DESIGN OF ROBUST GUARANTEED COST OBSERVER-BASED CONTROLLER FOR LINEAR UNCERTAIN SYSTEMS

by

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:DESIGN OF ROBUST GUARANTEED COST OBSERVER-BASED CONTROLLER FOR LINEAR UNCERTAIN SYSTEMS

DOĞRUSAL BELİRSİZ SİSTEMLER İÇİN GÖZETLEYİCİ TABANLI DAYANIKLI GARANTİLİ MALİYET DENETLEYİCİ TASARIMI

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4) Gözetleyici esaslı tasarım	4) Observer-based design
5) PD geri besleme	5) PD feedback

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ABSTRACT

DESIGN OF ROBUST GUARANTEED COST OBSERVER-BASED CONTROLLER FOR LINEAR UNCERTAIN SYSTEMS

In this thesis, robust guaranteed cost observer-based state feedback controller design problem has been investigated for a linear uncertain system with norm-bounded uncertainty parameters, which has been important for several systems. This is employed by Lyapunov stability theorem that the criteria of robust stabilization are proposed within the framework of linear matrix inequalities (LMI). The feasibility problem of the stabilization criteria with memoryless feedback is solved easily using the technique of cone complementary minimization algorithm. Also, the stability criteria of output feedback is convex in the shape of linear matrix inequalities. Moreover, minimization of guaranteed cost was obtained via cone complementary algorithm.

This thesis also proposes the design of the guaranteed cost of observer-based proportional derivative state feedback controller for linear nominal systems. The minimization of the performance index is satisfied to get a feasible solution than the proposed observer-based controller for the linear nominal system.

Finally, numeric examples have been presented to illustrate the stabilization that introduced approach provides considerable improvement.

Keywords: Robust control, Lyapunov stability theorem, guaranteed cost control, observer-based design, PD feedback

ÖZET

DOĞRUSAL BELİRSİZ SİSTEMLER İÇİN GÖZETLEYİCİ TABANLI DAYANIKLI GARANTİLİ MALİYET DENETLEYİCİ TASARIMI

Bu tezde, birçok sistem için önemli olan doğrusal belirsiz sistemler için gözetleyici tabanlı dayanıklı garantili maliyet denetleyici tasarımı incelendi. Lyapunov kararlılık teoremi, önerilen dayanıklı kararlılığın kriteri ile doğrusal matris eşitsizlik (LMI) çerçevesinde uygulandı. Hafızasız geri beslemeli ile kararlılık kriterinin geçerlilik problemi koni tamamlamalı minimizasyon algoritmasının uygulanmasıyla kolayca çözüm elde edildi. Ayrıca, doğrusal matris eşitsizlik bakımından geri beslemeli sonuç kararlılık kriteri dışbükeydir. Buna ek olarak garantili maliyet minimizasyonu koni tamamlamalı minimizasyon algoritması aracılıyla elde edildi.

Bu tez ayrıca doğrusal sistemler için garanti maliyet gözetleyici esaslı orantısal türev geri beslemeli kontrolör dizaynını sunmaktadır. Önerilen doğrusal sistem gözetleyici esaslı tasarımına göre performans indeks minimizasyon sonucu daha olumlu sonuç elde edilmiştir.

Sonuç olarak nümerik örnekler sunulmuş olup, tanımlanan kararlılık yaklaşımının önemli bir gelişme gösterdiği sonucu elde edilmiştir.

Anahtar kelimeler: Dayanıklı kontrol, Lyapunov kararlılık teoremi, garantili maliyet kontrol, gözetleyici esaslı tasarım, PD geri besleme

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LIST OF SYMBOLS / ABBREVIATIONS

LMI	Linear Matrix Inequalities
Diag	Diagonal Square Matrix
Tr	Trace

1. INTRODUCTION

The robust controller has drawn attention for linear uncertain systems since last decades because the purpose of the robust controller design is good steady-state and error modeling. One of the best application to obtain a feasible solution for uncertain models is Lyapunov stability theorem within the framework of linear matrix inequalities (LMIs). The Lyapunov design has been a significant improvement for linear uncertain systems. The aim of the Lyapunov stability theory is that the system's energy is dissipating, then the system reaches to equilibrium point which means the system will be stabilized for abundant systems. The basic idea has been showed several works to get a feasible solution in LMIs for linear uncertain systems.

The guaranteed cost controller for uncertain modeling has been a hot topic to remark in recent years. The purpose of design a guaranteed cost controller is that closed-loop system obtains stability with an upper bound parameter. We investigated that the design of the guaranteed cost robust observer-based state feedback controller design problem for a linear uncertain system with norm-bounded uncertainties, which has been a significant influenced on several dynamics of systems. This is applied by the approach of Lyapunov stability theorem that the criteria of robust stabilization are presented within the framework of linear matrix inequalities (LMI) via Schur complement [8]. The feasibility problem of the stabilization criteria with observer-based state feedback controller is solved easily due to the technique of cone complementary minimization algorithm [9]. Moreover, the output feedback stability criteria is convex within the shape of the linear matrix inequalities. Also, minimization of guaranteed cost has obtained via cone complementary algorithm.

We designed the stabilization of the observer-based PD state feedback controller to obtain a feasible solution for linear nominal systems. Then, we extended the findings that add the guaranteed cost function to minimize the performance index. Its purpose is that we compare the results both observer-based controller in linear nominal systems and observer-based PD state feedback for linear nominal modeling.

In this paper, a robust stability is analyzed with guaranteed cost observer-based controller for linear uncertain systems. Moreover, the guaranteed cost of observer-based proportion derivate state feedback controller has been studied because of the Lyapunov stability theorem. The criteria of stabilization are formulated within the context of LMIs. It is demonstrated the minimization guaranteed cost in terms of cone complementary theory. Finally, numerical examples which are both nominal and uncertain parameters have been illustrated proposed stabilization that introduced approach provides considerable improvement with an observer-based controller. The algorithm of the feasible solution and block diagrams is shown in Appendix A and B respectively. The last example, its contains results which are the design of an observer-based PD feedback controller for linear nominal systems. The feasible solution and block diagrams are illustrated with algorithm 1 and Lemma 3 in Appendix C.

1.1 Literature Reviews

All dynamical systems are related not only in physical but also in engineering area with the subject of uncertainties that are not well-known exactly because of the modeling errors. Robust controllers which apply to determine stability criteria for the linear uncertain system have gained considerably attention since last decades. A Riccati Matrix approach has been applied in [1] with the conception of quadratic guaranteed cost control for linear uncertain systems so as to control the robust controller with a guaranteed cost regard to the upper bounding by initial condition. A Lyapunov approach was introduced in [2] to improve the guaranteed cost controllers that lead to stability both in the time domain and in the frequency domain.

Recently, it has aroused a lot of interest in reducing abundant problems through optimization involving linear matrix inequality in [4]. Linear matrix inequality (LMI) approach has been proposed in [3] for the design of robust control that supports the minimization of guaranteed cost for linear uncertain systems with convex optimization within the framework of LMI conditions. A robust guaranteed cost controller system has been introduced for the linear uncertain time-delay system in [4] which guarantees to minimize upper bound of the cost with a convex optimization.

The design of the optimal guaranteed cost has been received in [5] for a class of linear time-delay system with norm-bounded uncertainties. A guaranteed cost is obtained via LMI. The design of robust control for the nonlinear uncertain system is obtained because of the fact that employing a guaranteed cost approach has been introduced in [6] LMIs optimization with off-the-shelf algorithms.

The condition for a guaranteed cost in [7] state feedback controller is converted within the framework of LMI conditions depend on the Lyapunov stability theorem for the linear uncertain systems and the guaranteed cost has been minimized. The problem of the decentralized robust guaranteed cost control has confirmed to minimize the performance index is given in approach of LMI conditions for linear uncertain systems [11]. A sufficient condition with the guaranteed cost controllers is proposed within the framework of LMIs. As long as this condition is found to be a feasible solution, the state feedback control law gain matrices can be obtained by means of convex optimization [12].

An observer design controller is useful to apply many the state of dynamic systems. Thus, a robust observer-based control is satisfied than a state feedback controller [13-17]. The problem of dynamic output observer-based state feedback controller is proposed for nonlinear delay systems. It has been provided to stabilize via LMIs approach [18].

Design of the observer-based controller for linear uncertain time-delay systems [19] has been minimized the optimal guaranteed cost by a framework of LMI approach.

There have been more researches about the PD state feedback to stabilize the dynamic of the system. Thus, the use of PD controller has been essential for achieving the control systems and the development of system performance [20-24]. It is considered that guaranteed cost is formulated to utilize to get minimization of the norm of gain controller [20]. Moreover, this paper proposed the PD feedback controller for the linear uncertain systems [24]. It has been illustrated to make the small gain controller.

The extended observer- based PD controller are useful for the stabilization of spacecraft [25]. It is obtained to succeed better control effect.

2. PRELIMINARIES and PROBLEM STATEMENT

2.1 Lyapunov Stability Theory

The Russian mathematician A. M. Lyapunov studied the problem of stability of dynamical systems around 1890. When he proposed significant works we call today Lyapunov Theory. There are two methods of which the second method has found extensive application in the study of the stability of control systems [8], [26].

The positive definite function is described as the following term,

$V\ R^n \rightarrow R$

a function $\forall x \in R^n$ for $V(x) \geq 0$

and $V(x) = 0 \iff x = 0$ so it is positive-definite.

The negative definite function is defined as

A function $V\ R^n \rightarrow R$ is negative definite functions

$V(x) \leq 0$ for all x

$V(x) = 0$ if and only if $x = 0$ all sublevel set of V is bounded

The positive semidefinite function is described,

If

$$\forall x \in R^n \ \ V(x) \geq 0$$

$V(x) = 0$ where $x \neq 0$ Thus, this function is positive semidefinite

This function is negative semidefinite is given by

if

$$V(x) \leq 0 \ \forall x \in R^n$$

$V(x) = 0$ where $x \neq 0$

$$\dot{x} = Ax\,; \quad x(t) = x_0 \tag{2.1}$$

The positive definite Lyapunov function is selected as

$$V = x^T(t)Px(t) \tag{2.2}$$

It is the nominal linear system of equations which is stable if there exists positive definite-matrix P such as

$$A^T P + PA < 0 \tag{2.3}$$

where $P > 0$

Hence, this system is asymptotically stable.

Proof

$$\dot{x} = Ax \tag{2.4}$$

$$\dot{x}^T = x^T A^T \tag{2.5}$$

$$V = x^T(t)Px(t) \tag{2.6}$$

Derive an equation

$$V = \frac{d}{dt}[x^T(t)Px(t)] \tag{2.7}$$

We get,

$$\dot{V}(x) = \dot{x}^T Px + x^T P\dot{x} \tag{2.8}$$

Substituting in (2.5) into (2. 8), we get

$$x^T A^T Px + x^T PAx \tag{2.9}$$

$$x^T[A^T P + PA]x < 0 \tag{2.10}$$

$$A^T P + PA < 0 \tag{2.11}$$

Hence, the nominal linear system is proved.

Indeed, it is selected $Q = Q^T > 0$ which is symmetric positive definite matrix we get,

$$A^T P + PA = -Q \tag{2.12}$$

for the matrix P which is guaranteed to be a positive-definite matrix

The aim of this theorem is that there is a constantly decreasing decisive positive function goes to zero. $\dot{V}(x)$ must be a negative matrix

Namely, with ever initial condition $t \to \infty$ for $x(t) \to 0$.

We cannot say that the system is unstable when the above condition is not satisfied. Maybe, we select other Lyapunov function $V(x)$ that may provide a feasible solution. However, $\dot{V}(x)$ is found positive definite matrix or positive semidefinite matrix we can say that this system is unstable.

2.2 Problem Statement

Considering a class of linear uncertain system is described

$$\dot{x}(t) = [A + \Delta A(t)]x(t) + [B + \Delta B(t)]x(t)u(t) \tag{2.13}$$

$$y(t) = Cx(t) \tag{2.14}$$

where , $x(t) \in R^n$ is the state vector, $u(t) \in R^{mxp}$ is the control input, $y(t) \in R^{nxp}$ is output vector, $A \in R^{nxn}$ and $\mathrm{B} \in \mathrm{R}^{\mathrm{nxm}}$ are constant system matrixes, $C \in R^{pxn}$ is the constant output matrix and $\Delta A(t)$ $\Delta B(t)$ represent the uncertainty matrix which are considered to be of following form

$$[\Delta A(t)\, \Delta B(t)] = DF(t)[E_a\, E_b] \tag{2.15}$$

where D, E_a , E_b are unknown constant matrices with relevant dimensions and $\mathrm{F(t)}$ is uncertain matrix with Lebesgue measurable elements satisfying

$$F^T(t)F(t) \leq I \tag{2.16}$$

A quadratic performance index for a linear system is introduced as follows

$$J = \frac{1}{2}\int_0^\infty [x^T(t)Sx(t) + u^T(t)Ru(t)]dt \tag{2.17}$$

where the S and R are positive-definite real symmetrical matrixes to be stated properly. Consider the system (2.13) with performance index (2.17) and observer-based control law (2.18) satisfies to a feasible solution where $P > 0$.Thus, this system is stable.

We assume that a state feedback controller is introduced by using state estimate of x

$$u(t) = K\hat{x}(t) \tag{2.18}$$

where $\mathrm{K} \in \mathrm{R}^{\mathrm{nxn}}$ is the control gain matrix and $\hat{x}(t)$ is the estimate of $x(t)$ governed by the following

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L[(y(t) - \hat{y}(t)] \tag{2.19}$$

where $L \in R^{nxp}$ is the observer feedback matrix [26]

Hence, the error dynamics is obtained as observer dynamics

$$\dot{e}(t) = (A - LC)\, e(t) \tag{2.20}$$

where

$$e(t) = x(t) - \hat{x}(t) \tag{2.21}$$

$$y(t) = Cx(t) \text{ and}$$

$$\hat{y}(t) = C\hat{x}(t) \tag{2.22}$$

Substituting (2.22), (2.21), (2.20) into (2.19) we obtain the following term

$$\dot{\hat{x}}(t) = A\hat{x}(t) + BK\hat{x}(t) + L(Cx(t) - C\hat{x}(t)) \tag{2.23}$$

We can rewrite the following term

$$\begin{pmatrix} \dot{x} \\ \dot{\hat{x}} \end{pmatrix} = \begin{pmatrix} A & BK \\ LC & A + BK - LC \end{pmatrix} \begin{pmatrix} x \\ \hat{x} \end{pmatrix} \tag{2.24}$$

The observer design dynamic system is obtained depending on the error dynamics. Our purpose select L so as to $e(t) \to 0$ as $t \to \infty$. Therefore, it is supposed that this system is equivalent to stability in (2.24).

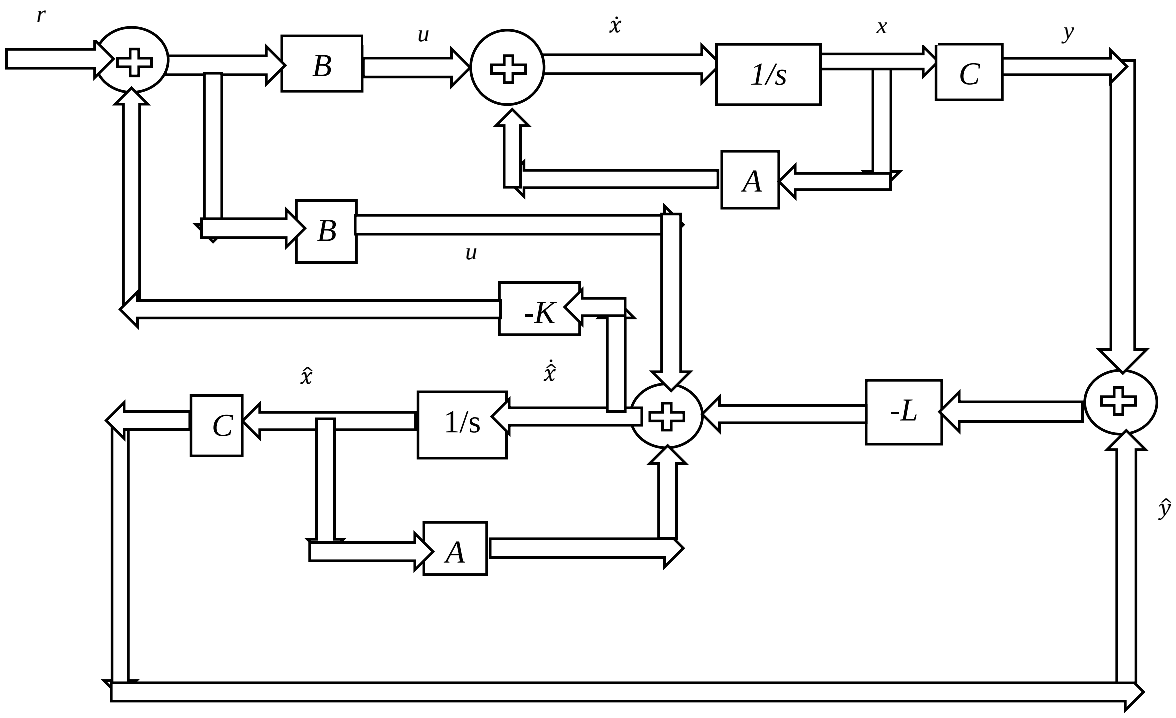

Figure 1– Observer design state feedback block diagram

3. DESIGN of ROBUST GUARANTEED COST OBSERVER-BASED STATE FEEDBACK CONTROLLER

We investigated the guaranteed cost observer-based state feedback controller for nominal linear system and linear uncertain system in this section.

3.1 Design of Robust Guaranteed Cost Observer-Based State Feedback Controller for Nominal Linear System

The nominal system is that there consists of no uncertainty parameters described in (3.1). Namely, $\Delta A(t) = 0$ and $\Delta B(t) = 0$

$$\dot{x}(t) = [A + \Delta A(t)]x(t) + [B + \Delta B(t)]x(t)u(t) \tag{3.1}$$

The following lemma 1 is demonstrated the result of design guaranteed cost observer-based controller for the nominal linear system

Lemma 1

Given real and symmetric positive definite matrices R and S, if there exist real and symmetric positive definite matrices X, Q ,matrices Y, W and positive scalar α all with convenient dimensions satisfying,

$$\Sigma = \begin{bmatrix} \Sigma_{11} & 0 & Y^T & X & -BY \\ * & \Sigma_{22} & 0 & 0 & 0 \\ * & * & -2R^{-1} & 0 & -Y \\ * & * & * & -2S^{-1} & 0 \\ * & * & * & * & -XZX \end{bmatrix} < 0 \tag{3.2a}$$

$$\begin{bmatrix} \alpha + 0.5e^T(0)Qe(0) & x^T(0) \\ * & -2X \end{bmatrix} < 0 \tag{3.2b}$$

where $\Sigma_{11} = AX + XA^T + BY + Y^T B^T, \Sigma_{22} = QA + A^T Q - WC - C^T W^T + Z$

then, a stabilizing observer-based state feedback controller gain is obtained as $K = YX^{-1}$.

Proof

We select the Lyapunov function for the nominal system in the following form

$$V(x(t), t) = x^T(t)Px(t) + e^T(t)Qe(t) \tag{3.3}$$

P and Q are positive symmetric matrixes or positive semidefinite matrixes and we can also add guaranteed cost function described (2.17) we obtain,

$$\dot{V}(x(t),t) + \frac{1}{2}x^T Sx + \frac{1}{2}\, u^T(t)Ru(t) \tag{3.4}$$

Substituting control law defined (2.18) into (3.4) allows calculating

$$2x^T(t)P\dot{x}(t) + 2e^T(t)Q\dot{e}(t) + \frac{1}{2}x^T Sx + \frac{1}{2}\hat{x}^T(t)K^T RK\hat{x}(t) \tag{3.5}$$

To substitute $\hat{x}(t) = x(t) - e(t)$, the equation in (3.5) is rewritten taking into consideration (3.1), (2.20) we get,

$$2x^T(t)P[\, Ax(t) - BKe(t) + BKx(t)] + 2e^T(t)\, Q(A - LC)\, e(t) + \frac{1}{2}x^T Sx \; + \frac{1}{2}\hat{x}^T(t)K^T RK\hat{x}(t) = \chi^T(t)\psi\, \chi(t) \tag{3.6}$$

where

$$\chi = [x^T \; e^T]^T$$

We get a bilinear matrix inequality (BMI) as the following term

$$\psi = \begin{bmatrix} \begin{matrix} PA + A^T P \\ + PBK + K^T B^T P \\ + \frac{1}{2} S \end{matrix} & -PBK - \frac{1}{2}K^T RK \\ * & \begin{matrix} QA + A^T Q \\ -WC - C^T W^T \\ + \frac{1}{2} K^T RK \end{matrix} \end{bmatrix} \tag{3.7}$$

With $\mathrm{L} = \mathrm{Q}^{-1}W$. In order to guarantee (3.2) to be less than zero, we need to be satisfying

$$\psi < 0 \tag{3.8}$$

If we apply Schur complement [8] to the BMI Then, we get an equivalent LMI

$$\psi = \begin{bmatrix} \begin{matrix} PA + A^T P \\ + PBK + K^T B^T P \\ + \frac{1}{2} S \end{matrix} & -PBK & K^T \\ * & \begin{matrix} QA + A^T Q \\ -WC - C^T W^T \end{matrix} & -K^T \\ * & * & -2R^{-1} \end{bmatrix} \tag{3.9}$$

To represent $KX = Y$ where $X = P^{-1}$ we implement a congruent transformation via pre-and post-multiplying (3.9) with $diag\ \{X, I, I\}$ to obtain

$$\begin{bmatrix} \begin{matrix} AX + XA^T \\ + BY + Y^T B^T \\ + \frac{1}{2} \quad XSX \end{matrix} & -BYX^{-1} & Y^T \\ * & \begin{matrix} QA + A^T Q \\ -WC - C^T W^T \end{matrix} & -X^{-1}Y^T \\ * & * & -2R^{-1} \end{bmatrix} < 0 \tag{3.10}$$

After we applied Schur complement [8] in (3.10) we get,

$$\Omega = \begin{bmatrix} \begin{matrix} AX + XA^T \\ + BY + Y^T B^T \end{matrix} & -BYX^{-1} & Y^T & X \\ * & \begin{matrix} QA + A^T Q \\ -WC - C^T W^T \end{matrix} & -X^{-1}Y^T & 0 \\ * & * & -2R^{-1} & 0 \\ * & * & * & -2S^{-1} \end{bmatrix} < 0 \tag{3.11}$$

Let us reexpress Ω in the following form

$$\Omega = \Omega_0 + {\Omega_1}^T + \Omega_1 \tag{3.12}$$

where

$$\Omega_0 = \begin{bmatrix} \begin{matrix} AX + XA^T \\ + BY + Y^T B^T \end{matrix} & 0 & Y^T & X \\ * & \begin{matrix} QA + A^T Q \\ -WC - C^T W^T \end{matrix} & 0 & 0 \\ * & * & -2R^{-1} & 0 \\ * & * & * & -2S^{-1} \end{bmatrix}$$

and

$$\Omega_1 = \begin{bmatrix} 0 & -BYX^{-1} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -YX^{-1} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Now we shall rewrite Ω_1

$$\Omega_1 = \begin{bmatrix} -BY \\ 0 \\ -Y \\ 0 \end{bmatrix} X^{-1} [0 \quad I \quad 0 \quad 0] \tag{3.13}$$

$$= \Pi^T X^{-1} \Theta$$

where

$$Ҁ = [Y^T B^T \quad 0 \quad -Y^T \quad 0]$$

$$\Theta = [0 \quad I \quad 0 \quad 0]$$

Substituting Ω_1 in (3.13) into (3.12) and applying the well-known bounding inequality, we get

$$\Omega = \Omega_0 + Ҁ^T X^{-1}\Theta + \Theta^T X^{-1} Ҁ$$

$$\leq \Omega_0 + (Ҁ^T X^{-1}) Z^{-1} (X^{-1}Ҁ) + \Theta^T Z\Theta \tag{3.14}$$

Applying Schur complement [8], we acquire inequality in (3.2). Therefore, this completes the proof. On the contrary, we realize that the equation in (3.14) is not in the form of convex LMI due to the nonlinear term which is $-XZX$. Thus, we now propose an iterative algorithm to get a feasible solution set (3.2).

Let us select a real symmetric and positive definite matrix $M^T = M > 0$ such that

$$-XZX < -M \tag{3.15}$$

After applying Schur complement, we get where $Z > 0$

$$\begin{pmatrix} M^{-1} & X^{-1} \\ * & Z \end{pmatrix} \geq 0 \tag{3.16}$$

Next, we present some new variables $M^{-1} = N, X^{-1} = T$. We obtain a feasible solution due to employing cone complementary technique [9] that provides the following nonlinear parameter minimization within the framework of LMI conditions.

$Minimize\ tr(MN + XT)$ [10].

$$\begin{pmatrix} M & I \\ * & N \end{pmatrix} > 0 \qquad \begin{pmatrix} X & I \\ * & T \end{pmatrix} > 0 \qquad \begin{pmatrix} N & T \\ * & Z \end{pmatrix} > 0 \quad \text{and}$$

$$\Sigma = \begin{bmatrix} \Sigma_{11} & 0 & Y^T & X & -BY \\ * & \Sigma_{22} & 0 & 0 & 0 \\ * & * & -2R^{-1} & 0 & -Y \\ * & * & * & -2S^{-1} & 0 \\ * & * & * & * & -M \end{bmatrix} < 0 \tag{3.17}$$

Next, the findings in the following terms (3.18) demonstrated the robust observer-based stabilization criteria for the linear uncertain system.

If there exist real and symmetric positive definite matrices P, Q and matrices Y, W all with convenient dimensions (3.17) and positive scalar α it will satisfy as the following form

$$\begin{bmatrix} \alpha + 0.5e^T(0)Qe(0) & x^T(0) \\ * & -2X \end{bmatrix} < 0 \tag{3.18}$$

If we now consider nominal linear system we shall show that performance index (2.17) has been upper bound as the following equivalent

$$V(x(0)) = \frac{1}{2}x^T(0)Px(0) + \frac{1}{2}e^T(0)Qe(0)$$

$$\leq \frac{1}{2}\lambda minP \ \parallel x(o) \parallel^2 + \frac{1}{2}\lambda min(Q) \ \parallel e(o) \parallel^2 = J^* \tag{3.19}$$

In order to minimize the guaranteed cost in (2.17), we introduce a positive scalar α such that

$$\frac{1}{2}x^T(0)Px(0) + \frac{1}{2}e^T(0)Qe(0) \leq -\alpha \tag{3.20}$$

We represent $X = P^{-1}$, implying that

$$\alpha + \frac{1}{2}x^T(0)X^{-1}x(0) + \frac{1}{2}e^T(0)Qe(0) \leq 0 \tag{3.21}$$

$$\begin{bmatrix} \alpha + 0.5\mathrm{e}^{\mathrm{T}}(0)\mathrm{Qe}(0) & \mathrm{x}^{\mathrm{T}}(0) \\ * & -2\mathrm{X} \end{bmatrix} < 0 \tag{3.22}$$

LMI of (3.22) can be obtained because of Schur complement. The performance index will have minimized upper bound once α is the smallest. This completes the proof based on that can be solved using the cone complementary approach outlined following Lemma 1. Thus, the minimization of the guaranteed cost is satisfied.

3.2 Design of Robust Guaranteed Cost Observed-Based State Feedback Controller for Linear Uncertain System

Consider linear uncertain system described as in (3.23), in (3.24) and in (3.25) respectively.

$$\dot{x}(t) = [A + \Delta A(t)]x(t) + [B + \Delta B(t)]x(t)u(t) \tag{3.23}$$

where

$$[\Delta A(t)\ \Delta B(t)] = DF(t)[E_a\ E_b] \tag{3.24}$$

$$F^T(t)F(t) \leq I \tag{3.25}$$

These equation (3.23), (3.24), (3.25) are described in section 2.2 completely. Next, the following theorem 1 is illustrated to propose approach of Lyapunov stability theorem.

Theorem 1

Given real and symmetric positive definite matrices R and S, if there exist real and symmetric positive definite matrices P, Q and matrices Y, W all with suitable dimensions and positive scalar α, $\varepsilon_1$and ε_2 are introduced satisfying,

$$\varphi = \begin{bmatrix} \varphi_{11} & 0 & Y^T & X & -BY & X{E_a}^T & 0 \\ * & \varphi_{22} & 0 & 0 & 0 & 0 & QD \\ * & * & -2R^{-1} & 0 & -Y & 0 & 0 \\ * & * & * & -2S^{-1} & 0 & 0 & 0 \\ * & * & * & * & -M & Y^T{E_b}^T & 0 \\ * & * & * & * & * & \varepsilon_1 I & 0 \\ * & * & * & * & * & * & \varepsilon_2 I \end{bmatrix} < 0 \tag{3.26}$$

$$\begin{bmatrix} \alpha + 0.5e^T(0)Qe(0) & x^T(0) \\ * & -2X \end{bmatrix} < 0 \tag{3.27}$$

where $\varphi_{11} = AX + XA^T + BY + Y^TB^T + \varepsilon_1 DD^T$, $\varphi_{22} = QA + A^TQ + -WC - C^T W^T + Z + \varepsilon_2 {E_a}^T E_a$

Then, a robust stabilizing controller gain is obtained as $K = YX^{-1}$.

Proof

If we now consider the norm-bounded uncertainty in (3.24) then, we replace A, B with $A + DF(t)E_a$ and $B + DF(t)E_b$ respectively to get

$$\Sigma_u = \begin{bmatrix} \Sigma_u(1,1) & 0 & Y^T & X & \Sigma_u(1,5) \\ * & \Sigma_u(2,2) & 0 & 0 & 0 \\ * & * & -2R^{-1} & 0 & -Y \\ * & * & * & -2S^{-1} & 0 \\ * & * & * & * & -XZX \end{bmatrix} < 0 \tag{3.28}$$

where $\Sigma_u(1,1) = AX + XA^T + DF(t)E_aX + (DF(t)E_aX)^T + BY + Y^TB^T + DF(t)E_bX + (DF(t)E_bX)^T$, $\Sigma_u(1,5) = -BY - DF(t)E_bY$

$$\Sigma_u(2,2) = QA + A^TQ + QDF(t)E_aX + Q(DF(t)E_aX)^T - WC - C^T W^T + Z$$

$$= \chi^T(t)\Sigma_u \chi(t)$$

where $\chi = [x^T \ e^T]^T$

We rewrite in (3.28) as follows

$$\Sigma_u = \Sigma_{u0} + \Sigma_{u1}{}^T + \Sigma_{u1} \tag{3.29}$$

Σ_{u0} is a linear part, and Σ_{u1} is the nonlinear part of the equivalent

$$\Sigma_{u1} = \begin{bmatrix} \Sigma_{u1}(1,1) & 0 & 0 & 0 & -DF(t)E_bY \\ * & \Sigma_{u1}(2,2) & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 \\ * & * & * & 0 & 0 \\ * & * & * & * & 0 \end{bmatrix} < 0 \tag{3.30}$$

where $\Sigma_{u1}(1,1) = DF(t)E_aX + (DF(t)E_aX)^T + +DF(t)E_bX + (DF(t)E_bX)^T$,$\Sigma_{u1}(2,2) = QDF(t)E_aX + Q(DF(t)E_aX)^T$

We rewrite Σ_{u1} in the following term as the

$$\Sigma_{u1} = \Pi_{u1}{}^T X^{-1} \theta_{u1} + \Pi_{u2}{}^T X^{-1} \theta_{u2} \tag{3.31}$$

where

$$\Pi_{u1} = \begin{pmatrix} D \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \ \theta_{u1} = (E_a X \quad 0 \quad 0 \quad 0 \quad -E_b Y)$$

$$\Pi_{u2} = \begin{pmatrix} 0 \\ QD \\ 0 \\ 0 \\ 0 \end{pmatrix}, \theta_{u2} = (0 \quad E_a \quad 0 \quad 0 \quad 0) \tag{3.32}$$

Hence, using the well-known bounding -inequality allows to rewrite (3.28) as follows

$$\Sigma_{u1} = \Pi_{u1}{}^T X^{-1} \theta_{u1} + \Pi_{u2}{}^T X^{-1} \theta_{u2}$$

$$\leq \Sigma_{u0} + \varepsilon_1 \Pi_{u1}{}^T \Pi 1 + \varepsilon_1{}^{-1} \theta_{u1}{}^T \theta_{u1} + \varepsilon_2{}^{-1} \Pi_{u2}{}^T \Pi_{u2} + \varepsilon_2 \theta_{u2}{}^T \theta_{u2} \tag{3.33}$$

Then, applying Schur Complement [8] one obtains (3.26)

If we now, consider nominal linear system we shall show that performance index (2.17) has been upper bound as the following equivalent

$$V(x(0)) = \frac{1}{2} x^T(0) P x(0) + \frac{1}{2} e^T(0) Q e(0)$$

$$\leq \frac{1}{2} \lambda min P \ \| x(o) \|^2 + \frac{1}{2} \lambda min(Q) \ \| e(o) \|^2 = J^* \tag{3.34}$$

Minimizing the guaranteed cost in (2.17), we introduce a positive scalar α such that

$$\frac{1}{2} x^T(0) P x(0) + \frac{1}{2} e^T(0) Q e(0) \leq -\alpha \tag{3.35}$$

To represent $X = P^{-1}$, implying that

$$\alpha + \frac{1}{2} x^T(0) X^{-1} x(0) + \frac{1}{2} e^T(0) Q e(0) \leq 0 \tag{3.36}$$

$$\begin{bmatrix} \alpha + 0.5 e^T(0) Q e(0) & x^T(0) \\ * & -2X \end{bmatrix} < 0 \tag{3.37}$$

Then, we apply Schur complement [8] we get in (3.27). The performance index will have minimized upper bound where α is the smallest. This completes the proof. The feasibility problem of theorem 1 can be solved by using the cone complementary technique [9-10] as well as linear nominal model. The results in the following theorem 1 illustrated the robust observer-based stabilization criteria for the linear uncertain system. Thus, the aim of the minimization of guaranteed cost is satisfied.

4. DESIGN of OBSERVER-BASED PD STATE FEEDBACK CONTROLLER

In this section, we divided into two parts. Firstly, we investigated the stability of observer-based PD state feedback controller for nominal system in the first part. Second part, we search the guaranteed cost minimization depending on observer-based PD state feedback controller for linear nominal system.

4.1 Design of Observer-Based PD State Feedback Controller for Nominal Linear System

We consider a class of nominal linear observer-based PD state feedback controller is described as well as in (3.1). However, the difference is that control law is described as the following term

$$u(t) = K_p\hat{x}(t) + K_d\dot{\hat{x}}(t) \tag{4.1}$$

Next, the following lemma 2 is showed to analyze design of the observer-based PD state feedback control.

Lemma 2

If there exist real and symmetric positive definite matrices X, Q and matrix Y with appropriate dimensions satisfying,

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & -BY \\ * & \Xi_{22} & 0 \\ * & * & -XZX \end{bmatrix} < 0 \tag{4.2}$$

where $\Xi_{11} = AX + XA^T + BK_pX + \left(BK_pX\right)^T - AY^TB^T - BYA^T - BY^TK_pB^T - BK_p{}^TYB^T$

$\Xi_{12} = -BK_p - (A - LC)$, $\Xi_{22} = Q(A - LC) + (A - LC)^TQ + Z$

Next, a stabilizing observer-based PD state feedback controller gain is described as

$K_d = YX^{-1}$, K_p is selected fixed as well as observer-based control gain for the linear nominal system.

Proof

State space system is described as

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{4.3}$$

We can select a Lyapunov function as the following term

$$V(x(t), t) = x^T(t)Px(t) + e^T(t)Qe(t) \tag{4.4}$$

P and Q must be positive real symmetric matrixes so we can derive in (4.4), we obtain as

$$\dot{V}(x(t), t) = 2x^T(t)P\dot{x}(t) + 2e^T(t)Q\dot{e}(t) \tag{4.5}$$

Substituting (4.1), (4.3) into (4.5) we get the following term to allow to calculate

$$2x^T P(I - BK_d)^{-1}(A + BK_p)x + 2e^T Q(A - LC)e = \chi^T \Gamma \chi \tag{4.6}$$

Assuming that $(I - BK_d)$ is nonsingular.

where

$\dot{x} = (I - BK_d)^{-1}[Ax(t) + BK_p x(t) - BK_p e(t) + BK_d(A - LC)]$,

$e(t) = x(t) - \hat{x}(t)$ and $\chi = [x^T \ e^T]^T$

We get the following term as

$$\Gamma = \begin{bmatrix} P(I - BK_d)^{-1}(A + BK_p) & -P(I - BK_d)^{-1}BK_p \\ +(A + BK_p)^T(I - BK_d)^{-1} & -P(I - BK_d)^{-1}BK_d(A - LC) \\ * & Q(A - LC) + (A - LC)^T Q \end{bmatrix}$$

We need to satisfy (4.7) to be less than 0

$$\Gamma < 0 \tag{4.7}$$

Multiplying via pre $(I - BK_d)X$ and post $X(I - BK_d)^T$ with $diag\,\{X, I\}$ where $P^{-1} = X$ and $K_d = YX^{-1}$ we get,

$$\Lambda = \begin{bmatrix} AX + XA^T + BK_pX + (BK_pX)^T & -BK_p \\ -AY^TB^T - BYA^T - BY^TK_pB^T - BK_p{}^TYB^T & -BYX^{-1}(A-LC) \\ * & Q(A-LC) + (A-LC)^TQ \end{bmatrix} < 0 \quad (4.8)$$

Let redefine Λ in the following term

$$\Lambda = \Lambda_0 + \Lambda_1{}^T + \Lambda_1 \quad (4.9)$$

where

$$\Lambda_0 = \begin{bmatrix} AX + XA^T + BK_pX + (BK_pX)^T & -BK_p - (A-LC) \\ -AY^TB^T - BYA^T - BY^TK_pB^T - BK_p{}^TYB^T & Q(A-LC) + (A-LC)^TQ \\ * & \end{bmatrix}$$

and

$$\Lambda_1 = \begin{bmatrix} 0 & -BY\,X^{-1} \\ 0 & 0 \end{bmatrix}$$

Let us rewrite Λ_1 as

$$\Lambda_1 = \begin{bmatrix} -BY \\ 0 \end{bmatrix} X^{-1}[0\ 1] = \Pi^T X^{-1}\Theta \quad (4.10)$$

where

$$\Pi = [-Y^TB^T \quad 0]^T, \Theta = [0 \quad I]$$

Following procedures are used to apply the well-known norm bounding inequalities, we obtain

$$\begin{aligned} \Lambda &= \Lambda_0 + \Pi^T X^{-1}\Theta + \Theta^T X^{-1}\Pi \\ &\leq \Lambda_0 + (\Pi^T X^{-1})\, Z^{-1}\, (X^{-1}\Pi) + \Theta^T Z \Theta \end{aligned} \quad (4.11)$$

After applying Schur complement [8] to get in (4.2). We realize the equation is not convex as well as observer-state feedback controller. Thus, an iterative algorithm is considered to obtain a feasible solution.

We can select matrix S which is positive definite and real symmetric matrix $S^T = S > 0$ as for the following term

$$-XZX < -S \quad (4.12)$$

Applying Schur complement to denote some new variables $S^{-1} = M, X^{-1} = J$ we obtain a feasible solution due to employing cone complementary technique [9] that provides the minimization of nonlinear parameters within the framework of LMI conditions in the following as described

$Minimize\ tr(SM + XJ)$ [10].

$$\begin{pmatrix} M & J \\ * & Z \end{pmatrix} > 0 \quad , \quad \begin{pmatrix} S & I \\ * & M \end{pmatrix} > 0 \quad , \quad \begin{pmatrix} X & I \\ * & J \end{pmatrix} > 0$$

$$\boldsymbol{\Xi} = \begin{bmatrix} \boldsymbol{\Xi}_{11} & \boldsymbol{\Xi}_{12} & -BY \\ * & \boldsymbol{\Xi}_{22} & 0 \\ * & * & S \end{bmatrix} < 0 \tag{4.13}$$

According to the cone complementary technique, we get in (4.13) because of the fact that *Kd* is found whereas *Kp* is selected fixed. All results on observer-based PD feedback controller are satisfied for linear nominal systems.

4.2 Design of Guaranteed Cost Observer-Based State Feedback PD Controller for Nominal Linear System

In this part, we searched the minimization of guaranteed cost observer-based PD state feedback controller for the nominal linear system. The process of the state space PD feedback controller which is described in (4.1) is taken fixed variables in order to get a feasible solution to Lemma 3.

Lemma 3

Given real and symmetric positive definite matrices R and S, if there exist real and symmetric positive definite matrices *X, Q* and the observer-based PD control law is chosen with fixed K_p and K_d obtained from Lemma 2 satisfying,

$$\Psi = \begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & -XK_p{}^T & -X \\ * & \Psi_{22} & 0 & 0 & 0 \\ * & * & -2R^{-1} & 0 & 0 \\ * & * & * & -2R^{-1} & 0 \\ * & * & * & * & -2S^{-1} \end{bmatrix} < 0 \tag{4.14}$$

where

$\Psi_{11} = (I - BK_d)^{-1}(A + BK_p)X + X(A + BK_p)^T(I - BK_d)^{-T}$

$\Psi_{12} = -(I - BK_d)^{-1}BK_p - (I - BK_d)^{-1}BK_d(A - LC)$

$\Psi_{13} = -X(A + BK_p)^T(I - BK_d)^{-T}{K_d}^T$

$\Psi_{22} = Q(A - LC) + (A - LC)^TQ + 0.5{K_p}^TRK_p + 0.5(A - LC)^T{K_d}^TRK_d(A - LC) + 0.5(A - LC){K_d}^TB^T(I - BK_d)^{-T}{K_d}^TRK_d(I - BK_d)^{-1}BK_d(A - LC)$

Proof

We consider the system in (4.3) and select Lyapunov function in the following term

$$V(x(t), t) = x^T(t)Px(t) + e^T(t)Qe(t) \tag{4.15}$$

If P and Q are positive symmetric matrixes and we add the guaranteed cost function described in (2.17) into (4.15) we obtain,

$$\dot{V}(x(t), t) + \frac{1}{2}x^TSx + \frac{1}{2}\,u^T(t)Ru(t) \tag{4.16}$$

We rewrite (4.16) as

$$2x^T(t)P\dot{x}(t) + 2e^T(t)Q\dot{e}(t) + \frac{1}{2}x^TSx + \frac{1}{2}\,u^T(t)Ru(t) \tag{4.17}$$

Substituting (4.1) into (4.17), we obtain as

$$2x^T(t)P\dot{x}(t) + 2e^T(t)Q\dot{e}(t) + \frac{1}{2}x^TSx + \frac{1}{2}[K_p\hat{x}(t) + K_d\dot{\hat{x}}(t)]^TR[K_p\hat{x}(t) + K_d\dot{\hat{x}}(t)] = \chi^T(t)\omega\,\chi(t) \tag{4.18}$$

where

$\dot{x} = (I - BK_d)^{-1}[Ax(t) + BK_px(t) - BK_pe(t) - BK_d(A - LC)e(t)]$

$e(t) = x(t) - \hat{x}(t)$,$\dot{e}(t) = (A - LC)\,e(t)$, and $\chi = [x^T\ e^T]^T$

Bilinear matrix inequality (BMI) is obtained in (4.19).

$$\omega = \begin{bmatrix} \omega_{11} & \omega_{12} \\ * & \omega_{22} \end{bmatrix} \tag{4.19}$$

where,

$$\begin{aligned} \omega_{11} = {} & P(I - BK_d)^{-1}(A + BK_p) + \ +(A + BK_p)^T(I - BK_d)^{-T}P + 0.5{K_p}^T RK_p \\ & + 0.5(A + BK_p)^T(I - BK_d)^{-T}{K_d}^T RK_d(I - BK_d)^{-1}(A + BK_p) + 0.5S \end{aligned}$$

$$\omega_{12} = -P(I - BK_d)^{-1}BK_p - P(I - BK_d)^{-1}BK_d(A - LC)$$

$$\begin{aligned} \omega_{22} = {} & Q(A - LC) + (A - LC)^T Q + 0.5(A - LC)^T {K_d}^T RK_d(A - LC) \\ & + 0.5(A - LC)^T {K_d}^T B^T (I - BK_d)^{-T}{K_d}^T RK_d(I - BK_d)^{-1}BK_d(A - LC) \\ & + 0.5{K_p}^T RK_p \end{aligned}$$

We need to guarantee the following inequality.

$$\omega < 0 \tag{4.20}$$

Let us apply congruent transformation via pre and post-multiplying (4.19) with $diag\{X, I\}$ to get where $X = P^{-1}$

$$\bar{\bar{W}} = \begin{bmatrix} \bar{\bar{W}}_{11} & \bar{\bar{W}}_{12} \\ * & \bar{\bar{W}}_{22} \end{bmatrix} < 0$$

$$\begin{aligned} \bar{\bar{W}}_{11} = {} & (I - BK_d)^{-1}(A + BK_p) + \ (A + BK_p)^T(I - BK_d)^{-T} + X0.5{K_p}^T RK_p X + \\ & X0.5(A + BK_p)^T(I - BK_d)^{-T}{K_d}^T RK_d(I - BK_d)^{-1}(A + BK_p)X + 0.5XSX \end{aligned}$$

$$\bar{\bar{W}}_{12} = -(I - BK_d)^{-1}BK_p - (I - BK_d)^{-1}BK_d(A - LC)$$

$$\begin{aligned} \bar{\bar{W}}_{22} = {} & Q(A - LC) + (A - LC)^T Q + 0.5(A - LC)^T {K_d}^T RK_d(A - LC) \\ & + 0.5(A - LC)^T {K_d}^T B^T (I - BK_d)^{-T}{K_d}^T RK_d(I - BK_d)^{-1}BK_d(A - LC) \\ & + 0.5{K_p}^T RK_p \end{aligned}$$

Employing Schur complement, we get (4.14). This completes the proof of lemma 3 to get a feasible solution. We could not apply cone complementary technique since the equation in (4.14) is convex. In addition, the observer-based PD control law is selected with fixed gain values to obtain a feasible solution. Moreover, this indicates that the minimization of the guaranteed cost has been achieved via using the inequality (3.27) from theorem 1.

5. NUMERICAL EXAMPLES

In this section, three numerical examples are shown due to the application of Lemma 1, Lemma 2, Lemma 3 and Theorem 1.

Example 1

The nominal form of guaranteed cost observer-based state feedback controller [4] is described as follows

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$y(t) = \ C\, x(t)$$

where $A = \begin{bmatrix} 0 & 0.5 \\ 0 & 1 \end{bmatrix}$,$B = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}$,.$C = [1 \quad 0]$

The initial condition is defined such as $x\_0 = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$. We can obtain a feasible solution in (3.17) with the following parameter

$$P = \begin{bmatrix} 9.8525 & 1.1911 \\ 1.1911 & 9.4857 \end{bmatrix},$$

$$Q = 10^8 * \begin{bmatrix} 3.2917 & -0.7656 \\ -0.7656 & 0.2493 \end{bmatrix}$$

The upper bound guaranteed cost function is calculated as $J^* = 6.7075$ .It shows the minimization of the guaranteed cost has been succeeded via theorem 1. The control gain of the state feedback obtained as $K = [-1.7309 \quad -10.1860]$. The control law observer-based controller for the nominal system is illustrated in Figure 2. It follows from Figure 3 and Figure 4 the method of estimating state vectors has been succeeded with a satisfactory form of steady state accuracy respectively. Based on Lyapunov stability theory, these figures show stability with an observer-based controller for linear nominal system.

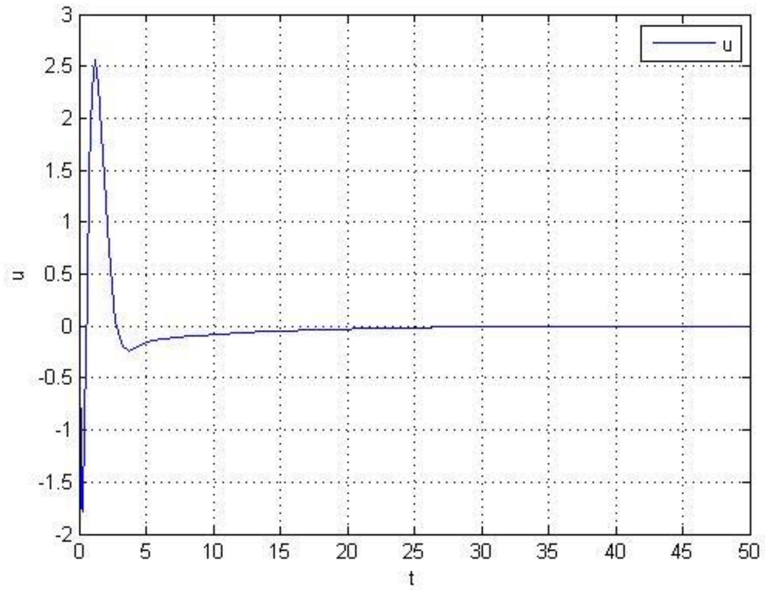

Figure 2 – Control law for the nominal system

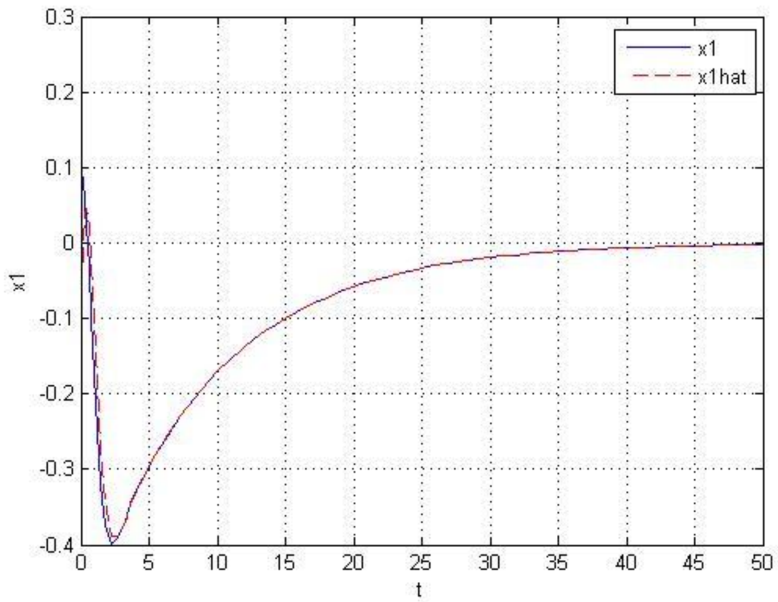

Figure 3 – A state feedback observer design variable of x1 for the nominal system

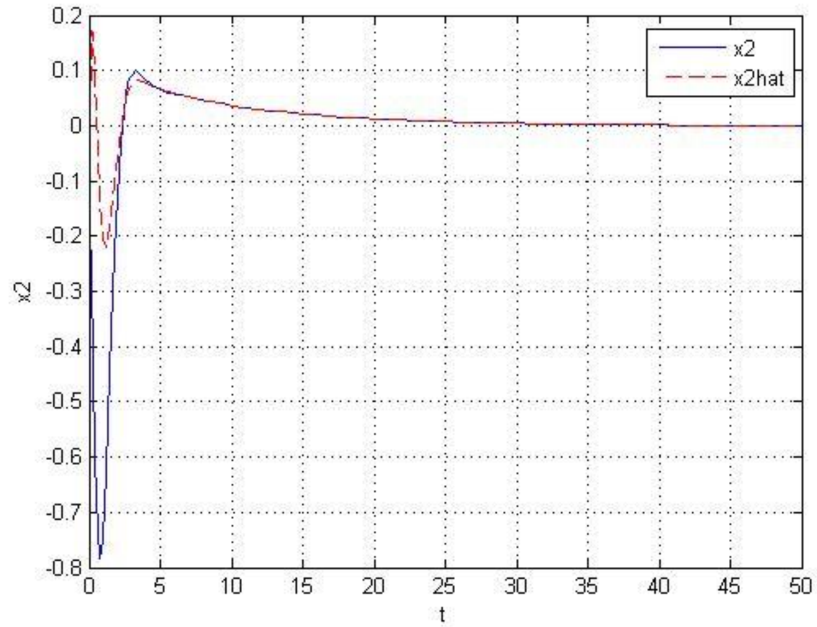

Figure 4 – A state feedback observer design variable of x2 for the nominal system

Example 2

We now consider linear uncertain system defined in (3.23), [4]

with $A = \begin{bmatrix} 0 & 0.5 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}, C = [1 \quad 0],$

$$D = \begin{bmatrix} 0.5 \\ 0.3 \end{bmatrix}, E_a = [0.1 \quad 0.2], E_b = 0.1 \; F = sin(t)$$

A feasible solution set is obtained for (3.26) with the following parameter results.

$$P = \begin{bmatrix} 42.5610 & 4.6125 \\ 4.6125 & 31.5982 \end{bmatrix}, \; Q = 10^7 * \begin{bmatrix} 1.7869 & -0.3948 \\ -0.3948 & 0.1141 \end{bmatrix}$$

The state feedback gain matrix is found as $= [-2.4085 \;\; -10.5507]$, The upper bound of performance index is achieved as $J^* = 27.5365$.It specifies the minimization of the guaranteed cost by theorem 1. The control law observer-based controller for uncertain system is illustrated in Figure 5. These graphics in Figure 6 and Figure 7 demonstrate that the process of performance index has been achieved by theorem 1 depend on the Lyapunov stability theorem within the framework of linear matrix inequalities.

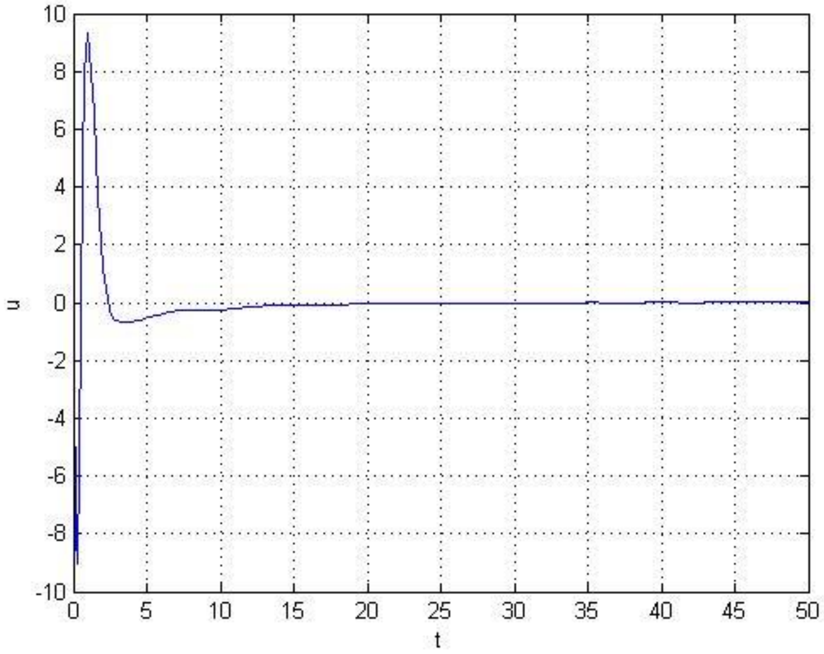

Figure 5 – Control law for an uncertain system

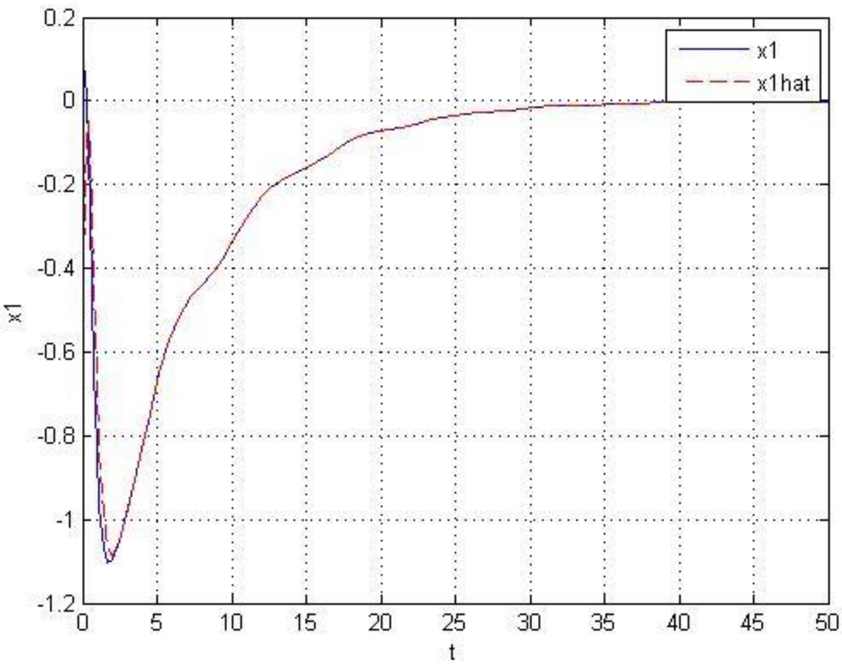

Figure 6 – A state feedback observer design variable of x1 for an uncertain system

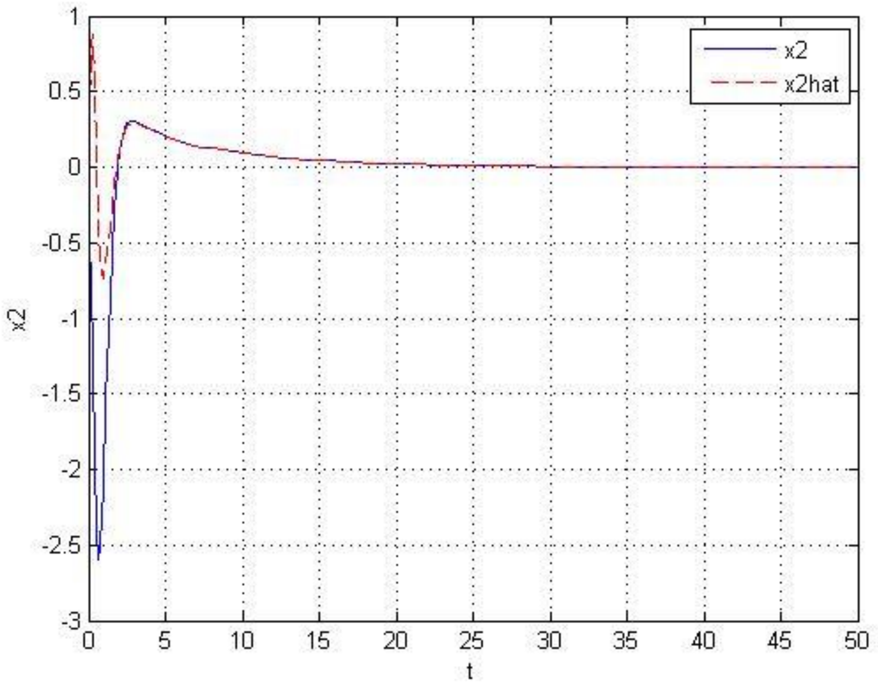

Figure 7 – A state feedback observer design variable of x2 for uncertain system

Example 3

We now consider system defined to design guaranteed cost observer-based PD state feedback controller for nominal systems [4].

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$y(t) = C\,x(t)$$

where $A = \begin{bmatrix} 0 & 0.5 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}$, $C = [1 \quad 0]$

We get a feasible solution set for (4.14) as the following term,

$$P = \begin{bmatrix} 3.5470 & 3.9180 \\ 3.9180 & 5.6958 \end{bmatrix},\ Q = 10^8 * \begin{bmatrix} 0.2465 & 0.6942 \\ 0.6942 & 6.8935 \end{bmatrix}$$

Based on lemma 2 observer-based PD state feedback control gain is obtained when the observer-based control gain (K_p) is fixed shown as

$$K_p = [-1.7309 \quad -10.1860],\ K_d = [-0.2376 \quad -0.8788]$$

Then, all variables are fixed in lemma 3 to get a feasible solution due to lemma 2. Moreover, the upper bound of performance index is accomplished as $J^* = 4.4445$. It indicates the minimization of the guaranteed cost has been achieved by means of theorem 1. It is concluded that the minimization guaranteed cost value is more satisfied to compare the minimization of the guaranteed cost observer-based controller for linear nominal system. The control law is demonstrated in Figure 8. The process of estimating the state vectors has been accomplished with a satisfactory form of steady state accuracy. It is also demonstrated in Figure 9 and Figure 10 respectively.

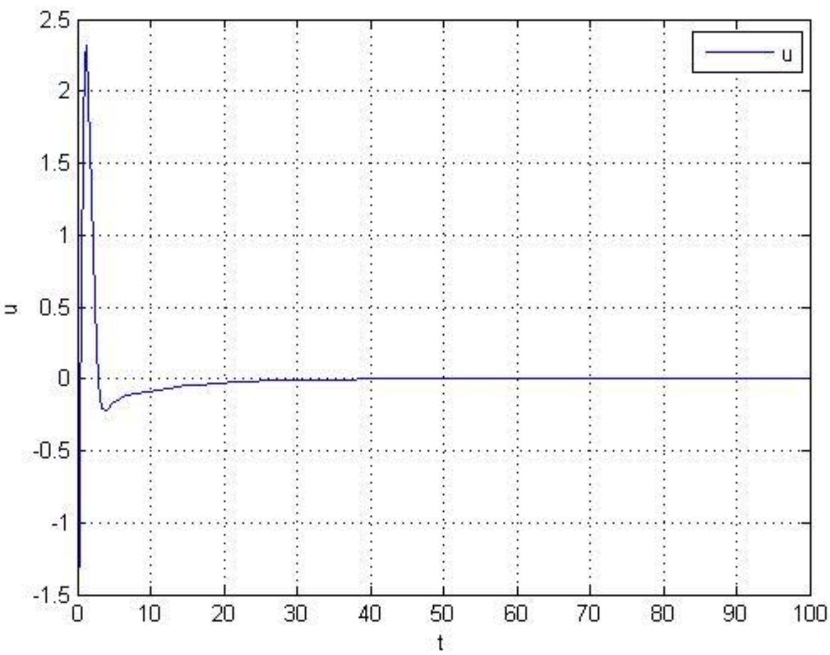

Figure 8 – Control law for the nominal system

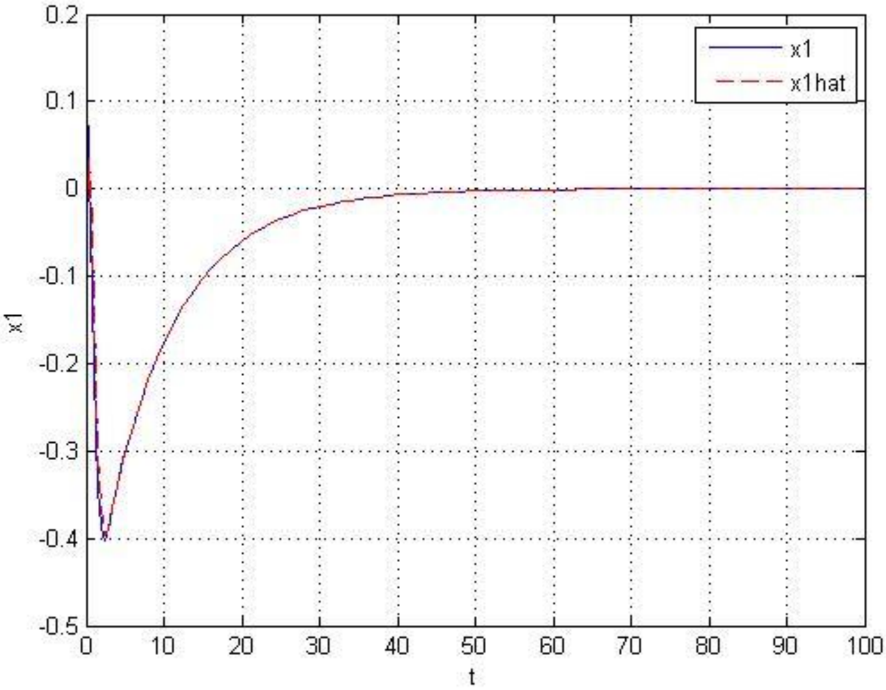

Figure 9 – A PD state feedback observer design variable of x1 for the nominal system

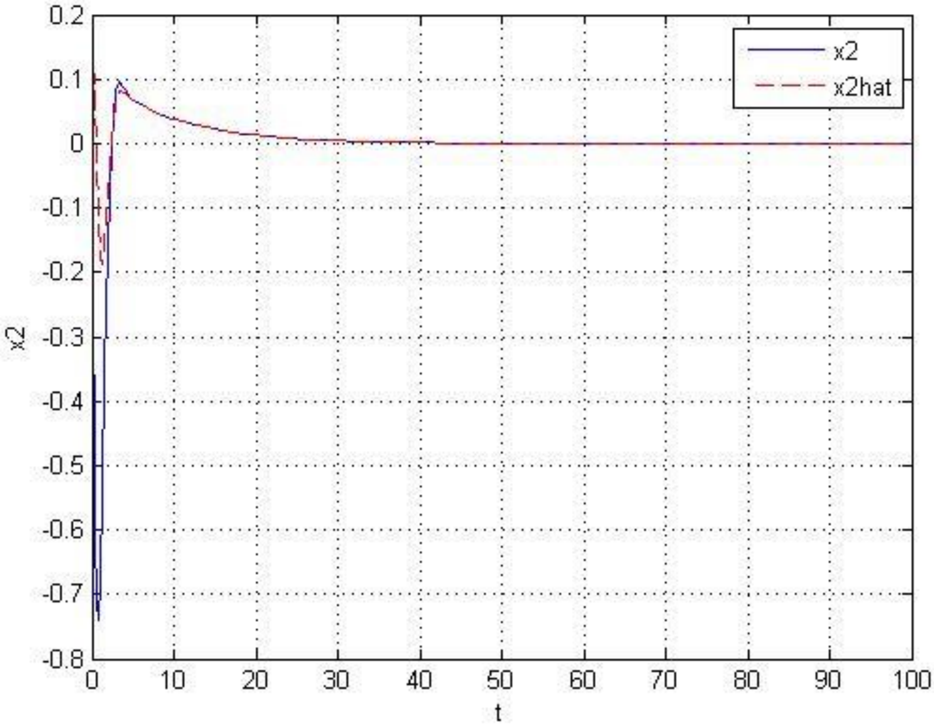

Figure 10 – A PD state feedback observer design variable of x2 for the nominal system

6.CONCLUSION

This study has performed an observer design based on robust control guaranteed cost for the linear uncertain system by introducing state feedback control law and using the estimated state from the observer design. This system has been investigated on the basis of employing Lyapunov stability theorem within the framework of LMIs. LMI conditions are shown to provide a feasible solution set by using a convex optimization through the cone complementarity linearization algorithm. Furthermore, this algorithm also employs the minimization of the guaranteed cost. Moreover, the proposed method has achieved a minimized performance index not only for nominal systems but also for linear uncertain system.

The design of observer-based PD feedback controller has been studied for nominal systems. When K_p which is the control gain of observer-based state feedback is selected fixed for the linear nominal system, K_d is found in the form of LMI. This feasible solution gives a suboptimal control action because K_p is chosen as a fixed value. Then, the guaranteed cost observer-based PD state feedback controller has been investigated for linear nominal systems. A feasible solution is obtained within the framework of linear matrix inequalities. The result indicates that the minimization of the guaranteed cost has been achieved to compare with design of the observer-based state feedback controller by approach of theorem1.

We can not investigate design of the guaranteed cost observer-based PD state feedback controller for the linear uncertain systems. Because the nonlinear parameters are risen up drastically for the linear uncertain systems.

Consequently, numerical examples have demonstrated that the introduced stabilization method provides guaranteed cost along with a satisfactory steady state accuracy.

APPENDIX A

Algorithm 1 is implemented in Matlab languages with lemma 1 for nominal and uncertain systems.

1.1 Nominal Linear System

File 1

```
A=[0 0.5;0 1];

B=[0;0.5];

C=[1 0];

R=0.5;
S=[1 0;0 1];
x_0=[1;0.5];
xhat_0=[1;0.5];
e_0=x_0-xhat_0;

[nn,m]=size(B);
[p,nn]=size(C);
K0=zeros(m,nn);
L0=zeros(nn,p);
k=1;
k0=0;
counter=0;
mySTOP=0;
setlmis([])

X=lmivar(1,[nn 1]);
Q=lmivar(1,[nn 1]);
Z=lmivar(1,[nn 1]);
M=lmivar(1,[nn 1]);
N=lmivar(1,[nn 1]);
T=lmivar(1,[nn 1]);
Y=lmivar(2,[m nn]);
W=lmivar(2,[nn p]);
alpha=lmivar(1,[1 1]);

lmiterm([1 1 1 X],A,1,'s')
lmiterm([1 1 1 Y],B,1,'s')

lmiterm([1 1 3 -Y],1,1)

lmiterm([1 1 4 X],1,1)
lmiterm([1 1 5 Y],-B,1)


lmiterm([1 2 2 Q],1,A,'s')
```

```
lmiterm([1 2 2 W],-1,C,'s')
lmiterm([1 2 2 Z],1,1)

lmiterm([1 3 3 0],-2*inv(R))

lmiterm([1 3 5 Y],-1,1)

lmiterm([1 4 4 0],-2*inv(S))

lmiterm([1 5 5 M],-1,1)

lmiterm([-2 1 1 M],1,1)
lmiterm([-2 1 2 0],1)
lmiterm([-2 2 2 N],1,1)

lmiterm([-3 1 1 X],1,1)
lmiterm([-3 1 2 0],1)
lmiterm([-3 2 2 T],1,1)

lmiterm([-4 1 1 N],1,1)
lmiterm([-4 1 2 T],1,1)
lmiterm([-4 2 2 Z],1,1)

lmiterm([-5 1 1 alpha],1,1)

lmiterm([6 1 1 alpha],-1,1)
lmiterm([6 1 1 Q],0.5*e_0',e_0)
lmiterm([6 1 2 0],x_0')
lmiterm([6 2 2 X],-2,1)

lmiterm([-7 1 1 Q],1,1)

 LMISYS=getlmis;

[copt,xopt]=feasp(LMISYS);
X=dec2mat(LMISYS,xopt,X);
Q=dec2mat(LMISYS,xopt,Q);
Z=dec2mat(LMISYS,xopt,Z);
M=dec2mat(LMISYS,xopt,M);
N=dec2mat(LMISYS,xopt,N);
T=dec2mat(LMISYS,xopt,T);
Y=dec2mat(LMISYS,xopt,Y);
W=dec2mat(LMISYS,xopt,W);
alpha=dec2mat(LMISYS,xopt,alpha);

evlmi=evallmi(LMISYS,xopt);


[lhs1,rhs1]=showlmi(evlmi,1);
[lhs2,rhs2]=showlmi(evlmi,2);
```

```
[lhs3,rhs3]=showlmi(evlmi,3);
[lhs4,rhs4]=showlmi(evlmi,4);
[lhs5,rhs5]=showlmi(evlmi,5);
[lhs6,rhs6]=showlmi(evlmi,6);

X_0=X;
Q_0=Q;
Z_0=Z;
M_0=M;
N_0=N;
T_0=T;
Y_0=Y;
W_0=W;
alpha_0=alpha;

D1=max(eig(lhs1))<0;
D2=min(eig(rhs2))>0;
D3=min(eig(rhs3))>0;
D4=min(eig(rhs4))>0;
D5=min(eig(rhs5))>0;
D6=max(eig(lhs1))<0;

myDecision=D1&D2&D3&D4&D5&D6;

save initialData6074 nn m p A B C x_0 e_0 S R k K0 L0 k0 counter mySTOP X_0 Q_0 Z_0
M_0 N_0 T_0 Y_0 W_0 alpha_0 myDecision
```

File 2

```
load initialData6074;

setlmis([])

X=lmivar(1,[nn 1]);
Q=lmivar(1,[nn 1]);
Z=lmivar(1,[nn 1]);
M=lmivar(1,[nn 1]);
N=lmivar(1,[nn 1]);
T=lmivar(1,[nn 1]);
Y=lmivar(2,[m nn]);
W=lmivar(2,[nn p]);
alpha=lmivar(1,[1 1]);

lmiterm([1 1 1 X],A,1,'s')
lmiterm([1 1 1 Y],B,1,'s')


lmiterm([1 1 3 -Y],1,1)
```

```
lmiterm([1 1 4 X],1,1)

lmiterm([1 1 5 Y],-B,1)

lmiterm([1 2 2 Q],1,A,'s')
lmiterm([1 2 2 W],-1,C,'s')
lmiterm([1 2 2 Z],1,1)

lmiterm([1 3 3 0],-2*inv(R))

lmiterm([1 3 5 Y],-1,1)

lmiterm([1 4 4 0],-2*inv(S))

lmiterm([1 5 5 M],-1,1)

lmiterm([-2 1 1 M],1,1)
lmiterm([-2 1 2 0],1)
lmiterm([-2 2 2 N],1,1)

lmiterm([-3 1 1 X],1,1)
lmiterm([-3 1 2 0],1)
lmiterm([-3 2 2 T],1,1)

lmiterm([-4 1 1 N],1,1)
lmiterm([-4 1 2 T],1,1)
lmiterm([-4 2 2 Z],1,1)

lmiterm([-5 1 1 alpha],1,1)

lmiterm([6 1 1 alpha],-1,1)
lmiterm([6 1 1 Q],0.5*e_0',e_0)
lmiterm([6 1 2 0],x_0')
lmiterm([6 2 2 X],-2,1)


lmiterm([-7 1 1 Q],1,1)

LMISYS=getlmis;

n=decnbr(LMISYS);
c=zeros(n,1);
for j=1:n
  [Mj,Nj,Xj,Tj,alphaj]=defcx(LMISYS,j,M,N,X,T,alpha);

     c(j)=alphaj+trace(M_0*Nj+N_0*Mj+X_0*Tj+T_0*Xj);
end

[copt,xopt]=mincx(LMISYS,c);
X=dec2mat(LMISYS,xopt,X);
```

```
Q=dec2mat(LMISYS,xopt,Q);
Z=dec2mat(LMISYS,xopt,Z);
M=dec2mat(LMISYS,xopt,M);
N=dec2mat(LMISYS,xopt,N);
T=dec2mat(LMISYS,xopt,T);
Y=dec2mat(LMISYS,xopt,Y);
W=dec2mat(LMISYS,xopt,W);
alpha=dec2mat(LMISYS,xopt,alpha);

evlmi=evallmi(LMISYS,xopt);

[lhs1,rhs1]=showlmi(evlmi,1);
[lhs2,rhs2]=showlmi(evlmi,2);
[lhs3,rhs3]=showlmi(evlmi,3);
[lhs4,rhs4]=showlmi(evlmi,4);
[lhs5,rhs5]=showlmi(evlmi,5);
[lhs6,rhs6]=showlmi(evlmi,6);

X_0=X;
Q_0=Q;
Z_0=Z;
M_0=M;
N_0=N;
T_0=T;
Y_0=Y;
W_0=W;
alpha_0=alpha;
D1=max(eig(lhs1))<0;
D2=min(eig(rhs2))>0;
D3=min(eig(rhs3))>0;
D4=min(eig(rhs4))>0;
D5=min(eig(rhs5))>0;
D6=max(eig(lhs1))<0;

myDecision=D1&D2&D3&D4&D5&D6;
save initialData6074 nn m p A B C x_0 e_0 S R k K0 L0 k0 counter mySTOP X_0 Q_0 Z_0
M_0 N_0 T_0 Y_0 W_0 alpha_0 myDecision
```

File 3

```
EX_2_18_10_17_6074
while 1
  EX_2_18_10_17_6074_2;
  if max(real(eig(M_0-X_0*Z_0*X_0)))<=0 && myDecision
    K0=Y_0*inv(X_0);
    L0=inv(Q_0)*W_0;
    k0=k;
    break;
end
```

```
k=k+1;
  if k>99
   break;
  end
  counter=counter+1;
  save initialData6074 nn m p A B C x_0 e_0 S R k K0 L0 k0 counter mySTOP X_0 Q_0 Z_0 M_0 N_0 T_0 Y_0 W_0 alpha_0 myDecision
end
```

1.2 Simulation Codes for Nominal Linear System

```
 function y = trying_addmfile_2(myInput)

x1=myInput(1);
x2=myInput(2);
x1hat=myInput(3);
x2hat=myInput(4);

 x=[x1;x2];

A=[0 0.5;0 1];
B=[0;0.5];
C=[1 0];
x_0=[1;0.5];
xhat_0=[1;0.5] ;
P=[42.5610    4.6125;
    4.6125   31.5982];

Q=1.0e+07 *[1.7869   -0.3948;
   -0.3948    0.1141];
K=[-1.7309  -10.1860];
L=[3.9500;15.6503];
e_0=x_0-xhat_0;

xhat=[x1hat;x2hat];
J=0.5*(x_0'*P*x_0+e_0'*Q*e_0);
u= K*xhat;
xdot=A*x+B*u;
y=C*x;
yhat=C*xhat;
xhatdot=A*xhat+B*u+L*(y-yhat);

x1dot=xdot(1,1);
x2dot=xdot(2,1);
x1hatdot=xhatdot(1,1);
x2hatdot=xhatdot(2,1);
y=[x1dot x2dot x1hatdot x2hatdot J u];
end
```

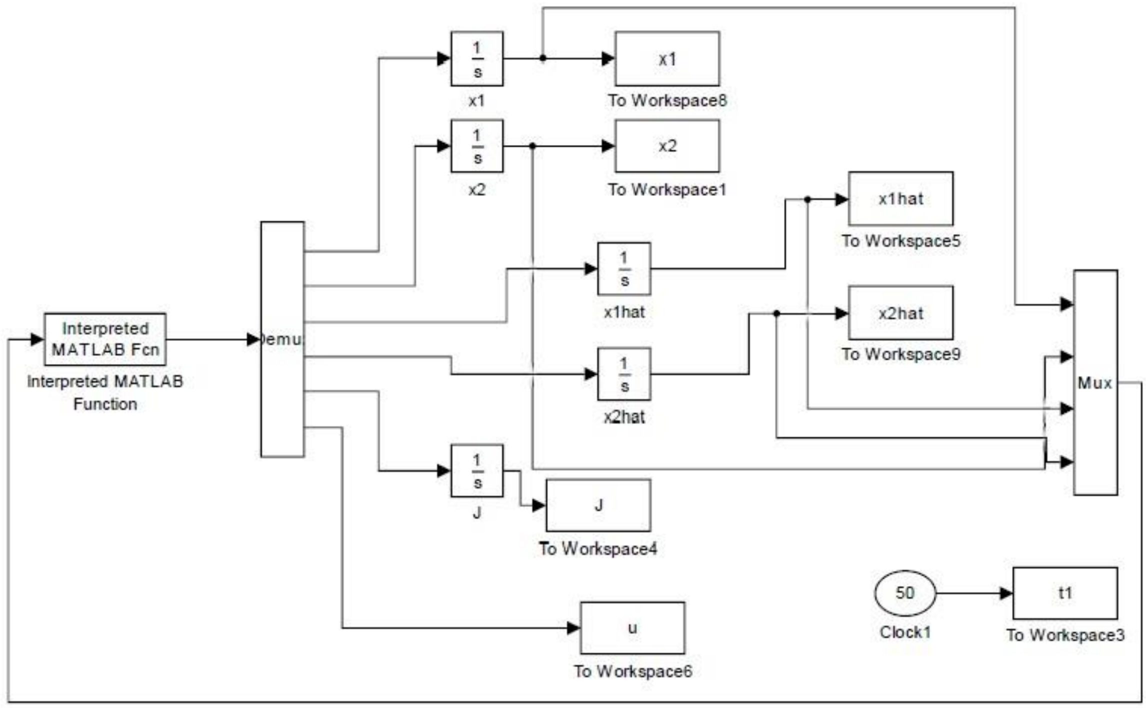

Figure 11– Block diagram of a state feedback observer design variables for the nominal system

1.3 Linear Uncertain System

File 1

```
A=[0 0.5;0 1];
B=[0;0.5];
C=[1 0];
D=[0.5;0.3];
R=0.5;
S=[1 0;0 1];
x_0=[1;0.5];
xhat_0=[1;0.5];
e_0=x_0-xhat_0;
Ea=[0.1  0.2];
Eb=0.1 ;

k=1;

k0=0;

counter=0;
 mySTOP=0;
```

```
[nn,m]=size(B);
[p,nn]=size(C);
K0=zeros(m,nn);
L0=zeros(nn,p);
setlmis([])

X=lmivar(1,[nn 1]);
Q=lmivar(1,[nn 1]);
Z=lmivar(1,[nn 1]);
M=lmivar(1,[nn 1]);
N=lmivar(1,[nn 1]);
T=lmivar(1,[nn 1]);
Y=lmivar(2,[m nn]);
W=lmivar(2,[nn p]);
eps1=lmivar(1,[1,1]);
eps2=lmivar(1,[1,1]);
alpha=lmivar(1,[1 1]);

lmiterm([1 1 1 X],A,1,'s')
lmiterm([1 1 1 Y],B,1,'s')
lmiterm([1 1 1 eps1],D',D)

lmiterm([1 1 3 -Y],1,1)
lmiterm([1 1 4 X],1,1)
lmiterm([1 1 5 Y],-B,1)

lmiterm([1 1 6 X],1,Ea')

lmiterm([1 2 2 Q],1,A,'s')
lmiterm([1 2 2 W],-1,C,'s')
lmiterm([1 2 2 Z],1,1)
lmiterm([1 2 2 eps2],Ea',Ea)

lmiterm([1 2 7 Q],1,D)

lmiterm([1 3 3 0],-2*inv(R))
lmiterm([1 3 5 Y],-1,1)

lmiterm([1 4 4 0],-2*inv(S))

lmiterm([1 5 5 M],-1,1)
lmiterm([1 5 6 -Y],1,Eb')

lmiterm([1 6 6 eps1],-1,1)

lmiterm([1 7 7 eps2],-1,1)
lmiterm([-2 1 1 M],1,1)
lmiterm([-2 1 2 0],1)
lmiterm([-2 2 2 N],1,1)
lmiterm([-3 1 1 X],1,1)
```

```
lmiterm([-3 1 2 0],1)
lmiterm([-3 2 2 T],1,1)

lmiterm([-4 1 1 N],1,1)
lmiterm([-4 1 2 T],1,1)
lmiterm([-4 2 2 Z],1,1)

lmiterm([-5 1 1 alpha],1,1)

lmiterm([6 1 1 alpha],-1,1)
lmiterm([6 1 1 Q],0.5*e_0',e_0)
lmiterm([6 1 2 0],x_0')
lmiterm([6 2 2 X],-2,1)


lmiterm([-7 1 1 Q],1,1)

LMISYS=getlmis;
[copt,xopt]=feasp(LMISYS);
X=dec2mat(LMISYS,xopt,X);
Q=dec2mat(LMISYS,xopt,Q);
Z=dec2mat(LMISYS,xopt,Z);
M=dec2mat(LMISYS,xopt,M);
N=dec2mat(LMISYS,xopt,N);
T=dec2mat(LMISYS,xopt,T);
Y=dec2mat(LMISYS,xopt,Y);
W=dec2mat(LMISYS,xopt,W);
alpha=dec2mat(LMISYS,xopt,alpha);
eps1=dec2mat(LMISYS,xopt,eps1);
eps2=dec2mat(LMISYS,xopt,eps2);


evlmi=evallmi(LMISYS,xopt);

[lhs1,rhs1]=showlmi(evlmi,1);
[lhs2,rhs2]=showlmi(evlmi,2);
[lhs3,rhs3]=showlmi(evlmi,3);
[lhs4,rhs4]=showlmi(evlmi,4);
[lhs5,rhs5]=showlmi(evlmi,5);
[lhs6,rhs6]=showlmi(evlmi,6);

X_0=X;
Q_0=Q;
Z_0=Z;
M_0=M;
N_0=N;
T_0=T;
Y_0=Y;
```

```
W_0=W;
alpha_0=alpha;
eps1_0=eps1;
eps2_0=eps2;

D1=max(eig(lhs1))<0;
D2=min(eig(rhs2))>0;
D3=min(eig(rhs3))>0;
D4=min(eig(rhs4))>0;
D5=min(eig(rhs5))>0;
D6=max(eig(lhs1))<0;

myDecision=D1&D2&D3&D4&D5&D6;
save initial1092017 nn m p A B C x_0 e_0 S R k K0 L0 k0 counter X_0 X Q_0 Q Z_0 Z M_0
M N_0 N T_0 T Y_0 Y W_0 W alpha_0 alpha eps1_0 eps1 eps2_0 eps2 myDecision
```

File 2

```
load initial1092017 ;

setlmis([])

X=lmivar(1,[nn 1]);
Q=lmivar(1,[nn 1]);
Z=lmivar(1,[nn 1]);
M=lmivar(1,[nn 1]);
N=lmivar(1,[nn 1]);
T=lmivar(1,[nn 1]);
Y=lmivar(2,[m nn]);
W=lmivar(2,[nn p]);
eps1=lmivar(1,[1,1]);
eps2=lmivar(1,[1,1]);
alpha=lmivar(1,[1 1]);

lmiterm([1 1 1 X],A,1,'s')
lmiterm([1 1 1 Y],B,1,'s')
lmiterm([1 1 1 eps1],D',D)

lmiterm([1 1 3 -Y],1,1)
lmiterm([1 1 4 X],1,1)

lmiterm([1 1 5 Y],-B,1)
lmiterm([1 1 6 X],1,Ea')

lmiterm([1 2 2 Q],1,A,'s')
lmiterm([1 2 2 W],-1,C,'s')
lmiterm([1 2 2 Z],1,1)
lmiterm([1 2 2 eps2],Ea',Ea)

lmiterm([1 2 7 Q],1,D)
```

```
lmiterm([1 3 3 0],-2*inv(R))
lmiterm([1 3 5 Y],-1,1)

lmiterm([1 4 4 0],-2*inv(S))

lmiterm([1 5 5 M],-1,1)
lmiterm([1 5 6 -Y],1,Eb')

lmiterm([1 6 6 eps1],-1,1)

lmiterm([1 7 7 eps2],-1,1)
lmiterm([-2 1 1 M],1,1)
lmiterm([-2 1 2 0],1)
lmiterm([-2 2 2 N],1,1)
lmiterm([-3 1 1 X],1,1)
lmiterm([-3 1 2 0],1)

lmiterm([-3 2 2 T],1,1)

lmiterm([-4 1 1 N],1,1)
lmiterm([-4 1 2 T],1,1)
lmiterm([-4 2 2 Z],1,1)

lmiterm([-5 1 1 alpha],1,1)
lmiterm([6 1 1 alpha],-1,1)
lmiterm([6 1 1 Q],0.5*e_0',e_0)

lmiterm([6 1 2 0],x_0')
lmiterm([6 2 2 X],-2,1)

lmiterm([-7 1 1 Q],1,1)

LMISYS=getlmis;

n=decnbr(LMISYS);
c=zeros(n,1);
for j=1:n

[Mj,Nj,Xj,Tj,alphaj]=defcx(LMISYS,j,M,N,X,T,alpha);
  c(j)=alphaj+trace(M_0*Nj+N_0*Mj+X_0*Tj+T_0*Xj);
end

[copt,xopt]=mincx(LMISYS,c);
X=dec2mat(LMISYS,xopt,X);
Q=dec2mat(LMISYS,xopt,Q);
Z=dec2mat(LMISYS,xopt,Z);
M=dec2mat(LMISYS,xopt,M);
N=dec2mat(LMISYS,xopt,N);
T=dec2mat(LMISYS,xopt,T);
```

```
Y=dec2mat(LMISYS,xopt,Y);
W=dec2mat(LMISYS,xopt,W);
alpha=dec2mat(LMISYS,xopt,alpha);
eps1=dec2mat(LMISYS,xopt,eps1);
eps2=dec2mat(LMISYS,xopt,eps2);

evlmi=evallmi(LMISYS,xopt);

[lhs1,rhs1]=showlmi(evlmi,1);
[lhs2,rhs2]=showlmi(evlmi,2);
[lhs3,rhs3]=showlmi(evlmi,3);
[lhs4,rhs4]=showlmi(evlmi,4);
[lhs5,rhs5]=showlmi(evlmi,5);
[lhs6,rhs6]=showlmi(evlmi,6);

X_0=X;
Q_0=Q;
Z_0=Z;
M_0=M;
N_0=N;
T_0=T;
Y_0=Y;
W_0=W;
alpha_0=alpha;
eps1_0=eps1;
eps2_0=eps2;

D1=max(eig(lhs1))<0;
D2=min(eig(rhs2))>0;
D3=min(eig(rhs3))>0;
D4=min(eig(rhs4))>0;
D5=min(eig(rhs5))>0;
D6=max(eig(lhs1))<0;

myDecision=D1&D2&D3&D4&D5&D6;
save initial1092017 nn m p A B C x_0 e_0 S R k K0 L0 k0 counter X_0 X Q_0 Q Z_0 Z M_0
M N_0 N T_0 T Y_0 Y W_0 W alpha_0 alpha eps1_0 eps1 eps2_0 eps2 myDecision ;
```

File 3

```
EX1_1_11_2017_1;
 while 1
 EX1_1_11_2017_2 ;

 if max(real(eig(M_0-X_0*Z_0*X_0)))<=0 && myDecision
      X_0=X;
      Q_0=Q;
      K0=Y_0*inv(X_0);
      L0=inv(Q_0)*W_0;
      Z_0=Z;
      M_0=M;
      T_0=T;
      Y_0=Y;
      W_0=W;
      alpha_0=alpha;
      eps1_0=eps1;
      eps2_0=eps2;
      k0=k;

      break;
  end
  k=k+1;
  if k>99

break;
end
  counter=counter+1;
  save initial1092017 nn m p A B C x_0 e_0 S R k K0 L0 k0 counter X_0 X Q_0 Q Z_0 Z M_0 
M N_0 N T_0 T Y_0 Y W_0 W alpha_0 alpha eps1_0 eps1 eps2_0 eps2 myDecision
en
```

1.4 Simulation Codes for Linear Uncertain System

```
function y =uncertain_addmfile(myInput)
x1=myInput(1);
x2=myInput(2);
x1hat=myInput(3);
x2hat=myInput(4);
t=myInput(5);
x=[x1;x2];

A=[0 0.5;0 1];
B=[0;0.5];
C=[1 0];
D=[0.5;0.3];
x_0=[1;0];
Ea=[0.1  0.2];
Eb=0.1 ;
```

```
F=sin(t);
P=[42.5610    4.6125;4.6125   31.5982];

Q=1.0e+07 *[1.7869   -0.3948;-0.3948    0.1141];
dela=D*F*Ea;
delb=D*F*Eb;

K=[-2.4085  -10.5507];

L= [5.7460;23.4037];
e_0=[0;0];
xhat=[x1hat;x2hat];
J=0.5*(x_0'*P*x_0+e_0'*Q*e_0);
u= K*xhat;
xdot=(A+dela)*x+(B+delb)*u;
y=C*x;
yhat=C*xhat;
xhatdot=(A+dela)*xhat+(B+delb)*u+L*(y-yhat);
x1dot=xdot(1,1);
x2dot=xdot(2,1);
x1hatdot=xhatdot(1,1);
x2hatdot=xhatdot(2,1);
y=[x1dot x2dot x1hatdot x2hatdot J u];
end
```

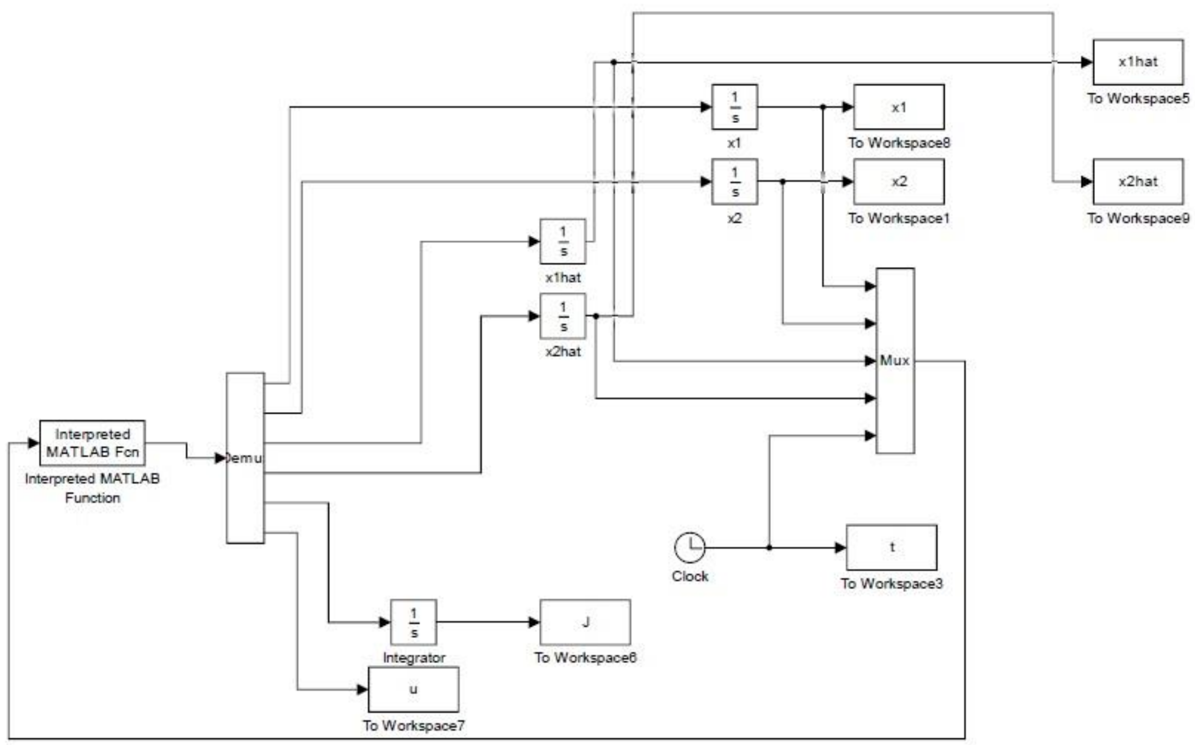

Figure 12 – Block diagram of a state feedback observer design variables for linear uncertain system

APPENDIX B

Theorem 1 is implemented with observer-based PD state feedback controller for linear nominal system.

File 1

```
A=[0 0.5;0 1];
B=[0;0.5];
Kp =[ -1.7309  -10.1860];
C=[1 0];
[nn,m]=size(B);
L=[3.95;15.6503];
k=1;
k0=0;

setlmis([])

X=lmivar(1,[nn 1]);
Y=lmivar(2,[m nn]);
Q=lmivar(1,[nn 1]);
S=lmivar(1,[nn 1]);
M=lmivar(1,[nn 1]);
J=lmivar(1,[nn 1]);
Z=lmivar(1,[nn 1]);

lmiterm([1 1 1 X],A,1,'s');
lmiterm([1 1 1 X],B*Kp,1,'s');
lmiterm([1 1 1 -Y],-A,B','s');
lmiterm([1 1 1 -Y],-B*Kp,B','s');

lmiterm([1 1 2 0],-B*Kp)
lmiterm([1 1 2 0],-A)
lmiterm([1 1 2 0],L*C)

lmiterm([1 1 3 Y],-B,1);
lmiterm([1 2 2 Q],A,1,'s');
lmiterm([1 2 2 Q],-L*C,1,'s');
lmiterm([1 2 2 Z],1,1);

lmiterm([1 3 3 S],-1,1);

lmiterm([-2 1 1 M],1,1);
lmiterm([-2 1 2 J],1,1);
lmiterm([-2 2 2 Z],1,1);

lmiterm([-3 1 1 S],1,1);
lmiterm([-3 1 2 0],1);
lmiterm([-3 2 2 M],1,1);

lmiterm([-4 1 1 X],1,1);
```

```
lmiterm([-4 1 2 0],1);
lmiterm([-4 2 2 J],1,1);

lmiterm([-5 1 1 Q],1,1);


LMISYS=getlmis;

[copt,xopt]=feasp(LMISYS);

X=dec2mat(LMISYS,xopt,X);
Y=dec2mat(LMISYS,xopt,Y);
Q=dec2mat(LMISYS,xopt,Q);
Z=dec2mat(LMISYS,xopt,Z);
M=dec2mat(LMISYS,xopt,M);
S=dec2mat(LMISYS,xopt,S);

evlmi=evallmi(LMISYS,xopt);
[lhs1,rhs1]=showlmi(evlmi,1);
[lhs2,rhs2]=showlmi(evlmi,2);
[lhs3,rhs3]=showlmi(evlmi,3);
[lhs4,rhs4]=showlmi(evlmi,4);
[lhs5,rhs5]=showlmi(evlmi,5);

X_0=X;
Q_0=Q;
Z_0=Z;
M_0=M;
J_0=J;
S_0=S;
Y_0=Y;
 Kd=Y*inv(X);
save initialDataObserver_PD_kpfixed  nn m k k0 A B C L X X_0 Q Q_0 Z Z_0 M M_0 J J_0
S S_0 Y Y_0 Kp
```

File 2

```
load initialDataObserver_PD_kdfixed;
setlmis([])
X=lmivar(1,[nn 1]);
Y=lmivar(2,[m nn]);
Q=lmivar(1,[nn 1]);
S=lmivar(1,[nn 1]);
M=lmivar(1,[nn 1]);
J=lmivar(1,[nn 1]);
Z=lmivar(1,[nn 1]);
lmiterm([1 1 1 X],A,1,'s');
lmiterm([1 1 1 X],B*Kp,1,'s');
lmiterm([1 1 1 -Y],-A,B','s');
lmiterm([1 1 1 -Y],-B*Kp,B','s');
lmiterm([1 1 2 0],-B*Kp)
```

```
lmiterm([1 1 2 0],-A)
lmiterm([1 1 2 0],L*C)
lmiterm([1 1 3 Y],-B,1);
lmiterm([1 2 2 Q],A,1,'s');
lmiterm([1 2 2 Q],-L*C,1,'s');
lmiterm([1 2 2 Z],1,1);
lmiterm([1 3 3 S],-1,1);
lmiterm([-2 1 1 M],1,1);
lmiterm([-2 1 2 J],1,1);
lmiterm([-2 2 2 Z],1,1);
lmiterm([-3 1 1 S],1,1);
lmiterm([-3 1 2 0],1);
lmiterm([-3 2 2 M],1,1);
lmiterm([-4 1 1 X],1,1);
lmiterm([-4 1 2 0],1);
lmiterm([-4 2 2 J],1,1);
lmiterm([-5 1 1 Q],1,1);
LMISYS=getlmis;
n=decnbr(LMISYS);
c=zeros(n,1);
for j=1:n
  [Mj,Jj,Xj,Sj]=defcx(LMISYS,j,M,J,X,S);
  c(j)=trace(S_0*Mj+M_0*Sj+X_0*Jj+J_0*Xj);
end
[copt,xopt]=mincx(LMISYS,c);
X=dec2mat(LMISYS,xopt,X);
Y=dec2mat(LMISYS,xopt,Y);
Q=dec2mat(LMISYS,xopt,Q);
Z=dec2mat(LMISYS,xopt,Z);
M=dec2mat(LMISYS,xopt,M);
S=dec2mat(LMISYS,xopt,S);
J=dec2mat(LMISYS,xopt,J);
evlmi=evallmi(LMISYS,xopt);

[lhs1,rhs1]=showlmi(evlmi,1);
[lhs2,rhs2]=showlmi(evlmi,2);
[lhs3,rhs3]=showlmi(evlmi,3);
[lhs4,rhs4]=showlmi(evlmi,4);
[lhs5,rhs5]=showlmi(evlmi,5);
X_0=X;
Q_0=Q;
Z_0=Z;
M_0=M;
J_0=J;
S_0=S;
Y_0=Y;
 save initialDataObserver_PD_kdfixed  nn m k k0 A B C L X X_0 Q Q_0 Z Z_0 M M_0 J
J_0 S S_0 Y Y_0 Kp
```

File 3

```
ex_observerkdsolved_5_3_2018

while 1
  ex_observedkdsolved_5_3_2018_2;
  if max(real(eig(S_0-X_0*Z_0*X_0)))<=0
    Kd=Y*inv(X);
    k0=k;
    break;
  end
  k=k+1;
  if k>99
   break;
  end
  counter=counter+1;
save initialDataObserver_PD_kdfixed  nn m k k0 A B C L X X_0 Q Q_0 Z Z_0 M M_0 J J_0
S S_0 Y Y_0 Kp
end
```

APPENDIX C

Robust guaranteed cost observer-based PD state feedback codes applied with algorithm 1 and lemma 3.

```
A=[0 0.5;0 1];
B=[0;0.5];
C=[1 0];
R=0.5;
S=[1 0;0 1];

%case3
Kp =[-1.7309  -10.1860];%Kp fixed from nominal system
Kd=[-0.2376   -0.8788];
[nn,m]=size(B);
%L=[-4;8];
L=[3.95;15.6503];
k=1;
k0=0;
x_0=[1;0.5];
xhat_0=[1;0.5];
e_0=x_0-xhat_0;
Ab=(A-L*C);
Ac=inv(1-B*Kd);
Ad=(A+B*Kp);

setlmis([])
X=lmivar(1,[nn 1]);
alpha=lmivar(1,[1 1]);
Q=lmivar(1,[nn 1]);
lmiterm([1 1 1 X],Ac*Ad,1,'s')
lmiterm([1 1 2 X],-Ac*B*Kp,1)

lmiterm([1 1 2 X],-Ac*B*Kd*Ab,1)

lmiterm([1 1 3 X],-1,Ad'*Ac'*Kd')
lmiterm([1 1 4 X],-1,-Kp')
lmiterm([1 1 5 X],-1,1)

lmiterm([1 2 2 Q],Ab,1,'s')
lmiterm([1 2 2 0],0.5*Ab'*Kd'*R*Kd*Ab)
lmiterm([1 2 2 0],0.5*Ab'*Kd'*B'*Ac'*Kd'*R*Kd*Ac*B*Kd*Ab)
lmiterm([1 2 2 0],0.5*Kp'*R*Kp)

lmiterm([1 3 3 0],-2*inv(R))
lmiterm([1 4 4 0],-2*inv(R))
```

```
lmiterm ([1 5 5 0],-2*inv(S))
lmiterm([-2 1 1 alpha],1,1)


lmiterm([3 1 1 alpha],-1,1)
lmiterm([3 1 1 Q],0.5*e_0',e_0)
lmiterm([3 1 2 0],x_0')
lmiterm([3 2 2 X],-2,1)


lmiterm([-4 1 1 Q],1,1)

LMISYS=getlmis;


n=decnbr(LMISYS);
c=zeros(n,1);

for j=1:n
  [alphaj]=defcx(LMISYS,j,alpha);
  c(j)=alphaj;
end

[copt,xopt]=mincx(LMISYS,c);

X=dec2mat(LMISYS,xopt,X);
Q=dec2mat(LMISYS,xopt,Q);
alpha=dec2mat(LMISYS,xopt,alpha);
evlmi=evallmi(LMISYS,xopt);

[lhs1,rhs1]=showlmi(evlmi,1);
[lhs2,rhs2]=showlmi(evlmi,2);
[lhs3,rhs3]=showlmi(evlmi,3);
```

1.1 Simulation Codes

```
function y = guaranteedcost_pd(myInput)

  x1=myInput(1);
  x2=myInput(2);
  x1hat=myInput(3);
  x2hat=myInput(4);
  x1hatdot=myInput(5);
  x2hatdot=myInput(6);

 x=[x1;x2];
```

```
A=[0 0.5;0 1];
B=[0;0.5];
C=[1 0];
x_0=[1;0.5];
xhat_0=[1;0.5] ;
xhatdot=[x1hatdot;x2hatdot];
P=[3.5470  3.9180;3.9180 5.6958];


Q=1.0e+08 *[0.2465  0.6942;0.6942 6.8935];

Kp=[-1.7309  -10.1860];
Kd=[-0.2376   -0.8788];

L= [3.9500;15.6503];
%L= [-4;8];
e_0=x_0-xhat_0;
xhat=[x1hat;x2hat];


J=0.5*(x_0'*P*x_0+e_0'*Q*e_0);
u= Kp*xhat+Kd*xhatdot;
%u= Kp*xhat;
xdot=A*x+B*u;
y=C*x;
yhat=C*xhat;
xhatdot=A*xhat+B*u+L*(y-yhat);

x1dot=xdot(1,1);
x2dot=xdot(2,1);
x1hatdot=xhatdot(1,1);
x2hatdot=xhatdot(2,1);
  y=[x1dot x2dot x1hatdot x2hatdot J u];
end
```

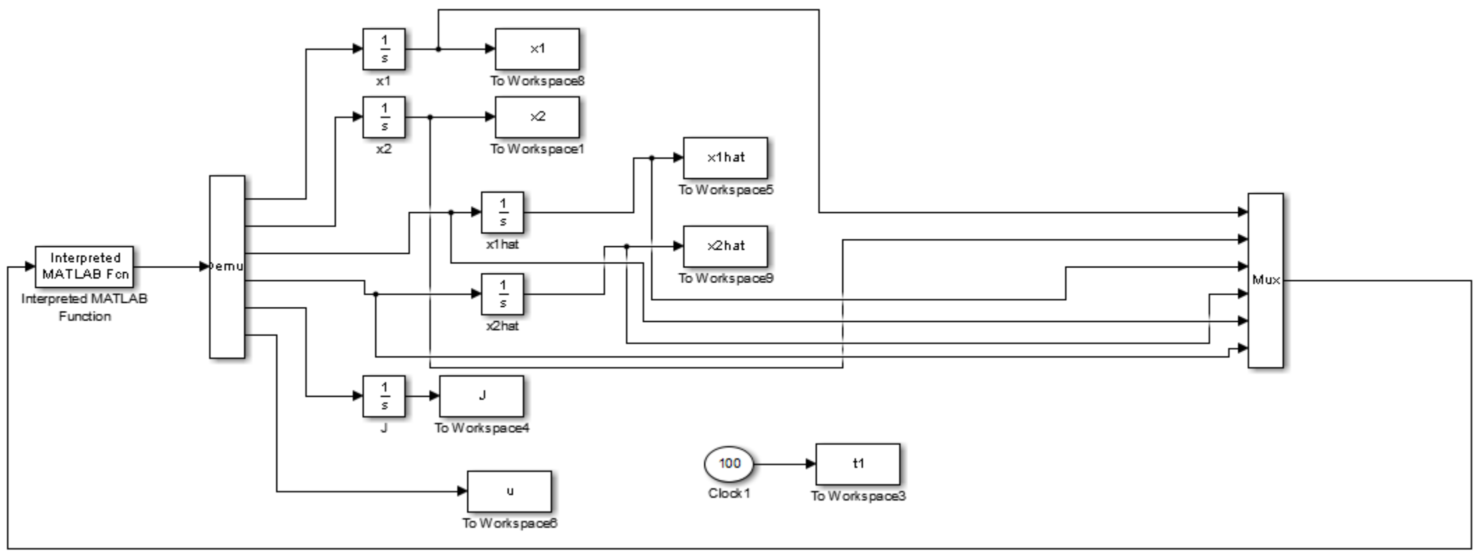

Figure 13 – Block diagram of a PD state feedback observer design variables for linear uncertain system.

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