Resource Constrained Two-Sided Assembly Line Balancing Problem

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ABSTRACT

RESOURCE CONSTRAINED TWO-SIDED ASSEMBLY LINE BALANCING PROBLEM

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Two-sided assembly lines are usually designed to produce high volume product like buses, trucks and automobiles. These types of assembly lines are used by both sides (left and right) in parallel. When large-size products are produced, sometimes they need a specific machine or qualified staff for using this machine. This also raises resource constrained assembly line balancing problem. Therefore, in this study mathematical model is presented for two sided assembly line balancing problem by considering generalized resource constraint. The presented mathematical model is solved using GAMS/CPLEX mathematical programming package. The mathematical model can find optimal solution for small–size test problems, but it doesn't give optimal solution within acceptable computation time for large scale test problems. For this reason, a heuristic approach based on COMSOAL is presented for solving large sized test problems. The validity of the proposed mathematical model and the heuristic algorithm are researched on known test problems**.**

Key Words: Resource constraint, Two-sided assembly lines, Mathematical model, Heuristic approach

ÖZ

KAYNAK KISITLI ÇİFT-TARAFLI MONTAJ HATTI DENGELEME PROBLEMİ

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Çift taraflı montaj hatları genellikle otobüs, traktör ve otomobil gibi büyük boyutlu ürünleri üretmek için tasarlanmıştır. Bu tip montaj hatları iki taraftan (sol ve sağ) da kullanılırlar. Büyük boyutlu ürünler üretildiğinde, bunları üretmek için bazen özellikli bir makineye veya bu makineyi kullanabilecek kalifiye bir personele ihtiyaç duyulabilir. Bu durum da kaynak kısıtlı montaj hattı dengeleme problemini ortaya çıkarmaktadır. Bu yüzden, bu çalışmada kaynak kısıtlarını da dikkate alarak çift taraflı montaj hattı dengeleme problemi için bir matematiksel model sunulmuştur. Sunulan matematiksel model GAMS/CPLEX matematiksel paket programlama ile çözülmüştür. Matematiksel model küçük test problemleri için en iyi sonuç bulabilirken, büyük test problemleri için makul hesaplama süresinde en iyi çözümü verememektedir. Bu nedenle, büyük boyutlu test problemlerini çözmek için COMSOAL tabanlı bir sezgisel sunulmuştur. Önerilen matematiksel model ve sezgisel algoritma literatür de bilinen test problemleri üzerinde incelenmiştir

Anahtar Kelimeler: Kaynak kısıtı, Çift taraflı montaj hatları, Matematiksel model, Sezgisel yaklaşım

To My Parents

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LIST OF ABREVIATIONS

ALB	Assembly line balancing
GALB	General assembly line balancing
TSALB	Two sided assembly line balancing
GRCTALB	Generalized resource constrained two sided assembly line balancing
CNF	Conjunctive normal form
DNF	Disjunctive normal form
SULB	Simple u-type line balancing
FFR	First fit rule
WR	Work relatedness
WS	Work slackness
ACO	Ant colony optimization
GA	Genetic algorithm
BA	Bee algorithm
TSUALBP	Two sided u-type assembly line balancing problem
CTALBP	Cost-oriented two sided assembly line balancing problem
ABCO	Artificial bee colony optimization
PSO	Particle swarm optimization
SA	Simulated annealing
TSA	Tabu search algorithm
RCALB	Resource constraint assembly line balancing

CHAPTER I

INTRODUCTION

An assembly line system is an arrangement of machines, employee and equipment for product being assembled passes successively from operation to operation until completed. The basic elements of assembly line systems are constituted by workstations, labors, products and tasks. The workstation is a place where workers perform some operations in order that raw material converts to product. Figure 1.1 shows an example of assembly line including four stations. The smallest and indivisible operation in assembly line system is named as a task. The time required to carry out a task is known as a task time or processing time. The task is performed according to some pre-specified limitations. These limitations are usually classified into two groups; demand satisfaction and precedence constraint. The precedence constraint defines technological requirement like some tasks can start only after completion of some other tasks. Demand satisfaction constraints pressure assembly lines to deliver a product at the end of the time period. This period, which is the time between completions of two products, is referred as cycle time of the assembly line. The rate of production is directly ascertained by the cycle time. Therefore, minimizing cycle time is equal to maximizing output rate of production.

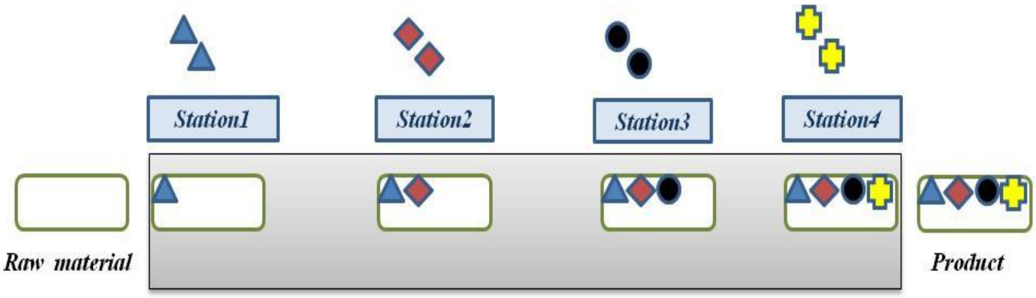

Figure 1.1. Example of assembly line including four stations

Assembly line balancing problem (ALB) is one of the well known manufacturing problems in the ALB literature. Objective of ALB problem is to minimize the cycle time, maximize the production rate, or minimize the line length and etc. Generally, ALB problem is conducted under the following constraints. Total of task times assigned to each of workstation cannot bigger than the cycle time (cycle time constraint). Each of tasks can be assigned to only one station (assignment constraint). The tasks must be assigned to the stations by taking into account the priority relationships between tasks (precedence relation constraint). These constraints are considered for simple assembly line balancing problem. On the other hand, market trends pressure companies in order to meet the requirement of the customer. To accomplish this goal, so many types of assembly line like parallel, two-sided and U-type lines are designed by considering technological restriction in production plants. In real industrial applications, these types of lines have so many restrictions such as side, zone and station load constraint.

The size of the product is important to produce a product. Products like car, truck and bus cannot assemble by one side, because operator at the other side of the line may not be accessible by opposite side of the product. Therefore, two sided lines are designed to produce large size product. Both of the sides are used by operators to work on the same work piece simultaneously. This type of lines ensures shorter line length, reduced output time, lower cost of tools and fixtures, and less material handling (Kim et al., 2009). When large size products are produced, sometimes they require a specific machine or a qualified staff to use this machine. This condition also exposes the problem of resource constraint. For this reason, this thesis is concerned on two sided assembly line balancing problem to consider generalized resource constraint.

The aim of thesis is to present a solution on two-sided assembly line balancing problem under resource constraints. The purpose of objective function is to minimize number of workstation and total resource cost for given cycle time and resources. To achieve this goal, a mathematical model for generalized resource constrained two sided assembly line balancing problem is presented to solve this problem in an optimal manner. For large size test problems, the mathematical model fails to give the optimal solution within reasonable computational times. Therefore, a heuristic approach based on the COMSOAL is proposed.

This thesis is separated into six chapters. In the first chapter, the main goal and subject of the study are given briefly.

In Chapter 2, an overview and classification of assembly lines balancing problem are presented. General literature review is presented and is tabulated according to methodology and type of problem.

In Chapter 3, mathematical model is presented to solve generalized resource constrained two sided assembly line balancing (GRCTSALB) problem. Objective functions of model are introduced. For verification of the model, an illustrative example is given and at end of the chapter, results of test problems are given and evaluated.

In Chapter 4, multi objective approach cases are examined. Computational analysis is performed for multi-objective situation. Proposed models are run and analyzed over several known test problems by considering three objective functions (Minimization number of station, number of position and resource cost). Results of the objective ranking are compared with each other. Also performance of the proposed models is compared according to average of solution time for objective ranking.

In Chapter 5, Heuristic approach is proposed for resource constraint two sided assembly line balancing problem. The objective of proposed based on COMSOAL heuristic is to minimize opened workstation cost and resource cost.

Finally in Chapter 6, conclusions and the possible future research directions about the problem are pointed out.

CHAPTER II

LITERATURE REVIEW

2.1 Classification of Assembly Line Balancing Problem

Assembly line production systems are presented in different industrial environments and are utilized to manufacture a large variety of products. Especially, they are used to produce consumer goods such as cars, engines, domestic appliances, television sets, computers and other electrical appliances. These products are rather different, and it is necessary to implement different production systems (Scholl, 1999). Figure 2.1 also shows main characteristic of assembly line balancing problem.

A remarkable amount of research is available for solving assembly line balancing problem. Simple assembly line balancing (SALB) problem is well-known and well-studied type of assembly line balancing problem in the literature. SALB is basic problem of ALB. Assembly line balancing problem also have several objective functions.

<u>Four different objective functions are presented as follows</u> (Becker & Scholl, 2006).
Type I: Cycle time is given in this type of objective function. The aim is to minimize number of workstation under determined cycle time.

Type II: In this type of problem, number of workstation and employees are known. The objective function is to minimize cycle time or maximize production rate under the determined workstation.

Type E: Objective of this problem type is maximizing line efficiency to consider cycle time and the number of workstations simultaneously. Maximizing line efficiency is also equally to minimize idle time of workstations.

Type F: Cycle time and number of workstation are given. The objective function is to find a feasible balance.

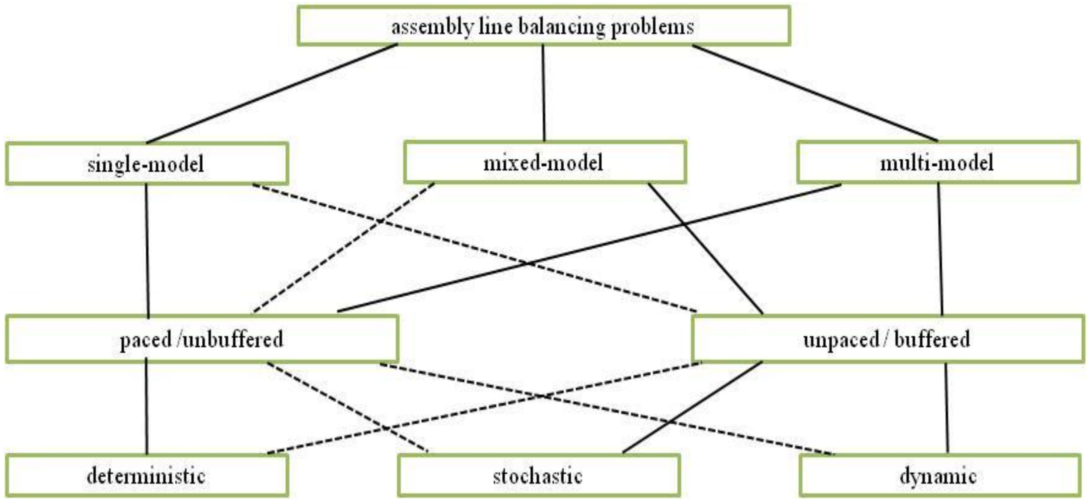

Figure 2.1. Classification of assembly line balancing problem (Scholl, 1999)

Assembly line system has very different range in the real manufacturing environment. Generally, assembly line system can be differentiated in terms of number of models, variability of task processing, line layout, assignment restrictions, type of stations and type of line etc. (Rekiek & Delchambre, 2006).

In the literature, the assumptions of SALB problem are relaxed and different model extensions are taken into account. Besides, alterations according to the objective are studied. A detailed classification of ALB problems was presented by the work of Boysen et al. (2007).

The most common alteration of assembly line system is demonstrated as follows.

2.1.1 Number of Models

The number and variety of assembled product in the line can be classified as single model, mixed model and multi-model. Figure 2.2 demonstrates classification of assembly line according to number of models.

Single Model line: Single model lines are suitable for large scale production of only one product. If a single product is assembled on assembly line in high volume, this type of line is named as a single model line.

Multi-model line: In most of the modern manufacturing environments, several products or different models of the same base product often share the same assembly line. In other words, these types of assembly lines are used to produce sequences of batches of one model with intermediate setup operations.

Mixed-model line: Mixed model assembly line provides sequential production by mixing more than one product on the same line. Range of product manufactures on the same line are quiet similar to the basic product.

2.1.2 Time of Processing

Another considerable characteristic defining different versions of assembly line system is the variability of operation times, and also it can be categorized with regard to nature of task operation time which is deterministic, stochastic and dynamic operation time. Because of worker's physical structure, psychology and social environmental factor, operation time of a task can show variability.

Deterministic operation time: When expected variance in task time has very small changes, the task operation time is defined as deterministic. Especially, automated production systems change in process time is to minimum.

Stochastic operation time: Stochastic operation time shows significant variability of in task times. It also affects performance of system. Generally, these types of operation times are originated machine breakdowns, worker's rate ability, and a lack of motivation.

Dynamic operation time: When processing time change over time due to learning effects or successive enhancement of the production processes, the task operation times are supposed to as dynamic.

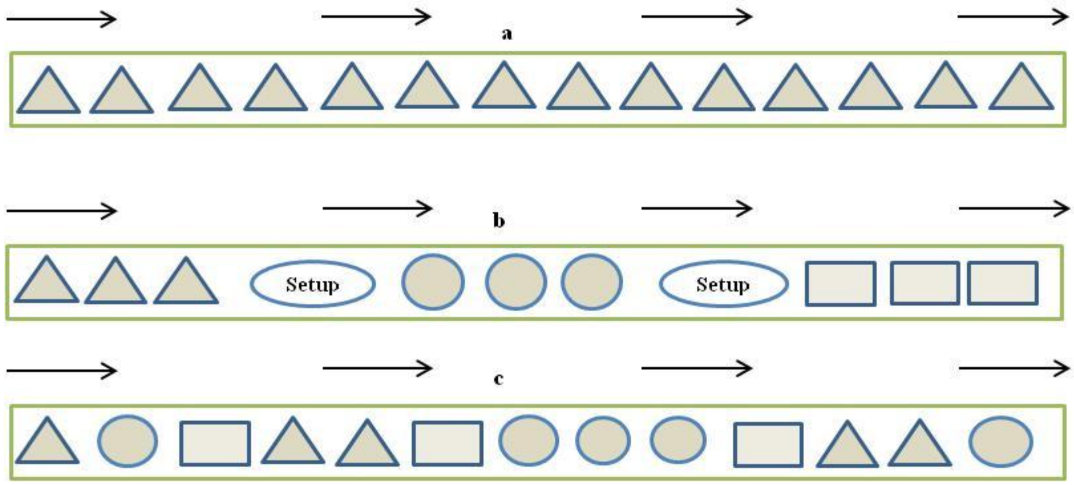

Figure 2.2. Assembly lines for a) single model b) multi-model and c) mixed-model

2.1.3 Paced and Unpaced Lines

Assembly lines can be distinguished by line control as paced and unpaced line (Groover, 2001).

In paced assembly line, each of workstation has a determined time, which is cycle time, for complete all task. These types of assembly line, the product follow to the next station at a constant speed because cycle time fixed for every workstation. Therefore, paced assembly lines have constant production rate equal to mutual of cycle time.

In unpaced assembly line has not fixed time for all of workstation. Unpaced assembly line control is usually applied if there is stochastic variability of processing time. In this type of assembly line, all stations operating at the individual rate so that the work parts will have to wait before the following stations. Hence, stations may have idle time when they have to wait for the next work element. For this reason, in unpaced lines allow to buffers between workstation for overcome this problem.

2.1.4 Layouts of Lines

Assembly lines can also be classified on account of line layout. Most common assembly lines layout can be explained as follows:

Straight (Serial) assembly line: Traditional assembly lines are known as serial or straight layout of assembly lines. In these type lines, stations are organized along a conveyor belt serially and workers carry out tasks on continues portion of the line (Scholl, 1999). Layouts of this type are generally preferred because it can be easily placement and it provides reduction of cost.

Parallel assembly line: parallel assembly lines layouts are designed by two serial lines placement as parallel. In this type of assembly line layout allows increase in flexibility and it gives advantage of complete all task with less employee. Generally parallel assembly line is shorter than straight line layout and this also provide less transportation cost. Moreover, the use of this line layout permits the expansion of cycle time and this gives some advantage like the risk of production stoppage. In parallel assembly lines, same or different products can be produced and cycle time can be same or different for each assembly line (Gökçen et al., 2006).

U-shaped assembly line: U-shaped line, the workstations are arranged around a U-shaped line, where legs are closely together (see Figure 2.3). Because of shape of this line, two work pieces at different position on the line can be managed at the same time. This also can result in better balances for cases with large number of tasks and stations. The number of stations needed for a U-type assembly line is never more than number of stations needed for serial assembly line type. Placement of U-shape assembly line layout is providing flexibility in number of workers because this line makes suitable optional and capacity change in customer demands (Scholl & Klein, 1999). U-shaped assembly line models have been popular in manufacturing system due to functional layouts and flexibility. Functional layouts involve easier production planning and control, lower stocks, simpler material handling, chances for teamwork and problem solving (Miltenburg & Wijngaard, 1994).

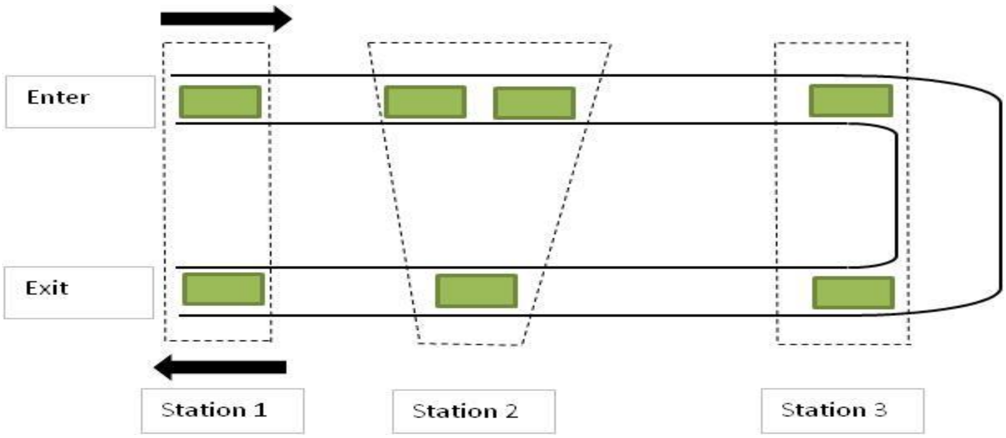

Figure 2.3. U-shaped assembly line layout

Two-sided assembly line: Two-sided assembly lines are usually designed to produce high volume and large size product such as buses, trucks and automobiles. Figure 2.4 demonstrated two-sided assembly line layout. The natures of physically large-sized products force side restriction on the tasks. For this reason, some task may only be executed on the right side of assembly line and some tasks may only be executed on the left side of assembly lines. Also some task may be executed both side (right side or left side) of assembly line. Therefore, in two-sided assembly line layout, tasks are grouped based on performable side on the line as left (L), right (R) or either (E).

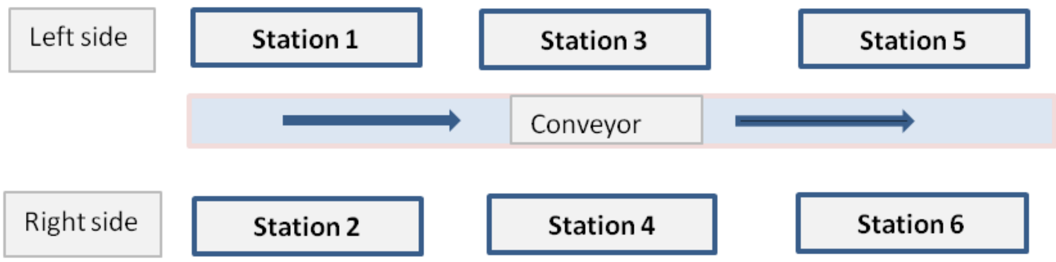

Figure 2.4. Two–sided assembly line layout

A more detailed on two-sided assembly lines is given at Chapter three

2.2 Literature on General Assembly Line Balancing Problem

The first mathematical model for ALB problem was defined by Salveson (1955), since that time, a large amount of academic literature has studied to balance of assembly lines. The problem was formulated as a simple assembly line balancing problem of type-1 by Salveson (1955). Bowman (1960) proposed simple assembly line balancing problem as integer programming problem with non-divisibility task constraint. This approach also provided a feasible assembly line balance with indivisible task. Subsequently, White (1961) improved integer programming model. To represent assignments some binary variables were added to model by White (1961) works.

Ranked positional weighted technique (RPWT) for SALB problem is presented by Helgeson & Birnie (1961) and also this technique is one of the first heuristic to solve SALB problem. RPWT heuristic assigns each of tasks to workstation according to weight rank value. The technique is simple to implement and it is also simple to understand its underlying principle. Heuristic algorithm to solve SALB-1 problem basis of precedence matrix is introduced by Hoffmann (1963).

Gutjahr & Nemhauser (1964) used shortest path concepts to balance the traditional single assembly line problem. Arcus (1966) developed a heuristic known as Computer Method of Sequencing Operations for Assembly Lines (COMSOAL). Procedure of this method assigns task to workstation from available task list according to predefined priority rules. COMSOAL heuristic approach is explained at Chapter five in detail. Patterson & Albracht (1975) developed 0-1 formulation of Bowman's model and also presented earliest and latest station concepts.

Talbot & Patterson (1984) presented an integer programming algorithm without binary variables. Johnson (1988) presented Fast Algorithm for Balancing Lines Effectively as known FABLE. This algorithm introduces branch and bound procedure to find an optimal solution to the large scale SALB-1 problem. An exact branch and bound method for SALB-1 problem was introduced by Hoffman (1992). Boctor (1995) introduced a multiple-rule heuristic approach to the SALB-1 problem.

Miltenburg & Wijngaard (1994) was the first authors to study U-type line balancing (SULB) problem. Dynamic programming model for solving SULB problem was developed by Miltenburg (1998). Urban (1998) improved the first integer programming formulation (IP) of SULB problem. The phantom precedence diagram concept is used in this formulation. Gokçen et al. (2005) presented a shortest route formulation for SULB problem and given a numerical example. Gokçen & Ağpak (2006) presented a goal programming approach for SULB problem to take into account several conflicting objectives. Their proposed model is based on the integer programming formulation developed by Urban (1998) for the SULB problem.

For more detail on general assembly line balancing problem literature can be found Baybars (1986), Ghosh & Gagnon (1989), Erel & Sarin (1998), Scholl (1999), Becker & Scholl (2006), Scholl & Becker (2006), Boysen et.al. (2007).

The studies on the two-sided assembly line balancing (TSALB) and resource constraint assembly line balancing problem are more relevant to our study. Hence, the literature on these areas will be introduced in more detail.

2.3 Literature on Two-Sided Assembly Line Balancing Problem

This section is devoted to literature on two sided assembly line balancing problem. The TSALB problem is a new area of concern and the studies are only a few. Research on one sided assembly lines are very extensive, although there are limited research on TSALB problem. Therefore in this section literature is examined and all of publication for TSALB problem is introduced in detail.

Bartholdi (1993) introduced the first study on two-sided assembly line balancing problem. Bartholdi (1993) developed an interactive computer software program which can be balance one sided and two sided assembly line problem. This software program's balancing procedure is based on "First Fit Rule" known as FFR. Bartholdi (1993) also compares one sided with two sided assembly lines and gives advantage of two sided line. These advantages are presented as follows.

- Operation times are smaller than one sided line.
- Length of the lines is shorter. This also provides to reduce transportation cost, because total length of the line is shorter according to one sided line.
- In case of same product or same cycle time, two sided lines always require equal or lesser workstation than one sided lines.
- Besides, the total cost of equipments and assembly vehicles is reduced.

Kim et al. (2000) proposed new genetic algorithm to solve TSALB problem. In their study, the objective function is to minimize the number of workstations. The algorithm also considers positional constraint when certain task assigned specific workstation. Proposed genetic algorithm approach present an encoding and decoding procedure of a solution to balancing problem.

Lee et al. (2001) presented a heuristic approach, which depends on group assignment procedure for TSALB problem. Two special performance measures are introduced to balance assembly line. One of them is work relatedness (WR) measure. WR measure depends on Agrawal (1985) formulation. The other performance measure is work slackness (WS). Slack time between finishing a task and successor's task starting time should be as possible as enough. If there is enough slack time between two tasks, the succeeding task will not affect because of delay. TSALB problem can be classified as follows by Lee et al. (2001).

TSALB-1: Objective of this type is to minimize length of assembly line for given cyle time. In other words, the objective is to minimize number of mated workstation for determined cycle time.
TSALB-2: The objective function is to minimize cycle time for given number of mated station.

An enhanced priority based heuristic was developed by Lapierre & Ruiz (2004) for TSALB problem. This heuristic is improved for the appliance manufacturing industry. They also examine assembly processes at two heights at both sides of the assembly line. MsAccess97 based software is used to find a solution for proposed heuristic. A problem having 400 precedence restrictions, 248 tasks and 4 task variables can be easily found a solution less than one minute on a computer by improved software program.

Baykasoglu & Dereli (2008) introduced ant-colony optimization heuristics, and by taking into account zoning constraint for TSALB problem. The objective is to minimize number of workstation for determined cycle time and maximize work relatedness. The proposed algorithm considers zoning constraint when balancing two sided assembly line. This publication also shows how ACO heuristic can be applied two sided assembly line balancing problem with zoning constraint and positional constraint.

Hu et al. (2008) presented a station-oriented enumerative algorithm depending on the concepts of earliest start time and latest start time. For TSALB-1(type-I) problems, a lower bound for the number of workers is presented. The proposed algorithm, which station oriented procedure, combined with Hoffman heuristic approach. The objective of this heuristic is to minimize total number of single workstation for determined cycle time. Station oriented procedure approach is tested and results shows that the procedure is effective.

Wu et al. (2008) presented a branch and bound algorithm approach to balance two sided assembly line problem. Firstly, they produce a mathematical model formulation and then branch and bound algorithm coded “C” computer programming language. Because size of TSALB enumeration tree is very large, they improved new task assignment rule for reduce the size of the tree. Developed task assignment rules are as follows.

- ✓ The task will be sorted with respect to its start time, which of them is earlier start, it will be started firstly.
- ✓ Ties broken, original left or right side operations are assigned firstly.
- ✓ Ties broken, which of tasks has greatest ranked positional weight are assigned firstly.
- ✓ Ties broken, which of tasks has maximal operating time are assigned firstly.
- ✓ Ties broken, assignment is selected randomly (Wu et al., 2008).

Özcan & Toklu (2009a) introduced a mathematical model and heuristic approach, which is simulated annealing, to balance mixed model TSALB problem. Proposed mathematical model has two objectives. First objective is to minimize mated station number as primary goal and minimize single station number as secondary objective. Two performance criteria are considered by proposed simulated annealing heuristic. One of them is to minimize weighted smoothness index and the other one is to maximize line efficiency.

Özcan & Toklu (2009b) presented a mixed integer goal programming approach to balance TSALB problem. In their publication, the objective is to minimize three different objects in lexicographic order. This lexicographic order is as follows

- ✓ Number of mated stations
- ✓ Cycle time
- ✓ Number of task to assigned a station

The second part of paper pursue with fuzzy goal approach. The objective is to maximize the weighted addition of fuzzy goals.

Özcan & Toklu (2009c) proposed a tabu search algorithm for TSALB problem. Smoothness index and line efficiency are considered as a performance criteria. Illustrative example is solved and outcomes demonstrate that proposed algorithm performance is well.

Kim et al. (2009) proposed a mathematical model to minimize cycle time for a given number of mated stations and also presented based on genetic algorithm heuristic approach. The mathematical model gives optimal solution for small test problems (T12, T16 and T24). Genetic algorithm approach is used for large test problem (T65, T148 and T205). The results of the proposed algorithm is compared with other algorithms, which are developed genetic algorithm by Kim et al. (2000) and first fit rule (FFR) algorithm by Bartholdi (1993). The outcomes of algorithm shows that proposed GA is well by means of solution quality and solution speed.

Simaria & Vilarinho (2009) proposed an ant colony optimization approach, which is known ACO, for mixed model TSALB problem. Firstly, they defined problem via mathematical model and then ACO heuristic approach presented to solution TSALB problem in their publication. They suppose that two ants work both of side (right and left) of the line simultaneously. Sub-colonies derive a beginning of ants. Each pair of ants belonging to these sub-colonies produces a solution for TSALB problem. The main objective minimizes number of workstation for determined cycle time. The mathematical model doesn't solve any test problems because of complexity of the proposed model. The end of the paper, illustrative example is presented and ACO heuristic approach performances are tested.

Özcan et al. (2010) developed heuristic algorithm, which is tabu search, for parallel two sided assembly line balancing problem. The objective is minimizing stations number. In this type of assembly line is one or more than one SALB line designed as parallel layout. In this paper test problems are solved. Results of test problems are matched with theoretical minimum number of station, and also outcomes of proposed algorithm for parallel TSALB problem are sufficient. This algorithm provides important improvement in TSALB problem, when the line is located as parallel.

Özcan (2010) proposed a mathematical model and simulated annealing approach which is based on COMSOAL heuristic to solve TSALB problem with stochastic task time. Therefore, piecewise-linear, chance constraint and mixed integer formulation was developed to find solution. For large test problems proposed heuristic is efficient and results show that the heuristic is sufficient for solving the problem. Proposed study is the first study to take into account stochastic task time for TSALB problem.

Özcan & Toklu (2010) presented a mixed integer mathematical model and heuristic approach for TSALB problem with regard to sequence-dependent setup time. Mathematical model is solved a small test problems. For large test problems proposed heuristic solves problems efficiently. Introduced study is the first study to consider setup time for TSALB problem.

Hu et al. (2010) proposed a new branch and bound approach for TSALB problem of type-1, which is minimization line length and number of position for given cycle time. In their study, reduction rule and dominance rule which use for one sided assembly line balancing problem develop to solve TSALB problem of type-1. Proposed algorithm is examined on test problems from the literature. Results of algorithm are efficient.

Yegul et al. (2010) introduced a new hybrid design for a specific case of assembly lines and used multi-pass random assignment algorithm to minimize number of station. In their study, two-sided lines and U-type lines are combined for new hybrid design.

Özbakır & Tapkan (2011) introduced a new meta heuristic approach to solve TSALB problem. Proposed meta-heuristic approach is known as Bee Algorithm. The algorithm is adopted TSALB problem to consider zone constraint. The objective of algorithm is to minimize number of station for determined cycle time. The new algorithm is tested with zone constraint and without zone constraint. The presented algorithm performs well, because it finds the best known number of workstation and gives the other aims effectively

Taha et al. (2011) proposed a genetic algorithm approach to solve TSALB problem. The presented GA indicates a novel method for generating the initial population. It performs a hybrid crossover and a modified scramble mutation operators. To assign tasks to mated-stations, proposed station oriented procedure is adopted. The proposed method especially finds effective results for large sized test problems.

Ağpak et al. (2012) presented a new line design and a mathematical programming model. In this study, U-shape line and two sided line are combined as a hybrid. Proposed new hybrid line design is called two sided U-type assembly line balancing problem (TSUALBP). New line design can be applied to any sector, which uses two sided line, and also can ensure benefit both of U-shape line and two sided line together. A binary integer programming model is presented to solve new hybrid design assembly line balancing problem.

Simulated annealing algorithm for cost oriented two sided assembly line balancing problem (CTALBP) was proposed by Roshani et al. (2012). The objective of presented simulated algorithm is to minimize total cost per product unit. Proposed algorithm is compared with mixed integer goal programming for small test problem and compared with first fit rule (FFR) algorithm for large size (T65, T148, T205) test problem from literature. The algorithm finds optimal solution in order to small size test problem and also can find efficient results for large size problem.

Tapkan et al. (2012) presented mixed integer non linear mathematical model to solve TSALB problem. Owing to complexity of the problem, artificial bee colony (ABC) and bee algorithm (BA) algorithm are introduced for large test problem. The objective function is to minimize total number of station and to achieve balanced line.

Chutima & Chimklai (2012) proposed a metaheuristic approach which is particle swarm optimization with negative knowledge to solve multi objective, mixed model TSALB problem. The proposed metaheuristic is known as PSO. The performance of presented algorithm is compared with some heuristic from literature with several scenarios. The results show that proposed algorithm is simple but strong, and demonstrates fast convergence speed when compared with the other algorithms.

The literature review for TSALB problem is summarized as shown in table 2.1. This table contains the published paper. These papers are examined, according to type of problem and solution methodology

Table 2.1. Literature on two sided assembly line balancing problems

Publications	***Type of Problem***	***Methodology***
Bartholdi (1993)	TSALB problem of type1	Interactive Computer Software Program and FFR Rule
Kim et al. (2000)	TSALB-1	Genetic Algorithm
Lee et al. (2001)	TSALB-1 and TSALB-2	Group Assignment Procedure
Lapierre & Ruiz (2004)	TSALB-1	Priority-Based Heuristic
Baykasoğlu & Dereli (2008)	TSALB-1 with zone constraint	Ant Colony Optimization
Hu et al. (2008)	TSALB-1	Heuristic (station-oriented enumerative algorithm)
Wu et al. (2008)	TSALB-1	Branch and Bound Algorithm
Özcan & Toklu (2009a)	TSALB-1	Mathematical Model and Simulated Annealing Algorithm
Özcan & Toklu (2009b)	TSALB-1 and TSALB-2 (Multiple Objective Approach)	Mathematical Model
Özcan & Toklu (2009c)	TSALB-1	Tabu Search Algorithm
Kim et al. (2009)	TSALB-2	Mathematical model and Genetic Algorithm
Simaria & Vilarinho (2009)	TSALB-1	Mathematical Model and Ant Colony Optimization

Özcan et al. (2010)	TSALB-1(Paralel two-sided assembly line)	Tabu Search Algorithm
Özcan (2010)	TSALB-1	Mathematical Model and Simulated Annealing Algorithm
Özcan and Toklu (2010)	TSALB-1(with setups)	Mathematical model and 2-COMSOAL/S Heuristic
Hu et al. (2010)	TSALB-1	Branch and Bound Approach
Yegul et al. (2010)	U-type line TSALB-1	Multi-pass Random Assignment Algorithm
Özbakır et al. (2011)	TSALB-1(with zone constraint)	Bees Algorithm
Taha et al. (2011)	TSALB-1	Genetic Algorithm
Ağpak et al. (2012)	U-type line TSALB-1 (with zoning constraint)	Mathematical Model
Roshani et al. (2012)	TSALB-1 (cost-oriented)	Mathematical Model and Simulated Annealing Algorithm
Tapkan et al. (2012)	TSALB-1	Bees Algorithm and Artificial Bee Colony Algorithm
Chutima & Chimklai (2012)	TSALB-1(multi-objective)	Particle Swarm Optimization
Purnomo et al. (2013)	TSALB-2	Heuristic and Iterative FFR Rule

2.4 Literature on Resource Constraint Assembly Line Balancing Problem

In this study, resource constraints are considered when assembly line balancing problem is solved. In the literature, most of papers considered same kind of resource such as equipment, workers, and all the operations need the same resources to be processed. Only a few papers examine different kind of resources with regards to time, cost etc. (for more detail Graves & Whitney, 1979; Faaland et al., 1992; Falkenauer, 1997; Buckhin & Rubinovitz, 2003; Amen, 2006 can be examined). Pinnoi & Wilhelm (1998) considered resource constraints in case of resources are limited situation (Corominas et al., 2011).

Ağpak & Gökçen (2005) defined resource constraint assembly line balancing problem. They developed 0-1 integer programming models to determine balance of the assembly line with minimum resources and number of station. Study is considered on account of traditional assembly line balancing problem. In their study, resources are examined two different ways. Firstly, each of tasks is performed by only one resource (resource A or B). For example, resource A can perform {1, 5, 7, 3, 9} tasks and resource B can perform {2, 6, 8, 4} tasks for 9 task assembly line balancing problem. Therefore, intersection of tasks is empty cluster. Secondly, some task can be carried out by two resources alternatively. For instance, task 2, 1, 5, 7, 9 can be done by resource A, and also task 2, 5, 6, 8, 4 can be done by resource B. As seen in example resource A or B can perform tasks 5 and 2. The problem is classified with two different ways. First case problem is called resource constraint assembly line balancing problem of type1 (RCALB1), and for second case problem is called RCALB of type 2.

Corominas et al. (2011) introduced a mathematical model to balance general resource constraint assembly line problem and presented an upper bound for number of usable resources. Proposed mathematical model solves small and medium size known benchmark test problem. Up to 60 tasks test problem is solved without one instance. In their study, they improve Ağpak & Gökçen (2005) approach with different way. A task is performed by considering different resource type and more than one resource simultaneously. In other words, a task can be done by multiple or simple, alternatively or simultaneous resource. For this reason, they consider resource constrained as a more general case. Two model types are defined according to sequence of resources. Objective function is defined for conjunctive normal form known as CNF, and for disjunctive normal form known as DNF.

Purnomo et al. (2013) proposed a mathematical model for TSALB-2 problem by considering assignment constraint. Objective of proposed model is to minimize cycle time for determined number mated stations. A heuristic and iterative FFR are improved to solve problem for large size test problem. Also they examined resource constraint for TSALB-2 problem as a simple.

The objective of this thesis is to present a solution method for being able to solve two sided assembly line balancing problem considering resource constraints. For this reason, a mathematical model is presented. The model is combined by Corominas et al. (2011) approach which is general resource constraint for simple assembly line balancing problem and Ağpak et al. (2012) mathematical model to adapt TSALB problem. While presented model can find optimal solution for small test problems, it doesn’t give a solution within admissible computation time for large size test problems. Thus, a heuristic approach which is based on COMSOAL is presented to solve large size test problems. Besides, presented model is examined as a multi objective function.

Thesis is continued with mathematical model at Chapter 3 and multi objective approach is given at Chapter 4.

CHAPTER III

MATHEMATICAL MODEL FOR GENERALIZED RESOURCE CONSTRAINED TWO-SIDED ASSEMBLY LINE BALANCING PROBLEM

In this chapter mathematical model is presented for generalized resource constrained two sided assembly line balancing problem. The objective function is to minimize number of station and resources under given cycle time according to cost. Chapter starts with basic terms of two sided assembly line balancing to pursue with assumption and notation. Subsequently, mathematical model presents with illustrative example. Finally, test results are evaluated.

3.1 Basic Terms of Two-sided Assembly Line Balancing

Basic term of two sided assembly line balancing is given as follows:

Assembly line: Assembly line is the production line in which assembly operations that performed on successive stations.

Tasks: Task is smallest and indivisible part of work element in assembly line systems.

Left Task (L): Task that should be carried out at left side of assembly line.

Right Tasks (R): Task that should be carried out at right side of assembly line. If a task has a right side constraint to performed, it shows that this task can be done just right side of the assembly line.

Either Task (E): Task that should be carried out either of side. A task can be performed right or left side, task of this type hasn't got side constraint.

Task Time (Operation Time): Task time is necessary time to perform a duty.

Station (workstation): Station is a part of the assembly line systems. In a workstation, one or more tasks are carried out along the work flow by one or more workers.

Mated Station: A pair of mutual station that is located on two sided assembly lines is named as mated station. A mated station symbolizes the length of assembly line.

Station Time: Total time of tasks, that is assigned workstation, is called as station time.

Cycle Time: The time between the leaves of two successive products from the line is named cycle time. In other words, available time to carry out each of task assigned to workstation. Cycle time cannot smaller than maximum task time.

Precedence Relations: There is some precedence restrictions owing to the nature of the task and product assembled. In other saying, precedence relations are the task sequence in which order tasks must be performed.

Precedence Diagram: Precedence diagram is graphical representation of precedence relations between tasks, task time and operation side of task. An example of a precedence diagram for 9-task is given in figure 3.1. Numbers in circle show task number. Task time and operation side are demonstrated in parenthesis. Precedence relations between tasks are symbolized by arcs.

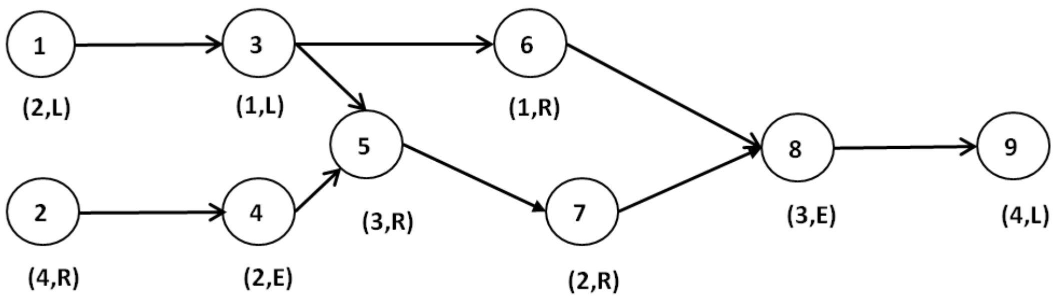

Figure 3.1. Example of precedence diagram

Precedence Matrix: Precedence matrix demonstrates precedence relations in a matrix form.

Predecessors of a Task: Set of tasks that must be performed before starting time of the task. For instance, the predecessors set of task 7 (see figure 3.1) is {1, 2, 3, 4 and 5}. Successors of a Task: Set of tasks that cannot be processed before the completion time the task. For instance, the successors set of task 4 (see figure 3.1) is {5, 6, 7 and 8}.

Start Time of a Task: A task start to perform according sequence of operation after all of predecessors are completed.

Finish Time of a Task: This term defines a task that is completed according to operation of sequence. Finish time of a task cannot be greater than cycle time.

Idle Time: Idle time is a positive difference between cycle time and workstation time. Total idle time for all station of assembly line is known as balance delay time.

3.2 Notations

In this part, notations are introduced for proposed model. The following notation and equations will be used to describe the problem characteristics.

Indices

$\boldsymbol{i, j}$ Task number i= 1… I; j=1… J

$\boldsymbol{k, g}$ Position number k=1…K; g=1…K

$\boldsymbol{r}$ Type of resource (r=1...R)

$\boldsymbol{s}$ Operation side of the line S= $\begin{cases} 1 & Left\ side \\ 2 & Right\ side \end{cases}$

$\boldsymbol{p}$ Clause

$\boldsymbol{c}$ Cycle Time

Parameters

$\boldsymbol{q}$ Amount of resource unit

$\boldsymbol{t_i}$ Operation time of task i; i=1...I

$\boldsymbol{CR_r}$ Cost of a resource unit of type “r”

$\boldsymbol{C_s}$ Cost of opening a station

$\boldsymbol{\alpha_{rpi}}$	Amount of "r" type resource unit in clause p of task "i" (r=1…R; p=1…C_i; i=1…I)
$\boldsymbol{\pi_{pi}}$	In case of $\alpha_{rpi} > 0$ number of resource type in clause 'p' for task 'i'. (p=1…L_i; i=1...I)
$\boldsymbol{L_i}$	Number of clause in logical expression of task "i" ($p = 1 \ldots L_i$).
$\boldsymbol{tt_i}$	Start time of task "i" (positive variable)
$\boldsymbol{Y_{rks}}$	Number of resource unit of type r assigned side s of position k
$\boldsymbol{RL_i}$	Logical expression of task "i" in CNF or DNF
$\boldsymbol{M1, M2}$	Big positive number
$\boldsymbol{side_i}$	Set of side task 'i' can be assigned
$\boldsymbol{side_s}$	Set of task that can be assigned side's'
$\boldsymbol{Ipred}$	Set of immediate predecessors of task 'i'
$\boldsymbol{Pred}$	Set of whole predecessors of task 'i'

<u>Decision Variable</u>

$\boldsymbol{X_{iks}}$	$\begin{cases} 1, & \text{if task 'i' is assigned side 's' of position 'k'} \\ 0, & \text{otherwise} \end{cases}$
$\boldsymbol{qc_{rpi}}$	0, if the primary propositions of resource type r of clause p of task i in Conjunctive Normal Form is satisfied. (i=1…I; p=1…L_i ; r $/\alpha_{rpi}$ >0)
$\boldsymbol{qd_{pi}}$	0, if the primary propositions of clause 'p' of task 'i' in Disjunctive Normal Form is satisfied (i=1…I; p=1…L_i)
$\boldsymbol{V_{rqks}}$	$\begin{cases} 1, & \text{if type 'r' of 'q' resource unit is assigned position 'k' of side 's'} \\ 0, & \text{otherwise} \end{cases}$
$\boldsymbol{St_{ks}}$	$\begin{cases} 1, & \text{if position 'k' of side 's' is opened} \\ 0, & \text{otherwise} \end{cases}$
$\boldsymbol{N_k}$	$\begin{cases} 1, & \text{if any of position 'k' of side 's' is opened} \\ 0, & \text{otherwise} \end{cases}$
$\boldsymbol{U_{ij}}$	0-1 indicator variable

3.3 Model Assumptions

These assumptions are generally shared for mathematical model. In this study, the generalized resource constraint two sided assembly lines consider to operate under the following assumptions:

- ✓ Task times are deterministic.
- ✓ All stations are equally skilled with respect to labor force.
- ✓ The precedence diagram for all problems are known and established
- ✓ Travel time of operators are disregarded
- ✓ Operation side of tasks are known
- ✓ For process one task may be needed more than one type or one unit of resource (for example (5A∨4B) ∧ 3C: to process this task 5 unit type A or 4 unit type B and 3 unit type C resources are required (Corominas et al., 2011).
- ✓ The numbers of available units of each type of resources are known.

3.4 Mathematical Modeling

Mathematical model is developed by considering resource constrained as conjunctive normal form (CNF) and disjunctive (DNF) normal form format. CNF is organized in two different ways by Corominas et al. (2010). Similarly, CNF is also modeled two different ways in this study. CNF-1, CNF-2 and DNF models are adapted two sided assembly line balancing problem and presented with different title.

3.4.1 Conjunctive and Disjunctive (CNF and DNF) Normal Form

The transformation of a logical expression with any binary operators into an equivalent one is well known. Conjunctive and disjunctive is special form for writing "and", "or" (Friedman, 1986). In case elementary proposition β is defined resource unit and λ is defined more than one β as conjunctive (∧) or disjunctive (∨) cluster:

Conjunctive Normal Form (CNF): A formula is in conjunctive normal form if it is a conjunction of one or more clauses.

$\lambda_1 \equiv (\beta_1 \wedge \beta_2)$

CNF shows concurrent resource situation like 2A ∧ 3B. In this example, tasks can be done by 2A and 3B resources concurrently.

Disjunctive Normal Form (DNF): A formula is in disjunctive normal form if it is disjunctions of one or more clauses.

$\lambda_2 \equiv (\beta_3 \vee \beta_4)$

DNF shows alternative resource situation like 2A ∨ 3B. And also task can be done by 2A or 3B resource alternatively

3.4.2 Resource Constraints for CNF-1 Model

Mathematical model will receive resource from logical arrangement of conjunctive normal form. Constraints for CNF1 are as follows:

$$\sum_{r \backslash \alpha_{rpi} \geq 0} qc_{rpi} \leq \pi_{pi} - 1 \quad \forall\, i \in I\,, r \in R, p \in P \tag{3.1}$$

$$\alpha_{rpi}.X_{iks} \leq Y_{rks} + \alpha_{rpi}.qc_{rpi} \quad \forall i \in I\,, r \in R, p \in P, k \in K, s \in S \; ve \; \forall r \backslash \alpha_{rpi} > 0 \tag{3.2}$$

In first inequality, at least one (logical expression of task i in clause p) qc_{rpi} must be equal to zero. This also shows that at least one resource of type r in clause p will be used.

Constraint (3.2) provided that if both task i is assigned side s of station k ($X_{iks} = 1$) and $qc_{rpi} = 0$, amount of type r resources assigned side s of station k (Y_{rks}) must be equal or bigger than amount of related resources (α_{rpi}) of task i in which are used.

3.4.3 Resource Constraints for CNF-2 Model

Arrangements of CNF-2 resources organize like CNF-1. Model takes resources from arrangement of CNF-1 format. CNF-2 resource constraint is developed as alternative situation of CNF-1

$$X_{iks} \leq \sum_{r \setminus \alpha_{rpi>0}} V_{r,\alpha_{rpi},k,s} \quad for \; \forall \; i,p,k,s,r \tag{3.3}$$

$$V_{r,q,k,s} \leq V_{r,q-1,k,s} \quad for \quad \forall \left(p = 1..L_i \, ; \forall r \setminus \alpha_{rpi} > 1; 2 \leq q \leq \alpha_{rpi} \, ; k \; and \; s\right) \tag{3.4}$$

Constraint (3.3) ensures that if task i is assigned side s of station k $(X_{iks} = 1)$ for clause p in logical expression of task i, at least one of type r resource must be assigned side s of station k (required resource to perform task i (α_{rpi})).

Constraint (3.4) provides consistent value of V_{rqks} which is binary variable. In other words, it satisfies that all of V_{rqks}value should take value of 1 from q=1 until q=α_{rpi}

3.4.4 Resource Constraints for DNF Model

In this model, resource requirement is organized as DNF format. This format of resources connect each other with “or” connective, and this also express alternative resource situation

$$\alpha_{rpi}.X_{iks} \leq Y_{rks} + \alpha_{rpi}.qd_{pi} \quad \forall i \in I \, , r \in R, p \in P, k \in K, s \in S \; ve \; \forall r \setminus \alpha_{rpi} > 0 \tag{3.5}$$

$$\sum_{p=1}^{C_i} qd_{pi} \leq L_i - 1 \quad for \; \forall \; p \; and \; i \tag{3.6}$$

Constraint (3.5) and constraint (3.2) have same structure. Constraint (3.2) is explained above. In this constraint (3.5), qd_{pi} represents each of clauses and for supplied all clause take 0 values. Constraint (3.6) is states that at least one of the variable in logical expression (RL_i) should take 0 value for qd_{pi}

3.4.5 Common Constraints for All Models

The following constraint is common restriction for all models.

$$\sum_{k=1}^{K} \sum_{s \in side_i} X_{iks} = 1 \; for \; \forall \; i = 1 \ldots . \mathrm{I}$$ (3.7

$$\sum_{i \in sides} t_i.X_{iks} \leq C.St_{ks} \quad \forall \; k = 1 \ldots . \mathrm{K} \; and \; s \in S \tag{3.8}$$

$$\sum_{g\in K}\sum_{s\in side_i} g\ .X_{jgs} - \sum_{k\in K}\sum_{s\in side_i} k\ .X_{iks}\ \leq 0\quad \forall\,(j,i)\in ipred \tag{3.9}$$

$$tt_j - tt_i\ + M.\sum_{k=1}^{K}\left(\sum_{s=1,s\in side_i}^{2}(K-k+1).X_{iks} - \sum_{s=1,s\in side_i}^{2}(K-k+1).X_{jks}\right) \geq\ t_i$$

$$\forall\ \ (i,j)\in ipred \tag{3.10}$$

$$tt_j - tt_i\ + M.\sum_{k=1}^{K}\left(\sum_{s\in side_i}\big((k-1).2+s\big).X_{jks} - \sum_{s\in side_i}\big((k-1).2+s\big).X_{iks}\right) + M2.U_{ij}\ \geq\ t_i$$

$$for\ \forall\,(i,j)\ \ ,\ i<j,(i,j)\notin pred,\big(side_i\cap side_j\big)\ \neq\emptyset \tag{3.11}$$

$$tt_i - tt_j\ + M.\sum_{k=1}^{K}\left(\sum_{s\in side_i}\big((k-1).2+s\big).X_{jks} - \sum_{s\in side_i}\big((k-1).2+s\big).X_{iks}\right) + M2.(1-U_{ij}\,)$$

$$\geq\ t_j for\ \forall\,(i,j)\,,i<j,(i,j)\notin pred,\big(side_i\cap side_j\big)\ \neq\emptyset \tag{3.12}$$

$$tt_i + t_i\ \ \leq C\ for\ \ \forall\, i\in I \tag{3.13}$$

$$\sum_{s=1}^{2} St_{ks} - 2.N_k\ \leq 0\ \forall\, k\ = 1\ldots\mathrm{K} \tag{3.14}$$

$$N_k - \sum_{s=1}^{2} St_{ks}\ \leq 0\ \ \forall\ \ k\ = 1\ldots\mathrm{K} \tag{3.15}$$

Constraint (3.7) defines assignment restriction and ensures that every task is assigned to only one side of only one station. Constraint (3.8) is cycle time restriction and provides total performance time of tasks, which are assigned workstation, cannot exceed cycle time. Constraint (3.9) is precedence relation restriction, and guarantees that precedence relations between all tasks are provided. Constraint (3.10), (3.11) and

(3.12) are sequencing restriction. During task assignment in a two sided assembly lines, tasks at positions require to be sequenced. For this reason, three constraint provide that task assigned to side (left or right) of any position are sequenced to fulfill precedence relations constraint. Constraint (3.13) provides that completion time of any tasks is not bigger than cycle time. Constraint (3.14) and (3.15) are positions restriction and guarantee that a position is opened whenever tasks are assigned one of its locations. In other words, with these constraints together provided that if any of station is opened in the related position, position variable takes value of 1.

3.4.6 Objective Functions

In this section, objective function will be minimized resource cost and total opened station cost simultaneously. Two objective functions are combined for minimization of total cost. These functions are minimization of resource cost and minimization of station cost. The objective functions are given as follows:

Objective function for CNF-1 and DNF

$$Min\left(\sum_{r=1}^{R}\sum_{k=1}^{K}\sum_{s=1}^{2} CR_r\,.\,Y_{rks} + \sum_{k}^{K}\sum_{s=1}^{2} Cs.St_{ks}\right) \tag{3.16}$$

Objective function for CNF-2

$$Min\left(\sum_{k=1}^{K}\sum_{s=1}^{2}\sum_{r=1}^{R}\sum_{q=1}^{\alpha_{rpi}} CR_r\ .V_{rqks} + \sum_{k}^{K}\sum_{s=1}^{2} Cs.St_{ks}\right) \tag{3.17}$$

3.5 An Illustrative Example

In this section, model is examined for small test problems (T9, T12, T16, and T24). These test problems are widely studied in the TSALBP literature. 16 task test problem set is obtained from the study of Lee et al. (2001). Three problem sets are achieved from Kim et al. (2001) which are 9-task, 12-task and 24-task problems. Results of all test problems will be given in section 3.6. One illustrative example which is 12-task problem is solved by using the proposed mathematical models. Data of 12-task problem is given in table 3.1

Some additional assumptions for presented models are given as follow:

- ✓ In this study, three resource type are used which are named as A, B and C type of resources.
- ✓ Cost of three type resource is determined as {A=10 unit, B=8 unit, C=12 unit}
- ✓ Cost of station is fixed and taken 10 units for all stations.
- ✓ Logical expression of resources and amount of resource type are selected randomly for all test problems.
- ✓ Upper limit of station is received number of optimal station +2 without resource.

Owing to there is no test problem with general resource constraint for TSALB in the literature; resources are selected randomly using mainly the same approach with Corominas et al. (2011).

From table 3.1, column 5 and 6 are shown resource requirement to done a task. Column 5 is designed for conjunctive normal form (CNF-1 and CNF-2) and column 6 is designed for disjunctive normal form (DNF).

Table 3.1. Data of 12-task problem

Task	Immediate Predecessors	Operation side	Operation time	CNF form	DNF form
1	-	L	2	$(2A \vee B) \wedge (2A \vee 2C)$	$(2A) \vee (B \wedge 2C)$
2	-	R	3	$(A \vee B) \wedge (A \vee C)$	$(A) \vee (B \wedge C)$
3	-	E	2	$(5A) \wedge (2B \vee 4C)$	$(5A \wedge 2B) \vee (5A \wedge 4C)$
4	1	L	3	$(A \vee 5B) \wedge (A \vee C)$	$(A) \vee (5B \wedge C)$
5	2	E	1	$(A \vee 5C) \wedge (B \vee 5C)$	$(A \wedge B) \vee (5C)$
6	3	L	1	$(4A \vee 4B) \wedge (5C)$	$(4A \wedge 5C) \vee (4B \wedge 5C)$
7	4,5	E	3	$(3A \vee 3B) \wedge (3A \vee 5C)$	$(3A) \vee (3B \wedge 5C)$
8	5	R	3	$(5A \vee 4C) \wedge (5B \vee 4C)$	$(5A \wedge 5B) \vee (4C)$
9	5,6	E	2	$(A \vee 3B) \wedge (A \vee 3C)$	$(A) \vee (3B \wedge 3C)$
10	7,8	E	2	$(A \vee 4C) \wedge (5B \vee 4C)$	$(A \wedge 5B) \vee (4C)$
11	9	E	2	$(5A \vee 2B) \wedge (3C)$	$(5A \wedge 3C) \vee (2B \wedge 3C)$
12	11	R	1	$(4A \vee 4B) \wedge (4C)$	$(4A \wedge 4C) \vee (4B \wedge 4C)$

L (left); R (right); E (either)

For example, task 3 can be done by 5 unit resources A and 2 unit resources B or 4 resource unit C regard of CNF form. In other words, task 3 also can be done by 5 unit A and 2 unit B resource or 5 unit A and 4 unit C resources regard of DNF form. Illustrative example is solved by three models which are CNF-1, CNF-2 and DNF. Three of models give same results, but with different CPU time.

Test problem is formulated with GAMS/ CPLEX (General Algebraic Modeling System) and run on a computer Intel Xeon 4 Core 2.40 GHz processor with 8 GB RAM. Solution of the models time is limited to 3600 seconds for the solver. 12-task test problem is solved for cycle time 5. In case of balance of 12-task problem, the result of test problem will be as follows.

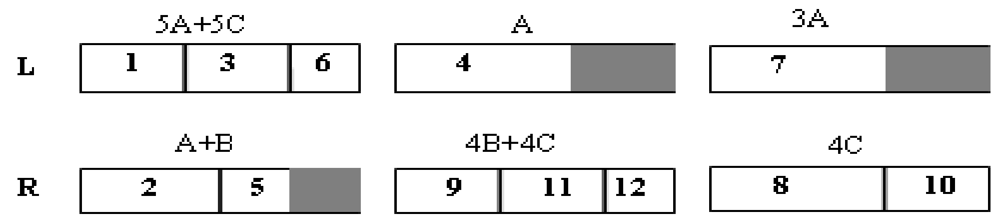

Figure 3.2. Balance of 12-task test problem

Total cost for illustrative example will be total resource cost and opened station cost.

Total cost= 10A+5B+13C+10*opened station number

= 10*10+5*8+13*12+10*6

=356 unit

Results of small test problems will be given next section.

3.6 Results of Test Problems

In this section, four literature problems are solved and results are given table 3.2. First column in table 3.2 is shown number of task. Third, fourth and fifth columns also demonstrate CPU time in seconds for CNF-1, CNF-2 and DNF models. In six column is represented cost of resource and cost of opened station number (total cost). Last column (NS) also shows number of workstation which is opened. The problems will be named as T9, T12, T16 and T24 and the data of the test problems can be found in Appendix A.

Table 3.2. Results of small test problems

Problems	Cycle Time	CPU Time for CNF-1	CPU Time for CNF-2	CPU Time for DNF	Total Cost	NS
T9	5	1.451	0.717	1.014	258	4
	6	0.296	0.578	0.562	230	3
T12	5	24.773	6.428	19.688	356	6
	6	31.980	2.808	24.025	318	5
	7	16.380	3.369	14.804	288	4
	8	5.163	0.749	6.240	260	4
T16	16	1180.070	314.389	3038.931	428	6
	18	248.619	57.611	692.442	396	5
	19	357.913	82.151	569.949	384	5
	21	23.213	14.711	28.236	360	4
	22	18.689	12.356	35.630	340	4
T24	20	+3600	+3600	+3600	**506***	**8**
	25	+3600	+3600	+3600	**434***	**6**
	30	448.066	78.188	1858.346	376	5
	35	45.132	7.082	57.829	316	4
	40	50.950	6.521	146.500	316	4

*Integer solution

Same total cost and number of stations are achieved from CNF-1, CNF-2 and DNF models. For this reason, we compared models according to performance of solution time (for representation of graphics, see figure 3.3). Obtained results from all test problems are optimal excluding T24 for cycle time 20 and 25 (see results in bold characters), but an integer solution is found for test problem T24/20 and T24/25.

Presented models can found an optimal solution for test problems T9/5 and T9/6 within one second (see table 3.2 or figure 3.3). When the numbers of tasks are increased, from the table 3.2 it is easily seen that solution times of proposed models are increased.

The performances of solution time for three models are compared in figure 3.3 for small (T9, T12, T16 and T24) test problems.

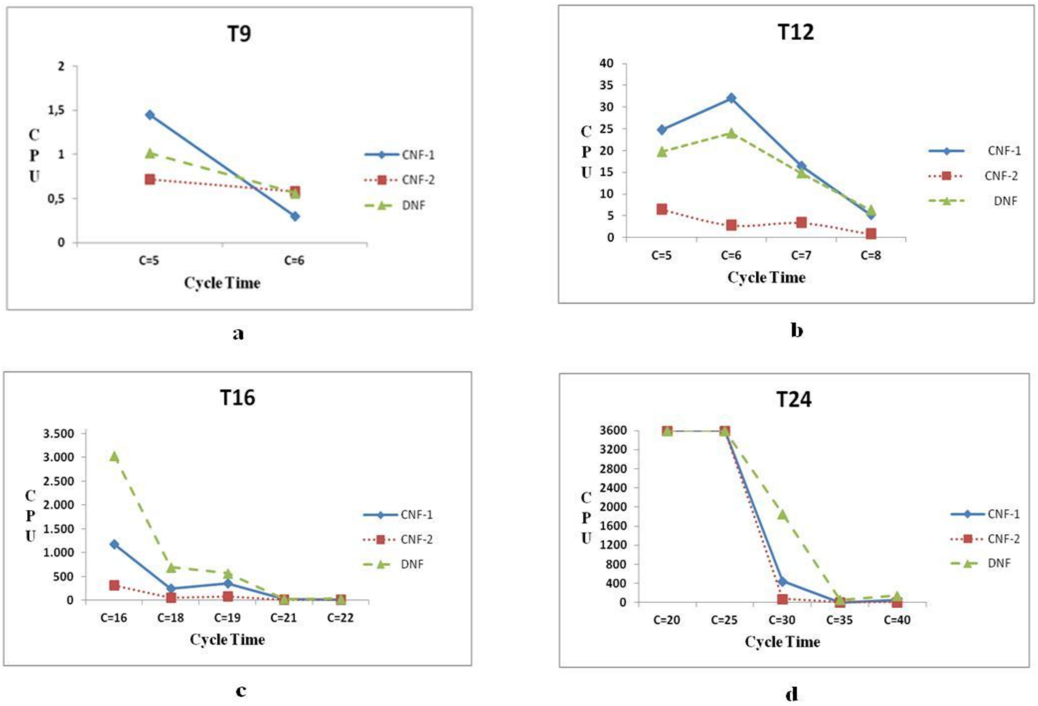

Figure 3.3. Speeds of CNF-1, CNF-2 and DNF models a) with 9-tasks b) with 12-tasks c) with 16-tasks c) with 24-tasks

Figure 3.3 shows speeds between models for four test problems according to CPU time in seconds for same cycle time. From figure, it can be claimed that CNF-2 is the faster than CNF-1 and DNF model. For example, while optimal solution for test problem T16/16 (see figure 3.3c) is obtained lesser than 500 seconds by CNF-2 model, CNF-1 model gives an optimal solution over than 1000 seconds, and also DNF model gives an optimal solution about 3000 seconds.

Figure 3.4 also shows speeds between models for all test problems on the same graph.

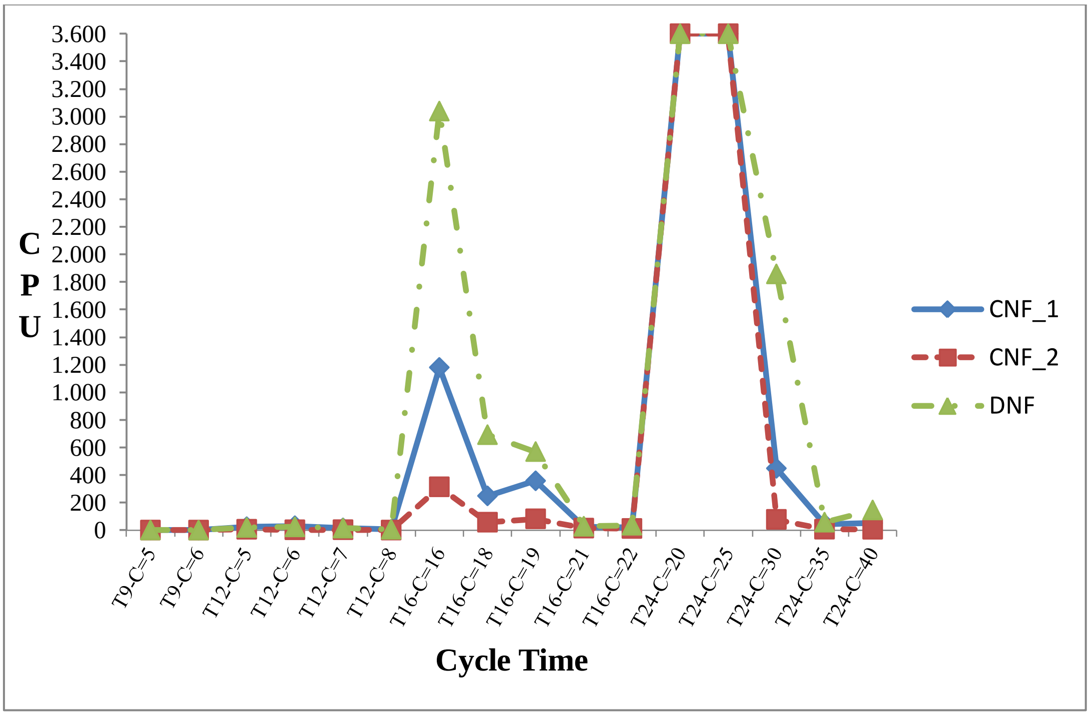

Figure 3.4. Comparison of three models on the same graph

CHAPTER IV

MULTI-OBJECTIVE APPROACH

In this chapter, the multi-objective generalized resource constraint TSALB problem is studied. Three different objective functions are presented. These functions are minimization of resource cost, minimization of station cost (in case of equal cost, number of station) and minimization number of position. Multi-objective approach is examined. In this part to be used assumptions, common constraints and test problem parameters are same as with Chapter three. Therefore, these data are not given again in this section.

4.1 Objective Functions

1. Objective functions**:** Minimization of resource cost

$$Min \sum_{r=1}^{R} \sum_{k=1}^{K} \sum_{s=1}^{2} CR_r \, . Y_{rks} \tag{4.1}$$

2. Objective functions: Minimization of station cost

$$Min \sum_{k}^{K} \sum_{s=1}^{2} Cs.St_{ks} \tag{4.2}$$

3. Objective functions: Minimization number of position

$$Min \sum_{k}^{K} N_k \tag{4.3}$$

For CNF-2 model 1. Objective functions: Minimization of resource cost

$$Min \sum_{k=1}^{K} \sum_{s=1}^{2} \sum_{r=1}^{R} \sum_{q=1}^{\alpha_{rpi}} CR_r \, . V_{rqks} \tag{4.4}$$

Formulations of objective functions are same for CNF-1 and DNF models, but formulation of the first objective function (minimization of resource cost) is different for CNF-2 due to nature of the model. Therefore, first objective function is organized again according to nature of CNF-2 model (see formulation 4.4).

4.2 Results of Test Problems for Multi-Objective Approach

Proposed models are analyzed over several known test problems by considering three objective functions (Minimization number of station, number of position and resource cost). The result of each objective is added as constraint for the next objective. Each of objectives is symbolized with their initial letter. Besides, these objectives are sequenced in different ways which are RPS, RSP, SPR, SRP, PRS, and PSR. For example, the abbreviation RPS is used to demonstrate objective ranking that is to minimize resource cost, minimize number of station and position respectively.

In case of using three resources (A, B and C), results of small test problems for each objective ranking are shown table 4.1

Table 4.1. In case of the use of three resource, obtained resource cost, number of station, number of position and total costs for each objective ranking

Problem		RPS	RSP	SPR	SRP	PRS	PSR
T9 _C5	Resource Cost	218	218	236	218	236	236
	Number of Station	4	4	4	4	4	4
	Number of Position	3	3	2	3	2	2
	Total Cost	**258**	**258**	**276**	**258**	**276**	**276**
T9 _C6	Resource Cost	200	200	200	200	200	200
	Number of Station	3	3	3	3	3	3
	Number of Position	2	2	2	2	2	2
	Total Cost	**230**	**230**	**230**	**230**	**230**	**230**
T12 _C5	Resource Cost	296	296	306	306	296	296
	Number of Station	6	6	5	5	6	6
	Number of Position	3	3	4	4	3	3
	Total Cost	**356**	**356**	**356**	**356**	**356**	**356**
T12 _C6	Resource Cost	268	268	268	268	268	268
	Number of Station	5	5	5	5	5	5
	Number of Position	3	3	3	3	3	3
	Total Cost	**318**	**318**	**318**	**318**	**318**	**318**

T12 _C7	Resource Cost	248	248	304	248	304	304
	Number of Station	4	4	4	4	4	4
	Number of Position	4	4	2	4	2	2
	Total Cost	**288**	**288**	**344**	**288**	**344**	**344**
T12 _C8	Resource Cost	220	220	228	220	228	228
	Number of Station	4	4	4	4	4	4
	Number of Position	3	3	2	3	2	2
	Total Cost	**260**	**260**	**268**	**260**	**268**	**268**
T16_C16	Resource Cost	368	368	400	368	400	400
	Number of Station	6	6	6	6	6	6
	Number of Position	4	4	3	4	3	3
	Total Cost	**428**	**428**	**460**	**428**	**460**	**460**
T16_C18	Resource Cost	338	338	346	346	360	360
	Number of Station	7	7	5	5	6	6
	Number of Position	5	5	4	4	3	3
	Total Cost	**408**	**408**	**396**	**396**	**420**	**420**
T16_C19	Resource Cost	326	326	348	334	348	348
	Number of Station	6	6	5	5	5	5
	Number of Position	5	5	3	4	3	3
	Total Cost	**386**	**386**	**398**	**384**	**398**	**398**
T16 _C21	Resource Cost	312	312	320	320	318	318
	Number of Station	5	5	4	4	5	5
	Number of Position	4	4	4	4	3	3
	Total Cost	**362**	**362**	**360**	**360**	**368**	**368**
T16 _C22	Resource Cost	298	298	300	300	300	300
	Number of Station	5	5	4	4	4	4
	Number of Position	4	4	2	2	2	2
	Total Cost	**348**	**348**	**340**	**340**	**340**	**340**
T24_C30	Resource Cost	326	326	326	326	326	326
	Number of Station	5	5	5	5	5	5
	Number of Position	3	3	3	3	3	3
	Total Cost	**376**	**376**	**376**	**376**	**376**	**376**
T24_C35	Resource Cost	276	276	276	276	276	276
	Number of Station	4	4	4	4	4	4
	Number of Position	2	2	2	2	2	2
	Total Cost	**316**	**316**	**316**	**316**	**316**	**316**
T24_C40	Resource Cost	276	276	276	276	276	276
	Number of Station	4	4	4	4	4	4
	Number of Position	2	2	2	2	2	2
	Total Cost	**316**	**316**	**316**	**316**	**316**	**316**

In this chapter, six different objective ranking (RPS, RSP, SPR, SRP, PRS, and PSR) are analyzed. Results of test problems for presented objective ranking have been seen in table 4.1. Also, first column is shown number of task and cycle time. The abbreviation "T" is used to indicate that is number of task, and the abbreviation "C" is used to symbolize for cycle time. Second column also show results of resource cost, number of station, number of position and total cost when a task is done according to each objective ranking.

From table 4.1, it can be seen that if the number of station or position is reduced, resource cost is generally increased. For instance, when T12_C5 test problem is analyzed, objective ranking of RPS or RSP are obtained 296 unit resource cost, 6 stations, 3 positions and 356 unit total cost. On the other hand, same example (T12_C5) are examined for objective ranking of SPR or SRP and is obtained 306 unit resource cost, 5 station, 4 positions and 356 unit total cost the same as before. In other words, when number of station is reduced, number of position and resource cost are increased. At this point, it is understood that there is interaction between resource cost with number of station and position.

When the all results in table 4.1 have been examined at the same time, it can be seen that the same solution has been found for objective ranking RPS and RSP. While the first objective is to minimize resource cost (RPS or RSP), there is no influence of the sequence of following objectives which are to minimize the number of station and position over ultimate results obtained. Similarly, while the first objective is to minimize the number of position (PRS or PSR), there is no influence of the sequence of following objectives which are to minimize the resource cost and the number of station over ultimate results obtained. For this reason, it can be generalized that if the first objective is whether to minimize resource cost or to minimize the number of position, the ultimate result is not affected from the sequence of the rest.

Several alternative objective ranking (RPS, RSP, SPR, SRP, PRS, and PSR) have been provided in this study. For instance, one can use the objective ranking of RSP or RPS in the case of limited resources. On the other hand, SPR or SRP can be preferred if minimum station number has been targeted.

Optimal solutions are found within reasonable times for given test problems as seen in table 4.1 by three models. Also, same results are obtained by CNF-1, CNF-2 and DNF models. Therefore, in this study, just CNF-2 and DNF models are compared each other according to solution time. For comparing models, solution time is taken from average of CPU time by objective ranking.

CNF-2 and DNF models are compared each other according to performance of solution time for two small test problems (see figure 4.1 and figure 4.2).

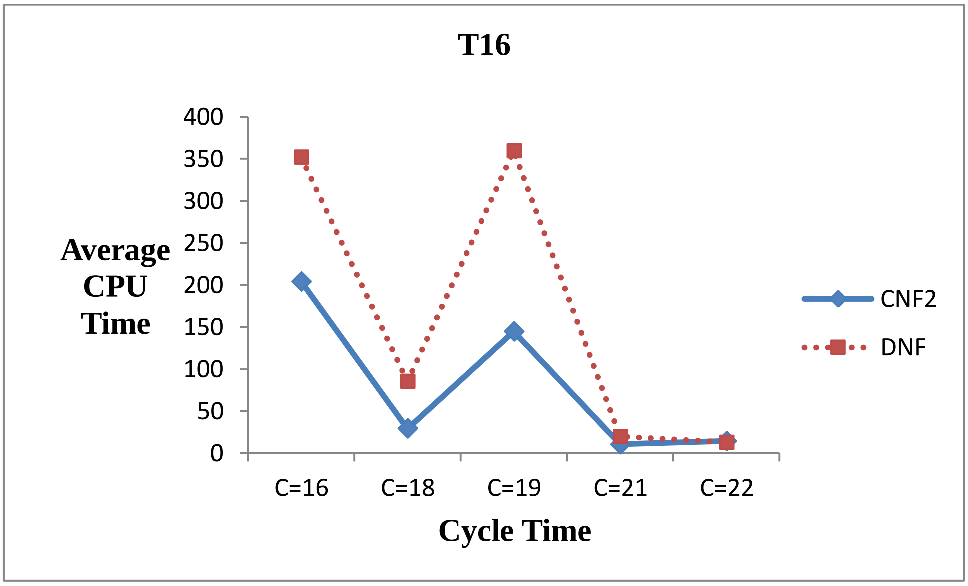

Figure 4.1. Speeds of CNF-2 and DNF models with 16-task

In figure 4.1, CNF-2 and DNF models are compared according to speeds of the solution time for 16-task test problem. CPU time is obtained by taking results of six objective ranking averages. It can be understood that CNF-2 model is better than DNF model according to solution time. For example, when solution time of T16/16 task problem is analyzed, CNF-2 model give an optimal solution about 200 in seconds for average CPU time of all objective ranking. On the other hand, with same example, optimal solution is obtained about 350 seconds by DNF model. Optimal solutions are achieved for T16 test problem with all cycle time and CNF-2 model is faster than DNF model as seen figure 4.1.

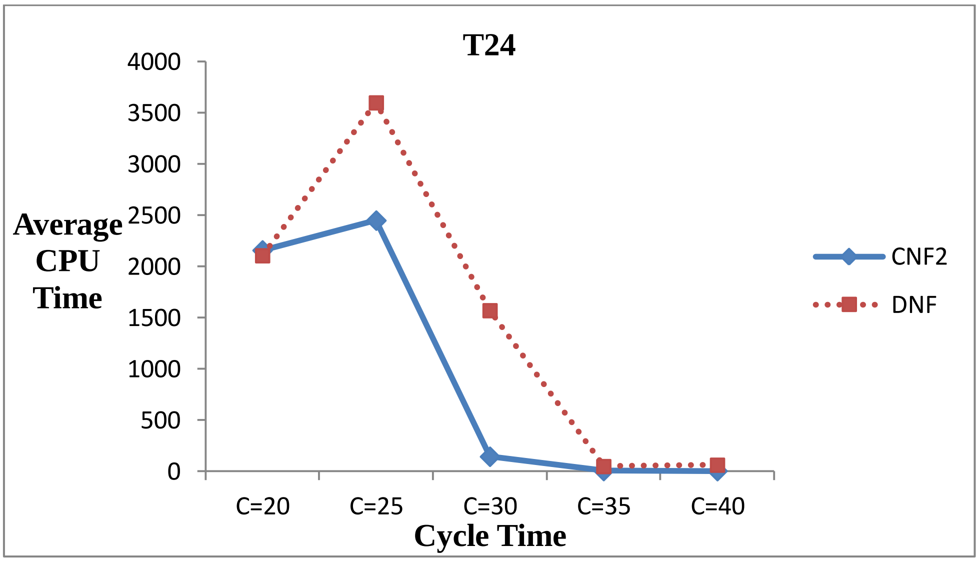

Figure 4.2. Speeds of CNF-2 and DNF models with 24-task

When results of T24 test problem are analyzed, it can be asserted that CNF-2 model is better than DNF model according to performance of solution time with same as result of T16 test problem as mentioned before. Average of solution time of all objective ranking for test problem T24/25 is about 3600 seconds for DNF model. It also mean that just an integer solution is found for all objective ranking by DNF model (see figure 4.2)

As mentioned before, proposed model for objective ranking can find optimal solution for small test problems, but it doesn't give optimal solution for large scale test problem within reasonable time. Therefore, presented models are tested for large size test problems.

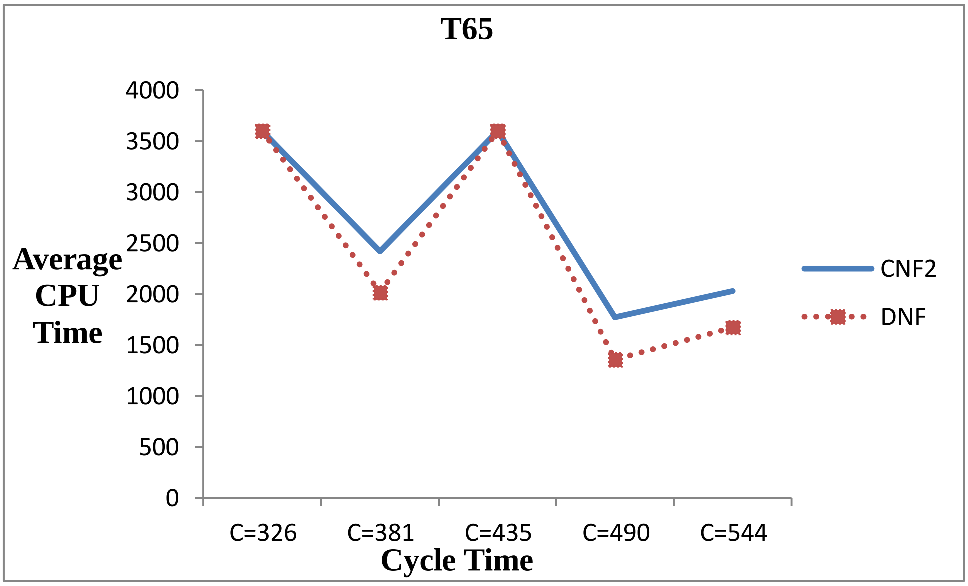

Figure 4.3. Speeds of CNF2 and DNF models with 65- task

The 65-task, 148-task and 205-task problems are solved, but just an integer solution is found for some objective ranking of T65 test problem (see figure 4.3). However, an integer solution also is not found for 148-task and 205-task problems owing to size of the problem. For this reason, results of 148-task and 205-task test problem aren't given. Average of CPU time from figure 4.3, is taken solution time of objective ranking which is given integer solution. It can be claimed that performance of CNF-2 model for large size test problem also is better than DNF model as seen figure 4.3.

CHAPTER V

HEURISTIC APPROACH

In Chapter three and four, proposed mathematical models can found a solution for small test problems within acceptable computation time. When the problem gets larger, the mathematical model fails to return optimal or integer solutions within reasonable computational times. Generally, as the number of tasks and number of workstations increase, the solution times of models increase. After conducting experimental runs on presented model, solution times are not acceptable for problems with large number of tasks. For this reason, in this chapter a heuristic approach based on COMSOAL is developed to solve especially the large size generalized resource constraint TSALB problems. Proposed heuristic is coded on Microsoft Visual C# computer programming language.

There are many studies in the literature proposing heuristic and meta heuristic approach to find a solution for TSALB problems. Some heuristic depend on behavior of population such as ant colony optimization (ACO) is presented by Baykasoglu & Dereli, (2008), artificial bee colony optimization (ABCO) is used by Tapkan et al. (2012). Genetic algorithm (GA) which is presented by Kim et al. (2000) is another important heuristic algorithm approach to solve TSALB problems. Some heuristic are also single solution based algorithm such as simulated annealing (SA) approach is proposed by Özcan (2010) and Tabu search algorithm (TSA) is used by Özcan et al. (2010) for TSALB problem.

This chapter is organized as follows: Firstly COMSOAL heuristic approach is explained. Then, the procedure of developed heuristic approach which is based on COMSOAL algorithm will be presented. An illustrative example will be given for understanding of proposed algorithm. Lastly, results of test problems will be introduced and evaluated.

5.1 Introduction COMSOAL Algorithm

The algorithm we propose in this study is based on COMSOAL (Computer Method of Sequencing Operations for Assembly Lines). This algorithm is a computer heuristic essentially reported as a solution approach to the assembly line balancing problem (Arcus, 1966). COMSOAL method has some advantages that are as follows:

- This method is fast and easy to apply assembly line balancing problem.
- Simplifies complex assembly line balancing problem.
- Solution quality could be improved by increasing the iterations –computing power makes this easy.
- Restrictions could be modeled into the algorithm easily.
- Easy to understand and implement.

These advantages also show that why COMSOAL heuristic in this study is used. The main idea of this algorithm generates so many numbers of feasible solutions randomly, and also selecting the best one among generated feasible solution. The traditional COMSOAL algorithm can be continued in 9 steps as follows:

Step 1: Determine cycle time

Step 2: Construct a list (list A), in one column, all the work elements, and in an adjacent column, operation time and number of immediate predecessors (see table 5.1)

Step 3: From list A, construct list B, constituted of the tasks which have zero predecessors.

Step 4: Activate a station, select a task from randomly list B for assignment to the station. Once more again, construct list A and list B.

Step 5: Select a task from list B and assign to station. If station time is lesser than cycle time, repeat same processes.

Step 6: If station time is bigger than cycle time, open new station.

Step 7: If unassigned tasks are not over, update list A and list B, return to Step 4

Table 5.1. COMSOAL list A

Number of task	Operation time	Number of immediate predecessors
1	6	0
2	2	1
3	5	1
4	7	1
5	1	1
6	2	1
7	3	3
8	6	1
9	5	1
10	5	1
11	4	2

Step 8: Calculate objective functions and save

Step 9: Repeat between Step 2 and Step 7, if objective function value is better than, deletes previous objective function and saved new one.

5.2 The Procedure of Proposed Approach

Proposed heuristic approach is based on COMSOAL by adding some constraint. Different from the traditional COMSOAL algorithm suitable for two sided assembly line balancing problem, the constraints of sides, mated station and resources constraints have been added.

Some assumption of the presented heuristic may be summarized as follows:

- Parameters of heuristic is known
- Objective function is to minimize total cost which are cost of resource and opened station.
- Problem can be classified as two sided assembly line balancing problem of type 1 (TSALB-1).
- Number of resources for each task is determined.
- Number of iteration is defined as iteration number = 10*number of tasks for each problem.
- All problems are considered for three resources (A, B and C).
- Requirement of resource for each of task is selected randomly (see Appendix A).
- Logical expressions of resources are designed just for conjunctive normal form (CNF) format.

The procedure keeps going until all variables are established, in other words, all tasks and resources are assigned. The general procedure steps of the proposed approach which is based on COMSOAL algorithm are summarized below:

Step 1: Read file.

Step 2: Start assignment process.

Step 3: Construct assignable list.

Step 4: If assignable list is empty, return to step 10; otherwise return to step 5.

Step 5: Select a task from assignable list randomly, and then continue.

Step 6: If selected task is either side task, then continue

Step 6.1: Choose a side randomly. If selected side is left, then go to Step 7, otherwise go to Step 8

Step 7: If selected task is left side task, then continue.

Step 7.1: Assign selected task left side station. If remaining station time is lesser then task time, go to step 9.

Step 7.2: Calculate minimum resources to perform assigned task, and also assign resource to station.

Step 7.3: Calculate remaining time of left side station.

Step 7.4: Return to step 3.

Step 8: If selected task is right side task, then continue.

Step 8.1: Assign selected task. If remaining station time is lesser then task time, go to Step 9.

Step 8.2: Calculate minimum resources to perform assigned task, and also assign resources to station.

Step 8.3: Calculate remaining time of right side station.

Step 8.4: Return to step 3.

Step 9: Open a new station in next position and go to Step 3.

Step 10: Calculate objective function. Objective function value is opened station cost and total assigned resource cost. Update the best assignment (Minimum cost assignment).

Step 11: If iteration number is satisfied then Stop; otherwise iteration number= iter+1 and go to Step 2.

For this study, our priority is to find solutions as good as the best existing procedures provide for the test problems and to keep the solution time at the minimum level. Large test problems (T65, T148 and T205) are solved. 148-task problem is taken by Bartholdi (1993), 65-task problem and 205-task problem are taken by Lee et al. (2001). Data of all task problems can be found in Appendix A. Also the results are presented in Appendix B.

5.3 An Illustrative Example

There is a trade-off between solution quality and the solution time for the heuristic procedures. If search time converges to infinity, the procedure also may be find optimal solution for all test problems. A balance should be determined between solution quality and solution time. For this reason, number of iteration is limited by 10*number of task. The procedure is coded in Microsoft Visual C# 2010 and run on a computer Intel Core™ i3 3.10 GHz processor with 4 GB RAM personal computer. Developed C# program interface for proposed heuristic approach is shown in figure 5.1.

When 65-task problem for cycle time C=381 is balancing, Results are given below. Position shows number of mated station.

Table 5.2. Results of T65/C381-test problem

Position	**Right Side**		**Left Side**	
	Assigned Tasks	**Assigned Resources**	**Assigned Tasks**	**Assigned Resources**
1	1,2,3,4,5,11,41	5B+5C	13,30,23,24	3A+5C
2	26,7,8,44	3A+2C	25,9,10,49,6	4A+4B+2C
3	42,43,12,14,22	4A+3B	45,46,29,47,48	5A+5B+C
4	27,28	2B+4C	15,16,17,20	3A+5C
5	18,19,21,31,59,55,60	2A+B+5C	-	-
6	61,32,33,34,52,62	3A+2B+2C	36,37,38,53	5A+4B
7	51,54,56,40	3A+B+4C	63,64,58,39	5A+5B
8	57,35,50,65	5A+4C	-	-
Total Cost: 1314 unit				

Total cost constitutes opened station cost and assigned resource cost. Table 5.2 shows tasks and resources to assign workstation. For example, if first position of right side station is examined , tasks{1,2,3,4,5,11,44} can be done by 5 unit resource B and 5 unit resource C. In this example 8 positions and 14 stations are opened. In this case, total cost=45*A+ 32*B+39*C+10*opened station number

=45*10+32*8+39*12+10*14

=1314 unit for T65/C381-test problem

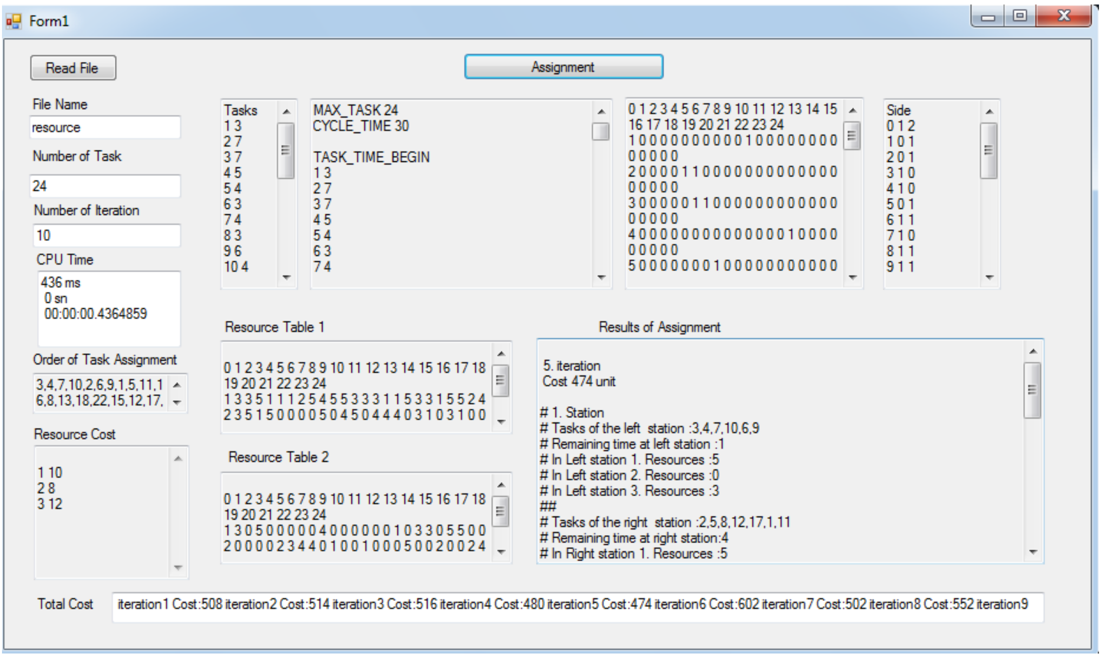

Figure 5.1. Developed C# program interface for proposed heuristic approach

5.4 Result of the Test Problems for Proposed Approach

In this section, the large size test problems are solved with different cycle time. 34-tests are done, and also compared with the results of Group assignment procedure which is presented by Lee et al. (2001), results of Ant colony optimization which is presented by Baykasoglu & Dereli (2008) and results of Bee algorithm which is presented by Özbakır & Tapkan (2011)

Table 5.3. Comparison of large size test problems according to number of station

Problems	Cycle Time	GA	BA	ACO	Proposed algorithm with resource constraint
T65	326	17	17	17	18
	381	15	15	15	**14**
	435	13	13	13	**13**
	490	12	12	12	**12**
	544	10	10	10	**10**
T148	204	27	26	26	28
	255	21	21	21	22
	306	18	18	18	**18**
	357	15	15	15	17
	408	14	14	14	**14**
	459	13	12	12	14
	510	11	11	11	**11**
T205	1133	23	24	24	24
	1322	20	20	22	21
	1510	20	18	18	**18**
	1699	16	16	18	17
	1888	16	15	15	16
	2077	14	13	14	15
	2266	13	12	12	14
	2454	12	12	12	14
	2643	12	10	11	14
	2832	10	10	10	14

Comparisons are done according to opened number of station by Group assignment procedure, ACO algorithm, Bee algorithm and proposed algorithm. Results on table 5.3 demonstrate that the proposed algorithm has given hope, although presented algorithm has resource constraint. The proposed algorithm found the same results for 7 problems, in 1 test problems it carried out better. For example, T65/C381 test problems are solved by Group assignment, ACO algorithm and Bee algorithm with 15 stations but it can be solved by proposed algorithm with 14 stations.

CHAPTER VI

CONCLUSION AND FUTURE RESEARCH DIRECTIONS

In this thesis, general resource constraint two sided assembly line balancing problem is discussed. First of all, a mathematical model is developed to solve the problems optimally and solve some example problems from the literature which is well known test problem. The model is combined by Corominas et al. (2011) approach which is general resource constraint for simple assembly line balancing problem and Ağpak et al. (2012) mathematical model to adapt TSALB problem. The purpose of objective function is to minimize number of workstation and total resource cost for given cycle time and resources. To achieve this aim, a mathematical model based on literature models for generalized resource constraint two sided assembly line balancing problem is presented to solve this problem in an optimal manner. For large size test problems, the mathematical model fails to give the optimal solution within reasonable computational times. Therefore, a heuristic approach based on the COMSOAL is proposed.

In Chapter 3, generalized resource constrained for TSALB problem is examined. Two objective functions are combined for minimization of total cost. These functions are minimization of resource cost and minimization of station cost. Presented models can be found optimal solution for small test problem within acceptable time. Three different models CNF-1, CNF-2 and DNF are tested and compared according to solution time. Speed of CNF-2 model is better than CNF-1 and DNF model.

In Chapter 4, the multi-objective generalized resource constraint TSALB problem is studied. Three different objective functions are presented. These functions are minimization of resource cost, minimization of station cost (in case of equal cost, number of station) and minimization number of position. Results of test problems show that the ultimate result is affected from objective ranking, and also it reveals that problems must examined as multi-objective. From the analysis, results of changing objective ranking are presented some alternative case for decision makers. Three objective ranking (resource cost, number of station and position) which are affected each other, objective ranking can be selected depend on requirement of decision-maker. For instance, one can use the objective ranking of RSP or RPS in the case of limited resources. On the other hand, SPR or SRP can be preferred if minimum station number has been targeted. While the first objective is to minimize resource cost (RPS or RSP), there is no influence of the sequence of the following objectives which are to minimize the number of station and position over ultimate results obtained. Similarly, while the first objective is to minimize the number of position (PRS or PSR), there is no influence of the sequence of following objectives which are to minimize the resource cost and the number of station over ultimate results obtained. For this reason, it can be generalized that if the first objective is whether to minimize resource cost or to minimize the number of position, the ultimate result is not affected from the sequence of the rest.

In chapter 5, heuristic approach is presented for generalized resource constraint TSALB problem. The heuristic which is based on COMSOAL algorithm is compare with other algorithm. Results of test problem provide that the proposed approach is efficient.

To the best of our knowledge, this study is the first attempt to solve generalized resource constrained two-sided assembly line balancing problem. We hope our study helps to open new research areas most substantial of which are debated as follow:

- In this study, the method of lexicographic optimization is used. For future study, different approaches can be used like weighting method. At the same time, amount of resources are not limited in this study. However, for a resource can be given restriction simply as follows:

$$\sum_{k=1}^{K}\sum_{s=1}^{2} Y_{rks} \leq \text{Amount of Resource}$$

Amount of resource deviation in certain value can be added as constraint. Also, by using goal programming approach can be minimizing this constraint (deviation variable).

- We used deterministic task times. Using stochastic task times may be taken into account for further study.
- We consider three resources case, four or more than resources can be used as a future study.
- Generalized resource constraint approach can be extended to include different line layout balancing problem like U-type or two sided U-type assembly line balancing problem.
- For large test problems, more efficient heuristic based on behavior of population such as ant colony optimization and bee colony algorithm can be improved to solve this problem.
- Practical applications of the problem are also left for future studies.
- Development of GRCTSALB problem like stochastic task times, multi or mixed model etc. can be examined in further studies.

To sum up, in this study different approaches are examined for generalized resource constraint two-sided assembly line balancing problem.

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APPENDIX A. DATA OF TEST PROBLEMS

Table A.1. Data of 9-task problem

Task	Operation Time	Side	Immediate Successors	Required Resources
1	2	L	4	$(A \vee 2B) \wedge 5C$
2	3	R	5-6	$(5A \vee 2B) \wedge (5A \vee C)$
3	2	E	6	$(3A \vee 3C) \wedge (B \vee 3C)$
4	3	L	7	$4A \wedge (B \vee 5C)$
5	1	R	7-8	$(2A \vee 5B) \wedge 3C$
6	1	E	9	$(3A \vee 4B) \wedge (3A \vee 5C)$
7	2	E	-	$A \wedge (B \vee 3C)$
8	2	L	-	$(3A \vee 4C) \wedge (3B \vee 4C)$
9	1	E	-	$(5A \vee B) \wedge (5A \vee C)$

Table A.2. Data of 12-task problem

Task	Operation Time	Side	Immediate Successors	Required Resources
1	2	L	4	$(2A \vee B) \wedge (2A \vee 2C)$
2	3	R	5	$(A \vee B) \wedge (A \vee C)$
3	2	E	6	$5A \wedge (2B \vee 4C$
4	3	L	7	$(A \vee 5B) \wedge (A \vee C)$
5	1	E	7-8-9	$(A \vee 5C) \wedge (B \vee 5C)$
6	1	L	9	$(4A \vee 4B) \wedge 5C$
7	3	E	10	$(3A \vee 3B) \wedge (3A \vee 5C)$
8	3	R	10	$(5A \vee 4C) \wedge (5B \vee 4C)$
9	2	E	11	$(A \vee 3B) \wedge (A \vee 3C)$
10	2	E	-	$(A \vee 4C) \wedge (5B \vee 4C)$
11	2	E	12	$(5A \vee 2B) \wedge 3C$
12	1	R	-	$(4A \vee 4B) \wedge 4C$

Table A.3. Data of 16- task problem

Task	Operation Time	Side	Immediate Successors	Required Resources
1	6	E	3-4	$(4A \vee 3B) \wedge (4A \vee 2C)$
2	5	E	5	$A \wedge (5B \vee C)$
3	2	L	6	$(2A \vee B) \wedge (2A \vee 2C)$
4	9	E	7	$(A \vee 5B) \wedge 4C$
5	8	R	7	$(4A \vee B) \wedge 5C$
6	4	L	8	$(A \vee 3C) \wedge (4B \vee 3C)$
7	7	E	8-9-10	$(3A \vee 3C) \wedge (3B \vee 3C)$
8	4	E	11	$2A \wedge (4B \vee 5C)$
9	5	R	12-13	$(5A \vee 4B) \wedge 4C$
10	4	R	13	$(4A \vee C) \wedge (4B \vee C)$
11	6	E	14-15	$(A \vee B) \wedge (A \vee 2C)$
12	5	L	15	$3A \wedge (5B \vee C)$
13	6	E	16	$(A \vee 3C) \wedge (4B \vee 3C)$
14	4	E	-	$3A \wedge (5B \vee 4C)$
15	3	E	-	$A \wedge (5B \vee 5C)$
16	4	E	-	$(4A \vee 2B) \wedge (4A \vee C)$

Table A.4. Data of 24-task problem

Task	Operation Time	Side	Immediate Successors	Required Resources
1	3	L	11	$(3A \vee 3B) \wedge (3A \vee C)$
2	7	L	5-6	$(3A \vee 5B) \wedge C$
3	7	R	6-7	$(5A \vee B) \wedge (5A \vee 4C)$
4	5	R	15	$(A \vee 5B) \wedge C$
5	4	L	8	$(A \vee 3C) \wedge (2B \vee 3C)$
6	3	E	9	$(A \vee C) \wedge (3B \vee C)$
7	4	R	10	$2A \wedge (4B \vee 3C)$
8	3	E	12	$5A \wedge (4B \vee 2C)$
9	6	E	12-13-14	$(4A \vee 5B) \wedge (4A \vee C)$
10	4	E	14	$(5A \vee 3C) \wedge (B \vee 3C)$
11	4	L	16	$(5A \vee 4B) \wedge 3C$
12	3	L	17	$(3A \vee 5B) \wedge 2C$
13	3	E	18-19	$3A \wedge (B \vee 4C)$
14	9	R	19	$(3A \vee 4B) \wedge 3C$
15	5	R	20	$(A \vee 4B) \wedge 4C$
16	9	L	21	$(A \vee 4B) \wedge (A \vee 2C)$
17	2	E	21	$5A \wedge (5B \vee 2C)$

Table A.4 (Continued)

18	7	E	22	$(3A \vee 3B) \wedge (3A \vee 3C)$
19	9	E	23	$(3A \vee B) \wedge (3A \vee C)$
20	9	R	23-24	$A \wedge (2B \vee 2C)$
21	8	L	-	$(5A \vee 3B) \wedge (5A \vee 3C)$
22	8	E	-	$(5A \vee B) \wedge (5A \vee 2C)$
23	9	R	-	$2A \wedge (2B \vee C)$
24	9	E	-	$(4A \vee 4C) \wedge (4B \vee 4C)$

Table A.5. Data of 65- task problem

Task	**Operation Time**	**Side**	**Immediate Successors**	**Required Resources**
1	49	E	3	$(5A \vee 4B) \wedge C$
2	49	E	3	$(2A \vee 2B) \wedge 5C$
3	71	E	4-23	$(5A \vee B) \wedge 3C$
4	26	E	5-6-7-9-11-12-25-26-27-41-45-49	$(4A \vee 2B) \wedge (4A \vee 2C)$
5	42	E	14	$(3A \vee 5B) \wedge 5C$
6	30	E	14	$(5A \vee 4B) \wedge C$
7	167	R	8	$(3A \vee 3B) \wedge (3A \vee 3C)$
8	91	R	14	$3A \wedge (5B \vee C)$
9	52	L	10	$(4A \vee 4B) \wedge (4A \vee 5C)$
10	153	L	14	$2A \wedge (4B \vee 5C)$
11	68	E	14	$(A \vee 3B) \wedge (A \vee C)$
12	52	E	14	$(4A \vee 5C) \wedge (3B \vee 5C)$
13	135	E	14	$3A \wedge (4B \vee 2C)$
14	54	E	15-18-20-22	$(2A \vee 5C) \wedge (2B \vee 5C)$
15	57	E	16	$(A \vee 4B) \wedge 4C$
16	151	L	17	$(A \vee B) \wedge 5C$
17	39	L	31	$A \wedge (4B \vee 5C)$
18	194	R	19	$(A \vee 4C) \wedge (5B \vee 4C)$
19	35	R	21	$(4A \vee 5C) \wedge (5B \vee 5C)$
20	119	E	21	$3A \wedge (4B \vee 3C)$
21	34	E	31	$(A \vee 3C) \wedge (5B \vee 3C)$
22	38	E	31	$(A \vee 2C) \wedge (3B \vee 2C)$
23	104	E	24	$(2A \vee 5B) \wedge 5C$
24	84	E	31	$A \wedge (2B \vee 5C)$
25	113	L	31	$(5A \vee B) \wedge (5A \vee 2C)$
26	72	R	31	$(3A \vee 5B) \wedge C$
27	62	R	28	$(5A \vee 2B) \wedge 3C$
28	272	R	50	$(3A \vee 4C) \wedge (3B \vee 4C)$

Table A.5 (Continued)

29	89	L	50	$(3A \vee 2B) \wedge (3A \vee 2C)$
30	49	L	50	$(3A \vee 4B) \wedge (3A \vee C)$
31	11	E	32-36-51-52-53-54-55-56-58-59-60-61-62	$(3A \vee B) \wedge (3A \vee 2C)$
32	45	E	33	$(A \vee 5B) \wedge (A \vee 4C)$
33	54	E	34	$(2A \vee 4C) \wedge (2B \vee 4C)$
34	106	E	35	$(2A \vee B) \wedge 2C$
35	132	R	50	$(2A \vee 3C) \wedge (4B \vee 3C)$
36	52	E	37	$2A \wedge (3B \vee 4C)$
37	157	E	38	$(4A \vee 4C) \wedge (2B \vee 4C)$
38	109	E	39-40	$(5A \vee 3B) \wedge (5A \vee 4C)$
39	32	L	50	$5A \wedge (5B \vee 3C)$
40	32	R	50	$(3A \vee 2C) \wedge (3B \vee 2C)$
41	52	E	42	$(2A \vee 5C) \wedge (2B \vee 5C)$
42	193	E	43	$4A \wedge (2B \vee 3C)$
43	34	E	62	$(2A \vee 5B) \wedge (2A \vee 2C)$
44	34	R	46	$(A \vee 5B) \wedge 2C$
45	97	L	46	$5A \wedge (4B \vee 4C)$
46	37	E	47	$(5A \vee 5B) \wedge (5A \vee 3C)$
47	25	L	48	$4A \wedge (5B \vee 2C)$
48	89	L	50	$(4A \vee 4B) \wedge C$
49	27	E	50	$(3A \vee 4C) \wedge (3B \vee 4C)$
50	50	E	65	$3A \wedge (B \vee 3C)$
51	46	R	65	$3A \wedge (B \vee 4C)$
52	46	E	65	$(2A \vee C) \wedge (2B \vee C)$
53	55	L	65	$(A \vee 4C) \wedge (4B \vee 4C)$
54	118	E	65	$(2A \vee 4B) \wedge 2C$
55	47	R	65	$(A \vee 4B) \wedge 2C$
56	164	E	57	$(A \vee 4C) \wedge (5B \vee 4C)$
57	113	E	65	$(5A \vee 2C) \wedge (2B \vee 2C)$
58	69	L	65	$3A \wedge (4B \vee 4C)$
59	30	R	65	$A \wedge (B \vee C)$
60	25	E	65	$2A \wedge (2B \vee 5C)$
61	106	R	65	$(3A \vee B) \wedge 2C$
62	23	E	63	$3A \wedge (3B \vee 2C)$
63	118	L	64	$(3A \vee 3B) \wedge (3A \vee 2C)$
64	155	L	65	$(4A \vee 3B) \wedge (4A \vee 4C)$
65	65	E	-	$5A \wedge (2B \vee 4C)$

Table A.6. Data of 148-task problem

Task	Operation Time	Side	Immediate Successors	Required Resources
1	16	E	5-6-7-8	$(5A \lor 4B) \land C$
2	30	E	3	$(2A \lor 2B) \land 5C$
3	7	E	4-5-6-7	$(5A \lor B) \land 3C$
4	47	E	8	$(4A \lor 2B) \land (4A \lor 2C)$
5	29	E	14	$(3A \lor 5B) \land 5C$
6	8	E	9	$(5A \lor 4B) \land C$
7	39	E	14	$(3A \lor 3B) \land (3A \lor 3C)$
8	37	E	10	$3A \land (5B \lor C)$
9	32	E	14	$(4A \lor 4B) \land (4A \lor 5C)$
10	29	E	14	$2A \land (4B \lor 5C)$
11	17	E	12	$(A \lor 3B) \land (A \lor C)$
12	11	E	13	$(4A \lor 5C) \land (3B \lor 5C)$
13	32	E	-	$3A \land (4B \lor 2C)$
14	15	E	15-16	$(2A \lor 5C) \land (2B \lor 5C)$
15	53	L	17	$(A \lor 4B) \land 4C$
16	53	R	17	$(A \lor B) \land 5C$
17	8	E	18-19	$A \land (4B \lor 5C)$
18	24	L	20	$(A \lor 4C) \land (5B \lor 4C)$
19	24	R	20	$(4A \lor 5C) \land (5B \lor 5C)$
20	8	E	21-22-23-24	$3A \land (4B \lor 3C)$
21	7	R	25-26-27-28	$(A \lor 3C) \land (5B \lor 3C)$
22	8	L	25-26-27-28	$(A \lor 2C) \land (3B \lor 2C)$
23	14	L	25-26-27-28	$(2A \lor 5B) \land 5C$
24	13	R	25-26-27-28	$A \land (2B \lor 5C)$
25	10	R	29	$(5A \lor B) \land (5A \lor 2C)$
26	25	R	29	$(3A \lor 5B) \land C$
27	11	L	29	$(5A \lor 2B) \land 3C$
28	25	L	29	$(3A \lor 4C) \land (3B \lor 4C)$
29	11	E	31	$(3A \lor 2B) \land (3A \lor 2C)$
30	29	R	-	$(3A \lor 4B) \land (3A \lor C)$
31	25	E	36	$(3A \lor B) \land (3A \lor 2C)$
32	10	L	34	$(A \lor 5B) \land (A \lor 4C)$
33	14	R	35	$(2A \lor 4C) \land (2B \lor 4C)$
34	41	L	36	$(2A \lor B) \land 2C$
35	42	R	36	$(2A \lor 3C) \land (4B \lor 3C)$
36	47	R	37	$2A \land (3B \lor 4C)$
37	7	R	38-45	$(4A \lor 4C) \land (2B \lor 4C)$
38	80	R	39	$(5A \lor 3B) \land (5A \lor 4C)$
39	7	R	40	$5A \land (5B \lor 3C)$

Table A.6 (Continued)

40	41	R	41-48-54	$(3A \vee 2C) \wedge (3B \vee 2C)$
41	47	R	-	$(2A \vee 5C) \wedge (2B \vee 5C)$
42	16	L	43	$4A \wedge (2B \vee 3C)$
43	32	L	44	$(2A \vee 5B) \wedge (2A \vee 2C)$
44	66	L	-	$(A \vee 5B) \wedge 2C$
45	80	L	46	$5A \wedge (4B \vee 4C)$
46	7	L	47	$(5A \vee 5B) \wedge (5A \vee 3C)$
47	41	L	48-49	$4A \wedge (5B \vee 2C)$
48	13	E	-	$(4A \vee 4B) \wedge C$
49	47	L	-	$(3A \vee 4C) \wedge (3B \vee 4C)$
50	33	E	51	$3A \wedge (B \vee 3C)$
51	34	L	53-69	$3A \wedge (B \vee 4C)$
52	11	L	53	$(2A \vee C) \wedge (2B \vee C)$
53	118	L	-	$(A \vee 4C) \wedge (4B \vee 4C)$
54	25	L	55-72-76-90-133	$(2A \vee 4B) \wedge 2C$
55	7	R	133	$(A \vee 4B) \wedge 2C$
56	28	E	73	$(A \vee 4C) \wedge (5B \vee 4C)$
57	12	L	82	$(5A \vee 2C) \wedge (2B \vee 2C)$
58	52	L	86-88	$3A \wedge (4B \vee 4C)$
59	14	E	75-89	$A \wedge (B \vee C)$
60	3	E	-	$2A \wedge (2B \vee 5C)$
61	3	E	62	$(3A \vee B) \wedge 2C$
62	8	E	63	$3A \wedge (3B \vee 2C)$
63	16	E	67	$(3A \vee 3B) \wedge (3A \vee 2C)$
64	33	R	65-71-72	$(4A \vee 3B) \wedge (4A \vee 4C)$
65	8	E	66-99	$5A \wedge (2B \vee 4C)$
66	18	E	67	$(5A \vee 4B) \wedge C$
67	10	E	68	$(2A \vee 2B) \wedge 5C$
68	14	E	95-98	$(5A \vee B) \wedge 3C$
69	28	R	79	$(4A \vee 2B) \wedge (4A \vee 2C)$
70	11	R	71	$(3A \vee 5B) \wedge 5C$
71	118	R	-	$(5A \vee 4B) \wedge C$
72	25	R	134	$(3A \vee 3B) \wedge (3A \vee 3C)$
73	40	E	86-88-89-90-96	$3A \wedge (5B \vee C)$
74	40	E	75	$(4A \vee 4B) \wedge (4A \vee 5C)$
75	101	E	90-97	$2A \wedge (4B \vee 5C)$
76	5	E	77	$(A \vee 3B) \wedge (A \vee C)$
77	28	E	78	$(4A \vee 5C) \wedge (3B \vee 5C)$
78	8	E	82	$3A \wedge (4B \vee 2C)$
79	111	E	80	$(2A \vee 5C) \wedge (2B \vee 5C)$

Table A.6 (Continued)

80	7	E	81	$(A \vee 4B) \wedge 4C$
81	26	E	82	$(A \vee B) \wedge 5C$
82	10	E	83-84	$A \wedge (4B \vee 5C)$
83	21	E	-	$(A \vee 4C) \wedge (5B \vee 4C)$
84	26	E	106	$(4A \vee 5C) \wedge (5B \vee 5C)$
85	20	E	-	$3A \wedge (4B \vee 3C)$
86	21	E	87	$(A \vee 3C) \wedge (5B \vee 3C)$
87	47	E	-	$(A \vee 2C) \wedge (3B \vee 2C)$
88	23	E	-	$(2A \vee 5B) \wedge 5C$
89	13	E	-	$A \wedge (2B \vee 5C)$
90	19	E	110	$(5A \vee B) \wedge (5A \vee 2C)$
91	115	E	105	$(3A \vee 5B) \wedge C$
92	35	E	135	$(5A \vee 2B) \wedge 3C$
93	26	L	-	$(3A \vee 4C) \wedge (3B \vee 4C)$
94	46	E	-	$(3A \vee 2B) \wedge (3A \vee 2C)$
95	20	E	101	$(3A \vee 4B) \wedge (3A \vee C)$
96	31	E	104	$(3A \vee B) \wedge (3A \vee 2C)$
97	19	E	-	$(A \vee 5B) \wedge (A \vee 4C)$
98	34	E	101	$(2A \vee 4C) \wedge (2B \vee 4C)$
99	51	E	100	$(2A \vee B) \wedge 2C$
100	39	E	101	$(2A \vee 3C) \wedge (4B \vee 3C)$
101	30	E	102-103	$2A \wedge (3B \vee 4C)$
102	26	E	127	$(4A \vee 4C) \wedge (2B \vee 4C)$
103	13	E	127	$(5A \vee 3B) \wedge (5A \vee 4C)$
104	45	E	-	$5A \wedge (5B \vee 3C)$
105	58	E	119	$(3A \vee 2C) \wedge (3B \vee 2C)$
106	28	E	107	$(2A \vee 5C) \wedge (2B \vee 5C)$
107	8	E	108	$4A \wedge (2B \vee 3C)$
108	43	E	109	$(2A \vee 5B) \wedge (2A \vee 2C)$
109	40	E	110	$(A \vee 5B) \wedge 2C$
110	34	E	-	$5A \wedge (4B \vee 4C)$
111	23	E	112	$(5A \vee 5B) \wedge (5A \vee 3C)$
112	162	L	113	$4A \wedge (5B \vee 2C)$
113	11	L	114-116-120-123-128	$(4A \vee 4B) \wedge C$
114	19	E	115	$(3A \vee 4C) \wedge (3B \vee 4C)$
115	14	E	125	$3A \wedge (B \vee 3C)$
116	31	E	117	$3A \wedge (B \vee 4C)$
117	32	E	118	$(2A \vee C) \wedge (2B \vee C)$
118	26	E	126	$(A \vee 4C) \wedge (4B \vee 4C)$
119	55	E	-	$(2A \vee 4B) \wedge 2C$

Table A.6 (Continued)

120	31	E	121	$(A \vee 4B) \wedge 2C$
121	32	E	122	$(A \vee 4C) \wedge (5B \vee 4C)$
122	26	E	126	$(5A \vee 2C) \wedge (2B \vee 2C)$
123	19	E	124	$3A \wedge (4B \vee 4C)$
124	14	E	125	$A \wedge (B \vee C)$
125	19	E	-	$2A \wedge (2B \vee 5C)$
126	48	E	-	$(3A \vee B) \wedge 2C$
127	55	E	-	$3A \wedge (3B \vee 2C)$
128	8	L	129	$(3A \vee 3B) \wedge (3A \vee 2C)$
129	11	L	130	$(4A \vee 3B) \wedge (4A \vee 4C)$
130	27	L	131-137	$5A \wedge (2B \vee 4C)$
131	18	L	-	$(5A \vee 4B) \wedge C$
132	36	E	135	$(2A \vee 2B) \wedge 5C$
133	23	L	135	$(5A \vee B) \wedge 3C$
134	20	R	135	$(4A \vee 2B) \wedge (4A \vee 2C)$
135	46	E	136	$(3A \vee 5B) \wedge 5C$
136	64	E	-	$(5A \vee 4B) \wedge C$
137	22	L	-	$(3A \vee 3B) \wedge (3A \vee 3C)$
138	15	E	139	$3A \wedge (5B \vee C)$
139	34	E	140	$(4A \vee 4B) \wedge (4A \vee 5C)$
140	22	E	-	$2A \wedge (4B \vee 5C)$
141	151	L	142	$(A \vee 3B) \wedge (A \vee C)$
142	148	R	143-146-147-148	$(4A \vee 5C) \wedge (3B \vee 5C)$
143	64	L	-	$3A \wedge (4B \vee 2C)$
144	170	L	145	$(2A \vee 5C) \wedge (2B \vee 5C)$
145	137	R	147-148	$(A \vee 4B) \wedge 4C$
146	64	R	-	$(A \vee B) \wedge 5C$
147	78	L	-	$A \wedge (4B \vee 5C)$
148	78	R	-	$(A \vee 4C) \wedge (5B \vee 4C)$

Table A.7. Data of 205- task problem

Task	Operation Time	Side	Immediate Successors	Required Resources
1	692	E	36	$(5A \vee 4B) \wedge C$
2	42	E	3-4	$(2A \vee 2B) \wedge 5C$
3	261	R	5	$(5A \vee B) \wedge 3C$
4	261	L	5	$(4A \vee 2B) \wedge (4A \vee 2C)$
5	157	E	7-13	$(3A \vee 5B) \wedge 5C$
6	90	E	36	$(5A \vee 4B) \wedge C$
7	54	R	8	$(3A \vee 3B) \wedge (3A \vee 3C)$
8	67	R	9	$3A \wedge (5B \vee C)$
9	30	R	10	$(4A \vee 4B) \wedge (4A \vee 5C)$
10	106	R	11	$2A \wedge (4B \vee 5C)$
11	32	R	12	$(A \vee 3B) \wedge (A \vee C)$
12	62	R	36	$(4A \vee 5C) \wedge (3B \vee 5C)$
13	54	L	14	$3A \wedge (4B \vee 2C)$
14	67	L	15	$(2A \vee 5C) \wedge (2B \vee 5C)$
15	30	L	16	$(A \vee 4B) \wedge 4C$
16	106	L	17	$(A \vee B) \wedge 5C$
17	32	L	18	$A \wedge (4B \vee 5C)$
18	62	L	36	$(A \vee 4C) \wedge (5B \vee 4C)$
19	56	E	36	$(4A \vee 5C) \wedge (5B \vee 5C)$
20	67	E	22	$3A \wedge (4B \vee 3C)$
21	86	E	22	$(A \vee 3C) \wedge (5B \vee 3C)$
22	37	E	23	$(A \vee 2C) \wedge (3B \vee 2C)$
23	41	E	24-34	$(2A \vee 5B) \wedge 5C$
24	72	E	26-27-28	$A \wedge (2B \vee 5C)$
25	86	R	28	$(5A \vee B) \wedge (5A \vee 2C)$
26	16	L	35	$(3A \vee 5B) \wedge C$
27	51	R	35	$(5A \vee 2B) \wedge 3C$
28	66	R	29	$(3A \vee 4C) \wedge (3B \vee 4C)$
29	41	R	30-33	$(3A \vee 2B) \wedge (3A \vee 2C)$
30	72	R	31-32	$(3A \vee 4B) \wedge (3A \vee C)$
31	51	R	35	$(3A \vee B) \wedge (3A \vee 2C)$
32	16	R	35	$(A \vee 5B) \wedge (A \vee 4C)$
33	15	R	35	$(2A \vee 4C) \wedge (2B \vee 4C)$
34	15	L	35	$(2A \vee B) \wedge 2C$
35	85	E	36	$(2A \vee 3C) \wedge (4B \vee 3C)$
36	59	E	37-40-41-42-62-69-72-75-83-110-111-112	$2A \wedge (3B \vee 4C)$
37	23	L	38	$(4A \vee 4C) \wedge (2B \vee 4C)$
38	13	L	39	$(5A \vee 3B) \wedge (5A \vee 4C)$

Table A.7 (Continued)

39	19	L	45	$5A \wedge (5B \vee 3C)$
40	108	E	43-54	$(3A \vee 2C) \wedge (3B \vee 2C)$
41	214	E	92	$(2A \vee 5C) \wedge (2B \vee 5C)$
42	80	E	43-54	$4A \wedge (2B \vee 3C)$
43	37	L	44	$(2A \vee 5B) \wedge (2A \vee 2C)$
44	84	L	45	$(A \vee 5B) \wedge 2C$
45	18	L	46-48-51-53	$5A \wedge (4B \vee 4C)$
46	12	L	47	$(5A \vee 5B) \wedge (5A \vee 3C)$
47	29	L	92	$4A \wedge (5B \vee 2C)$
48	37	L	49	$(4A \vee 4B) \wedge C$
49	13	L	50	$(3A \vee 4C) \wedge (3B \vee 4C)$
50	70	L	92	$3A \wedge (B \vee 3C)$
51	217	L	52	$3A \wedge (B \vee 4C)$
52	72	L	92	$(2A \vee C) \wedge (2B \vee C)$
53	85	L	92	$(A \vee 4C) \wedge (4B \vee 4C)$
54	43	R	55-133	$(2A \vee 4B) \wedge 2C$
55	97	R	56-59-61	$(A \vee 4B) \wedge 2C$
56	37	R	57	$(A \vee 4C) \wedge (5B \vee 4C)$
57	13	R	58	$(5A \vee 2C) \wedge (2B \vee 2C)$
58	35	R	92	$3A \wedge (4B \vee 4C)$
59	217	R	60	$A \wedge (B \vee C)$
60	72	R	92	$2A \wedge (2B \vee 5C)$
61	85	R	92	$(3A \vee B) \wedge 2C$
62	25	E	63	$3A \wedge (3B \vee 2C)$
63	37	E	64	$(3A \vee 3B) \wedge (3A \vee 2C)$
64	37	E	65-68	$(4A \vee 3B) \wedge (4A \vee 4C)$
65	103	E	66	$5A \wedge (2B \vee 4C)$
66	140	E	67	$(5A \vee 4B) \wedge C$
67	49	E	80	$(2A \vee 2B) \wedge 5C$
68	35	E	80	$(5A \vee B) \wedge 3C$
69	51	E	70	$(4A \vee 2B) \wedge (4A \vee 2C)$
70	88	E	71	$(3A \vee 5B) \wedge 5C$
71	53	E	73	$(5A \vee 4B) \wedge C$
72	144	E	73	$(3A \vee 3B) \wedge (3A \vee 3C)$
73	337	E	74	$3A \wedge (5B \vee C)$
74	107	E	76	$(4A \vee 4B) \wedge (4A \vee 5C)$
75	371	E	92	$2A \wedge (4B \vee 5C)$
76	97	E	77-78-79	$(A \vee 3B) \wedge (A \vee C)$
77	166	E	80-82	$(4A \vee 5C) \wedge (3B \vee 5C)$
78	92	L	80	$3A \wedge (4B \vee 2C)$

Table A.7 (Continued)

79	92	R	80	$(2A \vee 5C) \wedge (2B \vee 5C)$
80	106	E	81	$(A \vee 4B) \wedge 4C$
81	49	E	84	$(A \vee B) \wedge 5C$
82	92	E	92	$A \wedge (4B \vee 5C)$
83	371	E	92	$(A \vee 4C) \wedge (5B \vee 4C)$
84	87	E	85	$(4A \vee 5C) \wedge (5B \vee 5C)$
85	162	E	86-88-90	$3A \wedge (4B \vee 3C)$
86	96	E	87	$(A \vee 3C) \wedge (5B \vee 3C)$
87	79	E	92	$(A \vee 2C) \wedge (3B \vee 2C)$
88	96	E	89	$(2A \vee 5B) \wedge 5C$
89	42	E	92	$A \wedge (2B \vee 5C)$
90	88	R	91	$(5A \vee B) \wedge (5A \vee 2C)$
91	90	R	92	$(3A \vee 5B) \wedge C$
92	97	R	93-94-95-96-97-98-99	$(5A \vee 2B) \wedge 3C$
93	270	R	135	$(3A \vee 4C) \wedge (3B \vee 4C)$
94	452	E	135	$(3A \vee 2B) \wedge (3A \vee 2C)$
95	48	R	113	$(3A \vee 4B) \wedge (3A \vee C)$
96	338	E	113	$(3A \vee B) \wedge (3A \vee 2C)$
97	34	E	100	$(A \vee 5B) \wedge (A \vee 4C)$
98	65	E	100	$(2A \vee 4C) \wedge (2B \vee 4C)$
99	50	E	100	$(2A \vee B) \wedge 2C$
100	112	E	101-103-105-109-130-131-134	$(2A \vee 3C) \wedge (4B \vee 3C)$
101	48	E	102	$2A \wedge (3B \vee 4C)$
102	117	E	113	$(4A \vee 4C) \wedge (2B \vee 4C)$
103	50	E	104	$(5A \vee 3B) \wedge (5A \vee 4C)$
104	68	R	113	$5A \wedge (5B \vee 3C)$
105	232	L	106-107	$(3A \vee 2C) \wedge (3B \vee 2C)$
106	122	L	108	$(2A \vee 5C) \wedge (2B \vee 5C)$
107	151	E	108	$4A \wedge (2B \vee 3C)$
108	31	L	113	$(2A \vee 5B) \wedge (2A \vee 2C)$
109	97	E	113	$(A \vee 5B) \wedge 2C$
110	308	R	113	$5A \wedge (4B \vee 4C)$
111	116	L	113	$(5A \vee 5B) \wedge (5A \vee 3C)$
112	312	R	113	$4A \wedge (5B \vee 2C)$
113	34	E	114-115-116-117-118-119-120-121-122-123-124-161-162-163-169	$(4A \vee 4B) \wedge C$
114	128	L	160	$(3A \vee 4C) \wedge (3B \vee 4C)$
115	54	E	160	$3A \wedge (B \vee 3C)$

Table A.7 (Continued)

116	175	R	160	$3A \wedge (B \vee 4C)$
117	55	E	160	$(2A \vee C) \wedge (2B \vee C)$
118	306	E	126	$(A \vee 4C) \wedge (4B \vee 4C)$
119	59	E	126	$(2A \vee 4B) \wedge 2C$
120	59	E	126	$(A \vee 4B) \wedge 2C$
121	66	E	126	$(A \vee 4C) \wedge (5B \vee 4C)$
122	66	E	126	$(5A \vee 2C) \wedge (2B \vee 2C)$
123	23	E	126	$3A \wedge (4B \vee 4C)$
124	244	E	125	$A \wedge (B \vee C)$
125	54	E	126	$2A \wedge (2B \vee 5C)$
126	294	R	127-128-129	$(3A \vee B) \wedge 2C$
127	84	E	135	$3A \wedge (3B \vee 2C)$
128	61	E	135	$(3A \vee 3B) \wedge (3A \vee 2C)$
129	57	E	135	$(4A \vee 3B) \wedge (4A \vee 4C)$
130	38	R	136	$5A \wedge (2B \vee 4C)$
131	944	E	132	$(5A \vee 4B) \wedge C$
132	511	R	133	$(2A \vee 2B) \wedge 5C$
133	625	R	189	$(5A \vee B) \wedge 3C$
134	445	R	189	$(4A \vee 2B) \wedge (4A \vee 2C)$
135	68	L	136-137-138-139-140-141-142-144-145-148-149-150-151-152	$(3A \vee 5B) \wedge 5C$
136	53	L	189	$(5A \vee 4B) \wedge C$
137	49	E	160	$(3A \vee 3B) \wedge (3A \vee 3C)$
138	92	E	160	$3A \wedge (5B \vee C)$
139	236	E	160	$(4A \vee 4B) \wedge (4A \vee 5C)$
140	116	L	143	$2A \wedge (4B \vee 5C)$
141	265	L	143	$(A \vee 3B) \wedge (A \vee C)$
142	149	L	143	$(4A \vee 5C) \wedge (3B \vee 5C)$
143	74	L	160	$3A \wedge (4B \vee 2C)$
144	332	E	160	$(2A \vee 5C) \wedge (2B \vee 5C)$
145	324	E	146	$(A \vee 4B) \wedge 4C$
146	104	L	160	$(A \vee B) \wedge 5C$
147	51	L	160	$A \wedge (4B \vee 5C)$
148	58	R	160	$(A \vee 4C) \wedge (5B \vee 4C)$
149	67	R	160	$(4A \vee 5C) \wedge (5B \vee 5C)$
150	49	R	160	$3A \wedge (4B \vee 3C)$
151	107	E	160	$(A \vee 3C) \wedge (5B \vee 3C)$
152	38	L	160	$(A \vee 2C) \wedge (3B \vee 2C)$
153	27	L	154	$(2A \vee 5B) \wedge 5C$

Table A.7 (Continued)

154	68	E	155	$A \wedge (2B \vee 5C)$
155	207	E	156	$(5A \vee B) \wedge (5A \vee 2C)$
156	202	E	157	$(3A \vee 5B) \wedge C$
157	83	E	189	$(5A \vee 2B) \wedge 3C$
158	35	R	159	$(3A \vee 4C) \wedge (3B \vee 4C)$
159	58	R	189	$(3A \vee 2B) \wedge (3A \vee 2C)$
160	42	E	164-170-178-179-184	$(3A \vee 4B) \wedge (3A \vee C)$
161	68	R	167	$(3A \vee B) \wedge (3A \vee 2C)$
162	68	R	165	$(A \vee 5B) \wedge (A \vee 4C)$
163	68	R	164	$(2A \vee 4C) \wedge (2B \vee 4C)$
164	103	R	165	$(2A \vee B) \wedge 2C$
165	103	R	166	$(2A \vee 3C) \wedge (4B \vee 3C)$
166	103	R	167	$2A \wedge (3B \vee 4C)$
167	103	R	168	$(4A \vee 4C) \wedge (2B \vee 4C)$
168	103	R	177	$(5A \vee 3B) \wedge (5A \vee 4C)$
169	68	L	170	$5A \wedge (5B \vee 3C)$
170	103	L	172	$(3A \vee 2C) \wedge (3B \vee 2C)$
171	68	L	172	$(2A \vee 5C) \wedge (2B \vee 5C)$
172	103	L	173	$4A \wedge (2B \vee 3C)$
173	103	L	175	$(2A \vee 5B) \wedge (2A \vee 2C)$
174	68	L	175	$(A \vee 5B) \wedge 2C$
175	103	L	176	$5A \wedge (4B \vee 4C)$
176	103	L	177	$(5A \vee 5B) \wedge (5A \vee 3C)$
177	10	E	185-186-187-188-194-195	$4A \wedge (5B \vee 2C)$
178	187	E	180	$(4A \vee 4B) \wedge C$
179	134	L	180	$(3A \vee 4C) \wedge (3B \vee 4C)$
180	89	L	181-183	$3A \wedge (B \vee 3C)$
181	58	L	182	$3A \wedge (B \vee 4C)$
182	49	L	-	$(2A \vee C) \wedge (2B \vee C)$
183	134	L	-	$(A \vee 4C) \wedge (4B \vee 4C)$
184	53	L	-	$(2A \vee 4B) \wedge 2C$
185	334	E	189	$(A \vee 4B) \wedge 2C$
186	24	R	189	$(A \vee 4C) \wedge (5B \vee 4C)$
187	76	R	189	$(5A \vee 2C) \wedge (2B \vee 2C)$
188	76	L	189	$3A \wedge (4B \vee 4C)$
189	192	E	190-191-193	$A \wedge (B \vee C)$
190	98	E	-	$2A \wedge (2B \vee 5C)$
191	258	R	192	$(3A \vee B) \wedge 2C$
192	165	E	-	$3A \wedge (3B \vee 2C)$
193	38	R	-	$(3A \vee 3B) \wedge (3A \vee 2C)$

Table A.7 (Continued)

194	115	E	197	$(4A \vee 3B) \wedge (4A \vee 4C)$
195	83	L	196	$5A \wedge (2B \vee 4C)$
196	56	R	197	$(5A \vee 4B) \wedge C$
197	29	R	198-199	$(2A \vee 2B) \wedge 5C$
198	303	R	-	$(5A \vee B) \wedge 3C$
199	18	R	-	$(4A \vee 2B) \wedge (4A \vee 2C)$
200	29	R	-	$(3A \vee 5B) \wedge 5C$
201	154	L	-	$(5A \vee 4B) \wedge C$
202	90	L	-	$(3A \vee 3B) \wedge (3A \vee 3C)$
203	93	L	-	$3A \wedge (5B \vee C)$
204	94	E	-	$(4A \vee 4B) \wedge (4A \vee 5C)$
205	165	E	-	$2A \wedge (4B \vee 5C)$

APPENDIX B. RESULTS OF HEURISTIC APPROACH

Table B1.1. Results of T65/C326-test problem

Position	Right Side		Left Side	
	Assigned Tasks	**Assigned Resources**	**Assigned Tasks**	**Assigned Resources**
1	44,1,2,3,4,27,6	A+4B+3C	30,29,13,12	4A+3B+2C
2	5,11,49,41,23	3A+5C	45,46,47,48,9	5A+5B+C
3	24,42,43	4A+2B	25,10	2A+4B+2C
4	28	4C	-	-
5	26,7	3A+C	-	-
6	8,14,22,20	3A+4B+C	-	-
7	15,18,19,21	A+5C	-	-
8	-	-	16,17,31,53,58	3A+B+5C
9	51,55,59,62,60,54	3A+2B+2C	56,57,52	B+4C
10	36,37,38	5A+3B	63,64,32	4A
11	40,61,33,34	B+4C	39	5A+3C
12	35,50,65	5A+4C	-	-
Total Cost: 1482 unit				

Table B1.2. Results of T65/C381-test problem

Position	Right Side		Left Side	
	Assigned Tasks	**Assigned Resources**	**Assigned Tasks**	**Assigned Resources**
1	1,2,3,4,5,11,41	5B+5C	13,30,23,24	3A+5C
2	26,7,8,44	3A+2C	25,9,10,49,6	4A+4B+2C
3	42,43,12,14,22	4A+3B	45,46,29,47,48	5A+5B+C
4	27,28	2B+4C	15,16,17,20	3A+5C
5	18,19,21,31,59,55,60	2A+B+5C	-	-
6	61,32,33,34,52,62	3A+2B+2C	36,37,38,53	5A+4B
7	51,54,56,40	3A+B+4C	63,64,58,39	5A+5B
8	57,35,50,65	5A+4C	-	-
Total Cost: 1314 unit				

Table B1.3. Results of T65/C435-test problem

Position	Right Side		Left Side	
	Assigned Tasks	**Assigned Resources**	**Assigned Tasks**	**Assigned Resources**
1	2,1,3,4,6,5,11,44,49	5B+5C	13,30,23,24,9	4A+5C
2	7,8,26,27	3A+2B+3C	10,25,41,12,14	4A+4B+2C
3	42,43,22,20,46	5A+4B	15,45,16,17,29	5A+5C
4	18,19,21,31,61,55	A+B+5C	47,48	4A+2C
5	51,59,32,54,56,62	3A+B+4C	36,37,38,33,60,39	5A+5B
6	57,52,34,40,35	B+3C	63,64,58,53	4A+4B
7	28,50,65	5A+4C	-	-
Total Cost:1232 unit				

Table B1.4. Results of T65/C490-test problem

Position	Right Side		Left Side	
	Assigned Tasks	**Assigned Resources**	**Assigned Tasks**	**Assigned Resources**
1	1,2,3,4,44,5,6,27,41,12	5B+5C	13,30,23,9,25,49	4A+B+5C
2	7,8,26,11,24	3A+2B+C	42,43,45,46,47,48	5A+5B+C
3	28,14,22,20	3A+5C	29,10	3A+4B
4	15,18,19,21	A+5C	-	-
5	-	-	16,17,31,32,33,34,58	3A+B+5C
6	35,51,55,59,61,36,52,60	3A+2B+4C	54,56,57,53,62	3A+4C
7	37,38,40	3A+4C	63,64,39,50,65	5A+4C
Total Cost: 1150 unit				

Table B1.5. Results of T65/C544-test problem

Position	Right Side		Left Side	
	Assigned Tasks	**Assigned Resources**	**Assigned Tasks**	**Assigned Resources**
1	44,1,2,3,4,27,5,7,6	A+5B+5C	29,30,13,11,12,45,41	5A+4B+2C
2	23,24,46,42,49,43	5A+5C	25,9,10,47,48	4A+4B+2C
3	26,28,8,14,22	3A+3B+C	-	-
4	15,20,18,19,21	3A+5C	-	-
5	-	-	16,17,31,54,56,62,60	3A+B+5C
6	57,52,36,37,32,33,59,55	4A+3B+2C	63,64,58,53,38,39	5A+5B
7	40,51,61,34,35,50,65	5A+2B+3C	-	-
Total Cost: 1056 unit				

Table B2.1. Results of T148/C204-test problem

Position	Right Side		Left Side	
	Assigned Tasks	**Assigned Resources**	**Assigned Tasks**	**Assigned Resources**
1	56,59,2,3,4,74,61,60,11,12	4A+3B+4C	1,94,132,92,85,52,57,42,32	4A+4B+3C
2	30,33,7,8,6,9,70,5	4A+4B+5C	93,58,73,86,87,62,88	3A+5C
3	138,139,140,96	4A+4B+C	75,63,50,13,97	3A+4B
4	104,35,64,10,14,65,66,67	5A+5C	91,105	3A+2B
5	119,99,100,16	2A+5C	43,44,34,51,68	3A+B+3C
6	95,98,101,102,103,127	5A+3B	141,15	A+4C
7	142,17,69,79	A+2B+5C	144,18	2A+5B
8	80,81,146,19,20,24,21	3A+5C	143,53,23,22	3A+5C
9	71,26,25,29,31	5B+2C	28,27	2B+4C
10	145,36,37	2A+4C	-	
11	38,39,40,41,48,76	5A+2B+3C	45,46,47,49,54	5A+5B+2C
12	148,77,78,72,134,55	4A+5C	89,147,90,111	A+5B+3C
13	82,83,84,106,107,114	4A+5B	112,113,133,128	4A+B+3C
14	120,121,122,123,124,116,117,115	3A+4C	108,109,110,129,130,137,131	5A+4C
15	125,118,126,135,136	2A+5B+5C	-	-
Total Cost: 2702 unit				

Table B2.2. Results of T148/C255-test problem

Position	Right Side		Left Side	
	Assigned Tasks	**Assigned Resources**	**Assigned Tasks**	**Assigned Resources**
1	64,70,71,60,59,61,11,12,13,65	5A+5C	74,85,1,2,50,3,94,92,62,42	4A+4B+3C
2	5,4,30,33,35,6,56,63,9	5A+5C	57,52,58,93,141	3A+4B+2C
3	91,99,100,73	3A+3C	138,139,140,132,8,32	4A+4B+2C
4	96,104,88,7,10,14,16,66	5A+5C	43,34,44,51,86,15	3A+B+4C
5	69,67,68,75,87,95,79,97,80,81	4A+4B+5C	17,144,105	3A+4B
6	98,145,101,102,103	2A+3B+4C	53,119,18	A+5B+2C
7	142,127,19,20,24,21	3A+5C	-	
8	148,146	B+5C	147,143,23,22,28,27	3A+4B+5C
9	26,25,29,31,36,37,38,39,40	5A+5B+C	-	-
10	41,55,72,134,76,77,78,89,90,111	5A+3B+2C	54,45,46,47,49,133,135	5A+5C
11	82,83,84,106,107,108,48,114	4A+5B+C	112,113,116,117,128,129	4A+B+2C
12	115,123,124,125,120,121,122,118,126	3A+5B+2C	109,110,136,130,137,131	5A+4C
Total Cost: 2298 unit				

Table B2.3. Results of T148/C306-test problem

Position	Right Side		Left Side	
	Assigned Tasks	**Assigned Resources**	**Assigned Tasks**	**Assigned Resources**
1	1,61,62,132,94,92,138,74,60,59,33,35,64	4A+4B+3C	91,105,119,57,58,32	3A+4C
2	11,65,75,85,66,56,50,63,99,67	5A+5B+2C	139,97,2,12,13,34,42,52,3	4A+3B+3C
3	30,6,7,9,68,5,4,8,10,14,70	5A+5C	73,88,86,87,141,95,93	3A+5C
4	96,98,142,140,104,103	5A+5B	100,144,101,15	2A+5C
5	16,71,146,17,19,102,20	3A+4B+5C	43,44,51,53,18	3A+5B+2C
6	21,24,145,148,69,79,80,81	2A+2B+5C	23,143,22,27,28,127,147	3A+2B+5C
7	25,26,29,31,36,37,38,39,40,41	5A+3B+3C	-	-
8	-	-	45,46,47,54,89,76,77,78,90,111	5A+5B+2C
9	72,82,84,106,107,108,109,83,48	4A+5B+2C	112,113,120,121,122,128,129,114	4A+4C
10	55,134,110,115,123,124,125,116,117,118,126	5A+4B+2C	130,137,133,135,136,131,49	5A+2B+5C
Total Cost: 2016 unit				

Table B2.4. Results of T148/C357-test problem

Position	Right Side		Left Side	
	Assigned Tasks	**Assigned Resources**	**Assigned Tasks**	**Assigned Resources**
1	91,64,70,71,138,132,85,60,61	5A+5C	93,50,11,12,1,144,13,2,65,59	5A+5C
2	105,94,92,74,30,33,35,3,7,6,56,62	5A+2B+4C	51,52,53,66,75,5,57,9	3A+4B+5C
3	73,96,97,63,119,104,99,100,67,68,69	5A+3C	42,43,44,32,34,58,4,8,10,14	4A+4B+2C
4	16,86,87,88,79,80,81,95,98,145	3A+B+5C	15,141,17,18,101,102,103	2A+2B+5C
5	19,142,20,148,146,24,21	3A+5C	139,140,127,147,143	4A+4B
6	-	-	23,22,28,27	2A+2B+5C
7	26,25,29,31,36,37,38,39,40,41	5A+5B+C	-	-
8	-	-	54,90,111,89,76,77,78,82,45,46,83	5A+5B+2C
9	55,72,134,84,106,107,108,109,110,135,123,124,114	5A+5C	112,133,113,120,128,129,130,131,137,47	5A+2B+3C
10	115,116,125,121,122,117,118,126,136,48	5A+5B+2C	49	4C
Total Cost: 1858 unit				

Table B2.5. Results of T148/C408-test problem

Position	Right Side		Left Side	
	Assigned Tasks	**Assigned Resources**	**Assigned Tasks**	**Assigned Resources**
1	60,61,132,94,92,138,74,62,63,33,35,64,91	4A+2B+3C	85,59,56,58,57,93,11,12,50,13,2,1,3,5,4,65,66	5A+5B+5C
2	105,119,75,97,73,88,30,70,6,7,67,68	3A+4B+5C	52,141,144,42,51,32	4A+2B
3	145,142,86,95,96,87,14	3A+5C	8,9,10,99,100,139,98,34	4A+4B+2C
4	69,71,146,148,16,79,80,81,140	5A+5C	15,43,44,143,147,101,103,102,17,18	5A+5C
5	104,19,20,21,24,127,25,26,29,31,36,37,38,39,40	5A+5C	53,22,23,27,28	A+5B+5C
6	41,76,77,78,82,84,89,106,107,108,109,83	4A+5B+2C	54,90,111,112,113,120,121,122,116,123,124,114,128	5A+B+4C
7	55,72,134,110,115,117,118,125,126,135,136,48	5A+5C	129,130,137,133,131,45,46,47,49	5A+2B+4C
Total Cost: 1620 unit				

Table B2.6. Results of T148/C459-test problem

Position	Right Side		Left Side	
	Assigned Tasks	**Assigned Resources**	**Assigned Tasks**	**Assigned Resources**
1	11,12,85,74,91,59,60,56,132,105,119,94,61,62	4A+5B+2C	2,42,43,44,52,144,3,4,58,57,32	4A+2B+4C
2	70,63,73,86,87,13,50,1,7,96,104,92,33,35,64,6,65	5A+5C	34,93,141,8,10,75,88,5,97	3A+B+5C
3	71,142,145,9,14	4A+4B+4C	99,100,66,67,68,138,139,140,98,95,101,103,102	5A+4B+3C
4	148,127,146,30,16,17,19,69,79,80,81,20,24,21	4A+5C	15,51,147,143,53,18	3A+5C
5	-	-	23,22,28,27	2A+2B+5C
6	26,25,29,31,36,37,38,39,40,41	5A+5B+C	-	-
7	-	-	45,46,47,49,54,89,76,77,90,111,78,82,84	5A+5B+2C
8	83,72,134,55,106,107,108,109,110,48,135,136,123,124,120,121	5A+4B+5C	112,133,113,116,117,118,114,115,128,129,130,137,125,131	5A+2B+4C
9	122,126	B+2C	-	-
Total Cost: 1584 unit				

Table B2.7. Results of T148/C510-test problem

Position	Right Side		Left Side	
	Assigned Tasks	Assigned Resources	Assigned Tasks	Assigned Resources
1	59,94,92,91,70,1,64,2,30, 33,35,60,56,65,50,61,62, 63,66,3	5A+4B+5C	144,138,139,140,85,74,75	4A+4B+C
2	99,145,4,5,132,105,119, 100,8,11	3A+2B+5C	57,58,52,67,68,73,95,7,6,9, 86,87,98,97,96,104,88,51, 93,42	5A+B+5C
3	12,13,10,69,71,14,16,79, 80,81,101,102,103,127, 17,19	5A+5C	43,44,53,32,34,15,141,18, 20	3A+4C
4	21,24,142,146,148,26,25, 29,31,36,37	3A+2B+5C	22,23,28,27	2A+2B+5C
5	38,39,40,41,76,77,78,82, 90,111,89,84,106,107,83, 48	5A+5B+3C	143,45,54,46,47,49,147	5A+5C
6	55,72,134,108,109,110,1 14,115,116,117,118,120, 121,122,123,124,125,126	5A+5C	112,113,128,129,130,133,1 37,131,135,136	5A+2B+5C
Total Cost: 1432 unit				

Table B3.1. Results of T205/C1133-test problem

Position	Right Side		Left Side	
	Assigned Tasks	**Assigned Resources**	**Assigned Tasks**	**Assigned Resources**
1	19,6,20,1,2,200,158,204	3A+4B+5C	21,203,202,22,153,171,147,174,201,23,205,154,24,34,26	5A+5C
2	155,25,27,28,29,30,32,31,3,33,159,35,157	5B+4C	4,156	4A+C
3	5,7,8,9,10,11	4A+5C	-	-
4	-	-	13,14,15,16,17,18,12,36,83,72,69,70	3A+2B+5C
5	62,63,64,68,65,110,112,40,42,54	5A+4B+3C	75,71,73,74,76,77	4A+4B+C
6	79,41,55,59,60,61,66,67,82,56,57	2A+5B+2C	78,43,44,37,38,39,45,51,52,111,53,48,49,50,46,47	5A+4C
7	80,81,84,85,86,87,88,89,90,91,58,92,98,97	3A+2B+5C	-	-
8	93,95,94,96	3A+4C	99,100,105,106,101,102,107,103,108,109	5A+B+5C
9	130,134,104,113,163,162,115,117,161,121,122,120,123	5A+5B+2C	131,169,119	5A+5B+2C
10	132,118,116,125	3A+2B+4C	114,124	A+4C
11	126,127,128,129,133	4A+B+3C	-	-
12	-	-	135,144,139,145,138,136	5A+5C
13	148,149,150,137,151	3A+5C	140,152,146,142,141,143,160,178,184,170	3A+4B+5C
14	164,165,166,167,168,177,187,185,186,194,196	4A+4B+4C	172,173,175,176,179,188,180,181,182,183,195	5A+4B
15	197,198,199,189,191,193,190,192	3A+3B+3C	-	-
Total Cost: 2582 unit				

Table B3.2. Results of T205/C1322-test problem

Position	Right Side		Left Side	
	Assigned Tasks	**Assigned Resources**	**Assigned Tasks**	**Assigned Resources**
1	19,20,2,1,200,204,21,205,22,158	5A+5C	153,171,174,201,4,147,202,203,6,154,155	5A+5C
2	25,159,23,24,27,28,29,3,30,32,31,33,5,7,156,8	3A+2B+5C	-	-
3	157,9,10,11,35	4A+4B+3C	26,34,13,14,15,16,17,18,12,36,83,75,69,62	3A+2B+5C
4	70,71,63,110,112,40,41,42,54,64,68	5A+5C	72,73,74,76,77,78,37,38,39,111,82,43	5A+5B+C
5	55,79,61,65,66,67,80,81,84,59,60,85,56,57	5A+2B+5C	44,45,53,51,52,48,49,50,46,47	5A+4C
6	86,87,88,89,90,91,58,92,97,98,93,95,99,100,130,101	5A+2B+5C	-	-
7	134,94,102,103,104,109,113,117	5A+5B+2C	96,105,106,107,108	4A+3B
8	163,115,162,118,122,121,120,123,119,161,124,116,125	5A+5B+2C	169,114,131	5A+4C
9	126,127,128,132,129	4A+B+2C	-	-
10	133,145,144	A+B+5C	135,139,138,140,141,136,142,143,146,152,137	5A+5C
11	148,149,150,151,160,164,165,166,167,168,178	5A+5C	-	-
12	-	-	179,180,181,182,183,170,172,173,175,176,184,177,195,188,194	5A+4C
13	186,187,196,197,198,199,185,189,191	A+4B+4C	-	-
14	193,190,192	3A+3B	-	-
Total Cost: 2314 unit				

Table B3.3. Results of T205/C1510-test problem

Position	Right Side		Left Side	
	Assigned Tasks	**Assigned Resources**	**Assigned Tasks**	**Assigned Resources**
1	205,204,21,19,6,2,1,3	4A+5B+3C	153,171,174,201,203,20,22,23,202,147,34,24,26,4,154,155	5A+5C
2	5,200,158,159,27,156,157,25,28,29,30,32,31,33,35,7,8,9,10,11	4A+2B+5C	-	-
3	-	-	13,14,15,16,17,18,12,36,83,40,41,111,42,43,37,38,39,62	5A+2B+5C
4	54,55,112,110,63,64,65,66,61,59,60,67	5A+5C	69,70,72,75,71,73,74,76,68,77	5A+5C
5	56,57,58,79,82,80,81,84,85,88,89,86,87,90,91,92,95,97,98,99	3A+2B+5C	44,45,46,47,78,48,49,50,51,52,53	5A+4C
6	93,94,96,100,130,101,103,102,104	5A+2B+4C	-	-
7	109,134,131,123	3A+4B+2C	105,106,107,108,113,121,120,122,119,117,118,115,169,114	5A+5B+2C
8	124,125,163,162,161,132,126,129,128,127	4A+2B+2C	-	-
9	133,116,145,144,137	3A+2B+4C	135,139,140,141,136,142,143,152,138,151,146	5A+5C
10	149,150,148,160,178,164,165,166,167,168	5A+5C	-	-
11	-	-	179,180,181,182,183,170,172,173,175,176,184,177,195,194,188	5A+4C
12	196,197,199,198,185,187,186,189,190,193,191	2A+4B+4C	-	-
13	192	3A+3B	-	-
Total Cost: 2032 unit				

Table B3.4. Results of T205/C1699-test problem

Position	Right Side		Left Side	
	Assigned Tasks	**Assigned Resources**	**Assigned Tasks**	**Assigned Resources**
1	19,20,205,204,21,22,6,2,1,23,3,158,200	5A+5C	153,171,202,201,203,174,154,147,155,4,156,157,34,24,26	5A+5C
2	159,5,27,25,28,29,33,30,32,31,35,7,8,9,10,11	4A+2B+5C	-	-
3	-	-	13,14,15,16,17,18,12,36,83,75,72,69,111,40,37,38,39	5A+2B+5C
4	112,110,41,42,54,55,56,57,58,61,62,63,64,65,66,68,67	5A+5C	70,71,73,74,76,78,77,43,44,45,53,51,52,82,48,49,50,46	5A+5C
5	59,60,79,80,81,84,85,90,91,86,87,88,89,92,99,98,97,100,103	3A+3B+5C	47	4A+2C
6	109,95,104,130,134,94,93,101,102,113,161	5A+4C	105,106,96,107,108	4A+2B+2C
7	118,117,119,120,115,124,123,122,121,125,163,162,126,128,127,129	4A+5B+2C	169,131,114	5A+4C
8	132,133,116,151,139	4A+2B+3C	135,145,144,137,152,146,138,142,141,140,143,136	5A+5C
9	148,149,150,160,164,165,166,167,168,178	5A+5C	-	-
10	-	-	179,180,183,181,182,184,170,172,173,175,176,177,185,195,188,194	5A+4C
11	187,186,196,197,199,198,189,193,191,190,192	3A+4B+4C	-	-
Total Cost: 1930 unit				

Table B3.5. Results of T205/C1888-test problem

Position	Right Side		Left Side	
	Assigned Tasks	**Assigned Resources**	**Assigned Tasks**	**Assigned Resources**
1	158,159,25,19,6,20,2,200,1,21,22,204,23,24,28,3,27	3A+4B+5C	147,153,154,155,156,157,203,205,171,4,174,201,34,202,26	3A+5B+5C
2	29,30,31,32,33,5,7,8,9,10,11,35	4A+2B+5C	-	-
3	-	-	12,13,14,15,16,17,18,36,42,72,75,69,70,71,62,63,64,65,66,68,67,40,43,37,38,39	5A+5C
4	73,74,76,77,82,54,55,59,60,61,112,79,56,57,58	4A+5B+2C	41,111,44,45,48,49,50,78,53,83,51,52,46,47	5A+5B+2C
5	80,81,84,85,86,87,110,88,89,90,91,92,96,98,99,97,95	5A+5C	-	-
6	100,131,109,103,101,104,130,132	5A+5B+4C	94,105,106,107,108	4A+3B
7	93,133,134,102,113,161,162,119,117,120,121	5B+4C	-	-
8	118,115,163,122,116,123,124,125,126,127,129,128	5A+4B+2C	169,114	5A+4C
9	-	-	135,139,138,137,136,140,141,142,143,152,151,145,146	5A+5C
10	144,150,149,148,160,178,164,165,166,167,168	5A+5B+2C	-	-
11	-	-	170,172,173,175,176,179,180,181,182,183,184,177,188,195,185,194	5A+4C
12	186,187,196,197,198,199,189,191,190,193,192	3A+4B+4C	-	-
Total Cost: 1892 unit				

Table B3.6. Results of T205/C2077-test problem

Position	Right Side		Left Side	
	Assigned Tasks	**Assigned Resources**	**Assigned Tasks**	**Assigned Resources**
1	2,21,20,22,3,25,205,204, 23,24,158,159,28,27,29, 30,200,33,1,19	4A+4B+5C	203,202,201,174,171,153,6 ,154,155,4,26,34,147,5,13, 156,157,14,15,16,17,18	5A+2B+5C
2	7,8,9,10,11,31,32,35	4A+4B+C	-	-
3	-	-	12,36,62,63,64,68,42,41,65 ,66,67,75,72,69,70,71,73, 40,43,37,38	5A+B+5C
4	83,74,76,77,79,110,112, 54,55,59,56,57,58,60,82	5A+5C	44,111,39,45,51,48,49,50, 52,78,53,46,80,81,84,85,88 ,47,86,87,89	5A+5C
5	61,90,91,92,93,99,98,97, 100,131,109,130,95,101	5A+4B+3C	-	-
6	134,94,132,133	4A+B+3C	105,106,96,102,103,107, 108	5A+2B+2C
7	104,113,118,117,123,122 ,121,120,119,124,115,16 3,162,125,161,126,128, 116,127,129	5A+5C	-	-
8	-	-	114,135,136,145,144,139, 138,151,141,142,146,140, 143	5A+5C
9	148,150,137,149,160,178 ,164,165,166,167,168	5A+5C	169,152	5A+3C
10	-	-	179,180,181,182,183,170, 172,173,175,176,184,177, 194,185,195,188	5A+4C
11	196,197,199,198,187,186 ,189,193,190,191,192	3A+4B+4C	-	-
Total Cost: 1746 unit				

Table B3.7. Results of T205/C2266-test problem

Position	Right Side		Left Side	
	Assigned Tasks	**Assigned Resources**	**Assigned Tasks**	**Assigned Resources**
1	204,200,1,158,159,25,2,3,6,20,21,22,23,24,28,29,30,31,19,33,32,205,27	5A+5C	203,202,201,174,171,153,154,155,156,157,147,4,5,13,14,15,16,17,18,26,34	5A+2B+5C
2	7,8,9,10,11,35	4A+4B+C	-	-
3	-	-	12,36,37,38,39,42,62,63,64,65,72,69,70,71,73,74,68,75,66,67,41,40,76	5A+3B+5C
4	83,77,82,112,110,79,54,55,61,59,60,56,57,58,80,81,84	5A+5C	43,44,45,46,47,48,53,51,52,78,49,50,111	5A+4C
5	85,90,91,86,87,88,89,92,94,95,98,97,99,100,101,102,103,96,130,109	5A+5B+5C	-	-
6	134,104,131,132,93,123	5A+4C	105,107,106,108,113,122,121,169,115,117,118,114,119,120,124,125	5A+2B+4C
7	116,126,133,163,127,162,128,161,129	4A+2B+3C	-	-
8	-	-	135,152,137,141,142,140,143,138,139,144,145,146,151,136	5A+5C
9	149,150,148,160,178,164,165,166,167,168	5A+5C	-	-
10	-	-	184,179,180,183,181,182,170,172,173,175,176,177,194,185,195,188	5A+4C
11	196,197,199,198,187,186,189,190,193,191,192	3A+4B+4C	-	-
Total Cost: 1684 unit				

Table B3.8. Results of T205/C2454-test problem

Position	Right Side		Left Side	
	Assigned Tasks	**Assigned Resources**	**Assigned Tasks**	**Assigned Resources**
1	158,159,200,19,6,20,2,1, 25,21,22,23,204,24,28,29 ,30,31,32,27,205,33,3, 154,5	3A+5B+5C	202,203,153,147,171,174,4 ,201	5A+5C
2	7,8,9,10,11,155,156,157, 35,36,40,41,112,42,72,69 ,70,71,73,74,62	5A+4B+5C	26,13,14,15,16,17,18,12,34	3A+2B+5C
3	83,76,77,75,63,64,68,65, 66,54,55,59,60,56,82,67, 110,79,61	5A+B+5C	111,43,37,38,39,44,45,48, 49,50,53,78,51,52,46,47	5A+5B+2C
4	80,81,84,85,86,87,90,91, 88,89,57,58,92,99,98,97, 100,131,109,103,101	3A+4B+5C	-	-
5	94,134,96,132,133,95	4A+B+3C	105,106,102,107,108	4A+2B+2C
6	93,104,130,113,117,118, 119,115,120,162,163,161 ,121,122,116,123,124, 125,126,128,129,127	5A+5C	-	-
7	-	-	135,138,152,169,142,137, 141,145,144,151,139,146, 140,143,114,136	5A+5C
8	148,149,150,160,164,165 ,166,167,168,178	5A+5C	-	-
9	-	-	170,172,173,175,176,179, 180,181,182,183,184,177, 195,185,188,194	5A+4C
10	187,186,196,197,199,198 ,189,193,191,190,192	3A+4B+4C	-	-
Total Cost: 1684 unit				

Table B3.9. Results of T205/C2643-test problem

Position	Right Side		Left Side	
	Assigned Tasks	**Assigned Resources**	**Assigned Tasks**	**Assigned Resources**
1	25,158,159,6,19,2,1,204, 21,205,20,22,200,23,24, 27,28,29,33,30,32,31,3,5, 7,8,9,10	3A+4B+5C	202,201,203,174,4,153,171 ,154,155,156,157,147	5A+5C
2	11	A	26,13,14,15,16,17,18,12,34 ,35,36,72,69,70,62,63,64, 65,66,83,75,71,73,74,111,3 7,38,39	5A+3B+5C
3	40,112,110,41,42,54,55, 56,57,61,67,59,60,68,58, 76,79,77,82	5A+5C	-	-
4	-	-	43,44,45,48,49,50,51,52,78 ,80,81,84,85,86,87,88,89, 46,47,53	5A+5C
5	90,91,92,93,95,94,96,99, 98,97,100,131,103	3A+4B+4C	-	-
6	109,101,102,104,130,132 ,133,134,107	5A+3B+3C	105,106	2A+2B+2C
7	-	-	108,113,114,169,115,117, 118,121,122,120,123,119, 124,125	5A+5B+2C
8	161,163,162,126,129,128 ,127,116	4A+2B+2C	-	-
9	-	-	135,144,138,137,152,139, 142,145,151,146,141,140, 143,136	5A+5C
10	150,149,148,160,178,164 ,165,166,167,168	5A+5B+2C	-	-
11	-	-	184,179,180,183,181,182, 170,172,173,175,176,177,1 88,195,185,194	5A+4C
12	186,187,196,197,199,189 ,198,193,190,191,192	3A+4B+4C	-	-
Total Cost: 1652 unit				

Table B3.10. Results of T205/C2832-test problem

Position	Right Side		Left Side	
	Assigned Tasks	**Assigned Resources**	**Assigned Tasks**	**Assigned Resources**
1	205,204,200,1,2,25,3,158,19,20,6,21,22,23,24,159,28,29,33,30,27,31,32,154,155,156,157	4A+4B+5C	147,203,202,201,174,171,153	3A+5B+5C
2	-	-	4,26,34,5,13,14,35,15,16,17,18	4A+5C
3	7,8,9,10,11	4A+4B+C	-	-
4	-	-	12,36,42,62,75,72,69,70,71,63,64,68,65,66,67,37,38,39,111,40,41,73,74,76,78,77,43,44,45,46,47	5A+5C
5	112,110,82,79,80,81,83,54,55,59,60,61,56,84,85,88,89,86,87,57,58,90,91,92,99	5A+5C	48,49,50,51,52,53	3A+4B+C
6	98,97,100,131,109,130,132,133,95,96	5A+4B+3C	94,105,106,101,102,103,107,108	5A+3B
7	104,134,113,118,117,123,122,121,120,119,163,115,162,161,124,125,126,93,116,127,128,129	5A+5C	-	-
8	-	-	114,135,145,144,139,138,151,141,140,142,143,146,152,169,137,136	5A+5C
9	149,150,148,160,178,164,165,166,167,168	5A+5C	-	-
10	-	-	179,180,181,182,183,170,172,173,175,176,177,194,185,195,188,184	5A+4C
11	196,197,199,198,187,186,189,193,190,191,192	3A+4B+4C	-	-
Total Cost: 1610 unit				