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M. Sc. - Engineering of Physics

GÜLÇİÇEK DERE

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CALCULATION OF PROMPT NEUTRON ENERGY SPECTRUM IN LABORATORY SYSTEM FROM SPONTANEOUS FISSON OF *CF*-252

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Calculation of Prompt Neutron Energy Spectrum in Laboratory System from Spontaneous Fission of *Cf*-252

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Engineering Physics

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UNIVERSITY OF GAZİANTEP

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ABSTRACT

CALCULATION OF PROMPT NEUTRON ENERGY SPECTRUM IN LABORATORY SYSTEM FROM SPONTANEOUS FISSION OF CF-252

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In this work influence of the different forms of prompt fission neutron energy spectrums was discussed in the fission fragment center of mass system to the laboratory spectrum of neutrons in spontaneous fission of ^{252}Cf (Californium-252).

It is shown that the Le Couteur spectrum which takes into account multiple neutron emission of neutrons in the center of mass system well describes the observed neutron energy spectrum when transformed to the laboratory system.

The Le Couteur in laboratory spectrum, in addition to Watt spectrum, can be used for the representation of observed prompt neutron spectrum of ^{252}Cf spontaneous fission.

Key Words: Prompt neutron, spontaneous fission, center of mass system, laboratory energy spectra.

ÖZ

CF-252 NİN KENDİLİĞİNDEN BÖLÜNMESİNDE AÇIĞA ÇIKAN ANİ NÖTRONLARIN ENERJİ DAĞILIMLARININ LABORATUVAR SİSTEMİNDE HESAPLANMASI

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Bu çalışmada, fizyon ürün çekirdeklerinin kütle merkezindeki ani fizyon nötronlarının farklı enerji dağılımlarının kendiliğinden bölünen ^{252}Cf 'nin laboratuvar sistemindeki nötron enerji dağılımlarına etkisi tartışıldı.

Kütle merkezi sisteminde çoklu nötron emisyonlarını hesaba katan Le Couter spektrumunun laboratuvar sistemine dönüştürüldüğünde deneysel olarak gözlenen nötron enerji dağılımlarını çok iyi tasvir ettiği gösterildi.

Watt spektrumuna ilaveten Le Couter laboratuvar spectrumunun da, Cf-252'nin kendiliğinden fizyonunda gözlenen ani nötron dağılımını ifade eden bir kuramsal bağıntı olarak kullanılabileceğini doğrulamıştır.

Anahtar Kelimeler: Ani nötron, kendiliğinden fizyon, kütle merkezi sistemi, laboratuar enerji dağılımı.

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I would like to express my thanks to my dear supervisor, Assist. Prof. Dr. Mehmet Koçak, not only for his guidance, encouragement, and suggestions but also for his friendship in the construction of such a work. There is no doubt that our many discussions have always been construction and instructive.

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CHAPTER 1
INTRODUCTION

Nuclear fission is the process in which a heavy nucleus splits into two fragments, with the release of considerable energy and emission of neutrons and γ-rays. Fission is particularly important among nuclear reactions. Beside the fusion reaction in the sun, no other nuclear reaction has had a deep impact on the man. The discovery and developments of the fission have changed the world evermore and have influenced the human life quite rather. After the discovery of fission the exploitation of nuclear energy, especially in weapons of mass destruction has been of profound importance to humankind.

Nuclear fission was discovered by the chemists Otto Hahn and Fritz Strassmann in 1938 (Hahn and Strassmann, 1938). They were able show the irradiation of uranium led, not to many new elements as had been thought, but to products like barium, lanthanum, etc.

Fission is important for the nuclear power industry and the related fields of nuclear waste management and environmental cleanup. From the point of view of basic research fission is interesting as large scale collective motion of the nucleus, as an important exit channel for many nuclear reactions, and as a source of neutron-rich nuclei for nuclear structure studies and use as radioactive beams (Loveland et al., 2006).

It is noted that the fission process represents a very difficult problem. Although we understand many aspects of the fission process, there is no overall theoretical framework that gives a satisfactory account of the basic observations.

Fission occurs in two ways; by induced neutrons or spontaneously. In the case of neutron induced fission thermal neutron (or fast neutron, depends on the critical energy of fissile nucleus) is absorbed by the nucleus that cause to increase in energy level of nucleus to an amount equal to the binding energy of the neutron. If the total energy supplied to the nucleus by the incident neutron (neutron binding energy plus neutron kinetic energy) exceeds the critical energy, the nucleus may fission.

Spontaneous fission which is studied in this thesis for Californium-252 nucleus gives much the same result as induced nuclear fission. However, like the other forms of radioactive decay, such as alpha decay, it occurs due to quantum tunneling, without the atom having been struck by a neutron or other particle as in induced nuclear fission. As compared to alpha decay, spontaneous fission is much more complex and there are some uncertainties in this process, such as mass and charge numbers of the fragments, the number of emitted neutrons which is called neutron multiplicity and the released energy.

Spontaneous fissions release neutrons as all fissions do, so if a critical mass is present, a spontaneous fission can initiate a self-sustaining chain reaction. Also, radioisotopes for which spontaneous fission is not negligible can be used as neutron sources. For example Californium-252 can be used for this purpose.

^{252}Cf has a number of specialized applications as a strong neutron emitter, and each of microgram of fresh californium produces 139 million neutrons per minute (Martin et al., 1999). This property makes Californium useful as a “neutron startup source” for some nuclear reactors and as a portable (non-reactor based) neutron source for neutron activation analysis to detect trace amounts of elements in samples (Martin, 2000).

Without dependence of fission type any fission results in the emission of major pieces of the original nucleus, called fission fragments or fission products, plus the release of gamma rays, beta particles, neutrinos and neutrons. The emission of neutrons from fission makes it possible to achieve a self sustaining fission chain reaction. For the neutron induced fission this reaction can be written as

$$ {}_{0}^{1}n + {}_{Z}^{A}X \rightarrow {}^{A+1}_{\ \ \ Z}Y^{*} \rightarrow F_1 + F_2 + \ {}_{0}^{1}n's + \gamma + E $$

where ${}^{A}_{Z}X$ is fissile nucleus, ${}^{A+1}_{Z}Y^{*}$ is excited compound nucleus, F_1 and F_2 are fission (heavy and light) fragments, γ is gamma ray and E is the energy produced as a result of fission.

Neutrons are released by the fission fragments immediately after their formation; on the average 2.42 neutrons are released for each fission of a ^{235}U nucleus while for ^{252}Cf spontaneous fission it is 3.76 according to ENDF/B-VII nuclear data file (Chadwick et al., 2006). These neutrons have very high velocities, but because of their lesser mass have kinetic energy only on the order of 2 MeV.

These are called "prompt" neutrons, because they are promptly emitted by the fission fragments after their formation (within 10^{-14} seconds). These prompt neutrons are slowed by collisions and eventually one neutron from a previous fission causes another fission and the chain reaction is sustained.

In this thesis simple representations of prompt fission neutron spectrum is used in both center of mass system of fission fragments and the laboratory system. While we compare theoretical spectra in the center of mass system we also compare the laboratory spectra with the observed spectrum of prompt fission neutrons in spontaneous fission of ^{252}Cf.

In Chapter 2, a theoretical background for the neutron energy spectra is given. We firstly introduce the well known center of mass spectra then transform them into their concerned laboratory forms representing the observed neutron spectra. One of the center of mass spectrum usually used is the Le Couteur form of the spectrum. This spectrum, for the first time, has been transformed to the laboratory system and used for the description of experimental spectrum in our study. Because the transformations involve tedious calculations the related details are given in Appendix C.

In Chapter 3, we first calculate the parameters; temperature and fission fragment kinetic energies for each spectra for the spontaneous fission of ^{252}Cf, then give the comparisons of the spectra obtained theoretically using different methods together with the observed spectrum.

Final chapter involves some concluding words and outcome regarding the present work in this thesis.

CHAPTER 2
NEUTRON ENERGY SPECTRA

2.1 Introduction

Investigation of prompt fission neutron spectrum keeps to be up to date for application and scientific purposes in the related area (Hambsch et al., 2005; Iwamoto, 2008; Noy, 2010; Vorobyev et al., 2010; Zeynalov et al., 2011). Theoretical study of the problem prefers to have a simple representation of the laboratory energy spectrum, although the center of mass energy spectrum of neutrons depends on many effects such as multiple neutron emissions from fission fragments, initial excitation energy distribution of fission fragments, fission fragment charge and mass distribution, competition between neutron and γ-emissions from fragments with relatively lower excitation energies and so on (Terrell, 1959; Cluge, 1971; Saveliev, 1971; Browne and Dietrich, 1974; Madland and Nix, 1982; Ahmadov et al., 2001; Hambsch et al., 2005). Almost all of these effects reduce to the softening of center of mass neutron energy spectrum with respect to the spectrum of Weisskopf evaporation theory (Weisskopf, 1937; Blatt and Weisskopf, 1952). This theory gives the following expression for the neutron energy distribution in the range $\epsilon, \epsilon + d\epsilon$

$$\Phi(\epsilon)d\epsilon = const.\, \epsilon\sigma_c(\epsilon)W(\epsilon_{max} - \epsilon)d\epsilon \qquad (2.1)$$

where $\sigma_c(\epsilon)$ is the compound nucleus formation cross section at the energy ϵ, W is the energy level density of residual nucleus, ϵ_{max} is the maximum possible energy of emitted neutron. Relating the nuclear level density with the thermodynamic temperature (Weisskopf, 1937; Blatt and Weisskopf, 1952) of residual nucleus, $T(\epsilon_{max})$, Eq.(2.1) is transformed to the form of

$$\Phi(\epsilon)d\epsilon = const.\, \epsilon\sigma_c(\epsilon)e^{-\epsilon/T}d\epsilon \qquad (2.2)$$

If we take $\sigma_c(\epsilon)$ as constant then one comes to form of Weisskopf's neutron energy spectrum,

$$\Phi_{W_f}(\epsilon)d\epsilon = const.\,\epsilon e^{-\epsilon/T} d\epsilon \qquad (2.3)$$

where "W_f" refer to the Weisskopf spectrum. When $\sigma_c(\epsilon) \sim 1/\sqrt{\epsilon}$, ($1/v$ law), then the Maxwell form of energy spectrum is obtained,

$$\Phi_M(\epsilon)d\epsilon = const.\sqrt{\epsilon} e^{-\epsilon/T} d\epsilon \qquad (2.4)$$

Note that the spectrum in Eq. (2.4) is softer than in Eq. (2.3), i.e., the portion of low energy neutrons is greater in Eq. (2.4). This is due to pre-exponential energy dependence of spectrum which, in general form, is proportional to ϵ^{λ}, where λ is 1 in Eq. (2.3) while 1/2 in Eq. (2.4). So, lower λ leads to the softer spectrum. For the so called Le Couteur spectrum (Lang and Le Couteur, 1954; Le Couteur and Lang, 1959), $\lambda = 5/11$. This spectrum was calculated in the framework of cascade neutron evaporation from a highly excited nucleus and compared with experimental results for neutron evaporation mechanism in Bi(p,n)Po reaction with $190\ MeV$ incident protons. Le Couteur form of neutron energy spectrum is in this case expressed in the form

$$\frac{1}{\sigma_c(\epsilon)}\Phi(\epsilon)d\epsilon = const.\,\epsilon^{\ell-1} e^{-\epsilon/T} d\epsilon \qquad (2.5)$$

where $\ell \approx 16/11$, $T \approx (11/12)\,T_m$ within the Fermi-Gas model and T_m is the temperature corresponding to first neutron emission. The expression in Eq. (2.5) can be written in the usual form of

$$\Phi(\epsilon) = const.\,\epsilon^{5/11} e^{-\epsilon/T} \qquad (2.6)$$

Hence, we note that excitation energy distribution of initial fission fragments shows the softening effect (Piksaikin et al., 1978; Ahmadov and Stavinsky, 1979; Ahmadov et al., 2001) similar with cascade neutron evaporation from compound nucleus expressed in Eq.(2.6). Thus, the spectrum in Eq.(26) may be used in description of fission neutron spectrum.

In this thesis work, it is shown that the Le Couteur spectrum, which is softer than Weisskopf and Maxwell spectra, in its laboratory form gives better representation of the evaluated data (Mannhart, 2008) comparing with the Watt spectrum when calculation is done for the two average complementary fragments.

2.2 Center of mass and laboratory spectra

The energy spectra forms of Weisskopf, Maxwell and Le Couteur when normalized to unity in the energy range of emitted neutrons, $(0,\infty)$, are

$$\Phi_{W_f}(\epsilon) = \frac{\epsilon}{T_{Wf}{}^2} e^{-\epsilon/T_{Wf}} \qquad (2.7)$$

$$\Phi_M(\epsilon) = \frac{2\sqrt{\epsilon}}{\sqrt{\pi T_M{}^3}} e^{-\epsilon/T_M} \qquad (2.8)$$

$$\Phi_{LC}(\epsilon) = \frac{\epsilon^{5/11} e^{-\epsilon/T_{LC}}}{\Gamma(16/11) T_{LC}{}^{16/11}} \qquad (2.9)$$

where T_{Wf}, T_M *a*nd T_{LC} are the corresponding temperature parameters, respectively and $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$. Average energies of these spectra are, correspondingly

$$\bar{\epsilon}_{Wf} = 2T_{Wf}, \quad \bar{\epsilon}_M = \frac{3}{2}T_M, \quad \bar{\epsilon}_{LC} = \frac{16}{11}T_{LC} \tag{2.10}$$

Laboratory spectrum, for isotropic distribution of neutrons in the center of mass system of fission fragment having constant speed is

$$N(E) = \int_{(\sqrt{E}-\sqrt{E_f})^2}^{(\sqrt{E}+\sqrt{E_f})^2} \frac{\Phi(\epsilon)d\epsilon}{4\sqrt{\epsilon E_f}} \tag{2.11}$$

where E_f is the kinetic energy of fission fragment per nucleon. Laboratory spectra corresponding to center of mass system, given by Eqs. (2.7-2.9), have respectively forms of;

$$N_F(E) = \frac{1}{4\sqrt{E_f T}}\left[\left(-x_1 e^{-x_1{}^2} + \text{erf}\,(\text{x}_1)\right) - \left|\left(-x_2 e^{-x_2{}^2} - \text{erf}\,(x_2)\right)\right|\right] \tag{2.12}$$

where $x_1 = \sqrt{\frac{E}{T}} + \sqrt{\frac{E_f}{T}}$, $x_2 = \sqrt{\frac{E}{T}} - \sqrt{\frac{E_f}{T}}$ and $\text{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$

$$N_W(E) = \frac{1}{\sqrt{\pi T E_f}} e^{-E_f/T}\, e^{-E/T} \sinh\left(\frac{2}{T}\sqrt{EE_f}\right) \tag{2.13}$$

and

$$N_{LC}(E) = \frac{\Gamma(^{21}/_{22})}{\Gamma(^{16}/_{11})}\frac{1}{4\sqrt{E_f T}}\left[\Gamma\left(\frac{(\sqrt{E}+\sqrt{E_f})^2}{T}, \frac{21}{22}\right) - \Gamma\left(\frac{(\sqrt{E}-\sqrt{E_f})^2}{T}, \frac{21}{22}\right)\right] \tag{2.14}$$

where $\Gamma(x, a)$ is the incomplete gamma function defined by

$$\Gamma(x, a) = \frac{1}{\Gamma(a)}\int_0^x e^{-t}\, t^{a-1}dt \tag{2.15}$$

Here N_F is the Feather spectrum (Feather, 1942) while N_W is the Watt spectrum (Watt, 1952) and N_{LC} is the Le Couteur spectrum which is transformed to the laboratory system. The detailed calculation for the derivations of Eqs. (2.12-2.14) is given in Appendix A, B and C. Note that, throughout this work we use LCL abbreviation for the Le Couteur laboratory spectrum. It is also noted that, Terrell in his work (Terrell, 1959) has shown that single Feather spectrum cannot represent experimental data. This spectrum is averaged over the temperature distribution of fission fragment with seven temperature values for each of two complimentary fragments well approximates the data for ^{252}Cf spontaneous fission and thermal neutron induced fission of ^{235}U. We use Eq. (2.12) to compare only with other theoretical spectra. To the best of our knowledge the spectrum in Eq. (2.14) has not been used in the previous reports in the related literature for the description of observed neutron energy spectrum. Although a spectrum which is similar to that in Eq. (2.14) was given in the work (Brosa and Knitter, 1988), they however used this spectrum for the two limiting values of λ= 1 and ½.

CHAPTER 3

PARAMETER ESTIMATIONS AND CALCULATIONS

To compare different spectra with each other and with experimental spectrum for the spontaneous fission of ^{252}Cf we must define firstly the temperature parameters for each of Eqs. (2.7-2.9). These parameters are chosen due to the equality of average energies to that of observed laboratory spectrum, $2.13\ MeV$. Average energy in the laboratory system for the fragment of given mass, charge and kinetic energy is defined as

$$\bar{E}(Z,A) = \overline{E_f}(Z,A) + \bar{\epsilon}(Z,A). \tag{3.1}$$

This formula may be written for the average light and heavy fragments, approximately as

$$\bar{E} = \overline{E_f} + \bar{\epsilon} \tag{3.2}$$

Our calculations are carried out both for the average single fission fragment and for the two complementary average fission fragments. For the average single fission fragment we take $E_f = 0.768\ MeV$ as in the work (Madland and Nix, 1982). Then, the temperature parameters in equations (2.7) to (2.9) are being, $T_{Wf} = 0.681\ MeV$, $T_M = 0.91\ MeV$ and $T_{LC} = 0.936\ MeV$, respectively. Following first five figures, which are drawn by using these parameters. Figure 3.1 gives the center of mass spectra calculated from equations (2.7) to (2.9) and Figure 3.2 illustrates the ratio of this spectra to the Maxwell spectrum in the center of mass system. As it is expected, the Le Couteur spectrum in the center of mass is closer to the Maxwellian spectrum than the Weisskopf spectrum is to the Maxwellian.

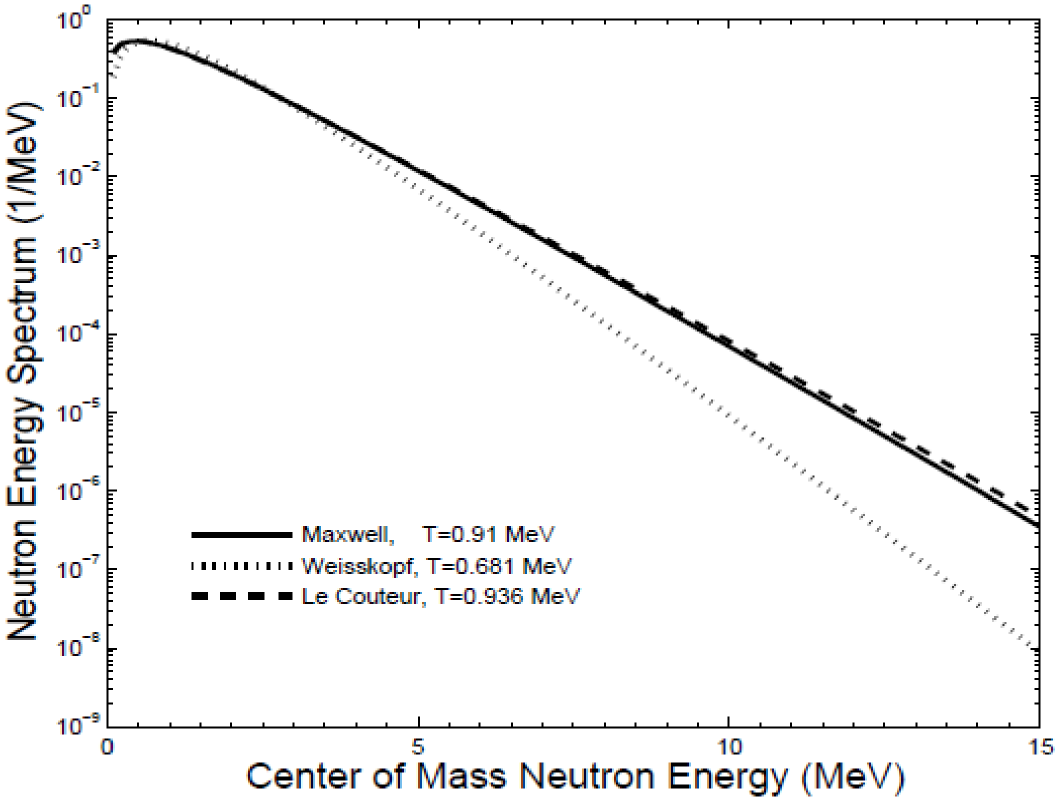

Figure 3.1 Comparisons of the center of mass neutron energy spectra.

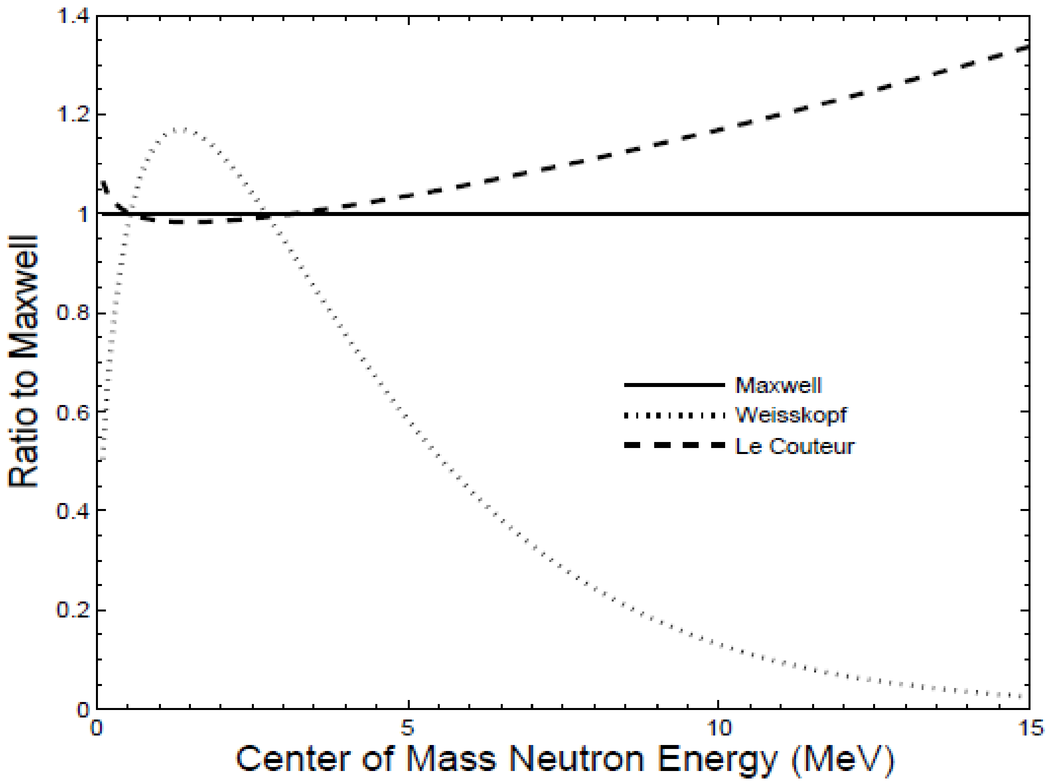

Figure 3.2 Ratio of the Weisskopf and Le Couteur spectra to the center of mass Maxwellian spectrum.

In Figure 3.3, the laboratory spectra, calculated by using Eqs. (2.12) to (2.14), are illustrated. Figure 3.4 shows the ratio of laboratory spectra to Watt spectrum. As seen in Figure 3.4, the LCL spectrum is closer to the Watt spectrum than the Le Couteur spectrum is to the Maxwellian spectrum in center of mass in Figure 3.2. This is due to the averaging effect in fission fragment motion. Figure 3.5 illustrates comparison of laboratory spectra given by Eqs. (2.12) to (2.14) with the Maxwell spectrum having $T = 1.42\, MeV$ temperature value which is recommended by ENDF (Evaluated Nuclear Data File) for spontaneous prompt fission neutron spectrum of ^{252}Cf. As it is evident from Figure 3.5, the LCL spectrum closer to the Maxwellian spectrum than the Watt spectrum is to the Maxwellian.

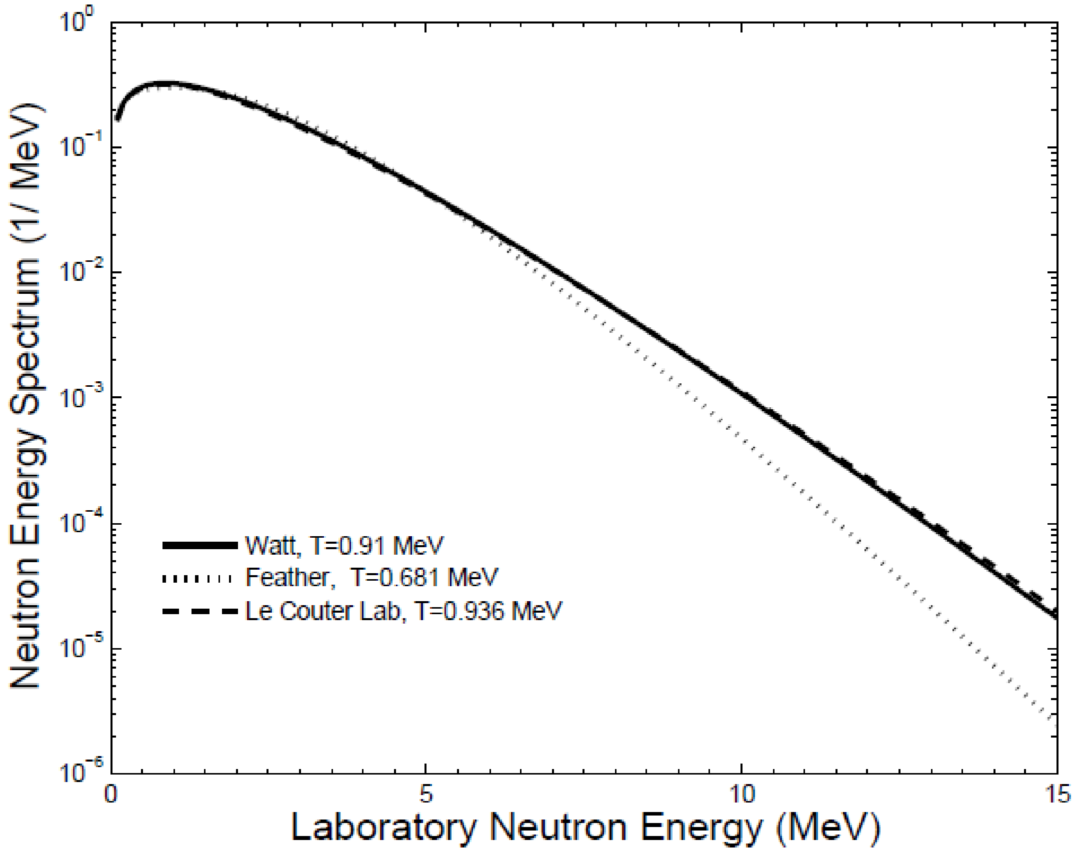

Figure 3.3 Comparisons of the laboratory neutron energy spectra for E_f =0.768 MeV.

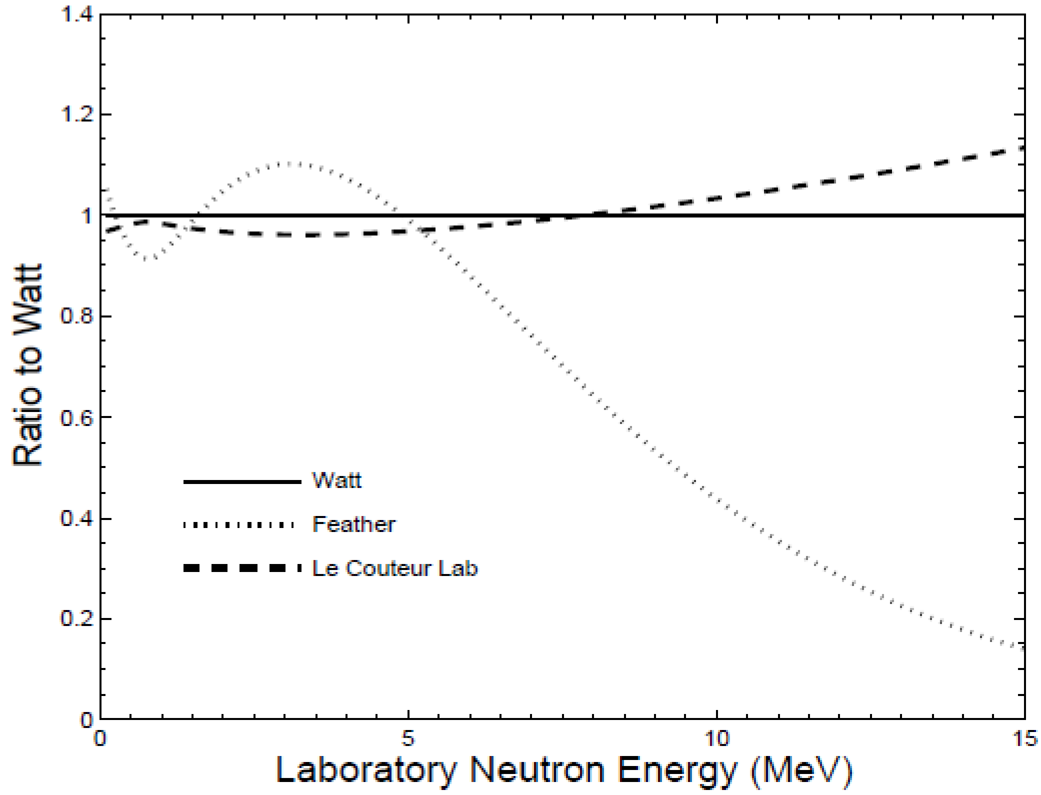

Figure 3.4: Ratio of the Feather and Le Couteur in lab spectra to the Watt spectrum.

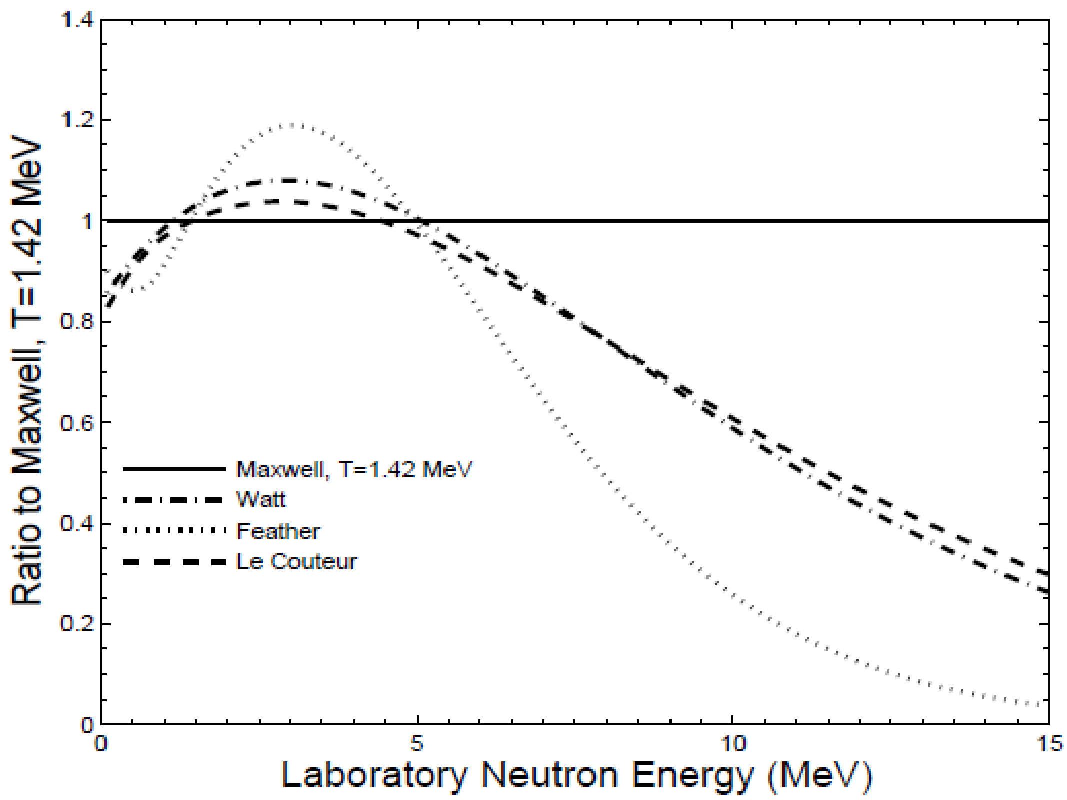

Figure 3.5 Ratio of the laboratory spectra to the Maxwellian spectrum with T = 1.42 *MeV*

Now, we perform the calculations for the two complementary average fission fragments with taking ${\bar{E}_f}^L = 0.984\ MeV$ and ${\bar{E}_f}^H = 0.553\ MeV$ as in (Madland and Nix, 1982) and using the expression for the average energy in laboratory system for the complementary fission fragments,

$$\bar{E} = \frac{1}{2}({\bar{E}_f}^L + {\bar{E}_f}^H) + \frac{1}{2}({\bar{\epsilon}_f}^L + {\bar{\epsilon}_f}^H) \tag{3.3}$$

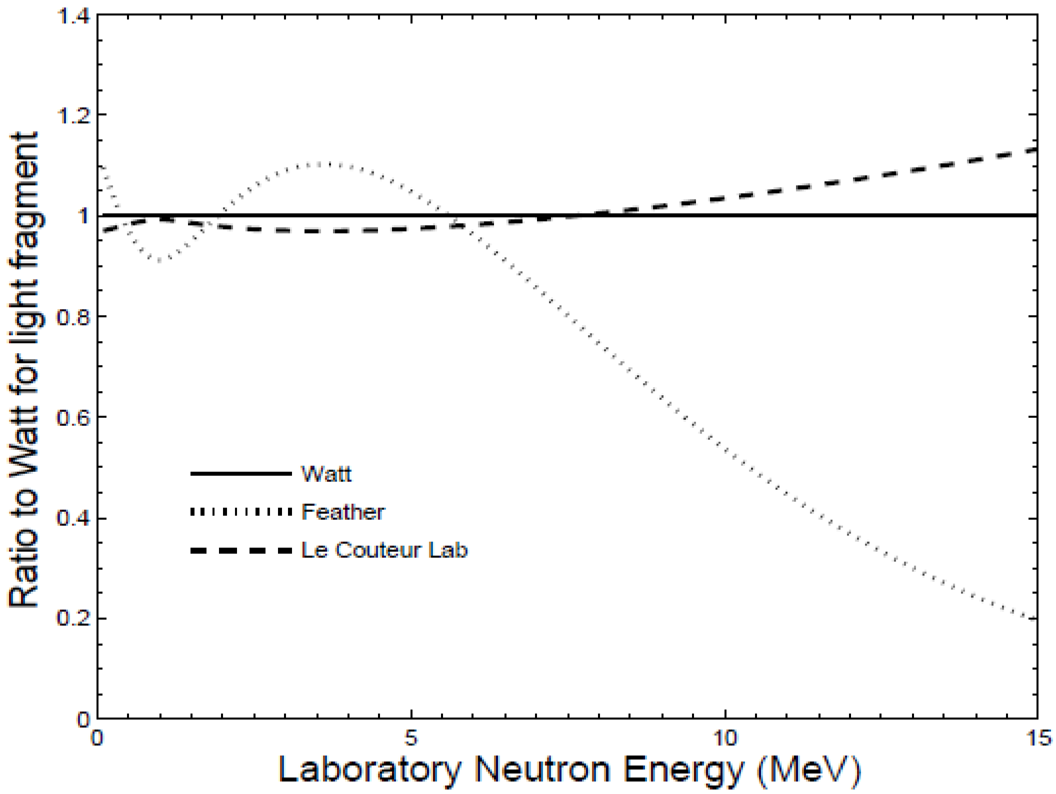

Figure 3.6 Ratio to the Watt spectrum for the light fragment in laboratory spectra.

where indices L and H indicate light and heavy fragments. Eq. (3.2) yields $\bar{\epsilon}^L + \bar{\epsilon}^H = 2.74\ MeV$. From experimental data of Bowman et al. (Bowman et al., 1962), $\bar{\epsilon}^L = 1.42\ MeV$ and $\bar{\epsilon}^H = 1.3\ MeV$ for the complementary average fission fragments $\bar{A}^L = 108$ and $\bar{A}^H = 144$, respectively. Interestingly, the sum of these data gives $2.72\ MeV$ which is very close to calculated result. From the average energies we define the temperature parameters for the light and heavy fragments, respectively as, ${T_{Wf}}^L = 0.71\ MeV$, ${T_M}^L = 0.947\ MeV$, ${T_{LC}}^L = 0.976\ MeV$ and ${T_{Wf}}^H = 0.65\ MeV$, ${T_M}^H = 0.87\ MeV$, ${T_{LC}}^H = 0.89\ MeV$

Figures 3.6 and 3.7 illustrate the ratio of the laboratory spectra for light and heavy fragments to the Watt spectrum, respectively, for the parameters discussed above. These figures illustrate more clearly, the difference between calculated spectra in the whole range of the neutron energy.

Figure 3.8 illustrates mean laboratory spectra calculated by

$$N = \frac{1}{2}(N_L + N_H) \tag{3.4}$$

where N_L and N_H are light and heavy fission fragment neutron spectra; calculated from Eqs. (2.12) to (2.14). Comparison of Figure 3.8 with Figure 3.5 shows that the calculated mean neutron spectra are closer to the Maxwell spectrum with $T = 1.42\ MeV$ than the calculated neutron spectra for average single fragment.

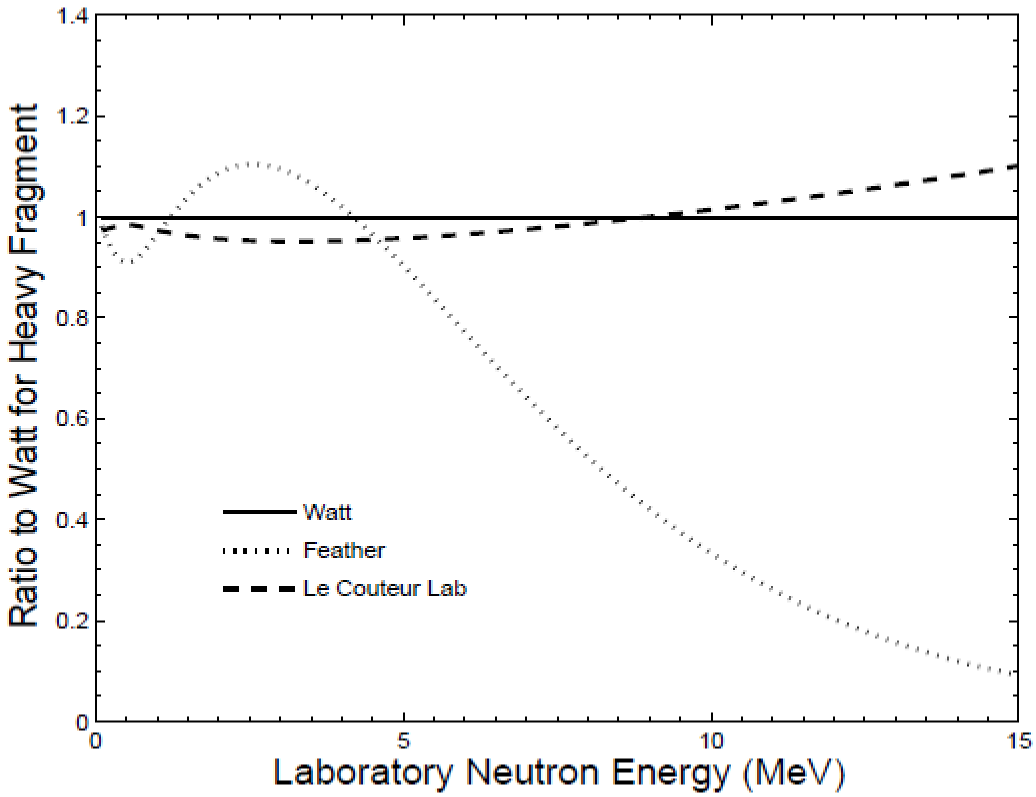

Figure 3.7 Ratio to the Watt spectrum for the heavy fragment in laboratory spectra.

Comparison of the calculated mean laboratory neutron energy spectra with the corresponding evaluated experimental data of Mannhart (Mannhart, 2008) are illustrated in Figure 3.9. As observed in Figure 3.9 the laboratory Le Couteur spectrum well approximates the data in a wide energy range.

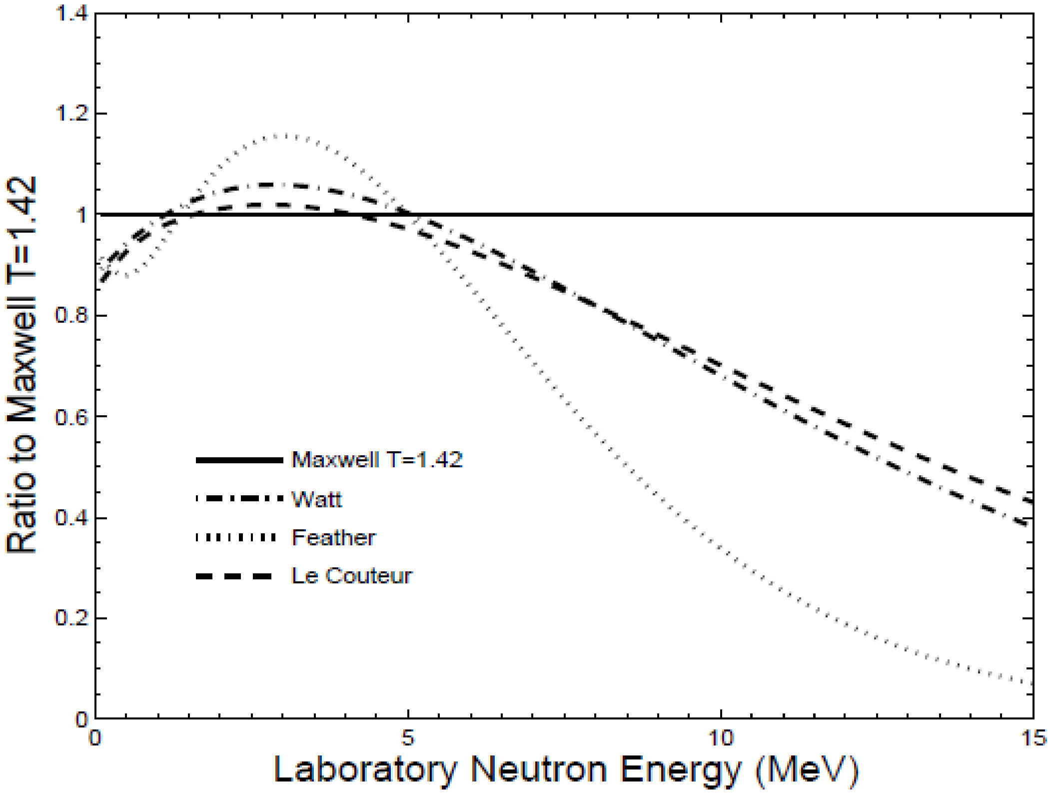

Figure 3.8 Comparison of the mean laboratory spectra as a ratio to the Maxwellian with $T = 1.42 MeV$.

In Figure 3.10, we also illustrate the comparison of the calculated mean laboratory Le Couteur spectrum with the related experimental data of Starostov et al. (Starostov et al., 1979), which was approximated with Maxwellian of $T = 1.428\ MeV$. The agreement between the data and calculation results is satisfactory in a wide region of neutron energies except in the low energy values ($< 0.7 MeV$).

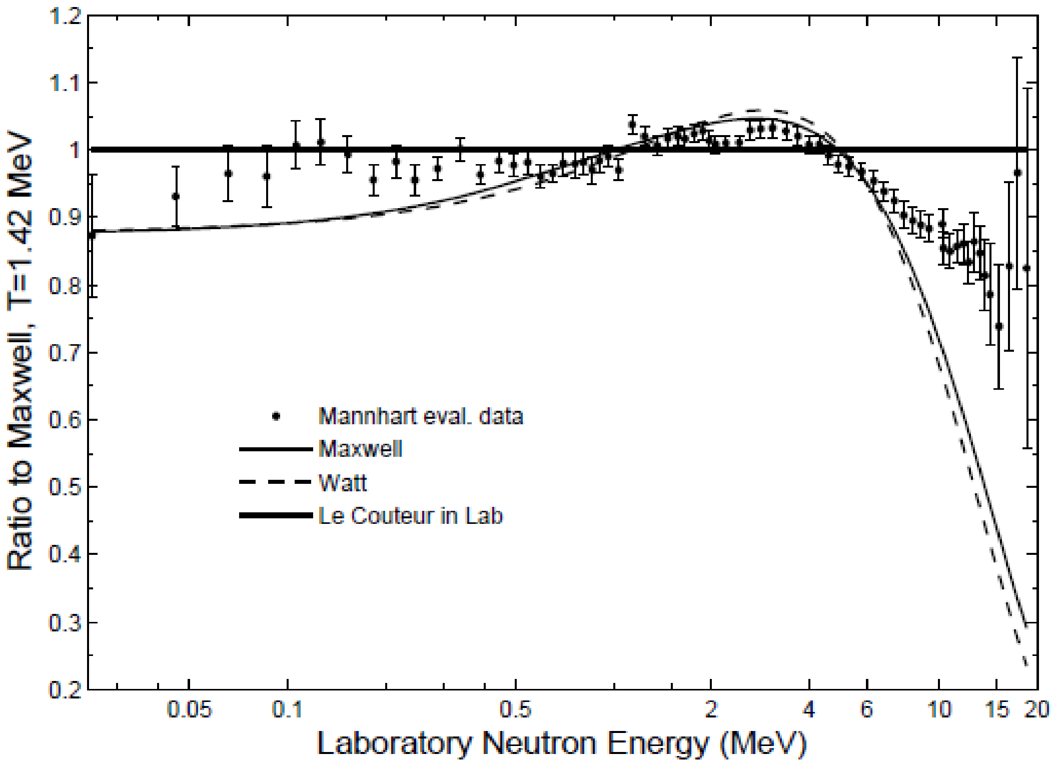

Figure 3.9: Comparison of the calculated mean neutron energy spectra with the evaluated experimental data of Mannhart as a ratio to the Maxwellian spectrum.

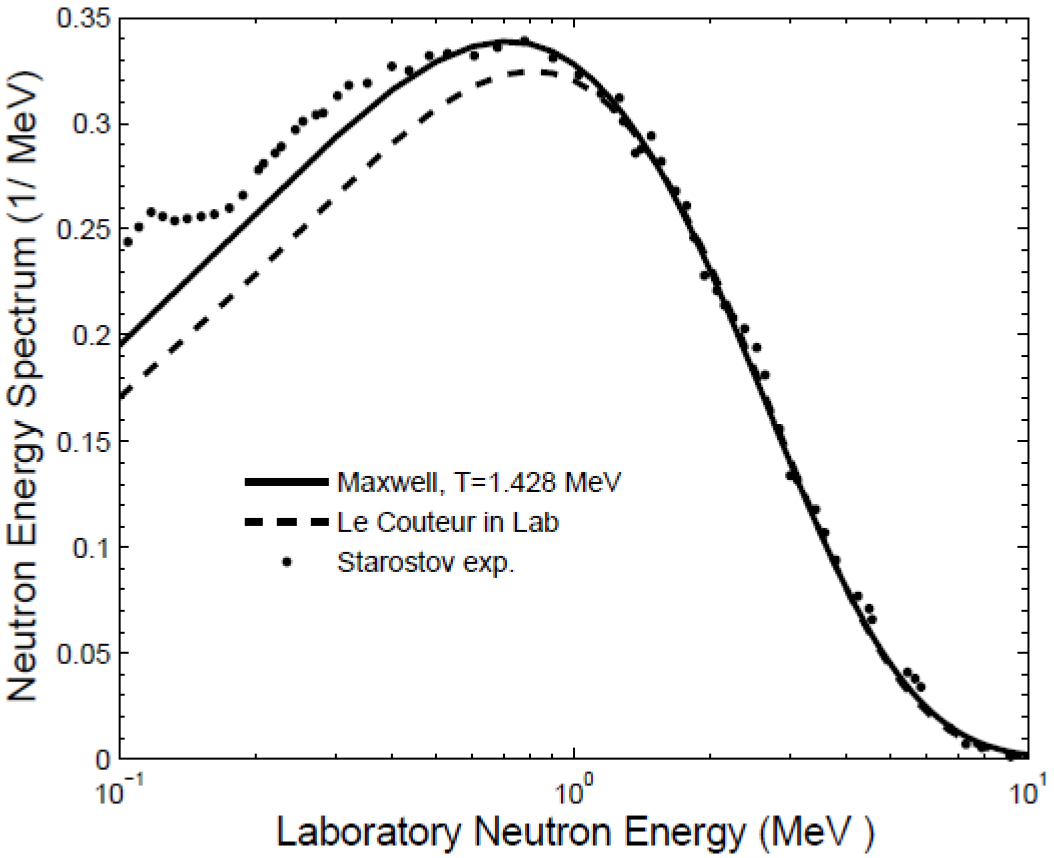

Figure 3.10 Comparison of calculated mean Le Couteur in lab spectrum with the experimental data of Starostov and Maxwellian spectrum with T =1.428.

CHAPTER 4

DISCUSSION AND CONCLUSION

Within the asymmetric fission mechanism model and the neutron isotropic emission mechanism in the center of mass system of fully accelerated complementary fragments, a theoretical analysis of prompt fission neutron spectrum in spontaneous fission of ^{252}Cf is presented both in the center of mass of fission fragments and the laboratory system.

Using simple representation models, like Maxwell and Watt spectrums, always desirable for application purposes. In this context, our calculation method considers simple representations of laboratory spectrum by use of the center of mass spectrum in the Le Couteur form. It is shown that while this spectrum has a similar behavior with Maxwellian in the center of mass, when transformed into the laboratory system it gives somewhat better representation for the observed laboratory spectrum than the Watt spectrum.

It should be noted that there is a direct relation between the softening effect and the description of experimental data. As any center of mass spectrum is described in the form of $\epsilon^{\lambda}e^{-\epsilon/T}$, the softening effect can be represented by the value of "λ". While "λ" is $5/11$ for Le Couteur spectrum it is $1/2$ for Maxwell spectrum. Because the smaller "λ" leads to the softer spectrum, it can be said that the Le Couteur spectrum is softer than the Maxwell spectrum. Hence, the laboratory form of Le Couteur spectrum well describes the observed neutron energy spectrum than the Watt spectrum which is the laboratory form of the Maxwell spectrum.

Our theoretical calculations are compared mainly with the evaluated experimental data of Mannhart. Because, these evaluated data consist of six different experiments and they well reflect the present status of neutron spectrum of spontaneous fission of Cf-252. Starostov's data is another important experimental data. Although it is not included in the Mannhart's evaluation it's importance increasing recently, especially in the reports of International Nuclear Data Center (INDC). Thus, we have compared our calculation results with also this data. It is observed that in both comparisons, with evaluated data and with Starostov's data, the LCL spectrum is in well agreement with the corresponding experimental data.

As a final remark, we particularly emphasize that the formula given by Eq. (2.14), which is the LCL spectrum, may be used in addition to the Watt spectrum, for representations of the observed prompt fission neutron spectra in the spontaneous fission of ^{252}Cf.

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APPENDIX A

Transformation from Maxwellian to Watt Spectrum:

The center of mass Maxwellian spectrum can be expressed as in Eq. (8),

$$\Phi_M(\epsilon) = \frac{2\sqrt{\epsilon}}{\sqrt{\pi T_M{}^3}} e^{-\epsilon/T_M} \quad (A.1)$$

Using the transformation function given in Eq. (11) for Maxwellian spectrum

$$N(E) = \int \frac{\Phi(\varepsilon)d\varepsilon}{4\sqrt{E_f\varepsilon}} = \frac{1}{4\sqrt{E_f\varepsilon}} \int \frac{2}{\sqrt{\pi}T^{3/2}} \frac{\sqrt{\varepsilon}e^{-\varepsilon/T}d\varepsilon}{\sqrt{\varepsilon}} d\varepsilon = \frac{1}{2\sqrt{\pi}\sqrt{E_f}T^{3/2}} \int_{(\sqrt{E}-\sqrt{E_f})^2}^{(\sqrt{E}+\sqrt{E_f})^2} e^{-\varepsilon/T} d\varepsilon \quad (A.2)$$

$$= \frac{1}{2\sqrt{\pi}\sqrt{E_f}T^{3/2}} (-T)(e^{-\varepsilon/T}) \Big|_{(\sqrt{E}-\sqrt{E_f})^2}^{(\sqrt{E}+\sqrt{E_f})^2}) = \frac{1}{2\sqrt{\pi E_f T}} (e^{-(\sqrt{E}+\sqrt{E_f})^2/T} - e^{-(\sqrt{E}-\sqrt{E_f})^2/T})$$

$$= \frac{(-1)}{2\sqrt{\pi E_f T}} (e^{-\left(E/T + E_f/T + 2\sqrt{E}\sqrt{E_f/T}\right)} - e^{-\left(E/T + E_f/T - 2\sqrt{E}\sqrt{E_f/T}\right)}$$

$$= \frac{(-1)}{\sqrt{\pi E_f T}} e^{-E/T} e^{-E_f/T} (\frac{e^{2\sqrt{E}\sqrt{E_f/T}} - e^{-2\sqrt{E}\sqrt{E_f/T}}}{2})$$

Then, we get,

$$\mathrm{N(E)} = \frac{e^{-E_f/T}}{(\pi E_f T)^{1/2}} e^{-E/T} \sinh\,[2(EE_F)^{1/2}/T] \quad (A.3)$$

This equation (distribution) is called as the Watt spectrum.

APPENDIX B

Transformation from Weisskopf's Spectrum to Feather Spectrum:

The center of mass Weisskopf spectrum can be expressed as in Eq. (7),

$$\Phi_{W_f}(\epsilon) = \frac{\epsilon}{T_{Wf}{}^2} e^{-\epsilon/T_{Wf}} \qquad (B.1)$$

Using the transformation function given in Eq. (11) for Weisskopf spectrum

$$N(E) = \int_{(\sqrt{E}-\sqrt{E_f})^2}^{(\sqrt{E}+\sqrt{E_f})^2} \frac{\phi(\varepsilon)d\varepsilon}{4\sqrt{E_f\varepsilon}} = \frac{1}{4\sqrt{E_f}} \int \frac{(\varepsilon/T^2)e^{-\varepsilon/T}}{\sqrt{\varepsilon}} d\varepsilon = \frac{1}{4\sqrt{E_f}} \int \frac{\sqrt{\varepsilon}}{T^2} e^{-\varepsilon/T}\, d\varepsilon \qquad (B.2)$$

$$N(E) = \frac{1}{4\sqrt{E_f}} \left[\frac{-\sqrt{\varepsilon}}{Te^{\varepsilon/T}} - \sqrt{\pi\varepsilon}\, erfc(\sqrt{\varepsilon/T}) \frac{1}{2T\sqrt{\varepsilon/T}} \right]_{(\sqrt{E}-\sqrt{E_f})^2}^{(\sqrt{E}+\sqrt{E_f})^2} \qquad (B.3)$$

$$N(E) = \frac{1}{4\sqrt{E_f}} \left[-\frac{\sqrt{E}+\sqrt{E_f}}{(\sqrt{E}+\sqrt{E_f})^2/T} - \sqrt{\pi}\left(\sqrt{E}+\sqrt{E_f}\right) erfc\left(\frac{\sqrt{E}+\sqrt{E_f}}{\sqrt{T}}\right) \frac{1}{2\sqrt{T}(\sqrt{E}+\sqrt{E_f})} \right] -$$

$$\left[-\frac{\sqrt{E}-\sqrt{E_f}}{(\sqrt{E}-\sqrt{E_f})^2/T} - \sqrt{\pi}\left(\sqrt{E}-\sqrt{E_f}\right) erfc\left(\frac{\sqrt{E}-\sqrt{E_f}}{\sqrt{T}}\right) \frac{1}{2\sqrt{T}(\sqrt{E}-\sqrt{E_f})} \right] \qquad (B.4)$$

where

$$erfc(x) = 1 - \mathrm{erf}(x) = \frac{2}{\pi}\int_0^\infty \exp(-t^2)\, dt$$

and

$$erf(x) = 1 - \mathrm{erf}(x) = \frac{2}{\pi}\int_0^x \exp(-t^2)\, dt$$

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}(\frac{1}{2}\int_{-x}^{x} \exp\,(-t^2)dt) = \frac{1}{\sqrt{\pi}}\int_{-x}^{x} \exp\,(-t^2)dt$$

as

$$t = \frac{u}{\sqrt{2}} \Rightarrow dt = \frac{du}{\sqrt{2}} \;\; ; \;\; t^2 = \frac{u^2}{2}$$

Then,

$$\mathrm{erf}(x) = \frac{1}{\sqrt{\pi}}\int_{-x}^{x} \exp\,(-\frac{u^2}{2})\frac{du}{\sqrt{2}} = \frac{1}{\sqrt{2\pi}}\int_{-x}^{x} exp\left(-\frac{u^2}{2}\right) du$$

$$erfc(x) = 1 - \frac{1}{\sqrt{2\pi}}\int_{-x}^{x} exp\left(-\frac{u^2}{2}\right) du$$

Therfore,

$$\mathrm{N(E)}=\frac{-1}{4\sqrt{E_f}}\frac{(\sqrt{E}+\sqrt{E_f})}{T} exp\left[-\,{}^{(\sqrt{E}+\sqrt{E_f})^2}\!/_{T}\right] - \frac{1}{2}\sqrt{\frac{\pi}{T}}\frac{1}{4\sqrt{E_f}}(1 - \frac{1}{\sqrt{2\pi}}\int_{-x_1}^{x_1} exp\left(-\frac{u^2}{2}\right) du \; +$$

$$\frac{1}{4\sqrt{E_f}}\frac{(\sqrt{E}-\sqrt{E_f})}{T} exp\left[-\,{}^{(\sqrt{E}-\sqrt{E_f})^2}\!/_{T}\right] - \frac{1}{2}\sqrt{\frac{\pi}{T}}\frac{1}{4\sqrt{E_f}}(1 - \frac{1}{\sqrt{2\pi}}\int_{-x_2}^{x_2} exp\left(-\frac{u^2}{2}\right) du \quad (B.5)$$

where $x_1 = \frac{(\sqrt{E}+\sqrt{E_f})}{T}$; $x_2 = \frac{(\sqrt{E}-\sqrt{E_f})}{T}$, which leads to

$$\mathrm{N(E)} = -\frac{1}{4\sqrt{E_f}}\frac{(\sqrt{E}+\sqrt{E_f})}{T}\exp(-{x_1}^2) - \frac{1}{8}\sqrt{\frac{\pi}{TE_f}} + \frac{1}{8}\sqrt{\frac{\pi}{TE_f}}\mathrm{erf}(x_1) + \frac{1}{4\sqrt{E_f}}\frac{(\sqrt{E}-\sqrt{E_f})}{T}\exp(-{x_2}^2) + \frac{1}{8}\sqrt{\frac{\pi}{TE_f}} - \frac{1}{8}\sqrt{\frac{\pi}{TE_f}}\mathrm{erf}\,(x_2) \qquad (B.6)$$

in which,

$$\exp(-{x_1}^2) = \exp\left[-\left(\sqrt{\frac{E}{T}}+\sqrt{\frac{E_f}{T}}\right)^2\right] = \exp\left[-\frac{1}{2}\left(\sqrt{\frac{2E}{T}}+\sqrt{\frac{2E_f}{T}}\right)^2\right]$$

and

$$\exp(-{x_2}^2) = \exp\left[-\left(\sqrt{\frac{E}{T}}+\sqrt{\frac{E_f}{T}}\right)^2\right] = \exp\left[-\frac{1}{2}\left(\sqrt{\frac{2E}{T}}+\sqrt{\frac{2E_f}{T}}\right)^2\right]$$

Finally, we have

$$N(E) = -\frac{\sqrt{2}}{8\sqrt{TE_f}}\frac{(\sqrt{E}+\sqrt{E_f})}{T}\exp\left(-{x_1'}^2\right)+\frac{1}{8}\sqrt{\frac{\pi}{TE_f}}\operatorname{erf}(x_1)+$$

$$\frac{\sqrt{2}}{8\sqrt{TE_f}}\frac{(\sqrt{E}-\sqrt{E_f})}{T}\exp\left(-{x_2'}^2\right)-\frac{1}{8}\sqrt{\frac{\pi}{TE_f}}\operatorname{erf}(x_2)$$

where $x_1' = \sqrt{\frac{2E}{T}}+\sqrt{\frac{2E_f}{T}}$ and $x_2'=\sqrt{\frac{2E}{T}}-\sqrt{\frac{2E_f}{T}}$

and,

$$\frac{1}{4\sqrt{E_f}}\frac{(\sqrt{E}\pm\sqrt{E_f})}{T}= \frac{1}{4\sqrt{E_f}\sqrt{T}}\frac{\sqrt{2}}{2}\left(\sqrt{\frac{2E}{T}}\pm\sqrt{\frac{2E_f}{T}}\right)= \frac{\sqrt{2}}{8\sqrt{E_f T}}\left(\sqrt{\frac{2E}{T}}\pm\sqrt{\frac{2E_f}{T}}\right)$$

which also equals to;

$$=\frac{1}{8}\sqrt{\frac{\pi}{TE_f}}\frac{2}{\sqrt{2\pi}}\left(\sqrt{\frac{2E}{T}}\pm\sqrt{\frac{2E_f}{T}}\right).$$

Hence,

$$N(E) = \left(\frac{\pi^{\frac{1}{2}}}{8E^{1/2}T^{1/2}}\right)\begin{bmatrix}-(2\pi)^{-\frac{1}{2}}\,2\left(\sqrt{\frac{2E}{T}}+\sqrt{\frac{2E_f}{T}}\right)\exp\left(\frac{-{x_1'}^2}{2}\right)+\operatorname{erf}(x_1)+\\ (2\pi)^{-\frac{1}{2}}\,2\left(\sqrt{\frac{2E}{T}}-\sqrt{\frac{2E_f}{T}}\right)\exp\left(\frac{-{x_2'}^2}{2}\right)-\operatorname{erf}(x_2)\end{bmatrix}$$

$$N(E) = \left(\frac{\pi^{\frac{1}{2}}}{8E^{1/2}T^{1/2}}\right)\left\{F\left[(\frac{2E}{T})^{1/2} + (\frac{2E_f}{T})^{1/2}\right] - F\left[\left|(\frac{2E}{T})^{1/2} - (\frac{2E_f}{T})^{1/2}\right|\right]\right\} \qquad (B.7)$$

in which,

$$\mathrm{F(x)} = 2x(2\pi)^{1/2}\exp\left(-{x^2}/{2}\right) + (2\pi)^{1/2}\int_{-x}^{x}\exp\left(-{t^2}/{2}\right)dt,$$

Eqn. (B.7) can also be written as

$$N_F(E) = \frac{1}{4\sqrt{E_f T}}\left[\left(-x_1 e^{-x_1{}^2} + \mathrm{erf}\,(\mathrm{x}_1)\right) - \left|\left(-x_2 e^{-x_2{}^2} - \mathrm{erf}\,(x_2)\right)\right|\right]$$

which is known as Feather spectrum

APPENDIX C

Transformation of the Le-Couter Spectrum to the LAB system

The Le Couter Spectrum in Center of Mass System is;

$$\Phi(\varepsilon) = C\varepsilon^{5/11} e^{-\varepsilon/T} \tag{C.1}$$

where $C = \frac{1}{\Gamma\left(\frac{16}{11}\right)\left(\frac{11}{12}T_m\right)^{16/11}} = (\Gamma(\frac{16}{11})T^{16/11})^{-1}$.

The relation between center of mass and LAB system is,

$$N(E) = \int_{(\sqrt{E}-\sqrt{E_f})^2}^{(\sqrt{E}+\sqrt{E_f})^2} \frac{\phi(\varepsilon)d\varepsilon}{4\sqrt{E_f\varepsilon}} \tag{C.2}$$

so,

$$N(E) = \int \frac{C\varepsilon^{5/11} e^{-\varepsilon/T}}{4\sqrt{E_f\varepsilon}} d\varepsilon = \frac{C}{4\sqrt{E_f}} \int_{(\sqrt{E}-\sqrt{E_f})^2}^{(\sqrt{E}+\sqrt{E_f})^2} \varepsilon^{-1/22} e^{-\varepsilon/T} d\varepsilon \tag{C.3}$$

and

$$\mathrm{I} = \int \varepsilon^{-1/22} e^{-\varepsilon/T} d\varepsilon = -\left(T\ \left(\frac{x}{T}\right)^{1/22} igamma\left(21/22, x/T\right)\right)/x^{1/22}\ . \tag{C.4}$$

As

$$F(a) = \int_0^\infty e^{-t} t^{a-1} dt$$

and

$\Gamma(\mathrm{x}, \mathrm{a}) = \frac{1}{\Gamma(a)} \int_0^x e^{-t} t^{a-1} dt$ or $gamma\ inc(x, a)$ is incomplete gamma function in (C.4), remembering the relation

$$igamma(a, x) = gamma(a)[1 - gammainc(x, a)], \tag{C.5}$$

we can rearrange Eqn. (C.4) as

$$\mathrm{I} = \int_{x^-}^{x^+} \varepsilon^{-1/22} e^{-\varepsilon/T} d\varepsilon = -T^{21/22}\ igamma(21/22\ \varepsilon/T)\Big|_{x^-}^{x^+} \quad \text{(C.6)}$$

$$\mathrm{I} = -T^{21/22}\left[igamma\left(21/22, \frac{\left(\sqrt{E}+\sqrt{E_f}\right)^2}{T}\right) - igamma(21/22, \frac{\left(\sqrt{E}-\sqrt{E_f}\right)^2}{T})\right] \quad \text{(C.7)}$$

where $x^+ = \left(\sqrt{E} + \sqrt{E_f}\right)^2$ and $x^- = \left(\sqrt{E} - \sqrt{E_f}\right)^2$

Substituting Equation (C.5) into (C.7) we have

$$\mathrm{I} = -T^{21/22}\left[\begin{array}{l} gamma(21/22)\left\{1 - gammainc(\frac{\left(\sqrt{E}+\sqrt{E_f}\right)^2}{T}, 21/22\right\} + \\ -gamma(21/22)\left\{1 - gammainc(\frac{\left(\sqrt{E}-\sqrt{E_f}\right)^2}{T}, 21/22\right\} \end{array}\right]$$

$$\mathrm{I} = -T^{21/22}\left[-gammainc\left(\frac{\left(\sqrt{E}+\sqrt{E_f}\right)^2}{T}, 21/22\right) + gammainc\left(\frac{\left(\sqrt{E}-\sqrt{E_f}\right)^2}{T}, 21/22\right)\right]$$

or basically,

$$\mathrm{I} = T^{21/22}\left[gammainc\left(\frac{\left(\sqrt{E}+\sqrt{E_f}\right)^2}{T}, 21/22\right) - gammainc(\frac{\left(\sqrt{E}-\sqrt{E_f}\right)^2}{T}, 21/22\right]$$

Thus,

$$\mathrm{N(E)}=\left(\frac{1}{4\sqrt{E_f T}}\right)\left(\frac{gamma(21/22)}{gamma(16/11)}\right)\left[gammainc\left(\frac{\left(\sqrt{E}+\sqrt{E_f}\right)^2}{T}, \frac{21}{22}\right) - gammainc(\frac{\left(\sqrt{E}-\sqrt{E_f}\right)^2}{T}, \frac{21}{22})\right]. \quad \text{(C.8)}$$

This equation can also be written as

$$N_{LC}(E) = \frac{\Gamma(21/22)}{\Gamma(16/11)}\frac{1}{4\sqrt{E_f T}}\left[\Gamma\left(\frac{\left(\sqrt{E}+\sqrt{E_f}\right)^2}{T}, \frac{21}{22}\right) - \Gamma\left(\frac{\left(\sqrt{E}-\sqrt{E_f}\right)^2}{T}, \frac{21}{22}\right)\right] \quad \text{(C.9)}$$

The equation in (C.9) is called the Le Couteur spectrum in Lab system or basically the LCL spectrum.

PUBLICATIONS

1. M. Koçak, H. Ahmedov and G. Dere, On Prompt Fission Neutron Spectrum in Spontaneous Fission of ^{252}Cf. Submitted to Annals of Nuclear Energy, 2013. Under review.

2. M. Koçak, H. Ahmedov and G. Dere, Prompt Fission Neutron Energy Spectrum of ^{252}Cf in Center of Mass and Laboratory Systems. Submitted to Balkan Physics Letters, 2013. Under review.

3. M. Koçak, H. Ahmedov and G. Dere, On Prompt Fission Neutron Spectrum in Spontaneous Fission of ^{252}Cf. Turkish Physical Society, 29th International Physics Congress, Bodrum, Turkey, September 2012.