EZGİ ŞENGÜL

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# COMPARISON OF INTERNATIONAL BACCALAUREATE PRIMARY YEARS PROGRAM AND NATIONAL CURRICULUM PROGRAM 4$^{TH}$ GRADE STUDENT'S MISCONCEPTIONS ON THE TOPIC OF FRACTIONS

A MASTER'S THESIS

BY

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THE PROGRAM OF CURRICULUM AND INSTRUCTION
İHSAN DOĞRAMACI BILKENT UNIVERSITY
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# COMPARISON OF INTERNATIONAL BACCALAUREATE PRIMARY YEARS PROGRAM AND NATIONAL CURRICULUM PROGRAM 4TH GRADE STUDENT'S MISCONCEPTIONS ON THE TOPIC OF FRACTIONS

The Graduate School of Education

of

İhsan Doğramacı Bilkent University

by

Ezgi Şengül

In Partial Fulfillment of the Requirements for the Degree of

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İHSAN DOĞRAMACI BILKENT UNIVERSITY
GRADUATE SCHOOL OF EDUCATION

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Ezgi Şengül

May 2015

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Curriculum and Instruction.

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# ABSTRACT

COMPARISON OF INTERNATIONAL BACCALAUREATE PRIMARY YEARS PROGRAM AND NATIONAL CURRICULUM PROGRAM 4$^{TH}$ GRADE STUDENS' MISCONCEPTIONS ON THE TOPIC OF FRACTIONS

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May 2015

The purpose of this study was to compare the misconceptions of fractions in IB Primary Years Program (IBPYP) to the misconceptions of fractions of Ministry of National Education (MoNE) 4$^{th}$ grade students. To measure this, the three most popular subtopics of fractions covered in 4$^{th}$ grade mathematics curriculum were selected. These subtopics were (1) partitioning, (2) ordering and (3) addition. Then, nine questions for each subtopics were developed. Accordingly, a fractions test that included 27 items total was developed and used in this research. Analyses were conducted to determine if different curricula cause any response patterns. Analysis showed that only 7 out of 27 items were answered statistically differently by the IBPYP and MoNE students. PYP students had higher correct answer and lower misconception rates in 6 out of these 7 items. However, in general, the correct answer and wrong answer patterns seemed to have no substantial difference across the two curricula. Also, the results proved that some fractions subtopics were more challenging for students than others. Some suggestions about how to address misconceptions were made in the present study.

Key words: Mathematics education, misconceptions, fractions, IBPYP, MoNE primary mathematics education.

# ÖZET

ULUSLARARASI BAKALORYA PROGRAMI VE MİLLİ EĞİTİM BAKANLIĞI İLKÖĞRETİM PROGRAMLARININ İLKÖĞRETİM 4.SINIF ÖĞRENCİLERİNİN KESİRLER KONUSUNDAKİ KAVRAM YANILGILARINA DAYANARAK KARŞILAŞTIRILMASI

Ezgi Şengül

Yüksek Lisans, Eğitim Programları ve Öğretim
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Mayıs 2015

Bu çalışmanın amacı Uluslararası Bakalorya İlk Yıllar Programı (UBİYP) ve Milli Eğitim Bakanlığı (MEB) 4. sınıf öğrencilerinin kesirler konusunda sahip oldukları kavram yanılgılarını karşılaştırmaktır. Bu amaçla, 4. sınıfta kesiler konusunda işlenen 3 alt başlık (1) kesirlerin bölümlere ayrılması, (2) kesirlerin sıralanması ve (3) kesirlerin toplanması olarak belirlenmiş ve her bir alt başlık için 9 soru geliştirilmiştir. Buna bağlı olarak, toplamda 27 sorudan oluşan bir Kesirler Testi ortaya çıkmıştır. İki farklı müfredatın öğrencilerinin kavram yanılgıları arasında anlamlı bir fark olup olmadığını anlamak için analizler yapılmıştır. Fakat araştırmanın sonunda 27 sorudan yalnız 7 tanesi istatistiksel olarak farklı cevap oranlarına sahip olduğu belirlenmiştir. Bu 7 sorunun 6'sında UBİYP öğrencileri MEB öğrencilerinden daha yüksek doğru cevap ve daha düşük kavram yanılgısı oranları göstermiştir. Yine de genel olarak doğru ve yanlış cevaplar arasında ciddi bir fark gözlenmemekle beraber, bazı alt başlıkların diğerlerine oranla daha az doğru cevap oranlarına sahip olduğu gözlemlenmiştir. Araştırmada ayrıca kavram yanılgılarının tespit ve önlenmesi konusunda bazı öneriler sunulmuştur.
Anahtar Kelimeler: Matematik eğitimi, kavram yanılgıları, kesirler, IB İlk Yıllar Programı ve Milli Eğitim Bakanlığı İlköğretim matematik eğitimi

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# CHAPTER 1: INTRODUCTION

## Introduction

Students' misconceptions can be simply defined as the partly incorrect or incomplete ideas that contradict with the scientific facts and are resistant to change (Steinle & Stacey, 2003; Leonard et al., 2014). Students' misconceptions have been one of the intensively studied research areas in mathematics education. They are mostly considered as one of the severe obstacles to students' complete learning. Research studies show that late correction of misconceptions of fundamental mathematics or science concepts could inhibit learning. Also, not correcting a misconception can make it more persistent in time (Strike, 1983; Micheal, 2002). Due to this, the diagnoses and the prevention of students' misconceptions are crucial in order to reach accurate and complete teaching and learning.

Fractions is one of the most important mathematics topics as it has wide real life reflections and connections with other mathematical and scientific concepts (Keijzer & Terwel, 2001; McLeod & Newmarch, 2006). In order for students to be able to apply fractions to real life and to other more advanced mathematical concepts, they should first be able to grasp fractions. Since the topic fractions is the first attempt of primary school students to work beyond whole numbers, students tend to apply their whole numbers knowledge to fractions (Hasemann, 1981; Baroody & Hume, 1991). For example, students might think that bigger denominator means bigger value. Such overgeneralization can cause misconceptions.

In Turkish schools, fractions teaching starts in second grade. Primary school students are taught according to the Ministry of National Education (MoNE) curriculum in Turkish schools today. On the other hand, International Baccalaureate Primary Years Program (IBPYP), which is an internationally recognized program, introduces fractions in the first year of education. The approach of the two curricula to fractions learning also differs in other ways. So, the frequencies and types of misconceptions can be observed differently for two different curricula.

## Background

Students' misconceptions have been one of the most intensively studied research areas in mathematics education because of their roles in interference with the meaningful and permanent learning of students (Köse, 2008). According to Çardak (2009) the source of misconceptions is generally the students' own interpretations or bias, and misconceptions often contradict with the reality. Before entering the formal education children already have their own perception of scientific ideas, which are based on their earlier experiences in life. These pre-existing experiences might lead them to develop partially formed and incorrect ideas about concepts and hence pre-existing knowledge becomes one of the most common reasons why students develop misconceptions (Johnston & Gray, 1999; Henriques, 2002).

Various researchers agreed on the severe function of misconceptions as obstacles to learning (Keijzer & Terwel, 2001; Yoshida & Sawano, 2002). For example, Çardak (2009) claimed that if misconceptions are not identified or not prevented, they can inhibit students' learning about related concepts. In a similar vein, Michael (2002) argued that one of the most important problems with misconceptions is that they are often persistent and severely prevent students' ability to learn the concept.

Misconceptions should be detected and corrected to supply better learning. Due to this, teachers should be aware of possible misconceptions students tend to exhibit. Knowing which stages of development or which part of curriculum are more likely to bring out misconceptions will give the opportunity to plan lessons accordingly and correct misconceptions if they still arise (Chick & Baker, 2005).

Fractions are often considered as one of the least popular mathematics topics by students at primary level. A high number of students find the concept of fractions challenging since the notation is quite different, and the operations in fractions require particular procedures that they often carry without enough reasoning (Lee, 2008).

After students complete their learning with whole numbers, they next move on to the number set that encompasses the whole numbers, which is the rational numbers (Hasemann, 1981; Baroody & Hume, 1991). Rational numbers are introduced with fractions and decimals, which have quite different notations and logic than whole numbers (Brown, 1993, Moss & Case, 1999). As students build their fractions learning on their prior knowledge of whole numbers, misconceptions could arise. Some of the commonly seen misconceptions are:

- Failing to understand the value of fractions as a part of a whole so, believing the denominators and nominators of fractions are separate whole numbers,
- Thinking that the shapes that are not equally-partitioned can define fractions,
- Failing to determine a common denominator in addition, subtraction or ordering hence adding, subtracting or ordering the denominators and nominators separately (Schifter, Bastable, & Russell, 1999; McNamara & Shaughnessy, 2010; Van de Walle et al., 2010, p. 287).

Some of these misconceptions could be predicted by teachers and they might help teachers to develop better lesson plans. In order to do this, teachers should be aware of the most common misconceptions of students, why and how these misconceptions occur and how they can be reduced or prevented.

Fractions teaching differs for MoNE and PYP curricula. The first confrontation with fractions and the way fractions are taught can alter from one curricula to another. So, this difference can affect the learning as well. This study attempted to figure if different curricula have an effect on the misconception rates.

## Problem

In today's world, the increasing role of globalization requires countries and educational organizations to revise their systems and make improvements to educate more people who are culturally and internationally aware (International Baccalaureate Organization, 2007). Hence more schools around the world have started to implement international education programs such as International General Certificate of Secondary Education (IGCSE) or IB to be recognized globally (Dağlı, 2007; Ateş 2011).

In Turkey there are 20 schools as of April 2015 that implement IBPYP and they are all private schools (www.ibo.org). Here, it should be also noted that students who attend private schools tend to have higher socio-economic background than those who attend public schools (OECD, 2012). Besides the existing difference in the philosophy and the educational approach of IBPYP to MoNE program, the quality of education in IB schools is also the result of being privately managed. From this point of view, literature needs more research about the effects of different educational approaches and different curricula on the quality of teaching and learning.

Even though students' misconceptions have been addressed by several researchers, most of them preferred to work on the diagnoses and the prevention of misconceptions. However, very few of these studies attempted to focus on misconceptions in a comparative manner between different curricula. For this reason, this study attempted to use misconceptions as a tool to compare MoNE and PYP curricula's education qualities. To this end, this study focuses on the comparison of the types and the frequencies of misconceptions that students who are taught with two different curricula have in the topic fractions.

## Purpose

The main purpose of this study was to compare the fraction misconceptions of MoNE and PYP 4$^{th}$ grade students and figure out if students from the two curricula showed different misconceptions patters. Also by comparison, research attempted to observe how frequent the misconceptions. The reason behind choosing 4$^{th}$ grade in particular was because the primary school is an important period in students' mathematical development in which students decide if they like mathematics or not. This grade is also significant since the first misconceptions are formed and they start to influence the following years such as middle school and high school years (Keazer, 2004). This study aimed to identify the first forms of misconception types before students build upon their primary fractions knowledge.

For this purpose, the most frequent misconceptions that students might have on topic of fractions were identified from the related literature and they were categorized under three sub-categories; misconceptions on partitioning, misconceptions on ordering and misconceptions on add tops-add bottoms. These misconceptions were also adapted to 4$^{th}$ grade students by considering the outcomes of the topic of fractions in both curricula. With regards to all these, a fractions test was developed by the researcher. With the aid of the results, the effects of the applied curricula on students' misconceptions are expected to be revealed. Yet, the present study only focused on the differences in misconceptions regarding fractions. Results should not be generalized to compare the two curricula in general.

This study also aimed to compare the frequencies of selected types of misconceptions without necessarily comparing the curricula. By doing that, research attempted to find out what particular sub-headings of fractions students most struggle with.

## Research questions

This study will address the following questions:

- Do $4^{th}$ grade students' misconceptions on the topic of fractions vary across MoNE and PYP curricula?
- Among some specific misconceptions on the topic of fractions, what are the most common ones that $4^{th}$ grade students struggle with regardless of their curriculum?

## Significance

Examining students' misconceptions provides chance to demonstrate students' understanding of a concept. On one hand, students' correct answers may not necessarily indicate their perceptions on a target topic completely because students can show a correct understanding by simple memorization of procedures or definitions. On the other hand, misconceptions point out the lack of knowledge or inappropriate connections (Li, 2006). If these misconceptions are identified and corrected, then the teaching and learning become more meaningful. The resolution of students' misconceptions leads to a more effective learning (Keazer, 2004). So, being aware of the fractions misconceptions enable teachers to be more careful.

Even though misconceptions are one of the major fields in mathematics education, there are a few studies that used misconceptions as a comparison tool. Due to this reason, this study aimed to fill this gap to some degree.

Moreover, in mathematics education, most students encounter challenges in grasping the concept of fractions (Lee, 2008). Since fractions are connected with many other algebraic topics such as number theory, greatest common divisor, least common denominator, and prime factorization, the misconceptions on fractions can function as an obstacle to learn all these related topics as well (Van de Walle et al., 2007, p. 319). Therefore, it is important to address students' misconceptions in the topic of fractions before they move on to other related topics. After the determination of problems and gaps in students' thinking, the suggestions can be given retrospectively.

Predicting the misconceptions of students on fractions will allow teachers to develop better lesson plans and hence provide a better learning and teaching even before any misconception occurs. For this reason, the study aims to contribute to literacy by addressing this critical point.

Furthermore, comparison of students taught with two different curricula are expected to give significant information for stakeholders such as policy makers, administrations of schools and teachers etc.

# CHAPTER 2: REVIEW OF RELATED LITERATURE

## Introduction

Students' misconceptions have been one of the most intensively studied research areas in mathematics education. Mathematics educators have defined misconceptions at the K-12 levels as the obstacles that prevent meaningful and permanent learning of a concept (Keijzer & Terwel, 2001; Yoshida & Sawano, 2002).

It is not a matter of debate that children already have developed their own perception about the world. Hence they already have some scientific knowledge before they actually start to receive formal education in classrooms (Henriques, 2002). This knowledge can and does affect their learning process in schools. In particular, it affects negatively if the knowledge is incorrect and resistant to change (Black & Lucas, 1993).

Misconceptions might guide researchers and teachers to understand the perceptions of students, how their minds work and what kind of connections they make while learning (Steinle & Stacey, 2003). Knowing how a student's mind works will eventually make the teacher's work easier. Due to this, teachers should pay special attention to find out students' possible misconceptions. In order to help teachers face with misconceptions sooner and more effective, the research studies that focus on misconceptions are of great importance (Wallace, 2007). Therefore, the aim of this literature review is pointing out the role of misconceptions in education, how misconceptions might be observed particularly in mathematics education and the most common misconceptions on the topic of fractions.

## Misconceptions

Misconceptions could be described as one of the leading factors that prevent students' meaningful and permanent learning. They do not match the scientific facts but instead contradict. Most of the time misconceptions are developed by individuals themselves often based on their own interpretations or bias (Johnston & Gray, 1999; Henriques, 2002; Çardak, 2009). Since they are substantial barriers against learning, the majority of studies carried out in the field of mathematics education now focus on students' misconceptions.

Even though misconceptions seem naive, they are actually extremely complex and have deeper effects on students' learning than expected (Wescott & Cunningham, 2005). They are widespread in formal education and considerably resistant to change. If they are not identified or if they continue for long term, misconceptions may prevent students' learning about related concepts (Çardak, 2009). Moreover, some students' misconceptions can spread to others while working in groups.

However, some researchers believe misconceptions are not always so severe and might be a natural step in learning. For example, Swan (2001) pointed out, "Frequently, a 'misconception' is not wrong thinking but is a concept in embryo or a local generalization that the pupil has made. It may in fact be a natural stage of development" (p. 154). From this point of view, misconceptions could also be considered a chance to elicit students' progress in learning and the way they perceive new information. This might lead us to think that misconceptions are not always critical obstacles to learning but also could be considered as a tool to elicit students' ways of perceiving new information and connect new knowledge with the old one.

Research evidence also indicates that the resolution of students' misconceptions leads to effective learning (Swan, 2001).

Whether misconceptions prevent meaningful learning or it is a natural step in improvement, it still needs to be understood to use them in the students' favor while teaching and planning.

**The significance of misconceptions**

If misconceptions go unnoticed, the new concepts that are built upon the previous ones will be incomplete or inaccurate. Even the increase of misconceptions on connected concepts might cause the sense of inadequacy and hence mathematics anxiety (Keazer, 2004).As a result, teachers need to know how a new learner's mind might work in order to promote deep and long-lasting learning. Being aware of what kind of misinterpretations might occur, gives teachers the opportunity to treat misconceptions and hence rebuild the mathematical understanding of students (Chick & Baker, 2005).

Moreover, Chen, Kirkby and Morin (2006) argued that teachers do not often spare time to identify students' misconceptions and since more often they focus on what kind of questions they may encounter while teaching, they do not pay attention to the ones they do not confront. A study that was conducted by Sadler, Sonnert, Coyle, Cook-Smith and Miller (2013) showed a surprising result. In a test that teachers took, they were asked to give both correct answers for questions and the most possible incorrect answers that students might give. Most of the teachers gave correct answer to questions while most of them failed to identify students' possible incorrect answers.

Yet, it is hard for teachers to diagnose misconceptions. How do teachers know that students have misconceptions or students are simply wrong? When teachers ask a question and encounter an odd or unexpectedly wrong answer they cannot conclude that students have misconceptions. However, if the same odd and unexpected, wrong answers follow the questions within a similar context, then teachers could suspect that there might be a possible misconception on this topic (Michael, 2002; Ball, Hill & Bass, 2005).

**Sources of misconceptions**

Misconceptions may occur for a variety of reasons. Some researchers agree that students' misconceptions are originated from their prior learning they informally developed before entering formal education. These early experiences, which can be considered as a natural development phase, lead children to have their own ideas about the outcomes of scientific facts (Johnston & Gray, 1999; Henriques, 2002; Çardak, 2009). Hence the observations and experiences that they bring into classrooms eventually can interfere with the formal education in schools.

Furthermore, Hanuscin (2007) claimed that misconceptions can occur when learner mixes more than one concept. As relations between the concepts in science and mathematics are inevitable, learners can develop their own links that might be incomplete or inaccurate and these links can eventually cause misconception.

Another possible scenario that has been suggested is that the common words that are used both in everyday life and in scientific concepts can cause misinterpretation and hence misconception (Hanuscin, 2007). So, misconceptions can arise from verbal confusion too.

Furthermore, Barrass (1984) and Kajander & Lovric (2009) claimed that textbooks might also be responsible to compound students' misconceptions about concepts. Especially when considering their major roles in education, as a significant tool for students to study and do homework and for teachers to see what to cover and how, the misconceptions they possibly raise become significant. The researchers claimed that textbooks have great potential to help students learn while they also have serious weaknesses and obvious mistakes.

Misconceptions also might oocur due to the pace of work, the slip of a pen, the lack of attention or knowledge or a misunderstanding. Apart from that, students' misconceptions may be reinforced by the lack of prior knowledge. Skelly and Hall (1993) stated that

> If the learner's prior knowledge needed to process new information is incomplete, the knowledge gaps will result in confusion, inaccurate reasoning, and eventually in the formation of misconceptions. If the learner's prior knowledge structure contains misconceptions, these can cause further faulty reasoning and incorrect concept formation (p.1504).

## The significance of fractions

Many students may wonder why learning fraction is essential in particular when they are first introduced. Fractions are considered important also because it is the first experience of a mathematical concept after learning the simple algebraic rules such as addition, subtraction, multiplication and division (Hasemann, 1981; Baroody & Hume, 1991; Mack, 1995, Lappan et al., 1998). If possible misconceptions about fractions are considered and the lessons are planned accordingly, students feel confident and comfortable with their learning of fractions. Hence, this successful experience of gaining a new concept in mathematics with comfort helps positively to their confidence and approach to mathematics. Even though the significance of misconceptions for teaching and learning are made explicit, the necessary attention is still not given.

The introduction of fractions could be considered as the first experience of students with a new mathematics concept beyond simple arithmetic operations (Mack, 1995). The topic of fractions is first introduced by the Ministry of National Education (MoNE) curriculum as early as second grade and it is taught through all grades up to grade 7. Because of its connection with other algebra topics, students should feel comfortable with their understanding of fractions in order to become capable of learning other related topics (D'Ambrosio & Mewborn, 1994, Chick, Tiemey & Storeygard, 2007). For example, understanding the concept of fractions would enable students to comprehend some of the essentials of number theory, such as greatest common divisor, least common denominator, and prime factorization (Bauman & Sauer, 1995; Burns, 2000). Predicting the misconceptions of students on fractions will allow teachers to develop better lesson plans and hence provide a better learning and teaching even before any misconception occurs (Stigler & Hiebert, 1999).

In addition to its connection with other topics, there are also several real life situations that people need to use their fraction knowledge. Fractions are used in a variety of examples from real life such as recipes, splitting costs, balancing budgets, and even in the world of sport. Due to this, students should be able to gain the ability of reasoning on fractions (Keijzer & Terwel, 2001; Parker, 2004).

### Challenges in learning fractions

Most students have difficulty to grasp the abstract symbols, terminology and visual representations of fractions (Saxe et al., 2005; Lee, 2008). The lack of correct and complete understanding of fractions might cause the difficulties with fractional computation, decimal and percentage learning and other algebraic concepts that use fractions as a tool (Tatsouka, 1984). Hanson (1995) claims that one of the main reasons students have difficulty to understand fractions is because they tend to memorize formulas and algorithms instead of understanding the logic behind them.

Another significant reason why fractions are considered confusing is that they break the rules students learned about whole numbers up to that point. Whole numbers are increased as they multiplied but for simple fractions the situation is quite the reverse. Other than that, students also have difficulty to understand the notation of fractions. This notation, one number over another, is quite different than whole numbers. So, this can be another reason of whole numbers' influence on fractions. Students naturally think the nominator and denominator of fractions are separate whole numbers (Small, 2008). So, they often carry out operations separately for nominators and denominators. This problem takes its source from not recognizing that denominators define the size of shares and nominators represent how many of these shares are considered. To avoid this problem, the values like $\frac{3}{4}$ should not be taught as "three over four" but instead "three fourths" should be used (Siebert & Gaskin, 2006).

Students also find it challenging to learn basic characteristics of fractions such as order or equivalence (Lamon, 1999; Yoshida & Sawano 2002). Both concepts are basic concepts of fractions curriculum. Even though most students do not have any difficulties in dealing with real numbers they can feel confused when fractions are involved. For example, when students are asked to order the fractions $\frac{1}{3}$ and $\frac{1}{4}$ they can say $\frac{1}{4}$ is greater than $\frac{1}{3}$ because 4 is greater than 3 (Nunes et al., 2006). In addition to ordering, the addition of fractions might seem challenging to some students. When they are asked to add two fractions they may add the denominators without making denominators equal.

Teachers need to make students realize that fractions are different from real numbers or natural numbers. Emphasizing that the denominators and nominators are not separate values instead they are used to represented a part of a whole is crucial for working with fractions (Steinle & Stacey, 2004). Constructing meaningful problem stories can be useful to overcome this problem. Visual representations that show how a whole is divided into pieces and how they are named, added or multiplied might also work with fractions (Ball, 1993; Streefland, 1993).

## Examining some specific misconceptions on fractions

Fractions is one of the leading topics in both MoNE and PYP curricula. Since fractions learning is core for many other topics in algebra and in other areas of subjects, fractions teaching starts with grade two and continues through almost all grades until high school (IBO, 2009; MEB, 2009). Among many subtopics of fractions the specifically partitioning, ordering and addition were examined for this research as these topics are both core for fractions teaching and are common for PYP and MoNE $4^{th}$ grade fractions curricula.

### Misconceptions on partitioning

Dividing a shape into equal-sized parts is called partitioning. Since the part-whole relationship is the core of the fractions teaching, fractions are generally introduced first with examples in which a part of a whole is shaded.

Siebert and Gaskin (2006) suggested that students' fractions misconceptions often arise from not being able to understand the relationship between nominator and denominator but instead believing they are separate two real numbers. In order to correct this thought, partitioning should be taught as "creating smaller, equal-sized amounts from a larger amount" or "making copies of smaller amount and combining them to create a larger amount" (p.395).Students tend to skip the importance of equal partitioning and think that unequally partitioned shapes or areas can also describe fraction (Empson, 2001; Cramer & Whitney, 2010). For example, for the below shape students may think the shaded region describes $\frac{3}{4}$ rather than $\frac{1}{2}$ of the whole (Van de Walle et al., 2012, p. 292).

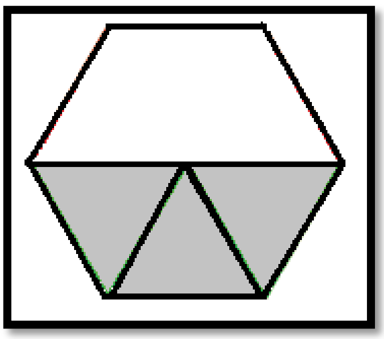


Figure 1. Example of misconception on partitioning

Due to this, students should learn the part-whole relationship and so, the focus should be on *equal parts*. These parts can have same shape or a different shape that has the same size, because too often students conclude that equal shares might not be the same shape, which is not correct (Van de Walle et al., 2012, p. 296).

An activity that explains this situation clearly have examples that are (1) same shape, same size; (2) different shape, same size; (3) different shape, different size; and (4) same shape, different size. Examples in number (1) and (2) are for the equally partitioned fractions while the examples in number (3) and (4) are for the parts that are not equivalent. A student whose partitioning knowledge is proper and complete should distinguish the figures that are correctly partitioned into four from the ones that are not partitioned equally (Van de Walle et al, 2012, p. 296).

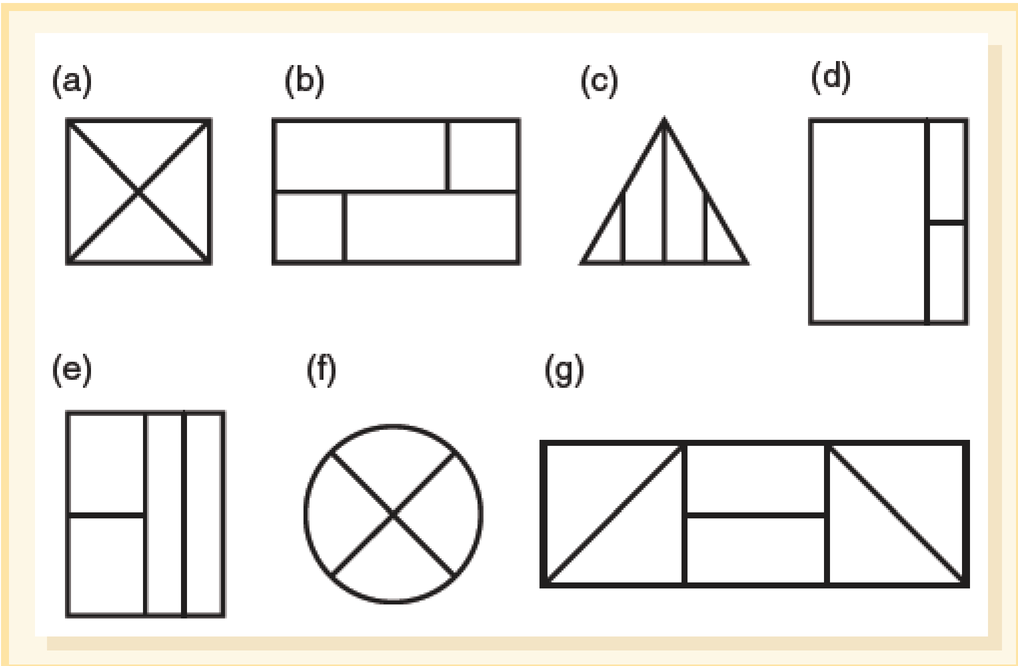



Figure 2. Correctly and incorrectly partitioned shapes (Van de Walle et al., 2012, p. 297)

The figures for the category (1) same shape, same size are figures (a) and (f) while the ones for category (2) different shape, same size are figures (e) and (g). These four figures should be selected as correct shares by students who learn partitioning well. On the other hand, the figures for the category (3) different shape, different size, were figures (b) and (c) and the figures for the category (4) same shape, different size, were the figure (d) were the ones that were not accurately partitioned. The students, who think one of these three figures illustrates the correct share, apparently have misconceptions on partitioning.

Another part-whole problem that leads misconception on partitioning is that students seeing three green and four blue counters think $\frac{3}{4}$ of counters are green (Bamberger, Oberdorf, &Schultz-Ferrell, 2010). This problem again takes its roots from not understanding completely what whole means and how a fraction describes a part of the whole with the aid of numerator and denominator.

Students should be told that partitioning fractions means dividing the whole into equal parts. Clearly explaining that the operations such as ordering, adding, or subtracting can be only carried out when two wholes are divided into same sized parts are crucial. To be able to comprehend what partitioning really means, the practices should be done on all kind of possible examples, not only on a pizza (McNamara & Shaughnessy, 2010). Area, length and sets should be used to diversify the examples. For partitioning a set, a class, counters, playing cards, marbles can be used while with length model partitioning a rope, a rode, or a ruler might work. For the area model, which is mostly the case, partitioning a pizza, a rectangular garden, etc. can be used.

**Misconceptions on ordering**

Being able to tell which fraction is greater is another aspect of number sense with fractions. Students have strong mind set about numbers such as thinking larger numbers mean more. This is valid for positive whole numbers such as $5 > 4$. Since students overgeneralize the whole numbers rules, they fail to understand the relative size of fractions and may think $\frac{1}{5} > \frac{1}{4}$ (Mark, 1995).

While ordering fractions students tend to think the bigger the number on the bottom, the bigger the fraction gets. As a result of this, students order unit fractions wrongly. For instance, they conclude that $\frac{1}{6}$ is bigger than $\frac{1}{2}$ (Nunes et al., 2006).

To prevent this, students should be told that the more parts there are in the denominator the smaller each portion will be. However, this logic should be given with plenty of visual representations and examples without having students to memorize the procedure that the bigger the denominator the smaller the fraction (Ball, 1993; Martinie & Bay-Williams, 2003). Teaching ordering with such rules could make students overgeneralize and conclude that $\frac{1}{6}$ is bigger than $\frac{5}{10}$ because 6 is smaller than 10 (Cramer, Wyberg, & Leavitt, 2008).

Also, not limiting the problems only with circle pieces but also using other context, models and mental imaginary may help students to enrich their understanding and they could be away from the risk of being too reliant on model (Bray & Abreu-Sanchez, 2010). Instead, deepen the problems with real world contexts that are meaningful to them is more useful. For example, asking students if they would rather have $\frac{1}{2}$ of marbles, $\frac{1}{4}$ of marbles, or $\frac{1}{10}$ of them. Letting them partition the marbles and then answer would make them realize the relationship between the denominator size and function size (Siegler et al., 2010).

**Misconceptions on add tops-add bottoms**

Another misconception that leads students to think that fractions are added together by adding the top numbers together and then adding the bottom numbers together. This misconception again takes it sources from the whole numbers knowledge influencing fractions (Lappan & Mouck, 1998; Cramer & Whitney, 2010). Students who have strong conceptual understanding of equivalence can easily move between fractions such as $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}$ etc. and adjust the fractions in order to make addition (Taber, 2009).

Teachers should focus more on part-whole concept instead of giving the rule of it right away. Students should be told that only the same sizes can be added or subtracted which implies that the denominators of the fractions should be equal first Using manipulative, modeling can help students to see which parts are equal and which parts are not (Mack, 2004; Cramer & Henry, 2002; Bamberger et al., 2010).

Comparing the addition with multiplication may be one of the reasons students get confused. Some students compare adding with multiplication and think why does the denominator stay same while adding and why does it multiply while multiplying (Huinker & DeAnn, 2002).

To prevent this misconception from occurring, students should be told that different denominators represent different sized shares and when we want to add or subtract different shares there won't be any equality. Students could be encouraged with questions like "Two fifths plus one fifths is how many fifths?" to think about the meaning of the denominator. Especially, doing that exercise before moving on unlike denominator would be helpful (Mack, 2004).

**Differences between PYP and MoNE schools**

In an increasingly globalized and rapidly changing world, the need for educated people who can think universally, culturally aware and competent to engage with other people increases as well (www.ibo.org). This leads schools all around the world to start to implement international education programs besides their national education programs in order to be recognized at international level (Dağlı, 2007; Ateş 2011). Due to this, starting from 1997 some Turkish schools started to implement PYP (Primary Years Program) which is one of the three programs that IBO (The International Baccalaureate Organization) offers as if January 2014. There are 19 schools in Turkey that offer PYP education and they are all private schools (www.ibo.org). PYP is designed for students aged 3 to 12. PYP is a program that creates intellectual challenges for students and aims to develop the whole child as an inquirer both inside and outside the school, to prepare them in their future career (International Baccalaureate Organization, 2007; www.ibo.org).

A study that was conducted in 2014 in Australia concluded that IB PYP students exhibit higher performance when comparing the national average in nationwide science tests (Campbell et al., 2014).

The distinction between national schools and private schools should also be investigated in terms of their education approaches, socio economic states of students and family backgrounds of their students.

Why do families in Turkey pay fees for private school, instead of sending their children to public schools? Dinler & Subası (2003) and Cinoglu (2006) stated that an increasing number of people prefer private schools since the education quality is higher due to the relationship between the market economy and education. They also pointed out that public schools are run by government bureaucracies so they cannot choose their curriculum or their teachers. Teachers have permanent status on public schools. On the other hand private schools feel obligated to monitor their own quality since parents as customers always monitor and judge the process. So private schools give great importance choosing the best for their schools in terms of teachers' quality, educational materials etc. Furthermore, researchers claim that since teachers in private schools do not have permanent status in schools, they are more concerned with their high performance when comparing with public school teachers.

Furthermore, for most of the countries, PISA results generally show that private schools are more advantageous than public schools in terms of student success (OECD, 2012). So as identified, the result of this study may eventually be affected by the quality distinctions of public (MoNE) and private (PYP) schools.

**How do MoNE and PYP curricula handle fractions?**

To be able to compare the results of the two curricula and hence make an interpretation about them, we need to know how the two curricula that we worked on handle the topic of fractions in their own frames. It is significant to learn how 4$^{th}$ graders' fraction teaching has developed throughout the grades 1 to 4.

*Fractions in MoNE curriculum*

The below information aimed to show how many teaching objectives and lesson hours were spared for fractions and how much percentage of whole curriculum was occupied by fractions (MEB, 2009). The below information was gathered from the curriculum framework of MoNE that was published in 2009. There have been some changes in the curriculum in 2013. However, the changes were applied to first graders and have followed them through consecutive years. So, in the time the data were collected, fourth graders were not affected by the changes. Due to this reason, new changes in the curriculum were not considered in the research.

2$^{nd}$ grade: 1 teaching objective, 3 lesson hours, percentage in whole curriculum: 2%

Teaching Objectives:

1. Students can explain the whole, half and the quarter.

3rd grade: 4 teaching objective, 10 lesson hours, percentage in whole curriculum: 7%

Teaching Objectives:

1. Students can partition a whole into equal parts and can express parts as the fractions.
2. Students can obtain fractions whose nominator is less than the denominator (proper fractions) by using natural numbers up to 2 digits.
3. Students can compare at most 3 fractions whose denominators are natural numbers up to 2 digits.
4. Students can identify any part of the whole that is expressed by a proper fraction.

4th grade: 13 teaching objective, 27 lesson hours, percentage in whole curriculum: 19%

Teaching Objectives:

1. Students can name the fractions as proper, improper, or mixed fractions whose nominators and denominators are natural numbers with up to 2 digits.
2. Students can place the fractions whose nominators and denominators are natural numbers with up to 2 digits on the number line.
3. Students can compare fractions.

4. Students can compare at most 4 fractions whose denominators are the same and the nominators are different from smallest to largest or from largest to smallest.
5. Students can compare at most 4 fractions whose nominators are the same and the denominators are different from smallest to largest or from largest to smallest.
6. Students can identify any part of the whole that is expressed by a proper fraction.
7. Students can add up two fractions whose denominators are same.
8. Students can subtract a fraction from another whose denominators are same.
9. Students can work out real world problems that include addition and subtraction on fractions.
10. Students can express a decimal number when a whole is divided into 10 or 100 equal parts.
11. Students can express decimal numbers by using decimal point.
12. Students can name the whole part and the decimal part of decimal numbers
13. Students can compare up to 2 decimal numbers by using <, > or = signs.

*Fractions in PYP curriculum*

Even though in MoNE curriculum the scope and sequence is clear in terms of grade levels and teaching objectives for each grade, the PYP sequence does not offer such information. According to International Baccalaureate Primary Years Program mathematics program (IB, 2009), the mathematics skills that students are expected to gain are split into different developmental processes that are called phases. Those phases that learners go through are not directly related with age and grade levels, so they are not linear. Also the way that PYP curriculum handles mathematics topics is different than MoNE's too. Primary school mathematics content is also split into five strands: numbers, measurement, data handling, shape and space and pattern and function. Since the topic fractions are dealt in chapter numbers, we are going to examine this chapter with its phases and learning outcomes.

Fraction teaching starts from phase 2. In this phase, students are expected to have an understanding of fractions as a part of a whole, to model fractions with part-whole relationship, and to use fraction names on a daily life base. In the following phase, in phase 3, students are able to understand the relation between fractions and decimals, model equivalent fractions and decimal fractions to hundredths and beyond. Also they are expected to model, read, write, compare and order fractions, and use them in real life situations. Also they learn to carry out basic operations, addition, subtraction, multiplication and division with fractions and solve problems involving fractions operations. Finally for the phase 4, students learn the relationship of fractions with decimals and percentages, they model, compare, read, write, order and convert fractions into decimals and percentage. They use mental and written strategies to solve problems that include fractions, decimals and percentages. The detailed explanations, the conceptual understanding and the learning outcomes of each phase were also given below.

An earlier mathematics programme which was published by International Baccalaureate in 2003 gives more detailed teaching objectives with their targeting age groups. Unlike the MoNE curriculum report, the total lesson hours and number of teaching objectives were not specified for PYP framework. Also instead of grade levels, the objectives are given according to the age groups. The details are as following:

Age group: 3-5 years

There is no fraction teaching between these ages.

Age group: 5-7 years

1. Read and write the time to the hour, half hour and quarter hour:

   - How can knowing about fractions help us to tell the time.

Age group: 7-9 years

1. Compare fractions using manipulative and using fractional notation:

   - Can different fractions be equal?

   - How can we know when one fraction is greater than, smaller than or equal to another?

2. Model addition and subtraction of fractions with the same denominator:

   - How can we add and subtract fractions?

3. Use mathematical vocabulary and symbols of fractions: numerator, denominator, equivalence:

   - How do mathematicians write fractions?

   - What is a numerator?

   - What is a denominator?

4. Understand and model the concept of equivalence to 1: two halves = 1, three thirds = 1:

- What is equivalence?
- Can you show fractions equivalent to 1?
- What patterns do you see in equivalence to 1?

Age group: 9-12 years

1. Read, write and model addition and subtraction of fractions with related denominators:
    - What is a fraction?
    - How does a fraction relate to a whole number?
    - How is a fraction represented?
    - How can we add and subtract fractions of different sizes?
2. Read, write and model improper fractions and mixed numbers:
    - What is an improper fraction?
    - What is a mixed number?
    - How are improper fractions and mixed number connected?
3. Compare and order fraction:
    - How do we know that a fraction is smaller/bigger than another?
    - How can two fractions be compared?
    - How do we compare two fractions with different denominators?

4. Model equivalency of fractions: 2/4 = ½

- Why are these two fractions the same?

- What patterns do you see in equivalent fractions?

5. Simplify fractions:

- Why do we simplify fractions?

- What mathematical understandings do we use to simplify fractions?

6. Use the mathematical vocabulary of fractions: improper, mixed number:

- What is the language of fractions?

- How is the language of fractions connected to other mathematical language?

7. Read, write and model the addition and subtraction of decimals to the thousandths:

- What is the connection between fractions and decimals?

- How is a decimal a fraction?

- How does addition and subtraction work with decimals?

- How is this connected to what you know about place value?

8. Read, write and model multiplication and division of decimals (with reference to money):

- What does the decimal point represent in money terms?

- What happens to the values when they are multiplied/divided by multiples of 10?

9. Round decimals to a given place or whole number:

   - Why do we want to round to decimal places?

   - When do we need to be less precise/more precise?

10. Read, write and model percentages:

    - What is a percentage?

    - To what do percentages relate?

    - What are real-life examples of percentages?

    - Why are percentages used in mathematics?

11. Interchange fractions, percentages and decimals:

    - How are percentages, fractions and decimals related?

    - Why can there be an interchange between there?

    - How can we work out how much we are saving when buying sales articles?

**How are MoNE and PYP different in teaching fractions?**

Some differences between the way MoNE and PYP curriculum handle fractions attract the attention. Some of these points are as following:

1. PYP curriculum starts teaching fractions earlier than MoNE curriculum. While MoNE introduces fractions firstly in second grade (at the age of 8) PYP students first encounter fractions at the age of 5-7.
2. In the first steps of fractions, PYP curriculum focuses on using the real life context such as telling the time as a tool to teach wholes, halves and quarters while there is no such a stress on MoNE curriculum.
3. Throughout the whole PYP framework, there are engaging and compelling questions that guide teachers such as “What is equivalence?”, “Can you show fractions equivalent to 1?” or “What patterns do you see in equivalence to 1?” On the other hand, MoNE curriculum gives no specific emphasis on the “equivalence to 1” concept and prefers to indicate this objective as “Students can explain wholes, halves and the quarters.”
4. PYP curriculum framework specifies the terms such as patterns, modeling and manipulative which are essential and significant on fractions teaching while MoNE only shares objectives and gives no suggestions about how to teach.

It should be also indicated that the curriculum cannot be the only parameter that affects teaching quality. Besides, teacher’s effort, family interventions and support, schools environment, etc. are some of the other factors that might affect the correct and permanent learning.

## Summary

In this chapter related literature on some topics were investigated such as, what a misconception is, how and why it occurs, why it is important to work on them, some specific types of misconceptions and why they were preferred to be investigated, the importance of fractions in the curricula, how PYP and MoNE schools can differ in terms of curricula content and other factors. Common results received from the related literature can be summarized as:

1. Misconceptions in mathematics exist and prevent students’ permanent and meaningful learning (Johnston & Gray, 1999; Swan, 2001; Henriques, 2002; Wescott & Cunningham, 2005; Çardak, 2009). The diagnosis and correction of misconceptions are important to prevent math anxiety and to promote deep and long lasting learning (Keazer, 2004; Chick and Baker, 2005).
2. Fractions are considered significant because of their connection with other algebraic topics and the wide applications in real life (D’Ambrosio & Mewborn, 1994; Mack, 1995; Keizjer & Terwel, 2001).
3. Fractions are also known as one of the topics that students tend to develop misconceptions about. In particular the attempt of applying whole number knowledge can cause fractions misconceptions (Saxe et al., 2005; Nunes et al., 2006; Lee, 2008; Van de Walle et al, 2010, p. 287).
4. Some specific sub topics of fractions such as partitioning, ordering and addition are the common topics that 4$^{th}$ grade MoNE and PYP students are taught (MEB, 2009; IBO, 2009).
5. MoNE and PYP curricula have some distinctions in their philosophies and objectives (MEB, 2009; IBO, 2009; OECD, 2012; Campbell et al., 2014). Private and public school difference is also another factor that might affect the variation between these the two curricula (Dinler & Subasi, 2003; Cinoglu, 2006). This variation between PYP and MoNE curricula might affect their education quality as well.

Based on the literature given in this chapter, significance of investigating the level of misconception of students who are taught with two different curricula was rationalized.

# CHAPTER 3: METHOD

## Introduction

This chapter describes the strategy of analysis and provides details about the design, sampling and participants. Chapter also explains how the researcher developed the instrument to detect the misconceptions of MoNE and PYP students taught at 4th grade. The information about the data collection from MoNE and PYP schools is also described. Finally, data analysis explains how the difference between MoNE and PYP students' response patterns were investigated.

## Research design

The present study only used a one-lesson-hour fractions test that was developed by the researcher to gather the quantitative data concerning the target sample. Due to this, it could be considered that the study uses cross-sectional design and provided a 'snapshot' of the frequencies and characteristics of misconceptions that 4th grade students had (Babbie, 1990; Creswell, 2003). The test that was used to collect quantitative data included 27 items, 9 items from each of the three categories of misconceptions, and the items in the test included both multiple choice items and open-ended problems that included real word context such as cake and pizza slices.

## Context

The present study was carried out at 4 schools in Ankara, Turkey. Among these four schools, two schools were private schools; Bilkent Laboratory and International School (BLIS) Ihsan Doğramacı Foundation Bilkent Primary School. Other two schools were public schools; National Education Foundation Batıkent Primary

School and Batıkent Primary School. The two private schools chosen for the present study were two of the only three PYP schools in Ankara. As for the two MoNE schools are concerned, they were chosen with the convenience sampling from all the schools situated in Batıkent, Ankara because of the ease of access.

It should also be noted that private schools and public schools may have some differences with regard to the students' profile. Students who attend private schools tend to have higher socio-economic background than those who attend public schools (OECD, 2012). With a few exceptions, in most of the PISA-participant countries and economies, including Turkey, more advantaged students seem to be attending privately managed schools (OECD, 2012).

## Participants

The research was conducted in April 2013 with 4$^{th}$ grade students from 4 schools (*n=264*). Among these 264 students, 112 were PYP students while 152 were MoNE students. 37 students participated from BLIS. Also, 75 students tested from I.D.F Bilkent Primary School. 58 students participated from N.E.F Batıkent Primary School and 23 of them were female and 35 of them were male. Additionally, 94 students that were tested from Batıkent Primary School consisted of 45 female and 49 male students. As their educational policy, two PYP schools did not prefer to share the additional gender information about students.

## Instrumentation

For the present study, a fractions test was developed to measure students' misconceptions on the topic of fractions. Partitioning, ordering and addition on fractions were included in the test since these sub-topics were the only ones that were covered by both MoNE and PYP curriculum at 4$^{th}$ grade level. The items were chosen after considering the related literature on misconceptions, MoNE and PYP textbooks. In particular, questions that students most tend to be mistaken were included in the test. Within this period, the book *Elementary and Middle School Mathematics: Teaching Developmentally* by Van de Walle, Karp and Bay-Williams provided handful tips with educational research studies and served as the main resource.

The misconceptions test that was developed by the researcher was shown to be valid based on expert opinions as well as quantitative analysis. The items 12 to 16 were used for validity analysis. Percentage of students who had misconceptions in at least 4 out of 5 items was found to be % 75.7. Students who provided responses with misconceptions in at least 3 items out of 5 had a percentage of 79.6. High level of misconceptions was detected by similar items. So, this can be considered as evidence for validity of instrument.

As for the expert reviews, the test was firstly checked by an expert who was a mathematics teacher. The expert who reviewed the instrument was experienced in primary school mathematics. He had a PhD degree in mathematics teaching and also was working as a mathematics teacher trainer at university. He advised to include fraction questions related to the sets, area and length to enrich the variety. He also suggested using active voice in question statements and supporting some questions with pictures. He also reviewed the language to make items clearer to students.

The items were also checked by another expert who works as a primary school mathematics teacher and a coordinator at a PYP school. She corrected some parts that caused contradictions, for example asking first about cakes and then about brownies, etc. She also asked to take the conversion in the last question out since asking ‘pounds’ to be changed to ‘kilograms’ would be irrelevant for a test that evaluated the misconceptions on the topic of fractions.

The test was composed of 27 items with all the sub-items. The number of the items was 35 at first; however it was reduced based upon the advice of the same expert who has experience in both primary school and university. After his feedback, considering the age group and the possible concentration time for this age group, the number of items was decreased to 27. Because of their young age, students might have developed anxiety or boredom towards the large number of items. So the sub-items were created and only the leading items were numbered. So, from the students' point of view, there were only 13 items in the test which was actually a more appropriate number of items for $4^{th}$ grade students. When the test was finished, students actually solved 27 items in total with the sub questions as well.

So, Appendix A and B represent the tests that students went through (English and Turkish versions respectively). Besides, Appendix C is the one that readers should follow since it includes the actual item numbers separately as the researcher used while analyzing the data.

Since the research was planned to be conducted both in Turkish public schools and private PYP schools that use English as a medium of education, the instruments Turkish and English versions were needed. The instrument was firstly prepared in English and 3 expert views were taken to validate the instrument's English version. The experts were all teachers, two mathematics and one statistics, who are fluent in English and also have teaching experience in both languages. The expert views made some corrections related to the comprehensibility of the language used in the problems. Also, through the agency of the feedback taken from them, the necessary add/drop changes were done. For example, changing pumpkin pie to apple pie since pumpkin pie would be too irrelevant to Turkish culture.

After this step, the researcher translated the instrument into Turkish for MoNE students. The Turkish version was checked out by one of the other expert, who was a native English and Turkish mathematics teacher.  The necessary changes were done through the feedback and the developed Turkish instrument was sent to other two experts who had also given feedback on the English version. They were asked to check the coherence between the Turkish and English version. Again, some changes were made with the aid of feedback and both Turkish and English instruments took their final forms. The English Fractions Test can be found in Appendix A, and the Turkish Fractions Test is in Appendix B.

27 items in the test were divided into three misconception categories as partitioning, ordering and addition. These sub-categories were determined from the related literature with regards to common objectives of MoNE and PYP curricula. However, the test did not contain any headings or parts that specify the categories in order not to interfere with students' thinking.

The first category among the 27 items, partitioning category, aimed to measure whether students know the importance of equal partitioning or not. Students, who failed to learn this, tend to think that a shape can be divided into non-equal-sized pieces and these pieces can state a fraction (Empson, 2001; Cramer & Whitney, 2010). Students were given 9 items for this category and asked to find out which figures express the given fraction values. The students who chose the non-equal-sized figures were considered as having a misconception on partitioning.

For the second category, ordering category, 9 items were developed. These 9 items aimed to measure students' understanding of ordering on fractions whose nominators are equal but denominators are different. The fractions that students were asked to order did not include the fractions that have the different denominators since $4^{th}$ grade objectives did not include it either in MoNE or PYP curricula. Since students attempt to continue with whole number ordering conception, they tend to choose the fraction with bigger denominators as the greater one among the fractions with equal nominators and different denominators (Nunes et al., 2006; Cramer, Wyberg, & Leavitt, 2008; Van de Wall et al., 2010, p. 300).

The third and the last category, add tops-add bottoms category, was included since most students carry out operation in fractions as they did in whole numbers (Lappan & Mouck,1998; Cramer & Whitney, 2010). Since they attempt to add fractions as they add whole numbers they may skip the fact that addition fractions do not mean adding the denominators of fractions straightforwardly. Similar with other two categories, 9 items were designed for this category.

## Method of data collection

The participants of MoNE schools were administered the test developed by the researcher. In both MoNE schools, firstly the administration and teachers of the school have been informed about the required permissions granted by MoNE. Having permission from class teachers to take over one lesson hour for each class, the test was administered by the researcher in one class after another. Students in each class were briefly informed about the aim, content, significance and the privacy of the study. The students were told that the results of the test will not be shared with teachers or parents. In all classes of MoNE schools students finished in almost 30 minutes. For the two PYP schools, the administrator and class teachers decided to deliver the test themselves. They were also asked to briefly inform students about the aim, content, significance and the privacy of the study as well. They were also asked to give students 30 minutes to complete the test. The administered tests were taken back by the researcher afterwards.

## Method of data analysis

After the data were collected from 4 schools, they were transferred into SPSS to carry out the necessary analyses. The curriculum types were coded as MoNE and PYP curriculum. All the students who participated to the study were asked to complete the whole test, yet there were some missing responses which were kept and any treatment was not done on data. Since the missing rates for the responses were not so high, no statistical procedure was conducted to handle them. Table 1shows the missing data numbers and percentages for every item.

Table 1
Missing rates for the items

| Item # | Number of missing responses | % of missing responses | Item # | Number of missing responses | % of missing responses |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 15 | 0 | 0 |
| 2 | 0 | 0 | 16 | 0 | 0 |
| 3 | 0 | 0 | 17 | 6 | 2.3 |
| 4 | 0 | 0 | 18 | 12 | 4.6 |
| 5 | 0 | 0 | 19 | 1 | 0.4 |
| 6 | 0 | 0 | 20 | 1 | 0.4 |
| 7 | 0 | 0 | 21 | 1 | 0.4 |
| 8 | 10 | 3.8 | 22 | 2 | 0.8 |
| 9 | 2 | 0.8 | 23 | 1 | 0.4 |
| 10 | 16 | 6.1 | 24 | 1 | 0.4 |
| 11 | 1 | 0.4 | 25 | 10 | 3.8 |
| 12 | 0 | 0 | 26 | 13 | 4.9 |
| 13 | 0 | 0 | 27 | 9 | 3.4 |
| 14 | 1 | 0.4 | | | |

The types of the 27 items were examined in three sub-categories; (i) misconceptions on partitioning, (ii) misconceptions on ordering and (iii) misconception on add tops-add bottoms. It should also be noted that the sub-categories were determined from the related literature with regards to frequencies and the age group that these misconception categories address.

Apart from the item categories, the responses of students were decided to be also categorized into three; (i) 'correct answer' which includes the full and correct answer without any misconception or operational mistakes, (ii) 'wrong answer with misconception' which stand for the wrong answers arising from the misconceptions that we expected to be encountered and (iii) 'wrong answer without misconception' which imply the wrong answers stemming from an operational mistake or from not being able to interpret what the items asked for. The reason for using 3 categories is to distinguish and identify the responses that include misconceptions from any other responses such as the correct ones or the ones that include errors that are not accepted as misconception. It should also be noted that, while transferring the test results into SPSS for each item, the *correct answer* was coded with "0", *wrong answer with misconception* was coded with "1" and finally *wrong answer without misconception* was coded with "2".

While deciding these response categories the distinction between the category 'wrong answer with misconception' from the 'wrong answer without misconception' category was made according to the misconceptions that the researcher expected to encounter according to the related literature. Those misconceptions that were expected to be encountered and included in the 'wrong answer with misconception' category were listed in the Appendix D. The mistakes that the researcher did not consider as misconception while evaluating students' responses and included in the 'wrong answer without misconception' category were also listed in Appendix D.

Since the responses were categorized without any justification for having the equal distance between categories, in other words the scales of the items were ordinal; the non-parametric statistical analyses were decided to be carried out (Gravetter & Wallnau, 2004). As a result, the chi square for homogeneity test was made to determine whether MoNE and PYP curriculum are similar or different in terms of the rates of three types of misconceptions.

First total scores of students were compared between the two curricula using Mann-Whitney test. The item level analyses were conducted using chi-square homogeneity test to investigate the distinction in response patterns across the two curricula. Lastly, ranks of the items were checked for each response categories.

Analyses were conducted at item level rather than total-score level due to the nature of scoring scheme.

However, since the chi square test for homogeneity only told us whether there was a difference between several populations or not, it had to support the findings with graphical representations. These graphs were aimed to help reader to figure out how different the two curricula were and how the answers of students from the two curricula vary between three types of responses, *correct answer, wrong answer with misconception* and *wrong answer without misconception.* While examining the results, the alpha level was assumed as 0.05 across all the analyses.

# CHAPTER 4: RESULTS

## Introduction

The main purpose of this study was to find out whether the types and frequencies of misconceptions that 4th grade students exhibited across MoNE and PYP curricula. With the aid of the results, the comparison of the two curricula with regards to misconceptions was expected to be revealed.

This chapter includes the results of analyses for chi square for homogeneity results that were conducted to investigate the differences between those three response patterns across MoNE and PYP curricula. It also includes the frequency graphs of students' responses for each of the 27 questions to help readers visualize these distinctions. Responses were categorized into 3 types; *correct answer* which included the full and correct answer without any misconception or operational mistakes, *wrong answer with misconception* which stood for the wrong answers arising from the misconceptions that we expected and *wrong answer without misconception* which implied the wrong answers stemming from an operational mistake or from not being able to interpret what the questions asked for.

## The overview of categories

Table 2 presents some information about misconceptions that were observed across the two curricula. Before elaborating on the information given in Table 2, we should recall that each of the three categories had nine questions. According to Table 2, all items of ordering category showed misconception sign for both curricula. For partitioning category the situation was quite same; except for only one item in PYP curriculum. The least percentages and number of questions that showed misconceptions belonged to the category add tops-add bottoms.

Table 2
Overview of misconception percentages for MoNE and PYP students

| | MoNE | | | PYP | | |
|---|---|---|---|---|---|---|
| | **The number of questions that misconception rate > 0** | **Minimum rate for misconception** | **Maximum rate for misconception** | **The number of questions that misconception rate > 0** | **Min rate for misconception** | **Max rate for misconception** |
| Partitioning | 9 | 2.6 | 27.6 | 8 | 0.9 | 18 |
| Ordering | 9 | 11.5 | 37.5 | 9 | 5.5 | 41.7 |
| Add tops- add bottoms | 1 | 0.7 | 0.7 | 5 | 1.8 | 2.7 |

It could be also concluded that for partitioning and ordering category the minimum rates for misconception were higher for MoNE curriculum than PYP curriculum. The situation was reverse for the add tops-add bottoms category. Besides, when maximum rates of misconceptions were examined, the PYP students exhibited higher rates for ordering and add tops-add bottoms categories. MoNE students showed higher maximum rate of misconception for partitioning category. It could be also observed that PYP students had more misconceptions on the category add tops-add bottoms.

The overall results of analysis showed that for all three categories, students' correct answers were more dominant than their wrong answers for both curricula type misconceptions (see Figure 3, 4 and 5). Furthermore, *wrong answer with misconception* percentages were also remarkable, especially for partitioning and ordering categories (see Figure 3 and 4). In particular for ordering category, the misconceptions had the highest percentages among all three with 30.6% for MoNE and 24.4% for PYP curriculum (see Figure 4). The second highest misconception percentage belonged to the partitioning category with 11.2% for MoNE and 7.6 for PYP curriculum (see Figure 3). Only for the last category, add tops-add bottoms, it was seen that the *wrong answer with misconception* percentages were less remarkable then the *wrong answer without misconception* percentages. For the partitioning and ordering, it can be concluded that wrong answers were mainly due to misconceptions rather than other factors such as misreading, operational mistakes, etc. However, for the add tops-add bottoms category the situation was quite reverse.

When we compared the results obtained from students of the two curricula according to the percentages, we noticed that for the *partitioning* and *ordering* categories, general correct answer percentages of PYP curriculum was higher than MoNE curriculum. Also, the *wrong answer with misconception* percentages of PYP curriculum was less than the percent values of MoNE curriculum (see Figure 3 and Figure 4). However, the situation was quite reverse for the *add tops-add bottoms* category: PYP students had slightly less percentage for correct answers and higher percentage for *wrong answer with misconception* (see Figure 5).

When we examined the correct answer percentages, the lowest correct answer percentages belonged to the *ordering* category (see Figure 4) whereas the highest correct answer percentages for both curricula belonged to the *add tops-add bottoms* category (see Figure 5). When we compared Figure 3, Figure 4 and Figure 5, we observed that the most challenging category for students seemed to be the *ordering* category with the highest *wrong answer with misconception* percentages among all. With a similar look, the category *add tops-add bottoms* had the least *wrong answer with misconception* percentages.

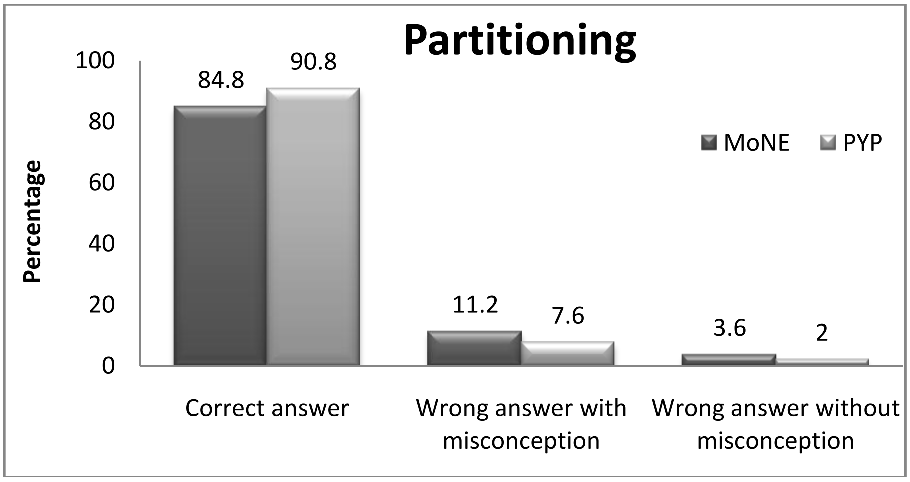



Figure 3. Comparison of response patterns for partitioning category

For the first category, *partitioning*, PYP students did better than MoNE students with higher correct answer percentages and lower *wrong answer with* and *without misconception* categories. Besides, for both curricula, the percentages for *wrong answer with misconception* seemed slightly higher than the percentage values of *wrong answer without misconception*.

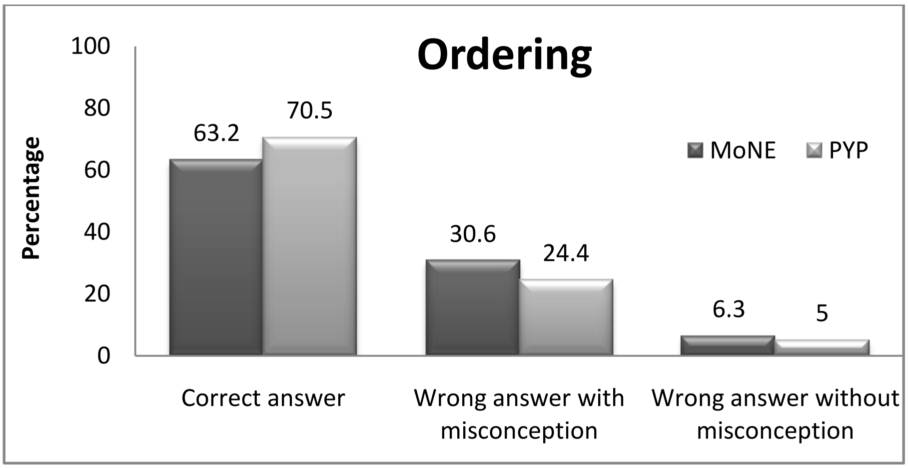



Figure 4. Comparison of response patterns for ordering category

The *ordering* category, which had the lowest ratios for correct answers, seemed to have similar results with *partitioning* category with regards to the curriculum comparison. PYP students exhibited higher percentages for correct answer while they had lower *wrong answer with* and *without misconception*. It can be also seen that the *wrong answer with misconception* percentages were much higher than the percentages of *wrong answer without misconception* for both curricula.

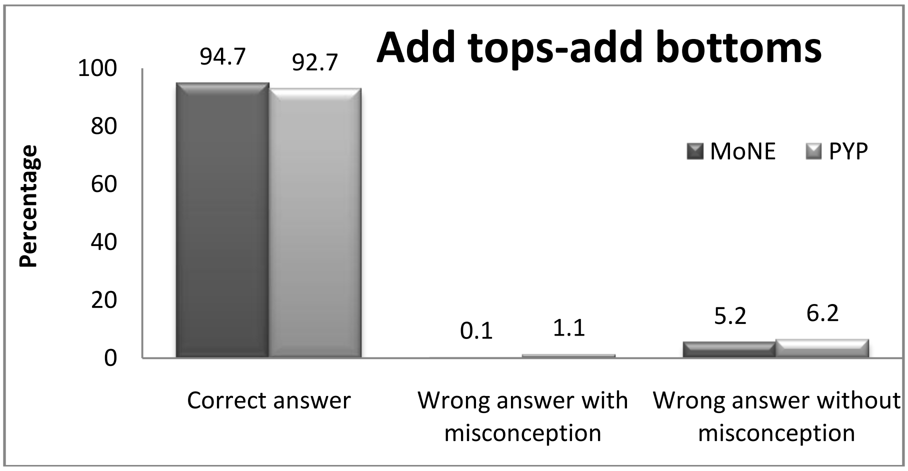



Figure 5. Comparison of response patterns for add tops-add bottoms category

Add tops-add bottoms category was the only category among three in which MoNE students did better than PYP students with higher correct answer percentages and less *wrong answer with* and *without misconception* percentages. This category was also the only category that the percentages values of *wrong answer without misconception* were higher than the percent of *wrong answer with misconception* for both curricula. In this category, most of the wrong answers were due to other factors than misconception.

Thus it can be concluded that among the three categories, partitioning, ordering and add tops-add bottoms, first two had higher misconception ratios, changing from 7.6% to 30.6. Wrong answers due to other reasons had relatively lower ratios changing between 2.0% and 6.3%. On the other hand, the last category, add tops-add bottoms, had extremely high ratios for correct answers (94.7 for MoNE and 92.7 for PYP) with very low ratios for wrong answers (between 5.2% and 6.2%). Furthermore, wrong answers given by students were mostly not due to misconceptions. After the overview of three categories, now we are going to identify how questions differed in response categories across the two curricula and we are going to elaborate on each category, item by item.

### Misconceptions on partitioning

9 items were developed to determine how well MoNE and PYP students were doing about the very basic idea of fractions. Students' ability to choose correctly partitioned figures was aimed to be detected. Also to students' knowledge to determine the correct fraction quantity from a figure or a problem were attempted to be evaluated. When Figure 3 was examined, it can be seen that the percentage values of *correct answer*s were greater than the percentages of both *wrong answer with misconception* and *wrong answer without misconception* categories for both curricula. When we compared the averages of *correct answer*s across the MoNE and PYP curricula, we observed that the correct answer percentages were 84.8 for MoNE curriculum and 90.8 for PYP curriculum. The percentage of *wrong answer with misconception* category was 11.2 for MoNE curriculum while it was 7.6 for PYP curriculum. We observed much lower percentages, even none for some questions, for *wrong answer without misconception* category in both curricula.

The percentages of MoNE and PYP students' responses according to three categories were given in Table 3. Table also included the chi square analysis for homogeneity results. Chi square for homogeneity was conducted to investigate which questions were answered with statistically significant difference between different responses (*correct answer*, *wrong answer with misconception* and *wrong answer without misconception)* with respect to MoNE and PYP curricula. For a better interpretation, the graphical representations of the percent values were presented afterwards.

Table 3
The percentages of responses to questions on partitioning and chi square for homogeneity analysis results

| Item number | MoNE: **Correct Answer (Co.)** | MoNE: **Wrong Answer with Misc. (Misc.)** | MoNE: **Wrong Answer without Misc. (No Misc.)** | PYP: **Correct Answer (Co.)** | PYP: **Wrong Answer with Misc. (Misc.)** | PYP: **Wrong Answer without Misc. (No Misc.)** | Chi-square | df | p |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 80.9 | 19.1 | 0.0 | 86.5 | 13.5 | 0.0 | 1.426 | 1 | .247 |
| 2 | 96.7 | 3.3 | 0.0 | 92.8 | 7.2 | 0.0 | 2.096 | 1 | .162 |
| 3 | 95.4 | 4.6 | 0.0 | 99.1 | 0.9 | 0.0 | 2.985 | 1 | .144 |
| 4 | 72.4 | 27.6 | 0.0 | 82.0 | 18.0 | 0.0 | 3.291 | 1 | .780 |
| 5 | 96.7 | 3.3 | 0.0 | 100.0 | 0.0 | 0.0 | 3.722 | 1 | .075 |
| 6 | 88.2 | 11.8 | 0.0 | 99.1 | 0.9 | 0.0 | 11.458 | 1 | .000 |
| 7 | 86.2 | 13.8 | 0.0 | 96.4 | 3.6 | 0.0 | 7.778 | 1 | .005 |
| 8 | 58.5 | 14.4 | 23.1 | 75.5 | 16.0 | 8.5 | 10.660 | 2 | .005 |
| 9 | 88.1 | 2.6 | 9.3 | 85.5 | 8.2 | 6.4 | 4.631 | 2 | .099 |

The first 7 questions of partitioning category were multiple choice questions. Students were asked to choose the figure(s) that correctly partitioned according to the given fractions. In the last two questions, we observed that the *wrong answers without misconception percentages are* much higher than others. These were open ended questions that asked students to find the fraction value of one quantity in the whole. Among these two questions, the 9$^{th}$ question was supported by a figure while the 8$^{th}$ question was not.

According to Table, only the question number 6, $\chi(1) = 11.458$; $p = .000$, question number 7, $t(1) = 7.778$; $p = .005$, and question number 8, $\chi(1) = 10.660$; $p = .005$ had statistically significant difference between MoNE and PYP curricula. Graphical representations can be seen in the Figures 7 (b), 7 (c) and 8 (a). Questions number 1, 2, 3 and 4 asked students to determine which figures correctly showed the fraction $\frac{1}{3}$ (see Appendix C for test questions). It can be seen that there was no statistical difference between the responses of MoNE and PYP students (see Table 3). The graphs of first four questions visually proved that the answer patterns of students from the two curricula were quite close. The difference in the correct answer and wrong answer with misconception values of the 4th question (see Figure 6 (d)) drew the attention, yet the statistical difference could not be observed for this item.

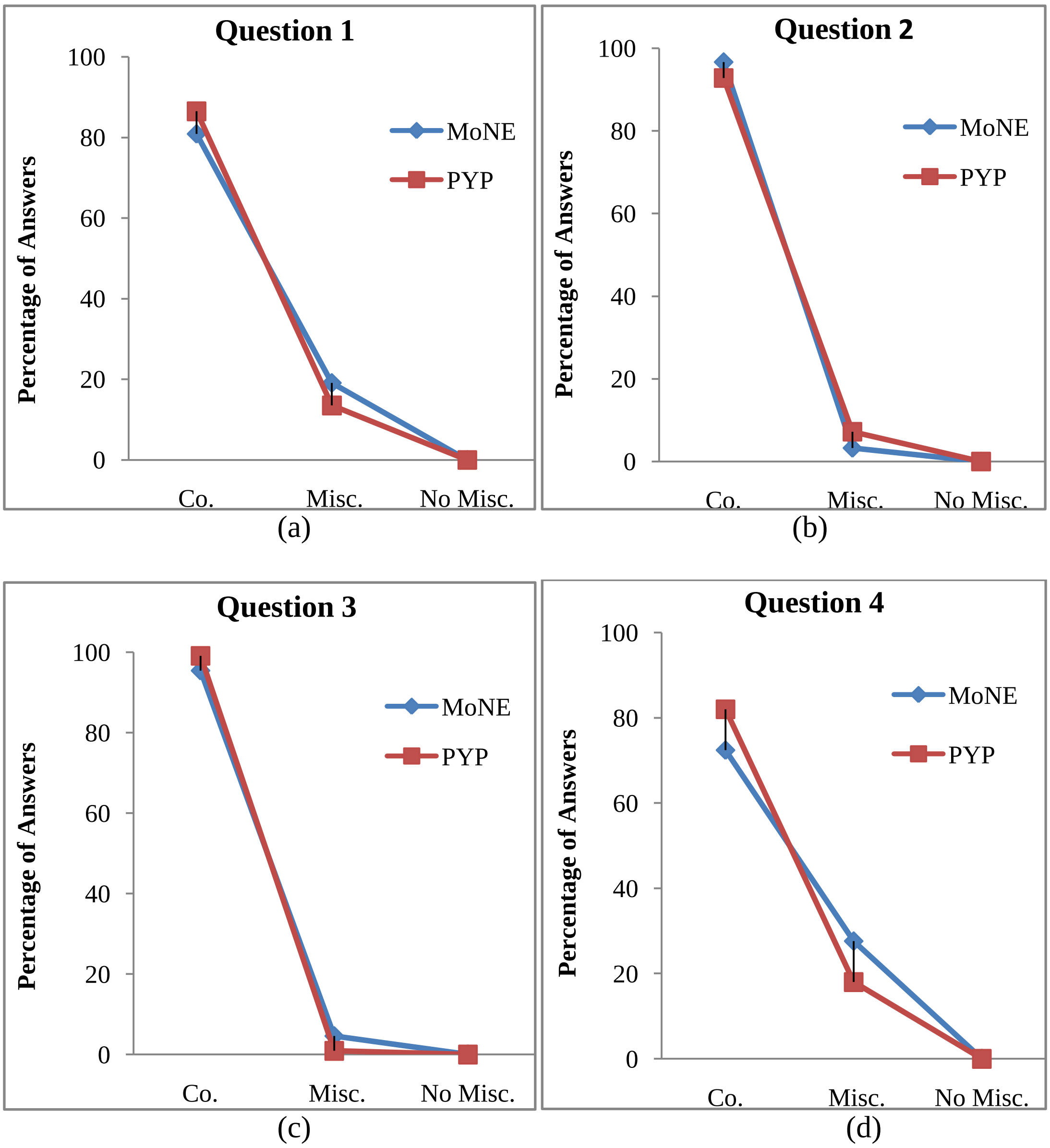
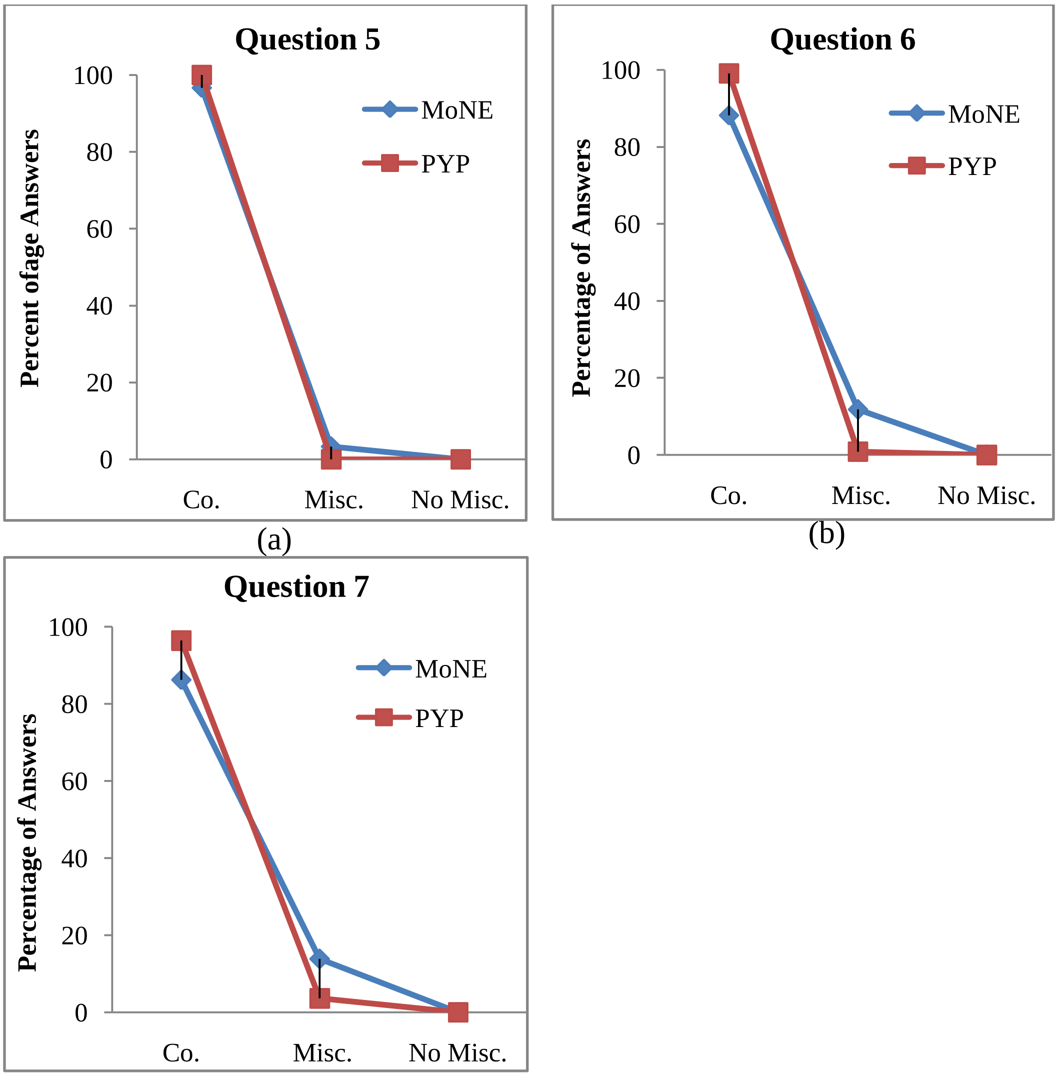



Figure 6. (a) Line graph of answers for question 1, (b) line graph of answers for question 2, (c) line graph of answers for question 3 (d) line graph of answers for question 4

Questions number 5, 6 and 7 were about identifying the figures which are correctly partitioned into fourths (see Appendix C for questions). The only figure which was correctly partitioned into fourths was question number 5. MoNE and PYP students seem to have no statistically different results for question 5 ($p > .05$). However, the remaining two questions, question number 6 and 7, were not supposed to be marked since they were not correctly partitioned. These were the questions that students who have misconceptions on partitioning would mark. For these questions, the responses of MoNE and PYP have differed statistically. These differences can be observed in chi square for homogeneity results (see Table 3) and besides it can also be verified with the figures in Figure 7 (b) and (c). In Figure 7 (b) wrong answer with misconception values of the two curricula were slightly the same however the wrong answer with misconception rates were different. Thus the correct answer percentages were affected too. In Figure 7 (c), the same thing could be observed. On the other hand, in Figure 7 (a), the rates of the two curricula seemed to be much closer.

In two of the three questions that statistically significant differences have been detected, in questions 6 and 7, the MoNE students had more misconception percentages than PYP students have. In question 6, MoNE students showed 11,8 percent misconceptions while PYP students have 0.9 misconception percentage. Also in question 7, MoNE students exhibited 13.8 misconception percentages while PYP students showed 3.6 misconception rates. These values can be traced in Table 3.

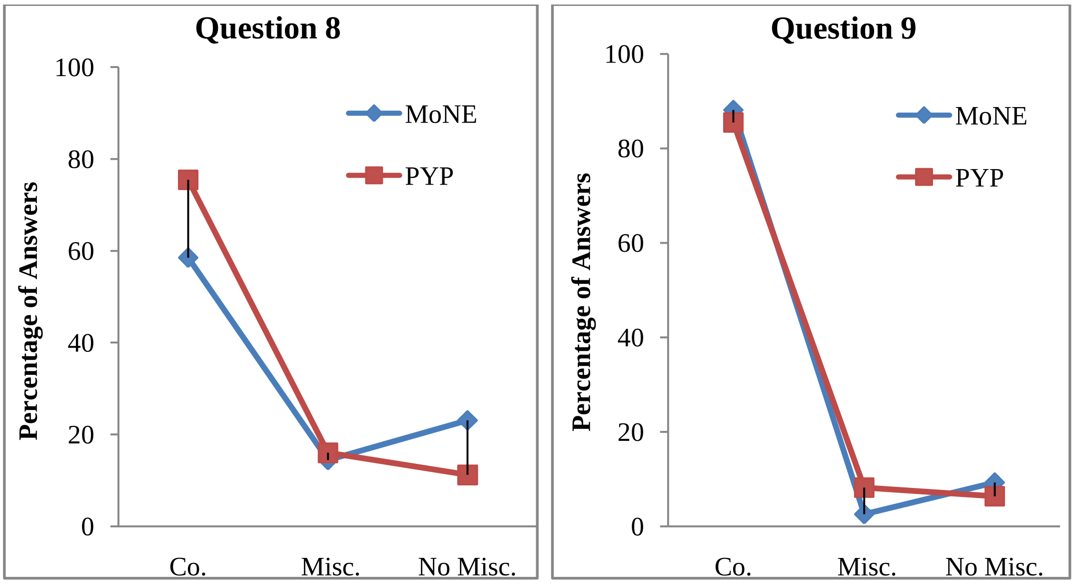
For the question number 8, the percentages of correct answers and the percentages of the wrong answer without misconception were much different for the two curricula. When comparing with MoNE students' percentages, the percentage of correct answer was higher for PYP students and the percentage of wrong answer without misconception was much lower for PYP students.




(c)

Figure 7. (a) Line graph of answers for question 5, (b) line graph of answers for question 6, (c) line graph of answers for question 7

Questions number 8 and 9 slightly asked the same thing but in a different way. The difference for the correct answer and wrong answer percentages across the 8$^{th}$ question which was not supported by a picture and the 9$^{th}$ question which was supported by a pictorial representation can be easily observed in the figures of question 8 and 9. Figure 8 (a) shows that misconception rates were similar for both curricula. So, the statistical difference could arise from the difference between the correct answer and wrong answer without misconception rates.




(a) (b)

Figure 8. (a) Line graph of answers for question 8, (b) line graph of answers for question 9, (c) line graph of answers for question 10

There is another point that draws the attention. Chi square for homogeneity analysis proved that only the questions number 6, 7 and 8 were statistically differed among the two curricula. Other than these 3 questions, questions number 1 and 4 drew the attention since they had high misconception rates. For both questions, students were asked to identify the figures which show $\frac{1}{3}$. The results showed that for question 1, 19.1 percent of MoNE students and 13.5 percent of PYP students and for question 2, 27.6 percent of MoNE students and 18.0 percent of PYP students had misconceptions about partitioning a whole into pieces correctly.

## Misconceptions on ordering

Similar with misconceptions on partitioning, 9 questions were developed to measure students' understanding of ordering on fractions whose nominators were equal but denominators were different. Since students tend to consider the fraction whose denominator is bigger as greater, the results of ordering category was expected to reveal whether they have this type of misconception or not. Table 4 shows the percentage values of MoNE and PYP students' responses to the following 9 questions that concern ordering on fractions. Similar with the partitioning table (Table 3), Table 4 includes the percentages of three response categories for both curricula. Table 4 also contains the chi square for homogeneity analysis to identify the questions that were statistically differently answered by MoNE and PYP student. Subsequently, the graphical representations of the percent values of 9 responses among MoNE and PYP curricula were presented.

Table 4
The percentages of responses to questions on ordering and chi square for homogeneity analysis results

| | MoNE | | | PYP | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Item number | **Correct Answer (Co.)** | **Wrong Answer with Misc. (Misc.)** | **Wrong Answer without Misc. (No Misc.)** | **Correct Answer (Co.)** | **Wrong Answer with Misc. (Misc.)** | **Wrong Answer without Misc. (No Misc.)** | Chi-square | df | p |
| 10 | 59.7 | 35.4 | 4.9 | 48.5 | 41.7 | 9.7 | 4.045 | 2 | .132 |
| 11 | 61.2 | 34.2 | 4.6 | 66.4 | 28.2 | 5.5 | 1.095 | 2 | .578 |
| 12 | 63.8 | 36.2 | 0.0 | 76.6 | 23.4 | 0.0 | 4.901 | 1 | .031 |
| 13 | 63.8 | 36.2 | 0.0 | 73.0 | 27.0 | 0.0 | 2.459 | 1 | .117 |
| 14 | 64.5 | 34.9 | 0.7 | 73.6 | 26.4 | 0.0 | 2.983 | 2 | .225 |
| 15 | 64.5 | 35.5 | 0.0 | 69.4 | 30.6 | 0.0 | 0.691 | 1 | .406 |
| 16 | 62.5 | 37.5 | 0.0 | 68.5 | 31.5 | 0.0 | 1.005 | 1 | .360 |
| 17 | 62.5 | 11.5 | 23.0 | 83.5 | 5.5 | 11.0 | 10.29 | 2 | .006 |
| 18 | 63.0 | 13.7 | 23.3 | 75.2 | 5.7 | 19.0 | 5.609 | 2 | .610 |

When Table 4 was examined, it could be observed that the *correct answer* and *wrong answer with misconception* percentages for both curricula was less different from each other than it was in the partitioning category. The average of percentages for *wrong answer with misconception* is 30.6% for MoNE students while it was 11.2% for the partitioning category. Similarly, the average of percentages for *wrong answer with misconception* is 24.4% for PYP students while it was 7.6% for the previous category. It can also be deduced that the average of percentages for *wrong answer* *with misconception* in the ordering category is higher for the MoNE students. Besides, an increase was observed in the percentage values of *wrong answer without misconception* for two groups of students in the last two questions which were open ended real world problems.

Even though the misconception rates were very high for both MoNE and PYP curricula in comparison with other categories, the chi square for homogeneity analysis could not determine a statistical difference between curricula. The reason behind this was because the misconception rates were close to each other among curricula.

Also, according to Table 4, only the question number 12, $\chi(1) = 4.901; p = .031$ and question number 17, $\chi(2) = 10.29; p = .006$ had statistically significant difference between MoNE and PYP curricula. Graphical representations can be seen in the following figures.

In question 12, where we asked students to compare $\frac{3}{6}$ with $\frac{3}{2}$, the statistical difference was detected. According to the figure of question 12 above, PYP students had less percentage of answers for the wrong answer, with and without misconceptions while they had greater percentages for the correct answer. For question 12, MoNE students had 36.2 misconception rates while PYP students showed 23.4 misconception percentages. The variation was also visually proved in Figure 9 (c).

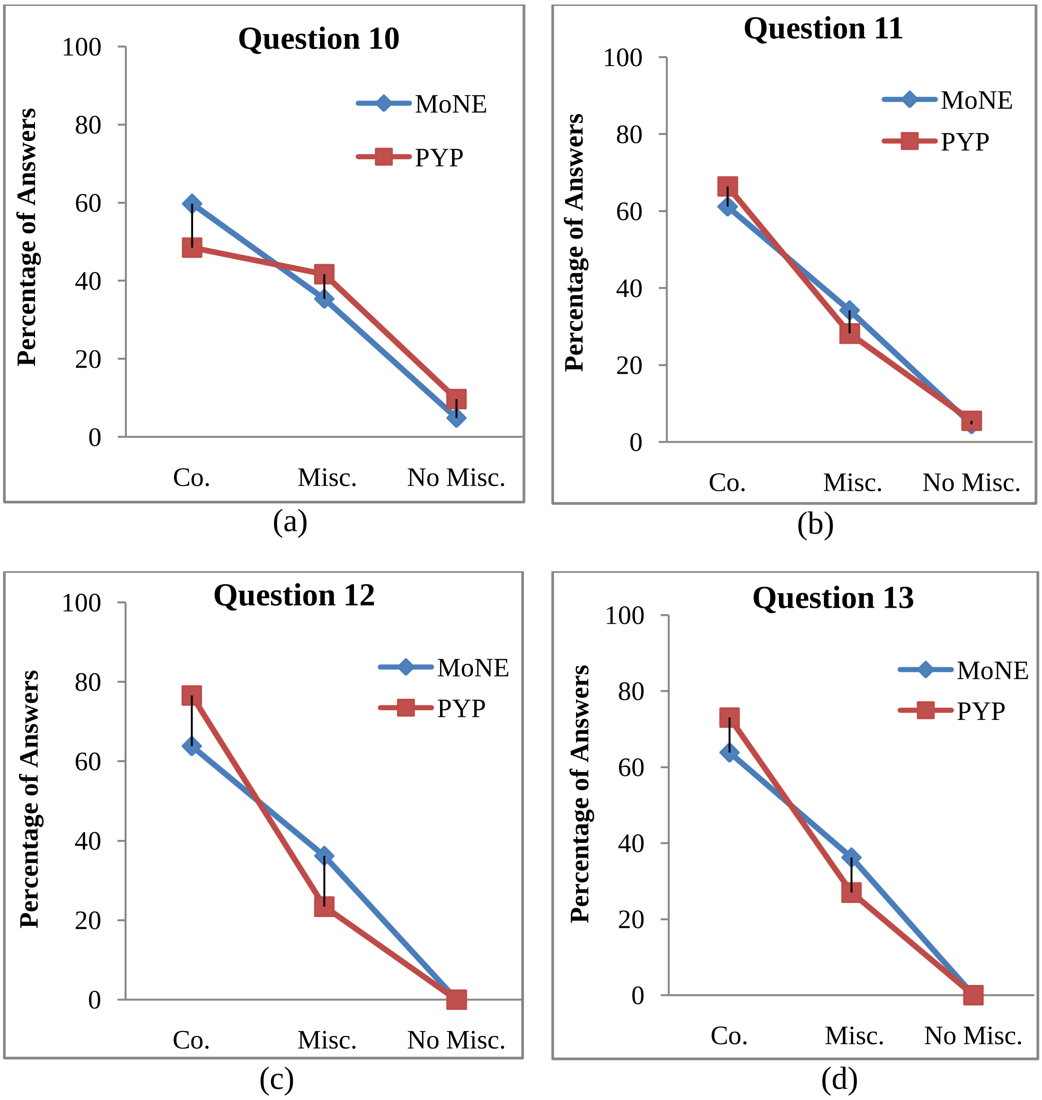



Figure 9. (a) Line graph of answers for question 10, (b) line graph of answers for question 11, (c) line graph of answers for question 12 (d) line graph of answers for question 13

The following 3 questions, questions number 14, 15, and 16 seemed to have no statistical difference among the answers of students from the two curricula ($p > .05$). Even though there was no statistical difference, the misconception rates were still very high for both curricula. For each of these 3 questions, MoNE students exhibited higher percentages for misconceptions than PYP students.

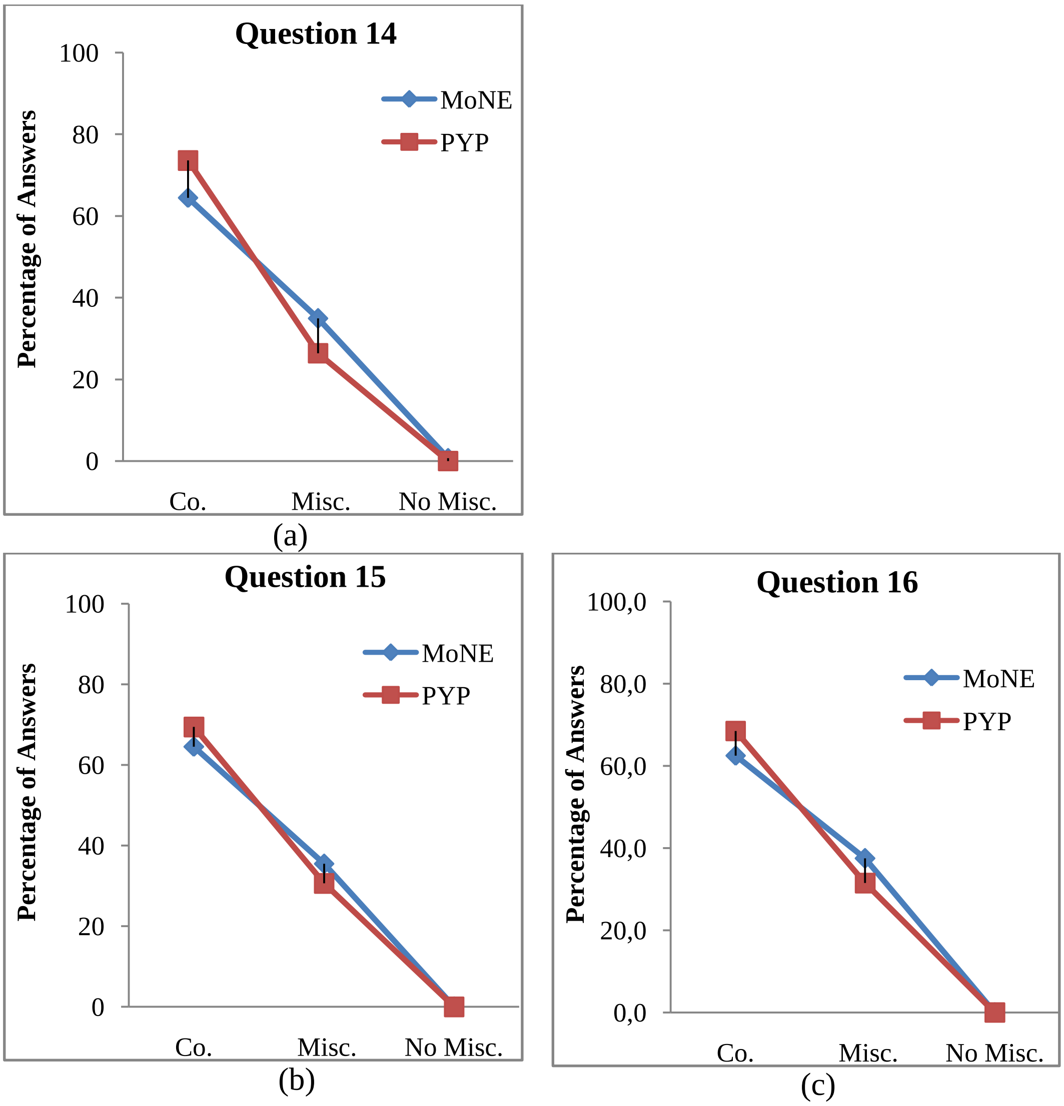



Figure 10. (a) Line graph of answers for question 14, (b) line graph of answers for question 15, (c) line graph of answers for question 16

Question number 17 was the other question that showed a statistically different result for the two curricula. In this question, we asked students to solve a real world problem and order the fractions that they figured from the question. PYP students had less percentage of answers for wrong answer with and without misconceptions while they had greater percentages for the correct answer for the question number 17. 11.5% of MoNE students had misconception while 5.5% of PYP students showed solutions that included misconception. Other than that, 23.0% of MoNE students and 11.0 percent of PYP student showed wrong answer without misconceptions. Figure 11 (a) also proves that MoNE and PYP students' misconception rates were close however the statistical difference could be traced in the correct answer and wrong answer without misconception rates.

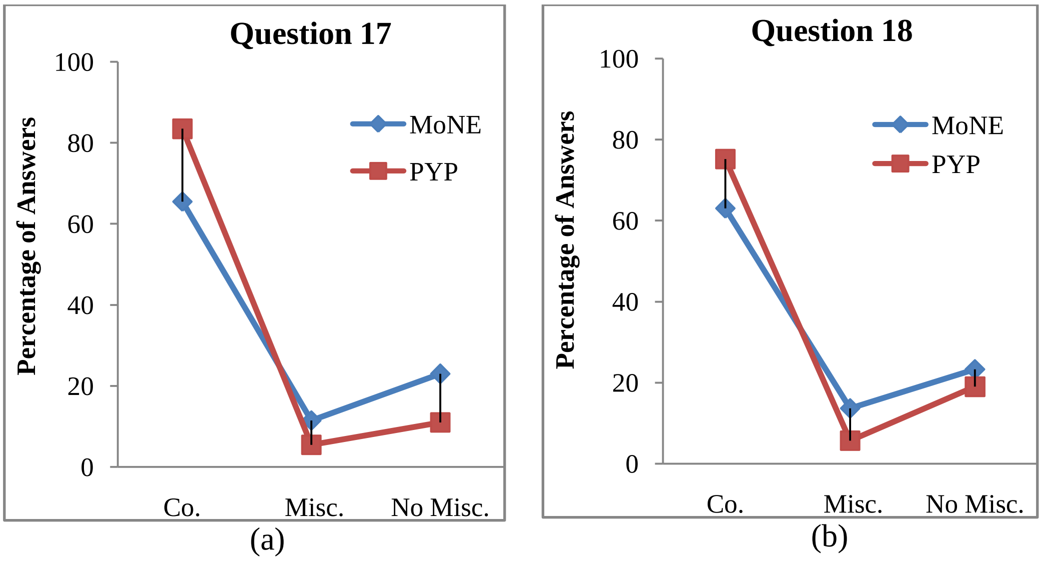



Figure 11. (a) Line graph of answers for question 17, (b) line graph of answers for question 18

### Misconceptions on add tops-add bottoms

Similar with the other two categories, 9 questions were developed for this category as well. With the help of this category, students' misconceptions about addition of fractions were aimed to be determined. The misconception that was considered while developing those items was adding the denominators of fractions while deciding the denominator of the sum. Due to this, 9 items of this category aimed to measure whether MoNE and PYP students' had this type of misconception or not.

Table 5 represents the percentages of frequencies of students' responses to the last 9 questions, questions number 19 to 27. Additionally, Table 5 includes the chi square for homogeneity analysis results that revealed the questions that were statistically differently answered by MoNE and PYP students. Again, the graphical representations of percentage values of responses from the two curricula were added.

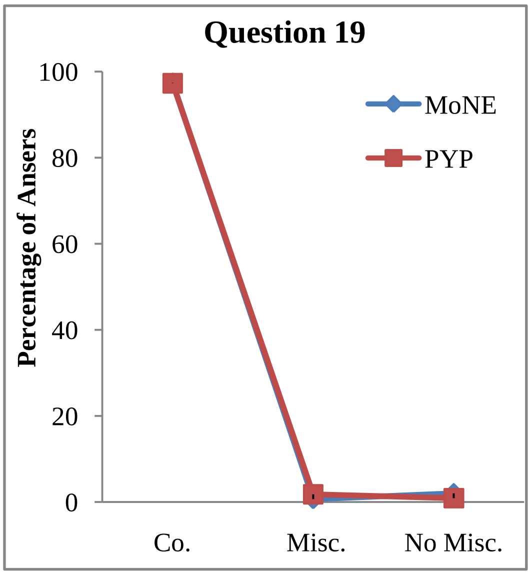
Table 5
The percentages of responses to questions on add tops-add bottoms and chi square for homogeneity analysis results

| Item number | MoNE | | | PYP | | | Chi-square | df | p |
|---|---|---|---|---|---|---|---|---|---|
| | **Correct Answer (Co,)** | **Wrong Answer with Misc, (Misc,)** | **Wrong Answer without Misc, (No Misc,)** | **Correct Answer (Co,)** | **Wrong Answer with Misc, (Misc,)** | **Wrong Answer without Misc, (No Misc,)** | | | |
| 19 | 97.4 | 0.7 | 2.0 | 97.3 | 1.8 | 0.9 | 1.224 | 2 | .542 |
| 20 | 98.7 | 0.0 | 1.3 | 97.3 | 2.7 | 0.0 | 5.606 | 2 | .061 |
| 21 | 100.0 | 0.0 | 0.0 | 97.3 | 0.0 | 2.7 | 4.193 | 1 | .073 |
| 22 | 100.0 | 0.0 | 0.0 | 98.2 | 1.8 | 0.0 | 2.811 | 1 | .173 |
| 23 | 100.0 | 0.0 | 0.0 | 98.2 | 1.8 | 0.0 | 2.785 | 1 | .175 |
| 24 | 99.3 | 0.0 | 0.7 | 98.2 | 1.8 | 0.0 | 3.496 | 2 | .174 |
| 25 | 74.3 | 0.0 | 25.7 | 85.7 | 0.0 | 14.3 | 4.812 | 1 | .029 |
| 26 | 88.4 | 0.0 | 11.6 | 80.8 | 0.0 | 19.2 | 2.773 | 1 | .106 |
| 27 | 94.6 | 0.0 | 5.4 | 81.1 | 0.0 | 18.9 | 11.413 | 1 | .001 |

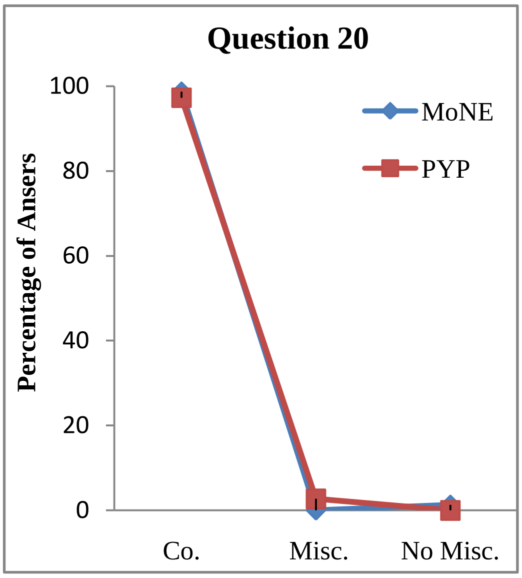
When we took a close look at Table 5, we realized that the percentages of correct answers were much higher than the percentages of wrong answers with and without misconception for both curricula. The average percentages for correct answer were 94.7% for MoNE curriculum and 92.7% for PYP curriculum which were the highest correct answer percentages among all three categories. The percentages for wrong answer with misconception were 0.1% for MoNE students and 1.1% for PYP students. These values had the least percentage values for wrong answer with misconception among three categories. Lastly, the percentages for wrong answer without misconception were 5.2% for MoNE students and 6.2% for PYP students.

In addition to these, according to Table 5, only the question number 25, $\chi(1) = 4.812; p = .029$ and the question number 27, $\chi(1) = 11.413; p = .001$ had statistically significant difference between MoNE and PYP curricula.

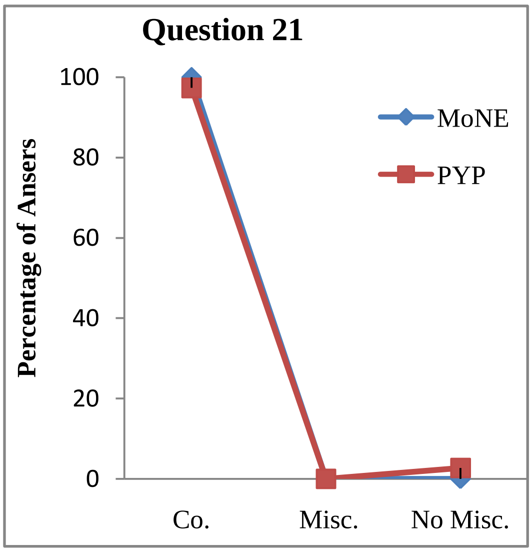
The first 6 questions, questions number 19 to 24, of this category directly asked students to carry out addition on fractions whose denominators were same but nominators were different. The misconception that was expected to be encountered was adding the denominators of two fractions instead of deciding the common denominator. The results in Table 5 show that the responses of MoNE and PYP students had no significant difference for these 6 questions. This conclusion also could be observed in the 6 figures below.



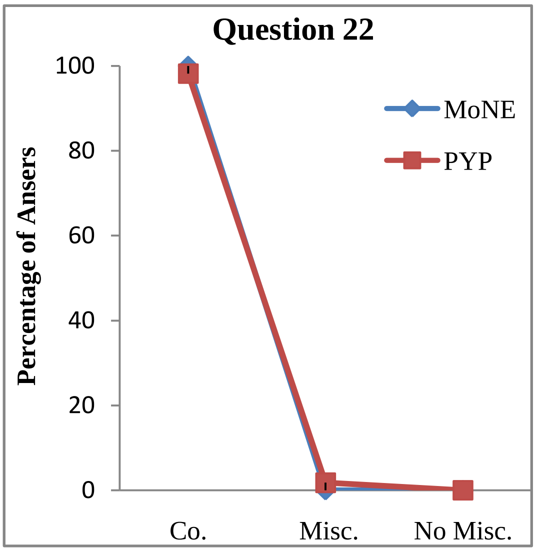

(a)



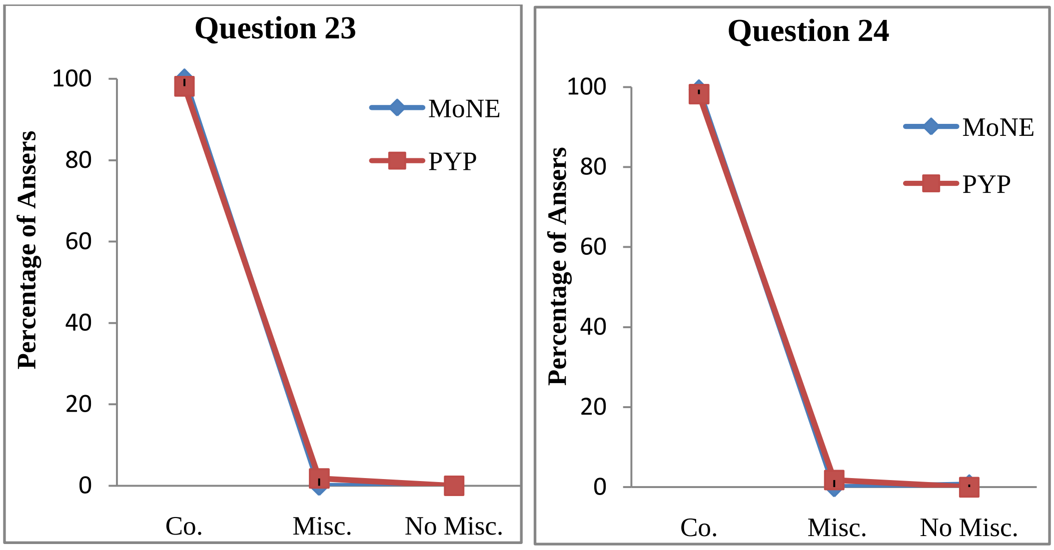

(b)




(c)




(d)




(e) (f)

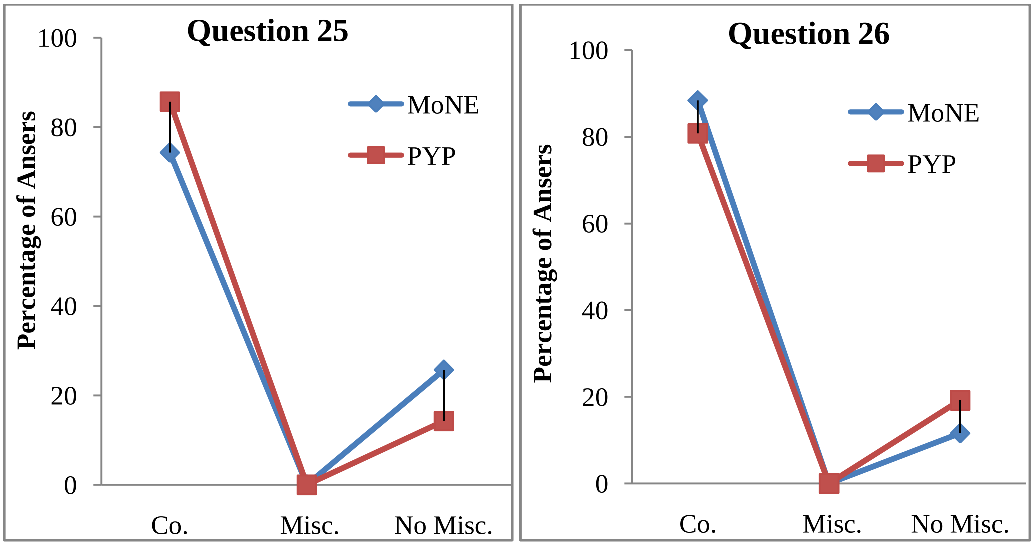
Figure 12. (a) Line graph of answers for question 19, (b) line graph of answers for question 20 (c) Line graph of answers for question 21, (d) line graph of answers for question 22, (e) Line graph of answers for question 23, (f) line graph of answers for question 24

The remaining 3 questions of the add tops-add bottoms category, questions number 25 to 27, were open ended real world problems. All three questions asked students to carry out addition operation in real world problem context.

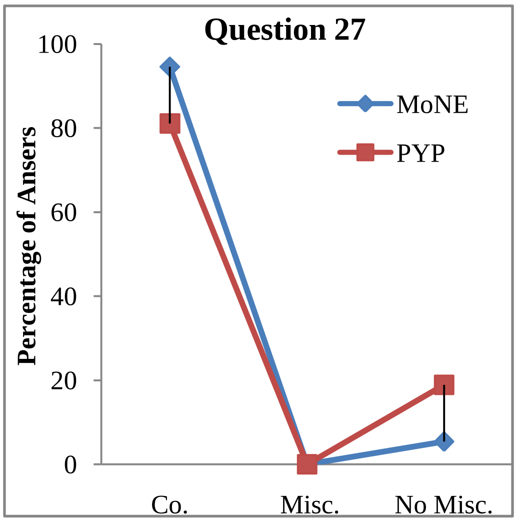
Chi square for homogeneity results showed the statistically significant mean difference for questions number 25 and 27. This result was in line with the figures of question 25 and 27 below (see Figure 13 (a) and (c)). For the item number 25, MoNE students had less correct answers percentage with 74.3% and greater wrong answers without misconception percentage with 25.7% than PYP students. Because, for the same question PYP students showed 85.7 correct answer percentage and 14.3 wrong answer without misconception rate.

It also drew the attention that for item number 25 the misconception rates were zero. So, the statistical difference resulted from the contrast between correct answer and wrong answer without misconception rates. This can be also seen in Figure 13 (a).

However, the situation was quite the reverse for question number 27. In the 27th question MoNE students seemed to have better results with less wrong answer and more correct answer than PYP students. 94.6 of MoNE students answered question 27 correctly while 81.1 of PYP students gave the correct answer. Same with the 25th question, misconception rates were zero for both curricula. However wrong answer without misconception rates drew the attention since MoNE students' rate was 5.4 while PYP students have 11.4. Similar with the 25th question, the statistical difference that was detected should be associated with the difference between correct answer and wrong answer without misconception rates. This claim can be also supported by Figure 13 (c).




(a) (b)




(c)

Figure 13. (a) Line graph of answers for question 25, (b) line graph of answers for question 26 (c) Line graph of answers for question 27

Other point that drew the attention was the high wrong answer without misconception rates for questions number 25, 26 and 27. The previous 6 questions of add tops-add bottoms category asked students to carry out addition for fraction couples while the last three questions, questions number 25, 26 and 27 asked them to do the same with a real world problem. For the items 25 and 27 significant differences were found due to the distinctions between wrong answers without misconceptions across the two curricula.

## Review of all items according to percentage ranks

Apart from the fraction categories, the questions should also be investigated according to the percentage values of correct answer, wrong answer with and without misconception. In this section, 27 items were ranked based upon the correct answer, wrong answer with and without misconception percentages for PYP and MoNE students separately. With the aid of the table, the achievement on item bases was attempted to be revealed.

Table 6
Correct answer percentages for MoNE and PYP students

| Item number | Correct answer percentages of MoNE | Item number | Correct answer percentages of PYP |
|---|---|---|---|
| 21 | 100.0 | 5 | 100.0 |
| 22 | 100.0 | 3 | 99.1 |
| 23 | 100.0 | 6 | 99.1 |
| 24 | 99.3 | 22 | 98.2 |
| 20 | 98.7 | 23 | 98.2 |
| 19 | 97.4 | 24 | 98.2 |
| 2 | 96.7 | 19 | 97.3 |
| 5 | 96.7 | 20 | 97.3 |
| 3 | 95.4 | 21 | 97.3 |
| 27 | 94.6 | 7 | 96.4 |
| 26 | 88.4 | 2 | 92.8 |
| 6 | 88.2 | 1 | 86.5 |
| 9 | 88.1 | 25 | 85.7 |
| 7 | 86.2 | 9 | 85.5 |
| 1 | 80.9 | 17 | 83.5 |
| 25 | 74.3 | 4 | 82.0 |
| 4 | 72.4 | 27 | 81.1 |
| 14 | 64.5 | 26 | 80.8 |
| 15 | 64.5 | 12 | 76.6 |
| 12 | 63.8 | 8 | 75.5 |
| 13 | 63.8 | 18 | 75.2 |
| 18 | 63.0 | 14 | 73.6 |
| 16 | 62.5 | 13 | 73.0 |
| 17 | 62.5 | 15 | 69.4 |
| 11 | 61.2 | 16 | 68.5 |
| 10 | 59.7 | 11 | 66.4 |
| 8 | 58.5 | 10 | 48.5 |

Evaluating the results in Table 6, we noticed all the top 6 questions that MoNE students did best belonged to add tops-add bottoms category while the 9 items out of 10 least successful items belonged to ordering category. When considering the fact that each category consisted of 9 questions, we quickly realized that the entire category for ordering laid in the bottom of the correct answer list of MoNE students.

On the other hand, among the top 9 questions of PYP students did best there were 3 partitioning items and 6 add tops-add bottoms items. The 7 questions at the end of the list that PYP students most struggled with were all ordering items.

Table 7
Wrong answer with misconception percentages for MoNE and PYP students

| Item number | Wrong answer with misconception percentage of MoNE | Item number | Wrong answer with misconception percentage of PYP |
|---|---|---|---|
| 16 | 37.5 | 10 | 41.7 |
| 12 | 36.2 | 16 | 31.5 |
| 13 | 36.2 | 15 | 30.6 |
| 15 | 35.5 | 13 | 27.0 |
| 10 | 35.4 | 11 | 28.2 |
| 14 | 34.9 | 14 | 26.4 |
| 11 | 34.2 | 12 | 23.4 |
| 4 | 27.6 | 4 | 18.0 |
| 1 | 19.1 | 8 | 16.0 |
| 8 | 14.4 | 1 | 13.5 |
| 7 | 13.8 | 9 | 8.2 |
| 18 | 13.7 | 2 | 7.2 |
| 6 | 11.8 | 18 | 5.7 |
| 17 | 11.5 | 17 | 5.5 |
| 3 | 4.6 | 7 | 3.6 |
| 2 | 3.3 | 20 | 2.7 |
| 5 | 3.3 | 19 | 1.8 |
| 9 | 2.6 | 22 | 1.8 |
| 19 | 0.7 | 23 | 1.8 |
| 20 | 0.0 | 24 | 1.8 |
| 21 | 0.0 | 3 | 0.9 |
| 22 | 0.0 | 6 | 0.9 |
| 23 | 0.0 | 5 | 0.0 |
| 24 | 0.0 | 21 | 0.0 |
| 25 | 0.0 | 25 | 0.0 |
| 26 | 0.0 | 26 | 0.0 |
| 27 | 0.0 | 27 | 0.0 |

When the item numbers in Table 7 were examined it was seen that the top 7 items that MoNE students had most misconceptions were all ordering questions. It drew the attention that the percentage values of misconceptions dramatically changed after the top 7 items which belonged to ordering category. Another interesting result that MoNE students showed was that all the 9 lowest misconception percentages belonged to the add tops-add bottoms category. Only one question out of 9 showed 0.7 percent misconception while the other 8 were answered with 0.0 misconception rate.

PYP students' misconception percentages were quite same with MoNE students. Again, the top 7 question that PYP students most struggled with were all ordering items. Similar with MoNE students' misconception results, the percentage values dropped after these items. On the other hand, among 13 questions in which PYP students exhibited lowest misconception rates there were 4 partitioning items and 9 add tops-add bottoms items which involve all category.

Table 8
Wrong answer without misconception percentages for MoNE and PYP students

| **Item number** | **Wrong answer without percentages of MoNE** | **Item number** | **Wrong answer without percentages of PYP** |
|---|---|---|---|
| 25 | 25.7 | 26 | 19.2 |
| 18 | 23.3 | 18 | 19.0 |
| 8 | 23.1 | 27 | 18.9 |
| 17 | 23.0 | 25 | 14.3 |
| 26 | 11.6 | 17 | 11.0 |
| 9 | 9.3 | 10 | 9.7 |
| 27 | 5.4 | 8 | 8.5 |
| 10 | 4.9 | 9 | 6.4 |
| 11 | 4.6 | 11 | 5.5 |
| 19 | 2.0 | 21 | 2.7 |
| 20 | 1.3 | 19 | 0.9 |
| 14 | 0.7 | 1 | 0.0 |
| 24 | 0.7 | 2 | 0.0 |
| 1 | 0.0 | 3 | 0.0 |
| 2 | 0.0 | 4 | 0.0 |
| 3 | 0.0 | 5 | 0.0 |
| 4 | 0.0 | 6 | 0.0 |
| 5 | 0.0 | 7 | 0.0 |
| 6 | 0.0 | 12 | 0.0 |
| 7 | 0.0 | 13 | 0.0 |
| 12 | 0.0 | 14 | 0.0 |
| 13 | 0.0 | 15 | 0.0 |
| 15 | 0.0 | 16 | 0.0 |
| 16 | 0.0 | 20 | 0.0 |
| 21 | 0.0 | 22 | 0.0 |
| 22 | 0.0 | 23 | 0.0 |
| 23 | 0.0 | 24 | 0.0 |

Table 8 gives us valuable information about students' approach to open ended problems. The top 5 questions that MoNE students did wrong without showing any misconception included all items from 3 categories (1 partitioning, 2 ordering and 2 add tops-add bottoms). Failing to find a common category for these 5 items, with a closer look we noticed that all these items were real world problems that did not ask students to follow a certain procedure but instead interpret the case. Considering the fact that the test included only 7 real world problems, it explained the situation better.

The situation was quite the same for PYP students. The top 6 questions included 3 ordering and 3 add tops-add bottoms questions. However the common thing for these items was again they are all open ended real world problems.

Apart from these, table showed plenty of 0 percentages at the bottom of the list. It should be explicit that some items on the test were evaluated as they were either correct or wrong with misconception. For example, for the first 7 partitioning category there were wholes that partitioned correctly or incorrectly. Since students who thought incorrectly partitioned shapes could represent fraction were already showing a sign of misconception. So, wrong without misconception option was disabled. There were 12 items that were evaluated like that. The other 5 were in the add tops-add bottoms. All these 12 items have automatically 0 percent for wrong answer without misconception as shown in Table 8

## Summary

In general, the results this study proved can be summarized as following:

1. In total 7 items out of 27 had statistically significant difference across the two curricula. Out of these 7 questions, ordering and add tops-add bottoms categories had 2 questions for each while 3 of the questions belonged to partitioning category.

2. For 6 of the 7 items that statistical difference has been detected, PYP students showed better results than MoNE students with higher correct answer percentages and lower misconception percentages.
3. Another thing that was observed at a glance was that among three categories the most misconceptions have identified in the ordering category while the lowest misconception rates existed in add tops-add bottoms category.
4. For partitioning and ordering categories great majority of wrong answers were associated with misconceptions.
5. Unlike the partitioning and ordering categories, add tops-add bottoms category was the only one in which most of the wrong answers were due to the factors other than misconceptions.

# CHAPTER 5: DISCUSSION

## Introduction

The present study aimed to find out how the frequencies and characteristics of some specific types of misconceptions vary across the MoNE and PYP curricula. First of all, the related literature was examined to find out the most popular fraction topics that students tend to have misconceptions most. Hence three categories were identified and named as partitioning, ordering and add tops-add bottoms. Then by using this information, a fraction test that includes 9 items for each three fraction categories which contained 27 items at total was developed by the researcher and implemented in two MoNE and two PYP schools to a total of 264 students. Students' answers were classified into three; *correct answers* which covered the correct and complete solution of the question, *wrong answers with misconceptions* that implied answers that include one or more of the expected misconception and *wrong answers without misconception* which covered the wrong answers that contained none of the misconceptions literature referred to.

The results showed that students' misconception varies among categories or curriculum types. Some items displayed statistically significantly different results for MoNE and PYP curriculum. Also, some categories showed more misconceptions than others. Chapter 4 explained such results with the help of graphs and tables. In this chapter, we will be interpreting the results and see if the research questions were answered.

This chapter includes the discussions of the major findings which will be closer argued according to the following outline: First, discussions according to misconception categories, partitioning, ordering and misconception on add tops-add bottoms, will be separately examined. Secondly, the discussion according to the 7 questions that displayed statistical difference between the two curricula will be carried. Then finally, the chapter will conclude with presentation of implications for practice and further research, and the limitations encountered in the present study

## Discussion according to misconception categories

The overall results indicated that among three misconceptions categories, students from both curricula seemed to most struggle with the ordering category (see Figure 3, Figure 4 and Figure 5). The ordering category had the least correct answer percentages among three categories for both curriculum types. Also, the wrong answer with misconception percentages for both curricula have dramatically increased only for this category.

On the other hand, add tops-add bottoms category was the one among three in which all students showed the best results with highest correct answer percentages and lowest wrong answer with and without misconceptions percentages.

### Discussion of misconceptions on partitioning

This category aimed to assess students' ability to decide if the equal-sized shapes could express a fractional value. When the related literature was examined, it was seen that students tend to misinterpret the relation between numerator and denominator. Hence, they fail to recognize denominator as whole and numerator as part of the whole (Siebert & Gaskin, 2006). Also another misconception that students demonstrate is skipping the importance of equal partitioning. The students who do not completely comprehend that the whole should be divided into equal parts may think shapes that are divided in anyway can express fraction (Empson, 2001; Van de Walle et al., 2012, p. 292).

At a first glance, the comparison of correct answer and wrong answer percentages of the two curricula in partitioning category revealed that PYP students had a better conceptual understanding of fraction partitioning. PYP students exhibited 90.8% correct answers while MoNE students showed 84.8% correct answer rates. Furthermore, PYP students seemed to have lower rates for wrong answer with and without misconception than MoNE students have. PYP students' wrong answer with misconception rate was 7.6 while it was 11.2 for MoNE students. Also, PYP students' wrong answer without misconception rate was 2.0 while it was 3.6 for MoNE students. This finding indicated that partitioning teaching should be carried out by emphasizing the importance of equal shares in particular for MoNE students (Cramer & Whitney, 2010; McNamara & Shaughnessy, 2010).

As for the partitioning category, the comparison of the wrong answer with to without misconception rates of both curricula led us to conclude that the misconceptions on partitioning constituted the high percentage of students' overall errors about fractions partitioning. When Table 3 in Chapter 4 was closely examined, it could be seen that for some specific items, such as questions number 1, 4, 6, 7 and 8, showed dramatically higher rates for MoNE students' misconceptions. At the same time, question number 1, 4 and 8 demonstrated the same situation for PYP students. We should separately elaborate on the questions numbers 6, 7, and 8 which were answered statistically differently by MoNE and PYP students.

The items number 5, 6 and 7 were about identifying the shapes that are correctly partitioned into fourths. Among these, number 6 and 7 answered statistically different between the two curricula. For item number 6, the statistical difference might arised from the misconception rates since it was 11.8% for MoNE students while it was 0.9% for PYP students. Similarly, for item number 7, MoNE students showed 13.8% misconception rate while PYP students' misconception rate was 3.6. These results showed that percentages for wrong answer with misconception in both questions have dramatically increased for MoNE students.  This may lead us to suggest that some MoNE students tend to ignore the shape of the pieces and they rather focus on the number of pieces which points out an incomplete understanding of partitioning.

For item number 8, the situation was different. Misconception rates were 14.4% for MoNE and 16.0% for PYP students. However, the statistical difference was observed between the rates of wrong answer without misconception with 23.1% for MoNE students and 8.5% for PYP students. Before interpreting these findings, we should state that among 9 questions of partitioning category the first 7 items asked students to choose the correctly partitioned shapes. Also, the last 2 items which were item number 8 and 9 asked students to do the same with a real world problem. The remarkable high percentages for wrong answers without misconception in both items number 8 and 9 showed that students from two curriculum types were facing difficulty to grasp real world problem situations. In particular, the higher rates of MoNE students' wrong answer without misconception showed that PYP students were better at interpreting the open ended real world problems than MoNE students.

For item number 8, as Bamberger, Oberdorf, &Schultz-Ferrell (2010) suggested, students tend to think that if Elena has 6 toys and Andre has 4 toys the fraction of the toys Andre has is $\frac{4}{6}$ rather than $\frac{4}{10}$. Similarly, for item number 9 students tend to think that the $\frac{3}{2}$ of the whole represents the dragons rather than $\frac{3}{5}$ which was the correct answer. Even though the answer $\frac{3}{2}$ represents a part that is bigger than the whole, still students failed to figure the real whole value. This misconception takes its source from failing to determine the whole by figuring what numerator and denominator actually represent (Bamberger, Oberdorf, &Schultz-Ferrell, 2010).

Another thing about the items number 8 and 9 was that while item number 8 had higher rates for wrong answers with and without misconception, it had lower correct answer rates in both curriculum types. Questions number 8 and 9 asked students to decide fraction of one quantity among all. So, why the success rate of students from both curricula was higher for item number 9 while the structure of both items were quite same? A closer look to the question types may reveal the answer. Question 9 have supported the problem statement with a pictural representation that might helped students to visualize the statement. On the other hand, item number 8 asked a very similar question only with words. This result may also lead us to conclude that problems that are supported with a pictural representation can be interpreted more easily.

Other than these items, the questions number 1 to 4 asked students to identify the alternative(s) in which the given fraction value was correctly partitioned. Among the correct answer percentages of these four questions, the low correct answer percentage and high misconception percentage for the two curricula in question number 4 drew the attention. The reason why students particularly had difficulty about question number 4 might be that they failed to compare the three unequal parts in the triangle and thought any shares could express a fraction (Cramer & Whitney, 2010).

Following question number 4, the first question had the second highest percentages for wrong answer with misconception for both curricula. In this question, students seemed to fail to understand if the first shape correctly partitioned, then it would represent $\frac{1}{4}$ rather than $\frac{1}{3}$. A student who could comprehend partitioning correctly should suggest, "If this shape was partitioned so that all pieces were the same, then there should be four pieces with the size of the piece shaded" not one third as the students who do not have partitioning conception would think (Van de Walle et al., 2012, p. 296).

**Discussion of misconceptions on ordering**

This category of the test aimed to find out whether MoNE and PYP students had competence to compare fractions whose nominators were same but denominators were different. Since students tend to use their whole number sense in ordering the fractions, they may suggest that the bigger denominator implies bigger fraction by ignoring the fact that ordering on fractions have the inverse relationship (Park, Güçler, & McCrory, 2013).

When we examined the percentages of MoNE and PYP curricula separately for ordering category, we concluded that PYP students exhibited better results than MoNE students (see Figure 4 in Chapter 4). PYP students had higher percentage for correct answers (70.5%) than MoNE students (63.2%). At the same time, PYP had fewer percentages for wrong answer with and without misconceptions (24.4% and

5.0%, respectively) while MoNE had higher rates for wrong answer with and without misconceptions (30.6% and 6.3% respectively).

At a glance, Figures 3, 4 and 5, it was seen that ordering category's misconception rates were the highest among all three categories which proved that students from both curriculum types had the lack of conceptual understanding of why one fraction was larger or smaller than the other. Other than that, Table 6 displayed the correct answer ranks of all items. In this table, we saw that almost all ordering items laid in the bottom of the list for both MoNE and PYP curriculum types. In a similar vein, Table 7 ranked the wrong answer with misconception rates and the top 7 items that had the highest misconception rates belonged to ordering category for both curricula.

Additionally, difference between the rates of wrong answer with and without misconceptions for ordering category was noticeable (see Figure 4). MoNE students exhibited 30.6% for wrong answer with misconception while they only showed 6.3% for wrong answer without misconception. Similarly, PYP students demonstrated 24.4% for wrong answer with misconception while they only showed 5.0% for wrong answer without misconception. Much higher rates for wrong answer with misconceptions in both curricula brought the idea that if students' misconceptions on ordering were addressed, almost complete learning of ordering could be assured.

When the responses were analyzed on questions based, we should first elaborate on the $10^{th}$ and the $11^{th}$ questions since they both asked students to compare fractions but one with a real world problem and the other asked this straightforwardly. The correct answer percentages for the one that included real world problem were much lower in the two curricula when comparing the question that asked students to compare fractions directly. Another thing that drew the attentions was that the MoNE students showed better results in the world problem when comparing with PYP students.

The next 5 questions, questions number 12 to 16, required students to compare fraction pairs and decide which one was larger or smaller. Among these five, only the 12th question answered statistically differently among the two curricula. The difference was in favor of PYP students with higher correct answer percentages and lower wrong answer with misconception percentages. It should also be pointed out that among these 5 questions that shared the same structure, the 16th question in which students were asked to decide if $\frac{20}{3}$ or $\frac{20}{40}$ was greater had the highest rate of wrong answer with misconception percentages for both curricula. This led us to think that students in general had difficulty to order fractions with equal numerators and different denominators but furthermore, it seemed they tend to be mistaken more when the change between denominators increases (Cramer, Wyberg, & Leavitt, 2008).

Another point that drew the attention about the questions number 12-16 was that the wrong answer without misconception percentages was zero for both curricula. This again led us to conclude that students' ordering mistakes took their sources from misconceptions.

The last 2 questions of this category, items number 17 and 18, were related in the sense that they both asked students to make comparisons between two fractions with similar world problems. Unlike the previous 7 ordering questions, the difference between the misconceptions rates of the two curricula were slightly close to each other, the situation has changed for last two items of ordering. The rates demonstrated that for these two questions the wrong answer with misconception rates radically decreased for MoNE students. Examining item number 17, we realized that MoNE students' misconception rate was 11.5% while PYP students only had 5.5% rates for misconception on ordering. Similarly, considering the 18$^{th}$ item we noticed that MoNE students' misconception rate was 13.7% while PYP students' misconception rate was 5.7%. This might led us think that MoNE students had more difficulty in interpreting open ended real world problems than PYP students did.

**Discussion of misconceptions on add tops-add bottoms**

Within this category, we aimed to compare MoNE and PYP students' addition misconception which was mostly observed as adding the nominators and denominators separately. Related literature proved that as a result of students' attempt of carrying their whole numbers knowledge into fractions, students tend to think numerators and denominators separate whole numbers. Thus, they fail to recognize denominators as wholes and numerator as their parts (Lappan & Mouck,1998; Cramer & Whitney, 2010). So, this category aimed to assess students' ability to carry out operation in fractions with equal denominators.

We also observed the relative frequency of this category by comparison with other two categories. At first glance, we saw that the highest correct answer rates belonged to the add tops-add bottoms category for both curriculum types. MoNE students had 94.7% correct answer rates while PYP students had 92.7%. This category was also the only one that PYP students' correct answer rates fell behind MoNE students' correct answer rates. MoNE students did slightly better than PYP students with higher correct answer percentages and lower wrong answer with and without misconceptions percentages. MoNE students' misconception rate was 0.1% while it was 1.1 for PYP students. Furthermore, MoNE students' wrong answer without misconception rate stayed at 5.2% while PYP students' rate was 6.2%.

This category was also the only category that MoNE and PYP students' wrong answer with misconception rates were lowest among others. Comparing the wrong answer with and without misconception rates, we quickly realized that students' mistakes did not generally arise from misconceptions. Because wrong answer without misconception rates were 5.2% and 6.2% respectively for MoNE and PYP students while it was only 0.1% and 1.1% for misconception rates.

These implications can be also proved with the help of Tables 6, 7 and 8 in Chapter 4. When we examined the correct answer ranks table (see Table 6), it was noticeable that the top items with highest correct answer rates resided in the addition category. Top 6 questions that MoNE students did best belonged to the add tops-add bottoms category. At the same time, among the top 9 questions of PYP students did best there were 3 partitioning items and 6 add tops-add bottoms items.  Also, Table 7 was in consistence with our results. MoNE students' all 9 lowest misconception rates belonged to the add tops-add bottoms category while among 13 questions in which PYP students exhibited lowest misconception rates there were 4 partitioning items and 9 add tops-add bottoms items which involved the entire category.

As a result, we could easily conclude that among three misconception types we examined in this research, the addition misconception was observed least. Both PYP and MoNE curriculum types seemed to address the importance of the relations between denominators and nominators. However, even though the misconception rates were low, the rates for other mistakes were remarkable especially for real world problems. Items such as 25, 26, and 27 were the only ones that included problem statements rather than asking to make addition straightforwardly. When Table 5 was examined, the wrong answers without misconception rates were noticeable for these items. This might led us think that even though students seemed to have they haven't got any misconception about addition; they were still facing difficulty to interpret the problem statements.

When the responses were analyzed on questions based, we could compare the two curriculum types better. Items number 19 to 27 consisted of the add tops-add bottoms questions. We should first checked the results of items number 25 and 27 as they were the only two items that chi square for homogeneity test found statistical difference between MoNE and PYP. Beforehand, it should be stated that both items were open ended real world problems about addition of fractions. For the 25$^{th}$ item we realized that the misconception rates were 0 for both curriculum types. So it seemed none of the students attempted to add the denominators of fractions described in the problem. So, the difference must be the result of correct answer or wrong answer without misconception rates. MoNE students' correct answer rate for item number 25 was 74.3% while PYP students' correct answer rate was 85.7% which proved that PYP students better comprehended the problem and reached the correct answer. Another interesting result was seen when the wrong answer without misconception rates of the two curricula were compared. MoNE students' wrong answer without misconception rate was 25.7% while PYP students showed 14.3%. The situation was quite reverse for the 27$^{th}$ item rates for wrong answer without misconception. MoNE students showed 5.4% wrong answer without misconception rate while PYP students demonstrated 18.9%. It was noticeable that for such similar question types, MoNE students showed low correct answer rate and high wrong answer without misconception rate in the 25th item while PYP students did the same for the 27th item.

Other than the last three items of add tops-add bottoms category, the wrong answer rates did not seem severe. For the first 6 items, MoNE students had at least 97.4% correct answer rate while PYP students had 97.3% which implied MoNE and PYP students were capable of carrying out addition on fractions with equal denominators and unequal nominators.

**Discussion in terms of PYP and MoNE curricula**

The results of the analysis can be discussed in terms of the differences between the two curricula as following:

1. The 6 out of 7 items that were statistically differently answer by PYP and MoNE students were in favor of PYP students. One of the reasons behind this could be the more engaging and real life-based teaching of fractions in the PYP curriculum. PYP curriculum outline gives specific importance to support learning with compelling question that might guide teachers and enable students to develop conceptual learning rather than procedural learning.
2. Other reason in PYP students' slightly better outcomes might be that they start to teach fractions a little earlier than MoNE (MEB, 2009; IB, 2009).
3. There are some studies that show PYP students in different countries showed higher performance when comparing to the national schools (Campbell et al., 2014). Despite the learning objectives, the teacher qualities, better school environment and high socio economic status of parents of PYP schools, MoNE schools are still not much different than PYP schools. Unlike the

expectations of the research, curricula did not create so substantial distinction for the fractions misconceptions of students.

4. Observing the difference of only 7 items out of 27, can be further studied since it may point out a lack of correct application of PYP in the Turkish schools. Generally PISA results show that Turkish students need to develop their mathematics literacy and focus on conceptual learning rather than procedural learning (OECD, 2012). This might be one of the reasons that PYP students in Turkey did not achieve a substantial success.

## Implications for practice

Despite the significant effort that they put into teaching, teachers sometimes may be astonished to see what kind of misinterpretations students have developed towards even the simplest concepts. This is because they skipped importance of addressing misconceptions in teaching and learning (Chick & Baker, 2005). Effective and urgent diagnosis and correction of misconceptions will promote a better learning (Swan, 2001). In particular, addressing misconceptions in early ages, as we worked on 4$^{th}$ graders, ensure more safety for the following related concepts.

Misconceptions should be first identified to be fixed. The results of this analysis can guide many teachers and educators to see what types of topics compel the students and what are some specific misconceptions they may develop. This can be used to anticipate the most common misconceptions and prevent if they still arise. Also as it was observed in the research, for some specific topics like addition students seem to have no serious misconception about the questions that require procedural knowledge. However, the success rate has dropped when it comes to questions demanding students to show their conceptual understanding of the topic. It cannot be safely guaranteed that students who give the correct answer to a procedural question learn the concept fully or they just memorized the formula or method. Due to this, the validity of teaching should be checked with open ended real world problems as well since students' conceptual understanding will be better evaluated.

Other than that, the results can be also used to reinforce MoNE and PYP curricula's approach to fractions learning as the research proves what type of deficiency they may have. By interpreting the comparison between the two curricula the stakeholders such as education policy makers, school administrations and teachers of the two curricula can learn from each other to enhance their frameworks and teaching-learning quality.

## Implications for further research

Since the research only worked on the misconceptions that can be observed in the topic fractions, similar research analysis can be done with other topics. Also, other grade levels can be tested through interviews that follow the tests to make sure what students thought about their misunderstandings. The instrumentation for other research studies can be designed as the comparison between the misconceptions in procedural questions and open ended problems will be more explicit.

## Limitations

There were several limitations of this study. First of all, the sample of this study was limited to 264 students from two private and two public schools in Ankara. Limited sample of the study might be considered as an obstacle to make generalizations. Besides, the participant numbers were 112 from PYP schools and 152 from MoNE schools. The uneven distribution of PYP and MoNE students can be also criticized as a limitation.

Also, the analysis of the research was challenging since the correct diagnosis of misconception was complicated. Even though analyzing the literature about fractions enabled us to observe some certain types of misconceptions, it was still hard to make a certain distinction between an expected misconception and a regular wrong answer.

It should also be pointed out that the two MoNE schools that were chosen for the research could not accurately reflect the MoNE schools population. Because, these schools are placed in the capital city of Turkey and the observations about the teacher quality, school environment, instructions quality were higher than Turkey's average. These factors can be considered in not detecting significant distinction between the two curricula.

In addition, the results may have slightly influenced by the translation differences between the Turkish and English instrument. Even though the multiple expert reviews, the challenge of guaranteeing the total and exact translation was difficult because of the nature of languages and translation.

Finally, it is important to emphasize that the outcome of this study is only limited with the comparison of the two curricula regarding fractions misconceptions. The findings should not be generalized to compare PYP and MoNE curriculum in all other aspects such as the quality of teaching, teachers or materials.

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# APPENDICES

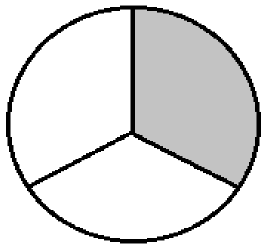
## Appendix A-Instrument (English)

**Fractions test**

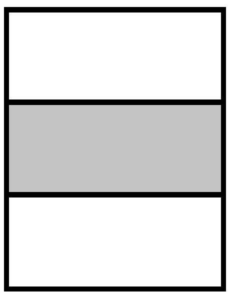
**1)** Which of the figures below show the fraction $\frac{1}{3}$ correctly? You can mark more than one alternative.

a)

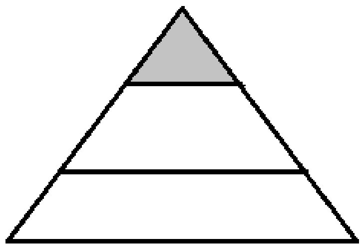
b)



c)

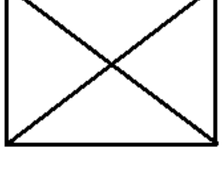


d)

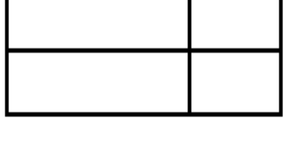


**2)** Which of the figures below are correctly partitioned into fourths? You can mark more than one alternative.

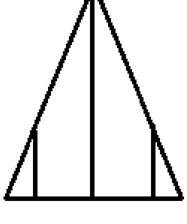
a)



b)

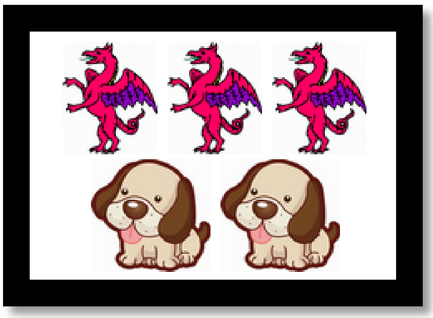


c)



**3)** Elena has 6 toy cars and Andre has 4 toy cards. What fraction of the toy cards does Andre have?

**4)** What fraction of the animals are dragons?



**5)** The friends below are playing a game in which the person who runs to the furthest distance wins. The fractions tell how much of the distance they have already moved. Place these friends on a line according to their distance to starting point from closest to furthest.

Mary: $\frac{9}{4}$ Harry: $\frac{9}{8}$ Larry: $\frac{9}{15}$ Han: $\frac{9}{3}$

**6)** Order the fractions you see below from the least to the greatest by using the < sign:

$$\frac{3}{5}\ ,\ \frac{3}{2}\ ,\ \frac{3}{9}\ ,\ \frac{3}{7}$$

**7)** Compare each pair of fractions placing < or > signs in the boxes.

$\frac{3}{6}\ \square\ \frac{3}{2}$ $\frac{6}{12}\ \square\ \frac{6}{2}$ $\frac{8}{11}\ \square\ \frac{8}{5}$

$\frac{12}{3}\ \square\ \frac{12}{7}$ $\frac{20}{3}\ \square\ \frac{20}{40}$

**8)** Jenny baked a pizza, she divided it into 8 equal slices and ate 3 of them. Kevin baked a pizza in the same size, but he divided his pizza into 4 equal slices and ate 3 of them. For both pizzas separately, express what fractions of pizza Jenny and Kevin ate and find who ate more pizza.

**9)** Kim made two pies that were exactly the same size. The first pie was a cherry pie which she cut into 9 equal slices. The second was an apple pie, which she cut into 12 equal slices. Kim takes her pies to a party and people ate 7 slices of both the cherry pie and apple pie. For both cakes separately, express what fraction of cakes people ate and find which pie people preferred more.

**10)** Find the result of each.

a) $\frac{3}{6}+\frac{2}{6}=$

b) $\frac{4}{10}+\frac{6}{10}=$

c) $\frac{1}{7}+\frac{1}{7}=$

d) $\frac{1}{8}+\frac{4}{8}=$

e) $\frac{2}{4}+\frac{2}{4}=$

f) $\frac{3}{5}+\frac{4}{5}=$

**11)** Ms. Rodriguez baked a cake for the sale and cut the cake into 8 equal-sized slices. In the morning, she sold 3 of the slices; in the afternoon, she sold 2 slices. <u>What fraction</u> of the brownies did she sell?

**12)** Jennifer practices $\frac{12}{5}$ hour of guitar on Wednesday and $\frac{3}{5}$ hour of guitar on Thursday. How many hours at total did she practice the guitar?

**13)** Sharon has $\frac{1}{4}$ kilograms of eggplants and $\frac{7}{4}$ kilograms of tomatoes. How many kilograms of vegetables does she have?

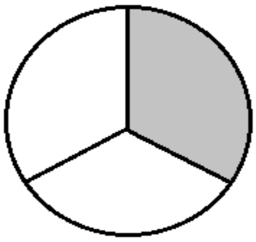
# Appendix B- Instrument (Turkish)

**Kesirler testi**

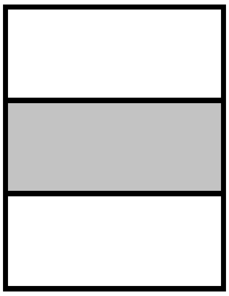
**1)** Aşağıdaki şekillerin hangilerinin $\frac{1}{3}$'ü doğru şekilde boyanmıştır? <u>Birden fazla seçeneği işaretleyebilirsiniz.</u>

a)

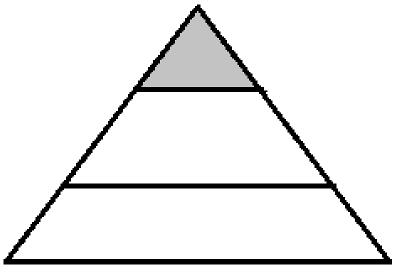
b)



c)

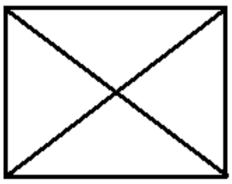


d)

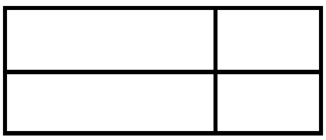


**2)** Aşağıdaki şekillerden hangileri 4 parçaya doğru biçimde ayrılmıştır? <u>Birden fazla seçeneği işaretleyebilirsiniz.</u>

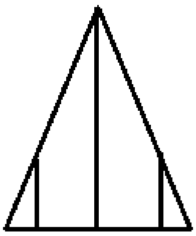
a)



b)

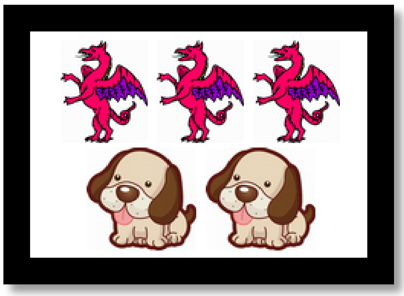


c)



**3)** Elif'in 6 oyuncak arabası, Ahmet'in ise 4 oyuncak arabası vardır. Ahmet'in oyuncak arabaları tüm arabaların kaçta kaçıdır?

**4)** Yandaki şekilde hayvanların kaçta kaçı ejderhadır?



**5)** Aşağıda isimleri belirtilen kişiler daha uzun mesafe koşanın kazandığı bir oyun oynuyorlar. Aşağıdaki kesirler hangisinin ne kadar yol aldığını ifade ediyor. Bu kişileri çizeceğiniz bir çizginin üzerinde başlangıç noktasına yakın olandan uzak olana doğru sıralayınız.

Ali: $\frac{9}{4}$ Ebru: $\frac{9}{8}$ Funda : $\frac{9}{15}$ Okan: $\frac{9}{3}$

**6)** Aşağıdaki kesirleri küçükten büyüğe doğru < sembolünü kullanarak sıralayınız:

$$\frac{3}{5}\ ,\ \frac{3}{2}\ ,\ \frac{3}{9}\ ,\ \frac{3}{7}$$

**7)** Aşağıda verilen kesirleri karşılaştırarak boş kutulara < veya > sembollerinden uygun olanı yerleştiriniz.

$$\frac{3}{6}\ \square\ \frac{3}{2} \qquad \frac{6}{12}\ \square\ \frac{6}{2} \qquad \frac{8}{11}\ \square\ \frac{8}{5}$$

$$\frac{12}{3}\ \square\ \frac{12}{7} \qquad \frac{20}{3}\ \square\ \frac{20}{40}$$

**8)** Aylin pişirdiği pizzayı 8 eşit dilime ayırır ve 3 dilimini yer. Mehmet ise aynı büyüklükte pişirdiği pizzayı 4 eşit dilime ayırır ve 3 dilimini yer. Aylin ve Mehmet'in pizzalarının kaçta kaçını yediklerini iki pizza için ayrı ayrı belirttiniz ve kimin daha çok yediğini bulunuz.

**9)** Gökçe eşit boyutlarda iki kek pişirdi. Portakallı keki 9 eşit dilime, üzümlü keki ise 12 eşit dilime ayırdı. Gökçe pişirdiği bu kekleri arkadaşının doğum günü partisine götürdü ve arkadaşları her iki kekten de 7şer dilim yediler. Keklerin ikisi için ayrı ayrı kaçta kaçının yendiğini belirtiniz ve Gökçe'nin arkadaşlarının hangi kekten daha çok yediğini bulunuz.

**10)** Aşağıdaki işlemlerin sonuçlarını bulunuz.

a) $\frac{3}{6}+\frac{2}{6}=$

b) $\frac{4}{10}+\frac{6}{10}=$

c) $\frac{1}{7}+\frac{1}{7}=$

d) $\frac{1}{8}+\frac{4}{8}=$

e) $\frac{2}{4}+\frac{2}{4}=$

f) $\frac{3}{5}+\frac{4}{5}=$

**11)** Anıl'ın annesi kermeste satmak üzere bir kek pişirip, keki 8 eşit dilime ayırmıştır. Sabah, kekin 3 dilimini, öğlen ise 2 dilimini satmıştır. Anıl'ın annesi günün sonunda kekin <u>kaçta kaçını</u> satmıştır?

**12)** Gülşah, Çarşamba günü $\frac{12}{5}$ saat gitar çalmış, Perşembe günü ise $\frac{3}{5}$ saat gitar çalmıştır. Gülşah iki günde kaç saat gitar çalmıştır?

**13)** Serpil, pazardan $\frac{1}{4}$ kilogram patlıcan, $\frac{7}{4}$ kilogram domates almıştır. Serpil'in kaç kilogram sebzesi vardır?

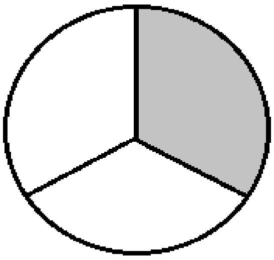
## Appendix C- Instrument with the actual item numbers

**Fractions test**

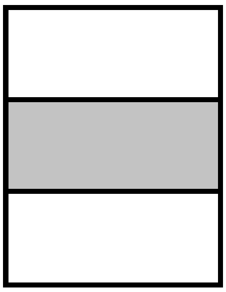
Which of the figures below show the fraction $\frac{1}{3}$ correctly? <u>You can mark more than one alternative.</u>

**1)**

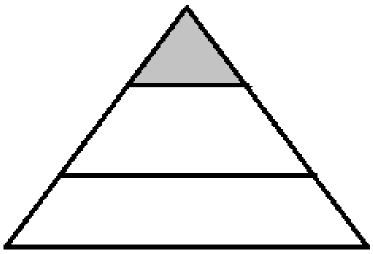
**2)**



**3)**

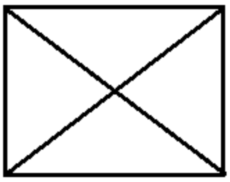


**4)**



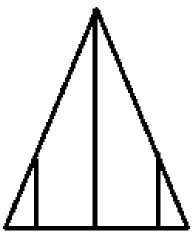
Which of the figures below are correctly partitioned into fourths? <u>You can mark more than one alternative.</u>

**5)**



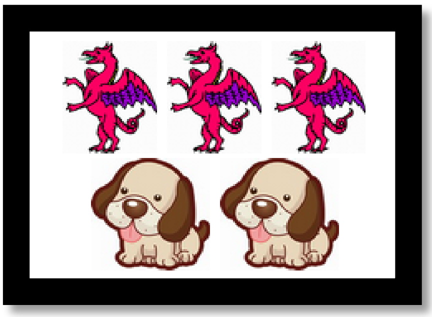
**6)**

**7)**



**8)** Elena has 6 toy cars and Andre has 4 toy cards. What fraction of the toy cards does Andre have?

**9)** What fraction of the animals are dragons?



**10)** The friends below are playing a game in which the person who runs to the furthest distance wins. The fractions tell how much of the distance they have already moved. Place these friends on a line according to their distance to starting point from closest to furthest.

Mary: $\frac{9}{4}$ Harry: $\frac{9}{8}$ Larry: $\frac{9}{15}$ Han: $\frac{9}{3}$

**11)** Order the fractions you see below from the least to the greatest by using the < sign:

$$\frac{3}{5}\ ,\ \frac{3}{2}\ ,\ \frac{3}{9}\ ,\ \frac{3}{7}$$

Compare each pair of fractions placing < or > signs in the boxes.

**12)** $\frac{3}{6}$ ☐ $\frac{3}{2}$

**13)** $\frac{6}{12}$ ☐ $\frac{6}{2}$

**14)** $\frac{8}{11}$ ☐ $\frac{8}{5}$

**15)** $\frac{12}{3}$ ☐ $\frac{12}{7}$

**16)** $\frac{20}{3}$ ☐ $\frac{20}{40}$

**17)** Jenny baked a pizza, she divided it into 8 equal slices and ate 3 of them. Kevin baked a pizza in the same size, but he divided his pizza into 4 equal slices and ate 3 of them. For both pizzas separately, express what fractions of pizza Jenny and Kevin ate and find who ate more pizza.

**18)** Kim made two pies that were exactly the same size. The first pie was a cherry pie which she cut into 9 equal slices. The second was an apple pie, which she cut into 12 equal slices. Kim takes her pies to a party and people ate 7 slices of both the cherry pie and apple pie. For <u>both</u> cakes separately, express <u>what fraction</u> of cakes people ate and find which pie people <u>preferred more</u>.

Find the result of each.

**19)** $\frac{3}{6}+\frac{2}{6}=$

**20)** $\frac{4}{10}+\frac{6}{10}=$

**21)** $\frac{1}{7}+\frac{1}{7}=$

**22)** $\frac{1}{8}+\frac{4}{8}=$

**23)** $\frac{2}{4}+\frac{2}{4}=$

**24)** $\frac{3}{5}+\frac{4}{5}=$

**25)** Ms. Rodriguez baked a cake for the sale and cut the cake into 8 equal-sized slices. In the morning, she sold 3 of the slices; in the afternoon, she sold 2 slices. <u>What fraction</u> of the brownies did she sell?

**26)** Jennifer practices $\frac{12}{5}$ hour of guitar on Wednesday and $\frac{3}{5}$ hour of guitar on Thursday. How many hours at total did she practice the guitar?

**27)** Sharon has $\frac{1}{4}$ kilograms of eggplants and $\frac{7}{4}$ kilograms of tomatoes. How many kilograms of vegetables does she have?

## Appendix D-Wrong answers classifying table

### Expected misconceptions and the wrong answers that are not considered as misconception*

| Question Number | Expected Misconceptions | Some examples of other errors and mistakes that are not misc. |
|---|---|---|
| 8 | $\frac{4}{6}, \frac{6}{4}, \frac{2}{3}, \frac{3}{2}$ | • $\frac{6}{10}, \frac{2}{6}, \frac{3}{5}, \frac{4}{4}, \frac{1}{4}, \frac{4}{10}, \frac{2}{10}, \frac{10}{10}$<br>• $6 \times 4 = 24,$<br>• $6 + 4 = 10 \;\; 10 \div 2 = 5$ |
| 9 | $\frac{2}{3}, \frac{3}{2}$ | • $\frac{5}{3}, \frac{2}{5}, \frac{3}{3}, \frac{1}{3}$<br>• 3 of them<br>• 2 dogs and 3 dragons |
| 10 | • $\frac{9}{15} > \frac{9}{8} > \frac{9}{4} > \frac{9}{3}$<br>• $\frac{9}{15}, \frac{9}{8}, \frac{9}{4}, \frac{9}{3}$<br>• Han, Mary, Harry, Larry<br>• Larry>Harry>Mary>Han<br>• Start — Han, Mary, Harry, Larry — Finish<br>• The closest is Han. Then Mary and Harry come. The furthest one is Larry. | • $\frac{9}{4} > \frac{9}{8} > \frac{9}{15} > \frac{9}{3}$<br>• $\frac{9}{8} > \frac{9}{4} > \frac{9}{3} > \frac{9}{15}$<br>• Start — Mary, Han, Harry, Larry — Finish<br>• Harry>Marry>Han>Larry<br>• Larry> Harry>Han>Marry<br>• Han, Harry, Larry, Marry |
| 11 | $\frac{3}{9} > \frac{3}{7} > \frac{3}{5} > \frac{3}{2}$ | • $\frac{3}{5} > \frac{3}{7} > \frac{3}{9} > \frac{3}{2}$<br>• $\frac{3}{2} > \frac{3}{9} > \frac{3}{5} > \frac{3}{7}$<br>• $\frac{3}{7} > \frac{3}{9} > \frac{3}{2} > \frac{3}{5}$<br>• $\frac{3}{5} > \frac{3}{2} > \frac{3}{9} > \frac{3}{7}$ |
| 12 | $\frac{3}{6} > \frac{3}{2}$ | |

| | | |
|---|---|---|
| 13 | $\frac{6}{12} > \frac{6}{2}$ | |
| 14 | $\frac{8}{11} > \frac{8}{5}$ | |
| 15 | $\frac{12}{7} > \frac{12}{3}$ | |
| 16 | $\frac{20}{40} > \frac{20}{3}$ | |
| 17 | • Jenny: $\frac{3}{8}$ , Kevin: $\frac{3}{4}$<br>Jenny ate more pizza<br><br>• Jenny: $\frac{3}{8}$ , Kevin: $\frac{3}{4}$<br>$\frac{3}{8} > \frac{3}{4}$ | • Kevin ate more.<br>• 8 - 3 = 5 , 4 - 3 = 1. So, Jenny are more.<br>• 8 ÷ 3 = 2, 4 ÷ 3 = 1. So, Jenny ate more.<br>• Jenny: $\frac{3}{8}$ , Kevin: $\frac{4}{8}$. So, Kevin ate more pizza.<br>• They ate equal amounts.<br>• Jenny: $\frac{4}{4}$ , Kevin: $\frac{1}{4}$. So, Kevin ate more pizza.<br>• $\frac{3}{8} + \frac{3}{4} = \frac{6}{12}$ |
| 18 | Cherry pie: $\frac{7}{9}$ , apple pie: $\frac{7}{12}$<br>They preferred apple pie more. | • 9 – 7 = 2, 12 -7 = 5. Cherry pie was preferred more.<br>• They ate apple pie more.<br>• Both pies were preferred equally.<br>• 9 – 7 = 2, 12 -7 = 5. Apple pie was preferred more.<br>• 12 × 7 = 84<br>• 9 – 7 = 2, 12 – 7 = 5, $\frac{5}{8}$. So, they ate apple pie more.<br>• 9 ÷ 2 = 4, 12 ÷ 4 = 3, 4 + 3 = 7. Cherry pie was preferred more. |
| 19 | $\frac{3}{6}+\frac{2}{6}=\frac{5}{12}$ | • $\frac{3}{6}+\frac{2}{6}=\frac{1}{6}$<br>• $\frac{3}{6}+\frac{2}{6}=\frac{6}{6}$ |
| 20 | $\frac{4}{10}+\frac{6}{10}=\frac{10}{20}$ | • $\frac{4}{10}+\frac{6}{10}=\frac{8}{10}$<br>• $\frac{4}{10}+\frac{6}{10}=\frac{9}{10}$ |

| | | |
|---|---|---|
| 21 | $\frac{1}{7}+\frac{1}{7}=\frac{2}{14}$ | • $\frac{1}{7}+\frac{1}{7}=\frac{1}{7}$ |
| 22 | $\frac{1}{8}+\frac{4}{8}=\frac{5}{16}$ | • $\frac{1}{8}+\frac{4}{8}=\frac{3}{8}$<br>• $\frac{1}{8}+\frac{4}{8}=\frac{6}{8}$ |
| 23 | $\frac{2}{4}+\frac{2}{4}=\frac{4}{8}$ | • $\frac{2}{4}+\frac{2}{4}=\frac{4}{6}$ |
| 24 | $\frac{3}{5}+\frac{4}{5}=\frac{7}{10}$ | • $\frac{3}{5}+\frac{4}{5}=\frac{9}{5}$<br>• $\frac{3}{5}+\frac{4}{5}=\frac{8}{5}$<br>• $\frac{3}{5}+\frac{4}{5}=1\frac{1}{5}$ |
| 25 | Morning: $\frac{3}{8}$, Afternoon: $\frac{2}{8}$<br>At the end of the day,<br>at total: $\frac{3}{8}+\frac{2}{8}=\frac{5}{16}$ sold. | • 3 + 2 = 5, $\frac{8}{8}+\frac{5}{8}=\frac{3}{8}$<br>• 3 + 2 = 5, she sold $\frac{8}{5}$ of cake.<br>• $\frac{3}{8}-\frac{2}{8}=\frac{1}{8}$ is left<br>• 8 – 5 = 3 slices were sold.<br>• 8 – 3 = 5, 5 – 3 = 2<br>• 3 + 2 = 5, 8 – 5 = 3, she sold $\frac{3}{8}$ of the cake.<br>• 8× 3 = 24, 2÷4 2 = 12 slices were sold. |
| 26 | Wednesday: $\frac{12}{5}$, Thursday: $\frac{3}{5}$<br>At total she practiced:<br>$\frac{12}{5}+\frac{3}{5}=\frac{15}{10}$ hours. | • $\frac{12}{5}-\frac{3}{5}=\frac{9}{5}$ total hours.<br>• 12 + 3 = 15, 15 + 10 = 25<br>• 24 ÷12 = 2, 2 × 5 = 10, 24 ÷3 = 8, 8 × 5 = 40, 40 + 10 = 50 hours practice. |
| 27 | Eggplants: $\frac{1}{4}$, tomato: $\frac{7}{4}$<br>At total $\frac{1}{4}+\frac{7}{4}=\frac{8}{8}$ kg vegetable | • $\frac{7}{4}-\frac{1}{4}=\frac{6}{4}$ kg<br>• $\frac{7}{4}-\frac{4}{4}=\frac{11}{4}$ kg<br>• 100 ÷4 = 25, 25 × 7 = 175, 175 + 25 = 200 kg |

* Misconceptions and errors were identified after PYP and MoNE students' papers were investigated. The related literature about fraction misconception guided this research to distinguish a misconception from other types of errors.

Questions 1 to 7 were not included in the above table. The students who marked the shapes in questions 1, 4, 6 and 7 were directly considered as they had partitioning misconceptions. On the other hand, marking the shapes in questions 2, 3 and 5 was assumed correct answer. So, for these 7 items, there were no answers that could be considered as wrong without misconceptions. Due to this, there was no need to include them in the table.