

**UNIVERSITY OF GAZİANTEP
GRADUATE SCHOOL OF
NATURAL & APPLIED SCIENCES**

**WAVELET BASED TUMOR
DETECTION AND ITS APPLICATION
ON MAMMOGRAMS**

**M. Sc. THESIS
IN
ELECTRICAL AND ELECTRONICS ENGINEERING**

**BY
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**Wavelet Based Tumor Detection and Its
Application on Mammograms**

**M.Sc. Thesis
in
Electrical and Electronics Engineering
University of Gaziantep**

**Supervisor
Assoc. Prof. Dr. Gülay TOHUMOĞLU**

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ABSTRACT

WAVELET BASED TUMOR DETECTION AND ITS APPLICATION ON MAMMOGRAMS

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The high incidence of breast cancer in women has increased significantly in recent years and it persists to be a significant health problem in the world. The reasons of breast cancer is still unknown so early detection of cancer is very important for medical treatment and decreasing the cases of death. Mammography is considered the most reliable and common method in early detection of breast cancer. However, it is difficult for the radiologists to provide accurate and fast evaluation of mammograms. The estimated sensitivity of radiologists in detecting tumors from mammograms is only about 75%. Breast cancer Computer Aided Diagnosis systems can be vital for radiologists for detection and diagnosing abnormalities earlier and faster. Several techniques can be used to accomplish the task of detection abnormalities to help radiologists having a second opinion.

In this thesis, digital mammograms are analyzed using mathematical morphology and lifting scheme of wavelet transform. Tumor detection algorithms follow preprocessing, enhancement and segmentation steps. The novelty or contribution of the thesis is usage of Lifting Scheme of wavelet transform in enhancement step. Thus, the modified tumor detection algorithm is called Lifting Scheme Based Enhancement Algorithm (LSBEA). Mathematical morphology used for contrast enhancement of digital mammograms before denoising. The lifting scheme of wavelet transform is used in denoising step. The lifting scheme has two extremely useful advantages over regular Discrete Wavelet Transform (DWT). Firstly, it

enables easy perfect reconstruction and secondly speeds up real time calculations, thus does not need extra memory allocation.

The evaluation of the modified detection algorithm LSBEA is carried out on approximately 100 digital mamaograms from Mammographic Image Analysis Society (MIAS) dataset. The lifting scheme based enhancement algorithm has provided superior image quality in enhancement step, and furthermore, fast and easy way to the detection of problematic areas. In the segmentation phase, the problematic areas defined clearly. Some of interesting applications are given illustratively and their results are discussed. It is concluded that the new algorithm LSBEA gives higher Peak Signal to Noise Ratio (PSNR) and Contrast Improvement Index (CII), values, quantitatively and better results visually for the tumor detected areas. Also, these results are discussed with the head of major discipline of radiology, Medicine Faculty, University of Gaziantep. His opinion is that it could be helpful to find the edges of masses for the evaluation of the digital mammograms.

Keywords: Tumor detection, lifting scheme, wavelet transform, mathematical morphology

ÖZET

MAMOGRAMLARDA DALGACIK TABANLI YÖNTEMLE TÜMÖR BELİRLEME

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Göğüs kanseri vakalarının kadınlarda görülme sıklığı son yıllarda önemli bir artış göstermektedir ve tüm dünyada önemli bir sağlık problemi olmaya devam etmektedir. Göğüs kanserinin sebepleri hala bilinmemektedir, bu yüzden kanserin erken teşhisi, tedavi ve ölüm oranının azaltılması için çok önemlidir. Mamografi, göğüs kanserinin erken teşhisinde en güvenilir ve yaygın metot olarak kabul görmektedir. Bununla birlikte, iyi yetişmiş radyolojist bulmak her zaman mümkün olamayabilir ve radyolojistlerin mamogramları kusursuz ve hızlı bir şekilde değerlendirmeleri oldukça zor olabilmektedir. Mamogramlarda ki tümör tespitinde radyolojistlerin tahmin hassasiyeti yalnızca %75 oranındadır. Göğüs kanserinde bilgisayar destekli sistemler, anormalliklerin erken ve hızlı tespiti ve tanısı, radyolojistler için bir gerekliliktir. Radyolojistlere ikinci bir fikir vererek yardımcı olmak için anormalliklerin belirlenmesinde çeşitli teknikler kullanılabilir.

Bu tezde, dijital mamogramlar matematiksel morfoloji ve kaldıraç düzenli dalgacık dönüşümü kullanılarak analiz edilmiştir. Tümör belirleme algoritmaları ön işleme, iyileştirme ve bölütleme aşamalarından oluşur. Bu tezin orijinalliği yada katkısı iyileştirme aşamasında kaldıraç düzenli dalgacık dönüşümünün kullanılmasıdır. Böylece, değiştirilmiş tümör belirleme algoritması, Kaldıraç Tabanlı İyileştirme Algoritması (KTİA) olarak adlandırılmıştır. Dijital mamogramlarda gürültü

arındırma işleminden önce matematiksel morfoloji kontrast iyileştirme işlemi için uygulanmıştır. Dalgacık dönüşümünün kaldıraç düzeni, gürültü arındırma aşamasında kullanılmıştır. Kaldıraç düzenli dalgacık dönüşümünün alışılmış kesikli dalgacık dönüşümüne (DWT) göre çok önemli iki avantajı vardır. Birincisi, kolaylıkla işaretin mükemmel bir şekilde yeniden oluşumuna olanak sağlaması, ikincisi ise gerçek zamanlı işlemleri hızlandırması, böylece fazladan bellek kullanımına gerek kalmamasıdır.

Uyarlanmış tümör belirleme algoritmasının (KTİA) uygulamaları Mamographic Image Analysis Society (MIAS) veri bankasından alınan yaklaşık 100 dijital mamogram üzerinde yapılmıştır. Kaldıraç tabanlı iyileştirme algoritması, iyileştirme aşamasında iyi resim kalitesi sağlamıştır; ek olarak problemlı bölgelerin hızlı ve kolay belirlenmesine olanak sağlamıştır. Bölütleme aşamasında ise problemlı bölgeler açıkça tanımlanmıştır. Bazı ilginç uygulamalar görsel olarak verilmiş ve sonuçları tartışılmıştır. Sonuç olarak yeni algoritma, daha iyi ölçülebilir tepe işaret gürültü oranı (PSNR) ve kontrast geliştirme indisi (CII) değerleri, ayrıca görsel olarak da tümör belirlenmiş alanlar için daha iyi sonuçlar vermektedir. Aynı zamanda Gaziantep Üniversitesi, Tıp Fakültesi, Radyoloji anabilim dalı başkanından görsel sonuçlar hakkında görüş alınmıştır. Bu görüş doğrultusunda kütlelerin kenarlarının belirlenmesinde ve dijital mamogramların değerlendirilmesinde adı geçen algoritmanın yardımcı olabileceği belirtilmiştir.

Anahtar Kelimeler: Tümör belirleme, kaldıraç düzenli algoritma, dalgacık dönüşümü, matematiksel morfoloji.

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LIST OF SYMBOLS/ABBREVIATIONS^{*}

CAD	Computer Aided Diagnosis
CII	Contrast Improvement Index
DCT	Discrete Cosine Transform
DWT	Discrete Wavelet Transform
FT	Fourier Transform
IWT	Inverse Wavelet Transform
LS	Lifting Scheme
LSBEA	Lifting Scheme Based Enhancement Algorithm
LSWT	Lifting Scheme Wavelet Transform
MIAS	Mammographic Image Analysis Society
MRA	Multi Resolution Analysis
MSE	Mean Square Error
P	Prediction Operator
PSNR	Peak Signal to Noise Ratio
ROI	Region of Interest
STFT	Short-time Fourier Transform
SURE	Stein's Unbiased Risk Estimate
U	Update Operator
WT	Wavelet Transform
$\langle \cdot, \cdot \rangle$	Inner product
$\psi(t)$	Mother Wavelet
$\varphi(t)$	Scaling function
$L^2(R)$	Square integrable functions space
\sum	Sum
\subset	Sub set
\cup	Union
\cap	Intersection
\Leftrightarrow	If and only if
\in	Is an element of
\notin	Does not belong to
$\int \dots dt$	Integration

^{*}First Latin, then Greek letters, both in alphabetical order.

CHAPTER 1

INTRODUCTION

The high incidence of breast cancer in woman has increased significantly in the recent years and it persists to be the top threat to women's health in the world. In Turkey, according to latest statistics of Turkish Association for Cancer Research and Control, cancer is the second death cause after heart and blood vessel diseases. Among these cancer types, breast cancer is the leading cause of cancer death occurrences, with a percentage of approximately 30% [1].

The etiologies of breast cancer are not clear, so there is no definite method yet, to prevent breast cancer. Therefore, early diagnosis represents a key role for beating breast cancer and reaching a high survival rate. Mammography -breast X raying- is the most common and the most effective method in early detection of breast cancer.

Mammography is reported to have a sensitivity of 70% to 90% [2]. In other words the false negative rate, which means that the mammogram is interpreted as negative although it has cancerous tissue, is between 10% and 30%. So, mammograms can miss over one quarter of all tumors. False negatives have seen most often with dense breasts that make the masses difficult to distinguish, contrarily, false positives occur when a mammogram is read as abnormal when no cancer is present. False positive results will lead the patient to go under medical procedures which would have been prevented with accurate screening results.

It has been proven that reading of mammograms by two physicians consecutively reduces these false results caused by visual fatigue. Although double reading has been shown to increase the sensitivity of mammogram results as much as 15%, it is a time consuming and costly procedure [3]. Computer Aided Diagnosis (CAD) is an

important subject because it brings benefits of double reading cost effectively and without wasting too much time.

Currently there are several image processing methods proposed for the detection of masses in mammograms. Generally, detection algorithms in mammograms include pre-processing, enhancement and segmentation steps [4-25]. Different methods used at these steps results in different algorithms. Techniques used at pre processing and enhancement steps include histogram stretching, histogram equalization, nonlinear mapping methods (local histogram technique, bi-linear, sigmoid, non-continuous, etc.), multiscale processing (wavelets) [4-5]. Segmentation techniques include global thresholding, statistical methods (region growing, region clustering ...), edge detection, stochastic relaxation, fuzzy technique and multiscale techniques [4-6]. Also in some studies, bilateral subtraction of mammograms (left and right) proposed to find asymmetries [7-9]. Wavelets mostly used for denoising and segmentation of mammograms [10-18]. In some studies a further classification step applied [19-20]. Sakellaropoulos et al. [21] presented a method for mammographic image denoising and contrast enhancement based on an over complete dyadic wavelet transform. Their method was based on local non-linear modification of multiscale gradient magnitudes provided by the wavelet transform. They introduced clipped local range transformation to enhance contrast of images. Tsai et al. [22] non-linearly mapped discrete wavelet transform (DWT) coefficients to a new set then inverse wavelet transformed for an efficient enhancement. They applied their algorithm to some medical images and noted an enhancement at the edges especially. They also noted that image obtained by using proposed DWT method is sharper and less noise compared to the image obtained using fast Fourier Transform based method. Hadhoud et al. [23] combined mathematical morphology and wavelet based thresholding algorithms to increase the contrast in mammograms. They proposed a level dependent statistical approach for the detection of threshold value. Zheng et al. [24] presented an algorithm that combines several artificial intelligent techniques with the discrete wavelet transform (DWT) for detection of masses in mammograms. They used fractal dimension analysis, multiresolution Markov random field, dogs-and-rabbits algorithm artificial intelligent algorithms then applied a tree-type classification strategy at the end to determine whether a given region is suspicious for cancer. Boccignone et al. [25] proposed an algorithm for the detection of microcalcifications based on wavelet transform. Unlike other techniques the

detection is directly accomplished into the wavelet domain and no inverse transform applied. Further background tissue is separated from microcalcifications using thresholding procedure.

In this thesis, it is focused on detecting cancerous masses from mammograms which is one of the major types of breast cancer, and locating suspicious regions in mammogram for more detailed examination. Our objective is to serve as a second opinion to the radiologist rather than constructing a high technology radiologist by using mathematical morphology and second generation wavelets. The tumor detection algorithm includes three major steps: preprocessing, enhancement and segmentation. It is observed that mammograms include unnecessary information which increases processing time. In preprocessing step, black borders and defining numbers of mammograms are removed to single out region of interest (ROI). A simple cropping operation used for this purpose. In the second step, enhancement of region of interest handled with mathematical morphology and Lifting Scheme (LS) of wavelet transform. In this step, first the image enhanced by using mathematical morphology then the wavelet transform coefficients are obtained, and mammogram denoised by applying thresholding to the detail coefficients. In literature, thresholding is done by different approaches such as soft, hard, adaptive [26]. The selection of threshold value made by NormalShrink [27] and applied by soft thresholding. Novelty of this study is the modification in the enhancement step by applying lifting scheme of wavelet transform. Lifting scheme provides a fast and efficient way for noise elimination compared to discrete wavelet transform. It also enables to easy perfect reconstruction. In segmentation step, problematic areas are defined clearly by using thresholding. The extracted wavelet coefficients by using lifting scheme are used to define threshold value in the following segmentation step. Stein's unbiased risk estimate (SURE) is used at the selection of threshold value [28]. In order to validate the effectiveness of the proposed method, we compare the results obtained by the proposed method to that obtained by using BayesShrink and VisuShrink.

After noting importance of the CAD systems to assist radiologist and mentioning about previous works in this section, the definition and known reasons of breast

cancer will be explained in the following section. Additionally, general properties of common breast tumor types will be given in subsequent sections.

1.1 Breast Cancer

It will be useful to define the breasts and its parts before giving definition of breast cancer.

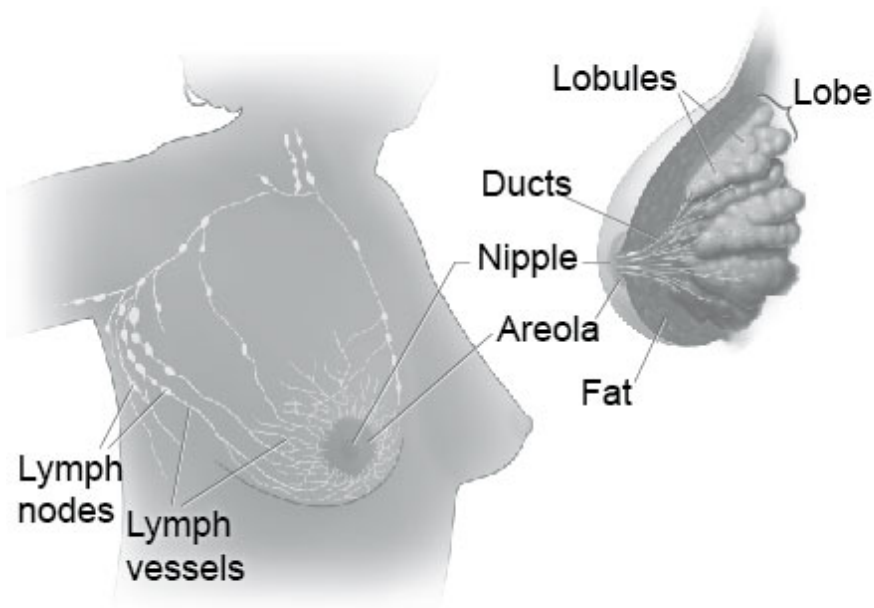


Figure 1.1 Parts of the breast and the lymph nodes and lymph vessels near the breast

The breasts sit on the chest muscles that cover the ribs. Each breast is made of 15 to 20 lobes. Lobes contain many smaller lobules. Lobules contain groups of tiny glands that can produce milk. Milk flows from the lobules through thin tubes called ducts to the nipple. The nipple is in the center of a dark area of skin called the areola. Fat fills the spaces between the lobules and ducts. The breasts also contain lymph vessels. These vessels lead to small, round organs called lymph nodes. Groups of lymph nodes are near the breast in the axilla (underarm), above the collarbone, in the chest behind the breastbone, and in many other parts of the body. These lymph nodes trap bacteria, cancer cells, or other harmful substances.

Definition of breast cancer can be given as cancer that forms in tissues of the breast, usually at the tubes that carry milk to the nipple and at the glands that make milk. Although male breast cancer is rare, it occurs in both men and women. Additional information needed for this section can be found at [29].

1.1.1 Risk factors

Exact causes of breast cancer are not clear yet. Researchers often can not explain why one woman develops breast cancer and another does not. They do know that bumping, bruising, or touching the breast does not cause cancer. And breast cancer is not contagious which means you can not "catch" it from another person.

Researches have indicated that women with certain risk factors are more likely than others to develop breast cancer. A risk factor is something that may increase the chance of developing a disease. Studies have found the following risk factors for breast cancer up to today, and researchers keep on finding new risk factors. Some risk factors are given below.

- a) **Age:** The chance of getting breast cancer goes up as a woman gets older. Most cases of breast cancer occur in women over 60. Cancer cases are not common before menopause.
- b) **Personal history of breast cancer:** A woman who had breast cancer in one breast has an increased risk of getting cancer in her other breast.
- c) **Family history:** A woman's risk of breast cancer is higher if her mother, sister, or daughter had breast cancer. The risk is higher if her family member got breast cancer before age 40. Having other relatives with breast cancer (in either her mother's or father's family) may also increase a woman's risk.
- d) **Certain breast changes:** Some women have cells in the breast that look abnormal under a microscope. Having certain types of abnormal cells increases the risk of breast cancer.
- e) **Gene changes:** Changes in certain genes increase the risk of breast cancer. Tests can sometimes show the presence of specific gene changes in families with many women who have had breast cancer. Health care providers may suggest ways to try to reduce the risk of breast cancer, or to improve the detection of this disease in women who have these changes in their genes.

- f) **Reproductive and menstrual history:**
- The older a woman is when she has her first child, the greater her chance of breast cancer.
 - Women who had their first menstrual period before age 12 are at an increased risk of breast cancer.
 - Women who went through menopause after age 55 are at an increased risk of breast cancer.
 - Women who never had children are at an increased risk of breast cancer.
 - Women who take menopausal hormone therapy with estrogen plus progestin after menopause also appear to have an increased risk of breast cancer.
 - Large, well-designed studies have shown no link between abortion or miscarriage and breast cancer.
- g) **Race:** Breast cancer is diagnosed more often in white women than Latina, Asian, or African American women.
- h) **Radiation therapy to the chest:** Women who had radiation therapy to the chest (including breasts) before age 30 are at an increased risk of breast cancer. This includes women treated with radiation for Hodgkin's lymphoma. Studies show that the younger a woman was when she received radiation treatment, the higher her risk of breast cancer later in life.
- i) **Breast density:** Breast tissue may be dense or fatty. Older women whose mammograms (breast x-rays) show more dense tissue are at increased risk of breast cancer.
- j) **Being overweight or obese after menopause:** The chance of getting breast cancer after menopause is higher in women who are overweight or obese.
- k) **Lack of physical activity:** Women who are physically inactive throughout life may have an increased risk of breast cancer. Being active may help reduce risk by preventing weight gain and obesity.
- l) **Drinking alcohol:** Studies suggest that the more alcohol a woman drinks, the greater her risk of breast cancer.

Researchers are currently studying the effect of diet, physical activity, and genetics on breast cancer risk. They are also studying whether certain substances in the environment can increase the risk of breast cancer [29 - 30].

1.1.2 Symptoms of breast cancer

Common symptoms of breast cancer will be classified as follows:

- i. **A change in how the breast or nipple feels:** It can be understood as a lump or thickening in or near the breast or in the underarm area and sometimes as nipple tenderness.
- ii. **A change in how the breast or nipple looks:** Shape or the size of the nipple of the breast can change also sometimes nipple turns inward into the breast. The skin of the breast, areola, or nipple may be scaly, red, or swollen. It may have ridges or pitting so that it looks like the skin of an orange.
- iii. **Nipple discharge (fluid):** Early breast cancer usually does not cause pain. Still, a woman should see her health care provider about breast pain or any other symptom that does not go away. Most often, these symptoms are not due to cancer, other health problems may also cause them.

1.1.3. Stages of the breast cancer

The stage is based on the size of the tumor and whether the cancer has spread. Staging may involve x-rays and lab tests. These tests can show whether the cancer has spread and, if so, to what parts of your body. When breast cancer spreads, cancer cells are often found in lymph nodes under the arm. The stage often is not known until after surgery to remove the tumor in your breast and the lymph nodes under your arm.

These are the stages of breast cancer:

- **Stage 0** is carcinoma in situ.
 - **Lobular carcinoma in situ:** Abnormal cells are in the lining of a lobule. Lobular Carcinoma in situ seldom becomes invasive cancer. However, having lobular carcinoma in situ in one breast increases the risk of cancer for both breasts.

- **Ductal carcinoma in situ:** Abnormal cells are in the lining of a duct. The abnormal cells have not spread outside the duct. They have not invaded the nearby breast tissue. Ductal carcinoma in situ sometimes becomes invasive cancer if not treated.

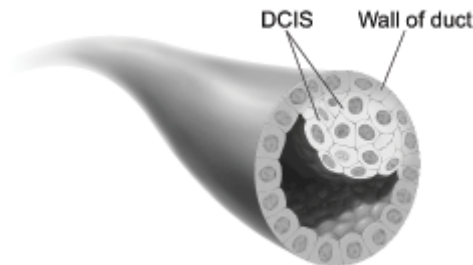


Figure 1.2 Ductal carcinoma in situ

- **Stage I** is an early stage of invasive breast cancer. The tumor is no more than 2 centimeters (three-quarters of an inch) across. Cancer cells have not spread beyond the breast.

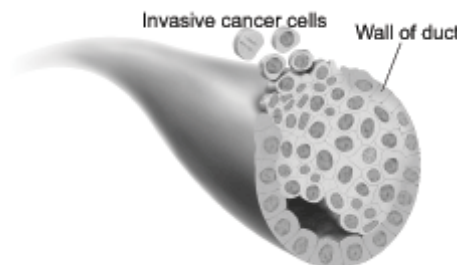


Figure 1.3 Cancer cells spreading outside the duct.

- **Stage II** is one of the following:
 - The tumor in the breast is no more than 2 centimeters across. The cancer has spread to the lymph nodes under the arm.
 - The tumor is between 2 and 5 centimeters. The cancer may have spread to the lymph nodes under the arm.
 - The tumor is larger than 5 centimeters. The cancer has not spread to the lymph nodes under the arm.

- **Stage III** may be a large tumor, but the cancer has not spread beyond the breast and nearby lymph nodes. It is locally advanced cancer.
- **Stage IIIA** is one of the following:
 - The tumor in the breast is smaller than 5 centimeters. The cancer has spread to underarm lymph nodes that are attached to each other or to other structures.
 - The tumor is more than 5 centimeters across. The cancer has spread to the underarm lymph nodes.
- **Stage IIIB** is one of the following:
 - The tumor has grown into the chest wall or the skin of the breast.
 - The cancer has spread to lymph nodes behind the breastbone.
 - Inflammatory breast cancer is a rare type of Stage IIIB breast cancer. The breast looks red and swollen because cancer cells block the lymph vessels in the skin of the breast.
- **Stage IIIC** is a tumor of any size. It has spread in one of the following ways:
 - The cancer has spread to the lymph nodes behind the breastbone and under the arm.
 - The cancer has spread to the lymph nodes under or above the collarbone.
- **Stage IV** is distant metastatic cancer. The cancer has spread to other parts of the body.
- **Recurrent cancer** is cancer that has come back (recurred) after a period of time when it could not be detected. It may recur locally in the breast or chest wall or it may recur in any other part of the body, such as the bone, liver, or lungs.

1.1.4 Screening techniques

Screening for breast cancer before the symptoms seen can be important. Screening can help doctors to find and treat cancer early. Treatment is more likely to work well when cancer is found early.

It may be suggested the following screening tests for breast cancer:

- **Screening mammogram**
- **Clinical breast exam**
- **Breast self-exam**

Screening mammogram

To find breast cancer early, it is suggested that

- Women in their 40s and older should have mammograms every 1 to 2 years. A mammogram is a picture of the breast made with x-rays.
- Women who are younger than 40 and have risk factors for breast cancer should ask their health care provider whether to have mammograms and how often to have them.

Mammograms (as well as dental x-rays, and other routine x-rays) use very small doses of radiation. The risk of any harm is very slight, but repeated x-rays could cause problems. During the operation shields must be used to protect parts of your body that are not in the picture. The benefits nearly always outweigh the risk.

Clinical breast exam

During a clinical breast exam, breasts are checked. The patient raises her arms over her head and let them hang by her sides, or press hands against the hips. Doctors look for differences in size or shape between the breasts. The skin of the breasts is checked for a rash, dimpling, or other abnormal signs. Nipples may be squeezed to check for fluid. Also the entire breast, underarm, and collarbone area will be checked. A lump is generally the size of a pea before anyone can feel it. The exam is done on one side, then the other. The lymph nodes near the breast are checked to see if they are enlarged.

Breast self-exam

Breast self-exams may be performed monthly to check for any changes in the breasts. It is important to remember that changes can occur because of aging, menstrual cycle, pregnancy, menopause, or taking birth control pills or other

hormones. It is normal for breasts to feel a little lumpy and uneven. Also, it is common for breasts to be swollen and tender right before or during the menstrual period.

Breast self-exams cannot replace regular screening mammograms and clinical breast exams. Studies have not shown that breast self-exams alone reduce the number of deaths from breast cancer [29-31].

1.1.5 Diagnosis

If a symptom or screening test result is seen that suggests cancer, it has to be find out whether it is due to cancer or to some other cause. A physical exam can be made. It may be a mammogram or other imaging procedure. These tests make pictures of tissues inside the breast. After the tests, the doctor may decide no other exams are needed or may suggest a follow-up exam later on. Alternatively, they may have a biopsy to look for cancerous cells.

Clinical Breast Exam

Doctor feels each breast for lumps and looks for other problems. If the patient has a lump, doctor will feel its size, shape, and texture. It will also be checked to see if it moves easily. Benign lumps often feel different from cancerous ones. Lumps that are soft, smooth, round, and movable are likely to be benign. A hard, oddly shaped lump that feels firmly attached within the breast is more likely to be cancer.

Diagnostic Mammogram

Diagnostic mammograms are x-ray pictures of the breast. They take clearer, more detailed images of areas that look abnormal on a screening mammogram. Doctors use them to learn more about unusual breast changes, such as a lump, pain, thickening, nipple discharge, or change in breast size or shape. Diagnostic mammograms may focus on a specific area of the breast. They may involve special techniques and more views than screening mammograms.

Ultrasound

An ultrasound device sends out sound waves that people cannot hear. The waves bounce off tissues. A computer uses the echoes to create a picture. Doctor can view these pictures on a monitor. The pictures may show whether a lump is solid or filled with fluid. A cyst is a fluid-filled sac. Cysts are not cancer. However, a solid mass may be cancer. After the test, the pictures can be stored on video or printed out. This exam may be used along with a mammogram.

Magnetic Resonance Imaging

Magnetic resonance imaging (MRI) uses a powerful magnet linked to a computer. MRI makes detailed pictures of breast tissue. These pictures can be seen on a monitor or on a printed film. MRI may be used along with a mammogram.

Biopsy

Fluid or tissue is removed from the breast to help find out if there is cancer. Some suspicious areas can be seen on a mammogram but cannot be felt during a clinical breast exam. Doctors can use imaging procedures to help see the area and remove tissue. Such procedures include ultrasound-guided, needle-localized, or stereo tactic biopsy.

1.1.6 Breast tumor types

After screening breasts, it is important to classify the tumor types if present. The most familiar tumor types seen are mass and microcalcification.

- Mass type tumor
- Microcalcificatin type tumor

Mass type tumor

Benign and malignant masses have different margin, density and location attributes. Round, low-density masses with smooth, sharply defined margins are considered benign. High-density, stellate, speculated masses with poor defined margins are

considered malignant [32]. Mammogram contains benign and malignant masses are shown in Figure 1.4.

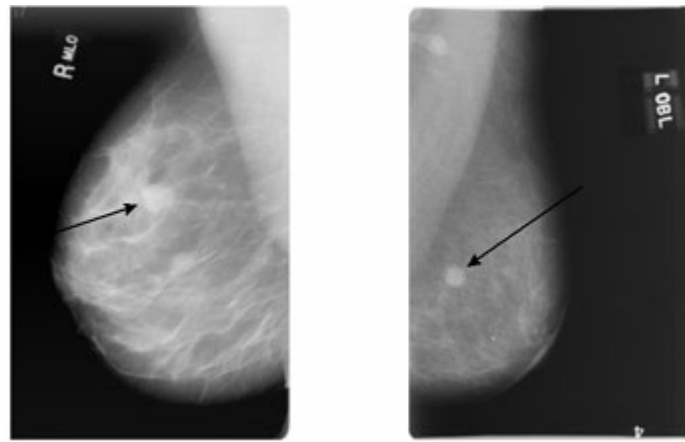


Figure 1.4 Mass type tumor

a) Benign mass

b) Malignant mass

Microcalcification type tumor

Benign and malignant microcalcifications can occur with or without a mass. When deciding if a microcalcification is benign or malignant size, shape, density, distribution pattern and number of microcalcifications are examined.

Benign microcalcifications are typically large, coarse, round or oval, and uniform in size and shape. Their distribution pattern is typically scattered or diffuse. If the microcalcifications are clustered, their number is less than 5 per cluster.

Malignant microcalcifications are typically microscopic and fine, linear branching, stellate-shaped, and varying in size and shape [32]. Their distribution pattern is grouped or clustered, and they are innumerable. The rule of malignancy that when the number of microcalcifications in a cluster is greater (usually more than 5); the likelihood of malignancy become greater. Typically, malignant microcalcifications present with a wide range in size, shape, and density. Mammogram contains microcalcification is shown in Figure 1.5.

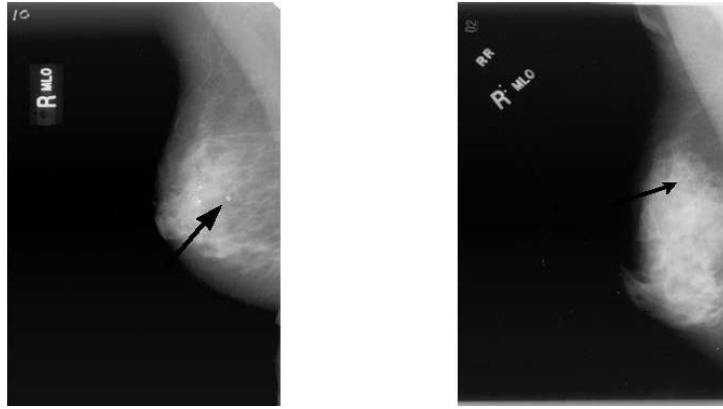


Figure 1.5 Microcalcification type tumor

a) Benign calcification

b) Malignant calcification

It is seen that early detection in breast cancer is very important for human health. In chapter two, theoretical aspects of wavelet transform and lifting scheme will be given. Moreover, advantages of lifting scheme over regular wavelet transform will be investigated in this chapter. In chapter three, tumor detection algorithm will be given in detail. Additionally, mathematical morphology which used before wavelet decomposition will be described. Finally, results will be discussed and conclusions have been expressed.

CHAPTER 2

WAVELET TRANSFORM

This chapter introduces wavelets beginning from their historical development to second generation wavelets. Concepts are revised related to the thesis topic. More detailed information about wavelets can be found at references [33 - 40].

2.1 History of Wavelets

Wavelet analysis is a new development in mathematics for almost twenty-five years. The theory of wavelets was developed from concepts and theories that already existed in various fields by Meyer, Mallat, Daubechies and others during the end of 1980's and beginning of 1990's. And the name "wavelet" or "ondelette" was first used by French researchers, including Morlet, Arens, Giard and Grossman [34-36].

In 1807, French mathematician Fourier established the partial differential equations governing heat diffusion and solved it by using an infinite series of trigonometric functions, named as the Fourier series. This work provided the foundation of modern time-frequency analysis and can be seen as the foundation of wavelets.

In 1909, Haar used a set of step functions to approximate a continuous function. The basis function was basically a scale and shift transformation of a single function and can be seen as the first wavelet basis in the literature. Now there is an entire wavelet family named after him.

Haar wavelet is discontinuous so that is not differentiable. But wavelets developed frequency analysis to scale analysis and lead to following discoveries. Most of the early works were done around 1930s.

Development of wavelets started mainly in three fields; mathematics, signal processing and image analysis. Due to the statistical properties of wavelets, they also have been applied in other fields recently.

In 1946 the first time-frequency wavelets, or Gabor wavelets, was introduced by D. Gabor [37]. The first definition of a wavelet was given by Morlet in 1982 and Morlet and Grossman in 1984 [34, 35]. Synthesis of quadrature mirror filtering and pyramidal algorithms was made by Mallat during the middle of 1980s [38]. Daubechies discovered compact supported wavelet at the end of 1980s [39].

The construction of second generation wavelets due to Sweldens [40] introduced the lifting scheme method, which enables multiscale analysis adaptive to irregular sampling, subdivision, and other constrains, whiling keeping all the aspects of the first generation wavelets.

The fact is that this method does not rely on classical Fourier method to build Wavelet basis. It uses the property that a new wavelet filter can be inferred from a simpler one simply by making linear combinations of the older coefficients. Starting from the simplest wavelet, this allows creating much more complex wavelet filters. The power of this construction is that the lazy wavelet and the lifting process do not rely on spatial propriety of the domain.

2.2 Introduction to Wavelets

A wave can be defined as an oscillating function of time or space, such as a sinusoid. Fourier analysis is a wave analysis. It expands signals or functions in terms of sinusoids. It is useful for time-invariant, periodic and stationary phenomena. A wavelet is a small oscillating wave, which has its energy concentrated in time to give a tool for the analysis of transient, non-stationary, or time-varying phenomena. Wavelets can be used in a series expansion of signals or functions much the same way as a Fourier series uses the wave or sinusoid to represent a signal or function.

2.3 What is Wavelet Transform

The fundamental idea behind the transform techniques is to transform a function or signal from one domain to another. There are well known transform techniques used in signal processing to transform time domain signals to frequency domain. Some of them are Fourier transform (FT) [41], Short-time Fourier transform (STFT) [42] and Discrete Cosine transform (DCT) [43]. In recent years researchers interest Wavelet Transform (WT) which solves resolution problems and provides perfect reconstruction [44 - 46].

In general, a signal or a function $f(t)$ can be better analyzed if expressed as a linear decomposition by

$$f(t) = \sum_l a_l \psi_l(t) \quad (2.1)$$

where l is the integer index for the summation, a_l are the real-valued expansion coefficients, and $\psi_l(t)$ are a set of real-valued functions of t called the expansion set. If the expansion (2.1) is unique then the set is called a basis for the same class of functions. If the basis is orthogonal

$$\langle \psi_k(t), \psi_l(t) \rangle = \int \psi_k(t) \psi_l(t) dt = 0 \quad (2.2)$$

then the coefficients can be calculated from the inner product

$$a_k = \langle f(t), \psi_k(t) \rangle = \int f(t) \psi_k(t) dt \quad (2.3)$$

For a Fourier series, the orthogonal basis functions $\psi_k(t)$ are $\sin(k\omega_o t)$ and $\cos(k\omega_o t)$ with frequencies of $k\omega_o$. However, its one dimensional structure does not provide time localization. To overcome this problem STFT introduced which uses windowing to deal with time localization. This reclamation solved time localization

problem but caused resolution problems. Wavelet transform not only solves time localization but also it enables Multiresolution Analysis (MRA) [46].

For the wavelet expansion, a two-parameter system is constructed, then equation (2.1) becomes

$$f(t) = \sum_k \sum_j a_{j,k} \psi_{j,k}(t) \quad (2.4)$$

where both j and k is integer indices and $\psi_{j,k}(t)$ is the wavelet expansion coefficients that usually form an orthogonal basis. The set of expansion coefficients $a_{j,k}$ is called the Discrete Wavelet Transform (DWT) of $f(t)$. Additionally equation (2.4) is called the inverse wavelet transform [47].

First generation wavelet systems are generated from a single scaling function or wavelet by simple scaling and translation. Two dimensional parameterization is achieved from the function ψ , which is called generating wavelet or mother wavelet, by

$$\psi_{j,k}(t) = s_o^{j/2} \psi(s_o^j t - k\tau_o) \quad j, k \in \mathbf{Z} \quad (2.5)$$

Convenient values for s_o and τ_o found to be “2” and “1”, respectively, where \mathbf{Z} is the set of all integers [48].

The MRA needs two closely related basic functions. In addition to the wavelet $\psi(t)$ that has been described another basic function called scaling function $\varphi(t)$ is needed. Reasons for needing this function will be discussed in detail in the next section. The multiresolution wavelet algorithm decomposes a signal $x(t)$ by the help of scaling functions $\varphi(t)$ and the wavelet functions $\psi(t)$. These two functions together resolve the signal into its coarse and detail components.

So the signal $x(t)$ can be described as

$$x(t) = \sum_{f=-\infty}^{\infty} c(k)\varphi(t) + \sum_{j=0}^{\infty} \sum_{k=-\infty}^{\infty} d(j,k)\psi_{j,k}(t) \quad (2.6)$$

where $c(k)$ and $d(j,k)$ are scaling and wavelet coefficients, respectively [48].

First part of the summation is the low resolution part of the signal which gives approximation information, and the second part represents high resolution to give detail information of the signal.

2.4 Multi-Resolution Analysis

Mainly multiresolution analysis is representation of a signal or function at various levels of resolution [46]. It allows us to decompose a signal into approximations and details. That is, we have bases $\varphi(t)$ and $\psi(t)$ and we use these bases to decompose our signal [48].

We can formulate the basic requirement of MRA by requiring a nesting of the spanned spaces as

$$\{0\} \dots \subset V_{-2} \subset V_{-1} \subset V_0 \subset V_1 \subset V_2 \dots L^2 \quad (2.7)$$

or

$$V_j \subset V_{j+1} \text{ for all } j \in \mathbb{Z} \quad (2.8)$$

which results into

$$V_{-\infty} = \{0\} \text{ and } V_{\infty} = L^2 \quad (2.9)$$

where L^2 is the space of all functions $f(t)$ with a well defined integral of the square of the modulus of the function. “ L ” signifies a Lebesgue integral, the “2” denotes the integral of the square of the modulus of the function [49].

In this space representation, high-resolution signals also contain the lower resolution information. In the dyadic case, each subspace V_j is twice as large as V_{j-1} . So this subspace leads to a natural scaling condition for any function

$$f(t) \in V_j \Leftrightarrow f(2t) \in V_{j+1} \quad (2.10)$$

(2.9) ensures elements of a function in a space are simply scaled versions of the elements in the next lower space.

Scaling functions can be defined in terms of integer translates of the basic scaling function.

$$\varphi_k(t) = \varphi(t - k) \quad (2.11)$$

These scaling functions spanning L^2 can be shown as

$$V_o = \overline{Span\{\varphi_k(t)\}} \quad (2.12)$$

The over bar denotes closure property where k in (2.11) and (2.12) denotes integers from minus infinity to infinity. So, two dimensional representation of (2.12) with scaling and translation is

$$V_j = \overline{Span\{\varphi_k(2^j t)\}} = \overline{Span\{\varphi_{j,k}(t)\}} \quad \text{where } \varphi_{j,k}(t) = 2^{j/2} \varphi(2^j t - k) \quad (2.13)$$

It is clear from (2.13) that if $\varphi(t)$ is in V_o , it is also in V_1 . This means $\varphi(t)$ can be expressed in terms of a weighted sum of shifted $\varphi(2t)$ as

$$\varphi(t) = \sum_n h_o(n) \sqrt{2} \varphi(2t - n), \quad n \in \mathbb{Z} \quad (2.14)$$

where $\sqrt{2}$ maintains the norm of the scaling function with the scale of two and $h_0(n)$ are called scaling function coefficients. This equation is fundamental to the theory of the scaling functions and can be called refinement equation, the multiresolution analysis equation or the dilation equation [50].

The important features of the signal can be better described by defining different set of functions named as wavelet functions $\psi_{j,k}(t)$ which are mentioned before. Let us define a new subspace W_j containing wavelet functions. Wavelet theory requires scaling functions and wavelet functions are orthogonal. So, this new subspace, called wavelet spaces, becomes orthogonal complement of V_j in V_{j+1} .

$$V_{j+1} = V_j \oplus W_j \quad (2.15)$$

where \oplus is direct sum. So as a general equation by using the property given in (2.15)

$$L^2 = V_0 \oplus W_0 \oplus W_1 \oplus \dots \quad (2.16)$$

If we expand (2.16) we get

$$L^2 = \dots \oplus W_{-2} \oplus W_{-1} \oplus W_0 \oplus W_1 \oplus W_2 \oplus \dots \quad (2.17)$$

so,

$$W_{-\infty} \oplus \dots \oplus W_{-1} = V_0 \quad (2.18)$$

These relations is shown schematically in Figure 2.1

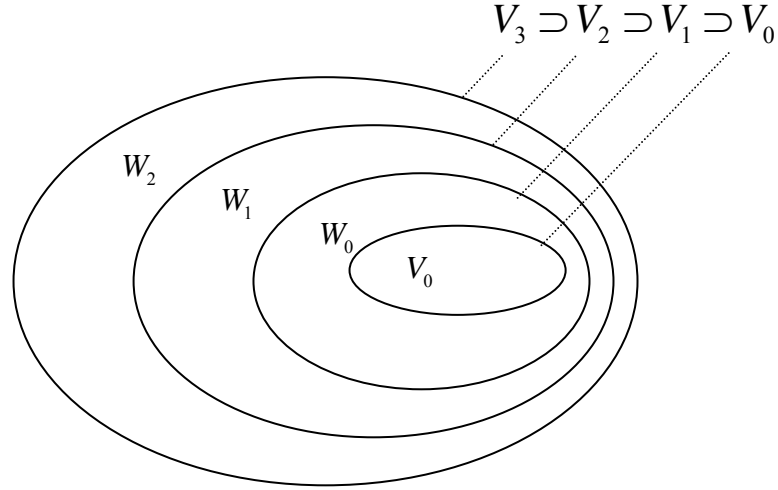


Figure 2.1 Scaling Function and Wavelet Vector Spaces

As can be seen from Figure 2.1 $W_0 \subset V_0$ so, wavelets can be represented by a weighted sum of shifted scaling function $\varphi(2t)$ as in (2.14)

$$\psi(t) = \sum_n h_1(n) \sqrt{2} \varphi(2t - n), \quad n \in \mathbb{Z} \quad (2.19)$$

where $h_1(n)$ are called wavelet function coefficients.

Eq. (2.19) generates a mother wavelet $\psi(t)$ for a class of expansion functions of the form

$$\psi_{j,k}(t) = 2^{j/2} \psi(2^j t - k) \quad (2.20)$$

Also the orthogonality relation between wavelet and scaling functions produces a relation between $h_0(n)$ and $h_1(n)$ as

$$h_1(n) = (-1)^n h_0(1 - n) \quad (2.21)$$

Now we have constructed a set of functions $\varphi_k(t)$ and $\psi_{j,k}(t)$, that can span all $L^2(\mathbb{R})$.

So a general equation for any $f(t) \in L^2$ can be written as [48]

$$f(t) = \sum_k c_{j_0}(k) \varphi_{j_0,k}(t) + \sum_k \sum_{j=j_0}^{\infty} d_j(k) \psi_{j,k}(t) \quad (2.22)$$

where j_0 can be positive negative or zero according to the usage. The choice of j_0 sets the coarsest scale whose space is spanned by $\varphi_{j_0,k}(t)$. Remaining part of $L^2(\mathbb{R})$ will be spanned by $\psi_{j,k}(t)$.

As mentioned at the introduction part the coefficients that we use in equation (2.22) are called DWT of the signal $f(t)$. If the wavelet system is constructed orthogonal, these coefficients can be calculated by inner products

$$\begin{aligned} c_j(k) &= \langle f(t), \varphi_{j,k}(t) \rangle = \int f(t) \varphi_{j,k}(t) dt \\ d_j(k) &= \langle f(t), \psi_{j,k}(t) \rangle = \int f(t) \psi_{j,k}(t) dt \end{aligned} \quad (2.23)$$

At the next section, we will investigate the usage of these formulations.

2.5 Filter Bank Structure

In many applications, it is not preferred to deal directly with scaling functions or wavelets. Only the filter coefficients $h_0(n)$ and $h_1(n)$ which are defined in (2.14) and $c_j(k)$, $d_j(k)$ from (2.23) will be considered [51].

In order to work directly with the wavelet transform coefficients, the relationship between expansion coefficients at a lower scale in terms of higher scale will be needed. The relation can be shown as

$$f(t) \in V_{j+1} \Rightarrow f(t) = \sum_k c_{j+1}(k) 2^{(j+1)/2} \varphi(2^{j+1}t - k) \quad (2.24)$$

at scale j same function can be represented both using wavelet and scaling functions as

$$f(t) = \sum_k c_j(k) 2^{j/2} \varphi(2^{j/2}t - k) + \sum_k d_j(k) 2^{j/2} \psi(2^j t - k) \quad (2.25)$$

As explained at the previous section inner product of $f(t)$ with scaling function at scale j will give

$$c_j(k) = \langle f(t), \varphi_{j,k}(t) \rangle = \sum_m h_0(m - 2k) c_{j+1}(m) \quad (2.26)$$

and the wavelet coefficients will be represented as

$$d_j(k) = \langle f(t), \psi_{j,k}(t) \rangle = \sum_m h_1(m - 2k) c_{j+1}(m) \quad (2.27)$$

It is clear from Eq. (2.26) and (2.27) that $j+1$ scale coefficients are down sampled and filtered with $h_0(n)$ and $h_1(n)$ to find corresponding lower level coefficients [51].

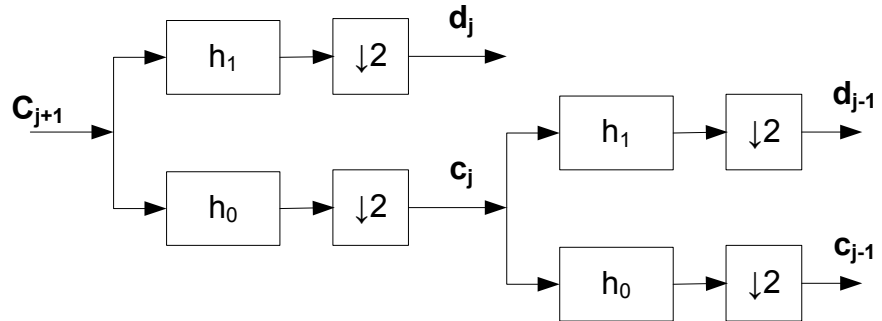


Figure 2.2 Two-Level Two-Band Analysis Tree

In Figure 2.2 analysis of a function by using h_0 and h_1 , which corresponds to low-pass and high-pass filter coefficients respectively, are given. Synthesis part can be thought as a reverse process with the same filter coefficients.

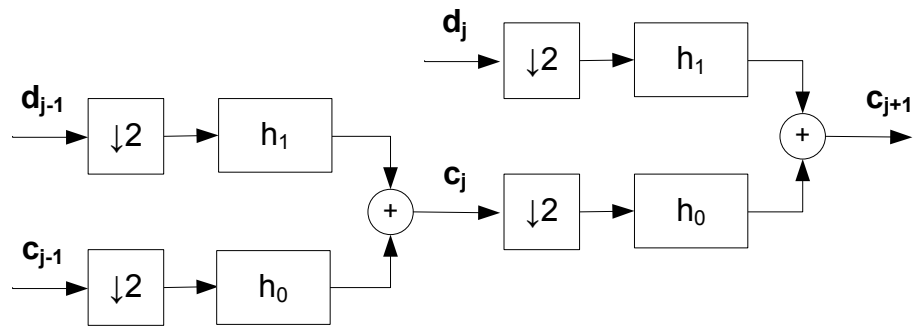


Figure 2.3 Two-Level Two-Band Synthesis Tree

For a one dimensional signal analysis and synthesis parts which are called Wavelet Transform (WT) and Inverse Wavelet Transform (IWT) are depicted above.

To use wavelet transform in image processing we must implement two- dimensional analysis and synthesis banks. For two-dimensional case, we use the same tree structure that is shown in Figure 2.3 and 2.4, and apply to columns and rows of the image. [48, 51]

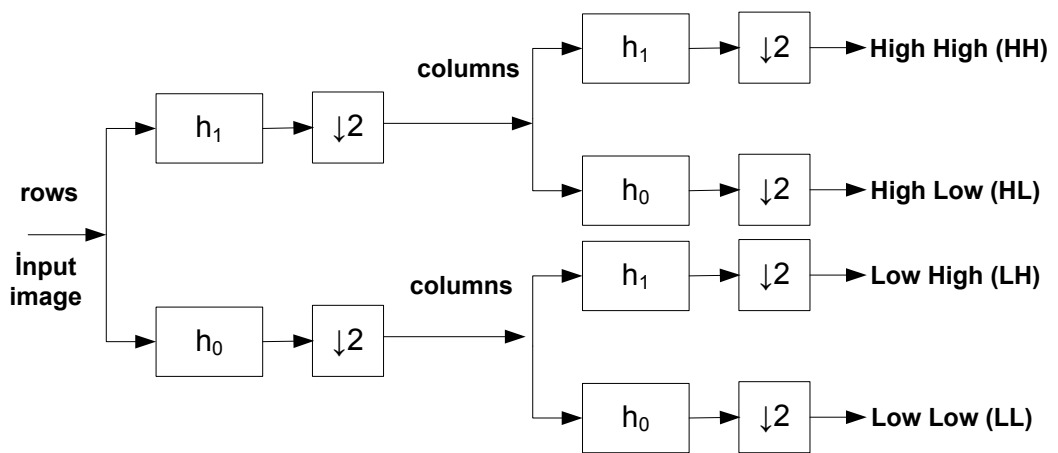


Figure 2.4 One-Level Two-Band Analysis Tree of a Two-Dimensional Signal

Figure 2.4 shows the block diagram for one-level decomposition. Note the terminology for the filter outputs after the column filtering. LL indicates that the image corresponds to the subband obtained by low-pass filtering and down sampling both the rows and the columns.

2.6 Lifting Scheme

There are various efficient techniques to construct wavelet bases or to factor existing wavelet filters into basic building blocks. The lifting scheme (LS) is one of those various methods [39]. Figure 2.5 illustrates analysis and synthesis parts of lifting scheme.

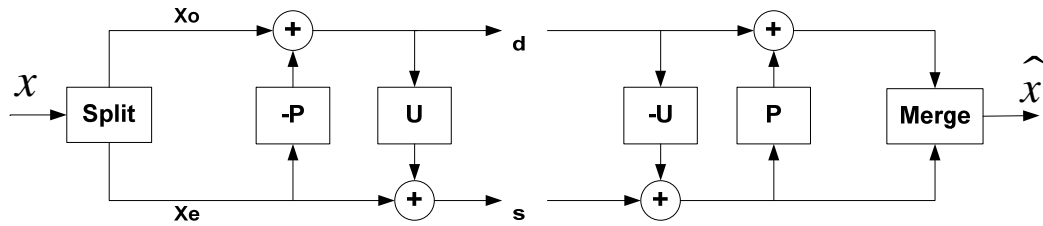


Figure 2.5 Lifting scheme

The first phase of lifting scheme is to split data into two maximally correlated phases. Preferred way is to split sampled data $x = \{x_k\}$ into even and odd samples in order to get two disjoint sets where

$$\{x_e\} = \{x_{2k}\} \quad \text{even samples}$$

$$\{x_o\} = \{x_{2k+1}\} \quad \text{odd samples}$$

These sets are strongly correlated between themselves, and even samples go before odd samples. Therefore, prediction of x_o by using linear or non-linear combination of x_e is possible.

After predicting x_o , an error or detail signal d between two sets are defined as

$$d = x_o - P(x_e) \quad (2.28)$$

where P represents prediction operator. In this equation if the prediction operator is accurate enough then d will be a very sparse set, which is a measure representing non-linearity of the logical portion of the signal. It corresponds to the high-pass filter

output of wavelet transform. After prediction, lifting scheme replaces x_o by d and it results in reduction of data.

Splitting operation to obtain two discrete sets, x_o and x_e , which takes place at the beginning of the lifting scheme, may not provide adequate spectral separation. Then it will lead to aliasing between x_e and x_o . To overcome this problem final step in lifting scheme is to update x_e and replace it with an aliasing free smoother set s which is the output of the low-pass filter. The smoother set yielded after applying update process to detail signal is given as

$$s = x_e - U(d) \quad (2.29)$$

In original lifting scheme the update operator U is linear combination of the detail signal. The P and the U are always invertible and are critical to guarantee perfect construction. By using s as the input of the next stage and cascading the scheme J times we yield a J level decomposition of the signal x [52].

In the synthesis part the prediction and update steps are employed just only their signs are changing. Finally, sub-signals are merged into the higher rate signal to recover the original data x [52, 53].

CHAPTER 3

TUMOR DETECTION ALGORITHM

There are many algorithms proposed for the detection of tumors present at mammograms. New transformation algorithms and enhancement techniques leads researchers in finding methods that are more effective. Currently, general procedure for detection algorithms includes pre-processing, enhancement and segmentation stages as can be seen in Figure 3.1.

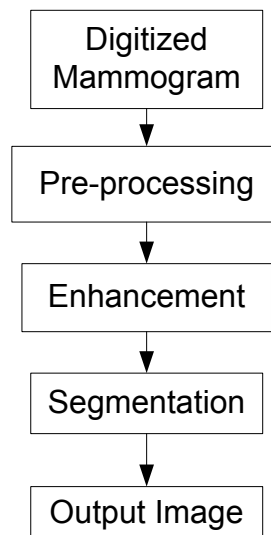


Figure 3.1 General CAD detection algorithm

Most of the proposed methods in the literature use the same procedure with different techniques at each block [54-57]. In this thesis we used the same algorithm proposed at [23] with some modifications at pre-processing, enhancement, and segmentation phases.

Our proposed method consists of three stages: pre-processing, enhancement and segmentation, which is shown in Figure 3.2.

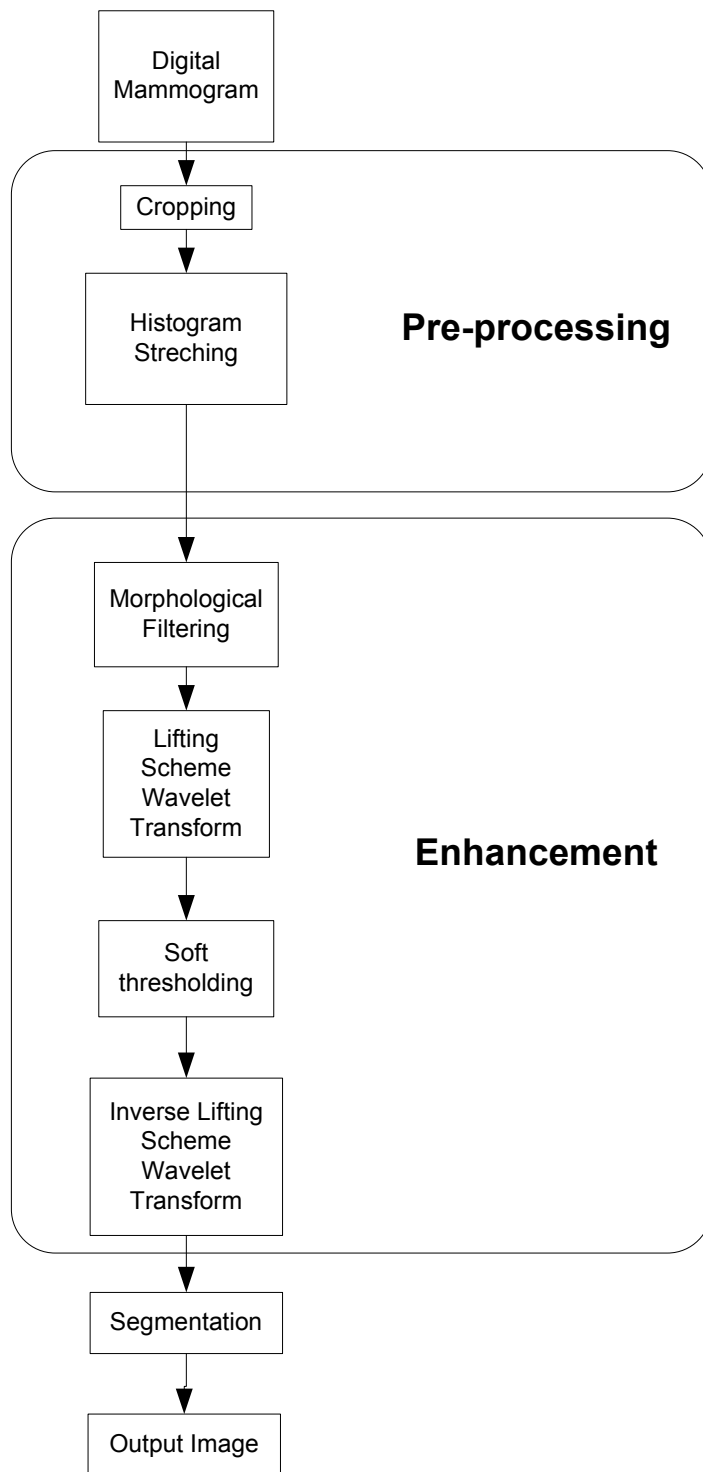


Figure 3.2 Block diagram of the detection method

3.1 Pre-Processing

This step can be thought as a preparation for the enhancement phase. Some applications are combining pre-processing and enhancement phases. In this study, it is better to investigate them separately.

Generally, mammograms include black borders and label information. To prevent processing unnecessary information and according to that to reduce the processing time it is applied a cropping operation to the sample image to obtain our region of interest. Then at the histogram stretching phase, we rescale the image such that it covers the whole dynamic range.

3.2 Enhancement

Pre-processed mammograms need to be enhanced before segmentation phase. We use mathematical morphology and lifting scheme wavelet transform at the enhancement phase. The isolation of gray-value objects that are convex can be accomplished with the top-hat transform [58]. For enhancing details, in the presence of shading, top-hat transform is applied as follows

$$h = f - (f \ominus b) \oplus b \quad (3.1)$$

where f and b represents image and constructing element respectively. In the equation, erosion \ominus , and dilation \oplus operators applied to the image, respectively. In the application of algorithm, it is obtained best results with cylindrical constructing element with a radius of 40 pixels [59].

A low pass filter reduces the high frequency content of the image. An ideal low-pass filter has the transfer function

$$H(u, v) = \begin{cases} 1 & \text{if } D(u, v) \leq D_o \\ 0 & \text{if } D(u, v) > D_o \end{cases} \quad (3.2)$$

where D_0 is a specified nonnegative number and $D(u, v)$ is the distance from point (u, v) to the center of the filter. The Gaussian low pass filter is defined as

$$H(u, v) = e^{-D^2(u, v)/2\sigma^2} \quad (3.3)$$

In Eq. (3.3) $D(u, v)$ is defined as

$$D(u, v) = \text{sqrt}\left(\left(u - \frac{M}{2}\right)^2 + \left(v - \frac{N}{2}\right)^2\right) \quad (3.4)$$

where N is the number of rows and M is the number of columns. After top-hat algorithm, the image is enhanced and the high frequency content is reduced with an experimentally determined low-pass Gaussian filter, but some noise still remains.

For denoising, the Lifting Scheme of Wavelet Transform is used to compute subband coefficients. For each subband level the detailed coefficients are determined whether significant or not with respect to a threshold value defined by NormalShrink [60]. The threshold value, which is adaptive to different subband characteristics, is given as

$$T = \frac{\beta \hat{\sigma}^2}{\hat{\sigma}_y} \quad (3.5)$$

where $\hat{\sigma}^2$ and $\hat{\sigma}_y$ are noise variance and standard deviation of the subband under estimation, respectively. β is the scale parameter which depends upon the subband size and the number of decompositions. The relation computes it once for each scale.

$$\beta = \sqrt{\log\left(\frac{L_k}{J}\right)} \quad (3.6)$$

where L_k is the length of the subband at the computed scale J . More information can be found in [60].

There are different types of thresholding methods in the literature like hard thresholding, soft thresholding, adaptive thresholding etc [4, 5, 26]. We use soft thresholding that prevents discontinuities in the image.

Soft thresholding is defined as

$$f(x) = \begin{cases} \text{sign}(x)(|x| - t) & \text{if } |x| > t \\ 0 & \text{if } |x| \leq t \end{cases} \quad (3.7)$$

where $\text{sign}(x)$ denotes the signum function. After thresholding, the image is reconstructed by using thresholded detail coefficients.

3.3 Segmentation

Enhancement phase helps to ease segmentation of our region of interest. The goal of the segmentation phase is to partition the image into regions that are homogeneous with respect to one or more characteristics or features. Most commonly used segmentation techniques can be classified into two categories: region based segmentation techniques, which look for regions satisfying a given criterion, and edge based segmentation techniques that detect edges between different characteristic regions [4, 5, 26].

In this work, it is preferred to use region based segmentation by global thresholding. The object can be extracted from the background by a simple operation that compares pixel values with a threshold value T . The thresholded image $f(x, y)$ is

$$f(x, y) = \begin{cases} 1 & \text{if } (x, y) > T \\ 0 & \text{if } (x, y) \leq T \end{cases} \quad (3.8)$$

The result of thresholding is a binary image, where pixels having intensity value of 1 or 0 corresponding to objects or background, respectively. It is an important parameter to compute threshold value. The principle of Stein's unbiased risk estimate (SURE) is used for computation of threshold value [61]. A threshold value is

computed with SURE at each scale of lifting decomposition for approximation coefficients and then it is taken mean of those values. The mean is normalized by multiplying a factor k having range $0 < k \leq 0.6$. In our applications depending on the evaluated database, best results are achieved by choosing $k= 0.55$.

We used Mammographic Image Analysis Society (MIAS) database in our experiments, which consist of 322 images, that contains normal, benign and malign cases [62]. Results of the experiments that we get will be discussed at the next chapter.

CHAPTER 4

EXPERIMENTAL RESULTS

It is an important subject to implement a CAD system for giving a second thought to the radiologists. To evaluate the performance of the proposed tumor detection algorithm MATLAB is used.

4.1 Results and Discussions

The proposed method experienced on over 100 mammograms obtained from MIAS database in order to compare the results given in the literature. Processing results of six sample mammograms are given illustratively in Figures 4.1 through 4.6.

As mentioned before they all have black borders and label information. After eliminating the black borders and label information, it is obtained that the region of interest, which is shown in Figures 4.2b through 4.6b.

The enhancement results can be seen in Figures 4.2c through 4.6c. As compared to the cropped mammograms, enhancement operation has been resulted in a superior image quality. Masses and lesions can be easily distinguished. Especially in Figure 4.4a, we have a low contrast mammogram, after the enhancement process where its output image shown in Figure 4.4c, the problematic areas can be seen clearly.

The segmentation results are given in Figures 4.1d through 4.6d. When the Figures 4.1c through 4.6c are compared with Figures 4.1d through 4.6d, respectively, the problematic areas obviously seen with strongly defined image contrast and shapes.

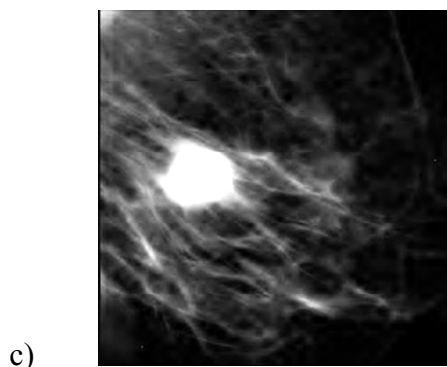
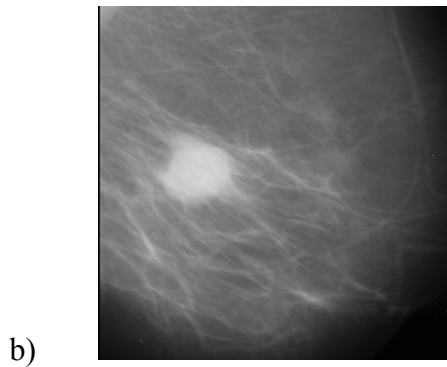
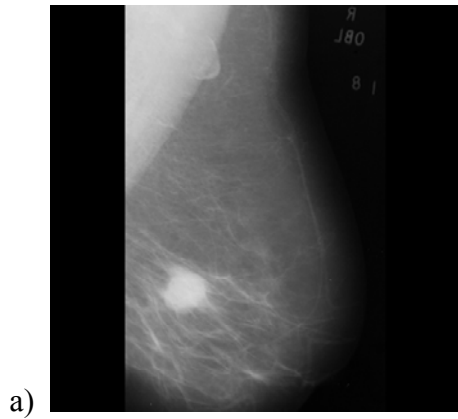


Figure 4.1 Input – Output
Mammogram (mdb028)
a) Original b) Cropped
c) Enhanced
d) Segmented Mammogram

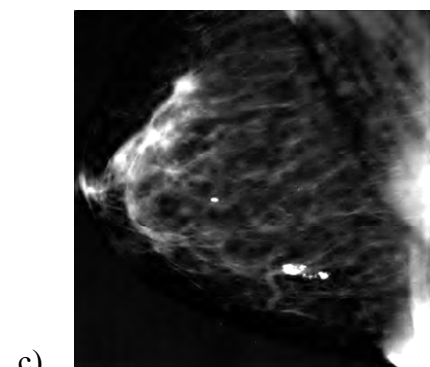
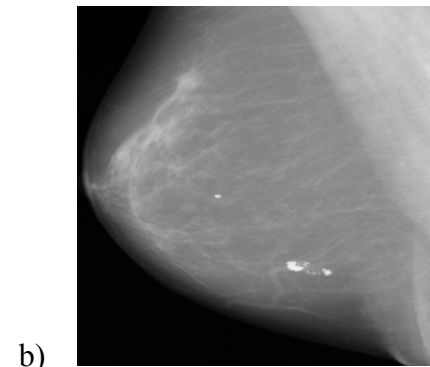
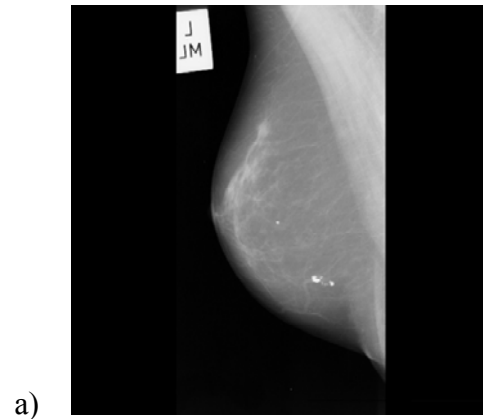


Figure 4.2 Input – Output
Mammogram (mdb075)
a) Original b) Cropped
c) Enhanced
d) Segmented Mammogram

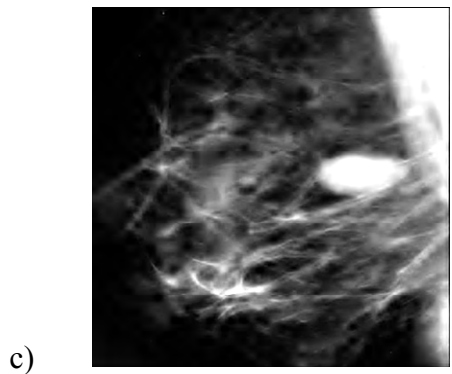
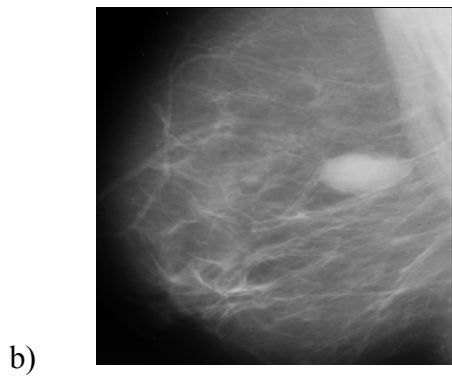
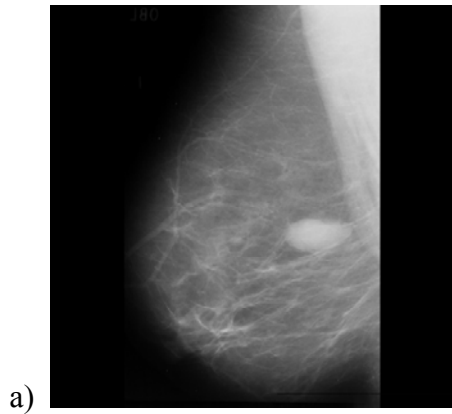


Figure 4.3 Input – Output
Mammogram (mdb025)
a) Original b) Cropped
c) Enhanced
d) Segmented Mammogram

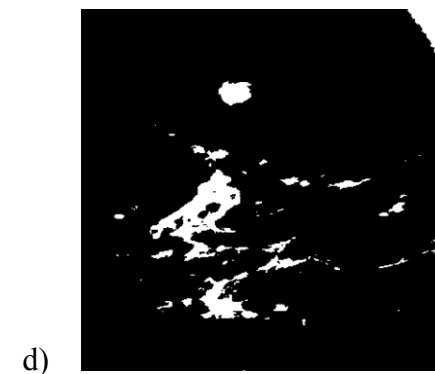
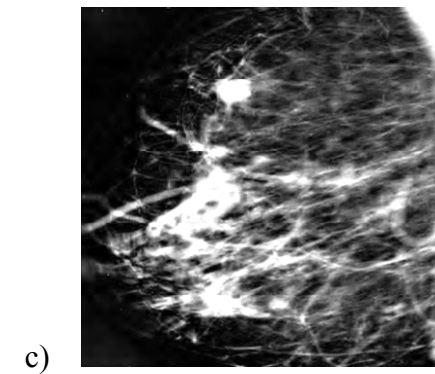
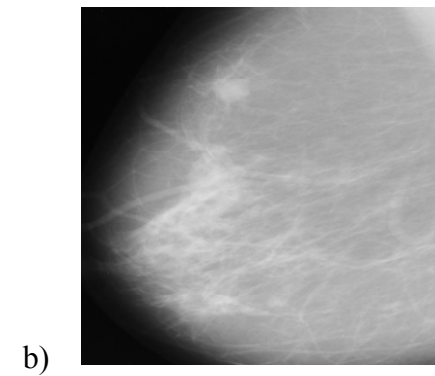
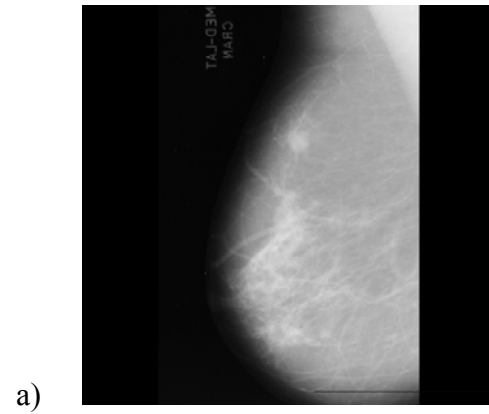


Figure 4.4 Input – Output
Mammogram (mdb023)
a) Original b) Cropped
c) Enhanced
d) Segmented Mammogram

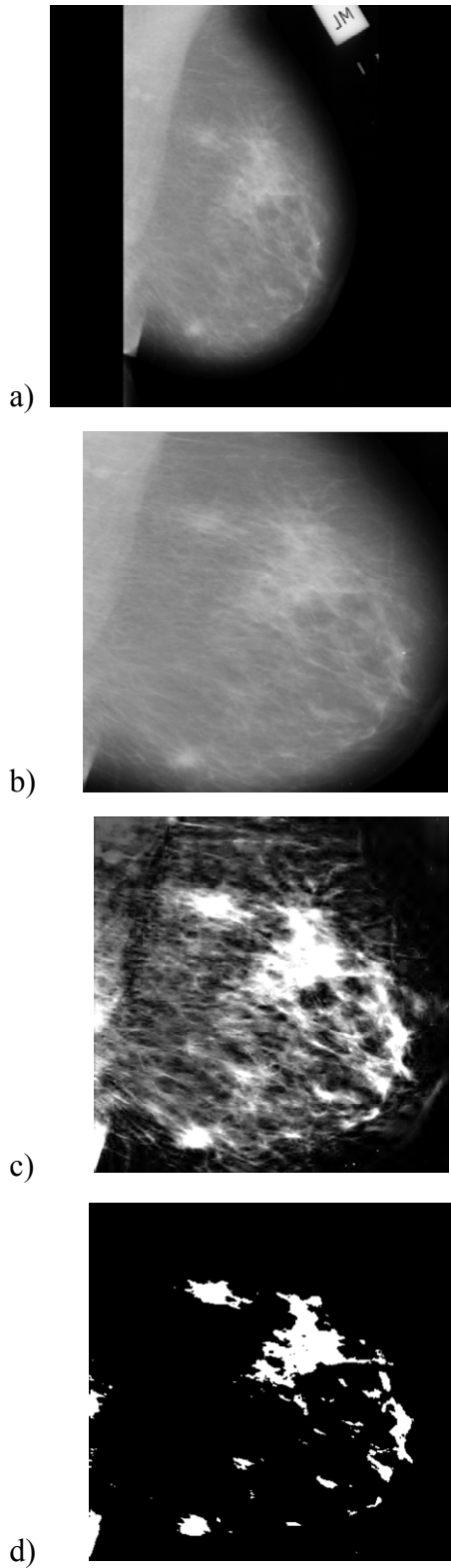


Figure 4.5 Input – Output
Mammogram (mdb206)
a) Original b) Cropped
c) Enhanced
d) Segmented Mammogram

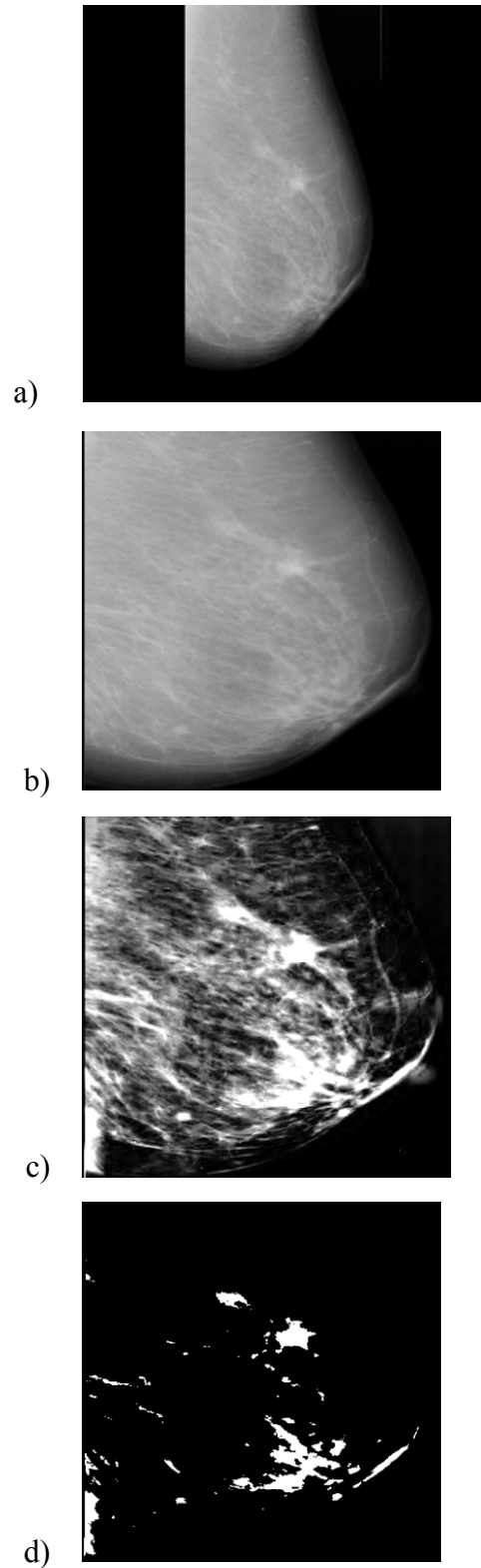


Figure 4.6 Input – Output
Mammogram (mdb158)
a) Original b) Cropped
c) Enhanced
d) Segmented Mammogram

To measure the improvement of the enhancement, Contrast Improvement Index (CII) and Peak Signal to Noise Ratio (PSNR) values are computed quantitatively.

A quantitative measure of contrast improvement which is the CII can be defined in [63] as

$$CII = \frac{C_{processed}}{C_{original}} \quad (8)$$

where $C_{processed}$ and $C_{original}$ are the contrasts for a region of interest in the processed and original images, respectively. The contrast of a region is described as

$$C = \frac{f - b}{f + b} \quad (9)$$

where f is the mean gray-level value of a particular object in the image, called the foreground, and b is the mean gray-level value of a surrounding region called the background.

Peak signal to noise ratio is a measure of similarity of an image that is computed by measuring the pixel difference between the original image and the enhanced image [64]. The PSNR can be computed as

$$PSNR = 10 \log_{10} \left(\frac{L^2}{MSE} \right) \quad (4.1)$$

where L is the dynamic range of pixel intensity defined as

$$L = 2^B - 1 \quad (4.2)$$

In equation given above B is the number of bits to represent a pixel so for eight bits per pixel gray scale image L will be 255.

Mean square error, also known as Euclidean distance or standard deviation [65]. It involves finding the square of the difference between the corresponding pixel values of the two scaled images. Mean square error of an image is defined as

$$MSE = \frac{1}{M * N} \sum_{i=1}^M \sum_{j=1}^N |x_{ij} - y_{ij}|^2 \quad (4.3)$$

In this equation x_{ij} and y_{ij} are the intensity values of the processed and the original image being compared where M and N are column and row sizes, respectively. The difference is squared in order to ensure that a negative difference between two corresponding pixels in the input and output images do not merely cancel out a positive difference between two other corresponding pixels. Multiplication of column and row sizes ($M * N$) takes place in the equation in order to prevent a larger image having larger MSE.

It is worth to note that the PSNR is expressed to give the mathematical similarity of two images in decibels. The PSNR is inversely proportional with MSE. Values for PSNR range between infinity for identical images and 0 for images that have no commonality. The PSNR values of the enhanced images shown in Figure 4.1 through Figure 4.6 are given in Table 4.1. In order to compare the Lifting Scheme Based Enhancement Algorithm (LSBEA) with other well known wavelet based denoising algorithms, PSNR values of BayesShrink and VisuShrink algorithms are evaluated.

Table 4.1 PSNR values of the enhanced mammograms

	Figure 4.1	Figure 4.2	Figure 4.3	Figure 4.4	Figure 4.5	Figure 4.6
BayesShrink	84,4900	77,1185	81,1411	72,9539	81,7010	73,1650
VisuShrink	59,1821	56,2247	58,2399	57,0640	58,1964	55,2786
Proposed Algorithm (LSBEA)	86,8074	85,8104	85,6388	75,4591	86,6117	85,6541

The greater PSNR values indicate better results. It is seen that the PSNR values obtained by the proposed algorithm for Figure 4.1 and Figure 4.5 are 86,8074 and 86,6117, respectively which gives the best results when compared by the PSNR

values for the Figures 4.2, 4.3 and 4.5. Also, when the LSBEA, BayesShrink, and VisuShrink are compared, the LSBEA has better results although the LSBEA and BayesShrink PSNR values are closed to each other but with VisuShrink, PSNR values has a great difference.

Another quantitative measure of contrast enhancement of image CII, has the advantage of being independent of the actual range of gray levels in the image. The CII values of the mammograms shown in Figure 4.1 through Figure 4.6 are given in Table 4.2. Also for comparison of the results, the CII values for VisuShrink and BayesShrink are included. The Bigger CII values indicate better contrast enhancement results because it is the ratio of the contrast of a particular object to the value of surrounding region called the background.

Table 4.2 CII values of the enhanced mammograms

	Figure 4.1	Figure 4.2	Figure 4.3	Figure 4.4	Figure 4.5	Figure 4.6
BayesShrink	0,8439	0,8738	0,8535	0,8817	0,8513	0,8502
VisuShrink	0,8278	0,8612	0,8325	0,8710	0,8377	0,8465
Proposed Algorithm (LSBEA)	0,9898	0,9870	0,9960	0,9336	0,9514	0,9998

From table 4.2 it can be seen that best CII values are obtained using the proposed algorithm. It is obviously seen that the CII value for Figure 4.6 computed as 0,9998 has the best value in the others. CII values of BayesShrink are better than VisuShrink but the difference is small compared to the difference with the proposed enhancement algorithm. The worst CII result yielded in Figure 4.1 for VisuShrink. That means the contrast difference of the problematic area and the surrounding tissue is the lowest at Figure 4.1 for VisuShrink. For the same figure proposed method provided much better contrast difference with a CII value of 0,9898.

It is important to notice that an objective comparison of the performance of different CAD methods is generally not easy because of the usage of different databases as The Mammographic Image Analysis Society: Mammography database [62], Washington University Digital Mammography Database [66], Digital Database for Screening Mammography [67], Nijmegen Digital Mammogram Database [68] and

UCSF/LLNL Digital Mammogram Library [69]. Even if a common database is used to test different methods, it could not guarantee that the comparison is valid and just. Best visual quality control must be done with the help of radiologists.

The provided results are discussed with the head of major discipline of radiology Prof. Dr. Metin BAYRAM from Medicine Faculty, University of Gaziantep in order to have a visual quality control. He told that it could be helpful especially to find the edges of masses for the evaluation of the digital mammograms. As a result, the output image of the LSBEA can give a second opinion to the radiologists for a second investigation of the digital mammogram.

CHAPTER 5

CONCLUSIONS AND FUTURE WORKS

In this thesis, we have introduced a modified tumor detection algorithm using Lifting Scheme of wavelet transform and mathematical morphology. In literature, wavelet Transform is used to denoising or segmentation of digital mammograms. In this study, it is used lifting scheme of wavelet transform for denoising and segmentation, firstly. Lifting Scheme provides easy perfect reconstruction. As a result, the proposed algorithm LSBEA provides a fast and easy way for the enhancement and segmentation of the digital mammograms. The lifting scheme based enhancement algorithm has provided superior image quality in enhancement step;. in the segmentation phase, the problematic areas defined clearly. It is concluded that the new algorithm LSBEA gives higher Peak Signal to Noise Ratio (PSNR) and Contrast Improvement Index (CII), values, quantitatively and better results visually for the tumor detected areas. Also these results are discussed with the head of major discipline of radiology, Medicine Faculty, University of Gaziantep. His opinion is that it could be helpful to find the edges of masses for the evaluation of the digital mammograms.

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As a future work, it can be studied to investigate the possibility of automatic or semi-automatic detection and classification of masses and image processing for regions of interest.

Currently, breast ultrasound imaging becomes an important adjunct to mammography in mass detection. As a further study, the breast ultrasound images will be used for the enhancement and segmentation by modifying the algorithm.

More generally, this study can be extended through for MRI images specifically brain MRI.

PUBLICATIONS

1. B. Tüysüz and G. Tohumođlu, “Breast Tumor Detection Using Lifting Scheme Wavelet Transform and Mathematical Morphology” 5th International Conference on Electrical and Electronics Engineering-ELECO’07- Bursa-Turkey (Accepted June 16, 2007).

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