UNIVERSITY OF GAZİANTEP
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A NEW SOLVABLE CLASS LINEAR TIME VARYING SYSTEM

M.Sc. THESIS
IN
ELECTRICAL AND ELECTRONICS ENGINEERING

BY

HASAN GÜNEYLİ

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A New Solvable Class Linear Time Varying System

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Supervisor

Prof. Dr. Arif NACAROĞLU

by

Hasan GÜNEYLİ

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Name of the thesis	:	A New Solvable Class Linear Time Varying System
Name of Student	:	Hasan GÜNEYLİ
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Prof. Dr. Ramazan KOÇ
Director

I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of science.

Prof. Dr. Celal KORAŞLI
Head of Department

This is to certify that we have read this thesis and that in our consensus/majority opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master.

Prof. Dr. Arif NACAROĞLU
Supervisor

Examining Committee Members

Asst. Prof. Dr. Sabri BİRLİK (Chairman)

Prof. Dr. Arif NACAROĞLU

Asst. Prof. Dr. Nurdal WATSUJI

ABSTRACT

A NEW SOLVABLE CLASS LINEAR TIME VARYING SYSTEM

GÜNEYLİ, Hasan

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In this thesis, a new definition of classification has been done, with regard to the time- varying linear systems can be solved and the solution method for the new proposed class has been described. There isn't a general analytical method of solution in time-varying linear systems and being solvable of these systems is related to their being able to turn into time-invariant systems, applying certain transformations. In the studies conducted to date, in general, it has been shown that two groups of the time-varying linear systems are solvable, in addition to this, some special systems that can be transformed into these groups, although not belonging to one of the these to groups, have been introduced. Determination of the system's being solvable is directly related to the structure of the system's eigenvalue and also, in this study, on the basis of eigenvalues, a new solvable group has been identified.

Keywords: Time varying linear system, Solvable linear system

ÖZET

ZAMANLA DEĞİŞEN DOĞRUSAL SİSTEMLERDE ÇÖZÜLEBİLİR YENİ BİR SINIF

GÜNEYLİ, Hasan
Yüksek Lisans Tezi, Elektrik ve Elektronik Mühendisliği
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Bu tez de, zamanla değişen doğrusal sistemlerin çözülebilir olmasına ilişkin yeni bir sınıflandırma tanımı yapılmış ve önerilen yeni sınıf için çözüm yöntemi açıklanmıştır. Zamanla değişen doğrusal sistemlerin genel bir analitik çözüm yöntemi yoktur ve bu sistemlerin çözülebilir olması belli dönüşümler uygulanarak sistemlerin zamanla değişmeyen sistemlere dönüşebiliyor olması ile ilişkilidir. Bu güne kadar yapılan çalışmalarda genel olarak iki grup zamanla değişen doğrusal sistemlerin çözülebilir olduğu gösterilmiş, buna ek olarak bu iki gruptan birine ait olmadığı halde, bu gruplara dönüştürülebilen bazı özel sistemler tanıtılmıştır. Sistemin çözülebilir olmasının belirlenmesi sistemin öz değerlerinin yapısı ile doğrudan ilişkilidir ve bu çalışmada da öz değerlerden yola çıkılarak yeni bir çözülebilir grup tanımlanmıştır.

Anahtar Kelimeler: Zamanla Değişen Doğrusal Sistemler, Çözülebilir Doğrusal Sistemler.

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LIST OF SYMBOLS

$x(t)$	Vector of state Variables
$x(t_0)$	Vector of State Variables at Initial Time
$y(t)$	Output
$y(t+\delta)$	Time Shifted Output
$z(t)$	A New Vector of State Variables of Constant System Matrix
$z(t_0)$	A New Vector of State Variables of Constant System at Initial Time
$T(t)$	Transfer Matrix
$T^{-1}(t)$	Inverse of Transfer Matrix
$h(t,t_0)$	Time Varying Function
$H(s,t)$	Time Variable System Function
A_1	A Class of Solvable Time Varying Linear System
A_h	A Class of Solvable Time Varying Linear System
$\overline{A}$	A Constant System Matrix Which is produced
A	Constant System Matrix
$A(t)$	Time Varying System Matrix
$B(t)$	Time Varying Input Matrix
$u(t)$	Input Vectors
$U(s)$	Laplace Transform of u(t)
$u(t+\delta)$	Time Shifted Input

$\emptyset(t, t_0)$	State Transition Matrix
$\bar{\phi}(t, t_0)$	State Transition Matrix of Constant System Matrix which is produced
λ	Eigenvalue of System Matrix
g(t)	Scalar Time Function
TVNL	Time Varying Non-Linear System
TINL	Time-invariant Non-Linear System
TV	Time Varying System
TVL	Time Varying Linear System
TI	Time-invariant System
TIL	Time-invariant Linear System
IC	Integrated Circuit
LTVS	Linear Time Varying System
LTIS	Linear Time-invariant System
$a_n(t)$	A known Continuous Function of Time
$b_n(t)$	A known Continuous Function of Time
RF	Radio Frequency

CHAPTER 1

INTRODUCTION

The solution of the dynamical systems in many diferent domains have been very important for the exact analysis of the physical events. The mathematical models under some assumption has result in differential equation forms with different structures in time domain. Depending on the complexity of the problem and the degree of the assumptions, differential equations are classified in higher or lower level of difficulties of the solution.

Although, almost all physical dynamic events show non-linear behaviour, the linearization of the problem may be sufficient to understand the all phenomena. The complexity of natural phenomena have get difficult mathematical system of that modeling him. The degree of difficulty or ease of mathematical model is related with degree of derivative of the system. It can be said that, if the degree of derivative of mathematical model is higher then the problem can solve more difficult. Coefficients of the variables are constant, the solution is relatively easier, when the variables are connected to a domain then the solution is more difficult. Of course, when we tried to solve these problems, we're looking at the form of state space. We reverse the system of differential equations to state space form then we lower degree of derivative to one. The degree of the derivative reflected in the size of matrices in state space form. when we reverse high order differential aquation to first order state space form then we have created the nxn matrices with high value of n.

The n-dimensional linear (time varying or constant) system is represented by

$$\frac{dx(t)}{dt} = \mathrm{A}(\mathrm{t})\,\mathrm{x}(\mathrm{t}) + \mathrm{B}(\mathrm{t})\ \mathrm{u(t)} \tag{1.1}$$

where x(t) is nx1vector of state variables, u(t) is 1x1 input vectors, A(t) is nxn time-varying system matrix, B(t) is nx1 time-varying input matrix. A(t) and B(t) are matrices with the elements directly related to the circuit elements and if the elements of circuit are time-dependent then all of the system is time-dependent.

If excitation u(t) is zero, the linear equations becomes

$$\frac{dx(t)}{dt} = \mathrm{A}(\mathrm{t})\,\mathrm{x}(\mathrm{t}) \tag{1.2}$$

and it is called as homogeneous systems.

There is a general and easy solution of linear time-invariant systems. If A(t) is constant matrix A or it is commutative with it is functions, then the homogeneous solution of A(t) is[1-3]

$$\begin{aligned} x(t) &= \phi(t,t_0)x(t_0) \\ &= L^{-1}[sI - A]^{-1}x(t_0) \\ &= e^{A(t-t_0)}x(t_0) \end{aligned} \tag{1.3}$$

However A(t) is a function of time in homogeneous time varying systems defined in equation $dx(t)/dt = \mathrm{A}(\mathrm{t})\mathrm{x}(\mathrm{t})$, a general solution method for these systems has not been defined yet.

The solution of this system is possible with the transformation of time-varying linear systems into the time-invariant systems. However, it is not shown yet that the general transformation matrix can be proposed for transformation of the system. Only for some limited time-varying linear systems, transformation matrices can be defined [4]. A general analytical solution for time-varying linear systems is not known yet.

All the time-varying systems subject to time-invariant systems can be solved if the suitable transformation matrix is found. Until now, two broad classes of the systems has been presented which are solvable time-varying linear systems (A_1 and A_h). Among the solvable classes of linear time-varying systems, the commutative class is probably the best known class. The solution method of A_1 class and A_h class is possible if the eigenvalue of the system matrix have some specific forms. The construction of the transformation matrix is then possible using the eigenvalues. In this case the transformation matrix must be invertible.

For time invariant systems the solution is

$$\frac{dx(t)}{dt} = A(t)x(t) \Rightarrow \frac{dx(t}{dt} = Ax(t) \Rightarrow In(x(t)) = \int_t^t Adt = A(t-t_0)+C$$

$$In(x(t)) = A(t-t_0)+C \Rightarrow e^{In(x(t))} = e^{A(t-t_0)+C} \Rightarrow x(t) = e^{A(t-t_0)}.e^{C} = e^{A(t-t_0)}.x(t_0) \qquad (1.4)$$

$$x(t) = e^{A(t-t_0)}.x(t_0)$$

where t_0 is initial time, $x(t_0)$ is initial state. If the system is not time-invariant linear system then it must checked for commutativity. If the system is commutative, the solution is

$$\phi(t,t_0) = e^{\int_{t_0}^{t} A(\tau)d\tau} \qquad (1.5)$$

where $\phi(t,t_0)$ is the state transition function which carries the initial state $x(t_0)$ to $x(t)$ at any time t.

If the system is time-varying linear system and not commutative then an analytical solution is not known. For solving, we must checked the eigenvalues. If eigenvalues are constant ($\lambda_1 = k_1, \lambda_2 = k_2, ..., \lambda_n = k_n$) then the system belongs to A_1 class. If eigenvalues are the multiple of the same time-varying function h(t) ($\lambda_1 = k_1 h(t), \lambda_2 = k_2 h(t), ..., \lambda_n = k_n h(t)$), then the system belongs to A_h class [3]. Otherwise, if eigenvalues are neither constant nor multiple of a function then there is no general methodical solution. But it does not mean that they are unsolvable.

Our main aim in this report is to define a new solvable time-varying linear system class. The propose class will be call as HG class and if the system eigenvalues are $\lambda = t^{k-1} - 1$ *ve* $\lambda = t^{k-1} + 1$, for integer k, the transformation matrix which converts the system into time-invariant case is easily formed.

CHAPTER 2

DYNAMIC SYSTEMS

2.1 Introduction

The Dynamic Systems are events of in the natural sciences that expressed in mathematical form. The dynamic system wins character according to the state of the event. These are very important and useful for analyzing events. A dynamic system may or may not be linear, or may not be time-dependent. Dynamic systems are available in all sciences. In Earth science, astronomy, energy science, biology, building science, control, economy and all other areas there are dynamic systems and these systems are mathematically modeled for the solution of these. In fact, all dynamic systems are nonlinear time-varying systems. However, in some places the system is made linear and modeled as a simple and understandable, after that it can be solved more easily. When we modeled as mathematically the system it is important to have a general idea and be able to offer a solution. The system is made linearization that is enough to get an idea about real system and to understand the real problem. For example, when we solve a problem about a car's location, speed, acceleration and time then the components of this problem is also solved with the car's mass is considered to be fixed. But in reality it varies the mass of the car. The fuel of car decreases with Car tries (the mass decreases) or moving mass of an object varies according to the theory of relativity. Ignored this and similar conditions which affect the mass. This is linearization for a system in order to facilitate. Problems solved in this way, the correct results are still 99.99%.

The general dynamic systems are as these four matrices completely specify the state-space model.

System	**Type**
$\frac{dx(t)}{dt} = \propto tx^2(t)$	TVNL(Time varying Non-linear System)
$\frac{dx(t)}{dt} = \propto x^2(t)$	TINL(Time-invariant Non-linear System)
$\frac{dx(t)}{dt} = \propto tx(t)$	TVL(Time Varying Linear System)
$\frac{dx(t)}{dt} = \propto x(t)$	TIL(Time-invariant Linear System)

In this work, only the linearly problems will be considered and they will be modeled as n dimensional problem for n-chosen state variables as $\frac{dx(t)}{dt} = \mathrm{A}(\mathrm{t})\mathrm{x}(\mathrm{t})$ and there for the first order coupled differential equations representing the j-th state becomes

$$\frac{dx_j(t)}{dt} = a_{j1}\mathrm{x}_1(\mathrm{t}) + a_{j2}\mathrm{x}_2(\mathrm{t}) + ... + a_{jn}\mathrm{x}_n(\mathrm{t}) \tag{2.1}$$

In system representation,

$$\mathrm{A(t)} = \begin{bmatrix} \mathrm{a_{11}(t)} & .. & \mathrm{a_{1n}(t)} \\ \vdots & \ddots & \vdots \\ \mathrm{a_{m1}(t)} & .. & \mathrm{a_{mn}(t)} \end{bmatrix} \tag{2.2}$$

$$\mathrm{x(t)} = \begin{bmatrix} \mathrm{x_1(t)} \\ \vdots \\ \mathrm{x_m(t)} \end{bmatrix} \tag{2.3}$$

$$\frac{dx(t)}{dt} = \begin{bmatrix} \frac{dx_1(t)}{dt} \\ \vdots \\ \frac{dx_m(t)}{dt} \end{bmatrix} \tag{2.4}$$

So,

$$\begin{bmatrix} \frac{dx_1(t)}{dt} \\ \vdots \\ \frac{dx_m(t)}{dt} \end{bmatrix} = \begin{bmatrix} a_{11}(t) & .. & a_{1n}(t) \\ \vdots & \ddots & \vdots \\ a_{m1}(t) & .. & a_{mn}(t) \end{bmatrix} \begin{bmatrix} x_1(t) \\ \vdots \\ x_m(t) \end{bmatrix} \tag{2.5}$$

2.2 Time-Invariant System(TI)

Like everything else, systems can be classified in a variety of ways according to their different properties. However, the more important modes of classification in system theory are dichotomies in the sense that they involve but two categories, say linear time-invariant systems and linear time-varying systems. In fact all physical systems are time-varying. The launching of the dynamics of an object is moving, dynamic system of a moving car, Central Heating Boiler thermo-dynamic system of a building, an operating system of electronic circuit, any a base station's efficiency, etc. In short, no system is really time-invariant and linear. But the linearization of systems analysis is made easier. For example, sometimes a problem is being solved, the system is considered linear and time independent. Thinking about an event that gives same result at different times, the problem is solved as these. This method is a method of solving a system over time, assuming no change.

A time-invariant (TI) system is one whose output does not depend explicitly on time. If the input signal u(t) produces an output y(t) then any time shifted input, u(t+δ), results in a time-shifted output y(t + δ). This property can be satisfied if the transfer function of the system is not a function of time except expressed by the input and output. This property can also be stated in another way in terms of a schematic If a system is time-invariant then the system block is commutative with an arbitrary delay.

$$\frac{dx_j(t)}{dt} = a_{j1}\mathrm{x}_1(\mathrm{t}) + a_{j2}\mathrm{x}_2(\mathrm{t}) + ... + a_{jn}\mathrm{x}_n(\mathrm{t}) \tag{2.6}$$

$a_{j1}(t)$, $a_{j2}(t)$, ... and $a_{jn}(t)$ are constant then that is Time-Invariant System, and it is represented as

$$\begin{bmatrix} \frac{dx_1(t)}{dt} \\ \vdots \\ \frac{dx_m(t)}{dt} \end{bmatrix} = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1(t) \\ \vdots \\ x_m(t) \end{bmatrix} \tag{2.7}$$

2.3 Time Varying System(TV)

A time-varying system is a system that is not time invariant and take an important place in modern technology. They are widely used as in communication systems, power electronic circuits, electrical machinery and electronics.

In communication system, the communication channels are time varying due to movement of the source, receiver or scatters. Therefore, the channel is acting like time varying filters. Besides that, parametric amplifiers, parametric converters, time varying filters, switched capacitor networks, mixers and RF circuits are also different types of time-varying systems.

In power electronic circuits high power semiconductors devices such as thristors, diacs, triacs are used and these devices are either triggered externally or controlled by the response signals; in either case the controlling signal is periodic and these devices behave as periodically time-varying components. Because of time-varying nature of power systems, time-varying system analysis methods are used in the 3 systems as, power system protection, power quality, power system transients, partial discharges, load forecasting, power system measurement [4].

In integrated circuits (IC) area, due to the heat generated by IC, circuit parameters are changing. The parameter variations need to be quantified in order to ensure a robust circuit.

The system approach is a widely used in modeling electronic and mechanical systems. Linear systems are highly popular models due to their simplicity and convenience for mathematical analysis. Thus, many systems can be modeled as linear time-varying systems at least for a limited range of operation. Figure 2.1 describes the general notion of an input-output system in a block diagram. The input is u and the output is y to describe physical quantities and their relations.

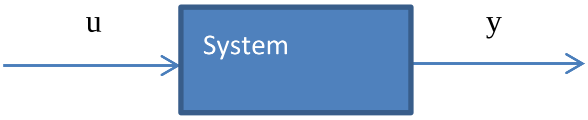

Figure 2.1 Input-Output System

A system is linear if it satisfies the property of superposition, that is, for any couple of inputs and outputs $y_1=f(u_1)$ and $y_2=f(u_2)$, the equation $ay_1 + by_2 = af(u_1) + bf(u_1)$ must be satisfied for any couple of scalars a and b.

A system is time-varying, if a system parameters changes with time, otherwise it is called a time-invariant system. If a system satisfies linearity property and it has at least a time-varying component, it is called a linear time-varying system (LTVS), otherwise linear time-invariant system (LTIS). A small class of LTVS is called periodically time-varying system, whose components change periodically with time.

The relation between the input and the output of a time-varying system can be expressed in a variety of ways. This forms “characterization” (representation) of the system. Basically, the input-output relation of a linear time-varying system may be expressed as

$$a_n(t)\frac{d^n y}{dt^n} + ... + a_1(t)\frac{dy}{dt} + a_0(t)y(t) = b_m(t)\frac{d^m u}{dt^m} + ... + b_1(t)\frac{du}{dt} + b_0(t)u(t) \qquad (2.8)$$

where $a_n(t)$ and $b_m(t)$ are known continuous functions of time. This equation is referred to as the fundamental equation of the system [5]. If there are more than one input and/or output in the system then, in general, we have more than one high order simultaneous differential equations containing multi-input, multi-output variables.

The classical differential equation solution techniques can be applied successfully to a small class of systems and corresponding basis functions can be found in. This small class contains the systems, which are characterized by the following equations: Bessel equations, Weber equations, Hypergeometric equations, Airy equations and others [5].

The equation (2.8) defines a periodically time-varying linear system if the coefficients of functions $a_n(t)$ and $b_m(t)$ are periodic with the system's fundamental period T_0. For periodically time-varying systems, the periodicity makes it possible to apply some special techniques such as Floquet theory and spectral analysis [5]. In spectral analysis fundamental differential equation of linear LTV system is expressed in terms of algebraic matrix-vector relation by defining operational matrices for derivative, integral, and any time-varying component behavior. The system equations are transferred to spectral domain. Thus, solution of the system equation can be easily obtained by using the matrix operations. The solution is computed in spectral domain in term of Fourier coefficients. Then it is carried to the time domain by applying inverse Fourier Transform. This method gives the steady-state analysis of periodically time-varying system. However the general analysis methods of LTV systems are still continuing to investigate.

Due to above mentioned difficulties of system representation by a single high order differential equation; state-space representation, has been developed. In modern system theory, it is preferred and found very convenient methods especially for computer simulations to use a set of N first order linear differential equations of the form Eq. (2.9) together with the expression Eq. (2.10) for the output.

$$\frac{dx}{dt} = A(t)x(t) + B(t)u(t) \tag{2.9}$$

$$y(t) = C(t)x(t) + D(t)u(t) \tag{2.10}$$

In these equations $x(t)\epsilon R^n$, $x(t)\epsilon R^n$, $u(t)\epsilon R^n$, $y(t)\epsilon R^m$ are the state, input and output respectively, at time $t\epsilon R^n$; A(t), B(t), C(t), D(t) are matrices of order compatible with x(t), u(t) and y(t), and their elements are known and they are piece-wise continues functions defined on R^+. It is well known that the state solution of Eq.

$$x(t) = \phi(t,t_0)x(t_0) + \int_{t_0}^{t} \phi(t,\tau)B(\tau)u(\tau)d\tau \tag{2.11}$$

where $\phi(t,t_0)$ is called the state transition matrix [3,5]. The $\phi(t,t_0)$ is the key to the solution of Eq.(2.10). Some solution techniques are given in [3] for different classes of linear system equations. The common one is commutative class. State-space representation of LTV system can be transformed into time-invariant representation through the commutative class by using transformation as,

$$x(t) = T(t)z(t) \tag{2.12}$$

Here, T(t) is the transformation matrix, which transforms the system representation into commutative or even a linear time-invariant system representation.

Although, It is concluded in [3], [6] that, the commutative property is not an inherent property of a dynamic system, but rather is just a system representation property it is difficult to find transformation matrix T(t) Eq. (2.12). Therefore it is not easy to get the solution of system if it is not in commutative class. Spectral analysis method [5] can be applied efficiently for this representation, if A(t), B(t), C(t), D(t) matrices are periodically time varying.

A LTV system is excited by an impulse function, that is the delta function, $\delta(t)$ and the system's response to the impulse function is called "impulse response" and denoted as $h(t,t_0)$. The system response y(t) to the input u(t) applied at the $t=t_0$ is given by the superposition integral

$$y(t) = \int_{t_0}^{t} h(t,\tau)u(\tau)d\tau \tag{2.13}$$

This superposition is expressed as convolution of input-output, that is

$$y(t) = h(t,t_0) * u(t) \tag{2.14}$$

However, a method for analytic expression of $h(t,t_0)$ is generally unknown and same difficulties mentioned in differential equation are valid for this representation.

Frequency domain approach for analysis of LTV is first developed by L.A. Zadeh, Zadeh's approach is essentially an extension of the frequency analysis techniques commonly used in LTI systems. He defines a time-variable system function H(s,t), for a variable linear network. This function possesses most of the fundamental properties of the transfer function of a fixed network. For this reason it is conveniently used to interpret the frequency domain behavior of systems and to realize the given frequency domain requirements in design problem. Further, once H(s,t) has been determined, the response to any given input can be obtained by treating H(s,t) as if it were the transfer function of a fixed network.

For a single-input, single-output time-varying linear system, which is initially, relaxed, the time-varying system function is defined by the relation

$$H(s,t)=\int_{-\infty}^{\infty} h(t,\tau)e^{-s(t-\tau)}d\tau \tag{2.15}$$

The response of linear system to any input u(t), $t \geq t_0 \geq 0$, can be derived by

$$y(t)=\frac{1}{2\pi j}\int_{-\infty}^{\infty} H(s,t)U(s)e^{-st}ds \tag{2.16}$$

where U(s) is the Laplace transform of u(t)

However, there are similar difficulties to determine H(s,t) involved in solving the fundamental equation or the state equations of the system. To overcome some of difficulties the system equations transformed to spectral domain to use the spectral analysis techniques. The spectral analysis method basically uses Fourier series expansion of variables in linear periodically time-varying systems.

Shortly, time-varying system is that the same system in different times and for different results. If the input signal u(t) produces an output y(t) then any time shifted input, $u(t + \delta_1)$, results in a time-shifted output $y(t + \delta_2)$,

where $\delta_1 \neq \delta_2$

Consider the nth-order time-linear state-space description

$$\frac{dx(t)}{dt} = A(t)x(t) + B(t)u(t) \tag{2.17}$$

The system matrix A(t) is not constant($A(t) \neq A$). It is changing with time.

$$\begin{bmatrix} \frac{dx_1(t)}{dt} \\ \vdots \\ \frac{dx_m(t)}{dt} \end{bmatrix} = \begin{bmatrix} a_{11}(t) & .. & a_{1n}(t) \\ \vdots & \ddots & \vdots \\ a_{m1}(t) & .. & a_{mn}(t) \end{bmatrix} \begin{bmatrix} x_1(t) \\ \vdots \\ x_m(t) \end{bmatrix} \tag{2.18}$$

2.4 Commutativity

In mathematics an operation is commutative if changing the order of the operands does not change the end result. It is a fundamental property of many binary operations, and many mathematical proofs depend on it. The commutativity of simple operations, such as multiplication and addition of numbers, was for many years implicitly assumed and the property was not named until the 19th century when mathematics started to become formalized. By contrast, division and subtraction are not commutative. The commutative property (or commutative law) is a property associated with binary operations and functions. Similarly, if the commutative property holds for a pair of elements under a certain binary operation then it is said that the two elements commute under that operation. In group and set theory, many algebraic structures are called commutative when certain operands satisfy the commutative property. In higher branches of mathematics, such as analysis and linear algebra the commutativity of well known operations (such as addition and multiplication on real and complex numbers) is often used (or implicitly assumed) in proofs.

Records of the implicit use of the commutative property go back to ancient times. The Egyptians used the commutative property of multiplication to simplify computing products. Euclid is known to have assumed the commutative property of multiplication in his book Elements. Formal uses of the commutative property arose in the late 18th and early 19th centuries, when mathematicians began to work on a theory of functions. Today the commutative property is a well known and basic property used in most branches of mathematics.

The first recorded use of the term commutative was in a memoir by François Servois in 1814, which used the word commutatives when describing functions that have what is now called the commutative property. The word is a combination of the French word commuter meaning "to substitute or switch" and the suffix -ative meaning "tending to" so the word literally means "tending to substitute or switch."

Mathematical Definitions

The term "commutative" is used in several related senses.

A binary operation $*$ on a set S is said to be commutative if:

$$\forall(x, y) \in S : x * y = y * x \tag{2.19}$$

An operation that does not satisfy the above property is called noncommutative.

One says that x commutes with y under $*$ if:

$$x * y = y * x \tag{2.20}$$

A binary function $f : A \times A \rightarrow B$ is said to be commutative if:

$$\forall(x, y) \in A : f(x, y) = f(y, x) \tag{2.21}$$

Commutative and Noncommutative operantions in Mathmetics

Two well-known examples of commutative binary operations are:

The addition of real numbers, which is commutative since

$$\forall(y, z) \in R : y + z = z + y \tag{2.22}$$

For example 4 + 5 = 5 + 4, since both expressions equal 9

The multiplication of real numbers, which is commutative since

$$\forall(y, z) \in R : yz = zy \tag{2.23}$$

For example, 3 × 5 = 5 × 3, since both expressions equal 15

Further examples of commutative binary operations include addition and multiplication of complex numbers, addition and scalar multiplication of vectors, and intersection and union of sets.

Some noncommutative binary operations are:

Subtraction is noncommutative since $0-1 \neq 1-0$ (2.24)

Division is noncommutative since $1/2 \neq 2/1$ (2.25)

Matrix multiplication is sometimes noncommutative since

$$\begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} . \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \neq \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} . \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \tag{2.26}$$

The vector product (or cross product) of two vectors in three dimensions is anti-commutative, i.e.,

$$b \times a = -(a \times b) \tag{2.27}$$

Commutativity at Linear Systems

The meaning of that described above as size of large has mean at dynamic systems. Linear time-invariant systems and time-varying linear systems is important whether or not commutative. The solution of commutative systems is easy. First of all we must note that all time-invariant linear systems are commutative. But the situation changes for time-varying linear systems. Some of the time-varying linear systems are commutative, some of them are not commutative. When two linear time-varying single input single output dynamical systems A and B are connected in cascade (or series), the input-output relation of the combined system depends on the parameters of both systems and on which appears first. If both of the connections AB and BA have the same input-output pairs irrespective of the applied input, then we say that these systems are commutative systems; in this case AB and BA are equivalent, i.e., AB=BA.[7] For the solution of noncommutative time varying linear systems, noncommutative situation must be transferred to a commutative situation.

For example we have two matrices A_1 and A_2. If $A_1 x A_2 = A_2 x A_1$ then matrices A_1 and A_2 are commutative [8]. Always a variable(or matrix) is commute with its own function, Txf(T)=f(T)xT. The all time-invariant systems and some time-varying systems are commutative. The linear time-varying system is said to be a commutative linear time-varying system if A(t) commutes with its integral, i.e.

$$A(t)\left[\int_{t_0}^{t} A(\tau)d\tau\right] = \left[\int_{t_0}^{t} A(\tau)d\tau\right]A(t) \tag{2.28}$$

It is well known that the commutative class of linear time-varying systems is a solvable class and its state transition matrix $\phi(t, t_0)$ can be computed by [3]

$$\phi(t, t_0) = \exp\left[\int_{t_0}^{t} A(\tau)d\tau\right] \tag{2.29}$$

CHAPTER 3

SOLVABLE CLASS LTV SYSTEMS

3.1 Introduction

The numerical solution of all kind of differential equation is possible by using many different technique under some acceptable deviations. But in some specific cases, because of some limit problems, analytical solutions become much more important. Depending on the chosen mathematical model analytical solutions are some times easy but in general requires some tests of solvability. For n dimensional dynamical systems, an eigenvalues of the systems give enough information about solvability of the system. Therefore either in n'th order differential equation form or n dimensional state space form the characteristic equation plays important role in the termination of solvability.

The functions structures of the eigenvalues determines the solvability of the system and the system are classified with their eigenvalues. In this chapter we will introduce some solvable classes of the systems. For example, we will introduce the well-known two broad classes that are solvable.

3.2. A_1 Class

3.2.1 Formulation

As it is mention above, one of solvable linear time-varying system is A_1 class. The eigenvalues of A_1 class systems must be constant. In this case it is always possible to define any constant nxn matrix A_1 which satisfies

$$A_1A(t) - A(t)A_1 = \frac{dA(t)}{dt} \tag{3.1}$$

The linear time-varying system considered is governed by

$$\frac{dx(t)}{dt} = A(t)x(t) + B(t)u(t) \qquad x(t_0) = x_0 \tag{3.2}$$

that the solution of (3.2) is given by

$$x(t) = \phi(t, t_0)x_0 + \int_{t_0}^{t} \phi(t, \tau)B(\tau)u(\tau)d\tau \tag{3.3}$$

For example, system is $\frac{dx(t)}{dt} = A(t)x(t)$ and A(t) has $\frac{dA(t)}{dt}$ and if there exists a constant matrix A_1 that satisfies [3]

$$A_1A(t) - A(t)A_1 = \frac{dA(t)}{dt} \tag{3.4}$$

then the system is in A_1 class.

Considering nxn matrix A(t), $f_A(\lambda) = \det(\lambda I - A(t))$ is a polynomial of degree n of the form

$$|\lambda I - A(t)| = 0 \tag{3.5}$$

$$\left| \begin{bmatrix} \lambda_{11} & .. & 0 \\ : & \lambda_{ij} & : \\ 0 & .. & \lambda_{nn} \end{bmatrix} - \begin{bmatrix} a_{11}(t) & .. & a_{1n}(t) \\ : & . & : \\ a_{n1}(t) & .. & a_{nn}(t) \end{bmatrix} \right| = 0 \tag{3.6}$$

$$\begin{vmatrix} \lambda_{11} - a_{11}(t) & .. & -a_{1n}(t) \\ : & . & : \\ -a_{n1}(t) & .. & \lambda_{nn} - a_{nn}(t) \end{vmatrix} = 0 \tag{3.7}$$

$$f_A(\lambda) = \lambda^n - tr(A(t))\lambda^{n-1} + ... + (-1)^n \det(A(t)) \tag{3.8}$$

$f_A(\lambda)$ is called the characteristic polynomial of A(t).

For A_1 class system, the system matrix must have constant eigenvalues.

$$\lambda_1 = k_1, \lambda_2 = k_2, \dots\dots\dots, \lambda_n = k_n \tag{3.9}$$

If the system matrix is

$$A(t) = \begin{bmatrix} a_{11}(t) & .. & a_{1n}(t) \\ : & . & : \\ a_{n1}(t) & .. & a_{nn}(t) \end{bmatrix} \tag{3.10}$$

the eigenvalues are found equating the determinant of $(\lambda I - A(t))$ to zero.

The transformation matrix

$$T(t) = \exp(A_1 t) \tag{3.11}$$

is used to transform

$$\frac{dx(t)}{dt} = A(t)x(t) \tag{3.12}$$

into

$$\frac{dz(t)}{dt} = \bar{A}(t)z(t) \tag{3.13}$$

In new state representation,

$$\overline{\mathrm{A}} = \mathrm{T}^{-1}(\mathrm{t})\mathrm{A}(\mathrm{t})\mathrm{T}(\mathrm{t}) - \mathrm{T}^{-1}(\mathrm{t})\frac{\mathrm{dT(t)}}{\mathrm{dt}} \tag{3.14}$$

and

$$\mathrm{x(t)} = \mathrm{T(t)z(t)} \tag{3.15}$$

3.2.2. Solution

In the section above it has been shown that any time-varying system with constant eigenvalues can be transformed into time-invariant form by means of transformation matrix T(t). In the new domain the state transition matrix can be found as

$$\overline{\phi}(\mathrm{t}, \mathrm{t}_0) = \exp(\overline{\mathrm{A}}(\mathrm{t} - t_0)) \tag{3.16}$$

This transition matrix takes the initial condition $x(t_0)$ to z(t) at any time t and the homogeneous solution may be represented to be

$$\mathrm{z(t)} = \overline{\phi}(\mathrm{t}, \mathrm{t}_0)\mathrm{z}(\mathrm{t}_0) \tag{3.17}$$

This solution is the analytical solution of the new state and the state may be retransformed into in original spaces as

$$\mathrm{x(t)} = \mathrm{T(t)z(t)} = \mathrm{T(t)}\overline{\phi}(\mathrm{t}, \mathrm{t}_0)\mathrm{T}^{-1}(\mathrm{t}_0)\mathrm{x}(\mathrm{t}_0) \tag{3.18}$$

In this back transformation state and initial values are transformed.

3.2.3 Example

Consider the system (3.2) with A(t) being

$$A(t)=\begin{bmatrix} -1+\alpha\cos^2 t & 1-\alpha\sin t\cos t \\ -1+\alpha\sin t\cos t & -1+\alpha\sin^2 t \end{bmatrix} \tag{3.19}$$

where α is constant.[3]

It can be checked that A(t) in (3.19) belongs to the A_1 class because the eigenvalues of the system matrix A(t) are

$$\begin{aligned} \lambda_1 &= \frac{(\alpha-2)-\sqrt{\alpha^2-4}}{2} \\ \lambda_1 &= \frac{(\alpha-2)+\sqrt{\alpha^2-4}}{2} \end{aligned} \tag{3.20}$$

As shown in (3.20), eigenvalues are constant. One simple constant matrix A_1 that satisfies (3.4) is

$$A_1=\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \tag{3.21}$$

At this point, we have to point out that the solution of

$$A_1A(t)-A(t)A_1=\frac{dA(t)}{dt} \tag{3.22}$$

has many possible A_1 and it is logical to chose the one which makes the solution of new state easier. The solution of (3.4) for the given example is shown as

$$\begin{aligned} &\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} -1+\alpha\cos^2 t & 1-\alpha\sin t\cos t \\ -1+\alpha\sin t\cos t & -1+\alpha\sin^2 t \end{bmatrix}-\begin{bmatrix} -1+\alpha\cos^2 t & 1-\alpha\sin t\cos t \\ -1+\alpha\sin t\cos t & -1+\alpha\sin^2 t \end{bmatrix}\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \\ &\quad=\begin{bmatrix} -2\alpha\cos t\sin t & -\alpha(\cos^2 t-\sin^2 t) \\ -\alpha(\cos^2 t+\sin^2 t) & 2\alpha\sin t\cos t \end{bmatrix} \\ &A_1=\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \end{aligned} \tag{3.23}$$

and related transformation matrix and state space matrix are found as

$$T = e^{A_1 t} = \begin{bmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{bmatrix} \Rightarrow \overline{A} = \begin{bmatrix} (\alpha - 1) & 0 \\ 0 & -1 \end{bmatrix} \tag{3.24}$$

The solution of the system $\overline{A}$ becomes

$$z(t) = e^{\overline{A}t} z(t_0) = \begin{bmatrix} e^{(\alpha-1)t} & 0 \\ 0 & e^{-t} \end{bmatrix} z(t_0) \tag{3.25}$$

Hence the transformation of the solution into the system A results as

$$x(t) = \begin{bmatrix} e^{(\alpha-1)t} \cos t & e^{-t} \sin t \\ -e^{(\alpha-1)t} \sin t & e^{-t} \cos t \end{bmatrix} x(t_0) \tag{3.26}$$

3.3. A_h Class

3.3.1 Formulation

The A_h class is a large group of linear time-varying systems as A_1 class. The solution of A_h class is similar to the A_1 class. The only difference is to find transformation matrix. To classify the class of the system if it belongs to A_h class or not, the eigenvalues of the system A(t) must be found and checked if they are the multiple of some differentiable function or not. Let us this function is h(t) and $\frac{\mathrm{dh(t)}}{\mathrm{dt}}$ exists. The solution of

$$\mathrm{A_1A(t) - A(t)A_1} = \frac{\frac{\mathrm{dA(t)}}{\mathrm{dt}}}{\mathrm{h(t)}} - \frac{\frac{\mathrm{dh(t)}}{\mathrm{dt}}}{\mathrm{h(t)^2}} \tag{3.27}$$

gives us many possible A_1 and then the best A_1 which makes the solution of the system $\overline{A}$ easier is chosen.

Since the eigenvalues of system A is multiple of function as

$$\lambda_1 = \mathrm{k_1h(t)}, \lambda_2 = \mathrm{k_2h(t)}, \ldots \ldots \ldots, \lambda_n = \mathrm{k_nh(t)} \tag{3.28}$$

and since

$$g(t) = \int_{t_0}^{t} h(\tau)d\tau \tag{3.29}$$

is satisfied then the transformation matrix is written as

$$T(t) = e^{A_1 g(t)} \tag{3.30}$$

3.3.2 Solution

After the new system $\frac{dz(t)}{dt} = A_2 h(t)z(t)$ is formulated where

$$A_2 = A_h(t_0) - A_1 \tag{3.31}$$

$$A_h(t_0) = \lim_{t \to t_0} \frac{A(t)}{h(t)} \tag{3.32}$$

the solution for system A can be easily performed as

$$\begin{aligned} x(t) &= T(t)z(t) = \phi(t, t_0)x(t_0) \\ &= e^{A_1 g(t)} e^{A_2 g(t)} e^{-A_1 g(t_0)} x(t_0) \end{aligned} \tag{3.33}$$

3.3.3 Example

Consider the system with A(t) being

$$A(t) = \begin{bmatrix} -3t^2 & 0 \\ 3t^5 & -6t^2 \end{bmatrix} \tag{3.34}$$

The eigenvalues of the system are $\lambda_1 = -3t^2$ and $\lambda_2 = -6t^2$. As it can be seen easily both eigenvalues are the multiple of $h(t) = 3t^2$ and it means the system belongs to A_h class where

$$A_1 = \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix} \tag{3.35}$$

satisfies equation (3.27) and the transformation matrix is written as

$$T(t) = \begin{bmatrix} e^{-t^3} & 0 \\ t^3 e^{-t^3} & e^{-t^3} \end{bmatrix} \tag{3.36}$$

which transforms the system into time-invariant form and the solution of the system is found to be

$$x(t) = \phi(t,t_0)x(t_0) = \begin{bmatrix} e^{-(t-t_0)^3} & 0 \\ ((t-t_0)^3 - 1)e^{-(t-t_0)^3} + e^{-2(t-t_0)^3} & e^{-(t-t_0)^3} \end{bmatrix} x(t_0) \tag{3.37}$$

CHAPTER 4

A NEW SOLVABLE CLASS LTV SYSTEM (HG Class)

4.1 Definition

In this section, we define a new solvable class of linear time-varying systems. It is known that A_1 class and A_h class are solvable by using suitable transformation operation. Noting that the eigenvalues play an important rule on the solution of the dynamic system, to find a new set of solvable class equations it must be focused on the eigenvalues. In the new solvable class of system presented in this work (HG class) is second order time-varying system with eigenvalues in the form of $\lambda = t^{k-1} - 1 \ \ and \ \ \lambda = t^{k-1} + 1$. Here, k is any integer and therefore eigenvalues are any order of t polynomials with conjugates. Some dynamical systems may result with these kinds of eigenvalues and it has been shown in this chapter that the transformation of the system into time-invariant case is possible. The mathematical model of the linear time-varying system are generally given by

$$\frac{dx(t)}{dt} = \mathrm{A}(\mathrm{t})\mathrm{x}(\mathrm{t}) \tag{4.1}$$

In this representation A(t) can be put in to the different forms using different definition of the state variables in canonic representation but the characteristic equation of the system and the eigenvalues remain unchanged.

Let us consider the eigenvalues of the second order dynamical system are as

$$\lambda_1 = t^{k-1} - 1 \ \ and \ \ \lambda_2 = t^{k-1} + 1 \tag{4.2}$$

As it is obvious, these eigenvalues are neither constant nor multiple of any differentiable function, these system does not belong to A_1 and A_h class.

The dynamical system represented by these eigenvalues can be represented in the state space form as

$$\begin{aligned}\frac{dx(t)}{dt} &= A(t)x(t) \\ &= \begin{bmatrix} t^{k-1}-t & 1 \\ -(t^2-1) & t^{k-1}+t \end{bmatrix} x(t)\end{aligned} \tag{4.3}$$

In this system the entries of A(t) are the k'th order polynomials or the time t. The transformation matrix which transforms the system into the time invariant form is in the form of

$$T(t) = e^{\frac{t^k}{k}} \begin{bmatrix} 1 & 0 \\ t & 1 \end{bmatrix} \tag{4.4}$$

The transformation matrix T(t) convert the system A(t) into $\overline{A}$ for any k as

$$\overline{A} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \tag{4.5}$$

Hence the solution of the new system

$$dz(t)/dt = \overline{A}z(t) \tag{4.6}$$

becomes

$$z(t) = e^{\frac{t^k}{k}} \begin{bmatrix} 1 & (t-t_0) \\ (t-t_0) & (t-t_0)^2+1 \end{bmatrix} z(t_0) \tag{4.7}$$

Applying the invers transformation to that solution including initial conditions, the solution of the system is found as

$$x(t) = T(t)\overline{\phi}(t,t_0)T^{-1}(t_0)x(t_0) \tag{4.8}$$

4.3 Example

Second order TVL mathematical equation is as

$$\frac{d^2y(t)}{dt^2} - (2t^2)\frac{dy(t)}{dt} + (t^4 - 1)y(t) = f(t) \tag{4.9}$$

State-space form for homogeneous system is [f(t) = 0]

$$\begin{aligned}\begin{bmatrix} dx_1(t)/dt \\ dx_2(t)/dt \end{bmatrix} &= A(t)x(t) \\ &= \begin{bmatrix} t^2 - t & 1 \\ -(t^2 - 1) & t^2 + t \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}\end{aligned} \tag{4.10}$$

The eigenvalues of the system are $\lambda_1 = t^2 - 1$ *and* $\lambda_2 = t^2 + 1$ As it can be seen easily these eigenvalues are not similar to eigenvalues or A_1 or eigenvalues of A_h.

We use the following transformation matrix

$$T(t) = e^{\frac{t^3}{3}}\begin{bmatrix} 1 & 0 \\ t & 1 \end{bmatrix} \tag{4.11}$$

and convert a new time-invariant system is

$$\begin{aligned}\frac{dz(t)}{dt} &= \overline{A}z(t) \\ &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} z(t)\end{aligned} \tag{4.12}$$

As it is seen easily $\overline{A}$ is

$$\overline{A} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \tag{4.13}$$

The solution of the new time-invariant system is

$$z(t) = e^{\bar{A}(t-t_0)} z(t_0) \tag{4.14}$$

After we find z(t), it must be transformed in original domain with transformation matrix T(t) and finally the result is obtained as

$$\begin{aligned} x(t) &= T(t)z(t)T^{-1}(t_0)x(t_0) \\ &= e^{\frac{t^3}{3}} \begin{bmatrix} 1 & (t-t_0) \\ (t-t_0) & (t-t_0)^2+1 \end{bmatrix} T^{-1}(t_0)x(t_0) \end{aligned} \tag{4.15}$$

$$x(t) = e^{\frac{(t-t_0)^3}{3}} \begin{bmatrix} 1 & (t-t_0) \\ (t-t_0) & (t-t_0)^2+1 \end{bmatrix} e^{-\frac{t_0^3}{3}} \begin{bmatrix} 1 & 0 \\ -t_0 & 1 \end{bmatrix} x(t_0) \tag{4.16}$$

If the initial time is $t_0=0$, then the homogeneous solution becomes

$$x(t) = e^{\frac{t^3}{3}} \begin{bmatrix} 1 & t \\ t & t^2+1 \end{bmatrix} x(0) \tag{4.17}$$

For excitation u(t), the general solution is

$$x(t) = \phi(t,t_0)x(t_0) + \int_{t_0}^{t} \phi(t,\tau)B(\tau)u(\tau)d\tau \tag{4.18}$$

CHAPTER 5

RESULT AND CONCLUSION

In this thesis, we have defined a new solvable class (HG class) of linearly time varying system. Unlikely to time-invariant system, unfortunately not all time varying systems are solvable. To classify the systems as solvable or not the basic parameter is the characteristic equation and hence the eigenvalues. The mathematical model of the dynamical system may be in differential equation form of state space form. In other cases the characteristics equations and eigenvalues are same. In the literature the possible transformation technics and solution methods are presented for the systems with constant eigenvalues and some specific functional eigenvalues. Our main purpose in this work is to extend the solvable class dynamic systems and to classify a new group of differential equations which are analytical solvable. Since all dynamical problems can be modeled as the second order system or the multiple of second order system. Although it is possible to model the dynamical system in different state space forms using different realization technics, here we have preferred the canonical form. So, the transformation matrix which transforms time varying systems into time-invariant case is constructed in methodical form and the transformed system is easily solvable. The solvable classes are well defined but it is not known that the system the solvable or not if it does not belong to solvable class. Therefore the works on the determination on solvability of some systems are still the work of future.

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CURRICULUM VITA

PERSONEL INFORMATION

Surname, Name	: GÜNEYLİ, Hasan
Nationality	: TC
Date and Place of Birth	: 1 May 1981, Silvan
Marital Status	: Maried
Phone	:+90412 2381074
Fax	: +90412 2381084
Email	:hasan@shginsaat.com

EDUCATION

Degree	**Institution**	**Year of Graduation**
B.S.	University of Gaziantep	2008
High School	Açık Öğretim Okulu	2001

WORK EXPERIENCE

Year	**Place**	**Enrollment**
2011-Present	KAYA Elek. İnşaat Ltd. Şti	Manager
2008-2011	SHG İnşaat Elektrik Petrol Tur. San. Ve Tic. Ltd Şti.	Manager

FOREIGN LANGUAGE

English