ÖZGE KESKİN

2020

A CASE-BASED APPROACH TO REFLECTIVE PRACTICE OF PRE-SERVICE SECONDARY MATHEMATICS TEACHERS FOR DEVELOPING A HOLISTIC PERCEPTION OF TEACHING

A DOCTORAL DISSERTATION

BY

ÖZGE KESKİN

THE PROGRAM OF CURRICULUM AND INSTRUCTION

İHSAN DOĞRAMACI BİLKENT UNIVERSITY

ANKARA

FEBRUARY 2020

To my beloved grandfather Halil Kabakcı

A Case-Based Approach to Reflective Practice of Pre-service Secondary Mathematics Teachers for Developing a Holistic Perception of Teaching

The Graduate School of Education

of

İhsan Doğramacı Bilkent University

by

Özge Keskin

In Partial Fulfillment of the Requirements for the Degree of

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GRADUATE SCHOOL OF EDUCATION

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Özge Keskin

January 2020

I certify that I have read this doctoral dissertation and have found that it is fully adequate, in scope and in quality, as a dissertation for the degree of Doctor of Philosophy in Curriculum and Instruction.

Prof. Dr. Alipaşa Ayas (Supervisor)

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Assoc. Prof. Dr. M. Sencer Çorlu, Bahçeşehir University (Examining Committee Member)

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Asst. Prof. Dr. Seyit Koçberber (Examining Committee Member)

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Approval of the Graduate School of Education

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ABSTRACT

A CASE-BASED APPROACH TO REFLECTIVE PRACTICE OF PRE-SERVICE SECONDARY MATHEMATICS TEACHERS FOR DEVELOPING A HOLISTIC PERCEPTION OF TEACHING

Özge Keskin

Ph. D. in Curriculum and Instruction

Supervisor: Prof. Dr. Alipaşa Ayas

February 2020

The aim of this study is to support learning to teach processes of preservice secondary mathematics teachers by providing them reflective practice opportunities using a case-based approach. The research was conducted in two stages: design and development of a case-based discussion module and implementation.

In the first stage, semi-structured interviews were conducted with ten teachers in order to ascertain the perceptions on early career challenges of secondary mathematics teachers who graduated from a MA program with a teaching certificate. It was determined that mathematics teachers experienced challenges related to different dimensions of teaching mathematics, and these challenges were associated with their beliefs, perceptions, and expectations before starting their careers. The findings indicate the need for providing preservice teachers with opportunities to reflect on challenges they may encounter, consider how different aspects of teaching interact, and elaborate on various reasons and possible solutions for those challenges. Aligned with the need, case-based pedagogy and productive reflection constituted the theoretical framework as two important elements. Within this framework, a case-based discussion module (CBDM) was developed. In the second stage, CBDM was implemented with eight preservice secondary mathematics teachers enrolled in the same program. The data collected was analyzed within a multi-dimensional analytical framework. The findings reveal that the CBDM provided a platform to discuss several aspects of teaching as well as to link these aspects and connect their reflections to their personal experiences and theory. Participants perceived this experience as a relevant, engaging, and awareness-increasing practice with potential positive reflections on their teaching.

Keywords: Challenges in early career, Mathematics teacher education, Productive reflection, Case-based pedagogy, Mathematical knowledge for teaching, Learning to teach

ÖZET

LİSE MATEMATİK ÖĞRETMEN ADAYLARININ BÜTÜNCÜL ÖĞRETİM ALGILARININ GELİŞTİRİLMESİ ÜZERİNE VAKA TEMELLİ YANSITICI DÜŞÜNME YAKLAŞIMI UYGULAMASI

Özge Keskin

Doktora, Eğitim Programları ve Öğretim

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Bu çalışmanın amacı, vaka temelli bir yaklaşımla, lise matematik öğretmen adaylarına yansıtıcı uygulama fırsatları tanıyarak öğretmeyi öğrenme süreçlerinin desteklenmesini sağlamaktır. Çalışma iki aşamalı olarak gerçekleştirilmiştir. Birinci aşama vaka temelli bir tartışma modülünün tasarım ve geliştirilmesi, ikinci aşama ise bu modülün uygulanmasıdır.

Çalışmanın ilk aşamasında, yüksek lisans düzeyinde öğretmen eğitimi yapan bir programdan mezun olan matematik öğretmenlerinin kariyerlerinin ilk yıllarında yaşadıkları zorluklar ile ilgili algıları ortaya çıkarılması amacıyla on öğretmenle yarı yapılandırılmış görüşmeler gerçekleştirilmiştir. Matematik öğretmenlerinin mesleğin ilk yıllarında öğretim süreçlerinin farklı boyutlarına ait sorunlar yaşadıkları ve bu sorunlarla, kariyerlerine başlamadan önceki mesleğe ilişkin inanç, algı ve beklentilerinin ilişkili olduğu belirlenmiştir. Bu durum, öğretmen adaylarına mesleğe başladıklarında karşılabilecekleri problemler ile ilgili düşünme, öğretmeye ilişkin farklı boyutların etkileşimini fark ederek, karşılaşılan problemlerin sebeplerini tartışma ve bunlara çözüm üretme fırsatı sağlanması gerekliliğini ortaya koymuştur. Bu gereklilik göz önüne alındığında, vaka temelli pedagoji ve üretken yansıtma iki önemli unsur olarak teorik çerçeveyi oluşturmuştur. Bu çerçeve kapsamında, çalışmanın ikinci aşamasında uygulanmak üzere, vaka temelli bir tartışma modülü geliştirilmiştir. Bu aşamaya, çalışmanın yapıldığı programda öğrenim görmekte olan sekiz lise matematik öğretmen adayı katılmıştır. Bu süreçte toplanan veriler çok boyutlu bir analitik çerçeve içinde analiz edilmiştir. Vaka temelli tartışma modülünün katılımcılara matematik öğretmeye ilişkin farklı aktörler ve matematik öğretmeye ilişkin beş ana boyutta bir çok konuyu tartışma imkanı sağladığı ve katılımcıların bu boyutları birbirleriyle ilişkilendirerek, yansıtmalarını kendi deneyimleriyle ve teoriyle bağlantılandırdıkları tespit edilmiştir. Katılımcılar, vaka temelli tartışma modülü deneyimini amaca uygun, ilgi çekici, öğretmenliklerine olumlu katkıda bulunma potansiyeli olan ve farkındalık yaratan bir süreç olarak tanımlamışlardır.

Anahtar Kelimeler: Mesleğin ilk yıllarındaki zorluklar, Matematik öğretmen eğitimi, Üretken yansıtma, Vaka temelli pedagoji, Öğretmek için matematik bilgisi, Öğretmeyi öğrenme

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ABBREVIATIONS

CBDM: Case-based Discussion Module
CCK: Common Content Knowledge
CMO: Classroom Management and Organization
CS (Number): Case Scenario (From 1 to 6)
D_C (Number): Discussion of Case Scenario (From 1 to 6)
FI: First Interview
HCK: Horizon Content Knowledge
IBDP: International Baccalaureate Diploma Program
KCC: Knowledge of content and curriculum
KCS: Knowledge of content and students
KCT: Knowledge of content and teaching
LI: Last Interview
MKT: Mathematical Knowledge for Teaching
PT (Number): Pre-service teachers involved in the research (From 1 to 8)
SCK: Specialized content knowledge
T (Number): Graduates involved in the research (From 1 to 10)
T_C (Number): Post-Discussion Written Task (From 1 to 6)
TI: Teacher Identity

CHAPTER 1: INTRODUCTION

Introduction

> To a music lover watching a concert from the audience, it would be easy to believe that a conductor has one of the easiest jobs in the world. There he stands, waving his arms in time with the music, and the orchestra produces glorious sounds, to all appearances quite spontaneously. Hidden from the audience, especially from the musical novice, are the conductor’s abilities to read and interpret all of the parts at once, to play several instruments and understand the capacities of many more, to organize and coordinate the disparate parts, to motive and communicate with all of the orchestra members. In the same way that conducting looks like hand-waving to the uninitiated, teaching looks simple from the perspective of others. (Bransford, Darling-Hammond, & Lepage, 2005, p.1)

It is widely acknowledged that student success in school is related to the quality and effectiveness of teaching. It is believed that high-quality teaching will ensure student success at school and will consequently help students be successful in later stages of their lives (Hattie, 2009; OECD, 2012; Sanders, Wright, & Horn, 1997). Therefore, educating teachers for high-quality teaching is the primary goal of all teacher education programs (Loughran, 2006; Yıldırım, 2013). The main question is how to accomplish the goal of helping the growth of high-quality teachers.

There is no straightforward answer to the question of educating effective teachers. It would be unrealistic to expect pre-service teachers to graduate from a teacher education program fully equipped to teach. *Learning to teach* is a continuous process and teacher education programs would not suffice for fully equipping teachers with the skills and knowledge needed to be qualified teachers by the time they graduated (Hammerness et al., 2005). Therefore, the critical goal of the teacher education process is to help pre-services to become adaptive experts. In other words, teacher education would aim for the development of pre-service teachers who would become aware of themselves as teachers (in terms of knowledge, beliefs, concerns, challenges, expectations, etc.) when they start their profession and take initiatives accordingly in the unpredictable and complex world of teaching.

Current research aims to shed light on the *learning to teach* processes of pre-service secondary mathematics teachers with a holistic approach. In this section, the background of the study is discussed, and problem statement, purpose, research questions and the significance are given.

Background

The issue of complexity

The analogy of an orchestra conductor given at the beginning of the chapter tells a lot about the perception related to the teaching profession from the perspective of others. This analogy gives a clear explanation of the idea that teaching is a complex profession in contrast to its deceptively simple perception (Grossman, Hammerness, & McDonald, 2009). As Sullivan and Mousley (2001) asserted, teaching is complex and multidimensional, requiring active decision making rather than just implementing standard directions, plans, and routines. Teachers have roles and responsibilities at the student level: initiating and managing learning processes, responding effectively to learning needs of individuals, integrating formative and summative assessments; at the classroom level: integrating students with special needs, cross-curricular emphasis; at the school level: working and planning in teams, evaluations and systematic improvement, information and communication technology usage in teaching and administration, management and shared leadership; finally at the community level: providing professional advice to parents, and building community partnership for learning (OECD, 2005). Regarding the complexity of the teaching profession, educating people for this profession is not an easy task (Clarke & Hollingsworth, 2002). Therefore, it is neccessary to scrutinize the teacher education practices with the complexity of the teaching profession in mind.

The developments in teacher education are parallel to developments of educational philosophies and theories. Korthagen (2017) categorized the trends into three models: theory-to-practice, practice-to-theory, and a realistic approach to teaching. The first model is defined as traditional, theory centered teacher education paradigm which is also called theory-to-practice approach (Carlsen, 1999). The idea behind theory-to-practice approach is the assumptions that providing the teachers relevant theory about teaching and learning would suffice and make a change in teachers' behaviors so that they would apply the theories in their classrooms. To name it differently, it is an approach that puts theory into the center and it creates a dichotomous view of theory and practice (Grossman, Hammernes, & McDonald, 2009). The dominance of theory-to-practice approach in educating teachers and the handicap of this approach as *theory-practice gap* provoked teacher educators to find strategies for making theory more meaningful to teachers. The second model, practice-to-theory approach, evolved as lessons derived from the failure of theory-to-practice approach. Therefore, practice was put at the center and teacher education took place more dominantly in the partner schools. However, this practice- to-theory approach also had pitfalls: the contexts of the schools were not the ideal places because they would generally be in traditional settings and without theorizing and relating the practice to guiding principles, practice-to-theory would result in a lack of rationale behind the teaching. Now, the question one needs to ask is how to define the central objectives of making sense of teaching. Lin and Cooney (2001) asserted the following principles of teacher education aligned with the mission of helping pre-service teachers making sense of teaching:

i. To elaborate on the complexity of teaching
ii. To represent and bring real teaching situation into teacher education programs,
iii. To motivate student teachers to the need and advantage of conceptualizing and theorizing teaching.
iv. To design strategies and develop tools for teachers to make sense of a particular aspect of teaching.
v. To design teacher education programs in which the research findings and processes facilitate teachers' professional developments
vi. To develop theories that help conceptualize the complexity of teaching (p.4).

These six principles lead to the idea that there should be a more balanced approach in terms of theory and practice, and the teacher, assuming that he/she had already had a knowledge base about teaching, should be at the center when *learning to teach* practices are to be designed. This approach brought us to realistic teacher education (Korthagen, 2011). Several factors affect teachers' behaviors or decision and these factors could be categorized under cognitive, affective, and motivational none of which the teacher would be aware. This view points out the unpredictable nature of learning outcomes of teacher education as any attempt for *learning to teach* would have a different effect on pre-service teachers, as they have different backgrounds, concerns, beliefs, strengths-weaknesses, and goals (Fullan, 2007). Therefore, professional development opportunities designed for pre-service teachers should neither follow a one-shot or one size fits all approaches (Korthagen, 2017). So putting the teacher, person into the center, the attempts that would make in the direction of integrating the theory and practice would be more meaningful. Realistic teacher education serves this idea, considering the gap between theory and practice. Korthagen (2001) asserted five guiding principles for realistic teacher education as:

> i. The approach starts with concrete practical problems and the concerns of student teachers in real contexts.
>
> ii. It aims at the promotion of systematic reflection by student teachers on their own and their pupils' wanting, feeling, thinking and acting, in the role of context, and the relationships between those aspects.
>
> iii. It builds on the personal interaction between the teacher educator and the student teachers and on the interaction amongst the student teachers themselves.
>
> iv. It takes the three-level model of professional learning into account, as well as the consequences of the three-level model for the kind of theory that is offered.
>
> v. A realistic program has a strongly integrated character. Two types of integration are involved: integration of theory and practice and the integration of several academic disciplines (p.38).

To have a more realistic stance, any *learning to teach* practice must be designed with an acknowledgment that beginning teachers are not empty vessels. They had a pre-existing schema about teaching which was formed over several years in their own schooling. Experiences in their own schooling would create the *apprenticeship of observation* problem (Lortie, 1975). This problem refers to the idea that pre-service teachers' images belong to past experiences has a significant influence on their *learning to teach* processes although they were tried to be equipped with theory-driven, novel and effective teaching by teacher education practices. Before digging into the problems in a reform-oriented context, it should first be investigated how the pre-service will react to real classroom situations with their existing and continuously evolving schemas.

The complex nature of teaching and the importance of making challenging, spur-of-the-moment decisions in unpredictable environments like schools should be introduced to pre-service teachers so they can digest and discuss with others systematically and develop habits of mind in perplexing situations. It goes without saying that neither all possible problematic situations nor what to do in those circumstances could be presented; however, one may help the pre-service teacher become aware of the complexities and develop reflective skills for more sound decisions regarding the unpredictable nature of teaching. As Mason (2002) asserted awareness is all educable and mentioned different levels of awareness both in mathematics and in mathematics teaching and relates them to the process of noticing that involves systematic reflection on acts or issues (Potari, 2013).

Practices should be shaped regarding the individual's backgrounds, beliefs, and needs. Rather than attempting to instill top-down concepts and beliefs, prospective teachers' beliefs and knowledge should be revealed and restructured for making sense of teaching. Otherwise, adopted beliefs would be abandoned at first and core-beliefs became dominant when faced with realities of the classroom. Hence, it is necessary to create awareness about the intertwined and complex structure of the profession and to allow discussion of the so-called duality of theory and practice. Consideration of the complexity of classroom practice situations can raise awareness and suggest alternative ways of resolving the situations. In this respect, pre-service teachers would come closer acknowledging teachers as intelligent, thoughtful, and decision-making professionals (Lin & Cooney, 2001). This acknowledgment would also have cultural reflections since, in many countries including Turkey, the teaching profession is perceived as a lower-status occupation (Ingersoll & Collins, 2018; Ünsal, 2018).

To shape teacher education in this direction, it has to be acknowledged what is missing in the current practices of educating future teachers. Although the quality of teaching is strongly related to initial teacher education, the experiences of beginning teachers after their initial teaching training stand as one of the important factors affecting teachers' performance throughout their career (Darling-Hammond, 1999; Hill, Rowan, & Ball, 2005; Wayne & Youngs, 2003). In addition, detailed analysis and consideration of the complexity of teaching situations drawn from classroom practice can both raise awareness of dynamic contexts and suggest alternative ways of resolving issues arising from the inherently intricate nature of teaching. To find the missing points or the so-called gap between theory and practice, investigating early career experiences of teachers would be a realistic step to start with. Now the question that should be asked is what challenges teachers face in their early careers and how these challenges could be used for the growth of future teachers.

Teachers' early career challenges

The literature on challenges that beginning teachers face showed that they had to cope with many difficulties at the same time (Fantilli & McDougall, 2009). Veenman's (1984) international review of perceived problems among beginning teachers comprised findings that included challenges in managing disruptive behavior in the classroom and overall classroom management, motivating students, dealing with individual differences, assessing students' work and relationships with parents. Related to these difficulties, the researcher indicated that consistency in these problems should be expected across both time and differently structured education systems. Lack of personal and emotional support, obtaining instructional resources and materials, planning and managing instruction were some of other findings when novice teachers' early career challenges were examined (Gordon & Maxey, 2000).

Moreover, similar studies conducted on novice teachers' early career experiences in Turkey revealed results consistent with the studies conducted elsewhere. These studies revealed that classroom management was one of the areas that challenged novice teachers (Akın, Yıldırım, & Goodwin, 2016; Gergin, 2010; Kozikoğlu, 2016; Taneri & Ok, 2014). For example, a comprehensive research that investigated the induction period of 465 novice teachers from randomly selected eight provinces of Turkey illustrated that the most frequently reported difficulties were heavy workload, low social status and perceived identity, problems in relationships with the school principals and inspectors, and problems in classroom management in that order (Öztürk & Yıldırım, 2013). In another study, it was found that novice teachers were challenged because of insufficient physical structure and facilities of the schools that they work in and classroom management. Besides, it was also highlighted that the novice teacher had to cope with a heavy workload (Kozikoğlu, 2016). Studies conducted specifically on the challenges that mathematics teachers face in Turkey were limited to middle school level. In addition to the complications that were found in other studies like classroom management or time management, challenges peculiar to a middle school novice mathematics teacher originated from the national curriculum context and its effect on teaching practices (Haser, 2010). Lack of content and pedagogical content knowledge, difficulty in implementing student-centered teaching practices and difficulty in use of alternative teaching methods were found to challenge the novice middle school mathematics teachers had to deal with (Yanik, Bağdat, Gelici, & Taştepe, 2016).

It is acknowledged that all these troubles have reflected negatively on many different aspects of the work of beginning teachers. To begin with, challenges that teachers faced during early career led to high attrition rates in many countries, including the U.S, Australia, England, and China (Department for Education and Skills, 2005; National Commission on Teaching and America's Future, 2003). However, attrition was not a major problem in Turkey as they chose to stay in the teaching pipeline with high burnout rates. Furthermore, the burnout syndrome experienced by novice teachers was mentioned in many studies (e.g. Fisher, 2011; Gavish & Friedman, 2010). Lack of appreciation and professional recognition from students and other stakeholders, and lack of support from colleagues were found to be the factors that contribute to burnout of teachers in their early careers (Gavish & Friedman, 2010). For instance, in Turkey, beginning teachers faced burnout due to several reasons, including lack of positive feedback from students and lack of support from colleagues (Bümen, 2010; Gündüz, 2005; Tümkaya, 1996). Another issue was that the quality of instruction and classroom learning environment were additional areas of concern in many Organization for Economic Co-operation and Development (OECD) countries, including Turkey. According to the results of Teaching and Learning International Survey (TALIS) 2008, beginning teachers reported lower levels of positive classroom climate, combined with greater losses of time during instruction when compared to experienced teachers (OECD, 2009).

Besides the short-term effects of early-career issues like attrition and burn out, it should be acknowledged that these challenges would have effects in the long run. As the early experiences of teachers shape their development, these challenges not only influence their effectiveness in their initial years but also their effectiveness throughout their careers (Gordon, Kane, & Stager, 2006). The problems that beginning teachers face in the classroom during initial years of teaching makes stakeholders question the effectiveness of teacher education programs in Turkey (Çakıroğlu & Çakıroğlu, 2003; Çorlu, Capraro & Capraro, 2014). After the year 1997, the Higher Education Council (HEC) in Turkey developed a new faculty-school partnership including school experience and teaching practice courses (Akşit & Sands, 2006). However, the amount of time spent in schools and number of lessons taught by pre-service teachers were still not adequate (Kocadere & Aşkar, 2013). Research conducted on the challenges that novice teachers faced in Turkey found inadequate preparation of pre-service teachers in terms of quality and quantity of school experiences that the novices had before entering the profession (Kozikoğlu, 2016). Özcan (2012) offered a two-year teacher preparation program together with a master's degree for carefully selected applicants who already had a bachelor's degree. In Turkey, the quantity of such programs is very limited. The *learning to teach* experiences of teachers who have graduated from a practice-based program accompanied by substantial theoretical courses would give insights to shape both teacher education courses and teacher education policies.

The early experience of teachers, therefore, shapes their development, not only influencing their effectiveness in their initial years but their effectiveness throughout their careers (Gordon, Kane, & Stager, 2006). Although teacher education programs provide theoretical training about some of the problematic areas discussed above (like classroom management, assessment etc.) or practicum course, it does not prevent them from experiencing many problems in these areas. The problem would be that many teacher education programs consist of a collection of separate courses in which theory is presented, in other words, it would be a problem of departmentalized structure of teacher education programs (Barone et al., 1996; Çorlu, 2012). It would be a difficulty for beginning teachers who graduated from these programs to link these separate parts of the theory during practice; in other words, there occurs a perceived gap between theory and practice (Korthagen, 2005). Beginning teachers had difficulties in transferring their knowledge to the practice. Given the variety of problems, the simultaneity of the occurrence of these and the consequences of these problems underline the complexity of teaching.

To make the pre-service teachers familiar with the realities and the complex nature of teaching, teacher education programs should investigate ways of raising awareness and giving opportunities to support *learning to teach* processes. Giving teacher education a more realistic stance, field experiences, reflective field logs, case methods and microteaching are currently being used. However, to help pre-service teachers engage in realities without keeping them away from theory, their existing beliefs, backgrounds, reasons for becoming a teacher, and concerns should also be taken into account.

In order to reach a theory-practice balance with positioning the pre-service teachers in the center, reflective practice should be thought as a glue that brings yin and yang, as an antidote to the perceived duality of theory and practice. Teacher education programs that can relate the theory with practice were found to be more successful in terms of raising good quality teachers and the link could be bridged by reflective practice (Korthagen & Kessel, 1999).

The roots of reflective practice in education go back to Dewey's construction of underlying mechanisms of thinking. John Dewey who is pivotal to the development of the idea of reflection, defined it as "[t]he active, persistent and careful consideration of any belief or supposed form of knowledge in the light of the grounds that support it and the further conclusion to which it tends" (1933, p.6). According to Dewey, there are two elements in reflective thinking: problems that puzzle and challenge the mind, and inquiry. Schön (1983) took this concept to professional practice and defined reflection-in-action and reflection-on-action. Reflection-in-action could be defined as kind of an unconscious reflection on the action while you are performing it, and making adjustments or changes according to the context or problem. However, reflection-on-action occurs after the practice and it is a more conscious reflection and a critical analysis of action. In order to reflect on problematic instances, one should notice it at first. Here this recognition involves identifying aspects in a teaching and learning situation, linking those to the broad principles of teaching and understanding it in its context (van Es & Sherin, 2002).

To provide a platform for reflection, the use of case-based pedagogy in teacher education was found to be effective (Levin, 1995; Merseth, 1996; Moore-Russo & Wilsey, 2014; Shulman, 2004). The case method provides a more demanding, engaging, intellectually exciting, and stimulating reflecting experience for pre-service teachers, all of which is effective in terms of bridging the gap between theory and practice. It helps pre-services think like a teacher (Shulman, 1992). Case-based pedagogy in teacher education stimulate personal reflection and develop habits of reflection and skills of self-analysis (Richert, 1992).

The perceived theory and practice gap as well as readiness levels of pre-services are a concern of many scholars in Turkey and abroad (Bulut, Demircioğlu, & Şimşek, 1995; Çakıroğu & Çakıroğlu, 2003; Korthagen & Kessel, 1999). There is a need to elaborate on early career challenges and make use of this knowledge to improve practices for pre-service teachers to help them reflect on the complexity of teaching during their training. Regarding the lack of studies focused on secondary mathematics teachers' early career challenges and utilizing these challenges for improving pre-service mathematics teachers, there is a need in providing evidence on the effectiveness of alternative applications aimed to improve the quality of secondary mathematics teacher education by taking a reflective and realistic lens gains importance.

Problem

Regarding the problem of making teacher education more realistic for pre-service mathematics teachers, which takes the individual at the center, there is a lack of practice and research highlighting these practices.

The first gap in the literature is to reveal the challenges that the early career mathematics teachers face in order to understand what reality should be presented to pre-service teachers.

The importance of bridging the link between theory and practice was underlined by many researchers (Korthagen & Kessel, 2005; Ball, 2000). However, there is a lack of research that provides evidence on how the problem of perceived theory-practice gap in mathematics teacher education might be solved. The problem addressed in this dissertation is how we can foster the development of pre-service mathematics teachers in a way to help them to start their teaching careers with an awareness of the complex nature of teaching.

Purpose

Regarding all the arguments mentioned above, in this dissertation, a case-based pedagogy with a productive reflection framework is proposed as a realistic teacher education practice. Firstly, the process of designing the case-based discussion module (CBDM) aligned to construct realistic teacher education practices was shared. Following the design, the implementation process and the experiences of the pre-service mathematics teachers in CBDM were presented to reveal the relevance and utility of this practice.

To accomplish the main aim of this dissertation which was to provide a comprehensive analysis on the reflective processes of pre-service mathematics teachers' *learning to teach* mathematics with the help of case-based pedagogy, firstly the profile of the group was portrayed with the help of semi-structured interviews and documents related to participants' characteristics. Second, in-depth and multi-faceted qualitative analysis was conducted to reveal the essence of the reflective experiences of the participants during the case-based pedagogical experience.

This dissertation sought evidence on how case-based pedagogy serves for a realistic teacher education which would help pre-service teachers to construct a more connected and complex schema of teaching mathematics.

Research questions

The research is organized in two stages: the design and development of the Case-Based Discussion Module (stage 1) and the implementation of the Case-Based Discussion Module (stage 2).

To provide a comprehensive analysis of reflective processes that pre-service mathematics teachers went through via case-based pedagogy, the following questions in this dissertation were explored with specific attention to how theory and practice balance might be established. The inquiry is bounded only with the participating in-service and pre-service secondary mathematics teachers.

Main Question

How does reflecting on case scenarios help pre-service secondary mathematics teachers to have a more holistic perception of teaching?

Stage 1: The Development of the Case-Based Discussion Module (CBDM)

How can a case-based discussion module be designed to implement as a complementary practice for pre-service mathematics teacher education?

Sub-questions:

1) What are the challenges that participating mathematics teachers face during the early career stage?

a) What are the prior beliefs and expectations of the participating mathematics teachers on the dimensions of teaching that they were challenged with in their early careers?

b) To what extent do they integrate different dimensions of teaching while reflecting on the reasons for those challenges?

Stage 2: The Implementation of the Case-Based Discussion Module (CBDM)

How do pre-service secondary mathematics teachers experience the process of the CBDM implementation?

Sub-questions:

1) To what extent are pre-service mathematics teachers' reflections productive?

a) On whom do the participating pre-service mathematics teachers reflect during the implementation of the CBDM?

b) What aspects of teaching mathematics are noticed by pre-service mathematics teachers during the implementation of the CBDM?

c) What are the characteristics of participants' reflections in terms of connectedness and complexity?

2) How do pre-service mathematics teachers' identities associate with their reflections in the CBDM process?

3) How did pre-service mathematics teachers perceive the CBDM experience?

Significance

The current study aims to contribute to the literature in several domains. The process unfolded participants' experiences of a case-based pedagogy enactment together with their background and the analysis involves a holistic approach to their experiences without disregarding their initial beliefs, expectations, and concerns related to teaching mathematics.

Teachers work in increasingly complex and diverse settings, and they have very different and changing professional learning needs. Current research involving the design of a case-based pedagogical practice stems from the idea that the learning needs of teachers may be very specific to teachers or to the context in which they work. In other words, teachers need professional learning opportunities that are tailored to their own needs (Livingston, 2017). Therefore, the CBDM was developed regarding early career challenges of secondary mathematics teachers who has graduated from a two-year master's program with a teaching certificate. The CBDM was finalized by taking the profiles the pre-service secondary mathematics teachers into account, who also attended the same program. Therefore, this study has the potential to add knowledge to the literature in terms of the following:

- Revealing the early career challenges of secondary mathematics teachers.
- Illuminating the process of producing a case-based discussion module from mathematics teachers' early career challenges.
- Developing relevant teacher education materials for the pre-service teachers' needs.
- Highlighting the experiences gathered during the implementation of a case-based discussion module.
- Exampling the attempts to link theory and practice.

The results of this study may be used to improve and support the curriculum of teacher education programs by adding a complementary platform like case-based discussions which would help pre-service mathematics teachers to have a more realistic and holistic view of teaching by linking theory and practice.

Definition of key terms

Case-Based Pedagogy: Case-based pedagogy is defined as the use of cases, "descriptive research document based on a real-life situation or event" (Merseth, 1996, p. 726), for preparing pre-service teachers for the complexities of teaching (Shulman, 1992).

Case-Based Discussion Module: Case-based discussion module consists of six written case scenarios together with their pre-case exercises, case-discussion plans and post discussion written tasks designed by the researcher.

Reflection: Reflection is defined as "Active, persistent, and careful consideration of any belief or supposed form of knowledge in light of the grounds that support it and the further consequences to which it leads" (Dewey, 1933, p.9). In this research, based on Dewey's definition of reflection, pre-service teachers' considerations on mathematics teaching case scenarios are considered as reflective statements.

Productive Reflection: Productive reflection is defined as reflection with considering and integrating multiple aspects of teaching and learning with an acknowledgment of personal experiences, others' perspectives, and educational theories. (Moore-Russo & Wilsey, 2014).

Program: The program in which this study was held refers to the two-year master's program with a teaching certificate that the Graduate School of Education of a non-profit foundation university in Ankara offered.

CHAPTER 2: LITERATURE REVIEW

Introduction

> As teachers talk about their work and name their experiences, they learn about what they know and what they believe. They also learn what they do not know. Such knowledge empowers the individual by providing a source for action that is generated from within rather than imposed from without… Then they become empowered to draw from the center of their own knowing and act as critics and creators of their world rather than solely respondents to it, or worse, victims of it. (Richert, 1992, p. 197)

In this section, the theoretical background of this study and related literature is presented. First, the theoretical underpinnings of the study are shared. Second, the components of *learning to teach,* and specifically *learning to teach mathematics* are elaborated. Third, case-based pedagogy and reflective practice, being at the core of this dissertation, are introduced in detail in terms of their definition, types and characteristics. In the end, a summary of the ideas which shaped this dissertation is provided.

Theoretical underpinnings

The nature of knowledge of teaching, how it is gained, and how it will be assessed are the main questions in a teacher education system. To establish a ground for a professional development opportunity for pre-service teachers, one needs to introduce the epistemological approaches possessed. This dissertation is built upon the following theories.

Constructivism and social constructivism

Constructivism is an epistemology based on the idea that individuals generate knowledge as a result of the interaction of the new phenomenon encountered with prior experiences, knowledge, and beliefs (Richardson, 2005). In the light of these, mathematics education is tried to be altered from a teacher-centered approach to a student-centered one on abroad and in Turkey (National Council of Teachers of Mathematics [NCTM], 2000; Talim Terbiye Kurulu [TTKB], 2006). However, it would not be achieved with an aligned mathematics teacher education (Artzt & Armour-Thomas, 2002). Therefore, teaching practices are tried to be shifted from a transmissive approach to constructivist in which students create their own meaning. Teacher education which aimed to help the growth of teachers who will teach under constructivist paradigm is expected to shape their practices accordingly. Teacher candidates should also be treated as learners who actively construct understandings about subject matter and pedagogy with attention to their existing beliefs, experiences and knowledge (Ball, 1988). From a Piagetian perspective, the conflict between new information and existing knowledge leads to cognitive disequilibrium, and learning could be defined as a state of tuning and equilibration of knowledge under these circumstances (Lin & Cooney, 2001). Therefore, problematic situations trigger mind to a state of cognitive dissonance and have potential to alter conceptions (Festinger, 1957). As it was discussed in the introduction, what realistic teacher education advocates is to bring an awareness of what *teacher as learner* brings to teacher education with them and building practices upon this insight (Feiman-Nemser, 1983).

Although research on cognition enlightens much of how learning occurs, approaching learning solely as an individual cognitive activity would lack an insight that the interaction of the individual with the environment and others would bring to learning. Knowledge construction through interacting with others and the environment surrounding oneself brings us to social constructivism (Vygotsky, 1962). Two approaches of social constructivism guided this research; situated cognition and social theory of learning.

Situated cognition and social theory of learning

Not contradicting what constructivism advocates about knowledge generation, but changing the focus from individual to context, situated cognition theory brings the idea that "knowledge is situated, being in part a product of the activity, context, and culture in which it is developed and used" (Brown, Colling & Duguid, 1989). The idea that knowledge is inseparable from the context brought situated learning approaches (Lave & Wenger, 1991). The situatedness of knowledge is also associated with learning as a result of social interaction, i.e. social theory of learning. It is tied to communities of practice in which personal knowledge evolved to shared knowledge and vice versa in a cyclic interaction. Communities of practice can be defined as groups of people who share a common concern for something they perform and learn better ways of doing it as they interact regularly (Wenger, 2005). Learning was tried to be explained as an increasing social participation of the novice, moving from peripheral to the center of the community of the practice, and shaping and reshaping identities while negotiating the meanings in the communities of practice (Lave & Wenger, 1991).

Learning to teach

The *learning to teach* process enlightens educational research on teachers' development and gives insights into the growth of teacher education practices as well as teacher development policies. Yet, the teacher's learning and development remain very complex domains. Hence, there have been several attempts to explain this complicated and never-ending path including the longitudinal studies that shed light onto learning-to-teach processes (Bullough, 1989; Clift & Brady, 2005; Fuller and Bown, 1975; Hollingsworth, 1989; Levin, 2003; Pigge & Marso, 1997). Various theories tried to explain teacher learning and many theories divided teachers' careers into phases by taking a developmental or psychological stance (Levin, 2003). In their longitudinal study, Fuller and Bown (1975) explained this process in terms of three concerns of novice teachers: survival concerns, teaching situation concerns, and pupil concerns. Similar to Fuller and Bown (1975), Ryan (1986) identified four developmental stages that novice teachers went through. These stages have been identified as fantasy, reality, master of the craft, and impact. Both pupil concerns and impact stage were noticed to be more complicated in terms of teachers' thinking. Another model with a cognitive psychology approach was offered by Hollingsworth (1989) as the model of complexity reduction to explain *learning to teach* processes of beginning teachers. Due to the complexity of the nature of *learning to teach* and because of the selective nature of attentional capacity of a human being (Bransford, 1979), teachers tend to focus on specific issues in the complexity of classroom issues. This focus of attention varies from teacher to teacher. Thus, beginning teacher learning has been examined in three dimensions in Lidstone and Hollingsworth's study (1992): the role of prior beliefs, areas of cognitive attention, and depth of cognitive processing. The results of the work of Lidstone and Hollingsworth (1992) lead to four assertions about what all beginning teachers need; i) opportunities to work, observe, collaborate with other teachers, ii) support from an induction program where other beginning teachers struggling similar challenges iii) support from people who has an understanding of what beginning teachers go through in their early career, iv) support from people who has an understanding of theories on teachers' change in the *learning to teach* process.

Teacher belief

The importance of prior beliefs was mentioned in many studies that investigate the learning of a teacher (Fuller and Bown, 1975; Kagan, 1992b; Levin, 2003; Pajares, 1992; Ryan, 1986). Beliefs and conceptions about teaching are lenses that influence the way teachers see problems and dilemmas in the classroom and consequently affect the way they take action (Richardson, 1996). Beliefs about teaching include teachers' expectations of teaching profession and they play an important role in the beginning teachers' experiencing reality shocks (DeRosa, 2016). It was revealed that pre-service teachers start the profession with a tendency to believe that they would have less difficulty compared to whatever a beginning teacher could face (Weinstein, 1988). Belief systems, in general, can be thought as a continuum that involves beliefs from central to peripheral (Rokeach, 1968). Core beliefs are central beliefs, which are resistant and the more central the belief is, the more likely a teacher act on these beliefs whenever a problematic and perplexing situation arises (Pajares, 1992).

Besides, mathematics teachers' beliefs on the nature of mathematical knowledge and mathematics teaching were also found to be determining factors in teachers' teaching practices (Baydar & Bulut, 2002; Dede & Karakuş, 2014; Haser & Star, 2009;

Raymond, 1997; Thompson, 1984). Epistemological beliefs about mathematics, in other words, beliefs about nature of mathematical knowledge are associated with instructional choices. Epistemological beliefs related to mathematics vary from static to dynamic. In other words, the beliefs about nature of mathematical knowledge range from "mathematics consisting of isolated facts and rules" to "mathematical knowledge being driven from problems and is continually developing" (Ernest, 1989). Teachers whose mathematical experiences in their own schooling was far from being student-centered, and limited to teacher telling and demonstrating mathematical facts, have difficulties in adapting a view of mathematics teaching and learning which places the learner into the center (Ball & Wilson, 1990; Dede & Karakuş, 2014). Creating a community that shares mathematical conjectures, discuss and construct mathematical knowledge would be difficult for a teacher who does not possess a constructivist mathematics learning.

Teacher knowledge

Discussions on *learning to teach* come along with the questions what will the teachers have to know to teach? This unavoidable question brings the inquiry to the knowledge frameworks that teachers need to possess to be competent in teaching. The duality of subject matter knowledge and pedagogical knowledge was destroyed by Shulman, as in his framework, subject matter knowledge and pedagogical knowledge merged and became pedagogical content knowledge. Shulman's (1987) categories of teacher knowledge consisted of:

> i) content knowledge;
> ii) general pedagogical knowledge, with special reference to those broad principles and strategies of classroom management and organization that appear to transcend subject matter;

> iii) curriculum knowledge, with particular grasp of the materials and programs that serve as "tools of the trade" for teachers;
> iv) pedagogical content knowledge, that special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding;
> v) knowledge of learners and their characteristics;
> vi) knowledge of educational contexts, ranging from the workings of the group or classroom, the governance, and financing of school districts, to the character of communities and cultures; and
> vii) knowledge of educational ends, purposes, and values, and their philosophical and historical grounds (p.8).

Developing upon Shulman's knowledge framework, Ball, Thames, and Phelps developed a research-based knowledge framework for mathematics teachers. Ball, Thames, and Phelps (2008) proposed Mathematical Knowledge for Teaching (MKT) framework based on Shulman's (1986) categories of knowledge of teachers, especially on pedagogical content knowledge as MKT served more integrated and complex framework specific to mathematics teaching. MKT is composed of two main parts; subject matter knowledge (SMK) and pedagogical content knowledge (PCK). In Figure 1, it can be seen that each main part is composed of three subunits. SAK consists of common content knowledge, specialized content knowledge, and horizon content knowledge. PCK, on the other hand, is composed of knowledge of content and students, knowledge of content and curriculum, and knowledge of content and teaching.

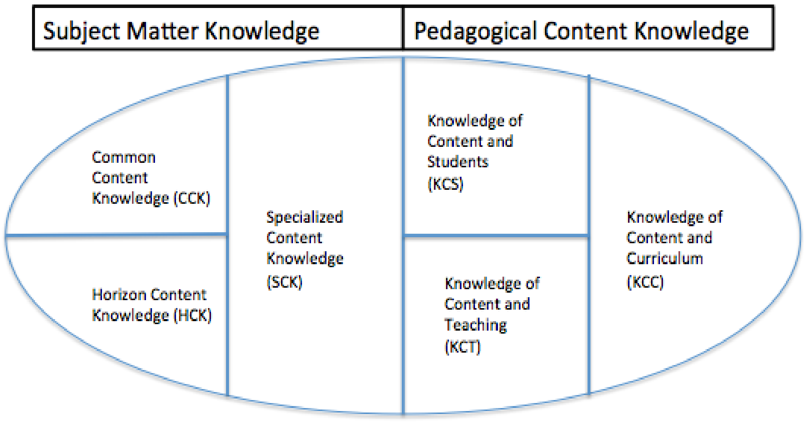

Figure 1. Model of mathematical knowledge for teaching (Ball et al., 2008)

Common Content Knowledge (CCK) is defined as mathematical knowledge and skills used in settings other than teaching; it is considered to be the problematic part of the MKT framework for secondary school mathematics teachers. The CCK differs for secondary school mathematics teachers who had a mathematics education in their undergraduate study. However, there are studies that use the MKT framework to assess mathematical knowledge of secondary school mathematics teachers (e.g. Khasaka & Berger, 2016). In this case, CCK will be taken for this study as knowledge of mathematics held in common with professionals in other mathematically intensive fields. Bearing in mind that this knowledge is not unique to teaching, teaching mathematics requires knowing how to solve a particular mathematics problem or knowing how to carry out a procedure as well as knowing the definition of a concept. However, specialized content knowledge (SCK) is the mathematical knowledge and skills used by teachers in their work but not generally possessed by well-educated adults, such as how to accurately represent mathematical ideas, provide mathematical explanations for common rules and procedures, and examine and understand unusual solution methods to problems (Hill et al., 2005).

Besides, horizon content knowledge (HCK) is an awareness of how mathematical topics are related over the span of mathematics included in the curriculum. As for the pedagogical content knowledge subunits, knowledge of content and students (KCS) includes cognizance of both mathematics and students. In other words, it is the knowledge of both content and what students know about the content in addition to how students know and learn that content. Knowing content and students requires understanding the difficult concepts for students to grasp, anticipating the common mistakes and misconceptions, finding the possible sources of students' errors, knowing how to eliminate those difficulties and misconceptions (Kılıç, 2011). Knowledge of content and teaching (KCT) combines the knowledge of mathematics and the knowledge of teaching. Finally, knowledge of content and curriculum (KCC) is about the identification of the purposes of teaching mathematics and relationships in the curriculum (Kim, 2013).

Teachers' mathematical knowledge was under investigation as a result of students' failure in mathematics. In secondary school mathematics, many foundational subjects were revealed to be unsubstantial in pre-service and in-service mathematics teachers' mathematical knowledge for teaching.

Teacher identity

Professional identity formation in pre-service teachers starts with their early histories as a student, goes under a continuous transformation as they construct a knowledge base about teaching and practice teaching. Starting from early schooling experiences, one began to construct mental images about teaching (Flores & Day, 2006). Having a dynamic and continually changing nature, professional identity of pre-service teachers is shaped with their experiences in teacher education programs with the evolving and changing knowledge and beliefs of teaching (Beauchamp & Thomas, 2009; Cooper & Olsen, 1996).

Gestalts, which could be defined as "feelings, images, role models, values, and so forth, may all play a role in shaping teaching behavior in the here-and-now of classroom experiences, and often unconsciously or only partly consciously" (Korthagen, 2001, p.6), has an important role in teacher identity. Gestalts could be considered as being mostly unconscious constructs in mind determines the beliefs and actions of the teacher. Without taking them as a starting point, teacher educators would less likely have an impact on pre-service teachers' development (Korthagen, 2001). Realistic teacher education practices should create opportunities to trigger images, emotions, needs and concerns which would lead to conflict and tensions. This tension would evoke these gestalts and may lead up to productive discussion about *learning to teach* by taking the pre-service teacher into center (Korthagen, 2001; Meijer & de Graaf, Meirink, 2011). Emotions are also associated to formation and transformation of the teacher identity (Zembylas, 2002). Emotions trigger teacher identity and with the help of emotions, it evolves. There is a cyclic relationship between emotions and teacher identity (Akkerman & Meijer, 2011).

Reasons for becoming a teacher is also linked to formation of teacher identity and could be considered as a starting point to investigate the teacher self (Olsen, 2008). Therefore, getting to know the reasons of choosing teaching as a career is important for understanding the complex relationship between personal history, prior beliefs and current practices of teachers with a focus on evolving teacher identity. Three main motives for becoming a teacher are defined in the literature as altruistic, intrinsic, and extrinsic (Kyriacou, Hultgren, & Stephens, 1999). Altruistic reasons are dealing with a perception of teaching as a socially worthwhile and important job with an intention to be beneficial to others' lives. On the other hand, intrinsic reasons are related to the joy of teaching, interest in subject matter knowledge and expertise. Extrinsic reasons are associated with the perceived benefits of the profession like job security, holidays, working hours, etc.

Case-based pedagogy and reflective practice constituted the two important elements of the theoretical framework of the study with an aim to support preservice secondary mathematics teachers' learning to teach processes. In the following sections, case-based pedagogy, reflective practice and use of case-based pedagogy to promote reflection are shared.

Case-based pedagogy in teacher education

To define case-based pedagogy and its applications in teacher education and mathematics teacher education, one has to define what a case is. A case can be defined as a descriptive research document based on a real-life situation with attempting to picture a balanced, multidimensional representation of the context, participants, and reality of the situation. Pioneers of using cases as a teaching material were law schools, followed by business and medicine (Merseth, 1991; Shulman, 1992). Cases are represented as teaching materials to provide a balanced, multidimensional representation of reality with three essential elements; they are real; they rely on careful research and study; they provide data for consideration and discussion by users (Merseth, 1994).

Shulman (1996) saw the heart of teaching was to respond to the unpredictable; case-based teacher education offered opportunities to reflect on variety of ways that unpredictability occurs with a safe environment to explore alternatives and judge their consequences. By interweaving context and theory, case-based pedagogy provides a platform to explore the perceptions, principles, theories, and frequently occurring practices as they actually occur in the real world (Darling-Hammond, 2012). Regarding the problem of deceptively simple perception of teaching, case-based pedagogy offers a window on multiple realities of classrooms as Shulman asserted:

> I envision case methods as a strategy for overcoming many of the most serious deficiencies in the education of teachers. Because they are contextual, local, and situated -as are all narratives- cases integrate what otherwise remains separated… Complex cases will communicate to both future teachers and laypersons that teaching is a complex domain demanding subtle judgments and agonizing decisions (Shulman, 1986, p.28).

Cases involve dilemmas of teaching, reflect the unpredictable nature of the profession, provokes cognition and emotions and provide opportunity to change tacit to explicit (Brown, Colling & Duguid, 1989; Harrington, Quinn-Leering & Hodson, 1996; Merseth, 2003).

Types and structures of cases

Cases would be categorized into three concerning their purposes; cases as exemplars, cases as opportunities to practice analysis and contemplate action, and cases as simulants to personal reflection (Merseth, 1994; Sykes & Bird, 1992; Shulman, 1986). The first type refers to the cases that present best practice and exemplifies theory, whereas the second type stems from the idea of teaching as a complex, context-specific activity and present problematic situations (Merseth, 1996). In the third type cases are used in order to provoke reflective practice, either on own experiences or others.

According to the lens of the practitioner who employs case-based pedagogy in teacher training, the type of cases would differ and would be presented in different formats accordingly. Cases can be in the form of written text, i.e. narratives, describing teaching practice, situated in a way that is significant for thinking them as texts for teacher learning (Brown, Colling, & Duguid, 1989). The cases would be shared with the audience in text, video, multimedia and animations. Although narrative cases are widely used, video cases start to dominate the practices. Multimedia offers a richer context to the participants. Animations are relatively new and bring a new dimension to case method (Chazan & Herbst, 2012; Moore-Russo & Wilsey, 2014). The advantages and disadvantages of each structure must be evaluated according to the purpose of use. For instance, video cases seemed to convey more about a classroom by reflecting the whole reality. On the other hand, narratives would be a better choice if the case is problematic and the teacher or students should not be shared to avoid labeling them with problematic practices (Clarke & Hollingsworth, 2000).

This use of cases works well with the conception of teaching as a complex, messy, and context-specific activity. The cases present problematic situations that require analysis, problem-solving, decision making, and action definition. With such cases, students can, within the confines and safety of a teacher education classroom, “practice such professional skills as interpreting situations, framing problems, generating various solutions to the problems posed and choosing among them" (Sykes & Bird, 1992, p. 482)

Literature prevailed that influence of case on how teachers think could be identified as the ability of cases to help develop problem-solving and decision-making skills, the ability of case to increase awareness of multiple perspectives' and other educational settings, the ability of cases to enhance beliefs about personal authority and efficacy, and the ability of cases to develop habits of reflection (Merseth, 1994).

Harrington (1995) aiming to provide insight about pre-service elementary teachers' development in their reasoned decisions, 26 pre-service elementary teachers were provided cases and asked to identify and discuss the following: the issues in the case; how they would prioritize the issues; based on that, what it was a case of; how different perspectives might inform the interpretation of the case; what the educator's solution should be; what the possible consequences to that solution might be; and how they would critique their solution and analysis. They were specifically asked to include substantiation and evidence when they considered other perspectives, made recommendations, addressed consequences, and critiqued their analysis. The development of pre-service teachers was examined in five different domains; problem identification, awareness of alternative perspectives, warranting of solutions, consideration of consequences, and reflectiveness. The results yielded that dilemma-based case analysis would foster pre-services thinking in these domains and it gave insight about their reasoned decisions.

A qualitative study exploring pre-service teachers' perceptions regarding case-based learning conducted in computer education and instructional technology department in Turkey with 38 pre-service teachers provide insight about applying case-based pedagogy in teacher education. The results of the study yielded that in general the cases provided a valued opportunity to engage developing teachers in solving real-life problems that would occur in actual teaching. The other evidences emerged from the study suggest that cases can help preservice teachers be prepared for their early teaching experiences in real classrooms by improving their understandings of how to respond to actual problems they will encounter in their fields and how to apply what they learned in classes to solve practical teaching issues (Çelik, Çevik, & Haşlaman, 2012).

Levin (1995) contributed to the literature of case-based methods in terms of the effectiveness of discussion on cases compared to writing and reading cases. Discussing cases were found to change reflective levels and thinking of beginning teachers positively compared to individual reading and writing on cases.

Reflective practice

Based on the constructivist paradigm of learning, reflection is thought to be a transforming practice in learning and the importance of reflective practice of teachers was emphasized by many educators (Dewey, 1933; Schön, 1983; Zeichner & Liston, 1996). Several studies held on the concept of reflection, ways to improve reflective skills, the impact of using critical reflective skills of teachers on their development. In order to examine the studies done on reflective practice, definition of it must be considered at first.

Definition

Many definitions are trying to explain reflection and reflective practice in detail. John Dewey and Donald. O. Schön are the two practitioners who contributed to the formation and development of the idea of reflection. According to Dewey (1933) who is pivotal to the development of idea of reflection, defined it as "the active, persistent and careful consideration of any belief or supposed form of knowledge in the light of the grounds that support it and the further conclusion to which it tends" (p.9). This persistent and careful consideration iterates and could be cyclic as the consequences would lead to another perplexing situation. According to Dewey, a perplexing moment- characterized as a difficulty, a troublesome event, or experience that cannot be immediately resolved- of teaching initiates the process of reflection. As was mentioned earlier in reforming the belief systems, the need for the *problematic* was again apparent as it has the potential to trigger conflicting beliefs and create a disequilibrium. Dewey (1933) made a distinction between reflective and routine action. Routine action occurs undertaken for granted beliefs, carried out spontaneously, difficult to be aware of the problematic parts and blind to alternative interpretations. Experiences dominated by this unconscious process may not lead to expertise in teaching. In order to learn from practice, this unconscious process should transform into a conscious one and reflective practice has the potential to change tacit knowledge in action to explicit as a metacognitive approach. Reflective action is required as it could be defined a holistic process that embraces teachers' emotions, intuition and personality and has potential to transform as everything taken for granted is under reframing. In order to engage reflective action, Dewey (1933), emphasized three attitudes one has to possess; open-mindedness, responsibility and wholeheartedness. To put differently, the reflective practice requires being committed to learning and being open to different perspectives, having a critical stance for evaluating the belief systems' strengths and weaknesses with a consideration of the consequences of an action.

As Schön (1983) defined reflection as idea formation for eventual testing through a recurring cycle of framing and re-framing process. According to Schön (1983), an effective practitioner is the one who recognizes and explores confusing or distinguishing events that occur during practice. However, an ineffective practitioner is the one who is more repetitive and routine in practice without thinking about what he or she is doing. Schön (1983) gave a new impulse to the definition of reflective practice by categorizing the reflection according to a time frame perspective. According to this categorization, there is reflection on action which could be regarded as looking back and commenting on what has been done and reflection in action which would be considered as attempting to solve the problems on the spot like a jazz improviser. What is missing in this time frame reflective model was the future since reflection on action refers to past and reflection in action refers to present. Van Manen's (1991) anticipatory reflection or Wilson's (2008) reflection for action refers to a reflection which includes thinking about the possibilities and strategies for future action.

There is a criticism of Schön's emphasis on reflection as an individual practice. As the difficulty of being critical to oneself is considered, reflection in collaborative and cooperative environments is regarded to have more potential for greater learning (Dewey; 1933; Osterman & Kottkamp, 1993), consistent with social learning and communities of practice theories (Lave & Wenger, 1991). One of the factors that determine the quality of reflective practice was mode of communication (oral or written) as it was revealed in Lee’s (2005) study with secondary mathematics pre-service teachers. In the same study, pre-service teachers differed in terms of the quality of reflections, writing was a more fruitful medium for some and oral reflection was more provoking for the others, therefore providing opportunities for both were suggested (Lee, 2005).

In the light of these definitions, the properties of a reflective teacher could be listed. Zeichner and Liston (1996) defined the attributes a reflective teacher as the one who

> examines, frames and attempts to solve the dilemmas of classroom practice; is aware of and question the assumptions and values he or she brings to teaching, is attentive to the institutional and cultural contexts in which he or she teaches; takes part in curriculum development and is involved in school change efforts; takes responsibility for his or her own professional development (p.6).

Characterization of reflective practice

As not every thinking on action should not be considered as reflective thought, not every reflection serves the same goal. Reflective thinking could be categorized and/or levelized according to ones’ definition of reflective practice.

Table 1 provides a picture for types of reflection (a concern based one) regarding the work of Schön’s reflection in action, reflection on action and technical rationality, as well as other scholars’ who extended the ideas of Schön on reflection.

Table 1
Types of reflections related to concerns

Reflection Type	Nature of Reflection	Possible content
"Reflection-in-action." (Schön, 1983) addressing impact concerns after some experience in the profession	5. Contextualization of multiple viewpoints drawing on any of the possibilities 1-4 below applied to situations as they are taking place	Dealing with on the spot professional problems as they arise (thinking can be recalled and then shared with others later)
Reflection on action (Schön, 1983; Smith &Lovat, 1990) Addressing tasks and impact concerns in the later stages of a pre-service program	4. Critical (social reconstructionist), seeing as problematic, according to ethical criteria, the goals, and practices of one's profession. 3. Dialogic (deliberative, cognitive, narrative) Weighing competing claims and viewpoints, and then exploring alternative solutions 2. Descriptive (social efficiency, developmental, personalistic) seeking what is seen as "best possible" practice	Thinking about the effects upon others of one's actions, taking account of social, political and/ or cultural forces (can be shared) Hearing one's own voice (alone or with another) exploring alternative ways to solve problems in a professional situation Analyzing one's performance in the professional role (probably alone), giving reasons for actions taken.
Technical rationality (Schön, 1983; Shulman, 1988; van Manen, 1977) addressing self and task concerns early in a program which prepares individuals for entry into a profession	1. Technical (decision making about immediate behavior or skills) Drawn from a given research/theory base, but always interpreted in light of personal worries and previous experience	Beginning to examine (usually with peers) one's use of essential skills or generic competencies as often applied in controlled, small scale settings

Note. From *Reflection in teacher education: Towards definition and implementation* (p. 45), by Hatton, N. & Smith, D. (1995), Teaching and Teacher Education, *11*(1), 33-49.

Hatton and Smith provided a reflective rubric to analyze reflective writing. The levels were similar to the ones in the Table 1; descriptive writing involving restating an event without any reasoning or justification is not counted as reflective action. Descriptive reflection is the lowest level of reflection, which is not only a description but there is an attempt to justify thinking. Dialogical reflection requires more elaboration and more critical judgment considering and integrating multiple aspects with an awareness of the consequences. The highest level is regarded as critical reflection which takes historical, cultural, political, and moral values and beliefs into account (Zeichner & Liston, 1996).

Jay and Johnson (2002) provided another typology which which had similar levels to what Hatton and Smith (1995) proposed. The descriptive level is again the lowest level of reflection and critical level is the highest level. However, descriptive level in this one is more superficial than Hatton and Smith's (1995). Recognizing alternative points of view was put under comparative category instead of descriptive.

Another categorization was made by Manouchehri (2002) and reflective thinking were considered in five levels, again which description is the lowest. Other levels were categorized as explaining, theorizing, confronting, and restructuring. Explaining moves beyond describing the event and refers linking interrelated events with an exploration of cause-effect. Theorizing requires explanation of the ideas with a reference to learning and teaching principles or past experience. Confronting requires suggesting alternative theories to explain events and actions. Restructuring, the highest level, re-organization of the action or curricular choices.

To have a holistic perception of teaching, one needs to notice and consider multiple aspects of teaching. Considering multiple aspects separately will not be sufficient to have a complex view of teaching. Regarding the complexities of teaching, one has to regard the connections between different aspects. Integrated knowledge is one of the indicators of productive reflection (Davis, 2006). Productive reflection goes beyond mere description. It requires noticing the aspects of teaching in the light of personal experiences, practical knowledge, educational theory, and professional development (Fund, 2010). Being open to different perspectives is important in terms of one of the characteristics of being a reflective practitioner. Dewey stated that one of the three important features of a reflective practitioner is to be open-minded, where the other two are responsibility and wholeheartedness (Dewey, 1933).

Based on the works of Davis (2006) and Fund (2010), Moore-Russo and Wilsey (2014) claimed that productive reflections should involve: considering the act of teaching, teaching and learning environment, students' thinking and learning, the nature of the subject, expectations of teachers, or some aspect related to the work of teaching; being comparative by acknowledging and building from past experiences, others' perspectives, educational theories, or educational research and; recognizing the complex nature of teaching by emphasizing and integrating multiple aspects of teaching.

There are different categories that reflective practice or critical reflection can be examined. Research paid attention to different categories of critical reflection. Reflective practice can be analyzed under two broad categories incident reflection and process reflection (Ricks, 2010). In Rick's study, by showing four prospective mathematics teachers' group deliberation in a Japanese lesson study activity, it was aimed to offer a new framework for reflection which is process reflection that is aligned to Dewey's and Schön's works on reflection. This qualitative study provided some episodes from group deliberations in order to explain how reflection cycle works for these four prospective students. This cycle consists of experiential event, idea suspension and problem creation, idea formation and idea testing. The results of this study showed that prospective teachers' reflective abilities improved by going through the cycle during lesson study.

Reflective practice can also be examined three category reflection; technical rationality which is more descriptive, practical action which focused on problematic situations, and critical reflection which indicated a high degree of open-mindedness (Collier, 1999). The purpose of Collier's study was to reveal novice teachers' reflective practices. In Collier's study four pre-service elementary teachers' reflective characteristics were examined through 8-week clinical field experience. This qualitative case study made use of four different opportunities to reveal reflection characteristics; reflective journals, reflective interviews, peer observation conferences, and group seminars. According the conclusion that the researcher reached, most of pre-service teachers fell into first and second categories. Another important message that could be driven from the article was the pre-service teachers' difficulty in shifting between a learner and a teacher.

A seven-stage reflective judgment model was offered by Collier (1999). This seven-stage model was related to the views of knowing. The first stage is concrete and limited to the senses. The second stage involves the justification of knowledge via observation and authorities. The pre-reflective stage ends with the third stage which integrates personal thought to what authorities say. The quasi-reflective stage starts with stage 4 and regards knowledge as a single abstraction and can be gathered by being uncertain. Stage 5 inquiry comes into the scene. Stage 6 knowledge is justified through an inquiry cycle which includes an evaluation of what expert says and personal values. Stage 7 involves a systematic evaluation of evidence using generalizable and evaluative criteria. Stage 6 and stage 7 are considered as a reflective stage. This seven-stage reflective judgment model was used to determine the level of pre-service teachers' reflections through an intervention process. The results indicated that the reflective judgment model provided an effective framework for distinguishing and enhancing pre-service teachers' critical reasoning about classroom practice (Collier, 1999).

Cyclic models of reflections were proposed by many scholars (Gibbs, 1988; Kolb 1984; Mezirow, 1981). Another cyclic reflective model is structured in order to reach a deeper reflection, defined as core reflection, and proposed as a realistic teacher education practice. Naturally people reflect on situation and find a rapid solution to the problems without thinking about the deeper reasons or the origins of the problems. A more structured way of reflection required. ALACT having the initials of a five phases cyclic model for core reflection was offered as a structured model. These five phases were action, looking back on the action, awareness of the essential aspects, creating alternative methods of action, and trial (Korthagen, & Vasalos, 2005).

To conclude, several reflective models offers different frameworks for researchers and teacher educators who were willing to enhance teachers' reflective skills. Different ways to improve reflective skills of teachers can be constructed on these frameworks.

Ways to engage pre-service teachers in reflective practice

Several pieces of research focused on the improvement of pre-service teachers' ability to reflect. Various methods were applied to increase the level of reflection of prospective teachers.

Reflective writing

Reflective writing is one of the most common methods to reveal and improve prospective teachers' ability to reflect. Writing helps to hold ideas or experiences still for reflection (Artzt, 1999). Artzt's study used reflective writing in many aspects and different periods of reflection. In order to promote the reflective behavior of pre-service secondary mathematics teachers, a structured model of reflection was implemented. Through the process, pre-service teachers had to write about specific parts of their thoughts and practices and interaction of both. This cognitive process revealed in their writing for pre-lesson and post-lesson. Journal writing is an example of written reflection which promotes reflection Erginel (2006) highlighted the importance of guidance and regular feedback to on-going journal articles of pre-service teachers. The aim of the study was to explore the perception of pre-service English teachers on reflection and how pre-service teachers improve their reflective ability through a reflective practicum course. This qualitative case study conducted with thirty pre-service English teachers in North Cyprus in an action research form since the content of the course exposed to intervention in order to enhance pre-service teachers' reflective abilities. At the end of video case discussion on various themes related to classroom interactions and reflections on school observations, data which are journals, interview records, video-taped class interactions and microteachings were analyzed both qualitatively and quantitatively. Results yielded that pre-service teacher's reflective level raised and they mainly focused on best ways of teaching. In addition to these, their self-awareness increased.

Another case study was conducted with six non-native pre-service teachers of English as a second language in an undergraduate program in Hong Kong, and the findings suggested that journal writing as a coursework assignment in pre-service teacher education program enhances teachers' critical reflectivity over time and promoted understanding of the nature, theories and praxis of teaching (Tsang, 2003).

Mentoring and supervision

An important factor affecting the reflective abilities of pre-service teachers is the amount and quality of mentoring or supervision provided (Crouch et al.,2012). Lack of supervision or mentoring makes it difficult for pre-service teachers to enhance their reflective skills (Valli, 1997). Regarding the problem of lack of supervision in field experiences of pre-service teachers, a web-based supervision framework provided, called e-supervision, was formed by Alger and Kopcha (2009). This technology integrated supervision mechanism provided results that showed progress in enhancing the role of mentors in the schools and supervisors in university. The results also yielded a bridge between practice and theory by enhancing communication with the university faculty and teachers. Artzt (1999) also emphasized the need for supervision by underlining the importance of support in order to construct new meanings on being an effective mathematics teacher. The role of supervisors and mentors is of great importance in terms of reflection opportunities that they have provided to the preservice teachers. Although the need for a mentor in the early career was emphasized by many scholars, teachers in Turkey and many countries could not work with a mentor when they start teaching. The focus of the current study is not the role of mentoring and supervision on reservice teachers' reflective practices,

Use of cases in reflective practice

As Loughran (2002) asserted, teaching reflective practice requires contextual anchors that would provide episodes of teaching, cases have the potential to trigger reflective practice as episodes of teaching. People learn best about the job when their own identity and growth are recognized as integral to that learning and these episodes of teaching potential to help pre-service teachers to engage in reflective practice (Boud & Walker, 1991).

The use of case-based pedagogy in reflective practice is varied. Video-cases and self-recording analysis were widely used for increasing the reflective skills of pre-service teachers. Many pieces of research supported the idea that analysis of video cases and self-recording helped teachers to develop reflective skills. In the research of Stockero (2008), participants who were pre-service mathematics teachers reflected on video-cases of class activities of other teachers' teaching on linear functions. In addition to the video cases, teachers visited some middle schools and applied the methods they fronted in video cases. They reflected on their teaching in the class discussions held after school visits. Results suggested that the use of video-case curriculum provided a way to develop reflective skills and transfer these skills to practice of teaching.

Another research which aimed to investigate the changes in prospective teachers' noticing skills by watching video-cases, had fifteen senior prospective mathematics teachers as participants in a public university. They watched and wrote reflection papers along with discussing on an online forum after watching six video cases of reform-minded teaching. The results of this qualitative case study were driven from reflections of participant pre-service teachers with an analytic framework that used *learning to notice*. The reflections showed that there was an increase in the number of participants who noticed reform-minded teaching issues in video cases. In addition to that, the number of issues noticed related to reform-minded teaching was also increased. The quality of the reflections improved throughout the process (Osmanoğlu, 2010).

Not only watching video-cases of other teachers but also self-recording is a viable tool for increasing reflective practice. An exploratory study where pre-service teachers were discussed video-recordings online provided that pre-service teachers improved in three dimensions of reflection; self-identity, type of reflection, provision of content for reflection (Melville, Bowen & Passmore, 2011). Another study that benefited from videotaped lessons of pre-service teachers to promote reflection was conducted to examine the impact of a three-phase collaborative coaching model (TCCM) on prospective language teachers' construction of knowledge. TCCM involves phases called pre-working, while-working and post-working sessions. A pre-working session aims to equip prospective teachers with theoretical knowledge related to instructional issues before their microteaching presentations. The while-working sessions involve peer-teaching presentations which were videotaped and their self-evaluations. The post-working session aimed to provide a collaborative discussion platform for teacher trainees on their videotape records. The analysis of data gathered from this three-phase model yielded positive results on raising trainees' awareness and reflection (Kuter, Gazi, & Aksal, 2012).

Narrative mathematics cases were used widely in mathematics teacher's education (Barnett,1991; Merseth, 2003; Stein, Smith, Henningsen & Silver, 2000). In the literature, there is a gap in narrative mathematics case materials designed and used for pre-service mathematics teacher education in Turkey. However, video-cases were used by a few number of scholars for pre-service middle school teachers' development (Çelikdemir, 2018; Osmanoğlu, 2010; Ulusoy, 2016) and pre-service secondary mathematics teachers' development (Didiş; 2014). However, these studies were mostly focused on video-cases and had a specific focus on students' work and students' understanding.

Summary: Theories revisited

This dissertation was based upon the theories shared, having a disposition that pre-service teachers bring their knowledge, beliefs and identity to the teacher education classrooms. Though any attempt to help them to grow as adaptive and competent experts in the field would be based upon constructivist theory of learning. Regarding the construction of knowledge tied to its context, case-based pedagogy was chosen as the method which promises as a stimulant for reflective practice. The choice of cases as problematic instead of exemplary stemmed from the idea that opportunities of reflective practice on problematic and perplexing situations could provoke conflicting beliefs thus create cognitive dissonance and had the potential to transform beliefs and create awareness about the complex nature of teaching.

Preservice teachers can not rely on their own practices for situate their learning and since they have limited teaching experiences from which to engage in discussions of pedagogical issues. Traditionally, teacher educators mostly use student teaching and field experiences as sites for learning.

The graduates and preservice teachers of the program could be considered as the member of same communities of practice. Case scenarios were developed with an effort to answer how the experience of graduate teachers of the program would be used and how to structure them in a way that it could be shared with the preservice teachers and use it for supporting their *learning to teach* processes.

The case scenarios were presented to the group of pre-service mathematics teachers, as a community of practice, believing that knowledge is socially constructed with the interaction with the others. However, there was room for individual practice on the case scenarios. With this purpose in mind, written tasks were given at the end of each discussion with an assumption of the power of writing on rethinking how to articulate and self-reflect to reconstruct meanings after interacting with others.

Pre-service teachers exposed to many opportunities to reflect with or without being aware. Unscaffolded opportunities may not lead to reflections, which have potential to contribute the quality of teaching and consequently learning of students. Therefore, the current research used a scaffolded approach opportunity for engaging preservice teachers' in reflective practice

From the design to enactment, CBDM experience is built upon *learning to teach* principles into account. *Learning to teach* mathematics were approached from looking at what pre-service teacher attend to as the knowledge of teaching in relation to the teacher identity as a dynamic construct. One of the reasons for conducting interviews with the participants before the CBDM was to reveal their lenses about being a mathematics teacher. This lens includes their beliefs about teaching, and specifically mathematics teaching, the motive behind becoming a teacher, their concerns and expectations about their career. These were all important in order to understand to whom, what and how they attended in CBDM.

Therefore, the idea that motivated the whole research as helping pre-service teacher developing a more holistic view of teaching, also broaden the lens of the researcher bringing different but strongly related components of mathematics teachers' *learning to teach* processes.

CHAPTER 3: METHODOLOGY

Introduction

This chapter consists of a detailed presentation of the research design, participants, context, instrumentation, data collection and analysis procedures, and credibility issues of this study. Researchers' background, role, and possible biases were also shared. A case study under the umbrella of naturalistic inquiry was utilized in order to answer the research questions.

Research questions

In order to provide a comprehensive analysis of reflective processes that pre-service mathematics teachers went through via case-based pedagogy, the researcher attempted to answer the following questions in this dissertation with specific attention how theory and practice balance established.

Main Question

How does reflecting on case scenarios help pre-service secondary mathematics teachers to have a more holistic perception of teaching?

Stage 1: The Development of the Case-Based Discussion Module (CBDM)

How can a case-based discussion module be designed to implement as a complementary practice for pre-service mathematics teacher education?

Sub-questions:

2) What are the challenges that participating mathematics teachers face during the early career stage?

a) What are the prior beliefs and expectations of the participating mathematics teachers on the dimensions of teaching that they were challenged with in their early careers?

b) To what extent do they integrate different dimensions of teaching while reflecting on the reasons for those challenges?

Stage 2: The Implementation of the Case-Based Discussion Module (CBDM)

How do pre-service secondary mathematics teachers experience the process of the CBDM implementation?

Sub-questions:

1) To what extent are pre-service mathematics teachers' reflections productive?

a) On whom do the participating pre-service mathematics teachers reflect during the implementation of the CBDM?

b) What aspects of teaching mathematics are noticed by pre-service mathematics teachers during the implementation of the CBDM?

c) What are the characteristics of participants' reflections in terms of connectedness and complexity?

2) How do pre-service mathematics teachers' identities associate with their reflections in the CBDM process?

3) How did pre-service mathematics teachers perceive the CBDM experience?

Research design

The main purpose of this study is to illuminate the reflective process that eight pre-service mathematics teachers experienced as they have involved in case-based pedagogy practices. In order to reach the main purpose, CBDM was constructed by gaining insight into mathematics teachers' early career problems. To explore the extent that reflecting on these real problem cases helps pre-service develop a more complex view of teaching, a case study approach within the paradigm of naturalistic inquiry was utilized. Seeking the truth about a phenomenon, especially in social sciences, moves us beyond positivism, which accepts the reality as a single, tangible and fragmentable nature (Lincoln & Guba, 1985).

On the other hand, naturalistic paradigm gives us that the lens of reality is multiple, holistic, and constructed. Examining individuals' experiences with a disposition that values their identity requires multiple perspectives with a holistic approach. This study was planned to gain an in-depth understanding of pre-service teachers' experiences with a case-based pedagogy implemented in a teacher education program. The endeavor of gaining an in-depth understanding brings the researcher to qualitative paradigm which is characterized as an interpretive, naturalistic approach to the world, studying things in their natural setting, trying to make sense of, interpret the phenomena regarding the insight that people bring (Cohen, Manion & Marrison, 2002; Denzin & Lincoln, 2005).

There are five axioms of researching naturalistic paradigm. These axioms and some of their characteristics and associative links to the current research shared below.

1) The nature of reality (ontology): "In the naturalistic inquiry, multiple constructed realities can be studied only holistically; inquiry into these multiple realities will inevitably diverge so that prediction and control are unlikely outcomes" (Lincoln & Guba, 1985, p.37). This research developed upon this axiom since the pre-service mathematics teachers' reflective processes are multi-dimensional and can be understood better with the knowledge of their teacher identity. Reality is subjective and multiple, as

seen by participants in the study. Many direct quotes and in vivo coding are utilized in this research to represent participants' perceptions.

2) The relationship of the knower to the known (epistemology): "The inquirer and the object of inquiry interact to influence one another. One of the characteristics of naturalistic inquiry is the human instrument. Human is the primary data-gathering instrument since only the human instrument is capable of grasping and evaluating the meanings of the differential interaction between multiple realities" (Lincoln & Guba, 1985, p. 37).
   In this research, the lens of the researcher is important in terms of understanding the reflective process that participants went through during the CBDM process. The role and background of the researcher were shared in order to provide the reader an understanding of the possible influences that could be resulted from the human instrument.
3) The possibility of generalization: "Inquiry aims to develop an idiographic body of knowledge in the form of working hypotheses that describe the individual case" (Lincoln & Guba, 1985, p. 38). This research focused on each participant in regards to their differences and similarities and resulted in assertions with an attempt to explain the phenomena without reaching any generalization.
4) The possibility of causal linkages: "All entities are in a state of simultaneous mutual shaping so that it is impossible to distinguish causes from effects" (Lincoln & Guba, 1985, p.38). In this study, multiple factors would affect pre-service teachers' view of teaching like the courses in the program, the schools that they have school experience in, mentoring or supervisory that they have been provided.

5) The role of values in inquiry: Inquiry is value bound in terms of values of the inquirer, choice of the paradigm and the substantive theory that guides the investigation and the value of the context (Lincoln & Guba, 1985, p. 38).

In the light of these axioms, the research focused on experiences of each pre-service mathematics teacher, which went through a discussion module built upon case-based pedagogy. There are different methods to conduct a study under naturalistic paradigm; phenomenology, ethnography, grounded theory, narrative study, and case study (Creswell, 2007). This research is a case study in many aspects, satisfying the characteristics shared below. The characteristics of a case study are referred as its particularistic, descriptive and heuristic nature. To list a few, case studies (Olson in Hoaglin et.al., 1982);

- Suggest to the reader what to do or what not to do in a similar situation. (Particularistic)
- May or may not be influenced by the authors' bias. (Particularistic)
- Illustrate the complexities of a situation- the fact that not one but many factors contributed to it. (Descriptive)
- Have the advantage of hindsight yet can be relevant in the present. (Descriptive)
- Show the influence of personalities on the issue. (Descriptive)
- Explain the reasons for a problem, the background of a situation, what happened, and why. (Heuristic)
- Explain why an innovation worked or failed to work. (Heuristic)
- Discuss and evaluate the alternatives not chosen. (Heuristic)

One of the intentions of this dissertation is to provide rich and thick descriptions about the process of design and implementation of CBDM and describe how different individuals experienced CBDM in order to provide readers a road map which could help implement a similar strategy in another teacher education program or any similar initiative for teacher development. In addition to that, the analysis is multi-layered and detailed in order to express the complexity of pre-service teachers' *learning to teach* processes with the aid of case-based pedagogy.

A case study could be defined as a comprehensive research strategy, which is an empirical inquiry that investigates a contemporary phenomenon within its real-life context, especially when the boundaries between phenomenon and context are not evident (Yin, 2003). As Merriam (1998) stated, a case study design is interested in process rather than outcome, in context rather than a specific variable, and exploration rather than confirmation. This study aims to provide a rich, descriptive and evaluative picture of pre-service mathematics teachers' experiences through the discussions built upon case-based pedagogy under a reflective framework. The study is context-bounded; people in the study were students of a unique teacher preparation program. The context is important in terms of its uniqueness and its relationship with the object of this study. Thus, the inquiry is value bounded and since all entities in the study are in a state of simultaneous mutual shaping, it is impossible to distinguish causes from effects (Lincoln & Guba, 1985).

Case studies are divided into some categories according to their intent and size (Creswell, 2007). A case may involve one individual or a single program. On the other hand, information about each relevant individual would be collected, and several such individuals or cases might be included in a multiple case study. This research is a single case study regarding its endeavor to explain how eight pre-service mathematics teachers experience a case-based discussion module. The single case is the design and implementation of a case-based pedagogy practice and it is bounded with the master's program that this design and implementation was held and with the in-service and pre-service mathematics teachers that were involved.

Context

The study has been conducted at a teacher education program in a non-profit English-medium foundation university. This program not only aimed to prepare future teachers but also targeted the growth of these teachers as educational leaders at an international level of excellence. With these intentions, the program serves to advance innovative and research-based teaching as well as raising adaptive professionals in curriculum and policy reform in education. This program is unique in terms of including an MA degree in Curriculum and Instruction with a teaching certificate program, awarding qualified teacher status with International Baccalaureate teaching-learning certificate. Pre-service teachers in this program, usually before they graduate, find teaching positions in Turkey's prestigious private schools, many of which implement the IB diploma program.

Opened in 2000, the program has produced more than 400 graduates. Before 2010, the teacher education program was a two-year non-thesis master program-accepting student having biology, mathematics, Turkish literature and language, history, and English majors. After 2010, the program became a Master's Program with a thesis in Curriculum and Instruction with a teaching certificate. The requirements to apply this program were a Cumulative Grade Point Average 2.5 out of 4 and an ALES (Academic Personnel & Graduate Education Exam) score of 60 (verbal). The candidates must assure their English proficiency with attaining required scores in TOEFL, IELTS or University's own proficiency exam. There is also a face-to-face interview and written essays in addition to an evaluation of their academic and language proficiency stated above. Face-to-face interviews were conducted in a way that participants' beliefs, attitudes and skills in teaching were examined. Carefully chosen students in this program are granted a scholarship, which includes tuition, school visits in Ankara, and as well as other cities and their internship in England. For those with homes outside Ankara, free accommodation is also available on campus.

This program differs from other graduate teacher education programs and other teacher preparation faculties in many ways. One of them is the opportunity of having teaching experience in a variety of schools, together with the length of time spent in these schools. The partner schools are in Ankara, İstanbul, Erzurum, and İzmir, as well as in the UK. Until 2010, student-teachers spent two months in the US. From 2010, student-teachers had overseas experience in England, attending classes at the University of Cambridge in the Postgraduate Certificate in Education program, and have experience in five different prestigious schools. In addition to these, graduates of the program gain an International Baccalaureate Teacher Award after 2011.

One of the most distinguishing parts of the program is the teaching practice in the first term of the second year. During teaching practice, preservice teachers spend six whole weeks in the school to which they are assigned. Different from the previous school experience courses, they do more teaching in addition to observing different teachers. During these six weeks, they teach approximately 30 periods and involve in departmental studies. Teaching practice is therefore a closer experience to real teaching career.

Having the specified properties above, this program is unique in terms of putting emphasis on practice in and outside Turkey, as well as providing the theory with a variety of courses given for two years. The content of the courses, together with school experiences, requires many reflections in different formats. Due to its uniqueness, the study, which was conducted in the context of the program, did not have an intention to generalize the results to the population of whole teacher education program students or graduates. This research was more of an offer of a methodology developed to help to achieve a more realistic teacher education in any teacher education context.

Sampling

This research is composed of two stages that were both belonged to the same context. The reason for choosing the same context was not arbitrary. Every teacher preparation program has its own features, strengths and weaknesses, and student profiles. In addition to that, graduates of different programs would experience different difficulties in their early careers. Hence, through the way to trying to help pre-service teachers developing a more holistic perception of teaching mathematics, the experiences of the graduates of the same program were used. In order to examine this, a unique teacher preparation program was chosen as the context of the study.

Sampling for stage 1

Purposive sampling was utilized under the umbrella of maximum variation sampling in which people could give rich information to document unique variations that emerged in adapting to different conditions (Lincoln & Guba, 1985). Participants were purposefully chosen in order to enrich the inquiry. The purposive sample of participants was determined by the criteria that participants would differ in terms of their characteristics such as year of experience, gender, and potential to give rich information about the challenges that they experienced openheartedly. In addition to personal characteristics, varieties in school contexts that the participants' work was also considered. Thus, the schools varied in terms of the city located, size, student profile and being well-established or newly established.

All of the participants were graduates of the same program offering a teaching certificate at the graduate level. In order to prevent any confusion with the participants of the second stage, participants of the first stage will be referred to as graduates from this point on. All graduates were working as full-time mathematics teachers in private schools. Detailed information is given in Table 2. Graduates were named T, followed by a number for anonymity. The vast majority of the program had female graduates and consistent with this fact that there was only one male teacher in the sample. In Table 2, it is shown in which grade levels they had experience.

Table 2
Information about participants in stage 1

Participant	Year of Teaching	Number of schools worked	Grade levels taught	Gender
T1	3	3	Secondary (9-10-11)	F
T2	3	2	Secondary (all)	F
T3	3	2	Secondary (9-10-11)	F

Table 2 (cont'd)
Information about participants in stage 1

T4	5	2	Secondary (all)	F
T5	8	3	Middle & Secondary (all)	F
T6	9	2	Secondary (all)	F
T7	2	1	Secondary (9-10)	F
T8	3	1	Secondary (all)	M
T9	3	1	Secondary (all)	F
T10	1	1	Secondary (9-10-11)	F

All of the graduates except one of them worked in schools, which offer the International Baccalaureate Diploma Program (IBDP) in addition to the national curriculum. T1, T4, and T10 had a mentor during their early career. T1 and T10 had an induction year, during which they only observed the lessons of experienced teachers or co-taught. The rest of the participants had the same responsibilities of experienced teachers in their early careers. They had 20-24 hours of teaching in a week. However, T2 had 30 hours of teaching per week in her first years. The schools that graduates worked or are currently working are all located in metropolitan cities of Turkey.

Sampling for stage 2

Purposeful sampling was utilized for stage 2 as well. Eight pre-service mathematics teachers who were enrolled in MA in Curriculum and Instruction with Teaching Certificate Program (class of 2016) were requested to participate in stage 2 and all of them gave consent to participate the study. Participants for stage 2 were named PT, followed a number for anonymity. There was only one male participant, PT 4. Some demographic information about participants of stage 2 is shared in Table 3. A more detailed table giving information about the participants is given in Chapter 4.

Table 3
Information about participants in stage 2

Participant	Gender	Undergraduate degree received from
PT 1	F	Galatasaray University
PT 2	F	Bilkent University
PT 3	F	Hacettepe University
PT 4	M	Bilgi University
PT 5	F	Akdeniz University
PT 6	F	Abant İzzet Baysal University
PT 7	F	Middle East Technical University
PT 8	F	Hacettepe University

Data collection instruments

The data was collected through mainly with semi-structured interviews, six case-based discussion sessions, written reflective tasks, and documents. The ethics committee approval was taken. (See Appendix H). Table 4 gives a summary of all the instruments used in the study by stating the type, the purpose, and the length.

Table 4
Instruments of the research

Data Collection	Instrument Type	Purpose	Length (Changes between approximately)
Stage 1			
Interviews with graduates (n=10)	Semi-Structured Interview Protocol	Reveal graduates' early career experiences	90-120 minutes per graduate
First interview with participants of CBDM (n=8)	Semi-Structured Interview Protocol	Establishing a rapport with participants Getting to know their existing beliefs, expectations, and concerns about being a mathematics teacher. Making necessary changes and revisions in the cases according to the data provided in these interviews.	60-120 minutes per participant
Stage 2			
Implementation of CBDM with the participants (n=8)	6 Narrative Case Discussions Pre-Case Exercises Post discussion tasks	Reveal participants' attentional foci and reflection characteristics. Reveal participants' perceptions related to CBDM process	90-120 minutes per discussion

Table 4 (cont'd)
Instruments of the research

Last Interview with the participants (n=8)	Semi-Structured Interview Protocol	Reveal participants' perceptions related with CBDM process Reveal the reflections of the CBDM process on Teaching Practice period.	70-150 minutes per participant
Other Documents	Teaching Practice Files Admission Essays	Complementary instruments	

Yin (2003) stated six commonly used data sources of case study evidence as documentation, interviews, archival records, direct observations, participant-observations, and physical artifacts. This study utilized mainly interviews and documentations. One of the reasons for using multiple data sources is for triangulation, which is important for credibility issues of qualitative research (Patton, 1999). Conducting case studies comes along with the strength of gathering multiple sources of data; triangulation (Yin, 2003). In this study, triangulation will be sustained by checking out the consistency of the findings generated by using different multiple data collection methods (methods triangulation), interviews, group discussion reflections, written reflections, and documentation (Patton, 1999).

Interviewing was a common means of conducting a case study (Merriam, 1998; Yin, 2003). Due to the purposes of the study, semi-structured interviews held with participants to get rich information regarding the fact that the interviewee's own definitions, wording, and perceptions. One of the characteristics of a semi-structured interview is its flexibleness and another one is that it allows the participants to answer the questions in their own way of using their own words was chosen for the purpose of the study (Matthews & Ross, 2010).

Three semi-structured interviews were conducted for the purposes of the current research. Ten mathematics graduates of the program were interviewed to reveal the challenges that they faced in their early careers in order to develop case scenarios. The interview protocol is shared in Appendix A.

The participants of CBDM experience who were the eight pre-service mathematics teachers in the program were interviewed twice. The first interview which was held before CBDM experience aimed to establish a rapport with participants, get to know their existing beliefs, expectations, and concerns about being a mathematics teacher and make necessary changes and revisions in the case scenarios according to the data provided in these interviews (See Appendix B). The second interview was conducted after the CBDM experience to reveal their perceptions of participants about the CBDM experience (See Appendix E).

Documents of all types helped the researcher uncover meaning, develop understanding, and discover insights relevant to the research problem (Merriam, 1998). This study utilized documents that were ready-made sources of data such as lesson plans, mentor meeting forms, and self-evaluation forms after each teaching as well as documents (such as reflective journals written by participants after case discussions) which were generated by the researcher after the research began.

Procedures

In this section, the data collection procedures consisted of two main stages. The cases produced in this study were both the product and the instrument of the research. Therefore, the production process of the Case-Based Discussion Module was shared, and then the implementation procedures were explained.

Stage 1: The design of CBDM

The idea of designing a case-based pedagogy as a complement to the development of pre-service mathematics teachers was originated in the readings done on teacher development. Although there were casebooks that are ready to use, the reason why the researcher produced the cases by the experiences of graduates of the program was to make the case scenarios more relevant and realistic for the participants who were pre-service teachers having their training at the same program. The results of this research in which the graduates were interviewed to reveal the challenges in their early careers were used to identify and elaborate on the issues of the case scenarios in CBDM.

To revise and finalize the writing up of the cases, participant pre-service teachers' beliefs, concerns and expectations were taken into account, and the researcher tried to make this module more relevant for their growth in the process of *learning to teach*.

As illustrated in Figure 2, the core of the design was based on literature. The inspiration for the issues of the cases was based on research and acquired through semi-structured interviews with graduates. Finally, the cases were finalized by taking participants' concerns and expectations into account. The followings were done to design and develop the cases:

1) Semi-structured interviews were conducted with the graduates of the program. The challenges in their early career pointed out the issues that has potential to take place in the case scenarios. Teachers' profiles and contextual details about the schools and the classrooms shared at the beginning of each case were also derived from findings.

2) A theoretical framework was utilized to design the cases together with the experiences of the graduates gathered from the interviews.

3) Semi-structured interviews were conducted with the participants of stage 2 to understand their existing beliefs, concerns and expectations. The case scenarios were finalized by taking the issues (especially in terms of mathematical content) that participants were concerned with. Teacher profiles were also written in a way that there were noticeable commonalities between participants' identities and case-teachers' profiles.

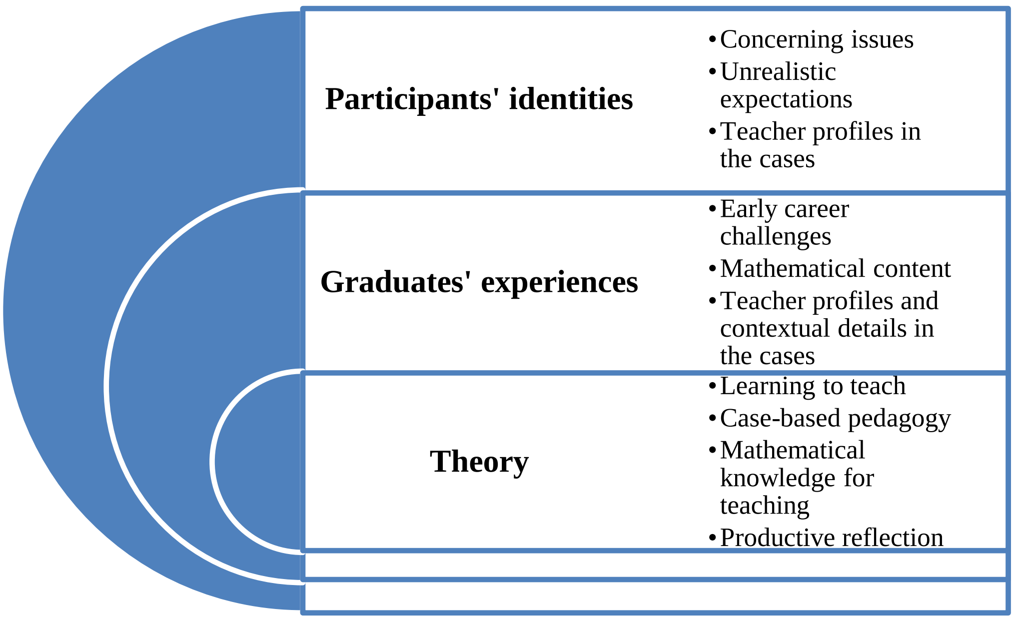

Figure 2. Framework for design process of CBDM

Semi-structured interviews with graduates

Potential participants who were among the graduates of the program were invited to participate in the study via phone or e-mail. Ten people responded to the invitation. Due to the purposes of the study, semi-structured interviews were held with these graduates to get rich information regarding the fact that the interviewee's own definitions, wording, and perceptions are central to qualitative research. The characteristics of semi-structured interviews allowed the participants to answer the questions in their own way using their own words (Matthews & Ross, 2010). An interview protocol was created. The expert view was taken for interview questions, and a pilot interview was conducted to ascertain if there was a need for further refinement that would make the participants understand the questions better. Each participant was asked to answer questions regarding the years of teaching that they did feel novice. However more or less these early career years corresponded to first three years of teaching and this is consistent with the literature (Veenman, 1984). Interviews were carried out face to face and lasted between 90 and 120 minutes. Sample questions from the interview protocol are shared below:

i. Why did you choose teaching as a profession?
ii. What were your expectations from this profession before and after the program?
iii. To what extent did your expectations match with the realities of teaching?
iv. What kind of problems and challenges did you face within your early career in teaching? (Sub-questions followed in order to dig into the problems that might have been faced related to different dimensions of teaching.)
v. What were the reasons for the challenges that you faced within your first years of teaching?

The construction of the cases was based on both research and theory. The analysis of early-career challenges revealed several issues that these graduates have faced in four dimensions: mathematical knowledge for teaching, classroom management and organization, assessment of students' learning, and the context of teaching. Also, teachers' prior beliefs and expectations showed a general lack of awareness, oversimplified beliefs, and unrealistic optimism about the teaching profession**.** The following assertions derived from the research conducted with graduates used to establish the foundation of CBDM. Examples of how the findings of stage 1 inspired the design and development of CBDM were given in the results section. Each assertion is further discussed in Chapter 5.

Assertion 1: Secondary mathematics teachers, in their early careers, mainly have challenges due to lack of mathematics knowledge for teaching, maintaining classroom discipline, dealing with overwhelming workload, lack of students' motivation towards mathematics lessons, maintaining curriculum pace, keeping records of students' progress, giving performance grades and preparing assessment materials suitable with the level of the students.

The case scenarios were constructed around these issues as minor or major challenges occurred in a classroom setting. In addition to these, mathematical concepts of the cases were supported by the literature since most of the issues like *undefined and undetermined terms, misconceptions related to limits, intuitive fallacies related to conditional probability* were also grasped interests of many researchers in mathematics education.

Assertion 2: Teachers have oversimplified beliefs, unrealistic expectations about teaching, and this is apparent in the challenges that they face.

It was found that before starting their career, teachers have oversimplified beliefs and unrealistic expectations from teaching. Reducing the complexity of teaching, oversimplifying has consequences like reality shock. Therefore, there is a need for a more connected and complex schema of teaching before starting the career**.**

Assertion 3: Teachers begin to have a more complex and connected schema about teaching when they begin to see how different dimensions of teaching affect each other when they start reflecting on the challenges.

CBDM was designed to reflect the complexity of teaching, and the reflective framework provided were combined to reflect on the connections of different dimensions of teaching in order to help pre-service teachers have a more connected schema.

Assertion 4: The mismatch between teachers' beliefs and practices indicates a theory-practice gap.

The perceived gap between theory and practice was open to discussion by the CBDM process. Discussions and reflective writings were formed in order to lead participants to consider theory in their reflections.

Each assertion was discussed in the discussion section. However, these assertions were shared here in order to state the research-based decision under the construction of the CBDM. These assertions of the research conducted with graduates illuminate the researcher's production of the CBDM by taking teachers' *learning to teach* principles into account (National Research Council, 2000).

Learning to teach principles

1) *Pre-service mathematics teachers have preconceptions and expectations about how teaching works. This "apprenticeship of observation" affects what they learn. If their initial understanding is not engaged, they may fail to grasp the new concepts or information.*

This principle implies a necessity. In that sense, it was obvious that the researcher should know the pre-service mathematics teachers' preconceptions, expectations, and concerns about mathematics teaching. The knowledge of pre-service mathematics teachers' preconceptions, expectations and concerns were gathered through conducting interviews with the participants of CBDM experience. Case scenarios were finalized according to the data gathered from the participants.

2) *In order to enact what pre-service mathematics teachers learn, they should i) have a sound knowledge of factual and theoretical knowledge, ii) understand facts and ideas in the context of a conceptual framework, and iii) organize knowledge in ways that facilitate retrieval and action.*

Cases should give opportunities for learning with developing insights, raise multiple perspectives into view, and create bridges between theory and practice (Darling Hammond, Hammerness, 2002). A combined theoretical framework for productive reflection, which was prepared by the researcher, has the potential to reinforce pre-service teachers' learning in the areas discussed in principle two. The productive reflection framework's emphasis on identifying multiple perspectives and integrating them, and through the analysis of cases, the use of theory would be reinforced. Therefore, this framework has potential to link theory to practice. Discussion plans were designed in a way that would help participants engage in productive reflection.

3) *A metacognitive approach to instruction (a reflective glance) can help pre-service mathematics teachers take control of their own learning. In order to do this, there must be opportunities for analyzing events and understand the complexities in teaching.*

One reason for the teaching profession being a complex practice is that many of the problems a teacher must address occur simultaneously, not one after another. Because of this simultaneity, several different problems must be addressed in a single action. Case scenarios should reflect the complexity of teaching. The productive reflection framework has the potential to fit this study in terms of its emphasis on recognizing multiple aspects of a teaching setting and integrating these aspects.

Semi-structured interviews with participants

Two semi-structured interviews were conducted with participants who involved in the CBDM process. The first interview was held before the CBDM process. The purpose of conducting the first interviews was to i) establish a rapport with participants ii) learn about their existing beliefs, expectations and concerns about being a mathematics teacher iii) make necessary changes and revisions in the cases according to the data provided in these interviews. At the end of CBDM process, another semi-structured interview was held. The aim of the last interviews was to i) reveal participants' perceptions related to CBDM process ii) get the reflections of CBDM process on their teaching practice period. For both of the interviews, interview protocols were prepared.

Theory, graduates' experiences, and participants' existing beliefs, concerns, and expectations were dripped from the filter of researcher and blended in order to design Case-Based Discussion Module. The reason why this collection of cases was called a module was the following characteristics it possesses (Donnelly & Fitzmaurice, 2005):

- The self-contained, formally structured learning experience with a coherent and explicit aim. All of the cases and the planned way of discussions follow the same structure.
- Not isolated but as a complementary to the courses in the program. Cases were designed not as a separate course material but as a glue that would help bring different dimensions of teaching mathematics together.

The structure of case scenarios in CBDM

In the module, there are six narrative cases (See Appendix C) with discussion plans and post discussion written tasks. The following components were employed in the case scenarios;

- Description of the teacher
- Teacher's background experience
- Description of school
- Teacher's feelings and intentions
- Students' feelings and intentions

- Dialogue
- Description of other relevant parties (e.g., parents, principals, other teachers) (Nichols, Tippins, & Wieseman, 1997, p.189)

The book *Windows on Teaching Math* and its facilitators guide edited by Merseth (2003) were used as reference books in designing the case scenarios and discuss plans. Therefore, each case scenario employed the components; pedagogy, mathematical content, assessment of students' thinking, and contextual dimensions (Merseth, 2003). All of the components belong to each case were shared in detail.

Pre-case exercise. The pre-case exercise was constructed in order to allow the participants to think about the mathematical issues raised in the case as an opening activity of the discussion session. Participants were given a certain amount of time at the beginning of each session to complete the task. Questions requiring specialized content knowledge were also asked in the pre-case exercise. Participants were expected not to discuss any solution before reading the case. Right after the time given for completing the task, they have started reading the case. Although examining subject matter knowledge of the participants was out of the scope of this dissertation, each solution and works in the papers became a data source that would enrich the findings of the CBDM process.

Description of context. Each case gave information about the school, the class that the case occurred, and the curriculum (national or international) that was provided to students. In the cases, a brief explanation was provided about the school and the class. The schools given in the cases were the ones that are described in graduates' interviews. In addition to these, in some of the cases, possible parent, colleague, and administration profiles and their communication with the case teacher were also shared.

Description of students. Cases are important means of data, providing the reader with students' real thinking. It gives students and teachers' dialogue in a natural way. So it becomes easier for the reader to reflect and discuss students' thinking and behaviors by relating it to the characteristics of the student. The cases in the CBDM involved information about students and their thinking. These were gathered through the experiences of the graduates' and researchers' observation and experience as a teacher.

Description of the teacher. Description of the teacher in the cases provided readers reasons for certain actions or emotions of the teacher in the case. The description involves clues about the case teachers' identity. Teacher Identity consists of information related to the following parts;

i) Teachers' reasons for entering the profession and aims for teaching mathematics

ii) Teachers' background experience in terms of

 a) schooling

 b) family and culture (related to teaching)

iii) Teachers' prior teaching experiences (private tutoring, tutoring in private teaching institutions (*dershane*), voluntary teaching in civil society organizations)

iv) Teachers' beliefs, concerns, and expectations about teaching mathematics.

Mathematical concepts. These concepts were chosen from the concepts that were shared by graduates as challenging topics to teach. In addition to that, the participants of CBDM experience were asked in the first interview about the topics that they feel uncomfortable to teach. Answers to this question were also taken into account while constructing cases. The details about mathematical concepts, teaching methods, and misconceptions were gathered by using the literature and the researcher's experience as a secondary mathematics teacher.

Pedagogy. Each case reflected the pedagogical choices of the teacher in order to examine the complexity in the cases. The case scenarios were designed to promote discussions about the links between pedagogical choices, and the problems occurred in the cases. Teachers' choices vary from classroom management strategies to responding students with different learning paces.

Assessment of students thinking. Case scenarios were designed to help the participants focus on interpreting students' thinking on challenging concepts in mathematics. Case scenarios also included students' reactions and behaviors when a classroom management gap occurs. It would also give an idea to the participant about identifying the origin of the problems that occurred.

In Table 5, a summary of the case scenarios in CBDM was shared. It involves the mathematical content and concepts of the cases, main issues, and sources that inspired the researcher to write these cases.

Table 5
Summary of the CBDM and the sources used during the construction process

	Mathematical Concepts	Main Issues	Source
Case Scenario 1 Undetermined	Domain of Functions Undefined and Indeterminate Concepts in Mathematics ($\frac{A}{0}, \frac{0}{0}$ where $A \neq 0$)	Lack of teacher's specialized content knowledge	Results of stage 1 *Theme 1. Lack of subject matter knowledge* Literature: (Ball & Wilson, 1990) (Cankoy, 2010) (Brahier, 2013)
Case Scenario 2 Construction	Topic: Probability Monty Hall Problem (Conditional Probability)	Lack of managing group work and its contextual consequences	*Graduates' interviews* *Participants' concerns for teaching probability* *Literature:* *(Merseth, 2003)*
Case Scenario 3 Performance task	Topic: Factorization-Completing the Square Performance tasks involving a real-life problem	Lack of assessing students' knowledge and parent-teacher communication	*Results of stage 1* *Assessment of students learning*
Case Scenario 4 Predictions and Realities	Topic: Statistics- Linear Regression	Lack of teacher's specialized content knowledge Working with graphing calculators in the classroom	Results of stage 1 *Lack of subject matter knowledge*
Case Scenario 5 Transformation	Topic: Transformation of Functions	Teachers' lack of common content knowledge	Results of stage 1 *Lack of subject matter knowledge*
Case Scenario 6 Pushing the Limit	Topic: Limits of functions	Lack of knowledge of content and teaching	Results of stage 1 *Lack of pedagogic content knowledge* *Literature:* *(Williams, 1991; Kula & Güzel, 2013)*

Stage 1 involves interviews with both graduates of the program and participants of CBDM. Stage 2 involves seven face-to-face sessions held with the participants and semi-structured interviews after case-based discussions. The first session was for introducing the study, describing what a case is, how the discussion would be held, and taking consent for their participation. The rest of the sessions were case discussions. Participants and the researcher met in a classroom at the Graduate School of Education. All the discussions were audio and video-recorded. The time-table of collecting data of both stages was given in Table 6.

Table 6
Time table of data collection procedures of stage 1 and stage 2

	Date or Time-Intervals
Stage 1	
Interviews with Graduates	From June 2014 to February 2015
Pre-Interview with participants	From March 2015 to July 2015
Stage 2	
Case Discussion 1	24th September 2015
Case Discussion 2	1st October 2015
Case Discussion 3	8th October 2015
Case Discussion 4	15th October 2015
Case Discussion 5	16th October 2015
Case Discussion 6	22nd October 2015
Last Interviews with Participants	From May 2016 to June 2016

Stage 2: The implementation of CBDM

The procedures of data collection during CBDM implementation composed of a number of sub-stages. The cases were written in Turkish and translated into English. The cases in Turkish were presented to the participants. All of the discussions were also in Turkish in order to prevent any language barriers that would lead to a less fruitful discussion. Participants were free to write post discussion tasks in Turkish or English. One of the participants preferred to write in English. English versions of the cases were given in Appendix C. Discussion plan and written task of case scenario 1 were given as an example in Appendix D.

Pre-case exercises

At the beginning of each session, participants were given pre-case exercises. These exercises are related to concepts or problems related to mathematical content of the case. Pre-case exercises were beneficial in terms of making a warm-up for the case; in order to understand to what extent the participants were competent about the mathematical content given in the case.

Reading the case

Participants were asked to highlight with a pen the issues that they found important to discuss while reading the case. This is an important piece of data to show how group discussion would affect the issues discussed rather than individual approaches to the cases. Before each discussion, in order to emphasize some important parts of the case, or in order to understand if all participants understand the dialogues in the cases, people shared roles and read the case aloud.

Case discussion part

The discussion was structured around four main leading questions which lead participants to think about and consider on issue identification, reasons of the issues, suggesting alternative viewpoints (possible viewpoints of other stakeholders such as parents, administration, colleagues, learners etc.), and solutions and consequences (Harrington, 1995). These parts were determined according to Harrington's (1995) study, which aimed to reveal pre-service teachers' development of professional reasoning with the use of dilemma-based cases. Besides these leading questions, there were case-specific questions, which would help the participants to notice and link different issues if not raised by themselves. The discussions were held with discussion plans prepared for each discussion (see Appendix D).

After the pre-discussion part, participants were given a couple of minutes to decide on the important issues and problems in the case. Each discussion started with the questions “What are the issues in this case and what is this a case of?”. Each participant raised issues that they found problematic regarding the case. Issues raised by participants were written on the board in order to discuss and find the reasons behind these problems and find solutions to them. Writing the issues on the board also helped to see the links between different problematic parts. If the participants did not raise some issues, researcher gave prompt to discuss those.

The discussion period (after pre-case exercise and reading the case) ranged from 90 minutes to 120 minutes. Most of the participants were active during the discussions. Two participants were much more silent during discussion. However, they continually took notes, and their reflections to post-tasks were very detailed and reflective.

After writing the issues, the reasons for the problems were started to be discussed. During discussions, participants were encouraged to think from different perspectives. They were asked how theory was linked to the problems occurred in the cases.

Post-discussion written tasks

After each discussion session, a reflective writing task was given to participants. Although each task was different, there were some common questions. Some questions differed according to the content and issues of the cases. Since group dynamics and personal traits would affect the contribution level of some participants, it was planned to give a post-task after the case in order to enrich the data. The post-discussion tasks were held with discussion plans prepared for each discussion (See Appendix D).

An online classroom was created in an online learning medium (EDMODO) to share post-reflective tasks and for communication. Participants turned back to the written tasks on this platform.

Last Interview

The last interviews after the teaching practice period bring us to the last data collection process of the study. The last interviews were conducted to examine participants' perceptions of case-based discussions and their teaching practice experiences. After all CBDM sessions completed, participants started their teaching practice period in two different schools in Ankara and one in Erzurum. These schools were also among the ones that the graduates in the first stage have worked in. Post interviews were conducted after the teaching practice process ended. Teaching Practice files were examined in order to pick critical incidents during their practice to ask specific questions about their experience. There were common questions, which were asked to every participant, and there were questions, which were participant-specific. (See Appendix E; an example of an interview protocol for PT1). Each participant had different experiences during teaching practice. For this reason, specific questions related to their teaching practice were written after examining their teaching practice files. The transcripts of the first interviews, case-based discussion tasks and discussions were also examined in order to especially look at some specific points that would deliver meaningful incidents to discuss on. One of the reasons to examine teaching practice files is to grasp incidents, which are worth to reflect on. Incidents like the ones in the cases have also been tried to identify in order to analyze how participants reflect on those situations. Interview questions were personalized after examining lesson plans, self-evaluation forms after each lesson, weekly reflections, and observers' evaluations. Each interview lasted between 75-150 minutes.

Data analysis

This section consisted of details of how the data analysis was conducted in both stages of the study separately. Qualitative data analysis methods were utilized in both of the stages. To be more specific, content analysis and constant comparative analysis approaches were used.

Data analysis for stage 1

All audio data gathered from semi-structured interviews with graduates were transcribed word by word. The researcher read all verbatim transcripts carefully. The constant comparative method (Lincoln & Guba, 1985) was utilized in order to analyze the data and gradually form categories by i) comparing incidents applicable to each category, ii) integrating categories and their properties, iii) delimiting the theory, and iv) writing the theory (Glaser & Strauss, 1967). Content analysis requires unitizing the data, which means finding the smallest unit of meaningful data (a word, a phrase, a sentence, or a paragraph) according to the area of research. After identifying these units in the data, each unit was assigned to a category. The researcher used software for the mechanical processing of data such as organizing, modifying and retrieving the data. It is still the researcher's skill (not the software) to interpret the data (as cited in Lincoln & Guba, 1985). In order to analyze the data, the researcher read the interview transcripts several times and unitized the data according to four dimensions: challenges, beliefs, expectations, and reasons. Then, these were grouped under categories of the main foci of attention of teachers in terms of challenges utilizing the theoretical framework. While creating categories, main foci of attention in the studies of Hollingsworth and Lidstone (1992) and Clarke and Hollingsworth (2002) were considered. Subject matter and pedagogy category were replaced with Mathematical Knowledge for Teaching (Ball et al., 2008) in order to provide more specific information related to the challenges in teaching mathematics.

The challenges that participants focused on in their early career were coded under four major categories: i) mathematics knowledge for teaching, ii) classroom management, and organization, iii) assessment of student learning and iv) context as indicated in Table 7. Participants' prior beliefs and expectations about the challenges and perceived reasons for these challenges were shared under each category. The prior beliefs and expectations that participants held about teaching had shown signs of lack of awareness of possible challenges, oversimplifying teaching, and unrealistic optimism about the profession. In addition to the prior beliefs and expectations, participants shared perceived reasons for experiencing the challenges. Through reflecting on the challenges, the source of the challenge in one category was attributed to the challenges experienced in other categories.

Table 7
Challenges in four main categories

Mathematical knowledge for teaching	Classroom management and organization	Assessment of students' learning	Context
Lack of subject matter knowledge	Difficulty in maintaining classroom discipline	Failure of the majority of the students in the exams	Overwhelming workload
Lack of pedagogical content knowledge	Students' misbehaviors	Difficulties in keeping track and recording students' progress	National curriculum load
	Students' lack of motivation		Lack of support from colleagues
		Challenges in giving performance grades	
	Lack of lesson planning and preparation		

Data analysis for stage 2

Although inductive reasoning was much more on the scene during the data analysis of the first stage, a combination of deductive and inductive reasoning shaped the second stage data analysis. The reason why stage 2 data analysis was more of a combination of both was that the development of the cases was subject to some theory and the discussions were tried to be examined by keeping some principles and "variables" in mind. Here the use of variable is not quantitative, and it represents the categories that the researcher specifically wanted to examine and the dimensions of the analysis. In this respect, coding the reflections in the discussions or the written tasks according to each category was the deductive part of the analysis and stood as directed content analysis. It allows predictions about the variables of interest, as well as the relationships between these variables, results in helping to determine the initial coding scheme referred as deductive category application (Mayring, 2000).

However, while each meaningful statement was categorized due to three variables and the subcategories that constitute the broad category, the meaning of each statement was also coded with the constant comparison technique, giving the data analysis an inductive stance. In order to understand the essence of the reflections in terms of actor, topic and level, quantification were also done. The idea here was not to reduce this intensive qualitative analysis into numbers. Quantification, regarded as an intrinsic part of qualitative research, was utilized as a complementary process to describe how participants went through during CBDM experience (Miles & Huberman, 2004). For example, while investigating the reflections, participants would have focused on an actor more than the others. So intrinsically, the researcher would look for the number of times that the participants reflect on each actor though the quantification process was held in order to make the data analysis more open to comment on the tendencies of participants' reflections in terms of actor, topic and level.

An analytic framework is developed in order to analyze the data gathered during the CBDM process while being open to new codes. Although a prefigured coding scheme was used in analysis, researcher was open to additional codes emerging during the analysis (Creswell, 2007).

Analytic framework

The analytic framework of this study consisted of many theories revised in Chapter 2. Productive reflection was taken as the framework of reflective practice in this study.

Based on the works of Davis (2006) and Fund (2010), Moore-Russo and Wilsey (2014) claimed that productive reflections should involve

a) considering the act of teaching, teaching and learning environment, students' thinking and learning, the nature of the subject, expectations of teachers, or some aspect related to the work of teaching
b) being comparative by acknowledging and building from past experiences, others' perspectives, educational theories, or educational research
c) recognizing the complex nature of teaching by emphasizing and integrating multiple aspects of teaching

It could also be related to integrating different aspects of teaching. In order to integrate different aspects of teaching, one has to notice them first. Noticing is important in teacher cognition and reflection, and the necessity of examining noticing skills brought the researcher to involve "Learning to Notice Framework." van Es and Sherin (2002) developed "Learning to Notice Framework" in order to help teachers learn to notice aspects of reformed classrooms. The study examines changes in teachers' thinking as they participated in a video club designed to help them learn to notice and interpret students' mathematical thinking. According to this framework there are three key aspects of noticing,

> a) identifying what is important or noteworthy about a classroom situation,
> b) making connections between the specifics of classroom interactions and the broader principles of teaching and learning they represent, and
> c) using what one knows about the context to reason about classroom interactions (van Es & Sherin, 2002, p. 573).

The first aspect of noticing, the ability to identify noteworthy events, is particularly important. Although the framework is developed for the cases that reflect reform practices, the framework would also be used for noticing the critical instances in a case of problematic teaching. Some of the dimensions of the framework fit into this study. The noticing framework was used in order to make use of the dimensions that the framework brought. The current research does not focus specifically on noticing the skills of teachers of students' mathematical understanding.

The original dimension of the study of van Es and Sherin (2002) was the *actor;* whom the participants focused on in their reflections. The second dimension is the *topic*; what the teachers noticed (mathematical thinking, pedagogy, climate, management or other). The third dimension is the *stance*; how the teachers analyzed the practice (describe, interpret, or evaluate). The fourth is *specificity* (general or specific) of the discussions of the thing that the participants involved. The last dimension, the fifth one, focuses on if the reflection is video-based or not.

This dissertation aims to provide an in-depth understanding of the participants' reflection processes during and after a case-based discussion experience. Multiple dimensions would provide rich information about their reflections. First three dimensions fit into the current study with some adjustments. The third dimension, *the stance*, points out the type of reflection that participants engaged in.

The participant's reflections were examined in terms of their attentional foci:

a) Actor of reflections on CBDM; who is the actor in the matter of reflection? The *actor* dimension involves *self (*participant), *teacher* (Case-teacher, teachers in general), *learner* (Learners in the cases or participants' own experiences), *school administration, parents,* and *colleagues.*

b) The topic of reflections on CBDM (What); what are the aspects of teaching and learning that the participants reflect on?

The topic dimension involves *Mathematical Knowledge for Teaching,* c*lassroom management and organization, assessment of students' learning, context, teacher identity* and *the case scenario.*

Mathematical knowledge for teaching

Although teachers' challenges can show similarities across disciplines, examining secondary school mathematics teachers' problems could be better analyzed with a framework specific to the knowledge of mathematics teachers. Therefore, in stage 1, the early career challenges of the graduates related to subject matter knowledge and pedagogy were investigated by the Mathematical Knowledge for Teaching framework. Similarly, the reflections of participants related to teaching mathematics in stage 2 were also investigated by the MKT framework. Ball, Thames and Phelps (2008). The sub-dimensions of Mathematical Knowledge for Teaching are common content knowledge (CCK), specialized content knowledge (SCK), horizon content knowledge (HCK), knowledge of content and teaching (KCT), knowledge of content and students (KCS), and knowledge of content and curriculum (KCC).

Classroom management and organization

Classroom management and organization consisted of: behaviors of students, classroom atmosphere, group dynamics, disciplinary procedures, time management, organization of the classroom, motivation, lesson planning, communication with students.

Assessment of students' learning

Assessment dimension is related to any attempt or concern related to checking student learning, formative assessments (homework, portfolios, quizzes, in-class tasks) or summative assessment (general exams), grading, keeping record of students' progress.

Context

Contextual factors consist of physical facilities and setting, types of students, parents, school and community characteristics, resource availability, classroom climate, school climate, degree of support provided by others, expectations, effects of standardized assessments, demands made on the teacher, and departmental guidelines (Grossman, 1990).

Teacher identity

Teacher identity dimension refers to teachers' reason for entry (to teaching profession), teacher education experience, current teaching context/practices, career plans, prior personal experience (incl. family, schooling), prior professional experience (incl. work) (Olsen, 2008). It also involves personality traits as a teacher.

Case scenario

The case referred to the foci when participants commented about their perceptions related to the case scenarios in CBDM.

c) Level of reflections on CBDM (How): How did the participants reflect on the issues in the cases?

Besides the *actor* and *topic* of the comments, reflection characteristics were also analyzed. The discussion was structured around four main parts: issue identification, reasons of the issues, suggesting alternative viewpoints (possible viewpoints of other stakeholders such as parents, administrators, colleagues, learners etc.), and solutions and consequences (Harrington, 1995). These parts were determined according to Harrington's (1995) study, which aimed to reveal pre-service teachers' development of professional reasoning with the use of dilemma-based cases. However, only focusing on these cases would lack a revealing source of reflections; in other words there was a need for an inquiry about these reflections on what they were based on i) individual opinions, ii) personal experiences iii) theory. With this two-dimensional reflective perspective which was explained in Table 8, the reflective dimension of the analysis was processed.

Each focus was matched with different reflection characteristic; individual viewpoint, personal connection and community connection (Moore-Russo & Wiley, 2014). Individual viewpoint: Sharing personal views or emotions without giving any evidence to personal experience or educational community's literature. Personal Connection: Connecting a viewpoint to a personal experience related as a student, as a pre-service teacher or as a tutor (or teacher). Community Connection: Connecting the viewpoint to educational community's literature or a commonly accepted theory held by many members of educational community.

Table 8
Explanation of productive reflection framework

	Individual Viewpoint	Personal Connection	Community Connection
Issue Identification	Identifying issues from a personal perspective. (I_I)	Recalling an event from own experiences (either as a student or a teacher candidate) while issues were shared. (I_P)	While describing issues, links to courses, theories commonly accepted by the educational community. (I_C)
Reason	Stating the reasons for the challenges based on personal opinions (R_I)	Stating the reasons for the challenges by giving examples from personal experiences (either as a student or a teacher candidate) (R_P)	Stating the reasons for the challenges by relating it to courses, theories commonly accepted by educational community. (R_C)
Alternative Viewpoints	Raising alternative voices (a stakeholder which would interpret the issue from a different perspective) (A_I)	Giving reasons from personal experiences in order to explain why an alternative voice may interpret the issue in the way participants reflected. (A_P)	Relate alternative voices to the principles of courses, theories commonly accepted by the educational community. (A_C)
Solutions and Consequences	Offering solutions or stating the consequence of the solutions (either their own or other solutions in the group) based on personal opinion. (S_I) (SC_I)	Offering solutions or stating consequences regarding their experiences (either as a student or a teacher candidate). These could be a solution that they offer by recalling their own teachers' practices or teachers that they have observed, or their own practices in their teaching experiences. (S_P) (SC_P)	Offering solutions or stating consequences based on what the courses offer in their teacher education or any commonly accepted theory by the educational community. (S_C) (SC_C)

Besides these, during the coding process, it was noticed that there were reflections, which were related to participants' future-oriented comments about their own careers. These could be regarded as reflection-for-action and then these were decided to be categorized as a different entity as future-oriented reflections. Future-oriented reflections included teachers' future plans as a teacher, their *note to self,* and/or *do* *and don’ts.* Although categories and variables initially guide the study, others are allowed and expected to emerge throughout the study (Altheide, 1987).

During the discussions, teachers also shared general beliefs and concerns related to teaching. Perceived changes and awareness moments were also categorized separately from their case-related reflections. In addition, as case discussions proceeds, teacher began to ask questions to the researcher during the discussions in order to get more information about her own experiences or contextual knowledge related to schools or recent curriculum. So, there were some other dimensions considered together with the reflection categories given in Table 8. These were future-oriented reflections, beliefs, general, concerns, changes/awareness and questions.

In order to accomplish analysis and reveal the relationships of different dimensions, the analyses were conducted on a spreadsheet, inspired by the methodology of a study conducted by Akşit (1998). Each meaningful statement in the discussions and discussion tasks were written in the excerpt column together with noting the participant, instrument that the excerpt belongs to. Then each excerpt was coded in terms of its foci (actor and topic) and level of reflection. Table 9 was shared in order to explain how the analysis process was held in detail.

Table 9
Sample coding

Order	Part.	Inst.	Who	What	How	Code	Excerpt	Memo
700	PT 1	D_ C4	S	A	S_I	Strategy for checking HW	Belki topladığımız ödev kağıtlarını değerlendiremey eceğimiz zamanlar oalbilir, PT 4 quizi söyledi, her zaman kağıda yapmalarını isterim, ilk hafta mutlaka kontrol ederim, arada bir kontrol eder, ona göre artı eksi veririm.	
301	PT3	D_ C2	T	KCT _TI	I_I	practicisi ng different strategies during teaching practices	Grup aktivitesini seviyor falan filan ama ikişerli tartışın dışında bir şey yok. Aslında bir özgüvensizlikle başlıyor.	emphasis on teachers' faults (criticism)
81	PT 8	D_ C1	S	KCT _Cont ext	R_C	Conceptu al teaching (constrai nt: curriculu m pace)	Biz mesela concept i anlatmak istiyoruz, conceptual olarak anlatmak istiyoruz, ama sınav var, müfredatı yetiştirmesi lazım, çelişiyoruz. Ayşe hocanın şu an yaşadığı şey gayet doğal ve hepimiz de yaşayacağız	
418	PT 1	T_ C3	S	A_Co ntext	S_I	Being accounta ble to parents	Cocugun velisine basitce anlatamayacagi miz bir etkinligi uygulanmamasi gerektigini dusunmekteyim.	In accordanc e with her teacher identity

The table summarizing the research questions together with data collection instruments, and analysis methods are provided in Table 10.

Table 10
A brief overview of the research question, data collection tools and data analysis

Main Research Questions	Sub-research Questions	Data Collection	Data Analysis
Stage 1: The Development of Case-Based Discussion Module (CBDM)			
How can a case-based discussion module designed in order to implement as a complementary practice for pre-service secondary mathematics teachers' education?	• What are the challenges that mathematics teachers face during the early career stage? • What are the prior beliefs and expectations of the mathematics teachers on the dimensions of teaching that they were challenged by in their early careers? • To what extent do they integrate different dimensions of teaching while reflecting on the reasons for those challenges?	Semi-structured Interviews with Graduates of the Program	Constant comparison method Citations from the participants
Stage 2: The Implementation of Case-based Discussion Module (CBDM)			
How do pre-service mathematics teacher experience the process of CBDM implementation?	To what extent are pre-service mathematics teachers' reflections productive? • On whom the pre-service mathematics teachers reflected during the implementation of CBDM? • What aspects of teaching mathematics are noticed by pre-service mathematics teachers during the implementation of CBDM? • What are the characteristics of participants' reflections in terms of connectedness and complexity?	Pre-Case Exercise Case Discussions Post Discussion Tasks	Directed Content Analysis Constant Comparison By citations from the participants

Table 10 (cont'd)
A brief overview of the research question, data collection tools and data analysis

	• How did pre-service mathematics teachers' identity associate with their reflections during the implementation of CBDM?	First-Interview Participants' Admission Essays Last-Interviews Teaching Practice Files
	• How do pre-service mathematics teachers perceive CBDM experience?	Case Discussions Post Discussion Tasks Last-Interviews

Credibility and trustworthiness

The terms of positivist paradigm like internal and external validity, reliability and objectivity leave its place to credibility, transferability dependability and conformability in the naturalistic inquiry.

The adoption of research methods well established: Theoretical framework of the study consisted of methods which were utilized by different scholars. Having the lens of this theoretical framework while designing the instruments, collecting the data and analyzing the data was built on well-established strategies followed by other scholar who studied teachers noticing, reasoning and reflection through case-based discussion.

The development of an early familiarity with the culture or participants "prolonged engagement" (Lincoln & Guba, 1985): which can be defined as spending adequate time in order to achieve the goals of the research such as learning the culture, testing the possible misinformation given by respondents or researcher, and building trust

(Lincoln & Guba, 1985). In stage 1, the researcher conducted interviews with the graduates of the program. Through being a graduate of the same program as the researcher, working in a private school, having knowledge of the culture of the schools that most of the interviewees worked, knowing some of them in person as a colleague, friend or mentor, a prolonged engagement was established. In this stage, the researcher's role was especially critical since interviewees were asked to share problems and challenges that they faced in their early careers. The researcher needed to make them feel comfortable to talk and give in-depth descriptions of their experiences.

Building trust was an important issue in establishing the credibility of qualitative data (Lincoln & Guba, 1985). The importance of the input provided by the participants to the inquiry process and anonymity of the participants was emphasized at the beginning of the study (Lincoln & Guba, 1985).

For stage 2, the prolonged engagement was sustained in two ways. First, researcher took the opportunity to establish a rapport with the participants by attending the school experience seminar hours as a guest in the second semester of the first year of the program. The researcher took the opportunity to establish a rapport with the participants. Participants' behaviors in-group discussions were observed with an understanding that getting to know the group dynamics would help researcher in order to get prepared to moderate the discussions in case analysis sessions. In addition to become familiar with the group, it was also important to get to know the participants in person. One of the purposes of the semi-structured interviews held before case-based discussion sessions were also getting to know the participants.

Moreover, during this process, the participants had spent School Experience 2 in the school that the researcher has been working. The researcher was also the mentor of three of them, trying to feel the others comfortable with asking help from her if needed. In conclusion, rapport was built before the discussions held with the participants. The researcher has also had a chance to learn about the culture of the group dynamics beforehand in order to take some precautions which would inhibit the fruitful discussions. The researcher was not a *stranger in a strange land* anymore before the data collections process related to case-discussions started. The investment of sufficient time to learn the culture and building trust was accomplished (Lincoln & Guba, 1985).

Triangulation: Triangulation could be achieved by multiple and different data sources, methods, investigators, and theories (Creswell, 2007). In this study, the use of multiple data sources (participants) and method triangulation were used. One of the strengths of the study was the detailed analysis of eight participants in order to check out bits of information across informants (van Manen, 1977). Participants' experiences related to the CBDM process were shared in detail in order to provide a comprehensive picture of use of cases.

Triangulation by differentiating methods was also utilized. Participants' reflections in case-based discussions were tried to be supported by the written tasks given after the discussions. In addition to the pre and post interviews, the documents such as admission essays to the program, and teaching practice files were also examined.

Tactics to help ensure honesty in informants: This study based on totally voluntary participation. Graduates in the first stage shared the challenges that they have experienced in their early careers openheartedly. Participants of the CBDM experience voluntarily participated in the study. Face-to-face discussions, interviews, post discussions tasks were held apart from the hours that they have spent in their courses in the program. The researcher has no role in affecting the participants' grades in the program. There was no obligatory feature of the study related to their teaching training process. In other words, independent status of the researcher helped to ensure honesty in participants.

Frequent debriefing sessions: The researcher has consulted the supervision committee in every important step evolving the dissertation. The researcher produced all the case scenarios and discussion plans, which were found to be suitable, used for the growth of learning of pre-service teachers by dissertation committee. In addition to that, an experienced mathematics teacher, who has PhD in secondary science and mathematics education and a part-time faculty of the program, had also commented positively on the relevancy for secondary mathematics teacher education as a person who knew the pre-service teacher profile of the program as she teaches them as a part-time faculty and being their mentor at the partner schools. In addition to that, she was consulted during the data analysis process and has seen the development of analytic framework development. She coded some reflections from the case discussion and written tasks and she found the last version of the analytic framework relevant and easier to code the data.

Peer scrutiny of the research project: The researcher also attended a workshop for PhD students in mathematics education from various universities in Turkey. In this workshop, PhD candidates presented their ongoing dissertations to a community of other PhD students and to the academics in mathematics education. In addition to that, the results of stage 1 were presented at a conference.

Thick description of the phenomenon under inquiry: Detailed description of each participants' experience with a multi-focused and multidimensional approach was a way to ensure credibility. Moreover, researcher tried to give examples from the real data as much as possible in order to enable the reader to assess to what extent the analysis matches the reality.

Transferability. The researcher tried to maintain transferability by giving detailed explanations about the process, analytic framework, the participants, and their experiences. Details related to the data collection method, length of data collection sessions and the time period the data was collected, detailed description of each of the participants in terms of their backgrounds, beliefs, expectations and concerns were shared. This was an attempt to give an opportunity to the reader to transfer the methodology in a similar setting or with similar types of pre-service teachers.

Dependability. This study has reported the details of the process in a way that enables a future researcher repeat the work and it also provided the reader to assess the extent to which proper methods have been used in order to answer research questions.

Researchers’ background, role, and possible bias

The researcher is the key instrument in the qualitative paradigm (Creswell, 2007). An inevitable consequence of being the key instrument is the reality that researcher may influence and shape the phenomena studied. Though in this dissertation, there should be information about the researcher who has multiple roles as a researcher, a teacher, and a mentor of teachers (Putnam & Borko, 2007).

Expecting the researcher to be totally objective would be impossible since an individual always has a stance. This does not mean it detaches us from scientific inquiry. Researchers should acknowledge the necessity and the responsibility to share the lenses they were looking at the data. One of these lenses was explicit since the conceptual and theoretical framework of the study was shared in detail. Therefore, it was shared that theories affected the interpretations of the researcher. However, there is also another important impact on researchers’ stance: her background, role in the study and possible biases.

The researcher is also a graduate of the program that this dissertation was held in. She is an insider to the context of the study. In the first and second stage, she was not a totally stranger to the participants, which provided possible advantages. On the other hand, there is risk of possible biases.

Schooling background

The researcher is a graduate of Ankara Anatolian High School. The preference of the graduates of this school was mainly medical schools or engineering faculties. She did not consider teaching for a career when she was in high school. She has memories of teachers who were not devoted to teaching except her primary school teacher that she highly respects. She went to private teaching institutions in order to have support for school and for the purposes of getting prepared for university entrance examinations. Mathematics was one of the majors that she wanted to study at university. She has placed in Middle East Technical University Mathematics Department as her third choice. None of the choices were educational faculties. She involved in voluntary education projects and taught students who were socio-economically disadvantaged during her undergraduate years. This experience became one of the cornerstones in her life and affected her career choice totally. Reasons to become a teacher was mainly altruistic since she wanted to have an impact on people's lives as much as possible. She applied to the graduate school of education where this research was held in. For the two years she had spent there, she believed that she had equipped with the skills to become a teacher.

Experience as a teacher

The researcher started to work in one of the partner schools that pre-services in the program went for school experience and teaching practice. Having worked there for three years, she pursued her career in other partner school of the program in Ankara. The researcher has been teaching IB DP high level and standard level courses and also has experience working with high achiever students who are enrolled to scholar development program in her school. However, she has also experienced many similar challenges in her early career with the other graduates she interviewed.

Summary

The current study is composed of two stages. The first stage is to reveal early career challenges of secondary mathematics teachers who graduated from a two-year master’s program. The results of these contributed to the construction a case-based discussion module which then presented to pre-service secondary mathematics teachers enrolled in the same program. Pre-service teachers’ identities, literature review and researchers’ experiences also shaped the development of the module. Pre-service teachers’ experiences of discussing and reflecting on the issues of the module were examined with the lens that they brought to the study. Their experiences were investigated with specific attention to the *actor* and *topic* of their reflections. In addition to that, the characteristics of their reflection in terms of connectedness and complexity were also examined. During the whole process and after the process, their perceptions about this experience were tried to be revealed. A conceptual and procedural map of the study which picturizes the whole process is given in Figure 3.

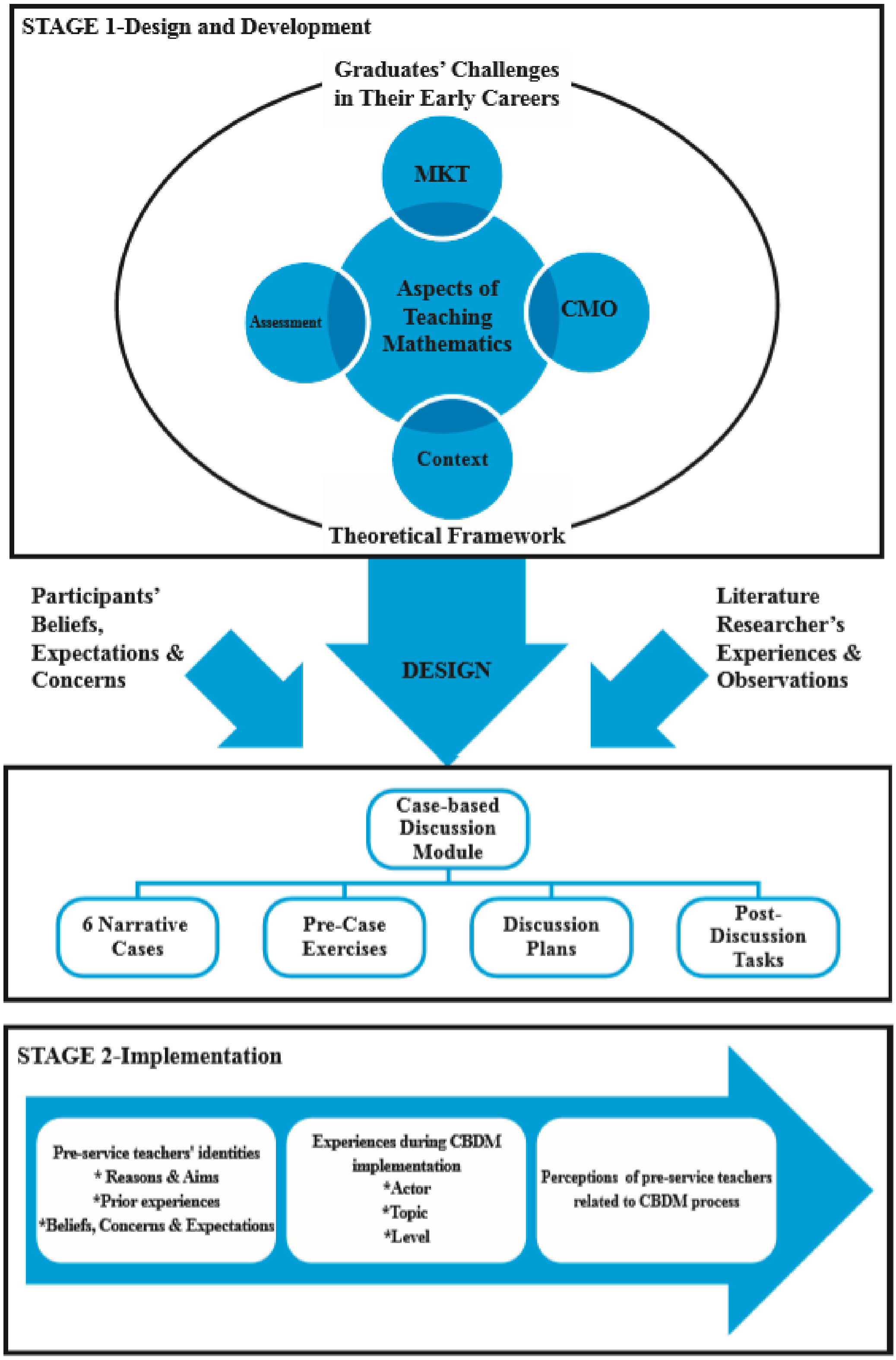

Figure 3. Conceptual and procedural map of the study.

CHAPTER 4: FINDINGS

Introduction

The findings chapter is composed of two sections. In the first section, the findings of the research for stage 1 (The Design of CBDM) conducted to reveal graduates' early career challenges are presented. The profile of the pre-service teachers, i.e., the participants to whom the CBDM was presented is shared at the end of stage 1. In the second section, the findings of the analysis related to the experiences that the pre-service teachers went through during stage 2 (Implementation of the CBDM) is portrayed.

The results of stage 1: The design of the CBDM

This part consisted of the major findings of the research, which intended to reveal the early career challenges of the mathematics graduates of the program. 10 participants were interviewed in order to understand the challenges with a holistic lens that involves their prior beliefs and expectations about teaching mathematics. The examples of how the researcher designed the case scenarios in CBDM were also shared in the following section. 6 case scenarios were produced by the researcher, and these were referred as CS followed by a number. (i.e. CS 1, CS 2, etc.). After the challenges of graduates are shared, the profile of participants to whom the CBDM was presented is given.

Challenges of graduates in the early careers

The challenges that graduates focused on in their early careers were coded under four major categories: mathematics knowledge for teaching, classroom management and organization, assessment of students' learning and context, as indicated in Table 11. Graduates' prior beliefs and expectations about the challenges and perceived reasons for these challenges were shared under each category. The prior beliefs and expectations that graduates held about teaching had shown signs of lack of awareness of possible challenges, oversimplifying teaching, and unrealistic optimism about the profession. In addition to the prior beliefs and expectations, graduates shared perceived reasons for experiencing the challenges. Through reflecting on the challenges, the source of the challenge in one category was attributed to the challenges experienced in other categories.

Table 11
Challenges of graduates in four main categories

Mathematical Knowledge for Teaching	Classroom Management and Organization	Assessment of students' learning	Context
Lack of subject matter knowledge Lack of pedagogical content knowledge	Difficulty in maintaining classroom discipline Students' misbehaviors Students' lack of motivation Lack of lesson planning and preparation	Failure of the majority of the students in the exams Difficulties in keeping track and recording students' progress Challenges in giving performance grades	Overwhelming Workload National Curriculum Load Lack of support from colleagues

Mathematical knowledge for teaching

All graduates mentioned a lack of sufficient mathematical knowledge for teaching. Most of the graduates affirmed concerns related to their lack of subject matter knowledge before they have started their careers. Graduates were aware that they had to work hard in order to recall some topics in secondary school mathematics during their initial years. As a matter of fact, their concerns related to teaching mathematics before they have started their career were limited with recalling the rules and concepts in secondary school mathematics curriculum or just being able to solve the questions that they would encounter. As T2 expressed, the prior beliefs that graduates hold about subject matter knowledge appeared to be related to mostly just common content knowledge.

> I used to believe that my content knowledge was sufficient for teaching. Of course, there were topics that I had concerns, but I realized that even for the ones I thought that I was competent, there were lots of missing parts. Without being aware of my inadequate knowledge related to where does a rule come from, I used to see myself sufficient in terms of content knowledge. However, I saw that there are many things that a student can question.

As could be understood from T2's expressions, mathematics teachers needed to have a specialized type of mathematical knowledge. Teachers' specialized content knowledge set them apart from other people who had knowledge of secondary school mathematics, and it consisted of answering "why" questions of students, being able to question students' conceptual understanding or being able to explain the reasons behind rules, formulae, and theorems. It was revealed that absence of specialized content knowledge challenged them when students questioned the concepts behind the given rules. They knew the rules and procedures in a topic but the reasons behind the rules were missing. To illustrate an example, T6 said:

> Teaching taking the square root of a number is easy when you just gave the definition or rule to the students and expect them to apply it. However, one of the students questioned me why we could write $\sqrt{3}$ as $3^{\frac{1}{2}}$. I had never questioned it before… There are lots of questions related to infinity. What does infinity mean? What does it mean to approach infinity or not reaching infinity? Similarly, they don't understand the difference between indeterminate and undefined. We have to think about it before we teach.

The challenges that occurred due to lack of specialized content knowledge inspired the researcher to write CS 1 which was related to a lesson about the domain of the functions, and students' questions why a number divided by zero is undefined and zero divided by zero is undetermined. The lack of mathematics teacher's knowledge related to operations with zero was found place in literature. (Ball & Wilson, 1990; Cankoy, 2010; Tsamir & Tirosh, 2002). The textbook that was used in methods course which was offered to the participants that CBDM was presented had also given place to the explanation of these concepts (Brahier, 2016).

Besides the shortage of specialized content knowledge, graduates shared challenges related to teaching the content that they were never taught during their schooling. For example, graduates mostly worked in IBDP schools and they were responsible for teaching statistics that they never undertook before. IBDP mathematics curriculum involved probability distributions and least square regression as well as descriptive statistics. Therefore, some graduates felt insecure about teaching statistics. The following excerpt from T8 stands as an example:

> Statistics... No matter how much we (teachers) study, we cannot internalize since we did not learn statistics before. I felt this lack of internalizing (in teaching statistics). I got prepared for the lesson by studying books and materials. However, there were questions beyond the books, the questions that arose in a students' mind. I was not prepared for this and the answers were not written in the books. I was anxious and asking myself, how could I answer? Where will I find the answer?

The articulated challenge of teaching statistics in international programs gave the idea to construct CS 4. Specific issues or concepts that challenged the graduates were asked, but there were no issues or concept-based answers in statistics. The researcher being a teacher in an IBDP school, observed colleagues discussing and struggling about teaching statistical concepts. Predicting x from y from a linear regression where y is the predicted and x is the predictor variable was a statistical mistake and the why of this was discussed in the department. Observing that even veteran teachers feeling discomfort in teaching these concepts inspired the researcher use this as the content of the CS 4. In addition to researcher's experience, the importance of teaching linear regression as one of the fundamental ideas in statistics used for exploring and modeling associations was emphasized (Batanero & Borovcnick, 2016)

Apart from the IBDP mathematics curriculum, national curriculum and national exams had also put pressure on how to handle the deficiency of subject matter knowledge. Students expected their teachers to solve the university preparation test questions and not being able to solve these questions quickly caused stress in initial years, as T6 expressed:

> There were problems with my content knowledge. I was teaching 12th grades in my first year of teaching, and I forgot the practical ways to solve the test questions. I was not as practical as I was in my high school years. That's why, in my first year, I had a need to study a lot.

Besides the topics or questions that graduates felt anxious about, they also had challenges with knowledge at the mathematical horizon. Graduates who felt ready to teach the topics in the levels that they were assigned to teach, realized that students' curiosity about the extension of the topic could be a challenge for them. For example, T7, who was responsible for only teaching 9th grade, had been challenged with the students' questions requiring extending the knowledge. One of the reasons for the problems related to knowledge at the mathematical horizon was asserted by T2 who had not prepared for the whole topic and its extension due to limited preparation time.

> For instance, when I was teaching the domain of functions, I had shown the case of square root that negative values could not go anywhere. They asked me what could be other situations of being undefined. I said when the denominator is 0, but I had not gone any further... I could not think of trigonometric functions such as tangent or logarithmic functions. I felt panic especially when I was going to teach at that class. It was like a traumatic experience.

This experience was used in the design of CS 1. It also brought up contextual issues such as the grade levels a teacher is assigned in the early career and the importance of preparing for a lesson by considering the related prior and future topics.

> A student asks a question, and the answer could be given by considering a further topic in the curriculum. The best thing is to get prepared for the whole topic. We can do it for the first topic early in the year when you have more time to get prepared. However, you cannot catch up afterward. You have no idea what to teach two weeks later.

These types of expressions were used at the beginning of the case scenarios where the profile of teacher and contextual details were given in order to enrich the discussions about the possible sources of lacking subject area knowledge.

Lack of subject area knowledge caused some other challenges as graduates indicated. Some of the graduates attributed the source of challenges in classroom discipline to their lack of subject area knowledge. A relationship between subject area knowledge and maintaining classroom discipline was not evident for graduates before they started teaching. However, they realized, after they have started their career, lack of subject matter knowledge damages the authority of the teacher as T1 confined:

> Your content knowledge is very important. When you cannot solve even a single question in front of the students, it can damage your authority and discipline. I could not foresee this before I became a teacher.

In the case scenarios, instances related to these were written in order to bring these type experiences into discussion. Beliefs about learning and teaching mathematics were constructed during schooling, and many teachers and students believed that being knowledgeable in mathematics is limited to solving questions with short ways and quickly.

On the other hand, although T10 also experienced similar challenges in classroom management due to insufficient subject matter knowledge, the response of the graduate to this situation differed from other graduates:

> There were questions that students asked, and I could not solve, and this affected my classroom management. Students asked sarcastic questions like, "Oh, so, you could not solve?" However, this was not a source of stress for me. I told students that there could be questions that make me struggle and this was normal.

In addition to the lack of subject matter knowledge, graduates shared problems related to pedagogical content knowledge. One of the major challenges was found to be the lack of knowledge of the content and students. The competencies of mathematics teaching require identifying why students do not understand a particular topic or being aware of potential misunderstandings and misconceptions. Some of the graduates realized how the pacing and flow of the lesson were connected to their knowledge of content and students. They started to relate mathematical knowledge for teaching to classroom management and organization after they started their teaching career. For example, T2 asserted:

> At the moment (during teaching), you may not understand where the student tackled. They may just stick on a meaningless point on the topic; they ask a question, which may confuse other students. Then chaos… There are many factors affecting this situation. However, the most realistic factor is your lack of knowledge of students.

During writing the cases, examples of students' ways of thinking like over-generalizations, mistakes, or misconceptions were tried to be embedded. Researcher's experience with students and common mistakes and misconceptions in the literature inspired the researchers to write students' mathematical comments or answers in the scenarios since graduates had difficulties exemplifying the fallacies in students' mathematical thinking. For instance, in CS 6, the student's misconception about limits, the limit value cannot be attained, was attained from the literature (Kula & Güzel, 2013).

In order to dig into the challenges related to graduates' knowledge of content and students, they were asked which topics were most unsettling to teach and which misconceptions students hold. Graduates' responses gave clues about the challenges that they had faced, not only in the knowledge of content and students. Probability was one of them since there were multiple ways of solving a probability question. T7 had expressed how she had got prepared for the lessons after she realized that students' methods would differ from hers:

> There are many ways to solve a problem, especially in probability. You might not understand the students' method at that moment. After I realized that, I began to get prepared much thoroughly. That's why I tried to think of many methods to solve a probability or combination, permutation problem as much as possible before the lesson.

In addition to probability theory, limits, derivatives, and integral in other words, calculus topics were also found to be challenging to teach, and in first years, graduates felt ill-equipped to teach these topics. Concepts like infinity, undefined and indeterminate terms were also challenging concepts that students ask for further clarification. Trigonometric functions, the concept of absolute value, domain, and range of functions were emphasized as the topics that the graduates reported as challenges. To illustrate, T3 had mentioned how teaching absolute value became a problem:

> I had really struggled with the absolute value concept. They were very confused how $|x| = -x$ could be possible.

Moreover, knowledge of content and the curriculum was also problematic in the early career of graduates, as T4 indicated.

> I was unable to decide what depth I should introduce the topics. Will I give details in everything I do, to what extent does the curriculum wants us to do? What is the output of an objective? Where are the boundaries?

Knowledge of content and teaching requires adapting teaching to students' levels and their needs. However, coping with the diverse needs of students also troubled the graduates. T1 related this challenge to the management problems by expressing " In my class, there are high and low achievers. When I tried to teach low achievers, the rest bored up and started to chat with the others."

Catering varying needs of students with different learning paces is an important part of the work of teaching mathematics. However, as it was noticed in graduate interviews, teachers had difficulties in differentiating their lessons according to their students' learning paces. For example, in CS 4, Arda was a student who got bored during the lesson since he had to wait several times for others to understand the content.

Illustrating the challenges that teachers faced in teaching, some student-centered practices were pointed out to be problematic. The program that graduates had graduated from designed mathematics teaching method courses around student-centered teaching practices. Relating mathematics contents to students' lives or solving problems related to real-life situations, cooperative learning, activities that students explore the content by themselves were frequently mentioned during the program as constructivist approaches to teaching. Accordingly, graduates were expected to plan their lessons with a student-centered approach during their internship, and they implemented constructivist lesson plans during the program. They were asked questions about their beliefs related to teaching mathematics before they have started their career. Most of the graduates shared how their teacher-centered beliefs related to teaching changed during the program. They utilized some of the approaches effectively after they have started to teach, like peer learning, pair work or helping students explore the rules themselves. However, they mentioned they did not continue to teach according to some of the constructivist approaches after they started their career. In this regard, T2 expressed that:

> After I graduated, I thought that I would be a different teacher (from her own teachers). I would teach with constructivist approaches; for example, group activities, performance tasks... You cannot do it with every class, or even you do it, you can not get the same response. You became demotivated. You fail because of the time limitations, process, and student profile. Hence, my expectations did not match with the realities of teaching.

Linked to what T2 expressed, there were graduates who tended to stop doing what they had found to be effective when they were pre-service teachers. However, graduates did not seem to reflect deeply on the reasons for the failure. They attributed the source of the failure to students' lack of motivation and behaviors. They seemed not to question if the activity was appropriate or not. In the end, the theory was found to be inapplicable in practice, as it could be understood in T10 statements:

> I had conducted a group activity only once. I had a plan beforehand. I arranged the groups. I gave directions. Although I carefully thought about every single detail, it did not work. I swore I would not do it again. They did not pay

> any attention except for a few students. They did not learn what I intended to teach.

Planning a group activity was thought to be determining the number of students in a group or the students in the group. Giving clear directions about what to do is important. However, determining students' roles in the group and planning how they would be accountable to other students or how the work will be assessed is also very important. Without including these in the plan and confronting chaos after the implementation was a source of giving up using the method and lead to overgeneralizations about the "theory-practice gap." This situation inspired the construction of CS 2.

Like cooperative learning, connecting topics to students' lives and the importance of relating topics to real-life situations for meaningful learning and for the motivation of students were frequently mentioned during the program. Graduate's beliefs about connecting topics to real-life before their teaching career, and how it changed during practice needed attention. T3 expressed that giving real-life examples was seemed to be something that could be dispensable for because of some contextual reasons:

> I was thinking like it would be easy to give real-life examples on each topic before I started teaching. However, during the rush to cover up the curriculum all the time, I could not focus on doing this, I could do it only at the beginning of each topic. Most of the time, I don't give the effort to do this.

In general, participants did not share major challenges due to using technology in the classroom except for infrastructural problems. However, teaching with graphing calculators in IB DP classrooms could be problematic as students were working with their own graphing calculators as the teacher was trying to lecture. T9 shared the chaos that she had during the lessons with the graphing calculator:

> One of the challenges with using graphing calculators with the whole class is that they constantly call me to look at their graphing calculators since it gives an error, or they can find something completely different than what the others found.

Challenges with using graphing calculators were used in CS 4, as it was also related to teaching linear regression. This lesson requires the use of graphing calculators.

In addition to these examples, it was noticed that while expressing their ideas about teaching, they used words “to give” and “to take” for teaching and learning. This language reflected graduates' view about the role of the teacher, as the giver of the information, and the student as the receiver, which indicated a transmissive theory of learning. To illustrate, T1 said:

> We have been taught about learning disabilities. However, experiencing it yourself is different from knowing these. You give; students don’t take. It is very different.

To conclude, graduates experienced challenges mostly in specialized content knowledge, knowledge of students and content, and student-centered teaching practices. Expectations about these challenges revealed a lack of awareness about these knowledge bases. Although graduates seemed to have beliefs that student-centered teaching should take place in teaching, they tended to be more teacher-centered in their early careers. In addition, it was revealed that they started to realize how different dimensions of teaching were integrated by relating their lack of mathematical knowledge for teaching to classroom management problems.

Classroom management and organization

All of the graduates expressed that they faced classroom management problems in their early careers. Some of the graduates said that challenges in classroom management were the most unexpected and the most demanding. These graduates stated that they had no concerns about classroom management issues before they entered the job. Graduates had observed classes with discipline problems during their internship, and they themselves experienced minor classroom management problems in their student teaching. However, they might not have reflected deeply on these and did not develop constructive strategies about what they would when issues raised in classroom management in their real teaching career.

During the program, graduates perceived classroom management as it was composed of only applying some specific strategies. Reducing classroom management to some strategies or tactics instigated a reality shock. In general, teachers' simplifications of teaching as a profession caused reality shock. An example of oversimplifying the process of classroom management was indicated by T7:

> …We thought that if you knew the rules to manage a classroom, you could just do it. In fact, it is not that easy…
>
> I had never experienced classroom management problems when I was a pre-service teacher. Maybe the presence of the mentor in the class was a factor. There was no moment that I needed to warn the students.
>
> I didn't expect to shout. I did not shout at all during the internship. Since I did not observe horrible classes, I haven't seen teachers shouting at students.

Unfortunately, expectations were not met. Their career started with many challenges related to classroom management. The major problems that they faced would be listed as the distractive behaviors of students, noise in the classroom, students' disrespectful behaviors towards the teachers and use of slang words among themselves. Students' disrespectful behaviors were again shocking for some of the graduates. Graduates seemed to have incomplete and unrealistic beliefs about students' attitudes. Quarreling with students and shouting were expressed as their dealing mechanism in the early career. The precarious part of the classroom discipline made graduates feel helpless and uncomfortable, as T3 and T2 explained:

> I had problems dealing with behavioral problems in the class. On many occasions, I have encountered attitudes that I never thought I would face… It was a really disappointing feeling incapable of managing the class.
>
> I turned out to be a person who usually shouts at students in order to control them. It did not work.

Noise in the classroom and the ways that the graduates tried to stop it was given a place in CS 6. The reasons for shouting as a way to stop disruptive behavior and the consequences of it was intended to be discussed.

When reasons for classroom management problems were questioned, one of the main factors expressed was students' inattentiveness to mathematics lessons. Actually, the disparity between expectations and realities related to students' attitudes towards mathematics was shocking for graduates. They also realized how the lack of motivation of students could affect their own motivation. As T7 expressed:

> I would never think that I would be worthless for students. I sometimes feel like this. They have no concern about learning something from you. They just want to come and sleep. I had never thought that type of student would demotivate me to this extent.

In the case scenarios, students who were inattentive to the lesson were shared with some dialogues with the teacher. For example, a failure avoiding student feels incapable of achieving and stop trying (Hardin, 2011), was shared in CS 4. These types of profiles of students were placed in the case scenarios in order to trigger elaborations on the reasons for the inattentiveness of the students.

When it was questioned why demotivated students were so shocking for the graduates, it was revealed that the schema they shaped during the program and the schema that they had as pupils again came onto the scene. T6 and T2 addressed the reasons as:

> I have been taught to teach to the students who are willing to learn; I realized that I haven’t known how to teach to the ones who do not want to learn. This was a huge shock.
> During the program, I had thought that students would be like me when I was a pupil.

When they were asked if they could observe a pattern in the situations in which they had management problems, most of the graduates expressed that whenever they did not feel ready for the lesson, there occurred classroom management problems. It might be very trivial for teacher educators and experienced teachers. However, it was noteworthy that graduates realized this not during their training but in their early careers. According to graduates, teachers’ inadequate lesson planning and consequent lack of confidence in teaching a particular topic reflected negatively in their classroom management. Graduates also have begun to recognize a link between mathematical knowledge for teaching and classroom management more lucidly after they had started their teaching career. As T2 and T7 addressed:

> Lesson planning affects your authority… If you feel secure (in terms of planning), If you are self-aware, you hold your ground before the students, your classroom management works better. But your schedule should be set in a way that enables you to prepare better. (During the program, the relationship between lesson planning and classroom management) were not considered.

> I knew that my content knowledge, pedagogic content knowledge, or lesson planning would reflect in classroom management, but I was not expecting that much of an effect.

Lack of planning and preparation for the lesson inspired most of the scenarios in CBDM. There were instances that the case teacher could not prepare for the lesson. In addition to that, there were situations that the case teacher thought enough preparation was made, but there were many dimensions that should be considered before the lesson was missed.

Lesson planning was mentioned as a possible cause for management problems. In order to have a better understanding of graduates' lesson planning, they were asked more questions about planning. It was revealed that none of them had a structured lesson plan as they had devised during the program. They had to prepare lesson plans during the program, which included the objectives of the lesson, and mainly listed the teacher and student activities with an estimated timing during the program. In their early career, there was no obligation related to written lesson plans. They mostly stated that they had a plan in their mind. If there were ready to use materials, books, or lesson notes provided by the department, they studied to retrieve the content from those, solved the problems that they assigned as homework. If they were not provided any notes, they tried to prepare lesson notes from different sources. A few graduates suggested that they imagined possible questions that students might ask during their preparation. During their internships, they mainly tried to include activities, which were student-centered, included an engaging introduction to the lesson. Some of the teachers expressed that they tried to continue this at the beginning, but they had less energy to search for engaging activities after the workload increased.

Contextual factors were also found to be among the reasons for classroom discipline problems. The timing of the lesson, especially being one of the last periods of the day, was conveyed as one of the factors that affect classroom discipline since students become tired and demotivated to learn. This issue was again an unexpected side of teaching. For example, T3 and T10 addressed:

> In the last period, I could not manage to keep students on task; maybe it's just me, I don't know.
>
> The size of the class does not matter. Students' attitudes change depending on the time of lessons. Sometimes, in the last periods, students want us just to stop. I was not expecting this.

This finding gave the idea of including the time of lessons that would enrich the discussions. CS 1 was planned to focus on a lesson that was in the last period of the day.

Although graduates did not mention many challenges in the relationship with the administration, a few had pointed out the attitudes of administrators as another contextual factor affecting the discipline of the classroom. More specifically, the lack of support from the administration when discipline problems arose was expressed. Inconsistency between teachers' and administrators' attitudes towards students damaged graduates' authority as T3 and T5 had expressed:

> In one of the lessons, I warned the students quite a few times, and in the end, I said, 'Don't you understand, are you stupid or what?'. I know that I should not have said it. The student told the assistant principal. The assistant principal told me that the student did not attend my classes because of what I had told him. The student was a problematic one, he and his friends don't attend most of the lessons, and vice-principal defends them.
>
> You take the students' phone when he/she plays with it during the class, you don't want to give it back in order to punish him/her or to change his/her behavior, but when the administration doesn't support you in this sense, you lose authority.

Moreover, the ages of the students were found to be a factor affecting classroom management. Besides, the age of the graduates was another concern. Parents' reactions also stiffed their concerns in their early careers. T2 and T3 addressed these issues with these expressions:

> We are too young (during the first few years), the age difference is not much… 12th grades wear down more, and they are more challenging.
> When you are young, parents automatically perceive you as inexperienced. At the meetings, they always point out how young we are.

In addition to these, a lack of knowing how to communicate with parents was also a problem in the early career. The dialogue of T3 with a parent was turned out to be an unexpected conversation of her:

> One day, one of my students' mother came to talk to me about the grades of her son. She was especially disappointed with the grade that her son got in the last exam. When she questioned the reasons, I told me that he did not listen. The class is a little bit noisy and her son is one the students who make noise. She said it was my job to stop the noise. I was quite shocked by her comment.

The source of parent-teacher dialogue in CS 3 grounded on the parents' reactions to novice teachers like the one given in the excerpts, like "how young you are" or the dialogue above.

One of the contextual factors that graduates addressed was that students had access to learning mathematics from different sources rather than school. Most of the students had private tutoring or attended university preparation courses. This had also challenged graduates in terms of classroom discipline since they were not the only source of information. However, this had also shown a sign of teachers' point of view about teaching; teachers as the source of information in teaching and learning. To illustrate, T3 expressed:

> Students who have private tutors or the ones at the university preparation courses don't pay much attention to my classes. They think that I am no good for them. I can't find solutions to the problems raised by these students.

The reality of private tutoring or private teaching institutions were shared in the cases. In CS 1, the student cannot get a satisfactory answer from her teacher and refuses her further help since there is no need, indicating that she will have a private lesson on the weekend anyway.

Assessment of students' learning

Graduates reported challenges related to assessing and evaluating students' learning from academic tasks. Failure of the majority of the students in the exams prepared by the graduates, challenges in keeping track and record of students' progress, and having difficulties in giving performance grades were mentioned.

To start with, the failure of the majority of the classroom in the exam was a problem for most of the graduates. This failure triggered the survival concerns of graduates and made them question their teaching practices and their prior beliefs about assessment. Apparently, formative assessment tools could not be used effectively to monitor students' learning and to adjust teaching practices accordingly. For example, T4 shared:

> …At first, you strongly believe that you taught them. I mean, I emphasized this (issue) several times, they can do it during the class. However, I might give them clues when I ask a question; I was not aware of this. When I asked the same problems in the quiz or exam, they can't solve. I questioned like "Can't I teach? Am I doing something wrong?

Graduates' lacking in assessing students' understanding by questioning in the classroom was placed in CS 1 in order to raise awareness about the teachers' questioning styles that would not enable the teacher to assess if the students could understand or not.

T1 had believed that the failure of the majority in the first exams she prepared was related to many difficult items that were chosen for the exam. The criteria that graduate set about the exam questions were not seemed to be related to learning objectives.

> In the first exam, I had chosen very good questions, which were very tricky or very challenging.

Prior beliefs related to assessing students were the decisive factors in some graduates' assessment practices. T2 had a belief that students should be asked questions they were not familiar with from classwork. Opposed to what others experienced, T7 had concerns about asking questions in the exams that students were not familiar with classwork. Assessing students turned out to be a pressure for the teacher:

> In my first exam, all my students' grades were low. I made a difficult one. I could not arrange the level. I have always thought like the questions in the exam should not be in the style of the ones you solved in the class.
>
> In my first exams, students got very high scores. I made it simple because I was anxious about students' reactions. Especially high achievers want you to teach every single thing, waiting in the wings. They saw a question in a practice book and come to you, "you did not teach us this." I could not take the risk... I don't feel any pressure from the administration about students' grades. However, I felt it from the students and their parents.

Besides exams, graduates were challenged with keeping track and record of students' progress. Checking students' homework was one of these challenges. T3 had mentioned the problem of unorganized checklists that she used and how she conveyed it to a more systematic way over time. T2 asserted the problem of being not clear about the way of keeping track of students' homework and their performances in the classroom, and how this situation caused problems about being fair in giving performance grades.

> I was not organized about checking homework and recording these. Now, I am checking homework more systematically. Previously, I was taking notes to pieces of paper, which was not organized. In my second year of teaching, I started to do it with a chart. I continually say students that doing homework is a part of performance grades. In the past, I was not mentioning it often. I was thinking like saying it once at the beginning was enough.
>
> In giving performance grades, a student asks how I gave those grades, comparing his grades with the others'. I looked at my list; I did not know why I put plus on the list, was it for doing homework or for contributing the lesson?

Challenges due to keeping a record of students' doing homework was given with the parent-teacher dialogue in CS 3. Accountability issues related to giving performance grades were associated with keeping records and opened to discussion in CS 3.

Moreover, teachers' beliefs about how homework should be used in order to enhance learning contradicted what they had done in practice. Number of students that the graduates were responsible for was an important factor in this discrepancy between their beliefs and practice, as T10 pointed out:

> In theory, I know and believe that homework is an important part of education; I knew the importance of giving feedback in order to encourage students to do their homework. However, I saw that it is hard to do it in reality. In my first year of teaching, I had more than 150 students. I was checking their homework and just asked the reasons if they did not do their homework. It was superficial. I don't think that theory is applicable in practice all the time.

Related to these challenges in keeping track and record of students' progress, graduates had been burdened with giving performance grades. Performance grades (formerly oral grades) are an important part of the evaluation in Turkey's national education system. These grades have the same weight as the exam grades in students' final mathematics grades. Teachers were not given any directives related to these oral grades in the past. Recently, Ministry of National Education (MONE) in Turkey attempted to change it in a way that these grades were supposed to be given according to objective criteria. Performance tasks were a part of those criteria. These were supposed to be class tasks that were constructed to emphasize the real-life applications of mathematics or interdisciplinary nature of mathematics. Graduates expressed challenges in preparing and implementing these performance tasks. For example, T2 had found a challenging real-life problem related to second-degree functions and equations while searching on the internet and thought it would be a good idea to give it as a performance task to students. However, it gave her a hard time with the students:

> I gave students a performance task, which was a part of the criteria of performance grades. A student preached down me, saying "you can't give grades from this, and you did not do something similar in class." I thought they had all the information and skills to solve the question since I taught it second-degree functions and equations. I was wondering to what extent they can use their skills in this topic to solve a real-life problem? I thought they could do it. In fact, it was really difficult.

This performance task is the one given in CS 3. The real student work is also shared in the case. It was also given as the pre-case exercise before reading the case.

During the program, formative assessment methods were taught, and the difference between summative assessments was mentioned. However, some graduates believed that students' signs of progress could be evaluated only by the results of summative assessments during their training. To illustrate, T10 mentioned:

> I was thinking like; I could give performance grades similar to what they have got from the exams. I was relieving myself by thinking like this before I started to teach. However, it could not be the case in practice.

Actually, in practice, giving performance grades became a very tough and stressful process for some graduates as T6 pointed out:

> Giving performance grades was very challenging. It even haunted my dreams. Giving homework, recording them, making them a part of the grad; but in the final decision, students' participation in class, behaviors, I was questioning if I am fair.

On the other hand, teachers who gave performance grades according to criteria that were decided upon collectively by the department reported that giving performance grades was not a big concern. All the criteria were set at the beginning of the semesters, and students were informed about the procedures. However, there were teachers who were not happy about pre-set criteria determining performance grades since this situation inhibited their freedom to increase the grades of the students.

Context

Overwhelming workload and national curriculum load were found as the major contextual challenges. The support that they could have received in these respects but did not was also mentioned as a challenge. Although these were expressed several times as major challenges own, they were also proclaimed to be the reasons for the difficulties in the previous categories; mathematical knowledge for teaching, classroom management and organization and assessing students.

To start with, most of the graduates implied that the workload was overwhelming, especially during the first years. They also perceived overwhelming workload as a reason for other challenges they faced. Out of school time was spent to prepare the lessons and lesson materials as well as to refresh their knowledge about the curriculum. Meetings and administrative reports were also exhausting in addition to the teaching hours. Prior beliefs and expectations related to the workload of teaching profession did not match with the realities of the graduates' early careers, as T2 expressed;

> …In my first year of teaching, I was teaching 30 hours... There were no materials to use right away, and I had to prepare everything on my own. I used to work approximately 6 hours on the computer after school time. This heavy workload demotivated me and even caused a dislike towards this job. I was looking for other jobs at the end of that year. I chose to teach since I don't want to work day and night and since I wanted a social life. However, I don't have a life after all in my first year.

Similar to what T2 experienced, most of the graduates had mismatches between their expectations related to the workload of teaching and the reality that they have faced in their early careers. Although some of the graduates had shared that the internships during the program had given an idea about the workload that a teacher could have, most of the graduates stated that there was an unexpected amount of work when they started their teaching career. To illustrate, T4 shared:

> I was not expecting to work this much out of school time and the fact that it (teaching) needs too much preliminary preparation. This is something I realized both during the program and during the early times of my career. One normally assumes things become automatic after some time, however, I realized it was not the case.

When it was questioned why graduates' perception of the teaching profession did not match with reality, it was revealed that the conviction that graduates hold about the teaching profession seemed to be affected by the public image of teaching as T8 addressed. Moreover, possible deterrents and obstacles were not considered. Instead, mostly idealistic beliefs and expectations about teaching were shared by graduates, as T1 asserted:

> I believed what had been told to me, that teaching job was easy, teachers worked fewer hours, and they worked less compared to another profession. However, it was on the contrary. Six days a week. Even on Sundays, you spend your time preparing for your lessons. You take work home and then you have to work at night as well. I feel like 24 hours in a day aren't enough to do all my work.
>
> I only wanted to help individuals gain some things and teach them some things. I never thought about the difficulties. Neither did I think about working hours nor the workload. This was a shock at the beginning.

At the beginning of each case scenario, information about the teacher is given. One of the information's about the teachers is about their working conditions. The overwhelming workload shared in the cases in order to raise awareness and discuss possible reasons for it and consequences on the practices of the teacher.

Although the vast majority of the graduates' perceived workload as unexpected, there were graduates who foresaw the intensity of the work-life before entering the profession. T7 even thought that the program was much more demanding in terms of workload compared to a teaching career.

> The work pace at the program prepared me for the idea that I have a similar future in the profession. I was not as exhausted after I started teaching compared to the times in the program.

Although the workload was a challenging aspect of teaching in the early career of the teachers, the heaviness seemed to change across schools. In some of the schools, there were no ready to teach materials or have no induction period for teachers. This situation increased some of the graduates' workload. The lack of collaborating with colleagues or the lack of ready-to-use materials that were prepared by the department affected the workload of graduates.

> It was not only improving myself academically. Every teacher had to prepare his or her own material and exercises. There was no common source or material among the teachers. When I started at another school, there was less work since the department had more teachers.

Lack of content knowledge was another reason for the long working hours for teachers outside the school. All of the graduates have mathematics degrees, and they had taken mathematics courses most of which were not directly related to what was taught in the secondary schools' mathematics curriculum. Apparently, they needed to review secondary school mathematics before they started their careers. Although the program provided curriculum review courses, it seemed that they needed to put in more effort in order to be compatible with the secondary school mathematics curriculum as T1 mentioned:

> I realized that the workload is not only teaching hours. Since I had focused on educational sciences during the program, not on content knowledge, I had to recall and upgrade my mathematical knowledge after I have started teaching. Most of the days, I was studying and solving questions for long hours after school.

Being a major challenge, overwhelming workload also interfered with teachers' classroom practices. They did not continue to use the methods that they thought were effective during their training. To illustrate, T4 and T2 mentioned:

> I used to solve a question related to previous lessons' topics, and I tried to give chance to students who were not quite comfortable with the topic. However, consistency is important. I started this way but stopped it after a semester or

> so… The reason could be because of me, fatigue, quizzes, heavy workload… you give up things.
>
> This much workload had affected my lesson planning. I did not think in-depth on my lessons as I used to do in the program. I could not use methods that we had learned during the program effectively.

In addition to an overwhelming workload, the national curriculum in secondary school mathematics in Turkey puts pressure on most of the graduates. The main challenge was to cover the entire curriculum in the allocated time given. In addition to curriculum load, lesson losses caused problems in terms of catching up with the annual plan. Teachers faced with many unplanned losses due to weather conditions, school activities, and general exams.

When prior beliefs and expectations about national curriculum were examined, it became clear that graduates were mainly concerned about delivering effectively the lesson that they had planned during the internships. They were also reflecting on whether they managed to achieve lesson objectives and covered all that they planned by writing self-evaluations after each lesson. Since they did not have the responsibility of the class, in the long run, they did not realize how covering the curriculum would be a challenge in their teaching career. Another important factor that affected their lack of concern about curriculum pace was their unrealistic beliefs about student learning. Accordingly, they did not think about the possible challenges related to implementing national curriculum for the whole year and how it would affect their teaching practices. Lack of concern, optimistic biases, and oversimplification related to curriculum load could be seen in T5 and T3’s expressions:

> In fact, I don’t have much idea about the curriculum or the reality of a rush to cover the curriculum. During the internship, I would think that it would be easy to cover the curriculum since students will understand what you teach immediately.
>
> I had never thought about (curriculum load). I thought I could manage to cover it when I have to. Although I sometimes have little concerns about teaching, whether I would succeed in this profession, I had always believed in myself. I had high self-confidence. I did not focus mainly on the negative sides or possible challenges I could face. For example, I had always thought I could modify the curriculum, after started teaching I realized that there are a lot of restrictions on it and now I really have difficulty in covering the curriculum.

On the contrary, there were a few graduates who realized that covering the curriculum was an issue for teachers during the observations in the program. Curriculum load was perceived to be something that restricts teachers’ practices during internships, as T6 mentioned:

> During the program and our internships, I noticed the reality of the national curriculum. How to enact it? What does it mean to cover it? Then I understood why teachers had a rush and skipped some topics quickly even though I thought it requires much time to teach.

Similar to an overwhelming workload, the curriculum load had also affected graduates’ choice of methods to teach. Maintaining curriculum pace was perceived to be an obstacle to implementing student-centered activities, as T4 asserted. Similarly, T6 had given up the use of technology in her lessons in the rush of covering the curriculum where T7 was very stressed after doing what seemed right to her in terms of teaching according to students’ needs.

> During internships, we always prepare activities rather than direct teaching since we believe that in that way, students could get the main point of the topic. However, if you do this after in real life, there occurs the problem of covering the curriculum. We have never thought about it during internships.

> I might use technology more, integrate it into my lessons, but we cannot do it because of rushing to cover the curriculum. Using technology becomes like wasting time.
>
> If it was needed, I spent my whole class on solving one or two questions. It is because I have idealistic views about teaching. I find it useful. However, this caused pressure and stress. Last year, through the end of the year, I lost sleep over covering the curriculum.

The rush to covering the national curriculum was given in CS 1 by pointing out the stress that it creates on teachers together with its effects on teachers' practices. The tensions that the teachers had about what they expected to do before their entry to teaching and what they ended up doing in reality were given in CBDM.

During the program, graduates were taught that motivation was key to success. Although their beliefs did not change, they found curriculum load as a restricting factor for doing activities increasing motivation. They tended to give up doing or even not to try what they found effective for students because of maintaining curriculum pace. For example, T2 had stated that the intention to deliver the lesson in an enjoyable way was not realistic because of maintaining curriculum pace. The choice of word "enjoyable" reflects the beliefs of graduates about student-centered practices and how it contradicted with their practices in their early careers.

> We have to do things to increase the motivation of the students. However, there are obstacles. Like maintaining the curriculum pace…You don't have to rush during the internship. You just focus on the method, which would make your lesson enjoyable and motivate your students. However, it is not the case in reality. If you teach the first period in an enjoyable way, in the next period you have to rush. What do I mean with enjoyable? For example, designing lessons according to multiple intelligence theory, or implementing a group work, or a performance task. You cannot do these with every class. They won't give the same reaction. You fail and become demotivated because of the insufficient time (curriculum pace) or the student profile you are working with. Hence expectations do not match with reality.

In addition to influencing graduates' teaching methods, the stress of maintaining curriculum pace affected teachers' communication with students and consequently classroom atmosphere as T2 mentioned:

> When I fall behind in the curriculum, I become anxious. I became totally intolerable, even a small interruption. Students also notice that I am nervous.

The pressure of covering the curriculum and general exams were shared in CS 1. The stress of the teacher and the wrong critical decision in the lesson due to the stress were shared in this scenario.

School context was a factor affecting the amount of stress when the graduate could not catch up with the curriculum. Working in crowded schools was found to be more stressful in terms of catching up with the other classes compared to working in less crowded schools. However, less crowded schools had some disadvantages in terms of support that a teacher could receive. For example, T3 was the only mathematics teacher in her department. Although she could get help from experienced teachers in other departments related to classroom management or communication, she could not get any support in terms of teaching mathematics. Moreover, some other graduates had also explained the attitudes of the colleagues as being not supportive. T2 had shared how some colleagues disappointed her in her first year.

> We were very idealistic when we first graduated. I was trying to decorate my classroom with mathematical stuff, or I was enthusiastic to try new methods. I was preparing reports in detail. However, some colleagues were coming and checking what I was doing and they were like "why do you do things like this? They (administration) would expect us to do the same?". They gave me the feeling that I was doing these in order to stand out among them. Over time, I started doing things more superficially.

Summary

The first stage of this study aimed to portray the early years' experiences of a group of mathematics teachers. The challenges faced by these participants were interpreted with a holistic approach. With this in mind, the beliefs and expectations behind the challenges and perceived reasons for those challenges were examined while vicariously revealing challenges. The results led to four main assertions.

i. Secondary mathematics teachers, in their early careers, mainly have challenges due to lack of mathematics knowledge for teaching, maintaining classroom discipline, dealing with overwhelming workload, lack of students' motivation towards mathematics lessons, maintaining curriculum pace, keeping records of students' progress, giving performance grades and preparing assessment materials suitable with the level of the students.
ii. Teachers have oversimplified beliefs, unrealistic expectations about teaching, and this is apparent in the challenges that they face.
iii. Teachers begin to have a more complex and connected schema about teaching when they begin to see how different dimensions of teaching affect each other when they start reflecting on the challenges.
iv. The mismatch between teachers' beliefs and practices indicates a theory-practice gap.

As was also mentioned in Chapter 3, CBDM was designed in the light of these assertions, researchers' experiences and the theoretical background. Examples that inspired the content of CBDM were tried to be shared in the results above. However, it was also important to improve CBDM with the identity of the participants. In the following part, the profile of participants in stage 2 (implementation of CBDM) is given.

Participants' teacher identities

Teaching related characteristics of participants with a lens that considered their reasons to choose teaching as a career, their aims and responsibilities that this choice brought, their beliefs related to mathematics and in specific learning and teaching mathematics were analyzed. Concerns and expectations associated with their future career as a teacher were also investigated. Therefore, participants' teacher identities, prior to the implementation of CBDM, were shared under two main sections.

i. Reasons, aims and responsibilities: Reasons for choosing teaching as a career, aims and anticipated responsibilities of the participants as future mathematics teachers.
ii. Beliefs, concerns, and expectations: Beliefs of the participant related to mathematics, mathematics teaching and learning, their expectations from the teacher education program, teaching and their concerns related to teaching mathematics or teaching in general.

Participants' background, experiences during their own schooling, their job experiences, their families are shared in order to give a detailed profile of the participant discussed under the subcategories mentioned above. Some descriptive information about the participants was given in Table 12. In this table, one can find out the participants' year of birth, undergraduate schools (college and high school), and prior experiences related to teaching.

Table 12
Participants background: The year of birth, schooling and prior working experiences

Participants	Date of birth	University (Graduation year-CGPA)	High School graduated	Prior Experience
PT 1	1991	Galatasaray University (2014, 2.72)	Yeşilköy Anatolian High School	Tango Tutor Data analyst (one year)
PT 2	1987	Bilkent University (2014, 2.59)	Başkent Ayşe Abla Private High School	None
PT 3	1991	Hacettepe University (2014, 3.1)	Kalaba Anatolian High School	Tutor at a PTI (1 year)
PT 4	1985	Bilgi University (2014, 2.85)	Kadıköy Anatolian High School	Volunteer in Şirince Mathematics Village and Matematik Dünyası magazine Private tutor
PT 5	1991	Akdeniz University (2013, 2.89)	Hürriyet Anatolian High School	Tutor at PTI Volunteer in Ankara Volunteer Team
PT 6	1988	Abant İzzet Baysal University (2013, 3.29)	Ümraniye High School	Tutor at PTI (1 year) Private tutor
PT 7	1990	METU (2014, 2.59)	Mustafa Saffet Anatolian High School	Private tutor Voluntary teacher at TEGV(Educational Volunteers Foundation of Turkey)
PT 8	1990	Hacettepe (2013, 2.61)	Eyüp Topçu Anatolian High School	Tutor at PTI (1 year)

Participants' ages varied from 24 to 30 when the study conducted. It was also noticed that participants were graduates of different universities, which are from various cities in Turkey. Fellow attendants of the program were mostly from similar universities except Galatasaray University and Bilgi University. Most of the participants were graduates of Anatolian High Schools in which generally high achiever students attend. There was only one participant who graduates from a private high school. PT 4 had also had a private middle school background as a student.

Considering the family background of the participants, we see that PT 4, PT 2, and PT 3's mothers are teachers. PT 7's family also has a teaching background. Her mother taught for a while and her father was a graduate of teacher institute, her three aunts were also teachers.

Moreover, PT 6 and PT 8 have teaching certificates from other universities. It was also important to note that having a teaching certificate and still trying to have a degree from the program were important. One of the reasons that they have chosen the program was the opportunities that they were offered in terms of field experience in Turkey and abroad. In addition to PT 6 and PT 8, PT 2 has her B.Sc. degree from the university that the program was offered. She has taken four education courses during her undergraduate studies.

In addition to these, they had teaching experiences before they have entered the program. Almost all of them had voluntary teaching during their undergraduate studies. The private tutoring was also very common as a teaching experience. Four of them worked in a private teaching institution. PT 3 who worked for one year after she graduated from university, frequently shared the experiences she had there during interview and CBDM experience.

Reasons and aims

Participants shared the reasons for their entry to a teaching career and these reasons could be grouped under three main categories; intrinsic, altruistic, and extrinsic reasons. One of the reasons for choosing teaching mathematics as a career was intrinsic. All of the participants shared intrinsic motives such as the joy of teaching, interest in mathematics, being good at mathematics, communicational aspects of teaching to list a few. Most of the participants emphasized that they were competent in mathematics in their schooling and this was one of the reasons that the participants preferred to study mathematics. For example, PT 8 defines her career choice of mathematics by saying that “In high school, my best course was mathematics. I wanted to be good in one certain field.” and this statement gave clues about their self-determination needs. Lacking the feeling of competency in mathematics and/or mathematics teaching would affect PT 8 more than the people who would not choose to define their motives as being good at something.

In the development of CBDM, teachers’ profile was composed of features of both the graduates and the participants. For example, in CS 5, the teacher represents a profile whose teacher identity is more of a subject matter expert as its foundations grounded on her prior experiences as a student. PT 8’s reason to become a teacher was used in order to create tension when the teacher made a mathematical mistake in the classroom in the given case. This was not peculiar to PT 8’s situation. For example, PT 6 and PT 2’s concerns, shared in the following part, also included making mathematical mistakes and teaching wrongly.

Altruistic reasons, on the other hand, have to do with the moral part of choosing the teaching career. The idea to help others and shape people’s lives was the main motive to choose the teaching career. Some participants shared altruistic reasons for becoming a teacher. To make a difference in students’ lives and the feeling of being beneficial to society were shared. As PT 2 compared teaching with other professions:

> You know, people do little things, but they endeavor as if they would save the world; just like this, we (teachers) do very little things, but their results could be majestic. (PT 2, FI)

Besides, there were extrinsic reasons for this career choice. As a matter of fact, the career choice of teaching was not their first choice after high school as they were graduates of mathematics department, not mathematics education departments. Their initial motives were not directly related to being a mathematics teacher. For most of the participants, teaching would be considered as a fallback career. When they could not be able to do what they want at first hand, like financial mathematics as PT 8 did or when they have compared it with other sectors like finance, they found teaching more approachable. For example, PT 1 had chance to work in a pharmaceutical firm as a data analyst in the year before she entered the program. As a result, she compared and contrasted her experiences there with the teaching career. One of the reasons that teaching became appealing for them was that they think teaching profession does not require accountability issues or at least it was not considered as serious as it is in many other occupations. As other extrinsic motives, the length of holidays and job security were listed.

The perception that teaching does not involve accountability issues was also given a place in CBDM. For example, in CS 1, the case teachers' expression "behind the closed doors of the classroom; no one would interfere" was inspired by the participants' perception of teaching profession.

When the sources of the reasons were also investigated, the effects of the parents and experiences in schooling became apparent in their career choices. Participants shared their memories of teaching their friends and how they got the feeling of satisfaction and being appreciated. PT 4, PT 3, PT 7, and PT 2 have teachers in their families. Being raised in a family having a teaching background would influence their choices and their beliefs of the teaching profession.PT 7's statements stand as an example:

> In fact, my decision was not a momentary decision. It was rather a process. When I think about the reasons for entry, I will go back in time. My father was graduated from a teaching institute and my three aunts are teacher. My mother had also worked as a teacher for a short period. I grove up directly in such an environment. Having experienced such a process, it is by itself that a kind of knowingness or the face of being teacher enters into your spirit. (PT 7, FI)

Their choices of teaching mathematics came along with the aims they set themselves. These aims include helping students grasp not only the applications but also the purity and beauty of mathematics, helping them understand the underlying ideas behind the content, learn how to think mathematically, leading them to be autonomous learners, making interdisciplinary links in the lessons, encouraging learners to ask why questions or motivating them to ask further questions. One of the goals was to be a non-traditional mathematics teacher. In PT 3's own words;

> As I said before, I would like to teach mathematics by using technology, not only with traditional methods. In fact, I want to be a contemporary teacher, not a teacher who forces the

> students to do test. I want to teach mathematics by making it concrete. I don't want mathematics remain something abstract. It could be possible via this modeling or games etc. (PT 3, FI)

Apart from goals related to mathematics teaching, the moral dimension of teaching was also stated as the aim of the teacher. PT 6, referring her own experiences as a student, aimed to be teacher who makes a change in students' lives as she asserted:

> Some people affected me; they added something to my life; they directed my way. I want to feel the same thing morally and I want to add something to someone's life. I want to make them say 'yes I learned something' when they look at their past. I don't know whether it is to satisfy my ego but I don't have such an aim; I just feel like this. (PT 6, FI)

PT 3 wants every student in her classroom to learn and she is afraid of being one of the traditional teachers. Moreover, like most of the participants, she emphasized the importance of constructivist methods. However, the beliefs which were claimed to be changed after they entered the program, could not be internalized. Her language reflects the beliefs of teachers as the transmitter of the knowledge, as it could be seen in the example:

> If a student does not understand the topic, I cannot ignore it; this is my duty. That is to say; I have to transmit it to the student one way or another. (PT 8, FI)

Apart from these idealistic views about their future career, they were further asked about their possible responsibilities in the future. Some participants again mentioned some ideals like "making students like mathematics" and "to reach every student" like PT 1:

> In general, the responsibility of the teacher is to discover the abilities and potentials of children and to motivate them. (PT 1, FI)

On the other hand, most of the participants (PT 3, PT 6, PT 4, PT 2, and PT 7) also referred to the responsibilities of a teacher in their job descriptions like duties, paperwork, leading school clubs, parent meetings etc.

Beliefs, expectations, and concerns

Participants' beliefs about mathematics, mathematics teaching, and learning were found to be intertwined. The realization that the nature of mathematical knowledge is beyond some rules and procedures made the participants imagine themselves as future teachers who would emphasize conceptual understanding. Helping students to explore mathematical ideas were shared. For example, PT 1 who had a dynamic problem driving orientation to mathematics as she expressed her appreciation of beauty of mathematics and the systematic way of thinking in mathematics and it was in accordance with her aims in teaching such as to show students the beauty of mathematics in itself. She has an aim of teaching mathematics parallel to her beliefs about the nature of mathematical knowledge as she mentioned:

> Mathematics is a challenging field but this is because of teachers. I have to change this. I want to show pupils that mathematics has a beauty in itself. I want to show them that they can do it by themselves being independent of their teachers. … Our responsibility is to teach how to think. To do this, we need logic more than mathematics to think and talk in a reasonable way. We learn mathematics in order to be able to think and speak logically. If we can manage the categorization system in our brain compatible with mathematics, we can reach knowledge more rapidly. (PT 1, FI)

Participants' beliefs related to learning were investigated from their comments both from the first interview and their admission essays to the program (An example is given in Appendix F). Apart from what participants shared as their prior experiences, the essays that they have written during the admission process to the program were also investigated in order to understand their approach to learning mathematics.

Participants generally associated learning with motivation, interest, and attracted attention. Accordingly, they have emphasized the teachers' role in making the content interesting and fun for the students. In addition to that, PT 5 gave the utmost importance to communication and stated that teachers would better make the lessons fun in order to grasp the attention of the students while PT 3 emphasized on the role of relating the mathematical content to real-life, using manipulatives in order to enhance factors for learning. Her experiences in private teaching institute gave her self-confidence as she claimed that she understood students' learning difficulties and many other aspects related to teaching mathematics.

> I worked as an intern teacher in a private teaching institute. So I know student's learning problems. I learned a lot of teaching methods, and I know how to manage classroom. (PT 3, E)

PT 4, in contrast to most of the participants, value individual intellectual effort given on problem-solving for learning mathematics. He mentioned how his experiences at Bilgi University and Şirince Mathematics Village changed his beliefs about learning mathematics.

> In Mathematics Village we took some courses, that is, some teachers there gave lectures to us. In six years, I learned how to deal with a mathematical problem at Bilgi University. If necessary, I have to think about the question for hours. I mean, I entered the program with a feeling that I am ready. (PT 4, FI)

Most of the participants had voluntary teaching experience, and one of the initial familiarities of performing teaching came along with these experiences. PT 7 stood out in terms of her devotion to voluntary teaching. In general, the amount of time and effort for these types of voluntary teaching experiences limited to going and giving a lecture without too much prior preparation. However, PT 7 shared in detail how she planned science and mathematics activities.

One of the most prominent issues that were revealed in relation to participants' beliefs about teaching and learning mathematics was their evaluation of their own schooling in terms of mathematics education. Participants were exposed to traditional teaching contexts in their own schooling. It was apparent in their memories and their statements of "change" after they entered the program. This perceived change was from traditional to a constructivist form of mathematics teaching. The memories related to past experiences as a student were being taught with a static orientation of source of mathematical knowledge which is the transmission of knowledge, rote-memorization, the reality of university entrance exam preparations and test preparation methods could be listed as examples. Participants realized these and shared how their orientation to learning mathematics changed, as PT 8 shared:

> When I prepare for the university entrance exam I learned procedurally. Our teacher used to teach well; I still remember and use what she taught us I didn't question the reasons behind the rules since we were exam-oriented. Yes, that part was missing since I didn't question the rationale when learning. But now, I search for the rationale in every topic. I have never been a student to ask why and how we learned this. Therefore, I think so many things changed in my beliefs. (PT 8, FI)

Change in beliefs related to learning affected their beliefs related to their ideal mathematics teaching. For example, PT 1 seemed to embrace a contemporary-constructivist orientation as she frequently mentioned the importance of discourse community that she was planning to have in her own classrooms and her endeavor of helping students being independent thinkers and problem solvers. However, her experiences in her own schooling made her think of teachers as the abstract source of true knowledge and she stated how her "good mathematics teacher" definition changed after starting the program. The following excerpt gave clues about her schema about teachers before the program.

> Before I entered the program, I used to believe that teachers were extremely knowledgeable creatures. I have not thought that teaching needs a preparation period before since I have never seen a hesitation in my teachers [in terms of content knowledge]… I had realized that the teachers which I identified as good when I was in high school turned out to be not as good after I have entered the program. Because according to my learning style, if a teacher gives the definition and solves a few challenging questions, he/she was the best. My definition of good mathematics teachers changed a lot from now and then. (PT 1, FI)

They were also introduced to the International Baccalaureate Diploma Program (IBDP). The program offered pre-services a teaching certificate that would enable them to teach in school implementing IBDP. Participants who encountered an international curriculum with a philosophy that promotes meaningful learning in mathematics and a constructivist disposition towards mathematics teaching and learning, they began to define themselves to be a future IBDP teacher. PT 5's words stood for an example of perceived change in her beliefs regarding mathematics teaching;

> In special teaching method courses, we learned how to teach by exploration, and it was completely IB that added this to me. I went through this phase very hard. In our own schooling, we always used to be given directions... But now, this is not so. We did some projects; we prepared worksheets in order to make the students explore the content step by step. (PT 5, FI)

PT 4, on the other hand, values pure mathematics and does not like to make it concrete. He frequently emphasized the abstract nature of mathematics and according to him, students should not be misguided with some applications or efforts of making the abstract concrete. His educational background in the university and in Şirince Mathematics Village seemed to have a strong influence as he gave examples from his experiences in there.

Participants have varying concerns related to their future careers. PT 3, PT 1 and PT 5 reported almost no self-concerns related to teaching. PT 3 only mentioned that she would struggle to teach in English in an early career. PT 5 and PT 4 had thought of the possible issues related to school climate, in terms of attitudes of administration to the teachers. PT 5 emphasized her prior concern related to teaching as a positive school climate, in her own words:

> Certainly, it is important to be happy in the school in terms of administration, communication, the rules of the school. The attitude of the school [administration] towards the teacher is also important. The profile of the students is not that important as the reason for your presence as a teacher to help them get better. This is your job. I don't have any difficulty with this issue. But I recognized that colleagues in the department, administrators, national education [ministry] or are incredibly important. (PT 5, FI)

PT 6, PT 2, PT 8, and PT 4 had some concerns in terms of sustaining discipline in the classroom. PT 8 and PT 7 had concerns related to catering students' needs with different learning paces. They had doubts if they can handle attending all students in class. PT 7 and PT 6 shared their concerns about creating mathematical misconceptions in students' minds. In PT 6's words:

> Did I teach what I supposed to teach? Did I cause something wrong while teaching? I always question these. This is why I generally did like this: If there are two similar examples that

> students possibly make generalizations, like if I gave two one-to-one functions as examples of functions, students might generalize all functions must be one to one, I tried to be careful about this. However, it was challenging to consider while you were selecting the examples. (PT 6, FI)

Lacking mathematical knowledge for teaching was a concern for most of the participants. PT 7 emphasized that she had concerns in terms of lacking conceptual knowledge that is required for meaningful learning. In other words, she is more concerned about specialized content knowledge rather than common content knowledge. On the other hand, PT 2 had concerns about making mathematical mistakes in front of the students as she asserted:

> It may be a situation like that I cannot transmit something correctly or that I teach something wrong. I am very afraid of doing this someday. I have to be aware that I did something wrong; if I don't, alas! Children may do everything wrong. I have to fix it immediately, in the following lesson. What will happen If I don't, the children learn it in the wrong way? (PT 2, FI)

When a lack of mathematical knowledge for teaching was shared, subjects that they would hesitate to teach were asked. One of the topics that the participant mentioned as a concern for teaching in the future was teaching statistics. One of the reasons for this was that they have no or little background in statistics. Except PT 8, PT 7 and PT 3, participants shared concerns related to teaching statistics. One of the reasons that they feel incompetent in teaching statistics is that they have never been taught statistics in their undergraduate years. In addition to that, in national curriculum that the participants were exposed to when they were in high school, they did not encounter topics related to statistics. Another frequently mentioned subject was probability. PT 4 shared his views about these subjects:

> Because I have never learned statistics. First of all, it is necessary to study in order to be good at a topic. Secondly, it is necessary to teach that topic. Thirdly, you have to write a book on it. At least I learned like this. For example, I am not good at probability. I was not good too when I was student. Most probably I wouldn't be good while teaching. I mean, it is one of the topics from that I avoid teaching. (PT 4, FI)

Other topics that were asserted to be a concern were Calculus topics. Participants especially emphasized limits as an abstract topic that would be difficult to teach. In addition to these, functions and graphing functions were also mentioned. Less frequent topics; geometry, matrices, logarithm, trigonometry. Mathematical contents of the case scenarios in CBDM were determined by several sources; graduates' challenges, participants' concerns, and literature. The concerns of the participants were also effective in choosing the mathematical content of the cases. (CS1: functions (zero concept), CS2: probability, CS3: quadratics and factorization, CS4: statistics, CS5: transforming functions and CS6: Limits)

In addition to these, PT 7 and PT 6 were seemed to be more concerned in terms of teaching; they differed from other participants with their disposition to reflection. Reflective practice (not a higher level) seemed to become a natural way of thinking about their experiences. PT 7's comments on her reflections were shared. It was also important to notice the effect of the language on reflective processes as she shared:

> Every evening, I try to think about what I have learned today. I especially try to write. After the class, I write these emotional changes since I am afraid of forgetting and being one of the teachers whom I criticized. I write my feelings and my questions like 'could it be like this?' or 'should I pay more attention to that?', etc. But what I write are feelings and emotions. By doing so, I can go back to that moment even a little. For example, I will definitely think about what I have learned in this interview and take some notes on it or the course that I took today. That is to say; I make an effort at least. But when they [reflection assignments in the program]

> force me to do this, I don't want. I am not comfortable especially when I write in English. I am more comfortable in expressing and criticizing my feelings in my own language. But [if it is in English] there exist some questions in my mind whether this grammar is true or not. I don't write all of them sincerely. (PT 7, FI)

Participants expected to implement what they have been exposed to in the program. They thought that they have enough theory and they were ready for just practicing it. They were exposed to a knowledge base regarding teaching and learning in the program and most of them adopted beliefs of the teacher education program. However, some of their comments seemed quite superficial and simplistic and expectations from their future career affected accordingly. PT 3's words could stand as an example;

> Letting children discover something on their own is one of the techniques; I recognized this in the program. Previously I used to consider education as something teacher-centered. But I noticed that the students' exploration of the content is also a technique and the trend in education is in this way. We took almost all theoretical courses; therefore, the only thing to do next year will be to practice this. (PT 3, FI)

In addition to being simplistic, dimensions of teaching seemed to be separated in participants' comments. For example, while talking about teachers' responsibilities related to keeping students on task, PT 8 claimed that good teaching and classroom management were different entities;

> For example, teachers must not let students playing with their mobile phones… I would not criticize her/his way of teaching if she/he may teach very well, but I care about some years of experience (in terms of being competent in classroom management). Are teaching very well and classroom management? I think they are of course different things. (PT 8, FI)

Apart from their idealistic expectations for their future career, PT 6 and PT 3 shared some possible negative constraints on achieving their goals as a teacher. Though they were expecting from the future that there would be parents' demands on students' success on high stakes exams, lack of students' intrinsic motivation, lack of materials, and school ethos incompatible with her ideal teaching. It was also noticed that participants were optimistic about their own teaching. Although she listed many constraints, PT 6 mentioned her expectation from herself as enacting what she has learned from the program, as she stated:

> In the future, I want to apply all that I did here [learned in the program]. I think there will be no positive outcomes products if we give up theory by claiming that we don't apply this. Spending so much time here [in the program] will become meaningless then. (PT 6, FI)

In the next sections, participants' experiences with CBDM were shared. These experiences were investigated by taking participants' characteristics related to teaching into account. Therefore, in the next section, more examples were shared in order to reveal the reflection of their teacher identities on CBDM experiences.

The results of stage 2: The implementation of CBDM

This section consisted of the findings of the second stage of this dissertation, which was implementing the CBDM. The results gave a detailed picture of 8 pre-service mathematics teachers' reflections on six case scenarios in terms of what (actor and topic) and how (characteristics) they reflected.

Actor of the reflections on CBDM

The results shared under this section answers the research question: On whom do pre-service mathematics teachers reflect during the CBDM process?

The participants focused on various actors during group discussions and written tasks. These actors are mainly teachers (the case teacher or teachers in general), learners, self, parents, administrators, colleagues and others. In some of their reflections, participants have made some generalizations about teachers or they have deduced some principles for teaching in general. These types of comments were coded as *teacher in general*. The comments, which did not have an actor in focus, were coded as *other*. There were few comments focusing on curriculum developers or education policy-makers. These were also coded under the category of *other*.

In general, it could be said that almost all of the participants have more or less a broad view about actors of teaching, which means none of the actors was dominant in more than half of the comments. CBDM provided an opportunity to raise issues on each stakeholder. However, the majority of the comments were on the teacher, self, and learner. These actors could be classified as *major* and *minor* in terms of teachers' foci. *Major actors* included the teacher (case or general), self and the learner. The rest could be classified as *minor actors*.

Major actors

Participants differed in terms of their main actor at the foci. PT 3 and PT 5 mostly commented on *the teacher in the case* where the others were mostly focused on *self* . Although these two participants showed similarity in terms of the primary actor that they have commented, the content of the reflections related to *teacher* actor has shown some differences.

PT 3's approach to case teachers was mostly full of criticism, which means her comments were beyond identifying an issue related to *the teacher*. She made comparison between themselves as pre-service teachers and the case teachers. She criticized case teachers harshly in terms of their preparation for the lessons, their lack of subject matter knowledge and poor classroom management. In the first interview, she has little concern about herself as a teacher. She personalized the issues less than other participants as she had focused on *self* less than the other participants. She thought she had all the skills except teaching mathematics in English fluently as she said:

> Being a mathematics teacher who is actively fluent in speaking English is the top for me. I'm ready for the rest. (PT 3, FI)

When PT 3's comments in the discussion and written tasks were compared, it was noticed that writing tasks involve more comments focused on *self* in density. One of the reasons for this could be that some of the questions in the writing tasks directly asked to comment on her own feelings or thoughts if they were in the shoes of the case teacher.

The rest of the participants mainly focused on *self*. It was a sign of identifying themselves with the case teachers or offering solutions by putting themselves in the place of case teacher. However, PT 7 differed from another participant with her particular focus on *self.* She is the only person who focused on *self* more than half of her comments. Actually, PT 7 gave clues about focusing on *self* as a teacher in her first interview. She has lots of concerns about herself since she has continuously reflected on her experiences. Compared to other participants, she has the most reflective comments about her experiences as a future teacher and a learner. In addition to these, her family also seemed to influence her teacher identity. Many members of her family as shared in the profile of participants were teachers and she had a childhood in a teacher dominant community. She had criticized many teachers in this community harshly in terms of being traditional and lacking effort. When she wanted to become a teacher, her biggest fear was to become one of them. This fear has also influenced her school experiences in the program. She had observed teachers and criticized them being more active than students. She reflected on her own teaching both in school experiences and her private tutoring about being teacher-centered. She realized that she has the tendency of adopting teacher-centered education which corresponds to her fears and the opposite is not as easy as it has been seen as she said:

> In the beginning, it was so [criticizing the teachers]. Regarding the experience in the school, I used to always see the same thing and criticize "S/he just lectures on the board." However, after my first lesson, my observation of the lesson was different. Why? Because I personally experienced the difficulties in teaching the lesson. I saw all the things such as how I call students to the board or how I say them "don't do it" or "shut up" I supposed I saw. But I didn't. I had just looked at. I started to comprehend them much more clearly. (PT 7, FI)

She had also reflected on her level of mathematical knowledge. She thought she did not have substantive knowledge in mathematics. Specifically, she had concerns about herself in terms of a lack of specialized content knowledge. PT 6 is the second person focusing mostly on *self*. She also has a lot of concerns about her *teaching*. In the first interview, she expressed her concerns about being able to teach students. The signs of beliefs of teaching as transmitting knowledge were seen in the participants' expressions. In her own words;

> I fear if I cannot do my job very well. If the lesson is not effective or if I cannot fulfill my responsibilities, there will appear some problems for both sides. If so, I cannot give anything to the students. Still, I say I cannot give. But I don't know how to do if the student is closed to learning. Shall we inject into their vessels? (PT 6, FI)

When the major actors were considered, all the participants seemed to have less focused on the *learner* compared to the *teacher* and *self*. However, PT 2 and PT 1 attended to *learners* as much as they attended to teachers. PT 2 draw a profile that was very empathetic with learners in the first interview. PT 1's attention to learner quite differed from PT 2's. One of the patterns found in the reflections related to the learner was that she tried to explain the reasons behind learners' inappropriate behaviors or struggles in terms of the problems occurred in other dimensions. Her learner-centered orientation was perceivable in her reflections about learners during the case discussions as well as her beliefs unfold during first interview. The following excerpts were from the case discussion and her first interview. The consistency between her beliefs related to learners before case discussions and her reflections during case discussions were noticeable.

> Please tell me why a student chats in class; either I send him/her off the class if she/he chats there is a problem. Did not he/she like the questions or is there a problem with my teaching? (PT 1, D_C6)
>
> Kids are learning-oriented creatures. We (teachers) enclosure them. (PT 1, FI)

Another detail noticed in her reflections was that she focused much more on the learners in the first two cases compared to the rest. Apparently, as focus on the *self* increased, focus on *learner* decreased.

Minor actors

Stakeholders outside the classroom, like parents, administrators and colleagues, which we can call as *minor actors*, are not the ones that teacher candidates usually focus on. In the first interviews, they have mentioned some factors related to *minor actors*. There was meeting with the administration in CS 2 and a parent meeting in CS 3, and the question "how would other stakeholders have interpreted this situation?" in the case discussion process led the prospective teachers to reflect on these stakeholders and possible connections of these actors to their practices such as assessment and evaluation or classroom management. They also expressed this while sharing their thoughts on CBDM, and this was also shared in the relevant section (perceptions of participants on CBDM). Although PT 5 participated less in discussions compared to other participants, she is more aware of the stakeholders outside the classroom. One of the reasons for this may be the fact that PT 5 is one of the participants who have teaching experience in a private teaching institute and in the first interview, she commented on administration or parents' expectations from the teachers and their possible consequences for teachers' practices.

> The exam reality of Turkey is also a problem. Maybe there is also the pressure from the parents. All the parents want their child's mathematics to be excellent. But they don't know why? So the matter is again with the exams. The reason is the weight of mathematics in the scores is high. I think that it is necessary to persuade the administration for different activities but then the administration will claim that the success in the university entrance exam will decrease. So it would be an obstacle for the teacher to employ different activities. (PT 5, FI)

PT 5 is not the only person who shared her concerns about *minor actors*. In the first interviews, PT 4 has also shared his beliefs and some of his concerns about his future colleagues. It could be associated with the fact that PT 4 is the participants who reflected on the actor colleague the most.

> I don't know how to say, and I fear it will be a bit donnish but I don't like department things. I should be there individually. I don't want to bring the issues in the classroom to the department. I don't want to involve in the issues of others' classes. My class is my class. (PT 4, FI)

There were similar findings in the first stage of this research. Graduates have similar expectations from the teaching profession. They have thought they would be less accountable compared to other professions. The teacher in CS 5 has a profile, which involves similar expressions. PT 4 was also inspired the researcher in terms of shaping the profiles of the teachers in the cases.

The topic of the reflections on CBDM

The issues that the participants noticed in CBDM were shared under five dimensions; Mathematical knowledge for teaching, classroom management and organization, assessment of students learning, context, and teacher identity. The results shared under this section are the answer to the research question: What aspects of teaching mathematics are noticed by pre-service mathematics teachers during the implementation of CBDM?

Mathematical knowledge for teaching

One of the most important dimensions of teaching was MKT since the cases were designed to keep mathematical content at the core, being a case-based discussion module developed for the growth of learning of pre-service mathematics teachers. Therefore, participants' comments on this dimension were of great importance.

MKT was the mostly commented dimension among the others. The issues that participants focused on in terms of mathematical knowledge for teaching were examined under two main titles; subject matter knowledge and pedagogical content knowledge.

Subject matter knowledge. Subject matter knowledge dimension of teaching consisted of common content knowledge (CCK), specialized content knowledge (SCK), and horizon content knowledge (HCK). All of the participants raised several issues under these dimensions, more specifically under CCK and SCK during CBDM process.

Participants focused on CCK by addressing the issues of *lacking common content knowledge*, *attitude toward topics in the curriculum*, and *nature of the topics in mathematics*. These were considered as in relation to the knowledge of mathematics that is not peculiar to mathematics teachers.

The issues raised in terms of *lacking common content knowledge* consisted of not being able to answer students' questions (in a limited time), making mathematical mistakes during instruction, and improper use of notations and terms. Having difficulty to answer the questions that the students asked was associated with forgetting it and could be resolved by devoting more time preparation process. On the other hand, making mistakes during instruction was found more serious and the discussions were focused on how to remedy after doing it. Making mistakes would stress teachers, especially beginning teachers. Discussing these were also important since PT 6 and PT 2 raised concerns about teaching wrongly in the first interview and it was shared in the profile of participants section.

Attitudes of teachers toward some topics were also discussed. For example, participants noticed that the case teacher in CS 4 did not like teaching statistics at all and the reluctance of the case teacher reflected the students. This situation made them think about their own attitudes toward some topics in mathematics. In the first interview, participants were asked if they have any concerns about teaching a particular mathematical subject. Probability, statistics, trigonometry, geometry (3D and conic sections), functions and calculus topics were addressed as a concern. Beyond being a concern to teach, there were attitudinal dispositions about some topics like statistics. PT 4, especially, emphasized his particular dislike. In CS 4, the case-teacher, Özgür has similar feelings about teaching statistics. In fact, in the process of revising CS 4, PT 4's concern was taken into consideration and added to the profile of the teacher in CS 4. As a consequence, PT 4 identified himself with the case teacher and commented as follows:

> As a pre-service mathematics teacher, I don't like teaching statistics., This is why I felt myself in that classroom when I read the text, and I felt suffocated since I may live in the same situation. I will probably live. (PT 4, T_C4)

Except PT 6 and PT 7, all other participants thought that the *nature of some mathematical topics* like being too abstract, having no real-life connections and being difficult was seen as one of the sources of problems occurred in the classroom. These reflections reveal beliefs of participants in terms of the nature of mathematical knowledge. They may be hesitant to teach these when they became teachers or they may devote more time to be prepared and help students overcome the difficulties that were rooted in the nature of the knowledge.

In terms of SCK, participants noticed *the lack of answering why questions of students*, *lack of conceptual understanding*, and *consequences of lacking specialized content knowledge*. They had discussed the strategies not to experience the consequences of lacking SCK. PT 7 differed from other participants in terms of her awareness and specific examples of differentiating common content knowledge and specialized content knowledge. PT 6 was also attentive to SCK. PT 7's frequent emphasis on lack of conceptual understanding in mathematics teachers including herself as a future teacher showed one of her needs as a pre-service teacher. Awareness of content knowledge, which is specific to mathematics teachers, separated her from other participants. She differentiated CCK from SCK, as it could be understood in one of her comments in CS 1 discussion.

> What we do is just to accept and continue. We think that it is related to content knowledge but in fact she could not make sense of it. I think it is not related to content knowledge but rather to know how to teach the difference between indeterminate and undefined terms. There are lots of books dealing with this. (PT 7, D_C1)

Although all of the participants noticed on the issues related to SCK and their lack of SCK, PT 7, PT 8, and PT 6 gave signs to improve it during CBDM. In post tasks, there were questions related to the challenge that the case teacher had in terms of lacking SCK. For example, in CS1 as a pre-case exercise, participants were asked to identify what "A/0 and 0/0" are and explain why they are so. Except PT 6, participants could not give a satisfactory explanation although they identified them as being undefined and indetermined. However, in the post-task, PT 8 gave a detailed and satisfactory explanation connecting her reflection to theories in learning. Her answer to pre-case exercise is also shared in order to show how she developed her understanding (see Figure 4.)

If I were the case teacher, rather than continuing through the examples, I would teach the concept of undefined first and solve a question regarding removing the value which makes the function undefined from the domain. Then I would teach indeterminate forms and solve a question regarding the subtraction of the value which makes the outcome indetermined and would show removing the value from the domain. Then I would solve challenging questions that include both cases. In the end, I would plan to solve some other questions involving square roots or trigonometry. In that stage, I would let them try themselves and then explain. In fact, this would be like an enactment of Vygotsky's scaffolding theory to my teaching. My teaching of “a number divided by zero is undefined.” For the number a that is different from 0, a/0 is undefined because if a/0=x, it means a=0.x
In this case, whatever x is, when it multiplies with 0, the result will be 0. But a was different from 0. Now let’s try to define the x value. We reach a definition like that “x is the number that will give a value different from 0 when it is multiplied with 0.” We cannot make such a definition, and so we say it is undefined. I had called it infinite when I first answered it in pre-case but after the case I said myself I should study on this and I did some research. Then I decided that this way of teaching will not confuse.

My teaching of Zero divided by zero; Again, if 0/0=x, then 0=x.0, and the result is 0=0. For any x value, this equality will be attained. That is, x can be any real numbers, so it is infinite. However, x is indeterminate. If there is a question like why we don’t term it as undefined instead, there is a number that would satisfy the equation, but this number is not unique. (PT 8, T_C1)

2) $\frac{A}{0} = \infty$ Bir sayının sıfıra bölümü sonsuzluk ifade etmektedir çünkü sıfır bir büyüklüğe sahip değildir ve $A \neq 0$ sayısının içerisinde sonsuz tane sıfır vardır.

3) $\frac{0}{0}$ = Belirsizlik belirtir. (2)'de belirttiğim gibi sıfır sayısı büyüklüğe sahip değil ve içerisinde büyüklüğe sahip olmayan sayıyı arıyoruz. Bu bir belirsizliktir.

Figure 4. PT 8’s answer to the pre-case exercise in CS 1

For the *horizon content knowledge*, participants except PT 1 and PT 8 did not consider any possible consequence of lacking HCK. However, in the early career, in teaching only one or two grade levels, teachers would lack helping students see the bigger picture; in other words having a less connected schema about mathematics.

Pedagogical content knowledge (PCK). Participants' reflections related to PCK were categorized under KCS, KCT, and KCC. Participants mostly attend to KCT and its link to other dimensions. Focus on KCS followed. However, there was little attention to KCC.

In terms of KCS, participants focused on anticipating students' possible questions, struggles, common mistakes and misconceptions. Participants varied in terms of their foci on KCS. PT 6 focused more on KCS compared to other participants. PT 3 followed her. PT 2, PT 1 and PT 4 showed similar patterns where PT 7 and PT 8 rarely touched the issue. PT 5 commented only once for KCS.

In all of the case discussions or written tasks of CBDM, participants noticed that students would ask unexpected questions all the time and the teacher must be prepared for these. The quality of the answer that the teacher gave to those unexpected questions was found important in terms of meaningful mathematics learning. Although experience in teaching was thought to be one of the important determinants in anticipating students' questions, some participants emphasized the importance of being ready about these in the early career. Therefore, it was addressed that the teacher should think of possible questions that students would ask during the lesson preparation process. For example, PT 6 claimed that CS 2 raised awareness in terms of lesson preparation by considering possible questions of students and possible challenges that may occur in the lesson.

> I should prepare the plan by considering possible questions and challenges; this affected my perception, in fact. (PT 6, T_C2)

On the other hand, PT 3 did not consider this process of anticipating students' questions and struggles as a part of the lesson preparation process. Relying on experiences in her own schooling seemed to be enough with an assumption and generalization about students' possible struggles does not change over time. In her own words;

> In fact, we should not relate it to teacher preparation for the lesson; in fact, all of us undergo high school and university education. Therefore [it is also important to think on] what type of troubles I experienced in learning a specific topic when I was a student. You don't need to have a mentor. I could ask these to myself. The challenges in learning mathematics is always the same; students do not change at all. We have good mathematics background; it is highly possible that we have understood some topics much more easily. But is it possible to foresee all the challenges? At least, we can ask to our close friends about their experiences in learning mathematics. Therefore, I think that my own schooling experiences support my teaching (PT 3, D_C1)

When the importance of planning was discussed in terms of anticipating students' questions, PT 4, who focused on contextual factors more than others, again looked at the constraints of work of teaching and stressed the importance of experience in teaching. He said:

> Let's suppose that you have 30 hours class in a week; it would be incredibly tedious to prepare lesson plans for each class by looking at the misconceptions, etc. I think it is something that could be gained by experience. (PT 4, D_C5)

Many participants articulated misconceptions held by students. However, PT 6, who had a pedagogical formation before she enrolled in the program, approached the misconceptions more consciously. She is distinguished from her knowledge and approach by having a more theoretical stance.

> The limit is one of the topics that are difficult to comprehend conceptually. Students will face various challenges while this topic was taught. For example, there is a mistake that limit value can never be reached; that the limit is the value which could be made much certain as; that calculating the limit means just evaluating the value of the function at that x value. I remembered these misconceptions from the notes that I had taken when I was reading some papers about misconceptions related with the limit concept while I was in Marmara University [She got a teaching license from Marmara University] (PT 6, T_C6)

Participants focused on several factors in *KCT*, especially on teachers' *approaches to teaching mathematics, instructional actions and decision, selecting tasks, and the teaching tools and strategies used.*

PT 8 focused on the KCT the most, and PT 6 followed her. The rest of the participants showed similar patterns in terms of frequency. One of the reasons for PT 6 and PT 8's particular focus on KCT would be that they both have pedagogical certifications from other universities.

The *approaches to teaching mathematics* were identified by the instructional decisions or styles of the teachers in the cases. CBDM gave an opportunity to discuss the approaches and poor instructional strategies that teachers employ. Procedural and conceptual teaching was discussed. Although direct teaching was a common source of criticism pointing to the teachers in the CSs, PT 8 seemed to understand that there would be some lecturing in a constructivist classroom. Her approach is more to remain focused on conceptual ideas while also helping students practice fundamental skills.

PT 3 and PT 4 noticed the lack of keeping students mentally active. Case teachers' questioning methods were found ineffective in terms of making students mentally active. PT 4 was alert to teachers or students' questions in the CSs.

PT 4 emphasized a more inductive approach in which the teacher should help students think and explore the concept by themselves. He pointed out that the case teachers do not make available the tools for helping students to explore the concepts. As he articulated:

> When the child asks the alternative ways, the case teacher immediately mentions the tree-diagram but I would not do this. If the students have grasped the main idea, then let them think themselves. I would not intervene at all. It is better to give a little clue and support them to discover themselves, rather than saying the solution directly. (PT 4, D_C2)

PT 8 stood out in terms of her attention to approach to teach mathematics. She made clear distinctions between the procedural and conceptual approach to teaching and linking it to assessing students by considering the consequences of procedural teaching. For example;

> I think in general, we should be aware of whether our solutions or shortcut solutions are consistent or not. Teaching the shortcut solutions is what the students like the most; however, if the system falls down, it could be the teacher who will pay the price for this, just like here in the case of teacher Gizem. For example, teacher Gizem cannot ask a difficult question or similar question in the exams. (PT 8, T_C5)

Instructional style of the case teachers in CBDM triggered conversations about the approaches to mathematics teaching. A procedural approach to teaching was identified with case teachers' language, as PT 6 noticed in the teacher phrases in CS 1. The case teacher said, "you will get used to as long as you drill and practice" to the students who were confused about the domain of functions. PT 6 asserted;

> She says 'you will get used to as long as you solve more questions' but by 'getting used to,' does she mean that the students will 'learn" or that they will just memorize the solutions of the similar questions. (PT 6, D_C1)

Most of the participants identified these types of phrases as leading to rote-memorization. On the contrary, the conceptual understanding was highly valued by participants and main emphasis was on adjusting instruction which would promote development of mathematical concepts in students' minds. Participants criticized the teacher-centered approach of the teachers in CBDM. Direct teaching was noticed and critiqued. Constructivism was underlined many times as participants' main orientation to mathematics teaching during the CBDM process like PT 5's comment;

> It could be one of the problems that I will live in my teaching life, and I should care about being clear in my definitions. The best solution, depending on the time and the level of the class is to ask some students to write what they understood, with their own words or however they like (they may be free to express themselves by words, questions or totally mathematics). This could be a self-assessment for the students while a pre-assessment for the teacher before question-solving. (PT 5, T_C6)

In terms of *KCT*, participants critiqued the *teaching tools and strategies used* by the teacher. One of the methods that were discussed was the group activity. Collaborative learning methods were discussed in terms of their pros and cons. In CS 2, there was a group activity that was turned out to be a lesson that not worked out. At the beginning of the discussion, some of the participants labeled the case as an example of theory-practice gap. However, as the discussion proceeded, participants mostly agreed that the problem occurred was due to lack of designing the activity properly or the task was chosen was found to be inappropriate. It was important that participants became aware that when some of the activities did not work in some lessons, they must not be in the tendency of quitting it for the rest of their careers. Reflecting on the pitfalls and trying again and again by making remediation were suggested as solutions.

> Teacher İpek makes an activity; she thinks that she fully planned. I had thought in a similar way at first. I hadn't recognized that there was much detail to consider. While I completed here, I understood that planning for so many group studies is much more different. (PT 8, D_C2)

In CS 2, there was no individual accountability of the students in the groups in the activity that the case-teacher designed. It was one of the most important issues in a collaborative learning environment, which differentiates a group of people working together from learning cooperatively. After the discussion on why the group activity did not work, in the task, PT 7 showed awareness of how to keep students more accountable for what they are doing in a collaborative learning environment.

> I think pair discussions will be more effective than group study. In the first stage, I would pair 2 students and give them time to discuss. I would ask the pairs to share their ideas and write their strategies and solutions step by step. In the second stage, I would ask the pairs to be 4 by combining with other pairs and share their ideas by showing what they wrote. Therefore, in the first stage, they would develop their own ideas and focus much easier and if they face challenges, they would be able to create new ideas by putting their heads together. In the second stage, they could assess the new ideas that they face. By doing so, I think each student will be more active. (PT 7, T_C2)

The use of technology in mathematics education was discussed, especially in CS 4 and CS 5. In CS 4, there were problems in monitoring the students' work and in giving clear instructions about the graphing calculator, and lesson time was lost due to these challenges. Therefore, all of the participants were attentive to the use of graphing calculators in a whole class activity. Participants criticized the method that the case teacher employed. All of the participants had suggestions for preventing the problems occurred. In general, virtual graphing calculators (projected on the board) were offered as a tool to help teacher to avoid problems occurred due to managing whole class while using graphing calculators. Group activity or working in pairs was another solution offered by some of the participants. They thought that the ones that were competent with the calculators would help others. The lack of preparation for graphing calculator was also mentioned. Therefore, a better lesson planning that works with graphing calculators was considered as a solution. In addition to that, they realized the importance of having a command of using the calculator and knowing its functions was very important since students would ask anything when they were dealing with it. For example, PT 2 experienced a similar problem to CS 4 during practice teaching and suggested some possible cautions in order to avoid similar challenges. She wrote:

> Make sure everyone has their graphing calculator with them beforehand. If not, use SmartView and projector. Make sure everyone has reset their calculators and then make the necessary arrangements (Diagnostic on, for example). Make sure you are completely familiar with calculator's functions. Because students ask a lot of questions. (PT 2, T_C4)

In CS 5 and CS 6, participants offered the use of graphing calculators in order to take advantage of graphing or visualizing to support students' understanding.

PT 4 complained using graphing calculators instead of computers. He has articulated some prejudices about the use of graphing calculator before CBDM process. During the discussion, he became aware of the reality that teaching in IBDP requires the use of graphing calculators and the teacher does not have the right to ignore its importance as he saw the consequences of it in CS 4.

Real-life connection was another issue that participants attend in terms of KCT. Connecting mathematics to real-life phenomenon was discussed with regard to meaningful learning and attracting students' attention. Most of the participants addressed the importance of mentioning real-life connections. Most of the participants agreed on the importance of introducing real-life connections of mathematical content to the students. PT 5 criticized the case teacher about not having enough preparation to give some examples of real-life, and she thought that it was related to a justification why the content was taught. In her own words:

> The teacher should search for real-life applications prior to the lesson. The teacher should have an answer to the questions, 'why do we learn this?'. I saw that any topic has a real-life example when you google it. (PT 5, D_C1)

PT 4 and PT 1 differed from other participants in this sense. PT 1 did not attend to real-life connections during CBDM process. It was also noticed that it might be related to her beliefs about mathematical knowledge and her personal goals as a future mathematics teacher. She thought that mathematics is beautiful beyond its connections to real life.

> I want to show that children, immanently and without depending on the teacher, may understand that mathematics is beautiful indeed. I think mathematics is beautiful in itself, not necessarily in its connections with real life. A certain formula or proof is beautiful in itself. (PT 1, FI)

PT 4 was much stricter in terms of real-life applications compared to others. During the first interview, he was mentioning about his views about connecting mathematics to real-life phenomena. Being a purist as his orientation towards mathematics, even though he was aware of the fact that students would eventually question real-life applications, he stated that;

> If we don’t use complex numbers in our real lives, then isn’t it ridiculous to give necessarily a real-life example? I mean, we don’t use them in fact. If so, why do you force it so much? This issue ultimately brings the issue of usefulness of mathematics, but I think it is nonsense. (PT 4, FI)

It was also noticed that teachers’ endeavors to related mathematical content to students’ every day realities would sometimes fail. In CS 6, the case teacher asked where students used the word limit in their daily life. It was not appropriate since the use of word limit in daily life is different from its use in mathematics and it may inhibit students’ understanding of limit concept. PT 4 and PT 7 recognized this and criticized.

PT 3 mainly focused on the variety of activities, not doing the same activity for the whole class, importance of wrapping up after a discussion. The variety of activities, not spending the whole period with one activity was addressed.

Participants noticed the significance of *selecting tasks.* Choosing worthwhile tasks and examples, i.e., tasks that would probe understanding and lead to meaningful learning were mentioned frequently. Most of the participants pointed out the importance of catering varying needs of students, advantages and disadvantages of challenging tasks, the tasks that are engaging and suitable for introducing the concepts.

PT 6 focused on sequencing the tasks. She emphasized starting from easiest to the most challenging one. She found that sequencing is also important not to confuse the students and avoid misconceptions. PT 6 showed awareness about this in the first interview. She considered how choosing examples would affect students' concept formation about mathematical content as she exemplified:

> For example, if I give only the example of one to one function in the topics of functions, children will suppose one to one function as the condition of being a function and think that the things other than one to one function are not function. Then too many works to correct this if you can do. (PT 6, FI)

In addition to PT 6's specific concerns about misconceptions, PT 1' learner-centered focus reflected on her views on the choice of tasks, and in this respect, she emphasized the importance of considering learners' needs.

> Once you direct the children to the questions that they can solve, you may make them feel that they can do, and their internal motivation would increase. I witnessed this in one of my observations in the school; the teacher consciously asked one simple question and the girl chose it; she held the functions in this question. In the previous class, she was crying for she couldn't do but not she can solve them. (PT 1, D_C1)

One of the common solutions for catering varying needs of the learners was differentiating the instruction. Although all of the participants mentioned differentiated instruction, their awareness of what counts as differentiated instructions were not common. PT 7 and PT 8 stood out in terms of referring differentiated instruction by giving examples. On the other hand, other participants make superficial comments about the issue.

Participants often called out the ideas that they have learned during the program in terms of KCT. Some participants showed that they had been affected by their teacher education positively in terms of methods used. PT 7 could be an example of this for she mentioned the strategies and techniques she has learned in the program. She emphasized on the concepts like equity and accessibility, lesson plans which enable teachers thinking deeply for catering students' variety of needs, as she stated:

> I think it is a basic criterion for the teachers to comprehend such concepts as equality and accessibility in mathematics if they would use different techniques. I can easily say that planning in accordance with 5E model promotes me to use differentiated models. You can use 2 minutes video and so address to the students of visual learning while you can employ a manipulative and so access to the students of kinesthetic learning. (PT 7, T_C4)

Contrary to PT 7, PT 4 stressed the importance of experiencing a method himself as a student in order to apply it when he starts teaching. He mentioned the role of the program, as he shared:

> In the program, the lessons are not differentiated. But as future teachers, we are asked to do. I need to see its practice in order to practice it. (PT 4, D_C1)

Teachers make numerous *instructional decisions* before, during, and after a lesson. During instruction, teachers make mostly spontaneous decisions due to time limitations. Case teachers' spontaneous decisions draw attention of most of the participants. PT 6 mentioned these as critical decisions and emphasized on their importance. In the first interviews, she had a concern for doing something wrong during instruction. Apparently, she had a tendency to attend on the issue of spontaneity of the profession and its risks in terms of students' learning or the climate of the learning environment.

Having an understanding of when to make clarifications, use some opportunities for discourse, or getting out of the lesson plan were considered as instructional decisions.

Some of the instructional decisions were found critical in terms of the flow of the lesson and promoting meaningful mathematics learning. Teaching requires some critical decisions during instruction and an understanding of the need to stop and make a clarification, use students' claims and questions to provoke meaningful discussions, or save some others for later not to affect the integrity of the lesson.

Participants noticed limited aspects of KCC. Sequencing of the topics in the curriculum, the depth of the mathematical contents in the national program, and the difference between IBDP and National program were the three main points addressed. PT 3, PT 4, PT 1 and PT 8 were attentive to the KCC.

> We have talked about it just today; I guess children read graphics in the science course for the first time; I don't know if it is too late. (PT 3, D_C6)

Classroom management and organization (CMO)

CBDM offered a platform to discuss several issues regarding classroom management and organization. At the beginning of each discussion, participants were asked to list the issues. It was noticed that, as an initial reaction, participants mentioned lack of classroom management or lack of teacher authority as one of the main issues of the case. During discussion process, these issues were expanded and associated with other dimensions of teaching. In terms of classroom management and organization, participants mainly discussed dealing with disciplinary issues, importance of communication and relationships in the classroom, engagement of students to the lesson, planning and preparation for the lesson, and time management.

Dealing with disciplinary issues. As CBDM involved disciplinary issues taking place during a lesson, participants noticed several issues regarding discipline. Participants mainly paid attention to students' inappropriate communication with other students and with their teacher like defiance, use of slangs, bullying, off-task behavior, and noise in the classroom. The participant discussed the possible sources of these problems and offered suggestions in order to not to confront these challenges.

One of the sources of these problems was teachers' lacking mathematical knowledge for teaching. Students who could not get satisfactory answers for their questions or students who lacked support from teachers in their learning process were considered to be less respectful. Participants realized that losing respect has some consequences like students' disobedience to the rules and this threatens the teachers' authority in the classroom. For example, in CS1, the teacher could not answer the question of a student who was already trying to manipulate the lesson. PT 7 noticed this situation and emphasized the importance of command of content knowledge, as she said:

> Erdem (the student in CS 1) manages the lesson. Due to the lack of her content knowledge, she could not provide a good example explaining the content, and she could not have the power to control the lesson. (PT 7, D_C1)

On the other hand, in CS2, there was a group activity, and the teacher could not manage to handle students. Referring to this situation, PT 5 recalled what her mentor said about the relationship between managing the classroom and content knowledge. This situation also revealed that CBDM discussions provided opportunity to discuss the interactions between the pre-service teachers and the mentors during school experience. In PT 5's words:

> In my first internship, my mentor was the head of the department, and she said to me “If one has content knowledge, everything could be managed.” If I become competent about my content knowledge, students will respect me. [But] I think this does not solve all the problems. I realized that lack of class management skills causes some troubles. (PT 5, D_C2)

Participants criticized the way that the teacher in CS6 tried to handle the misbehavior. There was noise in the classroom, and the teacher did not find a way to stop it at first. Shouting at and threatening the students with disciplinary punishment were found problematic by the participants. However, they were also honest in confessing that they would do the same thing when they became a teacher. As an example of participants’ disposition towards these, PT 2 asserted:

> There is an ongoing problem in classroom management regarding the students. I think it is a shame to threaten children firstly by saying that I will enforce disciplinary regulations, that is, “unless you shut up, you will suffer the consequences.” Maybe I will do the same thing in the future, but it is a shame. One of the children may challenge me there by saying, “let’s see what you can do.” (PT 2, D_C6)

In addition to these, they have also recalled the suggestions in their classroom management lessons. Ripple effect, correcting the misbehavior of one student and its positive influence on others were shared. PT 8 was one of the students who mentioned this. However, she thought to shout at one of the students in order to stop misbehavior will have ripple effect too. “Ripple effect means that everyone shuts up when you shout at one.” (PT 8, D_C6). On the other hand, PT 1, again by referring to their classroom management courses in the program, offered the opposite:

> In the Classroom Management courses in the program, we learned that we should not shout at and increase our voice. Instead of lowering the voice was offered (F, D_C6)

Upon the suggestion of directing questions to students to keep them active in the lesson, PT 2 talked about the possible consequences of this solution. PT 2 was one of the participants who mentioned negative memories about her teachers in her own schooling; she was more sensitive about students' possible reactions to teachers' actions as she asserted:

> My teachers in my own schooling period used to target the students who do not listen and ask them some questions. Actually, when you do such a thing, it is very difficult to regain these students. They may even be offended. In those ages, even saying something absurd in the class may make them cry for days. (PT 2, D_C1)

PT 6, reflecting on her future career, had also evaluated the shouting at or threatening the students as she expressed:

> In this case, shouting at and threatening cannot be a solution; on the contrary they result in bigger problems in the long run (as much as possible, I would like not to shout). Students get desensitized to this. I probably will shout at and threaten in the first years of my profession, but I should not do. (PT 6, T_C6)

They have also realized the importance of the consequences of students' misbehaviors. Giving sanctions like written warnings, time outs, detentions, sending students out of the classroom were the ones that were mentioned during the discussions. Sending students out of the classroom were discussed among the participants. PT 8 and PT 2 claimed that in the future they would send some students out of the classroom. PT 3 shared the possible consequences of sending students out of the classroom. The dialogue between them was an example of conflicting ideas about an important issue such as dealing with disruptive students in the classroom. PT 3's reaction to the idea of sending students out was a sign of thinking one step further; she thought that it was possible that students would obstinate with the teachers' decisions and this would place the teacher in an awkward position. In her own words:

> Sending the student out is a problem too. You say, "go out," she/he says "no I won't." What will you do at this moment? I could not understand this practice of sending the student out. (PT 3, D_C6).

Although sanctions were offered, they have also noticed the importance of having a proactive stance in classroom management with the idea "Prevention is better than cure." In order to prevent the possible problems in management, participants offered arriving to class early, engaging students, varying the activities to keep students on task. For example, in CS1, the teacher was stressed out since she had to cover up a topic before the general exam. General exam topics were decided by the department and all teachers were agreed to cover all the issues regarding exam in the assigned time. The teacher in the case was struggling to catch up with other teachers and last period of the day was triggering her stress since students tend to misbehave more in the last periods. She came after the bell rang and the students were not settled. Arriving early was emphasized by the participants as some type of "prevention" aiming for quickly settling up students and preventing possible disciplinary issues.

Participants also noticed the ineffective nature of threats without actions. Being consistent in this respect was emphasized several times. Most of the participants were good at anticipating how students would react to teachers' actions or decisions. They were aware of how things can change in the classroom instantly according to students' reactions. Although the complex, interactive nature of teaching makes it unpredictable, one may think of the consequences of teachers' actions on the students' reactions. This would lead to wiser decisions in the classroom.

Communications and relationships in the classroom. Some communicational aspects were also discussed. Both student-student and student-teacher communications in scenarios were examined. In terms of communication among students, participants mainly noticed students' bullying each other and slangs that they have used. In addition to that, they discussed the importance of grouping in regards to the dynamics of the classroom and the characteristics of the students.

CBDM represents different profile students in the classroom, such as attention-seeking, power-seeking, and/or failure-avoiding students. Participants noticed how these types of students might affect the lesson flow. Communication and relationship with these different types of students were also discussed.

Participants have realized how attention-seeking and power-seeking students may lead the lesson, and teacher can lost control of the lesson. There were students in the scenarios who are showing off like the ones in CS1 and CS6; some of the participants thought that these students should be suppressed. They did not think possible consequences which may result in power struggles or humiliating the students in front of others. However, there were also more positive approaches that tend to understand the students' needs and take action accordingly. PT 1 and PT 7's views about the same student in CS1 show two different approaches to student who was trying to dominate the lesson. It was also interesting that PT 1 was seemed to have a more student-centered approach compared to other participants in the other dimensions. Following excerpts could be shared as exemplars to this:

> I think the teacher might dominate over Erdem. This type of student with such confidence has potentiality to attack. We experienced this; it is necessary to suppress such types of

> children. He/she pretends to be a leader but I am the leader. The lessons from such teachers are listened. (PT 1, D_C1)
>
> From a different perspective, the sources behind such an attitude of Erdem could be considered. For example, if the student tries to draw attention, I would warn by saying that I see you and accept you have very nice ideas, but you hasten so much as to not give time to your friends. (PT 7, T_C1)

PT 2, who shared negative experiences in her own schooling showed more empathy towards the students who were talked down. She claimed that these types of instances would make the teachers lose the ones like her. For example, in CS2, Bora was a power-seeking student. When he learned that the teacher would do a group activity with them, he reactively wanted the teacher to give answer questions like "What is the reason for the group activity? Instead, why don't you tell us how to solve?". The case teacher scolded Bora with the words "It is not your business to tell me how to teach!". Upon this instance, PT 2 expressed:

> If I were Bora, I would never give an answer anymore after the teacher has treated me in such a way. (PT 2, D_C2)

Similarly, PT 5, who underlined the importance of communication for learning and teaching, noticed the fact that teachers dealing with confrontational students should not prolong the arguments, as she expressed:

> I think to obstinate with the student is very absurd; put aside the adolescent psychology, I will obstinate now if a teacher obstinate me. Instead, I would give up it by saying that let's have a talk for a couple of minutes. (PT 5, D_C6)

The need for failure avoiding students was also noticed by the participants. Especially the ones, like PT 8, who was sensitive about learning of every single individual in the classroom were more attentive to teachers' lack of taking care of needs of individuals. In the first interview, PT 8 defined her goal as a mathematics teacher to reach every single student in the classroom.

> I think my responsibility as a mathematics teacher is to provide everyone to understand what I teach. I am against the view that I should deal with those who are academically successful and ignore the others. Likewise, I am against too the view that I should teach to those who are academically bad and ignore the good ones. Still, I don't know what kind of method I should follow. (PT 8, FI)

In CS 4, the participant realized that a student who was not dealing with the lesson and gave up. There would be failure avoiding students in a classroom and teachers should be careful about these types students and know how to communicate and take action when they encounter these types of students in advance. In addition to that, students' off-task behaviors took attention of the participants and they criticized the case teacher for this. For example, a failure avoiding student type was given in CS 4. In CS 4, there were students who were inattentive to the lesson, sleeping on their desk and when the teacher approached one of them and asked the reason, he answered even if he tries he cannot succeed and he will leave the IBDP anyways so he did not want to pay attention to the topics which only belonged to IBDP curriculum. Participants not only discussed the student-teacher interaction in this instance, they also mentioned the disadvantages of implementing dual curricula in a school, especially a burden that the student goes under.

Engaging students. One of the highly noticed issues related to classroom management was the lack of engaging students in the lesson. One of the reasons for the disruptions or off-task behaviors in the classroom was associated with teachers' lack of engaging students in the lesson. There was an awareness of the link between unwanted behaviors and poor instruction.

In terms of lack of engagement, few students being active in the case scenarios attracted participants' attention. For example, PT 3, commenting on the student's behaviors in CS6, explained lack of withitness of the teacher by relating it to lack of engaging all and she also stated her opinion on how this situation affected the learning environment negatively:

> I think there is a lack of attention to other students. It is true that Ezgi is there and Burak appears sometimes. [But] There is no such method in classroom management; others pay no attention to the lesson, the teacher neither use ripple effect nor calling students by their name; instead he/she shouts only. In this situation, just like what PT 2 said, the higher the voice of the teacher is, the higher that of the students. (PT 3, D_C6)

Dealing with off-task students, involving all learners with different paces were the challenges and participants offered to direct questions to students in order to involve them in the lesson. Giving responsibility to students was also suggested. PT 1 connected her reflection to her personal experience in voluntary teaching with PT 3. Giving responsibility helped them to control the students as she expressed:

> We divided them into groups; there were students who have potential to lead so we gave them responsibilities. It was 3rd grade class. It was difficult to manage them but when they were assigned duties, they really got quiet and triggered the group work. (PT 1, D_C2)

In addition to these, participants recalled possible solutions for engagement by considering what they have been learning during the program. For example, PT 3 and PT 7 shared their suggestions for keeping all students active in the lesson.

> If the student had learned the topic in the private teaching institution, it is, of course, a problem when the teacher in the school talks about the same things in the lesson. In our classes [in the program] we study "differentiated instruction." It tells us that we should keep these students busy with something else. (PT 3, DC_6)

> I think pair discussions will be more effective than group work (in CS 2, Monty Hall Problem). I would want students to work in pairs (step 1) and give them time to discuss. I would like pairs sharing ideas and write their strategy or solution step by step. Then, I would like these couples to join together to form groups of 4 (step 2) and to share their ideas and show each other what they wrote. Thus, students (in step 1) will be able to develop their own ideas, focus more easily, but when they got stuck, they will be able to produce new ideas by putting their heads together. In the second step, I think that each student will be more active so that he/she can meet and evaluate new ideas. (PT 7, T_C2)

The motivation of students was mentioned several times as an important part of engaging students in the lessons. The ways to increase students' motivation to the lessons were discussed. The effect of the teachers' motivation and enthusiasm on students' motivation was also discussed.

Apart from these, PT 8 mentioned liking the teacher as a source of motivation to the lesson a few times in parallel to her beliefs that she shared during first interview as she asserted:

> We go outside the theory. You don't have to endear yourself to the students, but the matter seems related to whether the teacher is favored or not. If this student liked the teacher Erdem, he would continue listening even in the 8^{th} period even if teacher had rattled him. (PT 8, D-C1)
>
> If I don't like a teacher and find him/her unsatisfactory, I cannot learn from him/her; I study myself. (PT 8, FI)

PT 2 also connected her reflection on the effect of liking the teacher to her prior experiences as a student. She was a student who paid attention to the lessons of the teachers she liked. However, PT 2 expressed that it would not be the case for every student.

Giving the aim of the lesson and the importance of motivating students were two of the most emphasized factors that affected the CMO. The problems that occurred in CMO were mostly considered as the consequences of lack of motivating students. The solution suggestions to the problems mostly focused on preventing the challenges by increasing the motivation, either by giving the aim of the lesson properly or increasing the curiosity of students on the topic or activity. PT 1's beliefs about the role of motivation in teacher were also dominant during discussions. The following excerpts were from the discussions and the first interview and this also stood as examples of the possible benefit of CBDM discussion like bringing the beliefs and expectations of pre-service teachers to a discussion environment in the program.

> When you give a purpose to the students, everything has changed. Now all kids are listening. I think the motivation and the purpose are always the most important things. (PT 1, D_C2)

> When you give children something interesting, they will manage themselves. If you trigger their sense of wonder, they will not deal with other things. (PT 1, FI)

Planning and preparation. Participants wrote detailed lesson plans for each lesson during their training in the program. They were using a 5 E model (Engage, Explore, Explain, Extend, Evaluate), and this model enables a teacher to plan each step of the lesson in detail, considering multiple factors for an effective instruction. Most of the participants thought that preparing lesson plans was time-consuming and they were aware of the fact that teachers who they have observed did not have a lesson plan. Although they were quite opposed to written lesson plans, while reflecting on the problems occurred in the cases, they had to offer the solution of written lesson plans. They have thought that if the teacher had a well-written plan, the teacher might not have that bad experience. PT 4 was opposed to this idea since he did not think that written lesson plan would prevent experiencing the challenges in the CS. However, PT 1, who was thinking similarly before discussing CS1, changed her view as she expressed:

> In the beginning, I was thinking like PT 4. What is the need for a lesson plan? It is ridiculous since I know what to do, what to teach. However, she will have something in her hand when things went out of control. (PT 1, D_C1)

The discussion environment provided opportunities for considering consequences of the solutions offered. Written lesson plans were also found to be unrealistic as one considers the workload of the teachers. PT 2 thought that a teacher who has to teach 25 hours a week would not have time to write lesson plans for each. As an answer to this, PT 3 offered to prepare weekly, with her own words:

> It is problematic when we do our plans for 1 or 2 hours. Whereas, it is possible to see the big picture if the plans are weak. If it was so, everything would be on the right track. (PT 3, D_C1)

PT 6, who differed from other participants in terms of reflections that are connected to theory, justified her opinions with scholars' opinions in the field. She asserted that:

> The plan has some problems like lack of good instruction. Therefore, classroom management is also problematic. As Colberg or Kounin says, if there is no good plan, there will be no classroom management. (PT 6, D_C2)

Time management. Problems in the flow of the lesson, lack of using the lesson time effectively, lack of having time to wrap up the lessons, slow pace, and out of class time spent with the students were addressed in terms of problems due to time management.

In the case scenarios, case teachers had problems in keeping the pace of the lesson according to the varying needs of students. Participants also noticed that teachers in CBDM did not wrap the lesson; they do not have time to make a closure. They were frequently advised about the importance of a summary at the end of the lesson. They observed how students left the lesson without closure.

> There was chaos in the lesson due to using a graphing calculator. In these circumstances, we have to give clear instructions. We have to work with the whole group and make them listen our directions. We should directly tell the steps while using the TI. Otherwise, like this, the bell rang and she could not make a closure. (PT 6, D_C4)

They have also realized that teachers in the CSs mentioned the out of class time to help students who struggled to understand something. This was mentioned in two dimensions. One is keeping teachers' words; participants thought if the teacher said something like that, they should actualize their promises said to students. Another point was seen as a lack of using lesson time inefficient.

> It is possible to follow a way like keeping an agenda, so does our instructors [in the program] do in each lesson. At the beginning while giving instruction to the children, the duration was not defined and there is nothing related to the assignment that I thought. Neither he/she completed the topics nor controlled whether the children learned or not. (PT 6, D_C2)

Assessment of students' learning

CBDM triggered several issues in assessing students' learning dimensions for participants for reflection. Participants not only reflected on the issues that the CSs bring but also, they came up with new questions and future-oriented reflections related to the problems discussed. CBDM had also served as a platform that helps them to discuss their concerns related to assessment.

Six main issues emerged in the assessment dimension; choosing assessment items, using alternative assessment, determining homework (HW) policies, and monitoring students' learning and progress.

Choosing assessment items. Participants noticed several issues regarding the choice of assessment items. The choice of tasks was also elaborated regarding the aim of the assessment.

Blooms' taxonomy, skill attainment, aligning it to learning objectives, addressing all students from different levels of achievement were raised as the guides which would help teachers to choose assessment items. Sequencing the assessment items according to the difficulty level of assessment items was also mentioned frequently. PT 6 who connected her reflections to theory noticed the need for a taxonomical approach in terms of choosing assessment items, as she expressed:

> According to the steps of Bloom's taxonomy, the exam questions or homework or those assignments given repeatedly should be in the same steps of Bloom. (PT 6, T_C3)

It was interesting to notice how participants have different beliefs about choosing challenging tasks or problems as assessment items. Some of the participants reflected that challenging tasks were used for differentiating students.

PT 8, who frequently mentioned conceptual teaching and learning, favored challenging items for assessment in order to check if the students acquired conceptual understanding. On the other hand, PT 3 associated using challenging items with the dynamics of the classroom. Using difficult tasks to punish students' inappropriate behaviors in the classroom was articulated by her with a connection to her prior experiences as a student.

> If the class is a bit annoying, then I may ask difficult questions. Such cases used to happen in my high school; we didn't listen to the teacher, and he was angry and later on it was impossible to get 90 or 100 grades from the exam. (PT 3, D_C5)

Asking similar questions to what has been done while delivering the content was articulated as a suggestion to prevent students' reactions towards performance tasks. One of the advantages of the group discussions was the opportunity to discuss conflicting ideas. For example, PT 4 disagreed with this suggestion. Actually, PT 4's teacher identity became apparent in his comments related to asking challenging questions for assessment. His disposition towards learning mathematics was dominant in his approach to assess students. He thought that learning mathematics could occur when students think deeply about the problems and they should first learn to sit and think about something for several hours. This was how he used to learn mathematics in his undergraduate studies, especially going back to his experiences in Mathematics Village in Şirince and he generalized this belief for the learners whom he would teach. The following excerpts stood for examples of his views during the discussions and prior to CBDM process respectively.

> I would fail him by asking difficult questions since it is difficult to continue by giving the answers every time. I think that one challenging task should be given so that we force ourselves to think and study so much rather than by rote. (They also tried to solve the question themselves in pre-case exercises.) If we are in a mathematics lessons, we have to get used to mathematics in order to do mathematics; of course, it will not happen quickly but I have to spend time to deal with it. (PT 4, D_C3)

> In Mathematics Village, we also took a course. (In Bilgi University) In six years, I have learned how to deal with

> mathematical questions. If necessary, you have to think about this question for hours; we have to learn how to struggle with it. When I am a teacher, I would probably do such a bit maniacal thing: I would give a question to the student and leave him/her alone in a room to sit there and tackle the question. He/she can sleep there or walk around, I don't mind, but she/he would have thought on the issue. (PT 4, FI)

Although there was not a case related to the preparation of an examination, the CS 5 which was about a review lesson before an exam triggered discussion associated with teachers' beliefs about teachers' interference to an exam preparation process. Participants noticed several consequences of a review lesson. Linking to realities of Turkish Education System, participants realized that, in the context of general exams where all students at the same grade level have the same exams, the teacher who solved similar questions to the exam questions threatens justice. In CS 5, the teachers had a mistake in solving one of the questions that a student asked during review lesson and students' reaction was like "if you didn't solve, you would not expect us to solve such a question in the exam." This instance helped participants notice teachers' subject matter knowledge was directly associated with teachers' choice of an assessment item in an exam, as PT 3 articulated:

> We have to be aware of our solutions or shortcuts that we've developed are consistent in general or not. Students mostly like solving questions by using shortcuts but in a case where the system falls down, so as in the case of teacher Gizem, the cost of this may be on the shoulder of the teacher. For example, now teacher Gizem cannot ask difficult questions like this in the exam. (PT 8, D_C5)

Using alternative assessment. Challenges related to using alternative assessments were one of the issues that participants reflected on. CS 3 brought a teacher's experience of giving students a performance tasks that students did not encounter a similar task before. Students reacted to being graded with this task.

> I would try to give some necessary and meaningful tasks and then check them. I would ask them to save these assignments and any studies in their mathematics folder, and I would collect and check these folders regularly. I would assess them in accordance with previously defined assessment criteria and give feedback to the students when necessary. I would try to develop some individual and group studies relevant to the topics and I would assess them over their product. I would determine their performance grade by assessing all of these. I think both students and the parents should know how the performance grade is given by percentages. If students know our expectations, this will be instructive for them. (PT 7, T_C3)

One of the issues that participants noticed was that alternative assessment required detailed planning. In other words, it was not adequate to find a mathematical task which has real-life connection and assigns it to students. Many participants came up with a suggestion that the tension between the teacher and the students would resolve if the teacher has decided not to grade the performance task. PT 7 differed from other participants by considering consequences of not grading the task since students failed to accomplish.

> I would give up; I would not insist on the completion of the performance task; it is meaningless if I see that the students cannot do it. However, I would say to them that I will reevaluate this task, and they will face something more difficult in the next step. I may not say something now, I am not sure, I have to search for but I guess I would collect their ideas about the questions and the answers. I would think before on the condition that they cannot do. (PT 7, D_C3)

PT 8 offered using exit cards as an alternative assessment. It was one of the techniques that were recommended during their training in the program. PT 1 offered Khan Academy’s personalized tests as effective use of technology which would be effective in terms of catering students’ individual learning needs. PT 3 also underlined the importance of formative assessment and articulated her beliefs as she mentioned:

> First of all, I think a kind of rubric must be prepared before the applications aiming assessments such as performance tasks, homework or project. In this process, a table for grading or assessment should be prepared on the basis of the mistakes that the students may do or the problems that they may face. I think that an assessment including oral feedbacks together with the grades will be more advantageous for the students. Moreover, in a long-term process, the stages of development of the students must be controlled regularly (once a week or once in two days) and in this way, we can avoid a result-oriented assessment. (PT 3, T_C3)

Determining homework policies. Rationale, type, amount of homework, and checking homework were the main issues that were noticed and discussed. One of the triggers of these discussions was the teacher's experience with a parent-teacher meeting in CS 3. In this meeting, the teacher and the parent were discussing the students' progress, and the parent questioned her sons' performance about doing homework. The teacher was not ready to give feedback about students due to lack of keeping record for the homework.

This incident helped participants to discuss various important issues while giving homework. One of the important issues was the rationale for giving homework. Participants differed in terms of their beliefs about the rationale. Some of the participants thought that the main rationale is to help students become responsible individuals. On the other hand, other participants thought that it is important for rehearsal since students won't do it by themselves. PT 1 thought different from others since she did not believe that assigning homework is worthwhile since she supports that learners must be autonomous individuals and could take action without any compulsory assignment. In her own words;

> What if there is no homework? Isn't it a bother both for me and the students? There is no logical explanation for giving homework if I covered enough in my lesson. It seems to me

> that the student would do extra study if she/he has a sense of responsibility. (PT 1, D_C3)

Grading was another dimension of the discussions associated with homework. One of the components of performance grades was students' performance in accomplishing tasks on HW and submitting their HW on time. Thus, record keeping as a part of a justification of grading was highly noticed by participants, and it was realized that they had to be systematic and they should have a decision about record keeping and grading policy beforehand.

> In one of my internships, one of the mentors told me an anecdote; a parent come to talk with her with a tempered manner to talk about the low performance grade the students got, the mentor showed the records kept for the student which explain the missing homework etc. and the parent calmed down and respected to my mentors' decision. (PT 5, D_C3)

Although they agreed on the importance of keeping record, they realized that lack of time would be a problem. In CS 4, many participants tried to think about alternative ways to check homework since they think that it is time-consuming and it will reduce teaching time. During the discussions, suggestions were quite superficial and limited to how to grade the HW. However, some of the participants elaborated on this issue during the discussion task in more detail with an emphasis on checking HW to give feedback to students not solely giving grades. PT 7's future-oriented reflections on assigning and checking HW would stand as an exemplar;

> Assessment: Did my plan work? Did I reach my goals? What were the missing points, what should have been done? How can I give HW and check? Another note was on "How will I assign HW? I would ask them to solve the questions to a separate sheet to keep in their mathematics folders. The HW that I would assign must be meaningful, not just drill and practice type of work. I would check the HW randomly. So that the students take it seriously. (By the way, I can use some questions from the HW in a quiz.) I take these notes in order to remind myself the points where I have to be careful;

> I think they may be conceived as kind of precaution. (PT 7, T_C4)

In addition to that, in CS 4, the teacher gave HW which was important for the following lesson and only four of the students did the HW. The teacher gave full points to the ones who did their HW. Participants questioned this decision of the teacher. PT 3 noticed that grading should be beyond checking if they did or not, as she expressed:

> The case teacher gave full point to 4 students, but it means that he/she managed to solve all the questions in the HW; well, did he/she control all of them one by one or he/she gave directly full point just because the students did it. He/she gave full points to the ones who did their HW but gave extra time to those who did not. In that sense, those who did their HW on time could think why they finished it earlier. (PT 3, D_C4)

Participants also differed about their foci about giving HW. PT 1 generally focused on the rationale of giving an HW rather than the procedures of giving an HW. Contrary to PT 1, PT 6 was more focused on the procedural aspects of assigning and checking HW.

> According to my philosophy of education, the good teacher is not the one who gave lots of HW; a teacher who gave one but the efficient question might be better. However, controlling these HW on time efficiently and regularly is an important criterion for assessment and evaluation. The due date should not be changed easily, the submission after due date should be taken points off and the students should be informed all these procedures at the beginning of the year. (PT 5, T_C3)

Monitoring students' learning and progress. Participants noticed that the teachers in CSs faced challenges in monitoring students' learning and progress. The signs of lacking monitoring students' learning were articulated to be lack of effective questioning, being unaware of students' level, and not having a concern about students' learning. PT 6 noticed that the teacher in CS 2 did not seem to have a plan to assess students' learning. In her own words:

> Something could be done just like our instructors' [in the program] keeping agenda in each lesson. In the beginning, the students had not been informed while giving instructions; in course outline there is nothing related to the assessment that I thought; teacher could not complete the topics and control if the students learned or not. (PT 6, D_C2)

Participants did not only criticize the CS teachers in terms of lacking assessing students' learning. They also tried to reflect on their future career and suggest some possible ways to involve assessment in their teaching. For example, PT 5's reflective writing involved suggestions for checking students' understanding;

> Of course, this case might be one of the problems that I will face in my teaching career, and I should pay attention to be clear in my definitions to prevent this. The best solution is to ask some students repeat and write what they understood in their own words. Or it could be asked that everyone will write in their own style any definition that they understood in a piece of paper (they might be free to choose their ways of expression; oral, written or by using mathematical symbols). This may be self-assessment for the students and also pre-assessment for the teacher before solving the questions. (PT 5, T_C6)

It was also noticed that concern for students' learning was not a priority for CS teachers who were beginning teachers. Self-concern was more likely to be apparent in the early career compared to having impact concerns, whether students learned what was intended to be taught. PT 8 noticed the self-concern of the teacher in CS 1 as the teacher hesitated to answer a question of a student since she did not remember the answer.

> For example, she/he considers whether the students recognized the anxiety in her/his voice, but she/he does not consider whether they understood the topics or not. (PT 8, D_C1)

Context

Participants noticed several issues related to context and its association with other dimensions of teaching. *Relationships with other stakeholders, overwhelming workload, realities of Turkish Education System (TES) (Private teaching institutions (PTI), standardized exams, mathematics curriculum load), the school context* were addressed.

First of all, relationships with other stakeholders*, administrators, colleagues, parent and student profile and relationships with these stakeholders were identified as some of the main contextual issues that interfere with teachers' practices*.

To start with, all participants commented on the teacher-administrator relationship. The role of the administrator was perceived as an evaluator by some of the participants. Lesson observations are used as a part of teacher evaluation or for professional development purposes, especially in private schools. Therefore, the approach of teachers to lesson observations is important in terms of establishing a culture that cultivates teachers' growth of learning.

When lesson observations by administrators were discussed, participants differed in terms of their views on these issues. PT 1 and PT 3 found lesson observation of an administrator as an opportunity for professional development. On the other hand, PT 4 thought that it is unacceptable and in his own words: “This is an insult for the teacher; how dare you come and observe me in the class?”. (PT 4, D_C1)

In addition to that, participants noticed that administrators would question teachers' competencies and make decisions about renewing their contracts. They realized that administrators would also deal with parents' complaints about teachers.

In terms of *relationships with colleagues* in the department, participants noticed that departmental relationships could be key for support especially in the early career. However, some of the interactions in the department would also be noticed as impeding. Participants became aware of the fact that there would be colleagues who will not share similar aims or philosophies of teaching mathematics. As an example, In CS 2, a colleague of the case teacher was discouraging about the activity that the case-teacher designed. Although the profile of the colleague was not shared in detail, participants hypothesized about his teacher identity. PT 1 criticized him as being like a *PTI* teacher where PT 5 had a similar reaction and thought that he is like a MONE teacher. Actually, none of the participants wanted to be like most of their teachers in their own schooling. Participants wanted to work in a private school, implement different programs (like IBDP) and teach with an innovative, student-centered approach. The program also developed their visions in this sense. For this reason, they often criticize those who act as teachers in their own schooling.

In the first interview, her beliefs revealed that she did not want to be a MONE teacher, which is an expression for most of the participants as being traditional. After she was introduced with the IB diploma program, she was ambitious about becoming an IBDP teacher.

Participants differed in terms of perceiving people in the department as a threat or support mechanism. The need for a mentor in the early career was addressed by PT 6, PT 7, PT 8 and PT 1. On the other hand, PT 4 mentioned that mentors would not be helpful if they were traditional teachers. In CS 1, the case teacher began teaching without a mentor. PT 6, being one of the participants who were aware of the constraints due to contextual factors in the first interview, have unrealistic expectations from the schools she may work.

> Since she has no experience, she does not know so much about the questions that the students may ask, but probably she has no mentor to take advice; she is not at one of the schools where we would like to work. (PT 6, D_C1)

In terms of *teacher-parent relationships*, participants focused on parents' expectations from schools (especially private schools), attitudes towards teachers and the accountability issues related to parents. When talking about the attitudes of parents towards mathematics, participants stated that parents, in general, had a result-oriented approach rather than process. In other words, parents' concerns were grades of their children but not what they learn. Thus, it was noticed that the teachers felt the pressure of grade expectations of parents.

Participants compared the current status of parents in the existence of a conflict with a student. Some of the participants recalled their schooling and how their parents were taking a stance that time. For example, PT 2 thought that parents, in the past, took teachers' side but now it is vice versa. This was one of the issues that she also mentioned in the first interview. All of the participants agreed on the importance of being able to make explanations to the parents about the practices in the classroom when it is necessary. They acknowledged that teachers were accountable to parents about their assessment and evaluation. In this sense, keeping record of students' progress was found very important to hold firm against parents. Some perceived this as a threat, while others perceived it as a feedback mechanism and an opportunity for development.

Except for PT 4, none of the participants has a private school background in their schooling. However, teaching experiences prior to the program were effective in terms of forming beliefs about parents' profile in private schools. For example, PT 6 seemed to reflect her teacher identity in terms of belief about parents to CBDM discussions. She believed that there is a difference between public and private schools in terms of being accountable to parents. In her own words:

> I think Arda is one of the children who have no self-esteem; he is under stress because of his parents. Even when he becomes an adult, he will not take the fall. He complains about the teacher and provokes his parents about her. I have my own experience since I had worked at a PTI. Since parents pay for it, they feel justified to blame the teacher. (PT 6, D_C3)
>
> When there is a meeting in a state school, parents may not participate. This is not the case in a private school; there will be parents meeting and a continuous follow-up from parents. For example, last week there was a preparation for report card (in the partner school), and all these works were extra load for the teachers. (PT 6, FI)

PT 8, PT 5, and PT 3 did not mention any point about parents in the first interview. CBDM gave an opportunity to think and discuss on parents' involvement in schooling. For example, PT 3 articulates this awareness as:

> Our professional experiences are not long enough to live the student-teacher or parents-teacher relationships. After the discussions held here, I started to think that we may frequently face with such kind of events (dialogs between parents and teacher) in the beginning years. (PT 3, T_C3)

Another important contextual factor was noticed to be the *learners' profile*. Students' interest or disinterest detached from teachers' practices were found important factors in terms of effective teaching. Students' personal traits and group dynamics in a classroom were also considered as contextual factors that interfere effective teaching. Most of the participants just determined learners' profile but it was more of accepting the reality like what PT 4 did by saying “Nothing can change the reluctance of the children, right? How can it be possible?”. On the other hand, PT 1 offered alternative explanations to students’ profiles and tried to find reasons and stressed the importance of taking action accordingly.

> We should know sociologically with whom we work. We are Y, and we are more adaptable to change. The X generation experienced poverty; this is why they want their children have no difficulty. These are the parents of our students. But the children in front of us have wealthy lives. Moreover, they always see good things via social media and internet. This is why we have to treat differently to parents and students. Actually, we had a very difficult position in this situation. (PT 1, D_C3)

One of the issues that grasped participants’ attention was *overwhelming workload*. When the graduates’ challenges were considered, the workload was one of the unexpected ones. In the first interviews, participants, except PT 7, did not mention any concern about working hours. During CBDM process, participants began to understand the realities of teaching profession.

In CS 1, it was the teachers’ seventh period on that day. PT 1 asked “Can they (teachers) work for 7 hours?” In fact, this shows how pre-service teachers created incomplete schemas of teaching about some contextual facts before starting the profession. However, after the discussions, it was seen that they had increased their awareness of issues such as meetings, paperwork, course distributions, etc. These discussions revealed some of their beliefs about working hours. PT 5 was optimistic as she has some questions like “Why should I take a job home?”. In their early career, many graduates mentioned how much their social life was affected by the workload. There will be a difference in the sense of frustration between starting with this expectation and starting with expectations that she may not work at home. Contrary to these, PT 7 who was somehow aware of workload since many members of her family were teachers became more conscious about teachers' responsibilities and consequently amount of work required.

> A teacher has many responsibilities; first of all this responsibility is for himself / herself, then for the students, parents, administer, actually for anyone in relation and interaction. I think that it causes an increase in the workload while a decrease in motivation and desire. I had expected that the workload of teacher would be much less but by examples I realized that it was not so in reality. (PT 7, T_C3)

Another factor affecting teaching practices was the realities of the *Turkish Education System*. National examinations, intensive and one size fit for all types of curricula and PTIs which are generally exam-oriented formations were mentioned to be some of those realities.

The participants think that most of the students demanded to learn practical skills to solve questions on the test. Consequently, these types of demands were perceived as obstacles for teachers to implement learner-centered practices. Participants have also felt the pressure of covering the curriculum in a certain amount of time.

Although PTIs transformed, shadow education preserves its place in one form or another. The reality of dualism in education made participants thought of the students' comparison of teachers in formal schooling and the ones in shadow education. In addition to that, participants realized that students would come to the class by already knowing the topic. Here, many prospective teachers shared similar views. However, PT 4, who emphasizes teacher autonomy at every opportunity and believes that PTIs were more liberal and teacher-student relations, are more informal.

> I think that the practice of teaching in the state or private schools in Turkey is not any different from that in PTIs.

> Students naturally compare their school teachers with those in PTIs. Teachers working in PTIs know many more strategies for solving questions and they focus on the results rather than the processes. The students consequently like them much more. Moreover, in these PTIs there is no dress code or such rules and students like this. Everyone needs to feel independent (or free), not just teachers but also students. (PT 4, T_C1)

PT 4 commented more on context compared to others, and PT 3 followed him. PT 3 worked in private teaching institutions for a year and she often recalled her experiences there during the discussions. PT 4 has also associated private tutoring experiences with his choice of teaching profession. The teacher identity of these two participants was apparent in their comments in the discussions.

> I know this from my own experience. Parents hire a tutor, then the tutor starts to criticize in allusive way the teacher in the classroom. [Referring to the mathematical mistake of the teacher in CS 5] Such a little thing may become a big issue. (PT 4, D_C5)

> This is the reality of Turkey. For example, I will teach in the lesson, but the student may not understand it. Suppose that I did my best but the student will spend money and go to PTI or hire a tutor, just like I did in my schooling. She/he will learn there and then say that my tutor is better than my teacher in the school. This is a reality. I mean this is always like this. (PT 4, FI)

In grade levels with multiple classes, conducting the general examination is mandatory. Most of the participants thought that general exam was a source of pressure for the teachers. The source of stress could be summarized as rushing up for covering the curriculum in a limited time and catching up with other classes before the general exams. There were opposite views in general exam policy of MONE. PT 4 thought that this was interference to teachers' practices and disregarding teachers' autonomy, on the other hand, PT 1 stated that general exams would prevent injustice that would be sourced in teachers' differing practices and would lead to a fairer assessment.

School context; especially school culture, educational philosophy, and mission of the school and class size were the factors that the participants noticed and discussed as important school-related contextual factors affecting teaching and learning environment. Most of the participants discussed the importance of working in a school that complies ones' own educational philosophies. For example, PT 5 who was aware of the importance of school culture in the first interview, shared similar beliefs and suggestions aligned with her beliefs during the case discussions. PT 4 who was attentive to contextual factors, had also shared the importance of the consistency between the teachers' and the schools' philosophy of teaching.

> Maybe at the very beginning, even in the job interview, we should clearly express ourselves, we should state that we are teachers who are open to innovations and desire to apply theories to the practice of teaching in the class. This may enable us to work with students and colleagues more appropriate to us in terms of structure of school, administration and educational philosophy. (PT 5, T_C2)
>
> First of all, teachers should work in schools that are appropriate to their characteristics. A teacher who prefers traditional methods will not be considered as bad teacher. However, an innovative teacher in a school that adopted traditional methods might be considered as bad teacher. (PT 4, T_C2)

Participants noticed and discussed the class size and its possible consequences on their teaching practices. At first, some of the participants thought that small class size would be an advantage. PT 1 emphasized that the discussion culture would be more easily established compared to larger classes where PT 8 thought it would be much more stressful for the teacher if he/she made a mistake.

Participants' interest in contextual factors showed similarities and differences. PT 4 has shown much interest in contextual factors compared to other participants. It attracted attention not only in terms of focusing on contextual problems but also frequently seeing its connection with other problems. PT 3 and PT 2 followed PT 4 in terms of focusing on contextual factors without associating it with other dimensions. Other candidates showed similar profiles in terms of both content and intensity. However, when the reflection of these two participants on context was examined, it was noticed that the reflections differ in terms of their characteristics. Although PT 3 touched upon many contextual factors, she offered solutions or mentioned what the teacher should do under those circumstances. In other words, she saw this situation as a reality rather than an obstacle, which requires some adjustments. In the first interview, PT 4 gave the signals of focusing on external factors and his perceptions about contextual factors as a constraint. PT 1, who did not mention any contextual problems in her first interview, gained awareness about contextual issues.

PT 4 and PT 2 are older than the other participants and entered the mathematics department as their second or even third universities and then continued the program. Although this situation may make them more realistic about life, PT 4’s view about context was sometimes too pessimistic. PT 3, on the other hand, was interested in contextual situations, but maybe her self-confidence did not lead her to pessimism in this sense.

Teacher identity

The professional identity of new teachers passing through the phase of negotiations when encountered with realities, and these negotiations create tensions. Participants noticed several issues related to teacher identity and the role of teacher identity in teaching mathematics as each CS was designed with an intention to give participants a feeling of these tensions that a novice teacher would feel. During the program, participants continually construct their teacher identity. The desire to become a "non-traditional" teacher, caring each student and concerning their learning, using multiple methods and representations to teach was very common in participants' defining their future self as a teacher. However, they felt the tensions that the teachers in CBDM went through, the tension between whom the case teachers want to be as a teacher and whom they turned into. These tensions helped participants reflect on their own identities.

Perceiving *learning to teach* as it consists of acquiring a knowledge base would be decisive about performing it. In this sense, participants noticed and discussed the issues related to teacher identity alongside "know-how" of teaching. One of the prominent labels that were assigned to the CS teachers' identity was being inexperienced since the teachers in the CS were the ones in their early careers. Some of the challenges that the teachers in the CBDM faced were associated with being inexperienced. PT 6 was one of the participants who paid more attention to being a novice and stated that

> I don't know whether he/she could not manage the situation because he/she was inexperienced; I think the case is not big enough to test his/her field knowledge. If he/she had had some years of experience, if he/she had understood the topic before, it would be different. I think it was because of inexperience (PT 6, D_C5).

Being a novice was seen as a source of many problems occurred in the CBDM. Feeling vulnerable, lacking self-efficacy and self-confidence due to being inexperienced were mentioned regarding the case teachers. Was it also important to examine? to discuss? how culture identified them in the formation of teacher identity. They have noticed that parents' views also had an effect on their perception of self as a teacher. Statements of parents, like "You are too young" to the case teachers were perceived as negative connotations of being inexperienced. PT 5 agreed parents' view about being anxious about having their children educated by a novice. It is clear in her statements such as;

> I did the same thing: The child of my cousin goes to the private school. My cousin said that the class teacher was just graduated; it is his/her second year. I reacted that he/she was very young, 24, or 25 years-old. I felt anxious about my kid. (PT 5, D_C3)

The others asked her if the novice was a graduate of the program, would she react the same. Then it was argued among participants, and most of them trusted their teacher education program and identified themselves as a graduate of this specific program which made them feel differentiated from other novices in the sense of having a feeling of self-efficacy.

Besides this prominent factor that was perceived affecting teachers' practices, the following issues emerged during the discussions; the role of reasons for choosing teaching mathematics as a career and the aims of the teachers, the role of emotions in teaching career, the role of teacher education experience and the role of prior personal and professional experiences, the role of personality traits.

The role of reasons for entry and aims. Participants shared intrinsic, extrinsic, and altruistic reasons for choosing teaching as a career and these were similar to the reasons of teachers in CBDM. These reasons also had a role in how they defined their goals as a teacher. Thus, it is a part of their teacher identity. All of the participants shared intrinsic motivations to teach mathematics which could be summarized as joy of teaching, love of mathematics, being good at mathematics. Although it was not their main incentive, PT 4 and PT 1 stressed altruistic reasons for teaching as stating “having an effect on students’ lives.” It was also noticed that most of the participants shared some extrinsic reasons to choose teaching like being not able to continue with other career options. All three reasons would have an effect on their choice of entry. However, the main intention was also considered for each.

The reasons for participants’ career choice are in line with the graduates’ reasons for choosing the profession. Therefore, they were expected to find themselves in the profiles of the teachers given at the beginning of each CS. Some of the participants found similarities with the case teachers as expected. For example, choosing teaching profession for being autonomous and perceiving it as a profession that requires less accountability compared to other professions was articulated by some of the participants. In CS1, similar reasons for entry were shared in the case of teachers’ profiles. PT 4 who found similarities with his reasons for entry expressed his feelings on behalf of the lack of teacher empowerment in national curriculum context:

> It seems to me that teacher Ayşe chose teaching as a career to be independent, but she was totally confined to many obligations. I think such a situation disturbs her so much. So do I, since I also chose this profession to be independent a little bit. But I am sorry to see what I plan to do as profession refers in Turkey to nothing but being a robot (PT 4, D_C1).

The goals were also similar; making students like mathematics, teaching without making students memorize the rules, in other words, conceptual teaching, teaching mathematical thinking, teaching with innovative methods, using STEM, and using technology in their lessons. It is possible to claim that all of them refers to intrinsic reasons behind choosing mathematics teacher and in that sense the following statement of PT 1 seems summarizing:

> I think we should make our rationale clear in becoming teachers, then we should prepare for it. We should internalize the reason behind choosing to be a teacher. (PT 1, D_C3)

On CS 5, the case teacher defined herself as a subject matter expert. Participants noticed how this image could be shattered when the teacher could not answer students' questions. Then the question "Should the teachers be the ones who know everything in their subject area" was raised and discussed among them. There were conflicts of being a subject matter expert and being inexperienced and consequently need time to have a command of the mathematical knowledge for teaching.

The role of emotions and personality traits. In CBDM, participants were asked about how they would feel if they were in the place of the case teacher. Though they began to notice that they have to deal with some emotions, which would have potential to reform their teacher identity. Since the issues discussed in CBDM were problematic situations for teachers, mostly negative emotions were addressed. Being stressed out, feeling down, insufficient, vulnerable, desperate, and powerless were mentioned several times. Anger and panic were also articulated as a consequence or source of the problems. Though the importance of staying calm as a teacher, anger management or the tactics to reduce stress were discussed. Another important inference that the participants draw from the CSs is that, due to failures and problems, negative feelings will affect their motivation for teaching. CBDM provided a simulation (so to say) of the feeling of conflict between the ideals that a teacher has before starting the career and what they have faced within the reality of classroom.

Although they have shared the negative feelings and feel nervous about their future career during discussions, almost every task, they transformed the negative feelings to rational decisions and transformed or sustained their teacher identity. PT 7 was one of the participants who focused on emotions the most. In the analysis of the actor level, it was noticed that PT 7 was the teacher who has the greatest attention on *self*. It would be one of the consequences of her sympathy in terms of feelings and her awareness and attention to emotions during teaching. Although it could be an advantage for more opportunity to reflect and draw lessons for her from the case teachers, she was seen to be also the riskiest participant in terms of feeling less secure after seeing negative examples. However, especially in the written tasks, she came over those feelings and in order to leave that state of mind (with negative feelings), she began to propose solutions and relieved. As it could be seen in her comments, she also seemed to understand one of the central properties of becoming reflective teacher which is openheartedness:

> This event reminded me of the knowledge I derived from my experiences: Never give up when faced with a challenge. Instead, try to find the source of the problem honestly (be open to accepting that it could be due to your fault), think of an action plan suitable to the group you are teaching and the content. Don't forget that it is only you but no one else who can do this since it is you who knows the students the best as their teacher. (PT 7, T_C2)

It was evident in the reflections that participants felt the tension between being a subject matter expert and lacking competency in terms of knowledge. This was a serious conflict for beginning teachers since they have been given the responsibility of experienced teachers when they have started their careers. Although most of the participants felt the tension, it was noticed that they were optimistic about their own career although they anticipate that some contextual issues would challenge them. For example, PT 4 explained this situation as follows:

> I would like to have such a class. This case motivated me rather than scared. It seems possible that the classes where the students force the limits of the teacher could be conceived as difficult at the beginning. But after a while, such classes will advance the teacher's skills. As I wrote before, this is a case that everyone could face. Here I think the only important problem is that the students listen and study for the lesson just for success in the exam. In fact, mathematics is beautiful and they cannot see such beauty. I think the tragic thing is this. (PT 4, T_C5)

Participants also noticed the importance of some personality traits in the teaching profession. The prominent positive characteristics that were discussed were teachers' being idealist, open to development, honest, conservative, powerful, passionate, being reflective and determined. They have also noticed the effects of negative traits like teachers lacking enthusiasm and self-confidence in their teaching performances.

The role of teacher education experiences. The case teachers were the graduates of the program that the participants in CBDM enrolled in. Though reflecting on the challenges that the case teachers faced, participants noticed that they would experience similar problems when they have started their careers. This situation could be associated with participants' expectations of the program.

One of the expectations of the participants from the program was the knowledge base (perceived as theory) to become a teacher and when they had the knowledge, the next step is “just” to do some practice. This perceived distinction between theory and practice evolved to a perceived theory-practice gap when they have seen or experienced the failures of teachers’ attempts to be a good teacher in their cognitive schema.

The experience of teacher training program has a significant effect on the formation of teacher identities. They often define their own teacher identity as the teacher that the program aims to train. For example, a participant who has never heard of IBDP before coming to the program wants to express himself or herself as an IB teacher after entering the program. Because, in a sense, it would mean that she/he would be a reformist, student-centered, and conceptual oriented teacher. She/he hopes that she/he will be equipped with the required knowledge base to actualize this identity during her/his education and that she/he will start the profession as ready. However, candidates begin to show differences in this context. The teacher identity that they attribute to them in their images corresponds to the identity of the teachers in the cases they read. This would also create a conflict about identity. In fact, teachers often experience this in their early years, just as they do in stage 1 results. The perceived theory-practice gap was important in terms of their evolving teacher identity since it would affect their orientation to teach.

PT 4 revealed his beliefs about perceived theory and practice gap as he had fewer expectations from the program to prepare him for the real teaching career and seemed that his prejudice inhibits learning from the theory which is indispensable for making sense of teaching:

> Being a teacher isn't like what we were taught in faculties or institutes of education. Strictly speaking, I think faculties of education are meaningless for those who will become a teacher. It is painful to know that we learn so much theory which will be worse than useless. It is upsetting to see that our efforts will be wasted. (PT 4, T_C1)

Contrasting views during the discussions were very valuable in order for participants to face with different beliefs and opinions and to create a platform for active discussion. For example, contrary to PT 4, PT 1' disposition about the theory-practice gap was pointing teachers' lack of applying:

> At this moment, it just seemed to me that our training provides us so many things, but we anyhow apply what we remember. (PT 1, D_C2)

The tension between what they have been learning during the program and the realities in the classroom was one of the triggers for questioning their beliefs in teaching profession formed by prior personal experiences. PT 7 was quite critical about teachers' lack of commitment to teaching and did not find most of them as hardworking professionals since she has grown up in a family full of teachers. Her beliefs continued when she entered the program and consolidated when she started to observe teachers in various schools. However, during the discussions, her beliefs somehow evolved as she expressed:

> I think, in the second semester [in the first year of the two-year program] I used to criticize my mentors because they didn't apply what they had learned in their training and think that most of them were lazy. However, when I face the realities of teaching more in the teaching practice period, I started to look from another perspective. Although I still think that some of them are lazy but I have recognized the difficulty doing what I wanted to do in terms of teaching at

> the same time; instead, progressing step by step will be more reasonable. (PT 7, T_C3)

In this respect, participants developed an awareness of the professional identity defined during the program could be challenged continuously in their career. Actually, emotions during this period were both a result of these tensions and also a trigger to negotiate the identity again and again.

Prior personal and professional experiences. During discussions, participants realized how teachers' own schooling experience would be effective in their classroom decisions when they become teachers. In fact, teachers' prior personal experiences have a role in the formation of teacher identity. Participants noticed that the case teachers could also be affected by their own teachers in their own schooling. Shouting at students, threatening them with lowering their performance grades, in other words using assessment and evaluation for punishment purposes were shared as experiences that the participants have in their own schooling. PT 4 and PT 5 emphasized the idea that the reason of the case teachers' inappropriate behavior like shouting at or threatening the students with lowering grades were because of the behaviors that they have observed during their own schooling so they had an understanding of the effect of the apprenticeship of observation in teachers' identity.

They don't want to see themselves like their teachers in their own schooling, which they have associated with traditional teachers. Defining themselves as future teachers who were not traditional. When there were tensions between becoming a reformist teacher and constraints before this, they were a teacher or the context made them question the identity. The teacher in CS 2 who has defined herself as an idealist teacher who wanted to use what she had learned during the program; they have realized that the ideal teacher *self* could be challenged by some other factors. For example, PT 3, who was quite critical about the teachers in the CS in terms of their lack of applying what they have learned from the program and became traditional teachers, began to realize that it was not easy as it was considered. The following excerpts are an example of the change in the participants' schema of teaching during the CBDM process:

> During our internship in İzmir, we recognized that there are, in fact some teachers who could combine theory with practice and that the gap between theory and practice is in our minds. I personally observed lessons in which there is no lack of methodology, all the activities were accomplished well and the teacher could somehow engage the students even if they got bored. I think we have the gap between theory and practice; we have a kind of prejudice but we can recognize the possibility when we see it happening. Those who come here (the ones who enrolled the program) are the ones who want to be reformists; I mean, those who will study in state schools (under the control of Ministry of National Education) go to Public Personal Selection Examination, not come to here. (PT 3, D_C2)
>
> I started to think after the period of teaching practice when we as mathematics teachers give the rules of the topic by making a rapid generalization without allowing the students time to discover the topics themselves. I think this is a consequence of our own education, particularly mine (our own schooling years). Although we learned the opposite in this program, we, unfortunately, cannot become that student-centered when it comes to enacting what we learned. (PT 3, T_C6)

Level of reflections

The reflections of participants during CBDM experiences were examined in terms of two major characteristics of the reflections; connectedness and complexity. At first, an overview of the general characteristics of reflections of all participants was shared. Secondly, participants' similarities differences in reflection characteristics with a focus on connectedness and complexity were presented.

Overview of group dynamics in terms of reflection

CBDM discussions were organized under stages of resolving issues in the cases; issue identification, alternative viewpoints, reasons, and solutions (and consequences) to the issues. The characteristics that the reflections hold were categorized as individual viewpoint, personal connection and community connection. The reflections were examined with a two-dimensional analysis tool, the stage that the reflection belongs to and its connectedness. In addition to these, participants shared their general beliefs about the issues raised in the cases and the cases triggered some reflection-for-action. In other words, participants draw lessons from CBDM discussions and made comments about their plans for their future career as a teacher. Community connections were considered to be highest level of reflective practice that would contribute to productive reflection.

During CBDM discussion and written tasks, participants mainly identified the issues and proposed solutions or ways to avoid facing the problems occurred. Reasoning about the causal links and looking from other stakeholders' perspectives (alternative viewpoints) was lower in frequency compared to issue identification and offering solutions.

The issue identification process in case discussion led participants to elaborate on many dimensions of teaching mathematics. It was an initial step to recognize the links between these dimensions and consequently develop a more complex view.

Solution generating was also critical since the most connected reflections, in other words reflection with linking practice to theory occurred in this stage.

Besides the pre-determined categories involving the stages that discussions and written tasks structured around, CBDM discussions provided a platform that engaged participants to share their beliefs in general, reflects for their future actions as a teacher and ask questions about real teaching practices to the researcher who led the discussions. To start with, participants shared their beliefs about various aspects of teaching mathematics. The issues in the cases triggered discussions in which participants articulated themselves revealing some beliefs about learners, context, and nature of mathematical knowledge and teaching mathematics. Second, future-oriented reflections were also noticed. During the case discussions or post tasks, participants began to reflect on their actions as future teachers. These included concerns, awareness, expectations and planning for their future practices as a teacher. In addition to these, participants began to ask some questions to the researcher about her own practices or observations as a teacher.

In Table 13, the frequencies and the percentages of the categories that represent stages of the discussions were shared case by case. In the last two columns, frequencies and percentages were presented for written tasks and discussion sessions in total. When the characteristics of the discussions and written tasks were compared, it was revealed that participants focused on solutions and future-oriented reflections more in the written tasks. There were no questions directed to the participants about considering alternative viewpoints in the written tasks. Therefore, without any prompt, participants did not anticipate different stakeholders' viewpoints on the issues. In addition to that, beliefs of participants on various aspects of teaching showed up more during discussions whereas written tasks involved more case-specific comments. It could also be deduced that case scenarios have shown consistencies in terms of triggering different stages of discussion in similar fashion. For example, issue identification percentages did not show drastic differences among different case discussions. It was similar to the other stages of resolving the issues. However, case scenarios seemed to differ in terms of evoking beliefs and future-oriented reflections. Especially, CS 3, which was about assessing students and parent-teacher relationships, seemed to provide an opportunity to share beliefs. It was also noted that percentage of reasons is noticeably lower than the other cases.

Table 13
The frequencies of each stage of the discussion for each case

Stages	CS 1	CS 2	CS 3	CS 4	CS 5	CS 6	Task	Discussion
Issue Identification	84 (36)	84(36)	76(32)	66 (26)	59 (31)	69(30)	85(23)	353 (35)
Alternative Viewpoints	8(3)	20 (9)	12 (5)	13 (5)	13 (7)	29(13)	0(0)	95(9)
Reasons	37(16)	33 (14)	14(6)	23(9)	29 (15)	25(11)	49(13)	113 (11)
Solutions and Consequences	74(31)	54(23)	59(24)	97(38)	49 (25)	87(38)	168 (45)	249 (25)
Future oriented reflections	7(3)	20(9)	33(13)	29(11)	27 (14)	10(4)	55(15)	71 (7)
Beliefs (general)	25(11)	20 (9)	45 (18)	20(8)	10 (5)	7(4)	16 (4)	111 (11)
Questions	0	0	6(2)	6(2)	6 (3)	3 (1)	0 (0)	21 (2)
Total	235	231	245	254	193	230	373	1013

Note. Task: Post discussion written task; percentages are given in parenthesis.

Two-dimensional analysis of reflections revealed that participants typically shared their individual viewpoints, and this happened mostly during issue identification stage. Although these types of reflections could be identified as being beyond descriptive, could not be counted as connected. It should also be noted that participants called on theory more during offering solutions and considering consequences. Thus, the community-connected reflections were identified during this stage the most. Table 14 represented the frequencies and percentages of all entities of two-dimensional analysis in all of the reflective statements (beliefs and questions were excluded).

Table 14
The frequencies and percentages of reflection characteristics according to the stages.

Stages	Individual Viewpoint	Personal Connection	Community Connection	Total
Issue Identification	375 (30)	48 (4)	15 (1)	438 (35)
Alternative Viewpoints	84 (7)	11 (1)	0 (0)	95 (8)
Reasons	150 (12)	9 (1)	3 (0)	162 (13)
Solutions and Consequences	332 (27)	36 (3)	49 (4)	417 (34)
Future oriented reflections	114 (9)	5 (0)	7 (1)	126 (10)
Total	1055(85)	109(9)	74(6)	1238(100)

Connectedness

Participants varied about their reflective approaches to the cases. When connected reflections (personal or community) were considered, PT 8 has the highest percentage of connected reflections in her total number of reflections. PT 3 followed her in this respect.

PT 8 differed from other participants in terms of the connectedness of reflections with a specific focus on theory. She stood out in terms of her community connections, which could be considered as the highest level of reflection with regard to its contribution to productive reflection. Community connections stood for the reflection that involves commonly accepted theories by educational community. She had considered different methods that she learned during the program and offered alternative suggestions for preventing the issues that occurred in the cases. The following excerpt stood for an example of her reflections that has community connection.

> In each class, I will face different kinds of students and different ways of learning and each class has different atmosphere of learning. I think to use some of the methods that we learned in the course CI 515 in this semester. For example, when I want the class learn a major topic by group work, using Jigsaw could be pretty good. Moreover, I would like to use compacting method in order to challenge many more the better students in the class. In this teaching strategy, the students are challenged in accordance with their own level and they strengthen their new topics by virtue of their questions. For example, teacher Özgür could have used this method in his class. With graphing calculators, high achievers could have solved the more challenging questions while other students could have done studies so as to comprehend much more the steps. For this study, the readiness level of the students must be determined before the class. (PT 8, T_C6)

PT 6 and PT 7 followed PT 8 in terms of the density of their community connected reflections. PT 6 and PT 8 were the two participants who had received their teaching certificates before the program from other universities. One of the possible reasons for their attempts to connect theory into their reflections could be attributed to this commonality.

> Since I took a teaching certificate before, we completed 3 different books including a book on misconception and we as groups taught ourselves. (PT 6, D_C5)

PT 7 differed from other participants in terms of her commitment to the CBDM process. Her personality traits could also be dominant in this respect since she used to take reflective notes while she had voluntary teaching experience during her undergraduate years. Another discriminating feature of PT 7 was her reflections in written tasks. Compared to others, the reflective statements were not only differentiated from others' in terms of length but also with characteristics of reflective practice. When the written tasks and discussions were compared, only PT 7's reflections were more connected to theory in the written tasks compared to the discussions among all participants. The following two excerpts were examples of written reflective statements and her disposition to reflective writing before the CBDM process.

> Although we took the course "Differentiated Instruction" in this semester, I don't think that I have internalized and learned this topic adequately since your ability to answer the questions in theory or to assess the strong and weak dimensions of the techniques does not mean that you can use this technical knowledge in the right way at the right time. Group studies, Jigsaw, KWL can be applied. These seem to be techniques that could be justified for each class and different topics, but the class dynamics may not allow this. I think the meaningful thing is to understand the philosophy behind why we need 'differentiated instruction. (PT 7, T_C4)

> Every evening, I try to think about what I have learned today. I especially try to write. After the class, I write these emotional changes since I afraid of forgetting and being one of the teachers who I criticized. I write my feelings and my questions like 'could it be like this?' or 'should I pay more attention to that?', etc. But what I write are feelings and emotions rather than just the flow of events. By so, I can go back to that moment even a little. For example, I will definitely think about what I have learned in this interview and take some notes on it or the course that I took today. That is to say, I endeavor. (Gamze, FI))

PT 3 and PT 2 were the two participants who connected their views to their personal experiences more often compared to other participants. However, they have differed in terms of the type of the personal experience. PT 2 mostly linked the views to her own schooling experiences and teaching practices in the program. On the other hand, PT 3 mostly recalled and reflected on her teaching experiences in the private teaching institutions that she has worked for one year.

PT 2 shared the most negative experiences about her own schooling. The attitudes of some of her teachers became unforgettable memories and these memories become one of the factors that shape her teacher identity.

> One of my biggest aims related to mathematics is not to give children a trauma… In secondary school, I went through a trauma… The reason was that the teacher was very stern. I don't know how I have not hated mathematics yet. This is not valid only the mathematics teacher but it is generally mathematics that causes in traumas more than any other course. Okey, teacher may stern and disciplined but if the level of this is so much as to make 2-3 years of children, then they would never like mathematics. (PT 2, FI)

PT 2 was also one of the people who considered the possible consequences of the solutions offered by other participants. Participants differed in terms of their considering consequences of solutions. For example, PT 7 mostly considered the possible consequences of her own solutions to the issues raised in the case discussions.

In Table 15, participants' reflection characteristics in terms of connectedness were summarized. When personal connections and community connections were considered as productive reflection compared to individual viewpoint, PT 8's reflections could be identified as the most productive among other participants. The least productive ones belonged to PT 5 and PT 4. Being to context-oriented and focusing on the constraints too often, PT 4 may lack careful and deep thinking on the issues.

Table 15
Connectedness of participants' reflections

Participant	Individual Viewpoint	Personal Connection		Community Connection		Total
PT 1	132 (84)	18	(12)	6	(4)	156 (100)
PT 2	126 (84)	21	(14)	3	(2)	150 (100)
PT 3	157 (81)	26	(13)	12	(6)	195 (100)
PT 4	166 (89)	15	(8)	6	(3)	187 (100)
PT 5	90 (89)	7	(7)	4	(4)	101 (100)
PT 6	146 (87)	7	(4)	15	(9)	168 (100)
PT 7	101 (85)	7	(6)	11	(9)	119 (100)
PT 8	141(77)	19	(10)	23	(13)	183 (100)

Complexity

Another determinant of productive reflection was the complexity of reflections, i.e. the existence of reflections that integrated multiple aspects. Considering different aspects of teaching was a necessary but not sufficient on the way to *learning to teach* mathematics while developing a complex view about teaching mathematics. Integrating different aspects, on the other hand, was considered a sign of productive reflection.

In total, participants included all the aspects during all of the case discussions and written tasks. Participants integrated at most four of five dimensions. The complexity of reflections differed via participants. In Table 16, single dimension column and multi-dimension column (sum of all integrated reflections) were given in order to understand the complexity of the reflections of each participant. Participants did not differ drastically. However, it was noticed that PT 6 stood out with her complex reflections amongst others. PT 8 and PT 1 followed her.

When the reflection characteristics in terms of connectedness were also taken into account, PT 6 and PT 8's reflections could be considered more connected and complex. Thus, they engaged in more productive reflection compared to others. It was worth noticing that lack of connectedness in terms of reflections did not come along with less complexity. For example, PT 5 was one of the people whose reflections were the least connected ones. Despite the fact that PT 3's reflections were amongst the most connected ones, in terms of complexity, they were one of the least complex ones. However, in terms of complexity, she was mediocre. PT 2's reflections could be characterized as the least complex ones. On the other hand,

Table 16
The complexity of participants' reflections

Participant	Single Dim.	2 links	3 links	4 links	Multi_Dim	Total
PT 1	83-29	78	12	0	90 (52)	173
PT 2	119-46	45	6	1	52 (30)	171
PT 3	137-61	60	15	0	75 (35)	212
PT 4	151-63	67	11	4	82 (35)	233
PT 5	61-24	41	8	1	50 (45)	111
PT 6	85-18	70	12	3	85 (50)	170
PT 7	71-30	42	10	4	56 (45)	125
PT 8	106-46	60	22	3	85 (45)	191

Note. Multi_Dim refers to the total of the columns 2,3, and 4 links. The total frequencies are given and the percentages follow in parenthesis in the multi-Dim column.

During the issue identification process, participants were mentioning the issues in the cases mostly in one or two phrases; therefore, naturally these types of reflections mostly involved one or two aspects the most. Therefore, the single dimensions in issue identification were also given near the single dimension frequency. However, as the discussion continued with finding out reasons, suggesting alternative viewpoints and offering solutions, then the reflections became more complicated, i.e. involve two or more connections between aspects of teaching mathematics.

The complexity changed due to the stage of resolving the issues in the case. The stages except issue-identification were more complex. Moreover, discussions and written tasks also differed in terms of complexity. Regarding all discussion and tasks, percentage of the multi-dimensional to the all comments in discussions and written tasks were 34 % and 63 % respectively.

Table 17
Complexity and connectedness of participants' reflections

Participant	Individual Viewpoint		Personal Connection		Community Connection	
	SD	MD	SD	MD	SD	MD
PT 1	70	74	7	9	2	4
PT 2	96	49	18	4	1	0
PT 3	113	59	17	9	4	8
PT 4	138	70	9	6	1	5
PT 5	56	42	3	4	2	2
PT 6	67	78	4	3	10	5
PT 7	64	43	4	3	4	5
PT 8	83	66	9	10	12	11

Note. SD: Single dimension MD: Multi-dimensional

Further investigation was done in order to understand which aspects participants had more complex view. The main focus of the participants and the aspects that they have integrated with others the most were examined. It was also noticed that participants mentioned some aspects more when they have integrated those into other aspects.

First, PT 6 and PT 8 were the ones who integrated mathematical knowledge for teaching to other aspects the most. PT 6 also stood out with her integration of CMO. PT 1 did not see contextual issues in isolation to other aspects. However, it was noticeable how she frequently associated other aspects of context. In fact, PT 4 has focused on contextual factors the most. What differentiates PT 4 and PT 1 was, PT 1' did not pay attention to the factors that were beyond teachers' control. She had just tried to reflect on the contextual factors which would interfere teachers' practices. It must also be noted that she was not pessimistic about these contextual factors and offered solutions under the circumstances of the existing realities. PT 4's disposition was the opposite. He focused on contextual factors, mostly without integrating them into other aspects. When he integrated, he perceived that the contextual factors as being the reasons for the challenges faced and he thought many of these problems could not be solved or avoided by teachers unless the context changes. This was pessimistic way to look over the challenges. PT 7 was the one who mostly integrated assessment of other aspects of teaching.

Table 18
Topic and complexity of reflections

	MKT		CMO		Context		Assessment		TI	
	SD	MD	SD	MD	SD	MD	SD	MD	SD	MD
PT 1	26(15)	55(32)	22(13)	42(24)	14(8)	38(22)	6(3)	10(6)	26(15)	29(17)
PT 2	29(17)	33(19)	31(18)	34(20)	24(14)	19(11)	20(12)	8(5)	16(9)	9(5)
PT 3	44(21)	50(24)	29(14)	43(20)	22(10)	27(13)	18(8)	17(8)	19(9)	27(13)
PT 4	46(20)	43(18)	19 (8)	30(13)	40(17)	45(19)	22(9)	19(8)	30(13)	25(11)
PT 5	18(16)	24(22)	22(20)	21(19)	8(7)	22(20)	2(2)	5(5)	18(16)	18(16)
PT 6	34(20)	63(37)	22(13)	58(34)	10(6)	23(14)	14(8)	3(2)	19(11)	23(14)
PT 7	18(14)	37(30)	10(8)	31(25)	10(8)	17(14)	9(7)	15(12)	29(23)	11(9)
PT 8	40(21)	66(35)	22(12)	49(26)	11(6)	34(18)	17(9)	6(3)	22(12)	23(12)
Total	255	371	177	308	139	225	108	83	179	16
Grand Total	626		485		364		273		344	

Note. The numbers refer to frequencies of each dimension and the numbers in the parenthesis refers to the percentage. SD: Single Dimensional, MD: Multi-Dimensional

The perceptions of the participants about the CBDM experience

This section consists of perceptions of the participants about CBDM experience in relation to its content, process, and its perceived benefits to their teaching career. Case discussions, post-discussion tasks and last interviews were the main data sources to reveal the perceptions of participants about CBDM experiences Perceptions of the participants were organized under four main themes; content, process, perceived benefits to their teaching, and suggestions for improvement. Associated categories under each theme are presented in Table 19.

Table 19
Themes and associated categories for participants' perceptions about CBDM experiences

Themes	Categories
Content	Being real Relevancy of mathematical concepts
Process	Discussions as to the medium of reflection Scaffolding of discussions
Perceived benefits to their teaching	Increased awareness about -importance of planning and preparation -lack of competencies -contextual factors in teaching -the complexity of teaching mathematics Perceived influence on teaching practice Goals for professional development
Suggestions for improvement	Suggestions for the content Suggestions for the process

Content

One of the prominent opinions about the content of the cases of CBDM was the cases as being real. Participants have similar perceptions related to the cases as narratives that reflect classroom reality and complexity, and consequently, they were very likely occasions that would happen in their teaching career. Another noticeable similarity between participants' views on the content was the relevancy of mathematical content at the core of the cases.

Cases scenarios being real. All participants stated that case scenarios were realistic both in discussions, discussion tasks, and last interviews. They were informed about the design of the CBDM so that they were aware of the fact that the cases are combination of real-life experiences of graduates of the program. One of the criticisms of the participants related to their training in the program is that there are some practices and studies which are not pertinent in Turkish educational system.

For these reasons, it was important to reveal participants' perceptions about the cases being relevant to their future reality.

In all of the CSs in CBDM, they thought, they were likely to experience the challenges that the case teachers faced. In addition to that, they found some issues realistic since they have either experienced or observed similar challenges.
In CS 4, there occurred problems due to a lack of clear instructions related to use of graphing calculators. In the discussion task, PT 2 shared her experience as:

> I would definitely expect these kinds of problems. When I was teaching [school experience] at a leading private school in İzmir, I was unprepared to teach a lesson with graphing calculators, a lesson about functions. I felt embarrassed because I couldn't answer most of the questions well. (PT 2, T_C4)

It was also noticed that participants identified themselves with the case teacher. As one of the reasons for this, cases were revised intentionally to address their concerns. The profile of the case teachers shared in the cases was also intentionally matched up with the participants' teacher identity. As a consequence, some participants felt very close to at least one of the case teachers. The following two excerpts are examples of this kind.

> Damla (the teacher in CS 6) is me; I started out with the same goals. If I gave a lecture on limit, I would have a similar problem. (PT 8, D_C6)

> I did not want the teaching profession just to teach mathematics. As in CS 6. The teacher there is like that, she says that it is much more than just transferring the mathematics knowledge to others. To show that I can also help them in other areas, not just mathematics. (PT 8, LI)

Similarly, PT 7, as a person who is more reflective and open to self-evaluation compared to the other participants, was affected by case scenarios emotionally and the impact of the cases continued after the discussions.

> There were cases that I get out/finished satisfied. There were also other cases for which I felt bad, told myself that you are still incompetent as a teacher candidate… There were cases that I got carried away. The teacher [in the case] was me. I mean, I identified myself with the teacher excessively. I realized my academic incompetence plenty of times in the cases. (PT 7, LI)

On the other hand, some of them thought that they would less likely experience some of the issues in the cases. PT 3, giving harsh criticism about the case teacher, impressed less about the challenges. However, the teaching practice period changed her views. The following excerpt shows the changes in her beliefs about the content of the cases:

> It [CBDM] felt realistic, but I was saying that I wouldn't do these. Yes, people experience these but I wouldn't. Maybe, I don't know, it's because of the education we got, because of our self-confidence. However, I found it even more realistic after teaching practice. Maybe I didn't experience exactly the same challenges but some other challenges occurred. Other cases showed up. I mean I thought that these cases generally happen because of not having prepared so I was thinking like the problems would not occur if the teacher prepared for the lesson more intensely. I was criticizing the excuse that there was not enough time for it. But we recognized in teaching practice that it was true, there was really no such time. Still CBDM ultimately moves us one step further. Actually, CBDM was more realistic than us (participants); our approaches and perspective were so much optimistic. (PT 3, LI)

Relevancy of mathematical concepts. PT 1, PT 6, PT 2, and PT 4 specifically pointed out the mathematical content at the core of the CBDM. Before CBDM experiences, they were aware of the fact that they were going under a process that involved discussions related to challenges in teaching mathematics. However, some participants expressed that they have been expecting general cases like classroom management issues or challenges related to motivation of students. Finding out case scenarios that have mathematical content and built around mathematical issues engaged them more. Two examples of this are presented below.

> When we had the first interview since there were also general questions about teaching, I didn't expect such thing like that there will be specific things about mathematics. I thought it would be about the problems occurred during the lesson such as classroom management issues in general. Motivation, motivation of child and so forth. I liked it since mathematics was also involved. (PT 1, LI)

> I was expecting general things, rather than such specific things. I was thinking that we would mainly discuss problems related to classroom management but I wasn't expecting things like the problems we have while we teach limit or statistics. (PT 2, LI)

In addition to that, some of the mathematical concepts in the cases were also unpredicted for some participants. PT 6, being the most sensitive participant about specialized content knowledge, said that she is surprised, especially seeing some topics in the CBDM. She was realized how the lack of specialized knowledge or knowledge at horizon could create problems as she articulated:

> I was expecting discussing problems that occurred in a lesson. I never thought I would face anything about statistics or Monty Hall. In the CS 1, I was not expecting that the challenges occurred as the students did not understand the concept "undefined" or how the problems would be associated with the teachers' lack of knowledge for 11^{th} and 12^{th}-grade mathematics. (PT 6, LI)

PT 4, who is greatly inspired by his undergraduate education and especially from Ali Nesin, made the following comment regarding undefined and indeterminate terms which he did not find very important during CBDM. He realized that this was an important and common challenge is lack of specialized content knowledge when Ali Nesin asked the same problem in a mathematics teacher workshop.

> In the first case we discussed, there was an issue of a number divided by 0 and 0 divided by 0. At first, I was like, "why do we discuss this, what is the point?" I went to a workshop that secondary school teachers attend recently; teachers asked the same questions to Ali Nesin. I realized that it is a general question that teachers are confronted. (PT 4, LI)

Process

The participants commented on the effectiveness of the discussions and the stages that the face-to-face sessions were conducted.

Discussion as the medium of reflective practice. Face to face discussion was chosen as a medium for reflective practice. In fact, it was noticed that there was a tension in the group. The researcher was attended to seminar hours as a guest and interviewed all participants before the CBDM experience. Some of the participants expressed their concerns about group dynamics. It was anticipated that there would be some situations that the researcher must be careful about possible disruptions due to group dynamics. However, one of the most positive comments about the CBDM process was about the discussion. Different personalities and consequently different viewpoints seemed to be a plus for many participants even though they foresaw the opposite.

> Group dynamics were a little bit frightening for me. I didn't know how it is going to be. Would other person take offence if I say something opposite to his/her idea? Honestly, I was wondering whether there will be a war (laughing). Because this is an important thing (CBDM experience) to me, I really think there will be good effects of it. Therefore, I shouldn't offend the other person. In the beginning, I was worried that if I said something, it would be interrupted; I remember that I refrained and I didn't comment much in the first case. After the first case, for example, we, PT 6, PT 4 and I came together and discussed these again. How does it improve us? What should be done? We were discussing after our teaching experiences anyway. (PT 7, LI)

Discussions were found as positive in terms of being aware of different perspectives. PT 5, who frequently stressed the importance of communication both in interviews and discussions, found CBDM discussions effective in terms of helping them to simulate how to work with colleagues. She emphasized the contributions of the discussion process as:

> Two heads are better than one. During the discussions, I heard comments that I would never think of. There was also comments that I didn't agree at all. But still there comes a different perspective, after all we are not going to work alone in the future, we will work with a department. That's why I think that the discussions are important. (PT 5, LI)

Similarly, PT 6 emphasized the importance of discussions and perceived advantages compared to solitary reflection activity. The data was supported her argument as the participants were asked to highlight the issues that they want to discuss as they read the cases. The issues that were discussed during the case discussions were far more than the highlighted comments. In PT 6's words:

> Maybe I could count five issues on the case on my own, but others said other problems and I was saying like "Oh, there is that.". So, it was good to have group discussions. I think discussion parts were very useful for me. (PT 6, LI)

Stages of the discussion plans. Pre-case exercises were given just before the reading and discussing the cases. One of the intentions behind this was to identify participants' reactions towards the mathematical concepts without any preparation. PT 2 commented on how the pre-case exercises were delivered:

> The good thing was this: At first you were giving us a task, we were trying to answer it at that moment, without thinking about it before, that is better actually. I think it would be an unrealistic situation if we came prepared. Because it is better to see how we are going to do something that we haven't thought before, in terms of having the same difficulty as the case teacher. (PT 2, LI)

In addition to that, PT 6 thought that the part that they found solutions to the issues in the cases was effective with an emphasis on the possible contributions of the written tasks for examining her progress or change as she expressed:

> Making long discussions, discussing solution proposals, going deeper into the causes of problems, in-depth analyses contributed to me as a future teacher. I took notes there. Now when I look back to what I wrote at that time, I say, “oh I said these,” maybe I will say something different when I look at it three months later. This was really important to me; I didn’t expect anything like this. Its contribution was positive. (PT 6, LI)

There were discussion plans for each CS in CBDM. Each discussion plan involved four stages; identification of issues, reasons behind the issues, alternative viewpoints, solutions and consequences. All the issues raised by participants were first written to the board by the researcher in order to provide participants a view of seeing the whole picture. They got used to this process and compared the case discussion sessions and have some expressions like “the board is filled up quickly today (they came up with many issues in a short time)” or they were volunteered to share alternative viewpoints as “let’s wear a hat!” (think in the shoes of other stakeholders) although they were less active in sharing alternative viewpoints in the first few cases.

Perceived benefits to their teaching

One of the major benefits was to raise awareness about several issues: the importance of lesson preparation and planning, contextual factors affecting teachers’ practices, and teaching competencies.

One of the major impacts of the CBDM process was about lesson preparation process as most of the participants emphasized. The importance of lesson planning and preparation was underlined in all of the cases so it was one of the most frequent issues noticed and discussed.

> I was more conscious when I was planning, really more conscious. I fictionalize something about the lesson but what if, it is not going to be that way, then how is it going to be? Pretend to be in it, and visualize. To close our eyes and visualize it before the course. My visualization diversified. I said things like "Is it like this?", "This can also happen.", "This can also happen to me." much more. In terms of both subject area and communication with students. (PT 7, LI)

> Cases [in CBDM] are always in my mind when it comes to preparing and studying for the lesson beforehand. Noting down the questions of students, etc. I feel more insufficient on some issues in mathematics. I know that I don't have a professional approach right now. I thought I needed to overcome my deficiencies in that sense. Approach to students, approach to parents. Also, I thought I needed to study mathematics more. (PT 8, LI)

First of all, some of the participants acknowledge that teaching is much more complex than they have thought. CBDM is not the only factor that affected their beliefs about the complexity of teaching. However, it was one of the platforms that allowed them to discuss the issue without ignoring the complex structure of the teaching profession as PT 8 shared:

> My perceptions of teaching had already changed positively during my master's program here. I was aware of the fact that there is the possibility of such a case as that of Ayşe [case teacher]. But by virtue of such a case discussion, I think I understood the seriousness of the event better and I convinced myself that I needed to make precautions beforehand. Like "I have to study on how indeterminate expressions can be explained!". (PT 8, T-C1)

> I realized that there were too many wheels affecting each other when I started to get involve in it. We weren't watching the cases from outside, we were in the cases but not exactly. Cases helped us come closer to reality. I realized it again

> during teaching practice. When I encountered a problem, I remember saying that I wish I had taken my notes [her notes she took during case discussions and after the discussions] with me and looked at what were our solution proposals there. (PT 7, LI)

In addition to that, participants expressed that it raised awareness about some of the issues that the participants have never considered like teacher-parent or teacher-administration relationships.

> In some cases, there were situations related to management or parents. These were the details that we couldn't see unless we experience it in real teaching. (PT 5, LI)

Moreover, CBDM process was also found effective in terms of raising self-awareness about teaching competencies. As PT 4 stated:

> I hate statistics, it makes no sense to me. I mean, I have to fix this eventually, or the calculator issue… I can say that I saw my deficiencies on technology, statistics… (PT 4, LI)
>
> I don't want to do group activity for sake of doing it in my career. I guess these cases helped me to proceed step by step, without jumping into the things without careful planning. (PT 7, T_C2)

Besides unconsidered parts of teaching, CBDM helped some participants to recall some of the aspects that they have to reconsider. Although they knew that they have to keep records, seeing a consequence in the case where accountability to parents was noticed, alerts them to be careful about keeping track and recording systematically.

> I thought it is much more efficient and good for us to come across these types of things before we start our careers. I mean, I have never stayed back. Or I have never said "Alas! (Oh no!)" for the challenges. In the simplest term, for example, a parent comes for a meeting in a case. We have seen in one of our courses in the program, it is important to keep the records for homework well with dates, etc. It was completely out of my mind. After that case, I said "Ah, oh!" and I never forgot. I mean, I had this light-bulb moment. So, I think they were good reminders. (PT 5, LI)

Cases in CBDM were also found beneficial for utilizing other people experience and deduce lessons from these. Being able to learn what graduates of the program experienced was valuable in terms of feeling more connected with the alumni. PT 8 and PT 2 expressed their opinions as:

> The examples you brought made us know the stories of people we haven't met or seen. We learned about their experiences. I just have practice at school x and benefited from the experiences of the teachers there, whatever the person experiences at the moment I observe. I realized we haven't broken the bond with graduates. (PT 8, LI)
>
> These cases were somehow lessons for me. Like, I should design the activity and the lesson well; planning should be good since they will teach some topics for the first time, for example they will TI for the first time. I deduced "to do's to myself like "To use smart view, let them [students] follow you, etc. (PT 2, LI)

Similarly, PT 7 commented on her awareness about her goals as a future teacher, as she mentioned:

> As a teacher, I wanted to develop myself and I had some priorities (such as mathematical thinking, conceptual understanding, discovering the misconceptions student-centered education, etc.) but I realized that I could not develop all of them at the same time and I changed my priorities by adding new ones. For example, I thought that gaining authority and developing classroom management maybe my first aim in 1st year and later on (in 2nd and 3rd years) my aims may be to develop my own way of teaching and to establish it, etc. (PT 7, T_C1)

Some of the participants started to feel confident about teaching the issues in the cases even though they found it difficult when they have first encountered. In the first discussion task, participants were asked to rethink the pre-case exercise and after the discussion how they would explain undefined and indeterminate forms. PT 8, who gave inadequate answers in pre-case exercise, has written a detailed satisfactory explanation at the post-task and she felt confidence to teach it and expressed her wish as:

> I hope that they get the lesson without any question marks in their mind when the lesson is managed in this way. Actually, I wished that I would have a chance to teach this topic just like this during teaching practice. (PT 8, T_C1)

Perceived effects of CBDM on teaching practice

Shortly after the CBDM experience ended, participants started their six-week teaching practice in different schools at Ankara and Erzurum. These schools were also among the ones the graduates in the first stage worked at in their early career. The participants reported some positive influences of CBDM experience in their teaching practice period. These positive effects could be categorized as the effects on lesson planning and preparation, on the spot decisions during the lessons and their evaluations of their lessons.

Lesson planning and preparation. The results revealed that CBDM experiences influenced participants mostly in the lesson preparation process. Looking for ways for differentiating the lesson, careful planning for group activities, having additional plans for the lessons which they would use technology, and considering students' possible mistakes and misconceptions.

These were all emphasized during their courses in the program. Therefore, it would not be a direct influence of CBDM experience. However, they articulated that case scenarios triggered them to take actions in the aspects mentioned. For example, in CS 4, there was a lesson taught with graphing calculators and the chaos which was resulted from the lack of planning the lesson appropriate to work with the whole class. PT 2 who remembered the issues in that case, carefully planned her lesson which she engaged learners in an activity that requires graphing calculators. PT 2 articulated why she used a virtual calculator which was projected on the board so that she could demonstrate the steps to the students:

> I used a smart-view for the lesson I planned to do with graphing calculators. I directly thought of using smart-view, since I had some concerns related to using graphing calculators with the whole class. I remember the funny situation, displaying the wrong graph on the display screen in the case we discussed (CS 4). Before the lesson, I got prepared for the possible problems; I graphed the functions several times. (PT 2, LI)

During CBDM experience, participants had a chance to discuss the effect of some student profiles, like power or attention-seeking, to the lessons and they made suggestions for dealing with these types of students. PT 6 who had a power-seeking student in her class during teaching practice, shared that she utilized from her reflections that she wrote during CBDM experiences about similar student profiles.

> There was a dominant student in my class. I knew him from school experience 1. I had the idea of reaching him in advance. I used to see similar students in the cases we discussed. I knew that he had an interest in computer games. I searched for popular games and got information about them. I rearranged the questions of simultaneous equations according to these games. It attracted his attention and I moved him to the board to solve it. After this, he was always cooperative in my lessons. (PT 6, LI)

Another interesting reflection of CBDM in teaching practice period was PT 7's choosing the Monty Hall problem which was presented in CS 2 for her lesson which the school principal will observe. Although she was not comfortable with understanding the problem and she discussed the problematic planning of the Monty Hall activity in CS 2, she planned a lesson on Monty Hall by anticipating many possible students' questions. She also showed a simulation in her lesson during teaching practice. Showing a simulation was offered for probing students' probabilistic reasoning during CS 2 discussions. Examples of her lesson plan and lesson evaluation is in Appendix G.

It was also revealed that participants experienced many “Oh, I became a case teacher” moments. It means that they experienced something similar to the case teachers. It was a sign through reflecting on their experiences; case discussions became a source that they would consult. It was also very important to note that a few participants indicated some signs of reflection-in-action. For example, PT 6 shared how she stop and think for a moment after she shouted at students and the case discussion came to her mind at that point:

> I guess it was in the last case (CS 6). There was a problem with the teachers shouting at students. In teaching practice, this case happened to me. That day the teacher of a class did not come. I subbed the lesson and the students were making too much noise. I thought I would stop it by shouting at students. At first, it was effective. After five minutes, everything got back to same… When I first shouted and I heard my loud voice, I remembered the case and what we have discussed. I remember that it would not work. Then I chose to stay calm and gave up shouting. (PT 6, LI)

One of the reflections of the CBDM period was on the awareness of PT 1 on how the lesson went in general. She thought that the case raised awareness of the fact that there are multiple things to consider after a lesson, even it seemed to be going well, as she expressed:

> During teaching practice, I had thought that it was easy, but in the cases I realized that they are difficult. The problem emerges from that we think it will be easy because you are not aware. … I told the lesson, it finished very well. But when looking back and recognized that something left as untold or weak, then you see that it was not so easy. (PT 1, LI)

Suggestions for improvement

Although all of the participants found CBDM content and process satisfactory and helpful for their career, they were further questioned if they have any suggestions for the improvement of CBDM experiences for possible future implementation. After these questions, some participants came up with suggestions to improve in terms of process and content.

Suggestions for the process. During case discussions, there were suggestions for reading scholarly work after the discussions but they were not obligatory. Although participants admit that they did not have time to accomplish tasks, including reading, PT 2 and PT 6 thought that reflections could be much more productive. “It could be more effective both in cases accompanied by readings.” (PT 2, LI)

Suggestions for the content. There were no suggestions to enrich the content of the cases, except PT 6’s. PT 6 suggested the following;

> As content, I thought, there could be many more topics relevant to 9th grades or there could be cases relevant to middle school. I could consider these only when I decided to work in middle school. (PT 6, LI)

In fact, it was evident that she found the CBDM process effective so she thought it would be better to have more cases related to her future career or the areas that she felt less prepared. In fact, PT 6 was one of the participants who valued the process more compared to others as she took notes and write detailed post tasks,

> Maybe there could make a case regarding assessment or the teacher’s preparation of an exam and this case could be a case where the teacher cannot assess all levels taxonomically and the good and bad students get the same grade. For example, I think the assessment courses (in the program) of each subject area teachers should be separate. (PT 6, LI)

CHAPTER 5: DISCUSSION AND CONCLUSION

Introduction

> Good cases work as an antidote to oversimplification, moving students toward greater sensitivity to context and uniqueness. This technique exposes learners to differing interpretations of complex situations. (Merseth, 1991, p.7)

This study focused on developing and implementing a case-based pedagogy experience with a productive reflection framework under the principles of a realistic teacher education approach. Firstly, the process of designing a case-based discussion module (CBDM) aligned with the aim of constructing realistic teacher education practices was shared. Following the design, the implementation process and the experiences of the pre-service mathematics teachers were presented in order to reveal the relevance and utility of CBDM. These sections consisted of the discussions built around the experiences of designing and implementing CBDM. Several implications for practice and future research were also shared. The discussion section was built upon the assertions that the researcher derived based on the results of the two stages of this dissertation.

Discussion regarding stage 1: The design and development of the CBDM

One of the intentions of this study was to portray the early years' experiences of a group of mathematics teachers who were the graduates of the program in order to design and develop case materials. The challenges faced by the 10 graduates participated the research were interpreted with a holistic approach. With this in mind, the beliefs and expectations behind the challenges, and perceived reasons for those challenges were examined while vicariously revealing challenges. The results of

Stage 1 are discussed under four assertions, which also answers the following research questions: How can a case-based discussion module be designed to implement as a complementary practice for pre-service mathematics teacher education?

Sub-questions:

1) What are the challenges that participating mathematics teachers face during the early career stage?

a) What are the prior beliefs and expectations of the participating mathematics teachers on the dimensions of teaching that they were challenged with in their early careers?

b) To what extent do they integrate different dimensions of teaching while reflecting on the reasons for those challenges?

Assertion 1: Secondary mathematics teachers, in their early careers, mainly have challenges due to lack of mathematics knowledge for teaching, maintaining classroom discipline, dealing with overwhelming workload, lack of students' motivation towards mathematics lessons, maintaining curriculum pace, keeping records of students' progress, giving performance grades and preparing assessment materials suitable with the level of the students.

The findings of the first stage of the study indicate that teachers have challenges due to lack of mathematical knowledge for teaching. Evidently, teachers start their careers with inadequate and unsubstantive mathematical knowledge as many studies asserted (Carpenter, Fennema, Petersen, & Carey, 1998; Cooney, 1999; Feiman-Nemser & Parker, 1990). As a result, teachers felt insecure in the classroom because of this lack of content knowledge. Consequently, this situation both decreases the quality of mathematics instructions and causes classroom discipline problems. This research also shows that participant's belief about mathematical knowledge for teaching consists of facts and procedures, a view collaborated by other studies (Haser & Star, 2009; Noss & Baki, 1996). Pre-service teachers, who were once students in teacher-centered classrooms and were taught mathematics with a procedural approach, have constructed strong beliefs about how to be a teacher and teach mathematics (Baki, 2015). It is apparent that before they start their career, they did not realize the importance of substantive mathematical knowledge; in other words, its correctness, meaning and connectedness (Ball, 1991). Moreover, the results indicated that teachers' lack of knowledge of content and students affect their planning and organizing the lessons and result in the inefficient use of lesson time, and cause not covering the curriculum as required. In this regard, the results of this study confirmed the theories that emphasized the fact that utmost importance should be given to teachers' mathematical knowledge for teaching. For example, studies conducted with exemplary mathematics and science teachers provided important signs of quality teaching and stressed the importance of strong content knowledge and pedagogical content knowledge (Tobin & Fraser, 1989). The current study also revealed that reported challenges related to teaching mathematics were not mainly due to lack of procedural knowledge of teachers but mainly, they occurred as a result of lack of conceptual knowledge. However, teachers' specialized content knowledge, which requires conceptual knowledge and understanding, is important in terms of meaningful mathematics learning (Ball & McDiarmid, 1989). Otherwise, with the lack of conceptual mathematical knowledge, teachers tend to shift their practice from conceptual to procedural and traditional which they feel safe (Hutchinson, 1996).

This is, in fact, the way they have been educated during their own schooling and facing many difficulties at the same time in their early career, teachers tend to imitate what they have been exposed to in terms of teaching and learning. In other words, apprenticeship of observation becomes dominant in teachers' decision making when facing challenges without enough support and reflective practice (Lortie, 1975). Another important point is that, due to limited attention capacity, teachers who are dealing with lack of mathematical knowledge for teaching or having classroom management problems are less likely to direct their attention to students learning (Lidstone & Hollingsworth, 1992). The current study revealed that teachers in their early career mainly focused on being able to answer students' questions or dealt with students' misbehaviors. Students' learning was not the main concern for the teachers early in their careers. Instead, teachers had mostly task and self-concerns (Fuller & Bown, 1975; Ryan, 1986).

Some of the challenges the teachers face during their early careers should not be regarded as challenges peculiar to novices. Society in the information era changes rapidly, though the needs also change. Accordingly, necessary adjustments are made in the curricula in order to raise individuals answering those needs. For example, probabilistic reasoning and statistics literacy gained a lot of importance in mathematics education (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2010; National Council of Teachers of Mathematics [NCTM], 2002). Although statistics topics were added to Turkish elementary and secondary school curriculum, these topics were limited to simple descriptive statistics concepts. Participants in this study teaching international curricula had taught statistics in depth. However, it is apparent that teacher knowledge and beliefs about statistical and probabilistic reasoning are very limited since they lack education in these fields because not much importance is given to them during their own schooling. Coping with many challenges at the same time, teachers in their early careers had difficulties in the topics that they did have not enough experience as learners. In the light of these, it is important to become adaptive experts. Being an adaptive expert requires efficiency and innovation in teaching. Efficiency requires teaching without giving too many attentional resources to perform tasks and innovation requires being able to move beyond routines, being open to new ideas and practices with questioning existing ones critically (Hammernes et al., 2005). Professional development opportunities should be designed to help both the novice and the experienced teacher gain the skills to become adaptive experts in the rapidly changing society. Nevertheless, a routinized level of teaching is important to reduce the number of attentional foci and become open to novelty in teaching.

The findings also suggested that classroom management was found to be one of the dimensions that teachers confronted consistently as suggested in other studies (Gergin, 2010; Öztürk, 2008). It was revealed that most of the teachers were not expecting to experience substantial difficulty in classroom management. However, they faced classroom management problems during the initial years. As a matter of fact, classroom management was also recognized as an unexpected problem by novice teachers in Turkey’s national curriculum context as well as in international studies (Haser, 2010; Nahal, 2010). In-service teachers, before they have started their career, did not seem to have acknowledged that pedagogical content knowledge, lesson preparation or their physical or emotional mood would affect their power in managing their classroom.

In addition, the study revealed that participants had been challenged in keeping track of students' performances, giving performance grades, and preparing examinations. Similarly, none or non-systematic ways of keeping records of students' progress was revealed in a study conducted with Turkish mathematics teachers (Türnüklü, 2003). Hence, in this study, it was noticed that participants had quickly reorganized the way that they had used to keep track and give performance grades after they had seen the pitfalls of their early applications.

Moreover, it was revealed that heavy workload and lack of time management is a challenge affecting teachers' early careers (Fantilli & McDougall, 2009; Kozikoğlu, 2016; Öztürk, 2008). The heavy workload had caused burn-out syndrome, decrease in motivation for lesson preparation (Fullan & Hargreaves, 1991). The study also showed that maintaining curriculum pace was a factor that had affected participants' performances and teaching philosophy in their early careers. This overlaps with constructivist ideas of teaching mathematics replacing itself with direct teaching methods due to maintaining curriculum pace. Participants believed that they could maintain curriculum pace without direct teaching. Teachers had felt pressure under the rigid timeline of the curriculum, which was created to standardize instructional practices and ensured broad coverage (Brooks & Brooks, 1999). One of the areas that teachers' accountability was questioned in Turkey was related to covering the curriculum and it was interesting that the stress was not mentioned in the domains like students' achievement in the exams or their academic performances. The one-size-fits-all approach in the national curriculum restricted teachers to pre-determined times to cover each unit and this is a stress for not only novices but also experienced teachers.

Assertion 2: Teachers have oversimplified beliefs, unrealistic expectations about teaching, and these beliefs are apparent in the challenges that they face.

Reflections on challenges revealed another important finding as the mismatch between expectations and realities affecting teachers' early careers as a factor. In most of the areas that teachers face challenges, there exists a disparity between what teachers expected before they started their careers and what they really experienced afterward. One of the reasons that participants experienced reality shocks can be due to unrealistic high self-efficacy beliefs formed by vicarious experiences. Vicarious experiences are a source of self-efficacy belief, which is formed by modeling other people's behaviors. They persuade themselves that if others can do it, they should be able to achieve at least some improvement in performance (Bandura & Barab, 1973). However, without reflecting in depth about their own status of knowledge of teaching and competencies, it turns out to be oversimplifying the profession and become a source of reality shock. An alternative explanation could be teachers' main focus of attention at the beginning of the career. Teachers' expectations from themselves are to help students learn which could also be regarded as a part of fantasy stage and they may simply disregard possible challenges that would hinder their efforts in this fantasy world of teaching in their beliefs. This expectation is replaced quickly with the mode of survival causing a great deal of stress with a lot of concerns related to class control or content knowledge (Fuller & Bown, 1975). It is important for the preservice teachers perceive that teaching involves many complexities and the teachers must develop a more holistic view of teaching and classroom. Simplistic views about the profession and disregarding the complexity would lead to unrealistic optimism, which may have serious implications for teacher education. In addition to these, studies on exemplary teachers again revealed that exemplary teachers have a more complex view of teaching compared to non-exemplary teachers (as cited in Berliner, 2001). Simplistic views about the profession and disregarding complexity would lead to unrealistic optimism, which would have serious implications for teacher education. The results supported the findings of the study of Weinstein (1988), which revealed that pre-service teachers had optimistic biases in various domains of teaching and brought these biases to their early career. Either they did not have a concern, or even if they thought about the possible challenges, they simply were convinced that they could overcome them. However, these simplistic and unrealistic thoughts about teaching lead to the realization of how challenging and complicated teaching can be as teachers go through struggles in their early careers. Simplistic beliefs during pre-service teacher education would cause less reflective teaching, less focused observations of teaching and learning processes. Based upon these, it is important that preservice teachers perceive that teaching involves many complexities and the necessity of having a holistic perception of teaching.

Assertion 3: Teachers begin to have a more complex and connected schema about teaching when they begin to see how different dimensions of teaching affect each other when they start reflecting on the challenges.

The results indicate that teachers cannot envision the relationship between different dimensions of the teaching profession before they start their careers. In other words, they have the problem of integrating various components of complex classroom reality (Eilam & Poyas, 2009). They have vague ideas before they start their career about how their subject area knowledge can affect their classroom management or how lesson planning is crucial for their self-confidence in teaching. In contrast, skilled teachers know that management problems do not usually occur in isolation from the lesson being taught (Lidstone & Hollingsworth, 1992). Moreover, the effects of contextual factors are unclear and optimistic biases hinder the possible contextual challenges and their effect on other dimensions of teaching before they start their career. One of the reasons why pre-service teachers have a less connected schema of teaching could be about selective nature of attentional capacity (Bransford, 1979). The results showed that teachers, in their early career, had mostly self-concerns and an attention to students' learning and impact could not be found in the teachers' primary challenges they focused on since they had just started to newly develop a more interconnected schema of teaching and begun to realize why these challenges occur (Ryan, 1986). Although school experiences during teacher education programs were giving the opportunity to examine the complexity of real classrooms, it might not turn out to be a platform for integrating different components of teaching because of lack of productive reflection on the critical moments of the lessons they observed or taught. Another reason for lacking awareness of this complex nature of teaching could be the separation of the method courses and foundation courses, as well as the separation between course-work and field-work (Grossman, Hammerness, & McDonald, 2009). This situation was ambiguous in terms of realistic preparation since teaching experiences were not as fragmented as they were offered in the teacher preparation programs (Korthagen, 2001)

Assertion 4: The mismatch between teachers' beliefs and practices indicates a theory-practice gap.

One of the important findings of the current study is washed out experiences of teachers in their early career supporting various researchers. Teachers' perception of theory becomes somehow useless and inapplicable in their early career. There can be teacher-related and context-related reasons for this situation. Context-related reasons can be the existence of high-stake national exams, national curriculum constraints, and the lack of colleagues or administrative support. However, there are teacher-related constraints. The reasons for this can be the broad generalizations teachers make after the failure of a few attempts to integrate theory into practice so "the perceived gap between theory and practice originates not so much from demonstrable mismatches between ideal and practice but from the experience of being held accountable for them." (Elliot, 1997, p. 97).

The study showed that teachers come across contextual challenges that could result from the characteristics of the school they worked at, the national curriculum, the national exams, the students' profiles, or the overwhelming workload. When the contextual constraints interfere with a lack of knowledge and skills of teaching, teachers tend to attribute the sources of many challenges to contextual reasons. The causal perceptions may vary across different individuals and it is important in terms of the effort they would show in order to overcome the problems. Thus, whether the locus of control beliefs of teachers in other words, whether they believe the cause of events is internal or external is important in terms of the effort they put in their work (Strauser, Ketz, & Kim, 2002).

The results in stage 1 revealed that graduates gave up doing student-centered activities due to failure in a few attempts. It could be interpreted as although pre-service teachers may shift their traditional beliefs to non-traditional beliefs about teaching during teacher education programs, after they start their career, their practices become traditional. One of the reasons for this discrepancy between beliefs and practices would be teachers holding surface beliefs of non-traditional beliefs rather than deep beliefs (Kaplan, 1991). Under various contextual difficulties, teachers' surface beliefs do not reflect on their practices.

Teachers' beliefs and expectations stand as lenses for their educational courses and their practices. Teacher education programs should take into consideration the beliefs of the pre-service teachers and shape the teacher education programs in order to address the needs and to change the cognitive schema of teachers from a simpler, less connected form to a more complex and sophisticated schema of the teaching profession. The results of the current study confirm that pre-service teachers tend to disregard the complexities of teaching and underestimate the impact of the possible challenges that they may face with unrealistic optimism. However, one of the initial conditions of making sense of teaching lies at the heart of elaborating on its complexity (Lin & Cooney, 2001). Thus, they need to reflect more productively on the challenges that they may face. Field experience, which takes place during their training, is a good opportunity to become aware of these possible challenges. However, they may not be aware of the complexities when they observe expert teachers. In addition to these, teaching with the existence of a mentor would be different than being alone in the classroom because the full responsibility of a classroom differs from the experiences during internship.

The discussions above provided both the need and the inspiration for the design of a realistic teacher education practice which is case-based approach for giving pre-service teachers reflection opportunities to discuss realities of teaching mathematics. The reason for choosing case-based pedagogy was its potential to reflect the reality and complexity of the classroom in a way that would provide opportunities for reflective practices which found fruitful in terms of bridging links between theory and practice (Hammond & Collins, 1991; Jay & Johnson, 2002; Ünver, 2014). Moreover, creating reflection opportunities by using real teaching practices and cases would facilitate pre-service teachers linking theory to practice that would shape teachers' work explicitly and implicitly (Ball & Forzani, 2009). Regarding the importance of mathematical knowledge for mathematics teaching, the cases were designed as mathematics teaching cases to be more helpful if they reflected the specific reality of a mathematics classroom. Therefore, cases were designed with mathematical content in the core (Barnett, 1991; Merseth, 2003). Case-based discussions were also regarded as helpful in building learning communities. In other words, pre-service teachers would have the opportunity to understand how different points of view would open their minds and contribute to their reflective practice (Lin, 2002). Another advantage of case-based discussion would be the situated nature of the cases. Case scenarios that reflect the realities of possible teaching contexts would be valuable in terms of giving pre-service teachers the opportunity to reflect on the constraints that the context will bring before they personally experience them. As a matter of fact, they would have more realistic expectations about the context they would teach in. Research on teacher cognition also shows that teachers' knowledge and beliefs influence what they determine as important to attend to in complex situations (Schoenfeld, 1998). For example, mathematics teachers' reasoning about interactions in a mathematics classroom would differ from their reasoning about a literature or science classroom (van Es & Sherin, 2002). Similarly, a subject-specific professional development opportunity would be more relevant for both experienced and novice teachers.

It is unfair to expect effective teaching from novices when they have the same or even greater responsibilities relative to experienced teachers (Bartell, 2005; Feiman-Nemser, 2001). This research showed that teachers needed more time to prepare for their lessons in order to overcome the lack of knowledge and skills related to teaching. Thus, teachers' schedules should be arranged in a way that allows them time to prepare for their lessons, collaborate with other teachers, and reflect on their practices. In this process, effective induction was shown to be critically important in teachers' professional growth (Clift & Brady, 2005; Cohen & Fuller, 2006). Both school administrators and policy-makers must acknowledge the fact that teacher education should be supported with effective induction periods. There would be a smoother transition to the career if there was an induction period provided for novice teachers (Ingersoll & Smith, 2004). However, one of the reasons for the struggles of beginner teachers is the lack of induction periods. It would be a delusion to expect that pre-service education, no matter how intense, comprehensive, and practice-based, has fully prepared the teacher for this relentless career. Regarding the fact that *learning to teach* was a continuous process, beginning teachers should continue to learn about teaching via in-service professional development processes (Feiman-Nemser, 2001). Nevertheless, the induction period is crucial in terms of maintaining this continuum. Mentors in this period are of utmost importance in the development of a beginning teacher. However, it is reported that mentoring and induction are not well established in many schools, even in private, competitive schools (Özoğlu, 2010). Working with mentors, reflecting on their experiences, having time to be fully prepared for their lessons would decrease the amount of stress that they have and challenges would subside. However, they would turn these challenges into learning opportunities during an induction period. The Ministry of Education in Turkey launched an induction program in February 2016. A research was conducted with the first cohort who passed the MONE candidate teacher-training process. The results of this study revealed that although the participants endorsed the induction program, there were two important aspects reducing its effectiveness: the excessive number of forms that must be filled during the process; and the quality of the mentors (İlyas, Coşkun, & Toklucu, 2017). Most of the activities implemented in this induction process displayed similarities with the program that participants of the current study had undertaken. For this reason, the results of this study would provide insights for improving this induction program.

Expertise is not a natural consequence of experience. Teachers who begin their careers with lack of support and have limited professional development opportunities in their career, if do not quit the profession, will gain the experience but will neither become an expert nor turn out to be one of the exemplary figures in the profession (Berliner, 2001). The teachers, who are idealistic and self-motivated to improve themselves, would find opportunities to become better teachers. However, leaving it to teachers' characteristics should not be the only solution. Through professional development opportunities, teachers should be encouraged. The first years in a career have a considerable impact on the following phases of the career. Although the current study has several implications for teacher educators and policy-makers in terms of in-service and pre-service training, this study is limited to participants' verbal responses. Further studies would take a more longitudinal approach to early career experiences of mathematics teachers including classroom observations, other stakeholders' viewpoints besides teachers' own perceptions. Getting to know more about what teachers experienced would provide more opportunities to help pre-service teachers to develop a more realistic and holistic view of teaching mathematics and developing professional development programs to help in-services to become adaptive experts.

Discussion regarding stage 2: The implementation of CBDM

The second stage of this dissertation was the implementation process of CBDM, involving six case discussion sessions with post-discussion written tasks and a follow-up interview. This stage was accomplished with 8 pre-service secondary mathematics teachers enrolled in the program. The aim was to shed light on the reflective experiences that CBDM provided to pre-service mathematics teachers. The following research questions were answered in order to reach this aim:

How do pre-service secondary mathematics teachers experience the process of the CBDM implementation?

Sub-questions:

1) To what extent are pre-service mathematics teachers' reflections productive?

a) On whom do the participating pre-service mathematics teachers reflect during the implementation of the CBDM?

b) What aspects of teaching mathematics are noticed by pre-service mathematics teachers during the implementation of the CBDM?

c) What are the characteristics of participants' reflections in terms of connectedness and complexity?

2) How do pre-service mathematics teachers' identities associate with their reflections in the CBDM process?

3) How did pre-service mathematics teachers perceive the CBDM experience?

The discussion is built upon the assertions shared above. Each assertion is discussed within its own and linked to the assertions in stage 1 when they were associated.

Assertion 5: CBDM enabled participants discussing several actors in teaching mathematics.

Assertion 5 is claimed based on the results related to the section *actor of the reflections,* and under this assertion the research question "On whom do the pre-service mathematics teachers reflect during the implementation of CBDM?" is discussed.

The focus of the participants in terms of the actor of the reflection was mainly the case teacher during the discussions. When they focused on the teacher, the reflections were far from being emotional. However, when they have focused on self as the actor of their reflections, emotions were triggered and they also began to link their reflections to their background. Emotions were found influential in terms of triggering attention and motivation which would potentially create "a bottom-up" process leading an awareness (Vuilleumier, 2005; Bransford, Brown & Cocking, 2000). Linking what they have seen in the case scenarios to their own experience is valuable since people constructing knowledge when they have pre-existing categories about that subject (Correia & Bleicher, 2008). That's why this dissertation has its psychological roots in constructivism. CBDM experience is promising in terms of reaching the aim of putting the “pre-service teacher” in the center of the teacher education praxis as a realistic teacher education approach.

One of the intentions of designing the case scenarios in CBDM was to evoke awareness in pre-service teachers about being a mathematics teacher in the reality of their own context. Therefore, pre-service teachers’ focus on case teachers and their actions was expected. Although they were supported to put themselves in the place of the case teacher by scaffolding provided both in discussion and post-discussion writing tasks, participants’ tendencies in foci on self differed. The results shared under the *actor of reflections in the CBDM* revealed that when focusing on other teachers’ actions, some participants possessed a judgmental stance. It was also noticed that the ones who had less self-concerns had mainly reflected on the case teachers or teachers in general. Especially the tone of their language was didactic and judgmental. Levin’s study (1995) supported this finding as the study revealed that pre-service teachers were more judgmental and didactic compared to beginning and experienced teachers.

It was important to determine the circumstances that pre-services were considering learners as learner-centered beliefs would be an indicator of teaching with a learner-centered approach. In many studies, learners could be concern for pre-service teachers and experienced teachers but not beginning teachers as they were in their survival mode and had more self-concerns. However, in the complexity of the given cases, pre-services sometimes felt the survival concerns rather than pupil concerns.

Moreover, the results revealed that during case discussions, some participants' attention to the learners might leave its place to self as a focus. Facing the challenges of teaching through CBDM experience, pre-service teachers might feel survival or self-concerns that beginning teachers had during their early careers (Fuller & Bown, 1975; Ryan, 1986).

Although the major actors were in the foci of interest of the participants, professional awareness was observed in terms of noticing the role of parents, colleagues and administrators in teaching. Having an open-communication with, collaborating with colleagues were found to be critical in relieving their stress, and for their professional development. It was a positive outcome of the cases in CBDM that the participants reflected on collaborating with the department as a precaution for or solution of problems that they may have faced.

CBDM experience provided participants the opportunity to think from others' perspectives. It helped them to attend different stakeholders rather than the teacher. Although participants mainly focused on self, the teacher and the learner, they had a chance to see the roles of other actors like parents, colleagues and administrators. However, this could be a result of the scaffolding provided to participants since they were asked to comment on the issues if they were in the shoes of a learner, a parent, an administrator or a colleague.

The results revealed that pre-service teachers attended to parents and administrators if there was an issue of answerability. Thinking that teaching profession as a more autonomous one compared to other professions was a motive for choosing teaching as a career. Discussions during CBDM triggered pre-conceptions and make them rethink about the work of teaching in the framework of accountability. An awareness about moral, professional and contractual accountability was observed in pre-service teachers, especially for the ones who did not consider the accountability issues before. Participants varied in reactions to external control and external accountability. For example, general exams were perceived to be sustaining objectivity and increasing answerability by some participants. On the other hand, it was found to be a threat to the autonomy of the teacher by another participant. These discrepancies were also found in literature among veteran and novice teachers, where veteran teachers complain more about accountability (Wilkins, 2011). It was important to discuss these issues and face pre-services with the increasing accountability of teaching career in order to help them revise their schema. Otherwise, at the beginning of the teaching career, together with other challenges, feeling a lack of autonomy would cause a drop in job satisfaction (Brunetti, 2001)

Investigating the actors that the participants focused on gave strong clues about their teacher identity. When the participants were compared in terms of their foci, the ones who had self-concerns in the first interviews seemed to focus on self more than the others during the discussions. The reason for participants' varied attention could be associated with their personal backgrounds as students. Negative memories of ones' own schooling may affect their lens of analyzing the instants in the classroom as supported by a number of scholars (Lortie, 1975; Pajares, 1992).

Assertion 6: CBDM offered the opportunity to discuss several issues regarding teaching mathematics; Mathematical Knowledge for Teaching, classroom *management and organizations, assessment of students' learning, context and the role of teacher identity on teaching practices.*

Assertion 6 is claimed based on the results related to the section *topic of the reflections*, and under this assertion the research question "What aspects of teaching mathematics are noticed by pre-service mathematics teachers during the implementation of CBDM?" is discussed.

Mathematical knowledge for teaching

Mathematical content was at the core of each case together with multiple realities of teaching secondary mathematics. The results revealed that the challenges that the case teachers experienced due to lacking mathematical knowledge for teaching took the primary attention of participants. This was one of the most prominent implications of the current study. Although mathematical content was limited to six cases in CBDM, the issues discussed around these contents could be associated with the general principles of teaching secondary mathematics.

Teaching probability and statistics were the two major concerns of the participants. CBDM involved two case scenarios, CS2 and CS4, built around probability and statistics, respectively. It was not only the concerns of the participants, but also these were the challenging topics to teach for the graduates. The difficulties of teaching probability and statistics were emphasized in prior studies (Ben-Zvi & Garfield, 2004; Borovcnik, 2011). There were several reasons lying under the challenges of teaching. The case teachers' attitude towards teaching statistics and its possible effects on students' learning statistics were discussed by participants. This was important in terms of reconsidering their beliefs and attitudes towards teaching statistics. In CS 4, participants discussed teaching statistics and the role of the teachers' attitude towards teaching the subject. Having a purist orientation to mathematics, PT 4 had the most negative attitude towards statistics. During the discussions, he was quite conservative about his views on the role of teaching statistics in the curriculum. However, after the discussions, even he asserted that alternative views in the group made him question his beliefs about teaching statistics. Together with his teaching practice experience, he admitted that statistics could be helpful for modeling and therefore making interpretations about the data. As he began to value statistics, he became less strict on his views about teaching it (Wilson & Cooney, 2002). As Estrada, Batanero and Lancaster (2011) asserted "Positive attitudes increase when students have good learning experiences and perceive value for their own professional work or for their students' education. Negative attitudes are linked to perceived difficulty, lack of knowledge, and overly formal content." (p. 170).

Although assessing the participants' mathematical knowledge for teaching was not the aim of the current study, pre-case exercises gave clues about their status specifically in their specialized content knowledge. Except one participant, none of the participants could explain the concepts related to dividing a number by zero. This finding supported other studies that showed the lack of specialized content knowledge of pre-service mathematics teachers in teaching undefined and indeterminate terms (Ball & Wilson, 1990; Even & Tirosh, 1995; Cankoy, 2010). This also implicated that although participants had mathematics majors, it does not imply their adequacy in teaching mathematics in secondary schools. The need for enhancing the specialized content knowledge of pre-service mathematics teachers should be again emphasized as it was done in many other studies (e.g. Ball & McDiarmid, 1989; Aslan-Tutak & Ertaş, 2013). CBDM has the potential to evoke an awareness of the importance of specialized content knowledge of mathematics teachers in creating meaningful learning environments for the students and as the results revealed that careful thought and research enabled them to enhance their knowledge about underlying concepts behind definitions like in the example of undefined and indeterminate expressions.

Knowledge of content and students, another important dimension of MKT, was noticed and discussed in terms of its role in meaningful mathematics learning. CBDM involved examples of students' way of thinking and reasoning about mathematical concepts, as well as their unexpected questions to their teachers. The anticipation of students' possible questions, struggles and misconceptions found to be important by the participants. An awareness of lesson preparation was revealed. Lesson preparation does not only involve what to teach and how to teach but looking from a perspective of anticipating how students would think, struggle or further question. By giving examples of students' thinking, working on problems and questions they posed, it seemed that it helped in terms of focusing on students' thinking as it was intended in use of case-based pedagogy in mathematics teacher education (Merseth, 2003; van Es Sherin, 2002).

While discussing the challenges of teaching some concepts in mathematics, the nature of the topic was perceived as the source of the challenge. Abstract nature of mathematics was perceived as a reason for challenges of teaching and learning mathematics. At that point, participants realized the importance of pedagogical content knowledge. In terms of participants foci at the knowledge of content and teaching, procedural approaches of case teachers were criticized and the importance of conceptual understanding was emphasized. Pre-service mathematics teachers having a conceptual orientation is important in terms of their future practices (Thompson & Thompson, 1996).

It was revealed that participants thought that one of the sources of challenges that occurred in teaching some mathematical concepts like undefined and indeterminate terms or teaching limit concept is because of the abstract nature of the topics. Attributing the source of the difficulty to the nature of the topic (such as being abstract, or have no real-life connection) could be regarded as epistemological obstacles of learning mathematical concepts (Cornu, 1991). Although student-related causes of difficulties occurred in understanding the concepts were shared, pedagogical reasons (teacher-related) were shared more often. However, Bingölbalı, Akkoç Özmantar and Demir's (2011) study examining the views of pre-service and in-service teachers on the source of students' difficulties in learning mathematics revealed that pre-service and in-service teachers paid more attention to student-related causes rather than pedagogical sources of difficulties. One of the reasons for the participants in this dissertation considering pedagogical reasons more than the others would be the way that CBDM designed. Teachers' pedagogical decisions and their connections with students' learning were discussed in detail. Therefore, CBDM would be considered to help the pre-service teachers to associate the causes of students' struggles in learning mathematical concepts to lack of pedagogical content knowledge.

Although the aim of the current study was not to improve mathematical knowledge for teaching, the pre-case tasks and the mathematical content of the cases evoked an awareness for most of the participants. Probability was articulated to be one of the subjects that most of the pre-service teachers have hesitations for further teaching. For example, PT 2 could not give a correct explanation in pre-case exercise of CS 4 which was about a lesson built around the Monty Hall problem. Although she articulated that she did not feel comfortable with this problem during the discussions of CS 4, she designed an activity that she introduced students the Monty Hall's problem for a lesson that she will be observed by supervisor and school principal in her teaching practice. Some other participants also expressed that they would like to try to teach the content which they have discussed in CBDM. One of the reasons for this could be attributed to the increase in their confidence in teaching these as they discussed in detail considering many alternatives. Another explanation would be that they want to challenge themselves with the problems that graduates of the same program had problems with. In an effort to maintain self-esteem - to believe that "I'm not ordinary" - individuals may focus on ways in which they differ from their peers (Weinstein, 1988).

Classroom management and organization

Issues related to managing and organizing the classroom, especially discipline problems were one of the most problematic ones in the early career of teachers (e.g. Veenman, 1984; Haser, 2009; Akın, Yildirim & Goodwin, 2016). In *Assertion 1*, possible reasons were discussed and one of them was the participants' unawareness or having chosen to disregard these problems prior to their career. Almost all of the case scenarios in CBDM involve several instances of failures in classroom management and organization. Therefore, participants had chance to reflect on these during discussions and written tasks as well. Awareness of possible challenges, discussing the reasons and links, becoming aware of the consequences of teachers' decisions seemed to help participants a more realistic stance compared to graduates who experienced reality shock due to the challenges they experienced in managing students. Like it happened in other dimensions, participants had chance to recall what they covered in their coursework regarding classroom management and gave suggestions or examples under the given realistic circumstances. Reflection for their possible orientations for managing the classroom in their future career was revealed. Another important implication of CBDM was the raised awareness of the participants on management problems does not occur in isolation, which is a common belief by pre-service teachers (Ayers, 2001). Oversimplification of classroom management to applying set of techniques is conflicting with the ill-structured nature of the problems occurred in the classroom (Choi & Lee, 2009). Thus, it is important to discuss the issues of classroom management with pre-service teachers by situated tasks. Case-based pedagogy is found to be effective in terms of providing ill-structured problems solving opportunities for pre-service teachers in terms of classroom management (Ching, 2011; Choi & Lee, 2009; Merseth & Lacey, 1993; Wood & Nahmias, 2005).

One of the most prominent influences of CBDM on pre-service mathematics teachers is their increased awareness of the importance of lesson preparation process of a mathematics teacher on classroom management and organization. Planning should be explicit and detailed especially in the early career. However, beginning teachers who saw that the colleagues (mostly experienced) do not have detailed plans for their classes, skip the work of detail planning. Lack of planning, interacting with many other factors, complexity of teaching began to have a devastating effect on them leaving them with a feeling of “I can not survive in teaching”.

The importance of designing a lesson requires anticipating how students would respond to a task or an activity, which questions they would ask further, what to do if the plan does not work and doing all these while considering varying needs of students on creating meaningful learning environments were elaborated in the discussions. These conceptions related to classroom management were more of a continuous decision making and taking action process instead of focusing only on dealing with misbehavior. This view was found consistent with the definition of classroom management of Evertson and Harris (1999) as a “holistic descriptor of teachers’ actions orchestrating all that teachers do to encourage learning in their classrooms” (p. 121).

Context

One of the aims of using case-based pedagogy was the nature of cases reflecting the contextual realities of teaching. As it was discussed in *Assertion 3*, beginning teachers who faced with the contextual realities in their early career, tend to source the reasons for the challenges they experienced to contextual reasons. The reformist beliefs or the practices valued during training process would conflict with the resultant actions as a teacher due to intervening influence of external factors on the practices such as testing concerns, parents’ expectations or lack of time (Chaman, 2015).

Therefore, becoming aware of contextual factors prior to entry to the profession would help them to reconsider their goals, beliefs and attitudes towards teaching mathematics. Participants discussed many issues related to contextual challenges like relationships with other stakeholders, overwhelming workload, the realities of Turkish education system and the school climate. CBDM offered participants to discuss all and more of the contextual issues that were revealed in stage 1. Case-based pedagogy was found to be effective in terms of raising awareness about contextual realities of teaching (Çelik, Çevik & Haşlaman, 2012).

The contextual unawareness of pre-service teachers must be a concern for teacher-educators. Field experience could not be the only solution for raising contextual awareness. Karahasan (2008) worked with pre-service secondary mathematics teachers who were enrolled in the program that this current study held in. The study was about pedagogical content knowledge of three pre-service teachers on functions. One of the results were a lack of awareness of contextual factors and integrating knowledge of context to their teaching were revealed. In addition to that, in her research on middle school mathematics teachers' early career challenges in national context, Haser (2009) emphasized that contextual factors would be major source of pressure for beginning teachers and consequently, she advised that teacher education programs should provide opportunities to communicate the complexities, difficulties, and advantages of the contexts in which pre-service teachers will teach. It could be asserted that CBDM provided an opportunity for pre-service mathematics teachers to discuss contextual issues and link these issues to different dimensions of teaching mathematics. Other case-based pedagogy implementations in mathematics teacher education were more focused on the mathematical dimensions where contextual factors like relationship with other stakeholders or realities that Turkish education system brings were missing.

It was also revealed that pre-service teachers differed in terms of their attention to context. Having no concerns related to context would be undesirable for a holistic understanding of teaching mathematics. On the contrary, paying too much attention to contextual factors and seeing them as constraints for effective teaching and learning would point an external locus of control, which would inhibit the teacher from taking considerable action for good practice. It was revealed that pre-service teachers' characteristics became apparent in terms of their attention to contextual issues. This is one of the strengths of the CBDM experiences as teacher educators would find opportunity to assess the pre-service teachers' pre-conceived beliefs about themselves and teaching under the reality of the teaching (Melnick & Meister, 2008). One of the suggestions to strengthen theory-practice connection was integrating classroom reality to teacher education courses (Chong, Low & Goh, 2011). Participants in stage 2 discussed accountability issues to many stakeholders. This was important in terms of raising awareness since their schema about teaching was incomplete in terms of answerability to others (Sinnemena, Meyer & Atiken, 2017).

Assessment of students' learning

The results revealed that participants discussed several issues regarding assessing students' learning. The challenges that were discussed in *Assertion 1* were brought to the table by CBDM discussions. Especially, participants developed an awareness about keeping a systematic record of students' work and progress. Seeing the consequences in the case scenarios motivated them to think of possible ways of assessing students learning.

When the participants' solutions for the challenges occurred in assessing students were examined, it was revealed that they proposed multiple strategies of assessment with an approach of assessment for learning, not assessment of learning. This was a positive and promising finding in terms of participants' connecting their reflections to their coursework. Formative assessment is one of the aspects of teaching that even experienced teachers lack knowledge and skills, resulting in practices mainly based on the experiences gained through their own schooling or adopted from their colleagues (Ayas, Aydın, & Çorlu, 2013). Therefore, CBDM experience become valuable in terms of discussing several issues related to formative assessment as participants connected their reflections both to their personal experiences and to the theory.

The results shared in *assessing students' learning* in stage 2 also revealed participants' assessment preferences. Discussing the pros and cons of the orientations related to assessing students in a community of practice where the members had varying preferences about assessing students helped participants to discuss the advantages and disadvantages of assessment types. As Siegel and Wisseher (2017) asserted "Understanding the advantages and disadvantages of assessment types is a critical skill for teachers to develop" (p. 375).

Students' reactions to the tasks in the case scenarios, especially in CS 3, attracted the attention of pre-service teachers. One of the suggestions was to reduce the difficulty, decide not to grade or stop doing it. These could be explained by the effect of socio-cultural norms. In other words, students' behaviors which is avoiding tasks involving higher-order thinking skills, determine the teachers' decisions in mathematics teaching (Doyle, 1986).

Teacher identity

In the results of stage 2, pre-service teachers' reflections on teacher identity were shared. It was revealed that CBDM discussions triggered participants' negotiations of their own identity. Increased awareness on the importance of emotions, reasons for choosing teaching as a career, and the role of prior experiences as a student on the formations of "whom they will be as a teacher?" was observed. Beliefs of pre-service teachers about teaching and learning and self as teacher is important since the formation of their identities determine their meaning-making and affect their decisions. Teacher education programs were advised to acknowledge the role the evolving identities of pre-service teachers and employ practices that would possibly help pre-service teachers develop their identities (Bullough, 1997; Hong, 2010). Therefore, CBDM could be a viable tool for offering pre-service teachers to negotiate, shift and /or become aware of their professional identities in a safe environment. It was also an opportunity for teacher educators to observe and to interfere with this dynamic process of formation of teacher identity (Akkerman & Meijer, 2011).

Communities of practice

Assertion 7: CBDM served for building communities of practice in mathematics teacher education.

The profile of participants showed similarities and differences in terms of their reflections on learning and teaching mathematics. At the beginning of the study, group dynamics were observed by the researcher, and in the first interview, participants shared their concerns about coming together and discuss issues of mathematics teaching. The researcher was ready for possible arguments that would arise and tried to give equal chance for expressing different ideas. Contrary to the expectations, participants found group discussions as one of the most effective parts of CBDM. Beyond their perceptions, sharing of different ideas, finding consequences for each others' suggested solutions helped them learn in communities of practice by providing different perspectives. In this group, diversity which arose tensions, disagreements and conflicts served productive relationships (Wenger, 1998)

Building communities of practice and positive experiences of these practices would help pre-service teachers' collaborative practices in the future. For example, lesson study originated as a Japanese professional development model, is a collaborative work that teachers formulate goals, accordingly planning a research lesson, collecting data during research lesson, reflect on the various dimensions of the lesson and iterate the process (Lewis, Perry, & Murata, 2006). It has the potential to advance novice teachers' skills and expand their repertoire of teaching via the collaboration between experienced and novice teachers (Fernandez, Cannon, & Chokshi, 2003; Moir & Gless, 2001).

Productive reflection

Assertion 8: CBDM served for engaging pre-service mathematics teachers in productive reflection. Complexity and connectedness of the reflections differed among participants.

Assertion 8 is claimed based on an overall evaluation of the results shared in stage 2. Under this assertion, the research question, “To what extent are pre-service mathematics teachers’ reflections productive?” is discussed.

Considering multiple aspects of teaching together with focusing on various actors, integrating these, linking the views to personal experience or educational community, was considered as the signs of productive reflection (Davis, 2006; Moore-Russo, Wilsey, 2014). Two important dimensions of productive reflections were connectedness and complexity of the reflections. Connectedness was related to the extent that the person linking the opinions to personal experiences or accepted theories and principles of teaching and learning by educational community. Complexity, on the other hand, was associated with considering the multiple dimensions of teaching during elaboration of the classroom events.

The current study used a two-dimensional reflection framework for designing and analyzing reflective practices for pre-service mathematics teachers. The dimensions include the stages of problem-solving (Harrington, 1995) and the level of connections of reflections (Moore-Russo, Wilsey, 2014). The results revealed that the majority of the community connected reflections took place during offering solutions and/or consequences of these solutions. Personal connections were mainly seen during issue identification and offering solutions and consequences. One of the reasons for this situation could be due to lack of experience of pre-service teachers. Therefore, while offering solutions to the challenges occurred, since they could not rely on their teaching experiences, they tended to recall their theoretical knowledge. On the other hand, during issue identification, it could be expected that they linked the issues to personal experiences, an occasion they observed, they experienced either as a student or a pre-service teacher.

This study has two important implications in terms of the noticed issues, the variety and complexity of the issues that participants noticed as a result of CBDM experience, and pre-service teachers' attention and noticing characteristics became apparent for the teacher educator (Davis, 2006).

One of the aims of reflective practice on cases is framing and reframing the situation. Although one of the initial framings of participants to some of the cases (for example CS 2) was theory-practice gap, as the discussions proceed, they began to reframe it as lack of sufficient and adequate planning and preparation for the lesson. Loughran (2002) asserted that this framing and reframing process of the problematic situation is an important sign of effective reflective practice.

The important role of theory could not be perceived by the pre-service teachers unless the theory was presented in a way that would somehow address their personal concerns as a future teacher or they have some concrete problems (Korthagen, 2017). The results revealed that participants had a chance to discuss their concerns and actually become aware of new areas that need development with the triggering effect of concrete problems faced in actual teaching. Therefore, it could be asserted that CBDM provided a realistic teacher education experience for the participants. It was also a step taken for avoiding perceived theory-practice gap in pre-service teachers' minds. Other studies which utilized case-based pedagogy as a practice for building a bridge between the knowledge base the teacher education courses provided and the realities of teaching practice (Choi & Lee, 2009; Koç, Peker & Osmanoğlu, 2009).

Providing an opportunity for discussing the issues in CBDM in a community of practice and individual written reflection as a post-discussion task revealed that these two were complementary media for reflection. Providing different modes of communications for reflection (oral and written) was found to be important in terms of helping pre-service teachers express themselves in the way that they were more comfortable and revealed that the characteristics of their reflections differed in terms of mode of communication. Another research focusing on understanding pre-service teachers' reflections and use the same tool for analyzing the written and oral reflections, found similar findings to this study (Lee, 2005).

Dilemmas of teaching could not be approached as problems that can be resolved totally. In case discussions, pre-service teachers thought of many solutions for the problems raised but they were left with many others for further inquiry. Therefore, it was not a one-shot process that happens in two hours of in-class discussion. Writing tasks was also helpful to invade the inquiry for a longer period and the results revealed that the effects of discussions proceed as the participants articulated that they continued to discuss the issues among themselves after the discussions ended.

This could be another advantage of using mathematics case discussion for helping pre-service teachers' growth (Barnett & Friedman, 1997).

From a time-frame perspective, participants' reflections on the case-scenarios were mainly reflection on action. However, during case discussions, the participants reflected from time to time what type of teacher they will be in the future. They tried to find thumb rules for themselves, what they will do in specific situations. CBDM provided opportunities to think about future selves as teachers. There were many instances that the participants had future-oriented reflections such as expressing verbally or written the "do's" and "don'ts" for themselves in their future careers. Thus, it could be asserted that case-based discussions engaged participants in reflection-for-action as Wilson (2008) suggested analyzing and reflecting on the experiences of others performing an activity that one will perform in the future would be platform for reflection-for-action. Participants engaged in future-oriented reflection passing through the phases of reflection that Moon (1980) put forward: Develop awareness of the nature of current practice, clarify the new learning and how it related to the current understanding, integrate new learning and current practice, anticipate or imagine the nature of the improved practice (Moon, 1980, p. 180). Participants developed an awareness of the complex nature of teaching mathematics by reflecting on in-service teachers' challenges. Under the circumstances of the newly gained insights about teaching, they felt the tension of new conflict with the pre-existing. Therefore, they began to look for ways not to face similar challenges in the future and they thought of possible actions that would be thought of an improved practice of the teachers that they have reflected on.

Another implication of the CBDM was revealed during teaching practice period. In addition to reflection on action and reflection for action, there were also signs of reflection in action. Participants realized that they have been experiencing a problem similar to the ones they were encountered in the case scenarios at the moment of teaching. On the spot decisions at those instances were attributed to recalling the discussions in the cases. Reflection in action was perceived as one of the most challenging dimensions and puzzling phenomena regarding reflective practice (van Manen, 1995). As van Manen (1995) emphasized "It is true that at times, when there is a lull in the activity of teaching and when the teacher can momentarily stop from participating, hang back, or step away from the classroom situation in order to reflect on what needs to be done next, one can speak of reflection in a fuller sense of the term" (p.2), the significant influence of CBDM on participants reflective practice and especially seeing signs of reflection in action became apparent. Moreover, "I became a case" was one of the frequently expressed sentences in the last interviews with the participants. Together with reflection in action experiences due to CBDM experience, this statement could be interpreted as a sign for participants building a case knowledge that Shulman (1992) pointed in his foundational work for case-based pedagogy in teacher education.

Assertion 9: Participants' professional identities became apparent in their reflections as their tacit beliefs, concerns and expectations came into play by the tensions occurred in CBDM discussions.

Assertion 9 is claimed based on the overall results of stage 2 related, and under this assertion, the research question “How did pre-service mathematics teachers’ identity associate with their reflections in CBDM process?”

One of the reasons why cases were chosen to reflect the challenges and problems was to evoke a cognitive disequilibrium in pre-service teachers’ minds. The peripheral beliefs were shaken and the core beliefs came to play. During CBDM discussions, they were challenged with many issues in teaching which happen simultaneously, and they have faced the reality of teaching. As expected, the tacit knowledge about the cognitive schema of each pre-service teacher about teaching mathematics had a chance to be unfolded in CBDM experience. This was one of the intentions of designing the experience and the results revealed supporting results in many dimensions. This was supported by other researches as well (e.g. Gravett, Odendaal-Kroon & Merseth, 2017).

The first attempt was the idea that the theory does not work in practice. One of the reasons for this could be the participants' newly gained skills and adopted beliefs from the program. When they faced with the realities of teaching, the first attempt was to either stick to what they have adopted recently or to question what was not working there.

It is also important that they had this CBDM experience before their teaching practice in which they have spent most almost all week at schools like a real teacher. Mentoring and the experiences there had a tremendous effect on their beliefs, knowledge and practices. Without a doubt, they also questioned their existing teacher identity.

Similar to findings in graduates' early career problems analyzed in stage 1, participants in stage 2 have also shown signs of the effect of their schooling. Memories related with their teachers came into the scene in many discussions. It was important to see how they relate these to personal memories since core beliefs came into the scene. It is a valuable opportunity for teacher educators who wants to help future teachers' development. Pre-service teachers are not empty vessels that one can deliver them all theory and shape their whole teacher construct. It should be acknowledged that preservice teachers start training programs with a teaching schema and teacher identity. Current research revealed that preservice teachers differ in their attentions in the same situation and that what one notices is influenced by one's prior experiences, knowledge and beliefs (Olson et al., 1996). Research on teacher cognition also shows that teachers' knowledge and beliefs influence what they determine as important to attend to in complex situations (Schoenfeld, 1998). For example, teachers of mathematics will more accurately reason about a classroom interaction from a mathematics classroom than they will from a literature or science classroom.

Assertion 10: Participants perceived CBDM experience as a relevant, engaging and awareness increasing practice that had positive reflections on their teaching.

The way that CBDM designed was explicitly explained in Chapter 3, and it was aforementioned that using graduates' experiences of a teacher education program as an inspiration for the cases was mentioned. It was revealed that participants found cases very likely to happen. It was important for two dimensions. First of all, relevancy of case scenarios helped the participants identifying themselves with the case teachers.

Secondly, participants strongly emphasized that all case scenarios were very likely to happen. The content of the cases was relevant to their current and future practices. The complexity of the cases was also revealed by the variety of issues in multiple dimensions of teaching during the CBDM experience. Based on these, it could be said that CBDM satisfied the five essential elements of cases: relevant, realistic, challenging, engaging, and instructional. (Kim et al., 2006).

Researchers' reflection as the facilitator of the case discussions

The role of the case facilitator was also very important for productive discussions. As a novice case facilitator, the researcher felt the tension of controlling the group and directing them to more productive conversations. The process of facilitating the discussions was also a learning and self-awareness journey for the researcher. In the first case discussion, the researcher tried not to make any comment and tried to be completely neutral. However, as the process continued, it was revealed that the group dynamics and the needs at the moment created its own trajectories of interactions. After reading the cases, the immediate reaction of the participants such as "theory does not work in practice" was a source of disappointment in the researcher as the intention of the process was vice versa. However, the researcher then acknowledged the importance of trusting the process of voicing what the participants believe wright or wrong, intended or unintended, was very valuable. This was the opportunity itself to ask deeper questions to dig into the source of their beliefs. It was the only way that would have potential to create a change. Similar tensions were faced by other case facilitators and articulated this tension as balancing the autonomy of a learning community and allowing them to discuss freely, which has no focus and little opportunity to lead a productive discussion (Barnett & Friedman, 1997).

Implications for practice

There are several implications of this research for various stakeholders such as teacher educators, policymakers, teachers and administrators.

- First of all, it should be understood by the teachers themselves that the teaching profession is not being a technician. Pre-service teachers should understand that teaching is complex, multidimensional, requires spontaneous problem solving and continuous learning. Especially in countries like Turkey in which teaching is a low-status profession in terms of perceived cognitive demand, teachers themselves need to protect the reputation of their profession. Therefore, pre-service teachers should be aware of the complexity of the profession and take action accordingly during their training. CBDM stood for a promising material for the use of raising awareness of pre-service secondary mathematics teachers about the complexity of teaching mathematics.
- One of the prominent features of CBDM is being a subject-specific material for the process of *learning to teach*. These types of case-based approaches in teacher education would be part of a course or courses and this practice would be enriched with lesson plans, enacting lesson and reading tasks.

- In addition to these, pre-service teachers would decide to make action research during or after their teaching practices according their areas of concern revealed during case-based discussions. This process would enhance their growth as teachers as researches. This would be an important step to reach theory-practice nexus in the schools.
- CBDM would be used with novice mathematics teachers during their induction period. Communities of practice that involve novice and expert teachers would be formed to discuss the case scenarios. This process would help novice teachers to overcome possible challenges more easily during their early career when zone of proximal development as a learning theory considered (Vygotsky, 1978).
- Similar modules could be created in different fields with the cooperative work of in-service teachers and teacher educators.
- Case scenarios related to middle school mathematics classrooms would be developed. Cases which have a geometrical content at the core would also be designed.
- Cases which have a more assessment and evaluation focus would be produced and it would be used for pre-service and in-service teachers' development.
- Private schools would use CBDM in their interviews of recruiting mathematics teachers due to its potential to reveal the knowledge of the teacher of various dimensions of teaching together with the beliefs and attitudes. In this respect, it would be a relevant tool for recruiting teachers suitable for schools' vision and missions.

- CBDM or similar case-based materials would be used for the development of novice teacher educators. Discussing the content and use of CBDM or similar case materials with novice teacher educators would help them incorporate more theory-practice balanced and holistic practices with the pre-service teachers with cognizance of their prevalent teacher identity.

Implications for further research

Current study focused on the process of design, development and implementation of a case-based reflective module. The following could be investigated in further research:

- The method of designing and implementing case-based materials in this study would be replicated by other teacher education programs, and results of implementing such practices would add more knowledge to the field in terms of producing and developing case-based materials.
- This study did not focus on CBDM participants' early career experiences. Similar studies would be expanded with follow-up researches in the early career of pre-service teachers who participated in case-based pedagogy experiences.
- Current study made use of written cases. Further studies with similar intentions might make use of other case presentation styles like videos or animations. Advantages and disadvantages of using different structures would be revealed.

Limitations

In this research, the identified limitations are the low generalizability of the findings due to qualitative nature of the study. The aim of this dissertation was not producing a case-based teaching material that could be used for the growth of all secondary mathematics pre-service teachers in Turkey. Although there would be many relevant parts as the challenges shared in the case scenarios coinciding with the novice mathematics teachers not only in Turkey but also abroad, CBDM is designed specifically for the needs of a particular teacher education program. This dissertation is more of an offer for all teacher education programs to design/develop case-based discussion materials by taking the concerns and needs of their own pre-service teachers into account.

Due to the nature of the naturalistic paradigm, the data was filtered from the lens of the researcher. This lens is a combination of the researchers' theoretical background about the filed as well as her teacher identity as a graduate of the program as the context of the study. This limitation was tried to be eliminated by sustaining credibility and trustworthiness of the study.

Studying reflection was never easy, and this was also valid for this dissertation. Multiple parameters were directly or indirectly related to ones' reflective practice. Drawing a full and clear picture, without disrupting the integrity of the whole experience, was quite impossible. By using a multidimensional analysis for reflection and elaborating their reflections with a lens of the participants' teacher identity was attempts to understand the CBDM experience of participants more holistically.

The design of the CBDM was inspired by the experiences of the graduates of the program. Although the graduates were working both in middle and secondary schools, the scope of the CBDM is limited to secondary mathematics teaching.

Conclusion

This study sheds light on the experience of designing and implementing a realistic teacher education practice. Case-based pedagogy and productive reflection were utilized as the main approach to engage secondary mathematics pre-service teachers in realistic teacher education practices. The design and implementation were accomplished in two stages: Stage 1, the design of Case-based discussion module and stage 2, the implementation of Case-based discussion module.

In stage 1, the data were collected in order to design a case-based discussion material for pre-service mathematics teachers. To accomplish this aim, ten graduates of a teacher education program were interviewed in order to reveal their early career challenges regarding teaching mathematics. The results revealed several issues that teachers face in four dimensions: mathematical knowledge for teaching, classroom management and organization, assessment students' learning, and the context of teaching. Also, teachers' prior beliefs and expectations showed a general lack of awareness, oversimplified beliefs, and unrealistic optimism about the teaching profession. The findings of this study have shown a deficiency in teacher-education programs in terms of providing pre-service teachers with enough knowledge of the challenges they may encounter, naturally resulting in the inability to reflect on the various reasons for those challenges and how all four aspects of teaching interact with one another. These provided a basis and also justified the need for designing materials for enhancing pre-service teachers' reflections in order to help them to develop a more holistic view of teaching.

A case-based discussion module including six case scenarios, pre-case exercises, discussion plans for the facilitator, and post-discussion written tasks were designed. Theories and researches in literature together with the researchers' experiences added value to the scenarios. One of the prominent features of the case scenarios developed was the attempts made for making them relevant for the pre-service teachers to whom it was going to be presented. For this purpose, pre-service mathematics teachers in the program were interviewed in order to understand their profile as a future teacher, their beliefs and concerns about teaching mathematics. Case scenarios were developed by giving priorities to the issues that concerned participants. Similar beliefs and expectations held among graduates and pre-service teachers participated in the study. Therefore, the teacher profiles in the case scenarios involved these similarities in order to make the material more relevant and engaging for the participants.

The data collected during and after the implementation process revealed several issues shedding light on the experiences of the pre-service mathematics teachers with a case-based discussion module. CBDM provided a platform for participants to reflect on several actors and several issues regarding teaching mathematics. Moreover, CBDM highlighted the complex nature of teaching mathematics in several ways. Participants discussed the complexity of teaching mathematics by elaborating on the issues with a focus on reasons, possible solutions to the problems raised and consequences of solutions.

The results and the discussions lead to the conclusion that pre-service mathematics teachers need the opportunity to engage in realities of teaching in order to have a more connected and complex schema of teaching during their training and CBDM is a viable tool for helping secondary pre-service mathematics teachers to develop an awareness on several realities of teaching. Involving multiple dimensions, giving the opportunity to build links between these dimensions, CBDM offered a teacher education practice that has potential to help pre-service teachers a more complex schema of teaching.

An important contribution was also the opportunity of revealing and discussing what is tacit in pre-service teachers' knowledge, beliefs, and attitudes, in other words, in their evolving teacher identity. This study also showed the similarities and differences of pre-service mathematics teachers in terms of their focus of attention and how it was shaped with their prior experiences. Giving them opportunities to discuss their preconceived beliefs and schema of teaching in the safety of teacher education classrooms, a low-risk setting for novice learning as Schön (1987) named, is important for teacher educators to understand the status quo and would shape the theoretical courses and practicum in the light of revealed beliefs, concerns and expectations.

CBDM offered a window to the pre-service mathematics teachers' past, present and future. Although CBDM is designed for a specific teacher education program, it involves many issues common to secondary mathematics teaching. The use of CBDM or similar modules developed by taking participants' identities into account is offered to teacher education programs which aims to provide the future teachers constructivist, situated and realistic practices.

Researcher's reflection

The time that I spent during this dissertation process was long, tough but full of learning. One of the prominent values of this process was the raised awareness and mindfulness I had as a person, a learner and a teacher. Along the way, I turned out to be a person who asked foundational questions to my self, read more about everything, become more and more interested in psychology and philosophy.

The effects of this process to my teaching were invaluable. After 12 years of teaching, I can say that day by day I become more consistent with my intentions and expectations about my own teaching. All my readings, reflections on the case scenarios, and interacting with the evolving process of pre-service teachers, I began to reflect on my experiences more and more. I observed the changes in my own identity and my professional identity. I realized how my gestalts affected my practices unconsciously. I tried to change this unconscious process into a mindfull ones as much as I can.

My belief about the nature of mathematics changed in a way that altered my practices in the classroom. I used to help students explore what was already discovered. Day by day, I am becoming a teacher who helps them to construct their mathematical knowledge. I began to discuss the nature of mathematical knowledge as well as the role of reasoning, language, intuition in constructing mathematical knowledge. I started to give more information about historical developments of the concepts.

I had also begun to get to know myself as a person, my deficiencies as well as my strengths. I realized that the more you know about something, the more you understand that you know very little. Therefore, the learning process in this dissertation would never end as my inquiry about the issues discussed here still continues.

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APPENDICES

APPENDIX A: Interview Protocol for Graduates of The Program

Informed Consent for Participation in Semi-Structured Interview

Introduction and Purpose

I am Özge Keskin, Curriculum and Instruction PhD candidate at Bilkent University Graduate School of Education. You are invited to participate in research that investigates the challenges that mathematics teachers face during their first year(s) in their careers in order to add knowledge to the field for improving teacher education practices.

Procedures

1. The interview will last approximately 90 minutes.
2. The interview will involve questions about early career problems of teaching. With your permission, I will audiotape and take notes during the interview. The recording is to accurately record the information you provide and will be used for transcription purposes only.
3. If you choose not to be audiotaped, I will take notes instead. If you agree to be audiotaped but feel uncomfortable at any time during the interview, I can turn off the recorder at your request. Or if you don't wish to continue, you can stop the interview at any time.
4. The data gathered from the interviews will be handled as confidentially as possible. If the findings of this study are published or presented, individual names and other personally identifiable information will not be used.
5. Your participation in this study is strictly voluntary. You may withdraw at any time after signing this form.

By checking YES option below, you indicate that you have read and understood the information provided above and have decided to participate.

○ YES (I agree to participate.) ○ NO (I don't agree to participate.)

______________________________ Date

…../…../........

Name & Signature

Semi-structured Questions

Some of the questions that will guide these interviews are:

1. Why did you choose teaching as a profession?

2. What were your expectations about teaching before you have entered the master's program?

3. How did your expectations change, or is it revised during and after the program?

4. What were your expectations about your first year in your career right after you have completed the program?

5. How did it change during your early career?

6. What kind of problems did you face in your early career in teaching?

7. Could you please give specific examples of the problems you have faced? Could you please give details about the context.

- Have you experienced any problems in the classroom due to the lack of your knowledge about your subject area? How did you react when you faced one?

- Could you please share if you have experienced any moment in which you realized that you do not know how to teach a particular topic during instruction?

- Have you struggled integrating technology into your classroom, or any problem related to technology? If yes, could you please share it in detail?

-Have you experienced any problem related to the pace of the lesson?

-Have you experienced any problems in terms of completing the required curriculum (either national or international curriculum)? If you did, how did it affect your classroom climate?

-Have you faced any problem related to assessment and evaluation?

-Have you ever been blamed by your students for being unfair in grading? How did you react?

- Have you used any criteria for giving performance (oral) grades? If yes, what was your performance grade(oral) grade criteria? Did you have any difficulties with being consistent with those criteria? If you have faced any problems regarding this issue, could you please share them in detail?

-What were your classroom management strategies?

-Did you notice a pattern about the times in which you have experienced classroom management problems?

- Were you able to detect the origin of the problem when you faced one?

- Could you please share a though moment about classroom management in detail?

-Have you experienced a problem with colleagues, parents, or administrators? If yes, how did it reflect on your classroom? If it caused a problem, could you please give details.

8. What strategies did you apply to solve the problems you faced during your first year in teaching? Where did you learn these strategies?

9. In which ways has your training at Bilkent University GSE helped you to overcome the obstacles or problems that you have faced in your first year?

10. If any, could you please give examples to problems that you have faced in your first year of teaching, but you thought not to expect before you started your teaching career. (Any reality shock?)

11. To what extent could you reflect on your teaching and make changes accordingly? Could you please explain in detail the way you reflected on your teaching?

12. If you were to start the program again, what would you do differently?

Closure

i. Ask if there are any questions about the interview

ii. Ask if there is something else that he/she would like to add.

iii. Reiteration of the confidentiality aspect of the interview

iv. Thank them for their participation and their time for this study.

APPENDIX B: Interview Protocol for Pre-service Teachers (First Interview)

Informed consent form

This study is conducted by Özge Keskin, who is a PhD candidate at a non-profit foundation university in Ankara. The purpose of this research is to understand your perception of and expectation from the teaching profession as a future mathematics teacher. I believe your input will help us better understand the specific perceptions and expectations of Graduate School of Education students. As a participant of this study, you will be asked to spend an hour of your time to engage in an interview with me. All data acquired from the interviews will be kept confidential. Data will only be accessible to the researchers of this study. A pseudonym will be used during the writing stage and that will not allow any association to your identity.

By checking YES option below, you indicate that you have read and understood the information provided below and have decided to participate.

1. Your participation is strictly voluntary. There is no penalty or loss of benefits if you refuse to participate.
2. The interview will last approximately 30-45 minutes.
3. The interview will involve questions about your expectations of mathematics teaching.
4. With your permission, I will audiotape and take notes during the interview. The recording is to accurately record the information you provide and will be used for transcription purposes only.
5. Audio-records will be transcribed. Audio records and the typed transcription will be kept on the password-protected computer which is only accessible by the researcher.

6. The data gathered from the interviews will be handled confidentially. If the findings of this study are published or presented, individual names and other personally identifiable information will not be used.
7. You may withdraw at any time after signing this form.

○ YES (I agree to participate.)

○ NO (I don't agree to participate.)

____

Date .../.../...   Name &Signature

Contact information:

Interview Protocol

I. Preface

i. The interview will be pre-arranged so participants know in advance when and where they will be interviewed and for how long. Once introductions are made, we proceed with the interview.

ii. I will thank the interviewee for their participation and briefly explain the purpose of the interview. I will also explain that they can stop and ask for clarification of a question at any time. They may choose not to respond to a question, or they can stop the interview at any time.

iii. I will ask the interviewee to sign the Informed Consent Documents.

iv. I will ask permission for the interview to be taped, explaining it will serve as a means of recalling the interview information.

II. Descriptive information

Name:

Gender:

Age:

High School Diploma Received From:

University Degree Diploma Received From:

III. Semi-structured Interview Questions

1. How did you decide to become a mathematics teacher?

- What is your purpose in becoming a teacher of mathematics?
- Considering your purpose in becoming a teacher of mathematics, what are your expectations from your education?

2. Do you have any experience in teaching? If yes, are your past experiences in teaching similar to what you have observed during school experience so far?
3. What are the first things that come to your mind when I asked you to describe a mathematics lesson?

- What are the responsibilities of a mathematics teacher?
- What would be the factors that would affect the effectiveness of a mathematics lesson?
- What would be the factors that would affect the practices of a mathematics teacher?

4. What would be the challenges that are specific to mathematics teaching?
5. What would be the topics or concepts that you would feel anxious if you were asked to teach?
6. What, if any, concerns do you have about being a teacher?

IV. Closure

i. Ask if there are any questions about the interview

ii. Ask if there is something else that he/she would like to add.

iii. Reiteration of the confidentiality aspect of the interview

iv. Thank them for their participation and their time for this study.

APPENDIX C: Case-based Discussion Module

Case Scenario-1

Pre-case Exercise

How would you answer if a 9^{th} grader asks you the following? Please give reasons for your answers.

If A is a non-zero number, what are the following expressions?

a) $\frac{0}{A}$

b) $\frac{A}{0}$

c) $\frac{0}{0}$

Indeterminate

One of the reasons that lead Ayşe to choose the teaching profession was its interdependence. In other words, she thought that when a teacher closes the doors of her classroom, nobody could judge or intervene what she is teaching or how she is teaching. There were other options like being a banker or a software developer in a private company when she has graduated from mathematics department. However, she thought that there would be more accountability issues in these areas compared to teaching profession. Although she was aware that she would have responsibilities to students, colleagues and administrators, she thought that she would be more independent compared to other alternatives.

First year of her teaching career was full of rush. The school did not provide an internship program or a mentor, and she tried to adapt to school life by herself. During internships in the partner schools she was focused more on teaching the unit or the part of the lesson she had prepared, she was not concerned about covering the curriculum in a specific time. Curriculum load and curriculum pace were totally new to her. She was thinking about herself as a teacher who would use technology effectively and enrich her lessons with different activities. However, this was not the case. She realized that the only thing that she can do was stepping in front of the board and just lecturing.

On that day, it was the last period. Students were very tired and looking forward to finishing the last one and go home. Ayşe had already taught six periods that day. However, she was totally motivated to teach the last period since she had many things to cover before the exam. All the exams in the school were programmed as general exams. If she did not make it happen, this topic would not be included in the exam. Since all the other teachers in the department had covered, she would feel responsible and sorry for this situation.

Last Period of The Day

Ayşe: We had talked about the domain of functions in our last class. Now I will give you the rules of some functions and you are going to determine the largest possible domain of these functions.

My first example is $f(x) = x + 1$

Ela: All the values could be substituted.

Mehmet: Exactly. Isn't it real numbers?

Ayşe: Right, guys. What if $f(x) = \frac{1}{x-5}$ is given.

Ela: Isn't it again real numbers.

Erdem: Of course it is not Ela, be careful, she said: "what if".

Ayşe: Erdem, you got me. Yes, guys, there is something different here. What is that?

Ela: So we cannot say that our domain is real numbers if we have a function with a fraction?

Ayşe: In a way, yes.

Erdem: What if $f(x) = \frac{x}{3}$ is given. What would be the problem here?

Ayşe: Oh, you are right. I have just thought, there will be a x in the denominator when Ela said a function with a fraction. We will just exclude the value that makes the denominator zero. In our example, we will exclude 5 from real numbers in order to find the domain.

Ela: But why?

Ayşe: Because our expression becomes undefined if we substitute 5.

Ela: (silently to Merve who is sitting next to her) Why it is undefined, I could not remember.

Merve: (Silently) Since a number divided by zero is undefined.

Ayşe: Girls, why are you talking? Please ask me.

Merve: Teacher, she asks me why it is undefined.

Ayşe: Please ask me then. The reason is that it is undefined to divide a number with zero.

Ela: Ok, but why?

Ayşe: (It was the first time that she have to think about this). How can you define this? How can one divide a number with zero?

Ela: I don't know. I am confused. Whatever, please continue.

Ayşe: Ok, let's talk about it later. What about the domain of $f(x) = \frac{x+2}{x+2}$?

Can: It is a constant function, so all real numbers are its domain.

Erdem: So why did not she gave it like $f(x) = 1$?

Ayşe: Erdem got another good point.

Erdem: Of course I do, you know me, right.

(some of the students started to laugh)

Can: It is ridiculous. You divide one thing to itself; it is one.

Ayşe: Ok, Can. Please substitute -2 to the function and see what happens.

Can: It is $\frac{0}{0}$. Was not that 1? Was it 0? I don't remember.

Ela: This is also undefined then. We divide by zero. So we will exclude -2.

Ayşe: Yes, we will, but not it is not undefined; it is indeterminate.

Ela: Undefined or indeterminate. What is the difference? I don't get it. A moment ago, you said that a number divided by zero is undefined, so?

Ayşe: This is an exception, $\frac{0}{0}$ is indeterminate.

Ela: But why?

Ayşe: Let's think it like this. A number divided by 0 is undefined. 0 divided by a number is 0. So what are we going to call $\frac{0}{0}$, 0 or undefined.

Can: Cool. I liked it.

Ela: I got much more confused.

Ayşe: Ok, guys. Let's just skip this now. I had to finish this topic until the exam. Other classes had already finished with this.

Ela: What is the point that we covered these without understanding.

Ayşe: You are right, but I cannot do something about that. I had a curriculum to cover and a limited time.

Ela: Ok, let's continue.

(Ayşe looked at her notes, and since she ran out of time, she skipped the following examples in her notes; $b) f(x) = \sqrt{x-5}$ $c) f(x) = \frac{2}{\sqrt{x-4}}$ d) $g(x) = \sqrt{2x-7} + \sqrt[3]{x+1}$)

Ayşe: Let's look at this function

$f(x) = \frac{\sqrt{x-4}}{\sqrt[3]{x-5}}$

Erdem: 5 makes denominator zero, and 4 makes numerator 0. So let’s exclude 5 and 4.

Ayşe: Why did you exclude 4?

Erdem: Teacher, didn’t we say that numerator and denominator couldn’t be 0.

Ayşe: Did we? They cannot be 0 at the same time. If you substitute 4, the denominator becomes -1 which is ok.

Erdem: Ok, got it. May I come and solve it on the board.

Ayşe: Ok, Erdem. Come and write it on the board.

(Erdem wrote $R - 5$ on the board.)

Ayşe: There is something missing here.

Erdem: What is missing. I did it correctly.

Ayşe: There are expressions with roots. If you have a square root, it must not be negative. Did you remember? It would be undefined again.

Ela: Everything is undefined. So ridiculous. Why are we learning these?

Ayşe: Ela, please!

Ela: I did not understand a single point from the beginning of the lesson. I really hate mathematics.

(Students were chatting, and level of noise in the class increased)

Ayşe: Hey, guys, be quiet! I will work individually with you after the lesson, Ela. We should arrange some time to study.

Ela: It is ok. I don’t need help from you. I will have a private tutoring this weekend. No problem.

Ayşe: Ok, let’s continue then. I just meant to help you. Anyway let’s look at our example again.

Erdem: The bell will ring in a few moments. We got so tired. Can’t we just finish?

(Whole class started to beg)

Ayşe: No way. I had to finish this example.

Erdem: Could you please just solve it. Guys, let's stop talking, and she finishes the lesson.

Ayşe: A negative value cannot be in a square root. What should it be?

Can: Positive

Ayşe: Good. But it could also be 0, right?

Let's write it like this;

$x - 4 \geq 0 \rightarrow x \geq 4.$ We also cannot include 5. Since the expression would be undefined.

So the domain of the function is $(-\infty, 4] - \{5\}$.

Merve: Teacher, why did not we examine the denominator. Could it be negative?

Ayşe: Yes, Merve, it is root with odd degree. Negative numbers would be ok with that.

Merve: It is too difficult. Too many things to consider.

Ayşe: You will get used to it if you solve examples.

Erdem: I wonder what would be other cases that a function may be undefined.

Ayşe: You will learn that later. It is enough for this level.

Erdem: But I really wonder what would be other things to consider. Can't you give just a few examples, please?

Ayşe: I had to finish this topic before the exam, Erdem. Can we talk about it later?

Erdem: Ok. Seconds left before the bell rings

Ayşe: Ok, then, you may pack up.

Ayşe felt really exhausted as if she had fought against enemies. Why have they stuck on the issue of being undefined or indeterminate? She was behind her plan and she could not make the answer clear in students' minds. Ayşe was teaching only 9th and 10th graders so she was not comfortable with the other topics taught in 11th or 12th grades. She could not think of an example when a function could be undefined when Erdem asked. Were the students aware of her anxiousness in her voice? Why did Ela behave like this? She was very positive at the beginning of the school year. What did change? She could neither define nor determine.

Case Scenario-2

Pre-Case Exercise

Monthy Hall Problem: Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?

Please give mathematical reasoning for your answer.

Construction

İpek realized how concepts related to teaching changed during her preparation years for teaching. According to her, a good teacher was a person who was good at content knowledge and who can transfer it. She had easily learned from these types of teachers. However, last two years have changed her view. She realized that students have different learning styles and they must be active in order to construct the knowledge themselves. According to her, one of the most effective methods which help students to construct their own knowledge was group activity.

As soon as she started her career as a teacher, she planned to use the methods she had learned. She was working in a school that has about 300 students. Although she has tried letting students discuss with the ones sitting next to them about the questions or concepts, she has not done before a planned group activity. However, she was eager to do one since she had experienced its effectiveness during her internship. The theory also suggests that it is a good way to make students active in the classroom. While teaching probability in 10th grades, she had thought that she could make a group activity related to Monthy Hall Problem. Maybe in this way, she would grasp the attention of the students who are less motivated. In conducting group activities, she was aware of the fact that planning part is very important. She has arranged the groups according to students' academic levels. There were 16 students in the class. She arranged four heterogeneous groups according to their mathematical success. While she was planning that, one of her colleagues has come in. His reaction when he heard that İpek was preparing a group activity was very surprising;

Koray: I don't understand why you deal with these kinds of stuff. We can hardly cover the curriculum. Why should one lose time with group activities? I am really annoyed that administrators expect us to do similar things because they see you doing these kinds of activities. Anyway, I don't think that your enthusiasm will continue in the upcoming years. We were all like you in our first few years.

Ipek was really annoyed by Koray's attitude. He was not making any innovations in his classes. He also tries to demotivate others who work hard for their classrooms.

Group Activity

While planning the group activity, she had the students arrange their seats such that four people can see each other.

Bora: Why?

İpek: You will be working in groups today.

Bora: English teacher is also doing this. You have also started, ha?

İpek: Good news. You are doing it in other classes.

İpek had arranged the groups that she had decided before. Some students objected to this arrangement since they do not like some of the students that they are matched with. İpek had consistently warned them to move to their own groups. Although they had murmured complainingly, all of the students moved to their own groups.

Bora was a hyperactive one. He always wants to be on the front burner. He had also influence on other students. He did the distribution of the work in the group. Elif was the one who had the highest mathematics grade in the group. He has assigned the task of solving the question to Elif so that he can comfortably chat with his friends. Bora’s friends who see that Bora was not paying attention to the activity thought that they should also be uninterested like he does.

İpek has assigned the task to the groups;

Task: Suppose you're on a game show, and you're given the choice of three doors: There is a car behind one of them and goats behind the others. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?

Discuss in your groups. One of the group members should express group decisions on the board by giving mathematical reasoning.

Bora: Why don’t you just teach it? Why are we dealing with this?

(Students laughing)

İpek: It is not your business to tell me how to teach.

Bora: Why did you get so angry? What did I say?

İpek: Ok, just stop!

Bora: I did!

Bora was already uninterested in the lesson. After this argument, he did his best to disrupt the activity.

İpek has given 10 minutes for discussion. She was walking around them while they were discussing. First, she had visited Bora's group and asked them what they thought about the question.

Bora: It doesn't matter whether we discuss or not. The car is not going to change its place just because the host opens the door with the goat before.

Elif: Why would one ask a question about this then? It must have a point.

Bora: Ok, Elif, you are free to think about it.

İpek: You will decide within the group. All of you must involve in it.

Bora: Ok.

All discussions were very limited to just saying "I would change" or "I would not change" the door. İpek told the students that they could play the game. Two of the people in the group will be the host and the contestant.

The idea of acting the play was really tempting for the students to accept Emrah, a friend of Bora, who was still trying to be uninterested like Bora does.

Emrah: What if I don't want to find the car, but the goat?

Bora laughed aloud.

İpek: Guys, silence! Just do what you are supposed to do. Emrah, we all want to find the car!

They started to play the game, and in one group, the contestant who did not change his mind has won the prize. They have immediately called İpek.

Eren: Teacher, we did not change the door, but we won.

İpek: Eren, what did we talk at the beginning of the probability class? When we increase the number of trials, we approach to the theoretical probability. Right? Is it ok to decide with just one trial?

Eren: We won't play day and night. Right?

İpek: Of course not. Just try more. It will give you an idea.

When they increase the number of trials, they realized that switching the choice increase the change to win. However, some students were not convinced.

Elif: Is there another way rather than these trials?

İpek: Of course, there is. You can make a tree diagram. Let's do it for both options.

İpek draws a tree diagram that shows the place of the car. (Figure 1)

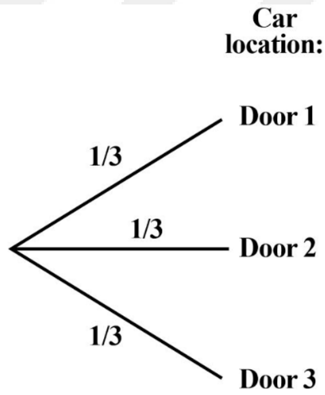


Figure 1. Initial tree diagram

İpek: Assume that the car is behind door 1 and you first choose door 1. Now please put in the tree diagram the probabilities that the host opens the other doors.

Some of the students who are already interested in the lesson, managed to complete the diagram and saw that by switching the door, the probability of winning the car is 2/3. (Figure 2)

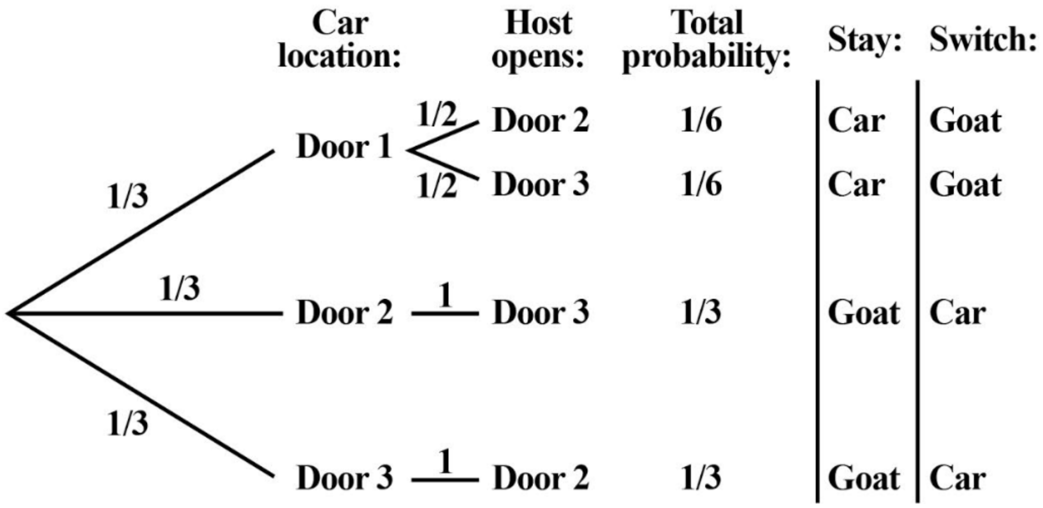


Figure 2. Complete tree diagram

The others were just playing the game without any aim related to the task. Bora was very busy trying to imitate a goat.

By the way, a group of administrators was passing through the corridor where İpek had this class. When they looked at the classroom, they saw that some students were just chatting and laughing and İpek was running from one desk to another desk. The school principal requested one of the vice principals to warn İpek about the noise in the classroom. They thought that this teaching environment is no good for students' learning. According to them, students must be silent and they must be listening to their teacher.

Through the end of the lesson, İpek wanted all groups to share their decisions with the class. Some students were very unwilling to move on the board when İpek call them. One of them has drawn the tree diagram that they have completed. However, there were still some students who think that switching the decisions had no point. The lesson ended before İpek managed to wrap up whole discussions.

At the Vice Principal's office

After the lesson, vice-principal called İpek to his office.

VP: İpek Hocam, can we talk for a couple of minutes? Please have a seat.

İpek: Yes, of course.

VP: İpek hocam, we were in the corridor in the last period. We appreciate your enthusiasm. However, the class was really noisy. Please be careful about your class discipline. Parents are very complainant about this.

İpek: In fact, I was doing a group activity. Students need to discuss it in groups.

VP: Whatever. Please be careful from now on. Thank you.

İpek: I will. Have a nice day.

İpek was dubious. Group activity did not work. The class was really messy. Neither could cover what she has planned nor check if the students could understand what she has tried to give. On top of that, she had a warning from administrators. She thought how certain things, in theory, are falling really far from real life. She would love the ones who support constructivism in her classroom and see what happens when one wants to use group activities in such classrooms. What was the probability that she will do another group activity? It was of course zero.

Case Scenario- 3

Pre-case Exercise

Suppose that you have a bus, and you're renting it to organizations. Your bus can seat 80 people. You decide to charge the first person \$30.00. If that person brings another person, you charge both of them \$29.75 each. If there are three people, you charge \$29.50 each. In other words, you charge EVERYBODY \$0.25 less for every person who joins in. The thing is, there will come the point when you've got enough people to ride that you'll start losing profit if you add more people to the deal.

a) Please find the number of people that you will start losing profit if you add more people to the deal.

b) Find how much you should discount (instead of $0.25 discount) per additional person so that you'll maximize your profit exactly on the 80th person on your bus.

Performance Task

Buket was working in a school, which has more than 2000 high school students. It was her second year of teaching. After two years of preparation for teaching, she was ready to implement alternative methods in her classroom. She did not want to turn out to be the type of teacher she used to have while she was a student herself. She thought that teaching mathematics by connecting it to real-life creates meaningful learning.

She was teaching 9th and 10th graders. That year, MONE changed oral grades to performance grades. Performance grades were supposed to be given according to objective criteria. Performance tasks were a part of those criteria. These were supposed to be in class tasks that are constructed to emphasize the real-life applications of mathematics or interdisciplinary nature of mathematics.

Teachers in departmental meetings continually discuss this issue. No one has a performance task material. Teachers tried to prepare performance tasks together and use them in all classrooms. Besides some teachers had prepared some performance tasks individually and apply these in their classrooms.

Buket had just completed the topic “Factorization” in her 10th grades. She thought it would be appropriate if she gave students a performance task before she had moved to next topic. She had searched on Internet and found the questions which she had liked and decided to give her classrooms.

Question: Suppose that you have a bus, and you're renting it to organizations. Your bus can seat 80 people. You decide to charge the first person $30.00. If that person brings another person, you charge both of them $29.75 each. If there are three people, you charge $29.50 each. In other words, you charge EVERYBODY $0.25 less for every person who joins in. The thing is, there will come the point when you've got enough people to ride that you'll start losing profit if you add more people to the deal.

a) Please find the number of the people that you will start losing profit if you add more people to the deal

b) Find how much you should discount (instead of $0.25 discount) per additional person so that you'll maximize your profit exactly on the 80th person on your bus.

She had thought that this question just fits the topic. She had not solved a similar example in the classroom. She believes that students can solve with the background knowledge that she had provided up to that time. According to her, students should not be given tasks or questions in the exams that are so similar to what they have done in the classroom. Students should comment on what they have learned and synthesize their knowledge.

She had not solved a real-life example while she was teaching factoring. In general, the type of questions was limited to simplifying rational expressions by using identities. Students had also learned how to complete an expression to a perfect square. By using this method, they know how to find the maximum or minimum value of that expression. For example, when $x^2 - 4x + 5$ is given, they can write in the form of $x^2 - 4x + 4 + 1 = (x - 2)^2 + 1$. After this, they can find that minimum value is 1. However, they have never seen a real-life example related to this method.

She had notified students that they would have a performance task in their upcoming lesson. At that class, she had distributed performance tasks for last 30 minutes. She had provided 20 minutes for the task, and shortly after, students started to murmur.

Arda: Have we done something similar to this?

Mert: I have no idea how to do this.

Buket: Guys, silence!

Arda: I have no idea how to do that. We haven't done anything similar in your lessons! (saying aloud)

Other students raised their voices after Arda's comment.

Buket became nervous and angry;

Buket: (shouting) If you do not do your task silently, I will give all of you 0.

Arda: You can't give a grade to us with this question. We have not done anything similar.

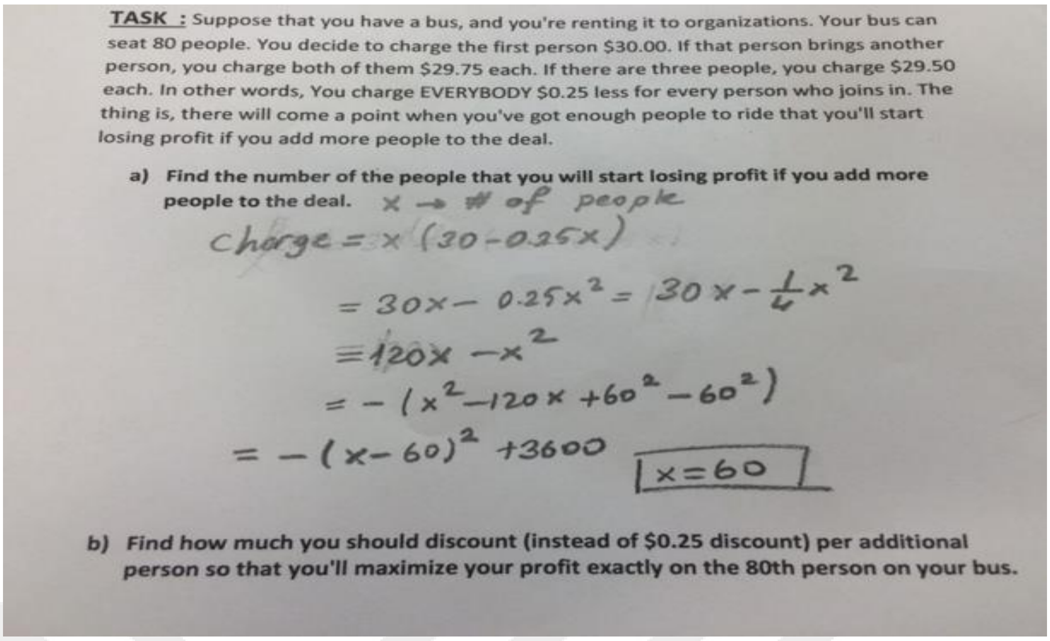
Arda had moved to the teachers' desk and left his paper.

Buket: How dare you! I will not learn grading from you.

Arda sat down murmuring. He was not one of the top students in the class. However, he had really high expectations from his grades. He was not a learning-oriented student. Exams were a source of stress for him.

The classroom was more silent. Buket was wandering around the students and looking at students' papers. Most of them either did not write something or tried to write the equation and could not go beyond.

Some students could only write $30 - 0.25(x - 1)$. Only one of the students managed to do more than what others could do. (Figure 1)



TASK : Suppose that you have a bus, and you're renting it to organizations. Your bus can seat 80 people. You decide to charge the first person \$30.00. If that person brings another person, you charge both of them \$29.75 each. If there are three people, you charge \$29.50 each. In other words, You charge EVERYBODY \$0.25 less for every person who joins in. The thing is, there will come a point when you've got enough people to ride that you'll start losing profit if you add more people to the deal.

a) Find the number of the people that you will start losing profit if you add more people to the deal. $x \rightarrow$ # of people

$$\text{charge} = x(30-0.25x)$$
$$= 30x - 0.25x^2 = 30x - \frac{1}{4}x^2$$
$$= 120x - x^2$$
$$= -(x^2 - 120x + 60^2 - 60^2)$$
$$= -(x-60)^2 + 3600 \quad \boxed{x=60}$$

b) Find how much you should discount (instead of \$0.25 discount) per additional person so that you'll maximize your profit exactly on the 80th person on your bus.

Figure1. A students' performance task paper.

She had extended the time that she had given for the task. At the end of the lesson, students submitted their papers while murmuring complainingly.

Parent Meeting

The next day, Buket had a parent meeting hour. When she had gone the place for parent meeting, Arda's mother was waiting for Buket.

Arda's mother: Hello, Buket Hanım. I am Arda's mother. By the way, you are younger than I had expected.

Buket: Thank you. Please have a seat.

Arda's mother: In fact, I planned to come before. You had done a performance task yesterday; students had really challenged as I heard from Arda. I wanted to talk about that besides Arda's general attitude and performance in the lesson.

Buket: I had given them a performance task yesterday as you heard. However, they should know everything required for completing the task. I expected them to apply what they know into the real-life context.

Arda's mother: Arda said you had not solved a similar one in the classroom before.

Buket: I had provided the knowledge to solve this question. They need to synthesize what they know. They should put the effort into it.

Arda's mother: Is there anyone who can solve it?

Buket: I haven't evaluated it yet. I will inform them later.

Arda's mother: I see. May I ask about Arda's performance in general? What would be his first oral grade? Is he doing his homework on time? We are continually checking him. Arda had to succeed in mathematics. The only lesson that his father gave importance is mathematics.

Buket: In general, Arda had a high potential in mathematics. However, I observe that he had lost motivation these days. He had some missing homework.

Arda's mother: How could it be? Believe me; we always control him. He barely leaves his room since he studies and does his homework. May I learn which is missing?

(Buket had a list for homework controls. However, she had not written the date or topic for each homework that she had checked. It was only a list just full of minuses and pluses.)

Buket: I had not my list with me. I will inform Arda about that later.

Arda's mother: Please. Arda's grades are very important for us. We don't want his grades to drop because of homework. You know how important that grade point average for university entrance. For that reason, I don't understand why teachers challenge students with the type of questions that they had not done in the classroom. We are expecting you to be more tolerant of these.

Buket: I understand you. We are doing our best to help them. However, students should also put some effort and show interest. Arda does not give all his attention to the class. Students are bubbly these days and classrooms can be noisy sometimes. Arda is one of the students who cause this.

Arda's mother: Maybe it is because they find you young. Please punish them, don't mercy!

Buket: I do what I supposed to do. Don't worry.

Arda's mother: If you say so. Thanks for your time. I try to come more often. Otherwise Arda doesn't study.

Buket: Ok. Thank you.

After talking to Arda's mother, Buket got really annoyed. She was questioning what she had experienced in the last two days. Was she really expected students to solve a question that was really over their capabilities? Should she grade this task? Arda's parents had really annoyed her. Will all parents pound at the door whenever their kids challenged in an exam unexpectedly? How can one expect a good performance from Buket after this demotivation?

Case Scenario-4

Pre-case exercise

A statistics professor conducts a study to investigate the relationship between the performance of his students on exams and their anxiety. Ten students from his class are selected for the experiment. Just before taking the final exam, the 10 students are given an anxiety questionnaire. Here are the final exam and anxiety scores for the 10 students:

Anxiety	28	41	35	39	31	42	50	46	45	37
Final Exam	82	58	63	89	92	64	55	70	51	72

Please use your graphing calculator to answer the following questions.

a. Construct a scatter plot of the paired scores. Use anxiety as the x variable.

b. Describe the relationship shown in the graph.

c. Assuming the relationship is linear, compute the value of Pearson correlation coefficient r. Comment on the value that you have found. What can you say about the relationship?

d. Determine the least-squares regression line for predicting the final exam score given the anxiety level.

e. Based on the data of the 10 students, if a student has an anxiety score of 38, what value would you predict for her final exam score?

f. By using the regression line for predicting the final exam score given the anxiety level, what value would you predict for the anxiety level of a student if the students have a final score of 85? (Pagano, 2012, p. 179)

(Pagano, R. R. (2012). *Understanding statistics in the behavioral sciences* (10th ed.). Belmont, CA: Wadsworth Cengage Learning.)

Predictions and Realities

Özgür had a hard time to decide what career path he will follow. However, he became totally sure that teaching was what he really wants. He had done some voluntary teaching and give private tutoring when he was a student. Having good communicative skills with teenagers and being very opposed to a desk job was only a couple of reasons that lead him to choose teaching as a career. He wished his students like mathematics as he does. He believed that if students gave adequate time and effort, they would be successful. He predicted himself as a teacher who can motivate students to study and be successful.

Özgür's first school had two programs, IBDP, and national curriculum. He was very happy since he had chance to have an IB class in his first year. He was supposed to teach 11th grade IB Standard Level Mathematics. It would be a valuable experience for him. There were students who were good enough to accomplish high-level mathematics and they could be easily noticed in the classroom. Özgür got challenged in this class because of individual differences in terms of learning pace. This was a concerning issue for him and he could not find a solution yet.

Although he really loves mathematics, some of the topics were not in his scope of interest. Especially Statistics was one of them. Students also did not like the topic. Besides the national curriculum, Özgür needs to cover "linear regression" in statistics.

In order to get prepared for this lesson, Özgür was studying the book that they have been using in the classroom. At the beginning of the year, he was preparing his lesson from different sources. However, as time passed and responsibilities increased, he began just to get prepared from just one source.

The use of graphing calculators was increased during statistics classes. Özgür was quite comfortable with teaching with graphing calculators since the education he got through during teacher preparation was very intense in terms of education technologies. He was not expecting to have a problem with technology.

Linear Regression Lesson

Özgür had taught correlation and line of best fit in his previous lesson and gave homework related to this. He had reminded students that they have to bring their calculators with them for the next lesson.

When Özgür started the lesson, he wanted to check the homework. However, only four of the students out of 16 had done it. They said they had an important exam for that day as an excuse. Özgür told them that he would give a plus to the ones who had done the homework and provide extra time for the ones who did not. He thought that it would be a bad idea to solve the questions that could not be done in the homework since majority of the class did not even have a chance to read the questions. He wanted students to open “least square regression line” part in their books. Arda, one of the top students at mathematics raised his hand;

Arda: What does regression mean?

Mehmet: Will this topic be in the exam? How many questions will there be from this topic?

Özgür: When we want to see the relationship between two variables, we do regression. Mehmet, of course, this topic will be included in the exam. Listen carefully.

Özgür: Just be a little bit patient, we will be finished with this after regression. In the previous lesson, we had mentioned that we draw a scatter plot to see the relationship between two variables. What could we figure out from scatter plots?

Mine: We could understand the strength of the relationship

Özgür: Good, what else?

Mine: Either it is linear or not?

Özgür: Exactly. I want to hear from other students too. We had mentioned that when we want to explain the relationship between two variables, we could write a line equation and we can use that for prediction purposes.

Arda: Why was it a line equation?

Özgür: Since the relationship is linear.

Arda: Can’t it be something else?

Özgür: It could be. However, we only deal with linear relationships at this level.

The students started to murmur when Arda asks a question. When Özgür heard that one of the student's murmur "He started his nonsense questions";

Özgür: Stop talking, guys!

Although Özgür realized that Arda was not satisfied with his answer, he had not had further knowledge at the moment.

Özgür: After we had graphed the scatter plot, we have drawn the line of best fit, right? However, we did not use all the data to draw that line. Now we have a different method. Least square regression method. Please follow me from your books. This method will provide us the most accurate line equation for prediction. However, it is beyond our scope to see how this line equation gathered. Graphing calculators will give us this equation. After we enter the data, I will teach you how we will find the line equation. Now I want you to read the question on the board and then enter the data to your graphing calculators.

Question

A statistics professor conducts a study to investigate the relationship between the performance of his students on exams and their anxiety. Ten students from his class are selected for the experiment. Just before taking the final exam, the 10 students are given an anxiety questionnaire. Here are the final exam and anxiety scores for the 10 students:

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e. Based on the data of the 10 students, if a student has an anxiety score of 38, what value would you predict for her final exam score?

Some students were just staring at the board; some had already started to deal with their calculators. Özgür realized that some students had not had calculators.

Özgür: Who do not have calculators with them?

Five students raised their hands.

Özgür: Did not I tell you to bring your calculators yesterday**?** The ones who did not have a graphing calculator move next to a person who has.

Mehmet: Hocam, could you please come? I totally forgot how we enter the data. I press "TABLE"; however I could not find where to enter the list.

Özgür: Coming. Do we enter data from "TABLE", Mehmet! Please! Press "STAT" and "EDIT"

Sema: Does it make any difference to enter the lists in an order? Can I enter final scores to "list 1".

Özgür: Please, enter anxiety scores to "L1" and final scores to "L2".

Sema: I did vice versa. Will it make a difference?

Özgür: Ok, leave it like that. We would manage to work with that as well.

Eda: I finished. What is next?

Özgür: Let’s wait for the others to finish. Now, first question asks us to do a scatter plot. You have learned that yesterday.

Eda: with “STAT PLOT”, right?

Özgür: Yes, after drawing, please wait.

Mehmet: I got this (figure 1)

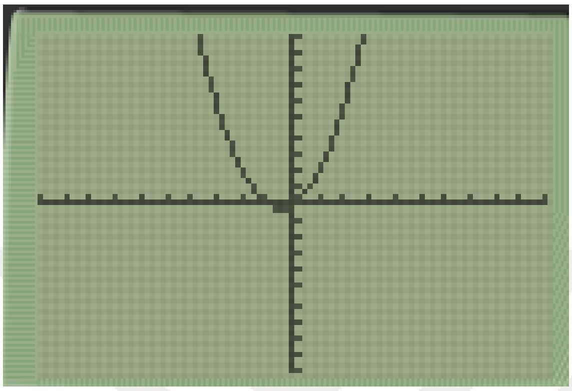


Figure 1. Screenshot from Mehmet’s graphing calculator.

Özgür: Does it look like a scatter plot?

Mehmet: I don’t know, I just press “GRAPH,” and I got this.

Özgür: You did not unhighlight the function you have entered before in the “$y =$” .

Eda: I did, but it still does not show scatter plot.

Özgür: Ok, I will come and check it.

Özgür was losing time for checking each students’ calculators. Although he had mentioned several times, students still make the same mistakes.

Özgür realized that Eda did not press “ZoomStat”.

Özgür: Ok, guys, please first enter stat plot and say on for your first plot and choose scatter plot there. After this, please enter zoom and choose ZoomStat.

After everyone draws it, he projected the correct one to the board. (Figure 2)

Özgür: Ok, what would you say about the relationship?

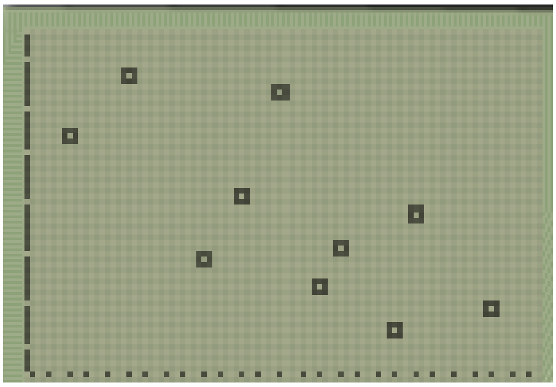


Figure 2. The graph on the board

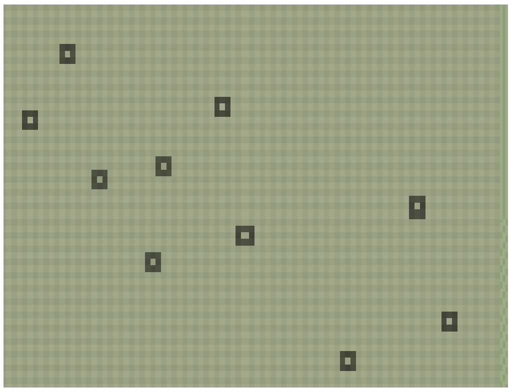


Figure 3. Eda’s screenshot

Arda: It seems linear.

Işıl: I don’t think it is strong.

Eda: Wait a minute. My graph is different. (Figure 4) How will I comment on this? But it seems linear and negative as well. So entering the lists in this way did not make a difference.

Özgür: Let’s see if it makes a difference or not for the next questions. One of you said we could not say that it is strong. Is there way that we can talk about strength rather than looking at the graph?

Arda: We have talked about r value, which is Pearson’s correlation coefficient.

Özgür: Exactly Arda.

Mehmet: Have we finished with the second question. I did not understand. What will I write as an answer?

Özgür: You can write “linear, negative and moderate”. How will we find r?

Eda: From “STAT” and “CALC”

Arda: There are two “linReg” options there, which one will we choose?

Özgür: Let me check (Özgür always used the first “LinReg function” that he had seen in the list and ignore the other one) I don’t think they have a major difference. Will I search for it and say it later?

Işıl: I could not have the r value. I had only these. What is the answer a or b? (Figure 4)

Özgür: Let me see Işıl.

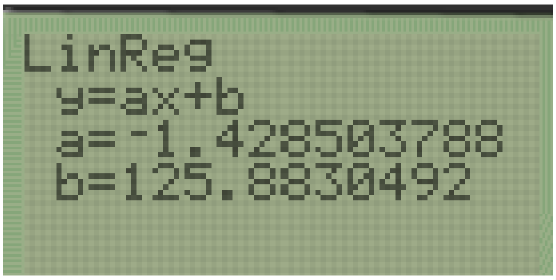


Figure 4. Screenshot from Işıl's graphing calculator

Özgür: (After looking at Işıl's graphing calculator) Remember what I told you yesterday. If you don't do "diagnostics" on, you can not see r.

Işıl: I was absent.

Özgür: Ok Işıl, now please find diagnostics from "Catalog".

Işıl: Will I move to the letter "D" one by one?

Arda: You can press "Alpha" and then "D".

Özgür: Thanks, Arda. Is everyone ok with that? What is r?

Arda: -0.690

Mehmet: Although I did diagnostics on, I can not find r.

Özgür: How could it that be? Let me look at it, Mehmet.

Mehmet: I hate calculators. I do not trust its accuracy.

Özgür looked at Mehmet's graphing calculator. By the way, Arda got really bored since he had already done what he was supposed to do. He was annoyed with this much interruption. He hates being stuck with this kind of easy stuff.

Özgür realized that Mehmet did not press "Enter after" he presses "Diagnostics On".

Özgür: Guys, I told you yesterday, you should press "Enter" after "Diagnostics on"

Mehmet: Ok, thank you.

Özgür realized that Barış was doing nothing but just staring at his friend's calculator.

Özgür: Why don't you pay any attention to the lesson?

Barış: I have a headache.

Özgür: Because I have a headache every day. I cannot do maths. Just leave me. I will leave IBDP anyway.

Özgür: Why do you just give up?

Barış: I studied night and day for our previous exam. You can ask my mother if you don't believe me. But I failed again.

Özgür: Ok

Others were waiting for Özgür to finish his conversation.

Arda: Teacher, can we please finish the last two?

Özgür: Ok, now, we have to find the equation of least square regression line. In fact, when we tried to find r, we had found value a and b. These are the coefficients of your regression equation.

Işıl: It is in the form of $y = ax + b$, right?

Özgür: Exactly, Işıl. We can use this to answer e. When we know the anxiety level of a student which is 38, can you predict final score?

Arda: If we substitute 38 in terms of x in the equation, we can find.

Özgür: That's true. What did you find?

Sema: Approximately 71.

Most of the class found the same thing. However, Eda had a different answer.

Eda: I found approximately 50.

When Özgür looked at Eda's calculator, he recalls that Eda enters the lists in a different order.

Eda: But you said, it is ok to do so.

Özgür: It is, but when you enter LinReg, you should enter the list order, first the list for x and then the list for y.

Eda: I should enter the lists correctly at the beginning. I am totally confused.

Arda: May I ask a question? Assume that I know the final score, and I want to predict the anxiety level. Let's say my final score is 85. If I substitute 85 in terms of y in the equation we found, will I find my anxiety score?

Özgür: You can do, I guess. It is a line equation at the end.

Arda: I guess so.

Arda thought that Eda had found the equation where y is the predictor variable and x is the predicted one. He checked if giving y score 85 in both equations will give the same result. But it did not. Meanwhile the bell rang.

Özgür: Ok, guys. Please make a revision when you go home. Your homework is to do the exercises in this section.

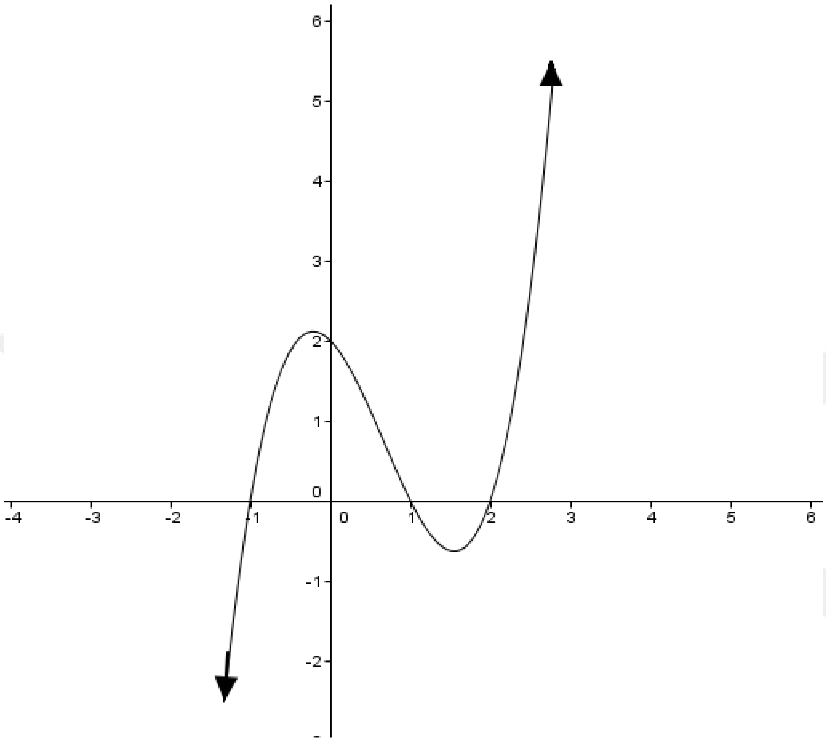
Arda: Teacher, may I ask you a question? I did check both equations to predict anxiety scores from final score. But I got different answers. Which one is correct?

Özgür checked the calculations. Arda was right at his point that results were completely different. But why? Özgür was completely dissatisfied with the lesson. He was surprised not only by the points that students were stuck at but also the students who totally shuts their doors to learn. He felt demotivated. In teaching, predictions do not coincide with realities.

Case Scenario 5

Pre-case exercise

Given the below graph of function f



Please draw the graph of the following functions;

a) $f(x+1)$

b) $f(x)-2$

c) $1-f(x)$

d) $f(-x)$

e) $f(|x|)$

f) $f(-x+1)$

TRANSFORMATION

Ayşe was the first person to be consulted by her high school friends when they have a topic to ask or a question to be answered about mathematics. She was impressed to be a person who seems to be having good mathematical knowledge and to be consulted by others. As a result of the combination of this situation with her love for mathematics, she decided to study mathematics and become a mathematics teacher.

In her first year of the profession, her school had IB Degree Program in addition to national curriculum. It was a private school with around 200 high school students. There was only one another teacher in her category who is able to teach for IB Program and Ayşe was trying to get help from him for the planning regarding the classes. Ayşe did not have a mentor in her first year. With the help of the program she was graduated from, she was teaching for 12. Level IB advanced mathematics even though it was her first year. She was teaching a class composed of 7 students with high motivation to learn and successful academic background.

In the first semester, she planned a pre-exercise class before the first exam. She was to solve some questions regarding the transformation of the functions. She checked the questions from previous years and solved the questions in the related books. She thought of the topics to discuss for each question. Also, every point where her students faced problems during the courses was in her mind.

She suggested the below questions to her students to solve. She thought that this question is well enough to check the knowledge of her students regarding the transformation of functions. By the way, she was to complete a general review before the exam

Question: Given the graph of function f, please draw the graphs for the below functions.

$a)\ f(x+1)$

$b)\ f(x)-2$

$c)\ 1-f(x)$

$d)\ f(-x)$

$e) f(|x|)$

$f)\ f(-x+1)$

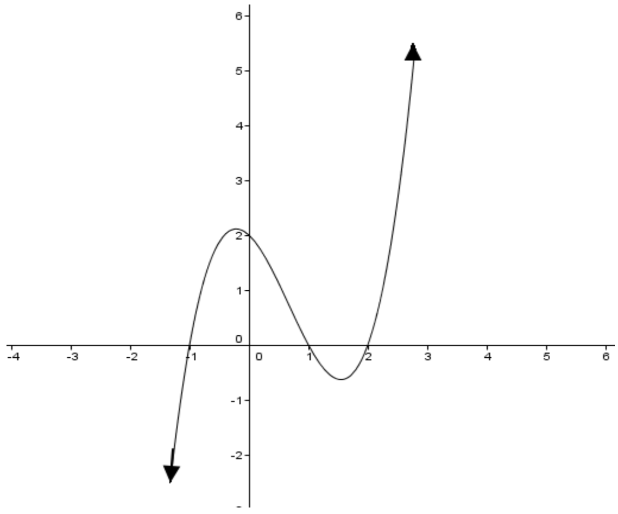


Figure 1. Graph of f(x).

Students solved many similar example questions before. They completed all 5 graphs without hesitation. Hakan was a student who asks the most questions as his learning practice. He was always thinking of alternatives for the given solutions and always asking “why?” if he meets with memorized information. After the solution of the question, Hakan started asking his “what if?” questions.

Hakan: What if we were asked $f(-x+1)$, what would we do?

Students started to discuss what they would do. It was an unexpected question for Ayşe. She checked and solved all the questions in the IB question bank during her preparation for the lesson. There was no such question. She tried to think quickly while the students were discussing loud.

Berk: Anyway, we know how to transform $f(-x)$; there is no problem. For $(-x+1)$, I first find $f(-x)$, I mean, I take symmetry of the graph with respect to y-axis and then I slide the graph 1 unit left since 1 is added to $-x$.

It seemed reasonable, and then İlayda whose mathematical knowledge is reliable, made a comment.

İlayda: I think it will not work. Because, in order to move the graph 1 unit left, I have to create a change for x. But here, we are adding 1 to $-x$. I think we may think as $f(-(x-1))$. Let us first move 1 unit to the left, and then take its symmetry with respect to y-axis.

Ayşe was quite confused. She could not think calmly while listening to the comments of the students. She was also stressed with the idea of being embarrassed in Case Scenario. She could not find the right answer, and this was avoiding her sound thinking. With a rush to give an answer, Ayşe thought the idea İlayda more reasonable and said:

Ayşe: İlayda is right. Let us first move the function one unit right and then take symmetry with respect to y-axis

Since the projector of the class was not working, she should draw by hand. If the projector was working, maybe she would draw by computer and show that. She did not even notice her calculator with graphing due to her panic.

Hakan: I could not understand. I am really confused. Why are we first moving 1 unit right and then symmetry with respect to the y-axis.

Ayşe: Let us check what happened to x in the function. I first subtracted 1 from x, and then I multiply it with minus 1. What do we do in this situation? Hakan?

Hakan: I do not know. Whatever I know is completely mixed. Would you please draw on the board?

Ayşe, first moved the function 1 unit on the board. (figure 2) and then took its symmetry with respect to y-axis (figure 3).

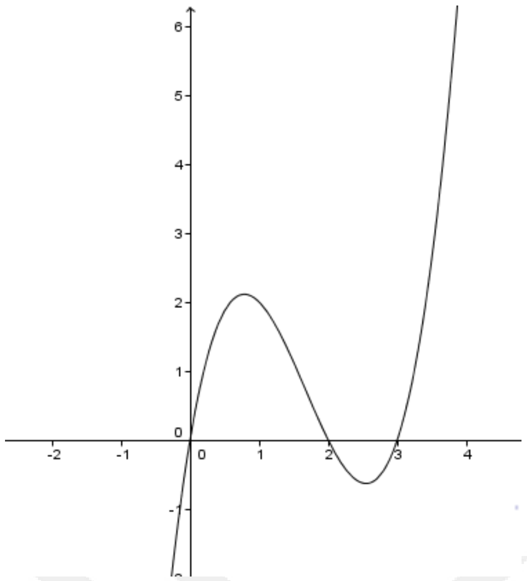


Figure 2. $f(x)$ moved 1 unit to the right

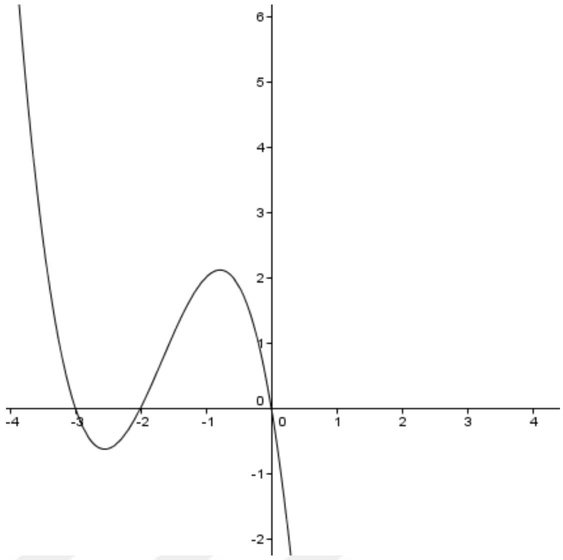


Figure 3. Symmetry of $f(x-1)$with respect to

Hakan: While you are drawing these, I tried to find the rule for the function. Since it is a cubic function, it has 3 roots, and I also know where it intersects the y-axis. I assumed $f(x) = a(x+1)(x-2)(x-1)$. At 0, it resulted 2. I replaced it. Since $(0) = 2a = 2$, it showed $a = 1$. Then my function is

$f(x) = (x+1)(x-2)(x-1)$. Now, I put $1 - x$ instead of x and graph it, I should find, right?

Ayşe: Well, Hakan. Of course you would find with that way, but it will be a loss of time, is not it right?

There were 2-3 minutes before the end of the class. Hakan created the new function; $f(-x+1) = (-x+2)(-x-1)(-x)$ and graphed it using TI (figure 4).

He was quite shocked when he saw the result in hand.

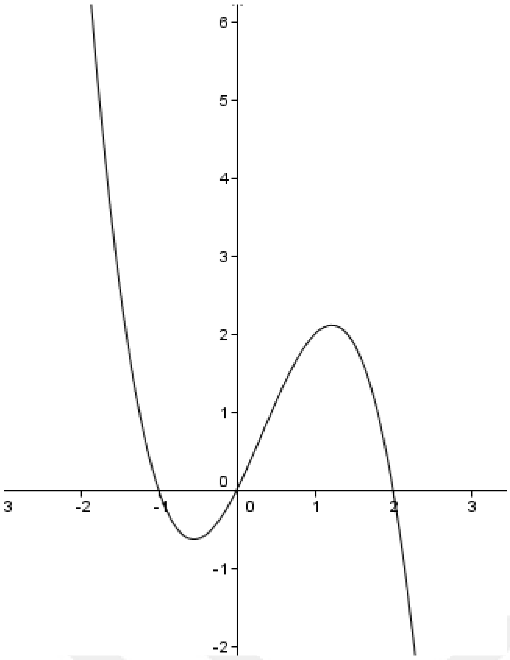


Figure 4: Graph of function $f(-x+1)$

Hakan: Teacher, may you look at this graph? It is not the same with yours, where did I make a mistake?

Ayşe, checked the function of Hakan, it was correct. After that, she checked the function thinking that he may enter it wrong to the calculator, but it was correct as well

Ayşe: There is no problem with your solution Hakan.

A few students from the class started murmuring. Ayşe was really confused. What was her mistake? She could not focus due to her feeling of embarrassment. And then the bell rang

Berk: Teacher, what about the result? Which one is correct?

Ayşe: Result found by Hakan is correct.

İlayda: Teacher, this will not be an exam question, right? As a result, even you solved it wrong.

Ayşe: We will discuss this issue in our next lesson. You should focus on our other exercises.

While leaving the class, Ayşe thought where she made a mistake. Also, last words of the students made her feel annoyed. Where did Ayşe make a mistake?

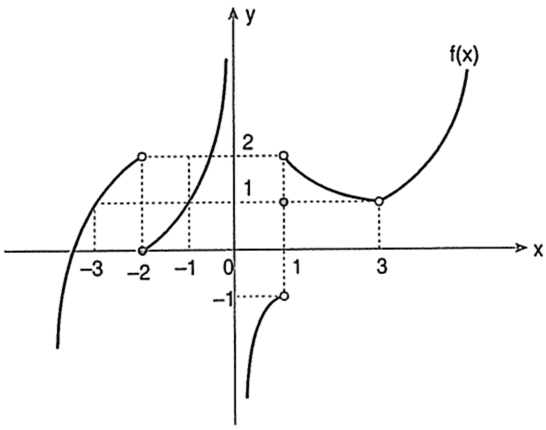
Case Scenario 6

Pre-case exercise:

1) How would you start a Limit introduction lesson?

2) How would you explain $\lim_{x \to a} f(x) = L$ to the students?

3) Please answer the following questions. At which parts would you think that students have a challenge in these questions?



a) $\lim_{x \to -3} f(x) = ?$

b) $\lim_{x \to -2} f(x) = ?$

c) $\lim_{x \to 0} f(x) = ?$

d) $\lim_{x \to 1} f(x) = ?$

e) $\lim_{x \to 3} f(x) = ?$

PUSHING THE LIMIT

Being a teacher was meant to Damla more than transferring mathematics knowledge to others. She chose this profession since she wants to have an effect on students' lives. She wanted to be someone beneficial to society and teaching just fits with her goals. She was a very calm, respectful and successful student when she was in high school. She was expecting student to be like her.

Damla started her teaching career in a private high school with 250 students. She was teaching five different classes. All of her classes involved 15 to 20 students. She tried to set the rules at the beginning of the semester. In all her classes, Damla told students that she expected them to obey the rules, and if one of them got out of line, she would apply the discipline procedure of the school which requires writing warning sheets for students who did not pay attention to verbal warnings.

For her 12th-grade mathematics class, she was given lesson notes from her department. She was quite happy as she didn't need to prepare notes because she was busy with other departmental works, and lesson notes preparation for her other classes. She was teaching the concept of limit for the first time in her life. She was quite comfortable with solving the questions related to limit. She did not feel the need to put much time and effort to get prepared for the lesson.

This class was academically an average one. In the first weeks, students were eager to learn and watching the lesson carefully. Most of them were started to study for university entrance examinations. They had already learned some of the topics that will be covered in this class. They usually followed major class rules and procedures, with most of the inappropriate behavior being limited to off-task socializing or inattentiveness. Student's mathematics performances in the previous years were not good. However, she was happy that they were paying attention to her lesson.

Introducing Limit Concept

Damla was thinking that it would be appropriate to construct students' own knowledge of their previous conceptions and existing knowledge. Thus she asked students what does limit means to them. Some students' expressions were like "speed limit", "credit card limit", "a value that you can not pass".

Damla: In mathematics, we will get closer and closer a x value and see what happens in the values of $f(x)$. We will denote it like $\lim_{x \to a} f(x) = L$.

Let's see it on a graph of a function.

Consider the graph of the function $f(x) = \frac{x^3 - 1}{x - 1}, x \neq 1$

(She drew it on the board)

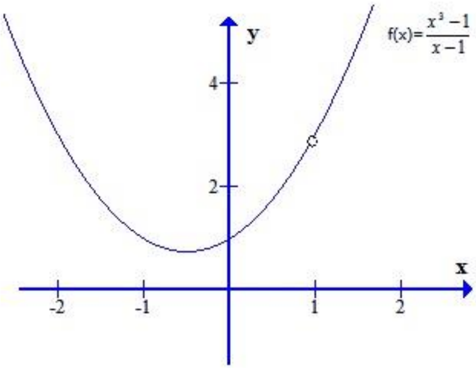


Figure 1: The graph of the function $f(x) = \frac{x^3-1}{x-1}$

Damla: It appears that the graph of f has a gap at the point (1,3). Although x cannot be equal to 1, you can move arbitrarily close to 1;

Let's look at this table

x approaches 1 from the left → ← x approaches 1 from the right

x	0.75	0.9	0.99	0.999	1	1.001	1.01	1.1	1.25
f(x)	2.313	2.710	2.970	2.997	?	3.003	3.030	3.310	3.813

f(x) approaches 3 → ← f(x) approaches 3

as a result, $f(x)$ moves arbitrarily close to 3. Using limit notation, we can write

$\lim_{x \to 1} f(x) = 3$ and we express it like "the limit of $f(x)$ as x approaches 1 is 3".

If $f(x)$ becomes arbitrarily close to a single number L as x approaches a from either side, the ***limit*** of $f(x)$, as x approaches a, is L. This limit is written as $\boxed{\lim_{x \to a} f(x) = L}$

After she had given these, Ezgi raised her hand.

Ezgi: I guess I did not understand. Can you show another example?

Meanwhile, a couple of guys were chatting. Although the noise was not distracting, Damla realized that they were not paying attention to the class.

Damla: Guys, listen! Ok Ezgi. Let’s draw another function.

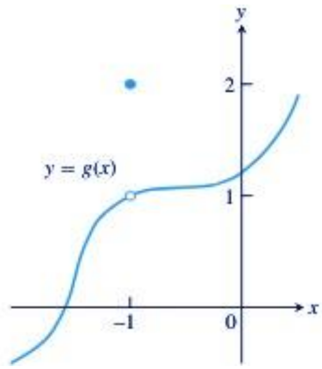


Figure 2: The graph of function $g(x)$

(While she was writing another example and try to teach it to Ezgi, boys at the back of the classroom started to chat again.)

Damla: I approach the value -1 but never be equal to -1, what does the function approach?

(By the way, the noise level built-up)

Ezgi: (turning to her friends) I can not listen to the teacher, please just shut up.

Barış: Ezgi, stop being a nerd. Hocam, we already know these.

Damla: You may know it, but you have to listen. Ezgi, please continue.

Ezgi: I guess it is 1. But from which value that I will start to approach. For example, will I give -3 then -2 and then a closer value to x.

Damla: You will be very close to -1. You don’t need to come from that further point.

Ezgi: Ok, I think I understood.

Damla: Let’s move on then.

Now we should talk about the right and left limits.

(She was writing on the board, and at the same time she was saying out loud what she writes.)

Students did not stop talking. Actually, new students added to the ones who are speaking.

At the end Damla yelled at the students;

Damla: Shut up! (almost like a scream) You will face the consequences if you don't stop talking.

Students were surprised by Damla's tone of speaking. It was the first time that she did this in that class. They immediately stopped the noise.

After being sure that everyone is quiet, she turned back to teaching.

Damla: Ok, let's move on.

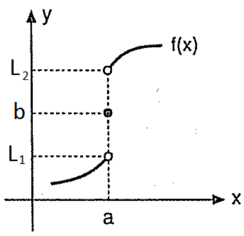
If $f(x)$ approaches L_1 as x tends toward a from the left $(x < a)$, we write

$\lim_{x \to a^-} f(x) = L_1$ (Limit from the left)

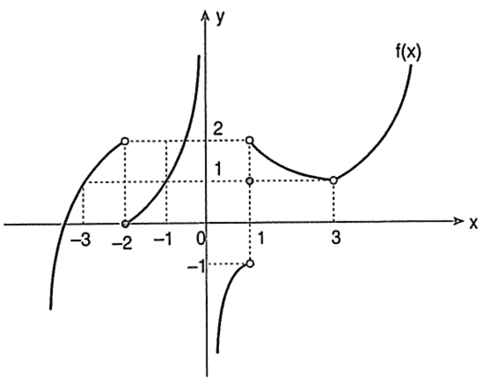
If $f(x)$ approaches L_2 as x tends toward a from the right $(x > a)$, we write

$\lim_{x \to a^+} f(x) = L_2$ (Limit from the right)

Look at this example, let me draw it x approaches to a from right, $f(x)$ approaches to L_2. On the other hand, when x approaches to a from left, $f(x)$ approaches to L_1. L_1 and L_2 are not the same. So we say that "the limit does not exist as x approaches to a".



Damla: Now, let's look at this graph of f and answer the questions.



a) $\lim_{x \to -3} f(x) = ?$
b) $\lim_{x \to -2} f(x) = ?$
c) $\lim_{x \to 0} f(x) = ?$
d) $\lim_{x \to 1} f(x) = ?$
e) $\lim_{x \to 3} f(x) = ?$

Figure 4. A question in students' notes.

Ezgi raised her hand to answer the question.

Ezgi: Only at $x = 3$, the function has a limit.

Damla was surprised by Ezgi's wrong answer since she was one of her best students.

Damla paused and turned to the class, and asked;

Damla: Who agrees with Ezgi's answer?

Barış who was not paying attention to the lesson at all answered the question;

Barış: I don't agree with Damla. Only it also has a limit at $x = -3$.

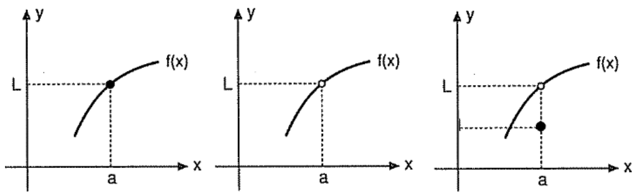
Some of the students agreed with Barış, and they said they were perfectly sure about the answer since they learned it during the private tutoring took for the university entrance exams. Damla approved the answer. Ezgi was surprised by Damla's approval.

Ezgi: How is that possible? I was carefully listening to you from the beginning of the lesson, and you said several times if a function has a limit as x approaches to a point, x come close to a value but never reaches it, so the function must be undefined at that value.

Damla was confused with what Ezgi had said. Ezgi was right in her point that Damla had emphasized the idea of x approaching a value but never become equal to it. While Damla was thinking on how to correct Ezgi's misconception, Barış said, "you see, I can answer correctly without listening to the teacher". The others laughed and approved him. After this, the noise level built up. Although she continually warned students to stop making noise, students did not stop talking.

Damla: Ezgi, I did not mean that the functions should be undefined in order to have a limit.

Let's look at these examples in your notes.



In the first graph, the function is defined at $x = a$ and $f(a) = L$. We look at the limit from the right and left, as you see from both sides we approach to L.

Ezgi: Ok. If you say so.

The lesson continued with Damla's struggle to continue to teach while trying to stop the noise. She was increasing her voice while she was teaching. She was also very anxious about the noise since the classroom was just near the vice-principal's office. When the bell rang, she felt guilty about Ezgi's misconception and she was disappointed regarding the loss of classroom management. This class was really pushed Damla's limit.

APPENDIX D: Discussion Plan Example of CBDM

Date: 16.09.2015 **Location: G160** **Facilitator:** Özge Keskin

Case Scenario 1: "Indeterminate"

Synopsis:

The case occurs during Ayşe's first year of teaching. General exams put pressure on her class activities. Her inadequate responses to "why" questions of students related to indeterminate and undefined expressions had decreased the motivation of some students who had already forced themselves to learn a subject that they hate. Ayşe rushed in order to catch up the curriculum. Some students could not grasp the ideas that were meant to be given. The last hour of the day was also a challenge for Ayşe to motivate students to learn.

Pre-Case Scenario Exercise:

Participants were expected to work on the pre-case exercise in order to be familiar with the mathematical content of the case. The answers will not be discussed.

Issues:

Mathematical Content	Mathematics Knowledge For Teaching	Classroom Management	Assessment	Context
Domain of functions	Procedural knowledge versus Conceptual knowledge ("Why?" questions of students)	Off-task students	Questioning skills	Departmental responsibilities
Undefined and indeterminate expressions	Being unaware of students' misconceptions	Students' problems with settling down at the beginning of the lesson	General exams	General exams
	Links to further topics	Students' language		Last period of the day
	Being aware of students' mistakes related to mathematical language and notation			Curriculum load and/or pace
				Private tutoring
				Students motivation

TIME TABLE FOR DISCUSSION SESSION

<table>
<tr><th>Time</th><th>Participants</th><th>Researcher</th></tr>
<tr><td colspan="3">PRE-DISCUSSION PART
Direction: Please work on the pre-case exercise individually for 5-10 minutes. If you need any further time, please don’t hesitate to ask.</td></tr>
<tr><td>15 minutes</td><td>Working individually on pre-case exercise.</td><td>Walk around the participants and observe their approaches to pre-case exercise.</td></tr>
<tr><td colspan="3">Direction: Would you please turn me the papers that you have worked on.
Now please read the case individually, highlight the sentences or phrases, which you think it is worth for discussing. (15 minutes)
After you have finished reading the case, could you please think for a minute about what this is a case of and then I will write your opinions on the board in order to discuss them.</td></tr>
<tr><td>10 minutes</td><td>Reading the case, highlighting the parts that are problematic, and /or any expression and instances that they think it is important and they want to discuss</td><td>Have a quick look at the participants’ pre-case exercise workings.</td></tr>
<tr><td>1-2 minutes</td><td>What is this a case of? Important Issues in the case?</td><td></td></tr>
<tr><td colspan="3">DISCUSSION PART</td></tr>
<tr><td>30 minutes</td><td colspan="2">Part 1</td></tr>
<tr><td colspan="3">Leading Questions
1) How did you feel?
2) What are the issues in this case? How would you prioritize these issues? What is it a case of?
3) What would be the reasons of the problems that Ayşe faced in this case?
Sub-questions

These will be prompts if the participants do not raise these issues by themselves
1) Did Ayşe provide a correct argument for the reason of being undefined or indeterminate?
2) What are the strategies that the teacher use to understand what students learn or how they learn?
Do you find those strategies effective?
3) Why did students have trouble with undefined and indeterminate expressions? Have you read anything about these, or have you noticed these during school experience or teaching practice?
4) What do you think about classroom management strategies of Ayşe?
5) What do you think about Ayşe’s reason to choose the profession? Is it any related to her concerns related to department?</td></tr>
</table>

<table>
<tr><td colspan="2">6) What are the critical decisions given by Ayşe during the lesson?</td></tr>
<tr><td>30 minutes</td><td>Part 2</td></tr>
<tr><td colspan="2">Leading Questions
How might different perspectives inform the interpretation of the case?
Please choose your role.
You can be a) a colleague in Ayşe’s department
b) a student in this class c) a parent of one the students (maybe Ela’s mother) d) an administrator of Ayşe.</td></tr>
<tr><td>30 minutes</td><td>Part 3</td></tr>
<tr><td colspan="2">Leading Questions
What solutions would you suggest to cope with the problems? What would you do differently?
How did you come up with these solutions?
What would be the consequences of your solution?</td></tr>
<tr><td colspan="2">CLOSURE</td></tr>
<tr><td colspan="2">5 minutes
Leading Question:
Overall what remainders would you have from this discussion session?</td></tr>
</table>

Post-Discussion Written Task

A) Reflective Journal:

1) How did you feel when you were in the shoes of Ayşe? Which situation would annoy you most? Please explain.
2) Would you expect to have similar problems like Ayşe faced in your teaching career? Which strategies you would apply in order to prevent or solve these kinds of problems?
3) Did you notice something new about teaching profession (or teaching mathematics) in general after analysing this case? If yes, please share details about this change.

B) Design Task

If you were Ayşe, how would you answer students’ questions related with undefined and indeterminate expressions?

Post-Discussion Written Task (Turkish Version)

Yansıtıcı Düşünme Raporu

A) Aşağıdaki soruları göz önünde bulundurarak bir yansıtıcı düşünme raporu yazınız. Raporunuzda, tartıştığımız örnek olaydan, grup tartışmamızdan örnekler verebilirsiniz.

1) Ayşe'nin yerinde siz olsaydınız, bu ders size neler hissettirirdi? En çok hangi durum sizi rahatsız ederdi? Nedenleriyle açıklayınız.
2) Öğretmenliğe başladığınızda, örnek olaydakine benzer problemler yaşamayı bekler misiniz? Bu problemleri önlemek ya da çözmek adına nasıl bir strateji izlersiniz?
3) Bu örnek olayın, öğretmenliğe ve spesifik olarak matematik öğretmenliğine ait algı ya da beklentilerinizle ilgili bir etkisi oldu mu? Eğer olduysa, bu etkiye ait detayları paylaşabilir misiniz?

B) Eğer Ayşe'nin yerinde olsaydınız, öğrencilerin tanımsızlık ve belirsizlik ile ilgili sorularını nasıl yanıtlardınız?

APPENDIX E: Semi-Structured Interview Protocol (Last Interview with Pre-service Teachers)

I. Preface

i. The interview will be pre-arranged so participants know in advance when and where they will be interviewed and for how long. Once introductions are made, we proceed with the interview.

ii. I will thank the interviewee for their participation and briefly explain the purpose of the interview. I will also explain that they can stop and ask for clarification of a question at any time. They may choose not to respond to a question, or they can stop the interview at any time.

iv. I will ask permission for the interview to be taped, explaining it will serve as a means of recalling the interview information.

II. Descriptive information

Name:

I. Semi-structured Interview Questions

General Questions

Regarding case discussion sessions

1) What were your expectations from the case discussion sessions before we have started these sessions?
2) How have you felt and thought about this experience? Does it match your expectations?
3) During the sessions, you have found that the cases are realistic. How would you evaluate these cases in terms of them being realistic after your teaching practice?

4) Have you experienced anything similar to the cases during your teaching practice? If yes, could you please give details about these experiences?
5) How did these case discussions affect your views on;
   a) Teaching profession
   b) Teaching mathematics
6) How would these discussions help you in terms of professional competencies?
7) Regarding the case discussions, what do you think about the practicality of theories in education?

Regarding your experiences during teaching practice;

1) Were there specific points that you looked for or wanted to observe during teaching practice? If yes, why did you choose to examine it further?
2) What were the main challenges that you have faced?
3) What kind of strategies did you use to overcome these challenges?
4) Would you please give details about the reflection process upon facing these challenges?

Specific Questions for PT1

1) You have noted "Interesting Points" in the first few observations. Then you gave up. Is there a reason for this?
2) What did you specifically pay attention to while you were planning your lessons? What challenged you the most during the preparations? I have seen that you had some extra parts in terms of differentiated instruction in your lesson plans? What were your incentive to add these?
3) You had a lesson on the Monty Hall Problem. Would you please give details about the planning process, the lesson itself, and your reflection after the lesson?

4) You noted, “I learned that students want to be in class when you behave them respectfully”. Which incident made you learn this?
5) You seemed that you could not cover what you had planned in some of the lessons? What would be the reasons for this?
6) In some lessons, you have faced with conceptual questions. How did you react to these kinds of questions? Had you considered these kinds of possible questions during your preparation?
7) You had a lesson with a graphing calculator. With regard to this, have you experienced any challenges, which impeded your delivery?

IV. Closure

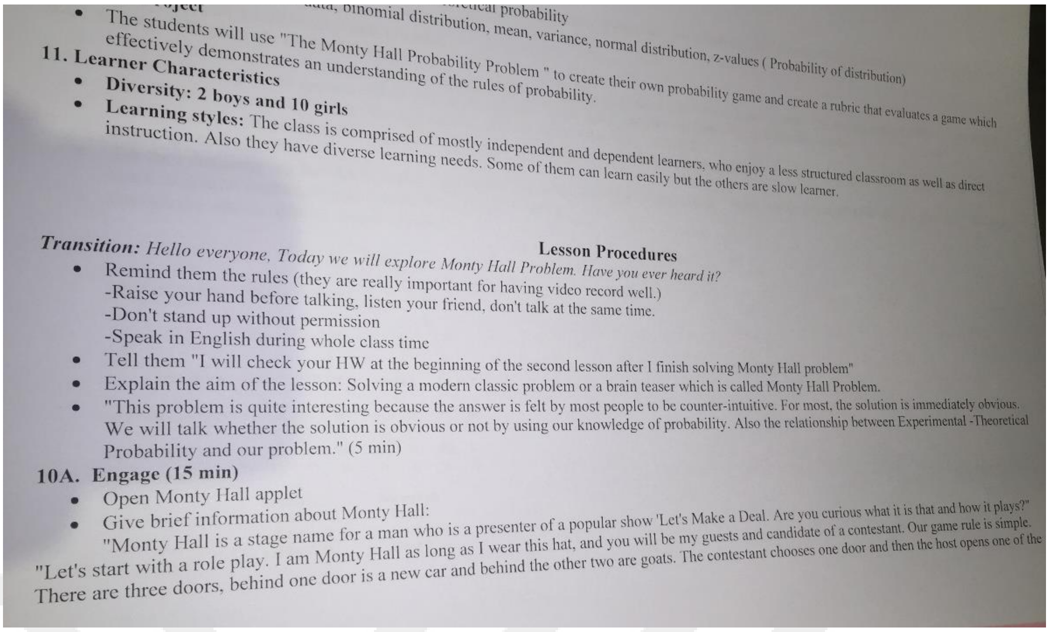
i. Ask if there are any questions about the interview

ii. Ask if there is something else that he/she would like to add.

iii. Reiteration of the confidentiality aspect of the interview

iv. Thank them for their participation and their time for this study.

APPENDIX G: Example of Lesson Plan of PT 7



- The students will use "The Monty Hall Probability Problem " to create their own probability game and create a rubric that evaluates a game which effectively demonstrates an understanding of the rules of probability.

11. Learner Characteristics

- **Diversity: 2 boys and 10 girls**
- **Learning styles:** The class is comprised of mostly independent and dependent learners, who enjoy a less structured classroom as well as direct instruction. Also they have diverse learning needs. Some of them can learn easily but the others are slow learner.

Lesson Procedures

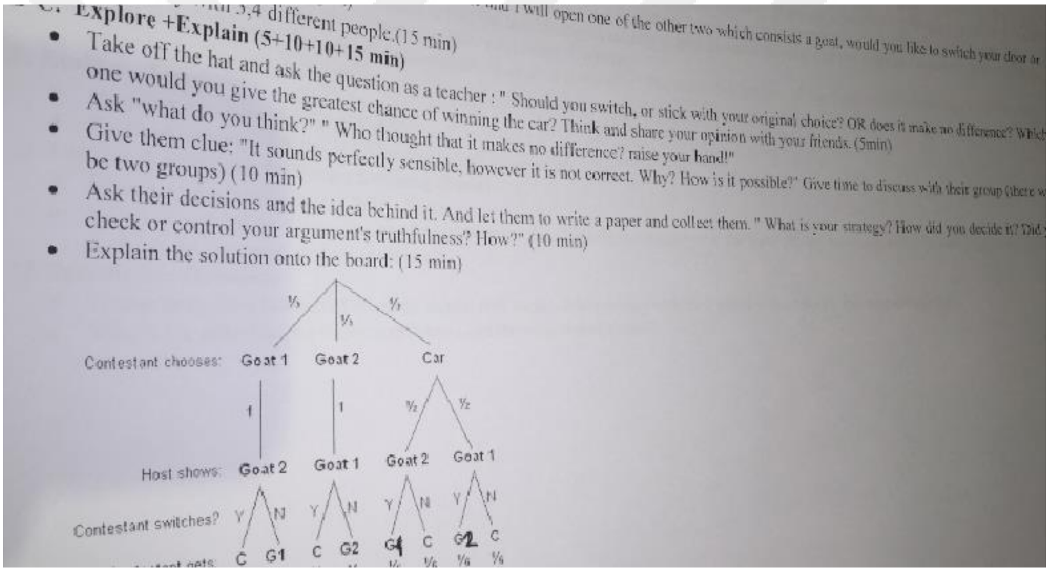
Transition: *Hello everyone, Today we will explore Monty Hall Problem. Have you ever heard it?*

- Remind them the rules (they are really important for having video record well.)
  -Raise your hand before talking, listen your friend, don't talk at the same time.
  -Don't stand up without permission
  -Speak in English during whole class time
- Tell them "I will check your HW at the beginning of the second lesson after I finish solving Monty Hall problem"
- Explain the aim of the lesson: Solving a modern classic problem or a brain teaser which is called Monty Hall Problem.
- "This problem is quite interesting because the answer is felt by most people to be counter-intuitive. For most, the solution is immediately obvious. We will talk whether the solution is obvious or not by using our knowledge of probability. Also the relationship between Experimental -Theoretical Probability and our problem." (5 min)

10A. Engage (15 min)

- Open Monty Hall applet
- Give brief information about Monty Hall:
  "Monty Hall is a stage name for a man who is a presenter of a popular show 'Let's Make a Deal. Are you curious what it is that and how it plays?"

"Let's start with a role play. I am Monty Hall as long as I wear this hat, and you will be my guests and candidate of a contestant. Our game rule is simple. There are three doors, behind one door is a new car and behind the other two are goats. The contestant chooses one door and then the host opens one of the

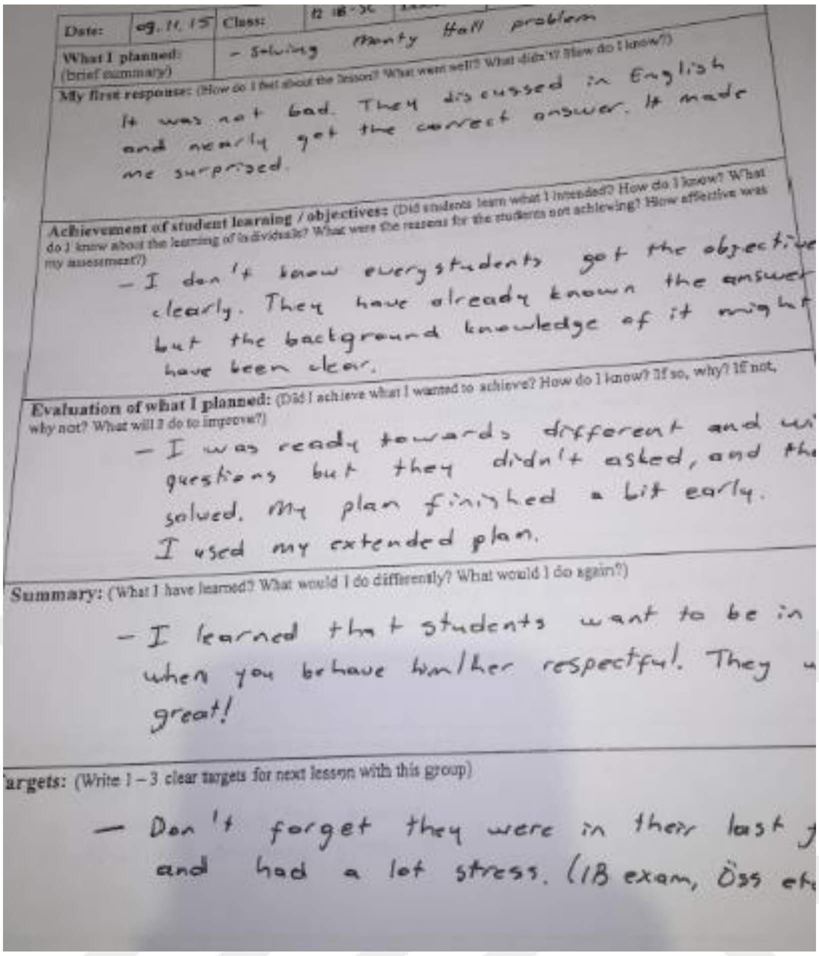


3,4 different people.(15 min) ... I will open one of the other two which consists a goat, would you like to swich your door or

Explore +Explain (5+10+10+15 min)

- Take off the hat and ask the question as a teacher : " Should you switch, or stick with your original choice? OR does it make no difference? Which one would you give the greatest chance of winning the car? Think and share your opinion with your friends. (5min)
- Ask "what do you think?" " Who thought that it makes no difference? raise your hand!"
- Give them clue: "It sounds perfectly sensible, however it is not correct. Why? How is it possible?" Give time to discuss with their group (there will be two groups) (10 min)
- Ask their decisions and the idea behind it. And let them to write a paper and collect them. " What is your strategy? How did you decide it? Did check or control your argument's truthfulness? How?" (10 min)
- Explain the solution onto the board: (15 min)

Pictures from PT 7's Teaching Practice File (Monty Hall Lesson)



Date: 09.11.15 | Class: 12 IB-3C

What I planned: (brief summary) - Solving Monty Hall problem

My first response: (How do I feel about the lesson? What went well? What didn't? How do I know?)

It was not bad. They discussed in English and nearly got the correct answer. It made me surprised.

Achievement of student learning / objectives: (Did students learn what I intended? How do I know? What do I know about the learning of individuals? What were the reasons for the students not achieving? How effective was my assessment?)

- I don't know every students got the objective clearly. They have already known the answer but the background knowledge of it might have been clear.

Evaluation of what I planned: (Did I achieve what I wanted to achieve? How do I know? If so, why? If not, why not? What will I do to improve?)

- I was ready towards different and wi questions but they didn't asked, and th solved. My plan finished a bit early. I used my extended plan.

Summary: (What I have learned? What would I do differently? What would I do again?)

- I learned that students want to be in when you behave him/her respectful. They w great!

Targets: (Write 1 – 3 clear targets for next lesson with this group)

- Don't forget they were in their last y and had a lot stress. (IB exam, ÖSS et

PT 7's self-evaluation after her Monty Hall lesson

APPENDIX H: Ethics Committee Approval

Bilkent Üniversitesi
Akademik İşler Rektör Yardımcılığı

Tarih: 28 Eylül 2015

Gönderilen: Özge Keskin

Tez Danışmanı: M. Sencer Çorlu

Gönderen: Hitay Özbay
Provost Yardımcısı

Konu: *"Through a More Realistic ... "* çalışması etik kurul onayı

Üniversitemiz İnsan Araştırmaları Etik Kurulu, 28 Eylül 2015 tarihli görüşme sonucu, *"Through a More Realistic and Holistic View of Teaching: A Reflective Framework for Mathematics Teacher Education by Using Case Analysis"* isimli çalışmanız kapsamında yapmayı önerdiğiniz etkinlik için etik onay vermiş bulunmaktadır. Onay, ekte verilmiş olan çalışma önerisi, çalışma yürütücüleri, ve bilgilendirme formu için geçerlidir.

Bu onay, yapmayı önerdiğiniz çalışmanın genel bilim etiği açısından bir değerlendirmesine karşı gelmektedir. Çalışmanızda, kurulumuzun değerlendirmesi dışında kalabilen özel etik ve yasal sınırlamalara uymakla ayrıca yükümlüsünüz.

Etik Kurul Üyeleri:

Ünvan / İsim	**Bölüm / Uzmanlık**	**İmza**
1. Prof. Dr. Hitay Özbay	Elektrik ve Elektronik Müh.	
2. Doç.Dr. Fatma Taşkın	İktisat	
3. Prof.Dr. Haldun Özaktaş	Elektrik ve Elektronik Müh.	
4. Prof.Dr. Tayfun Özçelik	Moleküler Biyoloji ve Genetik	
5. Prof.Dr. Erdal Onar	Hukuk	
Yd.1. Yrd.Doç.Dr. Ali Osmay Güre	Moleküler Biyoloji ve Genetik	(yedek üye)
Yd.2. Prof.Dr. Cemal Yalabık	Fizik	(yedek üye)

Kurul karar/toplantı No: 2015_09_28_01

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VITA

Özge Keskin was born in Ankara, Turkey, on August 31, 1984. She is a graduate of Ankara Atatürk Anatolian High School. She received her BSc degree in Mathematics from Middle East Technical University in 2006. After graduation, she pursued her education at Bilkent University Graduate School of Education to have a MA degree in Mathematics teaching. She started her teaching career at İhsan Doğramacı Foundation Bilkent High School (Ankara). She is currently teaching mathematics at TED Ankara College Foundation High School. She is experienced in enacting both national and international curricula and she contributes to curriculum development studies in her school.