ÇİĞDEM ÖZDEMİR

2017

THE DEVELOPMENT OF AN INQUIRY-BASED TEACHING UNIT FOR TURKISH HIGH SCHOOL MATHEMATICS TEACHERS ON INTEGRAL CALCULUS: THE CASE OF DEFINITE INTEGRAL

A MASTER'S THESIS

BY

ÇİĞDEM ÖZDEMİR

THE PROGRAM OF CURRICULUM AND INSTRUCTION
İHSAN DOĞRAMACI BİLKENT UNIVERSITY
ANKARA

SEPTEMBER 2017

THE DEVELOPMENT OF AN INQUIRY-BASED TEACHING UNIT FOR TURKISH HIGH SCHOOL MATHEMATICS TEACHERS ON INTEGRAL CALCULUS: THE CASE OF DEFINITE INTEGRAL

The Graduate School of Education

of

İhsan Doğramacı Bilkent University

by

ÇİĞDEM ÖZDEMİR

In Partial Fulfilment of the Requirements for the Degree of

Master of Arts

in

Curriculum and Instruction

Ankara

September 2017

İHSAN DOĞRAMACI BILKENT UNIVERSITY

GRADUATE SCHOOL OF EDUCATION

THE DEVELOPMENT OF AN INQUIRY-BASED TEACHING UNIT FOR TURKISH HIGH SCHOOL MATHEMATICS TEACHERS ON INTEGRAL CALCULUS: THE CASE OF DEFINITE INTEGRAL

Çiğdem Özdemir

September 2017

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Curriculum and Instruction.

Assoc. Prof. Dr. Erdat Çataloğlu (Supervisor)

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Curriculum and Instruction.

Prof. Dr. Salih Ateş (Examining Committee Member)

I certify that I have read this thesis and have found that it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Arts in Curriculum and Instruction.

Asst. Prof. Dr. Armağan Ateşkan (Examining Committee Member)

Approval of the Graduate School of Education

Director: Prof. Dr. Alipaşa Ayas

ABSTRACT

THE DEVELOPMENT OF AN INQUIRY-BASED TEACHING UNIT FOR TURKISH HIGH SCHOOL MATHEMATICS TEACHERS ON INTEGRAL CALCULUS: THE CASE OF DEFINITE INTEGRAL

Çiğdem Özdemir

M.A., Program of Curriculum and Instruction

Supervisor: Assoc. Prof. Dr. Erdat Çataloğlu

September 2017

The 2013 official national curriculum published by the Turkish Ministry of Education formally required high school mathematics teachers to actively incorporate computer software in their teaching. The primary purpose of this study was to demonstrate the development of an inquiry-based teaching unit especially geared for high school mathematics students and teachers for the general concept of integral calculus. The main theme chosen as a case for this proposed inquiry unit was on *definite integral* and *volumes of solids of revolution* of real life daily objects. As a result, the primary purpose was to provide the process of developing a practical example of using pedagogically driven dynamic mathematics software (GeoGebra), a 3D digital model coupled with hands-on real life examples, all embedded in a constructivist learning environment. Also, within this study, the perceived effectiveness of the developed teaching unit by in-service high school mathematics teachers based on their experiences was reported.

Key words: Mathematics education, inquiry-based learning, integral calculus, definite integral, constructivist learning, modeling, dynamic mathematics software, GeoGebra

ÖZET

LİSE MATEMATİK ÖĞRETMENLERİ İÇİN BELİRLİ İNTEGRAL KONUSU ÜZERİNDE ARAŞTIRMAYA DAYALI BİR ÜNİTE PLANI GELİŞTİRME

Çiğdem Özdemir

Yüksek Lisans, Eğitim Programları ve Öğretim

Tez Yöneticisi: Doç. Dr. Erdat Çataloğlu

Eylül 2017

2013 yılında Milli Eğitim Bakanlığı tarafından yayınlanan matematik öğretim programı, matematik öğretmenlerinin bilgisayar teknolojisini derslerine aktif bir şekilde entegre etmelerini gerektirmektedir. Bu çalışmanın öncelikli amacı özellikle matematik öğretmenleri ve öğrencileri için integral konusunda araştırmaya dayalı bir ünite planı geliştirme sürecini göstermektir. Bu çalışmanın ana teması olarak belirli integral ve günlük hayatta karşılaşılan dönel cisimlerin hacmini hesaplama olarak belirlenmiştir. Sonuç olarak, bu çalışmanın öncelikli amacı, lise matematik öğretmenleri için dinamik matematik yazılımı (GeoGebra), 3 boyutlu modelleme ve gerçek hayat örneklerinin yapılandırmacı bir öğrenme ortamında bir araya getirildiği pratik bir örneğin üretilme sürecinin ve bu örneğin sunulmasıdır. Bu çalışma ayrıca geliştirilen ünite planının geçerliliğini lise matematik öğretmenlerinin kendi deneyimlerine dayanarak değerlendirmelerini içermektedir.

Anahtar kelimeler: Matematik eğitimi, araştırmaya dayalı öğrenim, integral, belirli integral, yapılandırmacı öğrenim, modelleme, GeoGebra

ACKNOWLEDGEMENTS

I would like to offer my sincerest appreciation to Prof. Dr. Ali Doğramacı, Prof. Dr. Margaret K. Sands and Prof. Dr. Alipaşa Ayas and to everyone at Bilkent University Graduate School of Education for sharing their experiences and supporting me throughout the program.

I would like to express my appreciation and sincere gratitude to my supervisor Assoc. Prof. Dr. Erdat Çataloğlu for his encouragement and guidance throughout this research process. He made invaluable contributions to my thesis with his constructive feedback and endless patience. I would also like to thank the committee members Prof. Dr. Salih Ateş and Dr. Armağan Ateşkan for their suggestions about my thesis.

Besides, I would like to offer my acknowledgements to my colleagues in this program and dearest friends who participated in this study for their valuable time and support. I also appreciate my dear colleague Geoffrey Warren who spent his valuable time for doing the proofreading of my thesis.

Finally, I express my deepest gratitude to my parents Hayati Özdemir and Mihriban Özdemir for their endless love and support. And, I heartily thank to my sister Meryem Özdemir-Yılmazer who always encouraged me and academically contributed to this thesis. She has always been a guide to me in this life.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Figure Page

CHAPTER 1: INTRODUCTION

Technology has been used in mathematics education since the root of mathematics science. Therefore, the use of technology in teaching and learning of mathematics has become essential with the recent improvements in computer technology. Accordingly, technology is stated as one of the six principles (equity, curriculum, teaching, learning, assessment and technology) of teaching and learning of mathematics by National Council of Teachers of Mathematics (NCTM, 2000). This study aims to develop a teaching unit on integral calculus by means of definite integral (which is a particular mathematical concept) by enriching it with technology and inquiry. The target population for this unit plan is high school mathematics teachers and students.

Background

Since Dewey (1938) expressed that students improve their learning when they build their knowledge through their own experiences, researchers have become very interested in inquiry-based teaching and learning (Barrow, 2006). Many researchers acknowledge that inquiry based teaching leads to students learning in a variety of school subjects (Chapman, 2011; Hakverdi-Can & Sönmez, 2012; Engeln, Euler, & Maass, 2013; Hahkiöniemi, 2013). The reason behind the effectiveness of inquiry-based learning and teaching is that students investigate problems and construct their knowledge through observation and the synthesis of their own and others' ideas. Thus, students may achieve meaningful learning by understanding the logic behind the information through inquiry (Hahkiöniemi, 2013).

It has been proven by several studies that integrating technology into teaching promotes inquiry-based learning (Kubicek, 2005; Hohenwarter & Lavicza, 2007). Especially digital technologies, such as dynamic mathematics software encouraging students to extend more complex mathematical phenomenon (Hahkiöniemi, 2013). For example, Healy and Hoyles (2001) claim that with the help of these digital technologies, students achieve conceptual understanding as they analyze the geometrical relationships and produce their own proofs for conjecture. Moreover, these technologies provide students with opportunities beyond the need for memorizing formulas in mathematics (Salleh & Zakaria, 2012).

Integral calculus involving “definite integral” and “volumes of solids of revolutions” is one of those mathematics concepts that fits the concept mentioned above (Mofolo-Mbokane, 2011). This type of mathematics involves students with more complex mathematical procedures than are usually presented in a typical high school mathematics lesson. According to a research conducted by Mofolo-Mbokane (2011), there may be several reasons for students’ difficulties with volumes of solids of revolutions. As a result of their analyses, Mofolo-Mbokane (2011) concluded that the factors that cause those difficulties may be a lack of “three-dimensional thinking” “moving between discrete and continuous representations” and “consolidation and general level of cognitive development” (p. ii). Dynamic mathematics software seems to be able to solve these problems with their three dimensional features and concrete geometric representations.

When we analyze the history of calculating volumes of solids of revolution, we see that all those formal definitions and formulas were revealed by scientists as a result of actual geometrical problems (Boyer, 1949). However, in this century, some students are still taught these kinds of concepts as technical calculating abilities in high schools (Attorps, Björk, & Radic, 2013). That is why, students might experience struggles in solving problems of volumes of solids of revolution; they could not achieve conceptual understanding of the concept itself. In order to avoid these struggles and provide students with conceptual understanding, the National Council of Teachers of Mathematics (NCTM, 2000) included technology in the six principles of teaching and learning of mathematics. Moreover, a movement called technological pedagogical content knowledge (TPCK) was developed by Mishra and Koehler (2006) based on Shulman's (1986) framework of pedagogical content knowledge (PCK). Mishra and Koehler (2006) support that teachers should teach with integrating TPCK components: technology, pedagogy, and content knowledge.

As a result of the improvement of technology in education in developed countries, the Turkish Ministry of National Education (MoNE) also changed their perspective in teaching and learning in the way that teachers should use technology in classrooms effectively (MoNE, 2013). The new curriculum published in 2013 by MoNE explicitly promotes the use of technology in the classroom. Accordingly, MoNE (2013) promotes that teachers use dynamic geometry software, graphing software, spreadsheet software, graphing calculators, interactive smart boards and tablets, data acquisition devices, computer algebra systems, and dynamic statistics software and internet.

Problem

Integral calculus is perceived as a challenging concept by many high school students in developing conceptual understanding because of many complex formulations

(Orton, 1983). According to Orton (1983), students are struggling while solving problems; they do not notice integration as a limiting process of sums. Moreover, Attorps, Björk, and Radic (2013) suggest that some students perceive solving integral problems as a technical ability, therefore they could not achieve conceptual understanding despite doing all the calculations successfully.

The reason why students have difficulties in integral calculus is their failure to construct concrete meanings of the formal definitions in their mind. Although dynamic mathematics software has a potential to minimize these problems and encourage students to explore the relationship between abstract and concrete objects, this technology is not yet used effectively by high school mathematics teachers in Turkey (Baki, 2000). According to Baki (2000), the main problem for in-service mathematics teachers not using technology in their teaching is that they were not educated in the way of using technology in mathematics lessons when they were pre-service teachers.

Purpose

The primary purpose of this study is to develop an inquiry-based teaching unit on integral calculus that uses dynamic mathematics software technology for high school mathematics students and teachers. The definite integral and volumes of solids of revolution theme was chosen as an exemplary case for this inquiry-based unit plan to provide a practical example for teaching integral calculus by using mathematics software in a combination. With real life hands-on technology activities, this study aims to contribute to Turkish mathematics curriculum.

Research questions

With the main purpose of developing an inquiry-based technology-integrated teaching unit, the research questions of this study was defined as follows:

- What is the final version of technology integrated inquiry based teaching unit on integral calculus, in particular, the definite integral at the end of the development process?
- What are the views of in-service high school mathematics teachers regarding the developed technology-integrated inquiry-based teaching unit on integral calculus, in particular, the definite integral?
- What is the perceived effectiveness of the developed technology-integrated inquiry-based teaching unit on integral calculus by in-service high school mathematics teachers?

Significance

This study is significant in terms of developing a new technology-integrated inquiry-based learning plan on integral calculus for high school mathematics teachers and students. Thereby, in-service high school mathematics teachers are provided with a teaching unit that could be used in their teaching of integral calculus, and would enhance the learning of students through inquiry.

Definitions of key terms

NCTM: National Council of Teachers of Mathematics.

PCK: Pedagogical Content Knowledge (Shulman, 1986)

TPCK: Technological Pedagogical Content Knowledge (Mishra & Koehler, 2006).

GeoGebra: A dynamic mathematics and geometry software combining Dynamic Mathematics Systems (DGS) and Computer Algebra Systems (CAS).

Inquiry based learning: A concept which enables students to engage in conceptual understanding and to build students' ideas through inquiry (Chapman, 2011)

Integral calculus: A mathematics concept which was studied through the problem of finding the area of a region under a curve. The most contributions on this concept were done by Newton (1642-1727) and Leibniz (1646-1716) by finding "fundamental theorem of calculus".

CHAPTER 2: REVIEW OF THE LITERATURE

Introduction

The purpose of this chapter is to give the readers an insight about the background of this study. It reviews theory and discussions about the technological pedagogical content knowledge (TPCK) which is a framework that has become a requirement of mathematics teachers in developed countries (Mishra & Koehler, 2006). Accordingly, the current situation of Turkish high school mathematics teachers' abilities in integrating technology in their teaching is discussed. Also, information including the actions that have recently been taken by the Turkish Ministry of National Education (MoNE) related to use of technology in education was reported. This chapter summarizes the research related to inquiry-based teaching and learning technology integration in the teaching of mathematics. Moreover, this chapter discusses the concept of "solids of revolution" which is an application of integral calculus.

Technological Pedagogical Content Knowledge (TPCK)

Technological pedagogical content knowledge (TPCK) is a framework proposed by Mishra and Koehler (2006) (i.e. an extension of Shulman's (1986) formulation of pedagogical content knowledge (PCK)) with the integration of technology in teaching. According to Shulman (1986), PCK is a special type of professional interest, because it represents the integration of two distinctive frameworks of teaching: *pedagogy* and *content*. Hence, PCK has become a widely used notion, especially in the professional development of teachers.

According to Mishra and Koehler (2006), although technology was not considered unimportant by Shulman (1986), technology was not used widely as in classrooms in the 1980s-- as in today's classrooms. Therefore, Shulman did not comprehend technology in the framework of PCK.

Today the integration of technological knowledge into pedagogical content has become essential. Therefore, Mishra and Koehler (2006) proposed a new notion: technological pedagogical content knowledge (TPCK) is a requirement for developing good teaching. In this framework, Mishra and Koehler (2006) emphasize the "connections, interactions, affordances, and constraints between and among content, pedagogy, and technology" (p. 1025) as shown in Figure 1.

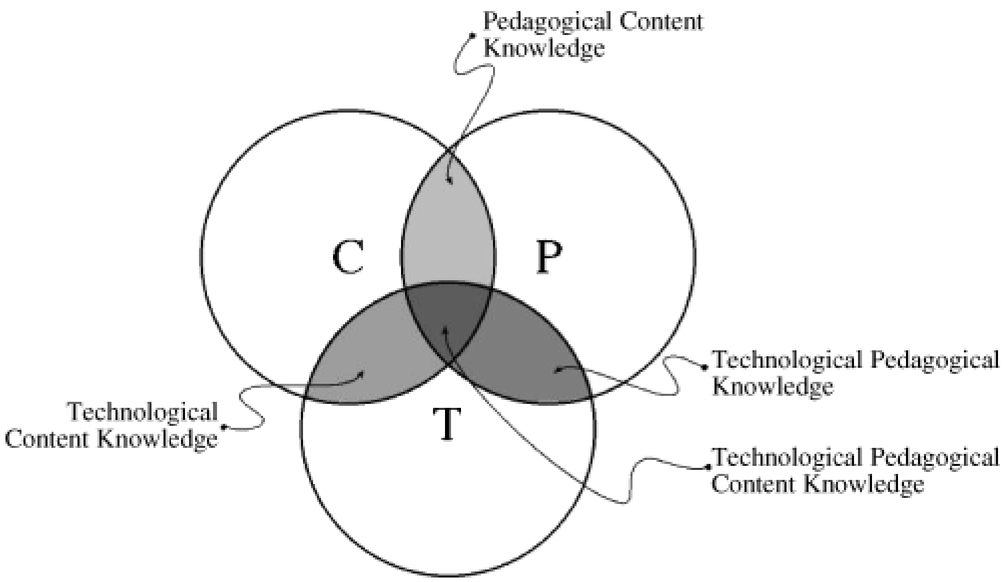


Figure 1. Technological pedagogical content knowledge (Mishra & Koehler, 2006, p.1025)

Despite the improvement of educational technology, technology is mostly used by Turkish mathematics teachers for administrative tasks, such as organizing scores of students or preparing for lectures and lesson plans instead of using technology to drive the learning process as an instructional necessity (Demiraslan & Usluel, 2008).

Furthermore, Baki (2000) indicates that although the need to educate new teachers to use technology in their lessons has been increasing, teachers training institutes in Turkey are insufficient in terms of the professional development of teaching staff. Therefore, when pre-service teachers graduate from their teacher education programs, they generally face the reality that they are not educated well enough to use educational technology in their classrooms (Baki, 2000). Research proves that technology is only successful in the classroom with excellent teacher training in technology. Consequently, Baki (2000) supports that teacher trainers in teacher education programs need to change their strategies as they prepare pre-service teachers to use technology in their teaching.

The Perspective of Turkish Ministry of National Education

The current official Turkish high school mathematics curriculum requires the use of technology in mathematics classrooms (MoNE, 2013). The Turkish MoNE mathematics curriculum indicates the fact that, both quality and quantity of teaching software related to mathematics education has increased as a result of the constant development of mathematical applications. Hence, the MoNE promotes mathematics teachers make use of technology in mathematics classrooms. It is also emphasized that the utilization of technology could provide new learning and teaching opportunities for both teachers and students alike. Accordingly, using information and communication technology effectively, students may work on mathematical problems related to real life; students may spend more time on reasoning and creative thinking, rather than time consuming computations that don’t connect with their own lives.

The new mathematics curriculum summarizes the main information and communication technologies to be used in mathematics classrooms such as dynamic geometry software, graphing software, spreadsheet software, graphing calculators, interactive smart boards and tablets, data acquisition devices, computer algebra systems, dynamic statistics software and the Internet. Correspondingly, Turkish MoNE expects students to use these technologies effectively in the new curriculum. Thus, students could explore the mathematical concepts through experiencing different types of thinking skills when teachers fulfill MoNE's recommendations effectively.

The use of technology has been essential in mathematics classrooms in terms of engaging students in learning (Brahier, 2000). Therefore, technology is emphasized as one of the six principles of mathematics education by the NCTM (2000). Although technology is not new in mathematics education, with the improvement of technology, new technological tools such as graphing calculators or dynamic mathematics software have been developed for mathematics education. In this regard, the Turkish Ministry of National Education (2012) developed the FATİH (Fırsatları Arttırma ve Teknolojiyi İyileştirme Hareketi) Project to improve the technology in education by providing technological devices such as interactive white boards and tablets.

Integral calculus

Integral calculus is a concept which has a wide coverage in world history (Boyer, 1949). Integral was studied through the problem of finding the area of a region under a curve. Although it is known that Newton (1642-1727) and Leibniz (1646-1716) made the most significant contributions to integral calculus by finding "fundamental theorem of calculus", the first steps of integral calculus were taken by Greek mathematicians (O'Connor & Robertson, 1996). Over 2000 years ago Eudoxus (408-355 B.C) and Archimedes (287-212- B.C) took the first steps of integral calculus as they stated and proved the "method of exhaustion" (Berkey & Blanchard, 1992). From those years many mathematicians such as Fourier (1768-1830), Gauss (1777-1855), Liouville (1809-1882), Hermite (1822-1901), Lebesgue (1875-1941) have contributed to the development of integral calculus ("History of integration," n. d.).

Integral calculus is perceived as challenging by many of high school students in developing conceptual understanding because of many formal definitions. According to Orton (1983), students are struggling while solving problems; they need to notice integration as a limit process of sums. Moreover, Attorps, Björk and Radic (2013) suggest that some students perceive solving integral problems as technical ability, and although they do all the calculations successfully, they may not achieve conceptual understanding. As a result, the reason why students have difficulties in integral calculus is that they fail to construct a concrete meaning of the formal definitions in their mind.

Calculating the volume of solids of revolution is an application of integral calculus concept. This concept was included in the high school mathematics curriculum as an introduction to calculus after the Second World War (Wurnig, 2009). The calculation of volumes of solids of revolution has been viewed as a good example of the application of integration in high school mathematics books.

Inquiry-based teaching and learning

Inquiry based learning is a concept which enables students to engage in conceptual understanding and to build students' ideas through inquiry (Chapman, 2011). The concept of inquiry based learning has grown since Dewey (1938) supported that students can learn better when they investigate the problems according to their own experiences (as cited in Barrow, 2006). Dewey's pioneering of social constructivism is all about creating meaning through doing. Several studies emphasize that inquiry-based learning plays a remarkable role in mathematics education (Chapman, 2011; Hähkiöniemi, 2013; NCTM, 2000). However, there are some challenges in implementing inquiry-based teaching in the mathematics classrooms (Dorier & Garcia, 2013). Teacher beliefs and attitudes towards inquiry-based mathematics might be the main reasons of those challenges (Engeln, Euler, & Maass, 2013). It is essential for teachers to become a technological advocate for students instead of being a technological adversary. Therefore, teachers should be trained to engage students in conceptual understanding through inquiry-based teaching (Hähkiöniemi, 2013).

Technology-enhanced inquiry-based learning and teaching

Many researchers support that integrating technology into mathematics classrooms enriches inquiry based learning (Hähkiöniemi, 2013; Hakverdi-Can & Sönmez, 2012; Wentworth & Monroe, 2011). According to Hahkiöniemi (2013), particularly dynamic mathematics software promotes students' investigation and exploration opportunities. Similarly, Healy and Hoyles (2001) suggest that, with the help of dynamic mathematics software, "students move from argumentation to logical deduction" (p. 235). Moreover, students could relate one geometrical representation of a formula to another. And they could build hypotheses through trial and error that they apply with dynamic mathematics software.

The effectiveness of visualization in teaching of mathematics has been recognized by most mathematics teachers (Gutierrez, 1996). Dynamic mathematics software enables students to see the visual outputs of mathematical calculations. Thus, with the help of dynamic mathematics and geometry software, students can be provided with a conceptual understanding in mathematical concepts such as integral calculus (Hähkiöniemi, 2013).

A dynamic mathematics software program: GeoGebra

There are powerful technological tools that help teachers to teach mathematics in a meaningful way, such as computer algebra systems (CAS) (e.g. Mathematica, Maple, and Matlab) or dynamic geometry systems (DGS) (e.g. Geometer's sketchpad and Cabri). According to Lavicza (2006), with the help of visualization features in these programs, students are encouraged to experiment and learn through inquiry. However, all those programs are expensive and might not be affordable for the entire student population. In contrast to these costly tools, GeoGebra is an open source dynamic mathematics system (DMS), combining the DGS and CAS (Hohenwarter, Kreis, & Lavicza, 2008).

GeoGebra was created by Marcus Hohenwarter (2001) to help students aged ten to eighteen achieve a better understanding of mathematics (Hohenwarter & Preiner, 2007). Another advantage of GeoGebra is that it is easy to use. As DGS packages could be used in early ages and CAS packages are used in upper level education.

Combining these two packages in high school, GeoGebra offers an easier and cost-free solution.

Since the development of GeoGebra software program, it has been used by thousands of students and teachers. A lot of activities, worksheets, and methods for a wide range of levels have been developed by teachers and researchers, and they have been shared on GeoGebraWiki-- which is a share point platform of those activities. GeoGebraWiki includes a large collection of activities related to calculus, and in particular, definite integral calculus as well. The activities available are growing exponentially. Therefore, to teach and learn definite integral, GeoGebra can be used with an easy-to-use interface.

CHAPTER 3: METHOD

Introduction

The primary purpose of this study was to develop an inquiry-based teaching unit on the general concept of integral calculus for high school mathematics students and teachers. In particular, the *volumes of solids of revolution* theme was chosen as an exemplary case for this inquiry based teaching unit. In order to develop the teaching unit, the "instructional design" method was used in this study. In the literature there are several instructional design models such as Gagne, Briggs, and Wagner's model; Dick, Carey, and Carey's model; Smith and Regan's model; Seels and Glasgow's model; Based on the general characteristics and common aspects of all these instructional design models, the instructional design model presented in Figure 2 was taken as a basis in the present study. In this chapter, general information about instructional design was presented. Accordingly, general and detailed process of developing the teaching unit informed through instructional design was stated in detail.

What is instructional design?

In a general manner, instructional design is defined as "a process of determining what to teach and how to teach it" (Dick, 1995, p.13). According to Smith and Ragan (1999) instructional design refers to "the systematic and reflective process of translating principles of learning and instruction into plans for instructional materials, activities, information resources, and evaluation" (p. 2). Determining the instructional problem, analysis and planning for development, development of the materials and evaluation form the steps of the instructional design process. Each step flows from the previous one where the associated revisions take place within the whole process (Dick, 1999). This linear process of instructional design indicates planning of instructional systems in which materials and procedures are arranged to support effective learning (Seels, 1995).

General procedure of developing instruction through instructional design

The first step of developing instruction informed through instructional design is determining what the learners will be able to do when they complete the instruction (Dick & Carey, 2001). The instructional goal may be derived from a problem or a gap in the current instruction of the related concept. Therefore, analyzing the instructional problem is needed first. Accordingly, finding a solution to the instructional problem and developing instructional materials form the following steps of the instructional design. Finally, with the evaluation process, the development is improved even further (Reiser & Dempsey, 2007).

Common aspects of instructional design models

Instructional design models share common aspects which are *empirical, iterative* and *self-correcting* (Reiser & Dempsey, 2007). In order to provide an *empirical* aspect, the instructional designer must grant that instructional materials include factual information. Therefore, throughout the designing process, an evaluation of the quality of the instructional materials should be held through data collection and data analyses.

By being *iterative,* an instructional design process refers a cyclic process specifically covering analysis and redesign; and an adaptive disposition to the changes derived from researchers and practitioners on the initial design. (Schwartz et al. 1999; van den Akker, 1999; Cobb et al. 2003).

With the *self-correcting* aspect, it is referred that the instruction could be improved through detecting the weak aspects of the designed instruction. The evaluation process helps the researcher to make modifications on the instructional material to provide that the instructional material has a more advanced form (Reiser & Dempsey, 2007).

The procedure of developing the teaching unit

Design of this study is arranged in a step-by-step order through scientific procedures. In this section, general and detailed procedures of developing the teaching unit is represented. The general framework of the process in this study is illustrated in Figure 2. Also, detailed procedure of the instructional design method study is represented in Figure 3.

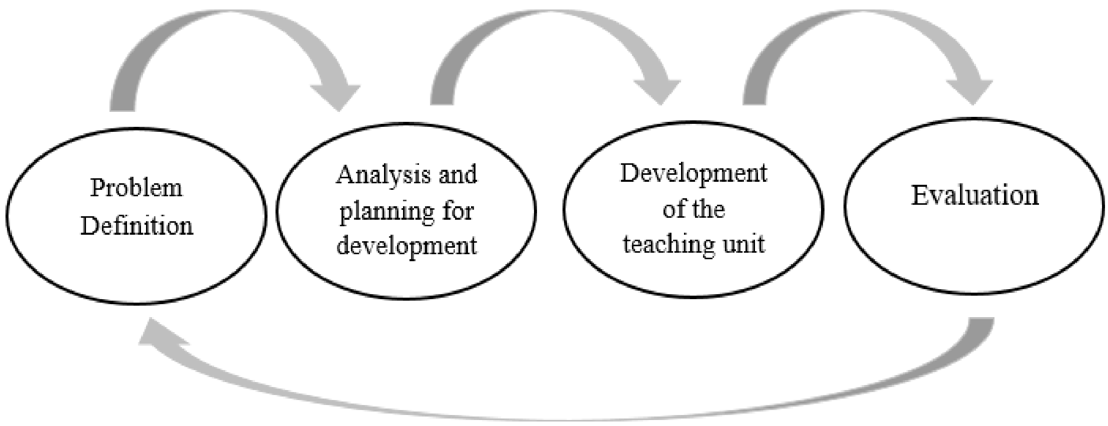


Figure 2. A general model of an instructional design model used to develop the teaching units

As it is shown in Figure 2, the general procedure of developing the teaching unit was separated into four main phases determined by the researcher. For each main phase, conducted activities and the obtained output were defined in detail in Figure 3. As a result of the cyclical process of the design, the next phase was organized under the light of obtained output at the end of each phase.

PHASES		WHAT HAD BEEN DONE	OUTPUT
1. Problem Definition	**Figuring out the problem**	**Elaboration on… (January, 2014)** MoNE high school mathematics curriculum High school course books Literature related to technology enhanced inquiry based teaching and learning Preliminary interview with a high school mathematics teacher	**Problem statement (March, 2014)** • There is a need for development of an inquiry-based, technology-enhanced teaching unit on definite integral for Turkish high school mathematics teachers.

Figure 3. Detailed procedure of the instructional design method study (cont'd)

<table>
<tr><th colspan="2">PHASES</th><th colspan="2">WHAT HAD BEEN DONE</th><th>OUTPUT</th></tr>
<tr><td rowspan="2">2. Analysis and planning for development</td><td rowspan="2">Preparing an action development plan</td><td>Theoretical foundation of the teaching unit (February - April, 2014)
Elaboration on 5E learning cycle model
Decision on the context of the teaching unit</td><td rowspan="2">Getting expert opinion (February - May, 2014)
Choosing a conference to get expert opinion
Writing a paper about the initial study
Presenting the initial study at the International Conference in Education of Mathematics, Science and Technology in Konya (May, 2014)</td><td rowspan="2">Outline of the teaching unit to be developed (June, 2014)</td></tr>
<tr><td>Creating GeoGebra activities (February, March, 2014)
Studies on GeoGebra
Creating GeoGebra activities</td></tr>
</table>

Figure 3. Detailed procedure of the instructional design method study (cont'd)

PHASES		WHAT HAD BEEN DONE	OUTPUT
3. Development of the teaching unit	**Survey development (July, 2014)**	Determining the characteristics of the teaching unit to be included in the survey Development of the survey questions	**Finalized form of the Teaching unit to be handed out to the participants (July, 2016)**
	Development of the teaching unit materials (July, 2014-July, 2016)	Setting the objectives of the teaching unit Teaching unit content development Development of the GeoGebra worksheets	

Figure 3. Detailed procedure of the instructional design method study (cont'd)

PHASES		WHAT HAD BEEN DONE				OUTPUT
4. Evaluation	**Finalizing the study**	Determining and choosing the participants (May, 2016)	Delivering the survey and the teaching unit to the participants (July, 2016)	Collecting data from the participants (August, 2016)	Data analyses (August-November, 2016)	• **Corrections on the teaching unit according to the participants' feedback** • **Ideas for further development**

Figure 3. Detailed procedure of the instructional design method study

Problem definition

The instructional problem of this study was that there was a need for developing an inquiry-based, technology-enhanced teaching unit for high school mathematics teachers in Turkey to teach definite integral informed through empirical instructional design model. In this section, the process of examining the problem of the lack of inquiry-based technology-enhanced teaching units available and then the initiations for finding a solution to the instructional problem was stated.

Figuring out the problem

In order to figure out the problem, a variety of resources were examined. The resources examined were national mathematics curriculum (MoNE, 2013) and high school course books written by Sevinik et al. (2012) and Ünlü et al. (2016). Moreover, a detailed analysis of literature was conducted within the scope of technology enhanced inquiry based teaching and learning. Although the problem was evidently put forward by the researcher, it was discussed and verified by an experienced high school mathematics teacher through a semi-structured interview having questions related to current teaching methods on integral calculus concept (see Appendix 1). This interview was conducted as the researcher needed to hear an experienced voice of a practitioner. It lasted one hour long and was held at the school of the teacher.

The problem was identified as a lack of instructional units on teaching the definite integral concept with the help of a dynamic mathematics software. Although MoNE mathematics curriculum (2013) explicitly promotes the use of technology, in particular, dynamic mathematics and geometry technologies in several mathematics concepts, including definite integral. It was observed that the integration of technology to teach definite integral was not covered in 12th grade mathematics course books written by Sevinik et al. (2012) and Ünlü et al. (2016). Instead, visual representations of two and three-dimensional objects were visualized in these books.

These books did not support the recognition of inquiry-based teaching and learning in playing a remarkable role in mathematics education (Chapman, 2011; Hähkiöniemi, 2013; NCTM, 2000). Moreover, there was a consensus by many researchers that integrating technology into instruction promotes inquiry-based learning (Hähkiöniemi, 2013; Hakverdi-Can & Sönmez, 2012; Wentworth & Monroe, 2011).

Finally, a preliminary interview with an experienced high school mathematics teacher was conducted to discuss these above mentioned identifications which were verified by the teacher. That is, the teacher expressed that there was currently an absence of use of dynamic mathematics software technology in mathematics instruction in Turkey, and so in his particular school. According to this teacher: teaching definite integral concept with the help of a dynamic mathematics software would promote meaningful learning and save time, as well.

Analysis and planning for development of the teaching unit

At this point of the study, the planning process including analyses in order to come up with a solution to the problem, was stated. Theoretical foundation of the teaching unit, creating GeoGebra activities, and combination of these two phases formed the actions in this section. At the end of this section, outline of the teaching unit to be developed was created.

Preparing an action development plan

The researcher prepared an action development plan through conducting two different studies simultaneously. Within the theoretical foundation of the teaching unit, the 5E learning cycle model was elaborated in order to put the teaching unit into a structure (Bybee, Taylor, Gardner, Van Scotter, Powell, Westbrook, & Landes, 2006). In addition, a context by which the teaching unit would lend itself to an inquiry-based teaching unit was decided. The second study was to create GeoGebra activities to be integrated into the teaching unit. Finally, these two studies were combined and a paper was written by the researchers and presented on an international conference in Turkey. As a result, an outline of the teaching unit to be developed was created.

The theoretical foundation of the teaching unit

The first step of developing the theoretical foundation of the teaching unit was to choose a model that put the teaching unit into a structure. 5E (engagement, exploration, explanation, extension, and evaluation) was determined as the flow of the teaching unit to be developed. The reason why 5E learning cycle model chosen was that this model of instruction would lead students to learn through inquiry and provide students with conceptual understanding (Liu et al., 2009). *Engage* stage of the 5E model helps students to become engaged in a new concept using their prior knowledge. Within *engage* stage, activities are used which promote students' curiosity and reveal students prior knowledge related to the new concept. *Explore* stage refers to the experiences of the students generate new ideas through using their prior knowledge on activities related to the new concept. *Explanation* stage focuses on the process that students demonstrate their conceptual understanding according to their engagement and exploration experiences. In this stage teachers could also provide a direct introduction to a new concept or a skill. Within *elaboration* stage, teachers challenge students to provide them with a deeper and a broader understanding on the concept. Finally, within *evaluation* stage students could assess their understanding and abilities as well as teachers could evaluate students process related to the educational objectives (Bybee et al., 2006, p. 90).

The second step of developing the foundation of the teaching unit was to decide the context of the teaching unit. Since this teaching unit was intended to be an inquiry based teaching unit, students were supposed to learn from their own experiences in real life contexts. Therefore, by this teaching unit it was decided to teach definite integral by defining its relation to area and volume concepts in which many real life situations could be generated. Moreover, those real life situations related to area and volume concepts could be visualized through dynamic mathematics software. Therefore, the context of this teaching unit was decided to be used as an example that is suitable for using a dynamic mathematics software.

Creating GeoGebra activities

A variety of dynamic geometry software such as Geometer's Sketchpad, Cabri and mathematical graphing programs such as Mathematica, Maple or Matlab has been used in education with the common aim to extend students' understanding in mathematical concepts. After an inquiry for dynamic mathematics and geometry software which would ease teaching and learning illustrating the formal definitions in definite integral concept with concrete objects by using 2D and 3D illustrations, GeoGebra was decided to be used because of the following advantages. The ability of showing both mathematical expressions and the geometrical visuals representing those mathematical expressions in one window, GeoGebra enables to see how the visuals change when we change numbers or variables in the mathematical expressions. Also, GeoGebra is able to visualize real life objects through using its 3D features, so it enables to illustrate real life situations. Moreover, being an open source dynamic mathematics software, any teacher or student could use GeoGebra legally without the need for license fees.

After deciding GeoGebra as the dynamic mathematics software to be used in this study, the activities to be integrated to the teaching unit was started to be created in coherence with the content of the teaching unit. Firstly, the researcher studied on GeoGebra to learn its basic features. The researcher developed the GeoGebra activities by using GeoGebra tools and features as a learner through trial and error method. Also, the researcher got help from *GeoGebra Tube* where you can watch the videos showing how the tools and features work, and from *GeoGebra Forum* where the GeoGebra experts and users reply quickly and help you when you have a question regarding GeoGebra tools and features.

Getting expert opinion

As a part of the instructional design process, expert opinion and evaluation about the initial study was needed. Therefore, the researchers chose the conference – International Conference on Education in Mathematics, Science and Technology

(ICEMST) - which was going to take place in Konya, Turkey. The conference was organized by the International Journal of Education in Mathematics, Science and Technology (IJEMST) with the aim of discussing theoretical and practical issues in the fields of education, science education, mathematics education and information and communication technologies by bringing academics, students and administrators from different countries together.

By combining the created GeoGebra activities with the theoretical foundation of the teaching unit, a paper was written by the researcher on this initial development of the teaching unit to present at the ICEMST. Through presenting the study at this international conference, the audience's perspectives and suggestions about the study were gathered. Also the paper was published on the ICEMST proceeding book. Consequently, the outline of the teaching unit to be developed was created at the end of the action development.

Development of the teaching unit

This section of the study presents the process of putting the teaching unit into its final form. In this section, materials needed for the teaching unit were developed. Also, a survey was developed as a checklist to evaluate the teaching unit. Developing the materials and the survey was held simultaneously as the survey helped the researcher to make sure that the teaching unit provides the criteria in the survey.

Survey development

The survey was developed by the researchers. In order to determine the questions in the survey, firstly the characteristics of a technology-integrated inquiry-based mathematics teaching unit were needed to be determined. So, in June 2014, the researcher made an inquiry in the literature related to a technology-integrated inquiry-based mathematics teaching and learning (Hakverdi-Can & Sönmez, 2012; Kubicek, 2005; Mishra & Koehler, 2006; Wentworth & Monroe, 2011). Accordingly, the characteristics of the teaching unit to be developed were determined so that it has *appropriate language and expression; has appropriate content; is practical; is mathematically accurate; is technologically accurate; and it is inquiry based*. In order to ensure the reliability of results, two experts in mathematics education field commented on the emerging characteristics in one hour time via oral discussions. These characteristics were integrated into the survey as sub-categories where *appropriate content* and *practical* characteristics were put together. Therefore, the survey was made up with five sub-categories with 65 questions in total and each question was a 4-point-Likert scale question (see Appendix 4). The sub-categories included the following number of items shown in Table 1.

Table 1
Number of items in each sub-category of the survey

Sub-categories of the survey	Number of items
Appropriate Language and Expression	12
Appropriate Content/Practical	18
Mathematically Accurate	15
Technologically Accurate	6
Inquiry Based	14

Development of the teaching unit materials

Materials of this teaching unit was developed through following four sections: *review of objectives, review of content, teaching unit content design and teaching unit cover* *design.* By completing these sections, the draft version of the teaching unit was developed. This process took a long time which was 20 months.

Review of objectives

In order to grant that this teaching unit has appropriate content, firstly the objectives were set linking them to the MoNE's and NCTM's standards addressing the definite integral concept. The objectives were determined by the researcher regarding what students would be able to do when they completed this teaching unit.

Teaching unit content development

The content of the teaching unit was generated in the light of the determined objectives. Also, conceptual framework was provided with the phases of 5E learning cycle model. Accordingly the teaching unit included three engagement, five exploration, four explanation, two elaboration parts and one evaluation part. Each phase of the 5E learning cycle model was illustrated by an image near the headings in the teaching unit.

Although the teaching unit was created to be a guide for primarily Turkish high school mathematics teachers, the language of the teaching unit was decided to be in English; with the aim that this teaching unit could be a guide for high school mathematics teachers in overseas countries as well. Moreover, by being in English it was provided that the teaching unit corresponded with this study's language. In order to ensure that this teaching unit has appropriate language and expression, proofreading and regular discussions on the teaching unit with the advisor of this study were done. Accordingly, several type of fonts and applications were used to enable a fluent and intelligible flow. For example, the text boxes in Figure 3 and Figure 4 were used for notes and GeoGebra instructions respectively; and bullet points in Figure 5 were used for instructions to teachers. Those different visual expressions were used with the aim that the reader has an easy understanding. Moreover the cover and the headings were designed by the researcher so that they were appropriate with the content and the activities.

Note:
Students must have already learned the limits topic. If needed, teacher must provide a brief revision on limits.

Figure 4. An example for notes in the teaching unit

GeoGebra Instructions

- Click on the drop down arrow which is located at the lower right side of the ABC *text* icon. Now select *pen* icon and draw the irregular shape by using your mouse so that it is a closed area.

Figure 5. An example for GeoGebra instructions in the teaching unit

- Ask students to use lower and upper rectangle sums for this function and generate a formula for these sums in the intervals $0 \leq x \leq 3$.
- Accordingly, ask students to generate a formula for upper and lower rectangle sums for a positive, continuous and increasing function on the interval $a \leq x \leq b$.

Figure 6. An example for instructions to teachers in the teaching unit

The main characteristic of this teaching unit was aimed to be inquiry-based. To determine the characteristics of an inquiry-based teaching unit, the question "What fosters a student to make inquiry?" was asked by the researcher and it was found that when students' curiosity was aroused, and students were leaded to investigate the problems according to their own experiences they tend to make inquiry as Dewey (1938) had stated. Thus, to foster students to make inquiry, firstly the teaching unit required the students to investigate the origin of the term *integral* by collecting information from a variety of resources. Then, a history of the *integral* concept was briefly mentioned in the teaching unit. Thereby, the students would get anxious about from what need the integral topic was generated. The exploration parts of this teaching unit were designed as students could use their knowledge in real life situations. While making inquiry in especially exploration parts of this teaching unit, it was aimed that students could search and collect information from various resources and share ideas with their friends and learn from each other. Therefore, all of those exploration parts were designed as group work activities.

Development of the GeoGebra worksheets

At the end of the teaching unit, GeoGebra worksheets were added which were giving directions to the students and teachers to be able to do the activities within the teaching unit. Each GeoGebra worksheet had an introduction which gives a brief information about what the worksheet was useful for. While creating the GeoGebra worksheets and instruction parts in the teaching unit, the flow was designed as step-by-step through bullet points. To provide students and teachers with an easy understanding while implementing GeoGebra worksheets, italic fonts for GeoGebra commands and images for GeoGebra icons were used. The researcher examined each

GeoGebra worksheet whether if they work or not after she created the worksheets and she did corrections if needed.

In addition to inquiries on the generation of the integral concept, this teaching unit aimed to make students realize the necessity of the concept of integral by application of *integration* to real life situations with the help of technology. As many researchers agree that integration of technology enhances inquiry-based learning, a mathematics and geometry software -GeoGebra, was integrated in this teaching unit (Hähkiöniemi, 2013; Hakverdi-Can & Sönmez, 2012; Wentworth & Monroe, 2011). Accordingly, the aim was that students could analyze mathematical expressions in integral concept through making connections between their two and three-dimensional visuals, as GeoGebra allows students to see visuals representing the mathematical expressions. So, students could see the relationship between mathematics and real life situations. By all these aspects of this teaching unit, the aim was that students would actively learn from their own experiences.

Evaluation of the teaching unit

In this section, the information about the participants who evaluated the teaching unit was given. At the end of this section, data collection and data analysis procedures were explained. Finally, further development action plans were discussed in Chapter 5 of this study.

Participants

Bilkent University Graduate School of Education offers a two year masters with a teaching certificate program which is called Master of Arts in Curriculum and

Instruction with Teaching Certificate. Each year, five to eight students who had a Bachelor of Science degree in mathematics, had proficiency in English are accepted to study in this program as pre-service mathematics teachers. The participants of this study consist of eight in-service mathematics teachers who had graduated from this program. Four of the participants were selected from the researcher's fellow teachers who studied in this program between 2013 and 2015, three of the participants who studied in this program in 2014 and 2016, and one participant who studied in this program between 2005 and 2007. So, all of the participants were considered as experts in teaching mathematics as they worked as mathematics teachers since they completed this two years program. Also these teachers had taken the course MTE 503, Computer Technology in Mathematics Education in which they learned to use GeoGebra. All of the participants were selected in May 2016 according to their willingness to study in this study.

Evaluation

The teaching unit was evaluated by the in service high school mathematics teachers through the survey regarding the determined characteristics of the teaching unit. During this process, the survey questions were written on a Google forms document so that data could be collected from the participants conveniently. The developed teaching unit and the Google form link of the survey were sent to the teachers with the written directions that they were: supposed to read the teaching unit, implement the GeoGebra activities in the teaching unit, answer the questions in the survey, and write written feedback on the teaching unit so that the teaching unit would be improved further. Also the latest version of GeoGebra link was sent out to the teachers via the same e-mail. The Google form link was sent out to the teachers in July 2016, and the teachers were given one week to fulfill the directions.

In order to make the analysis of teacher comments in accordance with the determined characteristics, the researcher originated the following themes for each characteristic shown in Table 2. Accordingly, each comment addressing one of these characteristics was linked to the appropriate theme by means of thematic analysis.

Table 2

Codes used for characteristics of the teaching unit

Code	Corresponding Characteristic
ALE	Appropriate Language and Expression
AC	Appropriate Content
P	Practical
MA	Mathematically Accurate
IB	Inquiry Based

Also, the screen shots showing participants' GeoGebra activities were added to Appendix 2. Finally, descriptive statistics showing the comments of each participants' scores on each sub-category of the survey were reported. The Likert scale statements were represented by numbers so that "strongly disagree" refers to the number "1", "disagree" refers to number "2", "agree" refers to number "3", and "strongly agree" refers to number 4. After analyses were done on the teachers' written feedback and their responses to the survey, necessary corrections were done and the teaching unit was finalized.

Summary

In this chapter, the development process of an inquiry-based teaching unit was indicated in detail informed through instructional design method. This process was divided into four main phases which were *problem definition, analysis and planning for development, development of the teaching unit,* and *evaluation.* Each phase were detailed by being shown through four figures and descriptive texts. Also the following timeline shows the summary of what had been done in this phases in one figure (see Figure 7).

PHASES	WHAT HAD BEEN DONE	DATE
Problem Definition	Elaboration on MoNE high school mathematics curriculum	January to March, 2014
	Elaboration on high school course books	January to March, 2014
	Literature review related to technology enhanced inquiry based teaching and learning	January to March, 2014
	Preliminary interview with a high school mathematics teacher	12th of March, 2014
Analysis and planning for development	Elaboration on 5E learning cycle model	February to April, 2014
	Decision on the context of the teaching unit	February to April, 2014
	Studies on GeoGebra	February to April, 2014
	Creating GeoGebra activities	February to April, 2014
	Choosing a conference to get expert opinion	February, 2014
	Writing a paper about the initial study	February to April, 2014
	Presenting the initial study at the International Conference in Education of Mathematics, Science and Technology in Konya	16th of May 2014
Development of the teaching unit	Determining the characteristics of the teaching unit to be included in the survey	June, 2014
	Development of the survey questions	July, 2014
	Setting the objectives of the teaching unit	July, 2014
	Development of the GeoGebra worksheets	September, 2014
	Teaching unit content development	September, 2014 to July, 2016
Evaluation	Determining and choosing the participants	May, 2016
	Delivering the survey and the teaching unit to the participants	26th of July, 2016
	Collecting data from the participants	August, 2016
	Data analyses (November, 2016)	August to November, 2016
	Corrections on the teaching unit	November, 2016

Figure 7. Timeline of the development process of the inquiry-based teaching unit

CHAPTER 4: RESULTS

Introduction

This chapter comprised the results regarding the research problem of the study and the following research questions:

- What is the final version of technology integrated inquiry based teaching unit on integral calculus, in particular, the definite integral at the end of the development process?
- What are the views of in-service high school mathematics teachers regarding the developed technology-integrated inquiry-based teaching unit on integral calculus, in particular, the definite integral?
- What is the perceived effectiveness of the developed technology-integrated inquiry-based teaching unit on integral calculus by in-service high school mathematics teachers?

The responses to these questions were explored by means of written resources such as the literature review, current curriculum documents, field experts and in-service mathematics teachers' comments on the technology-integrated inquiry-based teaching unit developed by the researcher. Moreover, perceived effectiveness of the developed technology-integrated inquiry-based teaching unit by in-service high school mathematics teachers was investigated through a survey of which analysis is presented in this chapter. The results were presented under the title for each of the relevant research questions.

Research Question 1: What is the final version of technology-integrated inquiry-based teaching unit on integral calculus, in particular, the definite integral at the end of the development process?

The main purpose of this research was to develop a technology-integrated inquiry-based teaching unit on integral calculus, in particular, the definite integral. The development process and content creation was demonstrated in the previous section in detail (see Chapter 3, pp. 22-40). The final draft of the developed teaching unit had the following content shown in Table 3 before it was handed out to the teachers.

Table 3
At a glance: Content in the final draft of developed teaching unit

Content	Number
Pages	47
Words	7316
Figures	16
Objectives	9
Engagement activities	3
Exploration activities	5
Explanation activities	4
Elaboration activities	2
Evaluation activities	1
GeoGebra instructions within the teaching unit	4
GeoGebra worksheets	4
Mathematical expressions and formulas	34
Time period (months)	22

In Table 3, the developed content is summarized by the given number of items within the teaching unit. As shown in the table, the development process of the final draft took approximately 22 months along with a cyclic process. Within this process, the researcher determined the objectives, did an interview with a high school mathematics teacher, wrote a paper for an international conference (ICEMST, 2014) and gathered expert opinions from the audience at the conference, created GeoGebra activities etc. (see Figure 7, p. 40). After all these cyclic process, the final draft of the teaching unit was ready to be handed out to the high school mathematics teachers. The final draft of the teaching unit was sent out to the teachers via e-mail including the survey questions and the directions about giving feedback on the teaching unit (see page 38 for the details). Through making corrections after gathering feedback from high school mathematics teachers, the teaching unit was finalized (see Appendix 3).

Research Question 2: What are the views of in-service high school mathematics teachers regarding the developed technology-integrated inquiry-based teaching unit on integral calculus, in particular, the definite integral?

To investigate the views of in-service high school mathematics teachers regarding the developed technology-integrated inquiry-based teaching unit on integral calculus, the developed teaching unit was sent to the teachers via e-mail and they were requested to provide written feedback on it with the given directions. After collecting the written feedback from the teachers, the researcher conducted a thematic analysis on them. The themes were pre-determined according to the defined characteristics of technology-integrated inquiry-based teaching unit in the scientific literature (Dewey, 1938; Hakverdi-Can & Sönmez, 2012; Kubicek, 2005; Mishra & Koehler, 2006;

Wentworth & Monroe, 2011). Therefore, the terminologies of themes or characteristics were used throughout the presentation of results. In the analysis procedure, any comments of teachers falling under any of the pre-determined characteristics were identified and their frequency was calculated. It should be noted that one statement of any of the teachers might fall under more than one theme, because some of those statements expressed more than one idea (all excerpts of the teachers and their categorization might be seen in Appendix 5). In order to increase the reliability of results, two experts in the field of mathematics education were asked to comment on the emerging themes and codes of the thematic analysis. The emerging themes and their frequency of mention were presented in Table 3.

Table 4

The summative frequency results of the thematic analysis

Themes	Frequency (f)
Appropriate Content (AC)	31
Appropriate Language and Expression (ALE)	28
Practical (P)	23
Inquiry Based (IB)	23
Technologically Accurate (TA)	22
Mathematically Accurate (MA)	12
Total	139

As it can be seen in Table 4, the most frequently emphasized characteristic (as a theme) of the teaching unit provided by the teachers was on *appropriate content* (seen 31 times) followed by *appropriate language and expression* (seen 28 times). The least mentioned characteristic of teaching unit was on *mathematically accurate* (seen 12 times) in the comments of teachers.

Below follows a discussion regarding the nature of feedback received for all six thematic domains listed in table 4. Example feedbacks provided by teachers are listed as an evidence towards the nature of the feedbacks and then an overall summary of the interpretation by the researcher for each of the theme domains was provided.

Appropriate Content (AC)

As the title implies these were comments regarding the content fit of the unit. Obviously the unit should reflect appropriate content. And one more way to ensure the validity and appropriateness of the content was to examine and adapt relevant criticism and feedback given by the teachers. In order to give some idea about the nature of the comments, three examples were provided for each characteristic listed below. In the excerpts, *italic* font was used to show teachers' comments. Following three excerpts are the example feedbacks provided by the teachers with 31 comments.

> "In "Exploration 1" part (page 9), teachers were expected to distribute a handout included regular geometric shapes with given lengths. *A sample handout would be given in the teaching unit.*" (Teacher 6)
>
> "In "Exploration 2" part (page 16), the students were expected to create a rectangle by using the rectangle tool. *More details would be given about how to create a rectangle."* (Teacher 4)
>
> "In "Exploration 2" part (page 15*), information about where the activity was going to be held (in classroom or in computer laboratory) was needed.*" (Teacher 8)

The above indicated examples form the general nature of the comments done for *appropriate content* theme. As it can be seen in the excerpts, simple missing information such as a sample hand out or information about the place where an activity was going to be held mentioned by the teachers. Moreover, some teachers thought that more details were needed at some points. The feedback provided by the teachers were classified as low level feedback by the researcher, because none of them highlighted a major missing content or inappropriate content with the teaching unit. Since the nature of a constructivist inquiry-based classroom involves grouped activities through peer and teacher instruction most of the feedback can easily be done through oral directions. Moreover, the unit was designed independent of an extra equipped classroom that is in order to execute the unit one does not need a computer lab or some other sort of classroom.

Appropriate Language and Expression (ALE)

In order to ensure that the teaching unit had a clear and understandable language and expressions, the researcher used headings, separate sections, different fonts, bullet points etc. For example, each "note" section was written in the same form in a box, or each command was shown by the same bullet points and so on. Following comments were about characteristic of having *appropriate language and expression* in the comments of teachers.

> In "Engagement 2" part on the "materials needed" section (page 14), "Worksheet 1" and "Worksheet 2" was included in the *materials needed* part as students were going to use those worksheets during the activity. *Were students expected to complete "Worksheet 1" and "Worksheet 2" before this activity, because it was written as "needed"?"* (Teacher 4)

"In "Explanation 3" part (page 24), the "formula" term was used. *Instead of using the term "formula", "formulae" could be used, because "formulae" is correct term in mathematics."* (Teacher 7)

"*On "Unit Objectives" part (page 3), the objectives would be determined as measurable objectives. For example, instead of the word "know" in the objective of "students will know the literal word meaning of integral" a more measurable word would be chosen.*" (Teacher 3)

As indicated in the comments of teachers, regarding the theme *appropriate language and expression,* it can be seen that the comments were formed by minor grammar and punctuation errors, teachers' misunderstandings, or their lack of knowledge in English mathematics terminology. For instance in the first example, Teacher 4 misunderstood that the worksheets should have been completed before the related activity when she saw the worksheets were listed in the *materials needed* part. However, those worksheets were needed during the activity. Also in the second example, Teacher 7 had a lack of knowledge in English mathematics terminology saying that "formulae" is the correct term instead of "formula". On the other hand, some minor grammar corrections or missing words were mentioned by the teachers. The rest of the other comments reflect similar concerns. So, overall we were provided with evidence that the language and related expressions were understandable by the mathematics teacher cohort.

Practical (P)

The following excerpts were the ones that the teachers commented on the practical aspect of the teaching unit. In order to ensure that this teaching unit was practical, the researcher designed the teaching unit in an easy to use flow and provided factual knowledge aiming that students would achieve conceptual understanding. Also, the researcher tried to ensure that this teaching unit would be used both by teachers and students without any additional information. And as the last step to ensure that this teaching unit was practical, the teachers' relevant feedback was analyzed by the researcher. Three examples for those comments were as follows:

> *"In the "Exploration 2" part (page 15), distributing both GeoGebra worksheet and the image of the inherited area on Google maps may cause confusion."* (Teacher 1)
>
> *"At the end of the teaching unit a references part was needed. So, the teacher could utilize from the resources which were utilized in this teaching unit."* (Teacher 3)
>
> "*For the "Evaluation" part (page 35), a more detailed rubric could be used in this part."* (Teacher 7)

As can be seen in the examples, the comments on the practical aspect of the teaching unit were formed by simple suggestions like adding a more detailed rubric, or a adding a references section. Also, some possible confusions were mentioned by the teachers. Since Teacher 4's comment indicated above were not supported by other teachers, the rubric used in the teaching unit was thought sufficient by the researcher. Therefore the comments were considered as minor advices for the researcher in order for further development of the unit regarding its being applicable.

Inquiry Based (IB)

As one of the main purpose to conduct this study was to develop an *inquiry-based* teaching unit, the researcher developed inquiry activities within the teaching unit so that students could learn from their own experiences. In order to ensure the validity of the teaching unit regarding its being inquiry based, relevant comments were analyzed. The following excerpts were the comments on the inquiry-based aspect of the teaching unit.

> "How to find the volume of a glass like in Figure 6 by using integration on "GeoGebra" would be asked as an inquiry question at the end of the teaching unit." (Teacher 4)

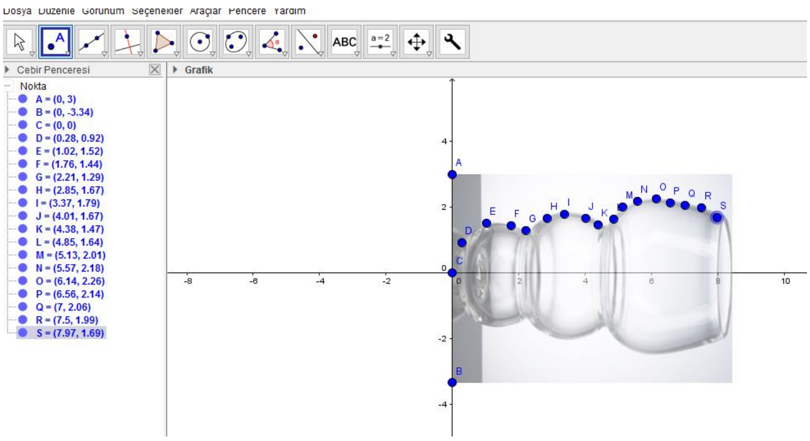


Figure 8*:* A glass to be calculated its volume by using Integration on GeoGebra suggested by Teacher 4

> "*In "Engagement 3" part (page 29), giving rationale to the students about the reason why we need to calculate the volume of a glass could be helpful to attract them. Accordingly a story or a scenario could be written in this part."* (Teacher 6)

> *"In "Exploration 2" part (page 17), the question "could you make a better estimation?" was a yes/no question, so instead of that, questions such as "What do you observe when you increase the number of rectangles? With how many rectangles would you make the best estimation" would be asked."* (Teacher 7)

Also, in these voices of the teachers, small advices to contribute the inquiry-based aspect of the teaching unit were given by the teachers. With this regard, additional examples to inquiry questions were provided by the teachers. These comments were viewed by the researcher as approvals to the given inquiry activities and some suggestions reflecting teachers' point of view on inquiry concept.

Technologically Accurate (TA)

The characteristic of being *technologically accurate* was in the fifth row among six themes in terms of its frequency in the comments of teacher on the teaching unit. As a main purpose the teaching unit was developed as a technology-integrated teaching unit. Accordingly, GeoGebra activities along with the inquiry activities were created by the researcher. Those activities were explained in a step by step order in the GeoGebra worksheets and GeoGebra instructions within the teaching unit so that both students and teachers could do those activities by following the given steps. Therefore, the teaching unit was aimed to reflect the *technologically accurate* aspect. Regarding the theme of *technologically accurate*, the following comments had been articulated by the teachers:

> *"In "Worksheet 3", (page 42, item 6) information about how to write the symbol π on GeoGebra was missed"* (Teacher 1)
>
> *"In "GeoGebra Worksheet 1" for item 4 (page 37), writing "click on a point on the graphics view-" caused a confusion, because there were not a point to be clicking on. So it would only be written as "click on graphics view-'"*"(Teacher 4)
>
> *"For the GeoGebra Instructions section on page 18, the Poly command did not work with the Turkish version, so the teachers and students must check that the language is in English."* (Teacher 6)

The comments of the teachers on technologically accurate theme were analyzed by the researcher with the assumption that all the teachers had done the GeoGebra activities. As it can be seen in above comments, the nature of the comments is derived from the problems in using previous GeoGebra versions, differentiation in command names in different languages, and the lack of technology literacy of the teachers. Therefore, it can be interpreted that the teaching unit was well developed regarding its being technologically accurate, and the problems indicated in the teacher comments were not major problems that assert a contrary to the technologically accurate aspect of the teaching unit.

Mathematically Accurate (MA)

Lastly, example comments on the least frequently mentioned characteristic of teaching unit-*mathematically accurate* were listed below. Developing a teaching unit in the field of mathematics education, mathematically accuracy was an essential aspect. Therefore the researcher paid attention to use correct mathematical expressions and formulas within the teaching unit. In order to ensure the validity in this aspect the relevant comments were analyzed and the following comments were listed as examples.

> *"Possible mathematical misconceptions would be more integrated in the teaching unit as tips for teachers."* (Teacher 3)
>
> *"In "Exploration 2" part (page 17), a reference to the limits concept would be given when students were expected to estimate the area by using rectangles."* (Teacher 5)
>
> *"In "Elaboration 1" part (page 28), the teaching unit gave the integration concept relating it to area concept, so while students were solving the exercise questions students could fall into a misconception thinking that a result of an integration cannot be negative. So an explanation would be made at this point."* (Teacher 7)

Viewing all the comments on mathematically accurate aspect, it can be seen that small suggestions such as integrating possible misconceptions, giving links to previously learned topics etc. However, some of those comments were found as out of content. The teaching unit was developed with the assumption that students had learnt previous concepts such as limits, so making links to the previously learnt concepts could cause confusions. After all, with the least number of comments, it can be said that the final draft of the developed teaching unit had a high degree of accuracy as far as mathematical correctness was concerned. This is also one more evidence that the prior work leading to the final draft was well executed.

Overall summary

Considering the final draft of the teaching unit that the teachers commented on (Table 3), the total number of comments falling under the characteristics (139 comments) shown in Table 4 could be regarded as minor suggestions to make corrections on the final draft of the teaching unit. As a result, it can be interpreted as the final draft of the developed teaching unit was almost finalized with the previous development process before it was sent out to the teachers.

As shown in Table 4, the least commented characteristic was “Mathematically Accurate” with 12 comments. So, it may be said that the least corrections was needed on the mathematical expressions in the teaching unit. On the other hand, the most commented characteristic was “Appropriate Content” with 31 comments. However, when we look at the number of comments of each teacher, it was seen that some outliers contributed to those numbers. Therefore, a more detailed analysis was needed to clarify the reasons behind the number of the comments.

Detailed analysis on the teacher comments for each characteristic of the teaching unit

In order to have a clear understanding of teachers' comments on the teaching unit, the themes and their frequency of mention were also identified for each teacher. The results were displayed in Table 5.

Table 5
Number of written feedback categorized by the characteristics of the teaching unit

	Appropriate Content (AC)	Appropriate Language and Expression (ALE)	Practical (P)	Inquiry Based (IB)	Technologically Accurate (TA)	Mathematically Accurate (MA)	Total
Teacher 1	2	3	2	2	7	2	18
Teacher 2	1	0	2	2	1	0	6
Teacher 3	11	4	7	9	0	4	35
Teacher 4	2	11	3	3	5	1	25
Teacher 5	2	1	0	1	1	2	7
Teacher 6	4	4	3	2	7	0	20
Teacher 7	4	1	4	3	1	3	16
Teacher 8	5	4	2	1	0	0	12
Total	31	28	23	23	22	12	139

As can be seen from table 5, the teachers usually provided around 3 comments for each category. Also another noteworthy point is that TA and MA categories were the ones that had fewer comments with respect to the other ones. Also in Table 5, the individual teachers were highlighted different characteristics of the teaching unit. For instance, Teacher 3 emphasized the characteristics of being *appropriate content* (11 times), while Teacher 4 mentioned about the unit's having *appropriate language and expression* (11 times) more than others. Further, some teachers had no comment for some characteristic such as Teacher 8 who had no mention of characteristics of *technologically accurate* and *mathematically accurate* in their comments on the teaching unit. Table 6 is a representation of the above data in an attempt to show the diversity and discuss sources of possible outliers.

Table 6
Descriptive statistics for teachers' comments on each characteristic

	N	Maximum	Minimum	Mean	Median	Mode	Standard Deviation
Appropriate Content (AC)	8	11	1	3.88	3	2	2,97
Appropriate Language and Expression (ALE)	8	11	0	3.50	3.5	1, 4	3.20
Practical (P)	8	7	0	2.88	2.5	2	3.61
Inquiry Based (IB)	8	9	1	2.88	2	2	2.42
Technologically Accurate (TA)	8	7	0	2.75	1	1	2.86
Mathematically Accurate (MA)	8	4	0	1.5	1.5	0	1.41

In addition, as shown in Table 6, the data show positive skewness for all the characteristics. The reasons for the skewness in the data could be explained by the background information of the teachers or the nature of the comments of the teachers. For example, it is clearly seen in Table 5 and Table 6 that the positive skewness for the *appropriate content*, *practical* and *inquiry based* characteristics was derived from the number of Teacher 3's comments with the number of 11, 7 and 9 respectively while the mean scores of those characteristics were 3.88, 2.88 and 2.88 respectively. So, Teacher 3 could be entitled as an outlier for those characteristics. When followed up the reason behind why teacher 3 made more comments on those characteristics than other teachers, it was seen that the nature of Teacher 3's most of the comments was approvals to the teaching unit, instead of suggestions for further development. For example, 4 comments out of 11 comments for *appropriate content* characteristic, 4 comments out of 7 comments for *practical* characteristic and 7 comments out of 9 comments were approvals while the other comments were suggestions to the researcher do make corrections on the teaching unit. It should be noted that some of those comments may fall under more than one characteristic. Following excerpts are the examples to those comments for *appropriate content, practical* and *inquiry based* characteristics respectively.

> "*The "note" parts of the teaching unit were appealing and they were really such as to a guide for teachers*."

> "*In "Exploration 4" part (page 26), making students to explore upper and lower sums by a GeoGebra applet which gives both geometrical and mathematical representation of upper and lower sum formula was a well-thought-of activity.*"

"In "Engagement 1" part (page 6), I definitely liked the exercise that the integral concept was associated with its literal word meaning. I had used this kind of activities in my lessons about polynomials and logarithm and I experienced it really draws students' attention. Also, referring the history of the integral concept is very important."

As a result, these comments are incompatible to the nature of the comments in general. That is, while the nature of the comments were formed by minor suggestions or corrections, these comments refer approvals to the activities. However, this result is another evidence that the final draft of the teaching unit was well developed through the protocol of the development process before which was reported in Chapter 3.

Teacher 3's another comment on *appropriate content* characteristic can be explained by her background knowledge that is Teacher 3 has a research interest in teaching to students with visual disabilities. The comment is;

"From beginning to this point ("Engagement 3" part (page 29)), of teaching unit, I started to think that if this teaching unit appeal to visually disabled learners, and could it be balanced by integrating activities appealing to other type of learners as well."

This comment could also be ignored as the developed teaching unit has not such aim.

For the *appropriate language and expression* characteristic, it could be viewed that the positive skewness was derived from the number of Teacher 4's comments with 11 comments while the mean score of the responses was 3.50. (Table 6). This difference shows that teacher 4 could be entitled as an outlier for the *appropriate* *language and expression* characteristic. The reason for the majority of Teacher 4's number of comments could be explained by her background knowledge that is Teacher 4's level of English grammar knowledge was better than other teachers. So, she focused on the grammar mistakes in the teaching unit more than other teachers. Following is an example to the Teacher 4' comments involving grammar mistakes.

> *"In the "note" section on page 26, instead of writing "students must have already learned the limits topic", it would be written as "It is assumed that students know the limits topic"".*
>
> *In "GeoGebra Instructions" section on page 21, the instructions would be given in more detail. For example, it was written that* "For upper sum, write *UpperSum* in the input bar and choose UpperSum[<Function>, <Start x-Value>, <End x-Value>, <Number of Rectangles>]" *This instruction would be rewritten as* "For upper sum, write UpperSum in the input bar so that alternative commands starting by UpperSum reveals in the input bar. Choose UpperSum[<Function>, <Start x-Value>, <End x-Value>, <Number of Rectangles>] from those alternative commands.
>
> *In "GeoGebra Worksheet 1" for item 4 (page 37), writing "click on a point on the graphics view-" caused a confusion, because there were not a point to be clicking on. So it would only be written as "click on graphics view-""*

So, the reason why Teacher 4 made the maximum number of comments on *appropriate language and expression* characteristic could be her background knowledge in English grammar.

Moreover, as shown in Table 5, the most contribution to the characteristic was made by Teacher 1, Teacher 4 and Teacher 6 with the highest number of comments 7, 5 and 7 respectively while the mean score of this characteristic was 2,75. This difference may also occurred because of the teachers' background knowledge in technology literacy.

In conclusion, with the help of thematic analysis, the characteristics that were mentioned by the high school mathematics teachers regarding the developed technology-integrated inquiry-based teaching unit on integral calculus were analyzed. Their frequency of mention for each of the pre-determined characteristic and descriptive statistics for teachers' comments on each characteristic was presented as a result of thematic analysis.

Research Question 3: What is the perceived effectiveness of the developed technology-integrated inquiry-based teaching unit on integral calculus by in-service high school mathematics teachers?

In order to explore the perceived effectiveness of developed technology-integrated inquiry-based teaching unit, a survey including 65 items was sent to teachers via Google Forms®. The items of the survey were generated under the light of the related studies in the literature aiming to investigate the perception of teachers regarding the unit. The survey was divided into five sub-categories each of which represents one of the characteristics of the inquiry-based teaching unit. That division also enabled the researcher to find out the perceived effectiveness of the unit in terms of each determined characteristic. The number of items in each sub category of the survey can be seen in Table 1 (Chapter 3, p. 33). The responses of teachers on this survey were analyzed through descriptive statistics. First of all, results of descriptive statistics operated on teacher responses to the items of the survey are shown (Table 7).

Table 7
Descriptive statistics for teachers' comments on the teaching unit

	# of Teachers N	Min.	Max.	Mean	Median	Std. Deviation
Perceived Effectiveness of the Unit	8	2.92	3.80	3.60	3,72	0.30

As shown in Table 7, the mean score of teachers' responses for the entire teaching unit is 3.60 which could be regarded as a relatively high score because the maximum score was 4.00. Therefore, it can be concluded that teachers' perception on the developed technology-integrated inquiry-based unit was mostly effective. The minimum score given by the participants for the teaching unit was 2.92, while the maximum score was 3.80. The standard deviation of the scores was 0.30.

Following the analysis of overall scores of the teachers for the teaching unit, their responses to the sub-categories of the survey were also analyzed through descriptive statistics. The results of the descriptive analysis conducted for 5 sub-categories of the survey were displayed in Table 8.

Table 8
Descriptive statistics for teachers' comments on the sub-categories

	N	Minimum	Maximum	Mean	Median	Std. Deviation
Appropriate Language and Expression	8	3.00	3.83	3.55	3.58	0.25
Appropriate Content/Practical	8	3.00	3.83	3.57	3.69	0.30
Mathematically Accurate	8	3.07	3.93	3.63	3.73	0.30
Technologically Accurate	8	2.83	4.00	3.60	3.83	0.45
Inquiry Based	8	2.57	4.00	3.64	3.78	0.45

As shown in Table 8, the sub-category of *inquiry-based* had the highest mean score of 3.64 (SD: 0.45) out of 4.00. Thus, it can be concluded that teachers found the inquiry-based part of the unit most effective. The *inquiry-based* sub-category was followed by categories of accuracy; namely, *mathematically accurate* ($\overline{X}$: 3.63, SD: 0.30); and *technologically accurate* ($\overline{X}$: 3.60, SD: 0.45). The sub-category of *appropriate content/practical* was in the fourth row among five sub-categories ($\overline{X}$: 3.57, SD: 0.30) followed by the least effectively found sub-category of *appropriate language and expression* ($\overline{X}$: 3.55; S: 0.25).

In conclusion, participating teachers perceived the technology-integrated inquiry-based teaching unit effectively in general as can be seen in the homogeneous scores. However, when further investigated with the small differences between the sub-categories, it was found out that they perceived the inquiry-based characteristic of the unit more effective than others. Further discussion regarding the results will be covered in the next chapter.

CHAPTER 5: DISCUSSION

Introduction

This study addressed the following research questions:

- What is the final version of technology integrated inquiry based teaching unit on integral calculus, in particular, the definite integral at the end of the development process?
- What are the views of in-service high school mathematics teachers regarding the developed technology-integrated inquiry-based teaching unit on integral calculus, in particular, the definite integral?
- What is the perceived effectiveness of the developed technology-integrated inquiry-based teaching unit on integral calculus by in-service high school mathematics teachers?

In this chapter of the study, major findings regarding the responses to research questions are reported under the light of previously conducted research. Moreover, pedagogical implications, limitations and suggestions for future research will be covered as well in this chapter.

Major findings

The idea of conducting this study originated in 2014 because of the gap in developed technology-integrated inquiry-based mathematics resources at high school level as well as the FATİH initiative (2012) by the Ministry of National Education of Turkey. By recent improvements in education with the help of technology integration, the Turkish Ministry of National Education (MoNE, 2013) mandated teachers to integrate technology into the high school mathematics lessons, and

MoNE incorporated some statements regarding integrating technology into the high school mathematics curriculum. Moreover, ever since Dewey (1938) expressed that students learn mostly through their own experiences by making inquiries, many researchers have come to a consensus that inquiry-based teaching enhances students' learning in many concepts, including mathematics (Chapman, 2011; Hakverdi-Can & Sönmez, 2012; Engeln, Euler, & Maass, 2013; Hahkiöniemi, 2013).

Therefore, in this particular study, it was intended to develop a relatively advanced teaching unit in mathematics at the high school level that would involve both technology-integrated and inquiry-based aspects. The developed unit at the end of this study would be considered as an example covering the needs of technology-integrated and inquiry-based instruction mentioned by the MoNE. As a method for development of the unit, methodology informed through instructional design was implemented. In the development process of the teaching unit, the general aspects of instructional design models, which are being *empirical, iterative*, and *self-correcting* aspects (Reiser & Dempsey, 2007) were taken into account. The content of the teaching unit was determined as "definite integral." Thus the empirical aspect of the developing teaching unit was provided by the factual information regarding "definite integral." As meaningful learning would be provided when the subject is taught with the help of a computer algebra system (Wurnig, 2009) and in an inquiry based context (Attorps, Björk, & Radic, 2013), the developing teaching unit on "definite integral" was decided to incorporate GeoGebra in an inquiry based context. After the need analysis and development of the final draft of the teaching unit within a cyclic process were completed, the iterative aspect was enabled through the feedback gathered from the high school mathematics teachers and the analysis of their written feedback. First, in this phase, a survey was developed covering items related to the characteristics of the technology integrated inquiry based teaching unit. These characteristics were determined under the light of the previous literature. After making revisions in the related literature, the characteristics of the developing unit were identified as having *appropriate language and expression;* having *appropriate content;* being *practical;* being *mathematically accurate;* being *technologically accurate;* and being *inquiry based* (Hakverdi-Can & Sönmez, 2012; Kubicek, 2005; Mishra, & Koehler, 2006; Wentworth, & Monroe, 2011).

With the help of this survey, the perceived effectiveness of the developing teaching unit, another research question of the study, was tried to be found out. As a result, participating teachers of mathematics generally perceived the unit effective. When further analyzed, it was found out that teachers mostly perceived the *inquiry based* characteristic of the unit effective. This result indicated that final draft of teaching unit was well developed before it was sent out to teachers regarding the aspect of being inquiry based. Although all of the characteristics were perceived as effective with close mean scores, the least effective perceived characteristics of the unit were identified as having *appropriate language and expression* and *appropriate content/practical* compared to other characteristics. This result may suggest that these characteristics of the teaching unit were needed to be corrected as indicated by the *self-correct* aspect of the instructional design models. Considering the written feedback of teachers on the final draft of the teaching unit, corrections on the final draft were done and the teaching unit was finalized.

In order to have a deeper insight into the corrected points of the developing teaching unit and triangulate the quantitative data (Creswell, 2009), qualitative data was collected through teachers' written feedback on the unit. The data were analyzed by means of thematic analysis in which the themes were pre-determined as the characteristics of technology-integrated inquiry based teaching unit.

In the thematic analysis of teachers' written feedback on the developing teaching unit, the individual differences across teachers' comments were observed. That is, while one teacher emphasized one characteristic more than others, another one highlighted a different characteristic more than others. The variation might stem from the different reasons such as prior experiences as learners, contextual factors, expertise and so on (Kagan 1992; Borg 2003). In order to clarify the variation in the number of comments of teachers, further analyses were done considering the nature of the comments of each teacher and background information of the teachers. As a result, while some of those variance were explained through teachers' nature of comments, some of them were explained through teachers' background knowledge. For example, while teachers usually provided around three comments for each characteristic, Teacher 3 provided 11, 7 and 9 comments respectively to *appropriate content, practical* and *inquiry based* characteristics. When the nature of Teacher 3's comments were analyzed, it was found that Teacher 3 mostly made approvals for the activities in the teaching unit instead of providing suggestions which form the nature of the comments in general. Therefore, these analyses strengthened the fact that the final draft of the teaching unit was achieved almost its final version through the cyclic development process explained in Chapter 3.

At the end of the systematic and reflective development process of the technology integrated inquiry based unit, the primary focus of the study was achieved (Smith & Ragan, 1999). That is, according to the needs of the context, technology-integrated inquiry-based teaching unit was developed after it had been corrected and revised under the light of comments of the teachers in the field. Although the process was cyclical in nature which signals the flexibility of the unit in process, a teaching unit on definite integral which is technology-integrated and inquiry-based was ready for implementation in practice.

Implications for practice

The most significant pedagogical implication of the study is its suggestion for practitioners to implement the developed technology integrated inquiry based unit on the definite integral. This teaching unit could be incorporated by high school mathematics teachers in 12th-grade classes in Turkey. Moreover, this unit might be considered to resolve the identified lack of instructional units on teaching the definite integral concept with the help of a dynamic mathematics software in an inquiry based manner. Furthermore, both practitioners and researchers who identified a need of technology-integrated inquiry-based unit on different topics can follow the steps implemented in this study. As the followed steps were described in detail throughout the study, it would be possible for practitioners/researchers to produce new technology-integrated inquiry-based instructional materials on the contents they feel the need. Also, the developed technology integrated inquiry based unit might raise awareness of mathematics course book writers regarding the development and inclusion of more technology-integrated inquiry-based units on different topics.

Moreover, preservice high school mathematics teachers could utilize from this study by means of using GeoGebra in an inquiry based context. In addition, they could develop new inquiry-based technology-integrated materials for their teaching.

Limitations

This study is not without some limitations. The first limitation of this study was its not having any evaluation about its actual use in practice. As participating teachers did not have enough time to implement the unit in their classes, it could not be possible to receive their reflections on the implementation of the unit. Therefore further research is recommended on an evaluation of the unit based on the reflection after action.

Moreover, the views of the 12th-grade students could not be gathered as the unit was not implemented in classes. As students are the main agent of the teaching/learning process, their comments were important. Thus, further development of the unit based on the views of the students is suggested.

Another limitation of the study was teachers' lack of knowledge in technology literacy. Unfortunately, teachers in Turkey do not effectively use mathematical technology especially Dynamic Mathematics Systems such as GeoGebra. As the developed teaching unit requires the use of GeoGebra effectively, teachers' background knowledge on the dynamic mathematics software is essential.

Another limitation of this study was a limited number of participants. As convenient sampling was used as a method sampling, only eight teachers were volunteered to comment on the developing unit. As the development process of an instructional unit is a cyclical process, the feedback from more teachers is expected to further revise the unit.

REFERENCES

Attorps, I., Björk, K., & Radic, M. (2013). Varied ways to teach the definite integral concept. *International Electronic Journal of Mathematics Education, 8*(2-3), 81-99.

Baki, A. (2000). Preparing student teachers to use computers in mathematics classrooms through a long-term pre-service course in Turkey. *Journal of Information Technology for Teacher Education, 9*(3), 343-362.

Barrow, L. H. (2006). A brief history of inquiry: From Dewey to standards. *Journal of Science Teacher Education*, *17*(3), 265–278. doi:10.1007/s10972-006-9008-5

Berkey, D. D. & Blanchard, P. (1992). *Calculus.* Saunders College Pub.

Boyer, C. B. (1949). *The history of the calculus and its conceptual development: The concepts of the calculus*. New York, NY: Courier Dover Publications.

Borg, S. (2003). Teacher cognition in language teaching: A review of research on what language teachers think, know, believe, and do. *Language Teaching, 36*(2), 81-109.

Brahier, D.,J. (2000) Teaching secondary and middle school mathematics. *Principles of mathematics education*. New York: Pearson.

Bybee, R. W., Taylor, J. A., Gardner, A., Van Scotter, P., Powell, J. C., Westbrook, A., & Landes, N. (2006). The BSCS 5E instructional model: Origins and effectiveness. *Colorado Springs, CO: BSCS, 5*, 88-98.

Chapman, O. (2011). Elementary school teachers' growth in inquiry-based teaching of mathematics. *ZDM*, *43*(6), 951–963. doi:10.1007/s11858-011-0360-3

Creswell, J. W. (2009). *Research design qualitative, quantitative and mixed-method approaches*. California, USA: Sage Publications.

Crowl, T. K. (1996). *Fundamentals of educational research* (2nd ed.).Madison, WI: Brown & Benchmark.

Demiraslan, Y., & Usluel Y. K. (2008). ICT integration processes in Turkish schools: Using activity theory to study issues and contradictions. *Australasian Journal of Educational Technology, 24*(4), 458-474.

Design-Based Research Collective. (2003). Design-based research: An emerging paradigm for educational inquiry. *Educational Researcher*, *32*(1), 5–8.

Dewey, J. (1938). *Experience and education*. New York, NY: Collier Books.

Dick, W. (1993). Enhanced ISD: A response to changing environments for learning and performance. *Educational Technology*, *33*(2), 12-16.

Dorier, J. L., & Garcia, F. J. (2013). Challenges and opportunities for the implementation of inquiry-based learning in day-to-day teaching, *ZDM—The International Journal on Mathematics Education*, *45*(6), 837–849. doi:10.1007/s11858-013-0512-8

Engeln, K., Euler, M., & Maass, K. (2013). Inquiry-based learning in mathematics and science: A comparative baseline study of teachers' beliefs and practices across 12 European countries. *ZDM*, *45*(6), 823–836. doi:10.1007/s11858-013-0507-5

Gutiérrez, A. (1996). Visualization in 3-dimensional geometry: In search of a framework. *Proceedings of the 20th Conference of the International Group for the Psychology of Mathematics Education, 1,* 1-19.

Hähkiöniemi, M. (2013). Teacher's reflections on experimenting with technology enriched inquiry-based mathematics teaching with a preplanned teaching unit. *The Journal of Mathematical Behavior, 32*(3), 295–308. doi:10.1016/j.jmathb.2013.03.007

Hakverdi-Can, M., & Sönmez, D. (2012). Learning how to design a technology supported inquiry-based learning environment, *Science Education International*, *23*(4), 338–352.

Healy, L., & Hoyles, C. (2001). Software tools for geometrical problem solving: potentials and pitfalls. *International Journal of Computers for Mathematical Learning*, *6*(3), 235–256.

History of Mathematics. (2016). Retrieved from https://www.math.uh.edu/~shanyuji/History/

Hohenwarter, M., Hohenwarter, J., Kreis, Y., & Lavicza, Z. (2008). Teaching and calculus with free dynamic mathematics software GeoGebra. *In 11th International Congress on Mathematical Education.*

Hohenwarter, M., & Lavicza, Z. (2007). Mathematics teacher development with ICT: Towards an international geogebra institute. *Proceedings of the British Society for Research into Learning Mathematics, 27*(3), 49-54.

Kagan, D. M. (1992). Professional growth among pre-service and beginning teachers. *Review of Educational Research, 62*(2). 129−169.

Kember, D., & Murphy, D. (1995). The impact of student learning research and the nature of design on ID fundamentals. *Instructional design fundamentals: A reconsideration* (pp. 99-112).

Kubicek, J. P. (2005). Inquiry-based learning, the nature of science and computer technology: New possibilities in science education. *Canadian Journal of Learning and Technology, 31*(1). Retrieved from http://cjlt.csj.ualberta.ca/index.php/cjlt/article/view/149/142.

Liu, T. C., Peng, H., Wu, W. H., & Lin, M. S. (2009). The effects of mobile natural science learning based on the 5E learning cycle: A case study. *Educational Technology & Society, 12*(4), 344-358.

Ministry of National Education. (2012). *Fatih Project.* Retrieved from http://fatihprojesi.meb.gov.tr/tr/index.php

Ministry of National Education. (2013). *Ortaöğretim matematik dersi 9-12. Sınıflar öğretim programı* [Secondary schools mathematics education program for grades 9-12]. Ankara, Turkey: Author.

Mishra, P., & Koehler, M. J. (2006). Technological pedagogical content knowledge: A framework for integrating technology in teachers' knowledge. *Teachers College Record, 108*(6), 1017–1054.

Mofolo-Mbokane, B. L.K. (2011). *Learning difficulties involving volumes of solids of revolution: A comparative study of engineering students at two colleges of further education and training in South Africa* (Doctoral dissertation). University of Pretoria, South Africa.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: National Council of Teachers of Mathematics.

O'Connor, J. J., & Robertson, E. F. (1996). *A history of the calculus.* Retrieved from http://www-history.mcs.st-and.ac.uk/HistTopics/The_rise_of_calculus.html

Orton, A. (1983). Student's understanding of integration. *Educational Studies in Mathematics, 14*(1), 1-18.

Özdemir, Ç. & Çataloğlu, E. (2014). The development of an inquiry-based learning unit for integral calculus: The case of volumes of solids of revolution. Proceedings of ICEMST *on Education in Mathematics, Science and Technology,* 241-247.

Reigeluth, C.M. & Frick, T.W. (1999). Formative research: A methodology for creating and improving design theories. In *C.M. Reigeluth (Ed.), Instructional-design theories and models – A new paradigm of instructional theory* (pp. 633-652). New Jersey: Lawrence Erlbaum.

Richey, R. C., Klein, J. D., & Nelson, W. A. (2004). Developmental research: Studies of instructional design and development. *Handbook of research for educational communications and technology*, *16*(2), 1099-1130.

Roytek, M. A. (2010). Enhancing instructional design efficiency: *Methodologies employed by instructional designers. British Journal of Educational Technology, 41*(2), 170-180.

Salleh, T. S. A., & Zakaria, E. (2012). The effects of integrating technology on students' conceptual and procedural understandings in integral calculus. *Canadian Center of Science and Education, 8*(16), 8-16.

Schwartz, D. L., Lin, X., Brophy, S., & Bransford, J. D. (1999). Toward the development of flexibility adaptive instructional designs. In C. M. Reigeluth (Ed.), *Instructional-design theories and models* (Vol. II, pp. 183–213). Mahwah, NJ: Lawrence Erlbaum.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4–14.

Smith, P. L., & Ragan, T. J. (1999). *Instructional design*. New York: Wiley.

Van den Akker, J. (1999). Principles and methods of development research.

In J. van den Akker, N. Nieveen, R. M. Branch, K. L. Gustafson & T. Plomp (Eds.), *Design approaches and tools in education and training* (pp. 1-14). Springer Netherlands.

Wang, F., & Hannafin, M. J. (2005). Design-based research and technology enhanced learning environments. *Educational technology research and development*, *53*(4), 5-23.

Wentworth, N., & Monroe, E. E. (2011). Inquiry-based lessons that integrate technology: Their development and evaluation in elementary mathematics teacher education. *Computers in the Schools*, *28*(4), 263–277. doi:10.1080/07380569.2011.620938

Wolfram Mathematica online integrator (n. d.). *History of integration.* Retrieved from http://integrals.wolfram.com/about/history/

Wurnig, O. (2009). Solids of revolution - from the Integration of a given function to the modeling of a problem with the help of CAS and GeoGebra. *Proceedings of the Tenth International Conference Models in Developing Mathematics Education.* HTW Dresden.

APPENDICES

Appendix 1: Interview with an experienced high school mathematics teacher who was experienced in teaching integral calculus topic

Soru 1: İntegral konusunu nasıl anlatıyorsunuz?

Cevap 1: Bu konuları hızlı bir şekilde bitirip alıştırma yapmaya daha çok zaman ayırıyoruz. Belirsiz integralden başlıyoruz. İntegralin türevin tersi olduğunu anlatıp sıralı bir şekilde integral alma kurallarını veriyoruz.

Soru2: İntegralin tanımından, alan ve hacim ile ilişkisinden bahsediyor musunuz?

Cevap 2: Bundan en son belirli integral konusuna geldiğimizde bahsediyoruz. Aslında en başında bahsetmemiz daha iyi olabilir ama çocukların kafası karışır diye bu şekilde ilerliyoruz.

Soru 3: Peki integralin nasıl hesaplanacağını bilmediği halde teknoloji (GeoGebra) kullanarak alan ve hacim hesaplaması ile derse başlansa daha iyi bir başlangıç olur mu?

Cevap 3: Evet, zaten böyle başlayamamamızın tek nedeni çocukların integral hesaplamayı bilmemesi. İntegrali otomatik olarak hesaplayan bir programla belirli integralden başlamak çok daha anlamlı ve güzel olur. Bir de integralden önceki konu türev olduğu için integrali türevin tersi olduğunu anlatıp çocuklar türev kurallarını unutmadan integral alma kurallarını veriyorum.

Bir öğrencim Ali Nesin'in Riemann İntegrali ile başladığı bir integral dersini örnek vererek "böyle anlatsanız daha iyi olmaz mı" demişti. Ama tabi bizim Ali Nesin Hoca'nın yaptığı gibi tahtaya tek tek dikdörtgen çizecek vaktimiz olmuyor ve bence çocukların kafaları işin içinde limit olduğu için karışabiliyor. Ayrıca integral konusu çocukların YGS'ye hazırlandıkları ve girdikleri zamana denk geliyor ve çocuklar LYS'ye (integral konusunun çıkacağı sınav) odaklanamıyor. Dolayısıyla integral konusuna motive olamıyorlar.

Appendix 2: Example screen shots showing the participants' work on GeoGebra

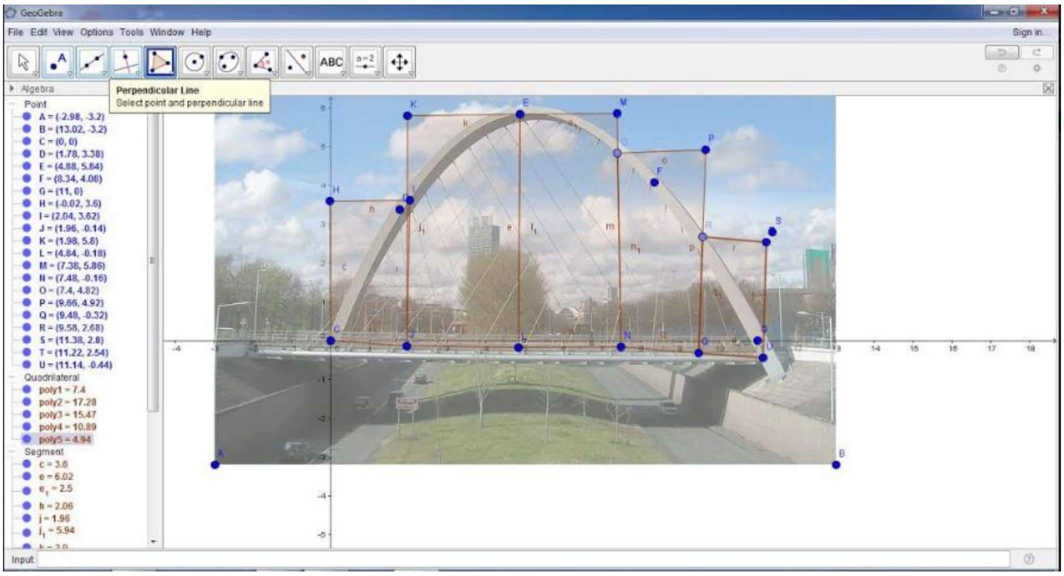


Figure 9. Participant 1's work on GeoGebra calculating Riemann sums on an image

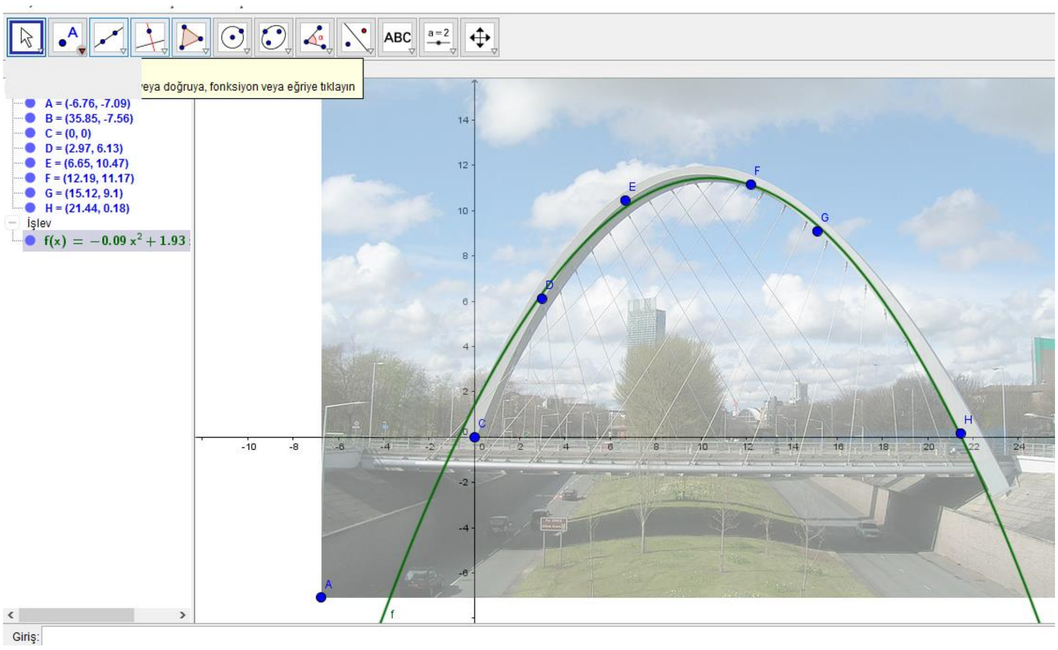


Figure 10. Participant 4's work on GeoGebra finding the curve of best fit of the points

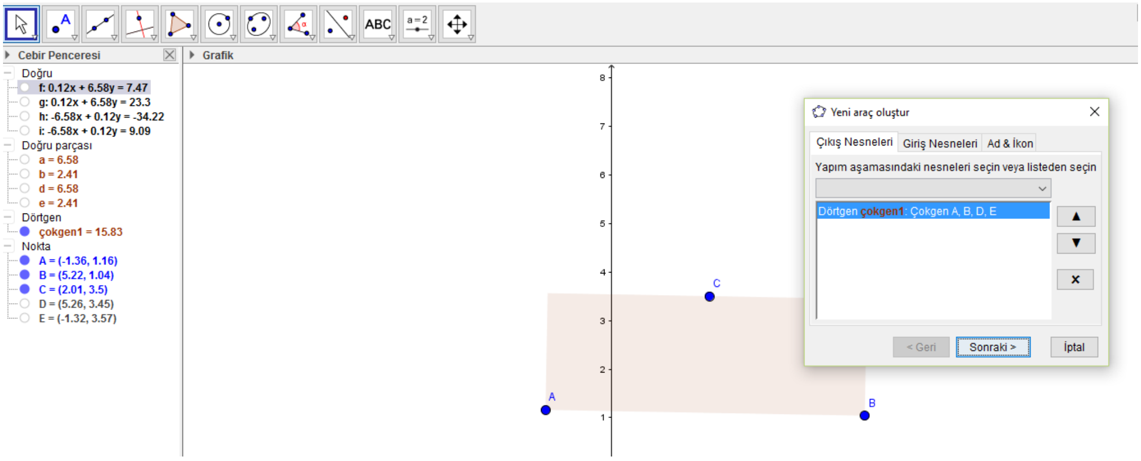


Figure 11. Participant 4's work on GeoGebra creating the "recrangle tool"

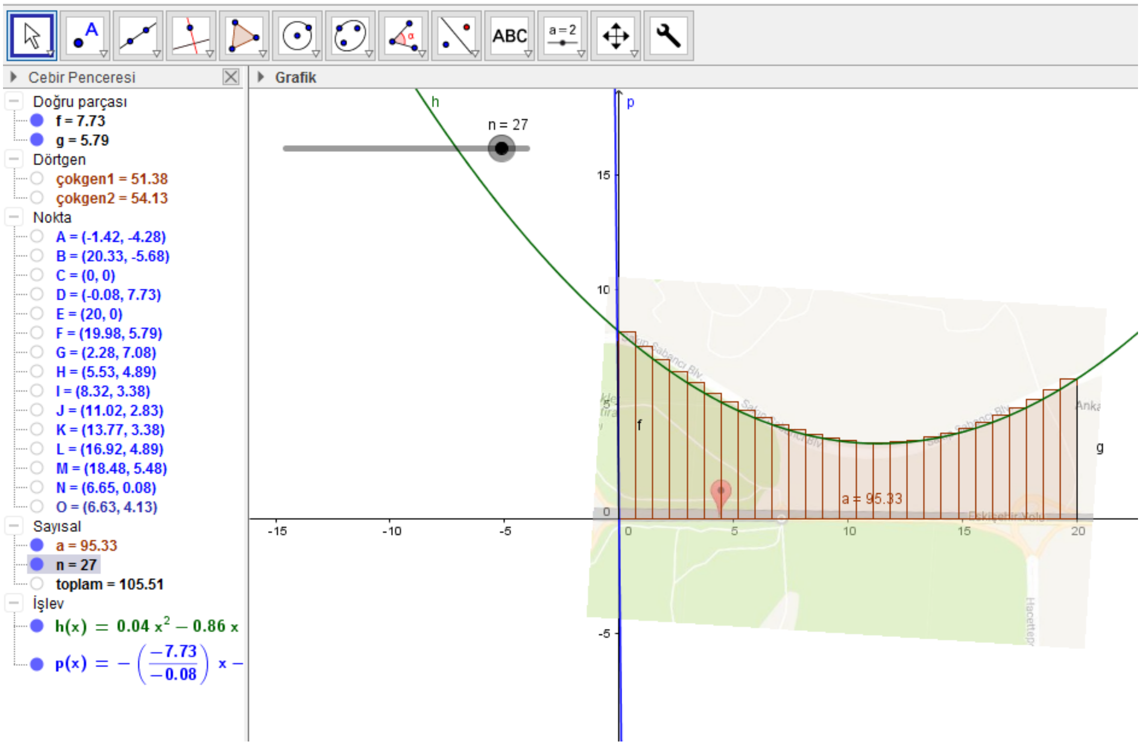


Figure 12. Participant 4's work on GeoGebra estimating the area by the upper sum command

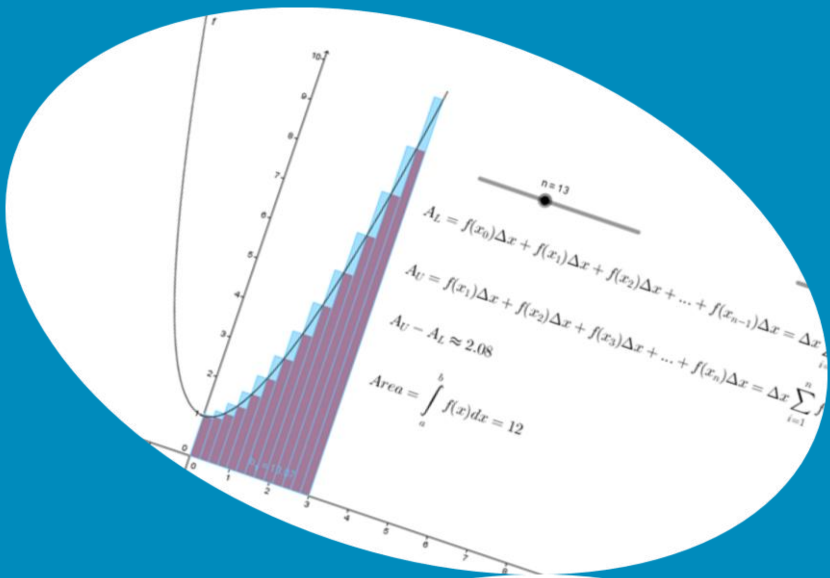
Appendix 3: Inquiry-based teaching unit

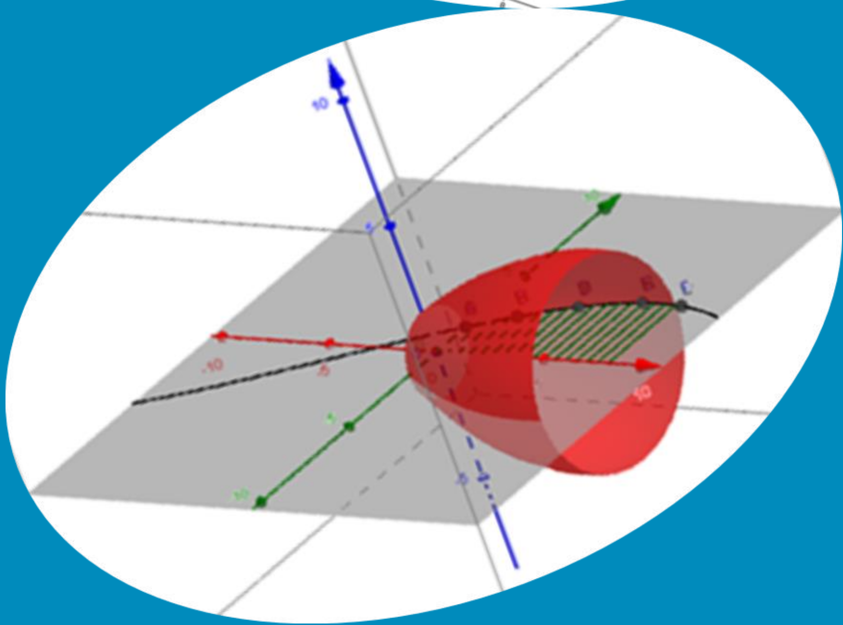
2017

Definite Integral

An inquiry based teaching unit

This teaching unit is a guide for high school mathematics teachers to teach definite integral. The activities in this teaching unit reference to an open source dynamic mathematics software, GeoGebra.





Çiğdem Özdemir

Introduction

Purpose

The purpose of this teaching unit is to provide an introduction to the integral calculus by means of definite integral to the students at the high school level. The teaching unit also provides students with real world mathematics concepts and problems that may come across in their life. The activities in this teaching unit rest on 5E Learning Cycle pedagogy which offers an effective inquiry based learning environment. The activities also reference to an open source dynamic mathematics software GeoGebra. This teaching unit is designed to be used by mathematics teachers who teach to MEB advanced level 12^{th} graders, but it can be used at any time as well. The activities in this teaching unit are planned to take about 10 teaching hours (45 minutes).

Mathematics Standards Addressed

MEB 2013 Standards

İD.12.2.1.1. Students should estimate the area between the graph of a function and x-axis by using Riemann Sums.

- Based upon real life situations, make feel the need of calculating the area between the graph of a function and x-axis.
- For some basic functions (such as $f(x) = ax$, $f(x) = ax^2$), the area is estimated by Riemann sums first by using the positive intervals of the function, then this method is extended by using the negative intervals of the function.
- The definite integral of a function is explained.
- Information and communication technology is used.

NCTM Standards

In grades 9–12 all students should–

Geometry

- establish the validity of geometric conjectures using deduction, prove theorems, and critique arguments made by others;
- draw and construct representations of two- and three-dimensional geometric objects using a variety of tools;
- use geometric models to gain insights into, and answer questions in, other areas of mathematics;
- use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture.

Number and Operations

- develop a deeper understanding of very large and very small numbers and of various representations of them;

Unit Objectives

As a result of this unit, students will-

- know the literal word meaning of integral.
- know that integration is used for the necessity of finding the area of irregular shapes.
- able to estimate the area between the graph of a function and x-axis by using upper and lower sums on GeoGebra, based upon a real life situation.
- able to calculate the definite integral of a function on GeoGebra.
- able to generate a formula to estimate the volume of solid of revolution of a function around x-axis by using Riemann Sums.
- able to calculate the volume of solids of revolution on GeoGebra.
- able to verify that integration helps to calculate the volume of solids of revolution by measuring volumes of several cups by volume measuring cups, and finding the volumes of those cups by using integration on GeoGebra.
- use GeoGebra fluently to calculate area and volume of real life objects, using definite integral formula.
- make an inquiry on maximizing the volume of s solid of revolution obtained by the same length of functions.

Table of Content

Engagement 1

Searching for the literal word meaning of "integral"

The aim of this activity is to make students conceptualize the word *integral* which is a new term for them. Group activity is chosen, because by this way students could share the ideas for the meaning of *integral* in small groups searching for different meanings of *integral*.

Materials needed

Three English dictionaries published by different companies for each group of three students:

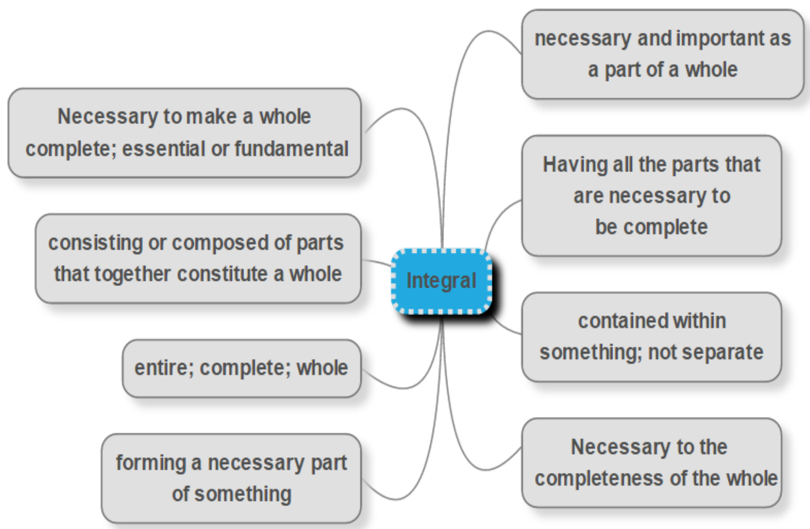
- Cambridge advanced learner's dictionary, fourth edition
- Oxford dictionary of English, third edition
- Online dictionary: http://dictionary.reference .com/

Note: Students may also use online versions of print dictionaries.

➢ Students will be formed as groups of three randomly and they will be engaged with the literal word meaning of "integral".

Note: *The group members will be chosen by the class teacher randomly. Because students' academic knowledge and skills are not determinant factors for this activity, random selection is chosen. The number of group members is chosen as three, because by this way students can establish an easy interaction within three students and they can discuss a wide range of different meanings of "integral" by searching from three different dictionaries.*

- Ask students the literal word meaning of *integral*. In groups of three, each member of the groups will search for the meaning of *integral* from dictionaries of different companies given in the materials part.
- Allow students to discuss the meanings of *integral* in their groups and finally express a meaning for "integral" in their own words within their groups.
- Create a mind map collecting students' definitions for the meaning of "integral".



Note:
The definitions on this mind map are the definitions of "integral" in provided dictionaries. Instead, write students' own definitions within their groups on the mind map.

Figure 1. Mind map for different definitions of "integral".

- Ask students to come up with a consensus among the definitions on the mind map, and highlight that definition on the board.

A brief history of integral calculus

In this part, students will be introduced that the topic of following lessons is *integration* which started to be worked on around 2400 years before now. Students will be introduced by providing a historical context through a general presentation:

"In ancient times, after people established the general methods for calculating the area of regular shapes such as triangle, rectangle, square, and other polygons, the challenge was to calculate the area of irregular shapes. As a first contribution to calculus, Greek mathematician Eudoxus (408 BC - 355 BC) stated the 'Method of Exhaustion' which refers to approximating the area of an object by dividing it in infinitely many pieces. Eudoxus had lived in Cnidus, a small village in modern Turkey.

Note: *Although the word "integral" is a new term for Turkish students, draw students' attention that the origins of integral calculus was laid in a small village in modern Turkey.*

Figure 2. The map of ancient Greece showing the part of modern Turkey.

Archimedes, 287 BC-212 BC was a great mathematician who was called as father of calculus, because he made very rigorous inventions by using 'Method of Exhaustion' that contributed to many mathematicians to work on integration in the history, and finally the invention of calculus by Newton and Leibnitz later in 17th century."

- Inform students that in the following lessons they will work as ancient mathematicians to find the area of irregular shapes and finding a bounded area under a curve on the coordinate system.

Exploration 1

Approximating the area of irregular shapes

The aim of this activity is to give students an insight that the area of irregular shapes can be calculated or approximated through using regular shapes with known areas.

Materials needed

- Handout for the geometric shapes with the given lengths of sides.
- A model irregular shape on GeoGebra (for teacher).

➢ Students will be formed as groups of two or three.

Note: *The group members will be chosen by the class teacher so that students in the same group have different skills and abilities. The number of group members is chosen as two or three, so that students in groups can easily work on one computer together.*

➢ Distribute handouts including the shapes below to groups of students and ask them to find the areas of those shapes.

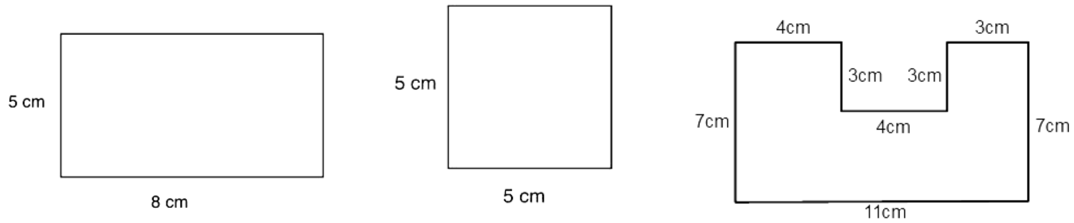


Figure 3. Geometric shapes to be found the areas of.

It is obvious that students will easily find the area of those shapes by calculation. For the last shape, ask students to find the area by using two different methods. By this way, some students may calculate the area by using areas of horizontal rectangles as their second method if their frst method is calculating the area by using areas of vertical rectangles, or vice versa. Some students may use the area that covers the shape and subtact the increment.

- Allow some students to present their method for finding the area of the last irregular shape to the whole class.

This activity leads to use of known areas to calculate the areas of irregular shapes.

- Ask students in groups to create an irregular shape such as the shape below on GeoGebra.

Figure 4. Model irregular shape.

- If needed provide students with the following instructions to create an irregular shape.

GeoGebra Instructions

- Click on the drop down arrow which is located at the lower right side of the  *text* icon. Now select  *pen* icon and draw the irregular shape by using your mouse so that it is a closed area.

- In order to avoid students to draw a regular shape such as a rectangle or a triangle, show your shape as a model.
- After drawing the irregular shape, ask students in groups to estimate the area of the irregular shape that they created by using any method they choose. The purpose of this activity is to lead to Riemann sum idea.
- Observe students while they are working.

Some students may use the grids of graphics view and count the squares, some may draw rectangles, triangles or other polygons within their shape that they know how to calculate the area of, and sum up the areas of those polygons. Some students may use the polygons that cover the irregular shape to estimate the area of the shape. Some students may use only one polygon that covers the irregular shape or inside the irregular shape by using as much vertices as they can. Even some students may draw the polygons both inside the irregular shape and covering the irregular shape, and make estimation between the areas of those two polygons.

➢ Allow groups of students to discuss their methods for estimating the area of their irregular shape with other groups.

Followings are possible student exercises:

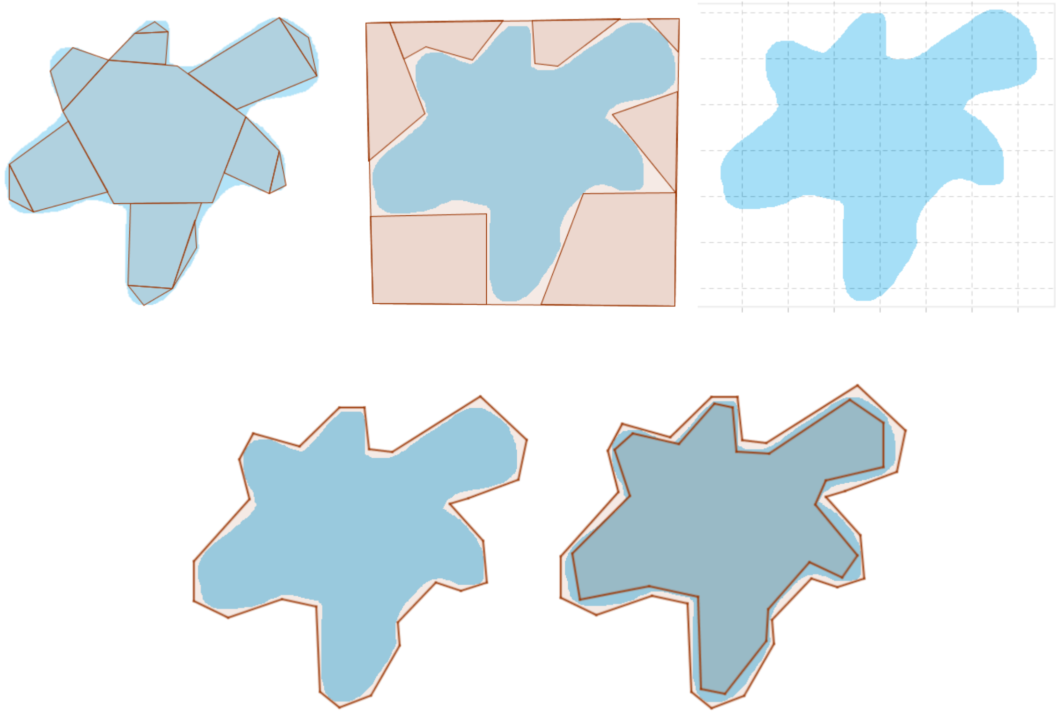
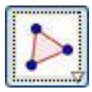


Figure 5. Possible student exercises.

As GeoGebra calculates the areas of polygons automatically by using  *Poly* commnad, students can easily calculate the sum of areas of polygons within the irregular shape or covering the irregular shape.

Explanation 1

Methods of approximating the area

- At the end of the exploration activity lead a whole class discussion by selecting some of the groups to explain their methods to the whole class.
- As a group of students share their method, ask questions to the groups of students in the class: “How your method differs from the other groups?” “Do you think other group’s method gives a better estimation than yours for the irregular shape? Why/Why not?”

Through this group discussion, students are expected to see that there may always be better estimations, and so this idea will lead to the limiting process of the area sums of regular shapes in an irregular shape.

Engagement 2

Riemann Sums

In the previous activity, students were expected to have a sense that a limiting process is needed to find the exact area of an irregular shape. The aim of this activity is that students use as many rectangles as they can to approximate the area of an irregular shape and finally use the limits of the sum of rectangles to find the exact area of an irregular shape as Bernhard Riemann did in 19th century.

Materials needed

- Digital image: Figure 5. Inherited estate.
- Worksheet-1
- Worksheet-2
- Rectangle tool on GeoGebra
- Internet connection to use Google Maps.

➢ Show the image below taken from Google Maps and pose students the following problem:

Figure 6. Inherited estate.

"Suppose that you have just inherited an estate between "Eskişehir Yolu" and "Sakıp Sabancı Bulvarı" and this land is between CD and EF lines. How would you determine the area of your estate? Make a preliminary estimation of the area of your estate."

Student responses will probably be similar to their responses for finding the area of their irregular shapes.

Exploration 2

Finding the area by using rectangles

- Students will be formed as groups of two or three as in the previous activity.
- For this problem, distribute the digital version of the image above and Worksheet-1 to the groups of students. Then, tell them to place "Eskişehir Yolu" on the x-axis, point D on the origin, and point F on point (20,0) on the coordinate system . Worksheet-1 will help students inserting and replacing the image on GeoGebra. After inserting and replacing the image, ask students to find the curve of "Sakıp Sabancı Bulvarı" by following the instructions in Worksheet-2.

By this way students will see that they need to find the area under a curve bounded by an interval to find the inherited area.

- As students had used different methods to approximate the area of an irregular shape at the previous activity, introduce students that in this activity they will only use vertical or horizontal rectangles to estimate the area.

➢ Ask groups of students to estimate the area by using only one rectangle that covers the whole area. For this activity, share the "Rectangle tool" as *rectangle.ggt* file with students and want them to upload it to their GeoGebra files by using *File* menu from the menu bar and selecting *Open* and clicking on *Open* after they choose the rectangle.ggt file from their computer.

Note: *The rectangle tool will be developed by the teacher using given rectangle tool instructions in Worksheet 4.*

➢ As students create the rectangle below, ask them if it is a good estimation for the area under the curve. Students will probably say "no" as they include quite a big area over the area under the curve.

Figure 7. Estimation of the inherited area by using only one rectangle that covers the whole area.

➢ Ask students if they do a better estimation when they use two rectangles covering the area under the curve. So students draw two rectangles similar to the rectangles below.

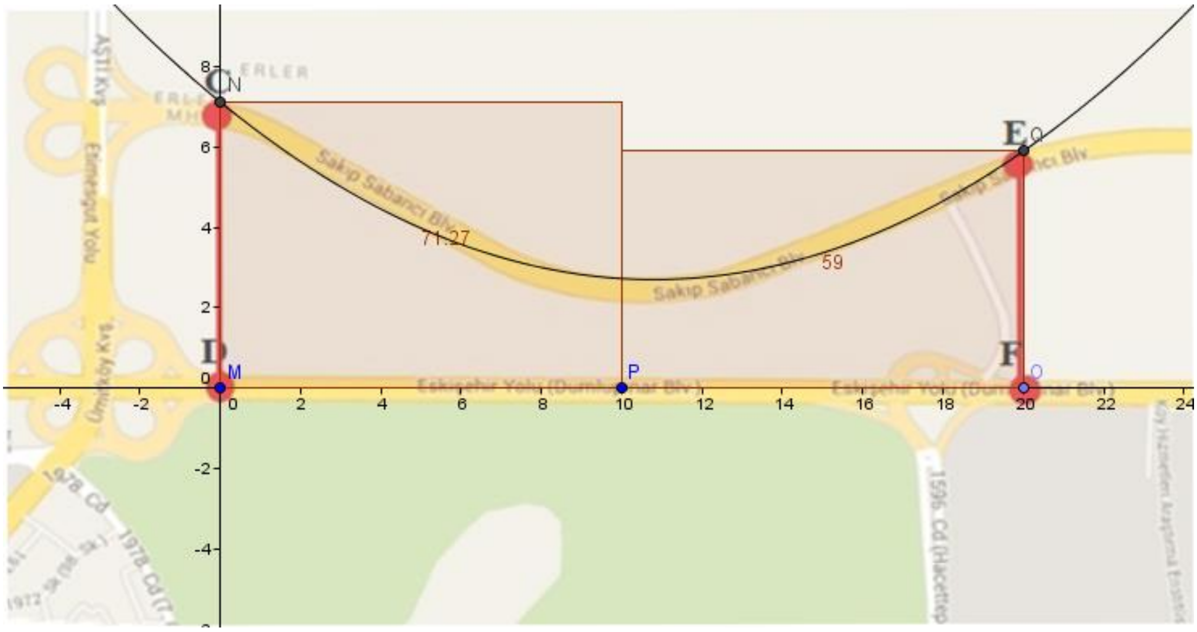


Figure 8. Estimation of the inherited area by using two rectangles that covers the whole area.

By this way students will see that this is a better estimation comparing to the first estimation.

➢ Ask students if they do better estimations when they increase the number of rectangles and allow them to work on creating as many rectangles as possible.

As students draw rectangles similar to the rectangles in the image below, they will see that they approximate to the area under the curve better and better when they increase the number of rectangles.

Figure 9. Estimation of the inherited area by using many rectangles that covers the whole area.

- Ask groups of students to make an estimation of the inherited area by using as many rectangles as they can and compare their estimations with other groups.
- Students will calculate the area by adding up all the rectangles. As it is too much work to add up all the rectangles, introduce students that they can add their rectangles by using *Sum* command easily on GeoGebra:

GeoGebra Instructions

- Create a list of rectangles by shading all “Poly”s in the algebra view and carry it to input bar. Press enter and you will see the created list in the algebra view.
- Write sum in the input bar and choose Sum[<List>] command.
- Write the name of your list for <List>. Press enter and you will see the sum in the algebra view.

➢ As groups of students find a result for the inherited area, ask questions such as “What is the unit of your result?” “Is your result sensible?” “How could you know if your result is sensible or not?” “How could you work out in order to make your result sensible?”

By these questions make students to think of how big the area is by comparing this area to several areas that they already know. Thus, students should understand that they should pay attention to the metric scale. Knowing that the picture is taken from Google maps, students could work on Google maps to get the distances between two points. Accordingly, they could use ratio and proportion by using the area that they have found through calculating the sum of rectangles.

➢ Walk around in the classroom and scaffold students to work on Google maps and find a meaningful result.

According to Google maps directions, the distance between point C and point D approximately seems to be 650 km as in the following image.

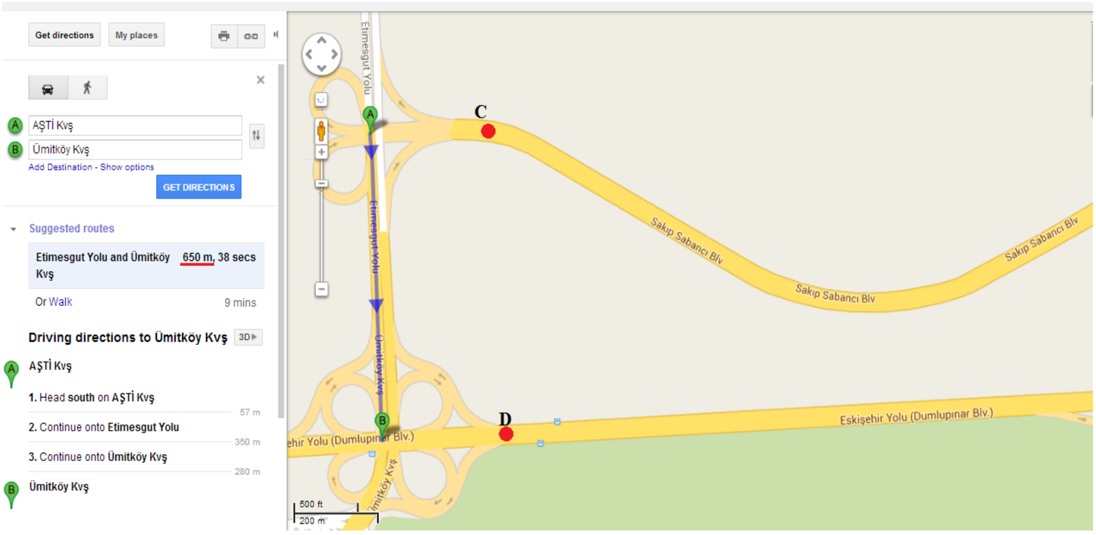


Figure 10. The approximate distance between point D and point C on Google maps.

- Explain students that you expect them to find an area between an interval like $\mathbf{600000\ m^2}$ and $\mathbf{750000\ m^2}$.

This expectation enables students to work on how their results approach to these values by regulating their scale, function of the curve and number of rectangles.

- Observe students while they are working and if you see that they find a result out of the interval that you expect, ask leading questions such as "Which information have you used to calculate the area from the beginning of the activity?" "Which of that information could have caused you to find a result out of these intervals? Why?" "How could you regulate your method to find a result between these intervals?"

Thus, students will work to find a result between the expected intervals by using trial and error method as they will regulate the information that they have used.

Explanation 2

Upper and lower sums

- Ask groups of students to explain the process for estimating the inherited area to the whole class, and gather other students' comments on that group's process.
- Explain students that:

  The sum of rectangles having the equal width that covers the area under a curve and they are as few as possible is called as "upper sum". So, upper sum gives a little more area than the area under a curve.

  The sum of rectangles having the equal width that is inside the area under a curve and they are as few as possible is called "lower sum". So, lower sum gives a little less area than the area under a curve.
- Explain students how to use upper sum and lower sum command on GeoGebra working on the image of inherited area:

GeoGebra Instructions

- For upper sum, write *UpperSum* in the input bar and choose UpperSum[<Function>, <Start x-Value>, <End x-Value>, <Number of Rectangles>]
- Write your function for <Function>, your start x- value and end x-value for <Start x-Value>and <End x-Value>, and write how many rectangles you want for <Number of Rectangles>
- Press enter and you will see the rectangles in the graphics view and the total area of the rectangles in both algebra view and graphics view.
- For lower sum, write *LowerSum* in the input bar and choose LowerSum[<Function>, <Start x-Value>, <End x-Value>, <Number of Rectangles>]
- Follow the same instructions as for upper sum.

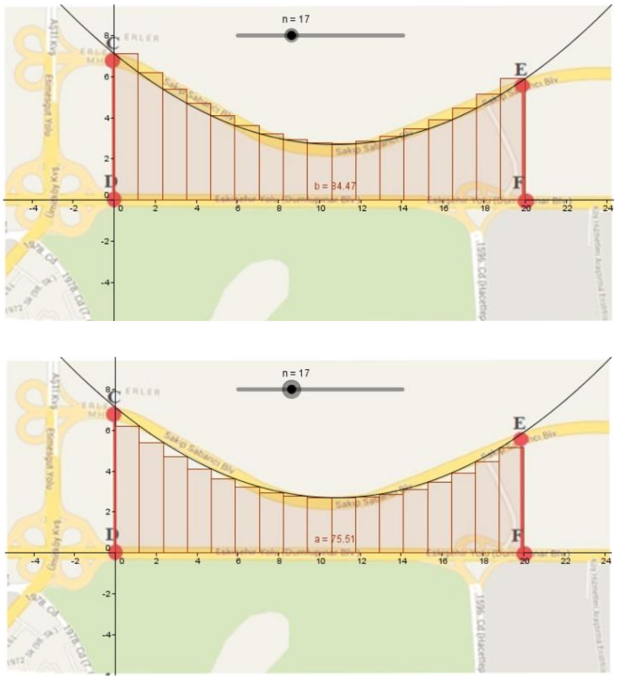


Figure 11. The figures produced by using *UpperSum* and *LowerSum* commands on Geogebra.

As students use upper and lower sums on GeoGebra, they will see that the difference between lower sum and upper sum will decrease as they increase the number of rectangles for both command, and the exact area will be between these two values.

Explain students that there are other methods for approximating the area under a curve which are left sum, right sum and these methods can be used similar to upper sum and lower sum.

Exploration 3

Upper and lower sums formula

- Ask students to draw the function of "*x^2+1*" in the graphics view.

Note: *Function is chosen to be a positive, continuous and increasing between an interval* $0 \leq x \leq 3$*, because when an interval is chosen including positive and increasing part of a function, students will simply take left or right sides of the rectangles as the heights of the rectangles for upper and lower sums respectively.*

x^2 is chosen as an increasing function in the intervals $0 \leq x \leq 3$*, and 1 is added not to make the function touching to the point (0,0), so that students could see upper and lower sums in the intervals* $0 \leq x \leq 3$ *accurately.*

- Provide students with the information for drawing the graph of the function in the graphics view:

GeoGebra Instructions

- Write x^2+1 in the input bar. Press Enter and you will see the graph of the function in the graphics view and algebraic representation of the function in the algebra view.

- Ask students to use lower and upper rectangle sums for this function and generate a formula for these sums in the intervals $0 \leq x \leq 3$.
- Accordingly, ask students to generate a formula for upper and lower rectangle sums for a positive, continuous and increasing function on the interval $a \leq x \leq b$.

- Do not give direct answers, but ask further questions such as "What is the width of one of your rectangles?" "What is the height of it?" "What is the relation between the function and the heights of the rectangles?"

  Thus, students can generate a formula by using the heights of rectangles by means of the function.

  Some student responses can be expected as:

For example dividing the interval [0,3] in three equal intervals, the lower sum:

$$f(0) * 1 + f(1) * 1 + f(2) * 1$$

General formula for dividing the interval [a,b] into n equal intervals, the lower sum:

$$f(a)\frac{b-a}{n} + f\left(a + \frac{b-a}{n}\right)\frac{b-a}{n} + f\left(a + 2 * \frac{b-a}{n}\right)\frac{b-a}{n} + \cdots$$
$$+ f\left(b - \frac{b-a}{n}\right)\frac{b-a}{n}$$

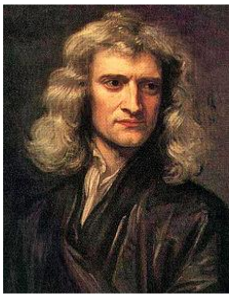
Where $\frac{b-a}{n}$ represents the widths of rectangles, $f(a)$ represents the height of the first rectangle, $f\left(a + \frac{b-a}{n}\right)$ represents the height of the second rectangle, and so on.

Explanation 2

Riemann Sums

- Gather student exercises and lead a class discussion allowing students to express their formulas.
- Introduce the following historical note to the class:

Isaac Newton (1642-1727) and Gottfried Wilhelm Leibniz (1646-1716) made the most significant contributions to integral calculus, and Leibniz used the notation of integration as long "s", $\int$ which is used today. After Newton and Leibniz's contributions to integral calculus, German Mathematician Bernhard Riemann rigorously formulized integration by using limits. Therefore, approximating the area enclosed by a function, x-axis and an interval $a \leq x \leq b$ by using rectangle sums is named as **Riemann Sums**.



Isaac Newton

Gottfried Wilhelm Leibniz

Bernhard Riemann

Exploration 4

Definite Integral

By doing this activity, students will explore the meanings of the Riemann Sum formulas by making connections between their own formulas in their groups that they created at the previous exploration task.

Materials needed

- GeoGebra applet: Riemann sum formulas

- Distribute students the following ***upper and lower sum applet*** and ask them to explore the meanings of the formulas in the applet as they change the value on the slider.

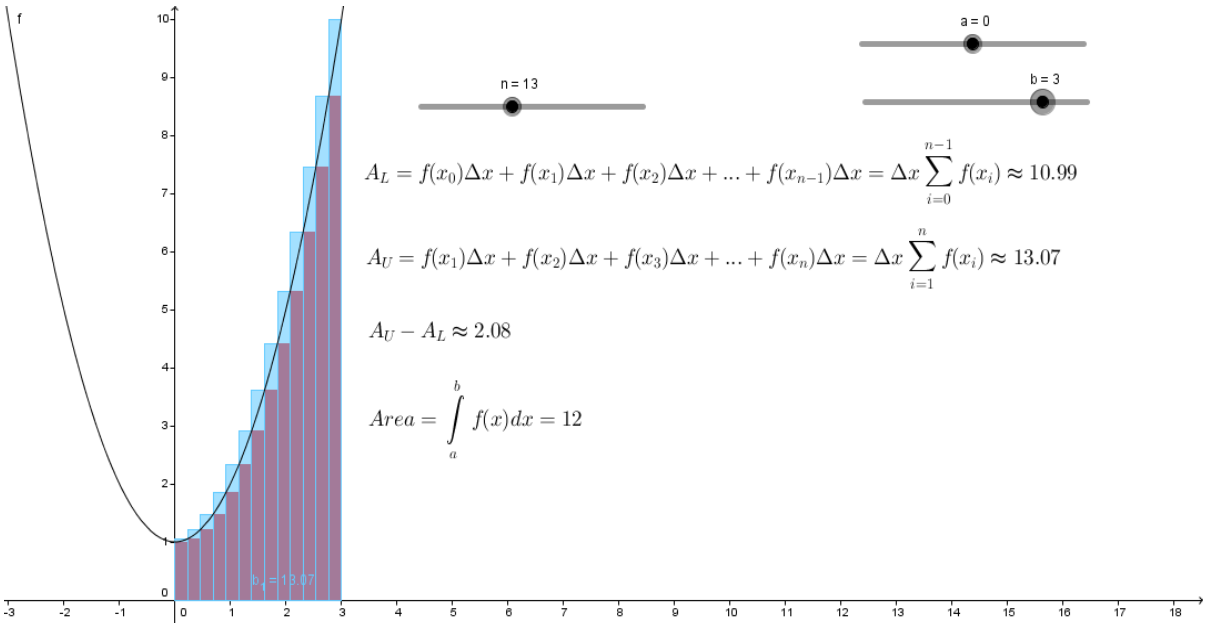


Figure 12. Geogebra applet in which students can explore the meanings of the Riemann Sum formulas by changing the values on the sliders.

- Accordingly ask the meanings of the symbols x_i, Δ x,dx in each formula after explaining that $\int_0^3 f(x)dx = 12$ gives the exact area under the following curve between the interval $0 \leq x \leq 3$.
- As students find out the meanings of the symbols in each formula in the applet, ask students which limiting process do they think that Riemann had done to find the exact area. Thus, students expected to see that:

> ***Note:***
> *Students must have already learned the limits topic. If needed, teacher must provide a brief revision on limits.*

$$\lim_{\Delta x \to 0} A_U - A_L = 0$$

So, $\lim_{\Delta x \to 0} A_U = Area = \lim_{\Delta x \to 0} A_L$

Explanation 3

Definite Integral

- According to students' exploration, explain that the area enclosed by a function f(x), x-axis, start x-value and end x-value is defined as the **definite integral** of f(x) from a start x-value "a" to end x-value "b", and it is written as:

$$Area = \int_a^b f(x)dx$$

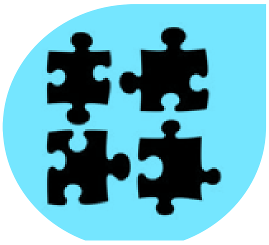
- Ask how students could demonstrate the formula

$$Area = \int_a^b f(y)dy$$

As it is not possible to write a function in y as f(y) on GeoGebra, ask students to demonstrate the integration above on the coordinate system by using paper and pencil.

Calculating definite integral on GeoGebra

- Introduce students that definite integral can be calculated on GeoGebra.
- Distribute Worksheet-3 to the groups of students and ask them to follow the instructions of calculating definite integral.



Elaboration 1

Definite Integral Applications

In this section, students will apply the definite integral formula given in the explanation part for different functions.

- Ask students to do the following exercises.

Exercises

1. Find the areas between the following functions and the x-axis and the given intervals first by using the formula for the area of a rectangle and a triangle, then by using the definite integral formula on Geogebra.

a. $y = 3$ between the interval $x = 0$ and $x = 4$
b. $y = x$ between the interval $x = 0$ and $x = 3$
c. $f(x) = \begin{cases} -x + 3, & -1 \leq x \leq 0 \\ x + 5, & -5 \leq x \leq -1 \end{cases}$
d. $y = x$ between the interval $x = -3$ and $x = 0$
e. $y = x$ between the interval $x = -4$ and $x = 2$

2. Find the areas between the following functions and the x-axis and the given intervals by using the definite integral formula on Geogebra.

a. $y = x^3$ between the interval $x = -3$ and $x = 3$
b. $y = (x - 2)(x + 2)(x - 1)$ between the interval $x = -3$ and $x = 3$
c. $y = x^3 - x$ between the interval $x = -3$ and $x = 3$

Note:

*For exercise **1.d** students must realize that definite integral gives negative value when graph line is below the x axis.*

So it is needed to take the absolute value of the definite integral of a function under the x axis in order to find the area.

Students also must realize that when the area partly above and partly below the x axis, they should calculate the definite integrals separately.

Engagement 3

Volumes of Solids of Revolution

By this engagement activity, students will draw attention that an accurate formula is needed to find volumes of solids of revolution through real life situations.

- Ask students to make an inquiry about why measuring volume became necessary in ancient times and in what forms it is necessary now. Make a brief discussion on students' findings.
- Bring glasses that represent solids of revolution and a volume measuring cup to the class.

Figure 13. Glasses that represent solids of revolution.

- Inform students that these glass shapes are called as solids of revolution, because they obtained by rotating a plane around a straight line.
- Show students the following ***3D applet*** showing how a glass represents a solid of revolution by changing the α slider.

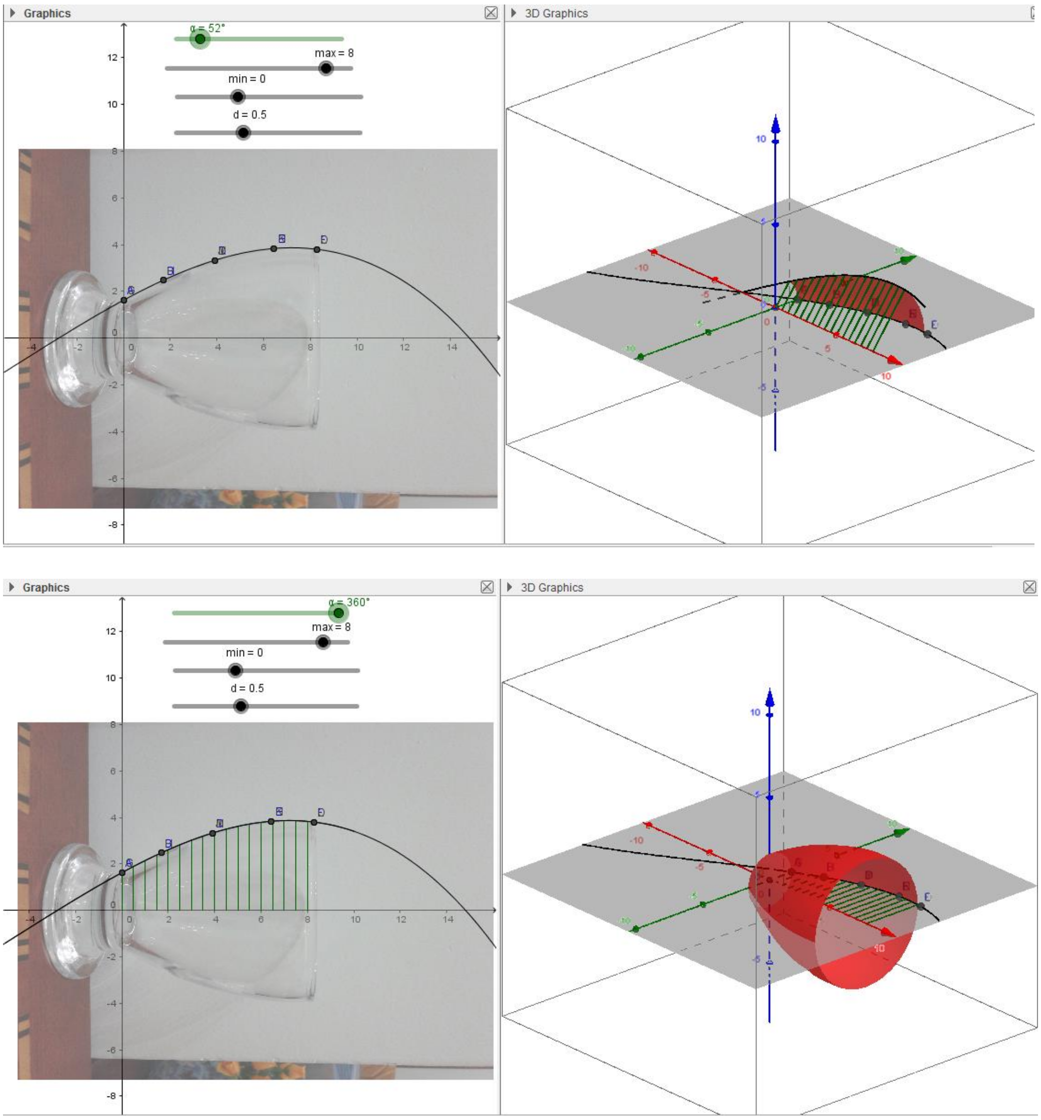


Figure 14. 3D applet showing how a glass represents a solid of revolution

- Ask students how to find the volume of those glasses. Students might answer this question as “by using volume measuring cup” as we intended to do.
- Fill one of the glasses with water and then measure how much water does that glass takes by using a volume measuring cup.
- Now ask students “how could we find the volumes of solids of revolution that are not fillable or that are too large to fill?”
- Inform students that integration also helps us to find the volume of solids of revolution.

Exploration 5

Riemann Sum on Volumes of Solids of Revolution

In this section, students will explore finding the volume of solids of revolution by recalling their knowledge of finding an area under a curve by Riemann Sum.

- Remind students that to estimate the area under a curve they used rectangles.
- Show the following image showing the lower sum under the curve of the glass, and ask students which shapes we should find the sum of when we rotate those rectangles 360 degrees around the x axis. By this way students are expected to find out that in order to estimate the volume of a solid of revolution it is needed to find the sum of the cylindrical disks.

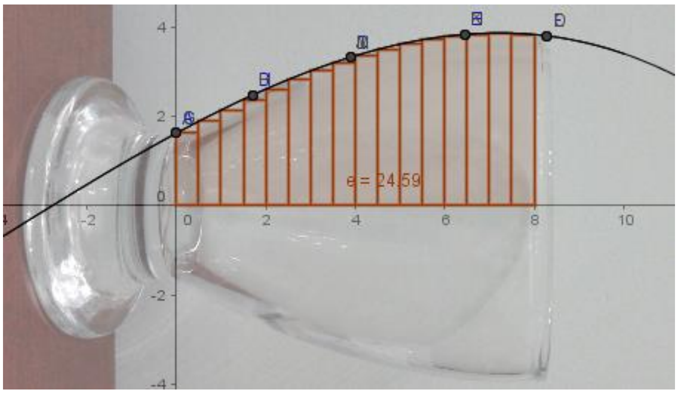


Figure 15. Lower sum under the curve of the glass

- Students should explore the volume of a disk obtained through the 360 degrees rotation of one rectangle in the Riemann sum around the x axis as follows:

$$\pi f(x_i)^2 \Delta x$$

- Thus students will find out that in order to find the volume of a solid of revolution the following limiting process is needed.

$$\lim_{\Delta x \to 0} \sum_{i=1}^{n} \pi f(x_i)^2 \Delta x = 0$$

Explanation 4

Volumes of Solids of Revolution

In this section, students will recall that they represented the limit of the sum of rectangles with the integration symbol as definite integral in *Explanation3* section. Thus students will transfer this knowledge to the representation of the limit of the sum of disks.

- Explain that the definite integral helps to find the volume of a solid that is obtained by revolving a region bounded by a function f(x), x-axis on the interval [a, b] about the x-axis, and it is written as:

$$Volume = \int_{a}^{b} \pi \, f(x)^2 dx$$

- Inform students that they can calculate volumes of solids of revolution on GeoGebra by following the instructions on Worksheet-3 as well, unless they do not forget to take the square of the function and multiply it by π .

Elaboration 2

Applications of Definite Integral on Volumes of Solids of Revolution

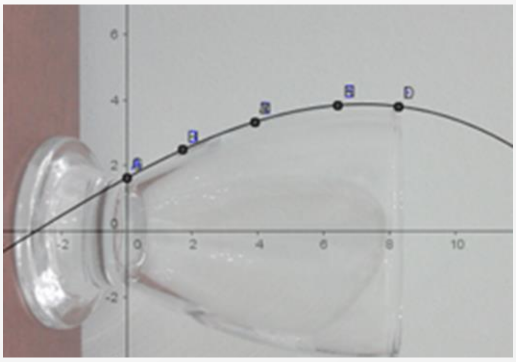
At the first part of this section, students will find the volumes of different glasses that represent solids of revolution by using volume measuring cup, and then they will calculate the volumes of those glasses on GeoGebra. Thus, students will compare their findings; thereby they will verify that integration helps to calculate the volume of solids of revolution.

At the second part of this section, students will calculate the volumes of large solids of revolution that they come across in their daily lives.

- As you found the volume of one of the glasses at *Engagement 3* section, now calculate the volume of that glass by inserting the image of it on GeoGebra and using definite integral formula. For example, if you found the volume as 220 ml by volume measuring cup, you must find the volume approximately 220 cm^3 on GeoGebra.
- Ask students in groups of three to find the volumes of different glasses that represent solids of revolution using both methods and compare their findings.

Note: *Students must be careful on resizing and repositioning of the image of the glasses when they insert them on GeoGebra. They must measure the height f the glass and make sure that the same height corresponds to the length between start x-value and end x-value on GeoGebra.*

For example, the following image shows repositioning the image of the glass that has 8 cm real height.



- Ask students in groups of three to find the volumes of large solids of revolution reminding them your engagement question: how could we find the volumes of solids of revolution that are not fillable or that are too large to fill? Students might find volume of a barrel, a building or a dome.

Figure *16*. Objects that represent the solids of revolution

Evaluation

Inquiry Presentation

In this section, students in groups of three will be asked to make an inquiry about how to get the maximum volume with the same length of functions. Students will use trial and error method on GeoGebra. After students make their inquiry they will give a presentation in the class showing their findings.

Students will be evaluated by themselves, by classmates and by the teacher regarding the following criteria.

- Presentation of adequate information in a logical sequence
- Clear presentation of graphics and illustrations on GeoGebra
- Mathematical accuracy
- Eye contact and elocution

GeoGebra Worksheets

Worksheet-1: Inserting an Image on Geogebra

This activity consists of four subtitles. By working on this activity, you will learn how to insert an image from your documents to Geogebra, and how to do manipulations on this inserted image. Thus, this activity will enable you to work on an image in GeoGebra.

A. Saving the image to local drive

1. Click on the link below and save the “Hulme Arch Bridge” image which is a parabolic arc.

   http://en.wikipedia.org/wiki/Hulme_Arch_Bridge#mediaviewer/File:Close_view_of_Hulme_Arch.jpg

   - Right click on the image, choose “save as” and save the image on your desktop.

B. Inserting the image on GeoGebra

2. Open a new GeoGebra window.
3. Click on the drop down arrow which is located at the lower right side of the 

text icon. Now select  *image* icon to insert the image.
4. Click on a point on the graphics view of your GeoGebra window where the lower left corner of your image will appear.
5. Choose your image from your desktop, and click “open”.
6. You can move your image by clicking on  *move* icon and dragging the image.

C. Resizing and repositioning the image

7. In order to make your image smaller or bigger, or change the position of your image;
   - Right click on the image and select object properties.
   - Select "position" on the menu bar of the object properties window and determine the points of only two corners of your image such as Corner1: (0.0), and Corner2: (5,0),
   - Close the window. (Other two corners will be determined automatically).

D. Color Manipulation

8. Make your image lighter, so that you can you can make your works on the image visible;
   - Right click on the image and select object properties.
   - Select "color" on the toolbar and change the opacity as between 50 and 75.
   - Close the window.

E. Exercises

9. Insert the following pictures in graphics view of GeoGebra.
   - http://www.clipartpanda.com/clipart_images/roller-coaster-2-999px-png-16400534
   - https://www.flickr.com/photos/doug_mall/8705540541/

Worksheet-2: Finding the equation of a curve

In GeoGebra, you can find the equation of a curve by finding the curve of best fit. All you need to do is to plot points on the curve that you want to find the equation in your image, and to use the *FitPoly* command. In this activity, use the parabolic image that you had already inserted in GeoGebra. By following the steps under three subtitles, you will learn plotting on an image, creating the list of points, and finally finding the line/curve of best fit of the points.

A. Plotting on the curve

1. Place your image by using  *move* icon so that the starting point of the parabola in your image will appear on the origin.
2. Plot five points where the distance between adjacent points is approximately same on the parabolic curve by using  *point* icon.

B. Creating the list of points

3. Click  *move* icon. Select the points that you plotted on the image either by dragging a rectangle around them or using Ctrl-Click.
4. Drag the selection from the Algebra View to the Input Line.
5. Press Enter, and you will see *List1* in the Algebra View.

C. Finding curve/line of best fit

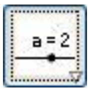
6. Write *FitPoly* in the input bar and choose FitPoly[<List of Points>, <Degree of Polynomial>]

7. As this curve looks like a curve of a quadratic equation the degree of your equation must be 2. So, write the name of your list instead of <List of Points>, and “2” instead of <Degree of Polynomial>.
8. Press Enter, and you will see if the curve of best fit that you found fits with the curve in the image on your graphics view.

Search for better results:

- ➢ How the curve of best fit changes when you plot more points on the image? (You can plot more by using the scroll wheel of your mouse or using the  *Zoom In* icon through clicking on the dropdown arrow on  *Move Graphics View* icon
- ➢ When the curve of best fit that you found fits better with the curve in the image? (By plotting more?, By changing the degree of the polynomial?)

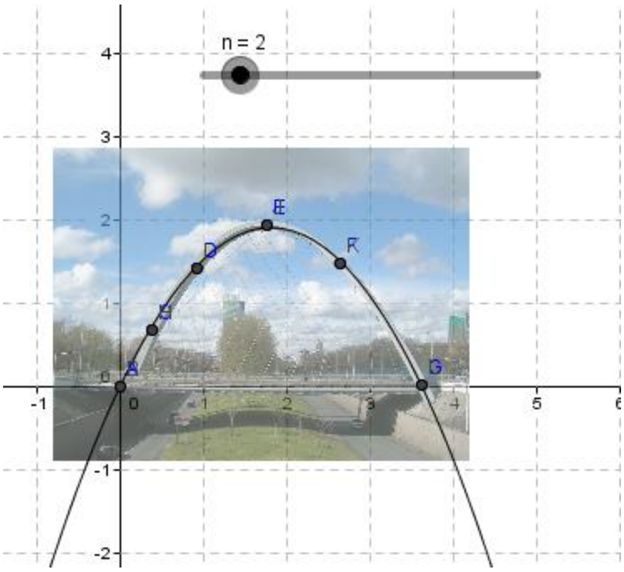
9. If you don’t know the equation with which degree best fits your curve in the image, then create a slider for the degree of your polynomial, so that you can see which degree fits best to your image by changing the degree on slider:

- Click on 

icon in the toolbar.
- Choose integer “n” in the slider window.
- Write “1” for the minimum interval, and “10” for the maximum interval in the slider window. Close the window.

10. Write *FitPoly* in the input bar and choose FitPoly[<List of Points>, <Degree of Polynomial>]

11. Write “list1” instead of <List of Points>, and “n” instead of <Degree of Polynomial>.
12. Change “n” on the slider and identify the equation best fits with the curve on your image.
13. Your final view must be similar to the image below.



Exercise: Insert the images given in the links below in GeoGebra graphics view and find the line or curve of best fits.

- http://www.clipartpanda.com/clipart_images/roller-coaster-2-999px-png-16400534
- https://www.flickr.com/photos/doug_mall/8705540541/

Worksheet-3: Integration on Geogebra

Geogebra easily calculates the area under a curve and volume of solids of revolution through *Integral* command. As you learned how the definite integration works for finding the area under a curve and finding solids of revolution at previous lessons, now you will do these calculations on Geogebra and see the visuals of your calculations. In this activity use the image that you already found the curve of in worksheet 2.

A. Finding the Area under a Curve

1. Write *Integral* in the input bar and choose Integral[<Function>, <Start x-Value>, <End x-Value>] command.
2. Identify your *Start x-Value* and *End x-Value* on your graphics view.
3. Write the name of function that you already found as the curve of best fit instead of <Function> , and your *Start x-Value* and *End x-Value* instead of <Start x-Value>, <End x-Value> respectively.
4. Press Enter and you will see the area under the curve between your *Start x-Value* and *End x-Value* in a different color.
5. In your algebra view, you will see the calculated area under the curve.

B. Finding the Volume of Solid of Revolution

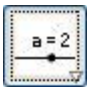
6. To find the volume of solid of revolution around x-axis, you need to use

$$\int_a^b \pi f(x)^2 dx$$

formula as you learned at previous lessons. In the input bar, write π first, and then write *Integral* and choose Integral[<Function>, <Start x-Value>, <End x-Value>] command.

7. Write the square of your function that you already found as the curve of best fit instead of <Function>, and your *Start x-Value* and *End x-Value* instead of <Start x-Value>, <End x-Value> respectively.
8. Press Enter and you will see the calculated volume of solid of revolution in your algebra view. However, Geogebra doesn't show the 3D visual by your calculation command as it does in calculating the area under a curve, but you can show it visually with another command.

C. Showing Volumes of Solids of Revolution Visually

9. To show the solid of revolution visually,
   - Click on 

 icon in the toolbar.
   - Choose angle "α" in the slider window.
   - Write "0°" for the minimum interval and "360°" for the maximum interval in the slider window. Close the window.
   - Write *Surface* in the input bar and choose Surface[<Expression>, <Expression>, <Expression>, <Parameter Variable 1>, <Start Value>, <End Value>, <Parameter Variable 2>, <Start Value>, <End Value>] command.
   - Write *a* instead of the first <Expression>, $f(a)cos(\beta)$ instead of the second <Expression>, $f(a)sin(\beta)$ instead of the third <Expression>, *a* instead of <Parameter Variable 1>, write *Start x-Value* and *End x-Value* instead of <Start Value> and <End Value> respectively, write β instead of

<Parameter Variable 2>, write *0* instead of the second <Start Value>, and α which is your slider name instead of the second <End Value>. Press Enter.

Note 1: Use the name of your function. For example if your function is g(x), then write g(a) instead of f(a).

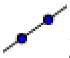
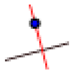
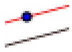
Note2: You can use different perimeters instead of a and β. Pay attention to the consistency in the command.

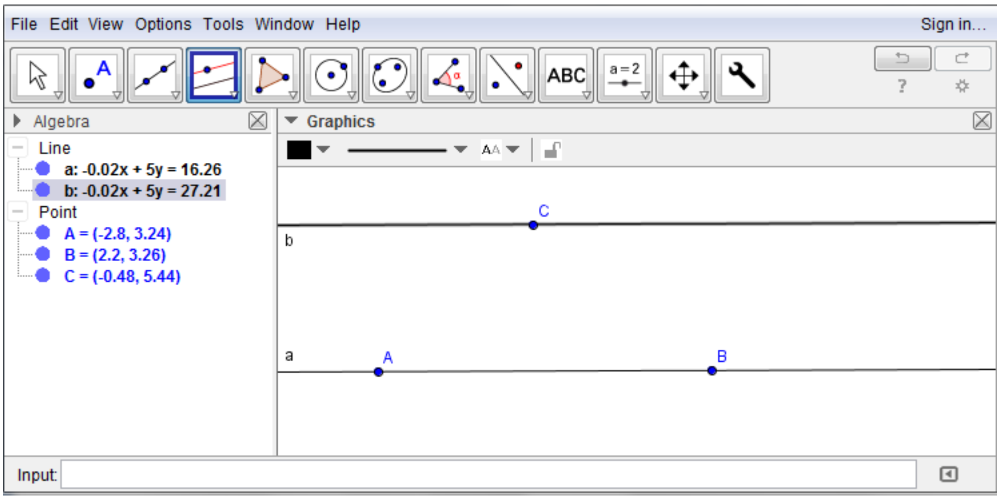
- Click on *View* in the menu bar and choose *3D Graphics.*
- You will see the *3D Graphics* view at the right hand side of your GeoGebra window at the same time with the graphics view. By changing the value of α on the slider in your graphics view, you will see the covered surface. By fixing the slider on 360°, you will see the solid of 360° revolution through the x-axis.

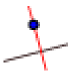
Worksheet-4: Rectangle Tool for Teachers

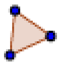
By yhis worksheet, teachers can create a rectangle tool where they select three points (first two points will determine the base and the third point will determine the height of the rectangle) and automatically find the area of the rectangle.

A. Constructing a rectangle

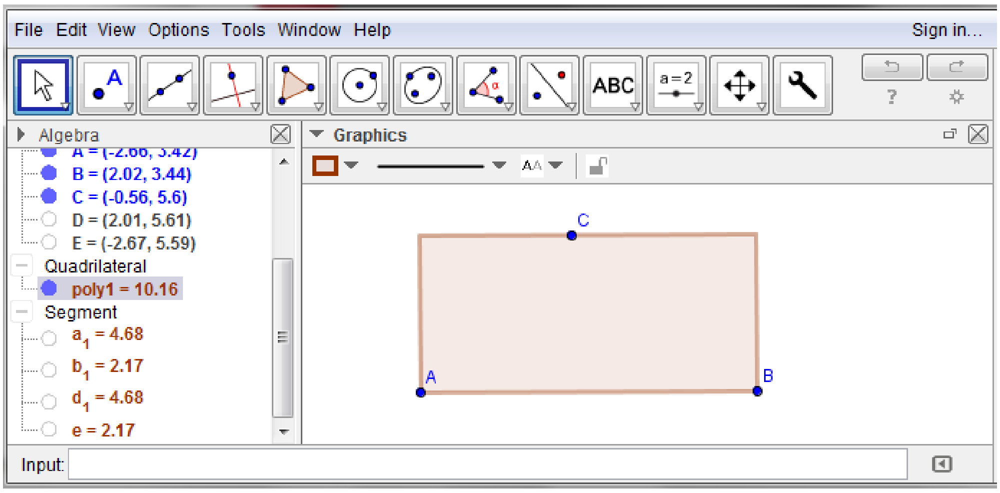
1. Select  *Line* command, and click on two points on the graphics view so that you draw a line.
2. Click on the drop down arrow which is located at the lower right side of the  *Perpendicular Line* icon and select  *Parallel Line* command.
3. Click on the line that you created to determine to which line you want to create a parallel line and click on a point on the graphics view where you want to create the parallel line as in the following image.



4. Click on  *Perpendicular Line* command and create perpendicular lines to the first line that you created clicking on it and then clicking on the two points on the line (on point A and point B in the image above) so that you create a rectangle.

5. Click on  *Poly* command and click on the four intersection points of the lines in order.

6. Hide the items clicking on the blue circles in front of them on the algebra view so that you only see the rectangle with points determining the base and the height as in the following image.



B. Creating a new tool for rectangle construction

1. Click on *Tools* menu on the menu bar and choose *Create New Tool* command.

2. For the output objects, click on drop and down arrow and select the poly that you created for the rectangle. Click on *Next.*

3. For the input objects, click on drop and down arrow and select points which determining the base and the height (Point A, Point B, and Point C in the image above). Click on *Next.*

4. For the *Name& Icon* window, write *rectangle* for both the *Tool name* and *Command name.* Click on *Finish.*

5. Now you can see the  *rectangle* tool on the tool bar.
6. By clicking on this  *rectangle* tool, now you can construct a rectangle by setting three points.
7. To make this rectangle tool valid for all your GeoGebra files click on *Options* menu on the menu bar and select *Save Settings.*
8. To create a file for the rectangle tool in order to distribute students, click on *Tools* on the menu bar and select *Manage Tools.*
9. On the *Manage Tools* window, select the *rectangle* tool, write rectangle for both tool name and command.
10. Click on *Save as* and save this file on your desktop as *rectangle.ggt* file.

Appendix 4: Survey

Biçim, Dil ve Anlatım	Kesinlikle katılıyorum	Katılıyorum	Katılmıyoru	Kesinlikle katılmıyoru
Kullanılan kağıt yeterli kalitededir				
Yazı karakterleri yeterli büyüklüktedir.				
Dış kapak dersin içeriğine uygundur.				
Ünite planında yer alan şekil, grafik ve fotoğraf gibi görsel öğeler net şekilde görünmektedir				
Konuların veriliş sırası konuların özelliklerine uygundur.				
Çelişkili ifadelerden kaçınılmıştır				
Vurgulanması gerekli kelime ve tamlamalar koyu olarak yazılmıştır.				
İmla kurallarına uyulmuştur.				
Yazım hatalarından kaçınılmıştır.				
Matematiksel semboller uluslar arası alanda kullanılan standart yazım şekillerine uygundur.				
Doğru ve uygun kelimeler kullanılmıştır.				
Gereksiz kelime kullanımından kaçınılmıştır.				
Anlatım bozukluklarından kaçınılmıştır.				

İçerik, müfredat, uygulanabilirlik				
Amaç ve kazanımlar ünite planında açıkça belirtilmiştir.				
Ünite planının içeriği belirtilen kazanımlara uygundur.				
Ünite planındaki kazanımlar MEB 2013 lise matematik müfredatına uygundur				
İçerik 12. Sınıf lise öğrenci düzeyi için uygundur.				
Ünite planının uygulanabilmesi için belirtilen zaman yeterlidir.				
Ünite planının uygulanabilmesi için belirtilen zaman MEB 2013 lise matematik müfredatı ile uyuşmaktadır.				
Ünite planının uygulanabilmesi için sınıfların gerekli teknolojik altyapıya sahip olması gerekmektedir.				
İçerik aşamalı ve birbirinin ön koşulu olacak şekilde sıralanmıştır.				
İçerikteki konular bir bütünlük içerisinde yapılandırılmıştır.				
Ünite planı mantıksal ve stratejik aktiviteler sunmaktadır.				
Ünite planı konunun öğrenilmesi için gerekli ön şart niteliğindeki bilgi ve becerileri ifade etmektedir.				
Ünite planı öğrenmeyi desteklemek için integral konusunda çoklu ve değişken olaylar sağlamaktadır.				

Ünite planı integral konusu ile ilgili olarak öğrencilerin kendilerinin yapabilecekleri aktiviteler (deneyler) içermektedir.				
İçerik öğrencinin elde ettiği bilgilere dayanarak yeni bilgi üretebilmesine olanak sağlayacak şekilde hazırlanmıştır.				
Ünite planı öğrencileri motive etmek ve anlamalarını sağlamak için genel amaç ve talimatları içermektedir.				
Ünite planı öğrencilerin kavramlar, deneyler ve olgular hakkında düşünmelerini, fikir yürütmelerini teşvik etmektedir.				
Ünite planı öğrencilerin muhakeme ve yorum yapmalarına rehber olmaktadır.				
Ünite planı öğrencilerin bilgiyi üretebilmeleri için ortam sağlamaktadır.				
Matematik doğruluğu, öğretmen ve öğrenci yeterliliği				
Matematiksel ifadeler doğru ve yerinde kullanılmıştır.				
Formüller doğru bir şekilde verilmiştir.				
Matematiksel ifadeler görsellerle desteklenmektedir.				
Yapılan deneyler doğru sonuç vermektedir.				
Örnek problemlerdeki veriler eksiksizdir.				
Ünite planında verilen şekiller doğru bilgiyi desteklemektedir.				
Bilimsel ifadeler öğrencilerde kavram yanılgısı uyandırmayacak şekilde, doğru ve anlaşılabilir kavramlarla sunulmaktadır.				
Ünite planı öğretmenlere öğrencilerin kavram yanılgıları konusunda uyarılar içermektedir.				
Ünite planı dersi öğretme konusunda öğretmene rehber niteliğindedir.				
Gerekli noktalarda öğretmenler için açıklamalara yer verilmiştir.				
Ünite planını uygulayabilmek için öğretmenlerin temel bilgisayar bilgisi ve becerisi sahibi olması gerekir.				
Ünite planını uygulayabilmek için öğrencilerin temel bilgisayar bilgisi ve becerisi sahibi olması gerekir.				
Ünite planını uygulayabilmek için öğretmenlerin GeoGebra programını kullanabiliyor olması gerekir.				
Ünite planını uygulayabilmek için öğrencilerin GeoGebra programını kullanabiliyor olması gerekir.				
Ünite planını uygulayabilmek için öğretmenlerin integral konusuna hakim olmaları gerekir.				

Araştırmaya dayalı öğretim				

Ünite planı öğrencilerin bilgiyi üretebilmeleri için araştırma yapmaya teşvik etmektedir.				
Ünite planı öğrencilerin bilgiyi bilmekten ziyade bilgiye ulaşma ve bilgiyi kullanma becerilerini desteklemektedir.				
Yüklenen bilgi yerine onun nerede, nasıl bulunacağını ve kullanılacağını bilme ön plana çıkmaktadır.				
Ünite planı öğrencilere bilim insanı gibi çalışmayı ve düşünmeyi öğretmeyi amaçlamaktadır.				
Ünite planı öğrencilerin bilimsel yöntemler ile yeni anlamlar, yorumlar ve bilgiler inşa etmelerini teşvik etmektedir.				
Ünite planı öğrencilerin gerçek hayat problemlerine çözüm getirmek için birbirleriyle etkileşim içerisinde tartışarak çalışmalarını teşvik etmektedir.				
Ünite planı öğrencilerin üst düzey düşünme becerilerinin gelişmesi için uygundur.				
Ünite planına göre öğrencilerin öğrenme sürecinde bizzat aktif olmaları gerekmektedir.				
Ünite planı öğrencilere uzun vadeli önemli kazanımları yerine getirmeyi hedeflemektedir.				
Ünite planı öğrencileri araştırmaya yöneltecek standart olmayan problemler içermektedir.				
Ünite planı öğrencilerin öğrenme sürecinde aktif olmalarını sağlayarak öğrencilerin konuya karşı pozitif tutum geliştirmesini sağlamaktadır.				
Ünite planı öğrencilerin kendi deneyimleri ve grup arkadaşlarının deneyimlerinden faydalanarak problemlerin çözüm yollarını araştırmaya yöneltmektedir.				
Ünite planı öğrencilerin kavramlar, deneyler ve olgular hakkında düşünmelerini, fikir yürütmelerini teşvik etmektedir.				
Ünite planı öğrencilerin muhakeme ve yorum yapmalarına rehber olmaktadır.				

Appendix 5: High School mathematics teachers' written comments on the final draft of the developed teaching unit

Teacher 1

1.1. The first comment of Teacher 1 was on the table of contents part (page 4). According to the 5E learning cycle, before "Explanation 2", there must have an "Explanation 1" part. "Explanation 2" was written two times while one of them must have been "Explanation 1". So the Teacher asked if there was an "Explanation 1" part. [ALE]

1.2. In the "Engagement 2" part (page 14), "materials needed" section included "rectangle tool on GeoGebra" which was going to be developed by teachers through given instructions in the next pages of the teaching unit. As it was not a familiar term at that part, Teacher 1 commented that there would be an image or an icon for rectangle tool, so it could be familiar for readers. [ALE], [TA]

1.3. In the "Exploration 2" part (page 15), the teacher commented that distributing both GeoGebra worksheet and the image of the inherited area on Google maps may cause confusion. But the worksheet was about inserting an image on GeoGebra. [P]

1.4. Another comment on page 15 was about the introduction of the exploration activity where students were going to estimate the area under a curve by using vertical and horizontal rectangles. The Teacher commented that a more detailed information would be given to make students understand how they will do the exploration activity. [AC],[TA]

1.5. In "GeoGebra Instructions" part (page 18) it was asked to create a list on GeoGebra by giving the instruction how to do it, but it seems that the expression caused a confusion. So, the Teacher asked whether the instruction about creating a list was given or did we assume that we know how to do it. [TA]

1.6. In the "Exploration 2" part (page 20), it was instructed to teachers to expect students to convert the area measurement that they estimated on GeoGebra to the real life area measurement using the scale that they used on Google maps. Accordingly it was instructed teachers to expect students to find an area measurement in an interval so that it becomes easy to evaluate. In that part the Teacher commented that this inference was a really good exercise. [IB], [MA]

1.7. In GeoGebra Instructions part on page 21, how to make a dynamic demonstration was instructed. The Teacher asked how to make a dynamic demonstration at that part, so a more detailed expression was needed. [TA]

1.8. In "Engagement 3" part (page 30), a comparison between finding the volume of a glass by volume measuring cup and finding the volume of the same glass by using integration on GeoGebra with the help of its 3D demonstration feature was given. The Teacher commented that this activity was really meaningful and inquiry based. [IB], [MA]

1.9. In "GeoGebra Worksheet 2" (page 40, item 7), on "finding the curve/line of best fit" part the Teacher commented that it would be better if an image showing how to do had been given. [TA]

1.10. In "GeoGebra Worksheet 2" (page 40, item 9), it was asked to choose integer "n" in the slider window. When integer was chosen in the slider window, it becomes automatically "n". The Teacher had a confusion on that part and expressed that she couldn't find "n". [TA]

1.11. For "GeoGebra Worksheet 2" the Teacher expressed that this worksheet was about finding the curve of best fit and it seems irrelevant about integral concept, so it would be given at first so that it wouldn't interrupt the alignment. [ALE],[AC],[P]

1.12. In "Worksheet 3", (page 42, item 6) how to write πon GeoGebra was asked by the Teacher. [TA]

Teacher 2

Teacher 2 gave an overall written feedback on the teaching unit after analyzing it and applying the GeoGebra activities in the teaching unit. The comments of Teacher 2 was given as follows:

2.1. The first comment of the Teacher was that it would be better if "Engagement 1" and "Exploration 1" were switched. Thus, students would be anxious when it was asked how to find the area of an irregular shape after reminding how to find the area of regular shapes. Then, the history of integral would be given by expressing that integral concept was generated because of a need. [IB], [P], [AC]

2.2. After applying GeoGebra activities, the Teacher stated that the instructions had worked, and integrating technology in the teaching unit contributed to an inquiry based teaching unit. [IB], [TA]

2.3. Teacher 2 also commented that she would use this teaching unit in her lessons. [P]

Teacher 3

3.1. The first comment of Teacher 3 was on “Unit Objectives” part (page 3). She suggested that objectives would be determined as measurable objectives. For example, instead of the word “know” in the objective “students will know the literal word meaning of integral” a more measurable word would be chosen. [AC], [ALE]

3.2. Another comment on the objectives part was that in addition to cognitive domain, affective and psychomotor domains would be included so that the teaching would be a more differentiated teaching unit. [AC]

3.3. For the “Engagement 1” part (page 6), the Teacher commented that she definitely liked the exercise that the integral concept was associated with its literal word meaning. She stated that she had used this kind of activities in her lessons about polynomials and logarithm and she experienced it really draws students’ attention. Also, she mentioned the importance of referring the history of the integral concept. [IB],[AC]

3.4. On the mind map for discussing the literal word meaning of integral (page 7), the Teacher commented that it is a useful strategy for students who have different learning profiles. [IB]

3.5. In “Engagement 1” part (page 7), a map of ancient Greece showing the part of modern Turkey showing the part where the origins of integral calculus was laid in. On this part, the Teacher stated that making connection between students’ cultural and historical background is really interesting. [IB]

3.6. In “Exploration 1” part (page 10), the Teacher suggested that using known areas to calculate the areas of irregular shapes activity is a good example which contributes to strategic competence. [IB],[MA]

3.7. In “GeoGebra Instructions” part on page 11, the Teacher said that GeoGebra commands were clear and easy to follow. [ALE], [P]

3.8. In “Exploration 1” part (page 12), teachers were going to ask students to create an irregular shape and estimate that area by using their own methods. Accordingly, some of possible student exercises were given by an image in the teaching unit. The Teacher commented that giving possible student exercises leads teachers to have a preliminary idea about what they would encounter. [IB], [AC], [MA]

3.9. In “Explanation 1” part (page 13), it was planned that students would share their findings about their methods of estimating the area of an irregular shape. The Teacher mentioned that this discourse which was oriented by remarkable questions was very important in terms of students’ expressing their knowledge by using mathematical language. [IB]

3.10. In “Engagement 2” (page 14), the Teacher found the real world setting as an interesting and attractive choice. [IB], [P]

3.11. In “Explanation 1” (page 20), teachers were going to make an explanation about what is called as “upper sum” and what is called as “lower sum”. The Teacher thought that students would make this inference. By this way, a more inductive teaching would be accomplished. [IB], [AC]

3.12. In “Exploration 4” part (page 26), making students to explore upper and lower sums by a GeoGebra applet which gives both geometrical and mathematical representation of upper and lower sum formula was valued by the Teacher as a well-thought-of activity. [IB], [P]

3.13. In “Engagement 3” part (page 29), the Teacher expressed that from beginning to this point of teaching unit, she started to think that if this teaching unit appeal to visual learners, and could it be balanced by integrating activities appealing to other type of learners as well. [AC]

3.14. Another comment on “Engagement 3” part (page 29) was about the activity about finding the volume of a glass through integration on GeoGebra by using an image of a real glass. The Teacher commented that working on an image of a real object would lead students an easy understanding, because some of the students could have difficulties in visualizing and interpreting three dimensional objects on two dimensional coordinate system. [AC], [MA], [P]

3.15. In “Evaluation” part (page 35), the Teacher suggested that ongoing assessment strategies would be integrated more during the teaching unit flow, maybe after each exploration activity. Also at this point the Teacher wondered that if they were expected to assess students’ ability on using GeoGebra, or only the mathematical outcomes would be assessed. [AC]

3.16. Another comment on “Evaluation” part was that a rubric sample would be integrated in the evaluation part. [AC]

3.17. For “GeoGebra Worksheets” section, the Teacher stated that she followed the given GeoGebra commands on those worksheets easily while applying the GeoGebra activities. [ALE]

3.18. At the end of the teaching unit, the Teacher suggested that teacher could utilize from the resources which were utilized in this teaching unit, so a references part was needed. [AC], [P]

As an overall feedback, the Teacher made the following suggestions:

3.19. Approximate time for each activity would be placed so that the teaching unit would guide teachers much easier. [P]

3.20. The “note” parts of the teaching unit were appealing and they were really such as to a guide for teachers. [ALE], [AC]

3.21. Possible mathematical misconceptions would be more integrated in the teaching unit as tips for teachers. [MA]

3.22. Being used of 5E’s learning cycle eases teachers to follow the flow. [P]

Teacher 4

4.1. The first comment of Teacher 4 was on the cover of the teaching unit. She made a suggestion for an expression on the introduction statement to be clearer to the readers. [ALE]

4.2. In “Engagement 2” part on the “materials needed” section (page 14), the Teacher had a confusion. “Worksheet 1” and “Worksheet 2” was included in the materials needed part as students were going to use those worksheets during the activity. The Teacher asked if students were expected to complete “Worksheet 1” and “Worksheet 2” before this activity, because it was written as “needed”. [ALE]

4.3. In “Exploration 2” part (page 15) teachers were expected to distribute the digital version of an image taken from Google maps. The Teacher suggested that students could find that area in expected scale on Google maps on their own which is a learning outcome as well, otherwise the Teacher asked how the teachers would distribute the image. [P], [IB]

4.4. In a “note” section of “Exploration 2” (page 16), it was stated that the rectangle tool was going to be developed by the teacher using the given instructions. The instructions were given in GeoGebra Worksheet 4, but the note did not refer to this worksheet so it caused a confusion. [ALE]

4.5. Again in “Exploration 2” part (page 16), the students were expected to create a rectangle by using the rectangle tool. The Teacher suggested that more detail would be given about how to create a rectangle. [AC]

4.6. In “Exploration 2” part (page 16), students were expected to estimate an area by using only one rectangle, the two rectangles and etc. The Teacher asked if there was a part in the worksheets that students could record their findings on, and then make discussions about those findings. [ALE]

4.7. Another comment in “Exploration 2” part (page 17), was that instead of asking students “could you make a better estimation?”, “how could you make a better estimation” would be asked, so that students would think critically. [AC], [IB]

4.8. In “Exploration 2” part (page 18), students were asked to make an estimation of the given area by using as many rectangles as they could and then compare their findings with each other. The Teacher suggested that a lower bound for the minimum number of rectangles would be given by the teacher, so that the teacher would avoid that while a student works on three rectangles and another one works on twenty rectangles. [IB], [MA]

4.9. In “GeoGebra Instructions” section on page 21, the Teacher suggested that the instructions would be given in more detail. [ALE], [TA]

For example, it was written that

For upper sum, write *UpperSum* in the input bar and choose UpperSum[<Function>, <Start x-Value>, <End x-Value>, <Number of Rectangles>]

This instruction would be rewritten as

For upper sum, writw *UpperSum* in the input bar so that alternative commands starting by *UpperSum* reveals in the input bar. Choose UpperSum[<Function>, <Start x-Value>, <End x-Value>, <Number of Rectangles>] from those alternative commands.

4.10. In “Exploration 4” part (page 28), teachers were expected to distribute a GeoGebra applet to students. The Teacher asked how the teacher was expected to distribute that applet. [ALE]

4.11. In a “note” section on page 26, the Teacher suggested that instead of writing “students must have already learned the limits topic”, it would be written as “It is assumed that students know the limits topic”. [ALE]

4.12. For the “note” section on page 28, the Teacher commented that English writing should have been revised. [ALE]

4.13. In “GeoGebra Worksheet 1” for item 4 (page 37), writing “click on a point on the graphics view-” caused a confusion, because there were not a point to be clicking on. So it would only be written as “click on graphics view-” [TA], [ALE]

4.14. In “GeoGebra Worksheet 2” (page 39), the Teacher commented that the first command in item 3 did not work. [TA]

4.15. In “GeoGebra Worksheet 1” (page 39), it was written “Input line” instead of “Input bar” in item 4, so it caused a misunderstanding. [ALE], [TA]

4.16. In “GeoGebra Worksheet 1” (page 39), an information as in the comment 4.9 was needed for item 6 as well. [ALE], [TA]

4.17. Teacher 4 suggested that “Worksheet 3” and “Worksheet 4” would be replaced, because “Worksheet 4” was used before the “Worksheet 3”. [P], [ALE]

4.18. Teacher 4 also suggested that “how to find the volume of a glass like in Figure 6 by using integration on “GeoGebra” would be asked as an inquiry question at the end of the teaching unit. [IB], [P]

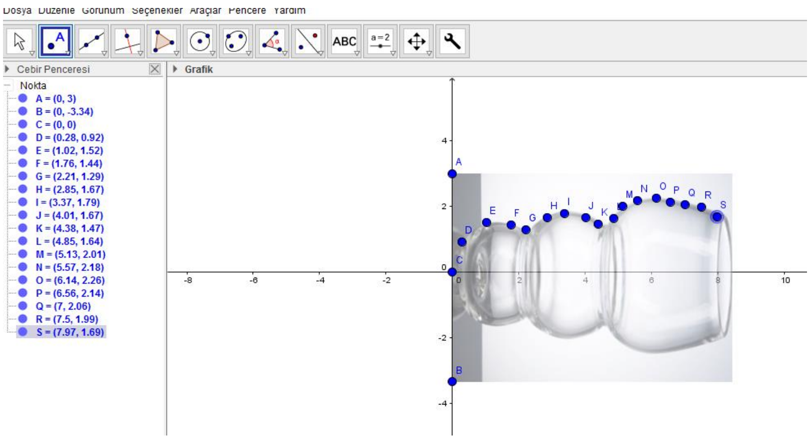


Figure 14: A glass to be calculated its volume by using Integration on GeoGebra

Teacher 5

5.1. In “Exploration 2” part (page 17), the Teacher stated that a reference to the limits concept would be given when students were expected to estimate the area by using rectangles. [MA]

5.2. In “Exploration 5” page (page 31), the Teacher suggested that a more concrete example would be given instead of a glass. The researcher had thought that a real glass was concrete enough. [AC]

5.3. Again in “Exploration 5” page (page 31), the Teacher suggested that the formula

$$\pi f(x_i)^2 \Delta x$$

would be examined by students. The Teacher didn’t think that students would not deduce that formula by the glass example. According to the Teacher, firstly the generation process of 3D shapes’ mathematical formula would be given by concrete materials and then this part of the teaching unit would be given. So the Teacher thought that to this point the flow was going fluently, but this part was a bit disconnected. [MA]

Overall, the Teacher 5 made the following comments:

5.4. The flow of the teaching unit was well planned. [AC], [ALE]

5.5. It seemed that generally the students follow the teaching unit by the teacher’s directions. It would be provided that students make more inquiry and think more critically. Teachers might ask only key questions to drive students make inquiry. [IB]

5.6. GeoGebra worksheets were designed as easy to follow. Teachers or students who had never used GeoGebra would be master in this subject with the help of those worksheets. [TA]

Teacher 6

6.1. The first comments of the Teacher 6 was on the table of contents section of the teaching unit. The Teacher thought that the order of 5E’s learning cycle phases were confusing. The flow would be designed so that each activity was main heading and 5E’s learning cycle phases would be reordered as they were subheadings of each activity. [ALE], [AC], [P]

6.2. In “Engagement 1” part (page 6), one of the instruction was written in a different font from the other fon used in entire teaching unit. The Teacher drew attention to that point. [ALE]

6.3. In “Engagement 1” part (page 8) the Teacher thought that it would be better if the historical note was written in italic font. [ALE]

6.4. In “Exploration 1” part (page 9), teachers were expected to distribute a handout included regular geometric shapes with given lengths. The Teacher suggested that a sample handout would be given in the teaching unit. [AC]

6.5. In “Exploration 1” part (page 12), it was stated that students would easily find the areas of polygons by using Poly command on GeoGebra. The Teacher suggested that more detail about how to use Poly command would be given. [TA]

6.6. In “Exploration 2” part (page 15), students were expected to insert an image taken from Google maps to GeoGebra and replace it as “Eskişehir yolu” was on x-axis. The Teacher commented that It was difficult to place “Eskişehir yolu” on x-axis, students need to resize the image by using the corners of the image. [TA]

6.7. In “Exploration 2” part (page 16), teachers were expected to create a rectangle tool through given GeoGebra worksheet. The Teacher suggested that it would be better if visuals for rectangle tool were given at this part of the teaching unit. [ALE]

6.8. In “Exploration 2” part (page 17), the curve of best fit of the road in the image was going to be a parabola. At this point, the Teacher asked “What if the curve was not a parabola? Maybe there were two different functions combined? What happens if this was the case?” The Teacher suggested that those questions would also be asked to students as inquiry questions. [IB], [AC]

6.9. For the GeoGebra Instructions section on page 18, the Teacher pointed that the Poly command did not work with the Turkish version, so the teachers and students must check that the language is in English. [TA]

6.10. In “Exploration 2” part (page 20), students were expected to calculate the real area by using the ruler in the Google maps. The Teacher asked if the students knew how to use Google maps, otherwise an explanation would be needed. [AC], [P]

6.11. In “Engagement 3” part (page 29), the Teacher suggested that giving rationale to the students about the reason why we need to calculate the volume of a glass could be helpful to attract them. Accordingly a story or a scenario could be written in this part. [P], [IB]

6.12. At the beginning of the “GeoGebra Worksheets” section (page 36), the Teacher proposed that more screen shots of GeoGebra window (such as in page 49 and 50) would be added to show teachers where they are and not to be lost with the instructions. [TA]

6.13. In “GeoGebra Worksheet 2” (page 39), the Teacher stated that the “*FitPoly*” command did not work with the Turkish version so the teachers and students must check that the language is in English. [TA]

6.14. In “GeoGebra Worksheet 2” (page 39), the Teacher suggested that an additional item would be added after item 5 stating that “The list of the points must be seen on the Algebra View Window”. [TA]

6.15. In “GeoGebra Worksheet 2” (page 39), the Teacher proposed that a note should be added which would express that “You need to write the points and degree of polynomial without “<” “>” symbols otherwise it does not work”. [TA]

Teacher 7

7.1. In “Engagement 1” part (page 7), the Teacher valued finding literal word meaning of *integral* as a good exercise. She suggested that she would spare a short time for this activity, so it would be better if approximate time for each activity was added. For the same activity, the Teacher had questions such as “Students will come up with a consensus related to what? What are the key questions to be asked by teachers while students were coming up with a consensus? Accordingly, the Teacher suggested that the questions such as “Which of those meanings are related to calculus?” “We are examining the rate of change in calculus, so which of those meaning refer to that aim?” [AC], [IB]

7.2. In “Exploration 2” part (page 17), as Teacher 4 stated in 4.7, Teacher 6 stated that the question “could you make a better estimation?” was a yes/no question, so instead of that, questions such as “What do you observe when you increase the number of rectangles? With how many rectangles would you make the best estimation” would be asked. [IB], [P]

7.3. In “Explanation 3” part (page 24), the “formula” term was used. The Teacher suggested that instead of “formula”, “formulae” could be used, because “formulae” is correct term in mathematics. [MA], [ALE]

7.4. In “Elaboration 1” part (page 28), the Teacher thought that the teaching unit gave the integration concept relating it to area concept, so while students were solving the exercise questions students could fall into a misconception thinking that a result of an integration cannot be negative. So an explanation would be made at this point. [MA], [AC], [P]

7.5. In “Exploration 5” part (page 31), the Teacher suggested that low ability students might not remember how to calculate the volume of a cylinder. So a reminder could be added. [MA], [AC], [P]

7.6. For the “Evaluation” part (page 35), the Teacher thought that a more detailed rubric could be used in this part. [AC], [P]

7.7. In “GeoGebra Worksheet 2” (page 39), the Teacher suggested that it would be more understandable when instructions in item 6 and item 7 were given in one item. [TA]

7.8. Overall, the Teacher stated that this teaching was a practical teaching unit which teaches students integral, area and volume concepts in a step by step flow. The Teacher also suggested that apart from the objectives given in the teaching unit, students could also be driven to explore displacement by a velocity-time graph, so that it would present coherence with derivative concept. [AC], [P], [IB]

Teacher 8

8.1. The first comment of Teacher 8 was on the introduction part (page 2). She corrected an expression according to American Psychological Association (APA) style. [ALE]

8.2. In the objectives part (page 3), the Teacher suggested that action verbs should have been used while writing the objectives. [AC], [ALE]

8.3. In “Engagement 1” part (page 7), the Teacher commented that after students found the literal word meanings of “integral”, a formal definition should have been given by the teachers. [AC]

8.4. Another comment in “Engagement 1” part (page 7) was that before giving formal information about history of integral, it could be asked to students. [AC], [IB]

8.5. In “Exploration 1” part (page 10) the Teacher stated that there was a fast transition between giving regular shapes and the irregular shape. So the Teacher suggested that a more difficult regular shape could have been given before giving irregular shape. [AC], [P]

8.6. In “Engagement 2” part (page 15), students were expected to answer the question “How would you determine the area of your estate?” after the image taken from Google maps was shown. The Teacher asked how the student answers were going to be evaluated. [AC], [P]

8.7. In “Exploration 2” part (page 15), the Teacher suggested that an information about where the activity was going to be held (in classroom or in computer laboratory) was needed. [AC]

8.8. In “GeoGebra Instructions” part (page 21), the teacher asked whether the teacher was going to give those instructions verbally, or was she expected to show the instructions on computer. So an additional information was needed about this issue. [AC]