EZGİ ÇALLI

A QUANTITATIVE INVESTIGATION OF MATHEMATICAL KNOWLEDGE FOR TEACHING AND SELF-EFFICACY: MIDDLE SCHOOL MATHEMATICS TEACHERS IN TURKEY

A MASTER'S THESIS

BY

EZGİ ÇALLI

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Ezgi Çallı

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Ezgi Çallı

May 2015

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Curriculum and Instruction.

Assoc. Prof. Dr. M. Sencer Çorlu

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Curriculum and Instruction.

Asst. Prof. Dr. Jennie F. Lane

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Curriculum and Instruction.

Asst. Prof. Dr. R. Didem Taylan

Approval of the Graduate School of Education

Director: Prof. Dr. Margaret K. Sands

ABSTRACT

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Ezgi Çallı

M.A., Program of Curriculum and Instruction

Supervisor: Assoc. Prof. Dr. M. Sencer Çorlu

May 2015

The purpose of this study was to investigate the mathematical knowledge for teaching and self-efficacy levels of middle school mathematics teachers in Turkey. The sample consisted of 42 teachers, working at 15 randomly-selected schools in the Çankaya district of Ankara. Data were collected by using the number concepts and operations scale and the mathematics teaching efficacy beliefs instrument. A multivariate analysis of variance was conducted, where the independent variables were age (novice or senior) and certification type (faculty of education or alternatively certified). The analysis revealed that there was 0.84 standard deviations difference between the mean self-efficacy levels of novice and senior teachers. No statistically significant difference was observed between the self-efficacy beliefs and mathematical knowledge of teachers with respect to their types of certification. Results were discussed in terms of subject-specific competencies for teaching, reform-oriented efforts in teacher education and recruitment, and quality of professional development for teachers.

Key words: Mathematical knowledge for teaching, self-efficacy, number concepts and operations, middle school.

ÖZET

ORTAOKUL MATEMATİK ÖĞRETMENLERİNİN MATEMATİK ÖĞRETMENLİK BİLGİSİ VE ÖZ YETERLİK SEVİYELERİ ÜZERİNE NİCEL BİR ÇALIŞMA

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Bu çalışmanın amacı, Türkiye'deki ortaokul matematik öğretmenlerinin matematik öğretmek için gereken alan bilgileri ve öz yeterlik düzeylerini incelemektir. Örneklem, Ankara'nın Çankaya ilçesine bağlı rastgele seçilmiş 15 farklı okulda görev yapan 42 matematik öğretmeninden oluşmaktadır. Veriler, *sayı kavramları ve işlemler bilgisi ölçeği* ve *matematik öğretimi öz yeterlik inancı ölçeği* kullanılarak toplanmıştır. Yaş grubu ve sertifika türü bağımsız değişkenleri ile çok değişkenli varyans analizi gerçekleştirilmiştir. Analiz iki farklı yaş grubundaki öğretmenlerin ortalama öz yeterlik düzeyleri arasında 0.84 standart sapmalık bir fark olduğunu ortaya koymuştur. Öğretmenlerin matematik öğretmek için gereken alan bilgileri ve öz yeterlik düzeyleri arasında sertifika türüne göre istatistiksel olarak anlamlı bir fark bulunmamıştır. Sonuçlar, öğretmenlerin öğrettikleri alana özgü yetkinlikleri, öğretmen eğitimindeki reform odaklı girişimler ve öğretmenlerin aldıkları hizmet içi eğitimlerin kalitesi göz önüne alınarak tartışılmıştır.

Anahtar Kelimeler: Matematik öğretmek için gereken alan bilgisi, öz yeterlik, sayı kavramları ve işlemler, orta okul.

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CHAPTER 1: INTRODUCTION

The study of occupations and professions has been a fundamental issue in sociology for decades. The degree of specialization and complexity of the work is regarded to be the most important characteristic to distinguish a profession from other kinds of works. *Knowledge-based* occupations are generally designated to be professions (Ingersoll & Perda, 2011). Professional work does not only require having a command on complicated or uncommon knowledge but also the mental capacity to put that knowledge into use. Ultimately, this mental capacity to anticipate upcoming problems and the motivation to uncover new solutions determines the impact a worker has in a particular profession. Both the knowledge and this mental readiness required in a profession need to be understood better in order to have people make a transforming impact in their work.

As in all other professions, there are hallmark requirements that apply well to teaching. Teacher quality has generally been attributed to teachers' knowledge and skills in content and pedagogy. Several organizations in Turkey and abroad have produced teaching standards (Darling-Hammond & Youngs, 2002; International Baccalaureate Organization, 2013; Ministry of National Education [MoNE], 2008b; National Council for Accreditation of Teacher Education, 2008; Türk Eğitim Derneği, 2009; UK Department for Education, 2011). These standards were required for the purpose of recruitment of teachers or for accountability purposes. However, more research is necessary in order to understand what teachers know and how they perform using this knowledge in the classroom.

In Turkey, teaching was recognized as an *expertise-requiring profession* by the Law of National Education (Millî Eğitim Temel Kanunu) in 1973*: Öğretmenlik, devletin eğitim, öğretim ve bununla ilgili yönetim görevlerini üzerine alan özel bir ihtisas mesleğidir*. The expected qualifications from teachers are also expressed in writing under the title of *teacher competencies* (öğretmen yeterlikleri) by the Ministry of National Education (MoNE). The rationale behind this document included supporting national education objectives, providing a benchmark framework for the quality of teachers, implementing the principle of transparency, and creating consistency in the social expectations about the status and the reputation of the teaching profession (MoNE, 2008b). Six main competency areas were defined in this document. These areas were: (a) personal and professional values and development; (b) student recognition; (c) teaching and learning process; (d) learning and development monitoring and evaluation; (e) family-school and community relation; (f) program and content knowledge. In addition to the general teacher competencies, subject specific teacher competencies were also prepared in separate reports. Although teaching has been recognized as an expertise-requiring profession, there are criticisms leveled at teacher education for not being based on empirical research. Thus, more research on subject-specific teacher competencies is needed.

One of the competency areas for teachers is *knowledge* and it is central for the teaching profession. Teachers are expected to know the subject-specific specific teaching methods, goals, objectives, principles, and curriculum approach (MoNE, 2008b). However, the relationship between knowledge and the profession of teaching is exclusively complicated (Rowland, 2014). The mainstay of teaching as a profession requires using advanced formal reasoning in practice, achieving high level of professional education, and acquiring knowledge specific to the area (Rowan, 1994). While professionalization seems promising for the teachers about their occupation, it is also a source of concern for accountability. Hence, teachers need to be well-equipped and knowledgeable to claim professional status.

Another competency area is teachers' professional values and development. In this domain, teachers are expected to demonstrate the attitudes and dispositions that they want to improve in their students (MoNE, 2008b). In order to facilitate knowledge construction in the classroom, teachers need to have a strong disposition in the teaching of their subject. A conceptual framework for studying teaching dispositions is *Bandura's self-efficacy theory* (1977, 1997). Teachers' self-efficacy, which is defined as teachers' beliefs in their own abilities to carry out necessary activities to achieve the desired results, has been repeatedly associated with positive student outcomes (Henson, 2001). Self- efficacious teachers demonstrate interest in and passion for their job. They are more likely to use reform-oriented teaching methods and enable students to build knowledge in the subject area. It was indicated by research that teachers' self-assurance in their own capacity as an effective teacher created positive effects on both the students' attitudes (achievement and motivation) as well as in their own practice (job satisfaction and burnout) (Klassen & Chiu, 2010). Thus, teacher qualifications in the affective domain are worth studying to build the standards for the profession.

Background

Turkish Context

The *quantity* problem has always been of the first priority in teacher education and recruitment in Turkey. Uncoordinated policies of MoNE, CoHE and faculties of education have resulted in lack or oversupply of teachers. A striking example to this problem was the acceptance of any four-year program graduates of universities for teaching positions in the public schools all over the country in 1997-1998 academic year (Çakıroğlu & Çakıroğlu, 2003). There are more than 800,000 teachers in formal education as of 2013-2014 academic year (MoNE, 2014). Because of the limited recruitment capacity of MoNE, more than 350,000 qualified teachers are waiting to be employed in public schools (Özoğlu, 2010). However, several universities currently offer "for-profit quick-fix [alternative] teacher certification programs" (Çorlu, Capraro & Capraro, 2014, p. 79).

A *quality* oriented attempt to elevate teacher education occurred through a progressive transformation between 1994 and 1999 by the Council of Higher Education (CoHE). This was part of a larger reform movement that was introduced as the World Bank-funded national education development project (NEDP). The pre-service teacher education section of the reform movement put emphasis on different teaching methods for 13 subject areas including mathematics, enhancement of teaching practices, and educational research (Grossman, Önkol, & Sands, 2007). In terms of containing noteworthy changes regarding the quality problem, this process has been one of the most important and progressive changes in the teacher education history of Turkey. Later, the curriculum of the middle school mathematics teacher education programs was revised in 2006 upon an agreement on the shortcomings of the teacher candidates in terms of subject matter knowledge, pedagogical content knowledge, and contemporary educational needs (Işıksal, Koç, Bulut, & Atay-Turhan, 2007).

Efforts of policy makers to address quality improvement issues have been a topic of debate during the years after 2006. For example, the middle school mathematics curriculum was restructured. This new curriculum has set forth renewed expectations from the teachers. Specific to mathematics education, the mathematics teachers were expected to create classroom environments in which students could express their thoughts and discuss mathematical ideas (MoNE, 2005; MoNE, 2013). Constructivist approaches for instruction and nontraditional authentic assessment methods were introduced (Ayas, Corlu, & Aydın, 2013). In this new environment, it became necessary to define the qualifications that mathematics teachers need to possess in order to implement the reformist curriculum. Thus, the Turkish educational reforms with roots in the teacher education reforms of 1999 have been transferred into school curricula in recent years and are still a topic of debate.

United States

Especially after the *No Child Left Behind* (NCLB) movement in the Unites States, student achievement was taken more seriously as an outcome of education quality. Studies showed that teacher quality has long been known as an important factor in student achievement (Darling-Hammond, 2000; Hill, Rowan, & Ball, 2005). At the elementary level, this relationship between achievement and teacher quality was strongest in school mathematics (River & Sanders, 2002). Teachers were found to have a greater impact than any other school related factor on students' learning (Rivkin, Hanushek, & Kain, 2005).

However, it was difficult to achieve a consensus on identification and measurement of teachers' effectiveness and quality of their teaching. Investigating knowledge as the major asset of mathematics teachers became an issue of debate. Research reviewed by the National Mathematics Advisory Panel (2008) supported the significance of the knowledge of mathematics teachers in increasing student achievement. The knowledge of mathematics teachers, which is difficult to identify and measure, was comprehensively discussed and elaborated by Hill, Schilling and Ball, (2004) and Ball, Thames and Phelps (2008) through the *Learning Mathematics for Teaching* (LMT) project. As a result of the project, one of the most widely accepted scales in the mathematics education community for measuring teacher knowledge has been produced. The constructs of the LMT project involved several aspects of mathematical proficiency (Kilpatrick, Swafford, & Findell, 2001).

Problem

In order to improve the quality of instruction, mathematics teachers should extensively know and understand the mathematics they teach (Hill, Schilling & Ball, 2004). Although there are studies related to understanding the *mathematical knowledge of teachers* in Turkey, they were mostly conducted at the pre-service level (Alpaslan, Işıksal, & Haser, 2014; Baki & Çekmez, 2012; Baki, 2013; Turnuklu & Yeşildere, 2007; Uçar, 2011; Ubuz & Yayan, 2010; Uygan, Tanışlı, & Köse, 2014). The common findings of those studies indicated that pre-service teachers need to improve their mathematics knowledge for teaching.

Turkish teachers have reported strong self-efficacy beliefs in terms of their instructional effectiveness (Dede, 2008; Şahin, Gökkurt, & Soylu, 2014). Having teachers with high self-efficacy levels is a desirable situation (Ashton & Webb, 1986; Tschannen-Moran & McMaster, 2009; Swars, Smith, Smith, & Hart, 2009). However, it is necessary to investigate how those self-reported beliefs overlap with their classroom performance. Moreover, the majority of the research in the teacher self-efficacy domain was conducted with primary school teachers or pre-service teachers (Bursal, 2010; Çakıroğlu, Çakıroglu, & Boone, 2005; Elkatmış, Demirbaş & Ertuğrul, 2013; Yenilmez & Kakmacı, 2008; Umay, 2002, Karakuş & Akbulut, 2010). Thus, there is a need to investigate the self-efficacy beliefs of in-service teachers specific to teaching mathematics.

Middle school mathematics is challenging for many young adolescents and mastering middle school mathematics is also substantial for their high school mathematics achievement (Wang & Goldschmidt, 2003). The area of *number concepts and operations* learning takes up most of the middle school curriculum and it is essential for stimulating an early interest in algebra and further mathematics. Hence, middle school teachers need to be resourceful and efficient while teaching these concepts. Thus, there is a need to investigate the subject-specific self-efficacy beliefs and knowledge of middle school mathematics teachers in Turkey.

Purpose

The primary purpose of this quantitative study was to better understand the subject-specific teacher competencies of the middle school mathematics teachers at both self-efficacy beliefs and mathematical knowledge for teaching in the number concepts and operations subdomain.

Research questions

The first null hypothesis in this study was that there are no differences in the self-efficacy belief and number concepts and operations mean scores of novice and senior

Turkish middle school mathematics teachers. The second null hypothesis was that there are no differences in the self-efficacy belief and number concepts and operations mean scores of faculty of education certified or alternatively certified Turkish middle school mathematics teachers.

Thus, the study addressed the following research questions:

- Is there any statistically significant difference on the average between self-efficacy beliefs and number concepts and operations knowledge of novice and senior Turkish middle school mathematics teachers?
- Is there any statistically significant difference on the average between self-efficacy beliefs and number concepts and operations knowledge of Turkish middle school mathematics teachers with different teaching certification types?
- Are the self-efficacy beliefs and number concepts and operations knowledge of Turkish middle school mathematics teachers on the average affected by the interaction of age groups (novice or senior) and teaching certification types (faculty of education certified or alternatively certified)?

Intellectual merit and broader impact

The study has the potential to provide empirical research evidence on the subject specific knowledge and self-efficacy beliefs of middle school mathematics teachers in Turkey. It also suggests a methodology that can be applied to other topics in middle school mathematics or topics at high school and undergraduate levels. In addition, a similar methodology can be used for research that extends to the other regions of Turkey. The findings have the potential to have a broader impact on the teacher recruitment and teacher performance evaluation system of MoNE with suggestions on subject specific teacher qualifications.

List of abbreviations

CoHE: Council of Higher Education

LMT: Learning Mathematics for Teaching

MKT: Mathematical Knowledge for Teaching

MoNE: Ministry of National Education

MTEBI: Mathematics Teaching Efficacy Beliefs Instrument

NCLB: No Child Left Behind

NCOP: Number Concepts and Operations

NEDP: National Education Development Project

OECD: Organization for Economic Co-operation and Development

PPSE: Public Personnel Selection Examination

TALIS: The OECD Teaching and Learning International Survey

CHAPTER 2: LITERATURE REVIEW

Introduction

The purpose of this chapter is to establish a theoretical framework for the current study. First, a systematic evaluation of the existing theory and research on teacher knowledge, with a special focus on mathematics teachers, is presented. In addition, literature related to mathematics teachers' knowledge specific to the *number concepts and operations* topic was explored. Second, different understandings and research about the self-efficacy construct and teachers' self-efficacy beliefs were analyzed. Finally, a brief overview of the Turkish teacher education and employment system was presented.

Teachers' knowledge

Foundations of teacher knowledge: Lee Shulman's perspective

In his presidential speech in the annual meeting of American Educational Research Association - 1985, Shulman established his understanding of teacher knowledge. In his speech, Shulman emphasized the lack of research and the need for elaboration about the potential role of teachers' subject matter knowledge on their teaching effectiveness (Shulman, 1986). Shulman proposed a frame of reference to explore the nature of teacher knowledge used in the classroom. By doing so, Shulman invited scholars to discuss questions such as:

- What are the sources of teacher knowledge?
- What does a teacher know and when did he or she come to know it?

- How is new knowledge acquired, old knowledge retrieved, and both combined to form a new knowledge base? (p.8).

Shulman's subject-matter knowledge categorization was comprised of three areas:

(1) content knowledge,

(2) pedagogical content knowledge, and

(3) curriculum knowledge.

Content knowledge stood for the scientific ground of the discipline in the teacher's mind along with its reasoning.

Pedagogical content knowledge referred to teacher's understanding of the difficulties and facilities in learning for students and teachers' ability to use appropriate representations. Shulman (1986) further explained pedagogical content knowledge as the distinctive property of a teacher from a scientist or a pedagogue. He viewed pedagogical content knowledge as the necessary knowledge for the successful implementation of activities, such as using proper representations, clarifying concepts, appraising student approach, criticizing textbooks' handling of certain subjects, and strong reasoning.

Curriculum knowledge specifies teachers' awareness about how subjects are aligned in the previous and subsequent school years in the curriculum and how to organize a coherent educational program for students including scope and sequence.

Shulman's framework on teacher knowledge was broadly recognized by other scholars even if there were some opposing views. To give an example to those opposing views, Fenstermacher (1994) argued that it was futile to use the notion of *types* of teacher knowledge, because he believed that the nomenclature in the literature for knowledge types did not necessarily mean different things, but rather referred to the same entity. Nevertheless, Shulman's classification provided an extensively recognized practical description for teacher knowledge. Shulman's inquiry continued to be the frame of reference for most of the studies that came after it (Ball, Thames, & Phelps, 2008; Carpenter et al., 1989; Corlu, 2012; Mishra & Koehler, 2006; Scheerens, 2013). The fields of mathematics education and mathematics teacher education were particularly influenced by Shulman's perspectives.

Building on Shulman: Mathematical knowledge for teaching

Ball, Thames and Phelps (2008) argued that the term *pedagogical content knowledge* of Shulman was immaturely used by many researchers and needed to be more comprehensively developed. Addressing this need in the field, a mathematics education research team developed the approach of *practice-based theory* to conceptualize teachers' subject matter knowledge and pedagogical content knowledge. After analyzing their initial observations in public school elementary mathematics classes, this research group focused on a specific kind of knowledge: mathematical knowledge for teaching (MKT). In order to extend the theory around MKT, the research team developed measures to test and enhance the domains of knowledge that are required in effective mathematics teaching based on Shulman's foundational frame of reference (Petrou, 2007).

Hill, Schilling and Ball (2004) argued that elementary teachers' MKT could be measured through paper-based tests if only all factors of MKT were conceptualized. Their main project for developing such measures was named *Learning Mathematics* *for Teaching* (LMT) (Hill & Ball, 2004). LMT focused on developing measures that represented classroom practices. The research group developed the measures after extensive fieldwork including interviews, observations, and structured tasks. By this way, the measures were intended to subrogate the fieldwork in order to reach a large number of teachers. The project team piloted their instruments with large samples time and again (Petrou, 2007). The validation process further was extended with a comparison of teachers' actual classroom performance to their performance on LMT items, interviews to monitor teachers' cognitive flow, and cross-referencing LMT items with the NCTM and state standards (Hill et al., 2004; Ball et al., 2008). As a result of this extensive effort and meticulous research design, items that span a range of topics, were well-received by the mathematics education research community. Today, LMT items and instruments are regarded as one of the most credible tools to measure mathematics teachers' knowledge. This influential project was funded by the National Science Foundation's Math-Science Partnership program in 2002 and project members continue to develop, test and disseminate the measures.

Ball, Thames and Phelps (2008) specified the required knowledge for teaching mathematics by building their study on Shulman's (1986) identification of teacher knowledge types. They grounded their model on classroom teaching practices that aimed to answer the following questions:

- What are the recurrent tasks and problems of teaching mathematics? What do teachers do as they teach mathematics?
- What mathematical knowledge, skills, and sensibilities are required to manage these tasks? (p.395).

They deconstructed Shulman's interpretation and established new domains around Shulman's three categories which were content knowledge, pedagogical content knowledge and curriculum knowledge.

Ball et al. (2008) segmented the subject matter knowledge into three sections; *common content knowledge*, *specialized content knowledge* and *horizon content knowledge.*

Common content knowledge was the fundamental mathematical knowledge that is required by any schooled person at the workplace or in daily life.

Specialized content knowledge, in contrast, was the kind of knowledge that differentiates teaching as a profession.

The emerging domains under pedagogical content knowledge were given as follows: *knowledge of content and students*, *knowledge of content and teaching* and *knowledge of content and curriculum* (Ball et al., 2008).

As a re-organization in Shulman's (1986) model, curriculum knowledge was dispersed as *horizon content knowledge* (a subcategory under subject matter knowledge) and *knowledge of content and curriculum* (a subcategory under pedagogical content knowledge). See Figure 1 for an overall representation.

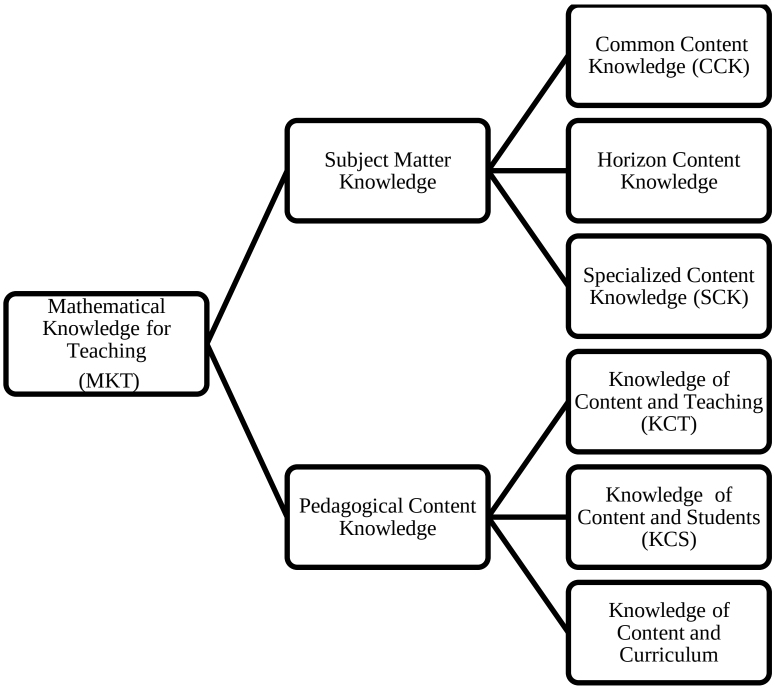

Figure 1. A representation of the MKT domains (based on Ball et al., 2008).

This new classification was found to be effective in terms of three different aspects: (1) understanding if there were components of content knowledge that better relate with student achievement than others; (2) clarifying which aspects of content knowledge were affected by the approaches towards teachers' professional development; (3) simplifying the design of teacher training and professional development activities and resources (Ball et al., 2008). The study offered a systematic way of analyzing mathematics teachers' knowledge for future studies.

An overview of other perspectives

Chapman (2015) summarized the major category-based models that conceptualized mathematical knowledge for teaching. The models in Chapman's category-based perspectives are given in Table 1.

Table 1
Category-based perspectives of mathematics teachers' knowledge*

Ball, Thames, & Phelps, 2008	Rowland, Turner, Thwaites, & Huckstep, 2009	Tatto et al., 2012	Krauss, Baumert, & Blum, 2008
Common content knowledge Specialized content knowledge Horizon content knowledge Knowledge of content and students Knowledge of content and teaching Knowledge of content and curriculum	Foundation knowledge Transformation Connection Contingency	Mathematics content knowledge Mathematics curricular knowledge Knowledge of planning Knowledge for enacting mathematics	Knowledge of mathematical tasks as instructional tools Knowledge and interpretation of students' thinking Knowledge of multiple representations and explanations of mathematical problems

Note. * Chapman (2015, p. 315).

Another model of mathematical knowledge for teaching by Rowland et al. (2009) is noteworthy. This model is based on transcriptions of notes taken during observations and videotape records of elementary pre-service mathematics teachers. They identified four dimensions of mathematics teachers' knowledge (see Table 1) which they named *knowledge quartet*. Although the approach of the researchers was not identical to Ball, Thames, & Phelps (2008), they established their framework by identifying how their model complemented the MKT framework (Speer, King, & Howell, 2014). For the *Teacher Education and Development Student in Mathematics* (TEDS-M) project, Tatto et al. (2012) focused on item development and testing for future primary and secondary mathematics teachers based on the MKT framework. Krauss, Baumert, & Blum (2008) investigated the validity of the content knowledge and pedagogical content knowledge items at the secondary level by implementing the constructs to various populations. The items were constructed as a result of the *Cognitively Activating Instruction* (COACTIV) project. Ultimately, the existing MKT framework provided a rigorous research trajectory for other research groups.

Factors that interact with mathematical knowledge for teaching

Hill, Rowan, and Ball (2005) conducted a project with the participation of classroom teachers and students from 89 different elementary schools in the US. The study was a longitudinal research, where data collection continued for three school years. The major finding of this study was that teachers' mathematical knowledge was an important predictor of student achievement at the primary school level. In addition, the study indicated that teachers' knowledge was more influential in the first grade than it was in the third grade, despite the expected belief that it would be more effective in higher grades. These results provided evidence for the importance of specialized content knowledge of teachers in improving student achievement in mathematics.

Hill (2010) investigated the relationship between elementary teachers' MKT and their educational experiences and found some statistically significant relationships between teachers' MKT and their experiences. The association between the number of content courses taken by a teacher and their MKT scores were negligible. However, when the teachers' mathematical leadership activities increased, their MKT scores were likely to increase. The results of the study revealed that professional development programs for the mathematics teachers should be centered around the specialized content knowledge and pedagogical content knowledge by identifying on which specific practical tasks and topics to focus. Another finding of the study was that mathematical knowledge might become misleading when the teachers themselves reported how knowledgeable they were (Hill, 2010).

While the discussions about mathematics teachers' knowledge were intense in the United States, an independent perspective called *didactique of mathematics* was developed in Europe, mainly in France. Margolinas, Coulange and Bessot (2005) preferred the term *observational didactic knowledge* instead of *pedagogical content knowledge*. According to these scholars, the observational didactic knowledge of mathematics teachers developed the most through recognizing the classroom activities of their students. Teachers' learning occurred when the teacher cautiously interacted with their surroundings. Further development of such knowledge required reflection upon teachers' actions in order to make them aware of their teaching-related biases and problematic aspects of their teaching. Such reflections were not limited to self-reflection, but included an external eye monitoring the classroom activities.

Mathematical knowledge for teaching number concepts and operations

Hill, Schilling, & Ball (2004) suggested that knowledge of teachers should be analyzed specific to mathematical subdomains rather than a single body of cognitive skills. One of those subdomains was *number concepts and operations*. Number concepts and operation was considered an important area because it is one of the fundamental learning areas that should be steadily and strongly developed starting at an early age (Van de Walle, Karp, & Bay-Williams, 2010). It is also important for students to achieve a computational fluency and this would be a foundational skill for their learning in algebra.

Throughout the numbers and operations area (NCTM, 2000), the term *number sense* was emphasized frequently. Howden (1989) explained number sense as follows:

> "…good intuition about numbers and their relationships. It develops gradually as a result of exploring numbers, visualizing them in a variety of contexts, and relating them in ways that are not limited by traditional algorithms" (p. 11).

Number sense is a basis for understanding number systems and operations and computational fluency. Other number concepts such as fractions, integers, decimals, percentages, ratio and proportion are also emphasized gradually throughout the middle school curricula (MoNE, 2013; NCTM, 2000). Baki (2014) advocated that teachers need to give particular importance to place value concepts and never turn the basic operations into algorithmic rules while teaching number concepts and operations. Therefore, teachers need to emphasize computational fluency without sacrificing the conceptual understanding (Corlu, Capraro, & Corlu, 2011).

Howden (1989) advocated that *doing of mathematics* is crucial for developing students' number sense. Doing of mathematics means being engaged in mathematical discussions, sometimes alone and sometimes in groups, rather than merely paper-pencil-textbook oriented or teacher-centered instruction. Therefore, an inquiry based environment and a capable teacher are considered critical in fostering a conceptual understanding of mathematics. Since the numbers and operations constitutes the majority of the mathematics curriculum in the middle school years, mathematics teachers should be knowledgeable and mentally ready to support their students and to encourage students to develop their own methods.

Numbers and operations are introduced as the first of the five main learning areas (numbers and operations, algebra, geometry and measurement, data processing, and probability) in the new 2013 Turkish middle school mathematics curriculum for grades five to eight (MoNE, 2013). Implementation of number concepts and operations in solving real life problems is an expected learning outcome (MoNE, 2013). In addition, number systems and relationships between numbers form the essence of the middle school mathematics curriculum. Therefore, mathematics teachers' knowledge regarding this topic is highly important for them to support their students. Teacher knowledge specifically in this domain has been predominantly investigated within the last two decades (Kim, 2014; Thanheiser, Browning, Edson, Lo, Whitacre, Olanoff, & Morton, 2014).

Studies in the United States

Stiegelmeyer (2012) investigated the numbers and operations knowledge of 82 pre-service elementary teachers. Data of this study also included participants' anxiety levels and completed content courses. The results showed that there was no statistically significant relationship between the number and operations knowledge score of the participants and their level of math-anxiety (r = -.29). The author also suggested that math-anxiety levels of the prospective teachers increased with the increasing number of completed higher level mathematics content courses. The researcher believed that teachers needed to spend more time to understand the fundamentals of mathematics that they would teach, rather than learn excessive amount of mathematics.

Carpenter, Fennema, Peterson, Chiang, and Loef, (1989) studied 40 first-grade teachers' understanding of children's solutions to arithmetic problems, in order to uncover the hidden links between student performance and teachers' knowledge. In this context, teacher knowledge was interpreted as knowledge of common mistakes and patterns in children's thinking and problem solving process. *Writing word problems* and *relative problem difficulty* was used to measure teachers' ability to distinguish between problem types. The teachers were asked to present how students from their class would solve different arithmetic word problems. The teachers were successful in demonstrating their knowledge about problem solving strategies and their distinctions. However, they were not successful in relating children's solutions to problem difficulty. Another result of this study was that teachers' ability to predict students' problem-solving strategies was not correlated with student achievement. In their analysis, Carpenter et al. discussed that teachers make instructional decisions based on their assessment of task difficulty, not based on the problem solving strategies that children use.

Khoury and Zazkis (1994) investigated understanding and problem-solving strategies of pre-service elementary mathematics teachers using different representations of fractions. In the study, responses to two questions were analyzed. First question asked whether 0.2 in base three was equal to 0.2 in base five, which included a similar numeric representation with different fractional values. The second question asked whether *one-half* in base three was equal to *one-half* in base five, which included similar fractional values but left out numeric representation in this case. According to the results, 37 out of 124 participants were unable to perform a correct conversion from the given base to base ten, leaving them unable to explain their answer that 0.2 in base three was not equal to 0.2 in base five. The responses revealed that a large number of the participants' answers were based on the belief that fractions changed their numerical value together with the change of symbolic representation. Strategies used for both correct and incorrect responses hinted that majority of the group had flaws in base conversion and fractional value concepts. The study revealed that pre-service teachers had a tendency to reach the answer by an algorithmic approach of conversion, accompanied by computational skills; however, ignoring the conceptual nature of the questions.

In Thanheiser (2010), analysis of a test administered to 33 pre-service elementary school teachers was presented. This group of pre-service teachers in their 4th year of a 5-year teacher education program in the US. The participants were given questions requiring them to explain the underlying place-value concepts while applying addition and subtraction algorithms. There were two separate tasks. The first one was explaining regrouped digits in a 3-digit addition and the second one was comparing values of digits in addition and subtraction. Only eight pre-service teachers provided the correct answer for the first task, and four pre-service teachers provided correct answer for the second task. Out of the 33 pre-service teachers, only three could provide correct explanations for both tasks. The study revealed that the pre-service teachers who failed at explaining underlying math concepts for operations before taking the content courses (Thanheiser, 2009), also failed after they had taken these content courses. Thanheiser (2010) further underscored that the way pre-service teachers learn to *teach* a subject would influence the way they would teach in the future. Thus, the researcher believed that teacher educators should explore and understand the existing perceptions of their teacher candidates before helping them to develop conceptual knowledge of mathematics.

Studies in Europe and Asia

Studies related to teachers' number concepts and operations knowledge was not limited to the US. In Yang, Reys, & Reys (2009), number sense abilities of pre-service elementary teachers in Taiwan were examined. In this descriptive study, 280 Taiwanese pre-service teachers from six different majors (all of whom took different courses in mathematics) were asked to work on a set of 12 questions. The focus of this exam in the topic of *fractions* was to identify two aspects of pre-service teacher reasoning: (1) using simple benchmarks such as 1, 1/2, 1/4 to work quickly with fractions and (2) using estimation to get a sense of final result. To test these abilities, pre-service teachers were asked to avoid applying an algorithm. Instead, the questions required estimation using different properties of numbers. Goals of these questions included checking the quality of fraction comparison, ability to order fractions in different forms, estimating decimal point of the result of a fractional operation, and estimating larger fraction without knowing its direct form. The results showed that only 20% of pre-service elementary teachers applied number-sense strategies. The rest of the group insisted on using rule-based algorithms. According to Yang et al. (2009), this result clearly indicated that pre-service elementary teachers had poor number sense.

Kaasila, Pehkonen, & Hellinen (2010) performed a qualitative comparison of reasoning strategies between Finnish pre-service elementary teachers and grade 11 upper secondary students to see if pre-service teachers had a deeper understanding of division operation than students. According to this study, division was the most complex operation to learn in elementary school, although it was perceived as an easy task by teachers in general. Kaasila et al. (2010) collected their data using only one question about division:

> Having knowledge of 498:6 = 83, how can one find what 491:6 is without using long-division algorithm?

Results contrasting the group of 269 pre-service elementary teachers to a group of 1,434 upper secondary students revealed that there was no statistically significant difference between their reasoning levels. Only 30% of both pre-service teachers and upper secondary students performed well on this division task.

Tanase (2011) qualitatively investigated four Romanian first grade teachers from two different schools to see how teacher knowledge affected student learning. One of the teachers from each school was a veteran and the others were less-experienced. Despite all four teachers having a good level of place value knowledge, only three of the teachers could see the relation of the concept to subsequent mathematics concepts. Each teacher had different lesson objectives, with only one veteran teacher supplementing the textbook to address the needs of different learning styles. Test scores showed that students performed poorly when the teachers followed the curriculum strictly and did not apply alternative strategies during instruction. Strategies such as distributing remedial worksheets to a group of students while giving additional material for improvement to others proved to work well. The conclusion was that content knowledge was not enough by itself in aiding students to perform better. Thus, understanding students' needs and adjusting materials accordingly was another key aspect in teacher performance. The conclusion was that student learning enhanced when the teacher carefully considered students' needs.

An, Kulm and Wu (2004) conducted a comparative study between Chinese and American middle school mathematics teachers. The authors constructed a network of pedagogical content knowledge with a reference to Shulman (1987). The network included three components, knowledge of content, knowledge of curriculum, and knowledge of teaching (See Figure 2).

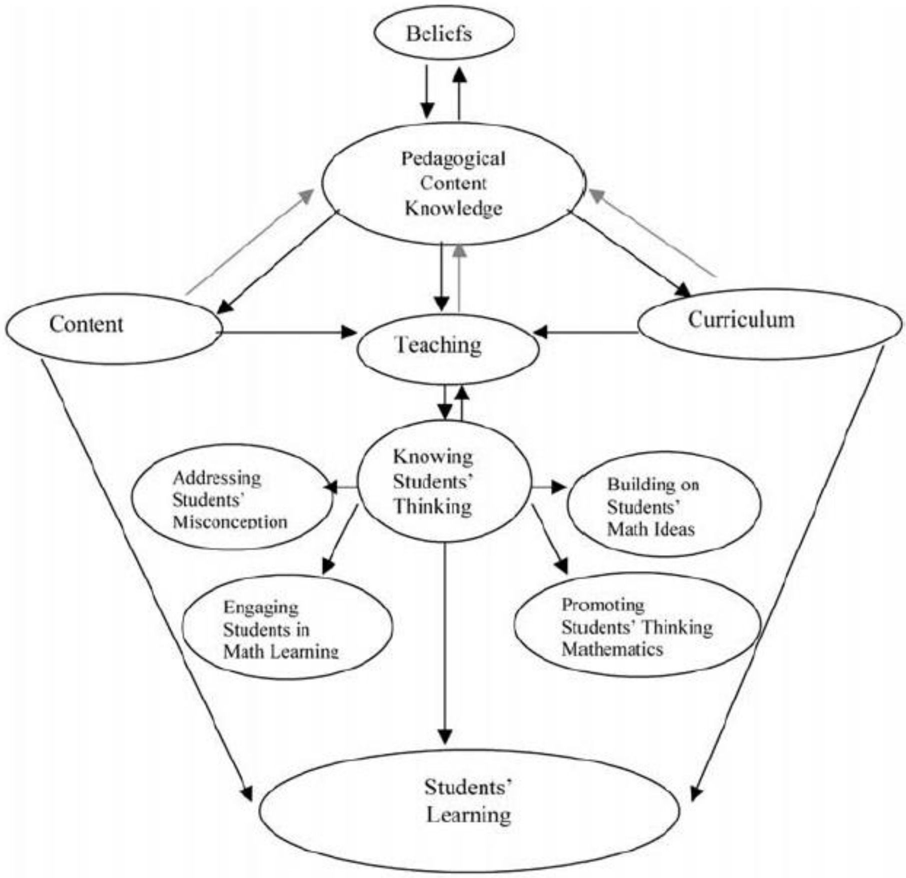

Figure 2. The network of pedagogical content knowledge.
(An, Kulm, &Wu, 2004, p. 147)

Placing the activity of teaching in the center of the network, the knowledge types were defined to be transitive and dynamic rather than stable and unchanging. As a finding of this study, it was stated that there was a remarkable difference in the pedagogical content knowledge of middle school mathematics teachers in both countries. The Chinese teachers based conceptual understanding upon traditional and unchanging procedures, whereas the American teachers based it upon various activities that foster ingenuity, but with a lack of connection between concrete activities and abstract thinking. The pros and cons of both approaches indicated that teachers’ pedagogical content knowledge had diverse requirements that could be systematically developed.

Studies in Turkey

The foremost effort in Turkey in terms of defining the knowledge required for teaching mathematics was presented by Baki (2010; 2014). According to Baki (2010), a teacher should be able to show the truth of a mathematical statement by using the language, applying algorithms, and showing the relationships between different concepts. The teacher should also know under which conditions the concepts, operations and properties are valid. Finally, the teacher should know why the concepts are important and how they are applied within and outside their discipline.

Pusmaz and Küpcü (2010) evaluated the pedagogical content knowledge of five pre-service elementary school teachers and the effect of a 4-hour teaching methods lesson on the weaknesses in their teaching approach. The lesson plans and problem solution techniques of the pre-service teachers were inspected before and after the 4-hour lesson. Before the lesson the teachers had weaknesses in associating the solution technique, process, and the goal of the course. The results were also aligned with Thompson (1992), who claimed that teachers have a tendency to teach in the same style as their own teachers. After the 4-hour lesson, however, the teachers used the knowledge they gathered (method of prime factorization) from the lesson and made an association between the numbers, their prime factors, and the solution process. Thus, teachers could improve their pedagogical content knowledge even after a short period of training.

Ubuz and Yayan (2010) investigated the mathematical knowledge of decimals of 63 primary school teachers from different cities across Turkey. The teachers were the participants of an in-service teacher training program. The study indicated that primary teachers' subject matter knowledge in mathematics needed major improvement. No relationship was found between teacher's years of experience and their subject matter knowledge. Researchers pointed out the responsibility of the teacher education programs in detecting and correcting the misconceptions of the teacher candidates.

Baki (2013) evaluated the quality of instructional explanations of pre-service elementary school teachers on the subject of division in natural numbers. One of the main targets of the study was to explore the conceptual understanding levels of teacher candidates and the difficulties that they experience during their teaching methods courses. The purpose was to improve the quality of the methods courses. The author stated that transforming the topic into an easily perceivable way for the student was located at the core of pedagogical content knowledge. Major indicators of sound pedagogical content knowledge were described as being able to use effective presentations, explanations, representations, illustrations and analogies. On that account, the author aimed to analyze if there existed a connection between pre-service teachers' content knowledge and their instructional explanations about the division algorithm. The results of this research indicated that the division knowledge of most of the participants was procedural rather than conceptual. Their explanations showed that they could not sufficiently internalize the underlying meaning of using *digit tables method* and transform their previous understanding to this new concept. Therefore, the participants' previous knowledge about the topic gave a shape to their instructional explanations.

To summarize, although there is still a lack of agreement in its definition or categorization (Speer, King, & Howel, 2014), mathematical knowledge required for teaching is subject to ongoing investigation in the mathematics education community. Number concepts and operations is a subdomain that is the subject of most research in this area. Predominantly conducted with pre-service teachers, the research indicates that teachers in Turkey and in the world need to develop their conceptual knowledge in number concepts and operations as well as improve their teaching methods; especially reasoning for basic operations, explaining place value concepts, understanding students' problem solving strategies, creating tailor-made and ad-hoc problems and activities to support their students. Because the teachers were influenced by their own educational experiences, the role of teacher education programs in reinforcing mathematical knowledge for teaching is particularly important.

Teachers' self-efficacy

Within the three domains of educational activities: cognitive, affective and psychomotor (Bloom & Krathwohl, 1956; Krathwohl, Bloom, & Masia, 1973), the affective domain was conceived as an internal representational system of human attitudes, beliefs, emotions and values (DeBellis & Goldin 2006). In the context of teachers, the affective factors have been examined with a special focus on teachers' beliefs, attitudes, and self-efficacy levels (Liljedahl & Oesterle, 2014). Beliefs were conceptualized as the "lenses through which one looks when interpreting the world" (Philipp, 2007, p.258). Attitudes represented the dispositions and manners of a person to react favorably or unfavorably to an entity (Ajzen, 1988). Self-efficacy is a concept that includes not only beliefs and attitudes but also involves emotional factors such as self-confidence or anxiety (Liljedahl & Oesterle, 2014).

Studies on teachers' self-efficacy beliefs were grounded on social cognitive theory (Bandura, 1979) and Albert Bandura's general definition of self-efficacy was:

> "beliefs in one's capabilities to organize and execute the courses of action required to produce given attainments" (Bandura, 1997, p. 3).

In other words, self-efficacy is not a function of a person's skills or adequateness, but the result of his or her judgments about what he or she can achieve. Although there is still no consensus on the measurement aspect, self-efficacy stands out as an important variable in educational research. Because, teachers' self-efficacy beliefs were found to be associated with positive teacher and student outcomes consistently in studies with various populations (Henson, 2001).

Researchers have found several correlates of teacher efficacy by using a variety of efficacy constructs. Riggs and Enochs (1989) suggested that elementary teachers' teaching efficacy beliefs have an influence on their classroom practices. Teacher efficacy beliefs were found to have an influence on their students' own self-efficacy beliefs (Tschannen-Moran & McMaster, 2009) and students' achievement (Ashton & Webb, 1986). Swars et al. (2009) indicated that teachers' efficacy beliefs were associated with the teachers' teaching approach. In addition, Bandura (1986) argued that teacher efficacy was specific to the subject taught and to the situation.

Studies about the self-efficacy levels of teachers in Turkey were conducted mostly with prospective teachers. A study conducted with pre-service mathematics teachers indicated that senior pre-service teachers had the highest self-efficacy scores in all the four grade levels (Çakıroğlu & Işıksal, 2009). A low level of positive correlation ($r = .11$) was observed between the self-efficacy beliefs and academic achievement at the university courses (GPA) of Turkish pre-service mathematics and science teachers (Azar, 2010). Gender effect was not found to be significant for pre-service primary teachers' self-efficacy scores in teaching science and mathematics (Bursal, 2009).

Turkish teachers were not more or less self-efficacious than other teachers in the world. Self-efficacy levels of in-service teachers in Turkey were found to be at a similar degree with the OECD average (OECD, 2009, p.112). Considering the subject specific nature of self-efficacy, Corlu, Erdogan, & Sahin (2011) analyzed the Teaching and Learning International Survey (TALIS) data, which was a country level representative educational data. The researchers drew a similar conclusion for the self-efficacy beliefs of mathematics teachers in Turkey. Dede (2008) found no statistically significant difference between the self-efficacy levels of in-service mathematics teachers in middle school and in high school.

Brief history of Turkish teacher education and employment system

Turkey has a long history of teacher education starting from the boys' teacher school *Darülmuallimin*, founded in 1848. Since the establishment of the Republic of Turkey in 1923, different policies have been implemented related to teacher education. The *Law of Unification of Education* gave the authority of all kinds of schools and teacher education programs to a single institution: Ministry of National Education (Gürşimşek, Kaptan & Erkan, 1997). A search for a new model started in 1920s and continued until the end of 1930s. As a continuation of the various trials in 1930s, *village institutes* were established to educate teachers in order to increase the literacy rates in rural Turkey. The institutes were integrated in the community life and offered practical skills for the teachers as well as academic skills. They were discontinued based upon the changes in the political conjuncture in 1954 (Çakıroğlu, & Çakıroğlu, 2003).

Until 1973, *teacher schools* provided secondary level education and *institutes of education* were providing two or three years of teacher education after secondary school. With the declaration of *Basic Law of National Education (Milli Eğitim Temel Kanunu) in* 1973, higher education became compulsory for all teachers. Faculties of education were founded at the universities and the responsibility of teacher education was conferred to the faculties of education by law in 1982. In 1989, completion of four years of an undergraduate program in a faculty of education became mandatory for all teachers along with the decision taken by the Council of Higher Education (CoHE) (Gürşimşek, Kaptan, & Erkan, 1997).

After the standards about the duration of teacher education were regulated, the policy focus was on transforming the quality of subject specific teaching during 1990s. Since specialized teacher education for middle schools was neglected for a long time, the shortage and hence the demand for middle school teachers were compensated by the teachers specialized for high school branches or faculty of arts and sciences graduates. Lack of teacher education for middle school teachers was also one of the reasons behind the reforms and the reorganization of the faculties of education in 1997 (Dursunoğlu, 2003). With the implementation of eight year compulsory education, a distinction between the middle school and high school teaching was specified. Middle school teacher education programs were modified in 2006 in order to produce better equipped graduates (Işıksal, Koç, Bulut, & Atay-Turhan, 2007). The modifications included integrating instructional technology courses and liberal education courses such as history and philosophy of mathematics, emphasizing problem solving and project based learning, putting an end to minor branch implementation (Council of Higher Education [CoHE], 2007)

Turkish education and teacher education systems are still exposed to rapid changes. CoHE declared that no prospective student quota was going to be given to the teacher education departments in the faculties of education effective from the 2013-2014 academic year. The justification behind this decision was that the supply of faculty of education graduates caused a crisis in recruitment and appointments of faulty of arts and sciences graduates. However, the decision was withdrawn as of 2014-2015 academic year as a result of the opposite reactions (Fen Eğitimi ve Araştırmaları Derneği [FEAD], 2012; Fen, Teknoloji, Mühendislik ve Matematik [FeTeMM] Eğitimi Çalışma Grubu, 2013; Matematik Eğitimi Derneği [MED], 2013; Middle East Technical University Faculty of Education, 2013).

Currently, there are two alternative strands for obtaining the qualified school mathematics teacher status in Turkey. The mainstream is a four-year teacher education program offered by faculties of education. The second alternative is the pedagogical formation programs offered for graduates of faculties of arts and sciences. Since 2002, the qualified teachers take a central exam: Public Personnel Selection Examination (PPSE) in order to be appointed to public schools. In addition to general knowledge and pedagogy knowledge, subject-specific content knowledge, and subject-specific pedagogical-content knowledge were integrated into PPSE effective from 2013 (Çatma & Corlu, 2015).

CHAPTER 3: METHOD

Introduction

This study investigates the subject specific teacher competencies of the middle school mathematics teachers for both self-efficacy and mathematical knowledge for teaching in the number concepts and operations subdomain. This chapter describes the research design, pilot study, sampling and participants, instrumentation, data collection and data analysis.

Research design

In the current study, quantitative research methods were used to explore the relationships among the variables. In non-experimental quantitative research, the researchers aim to quantify participant responses and interpret them without influencing the outcome (Arghode, 2012). The framework of the general steps in designing this study are based on Martella, Nelson, Morgan & Marchand-Martella (2013) and given in Figure 3.

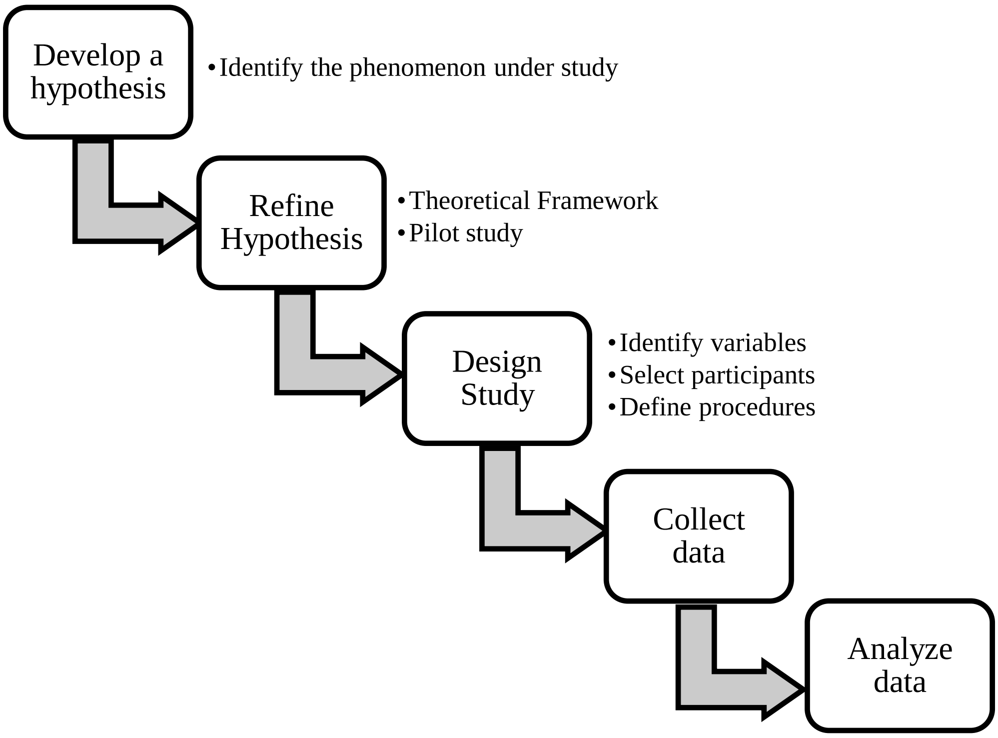

Figure 3. Procedures for designing the current study (adapted from Martella et al., 2013).

The hypothesis testing procedure in this study was carried out based on the 9-step version of hypothesis testing (Huck, 2011):

(1) State the null hypothesis (H_0),

(2) State the alternative hypothesis (H_a),

(3) Specify the desired level of significance (α),

(4) Specify the minimally important effect size,

(5) Specify the desired level of power,

(6) Determine the proper size of the sample,

(7) Collect and analyze the sample data,

(8) Refer to a criterion for assessing the sample evidence,

(9) Make a decision to discard or retain H_0 (p. 165).

Pilot study

A pilot study was conducted with teachers to increase the feasibility of the research. There were several motives for conducting a pilot study: (a) to check the wording of the items and instructions; (b) to get feedback about the type of questions and the format; (c) to monitor the time taken to complete; (d) to regulate the survey logistics (Cohen, Manion, & Morrison, 2005).

The pilot study used some of the items from the *Number Concepts and Operations Scale* (NCOP) and the Mathematics Teaching Efficacy Beliefs Instrument (MTEBI). Data collection for this pilot study was carried out online through Google® Forms—a free tool to collect data from the users and save the responses in a spreadsheet. As a result of the feedback from the pilot study, minor modifications were done in the translations from English to Turkish and the wordings of the items. The appropriateness of the instruments was affirmed and the length of the time to complete was estimated with the help of the pilot study.

For the pilot study, 19 participants were invited. In the first week, only three of the participants responded. After a reminder e-mail, four more participants responded. At the end of several reminder e-mails, phone calls and face-to-face reminders, the pilot study was finalized with 11 participants. The response rate in the pilot study and the long duration brought the researcher to the conclusion on administrating the actual survey on a face-to-face basis.

One of the most frequent motives to conduct a pilot study is to estimate the sample size (Cohen, Manion, & Morrison, 2005). This estimation is done by a procedure called a priori power analysis (Cohen, 1988). A priori power analysis is a useful way of examining the statistical power before the actual study is conducted (Faul, Erdfelder, & Buchner, 2007). In this analysis, the sample size is estimated as a function of the required level of power (1-β), previously determined level of significance (α), and the population effect size. However, it was not possible to find a similar study reporting the effect size in the Turkish context. For this reason, a power analysis was conducted by using a conventionally large effect size (Cohen's *d* = 0.75 standard deviations).

A power analysis software called G*Power 3 (Faul, Erdfelder, & Buchner, 2007) was used to estimate the minimum sample size required for the study. For the analysis, Cohen's *d* value was converted into Cohen's f^2 as 0.14 according to the conversion formulas given by Cohen (1988) as cited in DeCoster (2009). Thereby, the required sample size was found to be 69 in order to be 95% sure (α = 0.05) that there would be a statistically significant difference in our model.

Participants

This study was conducted with mathematics teachers working at different public middle schools in the Çankaya district of Ankara, Turkey. The school list was acquired from the official Ministry of National Education (MoNE) database. In total, 15 middle schools were evaluated as adequate to provide the researcher with the minimum number of teachers required. Therefore, the schools were randomly selected among the 51 public middle schools in the district by the help of random number generator software.

Within the designated 15 middle schools, there were 78 mathematics teachers in total. This number was slightly higher than the estimated sample size in the power analysis. All mathematics teachers at each school were invited to participate in the study on a voluntary basis. When the administration of the instruments was finalized, the response rate turned out to be 53.84% of the 78 teachers.

The participants in this study (*N* = 42) included 14 male and 28 female teachers. It was seen that the sample overwhelmingly consisted of female teachers which almost exactly corresponds to the MoNE statistics for middle school teachers in the city center of Ankara (MoNE, 2014, p. 40). The participants had 20.45 years of teaching experience on average with a standard deviation of 9.95 years and with a range from 2 years to 35 years. This does not show congruence with the overall mathematics teachers' population in Turkey described as young and early-career professionals (Corlu, Erdogan, & Sahin, 2011). Nevertheless, this can be explained by the unequal distribution of the experienced teachers nationwide since the experienced teachers have a tendency to be appointed to city centers (Özoğlu, 2010). Table 2 gives a comparison of the gender, age, and the advanced degrees of the participating mathematics teachers in addition to the population of Turkish mathematics teachers described in Corlu, Erdogan & Sahin (2011) with respect to TALIS data.

Table 2
Comparison of the values in the current sample with the population of Turkish mathematics teachers

Self-reported values by the teachers	Current sample	Turkey-OECD*
Percentage of female mathematics teachers	67%	45%
Percentage of teachers younger than 40	60%	75%
Percentage of teachers with advanced degrees (M.S. or Ph.D)	0%	6%

Note. OECD values indicated with (*) are used as cited by Corlu, Erdogan, & Sahin, (2011)

The participant teachers earned their bachelor's degrees from either mathematics education or mathematics departments. None of the participants had an advanced degree (M.S. or Ph.D). Table 3 presents the number of teachers with respect to their age groups and the institutions of teaching certification. Exactly half of the teachers had a bachelor's degree in mathematics education and half of them had alternative certification.

Table 3
Age distribution and teaching certification of the participants

	Age > 40	Age ≤ 40	Total
Faculty of Education certification	10	11	21
Alternative certification	7	14	21
Total	17	25	42

Instrumentation

Number concepts and operations knowledge of mathematics teachers

The first dependent variable in this study was the mathematical knowledge required for teaching (MKT) score of the middle school mathematics teachers. The instrument used for measuring the MKT was called the *Number Concepts and Operations (NCOP) Scale*[1]. The scale was developed by Hill, Schilling and Ball (2004) and further developed by Hill (2007) for the *Learning Mathematics for Teaching* (LMT) Project, University of Michigan. The developers of the instrument conducted workshops to elaborate on the use of the instrument. The researcher of the study received a briefing to use the instruments in advance.

 Measures development supported by NSF grants REC-9979873, REC- 0207649, EHR-0233456 & EHR 0335411, and by a subcontract to CPRE on Department of Education (DOE), Office of Educational Research and Improvement (OERI) award #R308A960003.

The researcher completed back to back translation of the items into Turkish for this study. The instrument consisted of 15 question stems with 30 items in total. The items were related to modeling, reasoning, evaluation of alternative methods for teaching and learning and definitions, properties and conjectures related to mathematical facts. The items were in multiple choice style. Table 4 represents a short description of each item in the *number concepts and operations* scale. Keeping respect for the copyright of the developers, the instrument could not be added as an appendix to this thesis. However, a few of the released items can be found in Appendix 1 to help the reader to gain an insight about the instrument.

Table 4
Item descriptions in the number concepts and operations scale

Item	Description
1	Use of base ten blocks for multiplication of decimal numbers
2	Ratio: gear turns
3a*	Properties of rational and irrational numbers: smallest positive rational number
3b*	Properties of rational and irrational numbers: sums and products
3c	Properties of rational and rational numbers: sums
3d	Properties of rational and irrational numbers: the number of rational numbers
4	Reasoning of standard long division algorithm
5	Ratio and proportion – correctness of methods
6*	Subtracting a negative number from a negative number using chips
7*	Explanation of why cross multiplying works
8*	Alternative approach to division of fractions
9*	Explanation of indeterminate fraction
10a	Modeling proportional relations: bacteria culture
10b*	Modeling proportional relations: ticket price
10c*	Modeling proportional relations: arts portfolio
10d	Modeling proportional relations: block steps
10e*	Modeling proportional relations: walking distance
11*	Explanation for understanding even numbers
12	Percentage – correctness of methods

Table 4 (cont'd)
Item descriptions in the number concepts and operations scale

13a*	GCF/LCM: LCM of relatively prime numbers
13b	GCF/LCM: Ordering GCF and both numbers
13c*	GCF/LCM: What happens to GCF/LCM when one number is increased/decreased
13d	GCF/LCM: Multiply both numbers by a constant
13e*	GCF/LCM: Relationship between GCF and LCM
14a*	Reasoning for ratio: density of a lemonade
14b	Reasoning for ratio: density of a lemonade
14c*	Reasoning for ratio: density of a lemonade
15a	Modeling subtraction of fractions
15b	Modeling subtraction of fractions
15c	Modeling subtraction of fractions

Note. * indicates the items that were removed because of the low item-total correlation values.

In the final version, the remaining 15 of the items in the NCOP scale were used in the analysis after investigating the item-total correlations. Responses for each item were coded as 1: True and 0: False with respect to the variable key. The sum of the 1's and 0's gave the *NCOP_total* value for each participant. *NCOP_total* represented the total score of each participant on the NCOP scale. Thus, the range of the *NCOP_total* data was from 0 to 15.

Mathematics teaching efficacy beliefs

The second dependent variable was mathematics teaching efficacy beliefs of the middle school mathematics teachers. An efficacy measure specific to mathematics teachers had evolved from Science Teaching Efficacy Beliefs Instrument (STEBI)

(Enochs & Riggs, 1990) to Mathematics Teaching Efficacy Beliefs Instrument (MTEBI) (Enochs, Smith, & Huinker, 2000) based on Gibson and Dembo's (1984) general teacher efficacy construct. The instrument used for measuring the mathematics teaching efficacy beliefs of middle school mathematics teachers in this study was an adaptation of the MTEBI (Enochs, Smith, & Huinker, 2000), which was translated into Turkish by Bursal (2010).

For the current study, minor modifications were made in order to specifically address the self-efficacy beliefs of middle school mathematics teachers (See Appendix 2 for the final version of the instrument). The instrument includes 13 five-point Likert-type items. (1: strongly disagree, 2: disagree, 3: neutral, 4: agree, 5: strongly agree). Five of the items were positively worded and eight of them were negatively worded. Negatively worded items were recoded before the statistical analysis in Predictive Analysis Software (PASW 22, formerly SPSS). By taking the arithmetic mean of the 13 items for each participant, *S_eff_ave* variable was created. Thus, the range was 1 to 5.

Data collection and variables

The researcher submitted a proposal to the Provincial Directorate for National Education of Ankara (Ankara İl Millî Eğitim Müdürlüğü) in order to get permission to administer the survey at Turkish schools. The written permission was granted on 06.06.2014. The data collection instruments were given on paper during the face-to-face meetings. The participating mathematics teachers were informed about the confidentiality of their personal information and the voluntariness. Some of the mathematics teachers immediately refused to participate. Some others started filling in but once they saw the MKT items, they withdrew immediately. The researcher developed a belief that taking a test made some of the teachers uncomfortable.

In accordance with the research questions, number concepts and operations scale total scores (*NCOP_total*) of the participants and the average mathematics teaching self-efficacy belief values (*S_eff_ave*) were the two dependent variables of the study. Certification (*Certification*) and age group (*Age_group*) were the two independent variables. The variables and corresponding data scales are presented below:

- *NCOP_total*: This variable denoted the total score of the participants in the number concepts and operations scale. It was measured in ratio scale. The possible range of the *NCOP_total* value was from 0 to 15.
- *S_eff_ave*: This variable denoted the average values of the participants in the self-efficacy instrument. It was measured in ratio scale. The score range for the *S_eff_ave* variable was from 1 to 5. Individual score for each participant was calculated by taking the average of responses to 13 items. The possible range of the S_eff_ave value was from 1 to 5.
- *Certification*: The type of institution, from which the participants received their teaching degree, was set in a nominal scale (0: Faculty of education, 1: Alternative certification).
- *Age_group*: The novice and senior teachers were split into two groups with respect to their ages with a cut-off value of 40 years (0: Below 40, 1: Above 40).

Reliability and validity

The reliability of the scores was estimated by using Cronbach's *alpha*—a commonly used method in assessment of internal consistency (Huck, 2011). Cronbach's *alpha* is identical to K-R 20 coefficient for dichotomously scored items. However, it can be used for not only dichotomous items (e.g. 0 for incorrect and 1 for correct) but also for the tests that are scored with three or more alternative options or Likert Type questionnaires (Huck, 2011).

Score reliability for the number concepts and operations knowledge

In the NCOP scale, 15 of the items out of 30, having extremely low corrected item-total correlation values, were removed. For the remaining 15 items, Cronbach's alpha was found to be 0.77. (1) *Corrected item-total correlations* and (2) *Cronbach's alpha if item deleted* values were estimated for the NCOP scale. While the former provided the correlation of each item with the total of all the other items in the same scale, the latter provided what the new Cronbach's alpha value would be when that particular item was deleted (Leech, Barrett, & Morgan, 2008). The corrected item-total correlations and Cronbach's *alpha* when each item was deleted are given in Table 5 for the NCOP scale.

Table 5
Item-total statistics for the remaining 15 items of the NCOP scale

Item	Corrected Item-Total Correlation	Cronbach's *alpha* if Item Deleted
NCOP_1	.41	.76
NCOP_2	.50	.75
NCOP_3_c	.24	.77
NCOP_3_d	.19	.77
NCOP_4	.15	.78
NCOP_5	.25	.77
NCOP_10_a	.51	.75
NCOP_10_d	.33	.76
NCOP_12	.61	.73
NCOP_13_b	.38	.76
NCOP_13_d	.33	.76
NCOP_14_b	.33	.76
NCOP_15_a	.50	.75
NCOP_15_b	.29	.77
NCOP_15_c	.49	.75

Score reliability for the mathematics teaching efficacy beliefs

For the MTEBI instrument, Cronbach's *alpha* was found to be 0.76. The corrected item-total correlations and Cronbach's *alpha* when each item was deleted are given in Table 6 for the MTEBI instrument.

Table 6
Item-total statistics of the MTEBI

Item	Corrected Item-Total Correlation	Cronbach's *alpha* if Item Deleted
S_eff_1	.57	.73
S_eff_2*	.59	.73
S_eff_3	.28	.76
S_eff_4*	.25	.76
S_eff_5*	.44	.74
S_eff_6	.36	.76
S_eff_7*	.59	.73
S_eff_8	.55	.73
S_eff_9*	.41	.75
S_eff_10*	.17	.80
S_eff_11*	.44	.74
S_eff_12	.42	.74
S_eff_13*	.39	.75

Note. * represents the negatively worded items

Validity

The feedback from the pilot study provided evidence for the validity of the study. In addition, expert views were taken from a mathematics education professor. Finally, it is accepted that reliability limits validity, the upper limits of validity was estimated by the square root of the Cronbach's alpha coefficients as 0.88 (NCOP) and 0.87 (MTEBI) (Angoff, 1988). A limitation of determining validity of the test results was that there were no observations of teaching or interviews conducted with teachers.

Data analysis

In order to address the research questions, a multivariate analysis of variance (MANOVA) was conducted for the dependent variables: *number concepts and operations scale* total score (*NCOP_total)* and self-efficacy belief scores (*S_eff_ave)* with the nominal independent variables: certification type (*Certification)* and age group (*Age_group)*. For each dependent variable, descriptive statistics such as skewness, kurtosis and graphical representations were analyzed. Standardized *z* values were checked. Univariate normality was checked. Mahalanobis distances and graphical representations were used to analyze multivariate normality (See Appendix 3). Correlation between two dependent variables was estimated by using Pearson's product-moment correlation coefficient *r* to describe the relationship between the dependent variables. Homogeneity of variance and covariance matrices and multicollinearity were checked. The assumptions of conducting MANOVA were accepted to be met.

CHAPTER 4: RESULTS

Introduction

In this chapter, results from the data analysis are reported to address the following research questions:

- Is there any statistically significant difference on the average between self-efficacy beliefs and number concepts and operations knowledge of novice and senior Turkish middle school mathematics teachers?
- Is there any statistically significant difference on the average between self-efficacy beliefs and number concepts and operations knowledge of Turkish middle school mathematics teachers with different teaching certification types?
- Are the self-efficacy beliefs and number concepts and operations knowledge of Turkish middle school mathematics teachers on the average affected by the interaction of age groups (novice or senior) and teaching certification types (faculty of education certified or alternatively certified)?

The analysis was performed on two dependent variables: *number concepts and operations* scale total scores (*NCOP_total*) and average mathematics teaching self-efficacy belief scores (*S_eff_ave*). The independent variables were certification (faculty of education or alternative certification) and age group (age ≤ 40 or age > 40).

Descriptive analysis of data

Number concepts and operations knowledge scores

Table 7 displays the participants' mean scores of total correct answers on the 15 number concepts and operations (NCOP) items, the standard deviations, and the number of participants in each category of age groups and certification. The range of the scores was from 0 to 15. The highest mean score was in the category of alternatively certified teachers who were younger than 40 years old (*M* = 9.57 points, *SD* = 2.94 points). On the other hand, the lowest mean score was observed in the category of faculty of education certified teachers who were older than 40 years old (*M* = 7.91 points, *SD* = 2.81 points); however, this mean value was more accurate than mean values in other categories because it had the smallest standard deviation.

Table 7
Descriptive statistics for NCOP_total scores for each category

Variable	Age Groups	Certification	N	Mean	SD
NCOP_total	Age ≤ 40	F. of Education	10	8.50	3.47
		Alt. Certification	7	9.57	2.94
		Total	17	8.94	3.21
	Age > 40	F. of Education	11	7.91	2.81
		Alt. Certification	14	9.00	3.46
		Total	25	8.52	3.18
	Total	F. of Education	21	8.19	3.08
		Alt. Certification	21	9.19	3.23
		Total	42	8.69	3.16

Table 8 displays the percentage of the participants, who gave the correct answer, for each item in the NCOP scale. The range of the percentage of correct answers was from 14% to 98% for all 15 items.

Table 8
Percentage of participants who gave the correct answer for each item

Item	Description	% correct answer
NCOP_1	Use of base ten blocks for multiplication of decimal numbers	38
NCOP_2	Ratio: gear turns	40
NCOP_3_c	Properties of rational and rational numbers	88
NCOP_3_d	Properties of rational and irrational numbers	98
NCOP_4	Reasoning of standard long division algorithm	14
NCOP_5	Ratio and proportion – correctness of methods	86
NCOP_10_a	Modeling proportional relations: bacteria culture	33
NCOP_10_d	Modeling proportional relations: block steps	76
NCOP_12	Percentage – correctness of methods	50
NCOP_13_b	GCF/LCM: Ordering GCF and both numbers	60
NCOP_13_d	GCF/LCM: Multiply both numbers by a constant	67
NCOP_14_b	Reasoning for ratio: density of a lemonade	24
NCOP_15_a	Modeling subtraction of fractions	38
NCOP_15_b	Modeling subtraction of fractions	81
NCOP_15_c	Modeling subtraction of fractions	76

Self-efficacy belief scores

Table 9 displays the participants' mean scores of self-efficacy beliefs on the 13 MTEBI items, the standard deviations and the number of participants in each category of age groups and certification. The range of the scores was from 1 to 5. The highest mean score was observed in the category of alternatively certified teachers who were younger than 40 years old (*M*=4.67 points, *SD*=0.35 points). This category was also the highest score achieving category in the NCOP section. The lowest mean self-efficacy belief score belonged to the category of alternatively certified teachers who were older than 40 years old (*M* = 4.20 points, *SD* = 0.43 points).

Table 9
Descriptive statistics for S_eff_ave scores for each category

Variable	Age Groups	Certification	N	Mean	SD
S_eff_ave	Age ≤ 40	F. of Education	10	4.47	.30
		Alt. Certification	7	4.67	.35
		Total	17	4.55	.32
	Age > 40	F. of Education	11	4.24	.55
		Alt. Certification	14	4.20	.43
		Total	25	4.22	.47
	Total	F. of Education	21	4.35	.45
		Alt. Certification	21	4.36	.46
		Total	42	4.35	.45

Table 10 displayed the distribution of participants' agreement on each MTEBI item. The lowest self-efficacy belief score was for the item *S_eff_10*: *If I had a choice, I would (not) want my math class to be observed and evaluated by the inspector*. In all other questions, teachers had relatively high self-efficacy belief scores; with a mode of five for ten of the items.

Table 10
Frequency of responses for each MTEBI item
(1: strongly disagree, 2: disagree, 3: neutral, 4: agree, 5: strongly agree)

Item	1	2	3	4	5	Total
S_eff_1	4	0	0	**22**	16	42
S_eff_2	1	0	1	11	**29**	42
S_eff_3	1	0	2	**22**	17	42
S_eff_4	1	1	3	15	**22**	42
S_eff_5	1	1	0	13	**27**	42
S_eff_6	0	0	0	11	**31**	42
S_eff_7	0	1	1	14	**26**	42
S_eff_8	1	0	0	11	**30**	42
S_eff_9	2	0	0	9	**31**	42
S_eff_10	**15**	11	4	5	7	42
S_eff_11	1	0	0	12	**29**	42
S_eff_12	2	0	0	12	**28**	42
S_eff_13	0	1	0	14	**27**	42

Bivariate correlations

The correlations between all four variables were analyzed using Pearson's correlation coefficient (See Table 11). It was observed that teachers' self-efficacy belief scores in teaching middle school mathematics were correlated to their NCOP score at the level of r = .26 (p < .05). When the R^2 (0.07) was compared with Hill's (2010) finding of teachers' mathematical self-concept and MKT correlation ($R^2 = 0.06$, p < .05), it was almost identical in terms of effect size.

Table 11
Bivariate correlation matrix for the variables

	Age_group	Certification	NCOP_total	S_eff_ave
Age_group	1	.15	-.07	-.37*
Certification		1	.16	.01
NCOP_total			1	.26
S_eff_ave				1

Note. *Correlation is significant at the 0.05 level

Inferential analysis of data

Analysis for the combined dependent variables

In order to answer the research questions, a multivariate analysis of variance (MANOVA) was conducted for the dependent variables: *NCOP_total* and *S_eff_ave* with the nominally-scaled independent variables: *Certification* and *Age_group*. The major advantage of using MANOVA over using multiple ANOVAs was known to avoid inflated type I error due to multiple tests of likely dependent variables (Tabachnick & Fidell, 2007).

In the 2x2 MANOVA, the assumption of homogeneity of covariance across groups was checked: Box's *M* = 7.31, *F* (9, 6611.24) = 0.73, *p* = .68. This indicated that there was no statistically significant difference between the covariance matrices. Therefore, the assumption of homogeneity of covariance was not violated and it was appropriate to use Wilk's lambda test. The test results were given in Table 12 (All other assumptions were checked and met, and presented in Appendix 3)

Table 12
Multivariate analysis of variance of combined dependent variables

Effect	Wilk's λ	F	Hyp df	Error df	p_{calc}	Par. η^2	Observed power
Intercept	.01	2050.42	2	37	< .05	.99	1.00
Age_group	.85	3.29	2	37	< .05*	.15	.59
Certification	.97	.60	2	37	.56	.03	.14
Age_group*Certification	.98	.41	2	37	.67	.02	.11

Note. *Computed using $\alpha \leq .05$.

The first step was to check the interaction effect of age group and certification (*Age_group*Certification*). With the use of Wilk's criterion, the *F* values were tested at α = .05 level. When Wilk's λ = .98 and $F_{calculated}$ (2, 37) = .41, *p* = .67 , the $F_{critical}(2\,,37) = 3.25$ at α = .05. Because $F_{calculated}$ was smaller than $F_{critical}$, the interaction effect was not statistically significant. A measure of effect size, *partial* η^2 (eta-squared), was calculated by using Wilk's λ:

$$\eta_i{}^2 = 1 - \lambda_i \tag{1}$$

Hence, the effect size was measured with partial η^2 = .02 for the interaction effect. For an easier interpretation, this value was converted into Cohen's *d* as 0.14 standard deviations according to Cohen's (1988) conversion formulas, cited by DeCoster (2009). Figure 4 shows that findings of the main effects were not distorted by the interaction. The graphs indicated that interaction was not statistically significant.

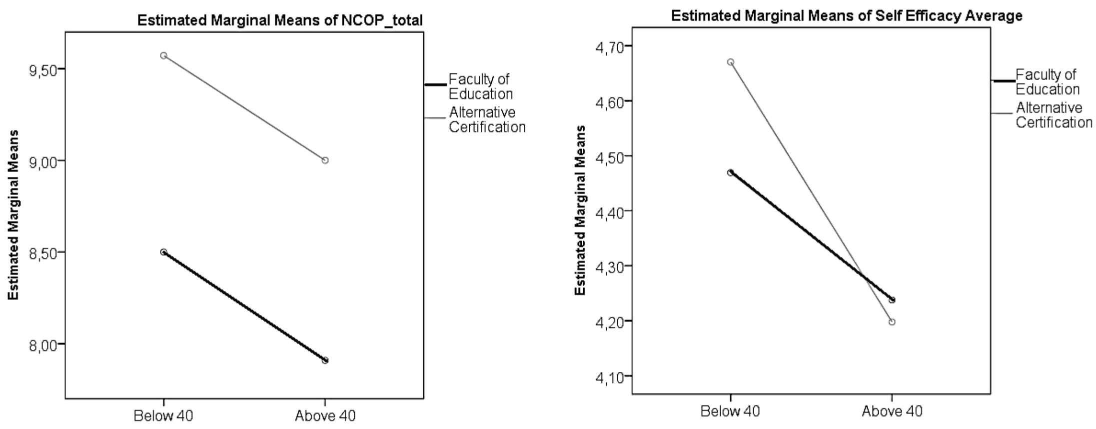

Figure 4. Graphical representations of interactions

Because the interaction effect was not statistically significant, main effects were analyzed. The combined dependent variables were statistically significantly affected by *Age_group,* Wilk's λ = .85, F (2, 37) = 3.30, p < .05, partial η^2= .15, Cohen's d = 0.84 standard deviations. This meant that teachers' age group contributed significantly toward discriminating the teachers' competencies and there existed a 0.84 standard deviation difference between the combined competencies of teachers who were younger than 40 years old and older than 40 years old.

Certification did not contribute significantly to distinguishing the competencies of the teachers, Wilk's λ = .97, F (2, 37) = .60, p = .56, partial η^2 = .03, Cohen's d = 0.36 standard deviations. There was no statistically significant difference between the faculty of education graduates and the alternatively certified teachers.

Analysis of between subjects effects

Because a significant multivariate *F* value was found in the MANOVA, the univariate ANOVA results were examined in order to understand which variables separately differed across the groups.

In Levene's test of equality of error variances, no statistical significance was found ($p > .05$) (See Table 13).

Table 13
Levene's test of equality of error variances

	F	df_1	df_2	p_{calc}
NCOP_total	0.52	3	38	.67
Self-Efficacy Average	2.59	3	38	.07

Sum of squares (SoS) was partitioned with type III method. The result of this analysis was summarized in Table 14. SPSS reports the *partial* η^2 as the effect size. However, it was also suggested to report the η^2 (Levine & Hullet , 2002).

$$\eta^2 = SoS_{effect}/SoS_{total} \quad (2)$$

Table 14
Multivariate tests of between subjects effects

Source	DV	Type III SoS	df	Mean Square	F	p_{calc}	Partial η^2	Observed power	η^2
Age_group	NCOP_total	3.33	1	3.33	.32	.58	.01	.09	.01
	S_eff_ave	1.22	1	1.22	6.76	.01*	.15	.72	.15
Certification	NCOP_total	11.54	1	11.54	1.11	.30	.03	.18	.03
	S_eff_ave	.06	1	.06	.35	.56	.01	.09	.01
Age_group* Certification	NCOP_total	.00	1	.00	.00	.99	.00	.05	.00
	S_eff_ave	.14	1	.14	.79	.38	.02	.14	.02
Error	NCOP_total	395.12	38	10.40					
	S_eff_ave	6.88	38	.18					
Total	NCOP_total	3581.00	42						
	S_eff_ave	803.55	42						
Corr. total	NCOP_total	408.98	41						
	S_eff_ave	8.20	41						

Note. Computed using $\alpha \leq .05$

The follow-up ANOVAs indicated that the effect of age group was statistically significant for the self-efficacy variable (partial η^2 = .15, Cohen's *d* = .84 standard deviations). This was a noteworthy result because this meant that there existed a 0.84 standard deviations difference between the mean self-efficacy levels of teachers who were younger than 40 years old and older than 40 years old. The younger teachers had higher self-efficacy belief levels than the older teachers.

Confidence intervals

Graphical representations of the 95% confidence intervals associated with the point estimates of means for each group are presented in Figure 5, Figure 6, Figure 7 and Figure 8. A 95% confidence interval means that if infinitely many random samples were drawn from the population, exactly 95% of the confidence intervals would enclose the parameter (Thompson, 2007; cf. Navruz & Delen, 2014). The upper and lower limits of the point estimate were calculated by using the following formula:

$$\mu = \bar{X} \mp t_{n-1} * \frac{S}{\sqrt{n}} \qquad (3)$$

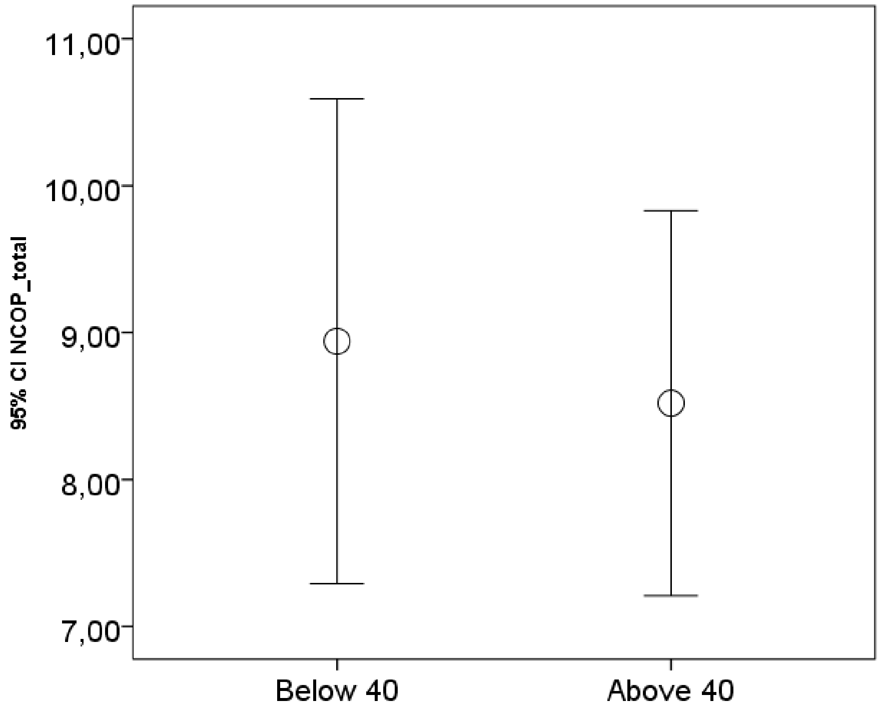

Figure 5. 95% confidence interval for NCOP_total for the age group

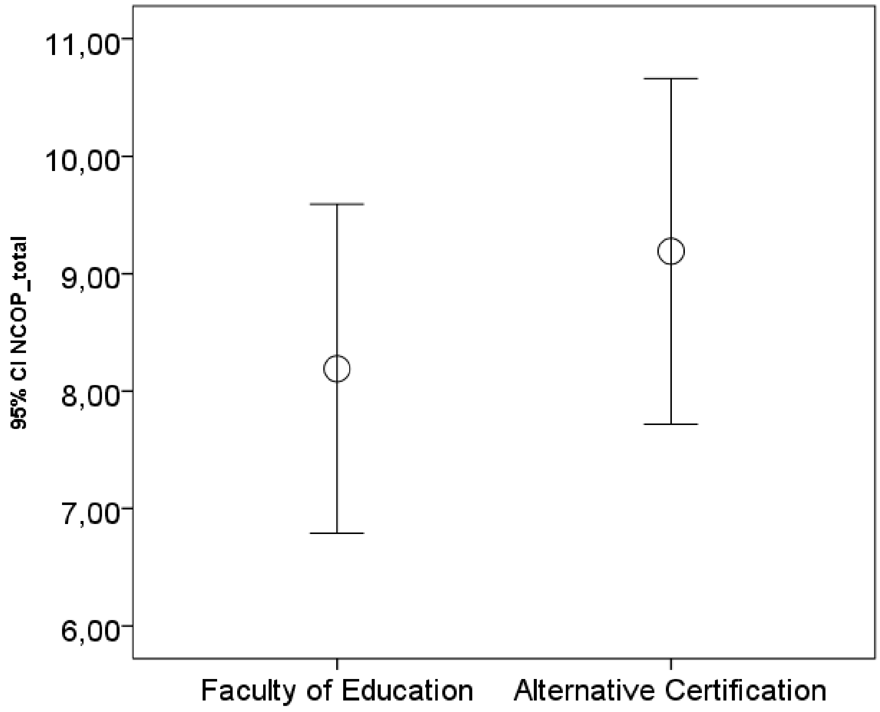

Figure 6. 95% confidence interval for NCOP_total for the certification groups

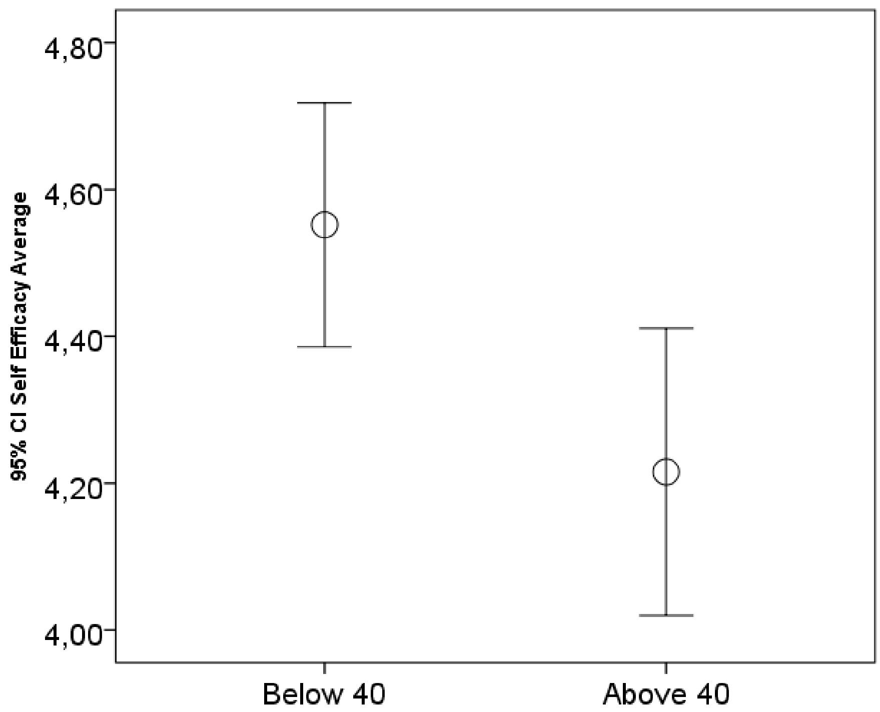

Figure 7. 95% confidence interval for S_eff_ave for the age groups

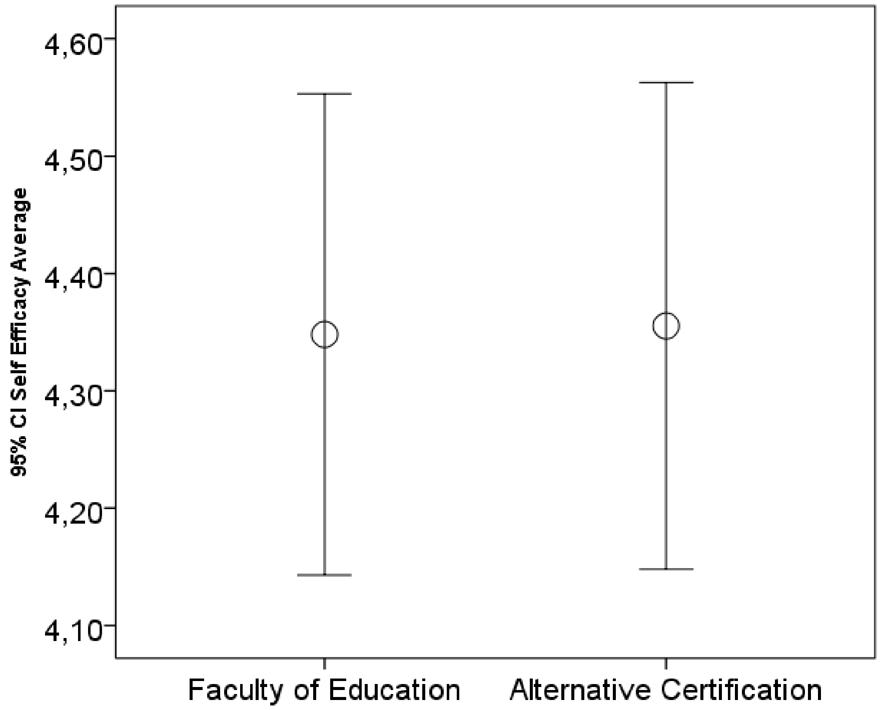

Figure 8. 95% confidence interval for S_eff_ave for the certification groups

The study examined the self-efficacy belief and number concepts and operations mean scores of novice middle school mathematics teachers in comparison to senior mathematics teachers. The null hypothesis, which indicated no differences in the self-efficacy belief and number concepts and operations mean scores, was rejected. Because a statistically significant difference was found between the self-efficacy beliefs of novice and senior teachers. In addition, the study examined the self-efficacy belief and number concepts and operations mean scores of faculty of education certified teachers in comparison to and alternatively certified teachers. The null hypothesis, which indicated no differences in the self-efficacy belief and number concepts and operations mean scores, was not rejected. Because no statistically significant difference was found between the self-efficacy beliefs or number concepts and operations knowledge of faculty of education certified teachers and alternatively certified teachers.

CHAPTER 5: DISCUSSION

Introduction

The current study addressed the following research questions:

- Is there any statistically significant difference on the average between self-efficacy beliefs and number concepts and operations knowledge of novice and senior Turkish middle school mathematics teachers?
- Is there any statistically significant difference on the average between self-efficacy beliefs and number concepts and operations knowledge of Turkish middle school mathematics teachers with different teaching certification types?
- Are the self-efficacy beliefs and number concepts and operations knowledge of Turkish middle school mathematics teachers on the average affected by the interaction of age groups (novice or senior) and teaching certification types (faculty of education certified or alternatively certified)?

In this chapter, the major findings are explained through previously conducted research on teacher knowledge, self-efficacy, and teacher education in Turkey. This chapter also provides implications for practice and for future research. Finally, the limitations and how they affected the current study are elaborated.

Major findings

- Middle school mathematics teachers younger than 40 years old are found to have higher self-efficacy beliefs than teachers older than 40 years old.
- No difference is found between the self-efficacy beliefs of middle school mathematics teachers who gained their teaching certification from a faculty of education or alternatively certified.
- Middle school mathematics teachers who are younger than 40 years are not more or less knowledgeable in number concepts and operations than teachers older than 40 years.
- Middle school mathematics teachers who gained their teaching certification from a faculty of education are not more or less knowledgeable in number concepts and operations than teachers who were alternatively certified.

Findings related to teachers' self-efficacy

It is evident from the current study that Turkish middle school mathematics teachers younger than 40 years old have higher self-efficacy beliefs than teachers older than 40 years old. Teachers who are younger than 40 years old were educated as a teacher after the faculties of education were restructured and the new programs were put into practice in 1998-1999. Therefore, this finding can be explained with the effect of the World Bank-funded national education development project (NEDP) and the transformation that it has created in the teacher education system.

The major emphasis in the reform-oriented NEDP was on the subject specific teaching methods and professional studies. The teacher education curricula were updated accordingly. Special teaching methods started to be regarded as a discipline (Grossman & Sands, 2008). Techniques that are applied in mathematics teaching methods courses such as *microteaching* were found to have a positive effect on the mathematics teaching self-efficacy beliefs of pre-service teachers (Bilen, 2015). The pre-service teachers understand the need to have their hands full with various methods before entering the classroom. Another explanation could be the increased amount of teaching practice times that was spent by the pre-service teachers at the schools after the NEDP (Grossman, Önkol, & Sands, 2007). The connection between theory and practice was addressed in order to improve the relationships between faculties of education and schools. With longer school experience, trainee teachers had opportunities to adapt to the professional life of a teacher, to prepare for a class and plan lessons and to gain classroom management skills during their education.

Furthermore, the teachers, who were educated after the initiation of NEDP in 1998, had the advantage of having access to contemporary resources. Instructional technology was also integrated into teacher education programs. Younger teachers were found to have more positive attitudes towards using instructional technologies in their classroom in Turkey (Çavaş, Çavaş, Karaoğlan, & Kışla, 2009). Since, teaching material preparation has gained importance for student centered teaching, better technology user teachers are expected to be more comfortable in providing resources for their students. Thus, it is not unusual that the young teachers having more self-confidence while using contemporary tools for teaching mathematics.

Alternatively to the reform results, the lower mean scores from the teachers older than 40 years old may indicate that teachers have a tendency to lose their motivation and take their job less seriously after spending a certain number of years in service (Beijaard, Verloop, & Vermunt, 2000; Bloom & Jorde-Bloom, 1987). Low self-efficacy beliefs of older mathematics teachers can also be associated to being exposed to stereotyped beliefs. Their competence and motivation can be negatively affected from the perceptions of their students or colleagues about their aging and loss of competence (Klassen & Chiu, 2010).

Another explanation for the high mean scores of younger teachers could be young teachers' lack of experience about the realities and complexities of classroom teaching scenarios with respect to older teachers (Hoy & Spero, 2005). In addition, efficacy beliefs of experienced teachers do not seem to change once established. However, the self-efficacy beliefs of novice teachers might still be open for being affected by external factors such as the amount of support they have received from administration, colleagues or community, and the quality of teaching resources they had access (Hoy & Spero, 2005).

Teachers' feeling of preparedness is found to be a determinative factor in their self-efficacy beliefs (Darling-Hammond, Chung, & Frelow, 2002). Teachers, who start teaching in a school with little professional education, were found to experience more difficulties in the classroom. Although faculty of education graduates have more professional education about teaching, the current study did not reveal higher self-efficacy beliefs for the faculty of education graduates than the alternatively certified teachers. Those beliefs may be circumstantial and related to individual differences (such as teachers' initial teaching context or available support) that were not measured in this study.

A notable result about the teachers' self-efficacy beliefs is that the agreement rating of the participants was explicitly the lowest for the item S_eff_10: *If I had a choice, I would (not) want my math class to be observed and evaluated by the inspector*.

However, this item should be interpreted with caution. Because, rather than teachers' self-efficacy beliefs, this item can be related to teachers' aversion towards being evaluated by an inspector or can be the result of their negative past experiences. It was found that Turkish teachers generally are not pleased with being inspected in general (Töremen & Döş, 2009).

Findings related to teachers' knowledge

The current study was noteworthy because it was one of the first implementations of the mathematical knowledge for teaching (MKT) measures in the Turkish context. The study revealed that the NCOP knowledge level of the Turkish middle school mathematics teachers can be assessed as inadequate. For example, an item *NCOP_4* from the instrument was about the explanation and reasoning of the standard division algorithm. Only 14% of the participant teachers gave the correct answer for this item. This result supported the claim that in service teachers as well as pre-service teachers fail to make instructional explanations about the underlying mathematical rationale in the sense of division algorithm. (Baki, 2013). The second item with relatively low performance was *NCOP_14_b* with 24% correct answers. This item was about the mixture of liquids and conceptually understanding what *strongest taste* means in terms of the ratio of volumes of liquids. As a result of the *number concepts and operations* scale data analysis, it is observed that teachers performed better at items that included factual or procedural knowledge and performed worse at items that mostly included reasoning and conceptual knowledge.

It is observed that there is no statistically significant difference in the number concepts and operations knowledge of teachers who entered the profession through two different pathways. Faculty of education certified teachers did not perform better in the MKT scale than the alternatively certified teachers. This situation can be interpreted by considering the mixed profile of the faculty members in the faculties of education. A majority of the faculty members in the faculties of education have alternative academic backgrounds, especially backgrounds in faculty of arts and sciences (Erginer, Erginer, & Bedir, 2009). Very few of the lecturers in the faculties of education had themselves taught mathematics in a school. Thus, they might not address the specialized knowledge required for teaching mathematics in the classroom. The teacher educators’ resistance to change their attitudes and behavior towards the reforms might also be an obstacle for the effectiveness of the reforms (Grossman, Önkol, & Sands, 2007).

Faculty of education certified mathematics teachers and mathematics majors who received alternative certification were not more or less knowledgeable than each other in the number concepts and operations domain. Although there is more mathematics content course in the faculty of arts and sciences than the faculty of education, it is observed that taking more mathematics courses does not lead to higher MKT scores for teaching middle school mathematics (cf. Hill, 2010). The assumption that mathematics majors know enough mathematics to teach and hence focusing only on pedagogy during the alternative certification programs seems to be a fallacy. In fact, it is commonly accepted that undergraduate mathematics programs do not address the kind of mathematics that teachers need (Ball, 1988). Because the numbers and operations is one of the basic concepts in school mathematics, it is difficult to speculate about the differences in the knowledge of teachers with different certifications. As a matter of fact, no relationship was found between pursuing the two alternative career pathways and good content knowledge in other studies, either (Cohen-Vogel & Smith, 2007; Hill, 2010; Tournaki, Lyublinskaya, & Carolan, 2009).

Unlike the findings in this current study, Safran et al. (2014) found that faculty of education graduates were more knowledgeable than the alternatively certified teachers when Safran analyzed 2013 Public Personnel Selection Examination (PPSE) data. This discrepancy can be explained best with the unique population used in the study, which does not represent all regions of Turkey nor does it represent pre-service teachers. Another explanation can be related to the scope of the Public Personnel Selection Examination (PPSE) which tests general aptitude, liberal arts, educational sciences and subject area and subject area teaching knowledge are used. However the MKT measures used in the current study included a scope of a specific subdomain of mathematics knowledge for teaching. The effect size in Safran et al.'s (2014) study was computed as Cohen's d = 0.1 standard deviations (Corlu, 2014). In the current study, the effect size for the certification in NCOP scores was Cohen's d = 0.35 standard deviations.

The sustainable improvement of teachers' knowledge depends on the continuous and sustainable *instructional* and *conceptual* professional development programs. However, in-service professional development programs are generally misconceived in the Turkish education system. They mostly include seminars such as first aid, national level exams, or use of interactive boards (Arıbaş & Göktaş, 2014). A striking situation is that when the middle school teachers were asked about their professional needs, more than 60% of the teachers reported that they *do not* need a professional development about teaching number concepts and operations (MoNE, 2008a). The results in the current study indicates that determining the professional development needs of the teachers solely based on teachers' own report might be misleading.

Hill (2010) found that there is a modest relationship between the MKT of the mathematics teachers and their years of experience. Senior teachers were found to have more MKT than novice teachers and this was associated to their learning on the job. However, older Turkish middle school teachers are not found to be more knowledgeable than novice teachers in this study. This might be interpreted in a way that teachers do not spend time for reflecting on their classroom experiences and think about ways to improve their teaching. Lack of instructional professional development might be another explanation for this finding.

Implications for practice

According to the results of this study, I believe that Turkish middle school mathematics teachers need to improve their mathematical knowledge for teaching. Throughout the data collection, it was observed that teachers were not aware of their shortcomings about their competencies. Teachers need to keep their knowledge fresh and frequently ponder about how to improve their teaching in order to claim a professional status. It is necessary for the Turkish teachers to receive content-related, long-term and research based professional development. The instructional professional development programs might help the teachers to realize their needs for development in this context. In order for the professional development programs to have noteworthy contributions to the learning of their students, collective participation of the teachers from the same school and subject should be assured.

Moreover, active and sustainable long-term learning opportunities should be provided for the teachers (Garet, Porter, Desimone, Birman, & Yoon, 2001).

I also suggest that in the teacher recruitment process, the weight of mathematics teaching knowledge questions be increased. Policy makers might consider strategies in order to encourage more talented people to select teaching mathematics as a profession while doing their university selections.

Implications for future research

The analysis reported in this study can be seen as a first step in understanding the mathematical knowledge for teaching and self-efficacy of Turkish mathematics teachers and their possible classroom implications. In this study, only quantitative research methods were used to investigate the mathematical knowledge of middle school mathematics teachers and their self-efficacy levels. The findings can be triangulated by classroom observations and interviews with the teachers in the future research, as well as longitudinal research on student performances in order to better understand this complex phenomena. Teachers' mathematical knowledge for teaching can be investigated in the other domains than number concepts and operations. Moreover, the research in the affective domain could be extended to other dimensions such as teachers' values and attitudes. Complex causal models are also needed. Thus, it may be also of interest to researchers to investigate the relationship of teacher competencies with the leadership roles such as being a head of department or coaching mathematics olympiad teams. Research with secondary level mathematics teachers is also warranted. The research can be replicated in other regions of Turkey with diverse populations, as well.

Limitations

It took a quite long time for the teachers to complete the tests with little motivation to do so during their busy working hours. Hill (2010) reported that when teachers were each paid $50 for completing their survey, they had a higher response rate. If the participants in this study were to take the tests in an exam-like setting, more accurate results could be obtained. The teachers did not want to be tested and some of them were anxious that the results might be reported to MoNE, although they were informed about the confidentiality of the research orally and by a written document. Therefore the sample was smaller than expected and the achieved power was low. The self-reported nature of the self-efficacy beliefs and demographic information was another limitation that could be barriers to obtain more accurate results.

Use of multiple choice items for measuring MKT can be another limitation. The distractors might have directed the participants to the correct answers or the items might have *taught* the subject. However, this is an unlikely situation due to the extensive fieldwork during item development process as well as the pilot study conducted prior to the actual data collection in the current study (Hill, Ball, & Schilling, 2008).

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APPENDICES

Appendix 1: Learning mathematics for teaching - sample released items

LEARNING MATHEMATICS FOR TEACHING

MATHEMATICAL KNOWLEDGE FOR TEACHING (MKT) MEASURES

MATHEMATICS RELEASED ITEMS 2008

University of Michigan, Ann Arbor
610 E. University #1600
Ann Arbor, MI 48109-1259
(734) 647-5233
www.sitemaker.umich.edu/lmt

 Measures development supported by NSF grants REC-9979873, REC- 0207649, EHR-0233456 & EHR 0335411, and by a subcontract to CPRE on Department of Education (DOE), Office of Educational Research and Improvement (OERI) award #R308A960003.

Study of Instructional Improvement

lmt learning mathematics for teaching project

December 26, 2008

Dear Colleague:

Thank you for your interest in our survey items measuring mathematical knowledge for teaching. To orient you to the items and their potential use, we explain their development, intent, and design in this letter.

The effort to design survey items measuring teachers' knowledge for teaching mathematics grew out of the unique needs of the *Study of Instructional Improvement* (SII). SII is investigating the design and enactment of three leading whole school reforms and these reforms' effects on students' academic and social performance. As part of this research, lead investigators realized a need not only for measures which represent school and classroom processes (e.g., school norms, resources, teachers' instructional methods) but also teachers' facility in using disciplinary knowledge in the context of classroom teaching. Having such measures will allow SII to investigate the effects of teachers' knowledge on student achievement, and understand how such knowledge affects program implementation. While many potential methods for exploring and measuring teachers' content knowledge exist (i.e., interviews, observations, structured tasks), we elected to focus our efforts on developing survey measures because of the large number of teachers (over 5000) participating in SII.

Beginning in 1999, we undertook the development of such survey measures. Using theory, research, the study of curriculum materials and student work, and our experience, we wrote items we believe represent some of the competencies teachers use in teaching elementary mathematics – representing numbers, interpreting unusual student answers or algorithms, anticipating student difficulties with material. With the assistance of the University of California Office of the President[2], we piloted these items with K-6 teachers engaged in mathematics professional development. This work developed into a sister project to SII, *Learning Mathematics for Teaching* (LMT). With funding from the National Science Foundation, LMT has taken over instrument development from SII, developing and piloting geometry and middle school items.

We have publicly released a small set of items from our projects' efforts to write and pilot survey measures. We believe these items can be useful in many different contexts: as open-ended prompts which allow for the exploration of teachers' reasoning about mathematics and student thinking; as materials for professional development or teacher education; as exemplars of the kinds of mathematics teachers must know to teach. We encourage their use in such contexts. <u>However, this particular set of items is, as a group, NOT appropriate for use as an overall measure, or scale, representing teacher knowledge.</u> In other words, one <u>cannot</u> calculate a teacher score that reliably indicates either level of content knowledge or growth over time.

We ask users to keep in mind that these items represent steps in the process of developing measures. In many cases, we released items that failed, statistically speaking, in our piloting; in these cases, items may contain small mathematical ambiguities or other imperfections. If you have comments or ideas about these items, please feel free to contact one of us by email at the addresses below.

These items are the result of years of thought and development, including both qualitative investigations of the content teachers use to teach elementary mathematics, and quantitative field trials with large numbers of survey items and participating teachers. <u>Because of the intellectual effort put into these items by SII investigators, we ask that *all* users of these items satisfy the following requirements:</u>

1) Please request permission from SII for any use of these items. To do so, contact Geoffrey Phelps at gphelps@umich.edu. Include a brief description of how you plan to use the items, and if applicable, what written products might result.

2) In any publications, grant proposals, or other written work which results from use of these items, please cite the development efforts which took place at SII by referencing this document:

[2] Elizabeth Stage, Patrick Callahan, Rena Dorph, principals.

Hill, H.C., Schilling, S.G., & Ball, D.L. (2004) Developing measures of teachers' mathematics knowledge for teaching. Elementary School Journal 105, 11-30.

3) Refrain from using these items in multiple choice format to evaluate teacher content knowledge in any way (e.g., by calculating number correct for any individual teacher, or gauging growth over time). Use in professional development, as open-ended prompts, or as examples of the kinds of knowledge teachers might need to know is permissible.

You can also check the SII website (http://www.sii.soe.umich.edu/) or LMT website (http://www.sitemaker.umich.edu/lmt) for more information about this effort.

Below, we present three types of released item – elementary content knowledge, elementary knowledge of students and content, and middle school content knowledge. Again, thank you for your interest in these items.

Sincerely,

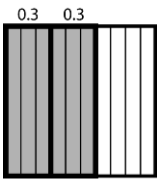
Deborah Loewenberg Ball
Dean, School of Education
William H. Payne Collegiate Professor
Education
University of Michigan

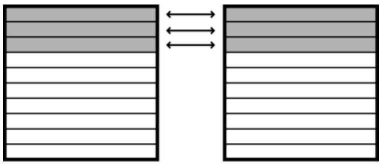
Heather Hill
Associate Professor
Harvard Graduate School of

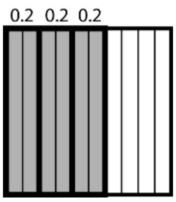
29. Ms. Austen was planning a lesson on decimal multiplication. She wanted to connect multiplication of decimals to her students' understanding of multiplication as repeated addition. She planned on reviewing the following definition with her class:

> The repeated addition interpretation of multiplication defines *a* x *b* as *b* added together *a* times, or *a* groups of *b.*

After reviewing this definition of repeated addition, she planned to ask her students to represent the problem 0.3 x 2 using the repeated addition interpretation of multiplication. Which of the following representations best illustrates the repeated addition definition of 0.3 x 2? (Circle ONE answer.)

a) 

b) 

c) 

d) These representations illustrate the repeated addition definition of 0.3 x 2 equally well.

e) Multiplication of decimals cannot be represented using a repeated addition interpretation of multiplication.

31. Ms. James' class was investigating patterns in whole-number addition. Her students noticed that whenever they added an even number and an odd number the sum was an odd number. Ms. James asked her students to explain why this claim is true for all whole numbers.

After giving the class time to work, she asked Susan to present her explanation:

> I can split the even number into two equal groups, and I can split the odd number into two equal groups with one left over. When I add them together I get an odd number, which means I can split the sum into two equal groups with one left over.

Which of the following best characterizes Susan's explanation? (Circle ONE answer.)

a) It provides a general and efficient basis for the claim.

b) It is correct, but it would be more efficient to examine the units digit of the sum to see if it is 1, 3, 5, 7, or 9.

c) It only shows that the claim is true for one example, rather than establishing that it is true in general.

d) It assumes what it is trying to show, rather than establishing why the sum is odd.

35. Ms. Williams plans to give the following problem to her class:

> Baker Joe is making apple tarts. If he uses $\frac{3}{4}$ of an apple for each tart, how many tarts can he make with 15 apples?

Because it has been a while since the class has worked with fractions, she decides to prepare her students by first giving them a simpler version of this same type of problem. Which of the following would be most useful for preparing the class to work on this problem? (Circle ONE answer.)

I. Baker Ted is making pumpkin pies. He has 8 pumpkins in his basket. If he uses $\frac{1}{4}$ of his pumpkins per pie, how many pumpkins does he use in each pie?

II. Baker Ted is making pumpkin pies. If he uses $\frac{1}{4}$ of a pumpkin for each pie, how many pies can he make with 9 pumpkins?

III. Baker Ted is making pumpkin pies. If he uses $\frac{3}{4}$ of a pumpkin for each pie, how many pies can he make with 10 pumpkins?

a) I only
b) II only
c) III only
d) II and III only
e) I, II, and III

Appendix 2: Adaptation of the MTEBI used in this study

Lütfen aşağıdaki her önermeye ne oranda katılıp katılmadığınızı belirtmek için, her öneri için seçeneklerden bir tanesini işaretleyiniz. Tüm önermeleri değerlendirdiğinizi kontrol etmeyi unutmayınız!

	Kesinlikle Katılmıyorum	Katılmıyorum	Çekimser	Katılıyorum	Kesinlikle Katılıyorum
1.Matematik dersini öğretmek için devamlı daha iyi yöntemler bulacağım.					
2.Ne kadar çabalarsam çabalayayım, matematik dersini iyi öğretemeyeceğim.					
3. Matematiksel kavramları etkili biçimde nasıl öğreteceğimi biliyorum.					
4. Matematikle ilgili sınıf etkinliklerini takip etmekte çok etkili olamayacağım.					
5. Matematik dersini genelde yetersiz öğreteceğim.					
6.Ortaokul matematik dersini etkili öğretmeye yetebilecek derecede matematiksel kavramları anlıyorum.					
7. Matematiksel işlemlerin nasıl sonuca ulaştığını öğrencilere açıklamak için sınıf etkinliklerini kullanmakta zorlanacağım.					
8. Öğrencilerin matematik ile ilgili sorularını genelde cevaplandırabileceğim.					
9. Matematik dersini öğretebilmek için gerekli becerilere sahip olabileceğimden emin değilim.					
10. Seçme şansım olursa, matematik dersimin müfettiş tarafından gözlenip değerlendirilmesini istemiyorum.					
11. Ne zaman bir öğrencim bir matematiksel kavramı anlamakta zorlansa, kavramı daha iyi anlamasına yardım etmek için çoğunlukla ne yapmam gerektiğini bilemeyeceğim.					
12. Matematik dersini anlatırken çoğunlukla öğrencilerin soru sormasını cesaretlendireceğim.					
13. Öğrencilerin matematiğe ilgilerini çekmek için ne yapmam gerektiğini bilemiyorum.					

Appendix 3: Assumptions for the statistical analysis of data

Univariate normality

Table 15
Descriptive statistics for the dependent variables

Statistic	*NCOP_total*	*S_eff_ave*
N	42	42
Mean	8.69	4.35
Median	9.00	4.46
Standard Deviation	3.16	0.45
Skewness	.01	-.73
Kurtosis	-1.32	-.05
Minimum	4	3.23
Maximum	14	5

Table 16
Descriptive statistics for the standardized z values

	N	Range	Minimum	Maximum	Mean	Std. Deviation
Z score S_eff_ave	42.00	3.96	-2.51	1.45	0.00	1.00
Z score NCOP_total	42.00	3.17	-1.49	1.68	0.00	1.00

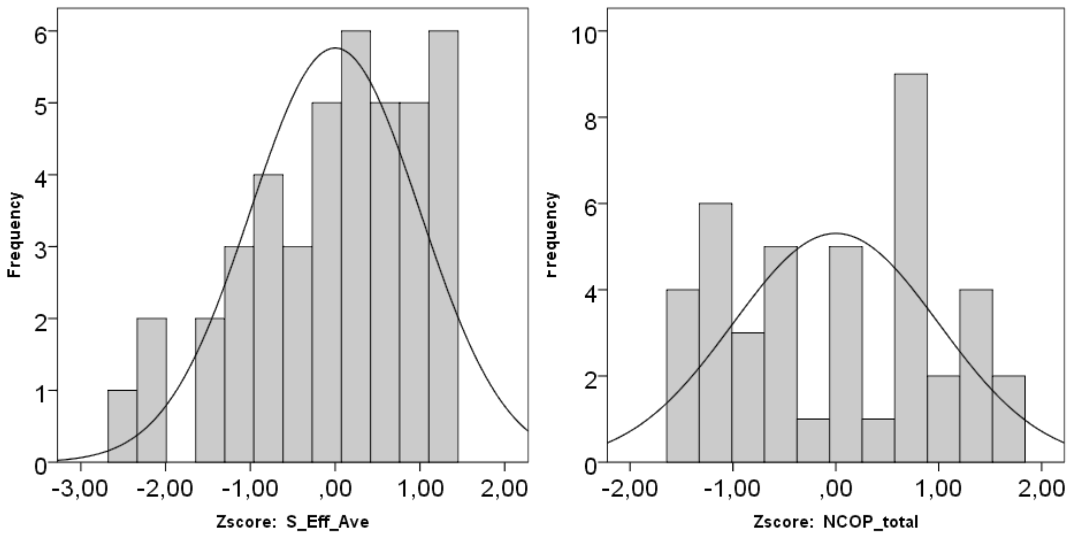


Figure 9. Frequency histogram and normal curve for the dependent variables

Multivariate normality

Mahalanobis distance was used to analyze multivariate normality and multivariate outliers. The maximum Mahalanobis distance score was 6.29 whereas the critical χ^2 value for df =2 was 13.82, i.e. no outlier existed. Graphical representations of multivariate normality are given in Figure 9 and Figure 10.

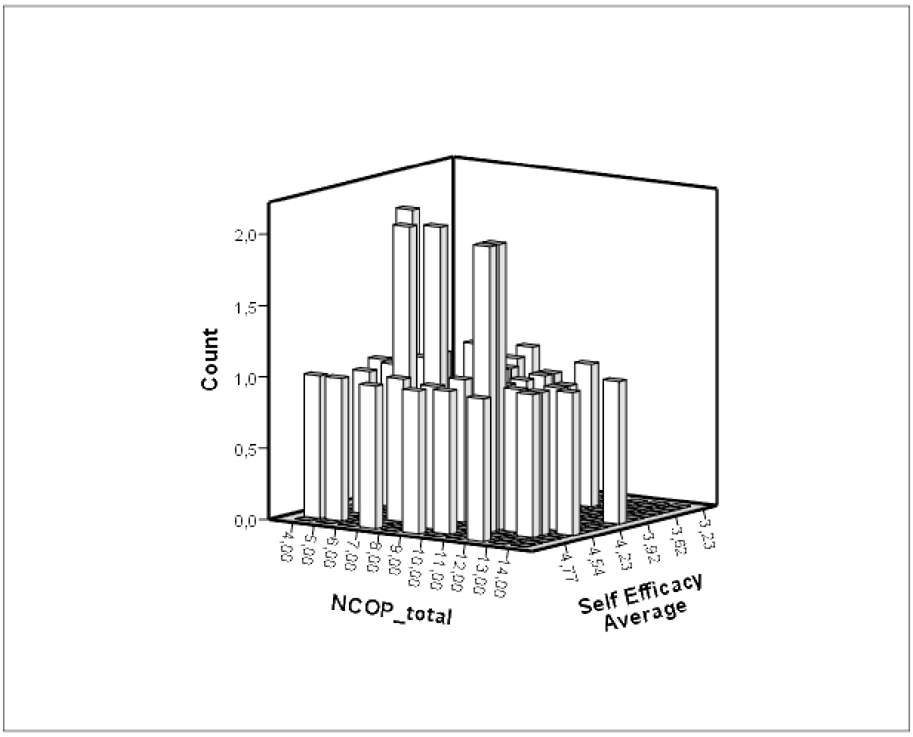


Figure 10. Three dimensional frequency histogram for the dependent variables.

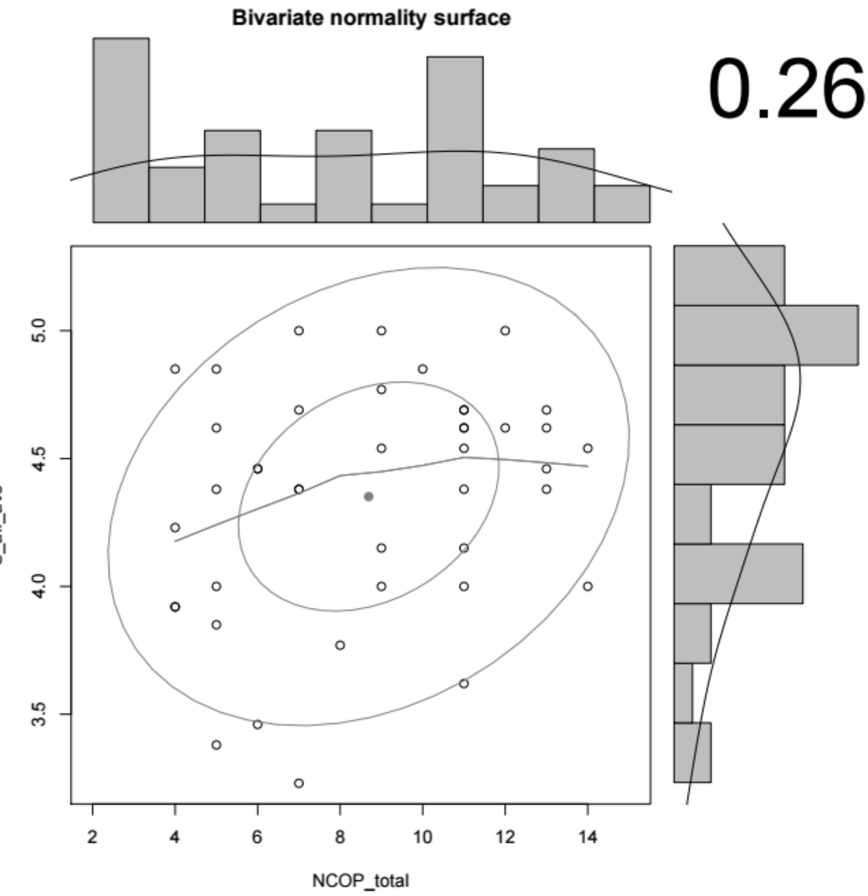


Figure 11. Bivariate normality distribution for the dependent variables.

Relationship between dependent variables

It was observed that teachers' self-efficacy belief scores in teaching middle school mathematics was correlated to their NCOP score at the level of *r* = .26 (p < .05), in the interval low to moderate as expected for the MANOVA (Leech et al., 2008).

Multicollinearity

Multicollinearity was not a threat since we have two independent variables. Variance Inflation Factor (VIF) is 1 which is the lower bound for VIF.

Coefficients[a]

Model		Collinearity Statistics	
		Tolerance	VIF
1	Faculty of Education or not	1,000	1,000

a. Dependent Variable: Below 40 or not

Coefficients[a]

Model		Collinearity Statistics	
		Tolerance	VIF
1	Below 40 or not	1,000	1,000

a. Dependent Variable: Faculty of Education or not