(10) = 0

y(11) = pi / 3

y(12) = 0

Joints ;

$$\left.\begin{array}{l} ef\,1 = 0 \\ ef\,2 = 1 \end{array}\right\} \text{Revolute joint}$$

$$\left.\begin{array}{l} ef\,3 = 0 \\ ef\,4 = 1 \end{array}\right\} \text{Revolute joint}$$

$$\left.\begin{array}{l} ef\,5 = 1 \\ ef\,6 = 0 \end{array}\right\} \text{Prismatic joint}$$

Forces for end of link 4 (joint 4);

fx = - gain * (dx – flx) – cgain * vx (for end of link 4)

fy = - gain * (dy – fly) – cgain * vy (for end of link 4)

Torque;

tt = 0

joint 4 is revolute joint.

5.3.5 The Scotch- yoke mechanism

Masses;

m2 = 1 kg. (for link2)

m3 = 1 kg. (for link 3)

m4 = 1 kg. (for link 4)

Mass moment of inertias;

mi2 = 0 (for link2)

mi3 = 0 (for link 3)

mi4 = 0 (for link 4)

P;

p2 = 0.1 m. (for link2)

p3 = 0.4 m. (for link 3)

p4 = 0.4 (for link 4)

q ;

q2 =0 (for link2)

q3 = 0 (for link 3)

q4 = 0 (for link 4)

length of links;

a2 = 0 (for link2)

a3 = 0 (for link 3)

a4 = 0.4 (for link 4)

Theta 1 = pi / 2

flx = 1 m. (position of end of link 4)

fly = 0 (position of end of link 4)

gain = 10000

cgain = 100

Initial valves for displacements and velocities;

y(1) = -0.6

y(2) = 0

y(3) = 0

y(4) = 0

y(5) = 0

y(6) = 0

y(7) = 0

y(8) = 0

y(9) = 0

y(10) = 0

y(11) = 0

y(12) = 0

Joints ;

$$\left.\begin{array}{l} ef\,1 = 1 \\ ef\,2 = 0 \end{array}\right\} \text{Prismatic joint}$$

$$\left.\begin{array}{l} ef\,3 = 1 \\ ef\,4 = 0 \end{array}\right\} \text{Prismatic joint}$$

$$\left.\begin{array}{l} ef\,5 = 0 \\ ef\,6 = 1 \end{array}\right\} \text{Revolute joint}$$

Forces for end of link 4 (joint 4);

fx = - gain * (dx – flx) – cgain * vx (for end of link 4)

fy = - gain * (dy – fly) – cgain * vy (for end of link 4)

Torque;

tt = 0

joint 4 is revolute joint.

5.3.6 The Oldham-coupling mechanism

Masses;

m2 = 1 kg. (for link2)

m3 = 2 kg. (for link 3)

m4 = 3 kg. (for link 4)

Mass moment of inertias;

mi2 = 0.1 (for link2)

mi3 = 0.2 (for link 3)

mi4 = 0.3 (for link 4)

P;

p2 = 0.1 m. (for link2)

p3 = 0.4 m. (for link 3)

p4 = 0.4 (for link 4)

q ;

q2 =0 (for link2)

q3 = 0 (for link 3)

q4 = 0 (for link 4)

length of links;

a2 = 0.2 m. (for link2)

a3 = 0.8 m. (for link 3)

a4 = 0.8 m. (for link 4)

Theta 1 = 0

flx = 1 m. (position of end of link 4)

fly = 0 (position of end of link 4)

gain = 10000

cgain = 100

Initial valves for displacements and velocities;

y(1) = -0.6 (B1)

y(2) = 0

y(3) = 0

y(4) = 0

y(5) = 0

y(6) = 0

y(7) = pi / 3

y(8) = 0

y(9) = 0

y(10) = 0

y(11) = - pi / 3

y(12) = 0

Joints ;

$$\left.\begin{array}{l} ef\,1 = 0 \\ ef\,2 = 1 \end{array}\right\}\text{ Revolute joint}$$

$$\left.\begin{array}{l} ef\,3 = 1 \\ ef\,4 = 0 \end{array}\right\}\text{ Prismatic joint}$$

$$\left.\begin{array}{l} ef\,5 = 1 \\ ef\,6 = 0 \end{array}\right\}\text{Prismatic joint}$$

Forces for end of link 4 (joint 4);

fx = - gain * (dx – flx) – cgain * vx (for end of link 4)

fy = - gain * (dy – fly) – cgain * vy (for end of link 4)

Torque;

tt = 0

joint 4 is revolute joint.

5.3.7 The Double slider mechanism

Masses;

m2 = 9 kg. (for link2)

m3 = 1 kg. (for link 3)

m4 = 1 kg. (for link 4)

Mass moment of inertias;

mi2 = 0 (for link2)

mi3 = 0 (for link 3)

mi4 = 0 (for link 4)

P;

p2 = 0.01 m. (for link2)

p3 = 0.4 m. (for link 3)

p4 = 0.01 (for link 4)

q ;

q2 =0 (for link2)

q3 = 0 (for link 3)

q4 = 0 (for link 4)

length of links;

a2 = 0 (for link2)

a3 = 0.8 m. (for link 3)

a4 = 0 (for link 4)

Theta 1 = pi / 2

flx = 0.8 m. (position of end of link 4)

fly = 0 (position of end of link 4)

gain = 10000

cgain = 100

Initial valves for displacements and velocities;

y(1) = 0

y(2) = 0

y(3) = 0

y(4) = 0

y(5) = 0

y(6) = 0

y(7) = 0

y(8) = 0

y(9) = 0

y(10) = 0

y(11) = 0

y(12) = 0

Joints ;

$$\left.\begin{array}{l} ef\,1 \;= 1 \\ ef\,2 \;= 0 \end{array}\right\} \text{Prismatic joint}$$

$$\left.\begin{array}{l} ef\,3 \;= 0 \\ ef\,4 \;= 1 \end{array}\right\} \text{Revolute joint}$$

$$\left.\begin{array}{l} ef\,5 \;= 0 \\ ef\,6 \;= 1 \end{array}\right\} \text{Revolute joint}$$

Forces for end of link 4 (joint 4);

fx = - gain * (dx – flx) – cgain * vx (for end of link 4)

fy = 0 (for end of link 4)

Torque;

tt = - gain * (y(11) - pi) –cgain * y(12),

joint 4 is prismatic joint.

5.4.8 Conchoidal motion mechanism

Masses;

m2 = 1 kg. (for link2)

m3 = 1 kg. (for link 3)

m4 = 1 kg. (for link 4)

Mass moment of inertias;

mi2 = 0 (for link2)

mi3 = 0 (for link 3)

mi4 = 0 (for link 4)

P;

p2 = 0.1 m. (for link2)

p3 = 0.4 m. (for link 3)

p4 = 0.4 (for link 4)

q ;

q2 =0 (for link2)

q3 = 0 (for link 3)

q4 = 0 (for link 4)

length of links;

a2 = 0.4 m. (for link2)

a3 = 0 (for link 3)

a4 = 0 (for link 4)

Theta 1 = pi / 2

flx = 0.4 m. (position of end of link 4)

fly = 1 (position of end of link 4)

gain = 10000

cgain = 100

Initial valves for displacements and velocities;

y(1) = 0

y(2) = 0

y(3) = 0

y(4) = 0

y(5) = 1

y(6) = 0

y(7) = 0

y(8) = 0

y(9) = 0

y(10) = 0

y(11) = pi / 2

y(12) = 0

Joints ;

$$\left.\begin{array}{l} \text{ef } 1 = 0 \\ \text{ef } 2 = 1 \end{array}\right\} \text{Revolute joint}$$

$$\left.\begin{array}{l} \text{ef } 3 = 1 \\ \text{ef } 4 = 0 \end{array}\right\} \text{Prismatic joint}$$

$$\left.\begin{array}{l} \text{ef } 5 = 0 \\ \text{ef } 6 = 1 \end{array}\right\} \text{Revolute joint}$$

Forces for end of link 4 (joint 4);

fx =0 (for end of link 4)

fy = - gain * (dy – fly) – cgain * vy (for end of link 4)

(for end of link 4)

Torque;

tt = - gain * (y(11) - pi) –cgain * y(12),

joint 4 is prismatic joint.

5.4 Verification Of Simulation Results For Correctness

Simulation results come up from a series of mathematical operations. If correctly prepared and executed, simulation programs should produce motion profiles, which can be obtained by experiments, that is they should define the natural motion of the system they are simulating. In preparing a simulation program for mechanical systems, the following points may be found important:

i) Often researchers find applying Lagrange Equation laborious. Firstly the kinetic and potential energy expressions must be derived correctly. Energies are scalar quantities and are meaningful to many calculated and checked for correctness. Once they are formulated correctly, a correct definition of the Lagrangian will be at hand, with which Lagrange formulation can advance.

ii) Intermediary steps in the application of the Lagrange equation is often found not too much meaningful. So in general the formulation must be done twice. If both derivations end up with the same formulas, they can be assumed correct. In case of discrepancy, a third and perhaps more derivations are required until derivation errors are detected and corrected.

iii) Programming may require a knowledge on how rounding-off errors accumulate. Program often contains many sine, cosine and logarithm function taker several hundred times greater cpu time than a summation. So, assigning variable names for them and using the values stored under these names will greatly improve computation efficiency and reduce solution time. A lengthy series of arithmetic should not be done as all multiplications first and all divisions next or vice versa. This may bring the calculated numerical value to too large or too small values where rounding-off errors becomes relatively important.

iv) The best way of testing the correctness of a simulation is repeating the same event at the laboratory and recording the data. If simulation results fit to that of experiments, simulation program can be assumed to be correctly working. If not, both simulation and experimentation must be checked through. This may be a very tiring and time consuming process.

v) Simulation programs may be made to solve for problems whose solutions are already known, like a free fall, static equilibrium, constant acceleration motions, linear oscillations etc. For example, the static equilibrium tests carried out in section (5.2) are aiming a verification of correctness. Solving statics problems are always simpler than that of dynamics problems and static condition is a sub set of dynamic condition.

Similarly in one of the examples in section (5.2), the system is put into the form of a simple pendulum and is allowed to display free oscillations of small amplitude. Formulas for linear oscillations are often well known and reliable.

vi) Static state is a subset of dynamic state, but dynamic forces are not existant. Being statically correct is a requirement but not a sufficient one. All the accelerations are calculated. Whether on inverse solution is correct or not, the best way to see is to solve the problem in reverse order, that is by forward dynamics. The resulting accelerations from the inverse dynamics solution are used in a forward dynamics solution and forces required are calculated. If the calculated forces come up equal to the forces we started with, this means that the simulation program and formulas incorporated are correct. The forward dynamics problem should be solved by the D'Alambert's approach. Position, velocity and acceleration analyses should be done first, the linear accelerations of all mass centers and angular accelerations of all moving bodies should be calculated. Then D'Alambert forces and moments must be calculated and added onto each moving link and the system forces must be solved statically. Correctness for dynamic forces of the formulation presented in this work has been checked many times. As the D'Alambert's quasi-static forward dynamics solution is not directly a part of the thesis, a sample solution is given in Appendix 1. for the dynamic force analysis of a 4-Bar mechanism.

Finally, a motion profile, in form of an animated graphics of the mechanism is very meaningful to experienced eyes. An experienced person can judge whether an animated motion is correct or not, whether the developing motion look, natural or not. In this final point, there is no mathematics, no solid rules. No such a thing is taught as part of a formal engineering education. Such a judgement, feeling of the motion is very important and equally valuable tool for a designer.

5.5 Programming Language

Programming language for this simulation program is QBasic. QBasic (a name derived from QuickBASIC, BASIC being an acronym for Beginner's All-purpose Symbolic Instruction Code) is a variant of the BASIC programming language. The source code is compiled to an intermediate form within the integrated development environment (IDE), and this intermediate form is immediately interpreted on demand within the IDE.

Like QuickBASIC, but unlike earlier versions of Microsoft BASIC, QBasic was a structured programming language, supporting constructs such as named subroutines and while loops. Line numbers, a concept often associated with BASIC, were supported for compatibility, but were not necessary and not considered good form, having been replaced by descriptive line labels. QBasic has limited support for user-defined data types (structures), and several primitive types used to contain strings of text or numeric data.

CHAPTER 6

CONCLUSION AND PROPOSAL FOR FUTURE STUDY

6.1 Conclusions

This dissertation presents software to simulate the dynamic behaviour of any kind of planar open or closed link chains, containing revolute or prismatic joints. From utility point of view, this is nothing new. There are many computer programs who can do this job available in the market. What is new is how a system is defined and formulated. All the programs known to the author of this dissertation separate a system into simpler and smaller units, which have their own transfer functions and interact with each other. In a linkage system, the indivisible unit is the link. Transfer function is Newton's second law and interaction between the individual units is governed by Newton's third law. This approach, which is totally so great, so correct, so systematic and so nice, generates three times degrees of freedom of the system. But main difficulty lies not in the number of equations, but in maintaining the correct interactions. In this dissertation Lagrange formulation is used and two different joint types assumed between each successive pair of links. This approach doubles the degree of freedom and one equation is generated for each movability. So, to simulate a 3 degree of freedom open chain 6 equations are required in contrast to 9 equations of other programs. This is an important contribution, but the more important contribution is the closed-from calculation of constraint forces accruing at the joints. As their exact values are known, solution of motion equations digitally becomes easy and fast.

The method presented here is novel. It was originally designed to model robotic manipulators, which are open loop linkages. What is done in this dissertation is to join the tip of the open chain to the ground by constraint forces calculated according to Newton's third law and convert the open chain into a closed link loop to model mechanisms. Application of this novel method to mechanisms is novel too.

As indicated in paragraph (vii) of section (5.4), mechanisms and machines are dynamic systems, highly mobile and difficult to control. In order to control something, we must know its dynamics. Machine designers must appreciate and feel the dynamics and preferable must have reached the perfection of correctly guessing how a motion will develop. Machines are complex systems composed of many mechanisms and hence, dynamics of a complete machine is surely much more complex than that of a mechanism. Fortunately, there is the friction-solid or liquid to calm down every sort of harsh motion and powerful motors to drive any sort of inertial load, such that machine design and control becomes simpler with them. This dissertation presents only a method and supports its applicability by examples. At this stage can not compete with commercially available programs. Never the less, it can be improved by further work as listed in the next section.

6.2 Proposal For Future Study

6.2.1 Minor Amendments

i) This program in the present form is not vary easily understandable and hence applicable. A user interface to get data from the user will be very helpful. Also the data outputting is not well organized. Required graphs can be selected from a menu, graphs can be labeled and scaled for proper presentation in a better user interface. An animated output displays the mechanism in motion, but the problem of scaling the figure to the screen of the computer still unsolved. A good user interface for data in and outputting is necessary and will greatly improve the program.

ii) This program in the present form does not contain the dynamics of motors. A mechanism must be driven by hydraulic, pneumatic or electric motors. Simulation of motors should also be included in the system if it should help in a larger area of the design work.

iii) This program in the present form neither contains any well known control strategies nor allows any control system to be designed and implemented. Modules containing the transfer functions of widely used and well known control strategies must be added. This will improve the usefulness of the program greatly. Further, there must be a user definable control module where not so much widely used control strategies can be implemented and used to control and drive the mechanisms under consideration. Also modules concerning transducers can be made. Various characteristics of the transducers can be put in there and measurement errors and their effect on control can be seen.

6.2.2 Major Amendments

The three proposals listed above are only amendments to the existing linkage simulation and are not major contributions or rather, need less effort. Some major changes can also be thought of as follows.

i) This programs in its present state can simulate only 4-link mechanisms. 4-link Grübler mechanisms are the most widely used systems in machine design, but incases where more complicated motion profiles are required, 6-link mechanisms become necessary. A 6-link Watt mechanism can be made as shown in figure (6.1).

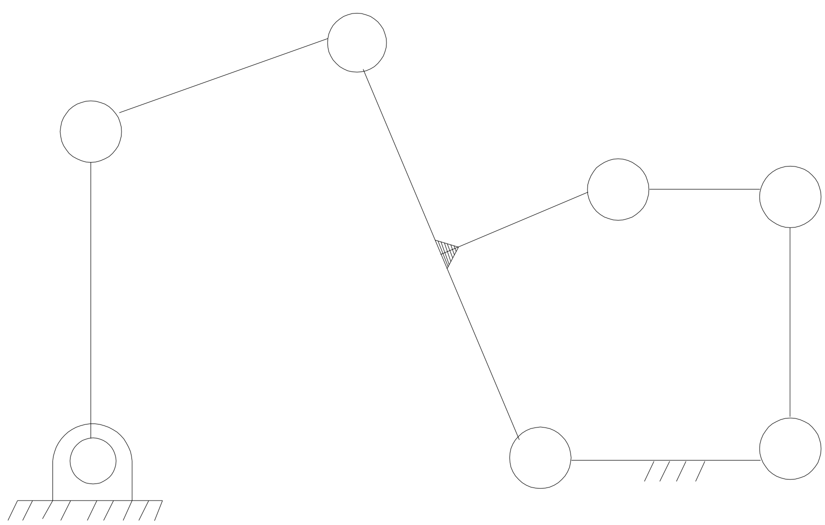


Figure 6.1 An all-revolute 6-link Watt mechanism

This is a really enjoyable task as if using a folding yard-stick to make a mechanism. Similarly, a 6-link Stephenson mechanism can be made as seen in figure

(6.2) by using and an articulation of 6 moving links. The program in its present state can handle only 3 moving links. With the some theory and some procedure of formula derivation, new formulas, capable of simulating 6 moving links can be made. This would be a major contribution to the presented work.

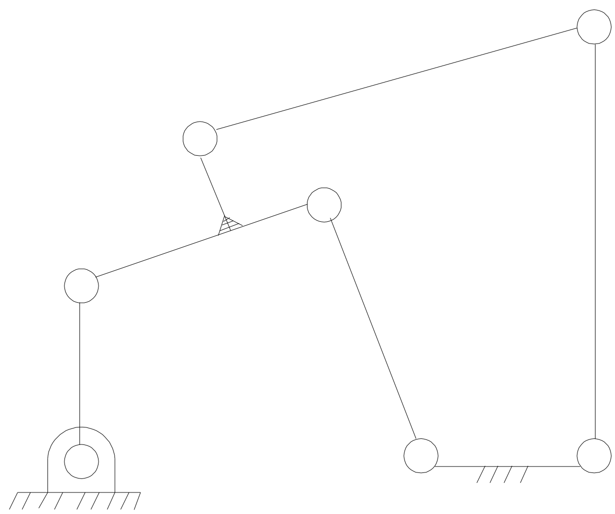


Figure 6.2 An all-revolute 6-link Stephenson mechanism

ii) The program in its present state can handle only planar mechanisms. Most of the mechanisms used in constructing machines are of 4-link planar mechanisms, because of simplicity in design, construction, manufacture and maintenance, but as indicated in paragraph (i) above, sometimes more intricate motion profiles are required. Such a problem is primarily attacked by introducing 6-link mechanisms. 8-link are not preferable at all. Second solution is that we can think of using spatial or 3 dimensional mechanisms. Three dimensional mechanisms are much more complex in kinematics and dynamics, though the systemacy in generating motion equations are not different. Some more joint types may be required to include like cylindrical and spherical joints, and a systemacy in defining positions in 3-dimensions are required. Denavit-Hartenberg representation of links and joints [9] can well be used as we see in robotics applications [10]. For a software handling 3 dimensional mechanisms, some further data input-output routines will be required as perspective appearance and animation of the mechanism.

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APPENDIX

1. Data of simulation

θ_2 = - 0,11929 rad. $\dot{\theta}_2$ = 2,2714 rad/sec $\ddot{\theta}_2$ = - 21,93 rad/sec²

θ_3 = 1,0711 rad. $\dot{\theta}_3$ = - 0,51486 rad/sec $\ddot{\theta}_3$ = 3,82267 rad/sec²

θ_4 = - 1,01924 rad. $\dot{\theta}_4$ = - 0,61032 rad/sec $\ddot{\theta}_4$ = 6,55760 rad/sec²

Linkage parameters:

m_2 = 1 kg. a_2 = 0,2 m. I_2 = 0,1 kg-m²

m_3 = 2 kg. a_3 = 0,8 m. I_3 = 0,2 kg-m²

m_3 = 3 kg. a_4 = 0,8 m. I_4 = 0,3 kg-m²

flx = 1 and fly = 0

Degree as:

θ_2 = - -6,835 deg.

θ_3 = 61,37 deg.

θ_4 = - 58,398 deg.

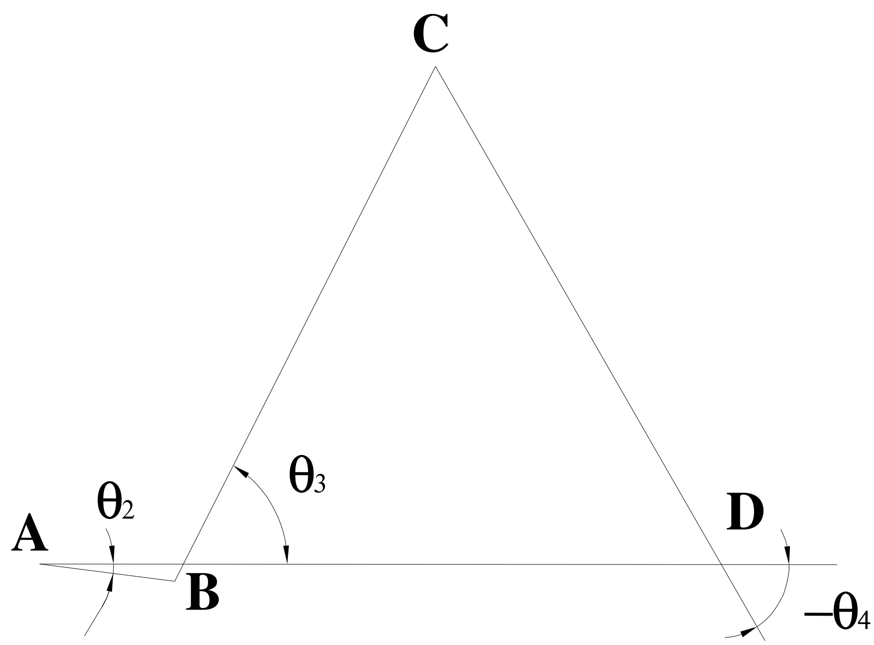


Figure A.1 Position of four bar mechanism in simulation (t=3 sec.).

2. Velocities

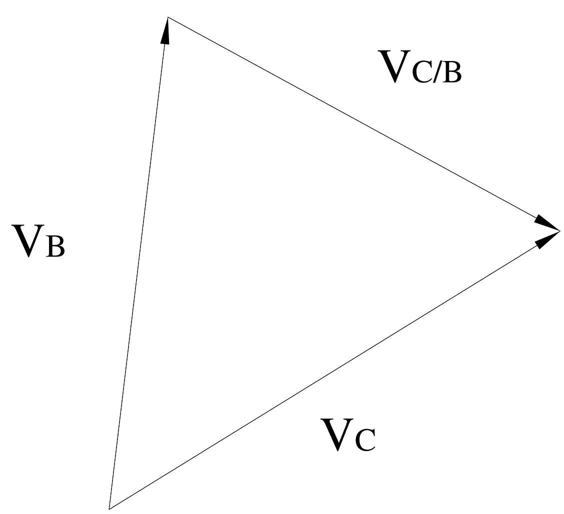


Figure A.2 velocity analysis of figure (A.1)

V_B = |AB| * W_2 = 0.45428 m/sec

$V_C = V_B + V_{C/B}$

V_C = 0,488 m/sec (obtained from graphical solution)

$V_{C/B}$ = 0,41 m/sec (obtained from graphical solution)

$V_{C/B}$ = |CB| * $\dot{\theta}_3$

$\dot{\theta}_3$ = 0,41 / 0,8 = 0,5125 rad/sec (cw)

$\dot{\theta}_4 = V_C$ / |CD| = 0,41 / 0,8 = 0,61 rad/sec (cw)

3. Accelerations

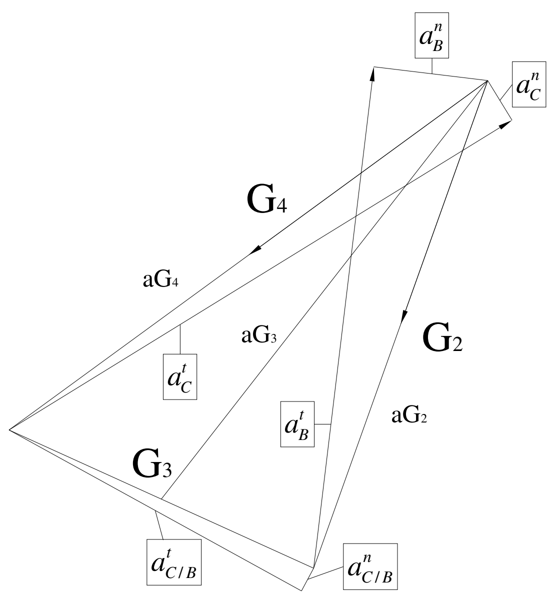


Figure A.3 Acceleration analysis of figure (A.1)

$a_C^n + a_C^t = a_B^n + a_B^t + a_{C/B}^n + a_{C/B}^t$

$a_B^n = w_2^2$ * |AB| = (2,2714)² * 0,2 = 1,032 m/sec²

$a_B^t = \alpha_2$ * |AB| = 21,93 * 0,2 = 4,386 m/sec²

$a_{C/B}^n = w_3^2$ * |CB| = (0,5125)² * 0,8 = 0,21 m/sec²

$a_C^n = w_4^2$ * |CD| = (0,61)² * 0,8 = 0,298 m/sec²

$a^t_{C/B}$ = 3,05 m/sec² (obtained from graphical solution)

a^t_C = 5,2 m/sec²

$\alpha_3 = a^t_{C/B}$ / |CB| = 3,05 / 0,8 = 3,8125 rad/sec² (ccw) (data is 3,82267)

$\alpha_4 = a^t_C$ / |CD| = 5,2 / 0,8 = 6,5 rad/sec² (ccw) (data is 6,5576)

4. D'alembert Forces

a_{G2} = 2,25 m/sec² (obtained from graphical solution)

a_{G3} = 4,6 m/sec² (obtained from graphical solution)

a_{G4} = 2,6 m/sec² (obtained from graphical solution)

- $m_2\ a_{G2}$ = 1 * 2,25 = 2,25 N
- $m_3\ a_{G3}$ = 2 * 4,6 = 9,2 N
- $m_4\ a_{G4}$ = 3 * 2,6 = 7,8 N

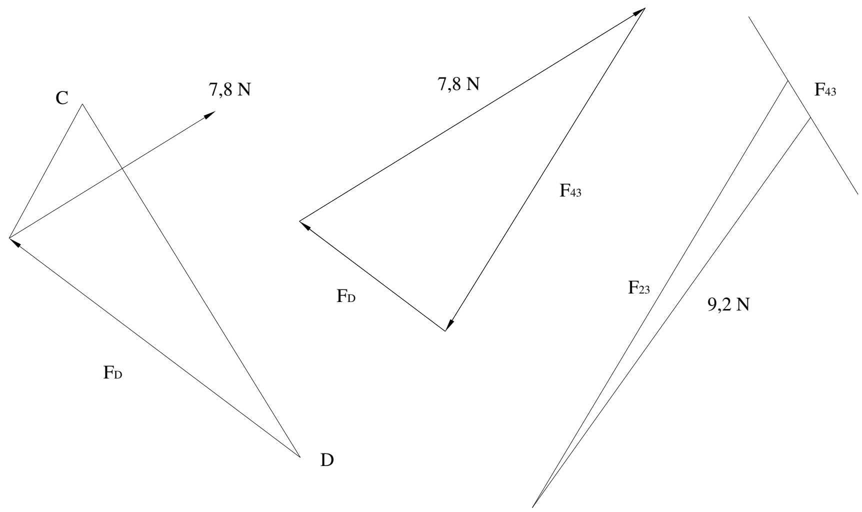
5. D'alembert Moments

- $I_2\ \alpha_2$ = 0,1 * 21,93 = 2,193 Nm (ccw)
- $I_3\ \alpha_3$ = 0,2 * 3,8226 = 0,7652 Nm (cw)
- $I_4\ \alpha_4$ = 0,3 * 6,5576 = 1,96728 Nm (cw)

$e_2 = I_2\ \alpha_2\ /\ m_2\ a_{G2}$ = 2,193 / 2,25 = 0,9746 m

$e_3 = I_3\ \alpha_3\ /\ m_3\ a_{G3}$ = 0,7652 / 9,2 = 0,083 m

$e_4 = I_4 \ \alpha_4 \ / \ m_4 \ a_{G4} = 1{,}96728 \ 7{,}8 = 0{,}252$ m



(a) (b) (c)

Figure A.4 Force polygons of figure (A.1)

$F_{43} = 7{,}215$ N

$F_{23} = 9{,}706$ N

$F_B = 2{,}25$ N

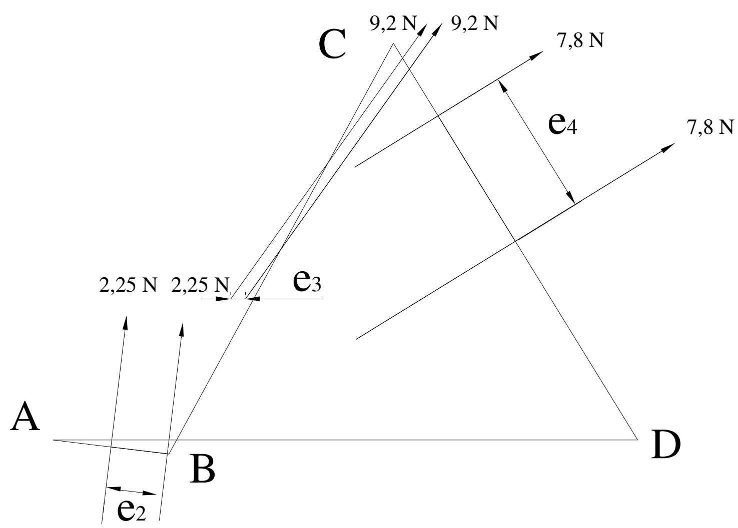


Figure A.5 inertia forces of figure (A.1)

$(\tau_1)_{7,8} = F_{43} * d_3$ = 7,215 * 0,183 = 1,320345 Nm (cw)

$(\tau_2)_{9,2} = F_{23} * d_2$ = 9,706 * 0,182 = 1,7665 Nm (ccw)

$(\tau_3)_{2,25} = F_B * d_1$ = 2,25 * 0,2 = 0,45 Nm (cw)

where d_1, d_2 and d_3 are the moment arms of the forces.

Total τ = (1,320345 + 0,45) – 1,7665 = 1,7703 – 1,7665 = 0,0038 N-m.

Initial torque value was zero, and the discrepancy is within tolerable limits.