MUSTAFA AYDOS

2015

# THE IMPACT OF TEACHING MATHEMATICS WITH GEOGEBRA ON THE CONCEPTUAL UNDERSTANDING OF LIMITS AND CONTINUITY: THE CASE OF TURKISH GIFTED AND TALENTED STUDENTS

A MASTER'S THESIS

BY

MUSTAFA AYDOS

THE PROGRAM OF CURRICULUM AND INSTRUCTION

İHSAN DOĞRAMACI BILKENT UNIVERSITY

ANKARA

JUNE, 2015

# THE IMPACT OF TEACHING MATHEMATICS WITH GEOGEBRA ON THE CONCEPTUAL UNDERSTANDING OF LIMITS AND CONTINUITY: THE CASE OF TURKISH GIFTED AND TALENTED STUDENTS

The Graduate School of Education

of

İhsan Doğramacı Bilkent University

by

MUSTAFA AYDOS

In Partial Fulfilment of the Requirements for the Degree of

Master of Arts

The Program of Curriculum and Instruction

İhsan Doğramacı Bilkent University

Ankara

JUNE 2015

İHSAN DOĞRAMACI BILKENT UNIVERSITY
GRADUATE SCHOOL OF EDUCATION

The Impact of Teaching Mathematics with GeoGebra on the Conceptual Understanding of Limits and Continuity: The Case of Turkish Gifted and Talented Students

Mustafa Aydos
JUNE 2015

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Curriculum and Instruction.

----------------------------
Assoc. Prof. Dr. M. Sencer Corlu (Supervisor)

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Curriculum and Instruction.

----------------------------
Assoc. Prof. Dr. Serkan Özel

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Curriculum and Instruction.

----------------------------
Assist. Prof. Dr. Jennie F. Lane

Approval of the Graduate School of Education

----------------------------
Prof. Dr. M. K. Sands (Director)

## ABSTRACT

THE IMPACT OF TEACHING MATHEMATICS WITH GEOGEBRA ON THE CONCEPTUAL UNDERSTANDING OF LIMITS AND CONTINUITY: THE CASE OF TURKISH GIFTED AND TALENTED STUDENTS

Mustafa Aydos

M.A., Program of Curriculum and Instruction
Supervisor: Assoc. Prof. Dr. M. Sencer Corlu

June, 2015

There is strong evidence in mathematics education literature that students benefit extensively from the use of technology that allows for multiple representations of mathematical concepts. The benefits include developing an advanced level of mathematical thinking and conceptual understanding. The purpose of this study was to investigate the impact of teaching limits and continuity topics in GeoGebra-supported environment on students' conceptual understanding and attitudes toward learning mathematics through technology. The sample consisted of 34 students studying in a unique high school for gifted and talented students in Turkey. This study followed a pre-test post-test controlled group design. Conceptual understanding of the topics of limits and continuity was measured through open-ended questions while attitudes toward learning mathematics through technology was measured using a Likert-type survey. The intervention was teaching with GeoGebra in contrast to using traditional instruction in the control group. Data were analyzed with an independent samples *t*-test on gain scores for control and experimental groups. In the conceptual understanding test, the gain scores of the experimental group was found to be 1.33 standard deviations higher than that of the control group on the average. This finding was evaluated noteworthy in terms of previously-conducted research on the impact of GeoGebra. Furthermore, the study found that student attitudes toward learning mathematics through technology improved, as well. The researcher concluded that Geogebra may be an effective tool for teaching calculus to gifted and talented students .

*Keywords*: limits and continuity concepts, dynamic geometry, computer algebra systems, GeoGebra, technology integration in mathematics education, gifted and talented students, affective domain, meta-analytical research.

# ÖZET

## MATEMATİĞİ GEOGEBRA İLE ÖĞRETMENİN LİMİT VE SÜREKLİLİK KONULARININ KAVRAMSAL ANLAŞILMASINA OLAN ETKİSİ: ÜSTÜN ZEKÂLI VE YETENEKLİ TÜRK ÖĞRENCİLERİ ÖRNEĞİ

Mustafa Aydos

Yüksek Lisans, Eğitim Programları ve Öğretim
Tez Yöneticisi: Doç. Dr. M. Sencer Çorlu

Haziran 2015

Matematik eğitimi literatüründe çoklu gösterime imkan sağlayan teknoloji kullanımının, öğrencilerin ileri seviye matematiksel düşünme gücünü ve kavramsal anlamalarını geliştirdiğine dair güçlü deliller vardır. Bu çalışmanın amacı, GeoGebra yazılımı yardımı ile limit ve süreklilik öğretiminin kavramsal anlama ve matematiği teknoloji ile öğrenme üzerine olan etkisini incelemektir. Çalışmanın örneklemi üstün zekâlı ve özel yetenekli öğrencilerin bulunduğu bir okulda okuyan 34 lise öğrencisidir. Ön ve son test kontrol gruplu araştırma deseni takip edilen bu çalışmada, limit ve süreklilik konusundaki kavramsal anlama açık uçlu sorular ile ölçülürken, matematiği teknoloji ile öğrenmeye karşı tutum Likert tipi anket ile ölçülmüştür. Ders anlatımı deney grubunda GeoGebra yardımıyla, kontrol grubunda ise geleneksel yöntemlerle yapılmıştır. Toplanılan data kontrol ve deney grubu ön ve son test arasındaki fark (gelişme) puanları için bağımsız örneklem *t* testi ile analiz edilmiştir. Deney grubunun fark kontrol grubuna nazaran 1,33 standart sapma daha fazla gelişme gösterdiği görülmüştür. Bu sonuç daha önce yapılmış olan GeoGebra çalışmalarına göre kayda değer olarak değerlendirilmiştir. Ayrıca benzer bir gelişme tutum ile ilgili sonuçlarda da görülmüştür. Sonuç olarak, analiz konularının GeoGebra yardımıyla öğretilmesinin üstün zekâlı ve özel yetenekli öğrenciler bağlamında etkili olabileceği düşünülmektedir.

*Anahtar Kelimeler:* limit ve süreklilik, dinamik geometri, bilgisayar cebir sistemleri, GeoGebra, matematik eğitiminde teknoloji entegrasyonu, üstün zekâlı ve üstün yetenekli öğrenciler, duyuşsal alan, meta-analiz.

## ACKNOWLEDGEMENTS

I would like to express my most sincere thanks to Prof. Dr. Ali Doğramacı and Prof. Dr. Margaret K. Sands for their support that allowed me to study at Bilkent University. I would also like to thank all other people at the Graduate School of Education for their sincere support.

I would like to extend my deepest thanks to my adviser Assoc. Prof. Dr. M. Sencer Corlu for constantly empowering and supporting me with his never ending patience and lending me his assistance and experiences throughout the authoring process of this thesis. I am very grateful for the time and energy spent by him for me. I am also indebted to Asst. Prof. Dr. Jennie F. Lane and Assoc. Prof. Dr. Serkan Özel, who participated in my dissertation defense as committee members, for their valuable comments and special advice. I would like to thank Mr. Cecil Allen, Asst. Prof. Dr. Jennie F. Lane, and Mr. Çağlar Özayrancı for helping me in the proofreading of my thesis.

In addition, I would like to extend my special thanks to the administration and teachers, particularly to my colleagues in the mathematics department, of the Turkish Education Foundation, İnanç Türkeş Private High School for always supporting me throughout my career.

Finally, I would like to express my most heartfelt thanks to my very dear father ÖMER AYDOS, my very dear mother HAVAGÜL AYDOS, my beloved little sister EDAGÜL AYDOS, and my precious wife KÜBRA KÖROĞLU AYDOS for their infinite love, support, and encouragement. I would not have succeeded in this process without them.

**TABLE OF CONTENTS**

## LIST OF TABLES

## LIST OF FIGURES

# CHAPTER 1: INTRODUCTION

## Introduction

A widely-accepted learning theory in the psychology of mathematics education is Bruner's (1966) stages of representations or *multiple representations* theory (MR theory). As a cognitive theory, Bruner's approach to learning is action-oriented and student-centered. Bruner's theory characterizes three stages of representation: enactive (representation through action), iconic (representation using visual images), and symbolic (representation using symbols) (Goldin, 2014; Goldin & Kaput, 1996; Tall, 1994). In mathematics education, "…representation refers both to process and to product—in other words, to the act of capturing a mathematical concept or relationships in some form and to the form itself…[including representations which are] observable externally as well as those that occur 'internally,' in the minds of people doing mathematics" (National Council of Teachers of Mathematics [NCTM], 2000, p. 67). Today, the MR theory is one of the most popular theories in mathematics education and has dominated the field of mathematics education since its introduction in 1960s during the *new mathematics movement* in the US.

Computer-algebra systems—CAS (see Artigue, 2002), dynamic geometry software—DGS (see Clements, 2000), and graphing display calculators—GDC (see Doerr & Zangor, 2000; Kastberg & Leatham, 2005) are some examples of modern technological tools that enable students to think mathematically in a variety of representations. There is strong evidence in mathematics education literature that, as an application of the MR theory, students benefit extensively from the use of technology in developing an advanced level of mathematical thinking and conceptual understanding (Özgün-Koca & Meagher, 2012). The MR theory is believed to be well-suited to explain the effective utilization of technology for conceptual understanding in mathematics.

Effectively integrating technology into mathematics education has been demonstrated through various software programs. Effectiveness in utilizing technology in mathematics education has been shown to be related to its capacity to allow for timely, efficient, and accurate transfer of external and internal mathematical thinking among enactive, iconic, and symbolic representations of mathematical concepts (Bulut & Bulut, 2011; Kabaca, Aktümen, Aksoy, & Bulut, 2010; Mainali & Key, 2012; NCTM, 2003). Research has provided educators with strong evidence that effective use of technology has resulted in noteworthy gains in conceptual understanding in a variety of mathematical topics, including:

(a) *geometry*; polygons, triangles, circles and Cartesian coordinates (Filiz, 2009; Gülseçen, Karataş, & Koçoğlu, 2012; İçel, 2011; Mulyono, 2010; Selçik & Bilgici, 2011; Shadaan & Eu, 2013; Uzun, 2014);

(b) *algebra*; functions, parabolas, trigonometry and real life problems (Aktümen & Bulut, 2013; Hutkemri & Zakaria 2012; Reis & Özdemir, 2010; Zengin, 2011; Zengin, Furkan & Kutluca, 2012); and

(c) *calculus*; limits and continuity, differentiation, and integration (Caligaris, Schivo & Romiti, 2015; Kepçeoğlu, 2010; Kutluca & Zengin, 2011; Taş, 2010).

The American-based NCTM (2003) states that DGS has emerged in recent years as an effective technological tool for visualizing abstract mathematical structures. The rationale was that mathematics uses everyday words with different meanings in different contexts (Mitchelmore & White, 2004) and DGS was successful in creating opportunities that would link real life and abstract mathematical concepts in a variety of contexts (Aktümen & Bulut, 2013; Saab, 2011). Based on the empirical evidence in favor of and policy-makers' support for the effectiveness of DGS in mathematics education, several types of DGS have become popular for teaching mathematics in both the United States and Turkey (Bakar, Tarmizi, Ayub, & Yunus, 2009; Güven & Kosa, 2008; Jones, 2000). Software, which can be categorized as CAS, has been recognized as another aide that allowed users to do computation with mathematical symbols (Aktümen, Horzum, Yıldız, & Ceylan, 2011). There has been some research evidence that supported this family of software for facilitating conceptual understanding, as well (Güven, 2012; Heugl, 2001; Pierce, 2005). Today, DGS and CAS are considered two of the most popular families of software that are used in teaching mathematics for conceptual understanding.

The GeoGebra software is equipped with features of both DGS and CAS. This particular software has established its place as a popular tool that can be used at all levels from primary school to university (Akkaya, Tatar & Kagızmanlı, 2011; Aktümen & Kabaca, 2012; Hohenwarter & Fuchs, 2004; Hohenwarter & Jones, 2007; Hohenwarter & Lavicza, 2007; Kutluca & Zengin, 2011). In addition to its functionality at all levels, GeoGebra is freeware and available in 45 different languages, including Turkish. GeoGebra, which is widely used to teach geometry, algebra, and calculus is an example of the effective use of MR theory in the classroom (Hohenwarter, Hohenwarter, Kreis, & Lavicza, 2008).

## Background

During the last decade, several educational reforms have been introduced in Turkey. The rationale behind these reforms was that Turkey needs to keep up with world-class standards in mathematics and science education. The effective use of technology for teaching mathematics has been particularly emphasized in curricular documents in mathematics education (Ministry of National Education [MoNE], 2005; 2013). In the latest curricular changes of 2013, MoNE has particularly advised mathematics teachers to use software, such as DGS, CAS, spreadsheets, GDC, smart boards, and tablets. This advise has revealed the need for Turkish mathematics teachers to learn how to use these programs and to learn how to use them effectively. In the meantime, the physical conditions and infrastructure of the classrooms needed to be improved and modernized. To this end, MoNE has developed and introduced several large-scale projects, the most important of which being the *Fatih* project—figuratively referring to the *Conqueror* title of Mehmet II (an Ottoman sultan who reigned between 1451 and 1481) and which can literally be translated as an acronym for *Fırsatları Artırma ve Teknolojiyi İyileştirme Hareketi* [movement to enhance opportunities and improve technology].

The Fatih project has been being piloted since 2010 in over 50 schools located in 17 different provinces of Turkey. The ultimate goal of the project was to increase the use of information and communication technologies (ICT) in the classroom; and thus, provide equal access to technology in schools across Turkey. The project website states the overall goal as that, "…42,000 schools and 570,000 classes will be equipped with the latest information technologies and will be transformed into computerized classes" (MoNE, 2012). In order accomplish this goal, there emerged a need to educate mathematics teachers who are capable of employing these technologies effectively. The first objective of the project was to provide smart boards, projectors, internet access, copiers, and printers along with a tablet computer for each student in every classroom in Turkey. The second objective was to deliver in-service training (professional development) for teachers of all subject areas. By ensuring the necessary physical conditions and through the delivery of extensive training on subject-specific technology use, mathematics teachers were expected to effectively integrate technology into their teaching.

Along with the support of the new curricula of 2013, the developments in technological infrastructure of the schools, and professional development activities for teachers, Fatih project is expected to create learning opportunities for students who encounter difficulties in understanding abstract mathematical concepts. Proceeding from this point, the need for affordable, user-friendly, and accessible (i.e., availability in Turkish language) software has been critical for the success of the project. As one of these software programs, GeoGebra is perceived by many as a promising technology (Aktümen, Yıldız, Horzum, & Ceylan, 2011; Kabaca, Aktümen, Aksoy, & Bulut, 2010; Tatar, Zengin, & Kağızmanlı, 2013).

## Problem

First, despite the supporting evidence deducted from studies investigating the impact of similar software programs with different populations of learners, including students in Turkey (Almeqdadi, 2000; Bulut & Bulut, 2011; İpek & İspir, 2011; Kabaca, 2006; Kepçeoğlu, 2010; Subramanian, 2005; Zengin, Furkan, & Kutluca, 2012), there is limited empirical research on the impact of these programs at the high school level or that measures the impact on conceptual understanding in calculus topics. Thus, there is a need to investigate the impact of GeoGebra on Turkish high school students' conceptual understanding of calculus topics.

Second, it is generally expected that around two percent of the individuals in the society are gifted and talented (G&T) when measured through IQ tests (MoNE, 2009). Yet, the number of studies conducted for G&T student populations in terms of teaching mathematics with technology is insufficient. Thus, there is a need to investigate the impact of GeoGebra on Turkish G&T students' conceptual understanding in mathematics and particularly in calculus.

## Purpose

The main purpose of this study was to investigate the impact of teaching mathematics using the GeoGebra software on 12th grade G&T students' conceptual understanding of limits and continuity concepts. A secondary purpose was to investigate the impact of GeoGebra on students' attitudes towards learning mathematics with technology.

## Research questions

The main research questions of the current study were:

a) What is the impact of using GeoGebra on G&T students' conceptual understanding of limits and continuity concepts?

b) What is the impact of using GeoGebra to teach limits and continuity concepts on G&T students' attitude towards learning mathematics with technology?

The null hypotheses can be stated as follows:

$$H_0: \mu_{Gain\_Experimental} = \mu_{Gain\_Control}$$

where $\mu_{Gain\_Experimental}$ stands for the mean of experimental group's gain scores, and $\mu_{Gain\_Control}$ stands for the mean of control group's gain scores. Gain scores are the difference between post-test and pre-test scores. The null hypothesis ($H_0$) states that there is no statistically significant difference between the mean gain scores of experimental and control groups. The alternative hypothesis ($H_A$) states that there is a statistically significant difference between the gain scores of experimental and control groups on the average.

## Significance

The use of technology has significantly increased in Turkey in recent years. These developments have led to certain innovations and reforms in the field of education. These innovations and reforms encourage both teachers and students to use technology in the teaching and learning process. This study contributed to such efforts that focus on increasing the quality and number of resources for students, teachers, and curriculum developers, as well as providing them with empirical evidence. It also serves as an example regarding investigations in other topics of mathematics and leads the way to further research on gifted and talented students.

## Definition of key terms

Multiple Representation (MR) theory emphasizes differentiation through representations and states that there are three stages of cognitive processes, which are enactive, iconic, and symbolic (Bruner, 1966).

Dynamic Geometry Software (DGS) refers to the family of software that assists teachers and students to teach/learn the relations between geometrical behaviors and shapes (Aktümen, Horzum, Yıldız, & Ceylan, 2011).

Computer Algebra System (CAS) refers to the family of software that allows teachers and students to do symbolic and algebraic operations in mathematics in a simpler and easier way (Kabaca, 2006).

Ministry of National Education (MoNE) is the state authority that regulates and allows the opening of all educational institutions from the pre-school level to the end of the 12th grade, that develops their curricula, and that incorporates all kinds of services in education and training programs in Turkey.

GeoGebra is a free and user-friendly mathematics software, which includes features of both DGS and CAS and has been translated into more than 40 languages. The software can be used from the primary school to university level. “GeoGebra brings together geometry, algebra, spreadsheets, graphing, statistics, and calculus” (GeoGebra Tube, 2015).

Conceptual understanding is about making connections between previously learned mathematical concepts and the concept which is being learned or the topics which will be learned in the future. Students with a conceptual understanding are assumed to be skilled in explaining concepts in depth.

Traditional instruction is assumed to be teacher centered where the teacher in the control group in this study used still (non-dynamic) graphs or power-point presentations. The instruction was mostly based on question-answer conversations with the students or paper-pencil activities.

# CHAPTER 2: LITERATURE REVIEW

## Introduction

This chapter establishes the theoretical framework for the study. The purpose is to present a synthesis of theory and research on multiple representations, the use of technology in mathematics instruction and learning, and the role of dynamic geometry software. Research on gifted and talented students in Turkey is included. First, multiple representations (MR) theory is introduced as a constructivist theory of mathematics education. Second, research on the use of dynamic geometry software (DGS) and computer algebra systems (CAS) in mathematics education is critically analyzed. Third, previous studies exploring issues relevant to the teaching and learning of calculus (particularly limits and continuity) concepts are investigated. Finally, a short summary of issues with regards to gifted and talented (G&T) students' education and related research is summarized.

## The multiple representations theory

Jerome Bruner, a prominent psychologist, proposed several theories in the field of education. Bruner's theories focused on cognitive psychology, developmental psychology, and educational psychology (Shore, 1997). Bruner's approach to learning was based on two modes of human thought: logico-scientific and narrative. In order for these modes of thought to be effective, Bruner emphasized the notion that learners would have a better understanding of abstract concepts if a differentiated learning strategy was planned and implemented according to the learner's individual strengths (Bruner, 1985).

Bruner's theory (1966), which emphasized differentiation through representations, stated that there were three stages of each mode of thought: enactive, iconic, and symbolic.

- *The enactive stage* focused on physical actions: Learning happens through movement or actions. Playing with a solid object and exploring its properties is an example of the enactive stage. In a virtual environment (such as DGS or GeoGebra), this stage is interpreted as manipulating the graphs by using pointers (mouse) or hand-held computers.
- *The iconic stage* fostered developing mental processes through vivid visualizations: Learning happens through images and icons. Investigating the properties of a solid shape from the text book images is an example of iconic stage. In a virtual environment, this stage is interpreted as observing teacher or peer demonstration on graphs or tables.
- *The symbolic stage* was characterized by the storage metaphor where information was kept in the form of codes or symbols: Learning happens through abstract symbols. Finding out a solid's surface area or volume by using mathematical symbols is an example of symbolic stage. In a virtual environment, this stage is interpreted as working with the symbolic equations.

Bruner's work on representations has been interpreted as MR theory in mathematics education. Many believed that MR theory would offer an explanation how students learn abstract mathematical concepts through a variety of mathematical representations (Cobb, Yackel, & Wood, 1992; Duval, 2006; Goldin, 2008), and that view was agreed upon by several other reformist mathematics educators (e.g., Brenner et al., 1997) along with some influential mathematics education organizations (e.g., National Council of Teachers of Mathematics [NCTM], 2000). Some prominent researchers advocated for MR theory due its ability to support students' cognitive processes in authentic, real-life problems and learning environments (e.g., Schonfeld, 1985). Some researchers proposed that learning environments that foster conceptual understanding through MR theory could be best created through the use of technology. According to these mathematics educators, technology offered several opportunities for students to learn abstract concepts in ways that are customized and based on students' individual learning styles and interests (Alacaci & McDonald, 2012, Kaput & Thompson, 1994; Özgün-Koca,1998, 2012). Some other researchers advocated for the use of the MR theory to establish the missing link between technology and mathematics education (Gagatsis & Elia, 2004; Özmantar, Akkoç, Bingölbali, Demir, & Ergene, 2010; Panasuk & Beyranevand, 2010; Pape & Tchoshanov, 2001; Swan, 2008; Wood, 2006). Today, there is a consensus among mathematics educators that MR theory is an integral part of reformist mathematics education and that technology plays an important role in achieving the desired outcomes of the reforms.

## Technology in mathematics education

Technology has been playing an increasingly important role in fostering conceptual understanding in mathematics education (Özel, Yetkiner & Capraro, 2008). According to the NCTM (2000), the use of technology has been an essential tool for teaching and learning mathematics at all grade levels as it improves student skills in decision making, reasoning, and problem solving. Similarly, several mathematics educators believe that teaching mathematics in technologically-rich environments was more effective than using paper-pencil based teaching methods (Clements, 2000; Schacter, 1999; Vanatta & Fordham, 2004).

Policy makers in Turkey have been encouraging teachers to integrate technology into mathematics classrooms. The Scientific and Technological Research Council of Turkey (TÜBİTAK, 2005) indicated that teachers at all levels needed to utilize new technologies into their teaching. Related to this point of view, TÜBITAK-initiated Vision 2023 document emphasized the smart use of technology in education (TÜBİTAK, 2005). In addition, The Ministry of National Education (MoNE, 2013) encouraged Turkish mathematics teachers to teach students the skills required to actively use information and communication technologies (ICT) in mathematics.

In accordance with the ideas proposed by influential policy making organizations, some research in the Turkish context supported the use of technology in mathematics education. For example, Baki (2001) argued that teachers could use innovative computer technologies not only for teaching content but also to help students learn mathematics by themselves. In another study, Baki and Güveli (2008) indicated that teachers could increase student success through creating well-prepared, technologically-rich learning environments. Bulut and Bulut (2011) found that Turkish mathematics teachers were open to adapting a variety of technologically-rich teaching methods when they believed that these methods would assist students to understand abstract concepts. Çatma and Corlu (2015); however, showed that Turkish mathematics teachers teaching high-ability students at specizalized schools were not more mentally prepared to implement Fatih project technologies than teachers at non-selective general schools.

**Dynamic geometry software**

The family of software that can be categorized as DGS has been considered by many as one of the most effective technological tools to foster conceptual understanding in mathematics education. Several researchers supported this view, claiming that such software would help students benefit from multiple representations of mathematical topics (Akkaya, Tatar, & Kağızmanlı, 2011; Hohenwarter & Fuchs, 2004; Kabaca, 2006; Zengin, Furkan, & Kutluca, 2012). The research of Kortenkamp (1999) encouraged instructors to use DGS in their teaching because of its capacity to foster understanding of multiple topics of advanced mathematics at both school and university levels, including different geometries, such as the Euclidean, linear space, and projective geometries, complex tracing and algebra, such as matrices, functions, limits, and continuity. Kortenkamp advocated that students who used DGS could explore multiple perspectives in a single construction.

Another evidence in favor of DGS was based on research that investigated the impact of DGS for developing mathematical skills exclusively at the school level. For example, Jones (2001) conducted a study to investigate the impact of DGS in learning *geometry* concepts. The researcher's sample included lower-secondary students (12 year olds). The findings showed that using DGS in mathematics classes had positive impacts on learning geometry concepts.

In another study, Subramanian (2005) investigated the impact of DGS on students' *logical thinking skills, proof construction*, and *general performances* in their mathematics courses. With a large sample of 1,325 high school students drawn from local schools in the United States, the researcher used a double pre-post test design to conclude that academically high achieving students benefited the most from using DGS in developing logical thinking skills.

In an empirical study by Bakar, Tarmizi, Ayub, and Yunus (2009), however, no statistically significant difference was reported for either conceptual understanding or procedural knowledge in *quadratic functions* between a control group taught with a traditional approach and a treatment group taught with DGS, in terms of student performance after an intervention with DGS. Researchers believed that their intervention, which was limited to six hours of instruction including the time spent to learn basic features of the DGS in the experimental group, needed to be longer for an impact to be observed.

Karakuş (2008) investigated student achievement in *transformation geometry* when DGS was used as the medium of instruction. The researcher conducted the study with 90 seventh-grade students in a school from Turkey. The research design included a pre-test and a post-test. Karakuş divided the students into four groups according to their pre-test scores (high-success experimental and control groups; low-success experimental and control groups). After the intervention, there was a statistically significant difference between the high-success experimental and the control groups, in favor of the group of students who were taught with DGS, while there was not a statistically significant difference between the low-success experimental and the control groups. This research was noteworthy because it showed that DGS might be an effective tool for high-success students with a large impact of 1.31 standard deviations.

İpek and İspir (2011) believed that DGS was essential both for students and teachers because such software brings about an environment that enables *discourse* and *exploration*. The researchers examined pre-service elementary mathematics teachers' algebraic proof processes and attitudes towards using DGS while making algebraic proofs. They designed a ten-week long course. The participants solved problems about algebra and proved some elementary theorems. The participants also wrote their reflections. At the end of the course, researchers interviewed a selected number of participants about their experiences with DGS. They found some pre-service elementary mathematics teachers believed that DGS was valuable for learning and teaching mathematics. Moreover, these informants reported a positive change in their feelings for using technology.

In their study, Bulut and Bulut (2011) showed that the DGS allowed teachers to observe and experience multiple teaching strategies. The purpose of their research was to investigate pre-service mathematics teachers' *opinions* about using DGS. They followed a qualitative research methodology with some forty-seven students at their sophomore year who reported a willingness to use DGS when they would become teachers.

*GeoGebra*

GeoGebra is a freely-available and open-source interactive geometry, algebra and calculus application created by Markus Hohenwarter in 2002. Hohenwarter and Jones (2007) believe that GeoGebra is a useful tool for visualizing mathematical concepts from the elementary to the university level. They emphasize that Geogebra integrates two prominent forms of technology; namely, CAS and DGS. GeoGebra, which offers dynamically connected multiple illustrations of mathematical objects through its graphical, algebraic, and spreadsheet views, also allows students to investigate the behaviors of the parameters of a function through its CAS component (Hohenwarter & Lavicza, 2009). The software is constantly being improved by an active team of researchers and teachers. The software has a large collection of activities which are developed and donated by users all over the world. In recent years, the software is being translated into a number of languages, making it available in 45 different languages as of 2015.

Some researchers have explored the impact of GeoGebra on achievement of objectives in different mathematical topics. Saha, Ayub, and Tarmizi (2010) used a quasi-experimental post-test only design to identify the differences on the average for high visual-spatial ability and low visual-spatial ability students after using GeoGebra for learning *coordinate geometry*. In their study, the sample consisted of 53 students who were 16 or 17 years old from a school in Malaysia. The researchers divided the sample into two homogeneous groups, where the experimental group students were taught with GeoGebra and the control group students were taught with traditional methods. Each group was categorized into two types of visual-spatial ability (high [HV] and low [LV]) by applying a paper and pencil test covering 29 items. They reported three main findings:

(a) students in the experimental group scored statistically significantly higher on the average than the students in the control group regardless of being HV or LV;

(b) in the HV group, there was no statistically significant difference on the average between experimental and control groups in favor of the experimental group;

(c) in the LV group, students in the experimental group scored statistically significantly higher on the average than the students in the control group.

This research was noteworthy because it showed that GeoGebra might be an effective tool for LV students, as well.

Another research study reflecting the positive impact of GeoGebra was conducted by Kllogjeri and Kllogjeri (2011) in Albania. The researchers presented some examples of how GeoGebra was used to teach the concepts of *derivatives*. In the study, they demonstrated three important theorems by using GeoGebra applets to explain: (a) the first derivative test and the theorem; (b) the extreme value theorem; and (c) the mean value theorem. The researchers used direct teaching method and measured GeoGebra's impact on students' conceptual understanding. They concluded that the multiple representation opportunities and the dynamic features of GeoGebra helped students' understand the mathematical concepts faster and at a deeper level.

Mehanovic (2011) wrote about GeoGebra that included two separate studies focusing on teaching *integral calculus* with GeoGebra. The first study was conducted with two classes from two different secondary school students in Sweden. The researcher observed students through regular classroom visits. After several classroom observations, individual interviews with students were conducted. For the second study, the researcher asked the participating teachers to prepare an introduction to the concept of integration and record their introductory presentations. The objective of the study was to investigate teachers' introductions to the subject of integrals in a normal classroom environment. After the preliminary analysis of the teacher presentations, individual interviews were conducted with the participating teachers. As a result of the first study, it was found that the students had some concerns, such as using GeoGebra was time-consuming. Furthermore, students seemed to believe that using GeoGebra was more confusing than their previous learning methods. In the second study, teachers reported some epistemological, technical, and didactical barriers for effective use of GeoGebra in the classroom. However, it was concluded in both studies that integrating a didactical environment with GeoGebra was complex and teachers needed to realize the potential challenges.

Some GeoGebra impact studies were conducted in Turkey, as well. For example, Selçik and Bilgici (2011) focused on the initial impact and the degree of retention of knowledge for *polygons*. The study was conducted with 32 seventh-grade students. Following a pre-test, the experimental group was instructed using GeoGebra and a constructivist face-to-face teaching was provided to the control group that did not have computer access. In the experimental group, one computer was given to every two students to create a collaborative environment and that enabled students to directly examine the prepared activities. Following an 11-hour long course, an identical post-test was applied. Students in the experimental group scored higher averages on the post-test than the students in the control group. When the test was administered for the third time a month after the intervention ended, the students in the experimental group performed better in terms of the amount of knowledge they retained.

Similarly, Zengin (2011) conducted a study with 51 high-school students to investigate the effect of the GeoGebra software in teaching the subject of *trigonometry* and to examine students' *attitude* toward mathematics. In this study, participants were divided into two equal groups, one experimental and one control.

Both groups were given a pre-test. While teaching was focused on using the GeoGebra software in the experimental group, the control group was taught with a constructivist teaching approach only. Both groups showed improvement in their achievement scores at the end of the study; although, the averages in the experimental group were statistically significantly higher when compared with those in the control group. However, according to the experimental groups' pre- and post-test scores, teaching mathematics through technology had negligible effect on students' attitudes toward mathematics.

A detailed analysis of effect sizes in selected and relevant impact studies is summarized in Table 31.

## Research on teaching and learning calculus

Calculus is a branch of mathematics that focuses on change. Calculus is taught both in high school as an advance mathematics course or at university level as a freshman (i.e., first year) course. Kidron (2014) stated that a usual calculus course consists of a combination of several topics including limits, differentiation, and integration, in which students are reported to experience difficulties in understanding. Students find calculus topics difficult because it includes abstract definitions and formal proofs (Tall, 1993).

Kidron (2014) asserted that the use of technology is one of the effective methods in teaching calculus. Hohenwarter, Hohenwarter, Kreis and Lavicza (2008) advocated that GeoGebra was a convenient software program for technology-supported mathematics (particularly calculus) teaching and argued that calculus education using

GeoGebra could be applied to courses in two ways: (a) presentation (teacher-centered approach); and (b) mathematical experiments (student-centered approach). Tall, Smith and Piez (2008) examined 40 graduate-level theses on this topic authored between 1998 and 2008. They concluded that most of the studied technologies showed positive contribution to learning of calculus topics.

Several studies about teaching calculus have also been conducted in Turkey. Kabaca (2006) instructed the *limits* topic using technology and traditional methods to freshman mathematics students ($n$ = 30). Dividing the sample into two as the control (the group using traditional methods) and experimental (the group using technological aids) groups and comparing the pre-test and post-test achievement scores, the researcher did not find a statistically statistical difference on the average between the group scores.

Aktümen and Kaçar (2008) instructed the concept of *definite integral* using technology to first year university students of a science education department ($n$ = 47). In their conclusion they stated that there was a statistically significant positive improvement in the attitudes of the students in the class where technology was used compared to the class where technology was not used.

Despite the growing knowledge-base, there is still a limited number of studies on calculus teaching conducted at the high school level.

### Limits and continuity concepts

Limits and continuity, which are the first steps to the subject of derivatives, are of great importance in such fields as engineering and architecture. Both topics are abstract concepts that confuse students when they first encounter them. In the new mathematics curriculum of 2013, MoNE encourages the use of certain DGS that may make such abstract topics accessible to students. MoNE prescribes 118 periods (of 40 minutes each) be dedicated to calculus in grade 12 (which is 54% of all the time assigned for all topics). Of these 118 periods, the national curriculum advised that 14 classroom periods be allocated to teach about limits and continuity concepts, this comprises 6% of the total contact hours in grade 12 mathematics (MoNE, 2013).

Because limits and continuity are abstract concepts that are difficult for teachers to instruct and students to comprehend, various studies on the limits topic exist in the literature. For example, Mastorides and Zachariades (2004) conducted a study to understand the content knowledge of secondary mathematics teachers about the concepts of limits and continuity. Fifteen secondary mathematics teachers, all attending master's degree programs in mathematics education, were enrolled in the study. They taught calculus, particularly limits and continuity concepts, for 12 weeks during their master's degree program and the researchers noted their challenges. At the end of the teaching period, participants were given a survey consisting of questions about the problems they had to overcome during the intervention. After the survey, the researchers conducted interviews with all the teachers. As a result, the researchers argued that the participating teachers had the greatest concern regarding their pedagogical content knowledge about the concepts of limits and continuity.

Another study about the limits concept was conducted by Blaisdell (2012) to investigate how students' answers change in terms of question and presentation format in the limits concept. The researcher applied a test to 111 calculus students at a university. The test questions focused on multiple representations such as graphs, mathematical notations, and definitions in the limits concept. The study indicated that students did best when the questions on limits were represented in graphs.

In Turkey, there are some similar studies focused on teaching and learning limits and continuity concepts. Baştürk and Dönmez (2011) conducted a study to understand pre-service mathematics teachers' knowledge of different teaching methods and representations of the limits and continuity topics. They gathered data from 37 pre-service high-school mathematics pre-service teachers from a public university in Turkey. In their research, the researchers used multiple research strategies to collect data such as observation, interviews, and document analysis. The survey consisted of questions to understand students' content knowledge related to the limits and continuity concepts.The researchers selected four students out of the 37 according to their responses to conduct interviews, microteaching observations, and document analysis. The interviews focused on about the teaching strategies for limits and continuity before they were requested to make a lesson plan and to teach in the form of microteaching. Although the students were aware that teachers should have made the concept of limits and continuity more concrete using teaching strategies such as drawing appropriate graphs or using technological devices, they all used question-answer methods in their microteachings and documentation. Researchers concluded that pre-service teachers should be encouraged to integrate innovative teaching methods and use them to concretize such abstract concepts such as limits and continuity.

Another study was conducted by Kabaca (2006) to understand the effect of CAS on teaching limits. In his PhD dissertation, Kabaca used an experimental design to examine a particular CAS named *Maple* while teaching limits to 30 pre-service mathematics teachers. Kabaca aimed to investigate whether teaching with Maple had any impact on student attitudes towards mathematics. The researcher divided students into experimental and control groups based on their scores of pre-attitude and pre-test on readiness for the limits concept. Then, Kabaca taught the limits concept in a 28 hour-course to the control group with a constructivist teaching method and to the experimental group with CAS-assisted constructivist approach. After the intervention, the post-test and post-attitude data were analyzed. In conclusion, the researcher deducted three major results comparing post test data for control and experimental groups:

(a) teaching with CAS had no statistically significant effect on students' total post-test score;

(b) teaching with CAS had a statistically significant effect on students' conceptual understanding of limits and continuity at the post-test level but no statistically significant difference was observed for procedural knowledge or problem solving skills;

(c) teaching with CAS had statistically significant positive effect on students' attitude towards mathematics.

Kepçeoğlu (2010) studied the effect of GeoGebra on students' achievement and conceptual understanding of the concepts of limits and continuity. Similarly, he designed an experimental study to conduct a study with 40 second-year pre-service elementary mathematics teachers. Kepçeoğlu divided the students into two groups (experimental and control) based on their pre-test scores. Researcher taught the limits and continuity concepts for a duration of six-lesson hours using traditional teaching methods to the control group, and using instructional methods along with GeoGebra to the experimental group. After the intervention, the researcher applied the same test as post-test to both groups; and compared the scores gathered from the pre- and post-tests. Kepçeoğlu concluded that teaching the limits concept to pre-service elementary mathematics teachers within the GeoGebra environment was more effective than the traditional teaching methods in terms of students' conceptual understanding. Although GeoGebra had a similar contribution in teaching the continuity concept, the effect was smaller compared to its impact on limits.

## Education for gifted and talented students

Individuals who are categorized as G&T are considered creative and productive people. They are assumed to learn faster compared to their peers and to have multiple interests (Karakuş, 2010). Identifying these individuals at an early age, providing them with appropriate developmental opportunities, and leading them to suitable careers are important. While measuring the level of intelligent quotient (IQ) was considered adequate to identify intellectual giftedness until 30-35 years ago; today, certain other tests (such as Progressive Matrices Test and performance evaluations) are used along with the tests that measure the IQ level (Bildiren & Uzun, 2007).

Turkey's experience with G&T individuals has a long history since the Enderun, world's first institution established for gifted and talented students during the 15th century in İstanbul (Corlu, Burlbaw, Capraro, Han, & Corlu, 2010). More recently, the Centers for Science and Art (Bilim ve Sanat Merkezleri—BİLSEM) were established to identify talented G&T students in Turkey. Working in close cooperation with schools around the country, BİLSEM has been instrumental in identifying talented G&T students and creating enriched learning environments appropriate for them. In addition, the Turkish Education Foundation has been operating the first and still the only school for such students in modern Turkey since 1993.

Preparing enriched and in-depth lessons that promote critical thinking and creativity in educating G&T students is one of the primary tasks of the teachers of G&T students. A tool that teachers can use in planning and preparation for this purpose is technology. In mathematics education, G&T students can be supported by technology, based on their areas of interest and mathematical abilities (Hohenwarter, Hohenwarter & Lavicza, 2010). In this regard, there are a few studies on G&T students' learning mathematics using technological aids. In their study conducted with gifted students, Duda, Ogolnoksztalcacych, and Poland (2010) stated that the use of graphing display calculators helped students produce creative solutions and provide them with opportunities to explore new mathematical concepts. Choi (2010) specified that GeoGebra increased interest in and motivation toward mathematics. Software programs that create environments of thinking creatively for G&T students direct students to explore and produce authentic mathematical knowledge.

# CHAPTER 3: METHOD

## Introduction

The main purpose of this study was to investigate the impact of teaching mathematics with GeoGebra on 12th grade gifted and talented (G&T) students' conceptual understanding of the limits and continuity concepts. The second purpose was to investigate the impact of GeoGebra on students' attitudes towards learning mathematics with technology. This chapter discusses the research design, pilot study, participants, instruments used in data collection, and data analysis.

## Research design

A pre-post test design was employed in the study to determine the impact of teaching with GeoGebra software on conceptual understanding of G&T students and their attitudes towards learning mathematics with technology. The participants of the study had already been divided into two classes by the school administration before the study—later determined randomly as an experimental group and a control group by the researcher. In this manner, the assignment of participants into the groups was not manipulated by the researcher. In order to correct for any possible difference in their ability and knowledge before the intervention, both groups were administered the limits and continuity readiness test (LCRT) along with the mathematics and technology attitude scale (MTAS). Following the pre-test, the limits and continuity concepts were taught to the experimental group in the GeoGebra environment; whereas the same concepts were taught with the traditional direct instruction methods to the control group. With the conclusion of the teaching process in two weeks, the limits and continuity achievement test (LCAT), a test closely similar to LCRT, was applied and the same attitude survey that was administered in the pre-test stage were given as a post-test. The research design is summarized in Table 1.

Table 1

A summary of the research design

| Group | Pre-tests | Intervention | Post-tests |
|---|---|---|---|
| Experimental Group | LCRT<br>MTAS | Teaching with<br>GeoGebra | LCAT<br>MTAS |
| Control Group | LCRT<br>MTAS | Teaching with<br>traditional method | LCAT<br>MTAS |

In quantitative research, the researcher states a hypothesis, tests this hypothesis, and generalizes the results to a larger population (Arghode, 2012). Huck (2011) stated a nine-step version of hypothesis testing which was followed in the current study:

(1) State the null hypothesis,

(2) State the alternative hypothesis,

(3) Specify the desired level of significance,

(4) Specify the minimally important effect size,

(5) Specify the desired level of effect size,

(6) Determine the proper size of sample,

(7) Collect and analyze the sample data,

(8) Refer to a criterion for assessing the sample evidence,

(9) Make a decision to discard/retain.

## Pilot study

Before the actual data collection, a pilot study with 12th grade high-school students of a private high-school in Ankara was conducted. The goals of this pilot study included the following:

(a) finalize the research questions and research design before the actual study with G&T students;

(b) review of the data collection process before the study;

(c) identification of possible problems that can be encountered during the course of the study;

(d) determination of the appropriate sample size for the study;

(e) identification of the shortcomings of data collection instruments and elimination of these shortcomings (Orimogunje, 2011).

The pilot study was conducted with a group of 26 students. The group was already divided into two sub-groups (the experimental and control groups) by the school administration. The experimental group was provided with the limits and continuity instruction (intervention) using GeoGebra whereas the control group was taught the same topic with traditional method by the same teacher. The instruction period was limited to 2 weeks (10 hours). Following the instruction, the limits and continuity post-test was applied.

The pilot revealed problems experienced during the intervention process. The researcher made the following arrangements and changes to ensure that the study would yield reliable data:

(a) The sample size was estimated through a prior power analysis with G*Power3;

(b) the procedures and duration of the intervention were considered appropriate for the main research upon feedback of school teachers and students;

(c) 12th grade students who were busy preparing for the university entrance exams during the course of the research could not attend intervention classes regularly. Given the fact that participants in the actual study were boarding students, this was not considered a serious concern.

One of the biggest outcomes of conducting a pilot study was to calculate the required sample size for the study (Teijlingen & Hundley, 2001). To calculate the required sample size, a special software named G*Power3 was used. Based on pilot data, the program estimated the effect size—strength of a relationship: Cohen's $d = 1.27$ in post test score differences between two groups. The magnitude of this effect, as well as effect sizes reported in similar studies on GeoGebra was used as a benchmark for meta-analytical purposes when assessing the effect of the intervention of the present study (See Table 31). Thus, it was estimated that the required sample size needed to be at least 22 in order to be 80% sure at an *alpha* level of .05 that there would be a statistically significant difference between the experimental and control group scores on the average.

### The context and participants

This study was conducted with 34 students in grade 12 of a private high school in Kocaeli, Turkey. This high school (grades 9 to 12) was founded to educate G&T students who were selected on merit from all over Turkey. This unique school established its vision as follows: To develop G&Tstudents who are suffering from economic and social difficulties; to offer them a proper learning environment; and to educate them as leaders of the society. In this sense, the participating students could be described as strong individuals in terms of both academic and social aspects. Because the school was a boarding school, students were staying in the school during the weekdays. The school followed the International Baccalaureate (IB) Diploma Programme (DP) in grade 11 and 12.

The school selects its students with several screening methods such as progressive matrices test, WISC-R's IQ test, interviews, and an observation camp that lasts for one week, all administered at the end of 8th grade. Some of the students are admitted with a full scholarship while others are provided with a partial scholarship. According to the school regulations, 30% of the students should have full scholarship, and the rest of the students get partial scholarship with respect to their parents' economic condition. The participating students of the current study reflected the school scholarship ratios. See Table 2 for gender distribution of the participant students.

Table 2

Gender distribution of participants

| | Experimental Group | Control Group | Total |
|---|---|---|---|
| Male | 6 | 10 | 16 |
| Female | 9 | 9 | 18 |
| Total | 15 | 19 | 34 |

Table 2 shows that male to female ratio was similar in both control and experimental groups. Table 3 provides data concerning the middle schools (before high school) they attended.

Table 3

Participants' primary school backgrounds

| | Experimental Group | Control Group | Total |
|---|---|---|---|
| Public School | 9 | 15 | 24 |
| Private School | 6 | 4 | 10 |
| Total | 15 | 19 | 34 |

Table 3 shows that most of the participants from both groups graduated from a public school. See Table 4 for their parents' occupation distribution in order to understand the socio-economic status of their parents.

Table 4

Parent occupations

| | Control Group | | | Experimental Group | | | Whole Group | | |
|---|---|---|---|---|---|---|---|---|---|
| | Father | Mother | Total | Father | Mother | Total | Father | Mother | Total |
| First Profile | 5 | 2 | 7 | 7 | 4 | 11 | 12 | 6 | 18 |
| Second Profile | 3 | 5 | 8 | 3 | 5 | 8 | 6 | 10 | 16 |
| Third Profile | 4 | 11 | 15 | 3 | 4 | 7 | 7 | 15 | 22 |
| Fourth Profile | 6 | 1 | 7 | 2 | 2 | 4 | 8 | 3 | 11 |
| Total | 18 | 19 | 37 | 15 | 15 | 30 | 33 | 34 | 67 |

*Note*. Profiles were determined by the researcher.

In Table 4, first profile consists of professions including doctors, engineers, architects, lawyers, directors, and financial advisors. Second profile jobs were civil servants, teachers, and nurses. Third profile includes parents who were retired or not working. Fourth profile parents' are accountants, self-employed, and painters. While parents of students in the experimental group were mostly employed in first profile jobs, parents from control group were mostly doing third profile jobs.

## Data collection

### Procedure

Two instruments, a *limits and continuity readiness test (LCRT)* and a *mathematics and technology attitudes scale (MTAS)*, were used during the pre-test period. After the pre-test, limits and continuity concepts were taught with two different methods. The researcher, also a teacher of the students, taught the concepts by using GeoGebra to the experimental group. A fellow teacher taught the control group. Traditional teaching methods were used to teach limits and continuity in the control group. After the intervention, the post-tests were adminsitered. The post-test used two instruments, a *limits and continuity achievement test (LCAT)* and *MTAS*. A written permission was granted by MoNE to conduct the study at this school. See Appendix 6.

### Instruments

*Limits and continuity readiness test*

This test originally consisted of 12 open-ended questions to test the readiness of students for the limits and continuity topics. The first item is an adaptation of a question that was asked in the university entrance exam in 1990 and this question was removed from further analysis due to negative item-total correlation. The second question was an adaptation of a question asked in the university entrance exam in 1997. The third question was adapted from a university entrance exam preparation workbook. These three questions required a low-level of cognitive demand with respect to the concepts of limits and continuity. The other nine questions were the same as the ones used by Kepçeoğlu (2010) in their study. The pre-test questions were evaluated to focus primarily on procedural knowledge. After the first question was excluded from the study—a decision made based on reliability analysis—the minimum score for the readiness test was 0 and the maximum score was 4 when total score was divided by 11 in order to find the final pre-test readiness score (See Appendix 1 for limits and continuity readiness test [LCRT] questions).

*Limits and continuity achievement test*

The limits and continuity achievement test (LCAT) was administered to both the experimental and control groups after teaching the topics of limits and continuity for two weeks for a total of six contact hours. The test consisted of 12 open-ended questions similar to the readiness test in terms of content. Question number 1, 2, 3, 5, 9, 10, 11 and 12 in the LCRT were changed. Instead of these questions that require mostly procedural knowledge of limits and continuity, the researcher modified these questions in order to test primarily the conceptual understanding. This modification was done in consultation with fellow teachers in the department of the school. The rationale behind this change was to control for procedural knowledge of some students who might have learned the content through private tutoring or by themselves. The minimum score for the achievement test was 0 and the maximum score was 4 when total score was divided by 12 in order to find the final post-test achievement score (See Appendix 2 for LCAT questions).

*Assessment criteria for LCRT and LCAT*

The following assessment criteria were used in grading responses to the limits and continuity readiness and achievement tests (cf. Kepçeoğlu, 2010). According to Table 1, the possible minimum score was 0, and the possible maximum score was 4. The answer key was prepared by the researcher and discussed with other teachers in the school, including the teacher of the control group.

Table 5

Assessment criteria for LCRT and LCAT

| Correct | Partially Correct | Wrong (1) | Wrong (2) | Unanswered |
|---|---|---|---|---|
| 4 marks | 3 marks | 2 marks | 1 mark | 0 mark |

*Correct:* The answer was totally correct.

*Partially Correct:* Some minor mistakes, including miscalculations.

*Wrong(1):* Error(s) were made at the very early stages of the steps required to reach the solution or the process was not specified.

*Wrong(2):* There was a meaningful attempt but the answer was wrong.

*Unanswered:* No answer to the question or no meaningful attempt was provided.

*The mathematics and technology attitudes scale*

The mathematics and technology attitudes scale (MTAS) developed by Pierce, Stacey, and Barkatsas (2007) was used to examine the effects of the GeoGebra software on student attitudes towards mathematics and technology. The researchers evaluated this instrument to be leaner, shorter, and more understandable compared to other scales. Furthermore, the survey avoided negative statements to prevent complexity in meaning and to protect students from delving into negative thoughts in the long term. The survey had five sub-scales.

(a) mathematical confidence (MC);

(b) confidence with technology (CT);

(c) attitude to learning mathematics with technology (MT);

(d) affective engagement (AE);

(e) behavioral engagement (BE).

For four of the sub-scales, MC, MT, MT and AE, a 5-point Likert-type with strongly agree to strongly disagree responses was used. For the sub-scale BE, a similar format—nearly always, usually, about half of the time, occasionally, hardly ever—was used (See Appendix 3 for MTAS items). All the sub-scales have been scored from 1 to 5 by computing the averages of responses within each factor.

**Intervention**

For the intervention, limits and continuity concepts were planned to be taught in GeoGebra supported environment (dynamic graphs) to the experimental group. The teacher was the researcher. Traditional teaching methods were used by a fellow teacher in the control group class involved using projecting the content (still non-dynamic graphs) to the board and in class discussions. Before the intervention period, both teachers prepared the lesson plans and materials together. Each lesson hour and activity was discussed in the department. Detailed explanation of each lesson hour is given as follows:

*First and second lesson hours*

In the first lesson, both teachers explained the difference between value of the limit of a function and a function converging to a particular value given that independent variable is manipulated.

GeoGebra software including basic tools were introduced to students in the experimental group in order for them to download and practice after school. This introduction lasted for abut five minutes and students reported that the program was user-friendly and they were able to use it with ease. In fact, students were observed to be skilled in adapting to the GeoGebra environment. During the in-class discussion, researcher created GeoGebra applets and worksheets for experimental group, whereas the traditional group used paper-based worksheets that included non-dynamic (still) graphs of the same functions. See Figure 1, Figure 2, and Figure 3 for materials used in experimental group:

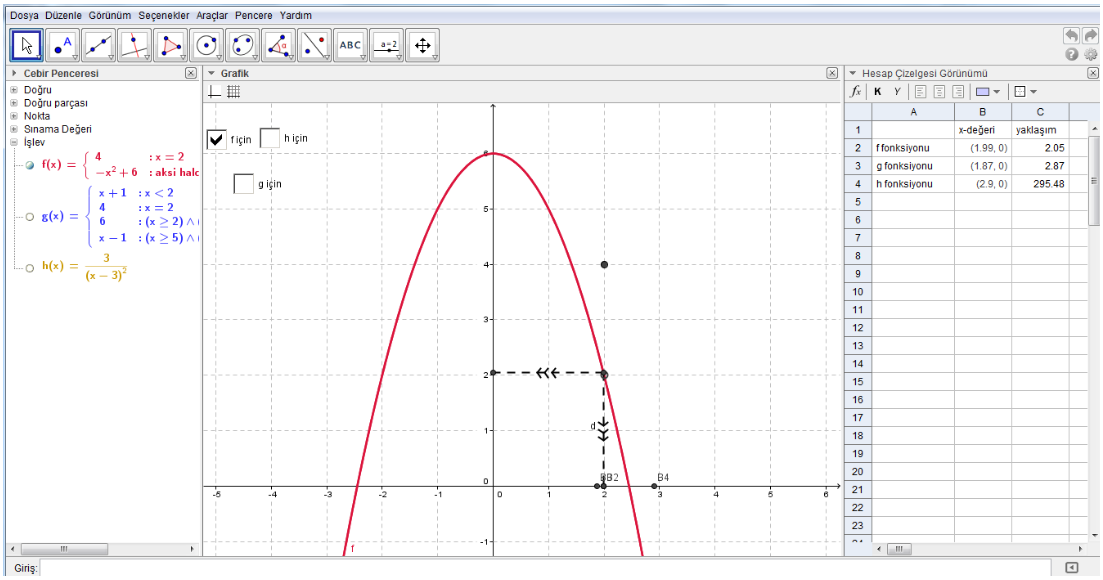



*Figure 1*. GeoGebra applet for limiting (first function)

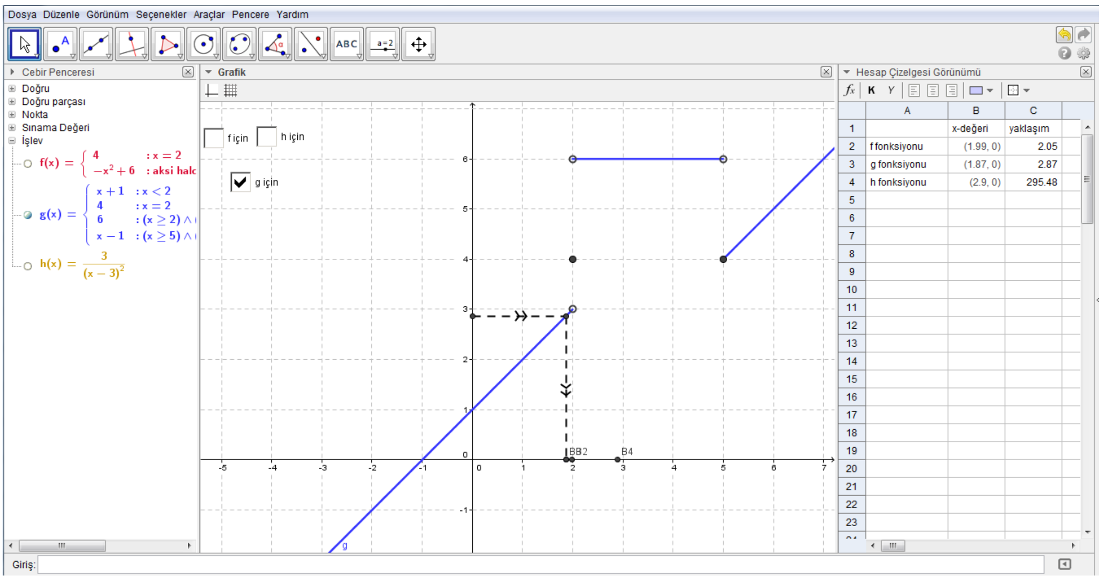



*Figure 2*. GeoGebra applet for limiting (second function)

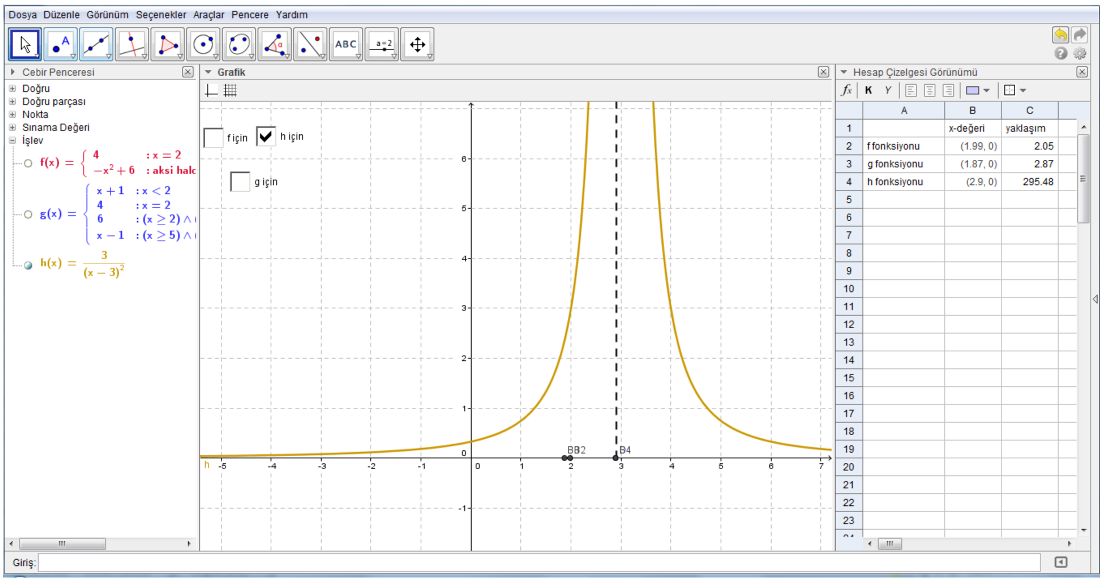



*Figure 3*. GeoGebra applet for limiting (third function)

First, the researcher explained to which value the function converges (left-hand and right-hand) in the first figure. Students were expected to estimate to which value was the function converging depending on the changing values of x. Some students used the computer and showed the process with the pointer (mouse). This was the *enactive stage* at which students attempted to show the left-hand and the right-hand limiting process by using their pointers. The researcher asked students to inquire the value where x = 2 while the function was jumping to y = 4. See Figure 1. Students discussed that the function was converging to y = 2 when approached to x = 2 from left or right, although the exact value of the function at that x value was y = 4. At the end, researcher stated the limit notation and that if these two values were not equal, limit would not exist.

Second, GeoGebra applet in Figure 2 was used to explain the limiting process for a piece-wise function. The students observed and followed the teacher showing the convergence on the graph. This observation stage in which the students were not actively required to manipulate the function was considered as an activity for the *iconic stage*. However, some students were engaged in Geogebra and showed the convergence by themselves. The researcher encouraged all students to manipulate the x values during their study time after school. Given the fact that the school was boarding and all students were required to attend these study hours, it would be expected that they used the program. Furthermore in the classroom, students were asked to observe the limits of the function when x values approach -1, 0, 2, 3, 5 and 6 from left or right. They were asked to use the formal notation and determine whether the limit existed or not. The students worked with the formal notation to connect enactive, iconic and symbolic representations.

Third, concept of infinity was discussed with the students on a GeoGebra applet with the help of the graphs in Figure 3. While x value was approaching to 3, they discovered that the value of the function had been getting bigger and bigger, getting closer to an idea, called the infinity. In addition, the students expressed their opinions about the relationship between the symbolic equation of the function and the idea of infinity. As the final activity of the first and second lesson hours, the researcher requested the students to fill out a table, which is shown in Figure 4:

| | | | | | |
|---|---|---|---|---|---|
| $f(x) = \begin{cases} 4 & , \quad x = 2 \text{ ise} \\ -x^2 + 6 & , \quad x \neq 2 \text{ ise} \end{cases}$ | $x$=1.9 | $x$=1.99 | $x$=2 | $x$=2.01 | $x$=2.1 |
| $g(x) = \begin{cases} x + 1 , & x < 2 \text{ ise} \\ 4 , & x = 2 \text{ ise} \\ 6 , & 2 < x < 5 \text{ ise} \\ x - 1 , & x \geq 5 \text{ ise} \end{cases}$ | $x$=1.9 | $x$=1.99 | $x$=2 | $x$=2.01 | $x$=2.1 |
| $h(x) = \frac{3}{(x-3)^2}$ | $x$=2.9 | $x$=2.99 | $x$=3 | $x$=3.01 | $x$=3.1 |

*Figure 4*. Table for algebraic calculations of the approaches

The students could use either a calculator or the GeoGebra program to calculate the values during this activity. The researcher allowed the students to use laptops while the researcher was providing feedback. At the end, the researcher draw the graphs of functions by using GeoGebra. Students had the opportunity to compare the values they have found and check their findings in the table.

In the meantime, the teacher in the control group used two hard-copy worksheets which included the same examples of functions. See Appendix 7 for the paper-based worksheets.

In the last ten minutes of the lessons, the limiting process was summarized in the same manner to both groups, including a conversation about Niels Henrik Abel, a prominent mathematician, whose contribution to limits and continuity concepts was remarkable. In addition, the teachers requested the students to think about the relationship between polygons and circles. This was given as homework to be discussed during the next lesson.

*Third and Fourth Lesson Hours*

In the beginning of the third lesson, a GeoGebra applet was used (GeoGebra Tube, 2015), to show the *relationship between polygons and circles* in the homework. The aim of this applet was to make students to understand that a polygon becomes to a circle when the number of its sides goes to infinity. See Figure 5 for an interface of that applet.

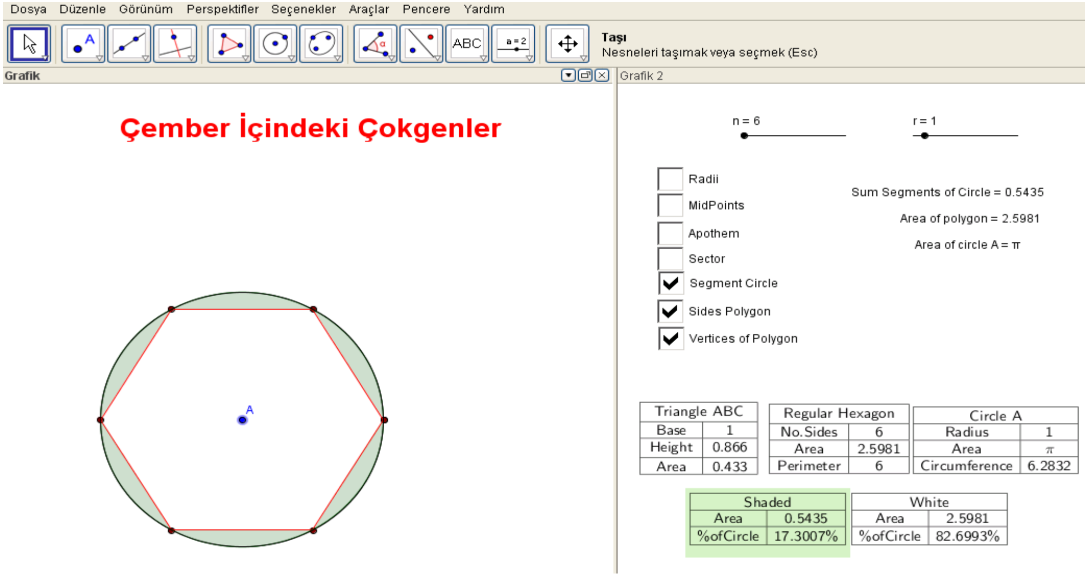



*Figure 5*. GeoGebra applet for investigating relations between circles and polygons

In that activity, *n* indicated the number of the sides of the polygon. The experimental group students were demonstrated that the polygon approached to a circle when the number of its sides increased. That could be considered the *iconic stage*. Students also observed in a tabular representation that the ratio of the area of the polygon to that of the circle would approach to 100%. The allocated time for this activity was ten minutes.

Second, another GeoGebra applet was used to enable students to explore the limiting process where the function was not undefined. Students initially investigated the limits value and the exact value of the function individually, and students explained their findings to their peers by using the GeoGebra applet. Figure 6 shows the snapshot of the applet.

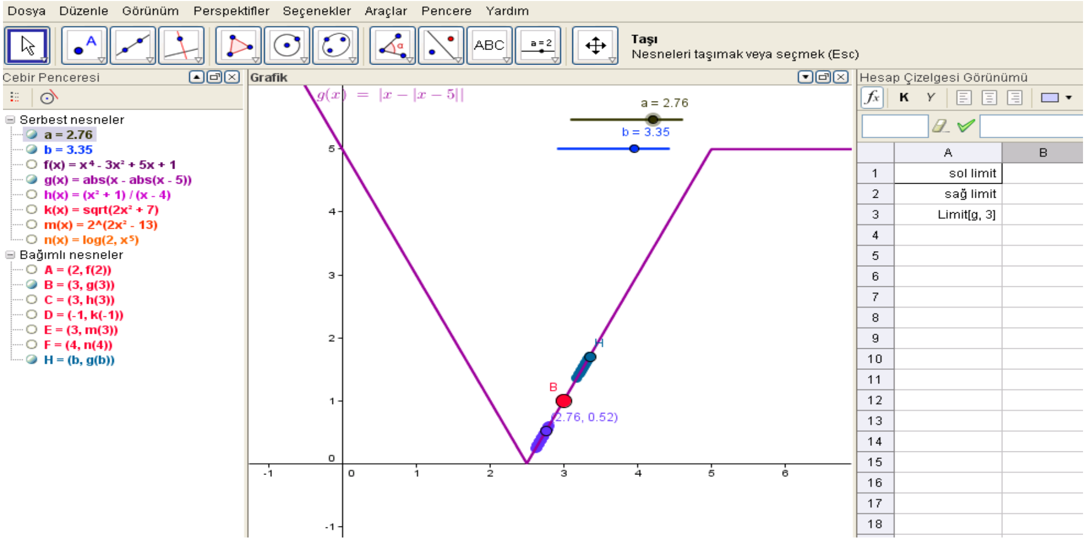



*Figure 6*. GeoGebra applet for limiting (six different functions)

The activity which is shown in Figure 6 was for the function g(*x*) = |*x*-|*x*-5||. The students used two separate sliders to manipulate the x values from left or right of *x* = *3*. The table at the far right side of the Figure 6 showed left-hand, right-hand and the exact values of the function around *x* = *3*. The students compared the values of the table and their initial estimations. This was considered as an activity for both *enactive* and *iconic stages*. The allocated time for this activity was 20 minutes.

In the third activity, the students discovered the basic limit properties for a variety of functions types including polynomial functions, radical functions, and absolute value functions. This was planned to be a group work activity. A GeoGebra applet, which can be seen in Figure 7, was used for this activity. The allocated time for this activity was 20 minutes. The students were also asked to fill out a table, which was shown in Figure 8.

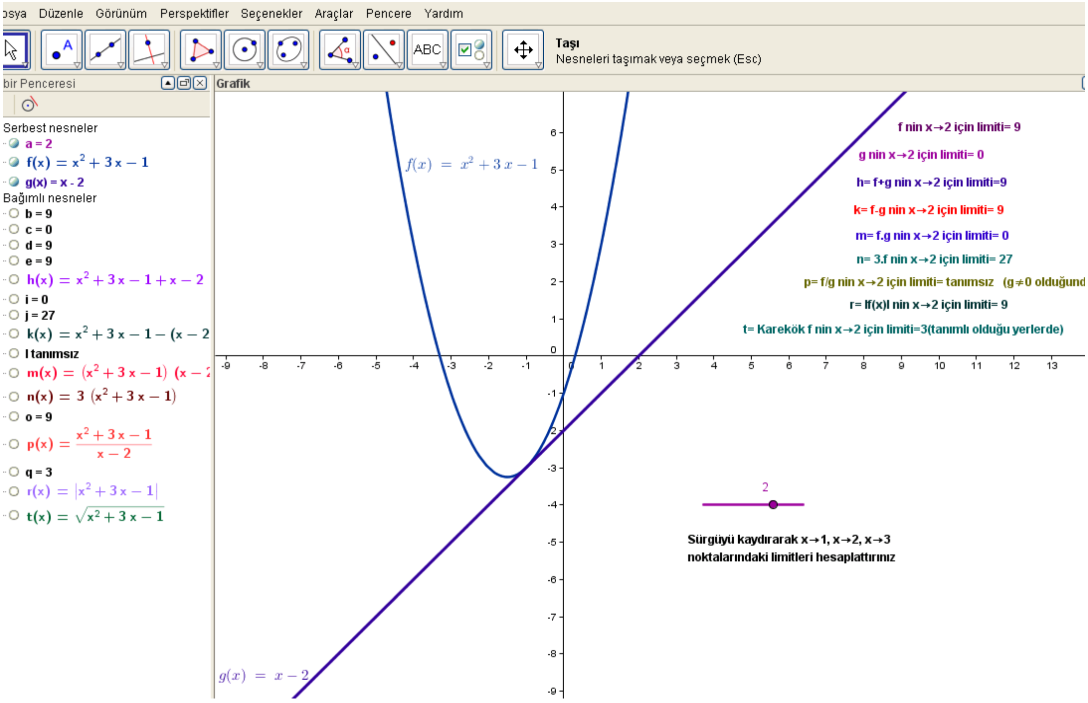



*Figure 7*. GeoGebra applet for explaining limits properties

| | | | | |
|---|---|---|---|---|
| $\lim_{x\to 2} f(x)$ | | | $\lim_{x\to 2}[f(x).g(x)]$ | |
| $\lim_{x\to 2} g(x)$ | | | $\lim_{x\to 2}\frac{f(x)}{g(x)}$, $g(x)\neq 0$ | |
| $\lim_{x\to 2}[f(x)+g(x)]$ | | | $\lim_{x\to 2}\lvert f(x)\rvert$ | |
| $\lim_{x\to 2}[f(x)-g(x)]$ | | | $\lim_{x\to 2}\sqrt{f(x)}$ | |
| $\lim_{x\to 2} 3.f(x)$ | | | | |

*Figure 8*. Table for algebraic investigations of limits properties

A closure to this part of the discussion included a video demonstration of justification of the area formula of a circle. Students discussed where the limit concept was used in this video. See Figure 9 for a screenshot of the video (https://www.youtube.com/watch?v=YokKp3pwVFc&hd=1).

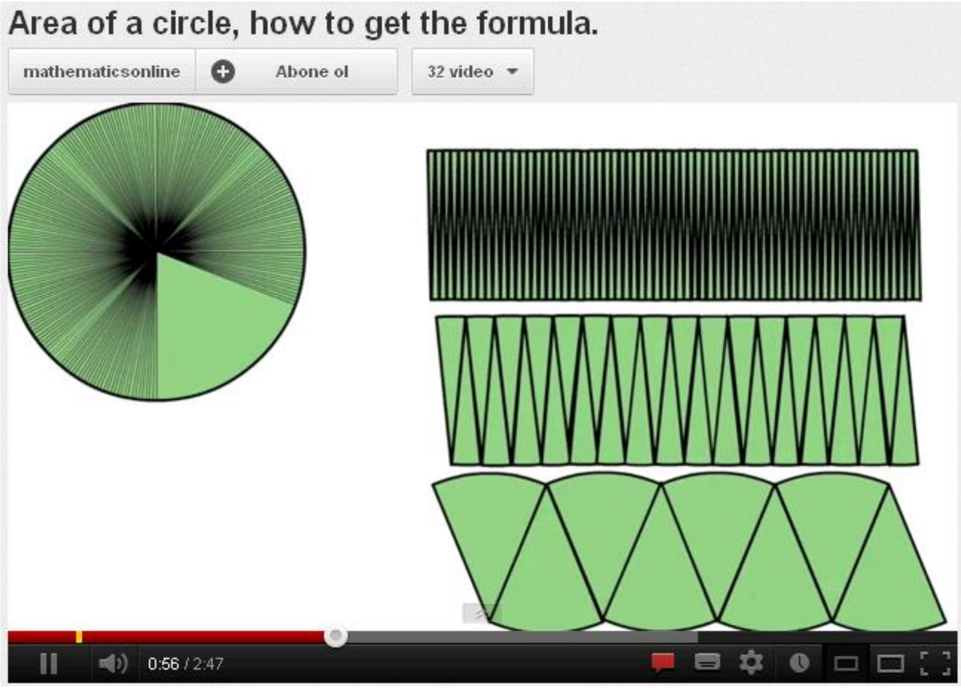



*Figure 9*. A picture of a video for the circle area formula

Next, a worksheet with several multiple choice questions about limit and its properties was distributed to the students. See the 3rd worksheet in Appendix 7. The students started to solve those questions during the class time, and finished them after the class time during their individual study time. This was an activity for the *symbolic stage* primarily. The researcher suggested the students to check and investigate their answers by using GeoGebra on their own laptops.

The same activities were prepared as a PowerPoint for the control group. Question-answer based direct instruction method was used to explain the properties of limits. The class ended with students watching and discussing the same video on deduction of the circle formula and individual study time for completing the identical questions in the worksheet.

*Fifth and sixth lesson hours*

At the first activity of the last two periods of intervention, continuity concept was explained, discussed, and explored on graphs prepared with GeoGebra. See Figure 10 for the first Geogebra applet for continuity.

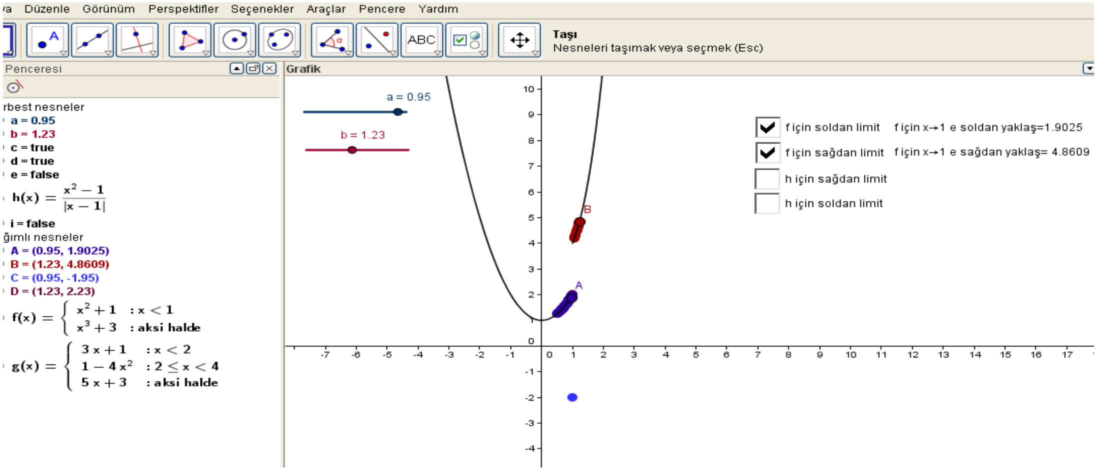



*Figure 10*. GeoGebra applet for continuity

The students were supposed to discover when a function was continuous or discontinuous by investigating the applet that included three different functions. At the end of the activity, the students were expected to discover that a function was continuous if and only if its left-hand limit and right-hand limit around an x value and exact value of the function at that value were the same. The allocated time for this *enactive stage* activity was ten minutes.

Second, several properties about limits of functions that approach to infinity were discussed with the students. For example, the sum of two functions that both had been going to infinity was also going to infinity. After that, the graph of the function *y=1/x* was drawn with GeoGebra in order to understand its behaviour when x approaches to $0^+$, $0^-$, $\infty^+$ and $\infty^-$. The allocated time for this *enactive stage* activity was ten minutes. See Figure 11 for an image of that GeoGebra activity.

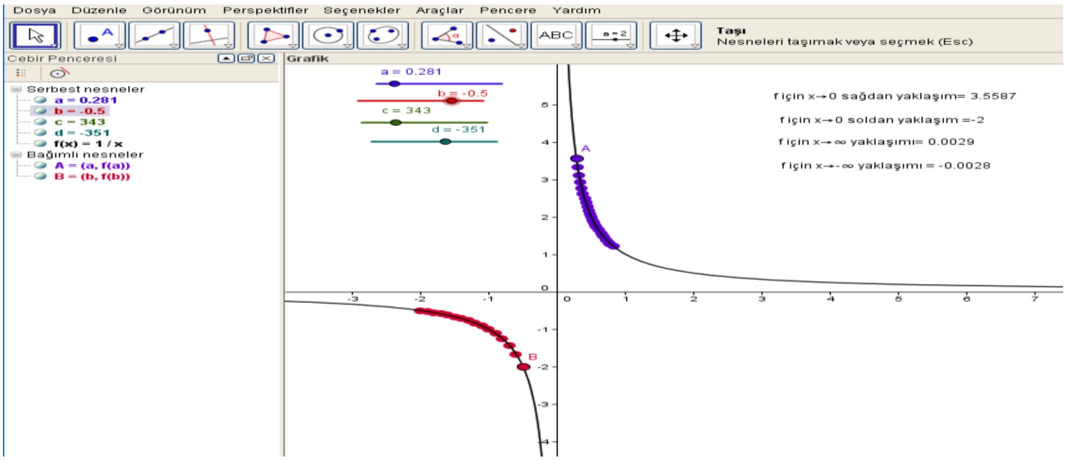



*Figure 11*. GeoGebra applet for investigation of the function *y=1/x* around zero

Third, some trigonometric functions were drawn with GeoGebra, and the students were expected to express their findings about the limits of trigonometric functions and their continuity. Figure 12 was an image of the graphs of two trigonometric functions drawn with GeoGebra.

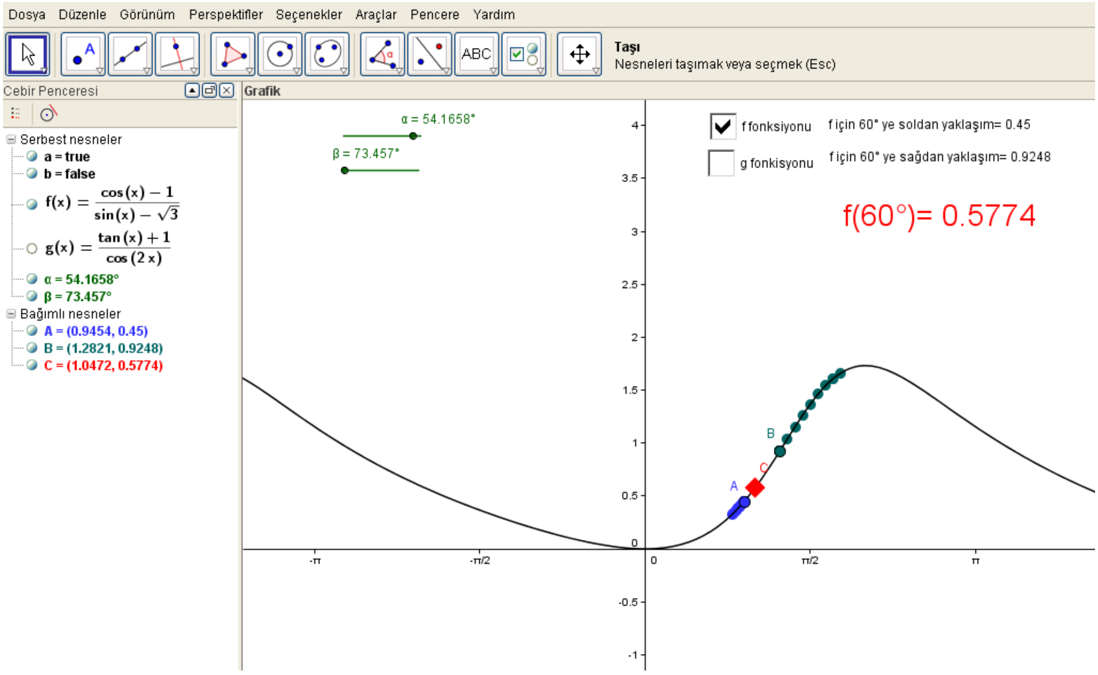



*Figure 12*. GeoGebra applet for trigonometric limits and continuity

In this activity, students were expected to understand that finding the value that functions converge to (the limit) was identical to finding the value of the function if the function was not undefined at that value of x. The students discussed whether there was an x value that made the function discontinous by comparing the functions' equations and graphs. Ten minutes were allocated for the activity.

After the students gained some general insights into limits of trigonometric functions with the help of the third worksheet (this was given as homework in previous lesson), students were asked to think about the *y=sinx/x* when *x* approaches to zero. In fact, some students claimed that they could resolve the 0/0 situation in sinx/x function through factorization. When they could not do so, the GeoGebra graphs were used to show that the limit was equal to 1. Figure 13 was an image of that activity.

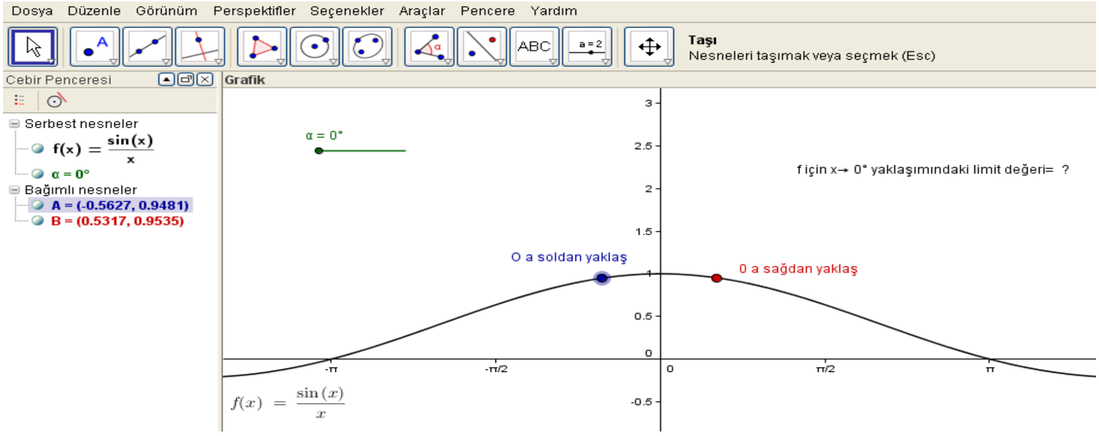



*Figure 13*. GeoGebra applet for investigation of the function *y=sinx/x* around zero

The students observed that the left-hand and right-hand limits of f(*x*)=*sinx/x* as x approaches to zero was equal to 1 despite their observation that f(0) was 0/0. In order to prove algebraically, the researcher gave the students an image as a clue which is shown in Figure 14 as an *iconic stage* activity.

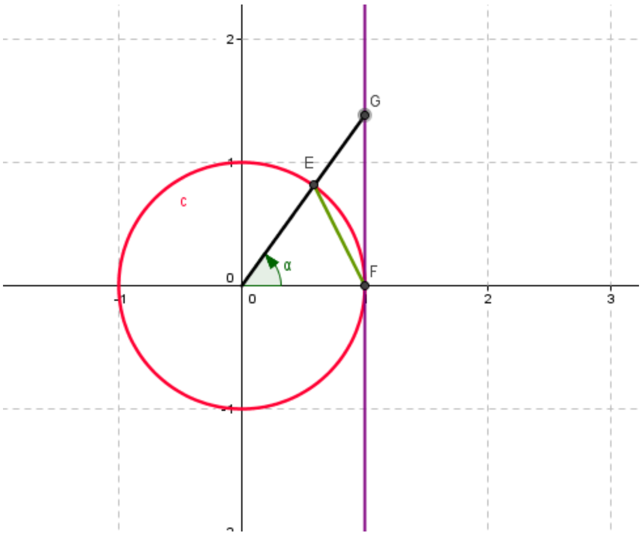



*Figure 14*. A picture given as a clue to prove the limit of *sinx/x* around zero.

The students tried to find out sinx $\leq$ x $\leq$ tanx as a group work by the help of that clue. The students had some difficulties during that activity and teacher had to help the groups. In the proof process, *symbolic stage* was the purpose of the activity. 20 minutes were given for students to prove this.

After the proof section of the lesson, the teacher explained several implications of sinx/x property, including $\lim_{x\to 0}\frac{tanx}{x} = 1$, $\lim_{x\to 0}\frac{sinax}{bx} = \frac{a}{b}$ or $\lim_{x\to 0}\frac{sinax}{tanbx} = \frac{a}{b}$. Then, the researcher distributed a worksheet including eight questions as a further *symbolic stage* activity. See worksheet 4 in the Appendix 7. The allocated time for those eight questions was ten minutes, and the students used GeoGebra by entering the equation of the functions if they needed to observe the behaviours of their graphs, which might be considered the *enactive* and *iconic stages*.

The last part of the lesson was based on continuity. In order to understand the discontinous points of given functions, the researcher used a GeoGebra applet, which is shown in Figure 15.

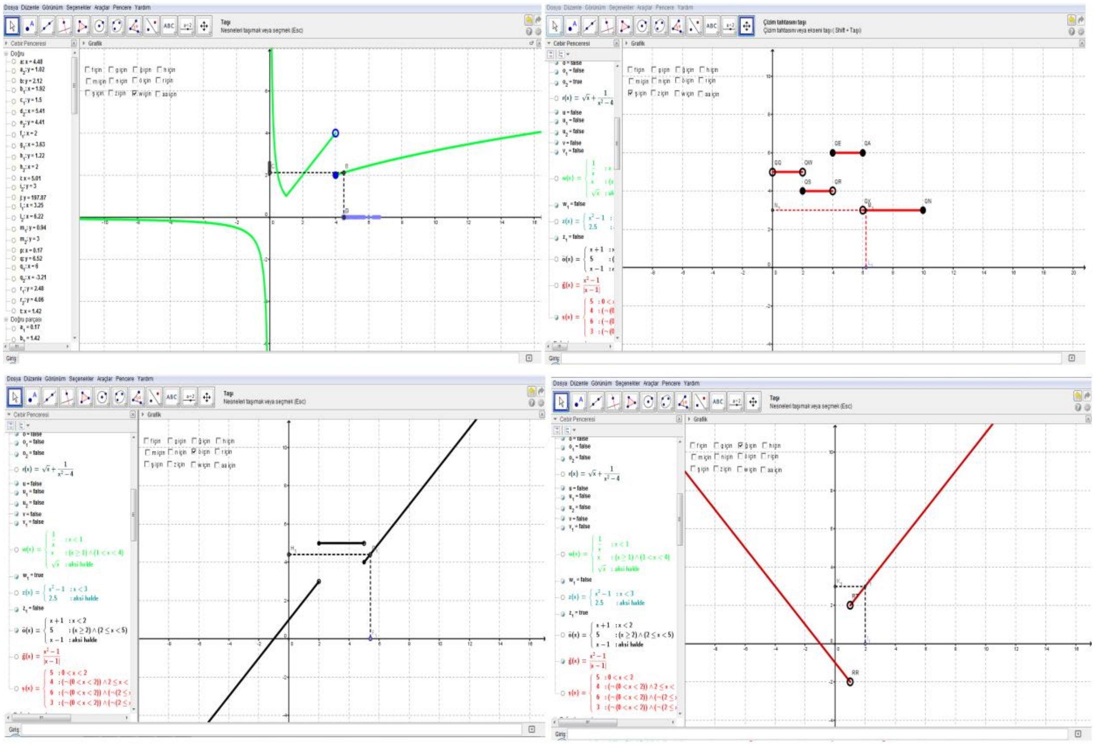


*Figure 15*. GeoGebra applet for continuity (twelve different functions)

On this applet, the students worked out twelve different functions to discover limits as x approaching some particular value, and continuous-discontinuous values and intervals. Four of the functions are shown in Figure 15. The students discussed the relations between the equations of the functions and *x*-values where the function was discontinuous. Students changed the *x* values with the pointer (mouse) to manipulate independently. At the end, there was a class discussion about the infinity and x values, which makes the function indefinite or undefined on the graphs of functions. The allocated time for this activity was about 25 minutes.

The researcher distributed a worksheet that included eight continuity questions. See Appendix 7 (worksheet 5). Some students could finish solving these questions in the class whereas others had work left during their individual study time after school.

For the fifth and sixth lessons, the control group students given the same questions, the same activities with still pictures.

After the intervention period was finished, four lesson periods were assigned to the groups to solve additional multiple choice and open-ended questions before the post-test. The teachers allowed students to solve questions from their own workbooks, or a worksheet that the teachers handed out. Some students from the experimental group used GeoGebra program during those extra lessons to investigate some graphs and limits. See Appendix 7 (worksheet 6) for the last set of worksheets given to both groups.

The teaching strategy for the intervention (experimental group) was to use enactive stage activities through the manipulation of variables on GeoGebra. That initial stage was used to help students move on to iconic (non-dynamic) graphs and finally to the symbolic representations of functions with respect to their limits and continunity. Students who needed to refer back to the enactive stage activities were allowed to do so in the class or during their individual study time. In the control group; however, only the iconic stage was emphasized before the symbolic stage. Thus, the difference was directly related to where students started their journey: enactive -> iconic -> symbolic in the experimental group; in contrast to iconic -> symbolic in the control group.

## Reliability and validity

The score reliabilities for LCRT, LCAT, and MTAS were estimated by using Cronbach's *alpha*, one of the most common internal consistency analysis methods. For MTAS, Cronbach's *alpha* was calculated for each factor separately. High alpha coefficients (above .7) are generally considered to indicate high internal consistency of the scores (Bryman & Cramer, 2005).

Table 6
Item total statistics of LCRT and LCAT scales

| | LCRT Scale | | LCAT Scale | |
|---|---|---|---|---|
| Question Number | Corrected Item-Total Correlation | Cronbach's *alpha* if Item deleted | Corrected Item-Total Correlation | Cronbach's *alpha* if Item deleted |
| LCT1 | -.21 | .58 | .52 | .85 |
| LCT2 | .15 | .52 | .34 | .86 |
| LCT3 | .37 | .47 | .34 | .86 |
| LCT4 | .19 | .51 | .45 | .86 |
| LCT5 | .19 | .51 | .52 | .85 |
| LCT6 | .16 | .52 | .67 | .84 |
| LCT7 | .15 | .52 | .64 | .84 |
| LCT8 | .40 | .48 | .71 | .84 |
| LCT9 | .30 | .48 | .56 | .85 |
| LCT10 | .31 | .48 | .53 | .85 |
| LCT11 | .27 | .49 | .73 | .84 |
| LCT12 | .28 | .49 | .61 | .85 |

In LCRT scale, the first question was needed to be removed because it had a negative corrected item-total correlation, meaning that the item was not measuring a construct similar to that of the rest of the questions (Pallant, 2001). Thus, the final version LCRT included 11 questions while the final version of LCAT was kept as a 12 question test.

Table 7
Item total statistics of MTAS pre-test and MTAS post-test scales

| MTAS Items | MTAS Pre-Test | | MTAS Post-Test | |
|---|---|---|---|---|
| | Corrected Item-Total Correlation | Cronbach's *alpha* if Item deleted | Corrected Item-Total Correlation | Cronbach's *alpha* if Item deleted |
| **Behavioral Engagement (BE)*** | | | | |
| I concentrate hard in mathematics | .06 | .32 | .11 | .63 |
| I try to answer questions the teacher asks | -.03 | .35 | .55 | .12 |
| If I make mistakes, I work until I have corrected them | .50 | -.20 | .35 | .35 |
| If I can't do a problem, I keep trying different ideas | .08 | .24 | .20 | .46 |
| **Technology Confidence (TC)** | | | | |
| I am good at using computers | .82 | .84 | .85 | .90 |
| I am good at using things like VCRs, DVDs, MP3s and mobile phones | .75 | .90 | .75 | .94 |
| I can fix a lot of computer problems | .82 | .84 | .29 | .88 |
| I am quick to learn new computer software needed for school | .80 | .86 | .85 | .90 |
| **Mathematics Confidence (MC)** | | | | |
| I have a mathematical mind | .77 | .86 | .86 | .95 |
| I can get good results in mathematics | .69 | .89 | .89 | .94 |
| I know I can handle difficulties in mathematics | .82 | .85 | .91 | .93 |
| I am confident with mathematics | .82 | .84 | .92 | .93 |
| **Affective Engagement (AE)** | | | | |
| I am interested to learn new things in mathematics | .77 | .67 | .74 | .72 |
| In mathematics you get rewards for your effort | .46 | .84 | .54 | .84 |
| Learning mathematics is enjoyable | .70 | .71 | .80 | .69 |
| I get a sense of satisfaction when I solve mathematics problems | .59 | .77 | .58 | .82 |
| **Learning Mathematics with Technology (MT)** | | | | |
| I like using DGS for learning mathematics | .47 | .58 | .65 | .82 |
| Using DGS is worth the extra effort | .09 | .84 | .57 | .86 |
| Mathematics is more interesting when using DGS | .70 | .45 | .81 | .75 |
| DGS help me learn mathematics better | .70 | .44 | .73 | .79 |

*Note:* *Items under this factor are not included in the final version.

According to the analysis of Cronbach's *alpha* for MTAS pre-test, the BE items needed to be removed in view of corrected item total correlation values which were low. In addition, the second item of MT scale was also removed due to low item-total correlation.

Table 8
Final Cronbach's *alpha* values

| | Cronbach's *alpha* | | *N* of items | |
|---|---|---|---|---|
| | Pre Test | Post Test | Pre Test | Post Test |
| LCRT (Pre Test)<br>LCAT (Post Test) | .59 | .86 | 11 | 12 |
| TC | .89 | .93 | 4 | 4 |
| MC | .89 | .95 | 4 | 4 |
| AE | .80 | .82 | 4 | 4 |
| MT | .84 | .86 | 3 | 3 |

*Note:* MC: Mathematics Confidence; TC: Technology Confidence; AE: Affective Engagement; MT: Learning Mathematics with Technology

Thus, as evidence of validity, the researcher used the following methods:

(a) a pilot study;

(b) expert views of a mathematics education professor;

(c) views of a mathematics teacher from outside the school where the study was conducted;

(d) views of mathematics teachers from within the school; and

(e) an external rating to assess the LCRT and LCAT.

## Data analysis

First, data were explored in terms of normality and outliers by using the statistics software SPSS version 15. Any divergence from normality was examined in terms of the standardized scores, skewness, kurtosis, and P-P plots (Tabachnick & Fidell, 2007). No outliers or missing scores were detected.

Second, descriptive statistics for each item were analyzed to have a better understanding of how the participants responded to each item on the average. In addition, non-parametric two-sample Mann-Whitney *U* test was aused to understand the mean rank differences between control and experimental groups at the item level. Effect sizes were estimated with the formula

$$r = z/\sqrt{n}$$

This *r* effect size was later converted to Cohen's *d* for an easier understanding of the size of the effect (DeCoster, 2009).

Third, to answer the research questions:

(a) paired sample *t*-tests were used to determine whether there were statistically significant pre-test and post-test score differences on the average within either control or experimental group internally;

(b) independent samples *t*-tests were used to determine whether there was a difference between the control and experimental group gain scores. Gain scores were computed by subtracting the pre-test score average from the post-test average as the number of items were not equal in LCRT and LCAT;

(c) Hence, effect sizes which helped researcher understand the sizes of the impact between control and experimental groups at factor level were estimated with Cohen's *d*.

Four, bivariate correlations for determining whether there was a statistically significant relationship between the students' learning and attitudes were applied at the factor level. Bivariate correlations were estimated between each pair of factors with Pearson's product-moment correlation coefficient *r*. Low correlations between MTAS, LCRT or LCAT scores provided the evidence to not conduct a multivariate analysis.

# CHAPTER 4: RESULTS

## Conceptual understanding in limits and continuity

### Impact at the question level

Data from LCRT and LCAT questions were analyzed in terms of percent distributions of student responses. Table 9 and Table 10 present the percent distribution for each the LCRT question.

Table 9

Percent distribution of responses to each LCRT question

| | Correct (4) | | Partially Correct (3) | | Wrong 1 (2) | | Wrong 2 (1) | | Unanswered (0) | |
|---|---|---|---|---|---|---|---|---|---|---|
| Question Number | EG* | CG** | EG | CG | EG | CG | EG | CG | EG | CG |
| LCRT2 | 86.7 | 57.9 | 6.7 | 0 | 0 | 10.5 | 6.7 | 21.1 | 0 | 10.5 |
| LCRT3 | 33.3 | 5.3 | 0 | 0 | 33.3 | 42.1 | 26.7 | 52.6 | 6.7 | 0 |
| LCRT4 | 0 | 5.3 | 6.7 | 5.3 | 0 | 5.3 | 80.0 | 47.4 | 13.3 | 36.8 |
| LCRT5 | 20.0 | 5.3 | 0 | 5.3 | 0 | 5.3 | 20.0 | 36.8 | 60.0 | 47.4 |
| LCRT6 | 0 | 0 | 0 | 0 | 86.7 | 78.9 | 13.3 | 15.8 | 0 | 5.3 |
| LCRT7 | 0 | 5.3 | 6.7 | 0 | 40.0 | 31.6 | 40.0 | 52.6 | 13.3 | 10.5 |
| LCRT8 | 0 | 5.3 | 0 | 0 | 46.7 | 42.1 | 40.0 | 47.4 | 13.3 | 5.3 |
| LCRT9 | 20.0 | 42.1 | 0 | 0 | 0 | 0 | 46.7 | 31.6 | 33.3 | 26.3 |
| LCRT10 | 26.7 | 57.9 | 0 | 0 | 0 | 0 | 46.7 | 21.1 | 26.7 | 21.1 |
| LCRT11 | 0 | 10.5 | 40.0 | 47.4 | 26.7 | 26.3 | 20.0 | 5.3 | 13.3 | 10.5 |
| LCRT12 | 0 | 10.5 | 46.7 | 26.3 | 26.7 | 21.1 | 13.3 | 31.6 | 13.3 | 10.5 |

*Note:* *EG stands for the experimental group. **CG stands for the control group.

Table 9 shows that there were six LCRT questions (4, 6, 7, 8, 11, and 12th questions) which none of the students from the experimental group could answer correctly; whereas there was only one LCRT question (6th question) which none of the control group students could answer correctly. Additionally, questions 6, 7, and 8 appeared to have been the most difficult questions because they had a low percentage of correctness for students in both of the groups. One other notable point was that 60% of the students from the experimental group did not answer question 5.

Table 10

Percent distribution of responses to each LCAT question

| | Correct (4) | | Partially Correct (3) | | Wrong1 (2) | | Wrong2 (1) | | Unanswered (0) | |
|---|---|---|---|---|---|---|---|---|---|---|
| Question Number | EG* | CG** | EG | CG | EG | CG | EG | CG | EG | CG |
| LCAT1 | 6.7 | 0 | 13.3 | 0 | 66.7 | 26.3 | 6.7 | 73.7 | 6.7 | 0 |
| LCAT2 | 66.7 | 47.4 | 0 | 5.3 | 33.3 | 26.3 | 0 | 21.1 | 0 | 0 |
| LCAT3 | 13.3 | 5.3 | 0 | 5.3 | 33.3 | 21.1 | 40.0 | 63.2 | 13.3 | 5.3 |
| LCAT4 | 73.3 | 36.8 | 6.7 | 21.1 | 0 | 15.8 | 6.7 | 26.3 | 13.3 | 0 |
| LCAT5 | 53.3 | 21.1 | 6.7 | 0 | 33.3 | 36.8 | 0 | 26.3 | 6.7 | 15.8 |
| LCAT6 | 73.3 | 21.1 | 6.7 | 15.8 | 0 | 10.5 | 13.3 | 52.6 | 6.7 | 0 |
| LCAT7 | 80.0 | 5.3 | 6.7 | 15.8 | 13.3 | 26.3 | 0 | 52.6 | 0 | 0 |
| LCAT8 | 66.7 | 10.5 | 0 | 15.8 | 6.7 | 36.8 | 26.7 | 31.6 | 0 | 5.3 |
| LCAT9 | 73.3 | 10.5 | 13.3 | 26.3 | 0 | 26.3 | 6.7 | 21.1 | 6.7 | 15.8 |
| LCAT10 | 93.3 | 47.4 | 0 | 26.3 | 6.7 | 21.1 | 0 | 0 | 0 | 5.3 |
| LCAT11 | 26.7 | 10.5 | 60.0 | 36.8 | 13.3 | 42.1 | 0 | 10.5 | 0 | 0 |
| LCAT12 | 33.3 | 15.8 | 46.7 | 26.3 | 0 | 0 | 13.3 | 47.4 | 6.7 | 10.5 |

*Note:* *EG stands for experimental group. **CG stands for control group.

Table 10 shows that the percentage of totally correct answers of the experimental group was higher than that of the control group for all LCAT questions. One remarkable point is that 80% of experimental group students answered the 7th question totally correct, as compared to only 5.3% of the control group students. Similar situation existed for questions number 8 and 9. The first question of the LCAT seems to have been the most difficult, in view of its low correctness percentage for both of the groups.

Question level location statistics were calculated in order to have a better understanding of how the students responded to the questions as a whole. See Table 11 and Table 12 for the item level location statistics.

Table 11

Question level location statistics for each LCRT item

| Question Number | Median | Mode | Range |
|---|---|---|---|
| LCRT2 | 4 | 4 | 4 |
| LCRT3 | 2 | 1 | 4 |
| LCRT4 | 1 | 1 | 4 |
| LCRT5 | 0 | 0 | 4 |
| LCRT6 | 2 | 2 | 2 |
| LCRT7 | 1 | 1 | 4 |
| LCRT8 | 1 | 1-2 | 4 |
| LCRT9 | 1 | 1 | 4 |
| LCRT10 | 1 | 4 | 4 |
| LCRT11 | 2.5 | 3 | 4 |
| LCRT12 | 2 | 3 | 4 |

The location of the data showed a variance for the LCRT in terms of the accuracy of the answers to the test questions. The first question and the last three questions of the LCRT were centered around modes of 3 and 4 which means that most of the questions were partially correct or totally correct. However, the rest of the questions except 5 were centered around the modes of 1 or 2 which means that they were mostly partially wrong or totally wrong. The fifth question was centered around a mode of 0 which means that it was mostly unanswered.

Table 12

Question level location statistics for each LCAT item

| Question Number | Median | Mode | Range |
|---|---|---|---|
| LCAT1 | 2 | 1-2 | 4 |
| LCAT2 | 4 | 4 | 3 |
| LCAT3 | 1 | 1 | 4 |
| LCAT4 | 4 | 4 | 4 |
| LCAT5 | 2 | 2-4 | 4 |
| LCAT6 | 3 | 4 | 4 |
| LCAT7 | 2.5 | 4 | 3 |
| LCAT8 | 2 | 4 | 4 |
| LCAT9 | 3 | 4 | 4 |
| LCAT10 | 4 | 4 | 4 |
| LCAT11 | 3 | 3 | 3 |
| LCAT12 | 3 | 3 | 4 |

The location of the data for the LCAT was centered around a mode of 4, indicating that the most of the students in the sample answered the questions correctly.

*Mann-Whitney U test statistics*

First, a non-parametric two-sample Mann-Whitney *U* test was conducted at the question level for each LCRT and LCAT item. Table 13 shows the statistical significance of the difference between the mean ranks of two independent groups—control group and the experimental group with regard to each question in the limits and continuity readiness test. The Mann Whitney test allows comparison between the mean ranks with critical *U* values. In addition, the *p*-calculated values column, which indicates the probability, provides assistance in rejecting or failing to reject the null hypothesis of no difference by considering its value to a pre-determined *alpha* value of .05 (Huck, 2011).

Table 13

The Mann-Whitney *U* test statistics between experimental and control groups in each LCRT item

| Question Number | Mann-Whitney *U* | Z | *p*-calculated |
|---|---|---|---|
| LCRT2 | 97.50 | -1.94 | > .05 |
| LCRT3 | 102.50 | -1.49 | .14 |
| LCRT4 | 122.50 | -0.80 | .42 |
| LCRT5 | 133.50 | -0.34 | .73 |
| LCRT6 | 130.50 | -0.62 | .53 |
| LCRT7 | 133.00 | -0.36 | .72 |
| LCRT8 | 132.00 | -0.40 | .69 |
| LCRT9 | 113.50 | -1.07 | .28 |
| LCRT10 | 104.00 | -1.43 | .15 |
| LCRT11 | 106.50 | -1.32 | .19 |
| LCRT12 | 131.50 | -0.40 | .70 |

The results of the non-parametric two-sample Mann Whitney *U* test showed that there was no statistically significant difference in any of the LCRT items between mean rank scores of the control and experimental groups (for all the questions $p > .05$).

Table 14

The Mann-Whitney U test statistics between experimental and control groups in each LCAT item

| | Mann-Whitney U | Z | p-calculated |
|---|---|---|---|
| LCAT1 | 56.00 | -3.30 | < .05 |
| LCAT2 | 107.50 | -1.36 | .17 |
| LCAT3 | 130.00 | -0.47 | .63 |
| LCAT4 | 102.00 | -1.53 | .13 |
| LCAT5 | 75.00 | -2.46 | .01 |
| LCAT6 | 74.50 | -2.53 | .01 |
| LCAT7 | 21.50 | -4.40 | < .05 |
| LCAT8 | 78.50 | -2.31 | .02 |
| LCAT9 | 51.50 | -3.28 | < .05 |
| LCAT10 | 79.00 | -2.66 | < .05 |
| LCAT11 | 79.50 | -2.35 | .02 |
| LCAT12 | 89.00 | -1.95 | > .05 |

Table 14 shows whether there was a statistically significant difference between the mean ranks of control group and the experimental group for each question in LCAT. The results of the non-parametric two-sample Mann Whitney *U* test showed that there was a statistically significant difference in 8 items between mean rank scores of the control and experimental groups (Question numbers 1, 5, 6, 7, 8, 9, 10, and 11). The test indicated that the experimental group's mean rank scores in those 8 items were statistically significantly higher than the control group's mean rank scores ($p < .05$).

Second, the *r* effect sizes were calculated and converted to Cohen's *d* for an easier interpretation. Table 15 shows the *r* effect sizes and Cohen's *d* equivalents for the LCAT items.

Table 15

Effect sizes in each LCAT item

| Items | *r* | Cohen's *d* |
|---|---|---|
| LCAT1 | .56 | 1.35 |
| LCAT2 | .23 | 0.47 |
| LCAT3 | .08 | 0.16 |
| LCAT4 | .26 | 0.54 |
| LCAT5 | .41 | 0.90 |
| LCAT6 | .43 | 0.95 |
| LCAT7 | .74 | 2.20 |
| LCAT8 | .39 | 0.84 |
| LCAT9 | .55 | 1.31 |
| LCAT10 | .45 | 1.00 |
| LCAT11 | .40 | 0.87 |
| LCAT12 | .33 | 0.70 |

Table 15 shows that some of the effect sizes could be considered to indicate a practical difference when they were compared to the pilot study's effect size or average effect size in similar studies (See Table 31). Thus, there was a very large difference between the groups in question 7, 1, and 9, in favour of the experimental group.

**Impact at the test level**

In order to determine whether there was a statistically significant improvement in both group of students' conceptual understanding, a paired sample *t*-test was used. See Table 16 for the groups' mean, standard deviation, *t*-value, *df*-value, and *p*-calculated values.

Table 16

Paired sample *t*-test statistics

| Group | | Mean | *SD* | *t* | *df* | *p* |
|---|---|---|---|---|---|---|
| Control Group (*n* = 19) | LCRT | 1.78 | 0.57 | -1.56 | 18 | .14 |
| | LCAT | 2.10 | 0.65 | | | |
| Experimental Group (*n* = 15) | LCRT | 1.76 | 0.50 | -8.97 | 14 | < .05 |
| | LCAT | 3.04 | 0.57 | | | |

*Note*. Only the correlation for the experimental group's pre-test and post-test scores was statistically significant ($r = .48$, $p < .05$).

Table 16 indicates that a statistically significant improvement was only observed for the experimental group: $t = -8.97$, $p < .05$. This finding helped the researcher reject the null hypothesis that there was not a significant difference between pre-test scores and the post-test scores for the experimental group.

In order to estimate the size of the impact of teaching with GeGebra, independent *t*-tests were conducted for LCRT, LCAT, and gain scores. Gain scores were computed as the difference between LCRT and LCAT scores. See Table 17 for independent samples *t* test statistics.

Table 17

Independent samples *t*-test statistics

| | Group | *N* | Mean | *SD* | *t* | *df* | *p* | Cohen's *d* |
|---|---|---|---|---|---|---|---|---|
| LCRT | CG** | 19 | 1.79 | 0.57 | .11 | 32 | .91 | 0.04 |
| | EG* | 15 | 1.76 | 0.50 | | | | |
| LCAT | CG | 19 | 2.10 | 0.65 | -4.42 | 32 | < .05 | 1.57 |
| | EG | 15 | 3.04 | 0.57 | | | | |
| Gain | CG | 19 | 0.31 | 0.87 | -3.74 | 32 | < .05 | 1.33 |
| | EG | 15 | 1.28 | 0.55 | | | | |

*Note.* Gain scores were computed by subtracting LCRT scores from LCAT scores. *EG stands for experimental group. **CG stands for control group.

*What is the impact of learning limits and continuity concepts with GeoGebra on G&T students' conceptual understanding?*

First, there was not a statistically significant difference between the control group and experimental group with respect to their LCRT scores on the average: $t = 0.11$, $p > .05$. According to this finding, the researcher did not reject the null hypothesis that there was not a statistically significant difference between the control and experimental groups in LCRT. Thus, students in both groups were assumed to have a similar level of knowledge on limits and continuity before the intervention.

Second, there was a statistically significant difference between the control group and experimental group with respect to the LCAT scores on the average: $t = -4.42$, $p < .05$, Cohen's $d = 1.57$. According to this finding, the researcher rejected the null hypothesis that there was not a significant difference between the groups in LCAT. Thus, it was found that the students in the experimental group had a more advanced conceptual understanding of limits and continuity at the end of the intervention. Finally, the results indicated that there was a statistically significant difference between the control group's gain scores and the experimental group's gain scores on the average: $t = -3.74$, $p < .05$, Cohen's $d = 1.33$. According to this finding, researcher rejected the null hypothesis that there was not a significant difference between the groups in gain scores. Thus, it was found that the students in the experimental group had developed a more advanced conceptual understanding of limits and continuity after the intervention.

The effect sizes for the comparison of the groups on the LCRT, the LCAT and gain limits scores were considered to indicate a practical difference when compared to the effect size estimated in the pilot study and average effect size in similar studies (See Table 31).

## Attitudes towards technology in mathematics education

### Impact at the item level

PreMTAS and PostMTAS items were analyzed in terms of percent distributions of students' responses.

Table 18

Percent distribution of responses to each MTAS pre-test item

| | Strongly disagree | | Disagree | | Not sure | | Agree | | Strongly agree | |
|---|---|---|---|---|---|---|---|---|---|---|
| | EG* | CG** | EG | CG | EG | CG | EG | CG | EG | CG |
| PreTC1 | 13.3 | 5.3 | 6.7 | 10.5 | 20.0 | 31.6 | 40.0 | 26.3 | 20.0 | 26.3 |
| PreTC2 | 0 | 0 | 6.7 | 0 | 6.7 | 21.1 | 53.3 | 52.6 | 33.3 | 26.3 |
| PreTC3 | 20.0 | 15.8 | 33.3 | 26.3 | 20.0 | 26.3 | 20.0 | 5.3 | 6.7 | 26.3 |
| PreTC4 | 13.3 | 21.1 | 13.3 | 15.8 | 33.3 | 15.8 | 13.3 | 15.8 | 26.7 | 31.6 |
| PreMC1 | 0 | 10.5 | 0 | 15.8 | 13.3 | 21.1 | 46.7 | 42.1 | 40.0 | 10.5 |
| PreMC2 | 0 | 5.3 | 0 | 5.3 | 0 | 31.6 | 53.3 | 47.4 | 46.7 | 10.5 |
| PreMC3 | 0 | 0 | 0 | 10.5 | 20.0 | 21.1 | 46.7 | 57.9 | 33.3 | 10.5 |
| PreMC4 | 6.7 | 5.3 | 0 | 5.3 | 20.0 | 36.8 | 40.0 | 42.1 | 33.3 | 10.5 |
| PreAE1 | 0 | 0 | 13.3 | 15.8 | 6.7 | 26.3 | 33.3 | 47.4 | 46.7 | 10.5 |
| PreAE2 | 0 | 10.5 | 6.7 | 15.8 | 33.3 | 26.3 | 46.7 | 26.3 | 13.3 | 21.1 |
| PreAE3 | 0 | 0 | 6.7 | 15.8 | 13.3 | 26.3 | 33.3 | 42.1 | 46.7 | 15.8 |
| PreAE4 | 0 | 0 | 0 | 5.3 | 13.3 | 5.3 | 26.7 | 36.8 | 60.0 | 52.6 |
| PreMT1 | 33.3 | 15.8 | 13.3 | 42.1 | 33.3 | 31.6 | 13.3 | 10.5 | 6.7 | 0 |
| PreMT3 | 20.0 | 10.5 | 26.7 | 42.1 | 40.0 | 42.1 | 6.7 | 5.3 | 6.7 | 0 |
| PreMT4 | 13.3 | 21.1 | 33.3 | 42.1 | 33.3 | 26.3 | 20.0 | 10.5 | 0 | 0 |

*Note:* *EG stands for experimental group. **CG stands for control group. PreTC: Technology confident pre test questions. PreMC: Mathematics confident pre test questions. PreAE: Affective engagement pre test questions. PreMT: Learning mathematics with technology pre test questions.

Table 18 shows that students from the experimental group did not strongly disagree with most of the items in the MTAS pre-test. However, in the *preMT*, none of the items were answered with strongly agree by either group. Table 19 indicates that the students from the experimental group were not generally in strong disagreement with the items in the MTAS post-test. On the other hand, none of the students from the control group strongly agreed with any of the items of the *postMT*.

Table 19

Percent distribution of responses to each MTAS post-test item

| | Strongly disagree | | Disagree | | Not sure | | Agree | | Strongly agree | |
|---|---|---|---|---|---|---|---|---|---|---|
| | EG* | CG** | EG | CG | EG | CG | EG | CG | EG | CG |
| PostTC1 | 0 | 5.3 | 40.0 | 15.8 | 13.3 | 26.3 | 33.3 | 26.3 | 13.3 | 26.3 |
| PostTC2 | 0 | 0 | 13.3 | 10.5 | 26.7 | 26.3 | 46.7 | 31.6 | 13.3 | 31.6 |
| PostTC3 | 33.3 | 10.5 | 6.7 | 36.8 | 33.3 | 15.8 | 20.0 | 10.5 | 6.7 | 26.3 |
| PostTC4 | 13.3 | 10.5 | 20.0 | 21.1 | 26.7 | 26.3 | 26.7 | 21.1 | 13.3 | 21.1 |
| PostMC1 | 0 | 10.5 | 0 | 15.8 | 26.7 | 15.8 | 40.0 | 42.1 | 33.3 | 15.8 |
| PostMC2 | 0 | 5.3 | 0 | 5.3 | 6.7 | 26.3 | 53.3 | 52.6 | 40.0 | 10.5 |
| PostMC3 | 0 | 5.3 | 0 | 10.5 | 20.0 | 26.3 | 33.3 | 42.1 | 46.7 | 15.8 |
| PostMC4 | 6.7 | 10.5 | 0 | 15.8 | 20.0 | 15.8 | 33.3 | 42.1 | 40.0 | 15.8 |
| PostAE1 | 0 | 5.3 | 6.7 | 15.8 | 6.7 | 26.3 | 46.7 | 42.1 | 40.0 | 10.5 |
| PostAE2 | 0 | 10.5 | 13.3 | 10.5 | 20.0 | 31.6 | 40.0 | 15.8 | 26.7 | 31.6 |
| PostAE3 | 0 | 5.3 | 0 | 10.5 | 13.3 | 36.8 | 40.0 | 31.6 | 46.7 | 15.8 |
| PostAE4 | 0 | 0 | 0 | 0 | 6.7 | 10.5 | 40.0 | 42.1 | 53.3 | 47.4 |
| PostMT1 | 0 | 36.8 | 33.3 | 31.6 | 20.0 | 26.3 | 40.0 | 5.3 | 6.7 | 0 |
| PostMT3 | 0 | 36.8 | 26.7 | 26.3 | 20.0 | 26.3 | 33.3 | 10.5 | 20.0 | 0 |
| PostMT4 | 6.7 | 31.6 | 20.0 | 10.5 | 26.7 | 42.1 | 40.0 | 15.8 | 6.7 | 0 |

*Note:* *EG stands for experimental group. **CG stands for control group. PostTC: Technology confident post test questions. PostMC: Mathematics confident post test questions. PostAE: Affective engagement post test questions. PostMT: Learning mathematics with technology post test questions.

Although Table 19 shows some similarities to Table 18, there were some slight differences. One of the most remarkable differences was in the *MT* items. In the MTAS pretest, some students from experimental group responded strongly disagree for the *preMT* items. However, in the MTAS post test, none of the students from the experimental group strongly disagreed for the items *preMT1* and *preMT3*.

Item level location statistics were calculated in order to have a better understanding of how the students responded to the MTAS items. See Table 20 and Table 21 for the item level location statistics.

Table 20
Item level location statistics for each MTAS pre-test item

| | Median | Mode | Range |
|---|---|---|---|
| PreTC1 | 4 | 4 | 4 |
| PreTC2 | 4 | 4 | 3 |
| PreTC3 | 3 | 2 | 4 |
| PreTC4 | 3 | 5 | 4 |
| PreMC1 | 4 | 4 | 4 |
| PreMC2 | 4 | 4 | 4 |
| PreMC3 | 4 | 4 | 3 |
| PreMC4 | 4 | 4 | 4 |
| PreAE1 | 4 | 4 | 3 |
| PreAE2 | 4 | 4 | 4 |
| PreAE3 | 4 | 4 | 3 |
| PreAE4 | 5 | 5 | 3 |
| PreMT1 | 2 | 3 | 4 |
| PreMT3 | 2.5 | 3 | 4 |
| PreMT4 | 2 | 2 | 3 |

*Note:* PreTC: Technology confident pre test questions. PreMC: Mathematics confident pre test questions. PreAE: Affective engagement pre test questions. PreMT: Learning mathematics with technology pre test questions.

Table 21

Item level location statistics for each MTAS post-test item

| | Median | Mode | Range |
|---|---|---|---|
| PostTC1 | 3.5 | 4 | 4 |
| PostTC2 | 4 | 4 | 3 |
| PostTC3 | 3 | 2-3 | 3 |
| PostTC4 | 3 | 3 | 4 |
| PostMC1 | 4 | 4 | 4 |
| PostMC2 | 4 | 4 | 4 |
| PostMC3 | 4 | 4 | 4 |
| PostMC4 | 4 | 4 | 4 |
| PostAE1 | 4 | 4 | 4 |
| PostAE2 | 4 | 5 | 4 |
| PostAE3 | 4 | 4 | 4 |
| PostAE4 | 4.5 | 5 | 2 |
| PostMT1 | 2 | 2 | 4 |
| PostMT3 | 3 | 2 | 4 |
| PostMT4 | 3 | 3 | 4 |

*Note:* PostTC: Technology confident post test questions. PostMC: Mathematics confident post test questions. PostAE: Affective engagement post test questions. PostMT: Learning mathematics with technology post test questions.

The location of data was centered around a mode of 4 which stands for agree for both MTAS pre-test and post-test items. However, the range values, which could help to understand measures of data dispersion, were quite large.

*Mann-Whitney U test statistics*

A non-parametric two-sample Mann-Whitney *U* test was conducted at the item level for each item in MTAS pre-test and post-test. Table 22 presents whether there was a statistically significant difference between the mean ranks of two independent groups—the control group and the experimental group—in attitudes towards use of technology in mathematics education.

Table 22

The Mann-Whitney *U* test statistics between experimental and control groups in each MTAS pre-test item

| | Mann-Whitney *U* | Z | *p*-calculated |
|---|---|---|---|
| PreTC1 | 139.50 | -0.11 | .91 |
| PreTC2 | 128.50 | -0.53 | .60 |
| PreTC3 | 120.50 | -0.78 | .43 |
| PreTC4 | 140.50 | -0.07 | .94 |
| PreMC1 | 72.00 | -2.58 | .01 |
| PreMC2 | 59.00 | -3.14 | <.05 |
| PreMC3 | 102.50 | -1.52 | .13 |
| PreMC4 | 100.00 | -1.56 | .12 |
| PreAE1 | 88.00 | -1.99 | > .05 |
| PreAE2 | 122.50 | -0.72 | .47 |
| PreAE3 | 90.00 | -1.91 | > .05 |
| PreAE4 | 134.00 | -0.33 | .74 |
| PreMT1 | 138.50 | -0.14 | .88 |
| PreMT3 | 136.50 | -0.22 | .82 |
| PreMT4 | 114.50 | -1.01 | .31 |

*Note:* PreTC: Technology confident pre test questions. PreMC: Mathematics confident pre test questions. PreAE: Affective engagement pre test questions. PreMT: Learning mathematics with technology pre test questions.

The results of non-parametric Mann-Whitney *U* test show that there was a statistically significant difference in two items between the experimental group's mean rank scores and the control group's mean rank scores (*preMC1* and *preMC2*). The results indicated that the experimental group's mean rank scores were statistically significantly higher than the control group's mean rank scores for those items. Both items belong to the domain MC: *have a mathematical mind* (MC1: $z$ = -

2.58, $p < .05$, $r = .44$ and Cohen's $d = 0.99$) and *get good results in mathematics* (MC2: $z = -3.14$, $r = .54$ *and* Cohen's $d = 1.28$).

Table 23

The Mann-Whitney *U* test statistics between experimental and control groups in each MTAS post-test item

| | Mann-Whitney *U* | *Z* | *p*-calculated |
|---|---|---|---|
| PostTC1 | 118.50 | -0.86 | .39 |
| PostTC2 | 122.00 | -0.74 | .46 |
| PostTC3 | 118.50 | -0.85 | .39 |
| PostTC4 | 134.00 | -0.30 | .76 |
| PostMC1 | 99.50 | -1.56 | .12 |
| PostMC2 | 76.50 | -2.50 | .01 |
| PostMC3 | 86.00 | -2.06 | .04 |
| PostMC4 | 99.50 | -1.55 | .12 |
| PostAE1 | 77.00 | -2.40 | .16 |
| PostAE2 | 125.00 | -0.63 | .53 |
| PostAE3 | 71.50 | -2.58 | .01 |
| PostAE4 | 132.00 | -0.40 | .68 |
| PostMT1 | 58.50 | -3.01 | <.05 |
| PostMT3 | 56.50 | -3.06 | <.05 |
| PostMT4 | 89.00 | -1.92 | >.05 |

*Note:* PostTC: Technology confident post test questions. PostMC: Mathematics confident post test questions. PostAE: Affective engagement post test questions. PostMT: Learning mathematics with technology post test questions.

There was a statistically significant difference between the mean rank scores of the experimental and control groups in five items: *postMC2*, *postMC3*, *postAE3*, *postMT1* and *postMT3*. The results show that the experimental group mean rank scores were statistically significantly higher than the control group mean rank scores for those items. Table 24 shows the effect sizes for all MTAS post-test items.

Table 24

Effect sizes in each MTAS post-test item

| Items | *R* | Cohen's *d* |
|---|---|---|
| PostTC1 | .15 | 0.30 |
| PostTC2 | .13 | 0.26 |
| PostTC3 | .14 | 0.28 |
| PostTC4 | .05 | 0.10 |
| PostMC1 | .27 | 0.56 |
| PostMC2 | .43 | 0.95 |
| PostMC3 | .35 | 0.75 |
| PostMC4 | .26 | 0.54 |
| PostAE1 | .41 | 0.90 |
| PostAE2 | .11 | 0.22 |
| PostAE3 | .44 | 0.98 |
| PostAE4 | .07 | 0.14 |
| PostMT1 | .52 | 1.22 |
| PostMT3 | .52 | 1.22 |
| PostMT4 | .33 | 0.70 |

**Impact at the factor level**

In order to determine whether there was a statistically significant effect of the intervention on the students attitude towards learning mathematics with technology, an independent *t*-test was conducted for MTAS pre-test, post-test, and gain scores (post-test average was subtracted from pre-test average) to find the size of the impact for control and experimental groups. See Table 25.

Table 25

Independent *t*-test statistics for MTAS

| | Groups | *N* | Mean | SD | *t* | *df* | *p* |
|---|---|---|---|---|---|---|---|
| PreTC | Control | 19 | 3.46 | 1.14 | 0.25 | 32 | .80 |
| | Experimental | 15 | 3.37 | 1.02 | | | |
| PreMC | Control | 19 | 3.49 | 0.86 | -2.64 | 32 | .01 |
| | Experimental | 15 | 4.20 | 0.66 | | | |
| PreAE | Control | 19 | 3.70 | 0.74 | -1.60 | 32 | .12 |
| | Experimental | 15 | 4.12 | 0.78 | | | |
| PreMT | Control | 19 | 2.35 | 0.76 | -0.61 | 32 | .55 |
| | Experimental | 15 | 2.53 | 1.00 | | | |
| PostTC | Control | 19 | 3.41 | 1.15 | 0.76 | 32 | .45 |
| | Experimental | 15 | 3.12 | 1.07 | | | |
| PostMC | Control | 19 | 3.46 | 1.08 | -2.14 | 32 | .04 |
| | Experimental | 15 | 4.17 | 0.77 | | | |
| PostAE | Control | 19 | 3.66 | 0.81 | -2.02 | 32 | >.05 |
| | Experimental | 15 | 4.20 | 0.73 | | | |
| PostMT | Control | 19 | 2.18 | 0.90 | -3.59 | 32 | <.05 |
| | Experimental | 15 | 3.29 | 0.90 | | | |
| TCgain | Control | 19 | -0.05 | 0.50 | 1.25 | 32 | .22 |
| | Experimental | 15 | -0.25 | 0.39 | | | |
| MCgain | Control | 19 | -0.02 | 0.66 | 0.04 | 28.00 | .97 |
| | Experimental | 15 | -0.03 | 0.34 | | | |
| AEgain | Control | 19 | -0.04 | 0.52 | -0.67 | 32 | .51 |
| | Experimental | 15 | 0.08 | 0.54 | | | |
| MTgain | Control | 19 | -0.18 | 0.86 | -2.96 | 32 | <.05 |
| | Experimental | 15 | 0.76 | 0.98 | | | |

*Note*. SD = standard deviation. $p = p_{\text{calculated}}$.

TC: Technology Confidence. MC: Mathematics Confidence. AE: Affective Engagement.

MT: Learning Mathematics with Technology.

*What is the impact of learning limits and continuity concepts with GeoGebra on G&T students' levels of attitude towards learning mathematics with technology?*

According to Table 25, there were statistically significant impacts on the following domains in favor of the experimental group:

preMC: $t$ = -2.64, $p$ <.05, Cohen's $d$ = 0.94;

postMC: $t$ = -2.14, $p$ <. 05, Cohen's $d$ = 0.76;

postMT: $t$ = -3. 59, $p$ <.05, Cohen's $d$ = 1.28;

MTgain: $t$ = 2.96, $p$ <.05, Cohen's $d$ = 1.05.

For the other domains of MTAS, the researcher did not reject the null hypothesis for no difference on the average. See Table 26 for the effect sizes of all MTAS domains.

Table 26

Effect sizes in each MTAS domain

| MTAS domains | *t* | Cohen's *d* |
|---|---|---|
| PreTC | 0.25 | 0.09 |
| PreMC | -2.64 | 0.94 |
| PreAE | -1.60 | 0.57 |
| PreMT | -0.61 | 0.22 |
| PostTC | 0.76 | 0.27 |
| PostMC | -2.14 | 0.76 |
| PostAE | -2.02 | 0.72 |
| PostMT | -3.59 | 1.28 |
| TCgain | 1.25 | 0.45 |
| MCgain | 0.04 | 0.01 |
| AEgain | -0.67 | 0.24 |
| MTgain | -2.96 | 1.05 |

## Bivariate correlations between variables

Bivariate correlations were calculated between the continuous variables. See Table 27 and Table 28 for bivariate correlations between *LCRT*, *LCAT* and MTAS factors according to the control and experimental groups separately:

Table 27

Bivariate correlations for control group (actual scores)

| | LCRT | LCAT | PreTC | PreMC | PreAE | PreMT | PostTC | PostMC | PostAE | PostMT |
|---|---|---|---|---|---|---|---|---|---|---|
| LCRT | 1 | -.01 | .13 | .08 | .12 | .57* | .19 | .01 | .23 | .31 |
| LCAT | | 1 | .11 | .38 | .06 | -.18 | -.01 | .59* | .38 | .13 |
| PreTC | | | 1 | .07 | -.56* | .20 | .90* | .03 | -.37 | -.22 |
| PreMC | | | | 1 | .40 | .25 | -.13 | .79* | .48* | .16 |
| PreAE | | | | | 1 | .16 | -.56* | .38 | .78* | .40 |
| PreMT | | | | | | 1 | .29 | -.01 | .03 | .57* |
| PostTC | | | | | | | 1 | -.23 | -.46* | -.18 |
| PostMC | | | | | | | | 1 | .66* | .07 |
| PostAE | | | | | | | | | 1 | .33 |
| PostMT | | | | | | | | | | 1 |

*Note.* *Correlation is statistically significant at the .05 level.

TC: Technology Confidence. MC: Mathematics Confidence. AE: Affective Engagement. MT: Learning Mathematics with Technology.

Some pairs of correlations, shown with an asterisk, were statistically significant for the control group at the $p < .05$ level. The strongest positive correlation was observed between the *preTC* scores and *postTC* scores and was evaluated as being very strong ($r^2 = .81$), indicating that students' technology confidence in the pretest was strongly correlated with students' technology confidence in the posttest for the control group. One other remarkable point in Table 27 was that the *preTC* (technology confidence) scores were moderately negatively correlated with the *preAE* scores, as were the *preAE* scores with the *postTC* scores ($r^2 = .31$) for the control group. Moreover, for the control group, only the pairs *LCRT* and *preMT*;

*LCAT* and *postMC* were positively moderately correlated; with the $r^2$ values .32 and .35, respectively.

Table 28
Bivariate correlations for experimental group (actual scores)

| | LCRT | LCAT | PreTC | PreMC | PreAE | PreMT | PostTC | PostMC | PostAE | PostMT |
|---|---|---|---|---|---|---|---|---|---|---|
| LCRT | 1 | .47 | -.01 | .51 | .25 | -.04 | -.07 | .44 | .36 | -.18 |
| LCAT | | 1 | .47 | .69* | .49 | .27 | .35 | .63* | .50 | .01 |
| PreTC | | | 1 | .70* | .62* | -.20 | .93* | .59* | .41 | -.45 |
| PreMC | | | | 1 | .69* | -.08 | .71* | .90* | .71* | -.26 |
| PreAE | | | | | 1 | .03 | .60* | .74* | .75* | -.31 |
| PreMT | | | | | | 1 | -.12 | -.02 | -.06 | .60* |
| PostTC | | | | | | | 1 | .61* | .49 | -.30 |
| PostMC | | | | | | | | 1 | .85* | -.10 |
| PostAE | | | | | | | | | 1 | .08 |
| PostMT | | | | | | | | | | 1 |

*Note.* *Correlation is statistically significant at the 0.05 level (2-tailed)

The Pearson's *r* values also were calculated for the correlations between the sum scores for the experimental group. The values in bold print show that those pairs of correlations were significant at $p < .05$ level. Two of highest positive correlations were observed between *preTC* and *postTC* scores ($r^2 = .86$), and between *preMC* and *postMC* scores ($r^2 = .81$), and was evaluated very as being strong, indicating that changes in one variable were strongly correlated with changes in the other variable. In addition, the pairs *LCAT* and *preMC*; *LCAT* and *postMC* were strongly positively correlated producing $r^2$ values of .48 and .40, respectively.

Bivariate correlations were also calculated for the gain scores for both of the groups separately. See Table 29 and Table 30 for bivariate correlations in order to see whether and how strongly pairs of gained scores were related.

Table 29

Bivariate correlations for control group (gain scores)

| | gainLimit | gainTC | gainMC | gainAE | gainMT |
|---|---|---|---|---|---|
| gainLimit | 1 | -.44 | .43 | .28 | .34 |
| gainTC | | 1 | -.39 | -.30 | -.08 |
| gainMC | | | 1 | .56* | .20 |
| gainAE | | | | 1 | .11 |
| gainMT | | | | | 1 |

*Note.* *Correlation is statistically significant at the 0.05 level (2-tailed). *GainLimit* is the gain scores between *LCRT* and *LCAT*.

Table 29 indicates that *gainMC* scores were moderately positively correlated with the *gainAE* scores ($r^2$ = .31), showing that changes in *gainMC* scores of students in the control group were moderately correlated with the changes in *gainAE* scores.

Table 30

Bivariate correlations for experimental group (gain scores)

| | gainLimit | gainTC | gainMC | gainAE | gainMT |
|---|---|---|---|---|---|
| gainLimit | 1 | -.16 | .08 | -.12 | -.16 |
| gainTC | | 1 | .10 | .34 | .20 |
| gainMC | | | 1 | .26 | .20 |
| gainAE | | | | 1 | .78* |
| gainMT | | | | | 1 |

*Note.* *Correlation is statistically significant at the 0.05 level (2-tailed). *GainLimit* is the gain scores between *LCRT* and *LCAT*.

In addition, Table 30 shows that *gainAE* scores were strongly positively correlated with the *gainMT* scores ($r^2$ = .61), indicating that changes in *gainAE* scores of students in the experimental group were strongly correlated with the changes in *gainMT* scores.

# CHAPTER 5: DISCUSSION

## Overview of the study

The main purpose of this study was to investigate the impact of teaching mathematics with GeoGebra on 12th grade gifted and talented (G&T) students' conceptual understanding of the limits and continuity concepts. The second purpose was to investigate the impact of GeoGebra on students' attitudes towards learning mathematics with technology. This chapter starts with a summary of the major findings and continues with a discussion of those findings. Discussion was written with a meta-analytical approach in mind; thus, comparing the effect sizes deducted from the current study to those deducted from other similar studies. The chapter ends with concluding remarks, implications for practice, implications for further research, and limitations of the study.

## Major findings

Major findings of the study are stated below:

1) Students in the experimental group outperformed their peers in the control group on the average in terms of gain scores in limits and continuity conceptual understanding test. When the gain scores (post-test scores subtracted from pre-test scores) were compared, there were 1.33 standard deviations difference between two groups. This difference was statistically significant ($p < .05$).

2) Students in the experimental group outperformed their peers in the control group on the average in terms of gain scores in attitudes towards *teaching mathematics with technology* (*MT* factor). There were 1.05 standard deviations difference between two groups. This difference was statistically significant ($p < .05$).

Some of the other findings were given as follows:

3) When post-test scores in limits and continuity test were compared, the experimental group outperformed the control group with 1.57 standard deviations. This difference was statistically significant ($p < .05$).

4) When the post-test scores and pre-test scores were compared within each group, only the experimental group showed a statistically significant improvement. The degree of conceptual understanding of limit and continuity concepts was limited for the control group.

Table 31 summarizes some of the most relevant GeoGebra studies in the literature and presents estimated effect sizes. These studies were selected based on similarity of their methodology to that of the current study and use of GeoGebra as an example of technology-supported mathematics education.

Table 31
Overall effect size for GeoGebra's impact with pre-test post-test research

| Researcher(s) (Year) | Research Area | Focus: Dependent variable | Sample Size | Level | Cohen's *d* |
|---|---|---|---|---|---|
| Pilot data in the current study | Limits and continuity | Achievement | 27 | 12th grade | 1.27 for achievement 1 |
| İçel (2011) | Triangles and Pythagorean theorem | Achievement | 40 | 8th grade | 1.09 for achievement 2 |
| Kepçeoğlu (2010) | Limits and continuity | Achievement | 40 | pre-service teachers | 1.27 for achievement 3 |
| Zengin (2011) | Trigonometry | Achievement<br>Attitude | 51 | 10th grade | 1.55 for achievement 4<br>0.12 for attitude 1 |
| Selçik & Bilgici (2011) | Polygons | Achievement | 32 | 7th grade | 1.43 for achievement 5 |
| Uzun (2014) | Circles and circular regions | Achievement<br>Attitude | 42 | 7th grade | 1.14 for achievement 6<br>0.85 for attitude 2 |
| Hutkemri & Zakaria (2012) | Functions | Conceptual Knowledge<br>Procedural Knowledge | 284 | high school | 0.55 for conceptual knowledge (achievement 7)<br>0.33 for procedural knowledge (achievement 8) |
| Reis & Özdemir (2010) | Parabola | Achievement | 204 | 12th grade | 0.94 for achievement 9 |
| Saha, Ayub & Tarmizi (2010) | Coordinate geometry | Achievement | 53 | high school | 0.63 for achievement 10 |
| | Average | | 1.02 for achievement<br>0.49 for attitude | | |

*Note*. All effect sizes are estimated with respect to control and experimental group difference in post-tests only.

**Discussion of the major findings**

1) *Students in the experimental group outperformed their peers in the control group on the average in terms of gain scores in limits and continuity conceptual understanding tests. When the gain scores (post test scores subtracted from pre-test scores) were compared, there were 1.33 standard deviations difference between two groups. This difference was statistically significant ($p < .05$).*

The best explanation to this finding is related to multiple representations theory. The intervention that facilitates learning abstract mathematical constructs such as limits and continuity, through multiple representations (enactive, iconic and symbolic stages) can be a reason why students in the experimental group outperformed their peers in the control group (Goldin, 2008). In fact, the evidence for dynamic geometry software and GeoGebra in particular, providing teachers with tools to use these stages simultaneously or one after the other is strong (Hohenwarter & Fuchs, 2004; Zengin, Furkan, & Kutluca, 2012). When the intervention in this study is examined more closely, it can be speculated that students benefit from the instruction that successfully enables them to move on to the symbolic stage after extensive exposure to activities that foster both the enactive and iconic stages. It can also be a possible explanation that students can see multiple representations at the same time, such as the one in lesson 1 and 2 (See Figure 2 and 3) where students can manipulate the graph and see how the tabular representation changes.

The alternative; but related explanation to this finding is based on Kortenkamp's (1999) claim that dynamic geometry software can provide students with tools to explore multiple perspectives in a single construction. For example, it is evident from lesson 5 and 6 during the intervention that teachers can show at which points the function is continous or not, while such software also allows for discussion on the limiting process at a particular x value or at infinity (see Figure 15). Another example can be given from the lesson 3 and 4 of the intervention of this study that students can connect the limits to area calculations (See Figure 5). One last example is related to an activity in lesson 1 and 2 where students can see the relationship between the value that makes the function undefined and the value that an asymptote appears (See Figure 3).

A final explanation is related to the characteristics of the population of the present study. Thus, the difference between the experimental and control groups may have originated from the fact that the participants of the study were gifted and talented (G&T) students whose potentials can be realized if appropriate opportunities are provided (Leikin, 2014), including the one provided by GeoGebra. Also, there is evidence in the literature that G&T students adapt to environments in which students have little prior experience, given the fact that students in the sample of this study did not have any previous experience with GeoGebra (Leikin, 2014). A related perspective is based on Renzulli's (1986) three-ring model, which indicates that G&T students' ability, task commitment, and creativity flourish when they are motivate. In fact, students in the study were encouraged to practice with GeoGebra after school hours in a structured way.

Overall, this impact has been evaluated as noteworthy when the effect size was compared to those estimated in İçel (2011), Kepçeoğlu (2010), Uzun (2014), Hutkemri & Zakaria (2012), Reis & Özdemir (2010), and Saha, Ayub, & Tarmizi (2010). The difference in size of the effect can be speculated to be based on the possibility that some students express negative reactions to learning mathematics with GeoGebra. It may be possible that participating students in other studies might have evaluated GeoGebra as time-consuming or just confusing (Mehanovic, 2011).

2) *Students in the experimental group outperformed their peers in the control group on the average in terms of gain scores in attitudes towards teaching mathematics with technology (MT factor). There were 1.05 standard deviations difference between two groups. This difference was statistically significant (p < .05).*

.

The possible reasons behind the positive impact of GeoGebra on student attitudes toward learning mathematics with technology are: (a) GeoGebra facilitates learning and students become aware of it; (b) GeoGebra enhances visuality and renders the topics tangible; and (c) GeoGebra is more appealing to students' learning styles due to its multiple representations features (Aktümen, Yıldız, Horzum, & Ceylan, 2010; Hohenwarter & Fuchs, 2004; Hohenwarter & Jones, 2007; Hohenwarter & Lavicza, 2009).

This finding can be considered important when compared to Zengin (2011) who did not report any practical positive gain in terms of attitudes toward mathematics (Cohen's $d$ = 0.12). However, it should be noted that the scope of the Zengin study did not include measuring attitudes toward learning mathematics with technology. Similarly in a study to investigate student attitudes toward geometry, Uzun (2014) found some practical effect of GeoGebra (Cohen's *d* = 0.85). However, neither of these two studies could reach the impact achieved in the current study. This finding can be explained with the different scopes of the studies. The current study focused on measuring attitudes toward learning mathematics with technology rather than attitude toward mathematics as a subject.

## Implications for practice

In light of the findings, the following recommendations are made:

- Subject-specific technology use should be encouraged at all schools across the country. Thus, technologies that foster conceptual understanding in mathematics such as GeoGebra should be the focus of Fatih project technologies and limited resources should be allocated to similar effective and freeware software.
- Gifted and talented students are versatile learners. Hence, the program developers and teachers of mathematics should consider that effective programs, curricula, or teaching methods can help G&T students fulfil their true potential. GeoGebra technology appears to offer an effective medium for G&T students in learning mathematics and learning mathematics with a applied focus. Thus, it is recommended that teachers of G&T students learn to use GeoGebra technology effectively and to use similar DGS technology with a student-centred approach.
- At the secondary school level, teaching to the test or a classroom teaching based on preparing students to high-stakes tests can be exclusively harmful for the development of G&T students. In order to overcome such harm, university admission system should be flexibly interpreted for these students.

- In order to ensure that GeoGebra and other similar DGS programs are used at the right times and for the right topics in the courses, more advanced and continuous (sustainable) professional development opportunities for mathematics teachers should be created. Forums and blogs through which teachers can exchange ideas can work as alternative professional development.
- The lesson plans of this study may be used for 12th grade mathematics course limits and continuity topics. However, similar resources are limited in number; particularly with a focus on Turkish mathematics curricula. A rich library consisting of detailed lesson plans in Turkish need to be developed. That would allow teachers to share their materials through MoNE's EBA infrastructure.
- Learning environments that support multiple representations should be prepared for gifted and talented students and such students should be supported by various other similar activities.

## Implications for further research

The following recommendations could be made for further studies:

Studies covering other mathematical topics within calculus can be designed and conducted. In fact, more research is needed in a variety of mathematical topics in algebra and geometry. Such research can be extended to both primary school and secondary school levels. However, the need for research at the undergraduate calculus education is critical and much needed, given the fact that calculus teaching at Turkish universities is mainly teacher-centered and that fosters procedural knowledge.

## Limitations

The limitations encountered during the research process were noted by the investigator and the following limitations were identified:

- Because of the obligation to deliver the lessons at concurrent hours in the school, the experimental group teacher and the control group teacher were different individuals,
- The course instruction time was limited to 6 hours, which required considering only certain sub-achievement targets of the limits and continuity topic.

# REFERENCES

Aizikovitsh-Udi, E., & Radakovic, N. (2012). Teaching probability by using GeoGebra dynamic tool and implemanting critical thinking skills. *Procedia-Social and Behavioral Sciences, 46*, 4943-4947.

Akkaya, A., Tatar, E., & Kağızmanlı, T. B. (2011). Using dynamic software in teaching of the symmetry in analytic geometry: The case of Geogebra. *Procedia-Social and Behavioral Sciences, 15*, 2540-2544.

Aktümen, M., Yıldız, A., Horzum, T., & Ceylan, T. (2011). İlköğretim matematik öğretmenlerinin geogebra yazılımının derslerde uygulanabilirliği hakkındaki görüşleri [Elementary mathematics teachers' opinions about geogebra practicality in the lessons]. *Turkish Journal of Computer and Mathematics Education, 2*(2), 103-120.

Aktümen, M., Horzum, T., Yıldız, A., & Ceylan, T. (2011). *Bir dinamik matematik yazılımı: Geogebra ve ilköğretim 6-8. sınıf matematik dersleri için örnek etkinlikler*. Retrieved from http://w3.gazi.edu.tr/web/aktumen/dosyalar/geo gebraturkiye.pdf.

Aktümen, M., & Kaçar, A. (2003). İlköğretim 8. sınıflarda harfli ifadelerle işlemlerin öğretiminde bilgisayar destekli öğretimin rolü ve bilgisayar destekli öğretim üzerine öğrenci görüşlerinin değerlendirilmesi [The role of computer assisted instruction in the teaching of expressions among primary education eight grade students and evaluation students' opinion about computer assisted instruction]. *Kastamonu Eğitim Dergisi*, *11*(2), 339-358.

Aktümen, M., & Kaçar, A. (2008). Bilgisayar cebiri sistemlerinin matematiğe yönelik tutuma etkisi [Effects of computer algebra systems on attitudes towards mathematics]. *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi, 35,* 13-26.

Aktümen, M., & Kabaca, T. (2012). Exploring the mathematical model of the thumbaround motion by Geogebra. *Technology, Knowledge and Learning, 17*(3), 109-114.

Aktümen, M., & Bulut, M. (2013). Teacher candidates' opinions on real life problems designed in GeoGebra software. *Anthropologist*, *16*(1-2), 167-176.

Alacacı, C., & McDonald, G. (2012). The impact of technology on high school mathematics curriculum. *Turkish Journal of Computer and Mathematics Education*, *3*(1), 21-34.

Almeqdadi, F. (2000). The effect of using geometer's sketchpad (GSP) on Jordanian students' understanding of geometrical concepts. *Proceedings of the International Conference on Technology in Mathematics Education* (pp. 163-169). Lebanon.

Arghode, V. (2012). Qualitative and quantitative research: Paradigmatic differences. *Global Education Journal, 2012*(4), 155-163.

Artigue, M. (2002). Learning mathematics in a CAS environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *International Journal of Computers for Mathematical Learning, 7,* 245-274.

Bakar, K. A., Tarmizi, R. A., Ayub, A. F. M., & Yunus, A. S. M. (2009). Effect of utilizing geometer's sketchpad on performance and mathematical thinking of secondary mathematics learners: An initial exploration. *International Journal of Education and Information Technologies, 3*(1), 20-27.

Baki, A. (2001). Bilişim teknolojisi ışığı altında matematik eğitiminin değerlendirilmesi [Evaluation of mathematics education in the light of information technology]. *Milli Eğitim Dergisi*, *149*, 26-31.

Baki, A., & Güveli, E. (2008). Evaluation of a web based mathematics teaching material on the subject of functions. *Computers & Education*, *51*(2), 854-863.

Baştürk, S., & Dönmez, G. (2011). Mathematics student teachers' misconceptions on the limit and continuity concepts. *Necatibey Faculty of Education Electronic Journal of Science and Mathematics Education*, *5*(1), 225-249.

Baydaş, Ö., Göktaş, Y., & Tatar, E. (2013). The use of GeoGebra with different perspectives in mathematics teaching. *Çukurova University Faculty of Education Journal*, *42*(2), 36-50.

Bildiren, A., & Uzun, M. (2007). Üstün yetenekli öğrencilerin belirlenmesine yönelik bir tanılama yönteminin kullanılabilirliğinin incelenmesi [An identification study for gifted students]. *Pamukkale Üniversitesi Eğitim Fakültesi Dergisi*, *22*(2), 31-39.

Blaisdell, R. (2012). Student understanding in the concept of limit in calculus: How student responses vary depending on question format and type of representation. In S. Brown, S. Larsen, K. Marrongelle, and M. Oehrtman (Eds.), *Proceedings of the 15th Annual Conference on Research in Undergraduate Mathematics Education* (pp. 348-353). Portland, Oregon.

Bryman, A., & Cramer, D. (2005). *Quantitative data analysis with SPSS 12 and 13: A guide for social scientists*. London, UK: Routledge.

Brenner, M. E., Mayer, R. E., Moseley, B., Brar, T., Duran, R., Reed, B. S., & Webb, D. (1997). Learning by understanding: The role of multiple representations in learning algebra. *American Educational Research Journal*, *34*(4), 663-689.

Bruner, J. S. (1966). *Toward a theory of instruction* (Vol. 59). Boston, MA: Harvard University Press.

Bruner, J. S. (1985). Vygotsky: A historical and conceptual perspective. *Culture, communication, and cognition: Vygotskian perspectives*, 21-34.

Bulut, M., & Bulut, N. (2011). Pre-service teachers ’ usage of dynamic mathematics software. *Educational Technology*, *10*(4), 294-299.

Caligaris, M. G., Schivo, M. E., & Romiti, M. R. (2015). Calculus & GeoGebra, an interesting partnership. *Procedia-Social and Behavioral Sciences*, *174*, 1183-1188.

Choi, K. S. (2010). Motivating students in learning mathematics with GeoGebra. *Annals. Computer Science Series*, *8*(2), 65-76.

Clements, D. H. (2000). From exercises and tasks to problems and projects: Unique contributions of computers to innovative mathematics education. *The Journal of Mathematical Behavior*, *19*(1), 9-47.

Cobb, P., Yackel, E., & Wood, T. (1992). A constructivist alternative to the representational view of mind in mathematics education. *Journal for Research in Mathematics education*, *23*(1), 2-33.

Corlu, M. S., Burlbaw, L. M., Capraro, R. M., Han, S., & Çorlu, M. A. (2010). The Ottoman palace school and the man with multiple talents, Matrakçı Nasuh. *Journal of the Korea Society of Mathematical Education Series D: Research in Mathematical Education*, *14*(1), 19–31.

Çatma, Z., & Corlu, M. S. (2015). How special are teachers of specialized schools? Assessing self-confidence levels in the technology domain. *Eurasia Journal of Mathematics, Science, and Technology Education.*

DeCoster, J. (2009). *Effect size calculator* [Excel Add-on]. Retrieved from http://www.stat-help.com

Doerr, H. M., & Zangor, R. (2000). Creating meaning for and with the graphing calculator. *Educational Studies in Mathematics, 41*(2), 143-163.

Duda, B. J., Ogolnokisztalcacych, Z. S., & Poland, Z.  (2010). Mathematical creative activity and graphic calculator. *International Journal of Technology in Mathematics Education, 18*(1), 3–14.

Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational studies in mathematics, 61*(1-2), 103-131.

Filiz, M. (2009). *Geogebra ve Cabri geometri II dinamik geometri yazılımlarının web destekli ortamlarda kullanılmasının öğrenci başarısına etkisi* [The effect of using GeoGebra and Cabri Geometry II dynamic geometry softwares in a web-based setting on students' achievement] (Unpublished master's thesis). Karadeniz Teknik Üniversitesi.

Gagatsis, A., & Elia, I. (2004). The effects of different modes of representation on mathematical problem solving. In M. Johnsen Hoines & A. Berit Fuglestad (Eds.), *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 447-454). Bergen, Norway: Bergen University College.

Goldin, G. A., & Kaput, J. J. (1996). A joint perspective on the idea of representation in learning and doing mathematics. In L. P. Steffe, P. Mesher,

P. Cobb, G. A. Goldin, & B. Greer (Eds.), *Theories of mathematical learning* (pp. 397-430). Hillsdale, NJ: Lawrence Erlbaum.

Goldin, G. A. (2008). Perspective on representation in mathematical learning and problem solving. In L. D. English (Ed.), *Handbook of international research in Mathematics Education, Second Edition* (pp. 176-201). New York, NY: Routledge.

Goldin, G. A. (2014). Mathematical representations. In *Encyclopedia of Mathematics Education* (pp. 409-413). Springer Netherlands.

Gülseçen, S., Karataş, E. K., & Koçoğlu, F. Ö. (2012). Can GeoGebra make easier the understanding of cartesian coordinates? A quantitative study in Turkey. *International Journal on New Trends in Education and Their Implications*, *3*(4), 19-29.

Güven, B., & Kosa, T. (2008). The effect of DGS on student mathematics teachers' spatial visualization skills. *The Turkish Online Journal of Educational Technology*, *7*(4), 100-107.

Güven, B. (2012). Using dynamic geometry software to improve eight grade students' understanding of transformation geometry. *Australasian Journal of Educational Technology, 28*(2), 364-382.

Heugl, H. (2001). The necessary fundamental algebraic competence in the age of CAS. *Proceedings of the 5th Austrian Center for Didactics of Computer Algebra Summer Academy.* Austria.

Hohenwarter, M., & Jones, K. (2007). Ways of linking geometry and algebra: the case of GeoGebra. In D. Küchemann (Ed), *Proceedings of the British Society for Research into Learning Mathematics*, *27*(3), 126-131.

Hohenwarter, M., & Lavicza, Z. (2007). Mathematics teacher development with ICT: Towards an International GeoGebra Institute. In D. Küchemann (Ed), *Proceedings of the British Society for Research into Learning Mathematics, 27*(3), 49-54.

Hohenwarter, M., Lavicza, Z. (2009). The strength of the community : How GeoGebra can inspire technology integration in mathematics teaching. *Mathematics, Statistics and Operations Research Connections, 9*(2), 3-5. doi: 10.11120/msor.2009.09020003

Hohenwarter, J., Hohenwarter, M., & Lavicza, Z. (2010). Evaluating difficulty levels of dynamic geometry software tools to enhance teachers' professional development. *International Journal for Technology in Mathematics Education, 17(3), 127-134.*

Hohenwarter, M., & Lavicza, Z. (2010, ). *GeoGebra, its community and future.* Paper presented at the Asian Technology Conference in Mathematics. Retrieved from http://www.unsam.edu.ar/

Hohenwarter, M., Hohenwarter, J., Kreis, Y., & Lavicza, Z. (2008). *Teaching and learning calculus with free dynamic mathematics software GeoGebra*. Paper presented at the 11th International Congress on Mathematical Education. Monterrey, Nuevo Leon, Mexico.

Hohenwarter, M., & Fuchs, K. (2004, May). Combination of dynamic geometry, algebra and calculus in the software system GeoGebra. *ZDM Mathematics Education, 20*(70), 1-5.

Huck, S. W. (2011). *Reading statistics and research* (6th ed.). Boston, MA. Pearson.

Hutkemri & Zakaria, E. (2012). The effect of GeoGebra on students' conceptual and procedural knowledge of function. *Indian Journal of Science and Technology*, *5*(12), 3802-3808.

İçel, R. (2011). *Bilgisayar destekli öğretimin matematik başarısına etkisi: GeoGebra örneği* [Effect of computer based teaching on students' mathematics achievements: Example of GeoGebra] (Unpublished master's thesis). Selçuk Üniversitesi Eğitim Bilimleri Enstitüsü, Turkey.

İpek, S., & İspir, O. A. (2011). Preservice elementary mathematics teachers' geometric and algebraic proof process with dynamic geometry software. *Turkish Journal of Computer and Mathematics Education*, *2*(1), 20-34.

Jones, K. (2000). Providing a foundation for deductive reasoning: Students' interpretations when using dynamic geometry software and their evolving mathematical explanations. *Educational Studies in Mathematics*, *44*(1-2), 55-85.

Jones, K. (2001). Learning geometrical concepts using DGS. In K. Irwin (Ed), *Mathematics education research: A catalyst for change* (pp. 50-58). Auckland: University of Auckland.

Kabaca, T. (2006). *Limit kavramının öğretiminde bilgisayar cebiri sistemlerinin etkisi* [The effect of computer algebra systems on teaching limit concept] (Unpublished doctoral dissertation). Gazi Üniversitesi, Ankara, Turkey.

Kabaca, T., Aktümen, M., Aksoy, Y., & Bulut, M. (2010). Matematik öğretmenlerinin avrasya geogebra toplantısı kapsamında dinamik matematik yazılımı geogebra ile tanıştırılması ve geogebra hakkındaki görüşleri [Introducing the in-service mathematics teachers with the dynamic mathematics software GeoGebra and their views about GeoGebra]. *Turkish Journal of Computer and Mathematics Education*, *1*(2), 148-165.

Kaput, J. J., & Thompson, P. W. (1994). Technology in mathematics education research: The first 25 years in the JRME. *Journal for Research in Mathematics Education*, *25*(6), 676-684.

Karakuş, Ö. (2008). *Bilgisayar destekli dönüşüm geometrisi öğretiminin öğrenci erişisine etkisi* [The effects on the students' success of computer based transformation geometry learning] (Unpublished master's thesis). The University of Eskisehir Osmangazi, Eskisehir, Turkey.

Karakuş, F. (2010). Üstün yetenekli çocukların anne babalarının karşılaştıkları güçlükler [Difficulties that families of gifted students face]. *Mersin Üniversitesi Eğitim Fakültesi Dergisi*, *6*(1), 127-144.

Kastberg, S., & Leatham, K. (2005). Research on graphing calculators at the secondary level: Implications for mathematics teacher education. *Contemporary Issues in Technology and Teacher Education*, *5*(1), 25-37.

Kepceoglu, I. (2010). *Geogebra yazılımıyla limit ve süreklilik öğretiminin öğretmen adaylarının başarısına ve kavramsal öğrenmelerine etkisi* [The effect of teaching limit and continuity with GeoGebra on pre-service teachers' achievement and conceptual understanding] (Unpublished master's thesis). Marmara Üniversitesi, İstanbul, Turkey.

Kidron, I. (2014). Calculus teaching and learning. In S. Lerman (Ed), *Encyclopedia of mathematics education* (pp. 69-75). London: Springer.

Kllogjeri, P., & Kllogjeri, Q. (2011). GeoGebra–a very effective tool for teaching mathematical concepts and properties. In D. Takači (Ed.), *Proceedings of the*

*International GeoGebra Conference for Southeast Europe* (pp. 90-98). Novi Sad: Stojkov.

Kortenkamp, U. (1999). *Foundations of dynamic geometry* (Unpublished doctoral dissertation). Swiss Federal Institute of Technology Zurih, Switzerland.

Kutluca, T., & Zengin, Y. (2011, September). *Belirli integral konusunda dinamik matematik yazılımı GeoGebra kullanarak çalısma yapraklarının gelistirilmesi [Developing worksheets for definite integral by using dynamic mathematics software with GeoGebra]*. Paper presented at the 5th international computer & instructional technologies symposium, Fırat University, Elazığ.

Leikin, R. (2014). Giftedness and high ability in mathematics. In S. Lerman (Eds.), *Encyclopedia of Mathematics Education* (pp. 247-251). Springer Netherlands.

Mainali, B. R., & Key, M. B. (2012). Using dynamic geometry software GeoGebra in developing countries: A case study of impressions of mathematics teachers in Nepal. *International Journal for Mathematics Teaching and Learning*. Retrieved from http://www.cimt.plymouth.ac.uk/journal/mainali.pdf/

Masteroides, E., & Zachariades, T. (2004). Secondary mathematics teachers' knowledge concerning the concept of limit and continuity. In M.J. Hoines, & A.B. Fuglestad (Eds.), *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 481–488). Bergen: PME.

Mehanovic, S. (2011). *The potential and challenges of the use of Dynamic Software in upper secondary mathematics: Students' and teachers' work with integrals in Geogebra based environments* (Unpublished doctoral dissertation). Linköping University, Sweden.

Mitchelmore, M. C., & White, P. (2004). Abstraction in mathematics and mathematics learning. In M. J. Høines & A. B. Fuglestad (Eds.), *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 329–336). Bergen, Norway: Program Committee.

Ministry of National Education [MoNE]. (2005). *Ortaöğretim matematik dersi öğretim programı* [Secondary school mathematics curriculum]. Ankara, Turkey.

Ministry of National Education [MoNE]. (2012). Fatih Project [Webpage]. Retrieved from http://fatihprojesi.meb.gov.tr/tr/english.php

Ministry of National Education [MoNE]. (2013). *Ortaöğretim matematik dersi öğretim programı* [Secondary school mathematics curriculum]. Ankara, Turkey.

Ministry of National Education [MoNE]. (2009). *Strengthening the vocational education and training system project in Turkey: Gifted and talented children.* Ankara, Turkey.

Moursund, D. (2006, March 30). Computers in education for talented and gifted students. Eugene , Oregon, United States of America. Retrieved from http://pages.uoregon.edu/moursund/Books/TAG/TAG.pdf

Mulyono, B. (2010). *Traditional teaching about angles compared to an active learning approach that focuses on students' skills in seeing, measuring and reasoning, including the use of dynamic geometry software: Differences in achievement* (Unpublished master's thesis). Universiteit van Amsterdam, Netherlands.

National Council of Teachers of Mathematics [NCTM]. (2000). *Principles and standards for school mathematics.* Reston, VA.

National Council of Teachers of Mathematics [NCTM]. (2003). *NCTM program standards: Programs for initial preparation of mathematics teachers.* Reston, VA: NCTM.

Orimogunje, E. (2011). *Pilot Study*. Retrieved from https://tr.scribd.com/doc/56221002/Pilot-Study

Özel, S., Yetkiner, Z.E., & Capraro, R. M. (2008). Technology in K-12 mathematics classrooms. *School Science and Mathematics, 108(2),* 80-85.

Özgün-Koca, S. A. (1998). *Students' use of representations in mathematics education*. Paper presented at the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Raleigh, NC.

Özgün-Koca, S. A. (2012). An analysis of prospective mathematics teachers' views on the use of computer algebra systems in algebra instruction in Turkey and in the United States. *Turkish Journal of Computer and Mathematics Education*, *3*(1), 1-20.

Özgün-Koca, S. A., & Meagher, M. (2012). Birth of a virtual manipulative. *North American GeoGebra Journal*, *1*(1), 27-32.

Özmantar, M. F., Akkoç, H., Bingölbali, E., Demir, S., & Ergene, B. (2010). Pre-service mathematics teachers' use of multiple representations in technology-rich environments. *Eurasia Journal of Mathematics, Science & Technology Education*, *6*(1), 19-36.

Pallant, J. (2001). *The SPSS survival manual: A step-by-step guide to data analysis using SPSS for Windows* (version 10). St Leonards, NSW: Allen & Unwin.

Panasuk, R. M., & Beyranevand, M. L. (2010). Algebra students' ability to recognize multiple representations and achievement. *International Journal for Mathematics Teaching and Learning*, *22*, 1-22.

Pape, S. J., & Tchoshanov, M. A. (2001). The role of representation(s) in developing mathematical understanding. *Theory into practice*, *40*(2), 118-127.

Pierce, R. (2005). Using CAS to enrich the teaching and learning of mathematics. In S-C. Chu, H-C. Lew, & W-C. Yang (Eds), *Proceedings of the 10th Asian Conference on Technology in Mathematics* (pp. 47-58). Blacksburg, VA: ATCM Inc.

Pierce, R., Stacey, K. & Barkatsas, A. (2007). A scale for monitoring students' attitudes to learning mathematics with technology. *Computers and Education*, *48*(2), 285-300.

Reis, Z. A., & Ozdemir, S. (2010). Using Geogebra as an information technology tool: parabola teaching. *Procedia-Social and Behavioral Sciences*, *9*, 565-572.

Renzulli, J. S. (1986). The three-ring conception of giftedness: A development model for creative productivity. In R. J. Sternberg, & J. E. Davidson (Eds.), *Conceptions of giftedness* (pp. 53-92). New York: Cambridge University Press.

Saab, M. A., (2011). *Implementing dynamic geometry software-based constructivist approach (DGS-CA) in teaching Thales' Theorem: Impact of students' achievement, problem solving skills and motivation* (Unpublished master's thesis). Lebanese American University, Lebanon.

Saha, R. A., Ayub, A. F. M., & Tarmizi, R. A., (2010). The effects of GeoGebra on mathematics achievement: Enlightening coordinate geometry learning. *Procedia-Social and Behavioral Sciences*, *8*, 686-693.

Schacter, J. (1999). The impact of education technology on student achievement: What the most current research has to say. *Journal of Educating Computing Research*, *20*.

Schoenfeld, A. H. (1985). *Mathematical problem solving*. Orlando, FL: Academic Press.

Scientific and Technological Research Council of Turkey. (2005). *Vizyon 2023 teknoloji öngörü projesi* [Vision 2023: Turkey's national technology foresight program]. Ankara, Turkey.

Selçik, N., & Bilgici, G. (2011). GeoGebra yazılımının öğrenci başarısına etkisi [The effect of GeoGebra software on students' academic achievement]. *Kastamonu Egitim Dergisi*, *19*(3), 913-924.

Shadaan, P., & Eu, L. K. (2013). Effectiveness of using GeoGebra on students' understanding in learning circles. *The Malaysian Online Journal of Educational Technology*, *1*(4), 1-11.

Shore, B. (1997). Keeping the conversation going: An interview with Jerome Bruner. *Ethos*, *25*(1), 7-62.

Subramanian, L. (2005). *An investigation of high school geometry students'proving and logical thinking abilities and the impact of dynamic geometry software on student performance* (Unpublished doctoral dissertation). University of Central Florida, Florida.

Swan, M. (2008). Designing a multiple representation learning experience in secondary algebra. *Educational Designer*, *1*(1), 1-17.

Tabachnick, B. G., & Fiedell, L. S. (2007). *Using multivariate statistics* (5th ed.) Boston, MA: Allyn and Bacon.

Tall, D. (1993). Students' difficulties in calculus. In C. Gaulin et al. (Eds.), *Proceedings of Working Group 3 on Students' Difficulties in Calculus, 7th International Conference on Mathematicaal Education* (pp. 13-28). Québec, Canada.

Tall, D. (1994). A versatile theory of visualisation and simbolisation in mathematics. *Invited plenary lecture at the CIEAIEM Conference*, Toulouse, France.

Tall, D. O., Smith, D., Piez, C. (2008). Technology and calculus. In M. K. Heid & G. W. Blume (Eds.), *Research on technology and the teaching and learning of mathematics: Research syntheses* (Vol. 1, pp. 207-258). Charlotte: Information Age Publishing.

Taş, M. (2010). *Dinamik matematik yazılımı GeoGebra ile eğrisel integrallerin görselleştirilmesi* [Visualization of curvilinear integrals using the dynamic mathematics software GeoGebra] (Unpublished master's thesis). Istanbul University, Turkey.

Tatar, E., Zengin, Y., & Kağızmanlı, T. B. (2013). Dinamik matematik yazılımı ile etkileşimli tahta teknolojisinin matematik öğretiminde kullanımı [The use of dynamic mathematics software and interactive whiteboard technology in mathematics teaching]. *Turkish Journal of Computer and Mathematics Education*, *4*(2), 104-123.

Teijlingen, E. R., & Hundley, V. (2001). The importance of pilot studies. *Social Research Update*, *35*, 1-4.

Uzun, P. (2014). G*eoGebra ile öğretimin 7.sınıf öğrencilerinin akademik başarılarına ve geometriye yönelik tutumlarına etkisi* [The effect of instruction with GeoGebra on 7th grade students' academic achievement and attitudes towards geometry] (Unpublished master's thesis). Kastamonu University, Turkey.

Vanatta, R. A., & Fordham, N. (2004). Teacher dispositions as predictors of classroom technology use. *Journal of Research on Technology in Education, 36*(3), 253–271.

Wood, K. (2006). *Effect of using multiple representations on student success in solving rational, radical, and absolute value equations and inequalities* (Unpublished doctoral dissertation). University of Victoria, Canada.

Zengin, Y. (2011). *Dinamik matematik yazılımı GeoGebranın öğrencilerin başarılarına ve tutumlarına etkisi* [The effect of dynamic mathematics software GeoGebra on students' achievement and attitude] (Unpublished master's thesis). Kahramanmaraş Sütçü İmam University, Turkey.

Zengin, Y., Furkan, H., & Kutluca, T. (2012). The effect of dynamic mathematics software geogebra on student achievement in teaching of trigonometry. *Procedia-Social and Behavioral Sciences, 31*, 183-187.

## APPENDICES

### Appendix 1: Limits and continuity readiness test [LCRT]

1) $\lim_{x\to 2}\frac{x^3-8x+8}{x^4-4x}$ limitinin değerini hesaplayınız.

2) $\lim_{x\to \frac{\pi}{6}}\frac{sinx-\frac{\sqrt{3}}{2}}{cosx-\frac{1}{2}}$ limitinin değerini hesaplayınız.

3) $f(t)=\frac{t^6-7t^3-8}{t^2-t-2}$ fonksiyonunun t → 2 için limitini hesaplayınız.

4) $f(x)=\frac{x}{|x|}$ fonksiyonu için $\lim_{x\to 0}$ f(x) limitinin değerini hesaplayınız.

5) $f(x)=\begin{cases} mx+n\,, & x<1\ ise \\ 5\,, & x=1\ ise \\ x^2+m\,, & x>1\ ise \end{cases}$ şeklinde verilen f fonksiyonu reel sayılarda sürekli ise n kaçtır?

*6, 7 ve 8. soruların doğru olup olmadığını aşağıdaki bilgiyi göze alarak belirtiniz ve nedenlerini kısaca açıklayınız.*

Bir f fonksiyonu için $\lim_{x\to 1} f(x)=0\ 'dır.$

6)

| İfade | Doğru/Yanlış | Neden |
|---|---|---|
| *f*(1)=1 olmak zorundadır | O Doğru<br>O Yanlış | |

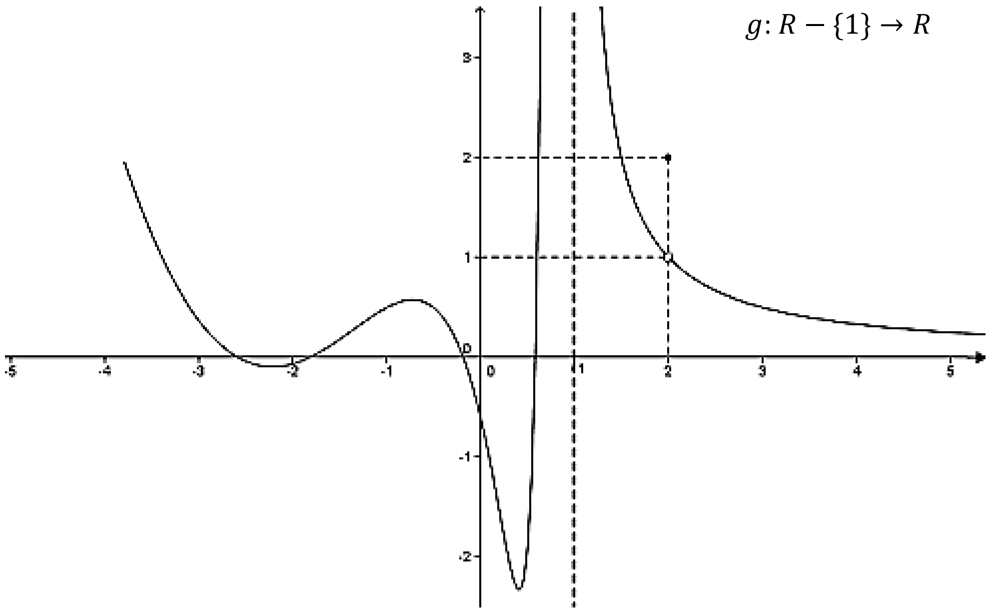
7)

| İfade | Doğru/Yanlış | Neden |
|---|---|---|
| *f* fonksiyonu x=1 noktasında sürekli olmak zorundadır | O Doğru<br>O Yanlış | |

8)

| İfade | Doğru/Yanlış | Neden |
|---|---|---|
| *f* fonksiyonu x=1 noktasında tanımlı olmak zorundadır. | O Doğru<br>O Yanlış | |

9,10,11 ve 12. soruları aşağıdaki grafikten yararlanarak cevaplayınız.




9) $\lim_{x \to 1} g(x) = ?$

10) $\lim_{x \to 2} g(x) = ?$

11) $g(x)$ fonksiyonu sürekli midir? Neden?

12) $g(x)$ fonksiyonu (0,2) aralığında sürekli midir? Neden?

## Appendix 2: Limits and continuity achievement test [LCAT]

1) $\lim_{x\to 0}\frac{2^x+1}{x^3-x^2}$ limitinin değerini varsa hesaplayınız. Yoksa nedenini belirtiniz.

2) $\lim_{x\to\infty}(15^{-x}+3^{-x}+4^{\frac{1}{x}})$ limitinin değerini hesaplayınız.

3) $f(t)=\frac{t^3+t^2-5t+3}{t^3-3t+2}$ fonksiyonunun t → 1 için limitini hesaplayınız.

4) $f(x)=\frac{x}{|x|}$ fonksiyonu için $\lim_{x\to 0} f(x)$ limitinin değerini hesaplayınız.

5) f: R → R fonksiyonu $\begin{cases}\frac{sin4x}{3x}, & x<0\\ k+3, & x=0\\ \frac{8x+n}{x+4}, & x>0\end{cases}$ şeklinde verilen $f$ fonksiyonu reel sayılarda sürekli ise k+n kaçtır?

*6, 7 ve 8. soruların doğru olup olmadığını aşağıdaki bilgiyi göze alarak belirtiniz ve nedenlerini kısaca açıklayınız:*

Bir f fonksiyonu için $\lim_{x\to 1} f(x)=0$ ′dır.

6)

| İfade | Doğru/Yanlış | Neden |
|---|---|---|
| *f*(1)=0 olmak zorundadır | O Doğru<br>O Yanlış | |

7)

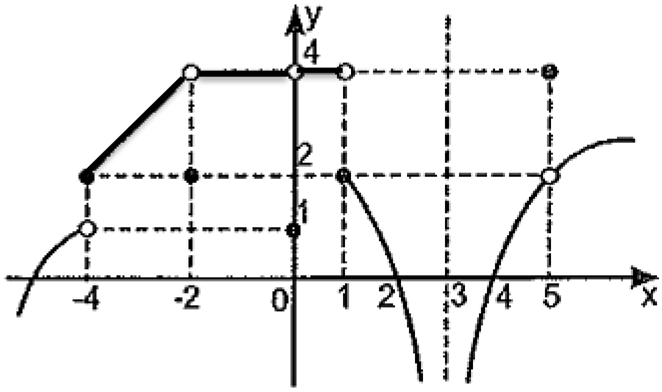
| İfade | Doğru/Yanlış | Neden |
|---|---|---|
| *f* fonksiyonu x=1 noktasında sürekli olmak zorundadır | O Doğru<br>O Yanlış | |

8)

| İfade | Doğru/Yanlış | Neden |
|---|---|---|
| *f* fonksiyonu x=1 noktasında tanımlı olmak zorundadır. | O Doğru<br>O Yanlış | |

9) $\lim_{x\to\infty} \frac{x!+3^{x-4}+x^{20}}{x^x-4^x-x^{40}}$ limitinin değeri kaçtır? Neden?

Aşağıdaki 10, 11 ve 12. soruları verilen grafiğe göre cevaplandırınız.




10) Verilen fonksiyonun hangi nokta(lar)da limiti yoktur? Nedenleriyle birlikte belirtiniz.

11) Verilen fonksiyon hangi nokta(lar)da sürekli değildir? Nedenleriyle birlikte belirtiniz.

12) Verilen fonksiyon hangi nokta(lar)da tanımsızdır? Nedenleriyle birlikte belirtiniz.

## Appendix 3: Mathematics and technology attitude scale [MTAS]

| | | | | | | |
|---|---|---|---|---|---|---|
| 1. | I concentrate hard in mathematics. | HE | Oc | Ha | U | NA |
| 2. | I try to answer questions the teacher asks | HE | Oc | Ha | U | NA |
| 3. | If I make mistakes, I work until I have corrected them. | HE | Oc | Ha | U | NA |
| 4. | If I can't do a problem, I keep trying different ideas. | HE | Oc | Ha | U | NA |
| 5. | I am good at using computers. | SD | D | NS | A | SA |
| 6. | I am good at using things like VCRs, DVDs, MP3s and mobile phones | SD | D | NS | A | SA |
| 7. | I can fix a lot of computer problems | SD | D | NS | A | SA |
| 8. | I am quick to learn new computer software needed for school | SD | D | NS | A | SA |
| 9. | I have a mathematical mind. | SD | D | NS | A | SA |
| 10. | I can get good results in mathematics | SD | D | NS | A | SA |
| 11. | I know I can handle difficulties in mathematics | SD | D | NS | A | SA |
| 12. | I am confident with mathematics | SD | D | NS | A | SA |
| 13. | I am interested to learn new things in mathematics | SD | D | NS | A | SA |
| 14. | In mathematics you get rewards for your effort | SD | D | NS | A | SA |
| 15. | Learning mathematics is enjoyable | SD | D | NS | A | SA |
| 16. | I get a sense of satisfaction when I solve mathematics problems | SD | D | NS | A | SA |
| 17. | I like using DGS for learning mathematics | SD | D | NS | A | SA |
| 18. | Using DGS is worth the extra effort | SD | D | NS | A | SA |
| 19. | Mathematics is more interesting when using DGS | SD | D | NS | A | SA |
| 20. | DGS help me learn mathematics better | SD | D | NS | A | SA |

HE: hardly ever    Oc: occasionally    Ha: about half the time    U: usually    NA. nearly always

SD: strongly disagree    D: disagree    NS: not sure    A: agree    SA: strongly agree

# Appendix 4: Written permission for LCRT and LCAT

| | |
|---|---|
| **Subject:** | Re: GeoGebra-Limit ve Süreklilik-Tez Çalışması |
| **From:** | "ibrahim.kepceoglu" <ibrahim.kepceoglu@marmara.edu.tr> |
| **Date:** | Wed, September 26, 2012 7:29 pm |
| **To:** | "mustafa aydos" <mustafa.aydos@bilkent.edu.tr> |
| **Priority:** | Normal |
| **Allow Sender:** | Allow Sender \| Allow Domain \| Block Sender \| |
| **Create Filter:** | Automatically \| From \| To \| Subject |
| **Options:** | View Full Header \| View Printable Version \| Download this as a file \| Ad |

Sayın Mustafa Aydos,

Öncelikle tezinizde başarılar dilerim ve tezinizde GeoGebra programını
kullanmayı tercih ettiğiniz memnuniyetimi bildiririm. Tezimden
dilediğiniz şekilde yararlanabilirsiniz. Herhangi bir yazılı izin
gerekli ise size böyle bir belge verebilirim. Ayrıca soru sormak ya da
görüş almak için de benimle iletişime geçebilirsiniz.

Başarılar dilerim.

İbrahim Kepceoğlu

2012-09-26 11:45, mustafa aydos yazmış:
> Sayın İbrahim Kepçeoğlu,
>
> Bilkent Üniversitesi Eğitim Bilimleri Fakültesi mastır programında
> öğrenim
> görmekteyim. Programı başarıyla bitirebilmem için araştırma tabanlı
> bir
> tez çalışmasında bulunmak ve savunmak durumundayım. Şu anda araştırma
> önerisi aşamasındayım ve GeoGebra yazılımının lise 12. sınıf
> öğrencilerinin limit ve
> süreklilik konularındaki öğrenmelerine etkisini araştırmak istiyorum.
> Bu
> ışıkta araştırmalarım sonucunda, sizin teziniz ile araştırmalarımın
> ortak
> birçok özelliğinin olduğunu ve tezinizden yararlanabileceğimi gördüm.

## Appendix 5: Written permission for MTAS

| | |
|---|---|
| **Subject:** | RE: MTAS permission |
| **From:** | "Robyn Pierce" <r.pierce@unimelb.edu.au> |
| **Date:** | Mon, September 1, 2014 2:04 am |
| **To:** | "mustafa aydos" <mustafa.aydos@bilkent.edu.tr> |
| **Cc:** | "Kaye Stacey" <k.stacey@unimelb.edu.au> |
| **Priority:** | Normal |
| **Allow Sender:** | Allow Sender \| Allow Domain \| Block Sender \| |
| **Create Filter:** | Automatically \| From \| To \| Subject |
| **Options:** | View Full Header \| View Printable Version \| Download this as a file \| Ad |

Dear Mustafa
Yes you have our permission to use the survey - subject of course to the usual referencing.
We hope your research goes very well.
Best wishes
Robyn

A/Prof Robyn Pierce
Melbourne Graduate School of Education | 234 Queensberry Street | University of Melbourne | Vic 3010 Australia| phone +61 3 83448519| email: r.pierce@unimelb.edu.au

Note: I am on campus on Tuesdays & Fridays

-----Original Message-----
From: mustafa aydos [mailto:mustafa.aydos@bilkent.edu.tr]
Sent: Saturday, 30 August 2014 10:47 PM
To: Robyn Pierce
Subject: MTAS permission

Hi Dr. Pierce,

I am a MA student in Bilkent University, Turkey. I am also a math teacher in a private high school. In my master thesis, I am studying effects of GeoGebra, a dynamic mathematics software, in terms of students' conceptual understanding on specific math concepts limit and continuity. On the other hand I am investigating GeoGebra's effect on students' attitude towards mathematics and technology. I and my supervisor Dr. M.Sencer Corlu agreed that your 'Mathematics and Technology Attitude Scale' is suitable and beneficial for my thesis. Therefore, I request your permission that I am able to use your MTAS as a part of my study.

**Appendix 6: Written permission of Ministry of National Education [MoNE]**

T.C.
**KOCAELİ VALİLİĞİ**
**İl Milli Eğitim Müdürlüğü**

**Sayı** : 99332089/605/4322371 30/09/2014
**Konu: Araştırma İzni**

VALİLİK MAKAMINA
KOCAELİ

İlgi: Millî Eğitim Bakanlığına Bağlı Okul ve Kurumlarda Yapılacak Araştırma ve Araştırma Desteğine Yönelik İzin Ve Uygulama Yönergesi.

Bilkent Üniversitesi Eğitim Bilimleri Enstitüsü Yüksek Lisans Öğrencisi Mustafa AYDOS' un, "Geogebra yazılımıyla limit ve süreklilik öğretiminin üstün zekalı ve özel yetenekli lise öğrencilerinin başarılarına ve tutumlarına etkisinin araştırılması" konulu araştırma çalışmasını İlimiz Gebze ilçesi TEV İnanç Türkeş Özel Lisesinde uygulama talebi, ilgili Üniversitenin 04/09/2014 tarih ve 13929 sayılı yazıları ile bildirilmektedir.

Adı geçenin söz konusu çalışmasına esas olmak üzere, ekte sunulan çalışmayı İlimiz Gebze ilçesi TEV İnanç Türkeş Özel Lisesinde uygulama talebi komisyonumuzca uygun görülmüş olup, Müdürlüğümüzce de uygun görülmektedir.

Makamlarınızca da uygun görüldüğü takdirde olurlarınıza arz ederim.

Fehmi Rasim ÇELİK
İl Millî Eğitim Müdürü

OLUR
..../09/2014

Derviş Ahmet SET
Vali a.
Vali Yardımcısı

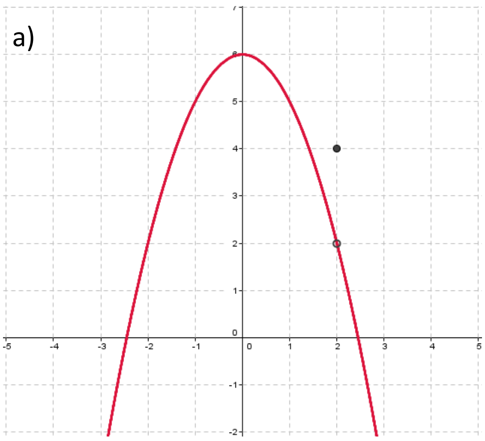
Güvenli Elektronik İmzalı
Aslı ile Aynıdır.
30.09.2014
EMEL SAĞLAM YAVUZ
ŞEF

## Appendix 7: Worksheets

Matematik Dersi Limit Konusu *[Mathematics Lesson Limits Concept]*

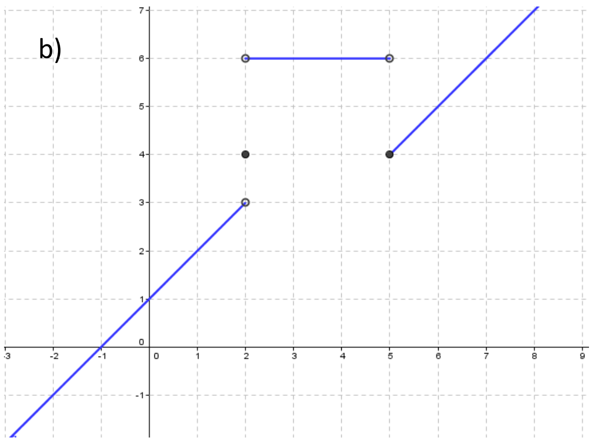
Çalışma Sayfası-1- *[Worksheet-1-]*

Aşağıda verilen grafikler için istenilen noktalardaki sağdan ve soldan limitleri bulunuz. Fonksiyonun o noktada limiti var mıdır? Tartışınız. Limit gösterimlerini kullanarak ifade ediniz. *[Find the left and right limits for the given graphs at the stated x values. Discuss whether there was a limits of the function at that exact x value. State the limits by using limits notations.]*



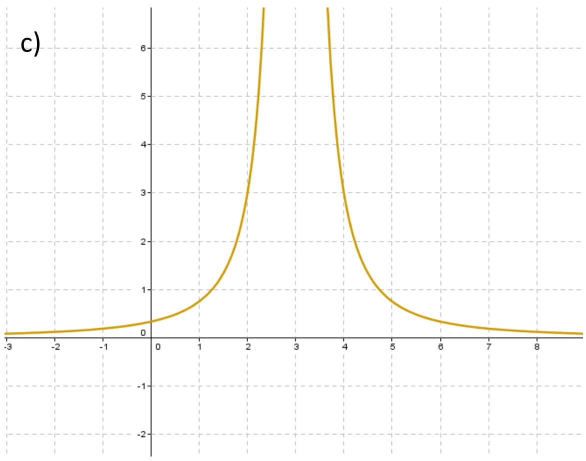

$$f(x) = \begin{cases} 4 & , \quad x = 2 \text{ ise} \\ -x^2 + 6 & , \quad x \neq 2 \text{ ise} \end{cases}$$

$x = -2, x = 0$ ve $x = 2$ için




$$g(x) = \begin{cases} x + 1 \ , & x < 2 \text{ ise} \\ 4 \ , & x = 2 \text{ ise} \\ 6 \ , & 2 < x < 5 \text{ ise} \\ x - 1 \ , & x \geq 5 \text{ ise} \end{cases}$$

$x = -1, x = 0, x = 2, x = 3, x = 5$ ve $x = 6$ için




$$h(x) = \frac{3}{(x-3)^2}$$

$x = 4,\ x = 2$ ve $x = 3$ için

Matematik Dersi Limit Konusu *[Mathematics Lesson Limits Concept]*

Çalışma Sayfası-2- *[Worksheet-2-]*

Aşağıdaki tabloyu verilen *x* değerleri için fonksiyon değerlerini bularak doldurunuz. Grafikleriyle karşılaştırınız. Hesap makinesi kullanabilirsiniz. *[Fill out the table below by finding values of the functions at the given x values. You can use calculator.]*

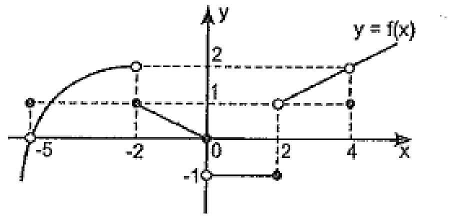
| | | | | | |
|---|---|---|---|---|---|
| $f(x) = \begin{cases} 4 & , \quad x = 2 \text{ ise} \\ -x^2 + 6 & , \quad x \neq 2 \text{ ise} \end{cases}$ | *x=1.9* | *x=1.99* | *x=2* | *x=2.01* | *x=2.1* |
| $g(x) = \begin{cases} x+1 \ , & x < 2 \text{ ise} \\ 4 \ , & x = 2 \text{ ise} \\ 6 \ , & 2 < x < 5 \text{ ise} \\ x-1 \ , & x \geq 5 \text{ ise} \end{cases}$ | *x=1.9* | *x=1.99* | *x=2* | *x=2.01* | *x=2.1* |
| $h(x) = \frac{3}{(x-3)^2}$ | *x=2.9* | *x=2.99* | *x=3* | *x=3.01* | *x=3.1* |

Matematik Dersi Limit Konusu *[Mathematics Lesson Limits Concept]*

Çalışma Sayfası-3- *[Worksheet-3-]*

Aşağıdaki test sorularını limit yaklaşımlarını ve limit özelliklerini kullanarak çözünüz. [*Solve the questions below by considering limiting process and limits properties.*]

1)




**Şekilde verilen $y = f(x)$ fonksiyonunun grafiğine göre, aşağıdakilerden kaç tanesi doğrudur?**

I. $\lim_{x \to -2^+} f(x) = 1$ II. $\lim_{x \to 0^+} f(x) = 0$

III. $\lim_{x \to 4} f(x) = 2$ IV. $\lim_{x \to 2^-} f(x) = -1$

V. $\lim_{x \to -5} f(x) = 0$ VI. $\lim_{x \to 1} f(x) = -1$

A) 2 B) 3 C) 4 D) 5 E) 6

2)

$$f: R \to R, \quad f(x) = \begin{cases} x^2 - 1, & x < 2 \\ 2x - 1, & 2 \le x < 4 \\ 3x - 5, & x \ge 4 \end{cases}$$

fonksiyonuna göre,

$\lim_{x \to 2} f(x) + \lim_{x \to 4} f(x)$ **toplamı kaçtır?**

A) 5 B) 6 C) 8 D) 9 E) 10

3)

$$f: R \to R, \quad f(x) = \begin{cases} 2 + \sin x, & x \le \frac{\pi}{6} \\ 1 - \cos x, & \frac{\pi}{6} < x < \frac{\pi}{3} \\ \sin 2x, & x \ge \frac{\pi}{3} \end{cases}$$

fonksiyonuna göre,

$\lim_{x \to \frac{\pi}{6}^-} f(x) + \lim_{x \to \frac{\pi}{6}^+} f(x) + \lim_{x \to \frac{\pi}{3}^+} f(x)$ **kaçtır?**

A) $1 - \frac{\sqrt{3}}{2}$ B) $1 + \frac{\sqrt{3}}{2}$ C) $\frac{1}{2}$ D) $\frac{5}{2}$ E) $\frac{7}{2}$

4)

$$\lim_{x \to 4} \frac{|x - 2| + |3 - x|}{x + |-x|}$$

**limitinin değeri kaçtır?**

A) $\frac{3}{8}$ B) $\frac{1}{2}$ C) $\frac{5}{8}$ D) $\frac{3}{4}$ E) $\frac{7}{8}$

5)

$$\lim_{x \to 5^-} \frac{x^2 - 25}{|5 - x|} + \lim_{x \to 1^+} \frac{|x - 1|}{x - 1}$$

**limitinin değeri kaçtır?**

A) 0 B) –9 C) –3 D) 5 E) 25

6)

$$\lim_{x \to 4} \sqrt{\frac{x^3 - 4}{x^2 + 4}}$$

**limitinin değeri kaçtır?**

A) $\sqrt{3}$ B) $\frac{\sqrt{5}}{2}$ C) $\sqrt{\frac{3}{2}}$ D) 2 E) $\sqrt{5}$

7)

$$\lim_{x \to \frac{\pi}{2}} (\sin x + \cos x)^x$$

**limitinin değeri kaçtır?**

A) –2 B) –1 C) 0 D) 1 E) 2

8)

$$\lim_{x\to -2} \left[ (x^2 - 3x + 1)\cdot g(x) \right] = 33$$

**olduğuna göre,** $\lim_{x\to -2} g(x)$ **değeri kaçtır?**

A) –2 B) –1 C) 1 D) 2 E) 3

9)

$\lim_{x\to 3} f(x) = 4$ , $\lim_{x\to 3} g(x) = -3$

$$\lim_{x\to 3} \frac{f(x)\cdot x - g(x)}{h(x) + x^2} = 2$$

olduğuna göre, $\lim_{x\to 3} h(x)$

**limitinin değeri kaçtır?**

A) $-\frac{3}{2}$ B) –1 C) $-\frac{2}{3}$ D) $-\frac{1}{2}$ E) $-\frac{1}{4}$

10)

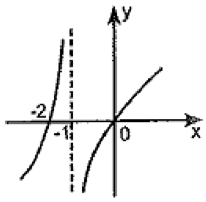
$$\lim_{x\to \frac{\pi}{2}} \left[ \cot\left(\frac{\pi}{6}\cdot \sin x\right) \right]$$

**limitinin değeri kaçtır?**

A) $\frac{\sqrt{3}}{3}$ B) $\frac{\sqrt{3}}{2}$ C) $\sqrt{3}$ D) $2\sqrt{3}$ E) $\frac{2\sqrt{3}}{3}$

11)

**Yanda grafiği verilen f(x) fonksiyonu için aşağıdakilerden hangisi yanlıştır?**




A) $\lim_{x\to -1^+} f(x) = -\infty$ B) $\lim_{x\to -1^-} f(x) = \infty$

C) $\lim_{x\to -2^+} f(x) = 0$ D) $\lim_{x\to 0^-} f(x) = 0$

E) $\lim_{x\to -1} f(x) = -1$

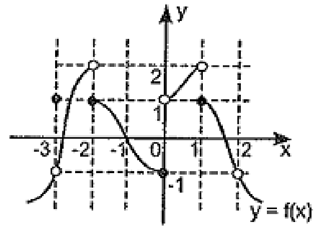
12)

$$f(x) = |x^3 - x| + |x - 4|$$

**fonksiyonunun** $x = 2$ **noktasındaki limit kaçtır?**

A) 4 B) 5 C) 6 D) 7 E) 8

13)




Şekilde verilen $y = f(x)$ fonksiyonunun grafiğine göre, **aşağıdakilerden kaç tanesi doğrudur?**

I. $\lim_{x\to -3} f(x) = 1$ II. $\lim_{x\to 2^+} f(x) = -1$

III. $\lim_{x\to 1^-} f(x) = 1$ IV. $\lim_{x\to 0} f(x) = 1$

V. $\lim_{x\to -1} f(x) = 0$ VI. $\lim_{x\to -2^-} f(x) = 2$

A) 2 B) 3 C) 4 D) 5 E) 6

14)

$$\frac{\lim_{x\to 0^+} \frac{x}{|x|}}{\lim_{x\to 1^-} \frac{|x-1|}{x-1}}$$

**limitinin değeri kaçtır?**

A) –5 B) –3 C) –1 D) 0 E) 2

15)

$$\lim_{x\to \pi} \frac{\sin\frac{x}{2} - \cos x}{4 + \sqrt{\cot\frac{x}{4}}}$$

**limitinin değeri kaçtır?**

A) $\frac{1}{4}$ B) $\frac{2}{7}$ C) $\frac{1}{3}$ D) $\frac{2}{5}$ E) $\frac{3}{4}$

Matematik Dersi Limit Konusu *[Mathematics Lesson Limits Concept]*

Çalışma Sayfası-4- *[Worksheet-4-]*

Aşağıdaki soruları $\lim_{x\to 0}\frac{sinx}{x}=1$ teoremini ve onun sonuçlarını kullanarak çözünüz. [*Solve out the questions by using the theorem and its corollaries.*]

1) $\lim_{x\to 0}\frac{\sin 6x}{2x}+\lim_{x\to 0}\frac{4x}{\sin x}+\lim_{x\to 0}\frac{\tan 8x}{\sin 4x}$

**limitinin değeri kaçtır?**

2) $\lim_{x\to 0}\frac{\tan^3 4x}{8x^3}$

**limitinin değeri kaçtır?**

3) $\lim_{x\to 0}\frac{\sin^2\frac{x}{5}}{\tan^2\frac{x}{3}}$

**limitinin değeri kaçtır?**

4) $\lim_{x\to 0}\frac{\tan^4 2x}{16x^3}$

**limitinin değeri kaçtır?**

5) $\lim_{x\to 4}\frac{\tan(3x-12)}{4x-16}$

**limitinin değeri kaçtır?**

6) $\lim_{x\to \pi}\frac{6x-6\pi}{\sin(\pi-x)}$

**limitinin değeri kaçtır?**

7) $\lim_{x\to 0}\left(\frac{8x-12\sin 2x}{\tan 3x+2x}\right)$

**limitinin değeri kaçtır?**

8) $\lim_{x\to 0}\left(\frac{\tan 6x-\sin^2 x}{2x}\right)$

**limitinin değeri kaçtır?**

Matematik Dersi Limit Konusu *[Mathematics Lesson Limits Concept]*

Çalışma Sayfası-5- *[Worksheet-5-]*

Aşağıdaki soruları çözünüz. [*Solve out the questions below.*]

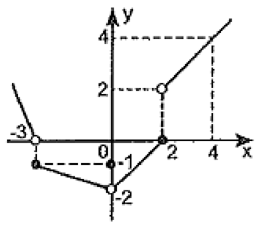
1) $f(x)=\begin{cases} ax-3 & , \ x<1 \\ 5 & , \ x=1 \\ x-b & , \ x>1 \end{cases}$

fonksiyonu R de sürekli ise a + b toplamı kaçtır?

2) $f(x)=\begin{cases} mx-4 & , \ x\geq 2 \\ \dfrac{nx+2}{x-2} & , \ x<2 \end{cases}$

fonksiyonu $\forall x \in R$ için sürekli ise m + n toplamı kaçtır?

3)

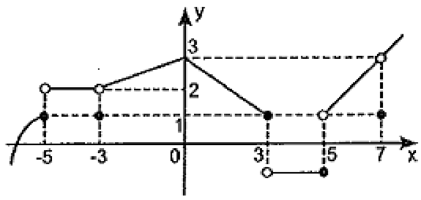



Yukarıda grafiği verilen y = f(x) fonksiyonunun süreksiz olduğu noktaların apsisleri toplamı kaçtır?

4) $f(x)=\dfrac{x^2-9}{x^2+4x+k}$

fonksiyonu x = −3 apsisli noktada süreksiz olduğuna göre, k kaçtır?

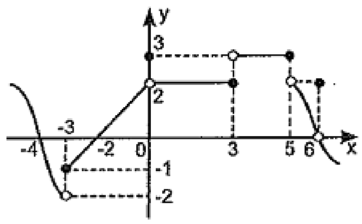
5)




Yukarıda grafiği verilen y = f(x) fonksiyonunun süreksiz ve limitinin olduğu noktaların apsisleri toplamı kaçtır?

A) 4 B) 5 C) 6 D) 7 E) 8

6)




Yukarıda grafiği verilen y = f(x) fonksiyonunun süreksiz olduğu ve limitinin olmadığı noktaların apsisleri toplamı kaçtır?

7) $f(x)=\begin{cases} \dfrac{x}{x+1} & , \ x<0 \\ 2x+5 & , \ 0\leq x<5 \\ \dfrac{2}{x^2-36} & , \ x\geq 5 \end{cases}$

fonksiyonunu süreksiz yapan x tam sayılarının toplamı kaçtır?

8) $f(x)=2^{\frac{x^2-4}{x^2-4x+3}}$

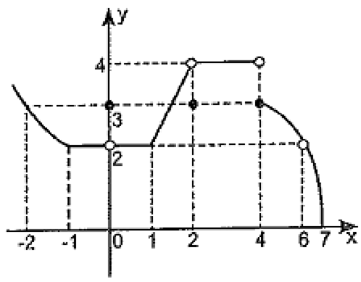
fonksiyonunun süreksiz olduğu noktaların apsisleri çarpımı kaçtır?

A) −8 B) −6 C) −4 D) 3 E) 4

Matematik Dersi Limit Konusu *[Mathematics Lesson Limits Concept]*

Çalışma Sayfası-6- *[Worksheet-6-]*

1)




Grafiği yukarıda verilen fonksiyonun x in

−2, −1, 0, 1, 2, 3, 4, 6

noktalarının bazıları için var olan limitlerin toplamı kaçtır?

A) 20  B) 19  C) 17  D) 14  E) 13

2) $a, b \in \mathbb{R}$ olmak üzere,

$$f(x) = \begin{cases} \dfrac{4x^2 - 1}{2x - 1}, & x > 0 \text{ ise} \\ ax + b, & x \leq 0 \text{ ise} \end{cases}$$

fonksiyonu $x = 0$ da sürekli ise b kaçtır?

A) −2  B) −1  C) 0  D) 1  E) 2

3) $\lim_{x \to 3} \sqrt[3]{3x^2 - 4x - 7}$

limitinin değeri kaçtır?

A) −3  B) −2  C) −1  D) 2  E) 3

4) $\lim_{x \to 2} \left(|x - 4| - |5 - x^2| - 2x\right)$

limitinin değeri kaçtır?

A) −3  B) −2  C) −1  D) 1  E) 2

5) Bir fonksiyon sürekli olduğu her noktada tanımlı olmak zorunda mıdır? Nedenini açıklayınız. [*Does a function have to be defined where it is continuous? Explain.]*

6) Bir fonksiyon sürekli olduğu her noktada limitli olmak zorunda mıdır? Nedenini açıklayınız. [*Does a function have limits where it is continuous? Explain.*]

7) Limit konusunun günlük hayatta nasıl kullanılabileceği konusunda araştırma yapınız. [*Investigate how limits concept could be used in real life.*]

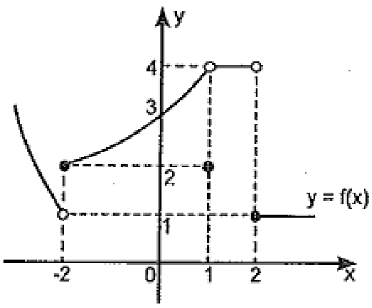
8) $f(x)=\begin{cases} x+2 & , \ x<1 \text{ ise} \\ 3 & , \ x=1 \text{ ise} \\ x^2-x & , \ x>1 \text{ ise} \end{cases}$

**f(x) fonksiyonu için aşağıdakilerden hangisi yanlıştır?**

A) $\lim\limits_{x\to1^+} f(x)=0$ B) $\lim\limits_{x\to1^-} f(x)=3$ C) $\lim\limits_{x\to4} f(x)=12$

D) $\lim\limits_{x\to1} f(x)=3$ E) $\lim\limits_{x\to0} f(x)=2$

9)




Şekilde y = f(x) fonksiyonunun grafiği verilmiştir.

**Aşağıdakilerden hangisi yanlıştır?**

A) $\lim\limits_{x\to1^+} f(x)=4$ B) $\lim\limits_{x\to-2^-} f(x)=4$

C) $\lim\limits_{x\to-2^+} f(x)=2$ D) $\lim\limits_{x\to1^-} f(x)=4$

E) $\lim\limits_{x\to2^+} f(x)=1$

10) $f(x)=\begin{cases} \dfrac{|3x-3|}{1-x} & , \ x>1 \text{ ise} \\ ax & , \ x=1 \text{ ise} \\ 3x-b & , \ x<1 \text{ ise} \end{cases}$

**f(x) fonksiyonu x = 1 de sürekli ise, a + b toplamı kaçtır?**

A) –9 B) –2 C) –1 D) 3 E) 5

11) Limit konusundaki yaklaşım kavramını kendi cümlelerinizle anlatınız. [*Explain limiting process with your own words.*]

12) Bir fonksiyonun tanım kümesi ile süreklilik arasındaki ilişkiyi açıklayınız. [*Explain the relations between the domain of a function and continuity of that function.*]

13) *x* değerleri sonsuza giderken fonksiyonun kendisinin sıfıra yaklaştığı bir fonksiyon örneği veriniz. [*Give an exemplary function that approaches to zero when x-value goes to infinity.*]

14) Belirsizlik ile tanımsızlık kavramlarının arasındaki fark nedir? Açıklayınız. [*What is the difference between indefinity and undefinety?*]

15) $\lim\limits_{x\to 4} \dfrac{x^3+1}{x^2-1}$

**limitinin değeri kaçtır?**

A) 5 B) $\frac{14}{3}$ C) $\frac{13}{3}$ D) 4 E) $\frac{10}{3}$

16) $\lim\limits_{x\to \infty} \dfrac{x^5+x^2}{x^3+1}$

**limitinin değeri kaçtır?**

A) $-\infty$ B) $-1$ C) 0 D) 1 E) $\infty$

17) $\lim\limits_{x\to 0} \left(\dfrac{\tan 4x}{5x}\right)$

**limitinin değeri kaçtır?**

A) $\infty$ B) $\frac{5}{4}$ C) 1 D) $\frac{4}{5}$ E) 0

18) $\lim\limits_{x\to 4} \left[f(x)+\dfrac{x-5}{x-3}\right]=12$

olduğuna göre, $\lim\limits_{x\to 4} f(x)$ **limitinin değeri kaçtır?**

A) 9 B) 10 C) 11 D) 12 E) 13

19) $\lim\limits_{y\to x} \dfrac{y^2-x^2}{\sin(x-y)}$

**limitinin değeri kaçtır?**

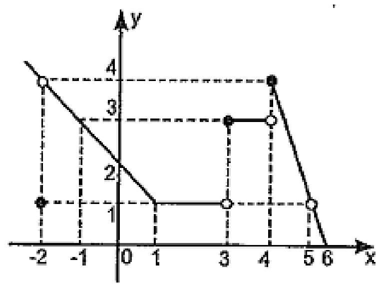
A) $-2y$ B) $-y$ C) $-2x$ D) $-x$ E) $2x$

20) $\lim\limits_{x\to 2} \left(\dfrac{4}{x^2-4}-\dfrac{1}{x-2}\right)$

**limitinin değeri aşağıdakilerden hangisidir?**

A) $-1$ B) $-\frac{1}{2}$ C) $-\frac{1}{4}$ D) $\frac{1}{4}$ E) $\frac{1}{2}$

21)




Grafiği yukarıda verilen fonksiyonunun x in $-2, -1, 0, 1, 2, 3, 4, 5$ noktalarının bazıları için limiti var ve süreksizdir.

**Bu noktaların apsisleri toplamı kaçtır?**

A) 11 B) 9 C) 6 D) 4 E) 3

22) $f(x) = 4x - 2$

ise $\lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$ **limitinin değeri kaçtır?**

A) $-4$ B) $4h$ C) $4x$ D) 4 E) 0