OBSERVER BASED FRICTION CANCELLATION IN MECHANICAL SYSTEMS

A THESIS

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MASTER OF SCIENCE

By

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September, 2014

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ABSTRACT

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In real life feedback control applications of mechanical systems, friction and time delays are two important issues that might have direct effects on the performance of systems. Hence, an adaptive nonlinear observer based friction compensation for a special time delayed system is presented in this thesis. Considering existing delay, an available Coulomb observer is modified and closed loop system is formed by using a Smith predictor based controller as if the process is delay free. Implemented hierarchical feedback system structure provides two-degree of freedom and controls both velocity and position separately. For this purpose, controller parametrization method is used to extend Smith predictor structure to the position control loop for different types of inputs and disturbance attenuation. Simulation results demonstrate that without requiring much information about friction force, the method can significantly improve the performance of a control system in which it is applied.

Keywords: Time Delay Systems, Adaptive Coulomb Friction Observer, Smith Predictor Based Controller, Hierarchical Position Control.

ÖZET

MEKANİK SİSTEMLERDE GÖZLEMCİ TABANLI SÜRTÜNME GİDERİMİ

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Sürtünme ve zaman gecikmesi, mekanik sistemler için geri beslemeli kontrol yapılarının gerçek hayat uygulamalarında performans üzerinde doğrudan etkili olabilmektedir. Bu sebeple, bu tezde belirli zaman gecikmesi bulunan sistemlerde adaptif doğrusal olmayan gözlemci tabanlı sürtünme giderme yöntemi sunulmaktadır. Gecikme göz önüne alınarak, varolan Coulomb gözlemcisi değiştirilmiş ve sistemin zaman gecikmesi yokmuş gibi olabilmesi için kapalı döngü sisteminde Smith kestirim tabanlı denetleyiciye yer verilmiştir. Uygulanan hiyerarşik kontrol yapısı hem pozisyon hem de hızı ayrı ayrı kontrol etmeye imkan veren serbestliği sağlamaktadır. Bu amaçla kontrolör parametrizasyonu kullanılarak farklı girdi isteklerini takip edebilmek veya bozucu etki regülasyonu sağlayabilmek için hız döngüsünde tasarlanan Smith kestirimi deneteyici yapısı pozisyon döngüsüne genişletilebilir. Elde edilen sonuçlar önerilen yöntemin sürtünme hakkında çok fazla bilgiye ihtiyaç duymaksızın uygulandığı sistemin performansını önemli ölçüde geliştirdiğini göstermektedir.

Anahtar sözcükler: Zaman Gecikmeli Sistemler, Adaptif Coulomb Sürtünme Gözlemcisi, Smith Kestirim Tabanlı Denetleyici, Hiyerarşik Pozisyon Kontrolü.

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Dedicated to my parents ...

Chapter 1

INTRODUCTION

In real life applications, performance of mechanical systems and theory might not completely overlap because of nonlinearities or imperfections of system models. While some of these mismatches are negligible, some of them might have severe effects on the performance. For instance, they might result in performance degradation or even instability. Existence of time delay leads system to be non-ideal. Similarly, friction is one of the substantial nonlinearities in servo systems. Thus, in order to satisfy desired performance criteria, one should also take into consideration them in designing feedback control systems.

1.1 Motivation

In this thesis, there are two main objectives, position tracking for plants with time delay and friction cancellation. Therefore, a hierarchical feedback structure with adaptive friction observer is employed for position tracking. Due to the fact that plant includes time delay, Smith predictor based controllers are utilized to the feedback system aiming to make the controller design as if the process is delay free. Thus, controller parametrization method is used to design the controller. Furthermore, in order to compensate the negative effect of the friction, adaptive observer is modified in order to estimate friction force in the presence of delay.

When estimated and real friction are equal to each other, complete cancellation is achieved and performance of the closed loop feedback system is increased. The general structure of studied system is depicted in Fig. 1.1

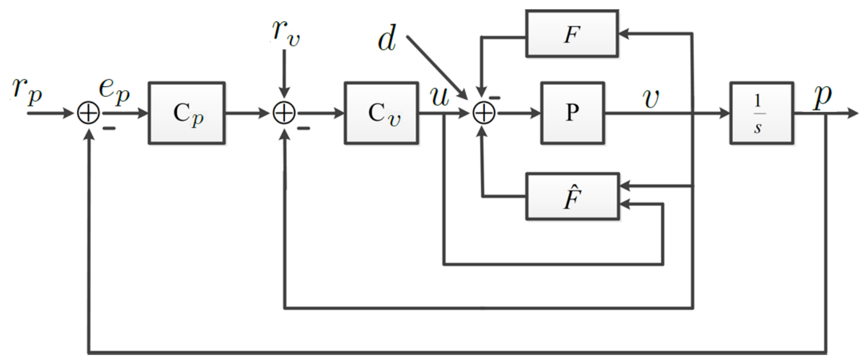

Figure 1.1: Block diagram of overall feedback system

In Fig. 1.1, P represents the plant, F represents the friction term whereas $\hat{F}$ represents an estimation of the friction; v is the velocity output whereas p stands for the position output. Furthermore, r_p and r_v represents position and velocity command inputs respectively and d symbolizes the disturbance term acting on the plant. Other than these signals, C_v is a controller which aims at stabilizing the first (inner) loop, which may be called as velocity loop, while C_p is another controller which aims at stabilizing the position loop.

In this system, if one aims at velocity control, the position loop should be switched off. Similarly, if one aims at position control, then one should choose $r_v = 0$.

1.2 Related Work

Proposed structure contains solutions for two different problems, controller design for delayed systems and friction cancellation. However, in the literature, generally time delays and friction compensation are investigated separately.

Firstly, time delay can appear very often in many systems due to transmission, computation or mechanical lags. In such cases, time delay may result in reducing system performance, or even cause instability. Since transport delay introduces infinitely many poles to the characteristic equation of the closed loop transfer function, in general it is more difficult to design a stabilizing controller for such systems as compared with the delay free case. Especially, stability margins of the closed loop system declines with increasing delay. PID controllers are preferred in many control applications. However, they are not so efficient when the time delay in system dynamics is sufficiently large. Hence, in 1957, Smith, [1], proposed a particular controller structure employing a feedback loop inside the controller. Therefore, although physically not the case, it is possible to remove time delay from the characteristic equation of the closed loop system mathematically. Hence, using proposed strategy one might design a controller as if the process is delay free. In this case system can be considered as given in Fig. 1.2

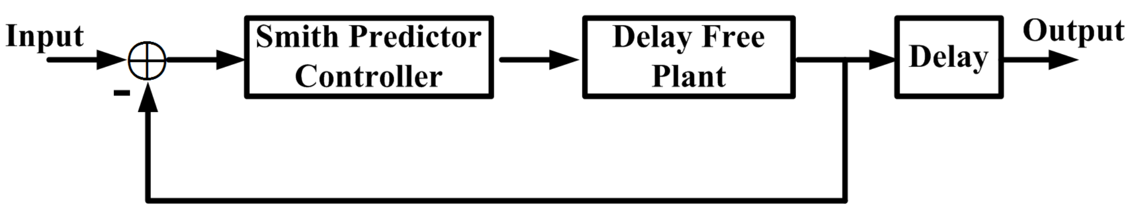

Figure 1.2: Equivalent block diagram of Smith predictor in terms of transfer function from input to output

Afterwards, many modifications have been proposed to this structure to meet a set of performance and robustness. On the other hand, in [2], it is proved that constant disturbance rejection cannot be achieved by using Smith predictor structure and steady state error cannot be avoided when there is a load change. Aiming to handle the challenge of a system with integrator and delay, [3] introduced a new form providing a separation between set point response and disturbance. Nevertheless, although usage of three new parameters; k_1, k_2 and k_3 in [3] increases the performance, parameter tuning is difficult since there is no apparent physical meaning. Another modification by [4] requires an extra feedback path from plant output and the model output to the controller input. Their strategy provides fast disturbance suppression for constant disturbance.

Furthermore, robustness of Smith predictor based controllers is addressed by some researchers since it is possible to destabilize the systems by minor changes on the dynamics even if the Smith predictor is nominally stable. Therefore, in [5] a geometric approach is proposed to diagnose influence of delay uncertainty on the stability. Likewise, [6] proposes a robust tuning method for the Smith predictor aiming to handle simultaneous uncertainties in all the parameters of the model. Finally, [7, 8] integrate Smith predictor with internal model principle. Therefore, controller has copies of the disturbance and reference signal generator. In this thesis, structure of controllers is determined by the same strategy proposed in [7, 8].

Secondly, friction force is one of the most important natural nonlinearities in mechanical systems. Mainly, friction compensation techniques can be grouped in two different ways, model based friction compensation and non-model based friction compensation. In model based control, the main idea is to use a model to predict the friction to compensate for it [9]. Although general approach to compensation is to use various friction models, acceleration feedback method described in [10] as a non-model based technique is perhaps one of the simplest and most robust ways of eliminating friction. Furthermore, [11] presents an adaptive compensation method which does not require a structured nonlinear friction model. In [12], local function estimation method is proposed for controlling a certain class of nonlinear, uncertain systems. In model based control, one can use a procedure proposed in [13] or [14] to estimate friction parameters. After estimation, it is possible to add required torque, which eliminates the friction, to the system. This approach is called fixed friction compensation. However, it is essential to adapt the friction model since its parameters can change with time due to temperature, lubricity, etc. or parameters cannot be identified because of model complexities or measurement errors [15]. Thus, [16] describes a self-tuning controller for velocity controller. In this paper, recursive least squares algorithm block estimating friction parameters and linear controller is used together. Likewise, passivity technique can be beneficial to design compensators with guaranteed stability [17]. Then friction observer and friction compensation method can be implemented with passivity based controllers as it is stated in [18].

There are different kinds of observers described in the literature. For instance, in [19], dual observer structure is utilized in order to identify LuGre friction parameters. Due to another approach, in [20], variable structure based observers are developed for friction estimation with or without information of velocity and sigmoidal functions are used instead of signum functions to reduce chattering. Although it can be applied for various purposes such as independent joint control, sensorless torque control, and fault diagnosis, nonlinear disturbance observer developed in [21]can be employed for friction compensation as well. However, in this thesis, an adaptive Coulomb observer described in [22] is modified in order to estimate the friction with the presence of delay in the process. Thanks to its simple structure, it is easy to modify the observer for the systems with time delay. Furthermore, no matter how complex it is, in most of friction models, Coulomb friction is one of the fundamental components of friction force. Although direct measurement of the velocity is assumed to be possible in this study, in [23], Friedland and Mentzelopoulou developed a combined observer for friction and velocity in the absence of direct velocity measurement in the system using Tafazoli modification. This modification requires employment of a low-pass differentiator and provides a better performance at the vicinity of zero velocity.

1.3 Contributions

A new Smith predictor based controller is presented in [7, 8] for a special class of plants. Note that friction terms are not considered in [7, 8] and the delay terms are not considered in the observer structure proposed in [22]. Motivated by the results obtained during this study, a friction cancellation scheme for a class of mechanical systems is proposed. Proposed friction cancellation scheme is based on hierarchical closed loop feedback system structure formed by Smith predictor based controllers and an adaptive friction observer similar to the one given in

[22]. Indeed, generalization of Friedlan-Park observer in [22] is presented and a modification in the original observer structure is provided for mechanical systems with time delay in order to estimate friction.

1.4 Organization of the Thesis

This thesis is organized as fallows. In Chapter 2, information on friction models is presented. Then, in Chapter 3, analysis of proposed closed loop system is given. Therefore, the structures of the plant and observer are defined. Furthermore, procedures of Smith predictor based controller designs for velocity and position control for different performance and robustness objectives are presented. Thus, step and ramp input tracking, step, ramp and sinusoidal disturbance rejection objectives are investigated. In Chapter 4, designed controllers' performances are evaluated under different scenarios. Especially, proposed hierarchical system is tested under friction with time varying parameters and more complex friction models rather than Coulomb friction. Finally, some concluding remarks are given in Chapter 5.

Throughout the thesis, notation in Table1.1 will be used.

Table 1.1: Notation used in the thesis

s	Laplace variable
r_p	Position demand
r_v	Velocity demand
P	Actual Plant
P_0	Delay free plant
u	Control Input
M	Mass of the moving object
p	Position of the moving obejct
v	Velocity of the moving object
a	Acceleration of the moving object
v_s	Stribeck velocity
σ_0	Stiffness coefficient
σ_1	Damping coefficient
z_d	Deflection of bristles
d	Disturbance
μ	Observer design exponent
a_c	Coulomb friction parameter in the observer
$\hat{a}_c$	Estimated Coulomb friction parameter in the observer
k	Observer gain
F	Real friction
F_a	Applied Force/Torque
F_s	Coefficient of stick friction
F_v	Coefficient of viscous frcition
$\hat{F}$	Estimated friction
$T_v(s)$	Closed loop transfer function including Smith predictor for velocity
$T_{0v}(s)$	Delay free part of $T_v(s)$
T_{v-PI}	Closed loop transfer function including PI controller for velocity
$T(s)$	Closed loop transfer function including Smith predictor for position
T_{PI}	Closed loop transfer function including PI controller for position
$C_{v(s)}$	Smith predictor based velocity controller
$C_p(s)$	Smith predictor based position controller
$C_{0v}(s)$	Internal controller in velocity control
$C_{0p}(s)$	Internal controller in position control
$C_{v-PI}(s)$	PI controller for velocity loop
$C_{p-PI}(s)$	PI controller for position loop
$sgn(.)$	Signum function

Chapter 2

BACKGROUND

In a simple mechanical system, applied force is control input whereas position and/or velocity is the output. Under the presence of friction, equation of motion can be written as,

$$M\ddot{x} = -F + u. \tag{2.1}$$

where M represents the mass of the moving object, x is the position, F is the friction force and u is the control input. In the absence of F, acceleration is simply generated by applied u. However, presence of F leads to performance degradation in the system. In model based compensation approach, in order to overcome this phenomenon, it is necessary to use a proper model of friction.

In the literature, friction is modeled in various complexities. Actually, there are mainly two different types of approaches, which are categorized as static friction models and dynamic friction models. Static models, also can be referred to as classical models, regard friction as static function of velocity [9].

Among all friction models, Coulomb friction is the most well known and simplest one. Coulomb model assumes that friction opposes motion and the friction force is independent of velocity and contact area. Therefore, it can be described by

$$F = a_c sgn(v). \tag{2.2}$$

In this equation a_c simply represents the magnitude of existing friction. However, because of signum function in the equation, friction is undefined at zero velocity. Therefore, stick friction concept is used to define the behavior of the system at rest. Friction force at rest is higher than the Coulomb friction level, called stick friction. Therefore, in order to trigger the motion, one should apply force more than stiction level, called as break-away force. Until this certain level, friction equals to applied force. In this case (2.2) can be re-written as,

$$F = \begin{cases} F_a & if \quad v = 0 \quad and \quad |F_a| < |F_s|, \\ F_s & if \quad v = 0 \quad and \quad |F_a| \geq |F_s|, \\ a_c sgn(F_a) & otherwise. \end{cases} \tag{2.3}$$

Even in sticking regime microscopic motion occurs. This is often called pre-sliding motion.

For most engineering applications lubrication is present in the friction interface. Hence, the Coulomb friction model can be improved by adding viscous friction. Then, (2.2) will be

$$F = a_c sgn(v) + F_v v. \tag{2.4}$$

In [24], Stribeck observed that at low velocities friction decreases continuously when velocity increases. This friction term is dominant at low velocities and called Stribeck effect and can be formulated as

$$F(v) = a_c + (F_s - a_c)e^{|v/v_s|} + F_v v. \tag{2.5}$$

In (2.5) v_s, is Stribeck velocity, where the friction force is minimal. Due to the fact that this model represents most of the key terms of existed friction, simulations in this thesis are based on this model.

There are also more sophisticated dynamic models in the literature. One of the most well known dynamic friction models is Dahl's model [25]. In 1968, Dahl developed a dynamic model for sliding and rolling friction that can be used in simulations of control systems with friction, especially in aerospace industry. The starting point for Dahl is modeling the stress-strain curve in classical solid mechanics by a differential equation. In this model the friction force is a function

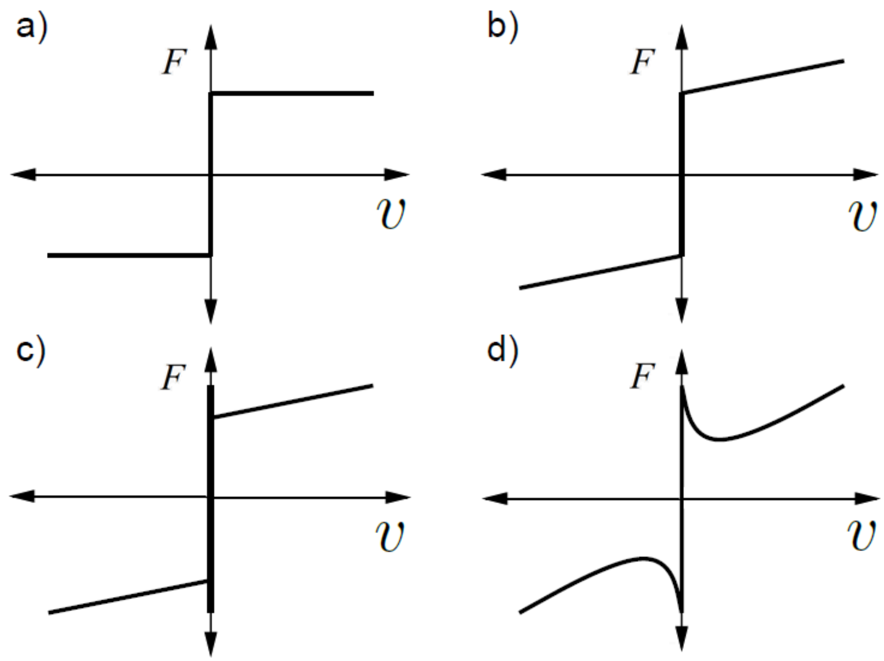

Figure 2.1: Illustrations of static friction models. a) Coulomb b) Coulomb+Viscous c) Coulomb+Viscous+Sticktion d) Stribeck Effect

of the displacement and the sign of the velocity. In other words, it has rate independence property. Nevertheless, it does not describe stick slip motion since it does not capture the Stribeck effect. General form of Dahl's friction model is,

$$\frac{dF}{dp} = \sigma_0 \left| 1 - \frac{F}{a_c} sgn\left(\frac{dp}{dt}\right) \right|^i sgn\left(1 - \frac{F}{a_c} sgn\left(\frac{dp}{dt}\right)\right). \tag{2.6}$$

where p is the position and i is a parameter of Dahl friction model. When referred to in the literature the Dahl model is often simplified, using $i = 1$ and (2.6) becomes

$$\frac{dF}{dt} = \sigma_0 \left(1 - \frac{F}{a_c} sgn\left(\frac{dp}{dt}\right)\right) \frac{dp}{dt}. \tag{2.7}$$

Another widely used dynamic model is LuGre friction model, see [26]. In this model, it is assumed that there is a contact between surface and rigid body through elastic bristles. When a tangential force is applied, the bristles deflect like springs and result in the friction force. If the force is sufficiently large randomly some of the bristles deflect so much that they slip off each other. During the motion new contacts are formed and slipped off again and again. Using this principle, friction can be modeled as fallows,

$$\begin{aligned} \frac{dz_d}{dt} &= v - \sigma_0 \frac{|v|}{g(v)} z_d, \\ F &= \sigma_0 z_d + \sigma_1(v) \frac{dz_d}{dt} + F_v v. \end{aligned} \tag{2.8}$$

In this equation, any velocity dependence can be obtained to capture Stribeck effect by a proper choice of the function $g(v)$ such that

$$g(v) = \frac{1}{\sigma_0} \left(a_c + (F_s - a_c) e^{-(v/v_s)^2} \right). \tag{2.9}$$

The steady-state relation between velocity and friction force for LuGre model is

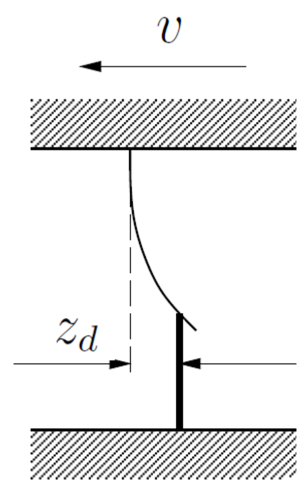

Figure 2.2: The elastic bristles deflect by the amount of z_d when a sufficiently large applied force is applied

obtained when $\frac{dz_d}{dt} = 0$. In this case (2.8) becomes

$$\begin{aligned} F(v) &= \sigma_0 g(v) sgn(v) + F_v v \\ &= a_c sgn(v) + (F_s - a_c) e^{-(v/v_s)^2} + F_v v. \end{aligned} \tag{2.10}$$

No matter how complex it is, in most of friction models, Coulomb friction is one of the fundamental components of friction force. Hence, an adaptive Coulomb observer could be utilized to estimate the friction force. In this study, an adaptive Coulomb observer considered in [22] is modified for the systems with time delay. Besides its simplicity, it works well enough for more complex cases and improves the closed loop performance.

Chapter 3

ANALYSIS OF CLOSED LOOP FEEDBACK SYSTEM

3.1 Plant Structure

From the physics in (2.1), transfer function of the plant should include an integrator. Furthermore, its velocity is inversely proportional to its mass for a fixed input. However, generally, time delay in the system appears due to sampling, sensor/actuator non-collocation, and signal transmission depending on the physical distance between the controller and the plant. Thus, transfer function from input to velocity is in the form

$$P(s) = \frac{1}{Ms} e^{-T_d s}. \tag{3.1}$$

Of course, in real systems some higher order dynamics can be occurred; nonetheless, an input filter can be used to suppress them. Hence, simulations are made using simple plant model in (3.1).

3.2 Observer Structure

In order to estimate Coulomb friction in a delay free system, an adaptive observer, whose aim is to estimate the Coulomb friction constant , is employed. Consider (2.1) and the controller structure given in Fig. 1.1. For simplicity, we will take $M = 1$. Then, we obtain

$$\dot{x} \;=\; v, \tag{3.2}$$

$$\dot{v} \;=\; -F(v, a_c) + \hat{F}(v, \hat{a}_c) + u, \tag{3.3}$$

where v is the velocity and $F(v, a_c) = a_c sgn(v)$. Note that in Fig. 1.1, we have $\hat{F} = F(v, \hat{a}_c) = \hat{a}_c sgn(v)$ and $p = x$. Likewise, note that in (3.3), the first term on the right hand side is the actual Coulomb friction, the second term is the estimated Coulomb friction which will be the output of the observer yet to be proposed, and u is the external input, of Fig. 1.1.

To design an observer, let $g(v) : \mathbf{R} \mapsto \mathbf{R}$ be an appropriate differentiable function yet to be determined. Let us propose the following observer structure

$$\dot{z} \;=\; g'(v)u, \tag{3.4}$$

$$\hat{a}_c \;=\; z - g(v). \tag{3.5}$$

Here, z is the internal state of the observer and $\hat{a}_c$ is the estimation of Coulomb friction constant. To analyze the performance of the observer given in (3.4), (3.5), let us define the following estimation error

$$e \;=\; a_c - \hat{a}_c. \tag{3.6}$$

After differentiation and assuming that a_c is a constant, by using (3.2)-(3.6) we obtain:

$$\begin{aligned} \dot{e} &= -\dot{\hat{a}}_c \\ &= -\dot{z} + g'(v)\dot{v} \\ &= -g'(v)u + g'(v)[F(v, \hat{a}_c) - F(v, a_c) + u] \\ &= -g'(v)[F(v, a_c) - F(v, \hat{a}_c)] \\ &= -g'(v)sgn(v)e. \end{aligned} \tag{3.7}$$

In order to obtain a stable error dynamics, obviously we need $g'(v)sgn(v) \geq 0$. Since the graph of $sgn(v)$ versus v is in the first and third quadrants, for this inequality hold, the graph of $g'(v)$ versus v should be in first and third quadrants as well. There are a large number of functions satisfying this property. Some of the examples are given below.

- $g(v) = klog\,(cosh(v))$ and $g'(v) = ktanh(v)$,

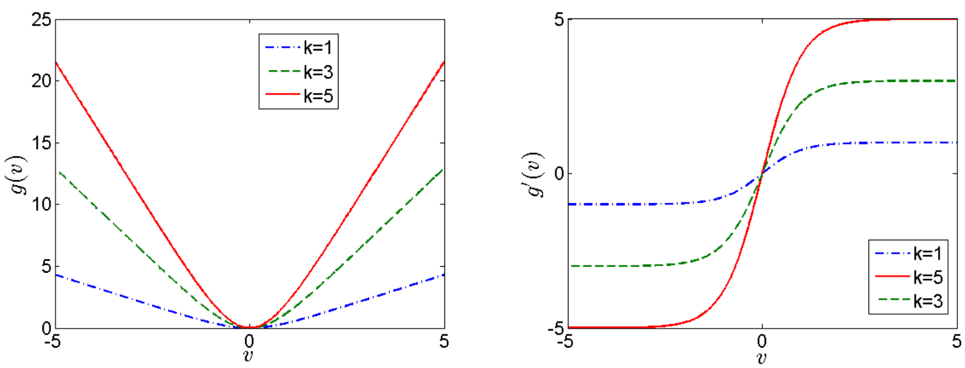

Figure 3.1: Graph of $g(v) = klog(cosh(v))$ and $g'(v) = ktanh(v)$ for different k values.

- $g(v) = kvtan^{-1}(v) - 0.5klog(v^2 + 1)$ and $g'(v) = ktan^{-1}(v)$,

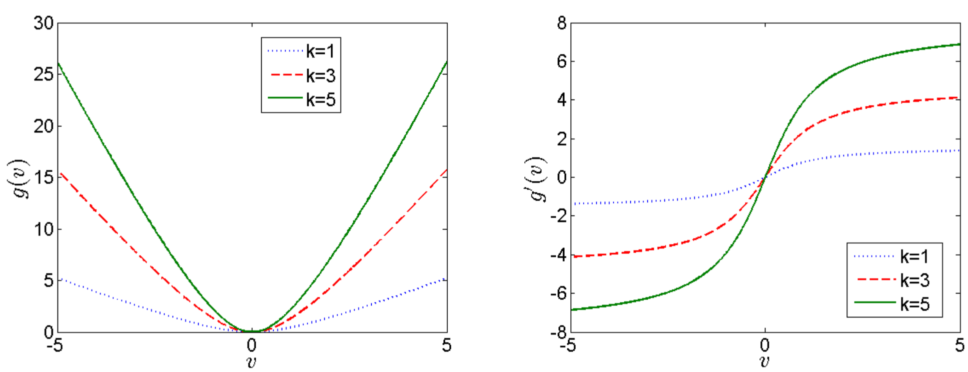

Figure 3.2: Graph of $g(v) = kvtan^{-1}(v) - 0.5klog(v^2 + 1)$ and $g'(v) = ktan^{-1}(v)$ for different k values.

- $g(v) = kv^{2n}$ and $g'(v) = 2nkv^{2n-1}$, where n $\in \mathbf{R}^+$,

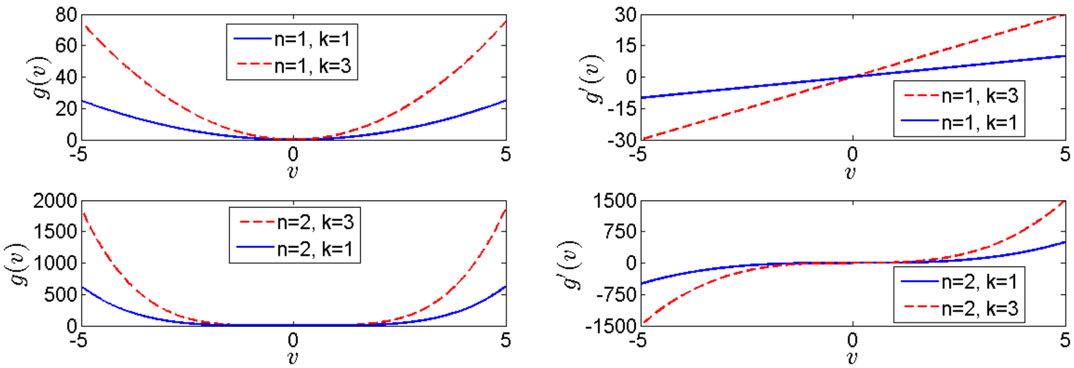

Figure 3.3: Graph of $g(v) = kv^{2n}$ and $g'(v) = 2nkv^{2n-1}$ for different k and n values.

- $g(v) = k|v|^{\mu}$ and $g'(v) = k\mu|v|^{\mu-1}sgn(v)$ etc.

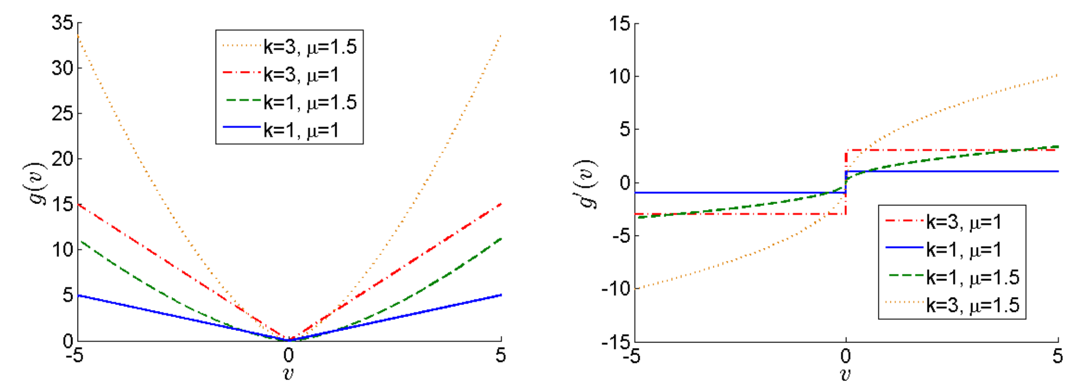

Figure 3.4: Graph of $g(v) = k\,|v|^{\mu}$ and $g'(v) = k\mu\,|v|^{\mu-1}\,sgn(v)$ for different k and μ values.

Actually, the last choice given above with $g(v) = k|v|^{\mu}$ results in Friedland-Park observer in [22]. Hence, the observer given in [22] can be considered as a special case of the observer given by (3.4), (3.5). If we rewrite (3.4) and (3.5) for this case, we obtain

$$\dot{z} = k\mu\,|v|^{\mu-1}\,sgn(v)u, \tag{3.8}$$

$$\hat{a}_c = z - k|v|^{\mu}. \quad (3.9)$$

and the error dynamics (3.8) becomes the following for this particular case

$$\dot{e} = -k\mu|v|^{\mu-1}e, \quad (3.10)$$

of [22]. For a constant friction coefficient a_c, the estimation error converges to zero provided that observer gain $k > 0$ and exponent $\mu > 0$ when $|v| \neq 0$, see [22].Although this result is not proven in [22], by using standard methods of Lyapunov stability analysis, it can be proven rigorously as follows:

Lemma 1 *Consider the system given by (3.2)-(3.7). Assume that $|v|$ is bounded and that $g'(v)$ is a noninear function whose graph versus v is in first and third quadrant with $g(v) \neq 0$ and $g'(v) \neq 0$ for $v \neq 0$. Under these constraints, if $|v| \geq \alpha\ \forall t$, then the error dynamics given by (3.7) is exponentially stable, i.e. $e(t) \longrightarrow 0$ with an exponential decay*

Proof Let $v(t)$ be the solution of (3.2)-(3.7) and define

$$G(t) = g'(v(t))\, sgn\,(v(t)). \quad (3.11)$$

Note that $G(t) \geq 0$ and since $|v| \geq \alpha{>}0\ \forall t$, there exists a $\beta{>}0$ such that

$$G(t) \geq \beta \ \ \forall t \geq 0. \quad (3.12)$$

Note that the error dynamics given by (3.7) can be written as

$$\dot{e} = -G(t)e. \quad (3.13)$$

Let us define the following Lyapunov function

$$V(e) = \frac{e^2}{2}. \quad (3.14)$$

Differentiating (3.14) and by using the fact $-G(t) \leq -\beta$, we obtain

$$\begin{aligned} \dot{V} &= e\dot{e} \quad (3.15)\\ &= -G(t)e^2 \\ &\leq -\beta e^2 = -2\beta V. \end{aligned}$$

Hence, we have

$$V(t) \leq e^{-2\beta t} V(0). \tag{3.16}$$

and therefore for (3.14), we have

$$|e(t)| \leq e^{-2\beta t} |e(0)| . \qquad \blacksquare \tag{3.17}$$

Remark 1 *Although the applicability of Lemma 1 seems to be limited since it requires that $|v| \geq \alpha > 0$; nevertheless, it could be utilized in various meaningful applications such as unit step tracking in velocity loop, ramp tracking in position loop. Note that in these cases, it is expected that condition $|v| \geq \alpha$ hold sufficiently long period of time.* $\blacksquare$

A somewhat less restrictive assumption on $v(t)$, which could be related to be persistency of excitation [27], could be given as follows.

Lemma 2 *Let $v(t)$ be the solution of (3.2)-(3.7) and let $G(t)$ be given as in (3.11). Assume that there exist some $\alpha > 0$ and $T > 0$ such that the following holds*

$$\int_{t}^{t+T} \mathrm{G(s)}\, \mathrm{d}s \;\; \geq \;\; \alpha, \qquad \forall t \geq 0 \tag{3.18}$$

Then for the error dynamics given by (3.7), we have $e(t) \longrightarrow 0$ as $t \longrightarrow \infty$; moreover, the decay is exponential.

Proof Note that the solution of (3.7) is given as

$$e(t) = e^{-\int_0^t \mathrm{G(s)}\, \mathrm{d}s} e(0). \tag{3.19}$$

Since the exponential term is always positive, we can rewrite (3.19) as

$$|e(t)| = e^{-\int_0^t \mathrm{G(s)}\, \mathrm{d}s} |e(0)| . \tag{3.20}$$

Let $t = nT + \tau$ for some integer $n \in Z^+$ and $0 < \tau < T$. Then we have

$$\int_0^t \mathrm{G(s)}\, \mathrm{d}s \;\; = \;\; \int_0^{nT} \mathrm{G(s)}\, \mathrm{d}s + \int_{nT}^{nT+\tau} \mathrm{G(s)}\, \mathrm{d}s \geq n\alpha. \tag{3.21}$$

where we used the fact that $G(s) \geq 0$. Using (3.21) in (3.20), we obtain:

$$|e(t)| \leq e^{-n\alpha} |e(0)|. \tag{3.22}$$

As $t \longrightarrow \infty$, we have $n \longrightarrow \infty$, hence $|e(t)| \longrightarrow 0$. As it can be seen from (3.22), the decay is exponential. In fact, since $n = \frac{t-\tau}{T}$ and $\tau < T$, we have

$$|e(t)| \leq e^{\alpha} e^{-\frac{\alpha t}{T}} |e(0)|. \quad \blacksquare \tag{3.23}$$

Remark 2 *For details on persistency of excitation of signals and its application to adaptive control systems, see e.g. [27]. Note that when $|v(t)| \geq \alpha > 0$, the condition given by (3.11) also holds. On the orher hand, when the signals are in the sinusoidal form, the condition $|v(t)| \geq \alpha$ may not hold whereas (3.11) may hold. Hence, Lemma (2) could be utilized in the sinusoidal signal tracking and/or rejection.* ■

It is obvious that the solution of the first order homogeneous differential equation in 3.10 is an exponential. Thus, increasing observer gain k provides a faster convergence. However, selecting larger values of it may result in instability. Moreover, as it is stated in [22], when high gain is combined with a relatively high value of μ, the response tends to ”ring” somewhat. According to Friedland and Park, this can be explained by an undesired sensitivity to disturbance signal existed in the system. Therefore, in this thesis, performance of the observer under the presence of high disturbance is also concerned. Simulations show that changing estimation function, performance of the system can be improved.

Unfortunately, [22] does not provide any criteria explicitly to determine values of k and μ. Generally, choice of these design parameters is determined by simulation, or on actual system. However, when desired velocity is a periodic signal, [28] gives a guideline for convergence rate and criteria for a choice of them. As a result, the observer performance can be improved to be faster for a proper range of k and μ.

Lastly, note that for the delay free case, i.e. when $P(s) = \frac{1}{Ms}$, block diagram of observer given by (3.4) and (3.5) can be depicted as in Fig. 3.5. However, when there is a delay term, i.e. $P(s) = \frac{1}{Ms}e^{-T_d s}$, then this observer structure cannot be utilized as it is since as velocity output $ve^{-T_d s}$ is obtained instead of v. To see that clearly, it is beneficial to consider Fig. 1.2 with a plant described by (3.1). As it can be observed from the figure, in order to estimate friction, observer needs to input, velocity output v and control input u. However, in delay case, if observer depicted in Fig. 3.5 is utilized, its inputs will be $ve^{-T_d s}$ and u. Hence, it is reasonable to modify the observer structure given above with a delay term as in Fig. 3.6 to make inputs synchronized and estimate friction correctly.

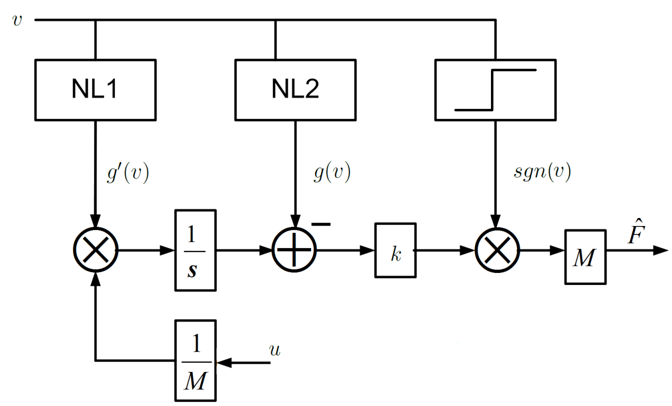

Figure 3.5: General structure of the Coulomb observer.

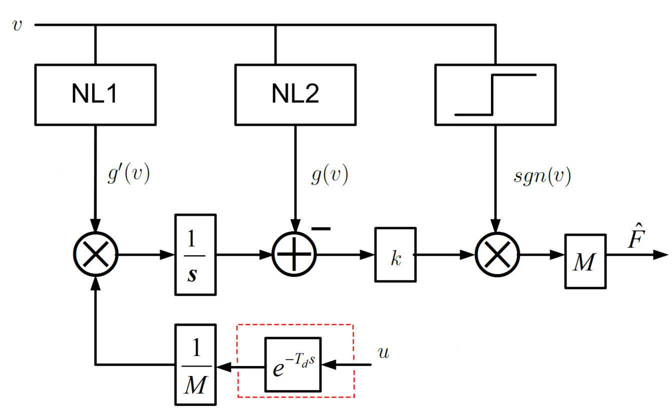

Figure 3.6: Modified Coulomb observer.

3.3 Smith Predictor Based Controller Design for Velocity Control

In this section, design methodology described in [7, 8] is utilized to satisfy different performance and robustness objectives. Although some examples presented here, it is also possible to apply this methodology to obtain controllers satisfying more complex requirements. The structure of $C_v(s)$ is illustrated in Fig. 3.7 and defined as

$$C_v(s) = \frac{MC_{0v}(s)}{1 + C_{0v}(s)\frac{1-e^{-T_d s}}{s}}. \tag{3.24}$$

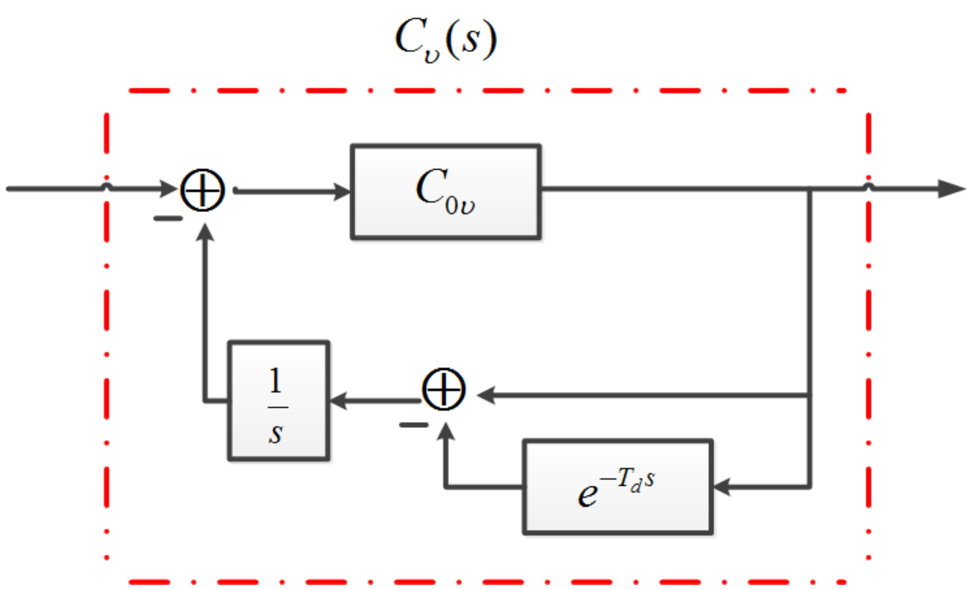

Figure 3.7: Internal structure of $C_v(s)$

With the defined structures of $C_v(s)$ and $P(s)$, characteristic equation of closed loop system is

$$1 + C_{0v}(s)\frac{1}{s} = 0, \tag{3.25}$$

which implies that $C_{0v}(s)$ must be designed to stabilize an integrator. To find $C_{0v}(s)$, controller parametrization technique, whose details are given at appendix part, is utilized. To make the design process simpler, inverse of the mass cascaded after controller block and controller parametrization is implemented for the plant with $M = 1$. Note that $Q(s)$ in the following denotes a proper and stable transfer function which is utilized in controller parametrization. For details, see appendix or [7, 8]

- If the aim is to track the step input without any disturbance in the velocity loop, then by using $Q(s) = 0$ particular choice would be

$$C_{0v}(s) = K_v, \tag{3.26}$$

which results in the closed loop as

$$T_v(s) = \frac{K_v}{s + K_v} e^{-T_d s}. \tag{3.27}$$

K_v in (3.26) is determined by pole placement method considering performance requirements.

- If the aim is to track the ramp input without disturbance in the velocity loop, then by using $Q(s) = K_v$ particular choice of C_{0v} and closed loop transfer function would be

$$C_{0v}(s) = \frac{2K_v s + K_v^2}{s}, \tag{3.28}$$

$$T_v(s) = \frac{2K_v s + K_v^2}{(s + K_v)^2} e^{-T_d s}. \tag{3.29}$$

- To design a controller suppressing constant disturbance and tracking step reference signal without a steady-state error, by using $Q(s) = K_v(1 + K_v T_d)$ a particular choice of C_{0v} is

$$C_{0v}(s) = \frac{(2K_v + K_v^2 T_d)s + K_v^2}{s - K_v^2 T_d}, \tag{3.30}$$

and corresponding closed loop transfer function is

$$T_v(s) = \frac{(2K_v + K_v^2 T_d)s + K_v^2}{(s + K_v)^2} e^{-T_d s}. \tag{3.31}$$

- To design a controller for step input tracking and both constant and ramp disturbance rejection, the structure of minimum degreed $Q(s)$ must be

$$Q(s) = \frac{\alpha_1 s + \alpha_2}{s + \alpha_4}. \tag{3.32}$$

In (3.32), α_4 is a free parameter determined by pole placement method like K_v whereas $\alpha_2 = Q(0)\alpha_4$ and $\alpha_1 = \dot{Q}(0)\alpha_4 + \frac{\alpha 2}{\alpha_4}$. Then, particular choice of C_{0v} and corresponding closed loop transfer function are

$$C_{0v}(s) = \frac{(K_v + \alpha_1)s^2 + (K_v l_1 + \alpha_2)s + al_2}{s^2 + (l_1 - \alpha_1)s + (l_2 - \alpha_2)}, \tag{3.33}$$

$$T_v(s) = \frac{(K_v + \alpha_1)s^2 + (K_v l_1 + \alpha_2)s + al_2}{(s + K_v)^2(s + \alpha_4)} e^{-T_d s} \tag{3.34}$$

where $l_1 = K_v + \alpha_4$ and $l_2 = K_v \alpha_4$.

- To design a step input tracking and both constant and sinusoidal disturbance rejection, the structure of minimum degreed $Q(s)$ must be

$$Q(s) = \frac{\alpha_1 s^2 + \alpha_2 s + \alpha_3}{s^2 + \alpha_4 s + \alpha_5}. \tag{3.35}$$

In (3.35), $\alpha_4, \alpha_5 > 0$ are free parameters. Then, other parameters are calculated considering frequency of periodic disturbance signal, which is w_d such that

$$\alpha_1 = \frac{Re\left(\left(\frac{jw_d + K_v - K_v e^{-jw_d T_d}(jw_d + K_v)}{jw_d e^{-jw_d T_d}}\right)(\alpha_5 + jw_d\alpha_4 - w_d^2)\right) - \alpha_4}{-w_d^2},$$

$$\alpha_2 = \frac{Im\left(\left(\frac{jw_d + K_v - K_v e^{-jw_d T_d}(jw_d + K_v)}{jw_d e^{-jw_d T_d}}\right)(\alpha_5 + jw_d\alpha_4 - w_d^2)\right)}{w_d}.$$

Then, particular choice of C_{0v} and corresponding closed loop transfer function are

$$C_{0v}(s) = \frac{(K_v + \alpha_1)s^3 + (K_v l_1 + \alpha_2)s^2 + (al_2 + \alpha_3)s + K_v l_3}{s^3 + (l_1 - \alpha_1)s^2 + (l_2 - \alpha_2)s + K_v l_3}, \tag{3.36}$$

$$T_v(s) = \frac{(K_v + \alpha_1)s^3 + (K_v l_1 + \alpha_2)s^2 + (al_2 + \alpha_3)s + K_v l_3}{(s + K_v)^2(s^2 + \alpha_4 s + \alpha_5)} e^{-T_d s} \tag{3.37}$$

where $l_1 = K_v + \alpha_4$ $l_2 = \alpha_5 + K_v\alpha_4$ and $l_3 = K_v\alpha_5$.

3.4 Smith Predictor Based Controller Design for Position Control

To design a position controller, structure of velocity controller in Fig. 3.7 is slightly changed. Fig. 3.8 is obtained by adding delay free closed loop transfer function for velocity loop to Fig. 3.7.

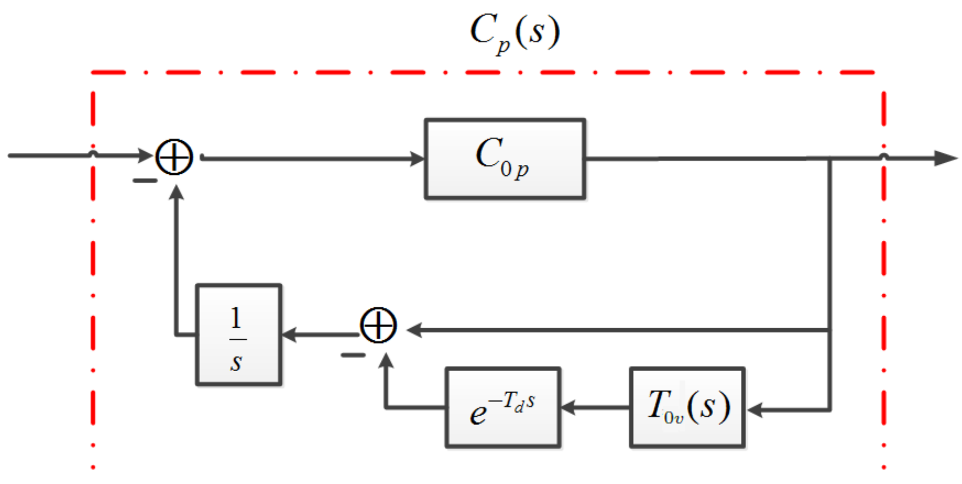

Figure 3.8: Internal structure of C_p(s)

For position controller, $C_p(s)$ given in Fig. 3.8, same controller structure given in previous section can be implemented according to design specifications. Let's consider step reference and constant disturbance rejection case presented in (3.30) which means

$$C_{0p}(s) = \frac{(2K_p + K_p^2 T_d)s + K_p^2}{s - K_p^2 T_d}, \tag{3.38}$$

where K_p is a free parameter to be chosen via pole placement like in the velocity loop. Then corresponding transfer function of the overall closed loop feedback system is

$$T(s) = \frac{(2K_p + K_p^2 T_d)s + K_p^2}{(s + K_p)^2} T_{v(s)}. \tag{3.39}$$

Chapter 4

SIMULATIONS AND EVALUATION

As an example we consider plant $P(s)$ given by (3.1) with $M = 5$ kg and $T_d = 0.2$ sec. Although mass is chosen arbitrarily, time delay is taken as same as in [22]. Furthermore, some of the results in this thesis also presented in [29]. Since real-world friction may not conform to only Coulomb friction, performance of the observer with a more representative model, (4.1), is investigated.

$$F(v) = (\beta_1 + \beta_2 e^{-\lambda|v|)} + \beta_3 |v|) sgn(v), \quad (4.1)$$

where β_1 term denotes Coulomb friction whereas $\beta_2 e^{-\lambda|v|)}$ and $\beta_3 |v|$ denotes Stribeck effect and viscous friction respectively. As a whole, (4.1) is a similar expression for steady state behavior of dynamical friction model, LuGre model given in [30].

For our simulations, let us choose $\beta_1 = 5, \beta_2 = 1, \beta_3 = 1$ and $\lambda = 1$ for simplicity. In this section, performances of different types of Smith predictor based controllers, P-P and PI-PI controllers are analyzed. Firstly, step input tracking without any disturbance, ramp input tracking without any disturbance and step input tracking with step disturbance are analyzed. In the following simulations, we considered three different situations.

- When there is no friction term in (2.1), in this case friction observer is not utilized,
- When friction exists but observer is not utilized,
- When friction exists and friction observer is also utilized.

4.1 Smith Predictor Based Position Control: Step input without disturbance

By choosing $K_v = 1$ and $K_p = 3$ in (3.26), corresponding controllers are obtained as

$$\begin{aligned} C_{0v}(s) &= 1, \\ C_{0p}(s) &= 3. \end{aligned}$$

while closed loop transfer functions for velocity and position control are

$$\begin{aligned} T_v(s) &= \frac{1}{s+1}e^{-0.2s}, \\ T(s) &= \frac{3}{s+3}T_v(s). \end{aligned}$$

As a friction observer, we choose Friedland-Park observer given by (3.8) and (3.9). Performance results of controllers for step tracking without disturbance are plotted in Fig. 4.1 and Fig. 4.2 for $k = 5$ and $\mu = 1.2$. As a result of friction, in friction cancellation, settling time is slightly increased compared to no friction existence case. However, as it can be observed in Fig. 4.1, without a compensation both settling time increases a lot and an oscillation, causing steady state error at the output, occurs.

Interestingly, although it seems that velocity becomes zero after a time in Fig. 4.3, direction of the friction in Fig. 4.2 changes. However, direction of friction can only change the direction of the velocity changes. Actually, the reason for this situation is modeling errors of the friction at low velocities in MATLAB. In Fig. 4.4, it is observed that at direction reversals of the friction, sign of velocity

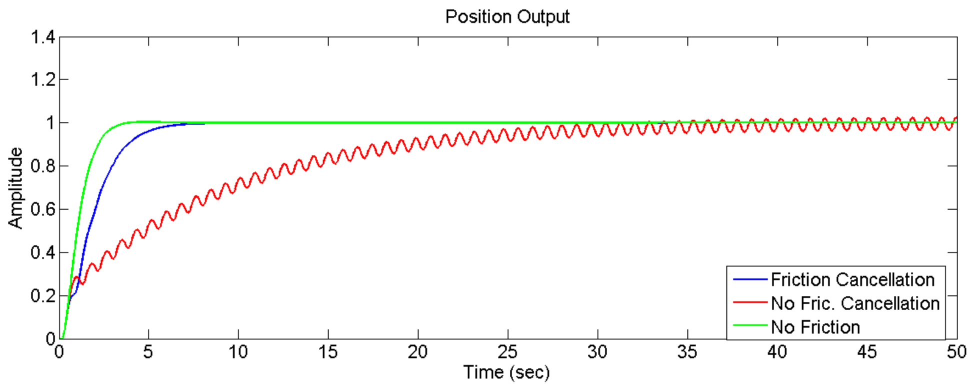

Figure 4.1: Unit step response of the system

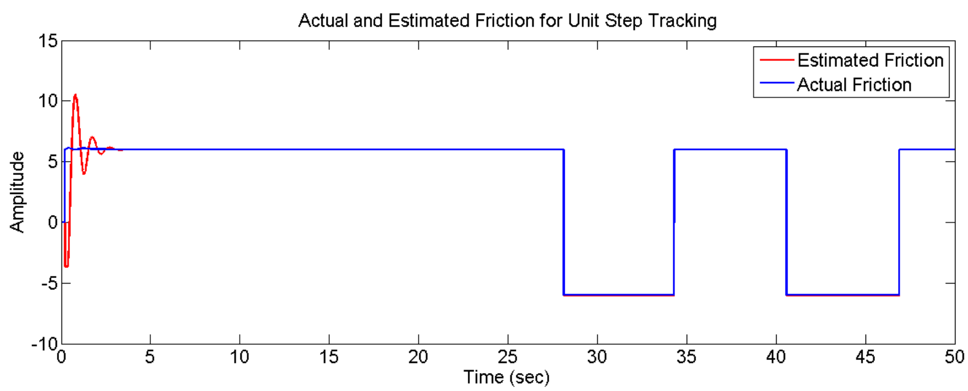

Figure 4.2: Actual and estimated friction

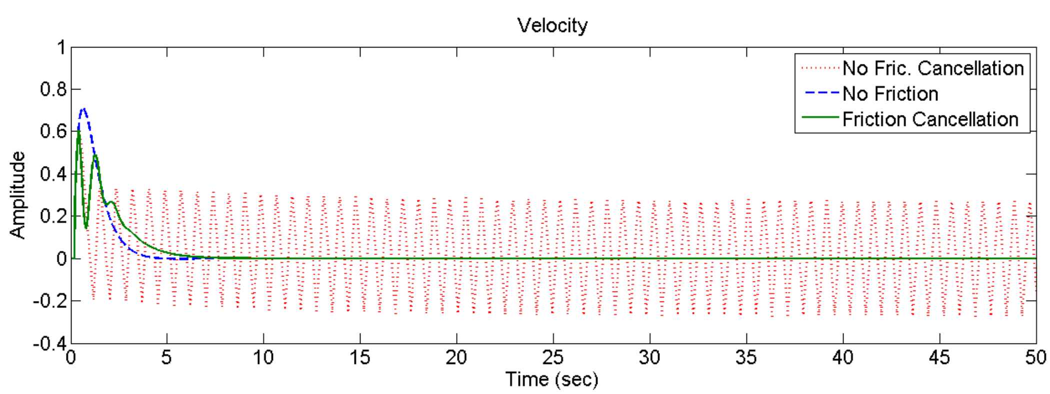

Figure 4.3: Velocity due to step response of the system

changes as well; however, amount of change is so little that it can be only realized when one zooms on it.

As a justification, same closed loop feedback system is utilized with an observer structure illustrated in Fig. 3.5. As it can be observed in Fig.4.5, without inclusion of time delay to the observer structure, it is not possible to estimate friction correctly and that is why system becomes unstable.

Moreover, from Fig. 4.2 it can be deduced that although actual friction does not only include Coulomb friction, observer is capable of estimating it reasonably well enough with a small amount of transition time. In observer parameters, gain k determines the speed of estimation. However, increasing it too much results in larger oscillations at the transient; therefore, it might cause to system to be unstable. Effects of the choices of these parameters on the tracking performance are investigated without any disturbance and plotted in Fig. 4.6 and Fig. 4.9.

Firstly, as it can be observed in Fig. 4.6, for a fixed value of μ, increasing observer gain k results in faster rise time. Nevertheless, it also makes deformations on the output when its value is so high.

Secondly, it is clear in Fig. 4.9 that, for a fixed value of k, increasing observer exponent μ provides a smoother transient response. When k is so large, at the transient response is not smooth. Thus, in order to make it smoother, μ should be also increased.

Although Fig. 4.6 and Fig. 4.9 give an insight into physical meanings of k and μ, there is not exactly one to one relationship. In Fig. 4.12, it can be deduced that settling time might change nonlinearly. Therefore, one should determine appropriate values of k and μ to obtain a region of desired settling time or change the estimation function $g(v)$ in (3.5).

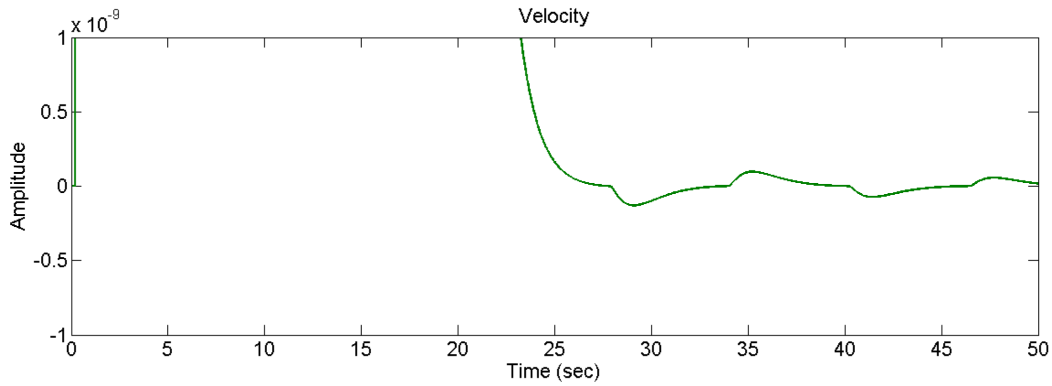

Figure 4.4: Zoomed version of Fig. 4.3

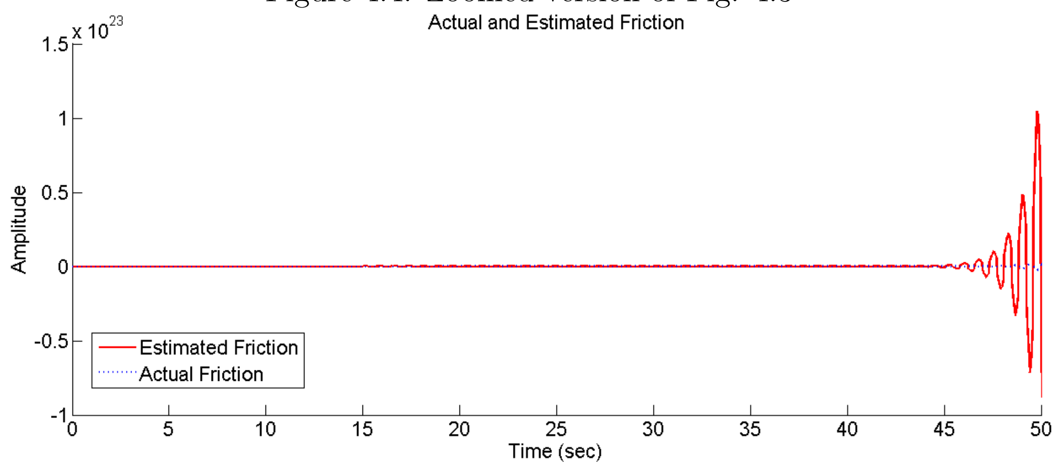

Figure 4.5: Friction estimation of an observer that does not include time delay

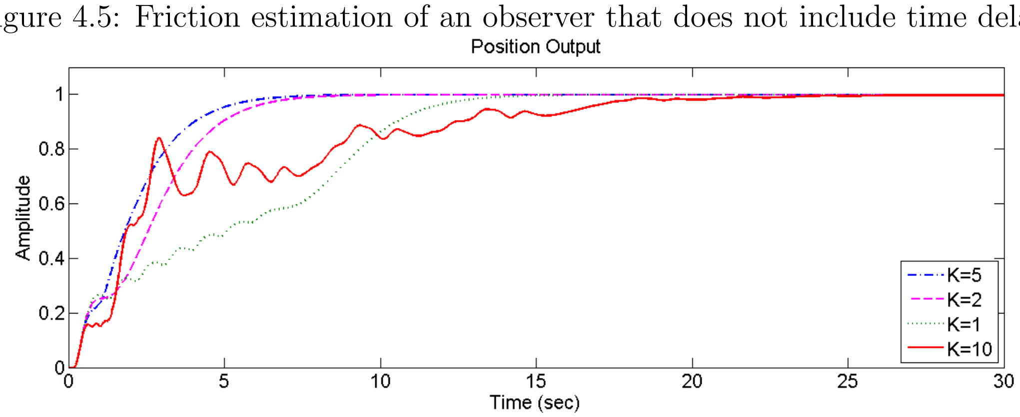

Figure 4.6: Effect of observer gain k to the position output when $\mu = 1$ is fixed.

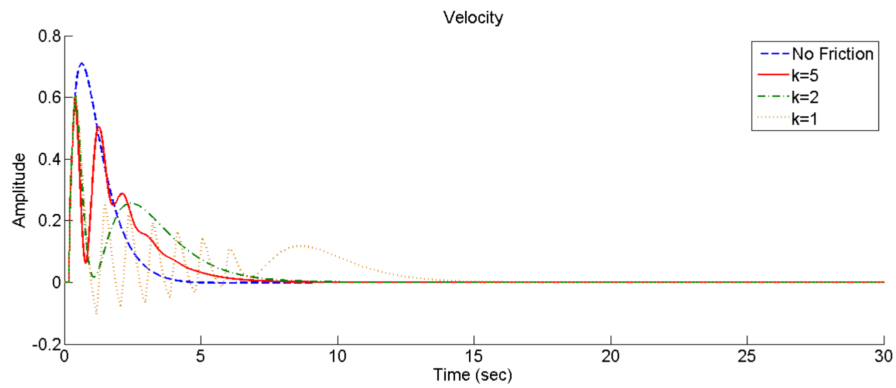

Figure 4.7: Effect of observer gain k to the velocity when $\mu = 1$ is fixed.

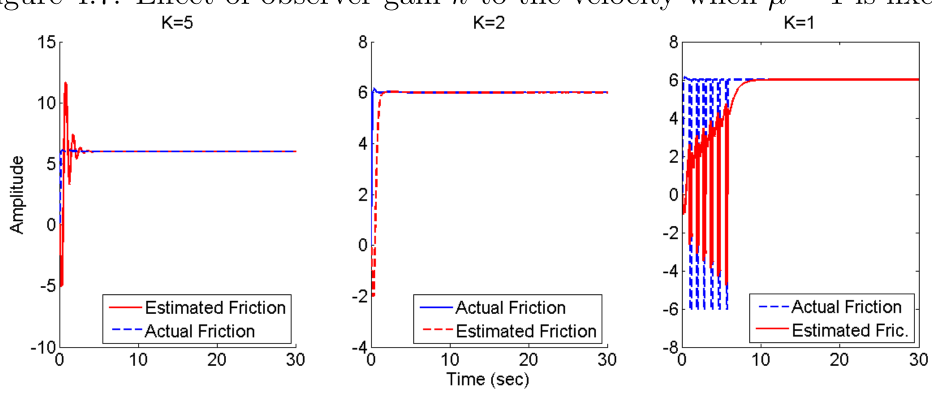

Figure 4.8: Effect of observer gain k to friction estimation when $\mu = 1$ is fixed.

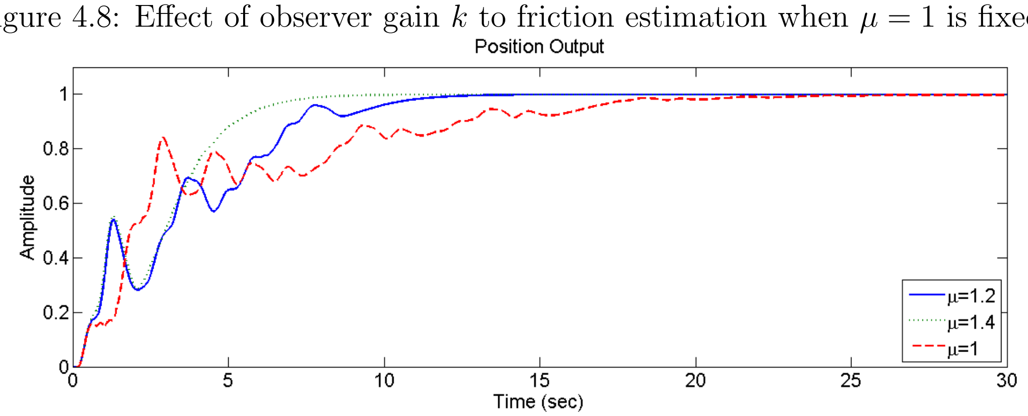

Figure 4.9: Effect of observer exponent μ to the position output when $k = 10$ is fixed.

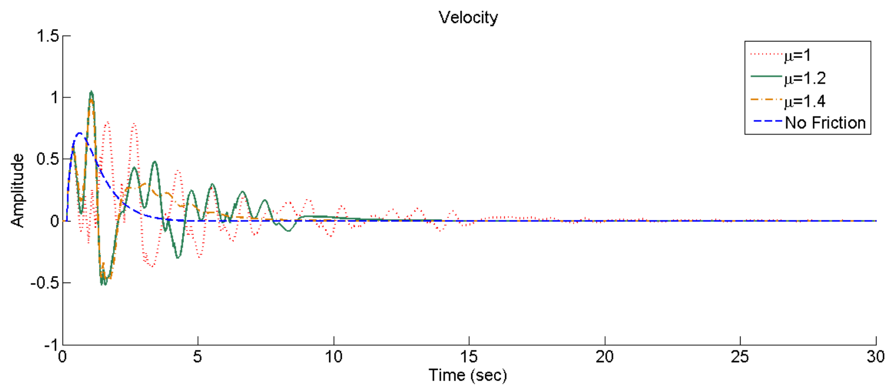

Figure 4.10: Effect of observer gain k to the velocity when $k = 10$ is fixed.

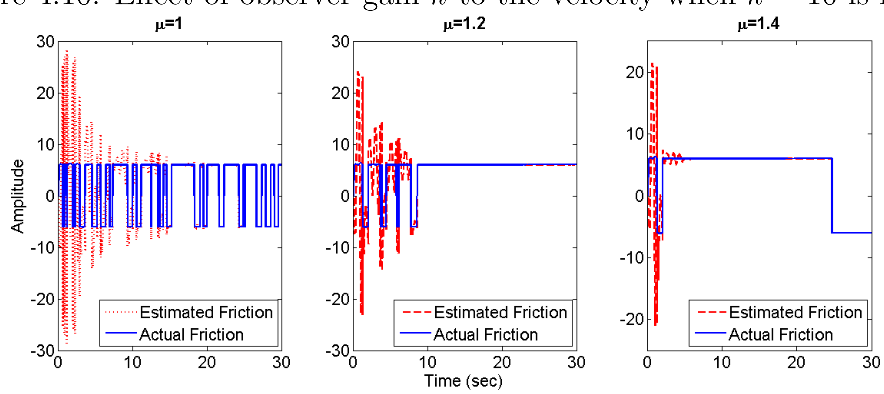

Figure 4.11: Effect of observer gain k to friction estimation when $k = 10$ is fixed.

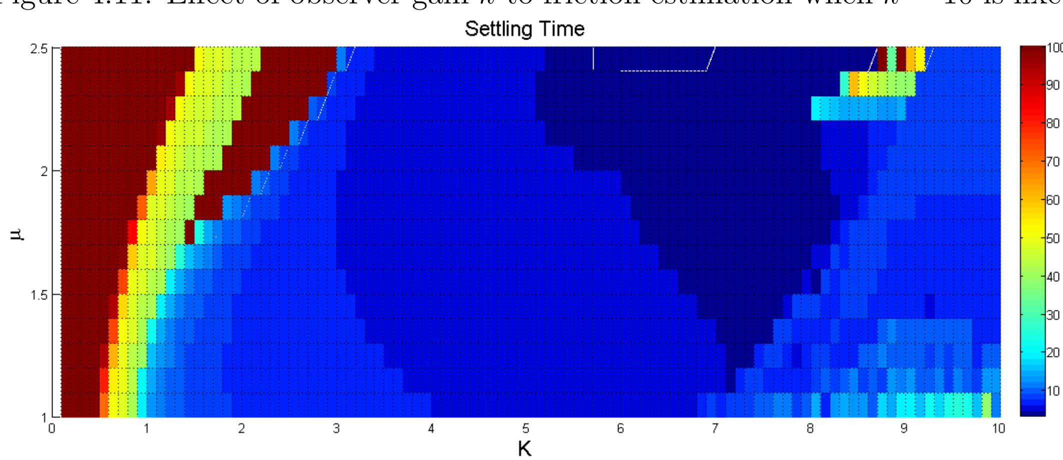

Figure 4.12: Settling time of the system desribed by (4.2) for different k and μ values.

4.2 P-P Controller Based Position Control: Step input without disturbance

Actually, controller designed to track step input without any disturbance is in the form of proportional controller. Therefore, it might be worthy to investigate the performance of modified observer with ordinary proportional controllers. Note that C_{v-P} and C_{p-P} are in the form of

$$C_{v-P}(s) = P_v, \tag{4.2}$$

$$C_{p-P}(s) = P_p. \tag{4.3}$$

If proportional controllers were utilized in a hierarchical feedback structure instead of designed Smith predictors to control an ordinary integrator, transfer functions of velocity and position loops without any delay would be

$$T_{v-P}(s) = \frac{P_v}{s + P_v}, \tag{4.4}$$

$$T(s) = \frac{P_v P_p}{s^2 + P_v s + P_v P_p} \tag{4.5}$$

When (4.4) and (4.2) are compared, it can be observed that it is possible to obtain exactly same poles choosing parameters appropriately. Therefore, $P_v = 4$ and $P_p = 3/4$ are determined in order to compare the performance of the controller with Smith predictor when $K_v = 1$ and $K_p = 3$ with $g(v) = 5 * \|v\|^{1.2}$. In this situation, although time delay is considered in the observer structure, gain of position and velocity controllers are designated without any delay concerns. In fact, when time delay is taken into consideration, (4.4) turns into

$$T_{v-P}(s) = \frac{P_v e^{-T_d s}}{s + P_v e^{-T_d s}}, \tag{4.6}$$

$$T(s) = \frac{P_v P_p e^{-T_d} s}{s^2 + P_v e_d^T s(s + P_p)}. \tag{4.7}$$

Clearly, compared to (4.4), (4.6) represents system dynamics much more realistically. Nonetheless, it is a more sophisticated expression with respect to (4.2) as well.

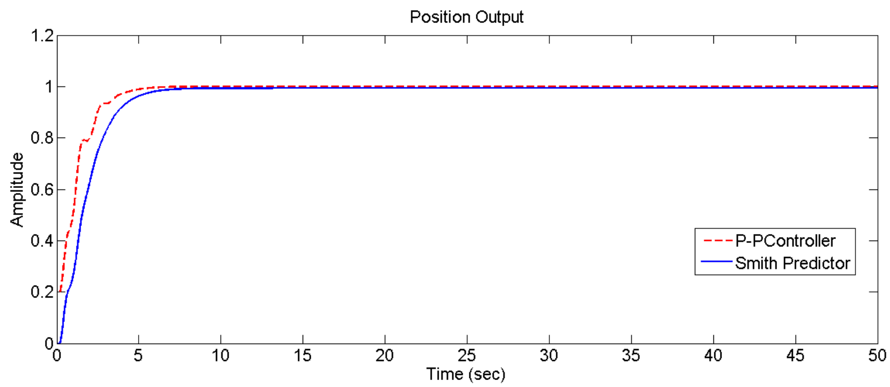

Figure 4.13: Position comparison of proportional controller and Smith Predictor.

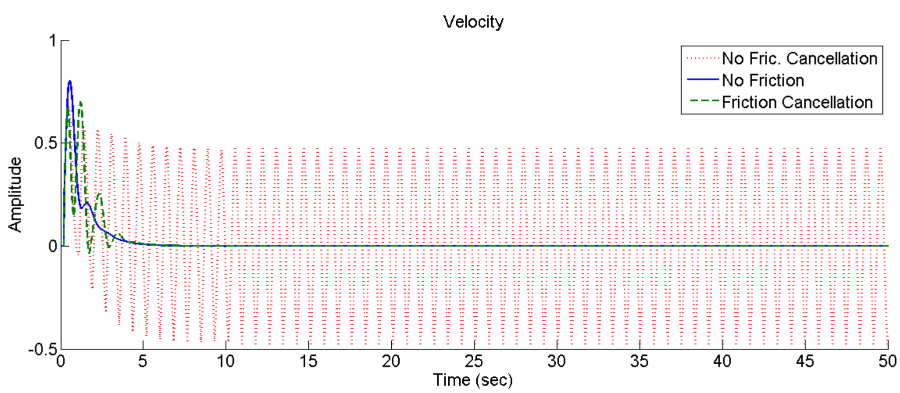

Figure 4.14: Velocity comparison of proportional controller and Smith Predictor.

As it can be observed in Fig. 4.13, similar position responses are obtained for $T_d = 0.2$ and $F = (5 + e^{-|v|} + |v|)sgn(v)$ while $K = 5$ and $m = 1.2$ again.

At first, it might be seemed that Smith predictor does not have much superiority than a standard position control; however, proportional controllers become unstable after a certain level of gain. Since increasing poles of the controller also leads to increase in magnitude of open loop transfer function of the system directly, stablity might be violated as in Fig. 4.16. In other words, gains of proportional controllers cannot be increased arbitrarily that is why one should pay attention to pole locations. On the other hand, Smith predictor assures freedom while determining pole locations.

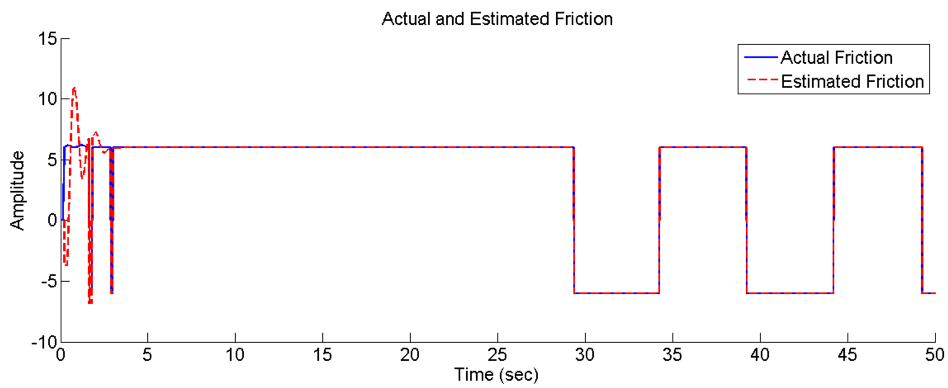

Figure 4.15: Friction comparison of proportional controller and Smith Predictor.

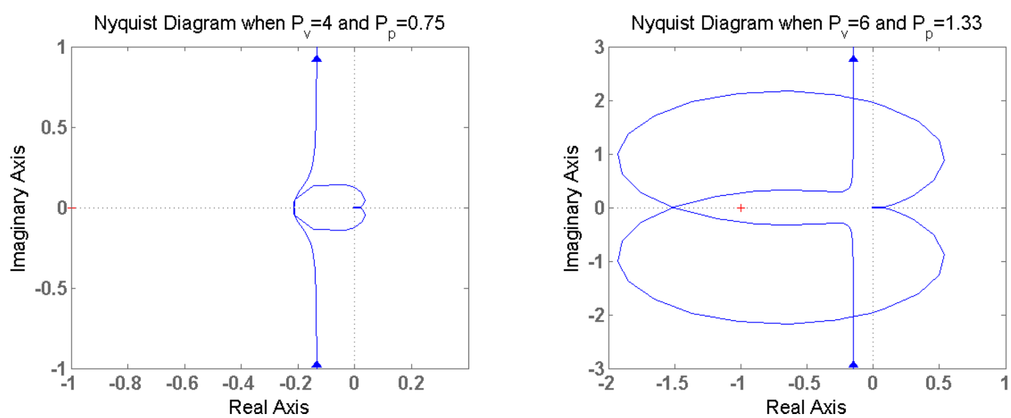

Figure 4.16: Nyquist Diagram of proportional controller for different poles.

4.3 Smith Predictor Based Position Control: Ramp input without disturbance

Likewise, tracking performance of proposed system for periodic triangular input with period of 20 sec. is considered with $K_p = 5, K_v = 1, k = 5$ and $\mu = 1.2$. Utilized controllers in velocity and position loops are

$$\begin{aligned} C_{0v}(s) &= \frac{2s+1}{s}, \\ C_{0p}(s) &= \frac{10s+25}{s}. \end{aligned}$$

Therefore, transfer functions for closed loop velocity and position control are

$$\begin{aligned} T_v(s) &= \frac{2s+1}{s^2+2s+1}e^{-0.2s}, \\ T(s) &= \frac{10s+25}{s^2+10s+25}T_v(s). \end{aligned}$$

In Fig. 4.17 it is clear that after an oscillatory transient, cancellation matches desired output and as expected, output is 0.2 sec shifted version of the input. However, at direction reversals it needs some time to track the signal in new direction. Nevertheless, without any cancellation there is an oscillatory output causing steady state errors again.

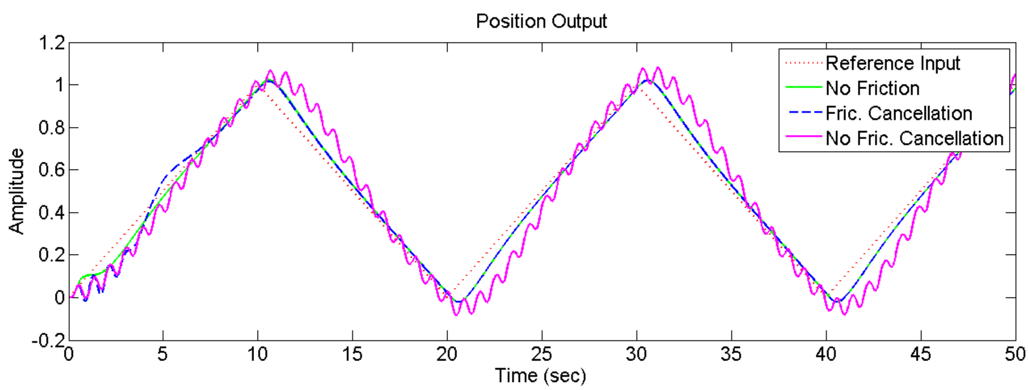

Figure 4.17: Position output when triangular input is applied

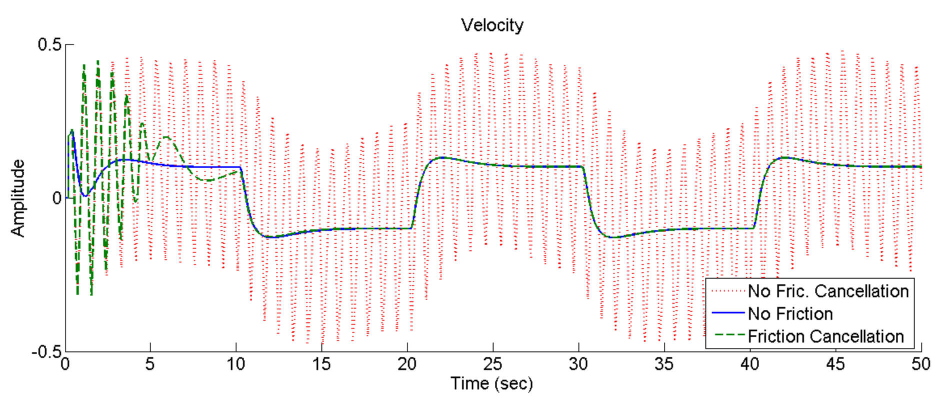

Figure 4.18: Velocity when triangular input is applied

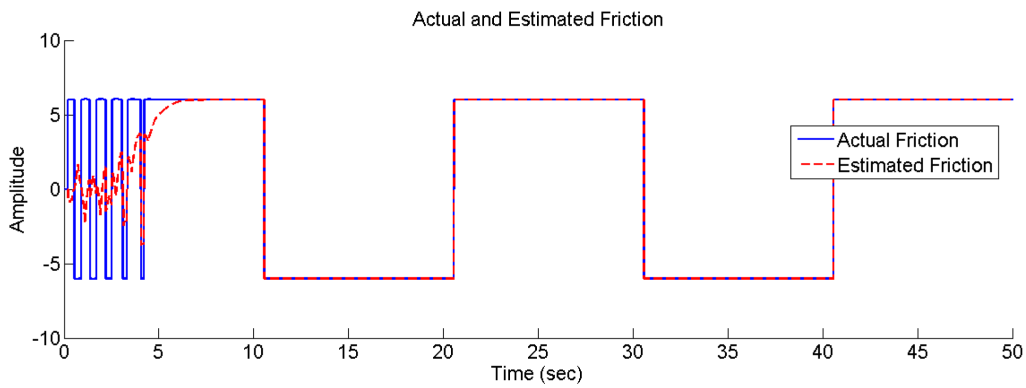

Figure 4.19: Friction estimation when triangular input is applied

4.4 PI-PI Controller Based Position Control: Ramp input without disturbance

Previously, performances of proportional controllers and Smith predictor based controllers are compared. Similarly, Smith predictor based controller designed to track ramp input without any disturbance can be compared to proportional-integral (PI) controllers. Note that C_{v-PI} and C_{p-PI} are in the form of

$$C_{v-PI}(s) = \frac{P_v s + I_v}{s}, \quad (4.8)$$

$$C_{p-PI}(s) = \frac{P_p s + I_p}{s}. \quad (4.9)$$

Afterwards, ignoring delay, corresponding closed loop transfer functions can be obtained as

$$T_{v-PI}(s) = \frac{P_v s + I_v}{s^2 + P_v s + I_v}, \quad (4.10)$$

$$T_{PI} = \frac{(P_v s + I)(P_p s + I_p)}{s^4 + P_v s^3 + (I_v + P_v P_p)s^2 + (I_p P_v + I_v P_p s + I_v I_p}. \quad (4.11)$$

Nevertheless, locating poles via pole placement method is much more difficult in this case. In order to obtain same pole locations with Smith predictor case, one should choose poles within a small range. When poles are so large, system becomes unstable due to time delay since closed loop transfer functions including time delay are

$$T_{v-PI}(s) = \frac{(P_v s + I_v)e^{-T_d s}}{s^2 + (P_v s + I_v)e^{-T_d s}}, \quad (4.12)$$

$$T_{PI} = \frac{(P_v s + I)(P_p s + I_p)e^{-T_d s}}{s^4 + (P_v s + I_v)(s^2 + P_p s + I_p)e^{-T_d s}}. \quad (4.13)$$

Although, controllers are designed without concerning friction and delay, it is necessary to utilize an observer in order to track the input precisely. Otherwise, performance of the system degrades dramatically due to friction. Moreover, delay term must be included in observer structure in order to estimate friction correctly even if controllers are designed without regarding time delay. Simulation results are obtained for an ordinary PI-PI control tuned to $P_v = 25.212, I_v = 2.225, P_p = 0.842$ and $I_p = 0.276$ by MATLAB in Fig. 4.20 for $g(v) = 5 * \|v\|$.

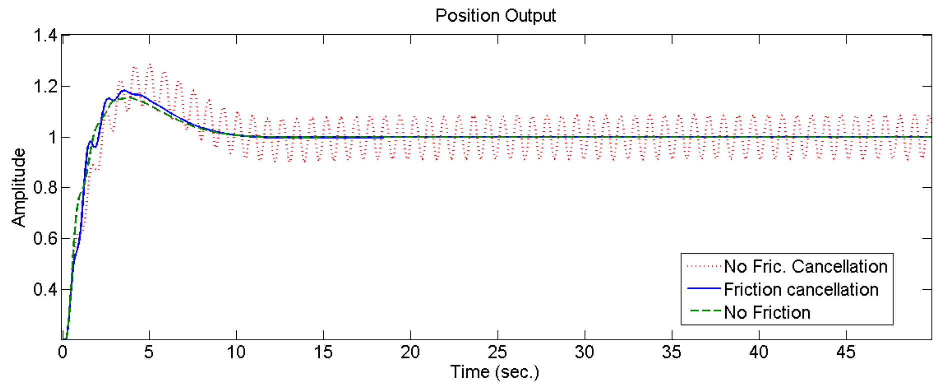

Figure 4.20: Step response of PI-PI control.

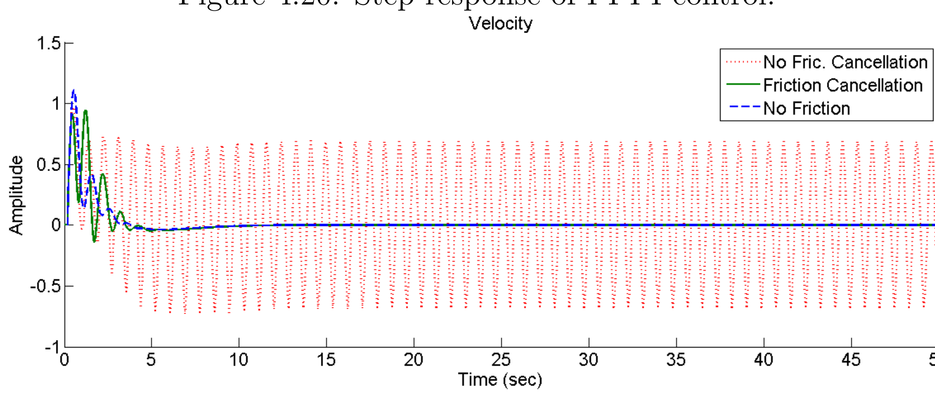

Figure 4.21: Velocity of PI-PI control.

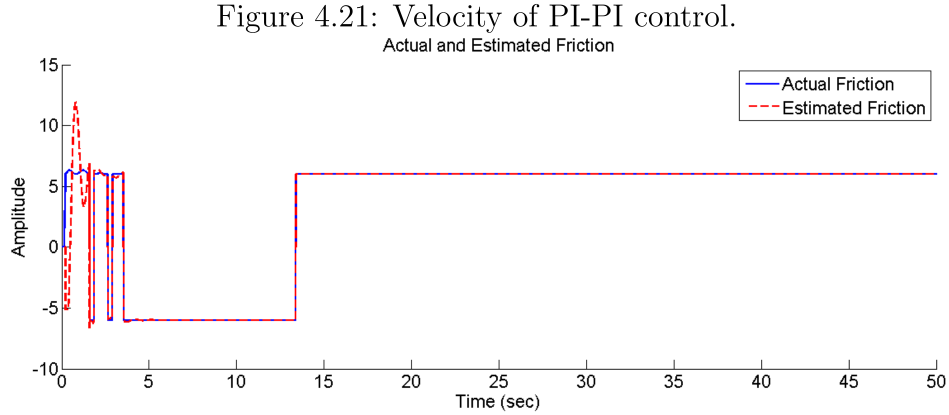

Figure 4.22: Friction estimation of PI-PI control.

4.5 Smith Predictor Based Position Control: Step input with step disturbance

In real mechanical systems, it is highly possible that some more disturbance signals other than friction having effects on the system performance. Hence, for a unit disturbance rejection and step input tracking controller given by (3.30) taking $K_p = 3, K_v = 2, k = 3$ and $\mu = 1.8$ is considered. With these design parameters, obtained controllers are

$$\begin{aligned} C_{0v}(s) &= \frac{4.8s+4}{s-0.8}, \\ C_{0p}(s) &= \frac{7.8s+9}{s-1.8}. \end{aligned}$$

Then, closed loop transfer functions for velocity and position control loops are

$$\begin{aligned} T_v(s) &= \frac{4.8s+4}{s^2+4s+4}e^{-0.2s}, \\ T(s) &= \frac{7.8s+9}{s^2+6s+9}T_v(s). \end{aligned}$$

When a relatively large value of k is combined with a large value μ, some "rings" occur at the output. This situation can be explained by an undesirable sensitivity to disturbance. Thus, considering step tracking case without disturbance, μ is slightly increased whereas k is decreased. From Fig. 4.23, it is clear that friction cancellation increases system performance under friction because without any compensation, both settling time increases and oscillation at the response occurs similar to 4.1. Compared to Fig. 4.1, overshoot at the transient time is observed due to the fact that controller structure is changed. In order to overcome this overshoot, a prefilter $H(s)$ can be utilized to cancel some of the higher dynamics in the closed loop transfer function as it is proposed in [7, 8]. Basically, a stable and strictly proper first order $H(s)$ with $H(0) = 1$ can be chosen for this purpose. Nonetheless, diminution of the cut-off frequency affects the rising time negatively.

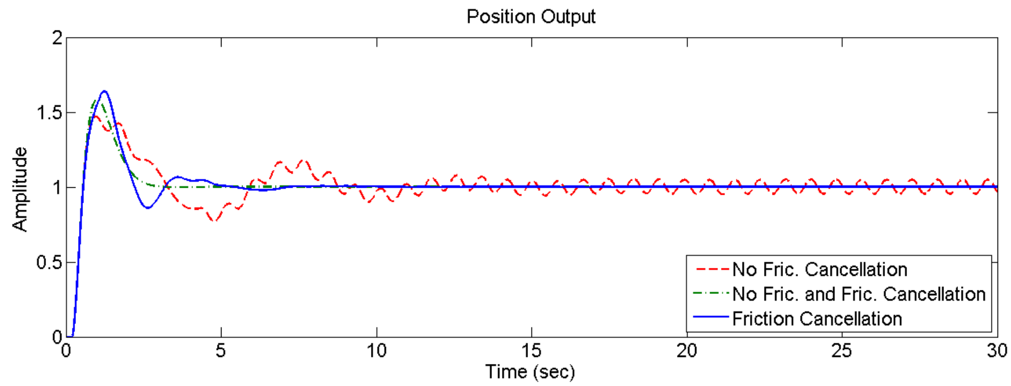

Figure 4.23: Step response of the system with unit disturbance

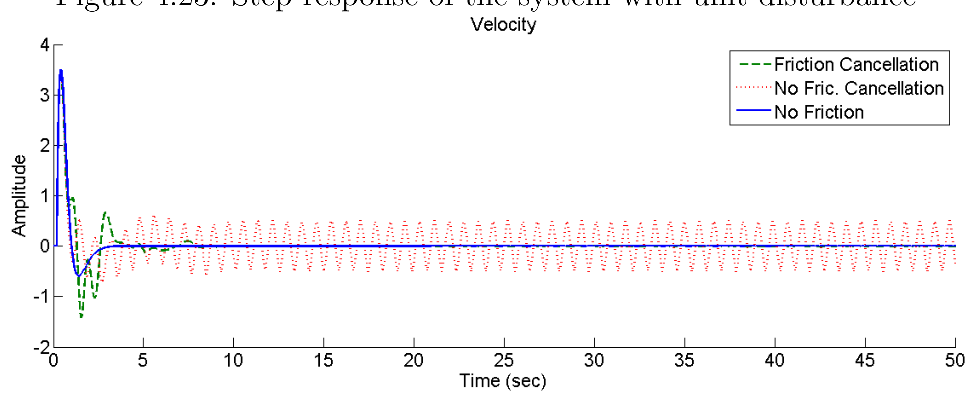

Figure 4.24: Velocity of the system for step input with step disturbance

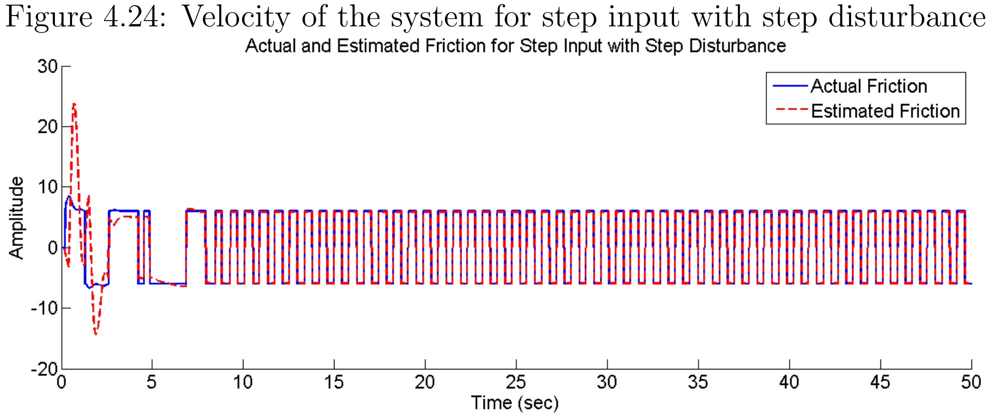

Figure 4.25: Estimated friction of the system for step input with step disturbance

4.6 Smith Predictor Based Position Control: Step input with step disturbance and general friction observer

Another approach might be changing estimation function in the observer structure. As it explained in 3.2, it is possible to use another function whose derivative is defined on first and third quadrant of the coordinate axis to estimate the friction instead of $k|v|^{\mu}$ in (3.9). Hence, performance of the same system is tested when estimation function in (3.5) is $g(v) = klog\,(cosh(v))$. For unit disturbance and unit step tracking case, performance of two functions are plotted in Fig. 4.26 and Fig. 4.29.

As it can be seen in Fig. 4.26, function $g(v) = 1 * |v|^{1.8}$ provides better performance compared to $g(v) = log\,(cosh(v))$. However, when k=5, as in Fig. 4.29 some rings occur for and $g(v) = 5log\,(cosh(v))$ provides a better performance. In other words, by changing estimation function $g(v)$, one can still have a well enough performance. Nonetheless, changing function may require changing observer gain as well for a smoother output. In fact, when magnitude of disturbance is getting larger for a fixed k and μ, $ktanh$ suppresses disturbance better and provides a better performance compared to classical estimation function.

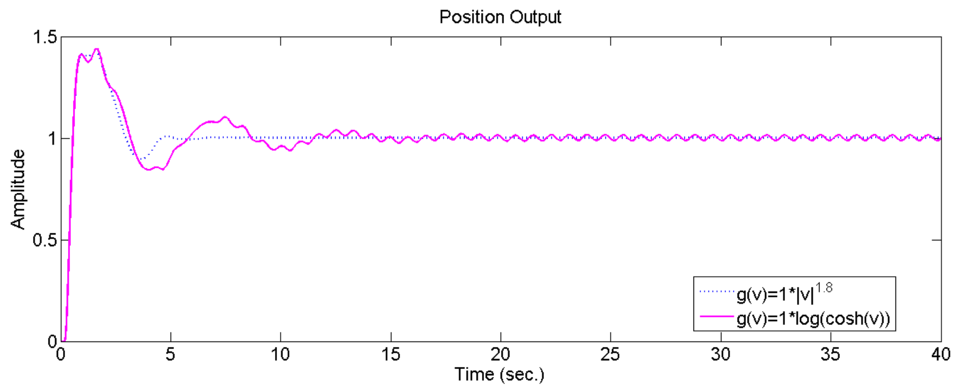

Figure 4.26: Comparisons of step responses of two estimation functions with unit disturbance when k=1

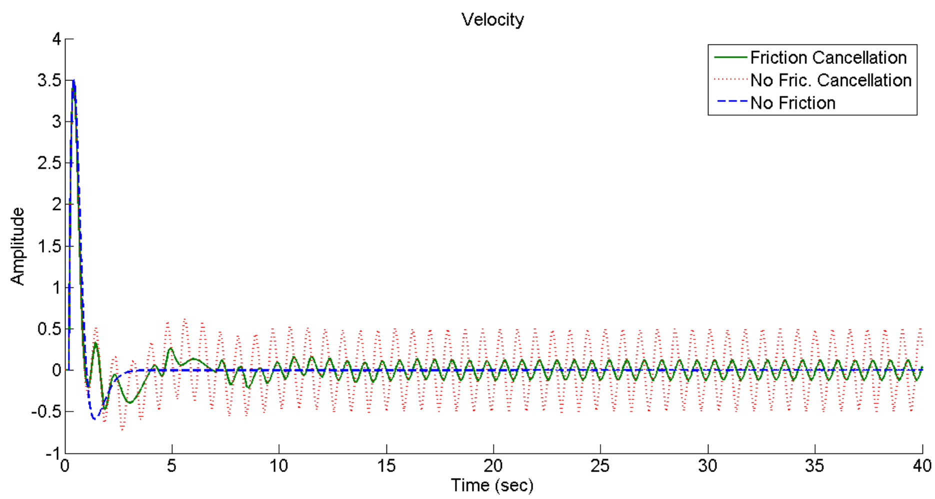

Figure 4.27: Velocity of $g(v) = log\,(cosh(v))$for step tracking with unit step disturbance

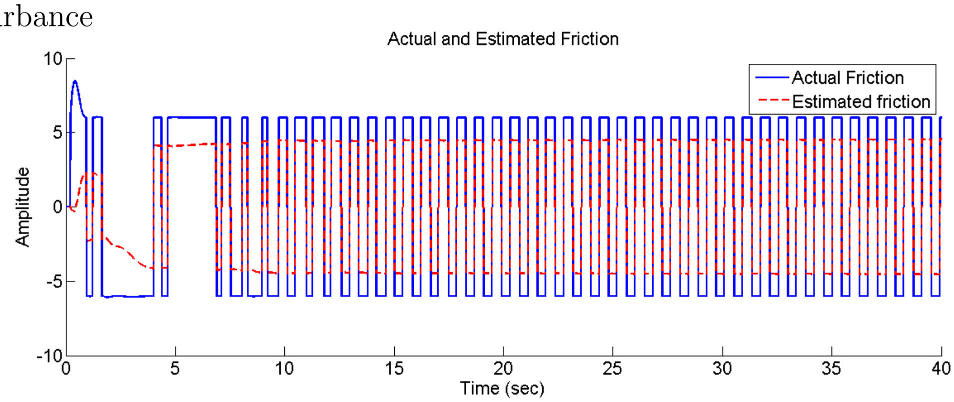

Figure 4.28: Friction estimation of $g(v) = log\,(cosh(v))$for step tracking with unit step disturbance

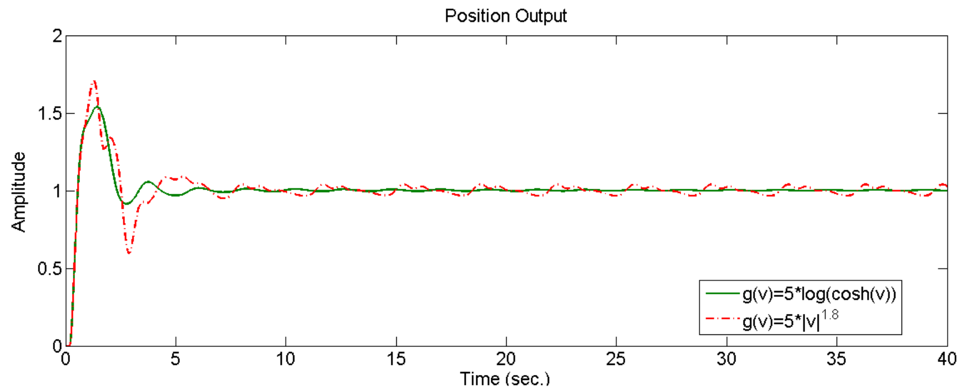

Figure 4.29: Comparisons of step responses of two estimation functions with unit disturbance when k=5

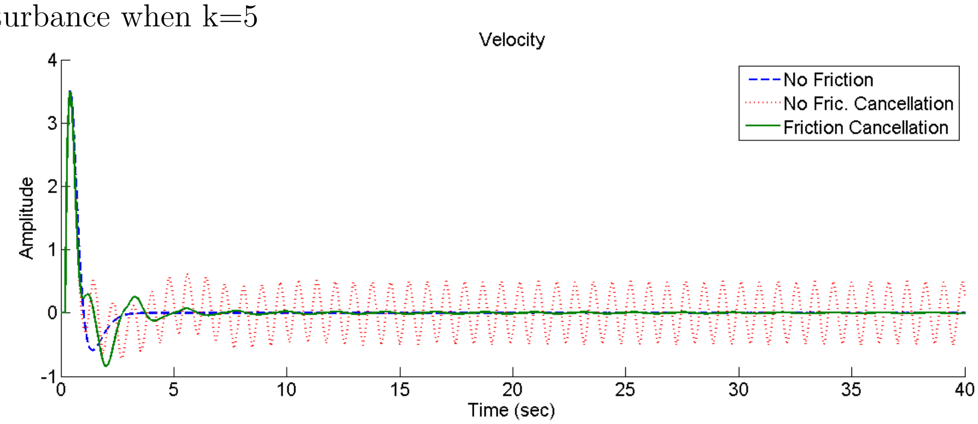

Figure 4.30: Velocity of $g(v) = 5log\,(cosh(v))$for step tracking with unit step disturbance

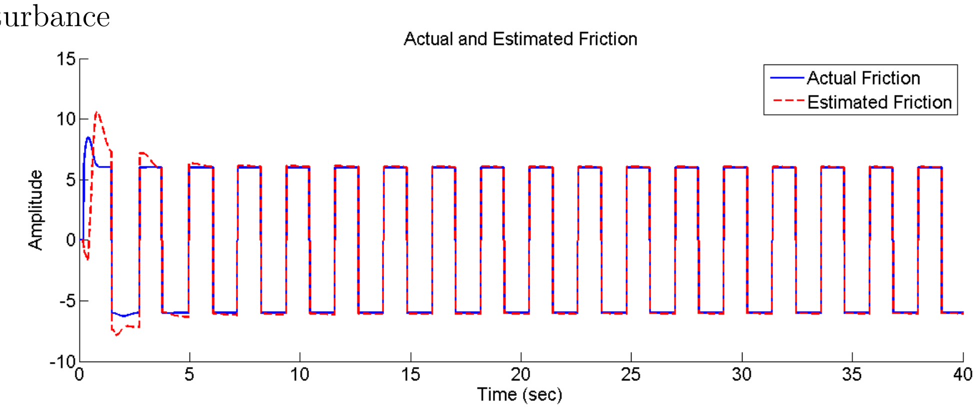

Figure 4.31: Estimated Friction of $g(v) = 5log\,(cosh(v))$for step tracking with unit step disturbance

4.7 Smith Predictor Based Position Control: Step input with triangular disturbance

Likewise, a triangular disturbance signal, whose amplitude is 1 and period is 20 sec., is applied to the system. In order to track step input in this case, controllers are designed using (3.33). For velocity loop $K_v = 1$ and $\alpha_4 = 0.5$ are selected whereas $K_p = 3.5$ and $\alpha_4 = 1.5$ are selected for position loop. Then, obtained controllers are

$$\begin{aligned} C_{0v}(s) &= \frac{2.91s^2 + 2.1s + 0.5}{s^2 - 0.41s - 0.1}, \\ C_{0p}(s) &= \frac{13.42s^2 + 26.43s + 18.38}{s^2 - 4.918s - 3.675}, \end{aligned}$$

and corresponding closed loop transfer functions are

$$\begin{aligned} T_v(s) &= \frac{2.91s^2 + 2.1s + 0.5}{s^3 + 2.5s^2 + 2s + 0.5} e^{-0.2s}, \\ T(s) &= \frac{13.42s^2 + 26.43s + 18.38}{s^3 + 8.5s^2 + 22.75s + 18.375} T_v(s). \end{aligned}$$

For $\mu = 1.9$ and $k = 1.5$, obtained results are plotted in Fig. 4.32.

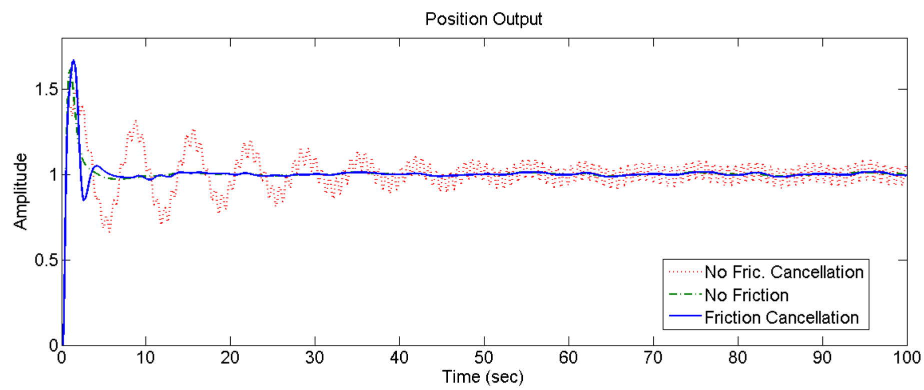

Figure 4.32: Step response of the system when disturbance is a triangular wave with the period of 20 sec and unit amplitude.

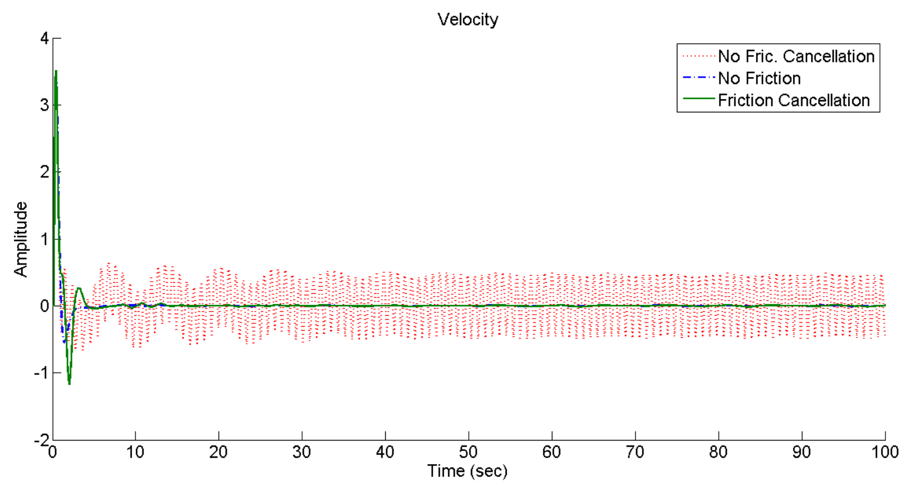

Figure 4.33: Velocity of the system when disturbance is a triangular wave with the period of 20 sec and unit amplitude.

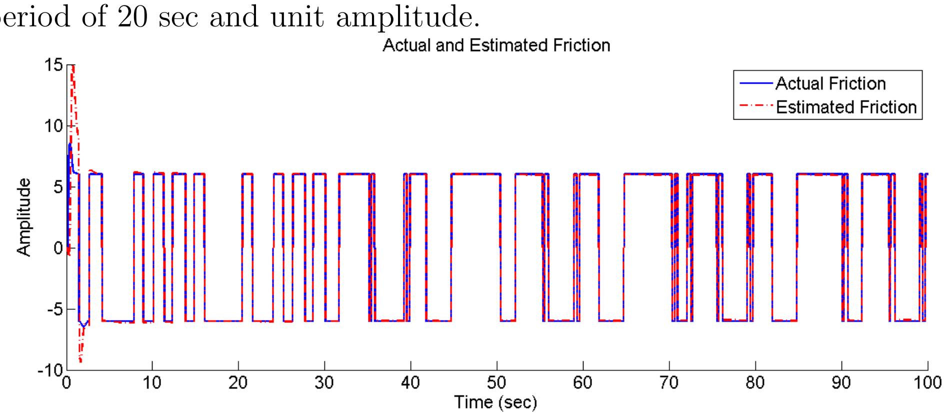

Figure 4.34: Estimeted friction of the system when disturbance is a triangular wave with the period of 20 sec and unit amplitude.

4.8 Smith Predictor Based Position Control: Step input with sinusoidal disturbance

Lastly, performance of the controller designed to track step input with suppressing sinusoidal disturbance is analyzed with $d = sin(1.5t)$. Using (3.36) controller for the velocity loop is designed for $K_v = 1, \alpha_4 = 2$ and $\alpha_5 = 3$. Similarly, another controller is designed in order to control position loop when $K_p = 4, \alpha_4 = 3$ and $\alpha_5 = 5$. Utilized controllers are

$$\begin{aligned} C_{0v}(s) &= \frac{5.117s^3 + 5.356s^2 + 8.6s + 3}{s^3 - 1.117s^2 + 2.644s - 0.6}, \\ C_{0p}(s) &= \frac{20.78s^3 + 54.89s^2 + 104s + 80}{s^3 - 9.78s^2 - 9.89s - 16}. \end{aligned}$$

Hence, closed loop transfer functions are

$$\begin{aligned} T_v(s) &= \frac{5.117s^3 + 5.356s^2 + 8.6s + 3}{s^4 + 4s^3 + 8s^2 + 8s + 3} e^{-0.2s}, \\ T(s) &= \frac{20.78s^3 + 54.89s^2 + 104s + 80}{s^4 + 11s^3 + 45s^2 + 88s + 80} T_v(s). \end{aligned}$$

For $g(v) = klog\left(cosh(v)\right)$ and $k = 5$, obtained results are plotted in Fig. 4.35. From the figure, it is clear that without any compensation, sinusoidal disturbance induces larger oscillations and steady state error at the output compared to constant disturbance. However, with a cancellation it is possible to improve the performance pretty much.

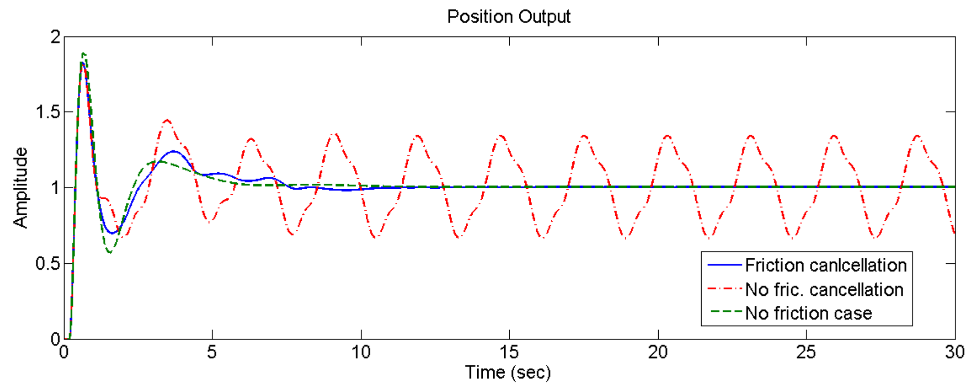

Figure 4.35: Step response of the system when disturbance is $sin(1.5t)$

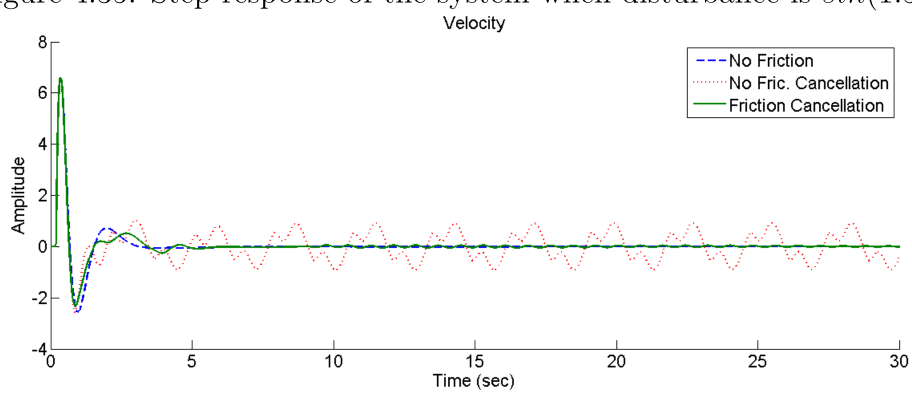

Figure 4.36: Velocity of the system for step tracking when disturbance is $sin(1.5t)$

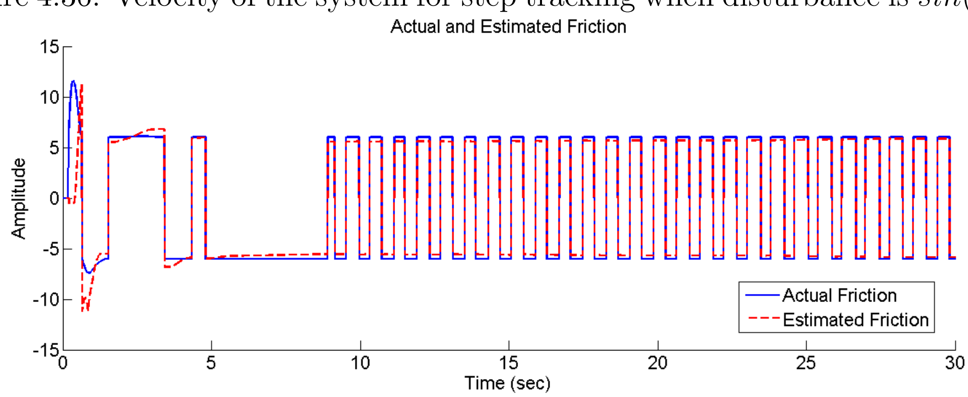

Figure 4.37: Estimated friction of the system for step tracking when disturbance is $sin(1.5t)$

4.9 Effect of Changing Friction Coefficients

For different reasons such as temperature or lubricity, friction coefficients can be time varying. Hence, observer performance under a friction force whose coefficients are slowly time-varying is also considered. Clearly, parameters of friction can change in time differently in different systems. Hence, in order to illustrate the response for time varying parameters in (4.1) is modified to $F = (6 + sin(0.05t))sgn(v)$. Step tracking without disturbance case is considered with $K_p = 4, K_v = 2, k = 4$ and $\mu = 1$. In Fig. 4.38, it is seen that adaptive observer can estimate time varying parameters as well.

4.10 Fixed Compensation versus Adaptive Compensation

Performances of fixed and adaptive friction compensation is compared. As it is stated, friction parameters change with time because of some physical reasons. Moreover, during identification process of friction parameters, some measurement or modeling errors can occur. Therefore, in Fig. 4.39, it is assumed that all parameters are estimated with a percentage error when controller designed to track step input without any disturbance with $K_v = 1$ and $K_p = 5$ is used. In Fig. 4.39, particularly cancellation with a positive identification error causes overshoot and steady state error. Furthermore, both negative and positive identification error lead to a larger settling time. Thus, simulation result verifies that adaptive compensation provides a better performance especially for small K_v.

4.11 Smith Predictor Based Velocity Control: Step input without disturbance

In velocity control case, velocity does not change its sign. Thus, Coulomb friction behaves like a constant disturbance. Hence, one aims only velocity control, he simply utilize a Smith predictor based controller rejecting constant disturbance without requiring any observer. Choosing $K_v = 1$ in (3.30), obtained results plotted in Fig. 4.41

However, note that this a special case for velocity control, as it is showed in above simulations, for position control, velocity may change its sign and observer may improve systems performance.

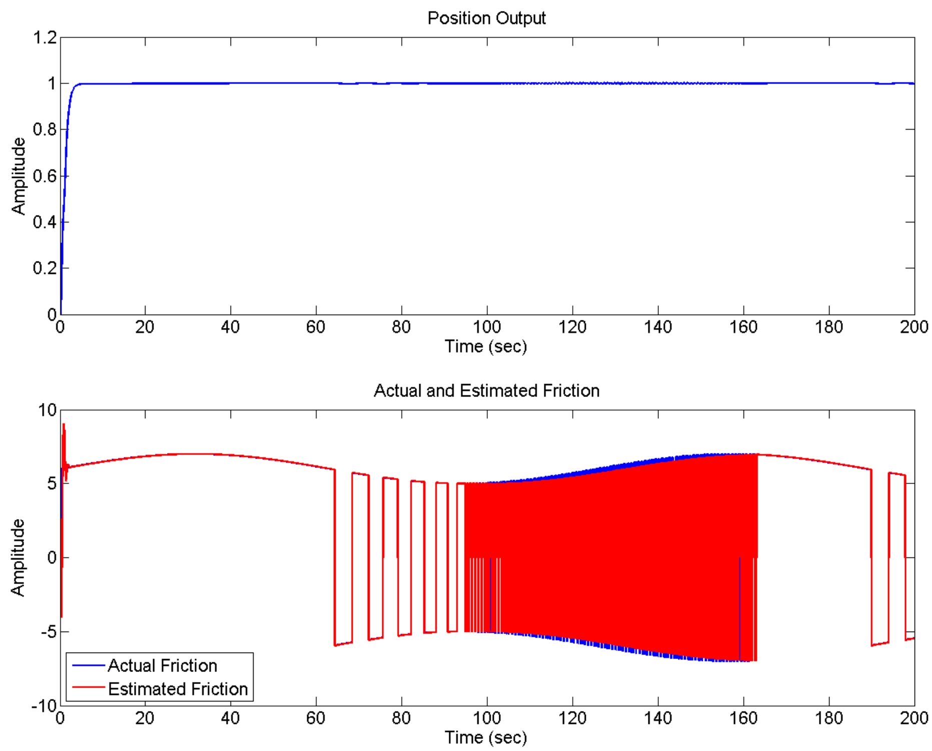

Figure 4.38: Actual and estimated friction with time varying parameters

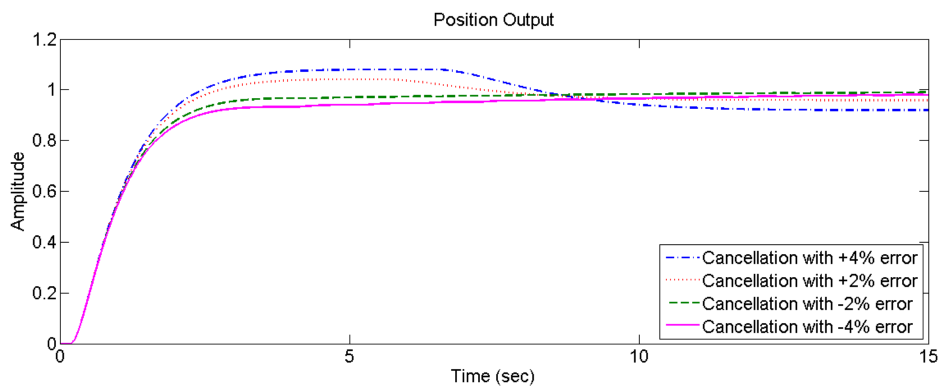

Figure 4.39: Position output when fixed friction cancellation with an error is applied

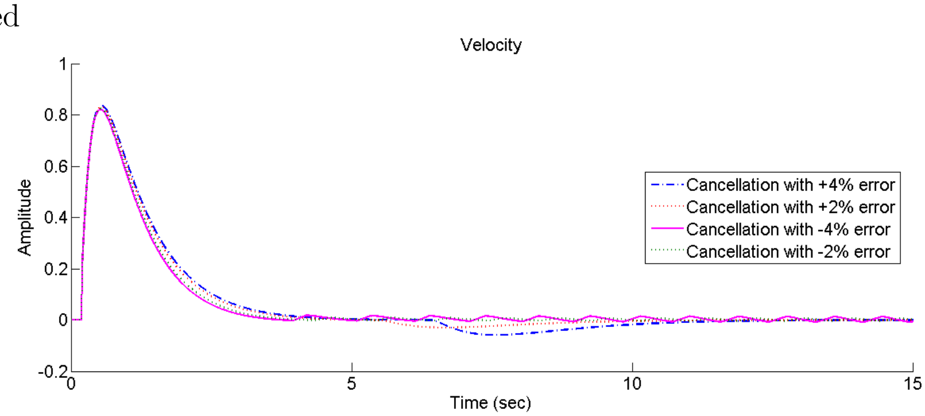

Figure 4.40: Velocity plot when fixed friction cancellation with an error is applied

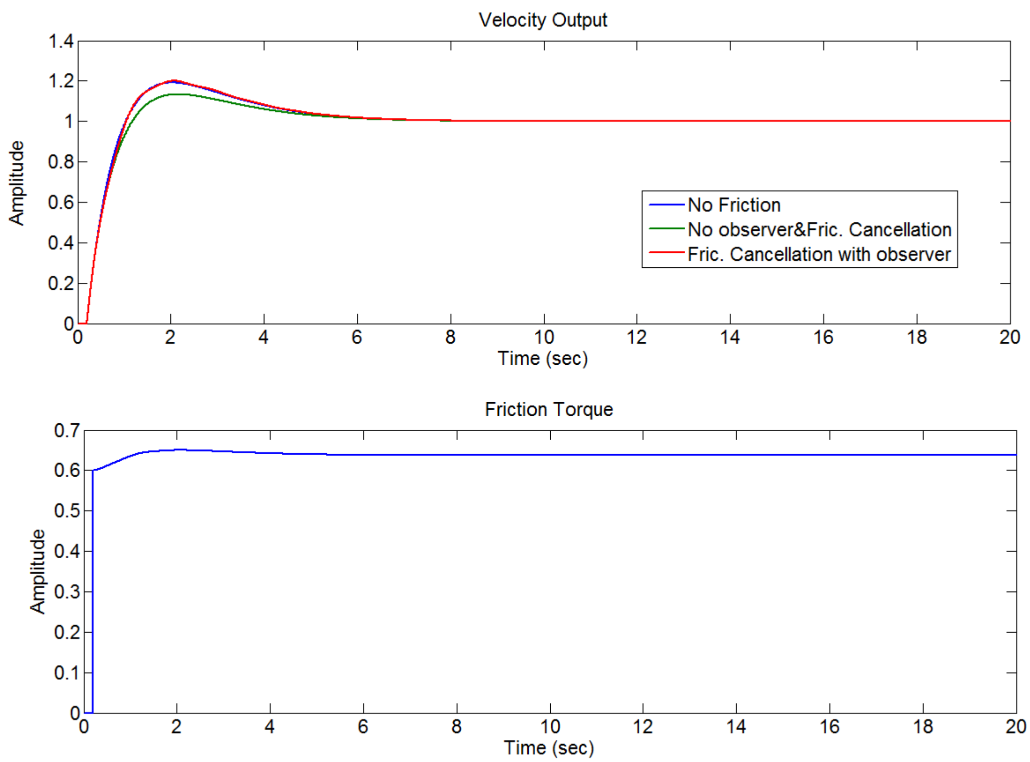

Figure 4.41: Velocity output and Friction Torque for velocity control

Chapter 5

CONCLUSION

In this thesis, in order to handle mechanical systems with time delay, Smith predictor based controllers, combined with an adaptive Coulomb observer, are employed. Based on interpolation conditions imposed by constant, ramp or sinusoidal disturbance rejection, internal controllers are designed. The order of resulting controllers changes according to robustness and performance demands. In this hierarchical structure, one should first design a controller for the velocity loops. Afterwards, it is easy to extend it to the position loop. Free parameters of these designs determine the closed loop system poles.

Compared to ordinary PI and P controllers, proposed Simith predictor based controllers make design process simpler. Using pole placement method freely, proposed design approach provides stable closed loop feedback systems. Furthermore, the main advantage of Smith predictor based controller is that it makes the process as if the process is delay free. In some of controller designs, overshoots are observed. Although it is not used in this work, a first order low pass filter might be used to filter reference signal. Therefore, tracking response is improved by eliminating overshoot without affecting disturbance attenuation.

As expected the mere inclusion of the delay in the modified observer works well when time delay exists in system dynamics. Simulations demonstrate that with a proper observer parameter selection it is possible to amend performance criterion such as steady state error or settling time. What is more, generalization of the original observer shows that, it is possible to change estimation function to obtain better performances especially when magnitude of disturbance is high. Besides, modified observer can work with other controller structures and capable of estimating friction parameters varying with time. Due to the fact that friction parameters can change because of some physical reasons, using such an observer to compensate friction is a plus. What is more, although it is designed to estimate Coulomb friction, results confirm that observer can work well enough for friction models rather than Coulomb friction. Thus, the compensator is predicated on the ability to the control system to generate a force exactly opposite to the estimated friction.

As a future work, other observer structures in the literature might be modified in order to capture more sophisticated friction models. In other words, more than a single friction parameter or a dynamical model might be estimated by improving observer structure. Furthermore, friction model should be improved especially to capture the behavior of friction at low velocities in a better way since model used in MATLAB simulations is limited at low velocities.

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Appendix A

Controller Parametrization Method

Controller parametrization method is an useful algorithm proposed by [31] in order to find all controllers $C(s)$ stabilizing plant $P(s)$ in a feedback system. Design procedure is as follows:

- Given transfer function of a plant can be written in terms of coprime rational and stable transfer functions, $N_p(s)$ and $D_p(s)$ such that

$$P(s) = \frac{N_p(s)}{D_p(s)}. \tag{A.1}$$

- These $N_p(s)$ and $D_p(s)$ must satisfy fallowing equation (Bezout equation).

$$X(s)N_p(s) + Y(s)D_p(s) = 1, \tag{A.2}$$

  where $X(s)$ and $Y(s)$ are two other stable functions. Suppose that $D_p(s)$ has n zeros at $s = z_1, z_2, ..., z_n$ then, $X(z_1) = \frac{1}{N_p(z_1)}, X(z_2) = \frac{1}{N_p(z_2)}, ..., X(z_n) = \frac{1}{N_p(z_n)}$.

- After finding $X(s)$ satisfying these conditions, $Y(s)$ can be found from

$$Y(s) = \frac{1 - N_p(s)X(s)}{D_p(s)}. \tag{A.3}$$

- Then, the set of all controllers are given by

$$C(s) = \frac{X(s) + D_p(s)Q(s)}{Y(s) - N_p(s)Q(s)}. \tag{A.4}$$

  where $Q(s)$ is a proper and stable transfer function such that $Q(s) \neq Y(s)N_p(s)^{-1}$.

In (A.4), depending on $Q(s)$, it is possible to obtain a controller satisfying different robustness and performance objectives. For instance, if one only concerns stability of feedback system without any additional objective, then $Q(s)$ is simply chosen as 0. Otherwise, minimum degree of $Q(s)$ satisfying interpolation conditions requirement is found such that

min degree of $Q(s)$= *Number of interpolations-1.*

After controller parametrization prosecure is applied, transfer function from input to output will be

$$T_{rp}(s) = N_p(s)\left(X(s) + D_p(s)Q(s)\right). \tag{A.5}$$

and transfer function from disturbance to output will be

$$T_{dp}(s) = N_p(s)\left(Y(s) - N_p(s)Q(s)\right). \tag{A.6}$$

Therefore, using final value theorem one can find the desired $Q(s)$ satisfying required conditions with zero steady state error such that

$$e_{ss} = \lim_{s \to 0}\left[\left(1 - T_{rp}\right)R_p(s) - T_{dp}(s)D(s)\right]. \tag{A.7}$$