REPUBLIC OF TURKEY GAZİANTEP UNIVERSITY GRADUATE SCHOOL OF NATURAL & APPLIED SCIENCES

MODELLING, SIMULATION AND CONTROL OF QUADRUPLE TANK PROCESS

M. Sc. THESIS IN ELECTRICAL AND ELECTRONICS ENGINEERING

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SERKAN ÖZKAN

Modelling, Simulation and Control of Quadruple Tank Process

M.Sc. Thesis

in

Electrical and Electronics Engineering Gaziantep University

Supervisor Assist. Prof. Dr. Tolgay KARA

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Approval of the Graduate School of Natural and Applied Sciences

Prof. Dr. A. Necmeddin YAZICI Director

I certify that this thesis satisfies all the requirements as a thesis for the degree of Master of Science.

Prof. Dr. Ergun ERÇELEBİ

Head of Department

This is to certify that we have read this thesis and that in our consensus opinion it is fully adequate, in scope and quality, as a thesis for the degree of Master of Science.

Asst. Prof. Dr. Tolgay KARA Supervisor

Examining Committee Members:

Prof. Dr. Ergun ERÇELEBİ Asst. Prof. Dr. Murat FURAT Asst. Prof. Dr. Tolgay KARA Signature

.....

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Serkan ÖZKAN

ABSTRACT

MODELLING, SIMULATION AND CONTROL OF QUADRUPLE TANK PROCESS

ÖZKAN, SERKAN

Msc in Electrical and Electronics Engineering Supervisor: Assist. Prof. Dr. Tolgay Kara

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Simple processes with only one output that may controlled by one input (variable) are known as single input single output process. But many processes are not such simple. They have more than one input (variable) and one output, which are called Multi Input-Multi Output (MIMO) processes. Common MIMO systems have some difficulties, such that they are large and complex. In addition, they have nonlinearities and also loop interactions which are between inputs and outputs. On the purpose of studying multivariable systems and designing controllers, the quadruple tank process (QTP), which has two inputs and two outputs, nonlinearities and loop interactions, is chosen as a benchmark. QTP is suitable for studying linear and nonlinear controllers and exhibits minimum and non-minimum system behaviors due to changing valve positions.

At first mathematical model of the system is obtained. Linearization of nonlinear process is delivered, than various control methods are applied and finally, the controlled system performance results are compared.

Keywords: Quadruple tank system, Control, Mpc, Decoupler, minimum phase, nonminimum phase.

ÖZET

DÖRTLÜ TANK SİSTEMİNİN MODELLENMESİ BENZETİMİ VE KONTROLÜ

ÖZKAN, Serkan

Yüksek Lisans Tezi, Elektrik ve Elektronik Mühendisliği Tez Yöneticisi: Dr. Öğr. Üyesi Tolgay KARA

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43 sayfa

Tek giriş (değişken) ile kontrol edilebilen tek çıkışlı basit prosesler; TGTÇ sistemler olarak tanımlanabilir. Gerçek hayatta karşılaştığımız sistemler maalesef bu kadar basit değildir ve genelde birden fazla girişe ve çıkışa sahiptirler. Böyle sistemlere Çok Girişli Çok Çıkışlı Sistemler (ÇGÇÇ) denir. ÇGÇÇ sistemler dinamiklerinin çeşitli ve karmaşık olmasının yanı sıra doğrusal olmayan davranışlar sergilerler. Bu tür sistemlerin bir diğer dezavantajı ise bir giriş birden fazla çıkışı etkileyebilir, bu da mevcut sistemlerin kontrolünü daha karmaşık hale getirebilir.

ÇGÇÇ sistemlerle çalışmak ve kontrolünü sağlamak üzere iki girişli ve iki çıkışlı olan dörtlü tank sistemi örnek proses olarak seçilmiştir. Bu sistem hem doğrusal hem de doğrusal olmayan denetleyici tasarlanmasına uygun olmasıyla beraber minimum fazlı ve minimum olmayan fazlı sistem özellikleri göstermektedir.

Bu amaç doğrultusunda prosesin matematik modeli elde edilmiştir. Dört farklı denetleyici tasarımı yapılmıştır. Son olarak sisteme eyleyici hatası uygulanarak sistem cevapları analiz edildi, ilgili çıkarımlar sonuçlar bölümünde ifade edildi.

Anahtar Kelimeler: MPC, Dörtlü tank sistemi, Ayırıcı, Minimum fazlı, Minimum fazlı olmayan.

"Dedicated to my dear family"

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TABLE OF CONTENTS

ABSTRACTv
ÖZETviii
ACKNOWLEDGEMETSviii
LIST OF FIGURES xi
LIST OF TABLESxiii
LIST OF ABBREVIATIONSxiv
CHAPTER I1
INTRODUCTION1
1.1 Review of Literature
1.2 Contribution of the Present Work
1.3 Organization of Thesis
CHAPTER II
CHAPTER II
CHAPTER II 4 PHYSICAL MODEL 4 2.1 Mathematical Model 5
CHAPTER II
CHAPTER II4PHYSICAL MODEL42.1 Mathematical Model52.1.1 The Nonlinear Model52.1.2 The Linearized Model7
CHAPTER II4PHYSICAL MODEL42.1 Mathematical Model52.1.1 The Nonlinear Model52.1.2 The Linearized Model72.2 Multivariable Zero10
CHAPTER II4PHYSICAL MODEL42.1 Mathematical Model52.1.1 The Nonlinear Model52.1.2 The Linearized Model72.2 Multivariable Zero102.3 Relative Gain Array12
CHAPTER II4PHYSICAL MODEL42.1 Mathematical Model52.1.1 The Nonlinear Model52.1.2 The Linearized Model72.2 Multivariable Zero102.3 Relative Gain Array12CHAPTER III13
CHAPTER II4PHYSICAL MODEL42.1 Mathematical Model52.1.1 The Nonlinear Model52.1.2 The Linearized Model72.2 Multivariable Zero102.3 Relative Gain Array12CHAPTER III13CONTROLLER DESIGN AND SIMULATIONS13
CHAPTER II4PHYSICAL MODEL42.1 Mathematical Model52.1.1 The Nonlinear Model52.1.2 The Linearized Model72.2 Multivariable Zero102.3 Relative Gain Array12CHAPTER III13CONTROLLER DESIGN AND SIMULATIONS133.1.1 Augmented Error State Feedback Control14

3.1.3 Decoupling Control	
3.1.4 Model Predictive Control	
3.1.5 Comparison of controller without fault	
CHAPTER IV	
COMPARISON OF CONTROLLERS DURING ACTUATOR FAULTS	5 29
4.1.1 Loss of effectiveness	
4.1.2 Leakage	
4.1.3 Lock-in-place	
4.1.4 Hard-over	
4.1.5 Stuck-open	
CHAPTER V	
CONCLUSION	
REFERENCES	41

LIST OF FIGURES

Figure 2.1 Quadruple-Tank Process
Figure 2.2 Single Tank Diagram
Figure 2.3. Basic system
Figure 3.1 Augmented error state feedback controlled system block structure 15
Figure 3.2 State feedback controlled system responses
Figure 3.3 PI-controlled system block structure
Figure 3.4 PI-controlled system simulation
Figure 3.5 Block diagram of system with decoupler
Figure 3.6 Decoupler controlled system block structure
Figure 3.7 Decoupler controlled system simulation
Figure 3.8 Basic structure of MPC
Figure 3.9 Matlab MPC toolbox
Figure 3.10 MPC block diagram
Figure 3 11 System simulation with MPC
Figure 3.12 Tank level for Decoupler – PI
Figure 3.13 Tank level for FB – MPC
Figure 4.1 Tank level with Loss of effectiveness, Decoupler – PI
Figure 4.2 Tank level with Loss of effectiveness, FB – MPC
Figure 4.3 Gain drop due to Loss effectiveness fault
Figure 4.4 Gain drop due to Leakage fault
Figure 4.5 Tank level with Leakage fault, Decoupler – PI
Figure 4.6 Tank level with Leakage fault, FB – MPC
Figure 4.6 Tank level with Leakage fault, FB – MPC33Figure 4.7 Valve 1 actuator output with Lock-in-place fault34
Figure 4.6 Tank level with Leakage fault, FB – MPC33Figure 4.7 Valve 1 actuator output with Lock-in-place fault34Figure 4.8 Tank level with Lock-in-place fault, Decoupler – PI35
Figure 4.6 Tank level with Leakage fault, FB – MPC33Figure 4.7 Valve 1 actuator output with Lock-in-place fault34Figure 4.8 Tank level with Lock-in-place fault, Decoupler – PI35Figure 4.9 Tank level with Lock-in-place fault, FB - MPC35

Figure 4.11 Tank level with Hardover fault, Decoupler - PI	37
Figure 4.12 Tank level with Hardover fault, FB - MPC	37
Figure 4.13 Tank level with Stuck-open fault, Decoupler - PI	38
Figure 4.14 Tank level with Stuck-open fault, FB - MPC	39



LIST OF TABLES

Table 2.1 Valve Setting. 11
Table 3.1 Physical parameters 13
Table 3.2 Operating points of process 13
Table 3.3 MSE values 27
Table 4.1 Mean squared error when loss of effectiveness fault is applied
Table 4.2 Mean squared error when leakage fault is applied 32
Table 4.3 Mean squared error of controllers when Lock-in-place fault is applied 34
Table 4.4 Mean squared error of controllers when Hard-over fault is applied
Table 4.5 Mean squared error of controllers when Stuck-open fault is applied

LIST OF ABBREVIATIONS

- MSE : Mean square error
- QTS : Quadruple tank system
- MIMO : Multi input multi output
- SISO : Single input single output
- MPC : Model predictive control
- QTP : Quadruple tank system
- RGA : Relative Gain Array

CHAPTER I

INTRODUCTION

Multivariable system involves at least two control loops, these loops interact with each other, in such a manner that single input of the process not only affects its own output but also affects other process outputs. These systems are called multi input multi output systems [1-5]. Some sort of tank systems have described before K.H. Johansson, but he introduced laboratory process that is called quadruple tank system which has interconnected four tanks, two pumps also two inputs and two outputs thus it is a MIMO system. Related laboratory process is shown in Figure 2.1.

The aim of the process is keeping the liquid level in the lower tanks at the desired values, but QTP has multivariable interactions, each output of the system has affected by two pumps. Even benchmark looks a simple water level control problem yet process shows nonlinearities, coupling and non-minimum phase characteristics and these reasons make system complex [6-7]. Multivariable interactions which are also known as coupling, it limits performances in MIMO control systems. Because of above reasons, QTS can be regarded as a prototype for many MIMO control applications in industry such as paper production processes, chemical processes, metallurgy and biotechnological areas, medical industries [8].

Transfer functions of QTP has multivariable zero that is interesting situation in order to study, because it makes system stable or unstable due to zero location which depends on three ways valve settings [9-12].

In literature various methods are used to control QTS. Some of approaches are PID, Decoupling [13-17], Model Predictive Control [18-22], Fuzzy Logic Control, Adaptive Control [23-27] for minimum or non-minimum phase conditions.

Because of above motivations; mathematical model of the system is obtained. Four different controllers are designed for without disturbance and no actuator faults.

Finally five kinds of actuator faults applied to QTS and behaviors of controllers are observed in this situation too.

1.1 Review of Literature

MIMO systems have more than one input and one output. Most of the industrial systems have more than one multivariable thus the design of a control system for MIMO systems is widely interested in by researchers [1-3].

The characteristics of process (linearity, phase, stability etc.) must be understood well in order to analyze, simulate and control. One of ways to reflect characteristics of process is modelling [4-6]. Systems can be modelled by two methods that may be *mathematical modelling* or *system identification*. The choice of modelling method is due to some properties of systems, such complexities and nonlinearities. For instance, simple system structure is more suitable for mathematical modelling method whereas system identification method is more suitable for the complex systems which require high computations and time in addition to hard nonlinearities that cannot simply linearize. Also if we have not sufficient knowledge, like how much noiseless reference input-output data we have about real system and how system works. For these situations, using system identification method is inevitable [7-8].

Quadruple tank system is one of the simple MIMO systems which has two inputs and two outputs. Process has four interconnected water tanks and two pumps. Pump voltages are inputs and tanks water levels are outputs of the system. Describing of system looks simple but process has multivariable characteristics which can be seen by mathematical modelling. QTS mathematical equations can be obtained by using Bernoulli's law and mass balance. Equations have square root term that causes nonlinearities in mass flow relationship, between flow and level of the tank [8-9].

In order to design controller, we supposed to decide which controller approach is appropriate, either linear or nonlinear. Nonlinear systems can be linearized by Taylor series expansion. System transfer functions can be obtained by Laplace transform of ratio of outputs and inputs. Roots of transfer function give knowledge about stability due to pole positions where roots are located on the s-domain. System may be minimum phase or non-minimum phase; it depends on the zero locations of transfer matrix. If at least one zero is on the right half plane that means system is nonminimum phase. QTS has two zeros and one of them is multivariable due to operating values of valves. One zero is always in the left half plane, but the multivariable one can be either in left or right half plane or at the origin [9-13].

QTS has loop interactions that can be solved by using centralized or decentralized controllers. Decentralize controller is more efficient than centralize methods by coping with loop interactions. One of the decentralized control method is decoupler which is a popular approach to eliminate coupling effects. The advantage of decoupler technic is to decompose a multivariable system into several independent SISO sub-systems [14-17].

Model predictive control strategy is another efficient, robust control method [15]. MPC deals with linear or non-linear dynamic behavior of the QTS and it can be applied to both centralized and decentralized of system [16-22].

1.2 Contribution of the Present Work

The aims of this study are obtaining mathematical model of quadruple-tank process, applying conventional control methods in both normal operation and actuator failure cases. To that end, firstly nonlinear mathematical model of system is obtained and validated with computer simulations. Than some conventional control methods such as augmented state feedback, decentralized PI control with and without decoupler are designed by considering coupling nature of multivariable system. In the final stage by considering multivariable interactions of tanks actuator failure effects, a robust control method is designed and the performers of controllers are compared numerically and graphically. Advantages and disadvantages of controllers are discussed in the case of actuator failures.

1.3 Organization of Thesis

The structure of thesis is organized as follows. In Chapter II, the physical and mathematical models of QTS are described. In Chapter III, firstly, four kind of controller are designed than simulations of system responses with controllers are discussed. In Chapter IV, five different actuators failures are applied to process and simulation results are discussed and performance comparison of controllers is presented by MSE and graphically. In Chapter V, the Conclusions of the thesis are given.

CHAPTER II

PHYSICAL MODEL

A schematic diagram of the process is illustrated in Figure 2.1. The system consists of four interconnected water tanks and two pumps. The pumps voltages are process inputs and water levels of bottom tanks are process outputs in Figure 2.1. For each tank the mathematical model is obtained by using Bernoulli's law yields and mass balance law. Tank numbers are represented by 'i', which may be 1,2,3,4.



Figure 2.1 Quadruple-Tank Process

The aim is to control the level of the two lower tanks with two pumps. The output of each pump is split into two using three-way valves. In quadruple-tank system given in Figure 2.1, water is pumped into each tank at the top through the pumps and at the bottom of tank the water flows out through a pipe.

2.1 Mathematical Model

2.1.1 The Nonlinear Model

Using by Bernoulli's law and conservation of the mass equation the mathematical equivalent of the system is as follows:

Rate of accumulation = (Flow Rate of in)-(Flow Rate of out)

$$\frac{d(\rho V)}{dt} = \rho q_{in} - \rho q_{out} \quad \text{(since } \rho = \rho_1 = \rho_2 \text{ as same liquid)} \\
A_i \frac{dh_i(t)}{dt} = q_{in} - q_{out}$$
(2.1)

- A_i = the area of related tank
- h_i = water level of the tank
- q_{in_i} = the inlet flow rate to tank
- q_{out_i} = the outlet flow rate to tank

The inlet flow of the tank (q_{in_i}) only depends on the input pump voltage and outflow of the tank (q_{out_i}) depends on the gravity and acceleration due to height of the water in the tank.

Based on Bernoulli's equation q_{out_i} can be determined as follows

$$q_{in_{1}} = k_{1}V_{1} \qquad q_{in_{3}} = k_{2}V_{2}(1 - \gamma_{2})$$

$$q_{in_{2}} = k_{2}V_{2} \qquad q_{in_{4}} = k_{1}V_{1}(1 - \gamma_{1})$$

$$(2.2)$$

where k_1 , k_2 are the pump constants; valve positions γ_1 , γ_2

$$q_{out_i} = a_i \sqrt{2gh_i(t)}$$
(2.3)

 a_i , cross sectional area of the outlet pipes;

g, acceleration of gravity



Figure 2.2 Single Tank Diagram

Help of Figure 2.2; conservation of mass [4]:

$$A_{1}\frac{dh_{1}(t)}{dt} = q_{in_{1}} + q_{out_{3}} - q_{out_{1}} = \gamma_{1}k_{1}V_{1} + a_{3}\sqrt{2gh_{3}(t)} - a_{1}\sqrt{2gh_{1}(t)}$$
(2.4)

The nonlinear equations of each tank for the QTP are obtained similarly (2.4) and given them as follows:

$$A_{1} \frac{dh_{1}(t)}{dt} = -a_{1}\sqrt{2gh_{1}(t)} + a_{3}\sqrt{2gh_{3}(t)} + \gamma_{1}k_{1}v_{1}$$

$$A_{2} \frac{dh_{2}(t)}{dt} = -a_{2}\sqrt{2gh_{2}(t)} + a_{4}\sqrt{2gh_{4}(t)} + \gamma_{2}k_{2}v_{2}$$

$$A_{3} \frac{dh_{3}(t)}{dt} = -a_{3}\sqrt{2gh_{3}(t)} + (1-\gamma_{2})k_{2}v_{2}$$

$$A_{4} \frac{dh_{4}(t)}{dt} = -a_{4}\sqrt{2gh_{4}(t)} + (1-\gamma_{1})k_{1}v_{1}$$

$$(2.5)$$

The parameters $\gamma_1, \gamma_2 \in (0,1)$ are defined according to our aim which may be minimum or non-minimum phase. The flow to Tank 1 is $\gamma_1 k_1 v_1$ and to Tank 4 is $(1-\gamma_1)k_1v_1$ likewise to Tank 2 $\gamma_2 k_2 v_2$ and to Tank 3 $(1-\gamma_2)k_2v_2$. The measured level signals are $k_c h_1$ and $k_c h_2$. The Bernoulli's law and conservation of mass equations can be represented as follows:

$$\frac{dh_{1}(t)}{dt} = -\frac{a_{1}}{A_{1}}\sqrt{2gh_{1}(t)} + \frac{a_{3}}{A_{1}}\sqrt{2gh_{3}(t)} + \frac{\gamma_{1}k_{1}}{A_{1}}v_{1}$$

$$\frac{dh_{2}(t)}{dt} = -\frac{a_{2}}{A_{2}}\sqrt{2gh_{2}(t)} + \frac{a_{4}}{A_{2}}\sqrt{2gh_{4}(t)} + \frac{\gamma_{2}k_{2}}{A_{2}}v_{2}$$

$$\frac{dh_{3}(t)}{dt} = -\frac{a_{3}}{A_{3}}\sqrt{2gh_{3}(t)} + \frac{(1-\gamma_{2})}{A_{3}}k_{2}v_{2}$$

$$\frac{dh_{4}(t)}{dt} = -\frac{a_{4}}{A_{4}}\sqrt{2gh_{4}(t)} + \frac{(1-\gamma_{1})}{A_{4}}k_{1}v_{1}$$
(2.6)

2.1.2 The Linearized Model

(2.4) have nonlinearity due to square root terms. So designing a controller becomes more challenging. (2.5) is solved using Taylor series after using Jacobian matrix transformation to get a state space form of the system.





$$\frac{dx_{1}}{dt} = f_{1}(h_{1}, h_{2}, h_{3}...h_{n}, u_{1}, u_{2}, u_{3}...u_{n})$$

$$\frac{dx_{n}}{dt} = f_{n}(h_{1}, h_{2}, h_{3}...h_{n}, u_{1}, u_{2}, u_{3}...u_{n})$$
(2.7)

The general vector form (x represents states)

$$H_e = h_e + \Delta h \qquad U_e = u_e + \Delta u \tag{2.8}$$

Linear approximation with Taylor series

$$\dot{x} = \frac{dx}{dt} f(H_e, U_e) = f(h_e + \Delta h, u_e + \Delta u)$$

$$f(x, u) = f(h_e, u_e) + \frac{df}{dh} (h_e, u_e) + \frac{df}{du} (h_e, u_e) + \underbrace{higher \text{ order terms}}_{0}$$
(2.9)

The higher order terms are neglected due to simplification.

$$\frac{dh_{1}(t)}{dt} = -\frac{a_{1}\sqrt{2gh_{1}}}{A_{1}} + \frac{a_{3}\sqrt{2gh_{3}}}{A_{3}} + \frac{\gamma_{1}k_{1}V_{1}}{A_{1}} \qquad (u_{1} = V_{1}; u_{2} = V_{2})$$

$$\frac{dh_{1}(t)}{dt} - \frac{dh_{10}}{dt} = -\frac{a_{1}}{A_{1}}\sqrt{\frac{g}{2h_{10}}}(h_{1} - h_{10}) + \frac{a_{3}}{A_{1}}\sqrt{\frac{g}{2h_{30}}}(h_{3} - h_{30}) + \frac{\gamma_{1}k_{1}}{A_{1}}(V_{1} - V_{0})$$
(2.10)

State space form of system as follows:

$$= -\frac{a_{1}}{A_{1}} \sqrt{\frac{g}{2h_{1o}}} x_{1} + \frac{a_{3}}{A_{1}} \sqrt{\frac{g}{2h_{3o}}} x_{3} + \frac{\gamma_{1}k_{1}}{A_{1}} u_{1}$$

$$\dot{x}_{2} = -\frac{a_{2}}{A_{2}} \sqrt{\frac{g}{2h_{2o}}} x_{2} + \frac{a_{4}}{A_{2}} \sqrt{\frac{g}{2h_{4o}}} x_{4} + \frac{\gamma_{2}k_{2}}{A_{2}} u_{2}$$

$$\dot{x}_{1}$$

$$\dot{x}_{3} = -\frac{a_{3}}{A_{3}} \sqrt{\frac{g}{2h_{3o}}} x_{3} + \frac{(1-\gamma_{2})}{A_{3}} k_{2} u_{2}$$

$$\dot{x}_{4} = -\frac{a_{4}}{A_{4}} \sqrt{\frac{g}{2h_{4o}}} x_{4} + \frac{(1-\gamma_{1})}{A_{4}} k_{1} u_{1}$$

$$(2.11)$$

and

$$\frac{dx}{dt} = \begin{bmatrix} -\frac{1}{T_1} & 0 & \frac{A_3}{A_1T_3} & 0\\ 0 & -\frac{1}{T_2} & 0 & \frac{A_4}{A_2T_4}\\ 0 & 0 & -\frac{1}{T_3} & 0\\ 0 & 0 & 0 & -\frac{1}{T_3} & 0\\ 0 & 0 & 0 & -\frac{1}{T_4} \end{bmatrix} x + \begin{bmatrix} \frac{\gamma_1k_1}{A_1} & 0\\ 0 & \frac{\gamma_1k_1}{A_1}\\ 0 & \frac{(1-\gamma_2)k_2}{A_3}\\ \frac{(1-\gamma_1)k_1}{A_4} & 0 \end{bmatrix} u$$
(2.12)
$$y = \begin{bmatrix} k_c & 0 & 0 & 0\\ 0 & k_c & 0 & 0 \end{bmatrix} x$$
(2.13)

the time constants are

$$T_{i} = \frac{A_{i}}{a_{i}} \sqrt{\frac{2h_{io}}{g}}, i = 1,..,4.$$

$$u = \begin{bmatrix} \Delta u_{1} \\ \Delta u_{2} \end{bmatrix}, x = \begin{bmatrix} \Delta h_{1} \\ \Delta h_{2} \\ \Delta h_{3} \\ \Delta h_{4} \end{bmatrix}, y = \begin{bmatrix} \Delta y_{1} \\ \Delta y_{2} \end{bmatrix}$$

$$(2.14)$$

$$(2.14)$$

Transfer function after linearization:

$$G(s) = \begin{bmatrix} \frac{\gamma_1 c_1}{1+T_1} & \frac{(1-\gamma_2)c_1}{(1+T_3 s)(1+sT_1)} \\ \frac{(1-\gamma_1)c_2}{(1+sT_4)(1+sT_2)} & \frac{\gamma_2 c_2}{1+sT_2} \end{bmatrix}$$
(2.16)

where $c_1 = T_1 k_1 k_c / A_1$ and $c_2 = T_2 k_2 k_c / A_2$. $\gamma_1 and \gamma_2$ are in the matrix.

The ratio of k_1/k_2 is approximately equal to 1.

2.2 Multivariable Zero

There are many definitions of zeros for MIMO systems. Transmission zero using is one of them and it can be defined as a pole of the inverse plant (for square matrix). In other words, transmission zeros are the values that input-output matrix lose rank. The zeros can be obtained from det[G(s)] = 0. In case system has a right half plane zero, system become unstable. For the QTS, zeros of system can be found by the following equation [9]:

$$\det[G(s)] = \frac{c_1 c_2}{\gamma_1 \gamma_2 \prod_{i=1}^{4} (1+sT_i)} \left[(1+sT_3)(1+sT_4) - \frac{(1-\gamma_1)(1-\gamma_2)}{\gamma_1 \gamma_2} \right]$$
(2.17)

The transfer matrix **G** has two finite zeros for values of $\gamma_1, \gamma_2 \in (0,1)$. One of them is in the left half plane as always, yet other zero can be placed in both side of plane either left or right, due to the valve positions. In order to find the roots of the numerator polynomial by introducing a parameter $\eta \in (0,\infty)$ as:

$$\eta = \frac{(1 - \gamma_1)(1 - \gamma_2)}{\gamma_1 \gamma_2}$$
(2.18)

If $\eta \to 0$, two zeros are approach to $-1/T_3$ and $-1/T_4$, sequentially. As $\eta \to \infty$, one zero approaches to $-\infty$ and other zero goes to $+\infty$. If $\eta = 1$ one zero is located at origin. That means $\gamma_1 + \gamma_2 = 1$. The system is non-minimum phase for $0 < \gamma_1 + \gamma_2 < 1$ and $1 < \gamma_1 + \gamma_2 < 2$ for minimum phase.

Physical interpretation of system according to multivariable zero is clear. Let q_i denote the flow through Pump *i* and assume that $q_1 = q_2$. Total flow to upper tanks is $[2-(\gamma_1 + \gamma_2)]q_1$ and total flow to lower tanks is $(\gamma_1 + \gamma_2)q_1$. According to this configuration, the flow to the top tanks is smaller than the flow to the bottom tanks if $\gamma_1 + \gamma_2 > 1$. It means system acts as minimum phase. If total flow to the top tanks is greater than the flow of bottom tanks the system is non-minimum phase $(\gamma_1 + \gamma_2 > 1)$. Controlling y_1 with u_1 and y_2 with u_2 is easier if the most of the flows goes

directly to the lower tanks. If the total flow to left tanks is equal to that of right tanks, the control problem is particularly hard. This case corresponds to $\gamma_1 + \gamma_2 = 1$, i.e., a multivariable zero in the origin [9].

The parameters $\gamma_1, \gamma_2 \in (0,1)$ are defined in prior to this study [5]. Process shows minimum or non-minimum phase characteristics due to values of values which are shown on Table 2.1.

Valve values	System Phase	Zero Location
$1 < \gamma_1 + \gamma_2 < 2$	minimum	Zero located in left half plain
$0 < \gamma_1 + \gamma_2 < 1$	non-minimum	Zero located in right half plain
$\gamma_1 + \gamma_2 = 1$		Zero is placed at the origin

Table 2.1 Valve Setting.

2.3 Relative Gain Array

The size and complexity of process makes it hard to approach using tools from the classical centralized control method. Researches in literature indicate that decentralized control using decomposition has been used widely [16].

Therefore, if decentralized control structure is chosen as a MIMO controller, an appropriate pairing of input and outputs is needed. In the case of a $m \times m$ plant transfer function there are m! different pairings. However, physical interpretation of system gives idea about which pairing is useful or which one can be ignored. Relative Gain Array is a method that can be used to suggest pairings through a known quantity. RGA is defined as a matrix Λ [2]:

$$\Lambda = G(0) \times G^{-T}(0)$$
(2.19)

where the asterisk denotes Schur product (element-wise multiplication) and -T inverse transpose. One usually aims to pick pairings such that the diagonal entries of Λ are large i.e., the transfer matrix of system is diagonally dominant and it may be possible to design a fine controller for each SISO loop. If diagonal entries of Λ is negative it means the system is particularly hard to control. A pairing with $0.67 < \lambda < 1.50$ in main diagonal entries generally gives good performance [4]. The RGA of the QTS is as follows:

$$\Lambda = \begin{bmatrix} \lambda & 1 - \lambda \\ 1 - \lambda & \lambda \end{bmatrix}, \quad \lambda = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 - 1}$$
(2.20)

For quadruple tank process, RGA only depends on valve settings. If valves are set such that $\gamma_1 + \gamma_2 < 1$, some diagonal elements become negative and system will be non-minimum phase. According to RGA analysis, input-output pairings must be permuted if a decentralized controller is aimed to design. Otherwise, the closed loop system is either unstable or it will become unstable if one of the SISO loops is broken.

CHAPTER III

CONTROLLER DESIGN AND SIMULATIONS

In order to construct system simulation, parameters of a real system are used. Real system parameters are given in Table 3.1. The inputs of process are v_1 , v_2 which represent water pumps input voltage (0-10V) and the outputs of process are y_1 and y_2 which represent measurement devices output voltages (0-10V). Process linear model is studied at two operating points, one for minimum-phase case, another one for non-minimum-phase. The chosen operating points correspond to the parameter values given in Table 3.2 [9].

Parameter	Value
Height of tanks, h_{max}	20 cm
Bottom area, Tank1, Tank3, A_1 , A_3	28 cm^2
3Bottom area, Tank2, Tank4, A_2 , A_4	32 cm^2
Cross sections of outlet hole, a_1 , a_3	0.071 cm^2
Cross sections of outlet hole, a_2 , a_4	0.057
Measurement device constant, k_c	0.500 V/cm
Gravity g	981 cm/s ²

Table 3.1 Physical parameters

Table 3.2 Operating points of process

Parameters	Minimum Phase	Non-minimum Phase	
(h_{1o}, h_{2o})	(12.26, 12.78) cm	(12.44, 13.17) cm	
(h_{3o}, h_{4o})	(1.63, 1.41) cm	(4.73, 4.99) cm	
(v_1, v_2)	(3.00, 3.00) V	(3.15, 3.15) V	
(k_1, k_2)	(3.33, 3.35) cm ³ /Vs	(3.14, 3.29) cm ³ /Vs	
(y_1, y_2)	(0.70, 0.60)	(0.43, 0.34)	

Due to operating point parameters of physical model, minimum phase (3.1) and nonminimum phase (3.2) transfer matrices are as follows:

$$G(s)_{-} = \begin{bmatrix} \frac{2.6}{1+62.3s} & \frac{1.48}{(1+22.8s)(1+62.3)} \\ \frac{1.4}{(1+30s)(1+90.6s)} & \frac{2.84}{1+90.6s} \end{bmatrix}$$
(3.1)

$$G(s)_{+} = \begin{bmatrix} \frac{1.5}{1+62.8s} & \frac{2.7}{(1+38.7s)(1+62.8)} \\ \frac{1.4}{(1+56.6s)(1+92s)} & \frac{1.61}{1+92s} \end{bmatrix}$$
(3.2)

3.1 Controller Design

In this part, various control methods are applied to the nonlinear system simulation to validate modeling and observe the system performance.

3.1.1 Augmented Error State Feedback Control

A state feedback controller is designed for quadruple tanks system which is suitable to minimum phase system. The QTS is controllable and observable due to calculations.

The goal is to obtain a level tracking controller for lower two tanks. It's clear to see by (3.1), the plant is a Type 0 system which has no integrator. The basic idea to design Type 1 servo system is adding an integrator in the feedforward path between the error comparator and the plant as shown in Figure 3.1.



Figure 3.1 Augmented error state feedback controlled system block structure

The system and controller equations are [5]:

$$\begin{cases} \dot{x} = Ax + Bu & \text{States} \\ y = Cx & \text{Output} \\ u = -Kx + k_I e & \text{Control signal} \\ \dot{\xi} = r - y = r - Cx \end{cases}$$
(3.3)

The output of the integrator is ' ξ ' and the reference signal is 'r'. By augmenting the states ξ with states x, integral action in controller can be obtained for better tracking of controller. The augmented system equations are as follows:

$$\begin{bmatrix} \dot{\tilde{x}}(t) \\ \dot{\tilde{\xi}}(t) \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ \tilde{\xi}(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(t)$$
(3.4)

Here an asymptotically stable system is desired to be designed such that $x(\infty)$, $\xi(\infty)$ and $u(\infty)$ approach constant values, respectively. Then at steady state, $\dot{\xi}(t) = 0$ leads $y(\infty) = r$. At steady state:

$$\begin{bmatrix} \dot{\tilde{x}}(\infty) \\ \dot{\tilde{\xi}}(\infty) \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x(\infty) \\ \xi(\infty) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(\infty) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(\infty)$$
(3.5)

(3.6) is obtained by subtracting (3.5) from (3.4):

$$\begin{bmatrix} \dot{\tilde{x}}(t) - \dot{\tilde{x}}(\infty) \\ \dot{\tilde{\xi}}(t) - \dot{\tilde{\xi}}(\infty) \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x(t) - x(\infty) \\ \xi(t) - \xi(\infty) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} \begin{bmatrix} u(t) - u(\infty) \end{bmatrix}$$
(3.6)

Define

$$\begin{cases} x(t) - x(\infty) = x_e(t) \\ \xi(t) - \xi(\infty) = \xi_e(t) \\ u(t) - u(\infty) = u_e(t) \end{cases}$$

$$(3.7)$$

(3.7) can be written as

$$\begin{bmatrix} \dot{\tilde{x}}_{e}(t) \\ \dot{\tilde{\xi}}_{e}(t) \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x_{e}(t) \\ \tilde{\xi}_{e}(t) \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u_{e}(t)$$
(3.8)

where

$$u_{e}(t) = -Kx_{e}(t) + k_{I}\xi_{e}(t)$$
(3.9)

and define a new (n+1) order error vector for n^{th} order system by

$$e(t) = \begin{bmatrix} x_e(t) \\ \xi_e(t) \end{bmatrix}$$

Then (3.8) becomes

$$\dot{e} = Ae + Bu_e \tag{3.10}$$

where

$$\hat{A} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}, \quad \hat{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}$$

Then (3.9) becomes

$$u_e(t) = -K x_{\hat{e}} \tag{3.11}$$

Where $\hat{K} = \begin{bmatrix} K & \vdots & k_I \end{bmatrix}$. The state error equation can be obtained by substituting (3.11) into (3.10):

$$\dot{e} = (\hat{A} - \hat{B}\hat{K})e \tag{3.12}$$

If the desired eigenvalues of matrix $(\hat{A} - \hat{B}\hat{K})$ are specified, then the state feedback gain matrix *K* and integral time gain constant k_I can be determined by the pole placement technique. The gain of the state feedback controller is calculated by Ackerman's method. Linearized system state space model is obtained at minimum phase operating point as:

$$\begin{cases} \frac{dx}{dt} = \begin{bmatrix} -0.016 & 0 & 0.044 & 0 \\ 0 & -0.011 & 0 & 0.033 \\ 0 & 0 & -0.044 & 0 \\ 0 & 0 & 0 & -0.033 \end{bmatrix} x + \begin{bmatrix} 0.0833 & 0 \\ 0 & 0.0628 \\ 0 & 0.0479 \\ 0.0312 & 0 \end{bmatrix} u \\ y = \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \end{bmatrix} x \end{cases}$$
(3.13)

Closed loop system poles are located at:

$$P = \begin{bmatrix} -0.0678 \pm 0.0683i & -0.0617 \pm 0.0591i & -0.0172 & -0.0562 \end{bmatrix}$$

The controlled system performance is observed via simulations. System simulation time is 25s system initially starts with operating point parameters given in Table 3.2. After 50s a unit step change in the reference signal is applied for Tank1 level. For Tank 2 a constant reference is chosen to track. The response of the system can be seen from Figure 3.2. The level of Tank1 is tracking the reference signal with zero

steady state error and a settling time approximately 100s. Tank2 level deviates from its reference a little between 60s and 120s as a result of interaction of tanks.



Figure 3.2 State feedback controlled system responses.

3.1.2 Decentralized PI Control

A decentralized PI controller is designed by using system transfer matrix [16]. Tank1 and Tank2 levels are the outputs supposed to be controlled. The control system as shown in Figure 3.3.Since operating minimum phase system configuration, the interaction between tanks are small in contrast to non-minimum phase model. Also it can be seen that by looking at diagonal elements of RGA matrix. From Table 3.2, valve positions γ 1 and γ 2 are 0.7 and 0.6 respectively. RGA matrix in this configuration is:

$$\Lambda = \begin{bmatrix} 1.4 & -0.4 \\ -0.4 & 1.4 \end{bmatrix} \tag{3.14}$$

RGA, in this case, suggests (y_1, u_1) and (y_2, u_2) pairings. PI controllers transfer function:

$$G_{ci}(s) = K_i (1 + \frac{1}{T_i s}), \quad i = 1, 2$$
 (3.15)

Controller parameters are tuned so that they give acceptable performance such as less than %10 overshoot and 60s settling time. The controller settings $(K_1, T_1) = (3.0, 30)$ and $(K_2, T_2) = (2.7, 40)$ give the response shown in Figure 3.4.







Figure 3.4 PI-controlled system simulation

3.1.3 Decoupling Control

MIMO problems can be converted to SISO problems by several methods. One of these methods is non-interacting or decoupling control schemes. This kind of control avoids the effects of loop interactions totally.

The loss of interaction between the control loop of QTS provide closed loop stability [28]. One of the other advantage is giving a reference point to one of the tanks will not affect the response of other tank systems even though at the same time given different set point values

The decoupler divides a MIMO process into a few independent single-loop subsystems. [14]. Fig. 3.5 shows the decoupling control plot. According to ideal decoupling procedure in [7]:

$$T(s) = \begin{bmatrix} T_{11} & T_{21} \\ T_{21} & T_{22} \end{bmatrix}$$
(3.16)

the diagonal elements, $T_{11} = T_{22} = 1$ (ideal decoupler case) and off diagonal elements,

$$\begin{cases} T_{12} = -\frac{G_{12}}{G_{11}} & T_{12} = -\frac{G_{21}}{G_{22}} \\ T_{12} = -\frac{0.57}{(1+22.8s)} & T_{21} = -\frac{0.5}{(1+30.07s)} \end{cases}$$
(3.17)

Comparison of PI and Decoupler controlled system is shown on Fig. 3.7.



Figure 3.5 Block diagram of system with decoupler



Figure 3.6 Decoupler controlled system block structure



Figure 3.7 Decoupler controlled system simulation

3.1.4 Model Predictive Control

MPC techniques are used not only in the process industry but also have applications to the control of a different processes ranging from robot manipulators to clinical anesthesia, PVC plants, steam generators [18]. MPC can deal with multivariable interactions such as coupling, and process nonlinearity systematically, it is also the only technique that is able to consider model constrains [16]. The biggest disadvantage of method is an appropriate mathematical model of the process supposed to be available [18].

The basic structure shown in Figure 3.8 is used in order to implement this technique. A mathematical model, which is corner-stone of the MPC, is used to predict the future plant outputs, based on previous and current values and on the proposed optimal future control actions.

All actions are computed by the optimizer with considering account the cost function (future tracking errors are included) as well as the constraints [18-20].



Figure 3.8 Basic structure of MPC

In MPC studies, the output variables are also referred to as controlled variables (CV's) which are liquid level of the tanks of our system, and the input variables are called as manipulated variables (MV's) which are pumps voltages of our system. The predictions of MPC computations are obtained at each sampling instant: set-point computations and control computations [17]. Inequality constraints of top and bottom limits can be included in either type of computations [18].

MPC basic theory has been shown in [19].

In this thesis, in order to implement MPC, Matlab/Model Predictive Control toolbox is used. MPC toolbox screen is showed in Figure 3.9 and related Simulink block structure is shown in Figure 3.11.

MPC parameters of QTS are as follows:

Prediction horizon =15; Control horizon =3; Sample time=0.1;







Figure 3.10 MPC block diagram



Figure 3 11 System simulation with MPC

3.1.5 Comparison of controller without fault

Four different controllers are used to control QTS in Chapter 3. In this section we will compare four of them not only by simulation but also mean squared error method and decide which one is more suitable for system.

Simulations on Figure 3.12 present outputs of the system which are water level of Tank1 and Tank2 versus time in seconds. Reference signal of Tank1 is shifted from 12.3cm to 14.5cm at 50 seconds and to see coupling effects clearly, reference level of Tank2 is kept constant at 12.78 cm. In order to reach desired water level, pump1 feeds Tank1 by valve 1 but valve 1 feeds Tank2 too through by Tank4. So one input affects two outputs which means, we have a multivariable system and we see coupling effects.

About responses of Tanks: by simulation, MPC is fastest and has minimum overshoot. PI and Decoupler PI controllers show similar responses, but PI is poorer

than Decoupler. Augmented error state feedback controller can't handle the process. All of them have no steady state error except state feedback.

Table 3.3 presents MSE of controllers. It's clear to see that MPC is the best.

Based on Fig 3.12 and Table 3.3, MPC and Decoupler are more effective controllers.

Table 3.3 MSE values

Controller	PI	Decoupler	State Feedback	MPC
Tank1	0.036	0.0187	6.2392	0.014
Tank2	0.001	0.000	0.4547	0.000



Figure 3.12 Tank level for Decoupler – PI



Figure 3.13 Tank level for FB – MPC

CHAPTER IV

COMPARISON OF CONTROLLERS DURING ACTUATOR FAULTS

On previous chapter, four controllers were introduced and compared without any faults or disturbances. But now, five kinds of actuator faults are applied on QTS system and the four states of system which are Tank1 to Tank4 liquid levels are observed. Controller behaviors are checked in regards to robustness and smooth reaction depend on faults. Each fault responses are observed by graphically and compared by mean squared error.

4.1.1 Loss of effectiveness

Loss of effectiveness type actuator fault applied to both lower tanks for Tank1 at 100 seconds and for Tank2 at 200 seconds. For valve 1 at Figure 4.3, gain drops 50% of its set value. And we can see how actuator output signal change with PI control due to fault signal.

Figure 4.3 shows us actuator output of valve 1 is changed due to reference signal, step signal is applied at 50s and the fault is applied at 100s. Figure 4.1, 4.2 present simulations and Table 4.1 states MSE values for all controllers. MPC has better responses than rest controllers by means of MSE for Tank1 and Tank2. PI and Decoupler PI controller have similar responses, but Decoupler slightly better. State Feedback controller is the poorest one.

 Table 4.1 Mean squared error when loss of effectiveness fault is applied

Controller	PI	Decoupler	State Feedback	MPC
Tank1	0.0526	0.0208	82.15	0.0154
Tank2	0.0020	0.0023	44.67	0.0018



Figure 4.1 Tank level with Loss of effectiveness, Decoupler – PI



Figure 4.2 Tank level with Loss of effectiveness, FB – MPC

Figure 4.3 Gain drop due to loss effectiveness fault

4.1.2 Leakage

Leakage type actuator fault is applied to Tank1 at 100 seconds and Tank2 at 200 seconds. Figure 4.5, 4.6 show simulations and Table 4.2 states MSE values for each controller. MPC has best responses by means of MSE and graphically for Tank1 and Tank2. PI and Decoupler PI controller have similar responses, but Tank1 decoupler was better than PI. And State Feedback controller was the poorest one and can't handle this kind of the actuator fault.

Controller	PI	Decoupler	State Feedback	MPC
Tank1	0.0380	0.0196	4.0400	0.0141
Tank2	0.0002	0.002	0.6180	0.0000

 Table 4.2 Mean squared error when leakage fault is applied

Figure 4.4 Gain drop due to Leakage fault

Figure 4.5 Tank level with Leakage fault, Decoupler – PI

Figure 4.6 Tank level with Leakage fault, FB – MPC

4.1.3 Lock-in-place

Actuator fault applied to just Tank1 at 100 seconds to see effects clearly. Figure 4.7 compares actuator output normal and faulty mode. At 100s valve is locked and kept constant value. Simulations on Figure 4.8, 4.9 and MSE values on Table 4.3 show which controller is suitable.

If fault is applied previous of settling time, the control will be more difficult even may not be possible.

Table 4.3 Mean squared error of controllers when Lock-in-place fault is applied

Controller	PI	Decoupler	State Feedback	MPC
Tank1	0.1753	0.0197	6.2479	00142
Tank2	0.001	0.0002	0.4577	0.0000

Figure 4.7 Valve 1 actuator output with Lock-in-place fault

Figure 4.8 Tank level with Lock-in-place fault, Decoupler – PI

Figure 4.9 Tank level with Lock-in-place fault, FB - MPC

4.1.4 Hard-over

Actuator fault applied to just Tank1 at 100 seconds to see effects clearly. Figure 4.11 and 4.12 shows simulations and Table 4.4 states MSE values. This kind of actuator faults can't be controlled by these controllers, unfortunately. As a result of actuator redundancy as well as analytic redundancy the overall system is out of control for the total failure cases.

Table 4.4 Mean squared error of controllers when Hard-over fault is applied

Controller	PI	Decoupler	State Feedback	MPC
Tank1	23.7722	23.7495	25.0570	23.7395
Tank2	0.8977	24.8063	1.6087	31.3508

Figure 4.10 Valve 1 actuator output with Hard-over fault

Figure 4.11 Tank level with Hardover fault, Decoupler – PI

Figure 4.12 Tank level with Hardover fault, FB - MPC

4.1.5 Stuck-open

Stuck-open type actuator fault applied to only Tank1 at 100. Figure 4.13 and 4.14 shows simulations and Table 4.5 states MSE values for each controller. MPC has best responses by means of MSE and graphically for Tank1 and Tank2. PI and Decoupler PI controller have similar responses, yet Decoupler acts better than PI. State Feedback controller was the poorest one as expected.

Table 4.5 Mean squared error of controllers when Stuck-open fault is applied

Controller	PI	Decoupler	State Feedback	MPC
Tank1	0.0375	0.0195	6.2479	0.0141
Tank2	0.001	0.0000	0.4577	0.0000

Figure 4.13 Tank level with Stuck-open fault, Decoupler - PI

Figure 4.14 Tank level with Stuck-open fault, FB - MPC

CHAPTER V

CONCLUSION

In this thesis, a multiple interacting coupled tanks system is chosen as a case study. The main objectives in this study are to obtain mathematical model of QTS, to design controllers which can not only control the level of the Tank1 and Tank2 but also cope with coupling effects, with or without actuator faults.

The linear system model is obtained and the conditions of the system are given for minimum or non-minimum phase. Minimum phase system structure is chosen for this study. Firstly, a centralized augmented state feedback controller is designed. For this controller, any further performance improvement is not considered. Then, a decentralized control system design is considered as another option since interactions between levels of tanks are in acceptable range. In this case, interactions between tanks may be considered as disturbance. Furthermore, for cases such as high changes in the reference signal for one tank, the disturbance will have more effect on the other tank level. In order to overcome the mentioned issue, a dynamical decoupler is designed and integrated to decentralized PI controller. Then, model predictive controller is designed because of its robustness property. Finally, in order to see effects of actuator faults various failure scenarios are applied, such as loss of effectiveness, leakage, stuck-open, hard-over and lock-in place. Performances of controllers are compared via simulations and numerically, by considering ability of coping with coupling effects and actuator fault rejection. The obtained results show that MPC scheme gives the best controller behaviors in terms of coping with tank interactions and actuator faults.

For future work, this study has the potential to be developed further. The QTS may be analyzed for non-minimum phase conditions. In addition to robust methods some adaptive approaches can be considered as advanced control methods for the varying parameter cases. Another important step would be that implementation of the model and its control algorithm to run real time applications.

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