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ANALYSIS AND OPTIMUM DESIGN OF CURVED ROOF STRUCTURES

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Galawezh Saber
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Galawezh SABER

ABSTRACT

ANALYSIS AND OPTIMUM DESIGN OF CURVED ROOF STRUCTUTURES

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M.Sc. in Civil Engineering
Supervisor: Asst. Prof. Dr. Nildem TAYŞI

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Curved roof structures are frequently designed to supply the users of the structure with ordinary light with a sense of capaciousness as well as grandness in public facilities such as stations, buying malls, leisure centers and airports.

This thesis' presents a method for analysis and optimum design of 2D and 3D curved roof structures subjected to static loading. Here the optimization refers to minimization of total weight of curved roof structures such that they can resist applied forces (stress constraint) and don't exceed certain deformations (displacement constraints). The finite element formulations are implemented for the static analysis of curved roof trusses to determine the stresses and displacements.

Optimization is an automated design procedure in which the computers are utilized to obtain the best results. A program was modified and used to automate analysis and optimization of the structure written in FORTRAN language based Finite Element analysis and Genetic Algorithm optimization technique. The developed method is tested on several examples and compared with previous researches or SAP2000 results. It is concluded that this method can serve as a useful tool in engineering design and optimization of curved roofs.

Keywords: curved roof structures, size optimization, finite element method, genetic algorithm.

ÖZET

EĞRİSEL ÇATI YAPILARI ANALİZİ VE OPTİMUM TASARIMI

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Eğrisel çatı yapıları, kullanıcılarına gün ışığından faydalanma imkanı sağladığı gibi istasyon, alışveriş merkezi, kültür merkezi ve havaalanları gibi halka açık alanlarda da sıklıkla kullanılmaktadır.

Bu tez statik yükler altındaki iki ve üç boyutlu eğrisel çatıların analizi ve optimizasyonu için yöntem sunmaktadır. Burada optimizasyon çatının toplam ağırlığının minimize edilmesine karşılık gelmektedir. Böylece yapı uygulanan yükleri güvenli bir şekilde taşırken gerilme ve yer değiştirme kısıtlarını aşmaz.

Otomatik olarak analiz ve optimizasyon yapabilmek için, sonlu elemanlar yöntemi ile genetik algoritmayı birleştiren FORTRAN dilinde yazılmış bir bilgisayar programı kullanılmaktadır. Geliştirilen yöntem pek çok örnekle test edilmiş ve sonuçlar önceki referans çalışmalarla veya SAP2000 program sonuçlarıyla karşılaştırılmıştır. Sonuç olarak, bu yöntemin mühendislik tasarımları ve eğrisel çatı yapılarının optimizasyonunda güvenli bir şekilde kullanılabileceği gözlemlenmiştir.

Anahtar kelimeler: eğrisel çatı yapıları, kesit optimizasyonu, sonlu şeritler metodu, genetik algoritma

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TABLE OF CONTENTS

CONTENTS **Page**

LIST OF FIGURES

Page

LIST OF TABLES

LIST OF SYMBOLS/ABBREVIATIONS

2D	Two Dimensional
3D	Three Dimensional
FE	Finite Element
GAs	Genetic Algorithms
MR	Mindlin-Reissner
F_H	Horizontal Component of Force
F_V	Vertical Component of Force
W	Uniform Distribution Load
M	Bending Moment

Scalar

A	area
D	global displacement
d_{all}	allowable displacement
$\hat{d}_{1x}$ $\hat{d}_{2x}$	nodal displacements of element
d_{max}	greatest permissible displacement
$\hat{f}_{1x}....\hat{f}_{2x}$	nodal force of element
m	Sinθ
l	Cosθ
E	Young's modulus
$F(\mathbf{x})$	objective function to be minimized
X	design variable
G	modulus of rigidity
h	Height of arch at mid span
$g_j(\mathbf{x})$	inequality constraint function
$h_k(\boldsymbol{x})$	E quality constraint function
x_i	i th design variable
x_i^{ℓ}	Lower bounds on a typical design variable
x_i^u	Upper bounds on a typical design variable
i, j and κ	quantity of design variables
n	number of bars

$\mathbf{L_i}$	length of the i-th bar
P_c	crossover probability
I	moment of inertia
K	effective length Factor
ℓ	Length
L	length of element
u_ℓ,, w_ℓ	Displacement components in ℓ, y and n-directions
u,w	Global displacement parameters
v_x, v_y, u and w	Global displacement parameters
W	Virtual Work
$\bar{u}_\ell$, $\bar{w}_\ell$ and $\bar{\theta}$	The corresponding displacement and rotation values at $\ell = \bar{\ell}$.
Q	shear force
N	axial force
D_m, D_b, D_s	membrane, bending and shear rigidities
q	traction force
q_x, q_y, q_z	distributed pressure loading along x, y, and z axes

Vector

$\mathbf{F}_{xi}$, $\mathbf{F}_{yi}$, $\mathbf{F}_{Zi}$	force components in the local x, y z direction for element of node i
Fi	fitness of the ith string in the population of size n
$\boldsymbol{\delta}_{xi}$, $\boldsymbol{\delta}_{yi}$	displacement components in the local x, y direction for element of
$\mathbf{d_j}$	displacement at the **j**th node

Matrix

$\mathbf{K}_{ij}$	Global stiffness matrix, with the ij component.
$\underline{\hat{k}}$	Element stiffness matrix
T	Matrix of direction cosine and **0**
T	Transformation matrix

Greek Symbols:

Θ	Angle between local and global axes
α	Angle between local and global axes, Newmark's Method
β	Arch opening angle, non-dimensional time parameter
$\varepsilon_m, \varepsilon_b, \varepsilon_s$	Membrane, bending and transverse shear strains
$\boldsymbol{\varepsilon}_m, \boldsymbol{\varepsilon}_b, \boldsymbol{\varepsilon}_s$	Membrane, bending and transverse shear strain vectors

$\sigma_{c\max}$, $\sigma_{t\max}$	maximum allowable compression and tension stresses
$\partial\ell$	Partial differential of ℓ
κ	Shear modification factor
σ_i	normal stress at the, i-th member
σ_{max}	greatest permissible stress
α1 with **α2**	quantities whose costs record the comparative significance of one
ν	Poisson's ratio
σ	Stress component
ε	Axial strains
σ_{all}	allowable stress
ρ	Mass density
Π	total potential energy

CHAPTER 1

INTRODUCTION

1.1General

Curved steel roofing is frequently designed to supply the users of the structure with ordinary light with a sense of capaciousness as well as grandness in public facilities such as stations, buying malls, leisure centers and airports. These structural techniques are advantageous in building costs to span over great distances.

Curved roofs of course have a number of important benefits, as well as they can be a superlative and long-lasting choice. That is where the attractiveness and price of the curved roof come into play. Balanced to the standard flat roof instatement, an arched roof can be far more durable, providing for superior charge for you.

In structural design, it is needed to obtain a suitable form in a structure so that it can carry the required loads safely and profitably. Traditional approaches to the job of discovery such shapes for structures have been using experimental models or by intuition with experience.

However, in many cases, the optimal size for structures is not obvious to experiments and experience. There is therefore, a required for best approaches, which submission a more public with reliable method for determining optimal size under static cases.

Bernoulli, Lagrange and Navier are only a great scientist who searched for the “best” forms for structural elements to content the given strength needs.

Optimization is an automated design procedure in which the computers are utilized to obtain the best results. The numerical methods of structural optimization with applications of computers automatically generate a near optimal design (converge to solve) in interactive manner. A program was modified and used to automate analysis and optimization of the structure written in FORTRAN language based Finite Element (FE) analysis and Genetic Algorithm (GA) optimization technique. After that, this direction increased and became an engineering area recognized as structural optimization, which searches for determine the most inexpensive geometrical forms satisfying the limitations (e.g. stresses and deflections) required for the design.

Nowadays computers are playing an important task in the analysis and design of such structures. In the past analysis and design were performed by manual calculations based on two dimension (2D) stress analysis which is time consuming and laborious. The high sophisticated software's have been developed to automate calculations of member forces based on three dimension (3D) FE analyses. Such software's find out critical member forces for a type of loading and a variety of possible curved roof structural combination, giving accurate results for analysis and design curved roof structures, are progressively used in the new built condition of the world in the final decades since they submission aerodynamically effective shapes as well as supply architects and designers with and substitute to regular rectangular structure types.

There are many systems developed by many researchers used for automated design of structures. However, most of these systems have same problems due to their mathematical origin (most of them are linear programing techniques). They tend to deal with structural optimization as a problem in which the search space is endless, when it's truly discontinuous. Only a small number of structural forms are available in the trade.

Although some structural optimization methods can deal with discontinuous search spaces, they support a native lack of generality and hence, can't be readily extended to different types of structures. The GA, for its part, is a problem independent. The code matured for this work can be reused to solve the unfinished framed structures (plane and space frames, plane grids and beams) among little mutation. Eventually, the GA has appearance similar to existing techniques, from time to time even surpassing them. In this thesis GA optimization techniques are used for the design of curved roof structures because GA gives yielding more realistic result when it is compared with linear programing methods.

1.2 Objectives

The ultimate motivation of this thesis is analysis and structural size optimization of

2D and 3D curved roofs structures applying optimization program combining FE analysis and GA optimization method. The aim of the structural size optimization is generally to obtain the minimum weight for the curved roofs; so that it can carry the imposed loads safely and economically.

The specific objectives of this thesis is the linear elastic analysis of 2D and 3D curved roof structures which consists of both curved trusses and solid arches and also optimum design of the same mentioned structures when they subjects to static external loads. Here the optimization refers to minimization of total weight of curved roof structures such that they can resist applied forces (stress constraint) and don’t exceed certain deformations (displacement constraints).

1.3 Curved Roof

Structural design is a part of engineering that pacts with systems consisted of a set of structural members. These members may be described as either truss or framework elements, combined by pined or fixed joints. Truss and arch type curved roof structures will be examined in this thesis.

Trusses: Engineering structures that are composed only of two-force members are called a truss structure. Trusses are triangular or pyramidal shapes that are used in the structure of buildings in order to make them more stable than structural components with 90 degree angles could.

Arches: An arch is occasionally explained as a curved structural member spanning an opening with helping like a support for the loads above mentioned the opening. Naturally arches have many important applications in mechanical and civil engineering.

1.4 Analysis

The FE is a powerful numerical method for solving all problem types in engineering and mathematical physics. In this study FE method is used for analyzing of 2D and 3D curved truss and solid arch roofing. The FE analysis of curved beam has been given significant attention by researchers in recent years mostly because it is a versatile method for solving structural and other mechanical problems [1]. All numeric analyses are carried out in this research have existed behaved using the FE software program.

1.5 Genetic Algorithm Optimization

In this thesis GA is used for optimization of truss type curved roofs and solid arch roofing. The advantages of applying GA to optimized design of structures include discrete design variables, open format for constraint statements and multiple load cases. A GA does not require an explicit relationship between the objective function and the constraints. Instead, the objective function for a set of design variables is adjusted to reflect any violation of the constraints [2].

1.6 Layout of the Thesis

In present work main attention is focused on structure optimization of curved roof under static loading condition. To do so FORTRAN based analysis and optimization tools is employed. The main goal of the study is minimize the total weight of curved roof without causing a strength base failure. The organization of the study and the layout of the thesis are now pronounced:

Chapter 2: Is the literature review in analysis as well as design optimization.

Chapter 3: Deals with various characteristics of 2D and 3D curved roof trusses and solid arch roofings.

Chapter 4: Illustrates the fundamental formulation for 2D and 3D static trusses and arches analysis methods. The matrix displacement approaches adopted are depicted. The primary assumption as well as matrix analysis is also presented.

Chapter 5: Deals with various static analysis examples about curved roof structures.

Chapter 6: Deals with various characteristics of the optimization process, involving the description with choice of the design variables as well as the GA method.

Chapter 7: This chapter deals with numerical applications of GA during various examples about curved roof structure are premeditated and demonstrated.

Chapter 8: Finally in this section some short conclusions are demonstrated with each other with some proposals for future work.

CHAPTER 2

LITERATURE SURVEY

2.1 Introduction

This literature is studied to have various ideas of the scientific researches about curved roof structures. It is a principal section of text that requires to review the important points of current knowledge, containing substantive findings as well as theoretical with methodological contribution to analysis and size optimization of the curved roof.

Curved beam buildings have been applied in much aerospace, mechanical and civil engineering implementations such like tire dynamic, wire, turbo machinery blade, curved girder bridges, with stiffeners in aircraft buildings. The thin walled cross sections, such as channel, angel and I section, are attractive since they submission a high performance in relations of minimum weight for presumption strength [3].

Arch's structures supply inexpensive results for crossing great spans with bear higher loads for a presumption volume of material when correctly shaped, balanced with beams shorter cross parts can be used in arches, like the membrane forces are dominant [4].

Overall curve can be executed in two various paths depending on the jointing method working. In structures by node connections, the curvature is manufactured in the shape of cutting kinks at the member-node connections. Though, on the condition that the structure contains unbroken members, the members themselves will be curved. The action in the latest case is more difficult, similarly the members are bent with endless rather than horizontal with pin-ended [5].

2.2 Static Analysis of Curved Roofs

FE methods are one of several approximate numerical techniques available for the solution of engineering boundary value problems. Problems in the mechanics of materials often lead to equations of this type and FE methods have a number of advantages in handling them. The method is particularly well suited to problems with irregular geometries and boundary conditions and it can be implemented in general computer codes that can be used for many different problems [6].

FE methods for Timoshenko beam, circular arch and Mindlin Reissner plate problems were discussed by Cheng et al. [7].

Engineers, executed mathematicians, with another researcher will certainly carry on building new implementations. For an extensive bibliography on the FE approach, refer to the work of, Clough [8] and Noor [9].

2.2.1 Trusses

Skeletal structures can be analyzed by a variety of hand-oriented approaches of structural analysis taught in start mechanics of Material's courses: the displacement with force approaches. They can also be analyzed by the computer-oriented FE method. That versatility causes those constructions a good selection to show the transformation from the hand-calculation approaches taught in undergraduate courses, to the completely automated FE analysis processes available in commercial programs.

The static analysis of trusses can be carried out surely, as well as the equations of even complex trusses can be collected in a matrix shape amenable to numerical solution. This access, from time to time named "matrix analysis," supplied the base of early FE development.

Detailed management of the matrix formulation for the analysis of linearly elastic trusses can be found in many textbooks such as Reddy [10], Bathe [11] and Kassimali [12].

FE codes are smaller hard than many of the remarks treating, in addition to application program packages demonstrate on modern microcomputers. Even so, they are difficult enough that mostly users do not find it effective in a schedule their own code. A sum of rewritten commercial codes is valid, indicating a broad price variety with compatible through machines from, microcomputers to supercomputers [13].

Standard FE matrix displacement methods are used in this thesis for 2D and 3D static analysis, such as those mentioned in Chandrupatla and Belegundu [14].

2.2.2 Arches

Many engineering constructions can be modeled like beam elements, because beams are the easiest as well as the majority usually used structural elements. Static analysis of naturally curved arch has got many significant implementations in mechanical as well as civil engineering. The problem is directly required to be thoroughly studied in applied science structures, especially in roof structures connected with curved beams.

The problem is immediately required to be deeply studied in engineering constructions, especially in roof structures associated with curved beams. The analysis of a curved beam is important also contains obtained considerable attention since the end of the nineteenth century.

Various scientists have tried to solve the arch problem by different methods. It seems that FE has been the major tool in this research. If the behavior of the curved beam is non-planar, then a usual beam FE model becomes very complicated with many degrees of freedom. In these cases, it is evident that a model, which, with a relatively few degrees of freedom, offers a good description of the behavior of the curved beam, is needed.

These analytical solutions widely need the solution of regular or partial differential equations that, sense of the difficult material properties, geometries, loadings and are not generally available. The FE formulation of the problem results in a technique of simultaneous algebraic equations for a result, rather than needing the solution of differential equations. Briefly, the solution to constructional problems normally relates to determining the displacements at each intersection also the stresses inside each member construction up to the building that is subjected to apply loads [15].

Closed form solutions for classical arch geometries under various boundary conditions have been gotten to institute rough guidelines for analysis of complicated arch structures various structural analysis processes established on Rayleigh-Ritz; FE as well as dissimilar quadrature element approaches has also been used by Tayşi et. al. [16].

There exists greatly literature on structural analysis of curved beam elements, Love [17], Parcel and Moorman [18], Washizu [19], Papangelis and Trahair [20], Rajasekaran and Padmanabhan [21], Murin and Kutis [22].

A number of curved beam elements have been advanced. In mainly of the earliest elements, the result of shear deformation was not pondered. The elements were advanced by applying finite-strain beam assumption of Reissner and geometrically exact beam theory.

Through the progression of computer technology with some programs, the FE has been used greatly to solve for further public geometry, also a number of curved elements have been advanced. If the action of the arch is non-planar, usual FE or finite dissimilarity model changes into very difficult [23].

2.3 Optimization Algorithms

The creation of optimization systems can be found over the years of Newton, Lagrange and Cauchy. The advancement of differential calculus systems of optimization was practicable since of the donations of Newton with Leibnitz to calculus. The basis of calculus of varieties, which deals through the minimization of functional, was established by Bernoulli, Euler, Lagrange and Weirstrass [24].

Spectacular forward movements followed, manufacturing a big literature on optimization methods. This progression also followed upon the appearance of some well-defined modern areas in optimization assumption. There is no unique approach valid for solving all optimization problems effectively. So a sum of optimization approaches has existed advanced for solving various kinds of optimization problems.

In general, optimization methods used in constructional engineering design can be classified into four different approaches: (1) mathematical programming; (2) optimality criteria methods; (3) heuristic search methods (GA); and (4) evolution strategies. These approaches as well as the literature reviews are demonstrated in the following. Between some recommendations on this subject, some of them are informed in this thesis.

The optimum searching for approaches are also recognized as arithmetical programming methods with is widely studied like a section of operation's study. Operation's investigation is a part of mathematics concerned through the implementation of scientific approaches as well as techniques to decision production problems in addition to with demonstrating the best otherwise optimal solutions.

Stochastic operation systems can be used to analyze problems specified by a set of random variables having recognized probability distributions. Statistical approaches empower one to analyze the experimental data plus construct empirical models to get the majority exact performance of the physical situation, [24].

Many approaches with algorithms have existed advanced for optimum design of structural methods in the final three decades. For the most part, of the approaches deal through continuous design variables as well as application arithmetical programming systems. In the majority of practical design problems, the design variables are discontinuous. This is due to the obtain ability of standard sizes plus their restrictions for construction in addition to construction reasons. A sum of approaches was announced for optimum deign of discontinuous structural systems, Templeman and Yates [25] and Zhu [26].

2.3.1 Genetic algorithm

GAs is one of the type search optimization that used for minimizing the objective function (the structure weight) and it is different from many algorithms of mathematical programming, is that they do not need the calculation of derivatives of the objective function and the constraints. It is search steps depends on the mechanics of natural genetics and selection. To represent a robust search mechanism it is working on the scheme of the artificial survival of the fittest with genetic operators founds from the nature.

The initial work employing the GA was complete by developing state machines in the 1960s. The GA is the oldest also the majority common shape of evolutionary computation. It obtains its action from ordinary evolution plus genetics, following Darwin's great rules of development. This approach depends on random behavior, trial with error, as well as survival of the fittest to develop solutions to optimization problems. It obtains its strength from the actuality that a large category of problems can be driven to extremely excellent solutions at recombining sections of earlier good solutions. While engineers with designer's examination for modern optimization systems, they determine that the GA can present results not at any time before possible, [27].

The principal characteristics of a GA are established on the principles of endurance of the fittest with adaptation. Since its establishment liked an intuitive idea, [28].

Many inventors have explored the applicability of GA as well as advanced many applicable supplements such as elitist GA. Gero, et al [29], improved augmented Lagrangian GA, Adeli and Cheng [30], hybrid algorithms of GA with fuzzy system. Tan, et al [31] and with applications of GA and neural network, Grierson and Hajela [32].

Best design applying GA has been manufactured for various structural elements such like beams and grillages. Baronet et al [33] dealt with planar and space trusses, Erbatur et al [34], Sarma and Adeli [35], though, extremely few implementations develop to have been demonstrated for shape optimization of arch structures, Vanderplaats and Han [36].

One characteristic of the GA that categorizes it from another optimization algorithm such like gradient-based mathematical programming otherwise simulated annealing is that the GA repeats on population of designs rather than on an individual design. This graces a present to the GA through the potential for converging to a multiplicity of good designs in the last generation [37].

2.3.2 Optimization of curved roof structures

In Saka and Kameshki [38], an algorithm is demonstrated for the best design of 3D strictly jointed frames, which obtains into account the nonlinear response due to the import of axial forces in members.

The problem of maximization of the critical load or else boundary point of changeableness of skin deep space truss of constant volume was presented in [39].

Pyrz [40] was discussed a discontinuous optimization of trusses considering stability restrictions and demonstrated examples of skin deep truss constructions when snap-through can happen.

Suleman and Sedaghati [41] were presented a structural optimization algorithm which is advanced for truss as well as beam structures bearing large deflections versus instability.

Saka [42] was presented an algorithm which takes into account the nonlinear response to the dome structure due to effect of axial forces on the flexural stiffness of members, in addition to the best solution to the design problem is gotten applying a coupled GA.

A method for the optimization of stability-constrained geometrically nonlinear shallow trusses through snap-trough habits is exhibited applying the arc length approach with a strain energy density approach with a discontinuous formulation, [43].

Degertekin et al [44] demonstrated an algorithm for the perfect design of geometrically nonlinear steel space frames applying tabu search plus GA.

CHAPTER 3

CURVED ROOF STRUCTURES

3.1 Introduction

There are many types of roofs and for many ages, the majority of roofs were flat. However, in the previous few years advancements in roofing engineering have led more and more householders, business owners as well as developers to think over the induction of a curved roof.

Curved steel structures are often designed to provide the users of the structure with natural light and a sense of capaciousness and grandeur in public facilities such as air ports, stations, shopping malls and leisure centers. This has led to forms of structures in which relatively light curved steel trusses or arched frames support substantial areas of glazing. Even with clad structures, exposing the arching steelwork to view can enhance the sense of internal space.

A lot of researchers have existed planned about curved structures, which examination their static result, aesthetic view, transporting the load, which materials may be used, etc.

The addition efficiency and user-friendliness of new computer engineering coupled with validity and versatility of lately advanced algorithms in calculative mechanics have begun up the probabilities of discovery efficient shapes of many structural kinds in a productive and reliable manner. In the field of optimum design of structures, the principal emphasis and the majority important advance has existed with the volume optimization of stress with arch problems.

Overall curve can be executed in two various paths depending on the jointing method working. In structures by node connections, the curvature is manufactured in the shape of cutting kinks at the member-node connections. Though, on the condition that the structure contains unbroken members, the members themselves will be curved. The action in the latest case is more difficult, similarly the members are bent with endless rather than horizontal with pin-ended, [5].

3.2 Advantages of Curved Roofs

There are several advantages of curved roofs and some of them are given below:

Curved steel roofing are frequently designed to supply the users of the structure with ordinary light with a sense of capaciousness as well as grandness in public facilities such as stations, buying malls, leisure centers and airports. These structural techniques are advantageous in building costs to span over great distances.

Curved roofs of course have a number of important benefits, as well as they can be a superlative and long-lasting choice. That is where the attractiveness and price of the curved roof come into play. Balanced to the standard flat roof instatement, an arched roof can be far more durable, providing for superior charge for you. Aesthetics, long span they submission aerodynamically effective shapes as well as supply architects and designers with and substitute to regular rectangular structure types.

Wide-span space buildings have existed more and more popular in coating great unclosed areas with several intermediary supports. Their famous applications subsist in all complete the world municipal halls, covering stadiums, show centers with many another structures. These buildings are usually curved in one or two directions.

When curved roof installation compared with the flat roof installation, curved roof can be extremely stronger, given for excellent protection for you.

In addition, a well arranged curved roof induction can supply a greatly higher degree of protection versus weather damage, also more and more significant feature to appear for any roof.

Due to the dissimilarities in design, a curved roof is frequently best able to resist high winds as well as drive rain than a comparable flat roof.

Even for industrial and distribution establishments, curved roofs can supply and productive results. Curved roofs avoid the aura of austerity that is often associated with “industrial warehouse” kind buildings with may supply a result that is appealing to local devisers. Adverse to some hopes, curved steel constructions require not be any further costly than another framed structures.

3.3 Disadvantage of Curved Roof

Curved roofs present a singular challenge. The roof inclines changes from dead flat at the highest point. Multiple adjacent, curved roofs produce great valleys where snow, water and ice can filter causing long-term roof, wall with structural problems. These troubles can be costly and disruptive to correct [45].

The extra payment of curving steelwork is generally small in connection to the all-inclusive cost of the structure and can often be offsetted at stores in ridge line detail as well as flashing costs and for spans nether nearly 25 m, by removing the require for a peak splice. Roof facing on curved roof beams sometimes does not require to be pre-curved, since many panels can go after the curvature of the roof during fixing without any specific manipulation.

A curved exterior appearance can be manufactured by employing faceted horizontal members, in addition to by changing depth junctions to the secondary members. However, the extra fabrication payments for the faceted solution usually denote that a curved solution is further cost-effective in addition to the decorative considerations, [46].

3.4 History of Steel Curved Roof

Although curved iron and steel structures have been in creation since the mid-19th Century, to time there has been short guidance covering the design of curved steel elements.

Before steel came into general use during the latter part of the 19th Century, curved structures were frequently built from metal, which was thrown in the liquid shape in an arched profile or constructed up from wrought-iron parts, either with formed web plates otherwise in the shape of grid trusses. Since wrought iron was extremely flexible, blacksmiths may curve little parts by hot forging, [46].

Throughout the time starting 1930 to 1950, short curved steel parts were as well as used in proportionately ordinary building structures. Nissen huts, Dutch barns and aircraft hangers generally contained a supporting structure of curved steel corners, small rolled **I** section or tees.

Apart from the express applies referred to previous, very several building structures applying arched steel were built up to the time of the late 1970s. However, through the final two groups of ten of the 20th Century, the request for curved steel members in building structures added to significantly.

3.5 Types of Curved Roof

There are two types of curved roof structures according to their shapes, one is concave and another is convex, as shown in the Figure 3.1.

Figure 3.1 Curved beams (concave &convex) shape

There are several possibilities of roof geometries frequently depending also on architectural considerations. In this thesis 2D and 3D trusses and solid arch roofing considered.

3.5.1 Two and three dimensional curved truss structures

Trusses determine great application in new erection, for instance, as towers, roofs, bridges, etc. In computation to their practical significance like useful structures, truss elements have a dimensional ease that will assist us elongate further the ideas of mechanics announced in the modules dealing with the uniaxial response, [47].

Trusses, whether 2D or 3D belong to the department of skeletal structures. These structures include of extended structural elements named members, joined at joints. One of the simplest ways to distinguish various truss kinds is to determine whether they exist in a singular plane (2D) or multiple planes (3D).

Plane trusses (2D), are frequently used in building, especially for roofing of residential with trading buildings, as well as in short-span bridges. A plane truss is idealized like a method of members lying on a plane and connected at hinged joints. As shown in Figure 3.2. All directed forces are adopted to act in the plane of the structure, in addition to all exterior pairs have their moment transmitters normal to the plane. The loads may include of concentrated forces directed to the joints, as well as loads that act as the members themselves. For aims of analysis, the latest loads may be put back by statically equivalent loads acting on the connections, [48].

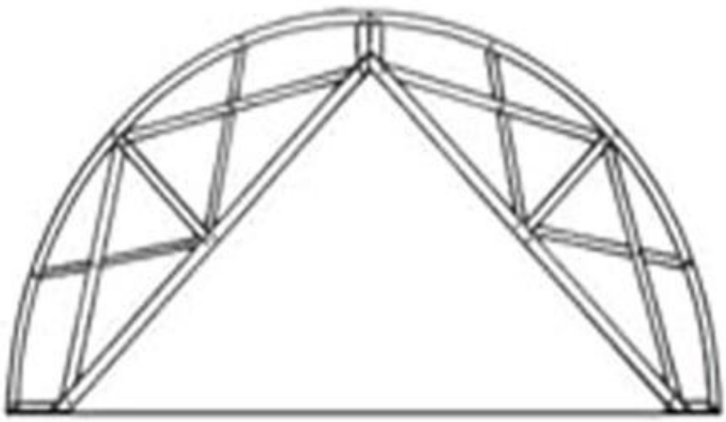


Figure 3.2 Hemispherical shape with 2D truss beams

Space truss (3D) a space truss is like to a plane truss other than that the members may have any instructions in space. The forces acting on a space truss may be in uncontrolled directions; however, any pair acting as a member must have its moment transmitter vertical through the axis of the member. The cause for this condition is that a truss member is unable to supporting a turning moment [49].A 3D dome over exhibition hall is shown in Figure 3.3.

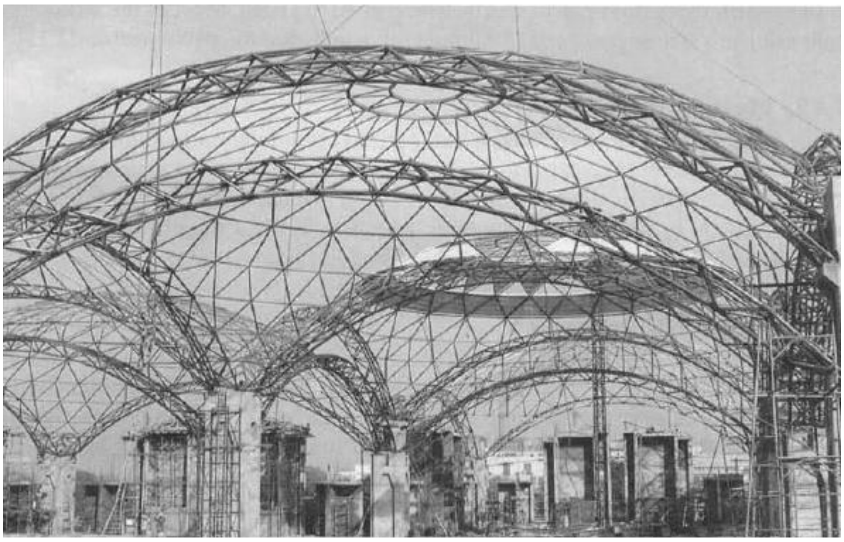


Figure 3.3 3D dome over exhibition hall

A 3D truss component contains pair local degrees of freedom with six global degrees of freedom and three translational degrees of freedom at each end of the member. Figure 3.4 indicates a 3D truss component with its local and global coordinate coordination's, grades of freedom, plus permissible forces.

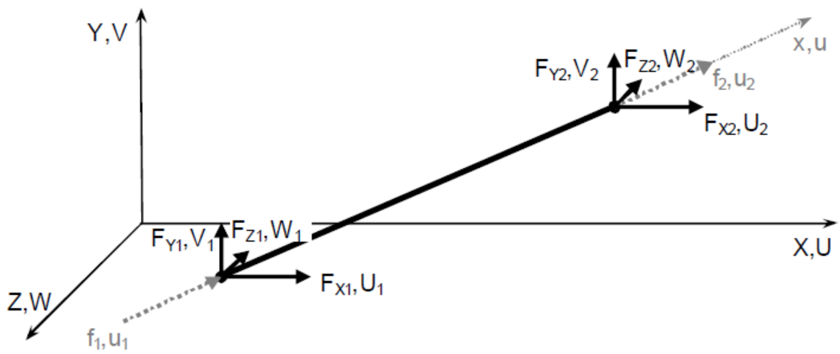


Figure 3.4. Truss element in local and global coordinate systems

3.5.2 Solid arch roofing

The arch is an unbelievable architectural invention, dating backward to old times. However, yet in large use today, as, up to the time of the 19th century; it was the just known approach for roofing a structure without the application of beams.

Arch's structures supply inexpensive results for crossing great spans with bear higher loads for a presumption volume of material when correctly shaped, balanced with beams shorter cross parts can be used in arches, like the membrane forces are dominant, [4].

An arch is occasionally explained as a curved structural member spanning an opening with helping like a support for the loads abovementioned the opening. This definition leaves out an explanation of what kind of constructional component, a moment as well as the axial force component, constructs up the arch. Typical metal arch bridge on Guilford Avenue in Baltimore is shown in Figure 3.5.

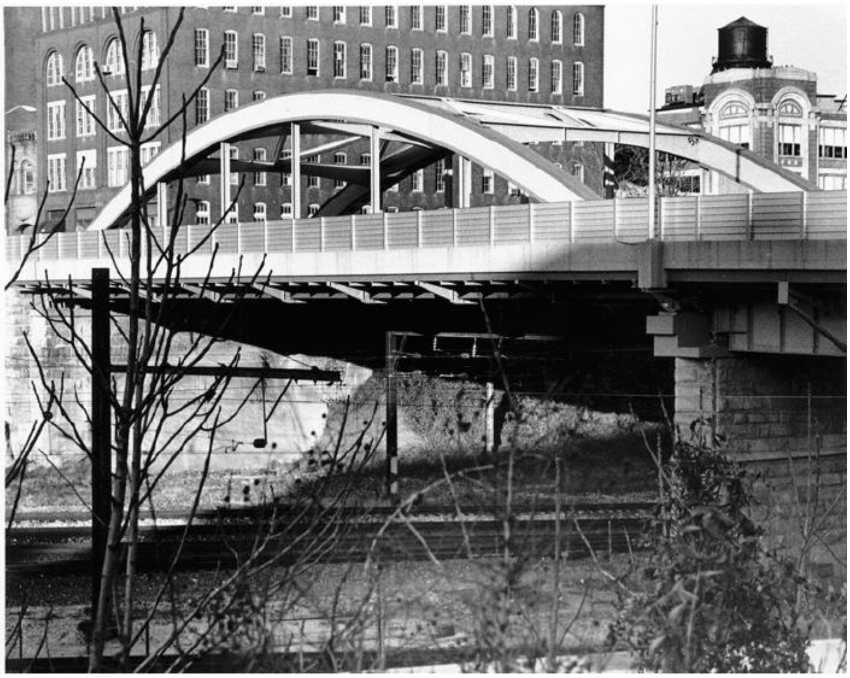


Figure 3.5 Typical metal arch bridge on Guilford Avenue in Baltimore

The actual or perfect arch, theoretically, is one in which just a compressive force act by the centroid of each member of the arch.

Many engineering constructions can be modeled like beam elements, because beams are the easiest as well as the majority usually used structural elements. For that reason, static analysis of the naturally arched beam contains many significant implementations in mechanical plus civil engineering.

The problem is immediately required to be deeply studied in engineering constructions, especially in roof structures associated with curved beams. The analysis of a curved beam is important also contains obtained considerable attention since the end of the nineteenth century.

Curved beam buildings have been apply in much aerospace, mechanical, with civil engineering implementations such like tire dynamic, wire, turbo machinery blade, curved girder bridges, with stiffeners in aircraft buildings. The thin walled cross-sections, such as channel, angel and I section, are attractive since they submission a high performance in relations of minimum weight for presumption strength [3].

Curved beams are further powerful in transfer of loads than horizontal beams since the move is infected by bending, shear also membrane activity. Some of the constructions such like arch bridges with arches are sculptural applying curved beam elements.

3.5.2.1 Additional stiffness requirements

Members curved in elevation are generally needed to function, at least in the region, like arches. Consequentially the deflecting moment in the member is smaller than that which would appear if the member enacted like a straight beam. It is very significant that the abutment rigidity indicated in Figure 3.6 is proportionately high. If this is not completed, the abutments will widen (forced separately by the horizontal response of the arch) also the head against the arch will drop importantly, as indicated in Figure 3.7.

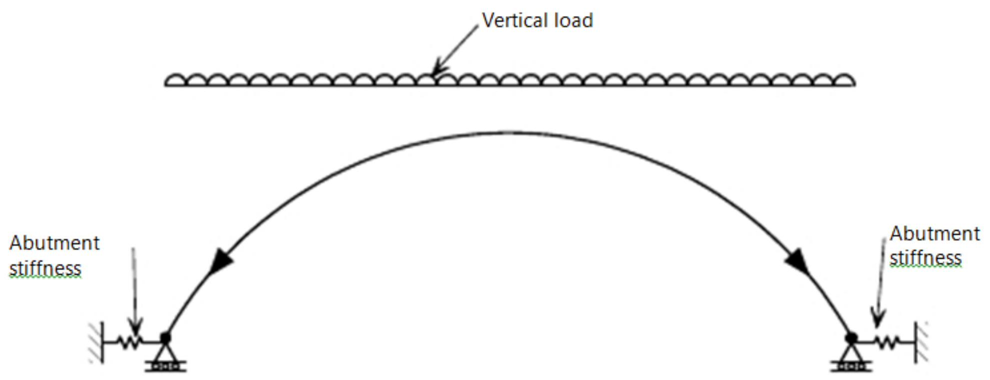


Figure 3.6.Curved member acting like an arch

The result of this widen is that the axial compression, with the turning over moment on the arch must addition to withstand the load, which might over-stress the arch member. In the inordinate, the expanding deflection of the arch head will reason expanding axial compression, which will cause growing to extend of the abutments, which might reason the arch to failure entirely.

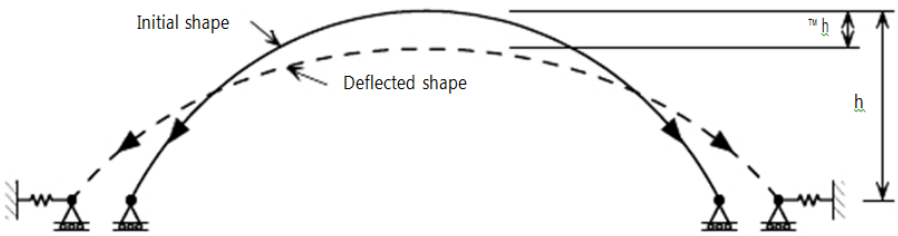


Figure 3.7 Reduction of arch height under load

Under axial compression, the member will shorten, which also reduces the height of the arch, again leading towards collapse unless the effect is accounted for in design. Both effects are more critical in arches with low rises compared with their span.

In the extreme, a flat arch with very low support stiffness behaves as a simply supported beam. This reduction in the height of the arch can be considered in two different ways.

The first and simplest approach is to consider how the reduction in height must be limited to ensure that normal first-order (small deflection) frame analysis is appropriate.

The second more complicated, approach is to find the actual deflection and axial compression resulting from the increasing spread of the abutments as the arch height reduces. This requires a second-order (large deflection) analysis. This publication considers only the first approach that of defining a limit such that first-order analysis is appropriate.

The simple check of arch deflection also assumes that a reasonable estimate of support stiffness has been used in the analysis, which is matched in the real structure. The importance of support stiffness increases as the rise: span ratio decreases.

Where the apex deflection is greater than 2.5% of the height of the arch, the increase in the arch force and the coincident bending moment should be considered.

3.6 Load Cases

The critical load case for an arch is not always easy to identify. It is important that the different cases are considered carefully, taking into account both the loading and the horizontal stiffness of the abutments. In many cases, the load case with maximum bending moment is the critical load case. To allow rapid initial sizing, the calculation of approximate maximum axial forces and moments is demonstrated below.

3.6.1 Maximum axial compression

The maximum axial compression in an arch normally occurs when there is a uniformly distributed load across the full span, as shown in Figure 3.8. If the arch is parabolic in elevation, with abutments "fixed" in position, the horizontal component, F_h, of the compressive force in the arch is approximately.

$$F_h = \frac{\text{Midspan bending moment of a simply supported beam}}{\text{Height of arch}} = \frac{wL^2}{8h} \quad (3.1)$$

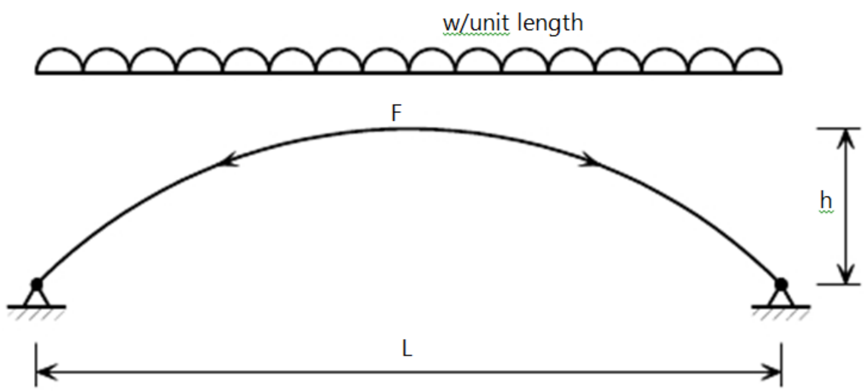


Figure 3.8 Parabolic arch under full span uniformly distributed load

The maximum compressive force occurs at the springing of the arch and is the vector sum of the horizontal component, F_h, of the compressive force and the vertical reaction, F_v:

$$F_{max} = \sqrt{F_h^2 + F_v^2} \quad (3.2)$$

Where F_v= wL/2 for the simplified loading shown in Figure 3.8.

3.6.2 Maximum bending for gravity loads

The maximum bending moment from gravity loads normally occurs from partial loading, as shown in Figure 3.9, in which the arch is loaded over only half of the span.

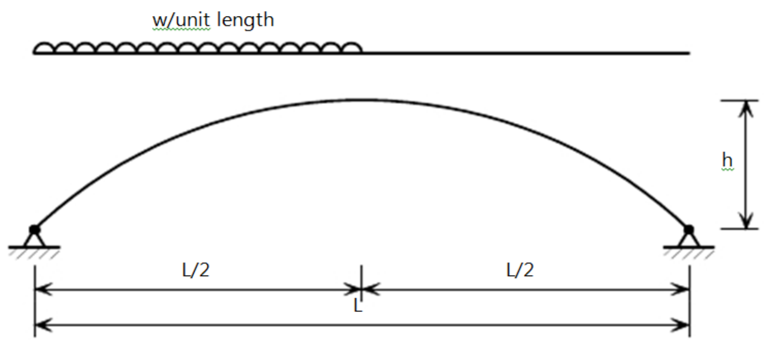


Figure 3.9 Parabolic arches with half span loading

The half span loading of *w*/unit length can be expressed as the sum of full span loading of (*w*/2)/unit length plus asymmetric loading of downward and upward loads of (*w*/2)/unit length. By considering the applied loads in this way, an approximate axial load and bending moment may be calculated for initial sizing. The compressive force in the arch is caused by the full-span component of the loading. The bending moment on a simply supported beam is shown in Figure 3.10.

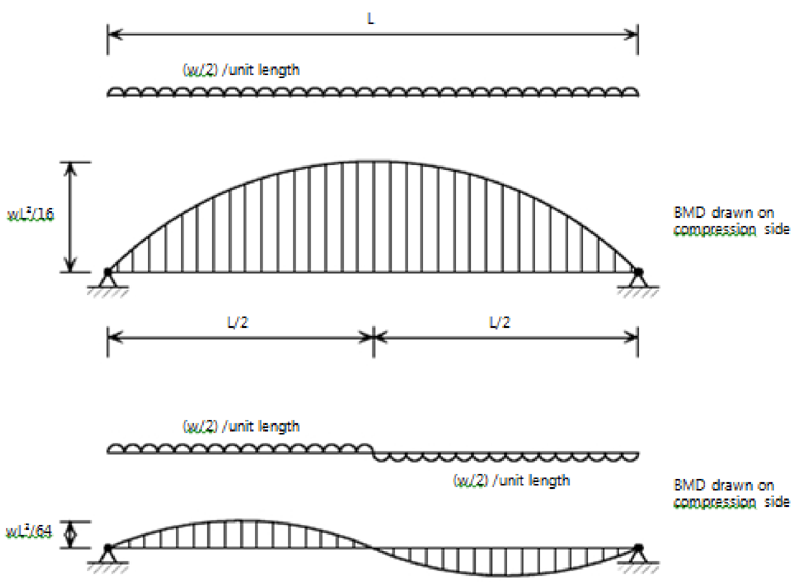


Figure 3.10 Bending moment diagram

Therefore, the horizontal component, F_h, of the compressive force in the arch is approximately:

$$F_h = \frac{\text{Midspan bending moment of a simply supported beam}}{\text{Height of arch}} \qquad (3.3)$$

The bending moment in the arch is approximately equal to the bending moment from the asymmetric component of the loads, as shown in Figure 3.10. This is at its maximum at the quarter points.

Therefore bending moment, $M= wL^2/64$.

3.6.3 Maximum bending from wind loads

Wind loads on arches are asymmetrical when the wind blows along the span of the arch. A possible wind pressure diagram is shown in Figure 3.11.

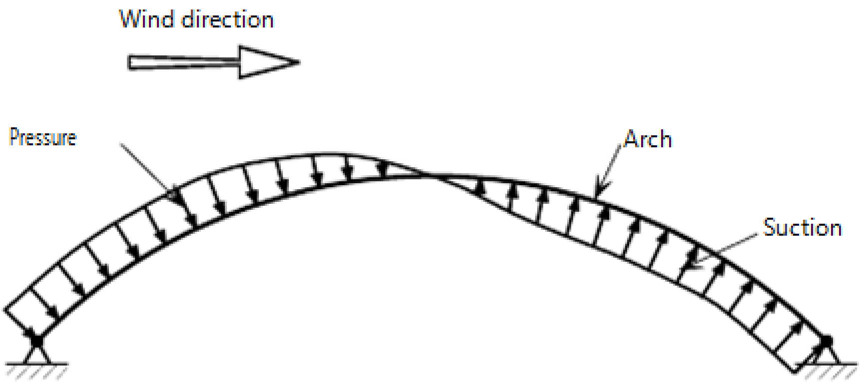


Figure 3.11 Possible wind pressure diagram

The point of change from pressure to suction is likely to be uncertain, so more than one case, within the envelope of cases shown in Figure 3.12, may need to be considered in sensitive designs.

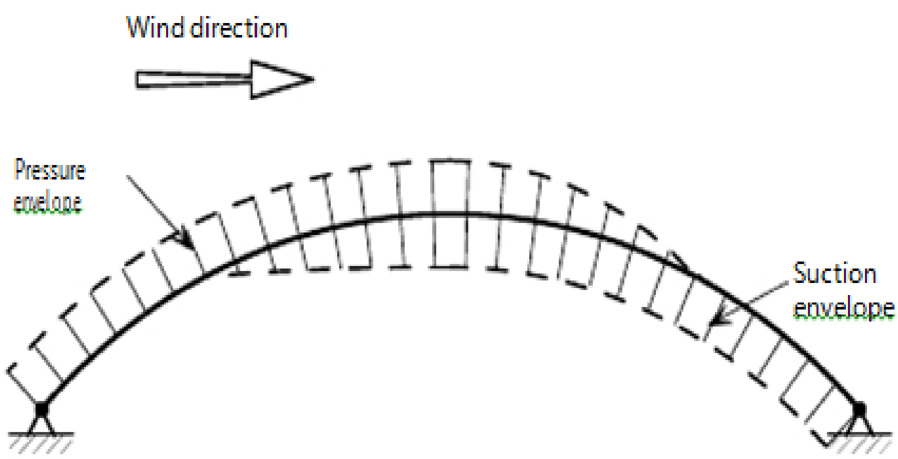


Figure 3.12 Possible envelope of wind pressure

As a guide to estimating the magnitude of the bending moments induced, a load case with equal pressure and suction is shown in Figure 3.13.

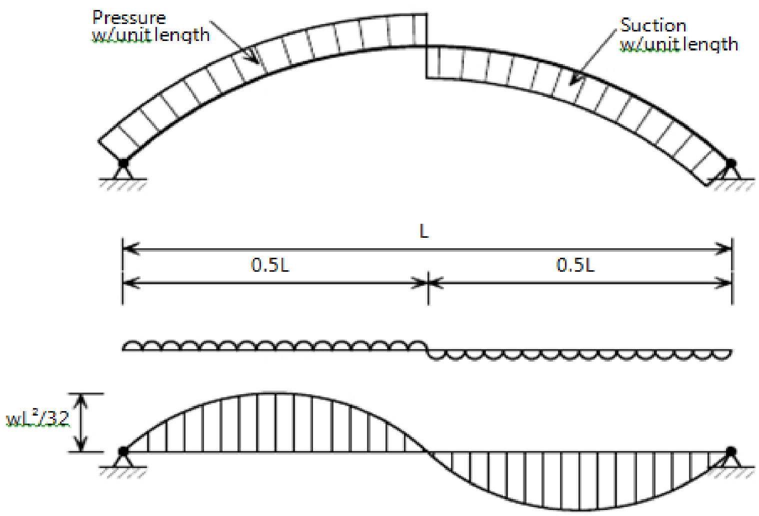


Figure 3.13 Approximate bending moment from wind loads from one direction

The length of the arch around its curve is taken as "L", so the approximate bending moment is given by: $M=WL^2/32$.

CHAPTER 4

STATIC ANALYSES OF CURVED ROOFS

4.1 Introduction

Although the subject of truss and arch roof analysis with optimization had been conversed frequently complete current years, this topic was contained to show the validity of an analysis program, which is used in the GA optimization program.

In this section, static matrix displacement analysis and FE methods are explained. For truss structures matrix displacement method and for arch structures FE method are used, each analysis program was examined versus known basis solutions, other literature and commercial package programs.

FE has become ordinary analyses method in current year. Numerical resolutions to be even very complicated structures can at present be analysis ordinarily using FE and the system is so significant that in spite of introductory treatments of engineers of materials such as these faculties should outline its main features, [6].

Most FE computer software is written in FORTRAN and in this thesis FORTRAN codes are also used.

A number of rewritten commercial codes are valid, denoting a broad price range and compatible with machines from microcomputers to supercomputers, [13]. Even so, users with specialized require should not be unavoidably wary far from code advancement; also may find the code origins available in such topics as that by, [49] to be a useful beginning point.

4.2 Steps of Finite Element Analysis

a. Preprocessing: The user concepts a model of the section to be broke down in which the geometry is segmented into a number of unattached sub elements, combined at discrete points named nodes. Sure of these intersections will have fixed displacements, also others will have recommended loads. These ideals can be very time consuming.

b. Analysis: Trade codes vie accompanied by one another to have the majority user-friendly graphic "preprocessor" to aid in this very tedious chore. Some of these preprocessors can overlay a mesh on a pre-existing CAD file, so that FE can be finished easily as section of the computerized drafting and design procedure.

The dataset arranged at the preprocessor is second-hand like input to the FE codes itself, which concepts and solves a method of linear otherwise nonlinear algebraic equations.

$$\mathbf{K}_{ij}\,\mathbf{d}_j = \mathbf{f}_i \qquad (4.1)$$

c.Postprocessing: In the earliest years of FE, the user would pore between reams of numbers whipped up by the code, listing displacements with stresses at discontinuous locations inside the model. It is simple to miss consequential trends and cool positions this way and new codes use graphical presentations to help in imagining the results. A characteristic postprocessor show overlay colored characters appearing for stress rungs on the model, presentation a full-field picture like to that of picture elastic or more trial results.

4.3 Stiffness Matrix Formulation

Stiffness matrix formulation is done by the principals of virtual work which is the area under the force-deflection curve and according linear behavior, displacements and forces are proportional by deflection, where single forces varies linearly with displacement from zero to its final intensity F_1 as shown in the Figure 4.1 [50].

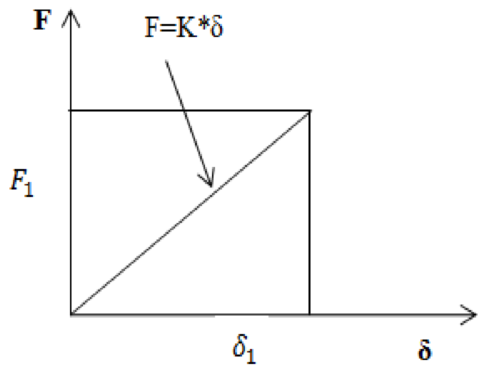


Figure 4.1 Force deflection relations

From Figure 4.1 by calculating the area under the triangle to represent the work done by F_1 can be written;

$$W = \frac{F_1 \times \delta_1}{2} \tag{4.2}$$

Matrix approaches are a required implement used in the FE approach for objectives of simplifying the formulation of the element stiffness equations, for aims of long hand solutions of difference problems, also, most significant, for use in scheduling the systems for high-speed electronic digital computers. Therefore, matrix comment denotes an uncomplicated and easy to use comment for writing as well as solving groups of simultaneous algebraic equations.

A matrix is a rectangular array of amounts arranged in rows with column that is adapted used like and helps in squeezing with solving a method of algebraic equations. Force vector (F_{1x}; F_{1y}; F_{1z}; F_{2x}; F_{2y}; F_{2z}; . . . ; F_{nx}; F_{ny}; F_{nz}) acting by the different nodes or points (1; 2; . . . ; n) on a structure in addition to the analogous set of nodal displacements (d_{1x}; d_{1y}; d_{1z}; d_{2x}; d_{2y}; d_{2z}; . . . ; d_{nx}; d_{ny}; d_{nz}) can both be squeezed like matrices:

$$\{F\} = \underline{F} = \begin{Bmatrix} F_{1x} \\ F_{1y} \\ F_{1z} \\ F_{2x} \\ F_{2y} \\ F_{2z} \\ \vdots \\ F_{nx} \\ F_{ny} \\ F_{nz} \end{Bmatrix} \qquad \{d\} = \underline{d} = \begin{Bmatrix} d_{1x} \\ d_{1y} \\ d_{1z} \\ d_{2x} \\ d_{2y} \\ d_{2z} \\ \vdots \\ d_{nx} \\ d_{ny} \\ d_{nz} \end{Bmatrix} \tag{4.3}$$

4.4 Matrix Analysis of Trusses

Static analysis of trusses can be carried out accurately; also the equations of even complex trusses can be collected in a matrix shape amenable to numeric solution. This approximation, now and then named “matrix analysis”, provided the basis of early FE advancement.

By considering the stiffness of each truss element one at a time matrix analysis of trusses acts and after that applying these stiffnesses by the displacements of the joints, generally named “nodes” in FE to determine the forces that are set up in the truss. Afterwards noting that the force that is externally contributed by each element to a node must equal the sum of force that is applied to that node, we can assemble a sequence. Of linear algebraical equations in which the applied nodal forces are known amounts, also the nodal displacements are the unknowns. These equations are comfortably written in matrix shape, which gives the system its name.

$$\begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} \tag{4.4}$$

Here $\mathbf{F}_i$ and $\mathbf{d}_j$ indicate the force at the $\mathbf{i}^{th}$ node and the deflection at the $\mathbf{j}^{th}$ node (these would actually be vector quantities, with subcomponents along each coordinate axis). The $\mathbf{K}_{ij}$ is global stiffness matrix, with the ij component. The matrix equations can be abbreviated as

$$\mathbf{K}_{ij}\ \mathbf{d}_j = \mathbf{F}_i \tag{4.5}$$

4.4.1 Stiffness matrix for a 2D truss element

As a first step in advancing a set of matrix equations that define truss methods, we require a connection among the forces as well as displacements at each end of an individual truss element. In Figure 4.2 consider 2D trusses element angle θ is measured positive in the counter clockwise direction as of the +x axis.

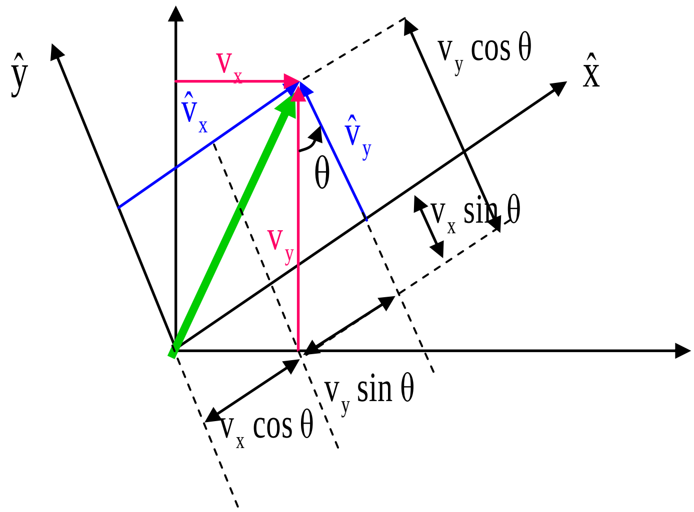


Figure 4.2 Global and local coordinates systems for a 2D truss element

The vector **v** has components (v_x, v_y) in the global coordinate system and ($\hat{v}_x$, $\hat{v}_y$) in the local coordinate system, from geometry.

$$\begin{aligned} \hat{v}_x &= v_x \cos\theta + v_y \sin\theta \\ \hat{v}_y &= -v_x \sin\theta + v_y \cos\theta \end{aligned} \tag{4.6}$$

or in matrix form

$$\begin{Bmatrix} \hat{v}_x \\ \hat{v}_y \end{Bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{Bmatrix} v_x \\ v_y \end{Bmatrix} \tag{4.7}$$

or

$$\boxed{\underline{\hat{v}} = \underline{T}^* \underline{v}} \tag{4.8}$$

Where **T** is transformation matrix, l is cosθ and m is sinθ

$$\underline{T}^* = \begin{bmatrix} l & m \\ -m & l \end{bmatrix} \tag{4.9}$$

Where

$$\underline{\hat{v}} = \begin{Bmatrix} \hat{v}_x \\ \hat{v}_y \end{Bmatrix} \text{ and } \underline{v} = \begin{Bmatrix} v_x \\ v_y \end{Bmatrix} \tag{4.10}$$

For the two-node 2D truss element, the relationship between local and global displacement is given in Figure 4.3.

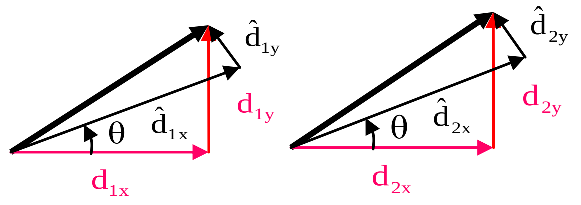


Figure 4.3 2D truss element relationship between local and global displacement

At node 1

$$\begin{Bmatrix} \hat{d}_{1x} \\ \hat{d}_{1y} \end{Bmatrix} = \underline{T}^* \begin{Bmatrix} d_{1x} \\ d_{1y} \end{Bmatrix} \tag{4.11}$$

At node 2

$$\begin{Bmatrix} \hat{d}_{2x} \\ \hat{d}_{2y} \end{Bmatrix} = \underline{T}^* \begin{Bmatrix} d_{2x} \\ d_{2y} \end{Bmatrix} \tag{4.12}$$

Putting these together

$$\underline{\hat{d}} = \underline{T}\underline{d} \tag{4.13}$$

$$\underbrace{\begin{Bmatrix} \hat{d}_{1x} \\ \hat{d}_{1y} \\ \hat{d}_{2x} \\ \hat{d}_{2y} \end{Bmatrix}}_{\underline{\hat{d}}} = \underbrace{\begin{bmatrix} l & m & 0 & 0 \\ -m & l & 0 & 0 \\ 0 & 0 & l & m \\ 0 & 0 & -m & l \end{bmatrix}}_{\underline{T}} \underbrace{\begin{Bmatrix} d_{1x} \\ d_{1y} \\ d_{2x} \\ d_{2y} \end{Bmatrix}}_{\underline{d}} \tag{4.14}$$

$$\underset{4\times4}{\underline{T}} = \begin{bmatrix} \underline{T}^* & \underline{0} \\ \underline{0} & \underline{T}^* \end{bmatrix} \tag{4.15}$$

Where **T** is the matrix of direction cosine and **0** is a 2×2 null matrix.

For the two-node 2D truss element, the relationship between local and global force is given in Figure below.

Figure 4.4 2D truss elements, the relationship between local and global force

At node 1

$$\begin{Bmatrix} \hat{f}_{1x} \\ \hat{f}_{1y} \end{Bmatrix} = \underline{T}^* \begin{Bmatrix} f_{1x} \\ f_{1y} \end{Bmatrix} \tag{4.16}$$

At node 2

$$\begin{Bmatrix} \hat{f}_{2x} \\ \hat{f}_{2y} \end{Bmatrix} = \underline{T}^* \begin{Bmatrix} f_{2x} \\ f_{2y} \end{Bmatrix} \tag{4.17}$$

Putting these together

$$\underline{\hat{f}} = \underline{T}\underline{f} \tag{4.18}$$

$$\underbrace{\begin{Bmatrix} \hat{f}_{1x} \\ \hat{f}_{1y} \\ \hat{f}_{2x} \\ \hat{f}_{2y} \end{Bmatrix}}_{\underline{\hat{f}}} = \underbrace{\begin{bmatrix} l & m & 0 & 0 \\ -m & l & 0 & 0 \\ 0 & 0 & l & m \\ 0 & 0 & -m & l \end{bmatrix}}_{\underline{T}} \underbrace{\begin{Bmatrix} f_{1x} \\ f_{1y} \\ f_{2x} \\ f_{2y} \end{Bmatrix}}_{\underline{f}} \tag{4.19}$$

Where

$$\underset{4\times4}{\underline{T}} = \begin{bmatrix} \underline{T}^* & \underline{0} \\ \underline{0} & \underline{T}^* \end{bmatrix} \tag{4.20}$$

Important property of the transformation matrix $\underline{T}$ is orthogonal; its inverse is its transpose.

Use the property that $l^2+m^2=1$

$$\underline{T}^{-1} = \underline{T}^T \tag{4.21}$$

Putting all the pieces together to obtain global stiffness matrix for 2D truss element

$$\hat{\underline{f}} = \underline{T}\,\underline{f} \tag{4.22}$$

$$\hat{\underline{d}} = \underline{T}\,\underline{d} \tag{4.23}$$

$$\begin{aligned} &\hat{\underline{f}} = \hat{\underline{k}}\,\hat{\underline{d}} \\ &\Rightarrow \underline{T}\underline{f} = \hat{\underline{k}}\,\underline{T}\underline{d} \\ &\Rightarrow \underline{f} = \underbrace{\left(\underline{T}^{-1}\hat{\underline{k}}\,\underline{T}\right)}_{\underline{k}}\underline{d} \end{aligned} \tag{4.24}$$

The desired relationship is

$$\underset{4\times1}{\underline{f}} = \underset{4\times4}{\underline{k}}\;\underset{4\times1}{\underline{d}} \tag{4.25}$$

Where

$$\underset{4\times4}{\underline{k}} = \underset{4\times4}{\underline{T}^{T}}\;\underset{4\times4}{\hat{\underline{k}}}\;\underset{4\times4}{\underline{T}} \tag{4.26}$$

The element stiffness matrix in the global coordinate system.

$$\underline{T} = \begin{bmatrix} l & m & 0 & 0 \\ -m & l & 0 & 0 \\ 0 & 0 & l & m \\ 0 & 0 & -m & l \end{bmatrix} \tag{4.27}$$

$$\hat{\underline{k}} = \begin{bmatrix} k & 0 & -k & 0 \\ 0 & 0 & 0 & 0 \\ -k & 0 & k & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{4.28}$$

$$\underline{k} = \underline{T}^{T}\hat{\underline{k}}\underline{T} = \frac{\mathrm{EA}}{\mathrm{L}}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix} \tag{4.29}$$

Where

$$l = \cos\theta = \frac{x_2 - x_1}{L}$$
$$m = \sin\theta = \frac{y_2 - y_1}{L} \quad (4.30)$$

For computation of element stress, strain and tension in 2D trusse element.

For element strain $\varepsilon = \frac{1}{L}\begin{bmatrix} -l & -m & l & m \end{bmatrix}\begin{Bmatrix} d_{1x} \\ d_{1y} \\ d_{2x} \\ d_{2y} \end{Bmatrix}$ (4.31)

For element stress

$$\sigma = E\varepsilon = \frac{E}{L}\left(\hat{d}_{2x} - \hat{d}_{1x}\right) = \frac{E}{L}\begin{bmatrix} -l & -m & l & m \end{bmatrix}\underline{d} \quad (4.32)$$

For element tension force

$$T = EA\varepsilon = \frac{EA}{L}\begin{bmatrix} -l & -m & l & m \end{bmatrix}\underline{d} \quad (4.33)$$

4.4.2 Stiffness matrix for a 3D truss element

Figure 4.5 indicates a 3D truss element. Employing the similarly ideas as those characterized in section 4.4.1 it can be easily exhibited that the element stiffness matrices for a 3D truss element.

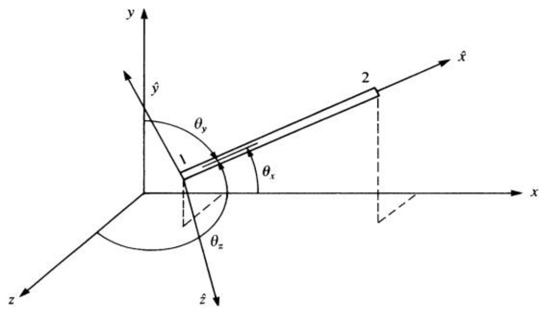


Figure 4.5 Bar in 3D space

The transformation matrix for a single vector in 3D

$$\underline{\hat{d}} = \underline{T}^{*}\underline{d} \quad (4.34)$$

$$\underline{T}^* = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix} \tag{4.35}$$

l_1, m_1 and n_1 are the direction cosines of x^

Where

$$\begin{aligned} l_1 &= \cos\theta_x \\ m_1 &= \cos\theta_y \\ n_1 &= \cos\theta_z \end{aligned} \tag{4.36}$$

In local coordinate system

$$\begin{Bmatrix} \hat{f}_{1x} \\ \hat{f}_{1y} \\ \hat{f}_{1z} \\ \hat{f}_{2x} \\ \hat{f}_{2y} \\ \hat{f}_{2z} \end{Bmatrix} = \begin{bmatrix} k & 0 & 0 & -k & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -k & 0 & 0 & k & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \hat{d}_{1x} \\ \hat{d}_{1y} \\ \hat{d}_{1z} \\ \hat{d}_{2x} \\ \hat{d}_{2y} \\ \hat{d}_{2z} \end{Bmatrix} \tag{4.37}$$

Transformation matrix $\underline{T}$ relating the local and global displacement and load vectors of the truss element.

$$\underline{\hat{d}} = \underline{T}\,\underline{d} \tag{4.38}$$

$$\underline{\hat{f}} = \underline{T}\,\underline{f} \tag{4.39}$$

$$\underset{6\times6}{\underline{T}} = \begin{bmatrix} \underline{T}^* & \underline{0} \\ \underline{0} & \underline{T}^* \end{bmatrix} \tag{4.40}$$

Element stiffness matrix in global coordinates.

$$\underset{6\times6}{\underline{k}} = \underset{6\times6}{\underline{T}^T}\ \underset{6\times6}{\underline{\hat{k}}}\ \underset{6\times6}{\underline{T}} \tag{4.41}$$

$$\underline{k} = \underline{T}^T \hat{\underline{k}} \underline{T} = \frac{EA}{L} \begin{bmatrix} l_1^2 & l_1 m_1 & l_1 n_1 & -l_1^2 & -l_1 m_1 & -l_1 n_1 \\ l_1 m_1 & m_1^2 & m_1 n_1 & -l_1 m_1 & -m_1^2 & -m_1 n_1 \\ l_1 n_1 & m_1 n_1 & n_1^2 & l_1 n_1 & m_1 n_1 & -n_1^2 \\ -l_1^2 & -l_1 m_1 & -l_1 n_1 & l_1^2 & l_1 m_1 & l_1 n_1 \\ -l_1 m_1 & -m_1^2 & -m_1 n_1 & l_1 m_1 & m_1^2 & m_1 n_1 \\ -l_1 n_1 & -m_1 n_1 & -n_1^2 & l_1 n_1 & m_1 n_1 & n_1^2 \end{bmatrix} \tag{4.42}$$

4.5 FE Analysis of Solid Arch Structures

Consider the Mindlin-Reissner (MR) curved beam element shown in Figure 4.6. The displacement components u_ℓ and w_ℓ, are associated with movements in ℓ and n directions respectively, expressed in terms of axes which are tangential and normal to the arch, may be written in terms of global displacements *u* and *w* in the *x* and *y* directions as

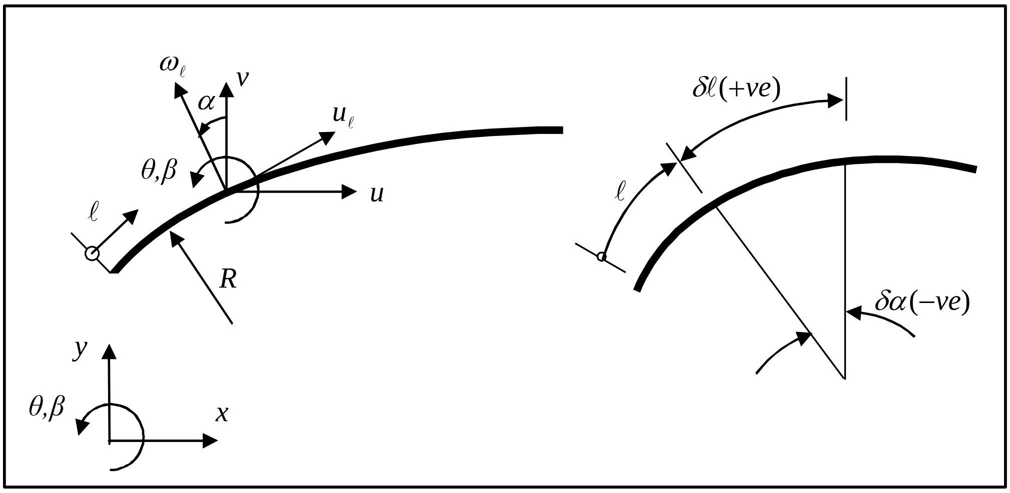


Figure 4.6 Definition of curved Mindlin-Reissner543 arch FE

$$u_\ell = u\cos\alpha + w\sin\alpha$$

$$w_\ell = -u\sin\alpha + w\cos\alpha \tag{4.43}$$

Where α is shown in Figure 4.6. The radius of curvature *R* may be obtained from the expression.

$$\frac{d\alpha}{d\ell} = -\frac{1}{R} \tag{4.44}$$

The total potential energy for a typical MR curved beam element resting on elastic Winkler type foundation of modulus k shown in Figure 4.6 is given in terms of the global displacements u and w and rotation θ of the mid surface normal in the ℓn plane by expression.

$$\begin{aligned}\Pi(u_\ell, w_\ell, \theta) = 1/2 \int \left([\varepsilon_m]^T D_m \varepsilon_m + [\varepsilon_b]^T D_b \varepsilon_b + [\varepsilon_s]^T D_s \varepsilon_s + k w_\ell^2 \right) d\ell \\ - \int u_\ell q_u d\ell - \int w_\ell q_w d\ell - \int \theta_\ell q_\theta d\ell - (M\overline{\theta} + N\overline{u}_\ell + \overline{w}_\ell)\end{aligned} \tag{4.45}$$

Where the membrane (axial) strain is given by the expression

$$\varepsilon_m = \frac{du_\ell}{d\ell} + \frac{w_\ell}{R} \tag{4.46}$$

or re-writing in terms of the global displacements

$$\varepsilon_m = \frac{du}{d\ell}\cos\alpha + \frac{dw}{d\ell}\sin\alpha \tag{4.47}$$

The bending (flexural) strain or curvature may be written as

$$\varepsilon_b = -\frac{d\theta}{d\ell} \tag{4.48}$$

and the shear strain is given as

$$\varepsilon_s = \frac{dw_\ell}{d\ell} - \theta - \frac{u_\ell}{R} \tag{4.49}$$

or

$$\varepsilon_s = -\theta - \frac{du}{d\ell}\sin\alpha + \frac{dw}{d\ell}\cos\alpha \tag{4.50}$$

Also, note that the membrane, bending and shear rigidities have the form

$$D_m = EA \qquad D_b = EI\,; \qquad D_s = \kappa GA \tag{4.51}$$

Where E is the elastic modulus, A is the cross-sectional area, I, is the moment of inertia,
G is the shear modulus and κ is the shear modification factor.

Note that the displacement field vector **u** has the form

$$\mathbf{u} = [u_\ell, w_\ell, \theta] \tag{4.52}$$

and the corresponding distributed loading **q** may be written as

$$\mathbf{q} = [q_u, q_w, q_\theta]^T \tag{4.53}$$

In which the distributed forces are q_u and q_w and the distributed couples q_θ .

The loading in Eq (4.45) consists of a distributed pressure loading **q**, as well as couples *M*, axial forces *N* or lateral forces *Q* applied at $\ell = \overline{\ell}$. Note that $\overline{u}_\ell$, $\overline{w}_\ell$ and $\overline{\theta}$ are the corresponding displacement and rotation values at $\ell = \overline{\ell}$ .The detailed definition about FE analysis of solid arch roof structure are given [51].

CHAPTER 5

ANALYSIS EXAMPLES

5.1 Static Analysis of 2D and 3D Truss

To perform the previous formulation of the stiffness matrix which is coded in FORTRAN, several examples are considered. FE model can be successfully used for the static analysis of the structures. Analysis is done by the FE method coded program for analysis of 2D and 3D curved truss roofing, results are compared with source program (SAP2000).

5.1.1 2D curved truss with 5.75 m height

This example consists of curved truss with 20 m span length and maximum height at the center 5.75 m with 71 elements and 37 joints as shown in Figure 5.1. The structure is loaded with a point load of 10 kN on all upper joints in the *Z* direction as shown in the Figure 5.2. The members of the structure are divided into 3 groups; first group is bottom chord, second group is top chord and third group is diagonals. Each group has same cross sectional area, cross-sectional areas for first group are A_1=0.000645 m^2, second group are A_2=0.0008973 m^2 and third group are A_3=0.00177 m^2. Material properties are: Young's modulus, $E = 2.0\times10^8$ kN/m^2 and material density, $\rho = 76.9729$ kN/m^3.

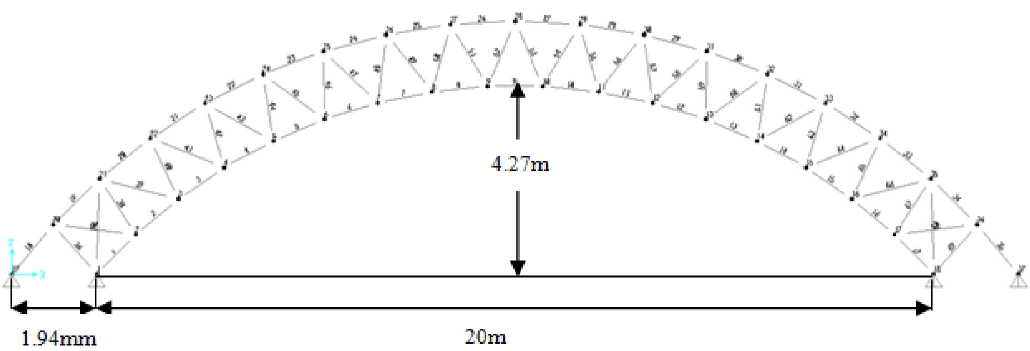


Figure 5.1 Dimensions of 2D curved truss with 5.75 m height

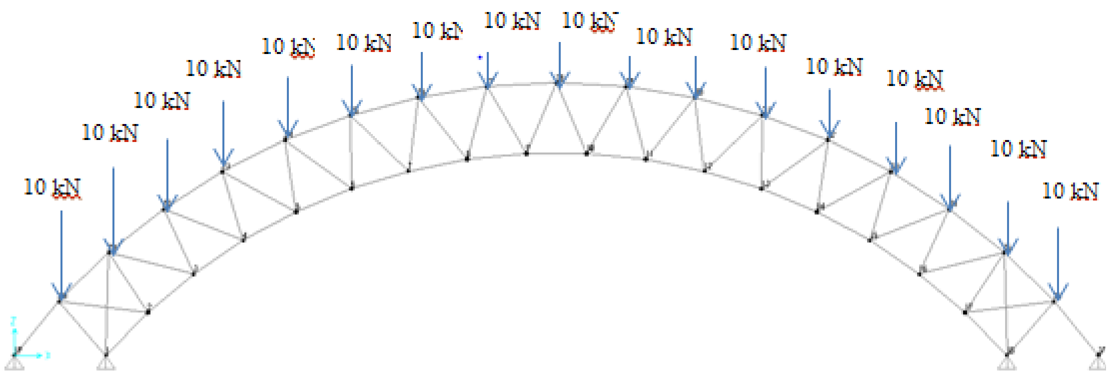


Figure 5.2 Loadings of 2D curved truss with 5.75 m height

Discussion of the results: Analyses are done by the present study program and results are compared with source program (SAP2000). Table 5.1 shows the forces of some selected members. Maximum compression forces are occurs in members (2, 16) and maximum tension forces are existing in members (41, 64). Table 5.2 shows the result of displacements in *x* and *z* directions. Maximum displacements occurred in joints (9, 10 and 28). The close agreement between results is seen.

Table 5.1 Member forces of 2D curved truss with 5.75 m height

Internal Force (kN)		
Element number	**SAP2000**	**Present work**
1	-51.032	-51.475
2	-65.911	-66.740
9	-7.034	-7.220
10	-8.735	-8.935
16	-65.911	-66.740
17	-51.032	-51.475
18	-28.583	-28.685
19	-7.796	-7.785
26	-54.763	-55.274
27	-54.763	-55.274
41	12.418	13.156
64	12.418	13.156

Table 5.2 Displacements of 2D curved truss with 5.75 m height

Displacements (m)				
Joint no	**X-direction**		**Z-direction**	
	Present	**Sap2000**	**Present**	**Sap2000**
2	-0.0004	0.0004	-0.0003	-0.0004
3	-0.0004	-0.0005	-0.0013	-0.0013
9	0.0000	-0.0001	-0.0068	-0.0069
10	0.0000	-0.0001	-0.0068	-0.0069
16	0.0004	0.0003	-0.0013	-0.0013
17	0.0004	0.0003	-0.0003	-0.0004
20	-0.0002	-0.0002	-0.0001	-0.0001
21	-0.0002	-0.0003	-0.0002	-0.0002
27	0.0004	0.0004	-0.0066	-0.0066
28	0.0000	-0.0001	-0.0068	-0.0069
29	-0.0004	-0.0005	-0.0066	-0.0066
35	0.0002	0.0001	-0.0002	-0.0002
36	0.0002	0.0001	-0.0001	-0.0001

5.1.2 2D curved truss with 7.5 m height

This example consists of curved truss with 30.0 m span length and maximum height at the center is 7.5 m, with 71 elements and 37 joints as shown in Figure 5.3. The structure is loaded with a point load of 10 kN on all upper joints in the *Z* direction as shown in Figure 5.4.

The members of the structure are divided into 3 groups; first group is bottom chord, second group is top chord and third group is diagonals. Cross-sectional areas for first group and second group A_1=A_2=0.000958 m^2 and third group A_3=0.000693 m^2. Material properties are: Young's modulus, $E = 2.0 \times 10^8$ kN/m^2 and material density, $\rho = 76.9729$ kN/m^3.

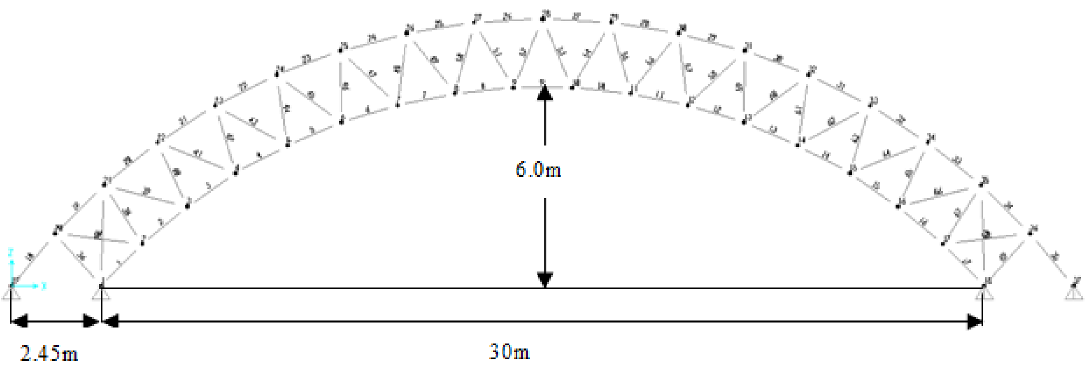


Figure 5.3 Dimensions of 2D curved truss with 7.5 m height

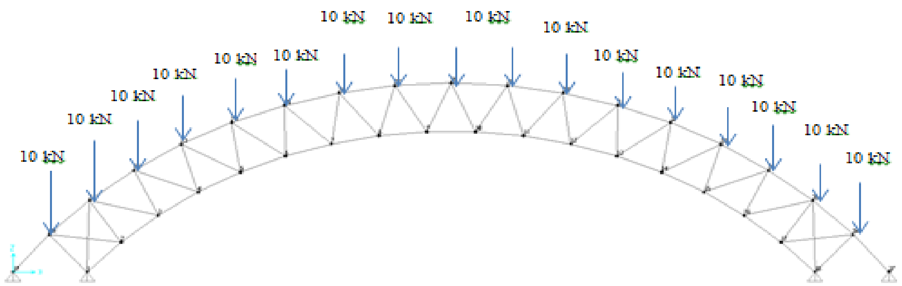


Figure 5.4 Loadings of 2D curved truss with 7.5 m height

Discussion of the results: Table 5.3 shows the forces of some selected important members by using present study program and SAP2000. Maximum compression force which is similar to previous example occurred in members (2, 16) and maximum tension force occurred in members (41, 64). Table 5.4 is the result of displacements in *x* and *z* directions. Maximum displacement occurred in joints (9, 10,). The close agreements between results are seen.

Table 5.3 Member forces of 2D curved truss with 7.5 m height

Internal Force (kN)		
Frame NO.	**Present work**	**sap2000**
1	-70.34	-70.33
2	-78.55	-78.50
9	-21.90	-21.87
10	-23.64	-23.61

16	-78.55	-78.50
17	-70.34	-70.33
18	-29.80	-29.77
19	-15.10	-15.09
26	-57.77	-57.76
27	-57.77	-57.76
41	9.09	9.03
64	9.09	9.03

Table 5.4 Displacements of 2D curved truss with 7.5 m height

Displacement (m)				
Joint no.	**X-direction**		**Z-direction**	
	Present	**Sap2000**	**Present**	**Sap2000**
2	-0.00055	-0.00055	-0.00046	-0.00046
3	-0.00054	-0.00054	-0.00184	-0.00184
9	0.00011	0.00011	-0.01125	-0.01125
10	-0.00011	-0.00011	-0.01125	-0.01125
16	0.00054	0.00054	-0.00367	-0.00367
17	0.00055	0.00055	-0.00184	-0.00184
20	-0.00033	-0.00033	-0.00015	-0.00015
21	-0.00037	-0.00037	-0.00037	-0.00037
27	0.00064	0.00064	-0.01082	-0.01082
28	0.00000	0.00000	-0.01122	-0.01122
29	-0.00064	-0.00064	-0.01082	-0.01082
35	0.00037	0.00037	-0.00037	-0.00037
36	0.00033	0.00033	-0.00015	-0.00015

5.1.3 2D curved truss with 288 inch height

This example consists of curved truss with 2000 inch span length with 41 elements as shown in Figure 5.5. The geometry of this example is taken from [52]. The structure is loaded with a point load of 50 kip on all upper joints in the Z direction. The members of the structure are divided into 3 groups; first group from element (1-22) and (40, 41), second group from element (26-36) and third group from element (23-25) and (37-39), each group have the same cross sectional area. Cross-sectional areas for first group A1=75.6 in^2, second group A2=14.7in^2 and third group A3=20in^2 Material properties are: Young's modulus, $E = 29000$ kip/in^2 and material density, $\rho =$ 2.386*10^{-4} kip/in^3.

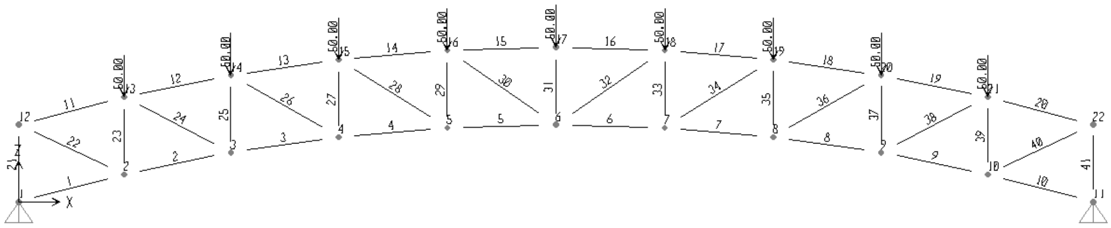


Figure 5.5 2D curved truss with 288 inch height

Discussion of the results*:* Analysis is done by the FE method coded program and source program (SAP2000). Table 5.5 shows the stress of some important selected members. Maximum tension stress occurs in elements (16, 26) and maximum compression stress occurs in elements (1, 10). Table 5.6 is the result of displacements in x and z directions. Maximum displacement occurs in joints (17, 6). The close agreements between results are seen.

Table 5.5 Comparison of stress for 41 bar 2D curved truss with 288 inch height

Stress (kips)		
Frame NO.	**SAP2000**	**Present work**
1	-6.4108	-6.3000
2	4.4588	4.3000
5	1.2089	1.2000
6	-1.2089	-1.2000
9	-4.4588	-4.3000
10	-6.4108	-6.3000
11	1.3440	1.3440
12	-2.0228	-2.0228
15	4.6429	4.6429
16	6.2846	6.2846
19	2.7162	2.7162
26	6.2846	6.2846

Table 5.6 Displacements for 41 bar 2D curved truss with 288 inch height

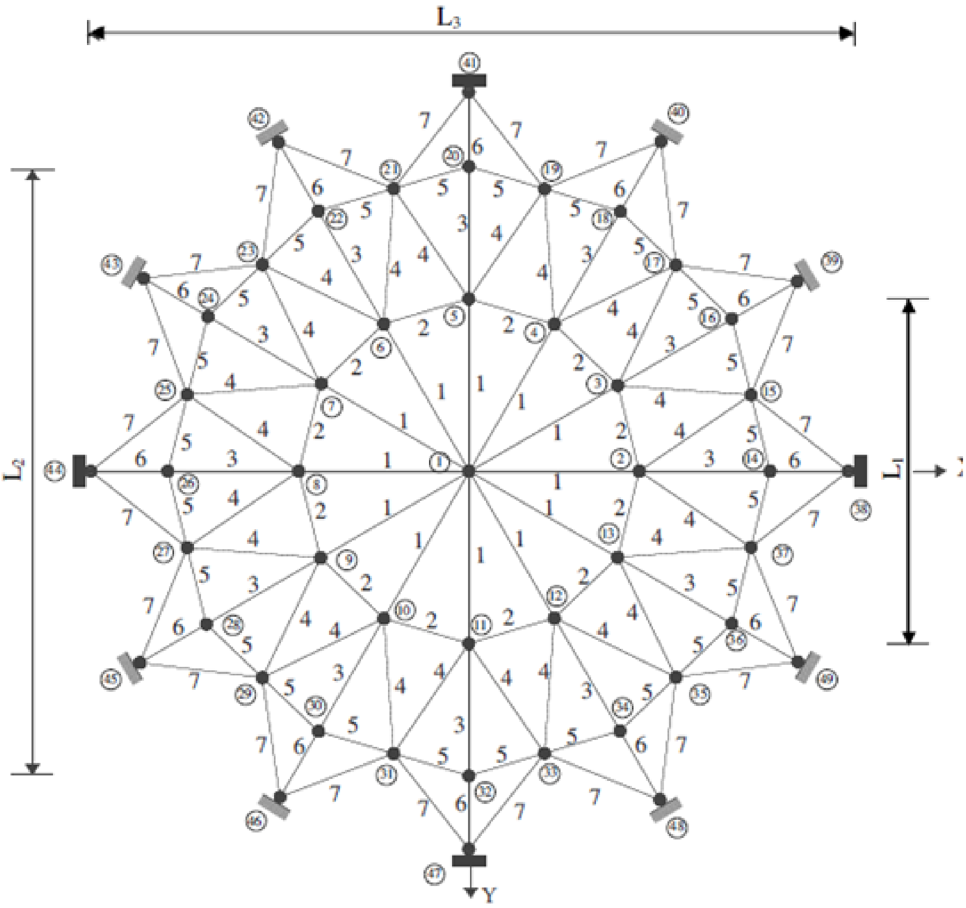
Displacements (in)				
Joint no	**X-direction**		**Z-direction**	
	Present	**Sap2000**	**Present**	**Sap2000**
1	0.0000	0.0000	0.0000	0.0000
2	-0.0169	-0.0169	-0.1120	-0.1120
3	-0.0048	-0.0048	-0.3281	-0.3281
4	0.0051	0.0051	-0.5414	-0.5414
5	0.0061	0.0061	-0.7038	-0.7038
6	0.0000	0.0000	-0.7800	-0.7800
7	-0.0061	-0.0061	-0.7038	-0.7038
8	-0.0051	-0.0051	-0.5414	-0.5414
9	0.0048	0.0048	-0.3281	-0.3281
10	0.0169	0.0169	-0.1120	-0.1120
11	0.0000	0.0000	0.0000	0.0000
12	0.0160	0.0160	-0.0067	-0.0067
13	0.0380	0.0390	-0.1400	-0.1400
14	0.0560	0.0560	-0.3400	-0.3400
15	0.0560	0.0550	-0.5500	-0.5500
16	0.0341	0.0340	-0.7000	-0.7000
17	0.0000	0.0000	-0.7800	-0.7800
18	-0.0340	-0.0340	-0.7000	-0.7000
19	-0.0560	-0.0550	-0.5500	-0.5500
20	-0.0560	-0.0560	-0.3400	-0.3400
21	-0.0380	-0.0390	-0.1400	-0.1400
22	-0.0160	-0.0160	-0.0067	-0.0067

5.1.4 3D dome

This example consists of space dome truss with 120-bars and 49 joints, dimensions of dome are shown in Figure 5.6 [53]. The members are collected into seven different groups. The truss joints are subjected to vertical loading except of supported joints. These are taken as 13.49 kips at node 1, 6.744 kips from nodes 2 to 14 and 2.248 kips at rest of the nodes, the dome span and total height shown in the Figure 5.7.

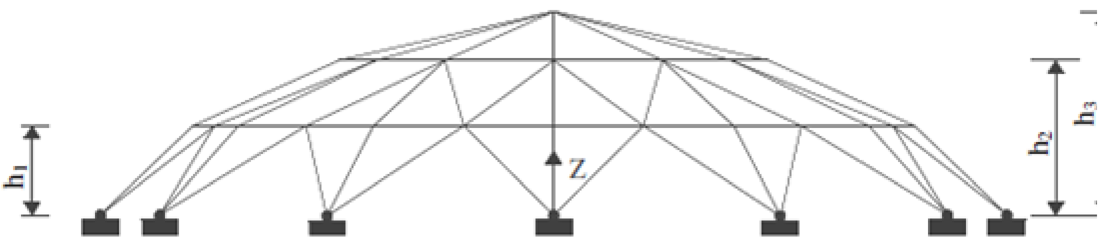
Used material properties are: density, $\rho = 29\text{x}10^{-4}$ kip/in^3, Young's modulus

$E = 29000$ kip/in^2. Cross-sectional areas for first group A_1= 6.11 in^2, second and third group A_2=A_3= 2.117 in^2, forth group A_4= 2.71 in^2, fifth group A_5= 1.51 in^2, sixth group A_6= 5.61 in^2 and seventh group A_7= 4.41 in^2.



L_1=546.61 in. (13883.89 mm) L_2=948.25 in. (24085.55 mm) L_3=1251.02 in (31775.91 mm)

Figure 5.6 Top view of 3D dome



h1=118.11 in. (3000 mm) h2= 230.32 in (5850.13 mm) h3= 275.59 in. (7000 m

Figure 5.7 Heights of 3D dome

Discussion of the results: Analysis is done and Table 5.7 shows the stress of members by using present study program and SAP2000. Maximum compression stress occurred in second group. Table 5.8 is the result of displacements in *x*, *y* and z directions. Maximum displacement occurred in joints (17). The close agreement between results is seen.

Table 5.7 Comparison of stress for 120 bar 3D dome with 275.59 inch height

Stress kip/in^2		
GROUP NO.	**Present**	**Sap2000**
1	-1.160	-1.130
2	-4.720	-4.820
3	-1.950	-1.830
4	-2.290	-2.230
5	-2.530	-2.530
6	-1.250	-1.290
7	-1.940	-1.910

Table 5.8 Displacements for 3D dome with 275.59 in height

Displacement (in)						
Joint no.	**X-direction**		**Y-direction**		**Z-direction**	
	Present	**Sap2000**	**Present**	**Sap2000**	**Present**	**Sap2000**
1	0.0000	0.0000	0.0000	-0.0002	0.0998	0.0947
2	-0.0432	-0.0440	0.0000	-0.0001	-0.0985	-0.1032
3	-0.0374	-0.0381	-0.0216	-0.0221	-0.0983	-0.1031
4	-0.0216	-0.0220	-0.0374	-0.0382	-0.0983	-0.1030
5	0.0000	0.0000	-0.0433	-0.0441	-0.0985	-0.1030
6	0.0216	0.0220	-0.0374	-0.0382	-0.0984	-0.1031
7	0.0374	0.0381	-0.0216	-0.0221	-0.0984	-0.1032
8	0.0432	0.0440	0.0000	-0.0001	-0.0985	-0.1034
9	0.0374	0.0381	0.0216	0.0219	-0.0985	-0.1036
10	0.0216	0.0220	0.0374	0.0380	-0.0985	-0.1036
11	0.0000	0.0000	0.0432	0.0439	-0.0985	-0.1036
12	-0.0216	-0.0221	0.0374	0.0380	-0.0984	-0.1036
17	-0.0542	-0.0518	-0.0542	-0.0518	-0.1286	-0.1252

5.1.5 3D curved truss with 7.6 m height

This example consists of 3D curved truss with 30 m span length and maximum height at the center 7.6 m, for modeling 138 elements and 56 joints is used as shown in Figure 5.8. The structure is loaded with a point load of 20 kN on all upper joints in Z direction. The members of the structure are divided into 3 groups; first group is bottom chord and top chord, second group is diagonal chord and elements between upper chords are the third group. Cross-sectional areas for first and second group $A_1=A_2=0.000958$ m^2 and third group $A_3=0.000693$ m^2 Material properties are: Young's modulus, $E=2.0\times10^8$ kN/m^2 and material density, $\rho=76.9729$ kN/m^3.

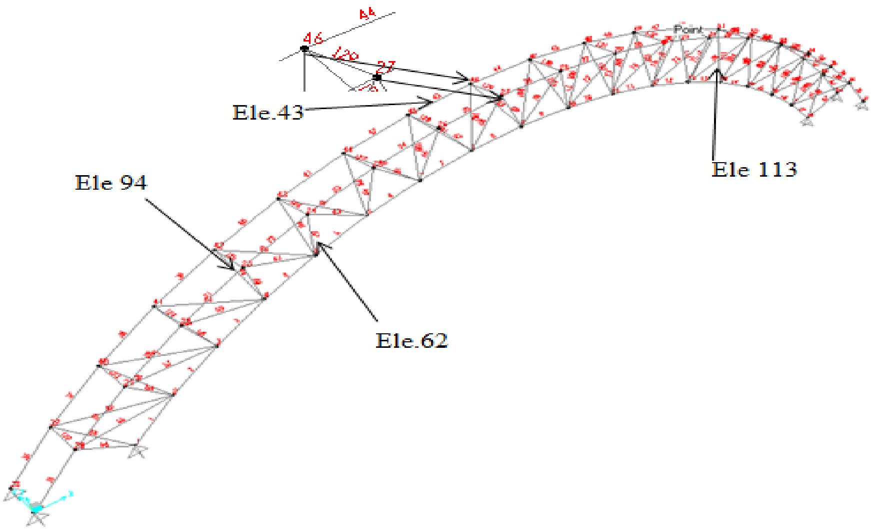


Figure 5.8 3D curved truss with 7.6 m height

Discussion of the results: Analysis is done by the FE method coded program and results are compared with source program (SAP2000). Table 5.9 shows the stress of some important selected members. Maximum compression stresses occur in elements (1, 17) and maximum tensions occur in elements (43, 62, 94 and 113). Table 5.10 is the result of displacements in x, y and z directions. Maximum displacement occurred in joints (46, 27). The close agreements between results are seen.

Table 5.9 Member stresses for 3D curved truss with 7.6 m height

Frame NO.	Stress (kN/m^2)	
	Present	SAP 2000
1	-163691.86	-163682.75
2	-153483.79	-153358.19
9	-56110.59	-56070.66
16	-153483.79	-153358.19
17	-163691.86	-163682.75
18	-72978.1	-72819.87
19	-66582.47	-66603.71
26	-115734.51	-115668.12
27	-115734.51	-115668.12
34	-66582.47	-66603.71
35	-72978.1	-72819.87
36	-31084.84	-31434.19
37	14611.09	14822.02
43	32211.42	32001.24
44	-50022.71	-49884.11
45	28227.47	28033.94
52	-12988.07	-13002.9
53	-12988.07	-13002.9
62	32211.42	32001.24
94	32211.42	32001.24
113	32211.42	32001.24

Table5.10 Displacements for 138 bar 3D curved truss with 7.6 m height

Displacement (m)						
Joint No	X-direction		Y-direction		Z-direction	
	Present	SAP2000	Present	SAP2000	Present	SAP2000
2	-0.0004	-0.0004	-0.0003	0.0000	-0.0019	-0.0019
3	-0.0003	-0.0003	-0.0008	0.0000	-0.0047	-0.0047
9	0.0003	0.0003	-0.0005	0.0000	-0.0220	-0.0220
10	-0.0003	-0.0003	-0.0004	0.0000	-0.0220	-0.0220

16	0.0003	0.0003	0.0001	0.0000	-0.0047	-0.0047
17	0.0004	0.0004	0.0001	0.0000	-0.0019	-0.0019
19	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
20	-0.0016	-0.0005	0.0055	0.0000	0.0005	-0.0006
28	0.0000	0.0000	-0.0003	0.0000	-0.0219	-0.0220
35	0.0002	0.0001	0.0006	0.0000	-0.0021	-0.0022
36	0.0003	0.0005	-0.0009	0.0000	-0.0008	-0.0006
39	0.0006	-0.0005	0.0055	0.0000	-0.0018	-0.0006
40	-0.0003	-0.0001	-0.0018	0.0000	-0.0018	-0.0022
46	0.0013	0.0012	0.0034	0.0000	-0.0225	-0.0222
54	0.0000	0.0001	0.0006	0.0000	-0.0024	-0.0022
55	0.0007	0.0005	-0.0009	0.0000	-0.0005	-0.0006

5.2 Static Analysis of Solid Arch

Analysis is done by the FE method coded program for analysis of solid arch roofing; results are compared with source program (SAP2000).

5.2.1 Arches under point load

This example involves a series of arches with rectangular cross-sections, which have been studied by Litewka and Rakowski [54]. The arches have a radius of curvature of $R = 4\,\mathrm{m}$, the opening angle $\omega = 2\pi/3$ (length $l = 8\pi/3$), thickness $t = 0.6\,\mathrm{m}$ and width $b = 0.4\,\mathrm{m}$ as shown in Figure 5.9. The following material properties are used: elastic modulus $E = 30\,\mathrm{GPa}$ and Poisson's ratio $\nu = 0.17$.

The analysis is repeated for, two different boundary conditions: fixed-fixed and hinged-hinged and three different loading cases; a) vertical point load at the crown, b) horizontal point load at the crown and c) moment at the crown.

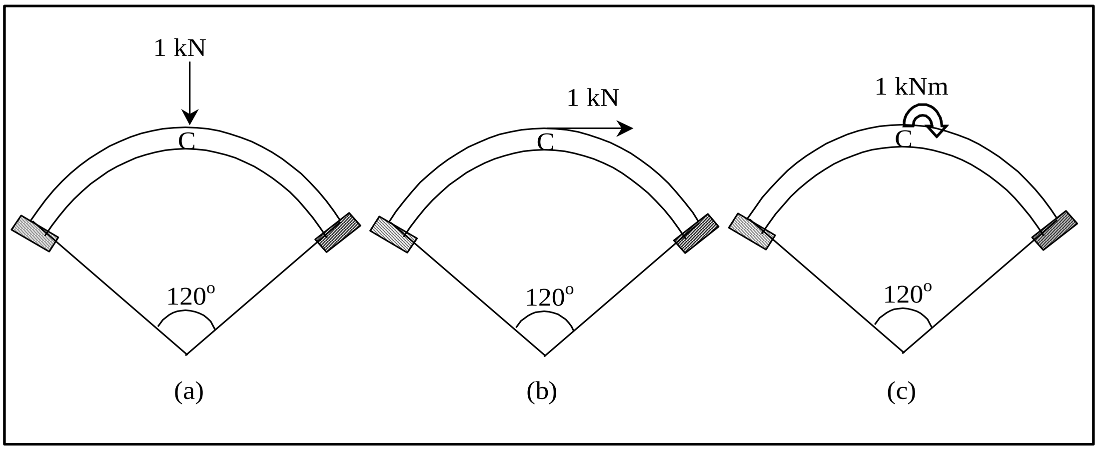


Figure 5.9 Loading conditions of uniform cross-section arch

Discussion of results: To avoid the possibility of significant discretization error, the arches are analyzed using 12 cubic elements. The results for maximum deflections are summarized in Tables 5.11 and 5.12 for fixed-fixed and hinged-hinged arches respectively. The results of the analyses compare very well with those obtained by Litewka and Rakowski [54] based on thick beam model.

Table 5.11 Displacements of uniform cross-section arches for fixed-fixed boundary condition $\times 10^{-7}$

Load	u_c/l (Disp. in *x*-direct.)			v_c/l (Disp. in *y*-direct.)			θ_c/ω (Rotation)		
	Present	**Ref. [54]**	**Sap2000**	**Present**	**Ref. [54]**	**Sap2000**	**Present**	**Ref. [54]**	**Sap2000**
a	0.006	0.00	0.00	-2.5100	-2.48	-2.380	0.216	0.00	0.00
b	1.230	1.25	1.55	-0.0060	0.00	0.000	3.620	3.78	3.18
c	0.909	0.94	-1.16	0.0543	0.00	0.000	10.700	10.80	34.90

Table 5.12 Displacements of uniform cross-section arches for hinged-hinged boundary condition $\times 10^{-7}$

Load	u_c/l (Disp. in *x*-direct.)			v_c/l (Disp. in *y*-direct.)			θ_c/ω (Rotation)		
	Present	**Ref. [54]**	**Sap2000**	**Present**	**Ref. [54]**	**Sap2000**	**Present**	**Ref. [54]**	**Sap2000**
a	0.09243	0.000	0.06432	-2.799	3.047	3.51	0.3741	0.000	0.000
b	2.7650	2.880	3.49000	-0.0924	0.000	0.00	3.6200	7.770	2.464
c	-1.9520	-2.016	-2.36000	-0.0939	0.000	0.00	1.3620	1.361	1.090

5.2.2 Solid arches under multi point load

This example involves analysis of an arches with circle cross-sections and similar dimensions of example 5.1.1, the geometry and loadings of arch which has uniform cross-section with 20 m span is considered shown in Figure. 5.10 The arches have a radius of curvature R = 11.547 m, the angle $\omega = 2\pi/3$ (span length l = 24.1383 m), the cross-section area= 0.008968 m^2. The following material properties are used: Young's modulus $E = 200 \times 10^6$ kN/m^2, material density $\rho = 76.9729$ kN/m^3.

Discussion of the results: Analysis is done by source program (SAP2000). Table 5.13 is the result of displacements in x and z directions. Maximum displacement occurred in the crown.

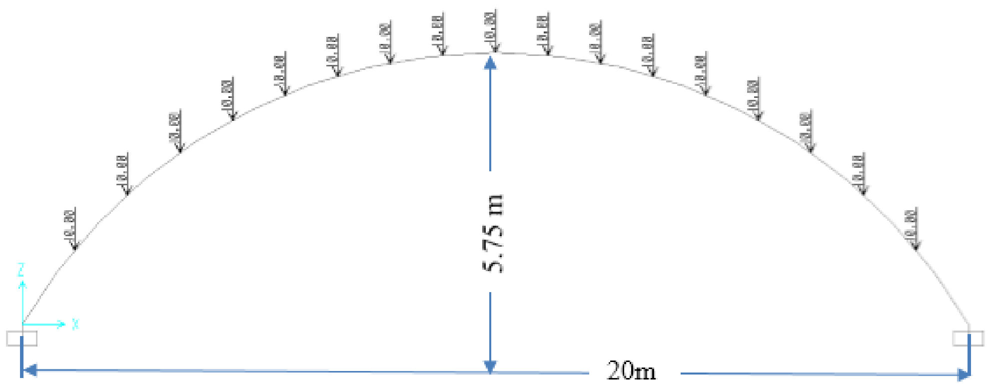


Figure 5.10 Loading condition of arch

Table 5.13 Displacements of uniform cross-section arches

Joint	u_c/l **(Disp. in x-direct.)**	v_c/l **(Disp. in z-direct.)**	θ_c/ω **(Rotation)**
Crown	0.00000	-0.00264	-0.000705

CHAPTER 6

OPTIMIZATION ALGORITHM

6.1 Introduction

Optimization characterized as the method of discovery the conditions that provide the minimum or else maximum value of a function, where the function appears upon the production needed or else the desired profit.

Structures are becoming lighter, tougher and cheaper as industry appropriates higher shapes of optimization. This kind and product improvement is now an important part of the design of problem solving system in today's engineering industry.

The subject of optimization has its arithmetical roots setting backward to the 1670s with the beginning of different calculus. Its earliest objective is to determine the optimum result in a problem given a set of environment. It was not up to the time of the early 1950s that computer-based optimization gotten under way itself into the engineering production. This was because the subject gives itself to numeric computation, which is the one job in which computers have the advantage over humans.

Software engineers shortly started establishing modern optimization approaches such as unconstrained optimization, multi-objective optimization and nonlinear programming. A new computation to the family of numeric optimization systems is that of evolutionary computation. This group of optimization contains the GA.

6.2 Structural Optimization Methods

There are three types of optimization: ad shown in Figure 6.1 size shape and topology. In structural optimization all three classes usually have the purpose of mass minimization with optional displacement or stress restrictions.

Size optimization variance of member cross sectional properties which may be unbroken or disconnected variables.

Shape optimization relocation of intersections to variation the form of the structure without varying the topology: The element-node connectivity remnants intact.

Topology optimization variation of element-node connectivity to determine an optimal design, Problems may appear when a variation truss topology causes the structure to change into a mechanism.

This study applies of size optimization techniques. Employing just size optimization avoids the problems related with topological as well as shape optimization while permitting substantial variations to the structure.

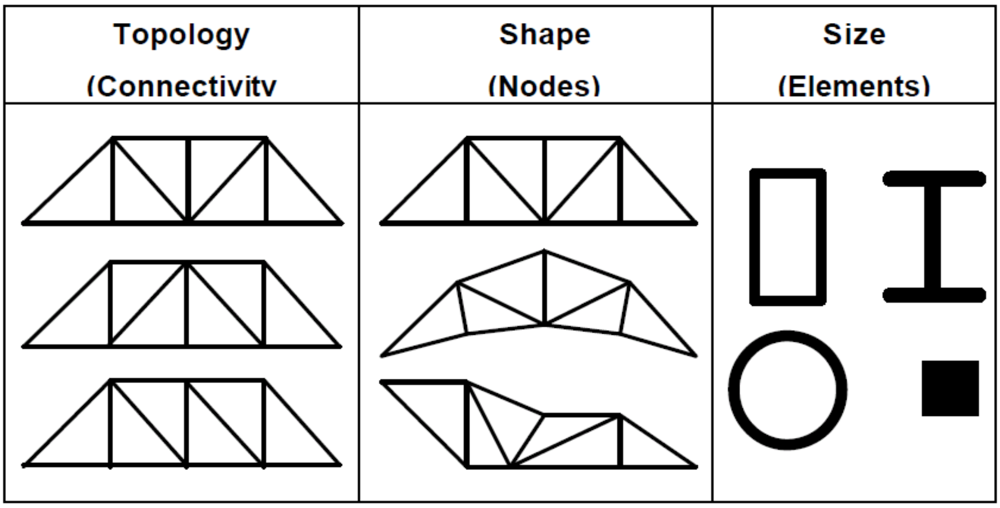


Figure 6.1 Three kinds of structural optimization patterns

6.3 Optimization Problem Definition

Problems of constructional optimization are described by various purposes with constraints, which are usually nonlinear functions of the design variables. The functions can be indirect, non-convex and discontinuous. Exact preparations of realistic optimization problems (i.e. object functions also constraints) change between every application.

6.4 Statement of an Optimization Problem

An optimization or else an arithmetical programming problem can be determined like follows:

Determine the design parameter **x** which maximizes (otherwise minimizes) the objective function **F(x)** subject to the behavioral restriction, $g_j(\boldsymbol{x}) \leq 0$**,** equality restrictions, $h_k(\boldsymbol{x}) = 0$ with explicit geometric restrictions, $x_i^{\ell} \leq x_i \leq x_i^{u}$.

The subscripts *i*, j and κ denote the quantity of design variables, behavioral restrictions, with equality restrictions respectively. The terms x_i^u and x_i^l refer to the stipulated upper with lower limits on the design variable quantities. For a presumption object functions **F(x),** where **x ∈ Rn** is the vector of design/decision variables, an example structural size optimization problem studies. Determine the set of regions x = {A_1, $A_{2.}$ A_n} which decreases the mass of the structure.

$$F(X) = \sum_{i=1}^{n} (A_i\, l_i) \tag{6.1}$$

Where **n** is the number of bars, as well as $\ell_{\boldsymbol{i}}$ is the length of the i^th bar of the truss. When size design variables are thought about the rates of $\ell_{\boldsymbol{i}}$ change, also the weight rests on not just on the rates of A_i, but as well as on the joint coordinates of the construction.

The problem is generally subject to difference constraints. $g_p(x) \geq 0$, p = 1, 2, ..,n and from time to time equality restrictions $h_q(x) = 0$, q = 1, 2, . . . , n. Furthermore, the variables are generally subject to bounds, $x_i^{\ell} \leq x_i \leq x_i^{u}$ i. The mainly common restrictions are normal stress restrictions:

$$\frac{\sigma i}{\sigma max} - 1 \leq 0, \text{ i} = 1, 2, \ldots, \text{n} \tag{6.2}$$

Where σ_{max} is the greatest permissible stress and $\boldsymbol{\sigma_i}$ is the normal stress at the, i^{th} member, displacements limitations can also be considered:

$$d_j\, |/d_{max} - 1 \leq 0, \text{ j} = 1, 2, .\,.\,, \text{n} \tag{6.3}$$

Where $\mathbf{d_{max}}$ is the greatest permissible displacement, $\mathbf{d_j}$ is the displacement at the j^{th} global degree of freedom.

6.4.1 Design vector

Refer to variables which express the design solutions and could be swapped during the optimization process with upper and lower limit. It represents the selected material or geometry. If it represents geometry, it may interact to a shape sophisticated interpolation or it may simply be the element cross-section, or the sheet thickness [55].

6.4.2 Objective function F(x)

The function is an arithmetical function communicated in relations of the design transmitter **x** which measures the worth of any design **x**. The selection of objective function is decided on the nature of the problem. The objective function for minimization is widely gotten like weight in aircraft plus aerospace constructional design problems.

For example, consider a structure, where the structure is subjected to design stress, displacement a the equation of optimization problem may be expressed as

$$\text{Minimize} \quad W = \sum_{i=1}^{N} \rho_i l i A_i \tag{6.4}$$

$$\text{Subjected to} \quad (\sigma^u) \geq (\sigma) \geq (\sigma^l) \tag{6.5}$$

$$(d^u) \geq (d) \geq (d^l) \tag{6.6}$$

Where σ , d , A are stress, displacement and cross sectional area and subscripts *u* and *l* refer to prescribed upper and lower boundaries of each constraints.

A_i Is the cross sectional area of element *i*.

In this study the optimization refers to weight optimization of curved roof structure by combining FE analysis program with GA optimization methods as shown in Figure 6.2.

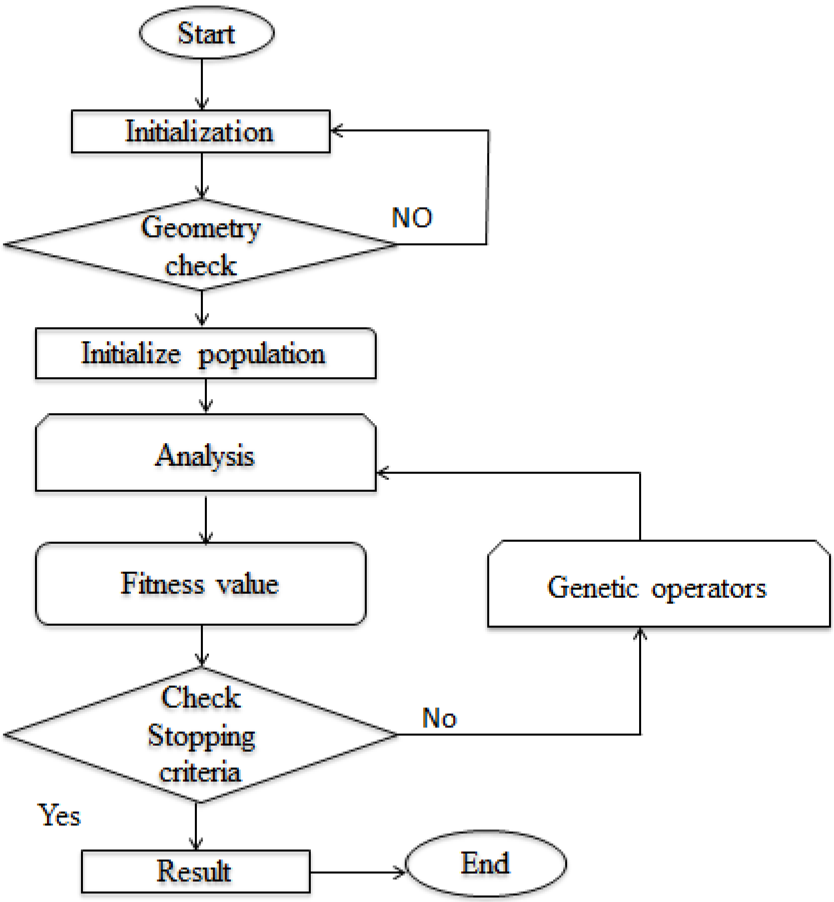


Figure 6.2 Structural optimization flow chart

An optimization problem requiring various objective functions is recognized as a multiobjective programming problem. With various objectives there arises from a probability of conflict, as well as one easy method to handle the problem is to build an overall objective function like a linear combination of the conflicting various objective functions. So if $f_1(X)$ also $f_2(X)$ indicate two objective functions, build a modern (overall) objective function for optimization as.

$$F(X) = \alpha_1 f_1(X) + \alpha_2 f_2(X) \tag{6.7}$$

Where $\boldsymbol{\alpha_1}$ with $\boldsymbol{\alpha_2}$ are quantities whose costs record the comparative significance of one objective function comparative to another.

6.5 GA Optimization Method

Randomized search with the optimization system guided by the principle of natural genetic systems. Since GA is established on the survival of the fittest principle of nature, they attempt to maximize a function named the fitness function. Thus, GAs are obviously suitable for solving unconstrained maximization problems. GAs are stochastic optimization methods founded upon the mechanism of natural evolution as well as endurance of the fittest strategy establishes in biological organizations.

Characteristics contain a survival of the fittest mechanism in which potential results in a population are pitted opposing each other, as well as recombination of solutions in a mating operation with random varieties. The unbelievable section is that this heuristic rule can "evolve" best and best solutions without any great understanding of the problem itself. GA can be executed to any problem that contains these two characteristics: **(i)** a result can be squeezed like a string, in addition to **(ii)** a rate denoting the valued at of the string can be computed.

GAs is successfully suited for solving such problems, within nearly all cases. They can determine the global optimal result by a high possibility. While GAs were initial presented methodically by [28].

Although randomized, GAs is not easy random exploration systems. They efficiently investigate the new combinations by means of the attainable knowledge to determine a new generation with best fitness or else objective function rate.

6.5.1 Advantages of GA's

GA optimization system does not require any prior knowledge such like derivatives of the objective function otherwise constraint functions.

The probability with flexibility of dealing among complicated structures under various loading conditions as well as constraints.

The capacity of dealing through parts from standard lists sorted like discontinuous design variables.

The ability of obtaining further than one design result.

The flexibility of formulating the engineer experiences also qualifications to the design optimization problem.

6.5.2 Genetic operators

There are three principal operators in a fundamental GA: *mutation, crossover* and *reproduction*. The solution of an optimization problem by GAs begins through a population of random strings indicating a number of (populations of) design transmitters. The population magnitude in GAs (n) is generally fixed. Each string (or else design transmitter) is estimated to determine its fitness cost.

The population (of designs) is acted at three operators-crossover mutation, also reproduction to manufacture a new population of points (designs). The new population is also estimated to determine the fitness rates in addition to experimented for the convergence of the operation.

Single cycle of variation, reproduction with crossover as well as the estimation of the fitness values is recognized as a generation in GAs. Whether the convergence criterion is not gratified the population is iteratively controlled by the three operators, as well as the resulting modern population is valued for the fitness rates. The system is continued during some generations up to the time of the convergence criterion is satisfied, also the operation is finished. The inside information of the three operations of GAs are presumption below.

6.5.2.1 The reproduction operator

The reproduction is equivalent to the "survival of the fittest" contest. It decides not just which solutions survive, although how many duplicates of each of the survivors to construct. This will be significant later for the time of the crossover action. The possibility of survival of a solution is symmetrical to its solution value; in addition, recognized like its fitness (the function that grants values to solution strings is as well recognized as the fitness function).

In generally used reproduction operative, a string is chosen as of the mating pool by means of a probability equivalent to its fitness. Thus if F_i indicates the fitness of the i^{th} string in the population of size n, the possibility for choosing the i^{th} string for the mating pool (P_i) is given by.

$$P_i = \frac{F_i}{\sum_{i=1}^{n} Fi} \qquad ; i=1, 2, \ldots, n \qquad (6.8)$$

Where F_i is the fitness otherwise objective function value of the i^{th} individual (design transmitter, x_i) also n is the volume of population. Thus, designs (individuals) by means of higher fitness values have a larger opportunity of being chosen for mating as well as following genetic action. As a result, greatly fit individuals. Live plus procreate and fewer fit individuals die (survival of the fittest).

6.5.2.2 Recombination or crossover

The crossover operation includes the changing of genetic bit strings (materials) between the two parents. Choosing randomly a place by this operator (along the two chromosomes a bit position) and swaps the sub strings before and after that point between two chromosomes to generate two offspring (new generation). There are various types of crossover can take place, hence these types are of the simplest is explained.

One-point crossover: The easiest kind of the crossover of all of the available kinds. When selecting a pair of individuals to exchange their first few bits and the results are a new pair of children. So suppose a selecting parent's pair from the mating pool is displayed in Figure 6.3, [55].

Parents1	0 0 0 0 1 1 1 1
Parents2	1 1 1 1 0 0 0 0

Figure 6.3 Parents before crossover.

Beside a randomly chosen between 1 and the string length of an integer place n along the string is taken. The established of two new strings by exchanging each character between positions 1 and n inclusively. Assume in selecting a random number between 1 and 8, taking n = 5, the results of crossover produces the two new strings displayed in Figure 6.4.

Parents1 1 1 1 1 0 1 1 1

Parents2 0 0 0 0 1 0 0 0

Figure 6.4 New strings after crossover operation.

The crossover probability (P_c) is an essential parameter in crossover performance. It is a parameter to explain how often crossover will be achieved. The off springs are exact copies of parents, if there is no crossover. The off springs create from portions of both parent's chromosomes, if there is crossover.

Then all offspring is created by crossover, if the crossover probability is 100%. All new generation is created from exact copies of parent chromosome from old population (but this does not mean that the new generation is the same), when it is 0%. The creation from the crossover is in expected that off springs will include good parts of the parents and therefore the off springs will be improved. Though, it is better to allow a specific portion of old population survives to the next generation.

6.5.2.3The mutation operator

The new strings gotten from crossover (off springs) are put in the new population, also the procedure is continued. The third operative of the ordinary GA is the variation which acts like a secondary part in the action of the GA. Variation is required since, in spite of though reproduction as well as crossover effectively examination with recombine having to be strings, to permit new genetic parents to be shaped progressing towards the search approach.

In artificial genetic techniques, the variation operator sometimes safeguards some effective genetic material imposingly loss. In GAs, mutation is the occasional arbitrary alternation of the rate of a string position.

The mutation operator is directed to the new string with a stipulated mutation probability. A mutation is the irregular random transposition of a binary digit (allele's value). Hence in mutation a 0 is changed to 1, as well as vice versa, at a random situation. When used sparingly in the company of the generation with crossover operators, variation serves like protect against a premature loss of significant genetic material by a particular situation.

6.5.3 Overview of the fundamental GA operation

At present, that we have seen the fundamental GA operators, we can place the full process with each other. In this place are the necessary steps:

0. Design the algorithm: select the population magnitude n as well as mutation rate; select the operators with the stopping surroundings (further on stopping conditions later).

1. Randomly generate an initial population (further on generating the initial population later) also estimate the fitness value for each string. Set the solution through the best value of the fitness function in the initial population.

2. Apply the reproduction operator to the current population to generate a mating pool of size **n**

3. Apply the crossover operator to the strings in the mating pool to generate a tentative new population of size **n**.

4. Apply the mutation operator to the experimental new population to produce the last new population. Compute the fitness values of the solution strings in the new population also renew the incumbent solution if there is a best solution in this population.

5. If the stopping conditions are met, then exit with the incumbent solution like the last solution. In other, respects go to Step 2.

CHAPTER 7

OPTIMIZATION EXAMPLES

7.1 2D Truss Optimization Examples

In this part 2D truss examples which are analyzed in chapter 5 are optimized under static loads. The objective function is weight minimization under stress and displacement constraints.

7.1.1 2D curved truss with 5.75m height

The 71 bar curved truss in Figure 5.1 of chapter 5 is to be optimized for minimum weight. The structure is loaded with a point load of 10 kN on all upper joints in the-*Z* direction. Three design variables are considered by the GA where design variable $s_1 = A_1, s_2 = A_2$ and $s_3 = A_3$. Material properties for the truss are: Young's modulus $E = 200 \times 10^6$ kN/m^2, material density $\rho = 76.9729$ kN/m^3, maximum tensile stress $\sigma_t = 120 \times 10^3$ kN/m^2, and maximum compressive stress $\sigma_c = -120 \times 10^3$ kN/ m^2 and maximum u_x and u_z displacement all nodes being 0.015 m.

Discussion of the results: Table 7.1 shows the displacement of some important selected members before and after optimization. It is seen that after optimization displacements are smaller than displacement constraints. From the results demonstrated in Table 7.2 it can be observed that the optimum values found for the final weights. The results got applying the GA for continuous design variable quantities. After 238 iterations, minimum weight design was obtained for continuous design variables. The weight of the truss is reduced from 11.107 to 2.1812 (80 % reduction) for continuous design variables. The iteration record of the optimization method is demonstrated in Figures 7.1 for continuous design variables.

Table 7.1 Joint displacements before and after optimization

Displacement (m)				
Joint no.	X-direction		Z-direction	
	Before optimization	After optimization	Before optimization	After optimization
2	-0.0004	-0.0008	-0.0003	-0.0002
3	-0.0004	-0.0007	-0.0013	-0.0015
9	0.0000	0.0002	-0.0068	-0.0125
10	0.0000	-0.0002	-0.0068	-0.0125
16	0.0004	0.0007	-0.0013	-0.0015
17	0.0004	0.0008	-0.0003	-0.0002
20	-0.0002	-0.0005	-0.0001	-0.0006
21	-0.0002	-0.0008	-0.0002	-0.0006
27	0.0004	0.0004	-0.0066	-0.0122
28	0.0000	0.0000	-0.0068	-0.0127
29	-0.0004	-0.0009	-0.0066	-0.0122
35	0.0002	0.0008	-0.0002	-0.0006
36	0.0002	0.0005	-0.0001	-0.0006

Table 7.2 Initial and optimum design variables of 2D curved truss with 5.75m height

Cross sectional area (m^2)		
Design variables	Initial	Optimum
S_1	0.000645	0.0008
S_2	0.000897	0.0001
S_3	0.001770	0.0001
Weight (kN)	11.107	2.1812
P.R		80 %

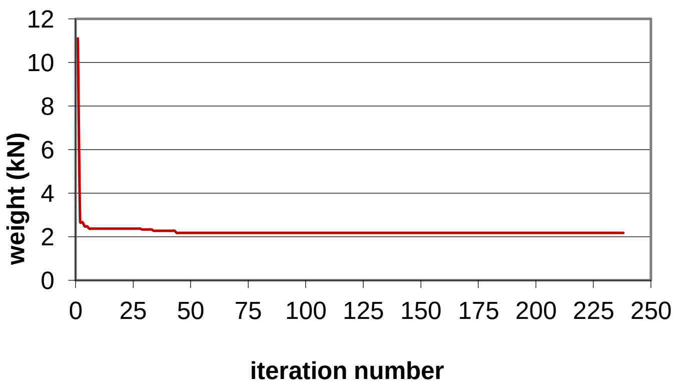


Figure 7.1 Convergence curve for 2D truss with 5.75 m height

7.1.2 2D curved truss with 7.5 m height

The 71 bar curved truss of Figure 5.3 in chapter 5 is to be optimized for minimum weight. The structure is loaded with a point load of 10 kN on all upper joints in the *Z* direction. Three design variables are considered by the GA where design variable $s_1 = A_1, s_2 = A_2$ and $s_3 = A_3$. Material properties for the truss are: Young's modulus $E = 200 \times 10^6$ kN/m^2, material density $\rho = 76.9729$ kN/m^3, maximum tensile stress $\sigma_t = 100 \times 10^3$ kN/m^2, and maximum compressive stress $\sigma_c = -100 \times 10^3$ kN/ m^2 and maximum u_x and u_z displacement all nodes is 0.015 m.

Discussion of the results: Table 7.3 shows the displacement before and after optimization of some selected members. The GA optimization satisfied displacement constraints.

From the results demonstrated in Table 7.4 it can be seen that the optimum values found for the final weights. The results got applying the GA for continuous design variable quantities. After 247 iterations, minimum weight design was obtained for continuous design variables. The weight of the truss is reduced from 9.1408 to 4.0919 (55.3 %reduction) for continuous design variables. The iteration record of the optimization method is demonstrated in Figures 7.2 for continuous design variables.

Table 7.3 Joint displacements before and after optimization

Displacement (m)				
Joint no.	**X-direction**		**Z-direction**	
	Before optimization	**After optimization**	**Before optimization**	**After optimization**
2	-0.00055	-0.00095	-0.00046	-0.00034
3	-0.00054	-0.00078	-0.00184	-0.00219
9	0.00011	0.00027	-0.01125	-0.01738
10	-0.00011	-0.00027	-0.01125	-0.01738
16	0.00054	0.00078	-0.00367	-0.00219
17	0.00055	0.00095	-0.00184	-0.00034
20	-0.00033	-0.00050	-0.00015	-0.00050
21	-0.00037	-0.00079	-0.00037	-0.00066
27	0.00064	0.00105	-0.01082	-0.01692
28	0.00000	0.00000	-0.01122	-0.01758

29	-0.00064	-0.00105	-0.01082	-0.01692
35	0.00037	0.00079	-0.00037	-0.00066
36	0.00033	0.00050	-0.00015	-0.00050

Table7.4 Initial and optimum design variables of 2D curved roof truss with 7.5 m height

Cross sectional area (m^2)		
Design variable	**Initial**	**Optimum**
S_1	0.000955	0.0010
S_2	0.000955	0.0001
S_3	0.000690	0.0003
Weight (kN)	9.1408	4.0919
P.R	55.3 %	

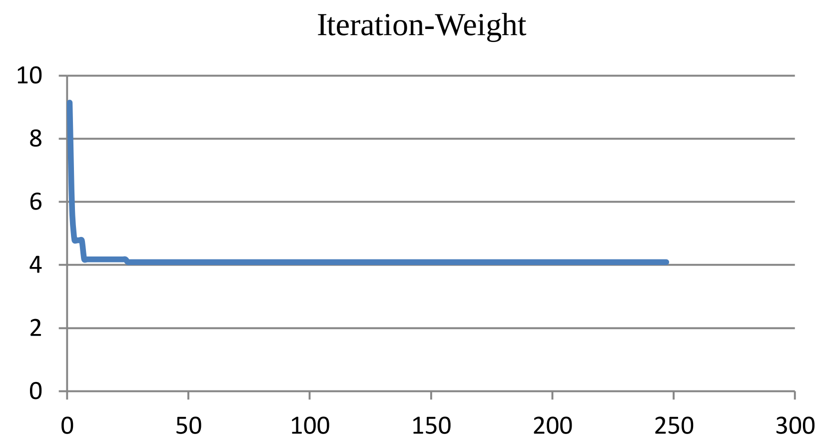


Figure 7.2 Convergence curve for 2D truss with 7.5 m height

7.1.3 2D curved truss with 288 inch height

This example consists of curved truss with 2000 inch span with 41 elements in Figure 5.5 in chapter 5. The structure is loaded with a point load of 50 kip on all upper joints in the *Z* direction. The members of the structure are divided into 3 groups, each one having one design variable.

The objective function is the weight (or volume) minimized. Material properties for the truss are: Young's modulus E=29000 kip/in^2, material density $\rho = 2.386 \times 10^{-4}$

kip/in^3, maximum tensile stress $\sigma_t = 20\text{ksi}$, maximum compressive stress $\sigma_c = -15\text{ksi}$ and maximum u_x and u_z displacement all nodes being 2 inch.

Discussion of the results: Table 7.5 shows the displacement before and after optimization of some selected members. The GA optimization satisfied displacement constraints. From the results demonstrated in Table 7.6 it can be observed that the optimum values found for the final weights. The results got applying the GA for continuous design variable quantities. After 65 iterations, minimum weight design was obtained for continuous design variables. The weight of the truss is reduced from 117.1699 to 53.721 (54 %reduction) for continuous design variables. The iteration record of the optimization method is demonstrated in Figures 7.3 for continuous design variables.

Table 7.5 Displacement result for all members before and after optimization

Displacement (in)				
Joint no.	**X-direction**		**Y-direction**	
	Before optimization	**After optimization**	**Before optimization**	**After optimization**
1	0.0000	0.0000	0.0000	0.0000
2	-0.0169	-0.0401	-0.1120	-0.2601
3	-0.0048	-0.0109	-0.3281	-0.7718
4	0.0051	0.0124	-0.5414	-1.2723
5	0.0061	0.0144	-0.7038	-1.6524
6	0.0000	0.0000	-0.7875	-1.8485
7	-0.0061	-0.0144	-0.7038	-1.6524
8	-0.0051	-0.0124	-0.5414	-1.2723
9	0.0048	0.0109	-0.3281	-0.7718
10	0.0169	0.0401	-0.1120	-0.2601
11	0.0000	0.0000	0.0000	0.0000
12	0.0160	0.0361	-0.0067	-0.0156
13	0.0380	0.0876	-0.1421	-0.3329
14	0.0565	0.1320	-0.3511	-0.8274
15	0.0561	0.1311	-0.5636	-1.3244
16	0.0342	0.0798	-0.7173	-1.6841
17	0.0000	0.0000	-0.7968	-1.8703
18	-0.0342	-0.0798	-0.7173	-1.6841

19	-0.0561	-0.1311	-0.5636	-1.3244
20	-0.0565	-0.1320	-0.3511	-0.8274
21	-0.0380	-0.0876	-0.1421	-0.3329
22	-0.0160	-0.0361	-0.0067	-0.0156

Table7.6 Initial and optimum design variables of 2D curved truss with 288 inch height

Crosses sectional area (in^2)		
Design variables	**Initial**	**Optimum**
S_1	75.6000	34.6667
S_2	14.7000	6.3330
S_3	20.0000	10.0000
Weight (kip)	117.1699	53.7210
P.R	54 %	

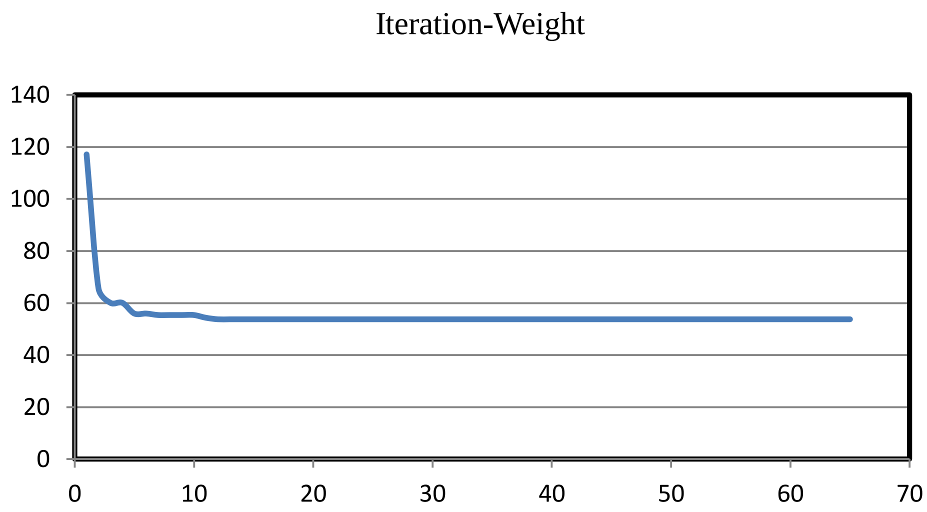
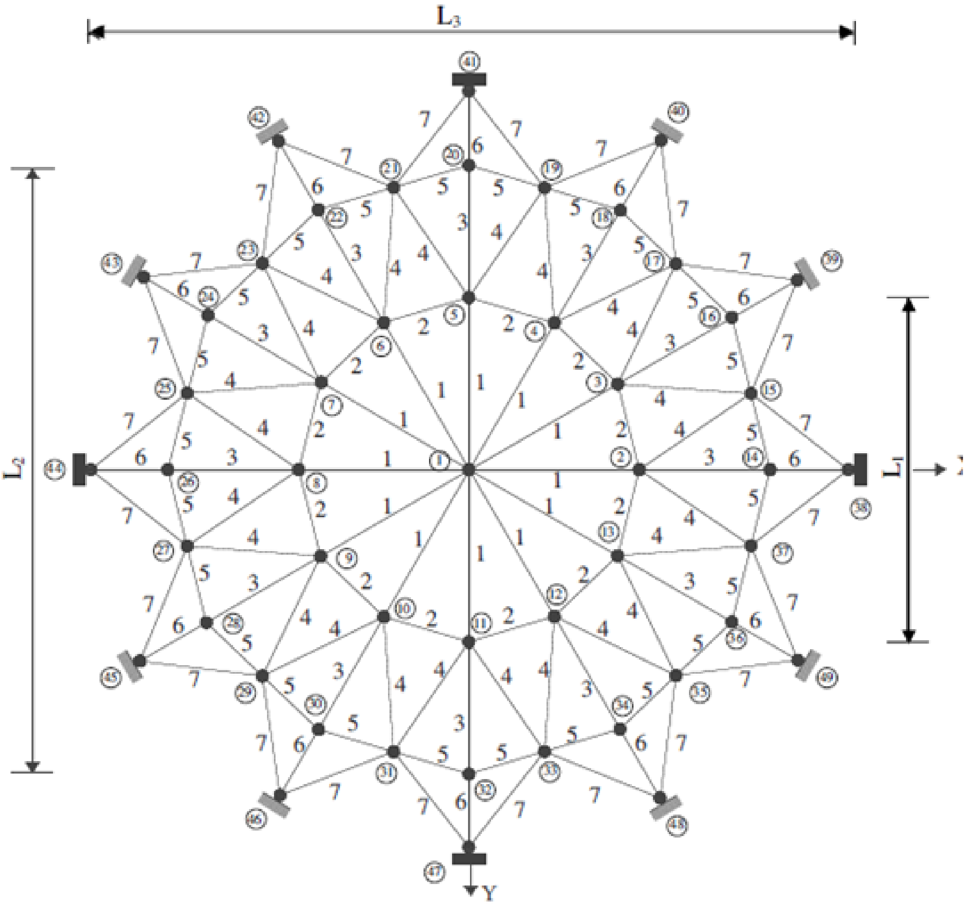


Figure 7.3 Convergence curve for 2D truss with 288 inch height

7.1 4 3D Truss Examples

In this part 3D truss dome example which is analyses in Chapter 5 are optimized under static loads. The objective function is weight minimization under stress and displacement constraints. As shown in Figure 7.4 the dome has 49 joints and 120 members which are collected into seven different groups. The truss is subjected to vertical loading at all the unsupported joints. These are taken as 13.49 kips at node 1, 6.744 kips from nodes 2 through 14 and 2.248 kips at rest of the nodes.



L_1=546.61 in. (13883.89 mm) L_2=948.25 in. (24085.55 mm) L_3=1251.02 in (31775.91 mm)

Figure 7.4 Top view of 3D dome

In addition to allowable tensile and compressive stresses, an upper limit for the displacement is taken as -0.19 in. at each node. The allowable compressive and tensile stresses are -15, 35 kip, Young modulus =290000 kip/in^2 and density ρ =0.00029 kip/in^3.

Discussion of the results: Table 7.7 shows the displacement before and after optimization of some selected members. The GA optimization satisfied displacement constraints. From the results demonstrated in Table 7.8 it can be seen that the optimum values found for the final weights. The results got applying the GA for continuous design variable quantities. After 136 iterations, minimum weight design was obtained for continuous design variables. The weight of the truss is reduced from 26.644 to 12.8765 (51.6 %reduction). The iteration record of the optimization method is demonstrated in Figures 7.5 for continuous design variables.

Table 7.7 Displacement result for all members before and after optimization

Displacement (inch)						
Joint no.	**X-direction**		**Y-direction**		**Z-direction**	
	Before optimization	**After optimization**	**Before optimization**	**After optimization**	**Before optimization**	**After optimization**
1	0.0000	0.000	0.0000	0.000	0.0998	-0.076
2	-0.0432	-0.045	0.0000	0.000	-0.0985	-0.179
3	-0.0374	-0.039	-0.0216	-0.022	-0.0983	-0.179
4	-0.0216	-0.022	-0.0374	-0.039	-0.0983	-0.179
5	0.0000	0.000	-0.0433	-0.045	-0.0985	-0.179
6	0.0216	0.022	-0.0374	-0.039	-0.0984	-0.179
7	0.0374	0.039	-0.0216	-0.022	-0.0984	-0.179
8	0.0432	0.045	0.0000	0.000	-0.0985	-0.179
9	0.0374	0.039	0.0216	0.022	-0.0985	-0.179
10	0.0216	0.022	0.0374	0.039	-0.0985	-0.179
11	0.0000	0.000	0.0432	0.045	-0.0985	-0.179
12	-0.0216	-0.022	0.0374	0.039	-0.0984	-0.179
13	-0.0374	-0.039	0.0216	0.022	-0.0984	-0.179

Table7.8 Initial and optimum design variables of 3D curved roof dome

Cross sectional area (inch^2)		
Design variables	**Initial**	**Optimum**
S_1	6.11	2.3032
S_2	2.21	2.1581
S_3	2.21	0.9806
S_4	2.71	2.1581
S_5	1.51	1.0355
S_6	5.61	1.1290
S_7	4.41	2.1581
Weight (kip)	26.644	12.8765
P.R	51.6%	

Figure 7.5 Convergence curve for 120 bar 3D truss

7.2 Arch Optimization Examples

7.2.1 Arch under point load

This example involves optimization of a series of arches with rectangle cross-sections, which have been analyzed by Litewka and Rakowski [54] and in chapter 5. The geometry and loadings of arch which has uniform cross-section with opening angle 120 is considered shown in Figure 7.6. The arches have a radius of curvature R = 4 m, the angle $\omega = 2\pi/3$ (span length $l = 8\pi/3$), the rectangular cross-section with depth h = 0.6 m and width b = 0.4 m. The following material properties are used: elastic modulus E = 30, GPa and Poisson's ratio v = 0.17.

Discussion of results: Table 8.9 shows the initial and optimal values of design variables and weight for the different loadings considered. In all loading cases there is considerable reduction in the magnitude of the weight. The percentage reductions obtained in the weight are 64.56, 62.09 and 72.68 % for loading cases a, b and c respectively.

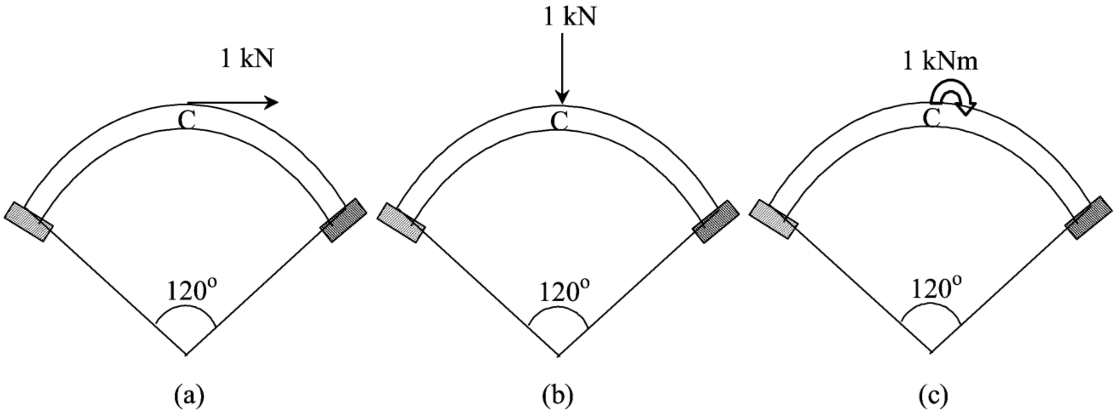


Figure 7.6 Loading conditions of arch

Table 7.9 For weight minimization of arch, initial and optimum values of design variables

Design variables				**Optimum**		
Type	**Minimum**	**Initial**	**Maximum**	**Case (a)**	**Case (b)**	**Case (c)**
t_1	0.1	0.6	1.0	0.1992	0.1501	0.2319
t_2	0.1	0.6	1.0	0.1743	0.1932	0.1704
t_3	0.1	0.6	1.0	0.4391	0.4853	0.1836
Weight		5.009		1.7752	1.8991	1.3682
P.R				64.56%	62.086%	72.68%

7.2.2 Solid arch under multi point load

This example involves optimization of an arch with circular cross-sections, the geometry and loading of arch which has uniform cross-section with 20 m span is considered shown in Figure. 7.7. The arches have a radius of curvature R = 11.547 m, the angle $\omega = 2\pi/3$ (span length l = 24.1383 m), the cross-section area = 0.008968. The following material properties are used: Young's modulus $E = 200\times10^6$ kN/m^2, material density $\rho = 76.9729$ kN/m^3, maximum tensile stress $\sigma_t = -120\times10^3$ kN/m^3, maximum compressive stress $\sigma_c = -120\times10^3$ kN/m^3 and maximum u_x and u_z displacement all nodes is 0.015 m.

Discussion of results: Table 7.10 shows the initial and optimal values of design variables and weight. After 42 iterations, minimum weight design was obtained for discrete design variables. The weight of the truss is reduced from 20.618 to 9.698 (53

%reduction).

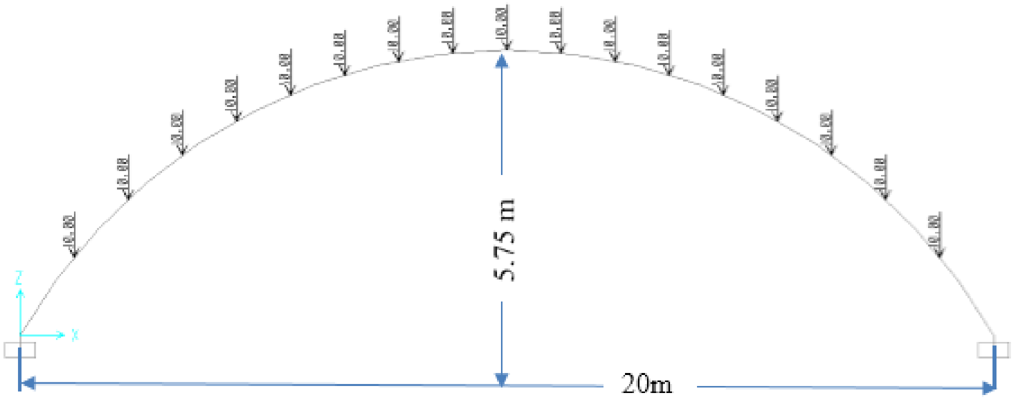


Figure 7.7 Loading conditions of arch

Table 7.10 Initial and optimum values of design variables

Crosses sectional area (m^2)		
Design variables	**Initial**	**Optimum**
S_1	8.968×10^{-3}	5.219×10^{-3}
Weight(kN)	20.618	9.698
P.R	53 %	

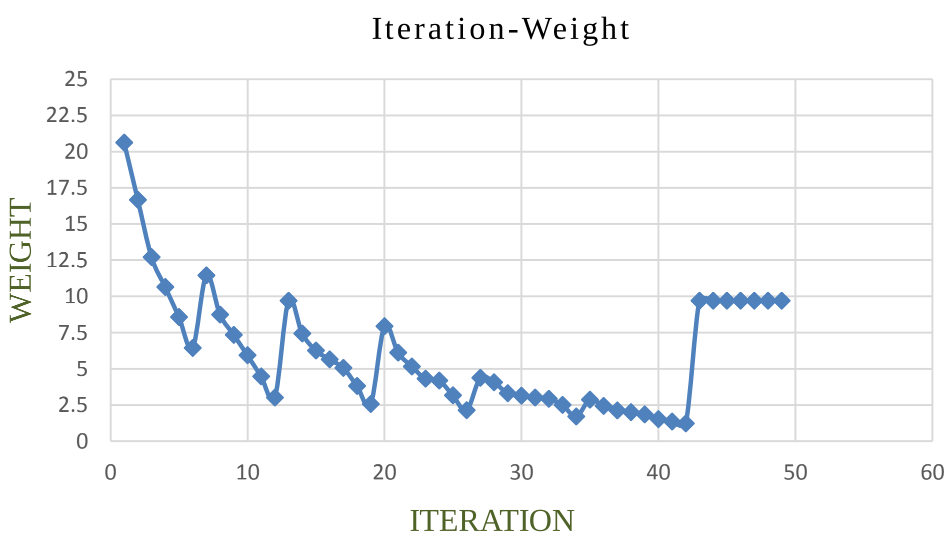


Figure 7.8 Convergence curve for solid arches under distributed load

CHAPTER 8

CONCLUSIONS AND FURTHER WORK

8.1 Conclusions

A design methodology of 2D and 3D curved roof trusses and solid arch roofing that combines stiffening sizing optimization is an important role in minimizing the amount of steel used in the construction of the structure for economic point of view. The optimization procedure implemented, combined with accurate FE simulation of steel curved roof and solid arch roofing, resulted in a robust and efficient optimization tool.

Optimization algorithm is starts following the implementation of the analysis of the structure. A FORTRAN program which uses the FEMs based numerical analysis was modified. To achieve size optimization based on GA to perform the analysis and design. The problem of choosing the sizes of the bars in order to minimize the weight of the structure while satisfying stress, displacement, stability.

To find the best solution under constrains of allowable displacement and stress GA searches all the available solution among all available results the best solution is selected. Design variables were considered corresponding to the sizing of the cross-sectional areas of the bars.

For all design variables significant decrease in weight of material with respect to the stress and displacement constraints were get. Finally, it must be emphasized that the algorithm proposed is capable of finding the optimum weight or volume with the least number of groups possible to make the design practical. Hence, the solution is feasible and the construction of the structure is easy. The results obtained on these typical problems showed that the accuracy of the concept presented is more than those of the other methods.

8.1.1 Structural analysis

The FE method is accurate and robust tools for analysis of curved roof structures were developed design variables were considered corresponding to the sizing of the cross-sectional areas of the bars. One can observe from the results of the analysis the importance of carry out the linear procedures since these analysis can lead to different final weights, mainly, the final values of the cross-sectional areas. The designer has to be attempt in order to choice the adequate analysis to be conducted.

The results obtained from the program are compared with other sources to prove the ability and accuracy. This comparison helps to improve the written program to give better solutions.
To perform the formulation of the stiffness matrix which is coded in FORTRAN, several examples are considered. FE model can be successfully used for the static analysis of the structures. Analysis is done by the FE method coded program for analysis of 2D and 3D curved truss roofing, results are compared with source program (SAP2000).

8.1.2 Structural optimization and design

A general methodology for structural size optimization of curved roof structures has been presented by integrating the tools developed for size definition, FE analysis with GA.

GAS seems to be a good choice for continuous structural optimization. They offer several advantages that other techniques lack such as generality and the ability to deal directly with discrete search spaces. GAs performed well on a plane truss and **3-D** truss problem as compared to more traditional techniques. We do not claim that GAs is the golden key to structural optimization that engineers have sought for nearly two centuries. There is no doubt, however, that GAs has potential for automating discrete structural optimization.

Various optimization examples were presented for minimizing the weight of the curved roof structures. Crosses section design variables was used. The influence of the number of design variable employed was also investigated.

Reductions of (80 %, 55.3 %, 54 %, 51.6 % and in arch case a, 64.56 %, for case b, 62.086 %, for case c, 72.68 %, 53 %) for the six illustrated examples respectively give great encouragement to optimize structures. These reductions are important to save extra materials in construction projects of curved roof structures consequently serving the economical point.

By comparing the results of two examples in section 7.1.1 and example 7.1.2 in optimization of arch with uniform cross section, it’s clear that the curved truss structure is stiffer than curved arch. As shown in tables (7.2 and 7.10) for the same constraints the curved truss structure is lighter than curved arch for about 77.5%.

The features of software in the analysis and design of special and complex structures with application of FEM is wide used. Using this application without a background in analysis and design decreases the degree of accuracy in modeling and obtaining correct results.

8.2 Future Work

Expanding the code in such a way that covers all types of structural members subjected to actual constraints of the American Institute of Steel Construction(AISC) Load and Resistance Factor Design (LRFD) and Allowable Stress Design (ASD) specifications (American Institute of Steel Construction) and also to use actual steel section as a discrete design variables.

Expanding the FORTRAN code in such a way that covers members buckling check, this concluded from comparing the results of the source program and GA.

Another drawback GAs requires large number of response (fitness) function evaluations depending on the number of individuals and number of generations.

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