AUGUST 2014 M.Sc. in Mechanical Engineering BURAK ŞAHİN

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DEVELOPMENT OF A USER FRIENDLY INTERFACE SOFTWARE FOR DESIGN AND ANALYSIS OF PARALEL AXES EXTERNAL GEARS INCLUDING QUASI-STATIC TRANSMISSION ERROR CALCULATIONS

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Including Quasi-Static Transmission Error Calculations

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This is to certify that we have read this thesis and that in our opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science.

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Burak ŞAHİN

ABSTRACT

DEVELOPMENT OF A USER FRIENDLY INTERFACE SOFTWARE FOR DESIGN AND ANALYSIS OF PARALEL AXES EXTERNAL GEARS INCLUDING QUASI-STATIC TRANSMISSION ERROR CALCULATIONS

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Gears are one of the most important power transmission elements required to transmit both torque and motion between shafts. The design or analyze of these very crucial elements of power transmission systems require too much time because of huge numbers of the parameters affect the gear static and dynamic performance. Gear design or analysis requires a good knowledge and engineering sense supported by the standards submitted by ISO (International Organization for Standardization), AGMA (The American Gear Manufacturers Association), DIN (The German Institute for Standardization). Even by using these international standards, designing a gear set, an engineer must carry out an iterative process because every geometrical or material property change of the gear influences the other parameters. Checking these influences by manual calculation is slow and prone to errors. These all increase the demand for software of designing gears, calculating parameters, redesigning (if required), determining geometry, size and material properties with high accuracy and minimum error in short time. In the scope of this thesis, a user friendly interface for software of gear design and analysis of parallel axes gears based on ISO, AGMA and DIN has been developed.

Key Words: Parallel axes gears, Gear design, Gear analysis, Design, Software, Spur gear, and Helical gear.

ÖZET

PARALEL EKSENLİ DİŞLİLERİN TASARIMI, ANALİZİ VE İLETİM HATALARI ANALİZİ İÇİN KULLANICI UYUMLU BİR BİLGİSAYAR PROGRAM GELİŞTİRİLMESİ

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Dişliler paralel ve paralel olmayan şaftlar arasında tork ve hareket aktarımında kullanılan güç aktarım elemanlarının en önemlilerindendir. Düz ve helis dişliler en sık kullanılan dişli türleridir. Bu kritik güç aktarım elemanlarının tasarım ve analizi büyük öneme sahiptir ve dişli statik ve dinamik performans parametreleri çokluğundan dolayı çok fazla zaman gerektirir. ISO, AGMA ve DIN gibi uluslar arası organizasyonlar tarafından hazırlanan standartlar ile desteklenen iyi bir bilgi birikimi ve mühendislik duyusu analiz ve tasarım sürecinin olmazsa olmazıdır. Bu standartları kullanarak bile bir dişli setinin tasarımı için mühendis pek çok kez tasarımı tekrarlamak zorunda kalabilir. Çünkü dişlinin geometrik veya performans parametrelerinden herhangi birindeki değişim diğer dişli parametrelerini ve dişli performansını da etkilemektedir. Bu parametrelerin el ile hesaplanması yavaş ve hatalara açıktır. Tüm bunlar dişli tasarım ve analizini azami hassasiyet ve asgari hata payı ile yapabilen bunun yanı sıra tüm dişli parametrelerini hesap edebilen bir yazılıma ihtiyacı artırmaktadır. Bu tez kapsamında paralel eksenli dişlilerin tasarım ve analizlerini ISO, AGMA ve DIN dişli standartlarına göre yapabilen, ayrıca dişli iletim hataları hesaplamalarını da içeren bir kullanıcı dostu dişli yazılımı geliştirilmiştir.

Anahtar Kelimeler: Paralel eksenli dişliler, Dişli tasarımı, Dişli analizi, Tasarım, Analiz, Yazılım, Düz dişli ve Helis dişli

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CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

Power transmission is a very crucial duty for many machines and systems which do work for industry. Because power generated by source is not directly usable by the machines and systems generally. It needs to be converted and transmitted from power source to power consuming equipments.

When two smooth cylinders are mounted on shafts with parallel axes and are pressed together lengthwise, it is possible to transmit power from one shaft to the other by friction drive. If there is no slippage during the contact, such rotating cylinders will ensure a smooth and accurate transmission of angular velocity. The angular velocities of these cylinders are inversely proportional to the diameters of the cylinders. This relation applies if the driving and the driven cylinders are perfectly accurate and the cylinders are said to produce uniform velocity transmission. However, this is impossible to achieve in practice because it does not produce a positive drive owing to the slippage which may be caused by various factors. To overcome the problem of slippage, toothed wheels or gears are used which produce positive drive with uniform angular velocity ratio [1].

Gears are one of the most important power transmission elements required to transmit both torque and motion between parallel and non-parallel shafts by means of progressive engagement of projections called teeth. There are three distinct types of gears in terms of shaft positions: power and motion transfer between parallel shafts including spur and helical gears, between intersecting shafts such as bevel gears and between neither parallel nor intersecting shafts such as worm, worm wheel, crossed helical gears and hypoid gears. Parallel axes spur and helical gears are the most widely used types of gears, and they are treated in the scope of this thesis.

Gear geometry is also an important concern to provide uniform motion transfer. Mating teeth of gears acting against each other to produce rotary motion are similar to cams. When the tooth profiles are designed so as to produce a constant angular velocity ratio during meshing, these are said to have conjugate action. Many tooth profile forms such as involute, cyclodial and circular can theoretically fulfill the basic requirements such as uniform motion transfer and constant angular velocity during gear meshing (conjugate action). Involute profile, which with few exceptions, is in universal use for gear teeth. It is insensitive to center distance change and allows different operating center distance without changing the velocity ratio.

Gear design criteria include two main points of static and dynamic performance. Static performance usually deals with global kinematic and geometric requirements and strength analysis of gear materials under load while dynamic performance usually deals with vibration and noise of gears.

Gears experience two kinds of stress mainly; contact stress at tooth surface and bending stress at tooth root. Gear tooth strength relates to the ability to resist tooth breakage under the effect of root bending stress; gear tooth surface durability relates to the ability to resist pitting under the effect of surface pressure between meshing teeth. Design criteria of gears determine some parameters like safety, reliability, service life, performance etc and all these parameters are affected by many geometrical and working condition parameters. Due to a vast amount of parameters and complexity of gear and gearbox design, designers need to have a comprehensive knowledge of gearing supported by well accepted equations and design procedures such as Lewis bending equation, AGMA stress equation and international standards. Some standards for parallel axes gears have been developed and presented by international organizations such as ISO (International Organization for Standardization) [2-10], AGMA (American Gear Manufacturers Association) [11-13], and DIN (Deutsches Institut für Normung) [14-20].

Even by using these equations and international standards, design or analysis of a gear set is a difficult task. An engineer must carry out an iterative process because every geometrical or material property change/variation (in gear surface hardness, surface roughness, pressure angle, helix angle, module, tooth numbers, input power, gear accuracy grade, oil type, bearing mounting conditions, gear materials and number of load cycles) influences most other properties and parameters (such as safe operation in terms of surface durability and tooth root bending resistance, center distance, pitch diameters, outside diameters, face width, etc). Checking these influences by manual calculation is slow and prone to errors. It requires turning back and changing any/suitable ones of design parameters to provide safe operation of gear pair. In such cases designer needs more and more time to redesign and check if operation is safe or not. These all increase the demand for gear design and analysis software that calculates and checks gear parameters, determines geometry, size and material properties, safety factors for surface contact stress, tooth root bending stress and scuffing resistance (if required) with high accuracy and minimum error in short time. Some commercial programs such as KISS Soft, Dontyne, Zar, Excellent, AGMA Gear Rating Suite, MitCalc and Fairfield Gear Program are used in industry for gear design and analysis processes

Gears are subjected to dynamic loads as well as static loads during operation. High dynamic load can lead to fatigue failure and affect the life and reliability of a gear transmission. Dynamic behaviour of gears is important because of its relation with static and dynamic transmission error hence vibration and noise. Transmission error (TE) is considered to be an important excitation mechanism for gear noise and vibration. It is defined as the difference between the actual position of the output gear and the position it would occupy if the gears were perfectly conjugate. Designer needs to construct loaded transmission error curve, this gives him to chance to simulate the quasi static behavior of gears at the design stage. It also allows the designer to design a proper tooth profile relief which produces a uniform motion transfer (with minimum TE) between driving and driven gear shafts.

The aim of this ***thesis is to develop a user friendly interface software for design and analysis of parallel axes gears according to international gear standards such as ISO 6336:2006 [2-6], AGMA 2101 D04:2010 [11] and DIN 3990:1987 [14-18] with some other auxiliary standards and make calculations of quasi static loaded transmission error in the light of well accepted theory of Harris Map by using Visual Studio C++.***

1.2 LAYOUT OF THESIS

Thesis lay out: the thesis consists of six chapters.

In the first chapter of the thesis, introduction and layout of thesis are presented.

Chapter 2 consists of literature survey for gear analysis, design and transmission error analysis for spur and helical gars.

Chapter 3 includes information about gear geometry, analysis/design of spur and helical gears. A brief explanation is given for basic terms of gear geometry. Parallel axes gear analysis and design is explained based on standards developed by international organizations such as ISO, AGMA and DIN. Influence and performance parameters which are very essential for gear power rating in terms of surface contact and tooth root bending stress are given.

Flowchart and software modules including gear analysis, design and transmission error are presented in Chapter 4. Visual studio C++ is used as programming tool to prepare/develop gear software to make analysis/design of spur and helical gears, and make a transmission error analysis of parallel axes gears.

Transmission error theory and construction is explained and flowchart of transmission error analysis module for involute external spur and helical gears is given in Chapter 5.

Chapter 6 includes case studies and results of spur and helical gear design/analysis and also transmission error analysis including transmission error curves.

Finally, discussion and conclusion on thesis and recommendation for further studies are included in Chapter 7.

CHAPTER 2

LITERATURE SURVEY

2.1 INTRODUCTION

A literature survey of design/analysis of parallel axes gears is presented here. Some researchers combined their gear design and analysis background with software. As a result, gear softwares have been developed as a gear engineering tool by using well accepted formulas; Lewis bending equations, AGMA equations, international standards such as ISO, AGMA and DIN completely or partially. Some researchers have analyzed gears by using finite element software and make calculations by using standards, formulas and equations. Then results were compared in their studies.

Some studies for calculation of transmission error of spur and helical gears are also presented here. Profile modification effects were also researched to provide a smoother operation of gears with fewer disturbances of noise and vibration.

2.2 LITERATURE SURVEY

Literature survey is presented based on gear standards, finite element method and transmission error analysis below:

2.2.1 Based on Gear Standards

Panchal [21] has used Mathcad software to calculate geometry, hardness and stress parameters of spur gear pair by calculating Lewis form factor and AGMA geometry factor. Tooth root bending stress and surface contact stress were calculated by using Lewis and AGMA equations. Bending stress calculation results for AGMA and Lewis, surface contact calculation for Hertzian and AGMA were presented and compared at the end.

Cetinkaya [22] has prepared a computer program to design single step spur gear pairs in GWBASIC Language. Computer program has two sections; first section contains gear design simulation and the second is composed of shaft design and the ball bearing selection. DIN Norms were used to design gears.

Avcil [23] prepared a computer program for involute cylindrical spur gears' design based on DIN gear standards. Design of gears was performed by using program developed in Visual Basic and Microsoft Excel; same gear pair was analyzed by using MitCalc software and results were presented. Solidworks was used to create 3-D model of gear pair to be used in finite element analysis for future studies.

Arikan [24] has developed a computer program to evaluate AGMA geometry J factor for external spur gears by using polynomial equations easily and with minimum process time in MathCAD. Results were compared with geometry factor values presented by AGMA.

Gopichand et al [25] have used Matlab to generate a software of design involute spur gear pairs for minimum center distance and standard module. After designing gear, factor of safeties for contact and bending stress were checked and tooth profile was generated.

A program was developed by Aycicek [26] in Excell to design a two stage speed gearbox. Module was calculated firstly and then other gear parameters were estimated. At second step, shafts were designed; bearings and keyways' selections were performed. Some examples of interval arithmetic's' applications for machine elements were presented.

Wu San Pan [27] developed a computer simulation program for the optimal design of high contact ratio gears with standard parameters. Static and dynamic characteristics were analyzed to determine optimal parameter values for gear sets by examining the effect of diametral pitch, gear ratio, addendum coefficient and operating speed. Programs were developed for calculations and a graphical presentation of optimal static design and DANST (Dynamic Analysis of Spur Gear Transmission, a FORTRAN Program) was modified for the optimal dynamic design study.

Nogay [28] has prepared a program in Ms Excell to design helical gear and dimension spur/helical gear with and without addendum modification. Gear was analyzed for contact stress and tooth root bending stress. The effects of some parameters on gear performance were checked based on DIN standards.

Ayyildiz M. et al [29] have aimed to obtain gear parameters required to the design of a spur gear by using reverse engineering techniques. They used a hybrid programming approach to generate and process the point cloud, extraction of the gear parameters were performed by Visual BASIC programming language. 3D modeling in the CAD environment was performed also by AutoLISP. In the study, Solidworks and AutoCAD were selected as CAD environments. With this study, a reverse engineering approach for the design of the spur gear was presented and modeling of the gears was carried out in the CAD environment.

Mahmoodullah [30] improved a spur gear design program by using AutoLIPS programming language. This software was prepared based on AGMA standards, some formulas and theories which were used to define required geometry firstly and then check safeties in terms of stresses for external spur gears at fixed center distance excluding change in center distance and profile shift.

2.2.2 Based on Finite Element Method

Karpat et al. [31] have developed a computer program in Visual Basic to size spur, helical, bevel and worm gears in terms of module values for surface contact and tooth root bending based on DIN gear standards. 2 D models of gears were developed in AutoCad 2000, Microsoft Excel, Bortland Pascal and Ansys were used to create 3 D model and this model was analyzed in Ansys.

Venkatesh et al. [32] have calculated bending stress and contact stress by using analytical and numerical methods. Modified Lewis beam strength method was used to estimate bending stress whereas contact stress calculation was performed according to modified AGMA contact stress method. 3-D model of helical gear was generated by using Pro Engineer Solid Modeling software. Contact and bending stresses were analyzed by Ansys. At the end, results were compared.

Gidado et al. [33] have designed helical gear for tooth bending stress condition according to AGMA bending stress equation by varying face width. Helical gear model was designed by using Pro Engineer design modeler and analyzed with Ansys finite element program. Then results were compared.

Kayimoglu [34] has analyzed involute spur gear model for tooth root bending stress by using Ansys finite element package. Basic was used to generate the coordinates of gear model to create model of gear in Ansys. Critical section in terms of tooth root stress was determined by finite element analysis. Graphics of stress on critical sections were generated.

Tiwari et al. [35] has used a 3D model of gear and finite element analysis to conduct root bending stress and surface contact stress calculation for mating involute spur gears. A pair of involute spur gear without tooth modification and transmission error was defined in a CAD system tool as CATIA V5 and Autodesk Inventor. Finite element analysis (FEM) was done by using finite element software ANSYS. Results were compared with theoretical and AGMA standard. Lewis formula and Hertz equation was used for quick stress calculation for gear, where as the AGMA standards and FEM was used for detailed gear stress calculation for a pair of involute spur gear.

2.2.3 Based on Transmission Error Calculations

Yildirim et al. [36] have introduced a systematic approach and applied it to low contact ratio (LCR) spur gears first with some design regions and to high contact ratio (HCR) spur gears with some new and promising design regions and rules. Several smooth transmission error curves at different loads were shown to be possible for the relief designed, hence allowing a range of loads with uniform motion transfer. The advantages of HCRG over LCRG in terms of smooth TE curves and tooth load values were noted, and compared with experimental results.

Yildirim [37] has studied on investigation and reducing transmission error of high contact ratio and low contact ratio spur gears. Theoretical values were verified with experimental results.

Tharmakulasingam [38] has modeled a spur gear pair by using finite element method, and the gear mesh was simulated and analyzed under static conditions. The main differences between the static and dynamic transmission error was investigated. Profile modification effect on the transmission error of the gear pair was studied. A combination of Finite-Element Analysis, hybrid numerical/analytical methodology and optimization algorithms was used to examine the dynamic behavior of the gear pairs under various operating conditions.

Kumar [39] has used spur gear to study the effect of the tooth profile geometry and their modification by using FEM method. The deflection of the teeth was calculated by using the bending stress and shear stress and principle stress. Tooth relief modification was taken into consideration for profile modification by using finite element method.

Paul [40] has used a model of spur gear to study the effect of intentional tooth profile modifications by using two dimensional FEM. The deflections of teeth were calculated by using the bending and shear influence function. Tooth relief modification was considered for profile modification, by using computational method.

There are many studies on gear design, gear analysis and transmission error calculations in literature. Some researchers have studied on design/analysis method of gears by using well accepted formulas, equations, international standards whereas some others combined gear engineering background with computer programming knowledge. Some of them prepared software to calculate gear parameters such as Lewis form factor, AGMA geometry factor instead of selection of these parameters from presented tables.

Most of the work referred to here is distinct studies dealing with either kinematic design/analysis, or geometrical design or sizing based on strength or similar isolated subjects such as profile relief design and transmission error curves only. Some other works intend to combine distinct subjects in one work and the work presented here is one of those studies. The work presented here is a combination of both developing a gear pair design/analysis software tool based on well known international gear standards like ISO, AGMA and DIN and also design of tooth profile relief and construction of loaded transmission error curves for the gears designed by the software developed herein.

In the chapters, following the discussion here, the general gearing theory, international standards gear design/analysis procedures, software flowchart construction, transmission error theory, software development based on C++ and some case studies for specific gear designs and relief designs with resulting transmission error curves are given.

CHAPTER 3

PARALLEL AXES GEARS

3.1 INTRODUCTION

Power and motion transmission is essential for many machines and systems because it is not always usable directly where it is produced. They are transmitted between parallel or non parallel shafts through machine elements such as belts, gears, ropes, chains, couplings and friction clutches.

Theoretically, when two smooth cylinders are mounted on shafts with parallel axes and are pressed together lengthwise, it is possible to transmit power from one shaft to the other by friction drive. In the case of no slippage during the contact, such rotating cylinders will ensure a smooth and accurate transmission of angular velocity. The angular velocities of these cylinders are inversely proportional to the diameters of the cylinders. This relation applies if the driving and the driven cylinders are perfectly accurate and the cylinders are said to produce uniform velocity transmission. However, this is not possible to achieve in practice because it does not produce a positive drive owing to the slippage which may be caused by various factors. To overcome the problem of slippage, toothed wheels or gears are used which produce positive drive with uniform angular velocity ratio.

3.2 GEARS

Gears are designed to fulfill functions of transmitting of power or angular motion between parallel and non parallel shafts or both by means of progressive engagement of projections called teeth. Power transmission can be in the application of speed decreasing (torque increasing) or speed increasing (torque decreasing) whereas transmission of angular motion requires elimination of motion lost or adjusting absolute position.

3.2.1 Gears Types

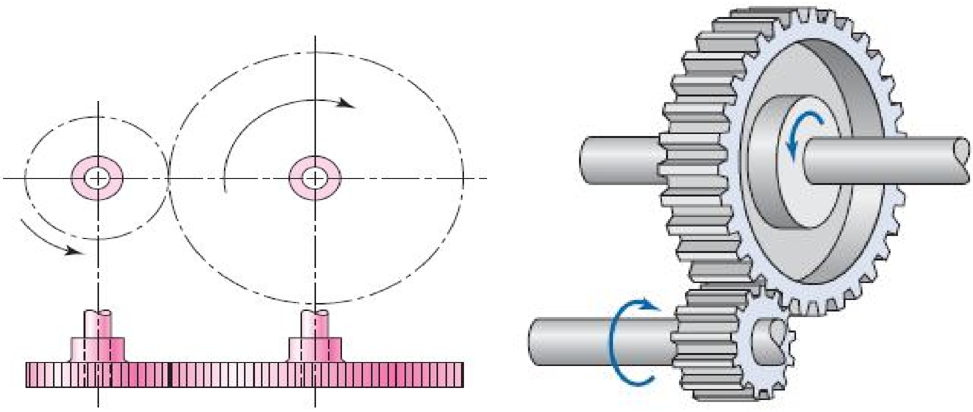
There are three distinct types of gears in terms of shaft positions; power and motion transfer between parallel shafts including spur and helical gears, gears between intersecting shafts such as bevel gears and neither parallel nor intersecting such as worm, worm wheel, crossed helical gears and hypoid gears. They are categorized according to shaft axes given in Table 3.1. Parallel axes spur and helical gears are the most widely used types of gears, and they are in scope of this thesis.

Spur gears are simplest and, hence, the most common type of gear. The teeth of a spur gear are parallel to the axis of rotation as shown in Figure 3.1a. Spur gears are used to transmit motion between parallel shafts, which encompasses the majority of applications. A pair of mating spur gears is illustrated in Figure 3.1b.

A rack is a special case of spur gear where the teeth of the rack are not formed around a circle, but laid flat. The rack can be perceived as a spur gear with an infinitely large diameter. When the rack mates with a spur gear, translating motion is produced. A mating rack and gear are illustrated in Figure 3.2.

Table 3.1 Gear types

Parallel Axes	**Intersecting Axes**	**Non-intersecting and Non-parallel Axes**
Spur External	Straight Bevel	Crossed-Helical
Spur Internal	Zerol Bevel	Single - enveloping worm
Helical External	Spiral Bevel	Double - enveloping worm
Helical Internal	Face Gear	Hypoid



(a)Top and side view of spur gear pair [41] (b) Spur gears [42]

Figure 3.1 Spur gear pair

Internal or annular gears have the teeth formed on the inner surface of a circle. When mating with a spur gear, the internal gear has the advantage of reducing the distance between the gear centers for a given speed variation. An internal gear mating with a traditional spur gear is illustrated in Figure 3.3.

Helical gears are similar to, and can be used in the same applications as, spur gears. The difference is that the teeth of a helical gear are inclined to the axis of rotation as shown in Figure 3.4a. The angle of inclination is termed the helix angle. This angle provides a more gradual engagement of the teeth during meshing and produces less impact and noise. Because of this smoother action, helical gears are preferred in high-speed applications. However, the helix angle produces thrust forces and bending couples, which are not generated in spur gears. A pair of mating helical gears is illustrated in Figure 3.4b.

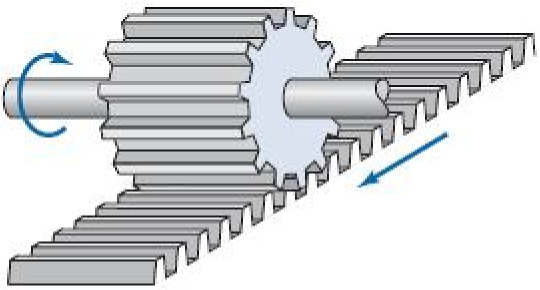


Figure 3.2 Rack and pinion [42]

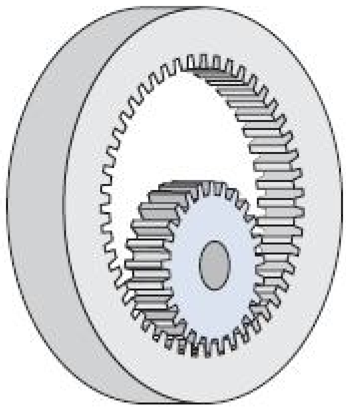
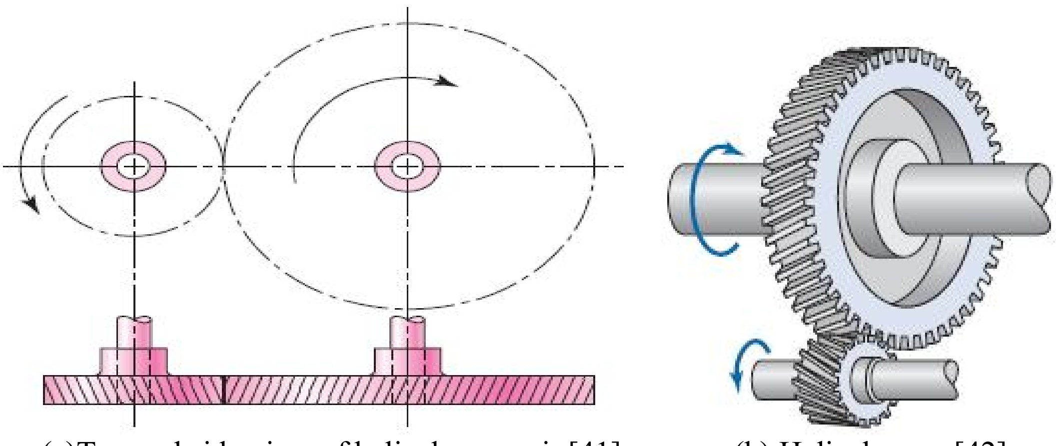


Figure 3.3 Internal spur gear pair [42]



(a)Top and side view of helical gear pair [41]

(b) Helical gears [42]

Figure 3.4 Helical gear pair

Herringbone gears are used in the same applications as spur gears and helical gears. In fact, they are also referred to as double helical gears. The herringbone gear appears as two opposite-hand helical gears butted against each other. This complex configuration counterbalances the thrust force of a helical gear. A herringbone gear is shown in Figure 3.5.

Bevel gears have teeth formed on a conical surface and are used to transmit motion between nonparallel shafts. Although most of their applications involve connecting perpendicular shafts, bevel gears can also be used in applications that require shaft angles that are both larger and smaller than 90°. As bevel gears mesh, their cones have a common apex. However, the actual cone angle of each gear depends on the gear ratio of the mating gears. Therefore, bevel gears are designed as a set, and replacing one gear to alter the gear ratio is not possible. A pair of mating bevel gears is illustrated in Figure 3.6.

Miter gears are a special case of bevel gears where the gears are of equal size and the shaft angle is 90°. A pair of mating miter gears is illustrated in Figure 3.7

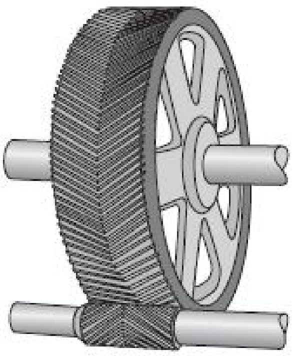


Figure 3.5 Herrignbone gear pair [42]

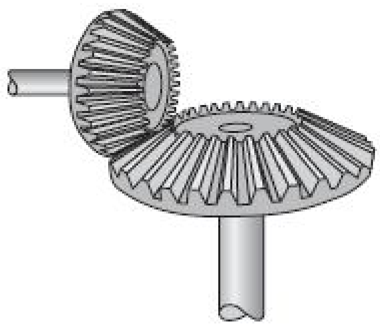


Figure 3.6 Bevel gear pair [42]

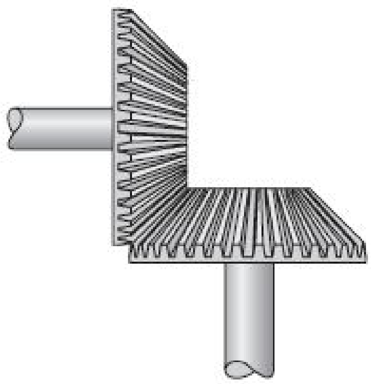


Figure 3.7 Miter gear pair (equal size bevel gear) [42]

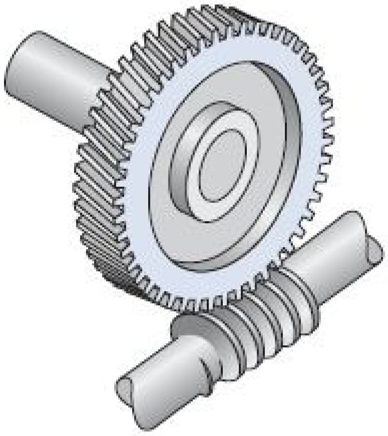


Figure 3.8 Worm and worm gear [42]

A worm and worm gear is used to transmit motion between nonparallel and nonintersecting shafts. The worm has one tooth that is formed in a spiral around a pitch cylinder. This one tooth is also referred to as the thread because it resembles a screw thread. Similar to the helical gear, the spiral pitch of the worm generates an axial force that must be supported. In most applications, the worm drives the worm gear to produce great speed reductions. Generally, a worm gear drive is not reversible. That is, the worm gear cannot drive the worm. A mating worm and worm gear are shown in Figure 3.8.

3.2.2 Conjugate Action and Principle of Transmission

When a pair of mating gear teeth act against each other, rotary motion is produced which is transmitted from the driver to driven gear. If such a pair of gears has tooth profiles which are so designed that a constant angular velocity ratio is produced and maintained during meshing, the two gears are said to have conjugate action and the tooth profiles are said to have conjugate curves. This requires that angular velocity ratio of driver and driven gear is constant.

Figure 3.9 shows two curved body-surfaces which are in contact with each other. Body 1, with centre at O, and having angular velocity of ω_1, is pushing body 2 of which the centre is at O. This produces rotary motion and body 2 rotates with an angular velocity of ω_2. The point of contact at this instant is at Q where the two surfaces are tangent to each other. The common tangent to the curves is T-T and the transmission of forces takes place along the common normal N-N which is also called the line of action. The line of action N-N intersects the line of centers $O_1 - O_2$, at P where is called the pitch point. Circles drawn through P, having centers at O_1, and O_2,is termed as pitch circles. The diameters of these circles, called pitch circles diameters (PCD), are the representative parameters of the two gears.

For producing a constant velocity ratio, the curved profiles of the mating teeth must be such that the law of gearing is satisfied. This law states that: In order to have a constant angular velocity ratio, the tooth curves must be so shaped that the common normal to the tooth profiles at the point of contact will always pass through the pitch point, irrespective of the position of the point of contact during the course of action [1].

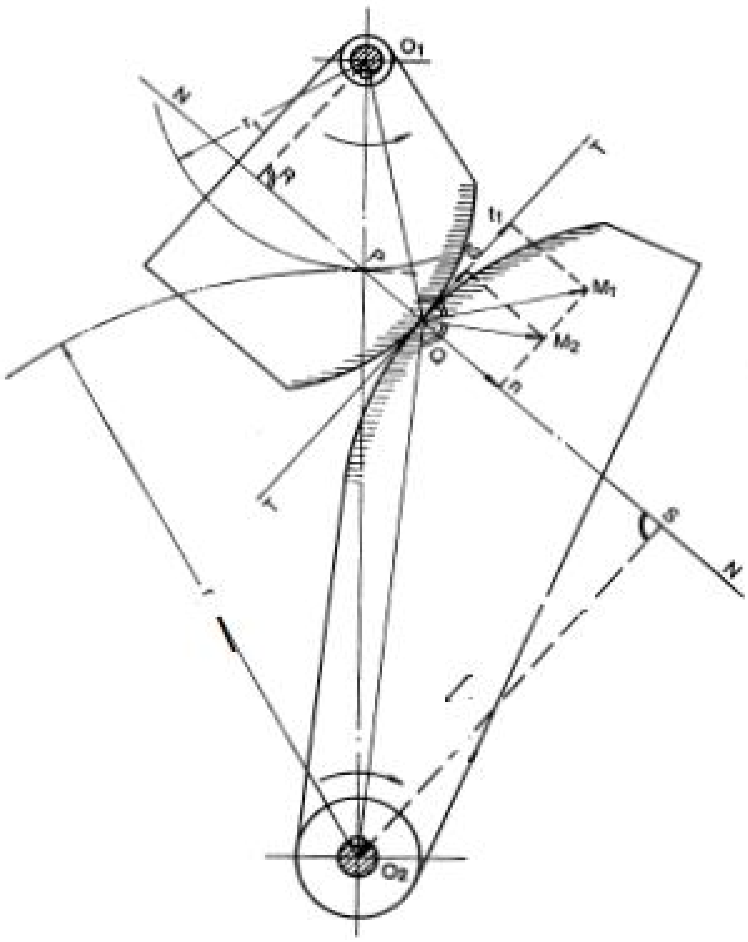


Figure 3.9 Transmission of motion and conjugate action [1]

3.2.3 Gear Tooth Profile

Gear tooth profile is expected to provide a uniform motion / power transmission between shafts with a nonslip and smooth drive. Due to these reasons, the shape and size of teeth profile is very essential.

Many tooth profile forms such as involute, cyclodial and circular can theoretically fulfill the basic requirements such as uniform motion transfer and constant angular velocity during gear meshing (conjugate action).

In theory, at least, it is possible arbitrarily to select any profile for one tooth and then to find a profile for the meshing tooth that will give conjugate action.

Involute profile, which with few exceptions, is in universal use for gear teeth. Involute gear tooth profile provides constant angular velocity and the meshing tooth forms specific geometrical characteristics. It is insensitive to center distance change and allows different operating center distance without changing the velocity ratio [43].

The involute is a spiral beginning from the periphery of a circle called the "base circle" which is the heart of the involute. A family of involute curves which are generated from points at equal distances on the same base circle is equidistant. An involute curve is generated by the point of a cord which is unwound from the periphery of a circle and which is a1ways held stiff, it naturally follows that the instantaneous radius of curvature of a point on the involute thus generated is the same as the stiff length of the cord which spans that point to the point on the circle from where the cord has just been unwrapped. It is illustrated in Figure 3.10. The involute of a circle is defined as the curve which is generated by the end point of a cord which is kept taut while being unwound from a circle. Any other point on the cord will also generate a similar involute curve as the cord is progressively unwrapped from the circle [1].

Involute profile has the advantages:

1) Easy production of gears with standardized cutters.
2) Insensitivity to gear center distance change, thus keeping the velocity ratio constant always.
3) Ensuring a constant velocity ratio from start of teeth engagement to the end of engagement.

In this project, the study is based on involute external spur and helical gears.

3.2.4 Basic Geometry

Spur gear is the simplest and basic one of the types of gears used universally as a power and motion transmission elements. There are many geometrical parameters and properties of gears, but some of these general geometrical properties are given in this section for spur gears below. A basic knowledge of spur gear is essential to understand gear geometry because they are common for all gear types.

3.2.4.1 Spur gear geometry

There are some similarities between spur gear and helical gear’s geometry as well as differences. Spur gear can be thought as a helical gear with a zero helix angle. Here, basic geometry will be discussed and then different properties of helical gears than spur gears are going to be explained.

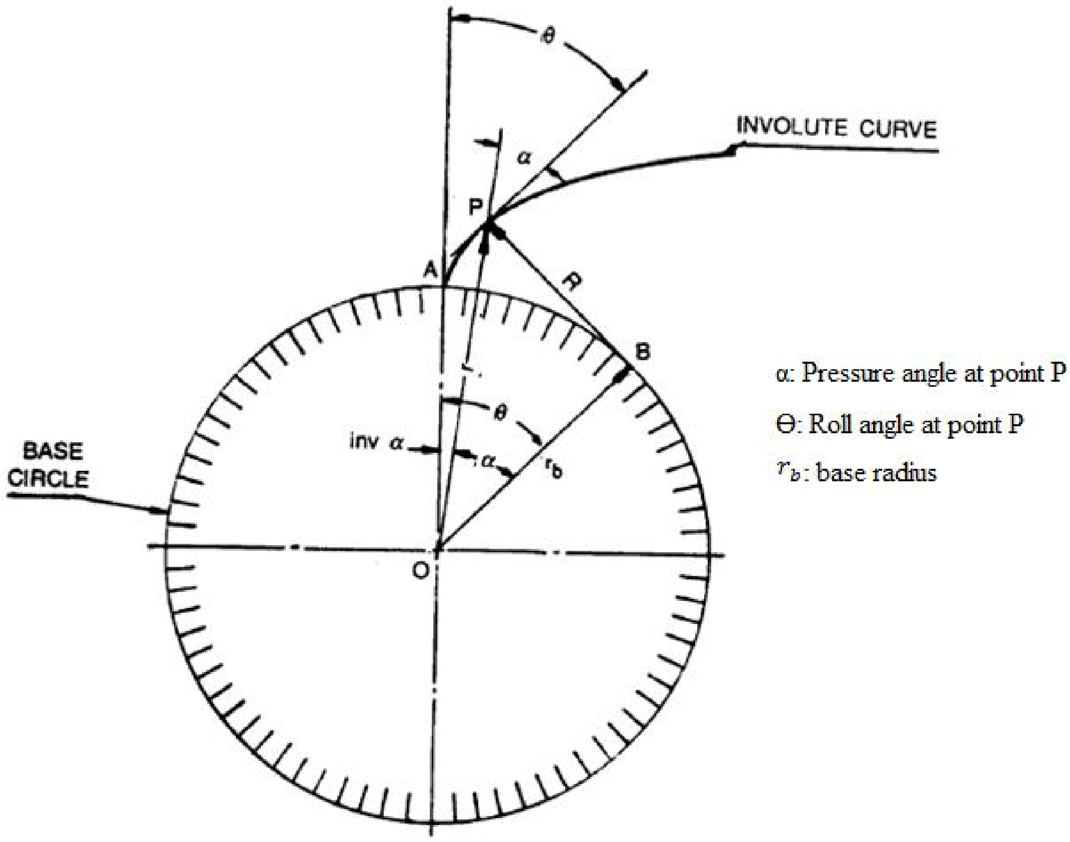


Figure 3.10 Involute curve [1]

Gear blank: The work piece used for the manufacture of a gear, prior to machining the gear teeth.

Active profile: The part of the gear tooth profile that actually comes in contact with the profile of the mating gear while in mesh (Figure 3.11a).

Top land: The surface of the top of a tooth (Figure 3.11b).

Fillet: The surface bounded by the form diameter and the root land and the ends of the teeth (Figure 3.11c).

Sides of gear: The surfaces and ends of the teeth in spur and helical gears (Figure 3.11d).

Root land: The surface bounded by fillets of the adjacent teeth and sides of the gear blank (Figure 3.11e).

Transverse section: A section through a gear perpendicular to the axis of the gear (Figure 3.11f).

Normal section: A section through a gear that is perpendicular to the tooth at the pitch circle (Figure 3.11g).

Tip round: The surface separates the active profile and the top land (Figure 3.11h).

End round: The surface separates the sides and the top land of the tooth (Figure 3.11i).

Fillet radius: The radius of the fillet curve at the base of the gear tooth (Figure 3.11j).

Edge round: The surface separates the active profiles of the teeth from the sides of the gear (Figure 3.11k).

Pinion is smaller one of two mating gear (Figure 3.12) while *gear* is the bigger one of gear pair and it is also called as wheel (Figure 3.12).

Module (m) is the ratio of the pitch diameter to the number of teeth. The module is the index of tooth size in SI. Diametral pitch (P_d) is the ratio of the number of teeth on the gear to the pitch diameter. Thus, it is the reciprocal of the module and relationship between diametral pitch and module is given in equation 3.1.

$$P_d = \frac{25.4}{m} \tag{3.1}$$

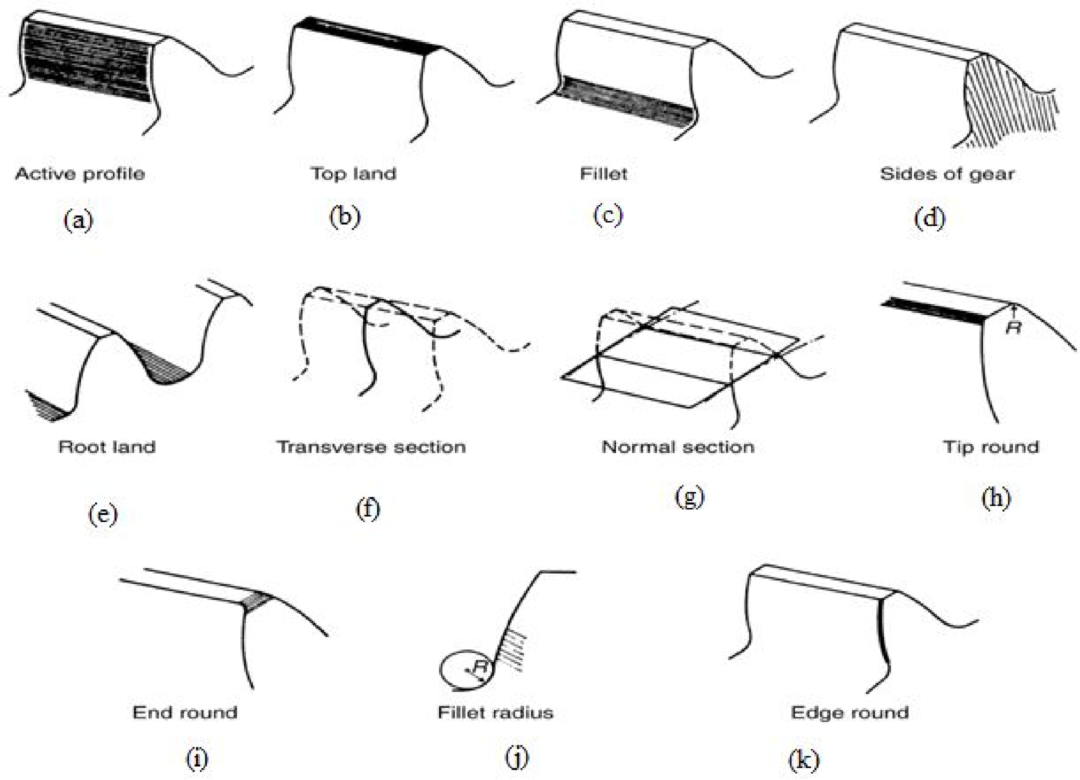


Figure 3.11 Nomenclatures of gear contact areas and boundary zones [44]

Figure 3.12 Spur gears [42]

<u>*Gear ratio*</u> *(GR)* is known as tooth number ratio and gear pair speed ratio of a gear train. It can be calculated directly from the numbers of teeth on the gears in the gear train as in equation 3.2.

$$GR = \frac{T_g}{T_p} \tag{3.2}$$

Addendum (a) is the radial distance between the top land and the pitch circle (Figure 3.13). It is equal to normal module if there is no additional information. Sometimes pinion is produced with a long addendum (Figure 3.14) to avoid undercut or balance strength through providing strong tooth pinion by increasing tooth thickness at the pitch line.

Dedendum (b) is the radial distance from the bottom land to the pitch circle (Figure 3.13). It is represented in terms of normal module given in equation 3.3 and is used as 1.25 of module if there is no information.

$$b = 1.25 \times m \tag{3.3}$$

Whole depth (h_t) is the sum of the addendum and the dedendum as represented in equation 3.4.

$$h_t = a + b \tag{3.4}$$

Clearance (c) is the amount by which the dedendum in a given gear exceeds the addendum of its mating gear (Figure 3.13). It can be calculated by using equation 3.5.

$$c = b - a \tag{3.5}$$

Circular pitch (P_c) is the distance from any point on a gear tooth to the corresponding point on the next tooth. It is also equal to the circumference of the pitch circle divided by the total number of teeth on the gear (Figure 3.13). It is calculated in accordance with equation 3.6.

$$P_c = \pi \times m \tag{3.6}$$

Base pitch (P_b) is arc distance measured around the base circle from the origin of the involute on one tooth to the origin of a similar involute on the next tooth. It can be calculated by using equation 3.7.

$$P_b = \pi \times m \times \cos\theta_n \tag{3.7}$$

Pitch Circle (Figure 3.13) is the imaginary circle that comes in contact with the imaginary circle of another gear when the two are in mesh. The pitch circles of a pair of mating gears are tangent to each other at pitch point.

Addendum circle is a circle which coincides with the top land of the tooth (Figure 3.13).

Dedendum circle is a circle which coincides with the bottoms of the tooth spaces (Figure 3.13).

Face width is the length of the teeth in an axial plane (Figure 3.13).

Clearance circle is a circle that is tangent to the addendum circle of the mating gear (Figure 3.13).

Face is the portion of the active tooth surface above pitch circle whereas flank is active tooth surface below the pitch circle (Figure 3.13).

Tooth thickness _(tt)_ is the distance between opposite faces of the same tooth measured around the pitch circle (Figure 3.13). It is theoretically calculated by using equation 3.8.

$$tt = \frac{\pi \times m}{2} \tag{3.8}$$

Width of space is the distance between faces bounding a tooth space, measured round pitch circle (Figure 3.13).

Base circle is the circle from where the involute tooth profiles are generated at (Figure 3.14).

Pitch diameter (d_p, d_g) is the diameter of pitch circle (Figure 3.15) and can be calculated based on equations 3.9 and 3.10 for pinion and gear respectively.

$$d_p = T_p \times m \tag{3.9}$$

$$d_g = T_g \times m \tag{3.10}$$

Base diameter (d_{bp}, d_{bg}) is the diameter of the base circle of gear and can be calculated based on equations 3.11 and 3.12.

$$d_{bp} = d_p \times \cos \emptyset_n \tag{3.11}$$

$$d_{bg} = d_g \times \cos \emptyset_n \tag{3.12}$$

Outside diameter (d_{ap}, d_{ag}) is measured from the tip circle of gear (Figure 3.15). It is calculated based on equations 3.13 and 3.14 for pinion and gear respectively.

$$d_{ap} = d_p + 2 \times a_p \tag{3.13}$$

$$d_{ag} = d_g + 2 \times a_g \tag{3.14}$$

Root diameter (d_{rp}, d_{rg}) is measured at the base of the tooth (Figure 3.15). It can be calculated based on equations 3.15 and 3.16 for pinion and gear respectively.

$$d_{rp} = d_p - 2 \times b_p \tag{3.15}$$

$$d_{rg} = d_g - 2 \times b_g \tag{3.16}$$

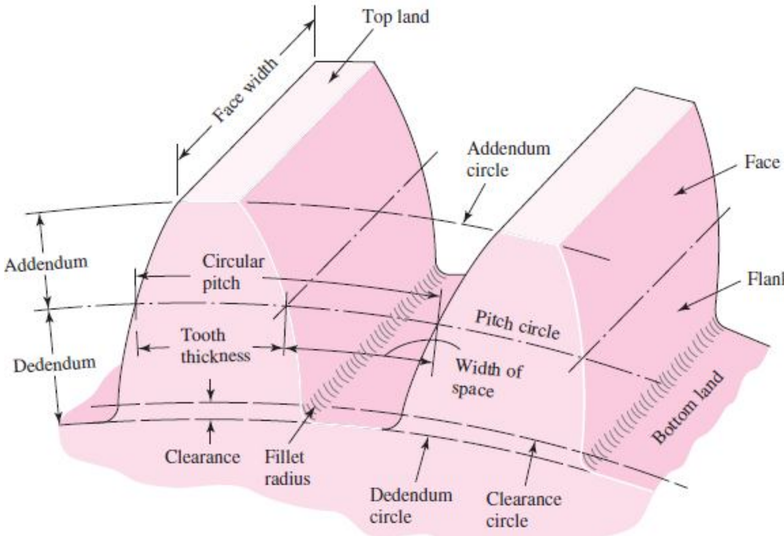


Figure 3.13 Gear tooth nomenclatures [41]

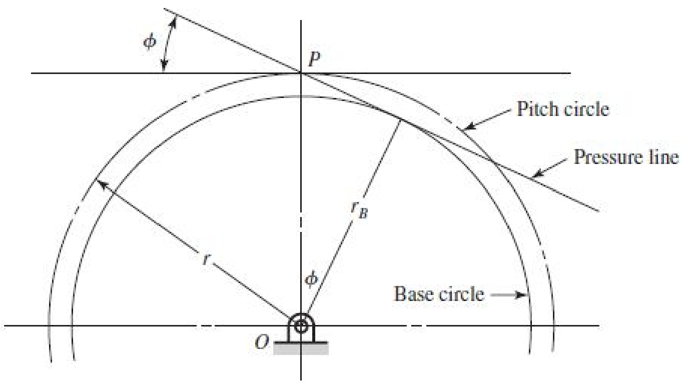


Figure 3.14 Base circle, pitch circle and pressure angle [41]

Limit diameter (d_{l1}) is the diameter at which the outside diameter of the mating gear crosses the line of action (Figure 3.16). It can be calculated by using equations 3.17-3.20.

$$\emptyset_2 = \cos^{-1}\left[\frac{d_{bg}}{d_{ag}}\right] \tag{3.17}$$

$$u_1 = r_{ag}\sin\emptyset_2 - 0.5 \times d_g \sin\emptyset \tag{3.18}$$

$$\emptyset_l = \left[\frac{[0.5\times \mathrm{d_p}]\sin\emptyset - u_1}{0.5\times d_b}\right] \tag{3.19}$$

$$d_{l1} = \frac{d_b}{\cos\emptyset_l} \tag{3.20}$$

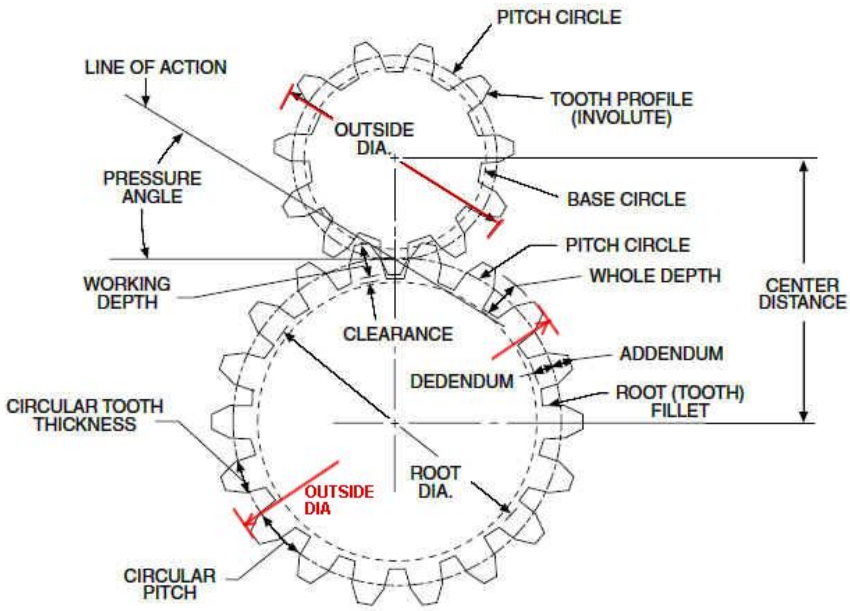


Figure 3.15 Pitch diameter, root diameter and outside diameter

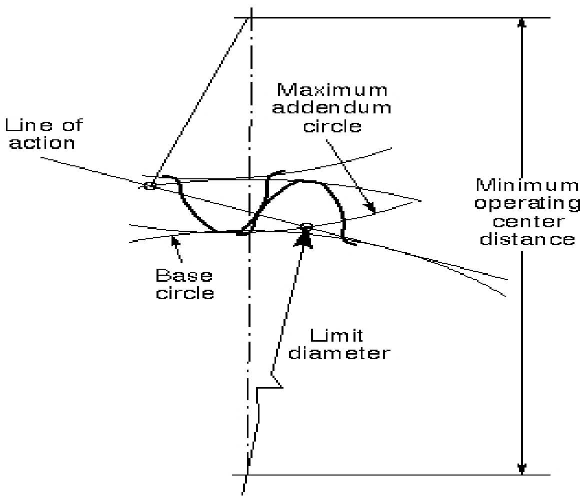


Figure 3.16 Limit diameter

Backlash: is the amount by which the width of a tooth space exceeds the thickness of the engaging tooth measured on the pitch circles. It is purposely built in, is very important because it helps prevent noise, abnormal wear and excessive heat while providing space for lubrication of the gears (Figure 3.17). Recommended backlash values with respect to diametral pitch are presented in Table 3.2.

Chordal tooth thickness: is the distance between symmetrical points on opposite faces of the same tooth measured along a chord (Figure 3.18).

Chordal addendum: is the radial distance from the circular tooth thickness chord to the top of the tooth. It is obtained by adding the rise of arc to the addendum (Figure 3.18).

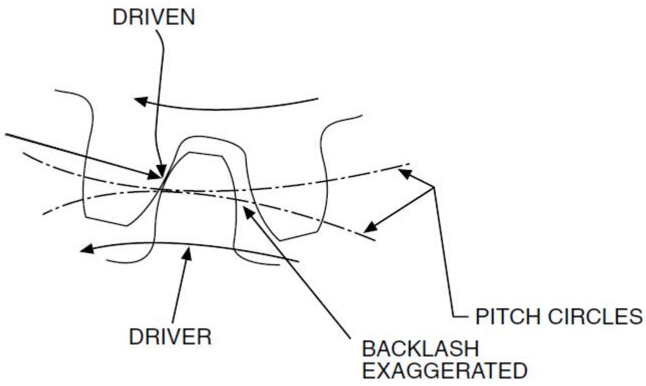


Figure 3.17 Backlash

Table 3.2 Recommended backlash values [45]

Normal Diametral Pitch, P_{nd}	Center Distance, in. (3.93701 × 10^{-2} mm) 0–5	5–10	10–20	20–30	30–50	50–80	80–120
0.5	–	–	–	–	0.045	0.060	0.080
1	–	–	–	0.035	0.040	0.050	0.060
2	–	–	0.025	0.030	0.035	0.045	0.055
3	–	0.018	0.022	0.027	0.033	0.042	–
4	–	0.016	0.020	0.025	0.030	0.040	–
6	0.008	0.010	0.015	0.020	0.025	–	–
8	0.006	0.008	0.012	0.017	–	–	–
10	0.005	0.007	0.010	–	–	–	–
12	0.004	0.006	–	–	–	–	–
16	0.004	0.005	–	–	–	–	–
20	0.004	–	–	–	–	–	–
32	0.003	–	–	–	–	–	–
64	0.002	–	–	–	–	–	–

Working depth (h_w) is the total depth of a tooth space (Figure 3.19). It is equal to the sum of addendum and dedendum and can be calculated based on equation 3.21.

$$h_w = a + b - c \tag{3.21}$$

Working clearance (c_w) is the distance from the working depth to the root circle (Figure 3.19). It can be calculated by equation 3.22 where $\mathrm{CD_w}$ is working center distance.

$$c_w = \mathrm{CD_w} - 0.5 \text{ x } (\mathrm{d_{rp}} + \mathrm{d_{ag}}) \tag{3.22}$$

Pressure angle (Ø) is the angle at which the pressure from the tooth of one gear is passed on to the tooth of another gear. Spur gears can be produced and operated in different pressure angles, two of standard values are 14.5° and 20° as shown in Figure 3.20a and 3.20b. But pressure angle is not limited to these values; it can be 17.5, 19, 22.5 or other values by depending on manufacturing facilities and cutting tool properties.

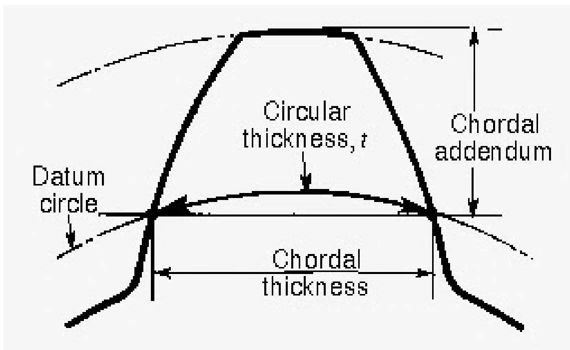


Figure 3.18 Chordal dimensions

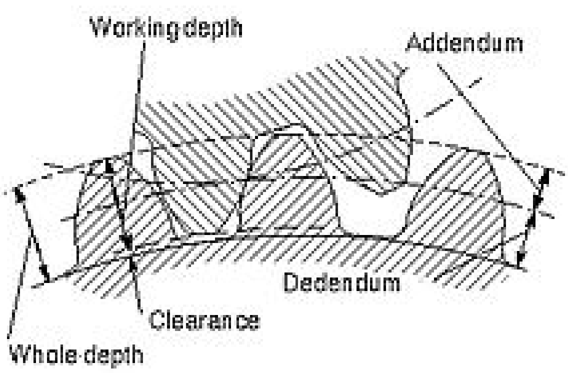
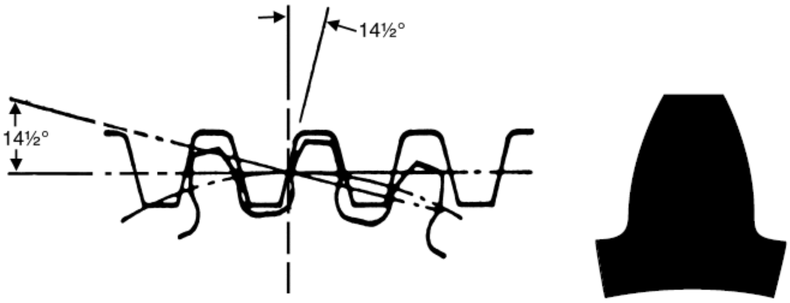
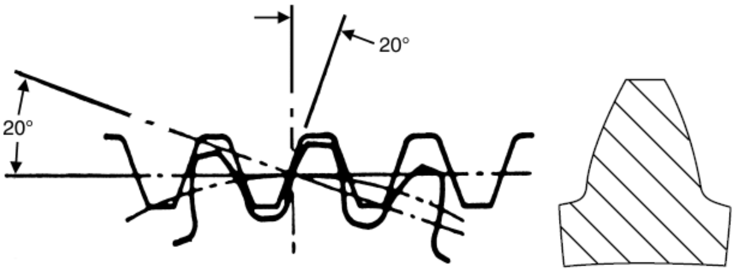


Figure 3.19 Working depth and working clearance



(a) Pressure angle 14.5°



(b) Pressure angle 20.0°

Figure 3.20 Pressure angle

<u>*Center distance*</u> *(CD)* is the distance between the centers of the driving and driven gears (Figure 3.21). Theoretically, it is equal to half of the sum of the pitch diameters of gears; it can be determined by using equation 3.23. When a pair of gear is mounted at a new (say extended, larger than theoretical) center distance the operating pressure angle, operating pitch circles and the contact ratio will all change but velocity ratio will not change. If the center distance is increased, two new operating pitch circles are created having larger diameters because they must be tangent to each other at the pitch point. Thus the pitch circles of gears really do not come into existence until a pair of gears is brought into mesh.

$$CD = \frac{d_p + d_g}{2} \tag{3.23}$$

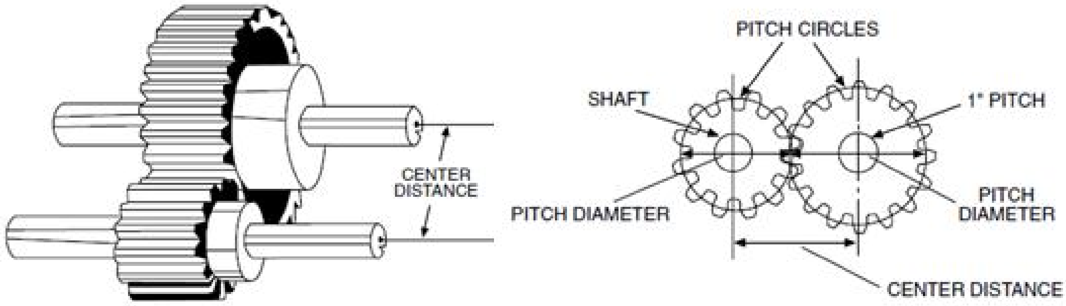


Figure 3.21 Center distance

<u>*Start of active profile*</u> *(SAP)* is the start of active part of length of contact and it can be determined depending on equation 3.24. It is shown in Figure 3.22.

$$SAP = CD_w \sin \emptyset_n - \sqrt{{r_{ag}}^2 - {r_{bg}}^2} \tag{3.24}$$

<u>*End of active profile*</u> *(EAP)* is the end of active part of length of contact and it can be determined by using equation 3.23. It is illustrated in Figure 3.25.

$$\mathrm{EAP} = \sqrt{\mathrm{rap}^2 - \mathrm{rbp}^2} \tag{3.25}$$

<u>*Lowest Point of Single Tooth Contact (LPSTC)*</u> is the point where contact initiates and full load is transmitted by one tooth of gear. Gear's contact stress is calculated with the load applied at this point (Figure 3.22). It is calculated using equation 3.26.

$$\mathrm{LPSTC} = \mathrm{EAP} - P_b \tag{3.26}$$

<u>*Pitch point*</u>: is a point on a gear tooth profile which lies on the pitch circle of that gear. The point of one gear contacts its mating gear, the contact occurs at the pitch point of the mating gear and this common pitch point lies on a line connecting the two gear centers (Figure 3.22).

<u>*Highest Point of Single Tooth Contact*</u> *(HPSTC)* is the highest point on a spur gear at which a single tooth is in contact with the mating gear. Gear's bending stress is determined with the load applied at this point (Figure 3.22). It can be determined depending on equation 3.27.

$$\mathrm{HPSTC} = \mathrm{SAP} + P_b \tag{3.27}$$

Line of action is a line normal to a pair of mating tooth profiles at their point of contact. It is common normal and tangent to both base circles of pinion and gear (Figure 3.22).

Length of contact (L_c) is the distance between beginning and leaving points of contact during the mesh of driver and drieven gears and can be determined by using equation 3.28.

$$L_c = \left(\sqrt{{r_{ag}}^2 + {r_{bg}}^2} + \sqrt{{r_{ap}}^2 + {r_{bp}}^2}\right) - \left(\sqrt{{r_{gw}}^2 + {r_{bg}}^2} + \sqrt{{r_{pw}}^2 + {r_{bp}}^2}\right) \quad (3.28)$$

where r_{pw} and r_{gw} are working pitch diameters of pinion and gear respectively.

Arc of action is the arc on the pitch circle through which a tooth travels from the beginning of contact with the mating gear tooth to the point where the contact ends. Since the two pitch cylinders are in rolling contact (without slippage as per the theory of gearing), the lengths of the arcs of action of the two gears are the same [1]. That is given in equation 3.29.

$$\text{Arc of action} = r_1\theta_1 = r_2\theta_2 \quad (3.29)$$

Where r_1 *and* r_2 are the pitch radii, and θ_1 *and* θ_2 are the angles subtended by the two arcs at their respective centers. The arc of action is divided into:

Arc of approach is the arc through which the tooth moves from the initial contact up to the pitch point (Figure 3.23).

Arc of recess is the arc through which the tooth travels from the pitch point to the end of contact (Figure 3.23).

Angle of approach and recess are the angles subtended at the center by the arc of approach and the arc of recess respectively (Figure 3.24).

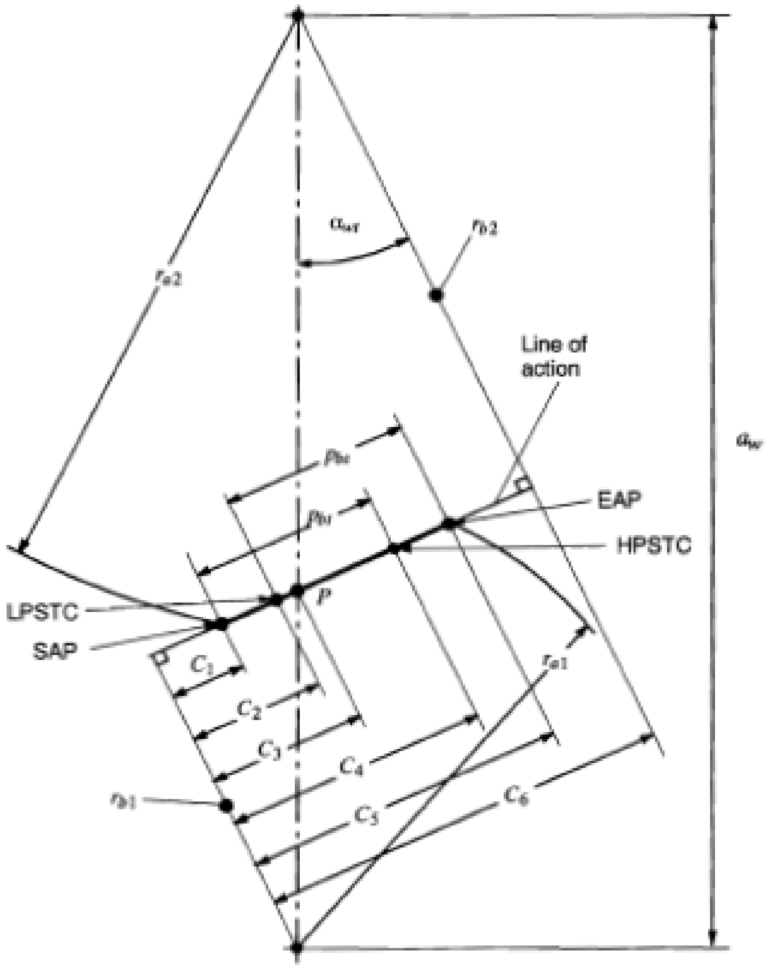


Figure 3.22 Distances along the line of action for external gear pair [12]

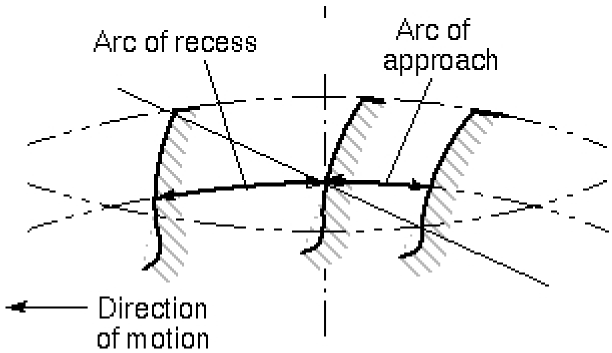


Figure 3.23 Arc of action

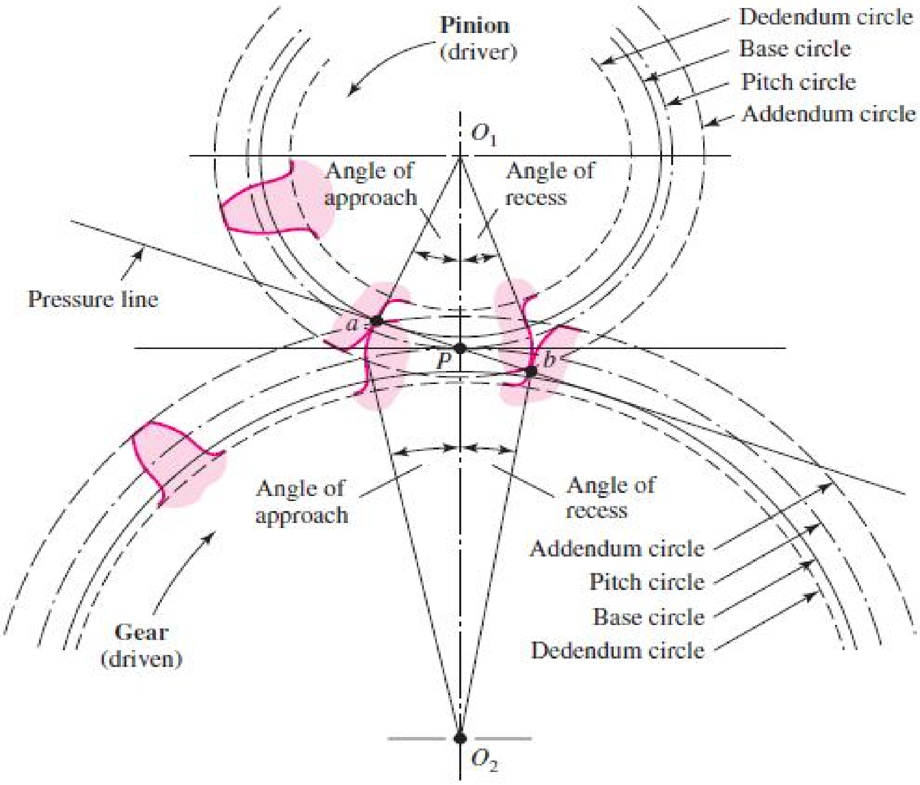


Figure 3.24 Arcs of approach and recess [41]

Contact ratio (CR) defines average number of teeth in contact at any instant of mesh and it is required to be larger than unity to provide continuity of motion. To assure smooth continuous tooth action, as one pair of teeth ceases contact a succeeding pair of teeth must already have come into engagement. It is desirable to have as much overlap as possible. The measure of this overlapping is the contact ratio. This is a ratio of the length of the line-of-action to the base pitch and is calculated using equation 3.30.

$$\mathrm{CR} = \frac{\mathrm{L_c}}{\mathrm{P_b}} \tag{3.30}$$

Profile radius of curvature is the radius of curvature of a tooth profile, usually at the pitch point or a point of contact. It varies continuously along the involute profile (Figure 3.25).

Roll angle is an angle whose arc on the base circle of radius unity equals the tangent of the pressure angle at a selected point on the involute (Figure 3.26).

Pitch line velocity: is a measure of the tangential or peripheral velocity of a gear set and is equal to rotational speed multiplied by the pitch circle circumference. It is a better indication of gear speed than rotational velocity. Because a large gear operating at relatively low rpm may experience the same velocity effects as a small gear operating at high rpm.

Gear teeth engage with each other during operation and experience two different kinds of motion; sliding and rolling. At the pitch point only rolling motion is effective and there is no sliding. But the motion consists of rolling and sliding when contact moves up or down the line of contact. Maximum sliding occurs at the beginning and end of contact; whereas its value is zero at the pitch point.

Rolling velocity is illustrated in Figure 3.27 as v_{r1} andv_{r2}. Gear teeth make a movement of combination of both rolling and sliding on one another. The rolling velocity of the pinion, v_{r1}, and the rolling velocity of the gear, v_{r2}, linearly increases from zero at the interference points to a maximum at each end of the path (SAP and EAP) of contact.

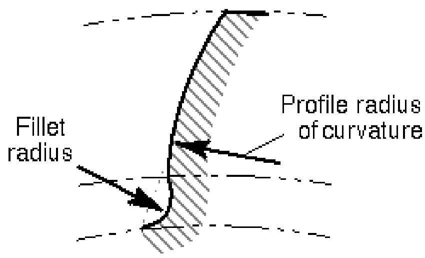


Figure 3.25 Profile radius of curvature

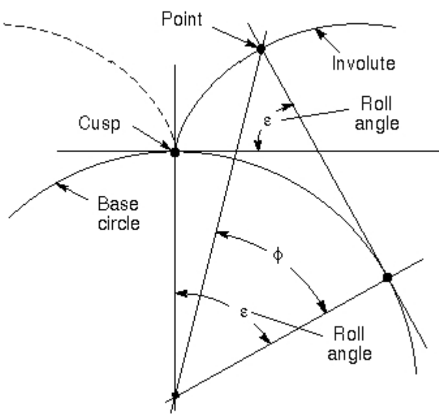


Figure 3.26 Roll angle

Sliding velocity is the linear velocity of the sliding component of the interaction between two gear teeth in mesh. It is the relative velocity in a transverse plane of a common contact point between mating gear teeth and the vectorial difference between the two rolling velocities that are tangential to the tooth profiles and perpendicular to the line of action. The rate of sliding changes constantly; it is zero at the pitch line and it increases as the contact point travels away from the pitch line in either direction. It is shown in Figure 3.27.

Profile shift refers to correction of gears and gears are corrected to avoid undercut, allow for non standard center distances and allow balancing strengths. It can be positive and negative shifting as shown in Figure 3.28. Profile shift has an effect on tooth thickness change; positive shift cause to increase in tooth thickness, whereas negative shift make tooth thickness narrower.

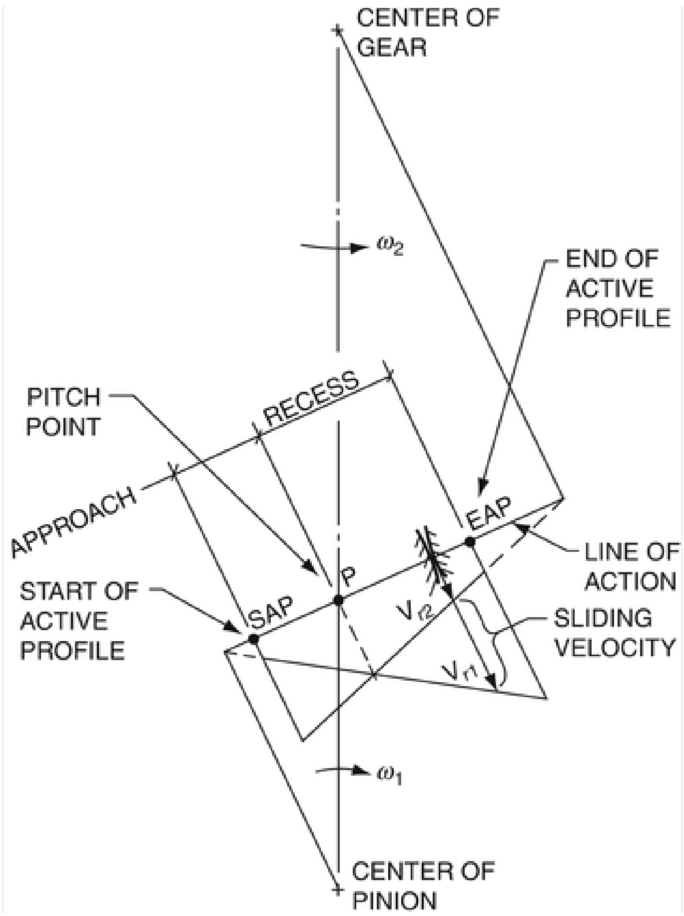


Figure 3.27 Rolling and sliding velocity

When part of the involute profile of a gear tooth is cut away near its base, the tooth is said to be undercut. If the cutter tries to generate below the base diameter, undercut is produced. Undercut is very bad for many reasons such as low in strength and easily wears. It is represented in Figure 3. 29.

In cases where the load comes upon the gears suddenly it is a wise procedure, if an exact ratio is not necessary; to have an odd number of teeth in one of gears, that is, provide a “hunting tooth”. This removes the risk of the same tooth or series of teeth continuously taking the load [46]. Hunting ratio is desirable to equalize wear of gear tooth with providing contact all of gear teeth on mating gear. As a general rule, tooth numbers should be selected so that there is no common factor between the number of teeth of a pinion and a gear that mesh together and there should be no common factor between the number of teeth of a gear and cutting tool that has a gear like meshing action with the part being cut [45].

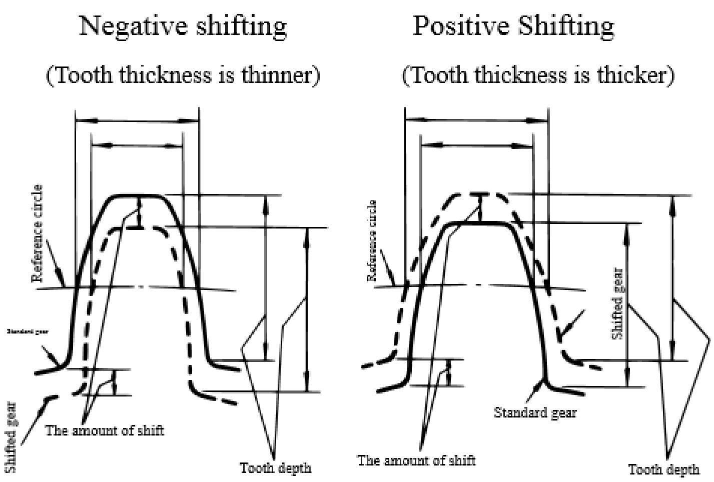


Figure 3.28 Negative and positive profile shifting

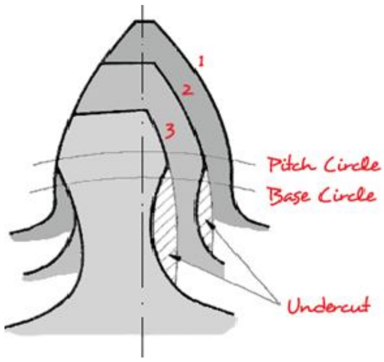


Figure 3.29 Undercutting

3.2.4.2 Helical gear geometry

Helical gears are special spur gears of which teeth are inclined to axis of gear at certain (helix) angles. They can be used for the same applications as spur gears with less noise and more gradual engagement of the teeth during meshing of gears. Helical gears have a disadvantage which does not occur in spurs gear operation. They create axial load which affects bearings, shafts and other neighboring machine elements.

Some geometrical parameters of helical gear different than spur gears are explained below:

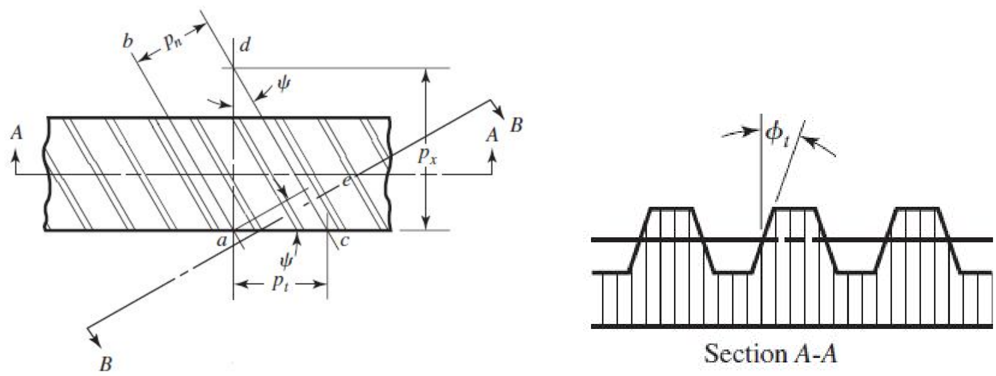
The teeth of the helical gears are not parallel to the axis of the gear but inclined at an angle ψ called helix angle as shown in Figure 3.30a.

Transverse pressure angle is the pressure angle in the plane of rotation. It is shown in Figure 3.30b. Transverse pressure angle can be calculated using equation 3.31.

$$\tan \emptyset_t = \frac{\tan \emptyset_n}{\cos \varphi} \tag{3.31}$$

Transverse module: is the module on the transverse plane as illustrated in figure 3.31 and given as in equation 3.32.

$$m_t = \frac{m}{\cos \varphi} \tag{3.32}$$



(a) Helix angle (b) Transverse pressure angle

Figure 3.30 Helix angle and transverse pressure angle

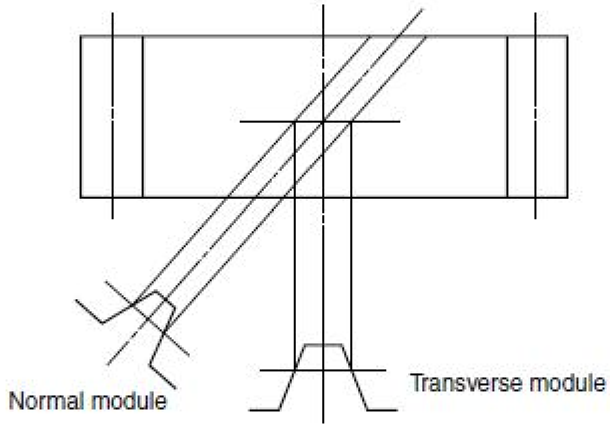


Figure 3.31 Normal and transverse modules

Transverse pitch (P_t) is the ratio of the number of teeth to inches of transverse pitch diameter. It can be calculated based on transverse module given in following equation 3.26.

$$P_t = \pi \times m_t \quad (3.33)$$

Transverse base pitch is the pitch on the base circle or along the line of action (in transverse plane). It can be calculated in equation 3.34.

$$P_{bt} = P_t \cos \emptyset_t \quad (3.34)$$

Axial pitch (P_x) is the distance in an axial plane surface between corresponding adjacent tooth profiles. It can be calculated as in equation 3.35.

$$P_x = \frac{P_t}{\tan \varphi} \quad (3.35)$$

3.3 SPUR AND HELICAL GEARS DESIGN AND ANALYSIS

Among large varieties of gears, the simplest form is external spur gears and they represent basics of many gear types. Helical gear design and analysis can be done with spur gear formulas based on virtual tooth number. So under the title of this chapter, spur and helical gear design/analysis can be explained together.

3.3.1 Spur and Helical Gear Design

The most important issue for designing a gear pair is to find a solution which fulfills load carrying for required power for long time service. Gears design is expected to provide enough mechanical strength to withstand force transmitted, enough surface resistance to overcome pitting failure and enough dynamic resistance to carry fluctuating loads without failure by taking limitations into consideration. There are some geometrical (space constraints), performance and economical limitations when designing a gear pair and other gearbox elements.

Gears are generally designed to operate at predefined center distance for many applications, because there is a space constraint and gearbox cases are manufactured at exact dimensions. Center distance change can be allowable for little amount or not. Designer should take this limitation and decide to apply profile shift to gears or use a standard dimension. Any change in operating center distance of gear pairs affects backlash amount which is usually very essential and sometimes undesirable depending on application. Those criteria are usually used for geometric design of gears. However gears should also be checked for a safe mechanical design.

Factor of safeties are taken into account to provide safe operation of gear pairs under loading for tooth root bending and surface contact stress. The underlying reason for purpose of using these values is to ensure that stresses occurs at gear root or surface should be less than strength values. Designing a gear pair much safer than required one is not economical. Due to this reason, engineer should be careful about economical design of gear pair.

Gear design criteria include two main points of static and dynamic performance. Static performance usually deals with global kinematic and geometric requirements and strength analysis of gear materials under load while dynamic performance usually deals with vibration and noise of gears. Although there are many different modes of gear and tooth failures (pitting, scuffing, wear, spalling, case crushing, frosting, scoring, fatigue breakage, overload fracture), there are two main causes of gear failures taken into consideration during gear mechanical designs; surface durability (pitting) due to surface contact stress and tooth breakage caused by tooth bending fatigue stress.

Gears are firstly designed according to geometrical limitations such as center distance, face width, backlash, etc and then checked against gear tooth root bending stress and surface contact stress. If it is required, they are redesigned to provide safe operation for required power, torque, life and reliability for both tooth bending and surface contact.

3.3.2 Spur and Helical Gear Analysis

Gears experience two kinds of stress mainly; contact stress at tooth surface and bending stress at tooth root under loading during operation. Gear tooth strength relates to the ability to resist tooth breakage; gear tooth surface durability relates to the ability to resist pitting and transmitted load between meshing gears.

There are possible failure types for gears under load and when they are rotating:

- Failure of tooth due to bending stress for static and fatigue (Figures 3.32 and 3.33)
- Failure of tooth surfaces due to contact stresses (Figure 3.34)

Thus, in terms of strength of gear teeth, design is based on

- Static (strength) failure due to bending stress
- Fatigue failure due to bending stress and
- Surface fatigue failure due to contact or Hertzian stresses

3.3.2.1 Bending stress analysis

There are few different calculation methods such as Lewis Equation, Heywood Equation, Kelly-Pederson Equation, AGMA Equation, and Modified Heywood Equation. But all at the end are based on simple cantilever beam bending theory with varying accuracies due to parameters included. Lewis Equation is the earliest attempt to calculate gear tooth root bending stress and uses the basic cantilever beam theory to calculate bending stress at tooth root with approximate root thickness. The classic method of estimating the bending stresses in a gear tooth is the Lewis equation.

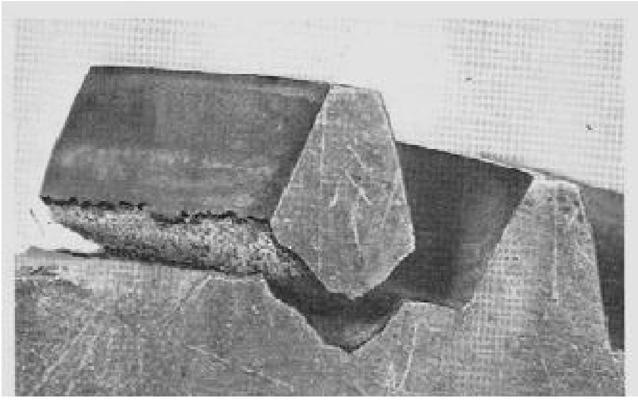


Figure 3.32 Full tooth breakage

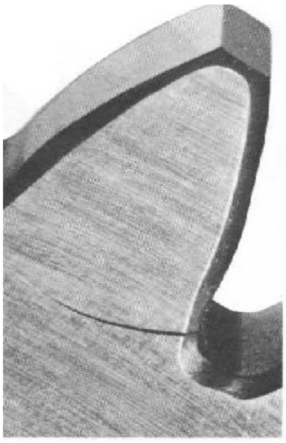


Figure 3.33 Crack due to bending stress

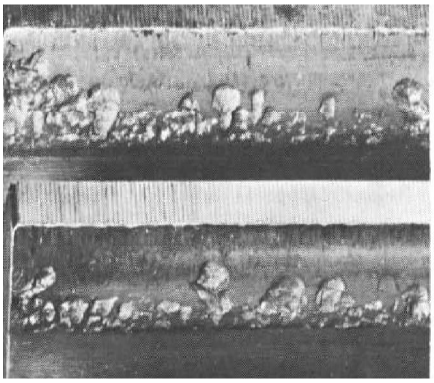


Figure 3.34 Surface failures due to contact stress

It models a gear tooth taking the full load at its tip as a simple cantilever beam as shown in Figure 3.35.

There are some assumptions made in the derivation:

1. The full load is applied to the tip of a single tooth in static condition.
2. The radial component is negligible.

3. The load is distributed uniformly across the full face width.

4. Forces due to tooth sliding friction are negligible.

5. Stress concentration in the tooth fillet is negligible.

Figure 3.36 shows clearly that the gear tooth is stronger throughout than the inscribed constant strength parabola, except for the section at 'a' where parabola and tooth profile are tangential to each other.

Lewis equation is given in equation 3.38 and it is derived by using equations 3.36 - 3.46.

At point 'a', bending stress is

$$\sigma = M * c / I \tag{3.36}$$

Here

$$M = W_t * h, \ c = t/2 \quad \text{and} \quad I = F * t^3/12, \tag{3.37}$$

F is the face width of tooth

By similar triangles,

$$\frac{t/2}{x} = \frac{h}{t/2} \quad \text{or} \quad \frac{t^2}{h} = 4x \tag{3.38}$$

$$\sigma = \frac{6F_t}{4bx} \tag{3.39}$$

$$y = \frac{2x}{3p} \tag{3.40}$$

$$\sigma = \frac{F_t}{bpy} \tag{3.41}$$

$$p = \pi \times m \tag{3.42}$$

$$\sigma = \frac{F_t}{b\pi ym} \tag{3.43}$$

$$Y = \pi \times m \tag{3.44}$$

$$\sigma = \frac{F_t}{bYm} \tag{3.45}$$

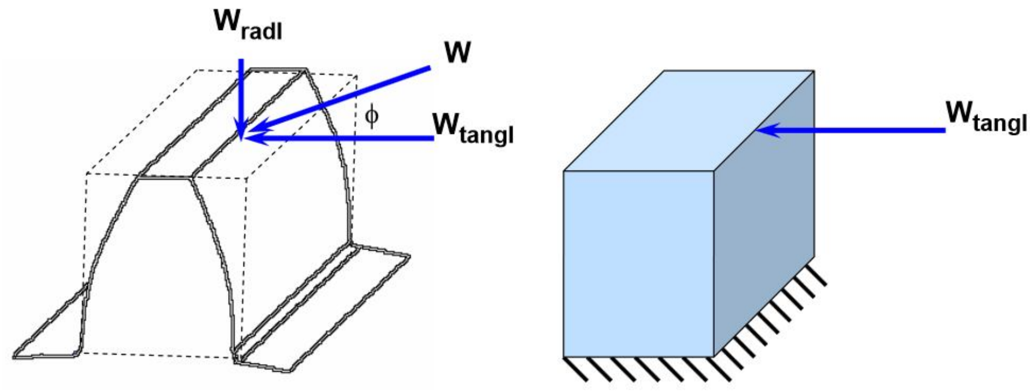


Figure 3.35 Gear tooth under load and cantilever beam

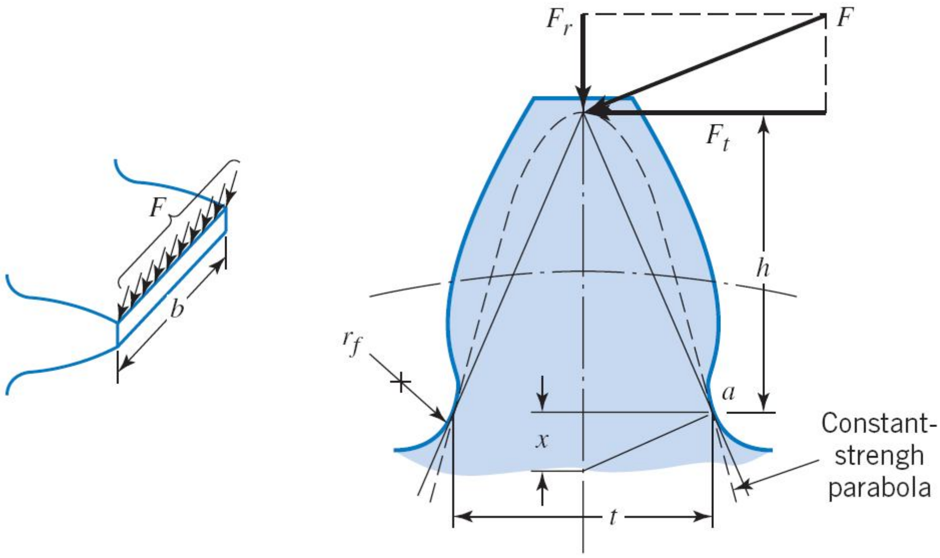


Figure 3.36 Constant strength parabola

The modified Lewis equation for bending stress is,

$$\sigma = \frac{F_t}{K_v b Y m} \tag{3.46}$$

K_v is known as velocity factor or dynamic factor. It depends on pitch line velocity V in m/s.

Y is Lewis form factor; it is tabulated in Table 3.3 and it is a function of the number of teeth, pressure angle, and involute depth of the gear.

Drawbacks of Lewis equation are:

1. The whole load is carried by single tooth at tip is not correct. Normally load is shared by teeth since contact ratio is near to 1.5.
2. The greatest force exerted at the tip of the tooth is not true as the load is shared by teeth. It is exerted much below the tip when single pair contact occurs.

3. The stress concentration effect at the fillet (fillet radius,r_f) is not considered.

Table 3.3 Lewis form factor Y

Number of teeth	Ø = 20° a = 0.8m* b = m	Ø = 20° a = m b = 1.25m	Ø = 25° a = m b = 1.25m	Ø = 25° a = m b = 1.35m+
12	0.335 12	0.229 60	0.276 77	0.254 73
13	0.348 27	0.243 17	0.292 81	0.271 77
14	0.359 85	0.255 30	0.307 17	0.287 11
15	0.370 13	0.266 22	0.320 09	0.301 00
16	0.379 31	0.276 10	0.331 78	0.133 63
17	0.387 57	0.285 08	0.342 40	0.325 17
18	0.395 02	0.293 27	0.352 10	0.335 74
19	0.401 79	0.300 78	0.360 99	0.345 46
20	0.407 97	0.307 69	0.369 16	0.354 44
21	0.413 63	0.314 06	0.376 71	0.362 76
22	0.418 83	0.319 97	0.383 70	0.370 48
24	0.428 06	0.330 56	0.396 24	0.384 39
26	0.436 01	0.339 79	0.407 17	0.396 57
28	0.442 94	0.347 90	0.416 78	0.407 33
30	0.449 02	0.355 10	0.425 30	0.416 91
34	0.459 20	0.367 31	0.439 76	0.433 23
38	0.467 40	0.377 27	0.451 56	0.446 63
45	0.478 46	0.390 93	0.467 74	0.465 11
50	0.484 58	0.398 60	0.476 81	0.475 55
60	0.493 91	0.410 47	0.490 86	0.491 77
75	0.503 45	0.422 83	0.505 46	0.508 77
100	0.513 21	0.435 74	0.520 71	0.526 65
150	0.523 21	0.449 30	0.536 68	0.545 56
300	0.533 48	0.463 64	0.553 51	0.565 70
Rack	0.544 06	0.478 97	0.571 39	0.587 39

* Stub teeth   + Large fillet

Heywood equation is based on photo-elasticity experiments with various fillet geometries of tooth like projections (Figure 3.38). Equation for root bending stress calculation includes both general tooth geometry and also the root fillet geometry hence stresses concentration effect. It gives more accurate results than Lewis equation. Tooth and root geometry are considered simultaneously.

Kelly-Pederson equation does also include both tooth form and geometry factors hence stress concentration in fillet in a way similar to Heywood equation.

AGMA suggests an equation (3.47) with geometry factors including fillet hence stress concentration effect. In addition, parameter of speed factor K_v is included in calculations to be able to design gears against fatigue failure at high speed runs. Parameters which are effective on bending stress calculation based on AGMA equation are given in Table 3.4.

$$\sigma = W^t K_o K_v K_s \frac{1}{bm_t} \frac{K_H K_B}{Y_J} \quad (3.47)$$

Table 3.4 Parameters used for AGMA bending stress equation

Symbol	Explanation	Unit
W^t	Tangential transmitted load	Newton
K_o	Overload Factor	-
K_v	Dynamic Factor	-
K_s	Size Factor	-
b	Face width	mm
m_t	Transverse module	mm
K_H	Load Distribution Factor	-
K_B	Rim Thickness Factor	-
Y_J	Geometry Factor for Bending Strength	-

Modified Heywood equation includes both tooth geometry and also stress concentration effect through fillet geometry. It also included an improved method of positioning the maximum root stress in root fillet.

3.3.2.2 Contact stress analysis

Gears fail by abrasion and wear as well as by tooth breakage. Pitting is the most common type of tooth surface failure. The main reason of pitting is repeating high contact stress. Mating tooth in contact under compression load can be represented two contacting cylinders under compression load as shown in Figure 3.37. The contact pressure is intensified near the pitch circle, where the contact is pure rolling with zero sliding velocity. This condition is modeled as a pair of cylinders in line contact, and a Hertzian contact stress analysis is used by using equations 3.39 - 3.41.

$$P_{max} = \frac{2W_t}{\pi bl} \quad (3.48)$$

$$b = \sqrt{\frac{2W_t}{\pi l} \left\{ \frac{\left(\frac{1-v_1^2}{E_1}\right)+\left(\frac{1-v_2^2}{E_2}\right)}{\left(\frac{1}{d_1}+\frac{1}{d_2}\right)} \right\}} \quad (3.49)$$

$$\sigma_c^2 = \frac{W_t}{\pi F \cos \emptyset}\left[\frac{\frac{1}{r_1}+\frac{1}{r_2}}{\left(\frac{1-v_1^2}{E_1}\right)+\left(\frac{1-v_2^2}{E_2}\right)}\right] \qquad (3.50)$$

$P_{max}: largest\ surface\ pressure$

$W_t: force\ pressing\ the\ tqwo\ cylinder\ together$

$l: length\ of\ cylinders$

$b: half\ width$

$v_{1,2}: Poission's\ ratio\ of\ pinion\ and\ gear$

$E_{1,2}: modulus\ of\ elasticty\ \ of\ pinion\ and\ gear$

$d_{1,2}: diameters\ of\ contacting\ cylinders$

$r_{1,2}: radius\ of\ curvature\ at\ pitch\ point\ of\ pinion\ and\ gear$

$F: face\ width$

$\emptyset: pressure\ angle$

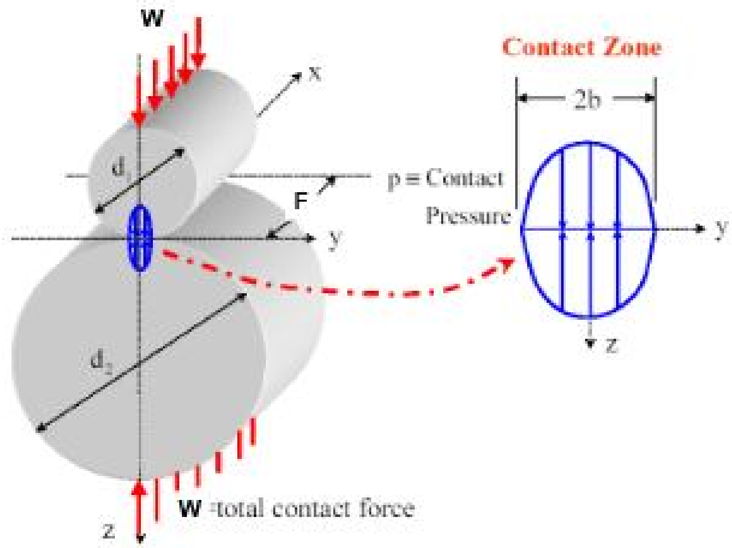


Figure 3.37 Contact stress between two cylinders

It is however, not only the contact stress which causes failure of tooth surface but also the relative sliding speed of the tooth surfaces which affects the surface failure in combination with the contact pressure or stress. Thus the product of contact stress and relative sliding speed at operating speed is an indication of likely risk of surface failure (pitting) and it may also be of the likely power loss in mesh.

3.4 PARALLEL AXES GEAR STANDARDS

Some parallel axes gear standards like ISO, AGMA and DIN have been developed by international organizations to design and analyze parallel axes gear. ISO 6336:2006, AGMA 2101 D 04 and DIN 3990: 1987 are some of main standards used in design / analysis of parallel axes spur and helical gear pairs. They are the same in general principle of gear design and analysis but there are some differences in methods used to calculate gear performance parameters, and some influence and performance parameters. Tooth root bending stress and surface contact stress are calculated and compared with allowable stress values for each stress type. All parts of this standard includes calculations, recommendations, tables and figures to calculate stresses at last and make comparison with allowable values possible.

3.4.1 ISO 6336: 2006 Calculation of Load Capacity of Spur and Helical Gears

ISO 6336:2006 consists five parts given in Table 3.5 under the general title of calculation of load capacity of spur and helical gears. These parts consist of calculations of basic principles, introduction and general influence factors, surface durability (pitting), tooth bending strength, strength and quality of gear materials and service life under variable load.

Table 3.5 ISO 6336:2006 parts

Standard	Part	Scope
ISO 6336:2006	1	Basic principles, introduction and general influence factors
ISO 6336:2006	2	Calculation of surface durability (pitting)
ISO 6336:2006	3	Calculation of tooth bending strength
ISO 6336:2006	5	Strength and quality of materials
ISO 6336:2006	6	Calculation of service life under variable load
ISO 53:1998	-	Cylindrical gear for general and heavy engineering -Standard basic rack tooth profile
ISO 54:1996		Cylindrical gear for general and heavy engineering - Module
ISO 1328:1997 Cylindrical gears-ISO system of accuracy	1	Definitions and allowable values of deviations relevant to corresponding flanks of gear teeth
	2	Definitions and allowable values of deviations relevant to radial composite deviations and run out information

3.4.1.1 ISO 6336:1-2006 Calculation of load capacity of spur and helical gears - basic principles, introduction and general influence factors

This part of the ISO 6336: 2006 [2] includes calculation of basic principles, introduction and general influence factors which are used in estimating of gear surface durability (pitting) and bending strength of cylindrical involute gears. Some of these general influence factors are explained and related formulae are presented below:

Dynamic factor is the factor, which takes load increments due to internal dynamic effects as related to gear tooth accuracy grade, speed and load into account.

Face load factor for contact stress takes uneven distribution of load over the face width, due to mesh misalignment caused by inaccuracies in manufacture and elastic deformations into account.

Face load factor for tooth root stress includes the effect of the load distribution over the face width on stresses at the tooth root.

Transverse load factor for contact stress takes into account uneven load distribution in the transverse direction resulting, for example, from pitch deviation.

3.4.1.2 ISO 6336-2-2006 Calculation of load capacity of spur and helical gears - calculation of surface durability (pitting)

Surface load capacity of cylindrical gears with involute external teeth is determined based on contact stress at the pitch point or at the inner point of single pair tooth contact. For this purpose, factor of safety for contact and many parameters such as nominal contact stress, permissible contact stress, zone factor, elasticity factor, contact ratio factor, helix angle factor are calculated by using formulae presented in ISO 6336:2-2006 [3].

Safety factor for surface durability (S_H, against pitting) is used to check whether gears are safe or not in terms of surface fatigue strength given in equation 3.51. The factor is needed to be bigger than required minimum factor of safety value.

$$S_H = \frac{\sigma_{HG}}{\sigma_H} \tag{3.51}$$

The permissible contact stress is calculated from following equation:

$$\sigma_{HG} = \sigma_{Hlim} Z_{NT}\, Z_L Z_V\, Z_R\, Z_W\, Z_X \tag{3.52}$$

Contact stress calculation of surface durability is based on the contact stress at the pitch point or at the inner point of single pair tooth contact. Its calculation for pinion and gear is given in equations 3.53 and 3.54 respectively.

$$\sigma_{H1}=Z_B\, \sigma_{H0} \sqrt{K_A K_V K_{H\beta} K_{H\alpha}} \tag{3.53}$$

$$\sigma_{H2}=Z_D\, \sigma_{H0}\sqrt{K_A K_V K_{H\beta} K_{H\alpha}} \tag{3.54}$$

Nominal contact stress number is required to estimate contact stresses for pinion and gear, and is calculated by using following equation.

$$\sigma_{H0}=Z_H Z_E Z_\varepsilon Z_\beta \sqrt{\frac{F_t}{d_1 b}\frac{u+1}{u}} \tag{3.55}$$

Single pair teeth contact factors account for the influence of tooth flank curvature on contact stress and is explained under conditions a, b, c for spur and helical gears. It is calculated by using equations 3.47- 3.53.

$$M_1=\frac{\tan\alpha_{wt}}{\sqrt{\left(\sqrt{\frac{d_{a1}^2}{d_{b1}^2}-1}-\frac{2\pi}{z1}\right)\left(\sqrt{\frac{d_{a2}^2}{d_{b2}^2}-1}-(\varepsilon_\alpha-1)\frac{2\pi}{z2}\right)}} \tag{3.56}$$

$$M2=\frac{\tan\alpha_{wt}}{\sqrt{\left(\sqrt{\frac{d_{a2}^2}{d_{b2}^2}-1}-\frac{2\pi}{z2}\right)\left(\sqrt{\frac{d_{a1}^2}{d_{b1}^2}-1}-(\varepsilon_\alpha-1)\frac{2\pi}{z1}\right)}} \tag{3.57}$$

a) Spur gears with $\varepsilon_\alpha > 1$:

If M1 ≤ 1 then $Z_B = 1$; if $M_2 \leq 1$ then $Z_D = 1$; (3.58)

If M1 $>$ 1 then $Z_B = M_1$; if $M_2 > 1$ then $Z_D = M_2$. (3.59)

b) Helical gears with $\varepsilon_\alpha > 1$ and $\varepsilon_\beta \geq 1$:

$$Z_B = Z_D = 1 \tag{3.60}$$

c) Helical gears with $\varepsilon_\alpha > 1$ and $\varepsilon_\beta < 1$:

Z_B and Z_D are determined by linear interpolation between the values for spur and helical gearing with $\varepsilon_\beta \geq 1$:

$Z_B = M_1 - \varepsilon_\beta (M_1 - 1)$ and $Z_B \geq 1$ (3.61)

$Z_D = M_2 - \varepsilon_\beta (M_2 - 1)$ and $Z_D \geq 1$ (3.62)

Zone factor (Z_H) accounts for the influence on Hertzian pressure of tooth flank curvature at the pitch point and transform the tangential load at the reference cylinder to normal load at pitch cylinder. It can be calculated by using base helix angle, working transverse pressure angle and transverse pressure angle based on equation 3.54.

$$Z_H = \sqrt{\frac{2\, cos\beta_b\, cos\alpha_{wt}}{cos\alpha_t^2\, sin\alpha_{wt}}} \quad (3.63)$$

The elasticity factor (Z_E) takes into account the influences of the material properties; modulus of elasticity and Poisson's ratio on the contact stress. It can be calculated by using equation 3.64 or read from Table 3.6 depending on the material of gears.

$$Z_E = \sqrt{\frac{1}{\pi(\frac{1-v_1^2}{E_1}+\frac{1-v_2^2}{E_2})}} \quad (3.64)$$

Table 3.6 Elasticity factor [3]

Wheel 1			Wheel 2			
Material [a]	**Modulus of elasticity, E** N/mm^2	**Poisson's ratio, ν**	**Material**	**Modulus of elasticity, E** N/mm^2	**Poisson's ratio, ν**	Z_E $\sqrt{N/mm^2}$
St, V, Eh, IF, NT, NV	206 000	0,3	St, V, Eh, IF, NT, NV	206 000	0,3	189,8
			St(cast)	202 000		188,9
			GGG, GTS	173 000		181,4
			GG	126 000 to 118 000		165,4 to 162,0
St(cast)	202 000		St(cast)	202 000		188,0
			GGG, GTS	173 000		180,5
			GG	118 000		161,4
GGG, GTS	173 000		GGG, GTS	173 000		173,9
			GG	118 000		156,6
GG	126 000 to 118 000		GG	118 000		146,0 to 143,7

Contact ratio factor (Z_{ε}) takes the influence of the transverse contact ratio and overlap ratios on the surface load capacity of cylindrical gears into account. It can be calculated for spur and helical gears by using equations 3.65 and 3.66 respectively.

For Spur gears

$$Z_{\varepsilon}=\sqrt{\frac{4-\varepsilon_{\alpha}}{3}} \tag{3.65}$$

For Helical gears

$$Z_{\varepsilon}=\sqrt{\frac{(4-\varepsilon_{\alpha})}{3}+\left(1-\varepsilon_{\beta}\right)+\frac{\varepsilon_{\beta}}{\varepsilon_{\alpha}}} \tag{3.66}$$

Helix angle factor (Z_{β}) which only depends on accounts for the influence of the helix angle on surface load capacity and it is calculated in following equation.

$$Z_{\beta}=\frac{1}{\sqrt{cos_{\beta}}} \tag{3.66}$$

Life factor (Z_{NT}) accounts for the higher contact stress, including static stress, which may be tolerable for a limited life (number of load cycles) as compared with the allowable stress at the point or knee on the curves of Figure 3.41 where factor is unity. It can be read from Table 3.7 or Figure 3.38.

The lubricant film between the tooth flanks influences surface durability. Viscosity of the lubricant in the mesh, sum of instantaneous velocities of the two tooth surfaces, loading, radius of relative curvature, relationship between the combined values of the surface roughness of the tooth flanks, and the minimum film thickness of the lubricant film have significant influence on surface durability.

Lubricant factor (Z_L) for mineral oils can be determined as a function of nominal viscosity at 40 °C (or 50 °C) and the value allowable contact stress number of softer of materials of the mating gear pair as in equations 3.67 - 3.70.

ν_{50} is nominal viscosity at 50 °C, mm^2/s
ν_{40} is nominal viscosity at 40 °C, mm^2/s

These viscosity values can be read from Table 3.8.

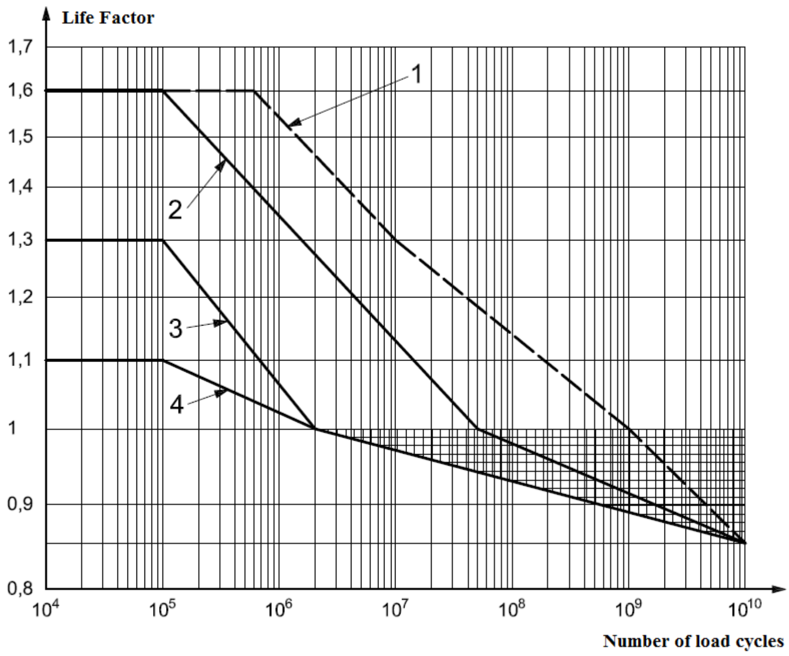


Figure 3.38 Life factor ($contact\ stress, Z_{NT}$) [3]

Table 3.7 Life factor ($contact\ stress, Z_{NT}$) [3]

Material [a]	Number of load cycles	Life factor, Z_{NT}
St, V, GGG (perl., bai.), GTS (perl.), Eh, IF; only when a certain degree of pitting is permissible	$N_L \leqslant 6 \times 10^5$, static	1,6
	$N_L = 10^7$	1,3
	$N_L = 10^9$	1,0
	$N_L = 10^{10}$	0,85 up to 1,0 [b]
St, V, GGG (perl., bai.), GTS (perl.), Eh, IF	$N_L \leqslant 10^5$, static	1,6
	$N_L = 5 \times 10^7$	1,0
	$N_L = 10^9$	1,0
	$N_L = 10^{10}$	0,85 up to 1,0
GG, GGG (ferr.), NT (nitr.), NV (nitr.)	$N_L \leqslant 10^5$, static	1,3
	$N_L = 2 \times 10^6$	1,0
	$N_L = 10^{10}$	0,85 up to 1,0
NV (nitrocar.)	$N_L \leqslant 10^5$, static	1,1
	$N_L = 2 \times 10^6$	1,0
	$N_L = 10^{10}$	0,85 up to 1,0

Table 3.8 Nominal viscosity [3]

ISO viscosity class (grade)		VG 32 [a]	VG 46 [a]	VG 68 [a]	VG 100	VG 150	VG 220	VG 320
Nominal viscosity, mm^2/s	ν_{40}	32	46	68	100	150	220	320
	ν_{50}	21	30	43	61	89	125	180
Viscosity parameter	ν_f	0,040	0,067	0,107	0,158	0,227	0,295	0,370

[a] Only for high speed transmission.

$$Z_L = C_{ZL} + \frac{4(1.0-C_{ZL})}{\left(1.2+\frac{80}{v_{50}}\right)^2} = C_{ZL} + \frac{4(1.0-C_{ZL})}{\left(1.2+\frac{134}{v_{40}}\right)^2} \quad (3.67)$$

C_{ZL} is determined according to σ_{Hlim} ranges given below:

In the range 850 N/mm^2 $\leq \sigma_{H\,lim} \leq$ 1 200 N/mm^2

$$C_{ZL} = \frac{\sigma_{Hlim}}{4375} + 0,6357 \quad (3.68)$$

In the range $\sigma_{H\,lim}$ < 850 N/mm^2

$$C_{ZL} = 0.83 \quad (3.69)$$

In the range $\sigma_{H\,lim}$ > 1 200 N/mm^2

$$C_{ZL} = 0.91 \quad (3.70)$$

The velocity factor (Z_v) can be determined as function of pitch line velocity and the allowable contact stress number of the softer of the materials of the mating gear pair. It is estimated by using equations 3.71-3.72.

$$Z_v = C_{ZV} + \frac{2\times(1.0-C_{ZV})}{\sqrt{0.8+\frac{32}{v}}} \quad (3.71)$$

$$C_{ZV} = C_{ZL} + 0.02 \quad (3.72)$$

Where v is pitch line velocity (m/sec).

Work hardening factor (Z_W) takes into account of the increase in the surface durability due to meshing a steel wheel (structural steel, through-hardened steel) with a hardened or substantially harder pinion with smooth tooth flanks. This factor is determined as in equations 3.73 - 3.75 depending on the ranges of gear surface hardness.

For 130 ≤HB ≤ 470, Z_W for reference and long life stress is calculated as

$$Z_W = \left(1.2 - \frac{(HB-130)}{1700}\right)\left(\frac{3}{Rz_H}\right)^{0.15} \quad (3.73)$$

HB is Brinell hardness number of the softer gear's tooth flanks.

For HB < 130

$$Z_W = 1.2\left(\frac{3}{Rz_H}\right)^{0.15} \tag{3.74}$$

For HB > 470

$$Z_W = \left(\frac{3}{Rz_H}\right)^{0.15} \tag{3.75}$$

The roughness factor (Z_R) can be determined in accordance with equation 3.76 as a function of the surface condition (roughness) of the tooth flanks, the dimensions (radius of relative curvature, ρ_{red}) and the $\sigma_{H\ lim}$ value for the softer material of the mating gear pair. It can be read from curves or calculated as a function of the mean relative roughness.

Mean peak-to-valley roughness of the gear pair is calculated in equation 3.68:

$$R_Z = \frac{Rz_1 + Rz_2}{2} \tag{3.76}$$

The mean roughness Rz_1 (pinion flank) and Rz_2 (wheel flank) shall be determined for their surface condition after manufacture, including any running-in treatment, planned as a manufacturing, commissioning or in-service process, when it is safe to assume that it will take place. Mean relative peak-to-valley roughness for the gear pair:

$$Z_R = \left(\frac{3}{Rz_{10}}\right)^{C_{ZR}} \tag{3.77}$$

$$Rz_{10} = R_z \sqrt[3]{\frac{10}{\rho_{red}}} \tag{3.78}$$

Radius of relative curvature:

$$\rho_{red} = \frac{\rho_1 \rho_2}{\rho_1 + \rho_2} \tag{3.79}$$

Where $\rho_{1,2} = 0.5\ d_{1,2} \tan \alpha_{Wt}$

In the range 850 N/mm² ≤ $\sigma_{H\ lim}$ ≤ 1 200 N/mm²:

$$C_{ZR} = 0.32 - 0.0002 \times \sigma_{Hlim} \tag{3.80}$$

In the range $\sigma_{H\ lim}$ < 850 N/mm²:

$$C_{ZR} = 0.15 \tag{3.81}$$

In the range $\sigma_{H\,lim}$ > 1 200 N/mm^2:

$C_{ZR} = 0.08$ (3.82)

3.4.1.3 ISO 6336-3-2006 Calculation of load capacity of spur and helical gears – Calculation of tooth bending strength

The maximum tensile stress at the tooth root which may not exceed the permissible bending stress for the material, is the basis for rating the bending strength of gear teeth. When the tooth loading is unidirectional and the teeth are of conventional shape, seldom propagate to failure. Crack propagation ending in failure is most likely to stem from cracks initiated in tension fillets. Safety factors for tooth root bending stress of pinion and gear ($S_{F1,}S_{F2}$) are calculated as following equations 3.83 and 3.84.

$$S_{F1}=\frac{\sigma_{FG1}}{\sigma_{F1}} \geq S_{Fmin} \quad (3.83)$$

$$S_{F2}=\frac{\sigma_{FG2}}{\sigma_{F2}} \geq S_{Fmin} \quad (3.84)$$

Nominal tooth root stress (σ_{FO}), is the maximum local principal stress produced at the tooth root when an error-free gear pair is loaded by the static nominal torque and without any pre-stress such as shrink fitting. It is calculated as in equation 3.85.

$$\sigma_{F0}=\frac{F_t}{b_{m_n}} Y_F Y_S Y_\beta Y_{DT} \quad (3.85)$$

Tooth root stress (σ_F) and tooth root stress limits (σ_{FG}) are important parameters to calculate factor of safeties by using equations 3.85 and 3.86.

$$\sigma_F=\sigma_{F0} K_A K_V K_{F\beta} K_{F\alpha} \quad (3.86)$$

$$\sigma_{FG} = \sigma_{Flim} Y_{ST}\, Y_{NT} Y_{\delta relT} Y_{RrelT} Y_X \quad (3.87)$$

Form factor (Y_F) is the factor by which the influence of tooth form on nominal tooth root stress is taken into account. The determination of the normal chordal dimension (S_{Fn}) of the tooth root critical section and the bending moment arm (h_{Fe}) relevant to load application at the outer point of single pair gear tooth contact. Equations between 3.88 and 3.108 are used to calculate form factor.

$$Y_F = \frac{\frac{6H_{Fe}}{m_n}\cos\alpha_{Fen}}{\left(\frac{S_{Fn}}{m_n}\right)^2 \cos\alpha_n} \quad (3.88)$$

Tooth root normal chord, S_{Fn}, radius of root fillet, ρ_F, bending moment arm, h_{Fe}

The auxiliary values are determined:

$$E=\frac{\pi}{4}m_n - h_{fP}\tan\alpha_n + \frac{s_{pr}}{\cos\alpha_n} - (1-\sin\alpha_n)\frac{\rho_{fP}}{\cos\alpha_n} \quad (3.89)$$

$s_{pr}=0$; when gears are not undercut

$$G = \frac{\rho_{fPv}}{m_n} - \frac{h_{fP}}{m_n} + x \quad (3.90)$$

$$H=\frac{2}{z_n}\left(\frac{\pi}{2} - \frac{E}{m_n}\right) - T \quad (3.91)$$

$$T=\frac{\pi}{3} \text{ for external gears} \quad (3.92)$$

$$\theta=\frac{2G}{z_n}\tan\theta - H \quad (3.93)$$

Tooth root normal chord:

$$s_{Fn} = m_n\left[z_n \sin\left(\frac{\pi}{3}+\theta\right) + \sqrt{3}\left(\frac{G}{\cos\theta} - \frac{\rho_{fPv}}{m_n}\right)\right] \quad (3.94)$$

Radius of root fillet:

$$\rho_f = m_n\left[\frac{\rho_{fPv}}{m_n} + \frac{2G^2}{\cos\theta(|z_n|\cos^2\theta - 2G)}\right] \quad (3.95)$$

Bending moment arm:

$$\frac{h_{Fe}}{m_n} = \frac{1}{2}\left[(\cos\gamma_e - \sin\gamma_e\tan\alpha_{Fen})\frac{d_{en}}{m_n} - z_n\cos\left(\frac{\pi}{3}-\theta\right) - \left(\frac{G}{\cos\theta} - \frac{\rho_{fPv}}{m_n}\right)\right] \quad (3.96)$$

All formulas are usable for helical gears but sometimes it is needed to calculate virtual tooth number to insert spur gear formulas. This ensures to assume helical gear as a spur gear.

$$\beta_b=\sin^{-1}(\sin\beta\cos\alpha_n) \quad (3.97)$$

$$z_n=\frac{z}{cos^2\beta b\cos\beta} \quad (3.98)$$

$$z_n\approx\frac{z}{cos^3\beta} \quad (3.99)$$

$$\varepsilon_{\alpha n}=\frac{\varepsilon_\alpha}{cos^2\beta b} \quad (3.100)$$

$$d_n=\frac{d}{cos^2\beta b} \quad (3.101)$$

$$p_{bn}=\pi m_n\cos\alpha_n \quad (3.102)$$

$$d_{bn}=d_n\cos\alpha_n \quad (3.103)$$

$$d_{an}=d_n+d_a-d \tag{3.104}$$

$$d_{en}=2\frac{z}{|z|}\sqrt{\left[\sqrt{\left(\left(\frac{d_{an}}{2}\right)^2-\left(\frac{d_{bn}}{2}\right)^2\right)}-\frac{\pi d\cos\beta\cos\alpha_n}{|z|}(\varepsilon_{\alpha n}-1)\right]^2+\left(\frac{d_{bn}}{2}\right)^2} \tag{3.105}$$

The number of teeth, z, is positive for external gears and negative for internal gears.

$$\alpha_{en}=\cos^{-1}\left(\frac{d_{bn}}{d_{en}}\right) \tag{3.106}$$

$$\gamma_e=\frac{0{,}5\pi+2x\tan\alpha_n}{z_n}+inv\alpha_n-inv\alpha_{en} \tag{3.107}$$

$$\alpha_{fen}=\alpha_{en}-\gamma_e \tag{3.108}$$

Stress correction factor (Y_S) is used to convert the nominal tooth root stress to local tooth root stress and, by means of this factor, the following are taken into account:

a) The stress amplifying effect of section change at the fillet radius at the tooth root.

b) That evaluation of the true stress system at the tooth root critical section is more complex than the simple system evaluation presented, with evidence indicating that the intensity of the local stress at the tooth root consists of two components, one of which is directly influenced by the value of the bending moment and the other increasing with closer proximity to the critical section of the determinant position of load application.

The calculation of the stress correction factor is made in accordance, which is valid in range: $1\leq q_s < 8$. The calculation is given in equations 3.109-3.111.

$$Y_s=(1.2+0.13L)q_s^{\left[\frac{1}{1.21+\frac{2.3}{L}}\right]} \tag{3.109}$$

$$L=\frac{s_{Fn}}{h_{Fe}} \tag{3.110}$$

$$q_s=\frac{s_{Fn}}{2\rho_F} \tag{3.111}$$

The tooth root stress of a virtual spur gear is calculated as a prelimary value, is converted by means of the helix factor to that of the corresponding helical gear. By this means, the oblique orientation of the lines of mesh contact is taken into account (less tooth root stress). The factor can be taken from Figure 3.39.

Rim thickness factor (Y_B) is a simplified factor used to derate thin rimmed gears when detailed calculations of stresses in both tension and compression or experiments are not available. Rim thickness factor can be taken from Figure 3.40.

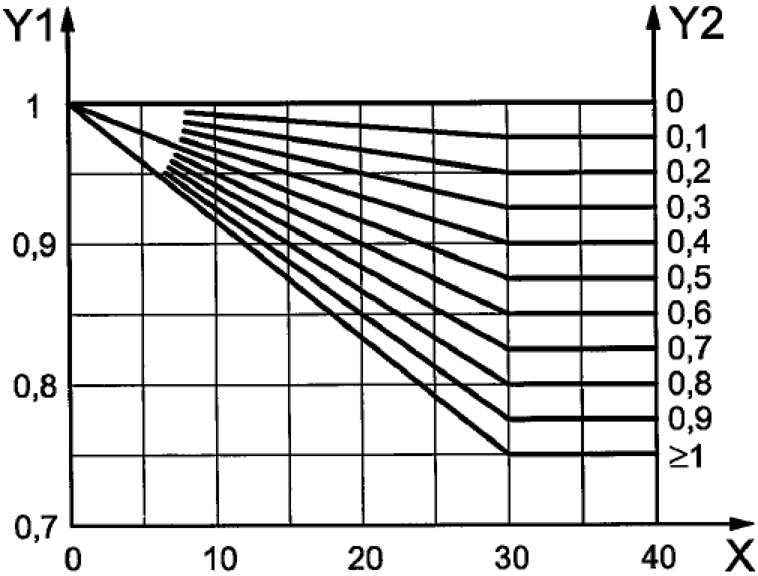


Figure 3.39 Helix angle factor [4]

Key

X reference helix angle, β, degrees

Y_1 helix factor, Y_β

Y_2 overlap ratio, ε_β

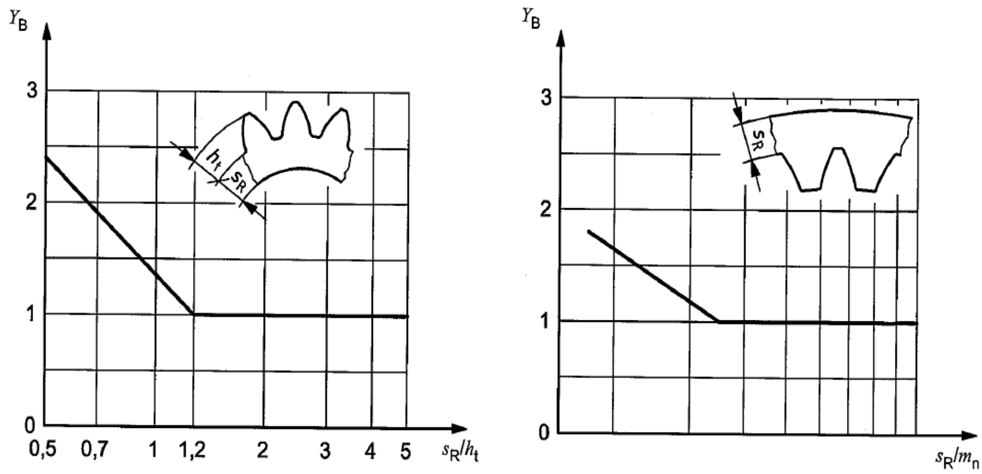


Figure 3.40 Rim thickness factor (Y_B) [4]

Deep tooth factor adjusts nominal tooth root stress for gears of high precision (accuracy grade $\leq$ 4) with contact ratios in the range of 2$\leq$ $\varepsilon_{\alpha n}$ $<$ *2.5* and with applied actual profile modification to obtain a trapezoidal load distribution along the path of contact. Deep tooth factor is calculated according to equations 3.112 - 3.114.

If $\varepsilon_{\alpha n} \leq 2.05$ or if $\varepsilon_{\alpha n} > 2.05$ and the accuracy grade > 4, then

Y_{DT}=1.0 (3.112)

If 2,05$< \varepsilon_{\alpha n} \leq 2.5$ and the accuracy grade ≤ 4 , then

Y_{DT}=-0.666*$\varepsilon_{\alpha n}$+2.366 (3.113)

If $\varepsilon_{\alpha n} > 2.5$ and the accuracy grade ≤ 4, then

Y_{DT}=0.7 (3.114)

Life factor (Y_{NT}) accounts for the higher tooth root stress, which may be tolerable for a limited life (number of load cycles), as compared with the allowable stress at 3×10^6 cycles. Determination of Y_{NT} for static stress and reference stress can be taken from Table 3.9.

The extent to which the calculated tooth root stress deemed to have caused fatigue or overload breakage exceeds the relevant material stress limit is indicated by the dynamic or the static factor. It characterizes the notch sensitivity of the material, and its values depend on the material and the stress gradient.

This applies to notch sensitivity factor in relation to breakage of a standard reference test gear tooth. It applies also to the relative sensitivity factors which relate the sensitivity of a gear of interest to that of a standard reference test gear (Figure 3.41).

Table 3.9 Life factor $(bending\ stress, Y_{NT})$ [4]

Material [a]	Number of load cycles, N_L	Life Factor, Y_{NT}
St, V,GGG (perl. bai.), GTS (perl.),	$N_L \leqslant 10^4$, static	2,5
	$N_L = 3 \times 10^6$	1,0
	$N_L = 10^{10}$	0,85 up to 1,0 [b]
Eh, IF (root)	$N_L \leqslant 10^3$, static	2,5
	$N_L = 3 \times 10^6$	1,0
	$N_L = 10^{10}$	0,85 up to 1,0 [b]
GG, GGG (ferr.), NT, NV (nitr.)	$N_L \leqslant 10^3$, static	1,6
	$N_L = 3 \leqslant 10^6$	1,0
	$N_L = 10^{10}$	0,85 up to 1,0 [b]
NV (nitrocar.)	$N_L \leqslant 10^3$, static	1,1
	$N_L = 3 \leqslant 10^6$	1,0
	$N_L = 10^{10}$	0,85 up to 1,0 [b]

Size factor (Y_X) is used to take into consideration the influence of size on the probable distribution of weak points in the structure of the material, the stress gradients, which, in accordance with strength materials theory, decrease with increasing dimensions, the quality of the material as determined by the extent and effectiveness of forging, the precesence of defects, etc. Size factor shall be determined from Table 3.10 separately for the pinion and wheel.

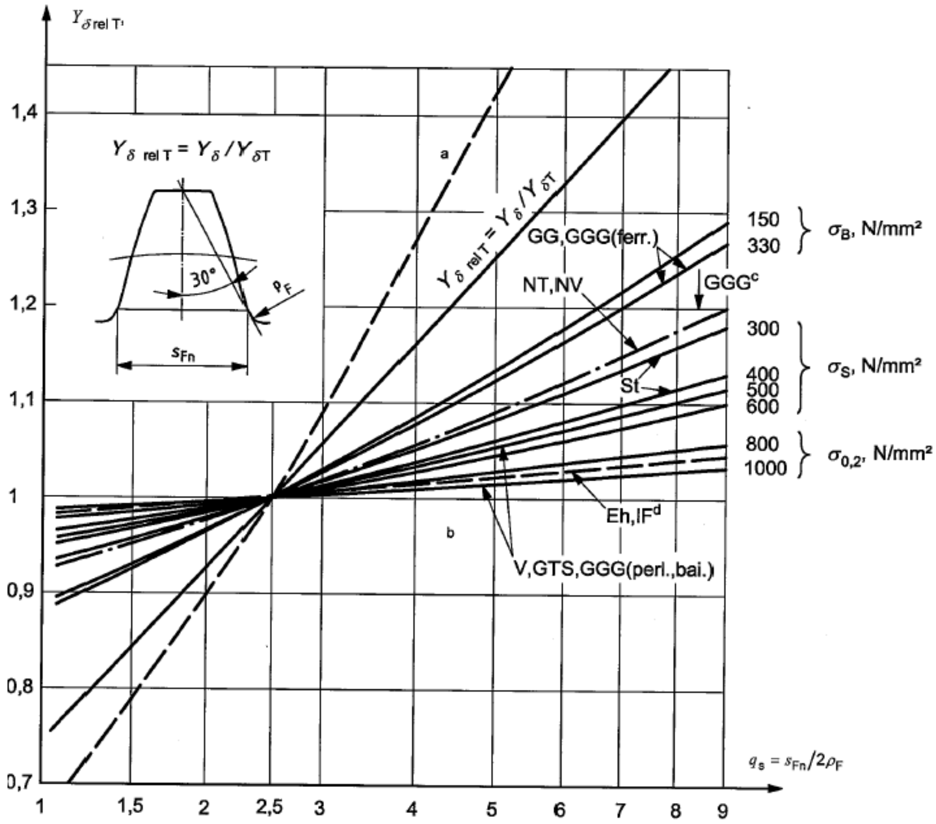


Figure 3.41 Relative notch sensitivity factor [4]

Table 3.10 Size factor (Y_X) [4]

Material [a]		Normal module, m_n	Size factor, Y_X
St, V, GGG (perl., bai.), GTS (perl.),	For 3×10^6 cycles	$m_n \leqslant 5$ $5 < m_n < 30$ $30 \leqslant m_n$	$Y_X = 1{,}0$ $Y_X = 1{,}03 - 0{,}006\ m_n$ $Y_X = 0{,}85$
Eh, IF (root), NT, NV		$m_n \leqslant 5$ $5 < m_n < 25$ $25 \leqslant m_n$	$Y_X = 1{,}0$ $Y_X = 1{,}05 - 0{,}01\ m_n$ $Y_X = 0{,}8$
GG, GGG (ferr.)		$m_n \leqslant 5$ $5 < m_n < 25$ $25 \leqslant m_n$	$Y_X = 1{,}0$ $Y_X = 1{,}075 - 0{,}015\ m_n$ $Y_X = 0{,}7$
All materials for static stress		—	$Y_X = 1{,}0$
[a] See ISO 6336-1:2006, Table 2 for an explanation of the abbreviations used.			

3.4.1.4 ISO 6336-5-2006 Calculation of load capacity of spur and helical gears - Strength and quality of materials

This part of ISO6336 [5] covers the most widely used ferrous gear materials and related heat treatment processes. Recommendations on the choice of specific materials, heat treatment processes or manufacturing processes are not included. This part of standard calculation of load capacity of spur and helical gears consist many tables and figures which are prepared after many experiments to define material allowable stresses including heat treatment, surface hardness, chemical composition and material properties. Allowable stress values for contact and tooth root stress are calculated by using equations 3.115 - 3.116. A part of table presented in the scope of this standard is given as Table 3.11.

$$\sigma_{Hlim} = A.x+B \tag{3.115}$$

$$\sigma_{Flim} = A.x+B \tag{3.116}$$

Table 3.11 Required parameters for calculation of allowable stresses [5]

No.	Material	Stress	Type	Abbre-viation	Fig.	Quality	A	B	Hard-ness	Min. hardness	Max. hardness
1	Normalized low carbon steels/cast steels [a]	contact	wrought normalized	St	1 a)	ML/MQ	1,000	190	HBW	110	210
2			low carbon steels			ME	1,520	250		110	210
3			cast steels	St	1 b)	ML/MQ	0,986	131	HBW	140	210
4				(cast)		ME	1,143	237		140	210
5		bending	wrought normalized	St	2 a)	ML/MQ	0,455	69	HBW	110	210
6			low carbon steels			ME	0,386	147		110	210
7			cast steels	St	2 b)	ML/MQ	0,313	62	HBW	140	210
8				(cast)		ME	0,254	137		140	210
9	Cast iron materials	contact	black malleable	GTS	3 a)	ML/MQ	1,371	143	HBW	135	250
10			cast iron	(perl.)		ME	1,333	267		175	250
11			nodular cast iron	GGG	3 b)	ML/MQ	1,434	211	HBW	175	300
12						ME	1,500	250		200	300
13			grey cast iron	GG	3 c)	ML/MQ	1,033	132	HBW	150	240
14						ME	1,465	122		175	275
15		bending	black malleable	GTS	4 a)	ML/MQ	0,345	77	HBW	135	250
16			cast iron	(perl.)		ME	0,403	128		175	250
17			nodular cast iron	GGG	4 b)	ML/MQ	0,350	119	HBW	175	300
18						ME	0,380	134		200	300
19			grey cast iron	GG	4 c)	ML/MQ	0,256	8	HBW	150	240
20						ME	0,200	53		175	275
21	Through hardened wrought steels [b]	contact	carbon steels	V	5	ML	0,963	283	HV	135	210
22						MQ	0,925	360		135	210
23						ME	0,838	432		135	210
24			alloy steels	V	5	ML	1,313	188	HV	200	360
25						MQ	1,313	373		200	360
26						ME	2,213	260		200	390
27		bending	carbon steels	V	6	ML	0,250	108	HV	115	215
28						MQ	0,240	163		115	215
29						ME	0,283	202		115	215
30			alloy steels	V	6	ML	0,423	104	HV	200	360
31						MQ	0,425	187		200	360
32						ME	0,358	231		200	390

3.4.1.5 ISO 6336-6-2006 Calculation of load capacity of spur and helical gears - Calculation of service life under variable load

This part of ISO 6336 [6] specifies the information and standardized conditions necessary for the calculation of service life (or safety factors for a required life) of gears subject to variable loading. While the method is presented in the context of ISO

6336 and calculation of the load capacity of spur and helical gears, it is equally applicable to other types of gear stress.

Application factor adjusts the nominal load in order to compensate for incremental gear loads from external sources. These additional loads are largely dependent on the characteristics of the driving and driven machines mainly. Designer can select factor value from Table 3.12 according to working characteristics of driven and driving machines.

3.4.1.6 ISO 53:1998 (E) Cylindrical gears for general and heavy engineering-Standard basic rack tooth profile

This standard [7] specifies the characteristics of the standard basic rack tooth profile for cylindrical external or internal involute gears. There are some recommendation for the values for addendum of standard basic rack tooth, bottom clearance between standard basic rack tooth and mating standard basic rack tooth, dedendum of standard basic rack tooth and fillet radius of basic rack as given in Table 3.13.

Table 3.12 Application factor [7]

Working characteristic of driving machine	Working characteristic of driven machine			
	Uniform	Light shocks	Moderate shocks	Heavy shocks
Uniform	1.00	1.25	1.50	1.75
Light shocks	1.10	1.35	1.60	1.85
Moderate shocks	1.25	1.50	1.75	2.00
Heavy shocks	1.50	1.75	2.00	≥2.25

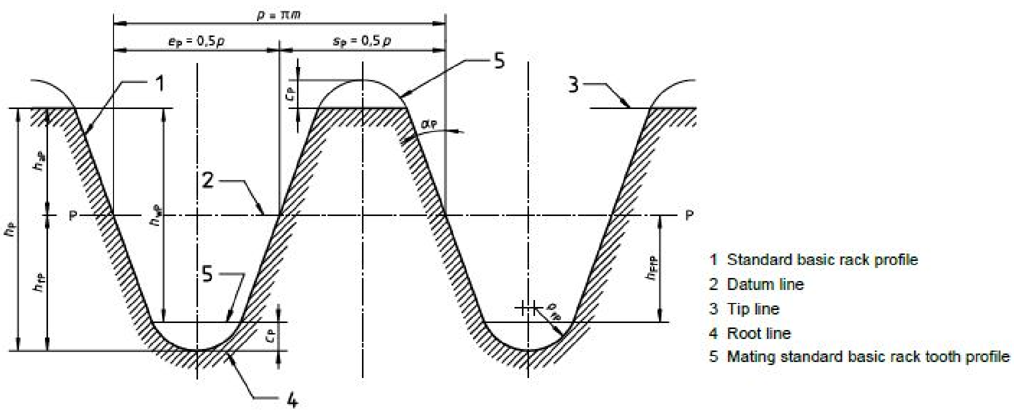


Figure 3.42 Standard basic rack tooth profile [7]

Table 3.13 Basic rack tooth profiles [7]

Symbol	Types of basic rack tooth profile			
	A	B	C	D
α_P	20°	20°	20°	20°
h_{aP}	1 m	1 m	1 m	1 m
c_P	0,25 m	0,25 m	0,25 m	0,4 m
h_{fP}	1,25 m	1,25 m	1,25 m	1,4 m
ρ_{fP}	0,38 m	0,3 m	0,25 m	0,39 m

3.4.1.7 ISO 54:1996 Cylindrical gears for general and heavy engineering-Modules

This standard [8] specifies the values of normal modules for straight and helical gears for general engineering and for heavy engineering. Modules suggested by ISO are given in Table 3.14 as series I and II.

3.4.1.8 ISO 1328, Cylindrical gears ISO system of accuracy - definitions and allowable values of deviations relevant to corresponding flanks of gear teeth

This standard [9] specifies appropriate definitions for gear accuracy terms, the structure of the gear accuracy system and allowable values of deviations such as pitch, total profile and total helix. Fig 3.43 and 3.44 shows allowable values of profile and helix deviations.

Table 3.14 Modules [8]

Series I	Series II	Series I	Series II
1.0	1.125	8.0	7.0
1.25	1.375	10.0	9.0
1.5	1.75	12.0	11.0
2.0	2.25	16.0	14.0
2.5	2.75	20.0	18.0
3.0	3.5	25.0	22.0
4.0	4.5	32.0	26.0
5.0	5.5	40.0	36.0
6.0	6.5	50.0	45.0

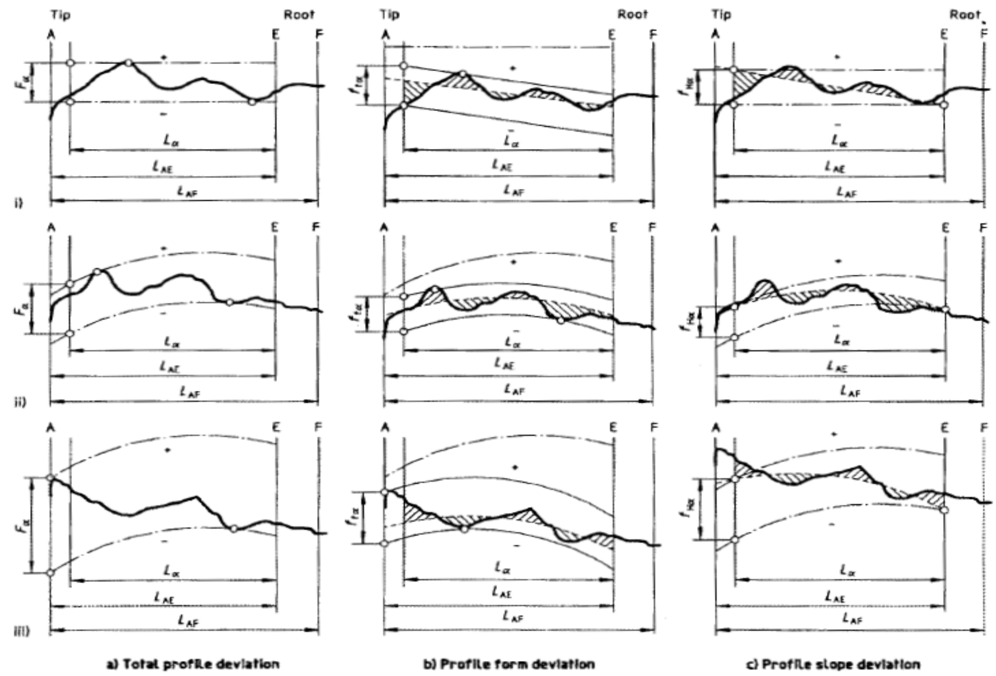


Figure 3.43 Profile deviations [9]

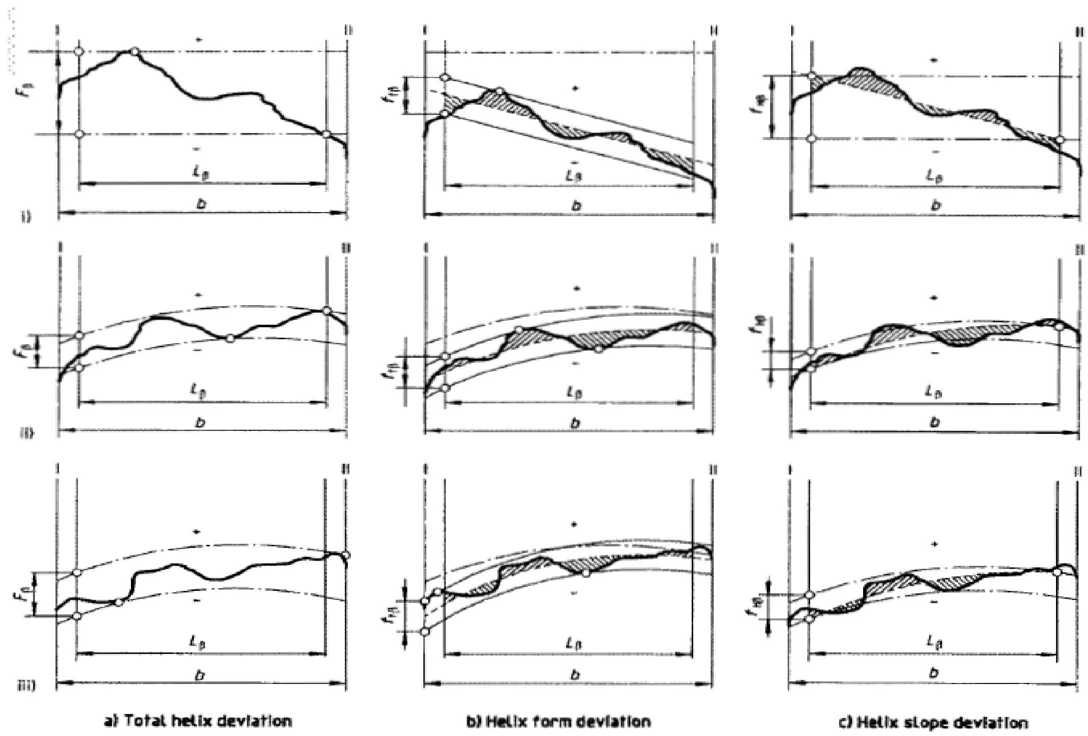


Figure 3.44 Helix deviations [9]

One of tables for allowable values of single pitch deviations presented in ISO 1328 is given in Table 3.15.

Table 3.15 Allowable single pitch deviations [9]

Reference diameter d mm	Module m mm	Accuracy grade 0	1	2	3	4	5	6	7	8	9	10	11	12
		$\pm f_{pt}$ µm												
$5 \leq d \leq 20$	$0,5 \leq m \leq 2$	0,8	1,2	1,7	2,3	3,3	4,7	6,5	9,5	13,0	19,0	26,0	37,0	53,0
	$2 < m \leq 3,5$	0,9	1,3	1,8	2,6	3,7	5,0	7,5	10,0	15,0	21,0	29,0	41,0	59,0
$20 < d \leq 50$	$0,5 \leq m \leq 2$	0,9	1,2	1,8	2,5	3,5	5,0	7,0	10,0	14,0	20,0	28,0	40,0	56,0
	$2 < m \leq 3,5$	1,0	1,4	1,9	2,7	3,9	5,5	7,5	11,0	15,0	22,0	31,0	44,0	62,0
	$3,5 < m \leq 6$	1,1	1,5	2,1	3,0	4,3	6,0	8,5	12,0	17,0	24,0	34,0	48,0	68,0
	$6 < m \leq 10$	1,2	1,7	2,5	3,5	4,9	7,0	10,0	14,0	20,0	28,0	40,0	56,0	79,0
$50 < d \leq 125$	$0,5 \leq m \leq 2$	0,9	1,3	1,9	2,7	3,8	5,5	7,5	11,0	15,0	21,0	30,0	43,0	61,0
	$2 < m \leq 3,5$	1,0	1,5	2,1	2,9	4,1	6,0	8,5	12,0	17,0	23,0	33,0	47,0	66,0
	$3,5 < m \leq 6$	1,1	1,6	2,3	3,2	4,6	6,5	9,0	13,0	18,0	26,0	36,0	52,0	73,0
	$6 < m \leq 10$	1,3	1,8	2,6	3,7	5,0	7,5	10,0	15,0	21,0	30,0	42,0	59,0	84,0
	$10 < m \leq 16$	1,6	2,2	3,1	4,4	6,5	9,0	13,0	18,0	25,0	35,0	50,0	71,0	100,0
	$16 < m \leq 25$	2,0	2,8	3,9	5,5	8,0	11,0	16,0	22,0	31,0	44,0	63,0	89,0	125,0
$125 < d \leq 280$	$0,5 \leq m \leq 2$	1,1	1,5	2,1	3,0	4,2	6,0	8,5	12,0	17,0	24,0	34,0	48,0	67,0
	$2 < m \leq 3,5$	1,1	1,6	2,3	3,2	4,6	6,5	9,0	13,0	18,0	26,0	36,0	51,0	73,0
	$3,5 < m \leq 6$	1,2	1,8	2,5	3,5	5,0	7,0	10,0	14,0	20,0	28,0	40,0	56,0	79,0
	$6 < m \leq 10$	1,4	2,0	2,8	4,0	5,5	8,0	11,0	16,0	23,0	32,0	45,0	64,0	90,0
	$10 < m \leq 16$	1,7	2,4	3,3	4,7	6,5	9,5	13,0	19,0	27,0	38,0	53,0	75,0	107,0
	$16 < m \leq 25$	2,1	2,9	4,1	6,0	8,0	12,0	16,0	23,0	33,0	47,0	66,0	93,0	132,0
	$25 < m \leq 40$	2,7	3,8	5,5	7,5	11,0	15,0	21,0	30,0	43,0	61,0	86,0	121,0	171,0
$280 < d \leq 560$	$0,5 \leq m \leq 2$	1,2	1,7	2,4	3,3	4,7	6,5	9,5	13,0	19,0	27,0	38,0	54,0	76,0
	$2 < m \leq 3,5$	1,3	1,8	2,5	3,6	5,0	7,0	10,0	14,0	20,0	29,0	41,0	57,0	81,0
	$3,5 < m \leq 6$	1,4	1,9	2,7	3,9	5,5	8,0	11,0	16,0	22,0	31,0	44,0	62,0	88,0
	$6 < m \leq 10$	1,5	2,2	3,1	4,4	6,0	8,5	12,0	17,0	25,0	35,0	49,0	70,0	99,0
	$10 < m \leq 16$	1,8	2,5	3,6	5,0	7,0	10,0	14,0	20,0	29,0	41,0	58,0	81,0	115,0
	$16 < m \leq 25$	2,2	3,1	4,4	6,0	9,0	12,0	18,0	25,0	35,0	50,0	70,0	99,0	140,0
	$25 < m \leq 40$	2,8	4,0	5,5	8,0	11,0	16,0	22,0	32,0	45,0	63,0	90,0	127,0	180,0
	$40 < m \leq 70$	3,9	5,5	8,0	11,0	16,0	22,0	31,0	45,0	63,0	89,0	126,0	178,0	252,0
$560 < d \leq 1\,000$	$0,5 \leq m \leq 2$	1,3	1,9	2,7	3,8	5,5	7,5	11,0	15,0	21,0	30,0	43,0	61,0	86,0
	$2 < m \leq 3,5$	1,4	2,0	2,9	4,0	5,5	8,0	11,0	16,0	23,0	32,0	46,0	65,0	91,0
	$3,5 < m \leq 6$	1,5	2,2	3,1	4,3	6,0	8,5	12,0	17,0	24,0	35,0	49,0	69,0	98,0
	$6 < m \leq 10$	1,7	2,4	3,4	4,8	7,0	9,5	14,0	19,0	27,0	38,0	54,0	77,0	109,0
	$10 < m \leq 16$	2,0	2,8	3,9	5,5	8,0	11,0	16,0	22,0	31,0	44,0	63,0	89,0	125,0
	$16 < m \leq 25$	2,3	3,3	4,7	6,5	9,5	13,0	19,0	27,0	38,0	53,0	75,0	106,0	150,0
	$25 < m \leq 40$	3,0	4,2	6,0	8,5	12,0	17,0	24,0	34,0	47,0	67,0	95,0	134,0	190,0
	$40 < m \leq 70$	4,1	6,0	8,0	12,0	16,0	23,0	33,0	46,0	65,0	93,0	131,0	185,0	262,0
$1\,000 < d \leq 1\,600$	$2 \leq m \leq 3,5$	1,6	2,3	3,2	4,5	6,5	9,0	13,0	18,0	26,0	36,0	51,0	72,0	103,0
	$3,5 < m \leq 6$	1,7	2,4	3,4	4,8	7,0	9,5	14,0	19,0	27,0	39,0	55,0	77,0	109,0
	$6 < m \leq 10$	1,9	2,6	3,7	5,5	7,5	11,0	15,0	21,0	30,0	42,0	60,0	85,0	120,0
	$10 < m \leq 16$	2,1	3,0	4,3	6,0	8,5	12,0	17,0	24,0	34,0	48,0	68,0	97,0	136,0
	$16 < m \leq 25$	2,5	3,6	5,0	7,0	10,0	14,0	20,0	29,0	40,0	57,0	81,0	114,0	161,0
	$25 < m \leq 40$	3,1	4,4	6,5	9,0	13,0	18,0	25,0	36,0	50,0	71,0	100,0	142,0	201,0
	$40 < m \leq 70$	4,3	6,0	8,5	12,0	17,0	24,0	34,0	48,0	68,0	97,0	137,0	193,0	273,0

3.4.2 AGMA 2101 D04 Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth

In the scope of this standard, there are fundamental rating formulas which are applicable for rating the surface pitting resistance and tooth root bending strength of internal and external involute parallel axes spur and helical gears. Analysis and design of these gears can be theoretically rated and results are compared. There is no intention to use to assure performance of assembled gears.

The formulas of this standard are not applicable when any of the following conditions exist:

- Damaged gear teeth.
- Spur gears with transverse contact ratio less than 1.0.
- Spur or helical gears with transverse contact ratio greater than 2.0.
- Interference exists between tips of teeth and root fillets
- Teeth are pointed.
- Backlash is zero.
- Undercut exists in an area above the theoretical start of active profile.
- The root profiles are stepped or irregular.
- The helix angle at the reference diameter is greater than 50 degrees.

3.4.2.1 Bending strength

The bending strength of gear teeth is related to gear's resistance to cracking at the tooth root fillet in external gears.

The fundamental formula for tooth root bending stress number in gear tooth is given as equation 3.117:

$$\sigma_F = F_t K_o K_v K_S \frac{1}{b m_t} \frac{K_H K_B}{Y_J} \tag{3.117}$$

σ_F: *bending stress number*, N/mm^2

F_t: *transmitted tangential load*, N

K_o: *overload factor*

K_v: *dynamic factor*

K_S: *size factor*

b: *net face width of narrowest member*, mm

d_{w1}: *operating pitch diameter of pinion*, mm

m_t: *transverse module*, mm

K_H: *load distribution factor*

K_B: *rim thickness factor*

Y_J: *geometry factor for bending strength*

Tooth root bending stress number (σ_F) should be less than allowable bending stress number (σ_{FP}) to ensure a safe design of gear pair. Factor of safety for bending stress (S_F) greater than unity is needed to provide a safe and reliable gear operation under load.
Allowable bending stress number varies for gear materials depending on material composition, cleanliness, residual stress, microstructure, quality, heat treatment and processing. Allowable values for tooth bending stress are determined from laboratory tests and gathered from gained experience, they are presented in Table 3.16 and Figure 3.45.

Gears can be overloaded due to reasons such as vibration, acceleration torques and over speeds. Overload factor is used to allow for all externally applied loads in excess of the nominal tangential load for a particular application. The factor can be selected according to working characteristics of driving and driven gear from Table 3.17.

Table 3.16 Allowable bending stress number [11]

Material designation	**Heat treatment**	**Allowable bending stress number[2)], σ_{FP} N/mm^2**		
		Grade 1	**Grade 2**	**Grade 3**
Steel[3)]	Through hardened	see figure 9	see figure 9	--
	Flame[4)] or induction hardened[4)] with type A pattern[5)]	310	380	--
	Flame[4)] or induction hardened[4)] with type B pattern[5)]	150	150	--
	Carburized & hardened[4)]	380	450 or 485[6)]	515
	Nitrided[4) 7)] (through hardened steels)	see figure 10	see figure 10	--
Nitralloy 135M, Nitralloy N, and 2.5% Chrome (no aluminum)	Nitrided[4) 7)]	see figure 11	see figure 11	see figure 11

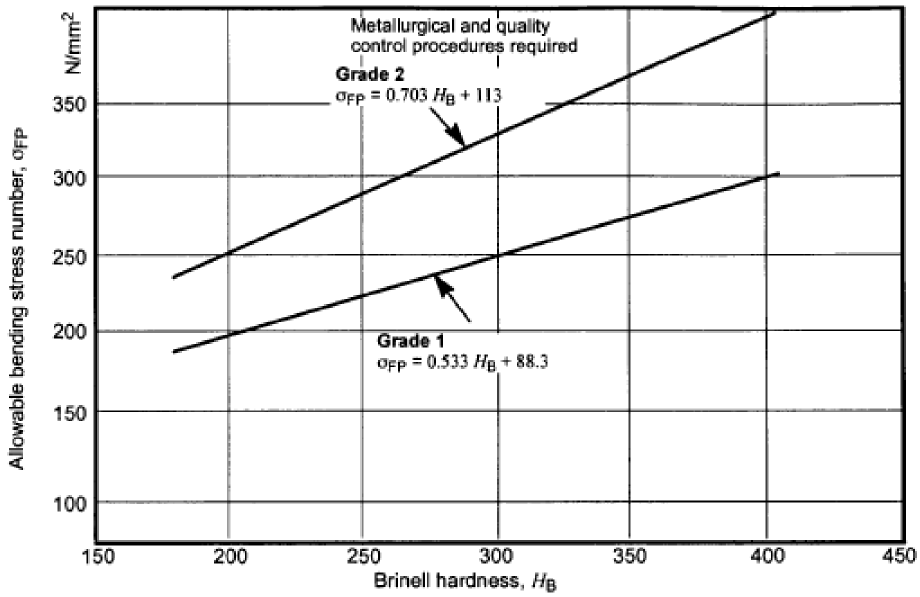


Figure 3.45 Allowable bending stress numbers [11]

Table 3.17 Overload factor

	Driven Machine		
Power source	Uniform	Moderate shock	Heavy shock
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25

Dynamic factor relates the total tooth load including internal dynamic effects to the transmitted tangential tooth load.

Size factor reflects non uniformity of material properties and depends on tooth size, diameter of parts, face width, etc.

Non uniform distribution of the load occurs due to manufacturing variation of gears, assembly variations of installed gears, deflections due to applied loads and distortions due to thermal and centrifugal effects. Load distribution factors ($K_{H\beta}$,$K_{H\alpha}$) modify the rating equations to reflect the non uniform distribution of the load along the lines of contact. Transverse load distribution factor ($K_{H\alpha}$) accounts for non uniform distribution of load among the gear teeth whereas face load distribution factor ($K_{H\beta}$) takes non uniform distribution of load across the gearing face width into consideration. It is calculated by using equation 3.118.

$$K_{H\beta} = 1.0 + K_{Hmc}(K_{Hpf}K_{Hpm} + K_{Hma}K_{He}) \quad (3.118)$$

K_{Hmc} : Lead correction factor

K_{Hpf}: Pinion proportion factor

K_{Hpm} : Pinion proportion modifier

K_{Hma}: Mesh alignment factor

K_{He}: Mesh alignment correction factor

Lead correction factor modifies peak load intensity when crowning or lead modification is applied. It is determined in accordance with

K_{Hmc} =1.0 for gear with unmodified leads, (3.119)

K_{Hmc} =0.8 for gear with leads properly modified by lead correction (3.120)

Deflections are normally higher for wide face widths (b) or higher b/d_{w1} ratios. Pinion proportion factor accounts for deflections due to load. It is determined by using equations 3.121 - 3.123 based on face width:

When b ≤ 25.0

$$K_{Hpf} = \frac{b}{10d_{w1}} - 0.025 \quad (3.121)$$

When 25.0 < b ≤ 432.0

$$K_{Hpf} = \frac{b}{10d_{w1}} - 0.0375 + 0.000492b \quad (3.122)$$

When 432.0 < b ≤ 1020.0

$$K_{Hpf} = \frac{b}{10d_{w1}} - 0.1109 + 0.000815b - 0{,}000000353b^2 \quad (3.123)$$

Pinion proportion modifier alters pinion proportion factor based on pinion location on shaft relative to centerline of bearing.

K_{Hpm}= 1.0 for straddle mounted pinion with $\frac{S_1}{S} < 0.175$. (3.124)

K_{Hpm}= 1.1 for straddle mounted pinion with $\frac{S_1}{S} \geq 0.175$. (3.125)

S_1 is the offset of the pinion, the distance from bearing span centerline to the pinion mid face, mm (Figure 3.46)
S is the bearing span; the distance between the bearing center lines, mm (Figure 3.46).

Mesh alignment factor accounts for the misalignment of the axes of rotation of the pitch cylinders of the mating gear elements from causes other than elastic deformations according to gearing assembly. The value can be read from Figure 3.47.

Rim thickness factor is used as a stress modifying factor. Bending fatigue failure may occur through gear rim, instead of at root fillet, if the rim thickness is not sufficient to ensure full support for the tooth root. The value of factor can be taken from Figure 3.48.

The geometry factor for bending strength (Y_J) evaluates the shape of the tooth, the position at which the most damaging load is applied, and sharing of the load between oblique lines of contact in helical gears. Its calculation method is based on reference [13].

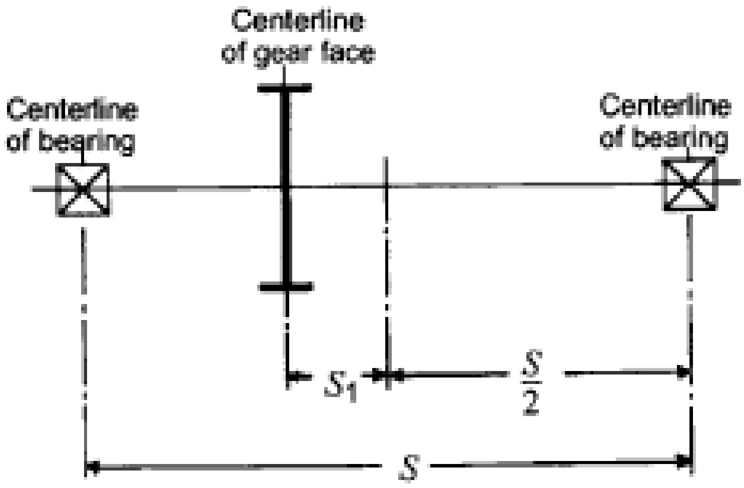


Figure 3.46 Evaluations of S and S_1 [11]

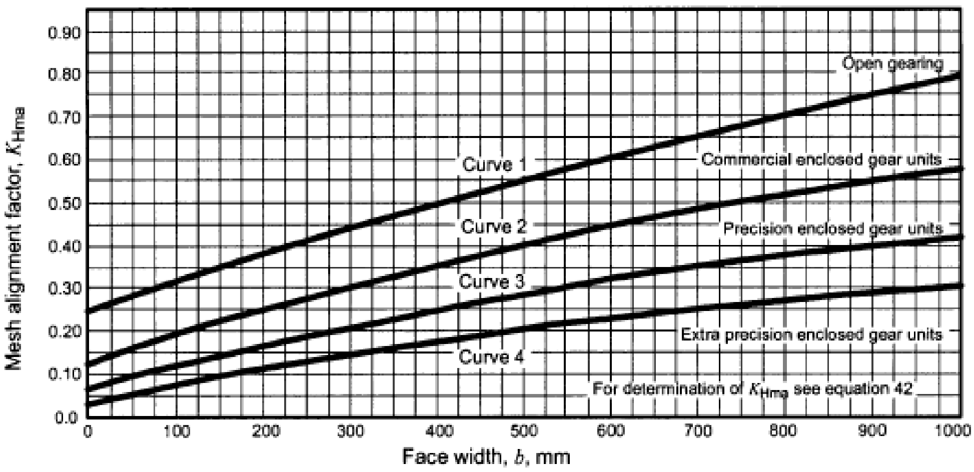


Figure 3.47 Mesh alignment factor [11]

3.4.2.2 Pitting resistance

The ratings of the pitting resistance are based on Hertzian contact stress between two curved surfaces. It is calculated depending on equation 3.113.

$$\sigma_H = Z_E\sqrt{F_t K_o K_v K_s \frac{K_H}{bd_{w1}}\frac{Z_R}{Z_I}} \tag{3.113}$$

σ_H: $contact\ stress\ number\ N/mm^2$

Z_E : $elastic\ coefficient\sqrt{N/mm^2}$

Z_R : $surface\ condition\ factor\ for\ pitting\ resisitance$

Z_I : $geometry\ factor\ for\ pitting\ resistance$

Elastic coefficient is defined as in following equation:

$$Z_E = \sqrt{\frac{1.0}{\pi(\frac{1-v_1^2}{E_1}+\frac{1-v_2^2}{E_2})}} \tag{3.114}$$

v_1 and v_2 : Poisson's ratio for pinion and gear

E_1 and E_2 : Modulus of elasticity for pinion and gear

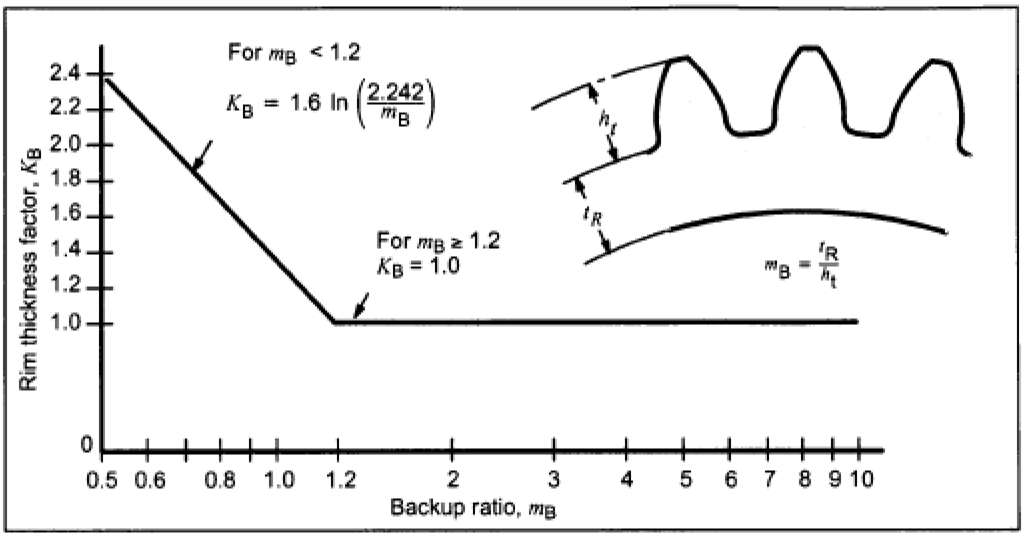


Figure 3.48 Rim thickness factor (AGMA) [11]

3.4.3 DIN 3990 Calculation of Load Capacity of Cylindrical Gears

This involute external and internal spur and helical gear standard contains factors, calculations, figures and tables which are used to evaluate gear rating in terms of tooth breakage, surface pitting and scuffing. DIN 3990 consists of parts from 1 to 6 [14-18]. These standards and some others which are used to developed gear analysis and design software are presented in Table 3.18.

3.4.3.1 DIN 3990-1 Calculation of load capacity of cylindrical gears - introduction and general influence factors

This part of standard [14] contains general influence factors for the calculations of load capacity of cylindrical gears. It includes general principles for a uniform calculation of the load capacity of involute cylindrical gears. There are some different methods to calculate various factors which are used to evaluate gear performance.

Dynamic factor (K_V) calculation is in the same manner as explained in the scope of general influence factor calculations by ISO 6336-1:2006 [2].

Table 3.18 DIN 3990:1987 parts

Standard	Part	Scope
DIN 3990:1987	1	Introduction and general influence factors
DIN 3990:1987	2	Calculation of pitting resistance
DIN 3990:1987	3	Calculation of tooth bending strength
DIN 3990:1987	4	Calculation of scuffing resistance
DIN 3990:1987	5	Endurance limits and material qualities
DIN 3990:1987	6	Calculation of service strength
DIN 867:1986	-	Basic rack tooth profiles for involute teeth of cylindrical gears for general engineering and heavy engineering
DIN 3962	1	Tolerances for of deviations of individual parameters
DIN 3962	2	Tolerances for tooth trace deviations
DIN 3962	3	Tolerances for pitch-span deviations

Longitudinal load factor for contact stress ($K_{H\beta}$) takes account of the effect of the load distribution across the face width on the contact stress.

Face load factor for tooth root ($K_{F\beta}$) takes the effect of the load distribution across the face on the stresses on the tooth root into account. It mainly depends on determinant variables and also face width / tooth height ratio.

Transverse load distribution factors for contact stress and tooth root stress (K_{Ha}, K_{Fa},) take account of the effect of the uneven distribution of load on several gear pairs meshing simultaneously on face stress, on the root stress and on the scuffing stress.

3.4.3.2 DIN 3990-2 Calculation of load capacity of cylindrical gears – calculation of pitting resistance

Hertzian stress is used as a basis for the calculation of the contact stress to evaluate pitting resistance of involute cylindrical gears. There are some other influence factors which affect gear performance such as the lubricant influence, coefficient of friction, the direction and magnitude of the slip and distribution of the pressure.

Safety factor for pitting (S_H) is required to be bigger than required minimum factor of safety value. It is calculated by using following equation.

$$S_H = \frac{\sigma_{HG}}{\sigma_H} \tag{3.115}$$

The limit of pitting resistance (σ_{HG}) is maximum allowable contact stress and is determined using equation 3.116.

$$\sigma_{HG} = \sigma_{Hlim} Z_{NT}\, Z_L Z_V\, Z_R\, Z_W\, Z_X \tag{3.116}$$

Contact stresses ($\sigma_{H1,}\sigma_{H2}$) are based on Hertzian stress and calculated depending on equations 3.117 and 3.118.

$$\sigma_{H1=} Z_B \sigma_{HO} \sqrt{K_A K_V K_{H\beta} K_{H\alpha}} \tag{3.117}$$

$$\sigma_{H2=} Z_D \sigma_{HO} \sqrt{K_A K_V K_{H\beta} K_{H\alpha}} \tag{3.118}$$

Where Z_B and Z_D are simple contact factors for pinion and gear respectively. They take accounts of the conversion of the contact stress in pitch point to the corresponding value at the contact point.

Nominal contact stress in pitch point (σ_{HO}) is required to estimate contact stresses for pinion and gear, and is calculated by using following equation.

$$\sigma_{HO} = Z_H Z_E Z_\varepsilon Z_\beta \sqrt{\frac{F_t}{d_1 b} \frac{u+1}{u}} \tag{3.119}$$

Zone factor (z_H) takes account of the influence of the tooth face curvature at pitch point on the face stress and the conversion of the tangential force on the reference cylinder to the normal force on the pitch cylinder. It can be calculated as in same manner with ISO standards or can be read Figure 3.49.

The elasticity factor (z_E), contact ratio factor (z_ε), helix angle factor (z_β), lubricant factor (z_L), velocity factor (z_V), roughness factor (z_R)' calculation are all based on same principles on ISO.

Life factors (z_{NT}), which are given for number of load cycles in the scope of DIN are presented in Table 3.19 and number of load cycle ranges differs from ISO.

Size factor (z_X), takes account of the static size influence on pitting resistance. Values are given in Table 3.20 with respect to material and normal modules of gear pairs.

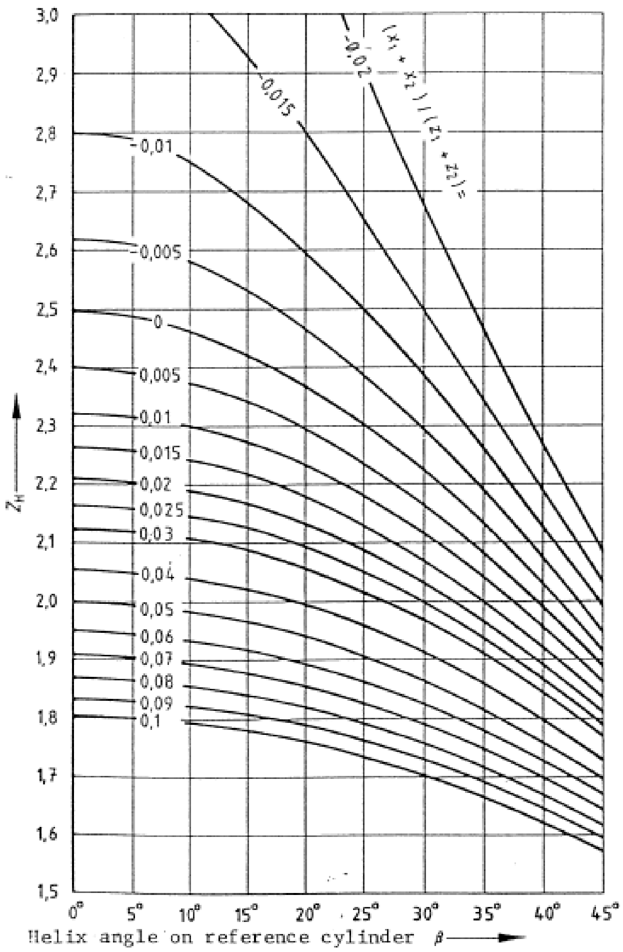


Figure 3.49 Zone factor (z_H) [15]

Table 3.19 Life factor ($DIN, contact\ stress,\ z_{NT}$) [15]

Material	Number of load changes N_L	Life factor Z_{NT}
ST, V, GGG (perl., bai.), GTS (perl.), Eh, IF, if a certain amount of pitting is permissible.	$N_L \le 6 \cdot 10^5$ static	1,6
	$N_L \ge 10^9$ endurance resistive	1,0
V, GGG (perl., bai.), GTS (perl.), Eh, IF	$N_L \le 10^5$ static	1,6
	$N_L > 5 \cdot 10^7$ endurance resistive	1,0
GG, GGG (ferr.), NT (nitr.), NV (nitr.)	$N_L \le 10^5$ static	1,3
	$N_L \ge 2 \cdot 10^6$ endurance resistive	1,0
NV (nitrocar.).	$N_L \le 10^5$ static	1,1
	$N_L \ge 2 \cdot 10^6$ endurance resistive	1,0

Table 3.20 Size factor ($DIN, contact\ stress, z_X$) [15]

Material		Normal module	Size factor
St, V, GG, GGG, GTS	For endurance resistance	All modules	1.0
Eh, IF		$m_n \leq 10$ $10 < m_n < 30$ $m_n \leq 30$	1.0 $1.05 - 0.005 m_n$ 0.9
NTV		$m_n \leq 7.5$ $7.5 < m_n < 30$ $30 \leq m_n$	1.0 $1.08 - 0.011\ m_n$ 0.75

3.4.3.3 DIN 3990-3 Calculation of load capacity of cylindrical gears – calculation of tooth strength

Tooth root stress values ($\sigma_{F1,}\sigma_{F2}$) should be less than tooth root stress limit values ($\sigma_{FG1,}\sigma_{FG2}$) for rating the bending strength of gear teeth. It is important to estimate it to provide a safe operation of gear drives in terms of tooth root bending stress from the point of designer.

$$s_{F1} = \frac{\sigma_{FG1}}{\sigma_{F1}} \geq s_{Fmin} \tag{3.120}$$

$$s_{F2} = \frac{\sigma_{FG2}}{\sigma_{F2}} \geq s_{Fmin} \tag{3.121}$$

Nominal tooth root stress (σ_{F0}), tooth root stress (σ_F) and tooth root stress limit (σ_{FG}) are calculated in equations 3.122, 3.123 and 3.124.

$$\sigma_{F0} = \frac{F_t}{b_{m_n}} Y_F Y_S Y_\beta \tag{3.122}$$

$$\sigma_F = \sigma_{F0} K_A K_V K_{F\beta} K_{F\alpha} \tag{3.123}$$

$$\sigma_{FG} = \sigma_{Flim} Y_{ST}\ Y_{NT} Y_{\delta relT} Y_{RrelT} Y_X \tag{3.124}$$

Form factor (Y_F), stress correction factor (Y_{ST}), helix angle factor (Y_β),, size factor (Y_S),, relative notch sensitivity factor ($Y_{\delta relT}$), and relative surface factor (Y_{RrelT}) are calculated as same with ISO 6336-3:2006 [4] principles to evaluate corresponding related parameters.

Life factor (Y_{NT}), accounts for the higher tooth root stress, which may be tolerable for a limited life (number of load cycles), as compared with the allowable stress at

3×10^6 cycles. Life factor (Y_{NT}) for static stress and reference stress can be taken from Table 3.21.

Table 3.21 Life factor ($DIN, bending\ stress$) [16]

Werkstoff	Anzahl der Lastwechsel N_L	Lebensdauerfaktor Y_{NT}
St, V, GGG (perl., bai.), GTS (perl.)	$N_L \leq 10^4$ statisch	2,5
	$3 \cdot 10^6 < N_L$ dauerfest	1
Eh, IF (grund)	$N_L \leq 10^3$ statisch	2,5
	$3 \cdot 10^6 < N_L$ dauerfest	1
NTV (nitr.), GG, GGG (ferr.)	$N_L \leq 10^3$ statisch	1,6
	$3 \cdot 10^6 < N_L$ dauerfest	1
NV (nitrocar.)	$N_L \leq 10^3$ statisch	1,1
	$3 \cdot 10^6 < N_L$ dauerfest	1

3.4.3.4 DIN 3990-5 Calculation of load capacity of cylindrical gears – endurance limits and material qualities

Allowable stresses for tooth root bending and surface contact are given in figures by DIN. These values can be read from following Figures 3.50 – 3.57 with respect to gear materials, heat treatment and surface hardness.

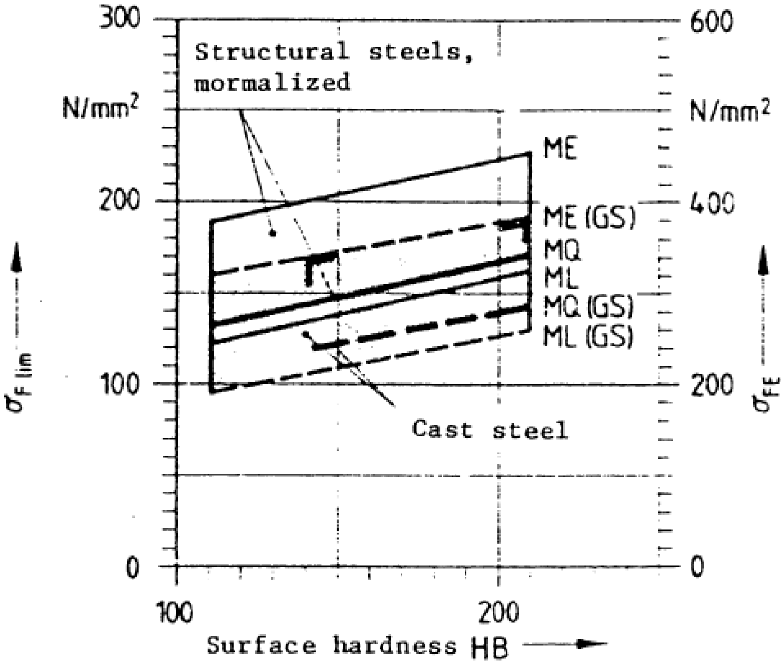


Figure 3.50 Allowable bending stresses for structural normalized steels and cast steel [17]

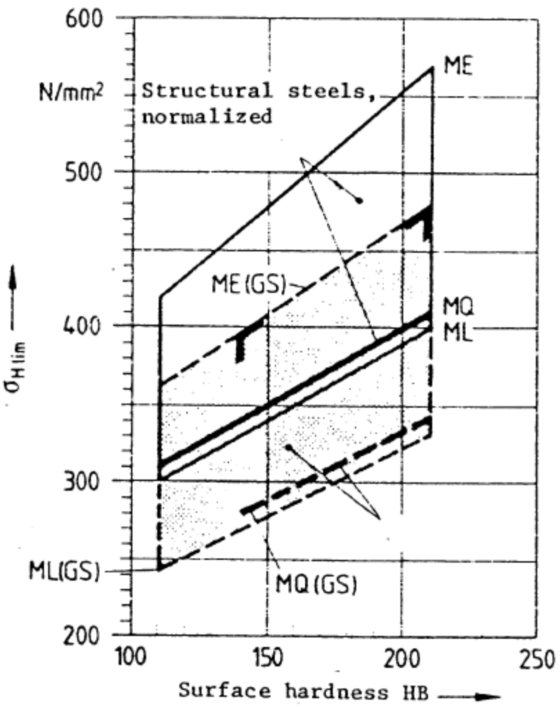


Figure 3.51 Allowable contact stresses for structural normalized steels and cast steel [17]

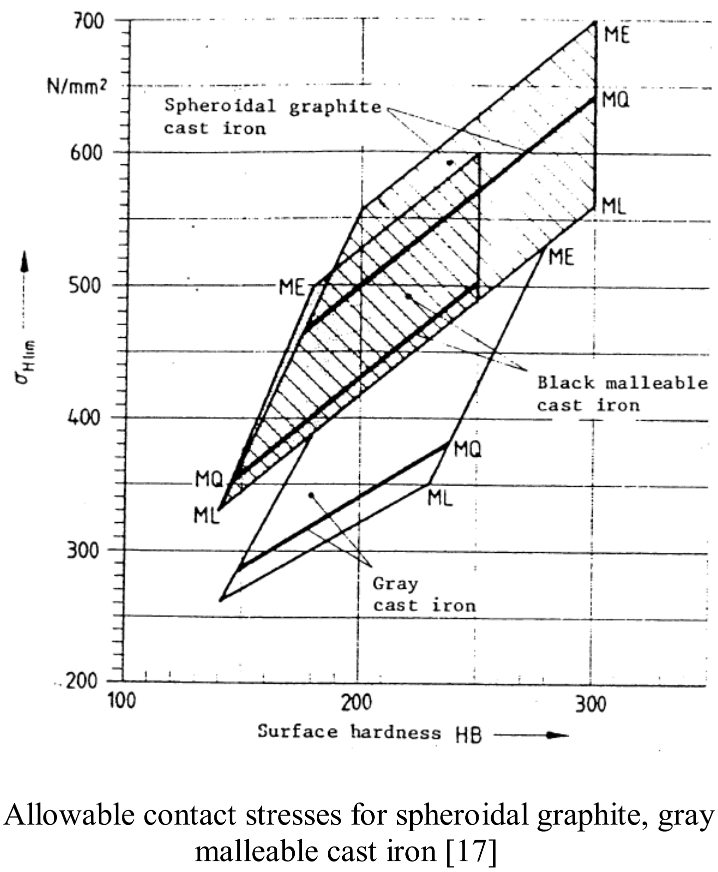


Figure 3.52 Allowable contact stresses for spheroidal graphite, gray and black malleable cast iron [17]

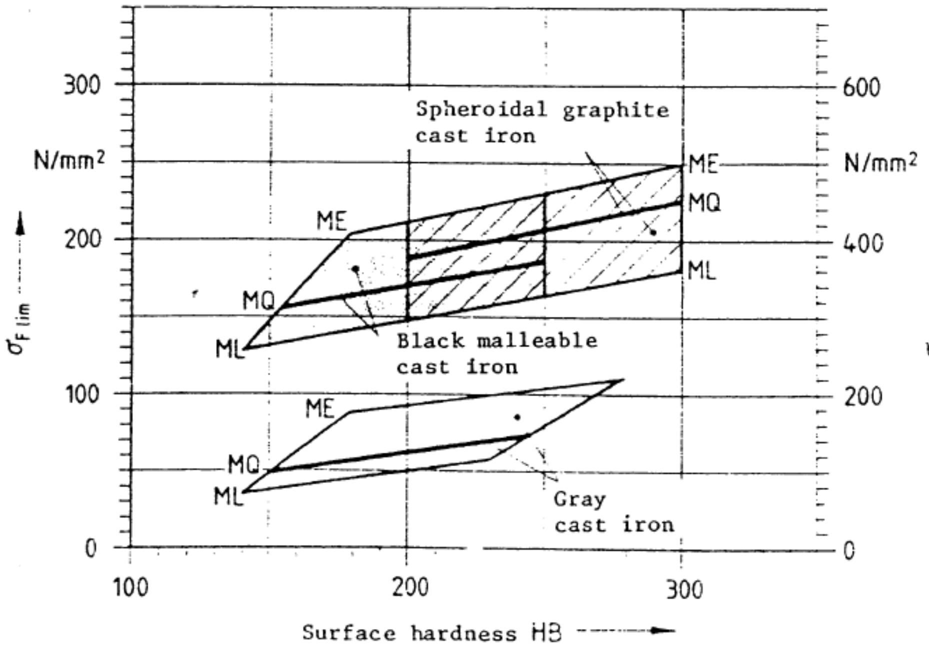


Figure 3.53 Allowable bending stresses for spheroidal graphite, gray and black malleable cast iron [17]

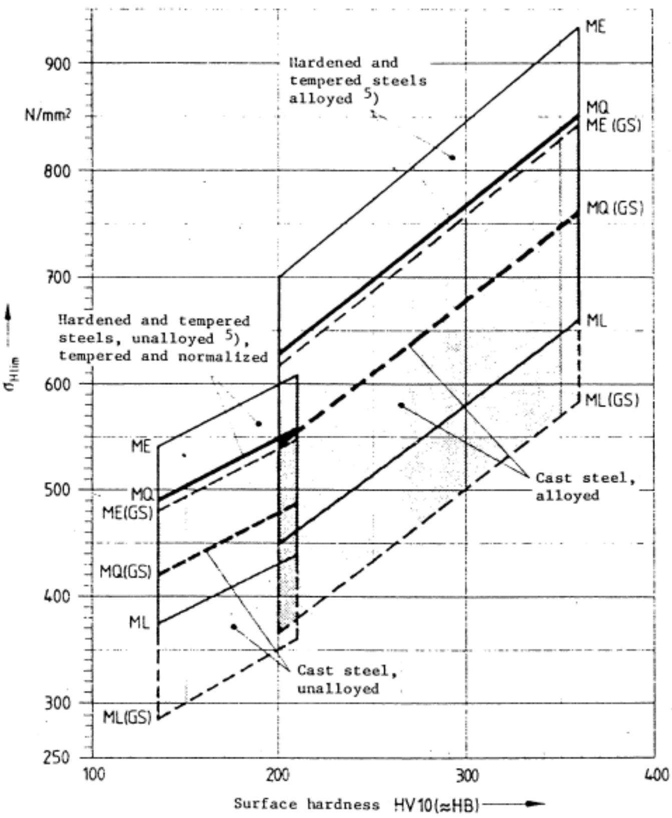


Figure 3.54 Allowable contact stresses for hardened and tempered steel (alloyed), hardened and tempered steel (unalloyed, tempered and normalized), cast steel alloyed and cast steel unalloyed [17]

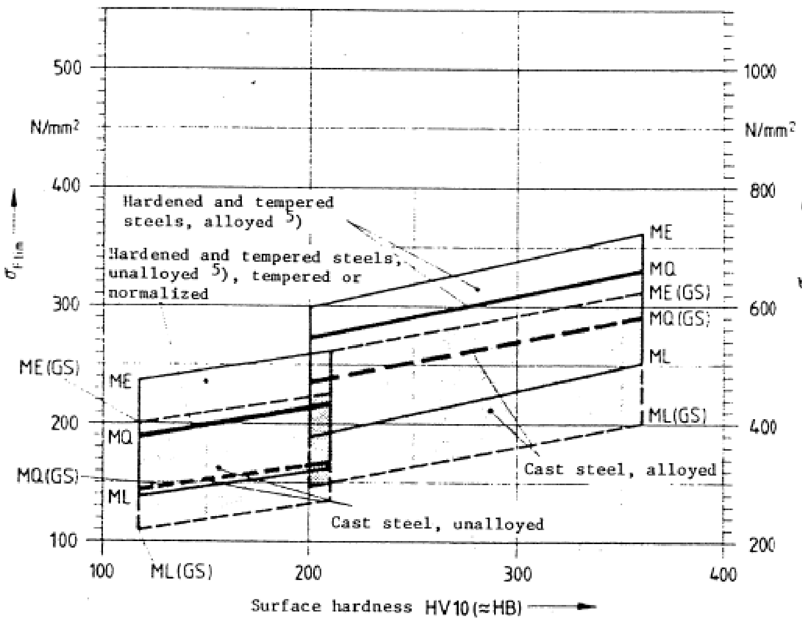


Figure 3.55 Allowable bending stresses for hardened and tempered steel (alloyed), hardened and tempered steel (unalloyed, tempered and normalized), cast steel alloyed and cast steel unalloyed [17]

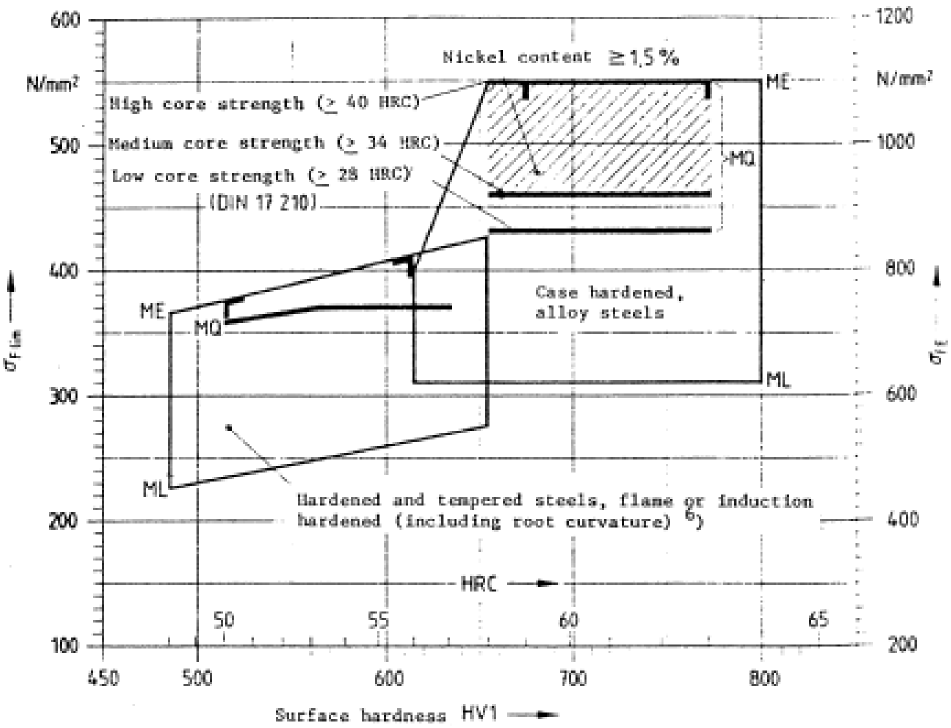


Figure 3.56 Allowable bending stresses for hardened and tempered steels flame or induction hardened and case hardened alloy steels [17]

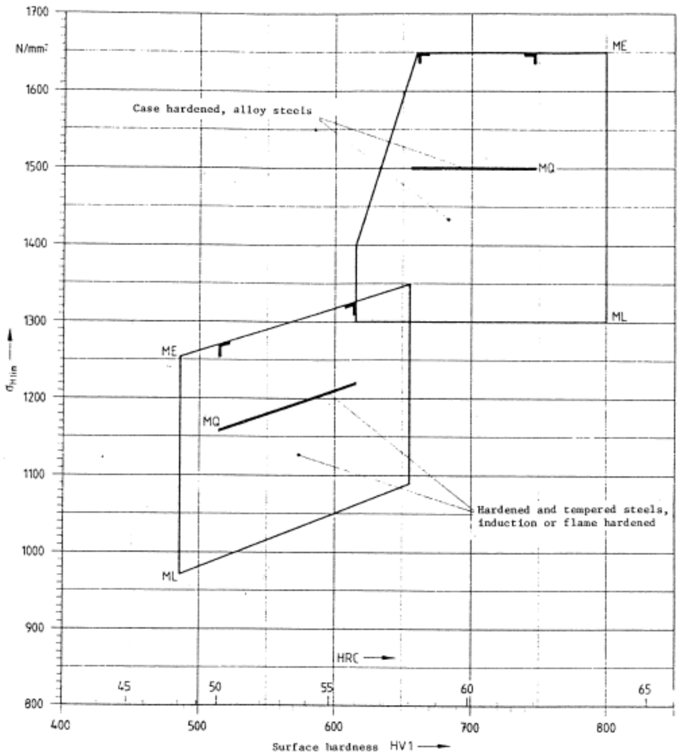


Figure 3.57 Allowable contact stresses for hardened and tempered steels flame or induction hardened and case hardened alloy steels [17]

3.4.3.5 DIN 3990-6 Calculation of load capacity of cylindrical gears calculation of service strength qualities

Application factor [18] takes all force into account, in addition to the nominal tangential load applied from outside. The values which are presented in Table 3.22 can be applied to only to gear drives which do not operate in resonance range and only where the power is uniform. It is necessary to test the gears, if extreme repeated shock loading or motors with high starting torques occur.

Table 3.22 Application factor (DIN) [18]

Working characteristic of driving machine	**Working characteristic of driven machine**			
	Uniform	Moderate shocks	Medium shocks	Heavy shocks
Uniform	1.00	1.25	1.50	1.75
Light shocks	1.10	1.35	1.60	1.85
Moderate shocks	1.25	1.50	1.75	2.00
Heavy shocks	1.50	1.75	2.00	≥2.25

3.4.3.6 DIN 867 Basic rack tooth profiles for involute teeth of cylindrical gears for general engineering and heavy engineering

This standard [19] includes the rules for the basic rack tooth profile which are preferred for involute teeth of cylindrical gears for general and heavy engineering (Figure 3.58). Bottom clearance and fillet radius coefficients usable ranges are given in the scope of this standard and illustrated in Figure 3.59.

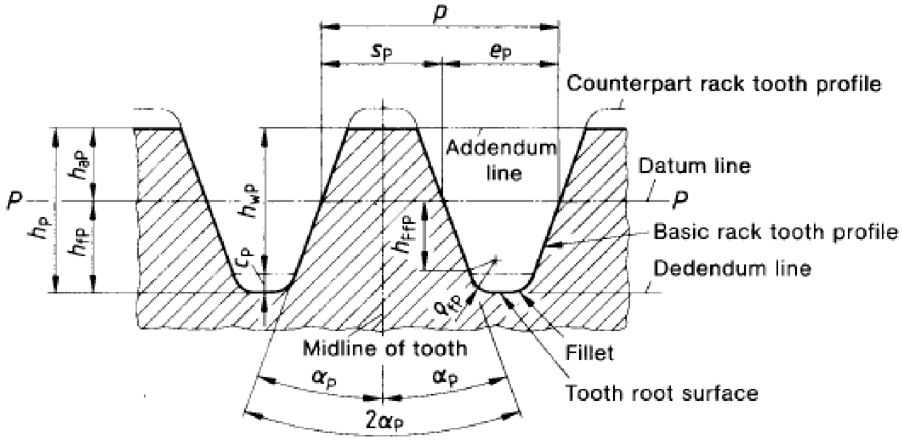


Figure 3.58 Basic rack tooth profile [19]

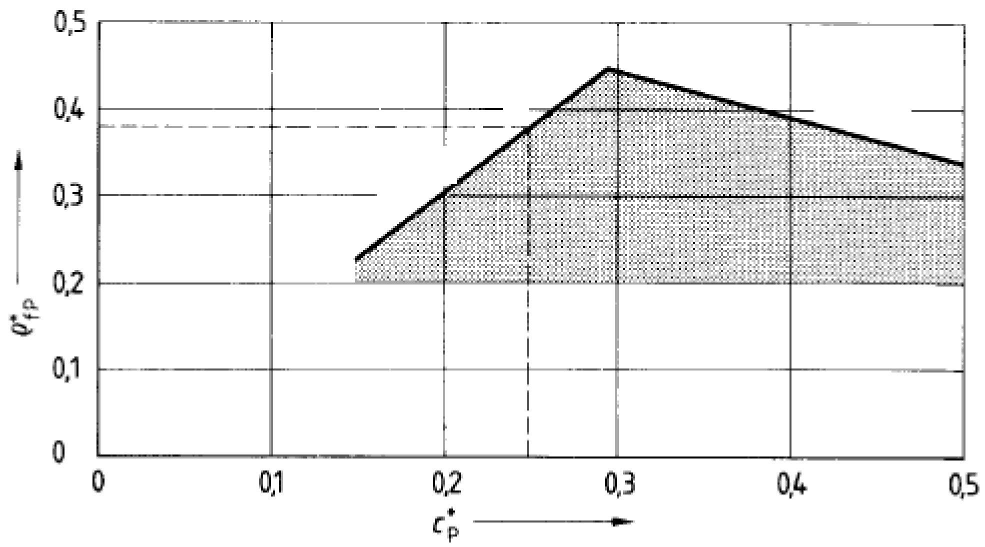


Figure 3.59 Relationship between bottom clearance coefficient and fillet radius coefficient [19]

Where

ρ^*_{fP}: *fillet radius coefficient*

c^*_p: *bottom clearance coefficient*

3.4.3.7 DIN 780 Module series for gears

Modules which are recommended for cylindrical gears for series I and series II are presented in Table 3.23.

Table 3.23 Modules [20]

I	II	I	II	I	II	II	II
0.05	0.055	0.5	0.55	3.0	3.25	3.5	3.75
0.06	0.07	0.6	0.65	4.0	4.25	4.5	4.75
0.08	0.09	0.7	0.75	5.0	5.25	5.5	5.75
0.1	0.11	0.8	0.85	6.0	6.5	7.0	-
0.12	0.14	0.9	0.95	8.0	9.0	-	-
0.16	0.18	1.0	1.125	10.0	11.0	-	-
0.2	0.22	1.25	1.375	12.0	14.0	-	-
0.25	0.28	1.5	1.75	16.0	18.0	-	-
0.3	0.35	2.0	2.25	20.0	22.0	-	-
0.4	0.45	2.5	2.75	25.0	27.0	28.0	30.0
				32.0	36.0	39.0	-
				40.0	42.0	45.0	-
				50.0	55.0	-	-
				60	70	-	

CHAPTER 4

FLOW CHART OF PARALEL AXES GEAR DESIGN AND ANALYSIS SOFTWARE

4.1 INTRODUCTION

Gears are very important machine elements which are very vital for power transmission by manipulating torque, rotational speed, and direction of rotation and axis of rotation. Gears are critical elements due to importance of their application areas such as ground wheel vehicles, airborne vehicles, energy transform turbines, satellites, space craft etc. Gear design and analysis are very comprehensive tasks due to complexity of gear and gear box elements so a good knowledge supported by well accepted standards submitted by ISO, AGMA, and DIN is required to provide safe operation of gears during meshing. Even by using gear standards, an engineer must carry out iterations for gear design because each property change influences other parameters. Checking influences manually is slow and prone to errors. These iterative calculations increase demand for design and analysis software for gear pairs based on international gear standards. In the scope of this study, a user friendly parallel axes gears' analysis and design software was developed by using Visual Studio C ++.

This chapter consists of brief information about Microsoft Visual Studio, design and analysis modules for spur and helical gears based on ISO, AGMA and DIN standards separately.

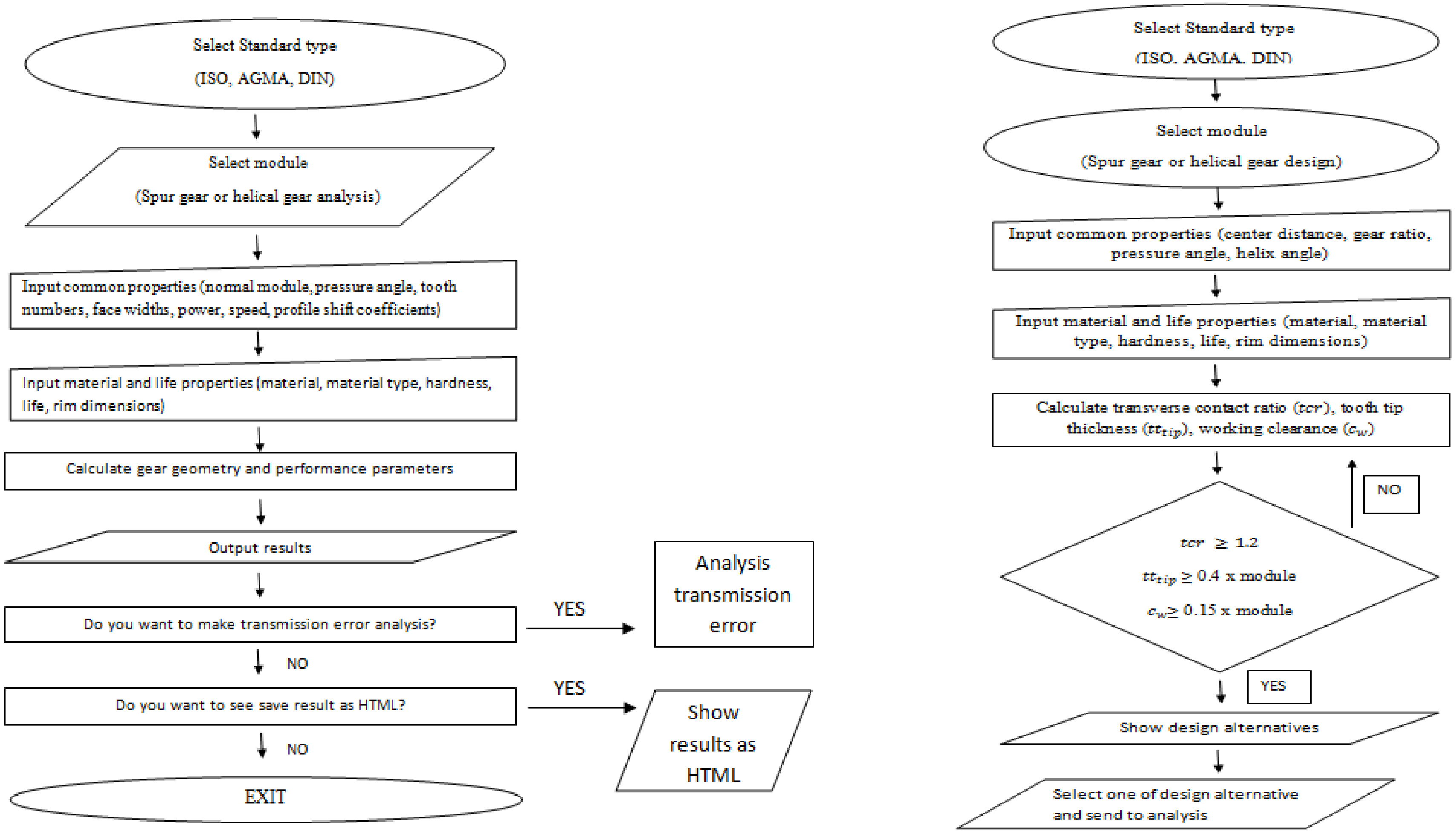
4.2 MICROSOFT VISUAL STUDIO C++

Visual Studio is a suite of applications created by Microsoft to give developers a compelling development environment for the Windows and .NET platforms. Visual Studio can be used to write console applications, Windows applications, Windows services, Windows Mobile applications, ASP.NET applications, and ASP.NET web services, in your choice of C++, C#, VB.NET, J#, and more.

Visual Studio also includes various additional development tools, such as Visual SourceSafe; which tools are included depends greatly on the edition of Visual Studio that you are using. Microsoft has a long history with development tools and Visual Studio is the natural culmination of these efforts [47].

4.3 FLOWCHART OF GEAR DESIGN AND ANALYSIS SOFTWARE

In the scope of this master thesis, user friendly gear design and analysis software was developed based on international gear standards presented by ISO, AGMA and DIN by using Microsoft Visual Studio C++ windows form applications including menu, button, radio button, textbox, combo box, numeric up down and list view. A brief flow chart of the software for parallel axes gear analysis and design are presented in Figure 4.1a and b. Software has a main page which includes ISO, AGMA and DIN modules. These all modules have sub modules such as helical gear analysis, spur gear analysis, helical gear design and spur gear design respectively as shown in Figure 4.2a, b and c. User can give a name of his analysis / design by himself when starting to work. If no name is given to study, software gives name by default to prevent confusion for results when user wants to come back and see results of software again. User can design and analysis spur and helical gears based on ISO, AGMA and DIN standards. Transmission analysis of both gear types and drawing of transmission error curves is also possible. User can save results as HTML and can open the file later to see again.



(a) (b)

Figure 4.1 Flowcharts of gear design and analysis software

(a)ISO (b) AGMA (c) DIN

Figure 4.2 Software main pages – ISO, AGMA and DIN modules

4.4 SPUR GEAR ANALYSIS MODULE OF SOFTWARE

Spur gears can be analyzed by using software modules based on ISO, AGMA and DIN separately. These modules are very similar in terms of input parameters. But there are differences for some required parameters to make analysis of spur gears and methods to be used for calculations of these parameters. Spur gear analysis modules of software are going to be explained below for ISO, AGMA and DIN standards respectively. Some of common properties are different for each standard type given in Table 4.1. Due to similarities of spur gear analysis modules based on ISO, AGMA and DIN, sections of spur gear analysis based on ISO module are only illustrated in this chapter.

Table 4.1 Comparison of common properties

ISO	DIN	AGMA
Pinion shaft diameter	Pinion shaft diameter	-
Bearing distance	Bearing distance	-
Distance of gear center from the shaft midpoint	Distance of gear center from the shaft midpoint	-
Application requirement	Bottom clearance coefficient	-
Surface condition	Surface condition	-
Oil type	Oil type	-
Pinion shaft type	Pinion shaft type	-
Helix modification	Helix modification	-
Contact pattern	Contact pattern	-
Accuracy grade	Accuracy grade	Transmission accuracy level number
-	-	Geometry factor
-	-	Lead correction condition
-	-	Gear unit condition
-	-	Mesh alignment condition
-	-	Mounting condition
-	-	Tip shortening
-	-	Tooth thinning
-	-	Reliability

4.4.1 Spur Gear Analysis Based on ISO

ISO Analysis Spur Gear module is selected to make analysis of spur gear pair based on ISO parallel axes gear standards. This module consists of three sections namely common properties, material and life properties, and results. Common properties and material / life properties include required parameters for spur gear analysis. Some of them are added by user and remaining is selected from software's related section. There are limitations for values and recommendations for usage of some parameters presented by ISO. These are given on software module as warning sections to make user's work easier. Some of required parameters namely normal module, pressure angle, input power, minimum required safety factors for bending and contact stress, tooth numbers, face widths, profile shift coefficients, surface roughness, accuracy grade, driver rotational speed, pinion shaft external diameter, bearing distance and distance of gear center from the shaft midpoint are added to data entry boxes by the user (Figure 4.3). Driver type, application requirement, working characteristics of pinion and gear, surface condition, oil type, pinion shaft type, helix modification and contact pattern situations are selected from menu bars (Figure 4.3). Determining these parameters is the first stage of spur gear analysis.

Normal module values recommended by ISO are given in Table 4.2. User can select any of these recommended values or define a nonstandard one by taking manufacturing facilities into consideration. Module is same for both pinion and wheel.

Minimum required safety factors for bending and contact stress are used to evaluate whether gears are in safe or not. These values are checked against calculated factor of safeties for both stress ratings for pinion and gear individually.

There is an explanatory note for contact pattern illustrated in Figure 4.3.

Table 4.2 Standard module values recommended by ISO (in mm)

1.000	2.000	4.000	7.000	14.000	28.000
1.125	2.250	4.500	8.000	16.000	32.000
1.250	2.500	5.000	9.000	18.000	36.000
1.375	2.750	5.500	10.000	20.000	40.000
1.500	3.000	6.000	11.000	22.000	45.000
1.750	3.500	6.500	12.000	25.000	50.000

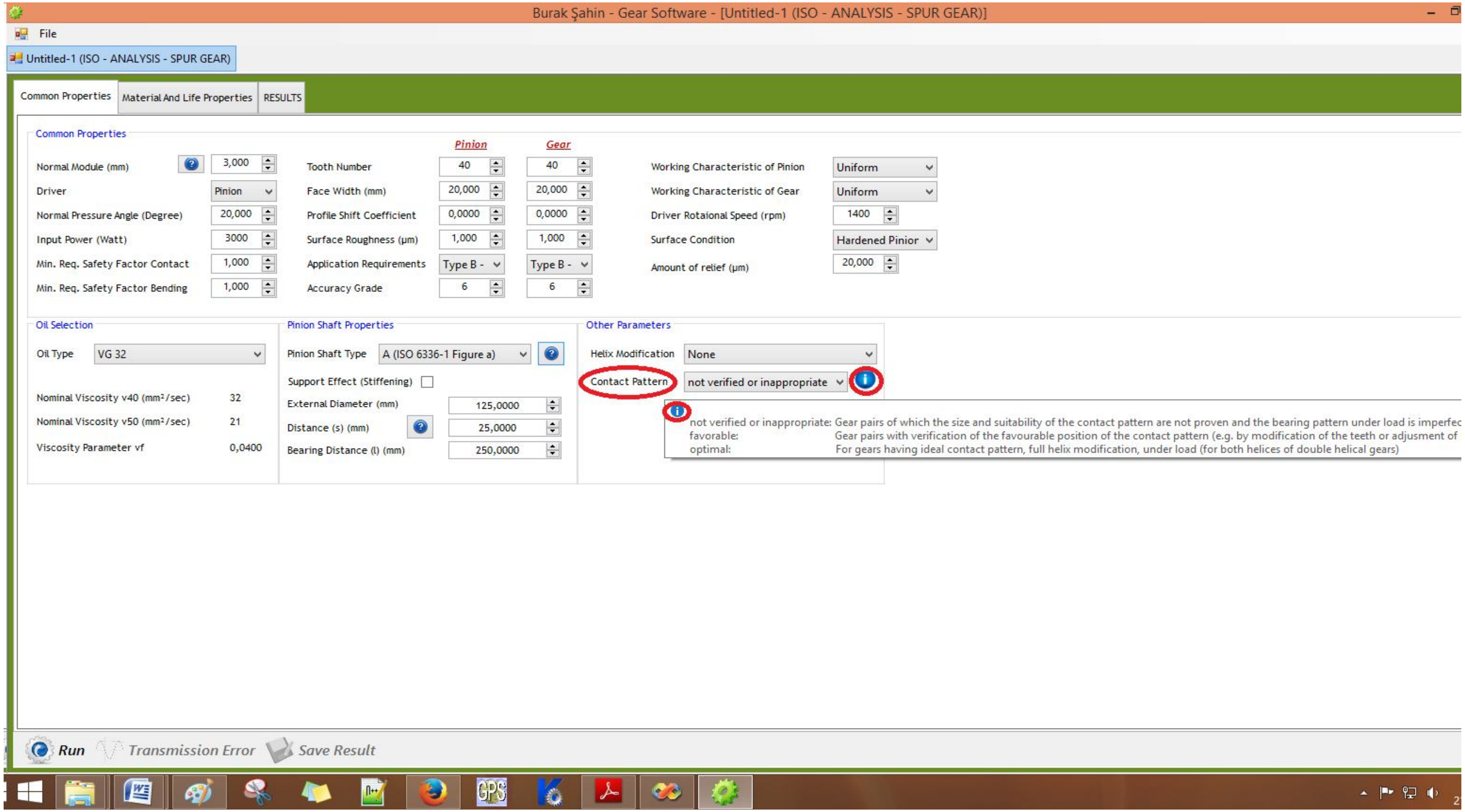


Figure 4.3 Spur gear analysis based on ISO – common properties

Surface roughness is an important parameter for gear surface quality and it is used to determine roughness factor which has significant effect on surface contact stress.

Profile shift coefficients are the amount of addendum modification in terms of normal module. Addendum modification is applied to gears to prevent undercutting and balance strength of gears. It can be different for pinion and gear, both of them can be positive, negative or sum of them can be zero to provide operation at reference center distance.

Gear accuracy grade is quality class of gears and allowable deviations such as pitch deviation, helix deviation, composite deviation, profile deviation, profile form deviation, and profile slope deviation are determined according to gear accuracy.

Driver type is pinion or gear; if pinion is driver gear pair increase torque decrease speed, if wheel drives gear pair vice versa.

Application requirements define the coefficients of addendum of basic rack, dedendum of basic rack, bottom clearance and root fillet radius for pinion and gear.

Pinion shaft external diameter, bearing distance, distance of gear center from the shaft midpoint, pinion shaft type, helix modification, and contact pattern are important parameters for load distribution through face width and tooth surface. They are used to calculate load distribution factors which are stress correction factor used in determination of contact and bending stress for gear teeth. There is information button of software to give information about bearing distance and pinion shaft type as shown in Figure 4.4.

Working characteristics of pinion and gear are important to define shock characteristics of systems driven by pinion and gear.

Surface condition defined by ISO has two situations such as surface hardened pinion with through-hardened gear and through-hardened pinion and gear. It is important to estimate work hardening factor which is one of stress correction factor of surface contact stress.

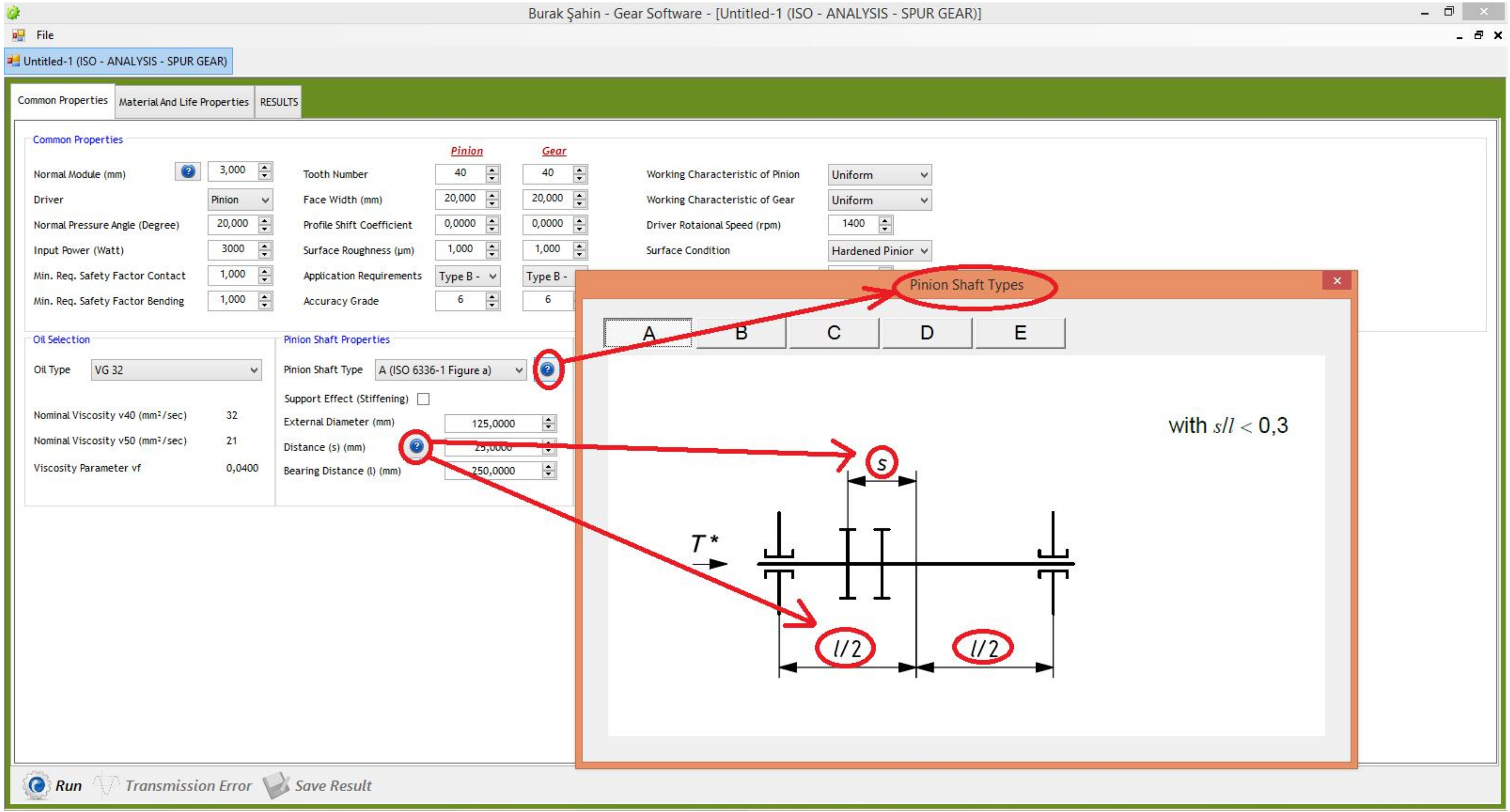


Figure 4.4 Pinion shaft type and bearing distance

Oil type is an important parameter in terms of lubrication of gear during operation. It has effects on lubricant factor, velocity factor and roughness factor. Software gives chance to select ISO oil grades such as VG 32, VG 46, VG 68, VG 100, VG 150, VG 220 and VG 320 and draw nominal viscosities and viscosity parameters from software database to calculate lubricant film influence factors for lubricant, velocity and roughness.

After common properties of gear pair, user needs to determine material and life properties from software as shown in Figure 4.5. In this connection, material, material type, strength type, strength value, quality, hardness, allowable contact stress, allowable bending stress, number of load cycles for bending, number of load cycles for contact, rim thickness, web thickness parameters which are given under the section of material and life properties should be determined by user.

There are three material quality grades ML, MQ and ME which refer to the allowable stress numbers for contact stress and bending stress. ML stands for modest demands on the material quality and on the material heat treatment process during gear manufacture. MQ stands for requirements that can be met by experienced manufacturers at moderate cost whereas ME represents requirements that must be realized when a high degree of operating reliability is required. These explanations which are aimed to give information to user are shown in Figure 4.5.

Rim thickness and web thickness are parameters which describes gear rim condition to define gear geometry and observe rim effect on gear performance.

There are some materials recommended as gear materials by ISO. These have different material types for each materials presented in Table 4.3.

Strength type and strength value are only used to calculate material slip layer thickness. This parameter is essential to determine relative notch sensitivity factor which is a stress correction factor for tooth root bending stress calculations.

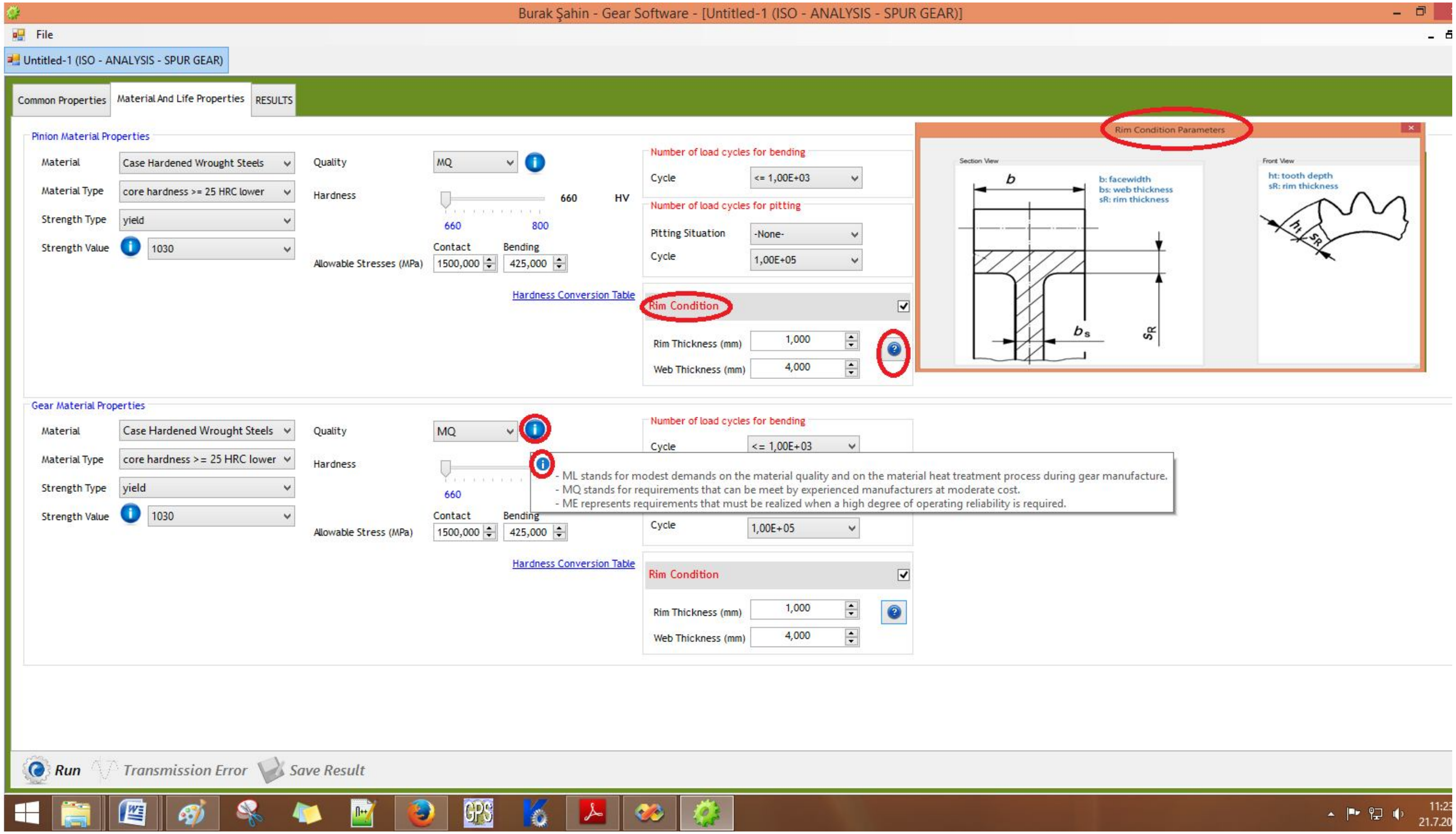


Figure 4.5 Spur gear analysis based on ISO – material/life properties, quality and rim condition

Table 4.3 Material and material types recommended by ISO [5]

Material	**Material Type**	**Material**	**Material Type**
Normalized low carbon /cast steels	Wrought normalized low carbon steels	Case hardened Wrought steels	-
	Cast steels		
Cast iron materials	Black malleable cast iron (perl.)	Flame or induction hardened wrought and cast steels	-
	Nodular cast iron		
	Grey cast iron		
Through hardened wrought steels	Carbon steels	Nitrided wrought steels/ nitriding steels/ through hardening steels nitrided	Nitriding steels
	Alloy steels		Through hardening steels
Through hardened cast steels	Carbon steels	Wrought steels nitrocarburized	Through hardening steels
	Alloy steels		

Hardness is one of important parameters to calculate allowable stresses in terms of surface contact and tooth root bending. After selecting material and material types of gears separately, software defines surface hardness values, allowable contact stress and allowable bending stress automatically according to data presented by ISO 6336-5:2006 [5]. Software gives chance to see corresponding hardness value for different hardness values such as Rockwell A, Rockwell C, Rockwell D, Rockwell HR15N, Rockwell HR30N, Rockwell HR 45N, Vickers, Brinell 10 mm standard 3000 kgf, Brinell 10 mm low carbide 3000 kgf, Knoop and Scleroscope by using ASTM related tables.

Allowable contact and bending stresses are determined by taking material, material type, quality and surface hardness into account. These values are taken from database by software and presented in material and life properties section. They are very essential to evaluate gear performance with respect to tooth root bending stress and surface contact stress. User can use these values for bending and contact stress or change them.

Life factor for bending stress and contact stress values are taken with respect to gear materials, number of load cycles for bending and number of load cycles for contact. So load cycle values are given for both stress types and each gear separately to calculate life factors.

In result section, many geometrical and performance parameters are calculated and presented in list view (Figure 4.6). These parameters are going to be given in Chapter 6 for case studies. In full results section consist of all geometrical and gear performance parameters whereas summary results section includes factor of safeties for bending and contact stress to give brief information about gear operation is safe or not.

4.4.2 Spur Gear Analysis Based on AGMA

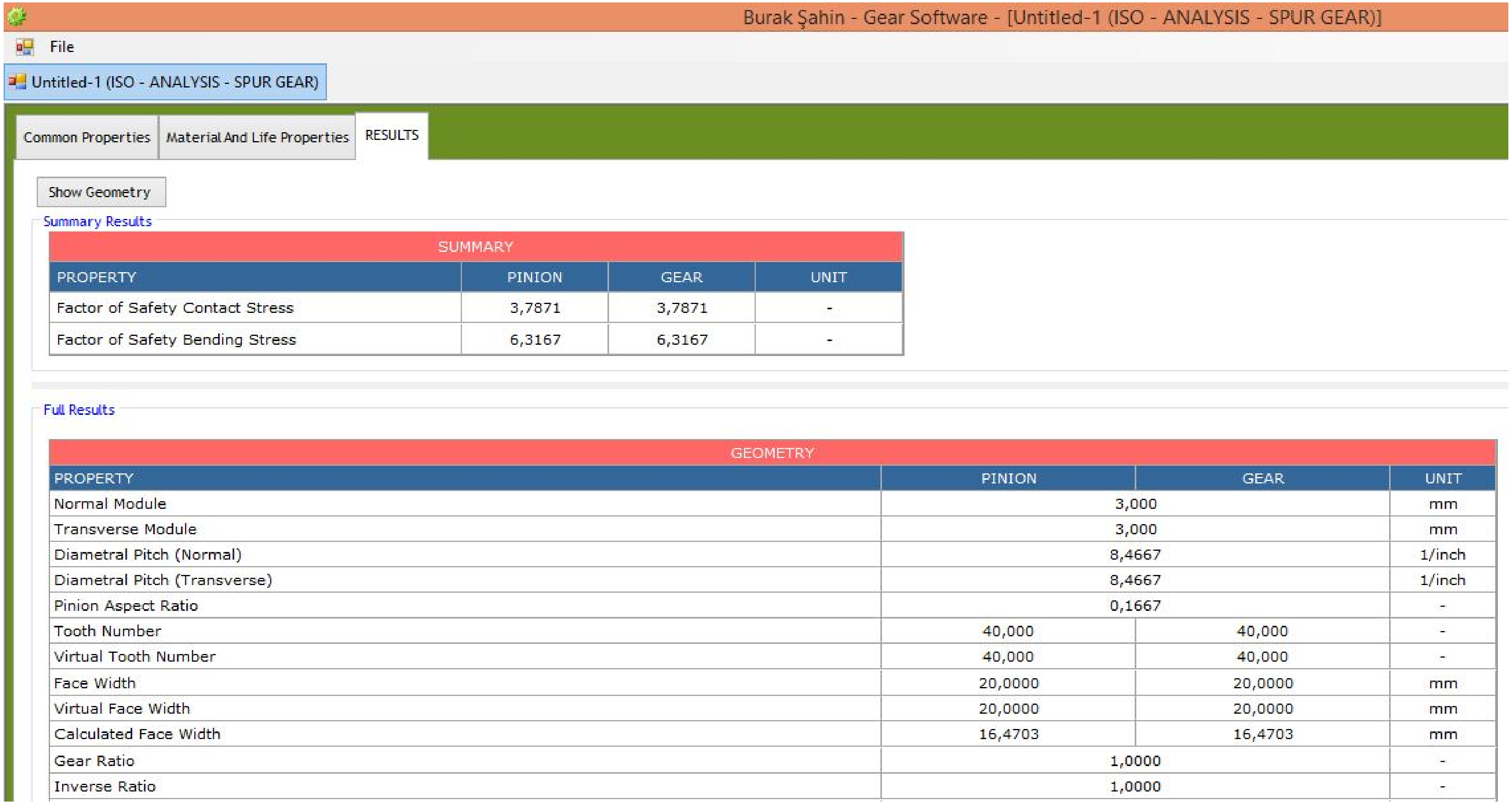
Spur gear analysis based on equations, formulas, figures and tables presented by AGMA is also possible as well as ISO, by using software module of AGMA Analysis Spur Gear. This module also has same sections; common properties, material life properties and results, but required parameters are little different. They are going to be explained briefly including their usage area.

Normal module, pressure angle, input power, minimum required safety factors for bending and contact stress, tooth numbers, face widths, profile shift coefficients, surface roughness values, driver rotational speed and working characteristics of power source and driven machine are required input parameters same as well as ISO analysis spur section.

Transmission accuracy level number is appropriate accuracy grade for calculation of dynamic factor and estimation of pitch and profile deviations.

Geometry factor for bending strength evaluates the shape of the tooth, the position where the most damaging load is applied.

Lead correction condition, gear unit condition, mesh alignment condition, work hardening condition, mounting condition, surface condition and reliability are general application properties of gear unit.



SUMMARY

PROPERTY	PINION	GEAR	UNIT
Factor of Safety Contact Stress	3,7871	3,7871	-
Factor of Safety Bending Stress	6,3167	6,3167	-

GEOMETRY

PROPERTY	PINION	GEAR	UNIT
Normal Module	3,000		mm
Transverse Module	3,000		mm
Diametral Pitch (Normal)	8,4667		1/inch
Diametral Pitch (Transverse)	8,4667		1/inch
Pinion Aspect Ratio	0,1667		-
Tooth Number	40,000	40,000	-
Virtual Tooth Number	40,000	40,000	-
Face Width	20,0000	20,0000	mm
Virtual Face Width	20,0000	20,0000	mm
Calculated Face Width	16,4703	16,4703	mm
Gear Ratio	1,0000		-
Inverse Ratio	1,0000		-

Figure 4.6 Spur gear analysis based on ISO – results

Lead correction condition modifies peak load intensity when crowning or lead modification is applied. Gear unit condition can be in four alternatives such as open gearing, commercial enclosed gear units, precision enclosed gear units and extra precision enclosed gear units. It is used to calculation of mesh alignment factor. Mesh alignment condition is given for calculation of mesh alignment correction factor and this factor is used to modify the mesh alignment factor when the manufacturing or assembly techniques improve the effective mesh alignment.

Mounting condition has two situations which are given by AGMA as straddle mounted pinion with $\frac{S_1}{S} < 0.175$ and straddle mounted pinion with $\frac{S_1}{S} \geq 0.175$. Mounting condition is important to define pinion proportion modifier which is dependent on the location of the pinion relative to its bearing centerline. There is an explanatory figure next to mounting condition and it is visible when user click button.

Surface condition is used to calculate work hardening factor which is a stress correction factor. Reliability value is taken to estimate reliability factor which accounts the effect of the normal statistical distribution of failure found in materials testing.

Tip shortening is a process which allows shortening gear teeth tip to maintain adequate tip to root clearance for gears operating on extended center distance. Tip shortening coefficient is key factor for this modification of gear teeth. There are three options such as tip shortening for full length teeth, tip shortening for full working depth and tip shortening for full tip to root clearance. When its radio button is selected, option can be visible and user can select any of them.

Tooth thinning for backlash is an intentional modification on tooth thickness to provide required amount of backlash for a smooth gear operation. It is applied by adjusting cutting tool position to thin the gear teeth. It is going to be visible when tooth thinning for backlash is selected.

Material and life properties are important to define gear pair' allowable stress values which are very vital for gear rating for surface contact and bending stress. Many of them are common for all gear standards whereas some of them are different. AGMA analysis spur module's material and life section have some important parameters such as material, material designation, heat treatment, grade number, hardness, allowable contact stress, allowable bending stress, number of load cycles for bending, number of load cycles for contact and rim thickness below the tooth.

Steel, iron and bronze are main materials recommended by AGMA with the material designation presented in Table 4.4 and 4.5 respectively.

Rim thickness below the tooth describes gear rim of gear geometry and evaluate gear blank rim thickness on the load carrying capacity of the gear tooth.

Software allows user to see corresponding hardness value for different hardness values such as Rockwell A, Rockwell C, Rockwell D, Rockwell HR15N, Rockwell HR30N, Rockwell HR 45N, Vickers, Brinell 10 mm standard 3000 kgf, Brinell 10 mm low carbide 3000 kgf, Knoop and Scleroscope by using ASTM related tables also for AGMA analysis and design modules.

Table 4.4 Material and material designation recommended by AGMA for steel [11]

Material	**Material Designation**	**Heat Treatment**
Steel	Steel	Carburized and Hardened
		Carburised and Hardened Bainite and Microcracks Limited
		Flame or Induction Hardened Flanks and Root Hardened
		Flame or Induction Hardened Flanks and Root Hardened
		Nitrided (Through Hardened)
		Through Hardened
	%2,5 Chrome (No Aluminum)	Nitrided
	Nitralloy 135M	Nitrided
	Nitralloy N	Nitrided

Table 4.5 Material and material designation recommended by AGMA for iron and bronze [11]

Material	Material Designation	Heat Treatment
ASTM A48 Gray cast iron	Class 20	As cast
	Class 30	As cast
	Class 40	As cast
ASTM A536 Ductile (nodular) iron	Grade 02	Quenched and tempered
	Grade 03	Quenched and tempered
	Grade 06	Quenched and tempered
	Grade 18	Annealed
	Grade 40	Annealed
	Grade 55	Quenched and tempered
	Grade 60	Annealed
	Grade 70	Quenched and tempered
	Grade 80	Quenched and tempered
	Grade 90	Quenched and tempered
	Grade 100	Quenched and tempered
	Grade 120	Quenched and tempered
Bronze	-	Sand cast
Bronze	ASTM B-148 Alloy 954	Heat treated

Results section and its content were explained in corresponding section of ISO. Parameters will be presented as a result later for case studies.

4.4.3 Spur Gear Analysis Based on DIN

Spur gear analysis based on DIN by software necessitates inputs as common properties and material / life properties. There are some warnings similar to ISO and AGMA modules for limitations and recommendations for some parameters.

All input parameters for common properties section of DIN is same as ISO corresponding module except bottom clearance coefficient. Bottom clearance coefficient is used by DIN analysis spur gear module instead of application requirement which is preferred by ISO module. Their usage purpose principle is same for ISO and DIN; to define addendum, dedendum, and clearance amount for gear pairs. Standard values of normal modules recommended by DIN have already been presented in Table 3.20, Chapter 3 [19].

Material and life properties have huge importance on gear performance due to effect on gear allowable stresses for surface contact and tooth root bending. All these properties are same used in ISO module of software but their calculations methods can differ used for DIN.

Similar to other modules of spur gear analysis abovementioned, results section of DIN analysis spur gear module presents many parameters for gear geometry and rating to light the way of gear engineer to observe effects of these parameters on gear performance.

4.5 SPUR GEAR DESIGN MODULE OF THE SOFTWARE

Spur gears can be designed based on ISO, AGMA and DIN parallel axes gear standards. Some constraints is taken into consideration as principle based on limitations of gear geometry and performance parameters. Tooth tip thickness, working root clearance and transverse contact ratio for spur gears are estimated and checked against limitations. Spur gear pair is designed alternatively. Design alternatives for redefined operating center distance and gear ratio are presented. User can select any of them for analysis of gear set.

4.5.1 Spur Gear Design Based on ISO

Spur gear design by using software is started and required minimum numbers of parameters are defined from the sections of common properties and material / life properties of ISO Design Spur Gear Module. Common properties section (Figure 4.7) of this module has same parameters as in ISO Analysis Spur Gear such as pressure angle, input power, minimum required safety factors for bending and contact stress, surface roughness values, accuracy grade values, driver rotational speed, pinion shaft external diameter, bearing distance, distance of gear center from the shaft midpoint are entered to software whereas driver type, application requirement, working characteristics of pinion and gear, surface condition, oil type, pinion shaft type, helix modification and contact pattern situations while there are some other additional parameters; center distance and gear ratio for design of gear pair.

Center distance is very essential to design gear drives, it is a geometrical constraint due to gearbox size. Designer should be aware of this constraint to design gears for predefined center distance. It allows sometimes little change with advantage of involute gear profile. This change can be tolerable by addendum modification for pinion and gear providing zero sums of profile shift coefficients. Gear ratio is indication of speed decrease or increase amount between gear pair with respect to driver gear which can be pinion or wheel. Gear ratio can be whole number or not. But tooth numbers must be integer and they have a relationship based on gear ratio.

Material and life properties (Figure 4.8) are all same as in analysis module of the software which was before mentioned.

Gear pair is designed according to limitations can be grouped into two; user defined and safe operation. User defined limitations such as center distance and gear ratio are important for dimensions and input/output speed or torque ratio whereas restrictions for tooth tip thickness, working clearance and transverse contact ratio is vital to provide a safe operation. For this purpose, software is based on some limitations given in equations 4.1-4.3.

$$tt_{tip} \geq 0.4\, m_n \tag{4.1}$$

$$c_w \geq 0.15\, m_n \tag{4.2}$$

$$t\, tcr \geq 1.2 \tag{4.3}$$

Where tt_{tip} refers to tooth tip thickness, c_w; working clearance and tcr; transverse contact ratio.

Software gives design results alternatively (Figure 4.8); this gives chance to user select any of them and sends to analysis. This allows observing different design parameter effect on gear performance in terms of tooth root and surface contact, and also gear geometry. User also selects another design alternative which is different than earlier one and evaluates results separately.

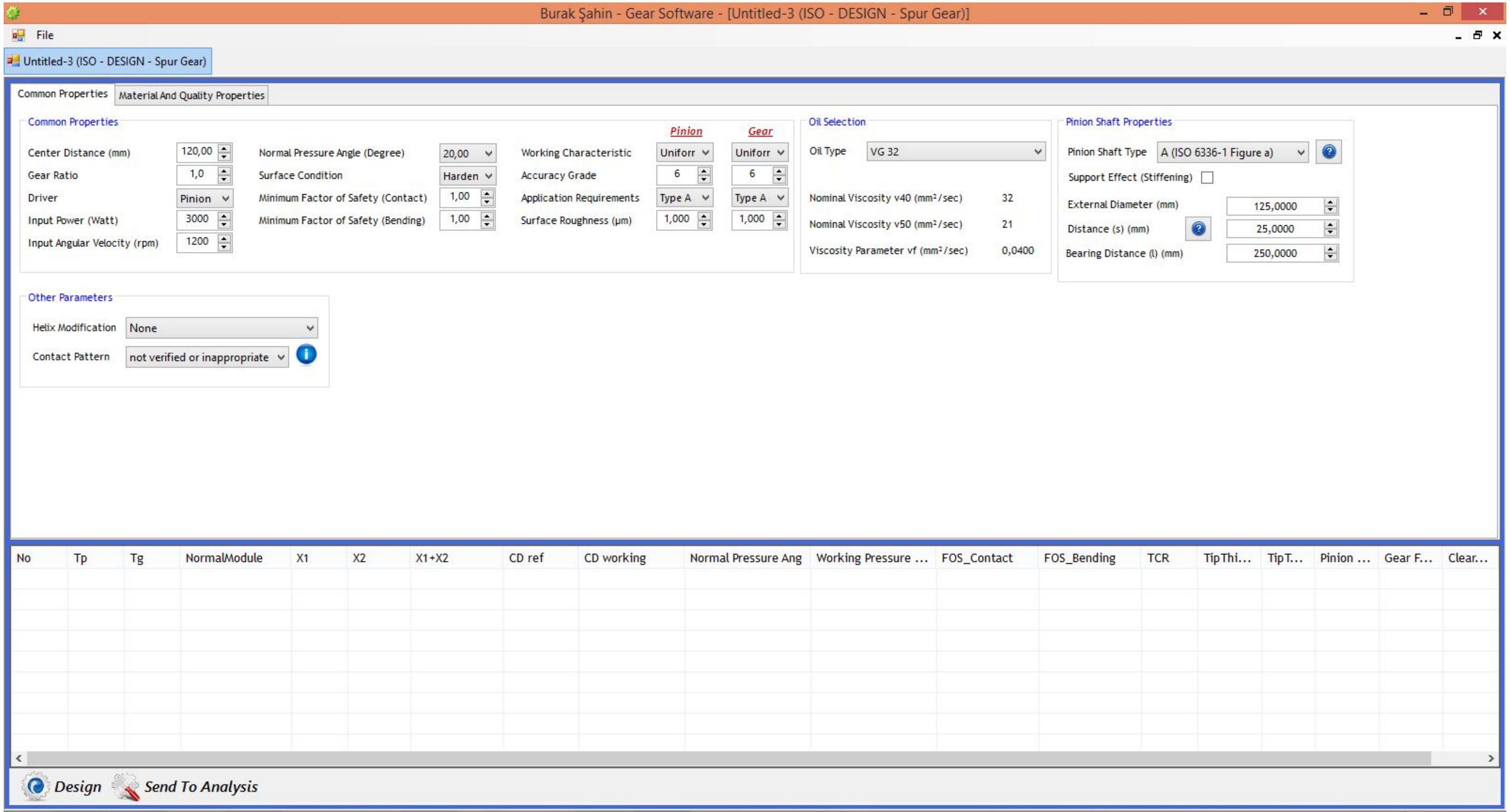
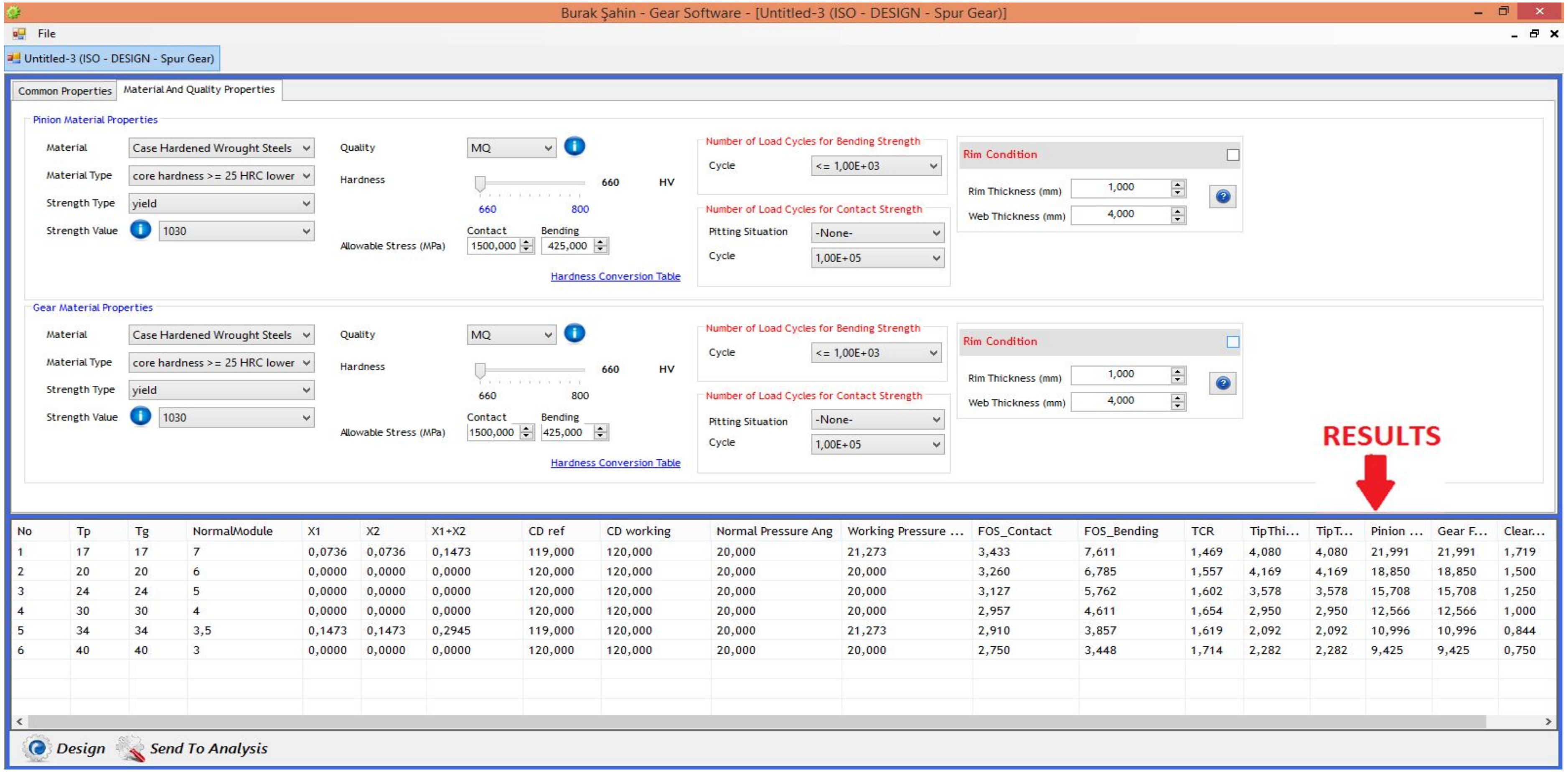


Figure 4.7 Spur gear design based on ISO – common properties



No	Tp	Tg	NormalModule	X1	X2	X1+X2	CD ref	CD working	Normal Pressure Ang	Working Pressure ...	FOS_Contact	FOS_Bending	TCR	TipThi...	TipT...	Pinion ...	Gear F...	Clear...
1	17	17	7	0,0736	0,0736	0,1473	119,000	120,000	20,000	21,273	3,433	7,611	1,469	4,080	4,080	21,991	21,991	1,719
2	20	20	6	0,0000	0,0000	0,0000	120,000	120,000	20,000	20,000	3,260	6,785	1,557	4,169	4,169	18,850	18,850	1,500
3	24	24	5	0,0000	0,0000	0,0000	120,000	120,000	20,000	20,000	3,127	5,762	1,602	3,578	3,578	15,708	15,708	1,250
4	30	30	4	0,0000	0,0000	0,0000	120,000	120,000	20,000	20,000	2,957	4,611	1,654	2,950	2,950	12,566	12,566	1,000
5	34	34	3,5	0,1473	0,1473	0,2945	119,000	120,000	20,000	21,273	2,910	3,857	1,619	2,092	2,092	10,996	10,996	0,844
6	40	40	3	0,0000	0,0000	0,0000	120,000	120,000	20,000	20,000	2,750	3,448	1,714	2,282	2,282	9,425	9,425	0,750

Figure 4.8 Spur gear design based on ISO – material / life properties and results

4.5.2 Spur Gear Design Based on AGMA

This module is same as module of "*AGMA Analysis Spur Gear*" in terms of common properties and material and life parameters. But normal module, number of teeth, face widths and profile shift coefficients are not predefined. Instead of them, center distance and gear ratio are necessary for design of gear pair.

Same design limitations as ISO are used here also to provide a safe operation of gear pair in terms of transverse contact ratio, working clearance and tooth thickness at the tip of tooth.

4.5.3 Spur Gear Design Based on DIN

This module is prepared for design of spur gear based on DIN standards. Same formulas, figures and tables in "*DIN Analysis Spur Gear*" module are used to calculate all required parameters for design. Design restriction in ISO and AGMA above mentioned are also valid here to ensure operation safety during meshing of gears in terms of continuity of motion, preventing pointed teeth and gear teeth entering and exiting from contact without jamming between other teeth.

4.6 HELICAL GEAR ANALYSIS MODULE OF THE SOFTWARE

Helical gears can be analyzed based on ISO, AGMA and DIN by using modules of *"ISO Analysis Helical Gear", "AGMA Analysis Helical Gear" and "DIN Analysis Helical Gear",* separately. These modules are very similar to spur gear analysis modules in terms of input parameters. Helical gears are modeled as virtual spur gears and analyzed by using modified spur gear analysis formulas depending on virtual parameters namely virtual tooth number.

4.6.1 Helical Gear Analysis Based on ISO

Helical gear teeth inclined with helix angle which is zero for spur gears. It has significant effect on gear geometry and rating parameters. ISO Analysis Helical Gear common properties section differs from corresponding section of spur gear analysis due to helix angle. Same warnings for many parameters are valid for helical gears as well as spur gear pairs. There are some information buttons to explain details in software.

4.6.2 Helical Gear Analysis Based on AGMA

Helical gear can be analyzed by using AGMA standards in a similar way as spur gear, but difference stems from helical gear geometrical parameters which reflect difference between spur and helical gears. There are also some differences in terms of standards and their application method to evaluate gear performance for tooth root and surface contact. These all have been already explained before in the scope of spur gear analysis based on AGMA, it is not going to be explained again here.

4.6.3 Helical Gear Analysis Based on DIN

As well as helical gear design modules based on ISO and AGMA, design of helical gears can be done based on DIN by using this module. This module is similar to *"DIN Analysis Spur Gear"* module of software in terms of equations, figure, and tables of gear rating based on root and contact stress.

4.7 HELICAL GEAR DESIGN MODULE OF THE SOFTWARE

Design of helical gears is based on same standards as spur gears; they are accepted as virtual spur gears. Parameters which are pertaining to helical gears are taken into account to evaluate power rating of helical gears in terms of tooth root bending and contact surface stresses. Design constraints in terms of safe operation of gear pair are valid as well as spur gears.

4.7.1 Helical Gear Design Based on ISO

Helical gear design based on ISO by software's module necessitates input of minimum number of required parameters from common, material and life properties sections of ISO Design Helical Gear Module. Common properties section is similar to analysis module of helical gear analysis for many parameters namely pressure angle, helix angle, input power, minimum required safety factors for bending and contact stress, surface roughness values, accuracy grade values, driver rotational speed, pinion shaft external diameter, distance between bearing distance, distance of gear center from the shaft midpoint are entered to software whereas driver type, application requirement, working characteristics of pinion and gear, surface condition, oil type, pinion shaft type, helix modification and contact pattern situations, but there are two additional parameters; center distance and gear ratio.

Material and life properties are completely same as in analysis module of the software.

4.7.2 Helical Gear Design Based on AGMA

Helical gears can be designed based on AGMA standards by using this module. Its material/life properties section is same as corresponding section of analysis module. But common properties section is different than analysis module; center distance and gear ratio are given instead of tooth numbers, face widths and module. Same design principles are used including AGMA standards for helical gears to provide a safe operation during meshing of gears.

4.7.3 Helical Gear Design Based on DIN

"DIN Design Helical Gear" is the last one of software helical gear design modules. This module works in same principle as other helical gear design modules; design alternatives are presented to allow user select any of these alternatives and analysis to observe all gear parameters.

In this chapter software modules of spur and helical gears analysis and design based on ISO, AGMA and DIN standards are explained. ISO modules are explained firstly, AGMA and DIN modules follow. There are some figures from software are presented for only ISO modules, because are similar to these modules. Different input parameters are explained for related sections.

CHAPTER 5

TRANSMISSION ERROR THEORY AND CONSTRUCTION

5.1 INTRODUCTION

Gears are machine elements used for power and motion transmission between parallel or non parallel shafts by manipulating rotational speed, torque, direction of rotation and axis of rotation. Spur gears are simplest among all gear types in terms of manufacturing, operation, and understanding of gears working principle and geometry. Involute gear tooth profile is the mostly widely used for all gear types due to easy manufacturing, constant velocity ratio (uniform motion transfer) and allowance for center distance change.

Spur gear design criteria include two main points of static and dynamic performances. Static performance usually deals with global kinematic requirements, geometrical requirements and strength analysis of gear materials under load. Dynamic performance of gears is just as important as performance under static conditions during meshing. It deals with gear noise and vibration which are key factors for smooth meshing and quietness. Any non uniformity in motion transfer cause gear noise and vibration under load.

Theoretically a pair of gear with the perfect involute tooth profiles and tooth spacing transmits a uniform motion between the shafts. But in reality motion is not transmitted uniformly. Due to deflection of the teeth under load, interference, in other words corner contact occurs between the teeth of the incoming pair and this causes impact loads on the teeth and instantaneous non uniform motion transfer between the gears. When the gears run at high speed these impact loads and non uniform motion transfer causes high dynamic tooth loads, vibration and the generation of noise.

The instantaneous error in uniform motion transfer is called the ***Transmission Error (TE)*** **(**Figure 5.1a) and it is defined as “the difference between the actual position of the output gear and the position it would occupy if the gears were perfectly conjugate”. This definition is valid for both loaded and unloaded gears. It is the TE under load which causes the problem at high speed whether it is due only to tooth deflection, manufacturing errors or both [37]. TE curves are constructed for different design loads (Figure 5.1b).

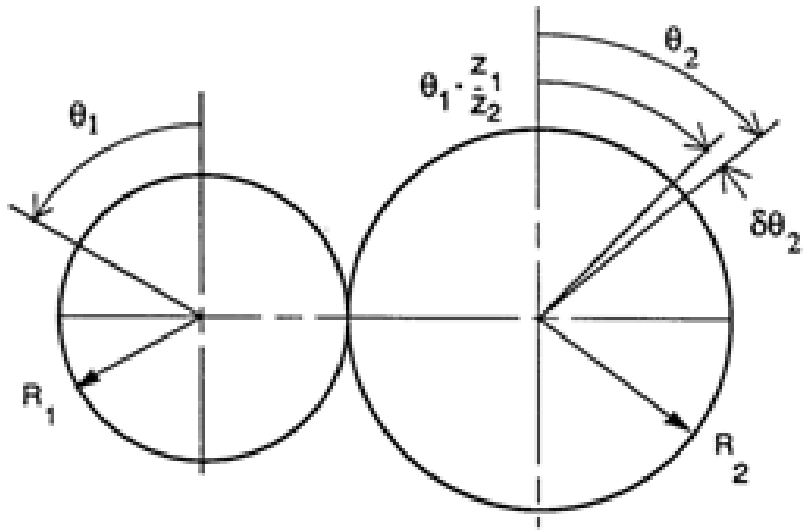


Figure 5.1a TE

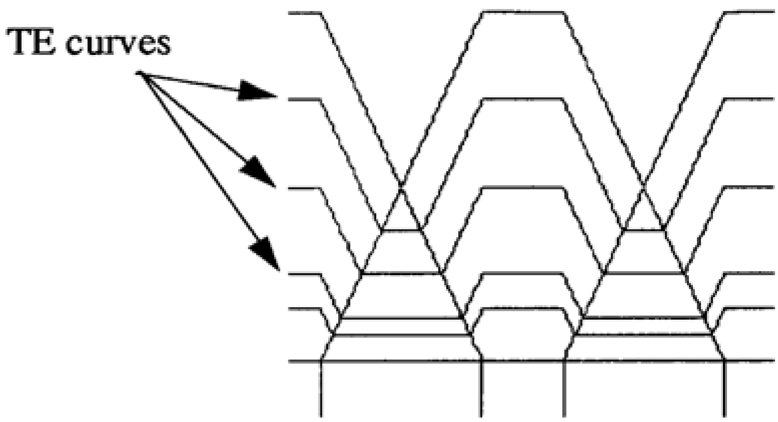
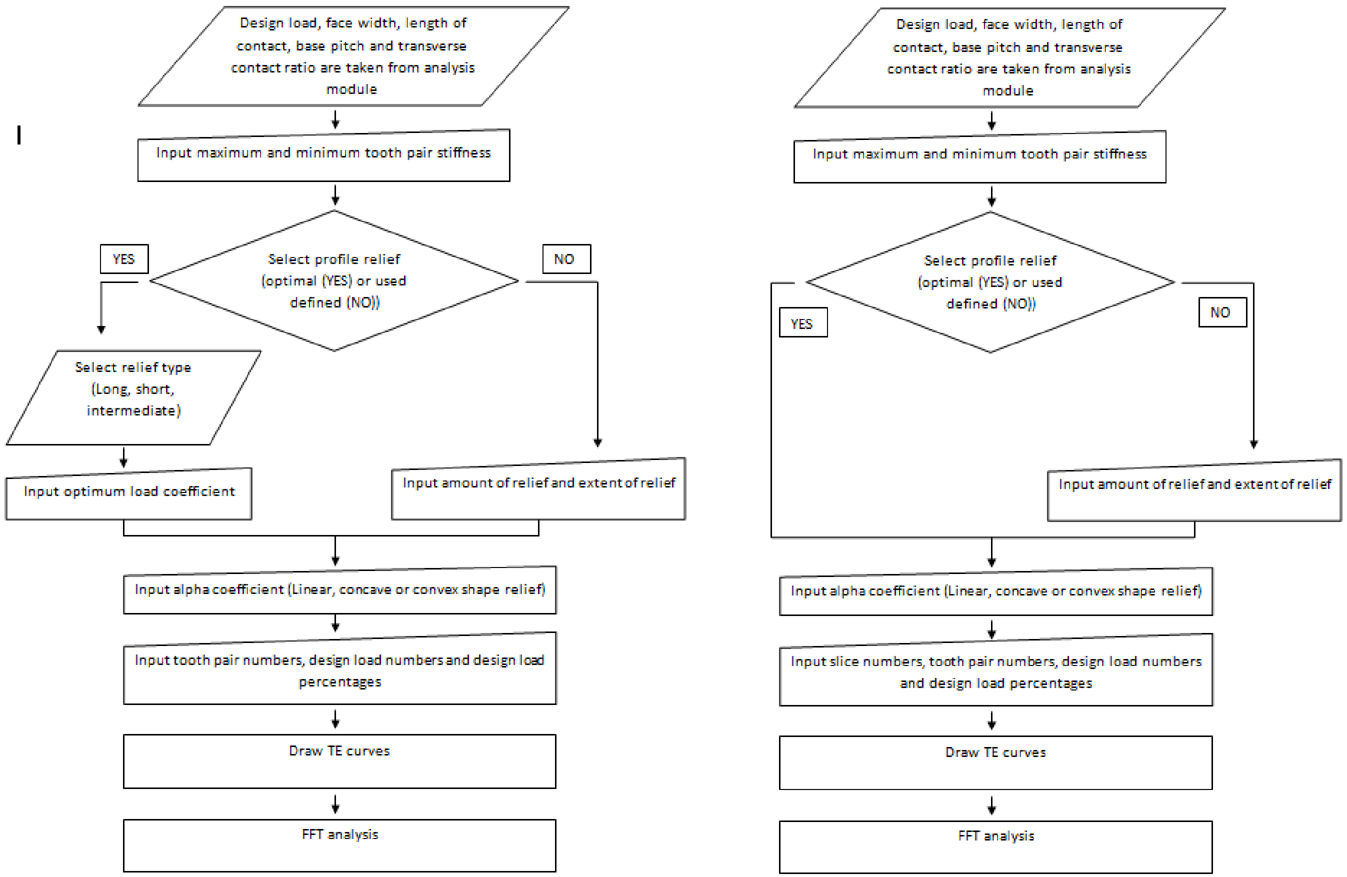


Figure 5.1b TE Curves

Figure 5.1 TE and TE Curves

5.2 FLOW CHART OF THE QUASI STATIC TRANSMISSION ERROR SOFTWARE

Software includes transmission error modules for spur and helical gears separately. Flowchart of transmission error analysis sections are presented in Figure 5.2a and 5.2b. Gears are analyzed according to international standards by using software, and then transmission error analysis modules (Figure 5.3) are going to be active. User can make analysis of transmission error. Design load, face width, length of contact, base pitch and transverse contact ratio are taken from analysis module. Some other parameters are input to transmission error analysis module to make analysis. Transmission error curves are drawn for different loads in percentages of design load and they can be compared on the screen of TE module. In transmission error module, no load transmission error curve, loaded transmission error curves, stiffness curves, single tooth pair stiffness curve, single tooth pair boundary curve and single tooth pair tooth load curves are created and illustrated (Figure 5.4). Later, Fast Fourier Transform (FFT) analysis can be done for any design load. FFT analysis curves are shown in Figures 5.5.



(a) Spur Gear (b) Helical Gear

Figure 5.2 Flow charts of transmission error modules for spur and helical gears

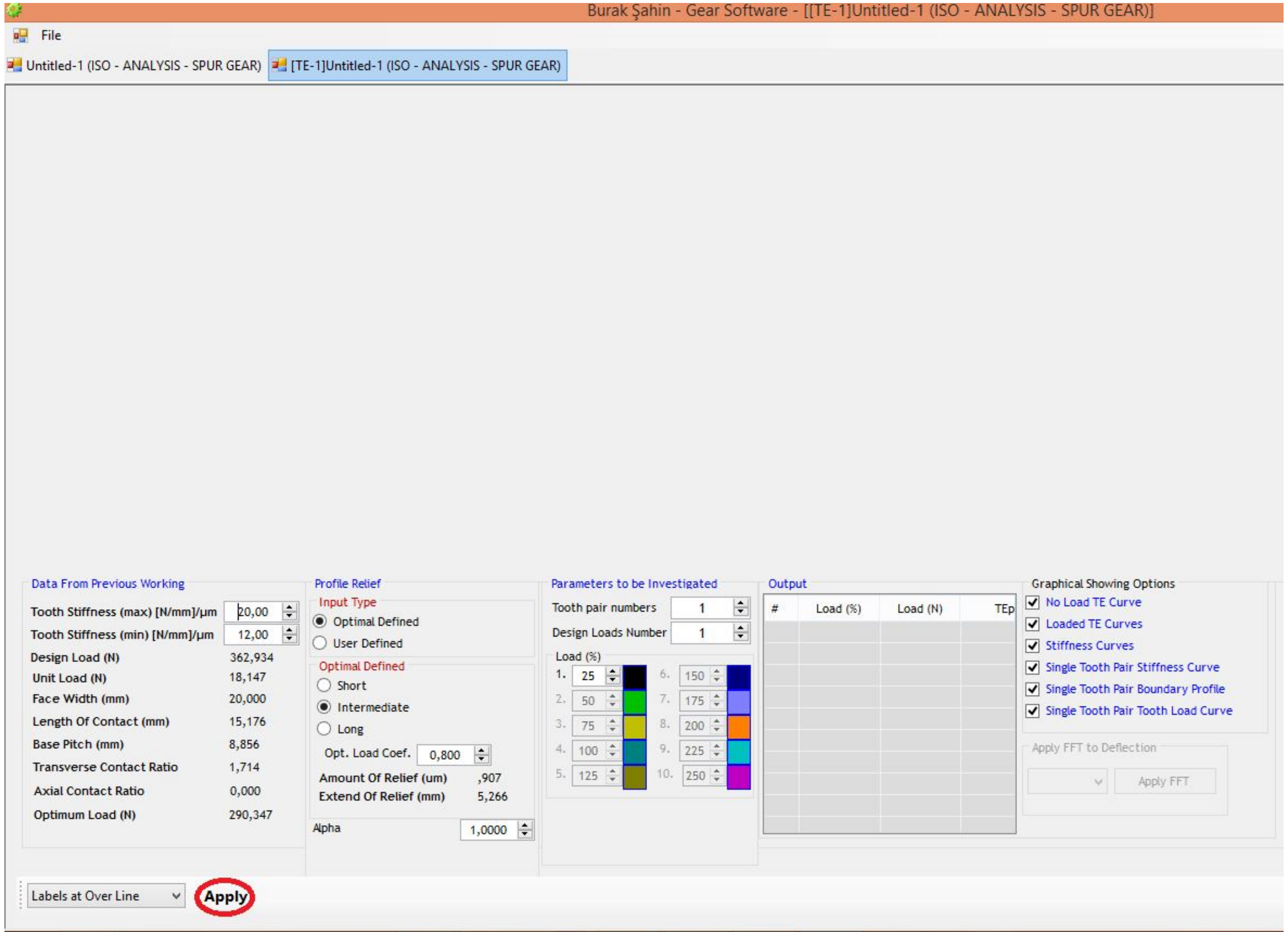


Figure 5.3 Transmission error module

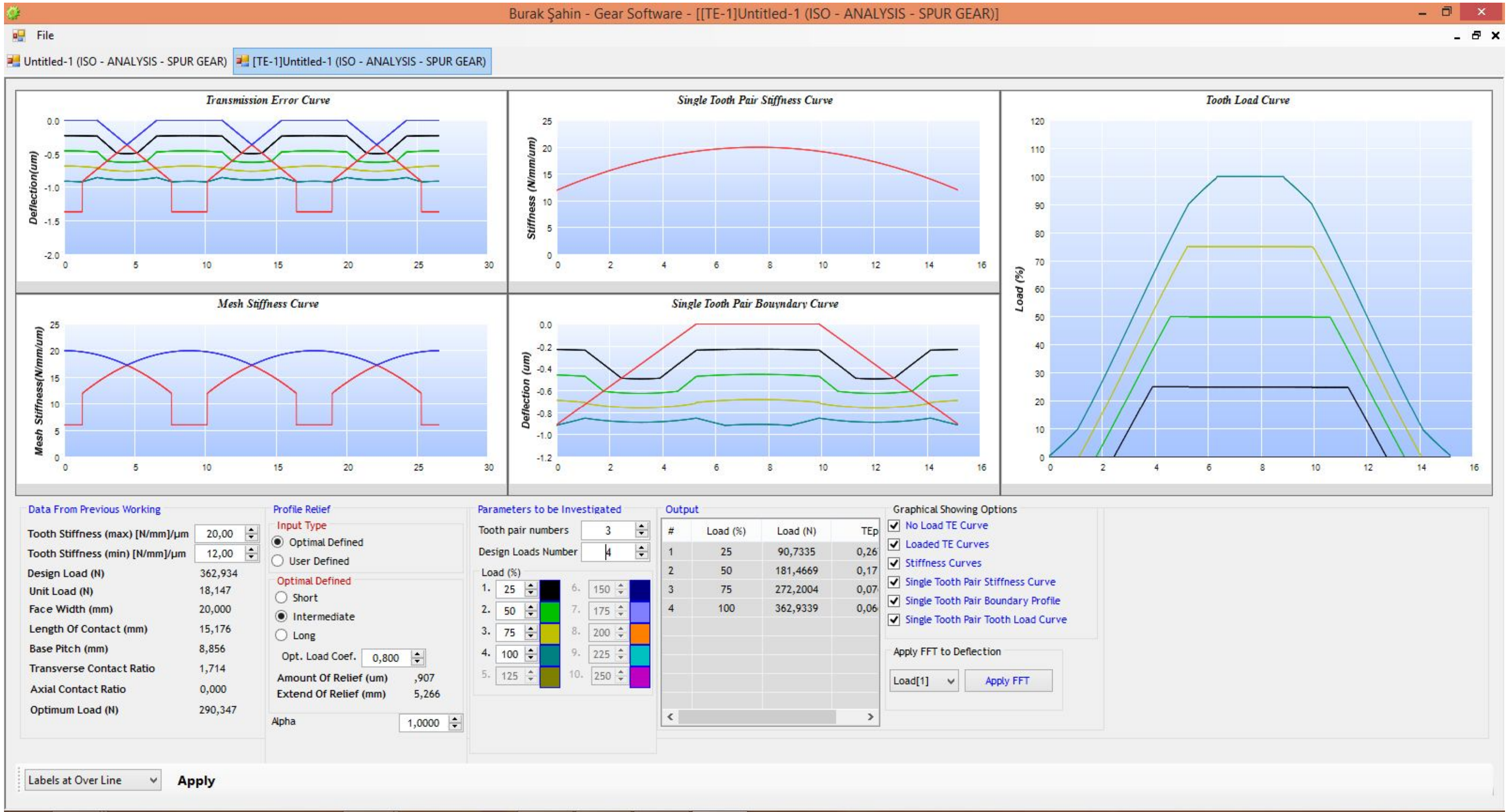


Figure 5.4 Transmission error curves

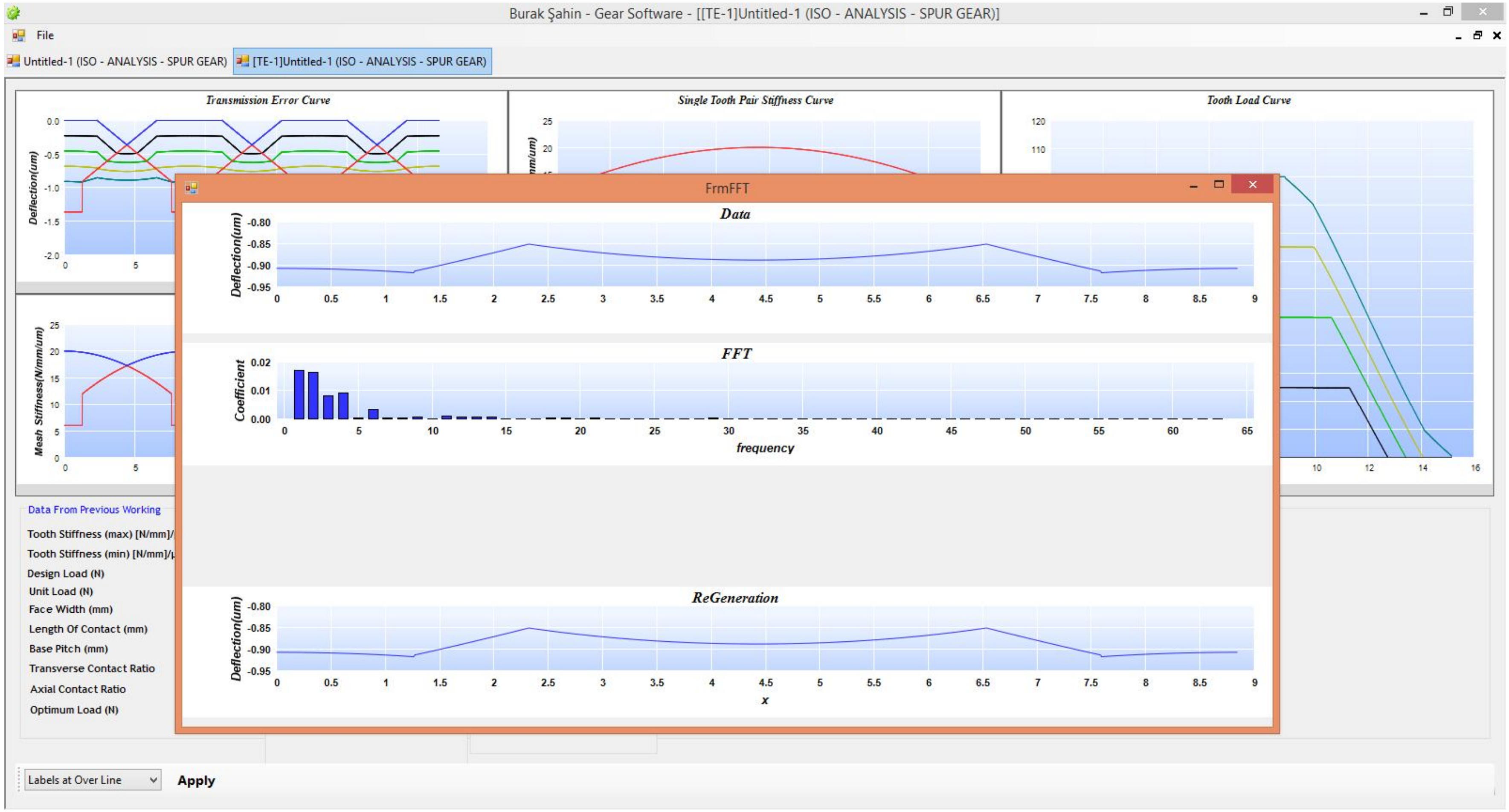


Figure 5.5 Fast Fourier Transform of design load

5.3 SPUR GEAR TRANSMISSION ERROR THEORY AND QUASI STATIC TRANSMISSION ERROR CURVES

Gear design is influenced by load carrying capacity under static conditions and dynamic performance of gear pairs. Dynamic performance of gears is dependent on vibration level during gear meshing due to loading conditions, manufacturing and mounting errors. Vibration cause to noise during operation and it is a proof of non uniform load distribution of tooth contact. Due to importance of gear application areas such as commercial vehicles, heavy industry machines, defense, aerospace, and military vehicles, high performance gears are required in terms of static and dynamic point of view. Gears should be designed based on minimum vibration and noise, long service life and high strength to weight ratio by minimizing transmission error which is cause of noise and vibration.

Transmission error (TE) is defined the difference between the actual position of the output gear and the position it would occupy if the gears were perfectly conjugate. It can be defined in angular units, or can be expressed in linear units along the line of action. The TE is generally converted to linear motion along the line of action which is tangential to the base circles of the mating gears. The mathematical formulation of TE in linear units is as follows:

$$TE = r_{b2} \, X \, (\theta_2 - \frac{T_1}{T_2} \, X \, \theta_1) \quad [37] \qquad (5.1.)$$

θ_1: Angular rotation of the input gear

θ_2: Angular rotation of the output gear

T_1: Tooth number of the driving gear

T_2: Tooth number of the driven gear

r_{b2}: Base radius of the output gear

The static transmission error is strongly related to the kinematic accuracy of gear trains, dynamic load of gear tooth, and gear noise. It is defined as the difference between the theoretical position of the output gear, with perfect geometric accuracy and rigid drive, and the actual output position at a low enough speed [49].

In a gear set, TE is defined as the difference between the effective and the ideal position of the output shaft with reference to the input shaft. The ideal position represents a condition of perfect gear box, without geometrical errors and deflections. TE can be expressed either by an angular displacement or, more conveniently, as a linear displacement measured along a line of action at base circle [49].

Transmission error is one of main excitation source of gears and it has possible negative effects on gear operation namely high vibration levels transmitted via gear body, shaft, bearing, housing and casing, high noise levels of both air borne and structure borne types, kinematic inaccuracy causing problems in accurate positioning applications. TE under design load is the main cause of all the negative effects and has to be minimized in amplitude and optimized in shape to get a smooth TE curve.

For a smooth meshing during gear operation, the production of gears with minimum geometrical errors is first step and very critical to reduce TE value. But loading effect or gear tooth flexibility under load cause to TE for an even error free gear pair. Adjacent pitch error which and tooth deflection both cause corner contact (CC) or can be called as interference (Figure 5.6). It is effective at the start and end of mesh. In addition, both adjacent PE and the teeth deflection will cause a position variation of the output gear hence a TE of gears.

Corner contact causes an instantaneous variation in TE curve as well as the physical damage of tooth surface under heavy loads. An intentional profile modification is very vital to avoid interferences. For this purpose, the tip of the driven flank or the root of the driving flank or both has to be modified. A definite amount of material is removed to provide a smooth meshing of gear pair. Intentional profile modifications of gears at tooth tips of both driving and driven gears are illustrated in Figure 5.7a and b respectively.

Profile modification for linear type can be characterized by two parameters namely amount of relief and extent of relief (Figure 5.8). Amount of relief is the thickness of material removed from tip of the tooth in a plane normal to the involute curve. Extent of relief shows how far the relief extends down the tooth from the tip (Figure 5.8).

Extent of relief, although can be measured in roll angle, is usually given as linear dimension between start and end points of the relief along the path of contact. Extent of relief in linear units is calculated by multiplying the required roll angle difference between start and end of relief by the base radius (r_b) in equation 5.2.

$$\text{Extent of relief} = r_b * (roll\ angle_{end} - roll\ angle_{start}) \tag{5.2}$$

Gear operation at speeds away from the system's resonant speeds is important. However, systems will generally be multiple degree of freedom systems with multiples of coupled and uncoupled resonant frequencies. In addition, the resonance is not hit by only the main excitation frequency but also by its harmonics and sub harmonics. There is a direct relationship between static TE values and vibration and noise levels. Smoothing the static TE curve under operating load, either by profile relief or some other means, comes front to avoid harmonics hitting problem and high vibration and noise levels.

Construction of loaded and unloaded TE curves in linear units for a pair of gears with profile relief is shown in Figure 5.8. For ease understand of topic, it is assumed that the gears in mesh have no adjacent pitch errors, tooth pair stiffness is constant and transmitted load is also constant. The vertical axis refers to TE in linear units while horizontal axis represents the nominal positions of contact along the line of action. When the contact point is in single tooth pair region (SC) all the load is carried by that one pair and when the contact point is in double tooth pair region (DC) the total load is shared by two pairs (regardless of whether equally or not).

Applying a proper relief on the gear teeth and also constructing the expected static TE curve under load to check the suitability of that relief is very essential. Loaded and no load static TE curve is constructed for a constant tooth load and constant tooth pair stiffness with a linear type of profile relief (Figure 5.9).

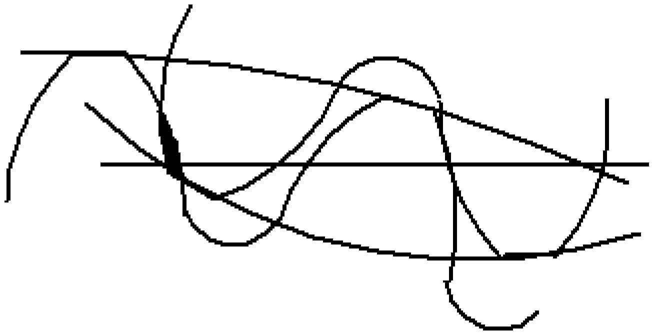
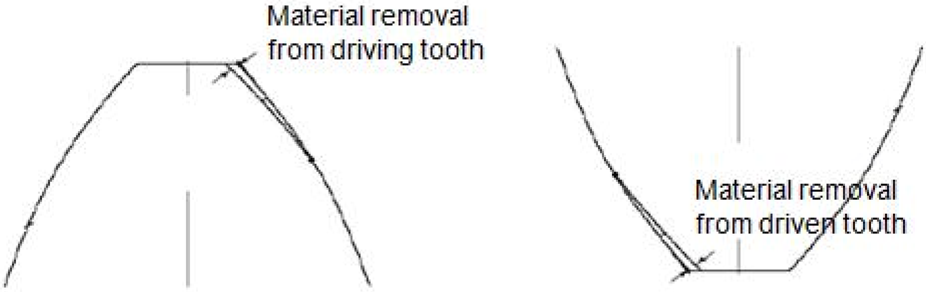


Figure 5.6 Corner contact



(a) Driving Gear (b) Driven Gear

Figure 5.7 Application of tip relief to both driving and driven teeth to prevent corner contact

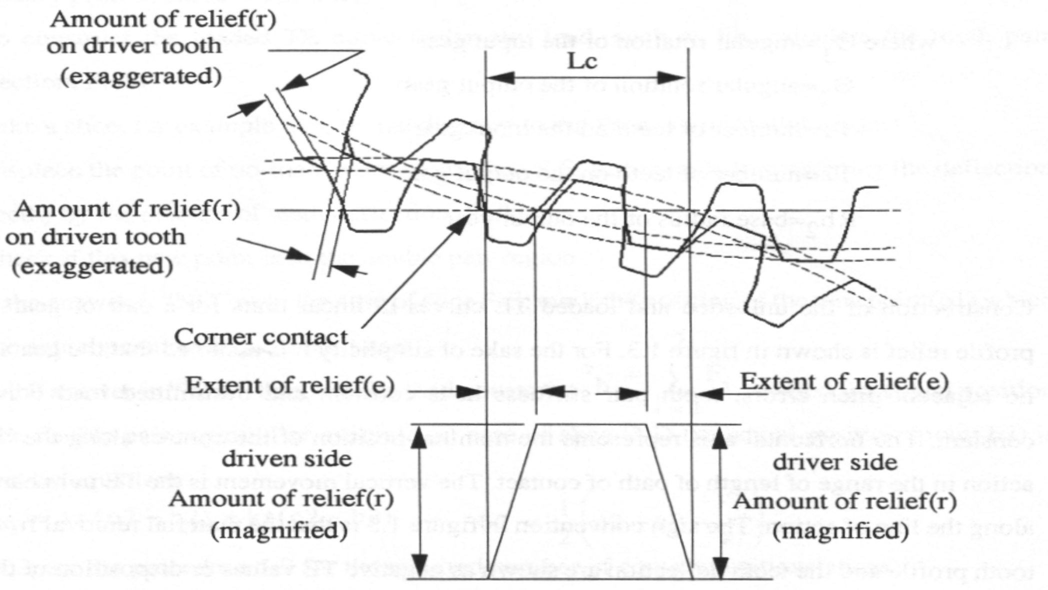


Figure 5.8 Geometry of the tooth contact and profile relief along path of contact

Relief shape is generally linear which the simplest form of relief is. There are other relief shapes mathematically, but manufacturing cost must be taken into account. It can be linear, concave or convex (Figure 5.10).

The procedure for constructing the unloaded and loaded TE curves as follows [49]:

a) Construct the boundary profiles of the mating tooth pairs such as curves f-g-h-k-l-m.
b) Displace them by the base pitch of the gears along the horizontal axis.
c) Construct the no load TE curve by following the top boundary borders of the each tooth boundary profile, curve d-e-h-k-n-r.
d) To construct the loaded TE curve under any load, such as design load F_b, calculate the tooth pair deflection, $\frac{F_b}{k_t}$
e) Take a slice, for example S-S, downwards representing the deflection of teeth) by the amount of teeth deflection, $\frac{F_b}{k_t}$
f) Check if this new point is in the double pair region
g) If answer is NO as in the case of slice S-S, mark the position of the new point b_1 which is the actual position of the output gear
h) If answer is YES then use the load constraint $F_b = \sum_{i=1}^{2} F_i$ to find the actual position in the double pair region. For instance, in case of slice D-D the actual position (point b_2) is founded by equations:
   $F_b = k_t(o_2 - b_2) + k_t(c_2 - b_2)$
i) Repeat the procedure 5-9 for the required number of contact positions (slices)
j) Connect the new marked points (b_1, b_2, etc) to construct the loaded TE curve

There are two cases of profile relief such as long and short relief (Figure 5.11). They define the useful limits beyond which is not profitable to go in terms of corner contact, tooth loading and TE. The amount of relief at teeth tip is same for short and long relief cases and it is equal to sum of the tooth pair deflection. There is difference for extent of relief values for both cases.

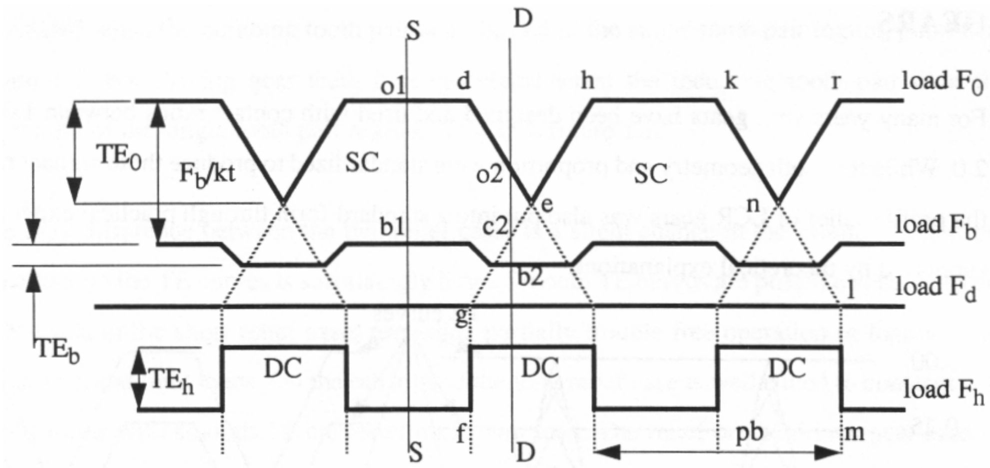


Figure 5.9 Construction of the TE curves for low contact ratio spur gears

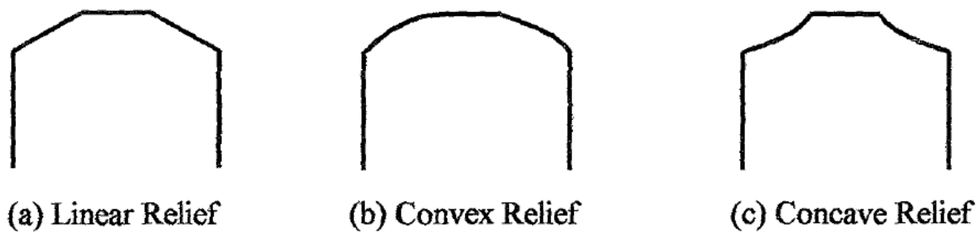


Figure 5.10 Relief types [49]

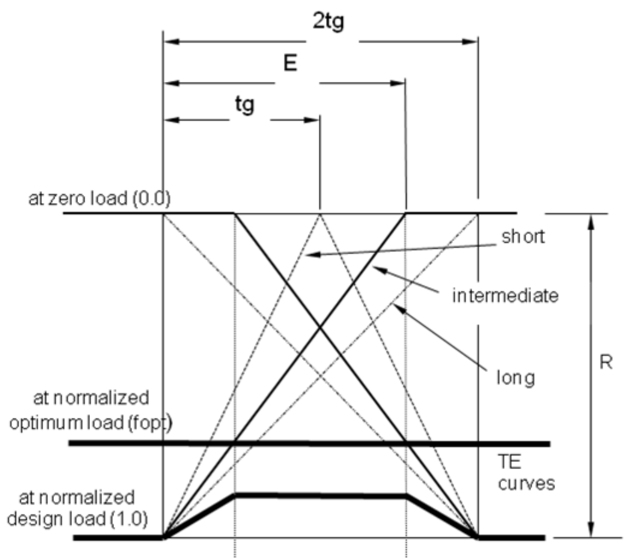


Figure 5.11 Profile relief geometry

5.4 HELICAL GEAR TRANSMISSION ERROR THEORY AND QUASI STATIC TRANSMISSION ERROR CURVES

Spur gear is special type of helical gear with zero helix angles. Spur gears are regarded as two dimensional objects, while helical gears are three dimensional objects.

Helical gears are analogous to a set of stepped gears which consist of a number of identical spur gears so arranged that the teeth of each individual member are slightly out of phase relative to each other. In such an arrangement, there is an overlap during successive engagement of teeth, that is, when two teeth are in mesh at the pitch line, other mating pairs of teeth are in different phases of contact including approach and recess contacts. A helical gear construction is approximated if a composite body is made up of an infinite number of such stepped gears, each of which is a lamination of infinitesimal thickness, placed side by side successively with a slight phase difference. This has been illustrated in Figure 5.12.

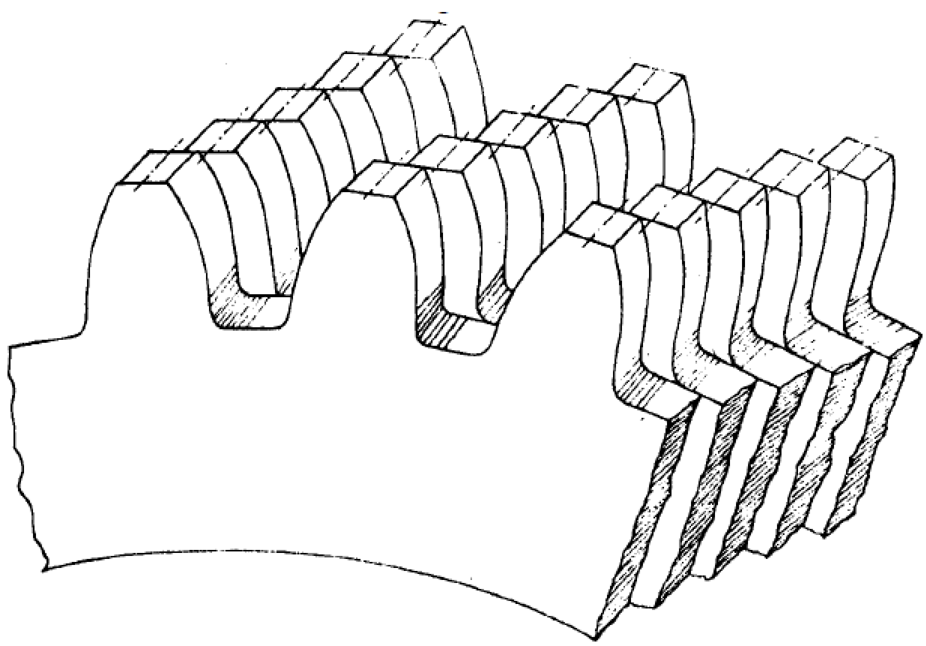


Figure 5.12 Formation of helical gear [1]

This approximation is called as thin slice theory which allows accepting helical gears as stepped spur gears based on helix angle and an amount of slice shift. In this case each slice of helical gears can be thought as spur gear and modeled in 2D. Thin slice theory helps to simplify helical gear analysis which is more difficult than spur gear analysis due to helical gear's complexity.

Helical gears can be considered as a combination of thin spur gear slices and these slices are brought together side by side with a slice shift (ss) based on helix angle (β), face width (b), and slice number (sn). Slice shift can be defined difference between slices (Figure 5.13). It can be calculated as in following equation.

$$ss = \frac{b}{n}\tan\beta \quad [49] \tag{5.3}$$

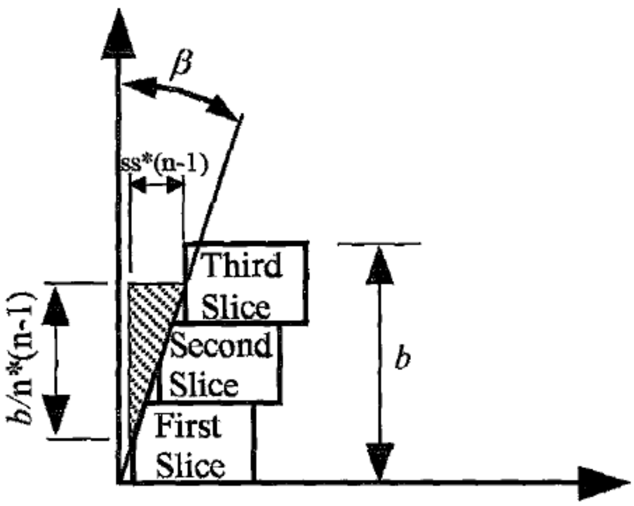


Figure 5.13 Top view of slice [49]

The procedure of constructing of helical gear transmission error curve is based on same principle with spur gears. TE curve of helical gear for no relief and relief situations is illustrated in Figures 5.14 - 5.15 respectively.

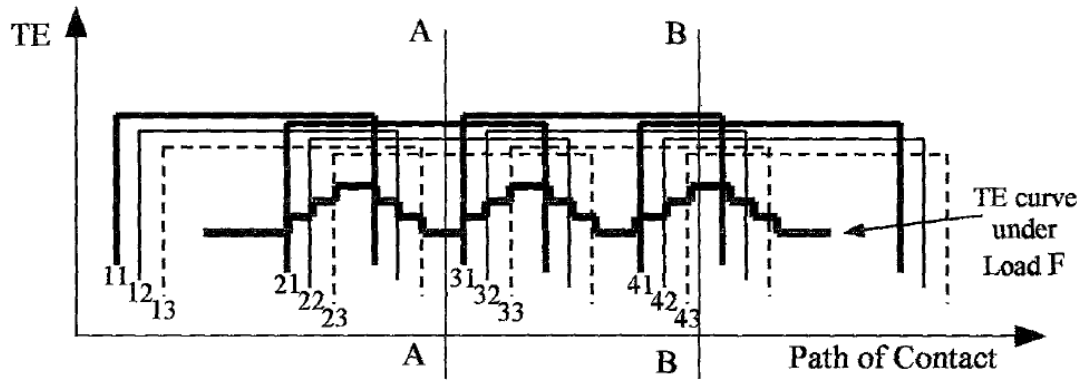


Figure 5.14 Formation of helical gear TE curve (no relief) [49]

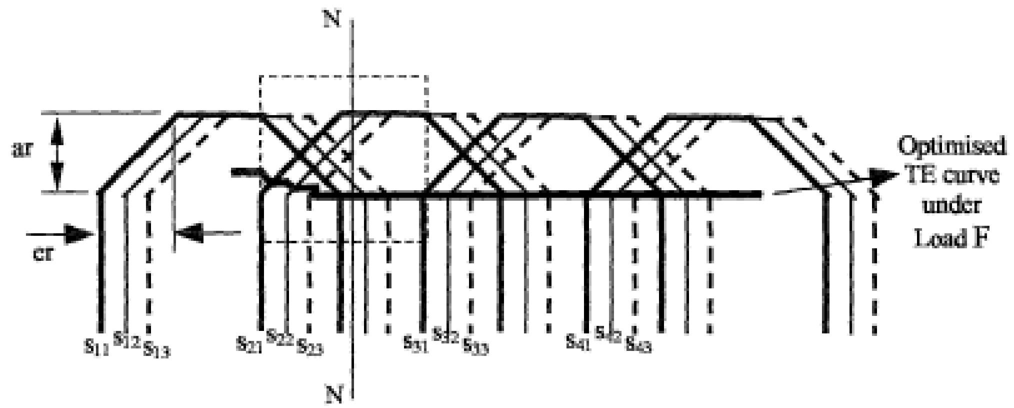


Figure 5.15 Formation of helical gear TE curve (relief) [49]

The procedure of constructing transmission error curves for no relief and relief situations [37] are given below:

a) Construct the boundary profiles of the each slice and place them with the distance (ss) in between, which is determined from equation 5.3.
b) Displace curves by the transverse base pitch of the gears along the horizontal axis.
c) No load TE curve is the upper most borders of each slice of Figure 5.15.
d) Take a section, for example N-N, to represent an instant of contact.
e) Determine which slices may be in contact, for section N-N.
f) Determine amount of relief values of all slices for any instant.
g) Put the relief values in ascending order ($r_{21} = 0 < r_{22} < r_{23} < r_{13} < r_{12}$)

h) Consider that all of the load (F) is carried by the slice with minimum relief value, determine deflection value of slice, $d_{21} = \frac{F}{(k*\frac{b}{n})}$

Where d_{21} is deflection of first slice of second tooth, k is stiffness value for the slice, b is face width and n is number of slices.

i) If $d_{21} > r_{22}$ then consider the slice 22 is in contact too

j) Write the load constraints to recalculate the deflection value

$F = \left(k * \frac{b}{n}\right)[(r_{22} - d_{21+22}) + (r_{21} - d_{21+22})]$

k) Repeat the procedure 9-10 for the required number of slices for instant N-N

l) Connect the marked points to obtain the Te curve for the load F.

CHAPTER 6

CASE STUDIES AND RESULTS

6.1 INTRODUCTION

In the scope of this thesis, spur and helical gears which are used on back to back test rig are designed and analyzed by software based on international parallel axes gear standards presented by ISO, AGMA and DIN respectively. These gears are designed alternatively by using software design module. Then one of them is selected and analyzed based on corresponding standard.

6.2 SPUR GEAR DESIGN AND ANALYSIS

Spur gear pair is designed based on international gear standards namely ISO, AGMA and DIN for constraints defined by user. Power, rotational speed, gear ratio and operating center distance are required design parameters and they are stated by designer before starting to design. Normal pressure angle is selected as 20° for all conditions.

6.2.1 Spur Gear Design and Analysis Based on ISO

These gears are used in a test rig to evaluate gear performance instead of speed decrease or increase so gear ratio is unity. Required design parameters and some other additional parameters such as material properties, surface quality, profile modification and geometrical constraints for spur gear pair are defined by designer (Table 6.1 and 6.2). Spur gear pair is designed based on ISO by using software depending on limitations for gear geometry and a safe operation during meshing. Design alternatives are presented in Table 6.3. First and fifth of alternatives are given with profile shift coefficient by software; this ensures that different reference center distance gears can be used for a predefined operating center distance. Remaining alternatives does not require profile modifications. Sixth of these alternatives (yellow colored) is selected and analyzed by using ISO analysis module for 32.0 mm face width instead of value given by design module as 31.42 mm. General influence factors, performance parameters for tooth root stress / surface contact stress and geometry parameters are calculated by using software. For analysis of selected design alternative, Kiss Soft (commercial machine elements software) is also used, and results are presented in Tables 6.4-6.6 respectively.

Table 6.1 Requirement design parameters

Operating center distance	120	mm
Gear ratio	1.0	-
Power	42	kW
Rotational speed	2500	rpm

Table 6.2 Input parameters for spur gear design based on ISO

Parameters	Pinion	Gear	Unit
Min factor of safety (contact)	2.0		-
Min factor of safety (bending)	2.0		-
Operating center distance	120		mm
Gear ratio	1.0		-
Power	42		kW
Rotational speed	2500		rpm
Normal pressure angle	20		deg
Driver	Pinion		-
Oil type	ISO VG 320		-
Pinion Shaft Type	ISO 6336-1 Figure E		-
Helix Modification	None		-
Contact Pattern	Optimal		-
Material	Case Hardened Wrought Steel		-
Material Type	Core hardness ≥25 HRC lower		-
Allowable contact stress	1550		MPa
Allowable bending stress	450		MPa
Quality	MQ		-
Surface Hardness	60		HRC

Table 6.3 Design alternatives for spur gears based on ISO

No	T_p	T_p	m_n	x_1	x_2	CD_{ref}	Φ_w	b_{cal}	$n_{contact}$	$n_{bending}$
1	17	17	7	0,0736	0,0736	119	21.27	43.98	2,031	4.542
2	20	20	6	0	0	120	20.00	43.98	2,078	5.082
3	24	24	5	0	0	120	20.00	36.65	2,095	5.337
4	30	30	4	0	0	120	20.00	33.51	2,056	5.451
5	34	34	3,5	0,1473	0,1473	119	21.27	29.32	2,024	5.204
6	40	40	3	0	0	120	20.00	31.42	2,024	5.311

There are differences between parameters which are calculated by software and Kiss Soft; general influence factors such as dynamic factor, face load factor for contact stress and face load factor for tooth root stress are different due to rounding off whereas application factor, transverse load factor for contact stress and transverse load factor for bending are same (Table 6.4).

Some of tooth root bending strength performance parameters (Table 6.5) such as relative notch factor, form factor, tooth root normal chord at root, radius of tooth root fillet, bending lever arm, stress correction factor, nominal tooth root stress and tooth root stress shows difference due to rounding off for last digit of parameters values while contact ratio factor, tooth root stress limit, permissible tooth root stress and factor of safeties for tooth root stress are different. Their difference stems from calculation approaches and methods used in software and Kiss Soft for these parameters [4].

There are different methods to calculate parameters of surface contact stress so they are different for software and Kiss Soft [3]. These parameters are contact ratio factor, nominal contact stress and tooth contact stress. All surface contact parameters are presented in Table 6.6.

Table 6.4 General influence factors for spur gears based on ISO

Parameter	Software	KISS soft	Unit
Application factor	1.0000	1.0000	-
Dynamic factor	2.0351	2.035	-
Face load factor (contact)	1.1834	1.183	-
Face load factor (bending)	1.1436	1.144	-
Transverse load factor (contact)	1.0000	1.0000	-
Transverse load factor (bending)	1.0000	1.0000	-

Table 6.5 Tooth root bending stress parameters for spur gears based on ISO

	Software		KISS soft		
Parameter	Pinion	Gear	Pinion	Gear	Unit
Life factor	1.0000	1.0000	1.0000	1.0000	-
Relative notch factor	0.9975	0.9975	0.998	0.998	-
Relative surface factor	1.1070	1.1070	1.1070	1.1070	-
Form factor	1.3106	1.3106	1.3100	1.3100	-
Tooth normal chord at root	6.3989	6.3989	6.4000	6.4000	mm
Radius of tooth root fillet	1.4260	1.4260	1.4300	1.4300	mm
Bending lever arm	2.9637	2.9637	2.9600	2.9600	mm
Stress correction factor	2.1121	2.1121	2.1100	2.1100	-
Size factor	1.0000	1.0000	1.0000	1.0000	-
Contact ratio factor	0.6877		1.0000		-
Deep tooth factor	1.0000		1.0000		-
Helix angle factor	1.0000		1.0000		-
Rim factor	1.0000	1.0000	1.0000	1.0000	-
Nominal stress number	450.000	450.000	450.000	450.000	MPa
Nominal tooth root stress	77.0995	77.0995	77.0800	77.0800	MPa
Tooth root stress	179.4302	179.4302	179.3900	179.3900	MPa
Tooth root stress limit	993.8662	993.8662	993.8100	993.8100	MPa
Permissible tooth root stress	469.9331	469.9331	496.905	496.905	MPa
Factor of safety (tooth)	5.5390	5.5390	5.54	5.54	-

Table 6.6 Surface contact stress parameters for spur gears based on ISO

	Software		KISS soft		
Parameter	Pinion	Gear	Pinion	Gear	Unit
Life factor	1.0000	1.0000	1.0000	1.0000	-
Single pair tooth factor	1.0019	1.0019	1.0000	1.0000	-
Velocity factor	1.0131	1.0131	1.0130	1.0130	-
Lubrication factor	1.0474	1.0474	1.0470	1.0470	-
Roughness factor	1.0926	1.0926	1.0930	1.0930	-
Work hardening factor	1.0000	1.0000	1.0000	1.0000	-
Size factor	1.0000	1.0000	1.0000	1.0000	-
Contact ratio factor	1.0000		0.8730		-
Zone factor	2.4946		2.4950		-
Helix angle factor	1.0000		1.0000		-
Elasticity factor	189.8117		189.812		$\sqrt{MPa}$
Nominal stress number	1550	1550	1550	1550	MPa
Nominal contact stress	558.7703	558.7703	487.81	487.81	MPa
Tooth contact stress	868.7164	868.7164	758.48	758.48	MPa
Pitting stress limit	1797.0713	1797.0713	1797.04	1797.04	MPa
Permissible contact stress	898.5357	898.5357	898.52	898.52	MPa
Factor of safety (contact)	2.0684	2.0684	2.37	2.37	-

6.2.2 Spur Gear Design and Analysis Based on AGMA

Spur gear pair is designed based on also AGMA parallel axes gear standards after ISO for specified design requirements given in Table 6.1. Design alternatives with and without addendum correction are given in Table 6.7. The fifth alternative is selected with 10 mm face width instead of 9.425 mm.

General influence factors, tooth root bending stress and pitting stress parameters are presented in Table 6.8, 6.9 and 6.10 respectively. There are differences for these parameters due to rounding and calculation approach [11].

Table 6.7 Design alternatives for spur gears based on AGMA

No	Tp	Tg	mn	x1	x2	CD ref	Φw	bcal	nc	nb
1	17	7	7.0	0.0736	0.0736	119.00	21.273	21.991	2.213	7.037
2	20	20	6.0	0.0000	0.0000	120.00	20.000	18.850	2.059	5.220
3	24	24	5.0	0.0000	0.0000	120.00	20.000	15.708	1.880	3.629
4	30	30	4.0	0.0000	0.0000	120.00	20.000	12.566	1.683	2.325
5	40	40	3.0	0.0000	0.0000	120.00	20.000	9.4250	1.458	1.310

Table 6.8 General influence factors for spur gears based on AGMA

Parameter	Software	KISS soft	Unit
Overload factor	1.0000	1.0000	-
Dynamic factor	1.1255	1.1250	-
Face load factor (contact)	1.0693	1.0690	-
Face load factor (bending)	1.0000	1.0000	-

Table 6.9 Tooth root bending stress parameters for spur gears based on AGMA

	Software		KISS soft		
Parameter	Pinion	Gear	Pinion	Gear	Unit
Geometry factor	0.6	0.6	0.423	0.423	-
Stress cycle factor	0.6577	0.6577	0.823	0.823	-
Rim thickness factor	1.0000	1.0000	1.0000	1.0000	-
Size factor	1.0000	1.0000	1.0000	1.0000	-
Helix angle factor	1.0000		1.0000		-
Temperature factor	1.0000		1.0000		-
Reliability factor	0.8500		0.8500		-
Load sharing ratio	1.0000		1.0000		-
Allowable stress number	450.00	450.00	450.00	450.00	MPa
Bending stress number	250.6555	250.6555	250.65	250.65	MPa
Factor of safety (tooth)	1.3892	1.3892	1.87	1.87	-

Table 6.10 Surface contact stress parameters for spur gears based on AGMA

	Software		KISS soft		
Parameter	Pinion	Gear	Pinion	Gear	Unit
Geometry factor	0.0803	0.0803	0.080	0.080	-
Stress cycle factor	1.0000	1.0000	0.7600	0.7600	-
Helix angle factor	1.0000		1.0000		-
Elasticity factor	189.8117		189.8100		$\sqrt{MPa}$
Hardness ratio factor	0.9022		1.0000		-
Surface condition factor	1.0000		1.0000		-
Temperature factor	1.0000		1.0000		-
Reliability factor	0.8500		0.8500		-
Contact load factor	4.4563		4.4550		-
Gear ratio factor	0.5000		0.5000		-
Allowable stress number	1550	1550	1550	1550	MPa
Contact stress number	1095.36	1095.36	1097.29	1097.29	MPa
Factor of safety	1.5020	1.5020	1.2600	1.2600	-

6.2.3 Spur Gear Design and Analysis Based on DIN

Spur gear design alternatives based on DIN 3990 Calculation of load capacity of cylindrical gear standards are given in Table 6.11. First, fourth, fifth and eighth alternatives are presented with profile shift for same reference center distance as operating value but others are based on no addendum modification for gears. The ninth alternative is selected and analyzed for spur gear pair. 18.85 mm face width is given for this alternative but 19 mm is selected for analysis. This increases factor of safeties for both surface contact and tooth root stresses by distribution of load on greater unit area of gears.

General influence factors and parameters of pitting and root stress are presented in Table 6.12-6.14 for software and Kiss Soft. General influence factors such as application factor, transverse load factor for surface and root are same but others are little different due to rounding off.

Tooth root and surface contact stress parameters differ from each other for software and Kiss Soft due to rounding off or calculation method.

Table 6.11 Design alternatives for spur gears based on DIN

No	Tp	Tg	mn	x1	x2	CD ref	Φw	bcal	nc	nb
1	17	17	7	0.0736	0.0736	119.00	21.273	29.322	2.028	6.941
2	20	20	6	0.0000	0.0000	120.00	20.000	31.416	2.255	7.929
3	24	24	5	0.0000	0.0000	120.00	20.000	20.940	2.001	5.355
4	25	25	4.75	0.1366	0.1366	118.75	21.580	19.897	2.043	4.985
5	28	28	4.25	0.1213	0.1213	119.00	21.273	22.253	2.111	4.878
6	30	30	4	0.0000	0.0000	120.00	20.000	20.944	2.076	4.739
7	32	32	3.75	0.0000	0.0000	120.00	20.000	19.635	2.050	4.352
8	34	34	3.5	0.1473	0.1473	119.00	21.273	18.326	2.029	3.724
9	40	40	3	0.0000	0.0000	120.00	20.000	18.85	2.050	3.546

Table 6.12 General influence factors for spur gears based on DIN

Parameter	Software	KISS soft	Unit
Application factor	1.0000	1.0000	-
Dynamic factor	1.6382	1.6380	-
Face load factor (contact)	1.1602	1.1600	-
Face load factor (bending)	1.1084	1.1080	-
Transverse load factor (contact)	1.0000	1.0000	-
Transverse load factor (bending)	1.0000	1.0000	-

Table 6.13 Tooth root bending stress parameters for spur gears based on DIN

	Software		KISS soft		
Parameter	Pinion	Gear	Pinion	Gear	Unit
Life factor	1.0000	1.0000	1.0000	1.0000	-
Relative notch factor	1.0053	1.0053	1.0050	1.0050	-
Relative surface factor	1.1070	1.1070	1.1070	1.1070	-
Form factor	1.3973	1.3973	1.3100	1.3100	-
Tooth normal chord at root	6.4175	6.4175	6.4000	6.4000	mm
Radius of tooth root fillet	1.0392	1.0392	1.4300	1.4300	mm
Bending lever arm	3.1780	3.1780	2.9600	2.9600	mm
Stress correction factor	2.3634	2.3634	2.1100	2.1100	-
Size factor	1.0000	1.0000	1.0000	1.0000	-
Contact ratio factor	0.6877		1.0000		-
Helix angle factor	1.0000		1.0000		-
Nominal stress number	450.00	450.00	450.00	450.00	MPa
Nominal tooth root stress	154.9079	154.9079	129.82	129.82	MPa
Tooth root stress	281.2710	281.2710	235.72	235.72	MPa
Tooth root stress limit	1001.604	1001.604	1001.580	1001.580	MPa
Permissible tooth root stress	500.802	500.802	500.790	500.790	MPa
Factor of safety (tooth)	3.5610	3.5610	4.25	4.25	-

Table 6.14 Surface contact stress parameters for spur gears based on DIN

	Software		KISS soft		
Parameter	Pinion	Gear	Pinion	Gear	Unit
Life factor	1.0000	1.0000	1.0000	1.0000	-
Single pair tooth factor	1.0019	1.0019	1.0000	1.0000	-
Velocity factor	1.0131	1.0131	1.0130	1.0130	-
Lubrication factor	1.0474	1.0474	1.0470	1.0470	-
Roughness factor	1.0919	1.0919	1.0920	1.0920	-
Work hardening factor	1.0000	1.0000	1.0000	1.0000	-
Size factor	1.0000	1.0000	1.0000	1.0000	-
Contact ratio factor	0.873		0.873		-
Zone factor	2.4946		2.4950		-
Helix angle factor	1.0000		1.0000		-
Elasticity factor	189.8117		189.812		$\sqrt{MPa}$
Nominal stress number	1550	1550	1550	1550	MPa
Nominal contact stress	633.0723	633.0723	633.07	633.07	MPa
Tooth contact stress	874.4572	874.4572	874.45	874.45	MPa
Pitting stress limit	1795.8389	1795.8389	1795.89	1795.89	MPa
Permissible contact stress	897.9194	897.9194	897.945	897.945	MPa
Factor of safety(contact)	2.0537	2.0537	2.060	2.060	-

6.3 HELICAL GEAR DESIGN AND ANALYSIS

Same design requirements (Table 6.1) are valid for helical gear pairs which are designed for back to back test rig. ISO, AGMA and DIN helical gear design and analysis modules are used to design and then analysis of same gear pair separately. Same gear pair is analyzed by Kiss Soft.

6.3.1 Helical Gear Design and Analysis Based on ISO

Helical gear pair is used in back to back test rig as well as spur gear pair to evaluate performance of gears. Helix angle is 18.2° for this gear pair. Design alternatives based on ISO are presented in Table 6.15 with and without addendum modification. The last alternative (yellow colored) is selected and sent to analysis module with a 15.71 mm face width. But 16 mm is selected in helical gear analysis module of software. This alternative is preferred due closeness of reference center distance to required operating center distance with 4 micron.

Table 6.15 Design alternatives for helical gears based on ISO

No	Tp	Tg	mn	x1	x2	CD ref	Φw	bcal	nc	nb
1	18	18	6.5	-0.2062	-0.2062	123.162	15.323	34.03	2.033	4.658
2	19	19	6	-0.0003	-0.0003	120.004	19.995	31.42	2.046	4.145
3	21	21	5.5	-0.1296	-0.1296	121.583	17.808	28.80	2.079	4.532
4	23	23	5	-0.0969	-0.0969	121.056	18.565	26.18	2.117	4.627
5	25	25	4.5	0.1743	0.1743	118.425	21.974	18.85	2.101	4.124
6	28	28	4	0.2654	0.2654	117.898	22.596	16.75	2.069	3.980
7	29	29	4	-0.2330	-0.2330	122.109	17.019	16.75	2.001	4.531
8	32	32	3.5	0.3033	0.3033	117.898	22.596	14.66	2.012	3.922
9	33	33	3,5	-0.2037	-0.2037	121.583	17.808	18.33	2.112	5.179
10	38	38	3	-0.0006	-0.0006	120.004	19.995	15.71	2.047	4.136

Table 6.16 General influence factors for helical gears based on ISO

Parameter	Software	KISS soft	Unit
Application factor	1.0000	1.0000	-
Dynamic factor	1.3970	1.3970	-
Face load factor (contact)	1.1198	1.1350	-
Face load factor (bending)	1.0815	1.091	-
Transverse load factor (contact)	1.1346	1.1350	-
Transverse load factor (bending)	1.1346	1.1350	-

Table 6.17 Tooth root bending stress parameters for helical gears based on ISO

	Software		KISS soft		
Parameter	Pinion	Gear	Pinion	Gear	Unit
Life factor	1.0000	1.0000	1.0000	1.0000	-
Relative notch factor	0.9981	0.9981	0.998	0.998	-
Relative surface factor	1.1070	1.1070	1.1070	1.1070	-
Form factor	1.2688	1.2688	1.2700	1.2700	-
Tooth normal chord at root	6.4551	6.4551	6.4600	6.4600	mm
Radius of tooth root fillet	1.4045	1.4045	1.4000	1.4000	mm
Bending lever arm	2.9195	2.9195	2.9200	2.9200	mm
Stress correction factor	2.1529	2.1529	2.1500	2.1500	-
Size factor	1.0000	1.0000	1.0000	1.0000	-
Contact ratio factor	0.6825		1.0000		-
Deep tooth factor	1.0000		1.0000		-
Helix angle factor	0.9196		0.9200		-
Rim factor	1.0000	1.0000	1.0000	1.0000	-
Nominal stress number	450.000	450.000	450.000	450.000	MPa
Nominal tooth root stress	139.9241	139.9241	139.900	139.900	MPa
Tooth root stress	239.877	239.877	242.000	242.000	MPa
Tooth root stress limit	994.397	994.397	994.410	994.410	MPa
Permissible tooth root stress	497.1985	497.1985	497.205	497.205	MPa
Factor of safety (tooth)	4.1454	4.1454	4.1100	4.1100	-

6.3.2 Helical Gear Design and Analysis Based on AGMA

Helical gears used in back to back test rig are designed based on AGMA standards alternatively. These all can be used for required design of test rig. The difference between them is presence of profile modification. The last alternative is selected to provide same values of normal module and tooth numbers used for ISO standard. Face width is used 10 mm instead of 9.43 mm.

Table 6.18 Surface contact stress parameters for helical gears based on ISO

	Software		KISS soft		
Parameter	Pinion	Gear	Pinion	Gear	Unit
Life factor	1.0000	1.0000	1.0000	1.0000	-
Single pair tooth factor	1.0040	1.0040	1.0000	1.0000	-
Velocity factor	1.0131	1.0131	1.0130	1.0130	-
Lubrication factor	1.0474	1.0474	1.0470	1.0470	-
Roughness factor	1.0939	1.0939	1.0940	1.0940	-
Work hardening factor	1.0000	1.0000	1.0000	1.0000	-
Size factor	1.0000	1.0000	1.0000	1.0000	-
Contact ratio factor	0.8443		0.844		-
Zone factor	2.3925		2.3920		-
Helix angle factor	1.0260		1.0260		-
Elasticity factor	189.8117		189.812		$\sqrt{MPa}$
Nominal stress number	1550	1550	1550	1550	MPa
Nominal contact stress	656.4614	656.4614	656.4600	656.4600	MPa
Tooth contact stress	878.1532	878.1532	882.01	882.01	MPa
Pitting stress limit	1799.2235	1799.2235	1799.18	1799.18	MPa
Permissible contact stress	899.6118	899.6118	899.59	899.59	MPa
Factor of safety (contact)	2.0489	2.0489	2.0400	2.0400	-

Table 6.19 Design alternatives for helical gears based on AGMA

No	Tp	Tg	mn	x1	x2	CD ref	Φw	bcal	nc	nb
1	19	19	6	-0.0003	-0.0003	120.004	19.995	18.85	2.327	7.721
2	23	23	5	-0.0969	-0.0969	121.056	18.565	15.71	2.135	5.415
3	28	28	4	0.2654	0.2654	117.898	22.596	12.57	1.884	3.379
4	29	29	4	-0.2330	-0.2330	122.109	17.019	12.57	1.919	3.500
5	38	38	3	-0.0006	-0.0006	120.004	19.995	9.43	1.648	1.937

Table 6.20 General influence factors for helical gears based on AGMA

Parameter	Software	KISS soft	Unit
Overload factor	1.0000	1.0000	-
Dynamic factor	1.1255	1.1250	-
Face load factor (contact)	1.0670	1.0670	-
Face load factor (bending)	1.0000	1.0000	-

Table 6.21 Tooth bending stress parameters for helical gears based on AGMA

	Software		KISS soft		
Parameter	Pinion	Gear	Pinion	Gear	Unit
Geometry factor	0.6	0.6	0.484	0.484	-
Stress cycle factor	0.6577	0.6577	0.823	0.823	-
Rim thickness factor	1.0000	1.0000	1.0000	1.0000	-
Size factor	1.0000	1.0000	1.0000	1.0000	-
Helix angle factor	1.0000		1.0000		-
Temperature factor	1.0000		1.0000		-
Reliability factor	0.8500		0.8500		-
Load sharing ratio	1.0000		1.0000		-
Allowable stress number	450.00	450.00	450.00	450.00	MPa
Bending stress number	169.454	169.454	210.19	210.19	MPa
Factor of safety (tooth)	2.0548	2.0548	2.0700	2.0700	-

Table 6.22 Surface contact stress parameters for helical gears based on AGMA

	Software		KISS soft		
Parameter	Pinion	Gear	Pinion	Gear	Unit
Geometry factor	0.0835	0.0835	0.101	0.101	-
Stress cycle factor	1.0000	1.0000	0.7600	0.7600	-
Helix angle factor	1.0000		1.106		-
Elasticity factor	189.8117		189.8100		$\sqrt{MPa}$
Hardness ratio factor	1.0000		1.0000		-
Surface condition factor	1.0000		1.0000		-
Temperature factor	1.0000		1.0000		-
Reliability factor	0.8500		0.8500		-
Contact load factor	4.4562		4.4550		-
Gear ratio factor	0.5000		0.5000		-
Allowable stress number	1550	1550	1550	1550	MPa
Contact stress number	1074.3223	1074.3223	975.55	975.55	MPa
Factor of safety	1.6973	1.6973	1.4200	1.4200	-

6.3.3 Helical Gear Design and Analysis Based on DIN

The last design alternative is selected based on DIN (Table 6.23) to provide integrity with other helical gear standards in terms of normal module and tooth numbers. Face width value is preferred as 26 mm. general influence factors and gear performance parameters for tooth root and surface contact are given in Tables 6.24-6.26 for software and Kiss Soft. Values show difference because of different calculation methods and rounding of last digit of parameters. This cause to difference occurs in factor of safeties for tooth root and surface contact stresses calculated for both gear software.

Table 6.23 Design alternatives for helical gears based on DIN

No	Tp	Tg	mn	x1	x2	CD ref	Φw	bcal	nc	nb
1	18	18	6.5	-0.2062	-0.2062	123.162	15.323	34.03	2.052	14.245
2	19	19	6	-0.0003	-0.0003	120.004	19.995	31.42	2.074	11.060
3	21	21	5.5	-0.1296	-0.1296	121.583	17.808	34.56	2.267	14.047
4	22	22	5.25	-0.1358	-0.1358	121.583	17.808	32.99	2.216	12.980
5	23	23	5	-0.0969	-0.0969	121.056	18.565	31.41	2.178	11.645
6	27	27	4.25	-0.0864	-0.0864	120.793	18.932	26.70	2.012	8.574
7	29	29	4	-0.2330	-0.2330	122.109	17.019	29.32	2.084	9.979
8	31	31	3.75	-0.2768	-0.2768	122.372	16.611	27.49	2.001	9.019
9	33	33	3,5	-0.2037	-0.2037	121.583	17.808	29.32	2.063	8.488
10	35	35	3.25	0.0383	0.0383	119.740	20.338	23.82	2.011	6.434
11	38	38	3	-0.0006	-0.0006	120.004	19.995	25.13	2.034	6.363

Table 6.24 General influence factors for helical gears based on DIN

Parameter	Software	KISS soft	Unit
Application factor	1.0000	1.0000	-
Dynamic factor	1.5181	1.5180	-
Face load factor (contact)	1.2186	1.219	-
Face load factor (bending)	1.1606	1.161	-
Transverse load factor (contact)	1.2251	1.225	-
Transverse load factor (bending)	1.2251	1.225	-

Table 6.25 Tooth root bending stress parameters for helical gears based on DIN

Parameter	Software		KISS soft		Unit
	Pinion	Gear	Pinion	Gear	
Life factor	1.0000	1.0000	1.0000	1.0000	-
Relative notch factor	1.0063	1.0063	0.9930	0.9930	-
Relative surface factor	1.1070	1.1070	1.1070	1.1070	-
Form factor	1.3518	1.3518	1.2700	1.2700	-
Tooth normal chord at root	6.4851	6.4851	6.4600	6.4600	mm
Radius of tooth root fillet	1.0100	1.0100	1.4000	1.4000	mm
Bending lever arm	3.1395	3.1395	2.9200	2.9200	mm
Stress correction factor	2.4261	2.4261	2.1500	2.1500	-
Size factor	1.0000	1.0000	1.0000	1.0000	-
Contact ratio factor	0.6825		1.0000		-
Helix angle factor	0.8693		0.8690		-
Nominal stress number	450.00	450.00	450.00	450.00	MPa
Nominal tooth root stress	97.730	97.730	81.380	81.380	MPa
Tooth root stress	210.9676	210.9676	175.68	175.68	MPa
Tooth root stress limit	1002.651	1002.651	989.53	989.53	MPa
Permissible tooth root stress	501.325	501.325	494.765	494.765	MPa
Factor of safety (tooth)	4.7526	4.7526	5.6300	5 .6300	-

Table 6.26 Surface contact stress parameters for helical gears based on DIN

Parameter	Software		KISS soft		Unit
	Pinion	Gear	Pinion	Gear	
Life factor	1.0000	1.0000	1.0000	1.0000	-
Single pair tooth factor	1.0040	1.0040	1.0000	1.0000	-
Velocity factor	1.0131	1.0131	1.0130	1.0130	-
Lubrication factor	1.0474	1.0474	1.0470	1.0470	-
Roughness factor	1.0919	1.0919	1.0920	1.0920	-
Work hardening factor	1.0000	1.0000	1.0000	1.0000	-
Size factor	1.0000	1.0000	1.0000	1.0000	-
Contact ratio factor	1.0000		0.8090		-
Zone factor	2.3925		2.3920		-
Helix angle factor	0.9747		0.975		-
Elasticity factor	189.8117		189.812		$\sqrt{MPa}$
Nominal stress number	1550	1550	1550	1550	MPa
Nominal contact stress	579.450	579.450	469.00	469.00	MPa
Tooth contact stress	875.872	875.872	706.44	706.44	MPa
Pitting stress limit	1795.841	1795.841	1795.89	1795.89	MPa
Permissible contact stress	897.921	897.921	897.945	897.45	MPa
Factor of safety(contact)	2.0503	2.0503	2.5400	2.5400	-

6.4 SPUR GEAR TRANSMISSION ERROR ANALYSIS

Spur gear transmission error is analyzed; transmission error curves, mesh stiffness curve, tooth load curve are drawn and peak to peak transmission error (TE pp) values are compared with studies in literature. FFT analysis of selected tooth load is done and curves are drawn.

Peak to peak transmission error value is given as 4.8 µm in reference study [50] and software estimates it as 4.82 µm for 100 % of design load based on required parameters given in Table 6.27. Loads are selected as percentage of 25%, 50%, 75%, and % 100 of design load in reference [50]. Related curves with them are represented by different colors selection as shown in Figure 6.1 to prevent confusion. Transmission error curves are given in Figure 6.2 for each design load. Tooth stiffness curve is presented in Figure 6.3. Peak to peak transmission error values are tabulated for different percentages of design load in Table 6.28. Tooth load curves for design loads are illustrated in Figures 6.4 and FFT analysis of 100 % of design load is given in Figure 6.5.

Table 6.27 Reference [50] spur gear transmission error analysis parameters

Parameter	Value	Unit
Normal pressure angle	20	Degree
Normal module	4.19	mm
Pinion tooth number	34	-
Gear tooth number	35	-
Design load	11700	Newton
Face width	28.45	mm
Amount of relief	26	µm
Extent of relief	10	mm

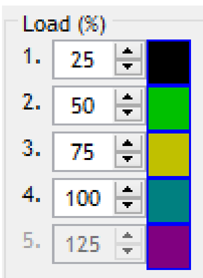


Figure 6.1 Colors corresponding to load from software

Figure 6.2 Transmission error curve from software for reference [50]

Figure 6.3 Tooth stiffness curve from software for reference [50]

Table 6.28 Peak to Peak TE values from software for reference [50]

Output

#	Load (%)	Load (N)	TEpp
1	25	2925,0241	12,4622
2	50	5850,0481	9,9159
3	75	8775,0722	7,3697
4	100	11700,0962	4,8235

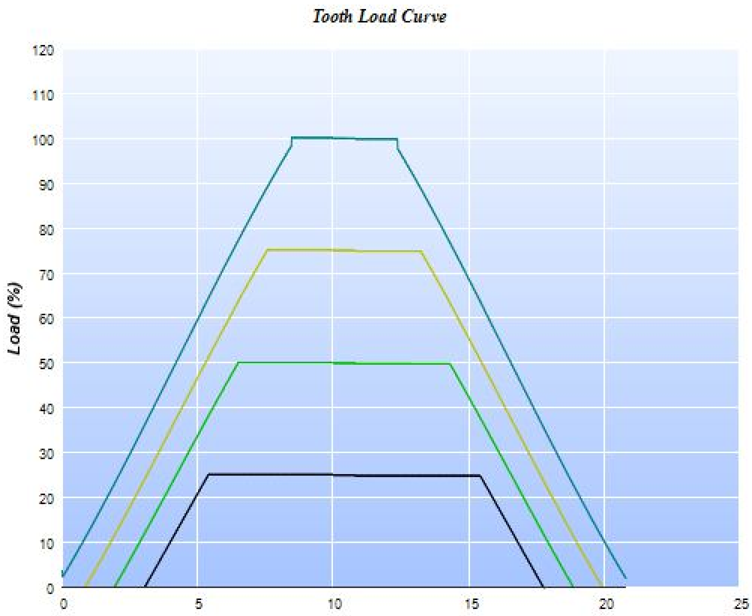


Figure 6.4 Tooth load curve from software for reference [50]

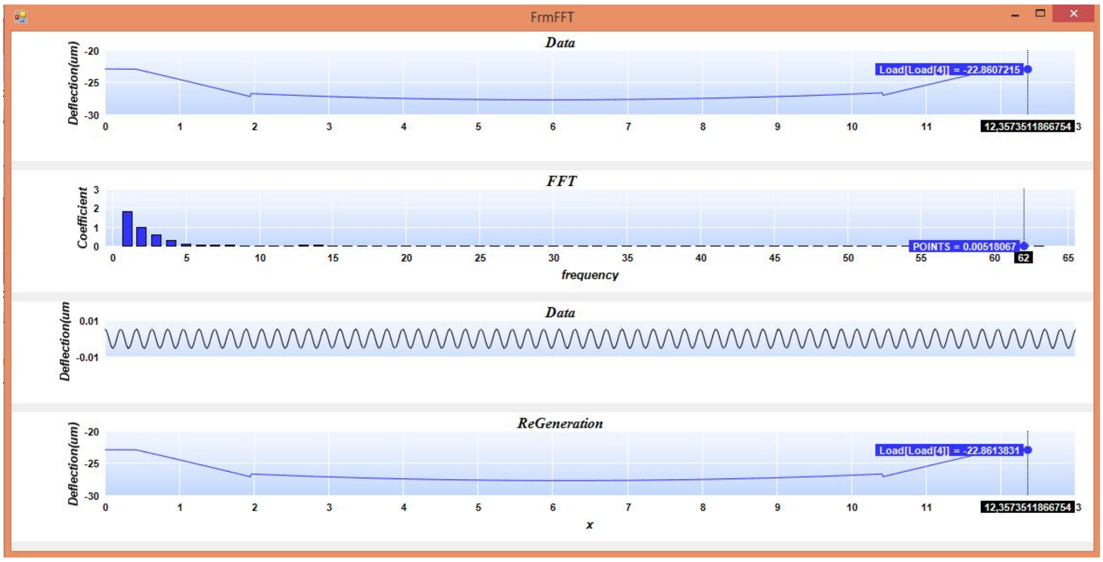


Figure 6.5 FFT analysis of %100 design load from software for reference [50]

For reference study [36] transmission error parameters are presented in Table 6.29. Peak to peak transmission error value is given as 1.0 µm by study while software calculates 1.2268 µm. transmission error curves for 25 %, 50 %, 75 % and 100 % of design load, tooth stiffness curve and tooth load curves are presented in Figures 6.6-

6.8 respectively. FFT analysis curves are given in Figure 6.9 for 100 % of design load. Peak to peak transmission errors for design loads are presented in Table 6.30.

Table 6.29 Reference [36] spur gear transmission error analysis parameters

Parameter	Value	Unit
Normal pressure angle	20	Degree
Normal module	6.35	mm
Pinion tooth number	32	-
Gear tooth number	32	-
Design load	300	N/mm
Face width	20.0	mm
Relief type	Long	-

Figure 6.6 Transmission error curve from software for reference [36]

Figure 6.7 Tooth stiffness curve from software for reference [36]

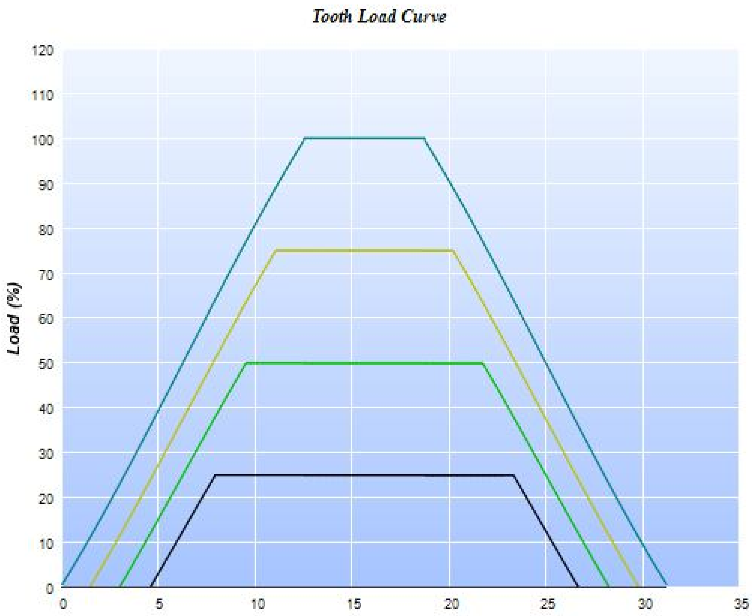


Figure 6.8 Tooth load curve from software for reference [36]

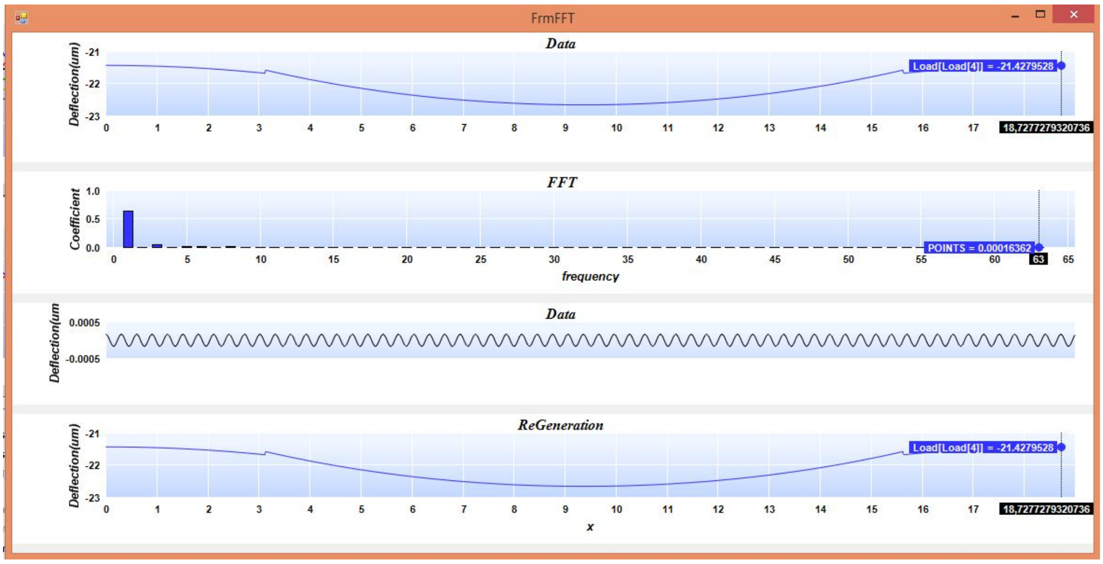


Figure 6.9 FFT analysis of %100 design load from software for reference [36]

Table 6.30 Peak to Peak TE values from software for reference [36]

Output

#	Load (%)	Load (N)	TEpp
1	25	1499,9561	8,3422
2	50	2999,9122	5,9704
3	75	4499,8683	3,5986
4	100	5999,8244	1,2268

6.5 HELICAL GEAR TRANSMISSION ERROR ANALYSIS

Transmission error of helical gear pair is analyzed by using software and LDP [51] (load distribution program). Required parameters are presented in Table 6.31 for gear geometry and load requirements. For same gear pair, different profile modifications are applied and results are presented in the scope of this study. Peak to peak transmission error (TEpp) values are estimated for five different design loads (and design torques) for same gear pair and presented in Table 6.32 for software and LDP respectively.

Loaded TE curves for each design load and mesh stiffness curves from LDP are presented while loaded TE curves for each design load and tooth stiffness curves from software are presented in Figures 6.10 - 6.29. Stiffness values are given for LDP and software in different units; N/m and N/mm/µm respectively. Mesh stiffness curve is given for LDP while software presents tooth stiffness curve.

Table 6.31 Helical gear transmission error analysis required parameters

Parameter	Value	Unit
Normal pressure angle	20.0	Degree
Helix angle	17.2	Degree
Normal module	4.0	mm
Pinion tooth number	65	-
Gear tooth number	65	-
Face width	25.4	mm

Profile Modifications	1st	2nd	3rd	4th	5th	Unit
Design Load	5080	7620	10160	12700	15240	N
First roll angle of start of EAP	23.19	23.19	22.90	22.90	22.90	Deg
First linear relief EAP modification magnitude	18	27	36	45	54	µm

Table 6.32 TE pp values for helical gear pair for software and LDP

Design Loads (Newton)	Design Torques (Nm)	TE pp (software) µm	TE pp (LDP) µm
5080	646	0.2743	0.2150
7620	969	0.4162	0.4016
10160	1292	0.5540	0.5933
12700	1615	0.6914	0.7600
15240	1938	0.9469	0.9400

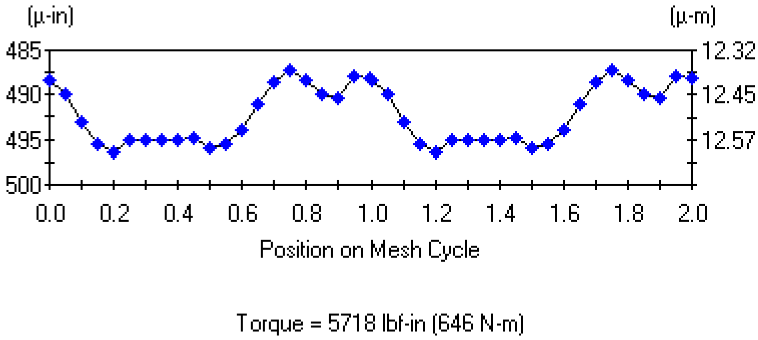


Figure 6.10 TE curve for 5080 N design load from LDP

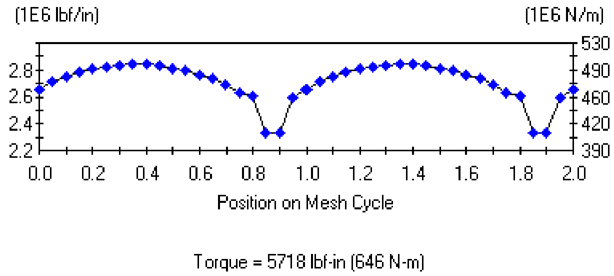


Figure 6.11 Mesh stiffness curve for 5080 N design load from LDP

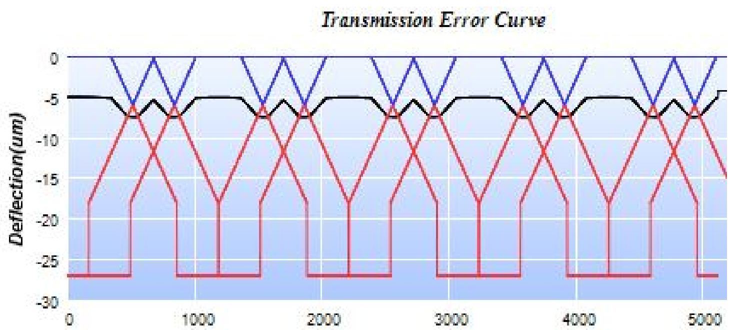


Figure 6.12 TE curve for 5080 N design load from Software

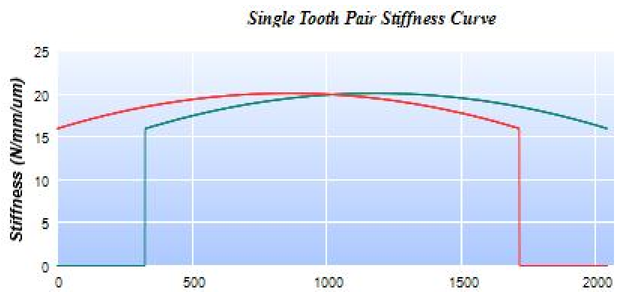


Figure 6.13 Tooth stiffness curve for 5080 N design load from Software

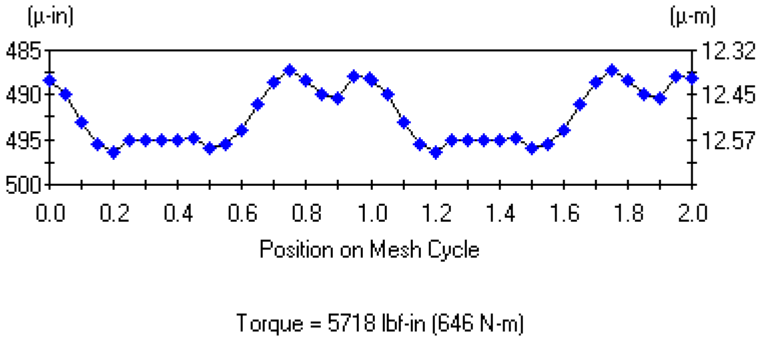


Figure 6.14 TE curve for 7620 N design load from LDP

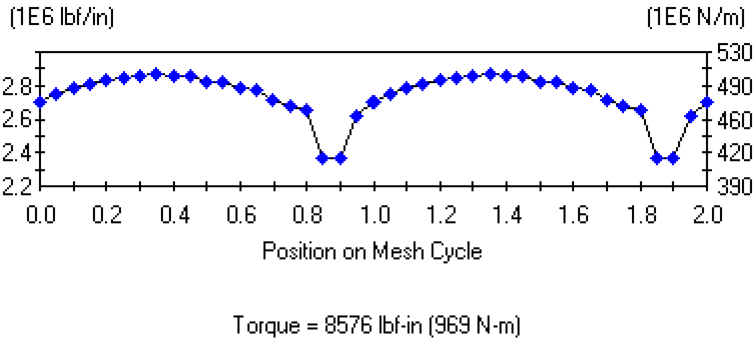


Figure 6.15 Mesh stiffness curve for 7620 N design load from LDP

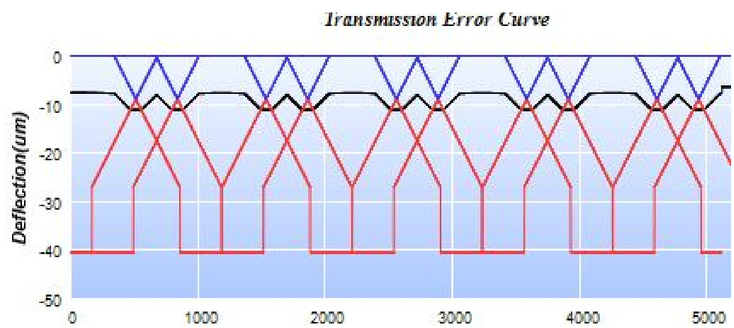


Figure 6.16 TE curve for 7620 N design load from Software

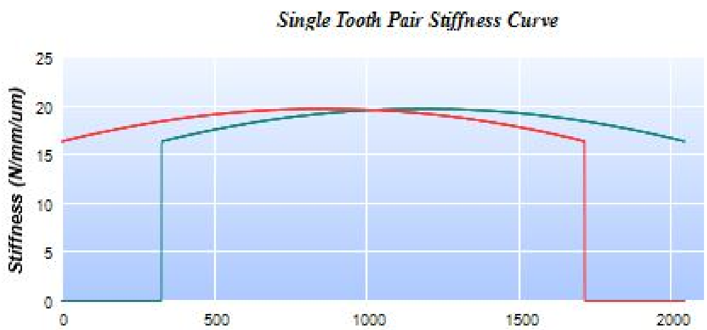


Figure 6.17 Tooth stiffness curve for 7620 N design load from Software

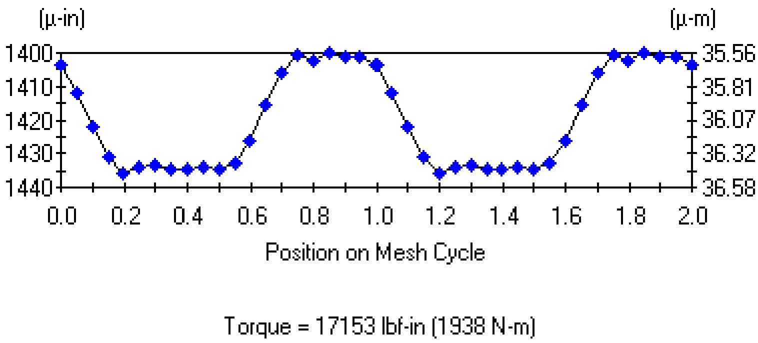


Figure 6.18 TE curve for 10160 N design load from LDP

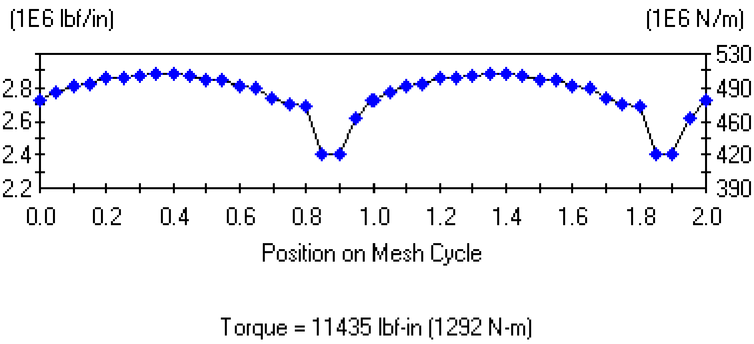


Figure 6.19 Mesh stiffness curve for 10160 N design load from LDP

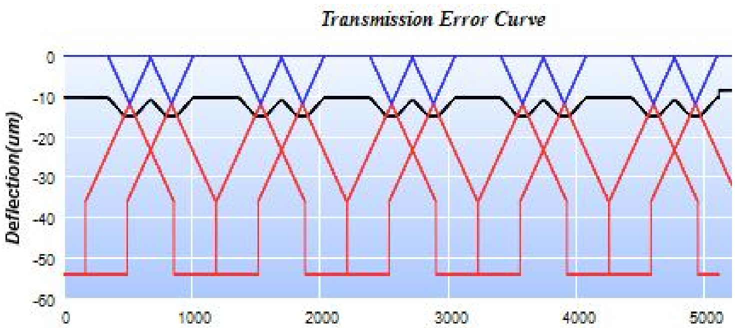


Figure 6.20 TE curve for 10160 N design load from software

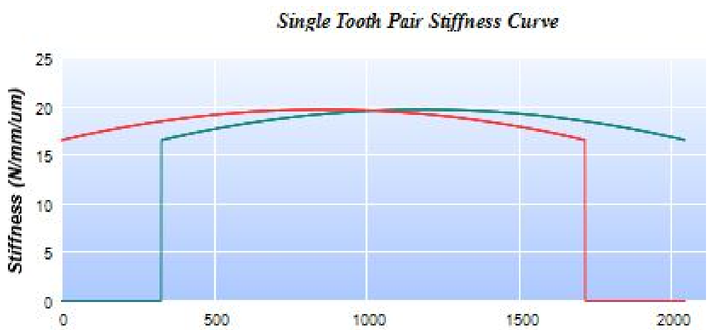


Figure 6.21 Tooth stiffness curve for 10160 N design load from software

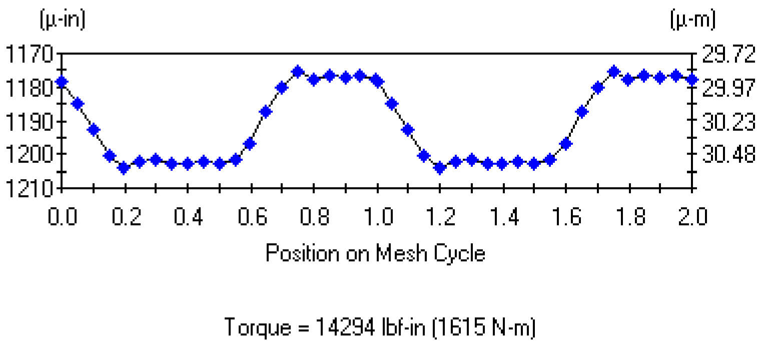


Figure 6.22 TE curve for 12700 N design load from LDP

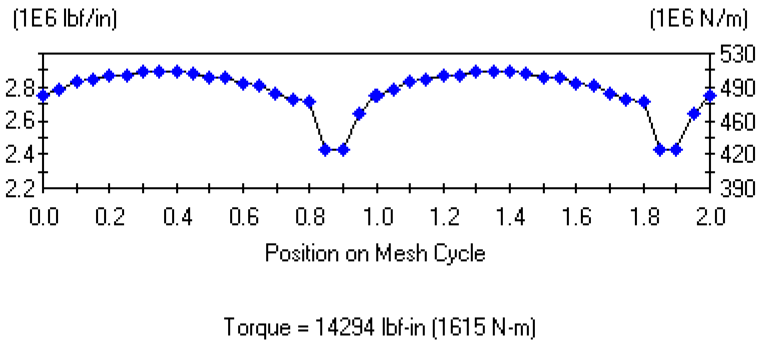


Figure 6.23Mesh stiffness curve for 12700 N design load from LDP

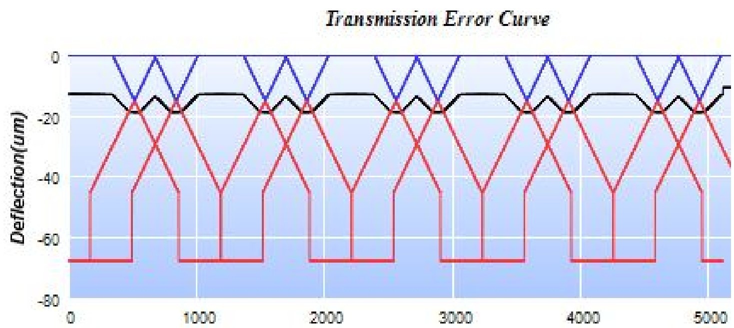


Figure 6.24 TE curve for 12700 N design load from software

Figure 6.25 Tooth stiffness curve for 12700 N design load from software

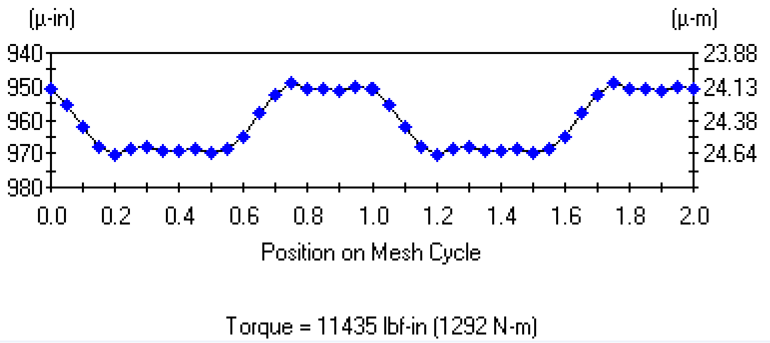


Figure 6.26 TE curve for 15240 N design load from LDP

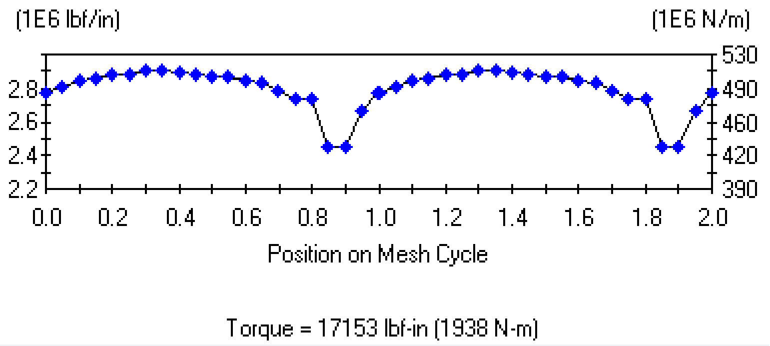


Figure 6.27 Mesh stiffness curve for 15240 N design load from LDP

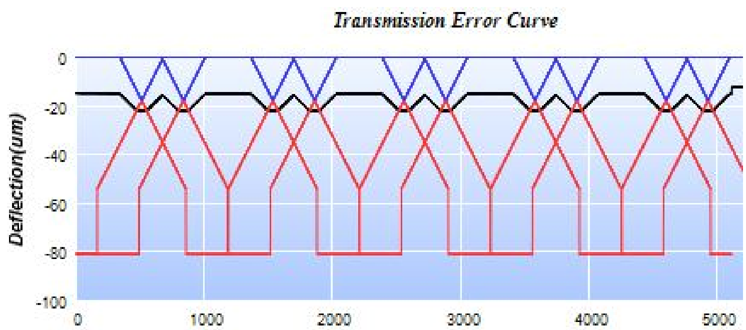


Figure 6.28 TE curve for 15240 N design load from Software

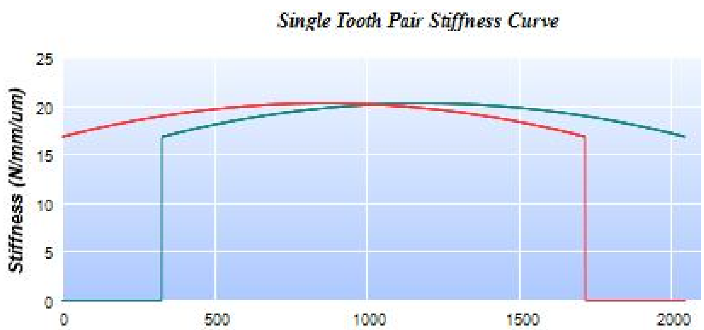


Figure 6.29 Tooth stiffness curve for 15240 N design load from Software

For same helical gear pair (Table 6.30), transmission error is analyzed by using Romax [52] and software. Peak to peak transmission error values are presented in Table 6.33 for five different design loads.

Loaded TE curves for each design load and mesh stiffness curves from Romax are presented while loaded TE curves for each design load and tooth stiffness curves from software are presented in Figures 6.30 - 6.49. Stiffness values are given for. Mesh stiffness curve is given for Romax while software presents tooth stiffness curve.

Table 6.33 TE pp values for helical gear pair for software and Romax

Design Loads (Newton)	Design Torques (Nm)	TE pp (software) µm	TE pp (Romax) µm
5080	646	1.2500	1.2300
7620	969	2.0000	2.1259
10160	1292	2.6000	2.8345
12700	1615	3.2500	3.5432
15240	1938	3.6000	3.6969

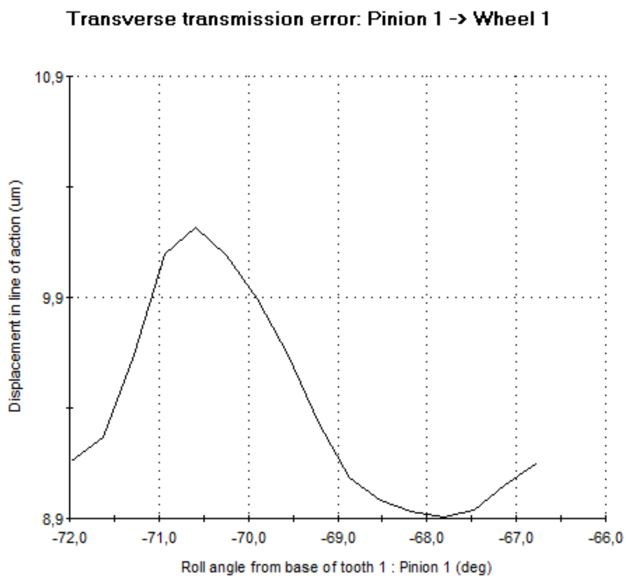


Figure 6.30 TE curve for 5080 N design load from Romax

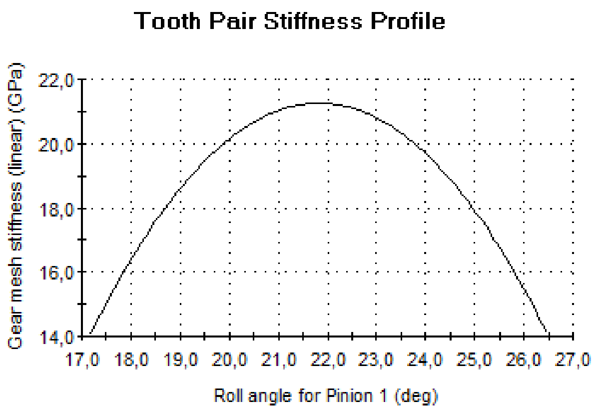


Figure 6.31 Mesh stiffness curve for 5080 N design load from Romax

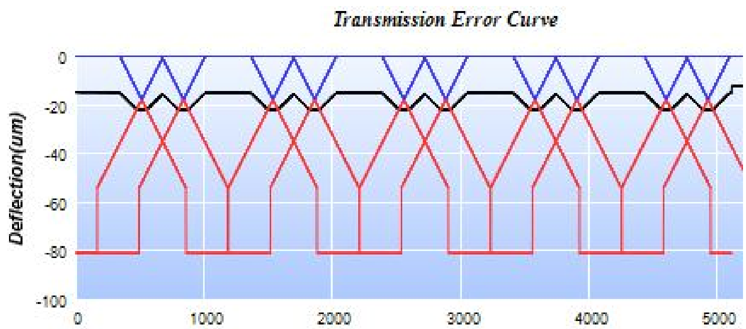


Figure 6.32 TE curve for 5080 N design load from software

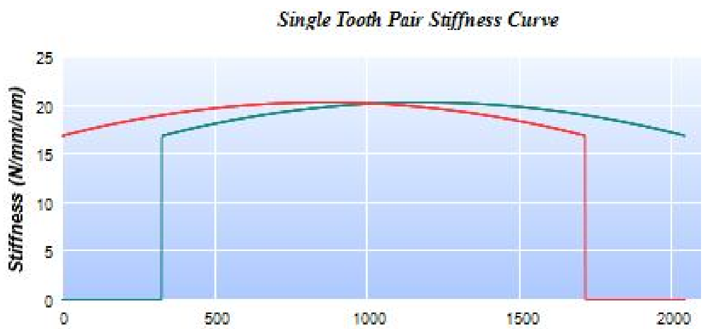


Figure 6.33 Tooth stiffness curve for 5080 N design load from software

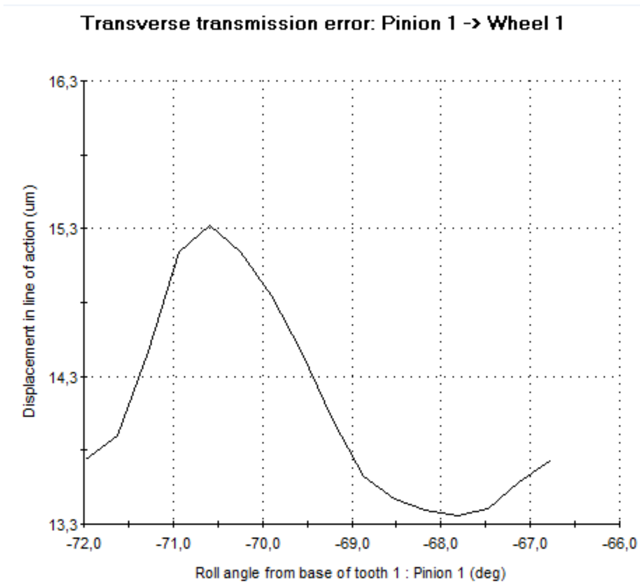


Figure 6.34 TE curve for 7620 N design load from Romax

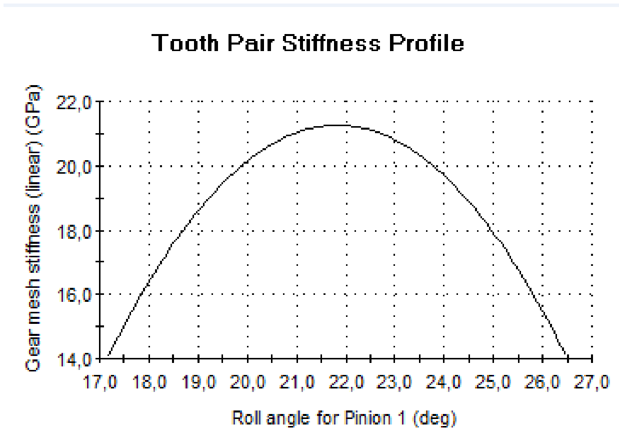


Figure 6.35 Mesh stiffness curve for 7620 N design load from Romax

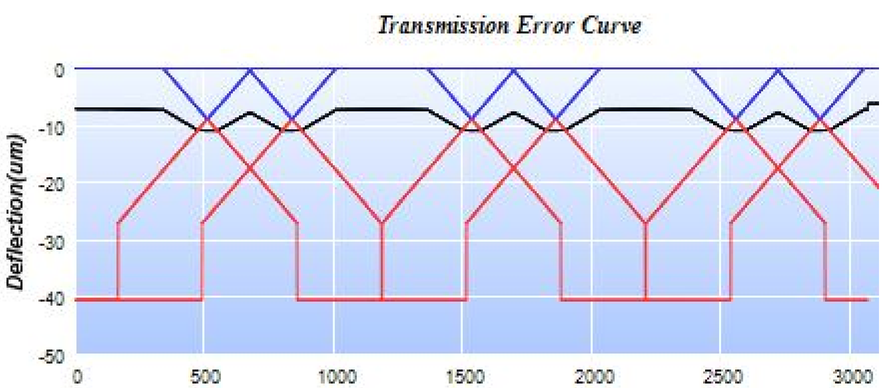


Figure 6.36 TE curve for 7620 N design load from Software

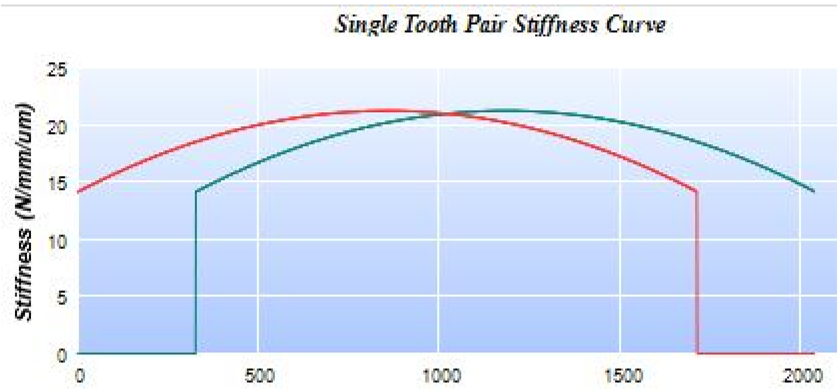


Figure 6.37 Tooth stiffness curve for 7620 N design load from Software

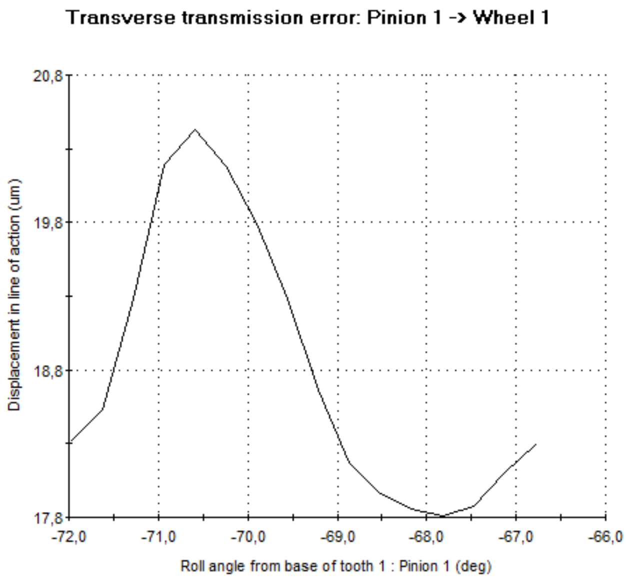


Figure 6.38 TE curve for 10160 N design load from Romax

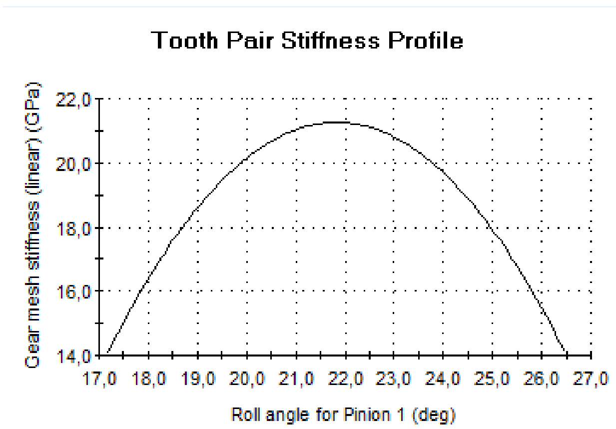


Figure 6.39 Mesh stiffness curve for 10160 N design load from Romax

Figure 6.40 TE curve for 10160 N design load from software

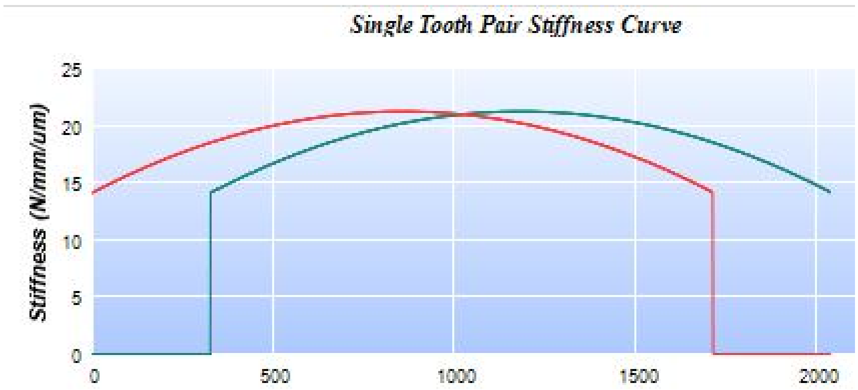


Figure 6.41 Tooth stiffness curve for 10160 N design load from software

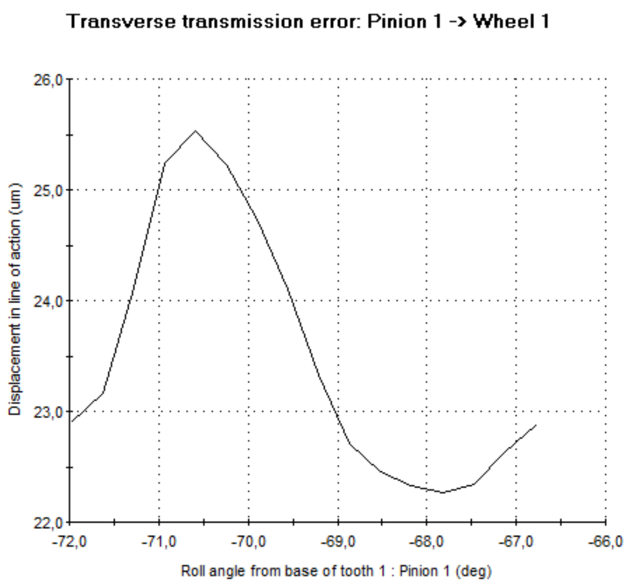


Figure 6.42 TE curve for 12700 N design load from Romax

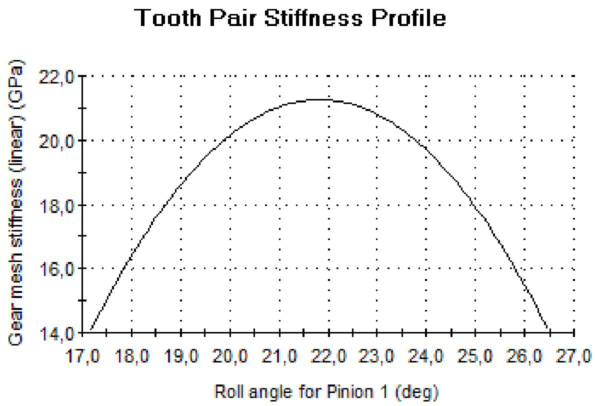


Figure 6.43 Mesh stiffness curve for 12700 N design load from Romax

Figure 6.44 TE curve for 12700 N design load from software

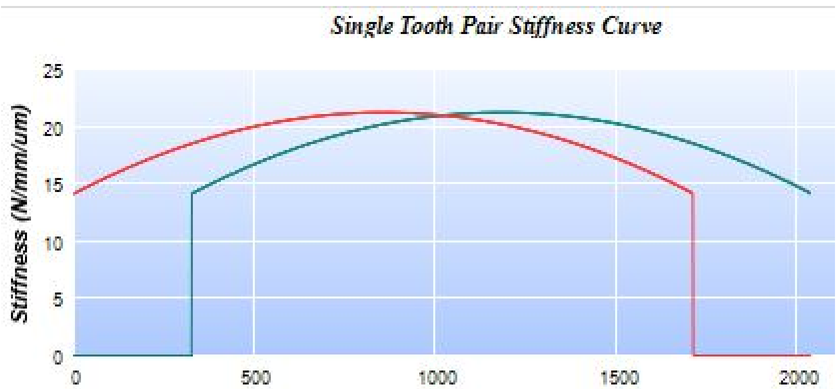


Figure 6.45 Tooth stiffness curve for 12700 N design load from software

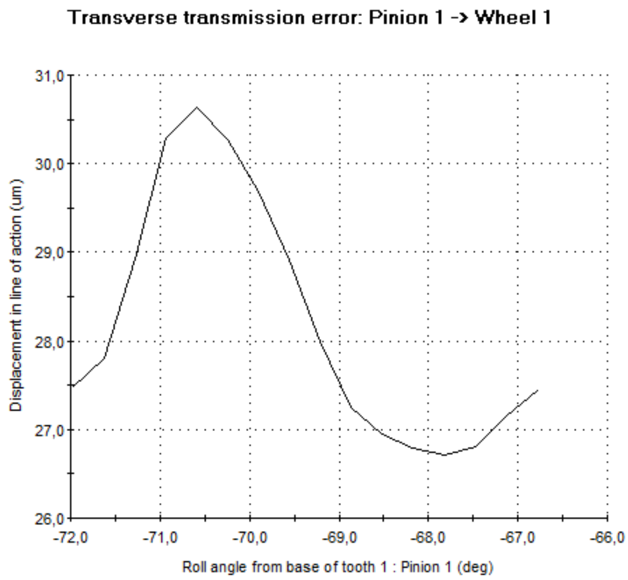


Figure 6.46 TE curve for 15240 N design load from Romax

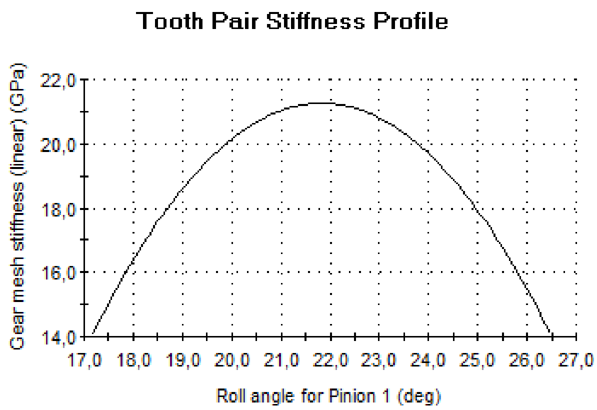


Figure 6.47 Mesh stiffness curve for 15240 N design load from Romax

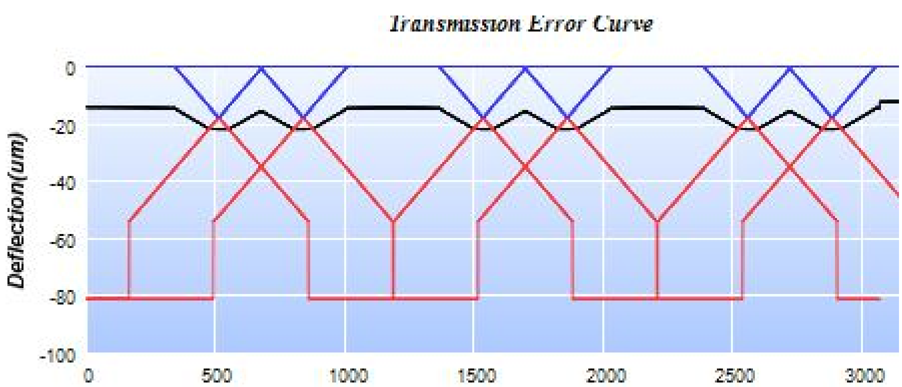


Figure 6.48 TE curve for 15240 N design load from software

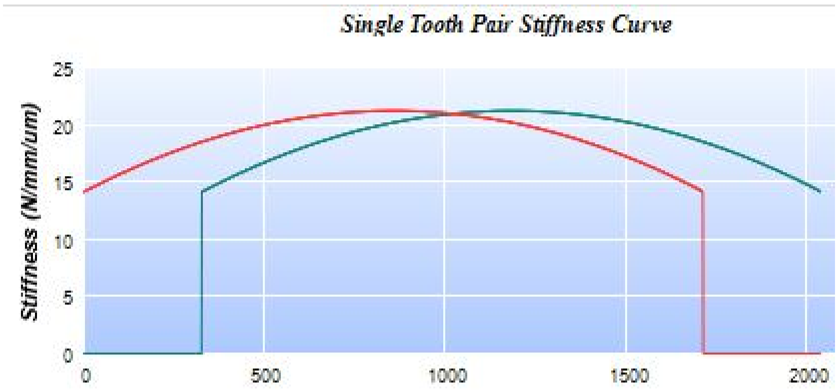


Figure 6.49 Tooth stiffness curve for 15240 N design load from software

6.6 CONCLUSION AND DISCUSSION

For spur gears, 3 mm of normal module, 40 of teeth number and 20 degree of normal pressure angle are selected to compare gear design and analysis for each standard type. Face width values are given 32, 10, and 19 mm for ISO, AGMA and DIN respectively. Minimum required factor of safeties are selected as 2.0 for ISO and DIN modules but it is 1.25 for AGMA module to provide working with same module, normal pressure angle and tooth numbers. Factor of safeties for surface contact and tooth root bending stresses in Table 6.34. If face width is selected 32 mm for AGMA and DIN modules by remaining all other parameters constant, factor of safeties are presented in Table 6.35.

For helical gears, 3 mm of normal module, 38 of teeth number, 20 degree of normal pressure angle and 18.2 of helix angle degree are selected to compare gear design and analysis for each standard type. Face width values are given 16, 10, and 26 mm for ISO, AGMA and DIN respectively. Minimum required factor of safeties are selected as 2.0 for ISO and DIN modules but it is 1.40 for AGMA module to provide working with same module, normal pressure angle and tooth numbers. Factor of safeties for surface contact and tooth root bending stresses in Table 6.36. For face width of 26 mm for ISO and AGMA modules by remaining all other parameters constant, factor of safeties are presented in Table 6.37.

Table 6.34 Factor of safeties for spur gear pair based on ISO, AGMA and DIN (For software and Kiss Soft)

Standard type		ISO	AGMA	DIN
Factor of safety (contact)	Software	2.0684	1.502	2..0537
	Kiss Soft	2.3700	1.260	2.0600
Factor of safety (bending)	Software	5.5390	1.3892	3.5610
	Kiss Soft	5.5400	1.8700	4.2500

Table 6.35 Factor of safeties for spur gear pair based on ISO, AGMA and DIN (32 mm face width)

Standard type	ISO	AGMA	DIN
Factor of safety (contact)	2.0684	2.6719	2.4323
Factor of safety (bending)	5.5390	6.1770	4.9760

Table 6.36 Factor of safeties for helical gear pair based on ISO, AGMA and DIN (For software and Kiss Soft)

Standard type		ISO	AGMA	DIN
Factor of safety (contact)	Software	2.0684	1.502	2..0537
	Kiss Soft	2.3700	1.260	2.0600
Factor of safety (bending)	Software	5.5390	1.3892	3.5610
	Kiss Soft	5.5400	1.8700	4.2500

Table 6.37 Factor of safeties for helical gear pair based on ISO, AGMA and DIN (26 mm face width)

Standard type	ISO	AGMA	DIN
Factor of safety (contact)	2.4352	2.4617	2.0537
Factor of safety (bending)	5.7345	5.3095	3.5610

Transmission error analyses for spur gears are done by using software and results are compared with reference studies [36, 50].

For helical gear pair, peak to peak transmission error values which are calculated by using software and LDP are close to each other but there are differences for each value refers to different design loads.

LDP and Romax calculate mesh stiffness values according to gear parameters. But software uses stiffness value as tooth stiffness value and they should be user defined. So mesh stiffness values which are calculated by LDP and Romax are used in software for tooth stiffness. There are also differences in peak to peak TE for software and Romax. This can be due to stiffness values.

CHAPTER 7

DISCUSSION AND CONCLUSION

7.1 DISCUSSION AND CONCLUSION

Gear design and analysis are complex tasks due to a large number of geometrical and performance parameters including strength, durability, kinematics, and dynamics and so on. All these interrelated parameters have relatively huge effects on each other and calculations of these parameters and evaluation of gear performance are therefore time consuming processes and are prone to errors. Gear engineers therefore need to have gear design/analysis software to perform gear design and analysis in much shorter time with minimum errors. Most of the time quasi-static analysis and design of gears need to be supported by another important phenomenon –called transmission error- to have some insight into gear pair dynamic behavior and noise level. Home-made software which can perform all these tasks is going to be a very helpful tool for gear designers. In the scope of this discussion, a user friendly home-made software has been developed for design and analysis of parallel axes gear. In addition, gear pair transmission error curve construction was also included in the software for further evaluation of dynamic performance of spur and helical gears.

International gear standards (ISO, AGMA and DIN) were the basis of design and analysis of parallel axes gears during development of the software. All information and calculation procedures regarding strength and quasi-static performance of gears were based on what is provided in those standards. Transmission error curve construction process, however, was based on the theory called Harris map.

Home-made software has been prepared / developed to be able to both design and analyze the gear pairs of parallel axes spur and helical types. The Software has modules of design and analyses for two types of gears namely the spur and helical. Based on which module is selected user is further asked to select which gear standards (namely the ISO, DIN and AGMA) to use for engineering calculations.

While analysis module is used to check validity of an already designed gear pair for a specific application, design module is used to select an alternative design option among the choices that software suggests based on the geometry, material and loading conditions defined by user as input parameters.

Most sub-modules like material selection module are based on what was provided by gear standards as gear materials. However, new gear materials with related mechanical properties to be used in engineering calculations can be added to the material list in material selection sub module.

Non-standard gears with profile shifts can also be designed and analyzed for non-standard center distances with an ability to observe sliding speeds and likely scuffing risk at start and end of engagement.

Not only the mechanical design and analysis of the gears, based on tooth root and surface contact stress, but also design and analysis of the involute profile modifications, based on transmission error, of gears can also be performed by using the software developed. Some case studies given in chapter 6 have shown good similarities between present software and other commercial softwares.

7.2 RECOMMENDATIONS FOR FUTURE WORK

The following studies may be considered as a future work.

i) Combining gear manufacturing knowledge with software engineering knowledge to develop a user friendly software interface for parallel axes gears. We have studied on parallel axes gears and C++ Microsoft Visual Studio. This software can be improved in terms of gear manufacturing data and software usefulness

ii) Gear tooth stiffness calculation based on different methods in literature can be added to software for transmission error modules.

iii) Addendum modification of gears can be studied in detail for avoiding undercut, balanced specific sliding, balanced flash temperature and balanced bending fatigue life to improve software related section.

iv) Gear model can be created in 2D and 3D by combining software with CAD CAM software.

v) High contact ratio spur and helical gears could also be included in the software.

vi) Other gear types (bevel, worm, etc) based on international gear standards for gear design; analysis (and possibly transmission error analysis) can be a future work to improve this software.

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