A COMPARATIVE ANALYSIS OF QUADRATICS IN MATHEMATICS TEXTBOOKS FROM TURKEY, SINGAPORE, AND THE INTERNATIONAL BACCALAUREATE DIPLOMA PROGRAMME

A MASTER'S THESIS

BY

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THE PROGRAM OF CURRICULUM AND INSTRUCTION
BILKENT UNIVERSITY
ANKARA

MAY 2012

To my dear parents, for their endless love and support

A COMPARATIVE ANALYSIS OF QUADRATICS IN MATHEMATICS TEXTBOOKS FROM TURKEY, SINGAPORE, AND THE INTERNATIONAL BACCALAUREATE DIPLOMA PROGRAMME

The Graduate School of Education

of

Bilkent University

by

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In Partial Fulfilment of the Requirements for the Degree of

Master of Arts

in

The Program of Curriculum and Instruction
Bilkent University
Ankara

May 2012

BILKENT UNIVERSITY

GRADUATE SCHOOL OF EDUCATION

THESIS TITLE: A COMPARATIVE ANALYSIS OF QUADRATICS IN MATHEMATICS TEXTBOOKS FROM TURKEY, SINGAPORE, AND THE INTERNATIONAL BACCALAUREATE DIPLOMA PROGRAMME

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May 2012

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Curriculum and Instruction.

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ABSTRACT

A COMPARATIVE ANALYSIS OF QUADRATICS IN MATHEMATICS TEXTBOOKS FROM TURKEY, SINGAPORE, AND THE INTERNATIONAL BACCALAUREATE DIPLOMA PROGRAMME

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The purpose of this study was to analyze and compare the chapters on quadratics in three mathematics textbooks selected from Turkey, Singapore, and the International Baccalaureate Diploma Programme (IBDP) in terms of content, organization, and presentation style through content analysis.

The analysis of mathematical content showed that the Turkish textbook covers a greater number of learning outcomes targeted for quadratics in the three mathematics syllabi, in a more detailed way compared to the other two textbooks.

The organization of mathematical knowledge reflects an inductive approach in the Turkish textbook from quadratic equations to functions, whereas the Singaporean and IBDP-SL (Standard Level) textbooks present mathematical concepts in a deductive way from quadratic functions to equations.

Regarding presentation style, the Turkish and IBDP-SL textbooks are rich in student-centered activities compared to the Singaporean textbook. While the IBDP-SL textbook gives opportunities to students to make investigations and reach generalizations, the Turkish textbook presents mathematical concepts in a ready-made way. The IBDP-SL textbook is also the one which uses real-life connections and technology the most. In addition, the IBDP-SL textbook uses problems with moderate complexity more frequently than the other two textbooks where the problems with low complexity are dominant.

This study revealed that each mathematics textbook has different priorities, and the approaches in the three textbooks were interpreted in the light of reader-oriented theory (Weinberg & Wiesner, 2011) and Rezat's (2006) model of textbook use.

Key words: Content analysis, international comparative studies, mathematics education, mathematics textbooks.

ÖZET

TÜRKİYE, SİNGAPUR VE ULUSLARARASI BAKALORYA DİPLOMA PROGRAMI'NIN MATEMATİK DERS KİTAPLARINDA İKİNCİ DERECEDEN DENKLEMLER, EŞİTSİZLİKLER VE FONKSİYONLAR KONUSUNUN KARŞILAŞTIRMALI BİR ANALİZİ

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Mayıs 2012

Bu çalışmanın amacı Türkiye, Singapur ve Uluslararası Bakalorya Diploma Programı'ndan (IBDP) seçilen üç matematik ders kitabında içerik analizi metoduyla ikinci dereceden denklemler, eşitsizlikler ve fonksiyonlar (İDDEF) konusunun içerik, organizasyon ve sunuş şekli açılarından incelenmesi ve karşılaştırılmasıdır.

Ders kitaplarının matematiksel içerik açısından analizi üç matematik müfredatında İDDEF konusu için hedeflenen öğrenme kazanımlarından en fazlasını Türk ders kitabının, diğer iki kitaba oranla daha detaylı bir biçimde işlediğini göstermiştir.

Matematiksel bilginin organizasyonu bakımından Türk ders kitabı denklemlerden fonksiyonlara tümevarımsal bir yaklaşımı yansıtırken, Singapur ve IBDP-SL (Standart Seviye) matematik ders kitapları matematiksel kavramları fonksiyonlardan denklemlere tümdengelimli bir yol izleyerek sunmuşlardır.

İDDEF konusunun sunuş şekli ile ilgili olarak, Türk ve IBDP-SL ders kitapları öğrenci merkezli etkinlikler açısından Singapur ders kitabına göre daha zengindir. IBDP-SL ders kitabı öğrencilere araştırma yaparak keşif yoluyla öğrenme fırsatları verirken, Türk ders kitabı kavramları hazır bir biçimde vermektedir. IBDP-SL ders kitabının aynı zamanda gerçek hayat bağlantılarını ve teknolojiyi en çok kullanan kitap olduğu ortaya çıkmıştır. Buna ek olarak, IBDP-SL ders kitabı orta zorluk seviyesindeki problemleri diğer iki kitaba nispeten daha sıklıkla kullanırken, Türk ve Singapur ders kitaplarında düşük zorluk derecesindeki problemler ağırlıktadır.

Bu çalışma üç matematik ders kitabının özgün öncelikleri olduğunu ortaya koymuş ve kitaplardaki yaklaşımlar okuyucu odaklı ders kitapları kuramı (Weinberg & Wiesner, 2011) ve Rezat'ın (2006) ders kitabı kullanım modeline göre yorumlanmıştır.

Anahtar Kelimeler: İçerik analizi, matematik ders kitapları, matematik eğitimi, uluslararası karşılaştırmalı çalışmalar.

ACKNOWLEDGEMENTS

I would like to express my deepest gratitude to my advisor Prof. Dr. Cengiz Alacacı for his invaluable support, guidance, and feedback throughout this study. I also thank him for his encouraging attitude as well as his voluntary effort and assistance to better the thesis. He served as an advisor to me during my education in this program except for the last semester. Even after leaving the program, he served as my thesis advisor informally up until the end.

I wish to express my love and gratitude to my parents, to whom this dissertation is dedicated. They always supported and encouraged me to achieve my goals during my life.

I would like to acknowledge and offer my sincere thanks to the committee members Prof. Dr. M. K. Sands and Dr. Minkee Kim for the time they spent reviewing my thesis and helping me improve it day by day.

I am grateful to many instructors in our department, Graduate School of Education, as they supported me during all the processes of the thesis. I wish to thank Prof. Dr. Alipaşa Ayas, Dr. Robin Martin, and Dr. Necmi Akşit for all the support they provided.

Finally, a great deal of thanks goes to Gökhan Karakoç for helping me get through the difficult times and for all the emotional support he provided.

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CHAPTER 1: INTRODUCTION

Introduction

Being the primary material of instruction used in mathematics classrooms by both students and teachers, textbooks play a vital role in mathematics education regarding the delivery of the curriculum, opportunities given to students to learn mathematics, and the contribution in teaching and learning activities. This study focuses on a comparison of the quadratics units in the three mathematics textbooks selected from Turkey, Singapore and the International Baccalaureate Diploma Programme (IBDP). Content, organization, and presentation style of the three chapters are analyzed in order to reveal characteristics of effective textbooks in light of *reader-oriented theory* (Weinberg & Wiesner, 2011) and Rezat's (2006) model of textbook use.

Background

Textbooks have been used as a basic resource of teaching in many countries with the aim of facilitating both student understanding and teacher instruction (Semerci, 2004). As they support teachers and instruction, textbooks constitute an integral element of mathematics education. The guidance they give includes giving teachers useful ideas about the content and depth of the mathematical knowledge to be taught to groups of students with varying stages of development and academic achievement, as well as the methods and time for teaching mathematics topics. While preparing content of their lessons, in-classroom activities, and homework, teachers make use of textbooks as primary materials to convey mathematics curricula in many schools (Apple, 1992; Ben-Peretz, 1990; Schmidt, McKnight, & Raizen, 1997).

Countries and educational programs use their own textbooks in order to meet the goals of their educational systems. Philosophies and needs of education in countries and programs shape the way textbooks are used, so curricular and pedagogical intentions are embedded through their usage. Regarding curricular and pedagogical intentions, textbooks may attempt to present a centrally assigned curriculum, enhance pedagogy within a curriculum, respond to new suggestions on pedagogy, and help to promote a new curriculum. Countries have one or more of these goals, being affected by the structure of their educational systems (Howson, 1993). Therefore, it is obvious that mathematics textbooks—with their distinctive curricular targets, content, organization, and presentation styles—are expected to reflect intentions and values of mathematics education in the countries and programs they are used (Schmidt, McKnight, Valverde, Houang, & Wiley, 1997).

There is not a prescriptive textbook theory dealing with the characteristics of effective textbooks and guiding how to write competent textbooks in the literature, although there are general theoretical attempts to do so such as reader oriented theory (Weinberg & Wiesner, 2011) and model of textbook use (Rezat, 2006).

Mathematics curriculum in Turkey has undergone reforms starting from the year 2005 to meet the requirements of the new century. Thus, from the year 2008; new curricula for primary and elementary education and secondary education have been put into effect (Alacacı, Erbaş, & Bulut, 2011). This improvement has the approach that "every student can learn mathematics" and advocates the importance of students' making connections between prior and new knowledge. In addition, this student-centered innovation has suggested that students need to be adapted to a new approach in learning mathematics with seven key components constituting the learning cycle *problem, discovery, hypothesizing, verification, generalization, making connection*s, and *reasoning*. Moreover, apart from traditional assessment methods, authentic assessment techniques have been necessitated for the assessment of students' learning processes (MEB, 2011).

Although development of mathematics curriculum has continued so far with minor changes made every year, the underlying traditional philosophy of mathematics education has not changed. As the curriculum has been reorganized every year since 2005, the textbooks being used have changed naturally in order to be consistent with the new curricular content and goals (Alacacı et al., 2011). There is a need to search if the changes in Turkish textbooks themselves are in useful directions or not, by comparing a Turkish textbook to others from different countries and programs.

Due to its high scores in international mathematics exams lately, Singapore has attracted the attention of people in education all around the world. For instance, in the United States, Singaporean textbooks are used in some school districts as teachers and mathematicians like them because of their simple approach (Hoven & Garelick, 2007). When the structure of secondary school mathematics education in Singapore is analyzed, it is clear that mathematical problem solving skill is located at the center of learning outcomes and supported by the following five crucial components of student learning: *concepts, skills, processes, attitudes,* and *metacognition* (Singapore Ministry of Education, 2006).

The IBDP is being used in many schools all around the world—including Turkey and Singapore—with its unique approach to mathematics education, which values students' realizing the power and significance of mathematics in real life together with its international and historical aspects, developing problem solving skill and using mathematics as a way of communication (IBDP, 2006).

As Singapore is an exemplary country in terms of the outcomes of mathematics education, and the IBDP is being used in twenty-six schools in Turkey, a Turkish mathematics textbook will be compared with two mathematics textbooks selected from Singapore and the IBDP (International Baccalaureate, 2012b).

Regarding the features of effective mathematics textbooks, Weinberg & Wiesner (2011) interpreted ideas of reader-oriented theory in the domain of mathematics textbooks by taking into account the student-textbook interaction as well as teachers' impact on shaping students' ways of using mathematics textbooks. Additionally, in his model of textbook use Rezat (2006) investigated the role of mathematics textbooks according to their suitability in the following sub-models: student-teacher-textbook, student-textbook-mathematical knowledge, teacher-textbook-mathematical knowledge (didactical aspects), and student-teacher-mathematical knowledge.

Problem

There are many studies in the literature about textbook analysis and evaluation which include comparisons of textbooks from different countries such as Ahuja's (2005) comparison of American and Singaporean mathematics textbooks and Haggarty and Pepin's (2001) comparison of English, French, and German mathematics textbooks. Current textbook analyses generally give information about what type of textbook properties give students the opportunity to learn mathematics effectively without making concrete judgments. This situation probably results from lack of a common textbook theory that deals with qualities of effective textbooks. In this study, reader-oriented theory (Weinberg & Wiesner, 2011) and Rezat's (2006) model of textbook use will serve as a general framework for the comparison of textbooks.

Purpose

The purpose of this study is to describe and compare the units on quadratics among the three mathematics textbooks from Singapore, Turkey and the IBDP from the perspectives of content, organization, and presentation style, by using content analysis as a research method. In addition, it is aimed to reveal what features of the three mathematics textbooks are supported by reader-oriented theory (Weinberg & Wiesner, 2011) as well as the sub-models in Rezat's (2006) model of textbook use.

Research questions

Questions of this study are given in the following:

- How do mathematics textbooks cover quadratics in Turkey, Singapore and the IBDP in terms of content (general structure of the three textbooks, position and weight of quadratics over the whole textbooks as well as the corresponding mathematics curricula, categorization of all the pages in the unit under titles based on learning outcomes, and the degree of each book's coverage of these titles)?
- How do mathematics textbooks cover quadratics in Turkey, Singapore and the IBDP in terms of organization (arrangement, categorization, order, frequency, and repetition of concepts and activities in the unit)?
- How do mathematics textbooks cover quadratics in Turkey, Singapore and the IBDP in terms of presentation style (place, order, and type of student-centered activities, topic explanations, real life connections, use of technology, problems and exercises, and historical notes)?

Significance

In this study, the purpose is to make a contribution to the findings of previous cross-national textbook analysis studies by investigating similarities and differences among the chapters on quadratics in the three mathematics textbooks from the different countries and programs. Approaches of the three different educational systems in terms of the opportunity given to students to learn mathematics are compared through reflections of the respective mathematics curricula in the textbooks.

The comparison of mathematics textbooks from a number of perspectives intends to uncover desired content, organization and presentation features of mathematics textbooks in terms of facilitation of teaching and learning during both the students' reading processes and the teachers' making use of teaching and learning activities in classrooms.

Another significant aspect of the study is the use of reader-oriented theory (Weinberg & Wiesner, 2011) while assigning values to characteristics of mathematics textbooks. In this way, it is aimed to render the study a guide for future textbook evaluation studies by giving researchers insights that may be useful in developing ideas for a new textbook theory, so that curriculum planners, teachers, and students can benefit from the outcomes of such research.

Lastly, the discussion on how to use distinct parts of the textbooks in order to maximize effectiveness in student learning in the light of Rezat's (2006) model of textbook use is expected to give teachers useful hints in planning lessons, especially in-class activities.

Definition of key terms

Mahmood (2010) defines *textbook* as “a source of potential learning as to what students learn”, by adding that “the practicality of that learning is mediated by the school context (teacher, peers, instruction, and assignments)” (p.16).

Reader-oriented theory is a part of literary criticism that attempts to explain how readers make meaning of a text through the whole reading activity, which is an effort shaped by the author’s intention, the reader’s perception and the qualities the text requires the reader to acquire (Weinberg & Wiesner, 2011).

The model of textbook use is a three dimensional tetrahedron shaped textbook usage representation which displays the relationships among the four corners: student, textbook, teacher, and mathematical knowledge/didactical aspects of the mathematical knowledge. Each set of three corners represents a triangular relationship consisting of the subject, the object and the mediated artifact (subject-mediator-object) (Rezat, 2006).

CHAPTER 2: REVIEW OF RELATED LITERATURE

Introduction

Textbooks are important indicators of an intended mathematics curriculum that affect both teaching and learning. Therefore, it is useful to review the importance and role of mathematics textbooks in learning mathematics.

The philosophies of mathematics education in Turkey, Singapore, and the IBDP—whose textbooks are being examined—have to be taken into account in the process of analyzing and comparing the three mathematics textbooks. It is necessary to consider the intended learning outcomes of these three mathematics education systems, what skills they aim for students to gain, and what they value in mathematics education.

As one main purpose of studies conducted in the area of textbook analysis is to identify the qualities of effective textbooks, the reader-oriented theory (Weinberg & Wiesner, 2011) and Rezat's (2006) model of textbook use will be discussed and used. They give two perspectives to comment on the design features of the three mathematics textbooks.

Although there are a large number of studies conducted on textbooks, including the role of mathematics textbooks in teaching and learning mathematics as well as the analysis of mathematics textbooks from various perspectives, there is a need for textbook studies that can help identify qualities of the better textbooks by making connections with textbook theories.

Textbooks in mathematics education

Importance and role of textbooks in mathematics education

Textbooks are essential components of mathematics education because they provide a guide for many teachers about what types of mathematical knowledge will be taught, to which group of students, when, and how. In other words, since the mathematics curriculum in many schools is presented through textbooks, teachers and students use textbook materials as primary sources for their preparation, class work, and homework (Apple, 1992; Ben-Peretz, 1990; Schmidt, McKnight, & Raizen, 1997). As Rock (1992) states succinctly, “it appears in most mathematics classes in most schools, that the curriculum for mathematical knowledge and learning is defined by the curricular materials, primarily by the mathematics textbook” (p.30).

Howson (1993) argues that textbooks have different roles in classrooms and in educational systems, including schools, classrooms, and students of a country. In the classroom, he suggests that a textbook provides a source of problems and exercises, and displays kernels—*theorems, rules, definitions, procedures, notations, and conventions to be learned as knowledge*—and explanations—*to prepare students for the kernels*—like a reference book.

Regarding the role of textbooks in an educational system, Foxman (1999) claims that this distinction between the classroom and the educational system indicates two of the three levels within the curriculum model used by TIMSS (Third International Mathematics and Science Study), which was used in the Second International Mathematics Study. According to this model, the three levels of curriculum are: the *intended* curriculum (the one stipulated in official documents), the *implemented* curriculum (what is fulfilled by students and teachers in classrooms), and the *attained* curriculum (acquired knowledge, skills, understanding, and attitudes by students) (Travers & Westbury, 1989).

Textbooks and other resource materials are included in the fourth level of the curriculum model above, *potentially implemented curriculum*, which was integrated into the model later (Johansson, 2003; Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002). At this point, Schmidt, McKnight, Valverde, Houang, and Wiley (1997) suggest that textbooks affect the intended and implemented curricula like a connector between them, by stating that "textbooks serve as intermediaries in turning intention to implementation" (p. 178).

Teachers' use of mathematics textbooks

Teachers benefit from textbooks as guides and sources to create their daily lesson plans by deciding on the content to be taught, the order of topics, classroom activities, and the homework to be given to students (Freeman & Porter, 1989; Nicol & Crespo, 2006; Haggarty & Pepin, 2001; Sosniak & Perlman, 1990).

Having investigated whether and how a new mathematics text could promote two fourth-grade teachers' learning in the first year of using the text, Remillard (2000) reports that teachers also learn from mathematics textbooks as textbooks affect their view of the nature of mathematics, mathematical content and students. Teachers shape their decisions in consultation with textbooks by exploring the content to prepare for teaching, predicting points of student confusion, and deciding on how to engage in mathematical thought with students (Russell, Schifter, Bastable, Yaffee, Lester, & Cohen, 1995).

Textbooks do not replace teachers since teachers have a crucial role in mediating the content to students (Love & Pimm, 1996). However, teachers are generally flexible in using textbooks; they adopt the content according to their intentions by rearranging the order of topics and selecting the parts to teach (Freeman, Kuhs, Porter, Floden, Schmidt, & Schwille, 1983; Stake & Easley, 1978).

Teachers' ability to decide how much time to allocate for each topic or activity in the textbook and their capability of integrating the textbook to their lessons appropriately affects students' ability to gain knowledge from textbooks (St. George, 2001). In their study with pre-service teachers, Nicol and Crespo (2006) suggest that since such teachers are novices in developing teaching methods to meet their students' needs, teacher education programs can increase their awareness about the roles and significance of texts in teaching and learning mathematics.

Studies show that teachers have a tendency to cover the content presented in the textbooks they use, and they rarely teach content which is not available in the textbook (Freeman & Porter, 1989; Reys, Reys, Lapan, Holliday, & Wasman, 2003). Therefore, textbooks are important indicators of how much opportunity is offered to students in terms of learning a mathematics topic comprehensively since the emphasis given to the topic in the textbook affects teachers' instruction in classrooms and the time they allocate to a topic (Törnroos, 2004). In fact, the frequency of teachers' using textbook materials increases remarkably as students get older; secondary school teachers are more dependent on textbooks than their colleagues in primary schools (Askew, Brown, Johnson, Millett, Prestage, & Walsh, 1993).

Teachers' using textbooks as a primary source results in their being used by students, too. That is, students make use of textbooks for many reasons such as reviewing what they learn in the classroom including exercises and tasks, analyzing worked examples, and pursuing mathematical coursework introduced by the teacher (Rezat, 2008).

Textbooks as indicators of mathematics education system

Mathematics textbooks reflect nations' perspective on mathematics education, their curricular goals, and way of understanding mathematics through cultural messages, emphasis on different outcomes and signs of thinking processes (Haggarty & Pepin, 2001). Since school textbooks reflect a curriculum, some teachers using textbooks actively follow the textbook as primary guides to form their instructional plans more than the curricular agenda or program of the course (Apple, 1986).

In fact, the precise purpose and role of textbooks differ according to the educational systems of countries due to the fact that their priorities vary in terms of the desired skills, competencies, theoretical and practical knowledge to be gained by students (Howson, 1993). Due to textbooks' reflecting distinctive curricular goals, content and organizational emphases and presentation styles, they can give hints about the way mathematics is taught within educational systems of countries where they are used (Schmidt, McKnight, Valverde, Houang, & Wiley, 1997).

Textbooks strive for meeting one or more of the following targets, based on the structure of the educational system: delivering a centrally prescribed curriculum in detail (i); struggling to amend pedagogy within a centrally prescribed curriculum (ii); following recent non-governmental suggestions on pedagogy such as National Council of Teachers of Mathematics (NCTM) Standards in the United States or the Cockcroft Report in Britain (iii); and contributing in defining a new curriculum (iv) (Howson, 1993). Using this approach, he claims that the Japanese texts can only intend to fulfill (i) and (ii) since the textbooks always strive for explaining elements of the national curriculum as well as increasing students' motivation, going ahead from the concrete to the abstract, and using contextualized problems. The French texts, on the other hand, display an improvement in enhancing the pedagogy within its centrally prescribed curriculum by benefiting from technological facilities, which points out (ii). Lastly, he argues that textbooks in England aim to fulfill (i) and (ii), and strive for getting ahead regarding (iv).

In summary, textbooks appear to be indispensable sources of education and more research should be conducted in order to explore features of effective mathematics textbooks through comparing textbooks of different countries and programs.

Philosophies of mathematics education in the Turkish, Singaporean, and IBDP contexts

Dowling (1996) exemplifies the close connection between the educational traditions of a country and its textbooks by arguing that school textbooks shape learners' view of mathematics, and the language of description and hidden messages delivered through textbooks affect learners' decisions about their future professions.

The literature supports the fact that textbooks, to some extent, reflect the nature of the mathematics education system, and therefore the curriculum of countries. Thus, it is necessary to evaluate textbooks from different countries by taking into account the philosophies of mathematics education in each country. The same situation is valid for the educational programs as they have their own perspectives, curricular goals and ways of perceiving mathematics. Therefore, the philosophy of mathematics education underlying the IBDP should be investigated besides those of the two countries: Turkey and Singapore.

Perspectives of Turkey, Singapore and the IBDP into mathematics education, their expectations from students in regard to learning of mathematics, their values in mathematics education, and what they aim to deliver by means of textbooks affect textbooks as well as their use in classrooms.

Mathematics education in the Turkish context

International examinations such as TIMSS-R (TIMSS-Repeat) 1999, PIRLS (Progress in International Reading Literacy Study) 2001, and PISA (Programme for International Student Assessment) 2003, 2006, and 2009 revealed that Turkish students' scores were lower than those of many countries (Karadağ, Deniz, Korkmaz, & Deniz, 2008). Outcomes of such international studies, educational reforms all around the world, and educators' efforts about increasing quality and access to education in Turkey resulted in a deep change in Turkish secondary school mathematics program, from the 2005-2006 academic year. The gradual transition to the new mathematics education system was completed in the 2008-2009 academic year, and the application of the new curriculum was started in company with the new textbooks, correspondingly prepared (Alacacı et al., 2011).

Turkey has a centralized mathematics curriculum prepared by the Turkish Ministry of National Education (*Milli Eğitim Bakanlığı* [MEB]), and the curriculum is based on the idea that "every student can learn mathematics." The program believes that learning mathematics is done not only to gain basic knowledge and skills but also to think mathematically, comprehend the strategies of solving mathematical problems, develop a positive attitude towards mathematics and believe in mathematics as a significant tool in daily life. At the end of the program, students are expected to comprehend concepts and systems, construct connections between concepts and use them in other subject areas, make inferences about induction and deduction, express their thoughts effectively by using tools of mathematical communication, benefit from mathematics to solve real-life problems, and appreciate the nature of mathematics (MEB, 2011).

As represented in Figure 1 below, the current approach in the mathematics curriculum emphasizes a transition from the teacher-centered traditional style to a newly adopted learning cycle, which is more student-centered and gives students the opportunity to start the mathematical processing with a problem case and finish it by reaching a mathematical inference. In this new approach, instruction starts with a problem, continues with discovery, hypothesizing, verification if applicable, generalization, and making connections. Supporting mathematical reasoning is the ultimate goal (MEB, 2011).

Traditional approach	Newly adopted learning cycle
Definition	Problem
Theorem	Discovery
Proof	Hypothesizing
Applications and test	Verification
	Generalization
	Connections
	Reasoning

Figure 1. The traditional versus the new approach in Turkish secondary school mathematics education in instructional explanation (MEB, 2011)

Students are expected to acquire the following skills within the current mathematics program: problem solving with their own solution styles, establishing relationships between mathematical concepts, using mathematics as an effective way of communication, benefiting from mathematical modeling to construct relationships between mathematics and real-life connections, and doing mathematics through reasoning (MEB, 2011). Regarding mathematical modeling, Bukova-Güzel (2010), who examined the approaches of pre-service mathematics teachers in constructing and solving mathematical modeling problems, suggests that for the first time in a Turkish public university a modeling course was put into practice in a secondary mathematics teacher training program in order to educate teachers according to the requirements of the new system. This shows that application of mathematical modeling is quite new in Turkish secondary schools.

Moreover, creating a new learning culture through the use of technology, not as a teaching but as a learning tool, aims to complement the system and turn mathematics classrooms into laboratories where students can investigate mathematical relationships (MEB, 2011). After analyzing students' views about using graphing calculators to learn functions and graphs, Ersoy (2007) states that although the use of technology in mathematics classrooms is becoming popular in developed countries, Turkish secondary schools still have difficulties in benefiting from technology appropriately, except for private schools.

The topics in the mathematics curriculum include six main areas (logic, algebra, trigonometry, linear algebra, probability-statistics, and fundamental mathematics) and 63 subtopics. The unit *quadratic equations, inequalities, and functions* is presented within the main topic *algebra* (MEB, 2011).

Apart from the formative and summative assessment applied in schools during each academic year, Turkish students have taken the University Entrance Exam at the end of high school since at least 1974 (ÖSYM, 2012b). The exam is prepared and assessed by the Measurement, Selection, and Placement Center (*Ölçme, Seçme ve Yerleştirme Merkezi* [ÖSYM]). According to their success in this exam and their preferences, students are assigned to universities and departments by ÖSYM for undergraduate education. Currently, the exam is conducted through two sessions: Passing to Higher Education and Undergraduate Placement Exams (*Yükseköğretime Geçiş Sınavı* [YGS] and *Lisans Yerleştirme Sınavı* [LYS] respectively). Both exams have mathematics sections that include only multiple choice items from the whole high school mathematics curriculum. The number of items and duration of the mathematics parts within the two exams are: 40 items and approximately 40 minutes in YGS, and 80 items and 2 hours in LYS (ÖSYM, 2012a). The same assessment system is also present while Turkish students are passing from elementary to secondary school. That is, Level Determination Exam (*Seviye Belirleme Sınavı* [SBS]) is applied at the end of the 8^{th} grade to place students in the appropriate secondary schools according to their level of achievement (MEB, 2012).

The selection of textbooks being used in Turkish secondary school classrooms is significant in students' fulfilling the requirements determined by MEB. Thus, the number of studies about textbook analysis has increased in Turkey. The goal of these research studies is to reveal deficiencies in current textbooks and help to develop more qualified textbooks which will contribute to teaching and learning.

Mathematics textbooks being used in Turkish high schools need to be improved as they are not effective in terms of facilitating student learning, meeting expectations of teachers, including sufficient real world applications of the topics, encouraging students to use technology, and arousing students' interest in mathematics (Şahin, 2005). At this point, comparing Turkish textbooks with those of other countries may be useful to identify qualities of better textbooks for high schools.

Mathematics education in the Singaporean context

International studies comparing students' achievement in mathematics education within the last twenty years have revealed that students from Asian countries such as Mainland China, Hong Kong, Taiwan, Singapore, Korea, and Japan got remarkably better results than students from other countries (Zhu & Fan, 2004).

As Mullis, Gonzalez, Gregory, Garden, O'Connor, Chrostowski, and Smith (2000) indicate, in TIMSS 1999, Singaporean students got the top average score out of 38 participant countries. Similarly, in TIMSS 2003, Singapore ranked first among 46 countries, which is significant evidence for Singapore's outstanding success in mathematics education. The topic *algebra,* where the unit *quadratic expressions and equations* is included, was among the mathematics content areas in which Singaporean students scored best (Mullis, Martin, Gonzalez, & Chrostowski, 2004). The difference between students' performance in these studies drew researches' attention to mathematics education in Singapore. While searching for the reason behind the consistent success of the country, many researchers analyzed mathematics textbooks used in Singapore with the idea that textbooks have an important place in teaching and learning mathematics (Zhu & Fan, 2004).

Analyzing the preferred qualities of the Singaporean mathematics education in comparison to that of the U.S., in terms of frameworks, textbooks, and assessments, the American Institutes for Research (2005) indicated that Singapore offers a reasonable and unique national framework for mathematics education that presents each topic more deeply than the U.S. does. In this framework, there is an alternative structure for students who have more difficulties in mathematics lessons. Moreover, Singaporean textbooks enable students to understand mathematics deeply by including step by step problems and illustrations that represent how to use abstract concepts to solve problems. Another difference is that the items within Singaporean high-stakes exams are more challenging than those in the U.S. Similarly, Hoven and Garelick (2007) argue that Singaporean textbooks have been appreciated by both teachers and mathematicians due to their simple approach and well-organized curriculum as well as their being logically structured and focused on the essential skills of mathematics.

Singaporean mathematics textbooks are globally considered as high-class, when crucial requirements of a textbook are taken into consideration, such as meeting the desired standards, containing ordinary and extraordinary problems, and using a pedagogically appropriate style. They facilitate students' understanding of mathematical concepts and acquisition of essential skills by avoiding redundant replication and simplifying abstract knowledge through the use of real-life connected examples and pictorial expressions (Ahuja, 2005).

Regarding mathematics education in Singapore, similar to the Turkish national education system, Singapore's public education is centralized, with common learning objectives and curricular programs for all schools in the country. Like Turkey's having two high-stakes examinations at the end of elementary and secondary school, Singaporean students are also assessed in the 6^{th}, 10^{th}, and 12^{th} years of their education through three high-stakes examinations. The leading institution for the centralized system is the Singapore Ministry of Education. The supportive institution in terms of constructing the structure of the frameworks within the national curriculum and conducting national assessment examinations is the Cambridge General Certificate of Education, just like Turkey's *MEB Talim Terbiye Kurulu Başkanlığı* and ÖSYM (National Center on Education and the Economy, 2008).

The results of Primary School Leaving Exam (PSLE), at the end of 6th year, are used to determine which of the three secondary levels Singaporean students will attend: express, normal academic or normal technical. As seen in figure 2, each of these levels contains mathematics to varying depths. Regarding the assessment of mathematics, at the end of four years of secondary education, special and express level students take the Singapore-Cambridge GCE O Level (General Certificate of Education Ordinary Level) Mathematics Exam, whereas normal academic (NA) and normal technical (NT) level students take the Singapore-Cambridge GCE N Level (General Certificate of Education Normal Level) Mathematics Exam. In addition, normal level students who have successful results from the GCE N Level Exam have the opportunity to study for a 5th year and then to take the GCE O Level Exam (Singapore Ministry of Education, 2011).

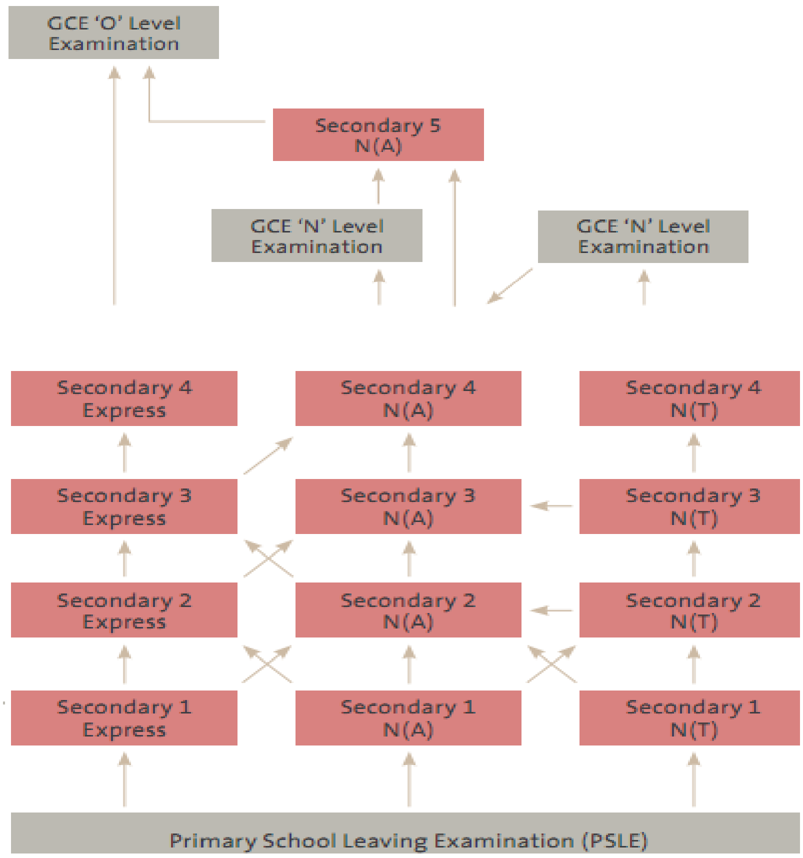


Figure 2. Flexibility between courses in the Singaporean secondary school education system (Singapore Ministry of Education, 2011, p. 7)

The mathematics framework within the Singaporean secondary school mathematics education is organized around mathematical problem solving, which represents attaining and applying mathematical knowledge and skills in a broad range of contexts, including real-life connected problems. Mathematical problem solving skills are expected to improve based on five elements: *concepts, skills, processes, attitudes*, and *metacognition*, as displayed in Figure 3 below (Singapore Ministry of Education, 2006):

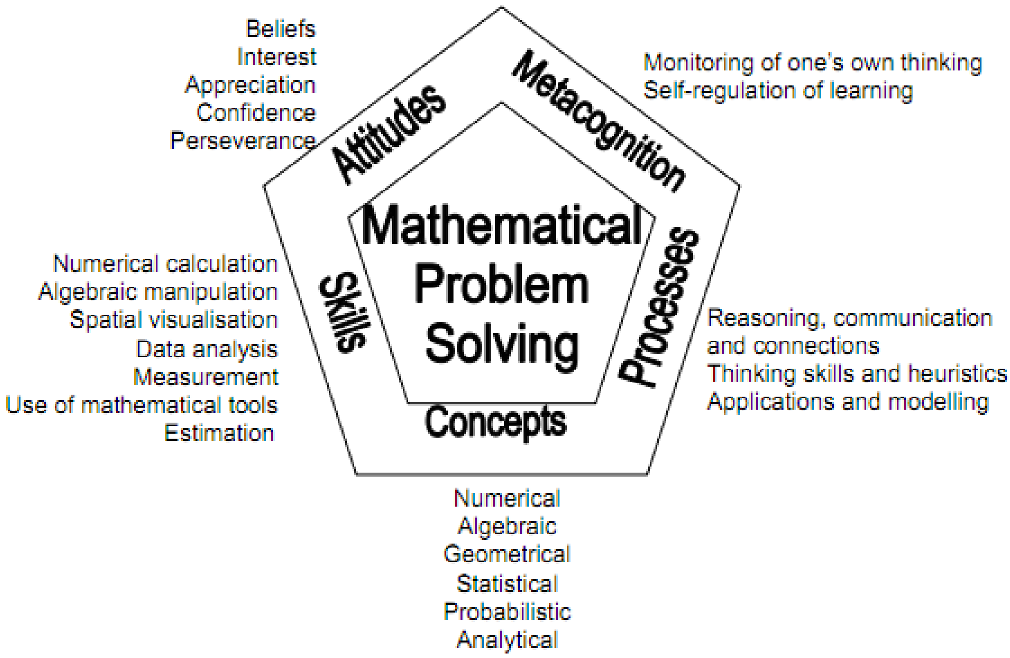


Figure 3. The mathematics framework within Singaporean secondary school mathematics (Singapore Ministry of Education, 2006, p. 2)

In the Singaporean mathematics education system, students are expected to attain the required mathematical concepts and skills for daily life, experience essential process skills for the attainment and implementation of mathematical concepts and skills, become proficient in problem solving and mathematical thinking, make use of connections among mathematical concepts and with other disciplines, appreciate the value of mathematics, benefit from mathematical tools appropriately, use mathematical ideas in a creative way, and develop independent thinking, cooperative learning, and reasoning skills (Singapore Ministry of Education, 2006).

GCE O level additional mathematics is offered in the last two years of the four-year secondary education in Singapore. The main mathematics topics included in syllabus of this course are algebra, geometry and trigonometry, and calculus. As stated before, the chapter *quadratic expressions and equations* is covered under the first main topic *algebra*. Additionally, the Singapore Examinations and Assessment Board (2012) indicates that the GCE O Level Mathematics Exam has the following learning objectives in assessment:

> i. Understand and use mathematical concepts and skills in a variety of contexts;
> ii. Organize and analyze data and information; formulate problems into mathematical terms and select and apply appropriate techniques of solution, including manipulation of algebraic expressions;
> iii. Solve higher order thinking problems; interpret mathematical results and makes inferences; reason and communicate mathematically through writing mathematical explanation, arguments, and proofs. (p.2)

Assessment of this course consists of two papers: Paper 1 and Paper 2. Regarding durations and number of questions contained, Paper 1 lasts 2 hours and includes 11-13 questions, whereas the duration of Paper 2 is 2.5 hours for 9-11 questions. Moreover, Paper 1 has a weight of 44% while Paper 2 constitutes 56% of the whole assessment of the course (Singapore Examinations and Assessment Board, 2012). Furthermore, use of scientific calculators is permitted in both Paper 1 and Paper 2.

Lastly, it is assumed that distinctive properties of a Singaporean textbook, which is prepared compatible with Singapore's popular mathematics education system, will be helpful in evaluating and comparing the textbooks in this study.

Mathematics education in the IBDP

The International Baccalaureate Diploma Programme (IBDP) is offered by the International Baccalaureate (IB), formerly International Baccalaureate Organization (IBO), which is an international education foundation founded in 1968, currently working with 3371 schools in 141 countries including Singapore and Turkey (IB, 2012a & 2012b).

The IBDP was first developed for international schools, which are generally private schools around the world that appeal to education needs of children living abroad. In those days, international schools were in search of a program that would enable students' entrance to universities both in their home or other countries, maybe with an extra idea of delivering international education (Peterson, 2003). Then, the IBDP has started to be used in many countries of the world with its global curriculum and own assessment style (Paris, 2003). Moreover, it has been implemented not only in international schools, but also in other types of schools within private and state contexts (Hayden, 2006).

The IBDP is a two-year, demanding, pre-university course of studies which aims to meet the needs of highly motivated secondary school students between the ages of 16 and 19, by including extensive student-centered opportunities (IB, 2012c; Van Tassel-Baska, 2004).

Students following the IBDP study six courses at two levels: higher level (HL) or standard level (SL). They are expected to enroll in one subject from each of the following five groups in the hexagonal framework: studies in language and literature (i), language acquisition (ii), individuals and societies (iii), experimental sciences (iv), and mathematics and computer science (v); as well as a sixth subject either from the sixth group, the arts, or from groups (i) to (v) again. Moreover, successful IBDP students fulfill the following three requirements in addition to these six subjects: the interdisciplinary theory of knowledge (TOK), the extended essay (an essay of approximately 4000 words about a topic of special interest with independent research, which can be a mathematics topic), and participation in creativity, action and service (CAS) activities (IB, 2012c).

Mathematics is taught as four different courses in the IBDP: mathematics standard level, mathematics higher level, mathematical studies standard level and further mathematics standard level. Students choose one of these four courses according to their individual needs, interests and abilities (IBDP, 2006).

In IBDP mathematics, it is aimed to enable students to appreciate the power and usefulness of topics together with multicultural and historical perspectives of mathematics, strengthen creative and critical thinking skills, have an understanding of the principles, and develop persistence in problem solving. Since international education is crucial in the IBDP mathematics curriculum, students' acquisition of an international understanding of mathematics is quite important; students are expected to learn mathematical notations, outstanding mathematicians' lives, mathematical discoveries, approaches of different societies towards mathematics, and mathematics as a way of worldwide communication (IBDP, 2006; Tilke, 2011).

IBDP students are expected to read, interpret and solve a given problem by using appropriate mathematical notations and terminology; formulate a mathematical argument and communicate it clearly by using appropriate mathematical strategies and techniques; and recognize and demonstrate an understanding of the practical applications of mathematics accompanied with technological devices (IBDP, 2006).

IBDP mathematics standard level (SL) includes the following seven main topics: algebra, functions and equations, circular functions and trigonometry, matrices, vectors, statistics and probability, and calculus. The unit *quadratic equations and functions* is contained within topic 2 (*functions and equations*). Students are assessed both internally by schools applying the IBDP and externally by IB. External assessment is conducted through Paper 1 and Paper 2, which are 90-minute examinations with short and extended response questions based on the whole syllabus. Each paper has a 40% weight in the overall assessment and the only difference between them is that students are allowed to use a graphing calculator in Paper 2, but not allowed in Paper 1. In addition to this, students are assessed internally with a weight of 20% through portfolio assignments, which are two pieces of work requiring mathematical investigation and mathematical modeling (IBDP, 2006).

Qualities of effective mathematics textbooks

An evaluation and comparison of textbooks ought to provide results that can be used in identifying features of effective mathematics textbooks. In this study, to obtain such results, reader-oriented theory (Weinberg & Wiesner, 2011) and Rezat's (2006) model of textbook use will be taken into consideration in the process of analyzing the three mathematics textbooks from Turkey, Singapore, and the IBDP. In this way, the research will provide clues about which property of each book could be more or less effective than that of the others in terms of giving opportunities to students in terms of facilitation of learning and reading.

Mathematics textbooks through reader-oriented theory

Student capacity to use textbooks effectively is one of the factors contributing in learning mathematics. In this sense, a mathematics textbook's power of guiding students to develop an understanding of mathematics is crucial.

Although the student-textbook relationship is known to be significant, there is little research explaining how students actually use textbooks in learning mathematics. For instance, in the study conducted by Weinberg, Wiesner, Benesh, and Boester (2012), undergraduate students are asked to explain the way they use mathematics textbooks. The outcomes of the study show that many students' use of a text is not compatible with the intended goals of the author. That is, instead of a consistent activity to improve mathematical understanding, students use textbooks as a tool to study for their exams through worked examples and to do their homework, which is also not intended by the instructors when they ask their students to follow the course from their textbooks. Moreover, this research argues that there are possible additional factors affecting students' use of textbooks such as class alignment with the textbook, perceived instructor requests, and student values.

Similarly, an investigation of three undergraduate students' ways of doing homework from a textbook revealed that most of their homework time is allocated to exercises and looking for similar solved examples and clues about procedures for solving problems, which shows that students pay insufficient attention to fundamental mathematical ideas and methods within the textbook (Lithner, 2003).

Weinberg and Wiesner (2011) support the arguments above by suggesting that students have difficulty in making appropriate use of textbooks as a mediator to increase their learning of mathematics. Thus, in their study, they investigate the elements shaping the ways students read textbooks. While doing this, they attempt to use reader-oriented theory as a tool.

Approaches used in the construction of textbooks can be put into two categories: *text-oriented* and *reader-oriented*. In text-oriented textbooks, readers are expected to decode the textbook. These textbooks act like a reference book including mathematical theorems, definitions, rules, procedures, and symbols. However, reader-oriented textbooks aim at students' internalizing mathematical concepts and methods by expecting them to construct meaning from the text, understand and appreciate the importance of what is learned, make connections with prior topics, and address and discuss exploratory questions asked by the author. Thus, reader-oriented theory attempts to explain how readers make meaning of a text through the activity of reading, by mainly dealing with the reader and the reading process (Weinberg & Wiesner, 2011).

Reader-oriented theory suggests that the productivity of a reader eliciting meaning from a text is identified by the following three factors: the 'intensions of the author', the 'beliefs of the reader', and the 'qualities the text requires the reader to possess'. Moreover, the three elements from reader-oriented theory should be taken into account: the intended reader, the implied reader, and the empirical reader. The *intended reader* symbolizes the reader that the author targets while writing the book, the *implied reader* is the one possessing all of the qualifications necessary for the reader in order to make meaningful inferences from the text, and *empirical reader* represents the actual individual reading the text. So, a textbook is regarded as a successful pedagogical mediator if the empirical reader coincides with the implied reader (Weinberg & Wiesner, 2011, p. 49-51).

Regarding the three types of readers in the reading process, the intended reader of a mathematics textbook is generally directed through a variety of supportive components such as solved examples and problems, which are not often used in academic texts. Thus, the difference between the intended reader of a textbook and that of an academic-level mathematics text can be identified by the author's assumptions and expectations implicit in the text. On the other hand, the implied reader is expected to have the *behaviors*, *codes*, and *competencies* necessary for the empirical reader to make use of the text appropriately. Imperatives such as 'work individually' or 'work with a partner' at the beginning of exercise sections, changes in the degree of difficulty among problems, and choices of places whether to use massed or independent practice orient student behavior while using mathematics textbooks. In addition, using codes in formatting, such as preference of particular heading styles for concepts, enables students to make inferences about the importance of concepts and their relationships with the previous and next topics. Moreover, the way mathematics textbooks explain concepts may help to ensure students' obtaining competencies such as comprehending components of concepts, constructing connections among these components, and acquiring necessary skills to understand them. Making use of such concepts from reader-oriented theory may help to create a framework to identify the features of mathematics textbooks that affect the ways students use textbooks to learn mathematics (Weinberg & Wiesner, 2011).

Through analyzing the properties of textbooks from Turkey, Singapore and the IBDP, a generalization can be reached about how much they facilitate the reading process of students by helping them develop behaviors, codes, and competencies that are essential for students to construct meaning from the text and, hence, understand mathematical concepts and procedures.

Rezat’s (2006) model of textbook use for mathematics textbooks

As textbooks occupy an important place in mathematics teaching and learning, the way they can be used most efficiently by students and teachers comes into question.

At this point, Rezat (2006) suggests that four questions need to be considered before looking for a model about textbook use.

> i. Is the textbook a pedagogical means or a marketed product?
> ii. Is the textbook an instrument for learning or the object of learning?
> iii. Is the textbook addressing the teacher or the student?
> iv. Is the textbook supposed to be mediated by the teacher or is its intention to substitute the teacher? (p. 410-411).

After an investigation of the questions above, the following results emerged: textbooks are produced with pedagogical purposes but are also a part of the market; they are perceived as instruments for learning or objects of learning by different educators; they address both students and teachers due to mathematical and informative essence of the knowledge; and there is a necessity for teachers’ mediating textbooks, although many textbooks seem to be a substitute for teachers (Keitel, Otte, & Seeger, 1980; Chambliss & Calfee, 1988; Griesel & Postel, 1983; Love & Pimm, 1996; Newton, 1990; Stein, 1995, as cited in Rezat, 2006).

Applying Vygotsky’s (1978) approach about activity theory, which explains the relationships between three components—subject, mediating artifact, and object—, in the context of teachers’ and students’ use of mathematics textbooks, three triangular models representing the relationships among the student, teacher, textbook, and mathematical knowledge are developed. These relationships include student-textbook-mathematical knowledge and student-teacher-textbook, as demonstrated in Figure 4.

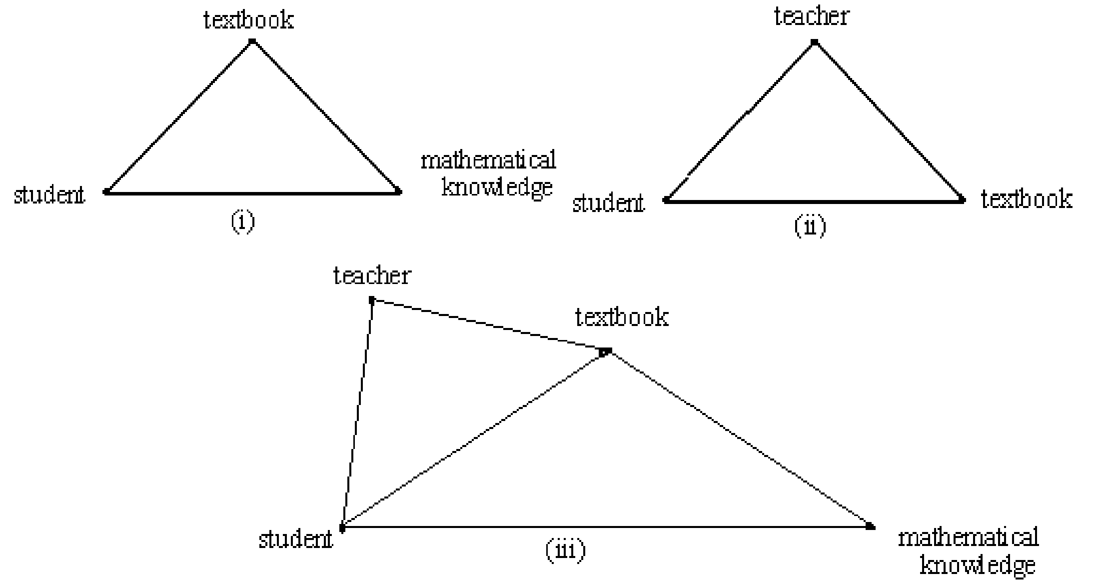


Figure 4. Triangular representations of the relationships among student, textbook, teacher, and mathematical knowledge (Rezat, 2006, p. 411-412)

Rezat (2006), after analyzing the models above, argues that the model of textbook use is best demonstrated by using the shape of a three dimensional tetrahedron, which is more comprehensive because of the four well-located corners: *student*, *textbook*, *teacher*, and *mathematical knowledge/didactical aspects of the mathematical knowledge*. Thus, this time, it displays all possible relationships among the components in textbook use, including the contexts teacher-textbook-mathematical knowledge (didactic aspects) and student-teacher-mathematical knowledge, as represented in Figure 5 below:

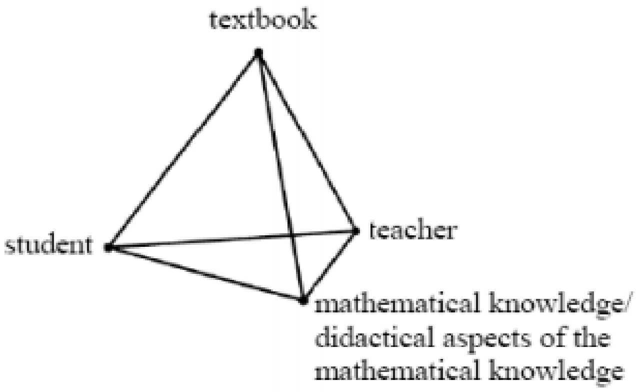


Figure 5. The model of textbook use (Rezat, 2006, p. 413)

In the model displayed above, each set of three corners represents a triangular relationship consisting of the subject, the object and the mediated artifact (subject-mediator-object). Namely, the set student-teacher-textbook represents the triangular sub-model where the student is the active subject, the textbook is the object used, and the teacher is the component who mediates the relationship. The triangle student-textbook-mathematical knowledge, on the other hand, represents the context where the textbook—as instrumentation—mediates between the student and the mathematical knowledge. The set teacher-textbook-mathematical knowledge (didactical aspects) means that the textbook mediates the teacher's use of mathematical knowledge or didactical aspects of mathematical knowledge within the textbook. Finally, the student-teacher-mathematical knowledge triangle displays the case where the teacher mediates the knowledge by applying it in the classroom with the aim of delivering it to students (Rezat, 2006).

Rezat's (2006) model of textbook use will be used in the process of deciding to what extent the mathematics textbooks from Turkey, Singapore and the IBDP fulfill their role of being mediating artifacts.

Summary and conclusions

Textbook evaluation and comparison studies have revealed differences among textbooks and respective curricular programs of different countries. However, in this research, after comparison of the textbooks according to specific criteria, results will be discussed on the basis of the reader-oriented theory (Weinberg & Wiesner, 2011) and Rezat's (2006) model of textbook use. At the beginning of the literature review, the importance of textbooks in mathematics education is discussed in order to understand the significance of the study for mathematics education in general. Then, philosophies of mathematics education in Turkey, Singapore and the IBDP are mentioned since the three textbooks to be analyzed are selected from these contexts. Finally, the two theories will be used in the process of discussing the results of the study in order to explain findings in regard to mathematics textbook development in a useful and realistic sense.

CHAPTER 3: METHOD

Introduction

This study used content analysis to address the following research questions:

- How do mathematics textbooks cover quadratics in Turkey, Singapore and the IBDP in terms of content (general structure of the three textbooks, position and weight of quadratics over the whole textbooks as well as the corresponding mathematics curricula, categorization of all the pages in the unit under titles based on learning outcomes, and the degree of each book's coverage of these titles)?
- How do mathematics textbooks cover quadratics in Turkey, Singapore and the IBDP in terms of organization (arrangement, categorization, order, frequency, and repetition of concepts and activities in the unit)?
- How do mathematics textbooks cover quadratics in Turkey, Singapore and the IBDP in terms of presentation style (place, order, and type of student-centered activities, topic explanations, real-life connections, use of technology, problems and exercises, and historical notes)?

A plan for systematic analysis of the three textbooks based on the criteria (content, organization, and presentation style) is described here, including design of the research, context of the study, and the method of data coding and analysis.

Research design

Content analysis was the most appropriate research design for this study, through which analysis of a body of texts could be made. In addition, counting key factors within the units, the comparison of the three quadratics units, and the interpretation of data according to content, organization and presentation style were all parts of this content analysis study. Data from this study is primarily qualitative, as qualitative methods can enable a study to analyze issues deeply and comprehensively (Patton, 1990). Furthermore, quantitative data such as the number of learning outcomes met in each textbook, the number of items within exercise sections, the number of student-centred activities, and the percent of pages allocated to specific aims within the three units was used to have an understanding of the issues.

Content analysis is defined as “a research method for the subjective interpretation of the content of text data through the systematic classification process of coding and identifying themes or patterns” (Hsieh & Shannon, 2005, p. 1278). It aims to produce “replicable and valid inferences from texts (or other meaningful matter) to the contexts of their use” (Krippendorff, 2004, p. 18). According to Weber (1990), “these inferences are about the sender(s) of the message, the message itself, or the audience of the message” (p. 9). Therefore, these three components of the reading process were taken into consideration throughout this study.

Context

The three units on *quadratics* were analyzed in the textbooks displayed in Table 1.

Table 1

Textbooks analyzed in the study

Turkey	Kaplan, E. (2008). *Ortaöğretim Matematik 10 Ders Kitabı* (pp. 55-121). PaşaYayıncılık Ltd.
Singapore	Thong, H. S., & Hiong, K. N. (2006). *New Additional Mathematics* (pp. 61-80). Singapore: SNP Panpac Publishing House.
IBDP-SL	Owen, J., Haese, R., Haese, S., & Bruce, M. (2008). *Mathematics for the International Student Mathematics SL* (pp. 149-194). SA, Australia: Haese & Harris Publications.

These three mathematics textbooks were referred to as the Turkish, Singaporean, and IBDP-SL mathematics textbooks throughout this study.

Textbooks in Turkish classrooms have undergone some changes, depending on the curricular arrangements in Turkish high school mathematics education made every year since 2005, to be consistent with the new student-centered approach (Alacacı et al., 2011). It is necessary to investigate if the innovations in Turkish textbooks are in beneficial directions or not, by comparing a Turkish textbook to other textbooks from different countries and programs. The research can also provide insights about features of effective textbook design and assumptions in the educational systems and curricula, by looking into multiple textbooks covering the same subject.

It is necessary to include the Singaporean textbook in this study due to a couple of facts. Singaporean students have been successful in international mathematics exams such as PISA and TIMSS lately. In addition, there is a widely-held view that Singapore's textbooks' are high-class in terms of meeting the desired standards, containing both ordinary and extraordinary problems, using an appropriate pedagogical approach, and being logically structured and focused on the essential skills of mathematics with a well-organized curriculum (Ahuja, 2005; Hoven & Garelick, 2007).

The IBDP, on the other hand, is applied in twenty-six Turkish high schools and students are expected to fulfill both the IBDP and the MEB curricula requirements in order to receive a high school diploma (Halıcıoğlu, 2008). Therefore, in these schools both IBDP and Turkish mathematics textbooks are used, which makes it reasonable to include an IBDP mathematics textbook in this study.

The Turkish textbook used in this study is the one approved by the Turkish Ministry of National Education and has the potential of both reaching most Turkish high school students and being representative of Turkish mathematics textbooks.

The Singaporean mathematics textbook is written for students who are preparing for GCE O Level Additional Mathematics Exam, which is conducted annually in Singapore and authorized by the Singapore Ministry of Education and the University of Cambridge International Examinations (CIE) (Singapore Examinations and Assessment Board, 2011). Hence, the textbook being used in this study is approved by the Singapore Ministry of Education. Additional mathematics is one of the O level subjects taken by students after the second year of secondary school in Singapore (Singapore Ministry of Education, 2006).

Turkey, with twenty-five schools applying the IBDP, is one of the countries where the program is becoming increasingly popular (IB, 2012b). Although there are many mathematics textbooks used in high schools with the IBDP, all of these textbooks relate to the same curriculum. Thus, the IBDP-SL mathematics textbook used in this study is selected because of its common use in Turkish schools as well as its being a representative textbook that reflects the philosophy of IBDP mathematics.

In this study, the unit *quadratics* was selected for analysis and comparison within the three textbooks. The reason for this selection is that the topic quadratics serves as an important bridge between basic mathematics topics such as linear functions and polynomials, and higher mathematics topics such as differentiation and integration. The topic has both algebraic and geometric features, which makes it a relatively good topic representative of the school mathematics field in general. Moreover, quadratics is one of the secondary school mathematics topics where use of a broad range of student-centered activities, real-life connections, technology, and historical notes are applicable during instruction. Thus, the three chapters on quadratics are expected to display differences among the Turkish, Singaporean and IBDP-SL mathematics textbooks in regard to the features above, too.

Method of data coding and analysis

The chapters on quadratics within the three mathematics textbooks were analyzed in terms of content, organization, and presentation style. The findings of this analysis were also used as fundamental resources to make inferences and reach generalizations about philosophies of the Turkish, Singaporean, and IBDP-SL mathematics curricula.

As a general rule, *inter-rater reliability* was ensured by double-coding the data through two researchers. Another reviewer (an MA student in the educational sciences) was trained to conduct the analysis of a significant subset of material from the textbooks. After comparing results of the reviewer and the researcher, a similarity above 80% was regarded as an acceptable level of agreement. When this is not the case, further training was performed to resolve disagreements.

While Microsoft Excel made it easy to construct checklists, charts, tables, diagrams and so on, a calculator helped to calculate simple values necessary to fill these tables, charts, and graphs.

Before describing the three components used in the method of data coding and analysis, the analytic framework representing the procedures followed in this section is given in Figure 6 as follows:

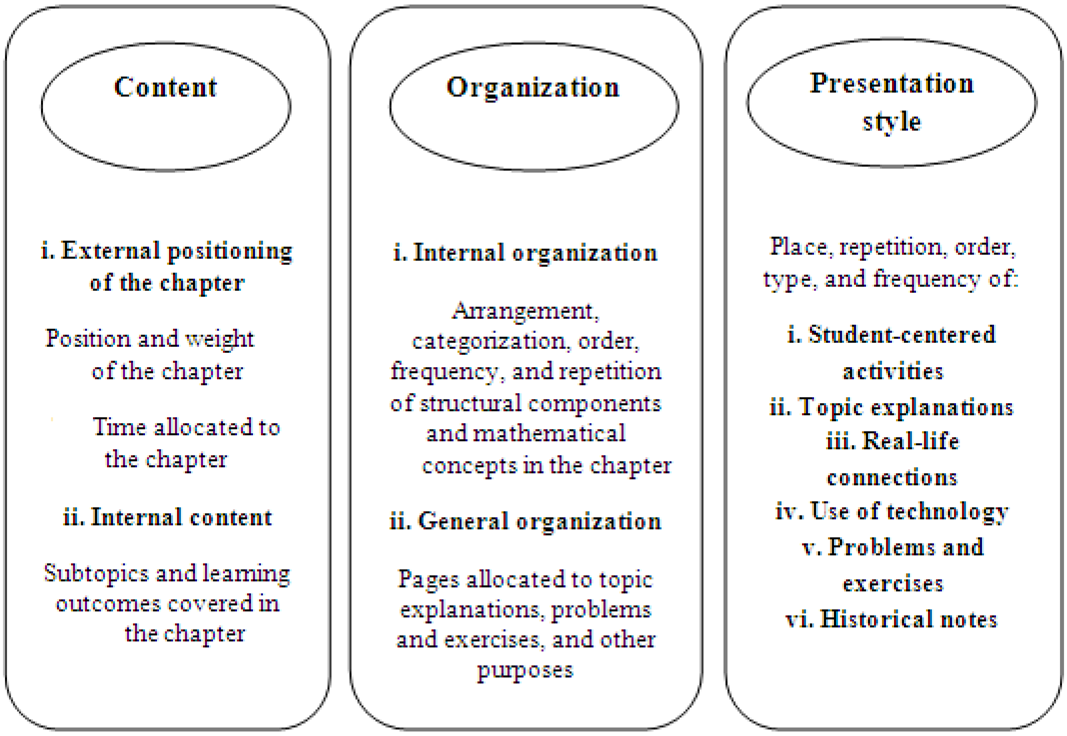


Figure 6. An overview of the analytic scheme used in data coding and analysis

Content

Contents of the three quadratics units were analyzed both externally and internally.

External positioning of the quadratics chapters

In this section, to describe position and weight of the unit *quadratics* in the three mathematics textbooks, the chapters coming before and after each quadratics unit were recorded first by creating tables of unit titles for each textbook to compare them practically. Then, the space occupied by quadratics within the totality of each textbook was computed by calculating the percentage of the number of pages allocated to the unit over the total number of pages in each textbook. Thus, a table—including number of units, total number of pages, and the number of pages allocated to quadratics and the percent weights of the quadratics unit over the whole textbooks—was constructed in order to see the differences and similarities among the textbooks.

Moreover, the time allocated to the unit in the Turkish, Singaporean, and IBDP-SL mathematics curricula and the grade when the unit is taught were explored by using the related parts of the Turkish Ministry of National Education’s Grade 10 mathematics syllabus (MEB, 2011), Singapore’s Secondary school mathematics syllabus (Singapore Ministry of Education, 2006), and the IBDP-SL mathematics syllabus (IBDP, 2006).

Analyzing internal content

While analyzing the unit quadratics in the three textbooks in regards to the unit’s internal content, three tables were constructed initially to list the learning outcomes for each of the quadratics chapters in the Turkish, Singaporean and IBDP-SL mathematics curricula through the use of the three syllabi mentioned above. Due to the fact that the Turkish Ministry of National Education’s Grade 10 mathematics syllabus (MEB, 2011) has a relatively comprehensive list of learning outcomes for the quadratics unit, it was used as a starting point and then extra learning outcomes within the Singaporean and IBDP-SL mathematics syllabi were added to this list. Thus, three all-inclusive tables (for *quadratic equations*, *inequalities*, and *functions* separately) containing all of the learning outcomes were created.

Afterwards, all of the pages included within the three chapters on quadratics were scanned, and a tick (or a cross) was put next to each learning outcome in all-inclusive tables when the learning outcome is covered (or not covered) in the corresponding mathematics textbook. Thus, the number of the learning outcomes met in each of the three textbooks was counted.

In addition, a bar chart showing the number of learning outcomes met in each textbook was constructed and discussed to compare the three quadratics units in terms of the number of the learning outcomes fulfilled within each of the subtopics quadratic equations, inequalities, and functions.

Furthermore, outcomes of internal content analysis were interpreted by discussing the whole data in a general resultant table containing the total number of learning outcomes addressed within each quadratics unit as well as the textbooks' percentage of covering these outcomes. Finally, the reasons which affected the outcomes of this analysis were explained for each of the Turkish, Singaporean and IBDP-SL mathematics curriculum.

Organization

Litz (2005) argues that "the layout and design of a textbook refers to its organization and presentation of language items and activities" (p.15). On the other hand, according to Alacacı et al. (2011):

> *Organization of a textbook* is considered as arrangement of units (orders and topics of units), methods of assigning units, lesson numbers, and page use with regard to topic explanations, activities, exercises and problems, and other goals (introduction, extra exercises, tests, and so on) in units. (p.8)

Another feature of *textbook organization* might be considered to be about categorization, order, number, frequency, and repetition of concepts and activities in a textbook (Stevenson, 1985).

In this study, the organization of the three chapters on quadratics was analyzed from two aspects: internal and general organization.

Internal organization

To explore internal organization of the three chapters on quadratics, pages within the quadratics units were scanned with two points of view in mind: organization of structural components (such as activities, topic explanations, and exercises) and mathematical content (quadratic equations, inequalities, and functions). During this process, arrangement, categorization, order, frequency and repetition of concepts and activities within the three chapters were described using a qualitative approach. Not only flow of activities, topic explanations, solved examples, and exercises but also logical links among mathematical concepts were depicted. As a result, the approaches used in the design structural components and mathematical concepts were explored by considering the layout of titles of subtopics within the quadratics units. In addition, general arrangements of structural components and mathematical concepts in the three chapters on quadratics were organized into tables.

General organization

To analyze general organization of the three quadratics units, number and percentage of pages allocated to the topic explanations, problems and exercises, and other purposes (table of content, historical note parts, large pictures and photos, topic summary pages) were counted and calculated for the three quadratics units. Then, the numbers and percents of pages allocated to different aims in the three chapters were compared.

Presentation style

Alacacı et al. (2011) state that

> *Presentation style* of concepts is considered to be the approach of analyzing place and order of the following components: student-centered activities, introduction of topics, real-life connections, use of technology and tangible materials, problems and exercises. (p. 9)

In a similar way, *presentation style* can be defined as instruction of topics by giving priority to the following issues: a variety of activities giving importance to learners' backgrounds and environments, tasks and assessments at the appropriate level, executable experiments and accessible tasks (Lemmer, Edwards, & Rapule, 2008).

In this study, during the evaluation and comparison of the three textbooks in terms of the presentation style of the unit, the focus was on each book's approach to presenting the student-centered activities, topic explanations, real-life connections, use of technology, problems and exercises, and historical notes.

Student-centered activities

To analyze student-centered activities, place, order, type, and repetition of such activities throughout the three quadratics chapters were described. In addition, examples of these activities from the chapters were put among the collected data. Then, the number of student-centered activities was counted for each textbook and the data was used for comparison of the three textbooks.

Topic explanations

To investigate topic explanations, the characteristic features of each quadratics chapter were described and examples of such sections from the chapters were put among descriptions for support. Then, the approaches representing styles of topic explanations in three textbooks were modeled.

Real-life connections

Similar to the data collection about student-centered activities, the place, order, and repetition of real-life connections within the three units was investigated. Afterwards, the numbers and types of such connections in the three textbooks were used for discussion.

Use of technology

Using the same approach in the analysis of real-life connections, the places and types of technological facilities used in the three textbooks were searched and examples of such sections from the chapters were included in the study. Then, incidences of technology usage together with types of technological support in each quadratics chapter were counted and interpreted for comparison of the three textbooks in terms of use of technology.

Problems and exercises

In analyzing the data about the problems and exercises within the three chapters on quadratics, the places and styles of the problems and exercises were explored first, and examples of the problems and exercises with different structures were then included in the study. Then, all of the problems and exercises were counted (for the items including sub-items, each sub-item was counted separately). Later, the items were classified into the three sections according to their level of mathematical complexity by using the categorization model in Table 2, which was compiled from National Assessment of Educational Progress (NAEP) (2007, p. 36-40). In addition, three sample items from the quadratics chapters with each level of mathematical complexity were included to exemplify the three levels in a more concrete way. Lastly, the number and percentage of items with low, moderate, and high mathematical complexity was calculated for each of the three units. Thus, a comparison of the three textbooks in terms of the numbers and complexity levels of items within the problems and exercises sections was done by interpreting the data.

Historical notes

The three units on quadratics were analyzed for the use of historical notes and references. When such information is noted, its number, place, and contribution to the instructional explanation were recorded. In this way, the historical notes and references in the three textbooks were examined and compared through their placements, numbers, and connections to the topic quadratics.

Table 2

Mathematical complexity of items

Low complexity • Recall or recognize a fact, term, or property. • Recognize an example of a concept. • Compute a sum, difference, product, or quotient. • Recognize an equivalent representation. • Perform a specified procedure. • Evaluate an expression in an equation or formula for a given variable. • Solve a one-step word problem. • Draw or measure simple geometric figures. • Retrieve information from a graph, table, or figure.
Moderate complexity • Represent a situation mathematically in more than one way. • Select and use different representations, depending on situation and purpose. • Solve a word problem requiring multiple steps. • Compare figures or statements. • Provide a justification for steps in a solution process. • Interpret a visual representation. • Extend a pattern. • Retrieve information from a graph, table, or figure and use it to solve a problem requiring multiple steps. • Formulate a routine problem, given data and conditions. • Interpret a simple argument.
High complexity • Describe how different representations can be used for different purposes. • Perform a procedure having multiple steps and multiple decision points. • Analyze similarities and differences between procedures and concepts. • Generalize a pattern. • Formulate an original problem, given a situation. • Solve a novel problem. • Solve a problem in more than one way. • Explain and justify a solution to a problem. • Describe, compare, and contrast solution methods. • Formulate a mathematical model for a complex situation. • Analyze the assumptions made in a mathematical model. • Analyze or produce a deductive argument. • Provide a mathematical justification.

CHAPTER 4: RESULTS

Introduction

In this chapter, results from the analysis of chapters on quadratics in the three mathematics textbooks were reported to address the following research questions:

- How do mathematics textbooks cover quadratics in Turkey, Singapore and the IBDP in terms of content (general structure of the three textbooks, position and weight of quadratics over the whole textbooks as well as the corresponding mathematics curricula, categorization of all the pages in the unit under titles based on learning outcomes, and the degree of each book's coverage of these titles)?
- How do mathematics textbooks cover quadratics in Turkey, Singapore and the IBDP in terms of organization (arrangement, categorization, order, frequency, and repetition of concepts and activities in the unit)?
- How do mathematics textbooks cover quadratics in Turkey, Singapore and the IBDP in terms of presentation style (place, order, and type of student-centered activities, topic explanations, real life connections, use of technology, problems and exercises, and historical notes)?

Content

The contents related to quadratics from each of the three textbooks were analyzed from two perspectives: externally and internally.

External positioning of the quadratics chapters

In this section, a report of the position of the units on quadratics within each of the three mathematics textbooks was presented. To do this, the following were reported: a general structure of the three textbooks, the position and weights of the units on quadratics within the totality of textbooks, and the time allocated to the units during the academic year based on respective curriculum frameworks and to which grades the units are taught.

Tables 3, 4, and 5 were created to display and compare the unit titles in the Turkish, Singaporean and IBDP-SL mathematics textbooks.

Table 3

Unit titles in the Turkish mathematics textbook

1. Polynomials
2. Quadratic equations, functions with one unknown, and inequalities
3. Permutations, combinations, binomials, and probability
4. Trigonometry

As seen in Table 3, in the Turkish textbook the unit *quadratic equations, functions with one unknown, and inequalities* is the second one following the polynomials unit and before the unit permutations, combinations, binomials, and probability.

As the Turkish textbook is approved by the Turkish Ministry of National Education as the mathematics textbook for the 10th grade and represents the national mathematics curriculum, the quadratics unit has the same order and a similar weight in the national curriculum. Since the textbook contains 256 pages in total and 70 (p.55-p.124) pages for the quadratics unit, the unit constitutes 27.3% of the whole textbook. Moreover, the quadratic equations, functions with one unknown, and inequalities unit occupies 30% of national mathematics curriculum of the 10th grade, and 44 lesson hours (one lesson hour means forty-five minutes) out of 144 in total are allocated to teach quadratics in the Turkish context (MEB, 2011).

Table 4

Unit titles in the Singaporean mathematics textbook

1. Sets	13. Permutations and combinations
2. Simultaneous equations	14. Binomial theorem
3. Indices, surds, and logarithms	15. Differentiation and its technique
4. Quadratic expressions and equations	16. Rates of change
5. Remainder and factor theorems	17. Higher derivatives and applications
6. Matrices	18. Derivatives of trigonometric functions
7. Coordinate geometry	19. Exponential and logarithmic functions
8. Linear law	20. Integration
9. Functions	21. Applications of integration
10. Trigonometric functions	22. Kinematics
11. Simple trigonometric identities and equations	23. Vectors
12. Circular measure	24. Relative velocity

Table 4 shows that in the Singaporean mathematics textbook, the unit *quadratic expressions and equations* is the fourth one coming after the unit indices, surds, and logarithms and before the unit remainder and factor theorems.

The Singaporean mathematics textbook has 623 pages, 22 of which (p.61-p.82) are devoted to the quadratics unit and nearly 5 of them are used in the quadratic function example sections in unit 9 (functions). So, in total 27 pages belong to quadratics, which means 4.3% of the whole textbook is occupied by quadratics topics. On the other hand, in the GCE O level additional mathematics curriculum, the unit quadratic expressions and equations is one of 11 topics in total and it is taught in last two years of secondary school (secondary three and secondary four). Different from the Turkish Grade 10 mathematics syllabus, Singapore’s GCE O level additional mathematics syllabus does not give a specified time allocation for the quadratics unit and, hence, a percentage value for the weight of the unit in the curriculum could not be computed (Singapore Ministry of Education, 2006).

Table 5

Unit titles in the IBDP-SL mathematics textbook

1. Functions	16. Vectors in 3-dimensions
2. Sequences and series	17. Lines in the plane and in space
3. Exponents	18. Descriptive statistics
4. Logarithms	19. Probability
5. Natural logarithms	20. Introduction to calculus
6. Graphing and transforming functions	21. Differential calculus
7. Coordinate geometry	22. Applications of differential calculus
8. Quadratic equations and functions	23. Derivatives of exponential and logarithmic functions
9. The binomial theorem	24. Derivatives of trigonometric Functions
10. Practical trigonometry with right angled triangles	25. Areas within curved boundaries
11. The unit circle	26. Integration
12. Non right angled triangle trigonometry	27. Trigonometric integration
13. Periodic phenomena	28. Volumes of revolution
14. Matrices	29. Statistical distributions
15. Vectors in 2-dimensions	

Table 5 shows that in the IBDP-SL mathematics textbook, the unit *quadratic equations and functions* is the eighth one coming after the unit coordinate geometry and before the unit the binomial theorem. The textbook is composed of 812 pages and 46 of them belong to the quadratics unit, which means 5.7% of the whole textbook is devoted to the quadratics unit. Regarding the position and weight of the unit in the IBDP-SL mathematics curriculum, the unit quadratic equations and functions is included in topic 2 (*functions and equations)*, which is one of the 7 main SL mathematics topics. Additionally, in the IBDP-SL mathematics curriculum, 24 teaching hours out of 140 hours in total are allocated to topic 2. When the review sets are excluded, learning outcomes of topic 2 are covered within approximately 90 pages and those of quadratics are met by using 42 pages. Therefore, the time allocated to quadratics in the IBDP-SL mathematics curriculum is about 11 teaching hours, which corresponds to approximately 15 lesson hours in the Turkish secondary school context (in the IBDP a teaching hour means 60 minutes whereas a lesson hour in Turkish high schools represents 45 minutes). Moreover, the unit is taught in the last two years of secondary school when the IBDP is applied (IBDP, 2006).

As a result, since both the Singaporean and IBDP-SL mathematics textbooks cover a two-year mathematics curricula, they include many topics while the Turkish mathematics textbook covers only 10th grade topics, which greatly increased the weight of the quadratics unit in the Turkish textbook, as shown in Table 6.

Table 6

Weight of the quadratics units in the three mathematics textbooks

Textbook	Number of units	Total number of pages	Number of pages allocated to quadratics	Percent weights of quadratics unit over whole textbook
Turkey	4	256	70	27.3%
Singapore	24	623	27	4.3%
IBDP-SL	29	812	46	5.7%

Table 6 demonstrates that the Turkish mathematics textbook is the one allocating the largest space to the quadratics unit (27.3% versus 4.3 % and 5.7 %). However, since the Turkish mathematics textbook covers the topics of only a one-year curriculum and the Singaporean and IBDP-SL mathematics textbooks represent two-year curricula, the percent weights of the quadratics unit over curricula in Singapore and the IBDP-SL can be multiplied by 2 (which gives 8.6 % and 11.4% respectively) in order to obtain a more reliable comparison.

Analyzing internal content

In this section, the three chapters on quadratics were analyzed in terms of their internal contents through the use of learning outcomes stated in the Turkish, Singaporean, and IBDP-SL mathematics curricula. In this way, the following question was addressed:

To what extent does each of the three approved textbooks conform to the existing criteria within the Turkish, Singaporean and IBDP-SL mathematics syllabi for learning outcomes?

First, the Turkish Ministry of National Education's Grade 10 mathematics syllabus was used in order to create Table 7, which summarizes the learning outcomes for the chapter quadratic equations, functions with one unknown, and inequalities into three subtopics.

Table 7

Topics and corresponding learning outcomes for quadratics in the Turkish mathematics textbook (compiled from MEB, 2011)

Topics	Learning outcomes
Quadratic equations	1. Identifies roots and solution set of a quadratic equation with one unknown. 2. Shows the formula giving roots of a quadratic equation with one unknown, and identifies presence of roots according to the sign of *discriminant*. 3. Shows the relationships between roots and coefficients in a quadratic equation. 4. Given a quadratic equation with one unknown including a parameter, finds the parameter according to desired restrictions. 5. Writes the quadratic equation whose roots are given. 6. Identifies solution set of an equation which can be turned into a quadratic equation with one unknown. 7. Explains systems of quadratic equations with two unknowns, and identifies solution set of systems of quadratic equations with two unknowns which can be turned into a quadratic equation with one unknown.
Inequalities	8. Analyzes sign of a binomial $ax+b$ by showing it in a table, and identifies solution set of a linear inequality with one unknown. 9. Analyzes sign of a trinomial ax^2+b+c by showing it in a table, and identifies solution set of a quadratic inequality with one unknown. 10. Identifies solution set of an inequality which is given as a product or a fraction of polynomials with first or second degree. 11. Identifies solution set of systems of linear or quadratic inequalities. 12. Identifies presence of roots and their signs in a quadratic equation with one unknown, without solving the equation. 13. Given a quadratic equation with one unknown expressed with a parameter, identifies presence of roots and their signs on a table based on the values of a parameter.

Quadratic functions	14. Explains a quadratic function, and finds out its maximum and minimum points. 15. Finds out vertex, *x* and *y* intercepts, and axis of symmetry belonging to graph of a quadratic function (parabola), sets up sign table of the function, and draws its graph. 16. Identifies the quadratic function when vertex and an arbitrary point on its graph are given, or when any three points on its graph are given. 17. Shows solution set of a quadratic inequality and solution set of systems of quadratic inequalities on the graph.

The Singapore Ministry of Education's Secondary school mathematics syllabus was used to list the learning outcomes for the unit quadratic expressions and equations in the Singaporean mathematics textbook. As the target of this textbook is to prepare students for GCE O Level Additional Mathematics Exam, the O level mathematics (secondary two, three, and four) and O level additional mathematics (secondary three and four) syllabi were used to create Table 8.

Table 8

Topics and corresponding learning outcomes of the unit on quadratics in the Singaporean mathematics textbook (Singapore Ministry of Education, 2006)

Topics	Learning outcomes
Solutions of quadratic equations and inequalities	1. Solves quadratic equations with one unknown by using four methods: factorization, use of formula, completing the square for $y= ax^2+bx+c$, and graphical methods. 2. Comprehends relationships between the roots and coefficients of the quadratic equation $ax^2+bx+c=0$. 3. Analyzes conditions for a quadratic equation to have: two real roots, two equal roots, and no real roots. 4. Analyzes conditions for ax^2+bx+c to be always positive (or always negative). 5. Solves quadratic inequalities and represents the solution set on the number line.
Graphs	6. Draws graphs of quadratic equations by representing their properties: positive or negative coefficient of x^2, maximum and minimum points, and symmetry. 7. Sketches graphs of quadratics given in the form: $y= \alpha (x-h)^2+k$, $y= \alpha (x-a)(x-b)$. 8. Draws graphs of linear equations in two unknowns. 9. Analyzes related conditions for a given line: to intersect a given curve, to be a tangent to a given curve, not to intersect a given curve.

In the third step, a similar table with the subtopics and learning outcomes of the unit quadratic equations and functions was formed based on the IBDP records. However, only a general view of the quadratics content was officially available within the IBDP-SL mathematics syllabus, where layout of the quadratics unit content is presented, as shown in Table 9.

Table 9

Overview of the quadratics unit content in IBDP-SL mathematics (IBDP, 2006)

Topic	Content
Quadratic equations and functions	1. The quadratic function $x \mapsto ax^2+bx+c$ (with rational coefficients only): its graph, *y*-intercept *(0,c)*. 2. Axis of symmetry $x=-\frac{b}{2a}$. 3. The form $x \mapsto \alpha\,(x-h)^2+k$: vertex *(h,k)*. 4. The form $x \mapsto \alpha\,(x-p)(x-q)$: x-intercepts *(p,0)* and *(q,0)*. 5. The solution of $ax^2+bx+c=0,\ a\neq 0$. 6. The quadratic formula. 7. Use of the *discriminant* $\Delta= b^2-4ac$.

Table 7, Table 8 and Table 9 show that the Turkish mathematics textbook has the most detailed list of learning outcomes for the unit quadratics. Therefore, while analyzing the contents of the three units, the list of learning outcomes in Table 7 was used as a starting point. Then, if there were extra subtopics and learning outcomes covered in the Singaporean and IBDP-SL mathematics textbooks, either available in Table 8 and Table 9, they were also added to the list. In this way, a comprehensive list of the learning outcomes was constructed to compare the contents of the three quadratics units. Thus, pages in these units were scanned and ticks or crosses were put near the learning outcomes in the list each time according to their being covered or not in these units. Then, the all-inclusive list of learning outcomes was presented through the three tables (Table 10, 11, and 12) arranged according to the three subtopics of quadratics: quadratic equations, inequalities, and quadratic functions.

Table 10

Comparison of internal content among the three quadratics units according to learning outcomes of quadratic equations (T: the Turkish mathematics textbook, S: the Singaporean mathematics textbook, IB: the IBDP-SL mathematics textbook)

Learning outcomes of *quadratic equations*	T	S	IB
1. Explains a quadratic equation with one unknown.	✔	✔	✔
2. Identifies roots and solution set of a quadratic equation with one unknown by factorization, completing the square, and using quadratic formula.	✔	✔	✔
3. Identifies roots and solution set of a quadratic equation with one unknown by using technology.	✘	✘	✔
4. Identifies presence and number of roots of a quadratic equation, as well as their being reel or not and their possible positions on curve according to the sign of *discriminant*.	✔	✔	✔
5. Shows the relationships between roots and coefficients in a quadratic equation.	✔	✔	✔
6. Given a quadratic equation with one unknown including a parameter, finds out the parameter according to desired restrictions.	✔	✔	✔
7. Writes the quadratic equation whose roots are given.	✔	✘	✘
8. Identifies solution set of an equation which can be turned into a quadratic equation with one unknown.	✔	✘	✘
9. Explains systems of quadratic equations with two unknowns, and identifies solution set of systems of quadratic equations with two unknowns which can be turned into a quadratic equation with one unknown.	✔	✘	✘
10. Formulates a pair of linear equations (a linear equation system) and turns them into a quadratic equation with one unknown according to given restrictions, and finds roots of obtained quadratic equation in problem solving.	✔	✘	✘
11. Formulates a quadratic equation in one unknown according to given restrictions and finds its roots in problem solving.	✔	✘	✔
Total number of coverage of the learning outcomes	**10**	**5**	**7**

Table 10 was used to compare the internal contents of the three quadratics units based on the learning outcomes covered for the subtopic quadratic equations. The table shows that while all the three textbooks cover quadratic equations as a crucial part of the quadratics chapter; they differ in the degree of detail given and number of learning outcomes intended. That is, the Turkish mathematics textbook covers a greater number of learning outcomes for quadratic equations compared to the other two mathematics textbooks.

Although the Singaporean mathematics textbook does not cover the way of solving quadratic equations with technology, it asks students to use a graph plotter in exercises since they are expected to use electronic calculators in GCE 0 Level Additional Mathematics Exam (Singapore Ministry of Education, 2006).

The Singaporean and IBDP-SL mathematics textbooks cover only the quadratic formula in regard to the relationships between roots and coefficients. However, in the Turkish mathematics textbook a separate title is allocated to show the direct relationships between roots and coefficients in a quadratic equation (e.g. direct formulas of addition and product of roots in terms of coefficients).

Table 11

Comparison of internal content among the three quadratics units according to learning outcomes of inequalities

Learning outcomes of *inequalities*	T	S	IB
1. Analyzes sign of a binomal $ax+b$ by showing it in table, and identifies solution set of a linear inequality with one unknown.	✔	✘	✘
2. Analyzes sign of a trinomial ax^2+b+c.	✔	✔	✘
3. Identifies solution set of a quadratic inequality with one unknown.	✔	✔	✘
4. Identifies solution set of an inequality which is given as product of polynomials with first degree.	✔	✔	✘
5. Identifies solution set of an inequality which is given as fraction of polynomials with first and second degree.	✔	✘	✘
6. Identifies solution set of systems of linear or quadratic inequalities.	✔	✘	✘
7. Identifies presence of roots in a quadratic equation with one unknown, without solving the equation.	✔	✔	✔
8. Identifies signs of roots in a quadratic equation with one unknown, without solving the equation.	✔	✘	✘
9. Given a quadratic equation with one unknown including a parameter, identifies presence of roots according to the values which the parameter gets.	✔	✔	✔
10. Given a quadratic equation with one unknown including a parameter, identifies signs of roots on table according to the values which the parameter gets.	✔	✘	✘
11. Solves a quadratic inequality by using the corresponding quadratic curve.	✘	✔	✘
12. Shows solution set of a quadratic inequality and solution set of systems of quadratic inequalities on the graph.	✔	✘	✘
Total number of coverage of the learning outcomes	**11**	**6**	**2**

Table 11 was used to compare the internal contents of the three chapters on quadratics in terms of the learning outcomes covered for the subtopic inequalities. It shows that although the IBDP-SL mathematics textbook does not include an inequalities part in the quadratics unit, it addresses 2 of the learning outcomes, which are related to presence of roots in a quadratic equation. On the other hand, the Turkish mathematics textbook is the one that presents the inequalities part in the most detailed way.

The comparison of the three chapters on quadratics in terms of their internal contents related to the quadratic functions parts is demonstrated in Table 12 below:

Table 12

Comparison of internal content among the three quadratics units according to learning outcomes of quadratic functions

Learning outcomes of *quadratic functions*	T	S	IB
1. Explains a quadratic function, and finds out its maximum and minimum points.	✔	✔	✔
2. Draws graph of a quadratic function (parabola) given in one of the forms: $y= ax^2+bx+c$, $y= \alpha (x-h)^2+k$, $y= \alpha (x-a)(x-b)$; and finds out vertex, x and y intercepts, and axis of symmetry.	✔	✔	✔
3. Sets up sign table of a quadratic function before drawing its graph.	✔	✘	✘
4. Analyzes x-intercepts, vertex, being either positive definite or negative definite of a quadratic function by using only *discriminant* and coefficients of $y= ax^2+bx+c$.	✔	✔	✔
5. Identifies the quadratic function when vertex and an arbitrary point on its graph are given, or when any three points on its graph are given.	✔	✘	✔
6. Draws graphs of linear functions, and analyzes three possible positions (cutting, touching, missing) of a linear and quadratic function on coordinate plane.	✔	✔	✔
7. Models problems that can be solved through a quadratic function, and solves them using its graph (parabola).	✔	✘	✔
Total number of coverage of the learning outcomes	**7**	**4**	**6**

Table 12 shows that the quadratic functions section is the most comprehensive one being covered by all three textbooks. For this section, the Turkish mathematics textbook covers all of the learning outcomes, and the IBDP-SL mathematics textbook transmits *nearly all* of them. In addition, although the Singaporean mathematics textbook never uses the expression *quadratic function* throughout the chapter, it addresses more than half of the learning outcomes in this part.

Even though the Singaporean mathematics textbook does not use the term quadratic function, it includes the quadratic forms ax^2+bx+c (by calling it as an *expression*), and $y= ax^2+bx+c$, $y= \alpha\ (x-h)^2+k$ and $y= \alpha\ (x-a)(x-b)$ (by calling them as *curves*).

While all three mathematics textbooks cover the way of finding maximum and minimum point of the quadratics, the IBDP-SL mathematics textbook gives the way of working out these extreme values of a quadratic function as a tool to solve optimization problems.

The Turkish and IBDP-SL mathematics textbooks present graphs of quadratic functions under a separate title (the graphical meaning comes after the algebraic meaning and then); however, in the Singaporean mathematics textbook, there is no special title for graphing quadratics (the algebraic and graphical representations go simultaneously from the beginning of the quadratics unit).

So, the results gathered from Table 10, 11 and 12 were summarized through the representation in Table 13 below:

Table 13

Results of internal content analysis conducted through learning outcomes

Textbook	Total number of learning outcomes addressed	Percentage of covering learning outcomes
Turkey	28	93.3%
Singapore	15	50.0%
IBDP-SL	15	50.0%

As Table 13 shows, the quadratics unit in the Turkish mathematics textbook addresses more learning outcomes compared to that of Singapore and the IBDP-SL.

These results can be explained in several ways. The Singaporean mathematics textbook covers quadratics in unit 4 without using the term quadratic function. Then, while introducing the function concept in unit 9, it gives the examples of linear and quadratic functions in detail to simplify each subtopic in the functions unit, which may be accepted as an indirect reference to the quadratic function concept. In addition, the IBDP-SL mathematics textbook does not contain inequalities in the quadratics unit. Furthermore, since the Singaporean and IBDP-SL mathematics textbooks include all of the O level additional mathematics and IBDP-SL mathematics topics respectively, they allocate a narrower space for the quadratics unit compared to the Turkish mathematics textbook, which is written for only students in the 10^{th} grade and cover only four comprehensive units.

Organization

In this section, findings from the analysis of layout and design of the three quadratics units were presented from two perspectives: internal and general organization.

Internal organization of the quadratics chapters

To analyze internal organization of each of the three units on quadratics, the units were investigated from the following points of view: the order and position of the subtopic titles, the way of introducing quadratics, the arrangement of topic explanation parts, the design of problem solving and exercise sections.

Table 14, 15, and 16 (representing the layouts of subtopic titles within the units on quadratics in the Turkish, Singaporean, and the IBDP-SL mathematics textbooks respectively) were used as starting points to describe and compare the internal organization of the quadratics units in the three mathematics textbooks.

Table 14

Layout of subtopic titles within the unit on quadratics in the Turkish mathematics textbook

Unit 2. Quadratic equations, functions with one unknown, and inequalities
2.1. Quadratic equations with one unknown
A. Solving a quadratic equation with one unknown
1. Solving by factorization
2. Solving by completing the square
3. General solution
2.1. Exercises-Part I
B. Relationships between roots and coefficients
2.1. Exercises-Part II
C. Writing a quadratic equation whose roots are known
2.1. Exercises-Part III
D. Equations which can be turned into a quadratic equation with one unknown, and systems of equations with two unknowns
2.1. Exercises-Part IV
2.2. Linear or quadratic inequalities with one unknown
2.2. Exercises
2.3. Presence and signs of roots
2.3. Exercises
2.4. Quadratic functions with one unknown and their graphs
A. The points where a parabola cuts axes
B. Axis of symmetry of a parabola
C. Writing the equation of a parabola whose several points are given
2.4. Exercises-Part I
D. Solution of inequalities with two unknowns by graphing
E. Solution of a system of inequalities by graphing
2.4. Exercises-Part II
Unit test

The quadratics unit, which is the second unit in the Turkish mathematics textbook, is divided into four main sections and further into eleven subsections, as shown in Table 14 above.

At the end of each of the main titles, there is an *exercises* section where students are expected to practice what they have learned throughout the topic. In addition, at the end of the unit, there is the *unit test* which contains mixed problems from the whole subtopics in the unit.

At the bottom of the first page of the quadratics unit, there is a pink box with the heading the subjects we will learn in this chapter, as well as four main titles in the unit with short summaries of what will be learned within each of these topics. Additionally, on the second page of the textbook, the significance of prior knowledge is emphasized by mentioning about linear and quadratics equations' being two special types of polynomial equations learned in the previous unit. Moreover, after explaining the key concepts to be learned in the quadratics unit, attributions to other subject areas and to the future topics are made. These connections are emphasized by explaining that graphs of linear functions will be covered in the analytical geometry course, and that third and higher degree polynomial functions will be analyzed in the coming years. In this way, the position of the new unit is shown within the big picture.

Each of the four main topics and some of the subtopics in the unit start with *activity* sections, which are intended to warm students up to the new concepts of the section by the help of opening problems or investigations. After these sections, topic explanations start with definitions and related conceptual information, and continue with at least two solved problems, solutions of which are demonstrated step by step.

Following the order in Table 14, the Turkish mathematics textbook starts the first part of the unit by defining the general form of quadratic equations as a special type of polynomial equations, which are covered in the previous unit, polynomials. Then, three different ways for solving quadratic equations are explained step by step using solved problems every time. In the first solution method, the use of factorization method—first introduced in the unit polynomials—is demonstrated. Additionally, prior knowledge on the absolute value and square root concepts is stated to be essential in the second and third way of solving quadratic equations.

Different from the Singaporean and IBDP-SL mathematics textbooks, the Turkish mathematics textbook allocates a separate subsection to the relationships between the roots and coefficients of a quadratic equation. This subsection includes many different operations on the roots such as addition and product of the roots, addition of squares of the roots and absolute value of the difference between roots. Results of these operations are expressed directly in terms of the coefficients in the given quadratic equation. In the examples of this subsection, quadratic equations including coefficients with parameters are used quite often, where students are expected to carry out operations on parameters. After that, a general formula is introduced to find a quadratic equation whose roots are known, by building on the factor concept in polynomials. In the final subtopic of the quadratic equations part, special types of equations—the ones that do not look like a quadratics at first sight but can be turned into a quadratic equation with one unknown—are explained through the substitution and factorization methods. Moreover, the solution set of a system of equations with two unknowns is worked out first by obtaining a quadratic equation from the system of equations through substitution and second by finding roots of the quadratic equation obtained.

In the second part of the quadratics unit in the Turkish mathematics textbook, linear and quadratic inequalities with one unknown are defined at the beginning. To work out the solution sets of linear and quadratic inequalities with one unknown, signs of the binomial $ax+b$ and the trinomial ax^2+bx+c are analyzed respectively by using sign tables. Thus, for a given system of inequalities, the common solution set for a quadratic and a linear inequality is demonstrated through sign tables that are arranged according to the given intervals.

The third part of the quadratics unit is about the analysis of the presence and signs of roots in a quadratic equation with one unknown. This analysis is performed without solving the quadratic equation but by investigating the signs of the *discriminant* and resultants of addition and product of roots, which was covered in the section relationships between roots and coefficients earlier. This extra section in the Turkish mathematics textbook facilitates students to analyze the presence and signs of roots in a quadratic equation in a practical way.

In the last part of the quadratics unit in the Turkish mathematics textbook, the formal definition of a quadratic function is first introduced. Then, the minimum and maximum points for a given quadratic function are discussed by giving a general formula for each. As the first way for drawing a graph of a quadratic function, the change table of the quadratic function is constructed. By plotting the obtained ordered pairs on the coordinate plane, the *parabola* concept is introduced. However, after explaining the significance of several key terms such as the *axis of symmetry*, *vertex*, *x and y-intercepts*, and *coefficient of* x^2 in the quadratic function, an alternative practical way for drawing graph of a quadratic function is demonstrated. Then, to draw a parabola whose several points are known, the general formula given in the section writing a quadratic equation whose roots are known and another quadratic formula that is written by using the vertex concept are used. Lastly, after explaining how to draw graphs of linear and quadratic inequalities with two unknowns, the ways of solving the systems of inequalities including two quadratic, a quadratic and a linear, or two linear inequalities are presented by graphing. In this way, the positions (cutting, touching, and missing) of two parabolas, a parabola and a line, or two lines on the coordinate plane are analyzed.

Table 15

Layout of subtopic titles within the unit on quadratics in the Singaporean mathematics textbook

Chapter 4. Quadratic expressions and equations
4.1. Maximum/minimum value of a quadratic expression
Exercise 4.1
4.2. Roots of a quadratic equation
Intersection problems leading to quadratic equations
Exercise 4.2.
4.3. Solving quadratic inequalities
Exercise 4.3.
Important notes
Miscellaneous examples
Miscellaneous exercise 4

In the Singaporean mathematics textbook, the quadratics unit has three main topics, as seen in Table 15. At the end of each of these main topics, there is an *exercise* section that gives students the chance to practice what they learned within that topic. At the end of the last topic, there begins the *important notes* part, where a practical summary of the whole quadratics unit is given by the help of a diagram, a table and some graphs. After this part, the three comprehensive examples that require knowledge from all of the topics in the unit are solved under the heading *miscellaneous examples.* Finally, at the end of the quadratics unit, the section *miscellaneous exercise 4* contains mixed problems from the whole unit.

Different from the Turkish mathematics textbook, the Singaporean mathematics textbook has neither an introduction for the quadratics unit nor activity sections at the beginning of the subtopics. Another difference between the two textbooks is that in the topic explanation parts of the Singaporean mathematics textbook, several examples are generally given at first, and then definitions and conceptual information to be reached are presented.

As shown in Table 15, instead of first presenting quadratic equations and then quadratic functions as in the Turkish mathematics textbook, the Singaporean mathematics textbook starts the quadratics unit by introducing the quadratic expression concept and the way for working out its maximum and minimum values. Although the textbook never uses the term quadratic function, $y= ax^2+bx+c$ notation is used for the quadratic expressions. In addition, calculations of the minimum and maximum points in such quadratic expressions are worked out by the help of curves (parabolas). Then, a transition from the extreme points to the *turning point*, *y-intercept*, and *symmetry line* concepts is made through solved problems. Furthermore, the prior knowledge on expanding an expression with perfect square is recalled in order to express the general form of a quadratic expression in the vertex form by completing the square method and calculating the *x-intercepts* of the quadratic expression. Moreover, in a solved example, the factorization method is described quite shortly as a way to find out the *x-intercepts* of a quadratic expression in an alternative way.

In the second subtopic of the unit (roots of a quadratic equation), the quadratic formula is delivered as a practical way to calculate the roots of a quadratic equation, and then the relationships between the roots and the *discriminant* are presented together with their graphical representations. Additionally, some intersection problems leading to quadratic equations are solved both algebraically and graphically, to construct a bridge between quadratic equations and quadratic inequalities.

In the last main topic of the quadratics unit, after defining the *quadratic inequality* concept, solution of a quadratic inequality is demonstrated by using the respective graphical representation (parabola), which was explained in detail during the first two sections of the unit.

Table 16

Layout of subtopic titles within the quadratics unit in the IBDP-SL mathematics textbook

Chapter 8. Quadratic equations and functions
A. Function notation $f: x \mapsto ax^2+bx+c$
Exercise 8A
B. Graphs of quadratic functions
Investigation 1: Graphing $y= a(x- \alpha)(x-\beta)$
Investigation 2: Graphing $y= a\ (x-h)^2+k$
Exercise 8B1 & Exercise 8B2
C. Completing the square
Exercise 8C
D. Quadratic equations
Exercise 8D1 & Exercise 8D2
E. The quadratic formula
Exercise 8E
F. Solving quadratic equations with technology
Exercise 8F
G. Problem solving with quadratics
Exercise 8G
H. Quadratic graphs (review)
Exercise 8H
I. The *discriminant*, Δ
Exercise 8I1 & Exercise 8I2
J. Determining the quadratic from a graph
Exercise 8J
K. Where functions meet
Exercise 8K
L. Quadratic modeling
Exercise 8L
Review set 8A, 8B, 8C, 8D, and 8E

In the IBDP-SL mathematics textbook, the quadratics unit is divided into twelve main titles (topics) that are labeled with capital letters from A to L, as shown in Table 16. In each of the main topics, the following order is followed: topic explanation, examples, definitions, *investigations*, and *exercise* sections, where some worked-out examples are given. Additionally, at the end of the unit, there are five *review sets* (from 8A to 8E) with mixed problems covering the whole topics within the unit. Moreover, the *investigation 1, investigation 2,* and *graphics calculator investigation* are the three activities designed to enable students to explore the new concepts step by step through the use of technology and critical thinking.

After giving the list of topics on the first page, the textbook does not start the quadratics unit directly. Instead, under the heading introduction, the focus point for this unit is explained after showing the position of quadratics among the other functions with different degrees—from simple to complex—by using the function notations (linear, quadratic, cubic, and quartic). Then, a modeling problem, a historical note and an opening problem is presented to introduce the new topic.

Following the structure in Table 16, the quadratics unit in the IBDP-SL mathematics textbook begins with the function notation $f: x \mapsto ax^2+bx+c$, where the general form of quadratic functions is introduced as a second degree function with the dependent and independent variables. Then, the *parabola* concept is presented through the investigation 1 and investigation 2. After giving general properties of the parabola as results of the investigation parts—through the *x-intercepts*, *axis of symmetry*, *coefficient of* x^2, and *vertex*—several practical ways of drawing graphs of quadratic functions are explained by using the two main quadratic function forms $y= a(x-\alpha)(x-\beta)$ and $y= a(x-h)^2+k$. Using the same approach in the Singaporean mathematics textbook, completing the square method is explained as a useful way to convert the general form of a quadratic equation ($y= ax^2+bx+c$) into its vertex form ($y= a(x-h)^2+k$).

Now that the quadratic function part is over, the textbook presents the quadratic equation concept, as well as four distinct methods for solving quadratic equations: factorization, completing the square, the quadratic formula, and using technology.

Different from the other two mathematics textbooks, the IBDP-SL mathematics textbooks allocates a special section to problem solving with quadratics, where quadratic equations are required to be constructed and solved for solving such problems. On the other hand, similar to the other two textbooks, definition of the *discriminant* is given by using the quadratic formula as a starting point, and then relationships between the roots and the *discriminant* of quadratics are presented.

Another special section is devoted to the determination of a quadratic function from its graph, where some key points of the parabola are given and the corresponding quadratic function is worked out by using the basic properties of parabola. Next, the three possible positions of a linear and a quadratic function on the coordinate plane (cutting, touching, and missing) are analyzed through the solution of the respective linear and quadratic equation simultaneously. Lastly, the extreme points of a quadratic function are used as a way to solve modeling problems by optimization.

The internal organization of each of the three quadratics units and the way each unit designed in terms of mathematical content are represented in Table 17 and Table 18.

Table 17

Internal organization of the three units on quadratics within the three textbook

Turkey	Singapore	IBDP-SL
• Activity sections as warm up activities • Definitions • Topic explanations • Solved problems • Exercises sections covering each topic separately • Unit test including mixed problems from all subtopics of the unit	• Brief definitions, through examples • Topic explanations • Solved problems • Exercise sections covering each topic separately • Important notes • Miscellaneous examples from all topics in the unit • Miscellaneous exercise, with problems from each topic covered in the unit	• Definitions • Solved problems • Topic explanations • Investigations, for some sections • Exercise sections, with worked-out examples • Review sets, with mixed problems covering all topics within the unit

Table 18

Order of mathematical content within the three units on quadratics

Turkey	Singapore	IBDP-SL
• Quadratic equations • Linear and quadratic inequalities • Quadratic functions • Graphing quadratic functions • Solving systems of linear and quadratic inequalities • Solving systems of inequalities by graphing	• Quadratic expressions and equations, with corresponding graphs • Solving systems of linear and quadratic equations • Solving quadratic inequalities through graphing	• Quadratic functions • Graphing quadratic functions • Quadratic equations • Problem solving with quadratics • Solving systems of linear and quadratic equations • Quadratic modeling

General organization of the units on quadratics

In this section, the number of pages devoted to the topic explanations (including conceptual information and worked-out examples to support explanation), problems and exercises (which are left unsolved for students, involving the activity and investigation parts) and other purposes (containing the tables of contents, historical note parts, large pictures and photos, and topic summary pages), as well as the percents of these pages over the whole quadratics units were analyzed. Table 19 summarizes the results below:

Table 19

Number and percent of pages allocated to the topic explanations, problems and exercises, and other purposes within the three quadratics units

Textbook	Pages allocated to topic explanations		Pages allocated to problems and exercises		Pages allocated to other purposes	
	Number	Percent	Number	Percent	Number	Percent
Turkey	46	65.7 %	20.5	29.3%	3.5	5.0%
Singapore	13.5	61.4%	7	31.8%	1.5	6.8%
IBDP-SL	22	47.8%	21	45.7%	3	6.5%

As represented in Table 19, while more than 60% of the pages within the quadratics units of the Turkish and Singaporean mathematics textbooks are used for the topic explanations, nearly half of the number of total pages is devoted to this purpose in the IBDP-SL mathematics textbook. On the other hand, the IBDP-SL mathematics textbook is the one that allocates the largest space to the problems and exercises. Finally, the percents of pages used for purposes such as the tables of contents, historical note parts, large pictures and photos, and the topic summary pages are close to one another in the three textbooks. Figure 7 gives a visual comparison of the three mathematics textbooks in terms of the percents of pages allocated to the topic explanations, problems and exercises, and other purposes within the quadratics units.

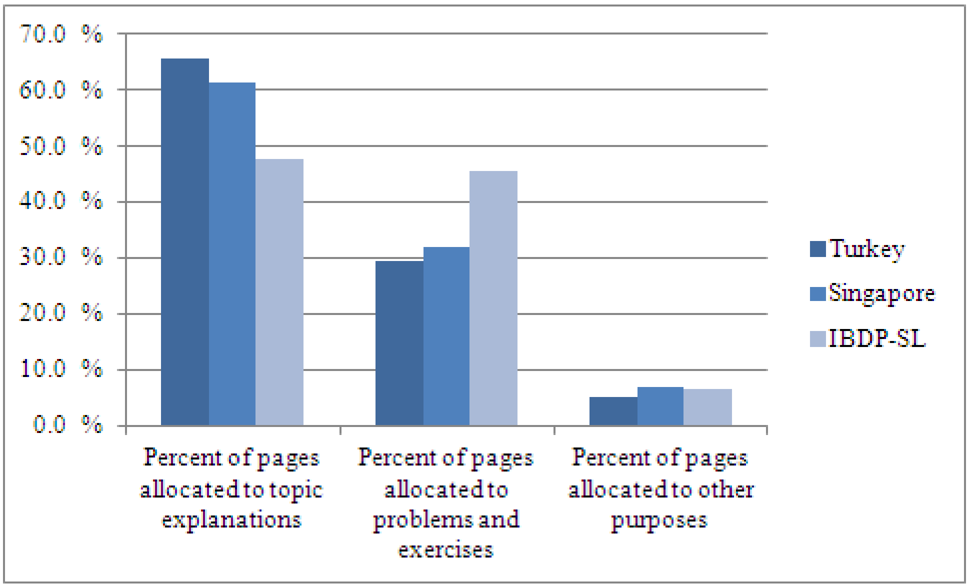


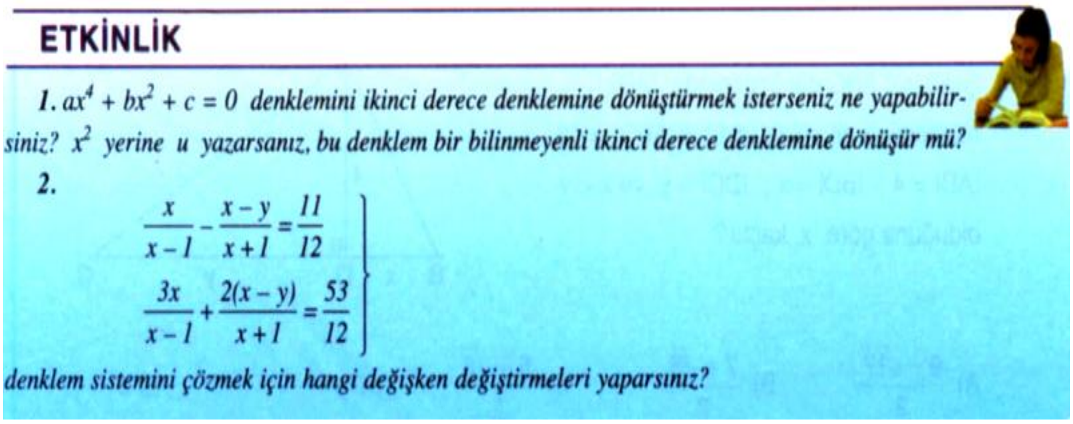
Figure 7. Percent of pages allocated to the topic explanations, problems and exercises, and other purposes within the three units on quadratics

Presentation style

The analysis of the three mathematics textbooks in terms of presentation style was done by looking into the place and order of the following six elements within the presentation of the contents: student-centered activities, topic explanations, real-life connections, use of technology, problems and exercises, and historical notes.

Student-centered activities

In the Turkish mathematics textbook, three of the four main topics start with pictures from real-life, as well as several related mathematical discussion questions in order to open the new topic with student inquiry. After these opening parts and at the beginning of most of the subtopics, the activities sections are placed in blue tables, which are designed to enable students to warm up to the new topic and remember the required background knowledge, and to arouse their interest and inspect their prior knowledge , as seen in Figure 8.



ETKİNLİK

1. $ax^4 + bx^2 + c = 0$ *denklemini ikinci derece denklemine dönüştürmek isterseniz ne yapabilirsiniz?* x^2 *yerine* u *yazarsanız, bu denklem bir bilinmeyenli ikinci derece denklemine dönüşür mü?*

2.

$$\left.\begin{aligned} \frac{x}{x-1} - \frac{x-y}{x+1} &= \frac{11}{12} \\ \frac{3x}{x-1} + \frac{2(x-y)}{x+1} &= \frac{53}{12} \end{aligned}\right\}$$

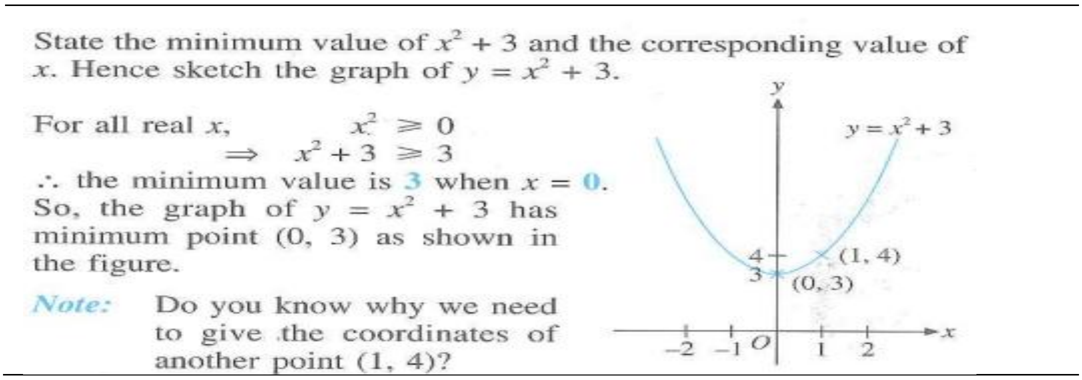
denklem sistemini çözmek için hangi değişken değiştirmeleri yaparsınız?

Figure 8. A sample student-centered activity from the Turkish mathematics textbook (Kaplan, 2008, p. 74)

Some of the activities sections are designed in the form of opening problems with real-life connections. Some others, on the other hand, look like investigation activities with a list of questions that intend to help students follow the solution process in order to reach the more complicated one coming next. If the new topic requires the use of prior knowledge or skills, some attributions are made to the related past topics through activities sections as well. In this case, students are expected to start the activity by applying their prior knowledge and then discovering several key points of the new topic. What is common in all types of the activities sections is the active participation of students.

During the topic explanation parts, student-centered activities are still available such as filling in a table designed in an inductive way, where several unknowns are expected to work out in order to reach a conjecture. Furthermore, sometimes after announcing the obtained results, several extension questions are asked immediately just like a teacher asking instant questions to students after explaining a new concept. Moreover, in some of the solved problems, the textbook offers to solve the problem together by giving the structure and clues for the solution with blanks to fill in and sub-questions to address. In this way, students are expected to think critically at crucial points and participate in the learning process actively.

Although the Singaporean mathematics textbook does not include student-centered activities at the beginning of the topics, sometimes direct questions are asked to students during the topic explanation parts and solved problems, as seen in Figure 9.


State the minimum value of $x^2 + 3$ and the corresponding value of x. Hence sketch the graph of $y = x^2 + 3$.

For all real x, $x^2 \geq 0$

$\Rightarrow x^2 + 3 \geq 3$

$\therefore$ the minimum value is 3 when $x = 0$.
So, the graph of $y = x^2 + 3$ has minimum point (0, 3) as shown in the figure.

Note: Do you know why we need to give the coordinates of another point (1, 4)?

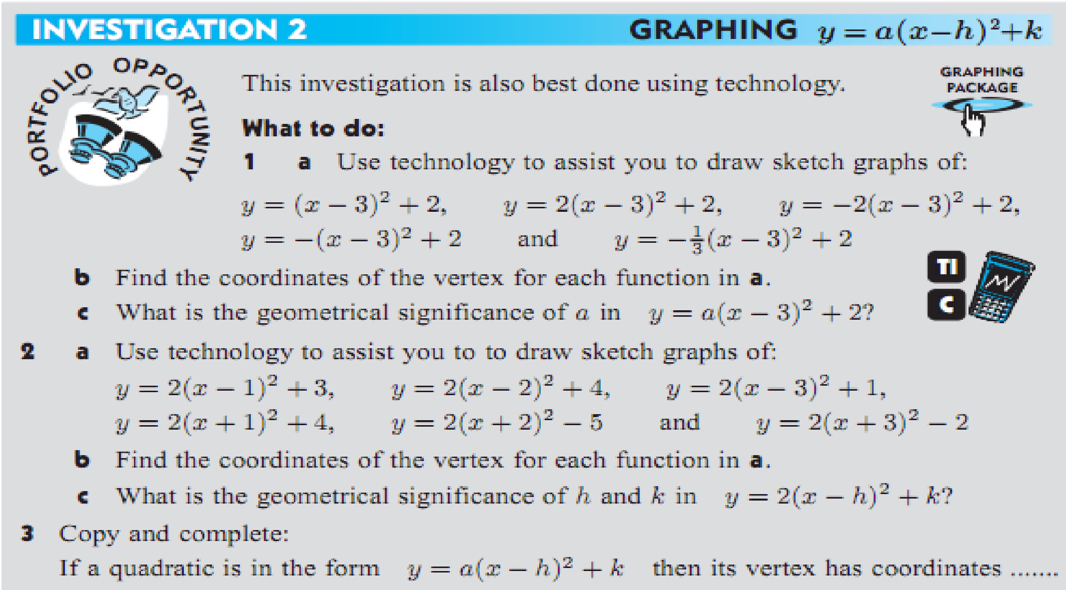
Figure 9. A sample student inquiry from the Singaporean mathematics textbook (Thong & Hiong, 2006, p. 61)

The inquiry questions in the Singaporean mathematics textbook ask students to think about why that knowledge is important or work out the solution for another case given. In this way, students are encouraged to look at the concepts from a broader perspective, question underlying reasons, and make inferences connections with coming topics.

In the IBDP- SL mathematics textbook, on the other hand, student-centered activities consist of the opening problem and the three investigation sections, two of which are presented under the topic graphs of quadratic functions and the third being at the end of the unit.

The opening problem is constructed with an interdisciplinary approach, where students are expected to warm up to the new chapter by considering five distinct steps of a physics motion problem and using quadratics in the solution process. The small picture of a semi-open window with the sun and sea outside represents the opening problem, and the picture on the right side of the problem portrays the story within the problem.

The investigation parts are also student-centered activities where students are led to explore the key mathematical concepts within quadratics, especially through the deliberate use of technology. That is, they require students to discover how to graph quadratic functions and identify the crucial points on parabolas through the active use of graphing packages or graphics calculators. They all include an icon of the graphics calculators and a step by step what to do list that direct students to reach the intended conjectures at the end of the activity. Furthermore, at the beginning of the investigation 1 and 2, the small picture with the text portfolio opportunity means that students can use these two activities in their IBDP-SL mathematics portfolio assignments, as seen in Figure 10. These assignments are two pieces of student work in the form of mathematical investigation and mathematical modeling and constitute 20% of student assessment in the IBDP-SL mathematics curriculum (IBDP, 2006).



INVESTIGATION 2 **GRAPHING** $y = a(x-h)^2+k$

PORTFOLIO OPPORTUNITY

This investigation is also best done using technology.

GRAPHING PACKAGE

What to do:

1 **a** Use technology to assist you to draw sketch graphs of:
$y = (x-3)^2 + 2$, $y = 2(x-3)^2 + 2$, $y = -2(x-3)^2 + 2$,
$y = -(x-3)^2 + 2$ and $y = -\frac{1}{3}(x-3)^2 + 2$

b Find the coordinates of the vertex for each function in **a**.

c What is the geometrical significance of a in $y = a(x-3)^2 + 2$?

TI C

2 **a** Use technology to assist you to to draw sketch graphs of:
$y = 2(x-1)^2 + 3$, $y = 2(x-2)^2 + 4$, $y = 2(x-3)^2 + 1$,
$y = 2(x+1)^2 + 4$, $y = 2(x+2)^2 - 5$ and $y = 2(x+3)^2 - 2$

b Find the coordinates of the vertex for each function in **a**.

c What is the geometrical significance of h and k in $y = 2(x-h)^2 + k$?

3 Copy and complete:
If a quadratic is in the form $y = a(x-h)^2 + k$ then its vertex has coordinates ....

Figure 10. Investigation 2 from the IBDP-SL mathematics textbook (Owen et al., 2008, p. 155)

Figure 11 is created to provide a general view about the comparison of the number of student-centered activities within the three quadratics units.

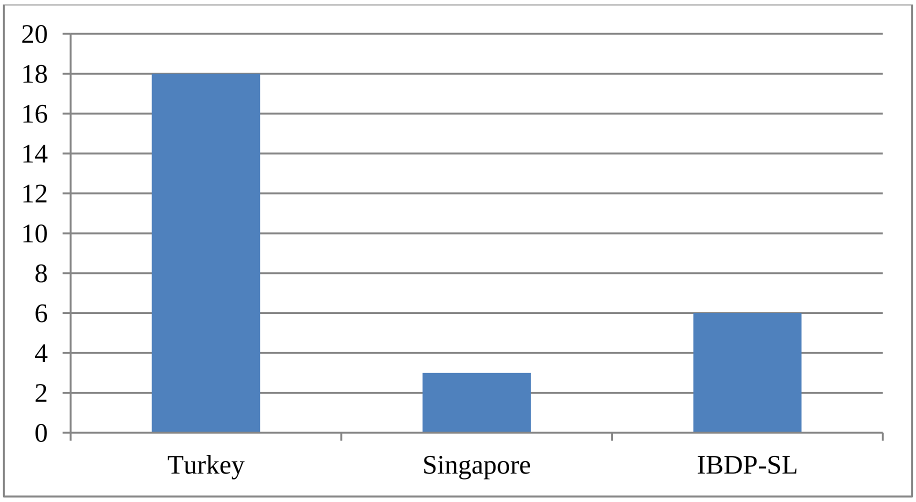


Figure 11. Number of student-centered activities within the three units on quadratics

The bar chart in Figure 11 demonstrates that the Turkish quadratics unit has the greatest number of student-centered activities with a clear difference compared to the other two textbooks. The IBDP-SL quadratics unit, on the other hand, is the second one in terms of including student-centered activities. Lastly the Singaporean quadratics unit has the least number of student-centered activities.

Topic explanations

In the Turkish mathematics textbook, after opening topics with activities sections, conceptual explanations generally start in a way that corresponds to the responses expected from students in the activities sections. That is, the key definitions and required conceptual information are given first, and then the solved problems are explained step by step to support student understanding in the introduction of each of the concepts. In the selection of the solved problems, mostly the order of a simple to complex problems is followed. Namely, while the beginning problems require only the comprehension of the concepts, the more complicated problems with the real-life and geometry connections are presented later. Generalizations and results that are obtained from the definitions, conceptual explanations, and activities sections and solved problems are given in the colorful tables and graphs (red, yellow, blue, and orange) or written in the form of the red colored bulleted items, as seen in Figure 12.

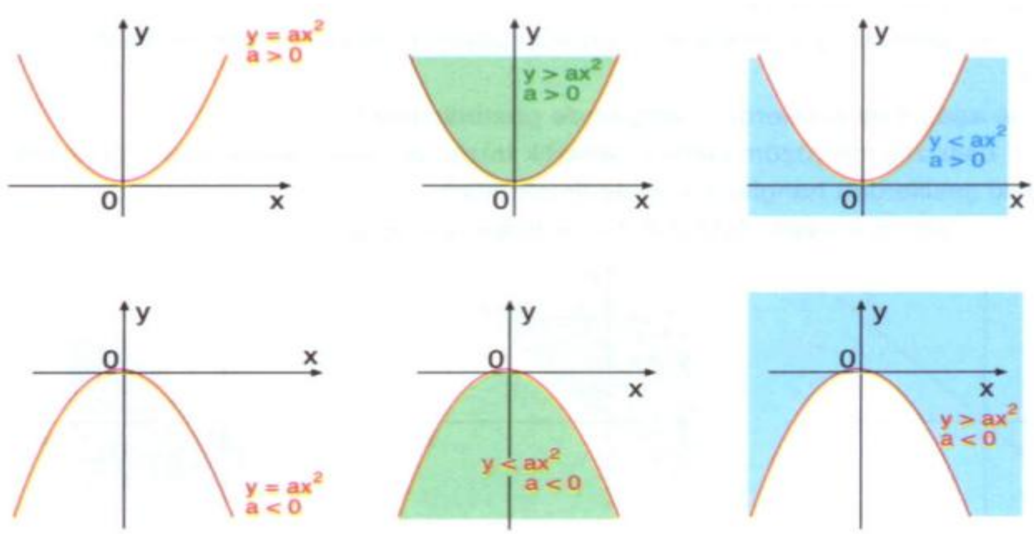


Figure 12. A sample generalization part from the quadratics unit in the Turkish mathematics textbook (Kaplan, 2008, p. 116)

After offering the acquired results and formulas, sometimes several advices about the way to follow in solving problems are given to students. Thus, the general deductive structure of the topic explanation parts in the Turkish mathematics textbook can be represented as in Figure 13.

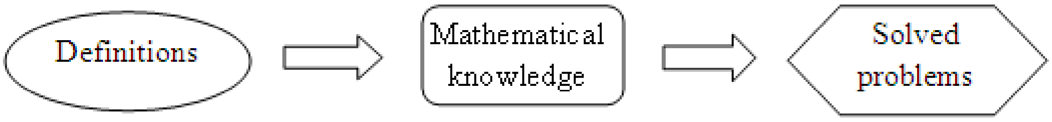


Figure 13. General structure of topic explanations in the Turkish quadratics unit

Finally, a quite informal *we* language is preferred to use in the textbook, which is similar to a teacher's or classmate's explaining the topic to a student, for example: "Therefore, we will perform two separate analyses according to the positive and negative cases of the variable *a*" (Kaplan, 2008, p. 82).

Compared to the other two textbooks, the Singaporean textbook has a more concise and plain style in the presentation of the topics in regard to not only what is delivered but also the way it is transferred, i.e., the summary tables are utilized very frequently, and only black and blue colors are used throughout the whole textbook. In addition, different from the Turkish textbook, it has relatively an inductive approach the in topic explanations. That is, after introducing the definitions quite briefly at the beginning of the topic, the solved problems are explained. Then, the kernels (theorems, rules, definitions, and procedures) of the mathematical knowledge that are obtained by making generalizations from findings of the problems considered thus far are given. These generalizations are presented in blue tables or written in blue color, in the form of practical conceptual information, as seen in Figure 14.

Note that the squared term $(x - h)^2 = 0 \Rightarrow x = h$ and $y = k$. Hence the turning point is at (h, k).
(a) When $a > 0$, (h, k) is a minimum point and k is the minimum value of y.
(b) When $a < 0$, (h, k) is a maximum point and k is the maximum value of y.
(c) The curve is symmetrical about the line $x = h$.

Figure 14. A sample result part from the quadratics unit in the Singaporean mathematics textbook (Thong & Hiong, 2006, p. 63)

All of the solved problems used in the topic explanation parts are abstract mathematical problems with no real-life connections and no use of technology. Sometimes, after giving step by step solutions, the concept under consideration is extended by questions directed to students.

At the end of the topic explanations, one and a half page-long summary of the chapter—under the heading important notes—is available in a blue table, where the crucial concepts within the chapter are presented with tables and graphical representations if necessary. Lastly, more solved problems are used to support and illustrate the obtained results.

As a result, the inductive structure of the topic explanation parts in the Singaporean mathematics textbook can be depicted as in Figure 15.

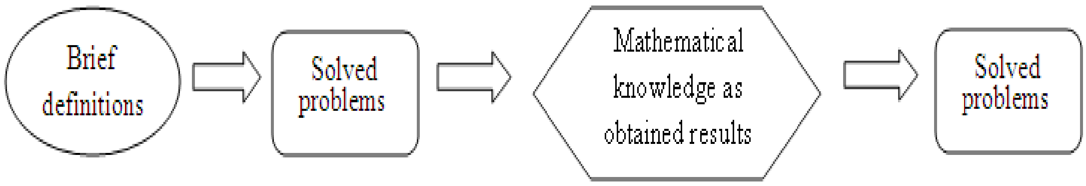


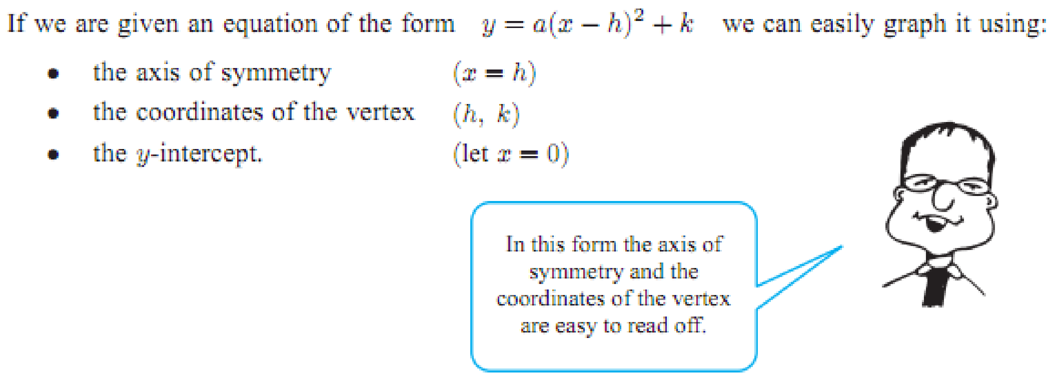
Figure 15. General structure of topic explanations in the Singaporean quadratics unit

Finally, although the *we* language is also used in the Singaporean mathematics textbook; the textbook has a more formal style in the explanation of the topics compared to the Turkish mathematics textbook since most of the mathematical knowledge is given in the tables as practical mathematical concepts.

As a remarkable difference from the Turkish and Singaporean mathematics textbooks, the IBDP-SL mathematics textbook has an interactive student CD. Thus, students have a PDF version of the textbook with a *background knowledge* icon on page 11. At the beginning of the quadratics unit, students are suggested to click on this icon and look at printable set of exercises and answers on algebraic expansion and factorization, which is a crucial topic that students need to review and practice before studying the new unit.

The IBDP-SL mathematics textbook displays a similar structure to the Turkish mathematics textbook in terms of the topic explanations, except for the activities and investigation parts. That is, at the beginning of each of the topics, the definitions and required mathematical knowledge are delivered through simple introductory problems in blue tables. These problems are abstract or real-life connected, often given with pictures representing the story contained. Then, the solved problems, including the word problems, the problems related to geometry, and the modeling problems, are presented from simple to complex within blue tables. Different from the other two textbooks, the solved problems are given within exercise sections so that after analyzing the solved one, students have an opportunity to apply the same procedural knowledge directly in the following problems.

Another apparent difference between the IBDP-SL mathematics textbook and the other two textbooks is that the IBDP-SL mathematics textbook uses small human figures with blue speech balloons in the form of cartoons quite frequently in the mathematical explanations, solved problems and exercise sections. In the texts within the speech balloons, a friendly language is used just like a classmate's giving clues to a student, which is similar to the language used in the Turkish mathematics textbook. Additionally, they have the objectives of making clarifications and emphases on the crucial points, reminding the significant mathematical knowledge and definitions, and giving advices about what kind of a way to follow in solving problems as in Figure 16.


If we are given an equation of the form $y = a(x-h)^2 + k$ we can easily graph it using:

- the axis of symmetry $(x = h)$
- the coordinates of the vertex $(h,\ k)$
- the y-intercept. (let $x = 0$)

Figure 16. A sample speech balloon from the IBDP-SL quadratics unit (Owen et al., 2008, p. 157)

On the other hand, similar to the other two textbooks, the IBDP-SL mathematics textbook uses summary graphs and tables quite often in order to provide students with the compact and practical summaries of the topics as in Figure 17.

Thus, the **discriminant** Δ, helps us to decide between the possibilities of:

- not cutting the x-axis $(\Delta < 0)$
- touching the x-axis $(\Delta = 0)$
- cutting the x-axis twice $(\Delta > 0)$.

Figure 17. A sample summary table from the IBDP-SL quadratics unit (Owen et al., 2008, p.180)

The general deductive structure of the topic explanations in the IBDP-SL mathematics textbook can be displayed with the same representation used for the Turkish mathematics textbook, as seen in Figure 18.

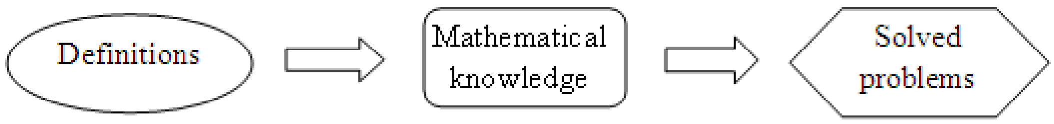


Figure 18. General structure of topic explanations in the IBDP-SL quadratics unit

Finally, as analyzed in Table 19 within the organization section, the Turkish and Singaporean quadratics units allocate more space to the topic explanations (65.7% and 61.4% of the total number of pages in the units respectively) than the IBDP-SL quadratics unit does (47.8%).

Real-life connections

In the Turkish mathematics textbook, on the first page of the quadratics unit, there is a picture that includes a parabolic object from real-life. Under the picture, students are informed that some objects from daily life have the shapes of different parts of the graph they will learn in this unit. In this way, it aims to arouse students' curiosity and interest at the beginning of the chapter.

The opening problems of the main topics, student-centered activities, solved problems, and exercise sections can be listed as the four places where the real-life connections are used frequently throughout the unit. That is, each of the opening problems has the picture of a bridge, a car factory, or a sprinkler from real-life. Under these pictures, there are some interesting questions directly asked to students which require students to make connections between the mathematical knowledge and real-life situations. Additionally, various problems in the student-centered activities, some of the problems solved to support the conceptual explanations and especially the problems asked in the exercise sections require a wide range of real-life connected mathematical thinking. The modeling and calculations of length in architectural structures, length or width of an object by using its area or volume, the profit and loss in manufacturing settings, the golden ratio, mechanics, especially projectile motion, and designing are included within these real-life connected word or mathematical modeling problems.

The Singaporean quadratics unit, on the other hand, includes only one real-life connection in one of the exercise problems, where the projectile motion of a marble is asked to model by using quadratic equations.

The IBDP-SL quadratics unit starts with the introduction part where a real-life connected modeling problem is solved step by step through the use of a graphical representation and the corresponding quadratic function. In addition, the opening problem of the unit is a mechanics problem that requires modeling of the vertical motion. Moreover, while explaining the graphs of quadratic functions, many every-day life examples of the parabolic shapes are given.

The quadratic equations, problem solving with quadratics, quadratic modeling, graphics calculator investigation, exercise and review set sections may be listed as the next places where the real-life connections are used very frequently through problems. Similar to the Turkish mathematics textbook, these problems are used as the opening, solved—as an immediate application of the mathematical knowledge—, exercise, and review set problems.

The real-life connected problems include the modeling and calculation of the income and number of products within a manufacture context, height in vertical motion, length and width and area of a box, length of a wire bent around an area, golden ratio, speed of a plane or train, as well as the maximum and minimum values of length, width, area, volume, temperature, cost, and height through optimization. Different from the Turkish mathematics textbook, there are no pictures from real-life accompanying to such problems. However, some black and white pictures that represent the context within the problems are used quite often.

Finally, in the graphics calculator investigation problem, a real-life connection is expected to make besides the modeling of the problem and use of technology.

Figure 19 represents the number of the real-life connections that are used in the three chapters on quadratics.

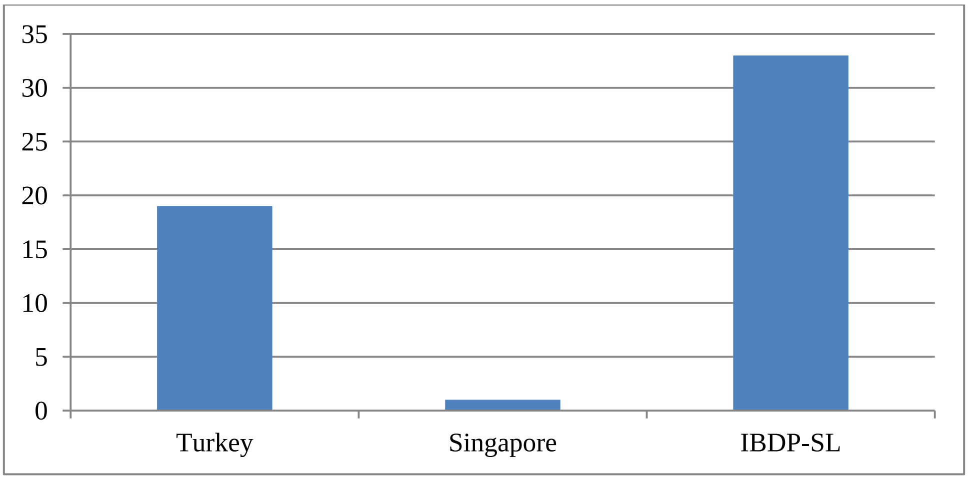


Figure 19. Number of real-life connections within the three units on quadratics

The bar chart in Figure 19 shows that the IBDP-SL quadratics unit is the one which employs the most examples with real-life connections (33), followed by the Turkish quadratics unit which has more than half of the real-life connection use in the IBDP-SL quadratics unit (19). The Singaporean mathematics textbook, on the other hand, has the least number of real-life connections (only one in a modeling problem).

Similar to the variance in the number of the real-life connections within the three quadratics units, types of the real-life connections differ in the Turkish, Singaporean and IBDP-SL mathematics textbooks. Therefore, Table 20 is created to demonstrate the real-life connection examples used each time within the three chapters on quadratics.

Table 20

The real-life connection examples used within the three quadratics units

Turkey	1. Example of parabolic shapes 2. Height and width of a bridge 3. Height and width of a bridge 4. Area of a land 5. Length and width of a book 6. Length and width of a cardboard 7. Width of a rectangular cardboard 8. Duration of manufacture 9. Golden ratio 10. Manufacture profit and loss	11. Motion of a sprinkler 12. Area of a garden 13. Manufacture profit 14. Motion of a sprinkler 15. Height of a bridge 16. Projectile motion 17. Designing a cell phone 18. Manufacture profit and loss 19. Manufacture profit

Singapore	1. Projectile motion	
IBDP-SL	1. Area of a chicken pen	18. Speed of an express
	2. Vertical motion	19. Payment for a trip
	3. Examples of parabolic shapes	20. Vertical motion
	4. Manufacture income	21.Vertical motion
	5. Vertical motion	22. Manufacture cost
	6. Vertical motion	23. Velocity of a car
	7. Manufacture profit	24. Profit of taxis
	8. Area and height of a box	25.Temperature in a greenhouse
	9. Dimensions of a tinplate	26. Area of a garden
	10. Bent wire to form an area	27. Height of a bridge
	11. Bent wire to form an area	28. Height of a cable
	12. Golden ratio	29. Height of tunnels and trucks
	13. Area of a triangular paddock	30. Area of a chicken enclosure
	14. Width of a concrete path	31. Size of a piece of tinplate
	15. Width of a timber paneling	32. Area of an animal pen
	16. Speed of a man	33. Volume and area of a gutter
	17. Speed of an airplane	

Use of technology

In the Turkish mathematics textbook, there is almost no technology usage except that in the calculation of square root of a big number, the use of a calculator is offered (in parentheses).

Although the Singaporean mathematics textbook asks students to use technology in the exercise sections, in the problems that require the use of the graph plotters, no active application of technology is performed during the topic explanation and solved problem parts. That is, the textbook does not demonstrate how to use such technology, but suggests using it within the exercise sections. When suggestions on the use of technology and open tools are made, they are indicated by a small laptop figure at the beginning of the technology related problems as in Figure 20 below:

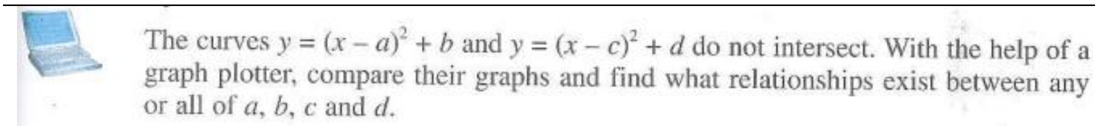
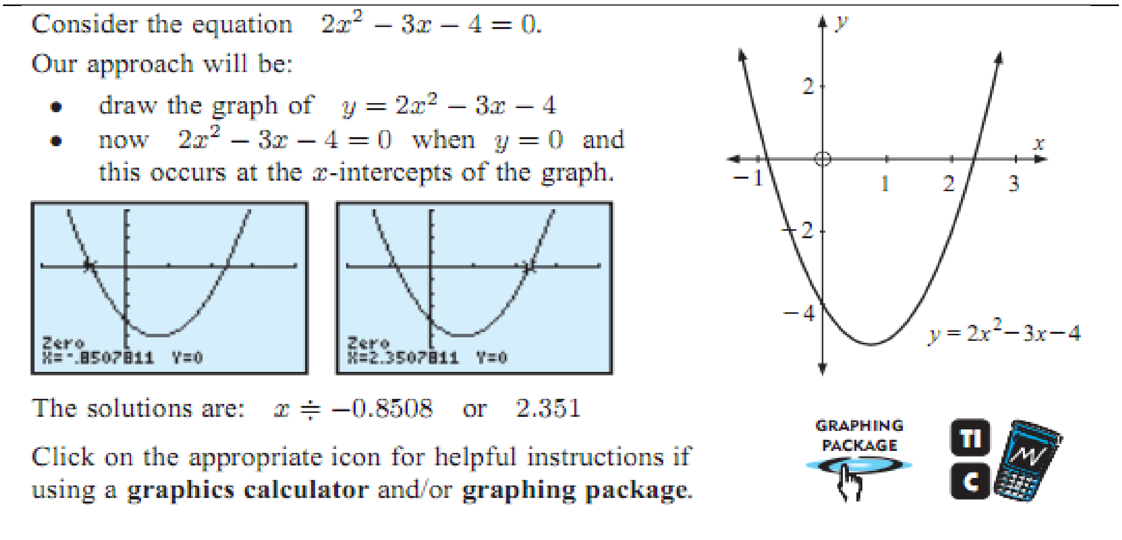

The curves $y = (x - a)^2 + b$ and $y = (x - c)^2 + d$ do not intersect. With the help of a graph plotter, compare their graphs and find what relationships exist between any or all of a, b, c and d.

Figure 20. A sample problem from the Singaporean quadratics unit where use of technology is required (Thong & Hiong, 2006, p. 81)

The IBDP-SL mathematics textbook, on the other hand, allocates a special title (solving quadratic equations with technology) in order to cover the use of the related technology in detail. In this section, students are expected to use a graphic calculator or the graphing package—with the options TI (Texas Instruments) and C (Casio)—, which is available in the PDF version of the textbook within the student CD. Namely, this version of the textbook has a graphics calculator and a graphing package icon where students can reach many helpful instructions about how to use these tools effectively to find out the roots of a quadratic equation or function, and intersection points of a quadratic function with other functions by drawing their graphs. Several quadratic equations are solved by using technology step by step and then their graphical solutions are represented in the same way as they appear on the screen of a graphics calculator as in Figure 21 below:


Consider the equation $2x^2 - 3x - 4 = 0$.

Our approach will be:

- draw the graph of $y = 2x^2 - 3x - 4$
- now $2x^2 - 3x - 4 = 0$ when $y = 0$ and this occurs at the x-intercepts of the graph.

The solutions are: $x \doteqdot -0.8508$ or 2.351

Click on the appropriate icon for helpful instructions if using a **graphics calculator** and/or **graphing package**.

Figure 21. A sample problem where technology is used in the IBDP-SL quadratics unit (Owen et al., 2008, p. 170)

Apart from the technology-special section, in the opening problem of the unit, several technological facilities such as the video clip and simulation of the problem are offered in the PDF version of the textbook so that students can comprehend the problem easily.

Moreover, the investigation 1, investigation 2, and graphics calculator investigation parts contain the usage of a graphics calculators and the graphing package before asking to write the conjecture to be reached. In these investigation activities, the technological tools are used to graph quadratic functions, find out their roots and vertex together with their geometrical significance. Furthermore, sometimes after a solved problem or a problem in the exercise parts, technology is used, or suggested, in order to confirm the results obtained from the algebraic solution of the problem. Lastly, there are some problems in the exercise sections where students are asked to use a graphics calculator and the graphing package in order to write the vertex form of a quadratic function from its general form, and to find out the intersection points of a linear and quadratic function or two quadratic functions.

Figure 22 shows the differences among the three textbooks in terms of the numbers and types of technology used.

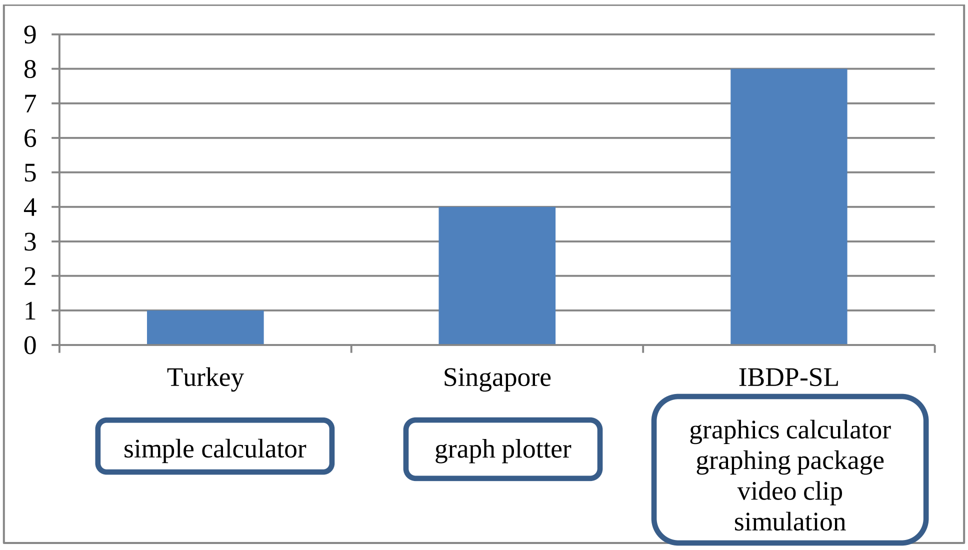


Figure 22. Number and types of technology usage within the three units on quadratics

As seen in Figure 22, the IBDP-SL quadratics unit has the greatest number of technology usage including the use of the graphics calculators, graphing package, video clip, and simulation. The Singapore quadratics unit, however, asks to use a graph plotter half of the times that the IBDP-SL includes application of technology. On the other hand, the Turkish quadratics chapter has almost nothing to do with technology. That is, only the use of a calculator is suggested within one of the solved problems during topic explanation.

Problems and exercises

The Turkish mathematics textbook has eight exercises sections: four in the first main topic (quadratic equations with one unknown), one in the second main topic (linear or quadratic inequalities with one unknown), one in the third main topic (presence and signs of roots), and two within the fourth main topic (quadratic functions with one unknown and their graphs). Some of the exercises sections come after two or three subtopics and cover the contents in these subtopics. However, some of them are delivered at the end of a main topic or a subtopic with problems that apply the acquired knowledge and skills about the main topic or subtopic. Hence, it can be said that the overall goal of each of the exercises sections is to provide students with problems about what they have learned after the previous exercises section.

The problems within the exercises sections are generally abstract mathematical problems which require students to use mainly procedural knowledge. Some of the problems have sub-problems so students can work out solutions within different contexts and internalize concepts by intensive practice. Some problems, however, contain matching or multiple choice items, and geometry or real-life connections, as in Figure 23. The unit test, on the other hand, has thirty multiple choice questions from each part of the unit, to assess students' overall knowledge about quadratics. The answer key of the test is presented just after the last problem.

Bir havan topu ateşlenince mermi parabol yayı çizerek hedefine doğru ilerler. Bu parabolün denklemi $y = \frac{-1}{56}x^2 + x$ olsun. Burada x, km; y, dam türündendir.

Bu topla yapılan atışta;

a. Top mermisi kaç metre uzağa düşer?

b. Mermi, topun bulunduğu noktaya göre kaç metre yükseklikteki bir dağı aşabilir?

Figure 23. A real-life connected exercise problem from the Turkish quadratics unit (Kaplan, 2008, p. 114)

The Singaporean mathematics textbook has a similar style to the Turkish mathematics textbook in terms of the exercise sections. That is, there are three exercise sections at the end of each of the three main topics which give students sufficient practice about the topics. In addition, the miscellaneous exercise 4 section at the end of the unit has twenty-six problems about each part of the chapter where students can apply the mathematical knowledge they acquired on quadratics within the textbook. Answers to the problems belonging to this part are given in the *answers* section at the end of the textbook.

There are four types of problems within the exercise parts: the ones where only practice of mathematical knowledge is aimed; the ones requiring the use of technology (indicated by a small laptop figure at the beginning of the problem), the ones necessitating critical thinking (marked by a small puzzle figure), and the challenging ones which have a star on the upper right side of the question number as in Figure 24.

***17.** Given that $y = kx^2 - 4x + 3k$, express y in the form $a(x - p)^2 + q$, where a, p and q are in terms of k. Hence find the value of k if the maximum value of y is 4.

Figure 24. A sample challenging problem from the Singaporean quadratics unit (Thong & Hiong, 2006, p. 68)

In the critical thinking problems, a deeper understanding of mathematics is expected where students are asked to complete a mathematical proof or create a new one by using technology and the induction method.

Similar to the structure in the other two textbooks, the IBDP-SL mathematics textbook has an exercise section at the end of each of the twelve subtopics where students can practice the newly learned mathematical concepts and skills. Additionally, each of the three topics graphs of quadratic functions, quadratic equations, and the *discriminant, Δ* has an extra exercise section.

Different from the other two textbooks, in the IBDP-SL mathematics textbook, several solved problems are given within the exercise parts in order to serve as an example for students. Moreover, in some problems, tips and suggestions are offered by using a small human figure with a blue speech balloon over his/her head. Sometimes, on the other hand, pictures are given next to the problems, especially the geometry and real-life connected ones, to represent the contexts within the problems, as in Figure 25 below:

A group of elderly citizens chartered a bus for \$160. However, at the last minute, due to illness, 8 of them had to miss the trip. Consequently the other citizens had to pay an extra \$1 each. How many elderly citizens went on the trip?

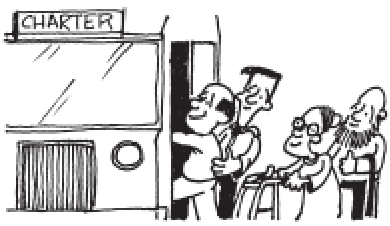


Figure 25. A real-life connected problem from the IBDP-SL quadratics unit (Owen et al., 2008, p. 174)

In the problems that require the use of technology, there are graphing package and graphics calculator icons in the PDF version of the textbook within the interactive student CD, so that students can make use of these tools effectively.

Finally, the five review sets appear at the end of the chapter with problems given in the same order followed in the organization of the topics. In this way, students are given the opportunity to revise and apply their knowledge about quadratics by working out the forty-five problems which are similar to the ones solved to support the topic explanations.

Lastly, all of the problems that are expected to be solved by students within the Turkish, Singaporean and IBDP-SL quadratics units are counted and classified into three categories—low, moderate, and high complexity—based on the mathematical complexity categorization in Table 2.

Three examples for each category of complexity from the three quadratics chapters are given in Figure 26, Figure 27, and Figure 28 below:

Aşağıdaki denklemleri çözmeden, bu denklemlerin köklerini söyleyiniz.

a. $x^2 + 3x + 2 = 0$ **b.** $x^2 - (a + b)x + ab = 0$

c. $x^2 - (\sqrt{3} + \sqrt{2})x + \sqrt{6} = 0$ **ç.** $x^2 + (\sqrt{3} + 1)x + \sqrt{3} = 0$

d. $x^2 - (a - b)x - ab = 0$ **e.** $(a^2 - 9)x^2 - 2ax + 1 = 0$

Use the discriminant to determine the nature of the roots of the following quadratic equations. When the roots are real, find these roots:

(a) $x^2 - 2x - 5 = 0$ **(b)** $4x^2 + 4x + 1 = 0$ **(c)** $3x^2 = 8x + 3$

(d) $x(1 - 3x) = 2$ **(e)** $(2x - 1)(x + 1) = 2$ **(f)** $2x(x - 3) = 3 - 4x$

Find the turning point (vertex) for the following quadratic functions:

a $y = x^2 - 4x + 2$ **b** $y = x^2 + 2x - 3$

c $y = 2x^2 + 4$ **d** $y = -3x^2 + 1$

e $y = 2x^2 + 8x - 7$ **f** $y = -x^2 - 4x - 9$

g $y = 2x^2 + 6x - 1$ **h** $y = 2x^2 - 10x + 3$

i $y = -\frac{1}{2}x^2 + x - 5$ **j** $y = -2x^2 + 8x - 2$

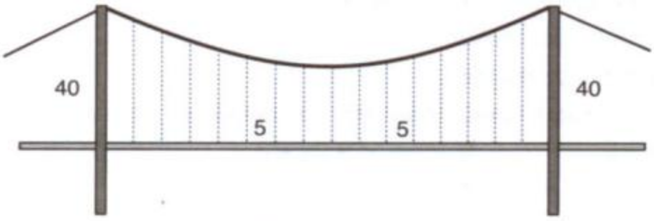
Figure 26. Low complexity examples from the three quadratics units (Kaplan, 2008, p. 72; Thong & Hiong, 2006, p. 73; Owen et al., 2008, p. 177)

Bir gömlek dikimevi belli bir sürede 360 gömlek dikmeyi planlamıştır. Her gün planlanandan 4 gömlek fazla dikilerek iş 1 gün önce bitirilmiştir. Başlangıçta bu işin kaç günde bitirileceği planlanmıştır?

If the equation $px^2 - p + 10 = 2(p + 2)x$ has real solution(s), show that p cannot lie between 1 and 2.

Chuong and Hassan both drive 40 km from home to work each day. One day, Chuong said to Hassan, "If you drive home at your usual speed, I will average 40 kmph faster than you and arrive home in 20 minutes less time." Find Hassan's speed.

Figure 27. Moderate complexity examples from the three quadratics units (Kaplan, 2008, p. 79; Thong & Hiong, 2006, p. 76; Owen et al., 2008, p. 174)

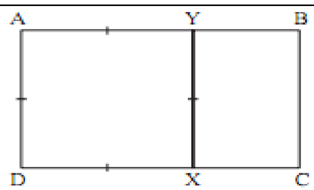


Bir asma köprünün taşıyıcısı olan iki çelik halattan her birinin köprünün iki ayağı arasında kalan kısmı (yaklaşık) bir parabolün parçasıdır. Köprü ayaklarının her birinin tepe noktası köprüden 40 metre yukarıdadır. Köprü iki yandan da beşer metre aralıklarla 15 adet çelik telle taşıyıcı halata bağlanmıştır. Köprünün bir yanında bulunan bu 15 çelik telin uzunluklarını hesaplayınız.

For each of the following pairs of quadratic curves, where $a \neq 0$, are there any relationships between their turning points, their intercepts with the axes and their shapes?
With a graph plotter, investigate these relationships for
(a) $y = ax^2 + bx + c$ and $y = ax^2 - bx + c$ by varying the value of b,
(b) $y = ax^2 + bx + c$ and $y = -ax^2 + bx + c$ by varying the value of a,
(c) $y = ax^2 + bx + c$ and $y = -ax^2 + bx - c$ by varying the values of a and c.
In the above investigation, we have only induced results from **certain** values of a, b and c. How can we prove that our results will hold for **any** values of these constants?

The *golden rectangle* is the rectangle defined by the following statement:

The golden rectangle can be divided into a square and a smaller rectangle by a line which is parallel to its shorter sides, and the smaller rectangle is **similar** to the original rectangle.



Thus, if ABCD is the golden rectangle, ADXY is a square and BCXY is similar to ABCD, (i.e., BCXY is a reduction of ABCD).

The ratio of $\frac{AB}{AD}$ for the golden rectangle is called the **golden ratio**.

Show that the golden ratio is $\frac{1+\sqrt{5}}{2}$. (**Hint:** Let AB $= x$ units and BC $= 1$ unit.)

Figure 28. High complexity examples from the three quadratics units (Kaplan, 2008, p. 114; Thong & Hiong, 2006, p. 82; Owen et al., 2008, p. 173)

Table 21 and Figure 29 represent the numbers and percents of items with low, moderate, and high cognitive complexity within problems and exercises in the three quadratics chapters.

Table 21

Number of exercise items at different levels of mathematical complexity within the three quadratics chapters

	Turkey	Singapore	IBDP-SL
Total number of items	357	145	546
Number of items with low complexity	202	72	237
Number of items with moderate complexity	144	60	276
Number of items with high complexity	11	13	33

Table 21 demonstrates that although the Turkish mathematics textbook allocates the largest space to quadratics compared to the other two textbooks, the IBDP-SL quadratics unit is the one with the greatest number of items, followed by the Turkish mathematics textbook. The Singaporean mathematics textbook, on the other hand, includes the least number of items among the three textbooks, which is parallel to the fact that it is the textbook where the quadratics chapter occupies the shortest space.

As seen in Figure 29, regarding mathematical complexity of the items, the Turkish and Singaporean mathematics textbooks mainly contain items with low complexity, followed by the items with moderate complexity whereas they have a very small number of items with high complexity. Even though the Singaporean mathematics textbook has a small number of items with high complexity compared to the other types of items within the chapter, it is the one having relatively the highest percent of items with high complexity compared to the other two textbooks. The IBDP-SL mathematics textbook, on the other hand, uses the items with moderate complexity the most, followed by the items with low complexity and high complexity.

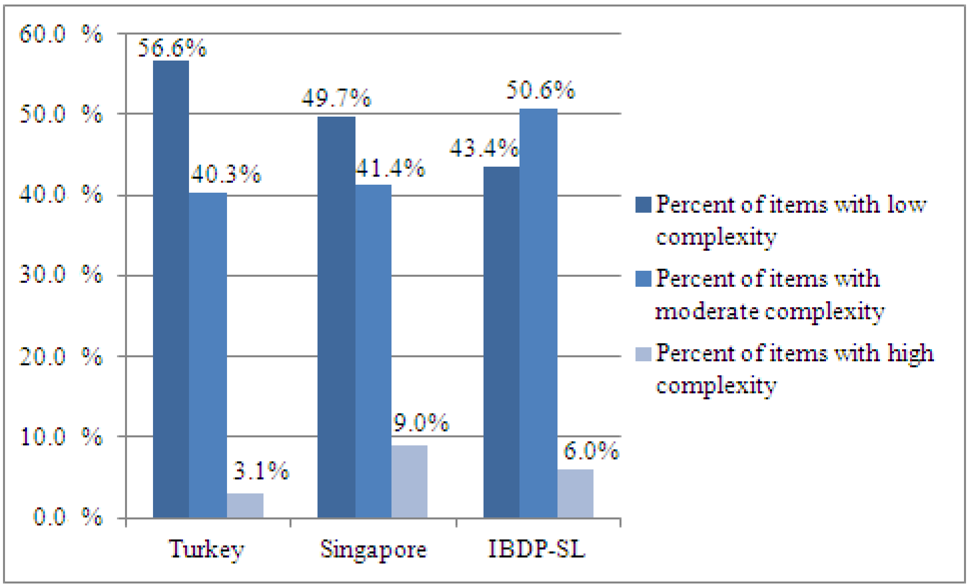


Figure 29. Percent of complexity of items within the three quadratics chapters

Historical notes

Although the Singaporean mathematics textbook has no historical notes within the quadratics unit, the Turkish textbook contains two different sections about two mathematicians: Carl Friedrich Gauss and El Harezmi. The first one is located at the end of the first exercises section and it gives information about Carl Friedrich Gauss, including the milestones of his life, his successes and contributions in mathematics, and the importance of mathematics in his life. Secondly, at the end of the first main topic of the chapter, another historical note about El Harezmi is presented that includes his life, the mathematics books he wrote, and his achievements in mathematics and the other fields. Different from the first historical note, here the connection between the scientist and quadratics is explained as he is the first mathematician who solved the linear and quadratic equations.

The IBDP-SL mathematics textbook, on the other hand, presents a historical note within the introduction part at the beginning of the chapter. In this note, the famous scientist Galileo's works on the paths of projectiles together with his approach to the experiments are explained without mentioning about the relationship between the projectile motion and quadratics. Nevertheless, the connection with quadratics is more evident in this section, compared to the historical notes in the Turkish mathematics textbook.

CHAPTER 5: DISCUSSION

Introduction

The purpose of this study was to compare the quadratics chapters in three mathematics textbooks from Turkey, Singapore and the IBDP, and investigate the features of textbook design in these chapters. The selected textbooks were analyzed from the perspectives of content, organization, and presentation style. In other words, by using content analysis, the three quadratics units were analyzed and compared in terms of their positions and weights within the mathematics curricula they belong, the learning outcomes they target, the layout of structural components and mathematical content, general organization, and the way of presentation with respect to the student-centered activities, topic explanations, real-life connections, use of technology, problems and exercises, and historical notes. The evaluation and interpretation of the results through reader-oriented theory (Weinberg & Wiesner, 2011) and Rezat's (2006) model of textbook use is also presented in this chapter.

Discussion of the findings

The findings of the study and corresponding discussions are presented below based on content, organization, and presentation style.

Content

External positioning of the quadratics chapters

The analysis of the three quadratics units from an external perspective revealed some similarities and differences among them in terms of the positions of chapters and weights of the units, the time allocated to them within the programs, and at which grades the units are taught.

The positions of the quadratics units in the three textbooks gave some hints about the continuity and connections among the topics intended in the corresponding mathematics syllabi (see Table 3-5). For instance, in the Turkish mathematics textbook, the quadratics chapter was given after polynomials, which shows that in the Turkish mathematics curricula, quadratics are considered as special types of polynomials. In this way, students are expected to make connections with their prior knowledge about polynomials and functions. Similarly, the Singaporean mathematics textbook presents quadratics before the chapter on remainder and factor theorems that includes polynomial identities and cubic equations. Thus, in the Singaporean mathematics syllabus, the polynomial concept is expected to explain with the assumption that students come across the quadratic and linear expressions (two types of polynomials) given earlier. Additionally, cubic equations are defined by using quadratic equations, showing an easy to complex approach. The IBDP-SL mathematics textbook, on the other hand, makes a connection with quadratics in the binomial theorem topic—the unit after quadratics—by expressing that the binomial expansion of $(a+b)^2$ is a quadratic expression and showing a move from a specific to general case. So, it became clear that in all three textbooks, the positions of the units are selected through a logical way that enables students to make connections among the topics and see the usefulness of each of the topics within a bigger picture. The intended connections contribute to students' use of textbooks according to reader-oriented theory (Weinberg & Wiesner, 2011). The reader will remember that according to reader-oriented theory, textbooks are better if they are organized to reflect an organization that will help the reader extract meaning and make connections among the concepts contained in the textbooks.

The Turkish mathematics textbook has a relatively small number of topics compared to the Singaporean and IBDP-SL mathematics textbooks, which probably results from its intention to be used by only the students in the 10th grade and the other two serving for two-year programs. Correspondingly, the percent weights of the quadratics unit over the three textbooks differ (see Table 6). It appears that the quadratics unit occupies a more dominant place in the Turkish mathematics textbook even if the percent weights of the chapters in the Singaporean and IBDP-SL mathematics textbooks are doubled—with the underlying reason that these two textbooks cover all of the topics in the two-year curricula.

Regarding the time allocated to quadratics within the programs, parallel to the percent weights of the units in the three textbooks, the quadratics topic is taught in a longer period of time in Turkey.

Analysis of internal content

The investigation of the three chapters on quadratics in terms of their internal contents revealed that that the Turkish mathematics textbook attempts to fulfill *nearly twice* of the learning outcomes covered by the Singaporean and the IBDP-SL mathematics textbooks (see Table 13). This finding seems parallel to the fact that the Turkish mathematics textbook has an approach to explain every mathematical issue directly under a separate title, instead of offering students group or individual work and portfolio opportunities to investigate these topics. However, this may lead students become used to absorbing ready-made mathematical knowledge without personally investing in it. Similarly, the Singaporean textbook appears to not explicitly encourage such activities and, hence, gives teachers insufficient support and material to enhance student learning pedagogically. This may result from the fact that the Singapore mathematics textbook is written for students who prepare for a written examination.

The IBDP-SL mathematics textbook, on the other hand, includes an opening problem and investigation sections where students are directed to explore the crucial points within the topics on their own or with their peers. It is clear that this approach of the IBDP-SL mathematics textbook would be supported by reader-oriented theory, as the theory advocates putting students at the center of the learning and reading process by enabling them to actively participate and make meaning from the textbook (Weinberg & Wiesner, 2011). Moreover, this feature of the textbook can give some practical ideas to teachers about the didactical aspects of the mathematical knowledge, as represented in Rezat's (2006) teacher-textbook-mathematical knowledge (didactical aspects) set in his model of textbook use. These hints may be useful for teachers since they benefit from textbooks to make decisions about how to teach a mathematics topic and engage students in a mathematical discovery (Russell et al., 1995).

Different from the Turkish and IBDP-SL mathematics textbooks, the Singaporean mathematics textbook presents both algebraic and geometric features of quadratics simultaneously from the start of the chapter, by exemplifying a synthetic approach in the mathematics curriculum. In addition, the textbook seems to have some epistemological assumptions. That is, although it aims to construct background knowledge and skills about quadratics in the quadratics chapter; it uses the quadratic function examples and their graphs in the functions chapter, which is positioned five chapters later. This reflects that different curricula may have different linkage assumptions within the inner organization of the topics.

Finally, among the three subtopics—quadratic equations, inequalities, and quadratic functions—in the all-inclusive list of learning outcomes, the greatest total number of learning outcomes covered by the three textbooks belongs to quadratic functions. This outcome is parallel to the weights of the learning outcomes aimed for the three subtopics in the Singaporean and IBDP-SL mathematics syllabi. Even if the Singaporean mathematics textbook does not use the quadratic function expression during the unit, graphs of quadratic expressions are indicators of the quadratic function concept, which is presented later on by the textbook. As a result, this similarity among the textbooks indicated the existence of an implicit international agreement in terms of the weight given to the quadratic function subtopic.

Organization

Internal organization of the quadratics units

The tables of contents with the names of the quadratics units and subtitles give students and teachers clues about in what order and how detailed the quadratics topic is covered in the Turkish, Singaporean and IBDP-SL mathematics textbooks.

All three textbooks have similar approaches in terms of the internal organization of the chapters on quadratics (see Table 17). It can be assumed that this relatively consistent pattern in the content presentation facilitates students to get used to the structure of the textbook, and, hence, gives them the opportunity to extract meaning from the textbook effectively (Weinberg & Wiesner, 2011).

The three chapters on quadratics differ in regard to the layout and progress of mathematical concepts by displaying two different approaches (see Table 18). These two styles of organization show that the Turkish mathematics curriculum attempts to teach the topic from a specific to general (inductive) approach as quadratic equations are special cases of quadratic functions when $y=0$ in $y= ax^2+bx+c$. However, the Singaporean and IBDP-SL mathematics curricula follow a general to specific (deductive) order in the instruction of quadratics. This differentiation among the textbooks shows that there is more than one way of organizing a mathematics topic in terms of the flow of the mathematical concepts.

General organization of the units on quadratics

The analysis of the three units on quadratics with respect to the general organization showed that each of the Turkish, Singaporean and IBDP-SL mathematics textbooks allocates a remarkable number of pages for the topic explanations (see Table 19). It seems that since the Turkish mathematics textbook has a tendency to explain each mathematical idea within the topics instead of leaving it for student discovery, it devotes the greatest percent of pages to the topic explanations. On the other hand, the IBDP-SL mathematics textbook has a relatively less space for this since it is the one which uses the largest space for the problems and exercises.

As the Singaporean mathematics syllabus is arranged around students' problem solving experiences, at first glance it may sound deceiving that the Singaporean mathematics textbook has a less space for the problems and exercises compared to the percent of pages with topic explanations. However, this situation results from the fact that the topic explanations are primarily presented through solved problems in this textbook (Singapore Ministry of Education, 2006).

Lastly, the Singaporean and IBDP-SL mathematics textbooks allocate greater percents of pages to other purposes—such as table of content, historical note parts, large pictures and photos, and topic summary pages—compared to the Turkish mathematics textbook. The underlying reason here is that while the Singaporean and IBDP-SL mathematics textbooks are in favor of presenting summary of the key points within topics on separate pages, the Turkish mathematics textbook seems to be contented with the result tables in the topic explanation parts. If the two approaches are evaluated according to reader-oriented theory, styles of the Singaporean and IBDP-SL mathematics textbooks can be accepted as more reader-oriented since in this case students have a chance to find summaries of the significant mathematical concepts more easily on special summary pages instead of searching them at multiple points within the topic explanations (Weinberg & Wiesner, 2011).

Presentation style

Student-centered activities

The Turkish mathematics textbook is the one that offers the greatest number of student-centered activities during the activities, topic explanation parts, and solved problems sections. This situation seems to be consistent with the philosophy of the national mathematics education in Turkey, which is changing from a teacher-centered to a student-centered approach (see Figure 1). In this new approach, students are given the opportunity to start from a problem and then discover, hypothesize, verify, and generalize the mathematical ideas through making connections and reasoning (MEB, 2011). This concurrency between the textbook and the curriculum in which the textbook is embedded is in line with the idea that mathematics textbooks are indicators of the perspectives of countries and programs on mathematics education.

The IBDP-SL mathematics textbook is the second one in terms of providing activities where students are placed at the center. The opening problem and investigation sections, where students are expected to perform mathematical discoveries and display an understanding of the mathematical applications with the support of technological facilities, reflect the philosophy of the IBDP mathematics education (IBDP, 2006). The Singaporean textbook, on the other hand, uses only student-inquiry during the topic explanations and worked examples without giving a place to student-centered activities, which is probably a result of the textbooks' being prepared to support students who study for an exam.

To sum up, the IBDP-SL and Turkish mathematics textbooks make use of student-centered activities to enable student discovery by discussing mathematical ideas with peers, making connections with prior knowledge, and raising questions. Additionally, such activities help to identify students' competencies, mathematical knowledge, skills and understandings to be able to work within the context, which is crucial in reader-oriented theory (Weinberg & Wiesner, 2011). Meanwhile, these activities can also be useful to direct teachers by giving them ideas about in-class activities and supporting them to serve as a bridge between students and textbooks, which is represented by the two subsets of Rezat's (2006) model of textbook use: student-teacher-textbook and teacher-textbook-mathematical knowledge(didactical aspects).

Topic explanations

During explanation of the topics, the Turkish and IBDP-SL mathematics textbooks have a deductive structure (definitions-mathematical-knowledge-solved problems) whereas the Singaporean mathematics textbook follows a relatively inductive approach (brief definitions-solved problems-mathematical knowledge as obtained results-solved problems). It is probable that a textbook with deductive approach is appropriate to the student-textbook-mathematical knowledge model of textbook use since it is likely to aim at replacing the teacher although Love and Pimm (1996) claim that the teacher's role is vital in mediating the textbook content to students in order to facilitate their learning. On the other hand, the student who uses a textbook with an inductive approach is expected to need the teacher to mediate the mathematical knowledge within the textbook.

All of the textbooks have result sections—generally in the form of tables or colorful texts given after the solved examples—which help students to generalize the mathematical ideas from the examples and revise the significant conjectures. This common feature of the three mathematics textbooks seems to be desirable according to reader-oriented theory (Weinberg & Wiesner, 2011).

The Turkish and IBDP-SL mathematics textbooks use an informal *we* language compared to the Singaporean mathematics textbook. That is, the topics are explained in a relationship with the student just like two peers' learning the new topic together with sentences such as 'let's find an answer for this' and 'let's see if this is correct.' In addition, warnings, suggestions, hints, reminders, and recommendations are used to prevent the possible misconceptions. As this way of communication in the Turkish and IBDP-SL mathematics textbooks aims to facilitate students during the reading process, it can be assumed as a contribution to effective reading according to reader-oriented theory (Weinberg & Wiesner, 2011). The Singaporean mathematics textbook, however, has the approach 'here is the mathematical knowledge, read and learn it', which indicates that the student is expected to reach the mathematical knowledge primarily by reading the text and following the solved problems.

Another similarity between the Turkish and IBDP-SL mathematics textbooks is that they use tables and pictures to represent the story within solved examples and facilitate meaning making. Furthermore, in the Turkish mathematics textbook, some questions are left unanswered for students to examine the necessary prior knowledge. Moreover, some problems that require graphical representations are half-solved in order to give the student the opportunity to transform the interpretations into drawings. All of these attempts target students' active participation in the reading process by developing an active habit in regard to the textbook use.

Real-life connections

The Singaporean mathematics textbook is different from the Turkish and IBDP-SL mathematics textbooks in terms of the real-life connections included. That is, although the Turkish and IBDP-SL mathematics textbooks make use of the real-life connections frequently, the Singaporean mathematics textbook has only one problem with a story from real-life in the whole chapter. In addition, regarding the types of real-life connections, the IBDP-SL mathematics textbook has a broader range of applications with more creative contexts and more attempts for student discovery compared to the Turkish mathematics textbook. This shows that there are both distinct degrees and ways of using real-life connections within mathematics textbooks.

These approaches of the Turkish and IBDP-SL mathematics textbooks are consistent with the philosophies of the respective mathematics curricula where students are expected to gain a positive attitude towards mathematics by appreciating the usefulness of mathematics in daily life (IBDP, 2006; MEB, 2011). Moreover, the pedagogical support given through real-life connections within the two textbooks show that the Turkish mathematics textbook struggles to amend pedagogy within a centrally prescribed curriculum and both the Turkish and IBDP-SL mathematics textbooks follow recent non-governmental suggestions on pedagogy (Howson, 1993). On the other hand, while the Singaporean mathematics syllabus has an objective for students to acquire the necessary mathematical concepts and skills for daily life, the textbook has an inconsistent application in terms of using real-life connections. This displays that not always the textbook reflects each single detail in the curriculum comprehensively (Singapore Ministry of Education, 2006).

Use of technology

The IBDP-SL mathematics textbook is the one that makes the most use of technology with the widest range of applications throughout the quadratics chapter. On the other hand, the Singaporean mathematics textbook does not explain how to use technology to solve and graph quadratics. Instead, it asks to use technological devices in the exercise sections, which demonstrates that teaching the procedure of technology use is expected from teachers during their instruction. It is clear that these implementations of the IBDP-SL and Singaporean mathematics textbooks are in line with the philosophies of mathematics education in the embedded curricula, which assume technology usage as an integral part of the mathematics education in secondary school (IBDP, 2006; Singapore Ministry of Education, 2006). Furthermore, since the IBDP and Singaporean students are expected to use graphing calculators in papers used in the assessment of mathematics, it is reasonable for these textbooks to include technology usage (IBDP, 2006; Singapore Examinations and Assessment Board, 2012).

The Turkish mathematics textbook, on the other hand, has nearly no applications with technology. It seems that although there is a tendency to use technology in the mathematics education according to the secondary school mathematics curriculum of Turkey, students need to be given more opportunity to use technology in learning mathematics. Thus, to take the full advantage of what technology can do for learning mathematics, the Turkish mathematics textbooks should include more applications of technology (MEB, 2011).

Problems and exercises

Regarding the comprehensive use of problems and exercises, all three mathematics textbooks seem to have the assumption that problem solving and exercises constitute an indispensable component of mathematics education (National Research Council, 1999). Although all of the textbooks make use of the problems and exercises within the chapters on quadratics, the IBDP-SL mathematics textbook includes the greatest number of questions, which is parallel to the percent weight of pages allocated to the problems and exercises in this textbook.

There are both similarities and differences with respect to the types of items used in the problems and exercises of the three mathematics textbooks. While the abstract mathematical, critical thinking, modeling, real-life and geometry connected problems are the common ones in the three textbooks, numbers of their usage differ. Additionally, the Turkish mathematics textbook is the only one that does not ask the use of technology but uses many multiple choice items within the exercise sections and the unit test. This seems to be parallel to the fact that the items in the university entrance exam in Turkey are all multiple choice ones without the requirement of technology use (ÖSYM, 2012a). These findings indicate that most of the time, the types of items used in the problems and exercises of the textbooks are in line with their way of explaining the topics within the chapters, as well as the targeted learning outcomes and ways of assessment in the respective mathematics curricula.

The findings revealed that there is an interesting pattern in the distribution of the problems and exercises in complexity. That is, the total numbers of the problems with low and moderate mathematical complexity constitute more than 90% of the whole problems in all three mathematics textbooks. The percent of the problems with low complexity is higher than the ones with moderate complexity in the Turkish and Singaporean mathematics textbook. On the other hand, the IBDP-SL mathematics textbook—with a higher cognitive demand in the problems within the chapter—allocates the greatest percent to the problems with moderate complexity (see Figure 29). This difference among the textbooks is not surprising and, hence, can be explained by the respective student populations that the textbooks are intended to serve. Turkey and Singapore have national mathematics curricula which appeal to students from all levels of academic aptitude in mathematics. The IBDP students, however, are selected among the students coming from relatively higher socio-economic backgrounds, with possibly higher motivation and academic aptitude (IB, 2012c, MEB, 2011; Singapore Ministry of Education, 2011). Finally, the Singaporean mathematics textbook is the one with the greatest percent of problems with high complexity, which results from the relatively frequent existence of problems that require higher level thinking skills such as analysis, synthesis, and evaluation of abstract mathematical concepts.

All in all, it is obvious that differences among the three textbooks in terms of cognitive complexity level of the problems and exercises seem to result from the fact that these three textbooks serve as transmitters of three different mathematics curricula with varying emphases on targeted learning outcomes including skills, competencies, theoretical and practical knowledge (Howson, 1993).

Historical notes

Different from the Singaporean mathematics textbooks, the Turkish and IBDP-SL mathematics textbooks make use of historical notes in the quadratics units through which students have the opportunity to learn the historical perspectives of mathematics, as well as outstanding mathematicians' lives. The use of historical notes also help students realize the significance of mathematics in daily life, which is encouraged in the Turkish and IBDP-SL mathematics syllabi (IBDP, 2006; MEB, 2011).

In conclusion, the findings and interpretations above revealed that the three mathematics textbooks reflect the philosophies of the countries and the program where they are used. This outcome is parallel with the idea in the literature. That is, textbooks with different content emphases, organizational preferences, and presentation styles provide insights about the way mathematics is taught and learned in the corresponding educational systems (Schmidt, McKnight, Valverde, Houang, & Wiley, 1997). In addition, each of the textbooks has different features that can be regarded as characteristics of effective mathematics textbooks according to reader-oriented theory (Weinberg & Wiesner, 2011). Similarly, it is now clear that each approach in the mathematics textbooks may represent different ways of textbook use in terms of the relationships among the student, teacher, textbook, and mathematical knowledge (didactical aspects) in Rezat's (2006) model of textbook use.

Implications for practice

Textbooks constitute an indispensable component of mathematics education by serving as main resources for teaching and learning activities. Therefore, curriculum planners, schools, and teachers should select textbooks carefully. At this point, the findings of the study indicated that there is a variety of textbook features that can enhance students' meaning making from the textbook and hence give them the opportunities to develop mathematical understanding and practice the mathematical knowledge and skills.

Organization of the units in a logical and constructivist way and taking into account intended students' level of cognitive development can be assumed as some of the characteristics of effective mathematics textbooks according to reader-oriented theory. Moreover, inclusion of a sufficient number of student-centered activities, use of special summary tables or parts that cover the crucial conjectures within the topics, integration of real-life connections and historical notes as much as possible, and use of an appropriate language with guiding suggestions and warnings are some other qualities of mathematics textbooks that seem to make them more reader-oriented (Weinberg & Wiesner, 2011). Thus, to make a good decision in the preparation and selection of mathematics textbooks, authors, curriculum and program planners, schools, and teachers may consider these characteristics of mathematics textbooks.

Teachers play a significant role in regard to the use of mathematics textbooks in the classroom as they occupy one of the four corners in Rezat's (2006) model of textbook use (see Figure 5). Therefore, they should be trained in in-service programs during pre-service education about how to use different parts of mathematics textbooks effectively. For instance, teachers ought to be aware of the intentions of textbooks about the mathematical knowledge and skills to be attained by students, as well as the underlying curricular philosophies in order to benefit from textbooks appropriately. Furthermore, teachers should make effective use of different parts of the textbook such as opening problems, investigation sections, student-centered activities, problems and activities that require the use of technology, and historical notes. That is, teachers can benefit from such sections of mathematics textbooks to identify and improve student competencies, knowledge, skills and understandings. They can also ensure student discovery by asking students to discuss the mathematical ideas in the textbook with their peers, increase student motivation by giving interesting examples of mathematical applications from daily life, and prepare creative and challenging homework. All in all, teachers need to be skillful about mediating the mathematical content in the textbook and turning these parts within the textbook into meaningful and practical teaching and learning activities that would increase student understanding and participation.

Implications for research

The findings of this study are delimited to the results of the analysis of one chapter from three perspectives (content, organization, presentation style) and the related interpretations made based on reader-oriented theory (Weinberg & Wiesner, 2011) and Rezat's (2006) model of textbook use, when applicable. Therefore, different textbooks should be analyzed from a broader scope to discover the properties of high quality textbooks that facilitate the student and teacher use. Additionally, students' and teachers' attitudes, preferences, and habits about the textbook use inside and outside the classroom should be studied. Thus, particular features of the textbooks that accomplish to meet the needs of students and teachers can be identified.

Moreover, the textbook qualities with the potential of maximizing student learning ought to be explored by investigating both the relationship between students and textbooks and the expectations of teachers, students and educational programs from textbooks. Then, more reliable criteria can be created to evaluate textbooks in terms of the degree of facilitating student learning and, hence, more practical suggestions can be made about the effective textbook use.

Limitations

There are several limitations of this study that affected the findings and, hence, called for a careful interpretation of the findings. First, the evaluation and comparison of the three textbooks was done focusing on the unit quadratics, which limited the amount of content. In addition, the features of the textbooks were delimited to the analysis conducted from the perspectives of content, organization, and presentation style. Further, more countries and programs may have been included.

Regarding the limitations faced during the analysis of three textbooks, the comparison of the three quadratics units in terms of external content was a little difficult for a few reasons. One is that the Turkish mathematics textbook was written only for students in the 10th grade, and the other two mathematics textbooks included the topics of two-year curricula. Another was the difficulty of accessing detailed syllabus information in Singapore.

Another limitation was about the comparing the mathematical complexity level of problems within the three mathematics textbooks, because of the different expectations of the three mathematics curricula, which affect the type of problems in the textbooks. The Turkish and Singaporean mathematics textbooks are used as a component of the national mathematics education of these two countries where students with different levels of mathematical achievement are included. However, the IBDP-SL mathematics textbook belongs to an international educational program that appeals to highly motivated secondary school students. The findings were interpreted with this context in mind.

REFERENCES

Ahuja, O. P. (2005). World-class high quality mathematics education for all K-12 American students. *The Montana Mathematics Enthusiast*, *3*(2), 223-248.

Alacacı, C., Erbaş, A. K., & Bulut, M. (2011). *A comparison of Turkish, Singaporean and American mathematics textbooks based on textbook design features* (Unpublished Manuscript).

American Institutes for Research. (2005). *What the United States can learn from Singapore's world-class mathematics system (and what Singapore can learn from the United States: An exploratory study.* Washington, DC: Ginsburg, A., Leinwand, S., Anstrom, T., Pollock, E., & Witt, E.

Apple, M. W. (1986). *Teachers and texts: Apolitical economy of class and gender relations in education.* New York, NY: Routledge.

Apple, M. W. (1992). The text and cultural politics. *Educational Researcher*, *21*(7), 4-11.

Askew, M., Brown, M., Johnson, D., Millett, A., Prestage, S., & Walsh, A. (1993). *Evaluation of the implementation of national curriculum mathematics at key stages 1, 2, and 3* (Vol. 1). London: SCAA.

Ben-Peretz, M. (1990). *The teacher-curriculum encounter: Freeing teachers from the tyranny of texts*. Albany: State University of New York Press.

Bukova-Güzel, E. (2010). An examination of pre-service mathematics teachers' approaches to construct and solve mathematical modeling problems. *Teaching Mathematics and Its Applications, 30*(1), 19-36.

Dowling, P. (1996). A sociological analysis of school mathematics texts. *Educational Studies in Mathematics, 31*(4), 389-415.

Ersoy, Y. (2007). Use of graphing calculators in high school mathematics-II: Students' views on comprehending functions and graphs. *Eurasian Journal of Educational Research, 28,* 55-65.

Foxman, D. D. (1999). *Mathematics textbooks across the world: Some evidence from the third international mathematics and science study (TIMSS)*. Slough: National Federation for Educational Research.

Freeman, D. J., Kuhs, T., Porter, A., Floden, R., Schmidt, W., & Schwille, J. (1983). Do textbooks and tests define a national curriculum in elementary school mathematics? *Elementary School Journal, 83,* 501-514.

Freeman, D.J., & Porter, A. C. (1989). Do textbooks dictate the content of mathematics instruction in elementary school? *American Educational Research Journal, 26* (3), 403-421.

Haggarty, L., & Pepin, B. (2001). Mathematics textbooks and their use in English, French and German classrooms: A way to understand teaching and learning cultures. *Zentralblatt for the Didactics of Mathematics, 33*(5), 158-175.

Halıcıoğlu, M. (2008). The IB diploma programme in national schools: The case of Turkey. *Journal of Research in International Education, 7*(2), 164-183.

Hayden, M. (2006). *Introduction to international education*. London: Sage Publications.

Hoven, J., & Garelick, B. (2007). Singapore math: Simple or complex? *Educational Leadership, 65*(3), 28-31.

Howson, A. G. (1993). *Mathematics textbooks: A comparative study of grade 8 texts.* (TIMSS Monograph, No.3). Vancouver, BC: Pacific Educational Press.

Hsieh, H. F., & Shannon, S.E. (2005). Three approaches to qualitative content analysis. *Qualitative Health Research, 15*(9), 1277-1288.

International Baccalaureate. (2012a). *About the International Baccalaureate.* Retrieved from http://www.ibo.org/general/who.cfm

International Baccalaureate. (2012b). *How to become an International Baccalaureate world school.* Retrieved from http://www.ibo.org/become/index.cfm

International Baccalaureate. (2012c). *The IB diploma programme*. Retrieved from http://www.ibo.org/diploma/

International Baccalaureate Diploma Programme. (2006). *Mathematics SL subject guide, first examinations 2008*. Cardiff, Wales, UK: International Baccalaureate Organization.

Johansson, M. (2003). *Textbooks in mathematics education: a study of textbooks as the potentially implemented curriculum* (Licentiate thesis). Department of Mathematics, Luleå University of Technology, Luleå.

Karadağ, E., Deniz, S., Korkmaz, T., & Deniz, G. (2008). Yapılandırmacı öğrenme yaklaşımı: Sınıf öğretmenleri görüşleri kapsamında bir araştırma. *Uludağ Üniversitesi Eğitim Fakültesi Dergisi, 21*(2), 383-402.

Krippendorff, K. (2004). *Content analysis: An introduction to its methodology*. Thousand Oaks, CA: Sage.

Lemmer, M., Edwards, J., & Rapule, S. (2008). Educators' selection and evaluation of natural sciences textbooks. *South African Journal of Education*, *28*, 175-187.

Lithner, J. (2003). Students' mathematical reasoning in university textbook exercises. *Educational Studies in Mathematics*, *52*(1), 29-55.

Litz, D. R. A. (2005). *Textbook evaluation and ELT management: A South Korean case study* (Master's thesis). UAE University: Al-Ain. Retrieved from http://www.asian-efl-journal.com/Litz_thesis.pdf

Love, E., & Pimm, D. (1996). ' This is so': a text on texts. In A. J. Bishop, K. Clements, C. Keitel, J. Kilpatrick & C. Laborde (Eds.), *International handbook for mathematics education* (Vol. 1, pp. 371-409). Dordrecht: Kluwer.

Mahmood, K. (2010). Textbook evaluation in Pakistan: Issue of conformity to the national curriculum guidelines. *Bulletin of Education and Research, 32*(1), 15-36.

Milli Eğitim Bakanlığı Talim ve Terbiye Kurulu Başkanlığı Türkiye. (2011). *Matematik dersi öğretim programı ve kılavuzu*. Ankara: MEB Yayınları.

Milli Eğitim Bakanlığı Yenilik ve Eğitim Teknolojileri Genel Müdürlüğü ve Ölçme Değerlendirme ve Yerleştirme Grup Başkanlığı. (2012). *MEB ortaöğretim kurumlarına geçiş sistemi seviye belirleme sınavı e-başvuru kılavuzu.* Retrieved from http://oges.meb.gov.tr/sbs_tkvm.htm

Mullis, I., Gonzalez, E., Gregory, K., Garden, R., O'Connor, K., Chrostowski, S., & Smith, T. (2000). *TIMSS 1999 International mathematics report: Findings from IEA's repeat of the third international mathematics and science study at* *the eighth grade*. Boston, Mass: TIMSS and PIRLS International Study Center, Lynch School of Education, Boston College.

Mullis, I., Martin, M., Gonzalez, E., & Chrostowski, S. (2004). *TIMSS 2003 international mathematics report.* Boston, Mass: TIMSS and PIRLS International Study Center, Lynch School of Education, Boston College.

National Assessment of Educational Progress. (2007). *Mathematics framework for the 2007 national assessment of educational progress*. Retrieved from http://www.nagb.org/

National Center on Education and the Economy. (2008). *International benchmarking Indiana's academic standards in mathematics.* Washington, DC.

National Research Council. (1999). *How people learn: Brain, mind, experience and school.* Washington, DC: National Academy Press.

Nicol, C. C., & Crespo, S. M. (2006). Learning to teach with mathematics textbooks: How pre-service teachers interpret and use curriculum materials. *Educational Studies in Mathematics*, *62*(3), 331-335.

Ölçme, Seçme ve Yerleştirme Merkezi. (2012a). *Öğrenci seçme ve yerleştirme sistemi (ÖSYS) kılavuzu*. Ankara: ÖSYM.

Ölçme, Seçme ve Yerleştirme Merkezi. (2012b). *Sınav arşivi*. Retrieved from http://www.osym.gov.tr/belge/1-4127/sinav-arsivi.html

Paris, P. (2003). The International Baccalaureate: A case study on why students choose to do the IB. *International Education Journal, 4*(3), 232-243.

Patton, M. Q. (1990). *Qualitative evaluation and research methods* (2nd ed.). Inc. Newbury Park, CA: Sage Publication.

Peterson, A. (2003). *Schools across frontiers: The story of the International Baccalaureate and the united world colleges*. Open court, La Salle, Illinois.

Remillard, J. T. (2000). Can curriculum materials support teachers' learning? Two fourth-grade teachers' use of a new mathematics text. *The Elementary School Journal*, *100*(4), 331-350.

Reys, R., Reys, B., Lapan, R., Holliday, G., & Wasman, D. (2003). Assessing the impact of standards-based middle grades mathematics curriculum materials on student achievement. *Journal for Research in Mathematics Education, 34*(1), 74-95.

Rezat, S. (2006). A model of textbook use. In J. Novotná, H. Moraová, M. Krátká & N. Stehlíková (Eds.), *Proceedings 30th conference of the international group for the psychology of mathematics education* (Vol. 4, pp. 409–416). Prague: PME.

Rezat, S. (2008). Learning mathematics with textbooks. In O. Figueras, J. L. Cortina, S. Alatorre, T. Rojano & A. Sepúlveda (Eds.), *Proceedings of the Joint Meeting of PME 32 und PME-NA XXX* (Vol. 4, pp. 177-184). Morelia: Cinestav-UMSNH.

Rock, S. (1992). *An examination of selected features of six mathematics textbooks at the seventh-grade level* (Unpublished doctoral dissertation). The University of Wisconsin, Madison.

Russell, S. J., Schifter, D., Bastable, V., Yaffee, L., Lester, J. B., & Cohen, S. (1995). Learning mathematics while teaching. In B. S. Nelson (Ed.), *Inquiry and the development of teaching: Issues in the transformation of mathematics teaching* (pp. 9-16). Newton, MA: Center for the Development of Teaching.

Schmidt, W. H., McKnight, C. C., & Raizen, S. A. (1997). *A splintered vision:An investigation of U.S. science and mathematics education.* Dordrecht: Kluwer Academic Publishers.

Schmidt, W. H., McKnight, C. C., Valverde, G.A., Houang, R.T., & Wiley, D. E. (1997). *Many visions, many aims: A cross-national investigation of curricular intentions in school mathematics* (Vol. 1). Dordrecht: Kluwer Academic Publishers.

Semerci, Ç. (2004). İlköğretim Türkçe ve Matematik ders kitaplarını genel değerlendirme ölçeği. *Cumhuriyet Üniversitesi Sosyal Bilimler Dergisi*, *28*(1), 49-54.

Singapore Examinations and Assessment Board. (2011). *Singapore-Cambridge GCE O-level.* Retrieved from http://www.seab.gov.sg/oLevel/generalInfo.html

Singapore Examinations and Assessment Board. (2012). *Additional mathematics GCE ordinary level (syllabus 4038).* Retrieved from http://www.seab.gov.sg/oLevel/GCEOsyllabus/privateCandidates/2012_GCE_O.html

Singapore Ministry of Education. (2006). *Secondary mathematics syllabuses.* Ministry of Education, Curriculum Planning and Development Division. Retrieved from http://www.moe.gov.sg/education/syllabuses/

Singapore Ministry of Education. (2011). *Secondary school education booklet*. Retrieved from http://www.moe.gov.sg/education/secondary/

Sosniak, L. A., & Perlman, C. L. (1990). Secondary education by the book. *Journal of Curriculum Studies, 22*(5), 427-442.

St. George, E. S. (2001). *Textbooks as a vehicle for curriculum reform* (Unpublished doctoral dissertation). The Florida State University, Tallahassee.

Stake, R. E., & Easley, J. A. (1978). *Case studies in science education*. Urbana, IL: Center for Instructional Research and Curriculum Evaluation University of Illinois at Urbana-Champaign.

Stevenson, H. (1985). *An analysis of Japanese and American textbooks in mathematics.* Office of Educational Research and Improvement, Washington, DC.

Şahin, S. (2005). Liselerde okutulmakta olan lise I. sınıf matematik kitaplarının değerlendirilmesi. *Journal of Gazi Educational Faculty*, *25*(2), 327-341.

Tilke, A. (2011). International Baccalaureate diploma programme: What it means to a school library and librarian. *School Library Monthly, 17*(5), 8-10.

Törnroos, J. (2004). Mathematics textbooks, opportunity to learn and student achievement. *Studies in Educational Evaluation, 31*(4), 315–327.

Travers, K. J., & Westbury, I. . (1989). *The IEA study of mathematics I: Analysis of mathematics curricula*. Oxford: Pergamon Press.

Valverde, G. A., Bianchi, L. J., Wolfe, R. G., Schmidt, W. H., & Houang, R. T. (2002). *According to the book: Using TIMSS to investigate the translation of policy into practice through the world of textbook.* Dordrecht: Kluwer Academic Publishers.

Van Tassel-Baska, J. (2004). The acceleration of gifted students' programs and curricula. In F. A. Karnes & K. R. Stephens (Eds.), *Fastback series*. Waco, TX: Prufrock Press.

Vygotsky, L.S. (1978). *Mind in society*: *The development of higher psychological processes.* Cambridge: Harvard University Press.

Weber, R. P. (1990). *Basic content analysis* (2nd ed.). Newbury Park, CA: Sage Publications.

Weinberg, A., & Wiesner, E. (2011). Understanding mathematics textbooks through reader-oriented theory. *Educational Studies in Mathematics, 76*(1), 49–63.

Weinberg, A., Wiesner E., Benesh, B., & Boester, T. (2012). Undergraduate students' self- reported use of mathematics textbooks. *Problems, Resources, and Issues in Mathematics Undergraduate Studies, 22*(2), 152-175.

Zhu, Y., & Fan, L. (2004). *An analysis of the representation of problem types in Chinese and US mathematics textbooks* (Paper accepted for ICME-10 Discussion Group): Copenhagen, Denmark.