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A CAPACITATED FACILITY LOCATION-ALLOCATION PROBLEM WITH LOCATION RISKS: A CASE STUDY FOR MILITARY WEAPON ALLOCATION

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BY

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A Capacitated Facility Location-Allocation Problem with Location Risks: A Case Study for Military Weapon Allocation

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Supervisor

Asst. Prof. Dr. Cihan ÇETİNKAYA

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Samer HAFFAR

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Samer HAFFAR

ABSTRACT

A CAPACITATED FACILITY LOCATION-ALLOCATION PROBLEM WITH LOCATION RISKS: A CASE STUDY FOR MILITARY WEAPON ALLOCATION

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This work introduces a mathematical model for solving the multi-product capacitated facility location allocation problem with location risks. The model allocates quantities of various product types to a set of candidate locations while minimizing total transportation and setup costs as well as the risk associated with candidate locations. The mentioned risk emerges from allocating the weapons for other military formations, thus becoming more vulnerable to attacks. A border city weapon allocation problem of Turkish Land Forces (containing 81 nodes) is solved to test the model. It is determined that the mathematical model solves the problem with minimum cost/risk so it can be used for the critical decision making processes during homeland defense issues.

Keywords: Homeland defense, location-allocation problem, weapon allocation, location risks.

ÖZET

KONUM RİSKLERİNE DAYALI KAPASİTELİ TESİS YERİ-TAHSİS PROBLEMİ: ASKERİ SİLAH SEVKİYATI İÇİN BİR VAKA ÇALIŞMASI

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Bu çalışmada, konum riskleri ile birlikte çok ürünlü ve kapasite kısıtlı tesis yeri-tahsis probleminin çözümü için matematiksel bir model sunulmaktadır. Bahsi geçen matematiksel model; toplam nakliye ve kurulum maliyetlerini ile aday konumlara ilişkin riskleri en aza indirirken, çeşitli ürün türlerini bir dizi noktaya sevk eder. Sözü edilen risk, silahların diğer askeri birliklere tahsis edilmesiyle ortaya çıkmaktadır ve sevk eden birlik böylece saldırılara karşı daha savunmasız hale gelmektedir. Modelin test edilmesi için Kara Kuvvetlerinin (81 düğüm içeren) sınır şehirleri için icra ettiği silah tahsisi problemi çözülmüştür. Sonuç olarak, geliştirilen matematiksel modelin silah tahsis problemi en az maliyet / risk ile çözdüğü tespit edilmiş, böylece ülke güvenliği sorunları sırasında kritik karar verme süreçleri için kullanılabilecek bir araç geliştirilmiştir.

Anahtar Kelimeler: Ülke güvenliği, tesis yeri-tahsis problemi, silah sevkiyatı, konum riskleri.

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CHAPTER 1

INTRODUCTION

The political instability in the middle east region signal a potential change in the global political ecosystem as we know it today. In this situation, one thing that countries need to pay special attention to is homeland defense. *Homeland defense*[1] is defined by the US Department of Defense as "the protection of a country's sovereignty, territory, domestic population, and critical defense infrastructure against external threats and aggression, or other threats" (USDOD, 2007). Due to the political instability and the potential changes to the world political ecosystem, it is essential that countries reevaluate their homeland defense mechanisms and the efficiency of those mechanisms for responding to potential threats.

The literature on the utilization of management science in military decision making processes is rare, therefore, it is believed that countries still rely on classical military decision making techniques for taking decisions on various issues. This work is intended to demonstrate how research on location problems can be utilized for solving homeland defense problems. This demonstration is achieved by introducing a mathematical model derived from location problems that helps in the decision making of weapon allocation for homeland defense purposes. To the best of our knowledge, this work is the first attempt to do so.

This thesis is organized as follows:

- Chapter 2 is a review of location problems and their various applications is presented as well as a review of weapon assignment problems.
- Chapter 3 introduces the mathematical notation and the mathematical model for this problem.
- Chapter 4 describes the case study (and its data sources and assumptions) that we tested the model with.

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[1] The definition is intended for the United States. However, the principle applies to other countries.

- Chapter 5 describes how the problem was solved using Excel and OpenSolver and summarizes the solution results.
- Chapter 6 presents the results of solving the model we introduced. This chapter also addresses this work's limitations and potential improvements.
- Chapter 7 is this work's conclusion.

CHAPTER 2

LITERATURE REVIEW

This review surveys the literature relevant to this work. Section 2.1 provides an overview of the facility location problems. Section 2.2 provides an overview of the military optimization problems. Section 2.3 describes the contribution of our work.

2.1. Facility Location Problems

The basic facility location problem investigates the determination of the location of one or more facilities to supply the demand of one or more demand points with the purpose of minimizing (or maximizing) some cost function. Over the past few decades, researchers proposed numerous variations and solution methods to the problem. The problem varies according to the factors considered in the location determination and the approach to supplying the demand of the demand points. Certain variations of the facility location problem are proved to be NP-Hard in (Kariv and Hakimi, 1979a, b). Solutions methods for facility location problems include solving mathematical models (Melkote and Daskin, 2001) as well as heuristics (Tran et al., 2017). In the following subsections, an overview of the various types of the facility location problem is provided.

2.1.1. Capacitated and Uncapacitated Problems

The difference between Capacitated Facility Location Problem (CFLP) and Uncapacitated Location Problem (UFLP) is that: a facility location problem is considered “uncapacitated” when there’s no limit on the amount of demand units a facility is able to supply; conversely, a facility location problem is considered “capacitated” when the amount of demand a facility is able to supply is limited Sridharan (1995). Recent research about this problem includes (Gendon et al., 2017; Tran et al., 2017).

2.1.2. Single- and Multi-Product Problems

The difference between Single- and Multi-Product Facility Location Problems (in the literature, called Multi-Commodity Facility Location Problem) is that in Single-Product problems, the facility locations are being determined to supply the demand of "one" type of goods and products (Ravi et al 2004). In Multi-Product problems, the facility locations are being determined to supply the demand of each of "several" types of goods and products (Ravi et al 2004).

2.1.3. Covering Problems

In *Covering Problems*, each facility has a "critical distance". If the distance between a customer and a facility is equal to or less than the facility's critical distance, the customer is said to be "covered" by that facility (Eiselt and Sandblom, 2013). (Schilling et al., 1993) classify Covering Problems into two categories: Set Covering Problems (SCP) and Maximal Coverage Problem (MCP). In *Set Covering Problems*, the objective is to cover all customers with a minimum number of facilities. Notable publications include: (Toregas et al, 1971) that introduced a model for the problem and (Beasley et al., 1996) that proposed a genetic algorithm for solving the problem. In *Maximal Covering Problems*, the objective is to cover as many customers as possible given a predefined number of facilities. (Eaton et al., 1985) applied the MCP in the determination of ambulances.

2.1.4. Center Problems

In *Center Problems* (also called p-Center Problems or Minmax), the locations of a p number of facilities is determined such that a number of demand points is clustered around each facility; i.e each cluster of demand points is served from a single facility (Eiselt and Sandblom, 2013). This is achieved by (i) determining a radius for each facility that equals to the maximum distance between the facility and a demand point (ii) A demand point whose distance is smaller than or equal to a facility's radius is served from that facility (Eiselt and Sandblom, 2013). The simplest Center Problem is the determination of the location of a single facility (i.e $p = 1$). (Dyer and Frieze, 1985) introduced a heuristic to solve the p-Center Problem. (Davidović et al., 2011) proposed a Bee-colony optimization meta-heuristic for solving the problem. Recent research in center problems includes (Irawan et al., 2016; Callaghan et al., 2017).

2.1.5. Median Problems

In *Median Problems* (also called p-Median Problems), the location of *p* number of facilities is determined where the objective is to minimize the total demand-weighted cost (Laporte et al., 2015). Goldman (1971) proposed an algorithm which solves the problem in polynomial time on a tree. A notable application of the problem is the allocation of schools by minimizing the total distance traveled by pupils (Ndiaye et al., 2012). Another heuristic for solving the p-Median Problem is proposed by (Dzator and Dzator, 2012) where the heuristic is applied to determine ambulance locations.

2.1.6. Location Allocation Problems

In *Location Allocation Problems*, a cost objective function is minimized to determine the number of facilities to open, the location of each facility, and the capacity of each facility, given the location and demand of each demand point, and, the transportation costs of regions Cooper (1963). A notable application of the problem is (Aboolian et al. 2007), where the location allocation problem was applied to determine the facility locations of a Web Service Provider, allocate servers to each facility and allocate customers to each facility.

2.1.7. Location Problem Models

(Revelle et al., 2008) classify models of location problems into four categories: analytic models, continuous models, network models and discrete models. In, *analytic models*, a large number of assumptions that simplify the problem are assumed. One example is provided for such assumptions is: a problem where the demand is distributed uniformly over the service area, a fixed location cost regardless of location area and a fixed shipping cost. *Continuous models* have demand points located at discrete demand locations and facilities are located anywhere in the service area. The Weber problem Weber (1929) was provided as an example of this model category. *Network models* study the location problem based on a network of nodes and links between the nodes. In this model, the demand points and the facilities are located on the nodes, however some research locates the facilities on the links as well. In, *discrete models*, there's a set of demand points and a set of candidate locations and the facility locations that get determined after solving the problem are a subset of the candidate locations set. These problems are generally formulated as integer or mixed integer programming problems.

2.2. Military Optimization Problems

2.2.1. Weapon Target Assignment Problem

In this problem, weapons are allocated to enemy targets in order to minimize the overall survival of those targets after completion of weapon engagements (Ahuta et al., 2007). There are two variations of the problem, static and dynamic. In static, the input to the problem (weapons, targets, etc) is known and the allocation is performed on a single stage. In the dynamic variation, the input to the problem is not fully known and allocation is performed on multiple-stages; the allocations made in a stage are considered in subsequent stages (Ahuta et al., 2007). Research on this problem include (Leboucher et al., 2014; Kalyanam et al., 2016; Yan et al., 2016).

2.2.2. Other Problems

In Jaiswal (1997), other types of military optimization problems are addressed as well, namely: weapon mix problem, weapon deployment problem, sortie allocation problem and airlift problem. *The weapon mix problem* is where the weapon mix (the weapon types and their quantities) used to take down an enemy aircraft is determined to maximize the average number of kills of an enemy aircraft in a vulnerable area. In the *weapon deployment problem*, the deployment of air defense weapons in sites is determined to maximize average number of kills of enemy aircraft. In the *sortie allocation problem*, aircraft sorties of various types are allocated to attack a group of targets of a particular type. And finally, in the *airlift problem*, the plan to airlift supplies in certain areas is determined. The plan is subject to various factors, including: availability of aircrafts, demand, and environmental conditions. In addition to the airlift plan, other decisions are made in this problem as well which include procurement.

2.3. Our Work

In this work, we introduce the concept of "location risks" to accommodate the potential risk that arises from locating product units to candidate locations. The mathematical model that we introduce in this work can be characterized as: a discrete location-allocation model for allocating multiple types of products with location capacities. The objective function in our model minimizes two terms: the demand-weighted transportation and setup cost, and, the location risks.

CHAPTER 3

MATHEMATICAL MODEL

In the formulation of this model, we use a notation based on the notation in Montoya (2016):

- Let N be the set of one or more candidate supply locations.
- M is the set of one or more demand points.
- $P$is the set of one or more product types.
- d_{jk} is the quantity of product type $k \in P$demanded by demand point $j \in M$.
- e_{ij} is the distance between candidate location $i \in N$ and demand point $j \in M$.
- x_{ijk} is the number of units of d_{jk} supplied by candidate location $i \in N$; this variable is is the decision variable in this model.
- r_{ij} is the risk of allocating a unit of demand of demand point $j \in M$ to candidate location $i \in N$.
- u_{ik} is the setup cost of allocating one unit of product type $k \in P$to candidate location $i \in N$.
- p_{ik} is the maximum quantity of units of product type $k \in P$allowed to be allocated to candidate location $i \in N$.

$$Min \sum_{j \in M} \sum_{i \in N} x_{ijk} e_{ij} r_{ij} + \sum_{j \in M} \sum_{i \in N} x_{ijk} u_{ik} \tag{1}$$

Subject to:

$$\sum_{j \in M} x_{ijk} \leq p_{ik};\ \forall k \in P \tag{2}$$

$$\sum_{j \in M} x_{ijk} = d_{jk}; \forall j \in M \tag{3}$$

$$x_{ijk}\ integer, x_{ijk} \geq 0 \tag{4}$$

The objective function (1) minimizes three terms: the total transportation cost, the total setup cost, and the allocation risk. The constraint set (2) is the capacity constraint, which ensures that the number of allocated units of a product type to a candidate location do not exceed the allowed limit. The constraint set (3) ensures that the allocation ensures that the entire demand of each demand point of all weapon types is satisfied. Constraints (4) ensure that weapon allocations are integers and non-negatives.

CHAPTER 4

CASE STUDY

This study addresses the scenario of Turkey being attacked by the entire land force military capabilities of all neighboring countries at the same time. In this study, the model in chapter 3 is used to determine the optimal locations for storing Turkey’s land force capabilities for the purpose of defending the country against such an attack with minimum transportation and setup costs as well as minimum risk associated with those locations.

4.1. Data Sources

The Military Capabilities of all bordering countries of Turkey were obtained from the Military Balance 2016 book. Due to ongoing war in Syria, the book doesn't provide sufficient details about Syria’s capabilities. Therefore, the capabilities of Syria were obtained from Military Balance 2010, an earlier edition of the same book.

This study considers two types of military capabilities, land force soldiers and land force weapons. Table 1 lists the weapon types considered in this study. In the Military Balance Book (both 2010 and 2016 editions), not all weapons have quantities. Further, for some weapon types, a range is provided (such as: quantity is more than 123). The weapon types that have no quantities are ignored, while the value of a weapon type’s that is defined as a range is assumed to be the floor value of the range (eg: if the quantity was specified as "more than 150", the quantity is assumed to be 150).

The City Elevation data used in the calculation of Setup Costs was obtained from a relevant geographical organization. The City and Country Population Data are obtained from Turkish Statistical Institute.

Table 1: Weapon Types and their abbreviations.

Abbreviation		Weapon Type
Category	Weapon	
AFV	MBT	Main Battle Tanks
	AIFV	Armoured Infantry Fighting Vehicle
	APC	Armoured Personnel Carrier
	ARV	Armoured Recovery Vehicles
	RECCE	Reconnaissance
AT	MSL	Missiles
	MSL SP	Self-propelled Missiles
	RCL	Recoilless Launchers
	GUNS	Guns
ARTY	SP	Self-Propelled Artillery
	TOWED	Towed Artillery
	MOR	Mortars
	MRL	Multiple Rocket Launchers

4.2. Assumptions

4.2.1. Attack Scenario

In the attack scenario being addressed, the land force weapon capabilities and land force soldiers of each bordering country are assumed to be distributed evenly across Turkish cities located on that country's border with Turkey. For example: Edirne is located on the border with Greece. Therefore, all Greece's land force capabilities (soldiers and weapons) are assumed to attack from Edirne. Another example: Ağrı, Iğdır and Van are located on the border with Armenia. Therefore, Armenia's land force capabilities are divided into three equal portions, and each portion is assumed to attack from one city, and, all attacks take place at the same time.

4.2.2. Nodes and Weapon Types

The set of candidate locations N is considered to be the set of Turkish cities to which quantities of weapons are allocated; throughout this document, these cities are referred to as *supply cities*. The set of demand points M is considered to be the set of bordering Turkish cities to which weapons must be supplied to defend the country against attacks; throughout this document, these cities are referred to as *bordering cities*. The set of product types P is considered to be the set of weapon types (in Table 1).

4.2.3. Demands

A demand point j's ($j \in M$) demand of a product type $k(k \in P)$, d_{jk}, is considered to be the total amount of a particular weapon type required to defend the country against one or more attacks; in other words, d_{jk} is the bordering city j's ($j \in M$) total demand of a weapon type $k(k \in P)$ that is required to respond to one or more attacks coming to the bordering city. d_{jk} is calculated and provided as input data as explained in section 5.4.

4.2.4. Risks

r_{ij}, which is considered to be the risk of allocating a unit of supply of demand of a bordering city $j \in M$ to city $i \in N$, is calculated in terms of the distance between city $i$and bordering city j.

- It is assumed that the farther city i from bordering city j, the riskier city j becomes for supplying bordering city j's demand of any weapon type.
- The lowest allocation risk is 1, which is assumed to be the shortest distance e_{ij} between bordering city $j$and all cities $i \in N$. City i's risk is, thus, calculated as multiples of the shortest distance between bordering city $j$and all cities $i \in N$.
- All risk values are rounded up to remove fractions.

For example, the distance between ADANA and bordering city AĞRI is 966; the shortest distance to AĞRI is 184; therefore, the risk of allocating one unit of AĞRI's demand of any weapon type to be supplied from ADANA is:

$$966 \div 184 \simeq 5.25 = 6.$$

4.2.5. Setup Costs

u_{ik}, which is the setup cost of allocating one unit of weapon type $k \in P$to be supplied from city $i \in N$. The setup cost for a weapon is usually incurred when the weapon is to be deployed in a city. The cost covers such matters as engineering and configuration time as well as testing.

- The setup cost of a weapon type in a supply city assumed to be proportional to the elevation of that city. Thus, the larger a city's elevation is, the higher the setup cost of a weapon type in that city becomes.
- In this study, only artillery weapons are considered to have setup costs. This is because these weapons require adjustment and configuration when deployed, while other weapon types, such as rifles, don't.
- The setup cost of a weapon type is calculated using a base value multiplied by the *elevation* of the supply city.
- The base values given for artillery weapons are as follows: 0.5 for Mortar, Towed Artillery is given 1, Multiple Rocket Launcher is given 1.5 and Self-propelled Artillery is given 2. Screenshots of these weapons are in figures 1 to 4.

Example of calculating setup costs is: the base value for Multiple Rocket Launcher is 1.5; the elevation of supply cities AKSARAY and ANTALYA is, respectively 900 and 43; the setup costs of Multiple Rocket Launcher in AKSARAY and ANTALYA, thus, becomes

$$1.5 \times 900 = 1350$$

and

$$1.5 \times 43 = 64.5$$

Again, values are rounded up to remove fractions, thus, the setup costs from the example becomes 1350 and 65.

Figure 1: The M224 60mm Mortar

Figure 2: The FH77B Twoed howitzer

Figure 3: The Valkiri multiple rocket launcher

Figure 4: Self-propelled howitzer T5-52

4.2.6. Supply City Capacities

p_{ik} is the maximum quantity of units of weapon type $k \in P$ allowed to be allocated to supply city $i \in N$,

- This value is assumed to be proportional to city population. The larger a city's population is, the larger that city's capacity becomes.

- The capacities of a weapon type for all supply cities are calculated by distributing the total demand $\sum_{k \in P} \; d_{jk}$ of that weapon type to all supply cities based on their population.
- Due to the fact that bordering cities are not considered as supply cities, their population is consequently not considered in the distribution.
- Since capacities are calculated based on total demand and each individual city's population, the total capacity available for allocation would be smaller than the total demand, and thus cause infeasibility. To overcome this issue, the total population of bordering cities is divided and added to the population of supply cities.

This example illustrates how capacities are calculated: In this study, the total demand for Main Battle Tanks (MBT), Self-Propelled Artillery (SP), and Towed Artillery (TOWED) are respectively: 9539, 1578 and 4895; The population of supply cities ANKARA, BURSA and BATMAN represented as a percentage of Turkey's population are respectively: 6.7%, 3.6%, 0.7%; The total population of bordering cities represented as a percentage of Turkey's population is 12.9%; The total number of candidate supply cities is 66; The capacity of ANKARA of MBTs is calculated as follows:

$$(\,6.7\% \;+\; (12.9\% \;\div\; 66)\,) \;\times\; 9539 \;=\; 657.76 \;\simeq 658\;;$$

The remaining capacities of supply cities ANKARA, BURSA and BATMAN of MBT, SP and TOWED are shown in Table 2:

Table 2: Capacities of Cities of weapon types MBT, SP and TOWED.

	ANKARA	**BURSA**	**BATMAN**
MBT	658	366	88
SP	109	61	15
TOWED	338	188	45

CHAPTER 5

SOLUTION METHOD

The problem is solved using Microsoft Excel and OpenSolver.

- Excel was used for defining and organizing the input data and OpenSolver (using the CBC Solver) was used to solve the problem and generate the solution. The XLSX spreadsheet that was created to solve this problem can be used to determine the locations and weapon quantity allocations for other scenarios. The steps to do so are explained in section 6.1.
- The problem is defined and organized in Excel using 12 tables, namely: Capabilities, Attacks, Responses, Cities, Demand Cities, Distances, Risks, Demand, Setup Costs, Capacities, Weapon Allocations, and Supply Allocations.
- The solution values generated by OpenSolver (allocations of quantities of weapon types to supply cities) is stored in predefined cells in the Supply Allocations table.

In the following subsections, the structure, uses, and Excel formulas of each table are described.

5.1. Capabilities, Attacks and Responses Tables

The *Capabilities* table contains a list of the military capabilities of all countries. The data in the table are obtained from the Military Balance book in its 2016 edition (Data for Syria is obtained from the 2010 edition). The data is specified in the form of table entries; each entry specifies the *Country*, *Category*, *Weapon* and *Quantity*. An entry specifies the total quantity of a weapon that a country has. For example, the first entry in screenshot in figure 1 (with the identifier CPT.126) means that Iran has a total number of 1663 Main Battle Tanks.

Country Capabilities

Entry Identifier	Country	Category	Weapon	Quantity
CPT.126	Iran	AFV	MBT	1663
CPT.127	Iran	AFV	RECCE	35
CPT.128	Iran	AFV	AIFV	610
CPT.129	Iran	AFV	APC T	340
CPT.130	Iran	AFV	APC W	300
CPT.131	Iran	ARTY	SP	292
CPT.132	Iran	ARTY	TOWED	2030
CPT.133	Iran	ARTY	MRL	1476
CPT.134	Iran	ARTY	MOR	5000
CPT.135	Iran	AT	MSL MANPATS	?
CPT.136	Iran	AT	RCL	200
CPT.175	Syria	AFV	MBT	4950
CPT.176	Syria	AFV	RECCE	590
CPT.177	Syria	AFV	AIFV	2450
CPT.178	Syria	AFV	APC W	1500
CPT.179	Syria	ARTY	SP	500
CPT.180	Syria	ARTY	TOWED	2030
CPT.181	Syria	ARTY	MRL	500
CPT.182	Syria	ARTY	MOR	410
CPT.183	Syria	AT	MSL SP	410
CPT.184	Syria	AT	MSL MANPATS	2190

Figure 5: A screenshot of the Country Capabilities table

The *Attacks* table is used to define attacks. An attack is defined by: choosing the attacking Country, the bordering City it's attacking from, weapon *Category*, attack *Weapon*[2], and then set the quantity of the weapon used in the attack. Each attack has an Attack Key. An Attack Key shows all the information about the attack (Attack City, Country, Weapon, Quantity) in one sentence. The cities a country is allowed to attack from are only those on the border with that country. The Attacks table enables defining an unlimited number of attacks for each country as long as the weapon stocks of that country is not exceeded. For example, the first entry in the Attacks table in the screenshot in figure 2 means that Greece is attacking the country from EDİRNE with 1354 Main Battle Tanks.

Attacks

Attack	Country	City	Category	Weapon	Quantity	Attack Key
ATK.1	Greece	EDiRNE	AFV	MBT	1354	EDiRNE: Greece: Weapon: AFV: MBT: 1354
ATK.2	Armenia	AĞRI	AFV	MBT	36	AĞRI: Armenia: Weapon: AFV: MBT: 36
ATK.3	Armenia	KARS	AFV	MBT	37	KARS: Armenia: Weapon: AFV: MBT: 37
ATK.4	Georgia	ARDAHAN	AFV	MBT	41	ARDAHAN: Georgia: Weapon: AFV: MBT: 41
ATK.5	Georgia	ARTViN	AFV	MBT	41	ARTViN: Georgia: Weapon: AFV: MBT: 41
ATK.6	Georgia	KARS	AFV	MBT	41	KARS: Georgia: Weapon: AFV: MBT: 41
ATK.7	Iran	HAKKARi	AFV	MBT	554	HAKKARi: Iran: Weapon: AFV: MBT: 554
ATK.8	Iran	VAN	AFV	MBT	554	VAN: Iran: Weapon: AFV: MBT: 554
ATK.9	Bulgaria	EDiRNE	AFV	MBT	40	EDiRNE: Bulgaria: Weapon: AFV: MBT: 40
ATK.10	Bulgaria	KIRKLARELi	AFV	MBT	40	KIRKLARELi: Bulgaria: Weapon: AFV: MBT: 40
ATK.11	Iraq	HAKKARi	AFV	MBT	135	HAKKARi: Iraq: Weapon: AFV: MBT: 135
ATK.12	Iraq	HAKKARi	AFV	MBT	135	HAKKARi: Iraq: Weapon: AFV: MBT: 135
ATK.13	Syria	GAZiANTEP	AFV	MBT	990	GAZiANTEP: Syria: Weapon: AFV: MBT: 990
ATK.14	Syria	HATAY	AFV	MBT	990	HATAY: Syria: Weapon: AFV: MBT: 990

Figure 6: A screenshot of the Attacks table.

The *Responses* table is used to define how Turkey defends itself against each of the attacks defined in the Attacks table. The defense against an attack is defined as the

[2] An attack involving Land Forces is defined by setting the Weapon column to "Land Forces".

weapons, and their quantities, that Turkey will use to respond to (defend itself against) an attack. The defense against each attack can be defined as one or more entries in the table. Thus, enabling the defense against an attack to consist of one or more weapon types. As mentioned in section 4.2.1, in this study, in is assumed that an attack is responded to with a weapon type and quantity identical to those of the attack, therefore, for each attack from the Attacks table, there's a single entry to define a response in the Responses table. A response is defined in the Responses table as follows: choose the *Attack Key*, choose *Category*, *Weapon*, and then set *Quantity* of the weapon used in the response. Defining additional responses for same attack can be made by adding more entries with the same Attack Key. For example, the first entry of the Responses table in figure 3 means that the attack with the key *EDIRNE: Greece: Weapon: AFV: MBT: 1354* is responded to with 1354 Main Battle Tanks.

Responses

Entry Identifier	Attack	Subcategory	Weapon	Quantity
RSP.1	EDİRNE: Greece: Weapon: AFV: MBT: 1354	AFV	MBT	1354
RSP.2	AĞRI: Armenia: Weapon: AFV: MBT: 36	AFV	MBT	36
RSP.3	KARS: Armenia: Weapon: AFV: MBT: 37	AFV	MBT	37
RSP.4	ARDAHAN: Georgia: Weapon: AFV: MBT: 41	AFV	MBT	41
RSP.5	ARTVİN: Georgia: Weapon: AFV: MBT: 41	AFV	MBT	41
RSP.6	KARS: Georgia: Weapon: AFV: MBT: 41	AFV	MBT	41
RSP.7	HAKKARİ: Iran: Weapon: AFV: MBT: 554	AFV	MBT	554
RSP.8	VAN: Iran: Weapon: AFV: MBT: 554	AFV	MBT	554
RSP.9	EDİRNE: Bulgaria: Weapon: AFV: MBT: 40	AFV	MBT	40
RSP.10	KIRKLARELİ: Bulgaria: Weapon: AFV: MBT: 40	AFV	MBT	40
RSP.11	HAKKARİ: Iraq: Weapon: AFV: MBT: 135	AFV	MBT	135
⋮	⋮		⋮	

Figure 7: A screenshot of the Responses table.

5.2. Cities and Distances Tables

The *Cities* table contains a list of all Turkish cities, which serve as the nodes (demand points and candidate locations) for this problem. For each city, the table provides three data items: the city's Population, the city's Population as a percentage of Turkey's population, and, the city's Elevation. The *Demand Cities* table contains the population of the demand cities as a percentage of Turkey's population. This data is used to define other input data for this problem as explained in previous sections. The *Distances* table contains the distance between each bordering city and all supply cities.

Supply Cities	Population	Pop Percentage	Elevation
Adana	2201670	2.8%	23
Adıyaman	610484	0.8%	669
Afyonkarahisar	714523	0.9%	1013
Aksaray	396673	0.5%	900
Amasya	326351	0.4%	392
Ankara	5346518	6.70%	870
Antalya	2328555	2.9%	43
Aydın	1068260	1.3%	70
Balıkesir	1196176	1.5%	101
Bartın	192389	0.2%	25

Figure 8: A screenshot of the Cities table.

Demand Cities	Population	Percentage
Ağrı	542255	0.7%
Ardahan	98335	0.1%
Artvin	168068	0.2%
Edirne	401701	0.5%
Gaziantep	1974244	2.5%
Hakkari	267813	0.3%
Hatay	1555165	1.9%
Iğdır	192785	0.2%
Kars	289786	0.4%
Kilis	130825	0.2%

Figure 9: A screenshot of the Demand Cities table.

Distances	ADANA	ADIYAMAN	AFYONKARAHİSAR	AKSARAY	AMASYA	ANKARA	ANTA
AĞRI	966	646	1312	967	738	1056	142
ARDAHAN	1036	759	1345	1008	779	1089	147
ARTVİN	1032	755	1236	1004	695	980	146
EDİRNE	1169	1438	684	904	901	683	91
GAZİANTEP	212	150	785	477	596	667	76
HAKKARİ	909	669	1482	1174	1139	1364	146
HATAY	191	318	764	456	694	681	74
IĞDIR	1069	749	1422	1077	848	1166	153
KARS	1009	732	1331	986	757	1075	144
KİLİS	248	209	821	513	655	726	80
KIRKLARELİ	1150	1419	665	885	882	664	92
MARDİN	537	296	1110	802	794	992	108
ŞANLIURFA	349	110	922	614	714	804	90
ŞIRNAK	720	480	1293	985	981	1175	127
VAN	895	575	1425	1060	967	1222	144

Figure 10: A screenshot of the distances table.

5.3. Risks Table

The *Risks* table contains the supply city allocation risks. The data is defined as follows: each row of data specifies the allocation risk of the a bordering city's demands in each of the supply cities. Each of the risk values is calculated using the following Excel formula:

=**ROUNDUP**(*supplyCityDistance* / **MIN**(*supplyCityDistances*), 0)

The MIN() function is used to calculate the shortest distance between the bordering city and all supply cities. The ROUNDUP() function is used to remove fractions and obtain the risk value as an integer.

Risks	ADANA	ADIYAMAN	AFYONKARAHİSAR	AKSARAY	AMASYA	ANK
AĞRI	6	4	8	6	5	6
ARDAHAN	5	4	6	5	4	5
ARTVİN	7	5	8	7	5	7
EDİRNE	9	11	5	7	7	5
GAZİANTEP	3	2	11	7	8	9
HAKKARİ	4	3	6	5	5	5
HATAY	2	3	6	4	6	6
IĞDIR	4	3	5	4	3	4
KARS	5	4	7	5	4	6
KİLİS	2	2	7	4	5	6
KIRKLARELİ	10	12	6	8	8	6
MARDİN	6	4	12	9	9	1

Figure 11: A screenshot of the Risks table.

5.4. Demands Table

The *Demands* table contains the total demand of each weapon type at each bordering city. The table also contains the total demand of each weapon type. A bordering city's demand of a weapon type is calculated using the following Excel formula:

=**SUMIFS**(responses[[Quantity]:[Quantity]], responses[[City]:[City]], *borderingCity*, responses[[wpKey]:[wpKey]], *weaponType*)

The SUMIFS() function sums the quantities from the Responses table for a particular bordering city and a particular weapon type. The total demand of a particular weapon type is calculated using the SUM() function.

Demand	AĞRI	ARDAHAN	ARTVİN	EDİRNE	GA
Turkey: AFV: AIFV	32	24	24	478	
Turkey: AFV: APC	43	63	63	2614	
Turkey: AFV: ARV	0	0	0	0	
Turkey: AT: GUNS	0	15	15	63	
Turkey: AFV: MBT	36	41	41	1394	
Turkey: ARTY: MOR	4	21	21	2427	
Turkey: ARTY: MRL	17	12	12	159	
Turkey: AT: MSL	0	10	0	0	
Turkey: AT: MSL SP	7	0	0	612	
Turkey: AT: RCL	0	0	0	4508	
Turkey: AFV: RECCE	0	1	1	239	
Turkey: ARTY: SP	12	22	22	611	
Turkey: ARTY: TOWED	44	23	23	565	
Turkey: Land Forces:	13950	5916	5916	101650	

Figure 12: A screenshot of the Demands table.

5.5. Setup Costs Table

The *Setup Costs* table contains the setup cost for each weapon type at each supply city. A weapon's setup cost at a particular supply city is calculated using the following Excel formula:

=**ROUNDUP**(**INDEX**(citiesTable[[Elevation]:[Elevation]], **MATCH**(*supplyCity*, citiesTable[[Supply Cities]:[Supply Cities]]), 0)**baseValue*,0)

The INDEX(MATCH()) functions retrieve the supply city's Elevation from the Cities table, which is multiplied by the Base Value specified for the weapon type to calculate the setup cost for the weapon type at the supply city. The ROUNDUP() function is used to remove fractions and obtain the setup cost value as an integer.

Setup Costs	BASE	ADANA	ADIYAMAN	AFYONK AI
Turkey: AFV: AIFV	0	0	0	0
Turkey: AFV: APC	0	0	0	0
Turkey: AFV: ARV	0	0	0	0
Turkey: AT: GUNS	0	0	0	0
Turkey: AFV: MBT	0	0	0	0
Turkey: ARTY: MOR	0.5	12	12	50
Turkey: ARTY: MRL	1.5	35	35	15
Turkey: AT: MSL	0	0	0	0
Turkey: AT: MSL SP	0	0	0	0
Turkey: AT: RCL	0	0	0	0
Turkey: AFV: RECCE	0	0	0	0

Figure 13: A screenshot of the Setup Costs table.

5.6. Capacities Table

The *Capacities* table contains the maximum allowed number of units of each weapon type at each supply city. The capacity of each supply city of each weapon type is calculated using the following Excel formula:

=**ROUNDUP**((INDEX(citiesTable[[Pop Percentage]:[Pop Percentage]], **MATCH**(*supplyCity*, citiesTable[[Supply Cities]:[Supply Cities]]), 0) + demandPopPercentages) * *weaponTypeTotalDemand*, 0)

The INDEX(MATCH()) functions retrieve the supply city's Population (as a percentage from Turkey's population) from the Cities table. The population percentage value is then added to the value calculated by the variable demandPopPercentages, which, as explained in section 4.2.6, equals 12.9% ÷ 66 ≃ 0.2%. Finally that value (total of population percentage and demandPopPercentages) is multiplied by the total demand of the weapon type to obtain the capacity value. The ROUNDUP() function removes fractions to obtain the capacity value as an integer.

Capacities	ADANA	ADIYAMAN	AFYONKARAHİS AR	AKSARAY
Turkey: AFV: AIFV	119	119	44	28
Turkey: AFV: APC	191	191	71	45
Turkey: AFV: ARV	7	7	3	2
Turkey: AT: GUNS	6	6	2	2
Turkey: AFV: MBT	282	282	105	67
Turkey: ARTY: MOR	265	265	98	63
Turkey: ARTY: MRL	67	67	25	16
Turkey: AT: MSL	76	76	28	18
Turkey: AT: MSL SP	32	32	12	8
Turkey: AT: RCL	140	140	52	33
Turkey: AFV: RECCE	29	29	11	7
Turkey: ARTY: SP	47	47	18	11

Figure 14: A screenshot of the Capacities table.

5.7. Weapon Allocations Table

The *Weapon Allocations* table calculates the total amount of a weapon type allocated to a supply city. The total amount of weapon type allocated to a supply city is calculated using this Excel formula =SUM(<u>*weaponCityRange*</u>), where the range weaponCityRange is the range of Excel cells at which all the quantities of weapon type allocated to the supply city are set. The Weapon Allocations table is used to define constraint (2), the capacity constraint.

Weapon Allocations	ADANA	ADIYAMAN	AFYONKARAHİS AR	AKSARAY
Turkey: AFV: AIFV	0	0	0	0
Turkey: AFV: APC	0	0	0	0
Turkey: AFV: ARV	0	0	0	0
Turkey: AT: GUNS	0	0	0	0
Turkey: AFV: MBT	0	0	0	0
Turkey: ARTY: MOR	0	0	0	0
Turkey: ARTY: MRL	0	0	0	0
Turkey: AT: MSL	0	0	0	0
Turkey: AT: MSL SP	0	0	0	0
Turkey: AT: RCL	0	0	0	0
Turkey: AFV: RECCE	0	0	0	0
Turkey: ARTY: SP	0	0	0	0
Turkey: ARTY: TOWED	0	0	0	0
Turkey: Land Forces:	0	0	0	0

Figure 15: A screenshot of the Weapon Allocations table before solving the problem.

Weapon Allocations	ADANA	ADIYAMAN	AFYONKARAHİS AR	AKSARAY	AMA
Turkey: AFV: AIFV	119	119	44	28	25
Turkey: AFV: APC	191	191	71	45	39
Turkey: AFV: ARV	7	7	3	2	2
Turkey: AT: GUNS	0	6	2	0	1
Turkey: AFV: MBT	282	282	105	67	58
Turkey: ARTY: MOR	265	265	98	63	55
Turkey: ARTY: MRL	67	67	25	16	14
Turkey: AT: MSL	76	76	28	18	16
Turkey: AT: MSL SP	32	32	12	8	7
Turkey: AT: RCL	140	140	52	33	29
Turkey: AFV: RECCE	29	29	11	7	6
Turkey: ARTY: SP	47	47	18	11	10
Turkey: ARTY: TOWED	145	145	54	34	30
Turkey: Land Forces:	23436	23436	8654	5494	479

Figure 16: A screenshot of the Weapon Allocations table after the problem was solved.

5.8. Supply Allocations Table

The *Supply Allocations* table is used to determine the supply cities from which the demand of bordering cities is supplied. The table consists of these columns: Demand

Point, Weapon Type, Supply City Columns, SUM, DEMAND, TRANSP, WEIGHTED TRANSP, and SETUP COST. The cells in columns *Demand Point* and *Weapon Type* are used to define an Allocation Row for each bordering city's demand of each weapon type. An *Allocation Row* is a row of cells that show the supply cities that supply a bordering city's demand from a particular weapon type. For each Allocation Row, the cells in each of the Supply City Columns show how many units of a bordering city's demand are supplied from each supply city.

Supply Allocations						
Demand Point	Weapon Type	ADANA	ADIYAMAN	AFYONKARAHİSAR	AKSARAY	AMASYA
HATAY	Turkey: ARTY: MOR	0	0	68	0	0
IĞDIR	Turkey: ARTY: MOR	0	0	0	0	0
KARS	Turkey: ARTY: MOR	0	0	0	0	0
KİLİS	Turkey: ARTY: MOR	0	0	0	0	0
KIRKLARELİ	Turkey: ARTY: MOR	0	0	0	0	0
MARDİN	Turkey: ARTY: MOR	68	0	0	0	0
ŞANLIURFA	Turkey: ARTY: MOR	68	0	0	0	0
ŞIRNAK	Turkey: ARTY: MOR	0	0	0	0	0
VAN	Turkey: ARTY: MOR	0	265	0	0	55
AĞRI	Turkey: ARTY: MRL	0	0	0	0	0
ARDAHAN	Turkey: ARTY: MRL	0	0	0	0	0
ARTVİN	Turkey: ARTY: MRL	0	0	0	0	0

Figure 17: A screenshot of the Supply Allocations table

The cells in the *SUM* column show the total quantity of the cells of an Allocation Row. The *DEMAND* column cells show each bordering city's demand of a weapon type. The values in the DEMAND column are the same values in the Demand table. The SUM and DEMAND columns are used to define constraint (3), which is the constraint that ensures that the entire bordering city's demand of a weapon type is satisfied. The cells in the *TRANSP* column show the transportation cost of the allocations made to the cells of each of the Allocation Rows. The cells in the *WEIGHTED TRANSP* column show the transportation cost of each Allocation Row weighted with supply city allocation risks. The SETUP COST column cells show the setup cost of each Allocation Row. The Excel formulas used to calculate TRANSP, WEIGHTED TRANSP and SETUP COST use the SUMPRODUCT() function to multiply the values associated with each supply city (distance, setup cost and risk) with the allocated quantity to that supply city and then add all the supply city values of an Allocation Row.

YOZGAT	ZONGULDAK	SUM	DEMAND	TRANSP	WEIGHTED TRANSP	SETUP COST
0	0	40	40	8440	16880	0
0	0	990	990	484878	2978886	0
0	0	990	990	745740	5538792	0
0	0	990	990	365460	1808773	0
0	0	554	554	452154	2486410	0
0	0	4	4	3320	16600	1596
0	0	21	21	17769	72225	371
0	0	21	21	15162	75810	336
0	0	2427	2427	725885	2037162	35654
0	0	68	68	14416	43248	816
15	0	2141	2141	2758584	14150630	706857
0	0	68	68	51952	311712	34476
0	85	1671	1671	2284970	11859391	209802

Figure 18: A screenshot of the Supply Allocations table

5.9. Defining Constraints and Generating Solution

OpenSolver is used to generate the solution. First, the objective function is set to "minimize" and a cell which contains a formula that calculates the sum of all the values of the cells of columns WEIGHTED TRANSP and SETUP COST is chosen as the objective function value. Then, the decision variables are chosen as the values of all the cells of the Supply City columns. Then, constraints (2) through (4) are defined in terms of the Excel tables described in sections 5.1 through 5.8. Finally, the "Solve" button is clicked to solve the problem. OpenSolver manipulates the decision variable values until the value of the objective function cell becomes optimal. The value of the objective function cell is.

ZONGULDAK	SUM	DEMAND	TRANSP	WEIGHTED TRANSP	SETUP COST
0	101650	101650	24429558	52241226	0
0	36666	36666	11047236	50361170	0
0	143666	143666	228462098	1396755120	0
0	36666	36666	35464016	270488816	0
7490	130616	130616	190114302	1064877059	0
0	19867	19867	21499704	118318734	0
0	36666	36666	29009102	177857234	0
0	8150	8150	1719650	3439300	0
0	36666	36666	16754249	90878073	0
0	36666	36666	24515686	161611172	0
0	63667	63667	19960190	86148730	0
0	116667	116667	86983054	454345719	0
			WT CST	4269312810	
			TL CST	755,129,528.00 ₺	

Figure 19: A screenshot of the Supply Allocations table with the Weighted Cost and Total Cost cells.

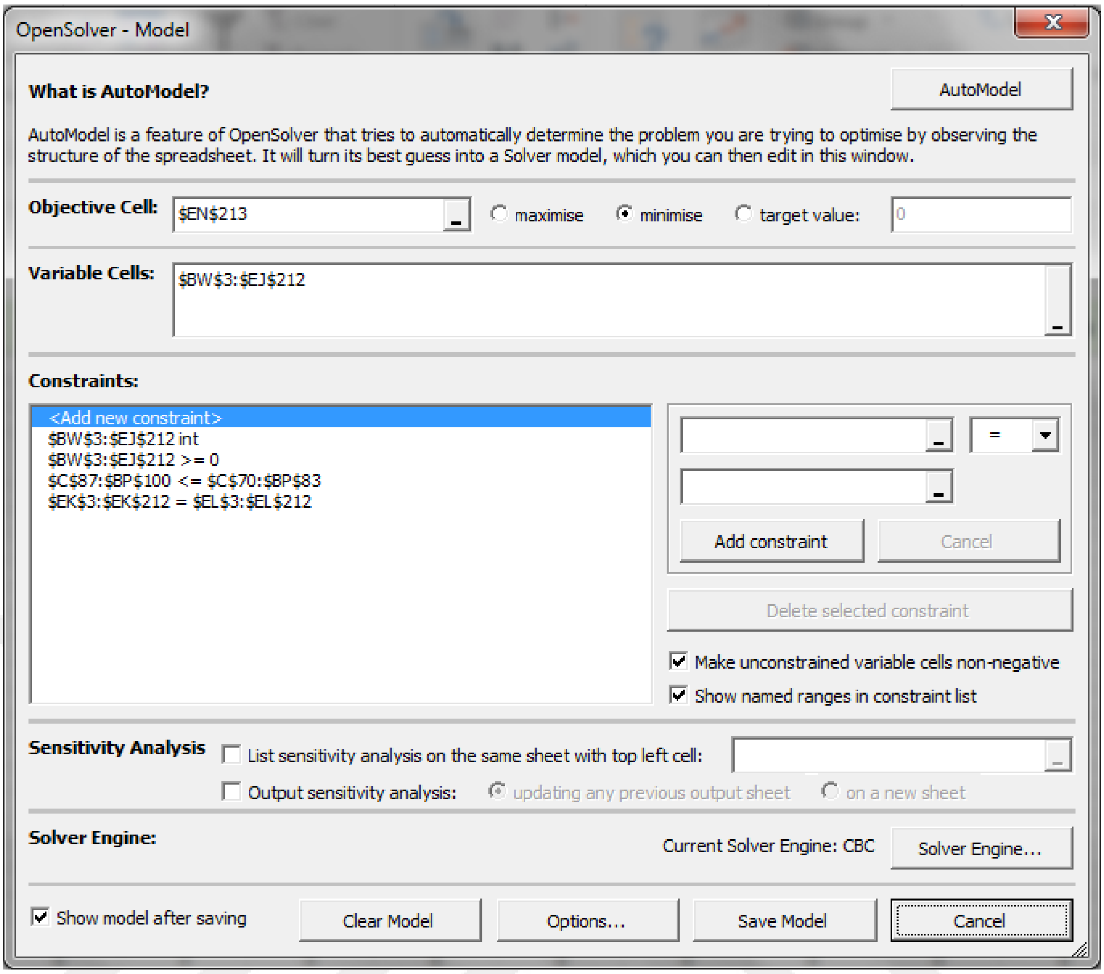

Figure 20: A screenshot of the OpenSolver window showing the Objective Cell, objective set to Minimize, Variable Cells and Constraints.

CHAPTER 6

RESULTS AND DISCUSSION

The case study was solved in approx. 4 minutes and the optimal total transportation and setup cost is 755,129,528.00. Table 3 shows the environment on which the problem was solved as well as the solution results. The results of this study can be used to establish facilities all over the country for storing weapons to guarantee that any attack is handled with the minimum cost and risk. Minimum transportation cost in this case also means minimum time because the transportation cost is calculated in terms of the distance traveled by each unit of a weapon type. The weapon allocations generated in this work considered only one scenario and are based on certain assumptions. More suitable weapon allocations can be achieved by conducting the study on different scenarios and changing the assumptions to accommodate those scenarios. One way of obtaining more suitable weapon allocations is to conduct the study on various attack scenarios (as described in section 6.1) and then analyze the resulting location data from all scenario studies to determine locations that suit all attack scenarios.

Table 3: Problem description, Solution infrastructure and results.

Problem Description and Solution Results	
Solution Time	~ 4 minutes
# Decision Variables	13,860 variable
# Bordering Cities	15
# Supply Cities	66
Total Cost	755,129,528.00
Solution Infrastructure	
Operating System	Windows 7 SP1
RAM	3 GB
CPU	Intel Core 2 Duo
Solver	CBC Solver

6.1. Solving Different Attack Scenarios

As mentioned earlier, this case study considers the scenario where Turkey is being attacked by all neighboring countries, and, these countries are using their entire land force capabilities. The Excel spreadsheet used to solve the model for this case study can be used to address other scenarios, such as the scenario where each neighboring country is attacking from a single bordering city, or, the scenario where a neighboring country is using only a portion of its land force military capability. In these scenarios, the spreadsheet can be copied, Attack and Response table entries deleted and new ones created to define the new scenario. If the spreadsheet is to be used for scenarios where the weapon types, bordering cities (or demand points), or supply cities are increased or decreased, such as the scenario where allocation is performed to only a portion of supply cities, then the spreadsheet needs to have some changes. The changes that need to be made are to the rows and columns of these tables: Distances, Risks, Setup Costs, Capacities, Weapon Allocations, and Supply Allocations.

6.2. Limitations and Potential Improvements

There are two known limitations for this work. The first limitation is that Turkish army capabilities (Turkey's arsenal of each weapon type) were not considered in the allocation. The second limitation is that the study focused only on land force weapons and attacks, and ignored other types of military forces and attacks such as the navy and the air force. The model can be improved in future works to consider these two limitations in planning weapon allocations.

This work determines weapon allocations to minimize cost and risk. However, there's no limit on where weapons can be allocated. It might be desired to limit the locations which certain weapon types can be allocated. Further, it might also be desired to limit the number of facilities opened in some way. One way of limiting the number of facilities would be to have the number of weapons allocated to a supply city exceed a predefined threshold for a facility to be opened in that city; otherwise, the allocated quantity of weapons can be reallocated to the nearest opened facility. Another way of limiting the number of facilities would be to limit the number of facilities for each weapon type; or, a facility gets opened in a supply city when there's at least three or more weapon types.

It is also possible to divide the country into regions (two or three) and conduct the study separately for each of the regions.

CHAPTER 7

CONCLUSION

Homeland defense is an important matter. Countries need to pay special attention to their defense mechanisms and ensure that they're ready for responding to threats. This work demonstrated how facility location problems can be utilized for military decision making. With this work, we contributed to sustaining Turkey's homeland defense decision making by introducing a mathematical model for determining the optimal locations for storing quantities of various weapon types to minimize the total transportation and setup cost as well as the risks associated with weapon locations. The model was tested on a case study where the scenario of Turkey being attacked by the entire land force of all countries on its border. The model was solved to determine the optimal locations for storing weapons in order to defend the country from such an attack with minimal cost and risk.

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