INVESTIGATING THE FOUNDATIONS OF TURKISH ELEMENTARY MATHEMATICS EDUCATION THROUGH AN ANALYSIS OF A LATE OTTOMAN TEXTBOOK

A MASTER'S THESIS

BY

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INVESTIGATING THE FOUNDATIONS OF TURKISH ELEMENTARY MATHEMATICS EDUCATION THROUGH AN ANALYSIS OF A LATE OTTOMAN TEXTBOOK

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I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Curriculum and Instruction.

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ABSTRACT

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M.A., Program of Curriculum and Instruction

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September 2015

Developing an understanding of the foundations of the educational tradition of the Turkish Republic is connected to an exploration of the specifics of Ottoman education. This qualitative study explored an Ottoman mathematics textbook published in the early twentieth century. Under the influence of naturalistic inquiry, the textbook was analyzed in terms of content, organization, and principles of elementary mathematics education. It was concluded that the textbook is successfully presented multiple representations and real-life examples while the development of content did not provide opportunities to develop reasoning skills.

Key words: Ottoman mathematics education, elementary school mathematics education, history of mathematics education.

ÖZET

TÜRKİYE'NİN KURULUŞ DÖNEMİNDEKİ MATEMATİK EĞİTİMİNİN OSMANLI'NIN SON DÖNEMİNE AİT İLKOKUL DERS KİTABI ÜZERİNDEN İNCELENMESİ

Esra Yaprak

Yüksek Lisans, Eğitim Programları ve Öğretim

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Türkiye'deki matematik eğitiminin dinamiklerini anlamak için, 19. ve 20. yüzyılda Osmanlı Devleti'nde meydana gelen gelişmelerin incelenmesi gerekir. Bu nitel çalışma yirminci yüzyılın başlarında yayınlanmış bir Osmanlı matematik ders kitabının sistematik ve eğitim tarihi bakış açısıyla incelenmesi üzerine kurulmuştur. Natüralistik araştırmanın etkisi altında, ders kitabı, içeriği, organizasyonu ve matematik eğitimi ilkelerine uygunluğu açısından analiz edilmiştir. Ana bulgular sonucunda ders kitabının görsel temsilleri ve gerçek hayattan alıntıları başarılı bir şekilde yansıttığı ancak sorgulama stratejilerinin geliştirilmesi konusunda yetersiz kaldığı görülmüştür.

Anahtar kelimeler: Osmanlı'da matematik eğitimi, ilköğretim matematik eğitimi, matematik eğitimi tarihi.

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Introduction

Mathematics has been useful in daily life throughout history, regardless of the nationalities of those using it. However, there were several specific uses of mathematics in the daily lives of Ottomans: for example, inheritance problems (a branch of law), finding the direction for prayer, times of prayer, annual calculations of Eid times, time calculations, astronomy, and so on. The Ottomans also used mathematics in the encryption of tax calculations (*siyakat*) and in other encryption system, such as the abjad (*ebced*) alphabet. This indicates that everyday life as an Ottoman to an extent depended on mathematics. Due to the importance of mathematics in Ottoman daily life, it was a core subject at the elementary school level (*maktab*). Students at this level gained basic numeracy skills, which prepared them for secondary school level mathematics (İzgü, 1997).

Background

The Ottoman education system consisted of institutions that were inherited by the Seljuq Turks up until the 18th century. Elementary schools (*maktab)* and secondary-higher education schools (*madrasa)* provided education to young people in the Empire. There were also palace schools (*enderun maktab*).

An elementary school (*maktab*) was based on a mosque-school system founded and supported by elite statesmen or sultans. Young learners began their education at those institutions with a ceremony called, literally translated, beautiful start (bed'i *besmele*; İhsanoglu, 2002). All children had the right to attend school. Those who were educated in secondary-higher education institutions (*madrasa*), along with certain literate mosque caretakers, were selected as elementary school teachers. *Maktabs* had mainly religious purposes. They taught reading and writing of the alphabet, handwriting (calligraphy), the basic principles of Islam and the Quran, basic counting, and the four basic arithmetical operations, known as black sentence (*kara cumle*; İhsanoglu, 2002). The basic principles of these institutions were based on the ideas of sharing and helping other people, being respectful to others' ideas and opinions, being more tolerant of others, and behaving rationally as educated people. All those principles were intended to encourage young learners to become well-educated citizens (Sönmez, 2013). The *maktab* was essential for students who wished to continue their educations at the *madrasa* level. The *madrasa,* which mainly refers to secondary-higher education, included both religious and secular subjects (İhsanoglu, Chatzis & Nicolaids, 2003). There were also palace schools (*enderun maktab*), which provided education for youngsters who were to become members of the administrative elite of the Ottoman society (Taşkın, 2008).

Towards the end of the 18th century, the performance of the *maktabs*, *madrasas,* and *enderun* began to fall, as a result of changes due to the influence of intellectual and cultural ideas in Western Europe during the 17th century (Akyüz, 1993). More emphasis was being given to reason, logic, and analysis in the West. There was also much talk about science, toleration, and skepticism. Those ideas spread throughout the continent. As a result, the Ottoman education was affected by those Western ideologies (Lewis, 2001).

Most traditional educational methods have been disputed during this era and modern educational philosophies have been developed, which are in contrast to traditional approaches. The importance of scientific knowledge and intellectual expression has increased (Mardin, 1960). Modern educational philosophies spread all around the world, including throughout the Ottoman Empire, in the 18th century (Lewis, 2001).

Modern educational ideas began to influence the worldview of Ottoman elites, causing concerns about the faith of the Empire. They identified country's main deficiency to lie in war technology and pressured Ottoman Sultans to reform the army. Changes then began, starting with military (Göçek, 1996; Lewis, 1968). A long period of reform also revealed deficiencies in qualified human resources for implementing reforms. This led the Ottoman educators to focus on educating new generations on the basis of contemporary education philosophies and principles. They believed that conventional education methods required changes, because traditional methods had not been satisfactory (Somel, 2001).

The *Tanzimat* period (1839-1856) was characterized by attempts to establish graded systems of schooling, which were different in many ways from traditional institutions (Kazamias, 1969). Traditional institutions were far from effective and sufficient for educating young people (Şanal, 2003). Many stakeholders (such as government, teachers, and parents) started to be aware of a need to increase the quality of teaching and learning, particularly in science, engineering, medicine, and mathematics (Cemaloğlu, 2005). Instead of abolishing ineffective institutions, policy makers decided to introduce Western style institutions alongside the traditional institutions, thus creating a dual system.

The dual system was initiated in 1869 by Mehmet Esad Safvet Pasha (1814-1883), then the Minister of Education. It was called the Education Act (*Maarif-i Umumiye Nizamnamesi*; Somel, 2001). The Ministry of Education started to open new institutions for training youngsters. The dual education system was divided into three parts: primary school education (*sibyan* schools and *rushdiyes*), secondary school education (*idadis* and *sultanis*), and higher education (*Darulfunun*; Göçek, 1996; Kazamias, 1969). New regulations gradually spread to the whole state; these became the foundation upon which later reforms were introduced during the early Republican period (1923-1938; Aslan & Olkun, 2011).

Problem

Despite increased interest in Ottoman life and language in recent years in Turkey, little research has been conducted on their educational systems. In particular, very few studies have focused on how textbooks were prepared to interact strategically with teachers and students in mathematics education. Only a couple of analyses have focused on the Ottoman mathematics textbooks published in the early period of 20th century. Thus, there is a need to understand the foundations of the educational tradition of the Turkish Republic (and perhaps other independent states) by exploring the specifics of Ottoman education. Mathematics education is one of the subjects that can reveal the development of educational traditions. Given the fact that written documents can serve as witnesses of historical periods, there is a need for analyses of Ottoman textbooks published in the 19th and 20th centuries.

Purpose

The purpose of this study was to explore a mathematics textbook published during the modernization period of the Ottoman Empire (1828-1908). This study analyzes the textbook in terms of its content and organization, as well as its instructional methods with no comparison to any other state or textbook.

Research questions

In order to achieve the purpose of the study, the researcher sought answers to the following research questions about teaching, learning, and assessment in an elementary school mathematics textbook:

- How does the textbook content take into account the developmental levels of students?
- How do the problems and exercises throughout the textbook address student’s developmental level?
- What evidence is there for multiple representations of mathematical structures?
- Does the author approach mathematics holistically, with a focus on investigations and reasoning, or in a more procedural fashion?

Significance

This study will contribute to the field by tracing the historical roots of mathematics education, and in so doing, it may provide insight to modern Turkish mathematics education. This investigation may help to make clear the milestones of the mathematics education, as it evolved from the Empire to the era of the Republic. In addition, a resource that is unlikely be known to mathematics educators in Turkey (due to language obstacle), has been revealed.

Definition of key terms

Maktab: Elementary school in Ottoman Empire (Somel, 2001).

Madrasa: Any type of institutions whether secular or traditional in the secondary and in higher schools of the Ottoman Empire (Somel, 2001)

Enderun maktab: Palace school mostly for the Balkanic peoples who were recruited for serving the Ottoman government as high administrators, military positions (Corlu, Burlbaw, Capraro, Corlu, & Han, 2010).

Hisab: Arithmetic in Ottoman language

Ulema: Madrasa and palace school teachers (İhsanoglu, 1992)

Rushdiye: Middle school in secular system of Ottoman Empire (Alkan, 2008)

İptidai: Primary school in secular system of in Ottoman Empire (Somel, 2001)

CHAPTER 2: LITERATURE REVIEW

Introduction

In this study, I explore mathematics education in Ottoman elementary schools by analyzing a textbook. This chapter firstly details how the Ottoman education system changed according to the needs of the society, secondly, the chapter reviews concepts and instructional strategies in elementary school mathematics. It is organized under two parts: Part 1 includes (a) traditional Ottoman education system, (b) educational changes in the Ottoman Empire after the 19th century, (c) elementary school mathematics education, (d) the impact of textbook in education while Part 2 includes contemporary mathematical instructional strategies in elementary school level.

Part 1. Traditional Ottoman education system

In the early stages of the Ottoman Empire, education was shaped around the social structure of Ottoman society. It was divided into two different groups: the elite group and the ordinary citizens (Mardin, 1960). At this time, education became an important criterion for social advancement. There were three educational structures, influencing the country's social profile: elementary school (*maktab*), secondary-higher education (*madrasa*), and palace school (*enderun maktab*; Somel, 2001). The majority of society was educated in *maktab* schools; these were important educational institutions in the Empire. The *maktab* were for children older than four years. The *maktab* curriculum helped young learners to develop their manual and writing skills, and introduced some religious subjects at early ages. In the following years, pupils could select subjects according to their interests (Mirbabayev, Zieme, &

Furen, 1996). Because every youngster had different kinds of skills, teachers and tutors at the *maktabs* paid utmost attention to the subject selection process (Uzunçarşılı, 1965). In *maktabs*, two school subjects were of particular importance, because of their necessity in daily life: the first one was writing (*kitab*) and the second one was arithmetic (*hisab*). The Ottomans expressed the importance of mathematics in *maktabs* by saying, "*Bil ki hisab ilmi, ilimlerin en üstünüdür* [the arithmetic science is the highest scientific subject]" (Akyüz, 1993, p. 191).

The *madrasas* were secondary-higher educational institutions. Those who studied in *maktabs* continued their education in *madrasas*. These institutions were placed in every city of the Empire. The administrative and legal classes of the Ottoman society had been raised in *madrasas* (İhsanoğlu, 2002). The courses at *madrasa* involved scientific and wisdom studies, grammar, syntax, logic, metaphysics, rhetoric, geometry, arithmetic, and geography. Through *madrasas*, Ottoman educational institutions continued to educate students holistically, emphasizing the development of virtue, talent, religion, and so on (Ahmed & Filipovic, 2004). In addition, *madrasas* occupied pivotal positions in Ottoman society because the teachers of *madrasa* and palace schools (*ulema)* were trained at these institutions (İhsanoğlu, 1992).

Palace schools (*enderun maktabs)* were unique with respect to being world's first educational institutions established for gifted and talented youth. Perhaps the most intriguing feature of palace schools was the student profile. There was a selection process and students were chosen based on their physical and intellectual skills

(Corlu, Burlbaw, Capraro, Corlu, & Han, 2010). Additionally, recruits were mostly preferred to be unmarried, Balkanic, and male youngsters. These schools were organized into seven grades, which were instructed by special teachers. The instructors were members of *ulema*, scientists, musicians, artists, and so on. The brightest youths were trained not just in subjects such as Turkish, Arabic, Persian, Islam, etiquette, mathematics, archery, riding, and so on, but also in knowledge about the protocols and rules of the palace (Ergin, 1977).

Private and religious institutions carried out the responsibility and administration of all schools, except the *enderun*. The education of ordinary citizens was led by the educated class of Muslim legal scholars (*ulema*), while elites were trained by community leaders. Thus, state-supervised public education was limited (Kazamias, 1969).

However, the state started to show interest in the educational system, and challenged the ideas of teachers (*ulema*) at the *maktabs* and *madrasas*. Certain necessities (public and state concerns about the overall quality when compared to schools in the West) may have led the state to desire control over all education in the Empire. The modern state school was introduced alongside the traditional education system, and both coexisted until the establishment of Republic of Turkey.

Educational changes in the Ottoman Empire during the 19th century

Beginning in the 18th century, there was a gradual decline in political and economic power in the Ottoman Empire. The Empire was challenged by several problems inside and outside. As a result, the Empire realized the necessity of modernization for the sake of its continuance. Attempts at modernization started in the educational sector because the performance of educational institutions had begun to decrease steadily below their levels in the 17th century (Akyüz, 1993). Consequently, the state decided to undertake some reform efforts in order to modernize the country. The first step was opening new institutions in the Empire (Weiker, 1968).

The Ottomans' interest in modernization movements dated back to before the *Tanzimat* era. The interaction between the Ottoman state and Western countries helped to modernize the state. The translation of European books, and the Ottoman ambassadors' visits to European countries, helped to put Ottomans in contact with the technological developments there. After the 19th century, Ottoman students were sent to Europe to study Western science. These innovations and changes led to the establishment of new types of institutions (Somel, 2001). Although technological and scientific developments resulted first from the need for new army and military techniques, after the *Tanzimat* era the state realized the need for innovation at large (İhsanoğlu, 1992).

The first modern educational institution was the Naval Engineering School (*Muhendishane-i Bahri-i Humayun*). This military school offered new courses on science (mechanics, astronomy), technology (technical drawing and design of military equipment), and mathematics (geometry, algebra, and logarithms), many of which were not taught at Ottoman schools before. The school aimed to educate people to become engineers and teachers of this engineering school. Even though the expectations of the military school did not satisfy the country, the school contributed to the development of scientific knowledge and thus had an impact in the society (Kaçar, 2007). French officers, technicians, and military experts who sought refugee in Ottoman Empire, were assigned to be instructors at military schools. The French language was made compulsory for all students (Lewis, 1968). Other military schools were opened in the following decades, to train medical officers and operators; these included Civil Engineering School (*Muhendishane-i Berri-i Humayun*) founded in 1795, Medicine School (*Tibbhane-i Amire*) in 1827, and Technical School (*Hendesehane*), which opened in 1773 (Somel, 2001).

After the opening of these new styles of schools, the need for educators increased. Consequently, the first teachers' school (*darulmuallimin*) in the history of Turkish education was opened in 1848. Traditional elementary schools were then revised, and in 1868 a school for elementary teachers was founded (Türkmen, 2007). This first elementary teacher school taught courses such as teaching methods, calculation, geography, Persian, Turkish language and grammar, history of the Ottoman Empire, algebra, and writing (Akyüz, 1993; Koçer, 1970).

Deficiencies at the primary education level became a challenge for modern military schools. Military school students were required to have basic proficiency in science and technology. The lack of science and technology courses in primary schools created a difficult situation for students who wanted to continue their educations in military schools. Primary schools offered only very basic courses, including reading and writing (Akyüz, 1993). Attempts at remedying this situation led to the opening of middle schools *(rushdiye)*. Middle schools were designed as an intermediate level, between the primary school and military school (Alkan, 2008).

The road to modernization: The education act

Modernization movements in the Empire continued through the 19th century. The *Tanzimat* era (1839-1876) constituted an important milestone in the modernization process of the Ottoman Empire. *Tanzimat* was a period during which the state's participation in Ottoman society increased (Shaw & Shaw, 1977). During this period, some of the goals of the educational system included: (1) the expansion of elementary level educational facilities; (2) the construction of middle schools (*rushdiye)* to link primary and secondary-higher education; (3) increasing the number of female students in secondary-higher education; and (4) the founding of modern universities (Kaçar, 2009). The government took initial steps toward these educational goals.

The first step in reorganizing public education was the creation of a route map, which was reported in the Council of Public Education (*Meclis-i Maarif-i Umumiyye*) in 1846. With regard to primary education, which was traditionally offered by *maktabs,* the Council established standardized organizations supervised by the state. Additionally, the inefficiency of elementary education and the gap between elementary and secondary education was to be filled by middle school (*rushdiye*; Ergin, 1977). The first middle school was opened in 1847 (Sakaoğlu, 2003).

In 1862, the state once again tried to reform primary education. Thirty-six *maktabs* were converted into primary schools (*iptidai*). Those schools were opened in twelve districts of Istanbul, with the aim of increasing the literacy rate (Ergin, 1977). The transition involved supplying each student with a slate, slate pen, and inkwell in order to facilitate learning how to read and write. This reform then spread throughout the Empire in due course (Somel, 2001). Another significant development in this period was the opening of middle school (*rushdiye*) for girls. Girls' educational opportunities were previously limited to primary schools. In the following years, the number of schools for girls gradually climbed. The need to train teachers for these new schools led to the opening of Teacher School for women (*Darulmuallimat*) in 1869 (Sakaoğlu, 2003).

The School of Civil Service (*Mekteb-i Mulkiye*) was also established around this time, to train new bureaucrats to take on administrative positions. The curriculum of this school included subjects such as law, economics, statistics, geography, and French. The state also required an agricultural school (*Ziraat maktab*) which was established in 1847, in combination with a mining school in 1859, an industrial school (*Mekteb-i Sanayi*) in 1864, a forestry school (*Orman maktab*) in 1859, and a telegraph school (*Telgraf maktab*) in 1860. These were all examples of vocational schools (Ergin, 1977).

The education act of 1869

The Education Act was issued on September 1st of 1869. It contained various items, including primary-secondary education, recruitment of teachers, organizational bodies, and financial matters (Evered, 2012). The Ottoman educational structure was divided into two categories—public and private school. While public schools were controlled by state, private schools were administered by religious communities, but supervised by the state. Public schools were divided into three tiers as well. The first level was composed of the elementary (*sıbyan* and *rushdiye)*. The second level included the preparatory secondary schools (*idadiye*) and academic secondary schools *(sultaniye*). The third tier was the high school (*Mekâtib-i Âliye*; Sanal, 2003). New instructional methods were adopted and compared to traditional methods. The *madrasas* were shifted into buildings that had much larger classrooms. Desks, maps, and blackboards were introduced. These new methods spread significantly throughout the Empire (Sakaoğlu, 2003).

Beginning in the 20th century, new systems of education spread to most provinces. Modern subjects such as Western philosophy and scientific inquiry were included in the form of a newly prepared curriculum. Translated Western style textbooks increased. Alongside these developments, the traditional school system still continued to educate young people (Kenan, 2014).

Elementary school mathematics in the Turkish Republic

Following the establishment of the Republic, reform movements continued in education. The educational system was unified under the Ministry of National Education (MoNE) and traditional schools, *maktab* and *madrasa,* were abolished. In 1924, some important decisions were made with regard to the curriculum: elementary schools spanned years, elementary school teachers' education increased from three to five years, and the curriculum was revised. Curriculum subjects were also secularized (Türkmen, 2007).

Aslan and Olkun (2011; 2013) investigated the first elementary school mathematics curriculum, which was used after the foundation of the Turkish Republic in 1923. They found that the overall scope and sequence of topics in the mathematics curriculum had been similar to the curriculum used in the Ottoman Empire. The new mathematics curriculum aimed to develop practical and real-life applications of the four basic operations, arithmetical thinking, interpretive and reasoning skills, and daily life issues; however, the available textbooks were not written in accordance to these goals of the curriculum (Aslan & Olkun, 2013). The topics in arithmetic (*hisab*) were the counting and writing of numbers, basic operations, measurement units, mental arithmetic, problems and exercises, and basic fractions. In addition, some old-fashioned measurement units were omitted from the curriculum (Aslan, 2011; Aslan & Olkun, 2011). Early emphasis on abstract structures was abolished with more emphasis on teaching methods that start with concrete objects and real-life applications (Binbaşıoğlu, 1995). The need for textbooks written in modern Turkish and in accordance with the new curriculum was critical.

The impact of textbooks on the Turkish education system

The textbook is an important material for understanding how the curriculum is taught at schools and how that affects the culture of a society. The representation of symbols and the techniques and language used to express concepts represent the teaching and learning cultures of a society. In this respect, textbooks influence the intellectual development of young people in addition to conveying the objectives of the curriculum (Pingel, 2010).

Haggarty and Pepin (2002) assert that the values and traditions of learning and teaching are not only made up of the perception of teachers towards subject learning and teaching, but also of official texts that are provided by authorities. Textbooks are commonly seen as the main sources for the content to be covered and the pedagogical styles to be used in classrooms (Apple, 1992).

In this context, analyzing a historical textbook helps to understand a curriculum and teaching styles and techniques of one country. Aslan and Olkun (2013) analyzed a report declared by the Inspection Commission of Arithmetic Textbooks in Primary School Education during the first years of the Turkish Republic in 1926. The report included the twenty textbook analysis and investigations. The findings showed that the arithmetic textbooks had some problems such as the content did not cover the real life situations, the educational methods did not satisfy the developmental level of students, and the exercises did not contain didactic approach. The content of the textbooks was considered as including too much information that is not compatible with the developmental level of students. In addition, more abstract and theoretical concepts has been showed. The teaching methods were criticized by concepts in elementary school textbooks was taught as if students were learning in higher mathematics. Moreover, the reporters suggested the usage of real life situations. Starting from grade 1 the children should be exposed to real life examples such as calculation sticks, nuts, walnuts, and so on. It was suggested that the graphic and pictorial representations should be put on the textbooks. The problems in the textbooks should not be explained in detail. Lastly, the report indicated the importance of teachers and it says that the textbooks should not steal the teacher's role (Aslan & Olkun, 2013)

Part II. The Development of number sense skills

Number sense is a term described as "referring to a person's general understanding of numbers and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgments and to develop useful strategies for handling numbers and operations" (McIntosh, Reys, & Reys, 1992, p. 26). Acquiring a number sense skill is an evolutionary process that starts from the pre-school years. Children begin to accumulate counting skills as well as a number sense at this developmental stage (Sowder, 1992).

At an early age, children build number concepts by gaining experiences from the environment and trying to make sense of them. Gallistel and Gelman (1990) proposed three principles that guide early counting: the *one to one correspondence principle* indicates that every number in a set must be assigned a unique tag; the *stable order principle* states that numbers used to count must be in a fixed order; and the *cardinality principle* states that the last number used in a count has a special status in the set. All principles must be satisfied in counting. Gallistel and Gelman (1990) concluded that children count objects using the same counting sequence, although sometimes they may make errors (e.g. 1, 2, 6, 9 instead of 1, 2, 3, 4). This implied that children understood the stable order principle. In addition, some researchers indicated that when children were asked how many objects there are, they tended to reply with the last number in the counting list; thus children appeared to follow the cardinality principle (Gallistel & Gelman, 1990; Gelman & Meck, 1986).

Mastery of counting refers to the understanding that counting results in a number. This number represents how many things are in the set that was counted. Having an understanding of the amount of things in a set was termed the cardinality principle (Fosnot & Dolk, 2001; Fuson & Hall, 1983). Anghileri (2006) stated that the *cardinality principle* indicates a transformation in knowledge, from counting in ones to representing a whole collection. Children first tend to develop counting skills (learn the procedure), then they begin to learn the underlying principles (Gelman & Meck, 1986).

In order to prove that children have an understanding of the *cardinality principle*, Wynn (1990) examined two and three half years old children. In this study, children were asked to count a list of objects; immediately after counting, they were asked how many objects were in the set in total. Only three and a half year old children answered correctly, by linking the counting with the whole set of objects. This result implied that only the elder children understood the *cardinality principle*. Nikoloska (2009) found that counting might play an important role prior to mastering the *cardinality principle*. After mastering counting, the *cardinality principle* can be developed by using pictured sets. Matching a number with a given object might be a useful way of developing this principle. Pattern recognition or matching activities help children's understanding of the *cardinality principle* (Fuson, et al., 1997).

Van de Walle et al. (2010) stated that representation of numbers is an important tool for communication and thinking. Most researchers agree that children can develop advanced understandings when they are able to perform flexible transfers between different representations (Boaler, 1993; Carraher, Carraher & Schlieman, 2000; Greeno & Hall, 1997). Huang and Yang (2004) have analyzed the performance differences among students in written computation, pictorial representation, symbolic representation, and number sense. Their results show that children who are highly skilled in written computation could not equally transfer their skills to symbolic and pictorial representation and number sense to solve similar exercises.

Two types of number relationships can be taught to children: *one and two more, and one and two less*; and *anchors or benchmarks of 5 and 10* (Van de Walle, Karp, & Bay-Williams, 2010).

One and Two More, and One and Two Less strategy refers to the realization of how one number is bound to another. For example, 6 is 1 more than 5, or 3 less than 9. The objective of this approach is to show the relationships among numbers, not just in terms of the ability to count, but also by representing the fact that numbers are related to one another in a variety of ways (Anghileri, 2006). One study indicate that most children had an experience in preschool years about counting numbers and more\less relations with less understanding of one number related to another. They can pick the set that is obviously different in number from another set (Baroody, 1987). In addition, children had opportunity to expose more comparing to less in daily life.

Relating *anchors or benchmarks of 5 and 10* with any number can enhance the learners' acquisition of mathematical concepts (Sowder & Schappelle, 1994). The number 10 plays an important role in number relations because of the numeration system established in *base-10*. Both 5 and 10 can count as useful benchmarks in understanding relationships between numbers less than 10. In addition, these relationships can help in the development of mental computation of larger numbers (Anghileri, 2006; Van de Walle et al., 2010).

Written and verbal representation is an essential part of understanding mathematics (Khisty, 1995). Children can derive mathematical knowledge from oral and written clues. Past research shows that different kinds of representation methods (verbal or written) may be efficient ways of supporting students' critical thinking (Aiken, 1972). The ability to articulate mathematical ideas in words can be considered a benchmark of deeper understanding (Carpenter & Lehrer, 1999). Oral and written expression might assist learners in making better connections among concepts (Meier, 2002). Studies reveal that students' written and verbal representations reflected their thought processes and promoted awareness and self-regulation in choosing the most suitable information and strategies (Artzt & Armour-Thomas, 1992; Carr & Biddlecomb, 1998).

The development of *place value* understanding is subject to the number concept and counting (Van de Walle et al., 2010). Ross (1986) stated that in order to understand place value numeration, children need to connect numeration knowledge with number concepts. Counting can help their understanding of the meaning of *tens* and *ones*. When children connect the idea of quantities, the grouping of numbers as tens and ones separately can be shown and *base-10 language* can be associated with daily language (Van de Walle et al, 2010). *Base-10* language can be beneficial for the explanation of the place value system (Fuson & Briars, 1990). *Place value* can be viewed as numbers being composed of other numbers (Resnick, 1983). The decomposition of numbers into ones and tens is a special case of grouping and regrouping numbers, particularly when performing number operations. Children should know that 32 is 3 tens and 2 ones (Sowder, 1992). Unfortunately, students can demonstrate some difficulties in place-value tasks. For example, in Jordan, Hanich, and Kaplan (2003) conducted a test designed to assess student's performance on a variety of task including place-value. The results showed that students who have difficulties with mathematical concepts performed lower scores on place-value tasks compared to the average students. They suggested that the amount of instruction time could be increased for young students who demonstrated problem in place value (Bryant, Smith, & Bryant, 2008).

Ross (1986) indicates that the place value of a number within a given multi-digit numeral can be determined by denoting the position of the digit in the numeral. The values of the places can be indicated depending on how the left-to-right order is written. However, especially in textbooks, modeling numbers in the following manner may not be an accurate way of constructing the idea of a place value system, because learners can easily copy down the numbers (Van de Walle et al, 2010), e.g. __5__ tens and __8__ ones is __58__ in all.

Many researchers believed that linking mathematics in *real-life situations* is beneficial for young learners. Children need to think about the purpose of the exercises, what information need to solve the problems. Griffiths (2001) stated that teachers sometimes even ignore the context when they present the exercises or problems.

Previous researches (Burkhardt, 1981; Mason, 1984) suggested that students have a tendency in finding practical, curious or unexpected solutions to problems. Pierce and Stacey (2006) analyzed two students' response through real world context problems. They both have made positive comments about the problems by finding interesting and relating with their own experiences.

Teaching the four basic operations

The operations *addition and subtraction* can be introduced through the combining or partitioning of objects (Anghileri, 2006). The most effective way of teaching the four operations can be using them in real life contexts. A highly integrated understanding of the four operations can be provided in real life settings (Van de Walle et al.,2010). Children can link objects, pictures, or words with numbers so that the operations would be carried out in a meaningful context (Abdi, Barrett, Fayol, & Lemaire, 1994).

Carpenter and Moser (1984) explained that direct modeling with fingers or objects using counting sequences, and recalling number facts, would be beneficial for teaching addition and subtraction. Two numbers to be added can be expressed by using fingers or physical objects to represent the union of the two sets. Counting sequences involve counting the sets starting from the first number given in the exercises, followed by the units of the second number. The final number counted is the answer. In addition, recalling number fact strategies involved in the present task may indicate the answer without counting. Carpenter and Moser (1984) also described subtraction strategies similarly to addition strategies. Counting down from a given number can be an effective strategy for developing an understanding of subtraction. Counting backwards from the larger number in units of the smaller one is called the *counting down strategy*.

Some studies (Baroody, 1987; Fuson, 1986; Thornton & Toohey, 1985) highlight the difficulties many children experience and conclude that learning subtraction is more difficult than mastering addition facts. When the numbers go beyond the three digits counting up become easier comparing to the counting back procedures (Baroody, 1984). Thorthon (1990) analyzed solution strategies (unknown fact/memorized fact) of two groups of first graders in subtraction problems. It was concluded that the group focusing on strategies to find the unknown fact showed significant difference from the group emphasized drill approach.

For two digit numbers and beyond, addition and subtraction strategies can be established on the basis of using a benchmark such as the number 2, 5 or 10. The benchmark is a useful strategy that children connect to easily. Pairs that make 10 can be for larger numbers, when counting is impractical (e.g., 41 – 29 = ?, 29 + 10 = 39, 39 + 2 = 41, so the answer is 10 + 2 = 12; Selter, 1998). Another strategy is the splitting method in which the units, tens, hundreds, and so on of both numbers are separated and handled differently (e.g., 86 – 32 = ? is decided by taking 80 – 30 = 50 and 6 – 2 = 4, answer 50 + 4 = 54; Fuson, 1986).

Understanding of mathematical operations is also developed when the provided connections between numbers are sufficient, for example, 3 *more* or less than 5 always results in 8 or 2. Anghileri (2006) suggested that language should be refined for expressing relationships between numbers; for example, 2 *more* than 3 can be changed to *adding 2 to 3* or take away 1 from 3 can be changed to *3 minus 1*. Young learners can conceptualize addition as the increasing of numbers, and subtraction as the difference between numbers (Blume, 1981).

Typically, the language of addition and subtraction may not be words that are commonly used outside school. Because of this, Anghileri (2006) suggested that it might take time for young learners to associate the words for operations with their existing knowledge. Especially before the introduction of the symbols plus (+) and (-), their oral representations such as *take away, minus, plus, add, subtract* and so on need to be mastered from an understanding of context.

Another useful strategy for teaching basic mathematical operations is story problems. Word problems are helpful for increasing children's computational skills. Another study indicated that children who were asked to solve arithmetic problems in real life contexts or in school-like settings performed better (Carraher, Carraher, & Schliemann, 2000). Moreover, several studies showed that algorithms may not be efficient strategies for children to use in solving numerical problems outside the classroom (Carraher, Carraher, & Schliemann, 2000; Cockcroft, 1986; Gingsburg, 1982). Further work showed that algorithm-based learning was not helping young people, because real-life situations required different kinds of procedures that were not taught in school (Carraher, Carraher, & Scliemann, 2000).

Furthermore, one study revealed that in nine textbooks out of ten, new topics were introduced with symbolic activities, and then story problems were offered as more challenging tasks (Nathan, Long, & Alibali, 2002). This study showed that readers understood symbolic representations more easily than story problems (Koedinger & Nathan, 2004). Another study by Zentall and Ferkis (1993) revealed that solving story problems might be difficult for learners, as they require both reading comprehension and mathematics skills. Difficulties encountered in solving story problems include choosing correct operations, determining the order of operations, or correctly handling extraneous information (Neef, Nelles, Iwata, & Page, 2003).

A common instructional strategy for operations used problem structures. Researchers, who divided problems into categories according to the relationships that they involved and who analyzed children’s solution processes as they related to the semantic structures of the problems, divided addition and subtraction problems into two different classes on the bases of the unknown quantities (Carpenter, Hiebert, & Moser, 1981; Carpenter & Moser, 1984; Jerman & Rees, 1972).

Van de Walle, et al. (2010) explained *joining and separating problems* with three quantities: an *initial*, a *change,* and the *resulting* amount. The starting point is referred to as the *initial,* the part being added or separated to or from the initial is called the *change*, and the total after the change is joined or removed from the initial amount is the *resulting* amount. Each of these three quantities can be unknown in a word problem. In *separate* problems, the initial amount is the biggest or whole number, whereas in *join* problems, the result is the whole. In *separate* problems, the amount of change is removed from the whole.

Before they develop *multiplication and division* skills, children should already have knowledge of addition and subtraction; they should be able to relate multiplication and division to the knowledge that they already have (Anghileri, 2006). In addition, the useful strategy for multiplication is *doubles*. Van de Walle et al. (2010) expressed that 2 as factor should be taught students. For example, practicing not only 2 × 5, but also 5 × 2 should be introduced for realization (Van de Walle et al., 2010). Another strategy is language representing. While young learners exposed the words (e.g. *lots of*, *each*, *times*, and *share*) in daily life before they formally learn multiplication and division, these words choices can be beneficial for students (Bell, Fischein, & Greer, 1984).

Studies of multiplication and division showed that even four year olds can perform division strategies by sharing or grouping concrete materials. The processes of grouping and sharing objects can help students to understand number relationships and related language. This research suggests that relationships between numbers should be the main focus of multiplication and division pedagogy; for instance, five cannot be grouped exactly into twos, so this problem can emphasize the idea of a remainder (Kouba, 1989). Another study about the relationship between number sense, multiplication, and division revealed that number relationships provide the ultimate key to successful multiplication and division. For example, understanding that the number 18 is related to 2, 3, and 6 can be a powerful strategy for understanding multiplication and division (Clark & Kamii, 1996).

One study focused on the relationship between multiplication and addition. Since the multiplication process involves repeated addition, the shorthand notation of multiplication might be confusing for young learners. Therefore, comparing addition and multiplication with its symbols can be an efficient strategy (Anghileri, 2006).

Many issues in addition and subtraction can also apply to multiplication and division. Conceptual or story problems, for example, may be beneficial strategies for developing children's understanding of multiplication and division (Mulligan, 1992). Recent studies that analyzed multiplication and division problems found that the development of an understanding of number relationships, addition strategies, and counting influence the solution strategies employed in solving multiplication and division problems (Mulligan, 1992; Steffe & Cobb, 2012). In addition, traditional multiplication and division algorithms may be inefficient mental strategies for performing these operations (Kamii, Lewis, & Livingston, 1993).

Another study focused on the remainder concept in division. According to Vergnaud's (1983) study, children had difficulties with division, especially division with remainders. The problems occurred because elementary school children generally do not consider the remainder to be a component of the division process. However, other researchers found that first graders were quite successful at solving multiplication and division problems, even if division included a remainder (Carpenter, & Lehrer, 1999). Multiplication and division problems are structured into three different parts, like addition and subtraction. Researchers identified these parts according to the unknowns in the problem (Greer, 1992).

Equal-group problem was divided the elements of multiplicative problems into three parts: the first *factor*, which represents how many sets or objects are involved in the problem, the second *factor*, which gives the size of each set or part, and the *whole* or *product*, which tells us the total of the parts. If one of these elements is missing, the problem is called an *equal-group problem* and it is a multiplicative problem. Alternatively, when the whole is known, and the size of the group or the number of group is unknown, the problem is a division situation (Van de Walle et al., 2010).

Developing children's measurement skills

The concept of measurement can be considered difficult in comparison to other curriculum topics. Thompson and Preston (2004) showed that students have some misconceptions about measurement units. Van de Walle et al. (2010) provides two different goals for measurement: the first is to understand the attribute they are going to measure—this is related to deciding which attributes measured are the same; the second objective is for to understand what units of measurement are appropriate for the particular attributes in question—this requires and understanding of unit models.

Because measurement units (units of length, weight, mass, time, money, and so on) are of different kinds, Anghileri (2006) suggested that schools provide students with rich experiences working with the metric or customary system of measurement. This would help to prepare students for diverse scientific and global issues involving measurement systems. Van de Walle et al. (2010) suggested that an understanding of the metric system could be developed by indicating the smallest and largest units, because they are designed systematically around powers of ten. Also, if there is a different customary system, it should not be compared with the metric system, in order to avoid confusion (Pumala & Klabunde, 2005).

The measurement system of the Ottomans varied greatly throughout the Empire (Anatolia, Egypt, and Balkans), perhaps because the Ottomans tried to respect the cultures and daily life issues of different regions. The Ottomans' traditional system of measurement was in use until they attempted to change it to facilitate trade, military, and political relationships with Europeans. The Ottomans adopted the metric weights and measures that were used in European countries in 1869 (Günergun, 1993).

Conclusion

In this chapter, I first emphasized the educational and social systems of the Ottoman Empire, in order to understand the contexts in which Ottoman textbooks existed. Traditional school types in the formal Ottoman education system were discussed. Secondly, I explored changes in education that accompanied the modernization of different areas of the Empire. The educational system became a dual system (traditional and secular schools) that aimed to teach more science. Next, I explored some of the textbook analyses that reveal information about the secular curriculum of the Ottoman Empire. Lastly, I presented a background and literature review relevant to the topics in the textbook that I will analyze (number sense, the basic operations, measurement).

CHAPTER 3: METHOD

Introduction

The purpose of the study was to accurately portray a historical mathematics textbook without making general conclusions. This study explored the content depth and instructional strategies of a mathematics textbook from the early 20th century Ottoman Empire. The four specific purposes of this chapter are to (1) describe the research methodology of this study, (2) explain the sample selection, (3) describe the procedure used in designing the analysis instrument, and (4) provide an explanation of the procedures used to analyze the data.

Naturalistic inquiry

A naturalistic paradigm of inquiry was used to carry out this study. Cohen, Manion, and Morrison (2007) stated that the selection of the design for a study should be led by identifying the problem and research purposes. Once the focus was shaped, the theoretical framework emerged from the inquiry and the methodology was designed. The naturalistic inquiry was the most appropriate strategy (Lincoln & Guba, 1985).

The focus of this study was to gain an understanding of mathematics education in the early 20th century, thereby providing a benchmark against which the development of modern mathematics education in Turkey can be analyzed. Because of the impacts of historical events on our lives, it is necessary to study the roots of modern Turkish mathematics education. In this vein, it is critical to analyze a historical textbook that contains an incredible amount of facts, data, and cultural information. As Schissler

(1990) indicated, “Textbooks convey a global understanding of history and of the rules of society as well as norms of living with other people” (p. 81).

The naturalistic paradigm is categorized by five axioms inherent in this study. The first axiom states that “there are multiple constructed realities that can be studied holistically” (Lincoln & Guba, 1985, p. 37). Taking into consideration the complexities of historical events, analysis of the textbook requires a methodology that “automatically assumes the existence of multiple realities” (Lincoln & Guba, 1985, p. 72). The second axiom is that there is mutual influence between a researcher and the research object during a research interaction. In this study, the researcher’s interpretations will constitute the results. The third axiom rejects generalization of this study (Lincoln & Guba, 1985). Indeed, the interpretation of data in the historical mathematics textbook cannot be assumed to be representative of mathematics textbooks used in early 20th century Turkey. The fourth axiom concerns “the possibility of causal linkages” (Lincoln & Guba, 1985, p. 37). This axiom asserts that it is not possible to separate causes and effects, because of the “mutual simultaneous shaping” among all entities (Lincoln & Guba, 1985, p. 38). Finally, the fifth axiom claims that “inquiry is value-bound” (Lincoln & Guba, 1985, p. 38). This study is influenced by the values of the stakeholders that lived during the early 20th century, and likewise, by the researcher’s own values.

Historical perspective

The historical analysis is generally discussed in terms of authenticity, meaning, and theorization. Firstly, the authenticity is provided by alerting some of the inconsistencies within the textbook itself. The researcher aware of there is no possibility of an informed judgment about the data. Secondly, different kind of pieces of evidence for the data investigated from historical source. The information in the textbook compared and considered together with other research. Thirdly, the textbook has its own language in terms of mathematical terms and particular Ottoman–Turkish words. The usage of these concepts have understood as its contemporaries would be understood it, rather than as it would be understood today. Lastly, the interpretive outlook stressed while analyzing the textbook. The interpretation of data established in terms of its symbolic structures and contextual determination of meaning (McCulloch, 2005).

Research design

Sampling

The researcher began with the assumption that context is critical and purposely selected a sample that was expected to provide a rich array of information (Lincoln & Guba, 1985). The purposeful sample was an Ottoman mathematics textbook that was published in the early 20th century. Purposive sampling is used for qualitative research studies (Lincoln & Guba, 1985). The sample textbook was selected based on several criteria: accessibility to the researcher, published date of the textbook, and number of pages due to time constraints (Cohen, Manion, & Morrison, 2007). Moreover, sample comes with supporting historical documents (Ottoman-Turkish/modern Turkish mathematics dictionary).

The sample was the third edition of a historical mathematics textbook to be used in elementary school (*Iptidai School*) in 1326 (1908 for Hijri calendar or 1910/1911 for Rumi calendar). It was published by Artin Asaduryan Mahdumlari Matbaasi in Istanbul, Turkey. The textbook was purchased from an online bookstore through auction. The textbook consisted of 81 pages. Because of circumstances beyond control during data collection (time constraints, limited the financial resources for translation), the current study includes only one textbook.

Instrumentation

The investigator was the primary data-gathering instrument, according to constructivist and naturalistic inquiry. Data from this study was interpreted according to the researcher's knowledge, background, and skills. In addition, the researcher was educated in qualitative research studies and studied mathematics education at the graduate level. Therefore, the quality of the study shaped by the researcher's interests and qualifications (Patton, 2002).

I graduated from the Department of Mathematics at Hacettepe University. My four-year-long education focused on (pure) professional mathematics. Despite my specialization, I had always personally been interested in history during these four years. I had read different kinds of Turkish and European history books. The period of the Ottoman Empire especially interested me. I attended several conferences and seminars to develop knowledge about the social, economic, and cultural life of the Ottomans. After graduating with a degree in mathematics, I decided to pursue a career in teaching mathematics. I applied to Bilkent University Graduate School of

Education, Curriculum, and Instruction's Teaching Certificate Program. During my master's program, I took several courses that improved my understanding of curriculum and teaching in mathematics education. However, my enthusiasm for Ottoman history did not subside. Then, at the beginning of a semester, my supervisor introduced me to his research on *Matrakçı Nasuh,* one of the prominent mathematics educators from the Ottoman Empire. The idea of integrating mathematics education with history came to my mind. I decided to conduct a research study to investigate the mysteries of Ottoman mathematics education, since there were few existing studies on this issue.

Data collection

I, the researcher, was the main data-gathering instrument. This was for several reasons: first, I have a special interest in all dimensions of the history of the Ottoman Empire. Second, during the analysis process I kept a journal to aid my reflection on the findings and my search for necessary supplementary information from history books. Third, I studied mathematics education at graduate level; this has made me informed on both theoretical and practical issues in mathematics education. Finally, I was qualified to carry out this study, as I was educated in qualitative research methods.

I employed several different methods to collect the data. Due to logistic reasons, including accessibility and availability of the document, data were collected via an online bookstore. The collection of the document lasted ten days. I first visited bookstores specializing in history in Ankara. I examined possible sample books for their accuracy, completeness, and usefulness in answering the research questions. After that, I searched online bookstores. NadirBook.com was chosen specifically because it contains rich resources in terms of Ottoman mathematics textbooks. I spent a considerable amount of time in bookstores in order to determine the most appropriate sample.

After I selected the convenience sample, I needed to translate the language of the textbook from Ottoman Turkish to the modern Turkish language. I searched for an expert in accurate translation. However, this process required finding an expert not just in the Ottoman-Turkish language, but also in mathematics. Therefore, finding an expert took a considerable amount of time. The expert was chosen from METU. She graduated from the Department of Mathematics in METU and she knew the Ottoman-Turkish language at a professional level.

Journal

I kept a reflective journal throughout the process in order to increase the reliability of my interpretations. The journal included notes on my meetings with my thesis supervisor, the translator, the history expert, and a peer debriefer, as well as notes on literature findings that I used to construct a working hypothesis. By keeping a journal, I was able to review each step in the research process while conducting the research. The journal helped me construct the research design, determine a working hypothesis, analyze data, and interpret the results.

Data analysis

Data collected (the translated history textbook) were subjected to qualitative analysis. This method uncovered the unforeseen and unexpected patterns that helped to give new insights about natural phenomena (Gall, Borg, & Gall, 2003). Data were analyzed using content analysis (Creswell, 2011). The content analysis steps followed in the current study based on Creswell's framework are: (a) unitizing data; (b) labeling the unitized data with codes; (c) categorization of the codes; (d) identifying themes (Creswell, 2011, p. 244).

Unitizing data

In order to unitize the data, the data—translated into modern Turkish—were first transcribed into Microsoft Word file. Second, I coded this transcript, and supplemented units with information from my literature review. I used memoing to represent my opinions and feelings about the findings. These data were transferred onto cards. The cards had two sides: the category and its information were written on the front, while a memo was recorded on the back (*Figure 1*). Data came from 81 pages of transcripts.

The aim of the unitization was to combine related content into temporary categories (Lincoln & Guba, 1985). By using this process, I could easily discover patterns. I used the following steps to form categories: I picked a card, studied it, and put down it in one category. If the relevant information differed from the established category, another category was formed. This process continued until each card was analyzed (Creswell, 2011).

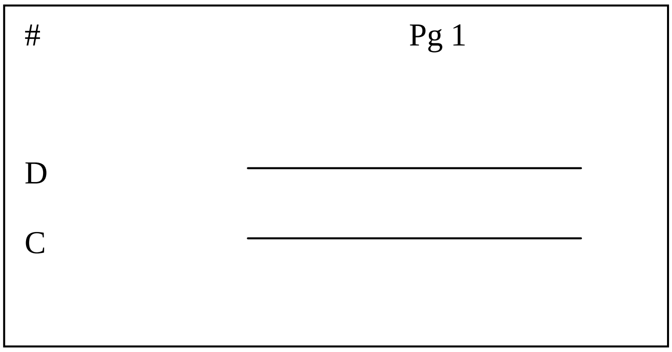


Figure 1. Example of a unit card; # = card number; Pg 1 = page number in the transcript; D = data; c = code.

Eventually, various categories were created. At the end, each category was reviewed and possible categories were reconsidered for accuracy. The final categories were: (1) the scope of the textbook; (2) procedural knowledge; (3) real life examples; (4) number concept; (5) multiple representations (6) problems with basic operations; (7) reasoning strategies; (8) measurement units; (9) addition facts; (10) multiplication facts; (11) subtraction facts; (12) division facts; (13) exercises with basic operations; (14) challenging tasks; (15) drill exercises.

During the categorization process, a peer debriefer was used, and major themes and patterns emerged. The themes that emerged were:

Theme I: Number sense skills

Theme II: Procedural mastery in basic arithmetic operations

Theme III: The concept of measurement

Theme IV: Procedural skills and level of challenge in questions

Ensuring reliability

In order to obtain reliable data, I incorporated several techniques into the study: I used multiple historical sources to increase the probability of producing credible findings, and the information in these sources provided external checks on the inquiry process. An expert in Ottoman historical textbooks was also consulted during interpretation of the data.

Researcher reflexivity

I kept a journal describing the analysis in detail throughout the study. This journal was kept to help ensure reliability and to document the process of the study. The journal included records of the conversations with the translator and the professor, as well as a weekly schedule. The description of people and logistics (when and where) were recorded for accountability of the study (Lincoln & Guba, 1985).

Peer debriefing

A debriefer is identified as “someone who is in every sense the inquirer’s peer, someone who knows a great deal about both the substantive area of the inquiry and the methodological issues” (Lincoln & Guba, 1985, p. 308). The chosen peer debriefer provided an external check of the data analysis process. The peer debriefer designed a process that helped me to discover my own biases, explore meaningful findings, and clarify interpretations as they occurred in the data (Creswell, 2011). The peer debriefer was a master student candidate who was in the process of finishing her thesis on another study with a similar methodology. Her familiarity with the dynamics of the university’s administration and her special interest in analyzing textbooks made her the ideal debriefer for this study. I met several times with the peer debriefer during the analysis of the document. Most sessions involved conversations, discussions, and question-and-answer periods.

CHAPTER 4: RESULTS

Introduction

This section includes the results of the study collected from the data and rich information connected to the findings of the data analysis. From the analysis, I identified four themes: numbers sense skills, procedural mastery in basic arithmetic operations, the concept of measurement, and problem solving skills.

Number sense skills

The first theme to emerge from the results is that of number concepts and number sense.

The first example involves the use of strategies to develop students' learning of the concepts of counting. I found that the concept of *less* and *more* was presented while introducing numbers, which was also supported with the question, which is *more/less*? As Broody (1987) stated, the relationship among numbers can be supported with these *more/less* activities, and the textbook used this strategy. However, Van de Walle et al. (2010) also emphasized the difficulty of the *less* concept compared to the *more* and I found that textbook allows students to get more exposure to the word *more* than *less*; which is not compatible with the developmental levels of the students.

A second example concerns the use of numbers as anchors or benchmarks. The author encouraged the use of 10 as a benchmark in order to develop the relationship among numbers. The textbook categorized numbers in groups of 10 (e.g., introducing numbers from 1–10 followed by 10–20, 20–30, etc); this sequence continues until 100. *Figure 2* illustrated the numbers from 1-10:

Figure 2. Numbers 1 to 10.

A third example concerns the use of real life examples or pictures. While numbers up to 10 were first symbolized with bird pictures—the zero concept was indicated with a no bird image. As Van de Walle et al. (2010) suggested that real life examples are useful to indicate numbers, including zero, which was followed by its symbolic representation. However, this characteristic of zero may mean for the student *a lack of a characteristic* and that may lead to a misinterpretation of all numbers that includes the zero symbol. For example, 10 (ten) is likely to be interpreted as 1 (Anghileri, 2006).

The textbook uses the cardinality principle. The way that this principle is used in the textbook allows the student to match counting words with objects one by one. This principle; however, is only used in smaller numbers. When the numbers got bigger, for example for 20 to 30, it is assumed that the student had already understood that the last number in a set has a special meaning: the last number represents the number of elements of the set. I believe that this is compatible with the developmental levels of the students (Nikoloska, 2009; Van de Walle et al., 2010). The *Figure 3* represented number ten as the whole in the pears set:

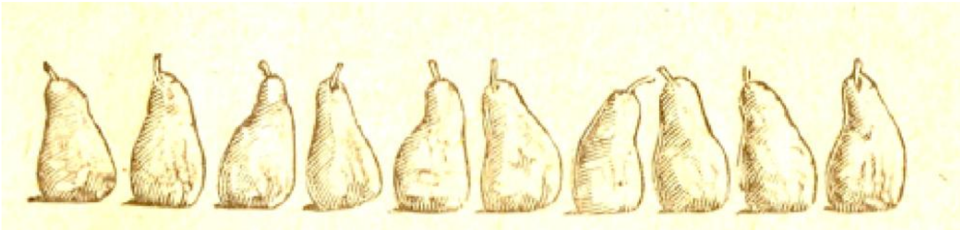


Figure 3. Ten (10) pears.

The textbook mentions the *base-ten system* in a developmental way; decimal places are presented while numbers are introduced in an increasing order. However, there is no comprehensive explanation about decimal places throughout the textbook. The textbook assumes that students can develop an overall understanding of decimal places from particular examples. These particular examples includes decimal places up to thousands which are introduced by using their location on a number—the ones on the right and the tens and hundreds places to the left. By doing so, the textbook shows the difference between consecutive decimal places. I do not find this approach trivial. For example, the author wrote, *The ones are located in the first column from the right*, which I think might be a useful way of avoiding confusion. Because my teaching experience in a high school showed me that even some of the high school students confused where the tens or hundreds were located in a number.

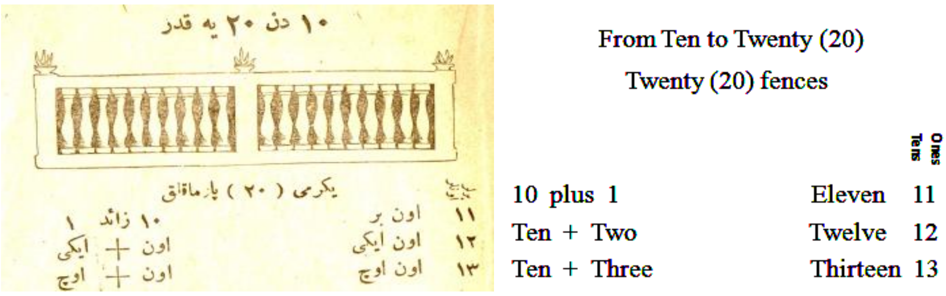


Figure 4. Units in numbers.

As illustrated in *Figure 4*, the textbook includes multiple representations for introducing numbers; this example was chosen from place values topic. Splitting the numbers into tens and ones, is expressed using images and written language as well as expressing eleven as comprised of 1 ten and 1 one. Van de Walle et al. (2010) stated that the more ways that are given to think about a concept, the more learners integrate the concepts in a meaningful manner. Thus, this repetition can be useful. In addition, the numbers (11, 12, 13, and so on) introduced in the *Figure 5* with written and verbal forms. Some researchers support this strategy that promotes students' awareness and their critical thinking abilities (Aiken, 1972; Carpenter & Lehrer, 1999). Therefore, textbook successfully integrates verbal and written representations.

Procedural mastery in basic arithmetic operations

The textbook combines the introduction of basic arithmetic operations with numbers. Numbers are constructed (1–1000) by indicating their relationships in terms of arithmetic operations. Particularly, addition and multiplication are used more than other basic operations.

The *addition* concept is combined with numbers by consecutive counting. Interestingly, in the textbook, addition is not explicitly defined first; instead, only some exercises and consecutive counting are initially employed, which harks back to Van de Walle et al. (2010), who stated that exemplifying concepts instead of providing direct definitions is effective to illuminate the logic behind the concepts. Thus, this strategy stimulates youngsters to explore the concepts independently.

Formalizing language is frequently used in the addition exercises. In the textbook, addition is represented as *one more than, two more than, make more, plus,* and *add*. It was interesting that the textbook introduced the mathematical notation, precisely the *plus* word and its (+) symbol in the second example, which followed an example with only written language. Furthermore, these two examples show that the verbal and symbolic representation are used in a developmental way. Symbolization help young children to remember more easily, as proposed by Carpenter and Moser (1984). The language used for the operations is instrumental in facilitating the early stages of conceptual understanding, as expounded by Anghileri (2006).

The following exercises from the textbook exemplify this concept:

1) *If there is one finger and one finger more, how many fingers do you have? (p. 9)*

2) *If there are two fingers and two fingers more (plus (+)), how many fingers do you have (p. 9)?*

The addition concept is properly defined only through the end of the textbook. It is illustrated in the textbook using the following example:

I am adding 3 walnuts to 6 walnuts. That means that I am doing addition. When I added those altogether, I had 9 walnuts. This is called the total (yekun). The symbol of addition is +, called plus or more (p. 11).

I remember that in elementary school, the instruction of mathematical concepts started with defining the definition, then exemplifying the structure, and finally demonstrating some examples, before asking students to solve problems. However, the structure of this textbook is different. Examples and problems constitute the first step of instruction in the textbook. In fact, as Brahier defined: “this method is often referred to as inductive teaching because the student thinks through several examples and then generalizes a rule at the end” (p.63). It is surprising to me to observe that the current textbooks used in Turkey generally follow a “deductive method of teaching; which states a rule or definition and then expects the students to apply it to invent some of their own rules and procedures” (Brahier, 2013, p.63). I think that the addition concept addressed students’ developmental level by using addition in an inductive way.

Once the *one more strategy* is employed, the *two more than strategy* is linked with even and odd numbers for two digit numbers. The concept of adding two is treated by researchers as a useful practical activity (Carpenter & Moster, 1984; Anghileri, 2006; Van de Walle et al., 2010), especially as larger numbers such as *30 and 2 more* or *57 and 2 more* establish proficiency in counting. Five is also used as *an anchor*; fingers in various counting and addition exercises demonstrate this while adding up to over 50. Therefore, using anchor is an effective tool in independently calculating other answers rapidly.

In addition, the question-answer teaching strategy is employed to specify certain elements of addition. The following is an example from the textbook:

What is addition? Addition is adding the same kinds of things together and reporting the result with the same kinds. How do you write the numbers that you will add together? Write the ones under the ones, the tens under the tens, and the hundreds under the hundreds (p. 39).

Therefore, I consider that the organization of the textbook seems expecting students to deduct the addition concept by themselves with addition elements and some examples.

Finally, the sum with zero refers to identity property in addition. For some students, it is a difficult structure (Anghileri, 2006). Adding with zero is not demonstrated anywhere in the textbook. I experienced that even some middle school students do not grasp identity property of zero in addition. I remember one student who asked why he added if he had nothing to add? In this respect, I would expect to see adding with 0 mentioned in the textbook.

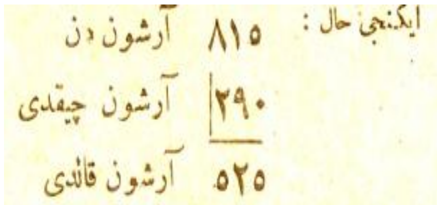
Subtraction is illustrated by employing the addition concept. The first reasoning strategy is the *one less strategy* that is used in number counting, which accompanies the *one more strategy*. The textbook compares these two strategies by using the same numbers; this approach is more effective because of the difficulties of subtraction, and children can grasp the concept more clearly (Thornton & Toohey, 1985).

Although adding zero is not specifically explained in numbers, zero is employed in many subtraction exercises. As previously mentioned in the addition section, zero is associated with the no bird scenario, which establishes an understanding of the conceptual meaning of zero. In addition, the *finger method* (using fingers for arithmetic operations) is implemented for subtraction by asking:

How many fingers are left if you close 3 out of 5 fingers (p. 9)?

The word *close* attracted me; instead of using the word *subtract*, the word *close* was used, emphasizing the effective use of the language (Van de Walle et al., 2010). Formalizing phrases such as *take away* and *decreasing* are other language preferences used in the textbook. However, those words should be used carefully. For example, 4 less than a number can be understood as 4 subtracted from any given number while at the same time it can be understood as any given number subtracted from 4 (Capraro, Capraro & Rupley, 2011) .

The textbook uses the *two less strategy* in the second stage. This helps students to understand the concept of subtraction while they use two as a subtrahend, and this reasoning strategy helps in solving multi-digit problems (Van de Walle et al., 2010). *The two less strategy* is employed for introducing odd and even numbers, and children can compare addition and subtraction when they are inverse and reversible.



Second Case: 815 cubits
| 290 cubits take away
525 cubits left

Figure5. Subtraction.

The process of subtraction is expressed by using the question-answer strategy again. The *Figure 5* explained as*:*

Subtract zero from five; it is five; I write five under the line; 1 is less than 9; the difference between 11 and 9 is 2. I write 2 under the line and you have 1. You have 2, which with the addend 1, equals 3. The difference between 8 and 3 is 5(p.44).

I thought that children might be confused about how the number 11 suddenly appears, as this is not explained in detail. The question might arise in young learners' minds: Why do you add 10 to 1 instead of any other number, or why do you add 1 to 2? This might be confusing.

Multiplication is introduced by using two; this strategy is called the *doubles strategy* (Van de Walle et al., 2010). Using two as a factor helps children to develop effective reasoning strategies. Moreover, zero is also represented as a factor. However, the zero effect in multiplication is not explained, and only the difference between addition and multiplication is demonstrated (e.g., 5 + 0 stays the same, 5 × 0 is always zero). Five, as a benchmark, is illustrated for the numbers from 1 to 50. It seems to me that the relationship between addition and multiplication is represented with the primary aim of showing the system of patterns, which is compatible with developmental level of students.

The symbol and its oral expression as well as the elements of multiplication in number introduction are demonstrated, and multiplication and addition are comparatively explained using the same numbers. The symbols along with their oral expressions are also applied to these numbers (Angliheri, 2006). The term *times* is used as a formalizing term in the textbook, and the examples are as follows:

How many fingers are there if I have 3 times 3 fingers? How many fingers are there if I have 2 times 3 fingers (p. 77)?

The procedure of multiplication is divided into two parts. In the first part, a three digit number is multiplied by one digit. In the second section, three digits are multiplied by two digits. I believe that separating the process into two different parts in this way is preferable, because they are two distinct processes. In addition, the multiplication algorithm is explained in a detailed stepwise manner. I assume that children can apply their knowledge to a given exercise when the multiplication operation algorithm is clearly depicted.

Multiplication tables constitute yet another important aspect of the textbook as well as an integral part of elementary classes; it is astonishing that multiplication tables have been used as a teaching method since the 20th century, continuing into the 21st century. The only recent change is that the tables now reach multiplication by 12, whereas in my school years, they only extended up to 10 (*Figure 6*).

ضرب جدولی — یاخود کرات (مابعدی)

٢٤	١٢	کره	٢	٢٠	١٠	کره	٢
٣٦	١٢	-	٣	٣٠	١٠	-	٣
٤٨	١٢	-	٤	٤٠	١٠	-	٤
٦٠	١٢	-	٥	٥٠	١٠	-	٥
٧٢	١٢	-	٦	٦٠	١٠	-	٦
٨٤	١٢	-	٧	٧٠	١٠	-	٧
٩٦	١٢	-	٨	٨٠	١٠	-	٨
١٠٨	١٢	-	٩	٩٠	١٠	-	٩
١٢٠	١٢	-	١٠	١٠٠	١٠	-	١٠
١٤٤	١٢	-	١٢				

Multiplication Table

2	times	12	24	2	times	10	20
3	-	12	36	3	-	10	30
4	-	12	48	4	-	10	40
5	-	12	60	5	-	10	50
6	-	12	72	6	-	10	60
7	-	12	84	7	-	10	70
8	-	12	96	8	-	10	80
9	-	12	108	9	-	10	90
10	-	12	120	10	-	10	100
12	-	12	144				

Figure 6. Multiplication table.

I found that, unlike the other three operations, the first task of the *division* operation was quite complex. The textbook began by asking:

Do you think 5 fingers can divide into two parts (p. 9)?

This is difficult, as compared to questions for other basic operations, such as*:*

If I add 3 fingers to 4 fingers, how many do I have? or if I close 3 fingers out of 5, how many fingers are left (p. 10)?

The numbers chosen for the division question imposes difficulties for beginners. Moreover, the textbook omits to explain the division concept in the previous pages; therefore, children need to consider how they can divide five into a whole number. In addition, formalizing language is used for division. In the exercises, generally *how many* or *half* refers to the division concept. For instance, the following questions are employed:

How many groups of 4 fingers are in 8 fingers? What is half of 19 (p. 70)?

The method of instruction for the division operation is the same as that for the other basic operations; that is, a question-answer strategy with an inductive teaching method. The elements of division are introduced using daily language first; followed by examples, and finally the mathematical names and the division algorithm.

The concept of measurement

There are many measurement instruments in the textbook, such as time, recipe ingredients, weight, distance, and size, at an elementary level. Units of measure are converted into others, and figures support visual images and daily life examples.

Time is the first concept. The textbook first depicts a clock. Using a question-answer technique, the units of time (i.e., seconds, minutes, and hours) are introduced. I found the explanation of time duration quite intriguing. Questions arose while introducing the names of the clock's hands. For instance, the textbook states: *The minute hand refers to what?* Then, it states: *It shows minutes*. Therefore, it is assumed that children know the minute concept. Youngsters might be expected to independently explore this concept.

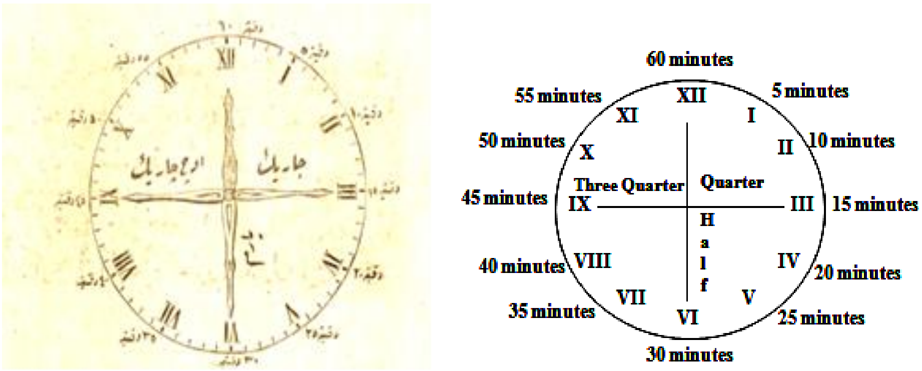


Figure 7. Time clock.

Figure 7 depicts the minutes on a clock in five-minute intervals, which is incorporated into a question.

The second measurement instrument is the meter, and a real life situation is presented as an example to illustrate it:

The meter is used for measuring fabric. What is the meter for? The meter measures the length of fabric, of a wall, or of a plank (p. 55).

Figure 8 shows that the metric system was already in daily use in the 20th century, which is surprising because I consider that the Ottoman Empire promoted its own, unique measurement system. Günergun (1993) revealed that a law regarding the measurement system was promulgated in 1869. According to this law, the metric system was prescribed for length measurements, and grams for weight measurements. In addition, because of complications in enforcing this system nation-wide, both systems are demonstrated in the textbook.

Figure 8. Measuring fabric.

Units are converted according to the *base-10 system*, and the multiples and sub-multiples of the units follow a decimal pattern, which the textbook depicts by using the phrases *bigger than* and *less than*. Pumala and Klabunde (2005) support the idea of designing the smallest and largest units around the powers of ten. Therefore, this strategy provides opportunity for both developing students’ number skills and measurement units. Moreover, the meter stick (*Figure 9*) precision is decimeters (there are ten of these per meter) and centimeters (ten of these per decimeter).

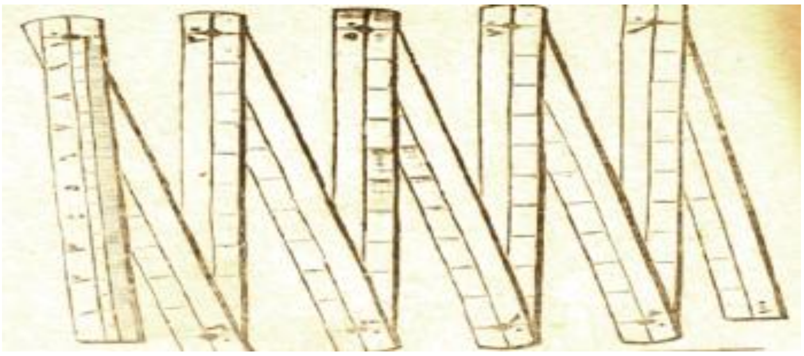


Figure 9. Meter stick.

Another unit of measurement is the liter. The sub-liters and multi-liters are illustrated according to the base-10 system. As Van de Walle et al. (2010) explained, the instruction regarding measurement units encourages pupils' place value understanding. This can be an effective method of relating concepts with numbers.

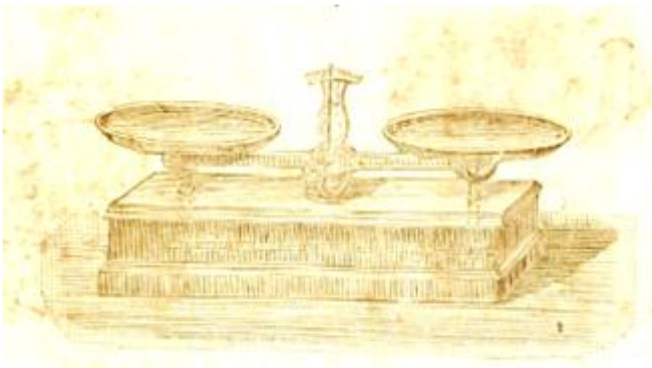


Figure 10. Scale (*terazi)*

The gram is the next measurement unit introduced in the textbook, and the figure above elucidates this measure of mass on a balance scale. Illustrated in *Figure 10*. The mass is pictorially represented in grams, kilograms, hectograms, and ten grams.

Günergun (1993) revealed that old and new measurement systems are depicted in mathematics textbooks, and this textbook also briefly presented old measurement units. The old system included various kinds of units that I had never encountered. For example, length was measured as, translates into English as, architecture's cubit (*Mi'mar arsını)*, bazaar's cubit *(carsı arsını)*, and ell (*endaze*).

Procedural skills and level of challenge in questions

The problems seemed to me to be the most interesting parts of the textbook. These included many challenging tasks and intriguing questions. There is a tendency to ask different kinds of questions before providing explanations. Considering the difficulty level of the questions, which is relatively high, the intended curriculum might require this approach. Further, the solutions require recalling previous knowledge and the ability to apply it in any given situation.

The questions regarding number concept mostly employ writing exercises:

Write the numerals above many times, write the following numbers five times neatly under each other, mentally calculate, memorize it, write what is written below, and complete the blanks (p. 7-38).

I realized that memorization is the predominant teaching practice, and I remember that in my elementary school, the same strategy was used. However, although continuously repeating the same sentences reinforces memory, it may lead to neglecting the logical structure during writing (Angliheri, 2006). Thus, procedural skills of children might be developed more than reasoning skills.

Different kinds of questions are asked in the counting exercises. I have seen real life situations such as:

Count the walnuts from 1 to 10(p. 3); if you have 21 clocks and 2 clocks, how many clocks you have (p.19) ?

The numbers chosen for the exercises seemed to me to be challenging, and there is an inconsistency between the instructions and the questions asked; for example, although numbers from 20–30 are introduced, the exercises include *71 minus 1, and 50 minus 3.*

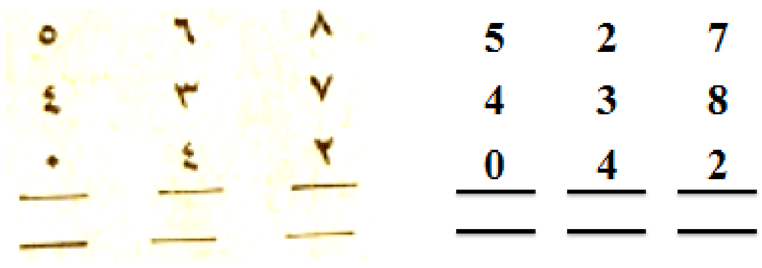


Figure 11. Addition of three numbers.

Another example of a challenging task concerns addition. As illustrated in *Figure 11*, three numbers are added. However, the instructions only contain numbers from ten to twenty. I thought that the relation might be explained when the outcome was calculated, the result is a number between ten and twenty. Moreover, zero is involved, which might pose a challenging problem, as there is no explanation in the textbook.

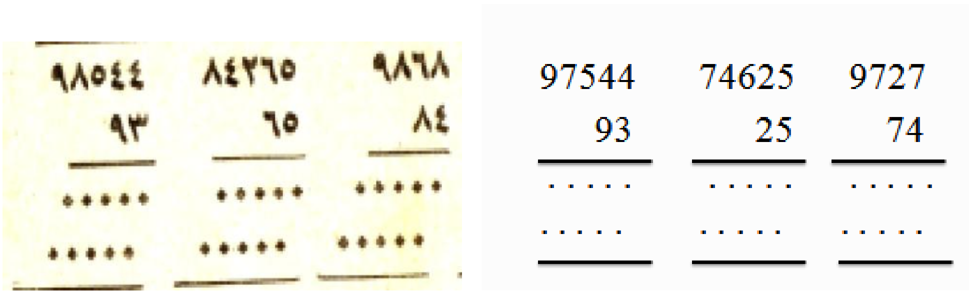


Figure 12. Multiplication with multi-digits.

Figure 12 is very challenging, to the point of being demoralizing. Since there is no explanation for five digit numbers and their multiplication by two, the children can have a difficulty calculating the answer to this question.

Further, problems are of an identical type, and repetitive drudgework is expected. Researchers call this procedure a *drill exercise*; this is used in exercises for numbers and basic operations, and measurement units are repeated many times (Van de Walle, 2010; Anghileri, 2006). It is redundant to ask the same type of question more than five times. Drill exercises help children to calculate the operations as a reflex action. Examples of *drill exercises* are:

20 subtracted by 1; how many are left? Subtracted by 3? Subtracted by 2? Subtracted by 4? Subtracted by 5? Subtracted by 2? and *5 deciliters is how many* *liters? 4 hectoliters? 9 hectoliters? 2 hectoliters? 16 hectoliters? 10 hectoliters? 20 hectoliters? 7 hectoliters (p. 19)?*

Moreover, there are many mental calculation questions. My teaching practice has convinced me that practicing mentally is not easy. Some examples are:

How many ones and tens are there in 56? In 42? In 63? In 68? In 70? In 48? In 26? In 50? In 16? In 18? In 13? In 15? In 3(p. 2?)?

Join problems: result unknown strategy is mainly used for addition problems (Van de Walle et al., 2010; Anghileri, 2006). This strategy employs with real life stories and asks about the whole by indicating the parts. For example,

What is the total of 20 books and 7 notebooks? Mr. Ahmet has 10 pens and if he takes 4 more pens, how many pens does he has (p. 72)?

The subtraction problems involve *separate problems: result unknown strategy*; these subtraction problems mostly ask about the part from the whole. In addition, an interesting problem type in subtraction involves combining addition and subtraction into one question. You can see some examples in the following sentences: *I have 843 grains of wheat in a bag.*

1) How many grains of wheat are left if I take 40 grains of wheat and add 100 more(p. 52)?

2) A merchant has 768 coins. He added 100 coins more. Then he took 10 coins. How many coins are left at the end (p. 76)?

The multiplication problems also include *the equal groups: whole unknown strategy*. I observe that multiplication and addition are used more frequently than the other two operations. However, in the arithmetic problems section, the author emphasized number sense skills and addition is much more than the other concepts. It seems that addition might be the most important operation considering real life situations. Moreover, the numbers chosen for the problems increased to seven digits, although the textbook did not explain millions or thousands. This approach could be difficult.

CHAPTER 5: DISCUSSION

Introduction

The findings of this study indicate that the instruction in the textbook was inductive, self-regulating, direct, but at times too challenging. The textbook provided opportunities to analyze number relationship tasks using real life connections and multiple representations yet procedural knowledge was fostered heavily. Thus, reasoning was not emphasized. This chapter explains the main findings in reference to the literature.

Major findings

There were two major findings in this study.

- There is evidence of a progression for multiple representations and real-life examples, which is compatible with the developmental level of students.
- The textbook focused too much on procedural skills and ignored problem solving or reasoning skills.

Discussion of major findings

Findings related to multiple representations

The findings with respect to the representation of mathematical concepts in the textbook can be explained by combining verbal and visual experiences to provide cognitive structure (Anghileri, 2006). In addition, using multiple representations is accepted as core methodology in mathematics education, and interacting with multiple representations promotes deeper understanding and elucidates meanings more effectively (Elia, Gagatsis, & Demetriou, 2007).

All topics in the textbook are related to real life issues and illustrate connections using the appropriate mathematical representations, which increases the flexibility of learning (Brahier, 2013). When situations are represented in various different ways, young learners will be able to independently explore and choose the most appropriate method for themselves (Fuson, 1986). Explaining basic operations and numbers using words, pictures, and images can help children in effective thinking and learning (Van de Walle et al., 2010).

The real-life word problems support the ideas conveyed by these findings. A highly integrated understanding can be achieved when contextual problems are involved (Carpenter, Hiebert, & Moser, 1999). Asking word problems with larger numbers can elucidate the operations (Jung, Kloosterman, & McMullen, 2007). Real life problems may capture learners' interest, allowing them to develop a positive disposition toward mathematics (Hanna, 2000).

The representation of measurement units in the textbook is consistent with extant studies (Thompson & Preston, 2004). Measurement activities help young learners to connect their ideas regarding numbers to the real world and provide them with number sense; further, using multiples of ten to show relationships among nonstandard measurements develops the idea of the *base-10 system* of numeration (Hiebert, 2013). However, the frequently used instrument for measuring time is the clock (Austin, Thompson, & Beckmann, 2005). Answering with units in measurement problems can help mitigate learners' misconceptions.

The findings in relation to language usage can be explained using Pimm's study (1987) on linguistics: using several semiotic systems simultaneously (symbols, oral language, written language, and visual representations) can help learners in recognizing mathematical concepts easily. According to Anghileri (2006), formalizing language is beneficial during the first stage of introducing mathematical concepts. Specifically, using the expressions and words develops children's understanding: *one more, two more, ten less, altogether, and leaves* (Anghileri, 2006). Besides, the mathematical symbolism and written language sufficiently help in constructing the meaning. In this context, traditional phrases such as *copy, write, memorize, and drill* require lower level thinking abilities, and these words do not sufficiently help learners in performing mathematical operations (Van de Walle et al., 2010).

Relating different topics to each other helps young learners in visualizing the coherent whole; linking the *base-10 models* with the written forms of numbers can increase a child's awareness (Kari & Anderson, 2003). Establishing the relationships between operations and numbers can also help children in arriving at accurate mathematical judgments (Anghileri, 2006), and in developing useful strategies for dealing with operations and numbers (McIntosh, Reys, & Reys, 1992). Representing the relationship among addition-subtraction and multiplication-division may establish useful connections for arithmetic problems.

Findings related to procedural skills

The textbook can be understood using the different keywords used throughout it (e.g., *less, more, times*) (Hegarty, Mayer, & Monk, 1995). The superficial usage of these words may lead young thinkers to believe that certain words will solve certain tasks (Schoenfeld, 1992). This restricted tendency in students' thinking as regards to different reasoning strategies may be one of the main causes of learning difficulties. Reasoning is established based on experiences, as problem solvers transfer and combine solution procedures from familiar situations or apply the same strategy with surface consideration (Lithner, 2000).

Challenging tasks can be determined according to the effectiveness of developing the creativity of young learners (Powell, Borge, Fioriti, Kondratieva, Koublanova, & Sukthankar, 2009). Memorization cannot help them with mastering basic concepts, and it damages their attempts to learn mathematics. Memorization and drill exercises might not be effective enough to justify the knowledge gained by using them. Therefore, challenging tasks can prepare students to create their own strategies. However, the task should be carefully designed because learners' experiences must be consistent with the given tasks (Schoenfeld, 1992); merely indicating in the textbook to *solve the following problems* without attempting to cement learners' knowledge is not an ideal approach for students' learning (Askey, 1999).

The findings with respect to the analysis can be clarified with the accumulation of knowledge of some procedures. Constructing knowledge begins at the implicit and procedural level and, then, it gradually attains the explicitly well-understood level

(Karmiloff-Smith, 1995). Learning begins with the procedures and, then, an understanding of the content is robustly constructed (Rittle-Johnson & Alibali, 1999). Building conceptual knowledge is at the reflective level and requires recalling previous knowledge while reflecting on the information being connected; it is a higher level of thinking (Schoenfeld, 1992). Considering the level of the textbook, the explanations of the arithmetic concepts are at the primary level.

The first part of the procedural knowledge form of mathematics includes familiarity with symbols and awareness of the syntactic rules. The knowledge of symbols and the syntax of mathematics generally refer to surface level features; this is not a higher level of understanding. The introduction of numbers and measurement in the textbook belongs to this category. The second part comprises rules and procedures that connect the textbook with basic operations. Systematic instruction may induce learners to recall certain kinds of procedures. In this context, basic facts and arithmetic problems of the textbook can be related to the second category (Hiebert, 2013).

The accumulation of information in the early phases of performing arithmetic operations may require the execution of many procedures; this might induce learners to memorize individual pieces of information. However, the mathematics education literature tends to cite the same kinds of problems in primary school, and these problems are not well-designed in terms of supporting higher level thinking (Star, 2002). Competent mathematical performance may not occur when these repetitive problems are continuously asked, and the understanding of procedures in two or three problems can lead the solver to think that proficiency has been achieved. Therefore, a student might erroneously believe that there is no need for more thinking (Mayer, 2002). I believe that this phenomenon in today's textbook can be applicable to explain the heavy emphasis on procedural knowledge in the textbook which was analyzed for this study.

According to Carpenter and Lehrer (1999), the procedures used by young children for conceptual knowledge are limited. Therefore, procedures lack the flexibility required to maintain the conceptual framework, and problems are solved by directly modeling the given processes and representations. Young children may be unable to undo their actions or to take apart the pieces; they follow a sequence of procedures to achieve a result, and they cannot reverse it.

The findings concerning *drill exercises* can be explained with memorization. According to Van de Walle et al. (2010), some textbooks move directly to memorization of facts after presenting concepts regarding basic operations. However, students face difficulties in the fourth and fifth grades since they have not mastered these operations. Then, in middle school, these students might still lack robust knowledge of the basic operations. This implies that memorization may not develop students' skills. Different strategies and processes for basic operations cannot be developed through drill exercises (Brownell & Chazal, 1935).

Implications for practice

According to the results of this study, I believe that mathematics education of Ottomans in the elementary level provides different kind of opportunities to connect mathematics with real life. Therefore, new textbook authors might provide this opportunity in the modern Turkish elementary level mathematics textbooks.

In addition, there was some evidence to traditional teaching methods such as memorization, and drill exercises. Therefore, the young learners might not be encouraged to develop higher mathematical thinking in the early 20th century. I strongly believe that the emergence of new studies might reveal the possible aspects of mathematics education that shape our societies. In this respect, textbook writers should review the historical textbook analysis studies.

Implications for future research

The investigation of the textbook may be informative for the enlightenment of historical mathematics education, and the results from this study add to the emerging body of literature on mathematics textbook analysis. The reported findings revealed that there was need to improve instruction in order to enhance the learning of students with disabilities. Traditional teaching methods (memorization, drill exercises) should not be more dominant than methods that foster reasoning and problem solving skills.

This study explored the historical bounds of the mathematics education analyzing one Ottoman elementary mathematics textbook. The study only focused on evaluating and revealing the mathematics textbook in terms of content, organization and instructional strategies. However, analyzing one mathematics textbook is insufficient for the comprehensive exploration of mathematics education. Therefore, future studies can focus on other mathematics resources. Moreover, because this study used the qualitative analysis method, quantitative analysis could be used for future historical textbook studies.

Limitations

The main limitation of the study concerns the number of textbooks, which were analyzed in this study. Insufficiency of the budget and other resources for a more accurate and multidimensional analysis were needed.

Thus, the findings should be interpreted with caution given the limited scope of the study, which did not include any comparison goal with other mathematics textbooks published in the first quarter of the 20th century. The sample was limited to one elementary school mathematics textbook, and the analyses should not be extended to elementary school mathematics textbooks published in the early 20th century.

Secondly, the study depended on a historical mathematics textbook. Therefore, the application of classroom teaching materials for analyzing the effectiveness of the instrument is not possible. In order to reduce this limitation, the data were enriched by investigating various kinds of research papers and books in the early 20th century. Moreover, the transcription required a large budget since it is not easy to find an expert in mathematics terminology, much less in the Ottoman-Turkish language.

Finally, the researcher is aware of making comparison of data by today's standards does not provide an accurate analysis. The circumstances of the first quarter of twentieth century should have been taken into a more rigorous consideration during data analysis process.

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