RISK-AVERSE OPTIMIZATION FOR MANAGING INVENTORY IN CLOSED-LOOP SUPPLY CHAINS

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Melis Beren ÖZER

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We certify that we have read this thesis and that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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ABSTRACT

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This thesis examines a closed-loop multi-stage inventory problem with remanufacturing option. A random fraction of used products is returned by consumers to the manufacturer after a certain number of stages. But the manufacturer may or may not collect any returned item. Demand can be satisfied through two channels: manufacturing new products and remanufacturing used products (cores). A control policy specifies the numbers of cores to collect and remanufacture, and the number of new products to manufacture, at each stage. The state space consists of the serviceable product and core inventory levels, and the amounts of future returns. We study this problem from the perspectives of risk-neutral and risk-averse decision-makers, in both cases of lost sales and backordering. We incorporate the dynamic coherent risk measures into our risk-averse problem formulation. We establish that it is always optimal to prefer remanufacturing to manufacturing under a mild condition. Numerical results indicate that a state-dependent threshold policy may be optimal for the core inventory. However, such a policy need not be optimal for the serviceable product inventory. We also conduct numerical experiments to evaluate the performance of several heuristics that are computationally less demanding than the optimal policy: a certainty equivalent controller (CEC), a myopic policy (MP), a no-recovery policy (NRP), a full-collection policy (FCP), and a fixed threshold policy (FTP). CEC, MP, and NRP have a distinct computational advantage over FCP and FTP, whereas FCP and FTP significantly outperform all the other heuristics with respect to objective value, in our numerical experiments.

Keywords: closed-loop supply chains, remanufacturing, inventory, risk-aversion, random returns.

ÖZET

KAPALI DEVRE TEDARİK ZİNCİRLERİNDE RİSKTEN KAÇINAN ENVANTER YÖNETİMİ OPTİMİZASYONU

Melis Beren ÖZER
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Bu tezde yeniden imalat opsiyonu içeren çok periyotlu kapalı devre envanter problemi incelenmiştir. Kullanılmış ürünler rassal oranla üreticiye belirli süre sonra geri dönmektedir. Talep yeni ürün üretimi ve kullanılmış ürünün yeniden imalatı ile karşılanır. Her periyotta kullanılmış ürünlerin toplanma ve yeniden imalat miktarları ve yeni ürün üretim miktarı belirlenmektedir. Durum uzayı satılacak ürün ve kullanılmış ürün envanterleri ile, gelecekte geri dönecek ürün miktarlarını içermektedir. Problem, riske duyarsız ve riskten kaçınan karar vericiler için, kayıp satış ve ardısmarlama durumlarında çalışılmıştır. Riskten kaçınan problemde tutarlı dinamik risk ölçütleri kullanılmış, üretim yerine yeniden imalata öncelik vermenin daima daha kârlı olduğu gösterilmiştir. Sayısal sonuçlar kullanılmış ürün envanteri için duruma göre değişen eşik değeri politikasının en iyi politika olabileceğini göstermiştir. Ancak böyle bir politika satılacak ürün envanteri için en iyi politika olmak zorunda değildir. Ayrıca, en iyi politikadan daha kısa sürede sonuç verebilen çeşitli sezgisel politikaların performansları sayısal analizlerle değerlendirilmiştir. Bu politikalar: kesinlik denkliği kontrolörü, uzakgörmez politika, geri kazanım yapmayan politika, tamamen toplama politikası ve sabit eşik değeri politikasıdır. Kesinlik denkliği kontrolörü, uzakgörmez politika ve geri kazanım yapmayan politikanın, tamamen toplama ve sabit eşik değeri politikalarından belirgin bir çözüm süresi avantajı olduğu gözlemlenmiştir. Tamamen toplama ve sabit eşik değeri politikalarının ise diğer sezgisel politikalardan objektif değer yönünden önemli ölçüde avantajlılığı gözlemlenmiştir.

Anahtar sözcükler: kapalı devre tedarik zincirleri, yeniden imalat, envanter, riskten kaçınma, rassal geri dönüş.

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To my family . . .

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Chapter 1

Introduction

Waste management is one of the top ten environmental issues facing humanity (Esty and Winston 2009). Most products end up in landfills after they reach the end of their life cycles. In order to mitigate the negative impact of those products on the environment, sustainability has gained an increasing attention over the last years. Closed-loop supply chains, on the other hand, have become a key aspect of environmental sustainability. By extending the scope of their supply chains to include used-product collection and recovery, today′s manufacturing firms aim not only to reduce their production costs, but also to meet stringent environmental regulations by reducing their waste of end-of-use products (Kiesmüller and Minner 2003).

Closed-loop supply chains involve the return of a used product back to the manufacturer as well as the delivery of a product to the final user, whereas traditional supply chains ignore the used product returns. The recovery of used products is appealing to manufacturers in various industries for numerous reasons: First, the manufacturer may greatly reduce its waste and operational costs by collecting and recovering its used products. Second, environmental legislations may mandate the used product recovery. Third, the manufacturer can extend its product line by offering "cheaper branded" products. Last, the recovered products may attract "green-sensitive" customers (Souza 2012).

Once a product is returned by its last user to the manufacturer, it can be reused, recovered, or disposed (Thierry et al. 1995). The well-known recovery options include incineration, recycling, parts harvesting, resale, and remanufacturing: Incineration refers to the process of igniting a product when the other options of recovery are not possible. Although the purpose is to disperse materials into the atmosphere in a clean way, generated heat can be used to produce electric power in some cases. Recycling refers to the process of converting waste materials for manufacturing products of different functionality. It is preferred when the returns have little economic value due to obsolescence. Parts harvesting refers to the recovery of only specific parts of a returned product. Resale happens when there exists a secondary market for the used product. Finally, remanufacturing refers to restoring a product to its originally manufactured quality and is often considered as the most profitable disposition decision (Souza 2012). This thesis focuses on an inventory system with remanufacturing option; see Figure 1.1.

The size of remanufacturing industry in the United States is estimated to be at least $53 billion, employing over 480,000 people (Souza 2012). Examples of remanufactured products include mobile phone parts, domestic appliances, toner cartridges, single-use cameras, automotive parts, and IT equipment. In addition, remanufacturing is a common practice in fashion, aerospace and defense industries (Dekker et al. 2004). Remanufacturing toner cartridges is a $3 billion industry and Xerox′s remanufacturing program saved nearly $200 million in material and part costs in less than five years (Ginsburg 2001). The annual sales volume in automotive remanufacturing industry, on the other hand, is reported to be $2.5 billion (Souza 2012).

Although the used product recovery is often very beneficial, it is quite difficult to effectively manage inventory in a closed-loop supply chain. This is because the quantity, timing, and quality of returns are highly variable, and the forward and reverse material flows of the supply chain impact each other. To handle such complexity, many authors assume that infinitely many products exist in the market so that the reverse material flow is not bounded by (and is independent from) the forward material flow; see, for instance, Simpson (1978), Buchanan and Abad (1998), and Zhou et al. (2011). But the amount of returns is in

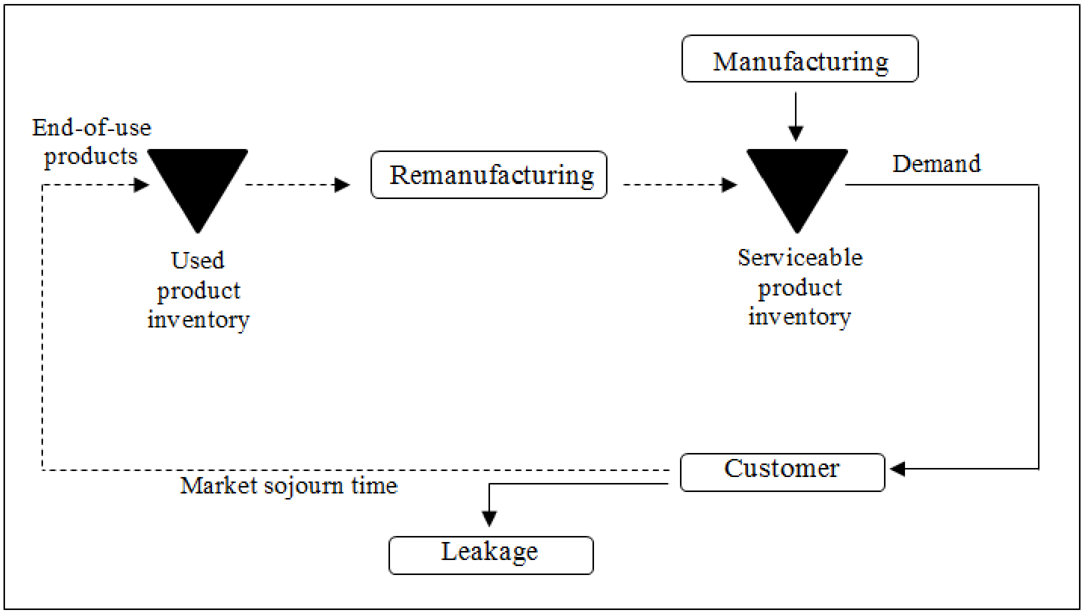

Figure 1.1: Illustration of an inventory system with remanufacturing option

general constrained by the total amount of past sales that is finite, especially if the product has a finite life-cycle (Geyer et al. 2007). Another key problem with much of the literature is that all returned products are collected; see, for instance, Inderfurth (1997) and Kiesmüller and Minner (2003). But a huge number of collected returns may lead to excess inventory and high disposal cost. Thus the manufacturer may want to collect only a certain amount of returns that will minimize its inventory costs. To our knowledge, the literature dealing with closed-loop inventory systems has not yet developed a comprehensive modelling framework that explicitly captures these two aspects of the problem. This thesis is the first attempt to fill this gap.

The literature on *closed-loop* inventory systems has also neglected to incorporate the concept of "risk" into decision-making. However, many decision-makers are willing to trade-off higher expected cost for protection against possible demand losses, especially in high-margin markets (Chen et al. 2007, and Schweitzer and Cachon 2000). In this thesis, we study not only the risk-neutral decision-maker′s problem but also the risk-averse decision-maker′s problem, which we model by employing the modern theory of risk. There are several different ways to incorporate risk into decision-making, such as expected utility theory and mean-risk approach. Although many authors in the literature dealing with *traditional* inventory systems use utility functions in their objectives as a measure of risk, it is often problematic to elicit the utility function of the decision-maker in practice. For this reason, we consider the law-invariant coherent risk measures in our study. Specifically, we take "mean-semi-deviation" as the risk measure in our risk-averse problem.

In this thesis we consider a single-product, closed-loop, multi-stage inventory system. A random fraction of the sold products in any stage becomes available for collection by the manufacturer after a certain number of stages, i.e., a market sojourn time. We label this fraction as return rate. A unit collection cost is incurred if the manufacturer collects a used product. But there is no cost if the manufacturer does not collect any used product. Demand and return rate are independent from each other and across time. The manufacturer satisfies the demand from the *serviceable* product inventory. Both the newly-manufactured and remanufactured products can be added to this inventory immediately. A control policy specifies how many new products should be manufactured, how many used products should be collected, and how many collected products should be remanufactured in each state and time period.

We consider two different objectives of the manufacturer: The risk-neutral objective is to minimize the expected total cost over a finite planning horizon (Chapter 3.1). The risk-averse objective is to minimize the weighted sum of the mean total cost and the expected excess from the mean total cost over a finite planning horizon (Chapter 3.2). We analyze the problem in both cases of backlogging and lost-sales. For both objectives and both cases, we are able to prove that remanufacturing should always be preferred to manufacturing at optimality if the serviceable product inventory is to be increased.

We formulate dynamic programming (DP) algorithms for both risk-neutral and risk-averse problems. The state space consists of the inventory levels of both serviceable and collected products, as well as the numbers of used products that will be returned over a certain number of stages in the future (a market sojourn time). Solving these DP algorithms to optimality is extremely problematic since both state and action spaces are unmanageably large. In order to reduce the computational burden of our DP algorithms, we develop several computationally-efficient heuristics: the Certainty Equivalent Controller (CEC), the Myopic Policy (MP), the No-Recovery Policy (NRP), the Full-Collection Policy (FCP), and the Fixed Threshold Policy (FTP).

- CEC finds the optimal policy in our DP algorithms by fixing the uncertain quantities at their "typical" values. Specifically, we set demand and return rate equal to their expected values, thereby eliminate randomness from our inventory system. The optimal policy within this heuristic class can be obtained from our DP algorithms in the absence of random disturbances.

- MP is a commonly used approach in the inventory literature. For a given state and stage, MP chooses the action that minimizes the expected total cost in that stage by ignoring the impact of future stages on the expected total cost. Because MP disregards the state evolution in future stages, it has the potential to greatly reduce the solution time.

- NRP never collects used products. The optimal policy within this heuristic class can be obtained from our DP algorithms by eliminating the collection and remanufacturing decisions from the action space. Note that the value of product recovery in our closed-loop inventory system can be measured by the optimality gap of NRP.

- FCP collects all *available* used products in the market. The optimal policy within this heuristic class can be obtained from our DP algorithms by setting the collection amount in each stage equal to the number of available used products in that stage.

- We describe FTP as follows: The used products (available in the market) are collected to bring the collected product inventory as close to a fixed target level as possible at each stage, if it is below it. And the collected products (available in inventory) are remanufactured to bring the serviceable product inventory up to a fixed target level in each stage, if it is below it. New products are manufactured only if remanufacturing is inadequate to bring the serviceable product inventory up to the target level. The optimal policy within this heuristic class can be obtained by running a DP algorithm under each possible pair of target levels and choosing the pair that yields the least cost in the first period.

We then conduct numerical experiments to provide insights into the optimal policy structure. Our numerical results suggest that a *state-dependent* threshold policy may be optimal for the core inventory in both cases of backlogging and lost-sales. However, we could not prove *discrete-convexity* of our optimal cost function, which is a standard method used in the inventory literature to establish the optimality of threshold policies. (In the case of backlogging we have found counter examples showing that discrete-convexity need not hold for our optimal cost function in general.) Hence whether state-dependent threshold policies are analytically optimal for the core inventory in our closed-loop inventory systems remains an open research problem.

We also conduct numerical experiments to examine the performance of each of our heuristic policies with respect to objective value and solution time. Numerical results show that although CEC has a computational advantage over all the other heuristics, it has the worst performance with respect to objective value. Unlike previous work showing that MP might be preferable in many closed-loop supply chains (Cohen 1980), MP performs worse than NRP, FCP, and FTP with respect to objective value. Although NRP performs better than CEC and MP in terms of objective value, it performs substantially worse than FCP and FTP, indicating a significant loss when products are not recovered. FCP and FTP surpasses the other heuristics and display similar performances with respect to the objective value. Last, FTP has a distinct computational advantage over FCP.

We contribute to the literature in several important ways: First, to our knowledge, our study is the first attempt to incorporate the coherent dynamic risk measures into a closed-loop inventory management problem. Second, we take the collection amount as a decision variable, as opposed to previous research collecting all cores and taking the disposal quantity as a decision variable. Third, we include all the information regarding future return quantities in our state space. We use this information to limit future collection quantities. Last, our numerical experiments reveal the practicality of fixed threshold policies for our closed-loop inventory problem. Our numerical results also lead to the conjecture that state dependent threshold policies may be optimal for the core inventory in our closed-loop inventory system.

The rest of the thesis is organized as follows. *Chapter 2* reviews the literature for the risk-neutral inventory problems with remanufacturing option and the risk-averse inventory problems. *Chapter 3* describes the inventory model under two different objectives (risk-neutral vs. risk-averse) in the cases of backlogging and lost-sales. *Chapter 4* describes the heuristics and their formulations. *Chapter 5* presents and interprets numerical results for the optimal policy structure and the heuristics. *Chapter 6* offers a summary and possible future research directions. Proofs of all analytical results are contained in the appendix.

Chapter 2

Literature Review

In this chapter, we review the literature dealing with the inventory control problem in closed-loop supply chains. To our knowledge, previous work has only focused on the risk-neutral decision maker′s problem (Chapter 2.1). The risk-sensitive decision maker′s problem has been studied in the literature only for traditional supply chains (Chapter 2.2).

2.1 The Risk-Neutral Problem

Many authors in the field of closed-loop supply chains assume that remanufactured products are the perfect substitutes of newly manufactured products. Geyer et al. (2007) investigate the profitability of remanufacturing under the following supply-loop constraints: collection capacity, limited component durability, and finite product life cycle. The fraction of used products that can be collected (i.e., the collection rate) is constant (which may be less than one). However, the collected products may have variable conditions and every collected product may not be remanufactured. The fraction of collected products that can be remanufactured and remarketed (i.e., the remanufacturing yield) is again constant (which may be less than one). They formulate the component durability constraint as a function of the maximum number of times the component can be used in production of the same kind of product, which limits the remanufacturing yield. They also model the market demand over the product life cycle as following an isosceles trapezoid, and relate the fraction of remarketable collected items to the remanufacturing yield. For the problem with finite product life cycle, Geyer et al. (2007) assume that there is a fixed time interval between the sale of a product and its resale after being collected and remanufactured (i.e., a fixed market sojourn time). They establish upper bounds for the average cost savings from remanufacturing in the cases of limited component durability and finite product life cycle. Unlike Geyer et al. (2007), we study the *inventory control* problem in a closed-loop supply chain with *random* returns. Furthermore, we take the numbers of used products to collect and collected products to remanufacture as decision variables.

Whisler et al. (1967) consider an inventory system in which products are rented to customers and returned after a stochastic market sojourn time. Any demand that is not satisfied immediately is lost. They seek an optimal policy that specifies the number of equipments to rent and the number of equipments to dispose over both finite horizon and infinite horizon. They establish the optimality of a *base-stock* policy with two critical levels under the assumption of linear costs: If the inventory level of equipments on hand is less than the lower limit, the optimal policy is to order up to the lower limit. If the inventory level is larger than the upper limit, the optimal policy is to dispose down to the upper limit. Since all rented equipments are returned in good condition (and thus remanufacturing is not needed), Whisler et al. (1967) do not incorporate remanufacturing of returned items into decision-making.

Simpson (1978) examines an inventory system with random demand and returns, under the discounted cost criterion. Any excess demand is backlogged. The state space consists of inventory levels of both the end-products and repairable items. Simpson (1978) establishes the optimality of a *base-stock* policy with three thresholds: repair-up-to level, purchase-up-to level, and scrap-down-to level. It is optimal to repair up to a certain limit, purchase up to a certain limit if repair is not possible, and finally scrap down to a certain limit if the inventory on hand exceeds this limit. Unlike Simpson (1978), our study takes into account the collection capacity and non-zero market sojourn time.

Buchanan and Abad (1998) consider an inventory system for containers with random returns and lost sales. A fixed fraction of the end products is destroyed or becomes unavailable. The state space consists of the number of containers available for sale and the number of containers in the field. The optimal policy specifies the number of containers that should be ordered at the beginning of each stage. They prove the optimality of a *base-stock* policy in this problem. Unlike Buchanan and Abad (1998), we allow for a market sojourn time for returns, and our returns are bounded by the past sales.

Galbreth and Blackburn (2006) consider a single-period inventory system in the cases of deterministic demand and random demand. Returned products may be in different conditions, which become known by the manufacturer upon collection. They seek the optimal number of used items to acquire and the optimal degree of selectivity during sorting operation after acquisition. They model the problem in both cases of linear and non-linear acquisition costs as the standard newsvendor problem. As the degree of selectivity increases, the remanufacturing yield decreases since more products are scrapped, but the cost of remanufacturing also decreases since the quality of selected products increases. They formulate the condition of a returned product as the remanufacturing cost: Returned products in a better condition lead to lower remanufacturing costs. Galbreth and Blackburn (2010) extend the model in Galbreth and Blackburn (2006) to allow for uncertain used product condition, establishing the optimal acquisition amount and the optimal sorting policy. Zikopoulos and Tagaras (2008) also study a variation of this problem in which defects may occur in sorting operations. See also Ferrer (2003), Guide et al. (2003), Bakal and Akcali (2006), and Zikopoulos and Tagaras (2007) for stochastic acquisition and sorting models. Unlike these papers, we consider a *multi-stage* inventory model with random returns (of the same condition) bounded by earlier sales.

Cohen (1980) considers an inventory system with random demand and lost sales. A fixed fraction of the sold products is returned to the manufacturer after a fixed number of time periods, and a fixed fraction of the products on hand decays. Cohen (1980) assumes all returned products can be resold with no remanufacturing effort. The state space consists of the inventory level of serviceable items as well as the number of previously sold items. Cohen (1980) then shows the optimality of a *base-stock* policy under the discounted cost criterion. Cohen (1980) also proves the optimality of a *myopic* base-stock policy when the market sojourn time is fixed at one period. Beltran et al. (2002) generalize the model in Cohen (1980) to allow for a fixed ordering cost, showing the optimality of an (s, S) policy. Unlike Cohen (1980) and Beltran et al (2002), our state space includes the inventory level of the collected products, and our control policy specifies the number of used items to collect and the number of collected items to remanufacture (in addition to the number of items to manufacture). We also allow the collection rate to be random in each time period, making our problem more realistic.

van der Laan et al. (1996) consider an inventory model in which demand and returns are independent from each other. Under the assumptions of backlogging and positive leadtimes, they show the optimality of (s, Q) policy in the average cost case. Fleischmann et al. (2002) extend this optimality result to the case with random returns. Bayındır et al. (2005) study a similar problem under the assumption of lost sales and zero leadtimes. Unlike these papers, in our model the number of demand at any stage impacts the number of returns at a later stage.

Inderfurth (1997) studies a multi-stage inventory control problem with remanufacturing option. The decision-maker faces stochastic demand and returns, and has two options to fulfill demand: remanufacturing and procurement. The decision-maker may also decide to dispose returned items. When procurement and remanufacturing have identical leadtimes, Inderfurth (1997) shows that the following policy is optimal at each stage: If the inventory level is below a certain lower limit, it is optimal to dispose nothing and remanufacture (or procure if remanufacturing is not possible or is inadequate) up to this lower limit. If the inventory level is higher than a certain upper limit, it is optimal to dispose down to this upper limit, remanufacture the remaining returned products, and procure nothing. Although a *base-stock* policy is optimal when leadtimes are identical, Inderfurth (1997) states that a base-stock policy need not be optimal when leadtimes are positive and non-identical. For this reason, Inderfurth (1997) suggests the use of heuristic algorithms that can perform well in the case of non-identical leadtimes, inspired by the threshold policy defined above. Kiesmüller and Minner (2003) also develop a heuristic algorithm for a periodic review inventory problem with random demand and returns. Unlike our study, Inderfurth (1997) neglects to consider a market sojourn time for product returns and assumes all returned products are collected. Kiesmüller and Minner (2003), on the other hand, assume that returns are independent from earlier sales and all returns are remanufactured.

Toktay (2000) examines a multi-stage inventory control problem in which the end products are returned to the manufacturer after a certain market sojourn time. Toktay (2000) studies the problem when backlogging is not allowed and the market sojourn time is fixed at one period. The problem consists of two decisions: how much to procure and how much to dispose. Similar to Whisler (1967) and Cohen (1980), the state space in Toktay (2000) consists only of the inventory level of serviceable products. Using the six-node closed queueing theory network, Toktay (2000) shows the optimality of a *base-stock* policy and proposes a heuristic procedure to construct a dynamic procurement policy.

Kiesmüller and van der Laan (2001) study an inventory model over a finite planning horizon with positive ordering lead times, and random returns that are dependent on demand stream. A sold item is returned to the manufacturer with a constant probability, and a returned item is either remanufactured with a constant probability or disposed. Any unsatisfied demand is backordered. They show the optimality of a *base-stock* policy. Unlike Kiesmüller and van der Laan (2001), in our study the state space contains two distinct inventory levels and the collection amount is a decision variable. We also study the cases of lost sales and backordering. Brito and van der Laan (2008) study a similar problem, establishing the optimality of a *base-stock* policy over an infinite planning horizon.

Zhou et al. (2011) study a multi-stage inventory control problem with random demand, random returns (cores), and multiple core conditions. The manufacturer holds different inventories for serviceable products and cores, and may dispose the excess amount of cores. The value function in their dynamic programming formulation involves two-layer optimization. They first solve the optimization problems sequentially across all types of cores and then choose the solution that minimizes the expected total cost. In the case of identical manufacturing and remanufacturing lead-times, they establish the optimality of a *threshold policy* with *state-dependent* manufacture/remanufacture-up-to levels and *state-dependent* dispose-down-to levels. They also formulate the problem in the case of non-identical lead times, developing a simple heuristic procedure to compute a near-optimal control policy. The main limitation of this study is that the impact of past sales on future product returns is ignored. However, in our study, the number of products available for collection is bounded by the amount of past sales. It is also important to note that Zhou et al. (2011) neglect to include the collection rate as a decision variable in their model. Tao et al. (2012) extend the model in Zhou et al. (2011) by allowing for random remanufacturing yield, in addition to random demand and returns.

2.2 The Risk-Sensitive Problem

As far as we are aware, the prior literature has not yet studied the risk-averse optimization of inventory systems in *closed-loop* supply chains. Therefore, we below review the literature dealing with the risk-averse optimization in *traditional* supply chains.

Schlesinger (1995) studies the newsvendor problem in a risk-averse setting. The objective is to maximize the expected utility, which is increasing, concave and thrice differentiable. Schlesinger (1995) shows that the optimal order quantity decreases as risk-aversion increases. When the decision maker is too risk-averse, he does not even order any newspapers due to the fear of losing money.

Agrawal and Seshadri (2000) consider a newsvendor setting in which the risk-neutral and risk-averse objectives are to maximize the expected utility, which is a concave function of the price. They develop two different formulations under two distinct assumptions: (i) a change in price affects the scale of the distribution and (ii) a change in price only affects the location of the distribution. They find that a risk-averse retailer prefers to charge a higher price and order less under assumption (i) whereas it prefers to charge a lower price under assumption (ii), in comparison with the risk-neutral case.

Chen et al. (2007) study a multi-stage inventory control problem in which the objective is to maximize the total expected utility over a flow of consumption. They introduce two models: In the first model, demand is exogenous, i.e., price is not a decision variable. In the second model, demand depends on price, i.e., price is a decision variable. Chen et al. (2007) show that when the utility function is exponential and the financial market is partially complete, the structure of the risk-averse optimal policy is almost identical to the structure of the risk-neutral optimal policy.

Choi and Ruszczyński (2011) extend the model in Chen et al. (2007) by allowing for multiple products, taking an exponential utility function of the profit as their objective. They prove that when the product demands are independent, and the ratio of the degree of risk aversion to the number of products approaches zero, the risk-averse optimal solution converges to the risk-neutral optimal solution. They also show that the risk-averse optimal order quantities are lower under positively correlated demands than under independent demands.

Although the expected utility approach has been widely adapted in the literature on the risk-averse optimization of inventory systems, the interpretation of such utility functions are quite difficult. An important limitation of the expected utility approach is that it is often very hard or not practical to elicit the utility function of the decision-maker. For this reason, Ahmed et al. (2007) examine the single-item inventory control problem with linear cost structure in both single-stage and multi-stage settings, incorporating the coherent risk measure into the objective function. They replace the expectation operation with the mean-absolute deviation risk measure in their objective for the multi-stage problem, and prove the optimality of a base-stock policy. They also show that as the risk-aversion increases, the decision-maker orders in higher amounts.

Choi and Ruszczyński (2008) use the general mean-risk model in order to solve the newsvendor problem. They find that the opposite results of Ahmed et al. (2007) hold in their model. Examples of the mean-risk models include semi-deviation (the risk model in our research) and weighted-mean-deviation from quantile. Using general law-invariant measures of risk, Choi and Ruszczyński (2008) show in the case of lost sales that as the newsvendor becomes more risk-averse, he prefers to order less. Choi et al. (2011) use law-invariant coherent risk measures to model the multi-product newsvendor problem in Choi and Ruszczyński (2011), obtaining the same results as in the expected utility case. In addition, they establish that as the number of products grows to infinity, the optimal solution converges to the risk-neutral optimal solution, i.e., risk-aversion becomes ineffective in the optimal policy.

Chapter 3

Problem Formulation

We formulate the closed-loop inventory problem in both cases of backlogging and lost-sales under two different risk-attitudes of the decision maker: (i) risk-neutral and (ii) risk-averse.

We consider a single product, closed-loop, finite-horizon inventory system. The manufacturer satisfies the demand through two channels: manufacturing new products and remanufacturing its own end-of-use products (cores). Demand for serviceable products at each stage t, D_t, is random. A random fraction C_t of the sold products at stage t, becomes available for collection and remanufacturing by the manufacturer after a fixed market sojourn time t_Δ, i.e., at stage $t + t_\Delta$. We label this fraction as return rate.

The order of the events at each stage is as follows: At the beginning of the stage, some or all of the previously sold products become available for collection. The decision-maker observes the serviceable product inventory, the core inventory, and the future returns. It then decides how many products to manufacture, how many cores to acquire (of the *newly* available cores at that stage), and how many products to remanufacture (of the *so far* acquired cores). Both newly-manufactured and remanufactured products are added to the serviceable product inventory. Finally, demand is observed and satisfied from the serviceable product inventory. Any excess demand is either *always* backlogged or *always* lost. The true fraction of the sold items at this stage that will be available for collection in the future is revealed to the decision-maker *at the end of this stage*.

We make several assumptions for analytical convenience: (i) Demand and return rate are independent from each other at each stage. Many papers in the closed-loop inventory literature have made this assumption; see, for instance, Simpson (1978), Buchanan and Abad (1998), Galbreth and Blackburn (2006), and Zhou et al. (2011). (ii) Both manufacturing and remanufacturing lead-times are zero. The same assumption appears in several papers; see, for instance, Inderfurth (1997), Galbreth and Blackburn (2006), and Zhou et al. (2011). (iii) All returned cores have the same level of quality; they are all identical. This assumption also appears in several papers; see, for instance, Cohen (1980), Inderfurth (1997), Galbreth and Blackburn (2006), and Geyer et al. (2007). (iv) Remanufactured products are the perfect substitutes of newly manufactured products. This is a standard assumption in the literature; see, for instance, Toktay (2000), Beltran (2002), Geyer et al. (2007), and Zhou et al. (2011). (v) Last, the cores that have been sold at stage t but have not been collected at stage $t+t_\Delta$ are lost. This allows us to keep the state space of the problem manageable. The uncollected cores correspond to those consumers who simply choose not to return their products and/or who dispose them (Buchanan and Abad 1998, and Geyer et al. 2007).

We define c_m as the unit cost of manufacturing a serviceable product, c_c as the unit cost of collecting a core, and c_r as the unit cost of remanufacturing a product. We denote by h_s and h_r the unit holding costs for serviceable products and cores per stage, respectively. We define p as the lost sale cost per unit of unmet demand, and b as the backlogging cost per unit of unmet demand per stage. There is no cost of having leftover items or being in shortage at the end of the planning horizon. Last, we denote by t_Δ the market sojourn time, i.e., the time interval between the sale of a particular product and its return.

We formulate a discrete-time stochastic dynamic program with T stages that determines the amount of products to manufacture Q_t, the amount of cores to collect Z_t, and the amount of collected cores to remanufacture R_t at each stage t.

The risk-neutral objective is to minimize the expected total cost that consists of the manufacturing, remanufacturing, and collection costs, the inventory holding costs, and the backordering or lost-sale costs, across all stages. The risk-averse objective, on the other hand, is to minimize the weighted sum of the mean cost and the expected excess from the mean cost.

The state space consists of the following state variables: X_t is the serviceable product inventory level at the beginning of stage t. Y_t is the core inventory level at the beginning of stage t. $< S_{t-1}, S_{t-2}, .., S_{t-t_\Delta} >$ is the vector of the numbers of cores that will become available for collection t_Δ stages later; S_t is the number of cores that will become available at stage $t + t_\Delta$. Note that the state space grows exponentially as t_Δ increases. Table 3.1 summarizes the notation that we use throughout the thesis.

3.1 The Risk-Neutral Problem

In this section we formulate the risk-neutral inventory control problem in both cases of backlogging and lost-sales.

3.1.1 The Case of Backlogging

We assume that any unmet demand is backlogged, incurring a unit backlog cost b per stage. Let $V_t(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta})$ denote the minimum expected total cost from stage t to the end of the planning horizon. Then the dynamic programming formulation of the problem for $t \in \{0, ..., T-1\}$ can be written as

Table 3.1: Summary of notation.

Decision variables	
Q_t	Number of serviceable products manufactured at stage t.
Z_t	Number of cores collected at stage t.
R_t	Number of cores remanufactured at stage t.
State variables	
S_t	Number of serviceable products that will become available for collection at stage $t + t_\Delta$.
X_t	Serviceable product inventory level at the beginning of stage t ($X_t \geq 0$ in the case of lost sales).
Y_t	Core inventory level at the beginning of stage t.
Parameters	
T	Number of stages.
c_m	Unit cost for manufacturing a serviceable product.
c_c	Unit cost for collecting a core.
c_r	Unit cost for remanufacturing a core.
h_s	Unit holding cost for serviceable products per stage.
h_r	Unit holding cost for cores per stage.
p	Lost sale cost per unit of unmet demand.
b	Backlogging cost per unit of unmet demand per stage.
t_Δ	Market sojourn time.
D_t	Customer demand for a serviceable product at stage t (random variable).
C_t	Return rate for stage $t + t_\Delta$ (random variable).

$$\begin{aligned} V_t(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta}) = \min_{Q_t, R_t, Z_t \geq 0} \Big\{ & c_m Q_t + c_r R_t + c_c Z_t \\ & + \mathop{\mathbb{E}}_{D_t, C_t} \Big[h_s [X_{t+1}]_+ + h_r Y_{t+1} + b[-X_{t+1}]_+ \\ & + V_{t+1}(X_{t+1}, Y_{t+1}, S_t, ..., S_{t-t_\Delta+1}) \Big] \Big\} \end{aligned} \tag{3.1.1}$$

$$\text{s.t.} \quad X_{t+1} = X_t + Q_t + R_t - D_t \tag{3.1.2}$$

$$Y_{t+1} = Y_t + Z_t - R_t \tag{3.1.3}$$

$$S_t = \left\lfloor C_t \left[\min\left\{ \max\{0, X_t + Q_t + R_t\}, D_t \right\} \right.\right. \\ \left.\left. + \max\left\{ 0, \min\{0, X_t + Q_t + R_t\} - X_t \right\} \right] \right\rfloor \tag{3.1.4}$$

$$Z_t \leq S_{t-t_\Delta} \tag{3.1.5}$$

$$Y_{t+1} \geq 0 \tag{3.1.6}$$

where V_T is a zero function. The objective function (3.1.1) consists of the manufacturing, remanufacturing, and collection costs, the expected holding and backlogging costs, and the future cost-to-go function V_{t+1}.

Constraint (3.1.2) ensures that the serviceable product inventory level at the beginning of the next stage is equal to the sum of the serviceable product inventory level at the beginning of the current stage and the numbers of newly-manufactured and remanufactured products at the current stage minus demand at the current stage. Note that the serviceable product inventory level can be negative in the case of backlogging.

Constraint (3.1.3) ensures that the core inventory level at the beginning of the next stage is equal to the sum of the core inventory level at the beginning of the current stage and the number of acquired cores at the current stage minus the number of remanufactured products at the current stage.

Constraint (3.1.4) calculates the number of sold products at the current stage that will become available for collection t_Δ stages later. The first part of the equation corresponds to demands that are immediately satisfied at the current stage, whereas the second part corresponds to backlogged demands that are satisfied at the current stage. The sum of these two parts yields the number of items sold at the current stage, which is multiplied by the return rate C_t to obtain the number of cores that will be available for collection t_Δ stages later.

Constraint (3.1.5) ensures that the number of acquired cores at any stage is no greater than the number of available cores at that stage. Constraint (3.1.6) ensures that the core inventory level is non-negative.

We are able to establish the following structural property of the optimal cost function V_t, under a mild condition:

Lemma 3.1. *Suppose that* $c_m \geq c_r$. *For the risk-neutral inventory problem with backlogging, the following inequality holds at each stage* t*:* $V_t(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta}) + c_m \geq V_t(X_t, Y_t - 1, S_{t-1}, ..., S_{t-t_\Delta}) + c_r$.

Proof. See Appendix A. □

Using Lemma 3.1, we obtain the following structural property of the optimal policy:

Proposition 3.2. *Suppose that* $c_m \geq c_r$. *For the risk neutral inventory problem with backlogging, it is optimal to prefer remanufacturing to manufacturing if the serviceable product inventory is to be increased.*

Proof. See Appendix A. □

3.1.2 The Case of Lost Sales

We now assume that backlogging is not allowed and any unmet demand is lost, incurring a unit lost-sale cost p. Then the dynamic programming formulation of the problem for $t \in \{0, 1, ..., T-1\}$ can be written as

$$V_t(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta}) = \min_{Q_t, R_t, Z_t \geq 0} \Big\{ c_m Q_t + c_r R_t + c_c Z_t + \mathop{\mathbb{E}}_{D_t, C_t} \Big[h_s X_{t+1} + h_r Y_{t+1} + p[D_t - X_t - Q_t - R_t]_+ + V_{t+1}(X_{t+1}, Y_{t+1}, S_t, ..., S_{t-t_\Delta+1}) \Big] \Big\} \tag{3.1.7}$$

$$\text{s.t.} \quad X_{t+1} = \max\{0, X_t + Q_t + R_t - D_t\} \tag{3.1.8}$$

$$Y_{t+1} = Y_t + Z_t - R_t \tag{3.1.9}$$

$$S_t = \left\lfloor C_t \left[\min\{X_t + Q_t + R_t, D_t\} \right] \right\rfloor \quad (3.1.10)$$

$$Z_t \le S_{t-t_\Delta} \quad (3.1.11)$$

$$Y_{t+1} \ge 0 \quad (3.1.12)$$

where V_T is a zero function. Unlike the formulation in the case of backlogging, (i) the objective function (3.1.7) includes the expected lost sale cost, disregarding the expected backlogging cost; (ii) the serviceable product inventory level at each stage is forced to be non-negative (constraint 3.1.8); and (iii) the amount of sales at any stage equals the minimum of the demand and the serviceable product inventory level at that stage (constraint 3.1.10).

Again, we are able to establish the following structural property of the optimal cost function V_t:

Lemma 3.3. *Suppose that $c_m \ge c_r$. For the risk-neutral inventory problem with lost sales, the following inequality holds at each stage t: $V_t(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta}) + c_m \ge V_t(X_t, Y_t - 1, S_{t-1}, ..., S_{t-t_\Delta}) + c_r$.*

Proof. See Appendix A. □

Using Lemma 3.3, we obtain the following structural property of the optimal policy:

Proposition 3.4. *Suppose that $c_m \ge c_r$. For the risk-neutral inventory problem with lost sales, it is optimal to prefer remanufacturing to manufacturing if the serviceable product inventory is to be increased.*

Proof. See Appendix A. □

3.2 The Risk-Averse Problem with Mean-Semi-Deviation

Our purpose in this section is to employ the modern theory of risk measures in our inventory control problem. First we briefly introduce the concept of risk measure. Then we incorporate mean-semi-deviation as a risk measure into our problem formulation in both cases of backlogging and lost sales.

Suppose that there exists a probability space (Ω, P). There exists a function $\mathcal{F} : \mathbb{R}^n \times \Omega \to \mathbb{R}$ and a set $\mathcal{X} = \{\mathcal{F}(x, .) | x \in X\}$. A *risk measure* is defined as a function $\rho : \mathcal{X} \to \mathbb{R}$ assigning a value corresponding to the assessment of the risk involved in holding the position defined by x to each random variable $\mathcal{F}(x, .)$. The risk averse problem has the objective:

$$\min_{x \in X} \ \rho(\mathcal{F}(x, \omega)).$$

Now let $(\Omega, \mathcal{F}, P)$ be the probability space, $X : \Omega \to \mathbb{R}$ be the random outcome (cost), and $\mathcal{Z} = \mathcal{L}_p(\Omega, \mathcal{F}, P)$ for $p \in [1, \infty]$ be the space of possible outcomes. A risk measure $\rho : \mathcal{Z} \to \mathbb{R}$ is a *coherent risk measure* if it satisfies the following four axioms (Artzner et al. 1999):

A1. *Convexity*: $\rho(\lambda W + (1 - \lambda)X) \leq \lambda\rho(W) + (1 - \lambda)\rho(X), \ \forall W, X \in \mathcal{Z}$ and $\forall \lambda \in [0, 1]$.

A2. *Monotonicity*: If $X \preceq W$ and $X, W \in \mathcal{Z}$, then $\rho(X) \leq \rho(W)$.

A3. *Translation Invariance*: $\forall a \in \mathbb{R}$, $X \in \mathcal{Z}$, $\rho(X + a) = \rho(X) + a$.

A4. *Positive Homogeneity*: If $\beta \geq 0$, then $\rho(\beta X) = \beta\rho(X), \ \forall X \in \mathcal{Z}$.

Ruszczyński and Shapiro (2009) give a further explanation of conditional and dynamic risk measures: Consider the probability space $(\Omega, \mathcal{F}, P)$ with filtration $\mathcal{F}_1 \subset \mathcal{F}_2 \subset \mathcal{F}_T \subset \mathcal{F}$ and the adopted sequence of random costs X_t for $t = 1, ..., T$. Assume $\mathcal{F}_1 = \{\Omega, \emptyset\}$. Thus X_1 is deterministic. Define the spaces

$\mathcal{Z}_t = \mathcal{L}_p(\Omega, \mathcal{F}, P)$ for $p \in [1, \infty]$, $t = 1, ..., T$, and let $\mathcal{Z}_{t,T} = \mathcal{Z}_t \times ... \times \mathcal{Z}_T$.

A *conditional risk measure* is defined as a mapping $\rho_{t,T} : \mathcal{Z}_{t,T} \to \mathcal{Z}_t$ where $1 \leq t \leq T$, if it satisfies the axiom of monotonicity. A *dynamic risk measure* is a sequence of conditional risk measures $\rho_{t,T} : \mathcal{Z}_{t,T} \to \mathcal{Z}_t$ for $t = 1, ..., T$. *One-step conditional risk measure* $\rho_t : \mathcal{Z}_{t+1} \to \mathcal{Z}_t$, $t = 1, ..., T-1$ is defined as

$$\rho_t(X_{t+1}) = \rho_{t,t+1}(0, X_{t+1}). \tag{3.2.1}$$

Using equation (3.2.1), we can retrieve the following recursive relation:

$$\rho_{t,T}(Z_t, ..., Z_T) = X_t + \rho_t(X_{t+1} + \rho_{t+1}(X_{t+2} + ... + \rho_{T-2}(X_{T-1} + \rho_{T-1}(X_T)))...) \tag{3.2.2}$$

The most significant examples of one-step conditional risk measures are *mean-semi-deviation* and *conditional average value at risk*. In our study we incorporate the risk into our problem via mean-semi-deviation. This enables us to formulate the problem as a parametric optimization problem and easily observe the trade-off between mean and risk.

Conditional mean-semi-deviation $\rho_t(X_{t+1})$ is defined as follows (Shapiro, Dentcheva, and Ruszczyński, 2009).

$$\rho_t(X_{t+1}) = \mathbb{E}[X_{t+1}|\mathcal{F}_t] + \kappa\, \mathbb{E}\left[\left((X_{t+1} - \mathbb{E}[X_{t+1}|\mathcal{F}_t])_+\right)^r |\mathcal{F}_t \right]^{\frac{1}{r}}. \tag{3.2.3}$$

The above equation calculates the sum of the expected upper deviation from the mean and the expected cost given a realization. r is the order of the one-step conditional risk measure and κ is the risk factor. Note that equation (3.2.3) simplifies into the risk-neutral case when $\kappa = 0$. The degree of risk-aversion rises as r or κ increases.

In Sections (3.2.1) and (3.2.2) we reformulate our inventory control problem for the risk-averse decision-maker, by incorporating mean-semi-deviation into the objective function.

3.2.1 The Case of Backlogging

The objective function of the risk-averse decision-maker with mean-semi-deviation risk measure takes the following form:

$$
\begin{aligned}
V_t(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta}) = \min_{Q_t, R_t, Z_t \geq 0} \Bigg\{ & c_m Q_t + c_r R_t + c_c Z_t \\
& + \mathop{\mathbb{E}}_{D_t, C_t} \Big[h_s[X_{t+1}]_+ + h_r Y_{t+1} + b[-X_{t+1}]_+ + V_{t+1}(X_{t+1}, Y_{t+1}, S_t, ..., S_{t-t_\Delta+1}) \Big] \\
& + \kappa \mathop{\mathbb{E}}_{D_t, C_t} \Bigg[\Big[h_s[X_{t+1}]_+ + h_r Y_{t+1} + b[-X_{t+1}]_+ + V_{t+1}(X_{t+1}, Y_{t+1}, S_t, ..., S_{t-t_\Delta+1}) \\
& - \mathop{\mathbb{E}}_{D_t, C_t} [h_s[X_{t+1}]_+ + h_r Y_{t+1} + b[-X_{t+1}]_+ + V_{t+1}(X_{t+1}, Y_{t+1}, S_t, ..., S_{t-t_\Delta+1})] \Big]_+^r \Bigg]^{\frac{1}{r}} \Bigg\}
\end{aligned} \tag{3.2.4}
$$

The dynamic programming formulation with mean-semi-deviation risk measure can be written as

$$
V_t(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta}) =
$$

$$
\min_{Q_t, R_t, Z_t \geq 0} \left\{ c_m Q_t + c_r R_t + c_c Z_t + \mu + \kappa \mathop{\mathbb{E}}_{D_t, C_t} \left[\Big([F_{t+1} - \mu]_+ \Big)^r \right]^{1/r} \right\} \tag{3.2.5}
$$

$$
\begin{aligned}
\text{s.t. } \mu = \mathop{\mathbb{E}}_{D_t, C_t} \Big[& h_s[X_{t+1}]_+ + h_r Y_{t+1} + b[-X_{t+1}]_+ \\
& + V_{t+1}(X_{t+1}, Y_{t+1}, S_t, ..., S_{t-t_\Delta+1}) \Big]
\end{aligned} \tag{3.2.6}
$$

$$
\begin{aligned}
F_{t+1} = & h_s[X_{t+1}]_+ + h_r Y_{t+1} + b[-X_{t+1}]_+ \\
& + V_{t+1}(X_{t+1}, Y_{t+1}, S_t, ..., S_{t-t_\Delta+1})
\end{aligned} \tag{3.2.7}
$$

$$
X_{t+1} = X_t + Q_t + R_t - D_t \tag{3.2.8}
$$

$$
Y_{t+1} = Y_t + Z_t - R_t \tag{3.2.9}
$$

$$S_t = \Bigg\lfloor C_t \Bigg[\min \Big\{ \max\{0, X_t + Q_t + R_t\}, D_t \Big\} + \max \Big\{ 0, \min\{0, X_t + Q_t + R_t\} - X_t \Big\} \Bigg] \Bigg\rfloor \tag{3.2.10}$$

$$Z_t \leq S_{t-t_\Delta} \tag{3.2.11}$$

$$Y_{t+1} \geq 0 \tag{3.2.12}$$

where V_T is a zero function. The objective function (3.2.5) minimizes the weighted sum of the expected cost and the expected upper deviation from the mean. Constraint (3.2.6) calculates the expected cost whereas constraint (3.2.7) calculates the cost for a given realization of random demand and collection rate at stage t. Note that the risk-averse problem in this section becomes equivalent to the risk-neutral problem in Section 3.1.1 when $\kappa = 0$.

We are able to establish the following structural property of the value function V_t:

Lemma 3.5. *Suppose that $c_m \geq c_r$. For the risk-averse inventory problem with backlogging, the following inequality holds for all coherent risk measures at each stage t: $V_t(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta}) + c_m \geq V_t(X_t, Y_t - 1, S_{t-1}, ..., S_{t-t_\Delta}) + c_r$.*

Proof. See Appendix A. □

Using Lemma 3.5, we obtain the following structural property of the optimal policy:

Proposition 3.6. *Suppose that $c_m \geq c_r$. For the risk-averse inventory problem with backlogging, it is optimal to prefer remanufacturing to manufacturing if the serviceable product inventory is to be increased, and this holds for all coherent risk measures.*

Proof. See Appendix A. □

3.2.2 The Case of Lost Sales

In the case of lost-sales, the objective function with mean-semi-deviation risk measure takes the following form:

$$
\begin{aligned}
V_t(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta}) = \min_{Q_t, R_t, Z_t \geq 0} \Bigg\{ & c_m Q_t + c_r R_t + c_c Z_t \\
& + \mathop{\mathbb{E}}_{D_t, C_t} \Big[h_s X_{t+1} + h_r Y_{t+1} + p[-X_{t+1}]_+ + V_{t+1}(X_{t+1}, Y_{t+1}, S_t, ..., S_{t-t_\Delta+1}) \Big] \\
& + \kappa \mathop{\mathbb{E}}_{D_t, C_t} \Bigg[\Big[h_s X_{t+1} + h_r Y_{t+1} + p[-X_{t+1}]_+ + V_{t+1}(X_{t+1}, Y_{t+1}, S_t, ..., S_{t-t_\Delta+1}) \\
& - \mathop{\mathbb{E}}_{D_t, C_t} [h_s X_{t+1} + h_r Y_{t+1} + p[-X_{t+1}]_+ + V_{t+1}(X_{t+1}, Y_{t+1}, S_t, ..., S_{t-t_\Delta+1})] \Big]_+^r \Bigg]^{\frac{1}{r}} \Bigg\}
\end{aligned}
\tag{3.2.13}
$$

The dynamic programming formulation with mean-semi-deviation risk measure can be written as

$$
V_t(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta}) =
$$

$$
\min_{Q_t, R_t, Z_t \geq 0} \Bigg\{ c_m Q_t + c_r R_t + c_c Z_t + \mu + \kappa \mathop{\mathbb{E}}_{D_t, C_t} \Bigg[\Big([F_{t+1} - \mu]_+ \Big)^r \Bigg]^{1/r} \Bigg\} \tag{3.2.14}
$$

$$
\begin{aligned}
\text{s.t. } \mu = \mathop{\mathbb{E}}_{D_t, C_t} \Big[& h_s X_{t+1} + h_r Y_{t+1} + p[D_t - X_t - Q_t - R_t]_+ \\
& + V_{t+1}(X_{t+1}, Y_{t+1}, S_t, ..., S_{t-t_\Delta+1}) \Big]
\end{aligned}
\tag{3.2.15}
$$

$$
\begin{aligned}
F_{t+1} = & h_s X_{t+1} + h_r Y_{t+1} + p[D_t - X_t - Q_t - R_t]_+ \\
& + V_{t+1}(X_{t+1}, Y_{t+1}, S_t, ..., S_{t-t_\Delta+1})
\end{aligned}
\tag{3.2.16}
$$

$$
X_{t+1} = \max\{0, X_t + Q_t + R_t - D_t\} \tag{3.2.17}
$$

$$
Y_{t+1} = Y_t + Z_t - R_t \tag{3.2.18}
$$

$$S_t = \left\lfloor C_t\left[\min\{X_t + Q_t + R_t, D_t\}\right]\right\rfloor \tag{3.2.19}$$

$$Z_t \leq S_{t-t_\Delta} \tag{3.2.20}$$

$$Y_{t+1} \geq 0 \tag{3.2.21}$$

where V_T is a zero function. Again, the risk-averse problem in this section becomes equivalent to the risk-neutral problem in Section 3.1.2 when $\kappa = 0$.

Again, we are able to establish the following structural property of the value function V_t:

Lemma 3.7. *Suppose that $c_m \geq c_r$. For the risk-averse inventory problem with lost sales, the following inequality holds for all coherent risk measures at each stage t: $V_t(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta}) + c_m \geq V_t(X_t, Y_t - 1, S_{t-1}, ..., S_{t-t_\Delta}) + c_r$.*

Proof. See Appendix A. □

Using Lemma 3.7, we obtain the following structural property of the optimal policy:

Proposition 3.8. *Suppose that $c_m \geq c_r$. For the risk-averse inventory problem with lost sales, it is optimal to prefer remanufacturing to manufacturing if the serviceable product inventory is to be increased, and this holds for all coherent risk measures.*

Proof. See Appendix A. □

We implement Propositions 3.2, 3.4, 3.6, and 3.8 into our fixed threshold policy, in Chapter 6.

We can solve each of the problems in Sections (3.1.1), (3.1.2), (3.2.1), and (3.2.2) to optimality with the backward dynamic programming algorithm. Let S denote the state space, and A the action space. Let

$V_t^{<Q_t,R_t,Z_t>}(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta})$ denote the cost function if actions Q_t, R_t, and Z_t are chosen at stage t. Also, let $V_t(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta})$ denote the minimum expected total cost at state $< X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta} >$, and $< Q_t^*, R_t^*, Z_t^* >$ the optimal decision at state $< X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta} >$. The algorithm is initialized with the zero function at stage T. State variables at stage $T-1$ are set to their initial values. In a given state, the expected total cost is calculated under each feasible action. After all the expected total costs are found for all feasible actions, the action with the least cost is the optimal decision in this state. The same procedure is repeated until the optimal decision is found in each possible state. The optimal *strategy* at stage $T-1$ is the mapping from all possible states to the optimal decisions. Once the optimal strategy is found at stage $T-1$, the algorithm proceeds backward in time to stage $T-2$, setting the cost function at stage $T-1$ equal to the optimal cost function under the optimal strategy at stage $T-1$. Proceeding similarly, the algorithm calculates the optimal strategy at each stage. The optimal strategies across all stages yield the optimal *control policy*. The pseudo code for this algorithm is given as follows.

Backward solution algorithm.

Initialization

Set $t \leftarrow T-1$

for all $t \in \{T-1, T-2, ..., 0\}$

Set $X_t \leftarrow 0, Y_t \leftarrow 0, S_{t-1}, ..., S_{t-t_\Delta} \leftarrow 0$

$Q_t \leftarrow 0, R_t \leftarrow 0, Z_t \leftarrow 0$

$V_t(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta}) \leftarrow 9999999$

for all $X_t \in S$

for all $Y_t \in S$

for all $S_{t-1} \in S$

...

for all $S_{t-t_\Delta} \in S$

for all $Q_t \in A$

for all $R_t \in A$

for all $Z_t \in A$

do

Solve the problem

if $V_t^{<Q_t, R_t, Z_t>}(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta})$ is less than

$V_t(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta})$ **then**

$V_t(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta}) \leftarrow V_t^{<Q_t, R_t, Z_t>}(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta})$

Set $< Q_t^*, R_t^*, Z_t^* > \leftarrow < Q_t, R_t, Z_t >$

end if

end for all

end for all

end for all

end for all

end for all

end for all

end for all

Chapter 4

Heuristic Policies

Optimal solutions for our closed-loop inventory control problem are computationally intractable since both the state and action spaces are extremely large. We thus consider five different heuristics that are computationally less demanding than the dynamic programming algorithm in Chapter 3, which can be used to find the optimal solution. In this chapter we describe all these heuristics along with their formulations.

First, we consider the following two heuristics that are widely used in the inventory literature: the Myopic Policy (MP) and the Certainty Equivalent Controller (CEC). MP minimizes the expected costs incurred only in the current period by disregarding the expected costs to be incurred in future periods. CEC minimizes the expected total cost by fixing both demand and return rate at their typical values and thus eliminating the stochasticity of the problem.

Second, we develop the following two heuristics that are specifically tailored to our inventory problem: the No-Recovery Policy (NRP) and the Full-Collection Policy (FCP). NRP *never* collects a core so that product recovery is not an option in fulfilment of the demand. FCP collects *all* available cores at each stage. Both NRP and FCP reduce the action space of our problem by eliminating the decision of how many cores to collect at each stage. Notice that the cost performance of

NRP relative to the globally optimal policy can be used to evaluate the economic viability of remanufacturing. Also, the cost performance of FCP relative to the globally optimal policy can be used to evaluate the cost of waste minimization.

Last, inspired by our numerical experiments, we propose the Fixed Threshold Policy (FTP) as a heuristic. Numerical results in Chapter 5 suggest that a state-dependent threshold policy may be optimal for the core inventory in our problem. But finding the optimal state-dependent threshold policy is extremely problematic due to the very large numbers of states and stages (and thus a very large number of state-dependent thresholds). FTP is a simpler form of a state-dependent threshold policy; it assumes fixed thresholds across all states and stages, making it computationally much more manageable.

4.1 Myopic Policy (MP)

MP minimizes the expected costs in each stage by ignoring the future expected costs. Myopic approach is very popular in the inventory literature since it is computationally less demanding and structurally less complex than many other heuristic approaches. Previous research has shown the optimality of MP in many stochastic multi-stage inventory problems; see for instance Cohen (1980), Cetinkaya and Parlak (1998), and Xu and Ningxiong (2013). However, ignoring the future expected costs may lead to results far from optimality in many other problems.

For our risk-neutral case, MP can be found by solving the following problem at each stage:

$$V_t(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta}) =$$
$$\min_{Q_t, R_t, Z_t \geq 0} \left\{ c_m Q_t + c_r R_t + c_c Z_t + \mathop{\mathbb{E}}_{D_t, C_t} \Big[H(X_t, Y_t, Q_t, R_t, Z_t) \Big] \right\} \quad \text{s.t.} \quad (Q_t, R_t, Z_t) \in \mathcal{G}'_{\mathcal{B}} \ \text{(or } \mathcal{G}'_{\mathcal{L}}) \tag{4.1.1}$$

where $H(X_t, Y_t, Q_t, R_t, Z_t)$ denotes the holding and lost sale/backlogging cost, and $\mathcal{G}'_{\mathcal{B}}$ and $\mathcal{G}'_{\mathcal{L}}$ denote the action spaces of the risk-neutral problem in the cases of backlogging and lost-sales, respectively.

For our risk-averse case, MP can be found by solving the following problem at each stage:

$$
\begin{aligned}
&V_t(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta}) = \\
&\qquad \min_{Q_t,R_t,Z_t \geq 0} \Bigg\{ c_m Q_t + c_r R_t + c_c Z_t + \mathop{\mathbb{E}}_{D_t,C_t} [H(X_t, Y_t, Q_t, R_t, Z_t)] \\
&\qquad\qquad + \kappa \mathop{\mathbb{E}}_{D_t,C_t} \left[\left([F_{t+1} - \mathop{\mathbb{E}}_{D_t,C_t} [H(X_t, Y_t, Q_t, R_t, Z_t)]]_+ \right)^r \right]^{\frac{1}{r}} \Bigg\} \qquad (4.1.2) \\
&\qquad\qquad \text{s.t.} \quad (Q_t, R_t, Z_t) \in \mathcal{G}''_{\mathcal{B}} \;\; (\text{or } \mathcal{G}''_{\mathcal{L}})
\end{aligned}
$$

where $F_{t+1} = H(X_t, Y_t, Q_t, R_t, Z_t)$, and $\mathcal{G}''_{\mathcal{B}}$ and $\mathcal{G}''_{\mathcal{L}}$ denote the action spaces of the risk-averse problem in the cases of backlogging and lost sales, respectively.

4.2 Certainty Equivalent Controller (CEC)

Certainty equivalent controller (CEC) is a suboptimal control scheme that builds upon the linear-quadratic control theory. CEC finds an optimal policy by fixing the uncertain quantities at some "typical" values, i.e., it assumes that the certainty equivalence principle holds. Reducing or eliminating uncertainty makes the problem computationally far less demanding (Bertsekas 1976). CEC is particularly useful in handling uncertainty in problems with imperfect state information (Treharne and Sox 2002).

In this study CEC fixes random demand and collection rate at their expected values at each stage, thereby converting our stochastic inventory problem into a deterministic inventory problem. Let $\overline{D}_t$ and $\overline{C}_t$ denote the expected values of demand and collection rate, respectively. Then S_t, X_{t+1}, and $H(X_t, Y_t, Q_t, R_t, Z_t)$ can be calculated as follows:

- For the lost sale case: $S_t = \left\lfloor \overline{C}_t \left[\min\{X_t + Q_t + R_t, \overline{D}_t\} \right] \right\rfloor$.

$H(X_t, Y_t, Q_t, R_t, Z_t) =$

$$h_s[X_t + Q_t + R_t - \overline{D}_t]_+ + h_r(Y_t + Z_t - R_t) + p[\overline{D}_t - X_t - Q_t - R_t]_+.$$

$$X_{t+1} = \max\{0, X_t + Q_t + R_t - \overline{D}_t\}.$$

- For the backlogging case: $S_t = \left\lfloor \overline{C}_t \left[\min\left\{ \max\{0, X_t + Q_t + R_t\}, \overline{D}_t \right\} + \max\left\{0, \min\{0, X_t + Q_t + R_t\} - X_t \right\} \right] \right\rfloor$.

$H(X_t, Y_t, Q_t, R_t, Z_t) =$

$$h_s[X_t + Q_t + R_t - \overline{D}_t]_+ + h_r(Y_t + Z_t - R_t) + b[\overline{D}_t - X_t - Q_t - R_t]_+.$$

$$X_{t+1} = X_t + Q_t + R_t - \overline{D}_t.$$

CEC can be found by solving the following deterministic problem for $t \in \{0, 1, ..., T-1\}$.

$V_t(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta}) =$

$$\begin{aligned} \min_{Q_t, R_t, Z_t \geq 0} \Big\{ & c_m Q_t + c_r R_t + c_c Z_t + H(X_t, Y_t, Q_t, R_t, Z_t) \\ & + V_{t+1}(X_{t+1}, Y_{t+1}, S_t, ..., S_{t-t_\Delta+1}) \Big\} \\ \text{s.t.} \quad & (Q_t, R_t, Z_t) \in \mathcal{G}'_{\mathcal{B}} \ (\text{or } \mathcal{G}'_{\mathcal{L}}) \end{aligned} \tag{4.2.1}$$

where V_T is a zero function.

Because uncertainty is eliminated from the problem, the risk-averse problem is equivalent to the risk-neutral problem for this heuristic. Notice that CEC is the same as the globally optimal policy obtained in the absence of random disturbances.

4.3 No-Recovery Policy (NRP)

NRP focuses on fulfilling the demand from newly-manufactured products by collecting and remanufacturing no core. NRP thus eliminates the decision of how many cores to collect and remanufacture at each stage. Let $H(X_t, Q_t)$ be defined as the following:

For the lost sale case: $H(X_t, Q_t) = hs[X_t + Q_t - D_t]_+ + p[D_t - X_t - Q_t]_+$.

For the backlogging case: $H(X_t, Q_t) = hs[X_t + Q_t - D_t]_+ + b[D_t - X_t - Q_t]_+$.

Then for our risk-neutral case, NRP can be found by solving the following problem:

$$V_t(X_t) = \min_{Q_t \geq 0} \left\{ c_m Q_t + \mathop{\mathbb{E}}_{D_t} \left[H(X_t, Q_t) + V_{t+1}(X_{t+1}) \right] \right\}$$
$$\text{s.t.} \quad X_{t+1} = X_t + Q_t - D_t \ \left(\text{or } X_{t+1} = \max\{0, X_t + Q_t - D_t\} \right) \tag{4.3.1}$$

where V_T is a zero function.

For our risk-averse case, NRP can be found by solving the following problem:

$$V_t(X_t) = \min_{Q_t \geq 0} \left\{ c_m Q_t + \mathop{\mathbb{E}}_{D_t} \left[H(X_t, Q_t) + V_{t+1}(X_{t+1}) \right] \right.$$

$$+\kappa \mathop{\mathbb{E}}_{D_t}\left[\left(\left[F_{t+1}-\mathop{\mathbb{E}}_{D_t}\Big[H(X_t,Q_t)+V_{t+1}(X_{t+1})\Big]_+\right)^r\right]^{\frac{1}{r}}\right\}$$

$$\text{s.t.}\quad X_{t+1}=X_t+Q_t-D_t\ \Big(\text{or } X_{t+1}=\max\{0,X_t+Q_t-D_t\}\Big)$$

$$F_{t+1}=h_s[X_{t+1}]_+ + b[-X_{t+1}]_+ + V_{t+1}(X_{t+1})$$

$$\Big(\text{or } F_{t+1}=h_sX_{t+1}+p[D_t-X_t-Q_t]_+ + V_{t+1}(X_{t+1})\Big) \tag{4.3.2}$$

where V_T is a zero function.

4.4 Full-Collection Policy (FCP)

FCP collects all available cores in the market at each stage; it minimizes the end-of-use product waste of the manufacturer and provides the maximum opportunity for remanufacturing. FCP thus eliminates the decision of how many cores to collect at each stage. Note that $Z_t = S_{t-t_\Delta}$ for all t within this heuristic class. For our risk-neutral case, FCP can be found by solving the following problem:

$$V_t(X_t,Y_t,S_{t-1},...,S_{t-t_\Delta}) =$$

$$\min_{Q_t,R_t\geq 0}\Big\{c_mQ_t+c_rR_t+c_cS_{t-t_\Delta}+\mathop{\mathbb{E}}_{D_t,C_t}\Big[H(X_t,Y_t,Q_t,R_t,S_{t-t_\Delta}) + V_{t+1}(X_{t+1},Y_{t+1},S_t,...,S_{t-t_\Delta+1})\Big]\Big\} \tag{4.4.1}$$

$$\text{s.t.}\quad (Q_t,R_t,S_{t-t_\Delta})\in\mathcal{G}'_{\mathcal{B}}\ (\text{or } \mathcal{G}'_{\mathcal{L}})$$

where V_T is a zero function.

For the risk-averse case, FCP can be found by solving the following problem:

$$V_t(X_t,Y_t,S_{t-1},...,S_{t-t_\Delta}) = \min_{Q_t,R_t\geq 0}\Big\{c_mQ_t+c_rR_t+c_cS_{t-t_\Delta}$$

$$
\begin{aligned}
&+ \mathop{\mathbb{E}}_{D_t,C_t}\left[H(X_t,Y_t,Q_t,R_t,S_{t-t_\Delta}) + V_{t+1}(X_{t+1},Y_{t+1},S_t,...,S_{t-t_\Delta+1})\right] \\
&+ \kappa \mathop{\mathbb{E}}_{D_t,C_t}\Bigg[\Bigg(\Big[F_{t+1} - \mathop{\mathbb{E}}_{D_t,C_t}\Big[H(X_t,Y_t,Q_t,R_t,Z_t) \\
&+ V_{t+1}(X_{t+1},Y_{t+1},S_t,...,S_{t-t_\Delta+1})\Big]_+\Bigg)^r\Bigg]^{\frac{1}{r}}\Bigg\} \\
&\quad \text{s.t.} \quad (Q_t,R_t,S_{t-t_\Delta}) \in \mathcal{G}''_{\mathcal{B}} \ (\text{or } \mathcal{G}''_{\mathcal{L}})
\end{aligned} \tag{4.4.2}
$$

where V_T is a zero function.

4.5 Fixed Threshold Policy (FTP)

We describe FTP as follows: (i) Collection decisions are governed by a fixed (state-independent) collect-up-to level δ_C: the core inventory is increased as close to δ_C as possible at each stage if it is below δ_C, by collecting the available cores in the market. (ii) Manufacturing and remanufacturing decisions are governed by a fixed (state-independent) produce-up-to level δ_P: the serviceable product inventory is increased to δ_P at each stage if it is below δ_P, by remanufacturing the collected cores, and by manufacturing new products in addition to remanufacturing if remanufacturing is inadequate. Remanufacturing takes priority over manufacturing in this heuristic. This is in line with our analytical results in Chapter 3 under the assumption of $c_m \geq c_r + c_c$. This assumption is often benign; see, for instance Zhou et al. (2011). Thus:

$$Z_t = \min\{\delta_C, Y_t + S_{t-t_\Delta}\} - Y_t, t = 0, ..., N-1.$$

$$Q_t = \Bigg(\max\{0, \delta_P - X_t\} - \min\Big\{Y_t + \max\Big\{0, \min\{\delta_C, Y_t + S_{t-t_\Delta}\} - Y_t\Big\}, \max\{0, \delta_P - X_t\}\Big\}\Bigg)$$

$$R_t = \min\Big\{Y_t + \max\Big\{0, \min\{\delta_C, Y_t + S_{t-t_\Delta}\} - Y_t\Big\}, \max\{0, \delta_P - X_t\}\Big\}$$

The following problem is solved for each combination of δ_P and δ_C in the risk-neutral case:

$$
\begin{aligned}
V_t^{\delta_P,\delta_C}(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta}) = c_m \Bigg(& \max\{0, \delta_P - X_t\} \\
& - \min\Big\{Y_t + \max\Big\{0, \min\{\delta_C, Y_t + S_{t-t_\Delta}\} - Y_t\Big\}, \max\{0, \delta_P - X_t\}\Big\}\Bigg) \\
& + c_r \min\Big\{Y_t + \max\Big\{0, \min\{\delta_C, Y_t + S_{t-t_\Delta}\} - Y_t\Big\}, \max\{0, \delta_P - X_t\}\Big\} \\
& + c_c \max\Big\{0, \min\{\delta_C, Y_t + S_{t-t_\Delta}\} - Y_t\Big\} \\
& + \mathop{\mathbb{E}}_{D_t, C_t}\Big[H(X_t, Y_t, \delta_P, \delta_C) + V_{t+1}^{\delta_P,\delta_C}(X_{t+1}, Y_{t+1}, S_t, ..., S_{t-t_\Delta+1})\Big]
\end{aligned}
\tag{4.5.1}
$$

where $V_T^{\delta_P,\delta_C}$ is a zero function.

The following problem is solved for each pair of δ_P and δ_C in the risk-averse case:

$$
\begin{aligned}
V_t^{\delta_P,\delta_C}(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta}) = c_m \Bigg(& \max\{0, \delta_P - X_t\} \\
& - \min\Big\{Y_t + \max\Big\{0, \min\{\delta_C, Y_t + S_{t-t_\Delta}\} - Y_t\Big\}, \max\{0, \delta_P - X_t\}\Big\}\Bigg) \\
& + c_r \min\Big\{Y_t + \max\Big\{0, \min\{\delta_C, Y_t + S_{t-t_\Delta}\} - Y_t\Big\}, \max\{0, \delta_P - X_t\}\Big\} \\
& + c_c \max\Big\{0, \min\{\delta_C, Y_t + S_{t-t_\Delta}\} - Y_t\Big\} \\
& + \mathop{\mathbb{E}}_{D_t, C_t}\Big[H(X_t, Y_t, \delta_P, \delta_C) + V_{t+1}^{\delta_P,\delta_C}(X_{t+1}, Y_{t+1}, S_t, ..., S_{t-t_\Delta+1})\Big] \\
& + \kappa \mathop{\mathbb{E}}_{D_t, C_t}\Bigg[\Bigg(\Big[F_{t+1} - \mathop{\mathbb{E}}_{D_t, C_t}\Big[H(X_t, Y_t, \delta_P, \delta_C) \\
& + V_{t+1}^{\delta_P,\delta_C}(X_{t+1}, Y_{t+1}, S_t, ..., S_{t-t_\Delta+1})\Big]\Big]_+\Bigg)^r\Bigg]^{\frac{1}{r}}\Bigg\}
\end{aligned}
\tag{4.5.2}
$$

where $V_T^{\delta_P,\delta_C}$ is a zero function.

The thresholds that minimize the expected total cost, i.e., $\underset{\delta_P,\delta_C}{\operatorname{argmin}} V_0^{\delta_P,\delta_C}(0, ..., 0)$ yield the optimal FTP.

The optimal cost of any heuristic policy can be found by solving the dynamic programming algorithm in Chapter 3 with the decision space restricted to that heuristic policy.

Chapter 5

Numerical Experiments

In this section, we conduct numerical experiments to provide insights into the optimal policy structure and evaluate the performance of each heuristic introduced in Chapter 4. We examine both cases of backlogging and lost-sales for different risk preferences of the decision-maker. Our experimental set up is based on the one proposed by Zhou et al. (2011): We consider instances in which $T = 6$, $c_m \in \{7, 10, 13\}$, $c_r \in \{2, 4, 6\}$, $c_c \in \{0.025, 1, 2\}$, $h_s \in \{1, 2, 3\}$, $h_r \in \{0.025, 1, 2\}$, $p \in \{12, 18, 24\}$, $b \in \{12, 18, 24\}$, $t_\Delta = 2$, and return rate C_t follows a discrete uniform distribution with support $\left\{\frac{1}{3}, \frac{2}{3}, 1\right\}$. We consider three different distributions for demand D_t: (i) discrete uniform distribution with support $\{0, 1, .., 5\}$; (ii) binomial distribution with parameters 5 and 0.5; and (iii) binomial distribution with parameters 5 and 0.75. Notice that the maximum possible demand is 5 at each stage under each of these distributions. In the case of lost sales we impose that $X_t \in \{0, 1, ..., 10\}$, $Y_t \in \{0, 1, ..., 10\}$, and $S_t \in \{0, 1, ..., 5\}$. In the case of backlogging we impose that $X_t \in \{-5, ..., 5\}$, $Y_t \in \{0, 1, ..., 10\}$, and $S_t \in \{0, 1, ..., 10\}$. All computations are performed on a computer with 8 GB of RAM, Intel(R) Core(TM) i7-4790 CPU @ 3.60 GHz and 64-bit operating system. All the tables and Figures 5.10–5.15 are available at the end of this chapter.

5.1 Analysis of the Optimal Policy

In this section, we aim to provide insights into the structure of optimal policy and to examine how risk-aversion and different demand distributions affect the optimal policy. We take $c_m = 10$, $c_r = 4$, $c_c = 1$, $h_s = 2$, $h_r = 1$, $b = p = 18$, and $S_{-1} = S_{-2} = 2$ in our sample problems. For the risk-averse problem, we take $\kappa = 1$ and $r \in \{1, 2\}$. Note that as r increases, the decision-maker becomes more risk averse. We then plot the optimal manufacturing quantity Q_0^*, the optimal remanufacturing quantity R_0^*, and the optimal collection quantity Z_0^* versus the serviceable product inventory level X_0 and the core inventory level Y_0; see Figures 5.10–5.15.

5.1.1 The Case of Lost Sales

First, we examine how Q_0^*, R_0^* and Z_0^* vary depending on X_0, Y_0, and the degree of risk-aversion, when demand follows a uniform distribution; see Figure 5.10.

We observe that Q_0^* is positive only when both X_0 and Y_0 are low. We also note that when both X_0 and Y_0 are low, Q_0^* tends to increase as risk-aversion increases. This is because the decision-maker seeks to avoid losing demand more by manufacturing more products as risk-aversion increases.

We observe that R_0^* is higher than Q_0^* at many values of X_0 and Y_0. This is because the sum of the unit remanufacturing cost and the unit collection cost is less than the unit manufacturing cost, and thus remanufacturing is a less costly channel in satisfying the demand. We also observe that R_0^* decreases as Y_0 decreases from 3 when X_0 is very low: The decision-maker wants to increase its serviceable product inventory, by giving priority to remanufacturing over manufacturing. If sufficient core inventory exists, manufacturing is not needed to increase the serviceable product inventory. But if there is no sufficient core inventory, i.e., if $Y_0 < 3$, manufacturing is necessary to increase the serviceable product inventory. Thus Q_0^* increases and R_0^* decreases as Y_0 decreases from 3. Last, we note that

R_0^* decreases or stays the same as X_0 increases, and R_0^* is not affected by the risk preference.

We observe that Z_0^* tends to increase as X_0 or Y_0 decreases. We also note that Z_0^* tends to increase as risk-aversion increases. This is because the decision-maker prefers to hold more cores in the inventory to avoid losing the remanufacturing opportunity in the upcoming stages.

Second, we examine how Q_0^*, R_0^* and Z_0^* vary depending on X_0, Y_0, and the degree of risk-aversion, when demand follows a binomial distribution with parameters 5 and 0.5; see Figure 5.11. We observe that Q_0^*, R_0^*, and Z_0^* are no greater than in the case of uniform demand. Because demand follows a bell-shaped distribution in Figure 5.11, medium values of demand occur with the highest probabilities, leading to less variability than in the case of uniform demand. Thus the decision-maker prefers to manufacture and remanufacture less. We again observe that both Q_0^* and Z_0^* tend to increase as risk-aversion increases, whereas R_0^* is not affected by the risk preference.

Last, we examine how Q_0^*, R_0^* and Z_0^* vary depending on X_0, Y_0, and the degree of risk-aversion, when demand follows a binomial distribution with parameters 5 and 0.75; see Figure 5.12. In this case, demand follows a left-skewed distribution and higher values of demand occur with the highest probabilities. Thus Q_0^*, R_0^*, and Z_0^* are no less than when demand follows the other two distributions. We again observe that Z_0^* tends to increase as risk-aversion increases and R_0^* is not affected by the risk preference. However, unlike the above two cases, Q_0^* does not vary with the degree of risk-aversion in this case: Q_0^* is now sufficiently high so that a high risk-averse decision-maker does not need to manufacture any further.

The structure of the optimal policy for core inventory in Figures 5.10, 5.11, and 5.12 can be specified as following a *state-dependent threshold policy*. This implies that if the core inventory level is below a state-dependent collect-up-to level $\delta_t^C(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta})$, then the core inventory level is increased to this threshold. If the core inventory level is above this threshold, then no core is collected. Specifically, $Y_t + Z_t = \min\{\delta_t^C(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta}), Y_t + S_{t-t_\Delta}\}$,

$t = 0, 1, ..., T-1$. We could not find any example violating this policy structure. However, the optimal policy for serviceable inventory need not follow a state-dependent threshold policy: At stage 4, while the optimal serviceable inventory level becomes $X_4 + Q_4 + R_4 = 4$ at state $(0, 2, 1, 2)$, this value becomes 5 at state $(1, 2, 1, 2)$.

Since we could not find any example violating the state-dependent threshold policy for core inventory, we wanted to analytically prove the optimality of this policy. A standard proof method to obtain such a result is to establish discrete-convexity of the optimal cost function. Despite our best efforts, however, we could not prove discrete-convexity of the optimal cost function. Hence whether a state-dependent threshold policy is analytically optimal for core inventory in the case of lost sales remains an open question in our research.

5.1.2 The Case of Backlogging

Most of the basic insights gained from the lost sales case remain valid in the backlogging case. Unlike the case of lost sales, we observe from Figure 5.14 that when demand follows a binomial distribution with parameters 5 and 0.5, R_0^* increases with the degree of risk-aversion. This difference between the lost sales case and backlogging case may ensue from the fact that in the backlogging case the decision-maker has the opportunity to fulfill a demand in the upcoming stages. Therefore, he/she may prefer to hold less serviceable product inventory by remanufacturing less.

The structure of the optimal policy for core inventory in Figures 5.13, 5.14, and 5.15 can again be characterized via a state-dependent threshold policy. We could not find any example violating this policy structure. However we again observe that the state-dependent threshold policy need not be optimal for serviceable product inventory: At stage 4, the optimal serviceable inventory level becomes 4 at state $(-5, 2, 5, 2)$. However, this level becomes 5 at state $(1, 2, 5, 2)$. We also found a counter example showing that discrete-convexity need not hold for our optimal cost function; see Figure 5.1. As in the case of lost sales, whether a state-dependent threshold policy is analytically optimal for core inventory in the case of backordering remains an open question in our research.

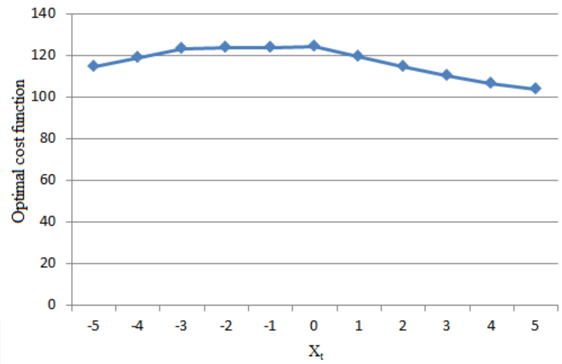

Figure 5.1: Optimal cost function V_0 when $Y_0 = 7$, $S_{-1} = 2$, $S_{-2} = 4$, $r = 1$, $\kappa = 0$, $c_m = 10$, $c_r = 4$, $c_c = 1$, $h_s = 2$, $h_r = 1$, $b = 18$, and $t_\Delta = 2$.

5.2 The Risk-Neutral Problem

Our primary goal in this section is to evaluate the performance of each heuristic in the risk-neutral case. We compare our heuristics with respect to (i) their percentage deviations from the risk-neutral optimal expected total cost at the initial stage when $X_0 = Y_0 = S_{-1} = S_{-2} = 0$ (ECPD) (ii) their percentage deviations from the risk-neutral optimal standard deviation of the total cost at the initial stage when $X_0 = Y_0 = S_{-1} = S_{-2} = 0$ (SDPD), and (iii) their computation times.

5.2.1 The Case of Lost Sales

Table 5.1 exhibits our numerical results in the case of lost sales: Although CEC has a distinct computational advantage over all the other heuristics, it has the worst performance with respect to the expected total cost. Unlike many paper dealing with traditional inventory problems, MP has a very poor performance in our closed-loop inventory problem. Despite its computational advantage, NRP leads to an increase of 12.67% in the expected total cost. FCP outperforms all the other heuristics with respect to the expected total cost. This is because FCP provides the maximum opportunity for remanufacturing, which takes priority over manufacturing at optimality. However, FCP performs worse than all the other heuristics with respect to computation time. This is because although the collection decision is eliminated from the problem since all the available cores are collected, the decision-maker still needs to decide on the amounts of manufacturing and remanufacturing. High standard deviation values in Table 5.1 validate that the risk-neutral decision-maker does not pay attention to cost variability.

5.2.2 The Case of Backlogging

Table 5.2 exhibits our numerical results in the case of backlogging: Our conclusions from the case of lost sales remain valid in this case. In addition, we observe that the solution times are much higher than in the case of lost sales. This is because S_t can take larger values in the backlogging case.

5.3 The Risk-Averse Problem

Our primary goal in this section is to evaluate the performance of each heuristic in the risk-averse case. We consider instances in which $\kappa \in \{0, 0.3, 0.5, 0.8, 1\}$ and $r \in \{1, 2, 3\}$. We again compare our heuristics with respect to (i) their percentage deviations from the risk-averse optimal expected total cost at the initial stage when $X_0 = Y_0 = S_{-1} = S_{-2} = 0$ (ECPD) (ii) their percentage deviations from the risk-averse optimal standard deviation of the total cost at the initial stage when $X_0 = Y_0 = S_{-1} = S_{-2} = 0$ (SDPD), and (iii) their computation times.

5.3.1 The Case of Lost Sales

Table 5.3 exhibits our numerical results for the optimal solution in the case of lost sales: The expected total cost gradually increases as the decision-maker becomes more risk-averse. This is because as the risk-aversion degree increases, the decision-maker tends to remanufacture/manufacture more to avoid losing sales. Also note that as risk-aversion increases, the standard deviation decreases substantially. This is because mean-semi deviation aims to minimize the difference between the mean and the realization, and higher risk-aversion leads to a lower standard deviation.

Figures 5.2 and 5.3 show how the expected total cost and the standard deviation vary depending on r and κ.

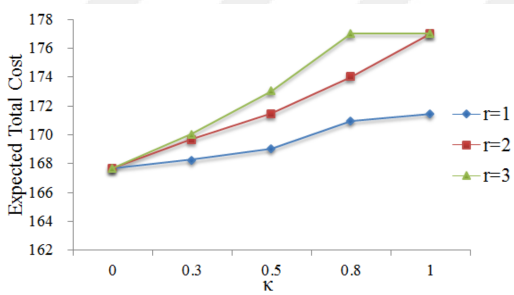

Figure 5.2: Expected total cost vs. κ

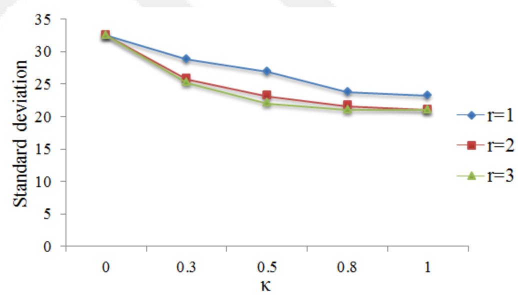

Figure 5.3: σ vs. κ

Table 5.4 exhibits our numerical results for MP in the case of lost sales. Both ECPD and SDPD values are quite large for almost all risk-aversion degrees. Such a poor performance of MP can be explained by the very complicated dynamics of state transitions and the high uncertainty involved in our closed-loop inventory problem.

Figures 5.4 and 5.5 show how the expected total cost and the standard deviation for MP vary depending on r and κ.

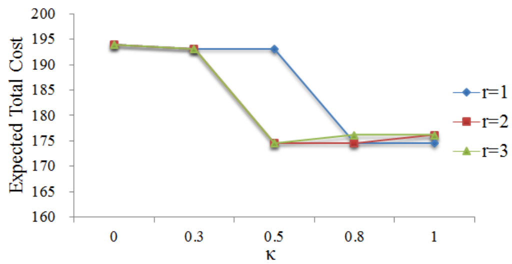

Figure 5.4: Expected total cost vs. κ

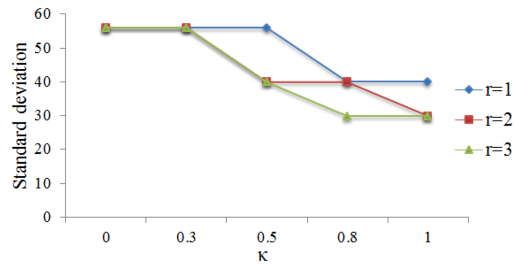

Figure 5.5: σ vs. κ

Table 5.5 exhibits our numerical results for NRP in the case of lost sales. We observe that NRP performs better than MP. NRP performs worse than the optimal solution by 12.11% with respect to the expected total cost and by 38.21% with respect to standard deviation, on average. Thus remanufacturing provides more than 10% decrease in the expected total cost on average.

Table 5.6 exhibits our numerical results for FTP in the case of lost sales. FTP performs better than MP and NRP with respect to both expected total cost and standard deviation. An important note here is that the optimal solution is the same when $r = 2, \kappa = 1$ and $r = 3, \kappa = 1$. This shows that further degrees of risk-aversion are not very crucial when $r \geq 2$.

Table 5.7 exhibits our numerical results for FCP in the case of lost sales: FCP outperforms all the other heuristics with respect to both expected total cost and standard deviation. Our choice of parameters ($c_m \geq c_r + c_c$), limited returns S_t, and the existence of an upper bound on Y_t can explain the success of this heuristic.

Note that the standard deviation when $r = 3, \kappa = 1.0$ is higher than when $r = 3, \kappa = 0.8$. This anomaly arises from the specified intervals for X_t and Y_t. As the degree of risk-aversion increases for this specific case, the global optimal solution goes beyond the specified bounds. For the purpose of more accurate comparison, we prefer not to increase the bounds when $r = 3, \kappa = 1$, keeping them the same as in the other cases.

Table 5.8 lists the average computation times for the heuristics. As stated before, CEC is the quickest heuristic in the risk-neutral case. MP, on the other hand, clearly has a distinct computational advantage in the risk-averse case. Such an advantage of MP over the other heuristics arises because it ignores the effect of future costs on any decision at the current stage. Although FCP surpasses all the other heuristics in terms of ECPD and SDPD, FCP has the worst performance with respect to computation times. Recall that collecting all available cores eliminates the collection decision but the decision-maker still needs to decide on the manufacturing and remanufacturing amounts.

Parameter Analysis. In this subsection, we examine how the heuristics behave under different parameter settings. For this purpose, we generate numerical instances by varying the value of each parameter as low, medium, and high. While solving the problem for a specific parameter value, we fix all the remaining parameters to their medium values. We label the decision-maker with $r = 1, \kappa = 1$ as *low risk-averse*, and the decision-maker with $r = 2, \kappa = 1$ as *high risk-averse*.

Table 5.9 exhibits the numerical results for the risk-neutral case. For low value of h_s, FCP has the best performance with respect to expected total cost, and FTP has the best performance with respect to cost variability. Although $h_s = h_r$, the decision-maker does not prefer to hold serviceable products in the inventory instead of cores since $c_m \geq c_r + c_c$. For high value of h_s, FCP has the best performance in terms of both criteria. Since $h_s > h_r$, the decision-maker prefers to hold cores instead of serviceable products in the inventory. When h_r is low, FTP is the exceeding solution approach. Although $h_r < h_s$, holding cores more than necessary causes the cost of FCP to be scattered in a wider range. When $h_r = 2$, FCP is the best heuristic in terms of the expected total cost. Although $h_s = h_r$, the decision-maker prefers to hold cores instead of serviceable products in the inventory.

When c_m is low, FCP outperforms the other heuristics in both criteria. Since we still have $c_m \geq c_r + c_c$, remanufacturing is still favored at optimality. When c_m is high, FCP again outperforms the other heuristics in terms of both criteria.

When c_r is low, FTP performs best with respect to standard deviation. Although c_r is now lower, holding excess cores in the inventory causes FCP instances to be more deviated. However, FCP is better with respect to expected total cost. When c_r is high, we observe the same behaviour. Since $c_m > c_r + c_c$, FCP is still less costly. Our results for c_c are similar to those for c_r.

When is low, FCP performs best with respect to expected total cost. When p is high, FCP is the best choice in both criteria. Our explanation for this result is that the decision-maker prefers to hold more cores in the inventory to avoid higher lost sales cost in the upcoming stages.

Next, we consider the risk-averse case in Table 5.10. Our conclusions for the risk-neutral case are valid for the low risk-averse case. When $r = 2, \kappa = 1$, FCP is a better choice than FTP. This is because now the decision-maker is more risk-averse and holding more cores in the inventory helps to reduce lost sales in the upcoming stages.

Pairwise Parameter Analysis. In this section, parameters that are thought to be related to each other are paired and the solutions for all heuristics are obtained for different combinations of these parameters.

Table 5.11 exhibits risk-neutral pair-wise analysis for c_r and c_c. For all possible combinations of c_c, c_r, FCP outperforms the other heuristics with respect to expected total cost. This result is logical since $c_m \geq c_r + c_c$ in every case so it is less costly to remanufacture as much as possible. When SDPD values are checked, FTP is the best choice. For the risk-averse cases, same intuitions follow and FCP performs best.

Table 5.13 exhibits the risk-neutral analysis for the h_r, c_c pair. For all combinations of h_r and c_c, FCP performs better than FTP. Because $c_m \geq c_r + c_c + h_r$, collecting and holding cores up to the upper bound becomes advantageous. Minimum SDPD values are provided by FTP in all cases. In low risk-averse case Table 5.14 displays the same results as in the risk-neutral case. The degree of risk-aversion may not be high enough to change the behaviours of heuristics.

In high risk-averse case, FCP gives the best performance for all combinations. Although the best heuristic performances did not change with increasing risk-aversion, being high risk-averse increases the performance of FCP with respect to standard deviation.

Table 5.15 exhibits our analysis for the h_r, h_s pair. Because $c_m \geq c_r + c_c + h_r$ in all cases, holding cores in the inventory is less costly. Thus FCP is the best heuristic in the risk-neutral case for all combinations. For the risk-averse cases, FCP performs best with respect to both criteria.

Lastly, Table 5.17 exhibits the risk-neutral heuristic performances for c_m, c_c, c_r. For all cases where $c_m < c_r + c_c$, solution of NRP becomes equal to the optimal solution. This result is logical since collecting and remanufacturing is now more costly than manufacturing and this makes the optimal policy equal to NRP policy. When $c_m > c_r + c_c$, FCP outperforms other heuristics. We observe the exact same intuitions for the risk-averse cases.

5.3.2 The Case of Backlogging

In this section, we solve the risk-averse problem for the backlogging case. The same intuitions in the lost sale case are still valid. It is easy to verify that the expected total cost increases as the decision-maker becomes more risk-averse. Table 5.19 shows that standard deviation gradually declines as expected. As it is seen by the percentage changes, a little increase in the cost can effectively decrease the percentage gap from the mean. In addition, after $r = 2, \kappa = 1$, expected total cost remains the same for the higher values of r. Consequently, we note that $r = 2$ case effectively provides the risk-averse optimal solution.

Figures 5.6 and 5.7 show how the expected total cost and the standard deviation vary depending on r and κ.

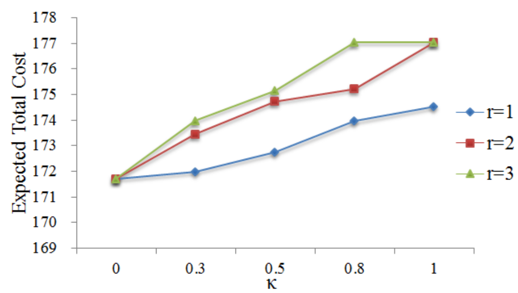

Figure 5.6: Expected total cost vs. κ

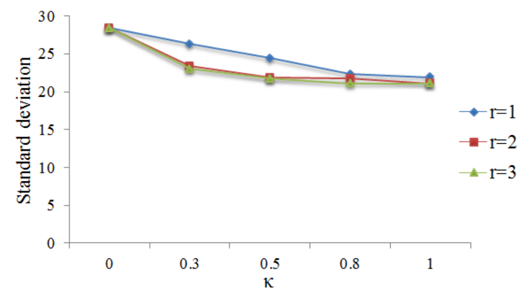

Figure 5.7: σ vs. κ

Table 5.20 exhibits our numerical results for MP. As in the case of lost sales, it is immediate that MP has a very poor performance. Both ECPD and SDPD values are quite large for almost all risk-aversion degrees.

Figures 5.8, and 5.9 show how the expected total cost and the standard deviation for MP vary depending on r and κ.

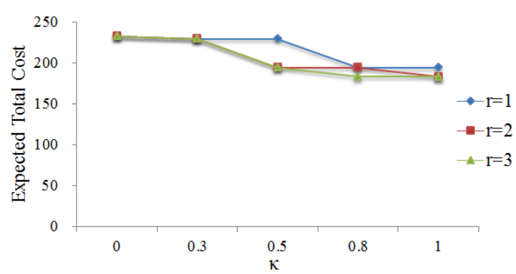

Figure 5.8: Expected total cost vs. κ

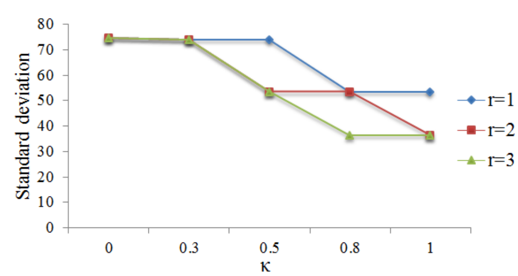

Figure 5.9: σ vs. κ

Table 5.21 exhibits our numerical results for NRP in the case of backlogging. The percentage gap from the expected total cost is 11.46 and standard deviation increases by 39.64%, on average. This shows that although NRP performs better than MP, absence of remanufacturing is reflected upon the expected total cost, and standard deviation. Note that if the problem was solved as a classical inventory problem with only manufacturing option, the optimal cost at first stage would be higher.

Table 5.22 exhibits our numerical results for FTP in the case of backlogging. In terms of the optimal solution, the optimal thresholds tend to increase with higher degrees of risk-aversion. An important insight is that the decision-maker prefers to manufacture/remanufacture more as the degree of risk-aversion increases. When we examine the results in terms of the expected total cost and the standard deviation, FTP has greater benefit compared to MP. Note that the optimal threshold levels are the same when $r = 2, \kappa = 1$ and $r = 3, \kappa = 1$. Thus further degrees of risk-aversion are not very crucial after $r \geq 2$.

Table 5.23 exhibits our numerical results for FCP in the case of backlogging. As in the case of lost sales, FCP outperforms all the other heuristics in terms of the expected total cost. However FTP outperforms FCP in terms of standard deviation. Because the decision-maker may now backlog a demand, he/she may want to increase the optimal threshold levels compared to the lost sales case. Thus the decision-maker may take optimal decisions closer to each other and this decreases standard deviation. Remark that the results show no change in optimality for $r = 2, \kappa = 1$ and $r = 3, \kappa = 1$. This shows that further degrees of risk-aversion are not very significant after reaching $r > 2$.

Table 5.24 lists the average computational times for the heuristics. Note that the same intuitions as in the case of lost sales are still valid. An important observation is that the solution times for the backlogging case are much higher than the case of lost sales. This increase arises from the wider ranges of S_t (a.k.a. future returns).

Parameter Analysis. In this section, we perform parameter analysis under the assumption of backlogging. Tables 5.25, and 5.26 exhibit our parameter analysis for both risk-neutral, and risk-averse cases.

When we analyze the risk-neutral results for different values of h_s, FCP outperforms other heuristics. Because $h_s \geq h_r$, the decision-maker prefers to hold cores in the inventory rather than stocking serviceable products.

For $h_r = 0.025$, FCP is the best policy since h_r value is low and hence, holding returned cores in the inventory is less costly. FCP is still the best policy when $h_r = 2$. Because now $h_s = h_r$, holding returned cores becomes equivalent to holding serviceable products.

When c_m is low, FCP performs best with respect to expected total cost. This result is logical since $c_m > c_r + c_c$ is still ensured. FTP outperforms other heuristics with respect to standard deviation. When c_m high, FCP performs best. This is because c_m is now higher than $c_r + c_c$ and the decision-maker prefers to remanufacture instead.

Analysis of c_r indicates that FCP performs best. Because $c_m \geq c_r + c_c$ still holds, FCP is still preferable. The same intuitions carry over to the analysis of c_c.

For different values of b, FCP outperforms other heuristics with respect to expected total cost. Standard deviation percentages, on the other hand, show that FTP is preferable over FCP.

When the degree of risk-aversion increases, FCP perform best in all cases. This result is justifiable because holding more cores in the inventory is now more appealing to the risk-averse decision-maker. In terms of SDPD, FTP values are closer to the mean for the low risk-averse case. When risk-aversion increases, FCP becomes preferable in terms of the standard deviation in nearly all cases.

Pairwise Parameter Analysis. Table 5.27 exhibits our numerical results for various combinations of c_r, and c_c for risk-neutral case. FCP performs best with respect to both criteria because $c_m < c_r + c_c$ in all cases. When the decision-maker becomes low risk-averse, FCP outperforms other heuristics with respect to expected total cost. However FTP performs best with respect to standard deviation. For the high risk-averse case, FCP performs best with respect to both criteria. The fact that the decision-maker wants to avoid backlogging any demand causes him/her to hold more cores in the inventory.

Next, Tables 5.29, and 5.30 examine the behaviour of heuristics for various values of h_r, and c_c. FCP performs best for all cases with respect to expected total cost. However FTP outperforms other heuristics with respect to standard deviation.

We next analyze the results for h_s, and h_r combinations in Tables 5.31 and 5.32. For the all cases, FCP outperforms other heuristics with respect to expected total cost. This behaviour is justified by the fact that the decision-maker wants to avoid backlogging as much as possible by holding cores in the inventory. FTP performs best with respect to standard deviation.

Last, we analyze computational results for various value combinations of c_m, c_r, and c_c. The same intuitions for the case of lost sales carry over to the backlogging case.

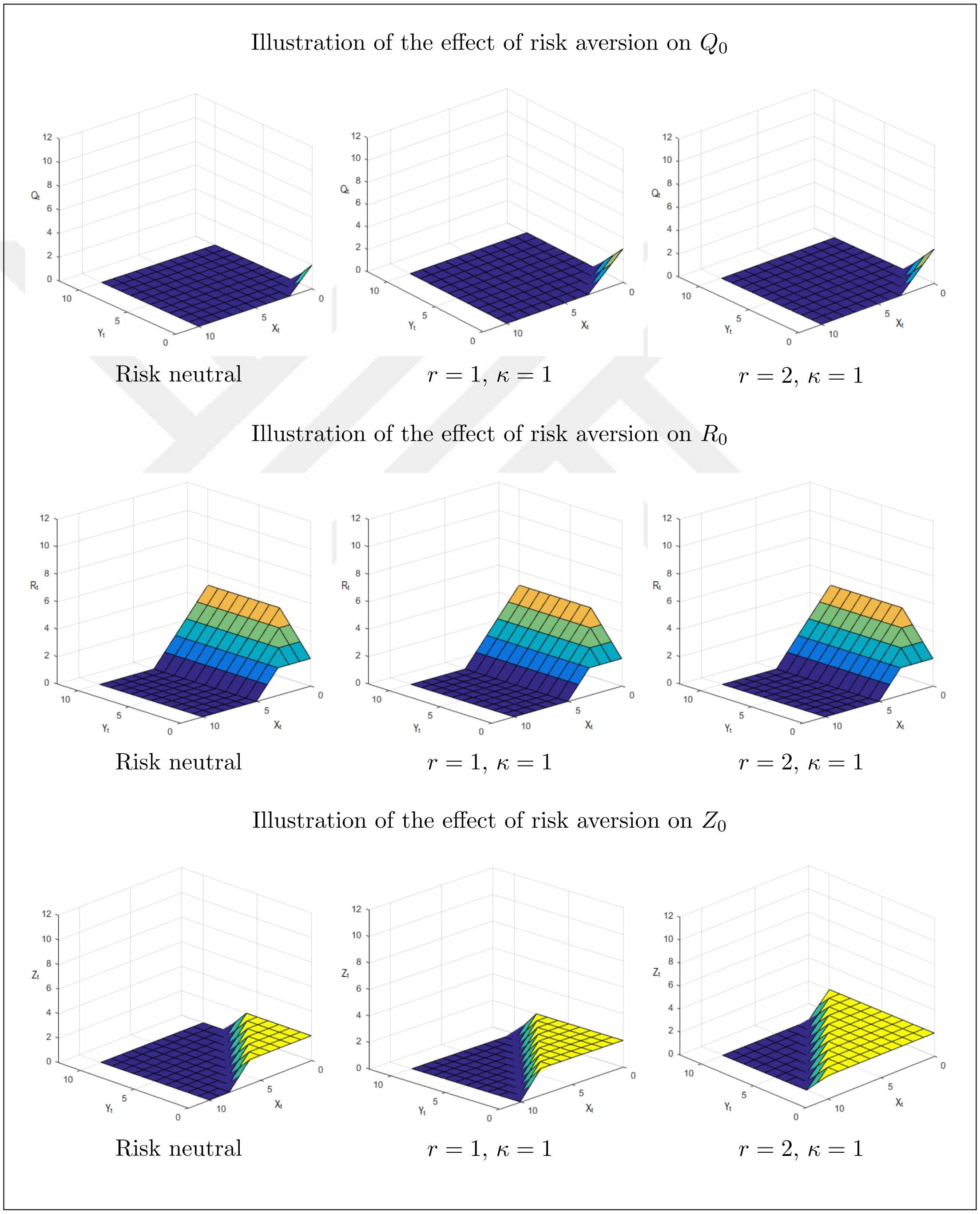

Figure 5.10: Illustration of the effect of risk aversion in the case of lost sales for $D_t \sim U(0, 5)$

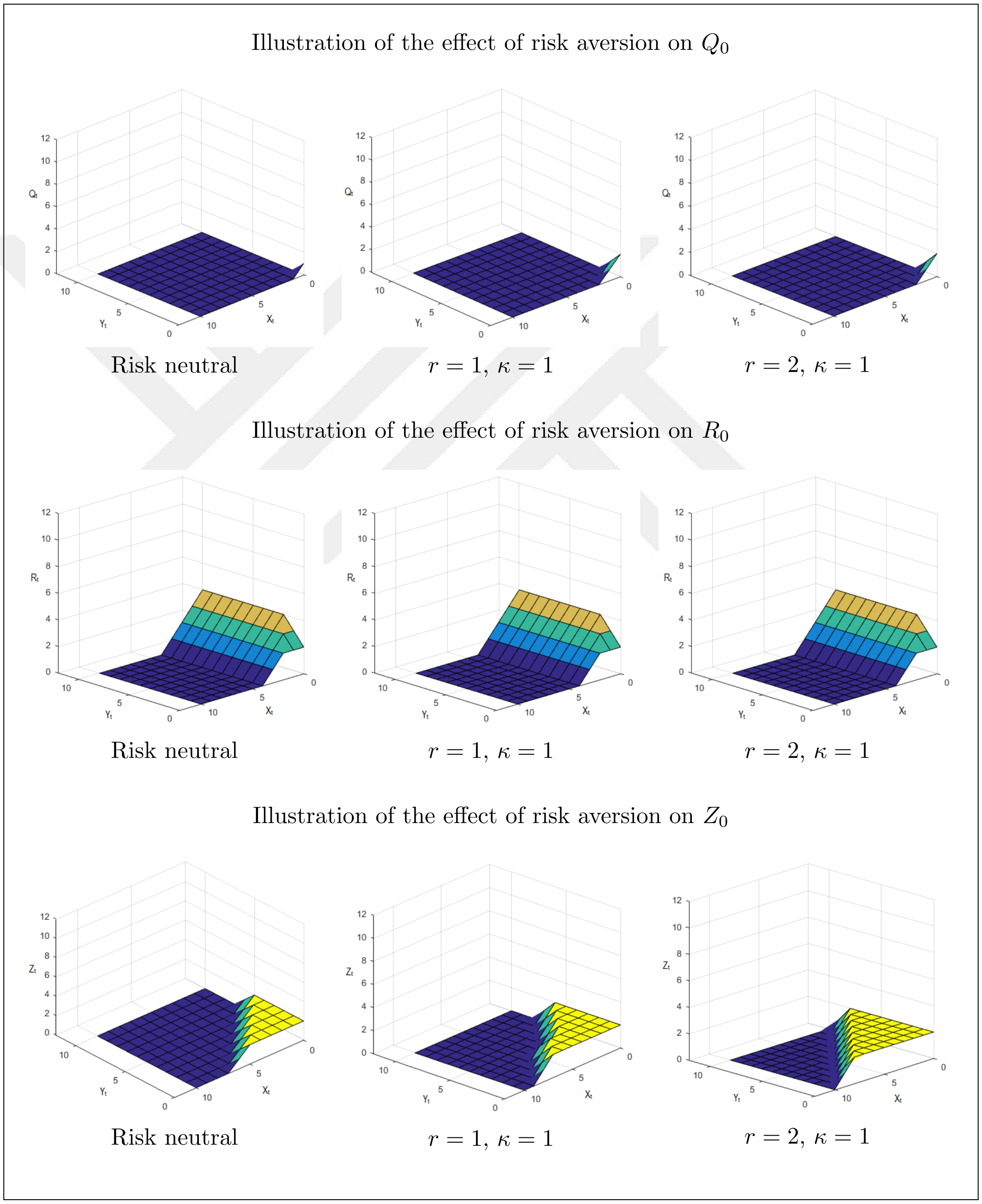

Figure 5.11: Illustration of the effect of risk aversion in the case of lost sales for $D_t \sim Bin(5, 0.5)$

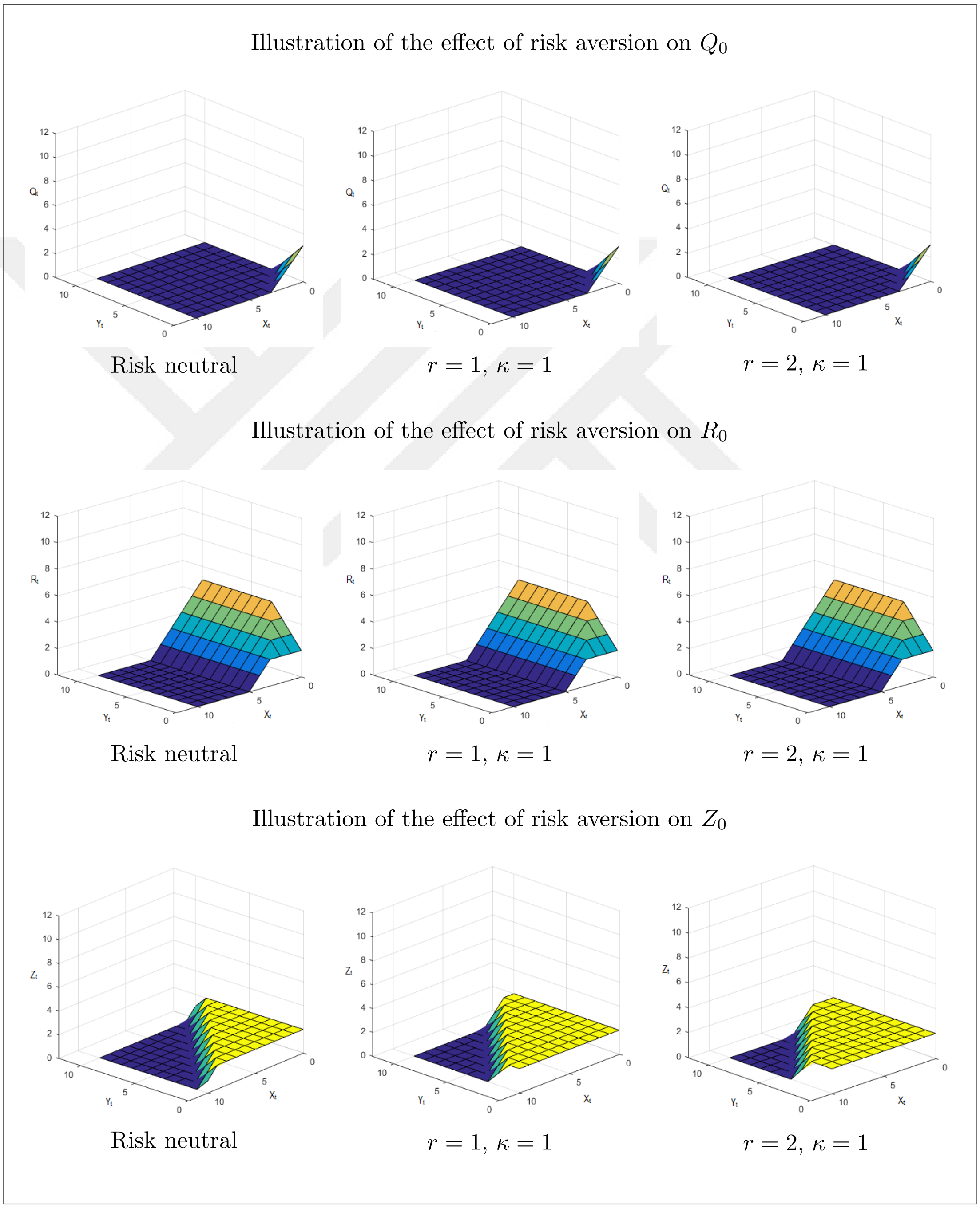

Figure 5.12: Illustration of the effect of risk aversion in the case of lost sales for $D_t \sim Bin(5, 0.75)$

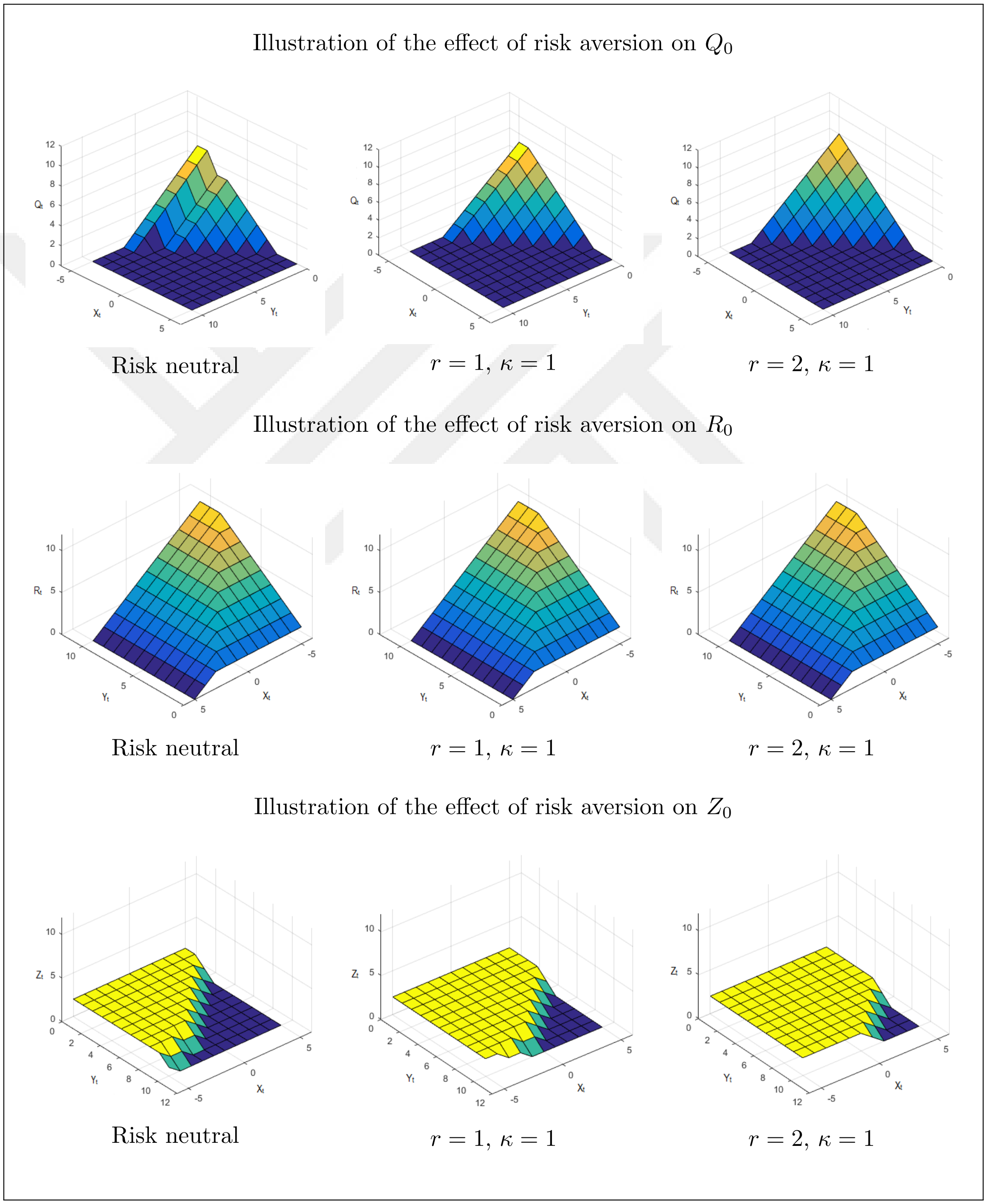

Figure 5.13: Illustration of the effect of risk aversion in the case of backlogging for $D_t \sim U(0, 5)$

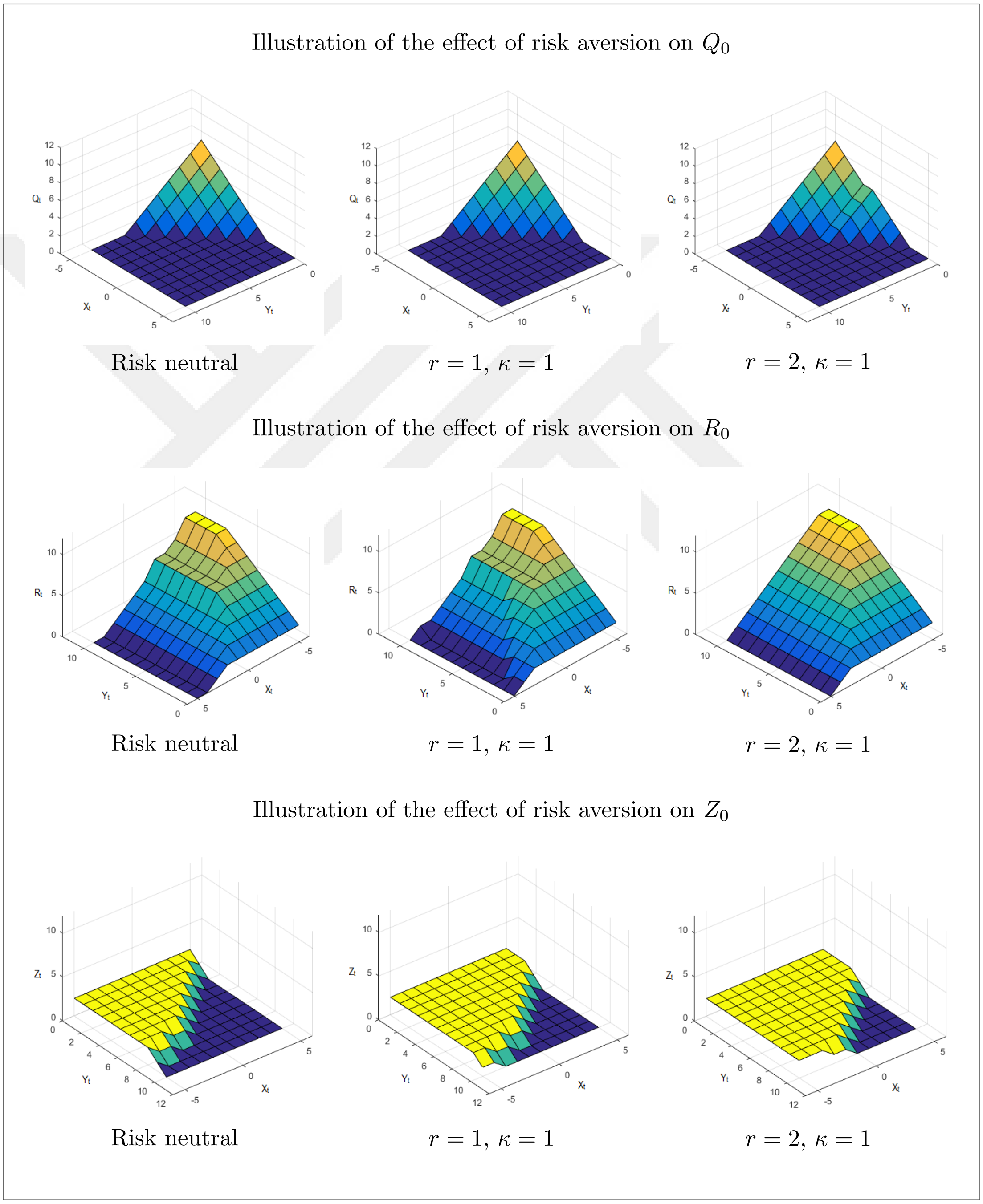

Figure 5.14: Illustration of the effect of risk aversion in the case of backlogging for $D_t \sim Bin(5, 0.5)$

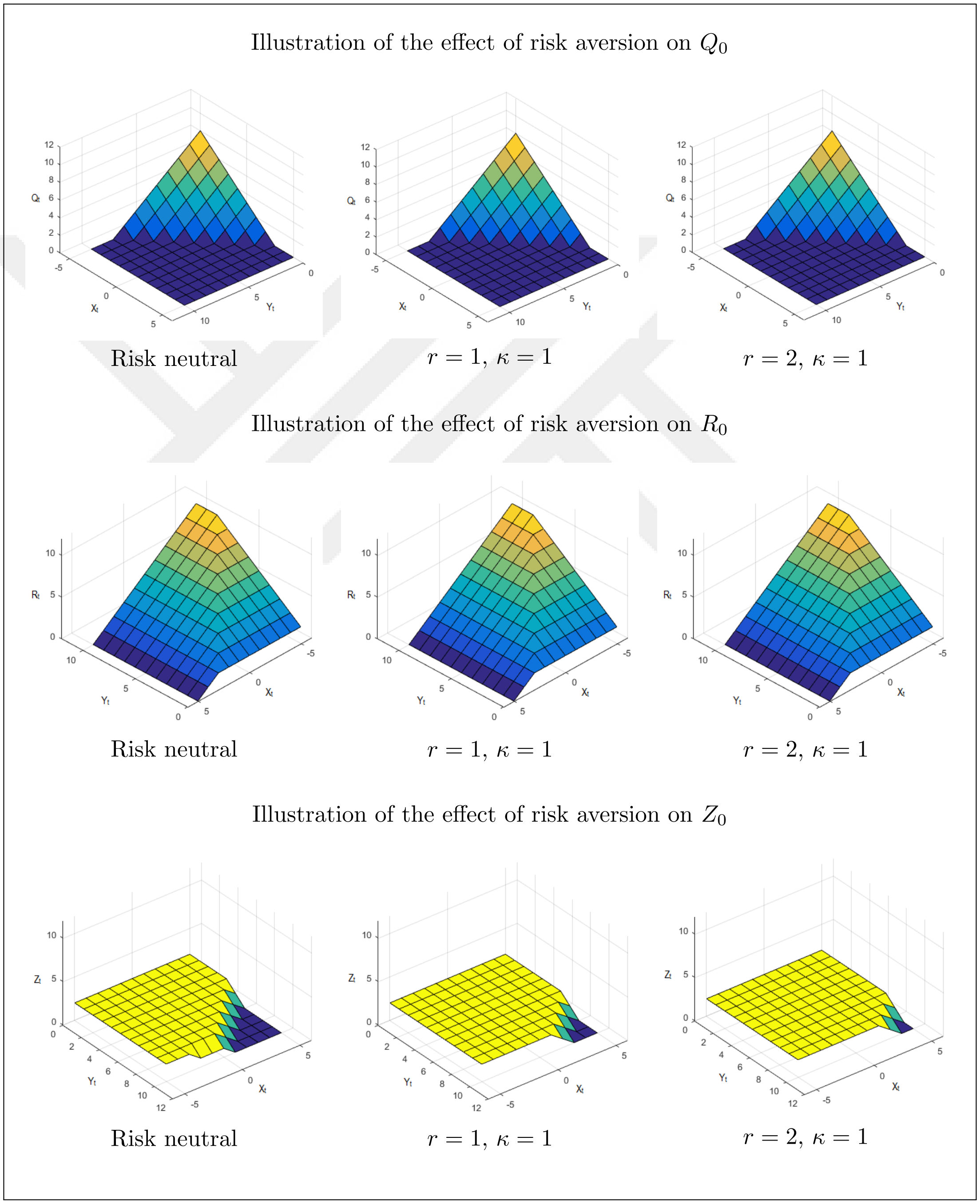

Figure 5.15: Illustration of the effect of risk aversion in the case of backlogging for $D_t \sim Bin(5, 0.75)$

Table 5.1: Numerical results for the risk-neutral problem with lost sales.

Solution method	Solution times (sec)	Expected total cost	ECPD (%)	Std. dev. of total cost	SDPD (%)
Exact	1698.34	167.644	-	32.568	-
CEC	45.02	196.192	17.03	56.116	72.31
MP	177.37	193.865	15.64	55.864	71.53
FTP	229.70	173.613	3.56	29.353	9.87
NRP	52.48	188.889	12.67	39.735	22.00
FCP	874.53	168.184	0.32	32.711	0.44

$c_r = 4$, $c_c = 1$, $c_m = 10$, $h_r = 1$, $h_s = 2$, $p = 18$, $t_\Delta = 2$, $T = 6$, $X_t \in \{0, 1, .., 10\}$, $Y_t \in \{0, 1, .., 10\}$, $S_t \in \{0, 1, .., 5\}$. Demand and collection rate follow discrete uniform distributions with supports $\{0, 1, .., 5\}$ and $\{1/3, 2/3, 1\}$, respectively.

Table 5.2: Numerical results for the risk-neutral problem with backlogging.

Solution method	Solution times (sec)	Expected total cost	ECPD (%)	Std. dev. of total cost	SDPD (%)
Exact	27311.45	171.689	-	28.414	-
CEC	63.49	239,497	39.49	76,215	168.23
MP	286.76	232.439	35.38	74.588	162.50
FTP	201.56	181.305	5.60	35.653	25.48
NRP	190.48	192.111	11.89	36.077	26.97
FCP	9832.98	172.840	0.67	28.743	1.16

$c_r = 4$, $c_c = 1$, $c_m = 10$, $h_r = 1$, $h_s = 2$, $b = 18$, $t_\Delta = 2$, $T = 6$, $X_t \in \{-5, -4, .., 5\}$, $Y_t \in \{0, 1, .., 10\}$, $S_t \in \{0, 1, ...10\}$. Demand and collection rate follow discrete uniform distributions with supports $\{0, 1, .., 5\}$ and $\{1/3, 2/3, 1\}$, respectively.

Table 5.3: Changes in the expected total cost and standard deviation for various values of r and κ for the case of lost sales.

		Expected total cost	ECPD (%)	Std. dev. of total cost	SDPD (%)
Risk-neutral		167.644	-	32.568	-
r	κ				
1	0.3	168.229	0.35	28.865	-11.37
	0.5	168.993	0.81	26.975	-17.17
	0.8	170.905	1.95	23.801	-26.92
	1	171.410	2.25	23.235	-28.66
2	0.3	169.650	1.20	25.838	-20.66
	0.5	171.410	2.25	23.235	-28.66
	0.8	174.004	3.79	21.765	-33.17
	1	177.011	5.59	21.118	-35.16
3	0.3	170.028	1.42	25.359	-22.13
	0.5	173.012	3.20	22.188	-31.87
	0.8	177.011	5.59	21.118	-35.16
	1	177.011	5.59	21.118	-35.16
Average		172.024	2.83	24.398	-27.17

$c_r = 4$, $c_c = 1$, $c_m = 10$, $h_r = 1$, $h_s = 2$, $p = 18$, $t_\Delta = 2$, $T = 6$, $X_t \in \{0, 1, .., 10\}$, $Y_t \in \{0, 1, .., 10\}$, $S_t \in \{0, 1, .., 5\}$. Demand and collection rate follow discrete uniform distributions with supports $\{0, 1, .., 5\}$ and $\{1/3, 2/3, 1\}$, respectively.

Table 5.4: Result comparisons between the the optimal value and myopic approach for the case of lost sales.

		Exact			Myopic Approach				
		Expected total cost	Std. dev. of total cost	Solution times(sec)	Expected total cost	Std. dev. of total cost	Solution times(sec)	ECPD (%)	SDPD (%)
Risk-neutral		167.648	32.568	1698.34	193.865	55.864	177.37	15.641	71.532
r	κ								
1	0.3	168.229	28.865	1676.79	193.096	55.876	160.38	14.781	93.578
-	0.5	168.993	26.975	1672.19	193.096	55.876	167.39	14.263	107.141
-	0.8	170.905	23.801	1690.74	174.444	39.931	171.13	2.071	67.768
-	1	171.410	23.235	1710.15	174.444	39.931	168.01	1.769	71.859
2	0.3	169.650	25.838	1682.11	193.096	55.876	169.79	13.820	116.255
-	0.5	171.410	23.235	1678.57	174.444	39.931	168.62	1.769	71.859
-	0.8	174.004	21.765	1627.69	174.444	39.931	170.76	0.253	83.462
-	1	177.011	21.118	1627.29	175.963	29.907	168.06	-0.592	41.699
3	0.3	170.028	25.359	1655.58	193.096	55.876	162.74	13.567	120.331
-	0.5	173.012	22.188	1678.31	174.444	39.931	165.86	0.827	79.966
-	0.8	177.011	21.118	1685.08	175.963	29.907	160.09	-0.592	41.619
-	1	177.011	21.118	1690.10	175.963	29.907	170.71	-0.592	41.619
Average		172.024	24.40	1674.84	182.028	43.75	167.76	5.922	77.591

$c_r = 4$, $c_c = 1$, $c_m = 10$, $h_r = 1$, $h_s = 2$, $p = 18$, $t_\Delta = 2$, $T = 6$, $X_t \in \{0, 1, .., 10\}$, $Y_t \in \{0, 1, .., 10\}$, $S_t \in \{0, 1, .., 5\}$. Demand and collection rate follow discrete uniform distributions with supports $\{0, 1, .., 5\}$ and $\{1/3, 2/3, 1\}$, respectively.

Table 5.5: No-Recovery Policy results in the case of lost sales.

		Solution times(sec)	Expected total cost	Std. dev. of total cost	ECPD (%)	SDPD (%)
Risk-neutral		52.48	188.889	39.735	12.67	22.00
r	κ					
1	0.3	51.23	188.889	39.735	12.28	37.66
	0.5	50.76	190.028	36.191	12.45	34.17
	0.8	51.08	192.000	32.769	12.34	37.68
	1	51.23	192.000	32.769	12.01	41.03
2	0.3	54.04	190.722	34.720	12.42	34.38
	0.5	52.01	192.000	32.769	12.01	41.03
	0.8	52.64	194.222	30.965	11.62	42.27
	1	53.73	197.778	30.268	11.73	43.33
3	0.3	58.68	190.722	34.720	12.17	36.91
	0.5	53.46	194.222	30.965	12.26	39.56
	0.8	60.37	197.778	30.268	11.73	43.33
	1	52.68	197.778	30.268	11.73	43.33
Average		53.41	192.848	33.549	12.11	38.21

$c_r = 4$, $c_c = 1$, $c_m = 10$, $h_r = 1$, $h_s = 2$, $p = 18$, $t_\Delta = 2$, $T = 6$, $X_t \in \{0, 1, .., 10\}$, $Y_t \in \{0, 1, .., 10\}$, $S_t \in \{0, 1, .., 5\}$. Demand and collection rate follow discrete uniform distributions with supports $\{0, 1, .., 5\}$ and $\{1/3, 2/3, 1\}$, respectively.

Table 5.6: Fixed-Threshold Policy results in the case of lost sales.

		Solution times(sec)	Expected total cost	Std. dev. of total cost	ECPD (%)	SDPD (%)
Risk-neutral		229.70	173.613	29.353	3.56	-9.87
r	κ					
1	0.3	243.75	173.613	29.353	3.20	1.69
	0.5	267.25	173.613	29.353	2.73	8.82
	0.8	204.75	173.613	29.353	1.58	23.33
	1	251.60	173.613	29.353	1.29	26.33
2	0.3	226.60	173.613	29.353	2.34	13.60
	0.5	265.65	173.613	29.353	1.29	26.33
	0.8	195.35	183.122	21.640	5.24	-0.57
	1	250.00	183.122	21.640	3.45	2.47
3	0.3	181.3	183.122	21.640	7.70	-14.67
	0.5	265.65	183.122	21.640	5.84	-2.47
	0.8	226.6	183.122	21.640	3.45	2.47
	1	273.5	183.122	21.640	3.45	2.47
Average		237.05	178.00	25.79	3.47	6.15

$c_r = 4$, $c_c = 1$, $c_m = 10$, $h_r = 1$, $h_s = 2$, $p = 18$, $t_\Delta = 2$, $T = 6$, $X_t \in \{0, 1, .., 10\}$, $Y_t \in \{0, 1, .., 10\}$, $S_t \in \{0, 1, .., 5\}$. Demand and collection rate follow discrete uniform distributions with supports $\{0, 1, .., 5\}$ and $\{1/3, 2/3, 1\}$, respectively. Produce-up-to level and collect-up-to level values are taken in intervals $\delta_P \in \{2, 3, 4, 5, 6\}$ and $\delta_C \in \{1, 2, 3, 4, 5\}$, respectively.

Table 5.7: Full-Collection Policy results in the case of lost sales.

		Solution times(sec)	Expected total cost	Std. dev. of total cost	ECPD (%)	SDPD (%)
Risk-neutral		874.53	168.184	32.711	0.32	0.44
r	κ					
1	0.3	896.04	168.743	29.189	0.30	1.12
	0.5	880.05	169.373	27.680	0.22	2.61
	0.8	904.42	171.492	24.207	0.34	1.71
	1	915.60	172.218	23.628	0.47	1.69
2	0.3	912.37	169.727	26.997	0.04	4.49
	0.5	911.08	172.238	23.411	0.48	0.76
	0.8	868.54	175.281	22.054	0.73	1.33
	1	940.76	178.409	21.172	0.79	0.26
3	0.3	991.96	170.804	25.442	0.46	0.33
	0.5	890.89	174.409	22.289	0.81	0.46
	0.8	879.88	178.702	21.191	0.96	0.35
	1	864.99	179.016	**21.349**	1.13	**1.09**
Average		902.39	172.969	24.998	0.54	1.30

$c_r = 4$, $c_c = 1$, $c_m = 10$, $h_r = 1$, $h_s = 2$, $p = 18$, $t_\Delta = 2$, $T = 6$, $X_t \in \{0, 1, .., 10\}$, $Y_t \in \{0, 1, .., 10\}$, $S_t \in \{0, 1, .., 5\}$. Demand and collection rate follow discrete uniform distributions with supports $\{0, 1, .., 5\}$ and $\{1/3, 2/3, 1\}$, respectively.

Table 5.8: Solution time comparison in the case of lost sales.

	Exact	CEC	MP	NRP	FCP	FTP
Average	1674.84	45.02	167.76	53.41	902.39	237.05
Std. Deviation	24.72	0	4.73	2.92	34.58	28.37
Min	1627.29	45.02	160.09	50.76	864.99	181.3
Max	1710.15	45.02	177.37	60.37	991.96	273.5

Table 5.9: Parameter analysis for the case of lost sales: Risk-neutral case

		Exact		ECPD(%)					SDPD(%)				
Risk-neutral		Expected total cost	Std. dev. of total cost	CEC	MP	NRP	FCP	FTP	CEC	MP	NRP	FCP	FTP
h_s	1	156.572	31.779	23.39	21.13	13.74	0.40	4.50	80.45	80.64	20.74	3.11	1.32
	2	167.644	32.568	17.03	15.64	12.67	0.32	3.56	72.31	71.53	22.01	0.44	-9.87
	3	176.530	34.441	12.84	11.69	11.81	0.29	3.01	59.46	58.18	14.49	0.63	13.07
h_r	0.025	166.806	32.809	17.27	16.22	13.24	0.20	2.86	71.76	70.27	21.11	-0.69	-9.13
	1	167.644	32.568	17.03	15.64	12.67	0.32	3.56	72.31	71.53	22.01	0.44	-9.87
	2	168.00	34.405	17.13	15.40	12.43	0.46	4.58	62.45	62.37	15.49	-0.45	-15.58
c_m	7	137.159	22.593	28.14	9.23	5.88	0.99	2.68	140.13	72.65	11.06	3.16	11.13
	10	167.644	32.568	17.03	15.64	12.67	0.32	3.56	72.31	71.53	22.01	0.44	-9.87
	13	194.105	42.301	11.60	23.44	17.84	0.12	3.81	37.84	64.58	25.56	-0.68	2.81
c_r	2	157.338	30.755	21.28	18.89	20.05	0.21	4.31	80.15	80.25	29.20	0.73	-10.47
	4	167.644	32.568	17.03	15.64	12.67	0.32	3.56	72.31	71.53	22.01	0.44	-9.87
	6	177.118	36.329	13.80	12.66	6.65	0.64	3.38	56.88	55.93	9.38	0.92	-12.36
c_c	0.025	162.655	30.921	18.98	16.90	16.13	0.12	3.54	80.39	79.93	28.50	2.51	-7.93
	1	167.644	32.568	17.03	15.64	12.67	0.32	3.56	72.31	71.53	22.01	0.44	-9.87
	2	172.447	34.769	15.35	14.07	9.53	0.61	3.77	62.51	61.74	14.28	1.70	-12.42
p	12	151.783	34.219	5.54	18.59	12.37	0.14	3.99	15.60	46.70	29.43	-0.35	-11.35
	18	167.644	32.568	17.03	15.64	12.67	0.32	3.56	72.31	71.53	22.01	0.44	-9.87
	24	173.627	29.371	33.73	9.74	12.55	0.49	3.45	148.09	71.39	20.85	2.96	15.67

While observing the effect of change for a specific parameter, all the remaining parameters are set to their medium values.

Table 5.10: Parameter analysis for the case of lost sales: Risk-averse case

r	κ	Exact		ECPD(%)				SDPD(%)			
1.0	1.0	ETC	σ	MP	NRP	FCP	FTP	MP	NRP	FCP	FTP
h_s	1	159.285	25.681	5.35	15.19	0.93	2.72	64.39	35.43	-0.47	-2.72
	2	171.410	23.235	1.77	12.01	0.47	1.29	71.86	41.03	1.69	26.33
	3	183.071	21.453	-1.09	9.07	0.02	0.30	76.09	62.42	4.01	24.49
h_r	0.025	170.061	23.374	2.58	12.90	0.34	0.89	70.83	40.19	-0.02	27.54
	1	171.410	23.235	1.77	12.01	0.47	1.29	71.86	41.03	1.69	26.33
	2	172.295	23.522	1.25	11.44	0.76	1.98	69.76	39.31	1.33	23.48
c_m	7	140.121	15.691	0.05	5.46	0.91	3.86	60.66	23.52	2.07	-1.96
	10	171.410	23.235	1.77	12.01	0.47	1.29	71.86	41.03	1.69	26.33
	13	199.749	31.153	7.17	15.84	0.06	3.32	86.36	53.43	1.16	10.95
c_r	2	161.425	21.396	2.61	18.94	0.38	1.67	78.28	53.15	0.01	28.69
	4	171.410	23.235	1.77	12.01	0.47	1.29	71.86	41.03	1.69	26.33
	6	181.213	26.085	1.12	5.95	0.55	1.04	61.37	25.62	1.24	22.06
c_c	0.025	166.356	22.278	2.28	15.42	0.35	1.24	75.10	47.09	-0.09	27.79
	1	171.410	23.235	1.77	12.01	0.47	1.29	71.86	41.03	1.69	26.33
	2	176.453	24.506	1.36	8.81	0.84	1.41	67.18	33.72	1.75	24.26
p	12	155.313	27.763	8.15	10.78	0.41	1.63	67.51	39.97	-0.42	9.26
	18	171.410	23.235	1.77	12.01	0.47	1.29	71.86	41.03	1.69	26.33
	24	177.071	21.914	2.76	12.16	0.73	3.42	57.44	40.22	-0.39	-1.25
2.0	1.0										
h_s	1	163.293	24.565	0.06	17.18	0.52	2.96	29.30	38.80	0.22	1.7
	2	177.011	21.118	-0.59	11.73	0.79	3.45	41.62	43.33	0.26	2.47
	3	188.520	18.665	-3.95	10.38	0.29	5.09	102.39	46.19	0.69	-0.13
h_r	0.025	174.756	21.664	0.69	13.17	0.14	3.35	60.23	62.16	0.14	2.03
	1	177.011	21.118	-0.59	11.73	0.79	3.45	41.62	43.33	0.26	2.47
	2	178.760	20.971	-1.57	10.64	0.97	3.88	42.61	44.33	0.30	2.52
c_m	7	140.669	15.316	-0.34	5.05	1.69	3.60	64.59	26.54	1.77	0.42
	10	177.011	21.118	-0.59	11.73	0.79	3.45	41.62	43.33	0.26	2.47
	13	205.997	28.015	-2.08	17.59	0.49	7.04	52.39	47.37	-0.42	1.60
c_r	2	166.906	19.073	-1.12	18.50	0.21	3.68	41.11	58.70	0.29	3.42
	4	177.011	21.118	-0.59	11.73	0.79	3.45	41.62	43.33	0.26	2.47
	6	186.069	24.187	-1.41	6.29	0.92	3.83	33.81	25.14	1.11	1.82
c_c	0.025	171.587	20.095	0.42	15.26	0.69	3.44	44.05	50.62	-0.46	2.54
	1	177.011	21.118	-0.59	11.73	0.79	3.45	41.62	43.33	0.26	2.47
	2	181.876	22.427	-1.19	8.74	1.08	3.86	38.49	34.96	0.64	2.65
p	12	163.502	22.675	2.74	8.80	-0.14	2.51	105.1	50.47	1.87	10.60
	18	177.011	21.118	-0.59	11.73	0.79	3.45	41.62	43.33	0.26	2.47
	24	178.373	21.476	0.48	11.35	0.63	2.66	53.10	43.09	0.94	0.76

While observing the effect of change for a specific parameter, all the remaining parameters are set to their medium values.

Table 5.11: Numerical results for various values of c_r and c_c: Risk-neutral case

	Exact		ECPD(%)					SDPD(%)				
(c_c, c_r)	Expected total cost	Std. dev. of total cost	CEC	MP	NRP	FCP	FTP	CEC	MP	NRP	FCP	FTP
(0.025, 2)	152.172	30.324	23.65	20.99	24.13	0.10	4.44	81.88	82.35	31.03	-0.33	-10.92
(0.025, 4)	162.655	30.921	18.98	16.90	16.13	0.12	3.54	80.39	79.93	28.50	2.51	-7.93
(0.025, 6)	172.528	35.035	15.29	14.06	9.48	0.31	3.12	61.49	60.54	13.42	1.07	-12.50
(1, 2)	157.338	30.755	21.28	18.89	20.05	0.21	4.31	80.15	80.25	29.20	0.73	-10.47
(1, 4)	167.644	32.568	17.03	15.64	12.67	0.32	3.56	72.31	71.53	22.01	0.44	-9.87
(1, 6)	177.118	36.329	13.80	12.66	6.65	0.64	3.38	56.88	55.93	9.38	0.92	-12.36
(2, 2)	162.607	31.182	19.03	16.89	16.16	0.32	4.21	78.66	78.41	27.43	2.12	-9.27
(2, 4)	172.447	34.769	15.35	14.07	9.53	0.61	3.77	62.51	61.74	14.28	1.70	-12.42
(2, 6)	181.540	38.066	12.53	12.64	4.05	1.11	3.71	50.94	51.21	4.38	-0.32	-6.62

While observing the correlation between a specific pair of parameters, all the remaining parameters are set to their medium values.

Table 5.12: Numerical results for various values of c_r and c_c: Risk-averse case

r	κ		Exact		ECPD(%)				SDPD(%)			
1	1	(c_c, c_r)	Expected total cost	Std. dev. of total cost	MP	NRP	FCP	FTP	MP	NRP	FCP	FTP
		(0.025, 2)	156.183	21.078	2.08	22.93	0.17	1.75	76.26	55.47	-0.26	28.16
		(0.025, 4)	162.655	22.278	2.28	15.42	0.35	1.24	75.40	47.09	-0.09	27.79
		(0.025, 6)	176.313	24.641	1.50	8.90	0.34	0.90	66.37	32.99	-0.01	24.41
		(1, 2)	157.338	21.396	2.61	18.94	0.38	1.67	78.28	53.15	0.01	28.69
		(1, 4)	171.410	23.235	1.77	12.01	0.47	1.29	71.86	41.03	1.69	26.33
		(1, 6)	177.118	26.085	1.12	5.95	0.55	1.04	61.37	25.62	1.24	22.06
		(2, 2)	166.610	22.105	2.06	15.24	0.49	1.71	76.37	48.24	1.90	27.98
		(2, 4)	172.447	24.506	1.36	8.81	0.84	1.41	67.18	33.72	1.75	24.26
		(2, 6)	184.837	28.541	2.15	3.88	1.59	1.95	56.60	14.81	-1.19	16.36
2	1											
		(0.025, 2)	161.710	18.659	-0.72	22.30	-0.05	3.53	41.14	62.22	0.24	3.60
		(0.025, 4)	171.587	20.095	0.42	15.26	0.69	3.44	44.05	50.62	-0.46	2.54
		(0.025, 6)	181.308	22.728	-0.83%	9.08	0.46	3.45	36.79	33.17	0.50	1.73
		(1, 2)	166.906	19.073	-1.12	18.50	0.21	3.68	41.11	58.70	0.29	3.42
		(1, 4)	177.011	21.118	-0.59	11.73	0.79	3.45	41.62	43.33	0.26	2.47
		(1, 6)	186.069	24.187	-1.41	6.29	0.92	3.83	33.81	25.14	1.11	1.82
		(2, 2)	171.787	20.014	0.25	15.13	0.72	4.10	44.51	51.23	-0.18	2.62
		(2, 4)	178.373	22.427	-1.19	8.74	1.08	3.86	38.49	34.96	0.64	2.65
		(2, 6)	190.540	25.981	-1.76	3.80	1.45	4.43	30.10	16.50	1.04	1.63

While observing the correlation between a specific pair of parameters, all the remaining parameters are set to their medium values.

Table 5.13: Numerical results for various values of h_r and c_c: Risk-neutral case

	Exact		ECPD(%)					SDPD(%)				
$(h_r,\ c_c)$	Expected total cost	Std. dev. of total cost	CEC	MP	NRP	FCP	FTP	CEC	MP	NRP	FCP	FTP
(0.025, 0.025)	161.787	30.742	19.27	17.53	16.75	0.44	2.84	82.23	80.98	29.25	-11.44	-5.70
(0.025, 1)	166.806	32.809	17.27	16.22	13.24	0.20	0.89	71.76	70.27	21.11	-0.69	-9.13
(0.025, 2)	171.702	35.195	15.52	14.56	10.01	1.02	3.03	61.20	59.78	12.90	-15.60	-12.36
(1, 0.025)	162.655	30.921	18.98	16.90	16.13	0.12	3.54	80.39	79.93	28.50	2.51	-7.93
(1, 1)	167.644	32.568	17.03	15.64	12.67	0.32	3.56	72.31	71.53	22.01	0.44	-9.87
(1, 2)	172.447	34.769	15.35	14.07	9.53	0.61	3.77	62.51	61.74	14.28	1.70	-12.42
(2, 0.025)	163.143	31.358	18.96	16.56	15.78	0.99	4.51	77.20	77.43	26.71	-9.95	-10.39
(2, 1)	168.00	34.405	17.13	15.40	12.43	0.46	4.58	62.45	62.37	15.49	-0.45	-15.58
(2, 2)	172.726	34.921	15.51	13.88	9.36	1.57	4.80	61.17	61.03	13.79	-11.54	-11.86

While observing the correlation between a specific pair of parameters, all the remaining parameters are set to their medium values.

Table 5.14: Numerical results for various values of h_r and c_c: Risk-averse case

r	κ		Exact		ECPD(%)				SDPD(%)			
			Expected total cost	Std. dev. of total cost								
1	1	(h_r, c_c)			MP	NRP	FCP	FTP	MP	NRP	FCP	FTP
		(0.025, 0.025)	165.002	22.430	3.12	16.36	0.74	0.83	73.91	46.09	-3.55	29.25
		(0.025, 1)	170.061	23.374	2.58	12.90	0.34	0.89	70.83	40.19	-0.02	27.54
		(0.025, 2)	175.233	24.587	2.06	9.57	1.24	0.96	66.63	32.28	-2.62	25.46
		(1, 0.025)	166.356	22.278	2.28	15.42	0.35	1.24	75.10	47.09	-0.09	27.79
		(1, 1)	171.410	23.235	1.77	12.01	0.47	1.29	71.86	41.03	1.69	26.33
		(1, 2)	176.453	24.506	1.36	8.81	0.84	1.41	67.18	33.72	1.75	24.26
		(2, 0.025)	167.385	22.517	1.65	14.71	2.21	1.86	73.24	45.53	-5.38	24.79
		(2, 1)	172.295	23.522	1.25	11.44	0.76	1.98	69.76	39.31	1.33	23.48
		(2, 2)	177.403	24.548	0.81	8.23	2.32	2.05	66.89	33.49	-2.02	23.04
2	1											
		(0.025, 0.025)	169.332	20.733	1.76	16.80	0.91	3.34	39.61	45.99	-1.02	2.33
		(0.025, 1)	174.756	18.665	0.69	13.17	0.14	3.35	60.23	62.16	0.14	2.03
		(0.025, 2)	179.751	22.870	-0.03	10.03	1.41	3.70	35.81	32.35	0.76	2.06
		(1, 0.025)	171.587	20.095	0.42	15.26	0.69	3.44	44.05	50.62	-0.46	2.54
		(1, 1)	177.011	21.118	-0.59	11.73	0.79	3.45	41.62	43.33	0.26	2.47
		(1, 2)	181.876	22.427	-1.19	8.74	1.08	3.86	38.49	34.96	0.64	2.65
		(2, 0.025)	173.286	19.893	-0.56	14.13	1.70	3.91	45.51	52.15	0.16	2.16
		(2, 1)	178.759	20.971	-1.57	10.64	0.97	3.88	42.61	44.33	0.30	2.52
		(2, 2)	183.333	22.342	-1.98	7.88	2.34	4.44	39.02	35.48	2.55	2.99

While observing the correlation between a specific pair of parameters, all the remaining parameters are set to their medium values.

Table 5.15: Numerical results for various values of h_r and h_s: Risk-neutral case

	Exact		ECPD(%)					SDPD(%)				
(h_r, h_s)	Expected total cost	Std. dev. of total cost	CEC	MP	NRP	FCP	FTP	CEC	MP	NRP	FCP	FTP
(0.025, 1)	155.549	30.838	23.83	21.92	14.49	0.22	3.88	86.75	86.16	24.42	1.72	5.95
(0.025, 2)	166.806	32.809	17.27	16.22	13.24	0.20	2.86	71.76	70.27	21.11	-0.69	-9.13
(0.025, 3)	175.500	33.305	13.17	12.34	12.46	0.22	2.71	65.59	63.58	18.39	0.51	18.05
(1, 1)	156.572	31.779	23.39	21.13	13.74	0.40	4.50	80.45	80.64	20.74	3.11	1.32
(1, 2)	167.644	32.568	17.03	15.64	12.67	0.32	3.56	72.31	71.53	22.01	0.44	-9.87
(1, 3)	176.530	34.441	12.84	11.69	11.81	0.29	3.01	59.46	58.18	14.49	0.63	13.07
(2, 1)	156.699	32.142	22.42	21.03	13.65	0.60	5.74	80.87	78.60	19.38	1.82	-0.88
(2, 2)	168.00	34.405	17.13	15.40	12.43	0.46	4.58	62.45	62.37	15.49	-0.45	-15.58
(2, 3)	176.879	33.945	12.94	11.47	11.59	0.41	3.72	61.15	60.49	16.16	3.33	13.79

While observing the correlation between a specific pair of parameters, all the remaining parameters are set to their medium values.

Table 5.16: Numerical results for various values of h_r and h_s: Risk-averse case

r	κ		Exact		ECPD(%)				SDPD(%)			
1	1	(h_r, h_s)	Expected total cost	Std. dev. of total cost	MP	NRP	FCP	FTP	MP	NRP	FCP	FTP
		(0.025, 1)	157.524	26.091	6.53	14.50	0.39	5.14	61.81	33.30	0.05	-2.49
		(0.025, 2)	170.061	23.374	2.58	12.90	0.34	0.89	70.83	40.19	-0.02	27.54
		(0.025, 3)	181.749	21.807	-0.37	9.86	0.29	-0.09	73.23	59.78	-.005	24.46
		(1, 1)	159.285	25.681	5.35	15.19	0.93	2.72	64.39	35.43	-0.47	-2.72
		(1, 2)	171.410	23.235	1.77	12.01	0.47	1.29	71.86	41.03	1.69	26.33
		(1, 3)	183.071	21.453	-1.09	9.07	0.02	0.30	76.09	62.42	4.01	24.49
		(2, 1)	159.637	25.601	5.12	12.98	0.63	6.92	64.90	35.85	2.18	-3.13
		(2, 2)	172.295	23.522	1.25	11.44	0.76	1.98	69.76	39.31	1.33	23.48
		(2, 3)	183.349	22.054	-1.24	8.90	0.08	1.28	71.29	57.99	5.99	19.88
2	1											
		(0.025, 1)	161.354	24.982	1.27	13.71	0.34	2.64	27.14	36.48	0.26	1.84
		(0.025, 2)	174.756	18.665	0.69	13.17	0.14	3.35	60.23	62.16	15.99	2.03
		(0.025, 3)	186.545	18.979	-2.93	11.55	0.08	4.86	99.05	43.77	0.47	0.64
		(1, 1)	163.293	24.565	0.06	17.18	0.52	2.96	29.30	38.80	0.22	1.7
		(1, 2)	177.011	21.118	-0.59	11.73	0.79	3.45	41.62	43.33	0.26	2.47
		(1, 3)	188.519	18.665	-3.95	10.38	0.29	5.09	102.39	46.19	0.69	-0.13
		(2, 1)	163.910	24.406	-0.31	11.93	0.92	4.14	30.14	39.70	0.76	1.61
		(2, 2)	178.759	20.971	-1.57	10.64	0.97	3.88	42.61	44.33	0.30	2.52
		(2, 3)	189.866	18.572	-4.63	9.59	0.60	5.70	103.41	46.93	1.79	-0.08

While observing the correlation between a specific pair of parameters, all the remaining parameters are set to their medium values.

Table 5.17: Numerical results for various values of c_m, c_r and c_c: Risk-neutral case

	Exact		ECPD(%)					SDPD(%)				
(c_m, c_r, c_c)	Expected total cost	Std. dev. of total cost	CEC	MP	NRP	FCP	FTP	CEC	MP	NRP	FCP	FTP
(3, 6, 0.025)	81.111	6.616	75.69	4.79	0.00	16.75	11.79	678.26	170.39	0.00	81.45	5.17
(3, 2, 2)	81.111	6.616	75.69	4.79	0.00	11.12	5.79	678.26	170.39	0.00	73.75	-17.00
(13, 6, 0.025)	199.084	42.977	10.18	20.95	14.89	0.14	3.29	36.58	62.02	23.58	0.42	3.80
(13, 2, 2)	189.078	41.314	13.17	26.11	20.97	0.13	4.45	40.31	68.51	28.56	-0.22	2.86

While observing the correlation between a specific pair of parameters, all the remaining parameters are set to their medium values.

Table 5.18: Numerical results for various values of c_m, c_r and c_c: Risk-averse case

r	κ		Exact		ECPD(%)				SDPD(%)			
1	1	(c_m, c_r, c_c)	Expected total cost	Std. dev. of total cost	MP	NRP	FCP	FTP	MP	NRP	FCP	FTP
		(3, 6, 0.025)	81.111	6.616	4.79	0.00	16.79	11.79	170.39	0.00	72.78	5.17
		(3, 2, 2)	81.111	6.616	4.79	0.00	11.84	5.79	170.39	0.00	36.67	-17.00
		(13, 6, 0.025)	204.852	32.388	6.01	12.95	0.18	2.84	79.97	47.58	-0.10	10.29
		(13, 2, 2)	194.210	30.630	8.67	19.14	0.33	4.12	88.97	56.05	0.18	9.75
2	1											
		(3, 6, 0.025)	82.500	5.123	0.00	0.00	16.51	9.91	0.00	0.00	107.65	35.82
		(3, 2, 2)	82.500	5.123	0.00	0.00	11.02	4.01	0.00	0.00	58.95	7.18
		(13, 6, 0.025)	210.980	29.262	-2.25	14.81	0.09	6.62	49.24	41.09	0.30	1.52
		(13, 2, 2)	200.663	27.091	-1.67	20.71	0.48	7.75	54.42	52.40	-0.23	1.62

While observing the correlation between a specific pair of parameters, all the remaining parameters are set to their medium values.

Table 5.19: Optimal values for the risk-averse problem in the case of backlogging.

		Expected total cost	ECPD(%) (%)	Std. dev. total cost	SDPD(%) (%)
Risk-neutral		171.689	-	28.414	-
r	κ				
1	0.3	171.975	0.17	26.333	-7.33
	0.5	172.724	0.60	24.435	-14.00
	0.8	173.940	1.31	22.333	-21.40
	1	174.513	1.64	21.929	-22.82
2	0.3	173.439	1.02	23.386	-17.69
	0.5	174.709	1.76	21.831	-23.17
	0.8	175.199	2.04	21.722	-23.55
	1	177.011	3.10	21.118	-25.68
3	0.3	173.969	1.91	22.977	-19.14
	0.5	175.124	1.99	21.721	-23.55
	0.8	177.011	3.10	21.118	-25.68
	1	177.011	3.10	21.118	-25.68
Average		174.486	1.81	22.957	-20.81

$c_r = 4$, $c_c = 1$, $c_m = 10$, $h_r = 1$, $h_s = 2$, $b = 18$, $t_\Delta = 2$, $T = 6$, $X_t \in \{-5, -4, .., 5\}$, $Y_t \in \{0, 1, .., 10\}$, $S_t \in \{0, 1, ...10\}$. Demand and collection rate follow discrete uniform distributions with supports $\{0, 1, .., 5\}$ and $\{1/3, 2/3, 1\}$, respectively.

Table 5.20: Solution results comparisons for the risk-averse problem in the case of backlogging.

		Exact			Myopic Approach				
		Expected total cost	Std. dev. of total cost	Solution times(sec)	Expected total cost	Std. dev. of total cost	Solution times(sec)	ECPD (%)	SDPD (%)
Risk-neutral		171.689	28.414	45482.75	232.439	74.588	190.27	15.64	71.53
r	κ								
1	0.3	171.975	26.333	45425.03	229.554	73.892	191.55	14.78	93.58
	0.5	172.724	24.435	44847.17	229.554	73.892	190.34	14.26	107.14
	0.8	173.940	22.333	45106.66	194.378	53.245	189.17	2.07	67.77
	1	174.513	21.929	44913.28	194.378	53.245	191.80	1.77	71.86
2	0.3	173.439	23.387	45992.90	229.554	73.892	189.46	1.27	76.29
	0.5	174.709	21.831	45211.91	194.378	53.245	189.15	-0.17	40.70
	0.8	175.199	21.722	44113.32	194.378	53.245	189.40	-1.28	43.83
	1	177.011	21.118	45543.48	183.463	36.189	189.19	-3.53	47.70
3	0.3	173.969	22.977	45020.22	229.554	73.892	192.97	-0.17	40.70
	0.5	175.124	21.721	45016.62	194.378	53.245	193.07	-1.28	43.83
	0.8	177.011	21.118	45674.77	183.463	36.189	192.97	-6.34	46.90
	1	177.011	21.118	45349.69	183.463	36.189	192.89	-10.29	41.49
Average		174.486	22.957	45207.52	205.610	57.304	190.94	2.06	65.87

$c_r = 4$, $c_c = 1$, $c_m = 10$, $h_r = 1$, $h_s = 2$, $b = 18$, $t_\Delta = 2$, $T = 6$, $X_t \in \{-5, -4, .., 5\}$, $Y_t \in \{0, 1, .., 10\}$, $S_t \in \{0, 1, ...10\}$. Demand and collection rate follow discrete uniform distributions with supports $\{0, 1, .., 5\}$ and $\{1/3, 2/3, 1\}$, respectively.

Table 5.21: No-Recovery Policy results in the case of backlogging.

		Solution times(sec)	Expected total cost	Std. dev. of total cost	ECPD (%)	SDPD (%)
Risk-neutral		190.48	192.111	36.077	11.89	26.97
r	κ					
1	0.3	187.36	192.111	36.077	11.71	37.00
	0.5	121.84	193.222	32.593	11.87	33.39
	0.8	185.33	194.222	30.965	11.66	38.65
	1	189.70	194.222	30.965	11.29	41.21
2	0.3	187.82	193.222	32.593	11.41	39.36
	0.5	183.46	194.222	30.965	11.17	41.84
	0.8	182.68	194.222	30.965	10.86	42.55
	1	168.64	197.778	30.268	11.73	43.33
3	0.3	182.68	193.222	32.593	11.07	41.85
	0.5	183.46	194.222	30.965	10.91	42.56
	0.8	160.37	197.778	30.268	11.73	43.33
	1	182.68	197.778	30.268	11.73	43.33
Average		177.42	194.487	34.295	11.46	39.64

$c_r = 4$, $c_c = 1$, $c_m = 10$, $h_r = 1$, $h_s = 2$, $b = 18$, $t_\Delta = 2$, $T = 6$, $X_t \in \{-5, -4, .., 5\}$, $Y_t \in \{0, 1, .., 10\}$, $S_t \in \{0, 1, ...10\}$. Demand and collection rate follow discrete uniform distributions with supports $\{0, 1, .., 5\}$ and $\{1/3, 2/3, 1\}$, respectively.

Table 5.22: Fixed-Threshold Policy results in the case of backlogging.

		Solution times(sec)	Expected total cost	Std. dev. of total cost	ECPD (%)	SDPD (%)
Risk-neutral		201.56	181.305	35.653	5.60	25.48
r	κ					
1	0.3	253.13	183.122	21.640	6.48	-17.82
	0.5	200.02	183.122	21.640	6.02	-11.44
	0.8	215.63	183.122	21.640	5.28	-3.10
	1	259.38	183.122	21.640	4.93	-1.32
2	0.3	223.44	183.122	21.640	5.58	-7.47
	0.5	225.00	183.122	21.640	4.82	-0.87
	0.8	243.38	183.122	21.640	4.52	-0.38
	1	245.31	183.122	21.640	3.45	2.47
3	0.3	257.81	183.122	21.640	5.26	-5.82
	0.5	264.06	183.122	21.640	4.57	-0.37
	0.8	223.44	183.122	21.640	3.45	2.47
	1	231.25	183.122	21.640	3.45	2.47
Average		234.11	182.982	22.718	4.88	-1.21

Problem parameters are specified as $c_r = 4, c_c = 1, c_m = 10, h_r = 1, h_s = 2, b = 18, t_\Delta = 2, N = 6$. Bounds for the state variables are specified as $X_t \in \{-5, -4, -3, ..., 5\}$, $Y_t \in \{0, 1, 2, ..., 10\}$. The intervals for previous sales take the values as explained before. Demand and collection rate are uniformly distributed over $D_t \in \{0, 1, 2, ..., 5\}$ and $C_t \in \{1/3, 2/3, 1\}$.

Table 5.23: Full-Collection Policy results in the case of backlogging.

		Solution times(sec)	Expected total cost	Std. dev. of total cost	ECPD (%)	SDPD (%)
Risk-neutral		9832.98	172.840	28.743	0.67	1.16
r	κ					
1	0.3	10072.97	173.190	26.209	0.71	-0.47
	0.5	9595.96	173.664	25.145	0.54	2.91
	0.8	9899.68	175.263	22.449	0.76	0.52
	1	9879.95	175.477	22.300	0.55	1.69
2	0.3	9831.65	174.264	24.099	0.48	3.04
	0.5	9905.55	175.866	22.119	0.66	1.32
	0.8	9732.42	176.997	21.952	1.03	1.06
	1	9671.64	179.016	21.349	1.13	1.09
3	0.3	9622.69	175.069	23.261	0.63	1.24
	0.5	9734.17	176.997	21.952	1.07	1.06
	0.8	9768.66	179.016	21.349	1.13	1.09
	1	9700.36	179.016	21.349	1.13	1.09
Average		9742.67	175.898	23.252	0.81	1.29

$c_r = 4$, $c_c = 1$, $c_m = 10$, $h_r = 1$, $h_s = 2$, $b = 18$, $t_\Delta = 2$, $T = 6$, $X_t \in \{-5, -4, .., 5\}$, $Y_t \in \{0, 1, .., 10\}$, $S_t \in \{0, 1, ...10\}$. Demand and collection rate follow discrete uniform distributions with supports $\{0, 1, .., 5\}$ and $\{1/3, 2/3, 1\}$, respectively.

Table 5.24: Solution time comparison in the case of backlogging.

	Exact	CEC	MP	NRP	FCP	FTP
Average	45207.52	111.8	190.94	177.42	9742.67	234.11
Std. Deviation	464.28	-	1.64	18.72	200.91	20.67
Min	44113.32	111.8	189.15	121.84	9238.98	200.02
Max	45992.90	111.8	193.07	190.48	10072.97	264.06

Table 5.25: Parameter analysis for the case of backlogging: Risk-neutral case

		Exact		ECPD(%)					SDPD(%)				
		Expected total cost	Std. dev. of total cost	CEC	MP	NRP	FCP	FTP	CEC	MP	NRP	FCP	FTP
h_s	1	158.944	29.814	48.79	41.99	12.92	0.75	5.77	159.51	152.73	33.14	1.30	-16.21
	2	171.689	28.414	39.49	15.64	11.89	0.67	5.60	168.23	71.53	26.97	1.16	25.48
	3	183.863	28.287	31.89	28.31	11.36	0.64	4.05	165.45	159.03	15.26	2.62	17.21
h_r	0.025	170.417	27.890	40.04	36.39	12.73	0.34	5.05	174.25	167.44	29.35	-1.05	28.75
	1	171.689	28.414	39.49	15.64	11.89	0.67	5.60	168.23	71.53	26.97	1.16	25.48
	2	172.585	29.032	39.28	34.68	11.31	1.12	6.34	161.69	156.92	24.27	7.75	22.54
c_m	7	138.667	20.760	50.18	20.00	5.45	1.24	4.94	235.28	138.91	1.86	-3.07	-24.40
	10	171.689	28.414	39.49	15.64	11.89	0.67	5.60	168.23	71.53	26.97	1.16	25.48
	13	203.143	38.130	33.27	55.50	16.46	0.39	6.32	118.07	159.77	22.50	-1.37	10.22
c_r	2	161.620	27.058	43.13	36.11	18.87	0.48	6.06	172.88	164.78	33.33	0.66	23.59
	4	171.689	28.414	39.49	15.64	11.89	0.67	5.60	168.23	71.53	26.97	1.16	25.48
	6	180.979	30.612	36.85	33.26	6.15	1.12	5.61	157.37	152.10	17.85	1.12	25.69
c_c	0.025	166.747	27.123	41.20	34.87	15.21	0.30	5.44	176.81	168.27	33.01	2.51	27.26
	1	171.689	28.414	39.49	15.64	11.89	0.67	5.60	168.23	71.53	26.97	1.16	25.48
	2	176.427	29.709	38.10	34.22	8.89	1.22	5.92	160.61	155.32	21.43	1.00	24.61
b	12	164.968	30.877	23.36	68.48	12.24	0.71	6.27	93.27	157.59	14.44	0.97	0.55
	18	171.689	28.414	39.49	15.64	11.89	0.67	5.60	168.23	71.53	26.97	1.16	25.48
	24	175.313	27.948	57.15	19.90	12.02	0.68	4.45	232.44	127.26	17.09	-1.07	-22.57

While observing the effect of change for a specific parameter, all the remaining parameters are set to their medium values.

Table 5.26: Parameter analysis for the case of backlogging: Risk-averse case

r	κ	Exact		ECPD(%)				SDPD(%)			
1.0	1.0	ETC	σ	MP	NRP	FCP	FTP	MP	NRP	FCP	FTP
h_s	1	160.613	25.319	16.84	12.30	1.00	4.68	118.72	37.36	0.15	-1.33
	2	174.513	21.929	1.77	11.29	0.55	4.93	71.86	41.21	1.69	-1.32
	3	187.775	19.155	7.10	10.82	0.57	5.51	167.42	42.45	2.50	-2.68
h_r	0.025	172.595	22.293	12.62	12.53	0.33	4.65	138.84	38.90	0.66	-0.85
	1	174.513	21.929	1.77	11.29	0.55	4.93	71.86	41.21	1.69	-1.32
	2	176.052	21.805	10.41	10.32	1.17	5.48	152.53	46.86	5.19	-1.40
c_m	7	140.333	15.508	3.81	5.31	1.40	3.70	93.69	24.97	1.65	-0.81
	10	174.513	21.929	1.77	11.29	0.55	4.93	71.86	41.21	1.69	-1.32
	13	206.826	28.845	25.83	15.91	0.35	6.62	179.98	45.93	1.27	-1.32
c_r	2	164.368	20.229	12.29	18.16	0.81	5.28	151.06	53.07	-0.77	-2.49
	4	174.513	21.929	1.77	11.29	0.55	4.93	71.86	41.21	1.69	-1.32
	6	183.735	24.956	11.13	5.71	0.93	5.15	124.58	24.08	0.75	-1.31
c_c	0.025	169.649	20.868	11.76	14.48	0.12	4.62	149.19	48.39	1.48	-1.26
	1	174.513	21.929	1.77	11.29	0.55	4.93	71.86	41.21	1.69	-1.32
	2	179.239	23.317	11.18	8.36	1.12	5.39	134.19	32.80	1.87	-1.27
b	12	168.010	22.737	37.94	10.42	0.81	4.34	210.78	43.87	0.49	36.54
	18	174.513	21.929	1.77	11.29	0.55	4.93	71.86	41.21	1.69	-1.32
	24	177.469	21.814	6.76	11.91	1.00	3.19	87.88	40.86	-0.77	-0.80
2.0	1.0										
h_s	1	163.293	24.565	3.87	12.36	0.68	2.96	51.22	38.80	0.74	1.70
	2	177.011	21.118	-3.53	11.73	1.13	3.45	47.70	43.33	1.09	2.47
	3	189.105	18.679	6.34	10.04	0.71	4.77	174.23	46.08	1.60	-0.20
h_r	0.025	174.756	21.664	4.98	13.17	0.46	3.35	67.05	39.72	0.52	2.03
	1	177.011	21.118	-3.53	11.73	1.13	3.45	47.70	43.33	1.09	2.47
	2	178.760	20.971	2.63	10.64	1.58	3.88	72.57	44.33	2.15	2.52
c_m	7	140.669	15.316	3.57	5.05	1.69	3.60	96.12	26.54	1.77	0.42
	10	177.011	21.118	-3.53	11.73	1.13	3.45	47.70	43.33	1.09	2.47
	13	209.924	27.534	8.16	15.39	0.91	5.04	115.88	49.95	0.84	3.38
c_r	2	167.828	19.091	1.85	17.85	0.43	3.11	68.43	58.55	0.30	3.33
	4	177.011	21.118	-3.53	11.73	1.13	3.45	47.70	43.33	1.09	2.47
	6	186.069	24.187	2.80	6.29	0.92	3.83	60.65	25.14	1.11	1.82
c_c	0.025	172.142	20.073	4.36	14.89	0.72	3.11	74.83	50.79	0.19	2.65
	1	177.011	21.118	-3.53	11.73	1.13	3.45	47.70	43.33	1.09	2.47
	2	181.876	22.427	3.02	8.74	1.60	3.86	67.01	34.96	2.18	2.65
b	12	172.019	21.320	34.36	9.68	0.79	6.45	230.47	42.95	1.66	1.50
	18	177.011	21.118	-3.53	11.73	1.13	3.45	47.70	43.33	1.09	2.47
	24	178.373	21.476	3.96	11.35	0.63	2.66	79.78	43.08	0.94	0.76

While observing the effect of change for a specific parameter, all the remaining parameters are set to their medium values.

Table 5.27: Numerical results for various values of c_r and c_c: Risk-neutral case

	Exact		ECPD(%)					SDPD(%)				
(c_c, c_r)	Expected total cost	Std. dev. of total cost	CEC	MP	NRP	FCP	FTP	CEC	MP	NRP	FCP	FTP
(0.025, 2)	156.463	26.511	45.25	37.61	22.78	0.22	6.05	174.59	166.49	36.08	0.95	23.06
(0.025, 4)	166.747	27.123	41.20	34.87	15.21	0.30	5.44	176.81	168.27	33.01	2.51	27.26
(0.025, 6)	176.447	29.585	38.07	34.27	8.88	0.54	5.24	162.14	156.50	21.94	0.09	25.24
(1, 2)	161.620	27.058	43.13	36.11	18.87	0.48	6.06	172.88	164.78	33.33	0.66	23.59
(1, 4)	171.689	28.414	39.49	15.64	11.89	0.67	5.60	168.23	71.53	26.97	1.16	25.48
(1, 6)	180.979	30.612	36.85	33.26	6.15	1.12	5.61	157.37	152.10	17.85	1.12	25.69
(2, 2)	166.815	27.164	41.16	34.74	15.16	0.79	6.13	175.93	167.76	32.81	3.34	27.36
(2, 4)	176.427	29.709	38.10	34.22	8.89	1.22	5.92	160.61	155.32	21.43	1.00	24.61
(2, 6)	185.414	32.486	35.81	35.88	3.61	1.79	5.97	146.72	147.19	11.05	0.26	24.01

While observing the correlation between a specific pair of parameters, all the remaining parameters are set to their medium values.

Table 5.28: Numerical results for various values of c_r and c_c: Risk-averse case

r	κ		Exact		ECPD(%)				SDPD(%)			
1	1	(c_c, c_r)	Expected total cost	Std. dev. of total cost	MP	NRP	FCP	FTP	MP	NRP	FCP	FTP
		(0.025, 2)	159.375	19.780	10.92	21.86	0.28	5.04	148.23	56.55	-0.16	-2.28
		(0.025, 4)	169.649	20.868	11.76	14.48	0.12	4.62	149.19	48.39	1.48	-1.26
		(0.025, 6)	179.103	23.427	11.34	8.44	0.40	4.72	133.24	32.18	0.57	-1.31
		(1, 2)	164.368	20.229	12.29	18.16	0.81	5.28	151.06	53.07	-0.77	-2.49
		(1, 4)	174.513	21.929	1.77	11.29	0.55	4.93	71.86	41.21	1.69	-1.32
		(1, 6)	183.735	24.956	11.13	5.71	0.93	5.15	124.58	24.08	0.75	-1.31
		(2, 2)	169.798	20.785	11.59	14.38	0.98	5.32	150.03	48.98	1.04	-1.18
		(2, 4)	179.239	23.317	11.18	8.36	1.12	5.39	134.19	32.80	1.87	-1.27
		(2, 6)	187.337	27.084	13.64	3.68	2.07	6.04	121.14	14.33	-0.37	-2.59
2	1											
		(0.025, 2)	162.724	18.629	2.17	21.54	0.12	2.88	68.48	62.48	-0.14	3.76
		(0.025, 4)	172.142	20.073	4.36	14.89	0.72	3.11	74.83	50.79	0.19	2.65
		(0.025, 6)	181.308	22.728	3.40	9.08	0.46	3.45	64.95	33.17	0.50	1.73
		(1, 2)	167.828	19.091	1.85	17.85	0.43	3.11	68.43	58.55	0.30	3.33
		(1, 4)	177.011	21.118	-3.53	11.73	1.13	3.45	47.70	43.33	1.09	2.47
		(1, 6)	186.069	24.187	2.80	6.29	0.92	3.83	60.65	25.14	1.11	1.82
		(2, 2)	172.344	19.983	4.18	14.76	1.15	3.76	75.49	51.47	0.66	2.78
		(2, 4)	181.876	22.427	3.02	8.74	1.60	3.86	67.01	34.96	2.18	2.65
		(2, 6)	190.540	25.981	2.44	3.80	1.58	4.43	55.41	16.50	1.34	1.63

While observing the correlation between a specific pair of parameters, all the remaining parameters are set to their medium values.

Table 5.29: Numerical results for various values of h_r and c_c: Risk-neutral case

	Exact		ECPD(%)					SDPD(%)				
$(h_r,\, c_c)$	Expected total cost	Std. dev. of total cost	CEC	MP	NRP	FCP	FTP	CEC	MP	NRP	FCP	FTP
(0.025, 0.025)	165.369	26.725	41.86	35.99	16.17	0.01	4.94	181.97	172.27	34.99	-0.01	30.36
(0.025, 1)	170.417	27.890	40.04	36.39	12.73	0.34	5.05	174.25	167.44	29.35	-1.05	28.75
(0.025, 2)	175.326	28.937	38.49	35.07	9.57	0.80	5.33	168.49	162.13	24.67	-0.63	28.48
(1, 0.025)	166.747	27.123	41.20	34.87	15.21	0.30	5.44	176.81	168.27	33.01	2.51	27.26
(1, 1)	171.689	28.414	39.49	15.64	11.89	0.67	5.60	168.23	71.53	26.97	1.16	25.48
(1, 2)	176.427	29.709	38.10	34.22	8.89	1.22	5.92	160.61	155.32	21.43	1.00	24.61
(2, 0.025)	167.837	28.154	40.56	33.99	14.46	0.65	6.13	165.37	158.45	28.14	0.93	22.41
(2, 1)	172.585	29.032	39.28	34.68	11.31	1.12	6.34	161.69	156.92	24.27	7.75	22.54
(2, 2)	177.161	30.221	38.01	33.67	8.44	1.72	6.69	155.82	151.00	19.38	7.64	23.98

While observing the correlation between a specific pair of parameters, all the remaining parameters are set to their medium values.

Table 5.30: Numerical results for various values of h_r and c_c: Risk-averse case

r	κ		Exact		ECPD(%)				SDPD(%)			
			Expected total cost	Std. dev. of total cost								
1	1	(h_r, c_c)			MP	NRP	FCP	FTP	MP	NRP	FCP	FTP
		(0.025, 0.025)	167.685	21.303	13.07	15.83	0.02	4.35	144.10	45.36	0.00	-0.40
		(0.025, 1)	172.595	22.293	12.62	12.53	0.33	4.65	138.84	38.90	0.66	-0.85
		(0.025, 2)	177.596	23.551	12.21	9.36	0.75	4.95	131.86	31.48	0.99	-0.90
		(1, 0.025)	169.649	20.868	11.76	14.48	0.12	4.62	149.19	48.39	1.48	-1.26
		(1, 1)	174.513	21.929	1.77	11.29	0.55	4.93	71.86	41.21	1.69	-1.32
		(1, 2)	179.239	23.317	11.18	8.36	1.12	5.39	134.19	32.80	1.87	-1.27
		(2, 0.025)	171.294	20.724	10.68	13.39	0.70	5.12	150.92	49.42	1.04	-1.93
		(2, 1)	176.052	21.805	10.41	10.32	1.17	5.48	152.53	46.86	5.19	-1.40
		(2, 2)	180.437	23.347	10.45	7.64	1.78	5.88	133.89	32.63	2.24	-1.67
2	1											
		(0.025, 0.025)	169.887	20.747	5.75	16.42	0.02	3.00	69.15	45.89	0.02	2.27
		(0.025, 1)	174.756	21.664	4.98	13.17	0.46	3.35	67.05	39.72	0.52	2.03
		(0.025, 2)	179.751	22.870	4.24	10.03	0.88	3.70	63.78	32.35	1.22	2.06
		(1, 0.025)	172.142	20.073	4.36	14.89	0.72	3.11	74.83	50.79	0.19	2.65
		(1, 1)	177.011	21.118	-3.53	11.73	1.13	3.45	47.70	43.33	1.09	2.47
		(1, 2)	181.876	22.427	3.02	8.74	1.60	3.86	67.01	34.96	2.18	2.65
		(2, 0.025)	174.146	19.891	3.16	13.57	1.04	3.40	76.43	52.17	0.81	2.17
		(2, 1)	178.760	20.971	2.63	10.64	1.58	3.88	72.57	44.33	2.15	2.52
		(2, 2)	183.333	22.342	2.20	7.88	2.20	4.44	67.65	35.48	3.42	2.99

While observing the correlation between a specific pair of parameters, all the remaining parameters are set to their medium values.

Table 5.31: Numerical results for various values of h_s and h_r: Risk-neutral case

	Exact		ECPD(%)					SDPD(%)				
(h_s, h_r)	Expected total cost	Std. dev. of total cost	CEC	MP	NRP	FCP	FTP	CEC	MP	NRP	FCP	FTP
(1, 0.025)	157.466	29.466	49.65	43.33	13.98	0.35	5.18	163.52	155.72	34.71	0.34	-13.66
(1, 1)	158.944	29.814	48.79	41.99	12.92	0.75	5.77	159.51	152.73	33.14	1.30	-16.21
(1, 2)	160.019	30.615	42.98	41.04	12.16	1.24	6.55	153.18	146.12	29.66	1.70	-18.52
(2, 0.025)	170.417	27.890	40.04	36.39	12.73	0.34	5.05	174.25	167.44	29.35	-1.05	28.75
(2, 1)	171.689	28.414	39.49	15.64	11.89	0.67	5.60	168.23	71.53	26.97	1.16	25.48
(2, 2)	172.585	29.032	39.28	34.68	11.31	1.12	6.34	161.69	156.92	24.27	7.75	22.54
(3, 0.025)	182.865	28.083	32.15	29.01	11.97	0.36	3.37	168.33	160.92	16.10	1.87	18.86
(3, 1)	183.863	28.287	31.89	28.31	11.36	0.64	4.05	165.45	159.03	15.26	2.62	17.21
(3, 2)	184.520	29.191	31.89	27.86	10.96	1.00	4.88	156.42	151.01	11.69	7.13	13.45

While observing the correlation between a specific pair of parameters, all the remaining parameters are set to their medium values.

Table 5.32: Numerical results for various values of h_s and h_r: Risk-averse case

r	κ		Exact		ECPD(%)				SDPD(%)			
1	1	(h_s, h_r)	Expected total cost	Std. dev. of total cost	MP	NRP	FCP	FTP	MP	NRP	FCP	FTP
		(1, 0.025)	158.633	25.777	18.29	13.70	0.46	4.40	114.83	34.92	0.20	-1.30
		(1, 1)	160.613	25.319	16.84	12.30	1.00	4.68	118.72	37.36	0.15	-1.33
		(1, 2)	162.116	25.237	15.75	11.25	1.55	5.29	119.43	37.81	0.16	-1.73
		(2, 0.025)	172.595	22.293	12.62	12.53	0.33	4.65	138.84	38.90	0.66	-0.85
		(2, 1)	174.513	21.929	1.77	11.29	0.55	4.93	71.86	41.21	1.69	-1.32
		(2, 2)	176.052	21.805	10.41	10.32	1.17	5.48	152.53	46.86	5.19	-1.40
		(3, 0.025)	185.978	19.453	8.13	11.89	0.37	5.18	163.32	40.27	-0.06	-1.81
		(3, 1)	187.775	19.155	7.10	10.82	0.57	5.51	167.42	42.45	2.50	-2.68
		(3, 2)	188.73	19.416	6.56	10.25	0.95	6.34	163.82	40.54	2.93	-4.42
2	1											
		(1, 0.025)	161.354	24.982	5.12	13.71	0.34	2.64	48.70	36.48	0.26	1.84
		(1, 1)	163.293	24.565	3.87	12.36	0.68	2.96	51.22	38.80	0.74	1.70
		(1, 2)	165.100	24.367	2.73	11.13	1.14	3.39	52.45	39.93	1.49	1.78
		(2, 0.025)	174.756	21.664	4.98	13.17	0.46	3.35	67.05	39.72	0.52	2.03
		(2, 1)	177.011	21.118	-3.53	11.73	1.13	3.45	47.70	43.33	1.09	2.47
		(2, 2)	178.760	20.971	2.63	10.64	1.58	3.88	72.57	44.33	2.15	2.52
		(3, 0.025)	187.130	19.024	7.47	11.20	0.43	4.53	169.26	43.43	1.08	0.40
		(3, 1)	189.105	18.679	6.34	10.04	0.71	4.77	174.23	46.08	1.60	-0.20
		(3, 2)	190.751	18.598	5.43	9.09	1.35	5.21	175.43	46.72	3.47	-0.22

While observing the correlation between a specific pair of parameters, all the remaining parameters are set to their medium values.

Table 5.33: Numerical results for various values of c_m, c_r and c_c: Risk-neutral case

	Exact		ECPD(%)					SDPD(%)				
(c_m, c_r, c_c)	Expected total cost	Std. dev. of total cost	CEC	MP	NRP	FCP	FTP	CEC	MP	NRP	FCP	FTP
(3, 6, 0.025)	81.111	6.616	94.18	7.88	0.00	16.79	11.79	787.32	205.89	0.00	72.78	5.17
(3, 2, 2)	81.111	6.616	94.18	7.88	0.00	11.63	5.79	787.32	205.89	0.00	56.26	-17.00
(13, 6, 0.025)	208.127	38.527	32.06	53.88	13.67	0.33	5.88	119.14	160.14	21.24	1.55	12.40
(13, 2, 2)	198.07	36.901	34.65	56.42	19.44	0.47	6.89	122.07	164.95	26.58	2.32	10.96

While observing the correlation between a specific pair of parameters, all the remaining parameters are set to their medium values.

Table 5.34: Numerical results for various values of c_m, c_r and c_c: Risk-averse case

r	κ		Exact		ECPD(%)				SDPD(%)			
1	1	(c_m, c_r, c_c)	Expected total cost	Std. dev. of total cost	MP	NRP	FCP	FTP	MP	NRP	FCP	FTP
		(3, 6, 0.025)	81.111	6.616	7.88	0.00	13.79	11.79	205.89	0.00	239.04	5.17
		(3, 2, 2)	81.111	6.616	7.88	0.00	11.94	5.79	205.89	0.00	35.85	-17.00
		(13, 6, 0.025)	211.524	30.141	25.35	13.33	0.35	6.35	171.83	39.66	0.02	-1.44
		(13, 2, 2)	201.861	27.955	26.55	18.76	0.59	7.11	185.02	50.58	0.58	-1.52
2	1											
		(3, 6, 0.025)	82.500	5.123	0.00	0.00	16.51	9.91	0.00	0.00	107.65	35.82
		(3, 2, 2)	82.500	5.123	0.00	0.00	11.02	4.01	0.00	0.00	58.95	7.18
		(13, 6, 0.025)	214.591	28.981	8.15	12.88	0.21	4.83	109.62	42.46	0.13	2.50
		(13, 2, 2)	205.292	26.566	8.21	17.99	1.06	5.32	119.24	55.41	1.14	3.62

While observing the correlation between a specific pair of parameters, all the remaining parameters are set to their medium values.

Chapter 6

Conclusion

In this study, we consider a closed-loop multi-stage inventory problem. Sold products return to the manufacturer after a specific market sojourn time, and collected based on a random collection rate. Demand is satisfied through two channels: manufacturing and remanufacturing.

We model our problem for two base cases assuming if not satisfied upon arrival, a demand is either backlogged or lost. We also study the problem from the perspective of risk-neutral and risk-averse decision makers. For this purpose, we incorporate the dynamic coherent risk measures into our problem formulation. The risk-neutral objective is to minimize the expected total cost. The risk-averse objective is to minimize the weighted sum of the mean total cost and the expected excess from the mean total cost.

We next conduct detailed numerical analysis and examine the optimal policy structure. The results indicate that a *state-dependent* threshold policy may be optimal for the core inventory. However, such a policy need not be optimal for the serviceable inventory. Although we could not find any example violating this policy for the core inventory, we could not prove discrete-convexity of the optimal cost function. Thus whether a state-dependent threshold policy is analytically optimal for this specific inventory level remains an open question in our research.

Numerical results also demonstrate the effect of risk-aversion over the optimal policy. We indicate that as the degree of risk-aversion increases, the expected total cost also increases.

We provide several heuristics that are computationally less demanding than the optimal policy: a certainty equivalent controller (CEC), a myopic policy (MP), a no-recovery policy (NRP), a fixed threshold policy (FTP), and a full-collection policy (FCP). CEC is a suboptimal control scheme which seeks to find the optimal policy by fixing the uncertain quantities at some "typical" values. In our problem, both demand and collection rate are fixed at their expected values and randomness is eliminated from the problem. MP is a commonly used approach in inventory management problems. It aims to minimize the expected cost at each stage while ignoring the impact of future stages. NRP aims to reduce the solution times by eliminating the collection and remanufacturing decisions. By using NRP, we evaluate the economic viability of remanufacturing. FTP assumes that there exist fixed (state-independent) thresholds, namely collect-up-to level, and produce-up-to level. Last, FCP eliminates collection decision by collecting all returned cores up to the core product inventory upper bound. FCP performance can be used to evaluate the cost of waste minimization.

We then conduct numerical analysis to assess the performance of each heuristic. CEC, MP, and NRP have a distinct computational advantage over FTP and FCP. However, FCP and FTP surpass all the other heuristics with respect to objective value.

For a future research direction, our problem can be extended by studying the case where demand and collection rate are dependent. Second, core product conditions can be assumed to vary instead of being in the same quality. Third, including a finite lifetime for the product, in other words, a specific number of times it can be remanufactured can be integrated into our problem. Assuming that the market sojourn time is random can also be a good extension. Last, disposal option can be introduced to the problem as another decision variable.

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Appendix A

Proofs of Analytical Results

A.1 Proof of Lemma 5.1

Suppose that $c_m \geq c_r$. We want to show that $V_t(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta}) + c_m \geq V_t(X_t, Y_t - 1, S_{t-1}, ..., S_{t-t_\Delta}) + c_r$, $\forall t$.

First we consider stage $T-1$:

(i) Suppose that $Q^*_{T-1}, R^*_{T-1}, Z^*_{T-1}$ is the optimal solution at state $(X_{T-1}, Y_{T-1}, S_{T-2}, ..., S_{T-1-t_\Delta})$ and $Q^*_{T-1}, R^*_{T-1}, Z^*_{T-1}$ is a feasible solution at state $(X_{T-1}, Y_{T-1} - 1, S_{T-2}, ..., S_{T-1-t_\Delta})$. Then it is easy to verify that the following inequalities hold.

$$
\begin{aligned}
&V_{T-1}(X_{T-1}, Y_{T-1}, S_{T-2}, ..., S_{T-1-t_\Delta}) = c_m Q^*_{T-1} + c_r R^*_{T-1} + c_c Z^*_{T-1} \\
&+ \mathop{\mathbb{E}}_{D_{T-1}, C_{T-1}} \Big[h_s [X_{T-1} + Q^*_{T-1} + R^*_{T-1} - D_{T-1}]_+ + h_r (Y_{T-1} + Z^*_{T-1} - R^*_{T-1}) \\
&+ p[D_{T-1} - X_{T-1} - Q^*_{T-1} - R^*_{T-1}]_+ \Big] \\
&\geq Q^*_{T-1} + c_r R^*_{T-1} + c_c Z^*_{T-1} + \mathop{\mathbb{E}}_{D_{T-1}, C_{T-1}} \Big[h_s [X_{T-1} + Q^*_{T-1} + R^*_{T-1} \\
&- D_{T-1}]_+ + h_r (Y_{T-1} - 1 + Z^*_{T-1} - R^*_{T-1}) + p[D_{T-1} - X_{T-1} - Q^*_{T-1} - R^*_{T-1}]_+ \Big] \\
&\geq V_{T-1}(X_{T-1}, Y_{T-1} - 1, S_{T-2}, ..., S_{T-1-t_\Delta}).
\end{aligned}
$$

Thus,
$V_{T-1}(X_{T-1}, Y_{T-1}, S_{T-2}, ..., S_{T-1-t_\Delta}) \geq V_{T-1}(X_{T-1}, Y_{T-1} - 1, S_{T-2}, ..., S_{T-1-t_\Delta})$. As we assume $c_m \geq c_r$, we must have $V_{T-1}(X_{T-1}, Y_{T-1}, S_{T-2}, ..., S_{T-1-t_\Delta}) + c_m \geq V_{T-1}(X_{T-1}, Y_{T-1} - 1, S_{T-2}, ..., S_{T-1-t_\Delta}) + c_r$.

(ii) Suppose that $Q^*_{T-1}, R^*_{T-1}, Z^*_{T-1}$ is the optimal solution at state $(X_{T-1}, Y_{T-1}, S_{T-2}, ..., S_{T-1-t_\Delta})$ and $Q^*_{T-1}, R^*_{T-1}, Z^*_{T-1}$ is not a feasible solution at state $(X_{T-1}, Y_{T-1} - 1, S_{T-2}, ..., S_{T-1-t_\Delta})$. Note that $Q^*_{T-1} + 1, R^*_{T-1} - 1, Z^*_{T-1}$ must be a feasible solution at state $(X_{T-1}, Y_{T-1} - 1, S_{T-2}, ..., S_{T-1-t_\Delta})$. Then it is easy to verify that the following holds.

$$
\begin{aligned}
&V_{T-1}(X_{T-1}, Y_{T-1}, S_{T-2}, ..., S_{T-1-t_\Delta}) + c_m = c_m Q^*_{T-1} + c_r R^*_{T-1} + c_c Z^*_{T-1} \\
&+ \mathop{\mathbb{E}}_{D_{T-1}, C_{T-1}} \Big[h_s[X_{T-1} + Q^*_{T-1} + R^*_{T-1} - D_{T-1}]_+ + h_r(Y_{T-1} + Z^*_{T-1} - R^*_{T-1}) \\
&+ p[D_{T-1} - X_{T-1} - Q^*_{T-1} - R^*_{T-1}]_+ \Big] + c_m \\
&= c_m(Q^*_{T-1} + 1) + c_r(R^*_{T-1} - 1) + c_c Z^*_{T-1} \\
&+ \mathop{\mathbb{E}}_{D_{T-1}, C_{T-1}} \Big[h_s[X_{T-1} + Q^*_{T-1} + 1 + R^*_{T-1} - 1 - D_{T-1}]_+ \\
&+ h_r(Y_{T-1} - 1 + Z^*_{T-1} - R^*_{T-1} + 1) \\
&+ p[D_{T-1} - X_{T-1} - Q^*_{T-1} - 1 - R^*_{T-1} + 1]_+ \Big] + c_r \\
&\geq V_{T-1}(X_{T-1}, Y_{T-1} - 1, S_{T-2}, ..., S_{T-1-t_\Delta}) + c_r.
\end{aligned}
$$

Next, we consider stage $t < T - 1$. Assuming $V_{t+1}(X_{t+1}, Y_{t+1}, S_t, ..., S_{t+1-t_\Delta}) + c_m \geq V_{t+1}(X_{t+1}, Y_{t+1} - 1, S_t, ..., S_{t+1-t_\Delta}) + c_r$, we will show $V_t(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta}) + c_m \geq V_t(X_t, Y_t - 1, S_{t-1}, ..., S_{t-t_\Delta}) + c_r$.

(i) Suppose that Q^*_t, R^*_t, Z^*_t is the optimal solution at state $(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta})$ and Q^*_t, R^*_t, Z^*_t is a feasible solution at state $(X_t, Y_t - 1, S_{t-1}, ..., S_{t-t_\Delta})$. Then it is easy to verify that the following inequalities hold.

$$V_t(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta}) = c_m Q_t^* + c_r R_t^* + c_c Z_t^*$$
$$+ \underset{D_t, C_t}{\mathbb{E}} \Big[h_s[X_t + Q_t^* + R_t^* - D_t]_+ + h_r(Y_t + Z_t^* - R_t^*)$$
$$+ p[D_t - X_t - Q_t^* - R_t^*]_+$$
$$+ V_{t+1}([X_t + Q_t^* + R_t^* - D_t]_+, Y_t + Z_t^* - R_t^*, S_t, ..., S_{t+1-t_\Delta}) + c_m \Big]$$
$$\geq$$
$$c_m Q_t^* + c_r R_t^* + c_c Z_t^*$$
$$+ \underset{D_t, C_t}{\mathbb{E}} \Big[h_s[X_t + Q_t^* + R_t^* - D_t]_+ + h_r(Y_t - 1 + Z_t^* - R_t^*)$$
$$+ p[D_t - X_t - Q_t^* - R_t^*]_+$$
$$+ V_{t+1}([X_t + Q_t^* + R_t^* - D_t]_+, Y_t - 1 + Z_t^* - R_t^*, S_t, ..., S_{t+1-t_\Delta}) + c_r \Big]$$
$$\geq V_t(X_t, Y_t - 1, S_{t-1}, ..., S_{t-t_\Delta}) + c_r$$

(ii) Suppose that Q_t^*, R_t^*, Z_t^* is the optimal solution at state combination $(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta})$ and Q_t^*, R_t^*, Z_t^* is not a feasible solution at state $(X_t, Y_t - 1, S_{t-1}, ..., S_{t-t_\Delta})$. Note that $Q_t^* + 1, R_t^* - 1, Z_t^*$, must be a feasible solution at state $(X_t, Y_t - 1, S_{t-1}, ..., S_{t-t_\Delta})$. Then it is easy to verify that the following holds.

$$V_t(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta}) + c_m =$$
$$c_m Q_t^* + c_r R_t^* + c_c Z_t^* + \underset{D_t, C_t}{\mathbb{E}} \Big[h_s[X_t + Q_t^* + R_t^* - D_t]_+$$
$$+ h_r(Y_t + Z_t^* - R_t^*) + p[D_t - X_t - Q_t^* - R_t^*]_+$$
$$+ V_{t+1}([X_t + Q_t^* + R_t^* - D_t]_+, Y_t + Z_t^* - R_t^*, S_t, ..., S_{t+1-t_\Delta}) \Big] + c_m$$
$$= c_m(Q_t^* + 1) + c_r(R_t^* - 1) + c_c Z_t^* + \underset{D_t, C_t}{\mathbb{E}} \Big[h_s[X_t + Q_t^* + 1 + R_t^* - 1 - D_t]_+$$
$$+ h_r(Y_t - 1 + Z_t^* - R_t^* + 1) + p[D_t - X_t - Q_t^* - 1 - R_t^* + 1]_+$$
$$+ V_{t+1}([X_t + Q_t^* - 1 + R_t^* + 1 - D_t]_+, Y_t - 1 + Z_t^* - R_t^* + 1, S_t, ..., S_{t+1-t_\Delta}) \Big] + c_r$$
$$\geq V_t(X_t, Y_t - 1, S_{t-1}, ..., S_{t-t_\Delta}) + c_r$$

Hence we showed that $V_t(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta}) + c_m \geq V_t(X_t, Y_t -$

$1, S_{t-1}, ..., S_{t-t_\Delta}) + c_r$, $\forall t$.

A.2 Proof of Proposition 5.2

Suppose that $c_m \geq c_r$. First we consider stage $T-1$. Pick arbitrary decision $Q_{T-1}, R_{T-1}, Z_{T-1}$ where $R_{T-1} > 0$. The expected cost at state $(X_{T-1}, Y_{T-1}, S_{T-2}, ...,_{T-1-t_\Delta})$ under this decision is given by

$$c_m Q_{T-1} + c_r R_{T-1} + c_c Z_{T-1} + \mathop{\mathbb{E}}_{D_{T-1}, C_{T-1}} \Big[h_s[X_{T-1} + Q_{T-1} + R_{T-1} - D_{T-1}]_+ \\ + h_r(Y_{T-1} + Z_{T-1} - R_{T-1}) + p[D_{T-1} - X_{T-1} - Q_{T-1} - R_{N-1}]_+ \Big]$$

Now pick another feasible decision $Q_{T-1} - 1, R_{T-1} + 1, Z_{T-1}$. The expected cost under this decision is given by

$$c_m(Q_{T-1} - 1) + c_r(R_{T-1} + 1) + c_c Z_{T-1} \\ + \mathop{\mathbb{E}}_{D_{T-1}, C_{T-1}} \Big[h_s[X_{T-1} + Q_{T-1} - 1 + R_{T-1} + 1 - D_{T-1}]_+ \\ + h_r(Y_{T-1} + Z_{T-1} - R_{T-1} - 1) + p[D_{T-1} - X_{T-1} - Q_{T-1} + 1 - R_{T-1} - 1]_+ \Big]$$

As we assume $c_m \geq c_r$, the expected cost under decision $Q_{T-1}, R_{T-1}, Z_{T-1}$ is no less than the expected cost under $Q_{T-1} - 1, R_{T-1} + 1, Z_{T-1}$:

$$c_m Q_{T-1} + c_r R_{T-1} + c_c Z_{T-1} \\ + \mathop{\mathbb{E}}_{D_{T-1}, C_{T-1}} \Big[h_s[X_{T-1} + Q_{T-1} + R_{T-1} - D_{T-1}]_+ \\ + h_r(Y_{T-1} + Z_{T-1} - R_{T-1}) + p[D_{T-1} - X_{T-1} - Q_{T-1} - R_{T-1}]_+ \Big] \\ \geq \\ c_m(Q_{T-1} - 1) + c_r(R_{T-1} + 1) + c_c Z_{T-1} \\ + \mathop{\mathbb{E}}_{D_{T-1}, C_{T-1}} \Big[h_s[X_{T-1} + Q_{T-1} - 1 + R_{T-1} + 1 - D_{T-1}]_+ \\ + h_r(Y_{T-1} + Z_{T-1} - R_{T-1} - 1) + p[D_{T-1} - X_{T-1} - Q_{T-1} + 1 - R_{T-1} - 1]_+ \Big]$$

Next we consider stage $t < T-1$. By Lemma 5.1, we know that $V_t(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta}) + c_m \geq V_t(X_t, Y_t - 1, S_{t-1}, ..., S_{t-t_\Delta}) + c_r$, $\forall t$. Thus the expected cost under decision Q_t, R_t, Z_t is no less than the expected cost under decision $Q_t - 1, R_t + 1, Z_t$:

$$
\begin{aligned}
&c_m Q_t + c_r R_t + c_c Z_t \\
&+ \mathop{\mathbb{E}}_{D_t, C_t} \Big[h_s [X_t + Q_t + R_t - D_t]_+ \\
&+ h_r (Y_t + Z_t - R_t) + p[D_t - X_t - Q_t - R_t]_+ \\
&+ V_{t+1}([X_t + Q_t + R_t - D_t]_+, Y_t + Z_t - R_t, S_t, ..., S_{t+1-t_\Delta}) \Big] \\
&\geq c_m (Q_t - 1) + c_r (R_t + 1) + c_c Z_t \\
&+ \mathop{\mathbb{E}}_{D_t, C_t} \Big[h_s [X_t + Q_t - 1 + R_t + 1 - D_t]_+ \\
&+ h_r (Y_t + Z_t - R_t - 1) + p[D_t - X_t - Q_t + 1 - R_t - 1]_+ \\
&+ V_{t+1}([X_t + Q_t - 1 + R_t + 1 - D_t]_+, Y_t + Z_t - R_t - 1, S_t, ..., S_{t+1-t_\Delta}) \Big] \\
&\geq V_t(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta})
\end{aligned}
$$

A.3 Proof of Lemma 5.3

Suppose that $c_m \geq c_r$. We want to show that $V_t(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta}) + c_m \geq V_t(X_t, Y_t - 1, S_{t-1}, ..., S_{t-t_\Delta}) + c_r$, $\forall t$.

First we consider stage $T - 1$:

(i) Suppose that $Q^*_{T-1}, R^*_{T-1}, Z^*_{T-1}$ is the optimal solution at state $(X_{T-1}, Y_{T-1}, S_{T-2}, ..., S_{T-1-t_\Delta})$ and $Q^*_{T-1}, R^*_{T-1}, Z^*_{T-1}$ is a feasible solution at state $(X_{T-1}, Y_{T-1} - 1, S_{T-2}, ..., S_{T-1-t_\Delta})$. Then it is easy to verify that the following inequalities hold.

$$V_{T-1}(X_{T-1}, Y_{T-1}, S_{T-2}, ..., S_{T-1-t_\Delta}) = c_m Q^*_{T-1} + c_r R^*_{T-1} + c_c Z^*_{T-1}$$
$$+ \mathop{\mathbb{E}}_{D_{T-1}, C_{T-1}} \Big[h_s[X_{T-1} + Q^*_{T-1} + R^*_{T-1} - D_{T-1}]_+ + h_r(Y_{T-1} + Z^*_{T-1} - R^*_{T-1})$$
$$+ b[D_{T-1} - X_{T-1} - Q^*_{T-1} - R^*_{T-1}]_+ \Big]$$
$$\geq Q^*_{T-1} + c_r R^*_{T-1} + c_c Z^*_{T-1} + \mathop{\mathbb{E}}_{D_{T-1}, C_{T-1}} \Big[h_s[X_{T-1} + Q^*_{T-1} + R^*_{T-1} - D_{T-1}]_+$$
$$+ h_r(Y_{T-1} - 1 + Z^*_{T-1} - R^*_{T-1}) + b[D_{T-1} - X_{T-1} - Q^*_{T-1} - R^*_{T-1}]_+ \Big]$$
$$\geq V_{T-1}(X_{T-1}, Y_{T-1} - 1, S_{T-2}, ..., S_{T-1-t_\Delta}).$$

Thus,
$V_{T-1}(X_{T-1}, Y_{T-1}, S_{T-2}, ..., S_{T-1-t_\Delta}) \geq V_{T-1}(X_{T-1}, Y_{T-1} - 1, S_{T-2}, ..., S_{T-1-t_\Delta})$. As we assume $c_m \geq c_r$, we must have $V_{T-1}(X_{T-1}, Y_{T-1}, S_{T-2}, ..., S_{T-1-t_\Delta}) + c_m \geq V_{T-1}(X_{T-1}, Y_{T-1} - 1, S_{T-2}, ..., S_{T-1-t_\Delta}) + c_r$.

(ii) Suppose that $Q^*_{T-1}, R^*_{T-1}, Z^*_{T-1}$ is the optimal solution at state $(X_{T-1}, Y_{T-1}, S_{T-2}, ..., S_{T-1-t_\Delta})$ and $Q^*_{T-1}, R^*_{T-1}, Z^*_{T-1}$ is not a feasible solution at state $(X_{T-1}, Y_{T-1} - 1, S_{T-2}, ..., S_{T-1-t_\Delta})$. Note that $Q^*_{T-1} + 1, R^*_{T-1} - 1, Z^*_{T-1}$ must be a feasible solution at state $(X_{T-1}, Y_{T-1} - 1, S_{T-2}, ..., S_{T-1-t_\Delta})$. Then it is easy to verify that the following holds.

$$V_{T-1}(X_{T-1}, Y_{T-1}, S_{T-2}, ..., S_{T-1-t_\Delta}) + c_m = c_m Q^*_{T-1} + c_r R^*_{T-1} + c_c Z^*_{T-1}$$
$$+ \mathop{\mathbb{E}}_{D_{T-1}, C_{T-1}} \Big[h_s[X_{T-1} + Q^*_{T-1} + R^*_{T-1} - D_{T-1}]_+ + h_r(Y_{T-1} + Z^*_{T-1} - R^*_{T-1})$$
$$+ b[D_{T-1} - X_{T-1} - Q^*_{T-1} - R^*_{T-1}]_+ \Big] + c_m$$
$$= c_m(Q^*_{T-1} + 1) + c_r(R^*_{T-1} - 1) + c_c Z^*_{T-1}$$
$$+ \mathop{\mathbb{E}}_{D_{T-1}, C_{T-1}} \Big[h_s[X_{T-1} + Q^*_{T-1} + 1 + R^*_{T-1} - 1 - D_{T-1}]_+$$
$$+ h_r(Y_{T-1} - 1 + Z^*_{T-1} - R^*_{T-1} + 1)$$
$$+ b[D_{T-1} - X_{T-1} - Q^*_{T-1} - 1 - R^*_{T-1} + 1]_+ \Big] + c_r$$
$$\geq V_{T-1}(X_{T-1}, Y_{T-1} - 1, S_{T-2}, ..., S_{T-1-t_\Delta}) + c_r.$$

Next we consider stage $t < T - 1$. Assuming $V_{t+1}(X_{t+1}, Y_{t+1}, S_t, ..., S_{t+1-t_\Delta}) +$

$c_m \geq V_{t+1}(X_{t+1}, Y_{t+1}-1, S_t, ..., S_{t+1-t_\Delta})+c_r$, we will show $V_t(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta})+c_m \geq V_t(X_t, Y_t - 1, S_{t-1}, ..., S_{t-t_\Delta}) + c_r$.

(i) Suppose that Q_t^*, R_t^*, Z_t^* is the optimal solution at state $(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta})$ and Q_t^*, R_t^*, Z_t^* is a feasible solution at state $(X_t, Y_t - 1, S_{t-1}, ..., S_{t-t_\Delta})$. Then it is easy to verify that the following inequalities hold.

$$
\begin{aligned}
& V_t(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta}) = c_m Q_t^* + c_r R_t^* + c_c Z_t^* \\
& + \mathop{\mathbb{E}}_{D_t, C_t} \Big[h_s[X_t + Q_t^* + R_t^* - D_t]_+ + h_r(Y_t + Z_t^* - R_t^*) \\
& + b[D_t - X_t - Q_t^* - R_t^*]_+ \\
& + V_{t+1}([X_t + Q_t^* + R_t^* - D_t]_+, Y_t + Z_t^* - R_t^*, S_t, ..., S_{t+1-t_\Delta}) + c_m \Big] \\
& \geq \\
& c_m Q_t^* + c_r R_t^* + c_c Z_t^* \\
& + \mathop{\mathbb{E}}_{D_t, C_t} \Big[h_s[X_t + Q_t^* + R_t^* - D_t]_+ + h_r(Y_t - 1 + Z_t^* - R_t^*) \\
& + b[D_t - X_t - Q_t^* - R_t^*]_+ \\
& + V_{t+1}([X_t + Q_t^* + R_t^* - D_t]_+, Y_t - 1 + Z_t^* - R_t^*, S_t, ..., S_{t+1-t_\Delta}) + c_r \Big] \\
& \geq V_t(X_t, Y_t - 1, S_{t-1}, ..., S_{t-t_\Delta}) + c_r
\end{aligned}
$$

(ii) Suppose that Q_t^*, R_t^*, Z_t^* is the optimal solution at state $(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta})$ and Q_t^*, R_t^*, Z_t^* is not a feasible solution at state $(X_t, Y_t - 1, S_{t-1}, ..., S_{t-t_\Delta})$. Note that $Q_t^* + 1, R_t^* - 1, Z_t^*$ must be a feasible solution at state $(X_t, Y_t -$

$1, S_{t-1}, ..., S_{t-t_\Delta})$. Then it is easy to verify that the following holds.

$$
\begin{aligned}
&V_t(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta}) + c_m = \\
&c_m Q_t^* + c_r R_t^* + c_c Z_t^* + \mathop{\mathbb{E}}_{D_t, C_t} \Big[h_s[X_t + Q_t^* + R_t^* - D_t]_+ \\
&+ h_r(Y_t + Z_t^* - R_t^*) + b[D_t - X_t - Q_t^* - R_t^*]_+ \\
&+ V_{t+1}([X_t + Q_t^* + R_t^* - D_t]_+, Y_t + Z_t^* - R_t^*, S_t, ..., S_{t+1-t_\Delta}) \Big] + c_m \\
&== c_m(Q_t^* + 1) + c_r(R_t^* - 1) + c_c Z_t^* + \mathop{\mathbb{E}}_{D_t, C_t} \Big[h_s[X_t + Q_t^* + 1 + R_t^* - 1 - D_t]_+ \\
&+ h_r(Y_t - 1 + Z_t^* - R_t^* + 1) + b[D_t - X_t - Q_t^* - 1 - R_t^* + 1]_+ \\
&+ V_{t+1}([X_t + Q_t^* - 1 + R_t^* + 1 - D_t]_+, Y_t - 1 + Z_t^* - R_t^* + 1, S_t, ..., S_{t+1-t_\Delta}) \Big] + c_r \\
&\geq V_t(X_t, Y_t - 1, S_{t-1}, ..., S_{t-t_\Delta}) + c_r
\end{aligned}
$$

Hence we showed that $V_t(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta}) + c_m \geq V_t(X_t, Y_t - 1, S_{t-1}, ..., S_{t-t_\Delta}) + c_r$, $\forall t$.

A.4 Proof of Proposition 5.4

Suppose that $c_m \geq c_r$. First we consider stage $T-1$. Pick an arbitrary decision $Q_{T-1}, R_{T-1}, Z_{T-1}$ where $R_{T-1} > 0$. The expected cost at state $(X_{T-1}, Y_{T-1}, S_{T-2}, ..., S_{T-1-t_\Delta})$ under this decision is given by

$$
\begin{aligned}
&c_m Q_{T-1} + c_r R_{T-1} + c_c Z_{T-1} + \mathop{\mathbb{E}}_{D_{T-1}, C_{T-1}} \Big[h_s[X_{T-1} + Q_{T-1} + R_{T-1} - D_{T-1}]_+ \\
&+ h_r(Y_{T-1} + Z_{T-1} - R_{T-1}) + b[D_{T-1} - X_{T-1} - Q_{T-1} - R_{T-1}]_+ \Big]
\end{aligned}
$$

Now, pick another feasible decision $Q_{T-1} - 1, R_{T-1} + 1, Z_{T-1}$. The expected cost under this decision is given by

$$
\begin{aligned}
&c_m(Q_{T-1} - 1) + c_r(R_{T-1} + 1) + c_c Z_{T-1} \\
&+ \mathop{\mathbb{E}}_{D_{T-1}, C_{T-1}} \Big[h_s[X_{T-1} + Q_{T-1} - 1 + R_{T-1} + 1 - D_{T-1}]_+ \\
&+ h_r(Y_{T-1} + Z_{T-1} - R_{T-1} - 1) + b[D_{T-1} - X_{T-1} - Q_{T-1} + 1 - R_{T-1} - 1]_+ \Big]
\end{aligned}
$$

As we assume $c_m \geq c_r$, the expected cost under decision $Q_{T-1}R_{T-1}, Z_{T-1}$ is no less than the expected cost under $Q_{T-1} - 1R_{T-1} + 1, Z_{T-1}$:

$$
\begin{aligned}
& c_m Q_{T-1} + c_r R_{T-1} + c_c Z_{T-1} \\
& + \mathop{\mathbb{E}}_{D_{T-1}, C_{T-1}} \Big[h_s[X_{T-1} + Q_{T-1} + R_{T-1} - D_{T-1}]_+ \\
& \qquad + h_r(Y_{T-1} + Z_{T-1} - R_{T-1}) + b[D_{T-1} - X_{T-1} - Q_{T-1} - R_{T-1}]_+ \Big] \\
& \geq \\
& c_m(Q_{T-1} - 1) + c_r(R_{T-1} + 1) + c_c Z_{T-1} \\
& + \mathop{\mathbb{E}}_{D_{T-1}, C_{T-1}} \Big[h_s[X_{T-1} + Q_{T-1} - 1 + R_{T-1} + 1 - D_{T-1}]_+ \\
& + h_r(Y_{T-1} + Z_{T-1} - R_{T-1} - 1) + b[D_{T-1} - X_{T-1} - Q_{T-1} + 1 - R_{T-1} - 1]_+ \Big]
\end{aligned}
$$

Next we consider stage $t < T - 1$. By Lemma 5.3, we know that $V_t(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta}) + c_m \geq V_t(X_t, Y_t - 1, S_{t-1}, ..., S_{t-t_\Delta}) + c_r$, $\forall t$. Thus the expected cost under decision Q_t, R_t, Z_t is no less than the expected cost under decision $Q_t - 1, R_t + 1, Z_t$:

$$
\begin{aligned}
& c_m Q_t + c_r R_t + c_c Z_t \\
& + \mathop{\mathbb{E}}_{D_t, C_t} \Big[h_s[X_t + Q_t + R_t - D_t]_+ \\
& + h_r(Y_t + Z_t - R_t) + b[D_t - X_t - Q_t - R_t]_+ \\
& + V_{t+1}([X_t + Q_t + R_t - D_t]_+, Y_t + Z_t - R_t, S_t, ..., S_{t+1-t_\Delta}) \Big] \\
& \geq c_m(Q_t - 1) + c_r(R_t + 1) + c_c Z_t \\
& + \mathop{\mathbb{E}}_{D_t, C_t} \Big[h_s[X_t + Q_t - 1 + R_t + 1 - D_t]_+ \\
& + h_r(Y_t + Z_t - R_t - 1) + b[D_t - X_t - Q_t + 1 - R_t - 1]_+ \\
& + V_{t+1}([X_t + Q_t - 1 + R_t + 1 - D_t]_+, Y_t + Z_t - R_t - 1, S_t, ..., S_{t+1-t_\Delta}) \Big] \\
& \geq V_t(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta})
\end{aligned}
$$

A.5 Proof of Lemma 5.5

Suppose that $c_m \geq c_r$. We want to show that $V_t(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta}) + c_m \geq V_t(X_t, Y_t - 1, S_{t-1}, ..., S_{t-t_\Delta}) + c_r$, $\forall t$.

In order to show that this inequality holds for the risk-averse inventory problem in the case of lost sales, we use dual representation of coherent risk measures (cf. Shapiro, Dentcheva and Ruszczyński). Let A represent a set of probability measures. Suppose the probabilities for the occurrences of demand and collection rate are given as below:

p_i : Probability of which demand i occurs, $i = 0, 1, ..., 5$

q_j : Probability of which collection rate j occurs, $j = 1, 2, 3$ for three different values of $\frac{1}{3}, \frac{2}{3}$, and 1.

Assuming that both demand and collection rate are independent, the joint probability of which demand i and collection rate j occurs becomes:

r_{ij} : Probability of which demand i and collection rate j occurs, $i = 0, 1, ..., 5$ and $j = 1, 2, 3$.

Finally let μ_{ij} be a function of r_{ij}.

Then the risk measure $\rho(F)$ is represented as:

$$\rho(F) = \max_{\mu \in A} < \mu, F > .$$

First we consider stage $T - 1$.

(i) Suppose that $Q^*_{T-1}, R^*_{T-1}, Z^*_{T-1}$ is the optimal solution at state $(X_{T-1}, Y_{T-1}, S_{T-2}, ..., S_{T-1-t_\Delta})$ and $Q^*_{T-1}, R^*_{T-1}, Z^*_{T-1}$ is a feasible solution at state $(X_{T-1}, Y_{T-1} - 1, S_{T-2}, ..., S_{T-1-t_\Delta})$. Then it is easy to verify that the following inequalities hold.

$$V_{T-1}(X_{T-1}, Y_{T-1}, S_{T-2}, ..., S_{T-1-t_\Delta}) = c_m Q^*_{T-1} + c_r R^*_{T-1} + c_c Z^*_{T-1} + < \mu^*, F^*_{T-1} >$$
$$\geq c_m Q^*_{T-1} + c_r R^*_{T-1} + c_c Z^*_{T-1} + < \overline{\mu}, \overline{F}_{T-1} >$$
$$\geq V_{T-1}(X_{T-1}, Y_{T-1} - 1, S_{T-2}, ..., S_{T-1-t_\Delta}).$$

Thus,
$V_{T-1}(X_{T-1}, Y_{T-1}, S_{T-2}, ..., S_{T-1-t_\Delta}) \geq V_{T-1}(X_{T-1}, Y_{T-1} - 1, S_{T-2}, ..., S_{T-1-t_\Delta})$. As we assume $c_m \geq c_r$, we must have $V_{T-1}(X_{T-1}, Y_{T-1}, S_{T-2}, ..., S_{T-1-t_\Delta}) + c_m \geq V_{T-1}(X_{T-1}, Y_{T-1} - 1, S_{T-2}, ..., S_{T-1-t_\Delta}) + c_r$.

(ii) Suppose that $Q^*_{T-1}, R^*_{T-1}, Z^*_{T-1}$ is the optimal solution at state $(X_{T-1}, Y_{T-1}, S_{T-2}, ..., S_{T-1-t_\Delta})$ and $Q^*_{T-1}, R^*_{T-1}, Z^*_{T-1}$ is not a feasible solution at state $(X_{T-1}, Y_{T-1} - 1, S_{T-2}, ..., S_{T-1-t_\Delta})$. Note that $Q^*_{T-1} + 1, R^*_{T-1} - 1, Z^*_{T-1}$ must be a feasible solution at state $(X_{T-1}, Y_{T-1} - 1, S_{T-2}, ..., S_{T-1-t_\Delta})$. Then it is easy to verify that the following holds.

$$V_{T-1}(X_{T-1}, Y_{T-1}, S_{T-2}, ..., S_{T-1-t_\Delta}) = c_m Q^*_{T-1} + c_r R^*_{T-1} + c_c Z^*_{T-1} + < \mu^*, F^*_{T-1} >$$
$$= c_m(Q^*_{T-1} + 1) + c_r(R^*_{T-1} - 1) + c_c Z^*_{T-1} + < \overline{\mu}, \overline{F}_{T-1} >$$
$$\geq V_{T-1}(X_{T-1}, Y_{T-1} - 1, S_{T-2}, ..., S_{T-1-t_\Delta}).$$

Next we consider stage $t < T - 1$. Assuming $V_{t+1}(X_{t+1}, Y_{t+1}, S_t, ..., S_{t+1-t_\Delta}) + c_m \geq V_{t+1}(X_{t+1}, Y_{t+1} - 1, S_t, ..., S_{t+1-t_\Delta}) + c_r$, we will show that $V_t(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta}) + c_m \geq V_t(X_t, Y_t - 1, S_{t-1}, ..., S_{t-t_\Delta}) + c_r$.

(i) Suppose that Q^*_t, R^*_t, Z^*_t is the optimal solution at state $(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta})$ and Q^*_t, R^*_t, Z^*_t is a feasible solution at state $(X_t, Y_t - 1, S_{t-1}, ..., S_{t-t_\Delta})$. Then it is easy to verify that the following inequalities hold.

$$\begin{aligned}
&V_t(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta}) + c_m = c_m Q^*_t + c_r R^*_t + c_c Z^*_t \\
&+ \max_{\mu \in A} \sum_{i=0}^{5} \sum_{j=1}^{3} \mu_{ij} \Big[h_s[X_t + Q^*_t + R^*_t - i]_+ + h_r(Y_t - 1 + Z^*_t - R^*_t) \\
&+ p[i - X_t - Q^*_t - R^*_t]_+ \\
&+ V_{t+1}([X_t + Q^*_t + R^*_t - i]_+, Y_t - 1 + Z^*_t - R^*_t, S_t, ..., S_{t+1-t_\Delta}) + c_r \Big] \\
&\geq V_t(X_t, Y_t - 1, S_{t-1}, ..., S_{t-t_\Delta}) + c_r
\end{aligned}$$

(ii) Suppose that Q^*_t, R^*_t, Z^*_t is the optimal solution at state $(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta})$ and Q^*_t, R^*_t, Z^*_t is not a feasible solution at state $(X_t, Y_t - 1, S_{t-1}, ..., S_{t-t_\Delta})$. Note that $Q^*_t + 1, R^*_t - 1, Z^*_t$ must be a feasible solution at state $(X_t, Y_t -$

$1, S_{t-1}, ..., S_{t-t_\Delta})$. Then it is easy to verify that the following holds.

$$V_t(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta}) =$$
$$c_m Q_t^* + c_r R_t^* + c_c Z_t^* + < \mu^*, F_t^* > + c_m$$
$$= c_m(Q_t^* + 1) + c_r(R_t^* - 1) + c_c Z_t^* + < \overline{\mu}, \overline{F}_t > + c_r$$
$$\geq V_t(X_t, Y_t - 1, S_{t-1}, ..., S_{t-t_\Delta}) + c_r.$$

Hence we showed that $V_t(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta}) + c_m \geq V_t(X_t, Y_t - 1, S_{t-1}, ..., S_{t-t_\Delta}) + c_r$, $\forall t$.

A.6 Proof of Proposition 5.6

Suppose that $c_m \geq c_r$. First we consider stage $T-1$. Pick an arbitrary decision $Q_{T-1}, R_{T-1}, Z_{T-1}$ where $R_{T-1} > 0$. The expected cost at state $(X_{T-1}, Y_{T-1}, S_{T-2}, S_{T-1-t_\Delta})$ under this decision is given by

$$c_m Q_{T-1} + c_r R_{T-1} + c_c Z_{T-1}$$
$$+ \max_{\mu \in A} \sum_{i=0}^{5} \sum_{j=1}^{3} \mu_{ij} \Big[h_s [X_{T-1} + Q_{T-1} + R_{T-1} - i]_+$$
$$+ h_r(Y_{T-1} + Z_{T-1} - R_{T-1}) + p[i - X_{T-1} - Q_{T-1} - R_{T-1}]_+ \Big]$$
$$\geq V_{T-1}(X_{T-1}, Y_{T-1}, S_{T-2}, ..., S_{T-1-t_\Delta})$$

Now, pick another feasible decision $Q_{T-1} - 1, R_{T-1} + 1, Z_{T-1}$. The expected cost under this decision is given by

$$c_m(Q_{T-1} - 1) + c_r(R_{T-1} + 1) + c_c Z_{T-1}$$
$$+ \max_{\mu \in A} \sum_{i=0}^{5} \sum_{j=1}^{3} \mu_{ij} \Big[h_s [X_{T-1} + Q_{T-1} - 1 + R_{T-1} + 1 - i]_+$$
$$+ h_r(Y_{T-1} + Z_{T-1} - R_{T-1} - 1) + p[i - X_{T-1} - Q_{T-1} + 1 - R_{T-1} - 1]_+ \Big]$$
$$\geq V_{T-1}(X_{T-1}, Y_{T-1}, S_{T-2}, ..., S_{T-1-t_\Delta})$$

As we assume $c_m \geq c_r$, the expected cost under decision $Q_{T-1}, R_{T-1}, Z_{T-1}$ is no less than the expected cost under $Q_{T-1} - 1, R_{T-1} + 1, Z_{T-1}$:

$$
\begin{aligned}
& c_m Q_{T-1} + c_r R_{T-1} + c_c Z_{T-1} + < \mu^*, F^*_{T-1} > \\
& \geq c_m (Q_{T-1} - 1) + c_r (R_{T-1} + 1) + c_c Z_{T-1} + < \overline{\mu}, \overline{F}_{T-1} >
\end{aligned}
$$

Next we consider stage $T - 1$. By Lemma 5.1, we know that $V_t(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta}) + c_m \geq V_t(X_t, Y_t - 1, S_{t-1}, ..., S_{t-t_\Delta}) + c_r$, $\forall t$. Thus the expected cost under decision Q_t, R_t, Z_t is no less than the expected cost under decision $Q_t - 1, R_t + 1, Z_t$:

$$
c_m Q_t + c_r R_t + c_c Z_t + < \mu^*, F^*_t > \geq c_m (Q_t - 1) + c_r (R_t + 1) + c_c Z_t + < \overline{\mu}, \overline{F}_t >
$$

A.7 Proof of Lemma 5.7

Suppose that $c_m \geq c_r$. We want to show that $V_t(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta}) + c_m \geq V_t(X_t, Y_t - 1, S_{t-1}, ..., S_{t-t_\Delta}) + c_r$, $\forall t$.

First we consider stage $T - 1$.

(i) Suppose that $Q^*_{T-1}, R^*_{T-1}, Z^*_{T-1}$ is the optimal solution at state $(X_{T-1}, Y_{T-1}, S_{T-2}, ..., S_{T-1-t_\Delta})$ and $Q^*_{T-1}, R^*_{T-1}, Z^*_{T-1}$ is a feasible solution at state $(X_{T-1}, Y_{T-1} - 1, S_{T-2}, ..., S_{T-1-t_\Delta})$. Then it is easy to verify that the following inequalities hold.

$$
\begin{aligned}
& V_{T-1}(X_{T-1}, Y_{T-1}, S_{T-2}, ..., S_{T-1-t_\Delta}) = c_m Q^*_{T-1} + c_r R^*_{T-1} + c_c Z^*_{T-1} + < \mu^*, F^*_{T-1} > \\
& \geq c_m Q^*_{T-1} + c_r R^*_{T-1} + c_c Z^*_{T-1} + < \overline{\mu}, \overline{F}_{T-1} > \\
& \geq V_{T-1}(X_{T-1}, Y_{T-1} - 1, S_{T-2}, ..., S_{T-1-t_\Delta}).
\end{aligned}
$$

Thus $V_{T-1}(X_{T-1}, Y_{T-1}, S_{T-2}, ..., S_{T-1-t_\Delta}) \geq V_{T-1}(X_{T-1}, Y_{T-1} - 1, S_{T-2}, ..., S_{T-1-t_\Delta})$. As we assume $c_m \geq c_r$, $V_{T-1}(X_{T-1}, Y_{T-1}, S_{T-2}, ..., S_{T-1-t_\Delta}) + c_m \geq V_{T-1}(X_{T-1}, Y_{T-1} - 1, S_{T-2}, ..., S_{T-1-t_\Delta}) + c_r$.

(ii) Suppose that $Q^*_{T-1}, R^*_{T-1}, Z^*_{T-1}$ is the optimal solution at state $(X_{T-1}, Y_{T-1}, S_{T-2}, ..., S_{T-1-t_\Delta})$ and $Q^*_{T-1}, R^*_{T-1}, Z^*_{T-1}$ is not a feasible solution at state $(X_{T-1}, Y_{T-1}-1, S_{T-2}, ..., S_{T-1-t_\Delta})$. Note that $Q^*_{T-1}+1, R^*_{T-1}-1, Z^*_{T-1}$ must be a feasible solution at state $(X_{T-1}, Y_{T-1}-1, S_{T-2}, ..., S_{T-1-t_\Delta})$. Then it is easy to verify that the following holds.

$$\begin{aligned}
&V_{T-1}(X_{T-1}, Y_{T-1}, S_{T-2}, ..., S_{T-1-t_\Delta}) = \\
&c_m Q^*_{T-1} + c_r R^*_{T-1} + c_c Z^*_{T-1} + < \mu^*, F^*_{T-1} > + c_m \\
&= c_m(Q^*_{T-1}+1) + c_r(R^*_{T-1}-1) + c_c Z^*_{T-1} + < \overline{\mu}, \overline{F}_{T-1} > + c_r \\
&\geq V_{T-1}(X_{T-1}, Y_{T-1}-1, S_{T-2}, ..., S_{T-1-t_\Delta}) + c_r.
\end{aligned}$$

Next we consider stage $t < T_1$. Assuming $V_{t+1}(X_{t+1}, Y_{t+1}, S_t, ..., S_{t+1-t_\Delta}) + c_m \geq V_{t+1}(X_{t+1}, Y_{t+1}-1, S_t, ..., S_{t+1-t_\Delta}) + c_r$, we will show $V_t(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta}) + c_m \geq V_t(X_t, Y_t-1, S_{t-1}, ..., S_{t-t_\Delta}) + c_r$.

(i) Suppose that Q^*_t, R^*_t, Z^*_t is the optimal solution at state $(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta})$ and Q^*_t, R^*_t, Z^*_t is a feasible solution at state $(X_t, Y_t-1, S_{t-1}, ..., S_{t-t_\Delta})$. Then it is easy to show that the following inequalities hold.

$$\begin{aligned}
&V_t(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta}) + c_m = c_m Q^*_t + c_r R^*_t + c_c Z^*_t \\
&+ \max_{\mu \in A} \sum_{i=0}^{5} \sum_{j=1}^{3} \mu_{ij} \Big[h_s[X_t + Q^*_t + R^*_t - i]_+ + h_r(Y_t - 1 + Z^*_t - R^*_t) \\
&+ b[i - X_t - Q^*_t - R^*_t]_+ \\
&+ V_{t+1}([X_t + Q^*_t + R^*_t - i]_+, Y_t - 1 + Z^*_t - R^*_t, S_t, ..., S_{t+1-t_\Delta}) + c_r \Big] \\
&\geq V_t(X_t, Y_t-1, S_{t-1}, ..., S_{t-t_\Delta}) + c_r
\end{aligned}$$

(ii) Suppose that Q^*_t, R^*_t, Z^*_t is the optimal solution at state $(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta})$ and Q^*_t, R^*_t, Z^*_t is not a feasible solution at state $(X_t, Y_t-1, S_{t-1}, ..., S_{t-t_\Delta})$. Note that Q^*_t+1, R^*_t-1, Z^*_t must be a feasible solution at state $(X_t, Y_t-1, S_{t-1}, ..., S_{t-t_\Delta})$. Then it is easy to verify that the following holds.

$$\begin{aligned}
&V_t(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta}) = c_m Q^*_t + c_r R^*_t + c_c Z^*_t + < \mu^*, F^*_t > + c_m \\
&= c_m(Q^*_t+1) + c_r(R^*_t-1) + c_c Z^*_t + < \overline{\mu}, \overline{F}_t > + c_r \\
&\geq V_t(X_t, Y_t-1, S_{t-1}, ..., S_{t-t_\Delta}) + c_r.
\end{aligned}$$

Hence we showed that $V_t(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta}) + c_m \geq V_t(X_t, Y_t - 1, S_{t-1}, ..., S_{t-t_\Delta}) + c_r$, $\forall t$.

A.8 Proof of Proposition 5.8

Suppose that $c_m \geq c_r$. First we consider stage $T-1$. Pick arbitrary decision combination $Q_{T-1}, R_{T-1}, Z_{T-1}$ where $R_{T-1} > 0$. The expected cost at state $(X_{T-1}, Y_{T-1}, S_{T-2}, ..., S_{T-1-t_\Delta})$ under this decision is given by

$$\begin{aligned}
&c_m Q_{T-1} + c_r R_{T-1} + c_c Z_{T-1} \\
&+ \max_{\mu \in A} \sum_{i=0}^{5} \sum_{j=1}^{3} \mu_{ij} \Big[h_s [X_{T-1} + Q_{T-1} + R_{T-1} - i]_+ \\
&+ h_r (Y_{T-1} + Z_{T-1} - R_{T-1}) + b[i - X_{T-1} - Q_{T-1} - R_{T-1}]_+ \Big] \\
&\geq V_{T-1}(X_{T-1}, Y_{T-1}, S_{T-2}, ..., S_{T-1-t_\Delta})
\end{aligned}$$

Now, pick another feasible decision $Q_{T-1} - 1, R_{T-1} + 1, Z_{T-1}$. The expected cost under this decision is given by

$$\begin{aligned}
&c_m (Q_{T-1} - 1) + c_r (R_{T-1} + 1) + c_c Z_{T-1} \\
&+ \max_{\mu \in A} \sum_{i=0}^{5} \sum_{j=1}^{3} \mu_{ij} \Big[h_s [X_{T-1} + Q_{T-1} - 1 + R_{T-1} + 1 - i]_+ \\
&+ h_r (Y_{T-1} + Z_{T-1} - R_{T-1} - 1) + b[i - X_{T-1} - Q_{T-1} + 1 - R_{T-1} - 1]_+ \Big] \\
&\geq V_{T-1}(X_{T-1}, Y_{T-1}, S_{T-2}, ..., S_{T-1-t_\Delta})
\end{aligned}$$

As we assume $c_m \geq c_r$, the expected cost under decision $Q_{T-1}, R_{T-1}, Z_{T-1}$ is no less than the expected cost under $Q_{T-1} - 1, R_{T-1} + 1, Z_{T-1}$:

$$\begin{aligned}
&c_m Q_{T-1} + c_r R_{T-1} + c_c Z_{T-1} + < \mu^*, F^*_{N-1} > + c_m \\
&\geq c_m (Q_{T-1} - 1) + c_r (R_{T-1} + 1) + c_c Z_{T-1} + < \overline{\mu}, \overline{F}_{T-1} > + c_r
\end{aligned}$$

Next we consider stage $t < T - 1$. By Lemma 5.7, we know that $V_t(X_t, Y_t, S_{t-1}, ..., S_{t-t_\Delta}) + c_m \geq V_t(X_t, Y_t - 1, S_{t-1}, ..., S_{t-t_\Delta}) + c_r$, $\forall t$. Thus the expected cost under decision Q_t, R_t, Z_t is no less than the expected cost under decision $Q_t - 1, R_t + 1, Z_t$:

$$c_m Q_t + c_r R_t + c_c Z_t + < \mu^*, F_t^* > \geq c_m(Q_t - 1) + c_r(R_t + 1) + c_c Z_t + < \overline{\mu}, \overline{F}_t >$$