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ARCHITECTURAL FORM DESIGN AND STRUCTURAL ANALYSIS OF TENSILE STRUCTURES

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ARCHITECTURAL FORM DESIGN AND STRUCTURAL ANALYSIS OF TENSILE STRUCTURES

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Supervisor
Prof. Dr. Mustafa ÖZAKÇA

by

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ABSTRACT

ARCHITECTURAL FORM DESIGN AND STRUCTURAL ANALYSIS OF TENSILE STRUCTURES

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This thesis deals with architectural form design and structural analysis of tensile structures. The literature survey of tensile structures about historical references, form findings and analysis methods are given. The design of tensile membrane structures is carried out in four steps. These are conceptual design, form finding, structural analysis and cutting pattern. The design procedure and numerical methods are given. Two tensile membrane structures such as, university main entrance and open sunshade lodge hall are designed. At the end, brief conclusions are presented together with some suggestions for future studies on the design of tensile structures.

Keywords: Tensile structures, conceptual design, form-finding, cutting pattern,

ÖZ

MİMARİ ŞEKİL TASARIMI VE ASMA GERME SİSTEMLERİN YAPI ANALİZİ

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Bu tez mimari şekil tasarımları ve asma germe sistemlerin analizi ile ilgilidir. Asma germe sistemlerin tarihsel gelişimi ile için form bulma ve analiz yöntemleri hakkında literatür araştırmaları verilmiştir. Asma germe yapıların tasarımları dört aşamada yapılmaktadır. Bunlar, kavramsal tasarım, form bulma, yapı analizi ve kesme kalıplarının oluşturulması. Formun bulunması, analizin yapılması ve kesme kalıpların çıkartılması için tasarım aşamaları ve sayısal yöntemler verilmiştir. İki adet asma germe yapı tasarlanmıştır, bunlar: Gaziantep Üniversite ana giriş kapısı ve kamelya. Son bölümde, asma germe yapıların tasarımı ile ilgili bu çalışma neticesinde elde edilen sonuçlar özetlenmiş ve gelecek çalışmalar için bazı öneriler verilmiştir.

Anahtar Kelimeler: Asma germe yapılar, kavramsal tasarım, form bulma, kesme kalıpları

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CHAPTER 1

INTRODUCTION

“Tomorrow’s architecture will again be minimal architecture, architecture of the self-forming and self-optimization processes suggested by human beings. This must be seen as part of the new developing ecological system of the people who have densely and peacefully settled the surface of the earth. It is an architecture that respects genuine traditions and the multiplicity of forms in animate and inanimate nature”

Frei Otto [1]

This saying marks the architecture in second half of the twentieth century. After the war, young architects, engineers, and entrepreneurs had been looking for new peaceful architectural forms, which should not counteract nature, but could be an integrated part of it instead. It was of prime importance, with the minimal material and energy, to rebuild the destroyed cities in a more effective and beautiful way at that time [2].

Nowadays, it is recognized for the first time that conservation and care of nature and a livable, sustainable environment should be taken into account in the process of selection and design of architectural forms. It is not only about the development of new structural technologies, construction methods, and materials to fulfill actual needs, but also about the realization and creation of new aesthetic environments. It is essential to conserve and protect the fundamental home of our mankind [3].

1.1 Introduction

Tension structure is a kind of structure considered to be born from these thought and recognition. Besides pursuing higher and higher buildings in crowd cities nowadays, human beings have been also making their consecutive efforts to find novel structural materials and forms, which can provide the same level of safety as the traditional structural ones, to facilitate the framing of large spaces and create bright, variegated interior spaces.

Tension structures can greatly extend architects' imagination because of their lightweight characteristics by making use of tensile materials, cables and/or membranes. This kind of structure can achieve the desired geometrical configuration and capacity of resisting external loads by the introduction of prestress. The last few decades have seen a rapid expansion in the growth of number and range of applications of them [3].

The tension structure has been recognized as an efficient and practical configuration for achieving various structural and architectural objectives. The primary goal in the design of a tension structure is to select and arrange the structural components so that loads are carried primarily in tension and the number of compression members is held to the minimum necessary to main stability. A design achieved by extensive use of high strength tension members with relatively small cross sections can offer advantages over a more conventional one. The potential for significant weight reduction is probably the most important, however, since, more often than not, weight is a limiting or controlling factor when a tension structure design is an alternate candidate to perform a specific function.

A common example of the use of tension structures is in the design of roof systems to span large areas enclosed by modern structures such as stadiums and shopping centers. Networks of slender cables provide the strength necessary to support the tremendous loads involved and the same time afford the architect the opportunity for imaginative creation. Space activities provide further applications for the tension structure concept since weight is a principal factor in determining feasibility [4].

1.2 Classification of Tension Structure

Kim [5] proposed a simple figure where various structural systems can be classified with ease, as shown in Figure 1.1. In this figure, first we can group into two parts according to materials. The first one is the hard structures which are made by reinforced concrete, steel frames and so on; initial structural shapes under unloading have been determined to be a fixed form. The other is the soft structures which are made by fabrics, cables and so on; initial structural shapes under unloading have not been determined. These two structural systems reveal different behaviors. In the former, structural stability changes from stable to unstable as the external load level gets higher and reaches a certain critical point. But the latter shows the opposite phenomenon, namely from unstable to stable in an un-tensioned state. In structural design, therefore, the hard structures need to check the critical load due to the bucking phenomena, but the soft structures need shape finding due to the introduction of initial stresses.

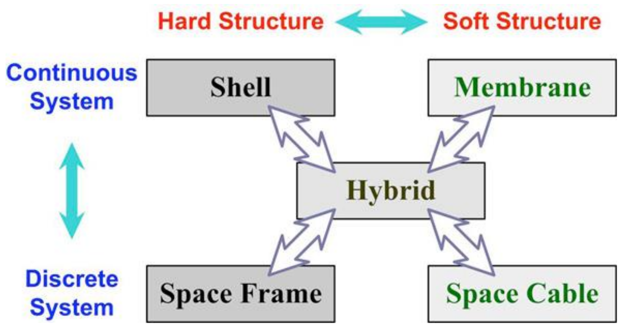


Figure 1.1 Classification of spatial structures [5]

Secondly, the spatial structures is grouped according to how to assemble; the one is built by continuous system such as shells and membrane structures, and the other is discrete system such as space frames and space cables.

The last two decades have seen a rapid expansion in the growth of the number and range of applications of tensile architecture [6]. Structural engineers and architects the design of lightweight structures is mysterious [7]. Most often, tensile structures are used for a dramatic effect over performance and outdoor exhibition areas, theatres, music stages, entrance driveways, parking areas, walkways, stadia, domes, airports. Designed as permanent or demountable canopies, they act as a foil for lighting and the projection of images, and can also be used for protecting audiences from the elements [8].

Tensile structure is made of tensile materials, cables and/or membranes, in which there is neither bending, compression nor torsion but only tension forces/stresses exist. It is usually used as a large-span structure because of the light-weight characteristics of the structural materials and absence of elements that may have buckle. Through tension structures come in varying size, scale, shape and form, all of them consists of the same basic elements:

- A lightweight and flexible fabric membrane, tensioned for stability and usually used as a roofing element,
- Flexible linear elements such as ties or cables, which are commonly used at boundaries or edges, and
- Rigid supporting members such as masts, frames, ring, arches and edge beams, which usually transfer loads in compression. These rigid supports are typically made from traditional building materials such as steel, concrete and timber.

Through there are variety of ways to categorize tensile membrane structures. Lewis [9] divides them into three main groups:

- Boundary tensioned membranes,
- Pneumatic or air supported structures,
- Cable-nets and cable-beams.

1.2.1 Boundary tensioned membranes

Boundary tensioned membranes are stressed by stretching the surface to meet the boundary made of flexible, tension cables, or a rigid frame/beam. The boundary elements are an important part of the tension structure as they determine the shape and nature of not just the boundary but the surface as whole. The former has zero excess pressure between the outer and inner surface [9,10].

The prestress level in the surface must be of an appropriate level, to ensure against a possible loss of tension during the life span of the structure on one hand, and to allow the material to stay within the elastic range of deformations on the other. Under imposed loads, such as wind and snow the stress in the surface can increase 6-8, even10-fold design of tension membranes is aimed at keeping the initial prestress at approximately 1/20 of the breaking strength of the cloth [9].

1.2.2 Pneumatic or air supported structures

Pneumatic structures or 'air houses' are thin membranes stressed by internal air pressure generated by fans. Their shape is very strongly affected by the difference between the external and internal pressures, which change continually, as a result of temperature variations, wind and snow load conditions. Air houses are designed to maintain an internal pressure between 0.2 kN/m^2 and 0.55 kN/m^2 [9, 11, 12].

1.2.3 Cable-nets and cable-beams

Cable nets can be stressed directly using rigid supports such as compression ring beams, or flexible edge cables with supporting mast and tie backs. They can take the form of suspended structure stabilized by means of a heavy roof cladding, or can be encased in concrete. The opportunity for creating new structural form is immense. If a prestressed cable net constitutes a part of a concrete shell, it is no longer a lightweight tension structure and the advantage of flexible is lost. However, because of their apparent visual lightness, the shapes of cable net structures are copied in to rigid forms, as was the case with concrete 'tents' constructed in the Middle East in 1991 [9,13,14].

The main difference between cable nets and fabric structures, from the structural view point, is that fabric structures are capable of sustaining membrane shear forces whereas cable nets are not, due to the deformability of the surface [18].

1.3 The Advantages and Disadvantages of Tensile Membrane Structures

The growing interest in tension structures can be attributed to following advantages:

- Provided appropriate and well designed structural forms are utilized these structure can become very attractive architectural landmarks.
- Many fabric architectural structures are designed so that the fabric can be removed if there is danger of a hurricane. This helps prevent damage to a

structure in a way that is not possible with roofed building structures. It also helps prevent damage to surrounding property from flying shingles or metal roofs.

- Adapts its own, unique shape. Because of their uniqueness and originality, fabric structures can attract extra attention to a business. Adding attractive signage and logos to help advertise businesses.
- They are light weight and can be transported at relatively low cost.
- These structures can be used to cover large areas at very competitive costs per unit area.
- These structural forms result in structures that can be fully stressed since there is no need to consider bending or buckling which results in very efficient use of the materials used.
- These structures may be prefabricated and can be manufactured in the most efficient method [15,17].

On the other hand, tensile membrane structures have some disadvantages. These are:

- Cannot take heavy weather conditions,
- Creep (stretch very slightly)
- Short economical life span of structures
- Loss of tension is dangerous for stability
- Thermal values limit use [16]

1.4 Objective of the thesis

This dissertation is based upon literal findings, historical references, the World Wide Web and two individual case studies. It is written as an introduction, to the wonderful and diverse world of tensile membrane architecture. The first half takes a brief retrospective look at the historical development of tensile membrane buildings, starting with the tent, moving to technology and through to the industrially produced high tech membrane buildings we see today. Briefly documenting the work, motivations and research of some of the most important architects and engineers involved with the development of this highly technological method of building throughout the twentieth century and beyond. The realization and subsequent understanding of this dissertation would be fruitless without the basic understanding of the historical perspective of tensile architecture. It is undoubtedly the understanding of the past which guides us towards a more informed opinion about the future.

The second half of this dissertation presents the analysis and design of two tensile membrane structures using commercial package program. The design of tensile membrane structures is carried out in four steps. These are conceptual design, form finding, structural analysis and cutting patterning. This dissertation is not meant to be a definitive engineering guide, but merely meant as an eye opener for an otherwise lesser understood method of building construction that has been growing in popularity and rationality for many years.

1.5 Thesis Outline

The coming contents of this thesis are organized as follows:

Chapter 2 gives the literal findings and historical references. The first part of the thesis presents pictorial references which are included to enable a better visual understanding of the subject matter. The historical reference studies are chosen firstly for their architectural relevance; secondly for their contrast to each other, and thirdly; for their design technology. The second part of the chapter discusses progress in the design procedure and its components such as form finding, analysis etc.

Chapter 3 presents the design procedure and numerical methods for form finding, analysis and cutting pattering.

Chapter 4 presents design of two tensile membrane structures. These structures are university main entrance and sunshade lodge in hospital park. The steps of each design are given in detail.

Chapter 5 concludes the study and gives some discussions of future study on the design of tensile membrane structures.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

Since the dawn of our on this planet, human kind has occupied its self with the creation of shelters. This is, of course, an in-built human instinct as a reaction to the extreme fluctuations in weather conditions and as a defensive barrier protecting us from predators or unwanted intruders. It could also be considered warm and safe nesting places for our young and loves ones. This instinctive need for the creation of shelter has, over many thousands of years, lead to many building techniques, styles, principles and technologies being past down from generation to generation, from culture to culture and country to country. This obsession with building has been prevalent in just about every period of history and this obsession is now affectionately called architecture [19].

Light, nonpermanent and early tensile architecture grew out of the wandering or nomadic life styles of the early barbarian people. These people developed portable, demountable buildings which were adequate for their physical, social and economical needs. The technical achievements of these early structures where perhaps underestimated in the past, by the majority of architectural historians, but fortunately the importance of these simple, light prefabricated and portable dwellings can style offer today designers inspiration. Tent architecture evolved out of a shortage of building materials and the concept of movability or non–permanentness, made the humble tent the ideal shelter for regions where lack of resources negated a nomadic life style. These types of tensile textile buildings also have an adaptability, which made them flexible to changing situations and conditions.

So to briefly sum up the qualities of these first primitive buildings is like writing a contemporary brief for a modern prefabricated housing project: Economic, flexible, limited material usage, prefabricated, transportable, environmentally friendly material, use of natural light, connection to the surrounding landscape and low energy consumption. Figure 2.1 summarized the development in building construction.

Figure 2.1 Frei Otto history of construction [19]

2.2 Tents

A membrane tent is probably the second most ancient building form in human history after caves. Evidence of mammoth bones and tusk used as supports for animal hides has been found at sites verified to be more than 40.000 years old in the Ukraine region [15,20].

In North America native tribes wandered and were able to carry their tents to new places as the seasons dictated. Of the three basic forms the conical form is oldest and saw widespread use across north Europe, Northern Asia and North America. The conical shapes ‘kibitlea’ is shown in Figure 2.2, shape dates to B.C. or earlier and has been the world’s most popular dwelling form [21].

The black tent is probably about as old as the kibitlea form and like it still much used today (see Figure 2.3). The tent has been the dwelling in one form or another for most nomadic peoples from the Ice Age to the present. Vegetation permitting, the most common supports for tents were tree branches or the trunks of saplings. The low profile and shallow slopes of the black tent make it resistant to the desert winds. Of the three basic shapes the black tent is the only one in which the form is not completely determined by its supporting framework [21].

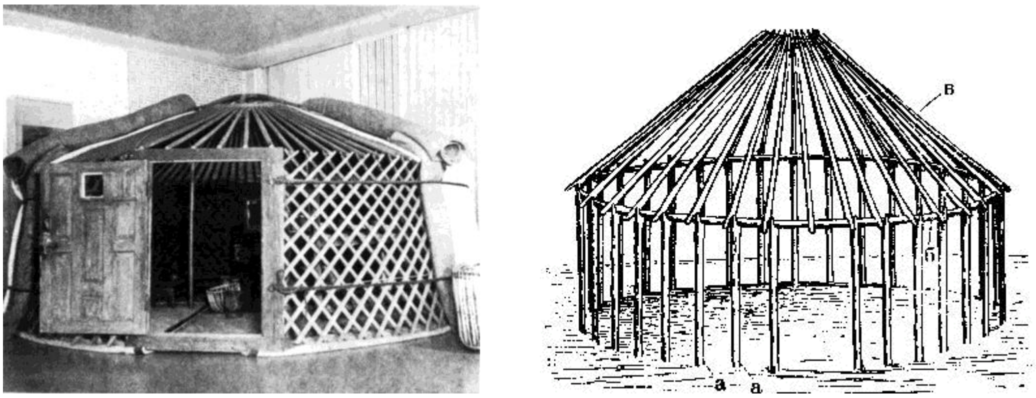


Figure 2.2 Kibitlea tent [21]

Another structurally interesting tent form is ‘envalet’ shown in Figure 2.4, popular in the Cataluña region of Spain for several decades near the turn of this Century. These had a clear span of about 30 meters, rectangular plan and were erected annually for village festivals.

Figure 2.3 Black tent [21]

The history of membrane structures in the late nineteenth century and early twentieth century is linked to the development of circus tents. The largest wall tents were the travelling circus 'big tops' popular in the U.S. from early 19 Century. In the 1950s these reached their maximum size covering more than one hectare [15,21].

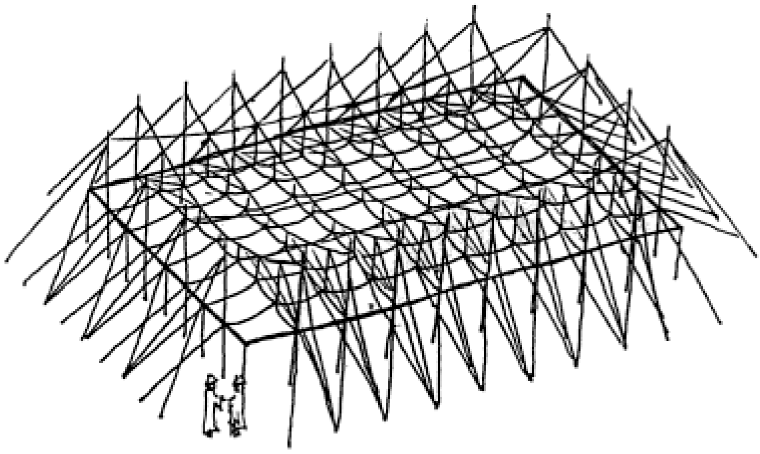


Figure 2.4 Envalet tent [21]

In the late nineteenth century the 'breakthrough' came as larger structures were required in a post–industrial society. Many significant developments took place, after the Second World War in the design procedures, materials, construction and erection methods, and of course computation [15].

2.3 Architectural Developments

Tensile membrane structures today still posses the same basic attributes of effective coverage and fast construction, but have evolved into large scale public projects, capable of sheltering huge crowds instead of small family units. Table 2.1 lists a number of notable tensile membrane structures constructed in Turkey and World. Each project brings with it a new set of design requirements and challenges. The following is brief of selected structures and their constructions to the advancement of tensile membrane structure design and constructions.

Table 2.1 Notable tensile membrane structures constructed in Turkey and World

Year	Structure	Architectures&Engineers	Design Style
1896	Nizhny Novgorod fair exhibition pavilions	Vladimir Shukhov	First tension structure
1946	Radome	Walter Bird (engr)	Air flated
1953	J. S. Dorton arena Raleigh,NC.US	Matthew Nowicki (arch) William Dietrick (arch) Fred Severud (engr)	First large cable-net structure
1957	D. S. Ingalls hockey rinks Yale University New Haven CT,US	Eero Saarinen (arch) Fred Severud (engr)	Cable-net
1958	McBac arts center theater Boston, MA, US	Carl Koch (arch) Weidlinger Assoc. (engr) Walter Bird (const)	Air flated
1958	French pavilion, World's fair Brussels, Belgium	Rene Sarger (arch)	Early cable net roof
1958	Sydney Myer music bowl Sydney, Australia	Robin Boyd (arch) Bill Irwin (engr)	Early cable net roof
1967	German pavilion, Expo Montreal,Canada	Frei Otto (engr) Rudolph Gotbrod (arch)	Tensile membrane structures
1972	Olympic stadium Munich,Germany	Behnich and Partners(arch) Frei Otto(engr) Leonhardt &Andra(engr)	Tensile membrane structures
1970	US pavilion, World's fair Osaka, Japan	Davis and Brody (arch) David Geiger (engr)	Air-inflated vinyl-coated fiberglass
1979	King Abdul Aziz university sport hall	Buro Happold (engr) Rudolph Gotbrod (arch)	
1981	Haj Jeddah airport terminal, Saudia Arabia	Skidmore-Owings Merrill(arch) Horst Berger (engr)	Largest fabric roof to date 47 hectares coverage
1982	Munich ice skating rink Munich, Germany	Jorg Schlaich	Cable net and trussed arch
1985	King Fahd stadium Riyadh, Saudia Arabia	Ian Fraser, John Roberts(arch) Horst Berger (engr)	288 m diameter plan
1986	Canada place Expo Vancouver, Canada	Zeidler/Roberts Partners Horst Berger (engr)	Ridge and valley design
1992	Georgia dome Atlanta, GA,US	Weidlinger Assoc. (engr) Walter Bird(const)	Largest cable dome to date
1993	Denver airport Denver, CO, US	Fentress and Brandburn Bergerand Severud	PTFE-coated fiberglass roof
1999	Millennium dome	Richard Rogers	Steel-tensioned fabric
2002	Stands roofing, Fenerbahce stadium / Turkey	Teschner, Kosel	PES/PVC fabric
2005	Stands roofing Formula 1 ring, Istanbul / Turkey	Teschner, Kosel	PES/PVC fabric

2.3.1 Nizhny Novgorod fair exhibition pavilions

Russian engineer Vladimir Shukhov was one of the first to develop practical calculations of stresses and deformations of tensile structures, shells and membranes. Shukhov designed eight tensile structures and thin-shell structures exhibition pavilions for the Nizhny Novgorod Fair of 1896 (see Figure 2.5) covering the area of 27000.0 m^2 [22].

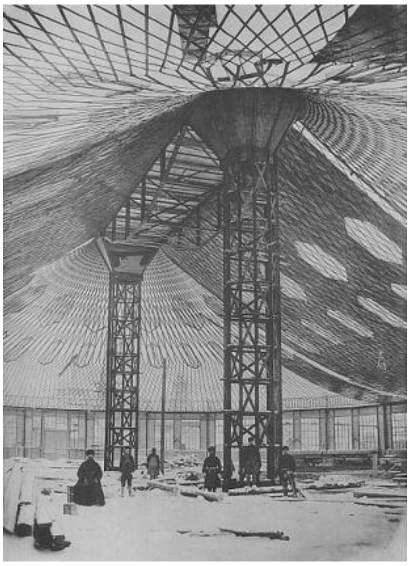


Figure 2.5 Nizhny Novgorod Fair exhibition pavilions

2.3.2 Dorton arena

The J.S. Dorton Arena which is shown in Figure 2.6 (known to its architect as the Paraboleum) is a 7610-seat multi-purpose arena in Raleigh, North Carolina, on the grounds of the North Carolina State Fair. It was opened in 1952.

The arena's bold parabolic design was conceived by Matthew Norwicki, a Polish architect who helped layout the rebuilding of Warsaw following World War II. Norwicki also assisted in designing the United Nations complex in New York. Local architect William Henley Dietrick supervised the completion of the arena using Nowicki's innovative design. Its design features a steel cable supported saddle-shaped roof in tension, held up by parabolic concrete arches in compression. The arches cross about 20 feet above ground level and continue underground, where the ends of the arches are held together by more steel cables in tension. The outer walls of the arena support next to no weight at all [23,24].

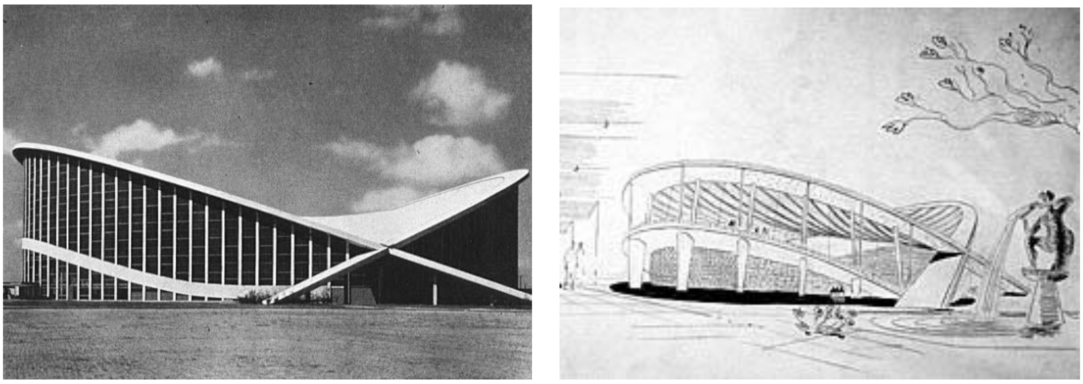


Figure 2.6 Dorton Arena

2.3.3 Ingalls rink

Yale University's Ingalls rink, informally known as the "Yale Whale" because of its signature humpbacked roof and arching 300 foot backbone, is one of the most visually arresting modern buildings of our time (see Figure 2.7). Designed by influential modern architect Eero Saarinen, the rink was designed in 1956 and opened in 1958.

Saarinen's sculptural masterpiece is deceptively simple in appearance. Although the ice rink itself is a standard rectangle with rounded corners, it is housed in an elliptical building. Moreover, the architect took a radical turn in creating the roof, conceiving a graceful concrete arch that forms the axial spine of the building. Roof-support cables are suspended laterally from this spine, extending to the perimeter wall of the building, where they are attached to a horizontal compression beam at the top of the wall. A curved plaster ceiling is suspended above the upper seating area and circulation ramps of the vast, column-free interior [25].

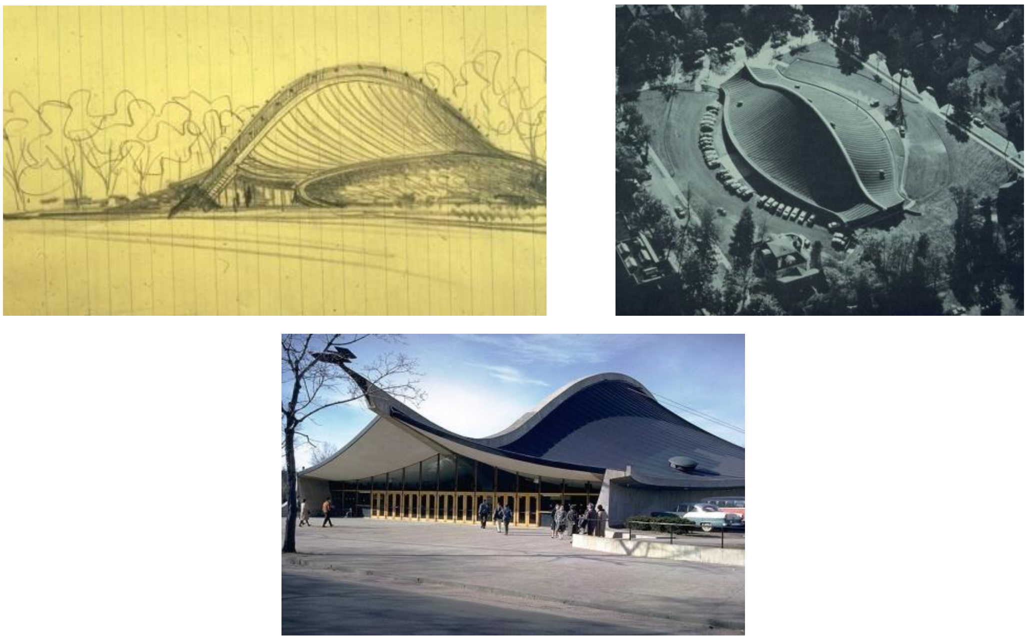


Figure 2.7 The Ingalls hockey rink by Eero Saarinen

2.3.4 French pavilion at the Brussels universal exhibition

Explicitly designed to express France's dynamism in a troubled period, this large pavilion with its complex forms undeniably marked the 1958 Universal Exhibition (see Figure 2.8). The general form of the roof is given by two joined lozenges in hyperbolic parabolic form balanced by an oblique mast, making it possible to cover 12000 m^2 without any internal support. The roof itself is comprised of thin metal sheet supported by a double network of tensioned wire. More of an expert in concrete, Guillaume Gillet who was building the Royan cathedral at the time, opted to use steel in this instance (Architecture: Guillaume Gillet, Consultant Engineers: Jean Prouve & Rene Sarger) [26].

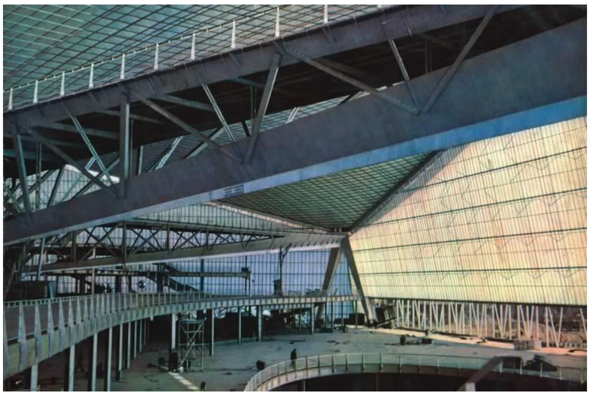


Figure 2.8 French Pavilions at the Brussels Universal Exhibition

2.3.5 Sidney Myer music bowl

The Bowl's canopy which is shown in Figure 2.9 consists of a thin membrane made out of half inch weather-proofed plywood sheeted on both sides with aluminum attached to a cobwebbed frame of steel cables and supported by 21.3 meter masts pivoted to the earth. The total area of the canopy is 4055 m^2. The main cable at the edge of the canopy comprises 7 ropes and 173 m long, anchored deep into the ground in concrete blocks. Longitudinal cables hold up the roof and transverse cables hold it down.

Project design was by Yuncken Freeman and Griffiths and Simpson during 1956. The project architect was Barry Patten. Construction commenced in 1958 with an innovative system of cables laced together and covered with aluminum faced plywood sandwich panels. To ensure the structure would be watertight yet aerodynamically stable and flexible, new construction techniques were developed. Ground anchors were required to be corrosion resistant. The shell also needed to be acoustically correct [27].

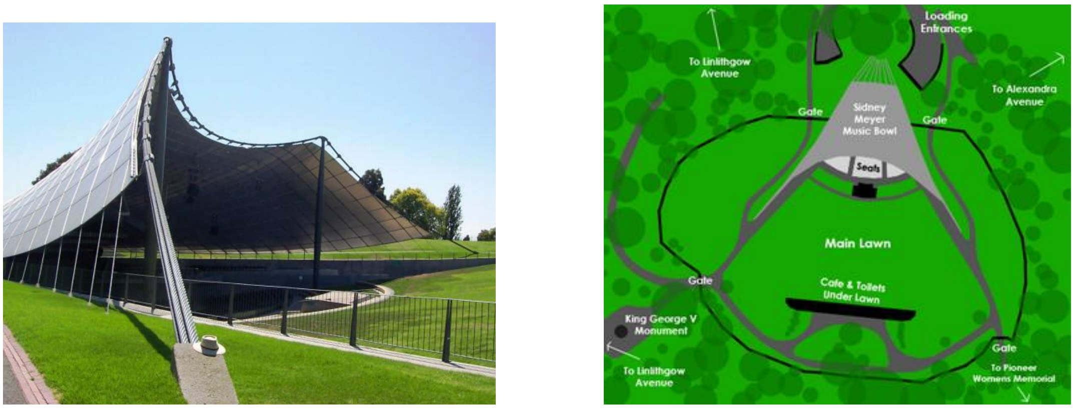


Figure 2.9 Sidney Myer music bowl

2.3.6 German pavilion Expo 67

Frei Otto and Rolf Gutbrod attempted, with this competition-winning project, to create a man-made landscape (see Figure 2.10). The cavernous interior contained modular steel platforms arranged at different levels. The entire area was covered by a single membrane of irregular plan and varying heights. Its contours were determined by the high points of the masts and the low points where the membrane was drawn, funnel- like, down to the ground. Eye loops filled with clear plastic material accentuated these points and the saddle surfaces they created. The prestressed membrane consisted of a translucent skin hung from a steel wire net, which, by eye, ridge, and edge ropes, was connected with the mast heads and anchor blocks.

Construction details: composite high-and-low-point net with 8 support points, 3 restraining points in combination with 3 continuous ridges, 31 perimeter anchor points maximum length: 130 m, maximum width: 105 m, covered area: 8,000 m^2, mast heights: 14 to 38 m [28].

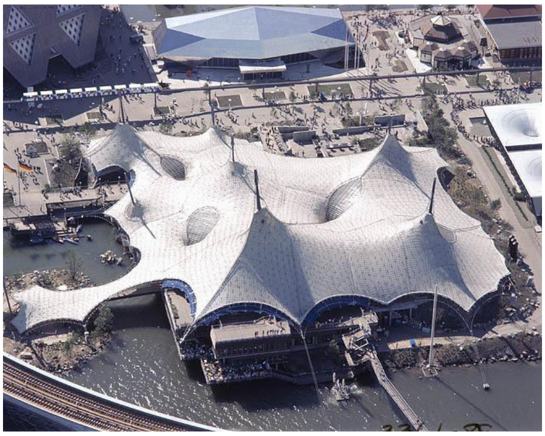
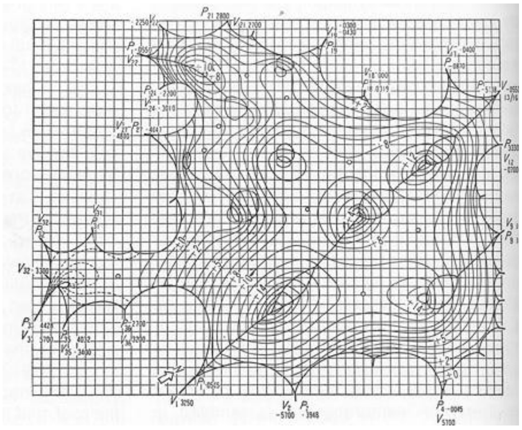



Figure 2.10 German pavilions Expo 67

2.3.7 United States pavilion 1970 World exposition

Sunk partly into the ground, the elliptical U.S. pavilion at Expo '70 (see Figure 2.11) was roofed by the largest clear-span air-supported cable roof ever built. Made of a translucent, vinyl-coated, fiberglass fabric, the roof provided filtered natural light by day, and glowed at night. The fabric weighed 1 lb/ft^2 and was kept inflated by an internal pressure of approximately three-Hollowed out of the earth; the interior structure housed seven major exhibits under the general heading "Image of America". It raised two levels with a combined floor area of 100000 ft^2. The project was unique collaboration between the architects and exhibit designers, Charmayeff & Geismar. The building form and its interior spaces were conceived by both offices.

Earlier major structural innovations (such as the Galerie des Machines, the Eiffel Tower, Brooklyn Bridge, hyperbolic parabolic and thin shell) have resulted in a highly visible and often startling change in physical form. This innovation was an exception. Like the sophisticated high-speed computer, its potential was not revealed by unusual physical form. It was a structural revolution barely visible to the professional and almost entirely invisible to the layman. Its impact may nevertheless be extremely visible, for this structure makes such schemes as totally enclosed urban areas believable and feasible. Its architects, Davis, Brody & Associates, with designers Chermayeff, Geismar, de Harak & Associates, of New York, envision entire regions, states and even the lunar surface contained within low-profile air structures [29].

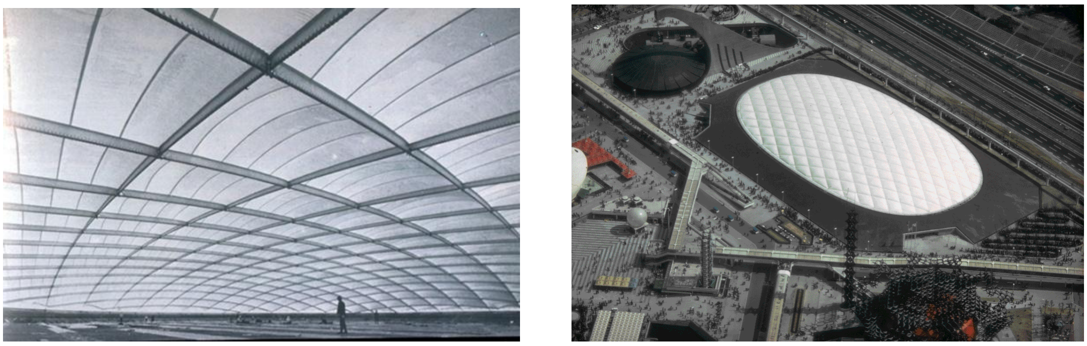


Figure 2.11 United States pavilion 1970 World exposition

The air-inflated structures design by David Geiger: Inspired by the success of the EXPO' 70 American pavilion, David Geiger developed several projects employing cable reinforced, insufflated membranes, for sport stadiums in the United States and Canada, from 1974 to 1984. The largest of these stadiums are the Pontiac Silverdome, in Michigan (1975), the Vancouver Amphitheater (1983) and the Minneapolis Metrodome (1982), all of them covering more than 40000.0 m^2, with capacities above 60000 persons. These roofs drastically reduced the cost per seat, compared with conventional stadium, and have worked satisfactorily, except for some operational problems, leading do deflations, in the Minnesota Metrodome, due to excessive accumulation of snow. It can be appointed as a paradox, that the main factor driving to construction of closed environments (harsh winter) is also the foulest enemy of the large pneumatic domes. Later domes such as the Tokyo "Big-Egg" Dome (1988) and the Akita Sky Metrodome, designed and built by Kajima Corporation (1990) avoided problems with snow using larger internal pressures, smaller distance between cables and higher profiles [29].

2.3.8 Munich Olympic stadium

The roof of the main stadium and indoor arenas for the 1972 Munich Olympic Games (see Figure 2.12), designed by Günter Behnisch with Frei Otto as roof design consultant, realized an entirely new scale for this type of structure, and led to the pioneering of purely mathematical computer-based procedures for determining their shape and behavior.

Figure 2.12 Munich Olympic stadium

The various tent and umbrella roof erected at Munich were the culmination of Otto's many years' experience. Earlier examples had indicated the potential of such temporary but economical large-span structures. But at Munich the scale was tremendous, involving the erection and linking of varied amoeba-shaped tents: the major areas covered included the main stadium, on one side only, linked to the arena and the swimming area, both wholly covered. The roof covering the main stadium consisted of a PVC-coated polyester fabric suspended on hangers independent of the cable net. The supporting masts held the main cables in tension, thus providing the necessary support for hanging roof areas.

Tent-like roofs consisting of the roof of the main stadium (34550.0 m^2), the gymnastics arena (21750.0 m^2), the indoor pool (11900.0 m^2), a large (5800.0 m^2)

and a small (800.0 m^2) intermediate roof. All roofs have translucent Plexiglas covering and are supported by a network of twin strands with a mesh width of 75 cm. The roof for the stadium is supported by eight cable-stayed towers, tower height up to 76 m, and tensioned by a curved cable consisting of 10 parallel-strand cables along the inner edge. The roof of the gymnastic arena is supported by four cable-trusses that are anchored to cable-stayed towers up to 70 m high. The roof of the indoor pool is supported by a single 80 m high tower [30-32].

2.3.9 Sport centre at the King Abdul Aziz University

Although it stands over 27.0 m high and covers an area exceeding 9.500 m^2, the sports hall at Jeddah's King Abdul Aziz University (see Figure 2.13) is unmistakably Arabian in architectural concept. Designed by Professor Frei Otto and the Stuttgart architectural practice of Buro Gutbrod, with engineering design by Buro Happold, the structure is a vast and beautiful tensile membrane structure which despite its scale, is strongly reminiscent of the traditional Bedouin tent form [33].

Figure 2.13 Sport centre at the King Abdul Aziz University

2.3.10 Haj terminal of the Jeddah international airport

During the conceptual design, concrete and metal roof schemes were abandoned because they absorb too much heat. Tensile membrane structure alternatives were studied in consultation with Horst Berger of Geiger Berger Associates whose fabric roof for the Bicentennial in Philadelphia had proven effective in improving comfort on hot days. PTFE coated glass fiber fabric reflects 70% of the sun's heat, radiates out during the night, eliminates electric light because of its translucency.

The detailed design of the Haj Terminal (see Figure 2.14) tensile roof was carried out by Geiger Berger Associates as consultants to Owens Corning Fiberglass in a design/construct contract. Horst Berger was principal-in-charge. This included form-finding of the membrane, sizing of components, detailing of connections, non-linear analysis of the structure, and engineering of the erection process of this record size roof which covers the area of 12 football stadiums. Each of the 210 tent units was the largest fabric tensile structure the firm had designed in 1976. The firm's new mathematical tools had to be expanded. Interaction of elements in a very large structure - each of the two five module halves is 427 x 320 m had to be considered [34].


Figure 2.14 Haj Terminal of the Jeddah International Airport

2.3.11 Ice skating rink (Olympic park Munich)

To enable the open ice-surface in the Olympic park (see Figure 2.15) to be used all round the year, independently of the weather, light roofing, naturally without supports, was required. Architects and engineers solved this problem together in an inspired fashion by means of a steel-trussed arch of three chords. With a span of 100 m and a height of roughly 19 m at its apex, the arch is capable of transmitting any thrusts to two large concrete abutments. Two sets of cables hang in opposing curves from the arch, stabilizing it by their anchorage and forming a net. These symmetrical nets of cable have a grid of 75 x 75 cm and support a wooden lattice, upon which is attached translucent plastic sheeting. At the roof's edges the cable nets are bordered by garland-shaped cables which pass over adjustable angled supports of steel being anchored fast.

The construction and form of the hanging from the arch correspond in the main to that of the roof edge, thus allowing the same components to be used and also producing a series of elliptically strung openings below the latticed arch. These are filled by "glass eyes" equipped with ventilators. The continuous, between about 3 and 5 m tall "facades" between the edge of the roof and the ground in the region of the angled supports incline from the eaves to the interior at an angle corresponding to that of these supports. The building thus has the first ever in itself mobile, horizontally barred glass "facade" which is able to participate in the formal changes allowed by the anchoring cables.

This extensive, and in itself symmetrical area-covering structure makes do with a minimum of materials. The logic of its construction, its beauty, elegance and exceptional boldness give the impression that genuine functionality, invented by technical intelligence, led to the real aesthetic effect [34].

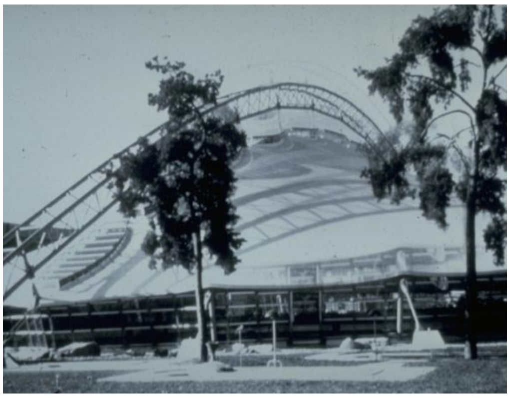
 

Figure 2.15 Ice skating rink (Olympic Park Munich)

2.3.12 King Fahd International Stadium

King Fahd International Stadium is a multi-purpose stadium in Riyadh, Saudi Arabia (see Figure 2.16). It is currently used mostly for football matches and it also has athletics facilities. The stadium was built in 1987. It also one of the largest stadium roofs in the world. It was a venue for matches of the FIFA World Youth Championship in 1989 including the final match.

The stadium's roof shades over 75000 seats and covering an area of 47000 m^2. The 24 columns are arranged on a circle with a 247 m diameter. The structure is made of

24 equal units. The entire roof has only two roof panel shapes which generate the rich sculpture of the roof umbrella. The huge umbrella keeps the sun off the seats and concourse slabs, providing shade and comfort in the hot desert climate [35].

Figure 2.16 King Fahd International Stadiums

2.3.13. Canada Harbor place

The sail-like tensile membrane structure units of the Canada Harbor (see Figure 2.17). Place convention center exhibit hall are skewed at 45° in their plan orientation to reflect the street grid of the city of Vancouver. This feature together with the peaks of the end-supported structural system gives the building its drama. It has become a symbol of the city. The structure has a clear span of 55 m and a length of 100 m. An inner liner fabric provides insulation and sound control [36].

Figure 2.17 Canada harbor place

2.3.14 Georgia dome

The Georgia dome is shown in Figure 2.18 became the largest cable-supported fabric roof in the world. Stretching more than 395000.0 ft^2, the Teflon-coated Fiberglas fabric roof is quite an engineering marvel. The roof weighs just 68 lb, but it is strong enough to support a fully loaded pickup truck. How? The answer lies with a fundamental engineering breakthrough, one that architect-engineer Buckminster Fuller dubbed "tensegrity". Put simply, tensegrity is a complex sequence of triangles. Short, vertical posts carry the weight of the Georgia dome roof. The posts are held in place by prestretched cables, attached to the top and bottom of each post with steel pins and welded connections. The cables pull on the posts with equal force in all directions to form strong, taut triangles. The cable roof is secured to a reinforced concrete ring along the perimeter of the dome. The 2750.0 ft concrete ring rests on slide-bearing Teflon pads that allow the roof to flex slightly during high winds.

It is this precise dance of pulling and pushing that allows tensegrity roofs like the Georgia dome to soar far above the stands and the playing field below Here's how this dome stacks up against some of the biggest domes in the world [37].

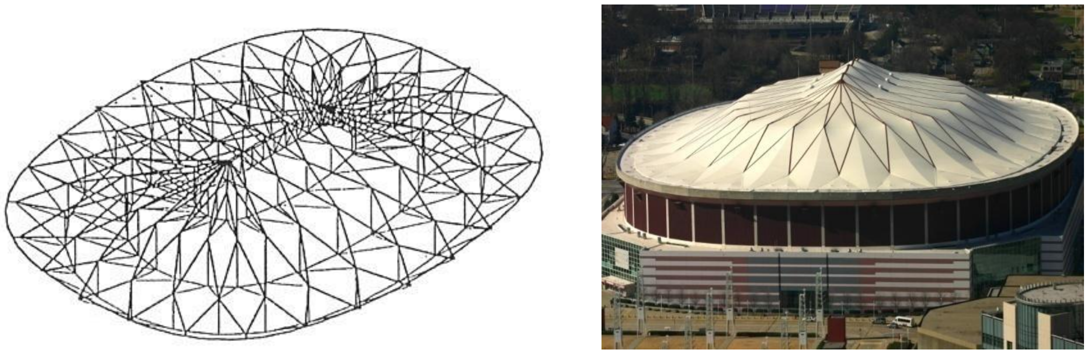


Figure 2.18 Georgia dome

2.3.15 Denver international airport

The airport's distinctive white tensile fiberglass roof is aesthetically designed to be reminiscent of the snow-capped Rocky Mountains in winter. The centenary steel cable system, similar to the Brooklyn Bridge design, supports the roof. Denver international airport is also known for a pedestrian bridge connecting the terminal to “Concourse A” that allows travelers to view planes taxiing directly underneath and provides sweeping views of the Rocky Mountains to the West and the high plains to the East.

Both during construction and after the opening of the airport, Denver international airport (see Figure 2.19) has set aside a portion of its construction and operation budgets for art. Gargoyles hiding in suitcases are present above the exit doors from baggage claim. The corridor from the Jeppesen Terminal and Concourse A usually contains additional temporary exhibits. Finally a number of different public art works are present in the underground train that links the main terminal with the concourses.

Mustang by New Mexico artist Luis Jimenez was one of the earliest public art commissions for Denver international airport in 1993. Standing at 9.8 m tall and weighing 4100.0 kg, "Mustang" is a blue cast-fiberglass sculpture with red shining eyes located between the inbound and outbound lanes of Pena Boulevard. Jimenez died in 2006 while creating the sculpture when a portion of it fell on him and severed an artery in his leg [38].

Figure 2.19 Denver International Airport

2.3.16 Millennium Dome

The dome is the largest of its type in the world (see Figure 2.20). Externally, it appears as a large white marquee with twelve 100 m high yellow support towers, one for each month of the year, or each hour of the clock face, representing the role played by Greenwich Mean Time. In plain view it is circular, 365 m in diameter (one meter for each day of the year) with scalloped edges. It has become one of the United Kingdom's most recognizable landmarks. It can easily be seen on aerial photographs of London. Its exterior is reminiscent of the Dome of Discovery built for the Festival of Britain in 1951.

The building structure was engineered by Buro Happold, and the entire roof structure weighs less than the air contained within the building. Although referred to as a dome it is not strictly one as it is not self-supporting, but is a mast-supported, dome-shaped cable network. For this reason, it has been disparagingly referred to as the Millennium Tent. The canopy is made of PTFE-coated glass fiber fabric, a durable and weather-resistant plastic, and is 52 m high in the middle (one meter for each week of the year). Its symmetry is interrupted by a hole through which a ventilation shaft from the Blackwall Tunnel rises [39].

Figure 2.20 Millennium Dome

2.3.17 Stadium roofing for the Turkish soccer club Fenerbahce Istanbul

In order to be able to offer space for an audience as large as possible, the supporting structure for stadium roofing is normally erected outside the stadium itself. Due to the tight traffic situation around the Fenerbahce stadium (see Figure 2.21), this was not possible. Therefore the decision was taken to do without some of the previous 32000 seats and make use of an internal supporting structure and build four massive concrete towers in the corners, which tower well above the seating area. These corner pillars, each with a surface of some 100 m^2 are the supports for the primary supporting structure for the roofing: the dimensions of the steel framework are 156 m on the longer sides by 123 m on the shorter sides, a four-belt carrier with a height of some 12 m.

The membrane which is in a typically curved structure consists of coated polyester fabric. It stretches between round steel cantilever girders with a diameter of 400 mm which can be unhitched upwards and downwards with steel cables using ascending forces. The membrane will firstly be welded together from individual perfectly fitting widths and later delivered as a complete field and assembled by crane.

Due to their material and construction, membranes are less susceptible to vibration and therefore ideal for earthquake areas. In order to achieve optimum earthquake protection, all bearings within the construction are made as elastomeric bearings and are completely reversible after changes in strain, in other words they take-on their original form. On top of this, special earthquake bearings are installed at the important points, so-called lead deformation bearings which can take deformation due to the relatively soft material as opposed to rigid bearings **[**40].

Figure 2.21 Stadium roofing for the Turkish soccer club Fenerbahce Istanbul

2.3.18 Grandstand roofing for the Formula One race course in Istanbul (Turkey)

A grandstand area for more than 30000 spectators with an area of approximately 17000.0 m² was completed right on time for the first Formula One race in Istanbul (see Figure 2.22). The roofing consists of 54 filler wall elements spanned by membranes, each with a size of 9.16×33.5 m^2. Due to the low design weight of the textile membrane, the roof structure could be given a light and filigree design despite its free overhang of approximately 30 m. Two large high-point canopies were constructed for the roof terrace areas of the two 25 m tre-high VIP towers. The membranes which are made of plastic-coated polyester fabric both have a diameter of approximately 30 m, thus each covering an area of approx. 800 m^2. The Istanbul facility has a pair of VIP towers, each having 37 m. height, 7 storey, settled on 1.056 m^2 each area and total settlement area of 7392.0 m^2each and two storey paddock buildings, having 11888.0 m^2 of seating area and 32616.0 $m^2$of usage area. Designed by Orion Istanbul and Teschner, Kosel Germany architecture offices, the structure is a vast and beautiful and is the first main remarkable and the biggest tensile membrane structures in Turkey [41,42].

Figure 2.22 Roofing of stands, Formula 1 ring,

2.4 Structural Developments

Since Frei Otto's pioneering works in the 1950's, taut structures (encompassing both cable and membrane structures) constitute an important research field in architecture and engineering. They are light, elegant and effective structures, whose applications range from large stadium roofs and high-rise building walls to pneumatic furniture or aerospace equipment. The design of taut structures is integrated to their analysis, in a process that includes procedures for shape finding patterning and load analysis. Some references on the design of taut structures are Haber and Abel [43], Knudson [44], Tabarrok and Qin [45], Moncrief and Topping [46], Barnes [47] and Pauletti and Pimenta [48].

Architects and engineers have to work together, to meet the proposed requirements. It is widely known that architects are focused on the geometric shape of the membrane, while engineers are concerned with the internal stress distribution of the membrane. Both approaches are directly related, since the geometric shape of the membrane depends on the given initial prestress of the membrane.

A diagram representing the tensile structure design process is shown in Figure 2.23. Some stages need to be completed, from the conception of the structure to its idealization. Computer based tools are used to help designers and engineers at each stage of the process. These tools can be divided in three groups [49]. These are:

- *Conception Tools*: equilibrium shapes can be obtained using form finding techniques.
- *Analysis Tools*: used for stress validation of the obtained equilibrium shapes under applied loads.
- *Cutting Pattern Tools*: the cutting patterns of the membrane are automatically generated.

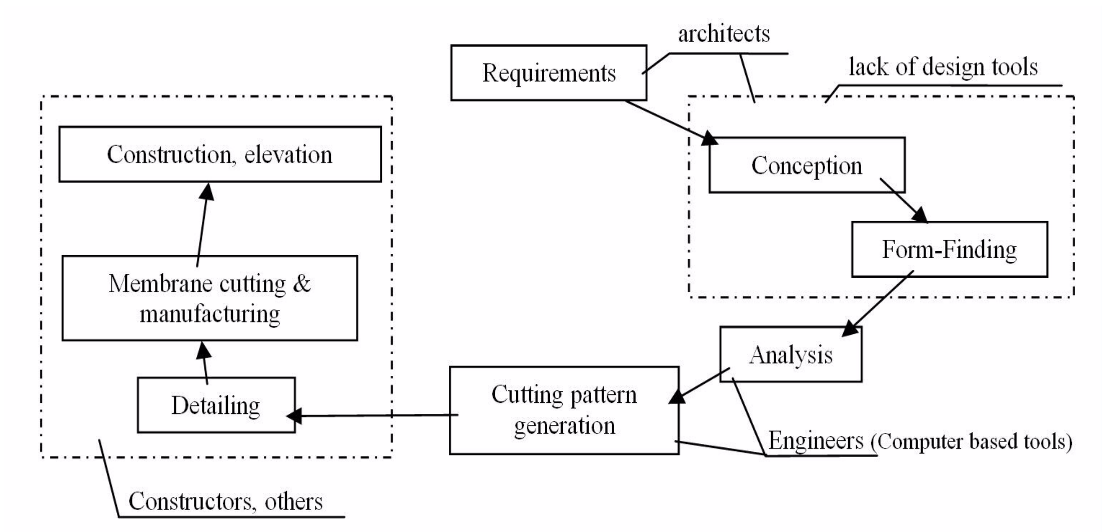


Figure 2.23 Tensile structure design process

2.4.1 Form finding developments

Although prestressed structures like tents belong to the first man-made structures, little was known about the analytical modeling of their load deflection behavior. Prestressed cable-nets and textile membranes are characterized by the inherent interaction between their geometry and stress distribution. This relationship between the form and forces makes it impossible to directly design such structures as is the case with conventional structures [50].

When in 1967 the German Pavilions for the Montreal EXPO was built, no practical analytical solution technique was available to determine the cable-net form, cutting pattern and behavior under external load. At that time the only way for the design and the realization of such nets was to use physical models [50].

Finding a feasible form requires the determination of a figure of equilibrium of inner forces and loads for the structure. This typically results in a doubly curved surface. Mathematically these surfaces could only be roughly approximated by differential equations. As doubly curved surfaces cannot be flattened without distortion, the generation of precise cutting patterns is required for fabrication. Finally the structure will undergo large deflections under acting loading conditions which means that analytical methods would have to be able to cope with that [50].

To design the cable-net roofs for the 1972 Munich Olympic Games stadium, Frei Otto built precise physical models which were intended to be the source of information for all relevant data. Linkwitz proposed to measure the models precisely applying close range photogrammetric methods which would allow for a simultaneous determination of the 3D-geometry of the model without touching it. It was realized however that the models were by no means precise enough to derive the cutting pattern for an equal mesh cable net made from steel [50].

The photogrammetric measurements of the physical models had to be modified in order to fulfill the constraints of equal unstressed mesh-width and of force equilibrium at each node. The analytical solution for this task was achieved by applying the method of least squares to the measured nodal coordinates [51,52] observing the boundary conditions above. Applying this technique, the cutting pattern for the stadium roof was created in a time consuming but successful procedure using all the computer power available at that time [50].

In 1971 Linkwitz and Schek [51] discovered the first formulation of the figure of equilibrium of forces, the force-density formulation. They realized that this was more appropriate for solving the problem, especially that of finding good initial geometry.

The design process of pretensioned structures such as cable nets and membrane structures is influenced by the development of computational methods. While the first methods of form finding had been physical modeling with fabric, wire nets or soap films, today several numerical methods of form finding are developed based on [53]:

- Conjugate Gradient /Force Density [52,54]
- Dynamic Relaxation [55,56]
- Modified Stiffness
- Principle of minimal surfaces [57,58]

or other approaches in fulfilling the three-dimensional equilibrium.

Force Density Method uses an analytic technique to linearize the form finding equations for a tension net. This linearization makes the method independent of the material properties of the membrane. Force density ratios (cable force divided by cable length) need to be specified for each element, and different ratios give different equilibrium shapes. The method is numerically robust, independent of the initial locations of the nodes and the equilibrium shape is found easily. The force density solution to applied loads is non-linear and requires iteration [59].

Surface Stress Density Method can be considered as a generalization of linear force density method to the bi-dimensional case and takes into account the shear stress. In this case, the surface stress density ratio is given by the stress divided by the area of the element [59].

Dynamic Relaxation method solves the geometric non-linear problem by equating it to a dynamic problem. Principles of dynamic are used to solve the analysis. Appropriate dynamic properties need to be defined, like the mass and damping characteristics of the membrane. A balance of forces is made at each node, giving a residual force that produces the movement of the node in the direction of this force, according to the dynamic behavior of the net. New positions for the nodes are calculated until the final equilibrium shape is reached. At this point the residual forces are sufficiently small [59].

Developments in mainland Europe have mostly used Conjugate gradient/force density solvers, Britain has concentrated on dynamic relaxation, and Japan and the USA have mainly used the modified stiffness method. Two element types are commonly used to model textile roofs. Cable net models using link elements have been popular in Conjugate gradient/force density systems, while triangular continuum elements have been typically used in dynamic relaxation and modified stiffness systems. It is important to highlight that the prevalence of using particular elements with particular solver algorithms does not have a theoretical or computational basis. Conjugate gradient/force density systems with triangular continuum elements are used when appropriate and modified stiffness and dynamic relaxation systems can also use link elements to model textile [7].

All methods have in common that no material laws are necessary finding an equilibrium of the three dimensional shape for given stress distributions, boundary conditions and supports. These shapes of equilibrium should ensure in the built structure a homogeneous distribution of the tension stresses. In reality the material behavior, process of cutting patterns, manufacturing and pretensioning on site influencing the stress distribution, wrinkles and regions of over stress are obvious, can be seen and measured [53].

2.4.2 Analysis

General analysis of membrane structures requires geometric non-linear techniques. A tensile structure generally consists of a membrane tensioned by cables or directly fixed to extremely rigid structural elements (walls, beams, posts or braced frames). The latter are often regarded as external structures. The mechanical behavior of the fabric and cables present a nonlinear nature which can be classified into two categories [60]:

1. Nonlinear nature due to large displacements (rotation and translation).
2. Nonlinear nature due to the fact that neither the fabric nor the cables can be subjected to compression loads.

Therefore, they do not present any compression stiffness. However, the strain field in the fabric and cables is considered small. The material associated with the fabric is considered to have orthotropic linear elastic properties. The same assumption is made for the cables. Finite membrane elements and cable elements have been implemented for the development of procedures able to analyze the mechanical behavior of a tensile structure under different static external loads [60].

Two mechanical aspects were studied. The first is based on form-finding through given stress or load fields. In this case, only the geometry can evolve towards a stable equilibrium state. The problem is then determined statically. Equilibrium no longer depends on the rheologic properties of the fabric. This approach has been widely adopted and uses two types of modeling [61]:

- Modeling with reinforced cable elements. In vector methods the conditions of equilibrium and compatibility are decoupled until convergence to an equilibrium solution. The most common of these are Dynamic Relaxation [61,62], the Scaled Conjugate Gradient Method [63], and Force Density method, described originally by Scheck [63], equivalent cable net models [64]. The former has gained the most acceptance for the analysis of tensile membrane structures because of its clear physical analogy and ease of implementation of the necessary controls and constraints. Individual element stiffness relations are held separately, which greatly eases the specification of stress controls at form-finding and non-linear, stress dependent, elastic properties under analysis. It is similarly straightforward to introduce a wide range of boundary conditions that can themselves be dependent upon the current deformed state of the structure [6].
- Modeling by triangular and quadrilateral membrane finite elements. This method was initially developed by Ishii [65, 66], Haber [67,68], D'Uston [69], Tsubota [70] and Nouri-Baranger [71-73].

The second aspect is based on a nonlinear elastic large deflection analysis and considers strained elements. Large deflections and stress factors constitute a set of parameters, which evolve during the iterations. This aspect was implemented in 1974 by Argyris et al. [61,74,75]. However, although it is efficient, it is the most difficult for the designer to conduct [61]. The procedure developed in this sense can be based on fabric modeling via a network of cables (Argyris [60], Gründig [76], Nishino [77] and Frei [78]). The disadvantage of this procedure rests in its inability to reproduce the actual behavior of the fabric: stress state, wrinkling and warp and weft direction orientations according to the pattern cutting. The alternative procedure uses modeling by triangular or quadrangular membrane finite elements. The nonlinear matrix methods are typically an application of more standard non-linear structural analyses such as the Newton–Raphson method [79]. The structure overall tangent stiffness matrix is solved incrementally until convergence is obtained. Special controls limiting the maximum incremental deflections and nodal residual forces may be required. The stress/strain relations for the individual components are coupled with the equilibrium and compatibility requirements for the complete structure [6].

Many tensile structural systems are strain hardening. A variety of common tensile structural systems are initially strain softening and begin to exhibit to strain hardening behavior once sufficient load is applied. Consequently, non-linear solution strategies that anticipate strain hardening have been employed with success and can speed convergence in a wide variety of commonly encountered problems. There are significant exceptions, such as a class of "tensegrity" type structures that become strain softening as load is increased. The dynamic relaxation method is also used with success for the general analysis of geometrically non-linear problems [80].

Most architectural/structural fabric materials exhibit non-linear behavior, as a consequence of being woven composites. Almost all architectural/structural fabrics in use today are coated composites. However, material non-linearity is rarely modeled. Mechanical behavior of textiles is primarily dependent upon the properties of both the yarn and the weave. Coating properties also have an effect upon the composite's mechanical behavior, albeit at a lesser extent than the properties of the base cloth. Fabric is commonly modeled utilizing linear constant strain element (finite element methods or a network of string elements. Both of these modeling approaches have been widely used with success while each has attendant limitations that the analyst must consider. Membrane elements that better simulate the non-linear behavior of woven composites have been developed [81]. While the fabric material non-linearity is typically no modeled, it will likely prove to be useful when the mechanics of fabric failures are better understood and utilized quantitatively in a limit state design approach [80].

2.4.3 Cutting Pattern Generation

The last step before arriving at an industrially achievable form, is the cutting process and the flattening of the free-form surface determined via the process described above. This surface is obtained by cutting plane fabric panels. The panels are then sewed or glued together. The cut panels should both minimize the fabric cuttings and take into account the warp and weft directions so the fabric may have optimum resistance capacity. Once cut, assembled and the membrane in place, the geometry of the structure and the internal loads should be as close as possible to those calculated and no wrinkles or pockets should appear [6].

Incorrect initial panel cutting can lead to or generate the appearance of a wrinkled surface and over-stress. This step therefore requires adequate definition of the surface cutting pattern and layout. The cutting pattern research process combines three major steps:

- the lines defining the different widths are generated on the free-form surface;
- the widths defined are then developed into a pattern;
- geometric corrections, referred to as compensations, are made to the cutting pattern since the fabric widths are not tensioned during the cutting and assembly process [6].

In 1986, Hangleiter carried out research in this field [82]. He applied geodesic surface patterning to a spatial cut patterning development process. In 1989, Tsubota [83] used the same method but made some modifications to take into account the strain generated on implementation. In 1989, Shimada [84] presented a method based on minimization of the strain energy calculated from the disparities between the three-dimensional form and the planar form requested. This method applied elastic formulation by finite element, within an iterative resolution estimation. In 1990 [85], the authors compared three cutting pattern methods. Two were based on a purely geometric technique to develop the triangles forming the width meshing.

The another method used the research carried out by Shimada, based on a representation of the surface in equilibrium via square NURBS surface cut into fabric widths according to one of the main directions of each square. These three methods produce cutting patterns that do not take into account the strain generated when the fabric is tensioned. After assembly, the fabric widths generate a free-form surface where the prestress field equals zero, therefore impossible to achieve. According to previous research, tension can only be achieved following a reduction in width size. Phelan and Haber [72] introduced the optimization concept; since the fabric is not tensioned when the cutting process is being carried out. Their method resides in the use of non-prestressed cutting patterns as the initial configuration for an equilibrium analysis and form optimization. They thereby attenuate the design problem by combining the form, equilibrium and cutting problems into one. A nonlinear equation is then to be resolved. However, to find the solution this method requires data on the following two initial geometries: the three-dimensional form and the cutting patterns. The two meshing should be topologically equivalent and carefully selected. Within the framework of this paper and following the research carried out by Shimada and Phelan, a cutting pattern tool has been developed. It can optimize the stress field generated in the structure after assembly, by finding the adequate cutting pattern shapes [6].

CHAPTER 3

ANALYSIS AND DESIGN OF TENSION STRUCTURES

3.1 Introduction

Traditional building design involves the definition of initial parameters such as member sizing and spacing, the analysis of these members under applied loads, and then the adjustment of the initial parameters under strength or displacement limitations. This procedure usually requires a few simple iterations and can be completed with classical (and often linear) analysis methods. Furthermore, as the majority of buildings in the last century have been designed in this way, the behavior of conventional building systems is well understood and documented. The design and analysis of tensile membrane structures, on the other hand, differs greatly from this traditional process, having a new set of design loads and considerations. Because initial geometry parameters can change so drastically, their analysis often requires multiple iterations with newer computational methods. In fact, the overall from of a tensile membrane structures changes so much that this process is commonly referred to as “shape” or “form” finding [87].

This chapter begins with general considerations for the design of tensile membrane structures and then explains the basics of how tensile membrane structures work. The rest of the chapter presents and discusses the design process and various numerical methods available for their analysis.

3.2 Design Considerations

As mentioned above, the design of tensile membrane structures comes with several considerations that need not be made for conventional structures. These include certain load and climate conditions, availability of material and labor, acoustic performance, fire protection, energy use and lights, as well as material maintenance, durability, and inspection [87].

3.3 How tensile membrane structures work

Conventional structures depend on gravity and internal rigidity for load transfer and for stability. Beams and columns in these structures can resist axial, shear, and bending stresses. Fabric structures, on the other hand, are so lightweight that gravity does not have any serious effect. Furthermore, fabric elements transfer all loads axially, as they have negligible bending and shear stiffness. They therefore gain stability from their curved form and internal axial prestressing alone.

As mentioned before, the basic structural element of a fabric membrane is set of cables running in perpendicular directions. Discussing the behavior of a single cable can therefore help to illustrate certain behavioral properties of a fabric membrane. To better understand this behavior, first consider the uniformly loaded beam and it's bending moment diagram in Figure 3.1.

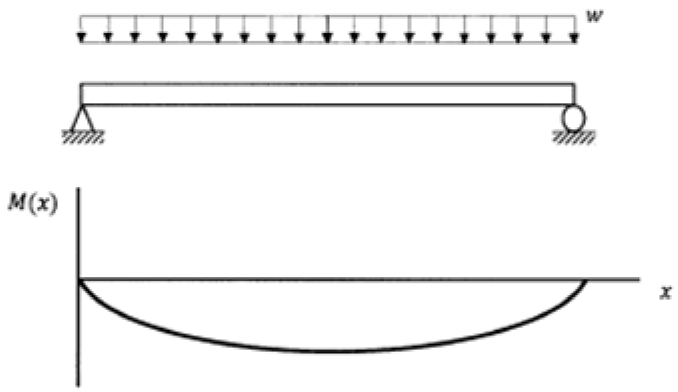


Figure 3.1 Uniformly loaded beam and moment diagram

The shape that a cable takes under this same load is considered optimal because it follows the bending moment diagram. When considering a cable's self weight only, the shape it takes called catenary (although this term is often used when describing the natural shape resulting from any applied load). Because a cable cannot resist shear and bending, it will therefore deflect in such a way as to carry applied loads in axial tension only, all the while doing so with much less material than a straight beam under the same loads. Now consider the uniformly loaded cable and deflected shape in Figure 3.2 w is the load per unit length, L is the total length of cable, h is the maximum vertical deflection at midspan, and *H* and *V* are horizontal and vertical reaction forces, respectively [87].

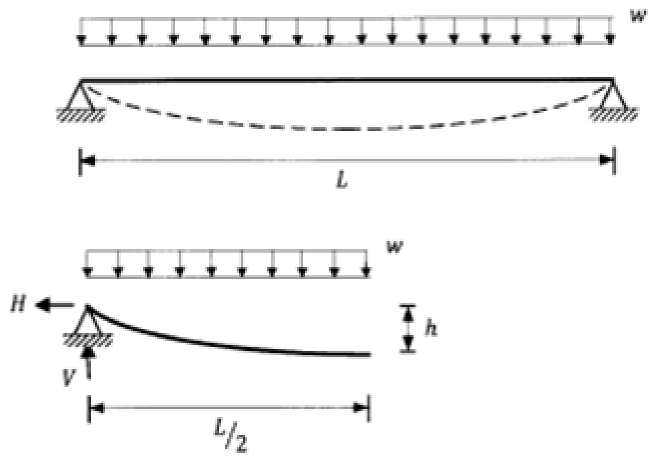


Figure 3.2 Uniformly loaded cable

Equilibrium in the vertical direction yields:

$$V = \frac{wL}{2} \tag{3.1}$$

Because cables cannot resist bending, sum of moments about any point will be equal to zero.

$$\sum M_{midpoint} = 0 \tag{3.2}$$

$$H(h) + w\left(\frac{L}{2}\right)\left(\frac{L}{4}\right) - V\left(\frac{L}{2}\right) = 0 \tag{3.3}$$

$$H = \frac{wL^2}{8h} \tag{3.4}$$

And the force in the cable is:

$$F = \sqrt{V^2 + H^2} \tag{3.5}$$

One can observe from these initial cable expressions that the sag or vertical deflection of a cable varies with the horizontal reaction force. This is the idea behind pretensioning in fabric membranes. Adjusting the horizontal tension force is the primary means of limiting membrane deflection. Applied loads will also be carried by the vertical “pins” (typically rigid supports such as mast or metal frames).

Depending on specified boundary conditions and internal pre-stressing, fabrics can either form in to an anticlastic shape with negative Gaussian curvature or a synclastic shape with positive Gaussian curvature. The term anticlastic refers to the opposing directions of perpendicular fiber elements. Joining together to form A saddle-like shape, these elements exert equal forces on each other and internally brace against themselves. Synclastic shapes consist of elements that are curved in the same direction like a balloon. In the design of tensile membrane structures, upwardly curved elements are usually called “ridge” cables while downwardly curved ones are “valley” cables.

The minimum number of anchor points needed for any section of fabric is four. Three points are insufficient because the resulting surface is a simple, flat triangle; as mentioned in the previous discussion about cables fabric elements gain stability with curvature. The four point structure is therefore the most basic element of a tensile membrane structure. It can be created with and endless number of boundary conditions and joined together to make a variety of interesting shapes and patterns. Figure 3.3 illustrates only small sampling of the types of structures that can be created with the four point structure (H. Berger) [88,89].

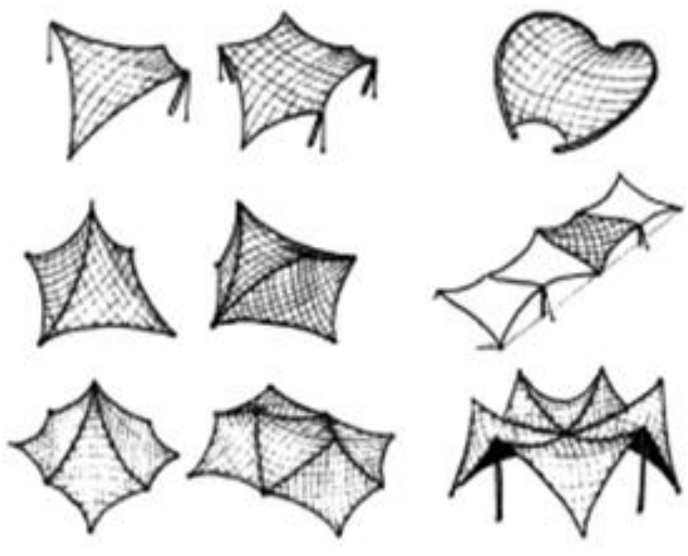


Figure 3.3 Examples of various 4 points structures [88,89]

3.4 Tensile Membranes Design Process, From Conception to Realization

A description of the tensile membranes design process is shown in Figure 3.4, from the preliminary sketches to the realization of the structure. Many tasks need to be completed to achieve the final structure [90].

Architects and engineers are involved in the study of tensile membranes structures [91-93]. Broadly speaking, architects tend to focus on the geometric shape and external appearance of the membrane, while engineers are more concerned with the internal stress distribution of the membrane and the viability of manufacturing it. Both approaches are directly related, since the geometric shape of the membrane depends on the given initial prestress of the membrane.

In Figure 3.4, the way in which architects and engineers work together within the tensile membrane design process is shown schematically.

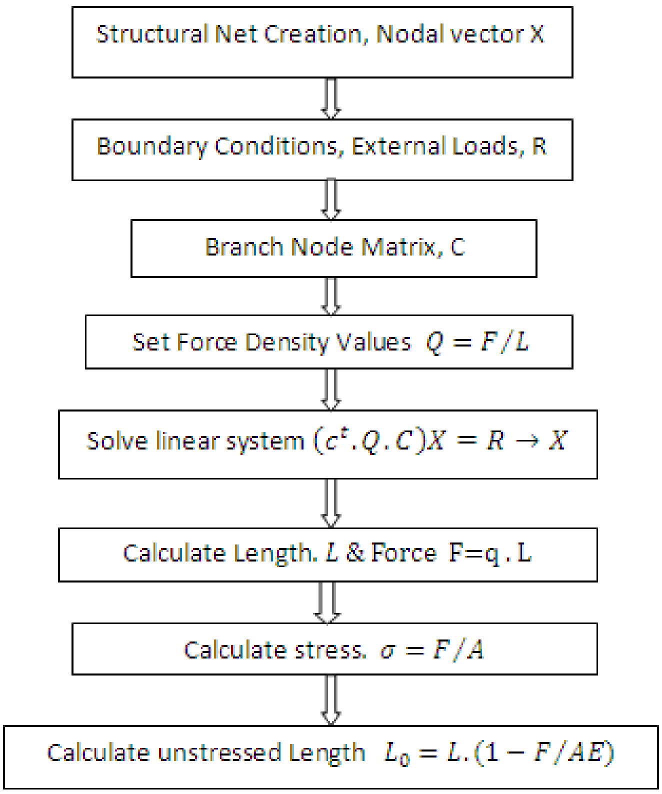


Figure 3.4 Force–density methods

- The initial steps of the design process are a task generally shared by the client and their architect together. The architect establishes the client's requirements, for example the area to cover, height clearance and data about the location of the future structure. It is important that the architect sketches the three dimensional shape of the structure considering all the requirements and some aesthetic criteria. For example the geological and geotechnical data relating to the site for the structure must be considered to position anchorage points and masts. The shape defined should provide all topological information on the structure, such as the position of cables, masts, anchorage points and areas covered by the membrane. Many objects and volumes are located in the same space and different solutions are proposed until the desired layout and shape are achieved. Furthermore the material type should also be selected at this phase. In the case of the materials selection several factors should be considered for example: translucency determines the required amount of lighting and heat insulation determines the heating requirements.

- The conceptual design stage is really the most important of the entire process because the initial conditions are defined and will be the basis of the rest of the project. Once the project has started, any change in these initial specifications affects the whole project.

- Form-finding consists of obtaining the equilibrium shape of a tensile structure. Many techniques exist to find the shape of a particular membrane [21,94-96]. Force–density methods are based on linearization techniques [97-100]. Other form-finding procedures include the dynamic relaxation method [56,101].

  There are different computer tools based on these techniques which help the designer to obtain the desired shape, although most existing computer applications require advanced structural skills, and too many tasks need to be completed before achieving the final shape. Thus, a design tool, in which the complete scene is shown (and redrawn) while the user is dragging a membrane vertex by means of a mouse, could be a very useful tool for designers. Currently, there is a lack of design tools covering this design stage, and present form-finding tools provide only a partial solution to the problem.

- Once the conception stage has been completed, a static analysis of the structure is carried out. Material properties are assigned to the membrane, and the stress distribution is obtained under different loads and boundary conditions to validate the initial shape of the membrane.

- The next step in the design process is called cutting pattern generation. This method allows the optimum way of cutting the membrane to be calculated. Based on a geometrical process, flat strips are obtained to comply with the desired shape. Many computer-based tools covering static analysis and pattern generation can be found, but specialized users are required to complete each task.

- Finally, detailing of the structure is carried out. Those details concerning foundation, structure supports, masts, connections, clamps, cable diameter selection, drawings and many others are defined at this stage. After detailing is completed, membrane cutting is carried out, according to the cutting pattern referred to in the previous step. Once the strips are joined, the completed membrane is prepared for the construction of the structure and the elevation of the membrane

3.4.1. Form-finding process

The shape analysis of tensile structures is a geometrically non-linear problem, and consists of finding an equilibrium shape compatible with given prestress conditions. The process of determining this initial equilibrium shape is known as form finding.

Non-linear methods are needed for design and analysis of tensile structures. Many linear methods of form finding have been implemented in the past in order to study the behavior of tensile membranes. The Force Density Method and Dynamic Relaxation Method are the most common methods used in the form-finding process. In the present study force density method is used.

Force Density Method: uses an analytic technique to linearize the form-finding equations of a tension net. This linearization makes the method independent of the material properties of the membrane. The force–density ratio (see Eq. (3.6)) that relates cable force to cable length needs to be specified for each element, and different ratios give different equilibrium shapes.

$$q = \frac{F}{L} \tag{3.6}$$

Where, q force-density ratio, F cable element force and L cable element length

A brief description of the force–density method is shown in the diagram of Fig. 3.4. The tensile membrane is divided into a discrete number of elements, which are then joined together at nodes. The position of the nodes is given by the Nodal Vector (X). Boundary conditions are defined, so some nodes (vertex nodes or edge nodes) may be restrained whilst others are free to translate. Loads are defined by the vector (R).

The branch-node matrix C describes the topological relationship between the nodes and the bar elements of the structure net. To linearize the problem, the force–density ratio (see Eq. (3.6)) is provided at each net element to relate the tension and corresponding length of respective elements. A constant value is given for the internal elements force density ratios, and different values are given for the boundary elements ratios.

$$(C^T.Q.C).X = R \tag{3.7}$$

Where, Q force-density matrix and X nodal displacement

The linear system is then solved (see Eq. (3.8)), and the position of the net nodes (X) is obtained. With the nodal coordinates of the net, the force, lengths, stress and the unstressed length of the element can be obtained using the following equations:

$$F = q.L \tag{3.8}$$

$$\sigma = \frac{F}{A} \tag{3.9}$$

$$L_0 = L.(1 - \frac{F}{AE}) \tag{3.10}$$

This method is numerically robust, independent of the initial locations of the nodes and enables the equilibrium shape to be found easily. With this linear method it is not possible to have control over the internal prestress of the net [90].

3.4.2 Analysis

Analysis models for conventional structures assume a linear between applied forces and displacements. These linear models can accurately describe a structure's shape, but are limited to a range of small displacements. These linear models can accurately describe a structure's shape, but are limited to a range of small displacements. Conversely, the design and analysis of tensile membrane structures requires a thoroughly non-linear approach, modeling large deformation behavior through the use of iterative numerical methods [87].

The Newton-Raphson method is a classical approach to the analysis of nonlinear structures, which does not apply well to behavior of fabric because convergence is slow and sometimes does not happen at all. However, Newton-Raphson works better when an initial estimate of shape or geometry is specified. Newer analysis methods have been developed for the direct application of analyzing cable-net and tensile membrane structures. These include the grid method and the force density method, which are both used to estimate initial system geometries before applying Newton-Raphson method. Another nonlinear analysis that can be applied to a tensile membrane structures is the dynamic relaxation method. The theory behind each of these methods is described in detail in the following sections [102].

Design Loads:

Through several building code stipulate design load requirements, the standards are usually intended for stiff and straight, conventional structures. ASCE is currently adapting their standards for application to tensioned fabric roofs. The following is an explanation of how load considerations will differ for tensile membrane structures [87].

Being much lighter than conventional structural members, tensile membrane roof structures and accompanying cables or ties incur only a small fraction of typical dead loading (generally less than 50 N/m^2). For this same reason, seismic loads, which vary with mass of a building, usually do not constitute a serious concern in the design of tensile membrane structures.

Roof live load code requirements were determined based on the assumption of heavy rooftop machinery and other usages that do not apply to a fabric roof. Indeed, with their curved forms and high deformability, fabric roofs are usually inaccessible to people except for maintenance and repair. However, current code provisions do not exempt fabric roofs and they are therefore designed with normal live loads [103].

Wind loads usually control over other types of loading in the design of tensile membrane structures. Though wind load specifications are available in local building codes, the behavior of fabric roofs under wind is unpredictable enough as to warrant the use of wind tunnel testing in many situations. Depending on the direction of wind and geometric properties of the fabric, wind can either act as inward pressure or outward suction, in which case the suction will tend to counteract downward gravity loads.

In some regions, snow and rain will govern over wind loading. The curvature and natural flexibility of fabric membranes can lead to a lot of uneven snow distribution. It is important to note that concentrated loads like high snow drift will often be more critical than large uniformly distributed pressures. Melted snow or rain also tends to pond in the downwardly curved sections of membrane. For this reason, drains and snow melting techniques are often incorporated into their design [104].

In general, temperature loads do not affect tensile membrane structures.

Tangent Stiffness Method:

Structural fabric exhibits large deformation and geometrically non-linear behavior. Because of this, it requires different methods of analysis, which tend to be numerical and iterative. The following describes the theory behind the tangent stiffness method (sometimes known as the transient stiffness method), which derives from the linear stiffness method discussed in the previous section.

The main reason that linear methods do not apply for large deformations is because the stiffness matrix depends on initial member geometries. When the members in a system deform by a significant amount, the stiffness matrix for the system will also change. The tangent stiffness method attempts to account for this discrepancy in the following way.

Begin by defining initial geometry vector, $\{X\}_i$ on which initial stiffness is based. The displacement resulting from this initial stiffness will be:

$$\{\delta\}_{i+1} = \{K\}_i^{-1}\{P\} \tag{3.11}$$

And the new system geometry is:

$$\{X\}_{i+1} = \{X\}_i + \{\delta\}_{i+1} \tag{3.12}$$

From this updated geometry, one can find a new stiffness matrix; $[K]_{i+1}$. This stiffness combined with displacement will yield a set of internal forces,$\{P_{in}\}$ which differ from the set of external forces.

$$\{P_{in}\}_{\boldsymbol{i+1}} = [\mathrm{K}]_{\boldsymbol{i+1}}^{\boldsymbol{-1}}\{\delta\}_{\boldsymbol{i+1}} \tag{3.13}$$

For large displacements, $\{P_{in}\} \neq \{P\}$. Define an error term or residual, $\{R\}$ and then an incremental displacement vector, $\{\Delta\delta\}$:

$$\{R\}_{\boldsymbol{i+1}} = \{P\} - \{P_{in}\}_{\boldsymbol{i+1}} \tag{3.14}$$

$$\{\Delta\delta\}_{\boldsymbol{i+1}} = [\mathrm{K}]_{\boldsymbol{i+1}}^{\boldsymbol{-1}}\{R\}_{\boldsymbol{i+1}} \tag{3.15}$$

At this point, the geometry vector is updated by the incremental displacement vector and more iteration can be performed until the residual vector converges on zero [87].

3.4.3. Cutting Pattern Generation

Usually the form-finding of a cable-membrane structure provides a non-developable surface. This means that the structure cannot be projected onto a plane explicitly, though this is necessary for the development of a production plan of the structure. The production plan of a cable-membrane structure is also called a cutting pattern. A model of a membrane structure with a cutting pattern marked on the surface [15].

The cable and membrane pieces are usually cut from a roll of material according to cutting pattern and stitched or welded to form the specified surface when appropriately stressed.

Today, the cutting pattern is usually generated from the equilibrium state of the structure by determining the stress free side lengths of the membrane elements in a plane. To reduce the wastage of material the most often used form is a strip of cloth with edges as straight as possible.

As a final step in the cutting pattern generation the size of the strips must also be "compensated". The reason is that the final structure is stressed, while the strips cut out of the membrane material are not. However the material of membrane structures is susceptible to creep due to temperature and loading. Creeping of the material causes permanent strain in the structure which should be considered or "compensated" in the determination of the stress-free size of the strips. Usually the compensation factor is determined by bi-axial material testing.

The cutting pattern generation can be performed in the following steps [105]:

1. Geodesic lines are created as seam lines.
2. Cutting procedures are used to cut the surface into different sub-surfaces according to these geodesic lines.
3. Ways of flattening are achieved: map projection, paper strip method.
4. Spline algorithms are applied to create equidistant points on the planar circumference.
5. Boundary adjustment is performed in order to produce identical seam lengths.
6. Compensation values are defined to compensate the strips.
7. Job-drawings are produced.

3.4.4 Detailing

Before the structure can be built several details should also be considered and designed for example:

- The type of connections between different parts of the structure;
- Gutters, water control systems; and
- Supporting structure as well as methods of fixing and anchoring cables.

The above list is not exhaustive and it may vary from structure to structure. For example the connections may require special attention from the design engineer. Most of modern cable-membrane structures are complex and visualization and virtual reality models can be very helpful in the design process. Well designed connections ensure the smooth flow of the loads from one component to another of the structure. Therefore particular care is required during the component design of fabric-fabric, fabric-rigid edge, fabric-cable and cable-cable connector as well as cable-mast or rigid edges [15].

For guidelines of the design of these components the reader is referred to the national building standard (for example Eurocode, British Standard) or to references. Recent work by Tensinet to establish standards can be found in reference.

Furthermore as a final step, the design of complex cable-membrane structures may require a prototype construction study. This study ensures that the structure can be built as intended, for example that the cranes are placed at strategic positions, the structure is stressed in an appropriate sequence without tearing and no structural component becomes entangled in the cables. For example references contain some information about the construction study for the Millenium Dome (London) [15].

CHAPTER 4

DESIGN EXAMPLES

This chapter contains two example problems which demonstrate the different aspects of the computer-aided design of tensile membrane structures, such as: form-finding, cutting pattern generation and analysis. These example problems are studied using commercial software ForTen 2000.

4.1 University of Gaziantep Main Entrance Gate

The first case study deals with the design of University of Gaziantep main entrance gate. The layout plan of the university main entrance is shown in Figure 4.1. The estimated area to be covered is approximately 1400 m^2.

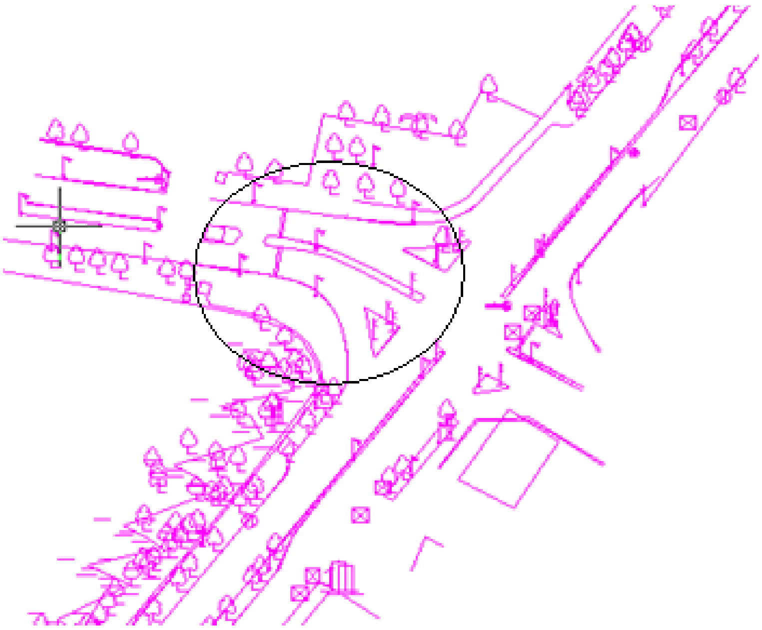


Figure 4.1 University of Gaziantep main enterance layout plan

The university main entrance gate should be well designed structural forms and reflect the university image. This structure should also become very attractive architectural landmarks and must offered wonderful design and structural efficiency. This gate must be departed from traditional constructions in order to emphasize the leading image of university. Because of their uniqueness and originality, tensile membrane structure which can attract extra attention to peoples is selected. The project is inspired by the wings of the butterfly. The covered area is 1335 m^2.

4.1.1 Conceptual design

Nature is beautiful and everyone loves nature. We find an infinite variety of shapes, colors and species in it. Figure 4.2 shows various butterfly found in nature. Our times demand lighter, more energy-saving, more mobile and more adaptable, in short more natural buildings, without disregarding the demand for safety and security. Overall, we believe that everybody are demanded to understand between nature and an ecological architecture through researching the minimal in nature in order to improve our health and nature.

Figure 4.2 Various butterfly from nature

How to achieve more with less, that is, less material and effort? Structural elements in designs have a modern building type capable of remarkably large spans from traditional constructions. Tension structures envisioned structures of extreme lightness as well as extreme strength, which were to make optimum use of new materials such as thin cables of high-strength steel or thin membranes of synthetic fabric.

A novel tensioned membrane structure of striking form named as the butterfly-shape structural system has been studied. Basic design concept, form finding procedures to create various structural forms are explained below. By combining either identical or different butterfly structures together, various structural forms of different shape and size for space enclosures can be created.

In the present study, a beautiful butterfly shown in Figure 4.3 is taken as a model for conceptual design. Butterfly structure is formed by three major components which are the inclined arches, the cables or struts, and the membrane. The key concept of the structure is to use inclined arches to form the membrane boundary.

Figure 4.3 Inspired butterfly

The conceptual butterfly design model is the one with two wings, which looks like a butterfly spreading its wings as shown in Figure 4.4. By taking the advantage of symmetry, only one wing is modeled. This wing is split into two parts which are named as upper wing and lower wing.

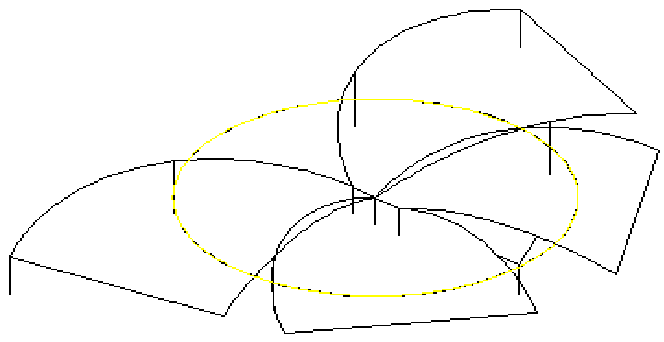


Figure 4.4 The conceptual butterfly design model

The geometry and dimension of upper and lower wings are given in Figure 4.5. The upper wing covers 380 m^2 area in plane and has a span length of 27.93 m. Moreover, the lower wing covers and 286 m^2 area in plane and has a span length of 20.48 m.

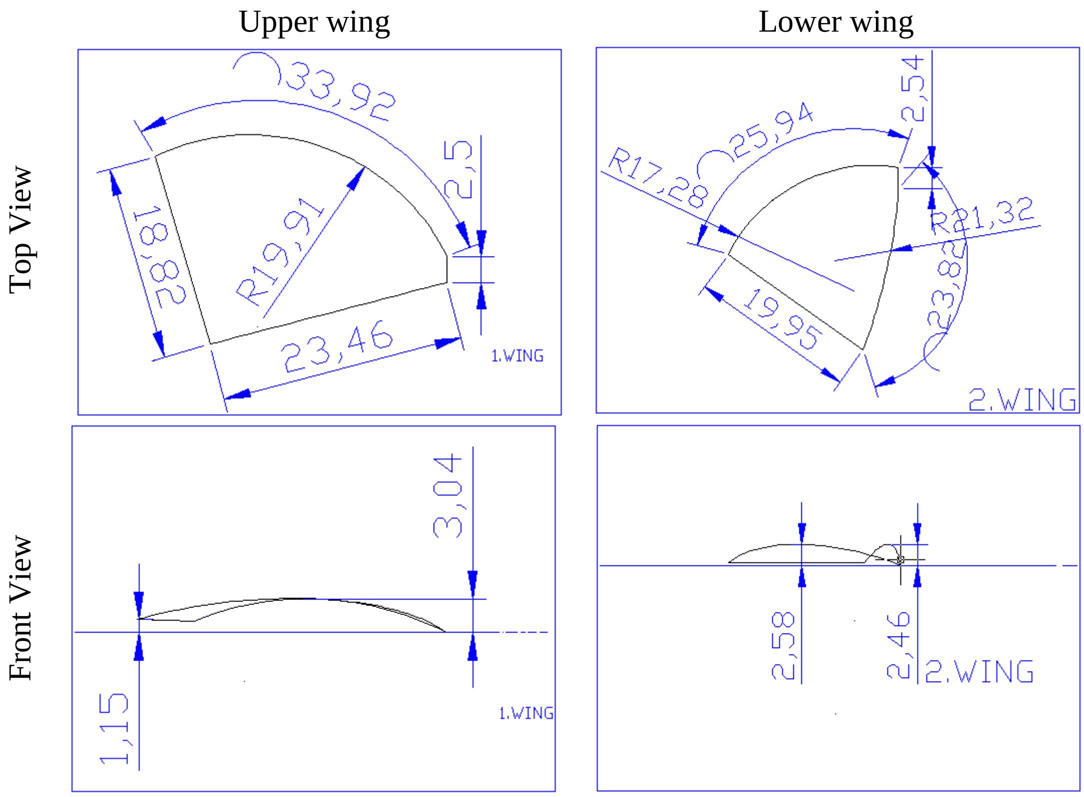


Figure 4.5 The geometry of upper and lower wing

4.1.2 Form-finding

In the both wing, the two opposite inclined edges are formed by arches. The arches are connected to each other by cable and beam at the outer and inner sides respectively. Membrane is stretched along these arches and cables, spreading between them to provide space enclosure. The upper and lower wings are supported by five and four columns respectively. Figure 4.6 show the boundaries of upper and lower wings

(a)

(b)

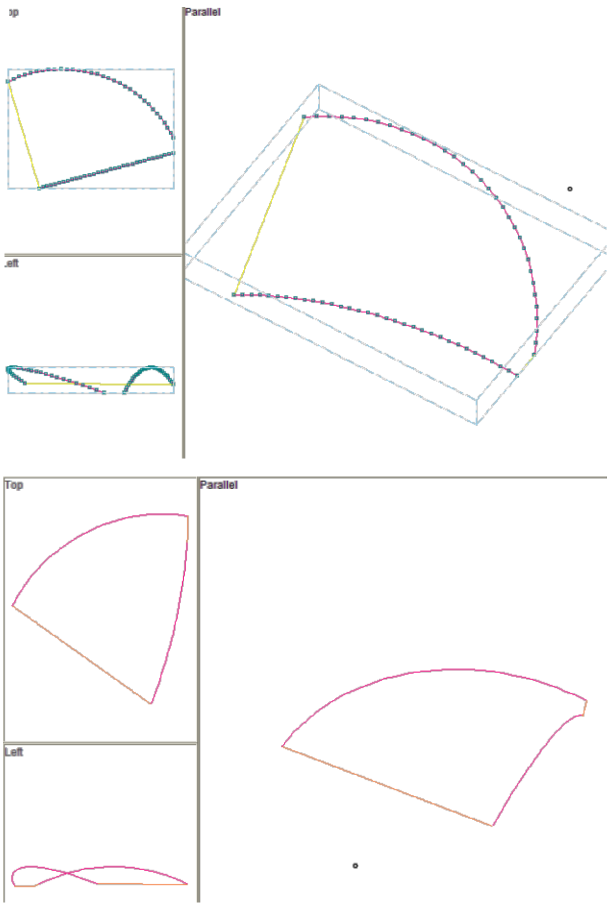


Figure 4.6 The boundaries of wings: (a) upper wing, (b) lower wing

The arches, cables and beams are made of steel and PVC is selected as membrane material. The material properties of steel and PVC are as follows:

- PVC (isotropic material): E_{warp} = 44 000kg/m, E_{weft} =39 500and density ρ = 0.9 kg/m^2.
- Steel : modulus of elasticity E = 210 GPa, Possion ratio ν =0.3 and density ρ = 7800 kg/m^3

The cable has a 16 mm diameter and a 157 mm^2 cross-sectional area. The cross-sectional dimension of steel tube which is used in arches and beams is shown in Figure 4.7.

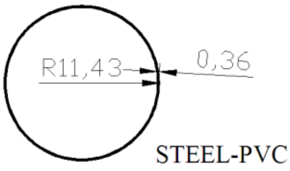


Figure 4.7 The cross sectional dimension of steel tube

The finite element meshing is made up of rectangular membrane elements to model the fabric, linear cable elements to model cables and linear beam elements to model beam and arches. The fine mesh is used in order to minimize the computation error. The hoops and fabric tips connected to the structure are considered rigid. For the boltropes, it is assumed that there is no slippage between the cable and the fabric. The meshing for form finding is presented in Figure 4.8.

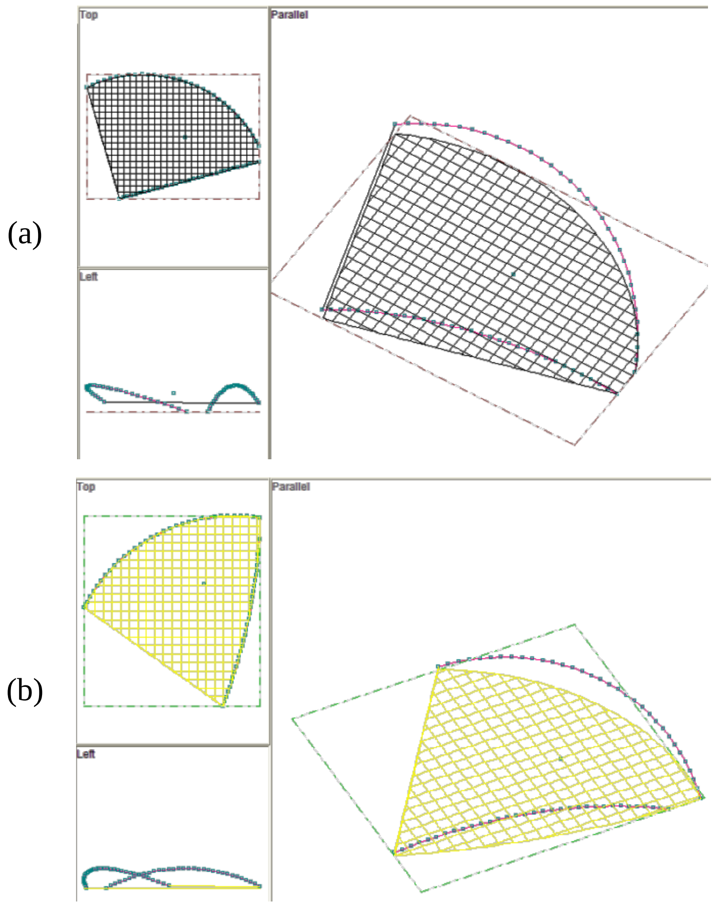


Figure 4.8 The boundaries and meshing for form finding: (a) upper wing, (b) lower wing

The form finding is performed by defining various initial prestress values for fabric (in warp and weft direction) and cable. After so many trials, an initial prestress of 75 kg/m, in warp and weft direction, is used to the fabric. The prestress value applied to cables is equal to 20 kN/m. The force density method is used to define a form in initial equilibrium. The equilibrium configuration of upper and lower wind is shown in Figure 4.9.

(a)

(b)

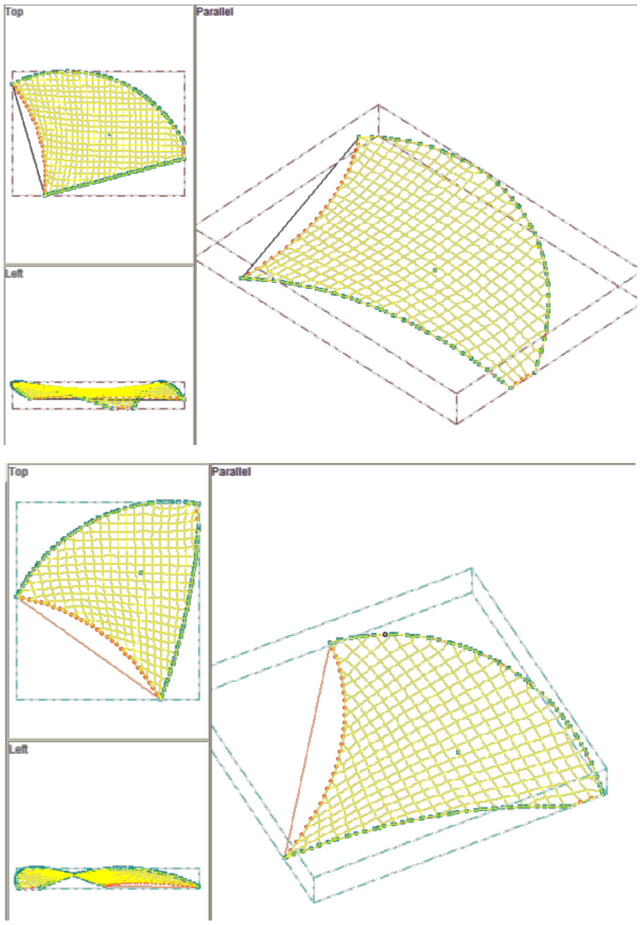


Figure 4.9 The general view of structure after form finding: (a) upper wing, (b) lower wing

Figure 4.10 shows the major stress distribution (σ_{11}) in the initial configuration. Their values vary in an interval of (0, 4.09) N/mm in upper wing and (0, 3.35) N/mm in lower wing, which is normal since this form does not a minimum surface area. Under the action of the prestresses, the maximum value of the stress can reach 21 N/mm in areas around the hoops. It is also noticed that a smooth appearance is obtained at the surface of the butterfly structure.

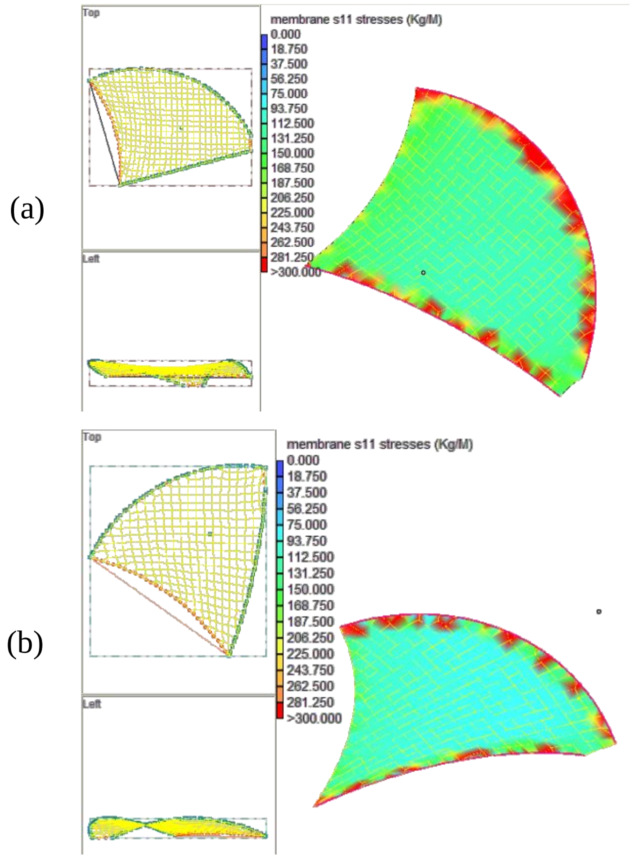
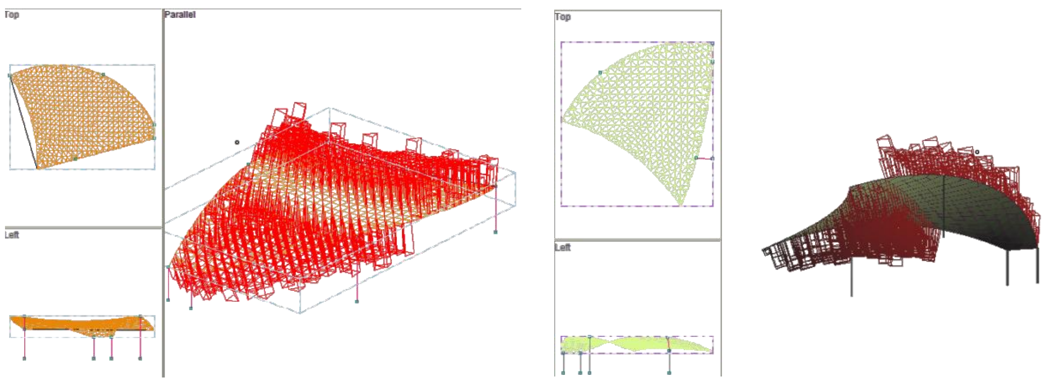


Figure 4.10 The membrane stress (σ_{11}) of form-finding: (a) upper wing, (b) lower wing

4.1.3 Structural Analysis

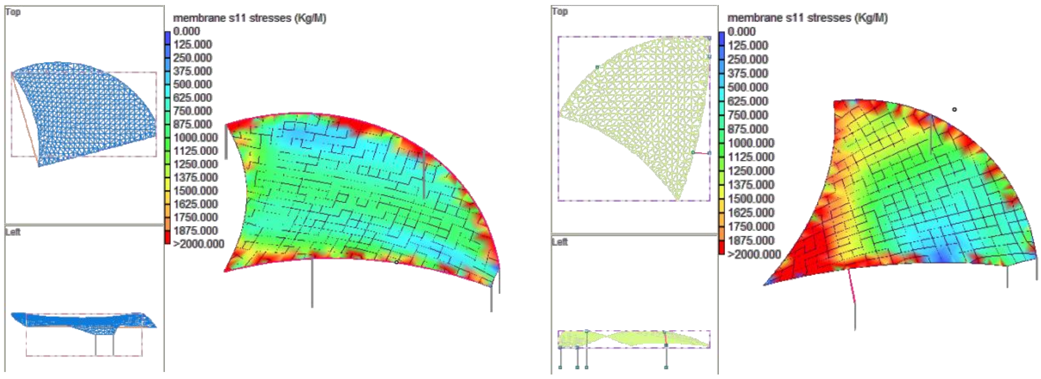
We will now study the analysis of butterfly structure for a single load case (wind load), as follows: prestress combined with an equivalent pressure of ±600 N/m see Figure 4.11.



Upper wing Lower wing

Figure 4.11 Distribution of wind load

Figure 4.12 shows the distribution of the principal stress component generated by the loads. The areas in red represent the maximum values reaching up to −13.18 kN/m in upper wing and 28.87 kN/m in lower wing.



Upper wing Lower wing

Figure 4.12 Principal stress distributions corresponding to nonlinear structural analysis

Our objective is translated by integrating limitations on the stress values, that is:

$$0 \text{ N/mm} < \sigma < 40 \text{ kN/mm}$$

We must emphasize that these limitations should not be taken in a strict sense but should be considered as objectives to be met. In the present study, the maximum principal stress values are within the specified stress limits.

If the principal stress values are above the critical level of stress, this could lead to premature fatigue of the material. Therefore, while guaranteeing minimum rigidity, we should reduce the prestress level of the fabric and cable.

4.1.4 Pattern generation

Figures 4.13 and 4.14 represent the cutting patterns for upper and lower wings respectively. The cutting patterns obtained depend on the materials used and generate the stress state as imposed in the spatial configuration.

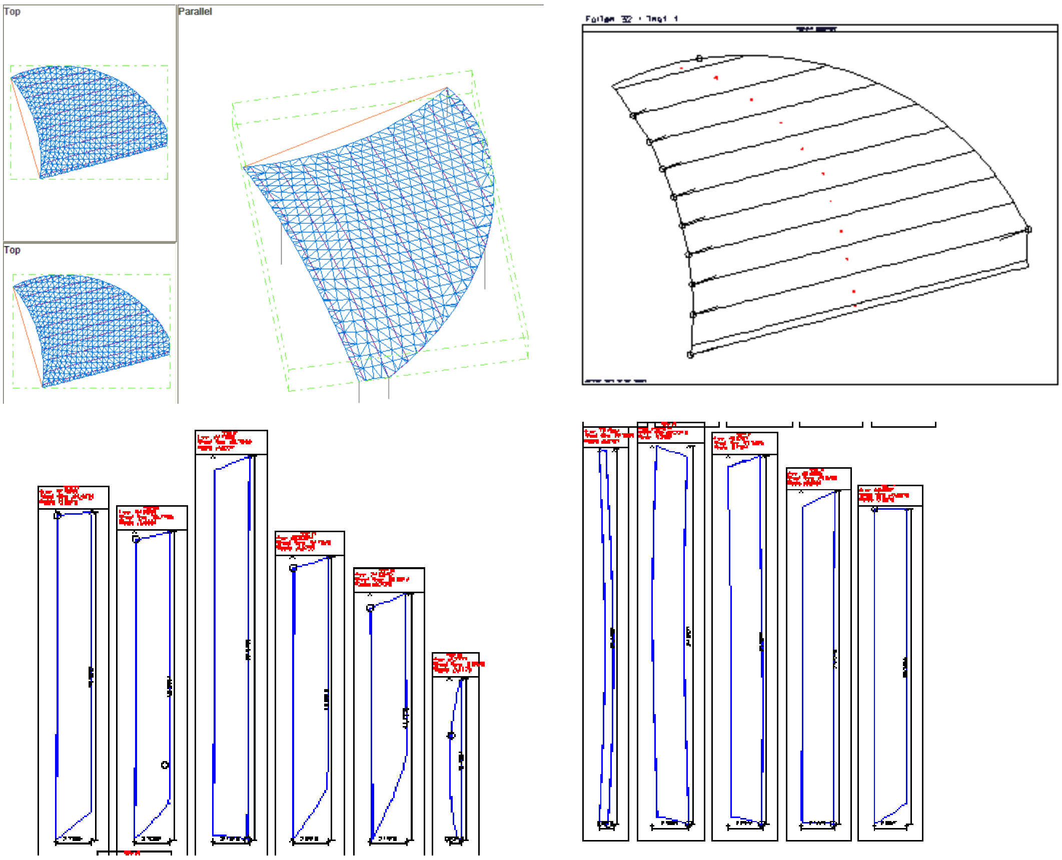


Figure 4.13 The cutting pattern for upper wing

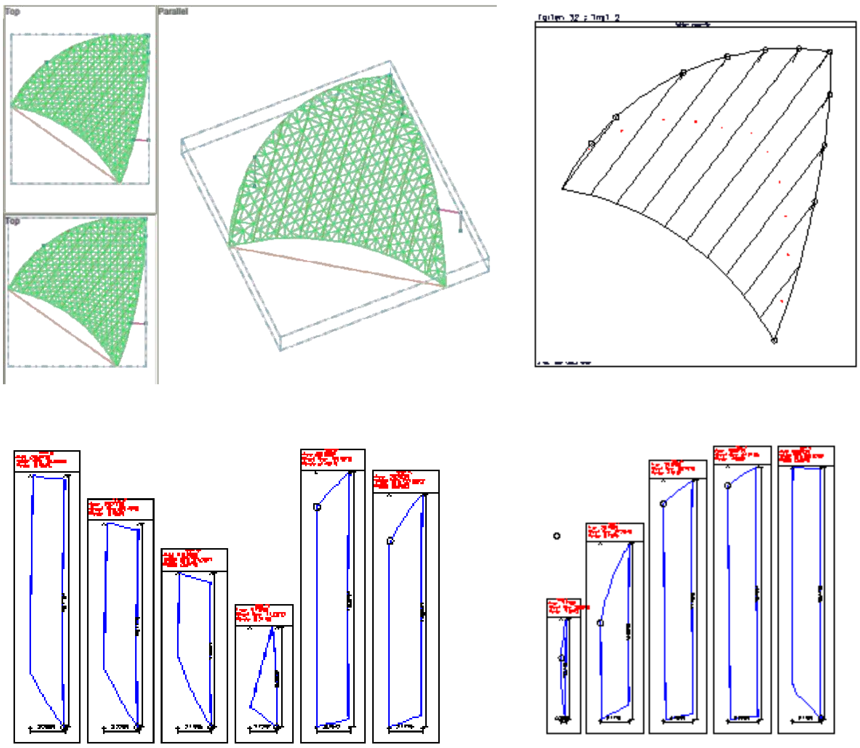


Figure 4.14 The cutting pattern for upper wing

4.1.5 Positioning of design on layout plan

The final design is placed on the layout plan. Figure 4.15 show the design on layout plan. It is concluded that all the objectives specified in conceptual design is satisfied. It is believed that final design can easily be fabricated and erected in site.

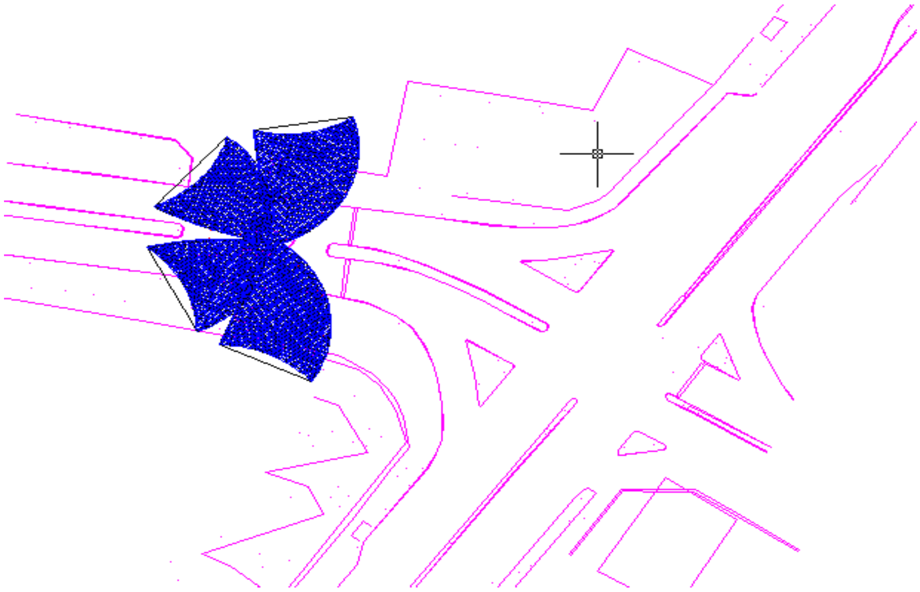


Figure 4.15 Placement of butterfly structure on layout plan

4.2 Sunshade Lodge in Hospital Park

The second case study deals with the sunshade lodge in hospital park. The layout plan of the university hospital park is shown in Figure 4.16. The estimated area to be covered is approximately 300 m^2.

The sunshade lodge in Hospital Park should be well designed structural forms and reflect the university image. The structures are designed so that the fabric can be removed if there is danger of a hurricane. This structure is selected to cover areas at very competitive costs per unit area. The sunshade lodge must protect the patient and their family from sun and rain. The project is inspired by chamomile flower. The exact covered area is 298 m^2

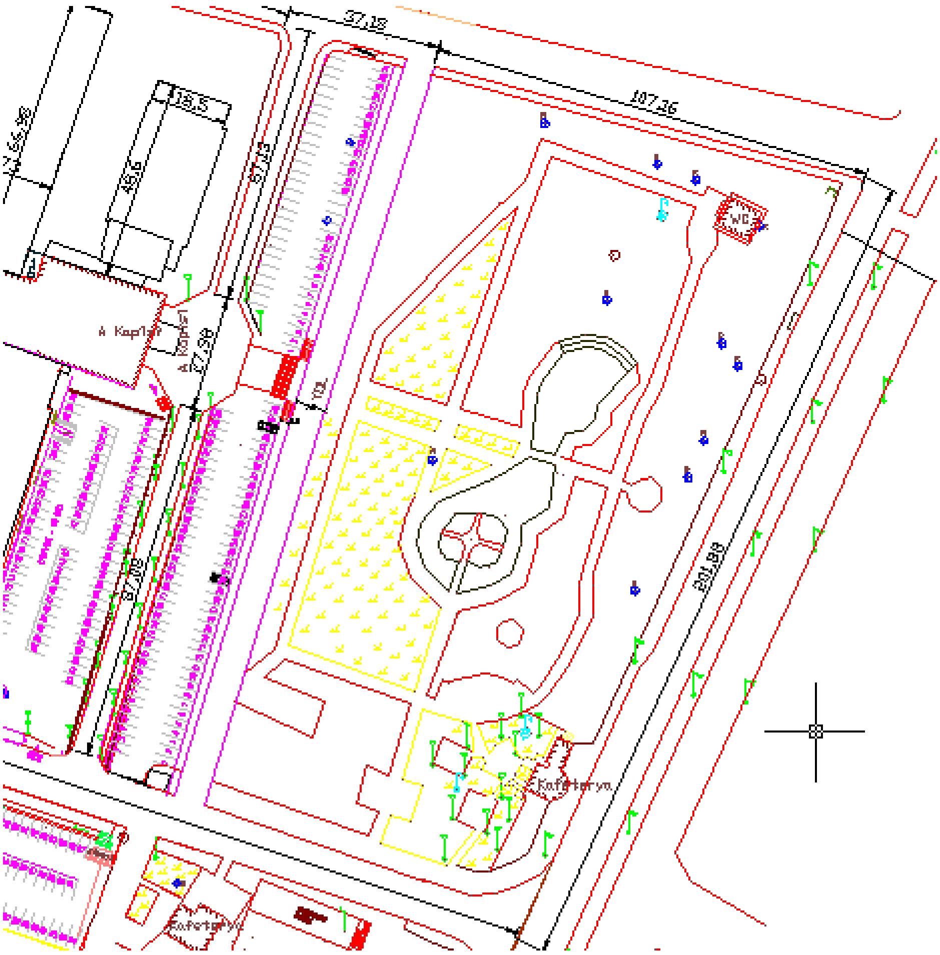
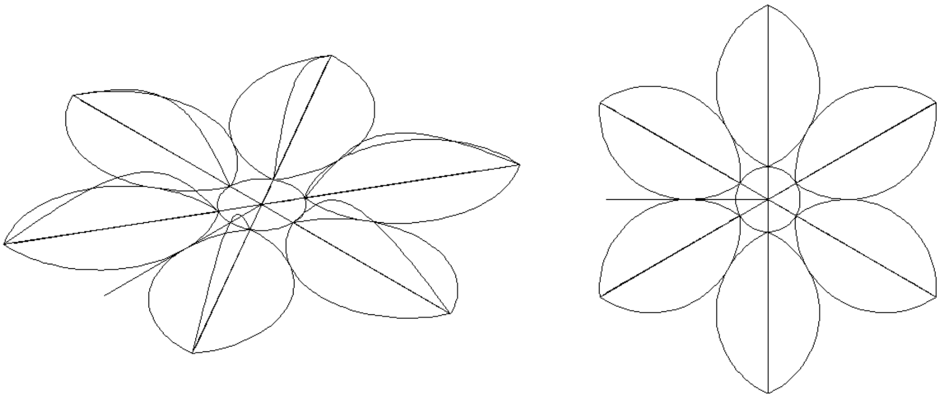


Figure 4.16 University of Gaziantep hospital park layout plan

4.2.1 Conceptual design

In this design, a chamomile flower is taken as a model for conceptual design. Chamomile structure is formed by arches and the membrane. The key concept of the structure is to use inclined arches to form the membrane boundary.

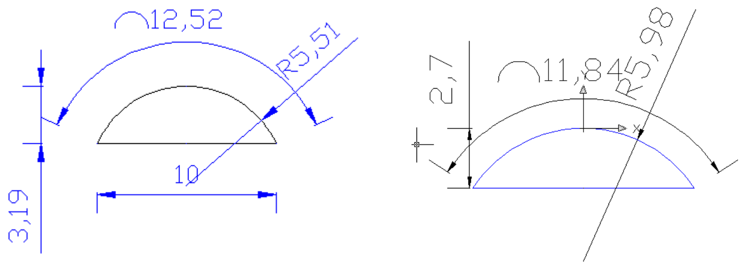
The conceptual chamomile design model is the one with six leafs and a circle as shown in Figure 4.17. By taking the advantage of symmetry, only one wing is modeled. This wing is split into two parts which are named as upper wing and lower wing.



Perspective view Top view

Figure 4.17 The conceptual flower design model

The geometry and dimension of a half of leaf are given in Figure 4.18. By taking the advantage of symmetry, only one half of the leaf is modeled. A leaf covers 49.67 m^2 area in plane and has a span length of 10.0 m, width of 6.38 m and height of 2.7 m.



Top view Front view

Figure 4.18 The geometry and dimension of flower

4.1.2 Form-finding

Each leaf consists of three arches and a membrane cover. The arches are connected to each other at the ends. The leaf has symmetry. Membrane is stretched along these arches to provide space enclosure. Each leaf is supported by two columns. Figure 4.19 show the boundaries of half of the leaf.

The arches are made of steel and PVC is selected as membrane material. The material properties of steel and PVC are as follows:

- PVC (isotropic material): E_{warp} =44 000 kg/m, E_{weft} =39 500 and density ρ =0.9 kg/m^2.

- Steel: modulus of elasticity E = 210 GPa, Possion's Ratio ν =0.3 and density $\rho = 7800$ kg/m^3

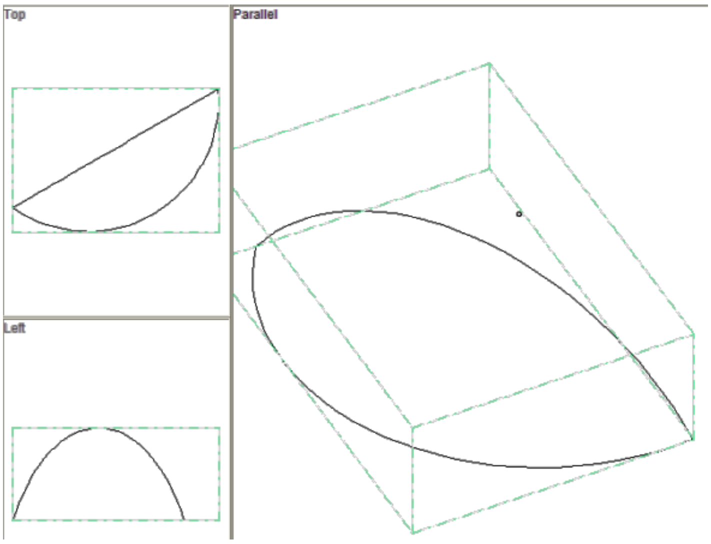
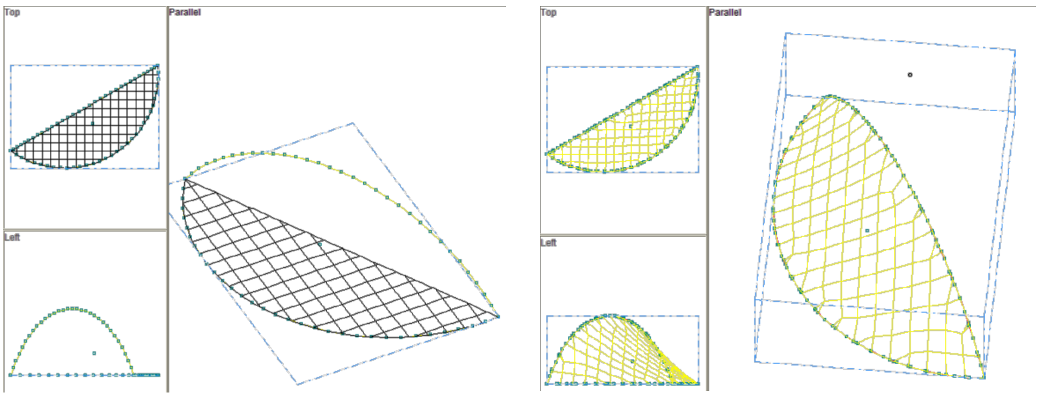


Figure 4.19 The boundaries and meshing for form finding

The cross-sectional dimension of steel tube which is used in arches is shown in Figure 4.7. The finite element meshing is made up of rectangular membrane elements to model the fabric and linear beam elements to model arches. The fine mesh is used in order to minimize the computation error. The hoops and fabric tips connected to the structure are considered rigid. The meshing for form finding is presented in Figure 4.20.



Gridding | Final shape after form finding

Figure 4.20 The general view of structure

The form finding is performed by defining various initial prestress values for fabric (in warp and weft direction). The prestress of 100 kg/m is applied to fabric and corresponding stress distribution and form of structure is shown in Figure 4.21. But it is found that stress values are above the 1/20 of tensile strength. After so many trials, an initial prestress of 50 kg/m, in warp and weft direction, is used. The force density method is used to define a form in initial equilibrium. The equilibrium configuration of structure corresponding to the prestress value of 50 kg/m is shown in Figure 4.22.

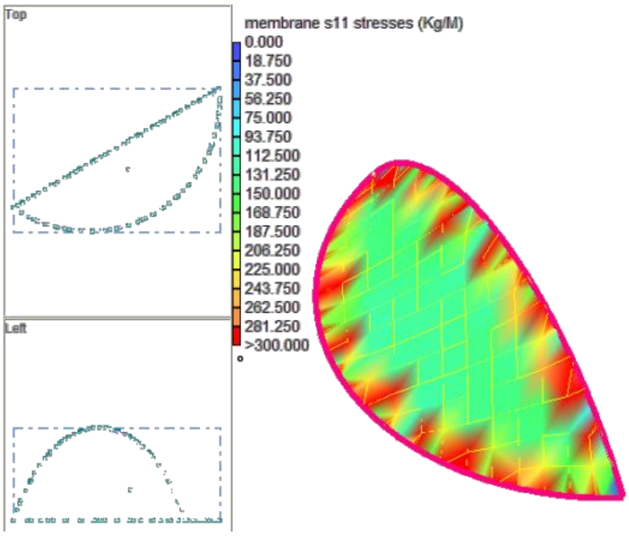


Figure 4.21 The general view of structure after form finding: Prestress value is equal to 100 kg/m

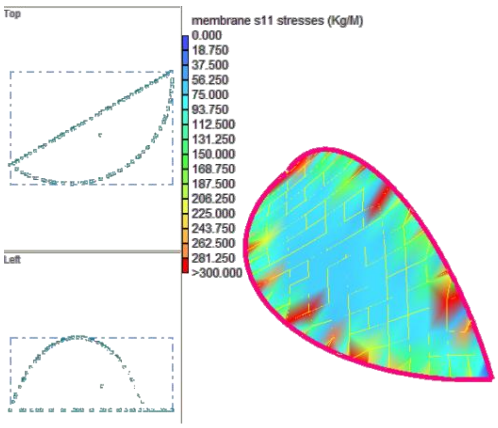


Figure 4.22 Final form of structure: Prestress value is equal to 50 kg/m

4.1.3 Structural Analysis

We will now study the analysis of leaf structure for a single load case (wind load), as follows: prestress combined with an equivalent pressure of ±600 N/m see Figure 4.23.

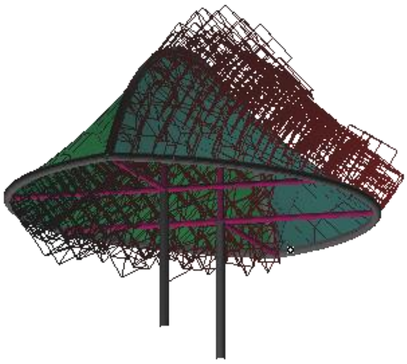


Figure 4.23 Distribution of wind load

Figure 4.24 shows the distribution of the principal stress component generated by the loads. The areas in red represent the maximum values reaching up to 0.88 kN/m. Our objective is translated by integrating limitations on the stress values, that is:

$$0 \text{ N/mm} < \sigma < 40 \text{ kN/mm}$$

We must emphasize that these limitations should not be taken in a strict sense but should be considered as objectives to be met. In the present study, the maximum principal stress values are within the specified stress limits

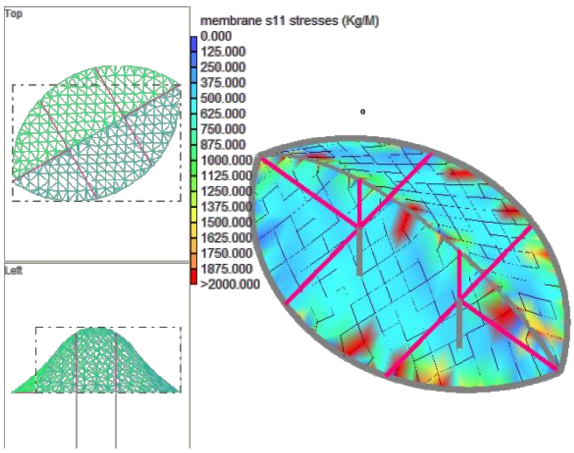


Figure 4.24 Principal stress distributions corresponding to nonlinear structural analysis

4.1.4 Pattern generation

Figure 4.25 represents the cutting patterns of leaf structure. The cutting patterns obtained depend on the materials used and generate the stress state as imposed in the spatial configuration.

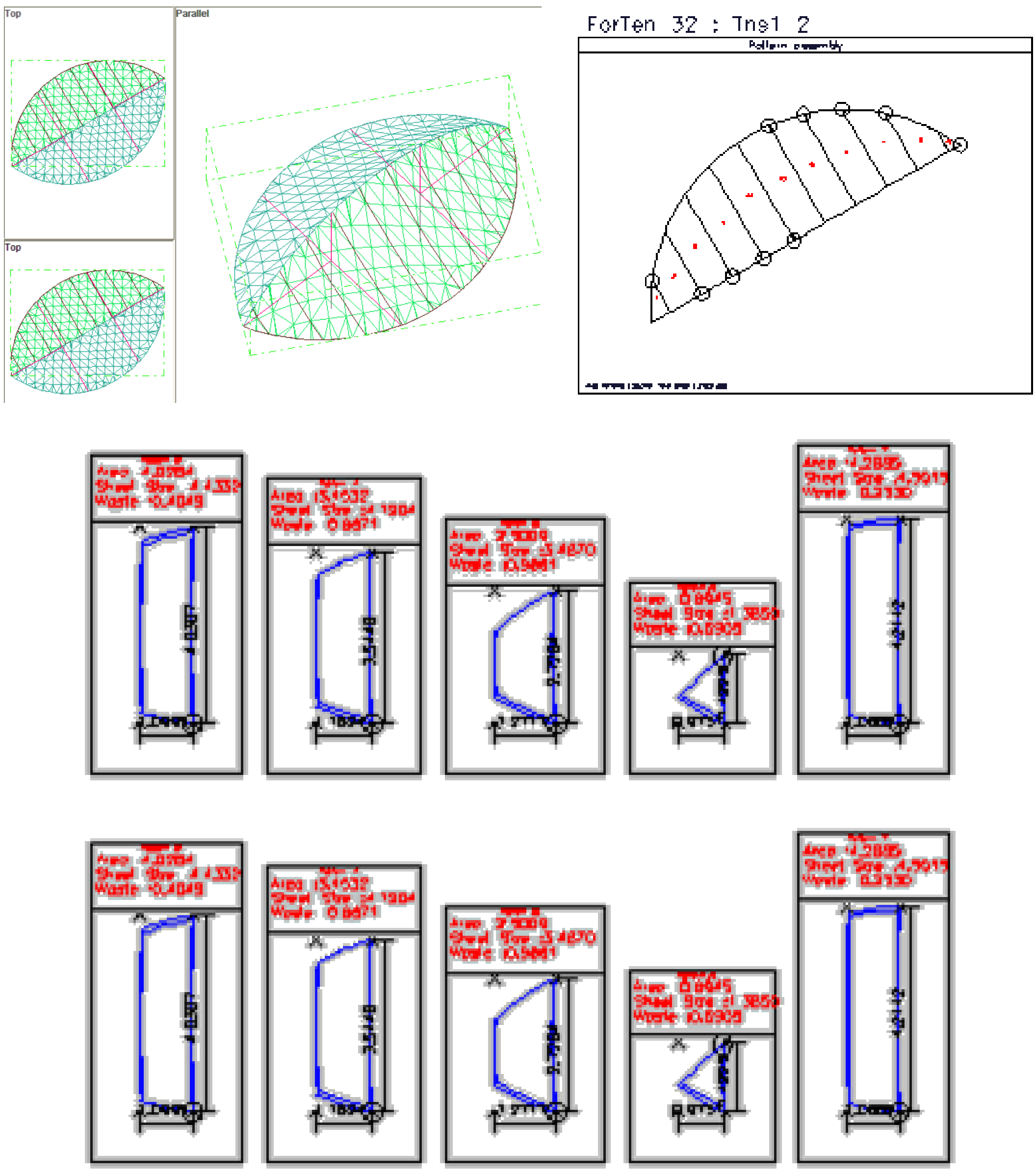


Figure 4.25 The cutting pattern

4.1.5 Positioning of design on layout plan

The final design is placed on the layout plan. Figure 4.26 show the design on layout plan. It is concluded that all the objectives specified in conceptual design is satisfied. It is believed that final design can easily be fabricated and erected in site.

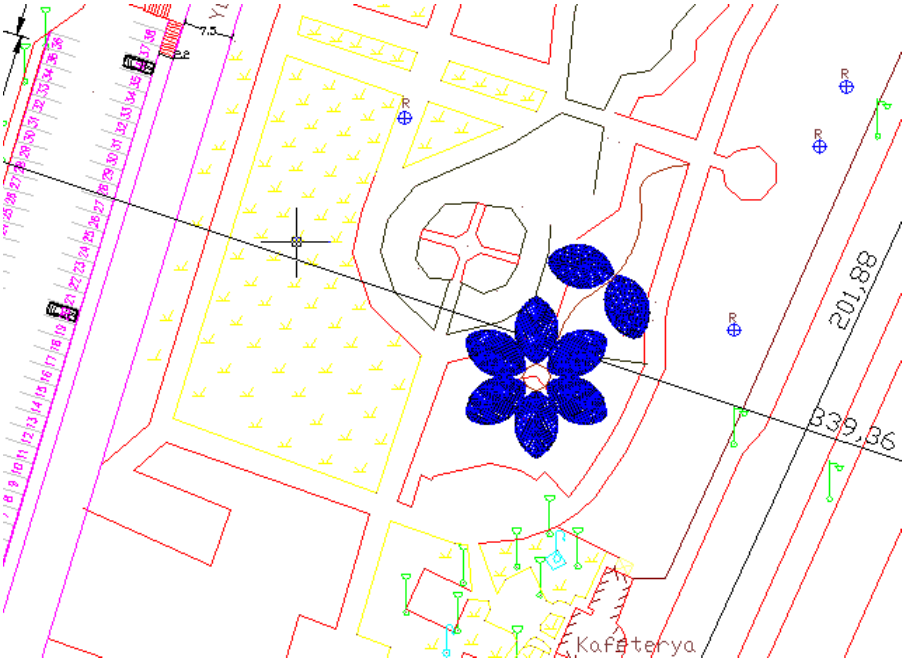


Figure 4.26 Placement of sunshade lodge on layout plan

CHAPTER 5

CONCLUSION

5.1 Summary

Tensile membrane structures represent a new chapter in the history of building structures. Capable of spanning large distances while incurring very little weight on supporting structure, developments in the design of tensile membrane structure can dramatically change the way we conceptualize permanent building construction. Though tensile membrane structure, computational analysis techniques, and construction methods have come a long way since the first modern fabric structure was built fifty years ago, there are still several challenges to be overcome before tensile membrane structure can be considered a viable option for the majority of new building projects.

Fabric materials and their associated properties continue to adapt to new and interesting problems. Though they exhibit drawbacks such as low durability due to UV degradation and fire resistance, they continue to improve in tensile and tearing strength capacities. Furthermore, increased light transmission and translucency allow for higher energy savings, a consideration that is coming ever more important to the green building industry.

The design and analysis process is perhaps the most limiting factor in the implementation of tensile membrane structure. Exhibiting highly nonlinear behavior, the behavior of fabric under applied loads is often difficult to understand and to model. Beginning with simple fabric models in the days of Frei Otto at the Institute of Lightweight Structures, analysis theories and methods have adapted into complex computational capable of quickly and accurately determining the form and behavior of a fabric membrane.

The advent of computer technology changed tensile architecture for ever. The possibilities of accurately calculating form and values for ever more complicated project proposals opened the door to an architectural language which we are still embracing end expanding to this day. Since then there has been a steady procession of projects which have pushed the boundary of this technology.

Small changes to the dimensions or specifications of a fabric structure usually result in major or complete re-design. Tensile membrane structures require many considerations, once designed any changes to tension shape require major and costly redesign. Unlike conventional structures, fabric structures do not allow changes at any point in the x, y, of z direction without complete redesign.

In this study, two tensile membrane structures are designed using commercial software. The program used in this study is quite simple to generate and modify shapes in real time, assign material, loads or modify the boundary conditions of the model. A designer can get accustomed to the application in very little time. However, program has certain limitation and does not have any optimization module.

5.2 Future works

The use of ‘black box’ software should be strongly resisted. It is necessary to develop an engineering analysis and design software for further study on tensile membrane structures. The key points in the development of engineering analysis software for tensile membrane structures are that it should not place any limitations on the design process, should have a clear physical analogy, should integrate computer aided geometry design, structural analysis and optimization methods, and most importantly, be fully understood by the engineer.

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