Design of Sharp Transition FIR Filters Using Frequency Response Masking Technique

M.Sc. Thesis
in
Electrical and Electronics Engineering
University of Gaziantep

Supervisor
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by

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August 2010

T.C.
UNIVERSITY OF GAZİANTEP
GRADUATE SCHOOL OF
NATURAL&APPLIED SCIENCES
ELECTRICAL&ELECTRONICS ENGINEERING

Name of the thesis: Design of Sharp Transition FIR Filters Using Frequency Response Masking Technique

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Exam date : 09.08.2010

Approval of the Graduate School of Natural and Applied Sciences

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ABSTRACT

Design of Sharp Transition FIR Filters Using Frequency Response Masking Technique

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August 2010, 38 pages

The design of FIR filters by using Fourier series coefficients takes an important place in the use of digital speech and signal processing applications. Since the number of coefficients has to be chosen as finite, gain curves cannot be obtained exactly. The deviation on the gain curves of designed and actual networks has been partially eliminated by applying the window functions on the coefficients. After getting partial filter characteristics by partitioning the gain characteristics at bandwidth and associating these partial filters gives total gain curve about FIR filter design and this what we are trying to present.

Key words: FIR filter, frequency masking method, recursive structure.

ÖZET

Frekans Tepkisi Maske Tekniği Kullanılarak Keskin Geçiş Bantlı FIR Süzgeç Tasarımı

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Ağustos 2010 38 sayfa

Fourier Seri açılımları kullanılarak ve katsayı benzetmesi yöntemi ile FIR süzgeç tasarımı sayısal sinyal işleme uygulamalarında önemli bir yer kaplamaktadır. Fourier seri katsayılarının sonlu seçilmek zorunda olması verilen kazanç eğrilerinin tam olarak elde edilmesini engellemektedir. Bu sorunların giderilmesi ve daha düşük dereceli, daha kaliteli FIR süzgeç tasarımı konusu, uygun pencere fonksiyonlarının uygulanması ile bir ölçüde giderilmiştir. Bu çalışmada kazanç fonksiyonlarının frekans bandında parçalanarak kısmi süzgeç tasarımı ve elde edilen kısmi süzgeçlerin birleştirilmesi ile toplam kazanç eğrisini gerçekleştiren FIR süzgeç tasarımına ilişkin bir çalışma sunulmuştur.

Anahtar Kelimeler: FIR filtre, frekans maskeleme metodu, yinelemeli yapı.

ACKNOWLEDGEMENTS

Firstly, I would like to express my deepest gratitude thanks to my supervisor, Prof. Dr. Arif NACAROĞLU for his advice and guidance in the preparation of this thesis.

I would like to thank to Prof. Dr. Rauf Mirzababayev who has grand effect and guidance on my university education.

Also, I would like to express my sincere gratitude to my friend Osman TURAN for his grand support.

Lastly, I wish to special thank my parents and also my sister.

CONTENTS

LIST OF FIGURES

LIST OF SYMBOLS

The following nomenclature defines the principal symbol used in the thesis.

Symbols	**Description**
ADC	Analog to Digital Converter
FIR	Finite Impulse Response
IIR	Infinite Impulse Response
FRM	Frequency-response Masking

CHAPTER 1

OVERVIEW

1.1 Introduction

In digital signal processing applications, the digital filters have great importance both in hardware and software. Depending on the type of the problem and signal shape, Finite Impulse Response (FIR) or Infinite Impulse Response (IIR) has superiorities and the filtering speed and time becomes significant.

IIR filters have mostly less length versus FIR filters but because of their feedback futures and their stability problems FIR filters are preferred in the application.

In electronics, computer science and mathematics, a digital filter is a system that performs mathematical operations on a sampled, discrete-time signal to reduce or enhance certain aspects of that signal. This is in contrast to the other major type of electronic filter, the analog filter, which is an electronic circuit operating on continuous-time analog signals. An analog signal may be processed by a digital filter by first being digitized and represented as a sequence of numbers, then manipulated mathematically, and then reconstructed as a new analog signal. In an analog filter, the input signal is "directly" manipulated by the circuit.

A digital filter system usually consists of an analog-to-digital converter (to sample the input signal), a microprocessor (often a specialized digital signal processor), and a digital-to-analog converter. Software running on the microprocessor can implement the digital filter by performing the necessary mathematical operations on the numbers received from the ADC.

Digital filters may be more expensive than an equivalent analog filter due to their increased complexity, but they can realize many designs that are impractical or impossible in an analog filters. Since the digital filters use a sampling process and discrete-time processing, they experience latency (the difference in time between the input and the response), which is almost irrelevant in analog filters.

Digital filters are commonplace and an essential element of everyday electronics such as radios, cell phones, and stereo receivers.

1.2 Characterization of digital filters

A digital filter is characterized by its transfer function, or equivalently, its difference equation. Mathematical analysis of the transfer function can describe how it will respond to any input. As such, designing a filter consists of developing specifications appropriate to the problem (for example, a second-order low-pass filter with a specific cut-off frequency), and then producing a transfer function which meets the specifications.

The transfer function for a linear, time-invariant digital filter can be expressed in the Z-domain as;

$$H(z) = \frac{A(z)}{B(z)} = \frac{a_0 + a_1 z^{-1} + \cdots + a_M z^{-M}}{1 + b_1 z^{-1} + \cdots + b_N z^{-N}} \qquad 1.1$$

where the order of the filter is the greater then N or M. This form is for a recursive filter, which typically leads to infinite impulse response behavior. But if the denominator is unity, then this is the form for a finite impulse response filter.

A variety of mathematical techniques may be employed to analyze the behavior of a given digital filter. Many of these analysis techniques may also be employed in designs, and often form the basis of a filter specification.

Typically, one analyzes filters by calculating how the filter will respond to a simple input. One can then extend this information to visualize the filter's response to more complex signals.

1.2.1 Filter design and realization

The design of digital filters is a deceptively complex topic. Although filters are easily understood and calculated, the practical challenges of their design and implementation are significant and are the subject of much advanced research. The design procedure is mainly classified as direct and indirect design methods. In indirect method the specifications required for the filter is first mathematically modeled as analog information and then the analog filter transfer functions are transformed in to the digital domain using the well-known transformation techniques. But in the direct realization, the information is processed directly using the curve fitting methods to find the coefficients of the proposed digital filter and checking the correctness of the gain and phase curves in acceptable error limits.

After a filter is designed, it must be realized by developing a signal flow diagram that describes the filter in terms of operations on sample sequences.

All realizations may be seen as "factorizations" of the same transfer function, but different realizations will have different numerical properties. Specifically, some realizations are more efficient in terms of the number of operations or storage elements required for their implementation, and others provide advantages such as improved numerical stability and reduced round-off error. Some structures are more optimal for fixed-point arithmetic and others may be more optimal for floating-point arithmetic.

1.3 Finite Impulse Response Filter

It is a well known fact that the filter length of a very sharp FIR filter is inversely proportional to the transition-width [1]. As a consequence, very sharp FIR filters are also very long filters. This causes serious implementation problems. First, the very large number of multipliers renders real time high-speed implementation impractical. Second, since the round off noise power is proportional to the number of rounding processes, the round off noise power generated by a filter with a large number of nontrivial coefficients will be unacceptable unless the word length of the registers and arithmetic units are sufficiently high. Third, it has been shown in [2] that the statistical frequency response deviation under finite coefficient word length condition is proportional to the square root of the number of coefficients. Hence, filters with a large number of nontrivial coefficients have high coefficient sensitivity. As a consequence, very sharp filters will have high hardware complexity, high coefficient sensitivity, and high round off noise unless the coefficient vector is sparse. The search of design and synthesis techniques which will produce efficient filters with sparse coefficient vector for the implementation of narrow transition-width FIR filters has attracted much research efforts in the past few years [1]-[l0]. One of the most successful techniques for the synthesis of very narrow transition-width filters is the frequency-response masking technique introduced in [2] and subsequently further developed in [3]-[5]. It was also demonstrated in [6] and [7] that the frequency-response masking technique is well suited to the implementation of linear phase filter bank for applications in such areas as graphic equalizer and tone control in digital audio systems.

In the hardware implementation of the IIR or FIR filters time delay elements, adders and multipliers are used in the implementation. Because of length limitation of the components the noise affects the overall transfer function in different ways. Either in software or in hardware implementation noise created by multipliers and adders can be reduced either increasing the length of registers or reducing the number of these elements.

Frequency-response masking (FRM) approach is a very efficient technique for drastically reducing the number of multipliers and adders in implementing sharp linear-phase finite-impulse-response (FIR) digital filters. It has been shown that further savings in arithmetic operations can be achieved by using the generalized FRM approach, where the masking filters have a new structure. In both the original and the generalized synthesis techniques, the sub-filters in the overall implementation are designed separately. The arithmetic complexity in the original one-stage FRM filter designs has been considerably reduced by using a two-step technique for simultaneously optimizing all the sub-filters. Such an efficient algorithm was also proposed for synthesizing multistage FRM filters.

It has been reported in several recent publications that the frequency response masking technique is eminently suitable for synthesizing filters with very narrow transition width. The major advantages of the frequency response masking approach are that the resulting filter has a very sparse coefficient vector and that the resulting filter length is only slightly longer than that of the theoretical (Remez) minimum. The system of filters produced by the frequency response masking technique consists of a sparse coefficient filter with periodic frequency response and one or more pairs of masking filters. Each pair of the masking filters consist of two filters whose frequency responses are similar except at frequencies near the band-edges.

The frequency response masking technique was first introduced in [l]. A very interesting extension of the frequency response masking technique for the design of band-pass filters was reported in [3] and [8]. It was reported in [4] that further savings in the narrow-band low-pass case can be achieved by combining the arbitrary bandwidth frequency response masking technique and the interpolated impulse response technique [9]. It was also demonstrated in [6] and [10] that the frequency response masking technique is well suited to the implementation of linear phase filter bank for applications in such areas as graphic equalizer and tone control in digital audio systems. Although the usefulness of the frequency response masking technique: has been reiterated many times [l]-[8], a few questions of interest to most filter designers remain unanswered. First, under what circumstances will the frequency response masking technique be effective? Second, how many stages of masking should be used if minimum complexity is desired? Third, what should the up-sampling ratio for each stage be? Fourth, can two-dimensional filters be realized using the frequency response masking technique?

In this study, some of the questions will be answered and some of them will be left as proposals. This thesis is organized as follows. In Chapter 2, basic principle of design and construction of recursive and non-recursive filters are received and frequency masking technique is presented. The substructures of the frequency masking block are explained in details with the required gain characteristics. In Chapter 3, the recursive structure is introduced and the gain and phase characteristic of each block and the overall network is obtained by using MATLAB toolboxes. The order of each block is chosen arbitrarily. The typical low-pass structure is realized in Chapter 4 as an example. The uses of tool windows are explained and the results are discussed comparing with the conventional design procedures.

All results and the masking method are discussed in Chapter 5 and some future proposals are given.

CHAPTER 2

FREQUENCY MASKING TECHNIQUE

2.1 Introduction

It has been reported in several recent publications [l]-[8] that the frequency response masking technique is eminently suitable for synthesizing filters with very narrow transition width. The major advantages of the frequency response masking approach are that it employs sub-filters with very sparse coefficient vectors and that the resulting effective filter length is only slightly longer than that of the theoretical minimum.

In the frequency response masking technique, the impulse response of a prototype filter and that of its complement are up-sampled (by inserting zeros) by a factor of M and then cascaded to a pair of interpolators [13]. The prototype filter itself may again he synthesized using the frequency response masking technique producing a multistage frequency response masking design.

2. 2 IIR Digital Filters

In this section the IIR filter structure will be given and transfer structure and the realization procedure of the IIR filter will be reviewed.

2.2.1 IIR Transfer Function

In general, the digital IIR filters are designed by using both analog design and then the analog digital conversion technique or by using numerical curve fitting methods (system identification).

In either case, the magnitude and/or phase filters result with the ratio of z-domain polynomials in frequency domains in equation

$$H(z) = \frac{A(z)}{B(z)} = \frac{a_0 + a_1 z^{-1} + \cdots + a_M z^{-M}}{1 + b_1 z^{-1} + \cdots + b_N z^{-N}} \tag{2.1}$$

where M and N are integers and N≥M the roots of numerator and denominator polynomials are the zeros and the poles of the transfer function, respectively and H(z) may written in polar form as

$$H(z) = \frac{(z - z_1)(z - z_2) \ldots \ldots (z - z_M)}{(z - p_1)(z - p_2) \ldots \ldots (z - p_N)} \tag{2.2}$$

H(z) is stable for sinusoidal excitation if the magnitudes of the poles are smaller than unity. For sinusoidal input ($z= e^{jwt}$) where t is the sampling, the complex transfer function is written as,

$$H\left(e^{jwT}\right) = \frac{(e^{jwT} - z_1)(e^{jwT} - z_2) \ldots \ldots (e^{jwT} - z_M)}{(e^{jwT} - p_1)(e^{jwT} - p_2) \ldots \ldots (e^{jwT} - p_N)} \tag{2.3}$$

Time domain input-output relation for H(z) is obtained by using inverse z-transformation as

$$y(nT) = \textstyle\sum_{k=0}^{M} a_k x(nT - kT) - \sum_{k=1}^{N} b_k y(nT - kT) \tag{2.4}$$

which shows that any sample of y(nT) at any instant of time depends on both input samples and some pre-output samples (feedback).

2.2.2 Realization

In signal processing applications, generally high speed computers are used as digital filters. Therefore the hardware structures of the computers are controlled by suitable software (like MATLAB tools) to design and the process the filters. But in some applications the digital system must be realized by using well known logic processors like adders, multipliers and unit delay elements or the integrated structures including these components. N-th order transfer functions may be realized using at least N delay units and the realization may be achieved either in direct canonic form as cascaded form (sometimes parallel connections). In the later case Nth order transfer function are decomposed as cascaded (parallel) 2nd order realizable transfer functions. Because of the availability of 2nd order IC chips, cascaded (or parallel) realizations are mostly preferable. The direct canonical and 2nd order cascaded structures are given in Figure 2.1 and Figure 2.2, respectively. In these figures the boxes D represents the unit delay elements (parallel shift registers) triggered by periodic pulses.

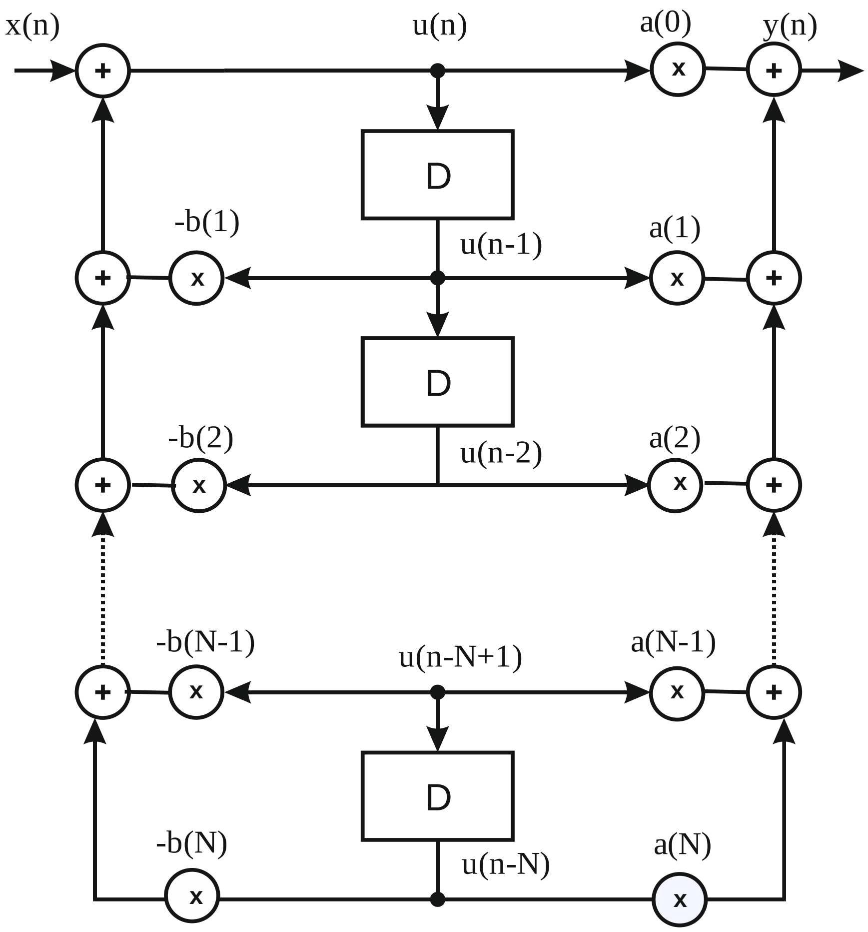

Figure 2.1 Direct canonic realizations of IIR Filters.

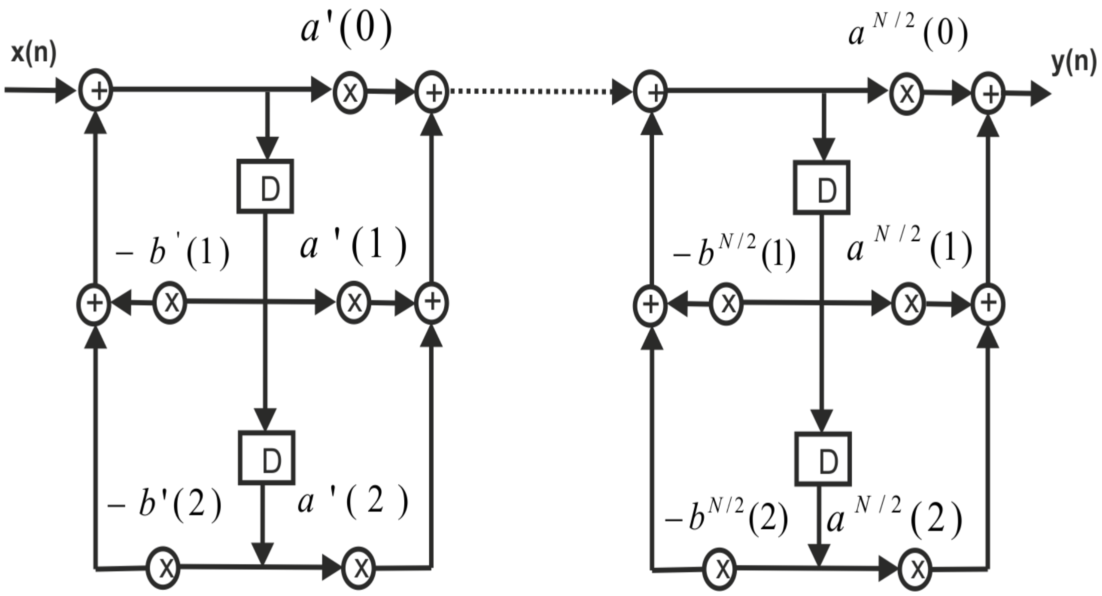

Figure 2.2 Cascaded Nth order direct canonic structures.

2.3 FIR Digital Filters

Recursive filters with constant group delay or with amplitude responses that are not piecewise constant are difficult to design in practice. Constant group delay can achieve by using the Bessel approximation in conjunction with the invariant-impulse-response method, but this approach is limited to low selectivity low-pass and band-pass filters. Non-recursive filters can be easily designed to have constant group delay, and a variety of amplitude responses can be readily achieved. Filters of this class are, in addition, naturally suited for certain applications, e.g. for the design of differentiator and interpolators. Furthermore, since the impulse responses of non-recursive filters is of finite duration, they can be implemented in terms of fast-Fourier transform.

An alternative approach is based on the use of the discrete Fourier transform. A third possibility is to use a powerful multivariable optimization algorithm known as Remez exchange algorithm. In all cases, the length of the digital filters must be limited by using the window functions. The well known window functions such as Rectangular Window, Hamming Window and von Hann Window all causes the ripples in the gain curves in pass-band and stop-band regions. If the required characteristics are sharp the length of the window functions should be kept as high as possible which causes the use of lots of circuit elements. In fact there is limit on the sharpness, i.e. the order does not affect the sharpness after some range of order.

2.3.1 FIR Transfer Function

A non-recursive (FIR) filter has a simpler transfer function which does not contain any denominator terms. The coefficient b_0 is regarded as being equal to unity, and all the other b coefficients are zero. The transfer function of a second-order FIR filter can therefore be expressed in the general form.

$$H(z) = \frac{A(z)}{B(z)} = a_0 + a_1 z^{-1} + \cdots + a_N z^{-N} \qquad (2.5)$$

This is obtained from the symmetrical form of the filter expression, and it allows us to describe a filter by means of a convenient, compact expression. The transfer function of a filter can be used to determine many of the characteristics of the filter, such as its frequency response.

2.3.2 Realization

Realizations of digital FIR filters can be obtained from IIR filter realizations by specializing these realizations to the case B(z)=1 However since FIR filters are usually either symmetric or anti-symmetric, we can save about half the number of multiplications by exploiting symmetry. The typical block diagram of the FIR structure is given in the Figure 2.3

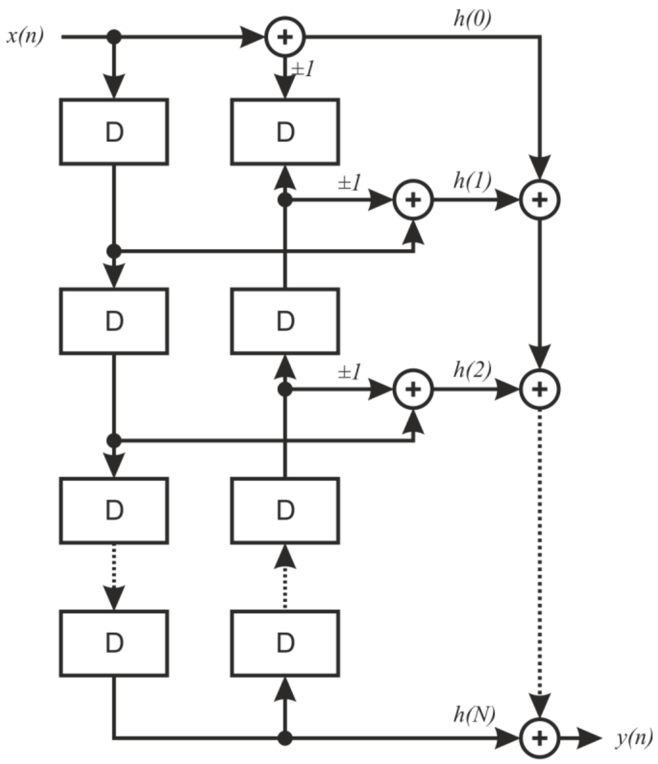

Figure 2.3 Direct realization of a symmetric or anti-symmetric FIR filter with odd N

2.4. Frequency Masking

Since the sharpness of the filter is limited by the order of the window function, frequency masking technique is proposed which contains the idea of using lower order filters in cascaded or parallel connections to achieve the sharper gain.

The block diagram of the technique is given in the Figure 2.4 In this block three FIR filters are designed separately as H'_a, H'_{Ma} , H'_{Mc} and the delay filter is used to create the masking case.

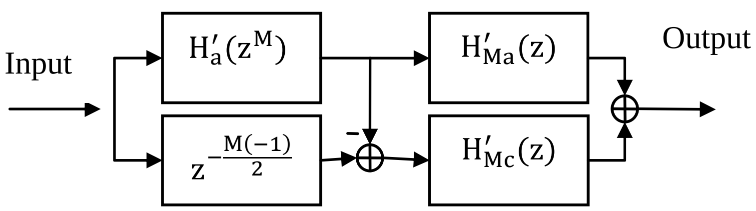

Figure 2.4 Main block diagram for frequency masking

The overall transfer function of the structure shown in Figure 2.4 is written as

$$H(z) = H'_{Ma}(z)H'_a(z^M) + H'_{Mc}(z)H'_c(z^M) \tag{2.6}$$

In Figure 2.4 each box is independent digital FIR structures and each has its own gain and phase characteristics. The transfer function of H_a is;

$$H'_a(z^M) = z^{-M_a} \sum_{n=0}^{N_{Ma}} h_a(n)\, z^{-n} \tag{2.7}$$

where the $h_a(n)$ is impulse response coefficient. It is a linear phase low-pass filter with length $N_{Ma.}$ The frequency response magnitude of $H'_a(e^{jw})$ is sketched for ideal gain as shown in the Figure 2.5 and this box is designed using suitable Fourier series approach or curve fitting techniques and suitable window. The sharpness of the overall characteristics is directly affected by the gain of $H_{a.}$ In this graph actual gain is very close to the ideal filter gain. Bandwidth is 0.25(normalized).

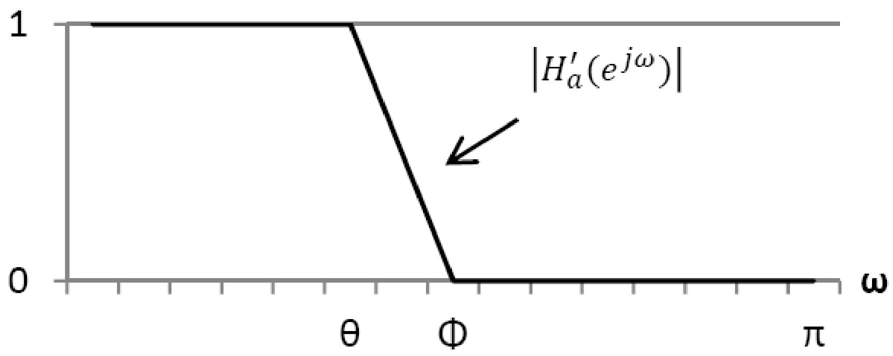

Figure 2.5 Ideal gain of $H'_a(e^{jw})$

Replacing each delay of this filter by M delays, a filter with z- domain transfer function $H'_a(z^M)$ and frequency response $H'_a\left(e^{jMw}\right)$ as shown in Figure 2.6 is obtained.

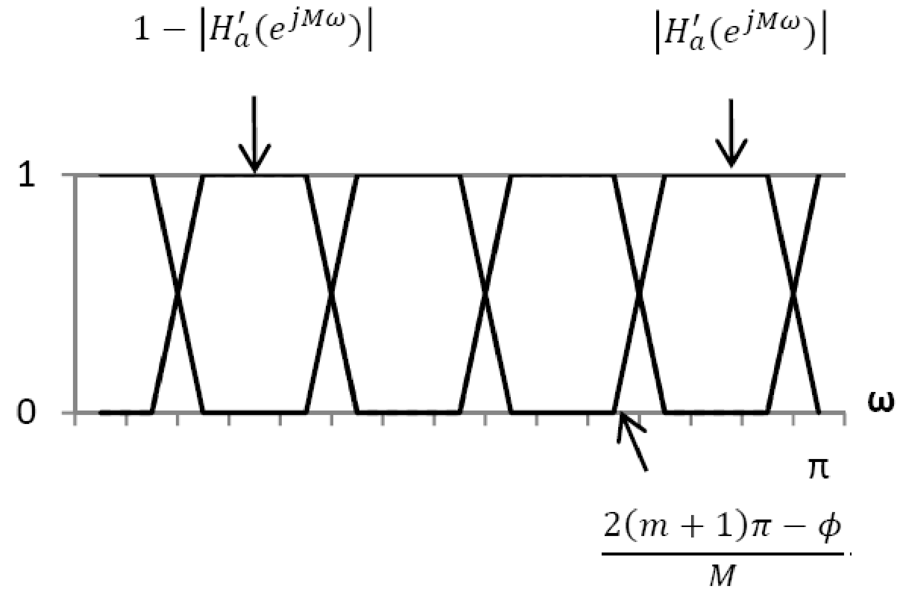

Figure 2.6 Ideal gain of $H'_a(e^{jMw})$ and 1-$H'_a(e^{jMw})$

The linear phase filter $H'_c(z^M)$ is said to be a complementary of $|H'_a(e^{jw}) + H'_c(e^{jw})| = 1$, where $H'_c(z^M)$ is the frequency response of H'_c. $H'_c(z^M)$ can be realized easily by subtracting the output of the realization of $H'_a(z^M)$ from M(N-1)/2 sample delayed version of the input. If the z- domain transfer function of H'_a is $H'_a(z^M)$, then the z- domain transfer function of $H'_c(z^M)$ is given by Eq. 2.8, as

$$H'_c(z^M) = z^{-N_a/2} - H'_a(z^M) \quad (2.8)$$

Two masking filters H'_{Ma} and H'_{Mc} with frequency responses $H'_{Ma}(e^{jw})$ and $H'_{Mc}(e^{jw})$ may be used to mask $H'_a(e^{jMw})$ and $H'_c(e^{jMw})$ respectively. If the impulse responses of H'_{Ma} (z) and H'_{Mc}(z) are both symmetrical then H'_{Ma}(z) and H'_{Mc}(z) must have the same length; if they are not in the same length, zero valued coefficients must be appended to shorter filter. The gain relations of these boxes

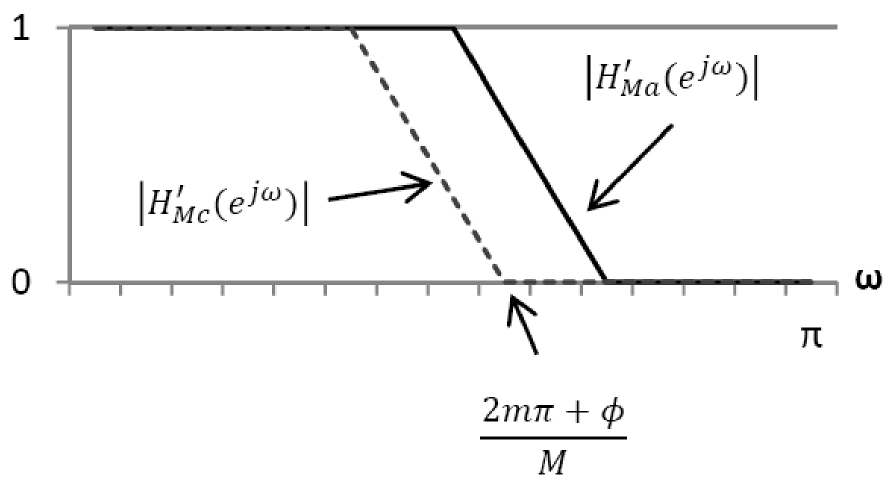

Figure 2.7 Ideal gain of $H'_{Ma}(e^{jw})$ and $H'_{Mc}(e^{jw})$

The transfer function $H'_{Ma}$is

$$H'_{Ma}(z^M) = z^{-M_{Ma}} \sum_{n=0}^{N_{Ma}} h_{Ma}(n)\, z^{-n} \tag{2.9}$$

and $H'_{Mc}$is

$$H'_{Mc}(z^M) = z^{-M_{Mc}} \sum_{n=0}^{N_{Mc}} h_{Mc}(n)\, z^{-n} \tag{2.10}$$

The gain curves of cascaded structure $H'_{Ma}(z)H'_a(z^M)$ is sketched in Figure 2.8

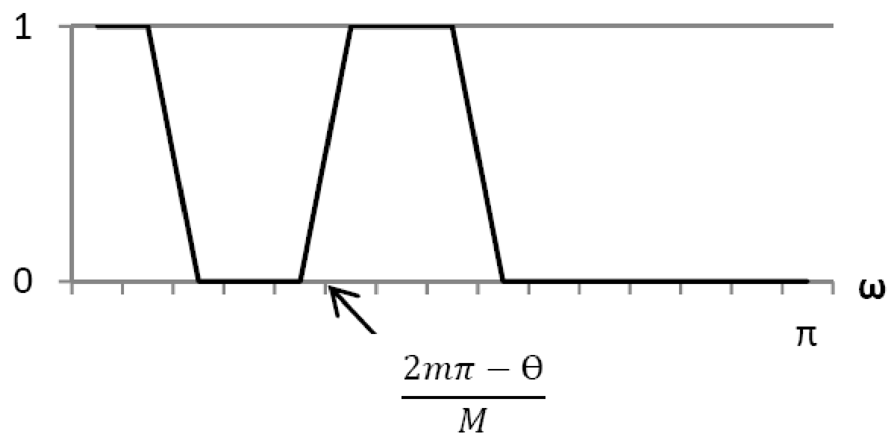

Figure 2.8 Ideal gain of $H'_{Ma}(z)H'_a(z^M)$

Figure 2.9 shows the frequency response magnitudes of cascaded $H'_{Mc}(z)H'_c(z^M)$

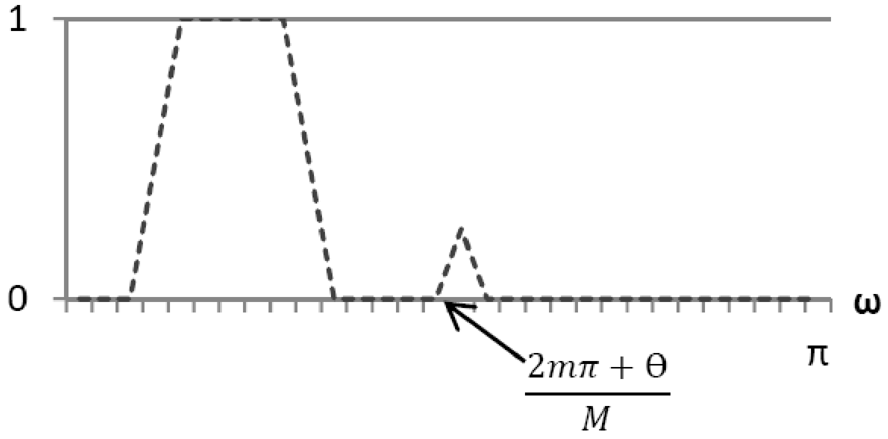

Figure 2.9 Ideal gain of $H'_{Mc}(z)H'_{c}(z^{M})$

The frequency response magnitudes of $H(z)$ will be as shown in Figure 2.10. The transition width of $H'(e^{jw})$ is (φ-θ)/M and is narrower than that of $H'_{a}(e^{jw})$ by a factor of M.

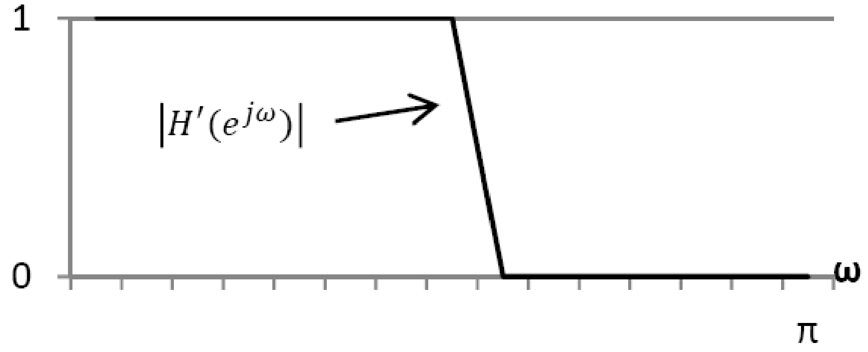

Figure 2.10 Ideal gain of $H'(e^{jw})$

The gain curves of each boxes are given above. The overall block will be substituted instead of 'BOX' as it will be shown in Figure 3.1 in Chapter 3.

The structure given in Figure 2.4 will be substituted instead of H'_{Mc}z) as it will be discussed in more detail in following chapter. The gain characteristic of the H'_{a} and H'_{Ma} remain same as it is shown in the Figure 2.5 and 2.7 but since the gain of the H_{Mc} becomes as the overall gain of the Figure 2.4 then the H_{Mc} change as in Figure 2.11 and 2.12.

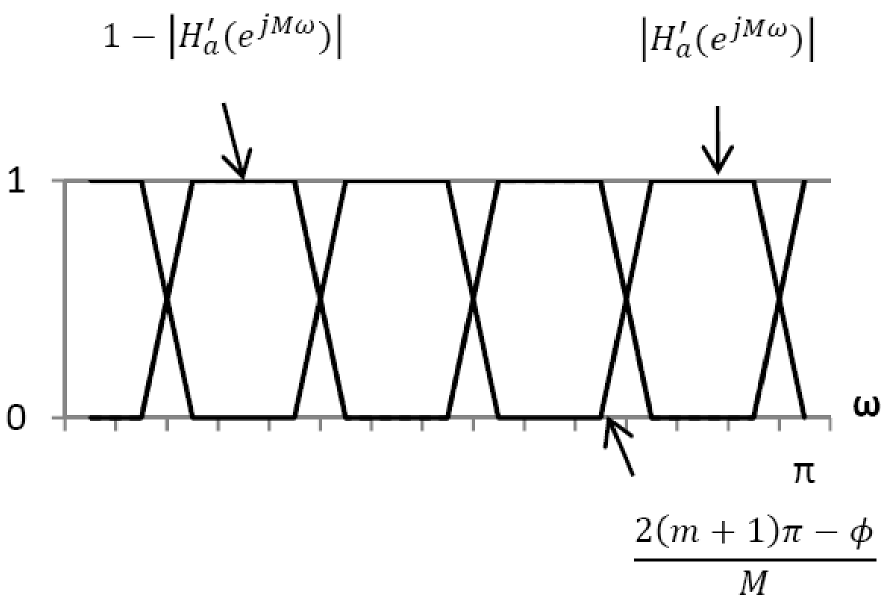

Figure 2.11 Ideal gain of $H_a\left(e^{jMw}\right)$ and 1-$H_a\left(e^{jMw}\right)$

In the Figure 2.11, as it will be seen easily, the gain curves are M times narrowed and repeated.

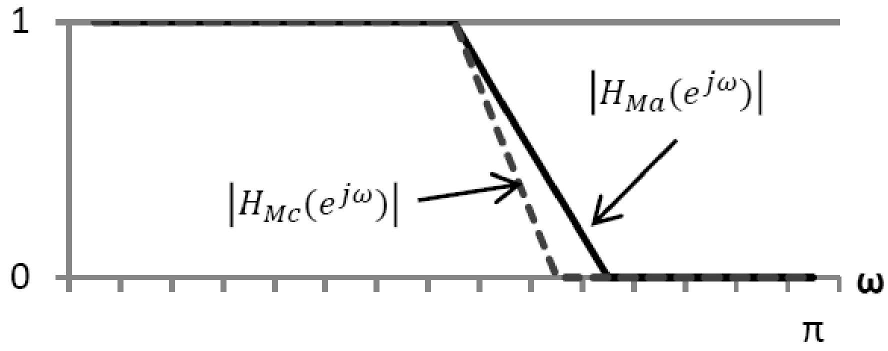

Figure 2.12 Ideal gain of $H_{Ma}(e^{jw})$ and $H_{Mc}(e^{jw})$

In Figure 2.12 it is obviously seem that H_{Mc} is sharper if we compare with the Figure 2.7 it means that it will help us to get sharper filter by using some cascaded filters.

Since the replacement of BOX with the overall structure does not effect the unchanged boxes, the gain characteristics of these boxes are not changing. So Figure 2.13 is identical Figure 2.8.

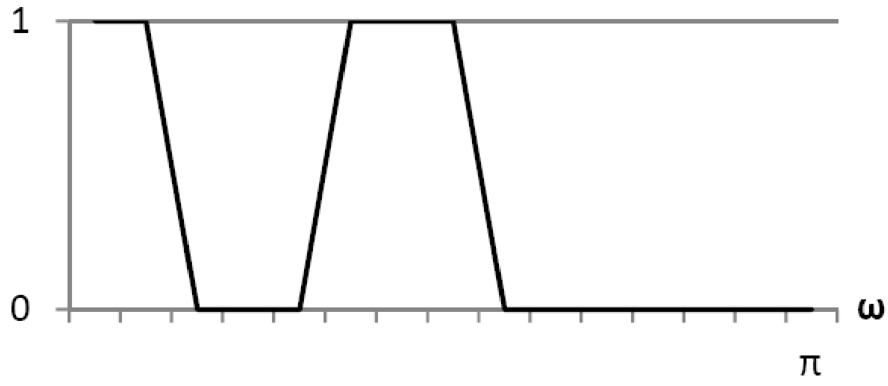

Figure 2.13 Ideal gain of $H_{Ma}(z)H_a(z^M)$

As it will be noticed by comparing the Figure 2.9 and Figure 2.14, the ripples in the gain curves are effected. In the later one the ripple is smaller. This is because of the recurrence substitution of the boxes.

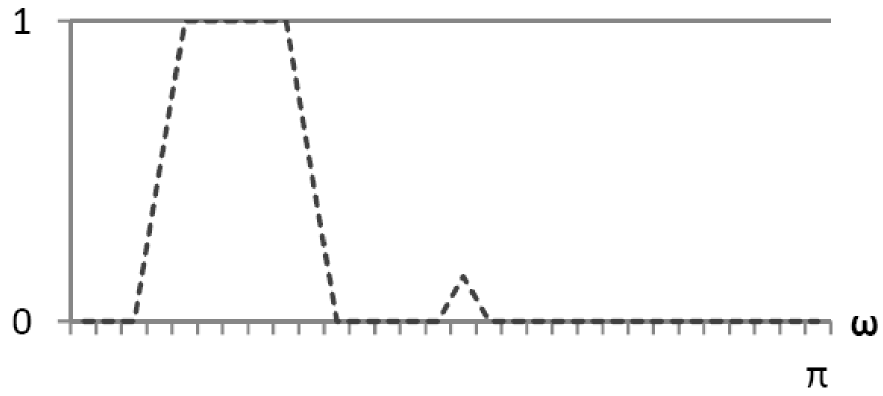

Figure 2.14 deal gain of $H_{Mc}(z)H_c(z^M)$

If box is inserted into 'BOX', Figure 2.15 will be the result. The figure shows that our way works well. Our aim is getting sharper transition band by using cascaded lower order filters. If we compare Figure 2.10 and 2.15, the difference can be easily seen. Figure 2.15 has sharper transition band by using same order filters.

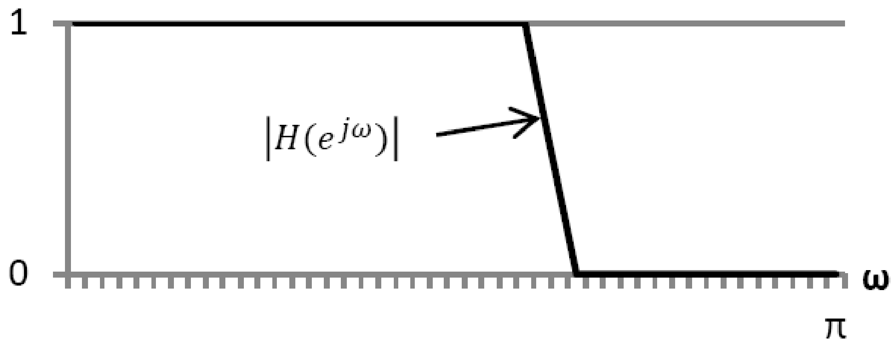

Figure 2.15 Ideal gain of $H(e^{jw})$

Since the gain structures of each block used in masking structure given in an ideal form in this chapter, of length and numerical method limitations, the more realistic gain characteristic will be slightly different than the one obtained above. In the following chapter these blocks will be designed and effect of these limitations will be obtained.

CHAPTER 3

RECURSIVE FREQUENCY MASKING

3.1 Introduction

The sharpness of the overall gain directly affects the pass-band and the transition region of the filter. Since the purpose of this study is to increase the sharpness of the filter using less degree sub-filters, the masking technique may be improved to create sharper characteristics for also the sub-filters. In this section, the procedure for designing of $\mathbf{H_{Mc}}$ by using similar block given in Figure 2.4 will be explained and structure will be analyzed**.**

3.2 Recursive Structure

In the previous chapter ideal gain characteristic of each block is given. The main block diagram which will be used for masking procedure is already given in Figure 2.4. the substitution of this block diagram into 'BOX' shown in Figure 3.1, results as in Figure 3.2 in this section first, each block will be designed by using MATLAB tool's and for each sub-blocks the gain and the phase of the transfer functions will be sketched**.**

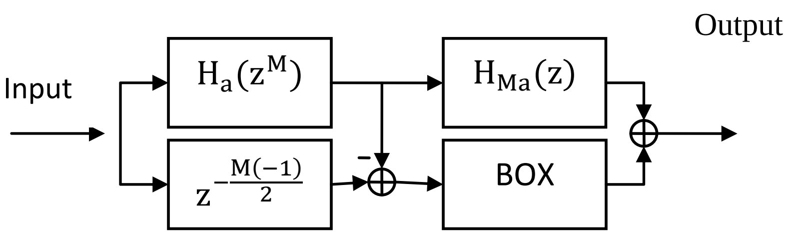

Figure 3.1 Block diagram for frequency masking

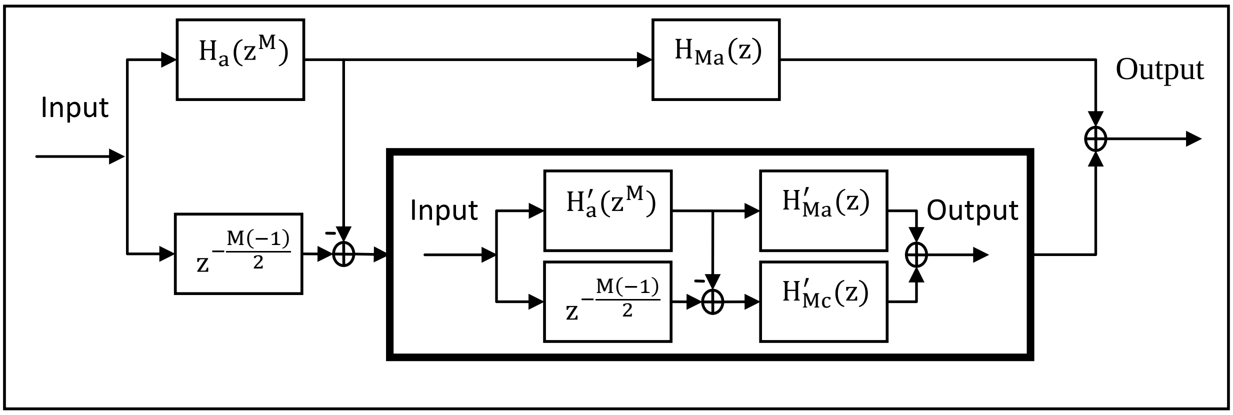

Figure 3.2 Total block diagram for frequency masking

In Figure 3.2, the transfer function of H_a and H'_a, H_{Ma} and H'_{Ma} are same and therefore the gains and phases of these blocks are identical.

As it is proposed in Chapter 2, $H_a(z)$ is low-pass filter with finite linear transition region. This box is designed for N= 200 and the gain and the phase of the box H_a is sketched as in Figure 3.3 and 3.4.

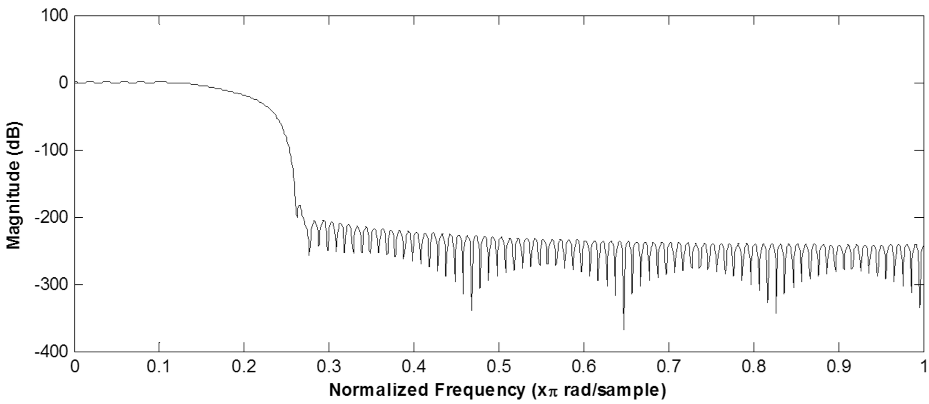

Figure 3.3 Gain curve of $H_a\left(e^{jw}\right)$

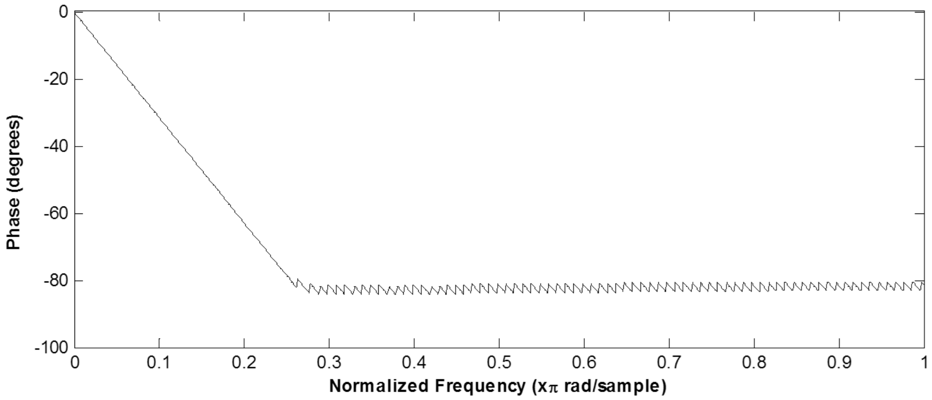

Figure 3.4 The phase of $H_a(e^{jw})$

As it is obvious from the gain curve of $H_a(z)$, in the pass-band region, the gain is almost smooth and then sharply decreases around the cut-off frequency of the pass-band region. The transition region is quite narrow and in the stop-band region, the gain is almost less than 200 dB, which is quite acceptable range for high quality low-pass filters. It is also obvious that the phase of the filter is linear function of the frequency in pass-band region which results with the constant delay. The ripples in phase in the stop-band region is negligibly small around -90 degree.

The gain of Ha given in the previous figures are used as modulated since the parameters of the transfer function z is replaced by z^m , therefore the gain for $H_a(z^m)$ is the multiples of the gain H_a, and H_{Ma} is used to mask the narrow part of the spectrum to obtain the narrower transition region. The transfer function H_{Ma} is designed for order 260 and the gain and phase of this masking block is sketched in the Figures 3.5 and 3.6.

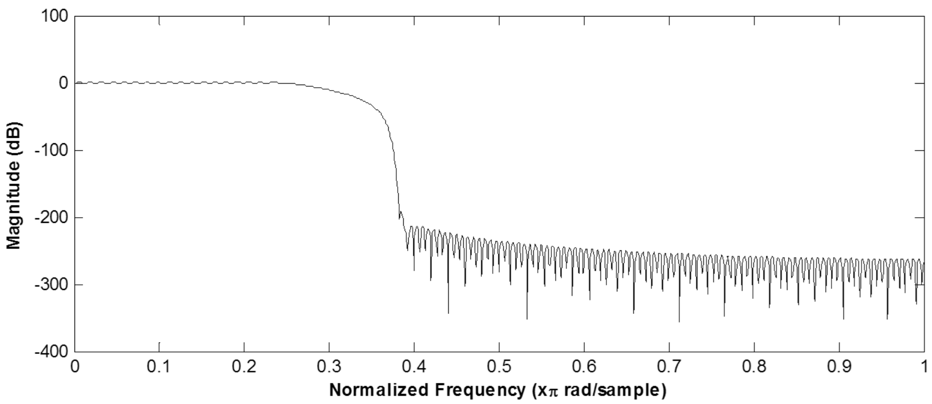

Figure 3.5 H_{Ma} gain curve

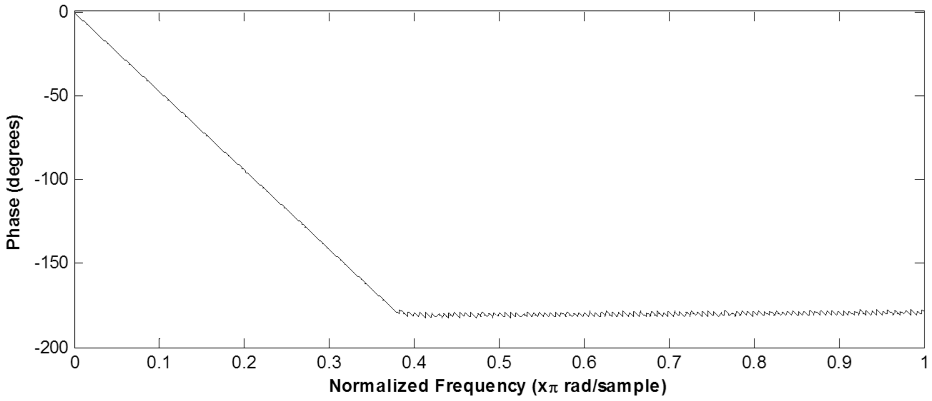

Figure 3.6 H_{Ma} phase graph

As it is seen from Figure 3.5, the width of pass-band region is much wider than the pass-band region for Ha given in Figure 3.3. The attenuation is big enough to stop the higher frequencies and the ripples are all below 200 dB. Since H_{Ma} is also designed as symmetrical FIR filter, its phase is perfectly linear in the pass-band region, as it is expected.

In the Figure 3.2, the sub-block containing four blocks is the overall structure given in Figure 3.1, therefore the 'BOX' in 3.1 is replaced for H'_{Mc} is used for designing BOX and the gain and phase of H'_{Mc} is calculated by using MATLAB tools and sketched in Figure 3.7 and 3.8. H_{Mc} is also chosen as low-pass filter with wider pass-band width and its order is taken as 260.The phase of H'_{Mc} is linear in both pass-band and stop-band region.

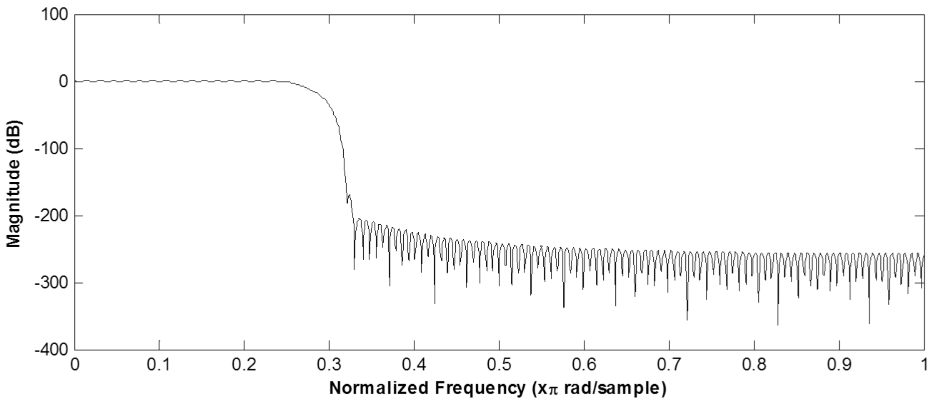

Figure 3.7 H'_{Mc} gain

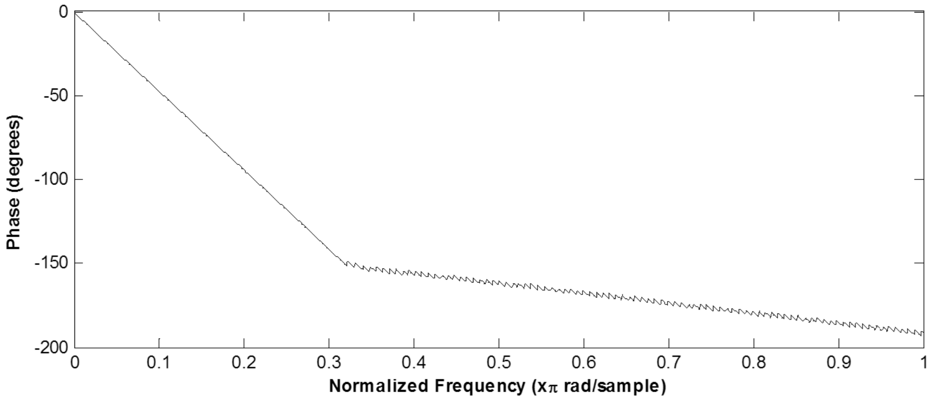

Figure 3.8 H'_{Mc} phase

3.3 Overall Transfer Function

The overall transfer functions $H(z)$ for the structure given in Figure 3.2.

$$H(z) = H_{Ma}(z)H_a(z^M) + H'_{Ma}(z)H'_a(z^M)H_c(z^M) + H'_{Mc}(z)H'_c(z^M)H_c(z^M) \quad (3.1)$$

In this equation the orders of the FIR sub-structures may be determined independently. For sinusoidal input signal ($z=e^{jwT}$) the gain and the phase of this transfer function are calculated.

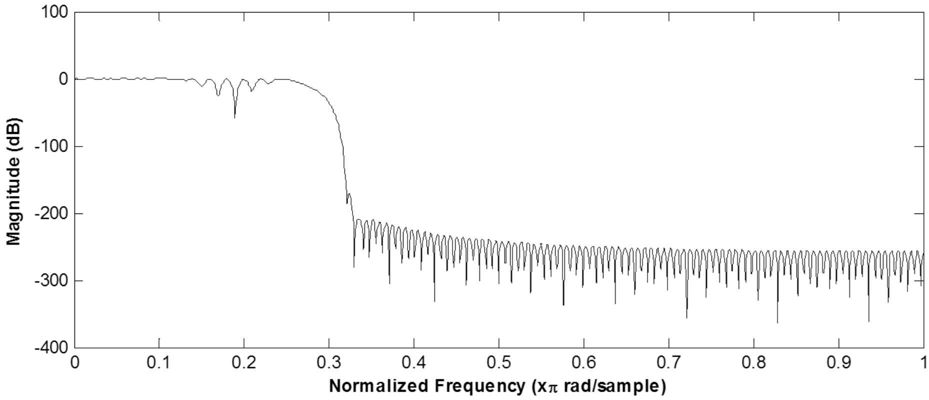

Figure 3.9 Total gain curve

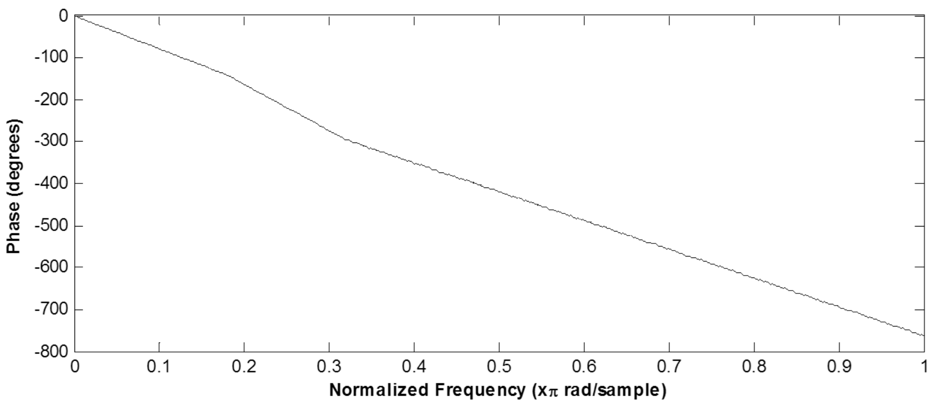

Figure 3.10 Total phase versus normalized frequency graph

Since the main purpose of masking technique is to get sharper and more ideal gain characteristics. Figure 3.9 shows that the gain of the overall structure is much sharper than the first one and it yields that the approach proposes yields considerably good result.

Figure 3.9 is gain curve, magnitude versus to normalized frequency. It is obviously seen that we have sharper transition. Our cut off is approximately 0.3 Hz*rad/sample. We can see the ripple at transition band because H_{Mc}. Cut off frequency is approximately 0.3 .Before cut off frequency we have ripples because of H_{Mc}.

3.4 K-th order recursive structure

In this study at our recursive structure there is only one box into ‘BOX’. But the number of boxes can be increased as infinity. The formula of this recursive structure is computed and written in 3.2, it is left a proposal for the future studies.

$$H(z) = \sum_{k=0}^{\infty}\left(H_{Ma}H_aH_c^{k}\right)+H_{Ma}H_c^{k+1} \qquad (3.2)$$

In this equation the upper limit of the summation is any integer which must be set to any optimum value to get sharp transition region with less order and less number of adder and multipliers in the hardware structure. Since all sub blocks in the structure are self designed FIR filters, the overall phase of the block is linear and therefore the overall delay is constant.

CHAPTER 4

DESIGN EXAMPLE

4.1 Low-pass Filter Design

A low-pass filter is a filter that passes low-frequency signals but attenuates signal with frequencies higher than the cut-off frequency. Actual amount of attenuation for each frequency varies from filter to filter.

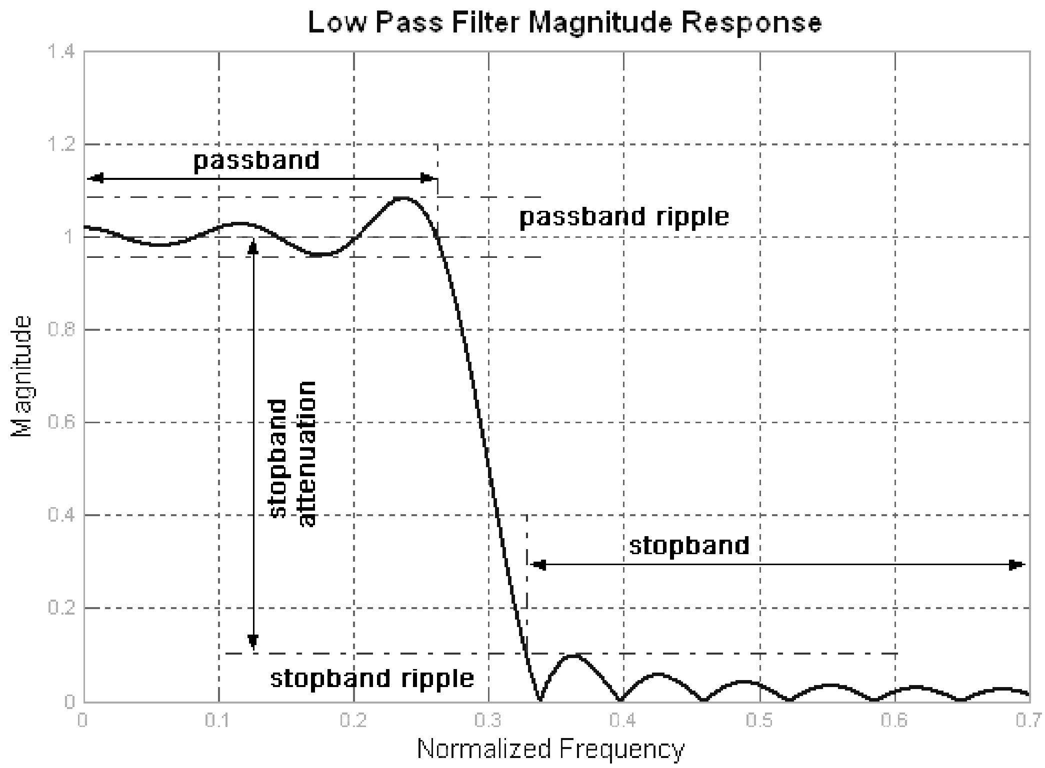

Figure 4.1 The gain characteristic of the elliptic low-pass filter

The terms which are shown in the Figure 4.1 are;

Pass-band: Range of frequencies throughout which a filter passes signals with no change.

Pass-band Ripple: The specified amount (typically in dB) of permissible amplitude variation in the pass-band.

Stop-band: Range of frequencies throughout which a filter attenuates signals by a defined amount.

Stop-band Attenuation: Specifies the minimum amount of attenuation a filter will exhibit at a designated frequency or range of frequencies, which lie outside the pass-band.

4.2 Practical Example

In this part a low-pass filter is designed by using MATLAB FDA tool like in the Figure 4.1. After that this filter is tried to get by using cascaded lower order filters by repeating the steps at Chapter 3.

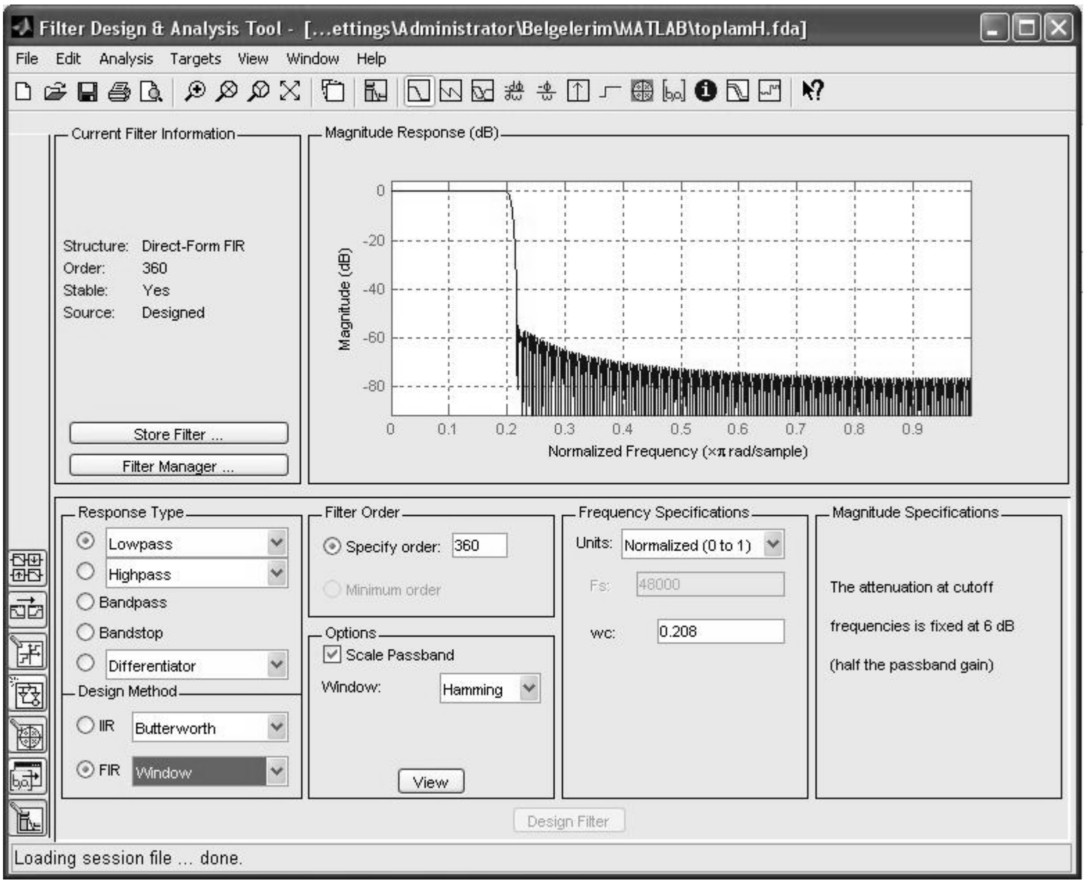

Figure 4.2 FDA tool

Since Hamming window is used at our previous works the Hamming is chosen window. 360th order and 0.208 rad. /sec. is preferred as cut-off frequency.

So, the plot in Figure 4.3 is got, it has a sharp transition band and there are ripples in stop-band region.

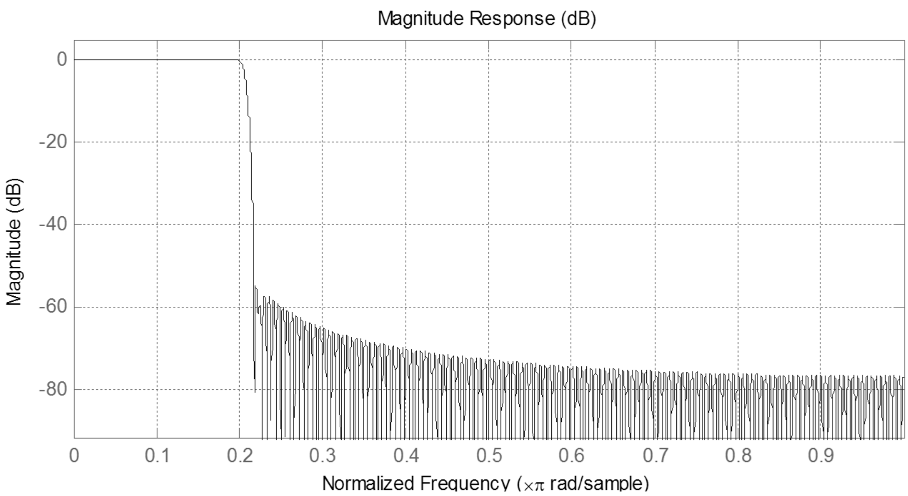

Figure 4.3 The magnitude response of filter

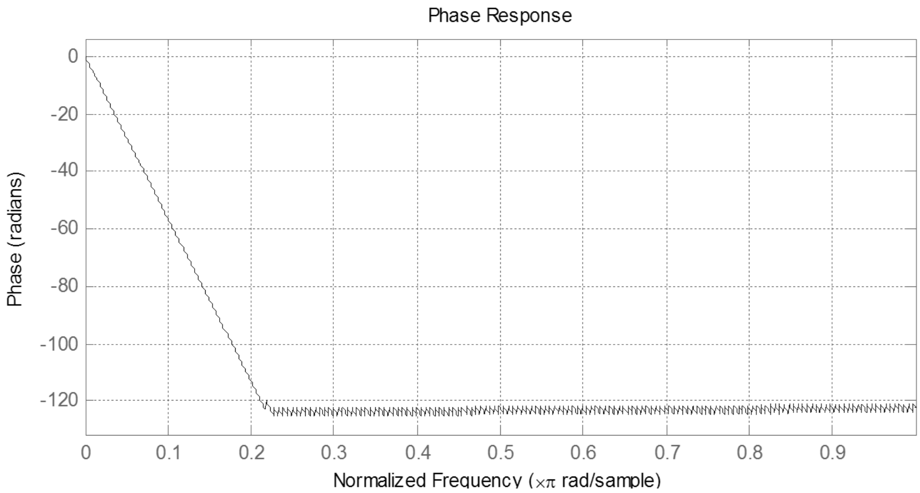

Figure 4.4 The phase response of filter

4.3 Design by Masking Method

In this part the magnitude response is tried to get which is in Figure 4.3 by using frequency masking method with lower order filters like in Chapter 3. Designing H_a can be starting point. We have to care about the total order of filters. It must not exceed 360th order so, the order is adjusted by taking care to total order which can be seen in Figure 4.5.

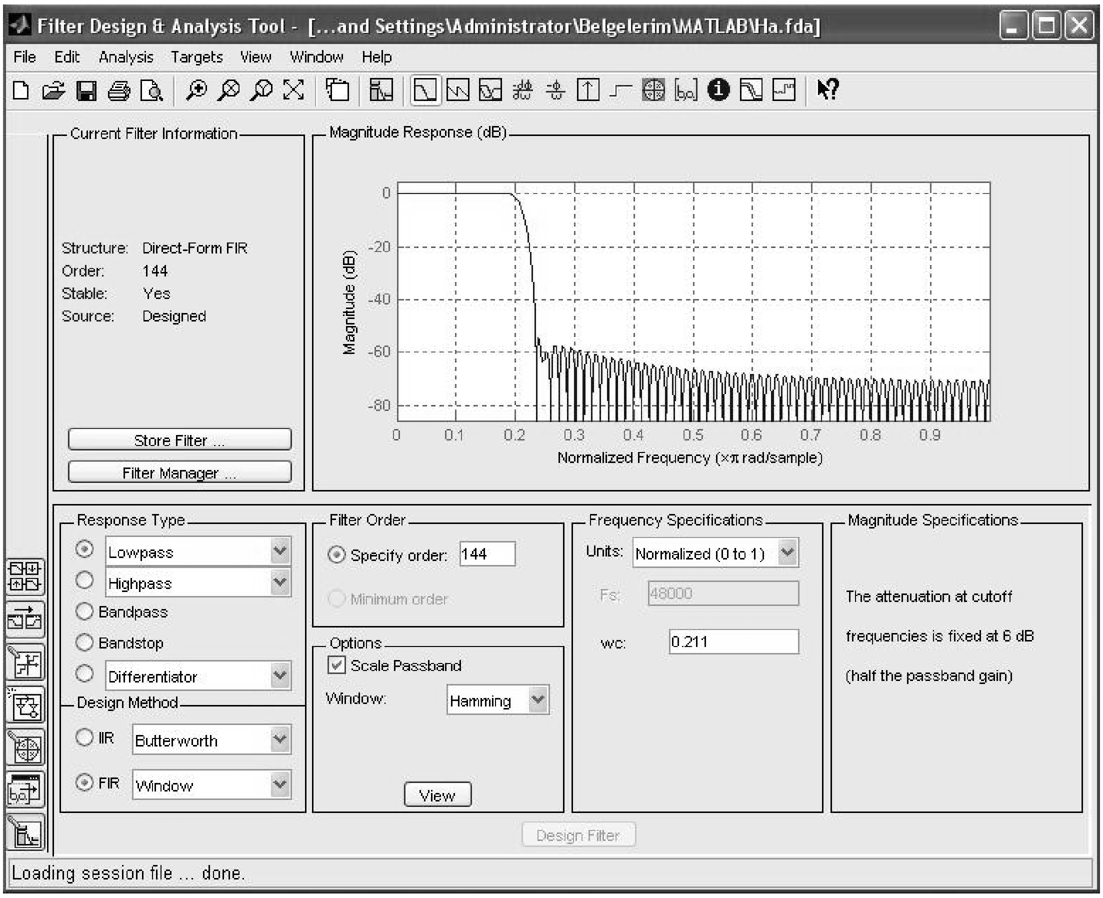

Figure 4.5 Values for H_a at FDA tool

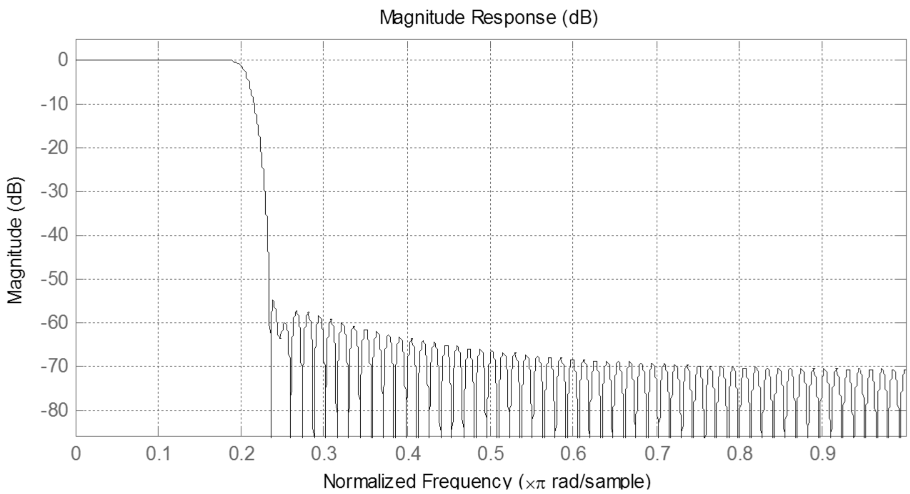

Figure 4.6 the magnitude response of filter H_a

H_a is designed with the order of 144^{th} and 0.211 rad. /sec. as cut-off frequency. From Figure 4.6 it is obviously seen that cut off frequency of H_a is higher than the cut of frequency of filter that we designed. Due to the selected lower order. Since our aim is getting sharper transition band with lower order, order is chosen like this.

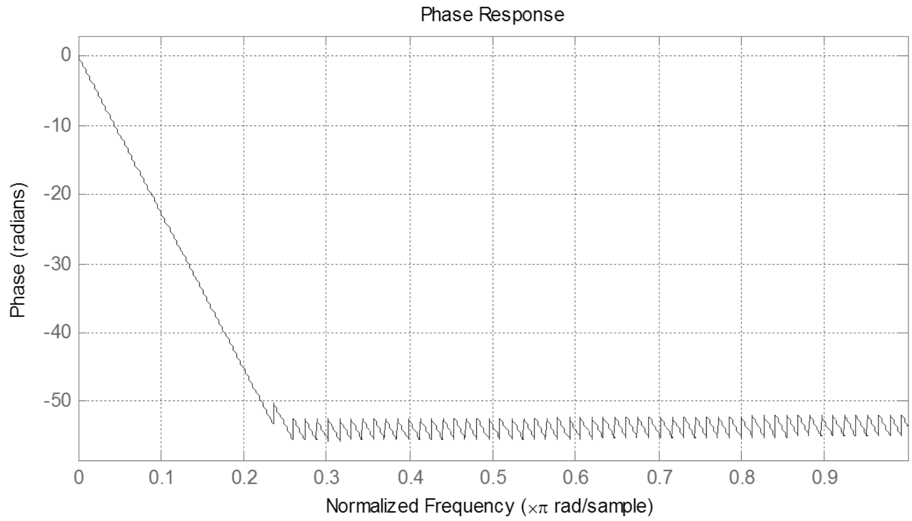

Figure 4.7 The phase response of H_a

The step of designing H_{Ma} is the most critical because it is really effective at this cascaded circuit. It has the highest order

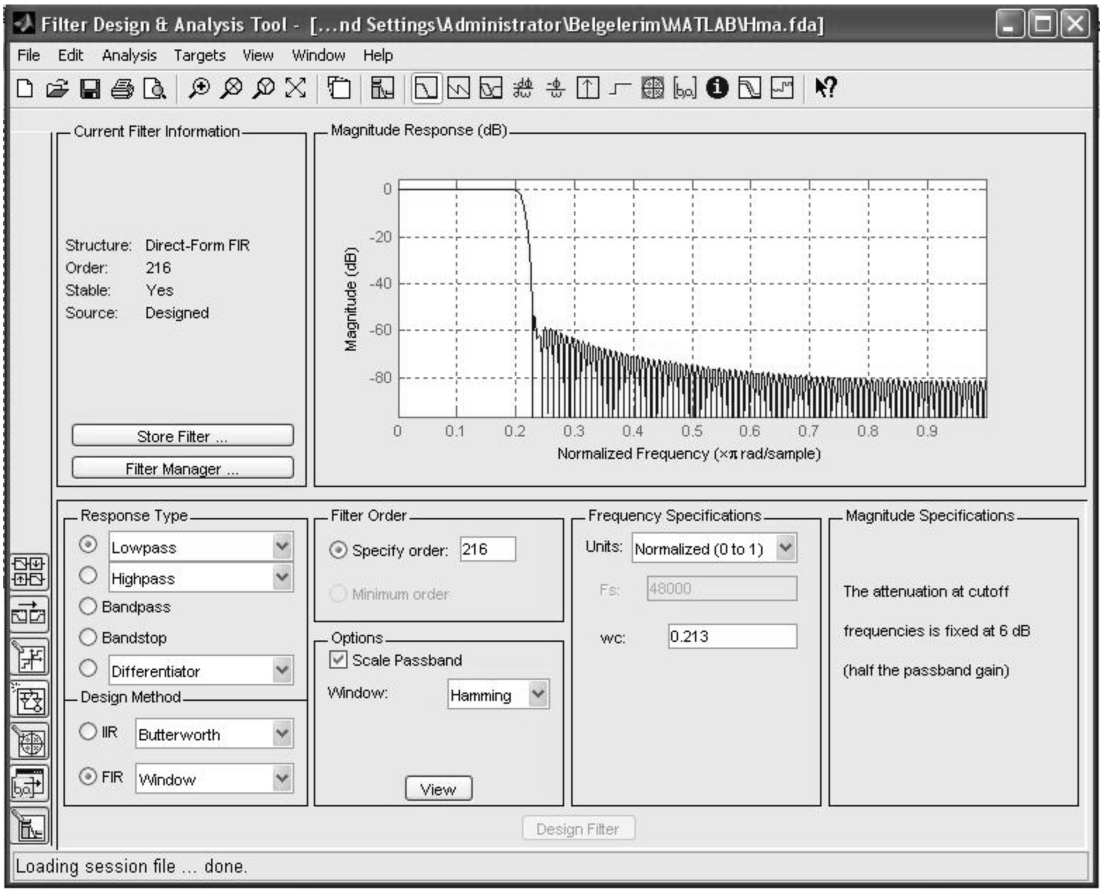

Figure 4.8 values for H_{Ma} at FDA tool

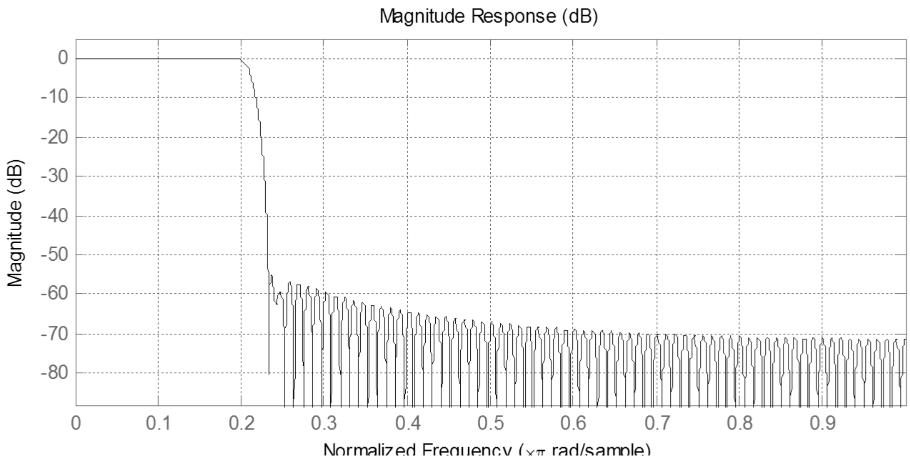

Figure 4.9 the magnitude response of filter H_{Ma}

H_{Ma} is designed with the order of 216th and 0.213 rad. /sec. as cut-off frequency. Since Hamming window is used at the pass-band region there is no ripple. But at the stop-band it has ripples.

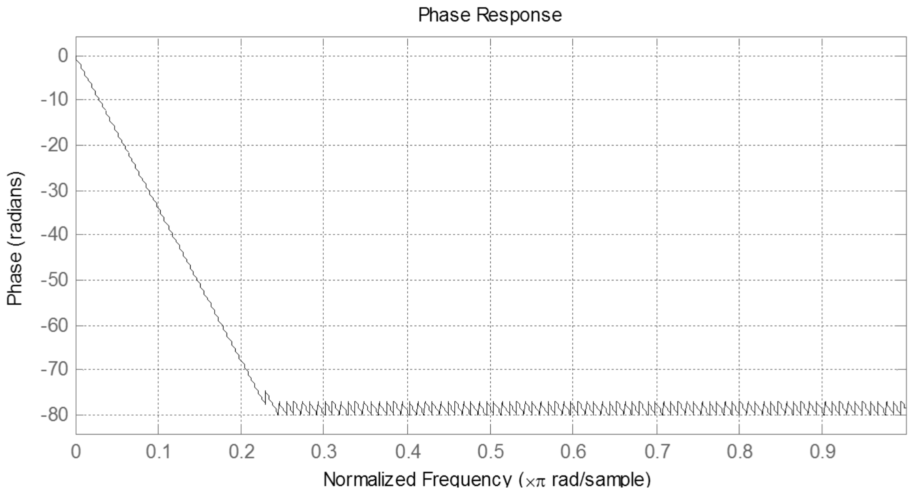

Figure 4.10 the phase response of H_{Ma}

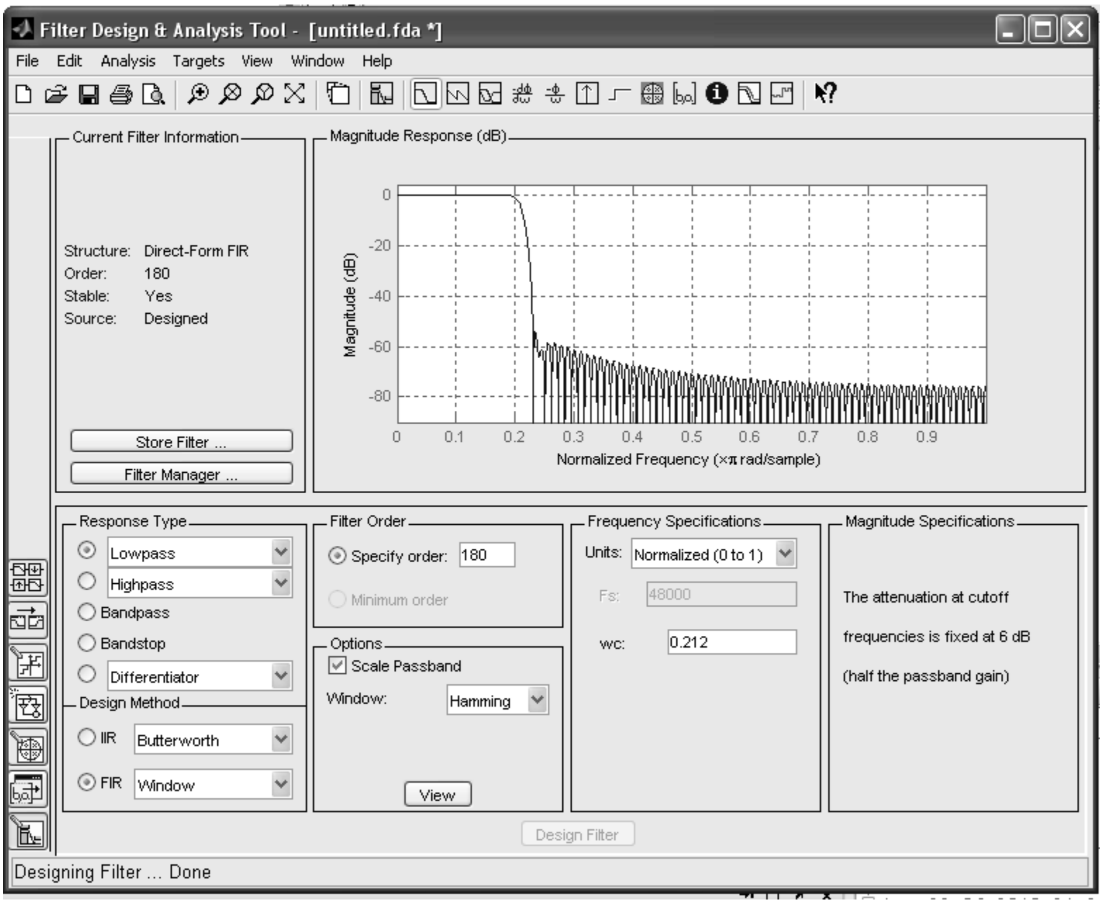

Figure 4.11 values for H_{Mc} at FDA tool

By using the same method H_{Mc} filter is got, also the gain curve and phase graph. Cut-off frequencies weren't chosen randomly. H_{Mc} has the cut-off frequency between Ha and H_{Ma} it can be seen from Figure 2.7.

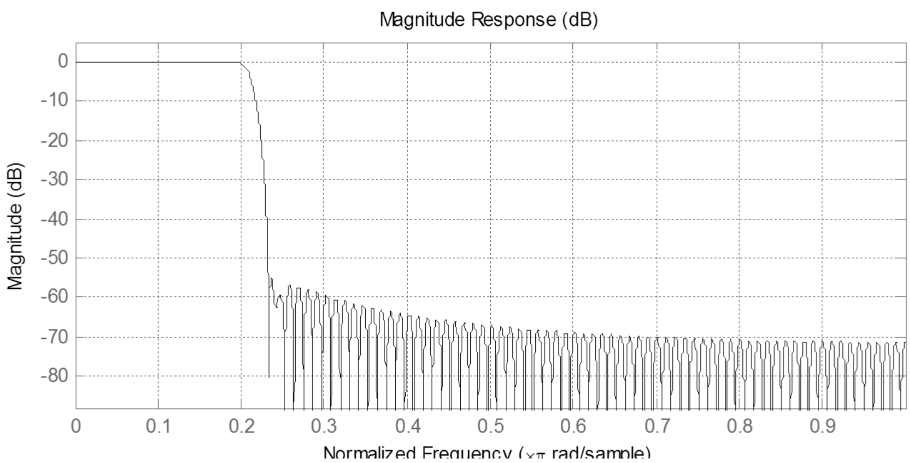

Figure 4.12 the magnitude response of filter H_{Mc}

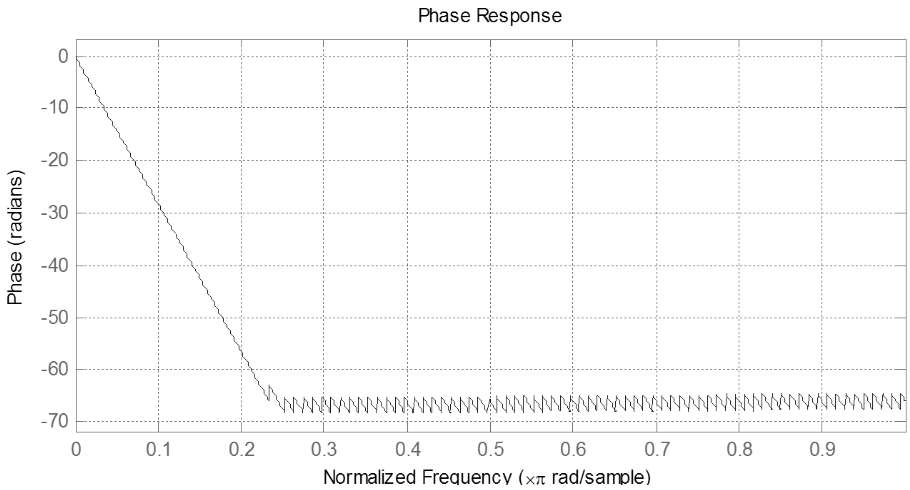

Figure 4.13 the phase response of $\mathrm{H_{Mc}}$

At the FDA tool the filters that are designed can be generated as M-file so, the MATLAB codes can be got. It helps us to get the total result of our masking technique like in Figure 3.2 at the total magnitude response of H(z) in Figure 4.14 it is obviously seen that method works well. It means that no need to use higher order filters to get sharp transition band.

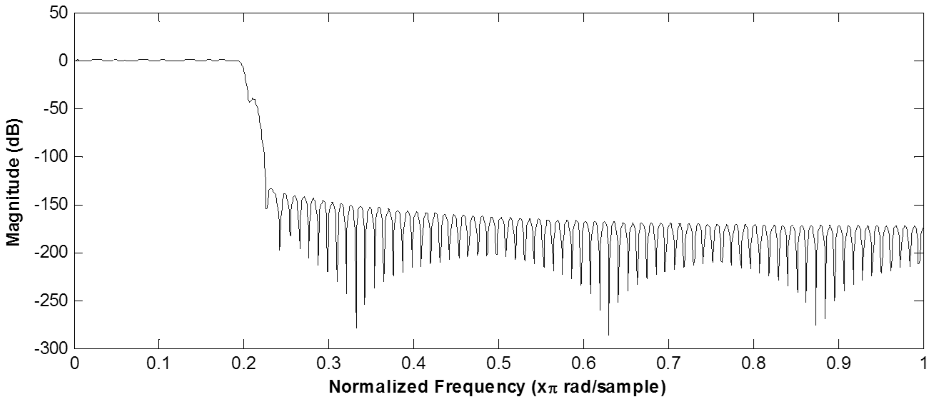

Figure 4.14 Total magnitude response of H(e^{jwT})

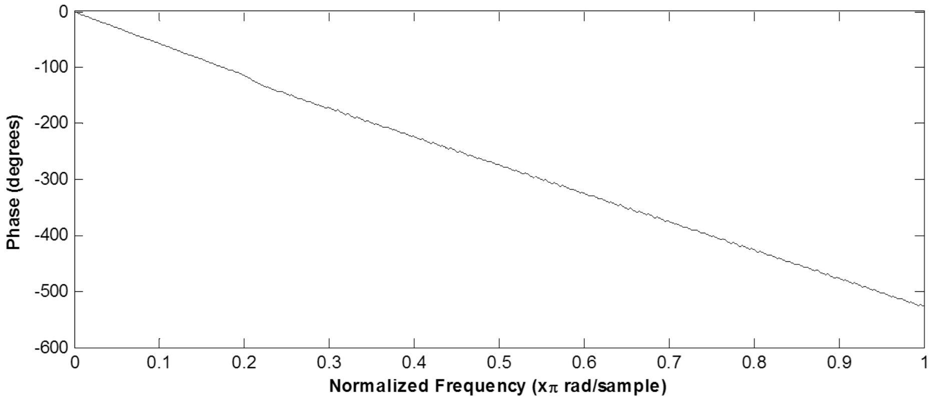

Figure 4.15 the phase response of H(e^{jwT})

CHAPTER 5

DISCUSSION

In the filter design theory the basic and most critical point is to obtain the sharp and narrower transition region, low pass-band ripples, high stop-band attenuation. Either in analog or in digital realization of such requirements, both circuits result with more circuit components due to the approach to the ideal requirements.

In analog circuit design the inductors are main problems owing their magnetic coupling and therefore active RC structures are replaced to simulate the terminal equation of inductors. Heating problems of the resistances are also reduced by using switched capacitors to eliminate resistors from the circuit and fully integrated pn structures may be realized to satisfy the filter characteristics in the analog form.

In the digital realization the circuit components are multipliers, adders and parallel shift registers. All of these components are either combinational or sequentional logic elements which are mainly constructed by using the gates and flip flops. This causes higher cost comparing to analog design but the design procedures are easier and the noise effect can be reduced using suitable algorithms.

In this work a technique for designing sharp linear phase digital filters using the masking technique is presented. This technique is based on the increase of the delay elements which results as the multiple similar gain structures in the same spectrum with sharper edges and to filter the higher harmonics of the gain structures. Because of secondary filtering, this technique is known as frequency masking technique. The outputs of the masking filters are combined to produce the desired output. In this work our aim was emphasized as getting sharp transition band by using lower order cascaded filters. FIR filter is chosen because it is always stable, it can have linear phase and it is easily implementable. Masking process is applied recursively to increase the efficiency of the technique and the results have shown that the technique is well applicable in FIR filter design.

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