

ABSTRACT

COMPUTER AIDED DESIGN OF ACTIVE AND PASSIVE ANALOG FILTER AND ITS APPLICATIONS

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In this thesis; a computer programming related with realizations of required filters with the help of approximation procedures are presented. The program contains Butterworth, Chebyshev and Inverse Chebyshev approximations to realize the required LC filters. For the elliptic approximation, the zeros and denominator polynomials of the low-pass and high-pass filter transfer functions are obtained.

Three types of approximations namely Butterworth, Chebyshev, Inverse Chebyshev will be used to get the realized network elements. For Elliptic approximation, the transfer function will be obtained. The characteristics of the specific filters; pass band frequency (ω_p), the maximum deviation of the magnitude characteristic (K_p), stop band frequency (ω_s), the minimum attenuation of the magnitude characteristic (K_s) and the type of approximation required will be entered to the computer program written in Visual C# by user, and the program will calculate the value of network elements that realize the specific filter. Also pole locations, quadratic factors and denominator coefficients of magnitude function $N(s)$ used for approximation will be shown in the display.

Key words: pass band frequency (ω_p), stop band frequency (ω_s), the maximum deviation of the magnitude characteristic (K_p), the minimum attenuation of the magnitude characteristic (K_s).

ÖZET

AKTİF ve PASİF FİLTRELERİN BİLGİSAYAR DESTEKLİ DİZAYNI ve UYGULAMALARI

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Bu tezde, yaklaşım metotları ile istenilen filtrelerin gerçekleşmesini sağlayan bilgisayar programı sunulmuştur. Çalışılan bilgisayar programı, istenilen filtrenin gerçekleşmesini elde etmek için, Butterworth, Chebyshev ve Ters Chebyshev türü yaklaşım metodlarını içermektedir. Eliptik tür yaklaşım metodu ile alçak ve yüksek geçiren filtrelerin transfer fonksiyonu elde edilecektir.

Filtre devresini gerçekleştirecek devre elemanlarına ulaşmak için 3 tip yaklaşım metodu kullanılmıştır. Bunlar Butterworth, Chebyshev ve Ters Chebyshev yaklaşımlarıdır. Eliptik yaklaşım kullanılarak transfer fonksiyonu elde edilmiştir. Özel filtreleri karakterize eden: geçiren frekans (ω_p), geçme bandı dalgalanma değeri (K_p), durduran frekans (ω_s), durdurma bandı dalgalanma değeri (K_s) ve üzerinde çalışılacak yaklaşım metodu, kullanıcı tarafından programa girdi olarak verilecektir. Visual C# ile yazılan program, özel filtreyi gerçekleştiren devre eleman değerlerini hesaplayarak, ekrana devre ile beraber verecektir. Yaklaşım metotlarında, devre eleman değerlerini vermeyi sağlayan matematiksel fonksiyonların sonuçları, ekranda görülebilecektir.

Anahtar kelimeler: geçiren frekans (ω_p), geçme bandı dalgalanma değeri (K_p), durduran frekans (ω_s), durdurma bandı dalgalanma değeri (K_s).

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I hope this thesis will be useful source studying in approximation methods.

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LIST OF SYMBOLS/ABBREVIATIONS

K_s	Maximum deviation of magnitude characteristic
K_p	Minimum attenuation of magnitude characteristic
$ N(j\omega) ^2$	Maximally flat magnitude squared function
Q	Quality factor
σ_k	Real part of the root
ω_k	Imaginary part of the root
ω_s	Stop band frequency
ω_p	Pass band frequency

CHAPTER 1

INTRODUCTION

In this chapter a brief overview of the filters will be given. In the last decades the progresses on the communication systems, especially high frequency communications networks [1], has required high quality filters. Since in the real time high frequency communication applications, analog filters have some advantages versus the digital filters [2], quick and correct determination of the analog network elements has been more important. Although there are some several indirect synthesis techniques to design the analog filters, in the study, mathematical approximation methods are investigated. This method works on any kind of filter types including the transformations.

1.1 Definition of Filter

The sinusoidal steady frequency responses of the network are of significance in many applications, especially in communications and control systems. A specific application is in electric filters that block out or eliminate signals with unwanted frequencies and pass signals of the desired frequencies. The filters are used in radio, T.V. and telephone systems to separate one broadcast frequency from another [3].

Owing to their electronic structures, they may be classified either in terms of their characteristics or types of the components. In many different applications, time domain dependence of their characteristics may also be used for some applications such as time-varying, periodically time-varying or adaptive filters.

1.2 Types of Filters

In terms of their frequency response characteristics, the filters may be classified in many different names. Although the phase filters have some application areas such as matching, equalization, etc., this study is focused on the magnitude characteristics and therefore the amplitude filters. Due to the amplitude characteristics versus the frequency domain, the filters, in general, may be classified as low pass, high pass, single or multi band pass and stop band filters.

1.2.1 Low-pass filters

Almost in all audio and video applications the low-pass filters are commonly used. They allow for easy passage of low-frequency harmonics of the signals from source to load, and difficult passage of high-frequency terms. An ideal low-pass filter magnitude characteristic is given in Figure 1.1. For low frequencies ($\omega \leq \omega_c$) the gain, ideally assumed to be unity (or more for active structures) and also it is assumed that the gain drops to zero as sharp as possible for frequencies greater than ω_c . The decaying interval (transition region) should be as short as possible [4].

1.2.2 High-pass filters

The high-pass filter has the property that low frequencies (stop-band) are blocked while high-frequencies (pass-band) are transmitted. As in the low-pass filters, the transition region between stop-band and pass-band regions must be as narrow as possible. Ideally the transition width is assumed to be zero and gain suddenly increases to unity (for passive filters) at the cut-off frequency and for higher frequencies. High-pass filters are also widely used in the communication and control systems [5].

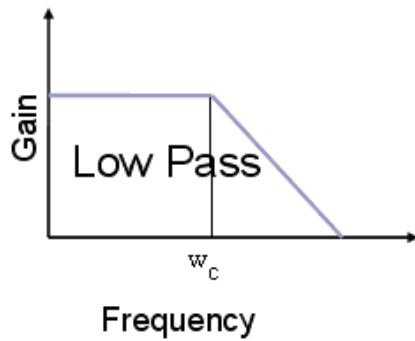


Figure 1.1 Ideal low-pass filter

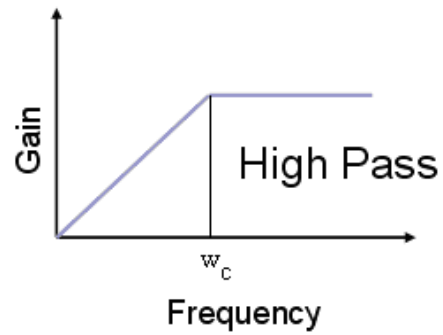


Figure 1.2 Ideal high-pass filter

1.2.3 Band-pass filters

The third general filters type that will be considered here is the band pass filter. It has the property that one band of frequencies (pass-band) is transmitted while two bands of frequencies, namely, those below and above the pass-band are blocked (the stop bands). In many communication applications the band-pass filters are used as frequency (band) selection networks and sometimes not only the unique pass band region but also more pass-band regions may be used for some specific applications. The range of frequencies that is passed is called the bandwidth (BW) and is defined the difference between the frequencies that define the edges of the pass band. Filter circuits can be designed to accomplish this task by combining the properties of low-pass and high-pass into a single filter. This case is only applicable for unique pass band region. An ideal band-pass filter is shown in Figure 1.3 [6].

1.2.4 Band-stop filters

The band stop filters, also called as *band-elimination filter*, *band-reject*, or *notch filter* has the property that are band of frequencies (the stop band) is blocked while two bands of frequencies, namely those below and above the stop band are passed (pass band). The range of frequencies that is blocked is called the bandwidth (BW) and is defined as the difference between the frequencies that define the edges of the stop band. In ideal case, as usually the transition band is assumed as narrow as possible. A band-stop filter is shown in Figure 1.4 [7].

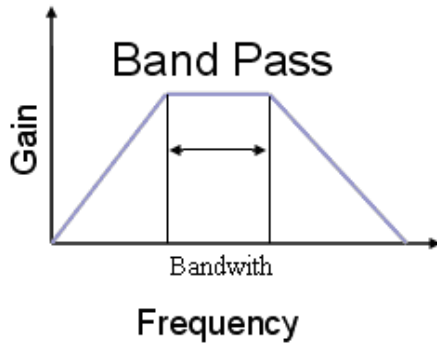


Figure 1.3 Ideal band-pass filter

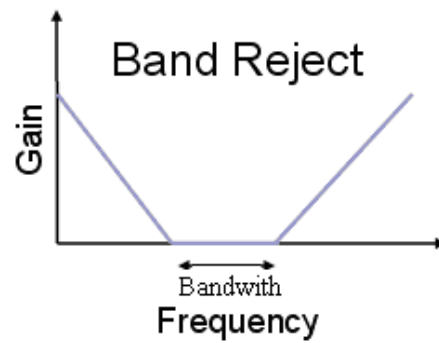


Figure 1.4 Ideal band-stop filter

1.3 Analog Filters

Analog networks [8] which operate on continuous time signals are widely used in communication, control and electronic applications. These networks use the analog components such that the terminal characteristics may or may not be linear. In particular, linear time-invariant (LTI) analog filters can be characterized by their (continuous) impulse response $h(t)$ where t is time. Since the input signals processed in analog networks are continuous, the analog networks do not require the use of analog-digital converters and since the sampling is not used, the speed of these networks are limited only by the time constants of the energy storage elements. For time varying and time invariant cases the Fourier Transformation is possible used to obtain the complex domain frequency characteristics which gives both magnitude and phase characteristics of the analog network. The magnitude characteristics similar to one shown in the previous section results as the analog filters. Instead of a difference equation, analog filters may be described by a differential equation. Instead of using the z transform to compute the transfer function, the Laplace transform can be used. Every aspect of the theory of digital filters has its counterpart in that of analog filters. In fact, one can think of analog filters as simply the limiting case of digital filters as the sampling-rate is allowed to go to infinity [9].

1.4 Filter Approximation Methods

The majority of practical filter specifications are based on steady-state performance requirements [10]. These are usually given for magnitude and/or phase characteristics as a function of a real frequency variable ω (radians per second) or f

(hertz). The actual synthesis techniques used to find active or passive filter realizations, however, invariably use the network function, a ratio of polynomials in the complex frequency variable s , as a starting point. The process of relating sinusoidal steady-state characteristics to a network function is called approximation. Figure 1.5 illustrates the difference between synthesis and approximation.

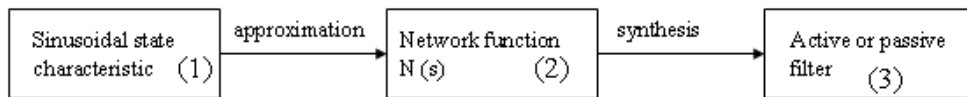


Figure 1.5 Difference between approximation and synthesis.

There are many mathematical approximation methods. In this study four of them will be reviewed and these approximations will be used for the computer aided design of the filters. These approximations methods are:

- 1) Butterworth Approximation
- 2) Chebeyshev Approximation
- 3) Inverse Chebeyshev Approximation
- 4) Elliptic Approximation

1.5 Use of Filter Approximation Methods

Especially in communication, when a filter is needed that separates some frequencies from others within mixed-frequency signals, the specific characteristics are given and with an approximation method given above, active or passive filter synthesis can be calculated within a set of mathematical calculations

The similar approach is also used for the design of the phase filters, delay filters, etc. but in general, gain approximation is more straight forward method to design an amplitude filter.

The basic principle of approximation method is to predict the suitable ω domain function which sketches the similar characteristics curve with the given properties. Since the proposed functions are well known and ω to s domain transformations

result with the realizable transfer functions, they are still widely used and most popular techniques for filter synthesis.

This thesis is organized as follows:

Basic filter parameter is given in Chapter 2. The general synthesis methods, approximation methods, normalization and denormalizations are reviewed. The procedure from s domain transfer functions analog filter implementation is given in this chapter.

Chapter 3 presents the parameters used in the computer aided design of the analog filters. Since the main aim of this study is to build up a computer program which is used to design a filter, an interface is introduced in this chapter. Computer interface is user independent and it is easy to use. The basic points of the prepared program and algorithm are also presented in this chapter.

Chapter 4 presents the program. The computer program is run for some specific examples and the results are given in this chapter.

The conclusion and future proposals of this work is given in Chapter 5. The program is attached to the thesis on a CD.

CHAPTER 2

REVIEW OF FILTER DESIGN

Approximation types of filters with their specifications are the main subject studied in this chapter.

2.1 Approximation

The majority of practical filter specifications are based on steady-state performance requirements. The process of relating sinusoidal steady-state characteristics to a network function is called approximation. One of the most frequently used types of approximation is that relating the magnitude $N(j\omega)$, specified either by a mathematical expression, a set of data values, or a plotted wave shape, to a rational function $F(s)$, so that in some specified sense $F(j\omega)$ approximates $N(j\omega)$. Ideally of course it is predicted the two magnitude functions must be identical and in many cases this is possible. Magnitude specification is usually given in decibels $[20 \log |N(j\omega)|]$, abbreviated dB. In fact the decibels of gain of a filter relate to the ratio between input and output voltages $dB = 20 \log(V_{out} / V_{in})$ [11]

2.2 Maximally Flat Magnitude Squared Function $|N(j\omega)|^2$

Firstly the properties the magnitude $N(j\omega)$ must be overviewed. The necessary properties beginning with considering the $|N(j\omega)|^2$ square root of $N(j\omega)$ are:

$$|N(j\omega)|^2 = N(j\omega)N^*(j\omega) = N(j\omega)N(-j\omega) \quad (2.1)$$

where the superscript asterix indicates the complex conjugate and where the justification for the right member of the equation is that, for rational functions with real coefficients, the conjugate of function is found by the replacing the variable by its conjugate, that is, by replacing jw with $-jw$. Now let $N(s)$ have the form

$$N(s) = \frac{c_0 + c_1s + c_2s^2 + c_3s^3 + c_4s^4 + \dots}{d_0 + d_1s + d_2s^2 + d_3s^3 + d_4s^4 + \dots} \quad (2.2)$$

The term $N(jw)$ will have the form:

$$N(jw) = \frac{c_0 - c_2w^2 + c_4w^4 - \dots + j(c_1w - c_3w^3 + \dots)}{d_0 - d_2w^2 + d_4w^4 - \dots + j(d_1w - d_3w^3 + \dots)} \quad (2.3)$$

Inserting this relation in the right member of equation (2.1), it is seen that first property of $|N(jw)|^2$, that it will be a ratio of even polynomials.

Evaluating (1) by letting $w = s/j$, the function $T(s^2)$ may be defined as

$$T(s^2) = |N(jw)|^2 \Big|_{w=s/j} = N(s) * N(-s) \quad (2.4)$$

In the right member of equation (2.4), the substitution of $-s$ for s as the argument of $N(s)$ simply reflects the pole and zero positions of $N(s)$ through the origin of s plane. As a result the product of $N(s) N(-s)$ has poles and zeros that symmetrically located with respect to both the real and the imaginary axes. This is called quadrantal symmetry. In general the numerator and denominator polynomials of $T(s^2)$ can have three types of factors:

- 1) $s^4 + as^2 + b$ where a and b may be positive or negative
- 2) $as^2 + b$ a and b have opposite signs
- 3) $as^2 + b$ where a and b have same signs.

The first and second types have the necessary quadrantal symmetry but the third type does not unless it has even multiplicity, that is, unless it appears $(as^2 + b)^2$, $(as^2 + b)^4$, and so on. In this case the resulting even-order $j\omega$ axis zeros have the necessary symmetry.

Now it is summarized the properties of $|N(j\omega)|^2$ to be the magnitude squared function of some rational function $N(s)$, it is necessary and sufficient that: [10]

- 1) The function $|N(j\omega)|^2$ be ratio of even polynomials in ω
- 2) In the function $T(s^2)$ defined in equation (2.4), any poles and zeros on the $j\omega$ axis are of even order.

The sufficiency of the two conditions given above is readily demonstrated by factoring $T(s^2)$ into the product $N(s)*N(-s)$, taking the left-half-plane poles and half of any even-order $j\omega$ -axis pole pairs from $T(s^2)$ as the poles of $N(s)$ and similarly assigning either right or left-half plane zeros and half of any even-order $j\omega$ -axis zeros from $T(s^2)$ as the zeros of $N(s)$. This restriction of using only the left-half-plane poles from $T(s^2)$ is of course simply a stability consideration.

The necessary and sufficient conditions developed above for magnitude-squared functions in general can be applied to specific filter characteristics. Considering the determination of magnitude-square function that, in the low-frequency range starting at zero, has as flat a characteristic as possible. One way of obtaining such flatness is to set as many derivatives of the function as possible to zero at $\omega = 0$ rad/s. frequency. Such a function is called maximally flat. This can be done by writing an expression for a general-squared function $|N(j\omega)|^2$ as follows:

$$|N(j\omega)|^2 = H^2 \frac{1 + b_1\omega^2 + b_2\omega^4 + \dots}{1 + a_1\omega^2 + a_2\omega^4 + \dots} \quad (2.5)$$

Dividing denominator into the numerator, it can be obtained

$$|N(j\omega)|^2 = H^2 [1 + (b_1 - a_1)\omega^2 + (b_2 - a_2 - a_1^2 - a_1 b_1)\omega^4 + \dots] \quad (2.6)$$

Considering a general Mac Lauren series, i.e., a Taylor series expansion at the origin, of an arbitrary function $F(\omega)$. This has the form

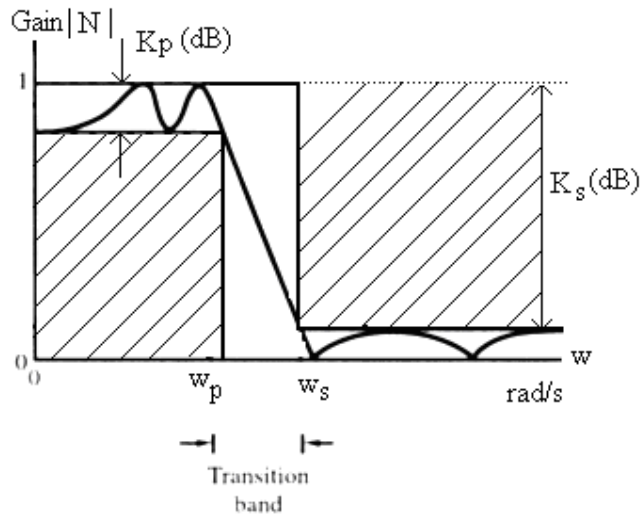
$$F(\omega) = F(0) + \frac{F^{(1)}(0)}{1!}\omega + \frac{F^{(2)}(0)}{2!}\omega^2 + \frac{F^{(3)}(0)}{3!}\omega^3 + \frac{F^{(4)}(0)}{4!}\omega^4 + \dots \quad (2.7)$$

where $F^{(i)}(0)$ is the i th derivative of $F(\omega)$ evaluated at $\omega=0$. Comparing this expression with the expansion for $|N(j\omega)|^2$ given in equation (2.6) and recalling that such an expansion must be unique, it is seen that due to the even nature of $|N(j\omega)|^2$, all its odd-ordered derivatives are already zero. In addition for the second derivative to be zero, it is required that the coefficients a_1 and b_1 be equal. Similarly for the fourth derivative to also be zero requires that, in addition, a_2 equal b_2 , etc. Thus the general maximally flat magnitude squared function $|N(j\omega)|^2$ is characterized by the restriction that

$$a_i = b_i \quad (2.8)$$

for as many coefficients as possible. [10]

2.3 Specifications



- ω_p : passband edge frequency
- ω_s : stopband edge frequency
- K_p : maximum deviation of magnitude
- K_s : minimum attenuation of magnitude

Figure 2. 1 Low-pass filter specification

As it is seen in Figure 2.1 filtering specifications consist of a pass band set and a stop band set. They have the form as below:

1. Passband: In the passband $0 \leq \omega \leq \omega_p$ radians per second ($0 \leq f \leq f_p$ hertz) the maximum deviation of the magnitude characteristic is K_p dB.
2. Stopband: In the stop band $\omega \geq \omega_s$ radians per second ($f \geq f_s$ hertz) the minimum attenuation of the magnitude characteristic is K_s dB. The attenuation is measured from the greatest passband value of the characteristic.

To meet the specifications, the actual magnitude characteristics must lie out of the shaded area.

It can be summarized that sets of specifications will be given are w_p (pass band frequency), K_s dB. (maximum deviation of magnitude characteristic), w_s (stop band frequency) and K_s dB (minimum attenuation of magnitude characteristic) [11]

Using the given parameters which defined the characteristics of the filter, the corresponding and necessary “order” is calculated. The order for any approximation method reveals the complexity of the filter. Higher order requires more components in the design. For different approximations since different mathematical formulation is used the various methods are used to find the polynomial characteristics (coefficients of the polynomials) which satisfy the given properties. The normalized transfer function $N(s)$ must be obtained in such way that the coefficients must satisfy the certain properties, especially the reliability. The algorithmic formulation of the four different approximation procedures are given below in Sections 2.3.1 to 2.3.4.

2.3.1 Butterworth approximation

Let’s consider a low-pass network. To approximate such a low-pass function, we can choose a magnitude squared function $|N(j\omega)|^2$ that satisfies the maximally flat criteria at $\omega = 0$ and will generate the desired flatness of the curve. To provide eventual drop-off of the characteristics at higher frequencies, the numerator of $N(j\omega)$ will be a constant and all the coefficients of equation (2.5) will be zero. From a maximally characteristics from equation (2.8), the coefficients a_i must also be set to zero, except the highest order one. The resulting magnitude-squared function has the form

$$|N(j\omega)|^2 = \frac{H^2}{1 + \omega^{2n}} \quad (2.9)$$

This function is called a normalized Butterworth function. It is also the simplest known rational function that gives a maximally flat low-pass filter characteristic. For high value of n , this function approximately unit for $\omega < 1$ and approximately zero for $\omega > 1$ which is the basic principle of the function being belonging to the normalized ideal case [12].

2.3.2 Chebyshev approximation

In the Butterworth approximation, the approximating effect is concentrated at a single frequency resulting not enough sharp transition from pass band to stop band. But now with a different type of approximation named equal-ripple characteristic the approximating effect is spread over the entire pass band.

The normalized low-pass equal-ripple magnitude approximation may be developed by writing the magnitude squared function $|N(jw)|^2$ in the form

$$|N(jw)|^2 = \frac{H^2}{1 + \epsilon^2 C_n^2(w)} \quad (2.10)$$

where $C_n(w)$ is a polynomial of order n. If these polynomials have properties

$$0 \leq C_n^2(w) \leq 1 \quad \text{for } 0 \leq w \leq 1 \quad (2.11.a)$$

$$C_n^2(w) \geq 1 \quad \text{for } w \geq 1 \quad (2.11.b)$$

sharp transition from pass band to stop band will be obtained. The value of ϵ determines the limit of variation in the pass band $0 \leq w \leq 1$ rad/s.

Functions having the form of equation (2.10) in which the quantities $C_n(w)$ are Chebyshev polynomials are called Chebyshev functions or equal-ripple function [13].

2.3.3 Inverse Chebyshev approximation

The Chebyshev magnitude approximation is characterized by equal-ripple behavior in the pass band and monotonic behavior in the stop band. A related type of magnitude characteristic named Inverse Chebyshev has a monotonic behavior in its pass band and an equal-ripple behavior in its stop band.

Consider a low-pass equal-ripple function, which in addition to being frequency normalized for a cut off frequency of 1 rad/s, has also been normalized so that its peak magnitude is unity. The Inverse Chebyshev characteristic

$$|N_{IC}(jw)|^2 = \frac{\epsilon^2 C_n^2(1/w)}{1 + \epsilon^2 C_n^2(1/w)} \quad (2.12)$$

will give this consideration. Here *IC* stands for Inverse Chebyshev. This characteristic has the desired monotonic pass band and equal-ripple stop band behavior [14].

2.3.4 Elliptic approximation

The maximally flat and equal ripple approximations both may be written in the form

$$|N(jw)|^2 = \frac{H^2}{1 + \epsilon^2 P_n^2(w)} \quad (2.13)$$

where P_n is a polynomial that for the maximally flat case is w^{2n} and for the equal ripple case is C_n^2 . Now let's replace P_n with a rational function R_n having both a numerator and a denominator polynomial. By choosing a specific function called a Chebyshev rational function, it is possible to produce a magnitude characteristic that is equal ripple in both pass band and stop band.

The determination of the form of the rational function $R_n(w)$ in general requires the use of Elliptic functions and Elliptic integrals, and the resulting network functions are referred to as elliptic functions [15].

The general form of the elliptic magnitude characteristics is specified as:

$$|N(j\omega)|^2 = \frac{H^2}{1 + \epsilon^2 R_n^2(\omega)} \quad (2.14)$$

The chebyshev rational function $R_n(\omega)$ can be shown to

$$R_n(\omega) = \begin{cases} M \prod_{i=1}^{n/2} \frac{\omega^2 - \omega_{pi}^2}{\omega^2 - \omega_{zi}^2} & \text{for } n \text{ even} \\ M\omega \prod_{i=1}^{(n-1)/2} \frac{\omega^2 - \omega_{pi}^2}{\omega^2 - \omega_{zi}^2} & \text{for } n \text{ odd} \end{cases} \quad (2.15)$$

2.3.5 Characteristics of 4 types of approximation

In the Figure 2.2, for 4 types of approximation pass band frequency, stop band frequency, the maximum deviation of the magnitude characteristic, the minimum attenuation of the magnitude characteristic can be seen and a comparison and a summary can be made. As it is obvious from these figures, the elliptic approximation gives sharper (close to ideal) characteristics from the others. But this property costs the ripples in both pass band and stop band region.

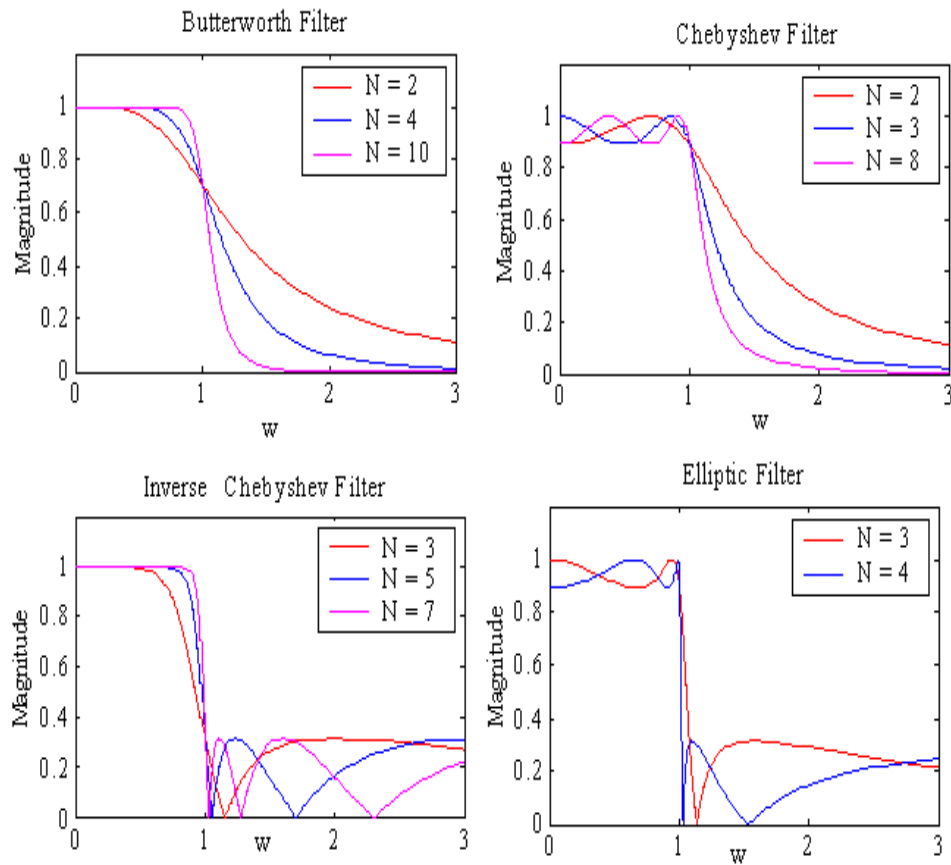


Figure 2.2 Behavior of Approximations

2.4 Normalization and Denormalization

The specifications usually given for the design of filters involve frequencies that have values of thousands of cycles per second. Synthesis calculations, however, are most easily done using frequencies of a few hertz or radians per second, since the numerical computations are simplified by not having to carry along various powers of 10. Such values are usually referred as normalized frequency values. Not only the conversion of the scaling of the frequencies but also the characteristics of the transfer functions such as low pass, high pass, band pass and band stop, requires the use of transformations. To convert such normalized values to the real world frequencies actually required in a given filter application including the change of the filter type or vice versa, the process used is called frequency denormalization and frequency transformation.

2.4.1 Low pass to low pass transformation

If we consider p as the normalized complex-frequency variable for the low pass equivalent and s as the denormalized one, then the frequency-denormalization process from lowpass to low pass is defined by the relation

$$s = \Omega_n p \quad (2.16)$$

where Ω_n is called the frequency-denormalization constant [16].

This transformation is also used in reverse case. In some applications, the normalization and denormalization may be applied on the circuit elements. In general all approximation techniques are modeled for normalized low pass filter case with 1 rad. cut off frequency. The implementation of the circuit is possible in first normalized structure and then circuit transformation results with the actual networks replacing all normalized L and C elements by their equivalent. The actual values of the capacitors and inductors are found by dividing their normalized values to Ω_n .

2.4.2 Low pass to high pass transformation

The frequency-denormalization process from lowpass to high pass is defined by the relation

$$s = \Omega_n / p \quad (2.17)$$

This transformation is also used in reverse case. The implementation of the circuit is possible in first normalized structure and then circuit transformation results with the actual networks replacing all normalized L and C elements by their equivalent. The all capacitors in the normalized low pass equivalent are replaced by inductors with the value of $1/\Omega_n C$ and all inductors in the normalized low pass equivalent are replaced by capacitors with the value of $1/\Omega_n L$ as seen in Figure 2.3.

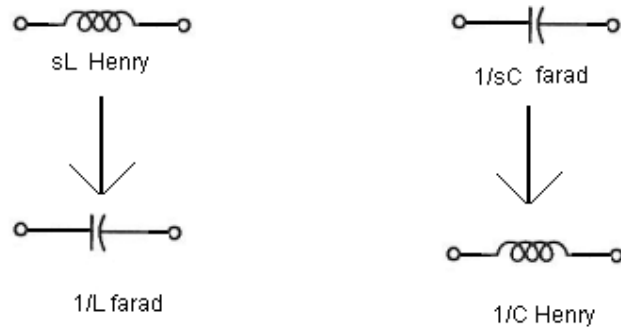


Figure 2.3. Low pass to high pass transformation of LC structures.

2.4.3 Low pass to band pass transformation

The frequency-denormalization process from lowpass to band pass and vice versa is defined by the relation

$$p = \frac{1}{\omega_{p2} - \omega_{p1}} \left[s + \frac{\omega_{p1} * \omega_{p2}}{s} \right] \quad (2.18)$$

The given band pass specifications such as ω_{p1} , ω_{p2} , ω_{s1} (edge of stopband region), ω_{s2} (edge of stopband region), K_p (maximum attenuation in pass band region), K_s (minimum attenuation in stop band region) are first transformed into normalized low pass equivalent using equation (2.16) and then normalized low pass transfer function and 1 ohm resistance terminated LC equivalent is obtained. The same equation is used for denormalization of the transfer function replacing p and the network is denormalized as actual band pass filter replacing inductors with series LC resonator and capacitors with parallel LC resonator. The element values of the resonators are determined using equation (2.18) as shown in Figure 2.4

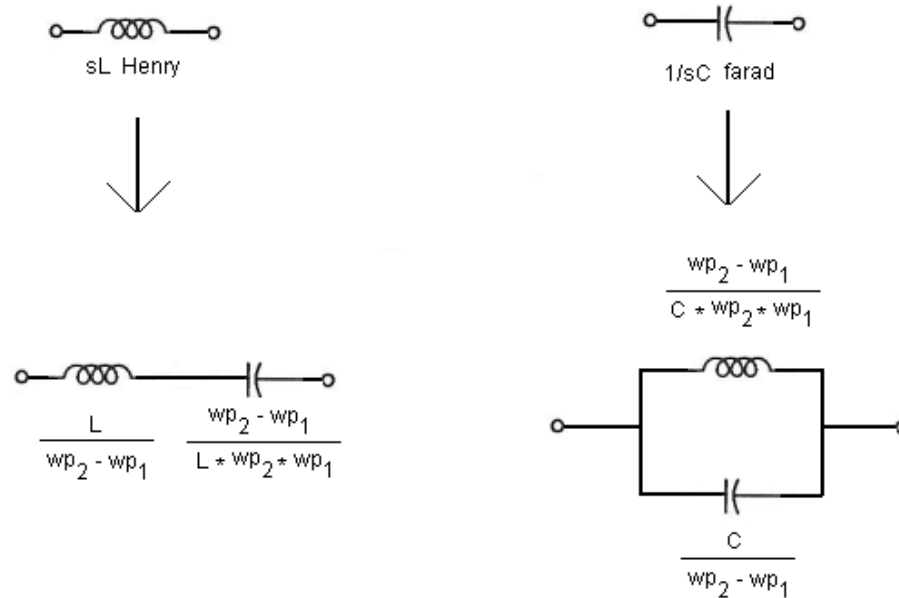


Figure 2.4 Low pass to band pass transformation of LC structures.

2.4.4 Low pass to band stop transformation

The given band stop specifications are converted into normalized low pass form in two steps. First the data is changed into band pass form and then low pass form. Therefore the equations

$$s = 1/p \quad (2.19)$$

and

$$p = \frac{1}{\omega_{p2} - \omega_{p1}} \left[s + \frac{\omega_{p1} * \omega_{p2}}{s} \right] \quad (2.20)$$

are both used. The network is denormalized as actual band stop filter replacing inductors with parallel LC resonator and capacitors with series LC resonator. The element values of the resonators are determined using equations (2.19) and (2.20), as shown in Figure 2.5.

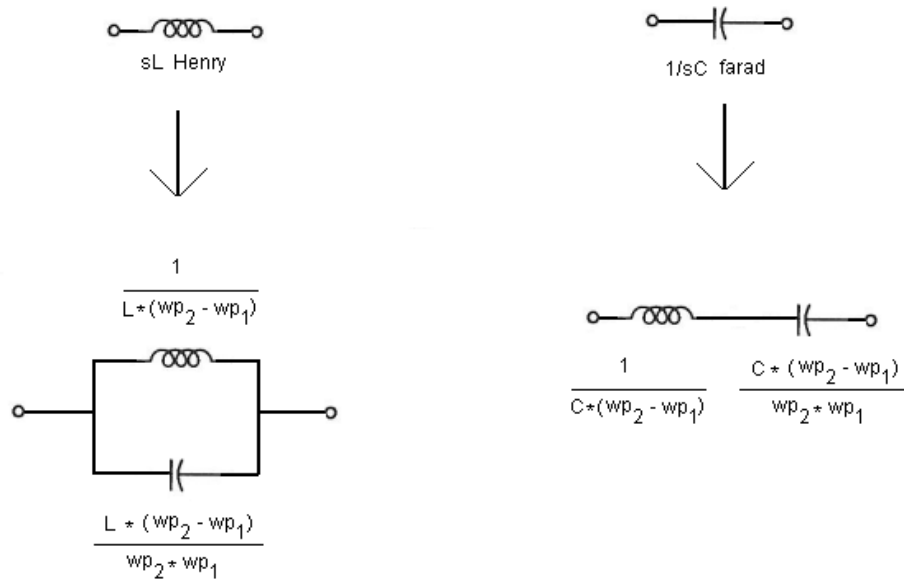


Figure 2.5. Low pass to band stop transformation of LC structure.

2.5 Implementation

In analog circuit implementation, basically well-known analog, linear (or non-linear) components such as resistors, capacitors, inductors, operational amplifiers, etc. are used [17]. Once the normalized or actual transfer function $N(s)$ is obtained, the maximum gain requirements answers whether the active or passive circuit elements will be used in the implementation. For every high frequency applications, since the gain of the operational amplifiers are not linear at high frequencies, the passive LC structures are preferred [18]. The critical frequencies such as cut off, resonance are cached by using series or parallel LC resonators in the passive structures. Voltage divider approach helps to calculate the maximum gain of the filters.

On the other hand, if the required gain is greater than unity (amplifier), then it is necessary to use the active circuit components such as transistor, operational amplifiers. For high frequency applications transistorized amplifiers should be designed while for considerably lower frequencies (audio, medium) operational amplifiers may be used. One and most important property of the active filters is not only their amplification properties but also the lack of the inductors in these circuits.

In fact last two decades, resistors are also eliminated by using the switched capacitor structures. In this study LC ladder implementation is computerized. In Figures 2.3.(a), (b) and (c) the low pass filter structures are given for the similar characteristics [19].

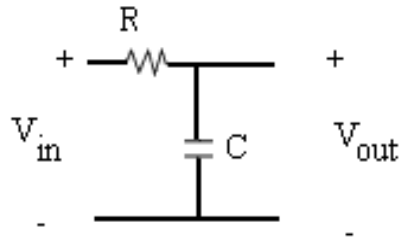


Figure 2.6.(a) Passive RC LPF

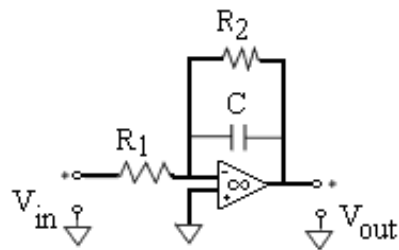


Figure 2.6.(b) Active RC LPF

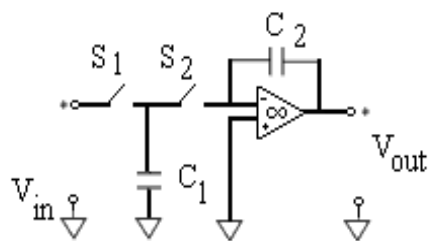


Figure 2.6.(c) SC – LPF (S_1 and S_2 are switches)

CHAPTER 3

COMPUTER AIDED DESIGN OF FILTERS

A set of computation is given for each type of the approximations in this chapter. They were described according to their squared magnitude function in Chapter 2. These computations using the given specifications of filters help us to get the element number, pole locations of the required filter. Hence quadratic factors and denominator coefficients are computed. Since computed values are normalized values, these values will be multiplied by denormalization constant w_p .

First of all, the mathematical models of the four different approximations are given. The details of how the poles and zeros of the transfer function are obtained will be then explained. The algorithm prepared to achieve all calculation steps will be given.

3.1 Butterworth Approximation

3.1.1 Pole locations, quadratic factors $(s^2 + a_1s + 1)$ and Q's of N(s)

The locations of the poles of a network function $N(s)$ that has a Butterworth magnitude characteristic may be found by using

$$|N(jw)|^2 = \frac{H^2}{1 + w^{2n}} \quad \text{and} \quad (3.1)$$

$$T(s^2) = |N(jw)|^2 \Big|_{w=s/j} = N(s) * N(-s) \quad (3.2)$$

Thus we obtain

$$N(s) * N(-s) = \frac{H^2}{1 + w^{2n}} \Big|_{w^2=s^2} = \frac{H^2}{1 + (-1)^n s^{2n}} \quad (3.3)$$

Setting the denominator polynomial of equation (3.3) to zero, we find that poles are located at the values of s that satisfy the relation

$$s = \left[-(-1)^n \right]^{1/2n} \quad (3.4)$$

Thus for even value of n $s = (-1)^{1/2n} = e^{j\pi k/2n}$ ($k=1,3,5,\dots,4n-1$), and for odd value of n $s = (1)^{1/2n} = e^{j\pi k/2n}$ ($k=0,2,4,\dots,4n-2$). From these relations, it is seen that the poles of $N(s)N(-s)$ are equal-angularly spaced around unit circle. Retaining only the left-half plane singularities, to satisfy the stability in the same time the poles of $N(s)$ are given as $p_k = \sigma_k + j\omega_k$ [2], where

$$\sigma_k = -\sin \frac{2k-1}{2n} \pi \quad k=1,2,3,\dots,n \quad (3.5)$$

$$\omega_k = \cos \frac{2k-1}{2n} \pi \quad k=1, 2, 3,\dots,n \quad (3.6)$$

By multiplying real part of root by 2, you can get a_1 ;

$$a_1 = \left(-\sin \frac{2k-1}{2n} \pi \right) * 2 \quad (3.7)$$

The inverse of a_1 is the quality factor Q ;

$$Q = 1/a_1 \quad (3.8)$$

3.1.2 Denominator coefficients of N(s)

The denominator polynomials characterized by the roots calculated above are called Butterworth polynomial. The values of the polynomial coefficients for a polynomial $P(s)$ where

$$P(s) = a_0 + a_1 s + a_2 s^2 + \dots + a_n s^n \quad (3.9)$$

are given by noting that, since all the poles are located on the unit circle, $a_0 = 1$. The other coefficients are determined by the iterative relation

$$a_k = \frac{\cos[(k-1)\pi/2n]}{\sin(k\pi/n)} a_{k-1} \quad k=1,2,\dots,n \quad (3.10)$$

Coefficients of the function is symmetric, so that

$$a_0 = a_n = 1$$

$$a_1 = a_{n-1}$$

$$a_2 = a_{n-2}$$

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3.1.3 Order of N(s)

The following parameters are defined to determine the order of function required to meet a given set of specifications

$$\Omega = \frac{\omega_s}{\omega_p} = \frac{f_s}{f_p} \quad (3.11)$$

$$M = \sqrt{\frac{10^{0.1K_s} - 1}{10^{0.1K_p} - 1}} \quad (3.12)$$

For the Butterworth case the required order n_b is given by

$$n_b = \frac{\log M}{\log \Omega} = \frac{\ln M}{\ln \Omega} \quad (3.13)$$

The required order of the function is the next highest integer greater than n_b . equation (3.13) satisfy the maximum attenuation at the pass band edge. Since the closest higher integer is chosen, the obtained transfer function always gives better results than the requirements.

3.2 Chebyshev Approximation

3.2.1 Pole locations and quadratic factors $(s^2 + a_1s + a_0)$ of $N(s)$

equation (3.1) is modified as

$$|N(j\omega)|^2 = \frac{H^2}{1 + \epsilon^2 C_n^2(\omega)} \quad (3.14)$$

to obtain the ideal low pass characteristics. Since ω^{2n} is replaced by $C_n^2(\omega)$, which is known as the Chebyshev function, the basic property of this function must be sharp change (high derivative) around normalized unity frequency.

Replacing the ω in equation (3.14) by s/j , the s domain complex transfer function is obtained as

$$N(s) * N(-s) = |N(j\omega)|_{s/j}^2 = \frac{H^2}{1 + \epsilon^2 C_n^2(s/j)} \quad (3.15)$$

Thus the poles of the product $N(s) * N(-s)$ are the roots of $C_n^2(s/j) = -1/\epsilon^2$ or $C_n(s/j) = \pm j/\epsilon$.

Using the trigonometric form for $C_n(\omega)$

$$C_n(\omega) = \cos(n \cos^{-1} w) \quad 0 \leq \omega \leq 1 \quad (3.16a)$$

$$C_n(\omega) = \cosh(n \cosh^{-1} w) \quad \omega \geq 1 \quad (3.16b)$$

$$C_n(\omega) = \frac{1}{2} \left[\left(\omega + \sqrt{\omega^2 - 1} \right)^n + \left(\omega - \sqrt{\omega^2 - 1} \right)^n \right] \quad \omega \geq 1 \quad (3.16c)$$

Hence

$$C_n\left(\frac{s}{j}\right) = \cos\left(n \cos^{-1} \frac{s}{j}\right) = \pm \frac{j}{\varepsilon} \quad (3.17)$$

To solve this equation, a complex function

$$\omega = u + jv = \cos^{-1} \frac{s}{j} \quad (3.18)$$

is defined.

Substituting this expression in equation (3.15)

$$\cos n(u + jv) = \cos nu \cosh nv - j \sin nu \sinh nv = \pm \frac{j}{\varepsilon} \quad (3.19)$$

is obtained.

Equating the real parts of the second and third members of this relation gives $\cos nu \cosh nv = 0$. Since $\cosh nv \geq 1$ for all values of nv , this equality requires $\cos nu = 0$. This may be written in the form

$$u_k = \frac{2k-1}{2n} \pi \quad k=1,2,3,\dots,2n \quad (3.20)$$

Equating the imaginary parts of equation (3.18) and recognizing that for the values of u defined by equation (3.19), $\sin nu = \pm 1$

$$v = \frac{1}{n} \sinh^{-1} \frac{1}{\varepsilon} \quad (3.21)$$

is obtained where only the positive value for v are retained. equation 3.17 may now put into the form

$$s = j \cos(u_k + jv) = \sin u_k \sinh v + j \cos u_k \cosh v \quad (3.22)$$

This relation specifies the poles of the product $N(s)N(-s)$. The left-half plane poles are assigned to $N(s)$ to complete the determination of the network function. Thus we see that the poles of $N(s)$ will be at $p_k = \sigma_k + j\omega_k$ [10], where

$$\sigma_k = -\sin u_k \sinh v \quad k=1,2,\dots,n \quad (3.23.a)$$

$$\omega_k = \cos u_k \cosh v \quad k=1,2,\dots,n \quad (3.23.b)$$

and where u_k and v are defined as in equations (3.20) and (3.21).

The coefficient ε is obtained by

$$\varepsilon^2 = \frac{1}{10^{0.1K_s} - 1} \quad (3.24)$$

After the pole locations are obtained in the form $(s - \sigma_1 + j\omega_1) * (s - \sigma_2 + j\omega_2)$ the quadratic factors are simply written in the form $(s^2 + a_1s + 1)$.

3.2.2 Denominator coefficients of N(s)

Finding the roots of the denominator polynomial, since the poles are already obtained, it is possible to write the denominator function either as the multiplication of the subsequent second order s domain functions or completely in the polynomial form. First case is very suitable for the RC-operational amplifier cascade realizations while the later case is good for passive LC-ladder realizations. The denominator obtained by using the given algorithm is Hurwitz and the coefficients are positive real numbers. So the overall transfer function is passive LC-ladder realizable.

3.2.3 Order of N(s)

To determine the order of function required to meet a given set of specifications

$$\Omega = \frac{\omega_s}{\omega_p} = \frac{f_s}{f_p} \quad (3.25)$$

$$M = \sqrt{\frac{10^{0.1K_s} - 1}{10^{0.1K_p} - 1}} \quad (3.26)$$

are defined as in the previous chapter.

For the Chebyshev case the required order n_c is given by

$$n_c = \frac{\cosh^{-1} M}{\cosh^{-1} \Omega} \quad (3.27)$$

The required order of the function is the next highest integer greater than n_c

3.3 Inverse Chebyshev Approximation

3.3.1 Pole and zero locations and quadratic factors $(s^2 + a_1s + a_0)$ of $N(s)$

In the Inverse Chebyshev approximation the reciprocal of the Chebyshev polynomial is used. This causes the generation of the zeros as well as the poles as in equation (3.28).

$$|N_{IC}(j\omega)|^2 = \frac{\epsilon^2 C_n^2(1/\omega)}{1 + \epsilon^2 C_n^2(1/\omega)} \quad (3.28)$$

The poles of the function are determined by finding the roots of the denominator polynomial after replacement of s/j for ω as in the previous approximations. Hence the magnitude squared network function becomes

$$|N(j\omega)|_{s/j}^2 = N(s)N(-s) = \frac{A(s)A(-s)}{B(s)B(-s)} \quad (3.29)$$

where $N(s)$ is the network function. $A(s)$ is the numerator polynomial and $B(s)$ is its denominator polynomial. Equating the numerator and denominator of equations (3.28) and (3.29), s domain polynomials are obtained as

$$A(s)A(-s) = \epsilon^2 C_n^2\left(\frac{j}{s}\right) \quad (3.30)$$

$$B(s)B(-s) = 1 + \epsilon^2 C_n^2\left(\frac{j}{s}\right) \quad (3.31)$$

First let consider the numerator. Setting equation (3.29) to zero and using

$$C_n\left(\frac{s}{j}\right) = \cos\left(n \cos^{-1} \frac{s}{j}\right) = \pm \frac{j}{\epsilon} \quad (3.32)$$

$$C_n\left(\frac{s}{j}\right) = \cos\left(n \cos^{-1} \frac{s}{j}\right) = 0 \quad (3.33)$$

is obtained.

The result is that the zeros $z_k = \alpha_k + j\beta_k$ [10] of the Inverse Chebyshev function are given by

$$\alpha_k = 0 \quad \beta_k = \frac{1}{\cos u_k} \quad k=1,2,\dots,n \quad (3.34)$$

$$u_k = \frac{2k-1}{2n} \pi \quad (3.35)$$

Now let consider the denominator polynomial of the Inverse Chebyshev function. The poles are found by setting equation (3.29)b to zero. The resulting equations are almost the same as that used for finding the poles of a Chebyshev function whose pass-band has the same attenuation (K_p) as the inverse Chebyshev but is defined for 0 to 1 rad/s and whose stop-band has the same attenuation (K_s) as the inverse Inverse Chebyshev but starts at $1/\omega_p$ radians per second. The only difference is that the argument of the Chebyshev polynomial $C_n^2(\cdot)$ is inverted and appears as j/s rather than as s/j . As a result, the poles of the Inverse Chebyshev function are simply the reciprocal of ones found for Chebyshev function. Thus they maybe specified as [10]

$$p_k = \frac{1}{\sigma_k + j\omega_k} \quad k=1,2,\dots,n \quad (3.36)$$

$$\sigma_k = -\sin u_k \sinh v \quad \omega_k = \cos u_k \cosh v \quad k=1,2,\dots,n \quad (3.37)$$

$$v = \frac{1}{n} \sinh^{-1} \frac{1}{\varepsilon} \quad \varepsilon = \sqrt{\frac{1}{10^{0.1K_s} - 1}} \quad (3.38)$$

and u_k is defined in equation (3.35)

After the pole locations are obtained in the form $(s - \sigma_1 + j\omega_1)^*(s - \sigma_2 + j\omega_2)$ the quadratic structure is also obtained as $(s^2 + a_1s + 1)$.

3.3.2 Denominator coefficients of N(s)

Finding the roots of the denominator polynomial, since the poles are already obtained, it is possible to write the denominator function either as the multiplication of the subsequent second order s domain functions or completely in the polynomial form. First case is very suitable for the RC-operational amplifier cascade realizations while the later case is good for passive LC-ladder realizations. The denominator obtained by using the given algorithm is Hurwitz and the coefficients are positive real numbers. So the overall transfer function is passive LC-ladder realizable.

3.3.3 Order of N(s)

To determine the order of function required to meet a given set of specifications

$$\Omega = \frac{\omega_s}{\omega_p} = \frac{f_s}{f_p} \quad (3.39)$$

$$M = \sqrt{\frac{10^{0.1K_s} - 1}{10^{0.1K_p} - 1}} \quad (3.40)$$

are defined as in the previous chapter.

For the Inverse Chebyshev case the required order n_{ic} is given by

$$n_{ic} = \frac{\cosh^{-1} M}{\cosh^{-1} \Omega} \quad (3.41)$$

The required order of the function is the next highest integer greater than n_{ic}

3.4 Elliptic Approximation

3.4.1 Order of N(s)

For the determination of the order of an Elliptic Function the specifications are:

1) Passband: For $0 \leq \omega \leq \omega_p$ radians per second ($0 \leq f \leq f_p$ hertz), the ripple of the magnitude characteristic is K_p dB

2) Stopband: For $\omega \geq \omega_s$ radians per second ($f \geq f_s$ hertz) , the equal-ripple attenuation varies between a minimum of K_s dB and infinity.

To determine the order M and Ω are used. The elliptic order n_E then is found by first determining the quantities.

$$C(M) = \frac{1}{16M^2} \left(1 + \frac{1}{2M^2} \right) \quad (3.42)$$

$$D(\Omega) = \frac{\sqrt{\Omega} - 1}{2(\sqrt{\Omega} + 1)} \quad (3.43)$$

The order n_E is given as

$$n_E = F_E(C)F_E(D) \quad (3.44)$$

where

$$F_E(x) = \frac{1}{\pi} \ln(x + 2x^5 + 15x^9) \quad (3.45)$$

The required order of the function is the next highest integer greater than n_E .

3.5 Algorithm

The overall equations given in the previous sections of this chapter are computerized and algorithm is prepared to process the given specification data to obtain the poles, zeros, order and some specific circuit structures. To communicate the computer, user independent easy interface is prepared.

The block diagram of the program code is given in Figure 3.1.

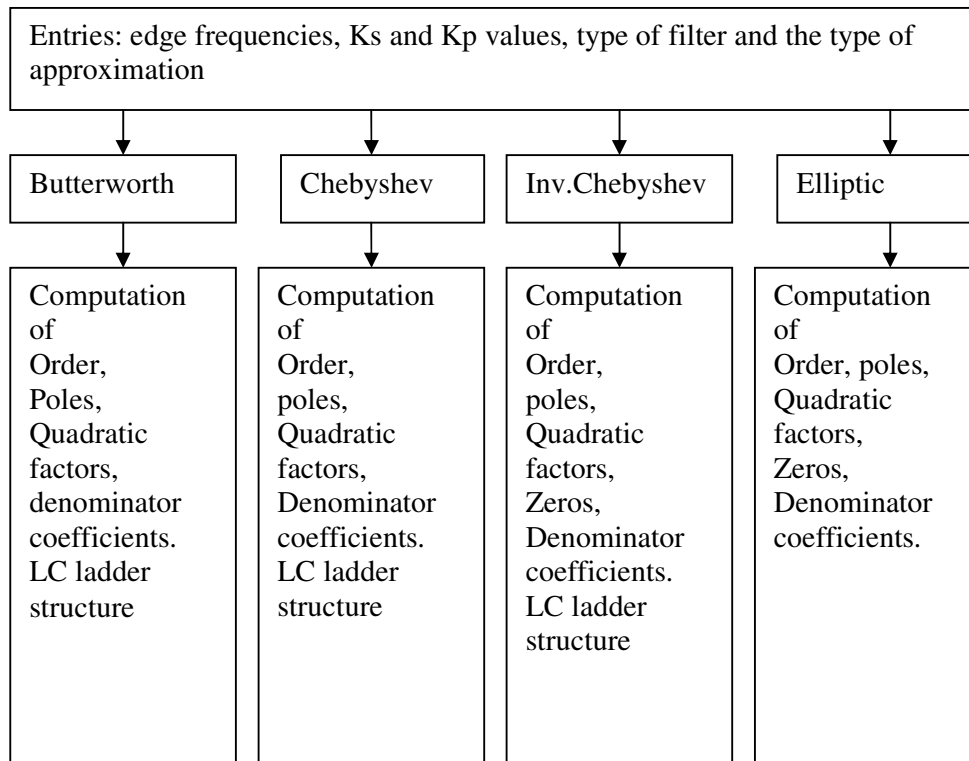


Figure 3.1 Program code flow diagram

The algorithm of the program for 4 different approximations is as:

Entries: edge frequencies (ω_s , ω_p for LP/HP filter $\omega_{s1}, \omega_{s2}, \omega_{p1}, \omega_{p2}$ for band pass/stop filter) K_s and K_p values and the type of approximation

Butterworth function computations:

Poles	: Equations (3.5) and (3.6)
Ao	: 1
A1 values	: Equation (3.7)
Denominator coefficients	: Equation (3.10)
Order	: Equations (3.11), (3.12), (3.13)
Denormalization	: Equation (2.16) (frequency-denormalization constant is ω_p)
Transformations	: Equations (2.17), (2.18), (2.19), (2.20)
Inductance and Capacitance values in actual LC Ladder structure	

Chebyshev function computations:

Poles	: Equations (3.21), (3.23.a), (3.23.b), (3.24), (3.32)
Ao and A1 values	: Multiplying every conjugate pair of roots by each of them.
Denominator coefficients	: Multiplying every pair of roots by each of them, store it at matrixes, and re-multiply until you reach the order computed.
Order	: Equations (3.25), (3.26), (3.27)
Denormalization	: Equation (2.16) (frequency denormalization constant is ω_p)
Transformations	: Equations (2.17), (2.18), (2.19), (2.20)
Inductance and Capacitance values in actual LC Ladder structure	

Inverse Chebyshev function computations:

Poles	: Equations (3.35), (3.36), (3.37), (3.38)
Ao and A1 values	: Multiplying every pair of roots by each of them.
Denominator coefficients	: Multiplying every pair of roots by each of them, store it at matrixes, and re multiply until you reach the order computed.
Order	: Equations (3.39), (3.40), (3.41)
Zero	: Equations (3.34), (3.35)

Denormalization : Equation (2.16) (frequency denormalization constant is ω_p)

Transformations : Equations (2.17), (2.18), (2.19), (2.20)

Inductance and Capacitance values in actual LC Ladder structure

Elliptic function computations:

Order : Equations (3.42), (3.43), (3.44), (3.45)

Ao and A1 values : Multiplying every pair of roots by each of them.

Denominator coefficients : Multiplying every pair of roots by each of them, store it at matrixes, and re multiply until you reach the order computed.

Order : Equations (3.39), (3.40), (3.41)

Zero : Equations (3.34), (3.35)

Denormalization : Equation (2.16) (frequency denormalization constant is ω_p)

Transformations : Equations (2.17), (2.18), (2.19), (2.20)

3.6 How The Program Works?

In the beginning window program asks for the filter type like high pass, low pass, band pass and band stop. After the selection of the type, to get the required network elements, according to the specifications needed for the filter design, critical edge frequencies and band attenuations (K_s , K_p) are entered. The approximation will be chosen from the following window by the user. Programming code take the values and use the selected approximation and values and make the set of computations to get order, poles, quadratic factors, denominator factors and actual LC network for Butterworth, Chebyshev, Inverse Chebyshev approximations. For elliptic approximation, the transfer function is obtained. On this window a basic circuit structure is presented and transfer function elements are listed. The all circuit element values listed are actual values of inductors and capacitors.

The values got from denominator and numerator polynomial is given to Matlab program as a.dat and b.dat files. These a.dat and b.dat values including denominator and numerator polynomials can be used in matlab program to see the behaviour of the filter resulted from approximation methods.

The flow diagram of interface is given in Figure 3.2.

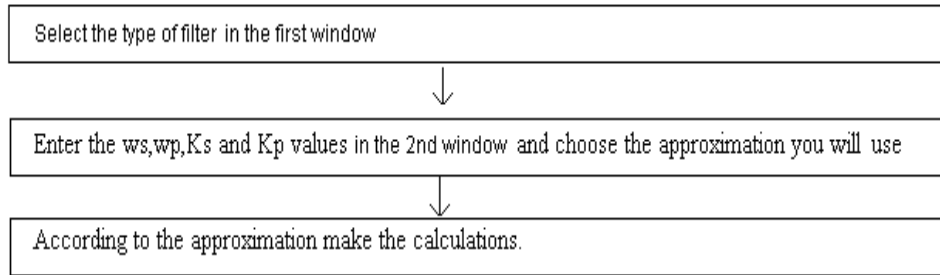


Figure 3.2. Interface flow diagram

The program runs for limited values as written below:

1) Actual LC structure elements calculation is limited by 10 elements in Butterworth and Chebyshev approximation. In Inverse Chebyshev approximation LC structure elements calculation is limited by 4 elements. Elliptic approximation is calculated for low-pass and high-pass filters and in the result transfer function is given as data. Here LC structure elements aren't given.

2) For low pass and high pass filters numerator and denominator coefficients (coefficients resulted from zeros and poles) are limited by 10 coefficients. In the band pass and band reject filters denominator coefficients are limited by 5 coefficients. Because for the order number 5, 10 coefficients will be get in the numerator and denominator polynomial.

Summary:

- 1) Choose filter type

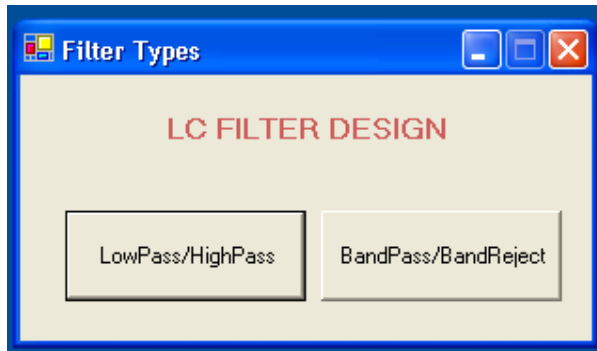


Figure 3.3. First interface to select the filter type

- 2) If you choose LowPass/High pass the window below is seen. Here choose low or high pass filter, then enter the values $\omega_s, \omega_p, K_s, K_p$ and select the type of approximation.

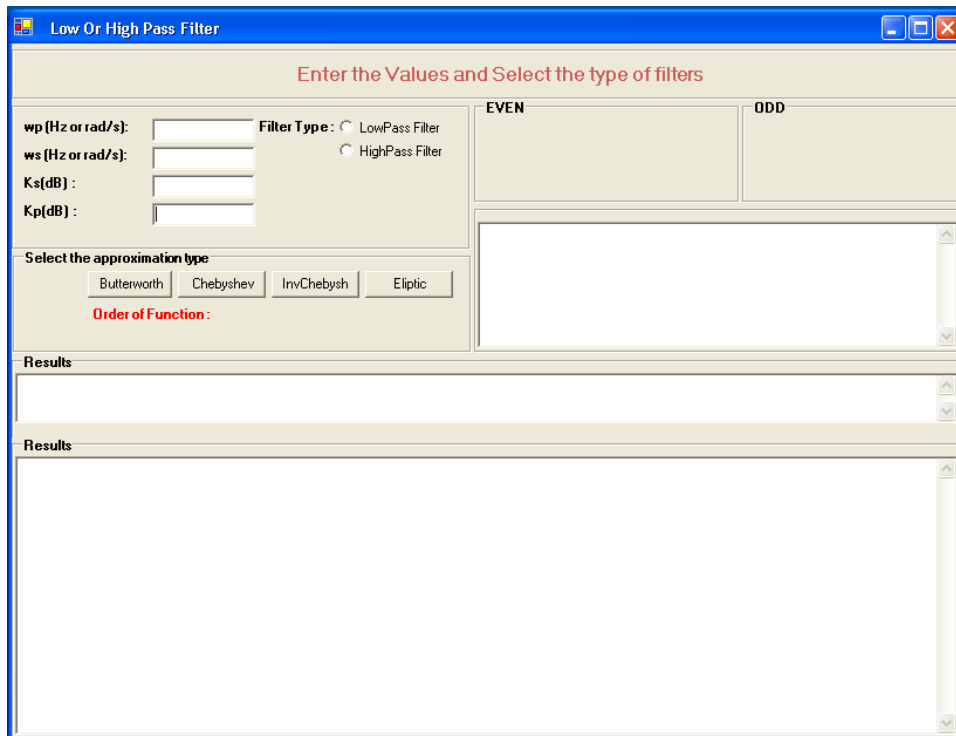


Figure 3.4. Second interface to enter datas and see results if you choose low/high pass filter

- 3) If you choose BandPass/stop filter the window below is seen. Here choose band pass or stop filter, then enter the values $\omega_{s1}, \omega_{p1}, \omega_{s2}, \omega_{p2}, K_s, K_p$ and select the type of approximation.

The screenshot shows a software window titled "Band Pass / Band Reject" with a blue title bar. The main area has a light beige background and contains the following elements:

- Header:** "Enter the Values and Select the type of filters" in red text.
- Input Fields:** Four text boxes for "wp1 (Hz or rad/s)", "wp2 (Hz or rad/s)", "ws1 (Hz or rad/s)", and "ws2 (Hz or rad/s)". Two more text boxes for "Ks(dB)" and "Kp(dB)".
- Filter Type:** Two radio buttons labeled "BandReject Filter" and "BandPass Filter".
- Order of Function:** A red label.
- Approximation Type:** A section titled "Select the approximation type" with three buttons: "Butterworth", "Chebyshev", and "InvChebyshev".
- Buttons:** Two buttons labeled "EVEN" and "ODD" are positioned above the approximation type buttons.
- Result Area:** A section titled "Result" with a scrollable text area.
- Denominator Coefficients:** A section titled "Denominator Coefficients" with a scrollable text area.

Figure 3.5. Second interface to enter datas and see results if you choose band pass/band reject filter

- 4) Enter the normalized polynomial coefficients into matlab to sketch the gain curve.

CHAPTER 4

PROGRAM

In this chapter the details of the prepared program will be given and it will be run for several examples.

4.1 Program

The program is written in visual C# [20, 21]. Program consists of many buttons, labels. Every button and label works with its special code. [22].The program gives the results in the screen [23, 24].

Since the programming code is so many pages, it is and the program exe files are given in CD as in appendix A.

4.2 Filter Design Examples

Example 1) A Butterworth function must meet the following specifications for low-pass filter.

$$\omega_s = 1500, \omega_p = 1000, K_s = 25, K_p = 1$$

Determine the

- 1) the required order
- 2) the denormalized pole locations
- 3) the denormalized quadratic factors
- 4) the normalized denominator coefficients.
- 5) the actual LC values.

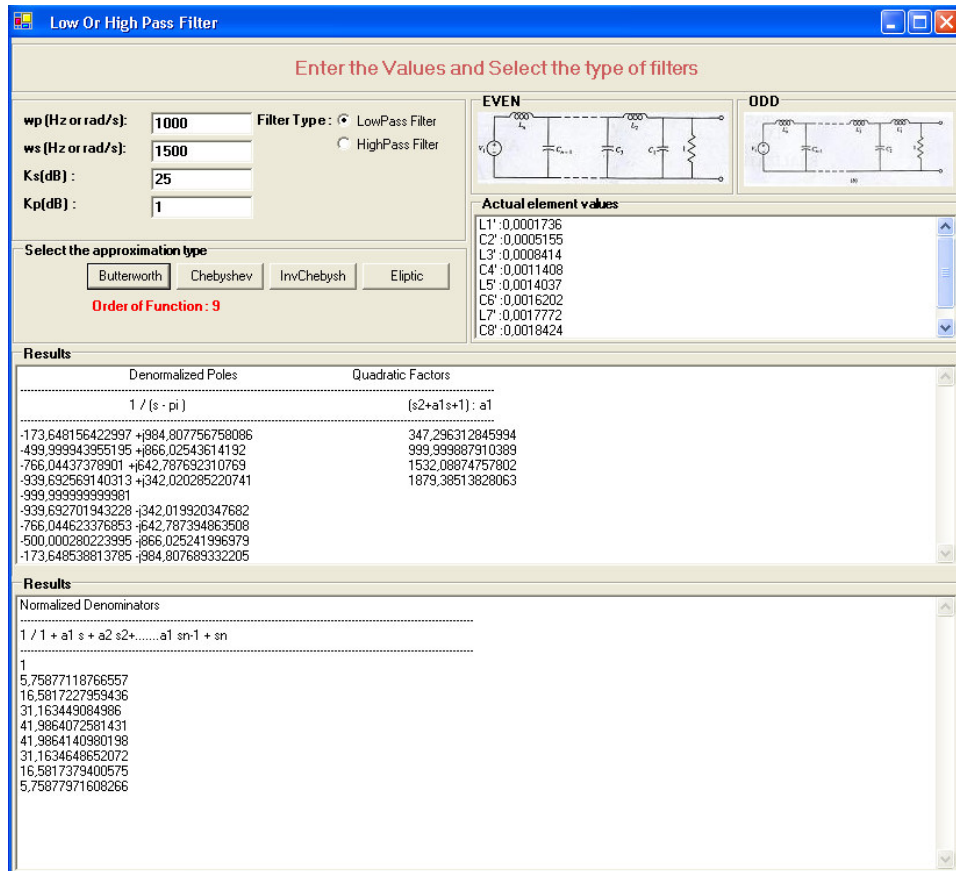


Figure 4.1. Entering the specifications and selecting the approximation. (The values computed and low-pass realizations of Butterworth elements)

To check the values, the program written below in MATLAB code gives us curve gain. The filter program written in C# gives the numerator and denominator coefficients to matlab/work/b.dat and matlab/work/b.dat. We must be careful here to check the a.dat and b.dat files so that the coefficients separators must be dot. From C# program's speciality, the coefficients sent to a.dat and b.dat files separators are comma, we must change them by dot. Also the filter program gives the Kp value entered as data from user to matlab/work/LHKp.dat and this value is used in the program written below in MATLAB.

```
clc
clear all
close all
```



```
a=load('a.dat')
b=load('b.dat')
w=[0 .1 .2 .3 .4 .5 .6 .7 .8 .9 1. 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2.];
h=freqs(b,a,w);
mag=abs(h); phase=angle(h);
%f=w/(2*pi);
%loglog(w,mag);
w1=[0 0.05 .1 .15 .2 .25 .3 .35 .4 .45 .5 .55 .6 .65 .7 .75 .8 .85 .9 .95 1.];
c=load('LHKp.dat')
plot(w,20*log10(mag),w1,c)
```

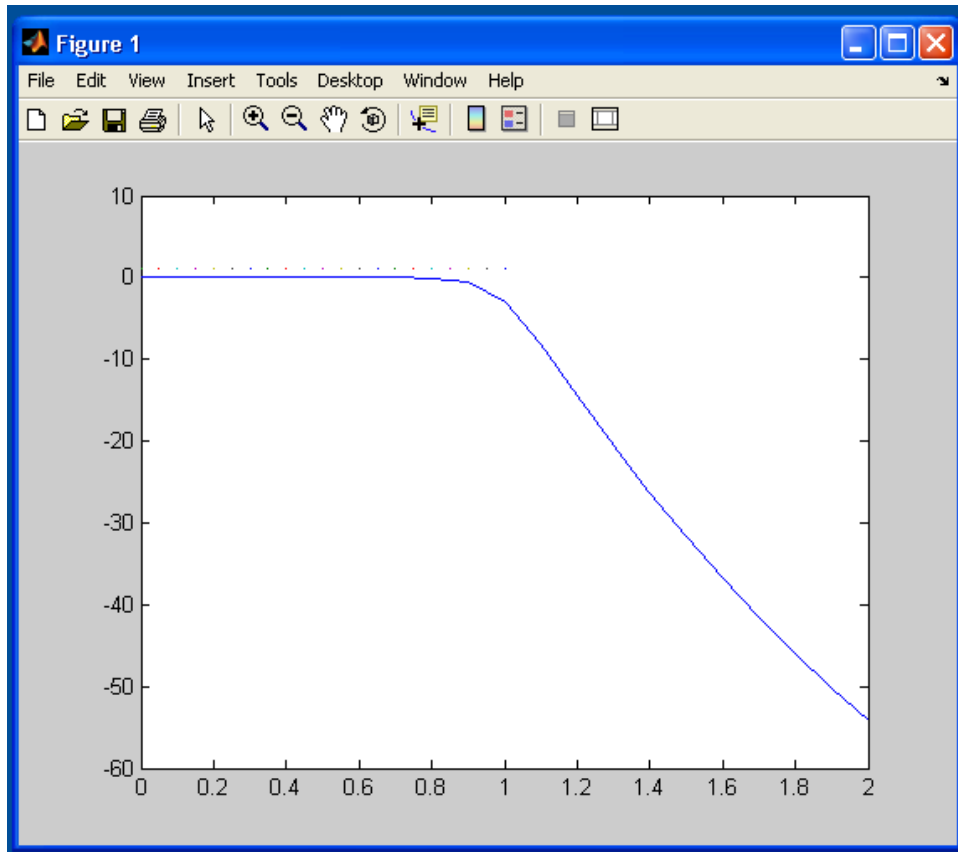


Figure 4.2. Gain curve for example 1

Example 2) A Chebyshev function must meet the following specifications for high-pass filter.

$$\omega_s = 1000, \omega_p = 1500, K_s = 25, K_p = 0.1$$

Determine the

- 1) the required order
- 2) the denormalized pole locations
- 3) the denormalized quadratic factors
- 4) the normalized denominator coefficients.
- 5) the actual LC values.

The screenshot shows a software window titled "Low Or High Pass Filter" with the following content:

Enter the Values and Select the type of filters

wp (Hz or rad/s): 1500 Filter Type: LowPass Filter
 HighPass Filter
ws (Hz or rad/s): 1000
Ks(dB): 25
Kp(dB): 1

Select the approximation type

Order of Function: 5

Actual element values

C1': 0.000781646930081682
L2': 0.00155363940029519
C3': 0.000367471429096388
L4': 0.00040586062746053
C5': 0.00043323802096872

Results

Denormalized Poles	Quadratic Factors
1 / [s - pi]	[s ² +a1s+a0] : a1 · a0
-135.774058731696 +j1502.72009939041	2276602.29213639 271.548117463391
-818.330187352105 +j2138.09498570925	5241114.46344679 1636.66037470421
-5181.46632871419	
-818.33112214531 -j2138.09568224317	
-135.774252340488 -j1502.72025682514	

Results

Normalized Denominators

s / a0 + a1 s + a2 s²+.....an-1 s(n-1)+ sn

8.14155194951229
7.62716326927002
13.7495838323149
7.93309637759945
4.72644988058955

Figure 4.3. Entering the specifications and selecting the approximation. (The values computed and high-pass realizations of Chebyshev elements)

To check the values, the program written below in MATLAB code gives us curve gain. The filter program written in C# gives the numerator and denominator coefficients to matlab/work/b.dat and matlab/work/b.dat. We must be careful here to check the a.dat and b.dat files so that the coefficients separators must be dot. From C# program's speciality, the coefficients sent to a.dat and b.dat files separators are comma, we must change them by dot. Also the filter program gives the Kp value entered as data from user to matlab/work/LHKp.dat and this value is used in the program written below in MATLAB.

```
clc
clear all
close all
a=load('a.dat')
b=load('b.dat')
w=[0 .1 .2 .3 .4 .5 .6 .7 .8 .9 1. 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2.];
h=freqs(b,a,w);
mag=abs(h); phase=angle(h);
%f=w/(2*pi);
%loglog(w,mag);
w1=[1. 1.05 1.1 1.15 1.2 1.25 1.3 1.35 1.4 1.45 1.5 1.55 1.6 1.65 1.7 1.75 1.8 1.85
1.9 1.95 2.];
c=load('LHKp.dat')
plot(w,20*log10(mag),w1,c)
```

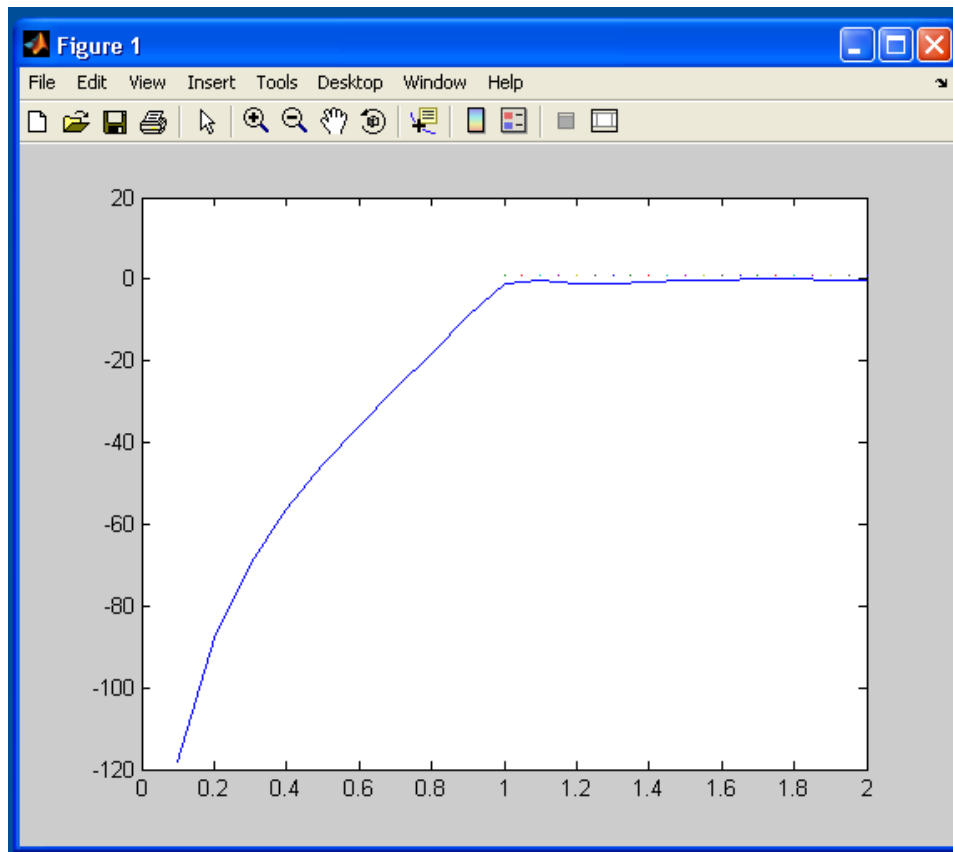


Figure 4.4. Gain curve for example 2

Example 3) An inverse Chebyshev function must meet the following specifications for band-pass filter.

$$\omega_{s1} = 1000, \omega_{s2} = 2500, \omega_{p1} = 1600, \omega_{p2} = 2000, K_s = 3, K_p = 1$$

Determine the

- 1) the required order
- 2) the normalized pole locations
- 3) the normalized zeros
- 4) the normalized numerator polynomials
- 5) the normalized denominator coefficients.
- 6) the actual element values

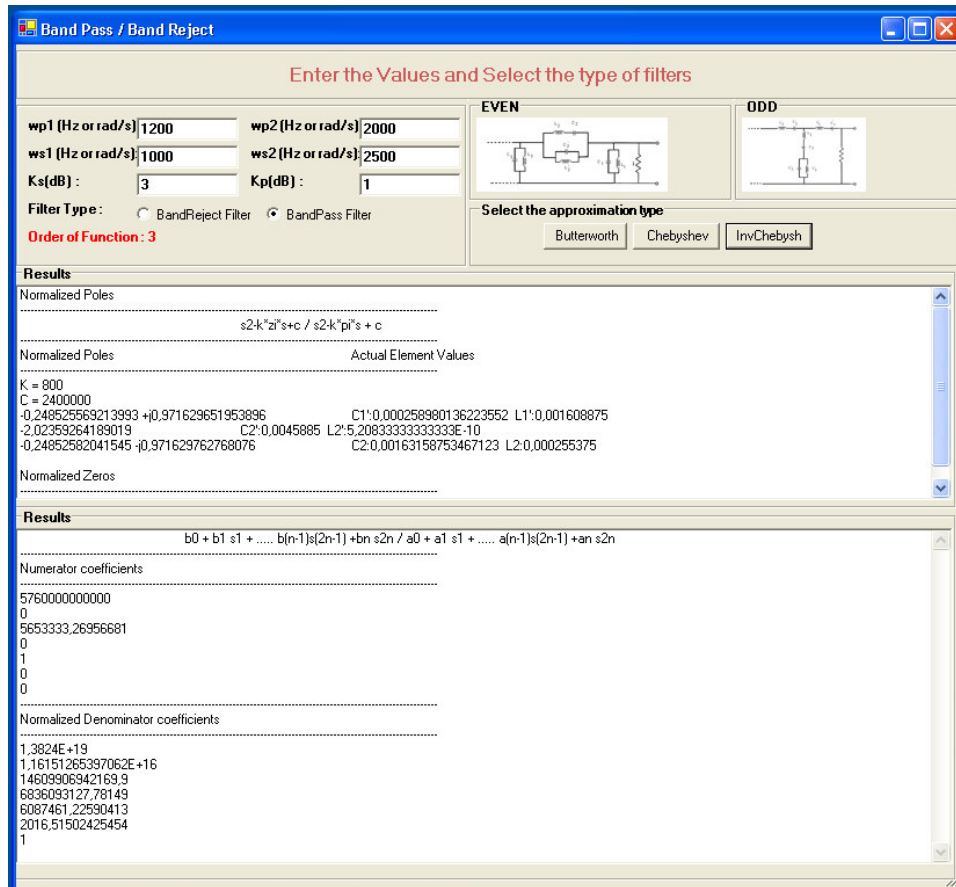


Figure 4.5. Entering the specifications and selecting the approximation. (The values computed and band-pass realizations of Inverse Chebyshev elements)

```

clc
clear all
close all
a=load('a.dat')
b=load('b.dat')
w=[100 150 200 250 300 350 400 450 500 550 600 650 700 750 800 850 900 950
1000 1050 1100 1150 1200 1250 1300 1350 1400 1450 1500 1550 1600 1650 1700
1800 1850 1900 1950 2000 2050 2100 2150 2200 2250 2300 2350 2400 2450 2500
2550 2600 2650 2700 2750 2800 2850 2900 2950 3000 3100 3150 3200 3250 3300
3350 3400 3450 3500];
h=freqs(b,a,w);
mag=abs(h); phase=angle(h);

```

```
1.85 1.9 1.95 2.];  
plot(w,20*log10(mag))
```

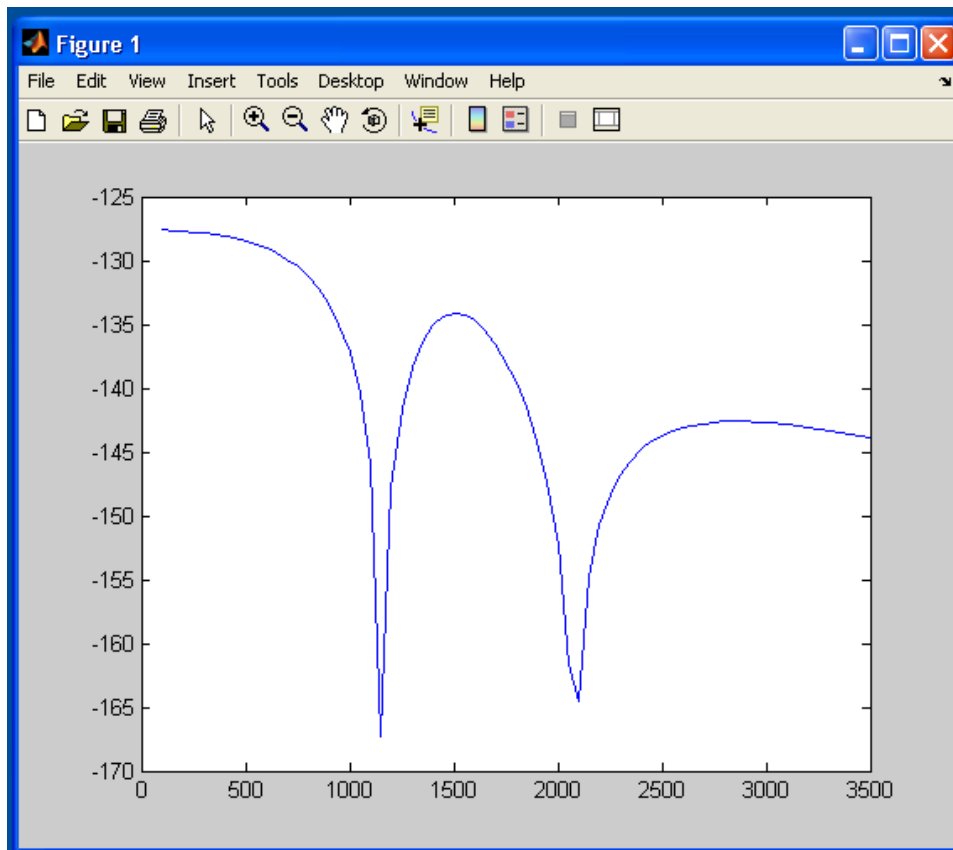


Figure 4.5. Gain curve for example 3

CHAPTER 5

DISCUSSIONS

In the communication and control applications, the elimination of the harmonics from the signal content has significant importance. In many analog or digital communication networks, the filters are widely used as well the modulators, convertors, oscillators, etc.

In both linear and non-linear electronics, the filter part of the network may be linear or non-linear filters. In this study the linear passive filter design is considered. Although the approximation theories used in *LC* filter design is very well known, here it is aimed to computerize the all complex mathematical expansions and to build up the package of finding filter design program.

In general although the actual filter specifications includes the edge frequencies and the attenuations of the stop band and pass band regions, the approximation algorithms convert the given data into the normalized low pass structure. It is some times useful to change the network elements by their actual values or actual transfer function is directly obtained by using the transformation.

Another basic purpose of this work is to prepare an interface for the users and to make the use of this program easy for the users. The program may be used as laboratory applications.

The inputs of the program are only the specification parameters (critical edge frequencies and corresponding attenuations) and the type of the approximation. In the program normalization and denormalization transformations are also included. Hence “ready to use” circuits may be obtained.

Since the elliptic filter realization has many different possible choices, in this study, only the order of the elliptic filter and transfer function is calculated for low pass and high pass filters.

As a future proposal; the computer program may be re-developed for the implementation of the obtained actual or normalized transfer function by using the *RC* operational amplifier, switched capacitor circuits or multi terminal structures.

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