Theoretical Analysis of Sudden Expansion Fittings in Pneumatic Conveying System

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ABSTRACT

THEORETICAL ANALYSIS OF SUDDEN EXPANSION FITTINGS IN PNEUMATIC CONVEYING SYSTEMS

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Theoretical analysis of minor loss coefficient for sudden expansions in pneumatic conveying system is the basic purpose of this thesis. The minor loss coefficient is an indication of the degree of permanent pressure or energy loss in the system. The flow structure in pneumatic conveying systems characterized as two phase flow (gas-solid particle) is complex and difficult to investigate by experimental methods. Therefore, theoretical analysis of this flow regime is important.

First, a literature survey on theoretical and experimental studies of sudden expansion pipe flows has been made. There are many studies on gas-liquid flows but scarcely on gas-solid flows. Models on gas-liquid flows can be adapted to gas-solid flows when gaseous phase is assumed incompressible. Experimental results are compared with theoretical models. The deficiencies in the theoretical analysis are defined. As a result, a method for the theoretical analysis of sudden expansion for gas-solid flows is presented.

Then, homogeneous flow and separate flow approaches are utilized for gas-solid flows. By applying global laws (mass, momentum) in both approaches three different theoretical models are formed for the calculation of pressure difference arising in sudden expansion pipe flows and also a theoretical model for gas-liquid flows available in literature are developed for gas-solid flows. Slip flow models in literature is also shown for the relative velocity between phases occurring during expansion.

All the theoretical models are compared with experimental data of some authors in literature. Results have shown that the separate flow model is the best among the existing models. The slip in velocity between phases is also considered and a hybrid model is formed by combining separate flow model with slip flow model and it has yielded very close agreement with experimental data. As a result, the hybrid model can be considered as the best suitable theoretical model applicable to two phase gas-solid turbulent flow regimes.

Finally, the pressure loss due to sudden expansion is divided as one due to gas alone and one due to particles so the pressure loss coefficient is formed as the sum of the pressure loss coefficient due to gas phase and additional pressure loss coefficient due to particles. It is resulted that addition of particles to the flow decreases the pressure loss along the expansion but this decreasing in pressure loss is related with the material type to be conveyed or with the relative velocity existing between phases. It is also concluded that pressure loss coefficient in theory is not a function of Reynolds number in case of constant solid loading ratio with the same particle.

Keywords: sudden expansion, pressure loss, gas-solid flow, two-phase flow

ÖZET

PÜNÖMATİK TAŞIMA SİSTEMLERİNDEKİ ANİ GENİŞLEME ELEMANLARININ TEORİK ANALİZİ

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Pünömatik taşıma sistemlerindeki ani genişlemelerde oluşan minor kayıp katsayısının teorik olarak incelenmesi bu tezin amacıdır. Minor kayıp katsayısı, sistemdeki enerji ve basınç kaybının bir göstergesidir. İki fazlı akış olarak karakterize edilen pünömatik taşıma sistemlerindeki akış yapıları oldukça karmaşık ve deneysel metotlarla araştırılması da zordur. Bu nedenle akış rejiminin teorik olarak analizi çok önemlidir.

İlk, ani genişlemeler için teorik ve deneysel çalışmaları içeren bir literatür araştırması yapılmıştır. Literatür çalışmalarının çoğu gaz-sıvı akışları üzerinde olduğu, katı-gaz akış üzerine çalışmaların az olduğu görülmüştür. Gaz-sıvı akışları üzerindeki teorik çalışmalar, gaz fazı “sıkıştırılamayan” olarak farz edildiği zaman, katı-gaz akışları için de uygulanabilir. Deneysel çalışma sonuçları ve teorik modeller incelenmiştir ve teorik modellerdeki eksiklikler belirlenmiştir. Bu deneysel çalışmaların ve teorik modellerin bir sonucu olarak, ani genişlemelerdeki gaz-katı akışlarının teorik analizi için bir çalışma metodu belirlenmiştir.

Sonra, homojen akış ve ayrık akış yaklaşımları katı-gaz akışları için tanımlanmıştır. Temel kanunların (kütle ve momentum) bu yaklaşımlarda uygulanmasıyla, ani genişlemelerdeki basınç farkı için üç ayrı teorik model tanımlanmıştır. Literatürdeki gaz-sıvı akışları için tanımlanmış bir teorik model gas-katı akışları için ayriyeten geliştirilmiştir. Ani genişlemeler boyunca oluşan fazlar arasındaki hız farkı için literatürde tanımlanmış teorik modeller belirtilmiştir.

Bu teorik modeller literatürdeki deneysel çalışmalarla karşılaştırılmıştır. Sonuçlar; ayrık akış modelin oluşan teorik modeller içerisinde en iyi olduğunu göstermiştir. Bu modelin fazlar arasında oluşan hız farkı için literatürde tanımlanmış bir model ile tanımlanmasıyla, sonuçların deneysel verilere daha da yakınlaştığı görülmüştür. Sonuç olarak; ayrık akış modeli gaz-katı türbülanslı akış rejimi için en uygun teorik model olarak gösterilebilir.

Ani genişlemelerde oluşan basınç kaybı biri gaz fazdan dolayı ve diğeri katı parçacıktan dolayı basınç kaybı olarak ikiye ayrılmıştır. Bu yüzden basınç kayıp katsayısı sadece gaz fazdan dolayı basınç kayıp katsayısı ve katı parçacıklardan dolayı ek basınç kaybının toplamı olarak tanımlanmıştır. Sonuçlar, katı parçacıkların akışa eklenmesi ani genişlemelerde oluşan basınç kaybını azalttığını göstermiş ve bu basınç kaybındaki düşüş miktarının taşınan malzemenin türüyle alakalı olduğu ve fazlar arasındaki bağıl hıza bağlı olduğunu göstermiştir. Ayrıca, sonuçlar, teorik olarak tanımlanmış basınç kayıp katsayısının, sabit bir katı yükleme oranıyla, Reynolds sayısının bir fonksiyonu olmadığı göstermiştir.

Anahtar Kelimeler: ani genişleme, basınç kaybı, gaz-katı akışı, iki fazlı akış

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CONTENTS

LIST OF SYMBOLS/ABREVIATIONS

A	Pipe cross-sectional area, (m^2)
A_s	Cross-sectional area occupied by solid particles, (m^2)
A_g	Cross-sectional area occupied by gaseous phase, (m^2)
A_p	Particle cross-sectional area, (m^2)
A_0	Specific surface area of solid particle per unit volume, (m^2/m^3)
a	Emprical constant defined in Eqn.(3.45)
a´	Acceleration, (m/s^2)
b	Emprical constant defined in Eqn.(3.45)
C_{Ds}	Superficial drag coefficient of solid particles
C_f	Friction coefficient
C	Apparent mass effect defined in Eqn.(3.51)
C_e	Expansion loss coefficient or pressure loss coefficient
C_{go}	Pressure loss coefficient of gas only flow
C_p	Additional pressure loss coefficient due to particles
D	Pipe diameter, (m)
d	Particle diameter, (m)
F	Force, (N)
F_{gs}	Volumetric drag force between phases, (N/m^3)
F_{wg}	Volumetric wall friction force on gas phase, (N/m^3)
F_{ws}	Volumetric wall friction force on particles, (N/m^3)
FM	Flow Model
f_{gp}	Drag force between particle and gaseous phase, (N)
f_{wp}	Particle wall friction force, (N)
f_{Gp}	Particle gravitational force, (N)
f_g	Gas friction factor
f_s	Solid friction factor defined in Eqn.(3.44)
$f_s{}^{´}$	Solid friction factor defined in Eqn.(3.46)
F_s	Average total surface force on particles, per unit volume, (N/m^3)
F_g	Average total surface force on gaseous phase, per unit volume, (N/m^3)
G	Mass flux rate, (kg/m^2s)
GLR	Gas Loading Ratio
g	Gravitational acceleration, (m^2/s)
H	Step height, ((D_2-D_1)/2), (m)
H_m	Minor head loss, (m)
h	Enthalpy, (kJ/kg)
K	Minor loss coefficient or pressure drop coefficient
L	Pipe length around enlargement, (m)
L_{tr}	Transition length, (m)
l	Transition length-pipe diameter ratio, (L_{tr} / D_2)
M	Solid-gas loading ratio, (W_s / W_g)
m_p	Mass of particle, (kg)
n	A parameter defined in Eqn.(3.46a)
P	Pressure, (N/m^2)
p	Perimeter, (m)
ΔP	Pressure difference, (Pa)
$\Delta P_{e, loss}$	Pressure loss of two phase flow due to expansion, (Pa)
ΔP_{fg}	Pressure loss due to gas wall friction, (Pa)
ΔP_{fs}	Pressure loss due to solid wall friction, (Pa)

Q	Volumetric flow rate, (m^3/s)
Re	Gaseous phase Reynolds number based on upstream pipe flow
T	Temperature, (^oC)
t	Time, (s)
S	Slip ratio
SLR	Solid loading ratio
V	Velocity, (m/s)
V_p	Particle velocity, (m/s)
z	Direction coordinate along the expansion section, (m)
Δz	Distance for the pressure recovery along the expansion, (m)
W	Total mass flow rate, (kg/s)
w	Work, (kJ/s)
x	Mass quality, (W_s / W)

Greek Symbols

α	Void fraction, solid particles concentration,(A_s/A)
Δ	Delta, means the difference between two quantity
ε	Void fraction defined in Eqn.(3.52) or gaseous phase concentration, (A_g/A)
θ	Angle between flow direction and gravitational acceleration, (Degree)
Λ	Volume
μ	Dynamic viscosity, ($N.s/m^2$)
ρ	Density, (kg/m^3)
σ	Expansion ratio or Area ratio, (A_1/A_2)
τ	Wall shear stress, (N/m^2)
Φ	Particle shape factor

Subscripts

1	Inlet to sudden expansion, just before expansion point
2	Exit of sudden expansion where pressure is recovered
A	Apparent mass
D	Drag
dp	Drop pressure
e	Expansion
F	Friction
f	Friction
fg	Friction gas
fs	Friction solid
G	Gravity
go	Gas only
m	Mean value
l	Liquid
g	Gas
P	Pressure
p	Particle
s	Solid
tr	Transition
w	Wall

CHAPTER 1

INTRODUCTION

Despite many years of research in the study of pneumatic conveying systems by manufacturers and investigators, the method of handling solids is still an area of interest. Much has been written on the theory of solid conveying is only applicable to a few selected materials having certain particle size range. The basic reason for the diversity of data on the pneumatic transportation of solids is the variety of the materials that can be conveyed. Moreover, possibility of each material can be conveyed by a broad range of air velocities at various materials to air loading. The conveying air velocities are themselves a function of the particle-size spectrum and of the density, shape and physical characteristics of the material as well as direction of flow being horizontal or vertical.

Pneumatic conveying systems have been widely used as a basic tool in many branches of industry such as food, textile, mining, cement, etc. Decreasing the pressure loss in the system therefore is very critical for increasing the system performance. Pressure loss in pneumatic conveying systems is mainly arising due to frictional and minor losses. The inevitable use of constructional fittings such as elbows, T-junctions, sudden-contractions, sudden expansions, causes the increase of minor losses in pneumatic conveying systems. Therefore, analyzing the constructional fittings in these systems from the point of view of the pressure loss is important since it affects the performance directly.

Theoretical evaluation of minor loss coefficients for sudden expansions in pneumatic conveying systems is the basic purpose of this thesis. The minor loss coefficient is an indication of the degree of pressure or energy loss in the systems. The flow structure in pneumatic conveying systems is characterized as two-phase (air-solid particle) flow, which is complex and difficult to investigate by experimental methods.

A literature survey related with two-phase flows for sudden expansion fittings of a duct is presented in the second chapter of the thesis. Researches on sudden expansion exist covering liquid-gas, solid-gas, liquid-solid and liquid-liquid two phase flows. For each case a different technique is applied to find a theoretical solution for sudden expansion flow.

In the third chapter, different theoretical approaches leading to analytical solutions for the pressure difference along the sudden expansion are presented. Besides, new theoretical analyses have been developed for the case of pressure difference in gas-solid two-phase flow over a sudden expansion through which the flow is gradiently upward. Briefly, the two methods are as follows: First, homogeneous flow model is applied which is based on momentum and continuity equations and subsequently analytical expressions are developed. In this simplified model, both phases are assumed having the same velocity and having a homogeneous mixture along the expansion section. In the second approach, separated flow model is applied and analytical expressions are developed. In this model, the phases are assumed to flow side by side and each phase is assumed having different velocities along the expansion section. Furthermore, when a solid particle accelerated in a fluid, it sets up additional force to the carrier fluid. The solid particle is assumed to behave as if it possesses an additional mass called "apparent mass". Here, in sudden expansion, the apparent mass due to acceleration of solid particles affects the gaseous phase which is reflected to the pressure at downstream flow. Also, the frictional interaction between phases due to relative velocity during decelerated flow causes the momentum transfer from solid particles to gaseous phase during expansion and this affects the pressure at downstream flow. So, an analytical expression is developed by taking the apparent mass effect and drag force effect into account for the pressure difference along expansion section. The theoretical models of gas-liquid flows formulated in literature are also examined and the best theoretical model one is adopted to gas-solid flows. There is always a relative velocity between phases during expansion and the relative velocity between phases has been assumed with a slip flow model in literature. Slip flow models in literature are also introduced in chapter 3.

In the fourth chapter, the agreement of theoretical models with available experimental data is examined. The analytical expressions are tested against experimental data of Marjanovic et al. (1999) and Tashiro&Tomita (1986), which is obtained for gas-solid two phase flows, respectively. Comparison show that all the theoretical models gets away from the data as solid loading ratio increases and close to data as loading ratio decreases. The separate flow model is resulted as the best one among existing models with near results to the experimental data than others. The slip flow models defined in chapter 3 are tested against the experimental data and it is seen that Kojasoy's slip flow model with a change in power of its equation is better than other model. When the separate flow model coupled with Kojasoy's slip flow model, very close agreement with experimental data is also observed. The separate flow model is also tested on the experimental data obtained from gas only flow and liquid only flow. In the case of gas only flows, very close agreement with experimental data are detected and in the case of liquid only flow, a little difference between data and model is also observed. Pressure loss due to sudden expansion of gas-solid two phase flow is divided into two parts as one due to gaseous and one due to particles. So the pressure loss coefficient is the sum of the pressure loss coefficient due to gas alone phase and additional pressure loss coefficient due to particles. It is observed that addition of particles to flow decrease the pressure loss in a decelerated flow which means that a momentum transfer from solid particles to gaseous phase occurs during expansion.

In the final chapter, conclusions reached from the theoretical analysis of gas-solid flows through sudden expansion are presented. It is shown that the separate flow model coupled with the slip flow model is the best among existing models since it yields minimum difference with the experimental data available in literature. This model has found to be the best to use in gas-solid two phase flow for turbulent pipe flow regimes within available ones.

CHAPTER 2

LITERATURE SURVEY

2.1 INTRODUCTION

Pneumatic transportation is one of the most important operations whose application is a vital and integral part in many industrial processes. It has been used for a long time to transport and distribute particulate materials such as grains, cement, etc. Due to its contained nature and the flexibility of operation, the pneumatic conveying of solids is often seen a standard practice in fluidized bed operations and in transferring solids in and between reactor vessels, bins, hoppers, etc. The so-called theory of pneumatic conveying in pipeline conveyors is still of the development stage.

The minor loss components have importance in pneumatic conveying systems. The minor loss along the sudden expansion in two phase flows is examined theoretically, numerically and experimentally by many authors. Much of investigations are focused on gas-liquid flows, but little is available on gas-solid flows.

In this chapter, a brief discussion on studies found in literature is given for two-phase flow through sudden expansion fittings. In some of the approaches, solution of sudden expansion problem is attempted both numerically and experimentally. Besides, some of the researchers have tried to solve the problem analytically as well.

2.2 LITERATURE SUMMARY

The first extensive investigation of a complete pneumatic conveying system is carried out experimentally by Gaserstödt as reported by Kraus (1968) using a wheat conveying unit which is set-up in the laboratory. He has presented his data graphically showing the variation in pressure along the flow direction under various material loadings at a constant air velocity. The most important observations were; (i) a large drop in pressure occurs at the beginning of the conveying line due to acceleration of the material while reaching to conveying velocity, (ii) a large drop in pressure takes place at the elbows and in vertical section of the line, and finally (iii) drop in pressure after the last elbow is observed which is caused by re-acceleration of the wheat grains.

Momentum transfer from particles to gas in the decelerated flow of suspensions is confirmed experimentally by Tomita et al. (1980) in a study of sudden expansion of the pipe by using different glass beads particles. In the case of fine particles of which the relaxation time is small in comparison with a characteristic time scale of the gas flow in the sudden expansion, the dynamic of the flow regime almost is similar to that of the gas alone case and the particles are passive additives. In the case of coarse particles of which the relaxation time is large enough in comparison with the characteristic time scale of the gas flow in the sudden expansion, the dynamic of the flow regime is affected by the particles through momentum transfer between phases and the particles are active additives.

Gas-solid two phase flow in sudden expansion of vertical downward flow and vertical upward flow is experimentally examined by Tashiro&Tomita (1984-1985) by using different expansion ratios and using PVC powder, Glass Beads and Iron solid particles as conveying materials. Results show that additional expansion loss due to particles takes negative values when flux Richardson number exceeds 0.0015 as in horizontal pipes under the condition that the velocity of solid particles is larger than that of air. Experiments show that the additional pressure loss coefficient due to particles in cases of all flow directions decreases as the expansion ratio increases. It is also observed that in cases of PVC powder and glass beads, the velocity of particles is larger than that of air in nearly all conditions but in cases of iron beads, the velocity of particles is smaller than that of air at the expansion interface. At the same diameter ratio the degree of decrease of the expansion loss coefficient is the largest in a vertical upward flow.

Diameter ratios ranging between 0.405 and 0.881, Tashiro&Tomita (1986) has examined additional expansion loss coefficient due to particles in a sudden expansion of a circular pipe with a vertical upward flow by using three different particles. They analyzed the two-phase flow pressure drop by dividing it one due to gas and another due to particles. Gas-solid flow sudden expansion pressure loss is primarily due to the gaseous phase-when the solid loading ratio is small-the loss coefficient for two phase flow is the same as that of gas alone. However, when the loading ratio exceeds some threshold value the loss coefficient decreases below that of gas alone flow case. This phenomenon is attempted to explain by the possible fact of momentum transfer from particles to gas in a decelerating flow.

An experimental investigation was undertaken by Arefmanesh&Michaelides (1988) to determine the pressure recovery when air-solid two phase mixture passes through a pipe enlargement. The solids used are medium size wax and lucite particles. Two different pipe diameter ratios were investigated, 0.63 and 0.76. The Reynolds number in the experiments varied between 45000 and 72000 and the loading ratios were low to intermediate (up to 6). It was shown that the contribution to the pressure recovery due to the flow of air can be expressed by the Borda-Carnot coefficient. The part of the pressure recovery due to the solid particles is expressed in terms of four parameters, namely, the Reynolds number, the loading ratio, the diameter ratio and the density ratio. The deviation between measured and correlated values of the total pressure recovery is very small. Of the contributions to the pressure recovery by the two phases the one due to the air is significantly higher than that due to the solids. This can be attiributed to fact that the solids interact with the main flow through the friction coefficient, which is a dissipative type of interaction.

Fesler et al. (1997) measured particle velocities and concentration through gas stream in a vertically downward planar sudden expansion flow for large-eddy particles. They showed that the smallest particles fill the recirculation zone and show strong response to the large eddies present in the flow. Particle streamwise fluctuating velocities were higher than those of the fluid, especially in the free-stream and recirculation region. In the shear layer, where the fluid fluctuations were the largest, wall-normal particle fluctuating velocities were lower than the fluid fluctuating velocities.

The effect of particle-to-particle collisions on the characteristics of the particle motion in a vertically downward sudden expansion flow was investigated both experimentally and computationally by Founti&Klipfel (1998). The investigated flow was nearly dense, laden with spherical glass particles at 5 % by volume. In the computational model, the instantaneous particle momentum equation contains the effect of drag force, added mass force, pressure gradient force, shear lift force, rotational lift force and gravitational force. They showed that the increase of particle loading induces a recovery of the mean reattachment point towards the single phase flow value due to the decrease of the particle turbulent kinetic energy associated with local particle-to-particle collisions. Particle-to-particle collisions were deemed to be responsible from the local decrease of the value of the particle turbulent kinetic energy in the regions of high fluid turbulence.

Numerical simulations of dispersion and deposition of particles in an axi-symmetric turbulent pipe flow with sudden expansion were performed by Goodarz and Qian (1998). The particle diameters ranging from 0.01 to 100 µm are used. The results have shown that small particles (submicron) are captured by the re-circulating flow where as large particles with high inertia can escape the re-circulating flow. The dispersion of particles of different sizes in the core region was similar in all cases. The effect of the lift force and gravity force was shown to be insignificant in particle deposition over the sudden expansion flow region while the turbulence has shown to be the dominating dispersion mechanism in the core region.

Founti et al. (1998) reported measurements of particle and fluid velocities for a turbulent, liquid-solid two-phase flow over a sudden expansion. The flow was in the direction of gravity and laden with solid particles at 1%, 2%, 3%, 4% and 5% by volume. They have shown that measured reattachment lengths in the case of a 1:2 sudden expansion two-phase flows with particle loadings mentioned above are always shorter than the corresponding single phase flow cases. Particles move faster than the carrier fluid nearly everywhere in the flow, apart from when crossing the shear layer zone in order to enter the recirculation zone. Particle dispersion is not affected by increasing the particle loading in the main "core" flow. Increasing loading reduces particle fluctuations in the near wall zone, where turbulent quantities tend to attain their single phase values.

An experimental study on gas-solid flows through sudden expansion has been performed by Marjanovic et al. (1999). It was shown that the solid particles have higher velocity than gaseous phase, so, a momentum transfer from solid particles to gaseous phase occur during expansion until a point where the pressure becomes maximum or the solid velocity becomes equal to gas velocity. The increase in pressure is higher than the theoretical value for gas only flow due to additional momentum transfer from solid particles which are flowing at higher velocity along transition zone after the step. Gas pressure reaches its maximum value at the point where gas and solid velocity equals at downstream. However, solid particles decelerate further downstream and gas-solid flow fully develops when slip velocity reaches its constant value. In this experimental work two transition lengths are defined as follows:

- From step to the point where gas pressure reaches it's maximum and average gas velocity it's minimum.
- From step to the point where both pressure gradient and slip velocity reach constant values.

Pressure drop caused by an abrupt change in flow area, either expansion or contraction, in small circular channels were experimentally investigated by Abdellal et al**.** (2005) by using air and water as the working fluids at room temperature and near-atmospheric pressure. The largest and smallest tube diameters were 1.6 and 0.84 mm, respectively. A homogeneous flow model is formed for the reversible pressure change along sudden expansion. Both gas and liquid phases were assumed to be incompressible. Flat velocity profile is assumed at downstream and upstream. The homogeneous flow model has shown to overpredicting the experimental data monotonically and significantly. Low experimental pressure drop indicate significant velocity slip in the vicinity of the flow disturbance. An increase in the agreement between the theory and experiment has been observed when a new model is formed by coupling the homogeneous flow model with slip flow model.

2.3 CONCLUSION

Experimental studies show that solid particles move faster than the carrier fluid during expansion so a momentum transfer from solid particles to carrier fluid occur during expansion and this causes the pressure at downstream to increase. The two phase flow pressure loss in decelerated flow is lower than that of gas alone value. This is due to the fact that the solid particles interact with the main flow through friction, which is a dissipative type of interaction. This effect is not taken into account in theoretical models formulated in the literature. It can be included in theoretical modeling of gas-solid flows as a drag force effect and apparent mass effect.

In the case of small size particles or at small solid loading ratios, the pressure recovery is not affected due to solid phase and the sudden expansion loss coefficient can be taken as the same gas alone flow case. In this case, the homogeneous flow approach can be used in the analysis of gas-solid flows.

The experimental studies show that the fully developed flow condition is broken down at upstream-just before expansion-and the flow develops to fully condition at downstream far from the reattachment length where the pressure gradient becomes constant. In theoretical analysis of gas-solid flows, the flow will be assumed in fully developed flow condition at upstream and along the downstream sections of the flow field.

The shear-lift and rotational-lift force are taken into account in the same theoretical models formulated in literature having contributing effect on the pressure loss term but these two terms can be neglected in analysis of gas-solid flows since they have negligible effect on pressure loss.

CHAPTER 3

THEORETICAL MODELLINGS OF A SUDDEN EXPANSION FITTING FOR GAS-SOLID FLOWS

3.1 INTRODUCTION

Because of its complexity, it is very difficult to compute the pressure loss analytically along the sudden expansion flow. This chapter aims to develop theoretical expressions giving the pressure recovery due to sudden expansion in gas-solid flow.

By coupling the global laws (mass, momentum), homogeneous and separate flow models were modified and two new models are suggested. Three different theoretical expressions for the pressure recovery along expansion section are derived from these models. Furthermore theoretical models formulated in literature, which were mainly developed for gas-liquid flows, are compared with gas-solid flow experimental data and the best theoretical model applicable to gas-solid two phase flow is selected. In all analysis, the inertial term, wall frictional term, gravity term are included as basic terms which affect the pressure recovery. Furthermore, the effect of turbulent component of inertial term and the effect of particle-to-particle collision on pressure loss along expansion are neglected and gaseous phase is assumed to be incompressible. The flow at upstream just before expansion and at downstream are assumed to be in fully developed condition.

Due to high inertia of the solid particles in comparison with gaseous phase during expansion, a momentum transfer from solid particles to gaseous phase occurs along the expansion. These effects also are taken into account in an analysis by the inclusion of a drag force term and an apparent mass term.

In gas-solid two phase flows, relative velocity between phases can not be found with analytical or experimental methods. The assumption to the slip in velocity between phases has been handled in literature broadly. Slip flow models in literature are also presented in this chapter.

3.2. DEFINITIONS and BASIC RELATIONS

As the fluid flows through the pipe, the friction on the side walls of the pipe creates resistance to the flow. This causes pressure loss along the pipe. For any pipe system, in addition to the channel wall friction losses, there are additional losses called as minor losses. These losses arise due to the geometry of the pipe entrance and exit, sudden expansion or contraction, bends, elbows and other fittings such as valves that may be present in the pipe system. The head losses for the pipe entrance and exit, bends, elbows, valves and other fittings in the case of single phase flows are calculated by using equation given below:

$$H_m = K \frac{V^2}{2g} \tag{3.1}$$

The minor loss coefficient (K) for valves and fittings in the case of single-phase flows may readily be found in any standard fluid mechanics textbook in tabulated forms. Minor losses in two-phase flows cannot be calculated simply by using the Eqn. (3.1) since it is defined for the single-phase flows.

So the aim in this thesis is to find the minor loss coefficient (K) for gas-solid two phase flow through sudden expansion of a duct.

Here, it is important to know that a phase is defined as a quantity of matter that is homogeneous throughout which can be either gas, liquid or solid. Multiphase flow is the co-existence of several phases in one flow. Two phase flow is the simplest case of multiphase flows.

3.2.1 Sudden Expansion

Fig. 3.1 shows a sudden expansion element placed in a duct. The flow can not follow the area change as suddenly as the geometry does. Any flow will find the sudden area increase difficult to negotiate. In fact a recirculating flow region develops which creates a separation of the flow from the wall. This gives rise to a loss which reflects itself as a reduction in total pressure at downstream.

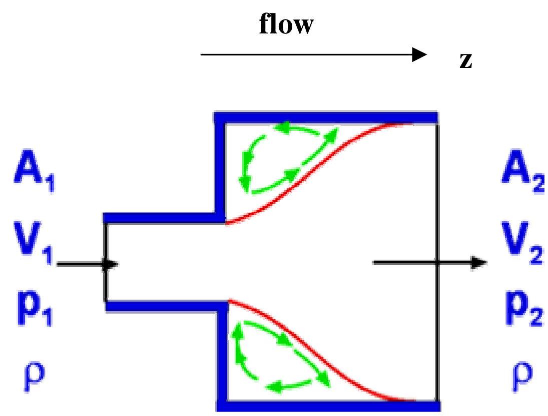

Figure 3.1**:** A sudden expansion element placed in a duct

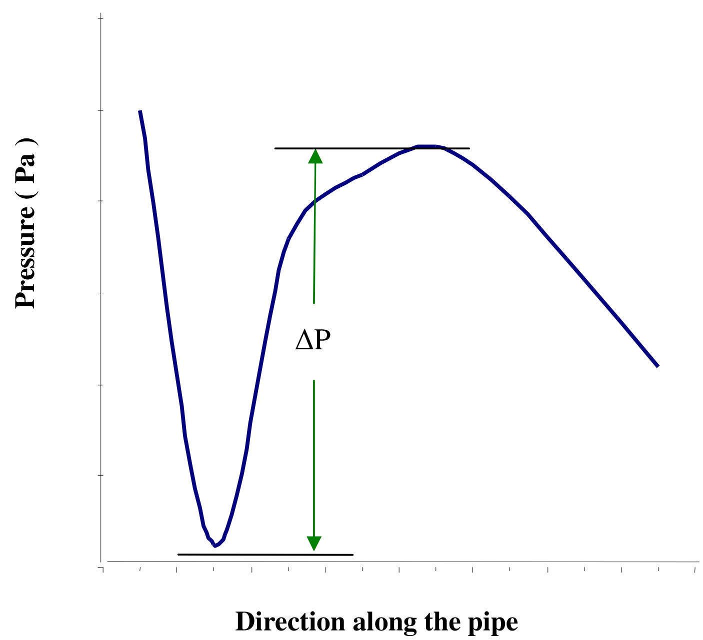

Figure 3.2**:** Pressure distribution for flow through a sudden expansion

Here, in Fig. 3.2, the static pressure at upstream decreases monotonically due to wall friction up to a point just before expansion point and then increases ubtruptly due to sudden expansion. Afterwards it attains its maximum at some distance away from the enlargement point and start to decrease linearly under frictional effect after a point where fully developed flow conditions is reached. Upstream velocity, V_1, decreases to V_2 upon expansion. Here, in this thesis, the aim is theoretically to find the pressure difference, ΔP, between before expansion and downstream where the pressure becomes maximum or the flow becomes fully developed.

3.2.2 Mass Flow rate

It is the quantity of mass flowing per unit time and defined as follows:

$$W = \rho\ V\ A \tag{3.2}$$

3.2.3 Volumetric Flow Rate

Volumetric flow rate is designated by the symbol Q and defined as follows:

$$Q = \frac{Mass\ \ flow\ rate}{Density\ of\ the\ fluid} = \frac{W}{\rho} \tag{3.3}$$

3.2.4 Mass Flux

It is the quantity of mass flowing through the unit cross-sectional area:

$$G = \frac{Mass\ flow\ rate}{Cross-\sec tional\ area} = \frac{W}{A} \tag{3.4}$$

3.2.5 Void Fraction

Void fraction which may also be termed as the volumetric concentration of the solid phase at any cross section is defined as follows:

$$\alpha = \frac{Cross-\sec tional\ area\ occupied\ by\ solid\ phase}{Total\ cross-\sec tional\ area} = \frac{Q_s}{Q_s + Q_g} \tag{3.5}$$

3.2.6 Quality

The definition of quality is:

$$x = \frac{Mass\ flow\ rate\ of\ solid\ phase}{Total\ mass\ flow\ rate} = \frac{W_s}{W} \tag{3.6}$$

3.2.7 Expansion Ratio (Area Ratio)

It is the ratio of the cross-sectional area of the upstream to that of downstream in sudden expansion.

$$\sigma = \frac{A_1}{A_2} \tag{3.7}$$

3.2.8 Density

In the case of single-phase flows, the density of the fluid can be easily calculated from established relationships for thermo physical properties of fluid. In two-phase flows, however, the density of two-phase mixture at any cross section in the flow path is calculated by using the following relation:

$$\rho_m = \alpha \rho_s + (1 - \alpha) \rho_g \tag{3.8}$$

The above expression indicates that for two-phase flows, it is necessary to predict void fraction accurately beforehand in order to determine density.

3.2.9 Transition Length (L_{tr}):

It is a distance over which some typical flow conditions at downstream is realized. Here, two transition lengths are defined along the expansion which is measured.

- From the step to the point where gas pressure reaches its maximum and at the same time average gas velocity is at its minimum.
- From the step to the point where both pressure gradient and slip velocity reach constant values or fully developed flow condition is reached

3.2.10 Global Conservation Laws

The global conservations laws (mass, momentum, energy etc.) are as follows:

3.2.10.1 Continuity:

$$W = \rho\, VA = const. \tag{3.9}$$

3.2.10.2 Momentum:

$$W \frac{\partial V}{\partial z} = -A \frac{\partial P}{\partial z} - p \tau_w - A \rho g \tag{3.10}$$

3.2.10.3 Energy:

$$\frac{\partial q_e}{\partial z}-\frac{\partial w}{\partial z}=W\frac{\partial}{\partial z}\left(h+\frac{V^2}{2}+g\,z\right) \tag{3.11}$$

3.2.11 Pressure Gradient:

Total pressure gradient due to sudden expansion consists of frictional, accelerational and gravitational pressure gradient terms:

$$\frac{\partial P}{\partial z}=\left(\frac{\partial P}{\partial z}\right)_F+\left(\frac{\partial P}{\partial z}\right)_A+\left(\frac{\partial P}{\partial z}\right)_G \tag{3.12}$$

Where F, A and G designates the frictional, accelerational and gravitational terms, respectively.

The following sections are related with theoretical models which are obtained for the pressure recovery along the expansion.

3.3 HOMOGENEOUS FLOW MODEL

Homogeneous flow theory provides the simplest technique for analyzing two-phase (or multiple phase) flows. Average properties are determined suitably and the mixture is treated as a pseudo-fluid obeying the usual equation of motion developed for single component flow.

Differences in velocity and temperature between the phases will promote mutual momentum and heat transfer. Often these processes proceed very rapidly, particularly when one phase is finely dispersed in the other and it can be assumed that equilibrium is reached. In this case the average values of velocity and temperature are the same for each component and we have on homogeneous equilibrium flow

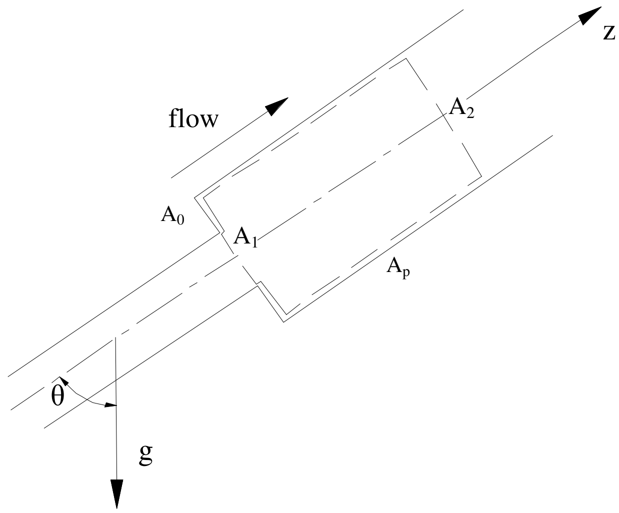

Figure 3.3: Gas-solid flow through a sudden expansion

General description of the flow case investigated is shown in Fig. 3.3, where, A_1 and A_2 indicate the upstream and downstream cross-sectional areas of the flow channel, respectively. A_p is the lateral area along the pipe wall starting from the point of sudden enlargement. A_o is difference between downstream and upstream cross-sectional areas.

By applying continuity, Eqn. (3.9), and momentum, Eqn. (3.10), principles to the flow described above, the following equation, giving the pressure gradient over the expansion, is obtained.

$$\frac{\partial P}{\partial z}=-\frac{p}{A}\tau_w-\frac{W}{A}\frac{\partial V}{\partial z}-\rho_m g\cos\theta \tag{3.13}$$

Each of the three terms on the right side of Eqn. (3.13) can be expressed as a function of basic variables as follows:

$$\frac{\partial P}{\partial z}=\left(\frac{\partial P}{\partial z}\right)_F+\left(\frac{\partial P}{\partial z}\right)_A+\left(\frac{\partial P}{\partial z}\right)_G \tag{3.14}$$

Frictional pressure gradient component can be expressed in terms of wall shear stress or equally well in terms of friction factor. Here, the wall shear stress belongs to the part of the flow starting from just downstream of sudden expansion section up to the part where the flow becomes fully developed. Thus,

$$\tau_w = C_f \frac{1}{2}\rho_m V^2 \tag{3.15}$$

$$-\left(\frac{\partial P}{\partial z}\right)_F = 4\frac{\tau_w}{D} = 2C_f \rho_m \frac{V_2^2}{D_2} \tag{3.16a}$$

$$V_2 = \frac{G_2}{\rho_m} = G_2\left(\frac{x}{\rho_s} + \frac{(1-x)}{\rho_g}\right) \tag{3.16b}$$

$$-\left(\frac{\partial P}{\partial z}\right)_F = \frac{2C_f\, G_2^2}{D_2\, \rho_m} \tag{3.16}$$

An expression for the inertial pressure gradient can be obtained as follows:

$$-\left(\frac{\partial P}{\partial z}\right)_A = G\,\frac{\partial V}{\partial z} \tag{3.17a}$$

$$-\left(\frac{\partial P}{\partial z}\right)_A = G\frac{\partial}{\partial z}\left(\frac{W}{A\rho_m}\right) \tag{3.17b}$$

$$-\left(\frac{\partial P}{\partial z}\right)_A = G^2\,\frac{\partial}{\partial z}\left(\frac{1}{\rho_m}\right) - \frac{G^2}{\rho_m}\frac{1}{A}\frac{\partial A}{\partial z} \tag{3.17c}$$

$$\frac{\partial}{\partial z}\left(\frac{1}{\rho_m}\right) = \frac{\partial}{\partial z}\left(\frac{x}{\rho_s} + \frac{(1-x)}{\rho_g}\right) = x\frac{\partial}{\partial z}\left(\frac{1}{\rho_s}\right) + (1-x)\frac{\partial}{\partial z}\left(\frac{1}{\rho_g}\right) \tag{3.17}$$

Finally,

$$-\left(\frac{\partial P}{\partial z}\right)_A = G^2\left\{\frac{\partial P}{\partial z}\left[x\frac{\partial}{\partial z}\left(\frac{1}{\rho_s}\right) + (1-x)\frac{\partial}{\partial z}\left(\frac{1}{\rho_g}\right)\right] - \frac{1}{\rho_m}\frac{1}{A}\frac{\partial A}{\partial z}\right\} \tag{3.18}$$

And the gravitational pressure gradient is simply:

$$-\left(\frac{\partial P}{\partial z}\right)_G = \rho_m\, g\cos\theta \tag{3.19}$$

Combining these three equations and regrouping yields,

$$-\frac{\partial P}{\partial z}=\frac{\left[\frac{2C_f G_2^2}{D_2\rho_m}-G^2\frac{1}{\rho_m}\frac{1}{A}\frac{\partial A}{\partial z}+\rho_m g\cos\theta\right]}{1+G^2\left[x\frac{\partial}{\partial P}\left(\frac{1}{\rho_s}\right)+(1-x)\frac{\partial}{\partial P}\left(\frac{1}{\rho_g}\right)\right]} \tag{3.20}$$

In the analysis given below the following assumptions are made

- There is thermal equilibrium between phases ($T_s = T_g$).
- The flow and state of the matter in the flow field are steady.
- The flow is fully developed at upstream and during downstream.
- Turbulent component of the inertial term is negligible.
- The gaseous phase is incompressible.
- Pipe is assumed smooth.

Under these assumptions, the total pressure gradient expression reduces the following simple form.

$$-\frac{\partial P}{\partial z}=\frac{2C_f G^2}{D\rho_m}-G^2\frac{1}{\rho_m}\frac{1}{A}\frac{\partial A}{\partial z}+\rho_m g\cos\theta \tag{3.21}$$

Integration of Eqn. (3.21) starting from step to a position at downstream where the flow becomes fully developed:

$$-\int_1^2 \partial P=\int_1^2 \frac{2C_f G_2^2}{D_2\rho_m}\partial z-\int_1^2 G^2\frac{1}{\rho_m}\frac{1}{A}\partial A+\int_1^2 \rho_m g\cos\theta\partial z \tag{3.22}$$

Resulting explicit expressions for each term are:

Frictional term:

$$\int_1^2 \frac{2C_f G_2^2}{D_2\rho_m}\partial z=2\frac{C_f G_2^2}{D_2\rho_m}\Delta z \tag{3.23}$$

Inertial term:

$$-\int_1^2 G^2 \frac{1}{\rho_m}\frac{1}{A}\partial A \Rightarrow -\int_1^2 \frac{W^2}{A^2\rho_m}\frac{1}{A}\partial A \tag{3.24a}$$

$$\Rightarrow -W^2 \frac{1}{\rho_m}\int_1^2 \frac{1}{A^3}\partial A \Rightarrow -\frac{1}{2}W^2 \frac{1}{\rho_m}\left(\frac{1}{A_2^2}-\frac{1}{A_1^2}\right) \tag{3.24b}$$

$$\Rightarrow -\frac{1}{2}\frac{W^2}{A_1^2}\frac{1}{\rho_m}\left(\frac{A_1^2}{A_2^2}-1\right) = -\frac{1}{2}\frac{1}{\rho_m}G_1^2\left(\sigma^2-1\right) \tag{3.24}$$

Finally, the total pressure difference over the expansion is:

$$\Delta P = -\frac{1}{2}G_1^2\frac{1}{\rho_m}\left(\sigma^2-1\right) - 2\frac{C_f G_2^2}{\rho_m}l_2 - \rho_m\, g\cos\theta\,\Delta z \tag{3.25}$$

Here, ρ_m, mean density can be computed by using Eqn. (3.8). l_2 is a dimensionless length and defined as,

$$l_2 = \frac{L_{tr}}{D_2} = \frac{\Delta z}{D_2} \tag{3.26}$$

The homogeneous friction factor, C_f, is differ from the single phase friction factor. It is advised that use a constant friction factor for turbulent flow, a good choice is by (Wallis, 1969)

$$C_f = 0.005 \tag{3.27}$$

For low concentrations, use Blasius equation for smooth pipe flow,

$$C_f = 0.079\,\mathrm{Re}^{-0.25} \tag{3.28}$$

Where, Reynolds number in Eqn. (3.28) is based on downstream pipe diameter.

Here, it is important to note that the homogeneous flow model, Eqn. (3.25), can only be applied for the range of concentrations below 5% since the change in viscosity is small where the viscosity of homogeneous mixture is given below by

Einstein equation and the flow is considered to be non-Newtonian for concentrations above 5% (Wallis, (1969)).

$$\mu = \mu_s (1 + 2.5\alpha) \qquad (3.29)$$

3.4 SEPARATED FLOW MODEL

The separated flow model takes into account of the fact that the two-phases can have differing properties and different velocities. The formulation may be varying degrees of complexity by inclusion of additional terms. In the most sophisticated version, separate equations of continuity, momentum, and energy are written for each phase and these six equations are then solved simultaneously together with rate equations which describe how the phases interact with each other and with the walls of the duct. In the simplest version, however, only one parameter, such as velocity is allowed to differ for the two-phases involved while the conservation equations are only written for the combined flow. When the number of unknowns to be determined exceeds the number of available equations, correlations or simplifying assumptions are introduced whenever is necessary.

The assumptions which is needed to simplify this model are as follow,

- Thermal equilibrium exists between phases ($T_g = T_s = T$).
- The turbulent component of inertial term is negligible.
- The flow at upstream and downstream of enlargement is assumed to be fully developed.
- Particle-particle collision effects can be neglected.
- Gaseous phase is assumed to be incompressible.
- The flow is assumed to be in steady state condition.

3.4.1 Continuity

Normally, no additional mass is injected to the flow from outside and the overall mass flow rate is constant throughout. Therefore,

$$W = W_s + W_g = constant \qquad (3.30)$$

If the cross-sectional area of the two streams are A_s and A_g then the continuity equation for both phases are as follows:

$$W_s = \rho_s V_s A_s \tag{3.31}$$

$$W_g = \rho_g V_g A_g \tag{3.32}$$

The mass flux of each stream is calculated as follows:

$$G_s = \frac{W_s}{A} = \rho_s V_s \alpha \tag{3.33}$$

$$G_g = \frac{W_g}{A} = \rho_g V_g (1-\alpha) \tag{3.34}$$

3.4.2 Momentum

If there is an interaction between two phases, the following momentum equations can be applied for two phases:

First, the forces upon on a spherical particle in the flow field are considered. By applying Newton's law, the following force relation is obtained

$$m_p \, \partial V_p = -\partial P \, A_p - f_{gp} - f_{wp} - f_{gp} \tag{3.35}$$

Here, the first term on the left side equation is the deceleration force during expansion and the terms on right side include pressure force, interaction force between particle and gaseous phase, wall friction force on particle and gravitational force of particle, respectively.

By multiplying and dividing Eqn. (3.35) with void fraction (α) and with volume of particle (Λ_p), respectively, the following equation is obtained.

$$\alpha \frac{m_p \, \partial V_p}{\Lambda_p} = -\alpha \frac{\partial P \, A_p}{\Lambda_p} - \alpha \frac{f_{gp}}{\Lambda_p} - \alpha \frac{f_{wp}}{\Lambda_p} - \alpha \frac{f_{gp}}{\Lambda_p} \tag{3.36}$$

Now, the forces becomes for unit volume of the flow field and the pressure term will be equal to pressure gradient force as follows,

$$\alpha \frac{\partial P \, A_p}{\Lambda_p} = \frac{\partial P}{\partial z} \qquad \Rightarrow \qquad \Lambda_p = \alpha \, A_p \partial z \tag{3.37}$$

The relation in Eqn. (3.37) gives the particle volume in unit volume of the flow field.

Now, the momentum equation of each phase can be derived from Eqn.(3.36). Let the drag force F_{gs}, for per unit volume of the flow, be acting on gas component in the direction of motion which is in the opposite direction on solid component. Here, the drag force F_{gs} contains components due to hydrodynamic drag, apparent mass effect during relative acceleration and particle-particle forces. Also, let the drag forces from duct wall to gas and solid components be F_{ws} and F_{wg} for per unit volume of the flow, respectively. Thus, the following momentum equation is obtained

$$(1-\alpha)\rho_g V_g \frac{\partial V_g}{\partial z} = -\frac{\partial P}{\partial z} + F_{gs} - F_{wg} - (1-\alpha)\rho_g g \cos\theta \tag{3.38}$$

$$\alpha\rho_s V_s \frac{\partial V_s}{\partial z} = -\frac{\partial P}{\partial z} - F_{gs} - F_{ws} - \alpha\rho_s g \cos\theta \tag{3.39}$$

By adding Eqn. (3.38)&(3.39),

$$\begin{aligned}(1-\alpha)\rho_g V_g \frac{\partial V_g}{\partial z} &+ \alpha\rho_s V_s \frac{\partial V_s}{\partial z} \\ &= -2\frac{\partial P}{\partial z} - g\cos\theta\left[\rho_g(1-\alpha) + \rho_s\alpha\right] - \left(F_{ws} + F_{wg}\right)\end{aligned} \tag{3.40}$$

Finally, the equation giving the pressure difference is as follows:

$$\frac{1}{A}\frac{\partial}{\partial z}\left(W_g V_g + W_s V_s\right) = -2\frac{\partial P}{\partial z} - g\cos\theta\left[\rho_g(1-\alpha) + \rho_s\alpha\right] - \left(F_{wg} + F_{ws}\right) \tag{3.41}$$

$$2\frac{\partial P}{\partial z} = -\frac{1}{A}\frac{\partial}{\partial z}\left(W_g V_g + W_s V_s\right) - g\cos\theta\left(\rho_g(1-\alpha) + \rho_s\alpha\right) - \left(F_{wg} + F_{ws}\right) \tag{3.41a}$$

Integration this equation along the expansion section, i.e., from step to the point where the flow is fully developed, is given below step by step:

$$2\int_1^2 \partial P = -\int_1^2 \frac{1}{A}\partial\left(W_g V_g\right) - \int_1^2 \frac{1}{A}\partial\left(W_s V_s\right) - \int_1^2 \rho_m g\cos\theta\,\partial z - \int_1^2 \left(F_{wg} + F_{ws}\right)\partial z \tag{3.41b}$$

$$2\Delta P=-\left[W_g\int_1^2\frac{V_g}{A_1V_{g1}}\partial V_g+W_s\int_1^2\frac{V_s}{A_1V_{s1}}\partial V_s+\rho_m g\cos\theta\Delta z+\int_1^2\left(F_{wg}+F_{ws}\right)\partial z\right] \quad (3.41c)$$

$$2\Delta P=-\frac{W_g}{A_1V_{g1}}\left(\frac{V_{g2}^2}{2}-\frac{V_{g1}^2}{2}\right)-\frac{W_s}{A_1V_{s1}}\left(\frac{V_{s2}^2}{2}-\frac{V_{s1}^2}{2}\right)$$
$$-\rho_m g\cos\theta\Delta z-\int_1^2\left(F_{wg}+F_{ws}\right)\partial z \quad (3\text{-}41d)$$

$$2\Delta P=-\frac{1}{2}\frac{W_g}{A_1}V_{g1}\left(\sigma^2-1\right)-\frac{1}{2}\frac{W_s}{A_1}V_{s1}\left(\sigma^2-1\right)-\rho_m g\cos\theta\Delta z-\int_1^2\left(F_{wg}+F_{ws}\right)\partial z \quad (3.41e)$$

$$2\Delta P=-\frac{1}{2}G_{g1}^2\frac{1}{\rho_g}\frac{1}{(1-\alpha)}\left(\sigma^2-1\right)-\frac{1}{2}G_{s1}^2\frac{1}{\rho_s}\frac{1}{\alpha}\left(\sigma^2-1\right)$$
$$-\rho_m g\cos\theta\Delta z-\int_1^2\left(F_{wg}+F_{ws}\right)\partial z \quad (3.42)$$

The final form of the expression giving the pressure difference along the expansion section is given by

$$\Delta P=-\frac{1}{4}\left(\sigma^2-1\right)\left(\frac{G_{g1}^2}{(1-\alpha)}\frac{1}{\rho_g}+\frac{G_{s1}^2}{\alpha}\frac{1}{\rho_s}\right)$$
$$-\frac{1}{2}\rho_m g\cos\theta\Delta z-\frac{1}{2}\int_1^2\left(F_{wg}+F_{ws}\right)\partial z \quad (3.43)$$

Here, all one needs to know is the wall shear stress to compute the pressure difference.

3.4.3 Wall Friction Loss Term

In pneumatic conveying systems, the wall friction loss term consists of two parts: (i) the gas-wall friction loss and (ii) solid-wall friction loss.

3.4.3.1 Gas-Wall Friction Loss Term

The gas-wall friction loss is frequently assumed to be equivalent to that due to gas alone and its definition is as follows:

$$\Delta P_{fg} = 2 f_g \rho_g V_g{}^2 \frac{L}{D} \tag{3.44}$$

The fanning gas friction factor f_g can be calculated either from Blasius's equation, Eqn. (3.27), for smooth pipe flow for turbulent flow regimes or following the suggestion by Koo (1981) which Wang, (1997) attiributed to him.

$$f_g = 0.0014 + 0.125/(\mathrm{Re})^{0.32} \tag{3.45}$$

Weber (1991), see ref. [2], has found that the gas friction pressure loss is also dependent on the solid loading ratio and has suggested the following correlation:

$$f_g = \left(0.1/\mathrm{Re}^{0.151}\right)\left[1/\left(1+M^{0.7}\right)\right] \tag{3.46}$$

Here, M is the solid loading ratio.

It is important to note that Reynolds number used in Eqns. (3.45)&(3.46) is based downstream pipe diameter.

3.4.3.2 Solid-Wall Friction Loss Term

Considerable effort has been devoted on modeling the solid-wall pressure loss encountered in the pneumatic transport systems. A good understanding is much needed, as large quantity of energy is used to overcome the solid-wall friction. The solid's contribution to the pressure loss function is normally given as:

$$\Delta P_{fs} = 2 f_s \rho_p \alpha V_p \frac{L}{D} \tag{3.47}$$

In contrast to its counterpart in gas-wall friction loss calculation, the solid-wall friction factor is often handeled differently by many researchers. According to Wang (1997), some investigators have indicated that the friction factor was inversely proportional to the particle velocity in the following form (Capes&Nakamura, 1973):

$$f_s = a V_p^{-b} \tag{3.48}$$

A summary of parameters "*a*" and "*b*" quoted by different researchers is listed in Table 3.1.

Table 3.1: Emprical constants to use in Eqn. (3.48)

Investigators	**a**	**b**
Reddy and Pei (1 969)	0.46	1.00
Van Swaaj et ai. (1970)	0.048	1.00
Capes and Nakamura(1973)	0.074	1.22
Leung and Wiles (1 976)	0. 046	1.00
Kmiec et al. (1978)	0.048	0.75
Quong (1983)	0.074	0.74

Wang (1997) provides us another approach which he attributed to jones et al. (1967). In this model a solid friction factor $f_s{}'$ defined and calculated as follows;

$$f_s' = 1.89x10^{-6} \frac{A_0}{\phi^{0.5}} M^n \tag{3.49a}$$

Where,

$$n = \left(20668/A_0\right)^{1/3} \quad \text{for} \quad A_0 > 20668 \quad \& \quad n=1 \quad \text{for} \quad A_0 < 20668 \tag{3.49b}$$

Where "A_0" is the total surface area of the solid particles per unit volume, "M" is the solid-gas loading ratio, and "ϕ" is the particle shape factor which is equal to unity for spherical particles (Wallis, (1969)).

Finally, the contribution of the solids to the pressure loss expression becomes:

$$\Delta P_{fs} = f_s' \rho_g V_g^2 \frac{L}{2D} \tag{3.49}$$

In this form, loss term is only based on gas density and velocity.

Using the correlations given above, the wall friction term can be written as:

$$\begin{aligned} \int_1^2 \left(F_{wg} + F_{ws}\right) \partial z &= \Delta P_{fg} + \Delta P_{fs} \\ &= 2 f_g \rho_g V_{g2}^2 \frac{L_2}{D_2} + \Delta P_{fs} \end{aligned} \tag{3.50}$$

Where, L_2 represents the length along the expansion section starting from enlargement up to the section where the flow attains its fully developed state.

It is suggested that for particles in the order of microns, ΔP_{fs} should be calculated by using Eqn. (3.47) and if particle sizes not in the order of microns, Eqn. (3.49) should be used instead.

Now, the final form of Eqn. (3.40) becomes:

$$\Delta P = -\frac{1}{4}\left(\sigma^2 - 1\right)\left(\frac{G_{g1}^{\ 2}}{(1-\alpha)}\frac{1}{\rho_g} + \frac{G_{s1}^{\ 2}}{\alpha}\frac{1}{\rho_s}\right) - f_g \rho_g V_{g2}^2 \frac{L_2}{D_2} - \frac{1}{2}\Delta P_{fs} - \frac{1}{2}\rho_m g \cos\theta \Delta z \tag{3.51}$$

Eqn. (3.51) is derived from separate flow model approaches and it is referred as "separate flow model".

3.5 ADDITIONAL FORCE COMPONENTS in RAPIDLY ACCELERATING FLOWS

The equation of motion of a particle, momentum, moving upward in steady flow through a sudden expansion fitting is given as,

$$m_p \frac{\partial U_p}{\partial t} = F_D + F_A + F_P + F_G \tag{3.52}$$

Here, F_A, F_D, F_P and F_G is apparent mass force, drag force, pressure force and gravitational force, respectively.

The following assumptions are made in the analysis given below:

- The turbulent component of inertial term is negligible.
- Thermal equilibrium exists between phases ($T_g = T_s = T$).
- The flow at upstream and downstream of enlargement is assumed to be fully developed.
- Particle-particle collision effects can be neglected.
- Gaseous phase is assumed to be incompressible.
- The flow is assumed to be in steady flow.

Apparent mass force and drag force terms are broadly introduced as follows,

<u>Apparent Mass Force</u>:

When a particle is accelerated or decelerated relative to surrounding fluid it sets up a two dimensional flow around it which possesses kinetic energy. Therefore, work must be supplied to move the particle in addition to that which is required to accelerate it alone. This extra energy requirement means that an additional force must be exerted on the particle. The acceleration of a sphere in a stationary inviscid fluid of large extent requires a force which is given by

$$F = \frac{1}{6}\pi d_s^3 \left(\rho_s + \frac{\rho_g}{2} \right) a' \tag{3.53}$$

The sphere is assumed to behave as if it possesses an additional "apparent mass" being equal to one half of the fluid mass which it displaces. Thus, the mass of particle is increased apparently by the amount of $\left[1 + C\left(\rho_g / \rho_s \right) \right]$; where C is a parameter which is a function of geometrical shape of particles. "C" is 1/2 for a spherical particle and is 1/5 for an ellipsoid particle (Wallis, (1969))

The effect of apparent mass is that it introduces an additional component in the forces F. This force is proportional to the relative acceleration as seen by a coordinate system moving with the particle.

$$F_s = -F_g \frac{\varepsilon}{1-\varepsilon} = -\frac{1}{6}\pi d_s^3 C \rho_g \left[\frac{\partial}{\partial t}\left(V_s - V_g\right) + V_s \frac{\partial}{\partial z}\left(V_s - V_g\right) \right] \tag{3.54}$$

For simplicity, the void fraction is defined in an alternative form:

$$\alpha = (1-\varepsilon) \tag{3.55}$$

Where, F_s and F_g represent the forces, per unit volume, on solid particle and its equivalent to the forces on the gas surrounding it. The force F_s acts to reduce the velocity lag.

Drag Force:

When a solid particle goes through a fluid, a resistance force sets up on solid particle due to friction between fluid and solid particles. This resistance force is known as drag force. Drag force is a dominating effect on pressure loss in sudden expansion pipe flow since the slip in velocity between phases occurs during expansion.

This drag force pressure drop due to solid particles is given by as (Wallis, 1969):

$$\Delta P_D = \frac{3}{4d_p}\varepsilon^{-1.7}\, C_{Ds}\,\rho_g\left(V_s - V_g\right)\left|V_s - V_g\right|\Delta z \tag{3.56}$$

Here, C_{Ds} is the superficial drag coefficient which varies with particle Reynolds number, Re_p and for $1 < Re_p < 1000$ is given by

$$C_{Ds} = \frac{24}{\mathrm{Re}_p}\left(1 + 0.15\mathrm{Re}_p^{0.657}\right) \tag{3.57}$$

and

$$\mathrm{Re}_p = \frac{\rho_g\left|\left(V_p - V_g\right)\right| d_p}{\mu_g} \tag{3.58}$$

3.5.1 Momentum:

By replacements of apparent mass force and drag force terms, Eqn. (3.54) and Eqn. (3.56), respectively, into the momentum equation yields the final form of equation of motion:

$$\begin{aligned}\rho_s V_s \frac{\partial V_s}{\partial z} = -\rho_s g\cos\theta - \frac{\partial P}{\partial z} - \frac{3}{4d_p}\varepsilon^{-1.7}\, C_{Ds}\,\rho_g\left(V_s - V_g\right)\left|V_s - V_g\right| \\ - C\rho_g V_s \frac{\partial\left(V_s - V_g\right)}{\partial z}\end{aligned} \tag{3.59}$$

and for the gaseous phase:

$$\rho_g V_g \frac{\partial V_g}{\partial z} = -\rho_g g \cos\theta - \frac{\partial P}{\partial z} + \frac{3}{4d_p}\frac{1-\varepsilon}{\varepsilon^{2.7}} C_{Ds}\,\rho_g \left(V_s - V_g\right)\left|V_s - V_g\right| + \frac{(1-\varepsilon)}{\varepsilon} C\rho_g V_s \frac{\partial\left(V_s - V_g\right)}{\partial z} \tag{3.60}$$

If the wall shear stress is also taken into account, the momentum equation for the gaseous phase may be rewritten as

$$\begin{aligned}\rho_g V_g \frac{\partial V_g}{\partial z} &= -\rho_g g \cos\theta - \frac{\partial P}{\partial z} \\ &\quad + \frac{3}{4d_p}\frac{1-\varepsilon}{\varepsilon^{2.7}} C_{Ds}\,\rho_g \left(V_s - V_g\right)\left|V_s - V_g\right| \\ &\quad + \frac{(1-\varepsilon)}{\varepsilon} C\rho_g V_s \frac{\partial\left(V_s - V_g\right)}{\partial z} - \frac{P}{A}\left(\tau_w\right)\end{aligned} \tag{3.61}$$

The momentum equation given above contains the body force, pressure gradient, drag force, apparent mass and the wall shear stress effects. The momentum equation for the gaseous phase can be rearranged to obtain an expression for the pressure gradient along the expansion section from step to the position where fully developed flow is reached as follows:

$$\begin{aligned}\frac{\partial P}{\partial z} &= -\rho_g g \cos\theta - V_g \rho_g \frac{\partial V_g}{\partial z} + \frac{3}{4d_p}\frac{1-\varepsilon}{\varepsilon^{2.7}} C_{Ds}\,\rho_g \left(V_s - V_g\right)\left|V_s - V_g\right| \\ &\quad + \frac{(1-\varepsilon)}{\varepsilon} C\rho_g V_s \frac{\partial\left(V_s - V_g\right)}{\partial z} - \frac{P}{A}\left(\tau_w\right)\end{aligned} \tag{3.61a}$$

$$\Downarrow$$

$$\begin{aligned}\partial P &= -\rho_g g \cos\theta dz - V_g \rho_g \partial V_g + \frac{3}{4d_p}\frac{1-\varepsilon}{\varepsilon^{2.7}} C_{Ds}\,\rho_g \left(V_s - V_g\right)\left|V_s - V_g\right| \partial z \\ &\quad + \frac{(1-\varepsilon)}{\varepsilon} C\rho_g V_s \partial\left(V_s - V_g\right) - \frac{P}{A}\left(\tau_w\right)\partial z\end{aligned} \tag{3.61b}$$

Integration of this equation term by term is given below:

$$\int_1^2 \partial P = \Delta P \tag{3.62}$$

$$\int_1^2 \rho_g\, g \cos\theta \, dz = \rho_g\, g \cos\theta \, \Delta z \tag{3.63}$$

$$\Rightarrow \int_1^2 V_g \rho_g \partial V_g = \rho_g \left(\frac{V_{g2}^2}{2} - \frac{V_{g1}^2}{2} \right) = \frac{1}{2} \rho_g V_{g1}^2 \left(\sigma^2 - 1 \right) \tag{3.64}$$

Integration of the following drag force term:

$$\Rightarrow \int_1^2 \frac{3}{4d_p} \frac{1-\varepsilon}{\varepsilon^{2.7}} C_{Ds}\, \rho_g \left(V_s - V_g \right) \left| V_s - V_g \right| \partial z \tag{3.65}$$

$$= \frac{3}{4d_p} \frac{1-\varepsilon}{\varepsilon^{2.7}} C_{Ds}\, \rho_g V_{g2}^2 \left(\frac{\rho_g}{\rho_s} \frac{x}{1-x} \frac{\varepsilon}{1-\varepsilon} - 1 \right) \left| \frac{\rho_g}{\rho_s} \frac{x}{1-x} \frac{\varepsilon}{1-\varepsilon} - 1 \right| \Delta z$$

$$\int_1^2 + \frac{(1-\varepsilon)}{\varepsilon} C\, \rho_g V_s \partial \left(V_s - V_g \right)$$

$$= \int_1^2 \frac{(1-\varepsilon)}{\varepsilon} C \rho_g V_s \partial V_s - \int_1^2 \frac{(1-\varepsilon)}{\varepsilon} C\, \rho_g V_s \partial V_g \tag{3.66}$$

$$\Rightarrow \int_1^2 \frac{(1-\varepsilon)}{\varepsilon} C \rho_g V_s \partial V_s = \frac{(1-\varepsilon)}{\varepsilon} C \rho_g \left(\frac{V_{s2}^2}{2} - \frac{V_{s1}^2}{2} \right)$$

$$= \frac{1}{2} \frac{(1-\varepsilon)}{\varepsilon} C \rho_g V_{s1}^2 \left(\sigma^2 - 1 \right) \tag{3.66a}$$

This last integral in Eqn. (3.66) can be evaluated by replacing the solid velocity in terms of gas velocity by using the following relation,

$$G = \rho_g V_g \frac{(1-\alpha)}{(1-x)} = \rho_s V_s \frac{\alpha}{x} \Rightarrow V_s = \frac{\rho_g}{\rho_s} V_g \frac{(1-\alpha)}{(1-x)} \frac{x}{\alpha}$$

Thus,

$$\Rightarrow \int_1^2 \frac{(1-\varepsilon)}{\varepsilon} C \rho_g V_s \partial V_g = \frac{1}{2} C \frac{\rho_g^2}{\rho_s} \frac{x}{(1-x)} V_{g1}^2 \left(\sigma^2 - 1\right) \tag{3.66b}$$

And finally, the gas-wall shear stress term is

$$\int_1^2 \frac{P}{A}(\tau_w) \partial z = 2 f_g \rho_g V_{g2}^2 \frac{L_2}{D_2} \tag{3.67}$$

Replacement of Eqns. (3.63) through (3.67) back to the original equation yields

$$\begin{aligned} \Delta P = & \frac{1}{2} \rho_g V_{g1}^2 \left(\sigma^2 - 1\right) \left[\left(\frac{\rho_g}{\rho_s}\right)^2 \left(\frac{x}{1-x}\right)^2 \frac{\varepsilon}{1-\varepsilon} C - \frac{\rho_g}{\rho_s} \frac{x}{1-x} C - 1 \right] \\ & + \frac{3}{4 d_p} \frac{1-\varepsilon}{\varepsilon^{2.7}} C_{Ds} \rho_g V_{g2}^2 \left(\frac{\rho_g}{\rho_s} \frac{x}{1-x} \frac{\varepsilon}{1-\varepsilon} - 1 \right) \left| \frac{\rho_g}{\rho_s} \frac{x}{1-x} \frac{\varepsilon}{1-\varepsilon} - 1 \right| \Delta z \\ & - 2 f_g \rho_g V_{g2}^2 \frac{L_2}{D_2} - \rho_g g \cos\theta \Delta z \end{aligned} \tag{3.68}$$

The computation of this expression is very hard due to the unknown the relative velocity between phases along the expansion.

3.6 EXISTING MODELS IN LITERATURE

The behavior of two-phase flow through a sudden enlargement has been the subject of several experimental and theoretical investigations. An important parameter which characterizes this type of singularity is the global singular pressure variation. Several analytical methods calculating this quantity exist in the literature. First, the theoretical analysis for gas-liquid flows is presented since bubbly flows are similar to gas-solid flows if the gaseous phase is assumed incompressible. Second, the attention will focus on gas-solid flows.

3.6.1 Studies on Gas-Liquid Flows

Two phase gas-liquid flow systems occupy an important place and many problems arise for the engineer during the design of the devices practical involving such flows. Some of the related literature gives analysis on gas-liquid flow through axi-symmetric sudden expansions. Attou et al. (1997) and Aloui et al. (1999) have studied bubbly flow along the sudden expansion section. Their aim was to find analytical expression for the pressure loss along the expansion.

Aloui et al. (1999) defined singular pressure drop coefficient based on the following assumptions:

- Both fluids are incompressible.
- The pressure in the cross-section (A_2-A_1) is the same as the pressure on A_1.
- The pressure inside a bubble is the same as the pressure of the surrounding liquid.

Aloui et al. (1999) measured pressure drop, wall shear stress, local and global void fraction, average and fluctuating velocities and bubble sizes in a experimental setup. The turbulent kinetic energy term was calculated from the experimental data. The two-phase singular pressure drop coefficient K_{dp} is written in the following form:

$$K_{dp} = K_{dp}^{1} + K_{dp}^{2} + K_{dp}^{3} \tag{3.69}$$

Where,

K_{dp}^{1} is the inertial term.

K_{dp}^{2} is the turbulent component of inertial term.

K_{dp}^{3} is the wall shear stress along the flow downstream of sudden expansion.

The inertial term is defined as follows,

$$K_{dp}^{1} = 2\sigma\left(\frac{1}{1-\alpha_1} - \frac{\sigma}{1-\alpha_2} + \left(\frac{\rho_l}{\rho_g}\right)\left(\frac{x}{1-x}\right)^2\left(\frac{1}{\alpha_1} - \frac{\sigma}{\alpha_2}\right)\right) \tag{3.70}$$

and the pressure difference due to enlargement can be calculated as follows,

$$\Delta P = \frac{1}{2}\rho_l V_l^2 K_{dp}^{1} \tag{3.71}$$

It was shown that the inertial term, Eqn. (3.70), have good agreement with experimental data. Their experimental work showed that the effect of the turbulent term and the wall shear stress term on the pressure drop coefficient, K_{dp}, is in the order of 1% and 2.5%, respectively.

Attou et al. (1997) presented a theoretical study on bubbly flow through a sudden expansion. In their study, the global formulations of the conservations laws (mass, momentum, energy) are applied to the two-phase gas-liquid flow through sudden expansion. The upstream section A_1 and the downstream section A_2 selected in such a way that the flow is fully developed in both. They have also used simplifying assumptions given below:

- The turbulent component of inertial term is negligible.
- The effect linked to the superficial tension is neglected ($P_g = P_l = P$).
- The pressure is approximately uniform over singularity section.
- The thermal equilibrium exists between phases ($T_g = T_l = T$).
- The gaseous phase is considered as an ideal gas and the liquid is incompressible.

The pressure difference along expansion, which is obtained from the momentum equation by using above assumptions, is as follows,

$$P_2 - P_1 = \sigma\left[\alpha_1 \rho_{g1} V_{g1}^2 + (1-\alpha_1)\rho_l V_{l1}^2\right] + \rho_m g \Delta z - \frac{\tau_w}{A_2} - \left[\alpha_2 \rho_{g2} V_{g2}^2 + (1-\alpha_2)\rho_l V_{l2}^2\right] \tag{3.72}$$

Comparison of Eqn. (3.70)&(3.72) with experimental data of gas-solid flows results to the best theoretical equation for gas-solid flows. The following table represents the comparison of Eqns. (3.70)&(3.72) with experimental data (Marjanovic et al., 1999). Material and gas properties used in this study were as follows,

<u>*Gas Properties:*</u>

Gas :……………………………………. Air

Dynamic Viscosity:……………………..$\mu = 17 x 10^{-6}$ kg/m.s

Density at ambient temperature:……….. $\rho = 1.2$ kg/m^3

<u>*Material Properties:*</u>

Material:……………………………….. Polyethylene pellets

Particle diameter:……………………… 3mm

Particle density:…………………………880 kg/m^3

Pipe Geometry and Flow conditions:

Pipeline bores:…………………………..D_1 =81 mm & D_2= 105 mm

(Area ratio: A_1/A_2=0.595)

Temperature:……………………………293 K

Solids Loading Ratio:………………….. Five values are used in the analysis

SLR: W_s/W_g=2, 4, 6, 8, 10

Transition Length L_{tr} / D_2: …………….. 27, 29, 32, 37, 39 across values in SLR respectively

Table 3.3: Comparison of pressure difference with experimental data of Marjanovic et al. (1999) for different solid loading ratios

W_g (kg/s)	SLR	ΔP (Pa) Aloui et al. (1999)	ΔP (Pa) Attou et al. (1997)	ΔP (Pa) (Exp. Data)
0.14	10	1582	1335	400
0.14	8	1329	1162	350
0.14	6	1018	933	300
0.14	4	707	738	250
0.14	2	428	422	200

Table 3.3 shows that Eqn. (3.70)&(3.72) have far values from the experimental data but Eqn. (3.72) have better result than Eqn. (3.70). So Eqn. (3.72) can be adopted to gas-solid flows under following assumptions.

- The turbulent component of inertial term is neglected.
- The pressure in the cross-section (A_2-A_1) is the same as the pressure on A_1.
- The thermal equilibrium exists between phases.
- The flow is assumed to be in steady state condition.
- Gas phase are assumed incompressible.

The pressure difference along the expansion, which is based on the above assumptions, are given as follows,

$$P_2 - P_1 = \sigma\left[\alpha\rho_s V_{s1}^2 + (1-\alpha)\rho_g V_{g1}^2\right] - \left[\alpha\rho_s V_{s2}^2 + (1-\alpha)\rho_g V_{g2}^2\right] + \rho_m g\cos\theta\Delta z - \frac{\tau_w}{A_2} \tag{3.73}$$

The wall shear stress term in equation can be defined in terms of gas and solid wall friction given as follows,

$$\frac{\tau_f}{A_2} = \Delta P_{fs} + \Delta P_{fg} \tag{3.74}$$

Now, Eqn. (3.70) can be written as follow,

$$\Delta P = \sigma\left[\alpha\,\rho_s V_{s1}^2 + (1-\alpha\,)\rho_g V_{g1}^2\right] - \left[\alpha\,\rho_s V_{s2}^2 + (1-\alpha)\rho_g V_{g2}^2\right] - 2 f_g \rho_g V_{g2}^2 \frac{L_2}{D_2} - \Delta P_{fs} - \rho_m\, g\, Cos\theta\Delta z \tag{3.75}$$

So far, in this chapter, the pressure difference along the sudden expansion is theoretically examined and different theoretical models are obtained. The homogeneous and separated flow approaches are applied separately and two different analytical expressions are obtained (Eqns. (3.25), (3.51)). Eqn. (3.68) is obtained by including the drag force effect and apparent mass effect. Finally, the theoretical models formulated in literature for gas-liquid flows are compared with experimental data and it is seen that all are far from predicting pressure difference which can be used in engineering analysis of such systems but also Eqn. (3.72) is seen as the best among others and developed for gas-solid flows as Eqn. (3.75).

3.7 SLIP FLOW MODELS

In pneumatic conveying systems, a relative velocity always occurs between phases and this relative velocity can show differences with the flow direction in horizontal or vertical in the case of same Reynolds number and same solid loading ratio. It is difficult to calculate this relative velocity with experimental methods and with analytical methods so the assumption of slip velocity between phases has been made in literature in broadly.

The fundamental void-quality relation with slip ratio, S, is displayed for theoretical models:

$$\frac{x}{1-x} = S\frac{\rho_s \alpha}{\rho_g (1-\alpha)} \tag{3.76}$$

Where, S is the slip ratio and is defined with the velocity ratio given below:

$$S = \frac{V_s}{V_g} \tag{3.77}$$

Many assumptions to the slip ratio in literature are presented by many authors and two of them are as follows,

A model for the slip ratio was presented by Abdellal et al. (2005) for gas-liquid flows:

$$S = \left(\frac{\rho_l}{\rho_g}\right)^{1/3} \tag{3.78}$$

Another model for the slip ratio was presented by Kojasoy et al. (1997) for gas-liquid flows and is given in following expression.

$$S = \left[1 + x\left(\frac{\rho_l}{\rho_g} - 1\right)\right]^{1/2} \tag{3.79}$$

Void fraction can be calculated from Eqn. (3.76) with a slip ratio assumption in Eqns. (3.78) or (3.79). The slip ratio introduces itself in the pressure recovery with a relation of void fraction in such way.

3.8 CONCLUSION

Theoretical modellings of sudden expansion in gas-solid two phase flow were carried out in this chapter. Here, assumptions are made for the flow through expansion and different theoretical expressions for the pressure difference are obtained from the momentum and continuity equations. The assumptions of homogeneous and separated two phase flow give three different theoretical expressions for the pressure difference. The inertial term and wall frictional term were considered dominant effect on pressure difference. The momentum transfer from solid particles to gaseous phase is taken into account by an introduction of a drag force term and apparent mass term into theoretical expression. The theoretical models formulated in literature for gas-liquid flows are also examined. Eqn. (3.72) is obtained for gas-solid flows from the theoretical expressions of gas-liquid flows. The slip flow models in literature is introduced for the relative velocity between phases with an assumption to the slip velocity since the slip ratio affect the pressure recovery by the relation of void fraction.

CHAPTER 4

EVALUATION OF THEORETICAL MODELS

4.1 INTRODUCTION

In this chapter, the theoretical models are compared with experimental studies of Marjanovic et al (1999) for gas-solid two phase flows. As a result of this comparison, a slip flow model is defined for the slip velocity between gas-solid phases. The separate flow model coupled with slip flow model is compared with experimental data of Marjanovic et al. (1999) and Tashiro&Tomita (1986), respectively. The separate flow model is compared with experimental data of gas only flows and liquid only flows; and also tested against gas-liquid flows by using experimental data of Abdellal et al. (2004). Finally, minor loss coefficient is divided as one due to gas only flow and one due to addition of solid particles to the flow and the minor loss coefficient is evaluated under different solid loading ratios and at various Reynolds numbers and either additional minor loss coefficient. All computations are carried out for turbulent pipe flow regime where Reynolds number is referred to small pipe.

4.2 COMPARISON of THEORETICAL MODELS with EXPERIMENTS

Here, the theoretical models will be compared with the experimental data of Marjanovic et al. (1999), Tashiro&Tomita (1986) and Abdellal et al. (2004). The theoretical models will also be compared with the experimental data of gas only flows of Tomita et al. (1980) and liquid only flows and gas-liquid flow of Abdellal et al. (2004) to see the applicability of the model to these cases.

4.2.1 Comparison with Marjanovic et al. (1999): Gas-Solid flows

The theoretical models of sudden expansion for gas-solid flows are compared with experimental data of Marjanovic et al. (1999). In their experimental study, the polyethylene pellets are horizontally conveyed by a pneumatic system through which the flow goes a sudden expansion section was monitored. The material and air properties and the geometry of pipe were as follows,

Gas Properties:

Gas:	Air
Dynamic Viscosity:	$17\,x10^{-6}$ kg/m.s
Density at ambient temperature:	ρ = 1.2 kg/m^3

Material Properties:

Material:	Polyethylene pellets
Particle diameter:	3 mm
Particle density:	880 kg/m^3

Pipe Geometry and Flow conditions:

Pipeline bores:	D_1 = 81 mm ; D_2 = 105 mm
	(Area ratio: A_1/A_2 = 0.595)
Temperature:	293 K
Pipeline Lengths:	$L_i = L_o$ = 6 m
Solids Loading Ratio (SLR):	W_s/W_g = 2, 4, 6, 8, 10
Reynolds Number:	88623 – 128647

Where, the subscripts “i” and “o” refer to upstream and downstream section, respectively.

For comparison of theoretical models with experimental data, the transition length obtained from the experimental study of Marjanovic et al. (1999) across different solid and gas mass flow rates is given in Fig. 4.1.

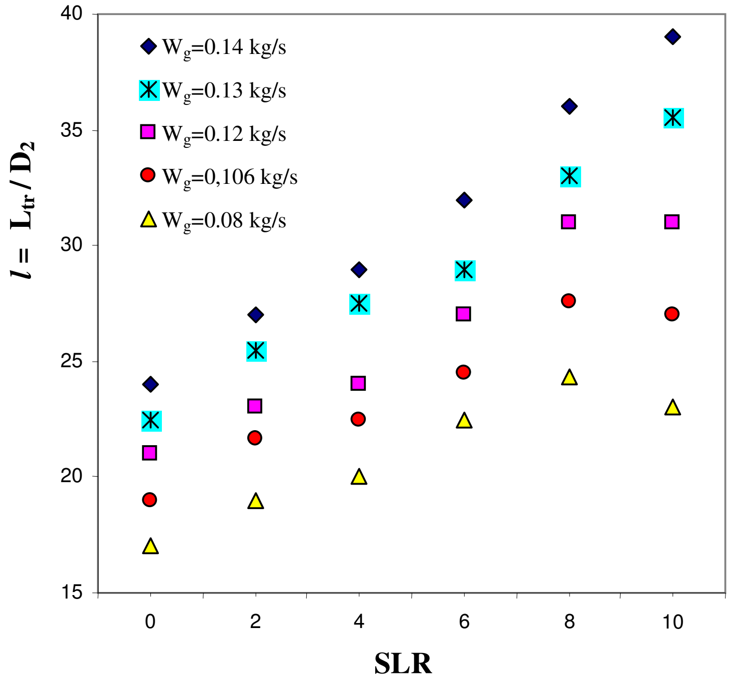

Figure 4.1: L_{tr} / D_2 versus SLR for different Reynolds numbers

Fig. 4.1 shows that the transition length increases nonlinearly as the solid loading ratio increases and has peak point for the SLR around 8 for gas mass flow rates between the range of 0.08 and 0.12 in gas mass flow rate. Transition length decreases as the gas mass flow rate decreases. Here, the transition length is defined from step to the point where the fully developed flow conditions occur for gas-solid flows. As it is stated in chapter 3.1, there are two transition lengths used in literature. Here, in Fig. 4.1, the second definition, which is defined in sec.3.2.9, is used.

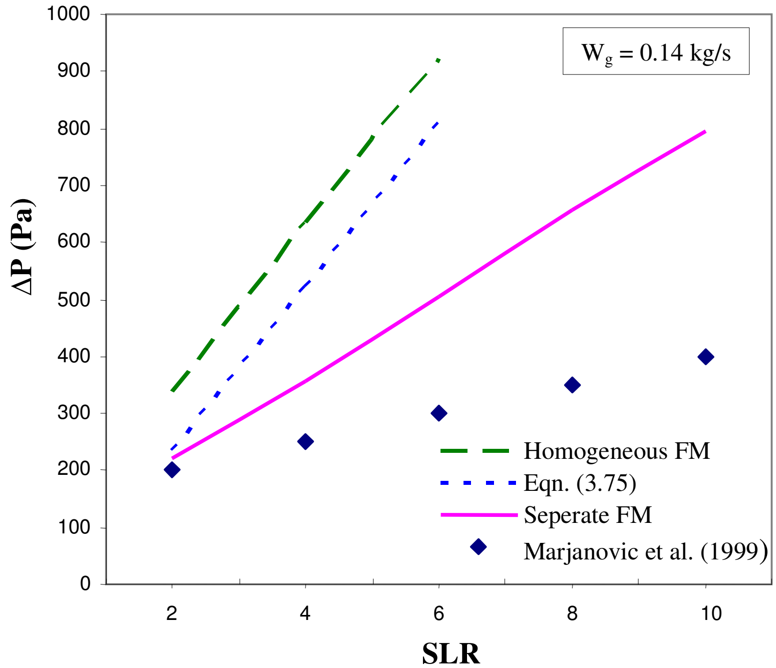

Figure 4.2: Comparison of pressure difference data with theory

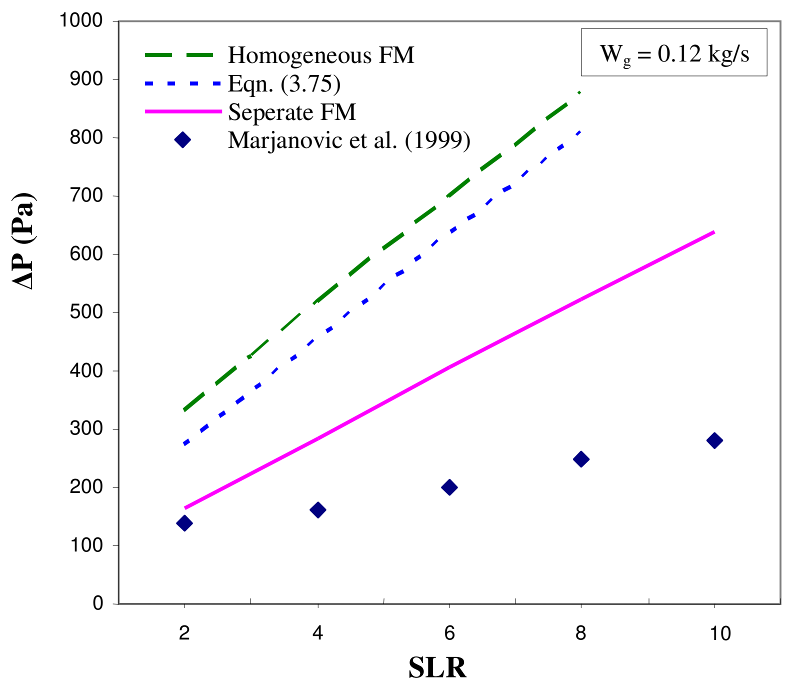

Figure 4.3: Comparison of pressure difference data with theory

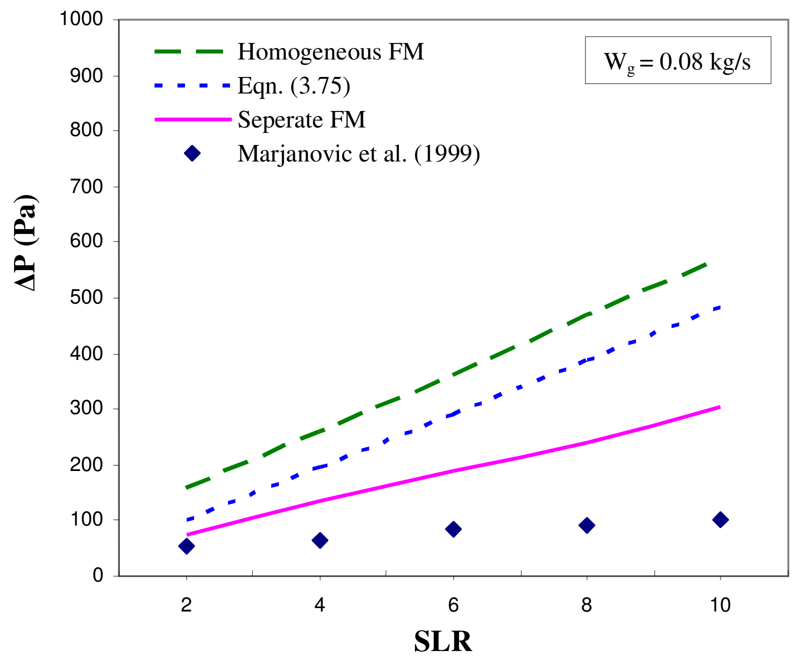

Figure 4.4: Comparison of pressure difference data with theory

Fig. 4.2 through Fig. 4.4 show that the pressure difference curves departs from the experimental data as solid loading ratio increases or in another word the error between theory and experimental data increases with an increase in solid loading ratio and goes to minimum with a decrease in solid loading ratio. It is seen that the error between theory and data increases with an increase in gas mass flow rate. Here, it is also seen that the separate flow model has near values to data than others for all ranges of gas mass flow rates and also the difference between separate flow model and data is in the range of around 200% in maximum and 10% in minimum for Fig 4.2 through Fig. 4.4.

Slip Flow Model:

Relative velocity between phases occurs during sudden expansion. The prediction of pressure difference in theoretical models coupled with Eqns. (3.75)& (3.76) is compared with experimental data of Marjanovic et al. (1999), respectively. It is seen that Kojasoy slip flow model, which is given in Eqn. (3.79), is better than the Abdellal slip flow model, which is given in Eqn. (3.78). Here, Kojasoy's slip flow model has a power as 1/2 but changing power between 1/2 and 1/4 and comparing with experimental data of Marjanovic et al. (1999) have shown that 2/5 power yields better agreement with experimental data and Kojasoy's slip flow model is changed to the following one.

$$S = \left[1 + x \left(\frac{\rho_g}{\rho_s} - 1 \right) \right]^{2/5} \tag{4.1}$$

Here, slip flow model in Eqn. (4.1) is an assumption for the relative velocity between phases. This model is introduced into the theoretical expressions with effect on void fraction (α) used in Eqn. (3.76).

By using the separate flow model together with the slip flow model the following two figures are obtained for various solid loading ratios and gas mass flow rates. Here, the pressure difference which is obtained from Eqn. (3.51) is compared with experimental data (Marjanovic et al., 1999). In this experimental study, the material and gas properties, flow conditions and pipe geometry were given in section 4.2.1.

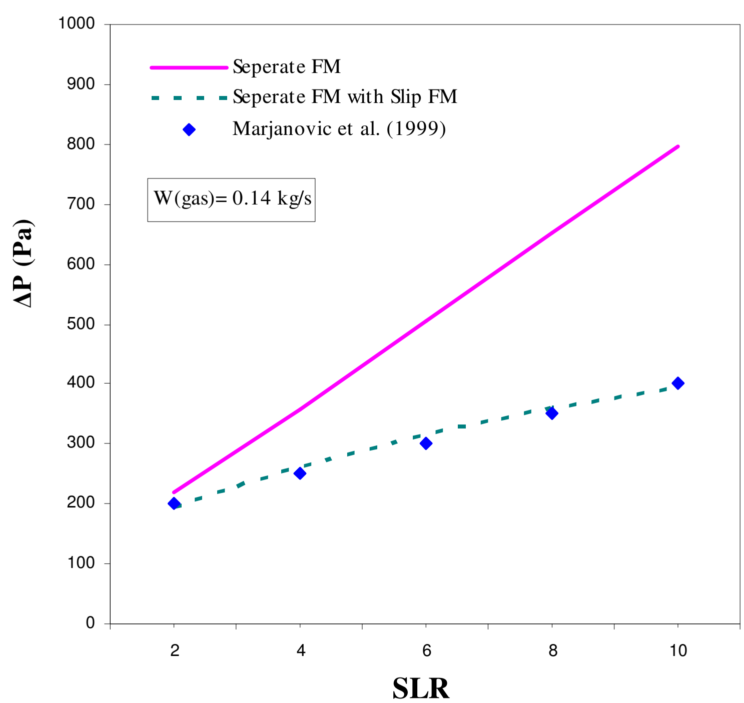

Figure 4.5: ΔP versus SLR, at Re = 129450

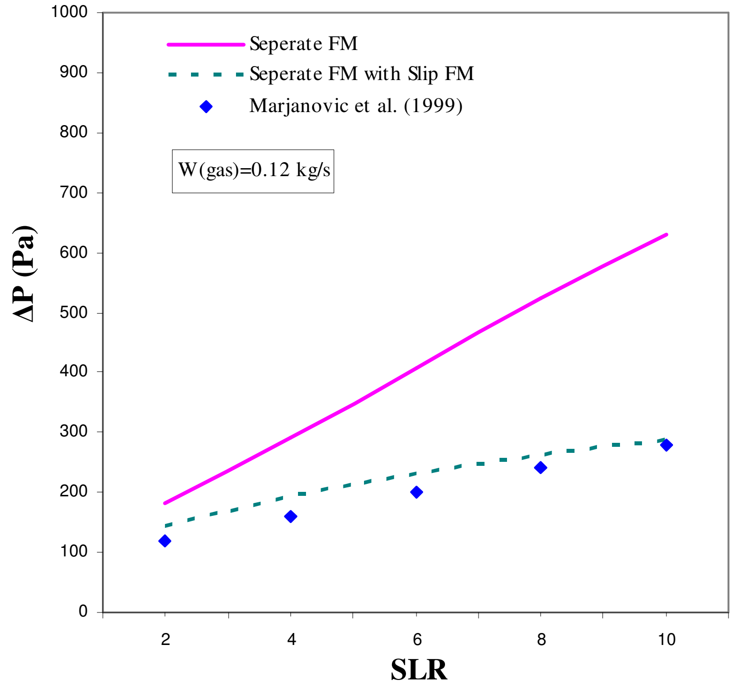

Figure 4.6: ΔP versus SLR, at Re = 110957

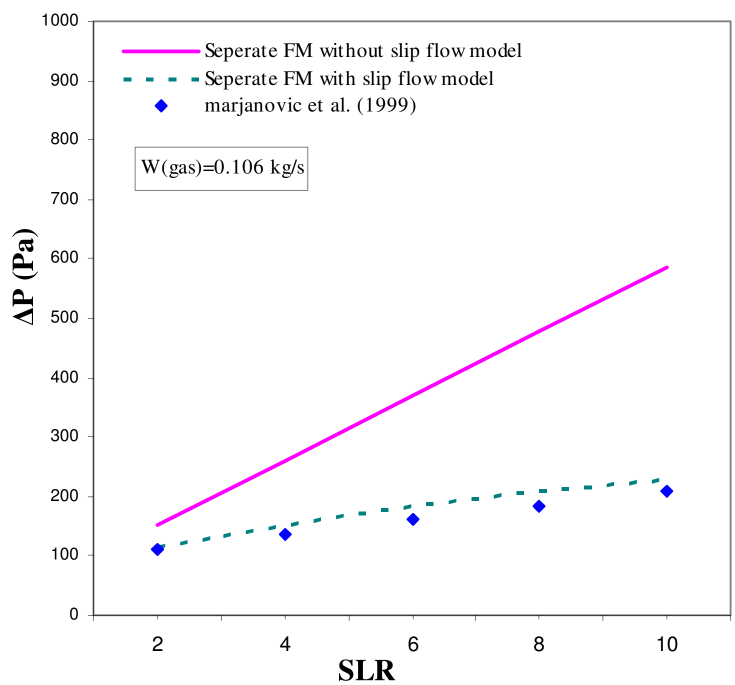

Figure 4.7: ΔP versus SLR, at Re = 98012

As can be seen from Fig. 4.5 through Fig. 4.7 the separate flow model coupled with the slip flow model is in very good agreement with experimental data. Here, figures show that the difference between data and the model with slip FM is changing between –3 % and +22 %. The differences may be attributed to the pressure losses arising because of particle-to-particle collision and due to turbulent inertial term during sudden expansion or the irreversibility's occurring in recirculation region.

Now, the separate flow model will be tested for various gas flow rates in a constant solid loading ratio flow against experimental data by Marjanovic et al. (1999). The material and gas properties and pipe geometry and flow condition were already given in section 4.2.1. Here, the transition lengths used in calculations are given in Fig. 4.1.

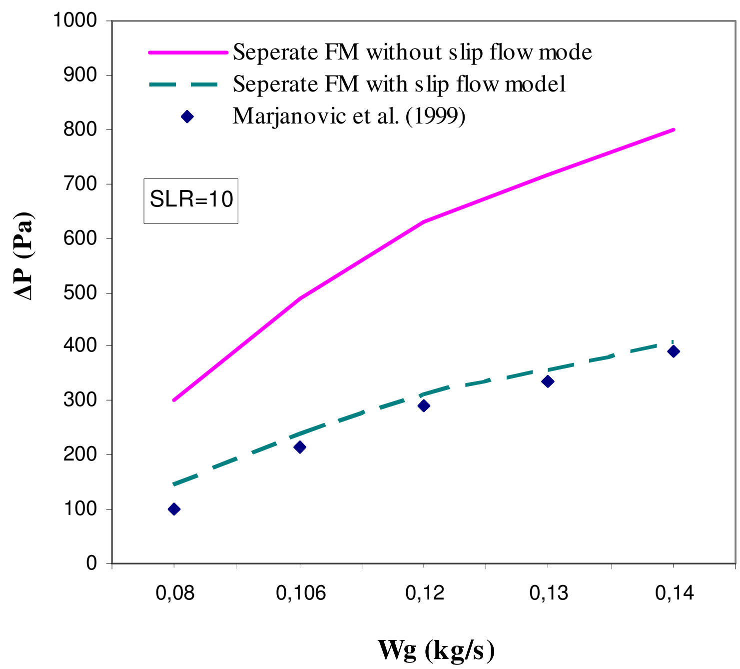

Figure 4.8: ΔP versus W_g, at SLR=10

In Fig. 4.8, the separate flow model coupled with slip flow model is in good agreement with experimental data than the model without one as the gas mass flow rate increases or the Reynolds number increases. The error between data and theory increases as the gas mass flow rate decrease.

4.2.2 Comparison with Tashiro&Tomita (1986): Gas-Solid flows

Now, theoretical models will be compared with the experimental data by Tashiro&Tomita (1986). In their experimental study, range of diameter ratio is between 0.405 and 0.881. The additional expansion loss coefficient due to particles in a sudden expansion of a circular pipe in vertical upward flow is examined by using three different particles. The particle and air properties, pipe geometry and flow conditions used in experiments are given as follows:

Gas Properties:

Gas:	Air
Dynamic Viscosity:	$17x10^{-6}$ kg /m.s
Density at ambient temperature:	ρ = 1.2 kg/m^3

Material Properties:

Table 4.1: Physical properties of particles

Particle	d (µm) – diameter	ρ (kg/m^3) – density
PVC powder	139	1330
Glass Beads 737	151	2480

Pipeline Bores and Flow Condition:

Table 4.2: Main dimensions of the pipeline and Reynolds Number

L_1 (m)	L_2 (m)	D_1 (mm)	D_2 (mm)	σ	Re	Material
2.2	2.8	20	30	0.444	38000	Glass Beads 737
2.2	2.8	20	24.5	0.666	38000	PVC powder

Here, in Table 4.2, D_1 and D_2 is referred to as the diameters of small pipe and large pipe and L_1 and L_2 is referred to their respective lengths.

Transition Length:

In experiment, the transition length is defined as the distance between the expansion interface and a station where the pressure reaches to a maximum in the large pipe is given in Fig. 4.9 below in terms of dimensionless length (l).

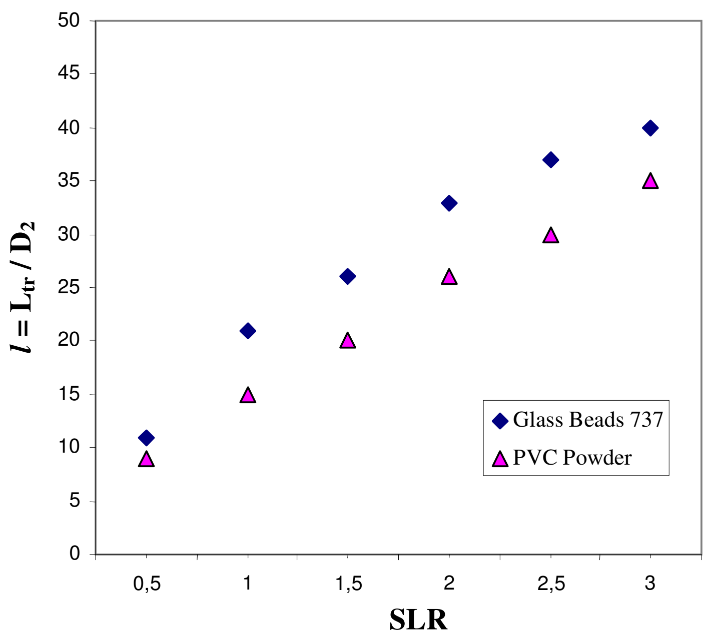

Figure 4.9: Transition length- pipe diameter ratio versus SLR for PVC Powder and Glass Beads 737

Predictions of pressure difference by theoretical models are compared with experimental data and results are given in Figs. 4.10&4.11.

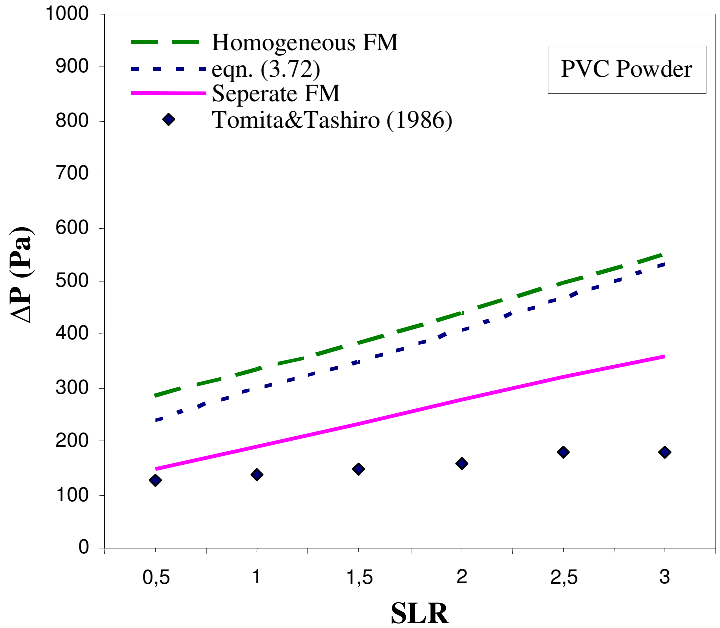

Figure 4.10: ΔP versus SLR, at Reynolds number =38000 for PVC Powder

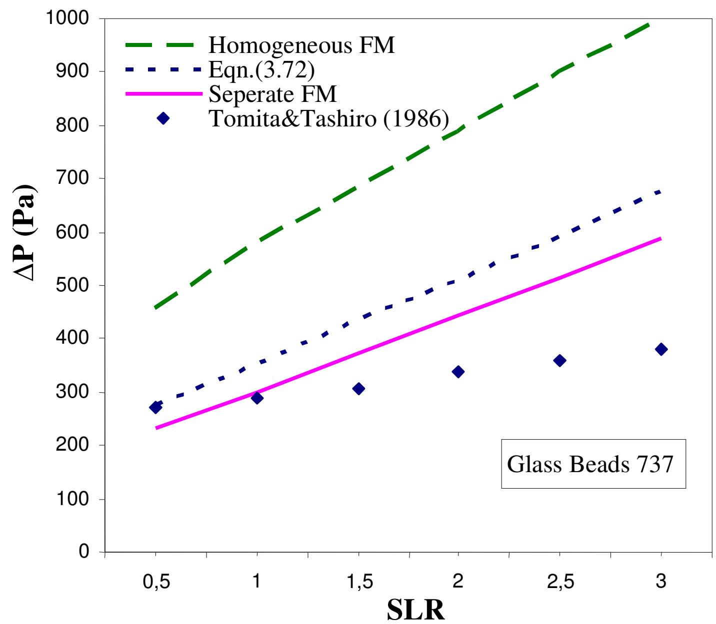

Figure 4.11: ΔP versus SLR, at Reynolds number =38000 for Glass Beads 737

Figs. 4.10&4.11 show that pressure difference curves are far apart from the data and the difference between models and data increases as solid loading ratio increases. Here, two figures show that the separate flow model is better among existing models.

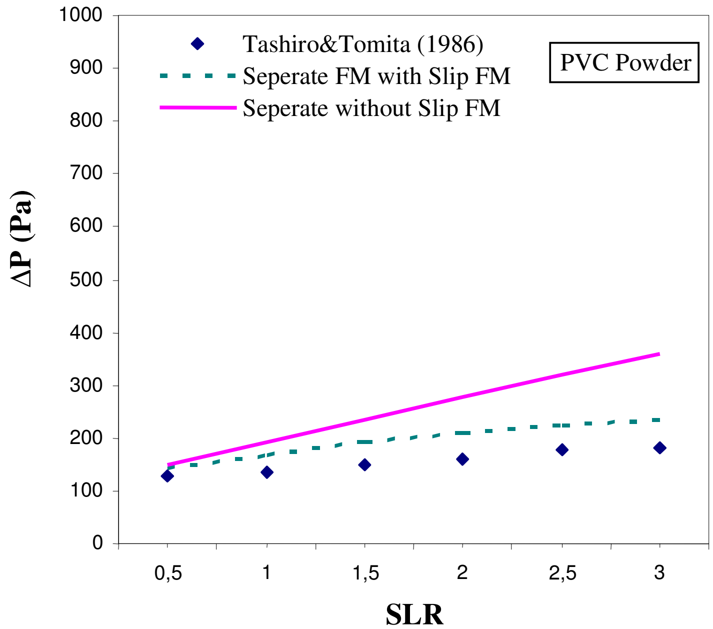

Figure 4.12: ΔP versus SLR for PVC Powder particles, at Re=38000

Figure 4.13: ΔP versus SLR for Glass Beads 737 particles, at Re=38000

Here, Fig. 4.12&4.13 show that the Separate FM with Slip FM agrees with experimental data than the model without slip flow model for both solid particles. The model with slip FM follow the data in parallel with high values than data for PVC Powder and the case in Glass Beads 737, it follow the data also in parallel but with lower results than the data. It is important to note that the difference between curve and data for PVC powder is in the range of +17% and +30% of data and for Glass Beads is in the range of -24% and -30% of data.

4.2.3 Comparison with Tomita et al. (1980): Gas only Flows

Comparison of theoretical models with experimental data of Tomita et al. (1980) for air only flows will be presented. In their experimental study the pressure difference is obtained for the case of turbulent pipe flow regimes at different Reynolds numbers. In experiment, the transition length required for the pressure difference for air only flow is taken approximately as $8.5D_2$ and the diameter and the length of the smaller pipe is 42 mm and 4 m, and those of the larger one is 80.3 mm and 6 m. Here, Borda-carnot equation, which is given by Eqn. (4.4), will also compared with experimental data as a reference to separate flow model. Here, in all calculations, wall friction term is neglected.

Borda-Carnot equation:

$$\Delta P = \rho_g V_{g1}^2 \sigma (1-\sigma) \tag{4.4}$$

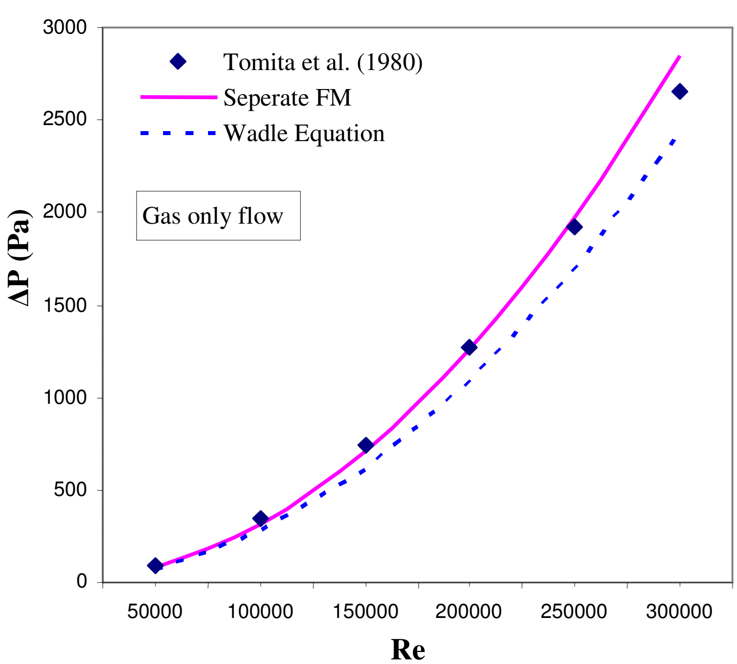

Figure 4.14: ΔP versus Re: gas only flows

Fig. 4.14 shows that the separate flow model is very close to experimental data with a little difference between curve and data consist in high Reynolds number. Whereas Borda-carnot equation follows the experimental data at low values which is very near to data and the difference between curve and data start to increase when Reynolds number increases. While the difference between separate flow model and data is +7% of data at maximum value, the case in Borda-carnot is -8% of data at maximum value.

4.2.4 Comparison with Abdellal et al. (2004): Liquid&Liquid-Gas Flows

Theoretical models are compared with the experimental study of Abdellal et al. (2004) at different gas flow rates and Reynolds numbers. There, the pressure drop caused by abrupt flow area expansion and contraction in small circular channels were experimentally investigated by using air and water as the working fluids at room temperature and near-atmospheric pressure. Large and small tube diameters were 1.6 mm and 0.84 mm, respectively. Experiments were performed using single-phase liquid and two-phase gas-liquid mixtures. In calculations, frictional pressure drop is neglected and the following two figures are obtained for this comparison.

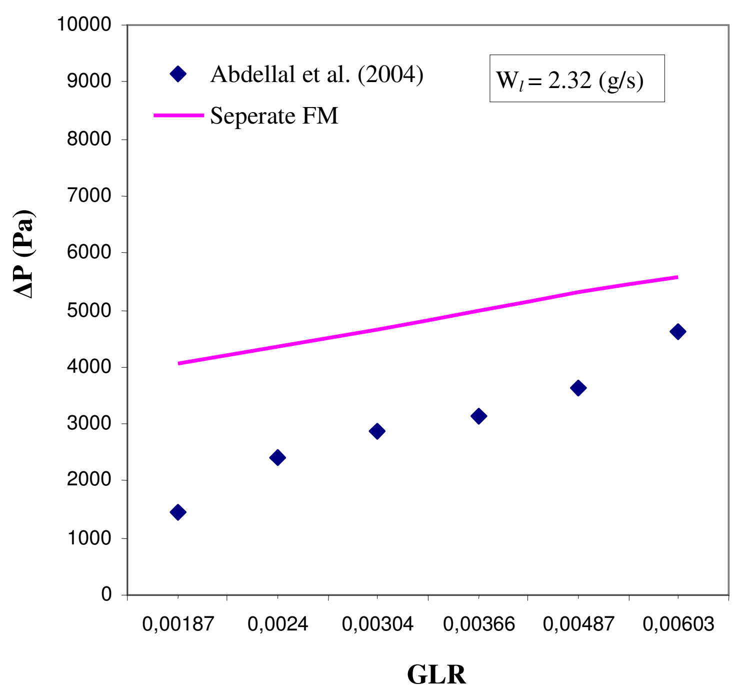

Figure 4.15: ΔP versus GLR, gas loading ratio, for gas-liquid two phase flow

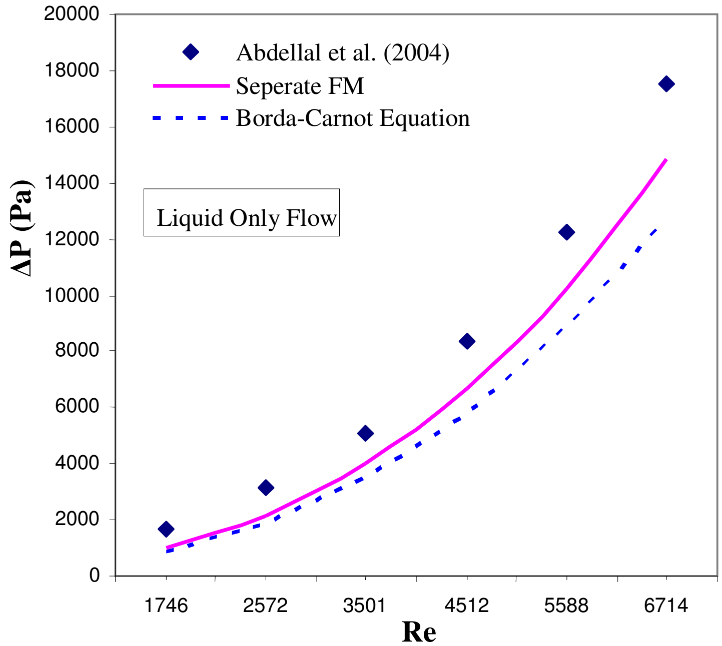

Figure 4.16: ΔP versus Re for liquid only flow

Fig. 4.15 shows that the separate flow model coupled with a slip flow model, which is given in Eqn. (3.75), gets nearer to experimental data as the gas loading ratio increases but the difference between model and data is still very high with a maximum value around 400 %. Fig. 4.16 shows that the separate flow model follows the data with near values and the difference between model and data increases as the Reynolds number increases. Borda-Carnot equation reflects the similar trend but the separate flow model is superior in any case. It is important to note that the difference between separate flow model and data is in the range of -15% and -49% of data and the difference between Borda-Carnot equation and data is in the range of -27% and -49% of data. As a result, the predictions of pressure recovery with separate flow model gives near results with experimental data in the case of liquid only flow but the same is not in the case of gas-liquid flows.

4.3 EVALUATION of PRESSURE LOSS COEFFICIENT

The pressure loss coefficient represents the pressure loss along the sudden expansion in dimensionless form. The pressure loss in two phase flows is divided into two parts, namely, one is due to gas and the other one is due to particles. It is important to note that the momentum transfer from solid particles to gaseous phase occurs in decelerated flows and this causes the pressure at downstream to increase.

4.3.1 Pressure Loss Coefficient (Expansion Loss Coefficient)

The pressure loss in two phase flow due to expansion will be handled as the difference between the pressure difference of gas only flow and the pressure difference of gas-solid flow:

$$\Delta P_{e,loss} = \Delta P_{(gas\ only\ flow)} - \Delta P_{(gas-solid\ flow)} \tag{4.2}$$

$$\Delta P_{e,loss} = \frac{1}{2}\rho_g\left(V_{go,1}^2 - V_{go,2}^2\right) - \Delta P\ \left(Seperate\ FM\ with\ slip\ FM\right) \tag{4.3}$$

The pressure loss coefficient or expansion loss coefficient is defined as the ratio of pressure loss in gas-solid flow due to expansion to inertial term at upstream of gas only flow.

$$C_e = \frac{\Delta P_{e,loss}}{\frac{1}{2}\rho_g V_{go,1}^2} \tag{4.4}$$

Expansion loss coefficient, Eqn. (4.4), can be rearranged in the following form:

$$C_e = \frac{1}{2}\left(1-\sigma^2\right)\left[1+\alpha\left(1-S^2\frac{\rho_s}{\rho_g}\right)\right] + \frac{\left(\Delta P_{fg} + \Delta P_{fs}\right)}{\rho_g V_{go,1}^2} + \frac{\rho_m\, g\cos\theta\,\Delta z}{\rho_g V_{go,1}^2} \tag{4.5}$$

Where, the subscript "*e*", "p" and "go" refer to expansion, particle and gas only.

It is important to note that the void fraction, "α", must be calculated by using Eqn. (3.73) and slip ratio, "S" must be calculated by using Eqn. (4.1). Gas-wall-friction and solid-wall-friction losses, ΔP_{fg} and ΔP_{fs} can be calculated by using Eqns. (3.40)&(3.43), respectively.

4.3.2 Additional Pressure Loss Coefficient

Here, it is important to note that the pressure loss due to expansion has two parts; one due to gas and one due to solid particles and so the pressure loss coefficient is defined as:

$$C_e = C_p + C_{go} \tag{4.6}$$

Here, C_p stands for the additional pressure loss due to particles and C_{go} stands for the pressure loss due to gaseous phase which would take place when there are no particles in the flow with the same gas mass flow rate.

The gas only flow expansion loss coefficient C_{go} consists of gas-wall friction loss during expansion and can be computed as follows,

$$C_{go} = \frac{\Delta P_{fg}}{\frac{1}{2}\rho_g V_{go,1}^2} = 4 f_g l_2 \sigma^2 \tag{4.7}$$

Thus, using Eqns. (4.5)&(4.7), the additional pressure loss coefficient can be calculated easily by using Eqn. (4.6).

4.3.3 Comparison with Marjanovic et al. (1999): Pressure Loss Coefficient

The pressure difference data, which is obtained from Fig. 4.5, 4.6&4.7 are used in the calculation of pressure loss coefficient. Calculation shows that the pressure loss coefficient is not a function of Reynolds number in the case of constant solid loading ratio so the following figure is valid for all gas mass flow rate.

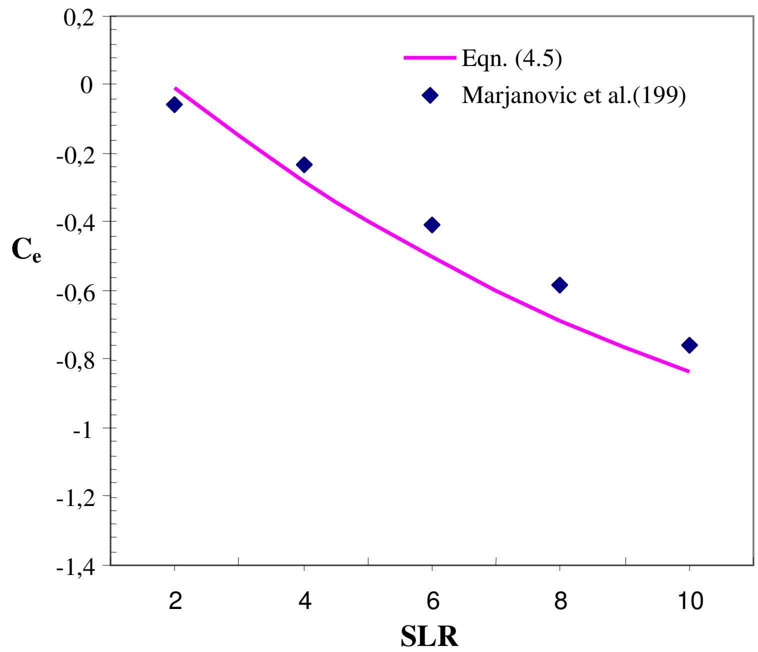

Figure 4.17: Pressure loss coefficient, C_e versus SLR, at W_g = 0,14 (kg/s)

Here, Fig. 4.17 shows that the pressure loss coefficient is in negative values for all solid loading ratio and decreases as the solid loading ratio increases. The model and data follows each other with near results also. Figure also reveals that the expansion loss tends to zero in very low solid loading ratios and there the flow resembles to gas only flow behavior. The difference between model and data is in the range of -12% and +12% of data.

4.3.4 Comparison with Marjanovic et al. (1999) and Tomita&Tashiro (1986): Additional Pressure Loss Coefficient

It is important to know that how much pressure loss is affected by the addition of solid particles to the flow. The additional pressure loss coefficient reflects this effect. Additional pressure loss coefficient presented in Fig. 4.18 is calculated by using the separate flow model and data which were already shown in Fig. 4.1 and the coefficient in Figs. 4.19&4.20 is calculated by using the model and data in Figs. 4.12&4.13, respectively. It is important to know that calculations show that pressure loss coefficient is the same for all Reynolds number for the same solid loading ratio. So the theory in Fig. 4.18 is valid for all gas mass flow rates.

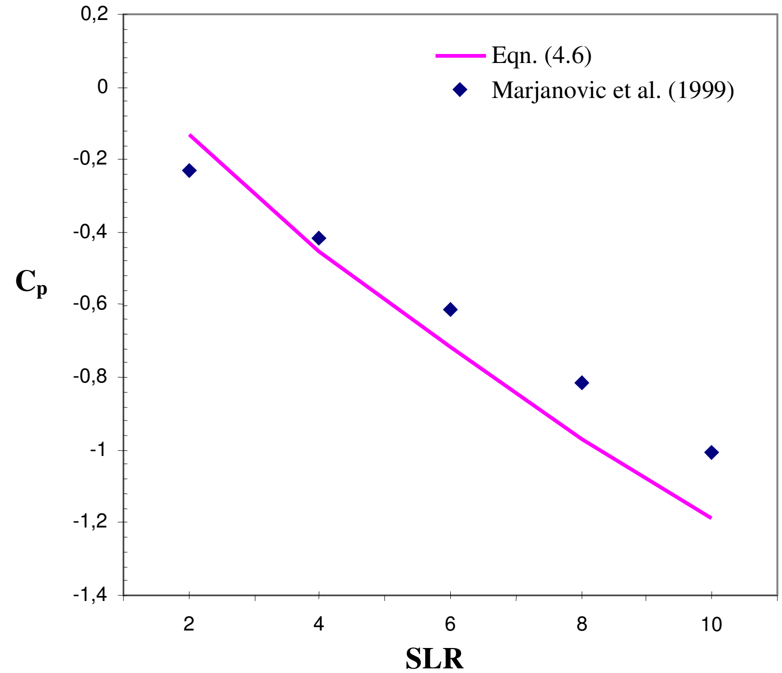

Figure 4.18: Additional pressure loss coefficient, C_p vs SLR, at W_g = 0,14

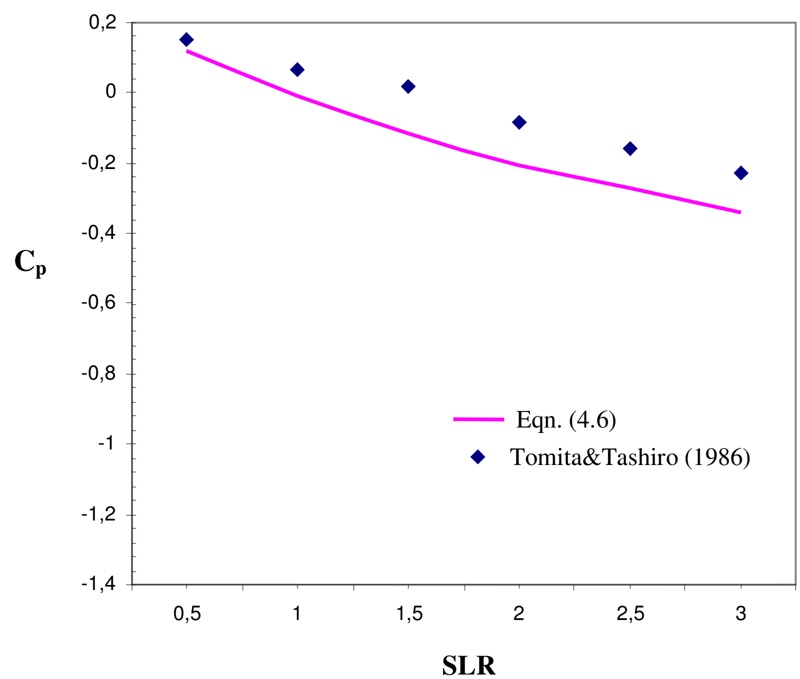

Figure 4.19: C_p vs SLR for PVC powder, at Re = 38000

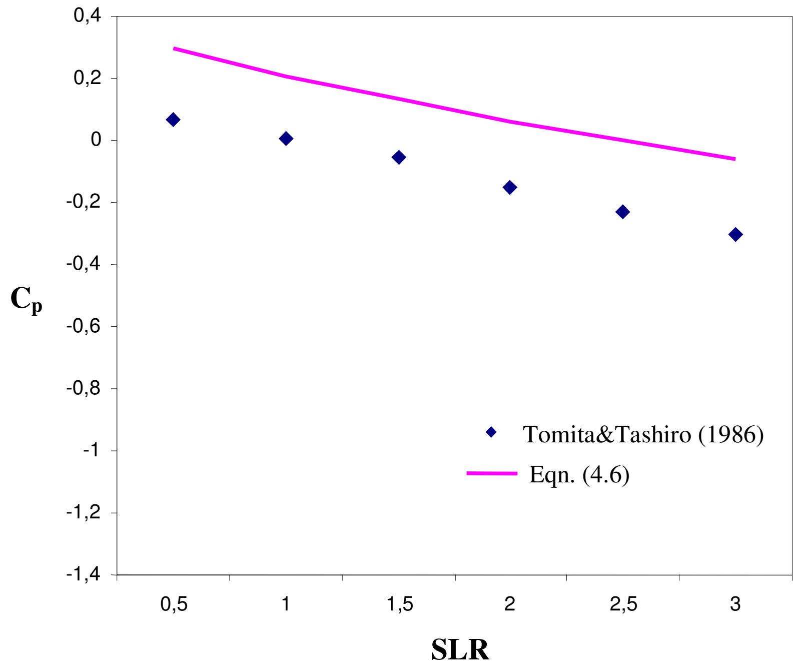

Figure 4.20: C_p vs SLR for Glass Beads 737, at Re = 38000

Here, Fig. 4.18 through Fig. 4.20 show that the additional pressure loss coefficient are in negative values and this negative values proves that solid particles decrease the pressure loss in a decelerated flow with a momentum transfer from particles to gas phase during expansion. The data in Fig. 4.18 is in negative values for all solid loading ratio but the data in Figs. 4.18&4.20 has positive values below solid loading ratio at around 1. Here, positive values shows that there is a pressure loss in the system but the case in negative values there is a pressure gain in the system due to solid particles. It is important to note that the pressure loss coefficient is in negative values in the case of velocity of solid particles is greater than the velocity of the carrier fluid during sudden expansion. The difference between model and data in Fig. 4.18 is in the range of -12% and +12% of data and the same case in Fig. 4.19 is in the range of -150% and -23% of data and also the case in Fig. 4.20 is in the range of +80% and +350% of data.

4.4 CONCLUSION

In this chapter, all theoretical models are compared with experimental data of Marjanovic et al. (1999) and Tomita&Tashiro (1986), respectively. Comparisons show that all theoretical models deviates from the data as solid loading ratio increases and closer to data as solid loading ratio decreases. It is seen that the separate flow model is the best among existing models. A hybrid model is derived by coupling the separate flow model with a slip flow model which is derived for the slip velocity existing between phases. Comparison of hybrid model with data shows that the model gives close results with data. It is also seen that for gas only flows, the separate flow model gives close results with a maximum difference at +7% of data. Comparison of separate flow model with liquid only flow data and gas-liquid flows data gives far away results and the model is not suggested for this flow behaviors.

Pressure loss coefficient is evaluated from separate flow model and divided as the pressure loss coefficient due to gas only flow and additional pressure loss coefficient due to solid particles. It is seen that the pressure loss coefficient in theory is not a function of Reynolds number in the case of constant solid loading ratios and pressure loss coefficient has negative values and decreases as the solid loading ratio increases. It can be said that there is a pressure gain in the system in the case of velocity of solid particles is greater than gaseous phase.

CHAPTER 5

CONCLUSION

5.1 Conclusion

Minor losses are one of the problems arising in pneumatic conveying systems and it is important to find these minor losses theoretically for the design operations.

In this project, the aim is to find the minor losses analytically arising due to sudden expansions for gas-solid two phase flows. The problem related with the pressure difference along the expansion is analyzed theoretically.

The assumption of flow mixture as homogeneous and separate one and applying momentum and continuity equation to the flow at both assumptions give three different theoretical models for gas-solid two phase flow along sudden expansion. A theoretical model is also obtained for gas-solid flow from the theoretical models in literature for gas-liquid flows. All theoretical models give the pressure difference arising in sudden expansion. Slip flow models in literature are also presented for the slip velocity between phases during expansion.

Four different theoretical models are compared with experimental data of some investigators available in literature. Comparisons are carried out for different solid loading ratios, different Reynolds numbers, different expansion ratios and different materials to be conveyed. Comparisons show that all theoretical models departs from the data as solid loading ratio increases and it is seen that separate flow model is the best among existing models with close results to data than others. Experimental studies show that the slip by velocity between phases occurs during expansion so that a hybrid model is formed by coupling separate flow model with a slip flow model formulated in literature. Comparisons of hybrid model with experimental data of some authors show that the model is in good agreement with data. The difference between data of Marjanovic et al. (1999) and model is in the range of -6% in minimum and +22% in maximum and the difference between model and data of Tomita&Tashiro et al. (1987) for PVC powder is found in the range of +17% and +30% of data and for Glass Beads is found in the range of -24% and -30% of data.. Separate flow model is also compared with experimental data of gas only, liquid only and gas-liquid two phase flows, respectively. It is important to note that the experimental study in gas only flow is carried out for Reynolds numbers between

50000 and 350000 and the case of liquid only flow is carried out for Reynolds numbers between 1746 and 6714. The difference between model and data in gas only flow is found to be in the range of 0% and +7% of data and in the case of liquid only flow the difference is found to be in the range of -15% and -49% of experimental data. The difference between data and model in the case of gas-liquid flows is found to be around 400% of experimental data.

As a result of comparisons, hybrid model (separate FM coupled with slip FM) gives minimum difference with data than other models and it is suggested that hybrid model can be applied to sudden expansion fittings in the case of gas-solid two phase flows for turbulent pipe flow regimes. The separate flow model gives very close results with data in the case of gas only flows and it is suggested that it can be applied to gas only flows through sudden expansion fittings for turbulent flow regimes but it is not suggested for liquid only flows and gas-liquid flows.

The pressure loss coefficient is obtained from separate flow model and divided as one due to gaseous phase and another due to solid particles and is compared with experimental data. It is seen that the difference between Eqn. (4.5) and data is in the range of -12% and +12% of data of Marjanovic et al. (1999). Comparison show that the pressure loss coefficient has negative values for all solid loading ratios, here, negative values show that there is a pressure gain in the system due to particles so the pressure loss coefficient in gas-solid flows is lower than the pressure loss coefficient in gas only flows in the case of velocity of particles is greater than the gaseous phase. It is seen that the pressure loss coefficient in theory is not a function of Reynolds number for constant solid loading ratio and for the same solid loading ratios. As a result, it is suggested that pressure loss in gas-solid flows through a sudden expansion fitting can be calculated from Eqn. (4.5).

5.2 Recommendation

The study in this thesis show that there is a deficiency in theoretical definition of relative velocity between phases and it is recommended to study theoretically on this subject. Transition length and recirculation length is measured with experimental methods in literature and there is no theoretical studies for both so it is important to study on this subject analytically

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