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M.Sc. in Mechanical Engineering

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UNIVERSITY OF GAZİANTEP

GRADUATE SCHOOL OF

NATURAL & APPLIED SCIENCES

THERMOELASTIC ANALYSIS OF THICK PRESSURIZED CYLINDERS

M. Sc. THESIS

IN

MECHANICAL ENGINEERING

BY

BEGÜM KANLIKAMA

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Thermoelastic Analysis of Thick Pressurized Cylinders

M.Sc. Thesis

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University of Gaziantep

Supervisors

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by

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This is to certify that we have read this thesis and that in our majority opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science.

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Begüm Kanlıkama

ABSTRACT

THERMOELASTIC ANALYSIS OF THICK PRESSURIZED CYLINDERS

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M.Sc. in Mechanical Engineering
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In this work, the thermoelastic analyses of the thick-walled cylinder and the nuclear reactor pressure vessel problems are studied using Finite Element Method. The two-dimensional finite element code for coupled thermoelasticity with conductive, convective, and radiative boundary conditions is generated, which is used to analyze the thick-walled cylinder and nuclear reactor pressure vessel cases. The elements are 8-noded quadrilateral for both thermal and mechanical solids. The mesh finery rates of the models for the thick-walled cylinder and the nuclear reactor pressure vessel are determined according to the agreement of the results with the ones obtained by ANSYS. To verify the obtained results, geometric models of the cases are generated within ANSYS and corresponding finite element types are utilized. Results are compared with ANSYS results.

Key Words: Finite Element Method, thermoelasticity, thick-walled cylinder, reactor vessel.

ÖZET

KALIN CİDARLI BASINÇLI SİLİNDİRLERİN TERMOELASTİK ANALİZİ

KANLIKAMA, Begüm
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Yrd. Doç. Dr. Ayşegül ABUŞOĞLU
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Bu çalışmada, kalın cidarlı silindir ve nükleer reaktör basınç kabı problemlerinin termoelastik analizi sonlu elemanlar yöntemi ile çalışılmıştır. Birleştirilmiş termoelastisite için iki boyutlu sonlu elemanlar kodu, iletim, taşınım ve ışınım sınır koşulları ile kalın cidarlı silindir ve nükleer reaktör basınç kabı problemlerinin analizinde kullanılmak üzere oluşturulmuştur. Elemanlar termal ve mekanik katılar için kullanılabilen 8 düğümlü dörtgen elemanlardır. Kalın cidarlı silindir ve nükleer reaktör basınç kabı modellerinin ağ sıklığı oranları, ANSYS'den elde edilen sonuçlarla uyuşmasına göre belirlenmiştir. Elde edilen sonuçları doğrulamak için problemlerin geometrik modelleri ANSYS'de oluşturulmuş ve tutarlı sonlu eleman tipleri kullanılmıştır. Sonuçlar ANSYS sonuçlarıyla karşılaştırılmıştır.

Anahtar Kelimeler: Sonlu elemanlar yöntemi, termoelastisite, kalın cidarlı silindir, reaktör kabı.

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LIST OF SYMBOLS

σ_r	Radial stress (Pa)
σ_t	Hoop (circumferential) stress (Pa)
σ_a	Longitudinal (axial) stress (Pa)
P_o	External pressure (Pa)
r_o	External radius of the cylinder (m)
r_i	Inner radius of the cylinder (m)
r	Radial distance from the cylinder center (m)
$\tau_{\max}$	Maximum shear stress (Pa)
ε_r	Radial strain
ε_t	Tangential strain
u_r	Radial displacement (m)
E	Modulus of elasticity (Pa)
ν	Poisson's ratio
T	Absolute temperature (˚C)
T_o	External wall temperature (˚C)
T_0	Reference temperature (˚C)
α	Linear thermal expansion coefficient (1 / K)
α_0	Coefficient of volumetric thermal expansion (m^3 / m^3 - K)
ΔT	Temperature change from the uniform reference temperature (˚C)
B	Body force intensity (N / m^3)
G	Modulus of rigidity (Pa)

β	Thermal shear modulus (MPa)
λ	Lamé modulus
λ_s	Isentropic Lamé elastic constant
ρ	Density (kg / m^3)
q_i	Area rate of the heat flux vector (W / m^2)
C_υ	Specific heat at constant volume per unit mass (J / kg - K)
C_υ^0	Specific heat at constant strain and reference temperature (J / kg - K)
s	Entropy (J / kg - K)
u	Internal energy (J / kg)
f	Helmholtz free energy (J / kg)
κ_T	Isothermal bulk modulus (Pa)
g	Gibbs energy (J / kg)
h	Enthalpy (J / kg)
k	Material conductivity (W / m - ˚C)

CHAPTER 1

INTRODUCTION

1.1 GENERAL INTRODUCTION

Thermoelasticity is the subdivision of science which is coping with the coupled effects associated with the deformation of a substance resulting from a variation of the internal heat and, thus, a variation of the temperature of the substance. A variation in the temperature leads to a strain. Hence, the internal energy of the substance becomes dependent on both temperature and deformation.

Engineering problems associated with thermoelasticity problems are given by PDEs for displacement, stress, strain, and conduction with known or assumed boundary conditions. It is quite challenging to obtain a solution which satisfies such differential equations through the problem domain.

In spite of the fact that there have been studies concerning thermoelasticity for over a century, many corresponding problems are difficult if their solutions are attempted by means of classical methods. Hence, researchers prefer to use numerical methods and finite element techniques in order to solve problems of thermal deformation. Initial studies of the method compassed the transient temperature distribution of the solid, and later on, the mechanical loading process of the body was carried on due to obtained temperature and mechanical loading results. These separate mechanical and thermal effects are the procedures for uncoupled thermoelasticity.

FEM is known as a protean method in handling most engineering problems. The finite element method is based on partitioning a region into a lot of smaller regions. These smaller regions are called as finite elements. Each of these smaller regions is governed by a simpler function that can be rather easily handled in comparison to the undivided region. The element boundary conditions of each finite element not only should be consistent with the boundary conditions of each of its neighbors, but also should satisfy overall region boundary conditions. Most consider the finite elements method as the most successful numerical method. But, it's most important drawback is excessive number of fruitless equations and elements that can be formed after the analyses, in particular for complex three-dimensional structures.

Cylindrical-shaped vessels can be considered as the most common type of vessels. Pressurized vessels, heat exchangers, reactors, and other nuclear equipments are some examples for the cylindrical vessels. Present study investigated the radial displacements, radial and circumferential stresses, temperature distributions, and thermodynamic potentials distributions throughout the cylindrical vessel cross-section using the Finite Elements Method. The present study considers the effect of the pressure and temperature for plane strain condition with conductive, convective, radiative, and mechanical boundary conditions, simultaneously. The present research proposes two sets of results obtained by numerical methods for two-dimensional analysis of a thick-walled cylinder and a nuclear reactor vessel. The latter solution considers the effect of outlet nozzles of cylinder.

1.2 RESEARCH OBJECTIVES AND TASKS

Main research objective of this study is to analyze the thick-walled cylinders in terms of thermoelasticity. Tasks can be listed as following;

I. To revise the thick-walled cylinders and thermoelasticity analyses in the literature.

II. To develop the theory and a computer code which can carry on finite element analyses in two-dimensional and axisymmetric options for transient coupled thermoelasticity with conductive, convective, and radiative boundary conditions with 10 different finite elements. The code should read the temperature distribution and use the data for stress analyses. Improved code must handle other loading conditions like pressure as well.

III. To adapt a wide range of thermal and structural data files to our case studies.

IV. To disentangle the thermoelasticity problems by utilizing written computer code and adapted data files. Gained results should be collated with analytical results.

V. To collate the results with finite element software ANSYS in order to certify improved computer code.

1.3 LAYOUT OF THESIS

- In chapter two, literature surveys for the thermoelasticity analyses and thermoelasticity with thermodynamics are presented.
- In chapter three, the main concepts of pressurized cylinders are presented.
- In chapter four, the main concepts for the coupled thermoelasticity are presented.
- In chapter five, thermodynamic potentials for linear thermoelasticity are considered.
- In chapter six, results and discussion for different case studies are presented.
- In chapter seven, conclusions are summarized.

CHAPTER 2

LITERATURE SURVEY

2.1. INTRODUCTION

In this chapter, the literature review concerning thick-walled pressurized cylinders and thermoelasticity analyses are presented. In section 2.2., some papers about pressurized cylinders are briefly reviewed. In section 2.3., some papers concerning thermodynamic potentials analysis are reviewed.

2.2. PRESSURIZED CYLINDERS

Since the thermal stresses in pressure vessels are very important in most applications, corresponding research in this field has a great significance for both theoretical and practical terms. When the thick-walled cylinders are exposed to internal and external temperatures and pressures, it becomes so important foreseeing the amount of deformation generated by applied load.

Shahani and Nabavi (2007) proposed an analytical solution for the thick thermoelastic cylinder using finite Hankel transform. Inside wall of the cylinder was subjected to transient thermal boundary conditions.

Simply supported beams whose thicknesses are changing along the lengths of the beams were subjected to thermal and mechanical loads simultaneously and their thermoelastic analyses in two-dimensional frame were carried out by Xu and Zhou (2012). The boundary conditions at their beam ends were the harmonic series with unknown coefficients and used to obtain the stress and displacement component series.

An experiment of a thermal stress analysis with one transient load was proposed by Fruehmann et al (2010). Specimens having different amount of damage were tested and their modified thermoelastic stress analysis method might be considered significant as a nondestructive evaluation tool.

A thick-walled cylinder made of a functionally graded material was studied in terms of its thermopiezoelectric behavior by Khoshgoftar et al. (2009). They subjected the thick-walled cylinder to internal and external pressures, and also a temperature gradient. Heat transfer equation was solved and a symmetric temperature distribution was obtained. Stress relations were derived and substituted into mechanical equilibrium equations to be solved as nonhomogeneous differential equations.

Analytical solution for thermoelastic finite annular cylinders with functionally graded materials which were subjected to internal pressure, axial loading and thermal loads were presented by Hosseini Kordkheili and Naghdabadi (2008). Differential equilibrium equations were simplified to linear algebraic equations by means of Fourier expansion series of the displacement field components. Those simplified equations were solved to obtain the thermoelastic behavior of the cylinder.

Baker and Webber (1982) described a method called *thermoelastic stress analysis* in order to obtain the stress distribution on a body subjected to loads and the relation between the stress distribution and strength of the body.

Oden and Kross (1968) dealt with the formulation of general thermoelasticity problem. Their study was relevant to the finite element formulations of coupled dynamic thermoelastic problems.

Carter and Booker (1989) presented a method in order to solve the developed governing equations of fully coupled thermoelasticity using finite element approximation. Analytical results were then compared with the results obtained by the numerical method for some trial cases. It is also shown that the fully coupled analyses have a significant effect on results according to the ones of semi-coupled analyses.

The uncoupled thermoelasticity behavior of a thick-walled cylinder of finite length with internal sources of radiation whose temperature was changing linearly was studied by Lamba and Khobragade (2012). The stresses and the temperature distribution for uncoupled thermoelasticity problem with radiative boundary conditions were obtained. Bessel functions of infinite series can be used in order to obtain the solutions.

An annular cylinder subjected to mechanical and thermal loads was studied by Furukawa and Takeuti (1986). According to their study, the mechanical loads have a significant effect on stress and temperature distributions that those effects cannot be neglected during the analysis. Material and physical properties were considered as nondependent on temperature.

Reddy and Chin (1998) carried on the dynamic thermomechanically coupled thermoelastic analysis of FG plates and cylinders by means of finite elements method.

Linear thermoelasticity analysis for FG cylindrical shells which are exposed to the transient thermal shock using finite elements method was studied by Santos et al. (2008). The three-dimensional equations of motion were simplified into 2D equations.

Jane and Lee (1999) presented a method based on the Laplace transform and finite difference method in order to investigate the transient thermoelastic behavior of multilayered hollow tubes with infinite lengths exposed to some known temperatures at internal and external walls. They found the transient results for the thermal stress distributions, displacements, and temperatures. The thermal stress, the displacement, and the temperature distributions were seen to change slightly as time increases.

Banas et al. (1987) proposed a finite elements formulation for a coupled thermoelastoplastic stress analysis of both a uniaxially loaded solid rod and a thick cylinder exposed to an internal pressure. Two numerical cases involving the coupling effect were analyzed. The numerical solutions show that the maximum effective strain in solid structures from uncoupled analyses may occur up to 10 % greater than that from coupled analyses under the same mechanical or pressure load.

In order to study a thermomechanical problem, a finite elements method which uses both the known computational FEM and the fast Fourier transform technique was presented by Cho and Ahn (2002). Since their introduced finite elements method reduces the three-dimensional disk brake computations to two-dimensional case, they can solve the transient thermoelastic problem simpler.

Eslami et al. (1994) investigated a thin-walled cylindrical shell subjected to general mechanical and thermal shocks. The field of temperature and the field of displacement were coupled by means of a system of dynamic thermoelasticity equations. They applied their solution technique to the average wall temperature, lateral deformation of thin-walled shell, and thermal gradient through the thickness of the shell.

A computational technique in order to solve a dynamic coupled thermoelastic problem for an annular long tube was investigated by Li et al. (1983). The external wall of the cylinder was exposed to convection heat transfer by an ambient fluid temperature. There was not any mechanical load applied on the outer surface. However, the internal wall of the cylinder was subjected to both mechanical and thermal boundary conditions. The thermoelastic problem was solved by finite elements method. Authors proposed the computational results for the stress and temperature distributions.

Shao et al. (2008) developed some analytical solutions for thermal-mechanical and nonaxisymmetric behavior of FG annular cylinders which were exposed to time-dependent thermal loads and nonaxisymmetric mechanical loads. They obtained numerical results of stress and temperature in functionally graded annular cylinder under proposed thermal-mechanical loads in graphical form. By solving the 2D thermal-mechanical problem, they used a complex Fourier series.

Time-dependent behavior of one-dimensional axisymmetric coupled thermoelasticity problems with initial interface pressure was dealt by Hung et al. (2001). Initial interface pressure in a multilayered cylinder was considered as initial boundary condition for the thermoelasticity problem during the solution.

Time-dependent thermal stresses in a thick-walled annular cylinder was investigated by Hosseini and Akhlaghi (2009) for a functionally graded cylinder whose material properties were assumed as nonlinear with a power law distribution through the radial distance of the cylinder. The thick-walled cylinder was modeled with infinite length and plane strain state condition. Stress and displacement distributions were found using the analytical solution of governing differential equations.

Pelletier and Vel (2006) investigated the steady-state behavior of a thick-walled hollow cylinder made of a functionally graded material, which was exposed to some steady-state mechanical and thermal loadings. The elastic material properties of the functionally graded thick-walled hollow cylinder were considered to be randomly variable through the radial direction of the cylinder from its center. The governing linear thermoelasticity and the governing steady-state three-dimensional heat conduction equations were simplified to the state condition of generalized plane strain deformations in axial (longitudinal) direction and then, analytical solutions were obtained.

Shao and Ma (2008) proposed a transient thermomechanical temperature and stress analysis for FG annular cylinders which were exposed to linearly increasing temperature distribution and mechanical loadings. They assumed the material properties as variable through the thickness and independent from temperature. Numerical results of thermomechanical stress and transient temperature distributions were obtained.

A general analytical solution for a thermoelastic problem exposed to 2D axisymmetric steady-state thermal and mechanical stresses in finite-length annular cylinder made of functionally graded material was derived by Jabbari et al. (2009). In this particular study, authors solved the temperature distribution in terms of axial and radial directions analytically. The thermoelastic analysis was carried out for two cases with different boundary conditions on the ends of cylinder. Their intended solution technique might be applied to other kinds of thermomechanical and mechanical problems of different geometries, i.e. plates, cylinders of infinite lengths, and spheres.

2.3. THERMOELASTICITY WITH THERMODYNAMICS

Lubarda (2004) derived four thermodynamic potentials including internal energy dependent on strain and entropy, Helmholtz free energy dependent on strain and temperature, Gibbs energy as a function of stress and temperature, and finally enthalpy as a function of stress and entropy by considering the linear dependence of specific heat on temperature. All four thermodynamic potentials were proposed in order to express them in terms of four possible pairs of independent state variables which are strain, entropy, temperature, and stress.

Some applications of thermoelasticity with developments in the irreversible thermodynamic techniques were presented by Biot (1956). General equations of the theory of thermoelasticity were given according to the principle of minimum entropy production in a variational form.

A method in order to compute the Gibbs energy and the Helmholtz free energy from an analysis of the enthalpy function and the internal energy was presented by Phillpot and Rickman (1991). A comparison of the Helmholtz and the Gibbs free energies computed by means of their methodology was carried out with the Helmholtz and the Gibbs free energies computed by means of a temperature-integration scheme.

2.4. CONCLUSIONS ON LITERATURE SURVEY

The following conclusions were obtained from the literature reviews;

1. Reactor vessels which cover the reactor cores are exposed to some nuclear materials. Hence, they should be constructed of materials which can withstand the bombardment with heat which is generated during fission and neutron irradiation. A few researchers dealt with the thermoelastic analysis of reactor vessels including convection and radiation.

2. The thermal and structural materials were generally analyzed by means of solely thermoelasticity or solely thermodynamics. However they can simultaneously be investigated by means of both. Coupled analysis of both improves the quality of the finite element analysis.

CHAPTER 3

PRESSURIZED CYLINDERS

3.1. INTRODUCTION

This chapter deals with the main concepts of pressurized cylindrical vessels. A brief overview of the cylindrical thin pressurized vessels will be given at first. Then expressions of stress, strain, and deformations in thick-walled cylindrical vessels will be introduced due to solely internal pressure load, solely external pressure load, both internal and external pressure loads, solely thermal load, and finally both internal and external pressure loads with temperature load.

Cylindrical pressurized vessels such as pipes, boilers, and hydraulic cylinders are commonly utilized in industrial applications in order to carry many types of pressurized fluids. Since damage of pressurized vessels can lead to loss of life and property, pressurized vessels are designed with special care. Brittle materials such as cast iron and ductile materials such as mild steel can be utilized as the pressure vessel material. Once an internal pressure is applied to the pressurized cylinder, the pressure vessel is subjected to stresses in all directions resulting from pressure. These normal stresses are dependent on the dimensions and the shape of the vessel, and the applied pressure also.

3.2. REVIEW OF CYLINDRICAL VESSELS

The pressurized cylinders can be exposed to several mechanical and thermal loads and these loads cause mechanical and thermal stresses of different intensities within the internal and external cylinder surfaces. Intensity and type of these stresses depend on cylindrical vessel geometry and type of applied loads. Some examples of the types of loads are the inner and outer pressures, the temperature gradients, weight of cylindrical vessel and its interior components, and the dynamic and cyclic reactions caused by the temperature and pressure changes.

3.2.1. Cylindrical Thin Pressurized Vessels

Consider a cylindrical vessel with internal diameter, d, and wall thickness, t as seen in Figure 3.1. An internal pressure, P, is resulted through the cylindrical vessel by the internal working fluid. An infinitesimally small element as seen in Figure 3.1 is under two types of tensional normal stresses: hoop (circumferential) stress, σ_t, and longitudinal stress, σ_a. Circumferential stress is a kind of mechanical stress in a cylindrical body generated by the inner or outer pressures. Circumferential stress is the average force generated circumferentially at any point on the cylindrical vessel surface. For the case of thin cylindrical vessels, $r_i / t > 10$, hence the change in the tangential stress in the radial direction is minimal. Radial stresses are the stresses that are perpendicular to the curved plane of the cylindrical vessel. For thin cylindrical vessels, their values can be ignored compared to the tangential and longitudinal stress values. Finally, the longitudinal stresses occur in the axis that lies through the length of the cylinder. The radial stress varies from $-P$ at the inner surface to zero at the outer surface, but again is small compared with the other two stresses, and so is taken to be zero.

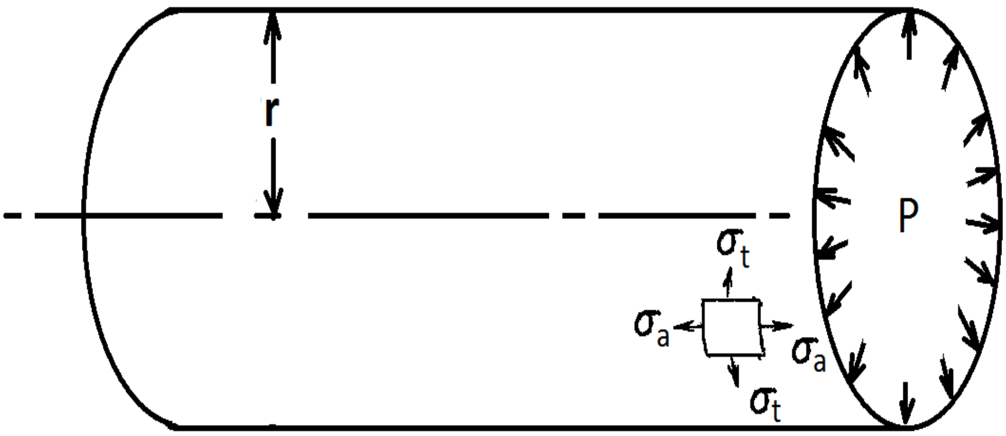

Figure 3.1. Cylindrical pressure vessel

$$\sigma_r = 0$$
$$\sigma_a = P\frac{r_i}{2t} \tag{3.1}$$
$$\sigma_t = P\frac{r_i}{t}$$

Therefore circumferential stress is nearly twice the longitudinal stress. According to strain and stress theory, corresponding tangential strain and longitudinal strain, respectively are:

$$\varepsilon_t = \frac{\sigma_t - \mu\sigma_a}{E}$$
$$\varepsilon_a = \frac{\sigma_a - \mu\sigma_t}{E} \tag{3.2}$$

3.2.2. Thick-Walled Cylindrical Vessels

French mathematician Lame was the first to determine the stresses through thick-walled cylindrical vessels in 1833. According to his sensible solution of determining the stresses, he assumed that thick-walled cylindrical vessels to consist of several thin-walled cylindrical vessels such that every single thin cylinder exerts pressure on the next other. His method of solution basically focused on three stress components at any point along the thick cylinder. Three stress components mentioned are tangential stress, or hoop stress, which is through the circumferential direction of the cylinder, longitudinal stress, or axial stress, which is along the direction of axis and remains unchanged for any cross-section, and finally, radial stress that is along the radial direction of the cylinder and it can be considered as pressure on outer or inner walls of the cylinder. The description for longitudinal stress is due to the assumption that any planar cross-section of the thick-walled cylindrical vessel will remain plane after the pressure loading and the material of the cylinder is homogeneous and isotropic. Hence, strains through the longitudinal axis remain the same.

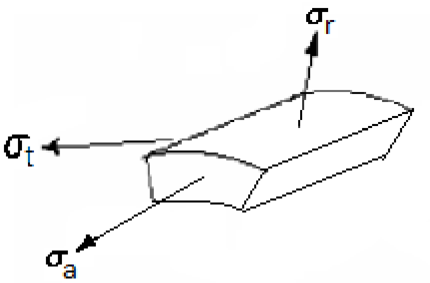

Figure 3.2. A small section from cylindrical wall

Three stress components along a thick-walled cylinder wall can be seen from a small section of the wall shown in Figure 3.2. σ_a denotes the axial (longitudinal) stress, σ_r denotes the radial stress, and σ_t is the tangential (hoop) stress. The equilibrium equation without the effect of body forces is

$$r\frac{d\sigma_r}{dr} = \sigma_t - \sigma_r \tag{3.3}$$

The expressions associating strains with corresponding stresses assuming that the planar cross-sections remain planar, and thus axial strain is constant through the radial direction, may be given as,

$$\begin{aligned}
\varepsilon_a &= \frac{1}{E}\left(\sigma_a - \nu(\sigma_t + \sigma_r)\right) = \text{constant} \\
\varepsilon_r &= \frac{1}{E}\left(\sigma_r - \nu(\sigma_a + \sigma_t)\right) \\
\varepsilon_t &= \frac{1}{E}\left(\sigma_t - \nu(\sigma_r + \sigma_a)\right)
\end{aligned} \tag{3.4}$$

3.2.2.1 Cylinder under Internal Pressure Only

Some applications for thick-walled cylinders for which there is no any outer pressure applied can be gun barrels, gas storage tanks, hydraulic cylinders and liquid or gas-carrying pipes. Submarines and pressure vessels are some examples for closed cylinders, while shrink fits and gun barrels are certain examples of open cylinders. If both an outer and an inner pressure are applied to a cylindrical vessel with constant thickness, the deformation through the cylinder will be symmetric with respect to its longitudinal axis, and so will not alter through its length. The perpendicularly applied pressure on the cylinder is symmetric, and so each point on the hollow thick-walled cylinder surface will be deformed by the same amount which is dependent on radius of thick cylinder. The radial shape of the thick cylinder will be maintained. As a result, there will not be any shear stresses on radial and circumferential planes and the stresses on these planes will be defined as principal stresses.

For pressurized cylinders of any wall thickness exposed only to an inner pressure as shown in Figure 3.3, the external pressure being zero ($P_o = 0$) and if the thick-walled cylinder is closed-ended, longitudinal stress expression should be added, so following expressions are obtained (Poworoznek, 2008):

$$\begin{aligned}
\sigma_r &= P\frac{r_i^2}{r_o^2 - r_i^2}\frac{r^2 - r_o^2}{r^2}(compressive) \\
\sigma_t &= P\frac{r_i^2}{r_o^2 - r_i^2}\frac{r^2 + r_o^2}{r^2}(tensile) \\
\sigma_a &= P\frac{r_i^2}{r_o^2 - r_i^2}
\end{aligned} \tag{3.5}$$

in which r is the radial distance from the cylinder center, r_o is the external radius, and r_i is the inner radius of the cylinder. The maximum shear stress occurs on inside wall of the cylindrical vessel for the case of inner pressure only. Both compressive and tensile stresses are their maximum on inside wall of the vessel. The maximum compressive stress is the radial stress, while the maximum tensile stress is the tangential one. Hence the maximum shear stress can be written as (Moss, 2004):

$$\tau_{\max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{\sigma_t - \sigma_r}{2} = P\frac{r_o^2}{r_o^2 - r_i^2} \tag{3.6}$$

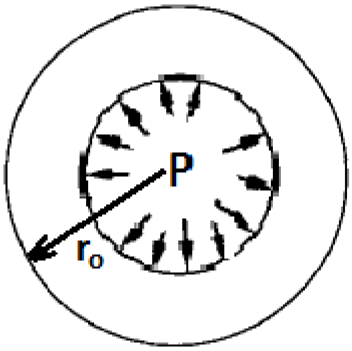

Figure 3.3. Cross-section of thick-walled cylinder

In equations (3.5), the tangential stress is numerically greater than the radial stress. Radial and tangential stresses are dependent on the radial distance, r. However, the longitudinal stress is not a function of the radial distance from the center and remains the same along the cylinder thickness. Figure 3.4 depicts the radial and tangential stress distributions through the cylinder thickness. For a thick cylindrical vessel that is exposed to an inner pressure, both radial and tangential stresses possess their maximum on internal wall of the vessel. Because inner pressure causes radial compression on the thick cylinder, it is considered as negative radial stress. Maximum numerical value for the circumferential stress occurs at the bore surface and minimum numerical value for the circumferential stress occurs at the external surface of the cylinder for the case of internal pressure only.

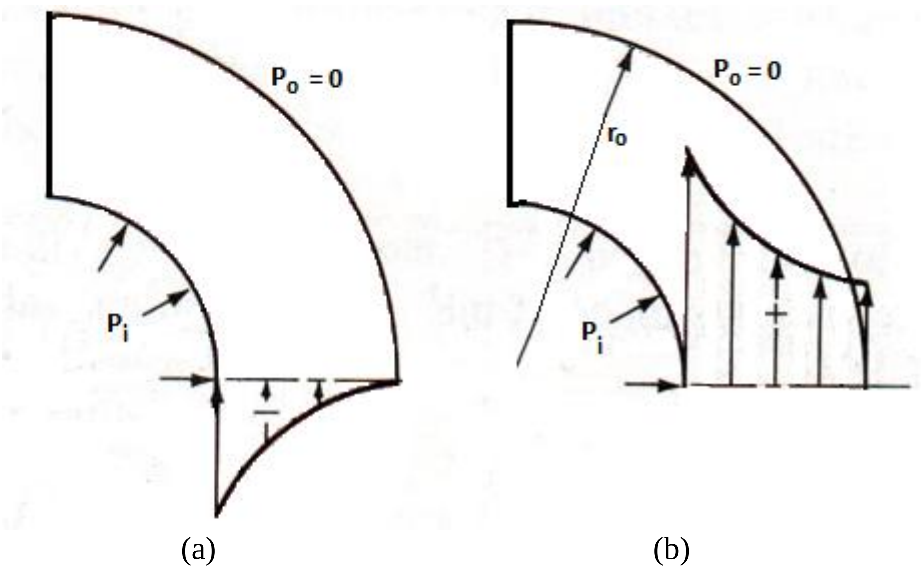

Figure 3.4. Radial and tangential stress distributions

Axial strain equals zero for the plane-strain case, and the remaining two strains are obtained by means of Hooke's law as (Poworoznek, 2008):

$$\begin{aligned}\varepsilon_r &= \frac{P}{E}\frac{r_i^2}{r_o^2 - r_i^2}\left(1-\nu-2\nu^2 - \frac{(1+\nu)r_o^2}{r^2}\right)\\ \varepsilon_t &= \frac{P}{E}\frac{r_i^2}{r_o^2 - r_i^2}\left(1-\nu-2\nu^2 + \frac{(1+\nu)r_o^2}{r^2}\right)\end{aligned} \tag{3.7}$$

Then, radial displacement can be calculated as:

$$u_r = \varepsilon_r r \tag{3.8}$$

$$u_r = (1+\nu)\frac{P}{E}\frac{r_i^2 r}{r_o^2 - r_i^2}\left(1+\frac{r_o^2}{r^2} - 2\nu\right) \tag{3.9}$$

3.2.2.2 Cylinder under External Pressure Only

If the thick-walled cylinder is exposed to an outer pressure P_o only, the tangential and radial stresses can be given as (Poworoznek, 2008)

$$\begin{aligned}\sigma_t &= -\frac{r_o^2 P_o}{r_o^2 - r_i^2}\left(\frac{r_i^2 + r^2}{r^2}\right)\\ \sigma_r &= -\frac{r_o^2 P_o}{r_o^2 - r_i^2}\left(\frac{r^2 - r_i^2}{r^2}\right)\end{aligned} \tag{3.10}$$

The tangential and radial stresses can be seen as compressive stresses from equations above. The tangential stress is numerically greater than the radial stress. Maximum compressive circumferential stress occurs at inner wall with magnitude

$$\sigma_{t,\max} = -\frac{2r_o^2 P_o}{r_o^2 - r_i^2} \tag{3.11}$$

However, maximum radial stress occurs at external wall of the cylinder and equals P_o. Thus, the maximum numerical values of the radial stress and the tangential stress do not occur at the same points throughout the thick-walled cylinder for the case of external pressure only (Fryer, 1998). If the pressure applied externally is reversed in the opposite direction (outward from the center), P_o is replaced by $-P_o$ in equations (3.10) and (3.11). If the thickness ratio r_o / r_i is very large, maximum stress approaches will be twice the magnitude of the pressure applied externally.

3.2.2.3 Cylinder under both Internal and External Pressures

For the case including both an inner and an outer pressure on the cylinder walls, equations (3.5) may be expanded as (Poworoznek, 2008)

$$\begin{aligned}
\sigma_r &= \frac{P_i r_i^2 - P_o r_o^2}{r_o^2 - r_i^2} + \frac{(P_o - P_i) r_o^2 r_i^2}{r^2 (r_o^2 - r_i^2)} \\
\sigma_t &= \frac{P_i r_i^2 - P_o r_o^2}{r_o^2 - r_i^2} - \frac{(P_o - P_i) r_o^2 r_i^2}{r^2 (r_o^2 - r_i^2)} \\
\sigma_a &= \frac{P_i r_i^2 - P_o r_o^2}{r_o^2 - r_i^2}
\end{aligned} \tag{3.12}$$

where P_i is the internal pressure and P_o denotes the external pressure. Radial displacement of any point in the surface of an open thick-walled cylinder with constant temperature is given as

$$u_r = \frac{P_i r_i^2 - P_o r_o^2}{r_o^2 - r_i^2} \frac{r(1-\nu)}{E} + \frac{r_o^2 r_i^2 (P_i - P_o)}{r_o^2 - r_i^2} \frac{1+\nu}{Er} \tag{3.13}$$

where E is the modulus of elasticity and ν is the Poisson's ratio. Each parameter is known in equations above except the radial distance vector r, whose value changes between the inner and outer radii. According to the equations (3.12), when the external pressure is greater than the internal one, stress increases as the radial distance approaches r_o. On the other hand, when internal pressure is greater than external one, the stresses increase as the radial distance approaches r_i. Since equations (3.12) do not possess any angular position variable, points which are located at the same radius but at different angles possess the same radial and circumferential stresses. But because of the fact that the radial and circumferential stress equations are dependent on the radial distance, points at different radial distances from the central line of the cylinder experience different stresses. In section

3.2.2.1, it has been stated the stresses at any point on the thick cylinder wall are principal stresses. Therefore, the maximum shear stress at any point will be given by

$$\tau_{\max} = \frac{\sigma_t - \sigma_r}{2} \tag{3.14}$$

i.e. half the difference between the tangential and the radial stresses which are the greatest and the least principal stresses, respectively, in the case of thick-walled cylindrical vessel.

3.2.2.4 Cylinder under Temperature Loads

If the natural expansion or shrinkage of a body resulting from some change in temperature is prevented, there will be thermal stresses induced throughout the body. This thermal stresses are induced due to the mechanical constraints applied. When internal wall of thick-walled cylinder is exposed to a uniform constant temperature and external wall is held at a temperature of T_o, temperature distribution as a function of the radial distance throughout the thick-walled cylinder is as follows (Poworoznek, 2008):

$$T(r) = \frac{T_i \ln(r_o / r)}{\ln(r_o / r_i)} + T_o \tag{3.15}$$

The analytical expressions for the radial, tangential, and axial stresses for the case of thermal loading can be calculated for plane-strain cylinder with fixed ends as (Poworoznek, 2008):

$$\begin{aligned}
\sigma_r &= -\frac{\alpha E(T_i - T_o)}{2(1-\nu)}\left(\frac{\ln(r_o / r)}{\ln(r_o / r_i)} + \frac{r^2 - r_o^2}{r^2}\frac{r_i^2}{r_o^2 - r_i^2}\right) \\
\sigma_t &= \frac{\alpha E(T_i - T_o)}{2(1-\nu)}\left(\frac{1-\ln(r_o / r)}{\ln(r_o / r_i)} - \frac{r^2 + r_o^2}{r^2}\frac{r_i^2}{r_o^2 - r_i^2}\right) \\
\sigma_a &= \frac{\alpha E(T_i - T_o)}{2(1-\nu)}\left(\frac{\nu - 2\ln(r_o / r)}{\ln(r_o / r_i)} - \frac{2\nu r_i^2}{r_o^2 - r_i^2}\right) - \alpha(T_o - T_0)E
\end{aligned} \tag{3.16}$$

Corresponding radial and circumferential strains are then:

$$\varepsilon_r = -\frac{(T_i - T_o)\alpha}{2(1-\nu)} \ldots$$
$$\left[\frac{\nu^2 + \nu - (1+\nu)\ln(r_o / r)}{\ln(r_o / r_i)} + \left(1 - 2\nu^2 - \nu - \frac{(1+\nu) r_o^2}{r^2}\right) \frac{r_i^2}{r_o^2 - r_i^2} \right]$$
$$\varepsilon_t = \frac{(T_i - T_o)\alpha}{2(1-\nu)} \ldots \tag{3.17}$$
$$\left[\frac{1 - \nu^2 + (1+\nu)\ln(r_o / r)}{\ln(r_o / r_i)} - \left(1 - 2\nu^2 - \nu + \frac{(1+\nu) r_o^2}{r^2}\right) \frac{r_i^2}{r_o^2 - r_i^2} \right]$$

Then, radial displacement expression can be given as:

$$u_r = \frac{r(T_i - T_o)\alpha}{2(1-\nu)} \ldots$$
$$\left[\frac{1 - \nu^2 + (1+\nu)\ln(r_o / r)}{\ln(r_o / r_i)} - \left(1 - 2\nu^2 - \nu + \frac{(1+\nu) r_o^2}{r^2}\right) \frac{r_i^2}{r_o^2 - r_i^2} \right] \tag{3.18}$$

3.2.2.5 Cylinder under Pressure with Temperature Change

Most applications include a cylinder with constant wall thickness through the length and exposed to both a uniform inner pressure and a uniform outer pressure with a temperature change shown in Figure 3.5.

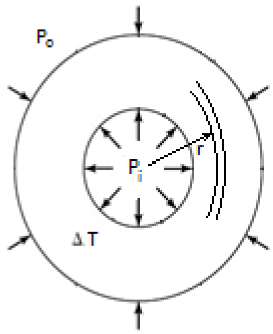

Figure 3.5. Cylinder under inner and outer pressure with temperature change

The stress-strain-temperature relations for the cylinder material that is assumed to be isotropic and linearly elastic are:

$$
\begin{aligned}
\varepsilon_r &= \alpha\Delta T + \frac{\sigma_r - \nu(\sigma_a + \sigma_t)}{E} \\
\varepsilon_t &= \alpha\Delta T + \frac{\sigma_t - \nu(\sigma_a + \sigma_r)}{E} \\
\varepsilon_a &= \alpha\Delta T + \frac{\sigma_a - \nu(\sigma_t + \sigma_r)}{E} = \text{constant}
\end{aligned}
\tag{3.19}
$$

where E is the Young's modulus, α is the linear thermal expansion coefficient, ΔT may be defined as the temperature change from the uniform reference temperature, and ν is the Poisson's ratio. The radial displacement, stresses and corresponding strains can be obtained by simply adding the expressions of solely pressure stresses and solely temperature stresses for the linear elastic cylinder concerned. For the plane-strain case, the radial stress, tangential stress and the radial displacement may be given as follows (Poworoznek, 2008):

$$
\begin{aligned}
\sigma_r &= P\frac{r_i^2(r^2 - r_o^{\,2})}{r^2(r_o^{\,2} - r_i^2)} + \ldots \\
&\quad \frac{E\alpha(T_i - T_o)}{2\ln\left(\dfrac{r_o}{r_i}\right)(1-\nu)}\left[\frac{r_i^2}{r_o^{\,2} - r_i^2}\left(\frac{r_o^{\,2}}{r^2} - 1\right)\ln\left(\frac{r_o}{r_i}\right) - \ln\left(\frac{r_o}{r}\right)\right] \\
\sigma_t &= P\frac{r_i^2(r^2 + r_o^{\,2})}{r^2(r_o^{\,2} - r_i^2)} + \ldots \\
&\quad \frac{E\alpha(T_i - T_o)}{2\ln\left(\dfrac{r_o}{r_i}\right)(1-\nu)}\left[1 - \ln\left(\frac{r_o}{r}\right) - \frac{r_i^2}{r_o^{\,2} - r_i^2}\left(1 + \frac{r_o^{\,2}}{r^2}\right)\ln\left(\frac{r_o}{r_i}\right)\right]
\end{aligned}
\tag{3.20}
$$

$$
\begin{aligned}
u_r &= r\frac{P(1+\nu)}{E}\frac{r_i^2}{r_o^{\,2} - r_i^2}\left(\frac{r^2 + r_o^{\,2}}{r^2} - 2\nu\right) + r\frac{\alpha(T_i - T_o)}{2(1-\nu)\ln\left(\dfrac{r_o}{r_i}\right)} \ldots \\
&\quad \left[\begin{aligned}&(\nu+1)\ln\left(\frac{r_o}{r}\right) + (1-\nu^2)\ldots \\ &\left(\frac{r_i^2}{r_o^{\,2} - r_i^2}\right)\left[(1+\nu)\frac{r_o^{\,2}}{r^2} + (1-\nu) - 2\nu^2\right]\ln\left(\frac{r_o}{r_i}\right)\end{aligned}\right] + \ldots \\
&\quad r(1+\nu)\alpha(T_o - T_0)
\end{aligned}
\tag{3.21}
$$

Stresses due to mechanical loads can be obtained from the same governing equations except that the temperature part is omitted.

CHAPTER 4

COUPLED THERMOELASTIC ANALYSIS

4.1. INTRODUCTION

This chapter deals with the development of formulations of general problems of coupled thermoelasticity.

4.2. GOVERNING EQUATIONS OF COUPLED THERMOELASTICITY

The deformation of an elastic body may be due to some change in the heat content within the body. Thermally loading of elastic bodies leads to not only deformation but also change in the temperature field within the body as time increases. The displacement of an elastic body is characterized by mutual interaction between deformation and temperature fields. The domain of science dealing with the mutual interaction of these fields is called the thermoelasticity.

The temperature field is expressed by the Fourier heat conduction equation and there is not any elastic term included in this equation for uncoupled thermoelasticity theory. The thermal stresses are due to the corresponding temperature field which is governed by the heat conduction equation that is separate from displacement-temperature field equation in uncoupled thermoelasticity.

Coupled thermoelastic cases consider the rate of change of the first invariant of strain tensor in time in first law of thermodynamics generating a relationship between strain and temperature fields. In this manner, the thermal and elastic fields become coupled (Hetnarski, 2009).

Due to the fundamental assumptions taken, it is considered to analyze homogeneous isotropic elastic bodies in this part. Under these assumptions, general governing equations of thermoelasticity are given. The elastic body shall possess a temperature, T_0, in its stress- and strain-free state without the effect of external loads. The initial state is defined as reference state. For reference state, entropy is assumed to be equal zero. Due to the effects of heating or cooling sources, external body loads, and tractions, elastic body is exposed to some amount of temperature change and deformation. Thus, displacements occur through the elastic body. The change in the temperature may be given as follows

$$\theta = T - T_0 \tag{4.1}$$

in which T is local absolute temperature at any point on the body. Resulting temperature change is expressed by occurred strains and stresses. Displacements, stresses, strains, and the temperature change are dependent upon the time and position. One of the governing equations of coupled thermoelasticity is the linear strain-displacement relations which define the strain components in terms of displacement components in matrix form as

$$\varepsilon = \begin{bmatrix} \partial/\partial x & 0 & 0 \\ 0 & \partial/\partial y & 0 \\ 0 & 0 & \partial/\partial z \\ \partial/\partial y & \partial/\partial x & 0 \\ 0 & \partial/\partial z & \partial/\partial y \\ \partial/\partial z & 0 & \partial/\partial x \end{bmatrix} u \tag{4.2}$$

in which the strain components in vectorial form is

$$\varepsilon = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \varepsilon_{xy} \\ \varepsilon_{yz} \\ \varepsilon_{xz} \end{bmatrix} \tag{4.3}$$

and the displacement components are

$$u = \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} \tag{4.4}$$

Basic equations can be categorized in three groups:

- Equilibrium equation
- Constitutive law
- Equation of heat conduction

4.2.1. Equilibrium Equation

Equilibrium of the isotropic thermoelastic solid including body force intensities is governed in Cartesian coordinates as

$$\begin{bmatrix} \partial/\partial x & 0 & 0 & \partial/\partial y & 0 & \partial/\partial z \\ 0 & \partial/\partial y & 0 & \partial/\partial x & \partial/\partial z & 0 \\ 0 & 0 & \partial/\partial z & 0 & \partial/\partial y & \partial/\partial x \end{bmatrix} \sigma + B = 0 \tag{4.5}$$

in which B represents the body force intensity and the stress components in vectorial form can be expressed as

$$\sigma = \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{xz} \end{bmatrix} \tag{4.6}$$

Stress components are accompanied by symmetry conditions as

$$\sigma_{ij} = \sigma_{ji} \qquad (i, j = x, y, z) \tag{4.7}$$

Positive quantities in stress specify the increase in stress compared to the free-stress state due to applied thermal effects and mechanical loads. Displacement-strain relationship for the case of small displacement is,

$$\varepsilon_{ij} = \frac{u_{i,j} + u_{j,i}}{2} \tag{4.8}$$

in which u_i are defined as the displacement components. Strain components are also in symmetry relations as

$$\varepsilon_{ij} = \varepsilon_{ji} \qquad (i, j = x, y, z) \tag{4.9}$$

4.2.2. Constitutive Law

The constitutive law for thermoelasticity is governed by the superposition of the separate effects of thermal strain which is generated due to the temperature gradient from the reference temperature, and mechanical strain which is generated by the stress field. So, the total strain may be given as following

$$\varepsilon_{ij} = \varepsilon_{ij,th} + \varepsilon_{ij,mech} \tag{4.10}$$

Strain due to applied temperature gradient effects, as mechanical response of the body assuming for thermal expansion, in general, may be written as

$$\varepsilon_{ij,th} = \alpha_{ij} \Delta T \tag{4.11}$$

Coefficient of thermal expansion, α, may be dependent on the temperature. For the thermoelasticity problem, the generalized linear elastic Hooke's law is assumed to be appropriate. However, generally the elastic components, as well, can be considered as dependent on the temperature. The mechanical strain components for the linear elastic homogeneous isotropic problem may be written with respect to the stress components as

$$
\begin{aligned}
\varepsilon_{x,mech} &= \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] \\
\varepsilon_{y,mech} &= \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right] \\
\varepsilon_{z,mech} &= \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right] \\
\varepsilon_{xy,mech} &= \frac{\tau_{xy}}{2G} \\
\varepsilon_{yz,mech} &= \frac{\tau_{yz}}{2G} \\
\varepsilon_{xz,mech} &= \frac{\tau_{xz}}{2G}
\end{aligned}
\tag{4.12}
$$

where

$$
G = \frac{E}{2(1+\nu)} \tag{4.13}
$$

In the equation above, G denotes the modulus of rigidity, E is the modulus of elasticity, and ν denotes the Poisson's ratio. The thermal strains are additive to elastic strains due to local stresses. Hence, the Hooke's law becomes

$$
\begin{aligned}
\varepsilon_x &= \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] + \alpha(T - T_0) \\
\varepsilon_y &= \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right] + \alpha(T - T_0) \\
\varepsilon_z &= \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right] + \alpha(T - T_0) \\
\varepsilon_{xy} &= \frac{\tau_{xy}(1+\nu)}{E} \\
\varepsilon_{yz} &= \frac{\tau_{yz}(1+\nu)}{E} \\
\varepsilon_{xz} &= \frac{\tau_{xz}(1+\nu)}{E}
\end{aligned}
\tag{4.14}
$$

Constitutive law for the thermoelasticity problem may, therefore, be written as following

$$\sigma_{ij} = E_{ijhk}\left(\varepsilon_{hk} - \alpha_{hk}\Delta T\right) \tag{4.15}$$

Hooke's law or the generalized stress-strain relationship for thermoelasticity in linear elastic homogeneous isotropic material can be expressed as (Sadd, 2005)

$$\sigma_{ij} = D_{ijhk}\varepsilon_{hk} - \beta_{ij}(T - T_0) \tag{4.16}$$

in which T represents the absolute temperature, T_0 represents the reference temperature, thus $T - T_0$ representing the excess temperature distribution, β is the thermal shear modulus and can be written as

$$\beta = \frac{E\alpha}{1-2v}\begin{bmatrix}1\\1\\1\\0\\0\\0\end{bmatrix} = \left(3\lambda + 2G\right)\alpha\begin{bmatrix}1\\1\\1\\0\\0\\0\end{bmatrix} \tag{4.17}$$

in which E represents the modulus of elasticity, v is the Poisson's ratio, α represents the thermal expansion coefficient, and D represents the elastic constants as

$$D = \begin{pmatrix}\lambda+2G & \lambda & \lambda & 0 & 0 & 0\\ \lambda & \lambda+2G & \lambda & 0 & 0 & 0\\ \lambda & \lambda & \lambda+2G & 0 & 0 & 0\\ 0 & 0 & 0 & G & 0 & 0\\ 0 & 0 & 0 & 0 & G & 0\\ 0 & 0 & 0 & 0 & 0 & G\end{pmatrix} \tag{4.18}$$

where G is the modulus of rigidity and λ represents the Lamé modulus as

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)} \tag{4.19}$$

4.2.3. Heat Conduction

For a body which has a constant thermal conductivity, k, and no internal heat generation in steady-state conditions, the conduction equation is the Laplace equation (Sadd, 2005). The Laplace equation is an elliptic type of differential equation. This equation is a harmonic equation.

$$\nabla^2\theta = \frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} + \frac{\partial^2\theta}{\partial z^2} = 0 \tag{4.20}$$

Instantaneous absolute temperature is depicted by T, and the difference between the instantaneous absolute temperature and the reference temperature $T - T_0$ by θ. Most materials nearly obey Fourier's law of heat conduction in which area rate of heat flux is linearly proportional to temperature gradient, $\frac{\partial\theta}{\partial x}$, which means the basic heat conduction equation of thermoelasticity is Fourier's law and can be given as

$$q_i = -k_{ij}\frac{\partial\theta}{\partial x_j} \tag{4.21}$$

where k_{ij} are thermal heat conductivity coefficients of the material. Coefficient of thermal heat conductivity is, in general, assumed as constant, while it is a function of the temperature for real materials. But, for the latter case, resulting nonlinearity will be remarkable particularly if temperature gradient is large. The thermal conduction equation is

$$\frac{\partial}{\partial x_i}\left(k_{ij}\frac{\partial\theta}{\partial x_j}\right) = \frac{\rho C_\upsilon}{T_0}T\dot{\theta} + T\beta_{ij}\dot{\varepsilon}_{ij}, \qquad \theta = T - T_0 \tag{4.22}$$

where T denotes the local temperature, ρ denotes the density, q_i denotes the area rate of the heat flux vector, and C_υ denotes specific heat at constant volume per unit mass. Equation (4.22) is nonlinear because of the terms on the right side of the equation. Replacing T by T_0 on right side of the equation makes it linear. The last term including strain on the right-hand side of the extended equation above is the part that contains coupling of temperature field with the deformation one. Dot sign above the strain indicates derivative of strain due to time. When heat sources are applied on material, there should be addition to equation (4.22) that specifies produced heat amount in unit volume and time. In addition to the equation above, there can occur some heat generation per unit volume of the material caused by a heat source like nuclear reaction. Hence, local temperature should satisfy

$$\rho C \frac{\partial T}{\partial t} + (3\lambda + 2G)\alpha T_0 \dot{\varepsilon}_{ii} = k\nabla^2 T + Q \tag{4.23}$$

in which C denotes the specific heat and ρ denotes density. Hence, multiplication of them gives heat amount that is needed in order to increase local temperature of unit volume one degree. The first term on the right side of the equation above denotes net heat flow to material and the latter one denotes heat generation rate per unit volume of the material. Their summation gives available heat in order to increase local temperature.

General Navier equation can be derived by implementing the strain-deformation expressions for small deformation and the general stress-strain expressions for thermoelasticity into the general equilibrium equation as

$$G\nabla^2 u_i + (\lambda + G)\frac{\partial}{\partial x_i}(\nabla \vec{u}) + B_i - \beta \frac{\partial \theta}{\partial x_i} = 0 \tag{4.24}$$

Utilizing the equation above is sensible solely when boundary conditions applied are in terms of displacement, u. The generalized Navier equation above is written in terms of displacement components, u_i through all three coordinate axes.

4.2.4. Heat Convective Boundary

For the convective boundary, if a region in the boundary must be in touch with the fluid which has a temperature of T_f outside the thermal boundary layer. The normal component of the heat flux vector can be given as (Al-Rushudi,1991):

$$\hat{q}_n = -k\frac{\partial T}{\partial n} = h(T_b - T_f) \tag{4.25}$$

where h is the convection heat transfer coefficient.

4.2.5. Radiative Boundary

For the radiative boundary, a body which has a temperature of T_b subjected to a heat source at temperature T_s may transfer heat from or to outside, by thermal radiation (Al-Rushudi,1991).

$$\hat{q}_{rad} = \frac{q}{A} = \in E_b(T_b) - \alpha G = \in \sigma_B(T_b^{\,4} - T_s^{\,4}) \tag{4.26}$$

where $\in$ is the emissivity and σ_B is the Stefan-Boltzmann constant and it's numerical value is $5.676 \times 10^{-8} W / m^2 K^{-4}$. The boundary condition can be given as:

$$k\frac{\partial T}{\partial n} + \in \sigma_B(T_b^{\,4} - T_s^{\,4}) = 0 \tag{4.27}$$

A body can possess multimode heat transfer conditions. The boundary condition for both convective and radiative conditions can be written as:

$$k\frac{\partial T}{\partial n} + h(T_b - T_f) + \in \sigma_B(T_b^{\,4} - T_s^{\,4}) = 0 \tag{4.28}$$

CHAPTER 5

THE THERMODYNAMIC POTENTIALS ANALYSIS

5.1. INTRODUCTION

Thermodynamic potentials include four quantities that are useful during thermodynamic analysis. The four thermodynamic potentials are called Gibbs energy, the internal energy, enthalpy, and Helmholtz free energy. The four thermodynamic potentials are extensive state variables of the dimensions of the energy. The objective in using these thermodynamic potentials is to provide the equilibrium for systems that are interacting with environment and to measure energy of the system in terms of different variables, thus measuring more state variables of the system. Once the thermodynamic potentials are known in terms of specified variables, all other variables characterizing system can be determined by partially differentiation of the thermodynamic potentials. The thermodynamic potentials are state functions which define system equilibrium in terms of natural independent state variables. These natural independent variables allow calculating other independent state variables by differentiating thermodynamic potentials.

Gibbs energy can be described as the subtraction of the energy obtained from the environment of the system by heating from the summation of the internal energy and the work to give the system final volume at a constant pressure P. Internal energy can be considered as energy needed in order to obtain system without any volume or temperature changes. The body is assumed to be in the reference state (free-stress and free-strain state) and have a uniform reference temperature of T_r, initially. The entropy, s, and the internal energy, u, of the unit element increase during heating and deformation. However, for the definition of enthalpy, if system is generated from a very small volume to generate room for system, additional amount of work should be carried on. For Helmholtz free energy, an environment at a constant temperature adds Ts amount of energy into the system, while decreasing overall energy required for generating a system.

The useful expressions for four thermodynamic potentials; internal energy, the Helmholtz free energy, Gibbs energy, and the enthalpy function in terms of their specific independent state variables are reviewed in the section 5.2.

5.2. EQUATIONS OF FOUR THERMODYNAMIC POTENTIALS

All thermodynamic potentials are the state functions derived from energetic form of fundamental equation by means of the Legendre transform, as a function of natural variables. Natural variables lead to calculate other state functions by partial differentiation of thermodynamic potentials. Thermodynamic potential for a system must depend on a thermal variable that may be temperature and entropy, and a mechanical one that may be stress and strain. Each of the four fundamental thermodynamic potentials provides the most convenient definition depending on the type of process involved. All of the four thermodynamic potentials functions have the units of energy. Any one of the four thermodynamic potentials may be utilized in order to define any system in equilibrium. However, some of them are more appropriate for a given system than the others. The expressions are, for all state functions, based on the assumption of linear dependence of specific heat upon temperature.

5.2.1. Internal Energy

The first state variable that may be considered as the first thermodynamic potential defined with the units of energy is the internal energy in thermodynamics. The starting point in order to describe the other thermodynamic potentials is going to be the internal energy (Lubarda, 2004).

$$du = \sigma_{ij} d\varepsilon_{ij} + Tds \tag{5.1}$$

Hence, strain and entropy are the natural independent state variables for internal energy. By means of the equation above, increment of the internal energy can be written in terms of increment of entropy and increment of strain. The simple form of the differential of the internal energy state function is given above and Maxwell relation gives

$$\left.\frac{\partial T}{\partial \varepsilon_{ij}}\right|_s = \left.\frac{\partial \sigma_{ij}}{\partial s}\right|_\varepsilon \tag{5.2}$$

The expression for stress, σ_{ij}, in terms of strain and temperature based on Hooke's law, can be given as (Lubarda, 2004)

$$\sigma_{ij} = 2\varepsilon_{ij}\mu + \varepsilon_{kk}\lambda_T\delta_{ij} - \alpha_0\kappa_T\delta_{ij}(T - T_r) \tag{5.3}$$

Specific heat, C_υ, is assumed to be linearly proportional to the temperature with the relationship

$$C_\upsilon = \frac{C_\upsilon^0 T}{T_r} \tag{5.4}$$

Using Maxwell relation and carrying on substitutions and joint integrations will lead (Lubarda, 2004)

$$T = T_r\left[1 + \frac{1}{C_\upsilon^0}\left(s - s_0 - \varepsilon_{kk}\kappa_T\alpha_0\right)\right] \tag{5.5}$$

Since a disadvantage of using internal energy as a thermodynamic potential is natural variable, entropy, is difficult to control, it is better to replace the entropy by the temperature. Dealing with other three thermodynamic potentials which may be obtained by utilizing Legendre transform is more convenient in practical manner. Other three thermodynamic potentials (enthalpy, Helmholtz free energy, and Gibbs function) are Legendre transforms of internal energy in terms of natural independent state variables. Expression for the internal energy in terms of its natural state variables, $u(\varepsilon_{ij}, s)$, can be written as (Lubarda, 2004)

$$u(\varepsilon_{ij}, s) = T_r(s - s_0) + \mu\varepsilon_{ij}^2 + \frac{T_r(s - s_0)^2}{2C_\upsilon^0} + \frac{\lambda_S \varepsilon_{kk}^2}{2} - \kappa_T \alpha_0 \varepsilon_{kk} \frac{T_r(s - s_0)}{C_\upsilon^0} \tag{5.6}$$

in which λ_S denotes the isentropic Lamé elastic constant and equals

$$\lambda_S = \alpha_0^2 \frac{\kappa_T^2 T_r}{C_\upsilon^0} + \lambda_T \tag{5.7}$$

5.2.2. Helmholtz Free Energy

One of the most widely used thermodynamic potentials is the Helmholtz function. Helmholtz free energy can be described as difference between the internal energy and the energy that can be obtained from the environment of the system by heating.

$$df = u - Ts \tag{5.8}$$

Internal energy can be considered as energy that is necessary in order to create system without any volume or temperature change. However, some amount of energy may be obtained by spontaneous heat transfer from environment into system, if system is generated in an environment having temperature T. The spontaneous heat transfer is Ts in which s denotes final entropy of system. For such a problem, there is no need to add in as much energy. That is, less work will be necessary in order to generate system, if a final state of higher entropy is generated. So, Helmholtz free energy is the amount of energy that is required to put in to generate a system, when instantaneous heat transfer from environment into system is accounted for. Helmholtz free energy is the Legendre transform of internal energy with respect to entropy, s. Differential of Helmholtz free energy is

$$df = \sigma_{ij} d\varepsilon_{ij} - sdT \tag{5.9}$$

and Maxwell relation for independent state variables

$$-\left.\frac{\partial s}{\partial \varepsilon_{ij}}\right|_T = \left.\frac{\partial \sigma_{ij}}{\partial T}\right|_\varepsilon \tag{5.10}$$

The entropy expression in terms of strain and temperature (Lubarda, 2004)

$$s = s_0 + \varepsilon_{kk}\kappa_T\alpha_0 + C_\upsilon^0 \frac{T}{T_r} - C_\upsilon^0 \tag{5.11}$$

Helmholtz free energy becomes constant at constant volume and constant temperature. The natural independent variables of the Helmholtz free energy are the strain and temperature. By replacing the independent variable, entropy, by temperature in internal energy expression using the relationship between the internal energy and temperature by means of a partial Legendre transform (Lubarda, 2004), we define the Helmholtz free energy, f :

$$f(\varepsilon_{ij}, T) = \mu\varepsilon_{ij}{}^2 + \frac{\lambda_T\varepsilon_{kk}{}^2}{2} - s_0T - \varepsilon_{kk}\kappa_T\alpha_0(T - T_r) - \frac{C_\upsilon{}^0}{2}\frac{(T - T_r)^2}{T_r} \tag{5.12}$$

in which μ and λ_T denote Lamé isothermal elastic constants, α_0 denotes volumetric thermal expansion coefficient in reference temperature T_r, s_0 denotes specific entropy at temperature T_r, and

$$\kappa_T = \lambda_T + \frac{2\mu}{3} \tag{5.13}$$

denotes isothermal bulk modulus, and, C_υ^0 denotes specific heat at constant strain and T_r.

5.2.3. Gibbs Energy

Gibbs free energy (also known as the Gibbs potential) is Legendre transformation of the internal energy with respect to entropy. It should be noted that Gibbs energy is naturally dependent on the independent variables stress and temperature. Differential form of the Gibbs energy

$$dg = -\left(sdT + \varepsilon_{ij}d\sigma_{ij}\right) \tag{5.14}$$

and the Maxwell relation for independent state variables reads

$$\left.\frac{\partial s}{\partial \sigma_{ij}}\right|_T = \left.\frac{\partial \varepsilon_{ij}}{\partial T}\right|_\sigma \tag{5.15}$$

Gibbs free energy becomes constant when pressure and temperature are constant. Constant pressure and constant temperature are the conditions that phase transitions occur. Lubarda (2004) derived an expression for the temperature dependence of the Gibbs free energy as:

$$g(\sigma_{ij},T) = -\left[s_0 T + \frac{\sigma_{kk}\alpha_0 (T-T_r)}{3} + \frac{1}{4\mu}\left(\sigma_{ij}{}^2 - \frac{\sigma_{kk}{}^2 \nu_T}{1+\nu_T} \right) + \frac{C_P{}^0}{2}\frac{(T-T_r)^2}{T_r} \right] \tag{5.16}$$

simply states that partial derivatives with respect to temperature and stress can be given by:

$$\left.\frac{dg}{d\sigma_{ij}}\right|_T = -\in_{ij}, \qquad \left.\frac{dg}{dT}\right|_\sigma = -s \tag{5.17}$$

It is obvious that independent variable, temperature, is the ideal variable that can be more easily controlled in the lab. Hence both the Helmholtz free energy and the Gibbs energy are two thermodynamic potentials that are mostly utilized.

5.2.4. Enthalpy

The natural independent variables of enthalpy are the stress and the entropy. Enthalpy, also, may be considered as a thermodynamic potential. The differential of enthalpy in terms of independent state variables is

$$dh = Tds - \varepsilon_{ij} d\sigma_{ij} \tag{5.18}$$

and the Maxwell relation holds

$$-\left.\frac{\partial T}{\partial \sigma_{ij}}\right|_s = \left.\frac{\partial \varepsilon_{ij}}{\partial s}\right|_\sigma \tag{5.19}$$

By partial differentiation, substitution, and joint integration, the temperature expression can be obtained as following (Lubarda, 2004)

$$T = T_r \left[1 + \frac{1}{C_P^0} \left(s - s_0 - \frac{\sigma_{kk}\alpha_0}{3} \right) \right] \tag{5.20}$$

Because a small change in length is one third of the change in volume of an isotropic substance, coefficient of volumetric thermal expansion, α_0, is three times greater than linear thermal expansion coefficient, α. The expressions above lead to the enthalpy expression in terms of its natural state variables of entropy and stress; that is (Lubarda, 2004)

$$h(\sigma_{ij}, s) = T_r (s - s_0) \left[1 + \frac{1}{C_P^0} \left(\frac{s - s_0}{2} - \frac{\sigma_{kk}\alpha_0}{3} \right) \right] + \frac{1}{4\mu} \left(\frac{\sigma_{kk}{}^2 \nu_S}{1 + \nu_S} - \sigma_{ij}^2 \right) \tag{5.21}$$

CHAPTER 6

CASE STUDIES AND RESULTS

6.1. INTRODUCTION

Main concepts of the analyses of thermoelastic cylinders with thermomechanical loads are summarized in this particular chapter. Analytical results obtained by the analytical formulations for thick-walled cylinder of plane strain with thermal and pressure loads are compared with the ones from the computational code and ANSYS. In the next section, thermoelasticity analysis of a thick-walled cylindrical geometry is presented in brief. In section 6.3., the analysis of the cross-section of the nozzle region of a nuclear reactor vessel is overviewed in brief.

6.2. COUPLED THERMOELASTIC ANALYSIS OF THICK-WALLED CYLINDER

Uncoupled thermoelastic analyses are more rustic than coupled thermoelasticity analyses. In uncoupled thermoelasticity, computation of heat flux and temperature for any internal node in the whole region exacts merely the heat flux and temperature at the surface nodes. But, in coupled theory of thermoelasticity by Biot (1956), the equations of elasticity and of heat conduction are coupled which deals with the first defect of the uncoupled theory.

In order to generate the finite element model, a typical thick-walled cylinder was built. Two quarter thermal and structural geometrical models were built with the 8-noded quadrilateral elements with the element behavior option of plane strain in the computer code according to the axisymmetric characteristic of the thick-walled cylinder and its boundary conditions. The displacement in x-direction is applied as zero for the x=0 line of the geometrical model and also, the displacement in y-direction is applied as zero for the y=0 line of the geometrical model. The numbers of elements and nodes were 480 and 1529, respectively.

The inside radius of the typical thick-walled cylinder was 0.1 m, whereas its outside radius is 0.2 m. The internal flowing fluid pressure was 15.7 MPa, and there is not any applied pressure externally. The outer surface of the thick-walled cylinder was held at a temperature of 232.7 °C, the temperature of the fluid flowing through the inside wall was 325 °C, and the temperature from the radiation source which affects internally to the cylinder was 1573 K.

The finite element software that was utilized in order to validate the results obtained by the computer code was ANSYS. The coupled thermal-stress analysis was carried out using the coupled field method [7] in order to compute the coupled thermomechanical stresses of the thick-walled cylinder.

A sequentially coupled problem analysis is the combination of analyses from different engineering disciplines which interact to solve a global engineering problem [7]. Solution procedures and the solutions themselves that are associated with a specific engineering discipline will be referred to as a *field analysis*. When the input of one field analysis depends on the results from another analysis, the analyses are coupled. Thereby, every aloof physics environment should be built loose. Each built physics environment might be utilized in order to obtain a coupled field solution. But for the whole model, it is fundamental to generate a unique nodal arrangement. The model is kept constant by utilizing the physical environment from the first one with the next coupled environments. During the analysis, the geometry was generated in the thermal environment which contains the thermal parameters and boundary conditions.

Element types might change albeit the model should remain constant. For example, structural elements are mandatory in order to determine the stresses in the cylinder while for thermal analysis thermal elements are mandatory. But, it is material that merely some of the element combinations might be used for coupled physics analyses.

Analysis exacts to generate all mandatory environments. At first hand, essentially in preprocessing part environments are generated and written to memory.

Then the environments might be combined in order to disentangle coupled analysis in the solution part. PLANE77 (2-D 8-node Thermal Solid) element type was used for the thermal environment. This 8-noded thermal element is applicable to 2D, transient or steady-state thermal analyses. After creating the geometry depicted in Figure 6.1 and presenting thermal properties, the thermal environment was already fully described. The thermal environment might now be saved in order to be used at a later time. In defining structural environment, as the model of the problem has already been characterized in thermal environment, merely detailing structural variables is mandatory. Because the geometry comprises PLANE77 element type and it is also be analyzed structurally, this element type must be switched into its corresponding structural element type. For this particular problem, the new element type is PLANE183 and merely material properties were required to be modified. Geometry was remaining constant. The structural environment was then fully described with structural material properties. After saving the structural environment and solving the thermal part, the thermal solution had then been obtained. If the steady-state temperature on the cylinder is plotted, it will be seen it is approximately a uniform 312 ˚C at the inner radius, as expected. This data is saved in a thermal results file which has an extension of ".rth". These results will be used in determining the structural effects.

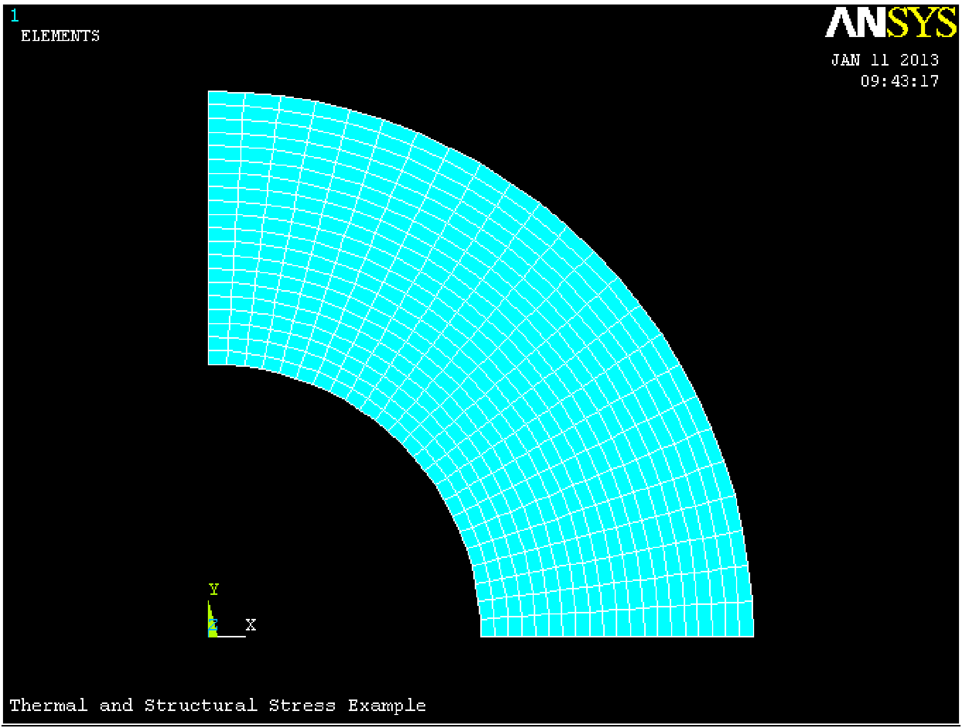

Figure 6.1. Finite element mesh for the cylinder consisting of 8-noded quadrilateral elements under thermal and pressure loads

When solving the structural part of the problem, the thermal effects were needed to be included. Including thermal effects coupled the data which prescribed in structural environment to the results obtained from the solution of thermal environment and used it during analysis. For this problem the reference temperature was fixed to 298 ^{0}K. The basic solution procedure for the coupled-field method was carried out as:

- The geometry was created and the thermal properties were defined for the thermal environment.
- The thermal environment which includes geometry and the thermal material properties is written to memory to be used at a later time.
- The physical properties were defined for the structural environment. The element types were switched from the thermal element PLANE77 to its corresponding mechanical element PLANE183. The element type was switched. The element material properties were defined.

- The structural environment was written to memory.

- In the solution phase, the thermal environment is read. The constraints were applied. The system was solved.

- The structural environment is read in. The structural constraints were applied. The thermal effects were included. The reference temperature was defined. The structural system was solved.

The temperature distribution, the radial displacement, the radial stress, and the circumferential stress distributions through the wall thickness that are obtained from the computer code, the finite element software ANSYS, and analytical formulations are shown in Figures 6.2., 6.3., 6.4. and 6.5., which proves the close agreement between different methods.

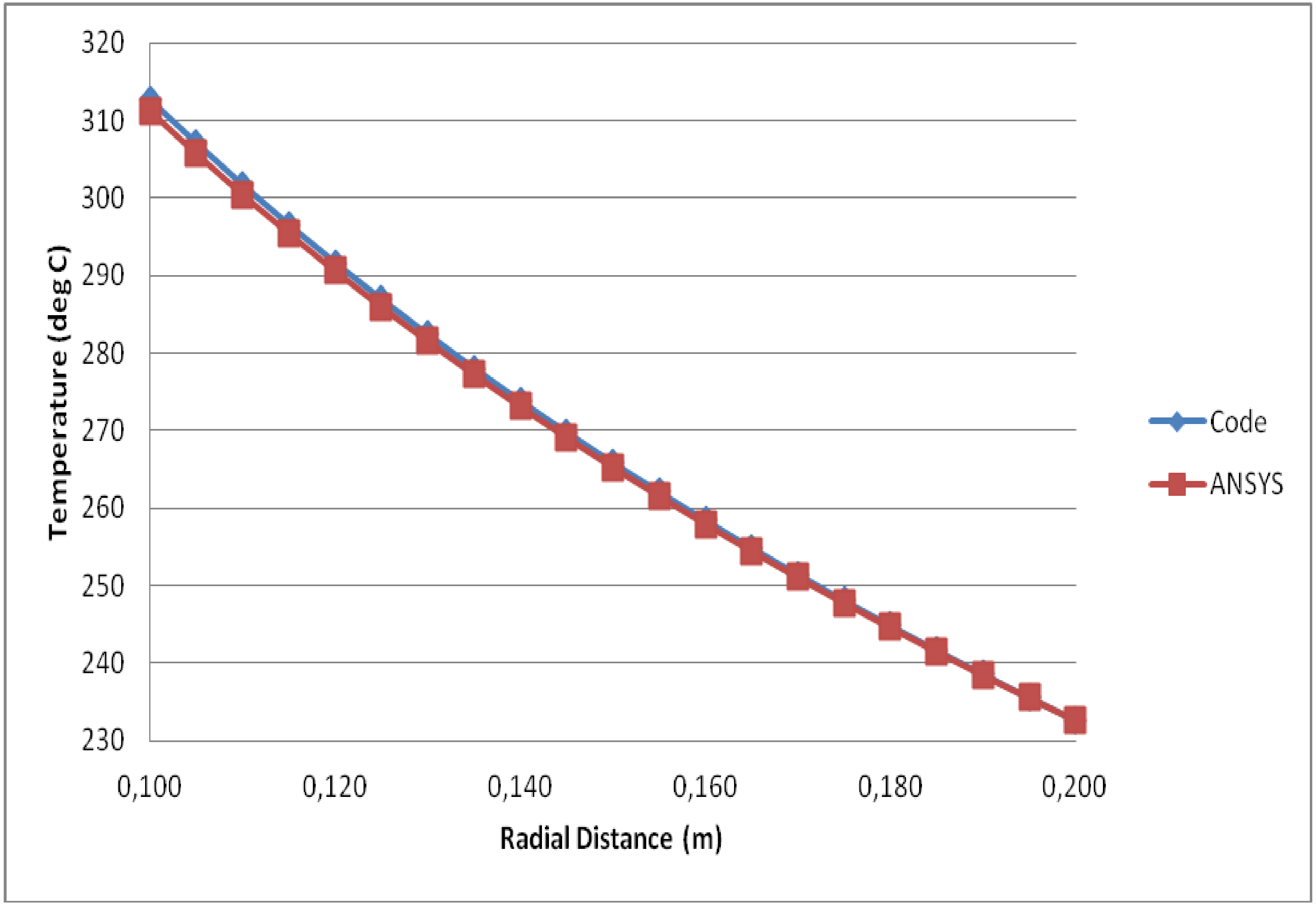

Figure 6.2. Temperature distributions through the radius of thick-walled cylinder

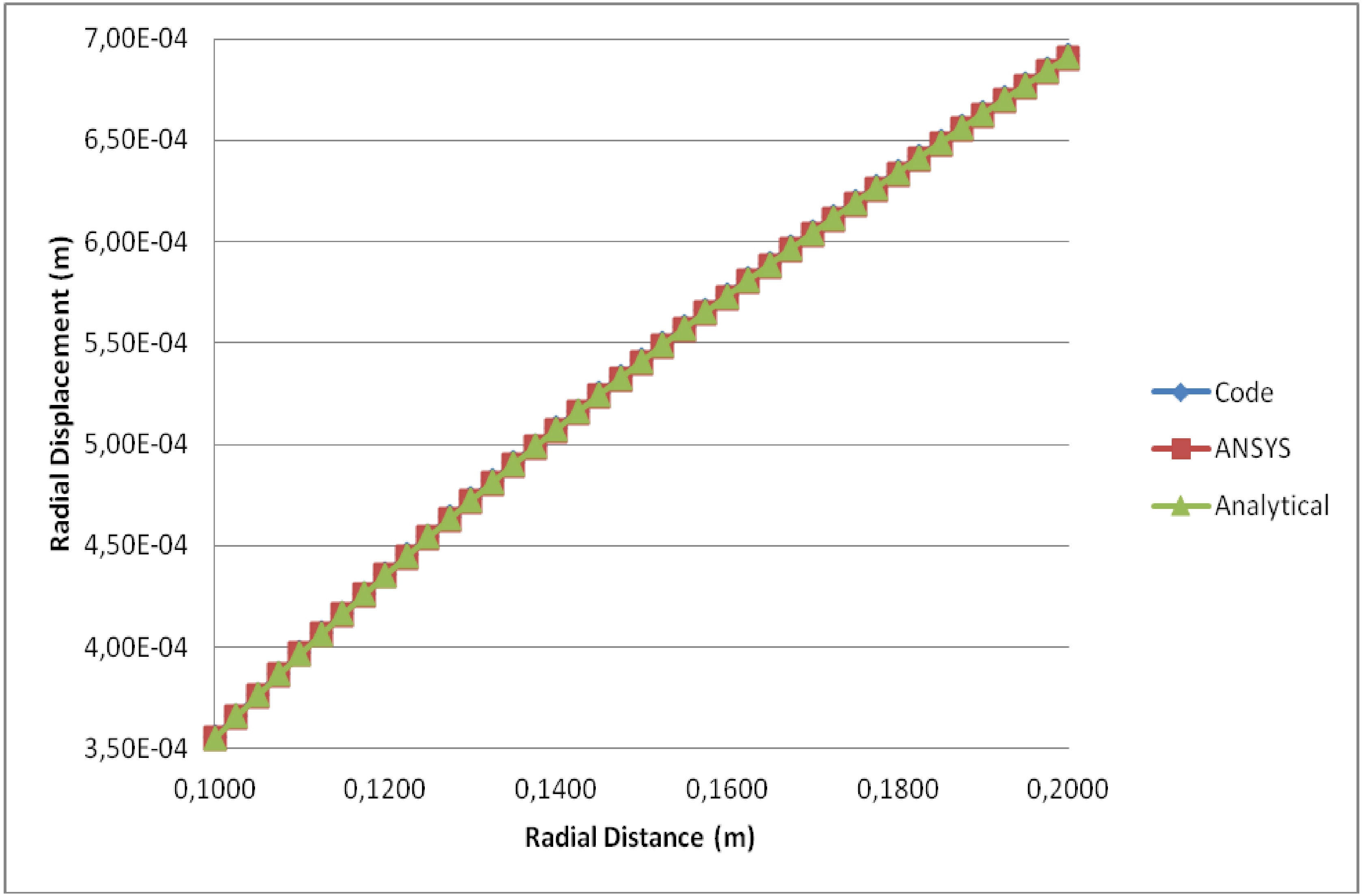

Figure 6.3. Variation of radial displacement of cylinder with radial distance in different methods.

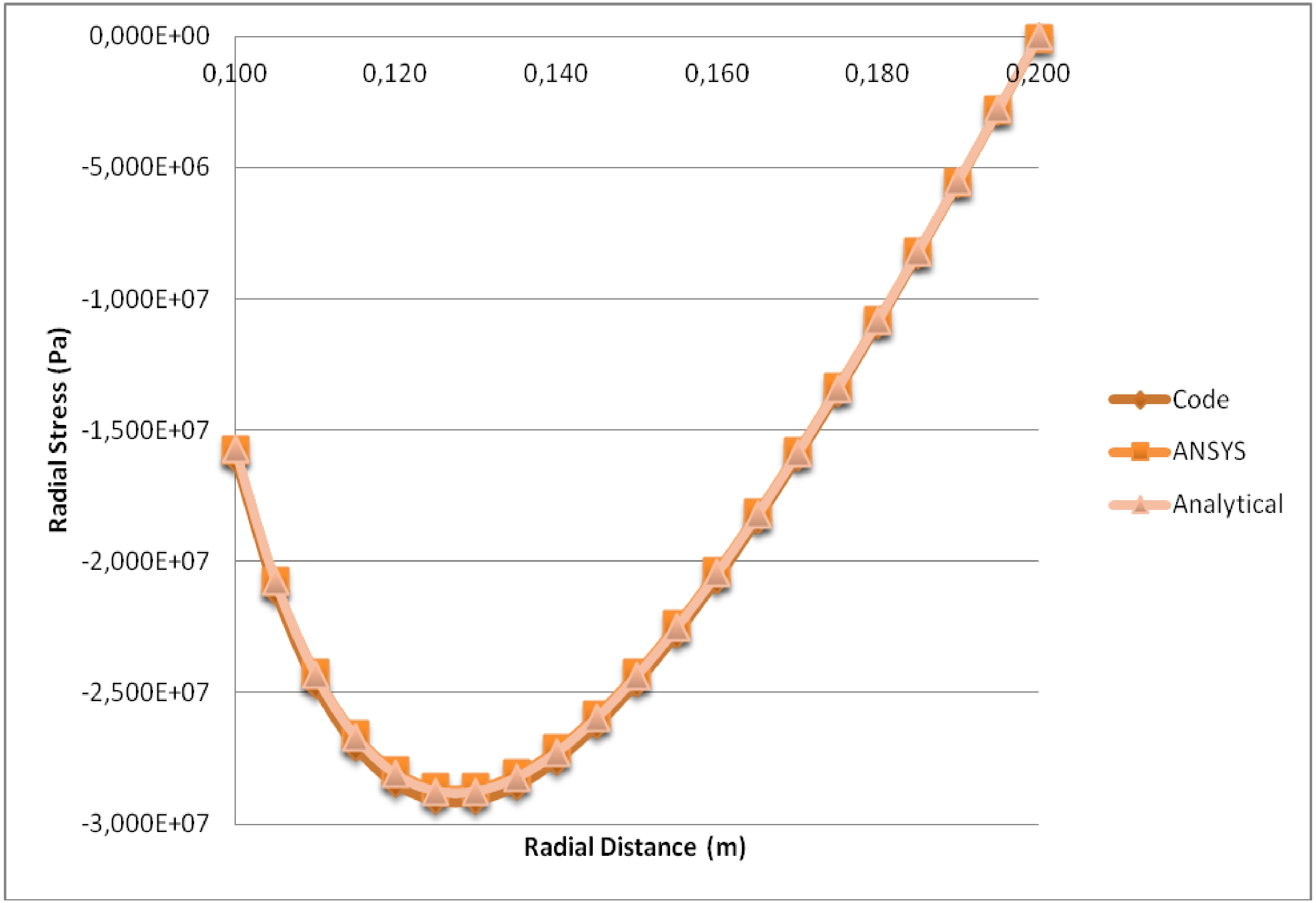

Figure 6.4. Steady-state radial stresses across the radius of a hollow cylinder subjected to pressure and heating on both internal and outer surface.

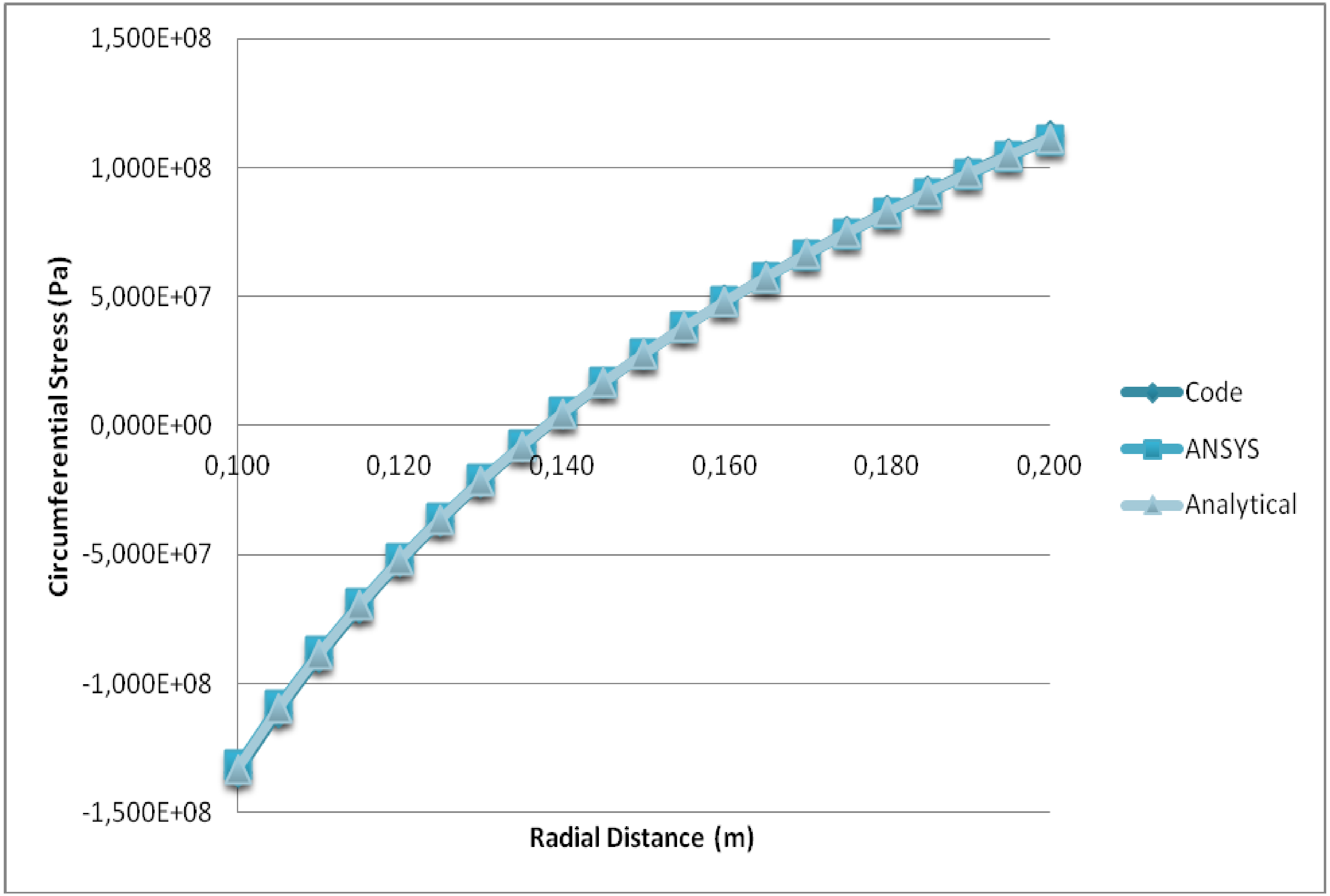

Figure 6.5. Steady-state hoop stress distributions across the radius of a hollow cylinder subjected to constant heating and pressure.

6.3. COUPLED THERMOELASTIC ANALYSIS OF THE NUCLEAR REACTOR VESSEL

The reactor coolant is transferred from the reactor pressure vessel to the steam generators by means of the primary piping system which consists of the *hot legs* or *outlet nozzles*. Each outlet nozzle connects reactor pressure vessel to the steam generator. The thermal loads in reactor vessel occur in normal operating and hot shutdown conditions. In this study, only thermal loads for normal operating conditions were examined. The reactor vessel can freely undergo some radial movement due to the pressure and thermal expansion. A reactor vessel overview including A-A which is the cross-section that the whole analysis is carried on within, is given in Figure 6.6.

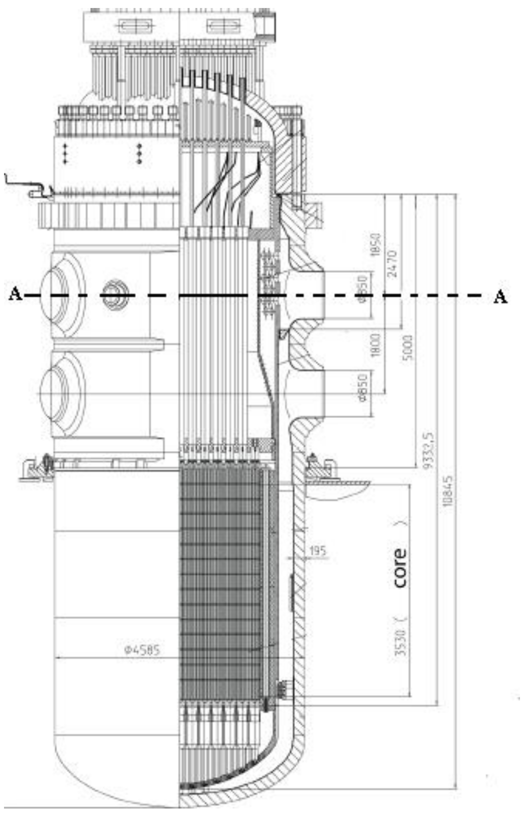

Figure 6.6. Reactor vessel [16].

A usual VVER nuclear reactor vessel was constructed to create FE geometry. Thermal and structural models were constructed separately with 8-noded quadrilateral elements with the element behavior option of plane strain in the computer code. Zero-displacement was applied to x=0 and y=0 lines of the finite element model. Node and element numbers used were 1185 and 336, respectively.

Inner radius of the usual VVER nuclear reactor vessel was 2.116 m and its outer radius is 2.3135 m, which leads to a wall thickness of 197.5 mm. External wall of the nuclear reactor vessel was held at 232.7 °C. This problem has the same convective boundary conditions as the thick-walled cylinder case which is in contact with a fluid at a temperature of 325 °C and having a heat convection coefficient of 550 $W/(m^2 - °C)$. Additionally, this problem has the same radiative boundary conditions as the previous problem which is exposed to radiation by a source with a temperature of 1573 ^{0}K. Internally flowing fluid pressure was 15.7 MPa. Outside of the reactor vessel was vacuum, thus there was not any pressure applied externally. Stephan-Boltzmann constant was taken as 5.67 $\times 10^{-8} W/(m^2 - K^4)$ and the emissivity was applied as 0.14.

Comparison of the results obtained from the computer code was carried out by means of finite element software package ANSYS. Coupled thermal-structural analysis was done by means of the coupled field method [7] of ANSYS. Hence, coupled thermomechanical stresses and displacements were determined for the nuclear reactor vessel.

Since element type of PLANE77 is suitable for two-dimensional, transient or steady-state thermal analyses, this element type was used during creating the thermal environment. After creating the geometry depicted in Figure 6.7 and describing the thermal material properties, thermal environment of our problem was ready to be saved. According to the thermal results file, the steady-state temperature on the inner surface was anticipated approximately as 355 ^{0}C.

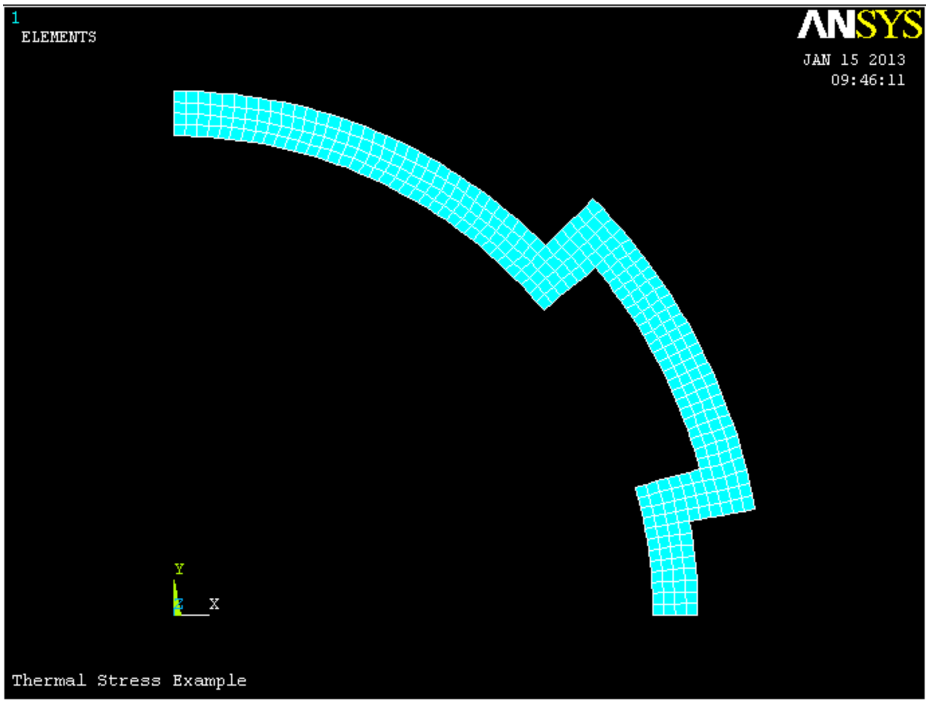

Figure 6.7. Finite element mesh for the nozzle region cross-section consisting of 8-noded quadrilateral elements

Thermal effects should be included during the solution of the structural problem. Including thermal effects were necessary in order to couple data of the structural environment to the thermal environment results. Reference temperature for the particular analysis was set to 298 ^{0}K.

The temperature distributions through the wall thickness that are obtained from the computer code and finite element software ANSYS are shown in Figures 6.8. and 6.9., which proves the close agreement between different methods.

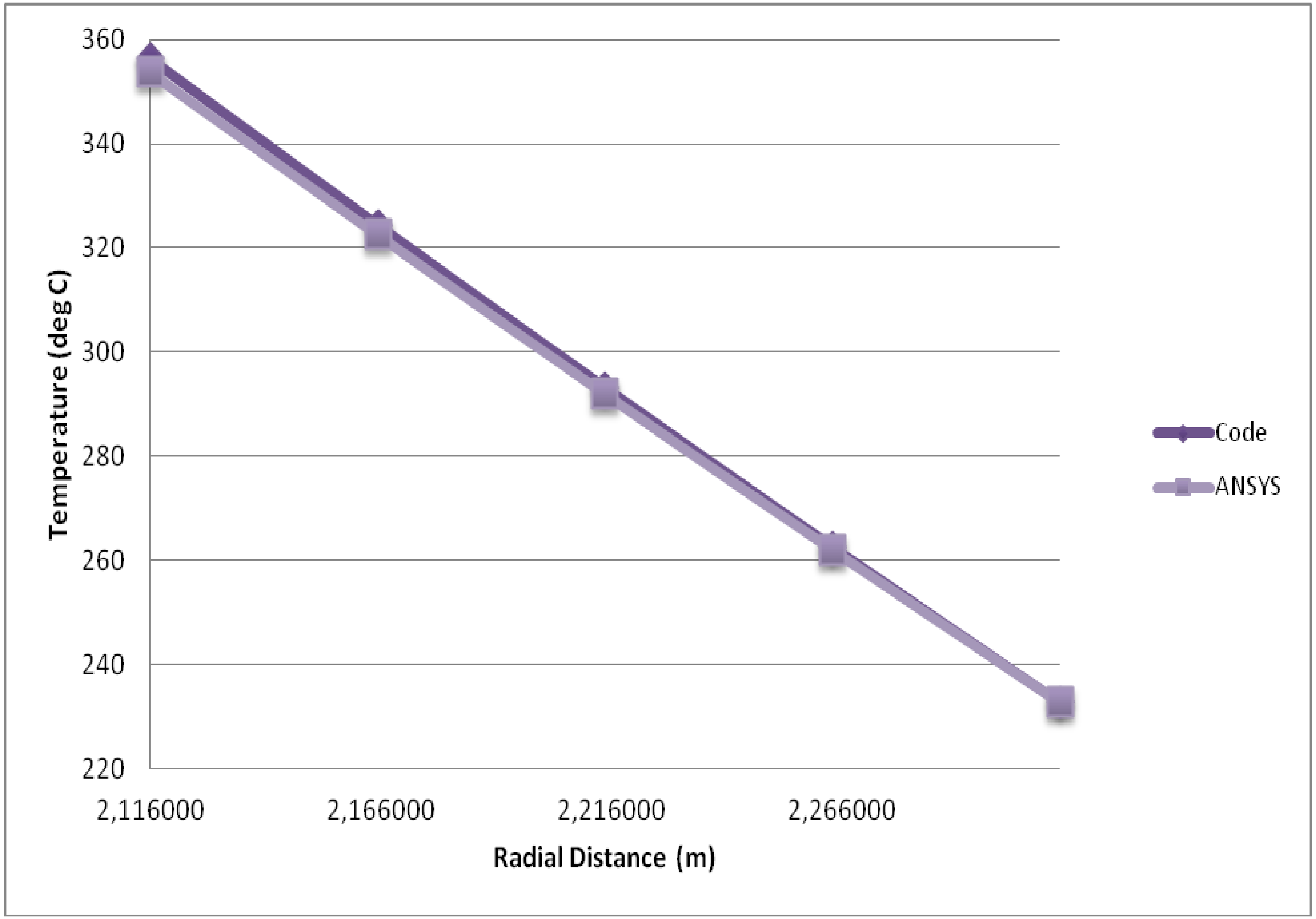

Figure 6.8. Comparison of temperature distributions through y=0 surface of the model.

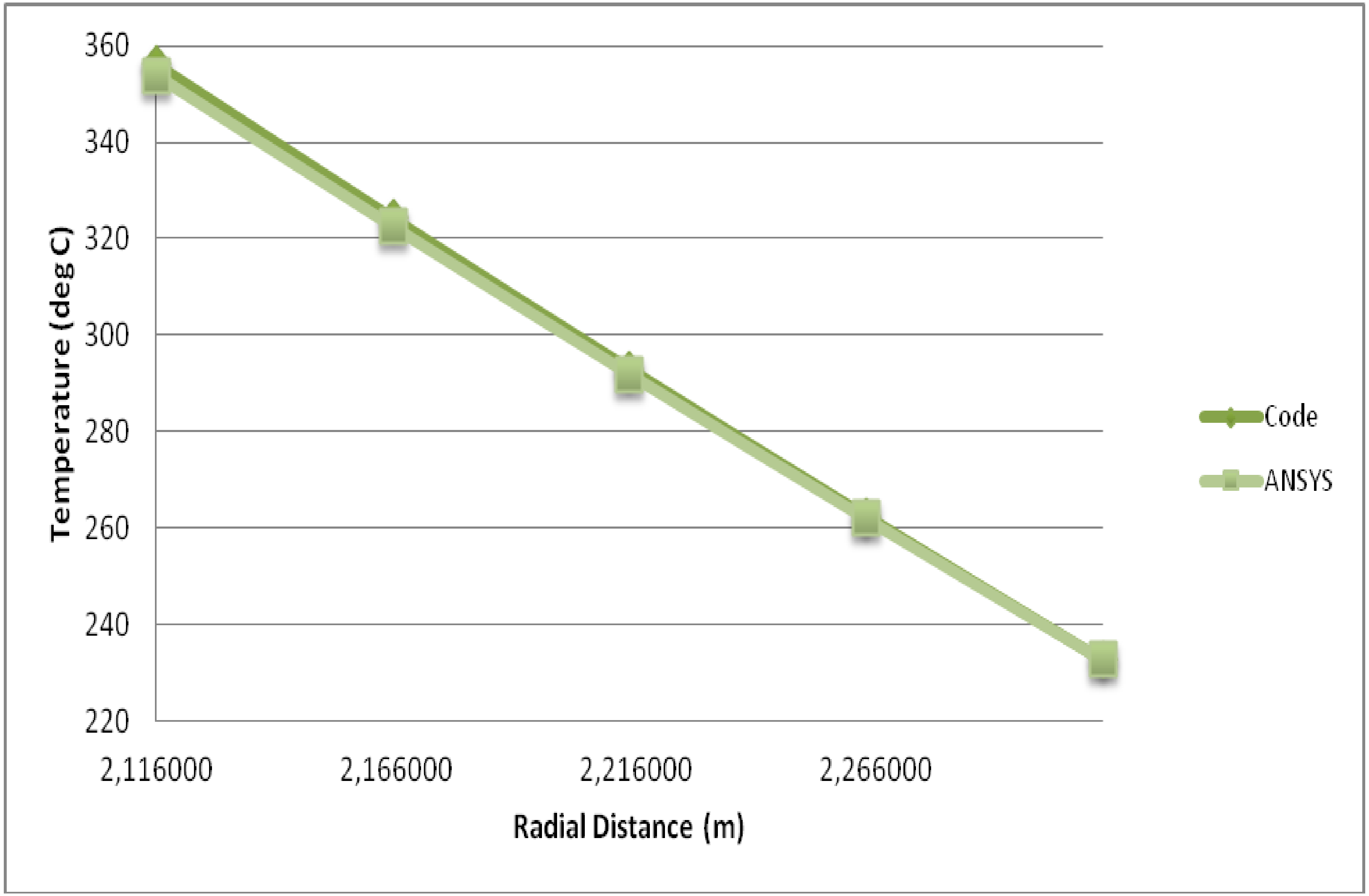

Figure 6.9. Temperature distributions through x=0 surface of the model for plane strain.

Radial distributions of displacement for y=0 and x=0 surfaces of the finite element model are depicted in Figures 6.10. and 6.11., respectively, and the radial stress distributions for y=0 and x=0 surfaces of the finite element model are depicted in Figures 6.12. and 6.13., respectively, and the hoop distributions of stress for y=0 and x=0 surfaces of the finite element model are depicted in Figures 6.14. and 6.15., respectively, and it is obvious that the results obtained from different methods are in close agreement with each other.

The results for the radial displacement, the radial stress, and the circumferential stress at the inner surface of the nozzle, that is the surface perpendicular to the flow, are depicted in Figures 6.16., 6.17., and 6.18., respectively.

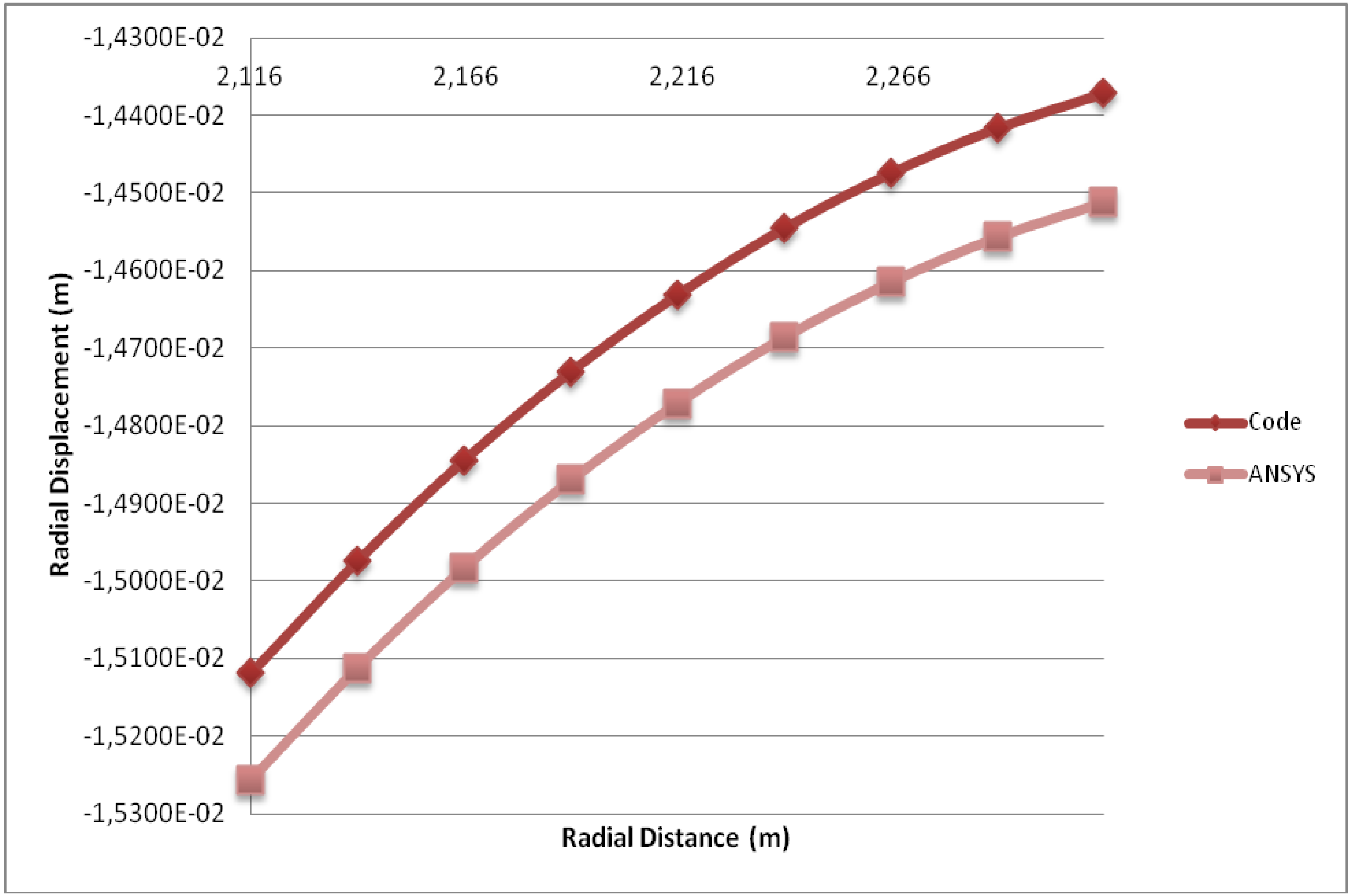

Figure 6.10. Radial displacement distributions through the thickness of the model at y=0 cross-section.

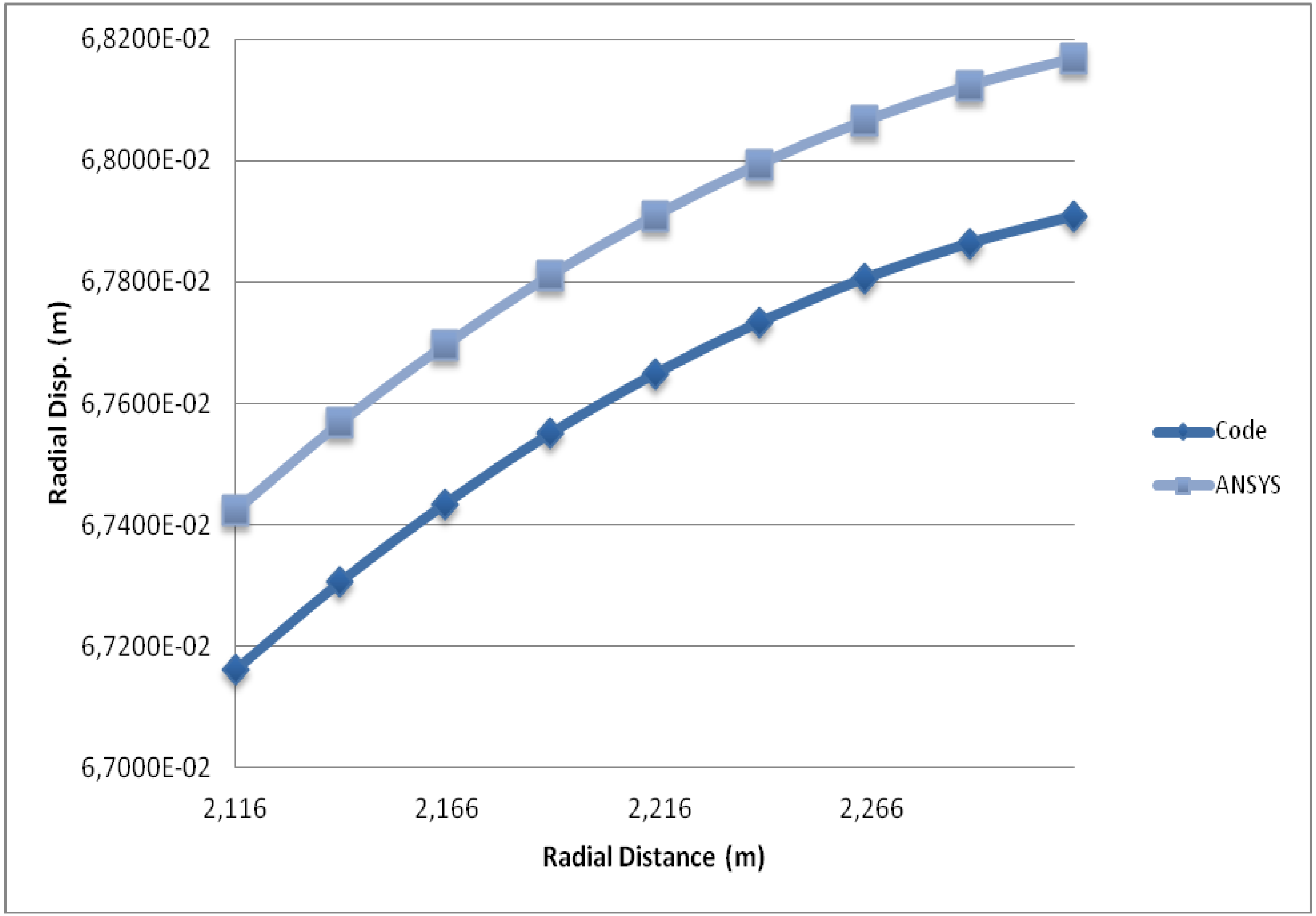

Figure 6.11. Radial displacement distributions at x=0 surface of the model by ANSYS and our computational code.

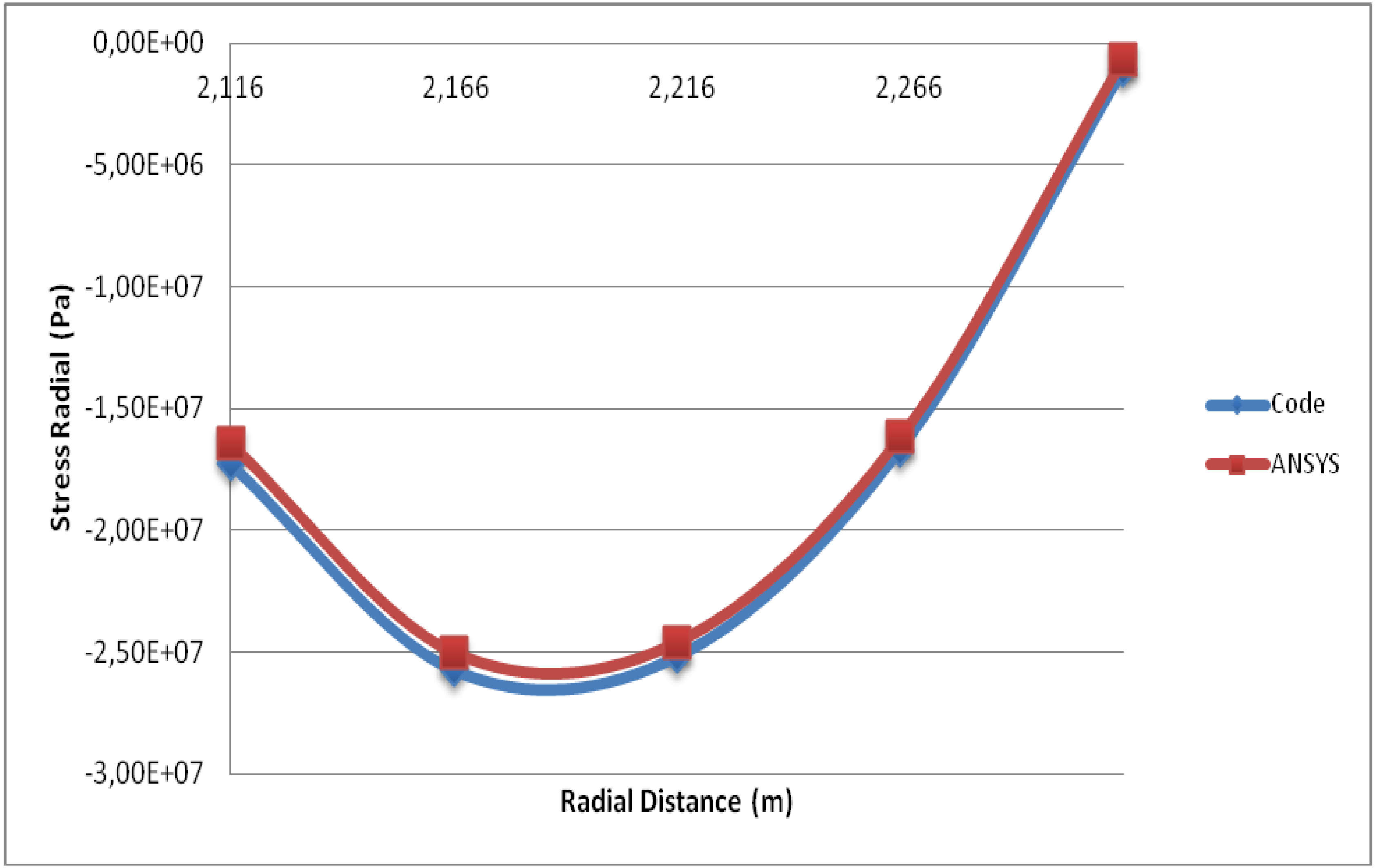

Figure 6.12. Radial stresses through the radial distance by ANSYS and our code at y=0 cross-section.

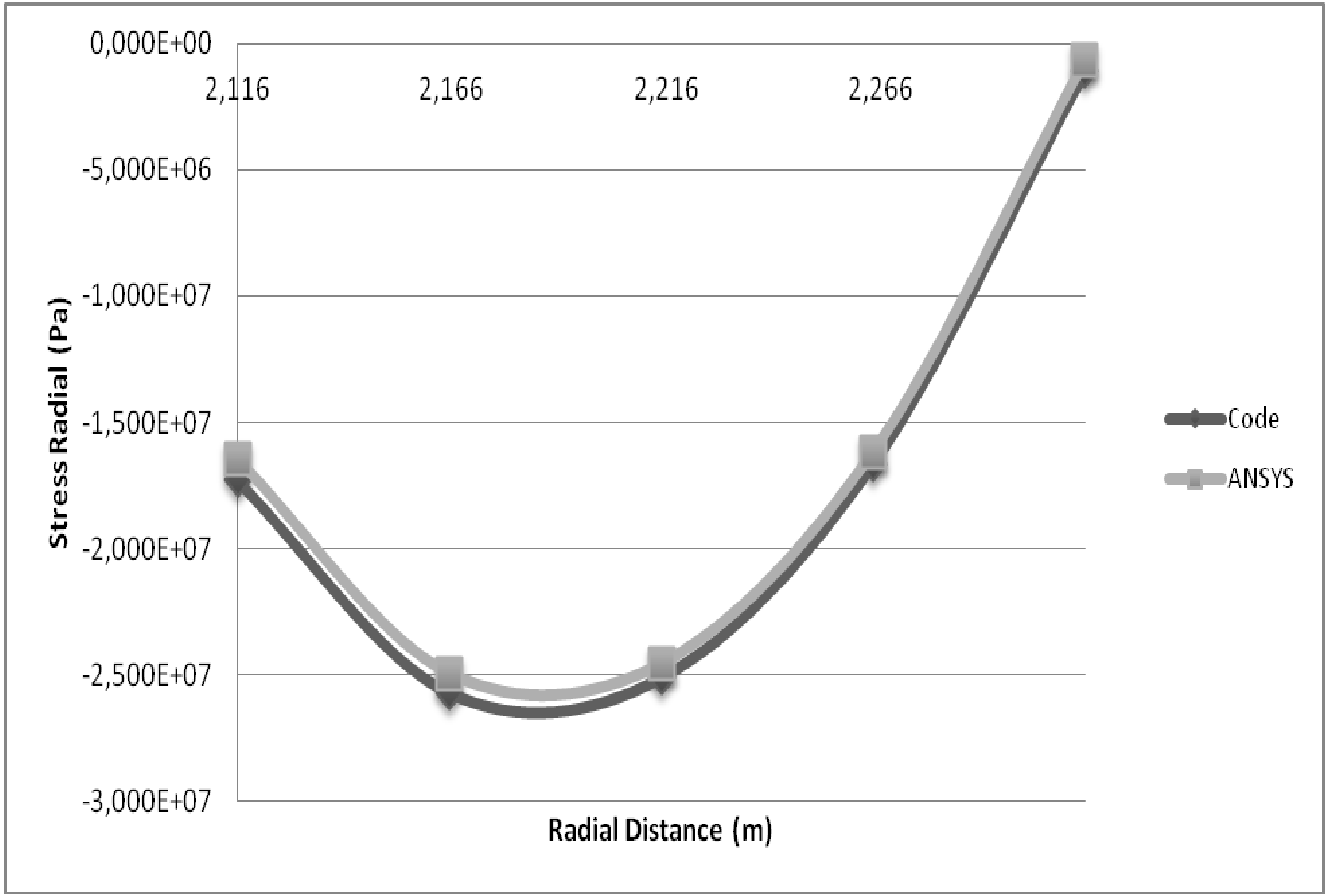

Figure 6.13. Comparison of radial stresses through the radial distance by ANSYS and our code at x=0 cross-section.

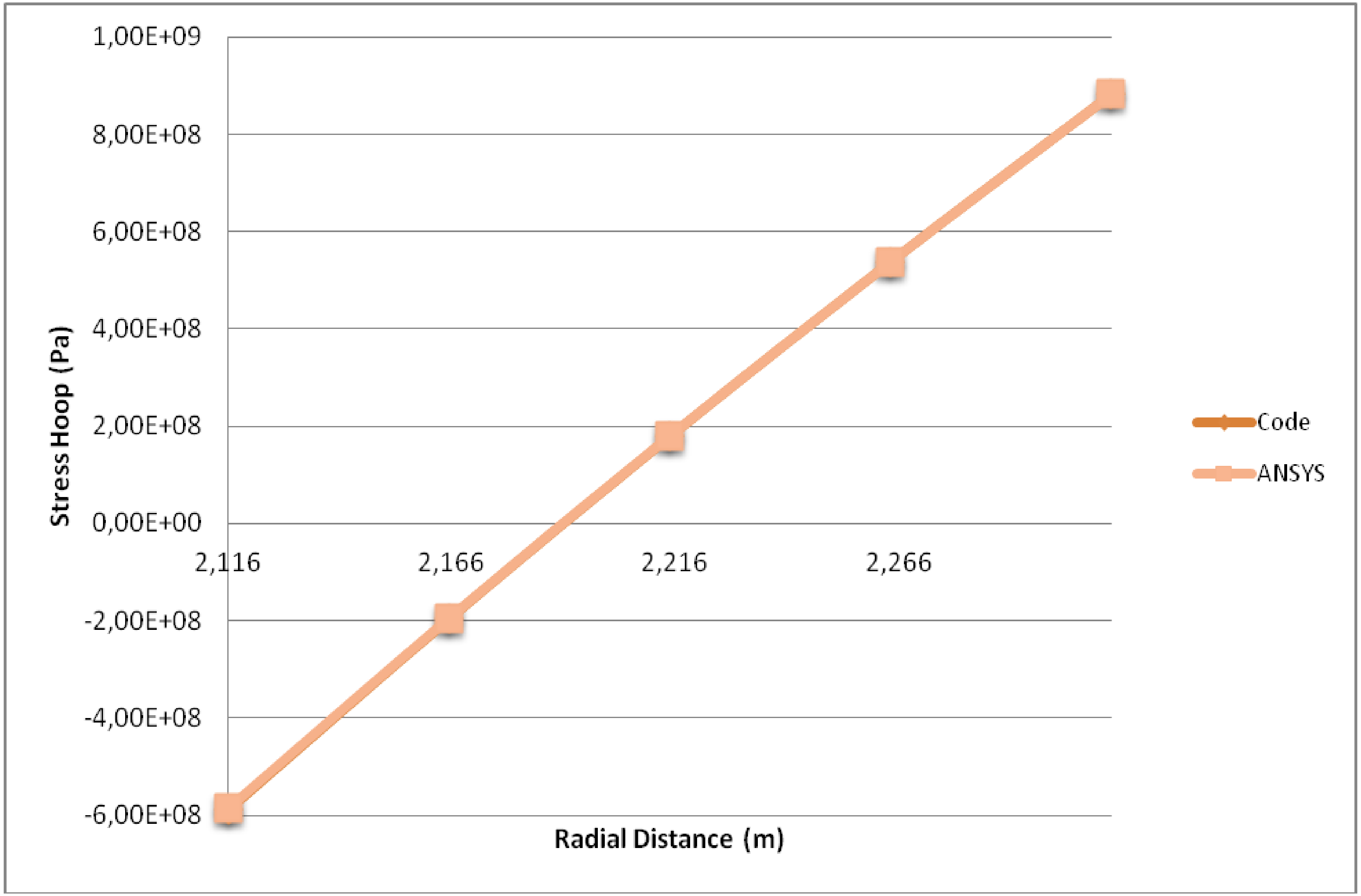

Figure 6.14. Comparison of hoop stresses through the radial distance by ANSYS and our code at y=0 cross-section.

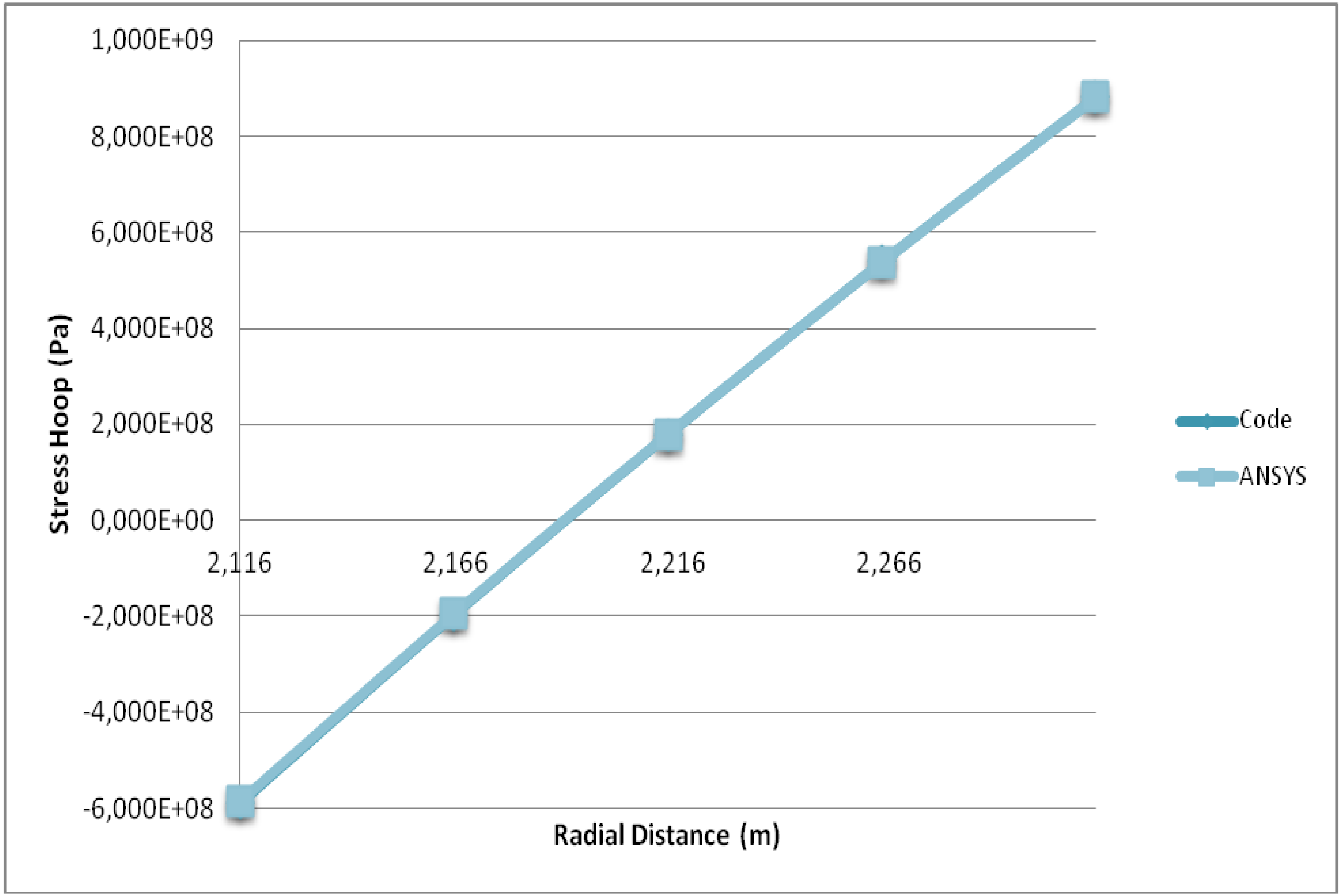

Figure 6.15. Hoop stresses through x=0 edge of the plane strain model.

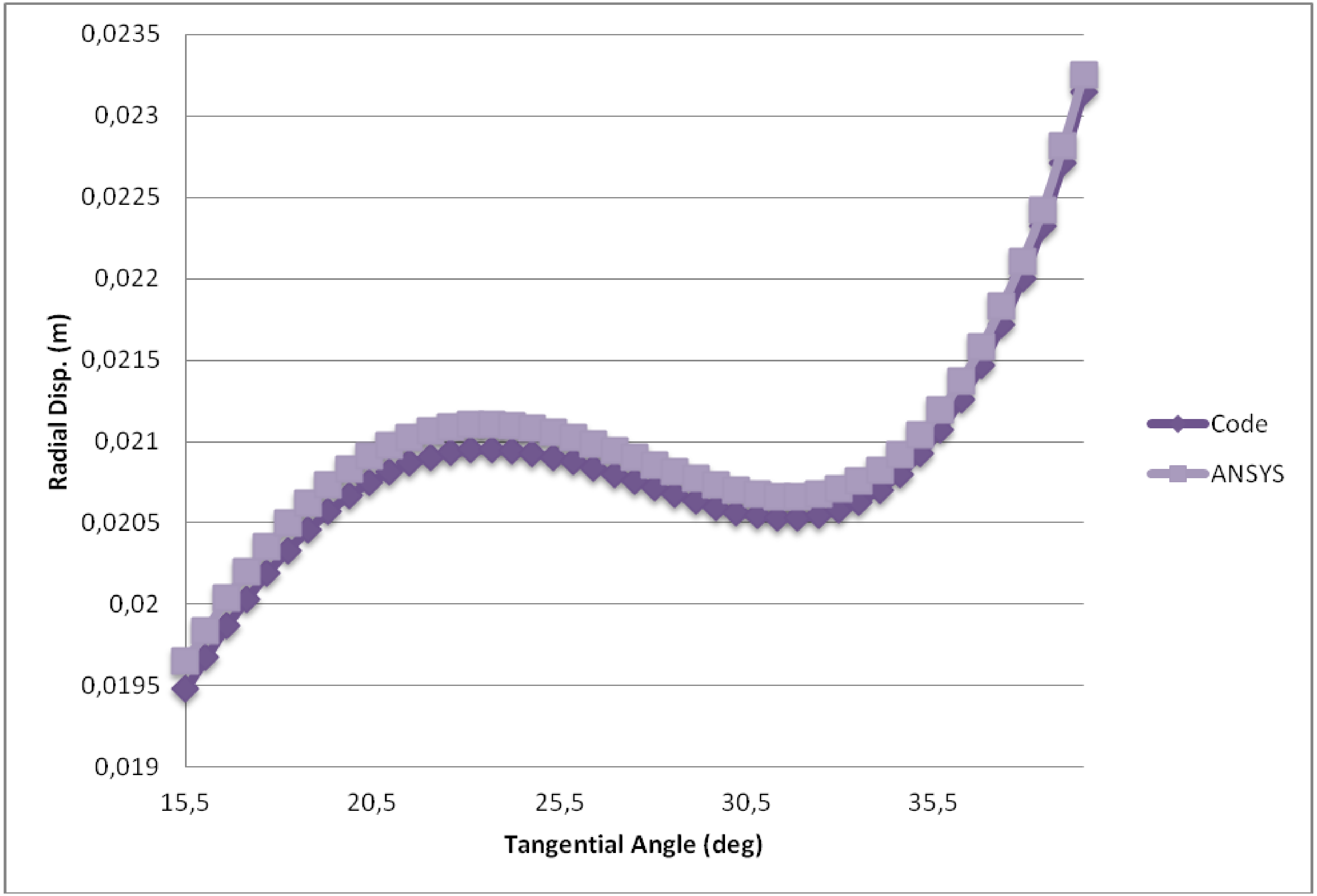

Figure 6.16. Radial displacements through the inlet nozzle region of the plane strain model.

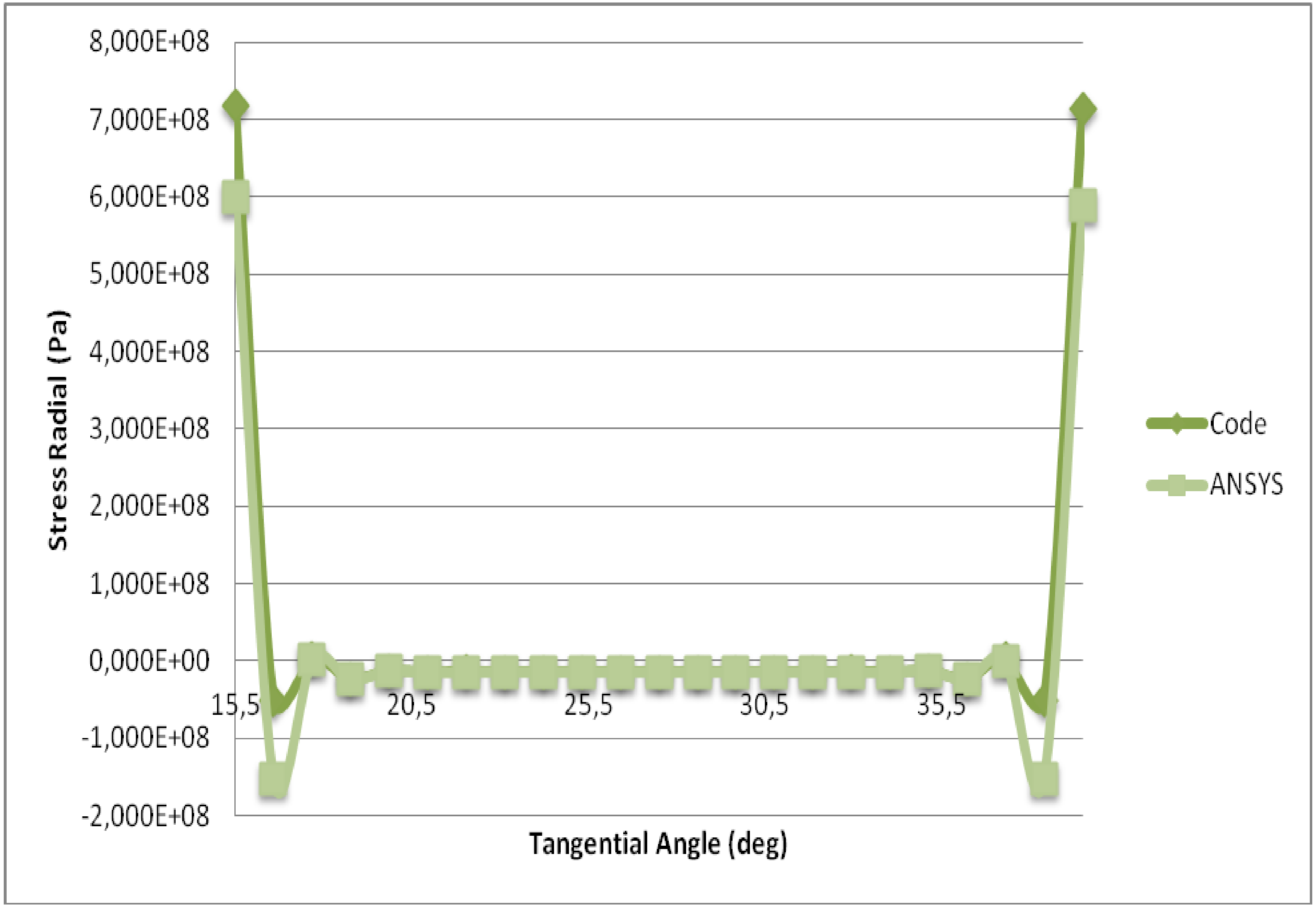

Figure 6.17. Radial stress distributions through the nozzle flow cross-section of the model by ANSYS and code.

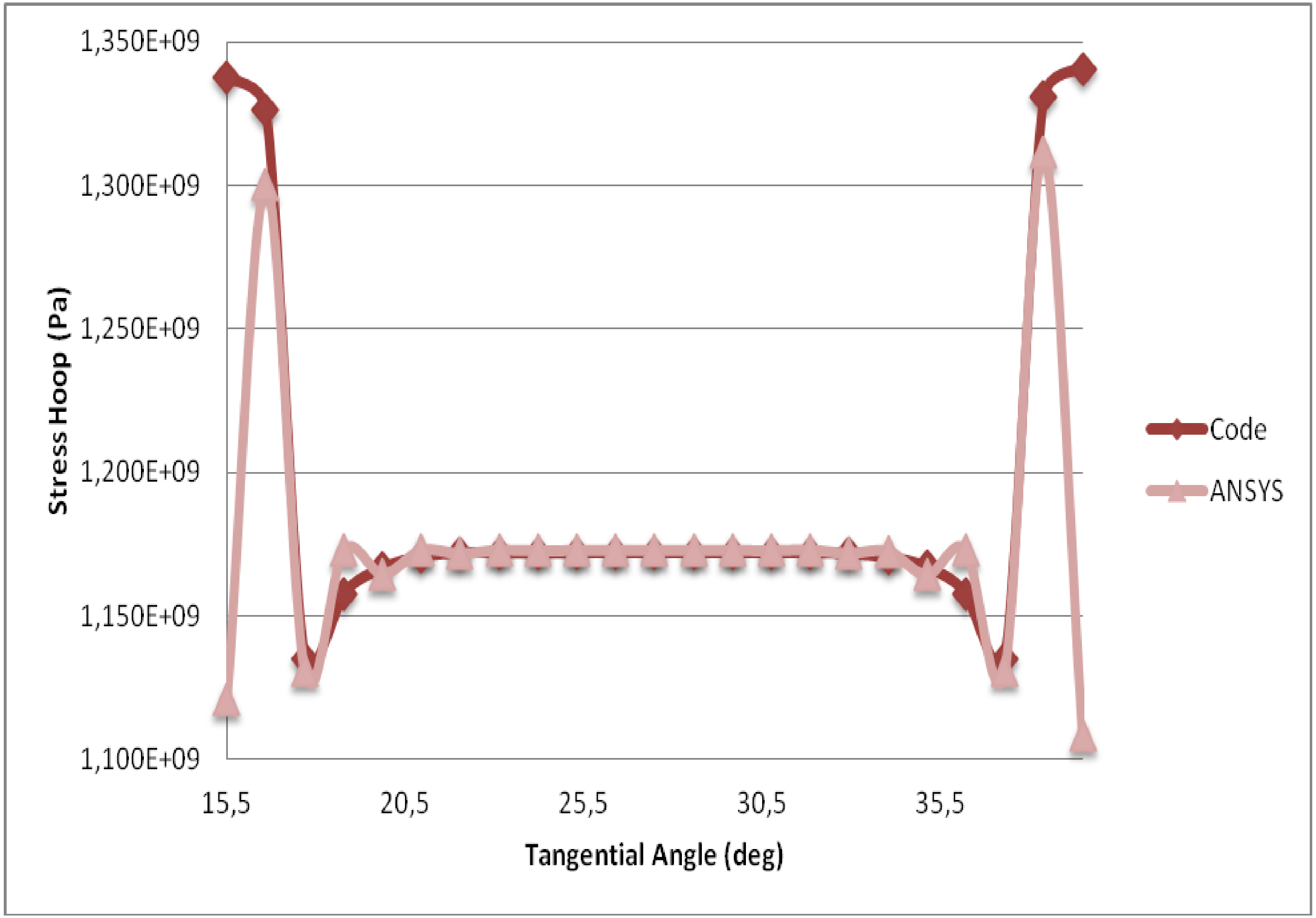

Figure 6.18. Hoop stresses through the nozzle flow cross-section of the plane strain model by ANSYS and our code.

6.4. THERMODYNAMIC POTENTIALS ANALYSIS OF THE NUCLEAR REACTOR VESSEL

This work demonstrates how four thermodynamic potentials; the internal energy, the Helmholtz free energy, the Gibbs energy and the enthalpy are calculated separately for a horizontal cross-section that contains four outlet nozzles of a nuclear reactor pressure vessel system when thermomechanical loads are applied. The finite element model showing a horizontal cross-section of the reactor vessel is generated by both our thermoelasticity code and finite element software ANSYS. Thermal and mechanical loads are applied to the overall system under plane strain condition. The transition to thermodynamic potentials is done by introducing the stress, strain, and temperature results from the thermoelasticity code into four thermodynamic potentials expressions. The thermodynamic analysis is presented based on the use of all thermodynamic potentials (internal energy, Helmholtz free energy, Gibbs free energy, and enthalpy) in terms of their natural independent variables; stress, strain, temperature, and entropy. The four energy functions commonly used in thermodynamics are used to provide results for thermodynamic analysis.

The expressions for all considered thermodynamic potentials are well-known from linear thermoelasticity. This section builds on previous work by Lubarda (2004) and provides specificity by including a simple case with both thermal and mechanical effects. The motivation for this study comes principally from the development of thermomechanical model for reactor vessel cross-section subjected to temperature and pressure loads. We believe that this framework is sufficiently general that realistic models of nuclear reactor vessels can be developed within it. The main objective is that once four independent state variables have been specified, the thermodynamic potentials distributions for the entire model can be obtained. The presented analysis is an extension of the classical thermodynamics analysis by including stress and strain tensors besides temperature and entropy. This was the first time when all four fundamental thermodynamic potentials have been evaluated for a reactor vessel system.

A thermomechanical finite element modeling of the cross-section of the nozzle region of a reactor pressure vessel was built in order to certify the solutions from our computational code. Due to the axisymmetry characteristic of the nozzle region cross-section and its boundary conditions, a one-fourth of the geometrical model as depicted in Figure 6.7 was built with the 8-noded thermal elements PLANE77 with the element behavior of plane strain and the 8-noded plane elements PLANE183 with the element behavior of plane strain by means of the finite element software ANSYS like in the first case. Again, zero-displacement in x-direction is applied for the x=0 line of the geometrical model and zero-displacement in y-direction is applied for the y=0 line of the geometrical model. The numbers of elements and nodes were 336 and 1185, respectively.

Inner radius of the nozzle region cross-section was 2.116 m, whereas its outside radius is 2.3135 m. Structural and thermal boundary conditions were applied in order to generate a realistic model. The internal flowing fluid pressure was 15.7 MPa, and there is not any applied pressure externally. The outer temperature was 232.7 °C, the inner fluid temperature was 325 °C, and the temperature from the radiation source which is subjected internally to the cylinder was 1573 K.

Some of the parameters of reactor pressure vessel are as follows:

- Inner diameter of the cylindrical shell is 4232 mm [16].
- Wall thickness of cylindrical shell is 197.5 mm [16].
- Operating pressure is 15.7 MPa.
- Operating temperature is 325 °C.
- Total inside height is 3855 mm [16].

The thermomechanical parameters of each case are listed in Table 1. Some thermomechanical properties for the case of reactor vessel were adopted from reference [1].

Table.1. Thermomechanical properties

	E (Pa)	ν	$\rho(kg.m^{-3})$	$k(W/m°C)$	$\alpha(1/K)$
Reactor pressure vessel	2.0577275×10^{11}	0.3	7850	48	12×10^{-6}

E is the Young's modulus, ν is the Poisson's ratio, ρ is the density, k is the material conductivity, and α is the coefficient of thermal expansion.

Once entropy is considered, the system must accomplish the equilibrium between maximizing the entropy while minimizing the Helmholtz free energy as far as possible. The Helmholtz free energy will be a minimum if the system reaches the equilibrium. It may be easily seen in Figures 6.19 and 6.21. Figure 6.20 shows the internal energy as a function of the thickness of the reactor vessel cross-section under prescribed boundary conditions. As stated before in Section 5.2.3, the Gibbs energy and the Helmholtz free energy are in a simple relationship by Legendre transform. Figure 6.22 depicts the Gibbs energy as a function of the radial distance of the reactor vessel cross-section under specified thermal and pressure boundary conditions. Figure 6.23 depicts the enthalpy function distribution with respect to the radial distance of the pressure vessel. Also, the inner surface of the nozzle, which is perpendicular to assumed direction of fluid, is investigated and results are shown in from Figure 6.24 to Figure 6.28 for entropy, internal energy, Helmholtz free energy, Gibbs energy, and enthalpy, respectively.

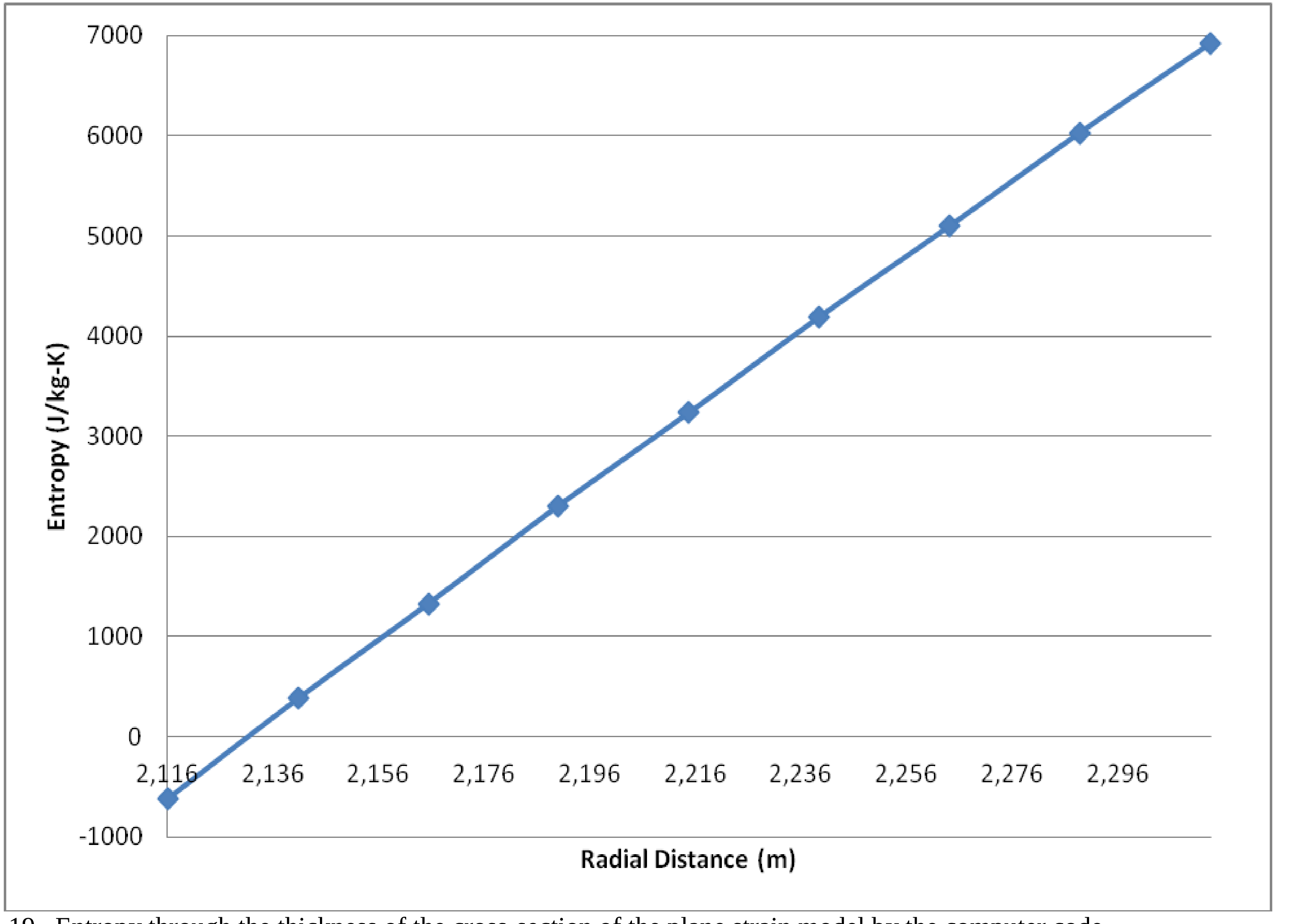

Figure 6.19. Entropy through the thickness of the cross-section of the plane strain model by the computer code.

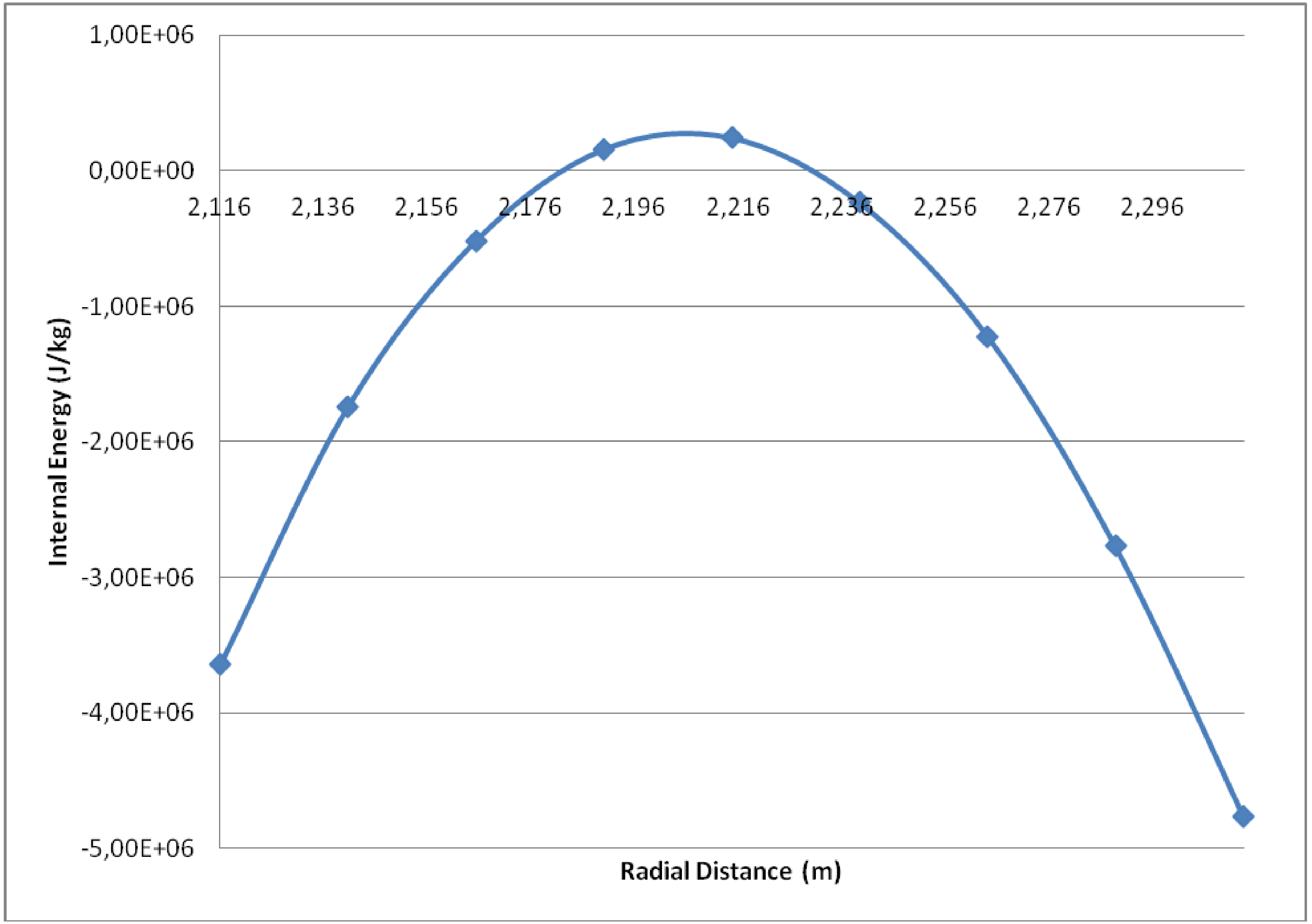

Figure 6.20. Internal energy through the thickness of the cross-section of the plane strain model by the computer code.

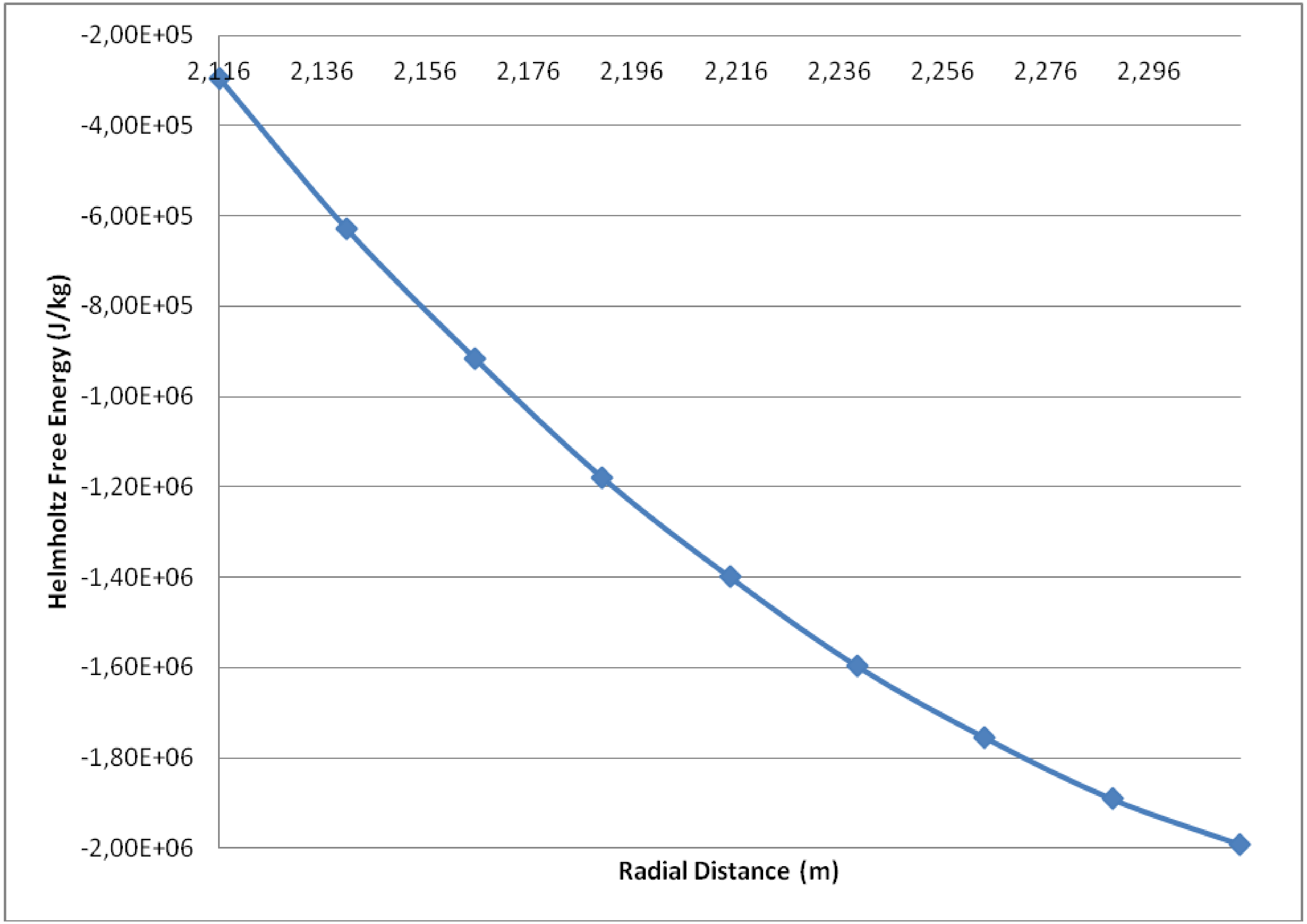

Figure 6.21. Helmholtz free energy through the thickness of the cross-section of the plane strain model by the computer code.

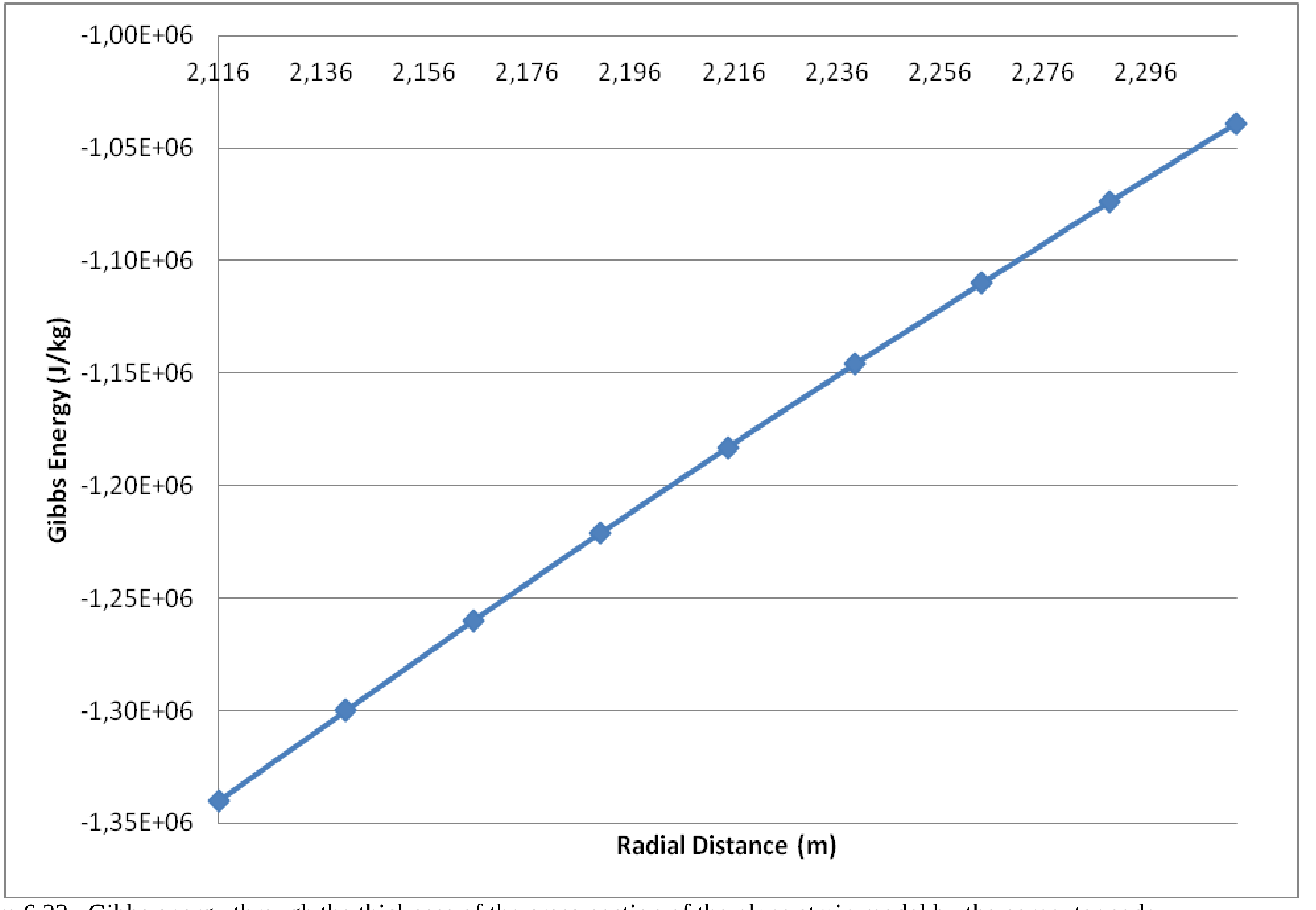

Figure 6.22. Gibbs energy through the thickness of the cross-section of the plane strain model by the computer code.

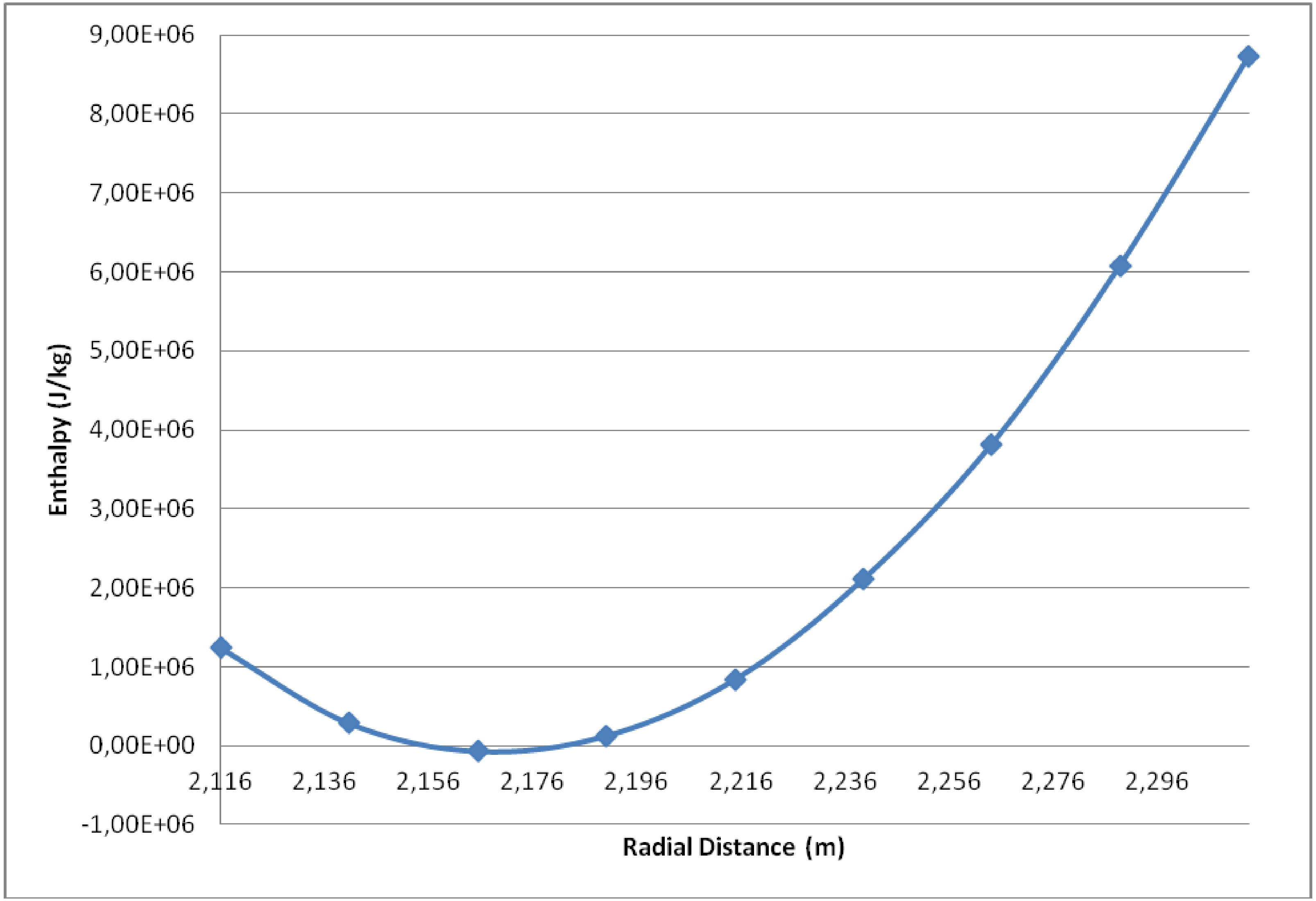

Figure 6.23. Enthalpy through the thickness of the cross-section of the plane strain model by the computer code.

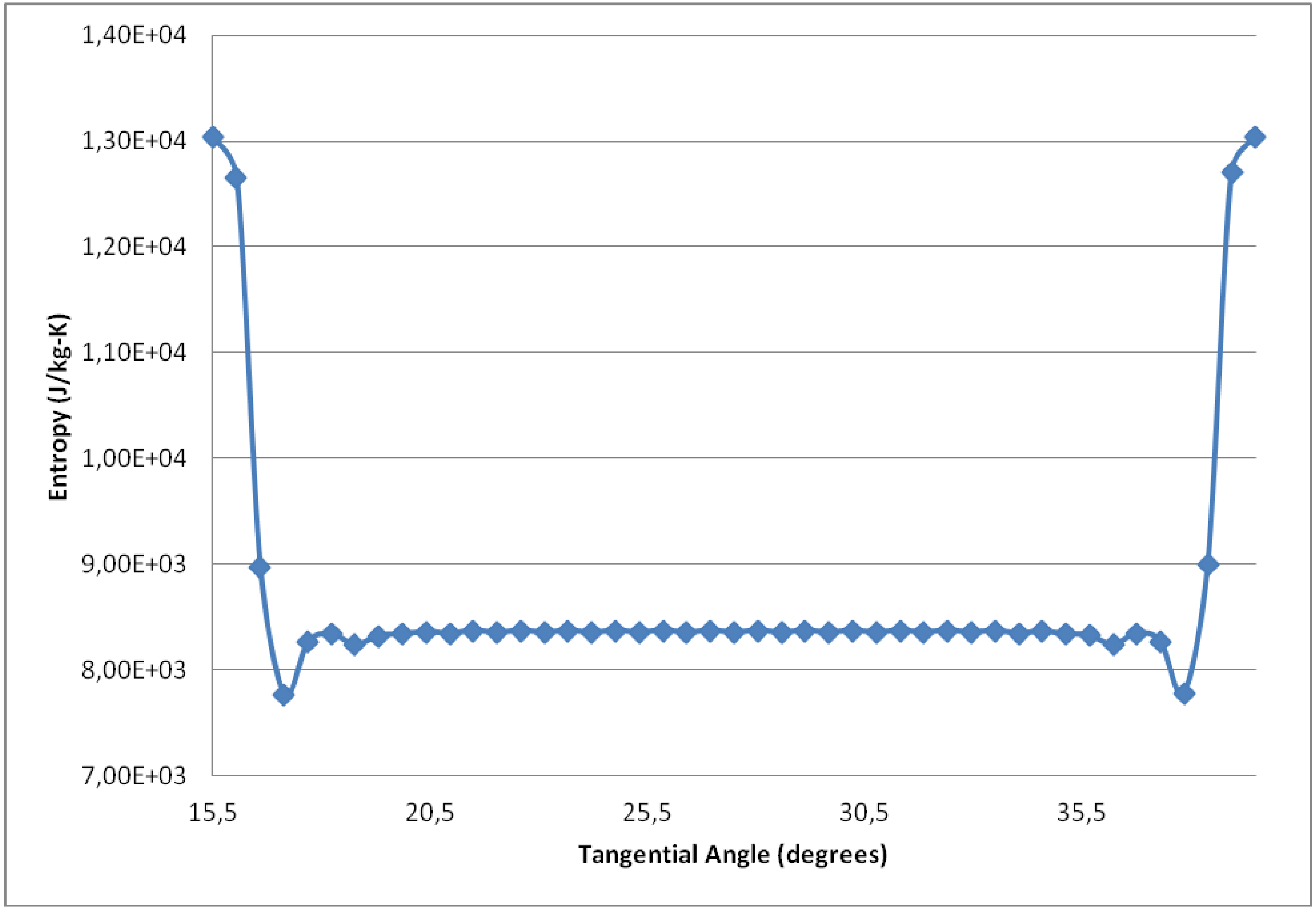

Figure 6.24. Entropy through the tangential angle of the nozzle for the plane strain model by the computer code.

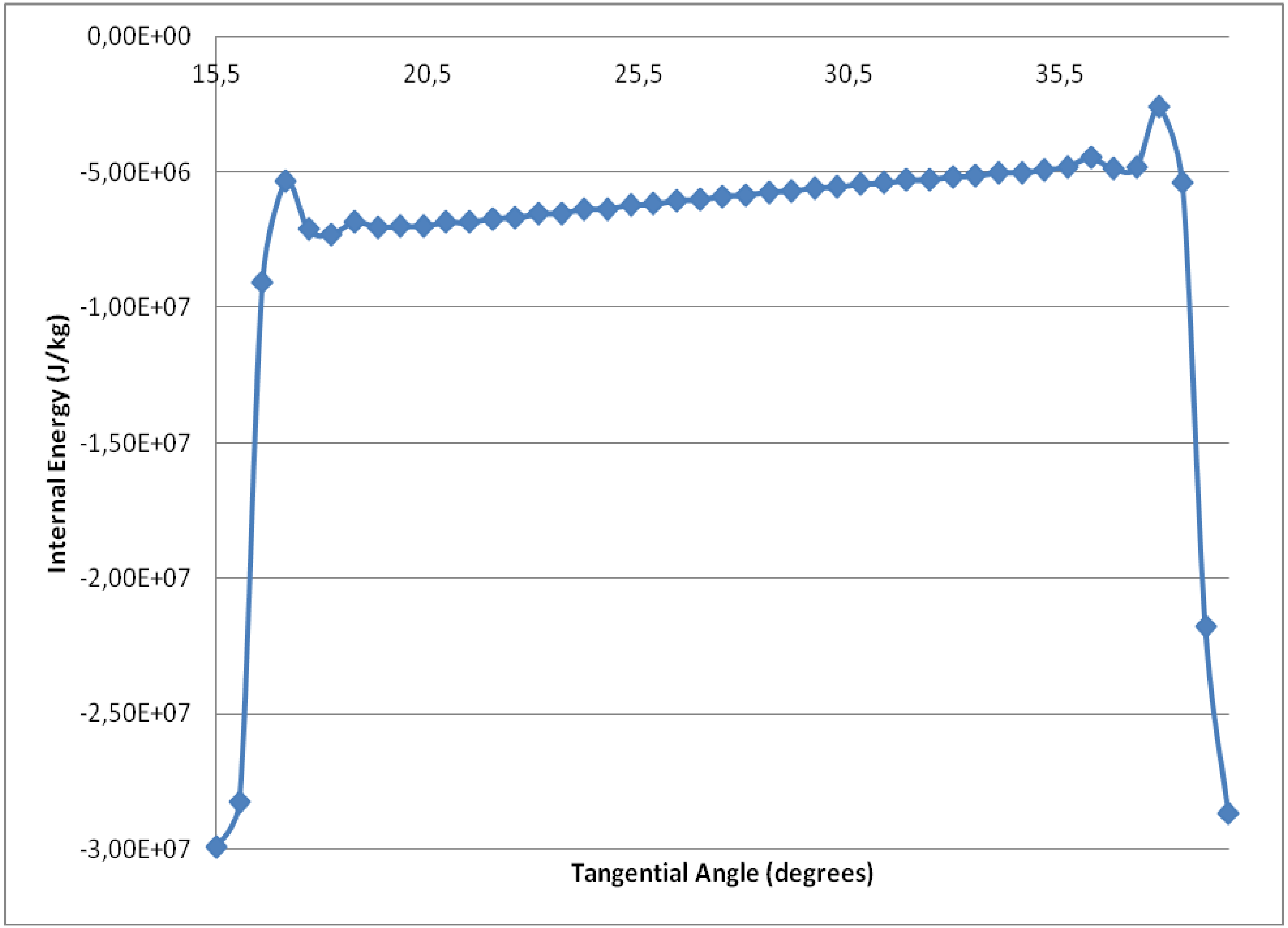

Figure 6.25. Internal energy through the tangential angle of the nozzle for the plane strain model by the computer code.

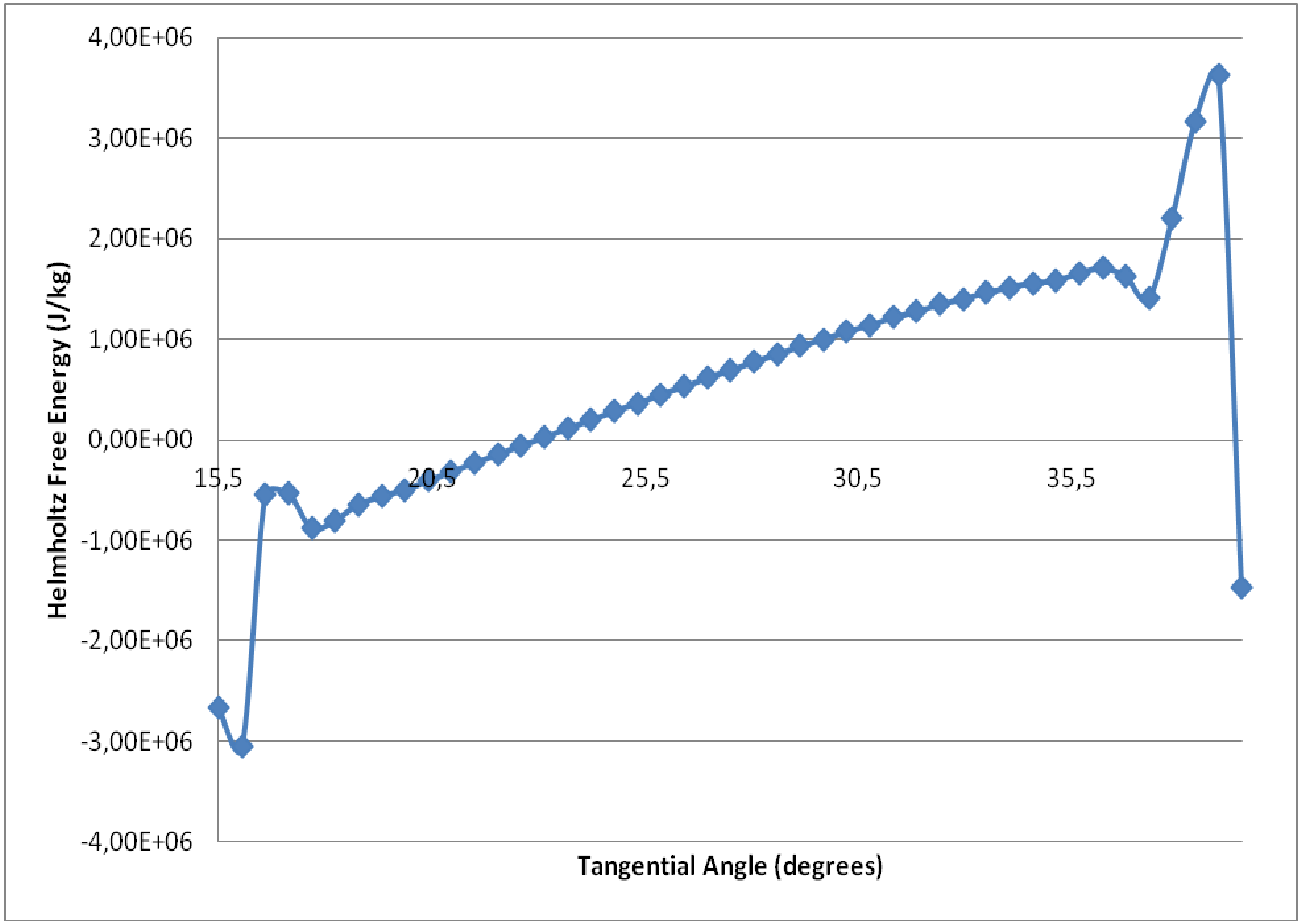

Figure 6.26. Helmholtz free energy through the tangential angle of the nozzle for the plane strain model by the computer code.

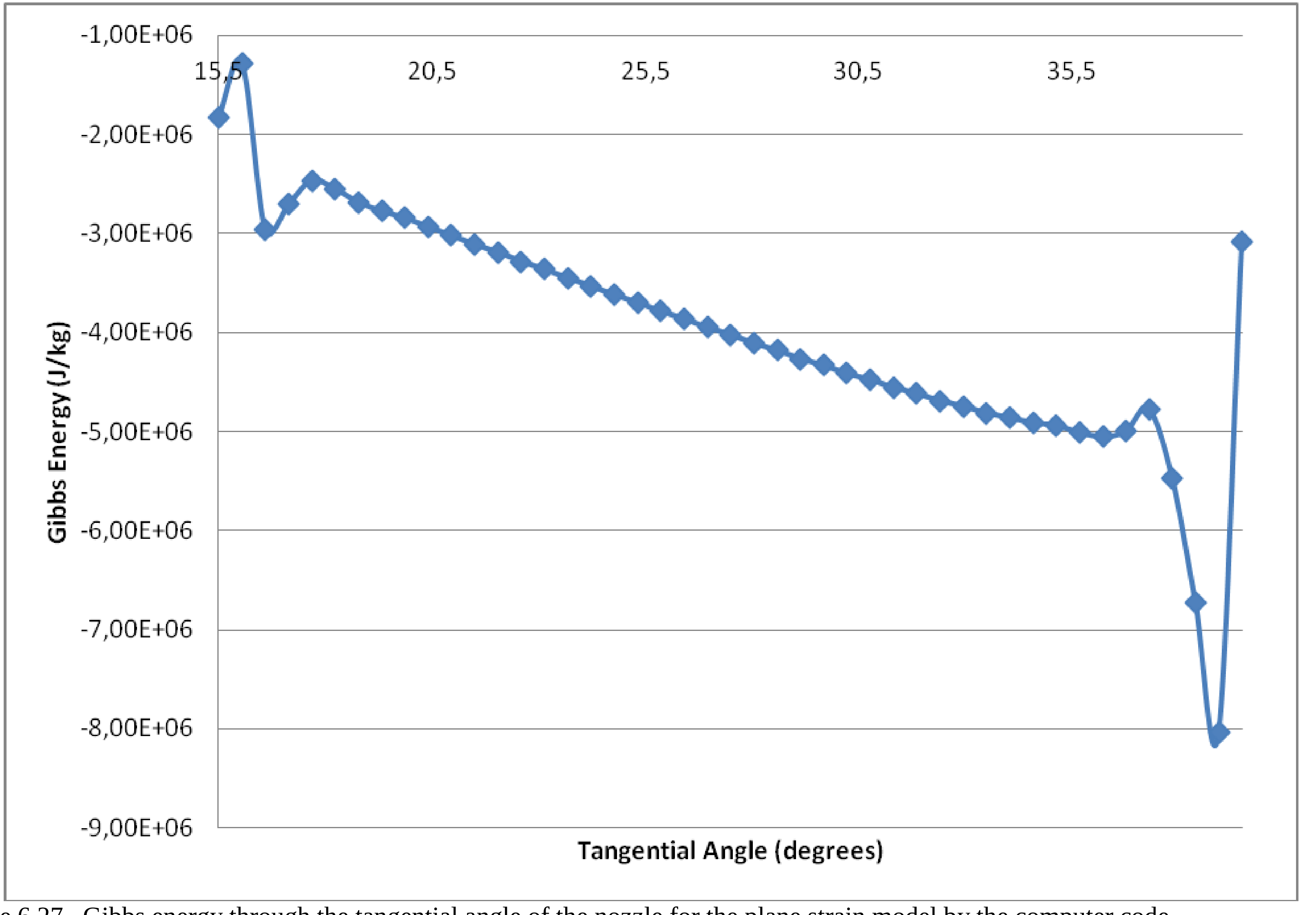

Figure 6.27. Gibbs energy through the tangential angle of the nozzle for the plane strain model by the computer code.

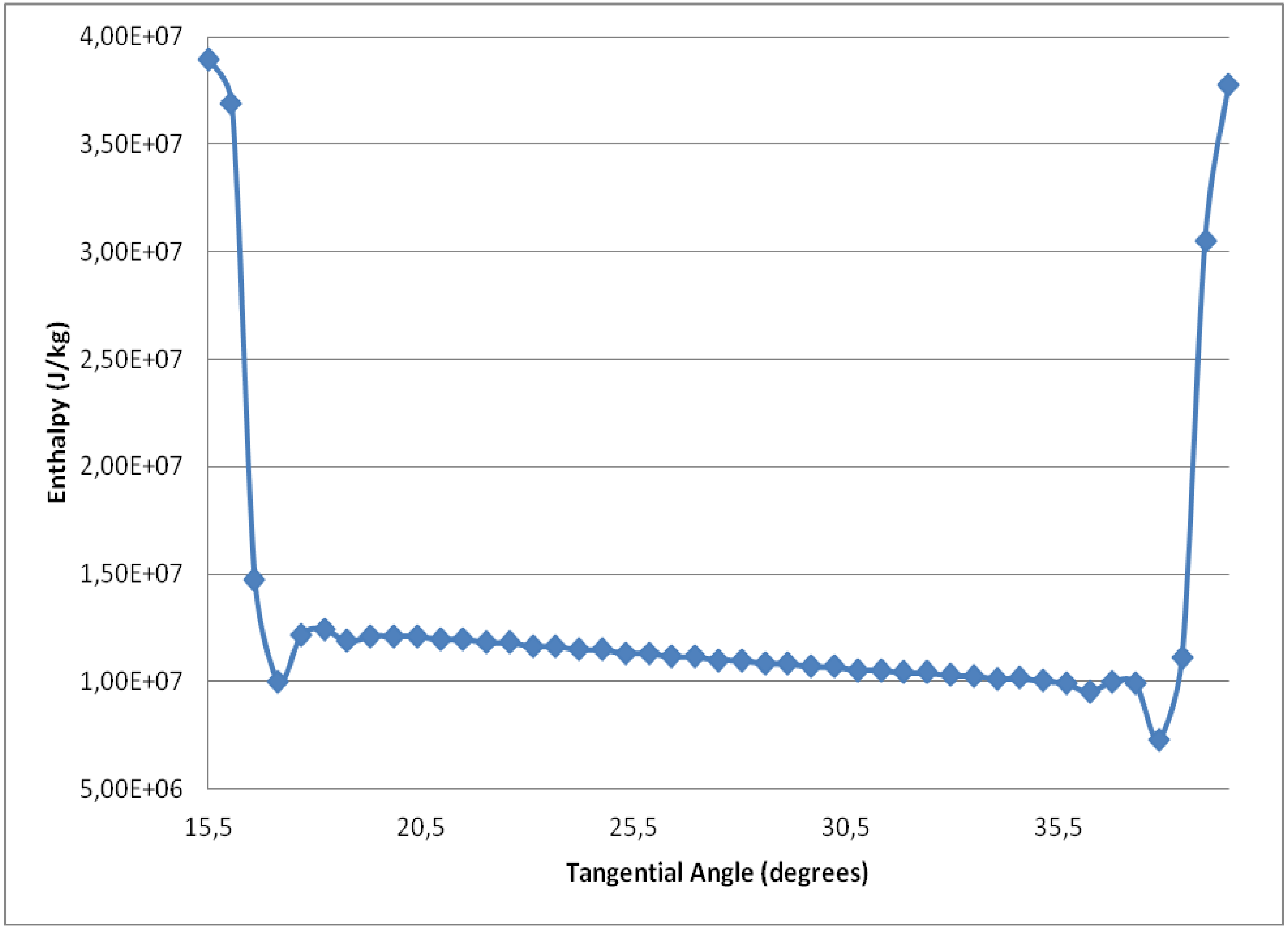

Figure 6.28. Enthalpy through the tangential angle of the nozzle for the plane strain model by the computer code.

CHAPTER 7

CONCLUSIONS

A finite element computer code was developed for this research. This finite element code can carry out coupled thermoelasticity analyses in two dimensional systems. The code is not only able to give results for the thermal analysis without structural loads or stress analysis without thermal loads, but also can take coupling effects into account. Therefore, the code includes three separate programs in it. One of them carries out the thermal analysis initially. The second program carries out the structural analyses by using the results from the first program. Finally, the last program is to couple the programs of thermal and structural analyses and generate a coupled thermoelasticity program.

Two different case studies have been studied and their results obtained by means of computer codes developed in this work were compared with those obtained by means of analytical solutions if they exist, otherwise they were compared with the results of the finite-element package ANSYS.

Main aims of the present study have been satisfied by generating two-dimensional finite elements analysis computer code for coupled thermoelastic analysis. The computer code satisfies accurate and close results compared to the ones obtained by finite elements package ANSYS for such kind of thermoelasticity analysis. Steady-state pressure and thermal such as conductive, convective, and radiative boundary conditions have been successfully dealt with for coupled thermoelasticity problems.

FUTURE WORKS

Although the analyses were only carried out for steady-state analysis, the idea can be extended in order to obtain results for transient analysis. So, the overall analysis would be more realistic and comprehensive.

Also, an addition to this study that contains the comparative analysis due the variation in the thickness of the reactor vessel and due to the variation in the temperature applied on the system concerned can be added for further studies.

In addition to these, an accident scenario of a nuclear reactor vessel can be modeled, simulated, and analyzed for further studies in order to provide a more realistic study.

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