Kenan Latifoğlu

UNIVERSITY OF GAZİANTEP
GRADUATE SCHOOL OF
NATURAL & APPLIED SCIENCES

HAMILTONIANS FOR SPIN QAUNTUM HALL EFFECT AND THEIR SOLUTIONS

M.Sc. Thesis in Physics Engineering

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KENAN LATİFOĞLU
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Hamiltonians for Spin Quantum Hall Effect and Their Solutions

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Supervisor

Prof .Dr. Ramazan KOÇ

By

Kenan LATİFOĞLU

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Name of the student: Kenan LATİFOĞLU
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Approval of the Graduate School of Natural and Applied Sciences

Assoc. Dr. Metin BEDİR
Director

I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of Science/Doctor of Philosophy.

Prof. Dr. A. Necmeddin YAZİCI
Head of Department

This is to certify that we have read this thesis and that in our consensus/majority opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science/Doctor of Philosophy.

Prof. Dr. Ramazan KOÇ
Supervisor

Examining Committee Members	Signature
Prof. Dr. Hayriye TÜTÜNCÜLER	.....
Prof. Dr. Ramazan KOÇ	.....
Ass. Prof. Dr. Eser ÖZKEKLİKÇİ	.....

I hereby declare that all information in this document has been obtained and presented in accordance with academic rules and ethical conduct. I also declare that, as required by these rules and conduct, I have fully cited and referenced all material and results that are not original to this work.

Kenan LATİFOĞLU

ABSTRACT

HAMILTONIANS FOR SPIN QUANTUM HALL EFFECT AND THEIR SOLUTIONS

LATİFOĞLU,Kenan

M.Sc. in Physics Engineering

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The main objective of this is to investigate physics behind the quantum Hall effect and solution of quantum Hall Effect Hamiltonians. Quantum integer and fractional Hall Effect appears in semiconductors and it is explained by kinetic energy quantization of electrons in a magnetic field or Landau quantization. In this work various Hamiltonians describing quantum Hall effect and including spin orbit coupling are also studied. We present solution of two level Hamiltonians.

In this thesis we propose an analytical approach to solve the Hamiltonians and we exhibit their eigenvalues and eigenfunctions in the analytical form. In order to investigate two level physical systems we introduced a general Hamiltonian in terms of Pauli matrices and ladder operators, including almost all spin-orbit coupling terms. The Hamiltonian consists of various parameters and under some specific conditions we obtained solution of the Hamiltonians. We show that the results are suitable to describe quantum Hall effect.

Finally, spin quantum Hall Effect is studied in detail. Using Mathematica program we solved various Hamiltonians concerned with quantum Hall Effect. At the end the results are concluded.

Key words: Quantum optics, Spin-Orbit Coupling, Spin Hall Effect, Hamiltonians.

ÖZ

HAMILTONIANS FOR SPIN QUANTUM HALL EFFECT AND THEIR SOLUTIONS

LATİFOĞLU,Kenan

Yüksek Lisans Fizik Müh.Bölümü

Tez Yöneticisi(leri): Prof. Dr. Ramazan KOÇ

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Bu tezin temel amacı kuantum Hall Etkisinin arkasındaki fiziği ve kuantum Hall Etkisi Hamiltoniyenlerinin çözümünü araştırmaktır. Tam ve kesirli kuantum Hall etkileri yarı iletkenlerde gözlenmekte olup manyetik alan içerisinde elektronun kinetik enerjisinin kuantumlanması, yani Landau kuantumlanmasıyla açıklanmıştır. Bu çalışmada kuantum spin Hall etkisini açıklayan ve spin-orbit çiftlerini içeren çeşitli Hamiltoniyenlerin çözümleri çalışılmıştır.

Bu tezde, Hamiltoniyenlerin çözümü için analitik bir yaklaşım tasarlanmış ve öz fonksiyonlar ile öz değerler analitik olarak elde edilmiştir. İki seviyeli fiziksel yapıları incelemek için, Pauli matrisleri ve merdiven operatörleriyle ifade edilen, spin orbit çiftlerini içeren genel bir Hamiltoniyen önerilmiştir. Bu Hamiltoniyen çeşitli parametreler içermekte olup, bazı özel durumlar için çözülmüştür. Sonuçların, kuantum Hall Etkisini açıklamak için uygun olduğu görülmüştür.

Son olarak, spin kuantum Hall Etkisi detaylı olarak çalışılmıştır. Mathematica programı kullanarak bu etki ile ilişkili bazı Hamiltoniyenlerin çözümü yapılmıştır. En son olarak sonuçlar özetlenmiştir.

Anahtar Kelimeler: Kuantum optik, Spin-Orbit çiftleri, Spin Hall etkisi.

To the my family. And to my Business friends

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LIST OF SYMBOL/ABBREVITION

ABBRE.	FULL TEXT
JC Model	Jaynes-Cumming Model
QHE	Quantum Hall Effect
FQHE	Fractional Quantum Hall Effect
IQHE	Integer Quantum Hall Effect
CF	Composite Fermions
SQHE	Spin Quantum Hall Effect
LL	Landau Level
a	Bosonic annihilation operator
a^+	Bosonic creation operator
b	Bosonic annihilation operator
b^+	Bosonic creation operator
$\sigma_{\mathbf{0}}, \sigma_+, \sigma_-$	Pauli matrices
ω	Effective frequency
κ	Coupling constant
ψ	Wavefunction
ω_c	Cyclotron frequency of electron
N	*SU* (2) algebra generators
E	Eigenvalue
B	Magnetic Field
E	Electric field

PERSONAL INFORMATION

Name and Surname: Kenan Latifoğlu

Natioality: TC

Birth place and date: Gaziantep/ 01.02.1984

Marial status: Single

Phone number: +90 506 706 5474

Fax:

Email: latifoglukenan@hotmail.com

EDUCATION

	Graduate school	**Year**
Master	----------	------
Bachelor	Abant İzzet Baysal Üniversitesi	2003-2008
High School	19 Mayıs Lisesi	1998-2002

WORK EXPERIENCE

	Place	**Enrollment**
2010-Present	Gezegenevi ve Bilim Merkezi	Uzman Eğitmen
2009-2010	----------	-----
2004-2009	----------	-----

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CHAPTER 1

INTRODUCTION

Classical Hall Effect was discovered by Edwin Hall in 1879, based on production of potential difference across an electrical conductor. The potential difference is generated by applying a magnetic field perpendicular to the current [1]. It is well known that Hall Effect is due to the movements of charge carriers, such as electrons, holes or ions under the influence of Lorentz force through magnetic field. The basic principle underline the Hall Effect is Lorentz force. Hall Effect occurs in metals and semiconductors.

In 1943, Klaus von Klitzing [2] discovered that Hall resistance is quantized to h/e^2, which is called von Klitzing constant. Mathematically we can write $R_H = \left(\frac{h}{e^2}\right)\frac{1}{n^2}$, where n is integer. Therefore, it is important to note that Hall resistance is a universal quantity, independent from material and geometry of material and its impurity distribution or concentration. Nowadays it is used as the resistance standard [3]. Origin of this quantization can be explained by kinetic energy quantization of electrons in a magnetic field (Landau quantization). This phenomenon is known as quantum Hall Effect or integer quantum Hall Effect.

After three years from the discovery of integer quantum Hall Effect, Tsui, Stormer and Gossard discovered that in the R_H integer n can take fractional value, such as $n = 1/3$ [4]. Origin of this fractional quantum Hall effect was explained by Laughlin in 1983. This production is due to incompressible electron liquid that is collective behavior of electrons because of condensation of electron gas [5]. Fractional number can be generalized as $n = p/q$, where p and q are integers with no common factors.

The other type of Hall Effect is known as quantum spin Hall Effect. This effect can be explained as follows [6]: when current flows through conductor or semiconductor

spin flux of the carriers directed from interior to exterior of the slap as a result spinning carriers collection the surface of the slap. They form a layer and this layer is known as spin layer. Thus transverse spin imbalance is produced and this generates a spin Hall voltage. This phenomena is called spin Hall Effect by Hirsch [7]. The spin Hall effect is due to spin orbit interaction. Using the same analogy as in the classical Hall Effect, Hirsch proposed an experiment to detect spin current and to measure Hall voltage in a paramagnetic metal. Various physicist observed spin orbit interaction using various experimental techniques [8-10]. Spin polarizations near the edge of strained and unstrained gallium arsenide was detected and imaged [8]. Experimental measurements of spin Hall effect in 2 dimensional hole system including spin orbit coupling was reported in [9]. Kane and Mele [10] was studied effect of spin orbit interaction in graphene surface under low energy. They reported effect of temperature, Rashba coupling, disorder and chemical potentials on spin orbit coupling.

The quantum spin Hall Effect is significantly different from the quantum Hall effect. First of all, the spinning charge carriers can be accumulated on spin layer without magnetic field. Spin polarization destroyed with magnetic field.

There are various physical and mathematical models for spin quantum Hall Effect. A first model Hamiltonian was introduced Laughlin [5], to describe condensation of two dimensional electron gas leads to the generation of Hall Effect. A quantum Hamiltonian to describe spin Hall Effect based on SU(2) symmetry recently proposed in [11]. The Hamiltonian consists of two types spin orbit coupling and Berry phase obtained from the wavefunction of the Hamiltonian to calculate spin Hall conductivity.

Another model Hamiltonian for quantum Hall Effect has been presented in [12], based on ladder geometry. J. Fröhlich, G. M. Graf and J. Walcher [13] proposed a Hamiltonian to study electron gases in uniform magnetic field. A review paper entitled "Hamiltonian theories of the fractional quantum Hall effect" deals with progress on fractional quantum Hall Effect [14]. Recently Yu and Zhang [15] presented super symmetric Hamiltonian for fractional quantum Hall Effect to study edge excitations for fraction number $\nu = 5/2$.

Meanwhile we point out application of the quantum (spin) Hall effect. In recent years, Hall Effect has found application in the areas; spin physics, information theory, quantum computers, and spintronics devices, sensors, magnetic field measuring devices, in various laboratory instruments, integrated circuits etc.

In this we obtained eigenvalues and eigenfunctions of quantum Hall effect Hamiltonians by using Mathematica program. We also investigate behaviour of electrons and spins in different magnetic fields. Obviously, simulating of eigenvalues for variety magnetic field will improve our understanding of spin quantum. We focus our attention to show eigenvalues and eigenfunctions in variety Hamiltonians on Mathematica.

In our work we deal with the algebraic solution of the Hamiltonians of the spin quantum hall effect and fractional quantum hall effect. We proposed a general Hamiltonian for two level systems. Depending on the choices of parameters various types Hamiltonians can be obtained. We concentrate our attention to the solution of Hamiltonians may be related to the Hall Effect.

The thesis is organized as follows. Chapter 2 provides a brief theory of Classical Hall Effect. This chapter include an overview of classical hall effect, involve classical equations, in this way, we calculated Hall coefficient, Hall voltage and other physical quantities. Hall coefficients of various materials are listed in this chapter. Classically, occurrence of Hall Effect in a semiconductor is described.

In chapter 3, we discussed the fractional quantum hall effect in detail. We have given a Hamiltonians and computed fractional coefficients for states. Fractional states are investigated by applying magnetic field.

Chapter 4 is devoted to study Hamiltonians of two level systems. We calculated eigenvalues and eigenfunctions of the Rabi and Jaynes Cummings type Hamiltonians.

In chapter 5, we presented spin quantum Hall Effect. Two different model Hamiltonian are given in this chapter. We discuss our results and solution of the Hamiltonians. The results are discussed and concluded all aspects of the energy eigenvalues of spin quantum hall effect in chapter 6.

CHAPTER 2

HALL EFFECT

As it is well known that a voltage difference causes Hall Effect, namely Hall voltage, across an electrical conductor and Edwin Hall discovered it when he was a graduate student [1]. It can be described as follows: Consider a thin flat and uniform conducting material immersed in a uniform magnetic field as in the figure. Suppose that a current flow through the conductor. It is obvious that positively charged particles deflect one side and negatively charged particles deflected to the opposite sides. Consequently a potential difference is produced between edges of the conductor. This potential difference is known as Hall potential and its formulation as follows.

Firstly we define the magnetic force on a given infinitesimal charge is of magnitude qv_dB, (infinitesimal charges each has a charge q and move along the plate with the drift velocity v_d), since the negative charges move in the plate actually at right-angles to the magnetic field. We know the steady-state, the electric force get balanced this force due to the set up of charges on the upper and lower edges of the plate. If the Hall voltage is V_H, and the width of the plate is w, then the elecric field pointing from the upper to the lower edge of the plate is of magnitude $E = V_H / w$. Now, qE on a infinitesimal charge is known the electric force. This force roles up opposition to the magnetic force. In a steady-state can be shown by the following,

$$qE = \frac{qV_H}{w} = qv_dB \tag{2.1}$$

Giving

$$V_H = v_d w B \tag{2.2}$$

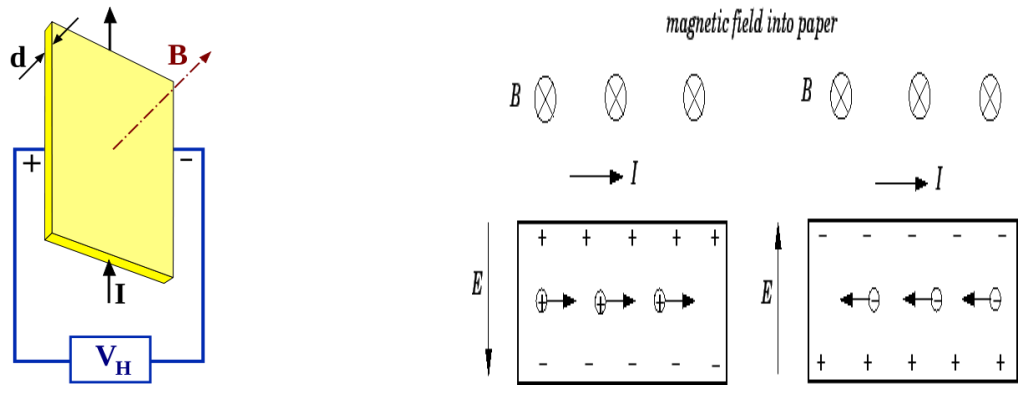
It means magnitude of the magnetic field is directly proportional to the Hall voltage. Normally, Hall current is a property of any material for each other, and we called it *probed of Hall*, which is used to measure of strength in magnetic field. Otherwise, suppose that any plate which is conducting the current has a thickness of *d*, and that it possessed *n* infinitesimal charge carriers per unit volume. So it ensures that the total current in the plate which is flowing through line can be described

$$I = qnwdv_d, \tag{2.3}$$

since all infinitesimal charges get located in a geometric shape that can be said rectangular a plate volume of lengthv_d, width w, and thickness *d*, flow past a given point on the plate in one second. Combining Eqns. (2) and (3), we get

$$V_H = \frac{IB}{qnd}, \tag{2,4}$$

It is clear that the Hall current is commensurate the voltage can be created by flowing the current in the plate, strength of magnetic field is definitely inversely proportional the number of density of infinitesimal charges in the plate, and surely the depth of the plate. In this way, in order to occur a tunable Hall button, we need to take a thin material that is made of plate which possesses same charges per unit volume (e.g., a semiconductor) according to each other, and then apply a large current through it. We can easily see Hall voltage and Hall Effect in the figure2.a and figure2.b



a b

Figure 2.1: In this figure we see the motion of charge carriers (negative and positive) and Hall effect is occurred.

We also have to mention about Hall coefficient for induced Hall Effect as classical in order to compute too. Now we can define the Hall coefficient as by the following formula;

$$R_H = \frac{E_y}{J_X B} \tag{2.5}$$

where j is the current density of the carrier electrons, and E_y is the induced electric field. In SI units, this becomes

$$R_H = \frac{E_y}{J_X B} = \frac{V_H d}{IB} = -\frac{1}{ne} \tag{2.6}$$

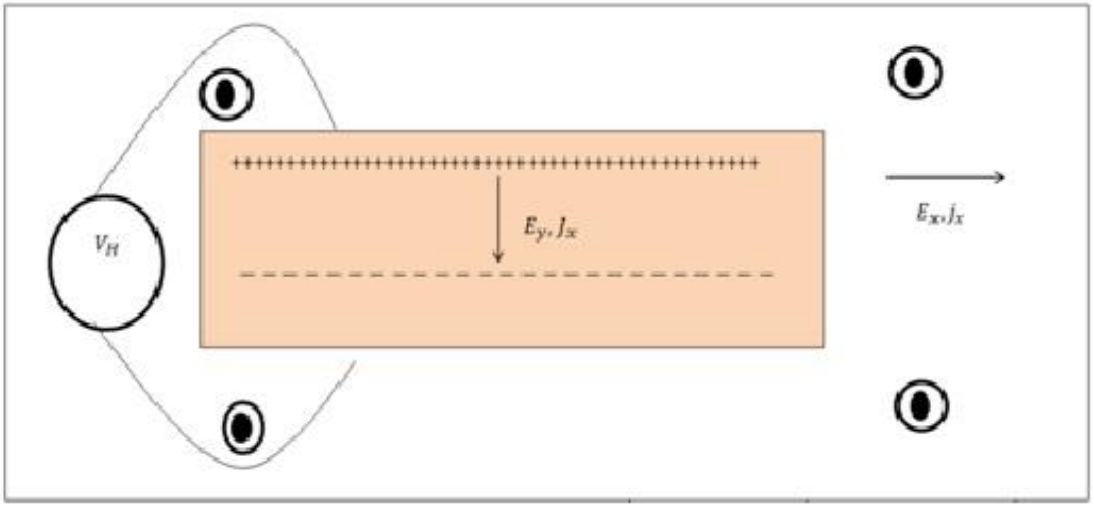


Figure 2.2. Hall effect diagram in a semiconductor

As final, the Hall Effect is very facilitator in terms of measuring the Hall current and Hall Voltage in any magnetic field or charge carrier densities of a semiconductor plate. Spin that, in atomic and molecular physics, quantum mechanics and particle physics, is very important a characteristic property of elementary particles, composite particles (hadrons), and atomic nuclei. After classical Hall Effect, we look at the other side of Hall Effect that is Unconventional Hall effect and Berry's

Table 2.1. Hall coefficient of the some materials

Metal	R_H $(-1/nec$
Be	-0.2
Li	0.8
Na	1.2
Rb	1.0
Ag	1.3

phase in order to grasp easily as quantum. We hereby see the classical Hall Effect and its calculation. Hall coefficient for each semiconductor or conductor is different each other. The Hall coeffients change fractionally numbers, so we also investigate the fractional and integer levels, quantum numbers and Landau Levels into the next chapters.

Experimental results classical Hall effect that, we can show a diagram and give some examples for Hall coefficient each material. In this diagram some examples for Hall coefficient each material.

2.1 Unconventional Hall Effect and Phases of Berry

Berry's phase π characterize charge carriers Dirac fermions in which we can get some results in a shifted position of Hall plateaus. Standard integer position, the Hall conductivity plateaus generally resulted in Landau quantization fermions, but missed the last plateau. In this section, the conventional zero-level anomaly, insulation stark contrast to the behaviour at low concentrations and high magnetic field, the limit is accompanied by metallic conductivity. No chiral fermions in the quantum-mechanical analogue and present an interesting case studies known to us.

Figure 2.3.a shows the quantum Hall Effect (QHE) as a picture that grabs very useful information about the behaviour of bilayer (2L) graphene observed in. So we can easily compare it with the conventional integer QHE in the standard theory. Hall conductivity makes a quantum $e^2/\hbar$ with Landau level which is each filled single-

degenerate as it contributes one conductance so that we obtain fractional and integer quantum numbers. The Conventional Quantum Hall Effect is shown in Figure 3a, where, lines in the graph, Hall conductivity σ_{xy} make up sustained ladder of that steps are equal to each other in 2L graphene, QHE tableland at zero temperature is unavailable. Unlikely, there is a conductivity of Hall continuous a couple-sized step in regime. After that, 2L graphene in terms of longitudinal conductivity σ_{xy} in 2L graphene remains of the order of $e^2/\hbar$, even at zeroσ_{xy}. In fact, we may explain that

the origin of the unconventional Quantum Hall Effect behaves going along in the coupling bilayer of graphene, which transforms massless Dirac fermions, characteristic of single-layer graphene, into a novel type of chiral quasiparticles. Such quasiparticles have an ordinary parabolic spectrum H(p)=p^2/2m with effective mass m but accumulate Berry's phase of 2π along cyclotron trajectories. The latter is shown to be related to a peculiar quantization where the two lowest LLs lie exactly at zero energy H, leading to the missing plateau and double step shown in Fig. 2.3.a

Bilayer films in this study getting a good formulation of microscopic forces with atomic and optical 2L crystals of pure graphite structures. It followed by the selection made by micromechanical cleavage. Many-terminal field-effect equipments (Fig. 2.3.b) structures were selected getting standard manufacturing techniques. Between graphite atomic structure and the substrate as a substrate is the gate voltage Vg us permission to apply for used. We studied on partially an oxidized silicon heavily doped devices, the concentrations of electrons and holes up to 10 13cm-2 can be induced by the effect of such an ambipolar electric field is displayed. Figure 2.3.b 2L graphene at a constant gate voltage (n fixed), and the magnetic field B ranging from 30T shows a typical behaviour of QHE. Pronounced plateaus are clearly σ_{xy} 3 high B Hall resistivity, and these are associated with zero longitudinal resistivity σ_{xx} to be completed. QHE double spin and double-valley degeneracy with the plateau observed in the sequence of two-dimensional (2D) the same sequence as expected for the free fermions system σ_{xy} = h/4Ne2 is described by. However, a clear difference between the conventional and reported QHE regime of a small fill-factor S <1 (see Fig. 2.3.c). This regime has been investigated by different concentrations of electrons and holes B and suitable for fixing flowing along the similarity point |n||0 where σ_{xy} changes its signal and, ordinal, Q=0. Also, because infinitesimal charge carriers P in

graphitic films are a little dependent on n, measurements show the in constant *B* are more effective. The quality of Landau quantization is defined previously we called that parameters correspond to a constant parameter PB, and this allows us to carry out snapshot taken datas several QHE plateaus through the double voltage instead of single voltage goes in same adjustments with magnetic fields (Fig. 2.3.a). The some periodic datas have been found by quantum oscillations in σ_{xx} as it described n is number of density with states 00 gB /I on each LL [1-10] In Fig. 2.3.c, for example, 'n|1.2˜10 12 cm-2 at B=12T, which gets around g=4 for 2L the calculations are clearly seen in Fig.2.3.c and shows that, all integer numbers are followed by Hall plateaus in 2L graphene in Fig.2.3.c confirmed the couple spin and couple valley degeneracy expected from atomic plateau structure.

ρ_{xy}=r (4e2/h) N for N, means any sign cannot be seen in zero-N plateau at ρ_{xy}=0, which may be hoped for 2D free-fermions systems (Fig. 2.3.c). In this way, the QHE for massless Dirac fermions is shown (Fig. 2.3.c), and they are behaviour resembles too. In Fig.2.3.c. Where a step reveal out, however, there is no plateau when ρ_{xy} flows the stability point. However, in 2L graphene, when we look up the graph, this time, that step possesses a couple height and is related to a central peak in σ_{xx}, that must be twice line border than all other peak (Fig. 2.3.c). In 2L graphene the feature is yielded peak which in 2L graphene for the penetration among the Quantum Hall Effect plateaus need double carriers due to the transition in lowest hole and electron Hall lines.

This four degeneracy represents that the Landau Level at zero level has double degeneracy $p^2\mu$ 4B /I , which easily can be seen two Landau Levels exceed together at n|0 (see LL charts in Fig. 2.3.c). Attendance records of laboratuvar results Q=0 as identified in Fig. 2.3.c unfortunately possesses numbers possible for conventional 2D systems where the lowest landau plateau in ρ_{xy}=σ_{xy} /(σ_{xy} 2+σ_{xx} 2) is inferred from a rapid increase in σ_{xx} >>h/e^2 with increasing B and decreased temperature T for factors called filling factors Q<1, it is indicated an insulating state. To be sure that we can compare directly the conventional QHE measurements, immediately we investigate the Fig. 2.3.c that figure shows σ_{xx} in 2L graphene with some peaks in variable coefficients also changing magnetic field and temperature around zero Q. 2L graphene presents that temperature dependent of Hall plateaus can be defined like

that ρ_{xy} in 2L graphene does not destroy in any domestic of Q and be measured zero just one point on the graph, where σ_{xy} orients just its direction and magnitude of number. We should know that interestingly assures a peak value |h/ge^2 in yields on to 20T and temperatures and peaks are fell down to 1K. If finite values taken as of σ_{xx} |h/4e^2 so lowest landau levels in the limit should be infinitesimal and low density of charge carrier concentrations measured in *B*. This was studied on already before 2L graphene [32]. These experimental results were convenient with theoretical ones which contribute the finite conductivity of metal and the spectrum of 1L graphene support the relativistic like with also destroyed any localization (see refs. in [30]). 2L graphene experimental results show us that the plateaus have the usual parabolic spectrum, and maximum resistivity is observed |h/4e^2 and, moreover, when the magnetic field B is going weaker and weaker, the system is more unpredictable. We understood from these experimental results that, the unconventional Quantum Hall Effect in 2L graphene, when charge carriers or mobile charges like fermions or chiral bosons are related with a finite mass, that time, it is clearly appeared from significant properties of its charge carriers as we interrupted in the Fig.2.3.c. Some calculations were used for the quasiparticles spectrum in 2L graphene, one of the calculation is standard nearest-neighbour approximation [12]. This technique is being used for computing the quasiparticles with distances as K-points, we find $\pm\frac{1}{2}\gamma_1 \pm \sqrt{\frac{1}{4}\gamma_1^2 + v_f^2 p^2}$, where$v_f = \frac{\sqrt{3}}{2}\gamma_0\, a/\hbar$, a is the atomic level period and γ_0 and γ_1 are the intra-and inter-layer coupling constants, respectively [13]. This dissipation is related (Fig. 2.3.c) as in agreement with the first-principle band-structure calculations [14] and, at low energies, becomes parabolic $\varepsilon = \pm p^2/2m$ with $m = \gamma_1/2v_f^2$ (sign $\pm$ refers to electron and hole states). Effective Hamiltonian is also other technique to calculate, so it helps us to analysis [15] quasiparticles in 2L graphene. [40].

$$\widehat{H}_2 = -\frac{1}{2m}\begin{pmatrix} 0 & \left(\hat{\pi}^{\dagger^2}\right) \\ \hat{\pi}^2 & 0 \end{pmatrix} \tag{2.7}$$

where $\hat{\pi} = \hat{p}_x + i\hat{p}_y$

$\widehat{H}_2$ plays a simple role in space of two-component Bloch functions are so helpful to find out the pseudo spins in order to describe the amplitude of electron waves on

weakly-coupled nearest sites F1 and F2 are belonged to two non-equivalent carbon sub lattices F1 and F2 are two graphene layers marked as 1 and 2. For a given direction of quasiparticles momentum p=(p cosM, p sinM), Hamiltonian HJ of a general form $\begin{pmatrix} 0 & (\hat{\pi}^{\dagger j}) \\ \hat{\pi}^{j} & 0 \end{pmatrix}$ can be rewritten as

$$\widehat{H_j} = \varepsilon(p)\vec{\sigma}\cdot\vec{n}(\varphi) \qquad (2.8)$$

where $\vec{n}$= -(cos$J\varphi$,sin$J\varphi$ and vector $\vec{\sigma}$ is made from Pauli matrices. For 2L graphene J=2 but notation J is useful to be allowed calculating Eq. (1b) to be signed with the case of 1L graphene where J=1. The eigenstates of $\widehat{H_j}$ correspond to pseudo spins negative charges (electrons) or anti parallel to the 'quantization' axis n. The rotation of momentum p $\vec{n}$ by angle $J\varphi$ which is related to an adiabatic evolution of such pseudo spin states in terms of the axis rotating by angleφ. Finally, if a quasiparticles like fermions that penetrate a closed contour in the momentum space (that is φ=2π), a phase shift $\emptyset$=$J\pi$ known as Berry's phase is gained by the quasiparticle's wavefunction. Rotation of pseudo spin arises like in the graph and Berry's phase is seen. Therefore quasiparticles again and again go between different carbon sub lattices (F1 and F2 for 1L graphene, and F1 and F2 for 2L graphene).

The phase of Shubnikov-de Hans oscillations (SdHO) is affected and separated to the semi classical quantization by Berry's phase, also cyclotron orbits are being completed for fermions. For 1L graphene, in the sequence of QHE plateaus is resulted in that a π-shift in SdHO and has a relation ½-shift, Berry's phase will be zero due to the conventional 2D systems have a comparison. For 2L graphene φ=2π and in the quasiclassical (N >>1) limit unfortunately must not be changed. One may also suppose that the quantum Hall Effect sequences may not be affected from the changing phase 2π. However, the exact analysis of the LL starts from the zero level and keep going the other degeneracy with excited levels like 2L. They have a combination J-fold degeneracy of the zero-energy Landau level. This is already known limit of quantum N=0). Sometimes Berry's phase is absent, it means there is free fermion in Quantum Hall Effect, $\varepsilon_N = \hbar\omega_c(N + 1/2)$ and we say the minimum state stays in same finite energy $\hbar\omega_c/2$, where $\omega_c = eB/m$. For 1L graphene (J=1; $\emptyset = \pi$), $\varepsilon_N = \pm v_F\sqrt{2e\hbar BN}$ and there would be single state ε_0 at zero energy. For 2L

graphene (J=2; ; $\emptyset = 2\pi$), $\varepsilon_N = \pm\hbar\omega_c\sqrt{N(N-1)}$ and two states $\varepsilon_0 = \varepsilon_1$ lies at zero energy.

We should say that the unconventional QHE found in 2L graphene can be defined by the existence of a double-degenerate LL. This LL reveals at all sides between electron and hole gases and, taking into account the quadruple spin and valley degeneracy, infinitesimal carrier density is shown as $8B/\emptyset_0$. As we mentioned above to Fig. 2.3.c, Quantum Hall Effect can be divided into two levels and they are seen in different magnetic fields, hence Landau Levels exist with like this phenomena in graphene. They are unsimilarity across the stable level. When Landau Level is seen, we should point out there is double degeneracy in 2L so, these degeneracy fold coupled numbers to fill it (all levels are in different regions), Also, the twice higher causes the step between the plateaus due to the energy levels, that is $8e^2$/h as compared to $4e^2$/h for the other steps must be more densities. These results are obtained by experimental and proved as theoretical. As conclusion, 2L graphene is a very good example to denote the Landau Levels as well as energy levels in an experiment. It contributes many double degenerate levels in order to observe the quantum Hall Effect system.

Berry's phase 2π is investigated widely and accompanied with reality of massive chiral fermions. Massive fermions are separated in other quasiparticles. All conductivities conductivity for metals e^2/h at same filling parameters ν=0 poses a serious challenge for theory.

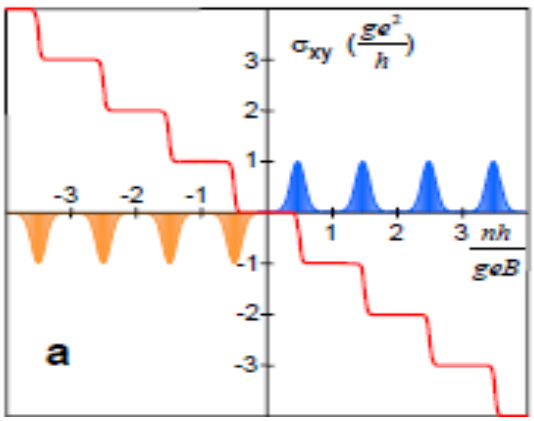


Figure. 2.3.a.

The integer quantum Hall Effect has three types in this schema. It is easily can be seen LL degrees that drawings illustrated for some values schematically conventional integer QHE found in 2D semiconductor systems(**a**) and the QHE in 2L graphene described in the present paper

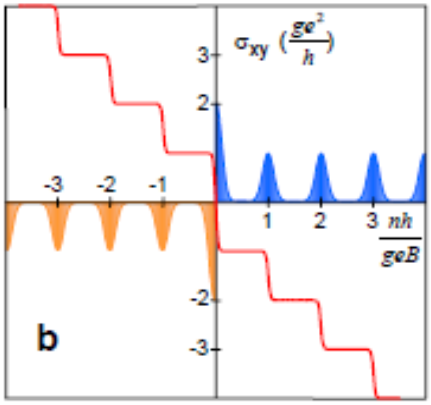


Figure. 2.3.b

Plateaus in Hall conductivity σ_{xy} occur at values $(ge2/h)N$ where N is integer, $e2/h$ the conductance quantum and g the system

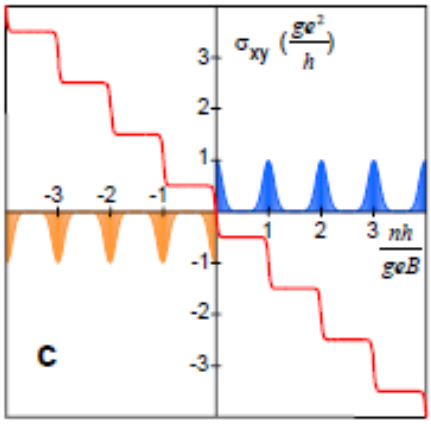


Figure. 2.3.c

This figure also shows the QHE behaviour for massless Dirac fermions in 1L graphene.

Degeneracy that is the density of states $gB/\emptyset_0$ on each LL define distance among steps along the concentration axis, in 2D spectrum had been observed independently each other. Here, B is magnetic field and $\emptyset_0=h/e$ the flux quantum. The sequences of

Landau levels are having a function of mobile carriers concentrations *n* are presented in blue and orange for electrons and holes, according to lines.

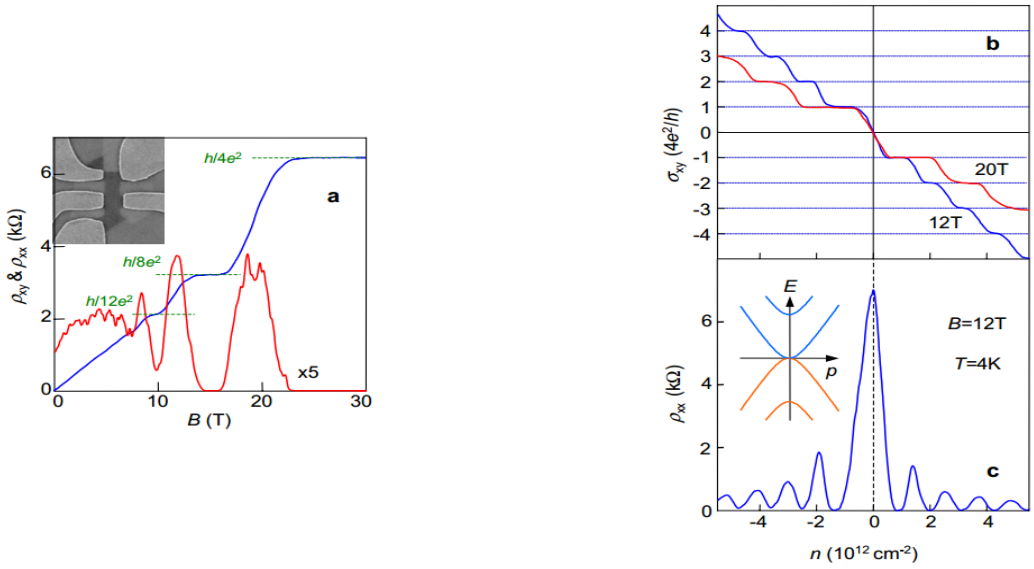


Figure.2.3.d QHE in bilayer graphene.

We can see changeable Quantum Hall Effect and their domains previously figure. The Hall conductivity σ_{xy}= p_{xy} /(p_{xy}^2+p_{xx}^2)2) is able to calculate directly from experimental curves for ρ_{xy} and $\rho_{xx.}$ σ_{xy} allows quantum Hall Effect plateaus followed respectively. σ_{xy} crosses zero without any sign of the zero-level plateau that would be expected for a conventional 2D system. Shows all calculations have different energy spectrums for 2L graphene, which is clearly seen as parabolic at low ε. Charge carriers conductivities μ in figure 2L devices were ordinary ≈3,000 cm2/Vs, which is nicer than for plates made from 1L graphene. There is an interesting situation in here. Because, we consider the mobile carriers are more damaged and more exposure than they are unprotected from both sides in case of 1L graphene at zero energy.

The Landau spectrum cannot be appeared due to the yields has no gaps, for a fixed *n* ≈0 and varying *B*, small magneto resistance was observed by us experimentally. The picks can be occurred by different values for adjustable equipments and configuration also will be varying notes. However, the observed magneto resistance overcome a factor of 2 in any theoretically for undoped 2L graphene.

We therefore separated Landau Levels into two parts as experimental results, energy spectrum for 1L behaves massless Dirac fermions unlikely, energy spectrum for 2L is Berry's phase and has a massive chiral fermions that results means their spectrum domains are different each other.

2.2 Integer Quantum Hall Effect

Before talking about the Fractional quantum Hall effect (FQHE) and Spin Quantum Hall effect (SQHE), we shortly mention about Integer Quantum Hall Effect (IQHE) which we need to compute these functions in quantum world. Therefore, we define a Hamiltonian for a system of electrons in **2**+**1** dimensions living in a magnetic field,

$$H = \sum_i \frac{1}{2m}\left(\overrightarrow{\boldsymbol{p}^2} - \boldsymbol{A}(\boldsymbol{x_i})\right)^{\mathbf{2}} \tag{2.9}$$

and try to solve Schrödinger's equation. We neglected interactions between electrons in order to figure out Hall Effect for fractional filling.

When we investigate the non-interacting particles, the operators $\boldsymbol{p}_i \; and \; \boldsymbol{x}_i$ commute for different *i*, hence assumption of non-interacting electrons thus are reducible a problem to solve and understand easily. We call these problems one particle problems. The particle label *i* is failed, *i*, from the operators for obvious. We point out electric field perpendicular to the plane. Invariant gauge tunes can be configurated in combining with this situation is, (0,-*Bx*, 0). Adding this, for one particle and its solution can be shown Hamiltonian yields,

$$H = \frac{1}{2m}\left(\overrightarrow{p_x^2} + \left(\overrightarrow{p_y} - Bx\right)^2\right) \tag{2.10}$$

The operator p_y are commuted with this Hamiltonian, so we immediately know the wave function can be constructed,

$$\emptyset(x, y) = e^{iky} v(x) \tag{2.11}$$

By obturation this approach there is a harmonic oscillator in the equation and we then solve the Schrödinger equation its potential shifted by $x_0 \equiv \frac{k}{B}$. We can write down our Hamiltonian with related to annihilation and creation operators, which we describe in the usual trend,

$$H = \omega_c \left(A^\dagger A + \frac{1}{2}\right), \tag{2.12}$$

$$A^\dagger = \sqrt{\frac{1}{2}}\,(x - x_0) \mp i\sqrt{\frac{1}{2}}p_x, \tag{2.13}$$

and *A* is the state in the ground state regarded to annihilation. In the equation for the Hamiltonian appears,$\omega_c = \frac{eB}{mc}$, which, is the cyclotron frequency can be shown like this. Moreover it is found a suitable to obtain *x* dimensionless a factor $\sqrt{B}$ is absorbed or in unconventional units $\sqrt{\frac{eB}{c\hbar}}$. This depth measurement is to know as the magnetic length. *A* is in the ground state which must be already there, we write down a Hamiltonian that we are able to compute to grant for *A* in the ground state wave function. Therefore normalized it reads,

$$\phi_{00}(x) = \left(\frac{B}{\pi}\right)^2 e^{-\frac{1}{2}(x-x_0)^2 e^{iky}} \tag{2.14}$$

We apply $\frac{1}{n!}(A^\dagger)^n$ to be occurred higher states in ground state. It is simple to understand this operation provides a polynomial (P_n) in front of the exponential. As we get the first state multiplies with $e^{+\frac{1}{2}(x-x_0)^2}$ from the left (we can neglect a few constants) we obturate a equation is generation by this polynomial.

$$P_n(x) = e^{\frac{1}{2}x^2}(A^\dagger)^n e^{-\frac{1}{2}x^2} = e^{\frac{1}{2}x^2}(x - \partial_x)^n e^{-\frac{1}{2}x^2} \tag{2.15}$$

This polynomial gets around assures the generating Hermite polynomials with equation. Properly normalized wave functions then read,

$$\emptyset_{mk}(x) = \left(\frac{\sqrt{B}}{2^m m!\sqrt{\pi}}\right)^{\frac{1}{2}} H_m(x - x_0) e^{-\frac{1}{2}(x-x_0)^2 e^{iky}} \tag{2.16}$$

The energy states after ground state are easily determined,

$$H\emptyset_{mk} = \omega_c \left(m + \frac{1}{2}\right)\emptyset_{mk}, \qquad m \in \mathbb{Z}. \tag{2.17}$$

We should note that each state has different number k, it can be chosen freely therefore energy levels have degeneracy like Zeeman Effect. If we presume our state is ground state, for describing we choose a four edges having geometry of

dimensions $L_x \times L_y$, the states' for all states values can be solved in finite levels but this is just theoretical assumptions. Firstly, eigenvalues of p_y are quantized, k=$\frac{2x}{L_y}n$. for the wave functions are shown (x_0) to lay on the disk, we must decelerate $0< x_0 < L_x$. this constrain k to the domain $0< k < \frac{BL_xL_y}{2\pi}$. Hence in a Landau level that the maximum number of states is $\frac{BL_xL_y}{2\pi}$.

The one- particle problem has been solved by us; hence their quantum numbers, state levels and energies are identified. The wave function entirely found which it was written for one particle and all wave functions were solved and we obtained non-interaction between electrons and holes. Other wave function is needed component is essential to write down for obtaining that wave functions would be accumulated. In fact component that refers to electrons are fermions. Pauli excursion principle always is applied to fermions that mean which their wave function has to be anti-symmetric. We can show the total wave function any more, these wave functions are obtained by Pauli matrices. the wave functions are subsequently same but opposite magnitude also they are solved for one-particle and we inferred them is the particle coordinates.

CHAPTER 3

THE FRACTIONAL QUANTUM HALL EFFECT

3.1 Background

There is a clear difference between Fractional Quantum Hall Effect and Integer Quantum Hall Effect, we can explain like that, level fillings and quantum numbers are fractional values in FQHE, and however, the level fillings and quantum numbers are exact numbers that means there are no rational values in IQHE [15]. Fractional fillings cannot say any information exactly for single-electron physics. The energy gaps in particle are many, so we would like to compute the fractional values for many-particle origin is inferred as within potentially degenerate, we can say Fermi energies have energy gaps as well as Landau or spin level.

In 1982, the fractional quantum Hall Effect was discovered as experimentally. So new Landau Levels were made recovery as well as spectrum levels. Then all old acknowledgments were rewritten and new values were added to Landau Levels with fractional numbers. These results are coming from electrical conduction in semiconductors. Initially enigmatic to theorists, it was expressed by R.B. Laughing as theoretical and it was so important to find out the wave functions for liquid. The fractional quantum Hall effect is an interaction between many-electron system, and between electrons that fractional values could be appeared, for example, to consist of a two dimensional gas of particles of charge which the numbers of fractional values are equal to one-third electron charge. Because of the fractional numbers and levels spectacular overviews, the theory of the fractional quantum Hall effect has been used to reveal out the properties of the electrons clearly development in many-body theory of this decade.

Early in the progression of experimental technique has been followed by reforms, in particular, semiconductor technology is advanced constantly improved the quality of the measurements experiments of quantum transport. Because of fractions of levels are obviously determined. Most of the currently observed quantum states, especially theoretical progress Laughling original work (but not all) from the picture hierarchy in the developing, as well as made. Much, however, has to be done. By the way, a lot studies on fractions has been continuing in laboratuvars. Experimental side, the other measurements, data development, transportation, multi-layered structures, extension, and even the most urgent clarification will be completed. theoretical side-dominator fractions, senior, and even denominators, the problems associated with finite temperature, nad a great need for multi-layered still in work.

Basically, we have been mentioning about the fractional quantum hall that, we get touched on the different parts. For the inhomogeneous state, the clearest example can be shown the many-body particle problems. All states are seen rational fractions within the high-field magnetic field. This state's includes 1L and 2L degenerates in terms of 2D spectrums and if we look up a unbounded-electron to the orbits gas and slowly investigate the interactions between the charge carries density, the interaction turns out to be potential energy from kinetic energy. In the Wigner crystal, we are aim to seek an arrangement of the electrons with at minimized the potential energy.

In the Hall case, due to quenching of kinetic energy, the Wigner crystal is appeared at lighter densities. It was initially thought by Fukuyama and Platzman in 1982 and this information was explained firstly. However, it was soon realized that state differs from the fractional quantum Hall States, because the fractional values belong to Hall conductance of levels. Unlikely, we see surprising that Hall Effect longitudinal and Hall conductance vanish at zero temperature and when the crystal is pinned by disorder leading to an insulating state. Further, the Wigner crystal state is not tied to any particular commensurate filling. Namely, the Wigner crystal state does not exactly equal to fractional Hall Effect.

At the next figure.3.1 shows that we demonstrate the fractional values of a semiconductor at the 80mK and increasing with magnetic field.

These findings are recorded at 80mK. Also resistivity components are diagonal includes, which these includes show regions are measurable at zero resistivity respectively to each Fractional Quantum Hall Effect plateau. The layers in the semiconductors, they play up very important roles to become fractional quantum numbers. Because distance between the layers affect minimise the interaction energy, it is therefore favourable to charge both layers equally.

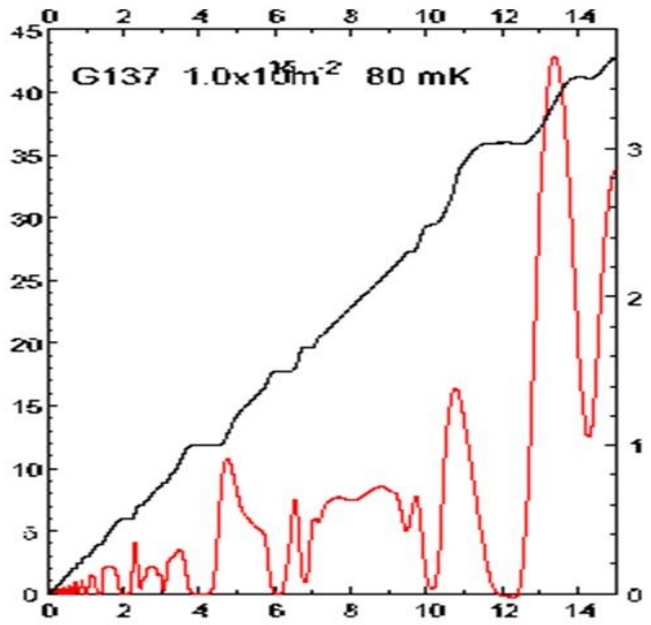


Figure 3.1 GaAs-GaAlAs fractional quantum Hall effect

The levels of fractional numbers have been seen in graphic as experimentally. They generally get weaker going from left to right when magnetic field is increased.

Likewise, as we try to explain another approach the quantum Hall effect with incompressibility that occurs when a two-dimensional electron gas becomes incompressible, it is provided that if the density of gases incompressible, it occurs magnetic field dependent. For the integer quantum Hall effect, the incompressibility can be easily understood. The quantization of the electron's cyclotron motion refers to only a discrete set of kinetic energies is allowed, and both the separation between allowed energies and the number of states of a given energy are proportional to magnetic field. (Landau levels known as given kinetic energy to the set of states). The incompressibility responsible for the integer QHE occurs at densities where an

integer number of Landau levels are filled and the chemical potential jumps from one allowed kinetic energy to another [3].

After the discovery of the Fractional Quantum Hall Effect with $n = \frac{1}{3^4}$, a plethora of other types of Fractional Quantum Hall Effect were observed and afforded so many times by early years 1980's. Each quantum number have to describe the 2/5 and 3/7 states (i.e with $n = \frac{2}{5}$ and $n = \frac{3}{7}$), that are Landau series $p/(2sp \pm 1)$, with the exact numbers called s and p. This series explained the compelling explication within the alleged *composite fermions* (CF) theory regarding to fractional quantum Hall Effect may be viewed as an integer quantum Hall Effect of a quasi-particle that contains of an electron that all integer number of flowing quantized. [13,14]. It is interesting quantum numbers with only states $n = p/q$ by the way fractional values observed in bilayer systems. In here, these quantum numbers origin from Coulomb interaction, because fractional quantum numbers facts observed in high magnetic fields. Every state, this interaction makes each other between the spin-orbit or electron and electron. Coulomb forces are already shown like-charged. After integer quantum numbers and conductivity values caused the fractional quantum Hall effect in 2D. These realities were found by 2D spectrum and many-particle states made quantum-mechanical future. Fractional quantum numbers and exactly fractional charged quasiparticles are probably the most gorgeous of its subsumptions.

The many-electron wave function is required by attachment of the magnetic field that is assumption of zeroes fields along the partial yield whereby there are magnetic flux quanta flowing through it. Each level "heals" in the parameter magnetic length ($l_0 = \sqrt{\hbar/(eB)}$) and, we can draw border of it to minimum energy level of Landau, each like "hole" in the electron around the atom mentions whole charge parts deficit of ve. In the magnetic field, charges also turn and get interaction a 2π phase strain to the wavelengths at the same position of each such lowest level, these terms are read vortices. In particular calculation, vortices are the solidified of flux in magnetic field quanta in an spin and electron system. A small plate can be conflict or get counter through the plane of the electrons loaded energy to harvest just one more magnetic flux quantum through its core could generate more interaction such hurricane. Hence, widely talking, hurricanes are often formulated with quantized energies.

Just like electrons, vortices are not landed in the plane. However, electron system combine a charge cumulative and vortices a charge scarcity, they attract perfectly each other. To consider Coulomb energy can be gained interaction between electrons by locating vortices onto electrons. At $v = 1/3$ there would be existed four times as many vortices as there are small negative charges, each vortex representing a local charge scarcity of $\frac{1}{3}e$.

For negative charge has to carry at least one vortex equivalent, Pauli principle always satisfied the all electron's wave function in one zero level. Coulomb energy would carry a high gain. Hurricane sticks in orbital to an electron, demonstrating a finitely consummation of electrons, filled energise. The condition is precisely is very hard and quantized in movement of charge.

So electrons are turning around the orbit and four holes to each electron is defined as Laughling wave function in prominent $v = \frac{1}{3}$ fractional quantum Hall Effect state expressed by Laughling's wave function as [14]

$$\psi_{1/3} = \prod_{i<j}^{n}(Z_i - Z_j)^3 \ exp(-\frac{1}{4}\sum_{k}^{n} Z_k^2 \qquad (3.1)$$

The $Z_{i,j,k}$ refers to coordinates of n electrons in a complex 2D plane, which complex interference the wave function more impact. To find the normalization of wave function, we set the polynomials and identify the functions. All electron-electron interaction would be derived second term, which is distance between electrons so, it would be a new item all over the complex pair. The each electron could be vortices in mathematical terms for four connections between electrons exponent 3 in each factor in position. It is occasionally known that, states at $v = 1/q$ (q=even numbers) consist of negative charges put up by q vortices, so, exponent comes to the upper of the wave function and they are so different, which are changed from 2 to q. Only even q are allowed, they are definitely not anti-symmetry, if they would be show the anti-symmetry feature, they have to become odd numbers, but wave function seem in here as symmetry feature since they are even numbers. When electron –electron interaction appears the anti-symmetry property, it would be recordable in order to observe the energy storage. Hurricane movement is increasing by induced the magnetic field, namely when magnetic field is going up steady, a new creation

movements we obtain for it. So it is required limitless amount of energy. The electronic state at this filling factor is varying with incompressibility by the energy blanks at $v = 1/3$

Electron freely wanders around the orbit like quasiholes, and when they got the energy or given the high magnetic field, they earn the action as carries +1/3 of. This can be seen in figure 3.1.

We again return the energy of the electrons in the gaps the $v = 1/3$ state, they are getting together with new elements. electrons to which three *vortex-generating* flux quanta have been attached. This approach makes it to understand electron's world. First, two flux quanta per electron does not cause the magnetic field, however, when electrons have the three flux quanta, magnetic field that we can got it. Second, if we write the wave functions of these electrons in the plate, we would calculate the flux quantized energies. Third, electrons which are exchanged among the each other and shifted the phase a bit of, then the Berry's phase could be obtained for the electrons around such hurricane.

After electrons, this results are valid for the fermions in the particles, they can be easily written down by wave functions. Whereas, the $v = 1/3$ state comes from the chiral bosons created by the stick of three flux quanta to each electron, that magnetic field is disappeared with the movement of the electrons. This feature, a new concept about the fermions are caused by developing of energy states.

Different scientists investigated the fractional values and energy states in different times, and they got the same results about the energy states of the fermions and electrons. Energy states are seem that filling factor $v = 1/q$, as well as $v = 1 - 1/q$ (odd numbers cause the electron-vanish symmetry), are understood so much. The scientists which their wave functions were researched repeatedly and interesting values recorded for science world and they helped the developing of the spintronics devices.

3.2 Composite Fermion in the Fractional Quantum Hall Effect

The fractional quantum Hall Effect has improved with the behaviour of electrons are understood, and new particles found recently years. New particles that can be said

composite fermions (CFs) are fresh particles and it provides us shortly investigating it. Their quantum numbers are odd and it means fractional filling and energy gaps can be possessed. First, the primary states at $v = 1/q$. This state notify the theorist to find out the fractional fillings and quantum numbers as at $v = 1/2$.

Higher-order FQHE states ($v = 2/5{,}5/9$ or $5/7$) are abundant in experimental results, and the these results produce very similar to other values at $v = 1/q$. That result may be for other state's energy gaps at $v = p/2p \pm 1$ which come together to half filling. Certainly, Coulomb energy is same in here, which informs thequantum Hall Effect subject at $v = 1/q$, is hoped to be at study. But, in this state, these fractional values are hard to identify the in many-electron and their wave functions related to higher-order $v = 1/q$. So physicists were used to start integer quantum states and then they calculated fractional ratios for the energy gaps. Whereby the electrons have a relation first states at $v = 1/q$, and filling factors can be derived from this situation. Electron charges or quasiholes may intercourse and make a formalism for new quantum liquids of quasiparticles at filling factors in terms of rational values. This alternatively can be used for odd fractions and after progression even exact values of filling factors.

The aim of the fractional values is to bring into the open feature for such states at $v = p/q$ of the Laughling wave function at $v = 1/q$, and Coulomb energy can be computed easily from this principle with fermions' wave functions for upper states.

This interaction as new model was introduced by using 2D, so higher energy states were built with new notations. This explanations $v = p/2p + 1$ set the new energy states and states were condensed with starting at $v = 1/3$ and approaches towards $v = 1/2$. For these states, electrons play important role for revealing out the wave functions of the states. this $v = 1/3$ state is formed with electrons and that means to many-electron system due to the 1/3 fractional ratio, and we call it Laughling wave function. By the following equation, the wave function can be written as,[14]

$$\psi_{1/2} = \prod_{i<j}^{n}(Z_i - Z_j)^2) \ \prod_{i<j}^{n} Z_i - Z_j \ exp(-\tfrac{1}{4}\textstyle\sum_k^n (Z_k)^2) \tag{3.2}$$

$$= \prod_{i<j}^{n}((Z_i - Z_j)^2 \ \psi_1. \tag{3.3}$$

Landau level must be filled with second factor, ψ_1, that vortex or movement provides fractions for each electron and Pauli excursion principle is valid for this calculation. In this equation, spin is neglected to non-interaction between many-particles. therefore, the equation will be reduced from fractional quantum Hall state to integer quantum Hall state, these reductions can be shown with at $v = 1/3$ $v = 1$. Unfortunately, every electron move with 2 vortices. New particle is interacted with flux-quanta per electron, purpose is to reduce the magnetic field and effect of the flux quantum per fermions, that is equal to the field at $v = 1$. These fermions carry the quantized flux with even number, treat as composite particle and not vanish the energy levels.

Landau level has valuable formal description in order to generalize the composite fermions and calculate the filling factor with new state as at n 51/3. Along the states for the composite fermions generate the new states in energy levels and filling factors, hence, many-particle problems can be solved with this technique. Wave functions of these particles can be written with these functions includes states. Like composite fermions particles produce perfect many-particle wave function for states n 5*p*/(2*p*11), it is implied to many-particle when comparison of lower –order particle calculations.

(Dev and Jain, 1992). Besides this explanation, these implications help to identify wave functions domain in the formulation. These states can be shown in(see Fig. 3.2).

Utilizing parameters between the integer quantum Hall Effect and fractional 5p/(2p11) quantum Hall Effect, the composite fermions act the filling factor at Landau level, it is demonstrated clearly the similarity between composite fermions gap energy and Landau gaps in the electron case (all gaps of the Landau guides external magnetic field), electrons are similar to this action and gap energies are prediction to start directly magnetic field. Unless the composite fermions introduce the function with gaps, the cyclotron mass could not appear and derive in the function. Electrons interact to composite fermions for many-particle interaction, and these interaction is represented electron-electron interaction as we said before. Hence, assume a fraction number, extends of the function of the electron density. Many-particle interaction is very weird a phenomena for infinitesimal masses. The

fractional numbers refer to quantum Hall effect due to the discrete lines. Fractional quantum states represent the different values at different magnitude of magnetic fields. For example the figure 3.2 that shows obviously fractional quantum Hall Effect is changing with different Tesla. It means also quantum interference would be avoid by resistance fluctuations. When theoretical calculations are so this reasons. When we calculate the fractional numbers in different magnetic fields, we obtain changing values for fractions. In the figure 3.2, the lines that their draws pick with different magnetic fields and it makes the matter can be classified with this way. The fractional quantum Hall state $\nu = 1/3$. In the high magnetic fields ,the semiconductors that are observable for fractional quantum Hall Effect, namely if we identify the properties of the each semiconductor or conductor, in fact we use this process. And then, fractional coefficients help us to compare their numbers with the Landau Levels. Landau-levels, as we said before, it is decisive for the fractional quantum numbers in order to *n* or *p* type of semiconductors, Lowest Landau-Levels (LLL) is already resulting of the fractional coefficient. Normally, we use the $\frac{e^2}{\hbar}$ as the fractional quantum levels. Moreover, integer quantum Hall State and fractional quantum Hall State is separated from each other with this way. The causes of the fractional quantum number is to edges of the plate that current carry negative charges and electrons are interaction between each other. We may infer also the quasiparticles may be explained with movement of the composite fermions and energy gaps of the orbits in quantum computer. These topics also are very interesting a study title for the physicist nowadays. It is the new step towards a full understanding of these new fractional charges like fermions.

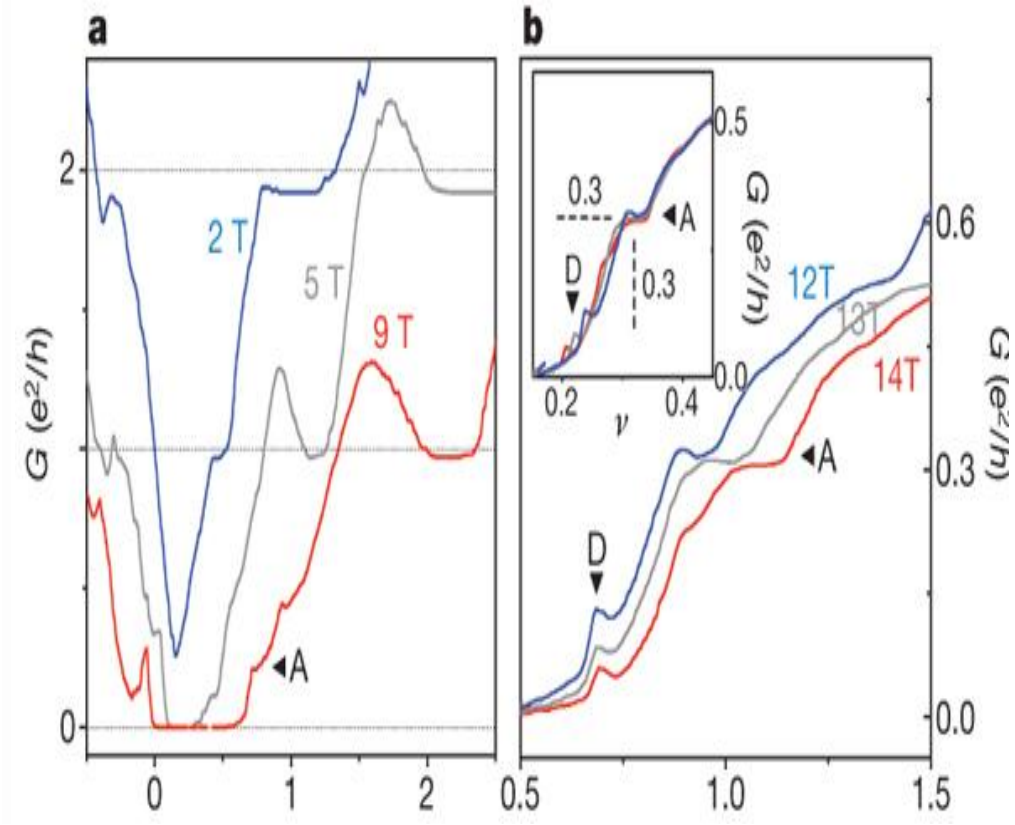


Figure.3.2. Quantum Spin Hall state

In this shema we represent Magneto transport at high magnetic fields and observation of Fractional Quantum Hall Effect in graphene [27].

The next chapter a general Hamiltonian of a two-level system is constructed also the present chapter emphasizes a general method to solve physical system.

CHAPTER 4

HAMILTONIAN SOLUTIONS FOR TWO LEVEL QUANTUM OPTICAL SYSTEM

In two dimensional geometry that we can write Hamiltonian of a two level system as generally[36]

$$H = H_0 + \beta\sigma_0 + (\kappa_1 a \kappa_2 a^+ + \kappa_3 b + \kappa_4 b^+)\sigma_+ + (\gamma_1 a + \gamma_2 a^+ + \gamma_3 b + \gamma_4 b^+)\sigma_- \quad 4.1$$

where

$$H_0 = \hbar\omega_1 a^+ a + \hbar\omega_2 b^+ b \quad (4.2)$$

a,b and a^+, b^+ are bosonic annihilation and creation operators, respectively and $\omega_i, \beta, \kappa_i$ and γ_1 are physical constants and $\sigma_0, \sigma_+, \sigma_-$ are usual Pauli matrices and they are given by,

$$\sigma_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad \sigma_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \quad \sigma_0 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad (4.3)$$

The Hamiltonian (4.1) contains different physical Hamiltonians depending on the choice of the parameters. If

a) $\omega_1 = \omega_2 = \omega, \quad \kappa_2 = \kappa_3 = \gamma_1 = \gamma_4 = 0$ and $\kappa_1 = \kappa_4 = \gamma_2 = \gamma_3 = \kappa$, the reducible the E$\otimes\varepsilon$ Jahn-Teller (JT) Hamiltonian by Hamiltonian, H,

b) $\omega_1 = \omega_2 = \omega$, $\kappa_2 = \kappa_3 = \gamma_1 = \gamma_4 = 0$ and , $\kappa_1 = -\kappa_4 = \gamma_2 = -\gamma_3 = \kappa$, the Hamiltonian, H, becomes the Hamiltonians of quantum dots including spin-orbit coupling

c) $\kappa_2 = \kappa_3 = \kappa_4 = \gamma_1 = \gamma_3 = \gamma_4 = 0$, $\kappa_1 = \gamma_2$ and $\beta = \frac{\omega_0}{2}$ the Hamiltonian,H, becomes JC Hamiltonian with RWA,

d) $\omega_2 = \kappa_3 = \kappa_4 = \gamma_3 = \gamma_4 = 0,\ \kappa_1 = \kappa_2 = \gamma_1 = \gamma_2 = \kappa,\quad \beta = \frac{\hbar\omega_0}{2}$ and $\omega_1 = \omega$ then we obtain JC Hamiltonian without RWA

e) $\kappa_2 = \kappa_4 = \gamma_1 = \gamma_3 = 0,\ \kappa_1 = \gamma_2 = \lambda_1, \kappa_3 = \gamma_4$, and $\beta = \hbar\omega_0$ the Hamiltonian, H, takes the form of modified JC (MJC) Hamiltonian.

Therefore Hamiltonians are demonstrated with first column and last column would help us to their algebras. So we computed the Hamiltonians and get the result of investigation.

The relationship between the suitable Lie algebra and Hamiltonian is like that they combine its bosonic and fermionic representation. We are inserted in the two-level system in a one and two-dimensional geometry, matrix-differential or bosons fermions give the Hamiltonians. Therefore, it is important to define a convenient function of the operators which Hamiltonians may be found by connection between boson-fermions and matrix differential. End of this, we can begin with by corresponding differential equations of the boson operators:

$$a^+ = \frac{l}{2}(x + iy) - \frac{1}{2l}(\partial_x - i\partial_y) \tag{4.4}$$

$$a = \frac{l}{2}(x - iy) + \frac{1}{2l}(\partial_x - \partial_y) \tag{4.5}$$

$$b^+ = \frac{l}{2}(x - iy) - \frac{1}{2l}(\partial_x - i\partial_y) \tag{4.6}$$

$$b = \frac{l}{2}(x + iy) + \frac{1}{2l}(\partial_x + i\partial_y) \tag{4.7}$$

where $l = \sqrt{\frac{m\omega}{\hbar}}$ is the length parameter and the boson operators obey the usual commutation relation

$$[a, a^+] = [b, b^+] = 1; \tag{4.8}$$

$$[a, b^+] = [b, a^+] = [a, b] = [a^+, b^+] = 0 \tag{4.9}$$

In principle, when boson operators introduce the Hamiltonian, number of particles would define the bosons operators that is steady on the known formula on a state

unless solving differential equations. We can separate the method from each other, occasionally transformation properties of the bosons could involved the Hamiltonians for putting a simple form.

4.1 Rabi Hamiltonian

We shortly mention about Rabi Hamiltonian which is useful to figure out the eigenvalues for the spin quantum hall effect, so in this thesis, we need to say something about Rabi Hamiltonian. The next chapter we carry the energy eigenvalues out with this Hamiltonian.

Fourfold axis symmetry refers to cyclooctatetraene molecular ion, which is a particular case, then a single mode would actualize the a doubly degenerate state. $E \otimes \beta$ Jahn-Teller system is this system for solving complex cases of the Jahn-Teller effect. The $E \otimes \beta$ Jahn-Teller system coupled to a system terminating harmonic oscillations whose energy eigenvalues differ by 2μ is characterized by the Rabi Hamiltonian [41]

$$H = a^{+}a + \kappa\sigma_3(a^{+} + a) + \mu(\sigma^{+} + \sigma^{-}) \tag{4.10}$$

where $\sigma^{\pm} = \frac{1}{2}(\sigma_1 \pm i\sigma_2)$ and $\sigma_1, \sigma_2, \sigma_3$ are Pauli matrices and the parameter κ is a linear coupling constant. Hamiltonian (4.1.1) that we easily define as a differential equation by using the realizations of the bosonic operators

$$a^{+} = z \qquad\qquad a = \frac{d}{dz} \tag{4.11}$$

In this derivation, we would set the linear first-order differential equation and it is useful to write the part of the Schrödinger equation. Equation (4.10) and equation (4.11) exchanged each other and (4.11) into (4.10) a system is obtained by two linear differential equations for the functions

$$\Psi_1(z) and\ \Psi_2(z) \tag{4.12}$$

$$(z+\kappa)\frac{d\Psi_1(z)}{dz}+(\kappa z-E)\Psi_1(z)+\mu\Psi_2(z)=0 \tag{4.13}$$

$$(z-\kappa)\frac{d\Psi_2(z)}{dz}-(\kappa z+E)\Psi_2(z)+\mu\Psi_1(z)=0 \tag{4.14}$$

where E is the eigenvalues of the Rabi Hamiltonian. We eliminate $\Psi_2(x)$ between the two equations, and substituting

$$z=\ \kappa(2x-1) \qquad\qquad \Psi_1(x)=e^{-2\kappa^2 x}R(x) \tag{4.15}$$

a second order differential equation is obtained

$$x(1-x)\frac{d^2R(x)}{dx^2}+[\kappa^2(4x^2-2x-1)+E(2x-1)-x+1]\frac{dR(x)}{dx}+$$

$$[\kappa^4(-4x+3)-E^2+2E\kappa^2(-2x+1)+\mu^2]R(x)=0 \tag{4.16}$$

For figuring out generators of the Lie algebra, we have to write down the Hamiltonians that are accompanied quasi exact solvability of (4.14).

4.2 Computation of Eigenvalues and Eigenfunctions of the Rabi Hamiltonian

We search a solution to obtain eigenvalues and eigenfunctions for quasi-exact solution. Since the function *R(x)*=$\{1,x,x^2,....x^{2j}\}$ forms a basis function for called *su* (1,1) algebra, we seek for a solution the polynomial of degree 2j

$$R(x)=\sum_{m=0}^{2j}a_m x^m \tag{4.17}$$

Energy polynomials are coming from production of the wavefunction. So, roots of the polynomials assist us to generate the eigenvalues. Hence the function can be followed as

$$R(x)=\sum_{m=0}^{2j}a_m P_m(\kappa)x^m \tag{4.18}$$

By substituted (4.18) into (4.17) and carrying out a straightforward calculation, we get the definition

$$R(x)=1+\frac{4P_{2j-1}(\kappa)(\kappa x)^{2j}}{\mu^2}+\sum_{m=0}^{2j}P_m(\kappa)x^m \tag{4.19}$$

Here $P_m(\kappa)$ satisfies the recurrence relation

$$(m+1)(m+1-2j)P_{m+1}(\kappa) + [(m-2j-4\kappa^4-m)+\mu^2]\,P_m(\kappa)$$

$$+4\kappa^2(m-1-2j)P_{m-1}(\kappa) = 0 \tag{4.20}$$

With the initial conditions $P_{-1}(\kappa) = 0$ and $P_0(\kappa) = 1$ certain properities of the polynomial $P_m(\kappa)$ can be debuted. Whether κ_j is a root of the polynomial$P_{m+1}(\kappa)$, the series (4.19) rounded to m *2j+1* and κ_j Rabi Hamiltonian has this polynomials. Therefore the solution given in (4.19) wage at m =2j and it becomes a polynomial of degree 2j. The first four term are presented by

$$P_1(\kappa) = \mu^2 \tag{4.21}$$

$$P_2(\kappa) = \mu^2(4\kappa^2+\mu^2-1) \tag{4.22}$$

$$P_3(\kappa) = \mu^2(32\kappa^4 + 4(3\mu^2-8)\kappa^2 + (\mu^2-5) + 4 \tag{4.23}$$

$$P_4(\kappa) = \mu^2(384\kappa^6 + 16(11\mu^2-54)\kappa^4 +$$

$$8(3\mu^4-29\mu^2+54)\kappa^2 + \mu^2(\mu^2-7)^2 - 36) \tag{4.24}$$

For j = $0, 1/2, 3/2$, respectively. The components of the eigenfunctions are expressed as

$$\psi_1(x) = N_1 e^{-\kappa^2 x} R(x) \tag{4.25}$$

$$\psi_2(x) = N_2 e^{-\kappa^2 x}(2(j-\kappa^2 x)R(x) - xR'(x)) \tag{4.26}$$

where N_1 and N_2 are normalization constants. It is clear that the degrees of polynomials in the expressions for $\psi_1(x)$ and $\psi_2(x)$ are 2_j and 2_{j+1}, respectively.

The eigenfunctions can be written for a given j. We give an instance review the *j*= $1/2$ case. The polynomial $P_m(\kappa)$=0. The zeros of the $P_2(\kappa)$ are given by

$$\mu = 0 \qquad \mu = \pm\sqrt{1-4\kappa^2} \tag{4.27}$$

In this condition $\mu \neq 0$ Rabi Hamiltonian can be obtained for the exact solution, and its normalized eigenfunctions may be written as follows

$$\psi_1(x) = \frac{(8+4\mu^2+\mu^4)e^{-\kappa^2 x}}{2\kappa^2\mu^4} \tag{4.28}$$

$$\psi_2(x) = \frac{728 - 288\mu^2 - 19\mu^4 + 6\mu^6 + \mu^8)e^{-\kappa^2 x}}{32\kappa^6}$$

$$\big((1+2\kappa^2)x - 1\big)(\mu^2 + 4\kappa^2 x) \tag{4.29}$$

With the eigenvalues

$$E = 1 - \kappa^2 \tag{4.29}$$

The functions give the condition normalizability as

$$Re(\kappa^2) > 0 \quad \text{and} \quad -\frac{\pi}{4} < arg(\kappa) < \frac{\pi}{4} \tag{4.30}$$

When *j*=1 the roots of the polynomial $P_3(\kappa)$ can be obtained from (4.23) and they read

$$\mu = 0 \qquad \mu = \sqrt{\frac{5}{2} - 6\kappa^2 \pm \sqrt{16\kappa^4 + 8\kappa^2 + 9}} \tag{4.31}$$

(4.28) and (4.29) functions give the eigenfunctions by evaluating the unnormalized eigenfunctions for j=1 are given by

$$\psi_1(x) = e^{-\kappa^2 x}(1 + (8\kappa^2 + \mu^2 - 4)(x + \frac{4\kappa^2}{\mu^2}x^2)) \tag{4.32}$$

$$\psi_2(x) = 2 + (14\,\kappa^2 + 2\mu^2 - 9)x - [8\kappa^2 + \mu^2 - 4] \times$$

$$[(1 + 2\kappa^2 - \frac{8\kappa^2}{\mu^2})x^2 - \frac{4\kappa^2(1+2\kappa^2)x^3}{\mu^2}] \tag{4.33}$$

Hereby we reached the our aim that is the eigenfunctions are normalizable regarding to their wave functions and eigenvalues are more easier under the situation given in (4.31).

CHAPTER 5

SPIN QUANTUM HALL EFFECT

5.1 Background

We study on Spin Quantum Hall Effect in this thesis and we investigate the relationship between classical Hall Effect and Spin Hall Effect. Spin Quantum Hall Effect (SQHE) is similar to classical Hall Effect in fact. However, as we research the spin Hall Effect, there is a different a current that is called spin current due to the movement of the spins.

The spin Hall Effect is observed recently in both *n* and *p* doped semiconductors, but, it is still unclear if the underlying mechanism is intrinsic or extrinsic. These investigations opened up a new technological developments which are like sensors, quantum spintronics devices, integrate information processing and storage units. All new sensors (pressure sensors, current sensors etc...) rule this Hall Effect. With this development of the electronic devices also help the understanding of the semiconductors well. On the other hand, all of the semiconductors are not same each other, they can be divided into two groups that are n type and p type.

N type semiconductors works for increasing the load on the valence of four semiconductor elements an impurity adding five-valent a doping operation, is obtained by carrying out carriers. Doping material is added, the semiconductor atoms release a poor-linked outer electrons. Give away some of its electrons, since the donor material of this type is called as doping agents. The purpose of the N type doping material, carrier to produce an abundance of electrons. To understand exactly how the n-type doping is performed, silicon (Si) can be shown an example. Four neighbouring Si atoms of Si atoms covalently connected with one of the each of the four valence electrons. Such periodic table group 15 (Former group VA, akanitrogen group) five valence electrons, such as an atom (e.g. phosphorus (P), arsenic (As), or antimony (Sb)), are included in the crystal lattice instead of a Si atom, the atom and

an electron-stick will have four covalent bonds. This is only weakly bound to the extra electron can be excited into the conduction band and easily. At normal temperatures, almost all of these electrons are excited into the conduction band.

This does not result in the formation of electron hole Since excitation such a material, the number of electron-hole far exceeds. In this case, the electron holes are the majority carriers and minority carriers. Five-electron atoms "donation" is not an extra electron, they that each moving electron donor atoms can be said. Semiconductors note is still never far from a positive ion dopant and N-doped material normally a zero net there is an electrical charge.

P type semiconductors are composed holes with wealth situation. Due to the silicon, a trivalent atom (group IIIA of the periodic table, such as boron or aluminium) is exchanged with other crystal structure. In this type, a dopant atom can accept an electron from a neighbouring atoms covalent bond to complete the fourth bond. So they gathers each other in order to compose a covalent bond. The other side, other atom loses an electron so, there will be a hole in the sequence of electron. This hole causes the electron which goes to near to negatively charge and hole wanders away into the lattice, one proton in the atom at the hole's location will be brought out and cancelled by an electron. The quantity of positive charge is assured by this hole [20].

In addition to these information, we now take a partially different approach to understanding *n* and *p* doped materials or intrinsic semiconductors, while we are informing about the intrinsic semiconductor where the electrons move toward the left and the holes move toward the right when the applied electric field is to the right. It is already become a Hall Effect in material.

It is important to point out that there are both negative and positive charge carriers in a semiconductor. Since a negative charge travels in the opposite direction according to positive charge, they compose a valence from the excitation state and it leaves out a hole, these electrons fill the valence band in atom. This hole appears as a positive charge, $+e$. The holes, as we said before, acts an important role for the interaction between the spin and orbit, electrons turn around the orbit, the same time spin turns around the itself, they cause an interaction due to the induced magnetic field. thereby electrons fill the holes and all electrons located stabile places. All intrinsic

semiconductors contains like this pairs and we call this pairs " electron-hole pairs." In this materials, holes move with the current in the electric field, unlikely electrons follow a opposite direction.

Since we are deal with the spin Hall Effect, so we shortly mentioned about semiconductors and their types. Now, we can start with the quantum spin Hall Effect with their eigenvalues and eigenfunctions in a semiconductor, spin Hall Effect is a distinguishing property for all materials and it is known as spin Hall State. Spin Hall state that is aimed to bring out to existence of spin current, two-dimensional, semiconductors with spin-orbit coupling.

Let me represent a figure about the spin Hall Effect and see the movement of the spins with electric current.

Figure 5.1. Spin Hall Effect demonstration

If we evaluate the spin quantum from another window and infer it as below figure;

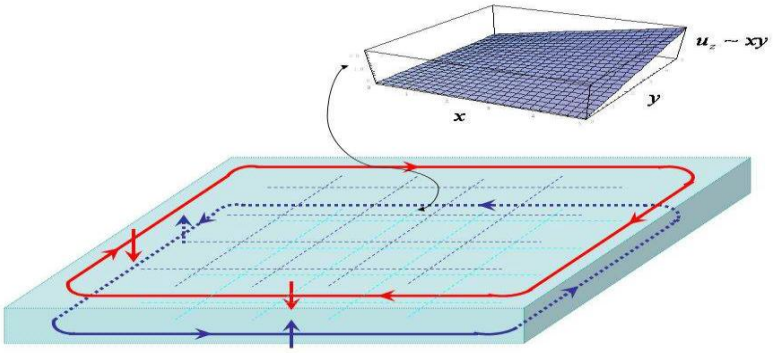


Figure 5.2 Opposite spin-orbit coupling force.

Total charge conductance vanishes but spin conductance is quantized. The Figure shows the lattice displacement leading to the strain configuration [12].

Bilayer system used for observing the spin current and Fig.5.2 obtained for it. This bilayer made the interaction between spins. So spins move different ways so that they are down and up direction. They get together each other edges of the plane, electric field is induced after they are separated. These two layers are located together. Negative charge of the electrons spin Hall conductivity is down 3 positive charges while electrons spin Hall conductivity.

Graphene plane in the direction of the edges of a strip of zigzag one-dimensional energy bands, which is able to show you some experimental results. K and K 'points in the range of one-dimensional projections bulk band are clearly seen. In addition, the two groups K and K' connecting points traverse the gap. These bands are localized to the edges of strip, and each band has completely degenerated copies for each edge. Since the states of the states propagate in both directions on each side edge is not chiral. However, the edge states with opposite spin electrons propagate in opposite directions here in after "spin filter" is. In this case, K and K' and k = 0 is similar to 1D projections of the edge states occurs because the edges of the seat. This zigzag edges Δ so $\rightarrow 0$ for an edge to make the states must insist on a completely flat striking. This is associated with zigzag edges of the Fermi energy density leads to an improved condition. Scanning tunnelling spectroscopy of graphite surfaces have recently seen this [34].

Whole system vanishes are filled the spin of electrons. Spin-orbit interaction is that induced magnetic field in the plane causes the interacting between the spin-up and spin-down. Then we can measure the current of spins. In here layers' direction is not important, because interaction is just between the spin and orbits. However, type of semiconductor is valuable for induced electric field due to the spin current. However, the spin Hall conductance does not remain infinite, as the chiral states are spin-up while the anti-chiral states are spin down, as shown something like that. Although it is so difficult to decide measuring as experimentally in spin Hall effect, the measurement of the charge quantum Hall effect has become relatively common.

Hall effect charge transport properties of solid-state scanning has proven to be a convenient and useful tool, and is used as a routine method of standard material characterization. It finds wide application in magnetic field sensors, and the two-dimensional systems and spin-dependent Hall effect in ferromagnetic systems, the integer and fractional quantum Hall effects of inconsistent Hall effect in a wealth of new phenomena, such as the way the spin Hall effect has been started up with it. Hall the effect of spin-orbit interaction with traditional analog also as a result of paramagnetic systems have been proposed, and in the absence of an electric field applied with a cross-flow generation means pure spin applied magnetic fields. A pure spin current in one direction, spin-up electrons and charged with no net current flow of spin angular momentum, resulting in a current flow in the opposite direction to spin down electrons can be considered as a combination of a Hall voltage of the Hall effect can be revealed. Conventional the edges of the sample caused by the accumulation of charge, similar to the edges of the spin accumulation is expected to sample the spin Hall effect. Early theoretical studies is referred to as an external spin Hall effect caused by the up and down spin asymmetries for the scattering of a spin Hall effect predicted. More recently, the band structure, even in the absence of scattering that occurs as a result has been pointed out that there may be an internal spin Hall effect. This idea has led to much theoretical debate, however, experimental evidence has been missing for the spin Hall effect.

5.2 Hamiltonians for Spin Quantum Hall Effect

To Start with describing free electrons in a magnetic field, one needs to replace the momentum by its gauge-invariant [38]

$$\Pi = \vec{p} + e\vec{A}(r), \tag{5.1}$$

Where $\vec{A}$ (r) is the vector potential that generates the magnetic field $\vec{B}$=$\nabla \times \vec{A}(r)$**.** That gauge-invariant momentum is proportional the electron velocity v, which must naturally be gauge-invariant because it is a physical quantity. Since $\vec{A}$ (r) is not gauge invariant, neither is the momentum $\vec{p}$. if we remind the adding the gradient of an arbitrary derivable function $\lambda(r)$, $\vec{A}(r) \rightarrow \vec{A}(r) + \vec{\nabla}\ \lambda(r)$, does not change the magnetic field because the rotational of a gradient is zero. By the way, the momentum transforms as$\vec{p} \rightarrow \vec{p} - e\vec{\nabla}\,\lambda(r)$ under a gauge transformation in order to

compensate the transformed vector potential, such that **Π** is gauge-invariant. The substitution (5.12) is called *minimal substitution.*

Electrons located to the lattice, this exchange is more deceptive because several bands can be occurred like this. Moreover, the vector potential is independent that we calculate the Hamiltonian, vector potential is added to equation in magnetic field. This becomes clear if one chooses a particular gauge, such as the Landau gauge $\boldsymbol{A}_L(\boldsymbol{r})$=$\vec{B}(-y,0,0)$, in this magnetic field, vector potential goes the largely as $\vec{B} \times \vec{L}_y$, where $\vec{L}_y$, is the macroscopic extension of the system in the *y*-direction. On the other hand, it may be shown that the substitution (5.1), which is called *Peierls substitution* in the context of electrons on a lattice, remains correct as long as the lattice spacing a is much smaller than the m*agnetic length*

$$l_B = \sqrt{\frac{\hbar}{eB}} \tag{5.2}$$

After induced magnetic field, we have a basic length parameter of the wave function. Because a is typically an atomic scale ($\sim 0.1\ to\ 10$ nm) and $l_B \cong 26\ nm/\sqrt{B[T]}$, magnetic field causes the lattice and all electrons fill the lattices with interaction.[22].

With equation exchange(5.1), we easily define the Hamiltonian for electrons in a magnetic field and now we show the this with differential equation.[39],

$$H(\overrightarrow{p}) \to H(\Pi) = H\left(\vec{p} + e\vec{A}\right) = H^B(\vec{p},\vec{r}). \tag{5.3}$$

We take a attention of the vector potential, the resulting Hamiltonian is independent in magnetic field anymore. Momentum $\vec{p}$ is a conserved quantity. We will limit the discussion to the *B*-field Hamiltonians corresponding to the Hamiltonians.

$$H_{\boldsymbol{S}}^{\boldsymbol{B}} = \frac{[\boldsymbol{p}+e\vec{A}(\vec{r})]^{\boldsymbol{2}}}{\boldsymbol{2}m_b} \tag{5.4}$$

for non-relativistic and

$$H_D^B = v\left[\vec{p} + e\vec{A}(\vec{r})\right] \cdot \boldsymbol{\sigma} \tag{5.5}$$

for relativistic 2D charged particles, respectively.

In equation, one particle can be analyzed with Hamiltonian (5.4) and (5.5) in a quantum mechanical fixing, the *canonical quantisation* that is called standard method which is used, where operators are shown physical quantities for inferred that they act on state vector in a Hilbert space. These operators do in general not commute with each other, i.e. the order matters in which they act on the state vectors that describe the physical system. Formally one introduces the *commutator* $[\mathcal{O}_1, \mathcal{O}_2] \equiv \mathcal{O}_1\mathcal{O}_2 - \mathcal{O}_2\mathcal{O}_1$ between the two operators $\mathcal{O}_1$ and $\mathcal{O}_2$, which are said to commute when $[\mathcal{O}_1, \mathcal{O}_2]$=0 or else not to commute. All processes are done in 2D, so our Hamiltonians help us to understand the basic physical quantities. Let show **r** = (*x*,*y*) and its canonical momenta **p** = (p_x, p_y), which satisfy the commutation relations

$$[x, p_x] = i\hbar, \quad [y, p_y] = i\hbar \quad \text{and} \quad [x, y] = [p_x, p_y] = [x, p_y] = [y, p_x] = 0, \qquad (5.6)$$

i.e. each component of the position operator does not commute with the momentum in the corresponding direction. Heisenberg inequality says that non-commutativity between the position and its momentum is the origin of the association of position., its momentum is following,

$$\Delta_x \Delta_{p_x} >_{\sim} h \text{ and } \Delta_y \Delta_{p_y} >_{\sim} h.$$

Commutation relations can be shown (5.6), the components of the gauge-invariant momentum no longer commute themselves,

$$[\Pi_x, \Pi_y] = [p_x + eA_x(\boldsymbol{r}), p_y + eA_y(\boldsymbol{r})] = e([p_x, A_y] - [p_y, A_x])$$

$$= e\left(\frac{\partial A_y}{\partial x}[p_x, x] + \frac{\partial A_y}{\partial y}[p_x, y] - \frac{\partial A_x}{\partial x}[p_x, x] - \frac{\partial A_x}{\partial y}[p_y, y]\right), \qquad (5.7)$$

where we have used the relation

$$[\mathcal{O}_1, f(\mathcal{O}_2)] = \frac{df}{d\mathcal{O}_2}[\mathcal{O}_1, \mathcal{O}_2] \qquad (5.8)$$

Between two arbitrary operators, the commutator of which is a c-number or an operator that commutes itself with both $\mathcal{O}_1$ and $\mathcal{O}_2$ [16]. With the help of the commutation relations (5.8), one finds that

$$[\Pi_x, \Pi_y] = -ie\hbar\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) = -ie\hbar\left(\vec{\nabla} \times \vec{A}\right)_z = -ie\hbar B, \qquad (5.9)$$

and, in terms of the magnetic length (5.8),

$$\left[\Pi_x, \Pi_y\right] = -i\frac{\hbar^2}{{l^2}_B} \tag{5.10}$$

After that, in order to compute (5.3) and (5.4), it is suitable to get the conjugate operators are pair Π_x, and Π_y to introduce *ladder operators* one-dimensional harmonic oscillator treats the quantum mechanical repairment. If we remind the old lecture notes about the quantum mechanics where the ladder operators can be done as the complex position of the one-dimensional in the phase space, which is prolonged the momentum and position

$$\tilde{a} = \frac{1}{\sqrt{2}}\left(\frac{x}{x_0} - i\frac{p}{p_0}\right) \text{ and } \tilde{a}^\dagger = \frac{1}{\sqrt{2}}\left(\frac{x}{x_0} + i\frac{p}{p_0}\right) \tag{5.11}$$

where $x_0 = \sqrt{\hbar/m_b\omega}$ and $p_0 = \sqrt{\hbar/m_b\omega}$ are normalization constants from the oscillator frequency ω. namely, conjugate variables refers to the position x and the momentum p. Normalization constants can be chosen significant numbers and commutation relation $[\tilde{a}, \tilde{a}^\dagger] = 1$ for the ladder operators[26].

The feature in 2D electron in a magnetic field, the ladder operators act like *complex* gauge-invariant momentum (or velocity), and they read

$$a = \frac{l_B}{\sqrt{2\hbar}}\left(\Pi_x - i\Pi_y\right) \text{ and } a^\dagger = \frac{l_B}{\sqrt{2\hbar}}\left(\Pi_x + i\Pi_y\right), \tag{5.12}$$

where the convenient normalization has been chosen by us such as to obtain the usual commutation relation

$$[a, a^\dagger] = 1. \tag{5.13}$$

The ladder operators assist us to solve for the equation next Hamiltonians to reverse the expression easily.(5.8),

$$\Pi_x = \frac{\hbar}{\sqrt{2}l_B}(a^\dagger + a) \quad \text{and} \quad \Pi_y = \frac{\hbar}{i\sqrt{2}l_B}(a^\dagger - a). \tag{5.14}$$

5.3 Hamiltonians for Spin-Orbit Coupling

Let me investigate in semiconductor structure, two-dimensional electron system. The spin-orbit interactions will be studied with different magnetic fields. A Hamiltonian is demonstrated and will consist of two terms. The following general model Hamiltonian with two types of spin-orbit coupling is considered [11]

$$H = \alpha\left(k_x\sigma_y - k_y\sigma_x\right) + \beta(k_x\sigma_x - k_y\sigma_y). \tag{5.15}$$

Where k= $(k_x k_y)$ calls wave vector with the Pauli matrices are given by

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},\ \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{5.16}$$

Matrix formation, and the Hamiltonian operator (1) can be written explicitly as

$$\text{H}=\begin{pmatrix} 0 & -\alpha\left(ik_x + k_y\right) + \beta(k_x + ik_y) \\ \alpha\left(ik_x - k_y\right) + \beta(k_x - k_y) & 0 \end{pmatrix}. \tag{5.17}$$

All eigenvalues can be calculated with this Hamiltonian H in (5.17) are given explicitly as

$$\lambda_{1,2} = \mp(\alpha^2\left(k_x^2 + k_y^2\right) + \beta^2\left(k_x^2 + k_y^2\right) - 4\left(\alpha\beta k_x k_y\right)^{\frac{1}{2}} =$$

$$(\alpha^2 + \beta^2 - 4\alpha\beta\frac{k_x k_y}{k_x^2 + k_y^2})^{\frac{1}{2}} \tag{5.18}$$

Where k=$(k_x^2 + k_y^2)^{\frac{1}{2}}$ and $\epsilon = \mp 1$. This denotes a band point in terms of k_x, k_y index. Describe the angle ϑ in the regarding to,

$$\text{Sin}(2\,\vartheta)=\frac{2k_xk_y}{k_x^2+k_y^2},\cos(2\vartheta)=\frac{k_x^2-k_y^2}{k_x^2+k_y^2} \tag{5.19}$$

From (5.19), tan ϑ is easily seen to be related to the componenst of k and simplifies to a very concise form,

$$\text{Tan}\,\vartheta=\frac{1-cos2\vartheta}{sin2\vartheta}=\frac{1-\frac{k_x^2-k_y^2}{k_x^2+k_y^2}}{\frac{2k_xk_y}{k_x^2+k_y^2}}=\frac{k_y}{k_x} \tag{5.20}$$

The required eigenenergies of H in (5.17) can be evaluated, and in terms of ϑ are,

$$E_\epsilon(k)=\epsilon k\gamma(\vartheta),\quad \gamma(\vartheta)=\sqrt{\alpha^2+\beta^2-2\alpha\beta sin2\vartheta} \tag{5.21}$$

Also, the eigenvectors of H in (5.17), which we write

$$\varphi_\epsilon(\text{k})=\underset{\varepsilon}{\left(\frac{\alpha k_y-\beta k_x+(\alpha k_x-\beta k_y)i}{k(\alpha^2\beta^2-2\alpha\beta\sin(2\vartheta)^{1/2}}\right)}=\underset{\varepsilon}{\left(\frac{1}{k\gamma(\vartheta)}(\alpha k_y-\beta k_x+(\alpha k_x-\beta k_y)i\right)} \tag{5.22}$$

Now, we look at the other Hamiltonian for spin quantum Hall effect and their eigenvalues that Hamiltonians include spin-orbit couplings in changing magnetic field in quantum optics. In this part, the most used mechanical model is known as Jaynes-Cummings (JC) that is also simple a formation [36].

$$H_c=v(\alpha^+\sigma^-+\sigma^+\alpha)+\Delta\sigma_z, \tag{5.23}$$

This Hamiltonian defines a single two-state atom, demonstrated by the Pauli matrices, that is interaction between the spin-orbit in electromagnetic field. α^+ α are the photon creation (annihilation) operators, and V is the coupling strength between the atom and electromagnetic field [37].

$$H_{JC}=V\left(a^+\begin{pmatrix}0&0\\1&0\end{pmatrix}+a\begin{pmatrix}0&1\\0&0\end{pmatrix}\right)+\Delta\begin{pmatrix}0&0\\0&-1\end{pmatrix} \tag{5.24}$$

Now we define a^{+}, α ;

$$\alpha^{+} = x, \alpha = \frac{d}{dx} \tag{5.25}$$

And we rewrite the equation (5.23)

$$H_{JC} = V\begin{pmatrix} 0 & \frac{d}{dx} \\ x & 0 \end{pmatrix} + \Delta\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} \Delta & \frac{Vd}{dx} \\ Vx & -\Delta \end{pmatrix}\begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} = E\begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} \tag{5.26}$$

$$\Delta\varphi_1 + V\frac{d}{dx}\varphi_2 = E\varphi_1$$

$$-\Delta\varphi_2 + Vx\varphi_1 = E\varphi_2$$

$$\varphi_1 = \frac{(E+\Delta)\varphi_2}{Vx}, \quad \Delta\,\frac{(E+\Delta)}{V}\frac{\varphi_2}{x} + V\frac{d\varphi_2}{dx} = E\frac{(E+V)}{V}\frac{\varphi_2}{x}$$

$$V\frac{d\varphi_2}{dx} + (E+\Delta)(\Delta - E)\frac{V}{x}\varphi_2 = 0$$

$$x\frac{d\varphi_2}{dx} + \frac{(E+\Delta)(\Delta - E)}{\propto}\varphi_2 = 0$$

$$\int \frac{d\varphi_2}{\varphi_2} = \int \frac{E^2 - \Delta^2}{V^2}\frac{1}{x}dx,$$

$$\frac{E^2 - \Delta^2}{V^2} = \gamma$$

and then,

$$\ln\varphi_2 = \gamma \ln x + \ln C$$

so,

$$\varphi_2 = Cx^{\gamma}$$

in here $\gamma =$ n integer

$$\frac{E^2 - \Delta^2}{V^2} = n \qquad n = 1,2,3 \ldots.$$

$$E^2 = \Delta^2 + V^2 n$$

$$E_{na} = a\sqrt{\Delta^2 + V^2 n} \tag{5.27}$$

Where n=1,2… non-negative integer , a=∓.

These interaction guide us that spin-orbit interaction and eigenvalues for these wave functions are found. By the way, these results may be represented many different ways. Because when gauge adjustment would be changed, all eigenvalues configuration will be varied. Energy between the electromagnetic field and the two-level system, known as Rabi oscillations. In terms of the Hamiltonian in Eq.5.34, the two-level atom in the pseudo spin ½ due to the two sub lattices in graphene, and the photon operators α^+ and α are translated to operators in graphene that are acting on Landau levels $\alpha^+ = {\pi^+}/{\sqrt{2eB}}$ and $\alpha = {\pi^-}/{\sqrt{2eB}}$

When we look at the another Hamiltonian that is [39]

$$H_0 = \frac{(\boldsymbol{p}+e\boldsymbol{A})^2}{2m} = \frac{\boldsymbol{\Pi}^2}{2m} \tag{5.28}$$

Here **A** refers to vector potential that guides to the magnetic field **B**=$\nabla \times \boldsymbol{A}$= - B$\hat{\boldsymbol{z}}$, **p** is the kind of momentum of the electron, -e is its charge, and m is its bare or band mass. Note that **B** points along the negative z axis.

The wave functions can be limited with light gauge shifted and we can infer the **A**. All spectrum are found by this way. Let us define a *cyclotron coordinate*

$$\boldsymbol{\eta} = \mathrm{l}^2 \hat{z} \times \Pi, \tag{5.29}$$

Where $\mathrm{l} = \sqrt{\hbar/_{eB}}$ is the magnetic length. Despite the name, the two components of $\boldsymbol{\eta}$ are not commuting but canonically conjugate variables:

$$[\eta_x, \eta_y] = i\mathrm{l}^2. \tag{5.30}$$

It follows that

$$H_0 = \frac{\boldsymbol{\eta}^2}{2m\mathrm{l}^4} \tag{5.31}$$

Describes a harmonic oscillator with energies

$$E = (n + \tfrac{1}{2})\omega_0, \tag{5.32}$$

Where n is the Landau-level index.

Electronic coordinates can be written with Hamiltonians and describing the composite fermions (CF), we focus on spin-polarized case for fractions of the form

$$\nu = \frac{p}{2ps+1} \tag{5.33}$$

The results contain to ν=$\frac{p}{2ps-1}$.

Let me look up band electrons with mass m and number density n, described by the following first quantized Hamiltonian:

$$H_{el} = \sum_i \frac{(p_i+eA)^2}{2m} + V \tag{5.34}$$

$$= \sum_i \frac{(\Pi_i)^2}{2m} + V \tag{5.35}$$

And then,

$$= \sum_i \frac{(\eta_i)^2}{2ml^4} + V \tag{5.36}$$

$$\eta = \frac{1}{2}\boldsymbol{r}+\mathrm{l}^2\hat{z} \times \boldsymbol{p} = l^2\hat{z} \times \boldsymbol{\Pi}$$

$$l^2 = \frac{1}{eB}$$

$$\nabla \times \boldsymbol{A} = -eB \tag{5.37}$$

where $\hbar = c = 1, \hat{z}$ refers to unit vector with the $\hat{z}$-axis, l is the magnetic length, B is the applied field, V is the inter-electron potential, and $\boldsymbol{\eta}$ is the *cyclotron coordinate,* whose components are canonically conjugate:

$$[\eta_x, \eta_y] = il^2.$$

Gradient of the *A* gives *–eB* and we propose the length of the magnetic. These are written in electronic coordinates. Therefore, if magnetic field is changed these results

would have different values. Fractional quantum numbers cause the generation of rational energy gaps. Landau levels provide the energy spectrums as followed.

$$E = \omega_0\left(n + {}^{1}/_{2}\right)\ \omega_0 = {}^{eB}/_{m} \tag{5.38}$$

CHAPTER 6

CONCLUSION

The goal of the Hamiltonian solutions are to start with the Hamiltonian for two-dimensional interacting electrons, final quasiparticles can be described in fractional quantum states and we can make some comments about this Hall Effect in a magnetic field and arrive at a comprehensive description. With Hamiltonians, we have shown the spin and its movement also energy eigenvalues and wave functions have been investigated by Rabi Hamiltonians. Different Hamiltonians have used in order to infer energy eigenvalues of spin quantum. While investigating these, we have used magnetic field and potentials so that Hall Effect occurred.

The work presented in this thesis also has been aimed to develop spintronics devices and some method to solve various kinds of physical Hamiltonians. After chapter 3, we have briefly mentioned and discussed some models and Rabi Hamiltonians. These models can be solved quasi-exact solvability.

A general Hamiltonian has been expressed by changing the parameters, so we can easily get a number of models can be obtained. After that, we also investigated quasi-exact solution of Rabi Hamiltonians. Because of some Hamiltonians has been used to compute the energy eigenvalues by Rabi Hamiltonians.

The most important results and achievements of this study are:

- Attentively studying Hall Effect, especially Spin Quantum Hall Effect for obtaining energy eigenvalues. Through energy eigenvalues and eigenfunctions is achieved and developed into special and different some Hamiltonians.
- Studying the new science of Spin Quantum and all its main components which help with the designing of many other different quantum hall effects.

- Obtaining the energy eigenvalues of Spin Quantum Hall Effect through familiarization with the Hamiltonians. Formulating all the above-mentioned processes in Spin Quantum Hall Effect by using Mathematica program, achieving remarkable results.
- All these will result in the development of Spin Quantum Hall Effect science and combining it with Mathematica program. Using this program for all the Hamiltonians of this science which have thus far been discovered.

Finally, we speculated that Quantum spin Hall effect with their energy eigenvalues by using different Hamiltonians.

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