GAZİANTEP UNIVERSITY GRADUATE
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INVESTIGATION OF
A NEW METHOD FOR DIRECT
IIR DIGITAL FILTER REALIZATION
USING FLATTENED COEFFICIENTS

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Investigation of a New Method for Direct IIR Digital Filter Realization Using Flattened Coefficients

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Supervisor

Prof. Dr. Arif NACAROĞLU

by

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Dedicated to

BERKAY , BERKE

and GÜLTEN

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This is to certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Doctor of Philosophy.

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ABSTRACT

INVESTIGATION OF A NEW METHOD FOR DIRECT IIR DIGITAL FILTER REALIZATION USING FLATTENED COEFFICIENTS

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A new direct method for the conversion of FIR transfer function to IIR transfer function using flattened coefficients is presented. The non-linear equations are obtained by the long division coefficients of the IIR transfer function. The given coefficients of the FIR filter are equated to these equations and solved by using the non-linear system solution algorithms. To reduce the error between the IIR filter and given FIR filter, an error minimization algorithm is used and the presented method is compared with the other approximation methods. The effects of binary flattened coefficients are investigated.

Keywords: FIR filter, IIR filter, long division

ÖZET

DÜZLEŞTİRİLMİŞ KATSAYILI IIR FİLTRELERİN DOĞRUDAN GERÇEKLEŞTİRİLMESİ İÇİN YENİ BİR METOT İNCELEMESİ

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Düzleştirilmiş katsayılarla FIR transfer fonksiyonunun, yeni doğrudan bir metot ile IIR transfer fonksiyonuna dönüştürülmesi anlatıldı. IIR transfer fonksiyonun katsayılarının uzun bölünmesi ile lineer olmayan denklemler elde edildi. Verilmiş FIR filtrelerinin katsayıları bu denklemlere eşitlendi ve lineer olmayan sistem çözüm algoritmaları ile çözüldü. FIR filtre ile IIR filtre arasındaki hatayı azaltmak için, hata azaltma algoritmaları kullanıldı ve sunulan bu yöntem diğer yöntemlerle karşılaştırıldı. İkili düzleştirilmiş katsayıların etkileri de araştırıldı.

Anahtar Kelimeler: IIR filtre, FIR filtre, uzun bölme

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LIST OF SYMBOLS

IIR	Infinite Impulse Response
FIR	Finite Impulse Response
DSP	Digital Signal Processor
DAC	Digital to analog converter
w_c	Transmitted highest frequency
$V_i(s)$	Laplace transforms of the input
$V_0(s)$	Laplace transforms of the output
$N(s)$	Nominator of transfer function
$L(w^2)$	Loss characteristic
$H_N(s)$	Normalized transfer function
$\theta(w)$	Phase characteristic
$\tau(w)$	Delay characteristic
OPAMP	Operational Amplifier
OTA	Operational Transconductance Amplifier
MOS	Metal-oxide semiconductor
$V_0(s)$	Laplace transforms of the input voltage
$V_i(s)$	Laplace transforms of the output voltage
$H(s)$	Transfer function
$A(w)$	The loss (or attenuation) of the filter
$\theta(w)$	Phase characteristic
$\tau(w)$	Delay characteristic
$H_A(s)$	Analog filter in s-domain
$H_D(z)$	Digital Filter in z-domain
$H_A(t)$	Analog filter in time domain
Ω	Frequency variable in the analog filter
Ω_p	The passband edge

$w(nT)$	A window function
$M_0(w)$	Amplitude response
H	Hessien Matrix
D	Differential operator
ξ_k	Unknown coefficients
β_j	Rate constants
ARMA	Autoregressive moving average
$E(w)$	Error function
$M_q(w)$	Amplitude response with quantization
h_i	Denotes the modified impulse response
ξ_k	Unknown coefficients
LSI	Least-square Inverse
$D(w)$	Desired frequency response
$W(w)$	Weighting function
$M_q(w)$	Amplitude response with quantization

CHAPTER 1

1. INTRODUCTION

Filtering is a process by which the frequency spectrum of a signal can be modified, reshaped, or manipulated according to some desired specification. It may entail amplifying or attenuating a range of frequency components, rejecting or isolating one specific frequency component, etc. The uses of filtering are manifold, e.g., to eliminate contamination such, to remove signal distortion brought about by an imperfect transmission channel or by inaccuracies in measurement, to separate two or more distinct signals which were purposely mixed in order to maximize channel utilization, to resolve signals into their frequency components, to demodulate signals, to convert discrete-time signals into continuous-time signals, and to bandlimit signals.

The digital filter is a digital system that can be used to filter discrete-time signals. It can be implemented by means of software (computer programs), dedicated hardware, or a combination of software and hardware. Software digital filters may be implemented using a low-level language on a general purpose digital processing chip or in terms of a high-level language on a personal computer or workstation. Both software and hardware digital filters can be used to process real time or non-real-time (recorded) signals, except that the latter are usually much faster and process signals whose frequency spectra extend to much higher frequency.

In the digital signal processing, in general, the digital filters are realized in two different ways [1]. Depending on its pole locations, these filters are called as Recursive (Infinite Impulse Response-IIR) or Nonrecursive (Finite Impulse Response-FIR). The main difference between these two filter types is that the poles of the FIR filters are located at the origin of the complex domain which guarantees the stability of this type of filters. Comparing the IIR realization, FIR realization is more convenient for the numerical design methods and relatively easier. However, one and most important advantage of the IIR filter is its less polynomial degree which requires less digital elements such as delay, multiplier and adder. In IIR filters, the approximation problem is usually solved through indirect methods. First, a continuous-time transfer function that satisfies certain specifications is obtained using the standard analog-filter approximations. Then obtained continuous time transfer function is transformed into the z-domain using any transformation methods presented in the later chapter of this thesis.

IIR filters are useful for a high-speed design because they typically require a lower number of multipliers compared to other type of digital filter [2]. Unfortunately, it is hard to design linear phase IIR filter and they can be unstable if not designed properly. IIR filters also are very sensitive to filter coefficient quantization errors that occur due to using a finite number of bits to represent the filter coefficients. Ones the required transfer function is found as IIR filter, because of its lower degree versus FIR degree; it is much faster to find the output samples of the filtered data. The output calculation softwares use the simple multiplications and addition operations.

The conventional techniques which are used to design a nonrecursive filters are well-known approximation techniques. The mostly used approximation method is to the Fourier Series Approximation. The given magnitude characteristics of the design filter is decomposed in the form of the Fourier Series and these coefficients are equated to the FIR coefficients after some simple mathematical operations. The best approximation requires infinitely long series representation which needs infinite number of circuit elements. Since it is impossible and very expensive, the length of series is adjusted either by truncation, which is well known to be Rectangular windowing or truncation and correction is applied at the same time, which are known to be general windowing of the Fourier Series Approach.

FIR filters have low sensitivity to filter coefficients quantization errors. This is an important property to have when implementing a filter a Digital Signal Processor or on an integrated circuit. FIR filters have the advantage of stability but usually have more coefficients and delay than the IIR filters for the same filter specification, that the required order in FIR filter can be as high as 5 to 10 times that in IIR filter.

In this Thesis, a new direct method for the conversion of FIR transfer function to IIR transfer function with flattened coefficients is presented. It is a direct realization method for IIR filters that only the designed FIR filter coefficients are used for IIR filter approximation. The long division process is used to find an FIR filter whose coefficients can be obtained directly from the coefficients of numerator and denominator of the IIR filter. Although the long division results are infinitely long series and hence infinite number of equations may be obtained, only dominant equations as much as the number of unknowns (coefficients of IIR) are used.

Non-linear equations obtained by the long division of IIR transfer function with the given coefficients of the FIR filter are solved by using the non-linear system solution algorithms. The large-scale algorithms are used for solving these non-linear equations. The solution of these non-linear equations gives the coefficients of IIR filter. Non-linear equation are be solved by using the Trust-region Dogleg method. IIR filter coefficients can be easily calculated from a given FIR filter the coefficients. The error between the FIR and IIR filter can be measured with en error function. In this study, a new algorithmic approach is used to reduce the error between the given FIR filter characteristics and approximated IIR equivalent.

The number of equations increases, the time for solving the nonlinear equations increase, but the error decreases. We use the other terms of the long division results for the new equations. The initial conditions or starting points of the unknown coefficients are changed before solving new set of nonlinear equations to approach result with less iteration.

The presented method is compared with the other approximation methods like Prony's [3] and Yule-Walker [4,5] method for the magnitude characteristics of lowpass, highpass, bandpass. Magnitude and Phase responses of the filters are compared and errors are calculated against the FIR filter. The coefficient quantization effects are investigated in this study.

In Chapter 2 introduces the types of filters. The two types of Digital Filters, Recursive and Nonrecursive filters are described in this chapter. Approximations for recursive filters are described by which a given continuous-time transfer function, e.g., impulse-invariant-response and bilinear transformation methods. Nonrecursive filter approximation methods are described in detail. Finite word length is investigated for digital filters. Coefficient quantization by MATLAB is described in this chapter.

In Chapter 3 methods for conversion FIR to IIR filters are represented. Optimization methods for approximation of recursive and nonrecursive filter are introduced e.g., Prony's Method, Yule-Walker Method.

In Chapter 4, a new direct method for the conversion of FIR transfer function to IIR transfer function is presented. The obtained coefficients are flattened by using roundation techniques to simplify the circuit structure.

In Chapter 5, this method is applied for different type of FIR filter. The IIR filter coefficients are obtained with the acceptable error and compared with other conversion methods. Phase and Magnitude and response of filters are plotted. The results are calculated with the help of MATLAB program in Appendix.

CHAPTER 2

2. LITERATURE SUMMARY

2.1. Introduction

Signals arise in almost every field of science and engineering, e.g., in acoustics, biomedical engineering, communications, control systems, radar, physics, seismology, and telemetry. Two classes of signals can be identified, namely, continuous-time and discrete-time signals.

Continuous-time signals are one that is defined at each and every instant of time. Typical examples are a voltage waveform and the velocity of space vehicle as a time function. A discrete-time signals, on the other hand, is one that is defined at discrete instants of time, perhaps every millisecond, second, or day. Examples of this type of signal are the closing price of a particular commodity on stock exchange and daily precipitation as functions of time.

A discrete-time signal, like a continuous-time signal, can be represented by a unique function of frequency referred to as the frequency spectrum of the signal. This is a description of the frequency content of the signal.

Frequency spectrum of a signal can be modified, reshaped, or manipulated according to some desired specification. It may entail amplifying or attenuating a range of frequency components, rejecting or isolating one specific frequency component, etc. The uses of filtering are manifold, e.g., to eliminate contamination such, to remove signal distortion brought about by an imperfect transmission channel or by inaccuracies in measurement, to separate two or more distinct signals which were purposely mixed in order to maximize channel utilization, to resolve signals into their frequency components, to demodulate signals, to convert discrete-time signals into continuous-time signals, and to bandlimit signals [2].

2.2. Types of Filters

There are two main kinds of filter, analog and digital. They are quite different in their physical makeup and in how they work.

An analog filter uses analog electronic circuits made up from components such as resistors, capacitors and op amps to produce the required filtering effect. Such filter circuits are widely used in such applications as noise reduction, video signal enhancement, graphic equalizers in hi-fi systems, and many other areas.

There are well-established standard techniques for designing an analog filter circuit for a given requirement. At all stages, the signal being filtered is an electrical voltage or current which is the direct analogue of the physical quantity (e.g. a sound or video signal or transducer output) involved.

A digital filter uses a digital processor to perform numerical calculations on sampled values of the signal. The processor may be a general-purpose computer such as a PC, or a specialized DSP (Digital Signal Processor) chip.

The analog input signal must first be sampled and digitized using an ADC (analog to digital converter). The resulting binary numbers, representing successive sampled values of the input signal, are transferred to the processor, which carries out numerical calculations on them. These calculations typically involve multiplying the input values by constants and adding the products together. If necessary, the results of these calculations, which now represent sampled values of the filtered signal, are output through a DAC (digital to analog converter) to convert the signal back to analog form. Note that in a digital filter, the signal is represented by a sequence of numbers, rather than a voltage or current [1].

2.3. Digital Filters

The digital filter is a digital system that can be used to filter discrete-time signals. It can be implemented by means of software (computer programs), dedicated hardware, or a combination of software and hardware. Software digital filters may be implemented using a low-level language on a general purpose digital processing chip or in terms of a high-level language on a personal computer or workstation. At the other extreme, hardware digital filters can be designed using a number of highly specialized interconnected VLSI chips. Both software and hard ware digital filters can be used to process real time or non-real-time (recorded) signals, except that the latter are usually much faster and process signals whose frequency spectra extend to much higher frequency.

2.3.1. Recursive Filters

A Recursive Filter or an Infinite Impulse Response (IIR) Filter is one whose impulse response is infinite duration. The response of a recursive filter is a function of elements in the excitation as well as the response sequence. In this case of a linear, time variant, causal filter

$$y(nT) = \sum_{i=0}^{N} a_i x(nT - iT) - \sum_{i=1}^{N} b_i y(nT - iT) \qquad (2.1)$$

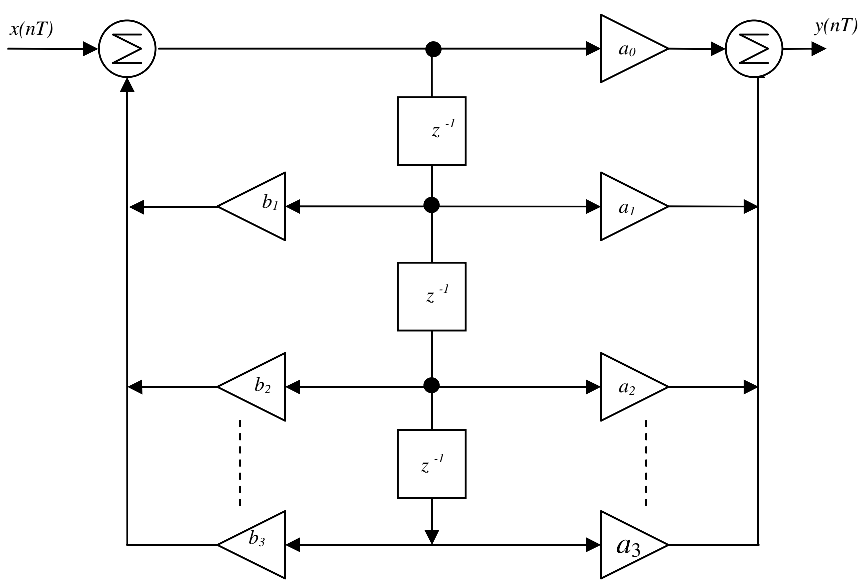

Figure 2.1. IIR filter structure

IIR filters are useful for a high-speed design because they typically require a lower number of multipliers compared to other type of digital filter. IIR filters can also be designed to have a frequency response that is a discrete version of the frequency response of an analog filter.

Unfortunately, IIR filters do not have linear phase and they can be unstable if not designed properly. IIR filters also are very sensitive to filter coefficient quantization errors that occur due to using a finite number of bits to represent the filter coefficients.

2.3.1.1. Approximations for Recursive Filters

In IIR filters, the approximation problem is usually solved through indirect methods. First, a continuous-time transfer function that satisfies certain specifications is obtained using the standard analog-filter approximations such as Butterworth, Chebyshev, Inverse-Chebyshev, Eliptic, and Bessel approximations.

Then corresponding discrete-time transfer function is obtained using one of the following transformation methods [6-10].

2.3.1.1.1. Invariant-Impulse Response Method

$H_A(s)$ is given analog filter, a corresponding digital filter, represented by $H_D(z)$, can be derived by using the following procedure [4].

-Deduce $H_A(s)$, the impulse response of the analog filter

-Replace t by nT in $H_A(t)$

-Form the z transform of $H_A(nT)$

2.3.1.1.2. Matched-z Transformation Method

An alternative approximation method for design of recursive filters is so called matched-z-transformation method [9]. The method is simple to apply and given reasonable results for highpass and bandstop filters, but intent to increase the amplitude of the passband error relative to that of analog filter.

In this method, each of the poles and zeros of analog filters are mapped directly from the s-plane to z-plane using the following equation

$$(s-a) \rightarrow (1-z^{-1}e^{aT}) \qquad (2.2)$$

2.3.1.1.3. Bilinear Transformation Method

The transfer function can be obtained by the Bilinear Transformation like

$$H_D(z) = H_A(s)\ \Big|_{s=\frac{2}{T}\left(\frac{z-1}{z+1}\right)} \tag{2.3}$$

Let w and Ω represents the frequency variable in the analog filter and the derived digital filter,

$$H_D(e^{j\Omega T}) = H_A(jw) \tag{2.4}$$

provided that

$$w = \frac{2}{T}\tan\frac{\Omega T}{2} \tag{2.5}$$

and, as a result, the digital filter the same frequency response as the analog filter. For higher frequencies, however, the relation between w and Ω becomes nonlinear, and there is a distortion for the high frequencies. This is known as the warping effect [11,12]. If only the amplitude response is of concern, the warping effect can for all practical purposes be eliminated by prewarping the analog filter [1].

2.3.1.2. Digital Filter Transformations

A normalized lowpass analog filter can be transformed to a denormalized lowpass, highpass, bandstop, or band pass filter employing the transformations shown in Constantinides [13].

The Constantinides transformations can be readily applied to filters with prescribed passband edges. The following procedure can be employed

1. Obtain a lowpass transfer Function $H_N(z)$ using approximation methods
2. Determine the passband edge Ω_p in $H_N(z)$.
3. Form $H_N(\bar{z})$ according to appropriate transformations.

2.3.2. Nonrecursive Filters

A nonrecursive Filter or a Finite Impulse Response (FIR) digital filter is one whose impulse response is of finite duration. FIR filters are simple to design and they are guaranteed to be bounded input-bounded output stable. By designing the filter tabs to be symmetrical about the center tap position, a FIR filter can be guaranteed to have linear phase. This is desirable property for many applications such as music and video processing.

The input-output of a nonrecursive filter can be expressed as

$$y(nT) = \sum_{i=0}^{N} a_i x(nT - iT) \tag{2.6}$$

This equation can be realized by many different topologies. The one of which is known to be canonic is shown in Figure 2.2.

FIR filters also have low sensitivity to filter coefficients quantization errors. This is an important property to have when implementing a filter a Digital Signal Processor or on an integrated circuit.

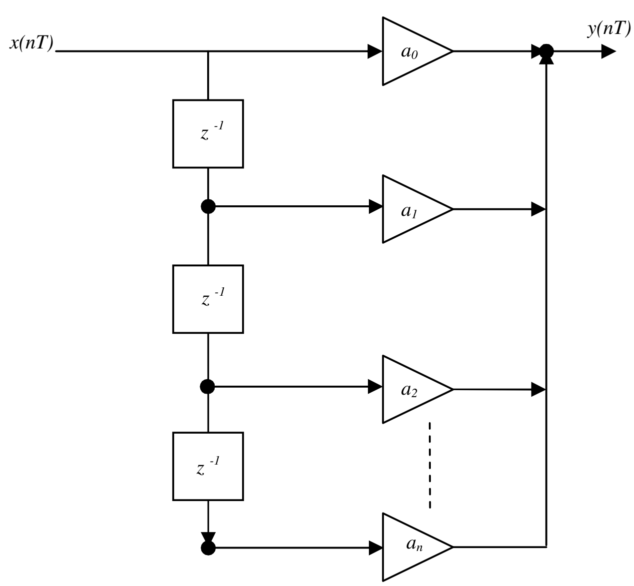

Figure 2.2. FIR filter structure

The approximation problem in non-recursive filter can be solved by using Fourier series or numerical analysis formula. The details of these methods are examined in this chapter. An alternative approach is based on the use of the Discrete Fourier Transforms. A third possibility is to use a powerful multivariable optimization algorithm known as Remez Exchance Algorithm.

The methods of this chapter are in terms of closed-form solutions an, as a result, they are straightforward and involve a minimal amount of computation. Unfortunately, the designs obtained are suboptimal; that is, the required filter order to satisfy a set of prescribed specifications is not minimum. On the other hand, the use of the Remez Exchance Algorithm yields optimal designs, but a large amount of computation is required to complete a design, as may be expected.

2.3.2.1. Approximation for Nonrecursive Filter

In nonrecursive filters, the approximation problem is solved through direct methods which can involve the application of Fourier series, window functions, numerical analysis formulas, or discrete-Fourier transform.

A nonrecursive causal filter can be characterized by the transfer functions

$$H(z) = \sum_{n=0}^{N-1} h(nT) z^{-n} \tag{2.7}$$

Its frequency response is given by

$$H(e^{jwT}) = M(w) e^{j\theta(w)} = \sum_{n=0}^{N-1} h(nT) e^{-jwnT} \tag{2.8}$$

where $$M(w) = \left| H(e^{jwT}) \right| \tag{2.9}$$

and $$\theta(w) = \arg H(e^{jwT}) \tag{2.10}$$

The phase and group delays of a filter are given by

$$\tau_p = -\frac{\theta(w)}{w} \quad \text{and} \quad \tau_g = -\frac{d\theta(w)}{dw} \tag{2.11}$$

For constant phase delay as well as group delay, the phase response must be linear,

$$\theta(w) = -\tau w \tag{2.12}$$

and thus from Eq.(2.8) and Eq.(2.10)

$$\theta(w) = -\tau \; w = \tan^{-1} \frac{-\sum_{n=0}^{N-1} h(nT)\sin wnT}{\sum_{n=0}^{N-1} h(nT)\cos wnT} \tag{2.13}$$

Consequently

$$\tan \; w\tau = \frac{\sum_{n=0}^{N-1} h(nT)\sin wnT}{\sum_{n=0}^{N-1} h(nT)\cos wnT} \tag{2.14}$$

and accordingly

$$\sum_{n=0}^{N-1} h(nT)(\cos wnT \sin wn\tau - \sin wnT \cos wn\tau) = 0 \tag{2.15}$$

or

$$\sum_{n=0}^{N-1} h(nT)\sin(w\tau - wnT) = 0 \tag{2.16}$$

The solution of this equation can be shown to be

$$\tau = \frac{(N-1)T}{2} \tag{2.17}$$

$$h(nT) = h[(N-1-n)T] \quad \text{for } 0 \le n \le N-1 \tag{2.18}$$

Therefore, a nonrecursive filter, unlike a recursive filter, can have constant phase and group delays over the entire baseband [1, 2].

2.3.2.2. Design Using the Fourier Series

As the frequency response of a nonrecursive filter is a periodical function of w with period w_s, it can be expressed as Fourier series. We can write

$$H(z)=\sum_{n=-\infty}^{\infty} h(nT)e^{-jwnT} \tag{2.19}$$

where $$h(nT)=\frac{1}{w_s}\int_{-w_s/2}^{w_s/2} H(e^{jwT})e^{jwnT}dw \tag{2.20}$$

and if $e^{jwT}=z$ Eq.(1) gives (2.21)

$$H(z)=\sum_{n=-\infty}^{\infty} h(nT)z^{-n} \tag{2.22}$$

Hence with an analytic representation for the frequency response available, a corresponding transfer function can be readily derived. Unfortunately, however, this is noncausal and of infinite order. For a finite-order transfer function, the series of Eq.(2.22) can be truncated by assigning

$$h(nT)=0 \qquad \text{for} \quad |n|>\frac{N-1}{2} \tag{2.23}$$

in which case

$$H(z)=h(0)+\sum_{n=1}^{(N-1)/2}\left[h(-nT)z^{n}+h(nT)z^{-n}\right] \tag{2.24}$$

Causality can be brought about by multiplying $H(z)$ by $z^{-(N-1)/2}$ so that

$$H^{'}(z)=z^{-(N-1)/2}H(z) \tag{2.25}$$

This modification is permissible since the amplitude response will remain unchanged and the group delay will be increase by a constant $(N-1)T/2$.

Note that if $H(e^{jwT})$ in Eq.(2.20) is even function of *w*, then the impulse response obtained is symmetrical about *n*=0, and hence the filter zero group delay [1]. Consequently, the filter represented by the transfer function of Eq.(2.25) has constant group delay equal to $(N-1)T/2$.

2.3.2.3. Use of Window Functions

The passband and stopband oscillations observed are due to slow convergence in the Fourier series, which in turn, is causal by the discontinuity at the passband edge. These are known as Gibbs' oscillations. As N increases, the frequency of these oscillations is seen to increase, and at both low and high frequencies their amplitude is decreased.

An alternative and easy-to-apply technique for reduction of Gibbs' oscillations is to precondition $h(nT)$ as given by Eq.(2.20) using a class of time domain functions known as window functions. Let

$$H(z) = Zh(nT) = \sum_{n=-\infty}^{\infty} h(nT) z^{-n} \tag{2.26}$$

$$W(z) = Zw(nT) = \sum_{n=-\infty}^{\infty} w(nT) z^{-n} \tag{2.27}$$

$$H_w(z) = Z[w(nT)h(nT)] \tag{2.28}$$

where $w(nT)$ represents a window function.

The windowing causes the change in the amplitude characteristics of the FIR filter but linear phase feature of the filter unchanged. The most frequently used window functions are Rectangular, Blackman, Dolph-Chebyshev, Kaiser.

2.3.3. Design of IIR Filters Using Optimization Methods

An alternative method approach for the solution of the approximation problem in digital filters is through the application of optimization methods [14,15]. In these methods, a discrete-time transfer function is assumed and an error function is formulated on the basis of some desired amplitude and/or phase response. A norm of the error function is then minimized with respect to the transfer-function coefficients. As the value of the norm approached zero, the resulting amplitude or phase response approaches, the desired amplitude or phase response. These methods are iterative and, as a result, they usually involve a large amount of computation. However, unlike the closed-form methods, they are suitable for the design of filters having arbitrary amplitude or phase response.

The application of optimization method of the design of recursive filters digital filters is considered. There is different kind of algorithms, such as quasi-Newton method which has been explored by Davidov, Fletched, Powell, Broyden and others [1].

Formulation

Assume that the amplitude response of a recursive filter is required to approach some specified amplitude response as closely as possible. Such a filter can be designed in two general steps, as follows:

1. An objective function which is dependent on the difference between the actual and specified amplitude response is formulated.
2. An objective function obtained is minimized with respect to the transfer function coefficients

An *N*th order recursive filter can be represented by the transfer function

$$H(z) = H_0 \prod_{j=1}^{N/2} \frac{a_{0j} + a_{1j} z + z^2}{b_{0j} + b_{1j} z + z^2} \tag{2.29}$$

where a_{ij} and b_{ij} are real coefficients, H_0 is positive multiplier constant. The amplitude response of the filter can be expressed as

$$M(x, w) = \left| H(e^{jwT}) \right| \tag{2.30}$$

where $$x = \left[a_{01}, a_{11}, b_{01}, b_{11}, \ldots. b_{1j}, H_0 \right]^T \tag{2.31}$$

is a column vector with $4J+1$ elements and *w* is the frequency.

Let $M_0(w)$ be the specified amplitude response and, for the sake of exposition, assume that it is piecewise continuous, as illustrated in Fig.2.3. The difference between $M(x,w)$ and $M_0(w)$ is, in effect, the approximation error and can be expressed as

$$e(x,w) = M(x,w) - M_0(w)$$

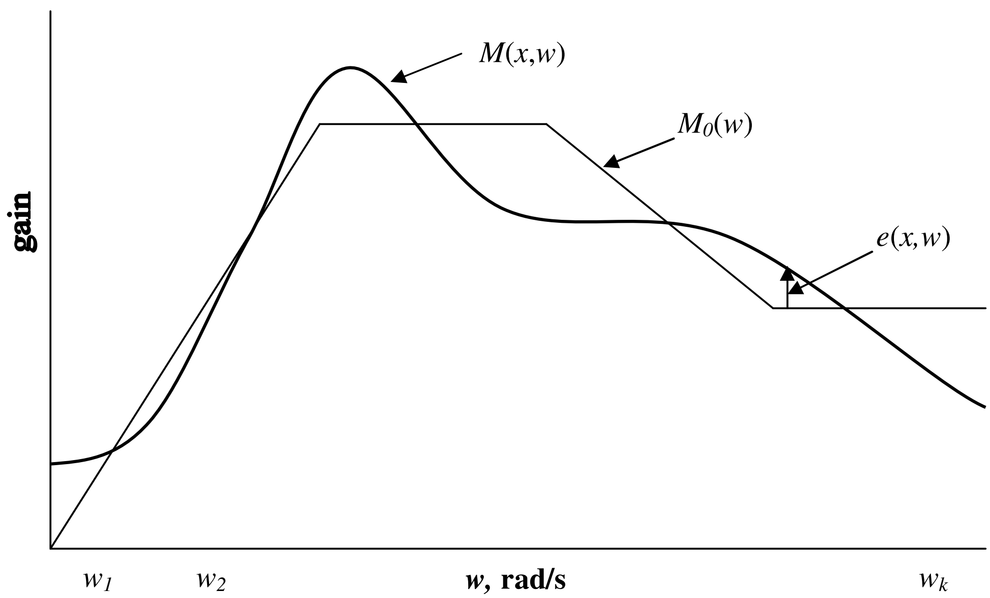

Figure 2.3. Formulation of error function

By sampling $e(x,w)$ at frequencies $w_1,w_2,\ldots.w_k$, as depicted in Figure 2.7, the column vector

$$E(x) = [e_1(x), e_2(x), \ldots\ldots\ldots e_k(x)]^T \tag{2.32}$$

and can be formed where

$$e_i(x) = e(x,w_i) \qquad \text{for } i = 1,2,\ldots.,K. \tag{2.33}$$

The approximation problem at hand can be solved by finding

$$e_i(\breve{x}) \approx 0 \tag{2.34}$$

2.3.4. Design of FIR Using Optimization Methods

The design of nonrecursive filters known as the weigthed-Chebyshev method. In this Method, an error function is formulated for the desired filter in the terms of linear combination of cosine functions and is then minimized by using a very efficient multivariable optimization algorithm known as the Remez Exchange Algorithm. When convergence is achieved, the error function becomes equiripple, as in other types of Chebyshev solutions. The amplitude of the error in different frequency bands interest is controlled by applying weighting the error function.

The weigthed-Chebbyshev method is very flexible and can be used to obtain optimal solutions for most non-recursive filters, e.g.., digital differentiators, Hilbert transformers, and lowpass, highpass, bandpass, bandstop, and multiband filters with piecewise-constant amplitude responses.

The development of the weighted-Chebyshev method began with a paper by Herrmann published in 1970 [16], which was followed soon after by a paper by Hofstetter, Oppenheim and Siegel [17]. These contributions were followed by a series of papers, during the seventies, by Parks, McClellan-Parks-Rabiner, and Herrmann [18-21]. These developments led, in turn, to the well-known McClellan-Parks- Rabiner computer program for the design of nonrecursive filters, documented in [22], which has found widespread applications.

The Remez Exchange Algorithm is an iterative multivariable algorithm which is naturally suited for solution of the minimax problem in Eq.(2.87). It is based on the second optimization method for [23] and involves the following basic steps:

2.4. Effects of Word Length in Digital Filters

In software as well as hardware digital filter implementations, numbers are stored in finite-length registers. Consequently, if coefficients and signal values cannot be accommodated in available registers, they must be quantized before they can be stored. Number quantization gives rise to three types of errors:

1. Coefficient-quantization errors
2. Product-quantization errors
3. Input-quantization errors

The transfer-function coefficients are normally evaluated to a high degree of precision during the approximation step. If coefficient-quantization is applied, the frequency response of the resulting filter may differ appreciably from the desired response, and if the quantization step is coarse, the filter may actually fail to meet the desired specifications.

Product-quantization errors arise at the output of multipliers. Each time a signal represented by a digit is multiplied by a coefficient represents. Since a uniform register length must, in practice, be used throughout the filter, each multiplier output must be rounded or truncated before processing can continue.

Input-quantization errors arise in applications where digital filters are used to process continuous-time signals. These are the errors inherent in the analog-to-digital conversion process.

2.4.1. Coefficient Quantization

Coefficient-quantization errors introduced perturbations in the zeros and poles of the transfer function, which in turn manifest themselves as errors in the frequency response. Product-quantization errors, on the other hand, can be regarded as noise source which gives rise to output roundoff noise. Since the importance of the two types of errors can vary considerably from application to application, it is frequently advantageous to use different word lengths for the coefficients and signal values. The coefficient word length can be chosen to satisfy a signal-to-noise ratio specification. Consider a digital filter characterized by $H(z)$ and let

$M(w) = \left|H(e^{jwT})\right|$ = amplitude response without quantization

$M_q(w)$ = amplitude response with quantization

The effect of coefficient quantization is to introduce an error in $M(w)$ given

$$\Delta M = M(w) - M_q(w) \tag{2.35}$$

The optimum word length can thus determined exactly by evaluating $|\Delta M|$.

An alternative approach is to employ a statistical method proposed by Avenhause [24,25]. This method yields a fairly accurate estimate of the required word length and is, in general, more efficient than the exact method outlined above.

A different approach for study of quantization effects are proposed by Jenkins and Leon [26]. In this approach a computer-aided analysis scheme is used to generate confidence-interval error bound on the time domain response of the filter. This method can be used to study the effects of coefficient or product quantization in fixed-point or floating-point implementations. Furthermore, the quantization can be by rounding or truncation.

2.4.2. Coefficient Quantization with MATLAB

One important issue that must be considered when IIR filters are implemented on a fixed-point processor is that the filter coefficients that are actually used are quantized from the "exact" (high-precision floating point) values computed by MATLAB. Although quantization was not a concern when we worked with FIR filters, it can cause significant deviations from the expected response of an IIR filter.

By default, MATLAB uses 64-bit floating point numbers in all of its computation. These floating point numbers can typically represent 15-16 digits of precision, far more than the DSP can represent internally. For this reason, when creating filters in MATLAB, we can generally regard the precision as "infinite," because it is high enough for any reasonable task.

The DSP, on the other hand, operates using 16-bit fixed-point numbers in the range of -1.0 to $1.0-2^{-15}$. This gives the DSP only 4-5 digits of precision and only if the input is properly scaled to occupy the full range from -1 to 1.

It is not difficult to use MATLAB to quantize the filter coefficients to the 16-bit precision used on the DSP. To do this, first take each vector of filter coefficients (that is, the *A* and *B* vectors) and divide by the smallest power of two such that the resulting absolute value of the largest filter coefficient is less than or equal to one. This is an easy but fairly reasonable approximation of how numbers outside the range of -1 to 1 are actually handled on the DSP.

Next, quantization in Matlab is achieved as first, quantizing the resulting vectors to 16 bits of precision by first multiplying them by 2^{15}=32768, rounding to the nearest integer (use `round`), and then dividing the resulting vectors by 32768. Then the resulting numbers are multiplied, which will be in the range of -1 to 1, back by the power of two that you divided out.

CHAPTER 3

3. RECENT METHODS FOR FIR TO IIR CONVERSION

Non-recursive digital filters (FIR) have the advantage of stability, but usually have more coefficients and delay than the IIR filters with the same specifications (impulse response or frequency response). For the same specifications the required order in a FIR filter can be as high as 5 to 10 times that the IIR filter. The need for recursive digital filters (IIR) arises from the usage of small digital computers and processors.

The problem of conversion from a FIR filters to reduced order IIR filters has surfaced from time to time in signal processing literature for a number of years [27-29]. IIR approximation of linear-phase FIR filters has received particular attention, especially in the context of the delay equalization.

One of the many possible avenues to FIR filters approximations via balanced state-space realization. This method of dynamic system reduction has been a big attraction in certain reaches of system theory for the past decade, with a lot of attention on focusing on state-space reduction [30,31]. The state-space realization ought to be in the balance form that is such that the space-space realization has controllability and observability grammians equal and diagonal.

In filter design and approximation, the concept of balance realization seems to have rarely used [32], although one especially pertinent application of concept may be found in [33]. A usual recipe for the reduction of a transfer function order is the following: Convert the transfer function into state-space form, calculate controllability and observability grammians, take the square roots of the singular values of their product (resulting in “the Hankel singular values”), determine necessary similarity transformation and balanced system realization, inspect the Hankel singular values and decide upon the reduced order state-space form into a transfer function if required. The original and reduced order system can then be compared against various criteria and a new order of approximation attempted if necessary. The primary principle used for system reduction lies in the elimination of subsystem associated with the smallest Hankel singular values.

Several of today's commercial software packages like MATLAB, C^{++} can be used to undertaken the various steps needed in the model reduction problem algorithms. Yule-Walker and Prony's Method is other algorithms in MATLAB.
There are several methods for FIR to IIR conversion. These are;

3.1. Yule-Walker Method

In recent years there has renewed interest in the development and application of autoregressive moving average (ARMA) techniques for high resolution spectral estimation. Some of these techniques are based on maximizing a likelihood function, using some iterative nonlinear optimization procedure [4,5]. Another class of techniques is based on estimation of autoregressive (AR) and moving average (MA) parameters in two separate steps: the AR parameters are obtained as the solution of the so-called modified Yule-Walker (MYW) equations. MA is the spectral parameters.

Yule-Walker performs a least-squares fit in the time domain. It computes the denominator coefficients using Modified Yule-Walker equations, with correlation computed by inverse Fourier transformation of the specified response. To compute the numerators, Yule-Walker takes the following step:

1. Computes a numerator polynomial corresponding to additive decomposition of the power frequency response
2. Evaluates the complete frequency response corresponding to the numerator and denominator polynomials.
3. Uses spectral factorization technique to obtain the impulse response of the filter.
4. Obtains the numerator polynomial by a least-squares fit to this impulse response.

3.2. Prony's Method

Prony's method is a technique for extracting the sinusoid or exponential signals by solving a set of linear equations for the coefficients of the recurrence equation that the signals satisfy. It is closely related to Pisarenko's method, which finds the smallest eigenvalue of an estimated covariance matrix.

Prony's method is an algorithm for finding an IIR filter with a prescribed time domain impulse response. It has applications in filter design, exponential signal modeling, and system identification (parametric modeling) Prony's implements the method described in [3].

This method uses *a* variation of the covariance method of AR modeling to find the denominator coefficients b and then finds the numerator coefficients a for which the impulse response of the output filter matches exactly the first $n + 1$ samples of x. The filter is not necessarily stable, but potentially can recover the coefficients exactly if the data sequence is truly an autoregressive moving-average (ARMA) process of the correct order [34].

3.3. Least Square Inverse Method in Time Domain

The corresponding introduces a method to approximation an FIR filter by an IIR filter in the time domain (the same approach applies to IIR filter order reduction). The method based on the Least-square Inverse (LSI), algorithm, which yields a stable IIR filter [35]. Unlike the other methods, the numerator of the approximated filter is part of FIR filter itself and no calculations and minimizations are needed to find the numerator coefficients.

The proposed method is based on the LSI algorithm [36], which was used in a context related to IIR in [37].

3.4. Balance Realization Technique

Another method for conversion of FIR to IIR filter is Balance Realization technique. This method convert single-input/single –output FIR filters to corresponding IIR filter approximations, simultaneously (and hopefully substantially) reducing system order [38]. The starting point of this algorithm is a canonical controllable form of an FIR filter. By using Hankel Matrix factorization a simple formula for similarity transformation is derived, which leads to a balanced state-space realization. However, in order to avoid calculation of a possibly ill-conditioned balanced realization, a reduced state-space form of the system is used that is input/output equivalent to balanced system, but which is obtained without inverting matrices.

3.5. Weighted Least Square Approximation

This method is based on frequency weighted least-square error optimization using the Boyden-Flether-Goldfarb-Shanno (BFGS) method [39] The gradient of the cost function with respect to the design parameters, required for the implementation of the BFGS method, is derived. This method start by obtaining an initial IIR filter design using reduction of linear phase FIR filter. Based on this initial design the cost function is minimized using BFGS method.

An initial IIR filter design is obtained from a linear phase FIR filter satisfying the design specifications. In most cases, frequency specifications are violated first in the transition band during model reduction [40]. To improve the approximation and obtain a low order IIR filter which satisfies the design specifications, a frequency weighted error function is optimized. The error function between the original specifications and the frequency response of the IIR design is minimized using the BFGS method, which is the ‘best’ quasi-Newton optimization method.

3.6. Optimal Hankel-norm Approximation

This method is based on the dominant part of the singular value decomposition of the Hankel Matrix formed from the modified impulse response of the desired filter, and those state components which are weakly coupled to both input and output are discarded [28,29,41] to reduce the model. The major objective of the proposed design approach is to minimize the error between the order-reduced filter’s response and the desired one in the Hankel norm sense [42-44]. The designed filter is obtained via the results of singular-value decomposition of the Hankel matrix, balanced realization and all pass functions, all in terms of the solutions to Lyapunov equations.

CHAPTER 4

4. A NEW DIRECT METHOD FOR FIR TO IIR FILTER CONVERSION WITH FLATTENED COEFFICIENTS

The main aim of this study is to obtain the corresponding IIR transfer function using the coefficients of the given (or calculated) FIR transfer function. The degree of the transfer function is defined and limited numbers of its coefficients are set to be unknown. To calculate these unknowns, the required number of equations must be obtained. The derivation of these equations are easily achieved using IIR to FIR conversion. An Infinite Impulse Response (recursive) filter can be represented by the following transfer function in the z-domain:

$$H_{IIR}(z) = \frac{a_0 + a_1 z^{-1} + \ldots + a_{N-1} z^{-(N-1)} + a_N z^{-N}}{b_0 + b_1 z^{-1} + \ldots + b_{N-1} z^{-(N-1)} + b_N z^{-N}} \tag{4.1}$$

If at least one of the coefficients $b_i, i = 1,....,N-1$ in Eq.(4.1) is non-zero, the corresponding system has infinite impulse response (IIR). If, on the other hand, all of these coefficients are zero, then all poles of the transfer function will be located in the origin of the z-plane and the corresponding system has finite impulse response (FIR). The transfer function of a FIR filter can simplified as follows:

$$H_{FIR}(z) = c_o + c_1 z^{-1} + c_2 z^{-2} + \ldots + c_N z^{-N} + \ldots \tag{4.2}$$

It is very obvious that FIR transfer function is infinite power series of z^{-1} with real coefficients c_i's. There are some possible techniques to deduce Eq.(4.2) from Eq.(4.1), but the simplest one is to divide the numerator of Eq.(4.1) to its denominator by using polynomial division technique (long division). Hence c_i's may be written in terms of coefficients of the IIR transfer function.

4.1. FIR to IIR Conversion

The long division process can be used to find an FIR filter whose coefficients can be obtained directly from the coefficients of numerator and denominator of the IIR filter. Consider an IIR filter with a transfer function given by Eq.(4.1). If $b_0 = 1$, the coefficients of a FIR filter of the form Eq.(4.2) obtained from long division can be computed as

$$c_k = \begin{cases} a_k - \sum_{l=1}^{k} c_{k-l} b_l & k \le N \\ -\sum_{l=1}^{N} c_{k-l} b_l & k > N \end{cases} \tag{4.3}$$

which turns out to be a recursive equation [45]. It can be concluded from Eq.(4.2) that the coefficients of the FIR filter can be generated from the coefficients of the numerator and denominator of the IIR filter and at most *N* recently generated coefficients of the FIR filter. The recursion can continue to obtain a FIR filter of any arbitrary order, typically greater then *N*. More specifically, the recursion can continue until the error between the resultant FIR filter and the original IIR filter sufficiently small.

It should be noted that since the coefficients of the filter obtained from long division represent the samples of the impulse response, the stability of the filter implies that the samples converge to zero.

Theoretically, the FIR filter of Eq.(4.2) converges to the original IIR filter as the order of the resultant filter *N* goes to infinity. In general the infinity norm of the error between the original FIR and IIR filter will depend on the truncated terms in the long division. The differences of these filters' impulse response can be written like,

$$\left| H_{IIR}(e^{jwT}) - H_{FIR}(e^{jwT}) \right| = 0 \tag{4.4}$$

If FIR filter coefficients are realized using one of the approximation method given in Chapter 2. The simplicity of this process is directly related to the chosen order of IIR filter. Although the long division results are infinitely long series and hence infinite number of equations may be obtained, only dominant equations as much as the number of unknowns (coefficients of IIR) are used.

Eq.(4.3) can be written as a matrix multiplication using k+1 terms of impulse response[46],

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ a_N \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} c_0 & 0 & 0 & \cdots & 0 \\ c_1 & c_0 & 0 & & \vdots \\ c_2 & c_1 & c_0 & & \vdots \\ \vdots & \vdots & & \ddots & \vdots \\ c_N & c_{N-1} & & & \vdots \\ \vdots & \vdots & & & \vdots \\ c_k & c_{k-1} & \cdots & \cdots & c_{k-N} \end{bmatrix} \begin{bmatrix} 1 \\ b_1 \\ b_2 \\ \vdots \\ b_N \end{bmatrix} \tag{4.5}$$

to compute a_i and b_i let us partition the matrices as

$$\begin{bmatrix} a \\ --- \\ 0 \end{bmatrix} = \begin{bmatrix} & C_1 & \\ -- & -- & -- \\ C_k & & C_2 \end{bmatrix} \begin{bmatrix} 1 \\ --- \\ b \end{bmatrix} \tag{4.6}$$

a: Column vector of the N+1 numerator coefficients of Eq.(4.1)

b: Column vector of the N+1 denominator coefficients of Eq.(4.1)

C_k: Column vector of the kast k-N terms of the impulse response,

C_1: (N+1) by (N+1) partition of the matrix in Eq.(4.5),

C_2: (k-N) b-by N partition of the matrix in Eq.(4.5).

The lower k-N equation s are written as

$$0 = C_k + C_2 b \tag{4.7}$$

b can be calculated by Eq.(4.7)

The upper N+1 equation of Eq.(4.6) can be written and a :

$$a = H_1 b \tag{4.8}$$

If $k=N+N$, C_2 is square. If C_2 is not singular, Eq.(4.7) and Eq.(4.8) can be solved respectively for a and b. If c_2 is singular, Eq.(4.7) may have many solutions.

4.1.1. Long Division for First Order IIR

For N=1 in Eq.(4.1) a first order IIR filter transfer function is,

$$H_{IIR}(z) = \frac{a_0 + a_1 z^{-1}}{b_0 + b_1 z^{-1}} \tag{4.9}$$

If $b_0 = 1$, the coefficients of a FIR filter obtained by using Eq.(4.3) as,

$$\begin{aligned}
c_0 &= a_0 \\
c_1 &= a_1 - c_0 b_1 \\
c_2 &= -c_1 b_1 \\
c_3 &= -c_2 b_1 \\
c_4 &= -c_3 b_1 \\
&\vdots \\
c_k &= -\sum_{l=1}^{1} c_{k-1} b_l
\end{aligned} \tag{4.10}$$

Since there are only 3 unknowns, first three of these equations are chosen as dominant equations and then, we can easily calculate coefficients of IIR filter from a given FIR filter.

$$a_0 = c_0 \ , \ b_1 = -\frac{c_2}{c_1} \ , \quad a_1 = c_1 - \frac{c_0 c_2}{c_1} \tag{4.11}$$

and the transfer function of FIR filter is

$$H_{FIR}(z) = a_0 - (a_0 b_1 - a_1) z^{-1} + (a_0 b_1^{\,2} - a_1 b_1) z^{-2} - (a_0 b_1^{\,3} - a_1 b_1^{\,2}) z^{-3} + \ldots \tag{4.12}$$

Hence the coefficients of the IIR filter transfer function may be obtained from the given FIR coefficients. It is obvious that a longer FIR series obtained by the long division will result smaller error between the IIR and FIR filters and longer FIR series closed form can be obtained by the higher order IIR filters.

4.1.2. Long Division for Second Order IIR

The same method is applied to the second order approximation. For N=2 in Eq.(4.1) a second order IIR filter transfer function is,

$$H_{IIR}(z) = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2}}{b_0 + b_1 z^{-1} + b_2 z^{-2}} \tag{4.13}$$

If $b_0 = 1$, the coefficients of a FIR filter obtained by using Eq.(4.3),

$$\begin{aligned}
c_0 &= a_0 \\
c_1 &= a_1 - c_0 b_1 \\
c_2 &= a_2 - c_1 b_1 - c_0 b_2 \\
c_3 &= -c_2 b_1 - c_1 b_2 \\
c_4 &= -c_3 b_1 - c_2 b_2 \\
&\vdots \\
c_k &= -\sum_{l=1}^{2} c_{k-l} b_l
\end{aligned} \tag{4.14}$$

It is clear that for second order or higher order IIR approximations, the solution of equations are becoming more difficult because of their non-linear behaviors. In Eq.(4.14), unknowns are b_i's and c_i's. The more explicit form of the Eq.(4.14) is

$$\begin{aligned}
c_0 &= a_0 \\
c_1 &= a_1 - a_0 b_1 \\
c_2 &= a_2 - a_0 b_2 - a_1 b_1 + a_0 b_1^2 \\
c_3 &= a_0 b_1 b_2 - a_1 b_2 - a_2 b_1 + a_0 b_1 b_2 + a_1 b_1^2 - a_0 b_1^3 \\
c_4 &= a_0 b_2 - a_2 b_2 + 2 a_1 b_1 b_2 - 3 a_0 b_1^2 b_2 + a_2 b_1^2 - a_1 b_1^3 + a_0 b_1^4 \\
&\vdots \\
c_k &= -\sum_{l=1}^{2} c_{k-l} b_l
\end{aligned} \tag{4.15}$$

and hence the coefficients of the *Nth* order FIR function are written in terms of unknown coefficients of the IIR filter as,

$$\begin{aligned} H_{FIR}(z) &= a_0 + (a_1 - a_0 b_1) z^{-1} + (a_0 b_1{}^2 + a_2 - a_0 b_2 - a_1 b_1) z^{-2} \\ &+ (a_0 b_1 b_2 - a_1 b_2 - a_2 b_1 + a_0 b_1 b_2 - a_0 b_1{}^3 + a_1 b_1{}^2) z^{-3} \\ &+ (a_0 b_2 - a_2 b_2 + 2 a_1 b_1 b_2 - 3 a_0 b_1^2 b_2 + a_2 b_1^2 - a_1 b_1^3 + a_0 b_1{}^4) z^{-4} + \ldots. \end{aligned} \tag{4.16}$$

The approach is based on the given (or calculated) FIR coefficients and hence the coefficients of FIR filter are known. For k=4, five terms of the impulse response a_i and b_i can be calculated by using Eq.(4.7) and Eq.(4.8)

$$b = -C_2{}^{-1} C_k \Rightarrow \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = -\begin{bmatrix} c_2 & c_1 \\ c_3 & c_2 \end{bmatrix}^{-1} \begin{bmatrix} c_3 \\ c_4 \end{bmatrix} \tag{4.17}$$

$$a = H_1 b \Rightarrow \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} c_0 & 0 & 0 \\ c_1 & c_0 & 0 \\ c_2 & c_1 & c_0 \end{bmatrix} \begin{bmatrix} 1 \\ b_1 \\ b_2 \end{bmatrix} \tag{4.18}$$

For k=N+N, C_2 is square and a and b can be solved easily. For k>N+N C_2 is singular, we may have many solutions.

4.2. Minimization of the Error

Non-linear equations can be solved by Trust-region method. IIR filter coefficients can be easily calculated from a given FIR filter the coefficients. The error between the FIR and IIR filter can be measured with en error function. An error function between the FIR and realized IIR filter can be written,

$$\left[\sum \left(\left| H_{IIR}(e^{jwT}) \right| - \left| H_{FIR}(e^{jwT}) \right| \right)^2 \right]^{1/2} = \varepsilon \tag{4.19}$$

Other terms of Long Division can be used to minimize the error between the FIR and IIR filter. If we organize a new set of non-linear equations by the other terms of Long Division result, the error between the FIR and IIR filter can be minimized. The new set of nonlinear equations can be solved by Trust-region method. If we use more terms of long division and more nonlinear equation set, the error between the FIR and IIR filter is minimized. To minimize the error, the long division terms related with higher terms of FIR filter are equalized to zero.

$$
\begin{aligned}
&c_0 = a_0 \\
&c_1 = a_1 - a_0 b_1 \\
&c_2 = a_2 - a_0 b_2 - a_1 b_1 + a_0 b_1^2 \\
&c_3 = a_0 b_1 b_2 - a_1 b_2 - a_2 b_1 + a_0 b_1 b_2 + a_1 b_1^2 - a_0 b_1^3 \\
&0 = a_0 b_2 - a_2 b_2 + 2a_1 b_1 b_2 - 3a_0 b_1^2 b_2 + a_2 b_1^2 - a_1 b_1^3 + a_0 b_1^4 \\
&\vdots \\
&0 = -\sum_{l=1}^{2} c_{k-1} b_l \qquad \text{for } k = 0, 1, 2, \ldots, \infty
\end{aligned}
\tag{4.20}
$$

If the number of equations increases, the time for solving the nonlinear equations increase, but the error decreases.

For higher terms of impulse response, the large-scale algorithms are used for solving these non-linear equations. The solution of these non-linear equations gives the coefficients of IIR filter. There are different methods for solving these equations.

4.3. Method for Solving Non-linear Equation

Solving a nonlinear system of equations $F(x)$ involves finding a solution such that every equation in the nonlinear system is 0. That is, there are n equations and n unknowns. The objective is to find x is an element of the n-dimensional real numbers $x \in R^n$ such that $F(x) = 0$ where

$$
F(x) = \begin{bmatrix} F_1(x) \\ F_2(x) \\ \vdots \\ F_n(x) \end{bmatrix} \tag{4.21}
$$

The assumption is that a zero, or root, of the system exists. These equations may represent economic constraints, for example, that must all be satisfied

4.4. Trust-region Dogleg Implementation

A non-linear system equation can be solved by using the Trust-region Dogleg implementation method. Firstly search direction is found. Newton's method says to solve for the search direction d_k such that

$$\begin{aligned} J(x_k)d_k &= -F(x_k) \\ x_{k+1} &= x_k + d_k \end{aligned} \tag{4.22}$$

where $J(x_k)$ is the n-by-n Jacobian

$$J(x_k) = \begin{bmatrix} \nabla F_1(x_k)^T \\ \nabla F_2(x_k)^T \\ \vdots \\ \nabla F_n(x_k)^T \end{bmatrix} \tag{4.23}$$

Newton's method can run into difficulties. $J(x_k)$ may be singular, and so the Newton step d_k is not even defined. Also, the exact Newton step d_k may be expensive to compute. In addition, Newton's method may not converge if the starting point is far from the solution.

Using trust-region techniques (introduced in Trust-region methods for nonlinear minimization) improves robustness when starting far from the solution and handles the case when $J(x_k)$ is singular. To use a trust-region strategy, a merit function is needed to decide if x_{k+1} is better or worse than x_k. A possible choice is

$$\min_d f(d) = \frac{1}{2} F(x_k + d)^T F(x_k + d) \tag{4.24}$$

But a minimum of $f(d)$ is not necessarily a root of $F(x)$.

The Newton step d_k is a root of

$$M(x_k + d) = F(x_k) + J(x_k)d \tag{4.25}$$

and so it is also a minimum of $m(d)$ where

$$\begin{aligned}\min_d m(d) &= \frac{1}{2}\left\| M(x_k + d)\right\|_2^2 = \frac{1}{1}\left\| F(x_k) + J(x_k)d \right\| \\ &= \frac{1}{2}F(x_k)^T F(x_k) + d^T J(x_k)^T F(x_k) + \frac{1}{2}d^T (J(x_k)^T J(x_k))d\end{aligned} \tag{4.26}$$

Then $m(d)$ is a better choice of merit function than $f(d)$, and so the trust-region sub problem is

$$\min_d = \left[\frac{1}{2}F(x_k)^T F(x_k) + d^T J(x_k)^T F(x_k) + \frac{1}{2}d^T (J(x_k)^T J(x_k))d\right] \tag{4.27}$$

This subproblem can be efficiently solved using a dogleg strategy. For an overview of trust-region methods, see Conn et al. [47] and Nocedal and Wright [48] studies.

The above non-linear equation in Eq.(4.15) can be solved by this method.

$$\begin{aligned}
c_0 &= a_0 \\
c_1 &= a_1 - a_0 b_1 \\
c_2 &= a_2 - a_0 b_2 - a_1 b_1 + a_0 b_1^2 \\
c_3 &= a_0 b_1 b_2 - a_1 b_2 - a_2 b_1 + a_0 b_1 b_2 + a_1 b_1^2 - a_0 b_1^3 \\
c_4 &= a_0 b_2 - a_2 b_2 + 2a_1 b_1 b_2 - 3a_0 b_1^2 b_2 + a_2 b_1^2 - a_1 b_1^3 + a_0 b_1^4 \\
&\vdots \\
c_k &= -\sum_{l=1}^{2} c_{k-1} b_l \qquad \text{for } k = 0,\ 1,\ 2, \ldots,\ \infty
\end{aligned} \tag{4.28}$$

If k goes to ∞, the error between the filters will be zero. Theoretically it is difficult to solve that much of equations. An error between the FIR and IIR filter must be defined to limit the number of equations.

The initial condition or starting point of the unknown coefficients must be change before solving new set of nonlinear equations. This decreases the computation time. A systematic algorithm is as follows;

1. Design a $H_{FIR}(z)$ filter transfer function and find the coefficients c_0, c_1,…,c_N of FIR in Eq.(4.2) from a given frequency response.
2. By using Eq.(4.14), get the long division terms from a desired IIR transfer function
3. Get the non-linear equations in Eq.(4.15),
4. Solve the non-linear equations by Trust-Region method
5. Get the coefficients of IIR filter $H_{IIR}(z)$
6. Compute the error $\varepsilon_{FIR-IIR}$ by Eq.(4.19)
7. If $\varepsilon_{FIR-IIR}$ is not small, repeat step 2 after using new set of non-linear equations by the help of other terms of Eq.(4.20).
8. End.

The first sets of nonlinear equations are equalized to the coefficients of given FIR filter. To minimize the error, new nonlinear equations are equalized to zero. All of these equations can be solved and the algorithms stops after reaching the desired error. Different examples are given in Chapter 5 and the MATLAB programs for these examples are in Appendix.

4.5. Coefficient Quantization for flattening

The transfer function coefficients are normally evaluated to a high degree of precision during the approximation step. If coefficient-quantization is applied, the frequency response of the resulting filter may differ appreciably from the desired response, and if the quantization step is coarse, the filter may actually fail to meet the desired specifications. Also the coefficient-quantization errors introduced perturbations in the zeros and poles of the transfer function, which in turn manifest themselves as errors in the frequency response.

The IIR filter is characterized by the following transfer function

$$H_{IIR}(z) = \frac{a_0 + a_1 z^{-1} + ... + a_{N-1} z^{-(N-1)} + a_N z^{-N}}{b_0 + b_1 z^{-1} + ... + b_{N-1} z^{-(N-1)} + b_N z^{-N}} \qquad (4.29)$$

The quantized IIR filter transfer function is

$$H_{qIIR}(z) = \frac{a_{q0} + a_{q1}z^{-1} + ... + a_{qN-1}z^{-(N-1)} + a_{qN}z^{-N}}{b_{q0} + b_{q1}z^{-1} + ... + b_{qN-1}z^{-(N-1)} + b_{qN}z^{-N}} \tag{4.30}$$

a_{qN} and b_{qN} coefficients are quantized quantity

$$a_{qN} = a_N + \Delta a_N\,; \;\; b_{qN} = b_N + \Delta b_N \tag{4.31}$$

One important issue that must be considered when IIR filters are implemented on a fixed-point processor is that the filter coefficients that are actually used are quantized from the "exact" (high-precision floating point) values computed by MATLAB. Although quantization was not a concern when we worked with FIR filters, it can cause significant deviations from the expected response of an IIR filter. In this section, the coefficients quantization errors for this direct method for FIR to IIR filter is investigated. General information about coefficient quantization was given in Chapter 2. An error can be defined to show the effect of coefficient quantization.

$$\varepsilon_q = \left[\, \sum \left(\left| H_{qIIR}(e^{jwT}) \right| - \left| H_{qFIR}(e^{jwT}) \right| \right)^2 \right]^{1/2} \tag{4.32}$$

$H_{qIIR}(e^{jwT})$ is frequency response of designed and quantizated IIR filter from a given FIR filter. $H_{qFIR}(e^{jwT})$ is the frequency response of given quantizated FIR filter. Error is calculated by MATLAB by the programs in Appendix. An optimum word-length can be found by the help of this method.

For real-time processing, finite word lengths (typically 8 bits, 12 bits, and 16 bits) are used to represent the input and output signals, filter coefficients and the results of arithmetic operations. In these cases, it is nearly always necessary to analyze the effects of quantization on the filter performance. The result of coefficient quantization effects can be seen at the end of the examples in Appendix.

CHAPTER 5

5. SIMULATION OF ALGORITMH WITH SOME EXAMPLES

In this chapter, the presented method is applied to the conversion of different (highpass, lowpass, bandpass) types of FIR filters, into the different (2^{nd}, 3^{rd}, 6^{th}) order of IIR filters. The results are compared with Yule-Walker and Prony's Methods. Magnitude and phase responses of filters are plotted.

Example 1

As a first example, lowpass FIR filter is considered. The cut of frequency of the filter is assumed to be one-tenth its sampling frequency and the order is taken to be 4. The FIR coefficients are windowed by using Hamming Window Function and the FIR transfer filter coefficients are calculated as,

$$
\begin{array}{lll}
c_0 = 0.028406470015011 & & c_3 = 0.237008213590703 \\
c_1 = 0.237008213590703 & & c_4 = 0.028406470015011 \\
c_2 = 0.469170632788571 & &
\end{array}
$$

In this example the 4^{th} order FIR transfer function is converted into the 2^{nd} order IIR transfer function and the coefficients are calculated with the program in Appendix. The equations are organized like in Eq.(4.20) and solved by the Trust Region Method. Six equations are solved to find the coefficients of IIR-long6. The 6^{th} term of the long division is equated to zero. Five terms of the long division result are equated to the FIR coefficients and starting points are calculated by using Eq.(4.7) and Eq.(4.8) for the solution the six non-linear equation. The starting points are

$$
\begin{array}{lll}
a_0 = 0.0284064700150 & & b_0 = 1.0000000000000 \\
a_1 = 0.2189081409886 & & b_1 = -0.637181339058 \\
a_2 = 0.3255770287588 & & b_2 = 0.2613350715492
\end{array}
$$

IIR-long7 coefficients are calculated by equating the 6^{th} and 7^{th} terms to zero. Same starting point is used for the solution of these equations. The coefficients of filters are calculated as given in Table 1.

Table 1. Coefficients of filters

	IIR –long6	IIR-long7	Prony's	Yule-Walker
a_0	0.0284060616657	0.0284060685785	0.0284064700150	0.3534142205801
a_1	0.2203492059878	0.2206308957142	0.2189081409886	0.3649129677769
a_2	0.3423836455649	0.3469322663466	0.3255770287588	0.0926306953779
b_0	1.0000000000000	1.0000000000000	1.0000000000000	1.0000000000000
b_1	-0.5700623384028	-0.553849004811	-0.637181339058	-0.178738579109
b_2	0.1783374648999	0.1618833695938	0.2613350715492	0.0159884965057

The error between the magnitudes of the given FIR and approximated IIR-long5, IIR-long6, Prony's, and Yule-Walker filters can be calculated by using the Eq.(4.19) and they are found to be

$$\varepsilon_{FIR-IIR6} = 0.249764024523196 \qquad \varepsilon_{FIR-IIR7} = 0.244048807890771$$
$$\varepsilon_{FIR-PRONY} = 0.465748387528172 \qquad \varepsilon_{FIR-YULE} = 0.315869985678026$$

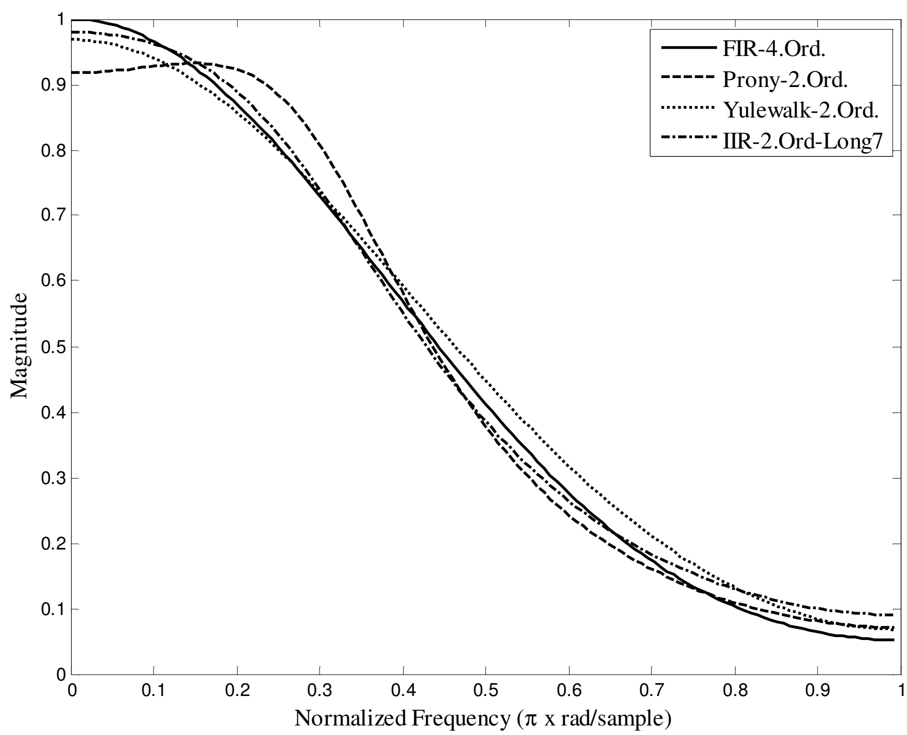

Figure 5.1. Frequency responses of filters

The normalized gain curve of the 4th order FIR characteristics and the gain of 2nd order IIR, Prony’s, Yule-Walker representations are plotted on Fig.5.1. The characteristics are plotted in logarithmic scale to compare the stopbands of the filters on Fig.5.2. The normalized phase response 4th order FIR characteristics and the gain of its 2nd order IIR, Prony’s, and Yule-Walker representations are plotted on Fig.5.3.

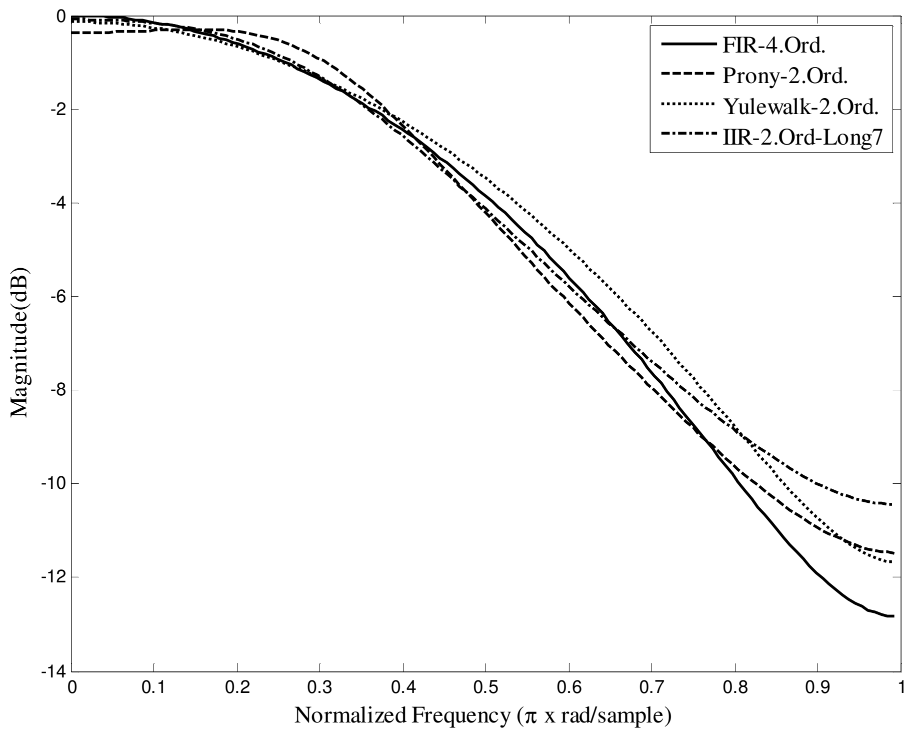

Figure 5.2. Frequency responses of filters

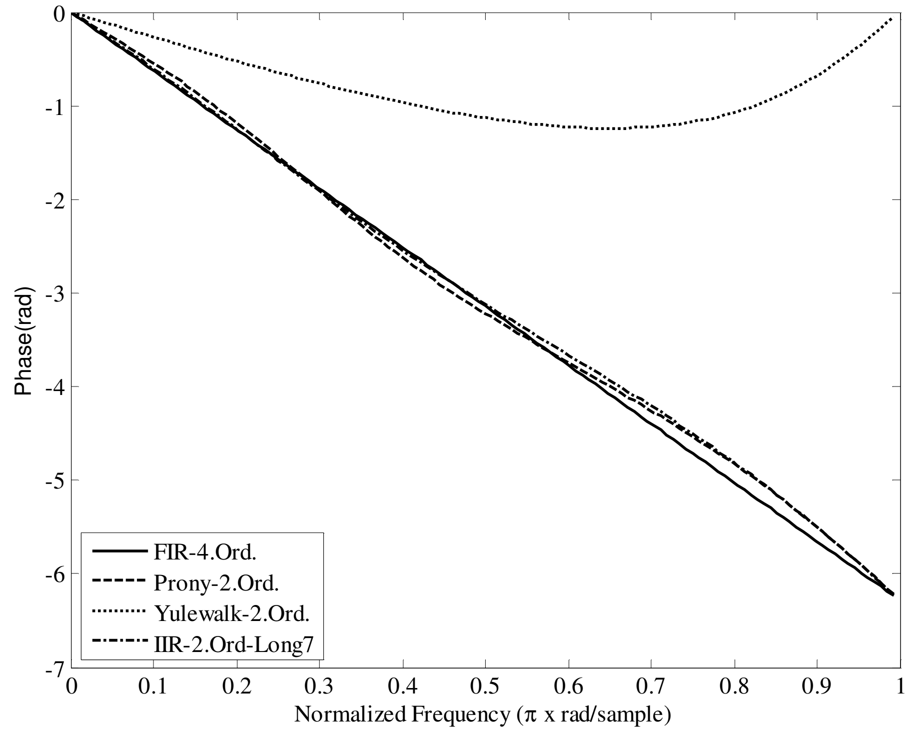

Figure 5.3. Phase responses of filters

The error of our new direct FIR to IIR conversion method is better than the results of the Prony's and Yule-Walker Method.

The error between the FIR and IIR characteristics may be more reduced by using higher term of the long division of IIR filter. The application of the correction algorithm revealed that inclusion of more equations into the solution, the error becoming less. The seven term of the long division is used for this example. The number of term can be increase to reduce the error. But after several loops, the amount of the reduction is also becoming less.

As it can be seen from the Fig.5.3. the passband phase responses of all three methods are nearly linear. Linearity is increasing by using higher order terms of long division of IIR filter's transfer function.

Example 2

As a second example, highpass FIR filter is considered. The cut of frequency of the filter is assumed to be one-tenth its sampling frequency and the order is taken to be 4. The FIR coefficients are windowed by using Hamming Window Function and the FIR transfer filter coefficients are calculated as,

$$\begin{aligned}
c_0 &= -0.012383557765435 \\
c_1 &= -0.103321704609266 \\
c_2 &= 0.818123706312338 \\
c_3 &= -0.103321704609266 \\
c_4 &= -0.012383557765435
\end{aligned}$$

The equations are organized with this new FIR filter coefficients and solved by the Trust Region Method. Seven equations are solved to find the coefficients of IIR-long7. The 6^{th} and 7^{th} term of the long division is equated to zero. Five terms of the long division result are equated to the FIR coefficients and starting points are calculated by using Eq.(4.7) and Eq.(4.8) for the solution the seven non-linear equation. The starting points are,

$$\begin{aligned}
a_0 &= -0.0123835577654 & b_0 &= 1.0000000000000 \\
a_1 &= -0.1049350414862 & b_1 &= 0.1302805629443 \\
a_2 &= 0.8042717023077 & b_2 &= 0.0315898040931
\end{aligned}$$

The coefficients of filters are found as in Table 2.

Table 2. Coefficients of filters

	IIR-long7	Prony's	Yule-Walker
a_0	-0.012383559938974	-0.012383557765435	0.806062166651525
a_1	-0.104923055295202	-0.104935041486200	-0.204599207918885
a_2	0.804391026636978	0.804271702307695	0.000491837357837
b_0	1.000000000000000	1.000000000000000	1.000000000000000
b_1	0.129649351551022	0.130280562944281	-0.007548545357371
b_2	0.030015220316801	0.031589804093057	0.009446692850085

The error between the magnitudes of the given FIR and approximated IIR-long7, Prony's, Yule-Walker filters can be calculated by using the Eq.(4.19) and they are found to be

$$\begin{aligned}
\varepsilon_{FIR-IIR7} &= 0.037849221444129 \\
\varepsilon_{FIR-PRONY} &= 0.039052945167387 \\
\varepsilon_{FIR-YULE} &= 0.315869985678026
\end{aligned}$$

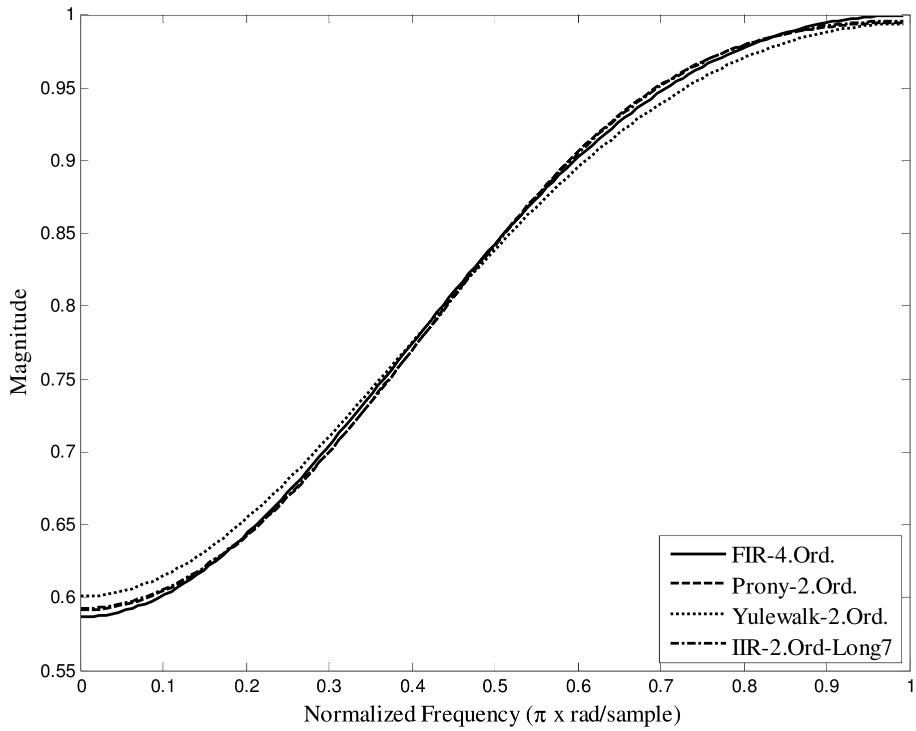

Figure 5.4. Frequency responses of filters

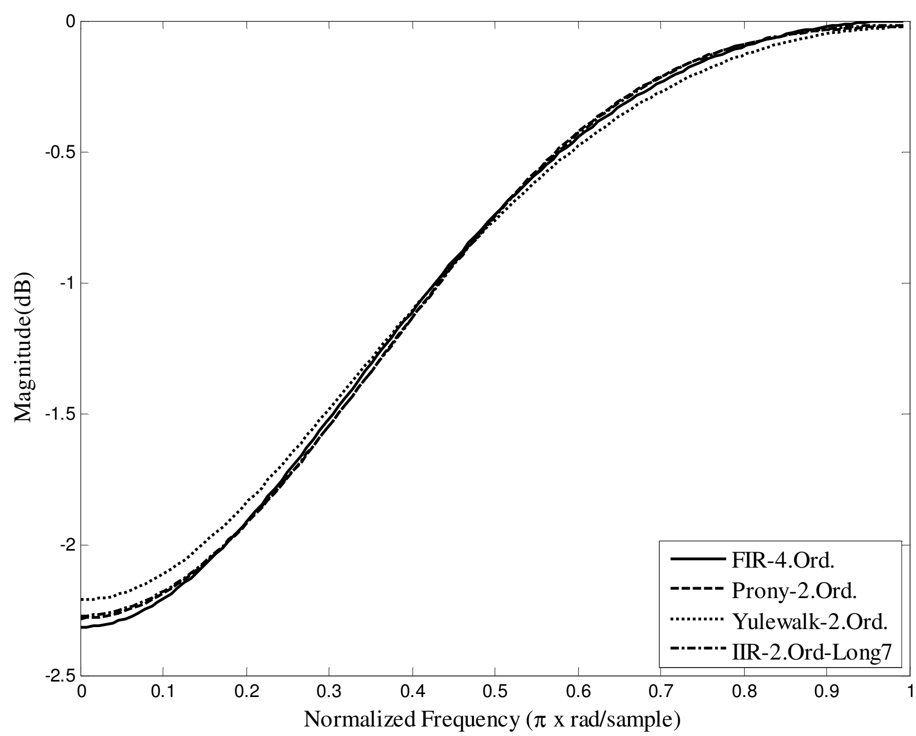

Figure 5.5. Frequency responses of filters

The normalized gain curve of the 4^{th} order FIR characteristics and the gain of its 2^{nd} order IIR, Prony's, Yule-Walker representations are plotted on Fig.5.4. The characteristics are plotted in logarithmic scale to compare the stopbands of the filters on Fig.5.5. The normalized phase response 4^{th} order FIR characteristics and the gain of its 2^{nd} order IIR, Prony's, Yule-Walker representations are plotted on Fig.5.6.

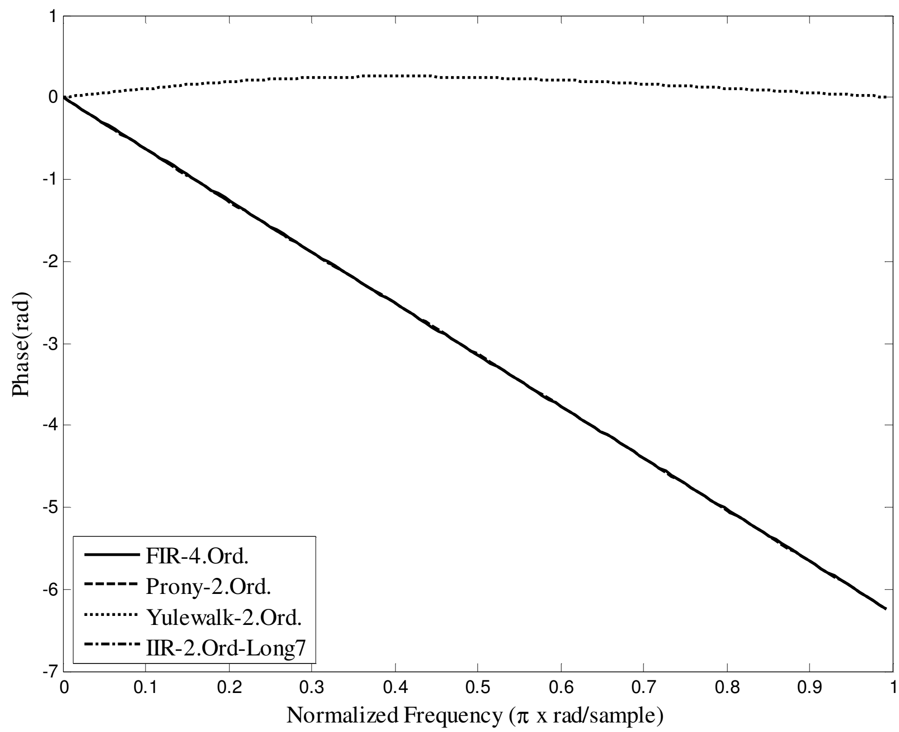

Figure 5.6. Phase responses of filters

The errors of our new direct and Prony's method for FIR to IIR conversion are better than the result of the Yule-Walker Method. In our method the error is the lowest one. Also the error is not changing so much for higher terms.

As it can be seen from the Fig.5.6. the passband phase responses of all three methods are nearly linear. Yule-Walker phase response is not linear, but other methods' phase responses are linear for all frequencies.

Example 3

In this example 6^{th} order lowpass FIR filter which has 0.2 cutoff frequency is converted to 2^{nd} order IIR filter and the phase and magnitude characteristics are compared with the 2^{nd} order Prony's and Yule-Walker Methods. FIR filter coefficients are

$$
\begin{aligned}
c_0 &= 0.013496923634082 \\
c_1 &= 0.078450868623102 \\
c_2 &= 0.240862474239631 \\
c_3 &= 0.334379467006371 \\
c_4 &= 0.240862474239631 \\
c_5 &= 0.078450868623102 \\
c_6 &= 0.013496923634082
\end{aligned}
$$

The equations are organized with these new FIR coefficients and solved by the Trust Region Method with the calculated starting point. Seven terms of the long division result are equated to the FIR coefficients and starting points are calculated by using Eq.(4.7) and Eq.(4.8). The coefficients of filters are found as in Table 3.

Table 3. Coefficients of filters

	IIR-long14	Prony's	Yule-Walk
a_0	0.013496923679214	0.013496923634082	0.155002715725143
a_1	0.048332833271608	0.060923891043628	0.256849242694650
a_2	0.221717798891762	0.147323006902478	0.154735399032099
b_0	1.000000000000000	1.000000000000000	1.000000000000000
b_1	-1.122584659408244	-1.298590556978121	-0.488594993325217
b_2	0.425031618949297	0.617628881183580	0.090414527194376

The error between the magnitudes of the given FIR and approximated IIR-long14, Prony's, Yule-Walker filters can be calculated by using the Eq.(4.19) as,

$$
\begin{aligned}
\varepsilon_{FIR-IIR14} &= 0.646677132795521 \\
\varepsilon_{FIR-PRONY} &= 1.277760616211978 \\
\varepsilon_{FIR-YULE} &= 0.488739970485410
\end{aligned}
$$

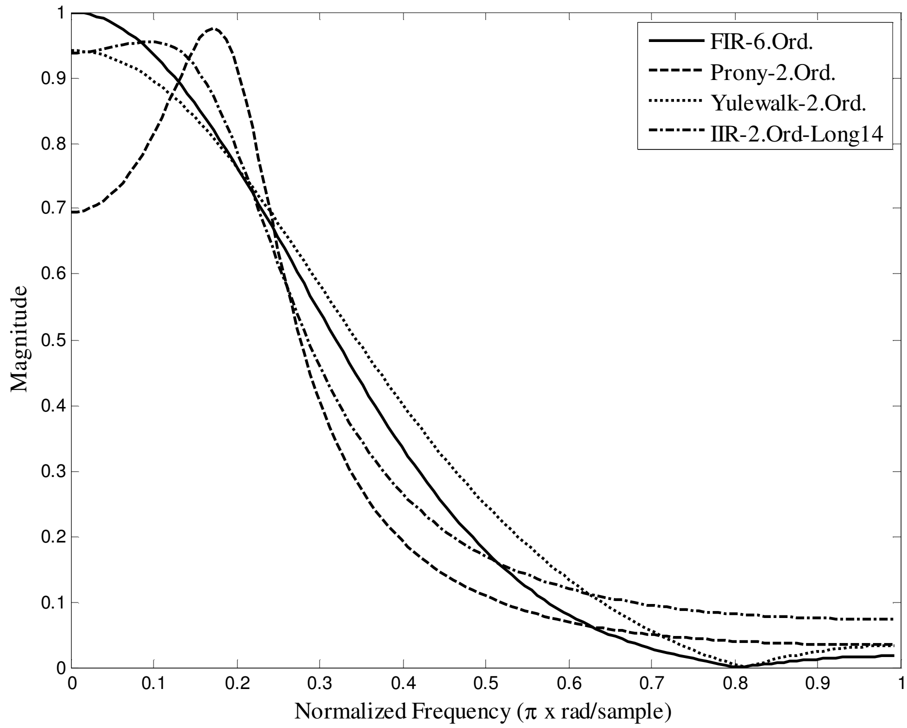

Figure 5.7. Frequency responses of filters

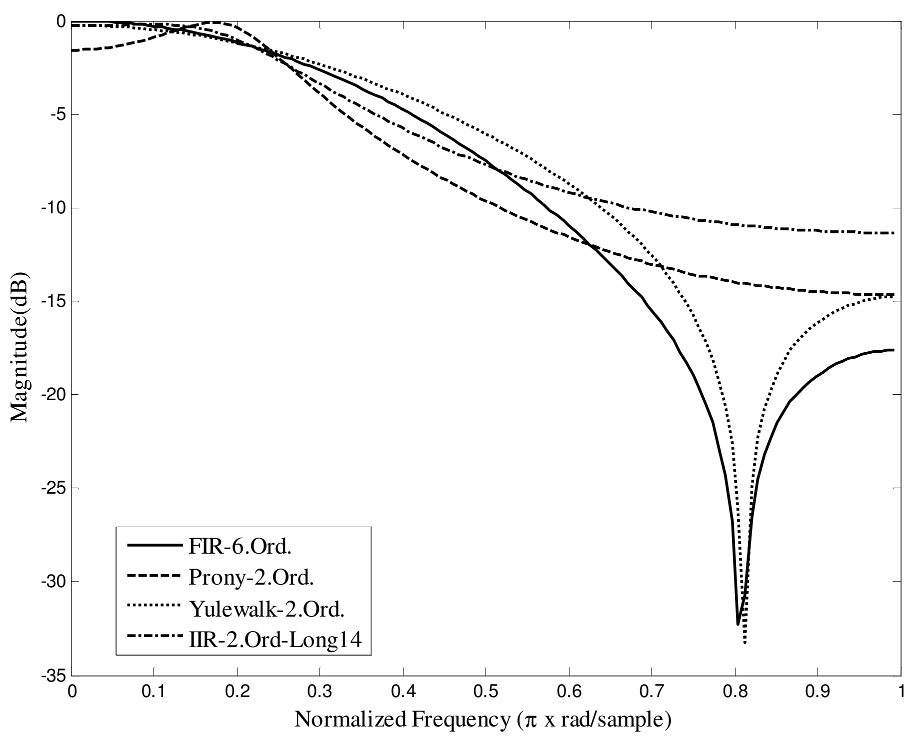

Figure 5.8. Frequency responses of filters

The normalized gain curve of the 6th order FIR characteristics and the gain of its 2nd order IIR, Prony's, Yule-Walker representations are plotted on Fig.5.7. The characteristics are plotted in logarithmic scale to compare the stopbands of the filters on Fig.5.8. The normalized phase response 4th order FIR characteristics and the gain of its 2nd order IIR, Prony's, Yule-Walker representations are plotted on Fig.5.9.

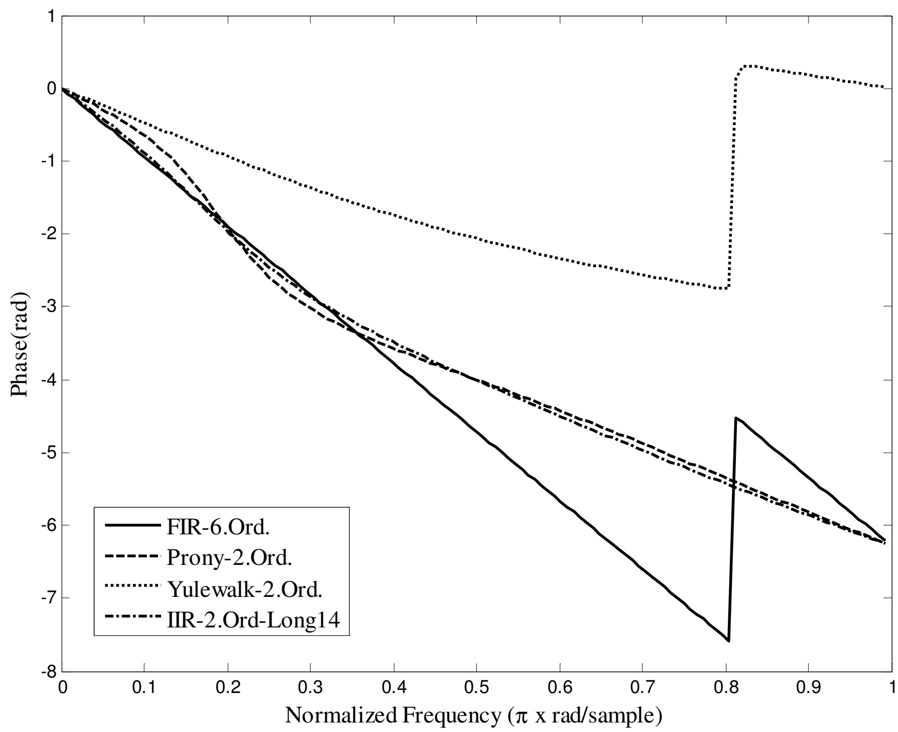

Figure 5.9. Phase responses of filters

FIR filter has a zero at 0.8 frequencies so the magnitude of FIR filter is equal to zero, hence phase response of FIR is undefined at this frequency. The error of magnitude responses of IIR and Yule-Walker Filters are nearly same but Yule-Walker error is better because the order of FIR filter was increased to 6th.

The passband phase responses of Prony's method are not linear. However, IIR and Yule-Walker filters' phase responses are nearly linear.

Example 4

In this example 6^{th} order highpass FIR filter which has 0.2 cutoff frequency is converted to 2^{nd} order IIR filter and the phase and magnitude characteristics are compared with the 2^{nd} order Prony's and Yule-Walker Methods. FIR filter coefficients are

$$
\begin{aligned}
c_0 &= -0.007989489486156 \\
c_1 &= -0.046438907638285 \\
c_2 &= -0.142578283594034 \\
c_3 &= 0.791742269116189 \\
c_4 &= -0.142578283594034 \\
c_5 &= -0.046438907638285 \\
c_6 &= -0.007989489486156
\end{aligned}
$$

The equations are organized with these new FIR coefficients and solved by the Trust Region Method with the calculated starting point. Seven terms of the long division result are equated to the FIR coefficients and starting points are calculated by using Eq.(4.7) and Eq.(4.8). The coefficients of filters are found as in Table 4.

Table 4. Coefficients of filters

	IIR-long14	Prony	Yule-Walker
a_0	-0.007989470323955	-0.007989489486156	0.766492910021975
a_1	-0.134160186837230	-0.049560598448024	-0.432834876170934
a_2	-0.515248455562996	-0.162335011992807	0.034519920421966
b_0	1.000000000000000	1.000000000000000	1.000000000000000
b_1	0.978833948597966	0.390724690876548	-0.195638425980272
b_2	0.459312301372243	0.201752636379674	0.042135146545861

The error between the magnitudes of the given FIR and approximated IIR-long14, Prony's, Yule-Walker filters can be calculated by using the Eq.(4.19) as,

$$
\begin{aligned}
\varepsilon_{FIR-IIR14} &= 2.971219867751848 \\
\varepsilon_{FIR-PRONY} &= 7.437283884000442 \\
\varepsilon_{FIR-YULE} &= 0.198187797648411
\end{aligned}
$$

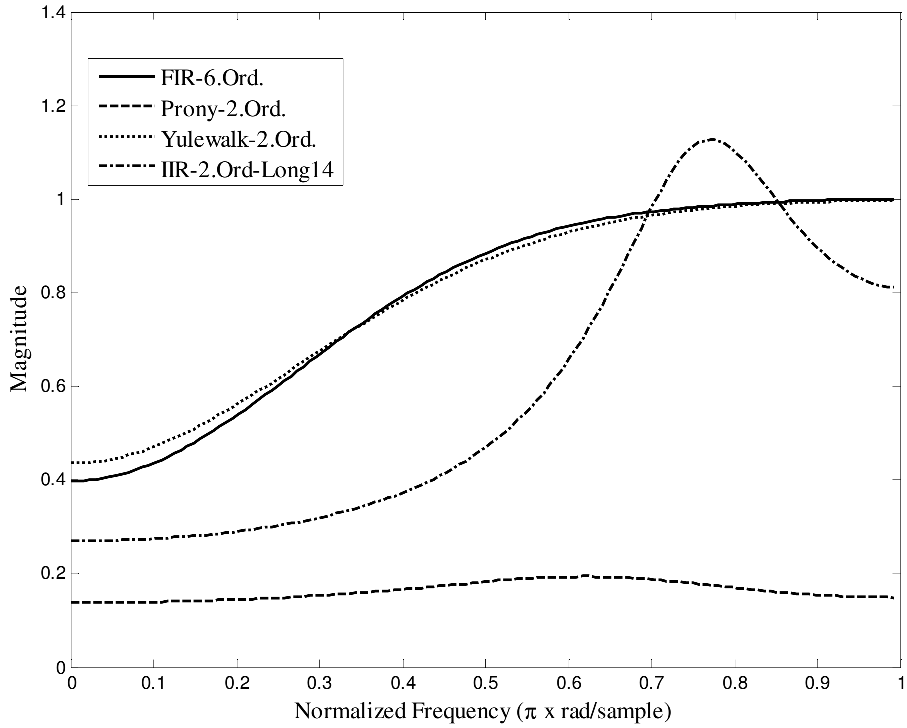

Figure 5.10. Frequency responses of filters

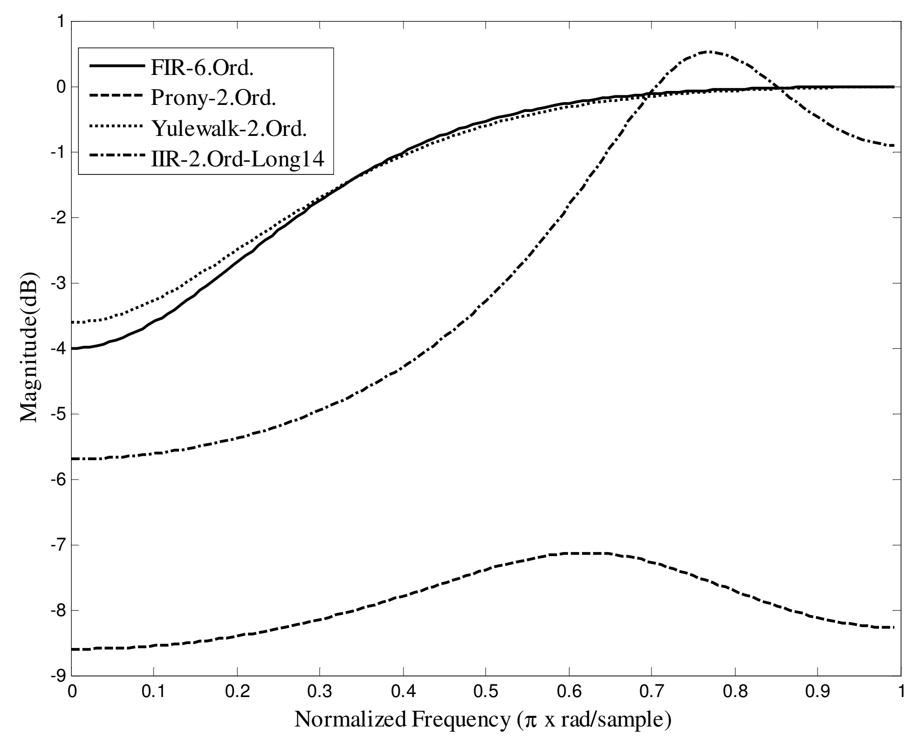

Figure 5.11. Frequency responses of filters

The normalized gain curve of the 6^{th} order FIR characteristics and the gain of its 2^{nd} order IIR, Prony's, Yule-Walker representations are plotted on Fig.5.10. The characteristics are plotted in logarithmic scale to compare the stopbands of the filters on Fig.5.11. The normalized phase response 4^{th} order FIR characteristics and the gain of its 2^{nd} order IIR, Prony's, Yule-Walker representations are plotted on Fig.5.12.

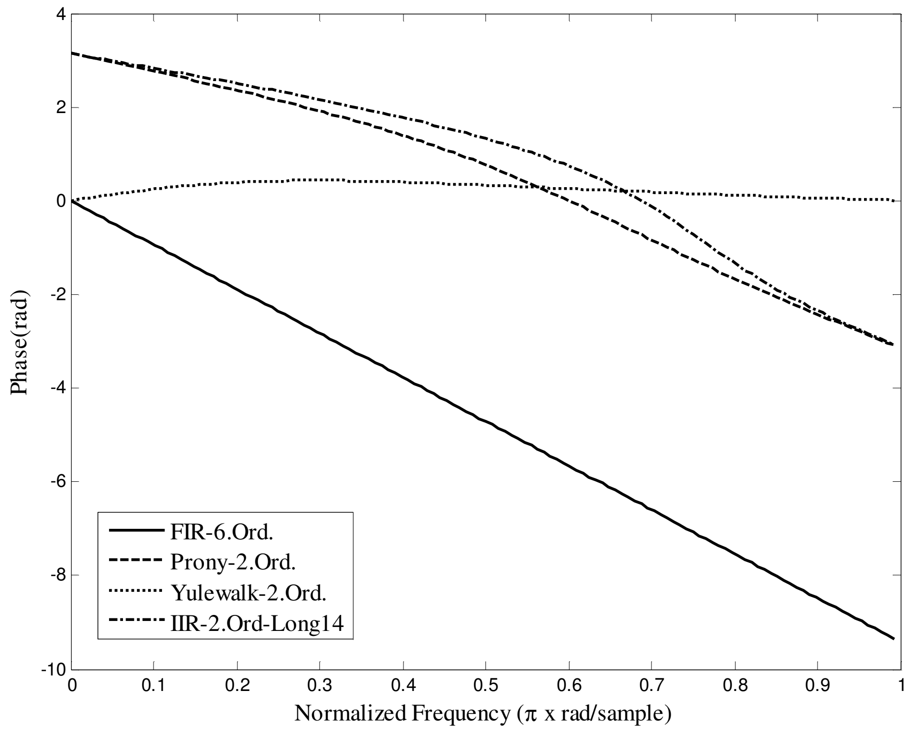

Figure 5.12. Phase responses of filters

The equations are solved with the coefficients of Yule-Walker as a starting point to reduced the error. The new IIR coefficients of filters are found as

$a_0 =$ 0.757078783755060 a_1= -0.442249002352499 a_2=0.025105794188449
$b_0 =$ 1.000000000000000 b_1=-0.191543817010068 b_2=0.046229745608049

The error is determined as

$$\varepsilon_{FIR-IIR14} = 0.073965278573571$$

6^{th} Order highpass FIR filter which has 0.8 cutoff frequency is converted to 2^{nd} order IIR filter and the phase and magnitude characteristics are compared with the 2^{nd} order Prony's and Yule-Walker Methods to show the effect of cutoff frequency. IIR filter coefficients are

a_0= -0.013496923620367 a_1=0.047508056792700 a_2= -0.224695245107251
b_0 = 1.000000000000000 b_1=1.114409516168226 b_2=0.419695846153642

And the errors

$$\varepsilon_{FIR-IIR14} = 0.645657132491772$$
$$\varepsilon_{FIR-YULE} = 0.450856563632776$$
$$\varepsilon_{FIR-PRONY} = 1.240948559393124$$

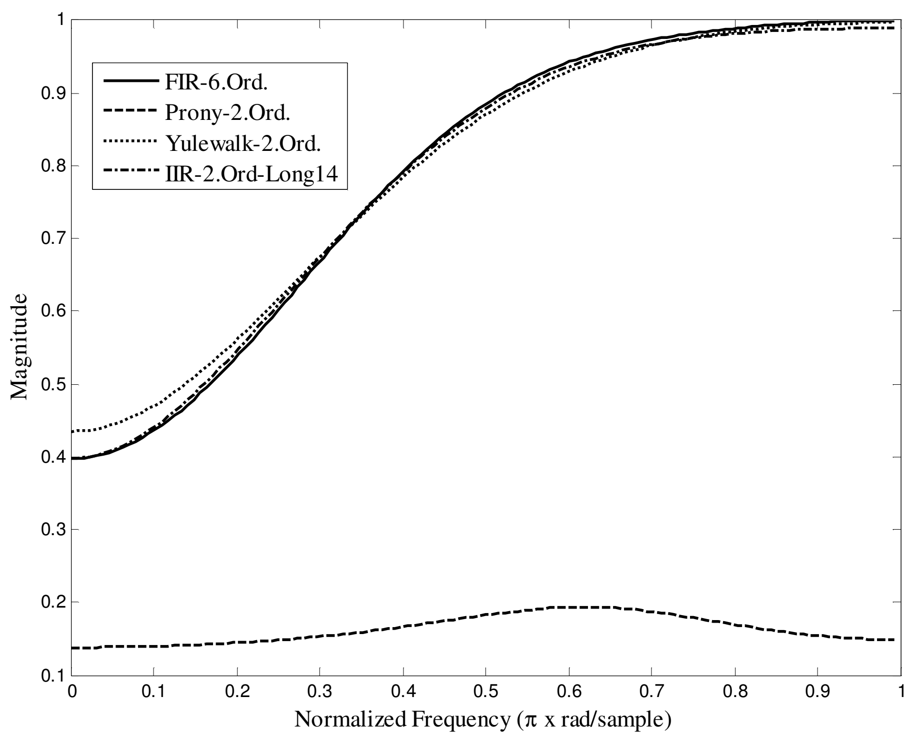

Figure 5.13. Frequency responses of filters for Yule-Walker starting point

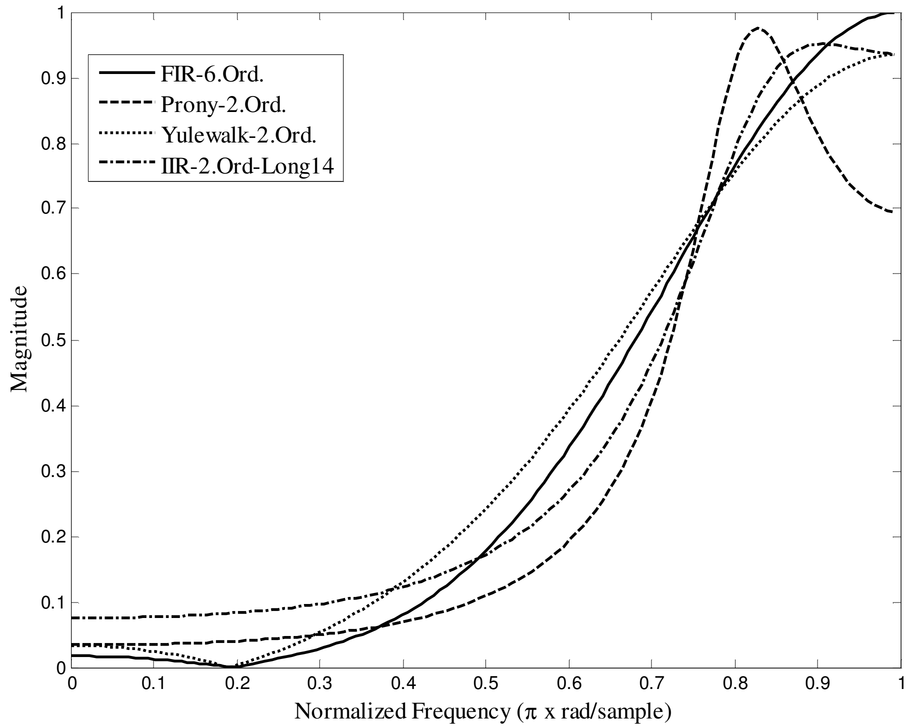

Figure 5.14. Frequency responses of filters for 0.8 cutoff frequencies

The errors with the calculated starting point were very bad because the converted FIR filter was a highpass filter. In highpass filters, magnitude values are dominant in high frequencies. The equations which are organized high terms of the long division can not be reduced with this starting point.

Yule-Walker starting point was used to solve the equations to improve the error, and the magnitude response is plotted in Fig.5.13. It can be seen that the error of IIR filter with the Yule-Walker starting point is better than the Yule-Walker result.

If we select the cut off frequency of the high pass filter as 0.8, all of the methods' errors change. Yule-Walker error increases but IIR and Prony's errors decrease. It shows that all of the methods success depend on the cutoff frequency. Another case for the high pass characteristics is that the sampling frequency is always limited with the assumed highest input frequency of the filter.

Example 5

In this example 8^{th} order lowpass FIR filter which has 0.2 cutoff frequency is converted to 2^{nd} order IIR filter and the phase and magnitude characteristics are compared with the 2^{nd} order Prony's and Yule-Walker Methods. FIR filter coefficients are

$$
\begin{aligned}
c_0 &= 0.005069883484836 \\
c_1 &= 0.029358162747516 \\
c_2 &= 0.110743791265797 \\
c_3 &= 0.219340680905496 \\
c_4 &= 0.270974963192709 \\
c_5 &= 0.219340680905496 \\
c_6 &= 0.110743791265797 \\
c_7 &= 0.029358162747516 \\
c_8 &= 0.005069883484836
\end{aligned}
$$

The equations are organized with these new FIR coefficients and solved by the Trust Region Method with the calculated starting point. Nine terms of the long division result are equated to the FIR coefficients and starting points are calculated by using Eq.(4.7) and Eq.(4.8). The coefficients of filters are found as in Table 5.

Table 5. Coefficients of filters

	IIR-long14	Prony	Yule-Walker
a_0	0.005069883581037	0.005069883484836	0.126238992913244
a_1	-0.005649667296852	0.021423454011406	0.181092043390114
a_2	0.165048217184271	0.068713204152368	0.125943463891234
b_0	1.000000000000000	1.000000000000000	1.000000000000000
b_1	-1.393987752498314	-1.565067276169670	-0.690049565579207
b_2	0.573208005176267	0.772584360742831	0.163224518650762

The error between the magnitudes of the given FIR and approximated IIR-long14, Prony's, Yule-Walker filters can be calculated by using the Eq.(4.19) as,

$$
\begin{aligned}
\varepsilon_{FIR-IIR14} &= 0.805999703942367 \\
\varepsilon_{FIR-PRONY} &= 2.078930344149698 \\
\varepsilon_{FIR-YULE} &= 0.776547050798376
\end{aligned}
$$

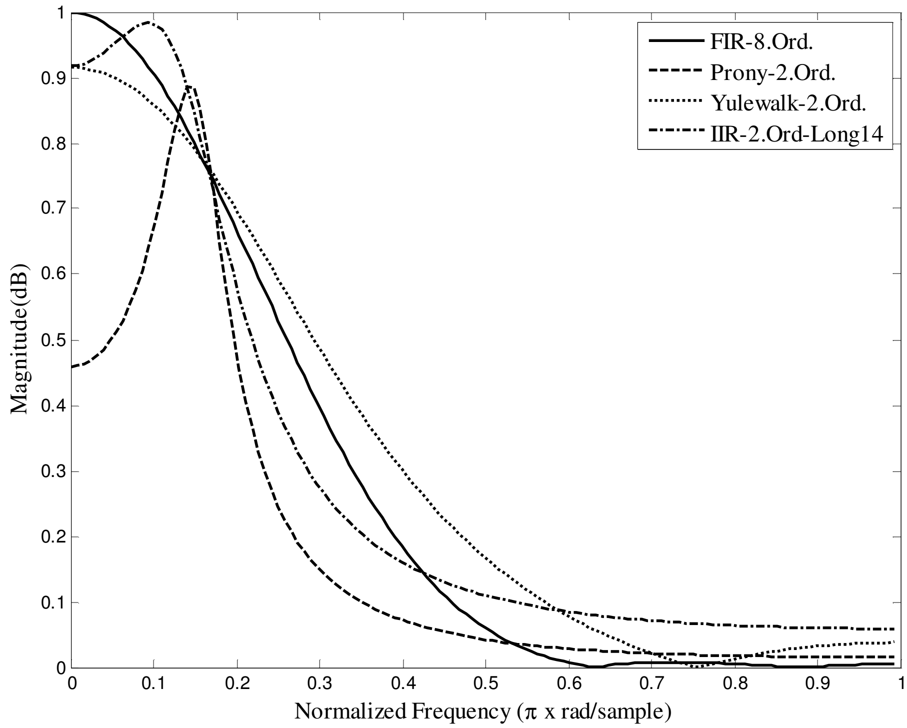

Figure 5.15. Frequency responses of filters

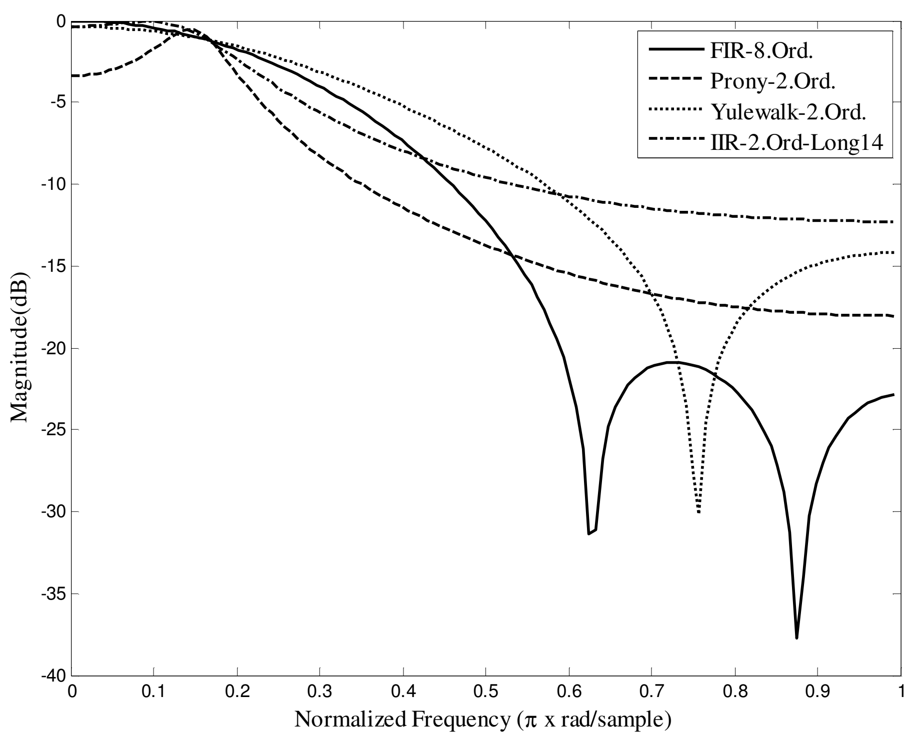

Figure 5.16. Frequency responses of filters

The normalized gain curve of the 8th order FIR characteristics and the gain of its 2nd order IIR, Prony's, Yule-Walker representations are plotted on Fig.5.15. The characteristics are plotted in logarithmic scale to compare the stopband of the filters on Fig.5.16. The normalized phase response 8th order FIR characteristics and the gain of its 2nd order IIR, Prony's, Yule-Walker representations are plotted on Fig.5.17.

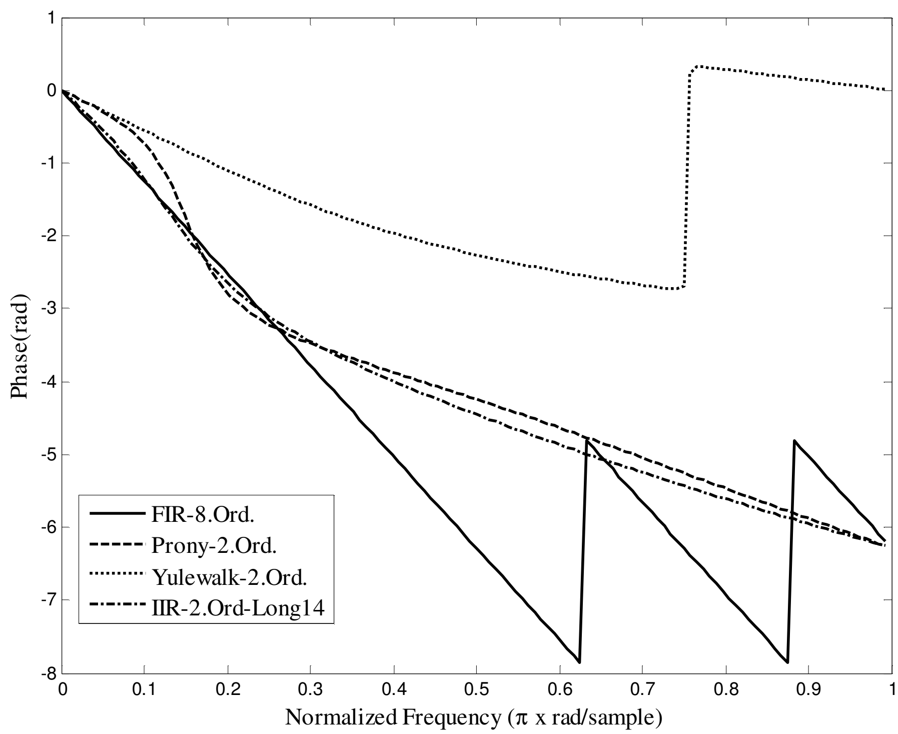

Figure 5.17. Frequency responses of filters

The error of Yule-Walker and IIR magnitude responses are nearly same but Prony's error is very bad. All of the methods get difficulties to approach for the higher order FIR filter. The order of IIR filter had to increase to improve the conversion.

Example 6

In this example 8th Order highpass FIR filter which has 0.2 cutoff frequency is converted to 2nd Order IIR filter and the phase and magnitude characteristics are compared with the 2nd Order Prony's and Yule-Walker Methods. FIR filter coefficients are

$$\begin{aligned}
c_0 &= -0.003756386945668\\
c_1 &= -0.021752101330026\\
c_2 &= -0.082052483665337\\
c_3 &= -0.162514281220032\\
c_4 &= 0.803084976121895\\
c_5 &= -0.162514281220032\\
c_6 &= -0.082052483665337\\
c_7 &= -0.021752101330026\\
c_8 &= -0.003756386945668
\end{aligned}$$

The equations are organized with these new FIR coefficients and solved by the Trust Region Method with the calculated starting point. Nine terms of the long division result are equated to the FIR coefficients and starting points are calculated by using Eq.(4.7) and Eq.(4.8). The coefficients of filters are found as in Table 6.

Table 6. Coefficients of filters

	IIR-long14	Prony	Yule-Walker
a_0	-0.003756242194791	-0.003756386945668	0.767692891415730
a_1	0.031369071746033	-0.023310816047878	-0.484334635810019
a_2	-0.344415554178154	-0.092101227929036	0.001021221829623
b_0	1.000000000000000	1.000000000000000	1.000000000000000
b_1	0.043993439124884	0.414950520379499	-0.197275743939053
b_2	0.737797777091902	0.272255897022458	0.052267136180276

The error between the magnitudes of the given FIR and approximated IIR-long14, Prony's, Yule-Walker filters can be calculated by using the Eq.(4.19) as,

$$\begin{aligned}
\varepsilon_{FIR-IIR14} &= 5.523751500237434\\
\varepsilon_{FIR-PRONY} &= 8.472225809521374\\
\varepsilon_{FIR-YULE} &= 0.356148021279132
\end{aligned}$$

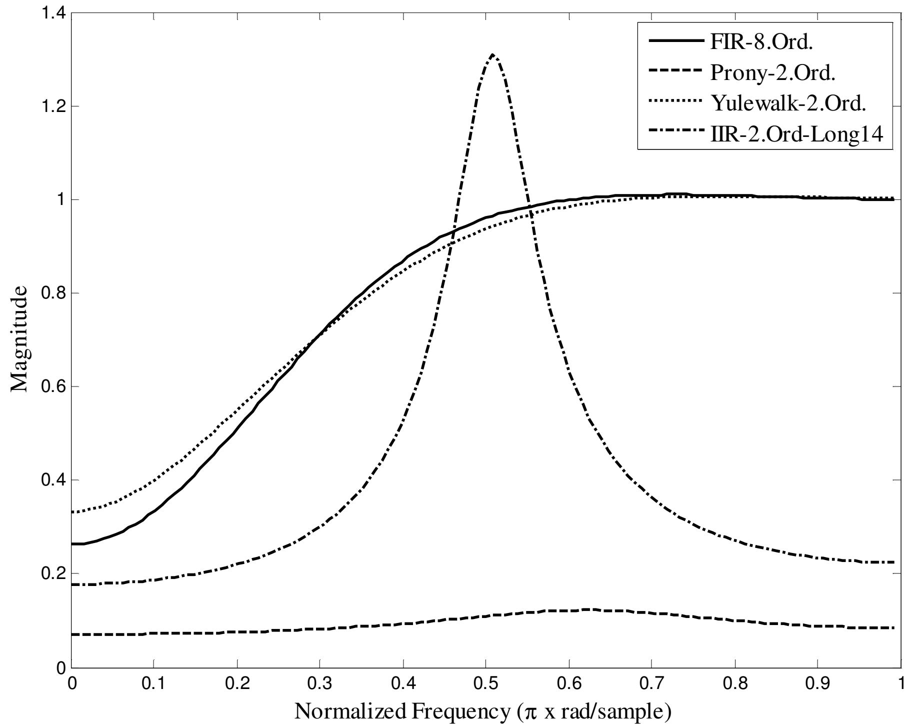

Figure 5.18. Frequency responses of filters

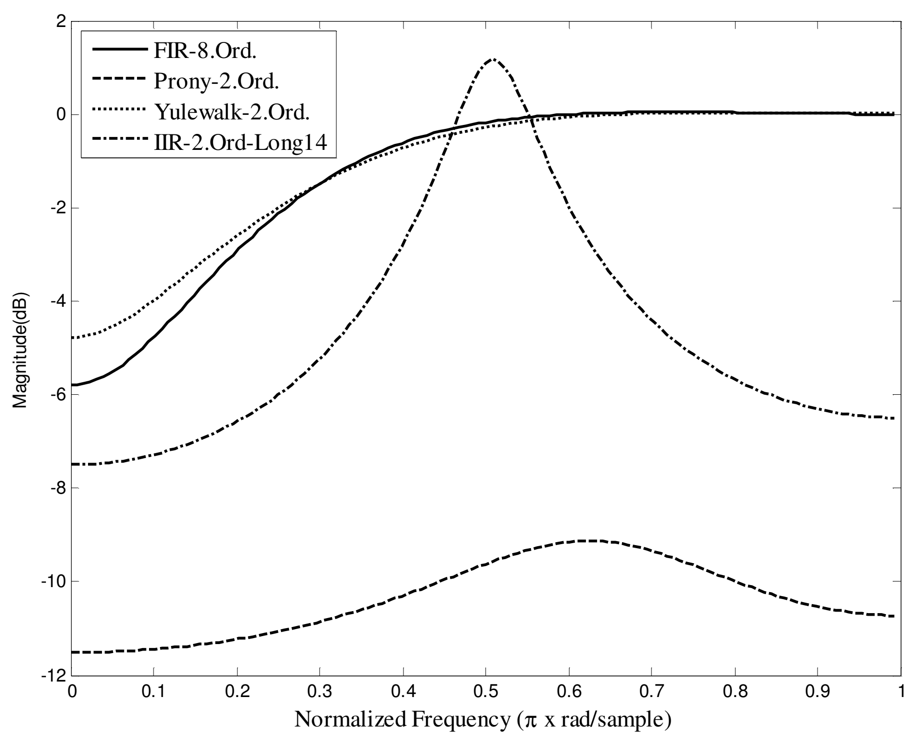

Figure 5.19. Frequency responses of filters

The normalized gain curve of the 8th order FIR characteristics and the gain of its 2nd order IIR, Prony's, Yule-Walker representations are plotted on Fig.5.18. The characteristics are plotted in logarithmic scale to compare the stopbands of the filters on Fig.5.19. The normalized phase response 8th order FIR characteristics and the gain of its 2nd order IIR, Prony's, Yule-Walker representations are plotted on Fig.5.20.

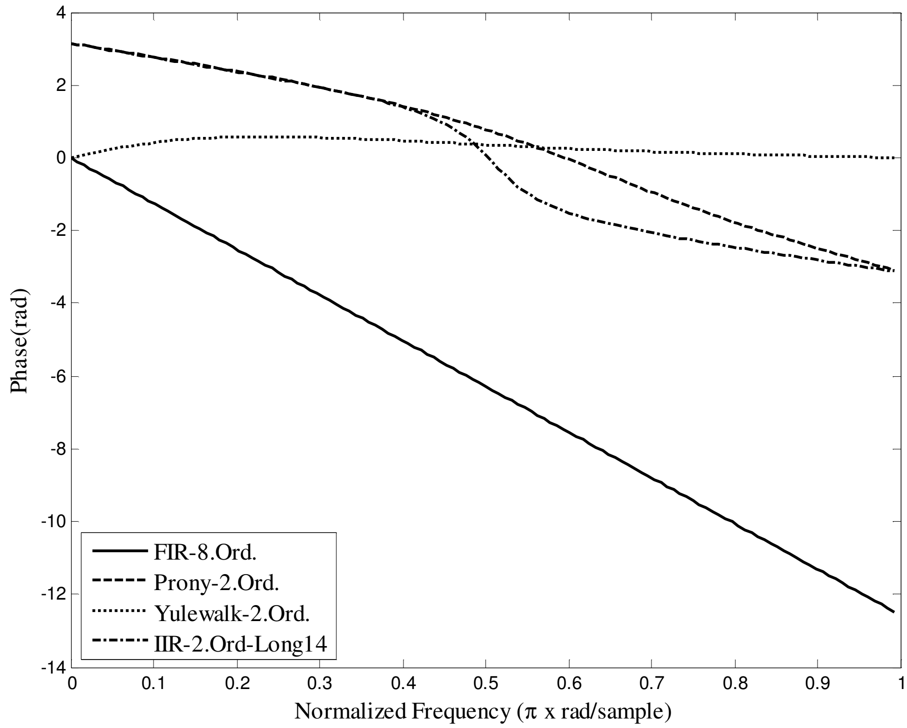

Figure 5.20. Phase responses of filters

The equations are solved with the coefficients of Yule-Walker as a starting point to reduced to error. The new IIR coefficients of filters are found as

a_1 = 0.748929205596222 a_2=-0.503098320075367 a_3=-0.017742460522346
b_0 =1.000000000000000 b_1=-0.191034977486918 b_2=0.058507823073133

and the error becomes

$$\varepsilon_{FIR-IIR14} = 0.124502034020608$$

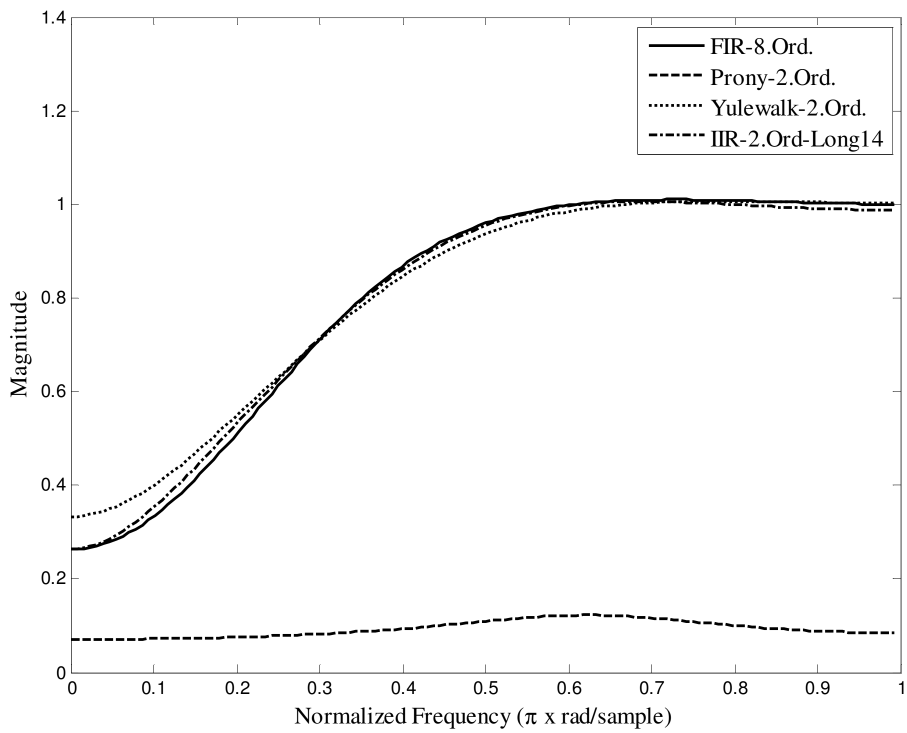

Figure 5.21. Frequency responses of filters with Yule-Walker starting point

The errors with the calculated starting point were not good enough. There are mainly two reasons for this error. First, the designed high pass filter pass band region lies at the higher frequencies and due to the application of the Prony's Method and since 8^{th} order characteristics is tried to be realized by 2^{nd} order IIR transfer function, the number of equations used to calculate the coefficients is not sufficient to determine the high frequency behaviors. For 2^{nd} order IIR conversion only 5 terms of given 8 terms of the FIR characteristics are used, therefore, Prony's results are becoming bad because of the cancellation of the higher terms which are more dominant at the high frequencies. To reduce the error, Yule-Walker starting point was used to solve the equations, and magnitude response is plotted in Fig.5.13. The error of IIR filter with the Yule-Walker starting point better than the Yule-Walker result.

Example 7

12^{th} order lowpass FIR filter which has 0.2 cutoff frequency is converted to 3^{rd} order IIR filter and the phase and magnitude characteristics are compared with the 3^{rd} order Prony's and Yule-Walker Methods. FIR filter coefficients are

$$
\begin{aligned}
c_0 &= -0.002719748296487 \\
c_1 &= 0 \\
c_2 &= 0.015808536973329 \\
c_3 &= 0.059408709991560 \\
c_4 &= 0.127068629704170 \\
c_5 &= 0.191410109532371 \\
c_6 &= 0.218047524190116 \\
c_7 &= 0.191410109532371 \\
c_8 &= 0.127068629704170 \\
c_9 &= 0.059408709991560 \\
c_{10} &= 0.015808536973329 \\
c_{11} &= 0 \\
c_{12} &= -0.002719748296487
\end{aligned}
$$

The equations are organized with these new FIR coefficients and solved by the Trust Region Method with the calculated starting point. 13 terms of the long division result are equated to the FIR coefficients and starting points are calculated by using Eq.(4.7) and Eq.(4.8). The coefficients of filters are found as in Table 7.

Table 7. Coefficients of filters

	IIR-long15	Prony	Yule-Walker
a_0	-0.002651689085539	-0.002719748296487	0.049869547920393
a_1	0.012081901525158	0.006599990408012	0.076961670281446
a_2	-0.017933505105523	0.009677152299683	0.076866541713083
a_3	0.074412946338609	0.023220375704049	0.049878697319975
b_0	1.000000000000000	1.000000000000000	1.000000000000000
b_1	-2.191181472193112	-2.426691623095490	-1.229315655517132
b_2	1.807415058604954	2.254394159035298	0.621356133718209
b_3	-0.556124340865500	-0.799379104996508	-0.125344799661899

The error between the magnitudes of the given FIR and approximated IIR-long15, Prony's, Yule-Walker filters can be calculated by using the Eq.(4.19) as,

$$
\begin{aligned}
\varepsilon_{FIR-IIR15} &= 0.450955878866840 \\
\varepsilon_{FIR-PRONY} &= 1.677070271323739 \\
\varepsilon_{FIR-YULE} &= 0.506913640450293
\end{aligned}
$$

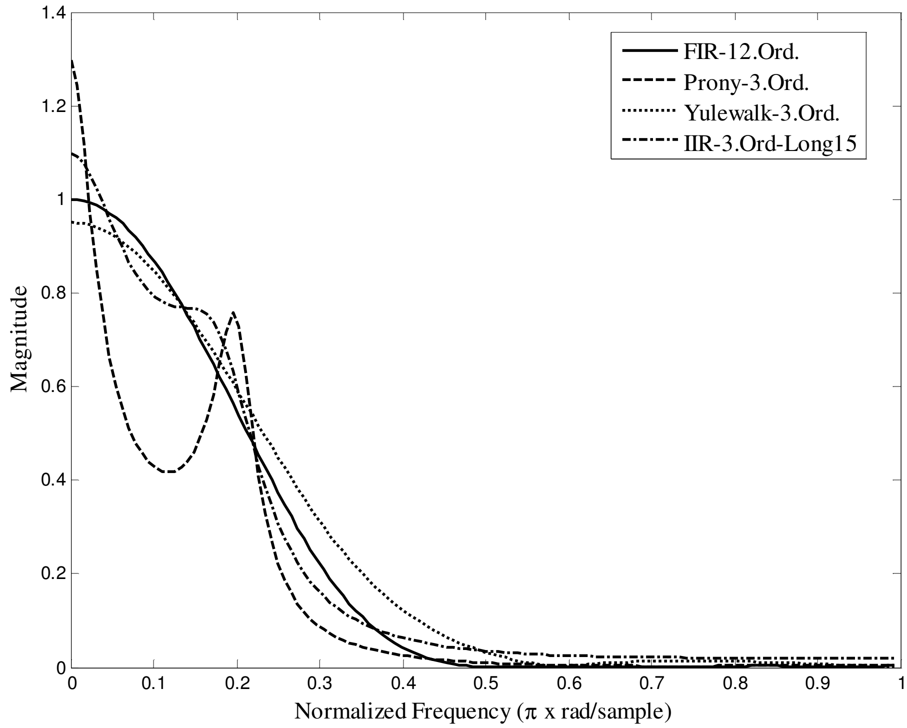

Figure 5.22. Frequency responses of filters

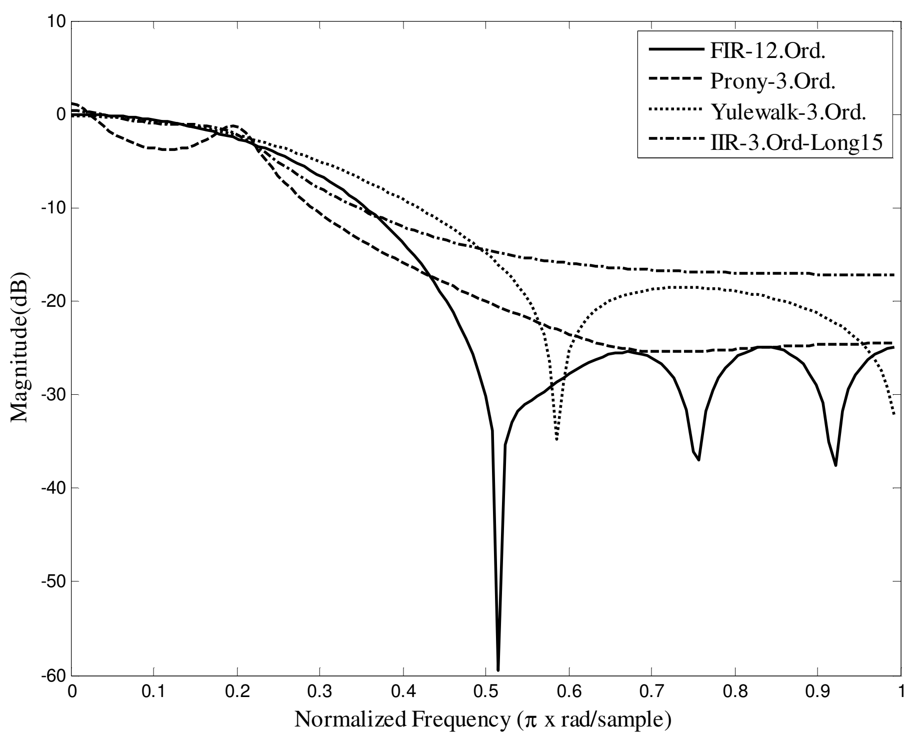

Figure 5.23. Frequency responses of filters

The normalized gain curve of the 12th order FIR characteristics and the gain of its 3rd order IIR, Prony's, Yule-Walker representations are plotted on Fig.5.22 and Fig.5.23. The normalized phase response 12th order FIR characteristics and the gain of its 3rd order IIR, Prony's, Yule-Walker representations are plotted on Fig.5.34

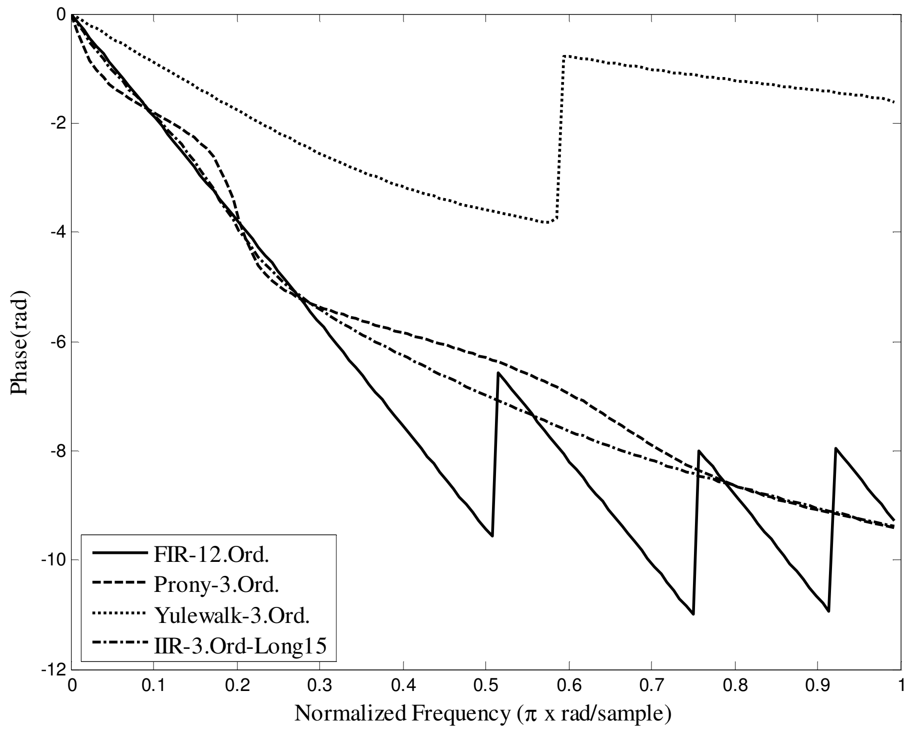

Figure 5.24. Frequency responses of filters

In this example the order of IIR filter was increased to 3rd and 12th FIR filter was converted with this method. 15 terms of 3rd order IIR filter is used and it has been observed that the error of IIR filters is better than the Yule-Walker and Prony's Metods. Phase responses of Yule-Walker and IIR filters are linear in the passband.

Example 8

In this example 12th order highpass FIR filter which has 0.2 cutoff frequency is converted to 3rd order IIR filter and the phase and magnitude characteristics are compared with the 3rd order Prony's and Yule-Walker Methods. Errors are found as,

$$\varepsilon_{FIR-IIR15} = 7.525858794869147$$
$$\varepsilon_{FIR-PRONY} = 8.908661599801572$$
$$\varepsilon_{FIR-YULE} = 0.332842444661047$$

Example 9

12th order highpass FIR filter which has 0.8 cutoff frequency is converted to 3rd 0rder IIR filter and the phase and magnitude characteristics are compared with the 3rd 0rder Prony's and Yule-Walker Methods. FIR filter coefficients are

$$
\begin{aligned}
c_0 &= -0.002719748296487 \\
c_1 &= 0 \\
c_2 &= 0.015808536973329 \\
c_3 &= -0.059408709991560 \\
c_4 &= 0.127068629704170 \\
c_5 &= -0.191410109532371 \\
c_6 &= 0.218047524190116 \\
c_7 &= -0.191410109532371 \\
c_8 &= 0.127068629704170 \\
c_9 &= -0.059408709991560 \\
c_{10} &= 0.015808536973329 \\
c_{11} &= 0 \\
c_{12} &= -0.002719748296487
\end{aligned}
$$

The equations are organized with these new FIR coefficients and solved by the Trust Region Method with the calculated starting point. 13 terms of the long division result are equated to the FIR coefficients and starting points are calculated by using Eq.(4.7) and Eq.(4.8). The coefficients of filters are found as in Table 8.

Table 8. Coefficients of filters

	IIR-long15	Prony	Yule-Walker
a_0	-0.002719897970076	-0.002719748296487	0.047710508823657
a_1	-0.012622116942944	-0.006599990408012	-0.072439281070350
a_2	-0.019458240567353	0.009677152299683	0.073012825838114
a_3	-0.076172364624734	-0.023220375704050	-0.047987541684431
b_0	1.000000000000000	1.000000000000000	1.000000000000000
b_1	2.183108160074164	2.426691623095489	1.254093600477427
b_2	1.794019981998517	2.254394159035297	0.640024875006352
b_3	0.549914721370090	0.799379104996508	0.130239819143839

The error between the magnitudes of the given FIR and approximated IIR-long14, Prony's, Yule-Walker filters can be calculated by using the Eq.(4.19) as,

$$
\begin{aligned}
\varepsilon_{FIR-IIR15} &= 0.431183888954051 \\
\varepsilon_{FIR-PRONY} &= 1.650292952478328 \\
\varepsilon_{FIR-YULE} &= 0.449274562496702
\end{aligned}
$$

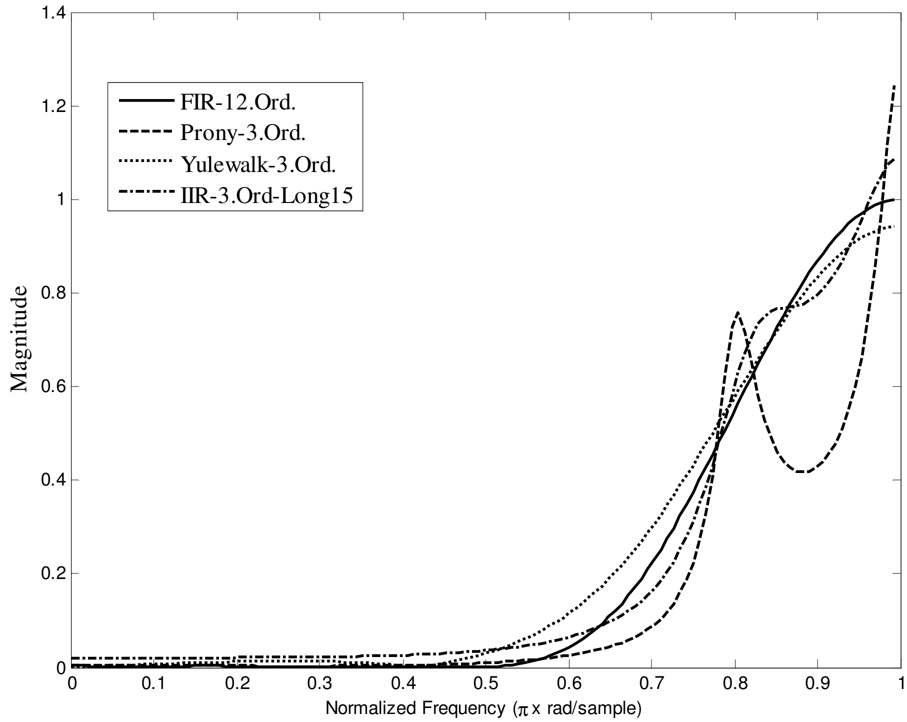

Figure 5.25. Frequency responses of filters

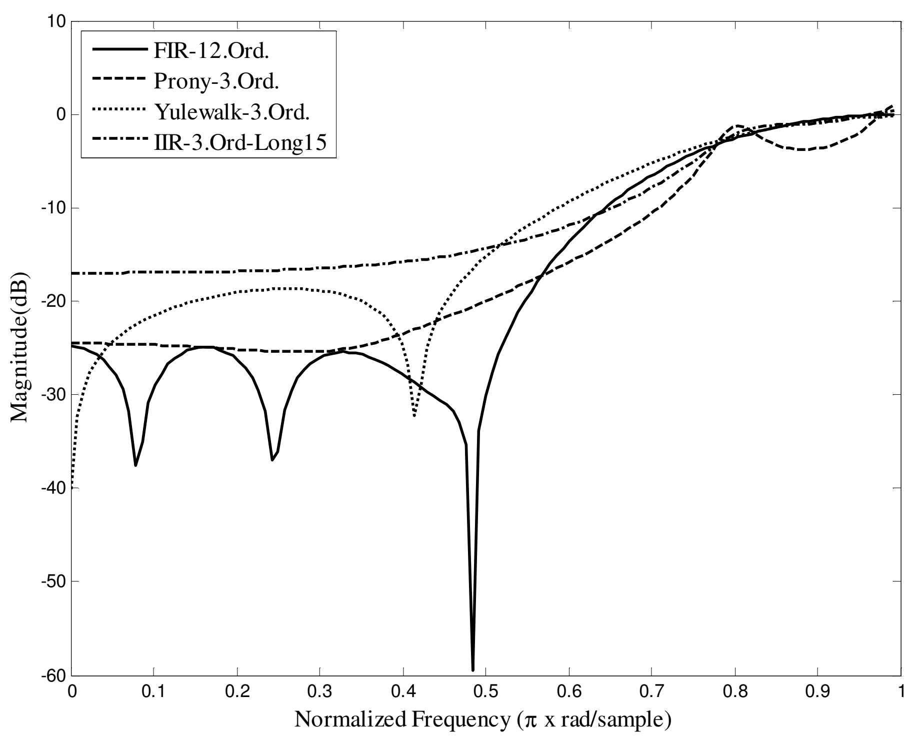

Figure 5.26. Frequency responses of filters

The normalized gain curve of the 12th order FIR characteristics and the gain of its 3rd order IIR, Prony's, Yule-Walker representations are plotted on Fig.5.25. The characteristics are plotted in logarithmic scale to compare the stopbands of the filters on Fig.5.26. The normalized phase response 12th order FIR characteristics and the gain of its 3rd order IIR, Prony's, Yule-Walker representations are plotted on Fig.5.27

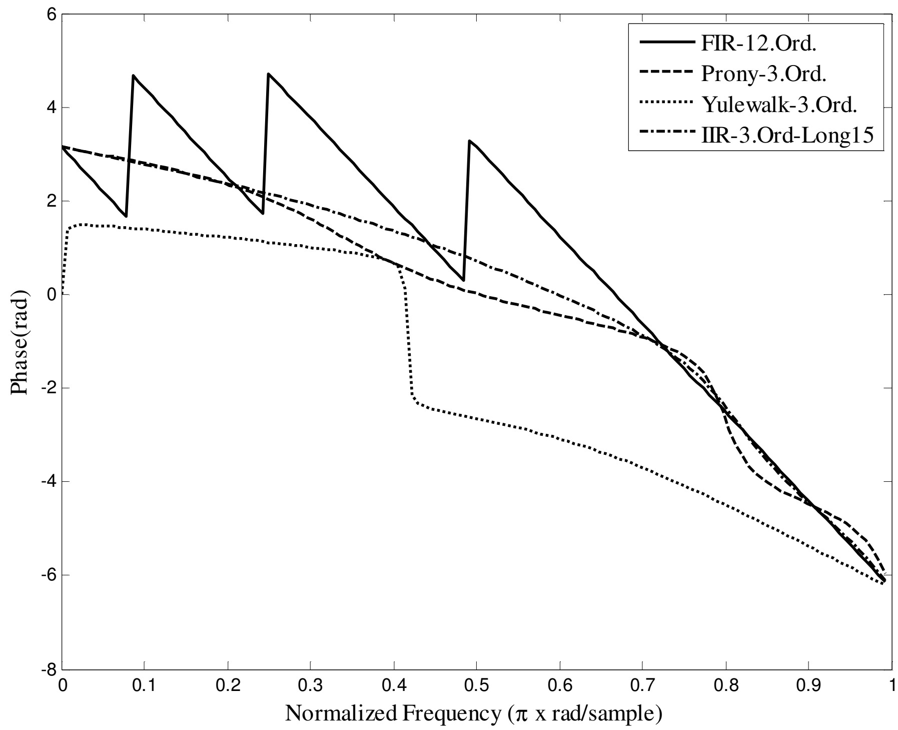

Figure 5.27. Phase responses of filters

In this example the order of IIR filter was increased to 3rd and 12th FIR filter is converted with this method. 15 terms of 3rd order IIR filter is used and the error of IIR filters is better than the Yule-Walker and Prony's methods. Phase responses of Yule-Walker and IIR filters are linear in the passband

Example 10

In this example 12th order lowpass FIR filter which has 0.2 cutoff frequency is converted to 6th order IIR filter and the phase and magnitude characteristics are compared with the 6th order Prony's and Yule-Walker Methods. FIR filter coefficients are

c_0= -0.002719748296487
c_1= 0
c_2= 0.015808536973329
c_3= 0.059408709991560
c_4=0.127068629704170
c_5= 0.191410109532371
c_6= 0.218047524190116
c_7= 0.191410109532371
c_8= 0.127068629704170
c_9= 0.059408709991560
c_{10}=0.015808536973329
c_{11}=0
c_{12}= -0.002719748296487

The equations are organized with these new FIR coefficients and solved by the Trust Region Method with the calculated starting point. 13 terms of the long division result are equated to the FIR coefficients and starting points are calculated by using Eq.(4.7) and Eq.(4.8). The coefficients of filters are found as in Table 9.

Table 9. Coefficients of Filters

	IIR-Long15	Prony	Yule-Walker
a_0	-0.002719748294461	-0.002719748296487	0.019262218977579
a_1	0.007540320455926	0.011837766607666	0.022919081826142
a_2	0.005286997169218	-0.008473783559840	0.030849118919432
a_3	0.024730853009768	0.020362980071623	0.007948328469209
a_4	0.018557771079155	-0.013258448559617	-0.002756875216664
a_5	0.016743484908016	0.006227987884736	-0.014031211785969
a_6	0.009314547974308	0.000149921133293	-0.007505293927384
b_0	1.000000000000000	1.000000000000000	1.000000000000000
b_1	-2.772820105672409	-4.352522850352664	-2.333618439170037
b_2	3.873155797760231	8.928149919070274	2.403996513856158
b_3	-3.392700070136967	-10.942662791834671	-1.419115179127580
b_4	1.935590152775038	8.416329792047295	0.486034357452320
b_5	-0.675772060250289	-3.847640145396304	-0.084352537729126
b_6	0.112276132636499	0.819518446973069	0.004462802663846

The error between the magnitudes of the given FIR and approximated IIR-long15, Prony's, Yule-Walker filters can be calculated by using the Eq.(4.19) as,

$$\varepsilon_{FIR-IIR} = 0.016633391319893$$
$$\varepsilon_{FIR-PRONY} = 2.232499545345478$$
$$\varepsilon_{FIR-YULE} = 0.127856728044188$$

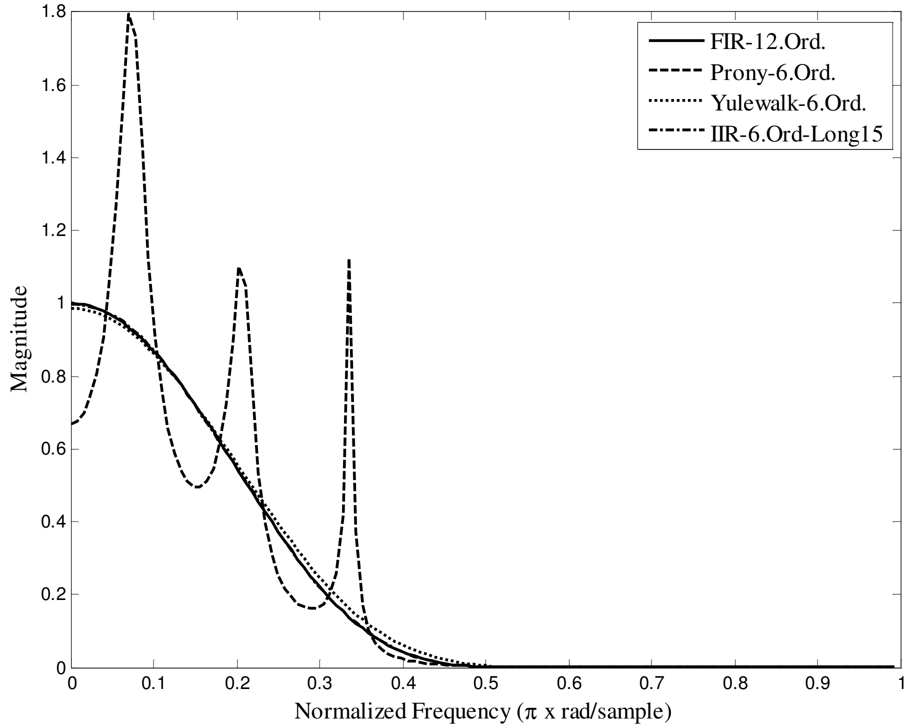

Figure 5.28. Frequency responses of filters

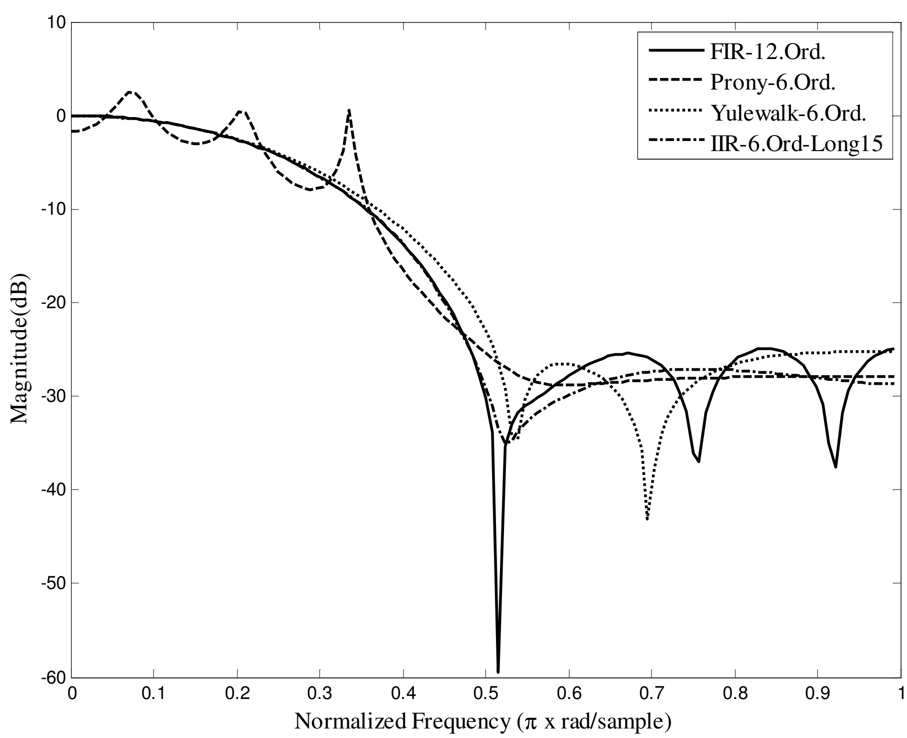

Figure 5.29. Frequency responses of filters

The normalized gain curve of the 12th order FIR characteristics and the gain of its 6th order IIR, Prony's, Yule-Walker representations are plotted on Fig.5.4. The characteristics are plotted in logarithmic scale to compare the stopbands of the filters on Fig.5.5. The normalized phase response 12th order FIR characteristics and the gain of its 6th order IIR, Prony's, Yule-Walker representations are plotted on Fig.5.6.

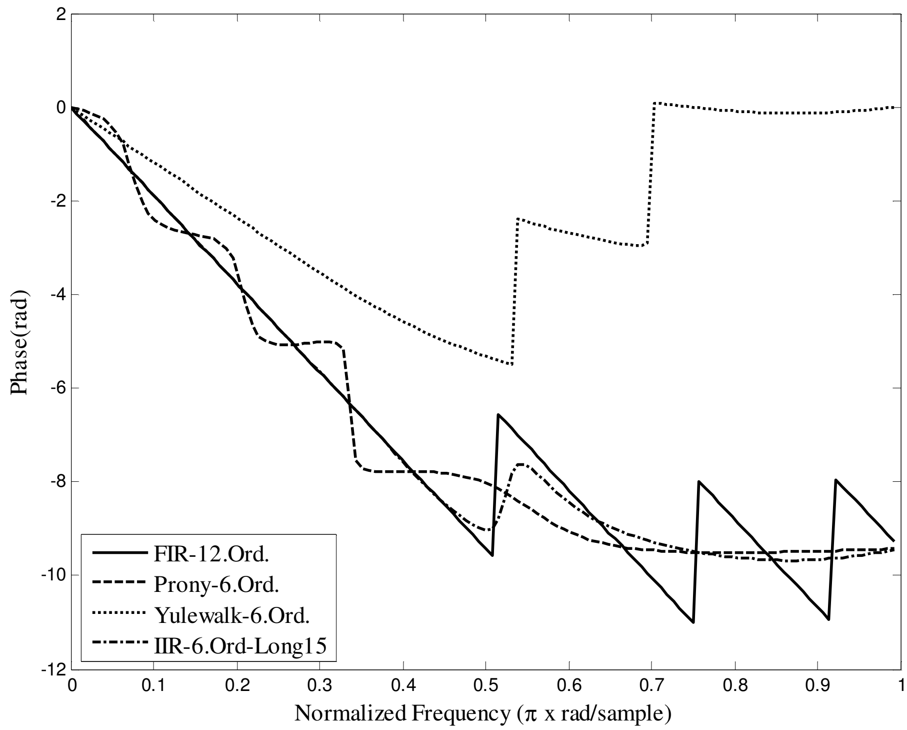

Figure 5.30. Phase responses of filters

In this example the order of IIR filter was increased to 6th and 12th FIR filter is converted with this method. 15 terms of 6th order IIR long division results are used and the error of IIR filters is better than the Yule-Walker and Prony's Metods. There is a significant difference between the errors of Yule-Walker and IIR filters.

Phase responses of Yule-Walker and IIR filters are linear in the passband.

Example 11

In this example 12th order highpass FIR filter which has 0.2 cutoff frequency is converted to 6th Order IIR filter and the phase and magnitude characteristics are compared with the 6th Order Prony's and Yule-Walker Methods. FIR filter coefficients are

c_0=0.002487166176877
c_1=0
c_2=-0.014456653403097
c_3=-0.054328312033057
c_4=-0.116202222959594
c_5= -0.175041473858523
c_6=0.797603848588469
c_7=-0.175041473858523
c_8=0.116202222959594
c_9=-0.054328312033057
c_{10}= -0.014456653403097
c_{11}=0
c_{12}=0.002487166176877

The equations are organized with these new FIR coefficients and solved by the Trust Region Method with the calculated starting point. 13 terms of the long division result are equated to the FIR coefficients and starting points are calculated by using Eq.(4.7) and Eq.(4.8). The coefficients of filters are found as in Table 10.

Table 10. Coefficients of filters

	IIR-Long15	Prony	Yule-Walker
a_0	0.002487166187841	0.005457371000672	0.693703838524968
a_1	0.000792602981558	0	-0.582167447105887
a_2	-0.013706853559393	0.036842536190572	-0.166303192018513
a_3	-0.058133867260105	0	0.021832098168283
a_4	-0.136297950864468	-0.221670656166991	0.063197091757782
a_5	-0.227837828911509	0	0.046723170091386
a_6	0.700543491436302	0.220397197020872	-0.011294496667860
b_0	1.000000000000000	1.000000000000000	1.000000000000000
b_1	0.313193439961132	0.000000000000001	-0.180509394991830
b_2	0.273342886072584	0.938467853575626	-0.080526259682802
b_3	0.200785644003198	0.000000000000001	-0.036532266477873
b_4	0.126142582310235	0.647307834964657	-0.011937699700733
b_5	0.068297735463401	0.000000000000001	0.013257928304138
b_6	0.028979384198530	0.296710397276996	-0.017690565980665

The error between the magnitudes of the given FIR and approximated IIR-long15, Prony's, Yule-Walker filters can be calculated by using the Eq.(4.19) as,

$$\varepsilon_{FIR-IIR} = 0.197178997967998$$
$$\varepsilon_{FIR-PRONY} = 0.320204191229605$$
$$\varepsilon_{FIR-YULE} = 0.092780986513754$$

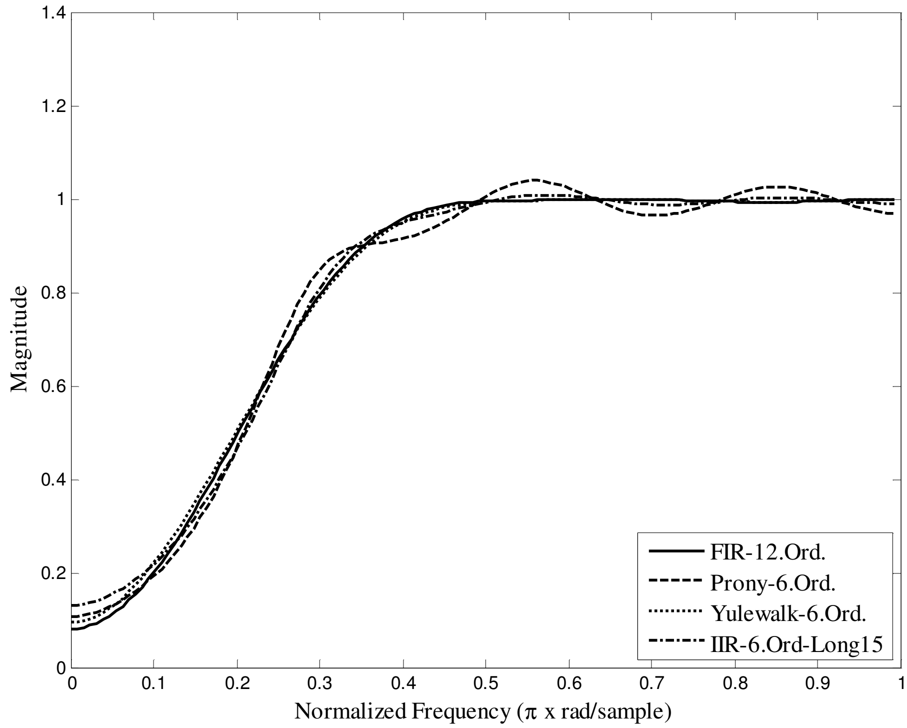

Figure 5.31. Frequency responses of filters

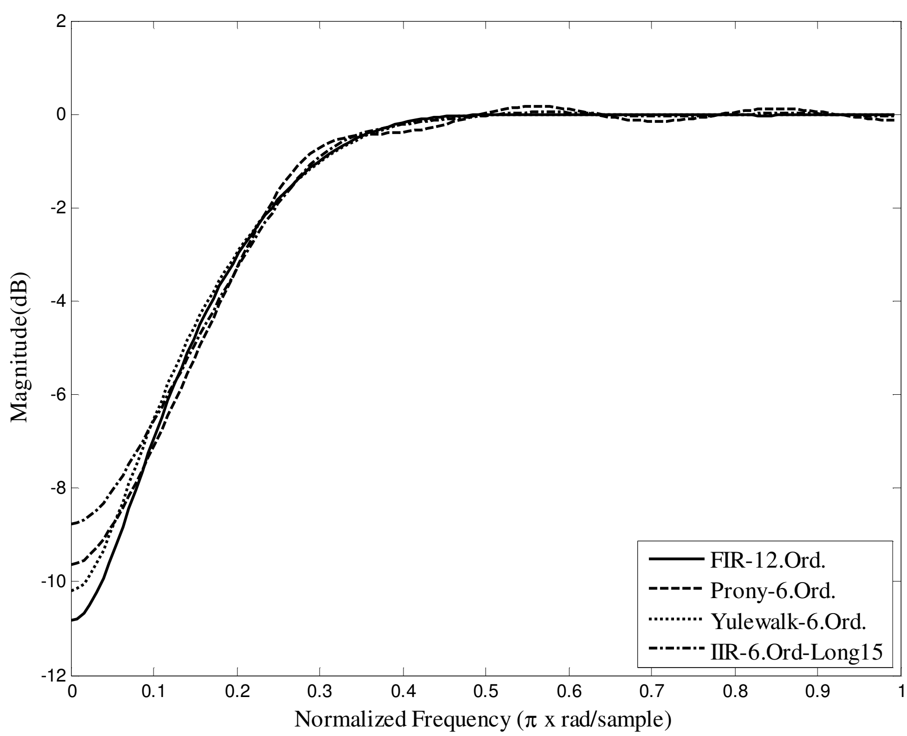

Figure 5.32. Frequency responses of filters

The normalized gain curve of the 12th order FIR characteristics and the gain of its 6th order IIR, Prony's, Yule-Walker representations are plotted on Fig.5.31. The characteristics are plotted in logarithmic scale to compare the stopbands of the filters on Fig.5.32. The normalized phase response 12th order FIR characteristics and the gain of its 6th order IIR, Prony's, Yule-Walker representations are plotted on Fig.5.33.

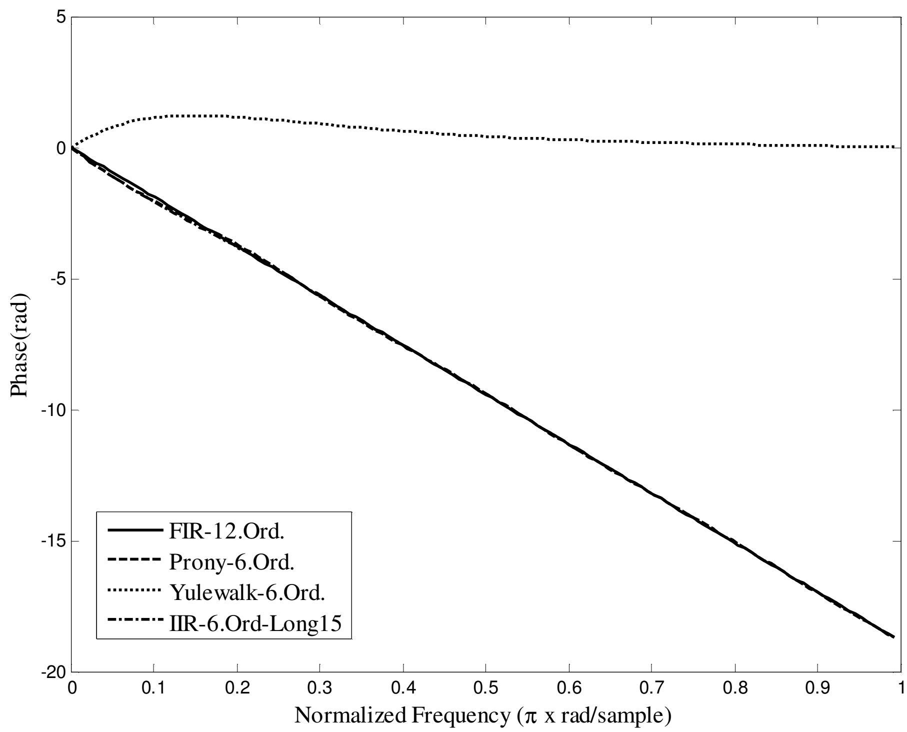

Figure 5.33. Phase responses of filters

In this example the order of IIR filter was increased to 6th and 12th FIR filter is converted with this method. 15 terms of 6th order IIR long division results are used and the error of IIR filters is nearly same with the Yule-Walker Metod.

Phase responses of Yule-Walker, Prony's and IIR filters are linear in the passband. Prony's and IIR filters are nearly linear in all frequencies.

Example 12

In this example 12^{th} order bandpass FIR filter which has passband between the 0.3 and 0.7 frequencies is converted to 6^{th} order IIR filter and the phase and magnitude characteristics are compared with the 6^{th} Order Prony's and Yule-Walker Methods. FIR filter coefficients are

c_0=0.005457371000672
c_1=0
c_2=0.031720968941406
c_3=0
c_4=0.254972364809816
c_5= 0
c_6=0.437528074498901
c_7=0
c_8=-0.254972364809816
c_9=0
c_{10}= 0.031720968941406
c_{11}=0
c_{12}=0.005457371000672

The equations are organized with these new FIR coefficients and solved by the Trust Region Method with the calculated starting point. 13 terms of the long division result are equated to the FIR coefficients and starting points are calculated by using Eq.(4.7) and Eq.(4.8). The coefficients of filters are found as in Table 11.

Table 11. Coefficients of Filters

	IIR –Long15	Prony	Yule-Walker
a_0	0.005457370997285	0.005457371000672	0.237884984115334
a_1	0.000000000003804	0	-0.020570930502208
a_2	0.036153565920184	0.036842536190572	-0.343739398225173
a_3	0.000000000036720	0	0.030511405180535
a_4	-0.226777616896603	-0.221670656166991	0.106467752436635
a_5	-0.000000001110893	0	-0.010641721985738
a_6	0.257385829891254	0.220397197020872	0.017903797213262
b_0	1.000000000000000	1.000000000000000	1.000000000000000
b_1	0.778160027354846	0.000000000000001	-0.039857361724509
b_2	0.000000003719506	0.938467853575626	0.419344686526277
b_3	0.000000004530558	0.000000000000001	-0.004051229999570
b_4	0.398952289306462	0.647307834964657	0.118112308761217
b_5	0.000000002265440	0.000000000000001	-0.000732375998110
b_6	0.112696003118024	0.296710397276996	0.018839515908309

The error between the magnitudes of the given FIR and approximated IIR-long15, Prony's, Yule-Walker filters can be calculated by using the Eq.(4.19) as,

$$\varepsilon_{FIR-IIR} = 0.188878628184022$$
$$\varepsilon_{FIR-PRONY} = 0.659527064041658$$
$$\varepsilon_{FIR-YULE} = 0.210240826900779$$

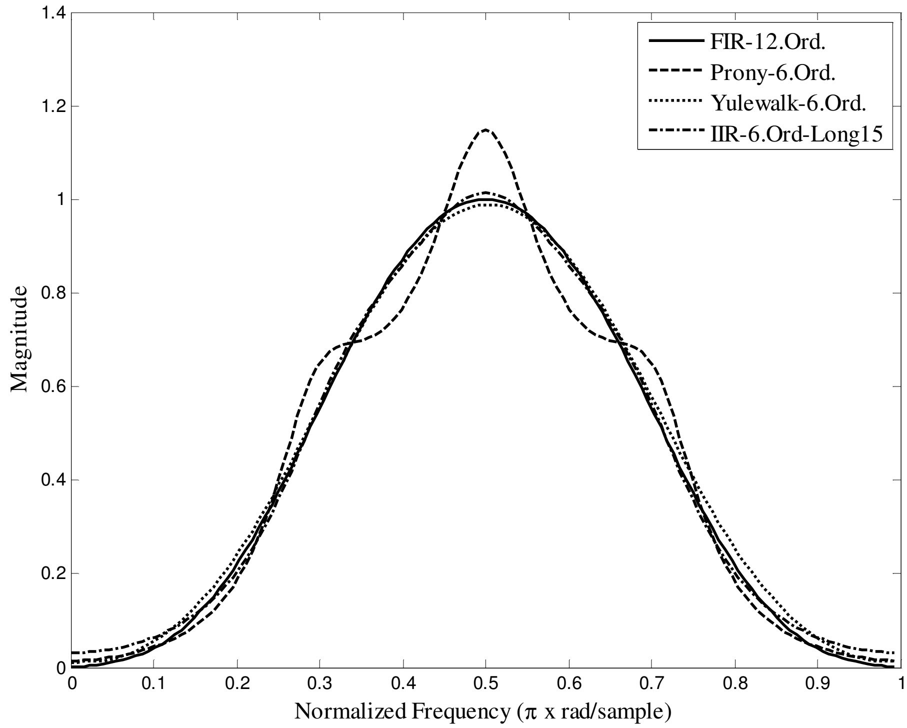

Figure 5.34. Frequency responses of filters

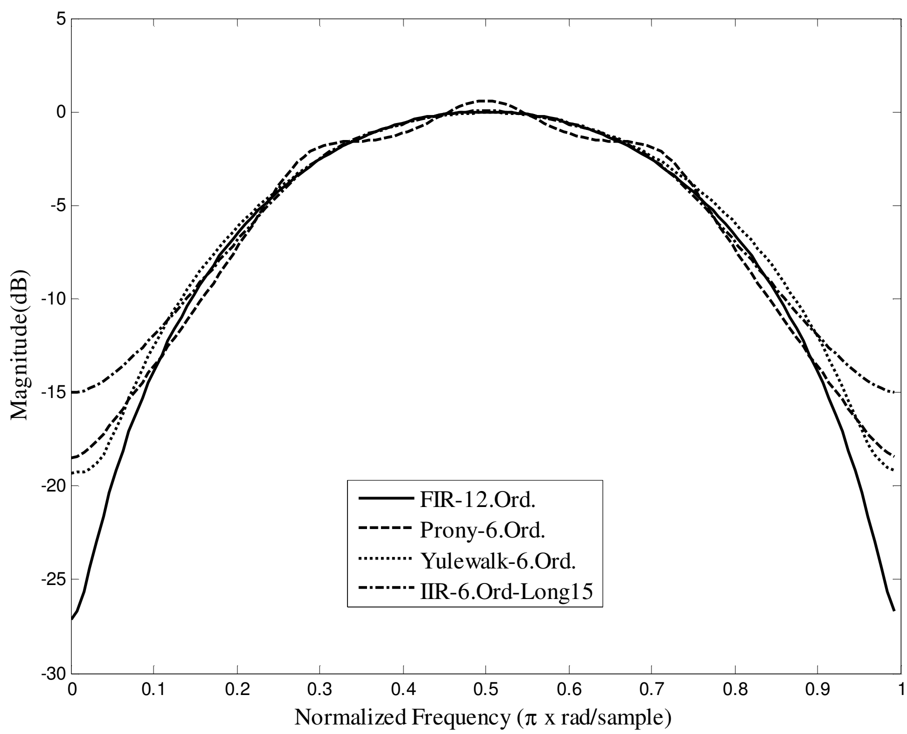

Figure 5.35. Frequency responses of filters

The normalized gain curve of the 12^{th} order FIR characteristics and the gain of its 6^{th} order IIR, Prony's, Yule-Walker representations are plotted on Fig.5.34. The characteristics are plotted in logarithmic scale to compare the stopbands of the filters on Fig.5.35. The normalized phase response 12^{th} order FIR characteristics and the gain of its 6^{th} order IIR, Prony's, Yule-Walker representations are plotted on Fig.5.36.

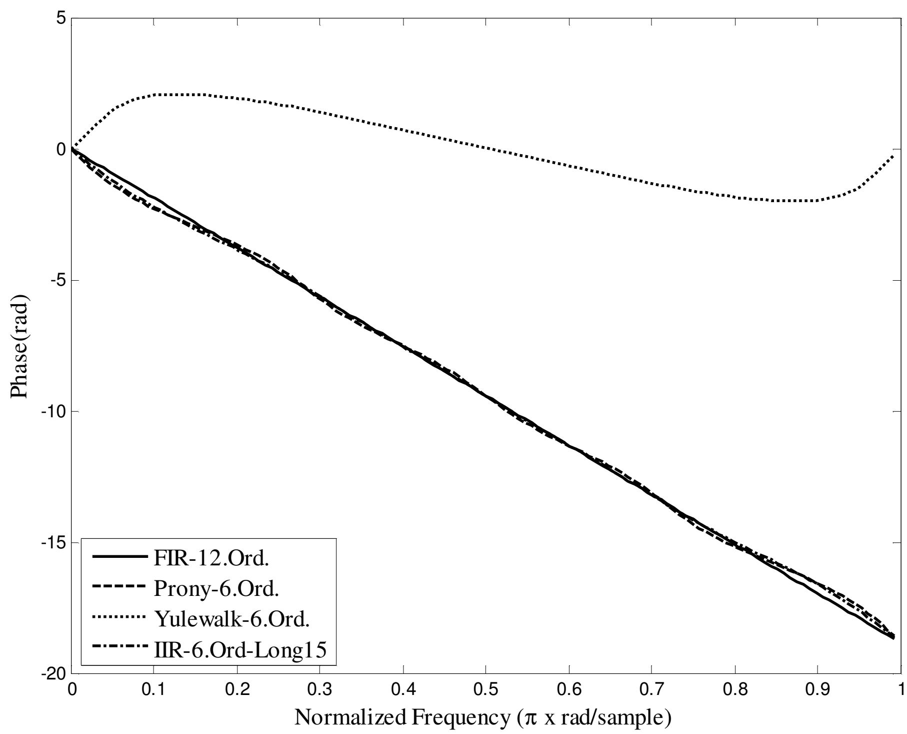

Figure 5.36. Phase responses of filters

In this example the order of IIR filter was increased to 6^{th} and 12^{th} FIR filter is converted with this method. 15 terms of 6^{th} order IIR long division results are used and the error of IIR filters is better than the Yule-Walker and Prony's Metods.

Phase responses of Yule-Walker and IIR filters are linear in the passband.

Example 13

In this example 12th Order arbitrary frequency response FIR filter is converted to 6th Order IIR filter and the phase and magnitude characteristics are compared with the 6th Order Prony's and Yule-Walker Methods. FIR filter coefficients are

c_0=0.012689734583583
c_1=0
c_2=-0.075479149435609
c_3=0
c_4=-0.088927318590436
c_5= 0
c_6=0.402343750000000
c_7=0
c_8=-0.088927318590436
c_9=0
c_{10}= -0.075479149435609
c_{11}=0
c_{12}=0.012689734583583

The equations are organized with these new FIR coefficients and solved by the Trust Region Method with the calculated starting point. 13 terms of the long division result are equated to the FIR coefficients and starting points are calculated by using Eq.(4.7) and Eq.(4.8). The coefficients of filters are found as in Table 12.

Table 12. Coefficients of Filters

	IIR-long15	Prony	Yule-Walker
a_0	0.012689734584362	0.012689734583583	0.367279445806776
a_1	0.000000000002885	0	0.004084890652672
a_2	-0.071988081319712	-0.071711428985743	-0.244979561019126
a_3	-0.000000000025012	0	-0.004476379015173
a_4	-0.106098612325383	-0.107888287670756	-0.076417541533428
a_5	-0.000000000067567	0	0.002171215998614
a_6	0.362457725202785	0.356490834081067	0.054388906025997
b_0	1.000000000000000	1.000000000000000	1.000000000000000
b_1	0.000000000298994	0	0.028939990676344
b_2	0.272026050090196	0.296910894790524	-0.114164324280983
b_3	0.000000000323808	0	0.002712165011677
b_4	0.234434626223041	0.271842779209678	0.028400613477566
b_5	0.000000000413867	0	0.000228687654152
b_6	0.047157236519489	0.084244123704320	-0.017172285450162

The error between the magnitudes of the given FIR and approximated IIR-long15, Prony's, Yule-Walker filters can be calculated by using the Eq.(4.19) as,

$$\varepsilon_{FIR-IIR} = 0.157746612352874$$
$$\varepsilon_{FIR-PRONY} = 0.204388573426533$$
$$\varepsilon_{FIR-YULE} = 0.156863565518397$$

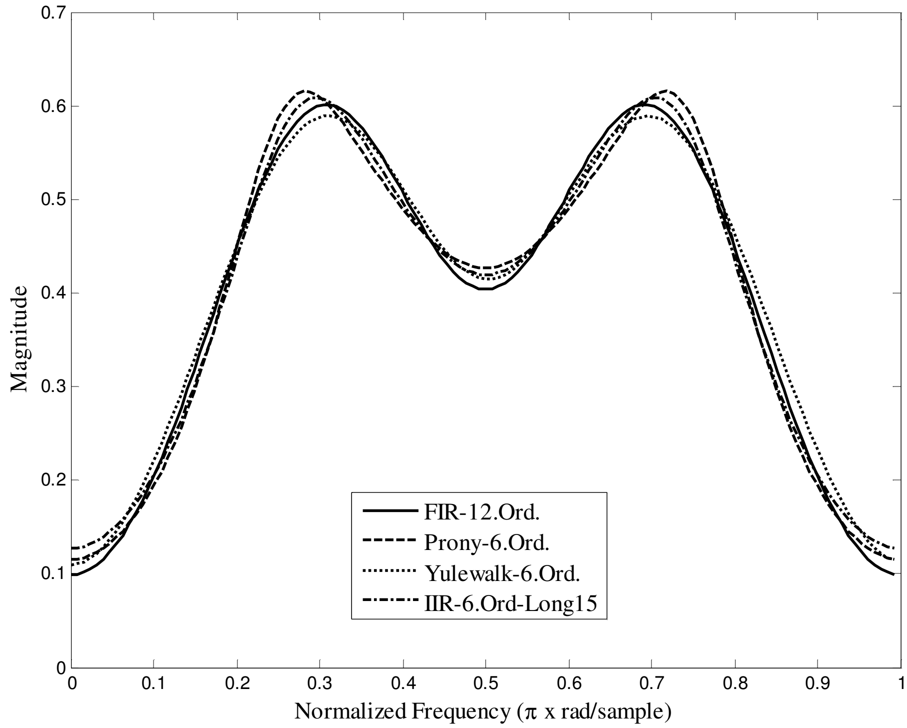

Figure 5.37. Frequency responses of filters

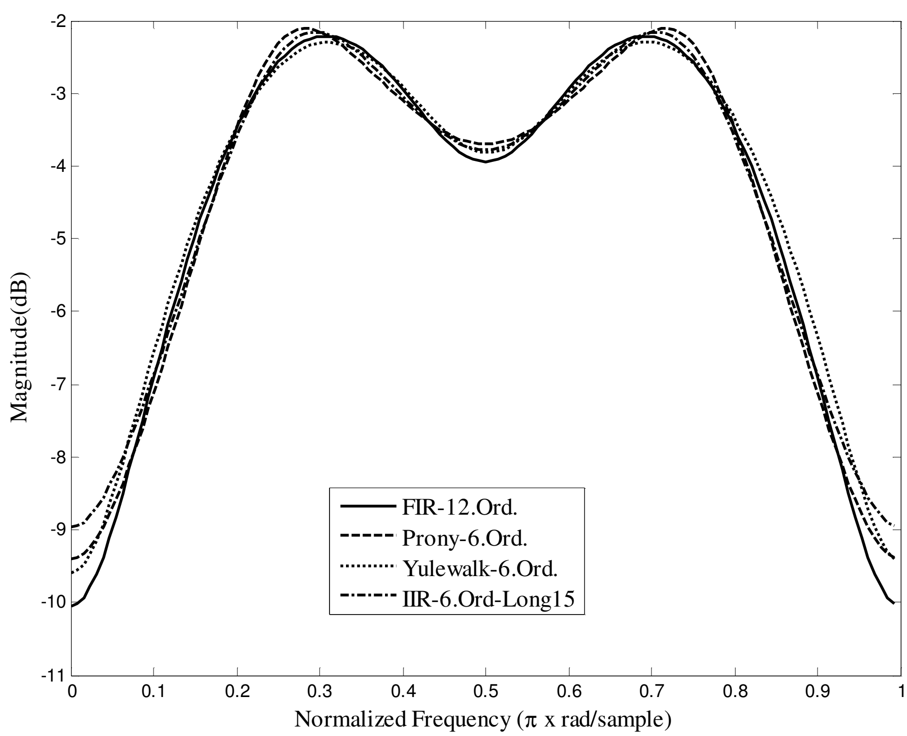

Figure 5.38. Frequency responses of filters

The normalized gain curve of the 12th order FIR characteristics and the gain of its 6th order IIR, Prony's, Yule-Walker representations are plotted on Fig.5.37. The characteristics are plotted in logarithmic scale to compare the stopbands of the filters on Fig.5.38. The normalized phase response 12th order FIR characteristics and the gain of its 6th order IIR, Prony's, Yule-Walker representations are plotted on Fig.5.39.

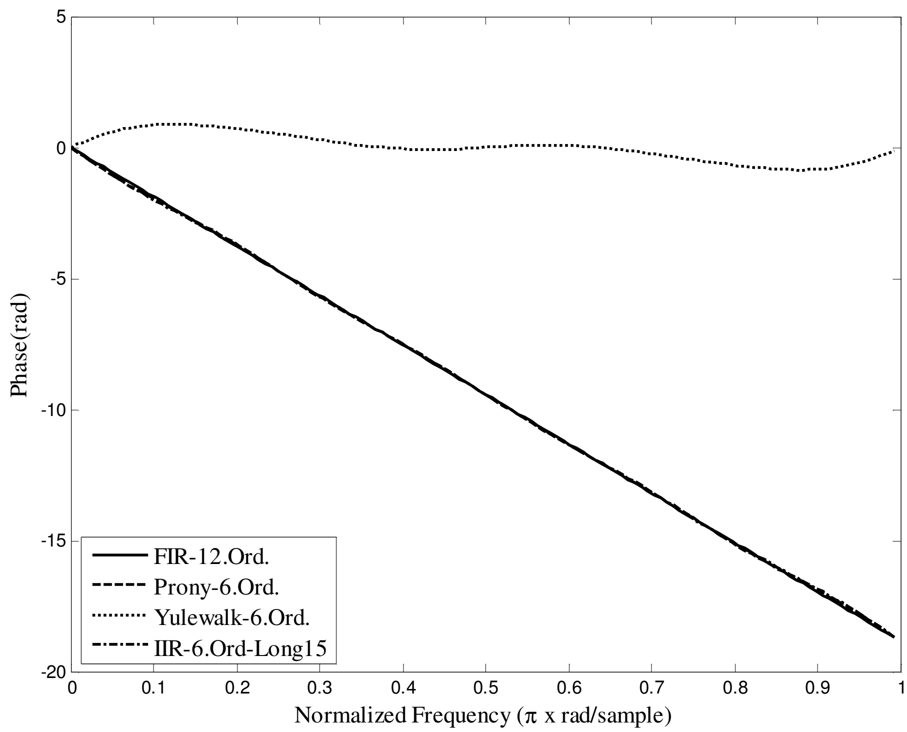

Figure 5.39. Frequency responses of filters

Example 14

In this example 15th order lowpass FIR filter which has 0.2 cutoff frequency is converted to 6th order IIR filter and the phase and magnitude characteristics are compared with the 6th order Prony's and Yule-Walker Methods. FIR filter coefficients are

c_0=-0.003471276445115	c_8=0.199106379157941
c_1=-0.004851203731090	c_9=0.160100278194831
c_2=-0.004245630750047	c_{10}= 0.100233107352696
c_3=0.008891029945241	c_{11}=0.044237316275544
c_4=0.044237316275544	c_{12}=0.008891029945241
c_5= 0.100233107352696	c_{13}=-0.004245630750047
c_6=0.160100278194831	c_{14}=-0.004851203731090
c_7=0.199106379157941	c_{15}=-0.003471276445115

The equations are organized with these new FIR coefficients and solved by the Trust Region Method with the calculated starting point. 16 terms of the long division result are equated to the FIR coefficients and starting points are calculated by using Eq.(4.7) and Eq.(4.8). The coefficients of filters are found as

Table 13. Coefficients of filters

	IIR-long19	Prony	Yule-Walker
a_0	-0.003358854462931	-0.003471276445115	0.019948272500534
a_1	0.003405139390548	0.011317653283646	0.016979826822949
a_2	-0.000807737808617	-0.016180435035325	0.027536020738919
a_3	0.009379029292630	0.023472987137416	0.017754439358695
a_4	0.023442456281692	-0.012225396685590	0.009700320418275
a_5	0.007512861472165	0.003903038256586	0.001383598012662
a_6	0.030584528591641	0.000205865938038	-0.001566005278995
b_0	1.000000000000000	1.000000000000000	1.000000000000000
b_1	-2.454872639421747	-4.657899556657375	-2.372922782859964
b_2	2.671142318992081	9.947702103118671	2.629042593767042
b_3	-1.619850697458879	-12.405973953133509	-1.676147628149048
b_4	0.615528132179041	9.506251153043911	0.628755331994045
b_5	-0.184450404500229	-4.241427118152392	-0.125542365948521
b_6	0.043848949570486	0.862051491620245	0.009601143253991

The error between the magnitudes of the given FIR and approximated IIR-long19, Prony's, Yule-Walker filters can be calculated by using the Eq.(4.19) as,

$$\varepsilon_{FIR-IIR} = 0.078826319159028$$
$$\varepsilon_{FIR-PRONY} = 2.528988481964258$$
$$\varepsilon_{FIR-YULE} = 0.138553927954529$$

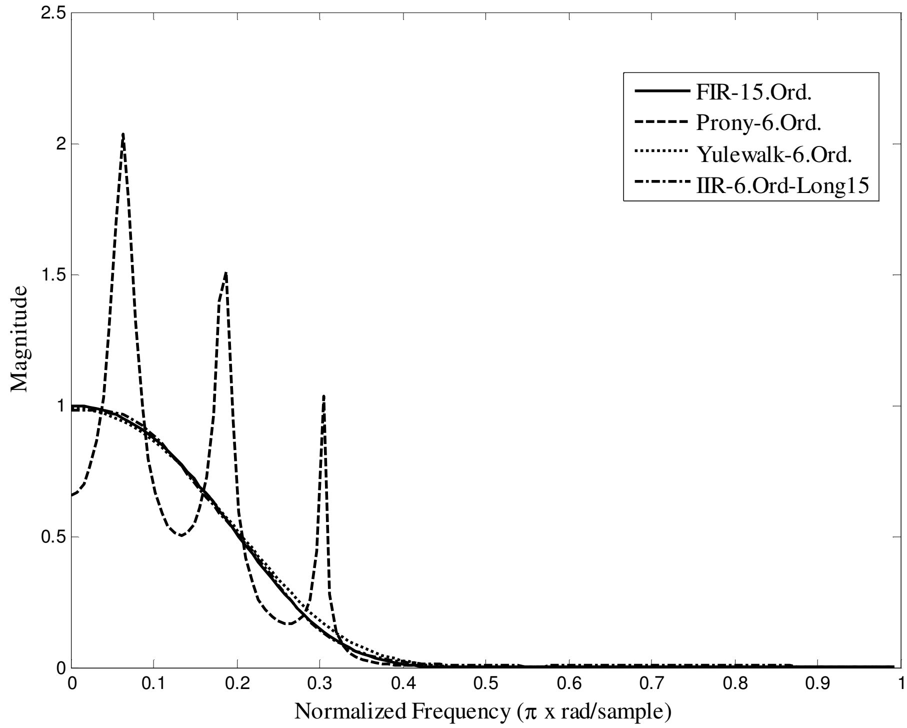

Figure 5.40. Frequency responses of filters

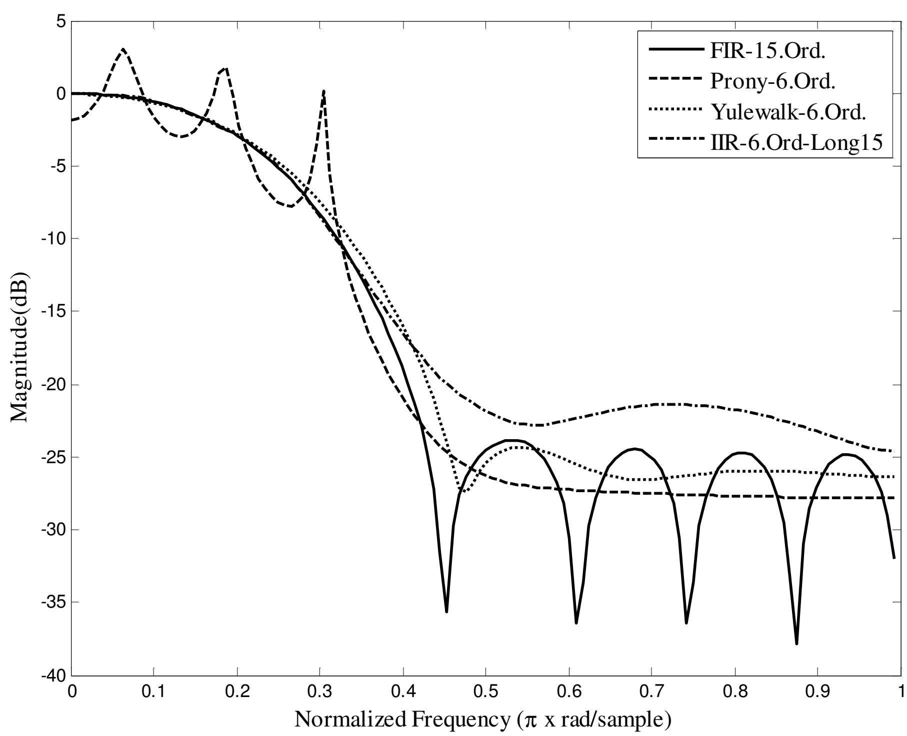

Figure 5.41. Frequency responses of filters

The normalized gain curve of the 12th order FIR characteristics and the gain of its 6th order IIR, Prony's, Yule-Walker representations are plotted on Fig.5.40. The characteristics are plotted in logarithmic scale to compare the stopbands of the filters on Fig.5.41. The normalized phase response 12th order FIR characteristics and the gain of its 6th order IIR, Prony's, Yule-Walker representations are plotted on Fig.5.42

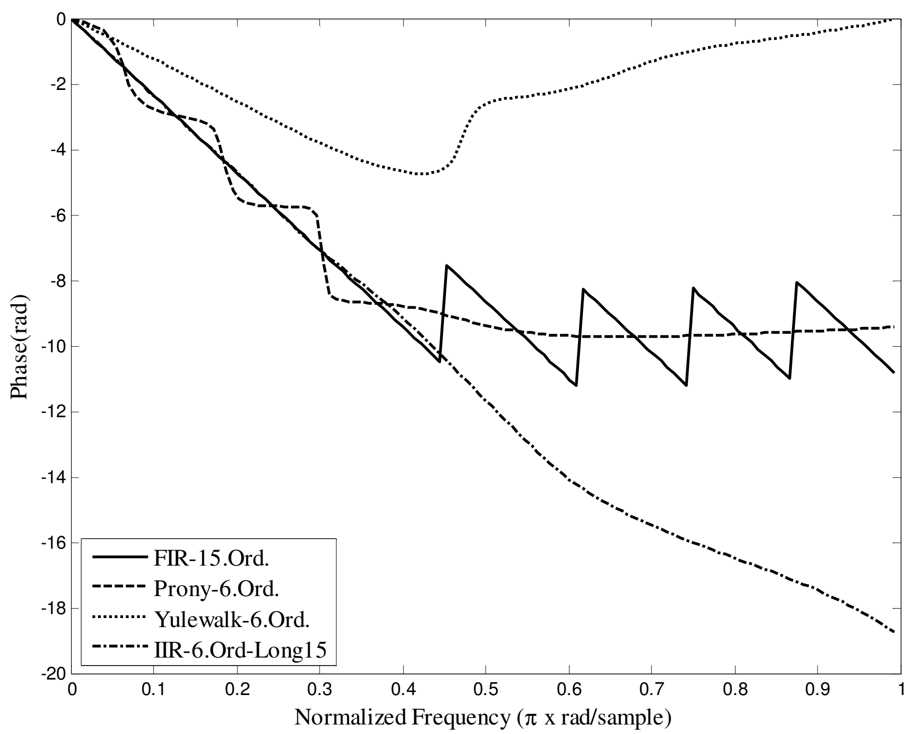

Figure 5.42. Phase responses of filters

Example 15

An algorithm is prepared for roundation of coefficients of IIR, FIR, Yule-Walker filters from 1 bit to 20 bit for the Example 13. QFIR, QIIR, QYULE coefficients are the quantized coefficients of the FIR, IIR, Yule-Walker filters. Errors are calculated like in Eq.(4.32) to show the effect of coefficient quantization

Table 14. Errors between the quantized and un-quantized filters

Bit	QFIR-FIR	QIIR-IIR	QYULE-YULE
1	4.5075	NaN	NaN
2	2.4051	NaN	4.4994
3	1.2804	NaN	4.4994
4	0.6566	4.506	4.4994
5	0.354	2.0209	2.6012
6	0.2034	0.6922	0.2353
7	0.1131	0.2295	0.2643
8	0.037	0.1095	0.1563
9	0.0265	0.1113	0.0352
10	0.0117	0.0466	0.0714
11	0.0052	0.0099	0.0155
12	0.0037	0.0046	0.0186
13	0.0017	0.0065	0.0085
14	0.0008	0.0034	0.0007
15	0.0004	0.0015	0.0008
16	0.0002	0.0008	0.0009
17	0.0001	0.0003	0.0002
18	0.0001	0.0001	0.0003
19	0	0	0
20	0	0	0.0001

Errors are plotted against the number of bit roundation in Fig.5.43

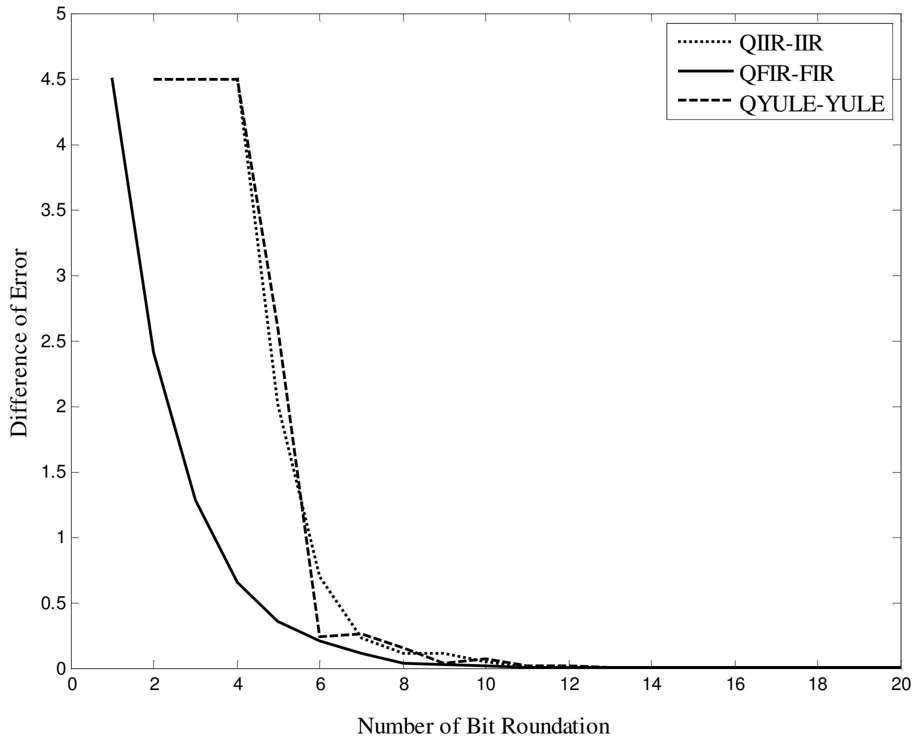

Figure 5.43. Bit roundation errors of filters

The results given in Table 15 for this example show that more bit results with less error. 10 and 12 bit roundation of coefficients gives a acceptable error. After the roundation of 12, the error is not so much changing.

Example 16

An algorithm is prepared for roundation of coefficients of IIR, FIR, Yule-Walker filters from 1 bit to 20 bit for the Example 11. QFIR, QIIR, QYULE coefficients are the quantized coefficients of the FIR, IIR, Yule-Walker filters. Errors are calculated like in Eq.(4.32) to show the effect of coefficient quantization

Table 15. Errors between the quantized and un-quantized filters

Bit	QFIR-FIR	QIIR-IIR	QYULE-YULE
1	4.2161	3.6364	2.3228
2	2.4451	2.022	1.5749
3	1.3208	1.3589	1.4378
4	0.398	0.3819	0.3953
5	0.398	0.2439	0.2037
6	0.176	0.125	0.1654
7	0.0688	0.0458	0.0642
8	0.0369	0.0365	0.0227
9	0.0265	0.0133	0.0101
10	0.0107	0.0082	0.0065
11	0.0059	0.0027	0.0042
12	0.0022	0.0023	0.0015
13	0.0011	0.0009	0.0008
14	0.0004	0.0004	0.0005
15	0.0004	0.0003	0.0003
16	0.0002	0.0001	0.0001
17	0	0.0001	0
18	0	0	0
19	0	0	0
20	0	0	0

Errors are plotted against the number of bit roundation in Fig.5.44.

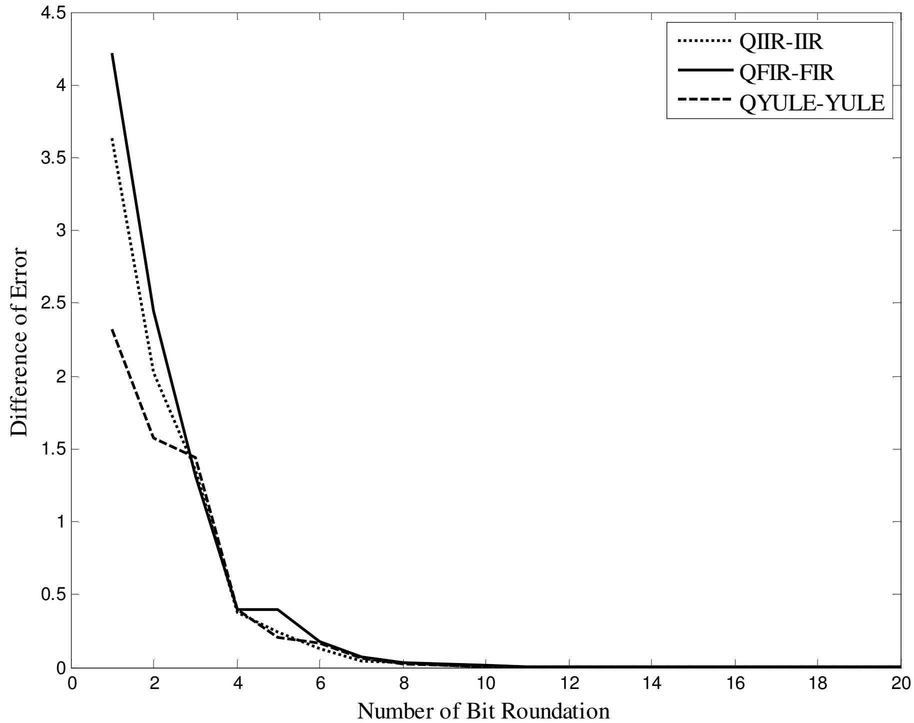

Figure 5.44. Bit roundation errors of filters

Example 17

An algorithm is prepared for roundation of coefficients of IIR, FIR, Yule-Walker filters from 1 bit to 20 bit for the Example 12. QFIR, QIIR, QYULE coefficients are the quantized coefficients of the FIR, IIR, Yule-Walker filters. Errors are calculated like in Eq.(4.32) to show the effect of coefficient quantization.

Table 16. Errors between the quantized and un-quantized filters

bit	QFIR-FIR	QIIR-IIR	QYULE-YULE
1	3.848	2.8269	2.1157
2	0.8781	1.7828	1.11
3	0.8781	0.3489	0.3421
4	0.5064	0.4067	0.471
5	0.1184	0.2337	0.2317
6	0.1184	0.0595	0.0404
7	0.0595	0.0523	0.0372
8	0.031	0.0179	0.0285
9	0.0173	0.008	0.0242
10	0.01	0.0099	0.0039
11	0.002	0.0036	0.0029
12	0.002	0.0016	0.002
13	0.0009	0.0013	0.0012
14	0.0007	0.0003	0.0006
15	0.0002	0.0001	0.0002
16	0.0001	0.0002	0.0001
17	0.0001	0.0001	0
18	0	0	0
19	0	0	0
20	0	0	0

Errors are plotted against the number of bit roundation in Fig.5.45

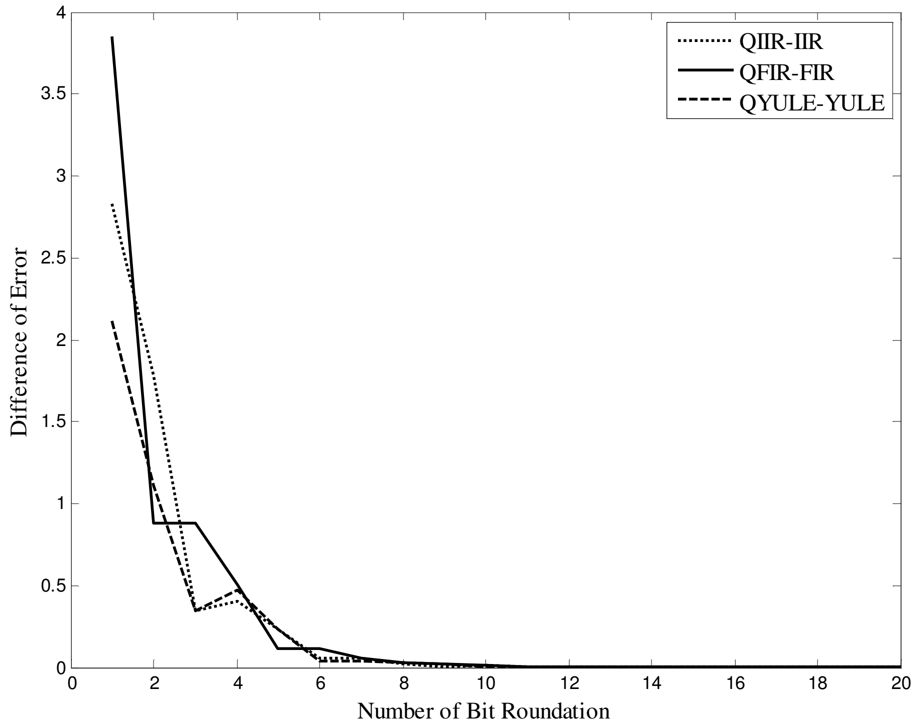

Figure 5.45. Bit roundation errors of filters

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APPENDIX

File name: myfilter5.m

%Non-linear equations is defined as a function of MATLAB in this program

%Coefficients of IIR filter is defined like

% a_0=x(1)

% a_1=x(2)

% a_2=x(3)

% b_1=x(4)

% b_2=x(5)

```
function f=flow20410(x)
a=0.028406470015011
b=0.237008213590703
c=0.469170632788571
d=0.237008213590703
e=0.028406470015011

f=[x(1)-a;
    x(2)-x(1)*x(4)-b;
    x(3)-x(4)*(x(2)-x(1)*x(4))-x(5)*x(1)-c;
    -x(4)*(x(3)-x(4)*(x(2)-x(1)*x(4))-x(5)*x(1))-x(5)*(x(2)-x(1)*x(4))-d;
    -x(4)*(-x(4)*(x(3)-x(4)*(x(2)-x(1)*x(4))-x(5)*x(1))-x(5)*(x(2)-x(1)*x(4)))-
x(5)*(x(3)-x(4)*(x(2)-x(1)*x(4))-x(5)*x(1))-e;
    -x(4)*(-x(4)*(-x(4)*(x(3)-x(4)*(x(2)-x(1)*x(4))-x(5)*x(1))-x(5)*(x(2)-
x(1)*x(4)))-x(5)*(x(3)-x(4)*(x(2)-x(1)*x(4))-x(5)*x(1)))-x(5)*(-x(4)*(x(3)-
x(4)*(x(2)-x(1)*x(4))-x(5)*x(1))-x(5)*(x(2)-x(1)*x(4)));
    -x(4)*(-x(4)*(-x(4)*(-x(4)*(x(3)-x(4)*(x(2)-x(1)*x(4))-x(5)*x(1))-x(5)*(x(2)-
x(1)*x(4)))-x(5)*(x(3)-x(4)*(x(2)-x(1)*x(4))-x(5)*x(1)))-x(5)*(-x(4)*(x(3)-
x(4)*(x(2)-x(1)*x(4))-x(5)*x(1))-x(5)*(x(2)-x(1)*x(4))))-x(5)*(-x(4)*(-x(4)*(x(3)-
x(4)*(x(2)-x(1)*x(4))-x(5)*x(1))-x(5)*(x(2)-x(1)*x(4)))-x(5)*(x(3)-x(4)*(x(2)-
x(1)*x(4))-x(5)*x(1)));
    -x(4)*(-x(4)*(-x(4)*(-x(4)*(-x(4)*(x(3)-x(4)*(x(2)-x(1)*x(4))-x(5)*x(1))-
x(5)*(x(2)-x(1)*x(4)))-x(5)*(x(3)-x(4)*(x(2)-x(1)*x(4))-x(5)*x(1)))-x(5)*(-
x(4)*(x(3)-x(4)*(x(2)-x(1)*x(4))-x(5)*x(1))-x(5)*(x(2)-x(1)*x(4))))-x(5)*(-x(4)*(-
x(4)*(x(3)-x(4)*(x(2)-x(1)*x(4))-x(5)*x(1))-x(5)*(x(2)-x(1)*x(4)))-x(5)*(x(3)-
x(4)*(x(2)-x(1)*x(4))-x(5)*x(1))))-x(5)*(-x(4)*(-x(4)*(-x(4)*(x(3)-x(4)*(x(2)-
x(1)*x(4))-x(5)*x(1))-x(5)*(x(2)-x(1)*x(4)))-x(5)*(x(3)-x(4)*(x(2)-x(1)*x(4))-
x(5)*x(1)))-x(5)*(-x(4)*(x(3)-x(4)*(x(2)-x(1)*x(4))-x(5)*x(1))-x(5)*(x(2)-
x(1)*x(4))));
    -x(4)*(-x(4)*(-x(4)*(-x(4)*(-x(4)*(-x(4)*(x(3)-x(4)*(x(2)-x(1)*x(4))-x(5)*x(1))-
x(5)*(x(2)-x(1)*x(4)))-x(5)*(x(3)-x(4)*(x(2)-x(1)*x(4))-x(5)*x(1)))-x(5)*(-
```

```
x(4)*(x(3)-x(4)*(x(2)-x(1)*x(4))-x(5)*x(1))-x(5)*(x(2)-x(1)*x(4))))-x(5)*(-x(4)*(-
x(4)*(x(3)-x(4)*(x(2)-x(1)*x(4))-x(5)*x(1))-x(5)*(x(2)-x(1)*x(4)))-x(5)*(x(3)-
x(4)*(x(2)-x(1)*x(4))-x(5)*x(1))))-x(5)*(-x(4)*(-x(4)*(-x(4)*(x(3)-x(4)*(x(2)-
x(1)*x(4))-x(5)*x(1))-x(5)*(x(2)-x(1)*x(4)))-x(5)*(x(3)-x(4)*(x(2)-x(1)*x(4))-
x(5)*x(1)))-x(5)*(-x(4)*(x(3)-x(4)*(x(2)-x(1)*x(4))-x(5)*x(1))-x(5)*(x(2)-
x(1)*x(4)))))-x(5)*(-x(4)*(-x(4)*(-x(4)*(-x(4)*(x(3)-x(4)*(x(2)-x(1)*x(4))-
x(5)*x(1))-x(5)*(x(2)-x(1)*x(4)))-x(5)*(x(3)-x(4)*(x(2)-x(1)*x(4))-x(5)*x(1)))-
x(5)*(-x(4)*(x(3)-x(4)*(x(2)-x(1)*x(4))-x(5)*x(1))-x(5)*(x(2)-x(1)*x(4))))-x(5)*(-
x(4)*(-x(4)*(x(3)-x(4)*(x(2)-x(1)*x(4))-x(5)*x(1))-x(5)*(x(2)-x(1)*x(4)))-
x(5)*(x(3)-x(4)*(x(2)-x(1)*x(4))-x(5)*x(1))));
    -x(4)*(-x(4)*(-x(4)*(-x(4)*(-x(4)*(-x(4)*(-x(4)*(x(3)-x(4)*(x(2)-x(1)*x(4))-
x(5)*x(1))-x(5)*(x(2)-x(1)*x(4)))-x(5)*(x(3)-x(4)*(x(2)-x(1)*x(4))-x(5)*x(1)))-
x(5)*(-x(4)*(x(3)-x(4)*(x(2)-x(1)*x(4))-x(5)*x(1))-x(5)*(x(2)-x(1)*x(4))))-x(5)*(-
x(4)*(-x(4)*(x(3)-x(4)*(x(2)-x(1)*x(4))-x(5)*x(1))-x(5)*(x(2)-x(1)*x(4)))-
x(5)*(x(3)-x(4)*(x(2)-x(1)*x(4))-x(5)*x(1))))-x(5)*(-x(4)*(-x(4)*(-x(4)*(x(3)-
x(4)*(x(2)-x(1)*x(4))-x(5)*x(1))-x(5)*(x(2)-x(1)*x(4)))-x(5)*(x(3)-x(4)*(x(2)-
x(1)*x(4))-x(5)*x(1)))-x(5)*(-x(4)*(x(3)-x(4)*(x(2)-x(1)*x(4))-x(5)*x(1))-
x(5)*(x(2)-x(1)*x(4)))))-x(5)*(-x(4)*(-x(4)*(-x(4)*(-x(4)*(x(3)-x(4)*(x(2)-
x(1)*x(4))-x(5)*x(1))-x(5)*(x(2)-x(1)*x(4)))-x(5)*(x(3)-x(4)*(x(2)-x(1)*x(4))-
x(5)*x(1)))-x(5)*(-x(4)*(x(3)-x(4)*(x(2)-x(1)*x(4))-x(5)*x(1))-x(5)*(x(2)-
x(1)*x(4))))-x(5)*(-x(4)*(-x(4)*(x(3)-x(4)*(x(2)-x(1)*x(4))-x(5)*x(1))-x(5)*(x(2)-
x(1)*x(4)))-x(5)*(x(3)-x(4)*(x(2)-x(1)*x(4))-x(5)*x(1)))))-x(5)*(-x(4)*(-x(4)*(-
x(4)*(-x(4)*(-x(4)*(x(3)-x(4)*(x(2)-x(1)*x(4))-x(5)*x(1))-x(5)*(x(2)-x(1)*x(4)))-
x(5)*(x(3)-x(4)*(x(2)-x(1)*x(4))-x(5)*x(1)))-x(5)*(-x(4)*(x(3)-x(4)*(x(2)-
x(1)*x(4))-x(5)*x(1))-x(5)*(x(2)-x(1)*x(4))))-x(5)*(-x(4)*(-x(4)*(x(3)-x(4)*(x(2)-
x(1)*x(4))-x(5)*x(1))-x(5)*(x(2)-x(1)*x(4)))-x(5)*(x(3)-x(4)*(x(2)-x(1)*x(4))-
x(5)*x(1))))-x(5)*(-x(4)*(-x(4)*(-x(4)*(x(3)-x(4)*(x(2)-x(1)*x(4))-x(5)*x(1))-
x(5)*(x(2)-x(1)*x(4)))-x(5)*(x(3)-x(4)*(x(2)-x(1)*x(4))-x(5)*x(1)))-x(5)*(-
x(4)*(x(3)-x(4)*(x(2)-x(1)*x(4))-x(5)*x(1))-x(5)*(x(2)-x(1)*x(4))))))]
```

File name: ahmet5.m

Solution of 5 non-linear equation

```
x0 = [1;1;1;1;1];           % Make a starting guess at the solution
options=optimset('Display','iter');   % Option to display output
[x,fval] = fsolve(@myfilter6,x0,options)  % Call optimizer
```

Filename Error.m

The results of ahmet5.m are used for calculation errors and plottings

format long

a=[0.028406470015011 0.218908140988611 0.325577028758799]
b=[1.000000000000000 -0.637181339058579 0.261335071549222]
[hlong5,w]=freqz(a,b,128);

aa=[0.028406061665725 0.220349205987817 0.342383645564858]
bb=[1 -0.570062338402709 0.178337464899862]
[hlong6,w]=freqz(aa,bb,128);

aaa=[0.028406068578503 0.220630895714169 0.346932266346568]
bbb=[1 -0.553849004810709 0.161883369593791]
[hlong7,w]=freqz(aaa,bbb,128);

d=fir1(4,.2);
[hfir,w]=freqz(d,1,128);

ahfir=abs(hfir);
t = 0:(1/127):1
f=t'
m =ahfir
[by,ay] = yulewalk(2,f,m);
[hy,w] = freqz(by,ay,128)

[bb1,aa1]=prony(d,2,2);
[hp,w]=freqz(bb1,aa1,128)

ahfir=abs(hfir);
ahlong5=abs(hlong5);
ahlong6=abs(hlong6);
ahlong7=abs(hlong7);

ahy=abs(hy);
ahp=abs(hp);

plot(w/pi,(abs(hfir)),'-',w/pi,(abs(hp)),'--',w/pi,(abs(hy)),':',w/pi,(abs(hlong7)),'-.');
ylabel('Magnitude')
xlabel('Normalized Frequency (\pi x rad/sample)')
legend('FIR-4.Ord.','Prony-2.Ord.','Yulewalk-2.Ord.','IIR-2.Ord-Long7',1);

plot(w/pi,10*log10(abs(hfir)),'-',w/pi,10*log10(abs(hp)),'--
',w/pi,10*log10(abs(hy)),':',w/pi,10*log10(abs(hlong7)),'-.');
ylabel('Magnitude(dB)')
xlabel('Normalized Frequency (\pi x rad/sample)')

```
legend('FIR-4.Ord.','Prony-2.Ord.','Yulewalk-2.Ord.','IIR-2.Ord-Long7',1);


plot(w/pi,unwrap(angle(hfir)),'-',w/pi,unwrap(angle(hp)),'--
',w/pi,unwrap(angle(hy)),':',w/pi,unwrap(angle(hlong7)),'-.')
legend('FIR-4.Ord.','Prony-2.Ord.','Yulewalk-2.Ord.','IIR-2.Ord-Long7',3);
ylabel('Phase(rad)')
xlabel('Normalized Frequency (\pi x rad/sample)')

ef5=abs(ahfir-ahlong5);
ef6=abs(ahfir-ahlong6);
ef7=abs(ahfir-ahlong7);
ef10=abs(ahfir-ahy);
ef9=abs(ahfir-ahp);

errorlong5=(ef5'*ef5)^0.5
errorlong6=(ef6'*ef6)^0.5
errorlong7=(ef7'*ef7)^0.5
erroryulewalk=(ef10'*ef10)^0.5
errorprony=(ef9'*ef9)^0.5
```

Filename: quantization.m

FIR IIR and Yule-Walker filter coefficients are quantized by rounding, and up to 1 to 20 digit

```
format long

a15=[-0.003358854462931 0.003405139390548 -0.000807737808617
0.009379029292630 0.023442456281692 0.007512861472165 0.030584528591641]
b15=[1 -2.454872639421747 2.671142318992081 -1.619850697458879
0.615528132179041 -0.184450404500229 0.043848949570486]
[hlong15,w]=freqz(a15,b15,128);

d=fir1(15,.2);
[hfir,w]=freqz(d,1,128);
ahfir=abs(hfir);
t = 0:(1/127):1
f=t'
m =ahfir
[by,ay] = yulewalk(6,f,m);
[hy,w] = freqz(by,ay,128)


ahy=abs(hy);
```

```
ahlong15=abs(hlong15);

for p=1:20
k(p)=p
    q=quantizer([40,p],'round');
    qa15=quantize(q,a15);
    qb15=quantize(q,b15);
     qay=quantize(q,ay);
    qby=quantize(q,by);

    qd=quantize(q,d);

     [qhy,w]=freqz(qby,qay,128);
    [qhfir,w]=freqz(qd,1,128);
 [qhlong15,w]=freqz(qa15,qb15,128);
 aqhlong15=abs(qhlong15);
 aqhfir=abs(qhfir);
 aqhy=abs(qhy);
s1=(aqhlong15-ahlong15);
s2=(aqhfir-ahfir);
s3=(aqhy-ahy);
FIRE(p)=(s2'*s2)^0.5
IIRE(p)=(s1'*s1)^0.5
YULEE(p)=(s3'*s3)^0.5
end
plot(k,IIRE,':',k,FIRE,'-',k,YULEE,'--')
legend('QFIR-FIR','QIIR-IIR','QYULE-YULE',1);
ylabel('Difference of Error')
xlabel('Number of Bit Roundation')
```

CURRICULUM VITAE

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EDUCATION

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B.S.	University of Dokuz Eylül	1991
High School	Gaziantep Anadolu Lisesi	1987

WORK EXPERIENCE

Year	**Place**	**Enrollment**
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1992-2001	University of Gaziantep- Gaziantep M.Y.O.	Instructor

FOREIGN LANGUAGES

English

PUBLICATIONS

1. Şekeroğlu, A.S., Nacaroğlu, A. İşlemsel İletili Yükselteç ile Yapılan Osilatörlerde Parazitik Elemanların Osilasyon Frekansına Etkilerinin Analizi. *Elektrik Müh. 8. Ulusal Kongresi*, 6-12 Eylül, Gaziantep,1999, 38-40.