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STATE SPACE ANALYSIS OF THE SHORT CIRCUIT FAULT FOR TRANSMISSION LINES

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State Space Analysis Of The Short Circuit Fault For Transmission Lines

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Neşe YANIÇ

ABSTRACT

STATE SPACE ANALYSIS OF THE SHORT CIRCUIT FAULT FOR TRANSMISSION LINES

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In this study, a real transmission line is modelled as a single phase lumped parameter transmission line (T-line model). State space equations are formulated for this T-line model and the system's stability and controllability are checked by using state space equations. The state equations for the T-line model are modified for short circuit case at any distance of the line and the T-line model's output voltage and short circuit current changing by time are plotted for 'before', 'during', and 'after' the fault case. These figures are obtained by solving the state equations using a computer program MATLAB. MATLAB program first calculates the state equations and then plots the output voltage and short circuit current figures. The output voltage and short circuit current frequency spectrums are plotted depending on the voltage and current time domain analysis.

Key Words: Transmission line, T-line model, State space equation, Short circuit fault

ÖZET

İLETİM HATLARINDA KISA DEVRE ARIZASININ 'DURAĞAN DURUM' TEORİSİYLE ANALİZİ

YANIÇ, Neşe

Yüksek Lisans Tezi, Elektrik Elektronik Mühendisliği Bölümü

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35 Sayfa

Bu çalışmada, gerçek bir iletim hattı tek fazlı T-devre modeline göre modellenmiştir. Hattın durağan durum denklemleri yazılmıştır ve bu durağan durum denklemleri kullanılarak sistemin kararlılığı ve kontrol edilebilirliği incelenmiştir. Hattın herhangi bir kilometresinde bir kısa devre arızası meydana geldiği zaman, T-devre modelinin durağan durum denklemleri tekrar düzenlenmiştir ve T-devre modelinin çıkış geriliminin ve kısa devre anında devreden akan kısa devre akımının zamanla değişimi arıza öncesi, arıza sırası ve arıza sonrası için şekiller üzerinde gösterilmiştir. Bu şekiller MATLAB isimli bilgisayar progamıyla çizdirilmiştir. Program önce durağan durum denklemlerini hesaplamış, sonra bu denklemleri kullanarak da şekilleri çizdirmiştir. En son olarak da, çıkış gerilim ve kısa devre akımının zaman domenindeki analizlerine bağlı olarak frekans spektrumları MATLAB programı kullanılarak çizdirilmiştir.

Anahtar Kelimeler: İletim hattı, T-devre modeli, Durağan durum denklemleri, Kısa devre arızası

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I would like to thank my husband ALPER YANIÇ for his grand help to motivate me about this study.

Lastly, I wish to special thank to my little daughter.

I hope this thesis will be useful source studying in approximation methods.

TABLE OF CONTENTS

LIST OF SYMBOLS

R	Resistance
L	Inductance
C	Capacitance
G	Conductance
I	Current
V	Voltage
t	Time
x	State vector
xdot	Derivative of x
y	Output vector
u	Input vector
A	System matrix
B	Input matrix
C	Output matrix
D	Feedthrough matrix

CHAPTER 1

INTRODUCTION

Power transmission and distribution lines are the important parts of the electrical power system that provides the contunity of electrical energy to the end users. Transmission lines connect the generating stations and load centers so when the generating stations are far away from the load centers they run over hundreds of kilometers.

Any abnormal conditions which causes flow of any abnormal current on the lines can be defined as a **fault**. Sudden changes in system conditions because of faults compose travelling waves on the transmission line and cause **transients** in the voltage and current waveforms.

A transient fault can be defined as an unexpected change in voltage or current by an unpredictable and sometimes unprecedented occurence. Most of the faults for transmission lines are transient in normal conditions. When a fault occurs, power system protection equipments run to insulate the fault area so after the fault finishes, power line apart from the fault area is returned to service. Transient currents and voltages are important factors that must be taken into account for reliable operation of power systems. A detailed voltage and current profile of the transmission line is needed for attaining an economical system insulation level, protection equipment design, test and development of relay algorithms and rapid fault detection.

Power transmission lines are modelled as distributed parameter systems. Current and voltage variations on a distributed parameter transmission line are both time and space dependent and these variations are mathematically defined in the time domain by partial ordinary equations known as **state space equation.**

This study is organized as follows. Following this introduction chapter, in the second section some studies, which are about short circuit faults on transmission lines, are reviewed to signify the similarities and differences between our study. In chapter 3, the transmission line model which consists of the connection of many time invariant lumped parameter sections is given. State equations are formulated by using lumped parameter model of transmission line. Then these equations are modified for a short circuit line model. Chapter 4 gives simulation and examples to show short circuit T-line model output voltage and current time domain and frequency domain responses by figures. The figures, that are plotted by using MATLAB program, show the short circuit T-line model output voltage and short circuit current changing by time and these voltages and currents frequency spectrums.

CHAPTER 2

REVIEW

Short circuit is a fault which depends on a low resistance connection occured by accident or intention between two or more than two points that have different potential in an electric circuit. This is an undesirable condition that from natural causes for example lighting, winding or human causes. Short circuits can occur between a phase and neutral, between two phases or between a phase and ground for a three phase transmission line. These short circuit faults result high currents and low voltages. Such short circuits are dangerous if the protection devices can not insulate the fault area as fast as possible.

In [3], the aim was to obtain the transient response of a transmission line in time domain by using three different methods, namely: eigenvector-based procedure, Vandermonde matrix vector, Lagrange interpolation formula. Our aim is the same as this study [3] but we use state space method to obtain the transient responses of the line.

In [4], the aim was to calculate transient fault current and voltages for a transmission line as our study. A travelling wave method based on Bewley lattice diagram was used for this calculation. But we use state space method to calculate the transients.

In [7], a single phase transmission line transient analysis was studied by using state space method. Then the transient responses of the transmission line were given for various sources and loads by using a computer program LPTLAP(Lumped Parameter Transmission Line Transient Analysis Program) which has been prepared specially for this study. We use also state space method like this study but we use a computer program MATLAB to show the transient responses.

Transmission lines can be represented as distributed parameter systems. In our study, lumped parameter model is used instead of distributed parameter model. In [8], both lumped parameter and distributed parameter models were used to find the state space equations and transient response of the line. In this study, theoretical error in the lumped parameter of a transmission line was calculated for state space and transient analysis. But in our study we don’t take into account the error about the lumped parameter model for analysing state space and transiets.

In [10], transient conditions caused by faults were searched for transmission lines. Voltage and current distibution were calculated in time domain considering a fault, and the voltage and current changing by time were shown in figures. The results and figures obtained by state space method were compared to the results that obtained by using frequency domain analysis. But in our study we analyse a transmission line considering a short circuit fault in time domain by using state space equations. We show how the line’s voltage and current change when a short circuit occurs.

In [12], the aim was to calculate the surge response of a transmission line with corona. State space method and lumped parameter line model were also used for this aim. We also use state space method and lumped parameter line but we didn’t calculate the surge response, we calculate only transients when a short circuit fault occurs on the line.

In [13], distributed parameter line model was used and then state equations are calculated to find the transients on transmission lines. These equations were converted to a different equation using the trapezoidal rule of integration and solved in time domain. Then a computer program FILT was used to show the transients by figures but we use MATLAB program to find the transients.

In [15], transient analysis was studied for an overhead line with a short circuit fault and transient overvoltages and overcurrents are obtained. But for analyzing these transient Fourier Transform Method was used in frequency domain. Unlike this method we use state space method to find the transients in time domain.

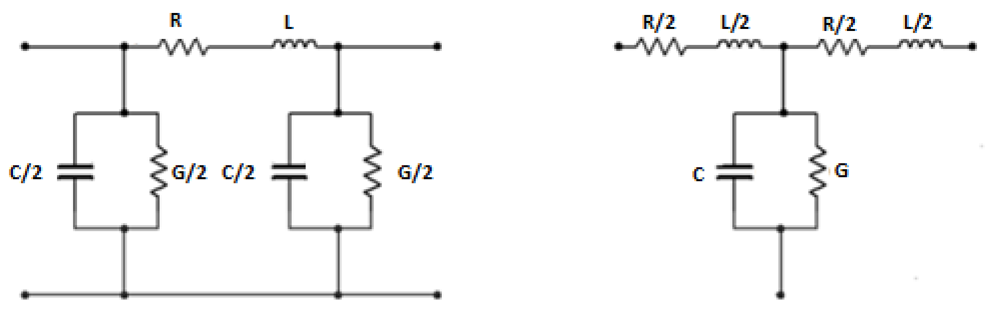
CHAPTER 3

STATE SPACE ANALYSIS OF TRANSMISSION LINES

3.1 Transmission line model

Transmission line can be showed as a distributed parameter line model. The distributed parameter line model assumes that the characteristics of the circuit (resistance, capacitance, inductance and conductance) are distributed continuously throughout the line. This is in contrast to the more common lumped parameter line model which assumes that these four electrical characteristics (R, C, L, G) are distributed perfectly along the line. And unlike the lumped parameter line model, distributed parameter line model assumes non-uniform current along each branch and non-uniform voltage along each node. So distributed element line model is more accurate and more complex than the lumped parameter line model.

Instead of distributed parameters, generally lumped parameter transmission line models are used for transient analysis. Lumped parameter line model can be represented as an interconnection of many lumped parameter identical circuits. These circuits are in the form Π and T and contains a series resistance and inductance, and a shunt conductance and capacitance as seen in Figure 3.1.



(a) **(b)**

Figure 3.1 Different lumped parameter line models a) Π line model b) T line model

When n T-sections are connected in cascade and some series elements are combined, the transmission line model shown in the Figure 3.2 is obtained. This model will be the fundemental basis for the state space analysis of the transmission line. Similar line models can be obtained by using other types of circuits.

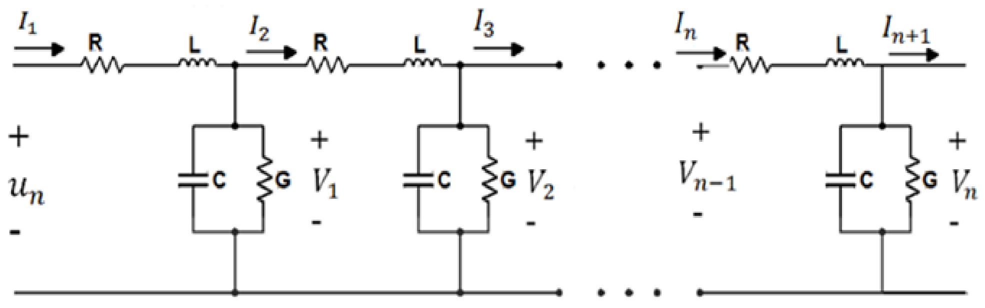


Figure 3.2 Single phase lumped parameter transmission line model

3.2 State space representation and solution

The most general state space representation of a linear system is in the following form:

$$\dot{x}(t) = A(t)x(t) + B(t)u(t); \qquad \dot{x}(t) = \frac{\mathrm{d}}{\mathrm{dt}}x(t)$$

$$y(t) = C(t)x(t) + D(t)u(t) \tag{3.1}$$

where

- **x** is called state vector
- **y** is called output vector
- **u** is called input vector
- **A** is called system matrix
- **B** is called input matrix
- **C** is called output matrix
- **D** is called feedthrough matrix(in case where the system model does not have a direct feedthrough, D is the zero matrix)

In this general formulation, all matrices are allowed to be time variant, however in the common continuous time invariant system, matrices will be time invariant.

To above approach provides a linear lumped parameter line can be represented as the continuous time invariant systems by the state space equations in the following:

$$\frac{dx(t)}{dt} = Ax(t) + Bu(t); \qquad x_0 = x(t0)$$
$$y(t) = Cx(t) + Du(t) \tag{3.2}$$

In the equation 3.2:

- The state vector *x* contains some capacitor voltages and inductor currents
- *x(t0)* is the initial value of the state vector at the initial time *t0*
- *A*, *B*, *C*, *D* are the constant matrices which depend on the lumped parameter values of transmission line

To obtain the state space equations for a single phase, the equivalent circuit shown in the Figure 3.2, is considered. The total number of the sections in this model is n. A sinusoidal source is assumed at the sending end. The first section state equations are calculated in the following.

$$V_R = R.I(t), \qquad V_L = L.\frac{dI(t)}{dt}, \qquad I_C = C.\frac{dV_C(t)}{dt} \tag{3.3}$$

When Equation 3.3 is known, Kirchoff's voltage and current law is applied to the first section of the circuit as in the following.

$$u(t) = V_R + V_L + V_C \qquad V_C = V_1$$

$$u(t) - R.I_1(t) - L.\frac{dI_1(t)}{dt} - V_C = 0$$

$$\frac{dI_1(t)}{dt} = -\frac{R}{L}I_1 - \frac{1}{L}V_1 + \frac{1}{L}u(t) \tag{3.4}$$

$$I_1 - I_2 = I_C + I_G$$

$$I_1 - I_2 = C.\frac{dV_1}{dt} + G.V_1$$

$$\frac{dV_1}{dt} = \frac{1}{C}I_1(t) - \frac{1}{C}I_2(t) - \frac{G}{C}V_1 \tag{3.5}$$

If the same rules are continued to apply for the n sections as first section, the state equations are obtained as follows Equation 3.6 .

$$\frac{d}{dt}\begin{bmatrix} I_1 \\ V_1 \\ I_2 \\ V_2 \\ \vdots \\ I_n \\ V_n \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & -\frac{1}{L} & 0 & \dots & \dots & \dots & 0 \\ \frac{1}{C} & -\frac{G}{C} & -\frac{1}{C} & 0 & \dots & \dots & 0 \\ 0 & \frac{1}{L} & -\frac{R}{L} & -\frac{1}{L} & \dots & \dots & 0 \\ 0 & 0 & \frac{1}{C} & -\frac{G}{C} & -\frac{1}{C} & \dots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & \dots & \dots & \frac{1}{L} & -\frac{R}{L} & -\frac{1}{L} \\ 0 & \dots & \dots & \dots & 0 & \frac{1}{C} & -\frac{G}{C} \end{bmatrix} \begin{bmatrix} I_1 \\ V_1 \\ I_2 \\ V_2 \\ \vdots \\ I_n \\ V_n \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \\ 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix} u(t) \qquad (3.6)$$

According to this equation

$$x = [I_1 \quad V_1 \quad I_2 \quad V_2 \quad \cdots \quad I_n \quad V_n]^T \qquad (3.7)$$

$$B = [1/L \quad 0 \quad 0 \quad 0 \quad \dots \quad 0 \quad 0]^T \qquad (3.8)$$

$$A = \begin{bmatrix} -\frac{R}{L} & -\frac{1}{L} & 0 & \dots & \dots & \dots & 0 \\ \frac{1}{C} & -\frac{G}{C} & -\frac{1}{C} & 0 & \dots & \dots & 0 \\ 0 & \frac{1}{L} & -\frac{R}{L} & -\frac{1}{L} & \dots & \dots & 0 \\ 0 & 0 & \frac{1}{C} & -\frac{G}{C} & -\frac{1}{C} & \dots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & \dots & \dots & \frac{1}{L} & -\frac{R}{L} & -\frac{1}{L} \\ 0 & \dots & \dots & \dots & 0 & \frac{1}{C} & -\frac{G}{C} \end{bmatrix} \qquad (3.9)$$

In this study, this time invarient state space model's dynamic characteristics will be also studied by checking the system stability and controllability.

3.2.1 Stability

Stability response characteristic of the continuous time invariant state space model can be studied from the eigenvalues of the matrix A. The eigenvalues of the A matrix are the poles of the system. By taking the determinant of $\lambda I - A$, the system poles (poles are the λ values) can be found.

$$poles = eig(A) = |\lambda I - A| \qquad (3.7)$$

The system poles are in the complex number form like, $\lambda = a + jb$. If all of the system poles are in the left-half plane, which means that the system is stable.

If the presented method is applied to our transmission line state space model which equations are given in 3.6, we can decide that the system is stabil or unstabil. We will check the system stability in Chapter 4 by using a computer program.

3.2.2 Controllability

A continuous time-invariant state-space model is controllable if and only if

$$rank[B \quad AB \quad A^2B \quad \dots \quad A^{n-1}B] = n \tag{3.8}$$

where rank is the number of linearly independent rows in a matrix.

If the presented method is applied to our transmission line state space model which equations are given in 3.3, we can decide that the system is controllable or not. We will check the system contrability in Chapter 4 by using a computer program.

3.3 State Space Analysis of a Short Circuit Line

The T-line model which is shown in the Figure 3.2 will be used again for writing the state space equations of a short circuit line. If there is a short circuit at the T-line model, it can be shown as the following Figure 3.3.

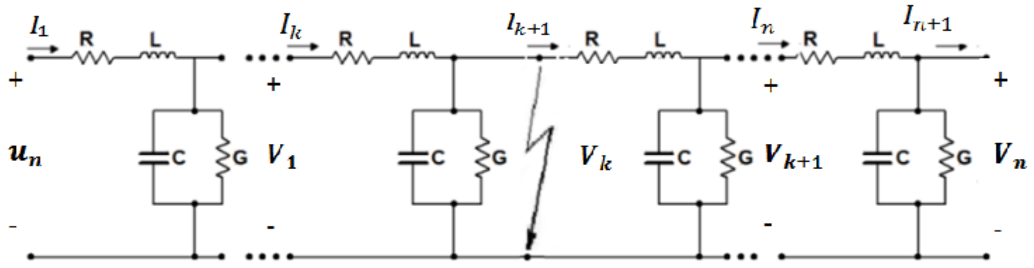


Figure 3.3 Single phase lumped parameter short circuit line model

As shown in the Figure 3.3, a short circuit fault occurs at n_k node so there are transient currents and voltages on the circuit until the fault finishes. If state space equations for this short circuit line model are desired to write, it can be seen that k.-th column of the A matrix will change. This k.-th column consists some R, L, C, G values which are related with the k.-th circuit. We will show how A matrix changes when a short circuit occurs on our T-line model in Chapter 4.

CHAPTER 4

SIMULATION AND EXAMPLES

In this chapter, the computer program MATLAB is used to illustrate the theoretical results which are given in Chapter 3. Three phase of 160 km, 400 kV transmission line with parameters *R=0.032W/km*, *L=0.88µh/km*, *G=0.042µS/km*, and *C=0.013µF/km* are studied. This line can be modelled as the T-line model which is shown in the Figure 3.2. We suppose that this line is approximated by 8 T-sections so every 20 km's line is modelled as one T-section. This T-line model will be used for the following examples.

4.1 Example 1: State Space Analysis of the T-line Model

In this example for the given T-line model, first state space equations are calculated by using equation 3.6 then stability and contrability characteristics are checked by using state equations. The input data for the MATLAB program are the line parameters, the line length and the number of sections.

4.1.1 Program of the State Space Analysis

```
s=input('enter the desired T-section number: ');
l=160; % l is the length of the line

for k=1:(2*s)
for m=1:(2*s)

R=0.032*(l/s);
L=(0.88e-3)*(l/s);
G=(0.042e-3)*(l/s);
C=(0.013e-3)*(l/s);

A(1,1)=-R/L , A(1,2)=-1/L;
```

```
for k=2:2:(2*s)
    if m==k-1
        A(k,m)=1/C;
    elseif m==k
        A(k,m)=-G/C;
    elseif m==k+1
        A(k,m)=-1/C;
    else
        A(k,m)=0;
    end
end
for k=3:2:((2*s)-1)
    if m==k-1
        A(k,m)=1/L;
    elseif m==k
        A(k,m)=-R/L;
    elseif m==k+1
        A(k,m)=-1/L;
    else
        A(k,m)=0;
    end
end
end
end
A

for i=1:(2*s)
    B(i,1)=0;
end

B(1,1)=1/L;
B

%for checking stability
poles=eig(A)

%for checking contrability
Co=ctrb (A,B)
rank=rank(Co)
```

After running the program, A matrix which has 16x16 dimensions and B matrix which has 16x1 dimensions are calculated like the following Equation 4.1 and 4.2.

$$A = \begin{bmatrix} -0{,}0364 & -0{,}0568 & 0 & \dots & \dots & \dots & 0 \\ 3{,}8462 & -0{,}0032 & -3{,}8462 & 0 & \dots & \dots & 0 \\ 0 & 0{,}0568 & -0{,}0364 & -0{,}0568 & \dots & \dots & 0 \\ 0 & 0 & 3{,}8462 & -0{,}0032 & -3{,}8462 & \dots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & \dots & \dots & 0{,}0568 & -0{,}0364 & -0{,}0568 \\ 0 & \dots & \dots & \dots & 0 & 3{,}8462 & -0{,}0032 \end{bmatrix} \quad (4.1)$$

$$B = [58{,}8182 \quad 0 \quad 0 \quad \dots \quad \dots \quad \dots \quad 0] \quad (4.2)$$

Also the program calculates the poles of the system for checking the stability. According to the system poles which are given in the following the system is **stable** because all of the poles are in left-half plane.

poles =
1.0e+002 *
 -0.1980 + 9.1888i
 -0.1980 - 9.1888i
 -0.1980 + 8.7165i
 -0.1980 - 8.7165i
 -0.1980 + 7.9474i
 -0.1980 - 7.9474i
 -0.1980 + 6.9074i
 -0.1980 - 6.9074i (4.3)
 -0.1980 + 5.6319i
 -0.1980 - 5.6319i
 -0.1980 + 0.8466i
 -0.1980 - 0.8466i
 -0.1980 + 2.5532i
 -0.1980 - 2.5532i
 -0.1980 + 4.1641i
 -0.1980 - 4.1641i

At the end, the program is checked the system controllability by calculating the rank of the controllability matrix Co in the following:

rank =
 5

As shown rank (Co) is not equal to n (n=16), we can say the system is **uncontrollable**.

4.2 Example 2: State Space Analysis of a Short Circuit Line for 3 msec.

The same line with the termination given in the previous example is studied again.

But in this example a short circuit fault occurs on the 100th kilometer of the line. As mentioned before every 20 km of the line is modelled one T-section so 100th km fault means that the fault is at the 5th T-section.

Line is energised for between t=0 msec. and t=100 msec. by a sinusoidal voltage source *u(t)*, *u(t)* is $\sqrt{2}\left(\frac{220}{\sqrt{3}}\right) sin(2 * W * t)\ kV$, and $W = 2\pi f, f = 50Hz$. Short circuit is assumed at t=45 msec. and continues for 3 msec. When a short circuit fault occurs at any T-line section, the voltage value will be zero.

Our aim is to show T-line model's output voltage and the T-line model short circuit current, which flows on the short circuit fault point, changing by time before the fault, during the fault and after the fault.

To plot the T-line's output voltage and short circuit current changing by time, first a function program in the following is written in MATLAB. This program calculates *xdot* (*xdot=dx/dt*) vectors. As defined equation 3.4, *x* is the state vector.

```
%the function calculates xdot as a function of t and x
% x contains some capacitor voltages and inductor currents
function xdot=output(t,x)

s=8;        % s is the line's T-section number
l=160;      % l is the length of the line
W=628;   % W =2*pi*f, f(frequency)=50 Hz

R=0.032*(l/s);
L=(0.88e-3)*(l/s);
G=(0.042e-3)*(l/s);
C=(0.013e-3)*(l/s);

 if t>=0 & t<45
   xdot= zeros((2*s),1);
   xdot(1)=((-R/L)*x(1))-((1/L)*x(2))+((1/L)*(2^(1/2))*(220/sqrt(3))*(sin(2*W*t)));
   xdot(2*s)=((1/C)*x((2*s)-1))-((G/C)*x(2*s));

   for k=2:2:((2*s)-2)
       xdot(k)=((1/C)*x(k-1))-((G/C)*x(k))-((1/C)*x(k+1));
   end
   for k=3:2:((2*s)-1)
       xdot(k)=((1/L)*x(k-1))-((R/L)*x(k))-((1/L)*x(k+1));
   end
 end

 if t>=45 & t<48
   xdot= zeros((2*s),1);
```

```
   xdot(1)=((-R/L)*x(1))-((1/L)*x(2))+((1/L)*(2^(1/2))*(220/sqrt(3))*(sin(2*W*t)));
   xdot(2*s)=((1/C)*x((2*s)-1))-((G/C)*x(2*s));

      for k=2:2:((2*s)-2)
         if k==10  %k is 2 times of the short circuit T-section number
        xdot(k)=0;
         else
        xdot(k)=((1/C)*x(k-1))-((G/C)*x(k))-((1/C)*x(k+1));
         end
      end
   for k=3:2:((2*s)-1)
      xdot(k)=((1/L)*x(k-1))-((R/L)*x(k))-((1/L)*x(k+1));
   end
 end

 if t>=48 & t<=100
   xdot= zeros((2*s),1);
   xdot(1)=((-R/L)*x(1))-((1/L)*x(2))+((1/L)*(2^(1/2))*(220/sqrt(3))*(sin(2*W*t)));
   xdot(2*s)=((1/C)*x((2*s)-1))-((G/C)*x(2*s));

   for k=2:2:((2*s)-2)
      xdot(k)=((1/C)*x(k-1))-((G/C)*x(k))-((1/C)*x(k+1));
   end
   for k=3:2:((2*s)-1)
      xdot(k)=((1/L)*x(k-1))-((R/L)*x(k))-((1/L)*x(k+1));
   end
 end
```

As shown in the program, when t=45 between 48 msec., a short circuit occurs on the line at the 5th T-section line model so this T-section voltage is showed zero in the program.

After calculating *xdot's*, the following is typed in the command window to solve *x* vector and plot output voltage and short circuit current changing by time. Based on the function program 'output', a differantial equation solver *ode45* command is used in the following. According to the following commands, first the *x* vector is calculated from t=0 msec. to t=45 msec. Then the *x* values for the t=45th msec. have been the initial values to calculate the *x* vector from the t=45 msec. to t=48 msec. At this time period, the short circuit fault has been occured and finished. And the *x* values at the 48th msec. have been the initial values to calculate the *x* vector from t=48 msec. to t=100 msec. So all the *x* vector values are calculated from t=0 msec. to t=100 msec. with considering a short circuit. These *x* vector values are the 8 T-line model section voltage and current values. So the desired voltage and current values of the T-sections can be calculated and the desired voltage and current changing by time can be shown by using the following commands. We showed the output voltage *x(16)* and short circuit current *x(9)* changing from t=0 msec. to t=100 msec. by the following figures.

```
s=input('enter the desired T-section number: ');
 for i=1:(2*s)
    x0(i,1)=0;
  end
  x0;

[t,x]=ode45('output',[0:1:45],x0);
x(46,1:(2*s));

[t,x]=ode45('output',[45:1:48],x(46,1:(2*s)));
x(4,1:(2*s));

[t,x]=ode45('output',[48:1:100],x(4,1:(2*s)));

[t,x]=ode45('output',[0:1:100],x0);
figure
plot(t,x(:,16))
xlabel('time, msec ');
ylabel('voltage, kV ');
figure
plot(t,x(:,9))
xlabel('time, msec ');
ylabel('current, kA ');
```

The program plots Figure 4.1, 4.2. The figures show T-line model's output voltage and short circuit current changing by time.

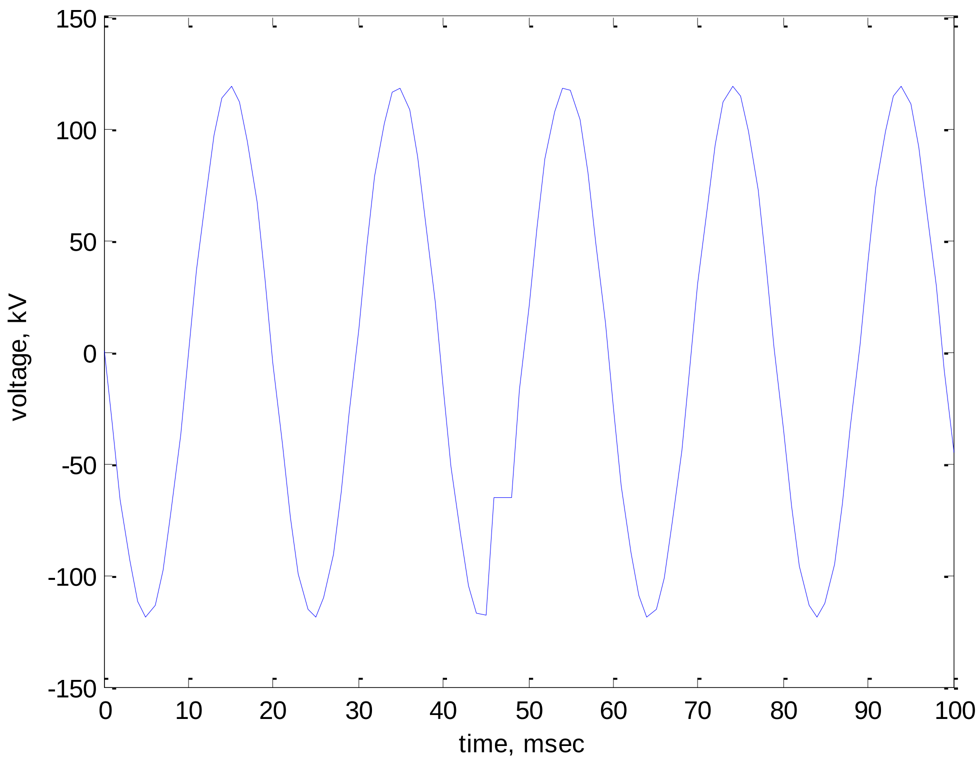


Figure 4.1 Output voltage for fault at 100th km of the line for 100 msec.

As shown in the Figure 4.1, when there is no fault on the line, the T-line model output voltage is as a sine wave that oscillating between -120kV and 120kV in 20 msec. periods. But at 45th sec. , the short circuit fault starting time, output voltage sinusoidal waveform is distorted. After the fault finishes the output voltage continues to change as the sine wave like before the fault.

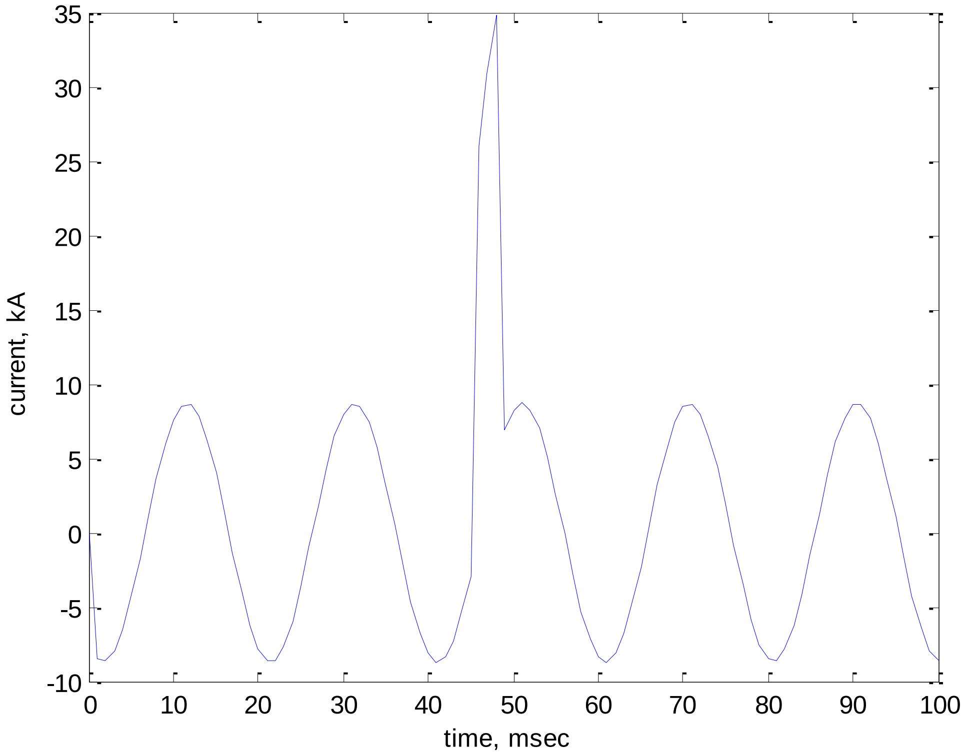


Figure 4.2 Short circuit current for fault at 100th km of the line for 100 msec.

As shown in the Figure 4.2, when there is no fault on the line, the T-line model output current is as a sine wave that oscillating between -8 kA and 8 kA in 20 msec. periods. But at 45th msec. , the short circuit fault starting time, short circuit current starts to increase and the short circuit current value reaches 35 kA at the short circuit fault finish time (48th msec). After the fault finishes the short circuit current continues to change as the sine wave like the before the fault.

4.3 Example 3: State Space Analysis of a Short Circuit Line for 3 msec. with Time Varying Short Circuit Resistance

When a short circuit fault occurs at any T-line section, the T-circuit line model output voltage and the short circuit current change depending on this T-section parallel circuit's *R* and *C* values.

In this example, we suppose that these *R* and *C* values first decrease 100 times from their original values at the fault beginning time, and after they will increase depending on the time until they reach to their original values when the fault finishes. The prepared program is capable to solve for different resistor and capacitor models.

To plot the T-line's output voltage and short circuit current changing by time, the same function program in the following is used. We changed only the 5th T-section parallel circuit's *G* and *C* values during the fault.

```
%the function calculates xdot as a function of t and x
% x contains some capacitor voltages and inductor currents
function xdot=output(t,x)

s=8;        % s is the line's T-section number
l=160;     % l is the length of the line
W=628;   % W =2*pi*f, f(frequency)=50 Hz

R=0.032*(l/s);
L=(0.88e-3)*(l/s);
G=(0.042e-3)*(l/s);
C=(0.013e-3)*(l/s);

 if t>=0 & t<45
   xdot= zeros((2*s),1);
   xdot(1)=((-R/L)*x(1))-((1/L)*x(2))+((1/L)*(2^(1/2))*(220/sqrt(3))*(sin(2*W*t)));
   xdot(2*s)=((1/C)*x((2*s)-1))-((G/C)*x(2*s));

   for k=2:2:((2*s)-2)
       xdot(k)=((1/C)*x(k-1))-((G/C)*x(k))-((1/C)*x(k+1));
   end
   for k=3:2:((2*s)-1)
       xdot(k)=((1/L)*x(k-1))-((R/L)*x(k))-((1/L)*x(k+1));
   end
 end

 if t>=45 & t<48
   xdot= zeros((2*s),1);
   xdot(1)=((-R/L)*x(1))-((1/L)*x(2))+((1/L)*(2^(1/2))*(220/sqrt(3))*(sin(2*W*t)));
   xdot(2*s)=((1/C)*x((2*s)-1))-((G/C)*x(2*s));

     for k=2:2:((2*s)-2)
        if k==10  %k is 2 times of the short circuit T-section number
           C1=100*(C/t);
           G1=100*(G*t);
       xdot(k)=((1/C1)*x(k-1))-((G1/C1)*x(k))-((1/C1)*x(k+1));

        else
       xdot(k)=((1/C)*x(k-1))-((G/C)*x(k))-((1/C)*x(k+1));
        end
```

```
      end
    for k=3:2:((2*s)-1)
        xdot(k)=((1/L)*x(k-1))-((R/L)*x(k))-((1/L)*x(k+1));
    end
  end

  if t>=48 & t<=100
    xdot= zeros((2*s),1);
    xdot(1)=((-R/L)*x(1))-((1/L)*x(2))+((1/L)*(2^(1/2))*(220/sqrt(3))*(sin(2*W*t)));
    xdot(2*s)=((1/C)*x((2*s)-1))-((G/C)*x(2*s));

    for k=2:2:((2*s)-2)
        xdot(k)=((1/C)*x(k-1))-((G/C)*x(k))-((1/C)*x(k+1));
    end
    for k=3:2:((2*s)-1)
        xdot(k)=((1/L)*x(k-1))-((R/L)*x(k))-((1/L)*x(k+1));
    end
  end
```

As shown in the program, when t=45 between 48 msec., a short circuit occurs on the line and the 5th T-section parallel circuit's *G* and *C* values are supposed to changes by time during the fault.

After calculating *xdot's*, the following is typed in the command window to solve *x* vector and plot output voltage and short circuit current changing by time.

```
s=input('enter the desired T-section number: ');
  for i=1:(2*s)
     x0(i,1)=0;
   end
   x0;

[t,x]=ode45('output',[0:1:45],x0);
x(46,1:(2*s));

[t,x]=ode45('output',[45:1:48],x(46,1:(2*s)));
x(4,1:(2*s));

[t,x]=ode45('output',[48:1:100],x(4,1:(2*s)));
[t,x]=ode45('output',[0:1:100],x0);
figure
plot(t,x(:,16))
xlabel('time, msec ');
ylabel('voltage, kV ');
figure
plot(t,x(:,9))
```

```
xlabel('time, msec ');
ylabel('current, kA ')
```

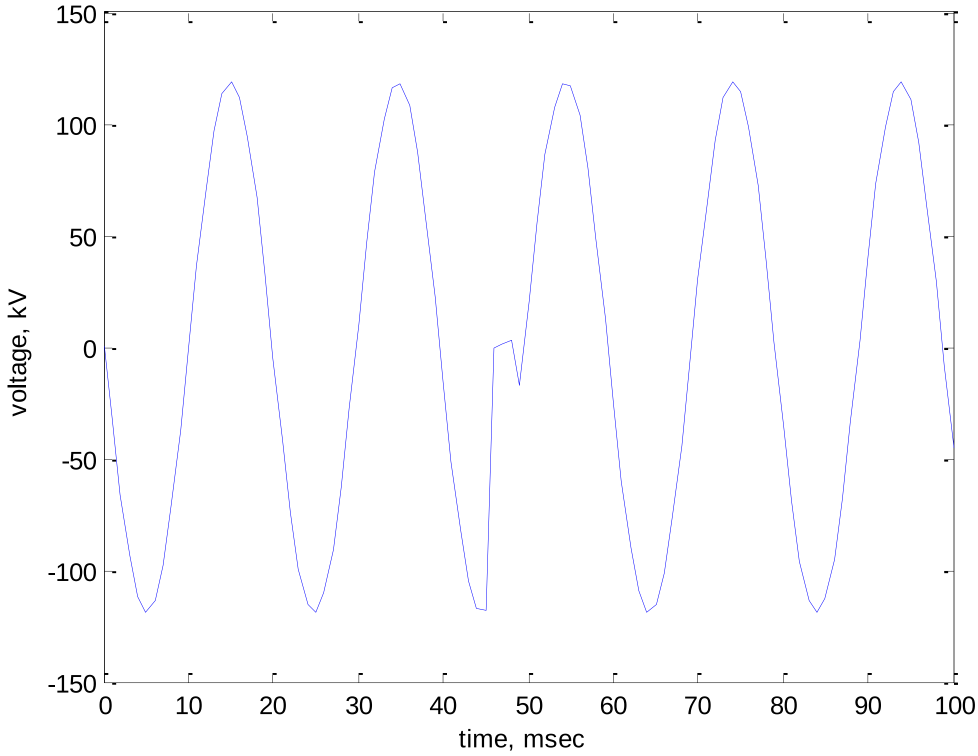


Figure 4.3 Output voltage for fault at 100th km of the line for 100 msec.

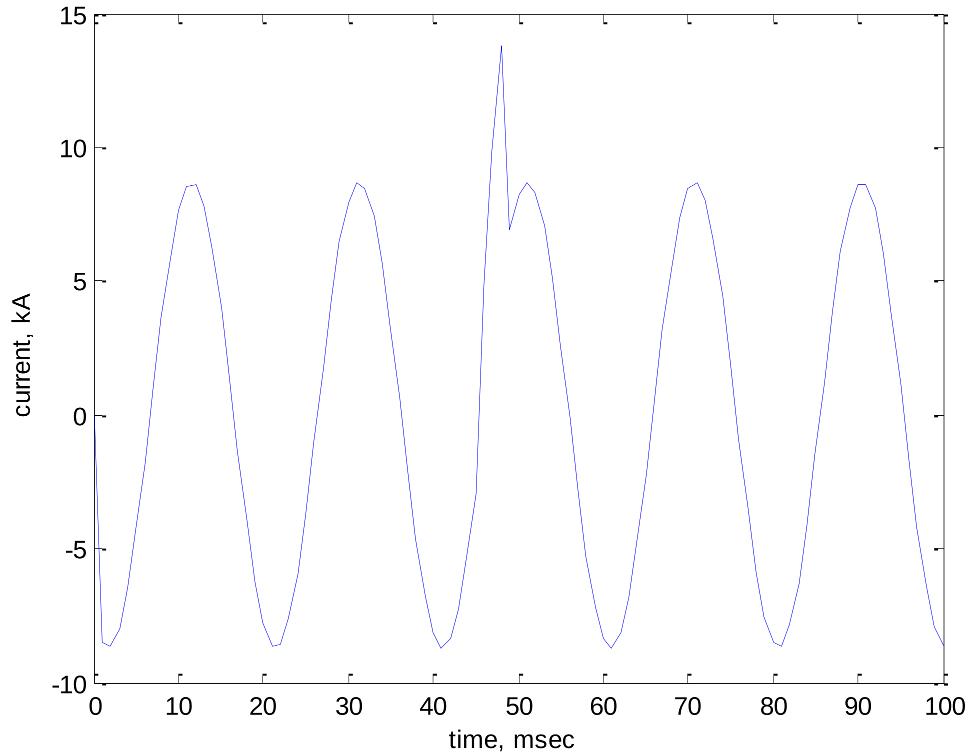


Figure 4.4 Time varying short circuit current for fault at 100th km of the line for 100 msec.

As shown in the Figure 4.4, we can limit the short circuit current by changing the short circuit T-section parallel circuit *G* and *C* values. In example 2, the short circuit reaches 35 kA but in this example we limit the current at 14 kA by changing the short circuit T-section parallel circuit *G* and *C* values depending on time.

4.4 Example 4: State Space Analysis of a Short Circuit Line for 10 msec

In this example, there is a short circuit fault on the 100th km of the line again. But for this example, the short circuit fault time period is changed. The short circuit fault starts at 40th msec. and finishes at 50th msec. As shown, it is a long period for a short circuit fault but we need long time period to show how the output voltage decreases to zero and how the short circuit current changes because of the zero voltage.

The following program named ‘output’ is written again, only by changing the time periods:

```
%the function calculates xdot as a function of t and x
% x contains some capacitor voltages and inductor currents
function xdot=output(t,x)

s=8;     % s is the line's T-section number
l=160;   % l is the length of the line
W=628;   % W=2*pi*f  (f=50Hz)

R=0.032*(l/s);
L=(0.88e-3)*(l/s);
G=(0.042e-3)*(l/s);
C=(0.013e-3)*(l/s);

 if t>=0 & t<40
   xdot= zeros((2*s),1);
   xdot(1)=((-R/L)*x(1))-((1/L)*x(2))+((1/L)*(2^(1/2))*(220/sqrt(3))*(sin(2*W*t)));
   xdot(2*s)=((1/C)*x((2*s)-1))-((G/C)*x(2*s));

   for k=2:2:((2*s)-2)
      xdot(k)=((1/C)*x(k-1))-((G/C)*x(k))-((1/C)*x(k+1));
   end
   for k=3:2:((2*s)-1)
      xdot(k)=((1/L)*x(k-1))-((R/L)*x(k))-((1/L)*x(k+1));
   end
 end
```

```
if t>=40 & t<50
  xdot= zeros((2*s),1);
  xdot(1)=((-R/L)*x(1))-((1/L)*x(2))+((1/L)*(2^(1/2))*(220/sqrt(3))*(sin(2*W*t)));
  xdot(2*s)=((1/C)*x((2*s)-1))-((G/C)*x(2*s));

     for k=2:2:((2*s)-2)
         if k==10  %k is 2 times of the short circuit T-section number
             C1=100*(C/t);
             G1=100*(G*t);
        xdot(k)=((1/C1)*x(k-1))-((G1/C1)*x(k))-((1/C1)*x(k+1));

         else
        xdot(k)=((1/C)*x(k-1))-((G/C)*x(k))-((1/C)*x(k+1));
         end
     end
  for k=3:2:((2*s)-1)
      xdot(k)=((1/L)*x(k-1))-((R/L)*x(k))-((1/L)*x(k+1));
  end
end

if t>=50 & t<=100
  xdot= zeros((2*s),1);
  xdot(1)=((-R/L)*x(1))-((1/L)*x(2))+((1/L)*(2^(1/2))*(220/sqrt(3))*(sin(2*W*t)));
  xdot(2*s)=((1/C)*x((2*s)-1))-((G/C)*x(2*s));

  for k=2:2:((2*s)-2)
      xdot(k)=((1/C)*x(k-1))-((G/C)*x(k))-((1/C)*x(k+1));
  end
  for k=3:2:((2*s)-1)
      xdot(k)=((1/L)*x(k-1))-((R/L)*x(k))-((1/L)*x(k+1));
  end
end
```

Then the following, that calculates the *x* vector for all times from 0 to 100 msec., is typed to the command window to plot the Figure 4.5 and 4.6.

```
s=input('enter the desired T-section number: ');
 for i=1:(2*s)
    x0(i,1)=0;
  end
  x0;

[t,x]=ode45('output',[0:1:40],x0);
x(41,1:(2*s));

[t,x]=ode45('output',[40:1:50],x(41,1:(2*s)));
x(11,1:(2*s));
```

```
[t,x]=ode45('output',[50:1:100],x(11,1:(2*s)));

[t,x]=ode45('output',[0:1:100],x0);
figure
plot(t,x(:,16))
xlabel('time, msec ');
ylabel('voltage, kV ');
figure
plot(t,x(:,9))
xlabel('time, msec ');
ylabel('current, kA ');
```

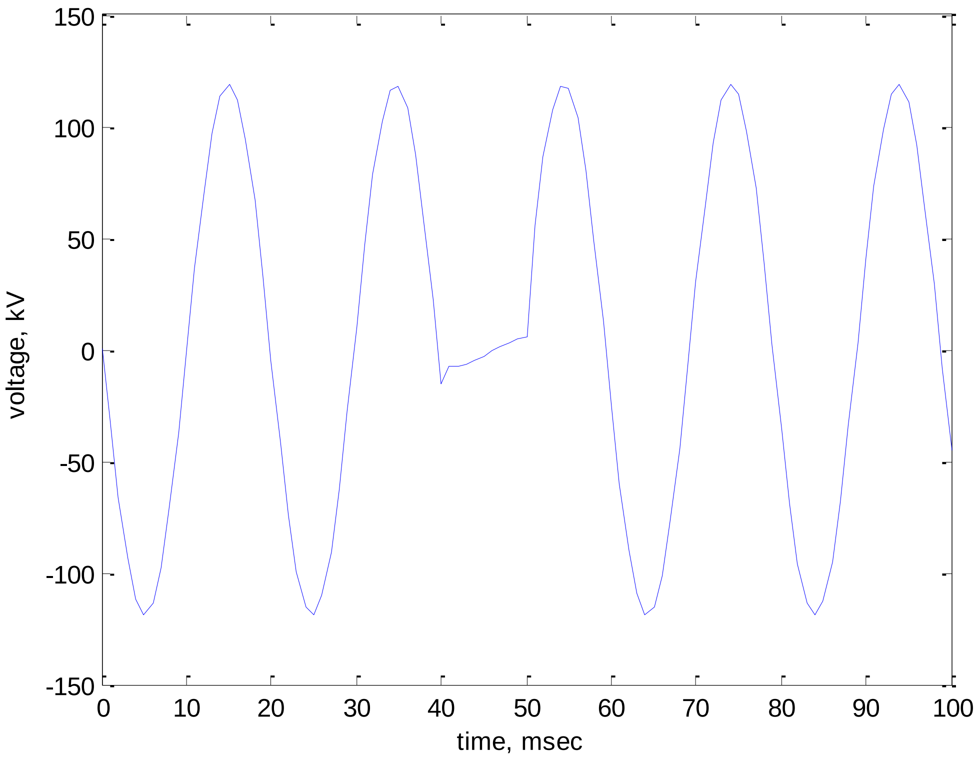


Figure 4.5 Output voltage for 10 msec. time period fault at the 100th km of the line

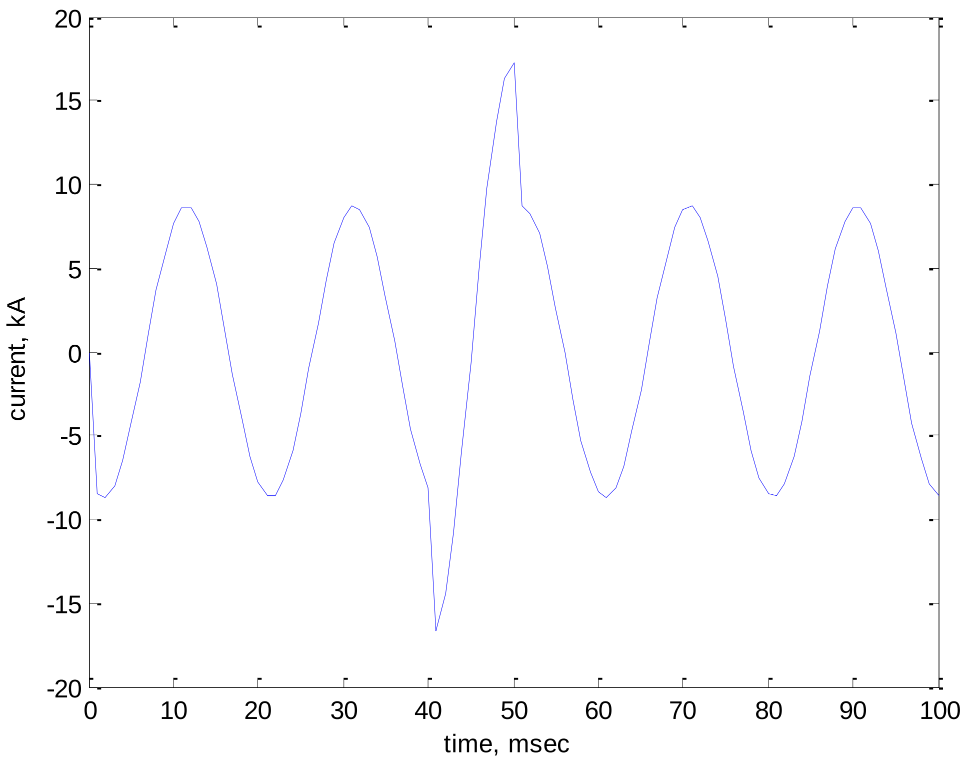


Figure 4.6 Short circuit current for 10 msec. time period fault at the 100th km of the line

As shown in the Figure 4.5, the short circuit fault which lasted about 10 msec. can cause to decrease the output voltage to zero. At the same time, the limited short circuit current, which is shown in the Figure 4.6, increases to the maximum peak value which the current can reach.

4.5 Example 5: Frequency Domain Analysis of a Short Circuit Line

In the example 2,3 and 4, the short circuit T-line model output voltage and short circuit current changing are analysed in time domain. But in this example, the short circuit T-line model output voltage and short circuit current are analysed in frequency domain. Fourier Transform method can provide to convert a time domain signal to the frequency domain. But Fourier Transform method mathematical equations are not calculated in this example. To generate the short circuit T-line model output voltage and short circuit current frequency spectrum ***fft (fast fourier transform)*** command in MATLAB is used.

The function program named ‘output’, which are written in Example 2 and 3, are used again for this example. As represented, the short circuit fault in the Example 2 and 3, continues for 3 msec. between 45th msec. to 48th msec. As defined before, unlike the Example 2, the short circuit current is limited by changing the T-section paralel circuit *G* and *C* values in the Example 3.

The following is typed in the command window to show the short circuit T-line model frequency spectrum of the output voltage and short circuit current that represented in the Example 2.

```
s=input('enter the desired T-section number: ');
 for i=1:(2*s)
    x0(i,1)=0;
  end
  x0;

[t,x]=ode45('output',[0:1:45],x0);
x(46,1:(2*s));

[t,x]=ode45('output',[45:1:48],x(46,1:(2*s)));
x(4,1:(2*s));

[t,x]=ode45('output',[48:1:100],x(4,1:(2*s)));

[t,x]=ode45('output',[0:1:100],x0);

w=x(:,(16));
fs=1000;  % fs is sampling frequency
y=abs((fft(w,fs)));
n=20*log(y);
figure
plot(n(1:500))
xlabel('frequency, Hz ');
ylabel('magnitude, dB ');

w=x(:,(9));
fs=1000;  % fs is sampling frequency
y=abs((fft(w,fs)));
n=20*log(y);
figure
plot(n(1:500))
xlabel('frequency, Hz ');
ylabel('magnitude, dB ');
```

After running the program Figure 4.7 and Figure 4.8 are plotted. Figure 4.7 shows the T-line model output voltage frequency spectrum for the short circuit fault at 100th km of the line for 3 msec. And Figure 4.8 shows the T-line model short circuit current frequency spectrum.

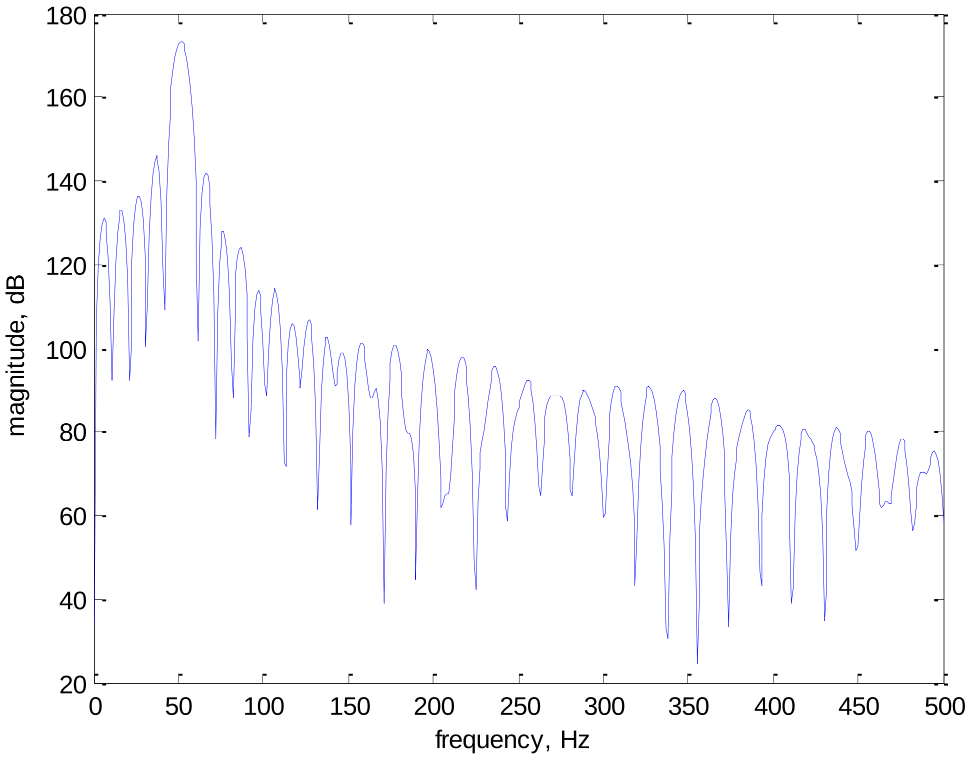


Figure 4.7 Output voltage frequency spectrum for the fault in Example 2

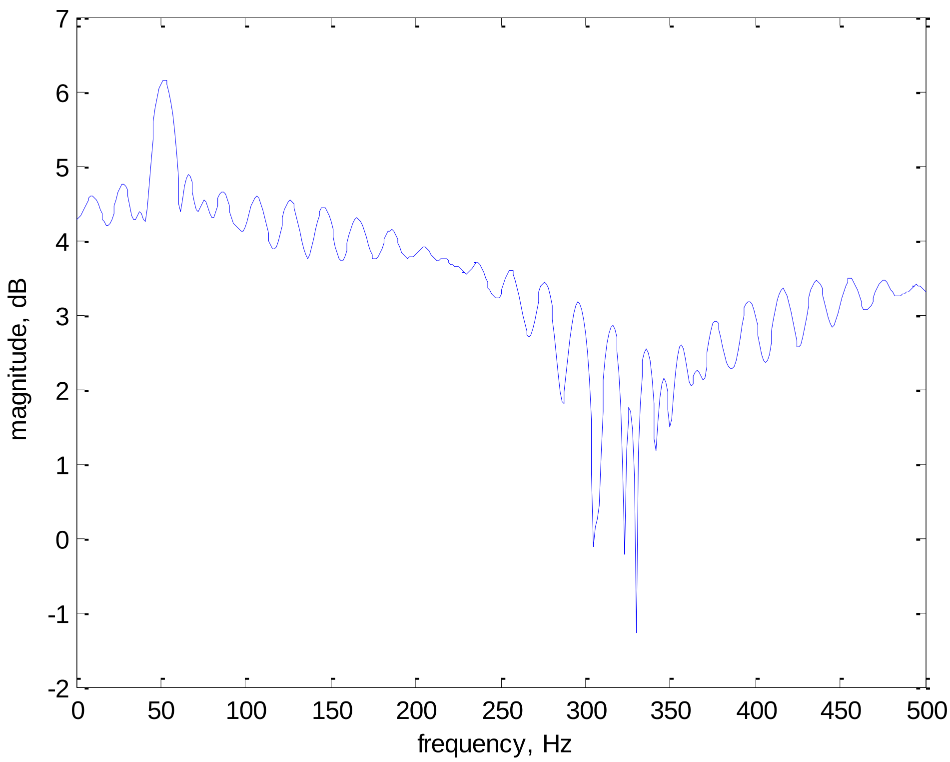


Figure 4.8 Short circuit current frequency spectrum for the fault in Example 2

We can see many peaks in the output voltage and short circuit current frequency domain response because the voltage and current sinusoidal waveform is distorted due to the short circuit fault. These peaks shows that the short circuit fault causes harmonics on the line.

Also the output voltage and short circuit current frequency spectrum are analysed for the short circuit fault in the Example 3 as in the following figures.

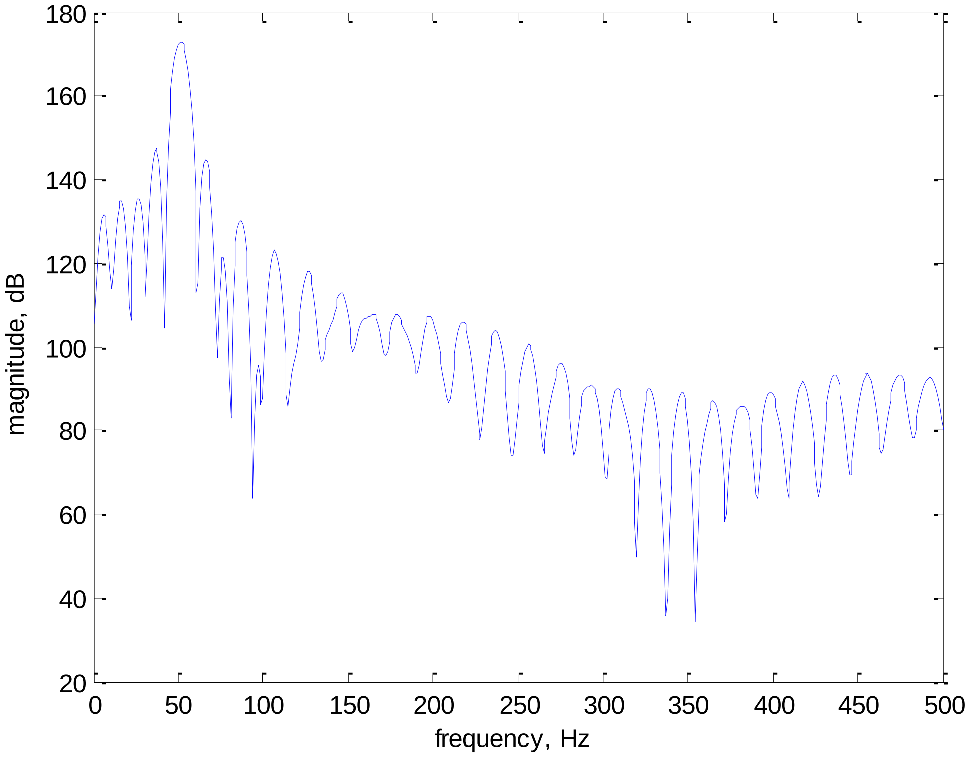


Figure 4.9 Output voltage frequency spectrum for the fault in Example 3

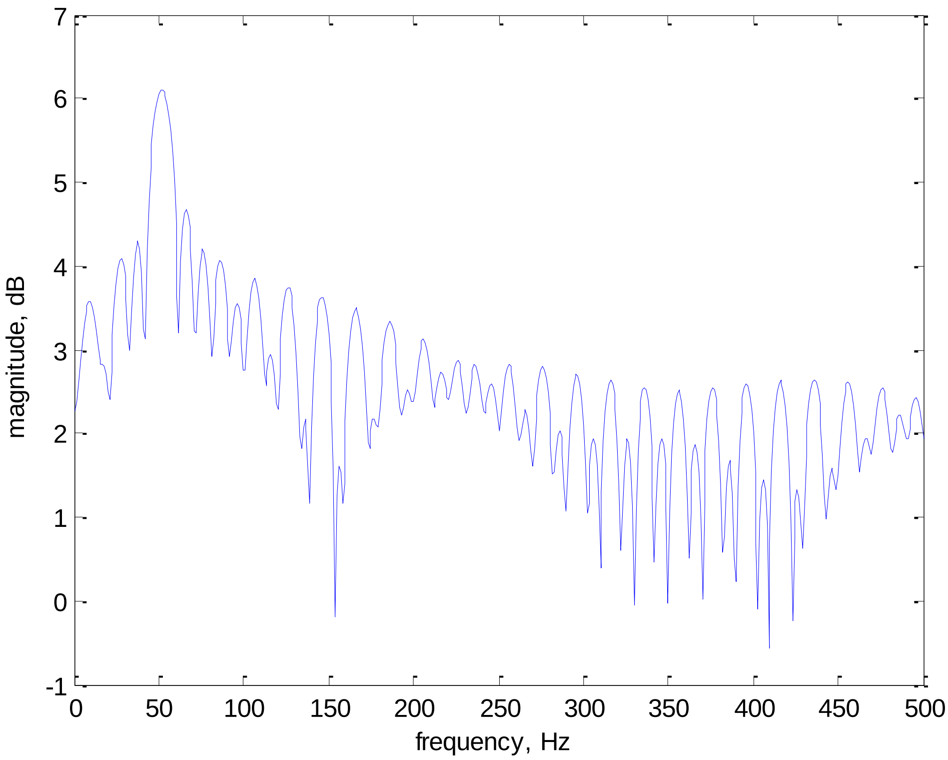


Figure 4.10 Short circuit current frequency spectrum for the fault in Example 3

4.6 Example 6: State Space Analysis of a Short Circuit Line for Different T-section Number

Up to this example, we suppose that the chosen line is approximated by 8 T-sections so every 20 km's line is modelled as one T-section. But in this example we modelled the same transmission line by 32 and 80 T-sections. The short circuit fault occurs 100th km of the line and continues 3 msec. for this example. When the fault occurs, the short circuit T-section output voltage is supposed to be zero as Example 2.

Figure 4.11 and 4.12 shows the output voltage and short circuit current changing by time for 32 T-sections circuit model and Figure 4.13 and 4.14 shows the voltage and current changing by time for 80 T-sections circuit model.

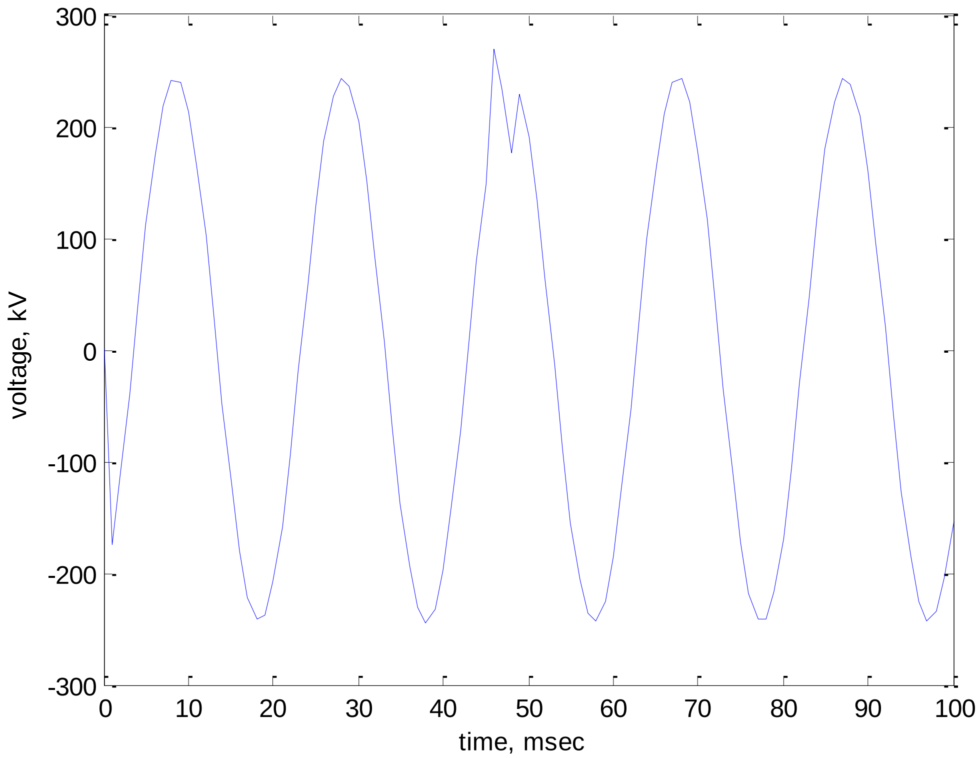


Figure 4.11 Output voltage for 32 T-section line model

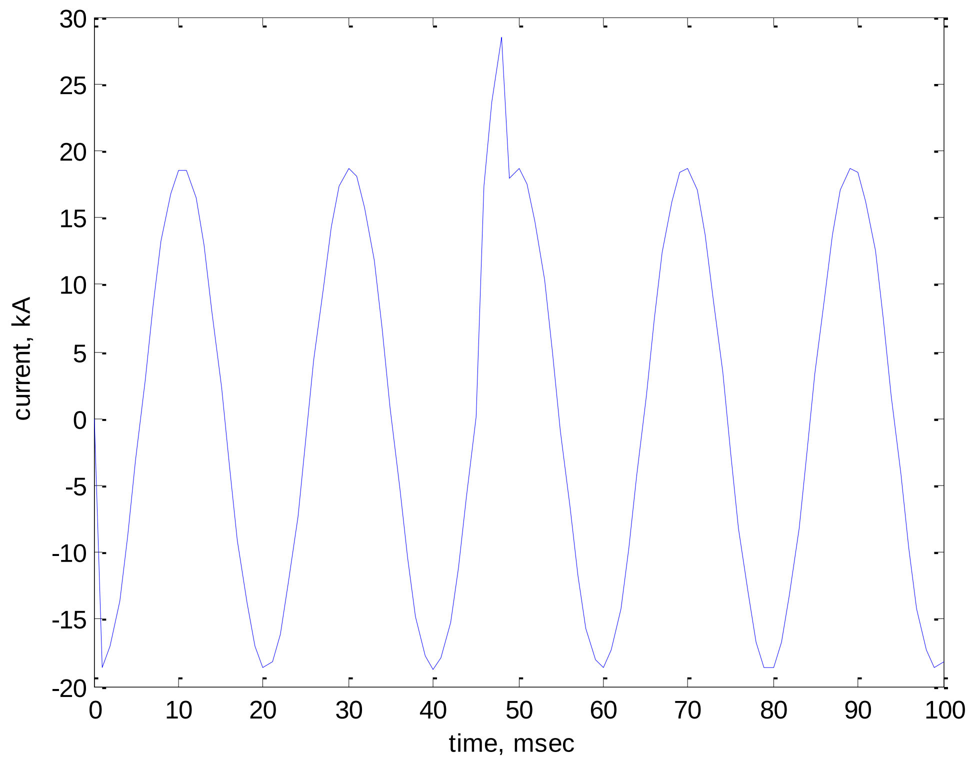


Figure 4.12 Short circuit current for 32 T-section line model

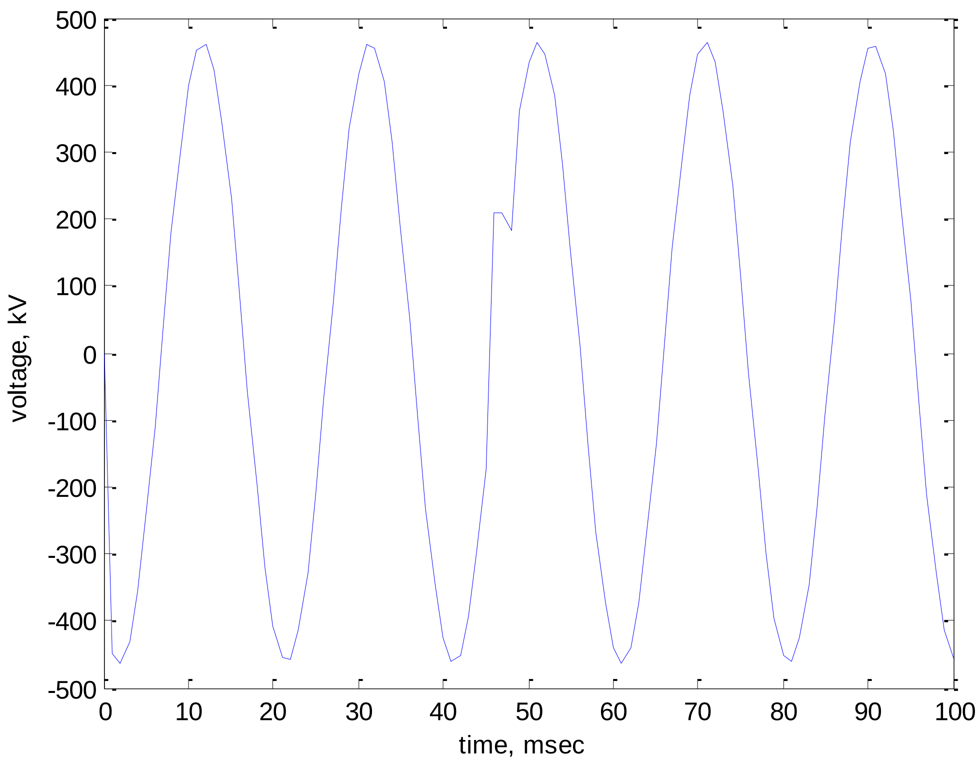


Figure 4.13 Output voltage for 80 T-section line model

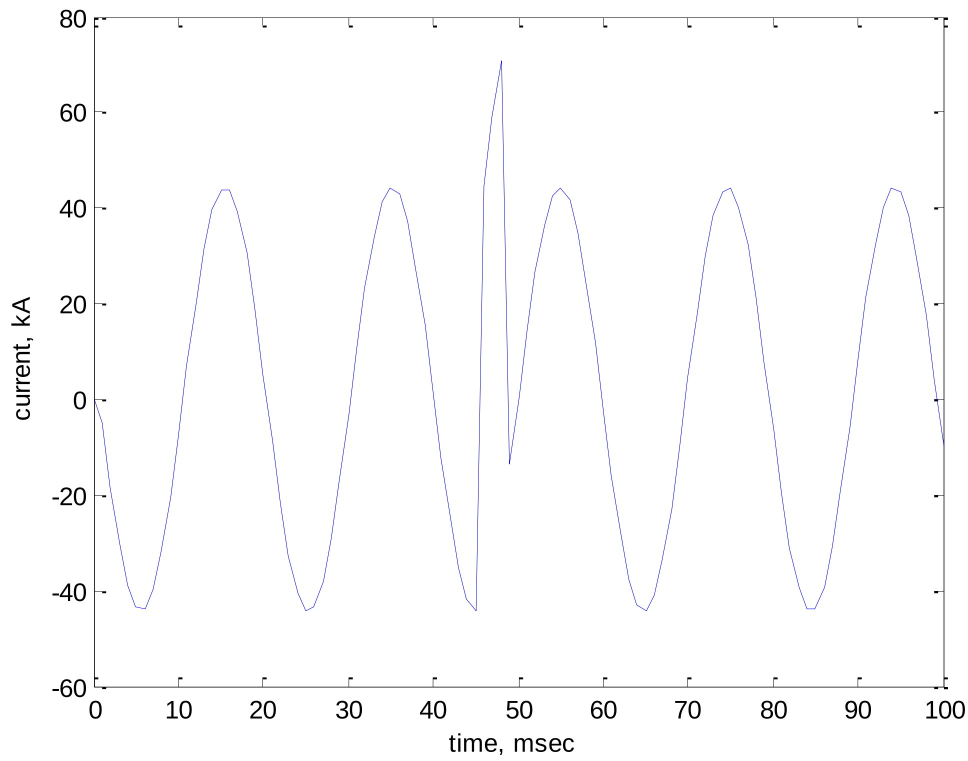


Figure 4.14 Short circuit current for 80 T-section line model

As shown in the figures, when the T-section number increases the short circuit current can not show the real changing of the current so we can say that, choosing the true T-section number is important to show the real changing of the short circuit current.

CHAPTER 5

RESULTS AND CONCLUSION

In this study, our aim is to show how a short circuit fault, that occurs on any distance of a transmission line, effects the line's voltage and current waveform. A transmission line, which *R*, *L*, *C*, *G* values are given for every kilometer of the line, is chosen. The chosen transmission line is modelled as a T-line circuit model. For this T-line circuit model first state space equations are formulated and then the state equations are modified for the transmission line by considering a short circuit on any T-section.

We assumed a sinusoidal voltage source to the T-line model sendig end for 100 msec. To show the voltage and short circuit current changing by time for short circuit T-line model, a computer program MATLAB is used. As shown by the figures in Chapter 4, when there is no fault on the line, the T-line model's output voltage and short circuit current are changing as the sinusoidal input. But when a short circuit occurs on the line, the output voltage starts to decrease and the current starts to increse during the short circuit fault. And if the short circuit is lasted long time period as the Example 3 in Chapter 4, the output voltage decreases to zero so when the voltage is zero, the short circuit current increases to the highest value that it can be reached. Also we showed the output voltage and short circuit current frequency spectrum to show the harmonic distortions that occurs because of the fault.

Using the developed MATLAB program, voltage or current distributions on a transmission line, at any time, for various transient conditions (for example short circuit or open circuit fault) can be obtained and frequency spectrum of these voltage and current can be analysed.

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