FLEET TYPE ASSIGNMENT AND ROBUST AIRLINE SCHEDULING WITH CHANCE CONSTRAINTS UNDER ENVIRONMENTAL EMISSION CONSIDERATIONS

A THESIS

SUBMITTED TO THE DEPARTMENT OF INDUSTRIAL ENGINEERING

AND THE GRADUATE SCHOOL OF ENGINEERING AND SCIENCE

OF BILKENT UNIVERSITY

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS

FOR THE DEGREE OF

MASTER OF SCIENCE

By

Özge ŞAFAK

August, 2013

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

Prof. Dr. M. Selim Aktürk (Advisor)

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

Assoc. Prof. Dr. Sinan Gürel (Co-Advisor)

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

Assoc. Prof. Dr. Oya E. Karaşan

I certify that I have read this thesis and that in my opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

Assist. Prof. Dr. Sakine Batun

Approved for the Graduate School of Engineering and Science:

Prof. Dr. Levent Onural
Director of the Graduate School

ABSTRACT

FLEET TYPE ASSIGNMENT AND ROBUST AIRLINE SCHEDULING WITH CHANCE CONSTRAINTS UNDER ENVIRONMENTAL EMISSION CONSIDERATIONS

Özge ŞAFAK
M.S. in Industrial Engineering
Supervisor: Prof. Dr. M. Selim Aktürk
Co-Supervisor: Assoc. Prof. Dr. Sinan Gürel
August, 2013

Fleet Type Assignment and Robust Airline Scheduling is to assign optimally aircraft to paths and develop a flight schedule resilient to disruptions. In this study, a Mixed Integer Nonlinear Programming formulation was developed using controllable cruise time and idle time insertion to ensure passengers' connection service level with the objective of minimizing the costs of fuel consumption, CO_2 emissions, idle time and spilled passengers. The crucial contribution of the model is to take fuel efficiency of aircraft into considerations to compensate for the idle time insertion as well as the cost of spilled passengers due to the insufficient seat capacity. The nonlinearity in the fuel consumption function associated with controllable cruise time was handled by second order conic reformulations. In addition, the uncertainty coming from a random variable of non-cruise time arises in chance constraints to guarantee passengers' connection service level, which was also tackled by transforming them into conic inequalities. We compared the performance of the schedule generated by the proposed model to the published schedule for a major U.S. airline. On the average, there exists a 20% total cost saving compared to the published schedule. To solve the large scale problems in a reasonable time, we also developed a two-stage algorithm, which decomposes the problem into planning stages such as fleet type assignment and robust schedule generation, and then solves them sequentially.

Keywords: fleet type assignment, airline scheduling, cruise time controllability, second order conic programming, chance constraints.

ÖZET

ÇEVRESEL EMİSYONU GÖZ ÖNÜNDE BULUNDURARAK ŞANS KISITLARI İLE DAYANAKLI HAVAYOLU ÇİZELGELEME VE FİLO TİPİ ATAMA MODELİ

Özge ŞAFAK
Endüstri Mühendisliği, Yüksek Lisans
Tez Yöneticisi: Prof. Dr. M. Selim Aktürk
Eş-Tez Yöneticisi: Doç. Dr. Sinan Gürel
Ağustos, 2013

Filo tipi atama ve gürbüz havayolu çizelgelemesi, uçakların rotalara optimal bir şekilde atanması ve aksamalara karşı dayanaklı bir uçuş çizelgesi geliştirilmesi anlamına gelir. Bu çalışmada; yakıt tüketimi, CO_2 emisyonu, atıl zaman ve taşan yolcu maliyetlerini en aza indirmeyi hedefleyen ve yolcuların bağlantı hizmet seviyelerini sağlamak amacıyla, kontrol edilebilen seyir zamanı ve atıl zaman kullanılarak, Karma Tamsayılı Doğrusal Olmayan Programlama formulasyonu geliştirilmiştir. Modelin kritik katkısı, yetersiz oturma kapasitesinden kaynaklı taşan yolcu maliyetiyle birlikte atıl zaman yerleştirmeyi telafi etmek amacıyla uçağın yakıt verimliğini hesaba katmasıdır. Kontrol edilebilir seyir süreleleriyle ilişkili yakıt tüketim fonksiyonundaki doğrusalsızlık, ikinci derece konik reformülasyonlarla işlenmiştir. Buna ek olarak, seyir dışı sürede bulunan bir raslantısal değişkeninden kaynaklanan belirsizlik, yolcu bağlanma hizmet seviyesini garanti etmek üzere şans kısıtlarnda ortaya çıkmaktadır ve bu da, konik eşitsizliklere dönüştürülerek ele alınmıştır. Önerilen model tarafından oluşturulan planlamanın performansını ABD'li büyük bir havayolu şirketi tarafından yayımlanan planla karşılaştırdık. Yayımlanan plana kıyasla toplamda ortalama 20%'lik bir maliyet tasarrufu sağlandı. Büyük ölçekli problemleri makul bir zamanda çözmek için de, problemi, filo tipi ataması ve gürbüz çizelgeleme gibi planlama aşamalarına ayıran ve sonra sırasıyla çözen iki aşamalı bir algoritma geliştirdik.

Anahtar sözcükler: filo tipi atama, uçuş çizelgeleme, kontrol edilebilir seyir zamanları, konik eşitsizlikler, şans kısıtları.

Acknowledgement

I thank my advisor Professor M. Selim Aktürk for his invaluable support during my M.S. study. He has always been an understanding advisor. I have learned many valueable lessons from him. I also thank my co-advisor Associate Professor Sinan Gürel for his time, help and patient. His advice contributed greatly to this work. It was and will be a pleasure to work with them.

I also would like to acknowledge the financial support of The Scientific and Technological Research Council of Turkey (TUBITAK) for the Graduate Study Scholarship Program they awarded.

I would like to thank my officemates. It has always been fun in the same office with them. Special thanks go to Damla Kurugöl and her family for offering me a room in their house whenever I got bored and for her invaluable friendship. We shared the same anxiety and excitement with Bilgesu Çetinkaya, Dilek Keyf, Ayşegül Onat, Gülce Çuhacı, Kumru Ada, Malek Ebadi and Ramez Kian as graduate students. I will also remember my homemate Nil Karacoğlu with her great friendship that made any boring times enjoyable at home.

My sincere gratitude goes to my aunt and her husband who enriched my time in Ankara. My aunt has always been an understanding family member. Her advice contributed in large measures to success in my life.

Lastly, I owe everything that I have achieved to my parents. Their support, love and encouragements through my life are most precious to me. I love them all.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

The goal of the Robust Airline Scheduling and Fleet Assignment Problem is to develop a flight schedule resilient to disruptions and assign optimally aircraft types to paths such that airline operating cost is minimized. It is a challenging problem with numerous parameters such as aircraft types, demand of flights, aircraft and passengers' connection information. Due to its complexity, it is hard to solve it manually, therefore an optimization tool is required. In this study, a mathematical model is developed and implemented in Java with a connection to a commercial solver, IBM ILOG CPLEX.

1.1 Motivation

After the U.S. airline deregulations, the competition was increased among not only the previous airlines but also new entrances. In order to survive, airlines had to manage their resources efficiently and apply operational methodologies effectively.

Airlines are one of the transportation industries who provide large scale network connections and use numerous resources to transport passengers. Therefore, they have to implement a proper planning to maintain a consistent profitability in the industry. Furthermore, airlines run in an uncertain environment which makes them vulnerable to unexpected changes. Consequently, robust and efficient planning tools are mandatory in order to handle the complexity of airline industry. This is the reason why airlines employ operations research methods.

Airline scheduling is to determine when and where to fly such that constraints related to the airline operations such as aircraft routing, crew assignment, maintenance planning and gate assignment are satisfied. Airlines have to solve an optimization model by integrating these operations to maximize profitability while guarantying passengers' connections in entire network. Consequently, it will result in millions of variables and constraints while ensuring the feasibility.

Besides airline scheduling, another crucial decision is to determine the assignment of aircraft types to flight legs such that operating cost is minimized. Since airlines operate large number of different aircraft, each having different characteristics, capacity, and fuel consumption, extremely many assignment possibilities occur. When the large scale network is taken into consideration, the problem size is increased by extremely. Thus, an optimization tool is required to solve more complex problems in a shorter period of time with saving millions of dollars.

Even though airlines make proper planning and manage their resources efficiently, they encounter some factors that cannot be controlled and result in passengers, crew and aircraft disruptions. Airlines are susceptible to unforeseeable flight delays due to inclement weather conditions, mechanical failure, congested airports, crew sickness or even strikes by pilots or airline personnel. Each disruption has different impact on the airline operations while having a different reasonable time to continue to the original schedule. To address this issue, robust optimization is required to capture uncertainties at airline operations while enabling airlines to recover at lower costs when disrupted.

However, robust airline scheduling is a challenging problem. Firstly, it is hard to quantify the value of robustness. Furthermore, airlines face difficulties to determine how much they are willing to pay for robustness in the planning stage since there is a trade-off between robustness and cost. Robustness is integrated into the model with two different ways. One of them is to capture the uncertainty with a stochastic model involving a delay cost coming from a probability distribution in the objective function. An alternative way is to incorporate additional terms such as idle times into the model such that propagation of delays and misconnection of passengers are minimized when disrupted.

1.2 Contributions

In our study, we integrated the fleet assignment along with the flight planning. We design a flexible schedule incorporating both idle time insertion and speeding up the aircraft to ease recovery by isolating the delay effects on the subsequent flights. Besides, we optimally assign the aircraft such that fuel and CO_2 emission costs coming from speeding up the aircraft, idle time and spill cost of passengers are minimized. However, a classical fleet type assignment approach assigns the aircraft in order to satisfy all passenger demand so that fuel and CO_2 emission costs may increase. In our study, we can compensate for the spill cost of passengers, who cannot be accommodated due to insufficient capacity of the aircraft with the conservation of fuel consumed and CO_2 emission. This is the crucial contribution of our study to the fleet assignment literature.

Regarding to robustness literature, idle time insertion is proposed to absorb large delays. However, it is not preferable that such expensive resources stand idle. Instead, the speed of the aircraft can be increased as necessary. It is obvious that speeding up the aircraft is more beneficial as opposed to the idle time insertion in terms of the aircraft utilization. On the other side, the speed of aircraft can only be increased until the cost of the fuel consumption and CO_2 emission is less than the cost of idle time of the aircraft. In the proposed model, the speed of the aircraft is only controlled during the cruise stage of the flight block times. The remaining stages such as take off and landing, which are viewed as non-cruise time, are represented by a random variable.

Another crucial contribution is to incorporate the congestion levels of airports in the random variable of non-cruise time to develop a robust schedule less susceptible to variability in paths. For each flight, variability of the path is separately calculated depending on the congestion factors of the origin and destination airport.

We integrated both passengers and aircraft connections into the model to maintain the feasibility. This gives us a more robust schedule and assignment while minimizing the delay propagations due to misconnections of both aircraft and passengers.

Another important contribution is that we tackled the chance constraints and nonlinear cost components by representing them as second order conic inequalities. More information about conic programming can be found in Ben-Tal and Nemirovski (1) and Günlük and Linderoth (2). We are able to solve a mixed integer second order conic programming formulation with a commercial solver, IBM ILOG CPLEX. As shown in computational results chapter, fleet assignment option indicates a significant cost saving in fuel consumption and CO_2 emission compared to the published schedule. Furthermore, controllability of cruise time results in drastic decrease in the idle time while ensuring the same passengers' service level.

To simplify the problem complexity and solve the large scale problems in a reasonable time, we also developed a two-stage algorithm. This two-stage algorithm decomposes the problem into planning stages such as fleet type assignment and robust airline schedule generation, then solves them sequentially.

1.3 Overview

In the next chapter, literature review is provided in detail. Extensive information about airline scheduling, fleet assignment, cruise time controllability and fuel consumptions of flights, methods to deal with the chance constraints and second order cone programming are given.

In Chapter 3, the framework of the problem is described and the proposed mathematical model parameters and variables are explained. The distribution of the random variable representing the non-cruise time in the model is analyzed in detail. Calculation of the fuel consumption and CO_2 emissions during the cruise stage are explained. In addition, the service level decisions for each connected flight and overall model service levels are characterized. Finally, a numerical example is given to show how the model works.

The proposed mathematical model is provided in Chapter 4. In addition, conic representations of chance constraints and nonlinear objective function are explained. Then, conic reformulation of the model is provided.

Chapter 5 is devoted to the two-stage algorithm to solve the problem in a reasonable time. First, our approach to simplify the problem is described. Then, we explain the two-stage algorithm step by step by giving the relation to proposed model.

We analyze the performance of the schedule developed by the proposed model and two-stage algorithm in Chapter 6. In two separate sections, we discuss the results for a schedule with 41 flights and a schedule with 114 flights, respectively. Computation time analysis is conducted for both two-stage algorithm and the proposed model. Finally, we conclude with extensions of the problem in Chapter 7.

Chapter 2

Literature Review

In this section, a detailed literature review is given about airline scheduling, fleet assignment, cruise time controllability and fuel consumption of flights, methods to deal with chance constraints and second order cone programming.

2.1 Airline Scheduling Process

Airline schedule planning process is to generate a schedule having the largest revenue under the consideration of fleet assignment, aircraft maintenance routing and crew assignment. Since it is a huge and complex problem, it is often divided into subproblems and solved sequentially. Airline schedule planning process consists of four stages such as schedule generation, fleet assignment, aircraft maintenance routing and crew assignment. In the first stage, which markets to be served, service frequencies to match the forecasted demand and departure times of flights are determined to generate an initial schedule. This stage affects every airline operation and has the biggest impact on the airline revenues. The second stage is to assign specific fleet types to every flight in the schedule such that airline operations costs are minimized by trying to match the seat capacity of aircraft to the demand of flights. The third stage is to determine the feasible set of routes for each aircraft such that the maintenance requirements of each aircraft are satisfied. The fleet assignment information is given into the maintenance problem as an input. In the last stage, crew assignment is done to achieve the minimum cost by considering some of the requirements. A detailed information about the airline schedule planning can be found in Barnhart and Cohn (3).

Integration of these four planning problems into a single model will result in millions of variables and constraints. Thus, some of the researchers try to solve these problems sequentially by dividing them into four sub problems. However, a sub optimal solution may not lead to an optimal solution, it may even lead to an infeasible solution for the overall problem. Therefore, another approach is to combine two of them into a single model to prevent some of the infeasibilities. Papadakos (4) solved integrated models by enhanced Benders decomposition method with column generation and results in less costs compared to the best known approaches in literature. Since airlines are susceptible to unforeseeable delay, any deterministic model may result in high operational costs. Therefore, robust schedule is required to capture the uncertainties.

2.1.1 Robust Airline Scheduling

Traditional airline planning approaches assume that flights arrive and depart as planned. However, airlines incurred billions of dollars losses due to the unexpected flight delays. Bureau of Transportation Statistics, BTS (5) tracked Airline On-Time Statistics and Delay Causes and reported that approximately 21% of U.S. domestic flights are delayed whose 5% is aircarrier delay, 5% is National Aviation System delay, 7% is aircraft arriving late, 1% is cancelled and left is weather delay, diverted and security delay. Therefore, robust airline scheduling is a crucial issue of airline operations to model a more flexible schedule to disruptions and continue with the original schedule as soon as possible when disrupted. Ageeva and Clarke (6) proposed a robust aircraft maintenance model to provide aircraft swap flexibilities. Therefore, the model facilitates the recovery strategies after flight delays.

Some of the researchers incorporate additional terms into the scheduling model to capture the uncertainties in the airline operations. One approach is to add slacks to minimize the effect of aircraft, passenger and crew delays on the subsequent flights. The delay of one flight may result in the delay to downstream flights and misconnections of the crew and passenger assigned to those flights, if there is not enough slack time between the consecutive flights. Lan et al. (7) proposed a mixed integer model which minimizes the delay propogation by allowing the changes in fleet type assignment to flights. In addition, another approach was developed to minimize the passenger misconnections by re-timing the departure times of flights such that changes in the fleet assignment is not allowed.

Some of the researchers conducted on slack re-allocation to build a robust schedule. Chiraphadnakul and Barnhart (8) proposed a different model minimizing the propagation delay in the entire network by redistributing the existing slacks. In order to analyse the performance of the schedule, they used some metrics such as propagation of delays and passenger delays. Significant improvements on the overall schedule performance was achieved with minor adjustments on the initial schedule.

Ahmedbeygi et al. (9) also conducted research on redistributions of slacks to minimize the delay propagations such that initial fleet and crew assignment are not changed. This study showed that downstream effects of delays are reduced by re-allocating the existing slacks to the connections of flights having a tendency to more delay propagations.

There occur few studies addressing the effects of delay propagations. Arıkan et al. (10) built a stochastic model to analyze the propagation of delays in the network by developing robustness measures. Dunbar et al. (11) introduced a new approach to minimize the cost of propagated delay while integrating both crew pairing and aircraft routing problems.

Some of the researchers analyzed the schedule performance and impacts of delays under the robust airline scheduling. Deshpande and Arıkan (12) modeled the total travel time distribution and provided a method for estimating the schedule ontime arrival probability. Burke et al. (13) developed a robust schedule with multiple-objectives of schedule reliability and schedule flexibility. They investigated the influence of robustness on the operational performance of the schedule.

Another approach to capture uncertainty in the flight block times is to use a stochastic programming. Sohoni et al. (14) proposed a model, that captures uncertainty related with block time through chance constraints and perturb flight schedule. The aim of the model is to maximize expected profit while ensuring both flight and passengers' service level. As a solution methodology, cut generation algorithm is developed based on the linearization of the chance constraints. Marla and Barnhart (15) studied three different approaches, extreme value based, probabilistic constraint programming and tailored approaches to robustness in aircraft routing. In this study, we also modeled variability through chance constraints.

2.2 Fleet Type Assignment

The fleet assignment problem deals with optimal assignment of aircraft types, each having different seat capacity and fuel consumption to the scheduled flight legs based on the availability of aircraft and operating costs. Assigning a smaller aircraft may result in spilled passengers due to insufficient seat capacity, on the other hand, assigning a larger aircraft may result in unsold seats and higher operational costs. Thus, fleet assignment constitutes a crucial part of the airline scheduling process. Due to the large number of scheduled flights and dependency of fleet assignment on the other airline schedule processes, fleet assignment problem is a challenging task for the airlines.

Basic fleet assignment model (FAM) is formulated as mixed integer program based on the network. Abara (16) was one of the first researcher who formulated the FAM using connection networks with arcs representing all feasible possible flight connections. Hane et al. (17) also formulated the FAM using time-space networks with arcs representing the flight legs and leaving the connection decisions to the model. They proposed some methods to reduce the size of the network and computational effort. The aim of both models is to maximize the profit under the schedule balance constraints for conservation of flow, flight coverage and aircraft availability constraints. However, both of the models do not incorporate the demand and spilled passengers.

Although earlier studies mentioned above solve the fleet assignment problem (FAP) independently of other airline operations, the FAP has an interaction with airline scheduling, aircraft maintenance and crew scheduling process. FAM gives an optimal assignment on the scheduled flights. Besides, the fleet types assignment information is fed into the aircraft maintenance routing process, so that routes for each aircraft are determined based on the maintenance requirements. Crews are assigned to flight legs by considering the capability of crews to fly with the assigned aircraft types to the flight leg. Thus, these dependencies have motivated researchers to solve the integrated models to obtain a better solution for the overall system. Due to the problem complexity, two or more sub problems have been integrated.

Lohatepanont and Barnhart (18) integrated leg selection decision among the optional flight legs with the FAM. Moreover, Barnhart et al. (19) proposed a Mixed Integer Programming model to solve the string-based fleet assignment and aircraft maintenance routing simultaneously, such that string is the sequence of legs flown by the same aircraft. In addition, Rosenberger et al. (20) proposed a robust FAM and aircraft rotation with short cycles to allow more aircraft swap opportunites when disrupted. Some of the researchers also integrated crew scheduling with the FAM. Sandhu and Klabjan (21) designed two solutions methodologies to solve the integrated model of crew pairing and fleet assignment. One of them is based on the Lagrangian relaxation and column generation, another is based on Benders decomposition.

In addition to integration of airline scheduling processes, some of the researchers incorporate passengers considerations. Barnhart et al. (22) proposed a passenger-mix model integrated with the fleet assignment. The aim of the model is to minimize net revenue lost due to spilled passengers on paths. Lohatepanont and Barnhart (18) integrated leg selection decision with the path based fleet assignment model. Jacobs et al. (23) presented a model that integrate the FAM model with the Origin and Destination revenue management model. These models do not take into account fuel burn of the aircraft and its adverse effect on environment such as CO_2 emissions, since it simply assigns the aircraft to match the seat capacity of aircraft to the demand while disregarding the fuel efficiency of aircraft. Isaac et al. (24) addressed environmental and economic considerations by developing a model determining the new and existing aircraft assignment such that all passenger demand is met.

2.3 Cruise Time versus Fuel Consumption and CO_2 Emission

Fuel has been the largest single cost term for the global airlines. According to IATA's (25) analysis on airline financial data, fuel expenses accounted for 30%-40% of total operating cost. While the share of the fuel cost was 12-13% between 2001 and 2003, it was 32.3% of the total airline cost in 2008. The reason for rise of the fuel share is the sharp increase in the fuel price. Moreover, each kilogram of fuel consumed generates approximately three kilograms of CO_2, which is a greenhouse gas. Thus, many studies have been conducted to decrease the airline fuel consumption under the environmental considerations.

In addition to the fuel cost, airlines have time related cost such as maintenance, crew and ownership or rental cost. When the aircraft is flown faster, more money is saved in terms of the time related cost. However, fuel burn increases by speeding up the aircraft, so that money will be lost. On the other hand, to decrease the fuel consumption aircraft should be flown slowly. Thus, airbus (26) presented a cost index function to balance these cost factors and help to select the best speed while minimizing the overall cost. Cost index is defined as the ratio of time related cost per minute of flight to the cost of fuel per kg. Cost index has two extreme points representing the minimum fuel mode for the maximum range when the cost index is set to minimum, and minimum time mode for maximum speed when the cost index is set to maximum. Thus, airlines optimize the cost by adjusting increased fuel consumption for reduced trip and vice versa.

The decision of cruise time versus fuel burn should not be locally made for each flight. The network effect should be considered in terms of passenger and aircraft connections. Therefore, an optimization tool involving the cost index and cruise time controllability is necessary. Under the area of recovery management and robust optimization, adjusting the cruise speed should be implemented by considering the environmental impact.

Cook et al. (27) also discussed the cost index parameter which quantifies the options of flying faster to recover when disrupted and flying slower for conservation of fuel. However, earlier researchers did not emphasize on adjusting the cruise speed instead of inserting idle time to capture the variability and alleviate the recovery options. Cruise speed controllability can be preferable to idle time insertion when the total cost of fuel consumed and CO_2 emitted by combustion of fuel is less than the cost of aircraft stand idle, and vice versa for the idle time insertion.

Aktürk et al. (28) proposed a recovery model using controllable cruise time with adjusting the aircraft speed. Arıkan et al. (29) also proposed a model for passenger and aircraft recovery problem by integrating cruise speed control along with retiming of the departure times of flights and swapping aircraft. In our study, we also consider the cruise speed controllability to develop a robust schedule by taking the cruise times as variables. They can be shortened to reduce the slack time in the schedule to ensure the desired passengers' service level, in contrast to increase in the fuel consumption and CO_2 emissions.

2.4 Chance Constraints

Chance constraint programming concerns random data which is represented via the constraints that prescribe a required level for the probability. The objective function is maximized or minimized over these probabilistic constraints. Probabilistic constraints programming was initiated by Charnes et al. (30) to deal with the uncertain conditions. For each stochastic constraint, they formulated the probabilistic constraints separately. The extension to joint probabilistic constraints was first developed by Miller and Wagner (31) for independent random variables. Later, Prékopa (32) permitted the multivariate distribution. For the general case, Prékopa (33) showed that if the probability distribution function is logarithmic concave, the convexity of the set of feasible solutions is guaranteed for the probabilistic constraint programming. However, even if the convexity is guaranteed, handling nonlinear constraints is usually more complicated than handling the nonlinear objective functions. Therefore, Komáromi (34) introduced a problem with concave objective function and linear constraints as a dual to the probabilistic constrained problem. A dual type algorithm was presented to solve both problems simultaneously. However, only a few papers exist to handle the probabailistic constraints involving discrete random variables. Dentcheva et al. (35) provided p-efficient point method to obtain lower and upper bounds for the optimal solution of the probabilistic constrained programming with integer valued random variables.

There are many studies on chance constrained programming with different approaches. Luedtke and Ahmed (36) obtained feasible solutions and optimality bound for the stochastic problem with probabilistic constraints by developing sample approximations based on Monte Carlo. Nemirovski and Shapiro (37) constructed convex approximations which are computationally tractable for the chance constrained programming. They extended their construction to the case where the distributions of the random variables are not known exactly but belong to a convex compact set.

In our study, to handle the chance constrained programming, we used an exact method as second order cone programming instead of obtaining an optimality bound using approximations. Detail literature review on the second order cone programming will be given in the following section.

2.5 Second Order Cone Programming

In our study, we tackled the chance constraints by representing them as second order conic inequalities. Therefore, it can be solved in an exact and fast way instead of approximation methods. In addition, we handled the non-linear cost function by transforming to second order conic equations. More information about conic programming and conic representable functions can be found in Ben-Tal and Nemirovski (1).

Second order cone programming has been applied in optimizations and operations research in recent years. For 0,1-mixed integer nonlinear programs, Günlük and Linderoth (2) proposed reformulation techniques to express the convex hull via conic quadratic constraints. Thus, relaxations can be solved via second-order cone programming. Aktürk et al. (38) studied conic quadratic reformulations to solve machine job assignment problem with separable convex cost functions.

2.6 Summary

Fuel cost is a significant cost factor, which constitutes the huge portion of the airline operation costs. The fuel burn is a characteristic property for each fleet type. In addition, seat capacity of aircraft differs for each of the aircraft types. Thus, each fleet assignment type for the scheduled flights leads to different operational cost by trading decreased fuel burn for increased cost of spilled passenger or vice versa. However, earlier studies on the fleet assignment tried to match the seat capacity of the aircraft to the forecasted demand of the flight while disregarding the fuel efficiency of the aircraft. On the recent years, few studies about fleet assignment have addressed the fuel burn and environmental emissions of the aircraft. We integrated the fleet assignment along with the flight planning by considering the fuel burn, CO_2 emission and spilled passengers.

Airlines assume that flights arrive and depart as planned. However, unforeseeable flight delays may lead to money loss while resulting in propagation of flight delays on the downstream flights and passenger misconnections. When the whole network is considered together with passenger, aircraft and crew connections, congestions of the networks make the delay effects significant for airlines. Thus, a flexible schedule that can absorb the delay effects and provide many recovery alternatives is needed.

There exists a growing literature on the robust scheduling to make the airlines resistant to unexpected flight delays. In our study, we develop a robust schedule by controlling the cruise time and adding idle time as necessary to ensure the desired passengers' service level and aircraft connections. We consider adjustment of the cruise speed together with the fleet type assignment while minimizing the total airline costs under the environmental emission considerations. Few studies have focused on the redistribution of the existing slacks instead of adjusting the cruise speed.

We model the uncertainty of the non-cruise time of the flights using chance constraints. Earlier studies handled the chance constraints with linear approximations and dual algorithms, whereas we tackled them with second order cone programming and obtained an exact solution.

Chapter 3

Problem Definition

The proposed model determines the aircraft types for each path involving sequence of flights operated by the same aircraft and generate a robust schedule by re-timing the departure time of flights in the initial published schedule. The objective of the model is to minimize the total cost of fuel consumption, CO_2 emission, idle times and unsatisfied demand. Sequences of flights, aircraft routings and passengers' connections are taken as input. The proposed model adjusts the departure time of the flights by controlling the cruise time and inserting idle time between flights such that desired passengers' connection service level is ensured.

Aircraft types are assigned to the set of routes by minimizing the total cost such that each route is assigned to exactly one aircraft type and assigned aircraft types do not exceed the available number of aircraft types. Speeding up the aircraft to shorten the cruise time has a tremendous impact on the assignment, since each aircraft has different fuel consumption and CO_2 emission at different speed. Passenger demand is another critical factor for aircraft type assignment, because each spilled passenger who cannot be accommodated due to insufficient seat capacity of the assigned aircraft will be costly for the airlines. Finally, assigned aircraft types are more or less affected by the idle times of the flights due to dependency of idle time cost on the aircraft type. Therefore, aircraft types are assigned by considering not only matching demand to the seat capacity of the aircraft as the classical fleet assignment approach but also consider gain from fuel consumption and CO_2 emission to compensate for the cost of spilled passengers as well as the idle time insertion.

Model block times are examined in two parts as cruise time, which can be controllable with speeding up the aircraft, and non-cruise times which are not controllable and represented by a random variable. Controllable cruise times with idle time insertion adjust the model departure times. Non-cruise times of the flights involve landing and takeoff stages of the flights which can be shorter or longer depending on the congestions of the origin and destination airports. Thus, congestion levels of the origin and destination airports are involved in the random variable, which represents the non-cruise time. Moreover, airport congestion factors have an impact on the turnaround time of the aircraft, which is a required time based on the aircraft type to be prepared for the following flights.

While developing a robust schedule and fleet assignment, passengers' connections are ensured with a desired service level and aircraft connections are guaranteed. An aircraft connection is possible between flights F1 and F2, if sum of the arrival time of F1 and required turnaround time for the aircraft at the destination airport of F1 is less than the departure time of F2 when the origin airport of F2 is the same as the destination airport of F1. It is guaranteed with a constraint in the proposed model. Passengers' connection is achieved at the desired service level via the chance constraints. It is also possible if the destination airport of F1 is same as the origin airport of F2 and departure time of F2 is later than and within a time interval of the arrival time of F1.

In the following section, the descriptions of model parameters are given. After that, a random variable representing the non-cruise time, passengers' connection service level, nonlinear cost function of the fuel consumption and cost of CO_2 emissions are described.

The notation is given below:

Parameters

T: set of aircraft types

J: set of flight legs

P: set of paths

J_p: set of flights in path $p \in P$

N^t: available number of aircraft of type $t \in T$

CAP^t: number of seats in aircraft of type $t \in T$

I^t: unit idle time cost of aircraft of type $t \in T$ in dollars per minute

$PAIR$: set of pairs of consecutive flights of the same aircraft

TA^t_{ij}: turntime needed to prepare aircraft $t \in T$ between flights $i, j \in PAIR$

$f_i^{t,u}$: original cruise time duration of flight $i \in J$ with aircraft $t \in T$

$[\mathrm{f}_i^{t,l}, f_i^{t,u}]$: time window for cruise time of flight $i \in J$ with aircraft $t \in T$

D_i: demand of each flight $i \in J$

Csp_i: opportunity cost of spilled passengers of flight $i \in J$

$[\mathrm{w}_i,\ v_i]$: time window for departure time of flight $i \in J$

P_i: set of flights that have a passenger connection with flight $i \in J$

TP_{ij}: turntime needed to connect passengers between flights $i \in J, j \in P_i$

PAS_{ij}: normalized passenger connection level between flights $i \in J, j \in P_i$

O_i: origin of flight $i \in J$

Dn_i: destination of flight $i \in J$

c_{fuel}: cost of fuel per kg of aircraft fuel consumption

c_{CO_2}: cost of emission per kg of aircraft CO_2 emission

B: set of airports

e_b: airport congestion coefficient for airport $b \in B$

γ^d_{ij}: minimum service level for each passenger connections between flights $i \in J$ and $j \in P_i$

Decision Variables

z_p^t: 1 if aircraft of type $t \in T$ is assigned to path $p \in P$, and 0, o.w
x_i: departure time of flight $i \in J$
f_i^t: cruise time of flight $i \in J$ with aircraft type $t \in T$
S_i^t: idle time after flight $i \in J$ with aircraft type $t \in T$
γ_{ij}: service level for passenger connections between flights $i \in J$ and $j \in P_i$
$NumPass_i$: accepted number of passengers for flight $i \in J$

In the model, T represents the set of aircraft types, each having different seat capacity represented by CAP^t, fuel consumption and CO_2 emission. The fuel consumption coefficients for different aircraft are provided in Table 6.3. The fuel efficient aircraft has less cost of fuel consumed and CO_2 emission, where the assignment of the aircraft having a larger seat capacity results in less cost of unsatisfied passengers. Thus, fleet type assignment has a significant impact on the airline total cost. The cost of fuel consumption is calculated by multiplying the amount of fuel consumed in kg with the fuel price in dollars per kg represented by c_{fuel}. The cost of emission is also calculated by multiplying the amount of CO_2 emission in kg with the unit cost of emission in dollars, c_{CO_2}. Moreover, each type of aircraft has different unit idle time cost in dollars per minute which is represented by I_t for each $t \in T$. Fleet type assignment is also based on the available number of aircraft type on hand which is represented by N^t for each $t \in T$.

J represents the set of flights. $f_i^{t,u}$ is the ideal duration of flight $i \in J$ with the aircraft of type $t \in T$, which is determined using the cost index ratio (Cook et al. (27)), corresponding the Maximum Range Cruise speed. [$\mathrm{f}_i^{t,l}$, $f_i^{t,u}$] is the time window for the cruise time of flight $i \in J$ with the aircraft type $t \in T$, where $f_i^{t,l}$ is determined by the maximum compression of the $f_i^{t,u}$. [w_i, v_i] is the time interval for the departure time of flights. Demand of each flight of $i \in J$ is represented by D_i. The opportunity cost of each unsatisfied passenger due to limited capacity of the aircraft is represented by Csp_i. O_i and Dn_i are the origin and destination airports of flights, respectively.

B is the set of airports. Each airport has different congestion coefficient, which is represented by e_b. Landing and takeoff times of the non-cruise times are affected by the congestion levels of the origin and destination airports. P is the set of routes involving sequence of flights operated by the same aircraft. J_p represents the set of flights in the route $p \in P$. $PAIR$ is the set of flights connected with the same aircraft. For each $(i, j) \in PAIR$, TA_{ij}^t is the turnaround time needed by the aircraft type $t \in T$ to be prepared between two consecutive flights. It depends on the congestion coefficient of the destination airport of flight $i \in J$ as well as the aircraft type.

P_i is the set of flights for passengers who have connection of flights at the destination airport of flight i. TP_{ij} represents the turntime needed by the passengers, having connection between flights i and j. PAS_{ij} represents the weighted passenger connection levels calculated by normalizing the connected number of passengers between flight i and j to total number of passengers. γ_{ij}^d is the minimum desired service level for each passengers' connections between flights $i \in J$ and $j \in P_i$.

For each route $p \in P$ and for each aircraft type $t \in T$, we have a binary assignment variable, represented by z_p^t. For each flight $i \in J$, model departure times x_i are determined by the proposed model. Moreover, for each flight $i \in J$ and for each aircraft type $t \in T$, we have decision variables f_i^t for the cruise times and S_i^t for the idle times after flight i. In addition, for each connected flight (i, j), decision variable, γ_{ij} represents the percentage of the satisfied passengers who have connection between flights (i, j).

3.1 Distribution of Non-cruise Times

In the model, flight duration is separated into two components as cruise and non-cruise time. Cruise time is controllable with speeding up the aircraft as necessary at cruise stage. However, there exists uncertainty at taxi-in and taxi-out stages of flights, especially variance increases at the congested airports. In addition, climb and descend are uncertain stages of flights due to air traffic and weather conditions. Therefore, we refer to the non-cruise time as a random variable and cruise time as a decision variable.

Arıkan and Deshpande (12) showed that the log-Laplace distribution provides a good-fit to the block time of a flight. Therefore, for each flight $i \in J$, random variable A_i, which represents the non-cruise time of flights is assumed to be log-Laplace distribution with two parameters, α and β_i. For each flight $i \in J$, β_i's are calculated by multiplying the parameter β with a function, g of origin and destination airports' congestion factors. It is given as:

$$\beta_i = \beta \cdot g\left(e_{O_i}, e_{Dn_i}\right) \tag{3.1}$$

where O_i and Dn_i are the origin and destination airports of flight $i \in J$ respectively. Therefore, the mean and variance of the random variable depend on the congestion factors of the origin and destination airports. It means that, if a flight arrives or departs from a congested airport, non-cruise stage of that flight requires more time.

The random variable A_i of non-cruise time arose in chance constraints to guarantee passengers' connection service level.

3.1.1 Log-Laplace Distribution

The probability density function and cumulative distribution function of Log-Laplace random variable X with a scale parameter, e^{α} and the tail parameter, $1/\beta_i$ is given as:

$$f_X(x) = \begin{cases} \frac{1}{2 \cdot \beta_i \cdot x} e^{\frac{(\ln(x)-\alpha)}{\beta_i}}, & \text{if } \ln(x) < \alpha \\ \frac{1}{2 \cdot \beta_i \cdot x} e^{\frac{-(\ln(x)-\alpha)}{\beta_i}}, & \text{if } \ln(x) \geq \alpha \end{cases}$$

$$F_X(x) = \begin{cases} \frac{1}{2} e^{\frac{(\ln(x)-\alpha)}{\beta_i}}, & \text{if } \ln(x) < \alpha \\ 1 - \frac{1}{2} e^{\frac{-(\ln(x)-\alpha)}{\beta_i}}, & \text{if } \ln(x) \geq \alpha \end{cases}$$

The quantile function of the log-Laplace distribution is given as:

$$F_X^{-1}(p) = \begin{cases} (2p)^{\beta_i} \cdot e^{\alpha}, & \text{if } \ln(x) < \alpha \\ \frac{e^{\alpha}}{(2-2p)^{\beta_i}}, & \text{if } \ln(x) \geq \alpha \end{cases}$$

Moreover, Duran et al. (39) provided a method to estimate the mean of the log-Laplace distribution for non-cruise time of the flights. It is stated that, mean is finite only if $\beta_i < 1$. For each flight $i \in J$, β_i's are calculated as in the Equation (3.1). The mean of the log-Laplace distribution with parameters α and β_i is given as:

$$E[X] = \frac{e^{\alpha}}{(1-\beta_i)\cdot(1+\beta_i)} \tag{3.2}$$

3.2 Fuel Cost

Fuel consumption is estimated based on fuel-flow of the aircraft which is determined in terms of thrust, true airspeed and altitude as described by the Base of Aircraft Data (BADA) fuel-flow model (40). The nominal fuel-flow is calculated by the multiplication of thrust specific fuel consumption in $kg/min \cdot kN$ specified as a linear function of true airspeed, V_{TAS} $(knots)$, and thrust, Thr as follows:

$$f_{nom} = C_{f1}\left(1 + \frac{V_{TAS}}{C_{f2}}\right) Thr \tag{3.3}$$

where
C_{f1}: 1^{st} thrust specific fuel consumption coefficient $(kg/mim \cdot kN)$
C_{f2}: 2^{nd} thrust specific fuel consumption coefficient $(knots)$

Note that, our notification does not involve the aircraft type in this section to simplify the presentation. Fuel consumption coefficients as well as mass of aircraft are listed in EUROCONTROL (41) for different aircraft types.

Fuel burn rate (kg/min) at the cruise stage is calculated using the nominal fuel-flow and the cruise fuel flow factor C_{fcr}:

$$f_{cr} = f_{nom} \times C_{fcr} \tag{3.4}$$

According to BADA *Total Energy Model*, we can claim that thrust, Thr, is equal to drag, D, at the cruise stage due to no change in altitude and true airspeed. Therefore, thrust, Thr, can be calculated as follows:

$$Thr = \frac{C_D \cdot \rho \cdot V_{TAS}^2 \cdot S}{2} \tag{3.5}$$

where

ρ: the air density (kg/m^3) at given altitude
S: the wing reference area (m^2)

Drag coefficient C_D is calculated as follows:

$$C_D = C_{D0,CR} + C_{D2} \times (C_L)^2 \tag{3.6}$$

The lift coefficient, C_L, is determined under the assumption that the flight path angle is zero.

$$C_L = \frac{2 \cdot m \cdot g_0}{\rho \cdot V_{TAS}^2 \cdot Scos\,(\phi)} \tag{3.7}$$

where

m: aircraft mass (kg)
g_0: gravitational acceleration (m/s^2)
ρ: bank angle

When we plug all the terms in the fuel burn rate formula, we obtain the following equation as a function of true air speed.

$$
\begin{aligned}
f_{cr}\left(V_{TAS}\right) = \tfrac{1}{2} \cdot C_{f1} \cdot C_{fcr} \cdot \quad & \Bigg(C_{D0,CR} \cdot \rho \cdot S \cdot V_{TAS}^2 \\
& + C_{D0,CR} \cdot \frac{\rho \cdot S}{C_{f2}} V_{TAS}^3 \\
& + C_{D2,CR} \cdot \frac{4 \cdot m^2 \cdot g_0^2}{\rho \cdot S \cdot cos\left(\phi\right)^2 \cdot V_{TAS}^2} \\
& + C_{D2,CR} \cdot \frac{4 \cdot m^2 \cdot g_0^2}{C_{f2} \cdot \rho \cdot S \cdot cos\left(\phi\right)^2 \cdot V_{TAS}} \Bigg)
\end{aligned}
$$

We assume that there is no wind, so true airspeed is considered as the speed of aircraft (V). We also assume that the distance flown at cruise stage is fixed d, the cruise time duration is expressed as d/V. Then, we can formulate the total fuel consumption as follows:

$$
F\left(V\right) = \frac{d}{V} \cdot f_{cr}(V)
$$

Then, total fuel consumption during cruise stage can be expressed as follows:

$$
\begin{aligned}
F\left(V\right) = \tfrac{1}{2} \cdot d \cdot C_{f1} \cdot C_{fcr} \cdot \quad & \Bigg(C_{D0,CR} \cdot \rho \cdot S \cdot V \\
& + C_{D0,CR} \cdot \frac{\rho \cdot S}{C_{f2}} V^2 \\
& + C_{D2,CR} \cdot \frac{4 \cdot m^2 \cdot g_0^2}{\rho \cdot S \cdot cos\left(\phi\right)^2 \cdot V^3} \\
& + C_{D2,CR} \cdot \frac{4 \cdot m^2 \cdot g_0^2}{C_{f2} \cdot \rho \cdot S \cdot cos\left(\phi\right)^2 \cdot V^2} \Bigg) \qquad (3.9)
\end{aligned}
$$

We can rewrite the fuel consumption in terms of the cruise time by replacing V by $\frac{d_i}{f_i^t}$ for each flight $i \in J$ and aircraft $t \in T$. Introduce four auxiliary parameters, c_1, c_2, c_3, c_4 for each flight $i \in J$ and aircraft $t \in T$ and define them as follows:

$$
\begin{aligned}
c_1^{i,t} &= \frac{1}{2} \cdot C_{f1}^t \cdot C_{fcr}^t \cdot C_{D0,CR}^t \cdot \rho \cdot S^t \cdot d_i^2 \\
c_2^{i,t} &= \frac{1}{2} \cdot C_{f1}^t \cdot C_{fcr}^t \cdot \frac{C_{D0,CR}^t \cdot \rho \cdot S^t \cdot d_i^3}{C_{f2}^t} \\
c_3^{i,t} &= \frac{1}{2} \cdot C_{f1}^t \cdot C_{fcr}^t \cdot \frac{C_{D2,CR}^t \cdot 4 \cdot m_t^2 \cdot g_0^2}{\rho \cdot S^t \cdot \cos(\phi)^2 \cdot d_i^2} \\
c_4^{i,t} &= \frac{1}{2} \cdot C_{f1}^t \cdot C_{fcr}^t \cdot \frac{C_{D2,CR}^t \cdot 4 \cdot m_t^2 \cdot g_0^2}{C_{f2}^t \cdot \rho \cdot S^t \cdot \cos(\phi)^2 \cdot d_i^2}
\end{aligned}
$$

For $i \in J$, and $t \in T$, total fuel consumption in kgs becomes,

$$
F_i^t\left(f_i^t\right) = c_1^{i,t} \cdot \frac{1}{f_i^t} + c_2^{i,t} \cdot \frac{1}{\left(f_i^t\right)^2} + c_3^{i,t} \cdot \left(f_i^t\right)^3 + c_4^{i,t} \cdot \left(f_i^t\right)^2 \qquad (3.10)
$$

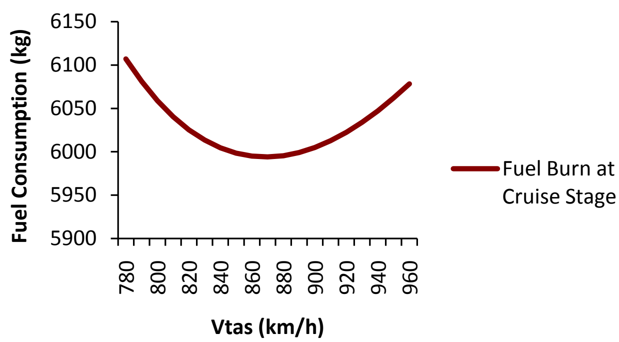

Figure 3.1: Fuel Cost Function at Cruise Stage

As an example, Figure 3.1 shows the fuel consumption of Airbus 320 212 type aircraft during the cruise stage. As it is seen, the minimum fuel consumption is obtained at the velocity, 868 km/h which represents the max-range cruise speed. It is obtained by taking the derivative of the fuel cost function in Equation (3.10). In the proposed model, upper bound of the cruise time is calculated under the assumption that aircraft flies with a constant speed corresponding to the max-range cruise speed. The proposed model tries to find an optimal speed which is greater or equal than the max-range cruise speed to compensate for the idle time insertion cost.

Let f_i^t be the cruise time variable, then we obtain the fuel cost for the flight i operated by the aircraft type t as the following:

$$FuelCost_i^t = c_{fuel}\left(F_i^t(f_i^t)\right) \tag{3.11}$$

There occurs a trade-off between the fuel consumption and demand satisfaction. Note that it can be cheaper to assign the fuel efficient aircraft to the long distances, even if it has not enough seat capacity for the passengers. Therefore, considering the fuel consumption of aircraft together with the seat capacity results in more savings in the fleet assignment problems. However, classical aircraft assignment approaches only try to maximize the match between demand of flights and seat capacity of the assigned aircraft.

3.3 CO_2 Emission Cost

As climate has changed considerably, it has become significant to control the green house gas emissions. The International Civil Aviation Organization ($ICAO$) developed standards for aircraft engine emissions, which are hydrocarbons (HC), carbon monoxide (CO), oxides of nitrogen (NO_x) and smoke. In addition, Swedish taxes were put on carbon dioxide (CO_2) emissions. As a result of these regulations and taxes, airlines put an emphasis on calculation of the aircraft engine emissions.

In the proposed model we are only interested in CO_2 emission calculator. Because the amount of HC emission is negligible when compared to other emissions and smoke vanishes into the air. Although analysis of Boeing (42) indicates that approximately 80% of emissions are NO_x, large amount of it occurs in non-cruise stages. Since we represent the non-cruise time with a distribution, we do not have any control over NO_x emission during the non-cruise stage. In the cruise stage, we speed up the aircraft as necessary; it directly affects CO_2 emissions.

Boeing (43) showed that the amount of CO_2 emission is proportional to the amount of fuel consumed and calculated this amount by Boeing Fuel Flow Method 2. According to EUROCONTROL (44) and ICAO (45), CO_2 emissions are approximately 3.15 times the weight of fuel consumed. Since the fuel consumption is expressed as a function of the cruise time of the flights, the cost of CO_2 emission can also be expressed as a function of the cruise time. It is formulated as follows

$$EmissionCost_i^t\left(f_i^t\right) = c_{CO_2} \cdot k \cdot F_i^t\left(f_i^t\right) \tag{3.12}$$

where c_{CO_2} is the cost of CO_2 emission (\$/kg) and k is CO_2 emission constant.

Controllable cruise time can be preferable than idle time insertion. In order to compensate for the delay due to the high variability of congested airports, we can speed up the aircraft as necessary, if total cost of fuel and CO_2 emission is cheaper than the idle time insertion cost. Therefore, it can be seen that there is also a trade-off between speeding up the aircraft and idle time insertion. To increase the overall saving, a mathematical model requires to decide optimal amount of idle time insertion and adjust the speed of the aircraft. Aircraft type assignment is also crucial, since each aircraft has different fuel consumption and CO_2 emission associated with speeding up the aircraft.

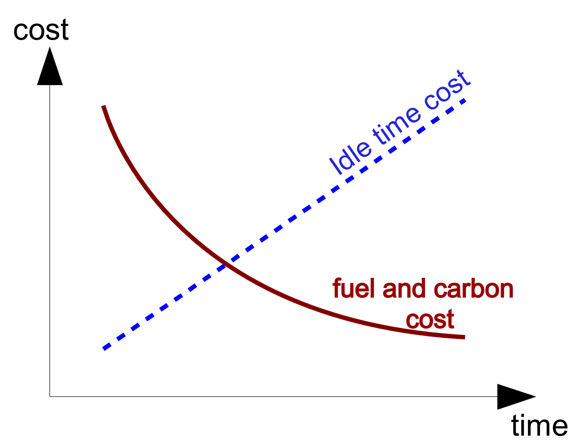

Figure 3.2: Idle Time versus Fuel and CO_2 Emission Cost Functions

In Figure 3.2, you can see the trade-off between speeding up the aircraft and idle time insertion. It is clear that, for an amount of slack that is needed, speeding up the aircraft is cheaper up to a point, and cover the rest of the time with idle time insertion. Note that, this intersection point differs among aircraft types, since each aircraft has different idle time cost and fuel burn rates.

We can represent the CO_2 emissions cost term in the objective via the second order conic inequalities as the fuel consumption cost is represented.

3.4 Service Level

Passengers' connections are taken into account in this study to develop a robust schedule such that misconnections of passengers are minimized when a disruption occurs. Between two flights i and j, if the origin airport of the flight j is same as the destination airport of the flight i and the departure time of flight j is later than the arrival time of the flight i, the time needed for the passengers' connection is TP_{ij}. The percentage of the passengers' connection satisfied between flights (i, j) is represented by the decision variable γ_{ij}.

The overall service level is calculated by the weighted average of the decision variables γ_{ij}. The weights, normalized number of passengers connected between flights (i, j) over whole passengers' connections, are represented by PAS_{ij}. In the model, for each connected flights (i, j), the probability of the departure time of the flight j is greater or equal than the sum of the arrival time of the flight i and passengers' connection time, TP_{ij} should be greater or equal than the service level, γ_{ij}.

3.5 Numerical Example

In this section, we provide a numerical example to give a better explanation of the model mechanics. Fleet type assignment and robust scheduling using controllable cruise time and idle time insertion is shown on a small schedule. Firstly, we give the network of the initial schedule which shows the original idle times and delays. In our approach, a schedule which is less susceptible to delays is generated by using better idle time distribution. It is assumed that, original fleet type assignment is constructed to match the demand of flights to the seat capacity of the aircraft. In our approach, we construct a new fleet type assignment, where the objective is to minimize the fuel cost, CO_2 emission cost, idle time cost and spilled passengers cost. In addition, we generate a new schedule along with the fleet type assignment by adjusting the speed of the aircraft and inserting idle time as necessary. We will also provide the network of the generated schedule which shows inserted idle times and compressed amount of cruise times with no delays.

The small schedule which will be used in the numerical example is given in Table 3.1. It includes only 2 paths operated by 2 different aircraft. The tail numbers of the aircraft are given in the first column, where type of the aircraft N531AA is B767 300 and type of the aircraft N4WPAA is A320 212. In the second column, the flight numbers are given. The following two columns give the information of the origin and destination airport of the flights. The next three columns represent the departure times, block times and arrival times of the flights, respectively. In the next column, actual departure times due to the delays are given. Turnaround times of the aircraft and demand of flights are given in the next two columns. Note that, there exists 2 flights with the same flight number, 336. It represents a through flight which is also called as a flight that includes one or more intermediate airports between the origin and destination airports.

Tail #	Flight #	From	To	Dep.Time	Duration	Arr.Time	Actual Dep.	TA Time	Demand
N531AA	2303	ORD	DFW	7:35	2:05	9:40	7:35	0:53	196
	2336	DFW	ORD	10:40	2:15	12:55	10:41	0:55	162
	1053	ORD	LGA	13:35	3:00	16:35	13:58	0:52	160
	336	LGA	ORD	17:20	3:00	20:20	17:57	0:28	190
	336	ORD	SAN	21:00	4:30	01:30	21:32		180
N4WPAA	2311	ORD	LGA	7:45	2:25	10:10	7:45	0:39	178
	2348	LGA	ORD	11:30	2:25	13:55	11:30	0:41	161
	1797	ORD	DFW	14:00	2:20	16:20	14:43	0:40	168
	1982	DFW	ORD	17:20	2:00	19:20	17:50	0:41	176
	1339	ORD	DFW	20:20	2:10	22:30	20:39		172

Table 3.1: Published Schedule

It can be seen that some of the actual departure times are different than the planned departure times of the flights, and it results in delays in the schedule. There exist some reasons to cause delays. One of them is the variability, which is represented in the congestions of the airport in our study. It may result in more flight block time than the expected, so that the departure time of the following flight with the same aircraft is shifted. In this study, it is assumed that, 20 minutes of the block times are given as the non-cruise time of the flights and the remaining times are given as the cruise time of the flights. For example, the flight block time of flight 1053 is 3 hours, where the 20 minutes represent the non-cruise time and 2 hours and 40 minutes represent the cruise time. However, non-cruise time has an expected value of 27 minutes due to the congestions of the origin and destination airports. This mean of the non-cruise time is calculated as in Equation (3.2) with α parameter of $\ln(20)$ and β parameter of 0.05. The airport congestion coefficients used to calculate this mean are also given in Table 6.4. Another reason of the delay is related to the turnaround time of the aircraft. In the schedule, some time is left between the arrival time of the flight and the departure time of the next flight with the same aircraft. If this time is not enough to prepare the aircraft for the following flight, it results in delay in the departure time of the next flight. If this time is longer than the turnaround time of the aircraft, there exist idle times between flights. It is also important that when a delay occurs on a flight, its effects are also seen on the subsequent flights, if there is not enough idle time to absorb the delay.

The time-space network of the published schedule is given in Figure 3.3. The continuous lines represent the actual departure times of the aircraft, where the dashed lines represent the planned departure times of the flights. The blue and red paths in the figure are for aircraft N531AA with type B767 300 and N4WPAA with type A320 212, respectively. Turnaround times of the aircraft are represented by the continuous ground lines and idle times are represented by the dashed ground lines. After the flight 2311, we can observe 34 minutes unnecessary time. It means that, idle times in the published schedule sometimes may cause the aircraft stand idle and sometimes may not capture the delay time. These delays may result in misconnection of passengers, since passenger connection time is needed to catch the next flight.

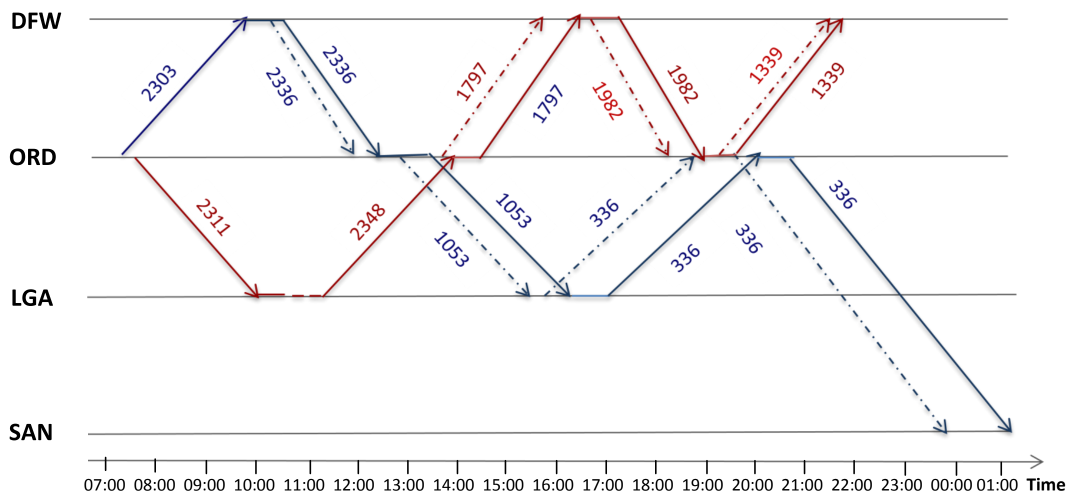

Figure 3.3: Time Space Network for the Published Schedule

The schedule delays need to be avoided. In addition, we can re-allocate the existing idle times so that unnecessary cost is minimized. Another approach is that aircraft can fly faster to compensate for the idle time cost. Fuel cost function is a nonlinear function of the speed of the aircraft. However, idle time cost increases linear with the increase in idle time. Therefore, a balance of speeding up the aircraft and idle time insertion is required to achieve a robust schedule. Moreover, the amount of fuel consumption and CO_2 emission together with the unit idle time cost depends on the aircraft type. The seat capacity of the aircraft is another crucial factor, since some of passengers may not be accommodated due to the insufficient seat capacity of assigned aircraft. On the other hand, assigning a fuel efficient aircraft to the long distances results in more fuel and CO_2 emission cost saving to compensate for the idle time insertion cost as well as the cost of spilled passengers due to the insufficient seat capacity. In order to obtain a better cost saving in total cost of airlines, our approach is to construct a better fleet type assignment with a robust schedule using controllable cruise times and idle time insertion, denoted as FA-RS.

The new schedule achieved by FA-RS is provided in Figure 3.4. In the new schedule, passengers' connection service level is taken same as the original service level. Thus, we can compare the results of the new schedule to the results of the initial schedule at the same service level corresponding to 94% realized

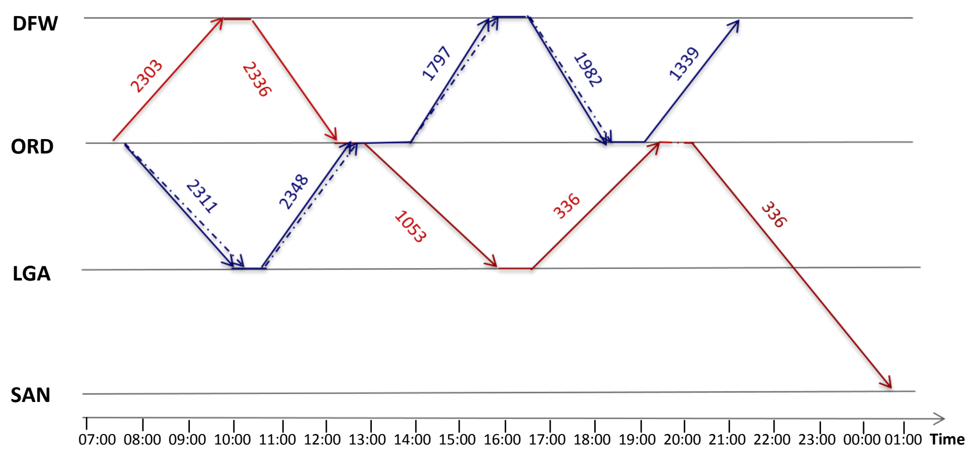

Figure 3.4: Time Space Network - After FA-RS

connections over all possible passengers' connections. Passenger connected flight pairs are 336-336 and 1982-336. In this study, passenger connections are possible between two flights i and j, if the original departure time of flight j is within 45 minutes or 180 minutes of the original arrival time of flight i and destination airport of flight i is same as the origin airport of the flight j. Turn back in a one way is not possible for the passenger connections.

We compare the performance of the new schedule to the initial schedule in terms of the improvement in total cost, which is the sum of the fuel consumption and CO_2 emission costs, idle time cost and spilled passengers cost. In addition, delay costs are added to the total cost of the original schedule. However, delays are not observed in the new schedule. Fuel consumption and CO_2 emission costs are calculated as explained in Equations (3.10) and (3.12), respectively. Idle time costs are calculated by multiplying the total idle time with the unit idle time cost of the aircraft, which is given in Table 6.3. The cost of spilled passengers are also calculated as multiplying the unit cost, which is adjusted using airport congestions level, with the total number of spilled passengers as in Equation (6.1). Lastly, delay cost of the original schedule is calculated in the same way, by multiplying the unit delay cost, 200$\$/min$ with the total delay times in the schedule. The costs of the published schedule and costs of the new schedule generated by FA-RS approach are given in Tables (3.2) and (3.3), respectively.

Tail No	Flight No	Fuel Cost	CO_2 Cost	Idle Cost	Delay Cost	Spilled Cost
N531AA	2303	10,936	576	0	0	0
	2336	11,978	631	0	123	0
	1053	16,665	878	0	4,645	0
	336	16,665	878	0	7,378	0
	336	26,038	1,371	0	6,312	0
N4WPAA	2311	6,509	343	4,944	0	0
	2348	6,509	343	0	0	0
	1797	6,249	329	0	8,553	0
	1982	5,208	274	0	6,036	0
	1339	5,728	302	0	3,818	0
Total		112,485	5,924	4,944	36,865	0

Table 3.2: Cost Calculation for Published Schedule

Tail No	Flight No	Fuel Cost	CO_2 Cost	Idle Cost	Delay Cost	Spilled Cost
N531AA	2303	5,518	291	0	0	434
	2336	6,044	318	0	0	0
	1053	8,410	443	0	0	0
	336	8,410	443	1,097	0	267
	336	13,134	692	0	0	0
N4WPAA	2311	13,015	685	0	0	0
	2348	13,015	685	0	0	0
	1797	12,495	658	0	0	0
	1982	10,412	548	0	0	0
	1339	11,353	598	0	0	0
Total		101,805	5,362	1,097	0	701

Table 3.3: Cost Calculation for FA-RS

Furthermore, the percentages improvement in cost terms of the schedule with FA-RS compared to the original schedule are calculated using the following formula:

$$\text{Cost Improvement} = 100 \times \frac{\text{Original Schedule - Schedule with FA-RS}}{\text{Original Schedule}}$$

When we analyze the new schedule, it is important to mention that the assignment of aircraft type among two paths is switched. In the new schedule, types of the aircraft N531AA and N4WPAA are A320 212 and B767 300, respectively. The red and blue paths in the Figure 3.4 are for the aircraft N531AA and N4WPAA, respectively. With this assignment, 22 passengers of total 1743 passengers are spilled due to the insufficient seat capacity of A320 212 type of the aircraft, which results in 701\$. In exchange for the cost of spilled passengers, there exists a 9% cost saving in fuel consumption and CO_2 emissions compared to the cost of initial schedule. Total fuel and CO_2 emission costs for the published schedule and new schedule with FA-RS are 118,409\$ and 107,169\$, respectively. The new fleet type assignment considers the fuel efficiency of the aircraft, so that improvement in the cost of fuel consumption and CO_2 emissions compensates for the cost of spilled passengers.

Cruise time durations of the flights 2311, 2348, 1797 and 1982 are compressed by the amount of 9, 9, 8 and 7 minutes, respectively. The reason of speeding up the aircraft is that, passengers of the flight 1982 have a connection to the second leg of the flight 336. Instead of inserting idle time before the flight 336, cruise times are compressed to ensure the passengers' connections, since speeding of the aircraft up to a point is cheaper than the idle time insertion. Another issue about the speeding is that total compression amount, 23 minutes are not imposed upon the one flight, since cost functions of the fuel consumption and CO_2 emission are nonlinear function of the aircraft speed.

In the new schedule, there exists one idle time slot inserted after the first leg of the flight 336 with 8 minutes. In the original schedule, after the delay is observed, there exist 34 minutes slack time after the flight 2311. It follows the fact that, new schedule results in 77% idle time cost improvement, where the costs of the idle time in the published schedule and new schedule are $4,944 and $1,097, respectively.

The overall results show that, total cost improvement is around 12% when we compare the cost of new schedule to the cost of initial schedule without considering the delay cost. When the delay cost is taken into consideration, there occurs a 31% cost savings compared to the initial schedule, where the total cost of the new schedule and published schedule are $108,965 and $160,218, respectively.

The model given in the following chapter works with these mechanics. The objective is to minimize the total airline costs in terms of fuel consumed, CO_2 emission, idle time and spilled passengers. The fleet type assignment is constructed along with the robust schedule generation by adjusting the speed of the aircraft and idle time insertion, such that significant cost savings are achieved in exchange for spilled passengers.

3.6 Summary

In this chapter, the problem definition is provided in detail. Parameter and decision variables of the problem are described. Some properties of the random variable, fuel consumption cost function and CO_2 emission cost functions are clearly explained. Moreover, to observe how the model works, a numerical example on a small initial schedule is provided.

Chapter 4

Problem Formulation

Basic fleet type assignment model tries to minimize the cost of spilled passengers due to the insufficient seat capacity of the aircraft by matching demand of flights to the seat capacity of the assigned aircraft. If it is the case that the opportunity cost of unsatisfied passengers is greater than the cost of fuel and CO_2 emission, assignment of larger aircraft is favorable. However, it is sometimes more profitable to assign the fuel efficient aircraft, having smaller seat capacity, to compensate for the spill cost of passengers.

Moreover, it is difficult to determine how much idle time insertion is required to deal with the variability at airports. We could also tackle with increasing the speed of the aircraft during the cruise stage to compensate for delays. Therefore, balancing the idle time insertion and speeding up the aircraft is a complex problem required a global optimization tools, even when the significant effects of the aircraft assignment on that balancing are considered too.

Therefore, we developed a mathematical model which determines the optimal aircraft type assignment to the routes together with the decisions of cruise times, idle times and departure times of flights by ensuring the overall desired service level. Sequence of flights in the routes, aircraft and passengers' connection information are taken as inputs.

4.1 Mathematical Model

In order to obtain the optimal policy for the large scale problems, we developed a mixed integer nonlinear programming formulation which includes chance constraints and nonlinear cost function. For each $p \in P, i \in J_p, t \in T$, we redefine the fuel cost and CO_2 emission cost as:

$$C_{fuel}^{i,t}(f_i^t) = \begin{cases} c_{fuel} \cdot (c_1^{i,t}\frac{1}{f_i^t} + c_2^{i,t}\frac{1}{(f_i^t)^2} + c_3^{i,t}(f_i^t)^3 + c_4^{i,t}(f_i^t)^2) & \text{if } z_p^t = 1 \\ 0 & \text{if } z_p^t = 0 \end{cases}$$

$$C_{CO_2}^{i,t}(f_i^t) = \begin{cases} c_{CO_2} \cdot k \cdot (c_1^{i,t}\frac{1}{f_i^t} + c_2^{i,t}\frac{1}{(f_i^t)^2} + c_3^{i,t}(f_i^t)^3 + c_4^{i,t}(f_i^t)^2) & \text{if } z_p^t = 1 \\ 0 & \text{if } z_p^t = 0 \end{cases}$$

so that if aircraft type t is not assigned to path p, then $C_{fuel}^{i,t}(f_i^t) = 0$ and $C_{CO_2}^{i,t}(f_i^t) = 0$.

The proposed nonlinear mathematical model is provided below:

$$\begin{aligned} \min \quad F1 : \quad & \sum_{i \in J_p} \sum_{t \in T} C_{fuel}^{i,t}\left(f_i^t\right) + \sum_{i \in J_p} \sum_{t \in T} C_{CO_2}^{i,t}\left(f_i^t\right) \\ & + \sum_{i \in J} Csp_i\left(D_i - NumPass_i\right) + \sum_{i \in J} \sum_{t \in T} S_i^t I_i^t \qquad (4.1) \\ \max \quad F2 : \quad & \sum_{i \in J} \sum_{j \in P_i} PAS_{ij} \cdot \gamma_{ij} \qquad (4.2) \end{aligned}$$

$$
\text{s.to} \quad \sum_{t \in T} min\left(CAP^t, D_i\right) \cdot z_p^t \geq NumPass_i \qquad p \in P, i \in J_p \tag{4.3}
$$

$$
\sum_{t \in T} z_p^t = 1 \qquad p \in P \tag{4.4}
$$

$$
\sum_{p \in P} z_p^t \leq N^t \qquad t \in T \tag{4.5}
$$

$$
Pr\left[A_i + \sum_{t \in T} f_i^t \leq x_j - x_i - TP_{ij}\right] \geq \gamma_{ij} \qquad i \in J, j \in P_i \tag{4.6}
$$

$$
f_i^{t,l} \cdot z_p^t \leq f_i^t \leq f_i^{t,u} \cdot z_p^t \qquad p \in P, i \in J_p, t \in T \tag{4.7}
$$

$$
x_j - x_i - \sum_{t \in T} TA_{ij}^t \cdot z_p^t - \sum_{t \in T} f_i^t - E\left[A_i\right] - \sum_{t \in T} S_i^t = 0 \qquad (i,j) \in PAIR \tag{4.8}
$$

$$
w_i \leq x_i \leq v_i \qquad i \in J \tag{4.9}
$$

$$
S_i^t \leq M \cdot z_p^t \qquad p \in P, i \in J_p, t \in T \tag{4.10}
$$

$$
NumPass_i \geq 0 \qquad i \in J \tag{4.11}
$$

$$
\gamma_{ij} \geq \gamma_{ij}^d \qquad i \in J, j \in P_i \tag{4.12}
$$

$$
S_i^t \geq 0 \qquad i \in J, t \in T \tag{4.13}
$$

$$
z_p^t \in \{0,1\} \qquad p \in P, t \in T \tag{4.14}
$$

The most common objective of the airlines is to minimize the total operating cost. However, a generated schedule that minimizes the total cost may not ensure a high service level of passengers' connections. Thus, in this study, we consider a bicriteria problem, where the objectives are jointly minimization of the total cost of airlines and maximization of the passengers' connection service level through the entire network. The sum of the cost of the fuel consumption, CO_2 emission, spilled passengers and idle time insertion over all flights in the network constitute the total airline cost. Moreover, passengers' connection service level is the weighted average of the service level of all passenger connections in the network. Constraint (4.3) is satisfied as equality, which means that the number of passengers for flight i is set to minimum value of demand of flight i and the capacity of aircraft assigned to the flight i, because the objective function $F1$ tries to minimize the cost of unsatisfied passengers. In (4.4), it is guaranteed that each route is assigned to exactly one aircraft type. In (4.5), total number of utilized aircraft type t cannot be larger than available number of aircraft type t. In (4.6), we require the probability, that the difference between departure time of flight j and arrival time of flight i is less than the turntime for passenger connection, to be greater or equal to service level variable associated with connected set of flights, P_i. In (4.7), we allow the cruise time to be at most the ideal block time determined by max-range cruise speed and the lower bound is determined by the maximum allowable compression amount on the ideal block time duration. In (4.8), we guarantee that there exists a minimum turnaround time between the departure and arrival time of two consecutive flights by the same aircraft using the idle time insertion, if it is necessary. In (4.9), we desire the departure time of each flight to be in given time window. We added the constraint (4.10) in order to eliminate the possible nonlinearity due to the multiplication of idle time of the aircraft with the decision variable of aircraft type assignment. Whenever z_p^t takes the value of zero, we force S_i^t to take the value zero too. For each passenger connection, we want the percentage of the satisfied passengers' connected be greater or equal than the desired upper bound, γ_{ij}^d in (4.12). The remaining constraints define the range of the variables.

There are different methods to handle the bicriteria problems. In this study, we use the epsilon-constraint method which is discussed in T'kindt et al. (46). In this approach, we will solve the problem minimizing $F1$ given a lower bound on $F2$. Thus, we add the following service level bound constraints into the proposed model.

$$\sum_{i \in J} \sum_{j \in P_i} PAS_{ij}.\gamma_{ij} \geq \gamma \tag{4.15}$$

In (4.15), we want the overall service level be greater or equal than desired service level, γ.

4.1.1 Challenges for Solving the Model

There occurs nonlinearity in the objective function due to the fuel consumption and CO_2 emission cost functions. In addition, there are chance constraints representing the random variable of non-cruise time. Earlier studies handled the chance constraints by linear approximations or dual algorithm, however our aim is to solve them in an exact form. We first transformed chance constraints into closed forms then transformed them into the second order conic inequalities. Besides, the nonlinearity in the objective function was also tackled by the second order conic inequalities. Even if the nonlinear objective function involves binary variables, second order conic inequalities with the binary variables are handled. In the following section, we explain the conic reformulation of the nonlinear objective function and chance constraints.

4.2 Conic Reformulation of the Model

Conic reformulation of the model provides an exact solution for chance constraints and nonlinear objective functions as opposed to approximation methods. Using second order cone programming, the conic reformulation is achieved by representing the nonlinear objective term and chance constraints with second order conic inequalities. To transform the chance constraints into the conic inequalities, we first expressed the chance constraints with closed forms. Afterwards, the closed form is transformed to the second order conic inequalities.

4.2.1 Closed Form Expressions for the Chance Constraints

The uncertainty coming from a random variable of non-cruise time arises in (probabilistic) chance constraints (4.6) to guarantee passengers' connection service level. It is assumed to be log-Laplace distribution. In order to represent the chance constraints with second order conic inequalities, the closed form of the quantile function of the log-Laplace distribution should be obtained. Duran et al. (39) derived a closed form expression for the quantile of order γ_{ij} for the log-Laplace distribution with parameters α and β_i as the following:

$$F^{-1}(\gamma_{ij}) = \begin{cases} 2^{\beta_i} \cdot e^{\alpha} \cdot \gamma_{ij}^{\beta_i}, & \text{if } 0 \leq \gamma_{ij} \leq \frac{1}{2} \\ \frac{e^{\alpha}}{2^{\beta_i} \cdot (1-\gamma_{ij})^{\beta_i}}, & \text{if } \frac{1}{2} \leq \gamma_{ij} \leq 1 \end{cases}$$

Also remember the chance constraints in the model:

$$Pr[A_i \leq x_j - x_i - TP_{ij} - \sum_{t \in T} f_i^t] \geq \gamma_{ij} \qquad i \in J, j \in P_i$$

They can be expressed using quantile function of the probability distribution of random variable A_i. The expression as follows,

$$x_j - x_i - TP_{ij} - \sum_{t \in T} f_i^t \geq F^{-1}(\gamma_{ij}) \qquad i \in J, j \in P_i$$

When we plug the closed form expression of quantile order of γ_{ij}, the expression becomes as following,

$$\begin{aligned} 2^{\beta_i} \cdot \gamma_{ij}^{\beta_i} \cdot e^{\alpha} \;&\leq\; x_j - x_i - TP_{ij} - \sum_{t \in T} f_i^t, \qquad \text{if} \quad 0 \leq \gamma_{ij} \leq \frac{1}{2}, \text{ i} \in J, j \in P_i \\ \frac{e^{\alpha}}{2^{\beta_i} \cdot (1-\gamma_{ij})^{\beta_i}} \;&\leq\; x_j - x_i - TP_{ij} - \sum_{t \in T} f_i^t, \qquad \text{if} \quad \frac{1}{2} \leq \gamma_{ij} \leq 1, \text{ i} \in J, j \in P_i \end{aligned}$$

Duran et al. (39) proved that the quantile function of non-cruise time, which is represented with the Log-laplace distribution, is convex when the passengers' connection service level is greater or equal than 50%. It is a desirable level in our model as well. They also proved that the mean of the log-Laplace random variable A_i is finite, only if $\beta_i < 1$. Therefore, we assumed that $\beta_i < 1$ to obtain the finite mean in our model, and transformed the chance constraints into second order conic inequalities under this assumption.

4.2.2 Conic Representation of the Chance Constraints

In the proposed model, random variable representing the non-cruise time of flights arises in chance constraints. It is shown that probabilistic constraints can be reformulated via second order conic constraints.

Proposition 4.1. *For* $i \in J, j \in P_i$, $(x_j - x_i - TP_{ij} - \sum_{t \in T} f_i^t) \geq F^{-1}(\gamma_{ij})$ *is SOCP representable if* $0 < \beta_i < 1$ *and* $\frac{1}{2} \leq \gamma_{ij} \leq 1$.

Proof. Replace

$$(x_j - x_i - TP_{ij} - \sum_{t \in T} f_i^t) \geq F^{-1}(\gamma_{ij})$$

in problem with

$$(x_j - x_i - TP_{ij} - \sum_{t \in T} f_i^t) \geq \frac{e^{\alpha}}{2^{\beta_i} \cdot (1 - \gamma_{ij})^{\beta_i}}$$

In our study, we work on the case where $\gamma_{ij} \geq \frac{1}{2}$. This lower bound is applied to achieve the convexity of the constraint as shown in Duran et. al (39) in Proposition 4.2. This is second order conic representable.

First, introduce two auxiliary variables $\sigma_{ij} \geq 0$ and $\overline{\gamma}_{ij} \geq 0$ and denote them as follows,

$$\overline{\gamma_{ij}} = 1 - \gamma_{ij} \qquad \text{i} \in J, j \in P_i \tag{4.16}$$

$$x_j - x_i - TP_{ij} - \sum_{t \in T} f_i^t = \sigma_{ij} \qquad \text{i} \in J, j \in P_i \tag{4.17}$$

Let the constant $\lambda = \frac{e^{\alpha}}{2^{\beta_i}}$. Then we can write:

$$\lambda \leq \sigma_{ij} \overline{\gamma_{ij}}^{\beta_i} \qquad i \in J, j \in P_i$$

β_i can be expressed as $\frac{a_i}{b_i}$ for integers a_i and b_i. This is written as:

$$\lambda^{b_i} \leq \sigma_{ij}^{b_i} \overline{\gamma}_{ij}^{a_i} \qquad i \in J, j \in P_i$$

Next, define $l_i = \lceil \log_2 (a_i + b_i) \rceil$

Then we can write:

$$\left(\sigma_{ij}^{b_i} \cdot \overline{\gamma}_{ij}{}^{a_i}\right)^{\frac{1}{2^l}} \geq \sqrt[2^l]{\lambda^{b_i}} \qquad \text{i} \in J, j \in P_i \qquad (4.18)$$

Due to Ben-Tal and Nemirovski (1), the hypograph of the geometric mean of 2^l variables is representable via the second order conic inequalities. In (4.18), it can be seen that b_i of the variables equal to σ_{ij}, a_i of the variables equal to $\overline{\gamma}_{ij}$ and the remaining $2^l - a_i - b_i$ variables can be set to 1. Hence, it is clearly observed that constraint (4.18) represents the hypograph of the geometric mean of 2^l variables. According to Ben-Tal and Nemirovski (1), the hypograph can be equivalently represented by hyperbolic inequalities of the form,

$$u^2 \leq v_1 v_2, \qquad u, v_1, v_2 \geq 0$$

which can be represented by the second order conic inequality below

$$\|(2u, v_1 - v_2)\| \leq v_1 + v_2$$

that concludes the proof. □

4.2.3 Conic Representation of the Fuel and CO_2 Emission Cost Functions

In the objective function, the cost functions involve nonlinearity due to controllable cruise time associated with the speeding up the aircraft. To handle nonlinearity, nonlinear mixed integer optimization often requires too much computation time. On the other hand, it may not result in exact solutions. In order to shorten the solution time and obtain an optimal solution, in this section we show the conic quadratic reformulation of the cost function as discussed in Aktürk et al. (38) and Günlük and Linderoth (2). To simplify the presentation, we drop the indices of the variables and parameters.

We combine the fuel cost and CO_2 emission cost functions and redefine them as:

$$K(f) = \begin{cases} (c_{fuel} + c_{CO_2} \cdot k)(c_1 \frac{1}{f} + c_2 \frac{1}{f^2} + c_3 f^3 + c_4 f^2) & \text{if } z = 1 \\ 0 & \text{if } z = 0 \end{cases}$$

$K(f)$ is discontinuous and therefore its epigraph $E_F = \{(f,t) \in R^2 : K(f)\}$ is nonconvex. In the next proposition, we describe how the convexity of E_F is obtained. A more detailed information can be found in Aktürk et al. (38) and Günlük and Linderoth (2).

Proposition 4.2. *The convex hull of E_F can be expressed as*

$$t \geq (c_{fuel} + k.c_{CO_2})(c_1 \cdot q + c_2 \cdot \delta + c_3 \cdot \varphi + c_4 \cdot \vartheta) \quad (4.19)$$

$$z^2 \leq q \times f \quad (4.20)$$

$$z^4 \leq f^2 \times \delta \times 1 \quad (4.21)$$

$$f^4 \leq z^2 \times \varphi \times f \quad (4.22)$$

$$f^2 \leq \vartheta \times z \quad (4.23)$$

in the constraint set. Moreover, each inequalities (4.20)-(4.23) *can be represented by conic quadratic inequalities.*

Proof. Perspective of a convex function $k(f)$ is $zk(f/z)$ (Hiriart-Urruty and Lemaréchal (47)). Since each of the nonlinear terms $\frac{1}{f}$, $\frac{1}{f^2}$, f^3 and f^2 is a convex function for $f \geq 0$, then epigragh of the perspective of each term can be stated as

$$\frac{z^2}{f} \leq q$$
$$\frac{z^4}{f^2} \leq \delta$$
$$\frac{f^3}{z^2} \leq \varphi$$
$$\frac{f^2}{z} \leq \vartheta$$

respectively. Since $z, f \geq 0$, they can be written as stated in the proposition.

Finally, observe that (4.20) and (4.23) are hyperbolic inequalities, (4.21) can be restated as two hyperbolic inequalities

$$z^2 \leq wf \qquad \text{and} \qquad w^2 \leq \delta.1$$

and (4.22) can be restated as

$$f^2 \leq wz \qquad \text{and} \qquad w^2 \leq \varphi.f$$

which can be written as a conic quadratic inequality as described in Section 4.2.2.

□

In the following section, we will explicitly restate each of the constraints (4.20)-(4.23) as second order conic inequalities.

For constraint (4.20)**:** Introduce two auxilary variables W_1 and $W_2 \geq 0$ and define them as follows,

$$\begin{aligned} W_1 &= q_i^t - f_i^t && \text{i} \in J_p, p \in P, t \in T && (4.24)\\ W_2 &= q_i^t + f_i^t && \text{i} \in J_p, p \in P, t \in T && (4.25)\\ 4\big(z_p^t\big)^2 &\leq (W_2)^2 - (W_1)^2 && \text{i} \in J_p, p \in P, t \in T && (4.26) \end{aligned}$$

For constraint (4.21)**:** It can be rewritten as follows,

$$\left(Q_i^t\right)^2 \leq \delta_i^t \times 1 \qquad \text{i} \in J_p, p \in P, t \in T \tag{4.27}$$

$$z_p^{t\,2} \leq Q_i^t \times f_i^t \qquad \text{i} \in J_p, p \in P, t \in T \tag{4.28}$$

Let, introduce two auxilary variables W_3 and $W_4 \geq 0$ and denote them as follows,

$$W_3 = \delta_i^t - 1 \qquad \text{i} \in J_p, p \in P, t \in T \tag{4.29}$$

$$W_4 = \delta_i^t + 1 \qquad \text{i} \in J_p, p \in P, t \in T \tag{4.30}$$

Then, the constraint (4.27), can be rewritten so as,

$$4\left(Q_i^t\right)^2 \leq (W_4)^2 - (W_3)^2 \qquad i \in J_p, p \in P, t \in T \tag{4.31}$$

Let, introduce two auxilary variables W_5 and $W_6 \geq 0$ and denote them as follows,

$$W_5 = Q_i^t - f_i^t \qquad \text{i} \in J_p, p \in P, t \in T \tag{4.32}$$

$$W_6 = Q_i^t + f_i^t \qquad \text{i} \in J_p, p \in P, t \in T \tag{4.33}$$

Then, the constraint (4.28), can be rewritten so as,

$$4\left(z_p^t\right)^2 \leq (W_6)^2 - (W_5)^2 \qquad i \in J_p, p \in P, t \in T \tag{4.34}$$

For constraint (4.22)**:** It can be rewritten as follows,

$$\begin{aligned} \left(Q_i^t\right)^2 &\leq \varphi_i^t \times f_i^t && \text{i} \in J_p, p \in P, t \in T && (4.35) \\ f_i^{t2} &\leq Q_i^t \times z_p^t && \text{i} \in J_p, p \in P, t \in T && (4.36) \end{aligned}$$

Let, introduce two auxilary variables W_7 and $W_8 \geq 0$ and denote them as follows,

$$\begin{aligned} W_7 &= \varphi_i^t - f_i^t && \text{i} \in J_p, p \in P, t \in T && (4.37) \\ W_8 &= \varphi_i^t + f_i^t && \text{i} \in J_p, p \in P, t \in T && (4.38) \end{aligned}$$

Then, the constraint (4.35), can be rewritten so as,

$$4\left(Q_i^t\right)^2 \leq (W_8)^2 - (W_7)^2 \qquad i \in J_p, p \in P, t \in T \qquad (4.39)$$

Let, introduce two auxilary variables W_9 and $W_{10} \geq 0$ and denote them as follows,

$$\begin{aligned} W_9 &= Q_i^t - z_p^t && \text{i} \in J_p, p \in P, t \in T && (4.40) \\ W_{10} &= Q_i^t + z_p^t && \text{i} \in J_p, p \in P, t \in T && (4.41) \end{aligned}$$

Then, the constraint (4.36), can be rewritten so as,

$$4\left(f_i^t\right)^2 \leq (W_{10})^2 - (W_9)^2 \qquad i \in J_p, p \in P, t \in T \qquad (4.42)$$

For constraint (4.23)**:** Introduce two auxilary variables W_{11} and $W_{12} \geq 0$ and define them as follows,

$$
\begin{aligned}
W_{11} &= \vartheta_i^t - z_p^t && \text{i} \in J_p, p \in P, t \in T && (4.43)\\
W_{12} &= \vartheta_i^t + z_p^t && \text{i} \in J_p, p \in P, t \in T && (4.44)\\
4\left(f_i^t\right)^2 &\leq (W_{12})^2 - (W_{11})^2 && \text{i} \in J_p, p \in P, t \in T && (4.45)
\end{aligned}
$$

4.2.4 Conic Formulation of the Model

After the representation of the chance constraints and nonlinear cost terms as second order conic inequalities, the model becomes:

$$
\begin{aligned}
\min \quad & \sum_{i \in J} \sum_{t \in T} c_{fuel} \cdot \left(c_1 \cdot q_i^t + c_2 \cdot \delta_i^t + c_3 \cdot \varphi_i^t + c_4 \cdot \vartheta_i^t\right) \\
& + \sum_{i \in J} \sum_{t \in T} c_{CO_2} \cdot k \cdot \left(c_1 \cdot q_i^t + c_2 \cdot \delta_i^t + c_3 \cdot \varphi_i^t + c_4 \cdot \vartheta_i^t\right) \\
& + \sum_{i \in J} Csp_i \left(D_i - NumPass_i\right) + \sum_{i \in J} \sum_{t \in T} S_i^t I_i^t \qquad (4.46)
\end{aligned}
$$

$$
\begin{aligned}
\text{s.to} \quad \sum_{t \in T} min\left(CAP^t, D_i\right) \cdot z_p^t \geq NumPass_i & \qquad p \in P, i \in J_p & (4.47)\\
\sum_{t \in T} z_p^t = 1 & \qquad p \in P & (4.48)\\
\sum_{p \in P} z_p^t \leq N^t & \qquad t \in T & (4.49)\\
\left(z_p^t\right)^2 \leq q_i^t \times f_i^t & \qquad i \in J_p, p \in P, t \in T & (4.50)\\
\left(z_p^t\right)^4 \leq \left(f_i^t\right)^2 \times \delta_i^t \times 1 & \qquad i \in J_p, p \in P, t \in T & (4.51)\\
\left(f_i^t\right)^4 \leq \left(z_p^t\right)^2 \times \varphi_i^t \times f_i^t & \qquad i \in J_p, p \in P, t \in T & (4.52)\\
\left(f_i^t\right)^2 \leq \vartheta_i^t \times z_p^t & \qquad i \in J_p, p \in P, t \in T & (4.53)\\
{\sigma_{ij}}^{b_i} \cdot {\overline{\gamma}_{ij}}^{a_i} \geq (\sqrt[2^l]{\lambda^{b_i}})^{2^l} & \qquad i \in J, j \in P_i & (4.54)\\
x_j - x_i - TP_{ij} - \sum_{t \in T} f_i^t = \sigma_{ij} & \qquad i \in J, j \in P_i & (4.55)\\
\overline{\gamma}_{ij} = 1 - \gamma_{ij} & \qquad i \in J, j \in P_i & (4.56)\\
\sum_{i \in J} \sum_{j \in P_i} PAS_{ij} \cdot \gamma_{ij} \geq \gamma & & (4.57)\\
f_i^{t,l} \cdot z_p^t \leq f_i^t \leq f_i^{t,u} \cdot z_p^t & \qquad p \in P, i \in J_p, t \in T & (4.58)\\
x_j - x_i - \sum_{t \in T} TA_{ij}^t \cdot z_p^t - \sum_{t \in T} f_i^t - E\left[A_i\right] - \sum_{t \in T} S_i^t = 0 & \qquad (i,j) \in PAIR & (4.59)\\
w_i \leq x_i \leq v_i & \qquad i \in J & (4.60)\\
S_i^t \leq M \cdot z_p^t & \qquad p \in P, i \in J_p, t \in T & (4.61)\\
NumPass_i \geq 0 & \qquad i \in J & (4.62)\\
\gamma_{ij} \geq \gamma_{ij}^d & \qquad i \in J, j \in P_i & (4.63)\\
S_i^t \geq 0 & \qquad i \in J, t \in T & (4.64)\\
z_p^t \in \{0,1\} & \qquad p \in P, t \in T & (4.65)
\end{aligned}
$$

The objective function (4.46) is slightly different than the original objective function of the proposed model. The original objective, $F1$ is represented by the new objective and conic constraints (4.50)-(4.53). Maximization of $F2$ is satisfied by a lower bound constraint (4.57) which is formed by the epsilon-constraint method. The probabilistic constraints, (4.6) are represented by the conic constraints (4.54)-(4.56). The remaining constraints (4.47)-(4.49) and (4.58)-(4.65) are same as the original constraints of the proposed model.

We can now solve this mathematical model and get an exact solution with commercial solvers due to the proposed second order conic formulation.

4.3 Summary

In this chapter, MINLP formulation of the problem is given. However, it is a complex bicriteria problem involving probabilistic constraints and nonlinear cost terms in the objective. Firstly, epsilon-constraint method is used to handle the bicriteria problem. Secondly, to obtain an exact solution in acceptable times, the model is reformulated by using second order cone programming.

The nonlinear cost functions of fuel consumption and CO_2 emissions in the objective are handled by representing them via second order conic inequalities. Moreover, using the closed form expression of the probabilistic constraints, they are also expressed by second order conic inequalities.

Chapter 5

Algorithm for Fleet Assignment and Robust Airline Scheduling

We solved a MINLP model for the fleet assignment and robust airline scheduling with chance constraints by using second order cone programming. As opposed to approximations and relaxations methods, we developed an exact method by transforming the chance constraints and nonlinear objective terms into the second order conic inequalities. We can solve the reformulated model for the airline network involving approximately 40 flights in a reasonable time as will be demonstrated in the next section. But, when a large scale network is considered, the problem becomes more complex due the increased number of binary variables and conic constraints. We can still handle the nonlinear cost terms and chance constraints using second order cone programming, but, computational time issues may arise. Therefore, to solve the large scale problems in less time than the proposed model, an algorithm which simplifies the problem is needed, called as two-stage algorithm.

5.1 Proposed Two-Stage Algorithm

One possible approach to simplify the proposed model is to decompose the problem into planning stages, such as robust airline scheduling with chance constraints and fleet assignment, then solve them sequentially. First, robust airline scheduling problem is solved. Then, decisions of the robust scheduling problem impose upon the fleet assignment model to decide the optimal types of aircraft of the scheduled flights. This approach unfortunately eliminates the dependency between two stages. Each different schedule may result in different optimal fleet type assignment. On the other hand, each fleet type assignment has different optimal schedule, since each aircraft type has different max range cruise speed minimizing the fuel cost. We cannot quantify how these dependencies affect the fleet type assignment and departure times for flights. Therefore, making a decision for planning stages sequentially might lead to higher overall cost than the proposed model.

The proposed algorithm is formed by construction and improvement parts. For the construction part, our approach is to solve the aircraft assignment model and robust airline scheduling model iteratively until no further reduction in total cost can be attained. Firstly, it is assumed that an initial schedule is given. Using the information of cruise time durations, $\vec{f}$, and idle times of flights, $\vec{S}$ in the initial schedule, we compute the total cost involving the cost of fuel consumption, CO_2 emission, spilled passengers and idle time for each path and for each aircraft type as the following:

$$\begin{aligned} TC_p^t(\vec{f},\vec{S}) = & \sum_{i \in J_p} (c_{fuel} + c_{CO_2} \cdot k \cdot)(c_1^{i,t} \frac{1}{f_i^t} + c_2^{i,t} \frac{1}{(f_i^t)^2} + c_3^{i,t} (f_i^t)^3 + c_4^{i,t} (f_i^t)^2) \\ & + \sum_{i \in J_p} Csp_i (D_i - min(CAP^t, D_i)) + \sum_{i \in J_p} S_i \cdot I^t \end{aligned} \tag{5.1}$$

Then, we solve the following aircraft assignment model (AAM) for given cruise times and idle time durations of the generated schedule.

$$AAM(\vec{f},\vec{S}): \qquad \min \quad \sum_{p\in P}\sum_{t\in T} TC_p^t(\vec{f},\vec{S}) \cdot z_p^t \tag{5.2}$$

$$\text{s.to} \qquad \sum_{t\in T} z_p^t \;=\; 1 \quad p \in P \tag{5.3}$$

$$\sum_{p\in P} z_p^t \;\leq\; N^t \quad t \in T \tag{5.4}$$

$$z_p^t \;\in\; \{0,1\} \quad p \in P, t \in T \tag{5.5}$$

The best fleet type assignment sequence $\vec{z}_1$, representing the assignment type information for all paths, is imposed upon the Robust Airline Scheduling Model (RASM). Optimal cruise time durations $\vec{f}$ and idle time durations $\vec{S}$ are determined by solving the RASM, where the objective is to minimize the costs of fuel consumption, CO_2 emission and idle time. RASM can be formulated as the following:

$$\begin{aligned} RASM(\vec{z}): \min \qquad & \sum_{i\in J_p}\sum_{t\in T} C_{fuel}^{i,t}\left(f_i^t\right) + \sum_{i\in J_p}\sum_{t\in T} C_{CO_2}^{i,t}\left(f_i^t\right) \\ & + \sum_{i\in J_p} Csp_i(D_i - min(CAP^t, D_i)) + \sum_{i\in J}\sum_{t\in T} S_i^t I_i^t \end{aligned} \tag{5.6}$$

$$\text{s.to} \quad Pr\left[A_i + \sum_{t\in T} f_i^t \le x_j - x_i - TP_{ij}\right] \ge \gamma_{ij} \qquad i \in J, j \in P_i \tag{5.7}$$

$$\sum_{i\in J}\sum_{j\in P_i} PAS_{ij}\cdot\gamma_{ij} \ge \gamma \tag{5.8}$$

$$f_i^{t,l}\cdot z_p^t \le f_i^t \le f_i^{t,u}\cdot z_p^t \qquad p\in P, i\in J_p, t\in T \tag{5.9}$$

$$x_j - x_i - \sum_{t\in T} TA_{ij}^t\cdot z_p^t - \sum_{t\in T} f_i^t - E\left[A_i\right] - \sum_{t\in T} S_i^t = 0 \qquad (i,j)\in PAIR \tag{5.10}$$

$$w_i \le x_i \le v_i \qquad i\in J \tag{5.11}$$

$$S_i^t \le M\cdot z_p^t \qquad p\in P, i\in J_p, t\in T \tag{5.12}$$

$$\gamma_{ij} \ge \gamma_{ij}^d \qquad i\in J, j\in P_i \tag{5.13}$$

$$S_i^t \ge 0 \qquad i\in J, t\in T \tag{5.14}$$

$$z_p^t \in \{0,1\} \qquad p\in P, t\in T \tag{5.15}$$

The conic formulation of this model using second order conic inequalities are solved as explained in Chapter 4. Then, the decisions of the cruise time durations with idle times are used to calculate the total cost TC_p^t for each path and for each aircraft type as defined in Equation 5.1. After that, to minimize the total cost over all paths, AAM model is solved again. The new best fleet type assignment sequence is given as input to solve RASM. This continues until no reduction in the total cost is obtained. It describes our construction algorithm.

When the construction algorithm is stuck in the same fleet type assignment sequence with no improvement, we construct new fleet type assignment sequences by making all pairwise interchange on the best fleet type assignment sequence found in the construction algorithm. Pairwise interchange means that fleet type assignment is changed between two paths operated by two different aircraft type. Each fleet type assignment sequence is generated by only one pairwise interchange on the previous best fleet type assignment sequence. All pairwise interchange means that all possible sequences with pairwise interchange are generated on the previous best assignment sequence. Let the best fleet type assignment sequence for four paths using the aircraft types 1, 2, 3 be $\vec{z}_0$:

$$\vec{z}_0 = (1,\ 2,\ 2,\ 3)$$

Then, all pairwise interchange produces the following five fleet type assignment sequences.

$$\begin{aligned}
\vec{z}_1 &= (2,\ 1,\ 2,\ 3)\\
\vec{z}_2 &= (2,\ 2,\ 1,\ 3)\\
\vec{z}_3 &= (3,\ 2,\ 2,\ 1)\\
\vec{z}_4 &= (1,\ 3,\ 2,\ 2)\\
\vec{z}_5 &= (1,\ 2,\ 3,\ 2)
\end{aligned}$$

For each constructed fleet type assignment sequence, the total cost TC_p^t is calculated as expressed in Equation (5.1). The best m fleet type assignment sequences, which give the minimum total cost over paths, are choosen among all pairwise interchanges. Then, for each of these best m fleet assignment sequences, RASM is solved to generate m schedules. The schedule giving the minimum total cost among these m generated schedules is selected as the best one. If the best schedule gives a lower total cost than the cost found so far, algorithm continues by the improvement part involving all pairwise interchange step. Note that, fleet type information of the selected pairwise interchange with path numbers is added to Tabu List (TL), whose size is 1, in order to prevent the backtracking in the following steps. It is defined as reverse move in the algorithm. If the best schedule gives higher cost than the cost found so far, the algorithm terminates.

Steps of Two-Stage Algorithm

Step 1: *Initialization.* Set the iteration index $k = 1$. Assume an initial schedule is given. Using the information of departure time $\vec{x}_1$, cruise time durations $\vec{f}_1$ and idle time $\vec{S}_1$ of the generated schedule, compute $TC_p^t(\vec{f}_1,\ \vec{S}_1)$ for each path $p \in P$ and for each aircraft type $t \in T$ as in Equation (5.1). Solve $\mathbf{AAM}(\vec{f}_1,\ \vec{S}_1)$ and set initial assignment sequence $\vec{z}_1 \rightarrow$ the fleet type assignment sequence found. For a given assignment sequence $\vec{z}_1$, RASM($\vec{z}_1$) is solved to generate a robust schedule. Set the objective of the RASM($\vec{z}_1$) to the $F(\vec{z}_1)$. Let $\vec{z}^* = \vec{z}_1$ and $F^* = F(\vec{z}_1)$. TL = {}

Step 2: *Construction Algorithm.* Increment the iteration index ($k = k + 1$). Update total cost $TC_p^t(\vec{f}_{k-1},\ \vec{S}_{k-1})$ for with respect to optimal solution of the RASM($\vec{z}_{k-1}$) as in Equation (5.1). Solve **AAM($\vec{f}_{k-1}$, $\vec{S}_{k-1}$)** and set the assignment sequence $\vec{z}_k \rightarrow$ the fleet type assignment found. Then, $\vec{z}_k$ is imposed upon the RASM. For a given assignment sequence $\overrightarrow{z_k}$, solve the **RASM($\vec{z}_k$)** to generate a new schedule. Set the objective of the RASM($\vec{z}_k$) to the $F(\vec{z}_k)$.

Step 2a: If $F(\vec{z}_k) \leq F^*$, set $\vec{z}^* = \vec{z}_k$ and $F^* = F(\vec{z}_k)$. Then go to Step 2.

Step 2b: If not, check whether the fleet type assignment sequence $\vec{z}_k$ is different than the fleet type assignment sequence $\vec{z}^*$. If so, go to Step 2 to find a different fleet type assignment, otherwise construction algorithm gets stuck in $\vec{z}^*$ and go to Step 3 to force the algorithm to obtain new fleet type assignment sequences.

Step 3: Update $TC_p^t(\vec{f}_k,\ \vec{S}_k)$ with respect to decisions of RASM($\vec{z}^*$). Using all pairwise interchange among the sequence $\vec{z}^*$, generate a neighborhood involving all possible fleet type assignment sequences. Then, for each of the sequences generated, calculate the $TC_p^t(\vec{f},\ \vec{S})$.

Step 4: Select *the best m fleet type sequences* $\vec{z}_1, \vec{z}_2, ..., \vec{z}_m$ among the neighborhood of $\vec{z}^*$, which give the minimum total cost calculated as in Equation (5.1). Set them as the candidate solutions.

Step 4a: If any of the moves $\vec{z}^* \rightarrow \vec{z}_1$, $\vec{z}^* \rightarrow \vec{z}_2 ...$, and $\vec{z}^* \rightarrow \vec{z}_m$ is prohibited by a move on the tabu list, eliminate that one from the neighborhood and select the $(m+1)^{th}$ best fleet type assignment sequence as one of the candidate solutions.

Step 4b: If not, set *the best m fleet type assignment sequences* as the candidate solutions.

Step 5: Generate m schedules by solving the RASM for given fleet type assignment sequences, $(\vec{z}_1)$, $(\vec{z}_2)$,...,$(\vec{z}_m)$. Set the objectives of the RASM to the $F(\vec{z}_1), F(\vec{z}_2), ..., F(\vec{z}_m)$. Set $F(\vec{z}_{k+1}) = min\,\{F(\vec{z}_1), F(\vec{z}_2), ..., F(\vec{z}_m)\}$ and $\vec{z}_{k+1}$ to the one whose objective is minimum among $(\vec{z}_1)$, $(\vec{z}_2)$,...,$(\vec{z}_m)$.

Step 6: Check for *Current Best Solution.* If $F(\vec{z}_{k+1}) \leq F^*$, set $\vec{z}^* = \vec{z}_{k+1}$ and $F^* = F(\vec{z}_{k+1})$. *Update Tabu List.* Enter the reverse move at the top of the tabu list. Delete the entry at the bottom of the tabu list if the list is full. Increment the iteration index ($k = k + 1$) and go to Step 3. Otherwise, the algorithm terminates.

5.2 Summary

In this study, we proposed a two-stage algorithm to solve the large problems in a reasonable time. Thus, we decomposed the problem into planning stages as robust airline scheduling and fleet type assignment and solve them sequentially. The approach depends on the idea that, the optimal solution of the robust airline scheduling problem is fed into the fleet type assignment problem. Then, a new schedule is generated by solving the robust airline scheduling problem for a given optimal fleet type assignment sequence. Recursively solving the problems continues until no improvement in total cost of airlines is observed. This approach describes the our construction algorithm which is defined in Steps 1 and 2.

When the construction algorithm gets stuck in the same fleet type assignment, we generate new fleet type assignment sequences using all pairwise interchange on the best fleet assignment sequence found so far. In other words, we change the aircraft of two paths operated by two different types of aircraft. Each new fleet type assignment sequence is generated by only one pairwise interchange. We generate all possible sequences from the best fleet type assignment sequence using all pairwise interchange. This generation mechanism is provided in Step 3. In Step 4, we choose the best m fleet type assignment sequences among the generated sequences. In Step 5, for given m fleet type assignment sequences, robust airline scheduling problem is solved to generate m schedules. Among the m schedules, the best schedule which gives the minimum cost among them is selected. If the selected schedule results in less total cost found so far, the algorithm continues with Step 3 to generate new fleet assignment. Otherwise, the algorithm terminates.

Chapter 6

Computational Results

In this study, we proposed a mixed integer second order conic programming formulation, which integrates the fleet type assignment along with the robust airline scheduling, called as integrated model. To handle time issues for the large scale problems, we also constructed a two-stage algorithm that sequentially solves the fleet assignment and robust airline scheduling problems. In this section, performances of the schedule developed by the integrated model are compared to the published schedule in terms of different airline cost components. Moreover, to analyze the performance of the schedule developed by the two-stage algorithm, we compare it to the schedule developed by the integrated model. In addition, analysis on the time performance of both the integrated model and two-stage algorithm is conducted.

In order to analyze the effects of parameters in the model, we made a 2^k full-factorial experimental design. There are three experimental factors and their corresponding levels are given in Table 6.1.

		Levels	
Factor	Description	Low(0)	High(1)
A	Fuel Cost	$ 600	$ 1,200
B	Base Spill Cost	$15	$ 60
C	β	0.01	0.05

Table 6.1: Factor Values

The fuel cost is the price of jet fuel per ton. A ton of fuel is equivalent to 1,254 liters which is approximately 331 gallons, or 7.8 barrels. According to the history of fuel prices obtained from IATA fuel price monitor (48), it is seen that price of a gallon fuel is fluctuating between \$1.5 and \$3.8. Average of the fuel price in 2013 is \$3.05 per gallon. In this study, the fuel prices are taken as \$1.8/gallon for the lower setting and \$3.6/gallon for the higher setting.

Base Spill Cost represents the opportunity cost for each of the unsatisfied passengers due to the insufficient seat capacity of the aircraft. It is adjusted for each flight using airport congestion coefficients in Table 6.4. Therefore, if a passenger flies from or to the airport, which has high number of visiting passengers, the spill cost of that passenger becomes more. Loosing a passenger, who flies from or to the airport with low market demand is more favorable than loosing a passenger, who flies from or to the airport, having high market demand. For each flight, the spill cost of each uncaptured passenger is calculated as follows,

$$Csp_i = BaseSpillCost \cdot (e_{O_i}) \cdot (e_{Dn_i}) \tag{6.1}$$

The third experimental factor, β, is used for the tail parameter of Log-Laplace distribution described in Chapter 3. The tail parameter is adjusted for each flight using β and airport congestion coefficients of origin and destination airports as described in equation (3.1). With a scale parameter, e^{α}, the tail parameter is used to adjust the mean and variance of random variable. In this study, the scale parameter, e^{α} is taken as 20 to have a non-cruise time deviating from 20 minutes.

In this study, the schedule is generated from the work of Aktürk et al. (28). The published schedule is provided in Table 6.2. The flight information were taken from BTS database. In the published schedule, each column represents the tail number, flight number, departure and arrival airport, departure time, flight block time and arrival time of each flight respectively. Thus, a group of flights under the same tail number represents a path in the published schedule.

In this study, there exist 6 aircraft types, each having different seat capacity, fuel consumption, turn time and unit idle time cost. Aircraft information is

Tail No	Flight No	Departure	Arrival	Departure Time	Flight Time	Arrival Time	Tail No	Flight No	Departure	Arrival	Departure Time	Flight Time	Arrival Time
N531AA	2303	ORD	DFW	6:45	2:35	9:20	N3DMAA	568	ORD	FLL	7:25	2:55	10:20
N531AA	2336	DFW	ORD	10:10	2:30	12:40	N3DMAA	711	FLL	ORD	11:10	3:15	14:25
N531AA	1053	ORD	AUS	13:25	2:50	16:15	N3DMAA	2021	ORD	SJU	15:25	4:35	20:00
N531AA	336	AUS	ORD	17:00	2:45	19:45	N544AA	2463	ORD	MCI	6:25	1:30	7:55
N531AA	336	ORD	LGA	20:40	2:14	22:54	N544AA	754	MCI	ORD	8:40	1:30	10:10
N598AA	1341	ORD	SFO	7:50	4:55	12:45	N544AA	2321	ORD	DFW	11:15	2:35	13:50
N598AA	348	SFO	ORD	13:30	4:25	17:55	N544AA	2356	DFW	ORD	14:40	2:30	17:10
N598AA	1521	ORD	TUS	19:15	3:55	23:10	N544AA	2487	ORD	DEN	17:50	2:45	20:35
N475AA	407	ORD	STL	6:20	1:10	7:30	N3EBAA	1565	ORD	MSP	6:40	1:30	8:10
N475AA	755	STL	ORD	8:35	1:15	9:50	N3EBAA	779	MSP	ORD	9:00	1:25	10:25
N475AA	755	ORD	SAT	10:45	3:00	13:45	N3EBAA	779	ORD	SAN	11:35	4:20	15:55
N475AA	408	SAT	ORD	14:30	2:40	17:10	N3EBAA	1358	SAN	ORD	16:45	3:55	20:40
N475AA	408	ORD	PHL	18:05	2:05	20:10	N3EBAA	1358	ORD	BOS	21:50	2:10	0:00
N3EEAA	876	ORD	BOS	6:35	2:10	8:45	N3ETAA	1704	ORD	EWR	6:35	2:05	8:40
N3EEAA	413	BOS	ORD	9:35	3:05	12:40	N3ETAA	1883	EWR	ORD	9:30	2:40	12:10
N3EEAA	413	ORD	SNA	13:45	4:35	18:20	N3ETAA	810	ORD	DCA	13:10	1:40	14:50
N3EEAA	1262	SNA	ORD	19:10	3:50	23:00	N3ETAA	2013	DCA	ORD	15:45	2:10	17:55
N4YDAA	451	ORD	SFO	9:45	4:55	14:40	N3ETAA	2013	ORD	LAS	19:00	4:05	23:05
N4YDAA	554	SFO	ORD	15:45	4:25	20:10	N3DYAA	1063	ORD	LAX	8:50	4:35	13:25
N3ERAA	496	ORD	DCA	6:45	1:40	8:25	N3DYAA	874	LAX	ORD	14:30	4:15	18:45
N3ERAA	1715	DCA	ORD	9:15	2:10	11:25	N3DYAA	874	ORD	BOS	19:45	2:10	21:55
N3ERAA	1715	ORD	LAS	12:25	4:05	16:30	N5DXAA	1048	ORD	MIA	7:35	3:00	10:35
N3ERAA	1708	LAS	ORD	17:20	3:40	21:00	N5DXAA	1763	MIA	ORD	11:55	3:20	15:15
N5CLAA	1425	ORD	SNA	8:25	4:40	13:05	N5DXAA	1899	ORD	MIA	16:20	3:00	19:20
N5CLAA	556	SNA	ORD	14:00	4:00	18:00	N454AA	2441	ORD	ATL	6:30	2:00	8:30
N5CLAA	1940	ORD	MIA	19:25	3:00	22:25	N454AA	1986	ATL	ORD	9:15	2:15	11:30
N535AA	2460	ORD	RSW	6:45	2:45	9:30	N454AA	1872	ORD	MCO	12:25	2:40	15:05
N535AA	564	RSW	ORD	10:20	3:05	13:25	N454AA	1131	MCO	ORD	15:50	3:05	18:55
N535AA	1446	ORD	EWR	14:55	2:45	17:40	N4YMAA	1137	ORD	MSY	8:20	2:25	10:45
N535AA	1411	EWR	ORD	18:45	2:45	21:30	N4YMAA	1768	MSY	ORD	11:30	2:30	14:00
N3DRAA	1021	ORD	LAS	8:30	4:05	12:35	N4YMAA	1768	ORD	PHL	15:05	2:05	17:10
N3DRAA	1544	LAS	ORD	13:25	3:40	17:05	N4YMAA	1697	PHL	ORD	18:00	2:35	20:35
N3DRAA	1544	ORD	DCA	18:00	1:40	19:40	N420AA	1686	ORD	RDU	6:50	1:50	8:40
N467AA	1823	ORD	PBI	9:20	2:55	12:15	N420AA	2435	RDU	ORD	9:25	2:15	11:40
N467AA	2067	PBI	ORD	13:00	3:20	16:20	N420AA	2435	ORD	PHX	12:35	4:00	16:35
N467AA	2067	ORD	STL	17:15	1:10	18:25	N420AA	1206	PHX	ORD	17:15	3:30	20:45
N467AA	1186	STL	ORD	19:10	1:15	20:25	N546AA	1462	ORD	EWR	8:00	2:45	10:45
N3DTAA	2363	ORD	HDN	9:50	2:50	12:40	N546AA	1387	EWR	ORD	11:25	2:45	14:10
N3DTAA	2318	HDN	ORD	13:40	2:50	16:30	N546AA	1397	ORD	MCO	15:00	2:40	17:40
N412AA	2345	ORD	DFW	17:15	2:35	19:50	N546AA	1221	MCO	ORD	18:25	2:55	21:20
N412AA	2374	DFW	ORD	20:40	2:20	23:00	N4WPAA	2311	ORD	DFW	9:05	2:35	11:40
N530AA	398	ORD	LGA	6:15	2:14	8:29	N4WPAA	2348	DFW	ORD	12:35	2:20	14:55
N530AA	319	LGA	ORD	9:25	2:50	12:15	N4WPAA	1797	ORD	STL	15:50	1:10	17:00
N530AA	2329	ORD	DFW	13:35	2:35	16:10	N4WPAA	1982	STL	ORD	18:00	1:15	19:15
N530AA	2364	DFW	ORD	17:00	2:30	19:30	N4WPAA	1339	ORD	SAN	20:15	4:30	0:45
N459AA	394	ORD	LGA	6:50	2:15	9:05	N439AA	2455	ORD	PHX	7:10	4:00	11:10
N459AA	321	LGA	ORD	10:00	2:50	12:50	N439AA	358	PHX	ORD	11:55	3:30	15:25
N459AA	366	ORD	LGA	13:55	2:20	16:15	N439AA	358	ORD	LGA	16:25	2:15	18:40
N459AA	347	LGA	ORD	17:15	2:50	20:05	N439AA	371	LGA	ORD	20:00	2:50	22:50
N4XGAA	2079	ORD	SAN	8:45	4:30	13:15	N5EBAA	2375	ORD	EGE	8:10	2:55	11:05
N4XGAA	1438	SAN	ORD	14:00	4:10	18:10	N4EBAA	2378	EGE	ORD	12:25	2:45	15:10
N4XGAA	346	ORD	LGA	19:50	2:15	22:05	N4EBAA	1677	ORD	SNA	18:40	4:40	23:20
N536AA	2305	ORD	DFW	7:45	2:35	10:20	N3DUAA	2099	ORD	LAX	7:00	4:35	11:35
N536AA	2344	DFW	ORD	11:35	2:30	14:05	N3DUAA	1972	LAX	ORD	12:40	4:15	16:55
N536AA	1201	ORD	STL	14:50	1:05	15:55	N3DUAA	1972	ORD	RDU	17:45	1:55	19:40
N536AA	1815	STL	ORD	17:00	1:20	18:20	N3ELAA	2057	ORD	SJU	8:30	4:35	13:05
N536AA	1815	ORD	SLC	19:15	3:40	22:55	N3ELAA	2078	SJU	ORD	14:25	5:35	20:00

Table 6.2: Published Schedule

given in Table 6.3. Mass, surface, $C_{D0,CR}$, $C_{D2,CR}$, C_{f1}, C_{f2} and C_{fcr} are used to calculate the fuel burn rate function of each aircraft type as described in Chapter 3. These parameters are obtained from the operations performance files provided by BADA (EUROCONTROL (41)). MRC speed is the speed of the aircraft which minimizes the fuel consumption of the aircraft. Base turntime will be used to calculate the aircraft turntimes. Finally, Idle Time Cost represents the unit idle time cost of each aircraft type. The cost of fuel consumption, CO_2 emission, idle time and spilled passengers depends on the aircraft type, since each aircraft has different characteristics. Thus, optimal assignment of fleet types is crucial to minimize the overall airline cost.

Original aircraft types which are assigned to paths in the published schedule

Aircraft type	B727 228	B737 500	MD 83	A320 111	A320 212	B767 300
Capacity	134	122	148	172	180	218
Mass (kgs)	74000	50000	61200	62000	64000	135000
Surface(m^2)	157.9	105.4	118	122.4	122.6	283.3
$C_{D0,CR}$	0.018	0.018	0.0211	0.024	0.024	0.021
$C_{D2,CR}$	0.06	0.055	0.0468	0.0375	0.0375	0.049
Cf_1	0.53178	0.46	0.7462	0.94	0.94	0.763
Cf_2	276.72	300	638.59	50000	100000	1430
cf_{CR}	0.954	1.079	0.9505	1.095	1.06	1.0347
MRC speed	867.6	859.2	867.6	855.15	868.79	876.70
Base Turntime	32	36	26	28	30	40
Idle Time Cost($)	150	140	142	136	144	147

Table 6.3: Aircraft Parameters

will be provided in the next sections. The total cost of the published schedule is calculated over paths flown by these types of aircraft. The costs of fuel consumption, CO_2 emission, idle time and spilled passengers are calculated based on these aircraft type information.

In the computational study, original fleet type assignment is taken as matching the seat capacity of the aircraft and demand of flights. When the original aircraft type of flights is B727 228, B737 500, MD 83, A320 111, A320 212 and B767 300, demand of the flights is taken uniformly between 110 and 134, 110 and 122, 110 and 148, 150 and 172, 160 and 180, 160 and 218 respectively.

Original cruise time duration is taken as 20 minutes less than the original flight block time of each flight in the published schedule. This is adjusted for each aircraft type and for each flight. Firstly, cruise length of each flight is calculated by multiplying the max-range cruise speed of the aircraft given in Table 6.3 with the original cruise time while assuming that speed is constant during the cruise stage. Then, the upper bound for the cruise time of flight i with aircraft type t, $f_i^{u,t}$ is calculated by dividing the cruise length of flight i with max-range cruise speed of aircraft type t. Delgado and Prats (49) state that the cruise speed can be varied by around 10% from max-range cruise speed. Thus, in the computational study, max allowable cruise time compression amount is taken as 15% of the $f_i^{u,t}$ for each flight i and for each aircraft type t. That is to say, $f_i^{l,t}$ becomes the 85% of the $f_i^{u,t}$ for each flight and aircraft type.

For the accuracy of the performance analysis, overall desired minimum service level of passengers' connections is taken as equal to the overall service level of the original published schedule. Therefore, we compare the performance of the schedule developed by model and published schedule at the same service level.

Passenger connection times are taken uniformly between 25 and 40 minutes. The weights of the passenger connections, PAS_{ij} are assigned adjusting the number of passengers connected. Passenger connections are possible between two flights i and j, if the original departure time of flight j is within 45 minutes or 180 minutes of the original arrival time of flight i and destination airport of flight i is same as the origin airport of the flight j. Turn back in a one way is not allowed for the passenger connections.

Airport congestion coefficients are normalized according to passengers' density at the airports. While the most congested airport has 1.4 congestion level, the least congested airport has 0.8 congestion level. Passengers' density is taken as the number of passengers visiting the airport, which is obtained from the T-100 market data of BTS (50). The data belongs the year of 2010, since the schedule we used in the computational study belongs the year of 2010, too. Airport congestion coefficients are used to calculate the turntimes of aircraft and β_i of the random variable A_i.

Airport	Location	Coefficient	Airport	Location	Coefficient
MIA	Miami, FL	1.40	DCA	Washington, DC	1.08
ORD	Chicago, IL	1.37	SAN	San Diego, CA	1.05
LAX	Los Angeles, CA	1.35	STL	St.Louis, MO	1.05
DEN	Denver,CO	1.35	MCI	Kansas City, MO	1.02
DFW	Dallas, TX	1.32	AUS	Austin, TX	1.00
LGA	New York, NY	1.30	RDU	Raleigh/Durham, NC	1.00
BOS	Boston, MA	1.30	MSY	New Orleans, LA	0.98
ATL	Atlanta, GA	1.28	SNA	Santa Ano, CA	0.98
PHX	Phoenix, AZ	1.25	SAT	San Antonio, TX	0.95
LAS	Las Vegas, NV	1.25	RSW	Fort Myers, FL	0.95
SFO	San Fransisco, CA	1.20	SJU	San Juan, PR	0.92
MSP	Minneapolis, MN	1.15	PBI	West Palm Beach, FL	0.90
PHL	Philadelphia, PA	1.15	TUS	Tuscan, AZ	0.88
EWR	Newark, NJ	1.12	MCO	Orlando, FL	0.85
FLL	Fort Lauderdale, FL	1.12	EGE	Eagle, CO	0.85
SLC	Salt Lake City, UT	1.08	HDN	Hayden, CO	0.80

Table 6.4: Congestion Coefficients

Aircraft turntimes are adjusted using airport congestion coefficients. If two consecutive flights i and j will be operated by the same aircraft, turnaround time needed to be prepared for the aircraft type t is calculated by using the following formula:

$$TA_{ij}^{t} = BaseTurntime^{t} \cdot (e_{Dn_i}) \tag{6.2}$$

Turntimes of the aircraft visiting the congested airports takes longer time compared to the turntimes of the aircraft visiting less congested airports. The calculated aircraft turntimes matches with the aircraft turntimes given in Arıkan et al. (10). Moreover, turntime between connecting flights are taken as 70% of the calculated turntimes in Equation (6.2), because boarding time of passengers and time to load and unload their cargos require less time since some of the passengers are already on the plane.

	Turn time (min.)	
Type	MIA	HDN
1	43.8	25.6
2	49.3	28.8
3	35.6	20.8
4	38.3	22.4
5	41.1	24
6	54.8	32

Table 6.5: Turnaround Time Study

In this study, the departure time of the first flight of each path is taken the same as the departure time in the published schedule not to loose market demand of that time interval.

6.1 Analysis on the Schedule with 41 Flights

In this study, the schedule is generated by taking the first 41 flights of the published schedule in Table 6.2. These 41 flights are operated by 12 aircraft with 3 different types.

In this study, we try to find the best aircraft assignment for each path in the schedule along with the robust airline scheduling while minimizing the total operating cost by considering the passenger and aircraft connection network for a given minimum service level. To ensure the overall service level for passengers' connections, some of the flights should be speeding up to arrive the airport before the departure time of the connected flights. Therefore, fuel consumption and CO_2 emission of the aircraft directly increase to fly faster. It means that, there exists a trade-off between the total cost of airline and desired service level of passengers' connections. For a given service level, the minimum total cost of the airline obtained by solving the integrated model is shown in Figure 6.1. It is clear that, as the desired service level increases, there exists a nonlinear increases in the total cost function. The increase in the total airline cost becomes significant, if the overall service level is desired to be greater than the 0.90.

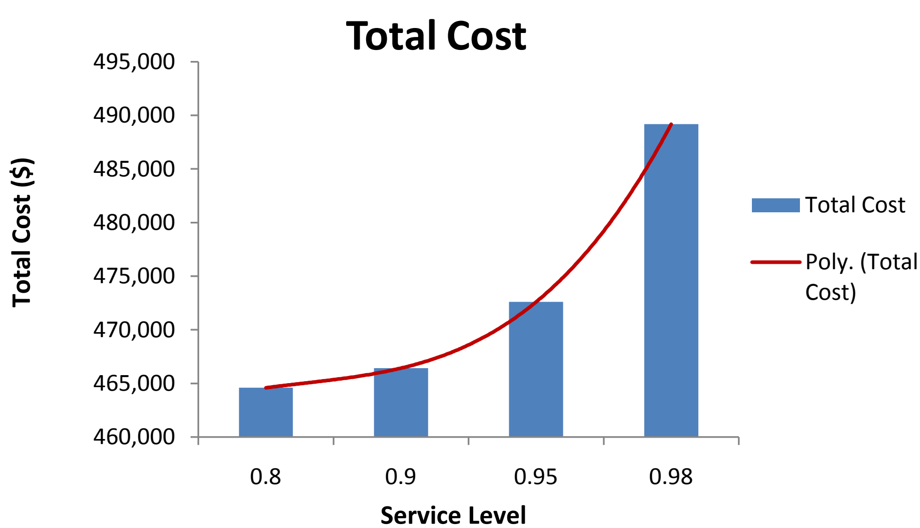

Figure 6.1: What if Analysis on the Service Level

The graph above clearly explains the bicriteria problem where the objectives are maximization of the service level and minimization of the total airline cost. As it is discussed in Chapter 4, we obtain the minimum total cost while setting a lower bound as desired on the service level of passengers' connections.

We divide the computational analysis in two parts as the analysis on the integrated model and analysis on the two-stage algorithm. In the computational analysis on the integrated model part, we first analyze the performance of the schedule developed by the integrated model by comparing the performance of the published schedule. We also analyze the computational time of the integrated model. In the computational analysis on the two-stage algorithm part, we compare the performance of the schedule developed by the two-stage algorithm and schedule developed by the integrated model while observing the optimality gap of the two-stage algorithm. Then, computational time analysis on the two-stage algorithm is also conducted.

6.1.1 Computational Analysis on the Integrated Model

We analyzed the performance of the schedule developed by the proposed model by comparing with the performance of the published schedule. For each cost component such as fuel consumption, CO_2 emission, idle time and spilled cost of passengers, we calculated the improvement in these costs compared to costs of the published schedule separately. In addition, the percentage improvement in total cost was calculated with two different methods by including the delay cost in the published schedule and not including it, since it may not be easy to determine the unit delay cost. However, even if the delay cost is not included, the performance of the schedule developed by the proposed model is still significantly better than the performance of the published schedule.

We analyzed the performance of the schedule developed by the integrated model compared to the initial schedule for five replications. For each factor level, we computed the minimum, average and maximum values of the improvement in each cost component. They are given separately in Table 6.7. The values in the table represents the percentage reduction in the corresponding cost component of the original schedule. They are calculated as follows:

$$\text{Cost Improvement} = 100 \times \frac{\text{Published Schedule - Proposed Model}}{\text{Published Schedule}}$$

Original types of the aircraft which are assigned to the paths in the published schedule are listed in Table 6.6. The original fleet type assignment is based on the macth between seat capacity of the aircraft and passengers' demand. In the published schedule, 100% passenger demand for each flight is satisfied and spilled cost of passengers becomes zero. Thus, the cost of the published schedule consists of the fuel consumption, CO_2 emission and idle time costs.

Tail No	**Aircraft Type**	**Tail No**	**Aircraft Type**
N531AA	B767 300	N5CLAA	B767 300
N598AA	B767 300	N535AA	B767 300
N475AA	A320 212	N3DRAA	A320 111
N3EEAA	A320 111	N467AA	A320 212
N4YDAA	A320 212	N3DTAA	A320 111
N3ERAA	A320 111	N412AA	A320 212

Table 6.6: Original Aircraft Types

The fuel consumption is calculated for each flight as the formula in (3.10) by replacing the published cruise time, f_i^t, with $f_i^{u,t}$ where t represents the assigned aircraft type in the published schedule in Table 6.6. After computing the total fuel consumed, the amount of CO_2 emission which is the k times of the fuel consumed is calculated. Then, the cost of CO_2 emission is calculated by multiplying the unit cost of the CO_2 emission with the amount of emission. The idle time cost of each flight is obtained by multiplying the unit idle time cost of the assigned aircraft with the idle time of each flight in the published schedule. The idle time between two consecutive flights i and j, operated by the same aircraft is the time between the arrival time of the flight i and departure time of the flight j after subtracting the cruise and non-cruise time of the flight i and turntime needed between flights i and j. In addition, delay cost of each flight is calculated also by multiplying the unit delay cost with the delay time of the flight. Unit delay cost is taken as 200$ in this study. After computing the each cost component for each flight, total cost can be obtained summing them up.

The effect of factor A, i.e. fuel cost per ton, can be seen in total cost improvement. When the fuel cost is set to a higher value, the fuel efficiency of the aircraft and cruise time controllability become more important. The proposed model results in more number of changes in the aircraft assignment types and changes in the cruise time duration of the flights to spell the increase in fuel prices with optimal arrangement of the fuel efficient aircraft and adjustment of the speed of the aircraft, so that the percentage reduction in the fuel consumption and CO_2 emission costs increases. On the other hand, passenger demand satisfaction decreases which is meant that additional spilled cost of passengers occurs. However, high setting of the fuel prices results in less cost saving since the increase in the cost of spilled passengers is greater than the reduction in the cost of fuel consumption and CO_2 emission. Moreover, improvement in the idle time is slightly affected by the fuel price.

The factor effect B, i.e. base spill cost, directly affects the percentage of uncaptured demand. When this factor is high, the proposed model gives more importance on the demand satisfaction to decrease the unsatisfied passenger cost. However, it increases the fuel consumption due to the optimal aircraft assignment having priority over the seat capacity not over the fuel efficiency. Since the fuel consumption cost consists the big portion of the airline operations, the proposed model results in less cost saving. In addition, unit uncaptured demand cost has a little effect on the idle time cost.

The factor C, i.e. β scale parameter of the log-Laplace distribution, indicates a significant improvement in idle time cost. When β is set to high value, the non-cruise times of the flights increase and fluctuate between 20 minutes and 37 minutes. Therefore, the proposed model has to insert more idle time to guarantee aircraft and passenger connections. Therefore, low setting leads to more total cost saving. Furthermore, the β factor slightly affects the fuel consumption and uncaptured demand costs.

Factors A and B lead the model in the opposite directions which indicates an inevitable trade-off between them. As it is seen Table 6.8, higher value of the factor, B, results in decrease in the percentage of the unsatisfied passengers, where the higher value of factor, A, results in increase in the percentage of the unsatisfied passengers. Decrease in the fuel consumption compansates for the increase in the percentage of unsatisfied passenger at the case of higher value of factor A. Thus, ensuring all the passenger demand satisfaction as a classical fleet assignment model results in higher fuel and emission cost. All the values in Table 6.7 represents the reduction in the cost components compared to the published schedule whose fleet type assignment is based on 100% demand satisfaction and flight cruise time is not controllable by adjusting the speed of the aircraft. In addition, integrating the airport congestions into the model indicates how the performance of the schedule changes as the value of the factor C, β changes.

		Fuel and Emission Cost Improvement			Idle Cost Improvement			Total Cost Improvement			Total Impr. without Delay		
		Min	Avg	Max	Min	Avg	Max.	Min	Avg	Max	Min	Avg.	Max
A	0	8.3	11.1	12.3	59.8	68.4	79.8	20.5	24.5	29.2	18.2	23.3	28.8
	1	10.5	11.7	12.3	59.4	68.1	79.8	16.6	19.3	22.4	15.2	18.5	22.2
B	0	12.1	12.3	12.3	59.5	67.9	79.8	19.2	23.1	29.2	17.8	22.1	28.8
	1	8.3	10.5	12.3	59.4	68.5	79.8	16.6	20.7	27.3	15.2	19.6	26.9
C	0	8.9	11.6	12.3	69.2	75.5	79.8	17.5	23.2	29.2	17.2	22.8	28.8
	1	8.3	11.1	12.3	59.4	61.0	66.3	16.6	20.7	24.9	15.2	18.9	22.7

Table 6.7: Comparison of Factor Effects

		Percentage of Unsatisfied Passengers (%)		
Factor	**Level**	**Min**	**Avg**	**Max**
A	0	0.6	2.1	3.5
	1	0.7	2.4	3.5
B	0	1.6	2.8	3.5
	1	0.6	1.6	2.4
C	0	0.7	2.3	3.5
	1	0.6	2.1	3.5

Table 6.8: Factor Effects on the Percentage of Uncaptured Passengers

Overall, it is important to observe that approximately 11% improvements in the cost of fuel consumption and CO_2 emissions, where the improvement in idle time cost is approximately is 68%. However, 2% of the passengers are not satisfied due to insufficient seat capacity of the aircraft. It shows that fleet type assignment and cruise time controllability result in great saving from the unneccassary idle times and fuel consumption by allowing a small percentage of the unsatisfied passengers. These lead to a 22% improvement in total cost of airlines.

Five replications are conducted for each factor combinations to observe if random values of flight demand and passenger connection times have any impact on objective values. For each replication, the minimum, average and maximum improvement in each cost component are seen in Table 6.9. The percentage of the unsatisfied passengers are also given in Table 6.10 for each replication. We could observe that there is no statistically significant randomization effect on the objective function values.

	Fuel and Emission Cost Improvement			Idle Cost Improvement			Total Cost Improvement		
Replications	Min	Avg	Max	Min	Avg	Max.	Min	Avg	Max
1	8.3	11.1	12.3	59.6	68.3	77.1	16.7	21.7	27.5
2	8.9	11.1	12.3	59.4	67.9	76.3	16.6	21.5	27.1
3	11.1	11.9	12.3	60.1	69.7	79.8	18.6	23.3	29.2
4	9.0	11.6	12.3	61.7	65.9	70.2	16.7	20.8	25.8
5	8.8	11.2	12.3	61.0	69.3	76.5	17.6	22.3	27.5

Table 6.9: Cost Comparison for Different Replications

	Percentage of Unsatisfied Passengers (%)		
Replication	**Min**	**Avg**	**Max**
1	0.9	2.2	2.9
2	1.1	2.5	3.3
3	0.7	1.3	1.6
4	2.4	3.0	3.5
5	0.6	2.0	2.9

Table 6.10: Percentage of Uncaptured Passengers for Different Replications

6.1.1.1 CPU Time Analysis of Integrated Model

We implemented a mixed integer second order conic programming formulation in the JAVA programming language with a connection to commercial solver IBM ILOG CPLEX Optimization Studio 12.5. Over the 5 replications, minimum, average and maximum values of CPU in seconds for each factor are analyzed in Table 6.11. High value of the factor A, i.e. fuel price per ton makes the problem harder to solve. Fuel efficiency of the aircraft and cruise time controllability become more crucial when the fuel price is high. Therefore, assignment posibilities by considering the fuel efficiency together with the nonlinearity in the fuel consumption due to the speed of the aircraft create a harder problem to be handled.

In addition, factor B has a significant impact on the computation time. High value of factor B, i.e. base spill cost eliminates some of the assignment possibilities by overrating the demand satisfaction as opposed to the factor A. Therefore, when the factor B is set to higher value, computation requires less time to solve the proposed model using second order conic inequalities.

Higher value of the factor C, i.e. variability, increases the congestions at the airport. Increase in the congestion of the airport directly affects non-cruise time of the flights. We include this variability of the paths in the tail parameter, β_i of the random variable representing the non-cruise time of the flights. Therefore, higher setting of β_i increases the non-cruise time so that ensuring the aircraft and passenger connection become more difficult as stated above. To satisfy the desired service level of passengers, some of the aircraft should fly faster to arrive the airport before the departure time of the connected flights. These complexities in the problem increase CPU time as expected.

		CPU Time (sec)		
Factor	Level	Min	Avg	Max
A	0	758	2250	6490
	1	606	3425	11290
B	0	606	3474	11290
	1	743	2201	6490
C	0	606	2350	7303
	1	743	3325	11290

Table 6.11: CPU Time Analysis of the Integrated Model

The overall, average time to solve the problem involving 12 paths operated by 12 aircraft with 3 different types is approximately 3000 seconds. However, the maximum time needed is 11290 seconds when the factors A and C are set to high values and factor B is set to low value. If the problem sizes become larger, it might be harder to solve the problem in a desirable time. Therefore, we solved larger problems with the two-stage algorithm and obtained an integer solution as an upper bound from the proposed model in 9000 seconds.

6.1.2 Computational Analysis on Two-Stage Algorithm

We obtain the exact solution of the fleet type assignment and robust scheduling problem with the proposed model by transforming the nonlinear cost function and chance constraints into the second order conic inequalities. In the computational analysis on the integrated model section, we observe that a significant total cost saving in the robust schedule, having 41 flights with 12 paths, developed by the proposed model compared to the published schedule. In this section, we can conduct a performance analysis on the schedule developed by two-stage algorithm by using the same generated schedule having 41 flights operated by 12 aircraft with 3 different types. The parameter, m to generate m schedules is set to three in the computational analysis. To analyze the performance of the schedule developed by the two-stage algorithm, the total cost of that schedule is compared to the total cost of the schedule developed by the integrated model. For each experimental factor combinations, we take the averages of the optimality gap over the 5 replications as seen in Table 6.12. The gap between the objectives of two-stage algorithm and integrated model is calculated as the following.

$$\text{Cost Increment} = \frac{\text{Two Stages Algorithm - Integrated Model}}{\text{Integrated Model}}$$

We perform a 2^3 full-factorial experimental design with 5 replications to evaluate the impact of different factors on the performance of the two-stage algorithm. When the factor A, i.e fuel price and the factor B, i.e base spill cost, are set to high, total cost of the schedule developed by the two-stage algorithm is approximately 0.001 times worse than the total cost of the schedule generated by the proposed model. The reason might be that two-stage algorithm may be not exactly capture the trade-off between the cost of fuel consumed and the cost of spilled passengers. For the remaining factor combinations, performance of the two-stage algorithm is approximately the same as the performance of the integrated model.

Factor			
A	B	C	Gap
0	0	0	0.000
0	0	1	0.000
0	1	0	0.000
0	1	1	0.001
1	0	0	0.000
1	0	1	0.000
1	1	0	0.001
1	1	1	0.000

Table 6.12: Gap Between Two-Stage Algorithm and the Integrated Model

In addition, the fleet type assignment generated by the two-stage algorithm is the same as the fleet type assignment generated by the proposed model for 34 runs among 40 runs of the 2^3 full-factorial experimental design with 5 replications. When the factors A, B, C are set to 0, 1 and 1, respectively, 2 of the fleet type assignments constructed by two-stage algorithm are different than the fleet type assignment of the integrated model among 5 replications. In the case that values of factors A, B and C are 1, 1 and 0 respectively, 3 of the fleet type assignments constructed by the two-stage algorithm differ from the fleet type assignment of the integrated model. 1 different fleet type assignment is constructed, when the values of the factors A, B and C are 0, 1 and 0, respectively, but, the gap between the objectives of the two-stage algorithm and the integrated model is negligible at that case. Therefore, we can conclude that the two stage algorithm shows a good performance for the schedule with 41 flights and 12 paths.

6.2 Analysis on the Schedule with 114 Flights

In this study, we use the published schedule in Table 6.2. These 114 flights include 32 paths operated by 32 aircraft with 6 different types. The types of aircraft which are assigned to these 32 paths are given in Table 6.13. The fuel consumption coefficients as well as the mass of aircraft, turntime of aircraft and unit idle cost of the aircraft are listed in Table 6.3.

Tail No	Aircraft Type	Tail No	Aircraft Type
N531AA	B767 300	N3DMAA	B737 500
N598AA	MD 83	N544AA	B767 300
N475AA	MD 83	N3EBAA	B737 500
N3EEAA	A320 111	N3ETAA	A320 111
N4YDAA	MD 83	N3DYAA	A320 111
N3ERAA	A320 111	N5DXAA	B727 228
N5CLAA	B767 300	N454AA	A320 212
N535AA	B727 228	N4YMAA	A320 212
N3DRAA	B737 500	N420AA	A320 212
N467AA	A320 212	N546AA	B767 300
N3DTAA	A320 111	N4WPAA	B737 500
N412AA	B737 500	N439AA	A320 212
N530AA	B767 300	N5EBAA	B767 300
N459AA	A320 212	N4EBAA	MD 83
N4XGAA	A320 212	N3DUAA	MD 83
N536AA	B767 300	N3ELAA	B727 228

Table 6.13: Original Aircraft Types

Firstly, we solve the problem with the integrated model by setting a time limit to solve in 9000 seconds. By solving the reformulated model using second order conic inequalities in 9000 seconds, we obtain an incumbent solution as an upper bound of the problem. In addition, we get the lower bound in 9000 seconds to see the optimality gap. We analyze the performance of the two-stage algorithm by comparing the objective of the two-stage algorithm with the lower and upper bound obtained by solving the integrated model. Then, we compare the performance of the schedule generated by two-stage algorithm with the published schedule.

6.2.1 Computational Analysis on the Two-Stage Algorithm

In this section, we first analyze the performance of the schedule developed by two-stage algorithm by comparing the total cost to the best objective found so far by solving the integrated model in 9000 seconds. In addition, we can obtain a best bound as a lower bound of the problem and compare it to the total cost of the schedule generated by two-stage algorithm. We perform a 2^3 full-factorial experimental design with 5 replications. For each run of two-stage algorithm, we calculate the optimality gap with the best lower bound found so far as the following:

$$\text{Gap with Lower Bound for Two-Stage} = 100 \times \frac{\text{Objective of Two-Stage - Best Bound}}{\text{Best Bound}}$$

The optimality gap with the best lower bound for the integrated model, which is solved in 9000 seconds can also be calculated as the following:

$$\text{Gap with Lower Bound for Integrated} = 100 \times \frac{\text{Best Objective of Integrated - Best Bound}}{\text{Best Bound}}$$

For each factor combinations, we take the minimum, average and maximum of the optimality gap over 5 replications as seen in Table 6.14. On the average, the gap with lower bound for the integrated model is 7.43%, where it was 6.14% for the two-stage algorithm. It is important to mention that, for each factor, minimum, average and maximum values of the gap with the lower bound for the integrated model are higher than the minimum, average and maximum values of the gap with lower bound for the two-stage algorithm.

		Gap with LB (%) for Integrated			Gap with LB (%) for Two-Stage		
Factor	**Level**	**Min**	**Avg**	**Max**	**Min**	**Avg**	**Max**
A	0	5.4	7.1	10.5	2.2	6.0	7.7
	1	4.9	7.8	10.0	2.6	6.3	8.1
B	0	5.9	8.6	10.5	4.0	7.1	8.1
	1	4.9	6.2	7.7	2.2	5.1	6.1
C	0	4.9	7.2	10.5	2.2	5.7	8.0
	1	5.4	7.7	10.0	5.0	6.6	8.1

Table 6.14: Factor Effects on the Gap with LB for Two-Stage Algorithm and Gap with LB for Integrated Model within 9000sec

For 18 of the 40 runs, the optimality gap with the lower bound for two-stage algorithm becomes less than 6%, where 6 of them result in 5% gap with the lower bound of the problem. However, we do not know whether the lower bound found so far is weak or strong. Thus, the optimality gap with the lower bound is not enough to analyze the performance of two-stage algorithm. To analyze the performance of the schedule developed by two-stage algorithm, we also calculate the total cost improvements compared to the published schedule.

We perform a 2^3 full-factorial experimental design with 5 replications to evaluate the impact of different factors on the performance of schedule generated by two-stage algorithm. The comparison is done for each factors A, B and C which are fuel cost, base spill cost and β scale parameter of the log-Laplace distribution respectively. The analysis on the performance of the schedule generated by two-stage algorithm is conducted by comparing the different cost components of the objectives of two-stage algorithm and published schedule. The savings in fuel and emission cost, idle time cost and total cost, which are obtained by two-stage algorithm compared to the published schedule are given in Table 6.15.

		Fuel and Emission Cost Improvement			Idle Cost Improvement			Total Cost Improvement			Total Impr. without Delay		
		Min	Avg	Max	Min	Avg	Max.	Min	Avg	Max	Min	Avg.	Max
A	0	2.1	3.7	4.9	69.4	72.5	78.4	23.3	25.2	28.0	20.1	23.0	26.7
	1	4.3	5.3	6.1	68.2	71.6	77.3	16.1	17.6	19.5	14.2	16.2	18.7
B	0	4.4	5.3	6.1	68.7	72.3	78.4	18.0	22.3	29.0	15.9	20.5	26.7
	1	2.1	3.7	4.8	68.2	71.9	77.6	16.1	20.6	25.9	14.2	18.7	24.7
C	0	2.4	4.6	6.1	68.2	72.5	78.4	16.1	21.8	28.0	15.2	20.7	26.7
	1	2.1	4.4	6.1	69.2	71.7	75.8	16.3	21.1	26.6	14.2	18.5	23.5

Table 6.15: Comparison of Factor Effects

It can be seen that, two-stage algorithm results in significant cost savings compared to the published schedule. Idle time cost savings are approximately 70%, where the fuel and emission cost reductions are approximately 4.5%. The overall cost saving is 21% compared to the published schedule. The results are very similar to the analysis on the schedule with 41 flights in terms of average cost savings in idle time cost and overall cost. However, improvement in the fuel and CO_2 emission costs are not as much as before. The results show that when the number of flights increase, improvement rates of idle time and overall costs are not changed, on the other hand, two-stage algorithm lead to lower cost savings in fuel and emission cost as the increase in data size.

Increase in fuel cost per ton results in more cost savings in fuel and CO_2 emission cost, since fuel efficiency of the aircraft becomes crucial for the fleet type assignment as the increase in the fuel prices. To spell the increase in fuel cost, the model optimally assigns the fuel efficient aircraft and adjusts the speed of the aircraft to decrease the fuel cost as possible as. Thus, percentage reduction in fuel and CO_2 emission costs increases. In contrast, the spill cost of unsatisfied passengers increases as the percentage of the spilled passengers increases. It is seen in Table 6.16. When the fuel price is set to higher value, speeding becomes more expensive. To achieve the robust schedule, the model uses more idle time instead of speeding up the aircraft. Therefore, improvement in idle time cost becomes slightly less. As the fuel price increases, total cost improvement decreases together with the increase in cost of spilled passengers and decrease in idle time cost improvement.

Analysis on the factor B shows the similar results in the analysis on the schedule with 41 flights as well. Increase in the base spill cost of unsatisfied passengers lead to decrease in the number of passengers who are not accommodated due to the insufficient seat capacity of the aircraft, as it is seen in Table 6.16. Thus, this strategy increases the total fuel consumption of the aircraft which is assigned to match the seat capacity to the demand of flights. Therefore, improvement in the fuel and CO_2 emission cost decreases compared to the lower case. Increase in level of factor B slightly decreases the percentage reduction of idle time cost. The decrease in fuel, emission and idle time cost improvements also reflect to decrease in total cost improvement.

The effects of factor C, i.e. β parameter of the random variable, are similar to the results in the schedule with 41 flights as well. Fuel and emission cost improvements slightly decrease in higher level of factor C. As previously stated, increase in the variability will result in more idle time insertion to achieve a robust schedule. Thus, when the factor C is set to the higher value, improvements in idle time cost decrease compared to the case of lower setting of the factor C. However, unnecessary idle time amount in the published schedule is decreased by the integrated model. Therefore, even if the variability is high, the improvement in the idle time cost is significant compared to the idle time cost of the published schedule. For the higher level, decrease in fuel, emission and idle time cost improvements will also result in decrease in total cost improvements.

Another measure of interest is the service level of the passengers' connection in the schedule. The results show that, the change in the values of β results in the

		Percentage of Unsatisfied Passengers (%)		
Factor	**Level**	**Min**	**Avg**	**Max**
A	0	0.5	1.3	2.3
	1	1.0	2.2	3.7
B	0	1.4	2.4	3.7
	1	0.5	1.0	1.5
C	0	0.5	1.7	3.4
	1	0.5	1.8	3.7

Table 6.16: Factor Effects on the Percentage of Uncaptured Passengers

significant change in the service level. For the lower case of β, the service level becomes 0.99, where it is 0.96 for the higher case of β. It is reasonable that, higher variability decreases passenger connections due to increase in non-cruise time of flights, so that the service level of the published schedule decreases. We desire the minimum service level of the schedule generated by two-stage algorithm to be equal to the service level of the published schedule to compare the performance of the schedules in equal conditions.

6.2.1.1 CPU Time Analysis of Two-Stage Algorithm

Computation times are very reasonable for each factor level. Minimum, average and maximum computation time values in CPU seconds are seen in Table 6.17 for each factor level over 5 replications.

		CPU Time (sec)		
Factor	Level	Min	Avg	Max
A	0	52	140	269
	1	48	113	187
B	0	48	137	269
	1	52	117	214
C	0	48	93	158
	1	80	160	269

Table 6.17: CPU Time Analysis of Two-Stage Algorithm

When we analyze the results, we can see each factor A, B and C have a significant effect on computation times. As previously stated, there exists a trade off between the fuel consumed and spill cost of passengers. When the unit fuel prices increases, the two-stage algorithm tries to minimize the fuel consumption of the aircraft by optimal assignment of fuel efficient aircraft. On the other hand, when the base spill cost of passengers increases, two-stage algorithm tries to minimize the uncaptured demand due to insufficient seat capacity of the aircraft by assigning the aircraft to match the seat capacity to the demand of flights. As it is seen, higher value of factors A and B, minimize either fuel consumed or spilled passengers. The trade-off between the fuel consumed and the cost of spilled passengers becomes significant, when the factors A and B are set to the lower value. Thus, computation times increase as the level of factors A and B decreases. Factor C, i.e. β of the random variable, increases the problem complexity as the increase in the variability. Therefore, for the higher value of factor C, computation times significantly increase.

Overall, the average computation time for all runs is 126 CPU seconds. This is a preferable time for a problem having 114 flights and 32 paths. It can be seen that, two-stage algorithm gives good improvements in a reasonable time.

6.3 Summary

This chapter was devoted to the computational experiments on our study. First, the parameters and experimental design factors, which are used in the study were described and their values are explained. The congestion coefficients of the airports were given. It was shown that turntimes of the aircraft, which are calculated using congestion coefficients match with the turntimes given in previous studies. In addition, aircraft specific parameters such as fuel consumption coefficients, seat capacity of the aircraft, mass of the aircraft and unit idle time cost were provided.

Afterwards, the computational studies that were conducted on the published schedule with 41 flights of an US airline were presented. The performance of the schedule developed by the integrated model was compared to the performance of the published schedule in cost terms. CPU time analysis was done for the integrated model. Furthermore, the optimality gap between two-stage algorithm and the integrated model was provided to see the performance of two-stage algorithm.

Finally, the computational studies that were conducted on the published schedule with 114 flights of an US airline were presented. The best integer solutions and best lower bound were obtained by solving the integrated model in 9000 seconds. Then, the optimality gap with lower bound for the integrated model and the optimality gap with lower bound for the two-stage algorithm was compared. In addition, to analyze the performance of the schedule generated by two-stage algorithm, it was compared to the published schedule in cost terms. The chapter is concluded with CPU time analysis on two-stage algorithm.

Chapter 7

Conclusions and Future Work

In the introduction, the motivation of this study was expressed as the fleet type assignment and robust airline scheduling. In this final chapter, the contributions of this study will be stated by describing approaches to capture the uncertainty in airline operations and solve the complex airline problems due to numerous parameters and variables. Then, some future research directions arising from this work will be discussed.

7.1 Summary of Thesis

We developed a global optimization tool to optimally assign aircraft to paths and generate a robust schedule which avoids delays while ensuring the desired service level of passengers' connection. The objective is to minimize the costs of fuel consumption, CO_2 emission, idle time and spilled passengers. Our approach captures the uncertainty in flight delays while speeding up the aircraft and idle time insertion as necessary. Fuel cost functions have a nonlinear structure of the aircraft speed, where the idle time cost functions are linear functions of idle time durations. Therefore, combination of these functions allow the aircraft to speed up until the intersection point of these function, then idle time insertion is preferable to minimize the total cost. Since speeding up the aircraft results in more fuel consumption and CO_2 emission, we also try to assign the fuel efficient aircraft to these flights which require more fuel consumption and CO_2 emission. On the other hand, this method may result in more number of spilled passengers due to the insufficient seat capacity of the aircraft. At that point, we only allow to use fuel efficient aircraft when the cost savings in fuel consumption and CO_2 emission compensate for the cost of spilled passengers as well as the idle time insertion.

Another contribution of this study is to model the variability due to the congestions of the airports. In this study, flight block times are divided into two separate parts as cruise and non-cruise times. Cruise time of flights are controllable by speeding up the aircraft. Non-cruise time are represented by a random variable which is assumed to have a Log-Laplace distribution. The variability in non-cruise times are modeled in chance constraints to guarantee the desired service level of passengers' connections. To handle the chance constraints, we use the conic quadratic inequalities. In the previous literature of the chance constrained programming, there exists different methods which provide a lower and upper bound for the problem. By using a conic reformulation, we can solve in a reasonable time and obtain an exact solution.

The congestion information of the airports are incorporated to adjust the aircraft turntime and variability of the non-cruise time of the flights. Turntime of the aircraft are adjusted using the congestion coefficient of airport where the aircraft is prepared for the next flight. In addition, for each flight, variability is seperately calculated depending on the congestion coefficients of the origin and destination airports. Thefore, this variability is used for better expectation of the non-cruise time durations.

We integrate both passengers and aircraft connection networks to maintain the feasible connections. Thus, a robust schedule is obtained that is less susceptible to delay propagations due to the misconnections of aircraft and passengers.

Another major contribution of this study is to tackle both chance constraints and nonlinear cost function by using conic quadratic programming. For the small size of the problems, we obtain an exact solution in a reasonable time. To solve the large scale problems, we also developed a two-stage algorithm which decomposes the problem into planning stages such as fleet type assignment and robust schedule generation, then solves them sequentially until no improvement in total airline cost is obtained. The fleet type assignment information is imposed upon the model which generates a robust schedule using controllable cruise time and idle time insertion. Then, the optimal solution of the robust scheduling problem is used to construct a new fleet type assignment. Solving these two planning problems recursively continues until no cost savings can be achieved.

7.2 Future Work

There are several research directions arising from this work that could be pursued. In our study, we assign the fleet types to the flight routes, which are assumed to be known a priori. The results show that the proposed model is solved in a reasonable time for a schedule with approximately 40 flights. In addition, significant cost savings in CPU seconds are obtained by running the two-stage algorithm. These are great advantages to extend the problem to involve the routing decisions as well as the fleet type assignment decisions. Moreover, aircraft maintenance routing can be considered in this enlarged problem. If the conic structure of our model is preserved, both of the integrated model and two-stage algorithm can be extended and optimal solutions or near optimal solutions can be obtained in acceptable times.

Airline schedule planning process consists of four stages such as schedule generation, fleet type assignment, aircraft maintenance routing and crew assignment. Solving each subproblem separately results in suboptimal solution, but may not result in global optimal solution when the total cost of airline is considered. Thus, it is crucial to integrate as much of these subproblem to get better solutions. In our work, we developed a two-stage algorithm that considers the fleet type assignment and robust schedule generation sequentially until no improvement in total cost is obtained. For each iteration, we obtain the suboptimal solution of one of the planning process and give this solution as an input to other planning process.

In this approach, we try to eliminate the dependencies between planning stages of airline scheduling. Since a significant cost improvement in fleet type assignment and robust schedule generation is obtained by the two-stage algorithm in CPU seconds, with a similar approach, four-stage algorithm that integrate all planning process can be developed for future studies.

Another possibility would be a modification of the random variable distribution representing the non-cruise time of the flights. We assumed that the random variable in this study has Log-Laplace distribution as discussed in Arıkan (12). However, other distributions, which have a closed form expression to be reformulated as conic quadratic inequalities, can be considered. Then, performance of the schedule generated by the model involving different random variables should be compared to the performance of the schedule generated by our model.

One such direction would be to capture the variability in non-cruise time of flights by using stochastic programming. The current method models the variability with chance constraints to ensure the desired service level of passengers' connections. The uncertainty would be also handled by a stochastic model where many of scenario cases are analyzed. On the other hand, considering many scenarios may lead to large decision trees that could require significant computation time to solve the overall stochastic model.

Bibliography

[1] A. Ben-Tal and A. Nemirovski, *Lectures on Modern Convex Optimization: Analysis, Algorithms, and Engineering Applications*. SIAM, 2001.

[2] O. Günlük and J. Linderoth, "Perspective reformulations of mixed integer nonlinear programs with indicator variables," *Mathematical Programming*, vol. 124, pp. 183–205, 2010.

[3] C. Barnhart and A. Cohn, "Airline schedule planning: Accomplishments and opportunities," *Manufacturing & Service Operations Management*, vol. 6, no. 1, pp. 3–22, 2004.

[4] N. Papadakos, "Integrated airline scheduling," *Computers & Operations Research*, vol. 36, pp. 176–195, 2009.

[5] BTS, "Airline on-time statistics and delay causes." `http://www.transtats.bts.gov/ot_delay/OT_DelayCause1.asp?pn=1`, 2013. Visited May 2013.

[6] Y. Ageeva and J. Clarke, "Approaches to incorporating robustness into airline scheduling," Master's thesis, Massachusetts Institute of Technology, 2000.

[7] S. Lan, J. P. Clarke, and C. Barnhart, "Planning for robust airline operations: Optimizing aircraft routings and flight departure times to minimize passenger disruptions," *Transportation Science*, vol. 40, no. 1, pp. 15–28, 2006.

[8] V. Chiraphadhanakul and C. Barnhart, "Robust flight schedules through slack re-allocation." Working Paper, 2011.

[9] S. Ahmadbeygi, A. Cohn, and M. Lapp, "Decreasing airline delay propagation by re-allocating scheduled slack," *IIE Transactions*, vol. 42, pp. 478–489, 2010.

[10] M. Arıkan, V. Deshpande, and M. Sohoni, "Building reliable air-travel infrastructure using empirical data and stochastic models of airline networks," *Operations Research*, vol. 61, no. 1, pp. 45–64, 2012.

[11] M. Dunbar, G. Froyland, and C. Wu, "Robust airline schedule planning: Minimizing propogated delay in an integrated routing and crewing framework," *Transportation Science*, vol. 46, pp. 204–216, 2012.

[12] M. Arıkan and V. Deshpande, "The impact of airline flight schedules on flight delays," *Manufacturing and Service Operations Management*, 2012.

[13] E. Burke, P. De Causmaecker, G. De Maere, J. Mulder, M. Paelinck, and G. Vanden Berghe, "A multi-objective approach for robust airline scheduling," *Computers & Operations Research*, vol. 37, no. 5, pp. 822 – 832, 2010.

[14] M. Sohoni, Y. Lee, and D. Klabjan, "Robust airline scheduling under block-time uncertainty," *Transportation Science*, vol. 45, pp. 451–464, 2011.

[15] L. Marla and C. Barnhart, "Robust optimization: Lessons learned from aircraft routing." Working Paper, 2010.

[16] J. Abara, "Applying integer linear programming to the fleet assignment problem," *Interfaces*, vol. 19, pp. 20–28, 1989.

[17] C. Hane, C. Barnhart, E. Johnson, R. Marsten, G. Nemhauser, and G. Sigismondi, "The fleet assignment problem: solving a large-scale integer program," *Mathematical Programming*, vol. 70, pp. 211–232, 1995.

[18] M. Lohatepanont and C. Barnhart, "Airline schedule planning: Integrated models and algorithms for schedule design and fleet assignment," *Transportation Science*, vol. 38, pp. 19–32, 2004.

[19] C. Barnhart, N. Boland, L. Clarke, E. Johnson, G. Nemhauser, and R. Shenot, "Flight string models for aircraft fleeting and routing," *Transportation Science*, vol. 32, pp. 208–220, 1998.

[20] J. Rosenberger, E. Johnson, and G. Nemhauser, "A robust fleet-assignment model with hub isolation and short cycles," *Transportation Science*, vol. 38, pp. 357–368, 2004.

[21] R. Sandhu and D. Klabjan, "Integrated airline fleeting and crew pairing decisions," *Operations Research*, vol. 55, no. 3, pp. 439–456, 2007.

[22] C. Barnhart, T. Kniker, and M. Lohatepanont, "Itinerary-based airline fleet assignment," *Transportation Science*, vol. 36, pp. 199–217, 2002.

[23] T. Jacobs, B. Smith, and E. Johnson, "Network flow effects into the airline fleet assignment process," *Transportation Science*, vol. 42, pp. 514–529, 2008.

[24] J. Isaac and A. William, "Impact of future generation aircraft on fleet-level environmental emission metrics." Working Paper, 2013.

[25] IATA, "Airline fuel and labor cost share." `http://www.iata.org/whatwedo/Documents/economics/Airline_Labour_Cost_Share_Feb2010.pdf`, 2010. Visited June 2013.

[26] Airbus, "Airbus flight operations support and line assistance, getting to grips with the cost index." `http://www.iata.org/whatwedo/Documents/fuel/airbus_cost_index_material.pdf`, 1998. Visited November 2012.

[27] A. Cook, G. Tanner, G. Williams, and G. Meise, "Dynamic cost indexing-managing airline delay costs," *Journal of Air Transport Management*, vol. 15, pp. 26–35, 2009.

[28] M. S. Aktürk, A. Atamtürk, and S. Gürel, "Aircraft rescheduling with cruise speed control." Working Paper, 2012.

[29] U. Arıkan, S. Gürel, and M. Aktürk, "Integrated aircraft and passenger recovery with cruise time controllability." 2013.

[30] A. Charnes, W. Cooper, and G. Symonds, "Cost horizons and certainty equivalents: An approach to stochastic programming of heating oil," *Management Science*, vol. 4, pp. 235–263, 1958.

[31] L. Miller and H. Wagner, “Chance constraint programming with joint constraints,” *Operations Research*, vol. 13, pp. 930–945, 1965.

[32] A. Prékopa, “On probabilistic constrained programming,” in *Proceedings of the Princeton Symposium on Mathematical Programming*, pp. 113–138, Princeton University Press, 1970.

[33] A. Prékopa, “Contributions to the theory of stochastic programming,” *Mathematical Programming*, vol. 4, pp. 202–221, 1973.

[34] E. Komáromi, “A dual method for probabilistic constrained problems,” *Mathematical Prgramming Study*, vol. 28, pp. 94–112, 1986.

[35] D. Dentcheva, L. Bogumila, and A. Ruszczyński, “Efficient point methods for probabilistic optimization problems,” 2003.

[36] J. Luedtke and S. Ahmed, “A sample approximation approach for optimization with probabilistic constraints,” *SIAM Journal on Optimization*, vol. 19, no. 2, pp. 674–699, 2008.

[37] A. Nemirovski and A. Shapiro, “Convex approximations of chance constrained programs,” *SIAM Journal on Optimization*, vol. 17, pp. 969–996, Dec. 2006.

[38] M. S. Aktürk, A. Atamtürk, and S. Gürel, “A strong conic quadratic reformulation for machine-job assignment with controllable processing times,” *Operations Research Letters*, vol. 37, pp. 187–191, 2009.

[39] A. S. Duran, M. S. Aktürk, and S. Gürel, “Robust airline scheduling with controllable cruise times and chance constraints.” Working Paper, 2012.

[40] EUROCONTROL, “Base of aircraft data (bada) aircraft performance modelling report,” Tech. Rep. 200-009, EEC Technical/Scientific, Eurocontrol, Eurocontrol Experimental Centre, B.P. 15, F-91222 Bretigny-sur-Orge, France, 2009.

[41] EUROCONTROL, “User manual for the base of aircraft data (bada) revision 3.10.,” Tech. Rep. 12/04/10-45, Eurocontrol, Eurocontrol Experimental Centre, B.P. 15, F-91222 Bretigny-sur-Orge, France, 2012.

[42] R. Martin, C. Oncina, and P. Zeeben, "A simplified method for estimating aircraft engine emissions. oral presentation at ASME, the Hague, 1994. reported as boeing method 1 fuel flow methodology description in Appexdix C of scheduled civil aircraft emission inventories for 1992. [baughcum, et al.]." 1996.

[43] D. DuBois and G. Paynter, "Fuel flow method 2 for estimating aircraft emissions," tech. rep., SAE Technical Paper Series 2006-01-198, Washington DC, USA, 2006.

[44] EUROCONTROL, "Forecasting civil aviation fuel burn and emissions in europe," Tech. Rep. 2001-8, Eurocontrol, Eurocontrol Experimental Centre, B.P. 15, F-91222 Bretigny-sur-Orge, France, 2001.

[45] ICAO, "Icao environmental report," tech. rep., International Civil Aviation Organization (ICAO), 2010.

[46] V. T'kindt and J.-C. Billaut, *Multicriteria scheduling: theory, models and algorithms*. Berlin: Springer, 2 ed., 2006.

[47] Hiriart-Urruty and C. L. Jean-Baptiste, *Fundamentals of Convex Analysis*. Berlin: Springer, 2001.

[48] IATA, "Fuel price analysis." `http://www.iata.org/publications/economics/fuel-monitor/Pages/price-analysis.aspx`, 2013. Visited May 2013.

[49] L. Delgado and X. Prats, "Fuel consumption assessment for speed variation concepts during the cruise phase.," in *Proceedings of the Conference on Air Traffic Management (ATM) Economics*, (Belgrade (Serbia)), 2009.

[50] BTS, "T-100 market data." `http://www.transtats.bts.gov/Fields.asp?Table_ID=258`, 2010. Visited June 2012.

Appendix A

Computational Results

A.1 Schedule with 41 flights

Table A.1: Costs for the schedule generated by the integrated model

Run	Factors			Replication	Costs				
#	A	B	C	#	Speeding	Emission	Idle Time	Spilled	Total
1	0	0	0	1	208,799	21,993	21,563	4,945	257,301
2	0	0	0	2	208,736	21,986	22,259	5,683	258,666
3	0	0	0	3	208,779	21,991	18,203	2,458	251,432
4	0	0	0	4	208,787	21,992	24,634	5,799	263,403
5	0	0	0	5	208,728	21,986	22,013	4,565	257,293
6	0	0	1	1	209,286	22,045	32,111	4,945	268,387
7	0	0	1	2	208,969	22,011	32,238	5,683	268,903
8	0	0	1	3	209,048	22,019	31,813	2,458	265,339
9	0	0	1	4	209,039	22,018	30,499	5,799	267,357
10	0	0	1	5	209,224	22,038	30,968	4,565	266,797
11	0	1	0	1	212,892	22,425	20,616	11,257	267,190

Continued on next page

Table A.1 – continued from previous page

Run	Factors			Replication	Costs				
#	A	B	C	#	Speeding	Emission	Idle Time	Spilled	Total
12	0	1	0	2	216,886	22,845	21,671	7,710	269,112
13	0	1	0	3	211,408	22,268	20,162	4,268	258,103
14	0	1	0	4	211,382	22,266	24,678	16,890	278,288
15	0	1	0	5	212,747	22,409	21,131	7,872	264,159
16	0	1	1	1	218,467	23,012	31,037	6,067	278,582
17	0	1	1	2	217,013	22,859	30,618	7,710	278,200
18	0	1	1	3	211,574	22,286	31,788	4,280	269,928
19	0	1	1	4	216,650	22,820	30,305	11,067	280,843
20	0	1	1	5	217,194	22,878	26,964	3,192	273,114
21	1	0	0	1	417,583	21,993	21,573	4,945	466,094
22	1	0	0	2	417,452	21,986	22,277	5,684	467,398
23	1	0	0	3	417,554	21,991	18,208	2,458	460,212
24	1	0	0	4	417,562	21,992	26,829	5,800	472,182
25	1	0	0	5	417,469	21,987	22,010	4,566	466,031
26	1	0	1	1	418,233	22,027	4,649	4,945	477,577
27	1	0	1	2	417,668	21,997	32,445	5,684	477,794
28	1	0	1	3	417,891	22,009	31,969	2,458	474,328
29	1	0	1	4	417,809	22,005	30,703	5,800	476,317
30	1	0	1	5	418,061	22,018	31,269	4,566	475,913
31	1	1	0	1	425,723	22,421	20,664	11,257	480,066
32	1	1	0	2	425,570	22,413	21,337	14,592	483,912
33	1	1	0	3	417,554	21,991	18,208	9,833	467,586
34	1	1	0	4	417,559	21,991	26,834	23,199	489,583
35	1	1	0	5	425,474	22,408	21,146	7,872	476,900
36	1	1	1	1	426,330	22,453	6,796	11,257	492,259
37	1	1	1	2	422,601	22,257	32,543	15,853	493,255
38	1	1	1	3	422,975	22,277	31,916	4,280	481,448
39	1	1	1	4	422,862	22,271	30,520	16,890	492,544

Continued on next page

Table A.1 – continued from previous page

Run	Factors			Replication	Costs				
#	A	B	C	#	Speeding	Emission	Idle Time	Spilled	Total
40	1	1	1	5	426,111	22,442	30,706	7,872	487,131

Table A.2: Costs for the schedule generated by two-stage algorithm

Run	Factors			Replication	Costs				
#	A	B	C	#	Speeding	Emission	Idle Time	Spilled	Total
1	0	0	0	1	208,796	21,993	21,562	4,945	257,296
2	0	0	0	2	208,735	21,987	22,261	5,684	258,666
3	0	0	0	3	208,779	21,991	18,207	2,458	251,435
4	0	0	0	4	208,780	21,992	26,826	5,800	263,398
5	0	0	0	5	208,736	21,987	22,007	4,566	257,296
6	0	0	1	1	209,286	22,045	32,119	4,945	268,396
7	0	0	1	2	208,969	22,011	32,245	5,684	268,910
8	0	0	1	3	209,046	22,020	31,823	2,458	265,348
9	0	0	1	4	209,041	22,019	30,505	5,800	267,364
10	0	0	1	5	209,224	22,038	30,974	4,566	266,802
11	0	1	0	1	212,892	22,425	20,614	11,257	267,188
12	0	1	0	2	216,884	22,845	21,672	7,710	269,111
13	0	1	0	3	211,409	22,268	20,162	4,268	258,107
14	0	1	0	4	211,366	22,264	27,704	16,890	278,224
15	0	1	0	5	216,840	22,840	22,092	3,192	264,965
16	0	1	1	1	213,387	22,477	31,893	11,257	279,014
17	0	1	1	2	217,015	22,859	30,622	7,710	278,205
18	0	1	1	3	211,576	22,286	31,792	4,280	269,935
19	0	1	1	4	211,571	22,285	30,321	16,890	281,068
20	0	1	1	5	217,195	22,878	29,857	3,192	273,122
21	1	0	0	1	417,583	21,993	21,570	4,945	466,091

Continued on next page

Table A.2 – continued from previous page

Run	Factors			Replication	Costs				
#	A	B	C	#	Speeding	Emission	Idle Time	Spilled	Total
22	1	0	0	2	417,449	21,986	22,277	5,684	467,396
23	1	0	0	3	417,554	21,991	18,211	2,458	460,215
24	1	0	0	4	417,551	21,991	26,837	5,800	472,179
25	1	0	0	5	417,467	21,987	22,011	4,566	466,031
26	1	0	1	1	418,241	22,027	32,375	4,945	477,589
27	1	0	1	2	417,669	21,997	32,454	5,684	477,804
28	1	0	1	3	417,897	22,009	31,975	2,458	474,340
29	1	0	1	4	417,809	22,005	30,712	5,800	476,325
30	1	0	1	5	418,067	22,018	31,271	4,566	475,921
31	1	1	0	1	425,725	22,422	20,662	11,257	480,066
32	1	1	0	2	422,640	22,259	23,667	15,853	484,419
33	1	1	0	3	422,795	22,267	20,180	4,268	469,510
34	1	1	0	4	422,711	22,263	27,764	16,890	489,628
35	1	1	0	5	425,475	22,408	21,147	7,872	476,903
36	1	1	1	1	426,335	22,454	32,225	11,257	492,271
37	1	1	1	2	422,599	22,257	32,553	15,853	493,262
38	1	1	1	3	422,979	22,277	31,921	4,280	481,457
39	1	1	1	4	422,869	22,271	30,526	16,890	492,557
40	1	1	1	5	426,112	22,442	30,714	7,872	487,140

Table A.3: Costs for the published schedule

Run	Factors			Replication	Costs				
#	A	B	C	#	Speeding	Emission	Idle Time	Spilled	Total
1	0	0	0	1	238,118	25,082	89,974	0	355,021
2	0	0	0	2	238,118	25,082	89,974	0	355,021
3	0	0	0	3	238,118	25,082	89,974	0	355,021
4	0	0	0	4	238,118	25,082	89,974	0	355,021

Continued on next page

Table A.3 – continued from previous page

Run	Factors			Replication	Costs				
#	A	B	C	#	Speeding	Emission	Idle Time	Spilled	Total
5	0	0	0	5	238,118	25,082	89,974	0	355,021
6	0	0	1	1	238,118	25,082	80,104	0	353,086
7	0	0	1	2	238,118	25,082	80,104	0	353,086
8	0	0	1	3	238,118	25,082	80,104	0	353,086
9	0	0	1	4	238,118	25,082	80,104	0	353,086
10	0	0	1	5	238,118	25,082	80,104	0	353,086
11	0	1	0	1	238,118	25,082	89,974	0	355,021
12	0	1	0	2	238,118	25,082	89,974	0	355,021
13	0	1	0	3	238,118	25,082	89,974	0	355,021
14	0	1	0	4	238,118	25,082	89,974	0	355,021
15	0	1	0	5	238,118	25,082	89,974	0	355,021
16	0	1	1	1	238,118	25,082	80,104	0	353,086
17	0	1	1	2	238,118	25,082	80,104	0	353,086
18	0	1	1	3	238,118	25,082	80,104	0	353,086
19	0	1	1	4	238,118	25,082	80,104	0	353,086
20	0	1	1	5	238,118	25,082	80,104	0	353,086
21	1	0	0	1	476,237	25,082	89,974	0	593,140
22	1	0	0	2	476,237	25,082	89,974	0	593,140
23	1	0	0	3	476,237	25,082	89,974	0	593,140
24	1	0	0	4	476,237	25,082	89,974	0	593,140
25	1	0	0	5	476,237	25,082	89,974	0	593,140
26	1	0	1	1	476,237	25,082	80,104	0	591,204
27	1	0	1	2	476,237	25,082	80,104	0	591,204
28	1	0	1	3	476,237	25,082	80,104	0	591,204
29	1	0	1	4	476,237	25,082	80,104	0	591,204
30	1	0	1	5	476,237	25,082	80,104	0	591,204
31	1	1	0	1	476,237	25,082	89,974	0	593,140
32	1	1	0	2	476,237	25,082	89,974	0	593,140

Continued on next page

Table A.3 – continued from previous page

Run	Factors			Replication	Costs				
#	A	B	C	#	Speeding	Emission	Idle Time	Spilled	Total
33	1	1	0	3	476,237	25,082	89,974	0	593,140
34	1	1	0	4	476,237	25,082	89,974	0	593,140
35	1	1	0	5	476,237	25,082	89,974	0	593,140
36	1	1	1	1	476,237	25,082	80,104	0	591,204
37	1	1	1	2	476,237	25,082	80,104	0	591,204
38	1	1	1	3	476,237	25,082	80,104	0	591,204
39	1	1	1	4	476,237	25,082	80,104	0	591,204
40	1	1	1	5	476,237	25,082	80,104	0	591,204

Table A.4: Service levels and CPU times

Run	Factors			Replication	Service Level of	CPU Time	CPU Time
#	A	B	C	#	Published Schedule	Integrated	Two-Stage
1	0	0	0	1	0.99	1,145	24
2	0	0	0	2	0.99	1,136	15
3	0	0	0	3	0.99	1,285	24
4	0	0	0	4	0.99	2,854	17
5	0	0	0	5	0.99	883	19
6	0	0	1	1	0.98	2,153	22
7	0	0	1	2	0.98	3,516	14
8	0	0	1	3	0.98	778	24
9	0	0	1	4	0.97	2,396	14
10	0	0	1	5	0.98	2,021	18
11	0	1	0	1	0.99	2,800	24
12	0	1	0	2	0.99	1,170	15
13	0	1	0	3	0.99	1,425	24
14	0	1	0	4	0.99	6,490	17
15	0	1	0	5	0.99	1,745	17

Continued on next page

Table A.4 – continued from previous page

Run	Factors			Replication	Service Level of	CPU Time	CPU Time
#	A	B	C	#	Published Schedule	Integrated	Two-Stage
16	0	1	1	1	0.98	4,751	22
17	0	1	1	2	0.98	1,146	14
18	0	1	1	3	0.98	758	14
19	0	1	1	4	0.97	2,449	14
20	0	1	1	5	0.98	4,098	14
21	1	0	0	1	0.99	2,548	25
22	1	0	0	2	0.99	2,943	15
23	1	0	0	3	0.99	606	24
24	1	0	0	4	0.99	7,303	18
25	1	0	0	5	0.99	2,996	16
26	1	0	1	1	0.98	8,082	21
27	1	0	1	2	0.98	11,290	14
28	1	0	1	3	0.98	2,252	22
29	1	0	1	4	0.97	4,300	14
30	1	0	1	5	0.98	8,994	21
31	1	1	0	1	0.99	1,519	25
32	1	1	0	2	0.99	3,384	15
33	1	1	0	3	0.99	1,029	24
34	1	1	0	4	0.99	2,147	16
35	1	1	0	5	0.99	1,584	16
36	1	1	1	1	0.98	2,957	20
37	1	1	1	2	0.98	1,786	13
38	1	1	1	3	0.98	743	14
39	1	1	1	4	0.97	1,194	13
40	1	1	1	5	0.98	838	13

A.2 Schedule with 114 flights

Table A.5: Costs for the schedule generated by two-stage algorithm

Run	Factors			Replication	Costs				
#	A	B	C	#	Speeding	Emission	Idle Time	Spilled	Total
1	0	0	0	1	502,924	52,975	71,423	6,414	633,736
2	0	0	0	2	501,834	52,860	78,370	6,576	639,639
3	0	0	0	3	503,476	53,033	78,799	7,731	643,038
4	0	0	0	4	504,099	53,098	57,267	7,565	622,030
5	0	0	0	5	503,268	53,011	71,595	6,064	633,937
6	0	0	1	1	501,991	52,876	67,606	7,998	630,472
7	0	0	1	2	504,249	53,114	67,551	6,039	630,954
8	0	0	1	3	503,273	53,011	55,427	9,347	621,058
9	0	0	1	4	504,468	53,137	60,219	9,086	626,910
10	0	0	1	5	504,115	53,100	65,072	7,823	630,111
11	0	1	0	1	514,679	54,213	71,613	8,046	648,551
12	0	1	0	2	503,511	53,037	76,977	15,298	648,823
13	0	1	0	3	513,590	54,098	80,197	12,336	660,221
14	0	1	0	4	514,923	54,239	59,352	10,822	639,347
15	0	1	0	5	510,455	53,768	75,540	13,766	653,529
16	0	1	1	1	516,419	54,396	62,897	8,046	641,758
17	0	1	1	2	513,169	54,054	69,862	10,156	647,241
18	0	1	1	3	514,294	54,172	58,494	10,721	637,682
19	0	1	1	4	516,275	54,381	60,397	10,822	641,875
20	0	1	1	5	512,044	53,935	68,571	13,766	648,316
21	1	0	0	1	990,689	52,176	72,229	12,081	1,127,175
22	1	0	0	2	991,533	52,221	78,002	10,494	1,132,250
23	1	0	0	3	991,263	52,207	83,059	13,616	1,140,145
24	1	0	0	4	992,508	52,272	60,514	14,065	1,119,359
25	1	0	0	5	991,847	52,237	72,926	13,033	1,130,043
26	1	0	1	1	991,314	52,209	68,764	13,167	1,125,454

Continued on next page

Table A.5 – continued from previous page

Run	Factors			Replication	Costs				
#	A	B	C	#	Speeding	Emission	Idle Time	Spilled	Total
27	1	0	1	2	992,983	52,297	68,959	10,494	1,124,733
28	1	0	1	3	990,768	52,180	57,450	13,618	1,114,016
29	1	0	1	4	992,751	52,285	65,220	15,628	1,125,884
30	1	0	1	5	993,234	52,310	67,727	13,033	1,126,305
31	1	1	0	1	1,008,020	53,089	73,063	19,283	1,153,456
32	1	1	0	2	1,005,013	52,931	77,764	15,717	1,151,425
33	1	1	0	3	1,005,351	52,948	84,353	24,427	1,167,079
34	1	1	0	4	1,008,802	53,130	60,148	21,596	1,143,676
35	1	1	0	5	1,004,490	52,903	77,278	16,791	1,151,463
36	1	1	1	1	1,004,259	52,891	67,253	25,183	1,149,585
37	1	1	1	2	1,006,099	52,988	69,551	15,717	1,144,356
38	1	1	1	3	1,004,970	52,928	59,764	24,279	1,141,941
39	1	1	1	4	1,009,456	53,165	64,723	21,287	1,148,631
40	1	1	1	5	1,005,924	52,979	70,338	16,791	1,146,032

Table A.6: Costs for the schedule generated by the integrated model

Run	Factors			Replication	Costs				
#	A	B	C	#	Speeding	Emission	Idle Time	Spilled	Total
1	0	0	0	1	502,556	52,936	69,659	12,544	637,695
2	0	0	0	2	501,924	52,869	77,935	9,153	641,880
3	0	0	0	3	500,921	52,764	83,320	39,232	676,237
4	0	0	0	4	503,299	53,014	57,120	14,153	627,586
5	0	0	0	5	502,074	52,885	72,649	9,460	637,069
6	0	0	1	1	511,689	53,898	64,693	10,545	640,825
7	0	0	1	2	500,090	52,676	66,159	15,187	634,112
8	0	0	1	3	505,985	53,297	56,378	14,045	629,705

Continued on next page

Table A.6 – continued from previous page

Run	Factors			Replication	Costs				
#	A	B	C	#	Speeding	Emission	Idle Time	Spilled	Total
9	0	0	1	4	503,979	53,086	60,904	13,330	631,299
10	0	0	1	5	508,493	53,561	62,858	11,616	636,528
11	0	1	0	1	513,203	54,057	72,807	28,703	668,770
12	0	1	0	2	511,364	53,864	78,446	9,639	653,312
13	0	1	0	3	511,617	53,890	81,262	20,237	667,007
14	0	1	0	4	512,916	54,027	57,250	20,134	644,327
15	0	1	0	5	516,339	54,387	69,645	17,766	658,138
16	0	1	1	1	513,409	54,079	64,170	12,221	643,880
17	0	1	1	2	504,274	53,117	68,173	20,802	646,366
18	0	1	1	3	510,858	53,810	58,628	18,331	641,628
19	0	1	1	4	512,187	53,950	63,653	15,783	645,574
20	0	1	1	5	506,303	53,331	70,247	18,687	648,568
21	1	0	0	1	991,994	52,245	73,662	29,457	1,147,357
22	1	0	0	2	1,001,543	52,748	78,937	12,533	1,145,761
23	1	0	0	3	1,007,285	53,050	82,746	17,646	1,160,728
24	1	0	0	4	1,000,051	52,669	59,708	16,741	1,129,169
25	1	0	0	5	991,705	52,230	74,028	23,133	1,141,096
26	1	0	1	1	990,586	52,171	68,087	28,838	1,139,682
27	1	0	1	2	997,786	52,550	70,708	19,301	1,140,345
28	1	0	1	3	991,181	52,202	55,569	25,266	1,124,218
29	1	0	1	4	1,008,156	53,096	75,427	24,623	1,161,303
30	1	0	1	5	996,929	52,505	69,770	25,819	1,145,023
31	1	1	0	1	1,004,339	52,895	77,730	44,145	1,179,109
32	1	1	0	2	1,012,314	53,315	78,775	18,177	1,162,581
33	1	1	0	3	1,023,471	53,903	83,394	16,155	1,176,923
34	1	1	0	4	1,009,082	53,145	61,534	29,096	1,152,856
35	1	1	0	5	1,011,496	53,272	81,637	17,712	1,164,117
36	1	1	1	1	1,002,705	52,809	68,893	37,916	1,162,323

Continued on next page

Table A.6 – continued from previous page

Run	Factors			Replication	Costs				
#	A	B	C	#	Speeding	Emission	Idle Time	Spilled	Total
37	1	1	1	2	1,011,701	53,283	73,662	17,740	1,156,387
38	1	1	1	3	1,014,657	53,439	62,256	28,531	1,158,883
39	1	1	1	4	1,012,922	53,347	62,772	36,883	1,165,925
40	1	1	1	5	999,527	52,642	67,027	37,987	1,157,183

Table A.7: Costs for the published schedule

Run	Factors			Replication	Costs				
#	A	B	C	#	Speeding	Emission	Idle Time	Spilled	Total
1	0	0	0	1	527,616	55,576	265,551	0	863,363
2	0	0	0	2	527,616	55,576	265,551	0	863,363
3	0	0	0	3	527,616	55,576	265,551	0	863,363
4	0	0	0	4	527,616	55,576	265,551	0	863,363
5	0	0	0	5	527,616	55,576	265,551	0	863,363
6	0	0	1	1	527,616	55,576	228,568	0	845,620
7	0	0	1	2	527,616	55,576	228,568	0	845,620
8	0	0	1	3	527,616	55,576	228,568	0	845,620
9	0	0	1	4	527,616	55,576	228,568	0	845,620
10	0	0	1	5	527,616	55,576	228,568	0	845,620
11	0	1	0	1	527,616	55,576	265,551	0	863,363
12	0	1	0	2	527,616	55,576	265,551	0	863,363
13	0	1	0	3	527,616	55,576	265,551	0	863,363
14	0	1	0	4	527,616	55,576	265,551	0	863,363
15	0	1	0	5	527,616	55,576	265,551	0	863,363
16	0	1	1	1	527,616	55,576	228,568	0	845,620
17	0	1	1	2	527,616	55,576	228,568	0	845,620
18	0	1	1	3	527,616	55,576	228,568	0	845,620
19	0	1	1	4	527,616	55,576	228,568	0	845,620

Continued on next page

Table A.7 – continued from previous page

Run	Factors			Replication	Costs				
#	A	B	C	#	Speeding	Emission	Idle Time	Spilled	Total
20	0	1	1	5	527,616	55,576	228,568	0	845,620
21	1	0	0	1	1,055,233	55,576	265,551	0	1,390,979
22	1	0	0	2	1,055,233	55,576	265,551	0	1,390,979
23	1	0	0	3	1,055,233	55,576	265,551	0	1,390,979
24	1	0	0	4	1,055,233	55,576	265,551	0	1,390,979
25	1	0	0	5	1,055,233	55,576	265,551	0	1,390,979
26	1	0	1	1	1,055,233	55,576	228,568	0	1,373,236
27	1	0	1	2	1,055,233	55,576	228,568	0	1,373,236
28	1	0	1	3	1,055,233	55,576	228,568	0	1,373,236
29	1	0	1	4	1,055,233	55,576	228,568	0	1,373,236
30	1	0	1	5	1,055,233	55,576	228,568	0	1,373,236
31	1	1	0	1	1,055,233	55,576	265,551	0	1,390,979
32	1	1	0	2	1,055,233	55,576	265,551	0	1,390,979
33	1	1	0	3	1,055,233	55,576	265,551	0	1,390,979
34	1	1	0	4	1,055,233	55,576	265,551	0	1,390,979
35	1	1	0	5	1,055,233	55,576	265,551	0	1,390,979
36	1	1	1	1	1,055,233	55,576	228,568	0	1,373,236
37	1	1	1	2	1,055,233	55,576	228,568	0	1,373,236
38	1	1	1	3	1,055,233	55,576	228,568	0	1,373,236
39	1	1	1	4	1,055,233	55,576	228,568	0	1,373,236
40	1	1	1	5	1,055,233	55,576	228,568	0	1,373,236

Table A.8: Service levels and CPU times

Run	Factors			Replication	Service Level of	CPU Time	CPU Time
#	A	B	C	#	Published Schedule	Integrated	Two-Stage
1	0	0	0	1	0.99	9,000	79
2	0	0	0	2	0.99	9,000	91

Continued on next page

Table A.8 – continued from previous page

Run	Factors			Replication	Service Level of	CPU Time	CPU Time
#	A	B	C	#	Published Schedule	Integrated	Two-Stage
3	0	0	0	3	0.99	9,000	116
4	0	0	0	4	0.99	9,000	108
5	0	0	0	5	0.99	9,000	115
6	0	0	1	1	0.97	9,000	152
7	0	0	1	2	0.97	9,000	174
8	0	0	1	3	0.96	9,000	194
9	0	0	1	4	0.97	9,000	200
10	0	0	1	5	0.97	9,000	269
11	0	1	0	1	0.99	9,000	83
12	0	1	0	2	0.99	9,000	52
13	0	1	0	3	0.99	9,000	158
14	0	1	0	4	0.99	9,000	104
15	0	1	0	5	0.99	9,000	115
16	0	1	1	1	0.97	9,000	148
17	0	1	1	2	0.97	9,000	95
18	0	1	1	3	0.96	9,000	142
19	0	1	1	4	0.97	9,000	196
20	0	1	1	5	0.97	9,000	214
21	1	0	0	1	0.99	9,000	77
22	1	0	0	2	0.99	9,000	82
23	1	0	0	3	0.99	9,000	108
24	1	0	0	4	0.99	9,000	48
25	1	0	0	5	0.99	9,000	112
26	1	0	1	1	0.97	9,000	131
27	1	0	1	2	0.97	9,000	130
28	1	0	1	3	0.96	9,000	183
29	1	0	1	4	0.97	9,000	177
30	1	0	1	5	0.97	9,000	187

Continued on next page

Table A.8 – continued from previous page

Run	Factors			Replication	Service Level of	CPU Time	CPU Time
#	A	B	C	#	Published Schedule	Integrated	Two-Stage
31	1	1	0	1	0.99	9,000	77
32	1	1	0	2	0.99	9,000	79
33	1	1	0	3	0.99	9,000	109
34	1	1	0	4	0.99	9,000	78
35	1	1	0	5	0.99	9,000	77
36	1	1	1	1	0.97	9,000	81
37	1	1	1	2	0.97	9,000	137
38	1	1	1	3	0.96	9,000	173
39	1	1	1	4	0.97	9,000	80
40	1	1	1	5	0.97	9,000	139

Vita

Özge Şafak was born on May 31, 1988 in Denizli, Turkey. She graduated from Denizli Science High School in 2006. She attended Sabanci University with honor scholarship and graduated from the Manufacturing System & Indusrial Engineering Program at Faculty of Engineering and Natural Science in 2011. In 2011, she attended Industrial Engineering Department of Bilkent University as a research assistant. Since then, she has been working with Prof. M. Selim Aktürk on her graduate study. She had been on the grant 2210 awarded by The Scientific and Technological Research Council of Turkey (TUBITAK) during her M.S. study.