UNIVERSITY OF GAZİANTEP
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FREE VIBRATION ANALYSIS AND STRUCTURAL OPTIMIZATION OF STIFFENED PANELS

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IN
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Free Vibration Analysis and Structural Optimization of Stiffened Panels

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Supervisor
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by
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ABSTRACT

FREE VIBRATION ANALYSIS AND STRUCTURAL OPTIMIZATION OF STIFFENED PANELS

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In this thesis, free vibration analysis and structural optimization of stiffened plates was studied. Different types of straight stiffeners were used. Examined plate types have common length, width and volume constraints. In each stiffener type, some combinations of pad elements, sub stiffener elements were used. Vibration analyses of plates were carried out using a Fortran computer code which is based on Finite Strip method and developed by Özakça[1]. Optimization of plates was carried out with same program, which uses SQP as optimization tool. By these applications the effectiveness of four plate types using straight stiffener types were investigated. Totally 168 runs were carried out for this purpose. The vibration optimization results are fluctuating in a wide range due to used elements combinations listed above and number of stiffeners. Improvements due to used element type and number of stiffeners are listed and compared according to stiffener types.

Key Words: Stiffened plates, Free vibration analysis, Structural optimization, Finite strip method.

ÖZET

TAKVIYELI PANELLERIN SERBEST TITREŞIM ANALIZI VE YAPI OPTIMIZASYONU

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Bu tezde takviyeli plakaların serbest titreşim analizi ve yapısal optimizasyonu çalışılmıştır. Farklı düz takviyeli eleman çeşidi kullanılmıştır. İncelenen plaka tiplerinin uzunluk, genişlik ve hacim ortak kısıtları vardır. Her çeşit takviyeli plaka tipinde yastık, ara takviye gibi bazı kombinasyonlar kullanılmıştır. Plakaların titreşim analizi Özakça[1] tarafından geliştirilen Sonlu Şeritler metodu tabanlı bir FORTRAN yazılımıyla gerçekleştirilmiştir. Plakaların optimizasyon işlemi de aynı program tarafından Ardışık Karesel Programlama algoritması kullanılarak gerçekleştirilmiştir. Bu çerçevede düz takviye elemanları kullanılarak dört tip plakanın serbest titreşim analizi incelenmiştir. Bu amaçla 168 tane plağın analizi ve optimizasyonu yapılmıştır. İncelenen plakların titreşim optimizasyon sonuçları kullanılan eleman kombinasyonuna ve takviye elemanları sayısına göre geniş bir aralık içerisinde dalgalanmaktadır. Eleman tiplerine ve takviye elemanlarının sayısına göre serbest titreşim analizindeki iyileşmeler gözlenip kullanılan eleman ve takviye eleman tipi ile ilişkilendirilip karşılaştırmalar yapılmıştır.

Anahtar kelimeler: Takviyeli plakalar, Serbest titreşim analizi, Yapısal optimizasyon, Sonlu şeritler yöntemi.

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LIST OF SYMBOLS/ABBREVIATIONS

FE	Finite Element
FS	Finite Strip
SQP	Sequential quatratic programming
DVs	Design variables

Scalar

A	Area
b	Length of strip
C_p, S_p	Cosine and sine fuction
C(0)	Order of continuity
d	Displacement
E	Young’s modulus
F(s)	Objective function to be minimized
$g_j(s)$	Inequality constraint function
$h_k(s)$	Equality constraint function
J	jacobian
ℓ	The arc length parameter of the curve
(m,n)	Mode and half-sine wave
N_i	Shape function associated with node *i*
R	Radius of curvature
S	Design variables

t	Thickness
u_ℓ, v_ℓ, w_ℓ	Displacement components in ℓ, y and n-direction
u, v, w	Global displacement parameters
u^p, v^p, w^p	Displacement amplitudes of p^{th} harmonic
V	Volume of the structure
U	Total strain energy
V_g	Potential energy of volume
P_{cr}	Critical buckling load
P_0	Initial load

Vector

$\mathbf{d}$	Vector of unknown displacements (eigenvector)
$\boldsymbol{d_i^p}$	Vector of nodal degrees of freedom
$\boldsymbol{d_i^e}$	Displacement (eigenvector) vector associated with element e and node *i*
$\boldsymbol{\bar{d}_i^p}$	Displacement (eigenvector) vector at node *I* and harmonic *p*
$\mathbf{s}$	Design variable vector
$\boldsymbol{\varepsilon_m}$	Inplane strains
$\boldsymbol{\varepsilon_b}$	Bending strain
$\boldsymbol{\varepsilon_s}$	Transverse shear strains
$\boldsymbol{\varepsilon_\ell^{nl}, \varepsilon_y^{nl}, \gamma_{\ell y}^{nl}}$	Second order strains
$\boldsymbol{\sigma_m^p, \sigma_b^p, \sigma_s^p}$	Membrane, bending and shear stress resultant vectors
$\boldsymbol{\sigma_\ell^0, \sigma_y^0, \tau_{\ell y}^0}$	Applied inplane stress

Matrix

B	Strain-displacement matrix
$\boldsymbol{B}_{mi}^{\ell}$	Membrane strain-displacement matrix for element *e* and node *i*
$\boldsymbol{B}_{bi}^{\ell}$	Bending strain-displacement matrix for element *e* and node *i*
$\boldsymbol{B}_{si}^{\ell}$	Shear strain-displacement matrix for element *e* and node *i*
$\boldsymbol{B}_{mi}^{p}$	Membrane strain-displacement matrix for node *i* and harmonic *p*
$\boldsymbol{B}_{si}^{p}$	Shear strain-displacement matrix for node *i* and harmonic *p*
$\boldsymbol{B}_{si}$	Shear strain displacement matrix
D	Matrix of rigidities
$\mathbf{D_m}$, $\mathbf{D_b}$, $\mathbf{D_s}$	Matrices of membrane, bending and shear rigidities
J	Jocobian matrix
$\boldsymbol{K}_{ij}^{\ell}$	Stiffness matrix associated with element *e* and node *I* and *j*
$[\mathbf{K}]^{\mathbf{pp}}$	Global stiffness matrix associated with harmonic *p*
$[\boldsymbol{K}_{ij}^{\ell}]^{\mathbf{pq}}$	Stiffness matrix linking nodes *I* and *j* and harmonics *p* and *q*
$\boldsymbol{K}_{mij}^{\ell}$	Membrane stiffeners matrix for element *e* and node *i* and *j*
$\boldsymbol{K}_{sij}^{\ell}$	Shear stiffeners matrix for element *e* and node *i* and j
N	Shape function matrix

Greek Symbols

α	Angle between local and global axes
δ_1	Mesh density
Δs_k	Small perturbation of design variables s_k
ε_ℓ, ε_ℓ	Strain in ℓ direction and longitudinal strain
$\gamma_{\ell y}$, γ_{yn}	Shear strain

κ	Shear modification factor
κ_ℓ	Curvature in the ℓ-direction
κ_y	Longitudinal curvature
$\kappa_{\ell y}$	Twisting curvature
λ^p	Load factor (eigenvalues)
ν	Poisson's ratio
$\text{Ø},\psi$	Rotation of the midsurface normal in the ℓ n and yn plane
$\text{Ø}^p,\psi^p$	Rotation amplitude for the p^{th} harmonic term
ξ	Isoparametric element natural (curvilinear) coordinate
σ	Stress component
$\partial\ell$	Partial differential of ℓ

CHAPTER 1

INTRODUCTION

1.1 General Information

In structural engineering, the crucial aim is weight saving for a structural element without loss of any strength. Two dimensional behaviors of flat plates is strengthened in third direction against vibration by adding longitudinal stiffeners to flat plate surface. Characteristics of stiffened plates which are vibration, stability, strength and deformation have been extensively studied since the applications of stiffened plates have been widely used in engineering structures, such as ships, aircrafts and bridges. This investigation has focused on optimization of stiffened plates under the free vibration.

When straight plates are stiffened with longitudinal stiffeners their response against free vibration become more complex. Analytical solutions for those types of structures become insufficient and tedious. In this regard, numerical solutions are the best approaches.

There are several numerical methods, which are applicable in structural analysis. The most effective of them is the Finite Element (FE) method, which is developed in 1960's. By the development of FE method, researchers attempt to apply this method in all parts of structural analysis. Observations verified that FE method gives excellent results compared with analytical solutions.

In the following years, Finite Strip (FS) method is developed by Cheung [2] and the method has the capability of solving structural analysis problems that have prismatic shape and simple supported boundary conditions. FS forges fewer equations to be solved than FE. As result of this reduction, FS method is faster than FE method. The extensively comparison between FE and FS is introduced in following chapter.

	Finite Element	Finite Strip
Applicability to structures	*Applicable to any geometry, boundary conditions and material variation. Extremely versatile and powerful.*	*In static analysis, more often used for structures with two opposite simply supported ends and with or without intermediate elastic supports, especially for bridges. In dynamic analysis it is used for structures with all boundary conditions but without discrete supports.*
Required equ. to be solved	*Usually large number of equations and matrix with comparatively large bandwith. Can be very expensive and at times impossible to work out solution because of limitation in computing facilities.*	*Usually much smaller number of equations and matrix with narrow bandwidth, especially true for problems with an opposite pair of simply supported ends. Consequently much shorter computing time for solution of comparable accuracy.*
Input data	*Large quantities of input data and easier to make mistakes. Requires automatic mesh and load generating schemes.*	*Very small amount of input data because of the small number of mesh lines involved due to the reduction in dimensional analysis.*
Output data	*Large quantities of output because as a rule all nodal displacements and element stresses are printed. Also many lower order elements will not yield correct stress at the nodes and stress averaging or interpretation of results.*	*Easy to specify only those locations at which displacements and stresses are required and then output accordingly.*
Required computer effort	*Requires a large amount of core and is more difficult to program. Very often, advanced techniques such as mass condensation or subspace iteration have to be resorted to for eigenvalue problems in order to reduce core requirements.*	*Requires smaller amount of core and easier to program. Because only the lowest few eigenvalues are required (for most cases anyway), the first two to three terms of the series will normally yield sufficiently accurate results matrix can usually be solved by standard eigenvalue subroutines.*

Table 1.1 Comparison between FE and FS methods [2]

1.2 Stiffened Plate Terminology

The structural elements of stiffened plates that examined in this thesis are shown in Figure 1.1.

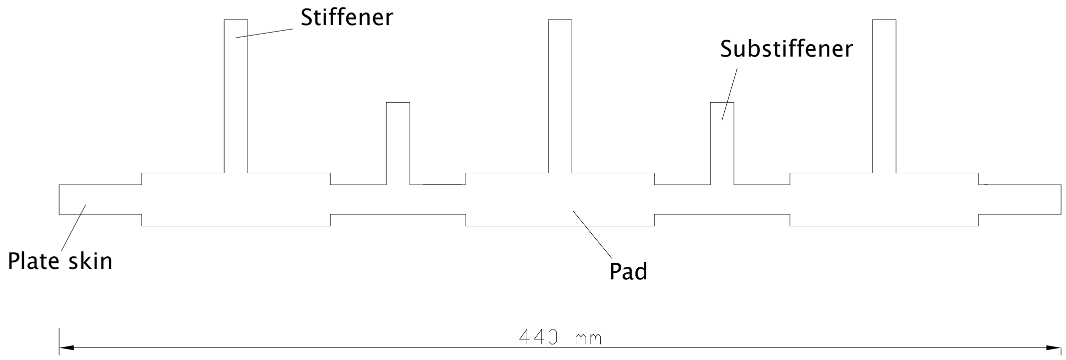

Figure 1.1 Structural Plate Elements

The structural plate elements are examined using the combinations of those elements. Number of stiffeners is also changed in the range of two to eight, for instance, Figure 1.1 shows stiffened plate with three stiffeners.

1.3 Principle Objectives

The critical motivation of the thesis is structural shape and structural size optimization of stiffened plates including some combinations of structural elements shown above, using a computer code. The specific objective may be expressed as follows:

- Maximizing the eigenvalue of considered stiffened plates.
- Investigating the performance of each structural element.
- Observing the change in element shape during optimization procedure to remark the efficiency of each structural element.
- Receiving the best geometric shape and thickness variation for the considered stiffened plate.

1.4 About Computer program

A free vibration analysis using FS method and shape optimization with (SQP) programs for straight plan form folded plates and shells were developed by Özakça [1] in FORTRAN programming language using double precision. The latest version of program is used in FS analysis and structural optimization of stiffened plates.

1.5 Layout of Thesis

The contents of each chapter are expressed as:

- Chapter 2 contains literature survey about free vibration analysis and shape optimization.
- Chapter 3 includes a condensed derivation of FS equations and examples.
- Optimization process, definition of elements and design variables, structural optimization flowchart are presented in Chapter 4.
- Chapter 5 deals with results of analyses and remarks according to results.
- In Chapter 6, conclusions based on the present thesis are underlined and suggestions for future work are discussed.

CHAPTER 2

LITERATURE SURVEY

2.1 Introduction

Stiffened panel are widely used in many engineering applications. A stiffened plate has low mass and high bending stiffness. The use of welding made it possible to produce different constructions. To increase the torsional rigidity, cellular plates have been introduced. Stiffened plates can be applied as roof structures of supermarkets, petrol stations, etc., orthotropic bridge decks, airplane wing structures, ship wall and deck structures, roof structure of tanks. By reason of two-dimensional actions of plates, they have widely use in engineering applications. Also two-dimensional behaviour of plates has several advantages as a structural element. This behaviour requires more complex analysis methods. Effectively methods such as FE and FS method should be performed according to problem behaviour and structural type.

2.2 Basics of Free-Vibration

Free vibration analysis of structures plays an important role in engineering applications as plates are widely used as structural components. Due to limitations of analytical methods for practical applications, numerical methods have become the most widely used tool for designing plate structures. One of the most popular numerical approaches for analysing vibration characteristics of plates is the Finite Element Method (FEM). Although the FEM provides a general and systematic technique for constructing basis functions, a number of difficulties still exist in the development of plate elements based on shear deformation theories. One is the shear locking phenomenon for low order displacement models based on Mindlin Reissner theory [3] as the plate thickness decreases.

2.3 Free Vibrational Analysis Methods

If the number of stiffeners is small, the stiffened plate can be divided into beam-like grid structures (Figure 2.1). This calculation is based on force method. The torsional stiffness can be neglected. The deflections at the nodes should be equal for the two orthogonal beams. The unknown internal forces can be calculated from the deflection equations. This is called grid calculation. [5]

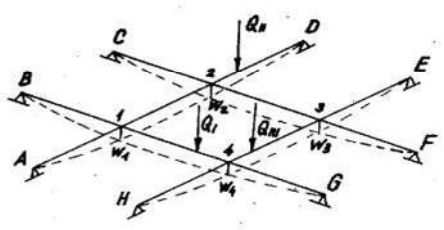

Figure 2.1 Grid Calculation

In Calculation as an anisotropic continuum [5]; There are some assumptions which are as follows: elastic stress and deformations, and deflections are small compared to the thickness of the plate, and normal stresses orthogonal to the plate can be neglected, and shear deformations can be neglected, and stresses from torsion can be calculated from Saint-Venant theory [5], and number of stiffeners in both directions is large enough to assume that the effective plate width is equal to the distance between stiffeners.

At Solution Methods; At least four different procedures have been employed for obtaining the structural behaviour of stiffened plate panels under normal (lateral) loading, each embodying certain simplifying assumptions: Orthotropic plate theory, Beam-on-elastic-foundation theory, Grillage theory, the FEM. [6]

Orthotropic plate theory [6] refers to the theory of bending of plates having different flexural rigidities in the two orthogonal directions. In applying this theory to panels having discrete stiffeners we idealize the structure by assuming that the structural properties of the stiffeners may be approximated by their average values, which are assumed to be distributed uniformly over the width or length of the plate. The deflections and stresses in the resulting continuum are then obtained from a solution of orthotropic plate deflection equation. The orthotropic plate method is best suited to a panel in which the stiffeners are uniform in size and spacing and closely spaced.

The beam-on-elastic-foundation [6] solution is suitable for a panel in which the stiffeners are uniform and closely spaced in one direction and more sparse the other. One of the latter members may be thought of as an individual beam having an elastic support at its point of intersection with each of the closely-spaced orthogonal beams. An average elastic modulus or spring constant per unit length may be determined by dividing the force per unit constant per unit length may be determined by dividing the force per unit deflection of one of these closely spaced members by the spacing. Using the average spring constant per unit length, the effect of the closely spaced members is then represented as an elastic support that is distributed evenly along the length of the widely spaced members. Each of these members is then treated individually as a beam on an elastic foundation.

In the grillage method [6] of Clarkson et al. (1959), each stiffener in the two orthogonal sets of members is represented as a simple beam. The external loading may be applied as a set of equivalent point forces at the intersections of the two beam systems. At these points of intersection conditions of equilibrium of the unknown reaction forces between the two beams, together with conditions of equal deflection, are required to be satisfied. The result is a system of algebraic equations to be solved for the deflections. From the solutions the forces in each set of beams and the resulting stresses may be obtained.

The FEM [6], is a versatile technique, may model the structure in a number of different ways. For example, each segment of stiffener between intersection points may be represented by a short beam, and the plating may be represented as a membrane capable of supporting in-plane stress as in the grillage technique. Conditions of equality of deflections and equilibrium of internal and external forces are then required to be satisfied at the points of intersection leading to the formulation of a system of simultaneous algebraic equations relating external loads to deflections. Machine computation is necessary in order to formulate and solve the large number of equations that are necessary in a practical situation. This procedure is the most general of the four, being virtually unrestricted in the degree to which complex structural geometry, variable member sizes, boundary conditions and load distributions can be represented.

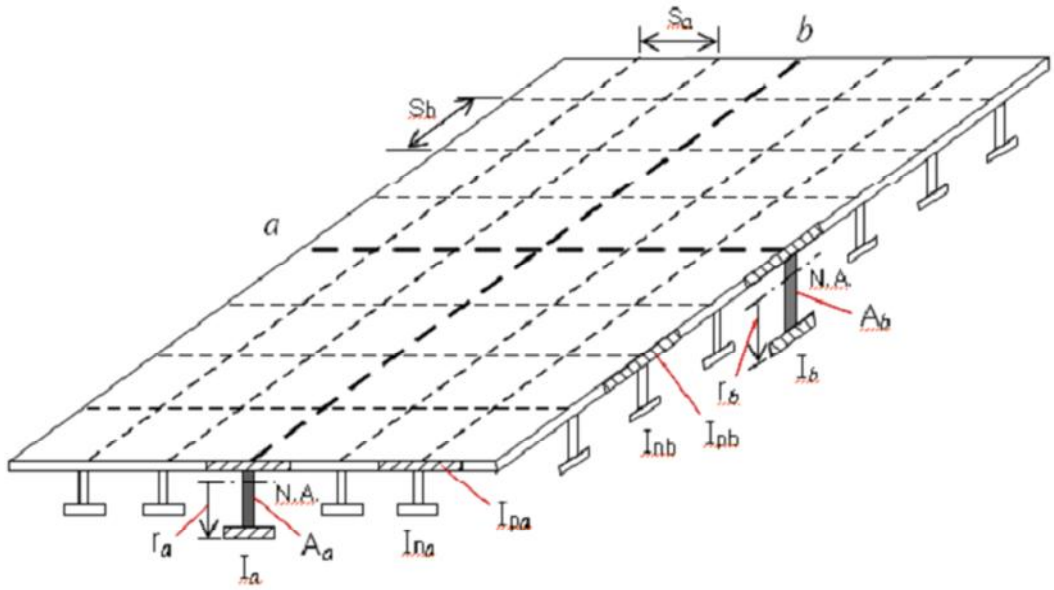

Figure 2.2 Stiffened plate nomenclatures

Z Canan Girgin; Konuralp Girgin [7] studied with A generalized numerical method is proposed to derive the static and dynamic stiffness matrices and to handle the nodal load vector for static analysis of non-uniform Timoshenko beam-columns under several effects. This method presents a unified approach based on effective utilization of the mohr method and focuses on the following arbitrarily variable characteristics: geometrical properties, bending and shear deformations, transverse and rotator inertia of mass, distributed and (or) concentrated axial and (or) transverse loads, and winkler foundation modulus and shear foundation modulus. A successive iterative algorithm is developed to comprise all these characteristics systematically. The algorithm enables a non-uniform Timoshenko beam-column to be regarded as a substructure.

L. Liu, G.R. Liu, V.B.C. Tan [8] investigated with both static deformation and free vibration analyses are considered. The formulation of the discrete system equations starts from the governing equations of stress resultant geometrically exact theory of shear flexible shells. Moving least squares approximation is used in both the construction of shape functions based on arbitrarily distributed nodes as well as in the surface approximation of general spatial shell geometry. Discrete system equations are obtained by incorporating these interpolations into the Galerkin weak form. The formulation is verified through numerical examples of static stress analysis and frequency analysis of spatial thin shell structures. For static load analysis, essential boundary conditions are enforced through penalty method and Lagrange multipliers while boundary conditions for frequency analysis are imposed through a weak form using orthogonal transformation techniques. The EFG results compare favorably with closed-form solutions and that of FE analyses.

Yao Koutsawa, El Mostafa Daya [9] developed static behavior and free vibration analysis of laminated glass beam on viscoelastic supports are performed. For the static case, an analytical way is developed for analyzing and optimization of laminated glass beam with general restraints at the boundaries. In the case of free linear vibrations, the modal properties of the glass are determined using a finite element method which is a powerful tool in the design of support damping treatment of a sandwich glass for passive vibration control.

G. Akhras, W. Li - Kingston, Ontario [10] studied with a spline FS method is developed for static and free vibration analysis of composite plates using Reddy's higher-order shear deformation theory. This analysis does not require shear correction coefficients, but yields improved accuracy for thick laminates. In this method, a consistent interpolation scheme is achieved for transverse shear strains. Thus, shear locking for thin plates is avoided. In addition, the selected shape functions can accurately simulate a linear variation of transverse bending moment in the transverse inplane direction for the laminates with bending and inplane coupling, so that the convergence is enhanced.

Guanghui Qing, Jiajun Qiu, Yanhong Liu [11] based on the semi-analytical solution of the state-vector equation theory, a novel mathematical model for free vibration analysis of stiffened laminated plates is developed by separate consideration of plate and stiffeners. The method accounts for the compatibility of displacements and stresses on the interface between the plate and stiffeners, the transverse shear deformation, and naturally the rotary inertia of the plate and stiffeners. Meanwhile, there is no restriction on the thickness of plate and the height of stiffeners.

Zhigang Yu, Xiaoli Guo, Fulei Chu [12] Formulations of a multivariable hierarchical beam element for static and vibration analysis are presented based on the generalized variational principle with two kinds of variables. Two forms of shifted Legendre hierarchical polynomials are used as interpolating basis functions of displacement and generalized force field functions for the beam element respectively, which will simplify the computations of the relevant matrices. The multi variable hierarchical beam element formulations, in which the displacement and generalized force field functions are independently constructed, are derived by applying the generalized variational principle with two kinds of variables. Since differential operations to obtain stress fields in conventional displacement based FEMs are not required, the present method has very high accuracy for the two kinds of independent variables simultaneously, especially for the generalized forces. Static and vibration numerical examples demonstrate the applicability of the proposed method. The proposed method can be easily extended to deal with structural analysis of shells or plates.

Cheung [2] tabulated a general comparison between FE method and FS method to detail applications, inputs and outputs of two methods that presented at Table 1.1

R.S. Srinivasan and V. Thiruvenkatachari [14] mention about a method for the static and dynamic analysis of eccentrically stiffened annular sector plates is presented. The plate is clamped on all the edges. The integral equation technique is adopted for the solution. In the static analysis the deflection and stresses at centre and the stresses at the edges are obtained and they are presented graphically. The results are compared for particular cases with those of other investigators who have used different analytical methods. The natural frequencies of stiffened clamped plates are also obtained for plates with different sector angles.

2.4 Optimization methods

The role of engineers, however, is not only to solve problems of analysis but mainly to provide designs, that is, to establish suitable solutions to physical problems, through proper choices of simplified models, structural schemes, materials to be employed, geometries to be adopted, and so on, in order to fulfil requirements. In common practice, this task is usually achieved by making use of the knowledge and personal experience of the designer who, more or less explicitly taking into account the environment (physical, economical, etc.) in which his work takes place, selects among several possible solutions that design which, when compared with all others, can be considered the best one with respect to parameters dictated by economy, performance, or other topical features.

For instance, it could be possible to define a proper mathematical model for the design problem, pointing out mathematical expressions for objectives, physical or behavioural limitations, constraints, and costs, and look for the best solution(s) as extreme values of functions (or functional). This is what in engineering is called optimization. [15]

In the mathematical formulation of optimal design problems, four basic elements must be taken into account: the objective function (or functional), design variables (or control variables), state equations and constraints. A short description of possible objective functions, design variables and constraints [15] will be given, in order to complete the general introduction on structural optimization.

The objective function [15] is in optimization an objective is supposed to represent a measure of the quality of a structure, but it is clear that the concept of quality is extremely general and it can assume different meanings in different situations or when different purposes are to be considered. For instance, the quality of a structure could be regarded as its economical quality or, from another point of view, as its mechanical quality or performance. More generally, in order to fulfil practical needs, the objective could be defined as the global quality of the structure, where both aspects, economy and performance, are taken into account. Starting from these considerations, in structural optimization the most common objective function is usually assumed to be the cost of the structure. The total cost of a structure can be considered as the sum of three terms: the cost of materials, the cost of manufacturing and the cost of the structure in service.

Of course, many other objective functions can be assumed, usually having a given total cost as the literature shows: structural compliance used to control the average stiffness of the structure, collapse load, maximum stress or strain, buckling load, fundamental frequency, and so on.

Design variables and constraints [15] are most cases variables are usually related to the geometry of the structure. Such a geometry can be defined through topological variables, which represent the number and spatial sequence of structural elements and joint locations, as well as through configurational variables adopted to describe the shape of centerlines (or midplanes) of structural elements. In the most general case, the shape of the structure, seen as the spatial domain defined by the body, can be assumed to be a design variable; in such a case the problem is usually called shape optimization.

From a physical point of view constraints can be distinguished between behavioural constraints and side (or technological) constraints. Behavioural constraints are typically related to the mechanical response of the structure. From a mathematical point of view, the constraints can be classified as equality constraints, expressed in form of equations (algebraic, differential or integral), or in the form of inequalities. The first class reduces the total number of independent variables and the number of constraint equations cannot be greater than the number of design variables. Inequality constraints, on the other hand, define the design space, i.e. the domain of the feasible solutions.

Ravi Shankar Bellur Ramaswamy [16] studied with a design methodology for the optimization of stiffened plates with frequency and buckling constraints is presented. The basic idea of the methodology is to consider a plate with a fairly dense distribution of stiffeners. Thickness of the plate and stiffeners, and the stiffener width are the design variables. Design variable linking is accomplished by the use of rational spline surfaces. The FEM is used for the analysis. The plate is modelled using linear Mindlin plate elements and the stiffeners by linear Timoshenko beam elements. Both the plate and beam elements are shear-locking free by formulation, without requiring any special techniques such as reduced integration. Results for a square stiffened plate with three different stiffener layout patterns and different stiffener density are presented. The best four stiffener configurations which give the lowest mass are chosen and applied to 2:1 and 3:1 rectangular plates. It is concluded that the present design methodology gives good results, and that the stiffener pattern and stiffener density play an important role in reducing the mass of a stiffened plate.

David Bushnell, Charles Rankin [17] investigated The capability of the computer program PANDA2 to generate minimum-weight designs of stiffened panels and cylindrical shells is enhanced to permit the adding of sub stiffeners with rectangular cross sections between adjacent major stringers and rings. As a result many new buckling margins exist that govern buckling over various domains and sub domains of the doubly stiffened panel or shell. These generally influence the evolution of the design during optimization cycles. The sub stiffeners may be stringers and/or rings or may form an isogrid pattern. The effects of local, inter-ring, and general buckling modal imperfections can be accounted for during optimization. Perfect and imperfect cylindrical shells with external T-shaped stringers and T-shaped rings and with and without sub stringers and sub rings and under combined axial compression, external pressure, and in-plane shear are optimized by multiple executions of a "global" optimizer called SUPEROPT. It is found that from the point of view of minimum weight there is little advantage of adding sub stiffeners. However, with sub stiffeners present the major stringers and rings are spaced farther apart at the optimum design than is so when there are no sub stiffeners. The weight of a cylindrical shell with sub stiffeners is much less sensitive to the spacing of the major T-shaped stringers than is the case for a cylindrical shell without sub stiffeners. The optimum designs obtained by PANDA2 are evaluated by comparisons with buckling loads obtained from a general-purpose FE program called STAGS. Predictions from STAGS agree well with those from PANDA2.

Levy and Ganz [18] analyzed plates that optimized using variational calculus to obtain the optimality condition which states the thickness is proportional to the strain energy density and truncated fourier series solution was used to obtain an optimal shape.

Hojjat and Kok [19] developed prototype knowledge based expert system for optimum design of steel plate girders used in highway bridges. They developed a mathematical optimization algorithm for minimum weight design of plate girders using generalized geometric programming technique.

Jarmai et al [20] investigated optimal design of cylindrical orthogonally stiffened shell member of an offshore fixed platform truss, loaded by axial compression and external pressure using various mathematical programming the methods. In optimization and design they used ring stiffeners of welded box section and stringers of halved rolled I-type sections.

Bedair [21] developed approaches for minimum weight design of stiffened plates. He described an alternative energy based approach for stability analysis of multi-stiffened plates under uniform compression and idealized the structure as assembled plate and beam elements are rigidity connected at their junctions. Then he derived strain energy components for the plate and the stiffener elements in terms of out of and in plane displacement functions and used SQP to find the buckling load of the structure for given plate/stiffener geometric proportions.

W. Akla, A. El-Sabbagha, A. Bazb,[22] arranged the orientation angles of stiffeners arranged in the form of isogrid configuration over a flat plate are selected to optimize the static and dynamic characteristics of these plates/stiffeners assemblies. The static characteristics are optimized by maximizing the critical buckling loads of the isogrid plate, while the dynamic characteristics are optimized by maximizing multiple natural frequencies of the stiffened plate. A FE model is developed to describe the statics and dynamics of Mindlin plates which are stiffened with arbitrarily oriented stiffeners. The model is used as a basis for optimizing separately or simultaneously the critical buckling loads and natural frequencies of the plates per unit volume of the plates/stiffeners assemblies. Numerical examples are presented to demonstrate the utility of the developed model and optimization procedures. The presented approach can be invaluable in the design of plates with isogrid stiffeners for various vibration and noise control applications.

Bisagni and Lanzi [23] investigated post buckling optimization procedure for the design of composite stiffened panels subjected to compression loads using neutral networks. To overcome too expensive analyses from a computational point of view, he developed an optimization procedure. It is based on a global approximation strategy, where the structure response is given by a system of neural networks trained by means of FE analyses, and on genetic algorithms that results particularly profitable due to presence of integer variables.

Kang and Kim [24] studied minimum weight design of compressively loaded composite plates and composite stiffened panels under constrained post buckling strength. As an optimization technique, they used a modified genetic algorithm to find optimum points.

There are lots of studies about stiffened plates, free vibration, buckling, static and dynamic analyses and also optimization of plates. These studies are shown on this chapter. Static and dynamic analyses of non- uniform beam-columns under several effects, static behavior and free vibration analysis of laminated glass beam and composite plates have been studied. In optimization methods, design methodology for the optimization of stiffened plates with frequency and buckling constraints, optimum design of stiffened steel plate girders and developed mathematical optimization algorithm for minimum weight design of plate and post buckling optimization at composite stiffened panels have been studied.

In this thesis, free vibration analysis and structural optimization of stiffened plates was studied. The critical motivation of the thesis is structural shape and structural size optimization of stiffened plates including some combinations of structural elements, using a computer code. The specific objectives are investigating the performance of each structural element, observing the change in element shape during optimization procedure to remark the efficiency of each structural element and receiving the best geometric shape and thickness variation for the considered stiffened plate.

CHAPTER 3

FREE VIBRATIONAL ANALYSIS OF PLATES

3.1 Introduction

Most of the structures are constructed using plates have regular geometric shapes along longitudinal direction. Analyzing such structures with classical methods or FE method is extravagant and the cost of the solutions can be very high as we discussed in previous chapter. Also designating the geometric positions of FEs and element connectivity properties of such structures to computer applications is time consuming and tedious. If such structures also have simple supported boundary conditions it is suitable that to apply FS method for free vibration analysis to simplify solution procedure.

3.2 Structural Plate Theories

The plate theories are divided in two groups; Kirchoff-Love (thin) plate theory and Mindlin-Reissner (thick) plate theory. The plate theories are also basis for shell, plates and also stiffened plates.

Kirchoff-Love shell theories neglect transverse shear and rotary inertia effects and consequently may yield incorrect results, especially for higher values of the ratio of the thickness to minimum span and also for higher modes. In addition, many structures may not be considered as a ‘thin plate’; in this regard transverse shear strains in plates cannot be ignored. Therefore, the plate theory is more suitable in general, and the elements developed based on the Mindlin-Reissner plate theory are more practical and useful for real life problems.

Mindlin-Reissner shell theory allows for transverse shear deformation effects. The main assumptions are that:

- Displacements are small compared to shell thickness,
- Stress normal to the mid-surface is negligible,
- Normals to the mid-surface before deformation remain straight but not necessarily normal to the mid-surface after deformation.

It is well known that displacement based Mindlin-Reissner finitestrips require only *C(0)* continuity of the displacements and independent normal rotations between adjacent elements. This provides an important advantage over FS based on classical Kirchhoff-Love thin shell theory where *C(1)* continuity is strictly required. Thus, it is simple to formulate Mindlin-Reissner shell elements. However, several difficulties can be emerged when Mindlin-Reissner shell elements are used in thin shell situations. The success of the Mindlin-Reissner formulation presented here for both thick and thin shell analysis lays in the use of reduced integration techniques for the numerical computation of stiffness matrix. This simply implies that the shear terms contributing to the stiffness matrix are numerically integrated with a lower order Gaussian quadrature than that needed for their exact computation, whereas the rest of the stiffness matrix is exactly calculate. Care has been taken to avoid mechanism or spurious zero-energy modes [25].

3.3 Finite Strip Formulation

In this section, the Mindlin-Reissner FS formulation for prismatic plates and shells in right plan form will be discussed.

3.3.1 Strain Energy

If consider the Mindlin-Reissner shell strip shown in Figure 3.1, translations in the ℓ, *y* and *n* directions can be represented by the displacement components u_ℓ, v_ℓ and w_ℓ. The displacement components u_ℓ, w_ℓ may be written in terms of global displacements u and win the x and z directions as

$$u_\ell = u \cos \alpha + w \sin \alpha$$
$$w_\ell = - u \sin \alpha + w \cos \alpha \qquad (3.1)$$

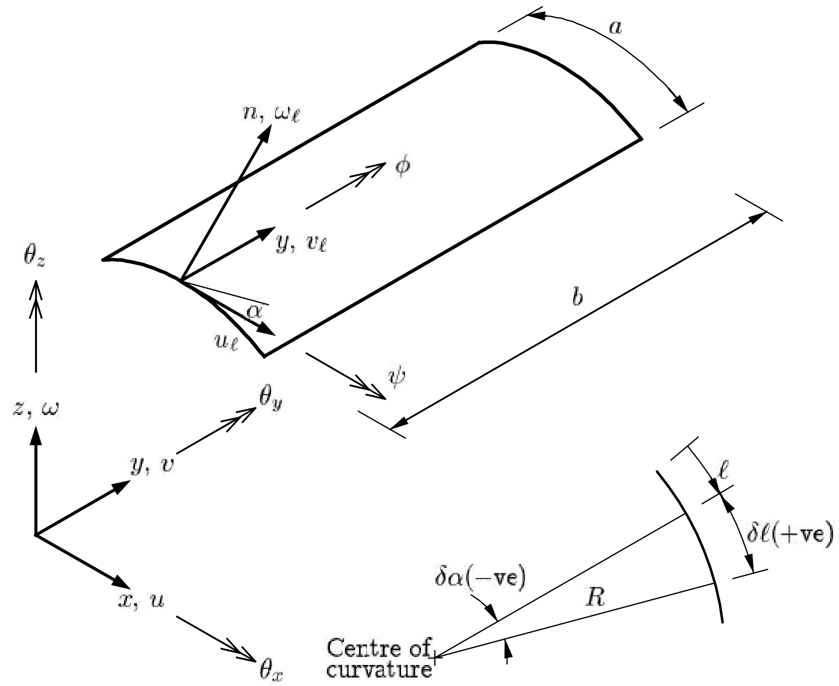

Figure 3.1 Definition of Mindlin-Reissner FS

The strain energy for a typical curved Mindlin-Reissner strip *e* of length *b* shown in Figure 3.1 is given in terms of the global displacements *u, v, w* and the rotations ø and ψ of the mid-surface normal in the ℓ_n and y_n planes respectively by the expressions (3.1).

$$U^e = \frac{1}{2}\int_0^b \int_{l^e} (\varepsilon_m^T D_m \varepsilon_m + \varepsilon_b^T D_b \varepsilon_b + \varepsilon_s^T D_s \varepsilon_s)\, dl dy \qquad (3.2)$$

The strain terms ε_m, ε_b and ε_s are in-plane strains, bending strains and transverse shear strains respectively. These strain terms are given in global coordinate system in Table 3.1.

Table 3.1 Strain terms and strain displacement matrices

Strain terms	Derived Equations
$\varepsilon_m = [\varepsilon_l, \varepsilon_y, \gamma_{ly}]^T$	$[\frac{\partial u}{\partial l}\cos\alpha + \frac{\partial w}{\partial l}\sin\alpha, \frac{\partial v}{\partial y}, \frac{\partial u}{\partial y}\cos\alpha + \frac{\partial w}{\partial y}\sin\alpha + \frac{\partial v}{\partial y}]^T$
$\varepsilon_b = [K_l, K_y, K_{ly}]^T$	$\left\|-\frac{\partial \square}{\partial l}, -\frac{\partial \varphi}{\partial y}, -(\frac{\partial \square}{\partial y} + \frac{\partial \varphi}{\partial l}) - (\frac{\partial u}{\partial y}\cos\alpha + \frac{\partial w}{\partial y}\sin\alpha)\frac{\partial \alpha}{\partial l}\right\|^T$
$\varepsilon_s = [\ell_{ln}, \gamma_{yn}]^T$	$[-\frac{\partial u}{\partial \ell}\sin\alpha + \frac{\partial w}{\partial \ell}\cos\alpha - \square, -\frac{\partial u}{\partial y}\sin\alpha + \frac{\partial w}{\partial y}\cos\alpha - \varphi]^T$

Considering an isotropic material has modulus of elasticity *E*, Poisson's ratio υ and thickness *t*, the matrix of membrane rigidities, flexural rigidities and shear rigidities are D_m, D_b, D_s respectively and they are given in Table 3.2.

Table 3.2 Membrane, flexural and shear rigidities

Rigidities	Derived Equations
D_m	$\frac{Et}{(1-v^2)}\begin{vmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & (1-v)/2 \end{vmatrix}$
D_b	$\frac{Et^3}{12(1-v^2)}\begin{vmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & (1-v)/2 \end{vmatrix}$
D_s	$\frac{k^2Et}{2(1+v)}\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$

k^2 is the shear modification factor and is usually taken as 5/6 for rectangular cross section. Detail information about derivation of FS method can be found in [25].

3.3.2 Potential Energy of the Applied in plane Stress

In plane strain energy of a structure converted to energy by applied in plane loads. The potential energy of the applied in plane σ_ℓ^0, σ_y^0 and $\tau^0_{\ell y}$ arises from the action of the applied stresses on the corresponding second order strains ε_ℓ^{nl}, ε_y^{nl}, $\gamma_{\ell y}^{nl}$ are taken from Dawe and Peshkam [26]. The potential energy of the shell of volume V_g is

$$V_g = \int_v \left(\sigma_\ell^o \varepsilon_\ell^{nl} + \sigma_y^o \varepsilon_y^{nl} + \tau_{\ell y}^0 \gamma_{\ell y}^{nl}\right) dV \tag{3.4}$$

Integrating though the thickness, this becomes

$$V_g^e = \frac{1}{2}\int_0^b \int_{\ell^e} \Big(t\Big\}\sigma_\ell^o \Big|\Big(\frac{\partial u}{\partial \ell}\Big)^2 + \Big(\frac{\partial v}{\partial \ell}\Big)^2 + \Big(\frac{\partial w}{\partial \ell}\Big)^2\Big| + \sigma_y^o \Big|\Big(\frac{\partial u}{\partial y}\Big)^2 + \Big(\frac{\partial v}{\partial y}\Big)^2 + \Big(\frac{\partial w}{\partial y}\Big)^2\Big|\Big\{ +$$

$$\frac{t^3}{12}\Big\}\sigma_\ell^0 \Big|\Big(\frac{\partial \square}{\partial \ell}\Big)^2 + \Big(\frac{\partial \varphi}{\partial \ell}\Big)^2\Big| + \sigma_y^0 \Big|\Big(\frac{\partial \square}{\partial y}\Big)^2 + \Big(\frac{\partial \varphi}{\partial y}\Big)^2\Big|\Big\{\Big) d\ell\, dy \tag{3.5}$$

3.3.3 Finite Strip Idealization

Using n-nodded, *C(0)* strips, the global displacements and rotations of strips may be interpolated within each strip in terms of truncated Fourier series along direction *y*, in which both the material and geometrical properties of the plate are taken to be constant, *i.e.*

$$u(\ell, y) = \sum_{p=p1}^{p2} u^p(\ell) S_p \quad ; \qquad v(\ell, y) = \sum_{p1}^{p2} v^p(\ell) C_p$$
$$w(\ell, y) = \sum_{p=p1}^{p2} w^p(\ell) S_p \quad ; \qquad \psi(\ell, y) = \sum_{p=p1}^{p2} \psi^p(\ell)\, S_p$$
$$\varphi(\ell, y) = \sum_{p=p1}^{p2} \varphi^p(\ell)\, C_p \tag{3.7}$$

Where C_p = Cos(pπy/b) and S_p= Sin(pπy/b), u^p, v^p, w^p, $ø^p$ and ψ^p are displacement and rotation amplitudes for the p^{th} harmonic term.

The next step is to discretize the displacement and rotation amplitudes (which are functions of the ℓ- coordinate only) using an n-noded FE representation so that within a strip *e* the amplitudes can be written as

$$u^p(\ell) = \sum_{i=1}^{n} N_i u_i^p \qquad v^p(\ell) = \sum_{i=1}^{n} N_i v_i^p \qquad w^p(\ell) = \sum_{i=1}^{n} N_i w_i^p$$
$$\psi^p(\ell) = \sum_{i=1}^{n} N_i \psi_i^p \qquad \varphi^p(\ell) = \sum_{i=1}^{n} N_i \varphi_i^p$$
$$u = \sum_{p=p1}^{p2} \sum_{i=1}^{n} N_i^p d_i^p \tag{3.7}$$

Where

$$u = [u, v, w, \psi, \varphi]^T$$
$$d_i^p = [u_i^p, v_i^p, w_i^p, \psi_i^p, \varphi_i^p]^T \tag{3.8}$$

and

$$N_i^p = \begin{bmatrix} N_iS_p & 0 & 0 & 0 & 0 \\ 0 & N_iC_p & 0 & 0 & 0 \\ 0 & 0 & N_iS_p & 0 & 0 \\ 0 & 0 & 0 & N_iS_p & 0 \\ 0 & 0 & 0 & 0 & N_iC_p \end{bmatrix} \tag{3.9}$$

$N_i(\xi)$ is the shape function associated with node i. These elements are essentially isoperimetric so that

$$x = \sum_{i=1}^{n} N_i x_i \qquad y = \sum_{i=1}^{n} N_i y_i \qquad t = \sum_{i=1}^{n} N_i t_i \tag{3.10}$$

Where x_i and y_i are typical coordinates of node i and t_i is the thickness at node *i*. The shape functions N_i used in this study is given in Table 3.3.

Table 3.3 Shape functions

Shape Functions	
Linear	$N_1 = \frac{1}{2}(1-\xi)$ $N_2 = \frac{1}{2}(1+\xi)$
Quadratic	$N_1 = \frac{\xi}{2}(\xi-1)$ $N_2 = 1-\xi^2$ $N_3 = \frac{1}{2}(1+\xi)$
Cubic	$N_1 = \frac{9}{16}\left(\frac{1}{9}-\xi^2\right)(\xi-1)$ $N_2 = \frac{27}{16}(1-\xi^2)\left(\frac{1}{3}-\xi\right)$ $N_3 = \frac{27}{16}(1-\xi^2)\left(\frac{1}{3}+\xi\right)$ $N_4 = -\frac{9}{16}\left(\frac{1}{9}-\xi^2\right)(\xi+1)$

Note also that Jacobian is defined as;

$$J = \frac{d\ell}{d\xi} = \left|\left(\frac{dx}{d\xi}\right)^2 + \left(\frac{dy}{d\xi}\right)^2\right|^{1/2} \qquad d\ell = Jd\xi \qquad (3.11)$$

Where

$$\frac{dx}{d\xi} = \sum_{i=1}^{n} \frac{dN_i}{d\xi} x_i \qquad \frac{dy}{d\xi} = \sum_{i=1}^{n} \frac{dN_i}{d\xi} y_i \qquad (3.12)$$

Also, it is possible to write that

$$sin\alpha = \frac{dy}{d\xi}\frac{1}{J} \qquad cos\alpha = \frac{dx}{d\xi}\frac{1}{J} \qquad (3.13)$$

and

$$\frac{dN_i}{d\ell} = \frac{dN_i}{d\xi}\frac{1}{J} \qquad (3.14)$$

3.3.4 Stiffness Matrix

Stiffness matrix K^e of strip elements can be evaluated considering the strain energy of the Midlin-Reissner strip. The strain energy of a strip element can be expressed as

$$U^e = \frac{1}{2} \sum_{p=p1}^{p2} \sum_{q=q1}^{q2} \sum_{i=1}^{n} \sum_{j=1}^{n} d_i^p \left[K_{ij}^e\right]^{pq} d_j^q \qquad (3.15)$$

Where the typical submatrix of the stiffness K^e of strip e linking nodes *i* and *j* and harmonics *p* and *q* has the form

$$\left[K_{ij}^e\right]^{pq} = \int_0^b \int_{-1}^{1} \left(\left[B_{mj}^p\right]^T D_m B_{mj}^q + \left[B_{bj}^p\right]^T D_b B_{bj}^q + \left[B_{sj}^p\right]^T D_s B_{sj}^q\right) \qquad (3.16)$$

The membrane strains ε_m may then be expressed as

$$\varepsilon_m = \sum_{p=p_1}^{p_2} \sum_{i=1}^{n} B_{mi}^m d_i^p$$

The flexural strains or curvatures ε_b can be written as

$$\varepsilon_b = \sum_{p=p1}^{p2} \sum_{i=1}^{n} B_{bi}^{p} d_i^{p}$$

The transverse shear strains ε_s are approximated as

$$\varepsilon_s = \sum_{p=p1}^{p2} \sum_{i=1}^{n} B_{si}^{p} d_i^{p}$$

Where B_{mi}, B_{bi} and B_{si} are the membrane, bending and shear strain matrices respectively and (strain displacement) matrices and given in Table 3.4.

Table 3.4 Strain displacement terms

	Derived Equations
B_{mi}^{p}	$\begin{vmatrix} (dN_i/d\ell)S_p cos\alpha & 0 & (dN_i/d\ell)S_p sin\alpha & 0 & 0 \\ 0 & -\bar{p}N_iS_p & 0 & 0 & 0 \\ \bar{p}N_iC_p cos\alpha & (dN_i/d\ell)C_p & \bar{p}N_iC_p sin\alpha & 0 & 0 \end{vmatrix}$
B_{bi}^{p}	$\begin{vmatrix} 0 & 0 & 0 & -(dN_i/d\ell)S_p & 0 \\ 0 & 0 & 0 & 0 & \bar{p}N_iS_p \\ (\bar{p}N_iC_p cos\alpha)/R & 0 & (\bar{p}N_iC_p sin\alpha)/R & -\bar{p}N_iC_p & -(dN_i/d\ell)C_p \end{vmatrix}$
B_{si}^{p}	$\begin{vmatrix} -(dN_i/d\ell)S_p sin\alpha & 0 & (dN_i/d\ell)S_p cos\alpha & -N_iS_p & 0 \\ -\bar{p}N_iC_p sin\alpha & 0 & \bar{p}N_iC_p cos\alpha & 0 & -N_iC_p \end{vmatrix}$

where $\bar{p} = p\pi/b$.

Using n-noded, $C(0)$ strips, the global displacements and rotations may be interpolated within each strip. The next step is to discretize the displacement and rotations amplitudes using n-noded FE representation.

If we list the nodal displacements and accelerations in a vector **d** and $\ddot{\mathbf{d}}$ respectively, then we distritize FS idelizaiton into (2.1) for all the strips and assuming simple harmonic motion we obtain the expression

$$\delta\mathbf{d}\left[\mathbf{K}\mathbf{d}+\mathbf{M}\ddot{\mathbf{d}}\right]=\mathbf{0} \tag{3.17}$$

where $\mathbf{K}$ and $\mathbf{M}$ are the global stiffness and mass matrices respectively and contain submatrices contributed from each strip e linking nodes i and j and harmonics p and q. These submatrices have the form

$$\left[\mathbf{K}_{ij}^{e}\right]^{pq} = \int_0^b \int_{-1}^{+1} \{[\mathbf{B}_{mi}^{p}]^T \mathbf{D}_m \mathbf{B}_{mj}^{q} + [\mathbf{B}_{bi}^{p}]^T \mathbf{D}_b \mathbf{B}_{bj}^{q} + [\mathbf{B}_{si}^{p}]^T \mathbf{D}_s \mathbf{B}_{sj}^{q}\} J d\xi dy \quad (3.18)$$

$$[\mathbf{M}_{ij}^{e}]^{pq} = \int_0^b \int_{-1}^{+1} \{[\mathbf{N}_i^{p}]^T \mathbf{P} \mathbf{N}_j^{q}\} J d\xi dy \quad (3.19)$$

where typically

$$\mathbf{N}_i^p = N_i \begin{bmatrix} S_p & 0 & 0 & 0 & 0 \\ 0 & C_p & 0 & 0 & 0 \\ 0 & 0 & S_p & 0 & 0 \\ 0 & 0 & 0 & S_p & 0 \\ 0 & 0 & 0 & 0 & C_p \end{bmatrix} \quad (3.20)$$

and $\mathbf{B}_{mi}^{p}, \mathbf{B}_{bi}^{p}$ and $\mathbf{B}_{si}^{p}$ are the membrane, bending and shear strain displacement matrices associated with harmonic p, node i and Jacobian J. $[\mathbf{K}_{ij}^{e}]^{pq}$ and $[\mathbf{M}_{ij}^{e}]^{pq} = \mathbf{0}$ if $p \neq q$ because of the ortogonality conditions. The matrix $[\mathbf{M}_{ij}^{e}]^{pp}$ is independent of the harmonic number p and therefore, the same matrix can be used for all the different harmonic equations as

$$[\mathbf{M}_{ij}^{e}]^{pp} = b/2 \int_{-1}^{+1} \{[\mathbf{N}_i]^T \mathbf{P} \mathbf{N}_j\} J d\xi \quad (3.21)$$

$\mathbf{N}_i = N_i \mathbf{I}_5$ in which $\mathbf{I}_5$ is the 5×5 identity matrix. Similar to buckling analysis, reduced integration is adopted to avoid locking behavior.

Since (2.3) must be true for any set of virtual displacements $\delta \mathbf{d}^p$, (2.3) may be written in uncoupled form for each harmonic p as

$$\mathbf{K}^{pp} \mathbf{d}^p + \mathbf{M}^{pp} \ddot{\mathbf{d}}^p = \mathbf{0} \quad (3.22)$$

The general solution of (2.8) is written as

$$\mathbf{d}^p = \bar{\mathbf{d}}^p e^{i\omega_p t} \tag{3.23}$$

where $e^{i\omega_p t} = \cos(\omega_p t) + i\sin(\omega_p t)$ and ω_p and $\bar{\mathbf{d}}^p$ are pth natural frequency and vibration mode (eigenvector). Thus (3.24) may be rewritten in the standard eigenvalue form for each harmonic p as

$$(\mathbf{K}^{pp} - \omega_p^2 \mathbf{M}^{pp})\bar{\mathbf{d}}^p = \mathbf{0} \tag{3.24}$$

In the present studies the eigenvalues are evaluated using the subspace iteration algorithm.

3.4 Free Vibration Examples

Several examples for which solutions are available have been considered and results are compared between design computer program.

3.4.1 Centrally Stiffened plate Example

In first example the simply supported stiffened plate in Figure 3.2 has been analysed by proposed method [27] the results are represented in Table 3.5. The properties of analized plate are Modulus of elasticity E = $2.07\text{x}10^5$ N/mm^2, mass density ρ = $7.83\text{x}10^6$ kg/mm^3, poisson's ratio ν = 0.3, geometric properties of plate can be shown on figure and also the stiffener is at the center of the plate. The analises model has number of four key point and three segments.

Table 3.5 Natural frequencies (Hz) of a simply supported plate having a centrally spaced stiffener

Mode	Ref [27]	Present Study	% Difference
1	254.94	256.34	0.55
2	269.46	272.4	1.09
3	511.64	520.3	1.69

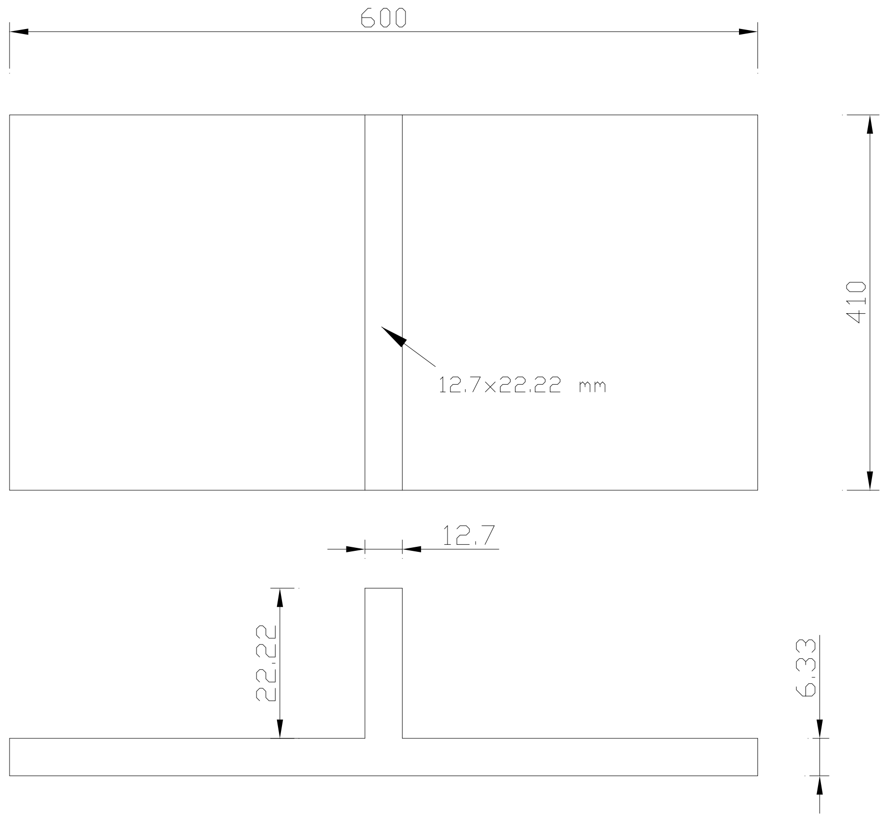

Figure 3.2 Centrally Stiffened Plate

The differences between two results are not considerable, in first mode the result is 254.94 at [27], in present analysis it is 256.34 and the difference between results is % 0.54, in second mode the result is 269.46 at [27], in our analysis it is 272.4 and the difference between results is % 1.08 and the third mode the result is 511.64 at [27], in present analysis it is 520.3 and the difference between results is % 1.66.

3.4.2 Three Stiffened Plate Example

The second example volume of stiffened plate is 691480.0 mm^3, width of all plate is 440 mm, length of plate is 590 mm, the following material properties are used; modulus of elasticity E= 73000 KN/mm^2, poison's ratio ν is 0.33, mass density ρ 8.0 x 10^6 kg/mm^3 thickness of plate is 2.236 mm, and thickness of stiffener is 2.236 mm, height of stiffener is 28.000 mm and number of stiffeners is three which is shown at Figure 3.3.

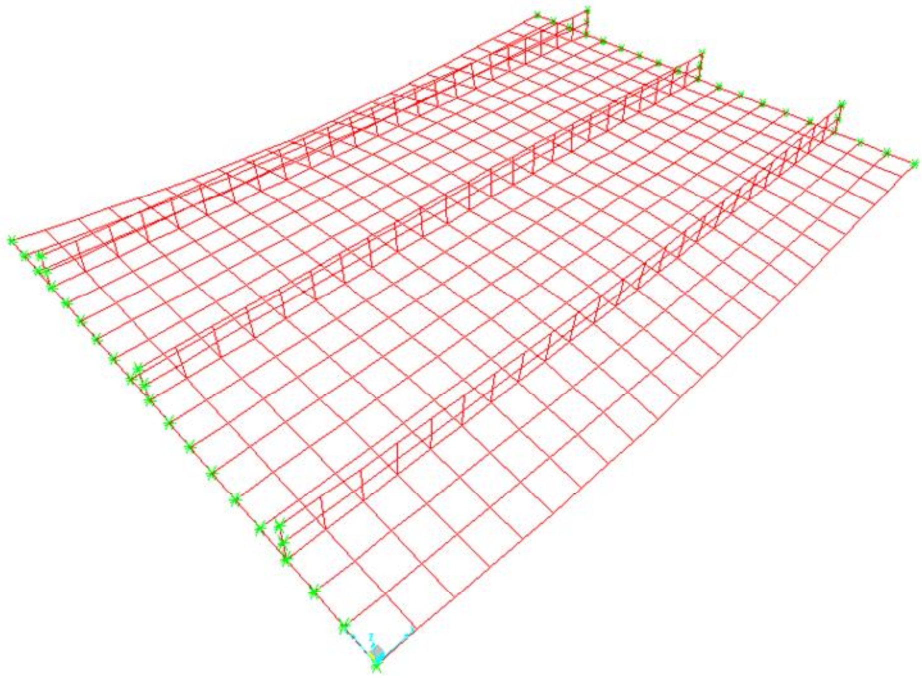

Figure 3.3 Geometry of three Stiffened Plate

Results of analysis and difference of Sap 2000 and present analysis are shown at Table 3.6.

Table 3.6 Results of analysis and difference of Sap 2000 and .Present analysis

Mode	Sap 2000	Present	% Difference
1	104.57	104.34	0.22
2	171.38	169.42	1.15
3	203.60	216.42	5.92
4	262.46	275.27	4.65

3.4.3 Four stiffened Plate Example

In the third example thickness of plate is 2.123 mm, thickness of stiffener is 2.123 mm, volume of stiffened plate is 691480.0 mm^3, material properties are used same previous example, width of plate is 440 mm, length of plate is 590 mm, and height of stiffener is 28.000 mm and number of stiffeners four which is shown at Figure 3.4.

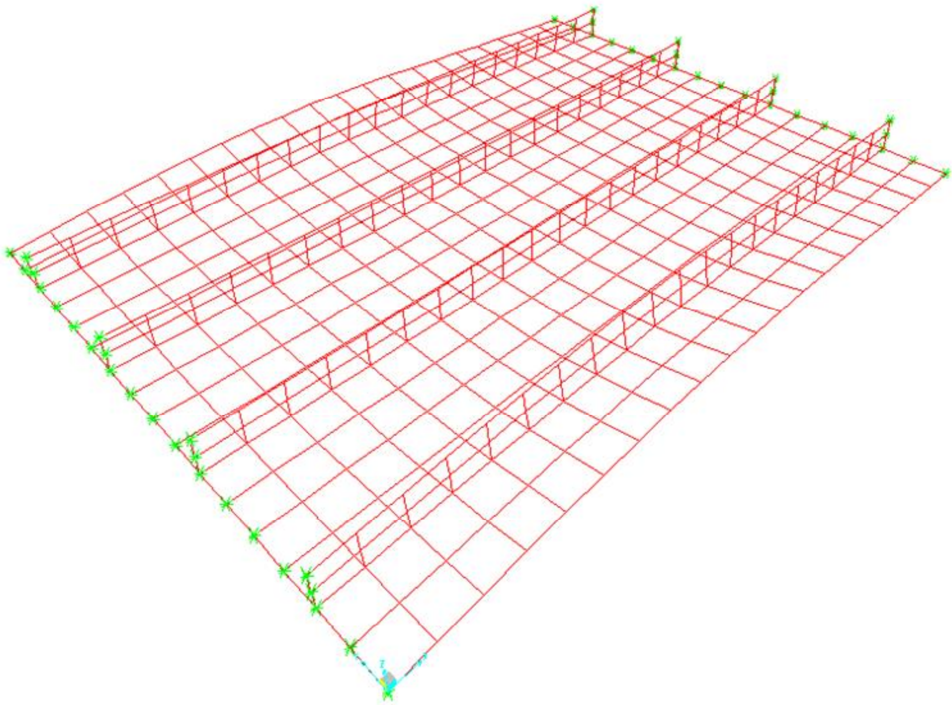

Figure 3.4 Geometry of four Stiffened Plate

Results of analysis and differences are tabulated in Table 3.7.

Table 3.7 Results of analysis and difference of Sap 2000 and .present analysis

Mode	Sap 2000	Present	% Difference
1	221.44	224.77	1.48
2	275.45	275.06	0.14
3	501.67	502.21	0.107
4	1174.20	1161.62	1.08

For the verification of computer code used in this thesis optimized plates also analyzed with SAP 2000 finite element structural analysis and design computer program. At Centrally stiffened plate example, results are compared with Chen [27]'s study at Table 3.5. Results are nearly closed and they can be obtained. At three stiffened plate example, results are compared with SAP 2000 at Table 3.6, results are closed and they can be obtained. At four stiffened plate example, also analyzed with SAP 2000 finite element structural analysis, they are closed and results can be obtained.

CHAPTER 4

OPTIMIZATION PROCEDURE

4.1 Introduction

The general principle by Maupertuis proclaims "*If there occur some changes in nature, the amount of action necessary for this change must be as small as possible"*. In this view, the main purpose of optimization is obtaining the best outcome of a given problem while assuring some restrictions. In this regard to consume limited resources that maximizes the objective. The objective varies depending on problem types and desired functions of problem.

The importance of minimum weight design of structures was first recognized by the aerospace industry where aircraft structural designs are often controlled more by weight than by cost considerations. In other words, industries dealing with civil, mechanical and automotive engineering systems, cost may be the primary consideration although the weight of the system does affect its cost and performance. A growing realization of the scarcity of raw materials and a rapid depletion of our conventional energy sources is being translated into a demand for lightweight, efficient and low cost structures [21].

Eigenvalue effects of stiffened plates can be decrease to very high values by using properly dimensioned stiffened plate elements. In this point, it is necessary to mention about the essentially of structural optimization procedure. This procedure involves iterative solutions and requires reanalyzing of problem several times before obtaining the optimum solution. In this study objective function is minimization of the eigenvalue of stiffened plates while satisfying constant volume constraint.

4.2 Structural Optimization Algorithm

The basic algorithm for structural shape optimization is given in Figure 4.1.

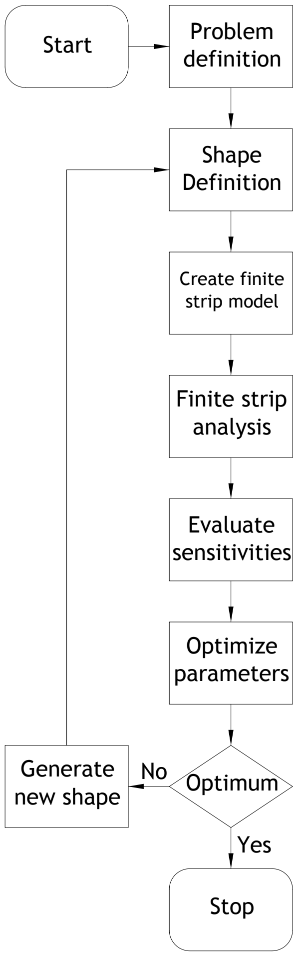

Figure 4.1 Structural Optimization Flowchart

Özakça *et al* [22] summarized the basic algorithm of structural optimization, using FS as an analysis method and SQP as an optimization method, in following steps;

1. Problem definition: Consider the case of the structural optimization of a plate structure in which we wish to maximize the eigenvalues subject to the constraints that the total volume of the plate should remain constant.

2. Shape definition: The shape of the plate cross section is defined in some convenient from that allows us to examine the sensitivities of the design to small changes in shape. Here, we describe the geometry of the plate cross section using parametric cubic spline segments with the coordinates specified at certain key points.

3. Create FS model: the next step is to generate a mesh of suitable FSs. Here, an unstructured mesh generator with mesh density specified at some key points and then interpolated though the segments appropriately are used. In order to ensure the accuracy of the FS model, it is necessary make sure refinement does not occur during the analysis in each optimization iteration. This means that, the strip size distribution (mesh density) remains unchanged during redesign. As the structural shape changes during the optimization process, the re-meshing is based on predetermined mesh density at all iteration. As with normal FS analysis also the boundary conditions and material properties must be defined.

4. FS analysis: Next we carry out a FS analysis and in the present work the structure is modeled using linear, variable thickness, Mindlin-Reissner, *C(0)* FSs.

5. Sensitivity analysis: The sensitivities of the eigenvalue and volume of the current design to small changes in the design variables are then evaluated. These design sensitivities are generally nonlinear implicit functions of the design variables and are therefore difficult and expensive to calculate. The numerical accuracy of sensitivity analysis affects the search directions that are used in optimization algorithms.

6. Optimize parameters: Using the objective and constant functions and their derivatives, SQP optimization algorithm is employed to optimize the parameters or design variables. The new set of values will result in a modified design. Furthermore, the constrains must be satisfied if the new design is to be demand acceptable. If a convergence criterion for optimization algorithm is satisfied, then the optimum solution has been found and the solution process is terminated.

7. Update optimization model: After the optimization, it is necessary to update the geometric model, *i.e.* the coordinates and/or thickness of the primary design variables in structural optimization. This is the only part of the original input data which has to be updated with for all optimization iteration. If convergence has not been achieved, the new geometry is sent to the mesh generator which automatically generates a new analysis model and the whole process is repeated from step2.

4.2.1 Mathematical Definition of Optimization Problem

Problems of structural optimization are characterized by various objectives and constraints, which are generally nonlinear functions of the design variables. These functions can be discontinues and non-convex. Each objective and constraint choice defines a different optimization problem, and solution can be found using several mathematical programming methods.

In general the constraint functions are grouped in to these classes: equality constraints h_j, inequality constraints g_i, and the geometric (regional) constraints defined by upper and the lower bounds of the design variables.

However, all optimization problems can be expressed in standard mathematical terms as: minimize (or maximize)

$$F(s) \tag{4.1}$$

Subject to:

$$\begin{aligned} &g_i(s) \le 0 && i = 1,\dots m \\ &h_j(s) = 0 && j = 1,\dots l \\ &s_k^l \le s_k \le s_k^u && k = 1,\dots n_{dv} \end{aligned} \tag{4.2}$$

The notion of improving or optimizing a structure implicitly presupposes some freedom to change the structure. The potential for change is typically expressed in terms of ranges of permissible changes of a group of parameters. Such parameters are usually called design variables in structural optimization terminology. Design variables can be cross-sectional dimensions or member sizes; they can be parameters controlling the geometry of the structure and its material properties, etc. In which, s is the design variables vector.

The notation of optimization also implies that are some merit function *F(s)* or functions $F(s) = [F_1(s), F_2(s), F_3(s), ...]$ that can be improved and can be used as a measure of effectiveness of the design. The common terminology for such functions is objective functions. For structural optimization problems, weight, displacements, stresses, vibration frequencies, eigenvalues and cost or any combination of these can be used as objective functions.

In optimization process of structures, there are limits about design variables. Sometimes design constraints may be dimensions of structural elements, weight of structure, vibration frequency and displacement of a point, $g_i(s)$ and $h_j(s)$ are the constraint functions. Finally, s_k^l and s_k^u represent the lower and the upper bounds of the design variables; *m* is the number of design variables used.

In this study objective function is minimizing the eigenvalue of stiffened plates. Design variables are stiffened plate cross sectional elements dimensions that are defined clearly in Chapter 5. When minimizing eigenvalue of stiffened plate first constraint is an equality constant material volume constraint. Optimized plates widths and lengths are constant. Also there are upper and lower limits inequality constraints of design variables.

Eigenvalue constraint g(s) can be expressed as

$$g\,(s) = 1 - \left(\frac{(\mathrm{Eig.})_i}{(\mathrm{Eig.})_{max}}\right) \tag{4.2}$$

where $(\mathrm{Eig.})_{max}$ defines the upper limit on eigenvalue and $(\mathrm{Eig.})_i$ describing the eigenvalue of the current design. Similarly

$$g(s) = \frac{V_i}{V_{max}} - 1 \tag{4.3}$$

Defines the volume constraint V_i and V_{max} are the current value and upper limit of the volume respectively.

4.2.2 Shape Definition

4.2.2.1 Structural Shape Definition

The designation of geometric model and control the parameters of optimization procedure for an appropriate flow algorithm are complex and require attention. The cross section of typical stiffened plate structure is shown in Figure 4.2.

To form cross section geometry of stiffened plates to introduce computer code the segments must be generated one by one. Generating a straight segment can be done by entering its two key points geometrical coordinates as input data.

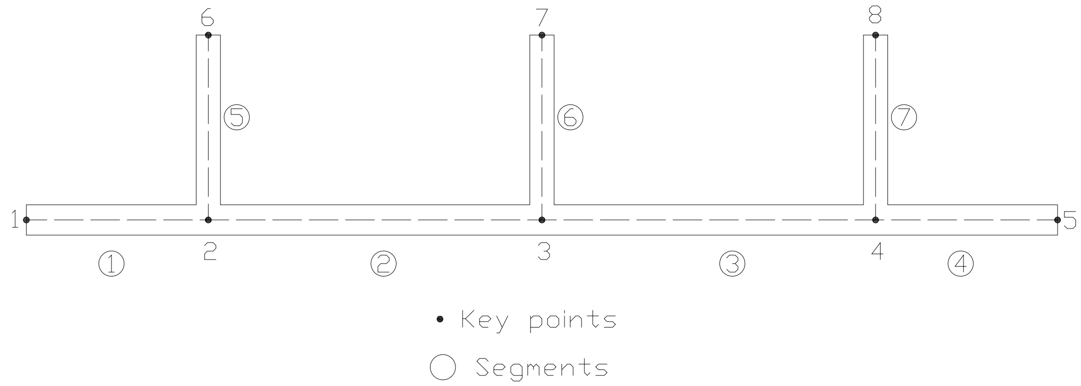

Figure 4.2 Geometric Representation of Stiffened Plate

Defined number of key points to form the cross sectional shapes of the stiffened plates which are important for computational algorithm. More key points mean more design variables for computer code. So increasing the defined number of key points cause increasing computational time.

For the applicability to real life, increasing the efficiency of computational effort and symmetrical behavior of structural elements it is a necessary situation to link the design variables at two or more key points. By linking of design variables, the length of a considered segment can be assigned as a design variable and symmetry of shape in an axis can be easily achieved. In this regard, the number of design variables for optimization is considerably reduced.

4.2.2.2 Structural Thickness Definition

The thickness of the stiffened plate elements are specified at some or all of the key points for the desired initial element shape of the structure and then interpolated by program.

4.2.3 Mesh Generation for Finite Strip Analysis

After defining the geometry, the next step is to generate a proper FE mesh for the cross section of stiffened plate. This meshing procedure can be carried out with an automatic mesh generator for desired mesh density. Automatic mesh generator has the capability of meshing the arbitrary complex geometry given no input other than the geometric representation of the domain to be meshed and an associated mesh density distribution. Mesh generation should be robust, versatile and efficient to obtain more accurate results. Here, we use a mesh generator which allows refinement of FE meshes. It also allows for significant variation in mesh spacing throughout the region of interest. The mesh generator can generate meshes of two three and four noded elements and strips.

It is very significant factor for obtaining more accurate results to mesh the cross section properly. In this regard mesh operation should be carried out considering critical points in cross section. Also meshes in segments should be compatible with each other. Figure 4.3 shows a mesh example of three stiffened plates.

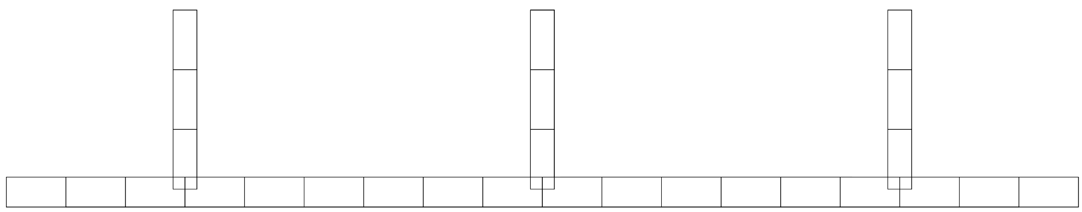

Figure 4.3 Mesh Representation of Plate

The mesh density is a piecewise linear function of the values of mesh size δ at some points along the mid-surface of structure.

4.2.4 Structural Finite Strip Analysis

It is the important factor for optimizing methods to reach optimum solution in minimum computational time. So efficiency of the optimization methods are based on the computational time required in the process. Most of the numerical optimization methods have iterative procedures. So the number of structural analyses required to complete the optimum solution is large. In this regard, to reduce the cost of problem the efficient and inexpensive structural analysis method should be used.

In such case, FS method is the best approach to the problems. As discussed in previous chapter the FS method has proven to be an inexpensive and useful tool in analysis of structures having regular prismatic type geometries and simple supported on diagrams at two opposite edges with the remaining edges arbitrarily restrained. Theory and implement of FS method for vibration analyses are given in previous chapter.

4.2.5 Sensitivity analysis

Sensitivity analysis is a crucial part of optimization procedure. After FS analysis completed the sensitivities of the current design should be evaluated to small changes in the design variables. We calculated the sensitivities of items such as vibration analyses based on finite differences.

Sensitivity analysis is dependent up on the systematic calculation of the derivatives of the response for the FS model with regards to parameters forming the model geometry *i.e.* the design variables which may be shape, thickness or length. The structural response quantities with respect to the shape (or other) variables at first partial derivatives and these variables provide the essential information required to couple mathematical programming methods and structural analysis procedures. The sensitivities of responses provide the mathematical programming algorithm with search directions for optimum solutions.

In the present study, PLATEV_1 code uses the finite difference to calculate sensitivities. For the numerically approximation of derivatives the finite difference method uses a difference formula. The finite difference scheme is accurate and computationally efficient.

4.2.6 Derivative of Volume

A forward finite difference approximation is used to evaluate the volume derivative

$$\frac{\partial V}{\partial S\mathrm{i}} \approx \frac{V(S\mathrm{i} + \Delta S\mathrm{i}) - V(S\mathrm{i})}{\Delta S\mathrm{i}} \tag{4.4}$$

Where the volume V of the whole structure (or cross-sectional area of the structure may also be used) can be calculated by adding the volumes of numerically integrated FS.

4.3 Mathematical Programming

SQP is used as a mathematical programming to generate shapes with improved objective function values using the information derived from the analysis and design sensitivities. No effort has been made to study the mathematical programming methods used for structural optimization procedures and the SQP algorithm is used here essentially as a 'black box'.

CHAPTER 5

OPTIMIZATION OF PLATES

5.1 Introduction

FS analysis and SQP optimization is to be used to find an optimal stiffened plate design using prismatic, rectangular sub stiffeners and pads. The starting point for these designs is the baseline plate from which the initial values of parameters are developed. A complete description of the baseline design is outlined in the following sections.

The main interest of this study is minimizing the eigenvalue of stiffened plates by optimizing the plate section dimensions under constant volume constraint.

Optimization is carried out for the following types of stiffened plates that are expressed below and plate types are shown on four stiffened plate template and given in Figure 5.1.

Types of stiffeners:

a) Straight stiffened plate
b) Straight stiffened plate with sub stiffeners
c) Straight stiffened plate and pads under main stiffeners
d) Straight stiffened plate with sub stiffeners and pads under stiffeners

5.1.1 Optimization Process

It is desired that two separate linear eigenvalue optimizations are run. The first design is carried out for obtaining thickness of initial values by providing constant cross sectional area. The second run will apply the design constraints associated with manufacturing process and other issues. Full details of the Design Variables *(dv)s* and constraints are outlined in the preceding sections.

5.1.2 Baseline Design

The baseline panel is the foundation for the stiffened plate design. The plate cross section is constant along its length. The baseline plate cross section has a total are of 1172 mm^2 of skin material available for manipulation.

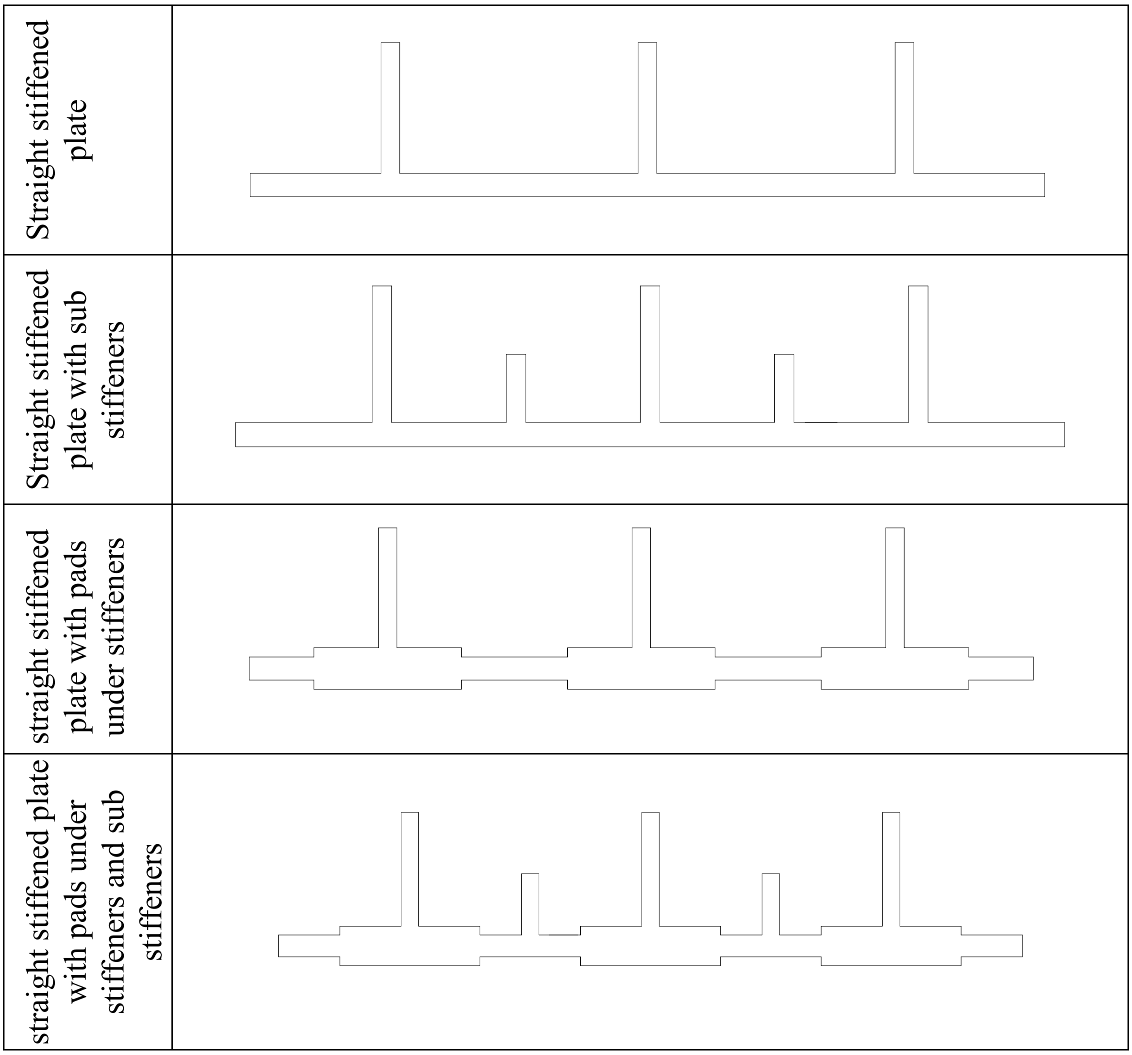

Figure 5.1 Examined Stiffened Plate Types

5.1.3 Parameter Definition

Figure 5.2 below describes the cross section and geometric (design variables) parameters associated with the prismatic blade sub stiffened panel.

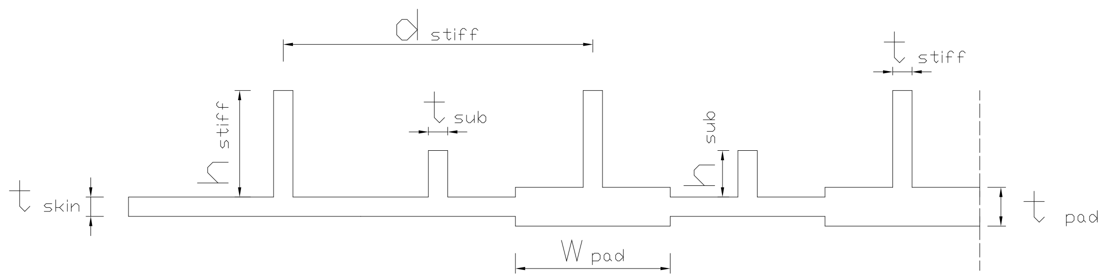

Figure 5.2 Plate Variable Parameters (Design Variables)

t_{skin}	$\longrightarrow$	Skin thickness
h_{stiff}	$\longrightarrow$	Primary stiffener height
t_{stiff}	$\longrightarrow$	Primary stiffener thickness
w_{pad}	$\longrightarrow$	Width of pad under stiffeners
t_{pad}	$\longrightarrow$	Thickness of pad under stiffeners
h_{sub}	$\longrightarrow$	Sub stiffener height
t_{sub}	$\longrightarrow$	Sub stiffener thickness
d_{stiff}	$\longrightarrow$	Distance between stiffeners
n_{stiff}	$\longrightarrow$	Number of stiffeners

5.1.4 Optimization Set up

This design has a number of sub stiffeners running parallel to primary stiffeners. Only variable parameters can be changed during the optimization process.

5.1.5 Design Constraints

There are a number of design constrains based on either the general design strategy or the manufacturing process as outlined below. All types of examined plates have the common fixed constraints as shown in Table 5.1. The common constraints are shown on a three dimensional aspect of five straight stiffened plate in Figure 5.3.

Table 5.1 Common Constraints

Plate width	440 mm
Plate length	590 mm
Total plate volume	691480 mm^3

Nevertheless, design variables have constrains (minimum and maximum limits) that are expressed in relevant sections.

5.1.6 Material Properties and Boundary Conditions

In this study eigenvalue vibrational analysis is considered. This analysis only requires elastic material properties. The used material properties are:

Modulus of elasticity (E): $73x10^9$ N/m^2

Poisson's ratio (v): 0.33

Boundary conditions are shown in Figure 5.4.

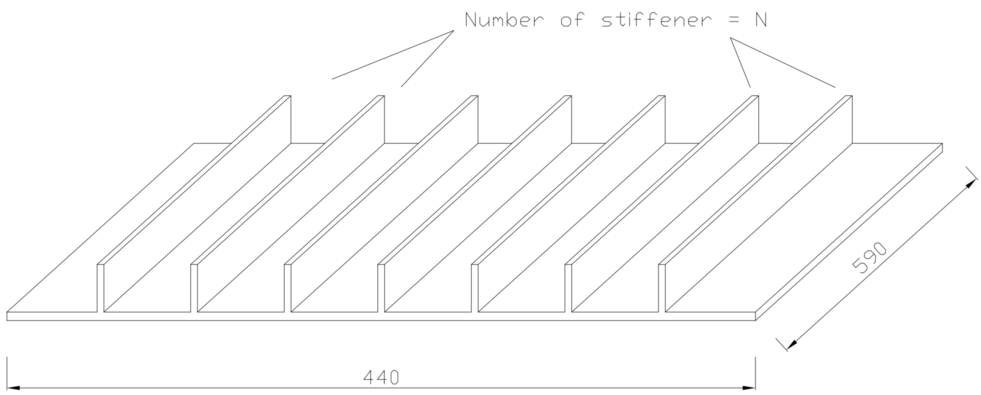

Figure 5.3 A Sample Three Dimensional Aspect of Stiffened Plate (Straight Stiffener with Seven Stiffeners)

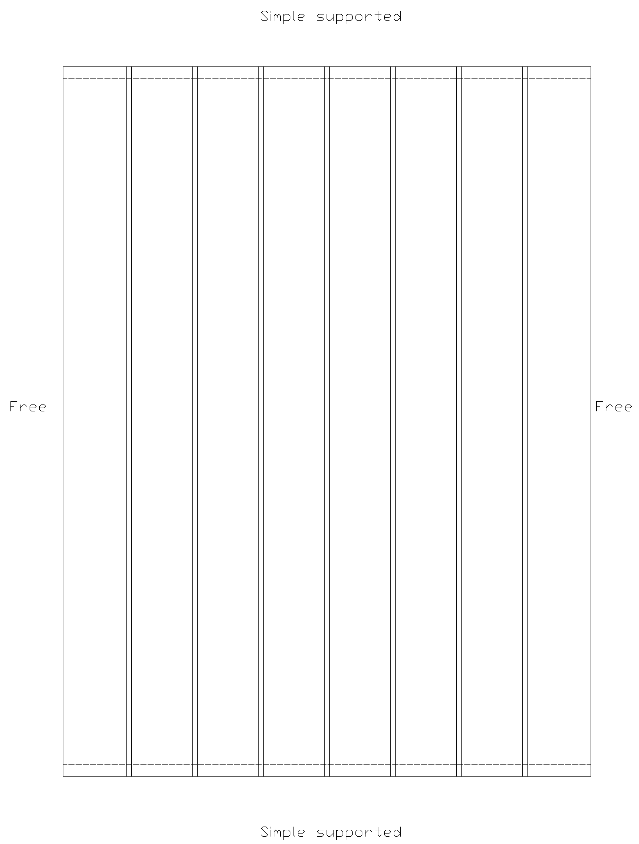

Figure 5.4 Boundary Conditions

5.2 Plate Types and Optimization Process

Straight stiffened plates defined in Section 5.1 are optimized. Optimization processes are defined, results of optimizations are presented and discussions are made in this section. All dimensions in tables are in mm in tables.

5.2.1 Straight Stiffeners

5.2.1.1 Straight Stiffened Plate

Figure 5.5 shows straight stiffened plate with three stiffeners

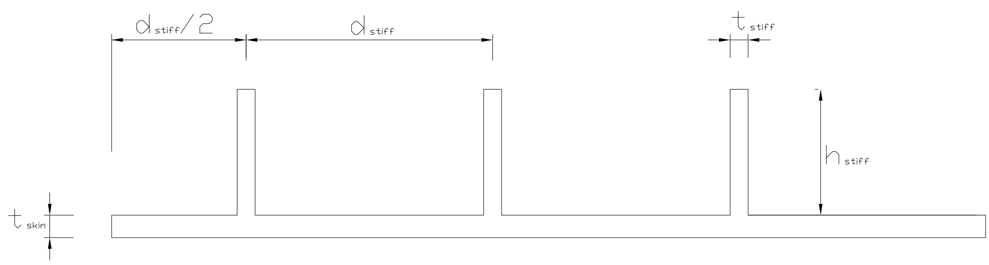

Figure 5.5 Straight Stiffened Plate

a) Optimization Process:

i) Size Optimization (Type1): Optimization is performed using thickness of plate skin (t_{skin}), thickness of stiffeners (t_{stiff}). During this stage height of stiffeners (h_{stiff}) have constant value of 28.0 mm (see Figure 5.5)

ii) Shape Optimization: Optimization is performed using all variables; thickness of plate skin (t_{skin}), thickness of stiffeners (t_{stiff}), and height of stiffeners (h_{stiff}) (see Figure 5.5).

iii) Shape Optimization (Type2): Optimization is performed using thickness of plate skin (t_{skin}), height of stiffeners (h_{stiff}). During this stage thicknesses of stiffeners (t_{stiff}) have constant value of what calculated at initial (see Figure 5.5).

Design constraints of three stages are specified in Table 5.2. Optimization process is repeated from two to eight stiffeners.

Table 5.2 Lower and Upper limits of design variables for stiffened plates

		Min (mm)	Max (mm)
Thickness of Plate	t_{skin}	1.1	3.0
Thickness of Stiffener	t_{stiff}	0.5	6.0
Height of Stiffener	h_{stiff}	8.0	40.0

b) Discussion of Results

Three types of optimization are performed. These are size optimization (Type 1) with two design variables (t_{skin}, t_{stiff}), shape optimization with three design variables (t_{skin}, t_{stiff}, h_{stiff}), and shape optimization (Type 2) with two design variables (t_{skin}, h_{stiff}). Effect of number of stiffeners is also observed. Number of stiffeners from two to eight is optimized. Optimizations are carried out for maximize of eigenvalue subject to constant volume constraint.

i) Size Optimization (Type 1): Thickness of plate and stiffeners are kept equal at initial design. The height of stiffeners is constant and equal to 28.0 mm. The optimum values of design variables and eigenvalues are given in Table 5.4. The highest improvement is obtained for four stiffeners case and approximately equal to 16.92 %. The stiffened panel analyzed using cubic strips. In order to obtain more accurate results the large number of degrees of freedom is taken an all analysis. The smallest eigenvalue is obtained in eight stiffeners case and equal to -52.003. The plate thickness is thinner than stiffeners in optimum results except two stiffeners case and by the increasing of the number of stiffeners skin thickness is going to be thinner and also stiffener thickness is going to be thinner except two stiffeners case.

Table 5.3 Size optimization (Type 1) of Straight Stiffened Plate

n	h_{stiff}	t_{skin}	t_{stiff}	Initial Eig.	Opt. Eig.	%
2	28	2.447	1.698	-18.101	-18.230	0.71
3	28	1.518	6.000	-30.299	-34.721	14.59
4	28	1.270	5.472	-37.642	-44.011	16.92
5	28	1.356	4.107	-41.860	-47.972	14.60
6	28	1.409	3.284	-44.628	-50.092	12.24
7	28	1.382	2.875	-46.558	-51.282	10.14
8	28	1.350	2.579	-48.018	-52.003	8.29

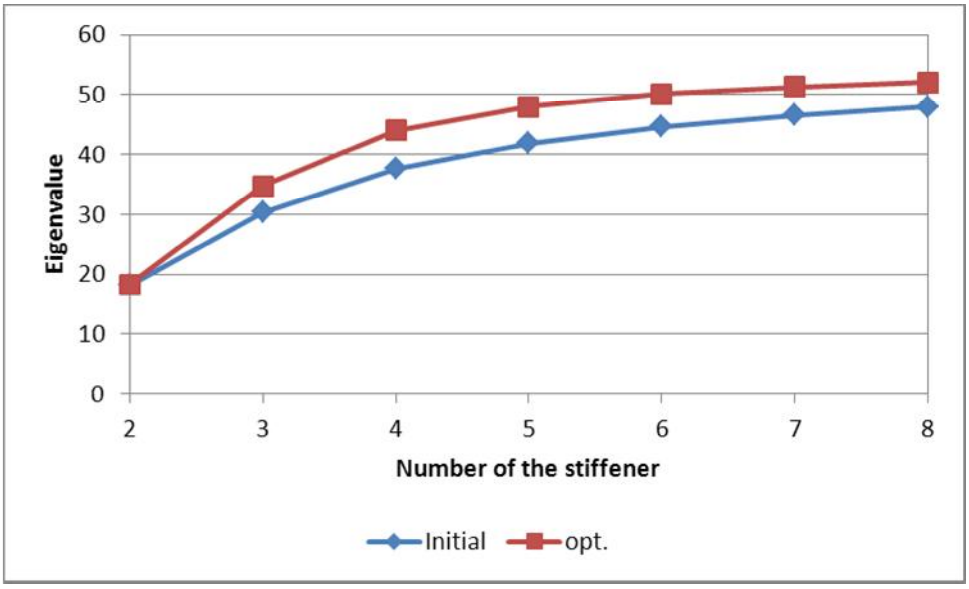

Figure 5.6 Comparison of Eigenvalues at Size Optimization (Type 1)

ii) Shape Optimization: Thickness of plate and stiffeners are kept equal at initial design. The optimum values of design variables and eigenvalues are given in Table 5.3. The highest improvement is obtained for six stiffeners case and approximately equal to 27.74 %. The stiffened panel analyzed using cubic strips. In order to obtain more accurate results the large number of degrees of freedom is taken an all analysis. The smallest eigenvalue is obtained in eight stiffeners case and equal to -60.209. Moreover, it is important to note that in optimum results skin thickness is thinner than stiffener thickness and the stiffener thicknesses are become less at two stiffeners plate toward eight stiffeners plate also the height of stiffeners increase and after five stiffener case thicknesses reach upper limits.

Table 5.4 Shape Optimization of Straight Stiffened Plate

n	h_{stiff}	t_{skin}	t_{stiff}	Initial Eig.	Opt. Eig.	%
2	30.439	1.833	6.000	-18.101	-20.460	13.02
3	28.806	1.485	6.000	-30.299	-34.737	14.64
4	32.717	1.154	5.074	-37.642	-45.620	21.19
5	40.000	1.731	2.051	-41.860	-51.296	22.54
6	40.000	1.584	1.978	-44.628	-57.008	27.74
7	40.000	1.502	1.824	-46.558	-58.648	25.96
8	40.000	1.430	1.695	-48.018	-60.209	25.38

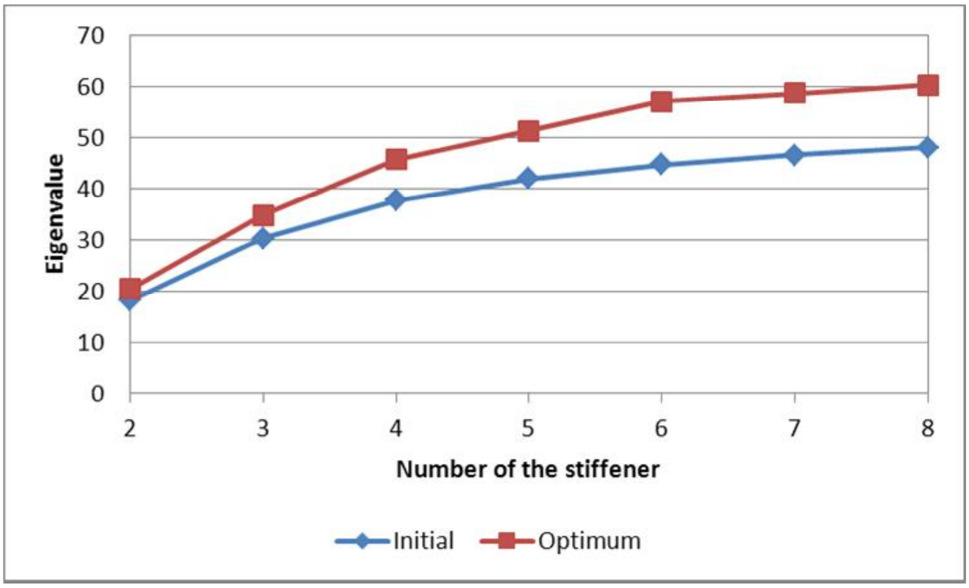

Figure 5.7 Comparison of Eigenvalues for Shape Optimization

iii) Shape Optimization (Type 2): Thickness of plate and stiffeners are kept equal at initial design. The thickness of stiffeners is constant and equal to initial value. The optimum values of design variables and eigenvalues are given in Table 5.5. The highest improvement is obtained for six stiffeners case and approximately equal to 28.13 %. The stiffened panel analyzed using cubic strips. In order to obtain more accurate results the large number of degrees of freedom is taken an all analysis. The smallest eigenvalue is obtained in eight stiffeners case and equal to -60.074. The plate thickness is thinner than stiffeners in optimum results except two stiffeners case and by the increasing of the number of stiffeners skin thickness is going to be thinner and also height of stiffener is going to be increase, the height of stiffeners reach upper limits after four stiffeners case.

Table 5.5 Shape Optimization (Type 2) of Straight Stiffened Plate

n	h_{stiff}	t_{skin}	t_{stiff}	Initial Eig.	Opt. Eig.	%
2	19.270	2.456	2.362	-18.101	-18.339	1.31
3	36.924	2.100	2.236	-30.299	-32.264	6.48
4	40.000	1.891	2.123	-37.642	-43.618	15.87
5	40.000	1.745	2.020	-41.860	-51.290	22.52
6	40.000	1.612	1.927	-44.628	-57.183	28.13
7	40.000	1.490	1.842	-46.558	-58.642	25.95
8	40.000	1.379	1.765	-48.018	-60.074	25.10

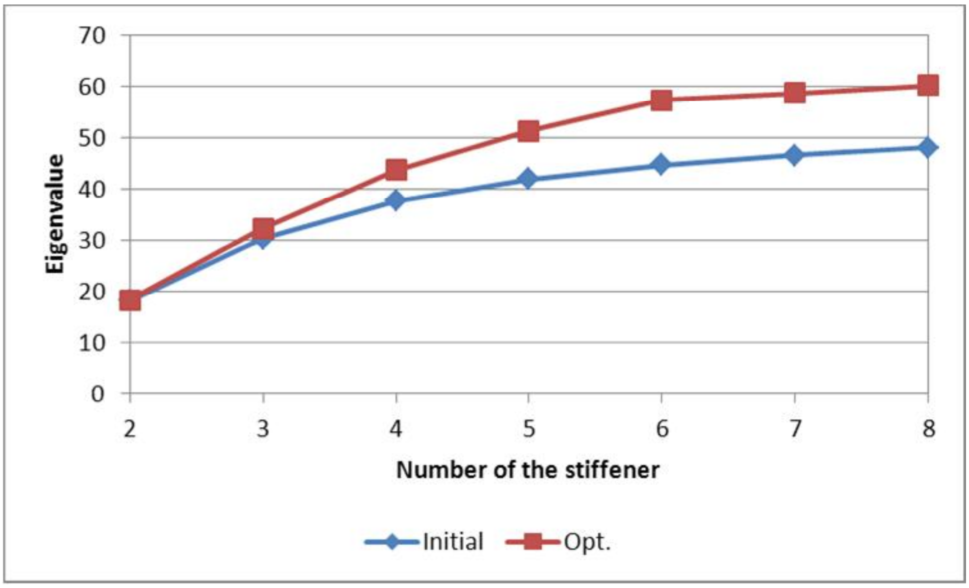

Figure 5.8 Comparison of Eigenvalues at Shape Optimization (Type 2)

Shape optimizations slightly gave better results compared to size optimizations (Type 1) and shape optimizations (Type 2) as shown Figure 5.9.For small number of stiffeners both optimizations give similar results. However when the number of the stiffeners increase shape optimization and shape optimization (Type 2) give better results. In shape optimization and shape optimization (Type 2) height of the stiffeners are increase; it is the fundamental causes of better eigenvalues. Shape optimization a little better than the shape optimization (Type 2) because of thickness of stiffeners.

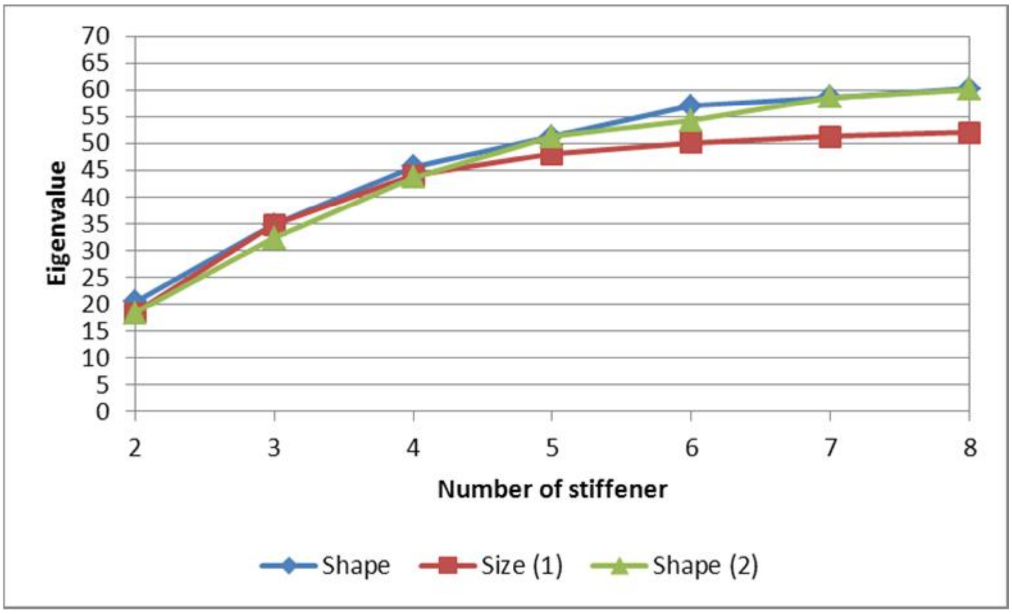

Figure 5.9 Comparison of Case Optimizations

5.2.1.2 Straight stiffened plate with sub stiffeners

Figure 5.10 shows straight stiffened plate with sub stiffeners. Sub stiffeners are attached between stiffeners, which divide the distance between two equal parts.

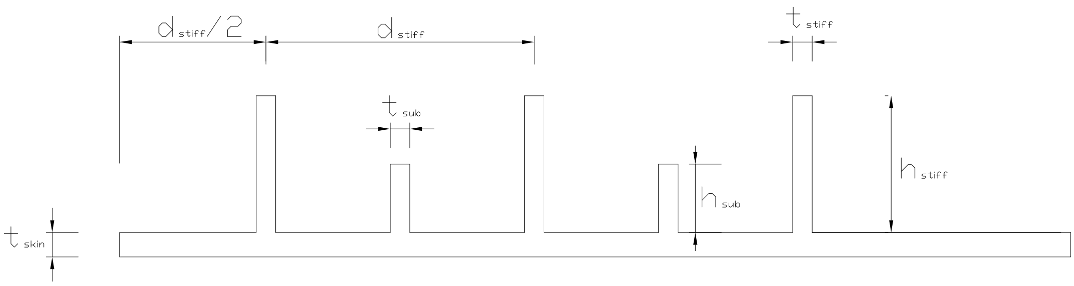

Figure 5.10 Straight Stiffened Plate with Sub stiffeners

a) Optimization Process:

i) Size Optimization (Type 1): Optimization is performed using thickness of plate skin (t_{skin}), thickness of stiffeners (t_{stiff}), thickness of sub stiffeners (t_{sub}). During this stage height of stiffeners (h_{stiff}) has constant value of 28.0 mm and height of sub stiffeners (h_{sub}) has a constant value of 14 mm (see Figure 5.10).

ii) Shape Optimization: Optimization is performed using all variables; thickness of plate skin (t_{skin}), thickness of stiffeners (t_{stiff}), thickness of sub stiffeners (t_{sub}), height of stiffeners (h_{stiff}) and height of sub stiffeners (h_{sub}) (see Figure 5.10).

iii) Shape Optimization (Type 2): Optimization is performed using thickness of plate skin (t_{skin}), thickness of stiffeners (t_{stiff}), thickness of sub stiffeners (t_{sub}) and height of sub stiffeners (h_{sub}). During this stage height of stiffeners (h_{stiff}) has constant value of 28.0 mm (see Figure 5.10).

iv) Shape Optimization (Type 3): Optimization is performed using thickness of plate skin (t_{skin}), height of stiffeners (h_{stiff}), height of sub stiffeners (h_{sub}). During this stage thickness of stiffeners (t_{stiff}) and thickness of sub stiffeners (t_{sub}) have constant value of what calculated at initial (see Figure 5.10).

v) Shape Optimization (Type 4): Optimization is performed using thickness of plate skin (t_{skin}), height of stiffeners (h_{stiff}), thickness of stiffeners (t_{stiff}). During this stage height of sub stiffeners (h_{sub}) has constant value of14 mm and thickness of sub stiffeners (t_{sub}) has constant value of what calculated at initial (see Figure 5.10).

Design constraints of five stages are specified in Table 5.6. Optimization process is carried out for two to eight stiffeners.

Table 5.6 Lower and Upper limits of design variables for stiffened plates with Sub stiffeners

		Min (mm)	Max (mm)
Thickness of Plate	t_{skin}	1.1	3.0
Thickness of Stiffener	t_{stiff}	0.5	6.0
Thickness of Sub stiffener	t_{sub}	1.0	3.0
Height of Stiffener	h_{stiff}	8.0	40.0
Height of Sub stiffener	h_{sub}	5.0	20.0

b) Discussion of Results

The effect of sub stiffeners between stiffeners to the effect of eigenvalues is examined. Five types of optimization are performed. These are size optimization (Type 1) with three design variables (t_{skin}, t_{stiff}, t_{sub}), shape optimization with five design variables (t_{skin}, t_{stiff}, t_{sub}, h_{stiff}, h_{sub}), shape optimization (Type 2) with four design variables (t_{skin}, t_{stiff}, t_{sub}, h_{sub}), shape optimization (Type 3) with three design variables (t_{skin}, h_{stiff}, h_{sub}), shape optimization (Type 4) with three design variables (t_{skin}, t_{stiff}, h_{stiff}). The effect of stiffeners is also observed similar to stiffened plate.

i) Size Optimization (Type 1): Thickness of plate and stiffeners are kept equal at initial design. The height of stiffeners is constant equal to 28 mm and the height of sub stiffeners is constant equal to 14 mm. The optimum values of design variables and eigenvalues are given in Table 5.9. The highest improvement is obtained for four stiffeners case and approximately equal to 17.63 %. The stiffened panel analyzed using cubic strips. In order to obtain more accurate results the large number of degrees of freedom is taken an all analysis. The smallest eigenvalue is obtained in seven stiffeners case and equal to -50.775. The plate thickness is thinner than stiffeners in optimum results and by the increasing of the number of stiffeners thickness of stiffener is going to be thinner and also skin thickness is going to be thinner except six stiffeners case, the thickness of stiffeners reach lower limits in all cases except two and seven stiffeners case.

Table 5.7 Size Optimization (Type 1) of Stiffened Plate with Stiffeners and Sub stiffeners

n	H_{stiff}	H_{sub}	t_{skin}	t_{stiff}	t_{sub}	Initial Eig.	Opt. Eig.	%
2	28	14	1.835	6.000	2.014	-19.490	-21.125	8.38
3	28	14	1.454	6.000	1.000	-29.618	-34.273	15.71
4	28	14	1.198	5.381	1.000	-36.826	-43.321	17.63
5	28	14	1.220	4.134	1.000	-41.712	-47.389	13.60
6	28	14	1.317	3.109	1.000	-45.056	-49.631	10.15
7	28	14	1.202	2.580	1.633	-46.636	-50.775	8.87
8	28	14	1.294	2.251	1.000	-47.067	-49.598	5.37

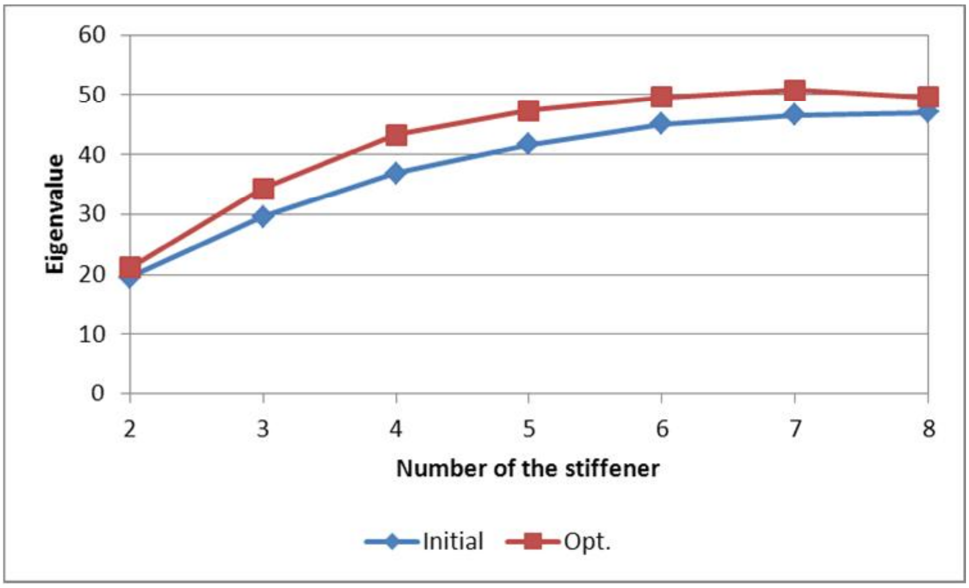

Figure 5.11 Comparison of Eigenvalues at Size Optimization (Type 1)

ii) Shape Optimization: Thickness of plate, stiffeners and sub stiffeners are kept equal at initial design. The optimum values of design variables and eigenvalues are given in Table 5.7. The highest improvement is obtained for eight stiffeners case and approximately equal to 26.7 %. The stiffened panel analyzed using cubic strips. In order to obtain more accurate results the large number of degrees of freedom is taken an all analysis. Also the smallest eigenvalue is obtained in eight stiffeners case and equal to -59.636. Moreover, in optimum results skin thickness is thinner than stiffener thickness, the stiffener thicknesses are become less at two stiffeners plate toward eight stiffeners plate also the height of stiffeners increase and after five stiffener case thicknesses reach upper limits, height of sub stiffeners are all decreases to lower limit, thickness of sub stiffeners are also decreases to lower limit except two stiffener case.

Table 5.8 Shape Optimization of Stiffened Plate with Stiffeners and Sub stiffeners

n	H_{stiff}	H_{sub}	t_{skin}	t_{stiff}	t_{sub}	Initial Eig.	Opt. Eig.	%
2	27.545	5	1.878	6.000	3	-19.490	-21.376	9.67
3	28.704	5	1.466	6.000	1	-29.618	-34.615	16.86
4	31.480	5	1.142	5.197	1	-36.826	-44.445	20.68
5	40.000	5	1.706	2.006	1	-41.712	-50.928	22.09
6	40.000	5	1.582	1.877	1	-45.056	-55.520	23.22
7	40.000	5	1.490	1.739	1	-46.636	-58.278	24.96
8	40.000	5	1.418	1.602	1	-47.067	-59.636	26.70

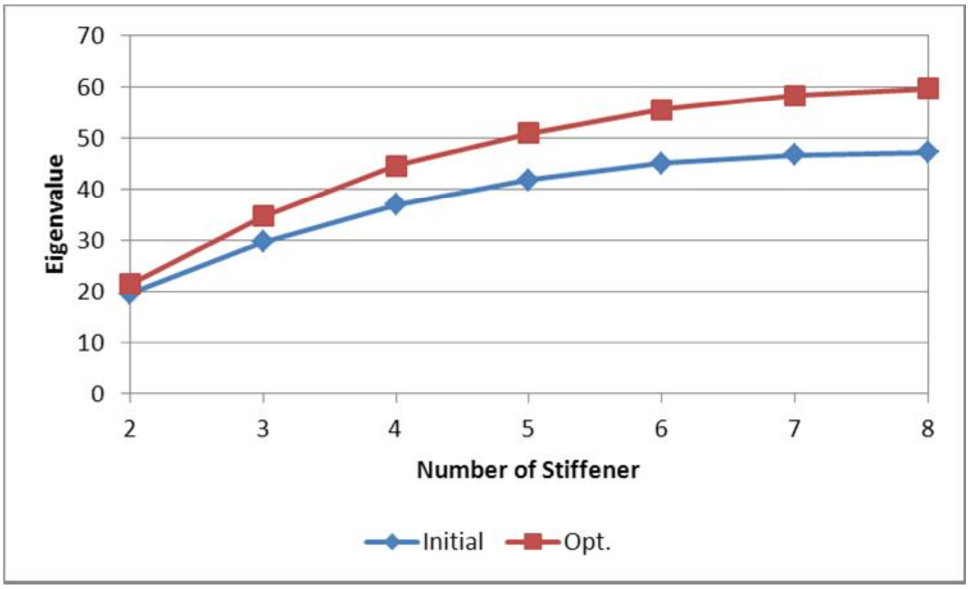

Figure 5.12 Comparison of Eigenvalues at Shape Optimization

iii) Shape Optimization (Type 2): Thickness of plate and stiffeners are kept equal at initial design. The height of stiffeners is constant and equal to 28.0 mm. The optimum values of design variables and eigenvalues are given in Table 5.8. The highest improvement is obtained for four stiffeners case and approximately equal to 19.09 %. The stiffened panel analyzed using cubic strips. In order to obtain more accurate results the large number of degrees of freedom is taken an all analysis. The smallest eigenvalue is obtained in seven stiffeners case and equal to -52.783. The plate thickness is thinner than stiffeners in optimum results and by the increasing of the number of stiffeners stiffener thickness is going to be thinner and height of sub stiffeners are all decreases to lower limit.

Table 5.9 Shape Optimization (Type 2) of Stiffened Plate with Stiffeners and Sub stiffeners

n	H_{stiff}	H_{sub}	t_{skin}	t_{stiff}	t_{sub}	Initial Eig.	Opt. Eig.	%
2	28	5	1.865	6.000	3.000	-19.490	-21.374	9.66
3	28	5	1.495	6.000	1.000	-29.618	-34.602	16.82
4	28	5	1.223	5.523	1.000	-36.826	-43.860	19.09
5	28	5	1.203	4.374	1.491	-41.712	-48.061	15.21
6	28	5	1.326	3.238	1.775	-45.056	-50.578	12.25
7	28	5	1.303	2.705	2.266	-46.636	-52.783	13.18
8	28	5	1.385	2.172	2.169	-47.0670	-52.140	10.77

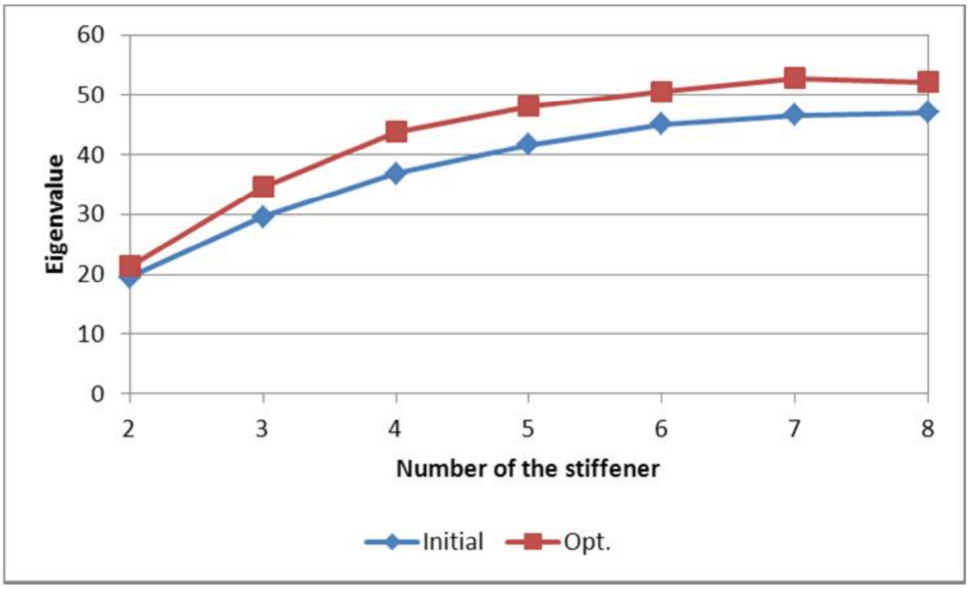

Figure 5.13 Comparison of Eigenvalues at Shape Optimization (Type 2)

iv) Shape Optimization (Type 3): Thickness of plate and stiffeners are kept equal at initial design. The thickness of stiffeners and sub stiffeners are constant and equal to initial value. The optimum values of design variables and eigenvalues are given in Table 5.10. The highest improvement is obtained for eight stiffeners case and approximately equal to 26.35 %. The stiffened panel analyzed using cubic strips. In order to obtain more accurate results the large number of degrees of freedom is taken an all analysis. The smallest eigenvalue is obtained in eight stiffeners case and equal to -59.469. The plate thickness is thinner than stiffeners in optimum results and by the increasing of the number of stiffeners thickness of plate is going to be thinner, the height of stiffeners become increase and after two stiffeners case reach upper limits, the height of sub stiffeners become decreases and after two stiffeners case reach lower limits.

Table 5.10 Shape Optimization (Type 3) of Stiffened Plate with Stiffeners and Sub stiffeners

n	H_{stiff}	H_{sub}	t_{skin}	t_{stiff}	t_{sub}	Initial Eig.	Opt. Eig.	%
2	39.13	13.08	2.186	2.298	2.298	-19.490	-19.925	2.23
3	40.00	5.00	2.036	2.123	2.123	-29.618	-33.047	11.57
4	40.00	5.00	1.878	1.973	1.973	-36.826	-43.349	17.71
5	40.00	5.00	1.742	1.842	1.842	-41.712	-50.500	21.06
6	40.00	5.00	1.622	1.728	1.728	-45.056	-55.103	22.29
7	40.00	5.00	1.516	1.627	1.627	-46.636	-58.023	24.41
8	40.00	5.00	1.422	1.538	1.538	-47.067	-59.469	26.35

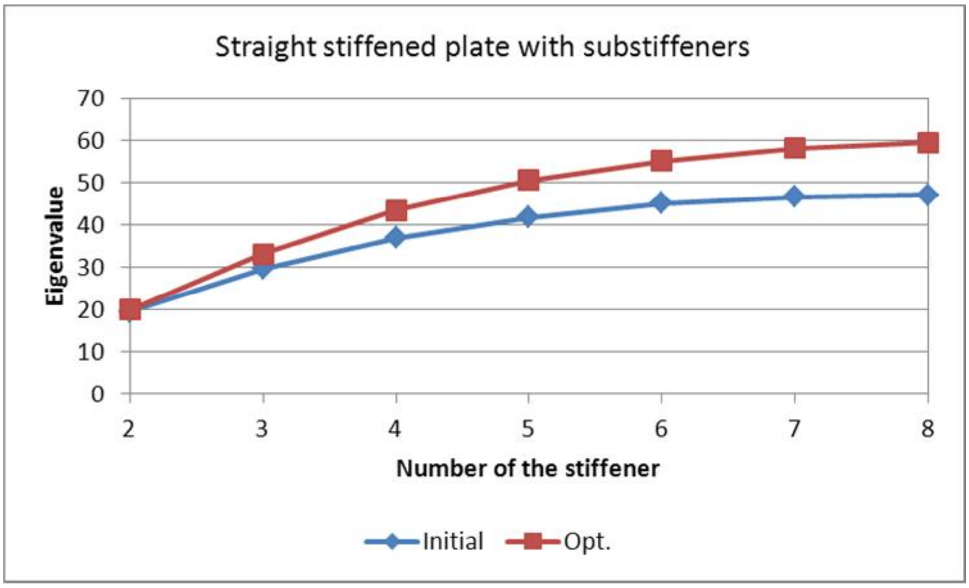

Figure 5.14 Comparison of Eigenvalues at Shape Optimization (Type 3)

v) Shape Optimization (Type 4): Thickness of plate and stiffeners are kept equal at initial design. The height of sub stiffeners is constant equal to 14 mm, the thickness of sub stiffeners is constant and equal to initial value. The optimum values of design variables and eigenvalues are given in Table 5.11. The highest improvement is obtained for eight stiffeners case and approximately equal to 22.20 %. The stiffened panel analyzed using cubic strips. In order to obtain more accurate results the large number of degrees of freedom is taken an all analysis. The smallest eigenvalue is obtained in eight stiffeners case and equal to -57.518. The plate thickness is thinner than stiffeners in optimum results and by the increasing of the number of stiffeners thickness of plate is going to be thinner except five stiffeners case, the height of stiffeners become increase and after five stiffeners case reach upper limits, the thickness of stiffeners become decreases

Table 5.11 Shape Optimization (Type 4) of Stiffened Plate with Stiffeners and Sub stiffeners

n	H_{stiff}	H_{sub}	t_{skin}	t_{stiff}	t_{sub}	Initial Eig.	Opt. Eig.	%
2	27.292	14	1.846	6.000	2.298	-19.490	-21.121	8.37
3	27.484	14	1.404	6.000	2.123	-29.618	-33.676	13.69
4	30.599	14	1.100	4.943	1.973	-36.826	-42.843	16.33
5	40.000	14	1.607	1.806	1.842	-41.712	-49.278	18.13
6	40.000	14	1.485	1.655	1.728	-45.056	-53.698	19.18
7	40.000	14	1.390	1.511	1.627	-46.636	-56.333	20.79
8	40.000	14	1.303	1.398	1.538	-47.067	-57.518	22.20

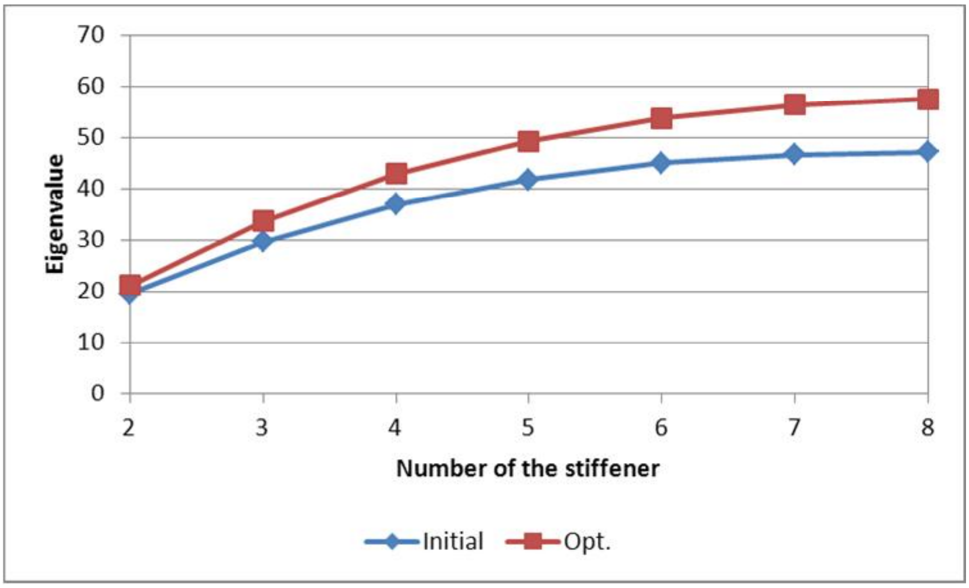

Figure 5.15 Comparison of Eigenvalues at Shape Optimization (Type 4)

Shape optimizations slightly gave better results compared to size optimizations as shown figure 5.16. For small number of stiffeners both optimizations give similar results. However when the number of the stiffeners increase shape optimization and shape optimization (Type 3) give better results. In shape optimization and shape optimization (Type 3) height of the stiffeners are not constant and can be increase; it is the fundamental causes of better eigenvalues. Shape optimization a little better than the shape optimization (Type 3) because of thickness of plate and thickness of stiffeners.

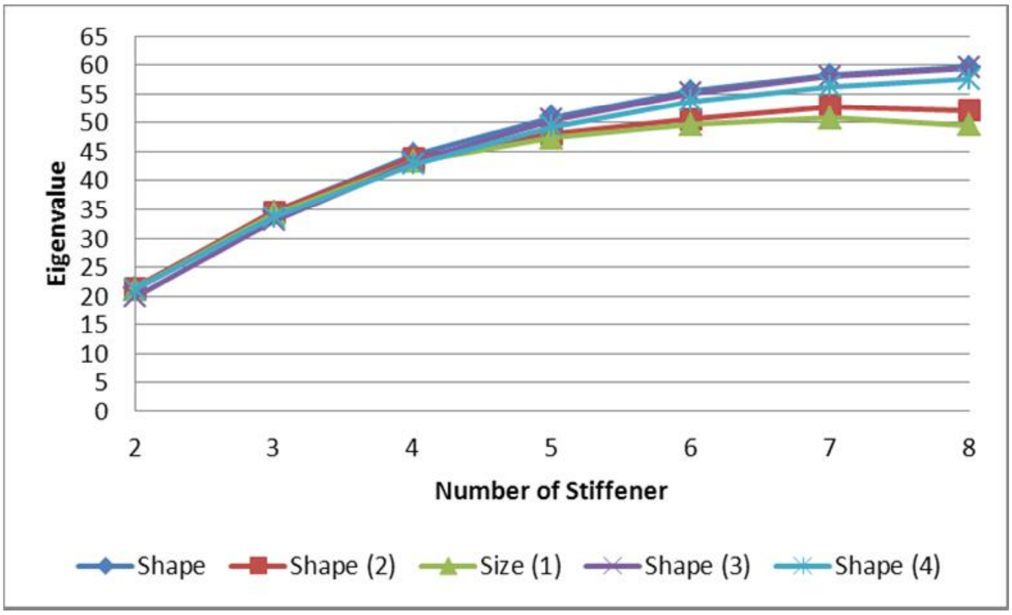

Figure 5.16 Comparison of Case Optimization

5.2.1.3 Straight stiffened plate and pads under main stiffeners

Pad elements are attached plate skin under straight stiffeners and Figure 5.17 shows straight stiffened plate and pads under stiffeners.

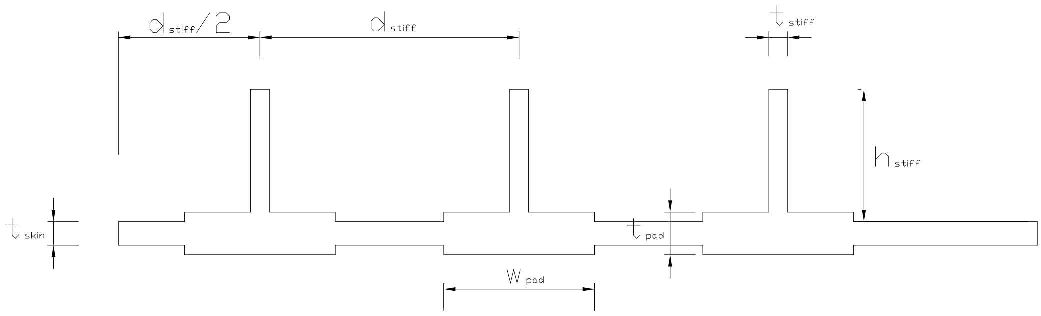

Figure 5.17 Straight stiffened plate and pads under stiffeners

a) Optimization Process:

i) Size Optimization (Type 1): Optimization is performed using thickness of plate skin (t_{skin}), thickness of stiffeners (t_{stiff}), thickness of pad (t_{pad}). During this stage width of pad (w_{pad}) has constant value of $d_{stiff}/2$ and height of stiffeners (h_{stiff}) has constant value of 28 mm (see Figure 5.17).

ii) Shape Optimization: Optimization is performed using all variables; thickness of plate skin (t_{skin}), thickness of stiffeners (t_{stiff}), thickness of pad (t_{pad}), height of stiffeners (h_{stiff}) and width of pad (w_{pad}) (see Figure 5.17).

iii) Shape Optimization (Type 1): Optimization is performed using thickness of plate skin (t_{skin}), thickness of stiffeners (t_{stiff}), thickness of pad (t_{pad}) and height of stiffeners (h_{stiff}). During this stage width of pad (w_{pad}) has constant value of $d_{stiff}/2$(see Figure 5.17).

iv) Shape Optimization (Type 2): Optimization: Optimization is performed using thickness of plate skin (t_{skin}), thickness of stiffeners (t_{stiff}), thickness of pad (t_{pad}), width of pad (w_{pad}). During this stageheight of stiffeners (h_{stiff}) has constant value of 28 mm (see Figure 5.17).

v) Shape Optimization (Type 3): Optimization is performed using thickness of pad (t_{pad}),width of pad (w_{pad}),height of stiffeners (h_{stiff}).During this stage thickness of plate skin (t_{skin}) and thickness of stiffeners (t_{stiff}) have constant values of initial values (see Figure 5.17).

Design constraints of five stages are specified in Table 5.12. Optimization process is carried out for two to eight stiffeners.

Table 5.12 Lower and Upper limits of design variables for Straight Stiffened Plate and Pads under Stiffeners

		Min (mm)	Max (mm)
Thickness of Plate	t_{skin}	1.1	3.0
Thickness of Stiffener	t_{stiff}	0.5	6.0
Height of Stiffener	h_{stiff}	8.0	40.0
Thickness of Pad	t_{pad}	2.0	6.0
Width of Pad	w_{pad}	$d_{stiff}/10$	$d_{stiff}/2$

b) Discussion of Results

In this type the effect of pad elements under stiffeners to the effect of eigenvalues is investigated. Five types of optimization are performed. The first one is size optimization (Type 1) with three design variables (t_{skin}, t_{stiff}, t_{pad}), the second one is shape optimization with five design variables (t_{skin}, t_{stiff}, t_{pad}, h_{stiff}, w_{pad}), the third one is shape optimization (Type 1) with four design variables (t_{skin}, t_{stiff}, t_{pad}, h_{stiff}),the fourth one is shape optimization (Type 2) with four design variables (t_{skin}, t_{stiff}, t_{pad}, w_{pad}), the fifth one is shape optimization (Type 3) with three design variables (t_{pad}, h_{stiff}, w_{pad}). The effect of stiffeners is also observed similar to stiffened plate.

i) Size Optimization (Type 1): Thickness of plate, stiffeners and sub stiffeners are kept equal at initial design. The width of pads is constant equal to $d_{stiff}/2$, the height of stiffeners is constant equal to 28 mm. The optimum values of design variables and eigenvalues are given in Table 5.15. The highest improvement is obtained for five stiffeners case and approximately equal to 38.52 %. The stiffened panel analyzed using cubic strips. In order to obtain more accurate results the large number of degrees of freedom is taken an all analysis. Also the smallest eigenvalue is obtained in seven stiffeners case and equal to -54.033. Moreover, in optimum results skin thickness is thinner than stiffener thickness and skin thickness reach lower limits except four stiffeners case, thickness of pads are also become less and after six stiffeners case reach lower limits.

Table 5.13 Size Optimization (Type 1) of Stiffened Plate and Pads under Stiffeners

n	h_{stiff}	t_{skin}	t_{pad}	t_{stiff}	W_{pad}	Initial Eig.	Opt. Eig	%
2	28	1.100	3.854	1.464	110	-19.747	-22.811	15.51
3	28	1.100	3.412	2.135	73.32	-29.813	-33.836	13.49
4	28	1.148	2.651	3.000	55	-34.256	-45.052	31.51
5	28	1.100	2.318	3.000	44	-37.362	-51.755	38.52
6	28	1.100	2.000	2.915	36.66	-39.638	-53.725	35.54
7	28	1.100	2.000	2.500	31.42	-41.457	-54.033	30.33
8	28	1.100	2.000	2.187	27.5	-43.034	-53.931	25.31

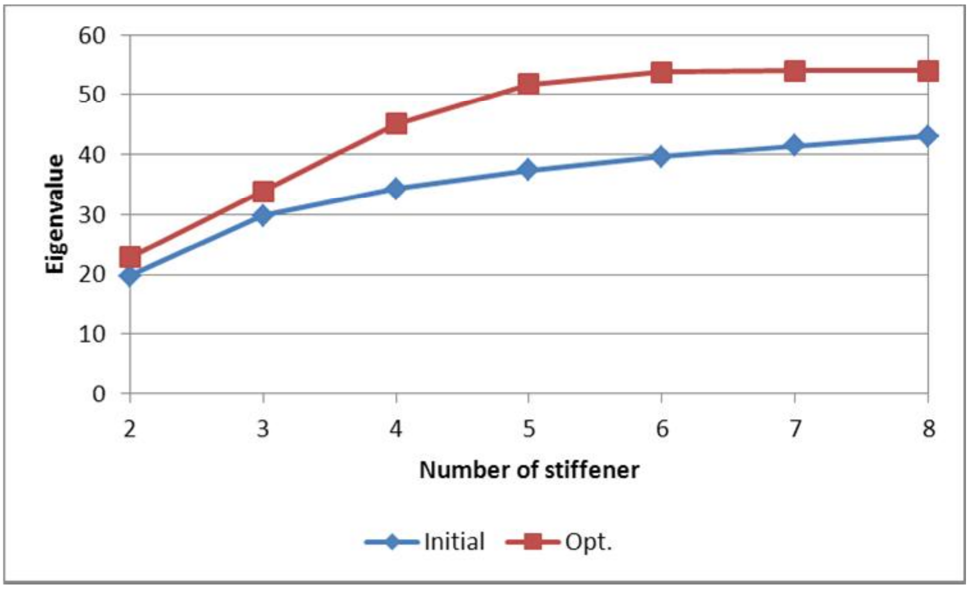

Figure 5.18 Comparison of Eigenvalues at Size Optimization (Type 1)

ii) Shape Optimization: Thickness of plate, stiffeners and sub stiffeners are kept equal at initial design. The optimum values of design variables and eigenvalues are given in Table 5.13. The highest improvement is obtained for six stiffeners case and approximately equal to 48.24 %. The stiffened panel analyzed using cubic strips. In order to obtain more accurate results the large number of degrees of freedom is taken an all analysis. Also the smallest eigenvalue is obtained in eight stiffeners case and equal to -61.498. Moreover, in optimum results skin thickness is thinner than stiffener thickness, the height of stiffeners increase except six stiffeners case , width of pads are become less except four stiffeners case, thickness of pads are also become less toward lower limits.

Table 5.14 Shape Optimization of Stiffened Plate and Pads under Stiffeners

n	h_{stiff}	t_{skin}	t_{pad}	t_{stiff}	W_{pad}	Initial Eig.	Opt. Eig	%
2	29.799	1.100	4.956	1.827	75.098	-19.747	-25.214	27.68
3	23.297	1.100	4.586	3.000	45.730	-29.813	-38.915	30.52
4	32.526	1.363	2.189	3.000	55.000	-34.256	-45.466	32.72
5	40.000	1.452	2.108	2.022	39.111	-37.362	-53.048	41.98
6	35.044	1.163	2.000	2.364	32.600	-39.647	-58.777	48.24
7	40.000	1.301	2.000	1.781	20.581	-41.457	-59.880	44.43
8	40.000	1.345	2.462	1.658	5.500	-43.032	-61.498	42.91

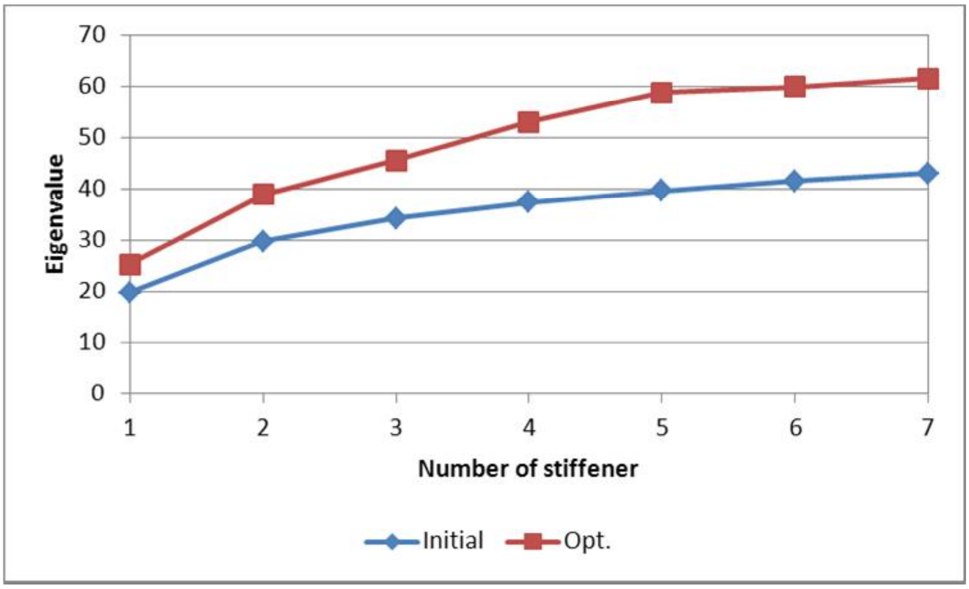

Figure 5.19 Comparison of Eigenvalues at Shape Optimization

iii) Shape Optimization (Type 1): Thickness of plate, stiffeners and sub stiffeners are kept equal at initial design. The width of pads is constant equal to $d_{stiff}/2$. The optimum values of design variables and eigenvalues are given in Table 5.14. The highest improvement is obtained for six stiffeners case and approximately equal to 46.12 %. The stiffened panel analyzed using cubic strips. In order to obtain more accurate results the large number of degrees of freedom is taken an all analysis. Also the smallest eigenvalue is obtained in eight stiffeners case and equal to -60.338. Moreover, in optimum results skin thickness is thinner than stiffener thickness, the height of stiffeners increase and after six stiffeners case reach upper limits, thickness of pads are also become less and after five stiffeners case reach lower limits.

Table 5.15 Shape Optimization (Type 1) of Stiffened Plate and Pads under Stiffeners

n	h_{stiff}	t_{skin}	t_{pad}	t_{stiff}	W_{pad}	Initial Eig.	Opt. Eig	%
2	36.029	1.100	3.899	1.000	110	-19.747	-23.032	16.63
3	17.882	1.100	3.496	3.000	73.32	-29.813	-34.807	16.75
4	32.538	1.372	2.180	3.000	55	-34.256	-45.458	32.69
5	33.582	1.125	2.000	2.884	44	-37.362	-54.291	45.30
6	40.000	1.289	2.000	1.867	36.66	-39.638	-57.922	46.12
7	40.000	1.179	2.000	1.688	31.42	-41.457	-59.344	43.14
8	40.000	1.100	2.000	1.530	27.5	-43.014	-60.338	40.27

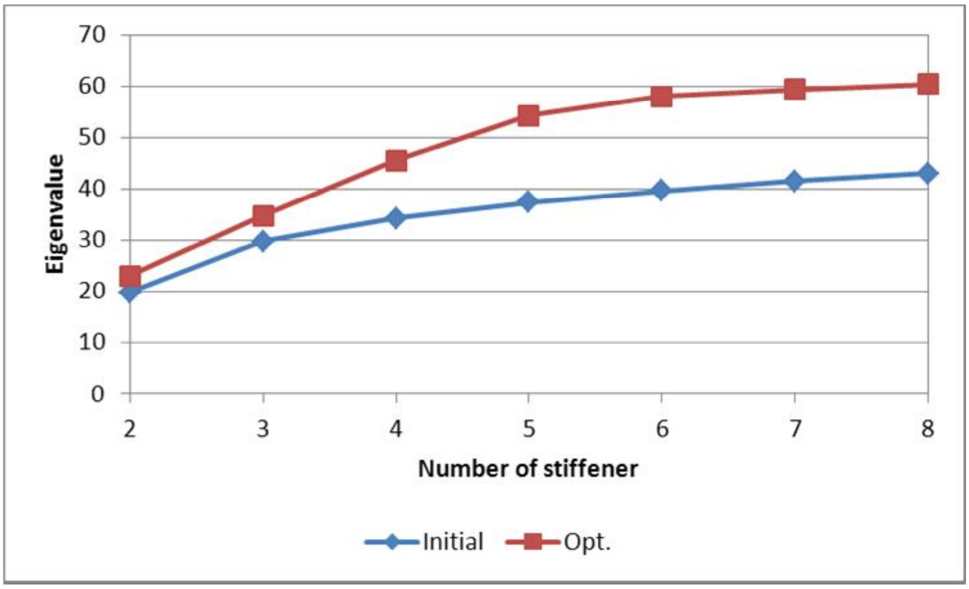

Figure 5.20 Comparison of eigenvalues at Shape optimization (Type 1)

iv) Shape Optimization (Type 2): Thickness of plate, stiffeners and sub stiffeners are kept equal at initial design. The height of stiffeners is constant equal to 28 mm. The optimum values of design variables and eigenvalues are given in Table 5.16. The highest improvement is obtained for seven stiffeners case and approximately equal to 50.50 %. The stiffened panel analyzed using cubic strips. In order to obtain more accurate results the large number of degrees of freedom is taken an all analysis. Also the smallest eigenvalue is obtained in eight stiffeners case and equal to -62.490. Moreover, in optimum results skin thickness is thinner than stiffener thickness and skin thickness reach lower limits in all stiffener case, thickness of stiffeners are increase toward near upper limit, width of pads become less.

Table 5.16 Shape Optimization (Type 2) of Stiffened Plate and Pads under Stiffeners

n	h_{stiff}	t_{skin}	t_{pad}	t_{stiff}	W_{pad}	Initial Eig.	Opt. Eig	%
2	28	1.1	3.855	1.525	109.420	-19.747	-22.868	15.80
3	28	1.1	5.886	2.925	30.797	-29.813	-39.571	32.73
4	28	1.1	2.636	3.000	55.000	-34.256	-45.065	31.55
5	28	1.1	2.318	3.000	44.000	-37.362	-51.755	38.52
6	28	1.1	3.053	3.000	15.678	-39.638	-58.428	47.40
7	28	1.1	3.804	2.903	6.284	-41.457	-62.396	50.50
8	28	1.1	3.474	2.604	5.500	-43.014	-62.490	45.27

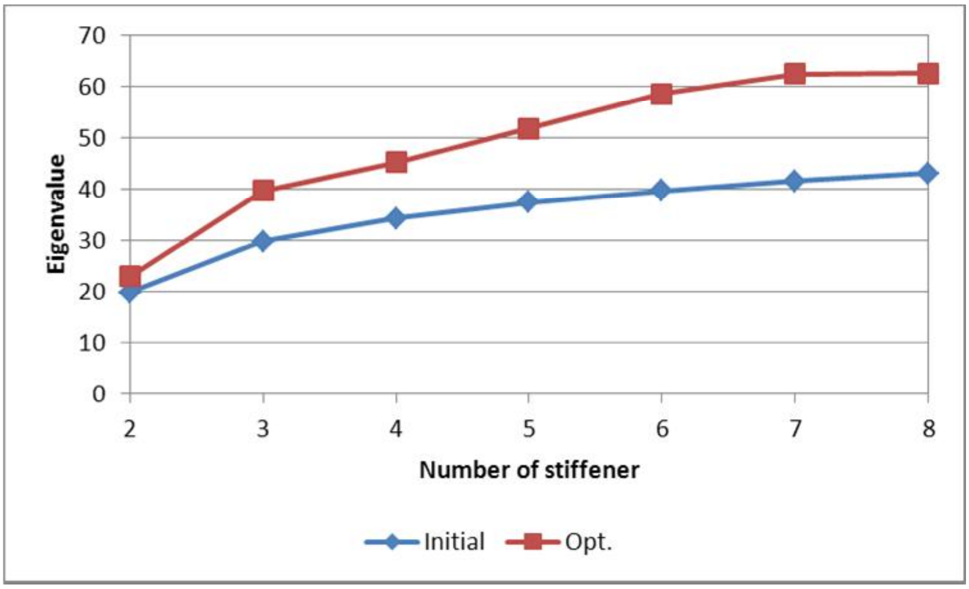

Figure 5.21 Comparison of Eigenvalues at Shape Optimization (Type 2)

v) Shape Optimization (Type 3): Thickness of plate, stiffeners and sub stiffeners are kept equal at initial design. The thickness of stiffeners and the thickness of plate are constant equal initial values. The optimum values of design variables and eigenvalues are given in Table 5.17. The highest improvement is obtained for six stiffeners case and approximately equal to 40.59 %. The stiffened panel analyzed using cubic strips. In order to obtain more accurate results the large number of degrees of freedom is taken an all analysis. Also the smallest eigenvalue is obtained in eight stiffeners case and equal to -58.908. Moreover, in optimum results height of stiffeners increase and after three stiffeners case reach upper limits, width of pads become less.

Table 5.17 Shape Optimization (Type 3) of Stiffened Plate and Pads under Stiffeners

n	h_{stiff}	t_{skin}	t_{pad}	t_{stiff}	W_{pad}	Initial Eig.	Opt. Eig	%
2	26.31	1.636	3.994	1.636	83.556	-19.747	-21.752	10.15
3	40.00	1.575	6.000	1.575	21.808	-29.813	-36.893	23.74
4	40.00	1.518	2.847	1.518	50.876	-34.256	-44.441	29.72
5	40.00	1.465	6.000	1.465	9.374	-37.362	-50.746	35.82
6	40.00	1.415	4.035	1.415	10.762	-39.647	-55.742	40.59
7	40.00	1.369	3.307	1.369	8.891	-41.457	-56.989	37.46
8	40.00	1.325	2.284	1.325	11.209	-43.034	-58.908	36.88

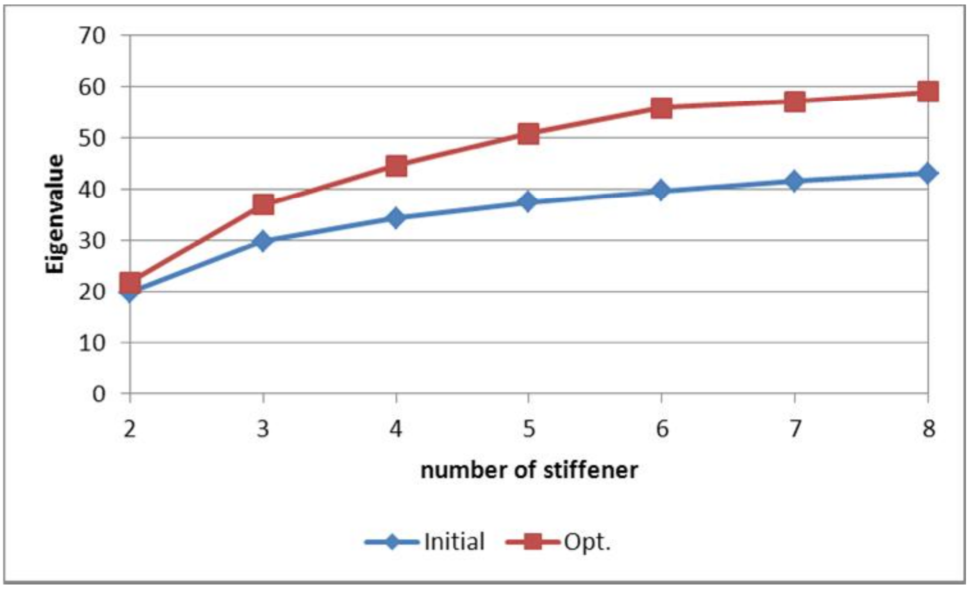

Figure 5.22 Comparison of Eigenvalues at Shape Optimization (Type 3)

Shape optimization (Type 2) and Shape optimizations slightly gave better results compared to other size optimizations as shown figure 5.23. For small number of stiffeners optimizations give similar results. However when the number of the stiffeners increase shape optimization and shape optimization (Type 2) give better results. At shape optimization (Type 2) height of the stiffeners is constant but results are nearly same. Shape optimization (Type 2) a little better than the shape optimization, at shape optimization (Type 2) case the thickness of stiffener and thickness of pad are bigger than the shape optimization case. The width of pad is constant at Size optimization (Type 1) this is only difference between shape optimization (Type 2) and it shows the common factor of optimization because the shape optimization (Type 2) gives the best solution, the size optimization (Type 1) gives the worst solution at this stage.

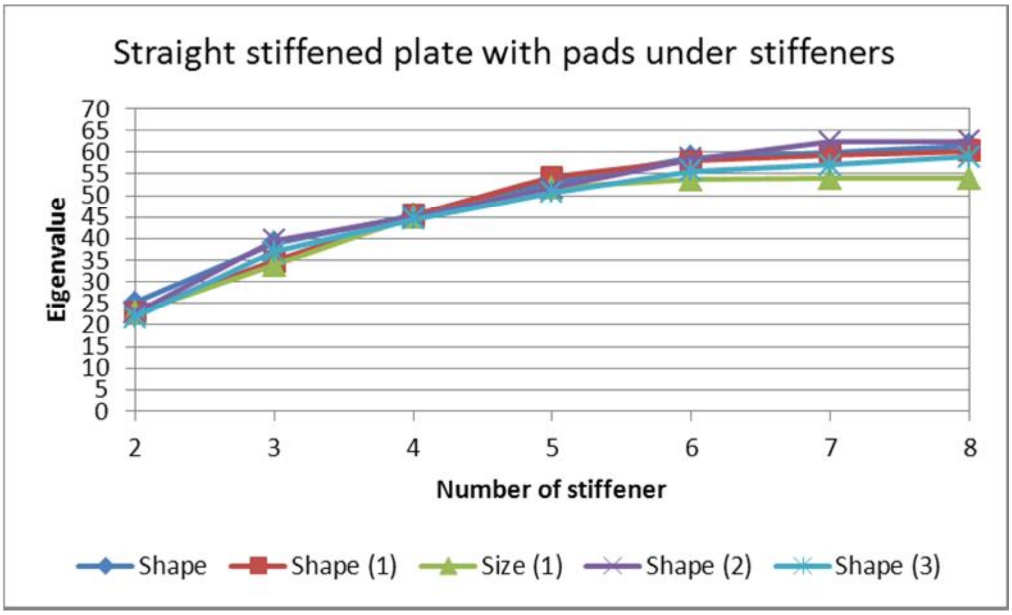

Figure 5.23 Comparison of Case Optimization

5.2.1.4 Straight Stiffened Plate with Sub stiffeners and Pads under Main Stiffeners

Sub stiffeners are added between stiffeners and pad elements are attached under stiffeners and Figure 5.24 shows straight stiffened plate with sub stiffeners and pads under stiffeners.

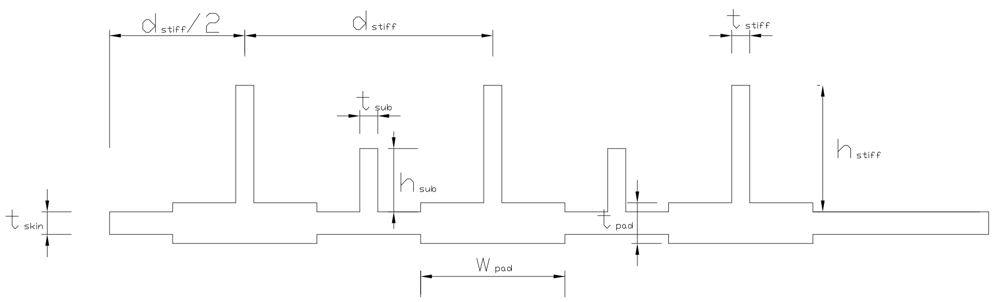

Figure 5.24 Straight Stiffened Plate with Sub Stiffeners and Pads under Stiffeners

a) Optimization Process:

i) Size Optimization (Type 1): Optimization is performed using thickness of plate skin (t_{skin}), thickness of stiffeners (t_{stiff}), thickness of sub stiffeners (t_{sub}) and thickness of pad (t_{pad}). During this stage height of stiffeners (h_{stiff}) has constant value of 28 mm and height of sub stiffeners (h_{sub}) has a constant value of 14 mm andwidth of pad (w_{pad}) has a constant value of $d_{stiff}/2$ (see Figure 5.24).

ii) Shape Optimization: Optimization is performed using all variables; thickness of plate skin (t_{skin}), thickness of stiffeners (t_{stiff}),thickness of sub stiffeners (t_{sub}), thickness of pad (t_{pad}), height of stiffeners (h_{stiff}),height of sub stiffeners (h_{sub}) and width of pad (w_{pad}) (see Figure 5.24).

iii) Shape Optimization (Type 1): Optimization is performed using thickness of plate skin (t_{skin}), thickness of stiffeners (t_{stiff}),thickness of sub stiffeners (t_{sub}), thickness of pad (t_{pad}), height of sub stiffeners (h_{sub}) and width of pad (w_{pad}). During this stage height of stiffeners (h_{stiff}) has constant value of 28 mm (see Figure 5.24).

iv) Shape Optimization (Type 2): Optimization is performed using thickness of plate skin (t_{skin}), thickness of stiffeners (t_{stiff}),thickness of sub stiffeners (t_{sub}), thickness of pad (t_{pad}), height of sub stiffeners (h_{sub}) and height of stiffeners (h_{stiff}). During this stage width of pad (w_{pad}) has constant value of $d_{stiff}/2$ (see Figure 5.24).

v) Shape Optimization (Type 3): Optimization is performed using thickness of plate skin (t_{skin}), thickness of stiffeners (t_{stiff}),thickness of sub stiffeners (t_{sub}), thickness of pad (t_{pad}), and width of pad (w_{pad}). During this stage height of stiffeners (h_{stiff}) has constant value of 28 mm and height of sub stiffeners (h_{sub}) has a constant value of 14 mm (see Figure 5.24).

vi) Shape Optimization (Type 4): Optimization is performed using, thickness of pad (t_{pad}), and width of pad (w_{pad}), height of stiffeners (h_{stiff}), height of sub stiffeners (h_{sub}). During this stage thickness of plate skin (t_{skin}), thickness of stiffeners (t_{stiff}),thickness of sub stiffeners (t_{sub}) have constant values of what is calculated at initial(see Figure 5.24).

vii) Shape Optimization (Type 5): Optimization is performed using, thickness of stiffeners (t_{stiff}), thickness of sub stiffeners (t_{sub}), width of pad (w_{pad}),height of stiffeners (h_{stiff}) and height of sub stiffeners (h_{sub}). During this stage thickness of plate skin (t_{skin}) and thickness of pad (t_{pad}) have constant values of what is calculated at initial (see Figure 5.24).

iix) Shape Optimization (Type 6): Optimization is performed using, thickness of stiffeners (t_{stiff}), thickness of sub stiffeners (t_{sub}), thickness of plate skin (t_{skin}), height of stiffeners (h_{stiff}) and height of sub stiffeners (h_{sub}). During this stage and thickness of pad (t_{pad}) has a constant value of what is calculated at initial, width of pad (w_{pad}) has a constant value of $d_{stiff}/2$ (see Figure 5.24).

Design constraints of eight stages are specified in Table 5.18. Optimization process is carried out for two to eight stiffeners.

Table 5.18 Lower and Upper Limits of Design Variables Straight Stiffened Plate with Sub stiffeners and Pads under Stiffeners

		Min (mm)	Max (mm)
Thickness of Plate	t_{skin}	1.1	3.0
Thickness of Stiffener	t_{stiff}	0.5	6.0
Thickness of Sub stiffener	t_{sub}	1.0	3.0
Height of Stiffener	h_{stiff}	8.0	40.0
Height of Sub stiffener	h_{sub}	5.0	20.0
Thickness of Pad	t_{pad}	2.0	6.0
Width of Pad	w_{pad}	$d_{stiff}/10$	$d_{stiff}/2$

b) Discussion of Results

The effect of sub stiffeners and pads are examined together in this type of plates. Eight types of optimization are performed. The first one is size optimization (Type 1) with four design variables (t_{skin}, t_{stiff}, t_{sub}, t_{pad}), the second one is shape optimization with seven design variables (t_{skin}, t_{stiff}, t_{sub}, t_{pad}, h_{stiff}, h_{sub}, w_{pad}), the third one is shape optimization (Type 1) with six design variables (t_{skin}, t_{stiff}, t_{sub}, t_{pad}, h_{sub}, w_{pad}), the fourth one is shape optimization (Type 2) with six design variables (t_{skin}, t_{stiff}, t_{sub}, t_{pad}, h_{stiff}, h_{sub}), the fifth one is shape optimization (Type 3) with five design variables (t_{skin}, t_{stiff}, t_{sub}, t_{pad}, w_{pad}), the sixth one is shape optimization (Type 4) with four design variables (t_{pad}, h_{stiff}, h_{sub}, w_{pad}), the seventh one is shape optimization (Type 5) with five design variables (t_{stiff}, t_{sub}, h_{stiff}, h_{sub}, w_{pad}), the eighth one is shape optimization (Type 6) with five design variables (t_{skin}, t_{stiff}, t_{sub}, h_{stiff}, h_{sub}) The effect of stiffeners is also observed similar to stiffened plate.

i) Size Optimization (Type 1): Thickness of plate, stiffeners and sub stiffeners are kept equal at initial design. The height of stiffener has a constant value equal to 28 mm, the height of sub stiffener has a constant value equal to 14 mm, The width of pad has a constant value equal to $d_{stiff}/2$. The optimum values of design variables and eigenvalues are given in Table 5.23. The highest improvement is obtained for five stiffeners case and approximately equal to 33.78 %. The stiffened panel analyzed using cubic strips. In order to obtain more accurate results the large number of degrees of freedom is taken an all analysis. Also the smallest eigenvalue is obtained in six stiffeners case and equal to -51.898. Moreover, in optimum results skin thickness is thinner than stiffener thickness, the thickness of plate reach lower limits in all cases and the thickness of pad and the thickness of stiffeners decreases.

Table 5.19 Size Optimization (Type 1) of stiffened plate and pads under stiffeners

n	h_{stiff}	h_{sub}	t_{skin}	t_{pad}	t_{stiff}	t_{sub}	W_{pad}	**Initial Eig.**	**Opt. Eig**	**%**
2	28	14	1.1	3.272	3.000	3.000	110	-21.692	-25.859	19.20
3	28	14	1.1	2.773	3.000	2.424	73.32	-29.212	-36.800	25.97
4	28	14	1.1	2.485	3.000	1.121	55	-34.202	-44.963	31.46
5	28	14	1.1	2.063	3.000	1.000	44	-37.832	-50.614	33.78
6	28	14	1.1	2.000	2.499	1.000	36.66	-40.498	-51.898	28.14
7	28	14	1.1	2.000	2.071	1.000	31.42	-42.551	-51.708	21.52
8	28	14	1.1	2.000	1.748	1.000	27.5	-44.152	-51.006	15.52

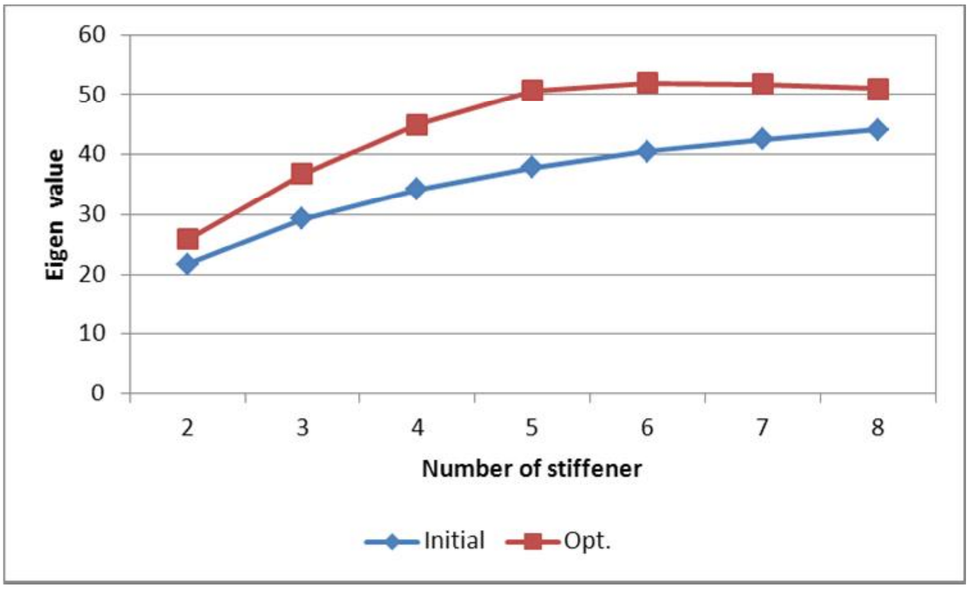

Figure 5.25 Comparison of eigenvalues at Size optimization (Type 1)

ii) Shape Optimization: Thickness of plate, stiffeners and sub stiffeners are kept equal at initial design. The optimum values of design variables and eigenvalues are given in Table 5.19. The highest improvement is obtained for four stiffeners case and approximately equal to 44.55 %. The stiffened panel analyzed using cubic strips. In order to obtain more accurate results the large number of degrees of freedom is taken an all analysis. Also the smallest eigenvalue is obtained in eight stiffeners case and equal to -60.511. Moreover, in optimum results skin thickness is thinner than stiffener thickness, the height of stiffeners increase except three stiffeners case and reach upper limits after five stiffeners case, height of sub stiffener decrease and reach lower limits after two stiffeners case, width of pads are become less except five and seven stiffeners case, thickness of pads are also become less, thickness of sub stiffener become less except eight stiffener case and reach lower limits after two stiffener case.

Table 5.20 Shape Optimization of Stiffened Plate and Pads under Stiffeners

n	h_{stiff}	h_{sub}	t_{skin}	t_{pad}	t_{stiff}	t_{sub}	W_{pad}	Initial Eig.	Opt. Eig	%
2	31.88	20	1.10	3.132	3.000	3.000	107.431	-21.692	-26.509	22.20
3	25.73	5	1.10	5.606	3.000	1.000	33.016	-29.212	-39.551	35.39
4	35.24	5	1.10	6.000	2.306	1.000	17.727	-34.202	-49.441	44.55
5	40.00	5	1.41	2.738	1.905	1.000	22.379	-37.832	-52.668	39.21
6	40.00	5	1.47	2.713	1.849	1.000	7.332	-40.498	-56.811	40.28
7	40.00	5	1.33	2.057	1.722	1.000	14.239	-42.551	-59.417	39.63
8	40.00	5	1.24	2.386	1.653	1.003	6.368	-44.152	-60.511	37.05

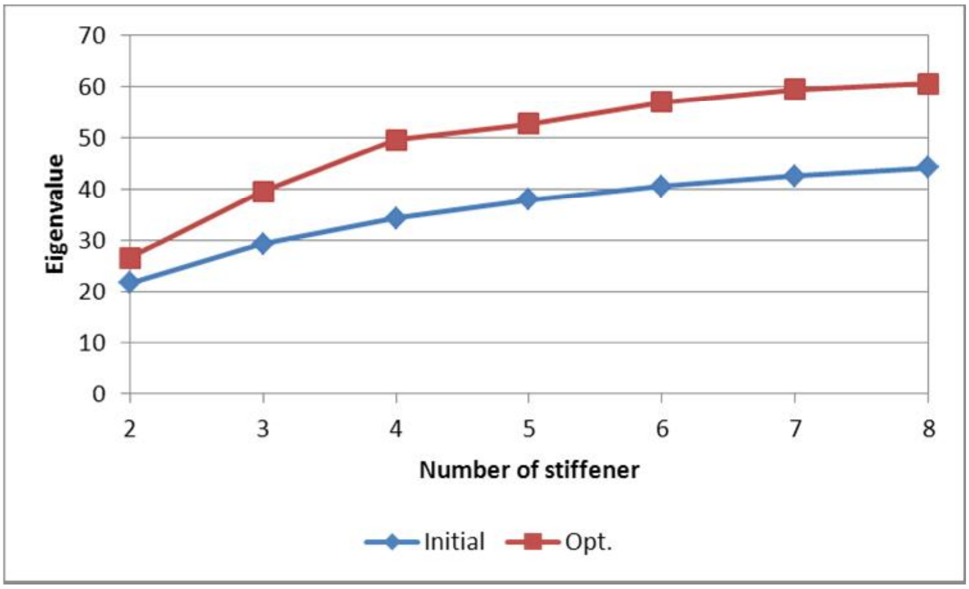

Figure 5.26 Comparison of Eigenvalues at Shape Optimization

iii) Shape Optimization (Type 1): Thickness of plate, stiffeners and sub stiffeners are kept equal at initial design. The height of stiffener is constant equal to 28 mm. The optimum values of design variables and eigenvalues are given in Table 5.20. The highest improvement is obtained for four stiffeners case and approximately equal to 46.37 %. The stiffened panel analyzed using cubic strips. In order to obtain more accurate results the large number of degrees of freedom is taken an all analysis. Also the smallest eigenvalue is obtained in seven stiffeners case and equal to -61.462. Moreover, in optimum results skin thickness is thinner than stiffener thickness, the height of sub stiffeners decrease and after two stiffeners case reach lower limits, width of pads are also become less and the thickness of sub stiffener become less and after two stiffener case reach lower value.

Table 5.21 Shape Optimization (Type 1) of Stiffened Plate and Pads under Stiffeners

n	h_{stiff}	h_{sub}	t_{skin}	t_{pad}	t_{stiff}	t_{sub}	W_{pad}	Initial Eig.	Opt. Eig	%
2	28	20	1.100	3.323	3.000	2.95	103.65	-21.692	-26.475	22.04
3	28	5	1.100	5.444	2.994	1.00	32.710	-29.212	-39.455	35.06
4	28	5	1.100	6.000	3.000	1.00	17.190	-34.202	-50.064	46.37
5	28	5	1.369	4.313	3.000	1.00	8.800	-37.832	-53.864	42.37
6	28	5	1.305	3.418	2.852	1.00	7.332	-40.498	-57.590	42.20
7	28	5	1.100	3.639	2.787	1.00	6.284	-42.551	-61.462	44.44
8	28	5	1.193	3.070	2.362	1.00	5.500	-44.152	-60.172	36.28

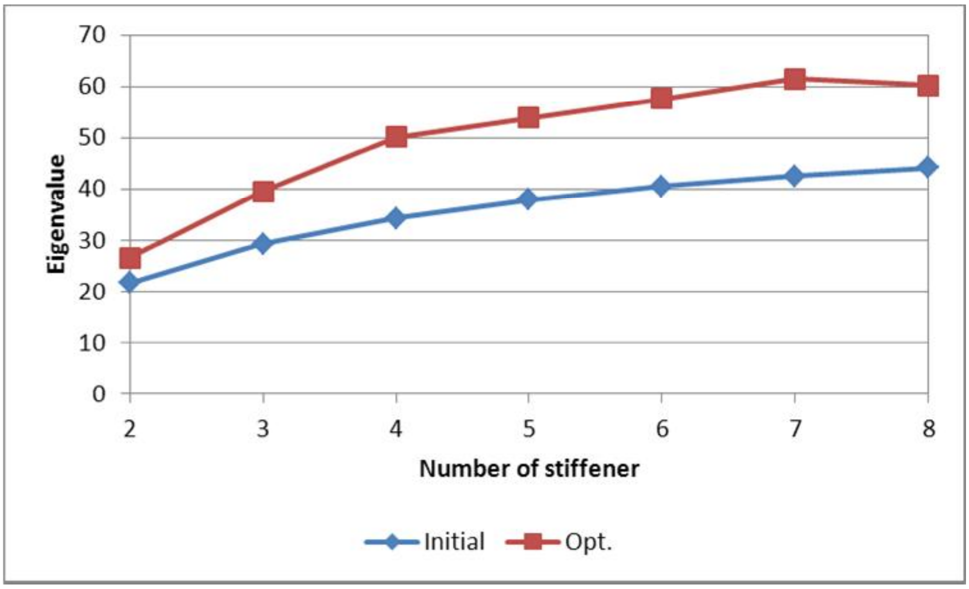

Figure 5.27 Comparison of Eigenvalues at Shape Optimization (Type 1)

iv) Shape Optimization (Type 2): Thickness of plate, stiffeners and sub stiffeners are kept equal at initial design. The width of pad is constant equal to $d_{stiff}/2$. The optimum values of design variables and eigenvalues are given in Table 5.21. The highest improvement is obtained for six stiffeners case and approximately equal to 41.17 %. The stiffened panel analyzed using cubic strips. In order to obtain more accurate results the large number of degrees of freedom is taken an all analysis. Also the smallest eigenvalue is obtained in seven stiffeners case and equal to -58.721. Moreover, in optimum results skin thickness is thinner than stiffener thickness except eight stiffener case, the thickness of sub stiffeners decrease and after five stiffeners case reach lower limits and the thickness of stiffener become less, the height of sub stiffener become less and reach lower limits.

Table 5.22 Shape Optimization (Type 2) of stiffened plate and pads under stiffeners

n	h_{stiff}	h_{sub}	t_{skin}	t_{pad}	t_{stiff}	t_{sub}	W_{pad}	Initial Eig.	Opt. Eig	%
2	32.10	20	1.100	3.078	3	3.000	110	-21.692	-26.487	22.10
3	31.30	5	1.123	2.787	3	3.000	73.32	-29.212	-37.503	28.38
4	33.84	5	1.144	2.167	3	2.474	55	-34.202	-46.582	36.19
5	40.00	5	1.369	2.738	1.241	1.000	44	-37.832	-50.763	34.17
6	40.00	5	1.213	2.000	1.831	1.001	36.66	-40.496	-57.168	41.17
7	40.00	5	1.154	2.000	1.599	1.000	31.42	-42.551	-58.721	38.00
8	40.00	5	1.193	2.386	1.090	1.000	27.5	-44.125	-55.361	25.46

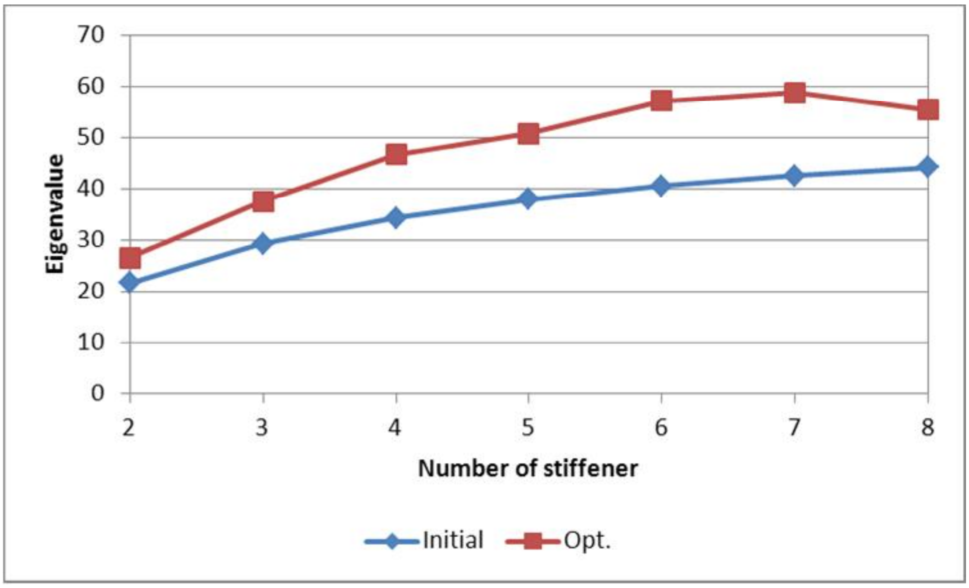

Figure 5.28 Comparison of eigenvalues at Shape optimization (Type 2)

v) Shape Optimization (Type 3): Thickness of plate, stiffeners and sub stiffeners are kept equal at initial design. The height of stiffener is constant equal to 28 mm, the height of stiffener is constant equal to 14 mm. The optimum values of design variables and eigenvalues are given in Table 5.21. The highest improvement is obtained for six stiffeners case and approximately equal to 44.44 %. The stiffened panel analyzed using cubic strips. In order to obtain more accurate results the large number of degrees of freedom is taken an all analysis. Also the smallest eigenvalue is obtained in seven stiffeners case and equal to -59.489. Moreover, in optimum results skin thickness is thinner than stiffener thickness, the thickness of sub stiffeners decrease and after three stiffeners case reach lower limits and the thickness of stiffeners almost near three, the width of pad become less.

Table 5.23 Shape Optimization (Type 3) of stiffened plate and pads under stiffeners

n	**h_{stiff}**	**h_{sub}**	**t_{skin}**	**t_{pad}**	**t_{stiff}**	**t_{sub}**	**W_{pad}**	**Initial Eig.**	**Opt. Eig**	**%**
2	28	14	1.10	3.682	3.000	2.93	92.854	-21.692	-26.162	20.60
3	28	14	1.10	5.710	2.955	1.00	29.779	-29.212	-38.957	33.35
4	28	14	1.10	6.000	3.000	1.00	15.804	-34.202	-49.364	44.32
5	28	14	1.36	3.495	3.000	1.00	8.800	-37.832	-52.635	39.12
6	28	14	1.10	3.943	2.934	1.00	7.332	-40.498	-58.497	44.44
7	28	14	1.10	3.309	2.585	1.00	6.284	-42.551	-59.489	39.80
8	28	14	1.19	2.749	2.144	1.00	5.500	-44.152	-57.426	30.06

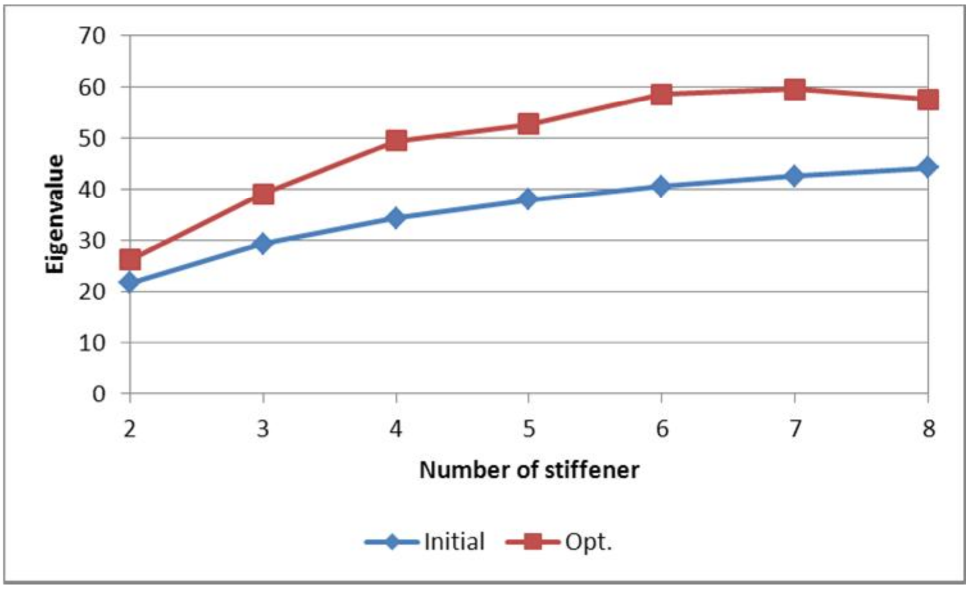

Figure 5.29 Comparison of eigenvalues at Shape optimization (Type 3)

vi) Shape Optimization (Type 4): Thickness of plate, stiffeners and sub stiffeners are kept equal at initial design. The thickness of plate, the thickness of stiffener and the thickness of sub stiffener has constant equal to what is calculated initial. The optimum values of design variables and eigenvalues are given in Table 5.22. The highest improvement is obtained for five stiffeners case and approximately equal to 35.34 %. The stiffened panel analyzed using cubic strips. In order to obtain more accurate results the large number of degrees of freedom is taken an all analysis. Also the smallest eigenvalue is obtained in eight stiffeners case and equal to -57.875. Moreover, in optimum results the height of sub stiffeners decrease and after three stiffeners case reach lower limits and the height of stiffeners reach lower limit at all cases, the width of pad become less.

Table 5.24 Shape Optimization (Type 4) of stiffened plate and pads under stiffeners

n	h_{stiff}	h_{sub}	t_{skin}	t_{pad}	t_{stiff}	t_{sub}	W_{pad}	Initial Eig.	Opt. Eig	%
2	40	20	1.605	3.028	1.605	1.605	107.25	-21.692	-23.530	8.47
3	40	5	1.518	6.000	1.518	1.518	22.804	-29.212	-37.106	27.02
4	40	5	1.439	6.000	1.439	1.439	15.707	-34.202	-45.760	33.79
5	40	5	1.369	6.000	1.369	1.369	11.589	-37.832	-51.204	35.34
6	40	5	1.305	6.000	1.305	1.305	8.9204	-40.498	-53.809	32.86
7	40	5	1.246	5.282	1.246	1.246	8.4182	-42.551	-56.123	31.89
8	40	5	1.193	4.823	1.193	1.193	7.4525	-44.152	-57.817	30.95

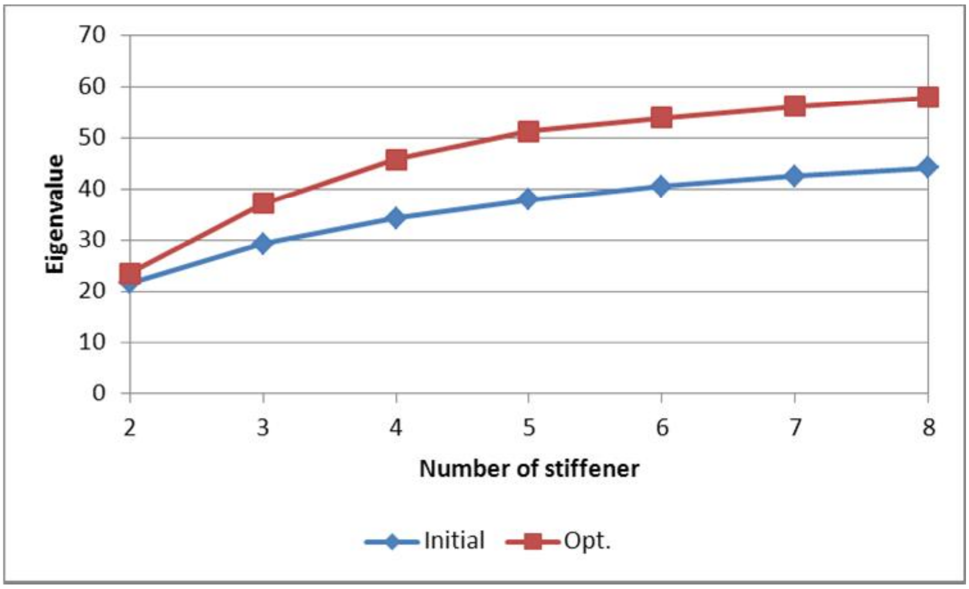

Figure 5.30 Comparison of eigenvalues at Shape optimization (Type 4)

vii) Shape Optimization (Type 5): Thickness of plate, stiffeners and sub stiffeners are kept equal at initial design. The thickness of plate, the and the thickness of pad has constant equal to what is calculated initial. The optimum values of design variables and eigenvalues are given in Table 5.24. The highest improvement is obtained for six stiffeners case and approximately equal to 40.44 %. The stiffened panel analyzed using cubic strips. In order to obtain more accurate results the large number of degrees of freedom is taken an all analysis. Also the smallest eigenvalue is obtained in eight stiffeners case and equal to -59.743. Moreover, in optimum results the height of sub stiffeners decrease to lower limits in all cases, and the width of pads and the thickness of sub stiffeners become less.

Table 5.25 Shape Optimization (Type 5) of stiffened plate and pads under stiffeners

n	h_{stiff}	h_{sub}	t_{skin}	t_{pad}	t_{stiff}	t_{sub}	W_{pad}	Initial Eig.	Opt. Eig	%
2	35.03	5	1.160	3.210	2.328	3.000	89.348	-21.692	-24.342	12.21
3	40.00	5	1.518	3.036	1.920	3.000	53.478	-29.212	-37.110	27.03
4	40.00	5	1.439	2.879	1.986	2.211	32.562	-34.202	-46.362	35.55
5	40.00	5	1.369	2.738	1.898	1.010	24.793	-37.832	-52.628	39.10
6	31.62	5	1.305	2.610	2.712	1.000	7.332	-40.498	-56.877	40.44
7	36.78	5	1.246	2.493	2.036	1.000	7.977	-42.551	-59.187	39.09
8	22.26	5	1.193	2.386	3.000	1.713	5.500	-44.152	-59.743	35.31

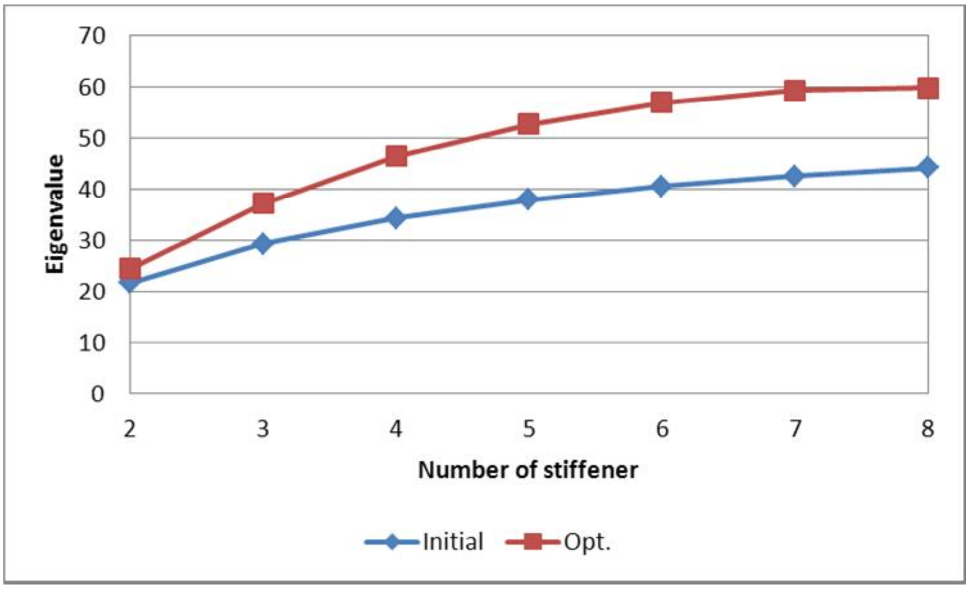

Figure 5.31 Comparison of eigenvalues at Shape optimization (Type 5)

iix) Shape Optimization (Type 6): Thickness of plate, stiffeners and sub stiffeners are kept equal at initial design. The thickness of pad has constant value equal to what is calculated initial and the width of pad has constant value equal to $d_{stiff}/2$. The optimum values of design variables and eigenvalues are given in Table 5.25. The highest improvement is obtained for four stiffeners case and approximately equal to 32.29 %. The stiffened panel analyzed using cubic strips. In order to obtain more accurate results the large number of degrees of freedom is taken an all analysis. Also the smallest eigenvalue is obtained in eight stiffeners case and equal to -55.361. Moreover, in optimum results the height of stiffeners increase to upper limits in all cases except three stiffeners case, the height of sub stiffeners decrease to lower limits in all cases except two stiffeners case and the thickness of stiffeners and the thickness of sub stiffeners become less.

Table 5.26 Shape Optimization (Type 6) of stiffened plate and pads under stiffeners

N	h_{stiff}	h_{sub}	t_{skin}	t_{pad}	t_{stiff}	t_{sub}	W_{pad}	Initial Eig.	Opt. Eig	%
2	40.000	20	1.100	3.210	2.044	3.00	110	-21.692	-26.466	22.00
3	35.786	5	1.100	3.036	2.161	3.00	73.32	-29.212	-36.936	26.44
4	40.000	5	1.100	2.879	1.671	1.94	55	-34.202	-45.247	32.29
5	40.000	5	1.369	2.738	1.241	1.00	44	-37.832	-47.707	26.10
6	40.000	5	1.305	2.610	1.188	1.00	36.66	-40.498	-50.984	25.89
7	40.000	5	1.100	2.493	1.255	1.00	31.42	-42.551	-55.221	29.77
8	40.000	5	1.193	2.386	1.090	1.00	27.5	-44.152	-55.361	25.38

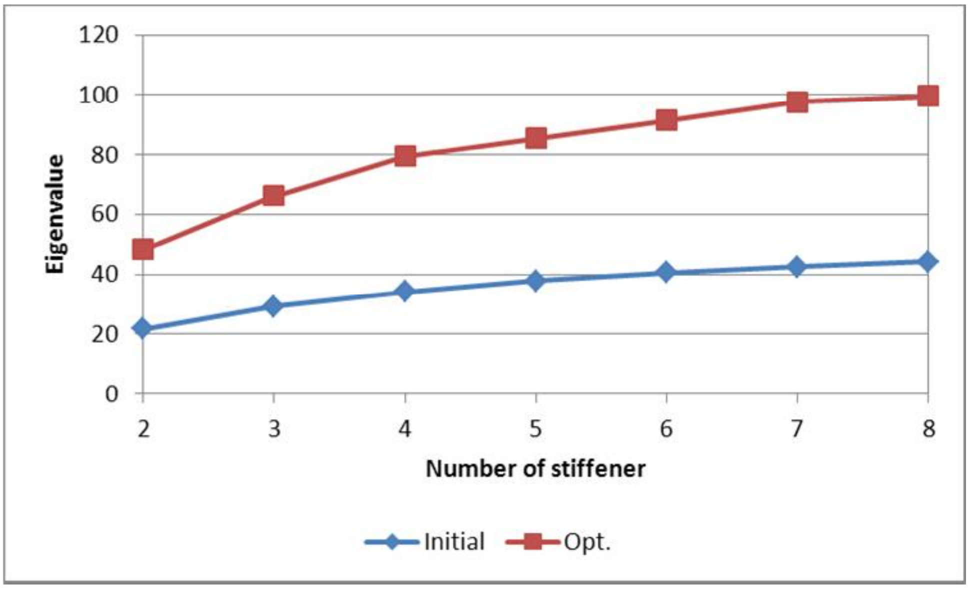

Figure 5.32 Comparison of eigenvalues at Shape optimization (Type 6)

Shape, shape optimization (Type 1) and shape optimizations (Type 5) slightly gave better results compared to other optimizations as shown figure 5.33. For small number of stiffeners optimizations give similar results. However when the number of the stiffeners increase shape optimization shape optimization (Type 1) and shape optimization (Type 5) optimizations give better results. At shape optimization (Type 1) height of the stiffeners is constant but results are nearly same with shape. Shape optimization a little better than the shape optimization (Type 1) and shape optimization (Type 5), at shape optimization (Type 5) case the thickness of plate is bigger than the shape optimization case. The width of pad and thickness of stiffeners and sub stiffeners are constant at Size optimization (Type 1) and the difference between shape optimization (Type 3) is only width of pads, it shows the common factor of optimization because the shape optimization (Type 3) gives the better solution than the size optimization (Type 1) which is the worst solution at this stage.

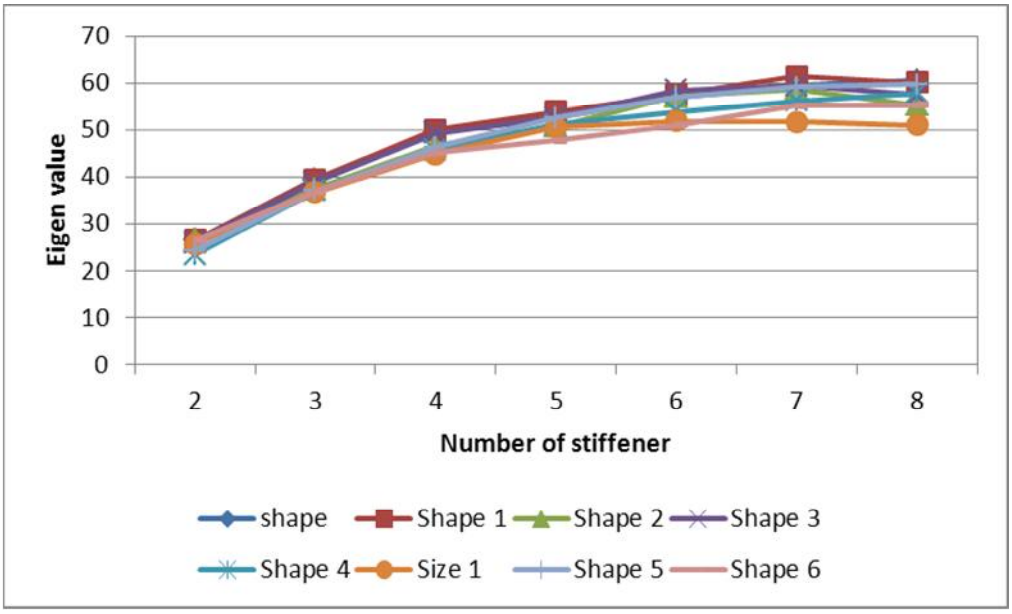

Figure 5.33 Comparison of Case Optimization

CHAPTER 6

CONCLUSION AND FURTHER WORK

6.1 Introduction

Structural optimization procedures are performed to obtain optimum sizes and shapes of stiffened plate types to gain min eigenvalue under constant volume constraint. For this purpose, totally 168 runs are carried out for considered plate types. The optimum results are obtained and detailed discussion of the efficiencies of plate types by interesting results that are presented in Chapter 5.

6.2 Achievements

During this thesis, PLATEV_1 (FS structural analysis and shape optimization program), which was developed by Özakça [27] was used. During the thesis, the following purposes were achieved.

1. *Geometric modeling of plate cross section:* The plate cross section is modeled by using coordinates of key points as defined in Chapter 4. The stiffener positions governed cross section modeling procedure. To satisfy initial baseline design values, thicknesses of elements and stiffener heights were arranged according to constant volume constraints.

2. *Mesh generation of cross section:* Mesh generation of stiffened plate sections were carried out by PLATEV_1 by an automatic FS mesh generator which was adapted to program.

3. *Static and free vibration analysis:* Eigen value free vibration analyses were carried out using FS analysis for all investigated plates. FS method was preferred in order to suitability of analyzing simply supported prismatic structures easily.

4. *Verify the accuracy of Free vibration analysis:* To prove the accuracy of computer code and formulation used in this study, results of three examples are compared with SAP2000 structural analysis and design computer package program's results. The SAP2000 program and PLATEV_1 gave very close eigenvalue results.

5. *Optimization:* SQP based algorithm was used as optimization method.

   a) Shape optimization: Shape optimizations were carried out to obtain maximum eigenvalues of plate under constraints. During this procedure only height, width and volume were kept constant, thicknesses and height of stiffener and height of sub stiffeners are used.

   b) Size optimization: In addition to some properties at used categories in constant categories to aim is developed the eigenvalues.

6. *Results and effectiveness of stiffened plate types:* By the steps mentioned above 168 runs were performed for considered plate types with desired element combinations. The obtained min eigenvalue of plate types fluctuate in a wide interval due to the used elements that forges plate cross section. The maximum eigenvalue for the desired combinations illustrate the effectiveness of element types on free vibration. These consequences orientated the comments on elements effectiveness and the suggestions about manufacturing of stiffened plates in conclusion section.

6.3 Conclusion

The effect of elements of plate cross section according to optimization results is discussed in previous chapter. This section deals with some suggestions about manufacturing and use of investigated plates.

The effect of sub stiffeners was examined in Chapter 5. According to optimization results in with sub stiffener plate gives a little better solution than without sub stiffener. Sub stiffener with pads under stiffeners plate gives better solutions than without pads under stiffener plate. Instead of sub stiffeners, plate skin and pads should be strengthened.

The effect of free vibration on pad elements is mentioned. However, including pad elements to plate cross section is difficult in practice; plates should be produced with pad elements if maximum eigenvalues are desired.

According to results at straight stiffened plate case, shape optimizations slightly gave better results compared to size optimizations. For small number of stiffeners both optimizations give similar results and when the number of the stiffeners increases shape optimization and shape optimization (Type 2) give better results. In shape optimization and shape optimization (Type 2) height of the stiffeners are increase; it is the fundamental causes of better eigenvalues. Shape optimization a little better than the shape optimization (Type 2) because of thickness of stiffeners.

At straight stiffened plate with sub stiffeners case, even so shape optimizations slightly gave better results compared to size optimizations. For small number of stiffeners both optimizations give similar results. However when the number of the stiffeners increase shape optimization and shape optimization (Type 3) give better results. In shape optimization and shape optimization (Type 3) height of the stiffeners are not constant and can be increase; it is the fundamental causes of better eigenvalues. Shape optimization a little better than the shape optimization (Type 3) because of thickness of plate and thickness of stiffeners.

At straight stiffened plate and pads under main stiffeners case, shape optimization (Type 2) and Shape optimizations slightly gave better results compared to other size optimizations. For small number of stiffeners optimizations give similar results. However when the number of the stiffeners increase shape optimization and shape optimization (Type 2) give better results. Shape optimization (Type 2) a little better than the shape optimization, at shape optimization (Type 2) case the thickness of stiffener and thickness of pad are bigger than the shape optimization case. . The width of pad is constant at Size optimization (Type 1) this is only difference between shape optimization (Type 2) and it shows the common factor of optimization because the shape optimization (Type 2) gives the best solution, the size optimization (Type 1) gives the worst solution at this stage.

According to results straight stiffened plate with sub stiffeners and pads under main stiffeners case, Shape, shape optimization (Type 1) and shape optimizations (Type 5) slightly gave better results compared to other optimizations. For small number of stiffeners optimizations give similar results. However when the number of the stiffeners increase shape optimization shape optimization (Type 1) and shape optimization (Type 5) optimizations give better results. At shape optimization (Type 1) height of the stiffeners is constant but results are nearly same with shape. Shape optimization a little better than the shape optimization (Type 1) and shape optimization (Type 5), at shape optimization (Type 5) case the thickness of plate is bigger than the shape optimization case. The width of pad and thickness of stiffeners and sub stiffeners are constant at Size optimization (Type 1) and the difference between shape optimization (Type 3) is only width of pads, it shows the common factor of optimization because the shape optimization (Type 3) gives the better solution than the size optimization (Type 1) which is the worst solution at this stage.

6.4 Furtherwork

In this thesis straight stiffened plates are investigated. It is necessary to examine this type of stiffened plate to possess general behaviors of free vibration and design structures that include axially compressive stiffened plates.

Investigated components of plate types also could be analyzed by different combinations. For instance in sub stiffened plate types only one sub stiffener considered between main stiffeners. Number of equally spaced sub stiffeners between main stiffeners may be increased.

Like the applicability of increasing number of sub stiffeners, number of pads between stiffeners may be increased too. Another case should be, investigated that the positions of main stiffeners. In this study, the distance between stiffeners is considered as d_{stiff} according to this, the distance between stiffeners and plate edge is taken $d_{stiff}/2$. What would be the effect of changing the positions of these distances symmetrically to plate axis?
In FS method, two opposite edges are simply supported and other two sides can be defined in any boundary condition. Some modifications can be made to apply any boundary conditions.

To possess general behavior of stiffened plates a wide search space like listed above should be investigated.

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