AN ANTI-WINDUP COMPENSATOR FOR SYSTEMS WITH TIME DELAY AND INTEGRAL ACTION

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By

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We certify that we have read this thesis and that in our opinion it is fully adequate, in scope and in quality, as a thesis for the degree of Master of Science.

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ABSTRACT

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Being one of the most popular saturation compensator methods, anti-windup mechanism is commonly used in various control applications. The problems arising from the system nonlinearities are prone to change the behaviors of the system adversely in time such as performance degradation or instability. Anti-windup schemes including internal model structure with the robust compensator are crucial in terms of preserving the system stability and minimizing the tracking error when controller operates at the limits of the actuator.

Saturation problem is further aggravated by the dead-time that appears frequently in the systems depending on processing of sensed signals or transferring control signals to plants. Smith predictor based controllers are efficient in the compensation of time delay, indeed the controller is designed by eliminating the delay element from the characteristic equation of the closed-loop system. We apply Smith predictor based controller design for the system incorporating time delay and integral action to achieve high performance sinusoidal tracking.

This study extends an anti-windup scheme via Smith predictor based controller approach by redesigning the transfer functions within the anti-windup structure. We present simulation studies on the plant transfer function including time delay and integrator to illustrate that our extended structure successfully accomplish accurate tracking under the saturation nonlinearity.

Keywords: Anti-windup, Saturation, Time Delay Systems, Smith Predictor Based Controller, Periodic Sinusoidal Tracking.

ÖZET

ZAMAN GECİKMELİ VE İNTEGRAL EYLEMLİ SİSTEMLER İÇİN İNTEGRAL YIĞILMASI DÜZENLEYİCİ

Dilan Öztürk
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En popüler satürasyon kompansatör yöntemlerinden biri olan integral yığılması önleme mekanizması çeşitli kontrol uygulamalarında yaygın olarak kullanılmaktadır. Sistemin doğrusal olmayan yönlerinden kaynaklanan sorunlar, performans düşüşü veya istikrarsızlık gibi zaman içinde sistemin davranışlarını olumsuz bir şekilde değiştirme eğilimindedir. Sağlam kompansatörlü dahili model yapısını içeren integral yığılması önleme tasarımları, sistem kararlılığını korumak ve denetleyici aktüatör sınırlarında çalışırken izleme hatasını en aza indirgemek açısından önemlidir.

Doygunluk sorunu, algılanan sinyallerin işlenmesine veya kontrol sinyallerinin sistemlere aktarılmasına bağlı olarak sıkça görülen zaman gecikmesi tarafından daha kötü bir hal almaktadır. Smith kestirim tabanlı denetleyiciler zaman gecikmesinin telafisinde etkilidir, aslında denetleyici, gecikme elemanını kapalı döngü sisteminin karakteristik denkleminden çıkararak tasarlanmıştır. Yüksek performanslı sinüzoidal izlemeyi elde etmek için, zaman gecikmesi ve integral eylemi içeren sistemde Smith kestirim tabanlı denetleyici tasarımı uygulanmıştır.

Bu çalışma literatürde önerilen bir integral yığılması önleme tasarımını, Smith kestirim tabanlı denetleyici yaklaşımı ile integral yığılması önleme yapısı içindeki transfer fonksiyonlarını yeniden tasarlayarak genişletmektedir. Genişletilmiş yapının doygunluk altında doğru izlemeyi başarılı bir şekilde gerçekleştirdiğini göstermek için, zaman gecikmesi ve integratör içeren transfer fonksiyonu üzerinde yapılan simülasyon çalışmaları sunulmaktadır.

Anahtar sözcükler: İntegral Yığılması Önleme, Satürasyon, Zaman Gecikmeli Sistemler, Smith Kestirim Tabanlı Denetleyici, Periyodik Sinüzoidal İzleme.

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to my precious grandmother and grandfather

Chapter 1

Introduction

This thesis concerns the design of a novel anti-windup scheme for systems involving time delay and integral action. The proposed control architecture tries to achieve high precision asymptotic tracking under the presence of saturation nonlinearity and to suppress the adverse effects of input saturation. We tackle problems regarding the stability and degradation in the performance of linear systems subject to nonlinearities. An anti-windup control architecture is extended for the dead-time system to improve the tracking performance of the feedback system under input saturation.

1.1 Motivation and Background

Actuator saturation emerges depending on the system specifications in most of the control approaches which causes performance degradations or even instability and this phenomenon is called as *windup* [1]. Rich variety of anti-windup control mechanisms have been developed to deal with actuator saturation since the 1950's [2, 3]. Anti-windup architecture mainly focuses on to preserve the system stability and enforces to minimize the tracking error when controller operates at the actuator limits. One of the primary advantages of anti-windup scheme is that it helps to recover from saturation type nonlinearity.

There have been novel and highly promising anti-windup solutions in literature. Campo et al. presents several anti-windup/bumpless transfer approaches for the linear time invariant systems in the existence of plant input saturation concerning that the system remains stable when limitations occur [4]. Internal model control (IMC) structure, observer based compensator and Hanus' conditioned controller are some of the proposed anti-windup/bumpless schemes in [4]. Besides, the analysis of the aircraft control system with the analytic-numerical solution for hidden oscillations is presented in [5]. Oscillations occurred at the aircraft control system under the effect of saturation are suppressed with the static anti-windup scheme. Another anti-windup based approach is illustrated in [6] to control the production rate of a manufacturing machine with saturation nonlinearities. To deal with the problem of integrator windup, which derives from the input saturation and integral action within the controller or plant, an anti-windup design based on convergent theory is applied on the manufacturing system [6].

Besides saturation nonlinearities and plant uncertainties, another inevitable problem that appears frequently in the systems is unfortunately time delay. It may occur depending on the physical distance between the process and the controller during the information flow or may occur because of the process or the controller itself [7]. Time delay element is represented as e^{-hs}, where $h > 0$ is the dead-time which makes the system infinite dimensional [7]. In addition to that, time delay element may cause performance degradation in the system or even cause instability by affecting the stability margins of the closed loop feedback system adversely. The transfer functions of time delay systems are also irrational, hence classical stability test methods such as Routh-Hurwitz or Kharitanov and root locus techniques can not be applied for this kind of systems directly [7].

There exist various control approaches to deal with infinite dimensional systems, in fact PID type controllers are the most preferred method in the controller design independently of time delay [8]. However, in the existence of long dead-time, these controllers may be inadequate to improve the system performance and to cope with instability. Hence, Smith predictor based type controllers are mostly used for the stable process of control systems involving time delay since these are very effective long dead-time compensators [9]. The most important advantage of Smith predictor approach is eliminating the time delay element from the closed loop system characteristic equation in the design of the controller [10]. Depending on the system requirements, design of the Smith predictor controller may vary and different applications can be found in [11–13]. In this thesis, the Smith predictor based controller proposed in [14], which aims to achieve a set of performance and robustness objectives, is used in the controller design of the time delay system.

Motivated by work in this area, this thesis presents a novel anti-windup structure based on the different solutions regarding system nonlinearity and time delay by combining anti-windup technique in [15] with the Smith predictor design proposed in [14]. The robust anti-windup compensator method we use in this study postulates an extension of the architecture in [15] to be applicable for the dead-time systems. Smith predictor method mentioned in [14] is inserted into the internal model units, stabilizer and compensator design of the anti-windup approach. The objective behind this design is to provide satisfying tracking performance of the reference signal under the saturation such that dead-time system output converges to the desired sinusoidal output.

1.2 Existing Work

Existing anti-windup methods exclusively focus on eliminating the effect of saturation for the stable performance of the control system without considering the specific challenges of the tracking [16–18]. In this regard, the internal model principle approach for the anti-windup compensator design is a significant technique for tracking and/or rejecting problems of the reference sinusoidal signal [19]. This approach is mainly based on a controller design to provide closed loop stability and to regulate the tracking error when specific system parameters are perturbed [20]. In contrast, there also exist internal model based solutions for the saturation control without aiming high performance tracking [21–23].

The ad-hoc solutions for the anti-windup compensator approach can be found in [24,25]. Hanus proposes a conditioning technique to overcome the deterioration existing in the closed-loop system performance due to input saturation for the multi-input multi-output nonlinear system [25]. However, in this method, neither internal model unit nor tracking capability are taken into consideration. The very first experimental study on the internal model control theory is presented by Francis in [20] for the multivariable nonlinear system including uncertainties and disturbances without tracking ability. On the other hand, Sun proposes a saturated adaptive robust control method for the nonlinear active suspension system to deal with the problem of vibration control in the presence of saturation [26]. This method mainly focuses on how the tracking performance under the saturation is maintained by suppressing the tracking error of the closed-loop system using an anti-windup compensator together with a robust nonlinear feedback block [26]. Hence, without utilizing internal model unit, the tracking error asymptotically converges to zero using adaptive robust control method in Sun's study.

Different than anti-windup approaches, the controller design for the dead-time systems is also investigated. As mentioned in Section 1.1, Smith predictor based controller design is a popular technique to overcome the damaging effects of systems with integral action and time delay. Smith proposed a dead-time compensator method first in [27] by giving a mathematical model of the process in the closed-loop feedback loop and then this technique became known as Smith predictor based method [12]. As an early study for the Smith predictor approach, a process-model control aiming to achieve zero steady-state error is built for the linear systems including time delay [12]. However, the proposed approach cannot provide constant disturbance rejection and zero steady-state error if the system includes integral action [12]. Moreover, the modified Smith predictor structure is introduced in [28] to yield better desired transient responses to step input and disturbance signals for the system with an integrator and dead-time. The load response is improved via additional transfer function involved in the modified structure, however the parameters of this function are adjustable which require tuning to obtain optimal values [28]. There also exist other modifications of the Smith predictor structure, see [29, 30] and the references therein, which simplify the tuning parameters within [28] in order to achieve a clear physical interpretation. Besides, faster disturbance suppression is reached in [30] with the additional derivative action preserving the same response.

1.3 Methodology and Contributions

The application of anti-windup mechanisms incorporating internal model principle and tracking capability of the desired signal for the dead-time systems is a challenging problem as discussed in the previous sections. Extension of the anti-windup structure with the combination of Smith predictor based controller design to be adapted for systems subject to time delay and integral action is proposed as a novel architecture in this study.

Robust anti-windup compensator based on [15] is presented in the first part of the thesis that includes internal model units together with the robust compensator. High performance sinusoidal tracking is achieved with parallel internal model control structure by designing the model units and robust stabilizer account for the desired system requirements. The stabilizer is optimized with $\mathcal{H}_\infty$ mixed sensitivity problem to provide robustness condition together with the sector bound criterion. In order to handle system nonlinearities and actuator saturations, anti-windup compensator design on top of the parallel internal model structure is introduced as discussed in [15].

Smith predictor based approach is preferred in the controller design of dead-time system with performance and robustness objectives. Similar to [14], the controller is designed to follow the sinusoidal reference signal assuming that the disturbance is negligible. Controller parametrization is applied in the design of free parameter within the Smith predictor controller based on the design criterion and interpolation conditions.

The extension is deployed depending on the relation between anti-windup based and Smith predictor based approaches by deriving the transfer functions for both techniques. The primary contribution of this thesis is the proposed new robust anti-windup control architecture extended with the Smith predictor method for the dead-time systems with integral term. Simulations with the proposed scheme on the time delay system under the existence of input saturation are presented. Our studies illustrate that the proposed structure can be used to minimize the tracking error in the presence of system nonlinearities.

1.4 Organization of the Thesis

An anti-windup scheme with the related block diagrams for the delay-free systems is given in Chapter 2. The design steps including parallel internal model structure and robust anti-windup compensation are explained in detail. Chapter 3 covers the basic steps of the Smith predictor-based controller design in order to extend the anti-windup mechanism to be applicable for the dead-time systems. This chapter presents how we achieve the relation between these two structures and how we postulate an extension of the anti-windup design.

A brief summary of the novel structure is given in the beginning of the second part of this thesis. We present the results of simulation studies we performed to evaluate the performance of our structure with and without anti-windup blocks in Chapter 4. The design steps are provided in detail on the time-delayed transfer function by utilizing the latest definitions of the anti-windup components. Finally in Chapter 5, we conclude the thesis with a summary of our study and propose a longer term goal as an extension of this structure for the unstable plants with more than one poles at the right-half complex plane and mention open research topics.

Chapter 2

Robust Anti-Windup Scheme

This chapter covers the novel architecture together with a parallel internal-model based control approach and a robust anti-windup control structure. High precision tracking is guaranteed via this control architecture while the system has saturation nonlinearities occurring at the actuators as well as uncertainties stemming from modeling the plant structure [15].

2.1 Preliminaries

Definition 2.1.1. *Let $G(s)$ denote the transfer function of a linear time invariant system G. We say that G is stable if $G(s)$ is analytic and bounded in the closed right half plane $(Re(s) > 0)$ [31], in which case we write $G \in \mathcal{H}_\infty$ and*

$$\|G\|_\infty = \sup_{Re(s)>0} \left|G(s)\right| < \infty \; . \tag{2.1}$$

Let $P(s)$ be a rational function of a plant given in Fig. 2.1. There exist coprime polynomials $N_p(s)$ and $D_p(s)$ such that

$$P(s) = \frac{N_p(s)}{D_p(s)}$$

where N_p, D_p are stable [7].

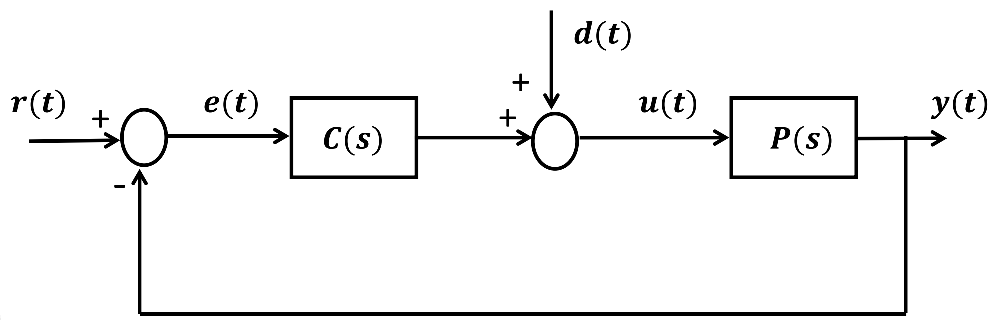


Figure 2.1: Closed-loop feedback system

Theorem 2.1.1. *(Controller Parametrization) Given $P(s)$, the set of all controllers, $C(s)$, satisfying the internal stability of the closed loop feedback system in Fig. 2.1 is characterized by the parametrization*

$$C(s) = \frac{X + D_p Q}{Y - N_p Q} \quad : \quad Q \in \mathbb{R}\mathcal{H}_\infty\,, \quad Q \neq Y N_p^{-1}$$

where $X(s)$ and $Y(s)$ are stable transfer functions obtained from the following Bezout equation

$$X N_p + Y D_p = 1\,.$$

Example : Let $P(s) = \frac{1}{s-a}$ where $a > 0$, to obtain stable coprime polynomials, $N_p(s)$ and $D_p(s)$ can be determined as

$$N_p(s) = \frac{1}{s+a}\,, \quad D_p(s) = \frac{s-a}{s+a}\,.$$

But we also need to find $X, Y \in \mathcal{H}_\infty$ such that $N_p(s)X(s) + D_p(s)Y(s) = 1$. Using chosen coprime polynomials, we can define

$$Y(s) = \frac{1 - \frac{1}{s+a}X(s)}{\frac{s-a}{s+a}} \in \mathcal{H}_\infty\,.$$

Giving $s = a$ results in $D_p(a) = 0$, hence we have to choose $1 - N_p(a)X(a) = 0$ to obtain stable $Y(s)$. While choosing $X(s) = 2a \in \mathcal{H}_\infty$, we can achieve $Y(s) = 1$ which is stable.

Consequently, corresponding stabilizing controller according to Theorem 2.1.1 can be written as

$$C(s) = \frac{2a + \frac{s-a}{s+a}Q(s)}{1 - \frac{1}{s+a}Q(s)} \quad : \quad Q \in \mathbb{R}\mathcal{H}_\infty \,.$$

2.2 Preface to Anti-Windup Control Structures

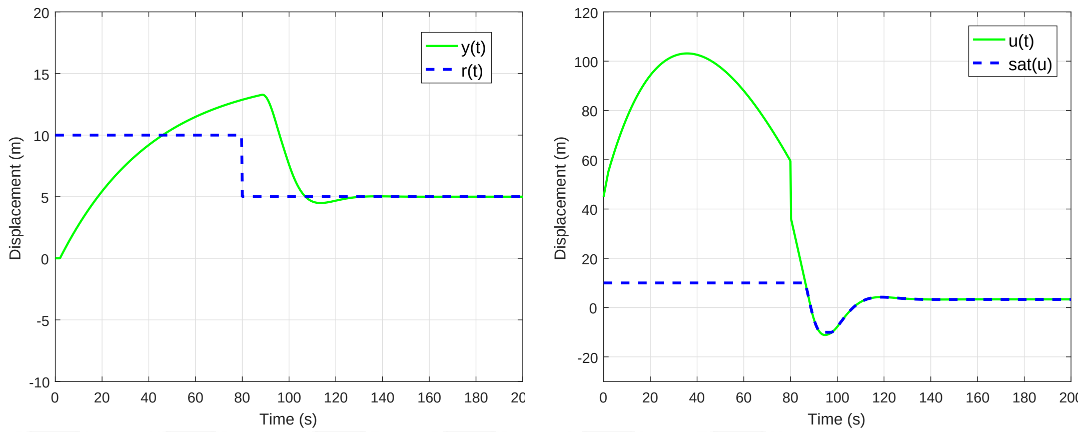
Various control approaches have been developed to deal with actuator saturations and anti-windup mechanisms are the most frequently used approach to recover this problem. The most widely used controller is the PID type since in the absence of nonlinearities, it is difficult to improve the performance with more complex controller structures [32]. Matlab also has an anti-windup control inside the PID block of Simulink to prevent the possible integration windup when the actuators are saturated. As a benchmark example, a low-order plant in Matlab with and without anti-windup control is implemented to compare the results and observe the effect of anti-windup mechanism. The back-calculation anti-windup method in Matlab discharges the PID controller's internal integrator when the controller exceeds the physical saturation limits of the system actuator [33]. At this time, the system enters in a nonlinear region where the controller is unable to immediately respond to the changes. When the effect of saturation is eliminated, the integrator in the PID block will be activated.

The feedback loop shown in Fig. 2.1 is implemented while assuming that the disturbance is negligible, i.e. $d(t)$ is zero. The plant is chosen as a first-order stable system with dead-time

$$P(s) = \frac{1.5}{40s+1}e^{-2s}$$

and has an input saturation limits of $[-10, 10]$. Similarly, the controller stabilizing the given plant is chosen as

$$C(s) = 0.5 + \frac{4.5}{s}\,.$$



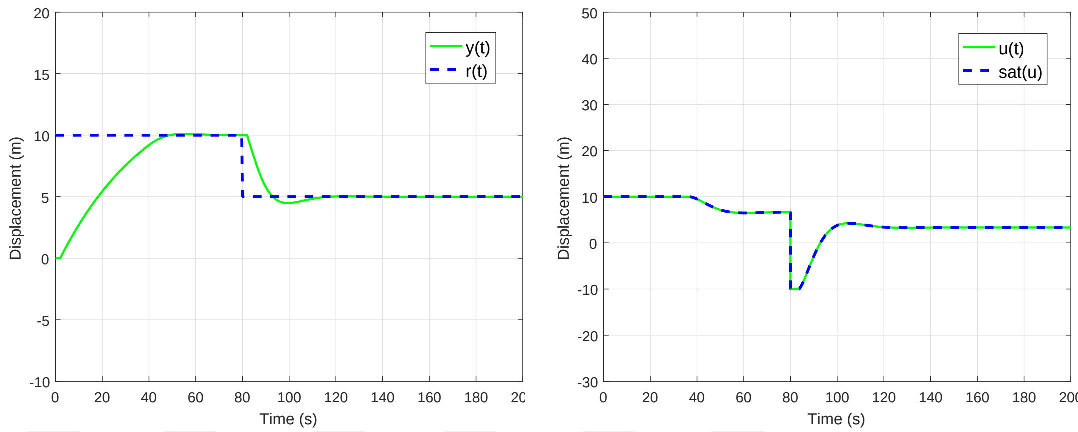
(a) r(t) : input signal
y(t) : measured output

(b) u(t) : controller output
sat(u) : saturated plant input

Figure 2.2: The comparison between input-output and controller output-saturated input signals under the presence of saturation when anti-windup in the PID block of Matlab Simulink is not activated.

In order to observe the effect of saturation, anti-windup in the PID block is not active at first. The controller output reaches a steady-state outside the range of the actuator as shown in Fig. 2.2b. In this case, controller is operating in a nonlinear region where increasing the control signal has no effect on the system output [33]. This means that the plant input is different from the controller output which causes a situation that the output of the controller can not drive the plant as required [34]. This condition is known as *winding-up* or *controller windup*. System output together with the given reference signal is also illustrated in Fig. 2.2a. There exists an overshoot at the measured output when anti-windup is not enabled.

When we enable the anti-windup in PID block, the controller operates under the specified saturation limits. As we can observe in Fig. 2.3b, controller output $u(t)$ and saturated input $sat(u)$ coincide with each other. Note that the output signal has no overshoot as depicted in Fig. 2.3a while we apply anti-windup mechanism.



(a) r(t) : input signal
y(t) : measured output

(b) u(t) : controller output
sat(u) : saturated plant input

Figure 2.3: The comparison between input-output and controller output-saturated input signals under the presence of saturation when anti-windup in the PID block of Matlab Simulink is activated.

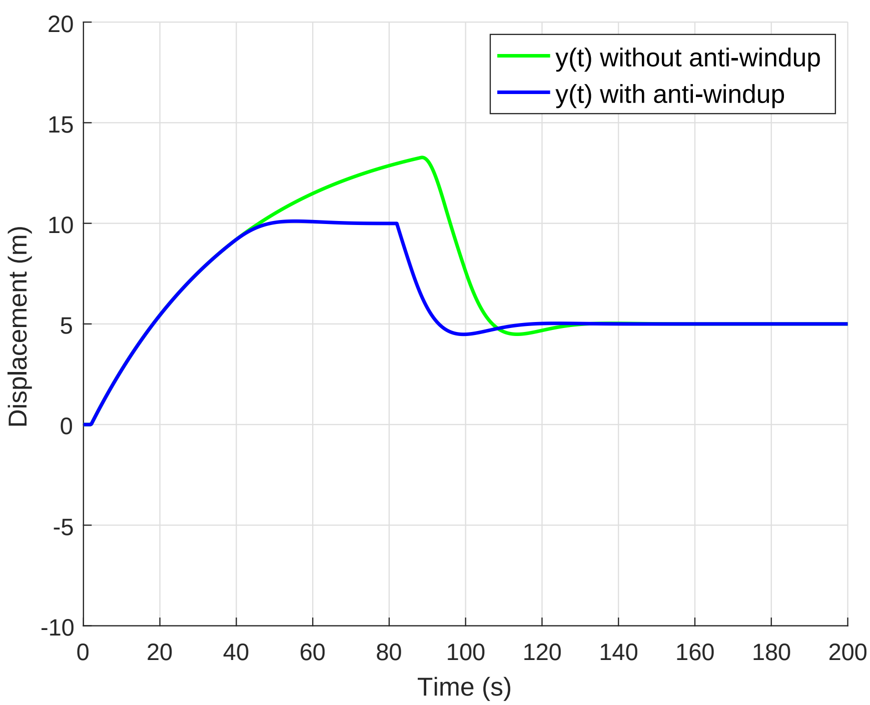


Figure 2.4: Comparison of the system output y(t) with and without anti-windup inside the PID block.

In order to better observe the effect of anti-windup on the saturated system, measured outputs are compared in Fig. 2.4. The system output (blue line in Fig. 2.4) reaches the steady-state faster when anti-windup is enabled without any overshoot.

As it can be understood from the benchmark example, the nonlinear characteristics of the actuators pose an obstacle in the tracking approaches. There exist different methods for the tracking control since it may depend on the system specifications, the actuator itself or specific tracking control challenges such as nonlinearity, system uncertainties, etc. J.She et al. [35] applies a repetitive control by using repeated learning actions of a given periodic reference signal to improve the tracking precision. After applying this approach gradually, the tracking error is reduced and system output tracks the desired reference signal. Another approach given in [36] is modeling-free inversion-based iterative feedforward control to achieve output tracking for a single-input single-output linear time-invariant systems by eliminating the dynamics modeling process.

Kanamaya [37] also proposed a method for a stable tracking of an autonomous mobile robot with abundant simulations results. He defines a control rule to determine the vehicle's linear and rotational velocities and applies Lyapunov function to solve the nonlinearity in the system equations. The desired input is determined as the reference posture and reference velocities. The output is observed by using proposed tracking control method aiming that the tracking error converges to zero [37].

Despite these different techniques, tracking performance is adversely affected by the actuator saturation arising form the large disturbances since saturation is the most widely encountered and most dangerous nonlinearity in control systems [38]. The saturation effects can be minimized by applying different control approaches. Zhou proposed a parametric discrete-time periodic Lyapunov equation based method for the stabilization of discrete-time linear periodic systems subject to actuator saturation [39]. He achieves this by generalizing the results obtained with time-invariant systems to periodic systems.

Hu et al. presented different controller design method for linear systems subject to input saturation and disturbance. The domain of attraction of a system with the existence of saturated linear feedback is estimated and a condition is derived for determining if a given ellipsoid is contractively invariant [40]. By extending this condition, linear matrix inequality based methods including all the varying parameters are developed to construct a feedback law both for closed-loop stability and disturbance rejection [40].

Besides these approaches, anti-windup control is a popular method for the saturating systems. The actuator saturation is ignored at first to design the stabilizing controller in the linear phase and then the adverse effects of the saturation on system performance is minimized via anti-windup compensation. There are various anti-windup techniques depending on the performance requirements and saturation nonlinearities existing at the limits of the actuators. Kothare et al. reported different anti-windup designs in [34] for the control of linear time-invariant (LTI) systems in the presence of saturation. He basically defines the general anti-windup problem and presents known LTI schemes listed as anti-reset windup, conventional anti-windup, Hanus conditioned controller, observer based anti-windup, internal model control, anti-windup design for internal model control and extended Kalman filter [34]. A general knowledge on the anti-windup compensator designs can be obtained with the help of this study.

Before explaining the method we use, several anti-windup implementations are provided. Wu proposed a generalized saturation control technique for the exponentially unstable LTI systems to guarantee the stability in the absence of input saturation [41]. The closed-loop system stability is reached by restricting the input nonlinearity to a smaller conic sector to stabilize the control system at certain limits. With this approach, improvement in the system performance and stabilization of the unstable LTI systems up to a specific size is achieved by restricting the controller input and using a dynamic anti-windup compensator. Linear conditioning approach is implemented in [21] to suppress the effect of nonlinearity by applying a linear transfer function during the saturation event to ensure the stability. While operating in the linear phase, this transfer function has no effect, however; while system is subjected to input saturation, the linear function modifies the system's behavior to remain stable [21]. This method differs from [34] because implementation of the controller and linear conditioning are decoupled in [21].

Different than these solutions, we apply internal model-based control method since our aim is to achieve high precision tracking of a sinusoidal reference input. Internal model-based approach is a fundamental technique in tracking/rejection problems because it contains the properties of the sinusoidal reference/disturbance signal to reproduce the desired/rejected signals in the feedback loop [19], [42]. However, applying only internal model control method is not effective in the tracking approaches for the systems in the presence of actuator saturation. In this sense, robust anti-windup compensator as a combination of internal model-based unit is applied both to handle the nonlinearities and to achieve a precise tracking. The details of how this process is performed can be found in the following sections.

2.3 The Proposed Robust Anti-Windup Architecture

A control architecture is introduced for high precision trajectory tracking including saturation compensation blocks, parallel internal model units as well as robust anti-windup compensator design. This section continues with the guidelines in the design of proposed architecture and introduces stability and robustness conditions with respect to the system uncertainties.

2.3.1 Parallel Internal Model Structure

The parallel internal model control structure introduced both in [15] and [42] is investigated in order to implement internal model-based control method for the sinusoidal tracking. Reference signal $r(t)$ illustrated in Fig. 2.5 is defined based on the exogenous dynamical system equation in the form

$$R(s) = \Lambda(s)^{-1} R_0(s) \tag{2.2}$$

where $R_0(s)$ represents Laplace transform of the signal $r_0(t)$, $R(s)$ represents Laplace transform of the reference signal $r(t)$ and $\Lambda(s)^{-1}$ represents the dynamics of the exogenous system. In the design, we are aiming to minimize the tracking error, denoted as $e(t)$ in Fig. 2.5, as much as possible such that the following conditions hold:

i. Considering zero tracking signal ($r(t) = 0$), the unforced closed-loop system is asymptotically stable,

ii. Considering any initial conditions of the plant, the closed-loop system satisfies $\lim_{t\to\infty} e(t) = 0$.

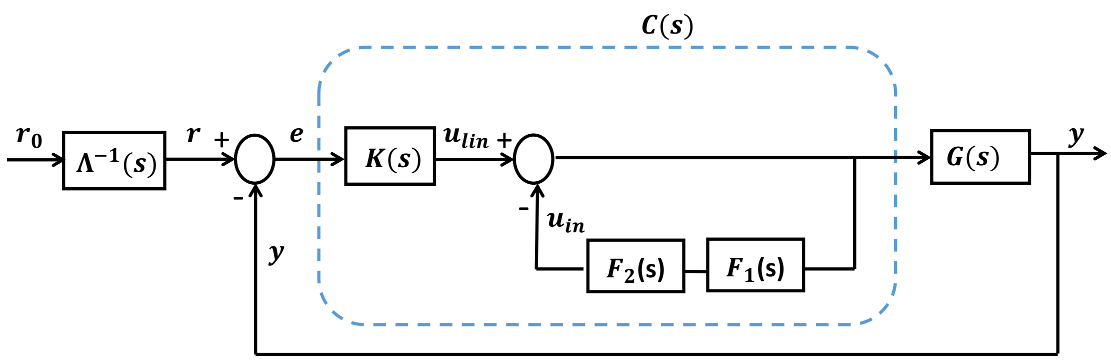


Figure 2.5: The block diagram of a parallel internal model control structure.

The numerator and denominator polynomials of the nominal plant $G(s)$ and internal model units $F_1(s)$ and $F_2(s)$ in Fig. 2.5 are defined as $A(s)$ and $B(s)$, $M(s)$ and $N(s)$, $P(s)$ and $Q(s)$ respectively:

$$G(s) = \frac{B(s)}{A(s)},$$
$$F_1(s) = \frac{M(s)}{N(s)} = \frac{B(s)}{A(s)} = G(s) \qquad and \qquad F_2(s) = \frac{P(s)}{Q(s)}. \tag{2.3}$$

Note that stable polynomial $A(s)$ implies the plant is stable.

Lemma 1. *[15] The controller that asymptotically stabilizes the unforced closed-loop system achieves asymptotic tracking performance if the condition*

$$(1 + F(s)) = A(s)^{-1}\Lambda(s) \tag{2.4}$$

holds, where $F = F_1 F_2$.

Proof. In order to give a complete picture, we provide the proof from [15]. $F(s)$ can be defined based on the polynomials given in (2.3)

$$F(s) = F_1(s)F_2(s) = P(s)Q(s)^{-1}M(s)N(s)^{-1}.$$

Error transfer function can be written as

$$E(s) = R(s) - Y(s) = R_0(s)\Lambda(s)^{-1} - U(s)G(s)$$

$$= R_0(s)\Lambda(s)^{-1} - E(s)K(s)(1 + F(s))^{-1}G(s)$$

which simplifies to

$$E(s) = \frac{(1 + F(s))A(s)R_0(s)\Lambda(s)^{-1}}{(1 + F(s))A(s) + K(s)B(s)}.$$

Hence, we have

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{s(1 + F(s))A(s)R_0(s)\Lambda(s)^{-1}}{(1 + F(s))A(s) + K(s)B(s)}.$$

For a stable feedback system, a sufficient condition to guarantee asymptotic tracking is that $(1 + F(s))A(s)$ includes a copy of the exogenous system. This means $(1+F(s))A(s) = \Lambda(s)$ which equals to (2.4). Also, by internal stability the term s is not canceled so that $\lim_{t\to\infty} e(t) = 0$ and this completes the proof.

□

Basically, $M(s)$, $N(s)$ and $Q(s)$ are chosen as

$$M(s) = B(s), \qquad N(s) = A(s), \qquad Q(s) = 1$$

and using (2.4), $P(s)$ can be found as

$$P(s) = \frac{\Lambda(s) - A(s)}{B(s)} \ . \tag{2.5}$$

An augmented system $G_A(s)$ composed of internal model units $(F_1(s), F_2(s))$ and nominal plant $(G(s))$ is defined in order to design the stabilizer $K(s)$ such that

$$G_A(s) = \frac{G(s)}{1 + F(s)} \tag{2.6}$$

where $F(s) = F_1(s)F_2(s)$.

Besides internal model units and augmented system, there also exist modeling uncertainties in the system since we intuitively claim that no mathematical system can exactly model a physical system. Hence, the performance of a control system might be adversely affected by the plant uncertainties. Considering the uncertainties in the system, the actual plant can be addressed as

$$\begin{aligned} G_\Delta =& G + \Delta_a = G(1 + \Delta_m) \ , \\ (\big|\Delta_a(jw)\big| < \big|W_a(jw)\big|,& \quad \big|\Delta_m(jw)\big| < \big|W_m(jw)\big| \quad \forall \omega \in \mathbb{R} \ ) \end{aligned} \tag{2.7}$$

where $\Delta_a(s)$ and $\Delta_m(s)$ represent additive and multiplicative uncertainties. Also, $W_a(s)$ and $W_m(s)$ are denoted as the additive and multiplicative uncertainty weighting functions respectively.

The anti-windup scheme with the internal model structure is designed based on a standard mixed sensitivity $\mathcal{H}_\infty$ problem in order to optimize the stabilizer design with the performance requirement and robustness against uncertainties. Accordingly, the aim is to find a stabilizing controller $K(s)$ for the mixed sensitivity minimization problem

$$\inf_{K stab. G_A} \left\| \begin{bmatrix} W_1(s)S(s) \\ W_2(s)T(s) \end{bmatrix} \right\|_\infty \tag{2.8}$$

where $S(s)$ and $T(s)$ are denoted as the sensitivity and complementary sensitivity transfer functions of the augmented system $G_A(s)$,

$$S(s) = \frac{1}{1 + G_A(s)K(s)} \ ,$$

$$T(s) = \frac{G_A(s)K(s)}{1 + G_A(s)K(s)} \, .$$

The optimal $\mathcal{H}_\infty$ index is also defined as

$$\gamma_{opt} = \inf_{K\, stab. G_A} \left\| \begin{bmatrix} W_1(s)(1 + G_A(s)K_{opt}(s))^{-1} \\ W_2(s)(G_A(s)K_{opt}(s))(1 + G_A(s)K_{opt}(s))^{-1} \end{bmatrix} \right\|_\infty$$

where $K_{opt}(s)$ is the optimal stabilizing controller.

Remark 1. *$W_1(s)$ is denoted as the performance weighting function and poles of $W_1(s)$ contain the poles of Laplace transform of the reference signal to be bounded. Besides, $W_2(s)$ is the robustness weight and defined as the upper bound of the multiplicative plant uncertainty.*

2.3.2 Robust Anti-Windup Compensation

For the systems including saturation nonlinearities and model uncertainties, the tracking performance and system stability are mostly deteriorated due to adverse effects of unmodeled dynamics and this causes loss of performance and limits the applicability of existing tracking algorithms. As mentioned, there are various control approaches have been developed to deal with this problem. In [18], Borisov et al. redesigns the consecutive compensator approach and adds an integral loop with the anti-windup scheme in order to avoid loss of performance in the saturated control for quadcopters. Their aim is to stabilize the quadcopter at the specified position with the specified orientation and they use back calculation approach anti-windup scheme for this purpose. Application of robust output controller with anti-windup loop including integral action removes the static error and reduces the overshoot in the bounded input quadcopter model.

Edwards et al. also uses model-based approach to anti-windup compensation in order to minimize the $\mathcal{H}_\infty$-norm of the transfer function around the saturation nonlinearities caused by actuators that limits the magnitude of the signal entering the plant by acting as a nonlinear saturation element between the controller and plant [38]. The saturation element is represented as a sector/conic nonlinearity and standard $\mathcal{H}_\infty$ controller design is modified based on the difference between the signal from the controller and the signal which enters the plant by applying corrective feedback to reduce the discrepancy.

In order to handle the system uncertainties and unmodeled dynamics including hysteresis nonlinearities, we apply robust anti-windup control architecture combined with the internal-model based tracking structure proposed in [15] which is illustrated in Fig. 2.6.

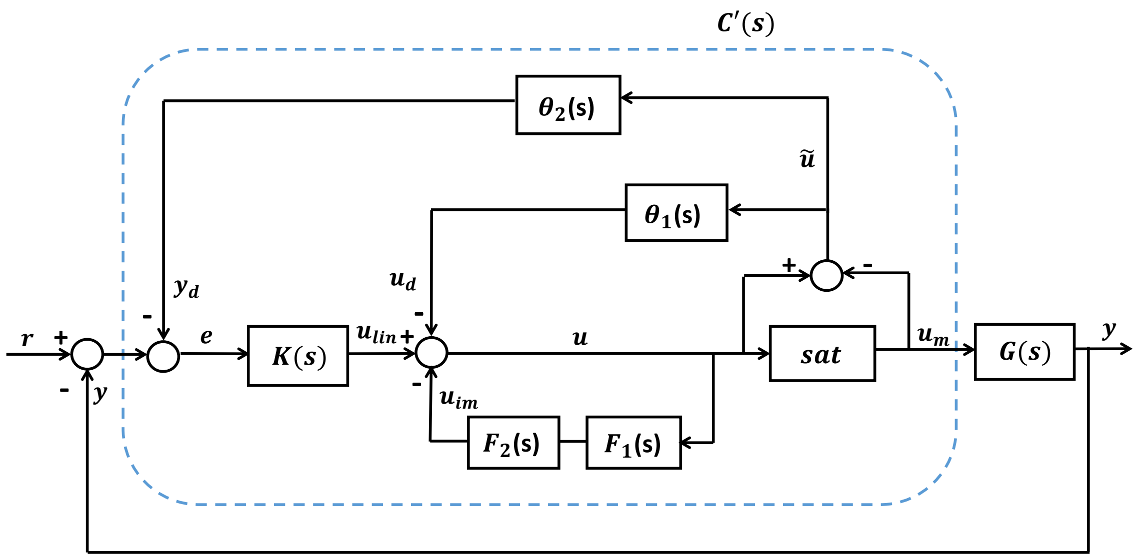


Figure 2.6: Anti-windup tracking control architecture with the internal model structure.

In the presence of saturation, the controller output u and the plant input u_m diverge from each other and the saturation can be expressed by the time-invariant relationship between u and u_m

$$sat(u) := u_m = \begin{cases} \sigma_1, \ u \leqslant \sigma_1 \\ u, \ \sigma_1 < u < \sigma_2 \\ \sigma_2, \ u \geq \sigma_2 \end{cases}$$

where $sat(.)$ is denoted as saturation operator and the saturation limits are determined based on the system specifications.

Lemma 2. *[15] The augmented systems shown in Fig. 2.5 and Fig. 2.6 are identical if and only if the following relationship is satisfied*

$$\begin{aligned} G_A(s) &= G'_A(s) \\ \frac{G}{1+F} &= \frac{\theta_2}{1+F+\theta_1} \,. \end{aligned} \tag{2.9}$$

Note that $G_A(s)$ is a transfer function from u_{lin} to y depicted in Fig. 2.5 and $G'_A(s)$ is a transfer function from u_{lin} to y_{lin} where y_{lin} equals to $y+y_d$ in Fig. 2.6.

Proof. To prove the lemma based on [15], first we define the input-output relationship in Fig. 2.6

$$y_{lin} = y + y_d \,, \qquad y = u_m G \quad and \quad y_d = \tilde{u}\theta_2$$

where $\tilde{u} = u - u_m$. Controller output u can be written as

$$u = \frac{u_{lin}}{1+F+\theta_1} + \frac{u_m\theta_1}{1+F+\theta_1} \,.$$

Hence, finally we have

$$\begin{aligned} y_{lin} &= u_m G + \Big(\frac{u_{lin}}{1+F+\theta_1} + \frac{u_m\theta_1}{1+F+\theta_1} - u_m\Big)\theta_2 \\ y_{lin} &= u_{lin}\,\frac{1}{1+F+\theta_1}\,\theta_2 - u_m\,\frac{(1+F)}{1+F+\theta_1}\,\theta_2 + u_m G \,. \end{aligned} \tag{2.10}$$

If the given condition (2.9) holds, then we have

$$G = \frac{(1+F)\theta_2}{1+F+\theta_1} \,. \tag{2.11}$$

Substituting (2.11) into (2.10) gives

$$y_{lin} = \frac{\theta_2}{1+F+\theta_1}\,u_{lin} \,. \tag{2.12}$$

The augmented system finally can be written as

$$G'_A(s) = \frac{y_{lin}}{u_{lin}} = \frac{\theta_2}{1+F+\theta_1} = \frac{G}{1+F} = G_A \,. \tag{2.13}$$

Remember that in the definition of the augmented system shown in Fig. 2.5, we define the input-output relationship as

$$y = G_A u_{lin} = \frac{G}{1+F}\, u_{lin}\,. \tag{2.14}$$

We suppose that $G_A(s)$ and $G'_A(s)$ are identical, so y and y_{lin} should be identical such that

$$\frac{G}{1+F}\, u_{lin} = u_{lin}\,\frac{1}{1+F+\theta_1}\,\theta_2 - u_m\,\frac{(1+F)}{1+F+\theta_1}\,\theta_2 + u_m G\,. \tag{2.15}$$

Therefore, to provide (2.15), we have to satisfy the conditions,

$$\begin{cases} \frac{\theta_2}{1+F+\theta_1} = \frac{G}{1+F} \\ \\ \frac{(1+F)\theta_2}{1+F+\theta_1} = G \end{cases}$$

Finally, the sufficient condition to satisfy the lemma can be characterized as

$$\frac{\theta_2}{1+F+\theta_1} = \frac{G}{1+F}$$

and this completes the proof.

□

The saturation nonlinearities based on the actuators in the system are compensated via anti-windup structure by adjusting the stabilizer output u_{lin} with u_d and the plant output y with y_d. Since we define the augmented system of the whole control structure depicted in Fig. 2.6 as $G'_A = y_{lin}/u_{lin}$, to fully eliminate the adverse effects of the saturations at the output, we have justified the ideality of the augmented system $G_A(s)$ shown in Fig. 2.5 with the augmented system $G'_A(s)$ shown in Fig. 2.6 via Lemma 2.

Besides the internal model units, there also exist the compensators θ_1 and θ_2 illustrated in Fig. 2.6. Anti-windup compensators are designed based on the criteria to guarantee the stability of the closed loop system with dead-zone nonlinearity. θ_1 and θ_2 is driven based on the equivalent representation of Fig. 2.6

with the dead-zone operator by the difference between u and u_m,

$$\tilde{u} = u - u_m = u - sat(u) := dz(u) \tag{2.16}$$

where $dz(.)$ represents the dead-zone operator.

Based on the definition of dead-zone operator (2.16) and Lemma 2, the equivalent representation of Fig. 2.6 can be re-drawn as Fig. 2.7.

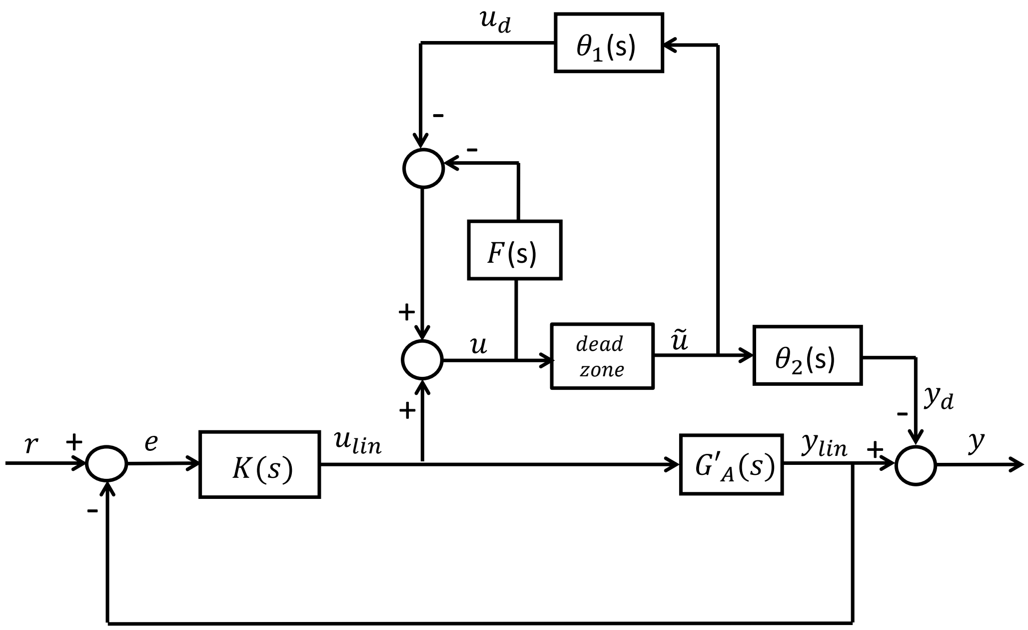


Figure 2.7: Equivalent anti-windup scheme with the internal model structure.

Theorem 2.3.1. *[15] In order to ensure the stability of the robust anti-windup tracking control architecture illustrated in Fig. 2.7, the following conditions should be satisfied:*

i. Equation (2.9) holds,

ii. For $\tilde{\theta_1} := \theta_1(1+F)^{-1}$, *there exists an* $\alpha > 0$ *such that*

$$Re(1 + j\alpha\omega)\tilde{\theta_1}(j\omega) + 1/k > 0 \qquad \forall\ \omega$$

where k *satisfies* $0 < u.dz(u) < ku^2$.

Proof. Lemma 2 indicates that the augmented systems shown in Fig. 2.5 and Fig. 2.6 are identical,

$$G_A(s) = G'_A(s) .$$

Hence, the robust stabilizer $K(s)$ given in Fig. 2.7 can stabilize the augmented system $G'_A(s)$. In (ii), we define $\tilde{\theta_1} := \theta_1(1+F)^{-1}$ and we can claim that the feedback structure including dead-zone operator and $\tilde{\theta_1}$ is stable if condition (ii) is satisfied, based on the Popov criteria [43, 44]. Also, the stability of $\theta_2(s)$ is dependent on the stability of $\theta_1(s)$ and $(1+F)^{-1}$ considering the relationship (2.9). Since (ii) includes $\theta_1(s)$ and $(1+F)^{-1}$, which are stable transfer functions, this condition is sufficient to imply the stability of closed loop system and completes the proof.

□

Based on Theorem 2.3.1, it is easy to define that

$$\theta_1(s) = \tilde{\theta_1}(s)(1+F) , \tag{2.17}$$

and based on Lemma 2, we can describe $\theta_2(s)$ as

$$\theta_2(s) = G\Big(1 + \frac{\theta_1}{1+F}\Big) = G(1+\tilde{\theta_1}) . \tag{2.18}$$

However, in the design of anti-windup compensator and augmented system, the system uncertainties were not considered while deriving the transfer functions. Remember that we define the additive uncertainty occurring in the system dynamics in the equation (2.7) and considering the additive uncertainty, Lemma 2 can be re-written as

$$\frac{G_\Delta}{1+F} = \frac{\theta_{2\Delta}}{1+F+\theta_1}$$

where $G_\Delta = G + \Delta_a$ and Δ_a represents the additive uncertainty.

Hence, we define

$$\theta_{2\Delta} = (G+\Delta_a)\Big(1 + \frac{\theta_1}{1+F}\Big) = (G+\Delta_a)(1+\tilde{\theta_1}) .$$

The difference between the definitions of θ_2 with and without system uncertainty can be written as

$$\Delta_{\theta_2} = |\theta_2 - \theta_{2\Delta}| = \left|\Delta_a(1 + \tilde{\theta_1})\right| ,$$

and our aim is to minimize this error by choosing (θ_1, θ_2) appropriately in order to eliminate the adverse effects of system uncertainties occurred modeling the physical system. Achieving robust stability and tracking the reference signal with the proper choices of (θ_1, θ_2) are the main subjects in the design which can be described as

$$\inf \left\|W_a(1 + \tilde{\theta_1})\right\|_\infty \tag{2.19}$$

over all $\tilde{\theta_1}$ satisfying the Theorem 2.3.1 with the additive uncertainty weighting function $W_a(s)$.

2.4 Simulation Studies

Robust anti-windup compensator is used in various applications on different systems and the references therein [15, 17, 34, 45, 46]. We designed the recommended anti-windup compensator for the stabilized antenna system which will be used in the satellite communication. The antenna system is three dimensional including elevation, cross-elevation and azimuth axes.

In order to derive the nominal plant representing the dynamics of this antenna, system identification tests were applied to the hardware by using 5V amplitude sine sweep signals between 10-200 Hz frequency range. Magnitude-phase values were obtained with signal analyzer and symbolic model nominal transfer functions for each axis were derived. This process is basically shown in Fig. 2.8.

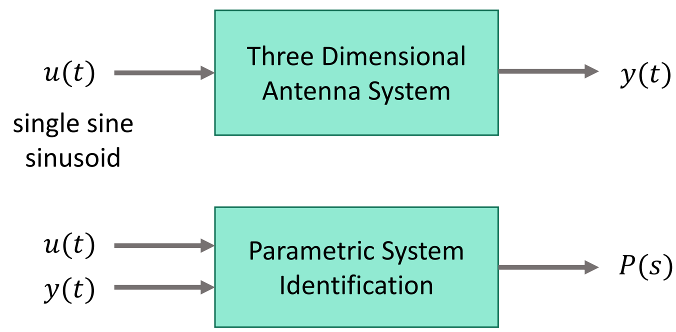


Figure 2.8: System identification structure

Elevation axis nominal transfer function is used as a plant to implement the proposed anti-windup scheme and design the appropriate robust controller to minimize the tracking error while following the desired trajectory. The transfer function is modeled as

$$P(s) = \frac{K\left(1 + 2\zeta_{n1}\frac{s}{\omega_{n1}} + (\frac{s}{\omega_{n1}})^2\right)\left(1 + 2\zeta_{n2}\frac{s}{\omega_{n2}} + (\frac{s}{\omega_{n2}})^2\right)}{s\left(1 + 2\zeta_{d1}\frac{s}{\omega_{d1}} + (\frac{s}{\omega_{d1}})^2\right)\left(1 + 2\zeta_{d2}\frac{s}{\omega_{d2}} + (\frac{s}{\omega_{d2}})^2\right)} e^{-hs} \tag{2.20}$$

with the parameters given in Table 2.1

Table 2.1: Elevation Axis Plant Parameters

K	ζ_{n1}	ω_{n1} (rad/sec)	ζ_{n2}	ω_{n2} (rad/sec)
7.1	0.08	175	0.04	930
h (ms)	ζ_{d1}	ω_{d1} (rad/sec)	ζ_{d2}	ω_{d2} (rad/sec)
8.1	0.02	285	0.1	960

Note that time delay is ignored in the following steps since the proposed architecture in this chapter is applicable for the plants without dead-time. The extension of this method for the systems including time delay will be explained in Section 3 and the simulation results will be provided in Section 4.

By considering the cumulative error differences between frequency response tests ($P_{EL,\ tests}(s)$) and nominal model ($P(s)$), an upper limit is calculated using the formula in (2.21).

$$|W_a(s)| \geq |P(s) - P_{EL}(s)| \qquad \forall \quad P(s) \in P_{EL,\ tests}(s) \tag{2.21}$$

This bound is denoted as $W_a(s)$ and limits the additive error as seen in Fig. 2.9. The transfer function representing this bound is computed as

$$W_a(s) = \frac{0.011\,(1 + s/20)}{\left(1 + 2\,\zeta_d\,(s/\omega_d) + (s/\omega_d)^2\right)} \tag{2.22}$$

where $\zeta_d = 0.01$ and $\omega_d = 280\ rad/sec.$

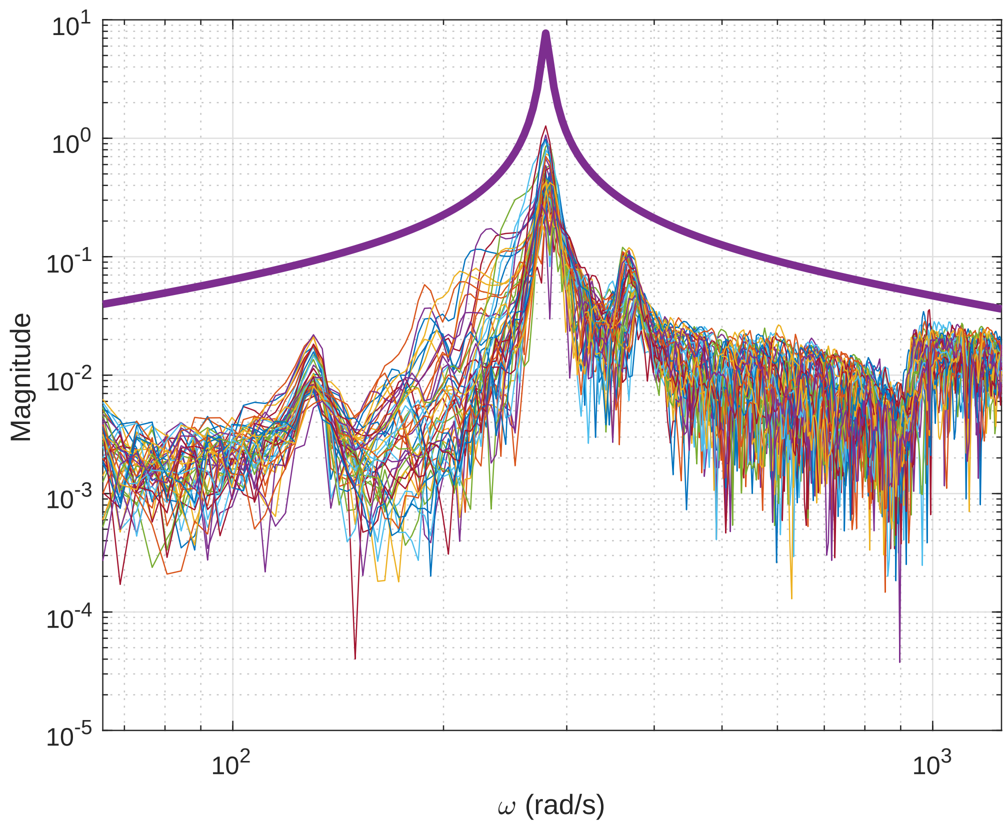


Figure 2.9: The additive error between nominal transfer function and frequency response test results with the upper bound $W_a(s)$.

The desired sinusoidal trajectory is chosen as $r(t) = 50\,sin(2\pi ft + \pi/2) - 50\,(m)$ with the period of 4 seconds. In order to calculate the exogenous system dynamics, Laplace transform of the reference input is found and $\Lambda(s)$ is defined via equation (2.2).

$$R(s) = \frac{s}{s^2 + \omega_1^2} \quad \Rightarrow \quad \Lambda(s) = (s^2 + \omega_1^2)$$

where $\omega_1 = 2\pi(1/T)$ for T equals 4 seconds.

Internal model units $F_1(s)$ and $F_2(s)$ are determined using the equation (2.3). Note that $F_1(s)$ directly equals the plant transfer function and $F_2(s)$ is achieved by applying the definition of the polynomial $P(s)$ defined in (2.5).

$$F_1(s) = G(s) = \frac{B(s)}{A(s)}$$

$$F_2(s) = \frac{P(s)}{Q(s)} = \frac{\Lambda(s) - A(s)}{B(s)}$$

where $B(s)$ is the numerator and $A(s)$ is the denominator polynomial of the nominal model transfer function defined in (2.20). Now, given the internal model units enable us to define the augmented system $G_A(s)$ as

$$G_A(s) = \frac{G(s)}{1 + F(s)}$$

$$G_A(s) = \frac{2.68 \times 10^{-10}(s^2 + 2.8s + 3.062 \times 10^4)(s^2 + 74.4s + 8.649 \times 10^5)}{(s^2 + \epsilon)(s^2 + 2.467)}$$

where $\epsilon = 0.0001$. Note that there exists additional term $(s^2+\epsilon)$ in the augmented system denominator polynomial which directly comes from the exogenous dynamics. We have to add this polynomial into the $\Lambda(s)$ to make augmented system transfer function proper. Besides, the weighting functions $W_1(s)$ and $W_2(s)$ are chosen similar with the given transfer functions in [15].

Another important subject in the design of anti-windup compensator for the systems in the absence of input saturation is robust stabilizer $K(s)$ illustrated in Fig. 2.6. To compute the optimal stabilizer, $\mathcal{H}_\infty$ mixed sensitivity minimization problem given in (2.8) is solved via Matlab *mixsyn* function and the optimal $\mathcal{H}_\infty$ index γ_{opt} is found as 0.0684. With these results, parallel internal model structure design is completed.

One of the central section in the implementation is the design of robust anti-windup compensator $\theta_1(s)$ and $\theta_2(s)$. Based on Theorem 2.3.1, $\tilde{\theta}_1$ is determined as

$$\tilde{\theta}_1 = \frac{\gamma}{(1+\alpha s)(1+\beta s)} \quad for \quad (\alpha > 0,\, \beta > 0) \tag{2.23}$$

and using this definition, the inequality in the Theorem 2.3.1 is satisfied if $\gamma > -\frac{1}{k}$. In order to solve the main problem defined in the equation (2.19), the function $f(\gamma)$ is described

$$f(\gamma) = \left\| W_a(s)(1+\tilde{\theta}_1) \right\|_\infty$$

where $W_a(s)$ represents the additive upper bound given in (2.22). The aim is to minimize $f(\gamma)$ by choosing the optimal values of α, β and γ. To simplify this optimization problem, α is chosen as $1/20$ to eliminate the numerator polynomial of additive uncertainty $W_a(s)$. For the other unknown parameters, this function is observed by using different β values in order to find the minimum f.

More generally, Fig. 2.10 illustrates the curves of function f with different β values from 0 to 1. We can claim based on Fig. 2.10 that the optimal β value minimizing the function $f(\gamma)$ is $1/40$ (green line in the figure). By choosing $\beta = 1/40$, we can determine the minimum value of this function with the corresponding γ as seen in Fig. 2.11.

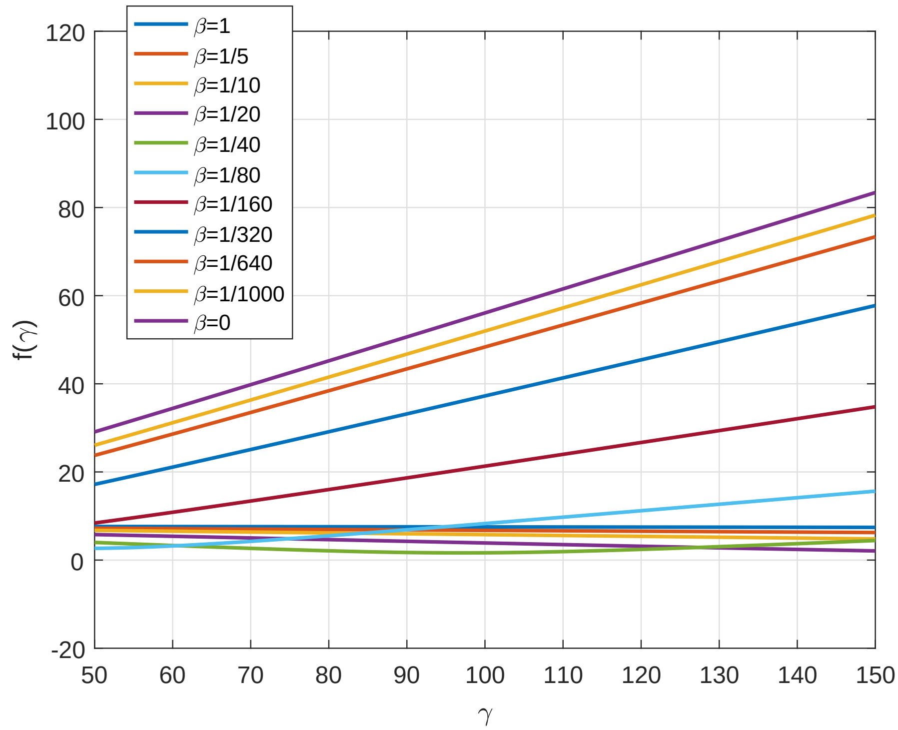


Figure 2.10: The optimization function $f(\gamma)$ versus γ for different β values. Our aim is to determine the optimum γ minimizing the function $f(\gamma)$.

The function f takes the minimum value when $\gamma = 97$ and $\beta = 1/40$ as illustrated in Fig. 2.11. Hence, all the parameters minimizing $W_a(s)\left(1 + \frac{\gamma}{(1+\alpha s)(1+\beta s)}\right)$ are specified.

Based on the optimal γ, α and β values, we can determine the robust anti-windup components by using the definitions (2.17) and (2.18) to finalize the design.

$$\theta_1(s) = \frac{5.687 \times 10^{15}\,(s^2 + 1.629 \times 10^{-12}s + 0.0001)\,(s^2 - 1.02 \times 10^{-10}s + 2.467)}{s\,(s+40)\,(s+20)\,(s^2 + 11s + 7.952 \times 10^4)\,(s^2 + 201.6s + 9.216 \times 10^5)}$$

$$\theta_2(s) = \frac{19.659\,(s^2 + 2.8s + 3.062 \times 10^4)\,(s^2 + 60s + 7.84 \times 10^4)}{s\,(s+40)\,(s+20)\,(s^2 + 11s + 7.952 \times 10^4)} \times \frac{(s^2 + 74.4s + 8.649 \times 10^5)}{(s^2 + 201.6s + 9.216 \times 10^5)}$$

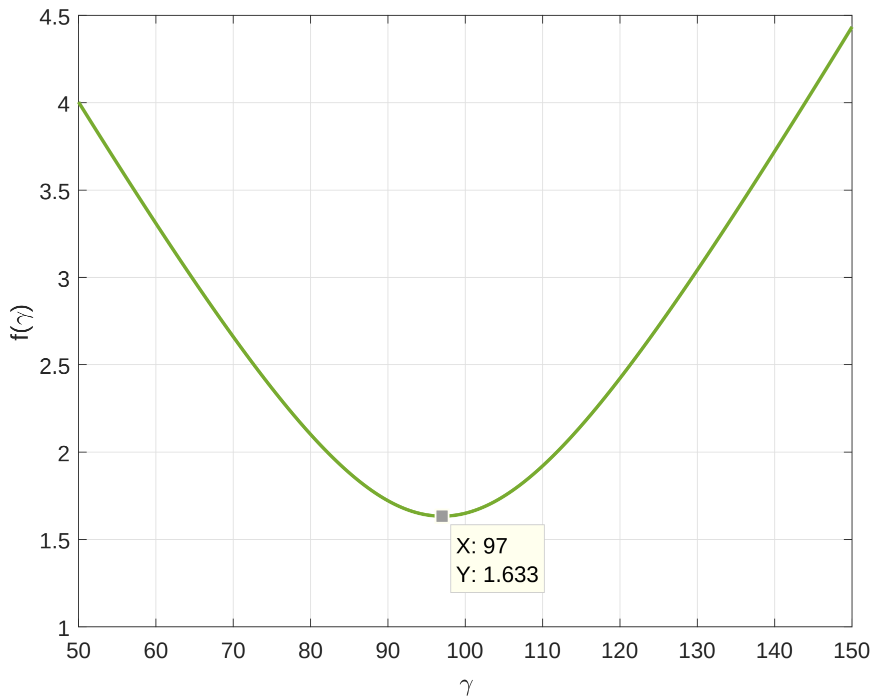


Figure 2.11: Blown-up image of Fig. 2.10 to determine the optimal γ minimizing the function $f(\gamma)$ when β equals 1/40.

Using internal model units $F_1(s)$ and $F_2(s)$, robust stabilizer $K(s)$, the augmented system transfer function $G_A(s)$ and robust anti-windup compensator $\theta_1(s)$ and $\theta_2(s)$, the equivalent tracking structure given in Fig. 2.7 is implemented both in the existence of anti-windup and without anti-windup schemes. Sinusoidal reference signal is described as $r(t) = 50\, sin(2\pi f t + \pi/2) - 50\,(m)$ for $f = 0.25 Hz$. Period of the reference signal is determined based on the specifications of the antenna system. Saturation limits of the actuator are $[-10, 10]$, hence when controller output is greater than the determined saturation limits, anti-windup compensator operates to handle the damaging effects of the situation of saturation.

We observe the performance of proposed anti-windup scheme in the presence of input saturation and expect that system output gets closer to the reference signal despite the saturation nonlinearity. As shown in Fig. 2.12, plant output pursues the desired reference approximately with 0.098% tracking error when system operates in the nonlinear region. The anti-windup compensator handle the saturation as successfully as possible taking into consideration the damaging effects of nonlinearities to the system. The main goal is to overcome the nonlinearity, and then to provide high accurate tracking while suppressing the saturation effect. The internal model units help to achieve asymptotic tracking, on the other hand, robust compensator parameters utilize to prevent the adverse effects of the actuator saturation.

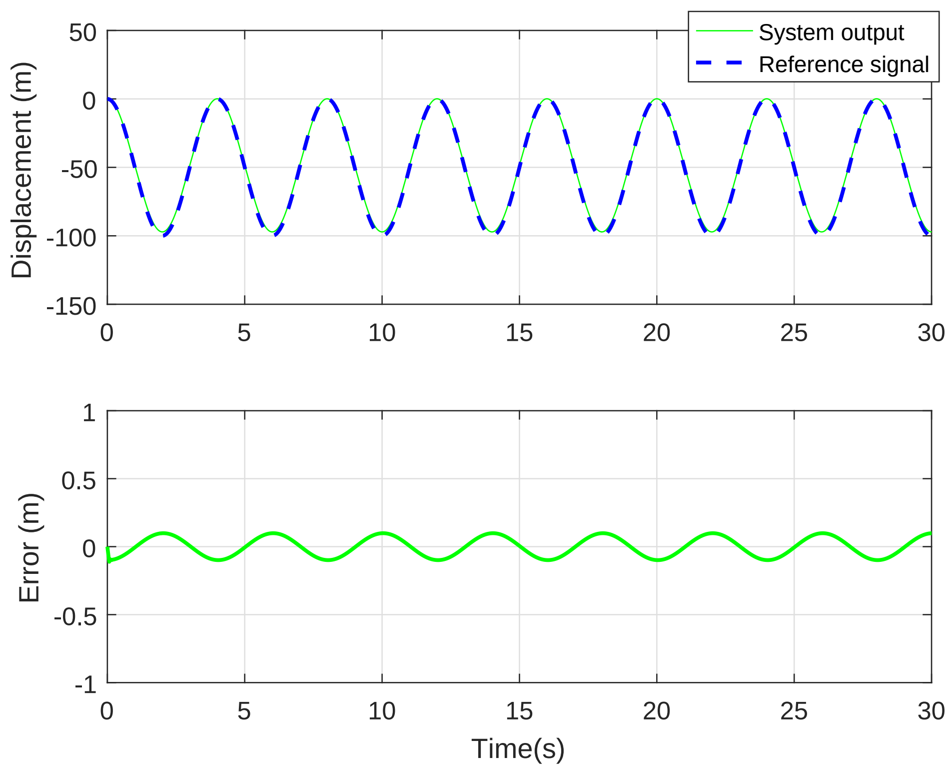


Figure 2.12: Tracking performance with input saturation using the proposed anti-windup structure. Tracking error is also given to observe the performance of the algorithm.

It can be clearly seen in Fig. 2.13 that the controller output is truncated at the limits of the saturation. As mentioned in Section 2.2, when *winding-up* effect occurs, the controller can not operate properly although increasing the control signal to overcome the input saturation. Therefore, the controller is limited by the anti-windup compensator to operate under the specified saturation region and drive the plant as expected.

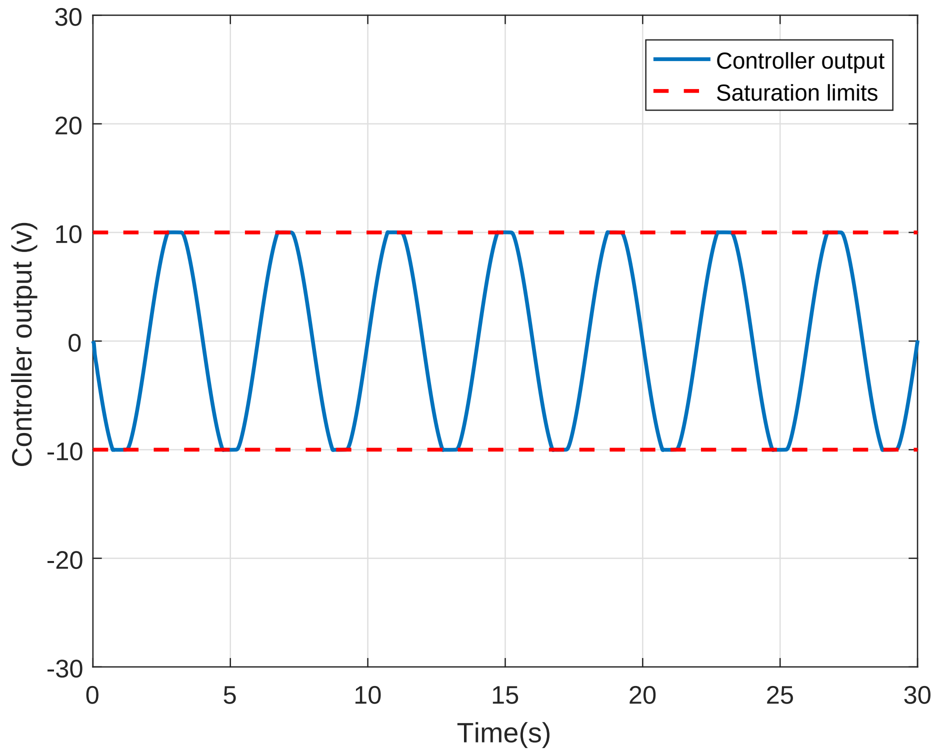


Figure 2.13: Controller output while there exists input saturation.

The performance of anti-windup structure is also observed by eliminating the anti-windup components from the loop and just applying controller and plant under the effect of saturation. The system output in this case is illustrated in Fig. 2.14. The negative effects of the system nonlinearities can be clearly observed with this result since the output can not follow the desired trajectory when anti-windup structure is not acting.

Comparing the results given in Fig. 2.12 and Fig. 2.14 demonstrates the acceptable performance of the proposed control architecture for the systems subject to input saturation. In the result depicted in Fig. 2.12, the system stays stable and overcomes the negative impacts of the saturation as good as possible, however the system becomes unstable when we eliminate the anti-windup structure.

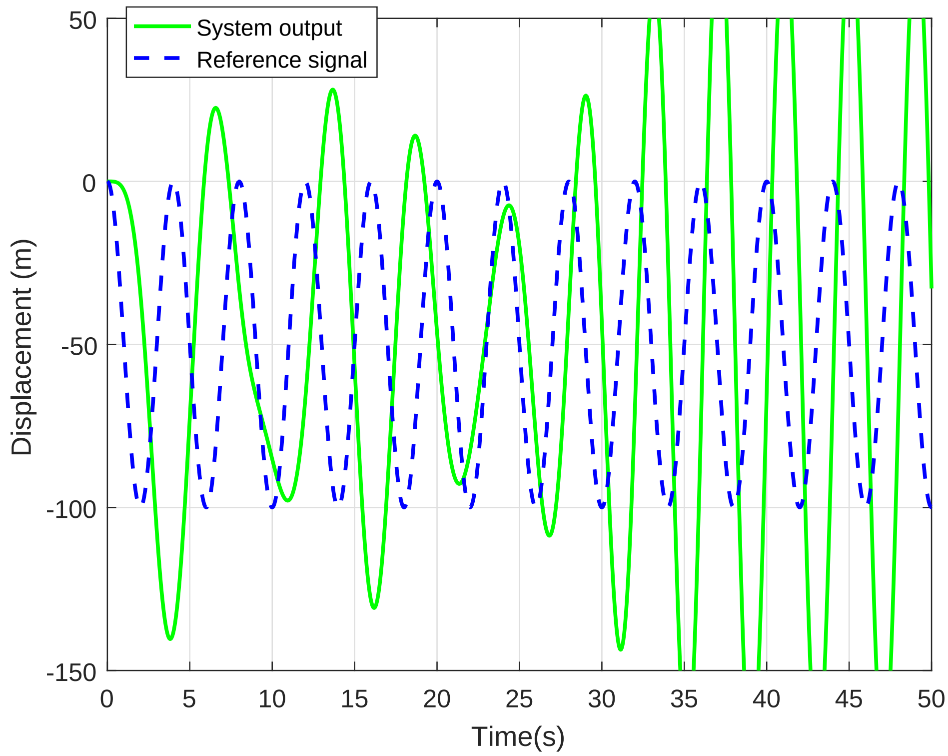


Figure 2.14: Tracking performance in the existence of input saturation without applying the proposed anti-windup structure.

Chapter 3

Extended Anti-windup Compensator via Smith Predictor-Based Controller Design for Plants with Time Delay and Integral Action

This chapter concerns the design of anti-windup compensator for plants including time delay in the face of actuator saturation. By the help of compensator method mentioned in Chapter 2, the dead-time control algorithm together with the robust anti-windup scheme is proposed in this chapter. The extension of the anti-windup structure is performed based on the Smith-predictor based controller design to allow high tracking performance for systems in the presence of saturation nonlinearities.

The details behind the Smith-predictor based design are first introduced in Section 3.1 with the essential milestones. We tackle the problem of controlling a system incorporating time delay and apply extended anti-windup compensator by ensuring the stability against possible nonlinear effects. The improvements on the anti-windup compensator based on the Smith predictor design are provided with the necessary details in Section 3.2. Consequently, we conclude this chapter with the summary of novel definitions of the anti-windup components.

3.1 Smith Predictor-Based Controller Design

Time delay appears frequently in the systems depending on processing of sensed signals and/or transferring control signals to systems [47]. Time delays in the feedback loop called as dead-time which causes inevitable problems such as loss of performance, instability, additional phase drop, etc. Moreover, feedback systems with dead-time in the loop generically have infinitely many poles which make the system analysis more complicated. Many control approaches have been developed to deal with infinite dimensional systems. In fact, PID controllers are able to provide good control system performance when there exist negligible or small time delay, but often are not so efficient when there is long dead-time in process dynamics [48].

The main advantage of the Smith predictor-based design for the dead-time systems is that time delay is effectively taken outside the characteristic equation of the closed loop system [9]. In the existence of long time delay, it is impossible to obtain sufficient information from output signal for the prediction. By using estimated parameters of the plant in the feedback loop of the controller, the prediction can be established on the control input via Smith predictor based structures.

There has been various Smith predictor structures for the time delay systems [10, 28, 30, 48–52]. Our purpose using Smith predictor approach is to design a controller for the antenna system described in Chapter 2 with performance and robustness considerations and then extend the anti-windup structure based on Smith predictor design for the systems with time delay and integral action in the presence of saturation.

The plant transfer function is in the form

$$P(s) = \frac{K}{s} R_0(s) e^{-T_d s} \tag{3.1}$$

where K is the gain of the nominal plant which is also proportional to the inertia, $T_d > 0$ is the time delay in the system and $R_0(s)$ represents the minimum phase transfer function which has the form

$$R_0(s) = \prod_{k=1}^{n} \frac{(s^2/\tilde{\omega}_k^2) + 2\tilde{\zeta}_k(s/\tilde{\omega_k}) + 1}{(s^2/\omega_k^2) + 2\zeta_k(s/\omega_k) + 1}$$

where $0 < \tilde{\omega}_k < \omega_k$ are the resonant and anti-resonant frequencies, and $\tilde{\zeta}$, ζ are the damping factors which take values between 0 and 1 [14]. All parameters are estimated based on the system identification studies we performed on the hardware structure. Note that $\frac{K}{s}R_0(s)$ is defined as a nominal plant $G(s)$ in Chapter 2. The difference between $G(s)$ described in (2.3) and the plant structure given in (3.1) is the time delay factor.

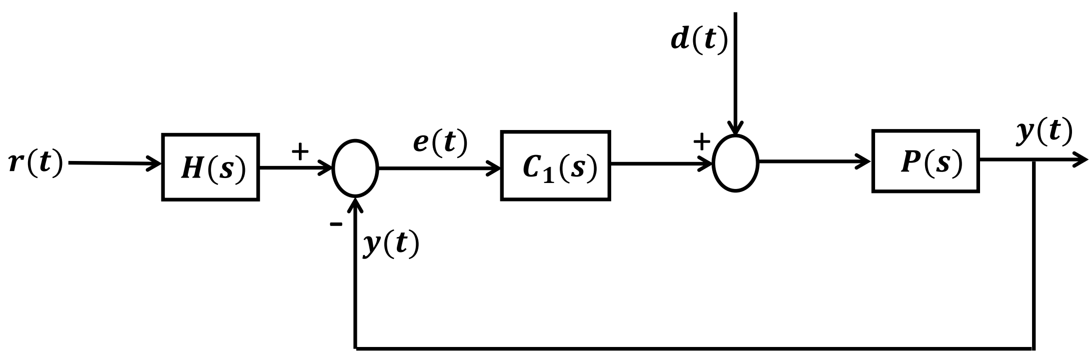


Figure 3.1: Smith predictor-based controller structure.

Proposed Smith predictor-based model controller structure is illustrated in Fig. 3.1 as well as the controller itself is given in Fig. 3.2.

Using the structure in Fig. 3.2, the Smith predictor-based controller can be defined as

$$C_1(s) = \frac{\hat{R}_\epsilon(s)^{-1}}{\hat{K}} \left(\frac{C_0(s)}{1 + C_0(s)\frac{1-e^{-\hat{T}_d s}}{s}} \right) \tag{3.2}$$

where $\hat{R}_\epsilon(s)^{-1} = \hat{R}_0(s)^{-1}/(1+\epsilon s)^2$ is the approximate inverse of the term due to flexible modes. $\hat{R}_0(s)^{-1}$ includes the estimated values of the parameters

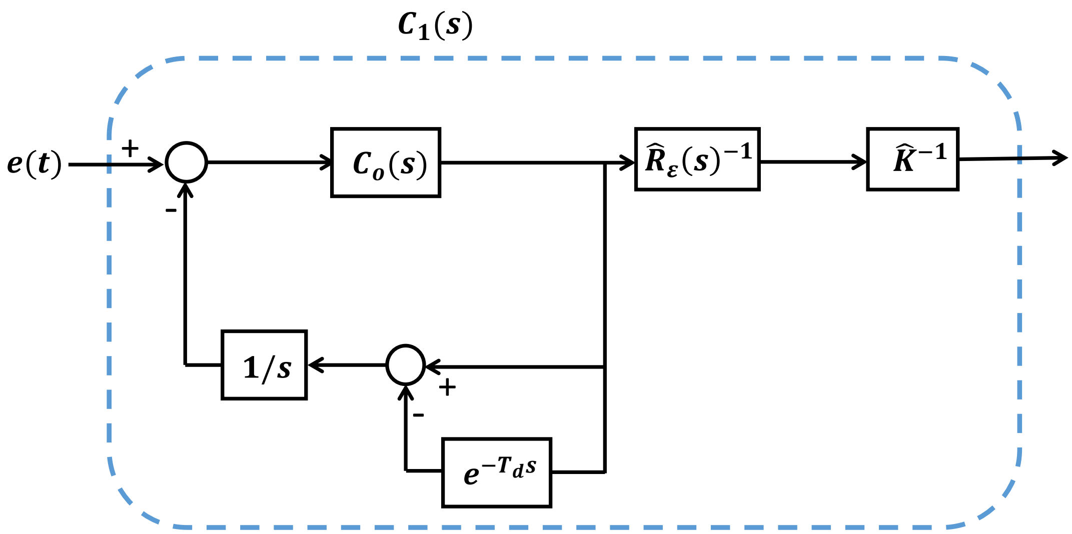


Figure 3.2: Smith predictor-based controller $:= C_1(s)$

ω_i, ζ_i, $\tilde{\omega}_i$, $\tilde{\zeta}_i$ for $i = 0, 1, ..., n$ whereas $R_0(s)$ consists of the real values of these parameters. We define $C_0(s)$ as the free part of the controller which is designed based on the delay free part of the nominal plant. In the stability analysis of the closed loop feedback system, typically $H(s)$ is chosen as 1 since it does not contribute to the system stability.

The aim is to achieve perfect steady-state tracking, hence in the design process of the Smith predictor controller, we consider that the system can successfully pursue the constant and sinusoidal reference input $r(t)$. In order to satisfy these requirements, $C_1(s)$ must have poles at $s = 0$ and at the periodic signal frequencies $s = \pm j\omega_d$.

- Steady-state tracking of a constant $r(t)$:

$$\lim_{s \to 0} C_1(s) = \infty \implies \lim_{s \to 0} \left(1 + C_0(s) \frac{1 - e^{-\hat{T}_d s}}{s} \right) = 0$$

Applying L'Hopital Rule, we can obtain

$$\lim_{s \to 0} \left(1 + C_0(s)' - C_0(s)' e^{-\hat{T}_d s} + C_0(s) \hat{T}_d e^{-\hat{T}_d s} \right) = 0$$

which means

$$1 + C_0(0)\hat{T}_d = 0$$

$$C_0(0) = -\frac{1}{\hat{T}_d} \tag{3.3}$$

- Steady-state tracking of a sinusoidal $r(t)$:

$$\lim_{s \to j\omega_d} C_1(s) = \infty \implies \lim_{s \to j\omega_d} \left(1 + C_0(s)\frac{1 - e^{-\hat{T}_d s}}{s}\right) = 0$$

Applying basic algebra, we can achieve

$$1 + C_0(j\omega_d)\frac{1 - e^{-\hat{T}_d j\omega_d}}{j\omega_d} = 0$$

$$C_0(j\omega_d) = -\frac{j\omega_d}{1 - e^{-\hat{T}_d j\omega_d}} \tag{3.4}$$

Besides design requirements, the stability of the closed loop system shown in Fig. 3.1 should be satisfied with the Smith predictor-based controller structure $C_1(s)$. Assuming that the plant is known, the characteristic equation of the closed loop system can be written as

$$1 + P(s)C_1(s) = 0 \implies 1 + \left(\frac{K}{s}R_0(s)e^{-T_d s}\right)\left(\frac{R_0(s)^{-1}}{\hat{K}}\frac{C_0(s)}{1 + C_0(s)\frac{1 - e^{-\hat{T}_d s}}{s}}\right) = 0$$

choosing $\hat{K} = K$ and $\hat{T}_d = T_d$,

$$1 + \frac{1}{s}\left(\frac{C_0(s)e^{-T_d s}}{1 + C_0(s)\frac{1 - e^{-T_d s}}{s}}\right) = 0$$

$$\implies \frac{1 + \frac{1}{s}C_0(s)}{1 + C_0(s)\frac{1 - e^{-T_d s}}{s}} = 0 \; .$$

Finally, the closed-loop system characteristic equation results as

$$1 + \frac{1}{s} C_0(s) = 0 \tag{3.5}$$

which means that $C_0(s)$ must be designed to stabilize the integrator $\frac{1}{s}$ [14]. If we assign $P_1(s) = \frac{1}{s}$, then the set of all controllers stabilizing the plant $P_1(s)$ can be determined using controller parametrization. We denote numerator and denominator polynomial of the plant as $N_p(s)$ and $D_p(s)$, where $N_p(s) = \frac{1}{s+a}$ and $D_p(s) = \frac{s}{s+a}$. The parameter $a > 0$ is determined based on the pole locations of the closed loop system. The set of all stabilizing controllers for $P_1(s)$ are parametrized as

$$C_0(s) = \frac{X(s) + D_p(s)Q(s)}{Y(s) - N_p(s)Q(s)} \tag{3.6}$$

where $Q \neq Y N_p^{-1}$ and $Q \in H_\infty$. Also, X and Y are the transfer functions satisfying the Bezout equation

$$N_p(s)X(s) + D_p(s)Y(s) = 1 \tag{3.7}$$

where $X, Y \in H_\infty$. Based on (3.7), we can define

$$Y(s) = \frac{1 - N_p(s)X(s)}{D_p(s)} \, . \tag{3.8}$$

Since we have a pole at $s = 0$, $D_p(s) = 0$ results in $X(0) = \frac{1}{N_p(0)}$. Using the definition, $N_p(0)$ can be written as $1/s$ and we can denote $X(s)$ as a, which is a stable transfer function already. Then, $Y(s)$ can be calculated basically

$$Y(s) = \frac{1 - \frac{1}{s+a} a}{\frac{s}{s+a}} = \frac{(s+a) - a}{s} = 1 \, .$$

Consequently, the stabilizing controller defined in (3.6) can be rewritten in the following form

$$C_0(s) = \frac{a + \frac{s}{s+a} Q(s)}{1 - \frac{1}{s+a} Q(s)} \tag{3.9}$$

Different designs of stable $Q(s)$ will be explained in detail in Chapter 4. After finding a stable $Q(s)$, the proposed Smith predictor-based controller allowing high tracking performance even in the presence of time delay can be calculated easily based on the formula (3.2).

3.2 Extension of Anti-Windup Scheme via Smith Predictor-Based Design

In Chapter 2, the design of a robust anti-windup compensation satisfying high precision tracking under saturation nonlinearity and model uncertainties is given. The proposed method uses an anti-windup tracking control architecture to effectively eliminate the steady state tracking error in the existence of saturation. As discussed in the simulation studies, the anti-windup block between the controller and plant operates as expected in the case of saturation and system output accurately follows the desired reference signal while minimizing the tracking error as much as possible.

In contrast, the proposed mechanism in Chapter 2 can not be applied on the systems including time delay in the feedback loop. Hence, we present a novel anti-windup compensator combined with the Smith predictor based controller design for the dead-time systems. The general idea behind this extension is to create a relationship between these two different approaches in order to redesign the proposed anti-windup structure in [15].

Back to the anti-windup compensator design, the controller $C(s)$ illustrated in Fig. 2.5 can be written as

$$C(s) := C_{aw}(s) = K(s)\,\frac{1}{1+F_1(s)F_2(s)} = K(s)\,\frac{1}{1+G(s)F_2(s)} \tag{3.10}$$

where $G(s)$ represents the nominal plant including integral action and time delay in the form of

$$G(s) = \frac{K}{s}R_0(s)e^{-T_d s} \tag{3.11}$$

and remember that the new plant structure is the same with (3.1).

Characteristic equation of the closed loop feedback system with $G(s)$ and $C_{aw}(s)$ can be described as

$$\Delta_{aw}(s) = 1 + G(s)C_{aw}(s)\,, \tag{3.12}$$

adding (3.11) and (3.10) to the characteristic polynomial equation, we obtain

$$\Delta_{aw}(s) = 1 + \left(\frac{K}{s}R_0(s)e^{-T_d s}\right)\left(K(s)\frac{1}{1+G(s)F_2(s)}\right).$$

Note that K is the gain of the plant transfer function as well as $K(s)$ is the stabilizer in the anti-windup scheme in Fig. 2.5.

By applying basic algebra, the characteristic polynomial can be written in the form

$$\Delta_{aw}(s) = 1 + \frac{\left(\frac{K}{s}R_0(s)e^{-T_d s}\right)}{1+\left(\frac{K}{s}R_0(s)e^{-T_d s}\right)F_2(s)}\,K(s) = 0$$

and we can conclude that

$$1 + \frac{G(s)}{1+G(s)F_2(s)}K(s) = 0 \tag{3.13}$$

which implies

$$K(s) \qquad \textit{stabilizes} \qquad \frac{G(s)}{1+G(s)F_2(s)}\,. \tag{3.14}$$

As previously stated, Smith predictor-based controller has a free part denoted as $C_0(s)$ which is designed from the non-delayed part of the plant by using controller parametrization. Similarly, based on (3.5), it can be observed that

$$C_0(s) \qquad \textit{stabilizes} \qquad \frac{1}{s}\,. \tag{3.15}$$

3.2.1 Comparison and Analysis of Anti-Windup and Smith Predictor-Based Transfer Functions

Before analyzing the relationship between anti-windup approach and Smith predictor-based design, sensitivity ($S(s)$), complementary sensitivity ($T(s)$) and the product of controller and sensitivity ($C(s)S(s)$) functions are given. Our aim is to find the similarity between these two different approaches in order to modify the robust anti-windup compensation applicable for the dead-time systems.

Smith Predictor-Based Design

- *Controller:*

$$C_s(s) = \frac{R_0(s)^{-1}}{K}\left(\frac{C_0(s)}{1 + C_0(s)\frac{1-e^{-T_d s}}{s}}\right)$$

- *Sensitivity Transfer Function:*

$$S_s(s) = (1+PC_s)^{-1} = \left(1 + \left(\frac{K}{s}R_0(s)e^{-T_d s}\right)\left(\frac{R_0(s)^{-1}}{K}\,\frac{C_0(s)}{1 + C_0(s)\frac{1-e^{-T_d s}}{s}}\right)\right)^{-1}$$

$$S_s(s) = \left(1 + C_0(s)\,\frac{1}{s}\,(1 - e^{-T_d s})\right)\left(1 + C_0(s)\frac{1}{s}\right)^{-1}$$

- *Controller-Sensitivity Transfer Function:*

$$C_s(s)S_s(s) = \left(\frac{R_0(s)^{-1}}{K}\,\frac{C_0(s)}{1 + C_0(s)\frac{1-e^{-T_d s}}{s}}\right)\left(1 + C_0(s)\frac{1}{s}\right)^{-1}\left(1 + C_0(s)\,\frac{1}{s}\,(1 - e^{-T_d s})\right)$$

$$C_s(s)S_s(s) = R_0(s)^{-1}K^{-1}C_0(s)\left(1 + C_0(s)\frac{1}{s}\right)^{-1}$$

- *Complementary Sensitivity Transfer Function:*

$$P(s)C_s(s)S_s(s) = T_s(s) = \left(\frac{K}{s}R_0(s)e^{-T_d s}\right) R_0(s)^{-1}K^{-1}C_0(s)\left(1 + C_0(s)\frac{1}{s}\right)^{-1}$$

$$T_s(s) = C_0(s)\,\frac{1}{s}\,\left(1 + C_0(s)\frac{1}{s}\right)^{-1}\, e^{-T_d s}$$

Note that the denominator of these transfer functions gives us the characteristic equation of the closed loop feedback system and we can clearly observe that $C_0(s)$ must be designed to stabilize $\frac{1}{s}$.

Anti-Windup Compensator Design

- *Controller:*

$$C_{aw}(s) = K(s)\,\frac{1}{1 + P(s)F_2(s)} = K(s)\,\frac{1}{1 + \left(\frac{K}{s}R_0(s)e^{-T_d s}\right)F_2(s)}$$

- *Sensitivity Transfer Function:*

$$S_{aw}(s) = (1 + PC_{aw})^{-1} = \left(1 + \frac{K(s)P(s)}{1 + P(s)F_2(s)}\right)^{-1}$$

- *Controller-Sensitivity Transfer Function:*

$$C_{aw}(s)S_{aw}(s) = K(s)\,\frac{1}{1 + P(s)F_2(s)}\left(1 + \frac{K(s)P(s)}{1 + P(s)F_2(s)}\right)^{-1}$$

- *Complementary Sensitivity Transfer Function:*

$$P(s)C_{aw}(s)S_{aw}(s) = T_{aw}(s) = P(s)K(s)\frac{1}{1 + P(s)F_2(s)}\left(1 + \frac{K(s)P(s)}{1 + P(s)F_2(s)}\right)^{-1}$$

Similarly, following the above equations, it can be claimed based on the characteristic polynomial that $K(s)$ must be designed to stabilize the function $\frac{P(s)}{1+P(s)F_2(s)}$.

After analyzing the closed loop transfer functions based on these two design approaches, the extension of anti-windup controller is firstly taken into consideration. As shown in Fig. 2.5, there is a term $\Lambda(s)^{-1}$ representing the dynamics of the exogenous system. In the design of the anti-windup controller, we are aiming to eliminate this term while choosing denominator polynomial of the controller C_{aw} as

$$1 + P(s)F_2(s) = \Lambda(s)^{-1}\,\frac{1}{s}\,Z_{aw}(s) \tag{3.16}$$

where $Z_{aw}(s)$ is a polynomial. Using the definition of plant transfer function given in (3.11) into this equation,

$$1 + F_2(s)K\,\frac{1}{s}\,R_0(s)e^{-T_d s} = \Lambda(s)^{-1}\,\frac{1}{s}\,Z_{aw}(s)$$

and we can easily eliminate the integrator term,

$$s + F_2(s)\,K\,R_0(s)e^{-T_d s} = \Lambda(s)^{-1}Z_{aw}(s)\,.$$

The internal model unit $F_2(s)$ can be derived with a simple algebra,

$$F_2(s) = \;K^{-1}\,R_0(s)^{-1}e^{T_d s}\,[\,\Lambda(s)^{-1}Z_{aw}(s) - s\,]\,.$$

By further manipulations, the simplest form of this equation can be derived as

$$F_2(s) = \;K^{-1}\,R_0(s)^{-1}W_{aw}(s) \tag{3.17}$$

where $W_{aw}(s)$ equals $e^{T_d s}\,[\,\Lambda(s)^{-1}Z_{aw}(s) - s\,]$.

As stated in the Smith predictor-based approach, the controller to be designed must have poles at $s = 0$ and $s = \pm j\omega_d$ where ω_d is the frequency of the reference signal to achieve high precision tracking. With the same strategy, the anti-windup compensator controller must also have the poles at these desired locations which means $Z_{aw}(s)$ must have roots at $s = \pm j\omega_d$ since the only unknown polynomial at the right half part of (3.16) is $Z_{aw}(s)$. We can define $Z_{aw}(s)$ as

$$Z_{aw}(s) = \Lambda(s)\,[\,W_{aw}(s)e^{-T_d s} + s\,]$$

and to have a root at $s = \pm j\omega_d$, the equality

$$W_{aw}(j\omega_d) = \frac{-j\omega_d}{e^{-T_d j\omega_d}} \tag{3.18}$$

must be satisfied. Note that $W_{aw}(s)$ is a stable transfer function.

Back to the definition of complementary sensitivity transfer function of the Smith predictor-based design :

$$T_s(s) = \frac{C_0(s)\,\frac{1}{s}}{\left(1 + C_0(s)\frac{1}{s}\right)}\, e^{-T_d s}\,.$$

Every stable transfer function can be factored as $H(s) = H_i(s)H_o(s)$ where $H_i(s)$ is all-pass inner and $H_o(s)$ is minimum phase outer transfer functions. Also note that

$$|H_i(j\omega)| = 1 \qquad \forall\omega \qquad \text{and} \qquad H_o(s) \text{ does not contain any zeros in } \mathbb{C}_+\ .$$

By applying inner-outer factorization, we can define the closed loop transfer function $T_s(s)$ as

$$T_s(s) = T_{so}(s)T_{si}(s) = \underbrace{\left(\frac{C_0(s)\,\frac{1}{s}}{\left(1 + C_0(s)\frac{1}{s}\right)}\right)}_{T_{so}} \underbrace{e^{-T_d s}}_{T_{si}} \tag{3.19}$$

provided that $C_0(s)$ does not contain zeros in $\mathbb{C}_+$. Note that

$$\begin{aligned} T_{so}(j\omega_d) &= \frac{C_0(j\omega_d)\,\frac{1}{j\omega_d}}{\left(1 + C_0(j\omega_d)\frac{1}{j\omega_d}\right)} = \frac{(e^{-T_d j\omega_d} - 1)^{-1}}{1 + (e^{-T_d j\omega_d} - 1)^{-1}} \\ T_{so}(j\omega_d) &= \frac{1}{e^{-T_d j\omega_d}}\,. \end{aligned} \tag{3.20}$$

As mentioned, we are modifying the anti-windup compensator design to be applicable for the dead-time systems. If pieces of information belong to these two different designs correlate, we can offer an extension of the anti-windup approach. Accordingly, we can rewrite (3.20) as

$$T_{so}(j\omega_d) = \frac{1}{e^{-T_d j\omega_d}} = \left(-\frac{1}{j\omega_d}\right)\left(\frac{-j\omega_d}{e^{-T_d j\omega_d}}\right) .$$

Note that $\left(\frac{-j\omega_d}{e^{-T_d j\omega_d}}\right)$ is defined as $W_{aw}(j\omega_d)$ in anti-windup design and given in (3.18). Hence, we can state the outer complementary sensitivity transfer function in Smith predictor based design by using the stable transfer function $W_{aw}(s)$ of anti-windup structure as

$$T_{so}(j\omega) = -\frac{1}{j\omega}\, W_{aw}(j\omega) \quad \Longrightarrow \quad T_{so}(s) = -\frac{W_{aw}(s)}{s} . \tag{3.21}$$

The closed loop transfer function can be represented based on the sensitivity transfer function in the system such that

$$T_s(s) = 1 - S_s(s) = P(s)C_s(s)S_s(s)$$

which gives us also the definition of outer function in terms of the plant, controller and sensitivity transfer functions

$$T_{so}(s) = \frac{T_s(s)}{T_{si}} = \frac{P(s)C_s(s)S_s(s)}{T_{si}} = P(s)C_s(s)S_s(s)\, e^{T_d s} \tag{3.22}$$

where T_{si} defined as $e^{-T_d s}$ in (3.19).

Meanwhile, we can define $W_{aw}(s)$ based on the plant, controller and sensitivity transfer functions of the Smith predictor design using (3.21) and (3.22)

$$W_{aw}(s) = -s\, P(s)C_s(s)S_s(s)\, e^{T_d s} .$$

Besides this equation, we have already defined $W_{aw}(s)$ as given in (3.17) and equalizing these two equations, we have

$$[\, \Lambda(s)^{-1} Z_{aw}(s) - s \,]\, e^{T_d s} = -s\, P(s)C_s(s)S_s(s)\, e^{T_d s} .$$

Applying basic algebra on this equality results in

$$\Lambda(s)^{-1} Z_{aw}(s) = s \, \big(1 - P(s)C_s(s)S_s(s)\big)$$

and using (3.16) into this equation, we have

$$s \, \big(1 + P(s)F_2(s)\big) = s \, \big(1 - P(s)C_s(s)S_s(s)\big).$$

Note that left hand part represents the anti-windup side including internal model unit $F_2(s)$ mentioned in Chapter 2 and right hand part represents the Smith predictor side including its controller and sensitivity transfer function. Then the internal model unit in the anti-windup design can be redefined as

$$F_2(s) = -C_s(s)S_s(s) \tag{3.23}$$

where $C_s(s)$ is the Smith predictor based controller and $S_s(s)$ is the Smith predictor based sensitivity transfer function.

Back to the definition of anti-windup compensation controller :

$$C_{aw}(s) = K(s) \, \frac{1}{1 + P(s)F_2(s)}$$

and putting (3.23) into this equation,

$$C_{aw}(s) = K(s) \, \frac{1}{1 - P(s)C_s(s)S_s(s)} \, .$$

Using the general definition of the sensitivity function we have

$$C_{aw}(s) = \frac{K(s)}{1 - \frac{P(s)C_s(s)}{1+P(s)C_s(s)}} \, .$$

Notice that $\frac{P(s)C_s(s)}{1+P(s)C_s(s)}$ equals the complementary sensitivity function $T_s(s)$ in the Smith predictor design. Using (3.19), we can rewrite the controller in the form

$$C_{aw}(s) = \frac{K(s)}{1 - \frac{C_0(s)\,\frac{1}{s}}{\left(1+C_0(s)\frac{1}{s}\right)} e^{-T_d s}} \tag{3.24}$$

where $C_0(s)$ is free part of the Smith predictor based controller. With this equality, we achieve the relationship between anti-windup compensator and Smith predictor controller designs which provides us to extend the anti-windup structure considering the time delay in the system. Note that the design of $C_0(s)$ is given in Section 3.1.

We define the sensitivity transfer function in the anti-windup design as

$$S_{aw}(s) = \frac{1}{\left(1 + P(s)C_{aw}(s)\right)}$$

and using the novel definition of the anti-windup controller (3.24), we can redefine the sensitivity function in the form

$$S_{aw}(s) = \frac{1}{1 + P(s)\frac{K(s)}{1 - \frac{C_0(s)\,\frac{1}{s}}{\left(1+C_0(s)\frac{1}{s}\right)} e^{-T_d s}}} \,. \tag{3.25}$$

The robust stabilizer $K(s)$ is also redesigned via Smith predictor technique. We consider $K(s)$ as two parts

$$K(s) = K_0(s)K_1(s) \tag{3.26}$$

and $K_1(s)$ is chosen as

$$K_1(s) = K^{-1}R_0(s)^{-1}$$

where K is the gain of the plant and $R_0(s)$ represents the minimum phase transfer function given in (3.1).

While using the definition of $K(s)$ and the plant structure in the equation (3.25), we have

$$S_{aw}(s) = \frac{1}{1 + \frac{K_0(s)\,\frac{1}{s}e^{-T_d s}}{1 - T_{so}(s)e^{-T_d s}}}$$

where $T_{so}(s)$ is the outer part of the closed loop transfer function in the Smith predictor-based designed defined as $\frac{\frac{1}{s}C_0(s)}{1+\frac{1}{s}C_0(s)}$ in the equation (3.19).

Besides, the complementary sensitivity transfer function in the control theory can be specified as

$$T_{aw}(s) = 1 - S_{aw}(s)$$

and using the sensitivity transfer function into this equality, we achieve

$$T_{aw}(s) = 1 - \frac{1}{1 + \frac{K_0(s)\,\frac{1}{s}\,e^{-T_d s}}{1 - T_{so}(s)e^{-T_d s}}} = \frac{1}{1 - T_{so}(s)\frac{e^{-T_d s}}{1+\,\frac{1}{s}\,K_0(s)e^{-T_d s}}}\,.$$

Denominator polynomial in the definition of closed loop transfer function gives the characteristic equation in the feedback system which can be written as

$$\Delta_{aw}(s) = 1 + P(s)C_{aw}(s) = 1 - T_{so}(s)\frac{e^{-T_d s}}{1+\,\frac{1}{s}\,K_0(s)e^{-T_d s}} \tag{3.27}$$

since the closed loop transfer function has the form $T_{aw}(s) = \frac{P(s)C_{aw}(s)}{1+P(s)C_{aw}(s)}$. In order to have a stable system, roots of the characteristic polynomial should be placed in the left half complex plane denoted as $\mathbb{C}_{-}$. Because, these roots are the poles of closed loop transfer function of the feedback system. The only unknown in the definition of characteristic equation is the stable polynomial $K_0(s)$ which is the part of robust stabilizer described in (3.26). Hence, the aim is to design $K_0(s)$ such that $(1 + P(s)C_{aw}(s))^{-1}$ is stable.

While putting the definition of $T_{so}(s)$ into (3.27), the inverse of characteristic polynomial equals

$$\Delta_{aw}(s)^{-1} = \left(\frac{1+\,\frac{1}{s}\,K_0(s)e^{-T_d s}\, - \left(\frac{\frac{1}{s}\,C_0(s)}{1+\frac{1}{s}C_0(s)}\right) e^{-T_d s}}{1+\,\frac{1}{s}\,K_0(s)e^{-T_d s}} \right)^{-1}$$

and choosing

$$K_0(s) = \frac{C_0(s)}{1+\frac{1}{s}C_0(s)} \tag{3.28}$$

makes $(1 + P(s)C_{aw}(s))^{-1}$ stable.

3.3 A Summary of the Extended Anti-windup Structure

The algorithm behind the structure of proposed extended design is briefly described here. The known parameters are the plant transfer function $P(s)$, additive upper bound $W_a(s)$, saturation limits of the actuator and desired reference input $r(t)$. Based on these parameters, we mainly focus on to redesign the internal model units $F_1(s)$ and $F_2(s)$, robust stabilizer $K(s)$, augmented system transfer function $G_A(s)$ and anti-windup compensator $\theta_1(s)$ and $\theta_2(s)$ given in Fig. 2.6 via Smith predictor-based design. Consequently, we obtain the following results

$$
\begin{aligned}
F_1(s) &= P(s) = \frac{K}{s} R_0(s) e^{-T_d s} \\
F_2(s) &= R_0(s)^{-1} K^{-1} W_{aw}(s) \quad \text{where} \quad W_{aw}(s) = -sT_{so}(s) \\
K(s) &= K_0(s) K_1(s) = \frac{C_0(s)}{1 + \frac{1}{s} C_0(s)} \left(K^{-1} R_0(s)^{-1}\right) \\
G_A(s) &= \frac{P(s)}{1 + F(s)} \quad \text{where} \quad F(s) = F_1(s) F_2(s)
\end{aligned}
\tag{3.29}
$$

$$
\begin{aligned}
\tilde{\theta}_1 &= \frac{\gamma}{(1 + \alpha s)(1 + \beta s)} \quad \text{for} \quad (\alpha > 0,\ \beta > 0) \\
\theta_1(s) &= \tilde{\theta}_1(s) \left(1 + F(s)\right) \\
\theta_2(s) &= P(s) \left(1 + \frac{\theta_1(s)}{1 + F(s)}\right) = P(s) \left(1 + \tilde{\theta}_1(s)\right)
\end{aligned}
\tag{3.30}
$$

and postulate an extension of anti-windup structure which is applicable for the dead-time systems. The new controller has the form

$$
C_{aw}(s) = \frac{K(s)}{1 - T_{so}(s) e^{-T_d s}} \tag{3.31}
$$

where $T_{so}(s) = \frac{C_0(s)\, \frac{1}{s}}{\left(1 + C_0(s) \frac{1}{s}\right)}$ represents the outer closed-loop Smith predictor-based transfer function. Note that $C_0(s)$ is a stabilizing controller for $\frac{1}{s}$ and an example for its design will be given in Section 4.

Chapter 4

Numerical Results on the Case Study

This chapter covers the simulation studies with the proposed new anti-windup compensator for the time delay systems. Different design procedures behind the stable polynomial $Q(s)$ within the controller parametrization formula are provided. Simulation studies conducted with and without proposed extended structure and their results are presented. Additionally, performance of the dead-time anti-windup compensator against saturation nonlinearities is discussed.

4.1 Design of Stable Function : Q(s)

Based on the results in (3.29) and (3.30), in order to achieve the extended internal model units, robust stabilizer and anti-windup controller, we have to design the free part $C_0(s)$ as mentioned in Section 3.1 equation (3.9). Since our aim is to achieve high tracking performance of the sinusoidal reference input, the controller or plant should include the poles at the periodic signal frequencies. Plant is already calculated based on the system identification methods and it has an integrator which means the pole at $s = 0$. Hence we have to design the controller including poles at $s = \pm j\omega$ where ω is the frequency of the reference signal. Note that only unknown parameter in the controller formula is $C_0(s)$ representing set of all stabilizing controllers for $\frac{1}{s}$ which are parametrized as

$$C_0(s) = \frac{X(s) + D_p(s)Q(s)}{Y(s) - N_p(s)Q(s)} \tag{4.1}$$

where $N_p(s) = \frac{1}{s+a}$, $D_p(s) = \frac{s}{s+a}$ and $Q(s) \in \mathcal{H}_\infty$ is a stable polynomial. $X(s)$ and $Y(s)$ are found as a and 1 respectively using Bezout equation. The internal controller has the form while using all these functions

$$C_0(s) = \frac{a + \frac{s}{s+a}Q(s)}{1 - \frac{1}{s+a}Q(s)} \tag{4.2}$$

and the problem reduces to designing a stable $Q(s)$ where degree of the polynomial is determined depending on the design requirements by satisfying the condition,

$$\text{Minimum degree of } Q(s) = \text{Number of interpolation conditions } -1\ .$$

In the design of $C_0(s)$, we have determined two interpolation conditions

$$\begin{aligned} C_0(0) &= -\frac{1}{T_d} \\ C_0(j\omega) &= -\frac{j\omega}{1 - e^{-T_d j\omega}} \end{aligned} \tag{4.3}$$

and from (4.1), the interpolation conditions are translated to

$$\begin{aligned} Q_0(0) &= a(1 + aT_d) \\ Q_0(j\omega) &= \frac{(j\omega + a - ae^{-j\omega T_d})(j\omega + a)}{j\omega e^{-j\omega T_d}}\ . \end{aligned} \tag{4.4}$$

Hence, the problem can be redefined as designing a stable $Q(s)$ which satisfies the interpolation conditions in (4.4) and finding the appropriate $C_0(s)$ defined in

(4.2) using the resulted $Q(s)$. Note that since $Q(s)$ is a rational function, it is sufficient to use only one interpolation condition for the complex roots.

We apply three different techniques in the design of this stable function which are listed as Lagrange interpolation methodology, designing the poles by estimating the zeros and designing the zeros by estimating the poles. The relative degree is changed according to the method we used, but degree of the denominator polynomial is always chosen based on the minimum degree structure. By considering all the roots $(0,\ +j\omega,\ -j\omega)$, we postulate the minimum degree of $Q(s)$ as two.

Lagrange Interpolation

Find a transfer function $X(s) \in \mathcal{H}_\infty$ such that $X(a_i) = b_i$ where $a_i \in \mathbb{C}_+$ and $b_i \in \mathbb{C}$. Let

$$X(s) = \frac{\alpha_1 s^{n-1} + \alpha_2 s^{n-2} + ... + \alpha_n}{(s+1)^{n-1}},$$

and using the definition, we have

$$\begin{bmatrix} s^{n-1}\, s^{n-2} \ldots\ s^0 \end{bmatrix} \times \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ .. \\ \alpha_n \end{bmatrix} = (s+1)^{n-1} \times b_i \quad \text{at} \quad s = a_i \quad \text{where} \quad i = 1, 2, ..., n$$

which implies

$$\begin{bmatrix} a_1^{n-1} & . & . & . & a_1 & 1 \\ a_2^{n-1} & . & . & . & a_2 & 1 \\ & & . & & & \\ & & . & & & \\ a_n^{n-1} & . & . & . & a_n & 1 \end{bmatrix} \times \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ . \\ . \\ \alpha_n \end{bmatrix} = \begin{bmatrix} (a_1+1)^{n-1}\, b_1 \\ (a_2+1)^{n-1}\, b_2 \\ . \\ . \\ (a_n+1)^{n-1}\, b_n \end{bmatrix}.$$

Hence, a stable transfer function can be easily calculated in the given form by choosing the relative degree depending on the parameter n via Lagrange interpolation.

We determine $Q(s)$ in the form

$$Q(s) = \frac{\alpha_1 s^2 + \alpha_2 s + \alpha_3}{(s+1)^2} \,,$$

and apply Lagrange interpolation technique to find the unknown parameters. Since a_i's are the roots, choose them as $a_1 = 0$, $a_2 = j\omega$, $a_3 = -j\omega$. Based on the definition, b_i equals to $Q(a_i)$ for the corresponding indexes where $i = 1\,, 2\,, 3$. Using the interpolation conditions defined in (4.4), we can calculate the unknown parameters $\alpha_1\,, \alpha_2$ and α_3. Their values are

$$\alpha_1 = \frac{Re\left(\frac{(-\omega^2+2j\omega+1)(j\omega+a-ae^{-j\omega T_d})(j\omega+a)}{j\omega e^{-j\omega T_d}}\right) - \alpha_3}{-\omega^2} \,,$$

$$\alpha_2 = \frac{Im\left(\frac{(-\omega^2+2j\omega+1)(j\omega+a-ae^{-j\omega T_d})(j\omega+a)}{j\omega e^{-j\omega T_d}}\right)}{\omega} \,,$$

$$\alpha_3 = Q(0) = a\,(1 + aT_d)\,.$$

In conclusion, a stable $Q(s)$ can be written via these parameters. Now, we achieve the internal controller $C_0(s)$ using this conceptually simple strategy.

Designing the poles of Q(s)

As mentioned in the previous sections, our aim is to achieve tracking of a given reference signal at the output while using anti-windup compensation. To achieve high performance tracking, we design the denominator polynomial such that $Q(s)$ has poles at the sinusoidal signal frequencies $s = \pm j\omega$. Relative degree of stable function $Q(s)$ may change depending on the degree of the numerator polynomial, however denominator always has order of 2 because of the minimum degree structure. Hence, the number of unknowns in $Q(s)$ differ by numerator degree.

When numerator is chosen as a constant,

$$Q(s) = \frac{b}{ds^2 + es + f}$$

for $s = 0$, we have

$$Q(s) = \frac{b}{f} = a\left(1 + aT_d\right)$$

implies

$$f = \frac{b}{a\left(1 + aT_d\right)}\,.$$

For $s = j\omega$ results in

$$Q(j\omega) = \frac{b}{-d\omega^2 + ej\omega + f},$$

and substituting f into this equation, we can calculate the remaining unknown parameters with the interpolation conditions

$$d = \frac{Re\left(\frac{b(j\omega)e^{-j\omega T_d}}{(j\omega + a - ae^{-j\omega T_d})(j\omega + a)}\right) - f}{-\omega^2},$$

$$e = \frac{Im\left(\frac{b(j\omega)e^{-j\omega T_d}}{(j\omega + a - ae^{-j\omega T_d})(j\omega + a)}\right)}{\omega}\,.$$

While increasing the degree of numerator polynomial, we apply the same procedure to design the poles of $Q(s)$ by estimating the free parameters (or constants in the numerator) depending on the sinusoidal reference signal and try to find a stable polynomial.

Designing the zeros of Q(s)

Another method in the design of $Q(s)$ is to determine the roots of numerator polynomial to guarantee the stability of $Q(s)$. The roots of denominator polynomial are chosen to place the closed loop system poles at the desired locations based on the given input. In this approach, $Q(s)$ has the form,

$$Q(s) = \frac{bs^2 + cs + d}{s^2 + es + f}\,. \tag{4.5}$$

Here $e, f > 0$ are free parameters; once these are chosen, the other parameters are determined by employing the interpolation conditions defined in (4.4). The free parameters are changed based on the frequency of the sinusoidal signal.

For $s = 0$, using the interpolation condition, the polynomial equals to

$$Q(s) = \frac{d}{f} = a\,(1 + aT_d)$$

which gives the first unknown parameter

$$d = f\,a\,(1 + aT_d)\,. \tag{4.6}$$

For $s = j\omega$, we can compute the other parameters by applying the interpolation condition again,

$$Q(j\omega) = \frac{-b\omega^2 + +cj\omega + d}{-d\omega^2 + ej\omega + f},$$

which is concluded as

$$\begin{aligned} b =& \frac{Re\left(\frac{(-\omega^2+ej\omega+f)(j\omega+a-ae^{-j\omega T_d})(j\omega+a)}{j\omega e^{-j\omega T_d}}\right) - d}{-\omega^2}, \\ c =& \frac{Im\left(\frac{(-\omega^2+ej\omega+f)(j\omega+a-ae^{-j\omega T_d})(j\omega+a)}{j\omega e^{-j\omega T_d}}\right)}{\omega}. \end{aligned} \tag{4.7}$$

4.2 Simulations and Results

In Chapter 2, the nominal model transfer function which is designed based on the system identification tests conducted on the hardware structure is provided. In the results given in Chapter 2, delay-free part of the plant model is taken into consideration and general anti-windup compensator structure is employed with and without saturation. Now, we simulate this plant model considering dead-time by using the extended anti-windup scheme. Remember the nominal model transfer function

$$P(s) = \frac{7.1\,\left(1 + 0.016\,\frac{s}{175} + (\frac{s}{175})^2\right)\,\left(1 + 0.08\,\frac{s}{930} + (\frac{s}{930})^2\right)}{s\,\left(1 + 0.04\,\frac{s}{285} + (\frac{s}{285})^2\right)\,\left(1 + 0.21\,\frac{s}{960} + (\frac{s}{960})^2\right)}\,e^{-8.1\times10^{-3}s}\,. \tag{4.8}$$

Note that the transfer function has the form described in the equation (3.1). K is the gain of the plant model which equals 7.1, s in the denominator represents the integral action and time delay is defined as $T_d = 8.1\,ms$. Remaining part in the definition of the plant belongs to the minimum phase transfer function $R_0(s)$.

Moreover, the additive uncertainty bound calculated previously again used in the simulations where the transfer function is given as

$$W_a(s) = \frac{0.011\,(1 + s/20)}{\left(1 + 0.02\,(s/280) + (s/280)^2\right)}\,. \tag{4.9}$$

The controller parametrization method is first applied in order to design the free part of the Smith predictor based controller $C_0(s)$ by choosing $N_p(s) = \frac{1}{s+a}$ and $D_p(s) = \frac{s}{s+a}$ as mentioned. In the design of a stable polynomial $Q(s)$, among all three methods, *designing the zeros of* $Q(s)$ approach is determined to be used in the simulation studies. *Lagrange interpolation* solution does not work appropriately for higher frequencies, on the other hand, *designing the poles of* $Q(s)$ technique only works using high frequencies. The parameter e sometimes becomes negative which makes $Q(s)$ unstable while using the poles design method; because this approach does not guarantee the stability of $Q(s)$ since we estimate the free parameters in the numerator.

$Q(s)$ is chosen as given in the equation (4.5) where the free parameters a, e and f are determined based on the sinusoidal reference signal. Note that a is used in the design of the free controller parameter $C_0(s)$.

Table 4.1: Free Parameters for the Stable Function $Q(s)$

Parameter :	Value
a	4
e	2
f	1

The desired input is described as $r(t) = 50sin(\omega t + \pi/2)$ for $\omega = 1.5\, rad/sec$ angular frequency and the estimated free parameters are provided in Table 4.1. The remaining parameters of stable $Q(s)$ can be computed based on the formulations in (4.6) and (4.7) which are calculated via interpolation conditions by using the values in Table 4.1 and the identified parameters are given in Table 4.2.

Table 4.2: Designed Parameters for the Stable Function $Q(s)$

Parameter :	Value
b	6.2496
c	6.89
d	4.1296

Finally, the free part of the controller $C_0(s)$ can be calculated using stable $Q(s)$ and Bezout equation polynomials $X(s), Y(s), N_p(s)$ and $D_p(s)$:

$$C_0(s) = \frac{10.25\,(s + 0.6618)\,(s^2 + 2.352s + 2.359)}{(s - 0.06176)\,(s^2 - 0.1879s + 2.098)}\,.$$

After completing the Smith predictor design parameters to be used in the calculations of extended anti-windup functions, we start with the design of novel internal model units $F_1(s)$ and $F_2(s)$. Remember that $F_1(s)$ is directly chosen as the plant transfer function, hence we only need to design the other internal model unit. In order to achieve $F_2(s)$, outer closed loop transfer function of the Smith predictor structure $T_{so}(s)$ has to be calculated. Then multiplying two internal model units, the transfer function $F(s)$ is obtained as

$$\begin{aligned} F(s) &= F_1(s) \times F_2(s) \\ &= \left(\frac{K}{s}\, R_0(s) e^{-T_d s}\right) \left(R_0(s)^{-1}\, K^{-1}\, (-s\, T_{so}(s))\right) \end{aligned}$$

which results in

$$F(s) = \frac{-10.25\,(s + 0.6618)\,(s^2 + 2.352s + 2.359)}{(s + 4)^2\,(s + 1)^2} \times e^{-8.1\times 10^{-3} s}.$$

In order to design $K(s)$, robust stabilizer, time delay in this equation is replaced with its rational equivalent obtained via second order Pade approximation (tough there are direct design methods for systems with delays [53]). This technique is commonly used in many control applications to deal with irrational delay parameter $e^{-T_d s}$ by representing it with a stable rational transfer function [54]. The time delay element can be approximated with the function

$$e^{-T_d s} \cong \frac{1 - k_1 s + k_2 s^2 - ... \pm k_n s^n}{1 + k_1 s + k_2 s + ... + k_n s^n},$$

where n is the approximation order. Larger n gives more accurate equivalent of the dead-time; however, numerical problems may occur while computing the polynomial coefficients [7]. Second order Pade approximation is used to represent the time delay in our simulations :

$$e^{-T_d s} \cong \frac{1 - \frac{T_d}{2} s + \frac{T_d^2}{12} s^2}{1 + \frac{T_d}{2} s + \frac{T_d^2}{12} s^2}.$$

We continue with the design of stabilizer $K(s)$ which has two functions denoted as $K_0(s)$ and $K_1(s)$. Utilizing the definitions of these transfer functions given in (3.29), the following results are obtained :

$$K_0(s) = \frac{10.25\, s\, (s + 0.6618)\, (s^2 + 2.352s + 2.359)}{(s+4)^2\, (s+1)^2},$$

$$K_1(s) = \frac{0.050867\, (s^2 + 11s + 7.952 \times 10^4)\, (s^2 + 201.6s + 9.216 \times 10^5)}{(s^2 + 2.8s + 3.063 \times 10^4)\, (s^2 + 74.4s + 8.649 \times 10^5)}.$$

Moreover, the augmented transfer function $G_A(s)$ is computed by using the result of $F(s)$ found with the new definitions of the internal model units

$$G_A(s) = \frac{G(s)}{1 + F(s)}$$

$$G_A(s) = \frac{19.659\, (s+4)^2\, (s+1)^2\, (s^2 + 2.8s + 3.063 \times 10^4)}{s^3\, (s^2 - 5.348 \times 10^{-12} s + 2.25)\, (s^2 + 11s + 7.952 \times 10^4)}\, e^{-8.1 \times 10^{-3} s}$$

and time delay is approximated again 2^{nd} order Pade.

Finally, extended anti-windup controller $C_{aw}(s)$ is achieved based on the stabilizer $K(s)$ and product of the internal model units $F(s)$

$$C_{aw}(s) = \frac{0.52137\,(s+0.6618)\,(s^2+2.352s+2.359)\,(s^2+11s+7.952\times10^4)}{s\,(s^2+2.25)\,(s^2+2.8s+3.062\times10^4)}\,.$$

Besides, the anti-windup compensator $\theta_1(s)$ and $\theta_2(s)$ are also calculated by applying their definitions provided in (3.30) via extended $F(s)$. The parameters α, β and γ are chosen the same with the previous simulation studies in Section 2.4.

The anti-windup scheme illustrated in Fig. 2.6 is implemented with the extended internal model units, stabilizer, augmented system and compensator in the absence of saturation. We aim to tackle the problems of system nonlinearities which cause instability and performance degradation in linear systems and aim to achieve high tracking performance.

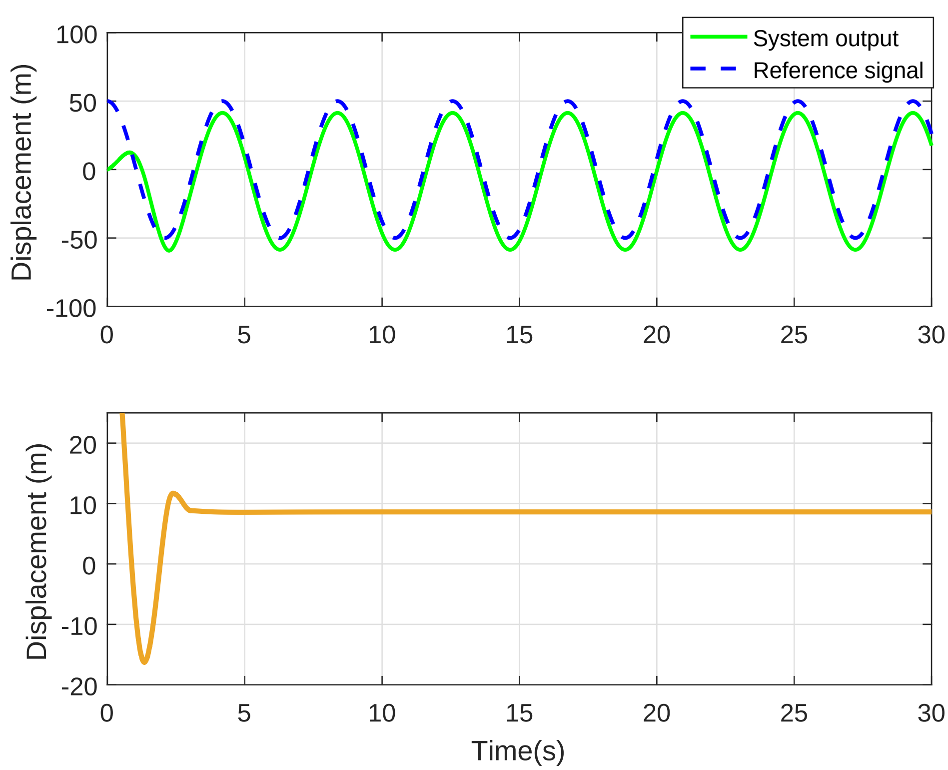


Figure 4.1: System output under the effect of input saturation when there is no anti-windup structure. The tracking error is also represented in the second graph.

The results of the simulation studies are performed both using the anti-windup structure depicted in Fig. 2.6 and using only controller and plant without anti-windup scheme. First we examine the system behavior in the effect of input saturation without anti-windup controller structure. System output together with the reference sinusoidal signal is illustrated in Fig. 4.1. Note that there exist a difference between the output and desired input which can be seen in the second graph. Tracking error is approximately found as $8.6\,m$ from this plot.

The output of the controller is shown in Fig. 4.2 to observe the situation of saturation. The red lines represents the saturation limits, hence until around 4 seconds, the system operates in the nonlinear region. The second graph given in Fig. 4.2 belongs to the saturated plant input.

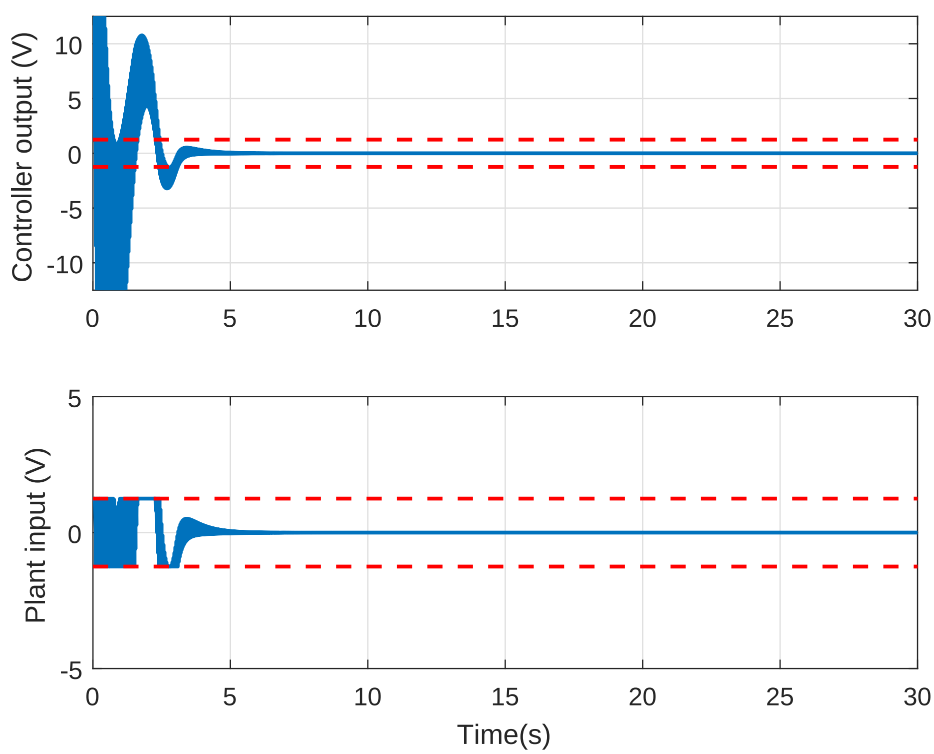


Figure 4.2: Controller output and plant input under the effect of input saturation when there is no anti-windup structure. Limits of the saturation is represented as red dashed lines.

Fig. 4.3 illustrates the system output when we use the proposed anti-windup architecture. The output recovers from the nonlinearity after around 4.46 seconds and tracking error converges to zero accurately which means that the extended robust anti-windup compensator for the systems including time delay and integral action works as expected. The system output depends on the design specifications such as saturation limits, desired sinusoidal signal, dead-time in the system, etc. Hence, in the design of anti-windup compensator, these specifications have to be taken into account.

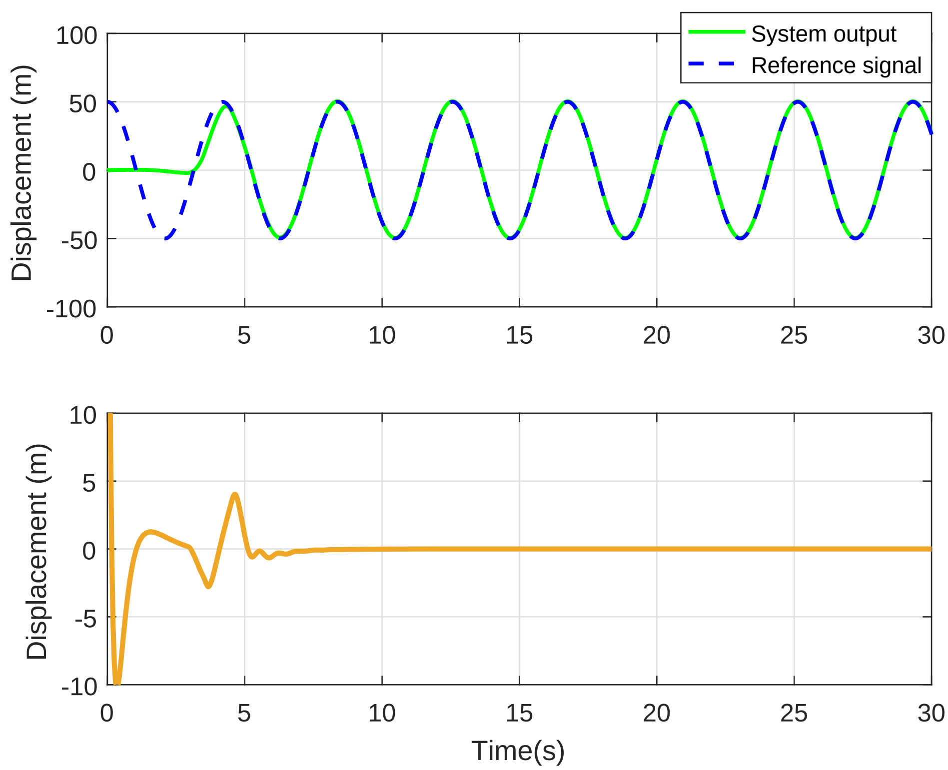


Figure 4.3: System output under the effect of input saturation when extended anti-windup structure is operating. The tracking error is also represented in the second graph.

The output of the controller in this case is depicted in Fig. 4.4 with the corresponding plant input. The control signal is limited by the actuator, hence proposed controller can not operate properly which means that there exists a difference between the controller output and plant input. Applying the extended structure into this plant, the damaging nonlinear effects are resolved.

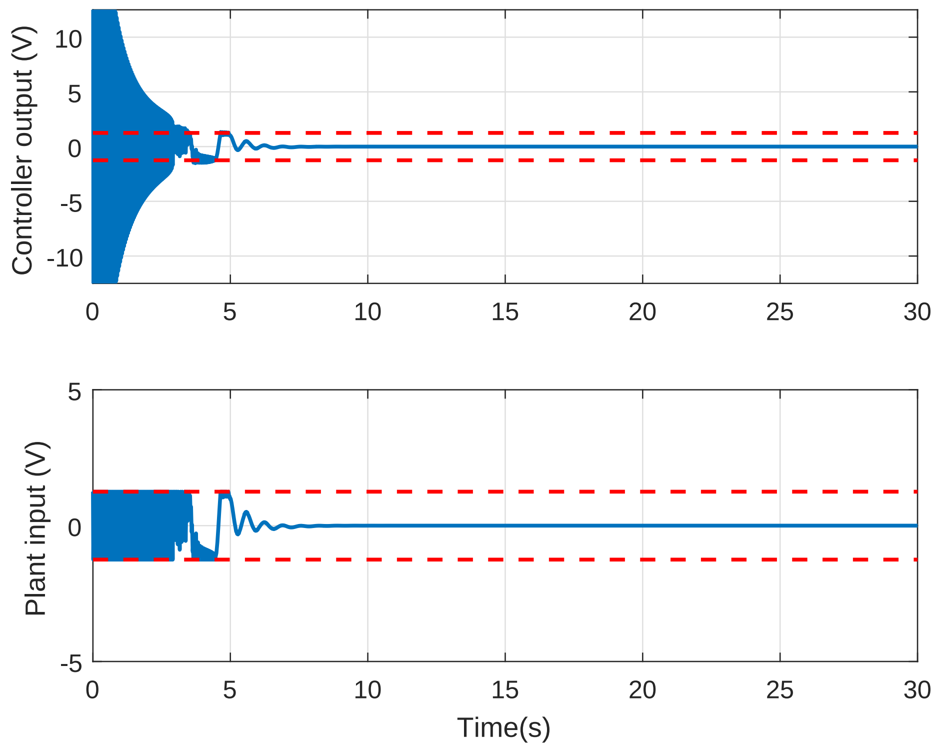


Figure 4.4: Controller output and plant input under the effect of input saturation when extended anti-windup structure is operating. Limits of the saturation is represented as red dashed lines.

By comparing the system output results depending on the anti-windup and without anti-windup studies, the system successfully suppresses the nonlinear effects after a time and minimizes the tracking error when we apply the extended architecture. Time delay is considered in the proposed structure, hence the delay element may also affect recovering the nonlinearity in Fig. 4.3. However, the measured system output follows the desired sinusoidal reference with the acceptable performance despite the saturation nonlinearity and time delay existing in the system dynamics.

Chapter 5

Conclusion and Future Work

One can intuitively assert that an unknown physical system can never be modeled as exactly as accurate which allows crucial problems to occur in the system dynamics. The *winding-up* effect causes a situation that the designed controller can not operate as expected and can not drive the plant as required. An anti-windup scheme is a preferred method which helps to suppress the degradations in the system performance and to preserve the system stability. To this end, the solution for the problem of saturation type nonlinearities in control systems based on anti-windup scheme is presented in this thesis.

Based on aforementioned advantages of the anti-windup design, we can conclude that designing the controller to account for the effect of saturation and other system uncertainties improves the performance of the controller by allowing it to operate in the linear region most of the time. The proposed anti-windup mechanism in [15] including internal model structure together with the robust anti-windup compensator is used to allow high tracking performance, however, this method was not applicable for the dead-time systems. The present work fills this gap by focusing on how the adverse effects of actuator saturation can be suppressed independently of time delay in the system.

We also presented Smith predictor based method for the controller design of the dead-time system with integral term. Motivated by the Smith predictor-based design strategy of [14], we employed a new anti-windup mechanism applicable for the dead-time systems by extending the used anti-windup architecture. Our main goal in this extension was to improve the anti-windup control strategy to enhance the sinusoidal tracking performance of the closed-loop feedback system under the saturation effect.

Consequently, the applicability of extended anti-windup architecture was shown on the time delay plant incorporating integral action with the presence of saturation nonlinearity. Starting from the definitions of components in the extended architecture, our future work includes improvements of this extension for the unstable dead-time systems. The controller stabilizing the saturated systems with integral action and time delay is designed by utilizing the proposed method. The longer term goal of this study is to design a Smith predictor-like controller based on the extended anti-windup scheme for the plants including more than one pole at $\mathbb{C}_+$.

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