

EVALUATION OF TRACKING CONTROL OF
RACK POSITION OF A STEER-BY-WIRE SYSTEM

by
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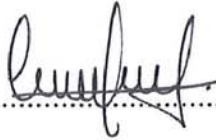
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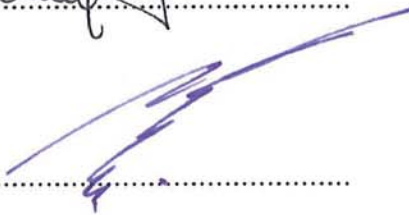
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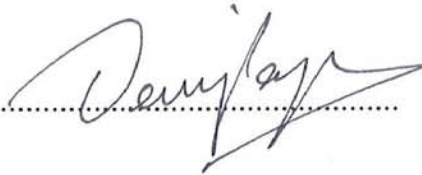
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ABSTRACT

EVALUATION OF TRACKING CONTROL OF RACK POSITION OF A STEER-BY-WIRE SYSTEM

In this thesis, the purpose is to track the desired rack position of a vehicle in an accurate manner when the parameters of the driver interaction and the vehicle directional units of a steer-by-wire system are varied. An impedance control for the driver interaction unit of a steer-by-wire system has been used to obtain desired rack position and a Proportional-Derivative (PD) controller has been used for the vehicle directional control unit of the same steer-by-wire system to track the desired rack position in an accurate manner. Various tests have been performed to evaluate the tracking performance.

ÖZET

ELEKTRİKLİ DİREKSİYON SİSTEMİNİN DİREKSİYON KUTUSUNDAKİ DİŞLİNİN HAREKETİNİN İZLENMESİ

Bu tezde elektrikli direksiyon sisteminin (steer-by-wire) sürücü etkileşim (direksiyon) ve direksiyon kutusu parametreleri ayarlanarak bir aracın direksiyon kutusundaki dişlinin hareketi izlenmiştir. Direksiyon kutusundaki dişlinin istenilen doğru hareketi yapması için sürücü etkileşim kısmında empedans kontrolü ve aracın yönünün (direksiyon kutusu) kontrolü için de PD kontrol sistemi kullanılmıştır. Birçok test yapılarak direksiyon kutusundaki dişlinin performansı değerlendirilmiştir.

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LIST OF SYMBOLS / ABBREVIATIONS

a_R	Rack viscous damping (Ns/rad)
a_{SW}	Steering-wheel viscous damping (Nms/rad)
b_R	Rack input gain (N/A)
b_{SW}	Steering-wheel input gain (Nm/A)
B_C	Steering-column viscous damping (Nms/rad)
B_H	Hydraulic cylinder viscous damping (Ns/m)
B_R	Rack viscous friction including the driver interaction motor (B_M) and gear box (B_G) viscous frictions (Ns/m)
B_{SW}	Steering-wheel viscous damping (Nms/rad)
B_{TB}	Torsion-bar viscous damping (Nms/rad)
c_{SW}	Steering-wheel driver gain
f_{SW}	Steering-wheel static friction (Nm)
f_R	Rack static friction (N)
F_H	Hydraulic assist force
F_R	Static friction (N)
F_{SW}	Static friction (Nm)
i_G	Driver interaction motor gear box ratio
i_M	Current drawn by the motor (A)
J_C	Steering-column inertia in hydraulic system (kgm^2)
J_{SW}	Steering-column inertia (kgm^2)
J_{TB}	Torsion-bar inertia (kgm^2)
k_A	Torque constant of the driver interaction motor (Nm/A)
K_C	Steering-column stiffness (Nm/rad)
K_{TB}	Torsion-bar stiffness (Nm/rad)
K_{TR}	Spring rate connecting the left and right rack ends to the fixture bas (N/m)

l_{SA}	Moment arm length (m)
M_R	Rack mass including driver interaction motor and gear box inertia (kg)
r_p	Pinion radius (m)
x_1	Steering-wheel angular position (rad)
x_2	Steering-wheel angular velocity (rad/s)
x_3	Rack position (m)
x_4	Rack velocity (m/s)
x_R	Rack position (m/s)
θ_C	Steering-column angular position of the hydraulic steering system (rad)
θ_{SW}	Steering-column angular position (rad)
θ_{TB}	Torsion-bar angular position (rad)
δ_{FL}	Left wheel angles (rad)
δ_{FR}	Right wheel angles (rad)
τ_D	Driver torque (Nm)

1. INTRODUCTION

One of the greatest developments is about steering-control system, where the driver interacts to guide it in vehicle technology [1]. The steering systems consist of mechanical steering system, hydraulic- and electro-hydraulic-power-assisted steering system and electric-power-assisted steering system (EPAS) [1]. Besides these steering systems, a new technology called the steer-by-wire system which can be thought as a subsystem of electric power assisted steering system is developed. There is not a mechanical link available between the steering wheel and the steering gear in steer-by-wire systems. The direction of the vehicle is controlled via servo unit installed on the steering gear in a similar way as rack assisted electric power assisted steering system. The steer-by-wire system is composed of the vehicle directional control unit and driver interaction unit. The vehicle directional control unit, which is shown in Figure 1.1, is composed of steering gear, servo unit, and tie-roads connecting rack ends to wheel. The driver interaction unit, which is shown in Figure 1.1, is composed of steering wheel, steering column, and servo unit.

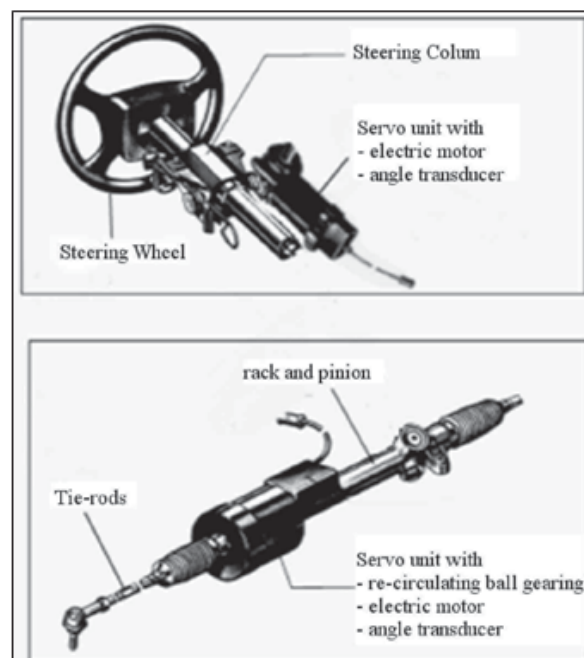


Figure 1.1. Steer-by-wire system [2]

Steer-by-wire systems have advantages compared to the conventional steering systems. Steering column is absent in steer-by wire system which simplifies the interior car design [3]. Additionally, the absence of steering shaft, column and gear reduction mechanism allows much better space utilization in the engine compartment [3]. The steering mechanism can be designed and installed as a modular unit [3] and the injury risk caused by the steering column in an accident can be diminished [3]. The main disadvantage of steer-by-wire system is that there is no direct mechanical feedback to the operator about the steering conditions, which must be emulated by an active control system [3].

Various controllers have been developed for steer-by-wire system. The main purpose of the controllers is to improve the stability of the vehicle and the comfort of the driver. Yaw rate feedback has been used as an understeer gradient in addition to the steering-wheel and front-wheel controls on a hardware-in-the-loop (HIL) system with a steer-by-wire controller to improve vehicle safety at high speeds [1]. A nonlinear tracking controller is used for steer-by-wire systems to asymptotically force the tracking errors to zero under parameter uncertainty [1]. The impedance control strategy is used to build a reference model, including the driver torque and the force on the rack arising from the tire-road interaction. A model reference adaptive controller for steer-by-wire applications has been proposed for two-wheel-steering vehicles that can treat nonlinearities and uncertainties between the slip angles and the lateral forces on tires [1]. An HIL system has been developed to investigate the effect of steer-by-wire actuator bandwidth and saturation limits on the yaw-stability controller built on a test setup, including a steering wheel, gas and brake pedals, a steering actuator, and the road load actuator equipped with a steering controller [1].

In this thesis, the purpose is to track the desired rack position of a vehicle in an accurate manner when the parameters of the driver interaction and the vehicle directional units of a steer-by-wire system are varied. A steer-by-wire system has been previously developed and the tracking of the desired rack position has previously been evaluated using this steer-by-wire system [2]. However, the effects of the changes in the parameters on control performance have not been performed. Thus, in this thesis various tests have

been performed to evaluate the tracking performance when the parameters of the driver interaction and the vehicle directional units of the same steer-by-wire system are varied.

The details of steer-by-wire system are presented in Section 2. The controllers that are used to obtain the desired rack position and to track this position are given in Section 3. In Section 4, the tracking results when the parameters of the driver interaction and the vehicle directional units of the same steer-by-wire system are varied are presented. In Section 5, the conclusion and discussion of this work are presented.

2. SYSTEM ARCHITECTURE

2.1. MECHANICAL STEERING SYSTEM

The steering system is the most important part of a vehicle where is manipulated by the driver to guide the vehicle. The steering system is shown in Figure 2.1. The components of the steering system, which are the steering wheel, steering column, steering gear, bushings, tie-rod joints, and tires, are not linear. Therefore, the driver needs to adjust the steering wheel at an angle to keep the deviations of the vehicle low from the required course.

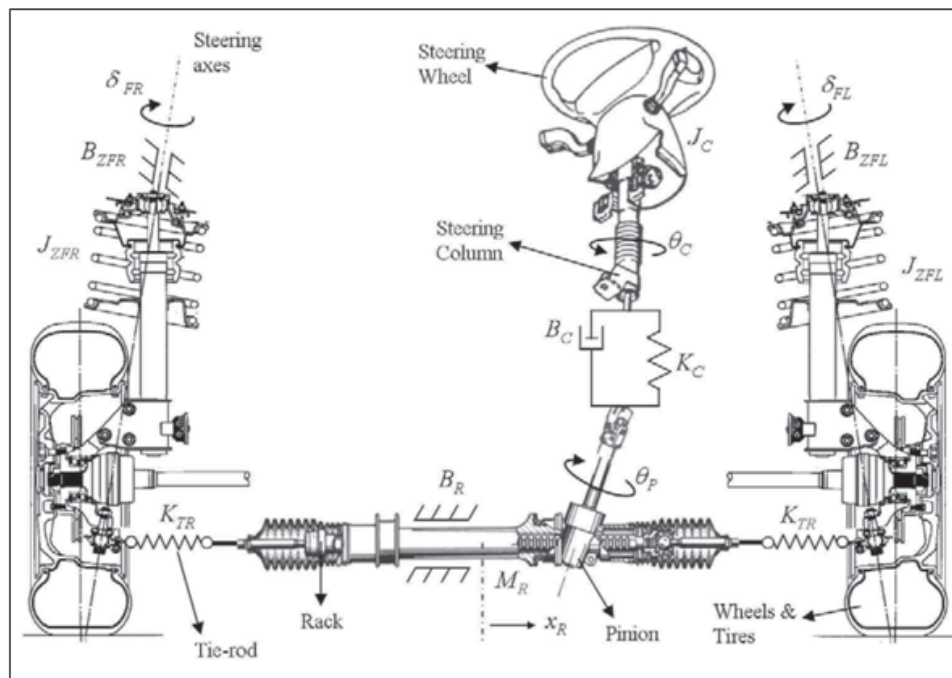


Figure 2.1. Mechanical steering system model [2]

2.2 STEERING SYSTEM ARCHITECTURE

The blocks of mechanical steering system model are shown in Figure 2.2. There are five blocks which are driver interaction unit, torsion bar dynamics, rack and pinion dynamics, impedance controller, Proportional-Derivative (PD) control, and vehicle directional control unit.

The steering system architecture used in this thesis is shown in Figure 2.2. A torque is applied by the driver to steer wheel column and steer wheel angle. This torque is the input of the driver interaction unit. Then, the steering wheel starts to rotate with an angle. The outputs of the driver interaction unit are the steer wheel position and velocity. The steer wheel position and velocity are given as an input to the torsion bar, rack and pinion dynamics of the vehicle to obtain the torsion bar and rack and pinion position and velocity. PD control is used to track this desired rack position.

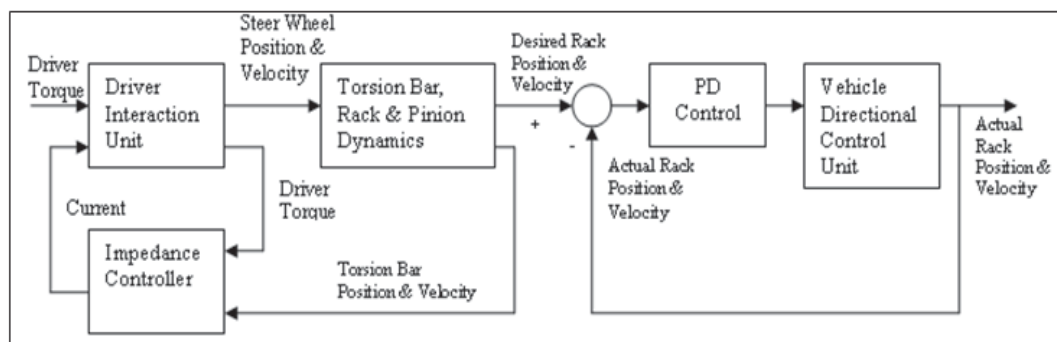


Figure 2.2. Steering system architecture

Steering system has two main units which are driver interaction unit and vehicle directional control unit. In this section, the dynamics of the driver interaction unit and vehicle directional control unit details are presented. The torsion bar and rack and pinion dynamics is taken from the paper [1]. The impedance controller and PD control are given in the next section.

2.3. STEERING SYSTEM DYNAMICS

2.3.1. Driver Interaction Unit

The steer-by-wire driver interaction unit is illustrated in Figure 2.3.

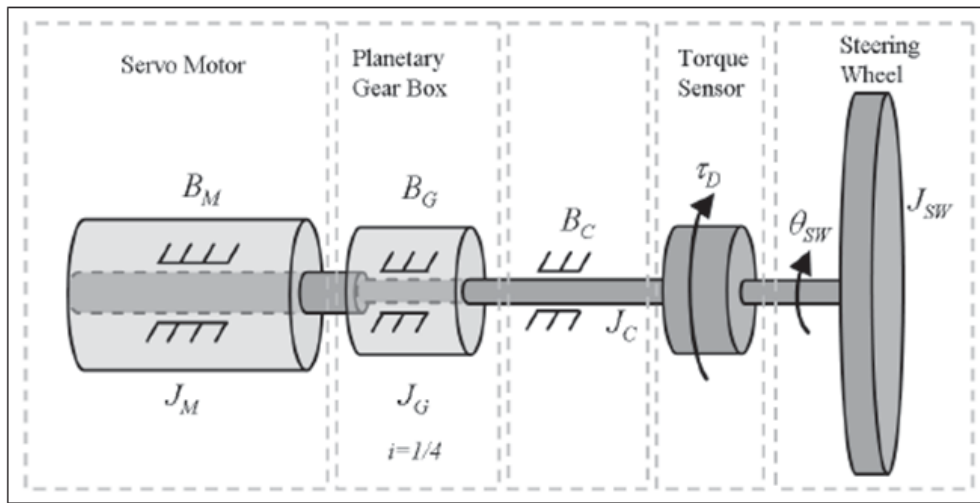


Figure 2.3. Driver interaction unit [2]

A torque by the driver is applied to the driver interaction unit. The steering wheel starts to rotate with an angle and the driver interaction motor generates current. Therefore, the dynamic of the driver interaction unit can be given as

$$J_{SW} \ddot{\theta}_{SW} + B_{SW} \dot{\theta}_{SW} + F_{SW} \operatorname{sgn} \dot{\theta}_{SW} = i_G k_A i_M + \tau_D \quad (2.1)$$

where J_{SW} , $\ddot{\theta}_{SW}$, B_{SW} , $\dot{\theta}_{SW}$, F_{SW} , i_G , k_A , i_M , and τ_D are steering column inertia, steering column angular acceleration, steering wheel viscous damping, steering column angular velocity, static friction, driver interaction motor gear box ratio, torque constant of the driver interaction motor, current drawn by the motor, and driver torque, respectively. Then, equation 2.1 is organized to obtain the steering-column angular acceleration.

$$J_{SW}\ddot{\theta}_{SW} = i_G k_A i_M + \tau_D - B_{SW}\dot{\theta}_{SW} - F_{SW} \operatorname{sgn} \dot{\theta}_{SW} \quad (2.2)$$

$$\ddot{\theta}_{SW} = \frac{i_G k_A i_M}{J_{SW}} + \frac{\tau_D}{J_{SW}} - \frac{B_{SW}\dot{\theta}_{SW}}{J_{SW}} - \frac{F_{SW} \operatorname{sgn} \dot{\theta}_{SW}}{J_{SW}} \quad (2.3)$$

$$\ddot{\theta}_{SW} = b_{SW}i_M + c_{SW}\tau_D - a_{SW}\dot{\theta}_{SW} - f_{SW} \operatorname{sgn} \dot{\theta}_{SW} \quad (2.4)$$

where $b_{SW} = \frac{i_G k_A}{J_{SW}}$, $c_{SW} = \frac{1}{J_{SW}}$, $a_{SW} = \frac{B_{SW}}{J_{SW}}$, $f_{SW} = \frac{F_{SW}}{J_{SW}}$. a_{SW} is steering-wheel viscous damping, f_{SW} is steering-wheel static friction, b_{SW} and c_{SW} are input gains.

The driver interaction unit state equations can be written as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -a_{SW}\dot{\theta}_{SW} - f_{SW} \operatorname{sgn} x_2 + b_{SW}i_M + c_{SW}\tau_D \end{aligned} \quad (2.5)$$

where the state variable $x_1 \in \mathfrak{R}^1$ is the angular position (θ_{SW}) of the steering wheel, x_2 is the angular velocity ($\dot{\theta}_{SW}$) of the steering wheel. Neglecting the static friction, the matrix form of the equation is

$$\dot{x}_{SW} = \begin{bmatrix} 0 & 1 \\ 0 & -a_{SW} \end{bmatrix} x_{SW} + \begin{bmatrix} 0 & 0 \\ b_{SW} & c_{SW} \end{bmatrix} \begin{bmatrix} i_M \\ \tau_D \end{bmatrix} \quad (2.6)$$

where, $x_{SW} \in \mathfrak{R}^2$ is the state vector, that is $x_{SW} = [x_1 \quad x_2]^T$.

2.3.2. Vehicle Directional Control Unit

The driver interaction motor generates the motor angular position, the motor angular velocity, and the current drawn by the motor. Thus, all the state variables, x_1 and x_2 , and

the input of the system i_M are available for measurement. The torque sensor installed between the steering wheel and the column is feeding back the driver torque τ_D as well.

The steering column transfers the angular input of the steering wheel to the pinion. Then, the angular motion of the pinion is transferred to a lateral motion of the rack. Considering the rack and pinion steering gear, the lateral translation of the rack is transmitted to the left and right steering arms through the tie rods. As a result of the lateral translation of the rack, a steering moment is generated turning the wheels around the steering axes. Steer-by-wire vehicle direction control unit is illustrated in Figure 2.4.

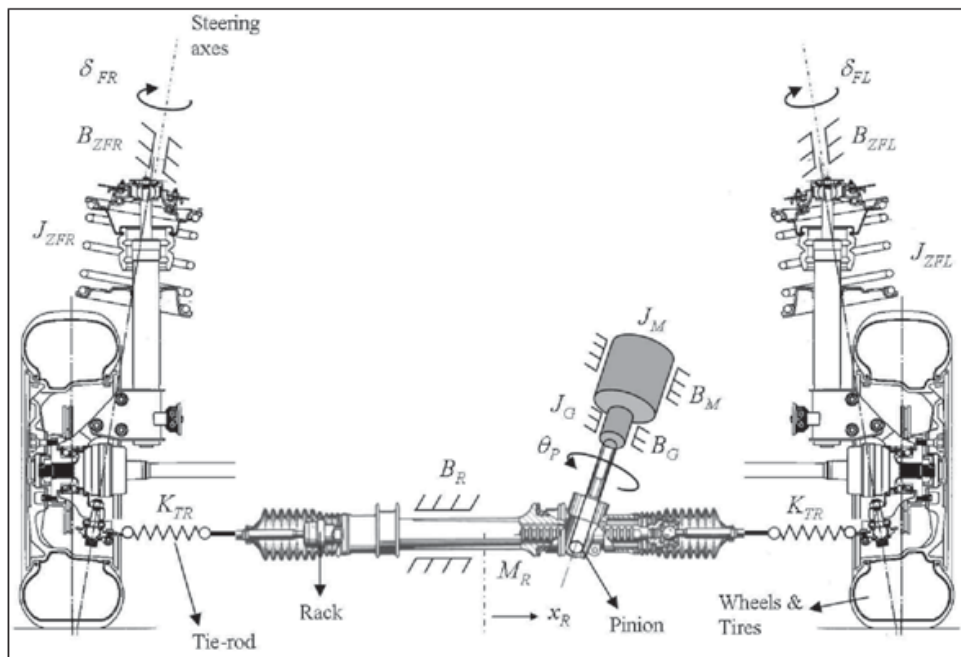


Figure 2.4. Vehicle directional control unit [2]

The dynamics of the vehicle directional control unit is given as

$$M_R \ddot{x}_R + B_R \dot{x}_R + F_R \operatorname{sgn} \dot{x}_R + 2K_{TR} x_R = i_G k_A i_M / r_P \quad (2.7)$$

where M_R , \ddot{x}_R , B_R , \dot{x}_R , F_R , K_{TR} , x_R , i_G , k_A , i_M , and r_P are rack mass including driver interaction motor and gear box inertia, rack acceleration, rack viscous friction including the

driver interaction motor and gear box viscous frictions, rack velocity, static friction, spring rate connecting the left and right rack ends to the fixture base, rack position, driver interaction motor gear box ratio, torque constant of the driver interaction motor, current drawn by the motor, and pinion radius, respectively. Then, equation 2.7 is organized to obtain the rack acceleration.

$$M_R \ddot{x}_R = i_G k_A i_M / r_P - B_R \dot{x}_R - F_R \operatorname{sgn} \dot{x}_R - 2K_{TR} x_R \quad (2.8)$$

$$\ddot{x}_R = \frac{i_G k_A i_M}{M_R r_P} - \frac{B_R}{M_R} \dot{x}_R - \frac{F_R}{M_R} \operatorname{sgn} \dot{x}_R - \frac{2K_{TR}}{M_R} x_R \quad (2.9)$$

$$\ddot{x}_R = b_R i_M - a_R \dot{x}_R - f_R \operatorname{sgn} \dot{x}_R - 2k_{TR} x_R \quad (2.10)$$

where $b_R = \frac{i_G k_A}{r_P M_R}$, $a_R = \frac{B_R}{M_R}$, $f_R = \frac{F_R}{M_R}$, $k_{TR} = \frac{K_{TR}}{M_R}$. a_R is viscous friction, f_R is static friction, b_R is input gain and k_{TR} is spring rate.

The state equation of the vehicle directional control unit is given as

$$\begin{aligned} \dot{x}_3 &= x_4 \\ \dot{x}_4 &= -a_R x_4 - f_R \operatorname{sgn} x_4 - 2k_{TR} x_3 + b_R i_M \end{aligned} \quad (2.11)$$

where the state variable $x_3 \in \mathfrak{R}^1$ is the angular position x_R of the rack, x_4 is the velocity \dot{x}_R of the steering wheel. Neglecting the static friction, the matrix form of the equation is

$$\begin{bmatrix} \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2k_{TR} & -a_R \end{bmatrix} \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ b_R \end{bmatrix} [i_M] \quad (2.12)$$

where, $x = [x_3 \quad x_4]^T$ is the state vector.

The driver interaction motor generates the motor angular position, the motor angular velocity, and the current drawn by the motor. Thus, all the state variables, x_3 and x_4 , and the input of the system i_M are available for measurement.

3. CONTROLLERS

3.1. CONTROL OF DRIVER INTERACTION UNIT

Impedance control is used to force the steer-by-wire system to behave as a hydraulic steering system because of its robustness [1]. Additionally, impedance control is used to tune the dynamic behavior of the steering system for passenger safety and driver comfort.

If we select the input i_M in equation 2.5 as,

$$i_M = \frac{a_{SW}x_2 + f_{SW} \operatorname{sgn} x_2}{b_{SW}} - \frac{B_C}{J_C b_{SW}}(x_2 - \dot{\theta}_{TB}) - \frac{K_C}{J_C b_{SW}}(x_1 - \theta_{TB}) + \frac{\tau_D}{b_{SW}} \left(\frac{1}{J_{SW}} - c_{SW} \right) \quad (3.1)$$

and substitute into the equation 2.5, the following closed loop system equation is obtained

$$J_C \dot{x}_2 + B_C(x_2 - \dot{\theta}_{TB}) + K_C(x_1 - \theta_{TB}) = \tau_D \quad (3.2)$$

and substituting $\dot{x}_2 = \ddot{\theta}_{SW}$, $x_2 = \dot{\theta}_{SW}$, and $x_1 = \theta_{SW}$ into equation 3.2

$$J_C \ddot{\theta}_{SW} + B_C(\dot{\theta}_{SW} - \dot{\theta}_{TB}) + K_C(\theta_{SW} - \theta_{TB}) = \tau_D \quad (3.3)$$

which is equivalent to hydraulic steering system. The other equations of the hydraulic steering system related to torsion bar and rack and pinion are given respectively as,

$$J_{TB} \ddot{\theta}_{TB} + B_{TB}(\dot{\theta}_{TB} - \dot{x}_R / r_P) + B_C(\dot{\theta}_{TB} - \dot{\theta}_C) + K_{TB}(\theta_{TB} - x_R / r_P) + K_C(\theta_{TB} - \theta_C) = 0 \quad (3.4)$$

$$M_R \ddot{x}_R + B_R \dot{x}_R + B_H \dot{x}_R + B_{TB} r_P (\dot{x}_R / r_P - \dot{\theta}_{TB}) + K_{TB} r_P (x_R / r_P - \theta_{TB}) + K_{TR} (x_R - l_{SA} \sin \delta_{FL}) + K_{TR} (x_R - l_{SA} \sin \delta_{FR}) = F_H \quad (3.5)$$

where J_{TB} , $\ddot{\theta}_{TB}$, B_{TB} , $\dot{\theta}_{TB}$, \dot{x}_R , r_p , B_C , $\dot{\theta}_C$, K_{TB} , θ_{TB} , x_R , K_C , θ_C , M_R , \ddot{x}_R , B_R , B_H , K_{TR} , l_{SA} , δ_{FL} , δ_{FR} , and F_H are torsion bar inertia, torsion bar angular acceleration, torsion bar viscous damping, torsion bar angular velocity, rack velocity, pinion radius, steering column viscous damping, steering column angular acceleration of the hydraulic steering system, torsion bar stiffness, torsion bar angular position, rack position, steering column stiffness, column angular position of the hydraulic steering system, rack mass including driver interaction motor and gear box inertia, rack acceleration, rack viscous friction including the driver interaction motor and gear box viscous frictions, hydraulic cylinder viscous damping, spring rate connecting to the left and right rack ends to the fixture base, moment arm length, left wheel angle, right wheel angle, and hydraulic assist force, respectively.

As a result, we obtain a hardware in the loop system behaving similar with the hydraulic steering system. This is referred as impedance control in literature which is proposed by Hogan [4].

3.2. CONTROL OF VEHICLE DIRECTIONAL UNIT

A Proportional-Derivative PD controller is used to track the position of the hydraulic rack and pinion [2]. The control law input i_M in equation 2.11 is selected as,

$$i_M = K_V(\dot{x}_R - x_4) + K_P(x_R - x_3) \quad (3.6)$$

where K_V and K_P are the derivative and proportional gains.

4. TESTS AND CALCULATIONS

In this thesis, the purpose is to track the desired rack position in an accurate manner when the parameters of the driver interaction and the vehicle directional units are varied. The details of driver interaction and vehicle directional control units have been previously given in Section 2. In this section various tests are performed to evaluate the tracking accuracy using MATLAB/Simulink.

As it has been mentioned in Section 2, a driver torque is applied to the driver interaction unit to obtain the desired rack position. The driver torque is selected as a sinusoidal signal using following equation.

$$\tau_D = A \sin wt \quad (4.1)$$

The amplitude of the driver torque A is selected according to the rotation of steer wheel. Thus, test was performed with a car Mercedes-Benz B150 in order to decide driver torque value. Experimental setup is shown in Figure 4.1. Initially, value of tour of steer wheel and the degree of rotation were found. When tires of Mercedes-Benz B150 were in straight position, the steer wheel rotated approximately maximum 1.3 tours from left to right position, or vice versa. 1.3 tours correspond to 468 degree. If the engine of Mercedes-Benz B150 was running at an idle and steer wheel was rotated by the driver, then the value of A took any value between 0 and 7.5 Nm written in Table 4.1.



Figure 4.1. Experimental setup

Table 4.1. Torque values with respect to rotation degree

	Torque (Nm)	Rotation Degree
1	1	62.5
2	2	125
3	3	187
4	4	250
5	5	312
6	6	374
7	7	437
8	7.5	468

In this thesis, we selected the amplitude of the driver torque as 3 and the frequency was 1 rad/s. Thus, the driver torque was selected as $3\sin t$ Nm shown in Figure 4.2 to drive the interaction system.

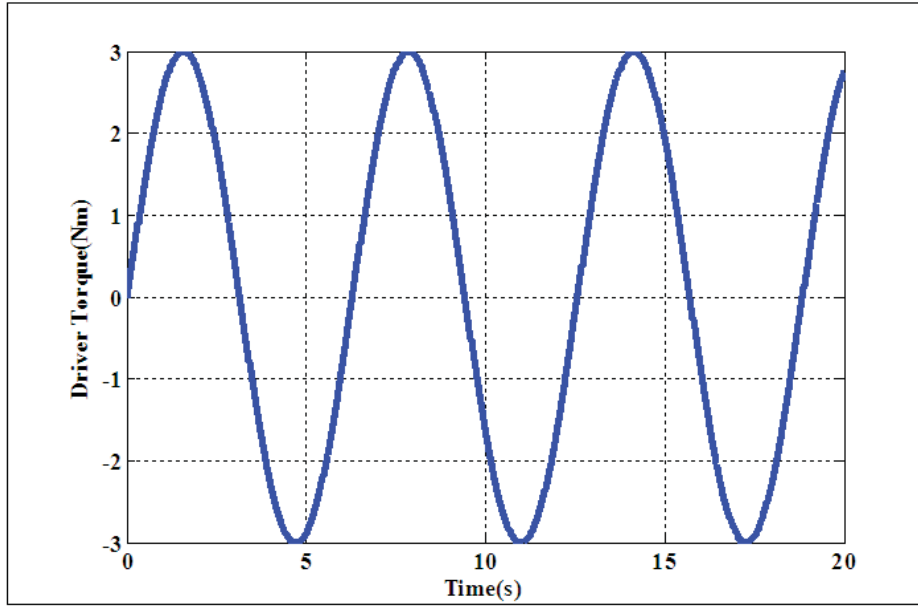


Figure 4.2. Driver torque

When the driver torque was selected as $3 \sin t$ Nm, then the steer wheel angular position was required to be approximately 187 degree or approximately 3.26 radians. The rack position was calculated as approximately 0.0248 m using the following equation

$$x_R = \theta_{SW} * r_{R-SW} \quad (4.2)$$

where θ_{SW} was steer wheel angular position with a value of 3.26 rad. r_{R-SW} was rack and steer wheel ratio with a value of $47.7966 * 10^{-3} / (2 * 3.1416)$ [2].

When the driver torque was determined, then the best a_{SW} , b_{SW} , c_{SW} , and f_{SW} parameters and J_C were determined to obtain the desired rack position 0.0248 m. It was noticed that the parameters a_{SW} , c_{SW} , and f_{SW} did not affect the rack position. These parameters only influenced the current of driver interaction unit motor. In this thesis, we did not use real time system, thus we did not need to track the current. Thus, we only varied b_{SW} and J_C parameters to obtain the desired rack position.

Initially, the values b_{sw} effects were evaluated by keeping $J_C = 0.16 \text{ kgm}^2$ shown in the Table 4.2. J_C value had been taken as a reference from the paper [2]. The steer wheel input gain b_{sw} is $i_G k_A / J_{sw}$, which depends on driver interaction motor gear box ratio (i_G), torque constant of driver interaction motor (k_A), and steering column inertia (J_{sw}).

Table 4.2. The values of b_{sw} when $J_C = 0.1 \text{ kgm}^2$

Test	$b_{sw} (Nm / A)$
1	1
2	10
3	20
4	30
5	50
6	75
7	100
8	180
9	300
10	500
11	1000
12	1500
13	2000

The initial value of b_{sw} was selected as 180 [1] and it was observed that the desired position value reached to 0.0001m, which was not close to the desired value 0.0248m show in Figure 4.3.

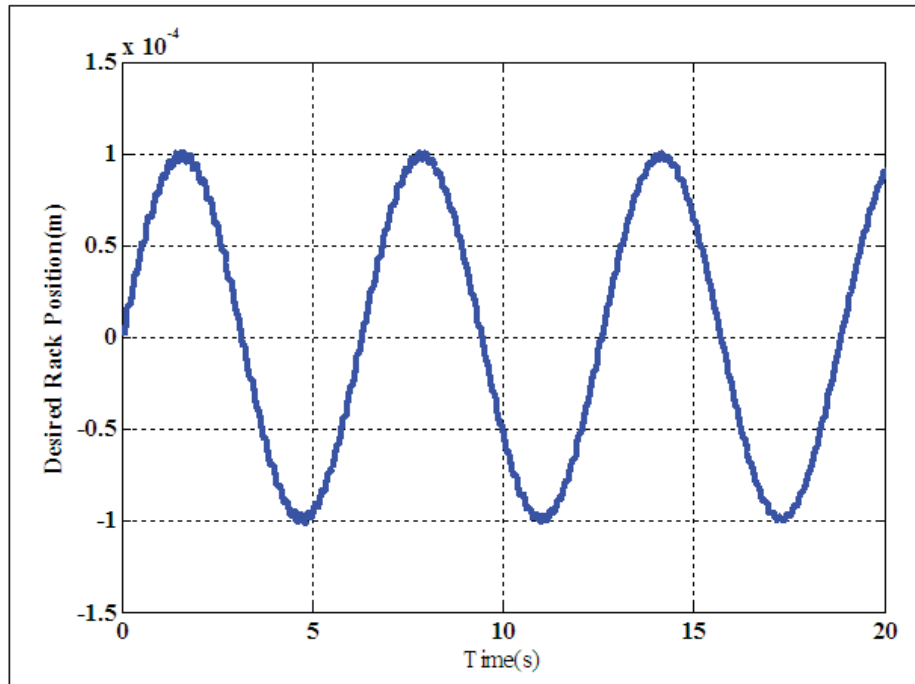


Figure 4.3. Desired rack position when $b_{sw} = 180$ and $J_C = 0.16$

Then, the value of b_{sw} decreased until the value of 10 shown in Test 2. However, the oscillation at the desired rack position was noticed when b_{sw} decreased. This oscillation was not desired because it meant that the vehicle rotated itself to left or to right. Therefore, we decided to increase the value of b_{sw} till 2000 shown in Test 13. If the value of b_{sw} was increased, it was observed that the rack position moved away from the desired value, while decreasing the oscillation. When $b_{sw} = 500$ shown in Test 10 was selected, then the desired rack position went approximately to 0.000035m like in Figure 4.4, which was not close to the desired value 0.0248m, however oscillation increased. If b_{sw} was selected 500, the oscillation was approximately 0.0009m. We noted that when $b_{sw} = 500$ was selected, oscillation did not significantly influence the desired rack position, thus b_{sw} was kept at 500 shown in Figure 4.4. As a result, when the value of b_{sw} was increased, the rack position did not go to the desired value and if the value of b_{sw} was decreased, the oscillation at the desired rack position was observed. Thus, we selected b_{sw} as 500 because

the oscillation was decreased. According to these results, it was said that b_{sw} affects the vibration at the steer wheel and the ability of rotation of steer wheel.

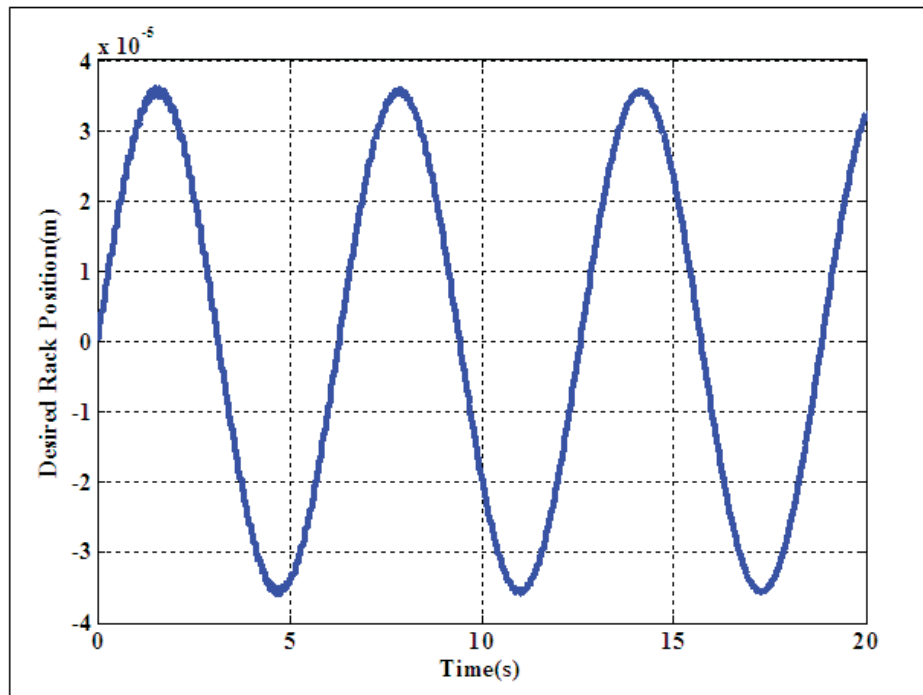


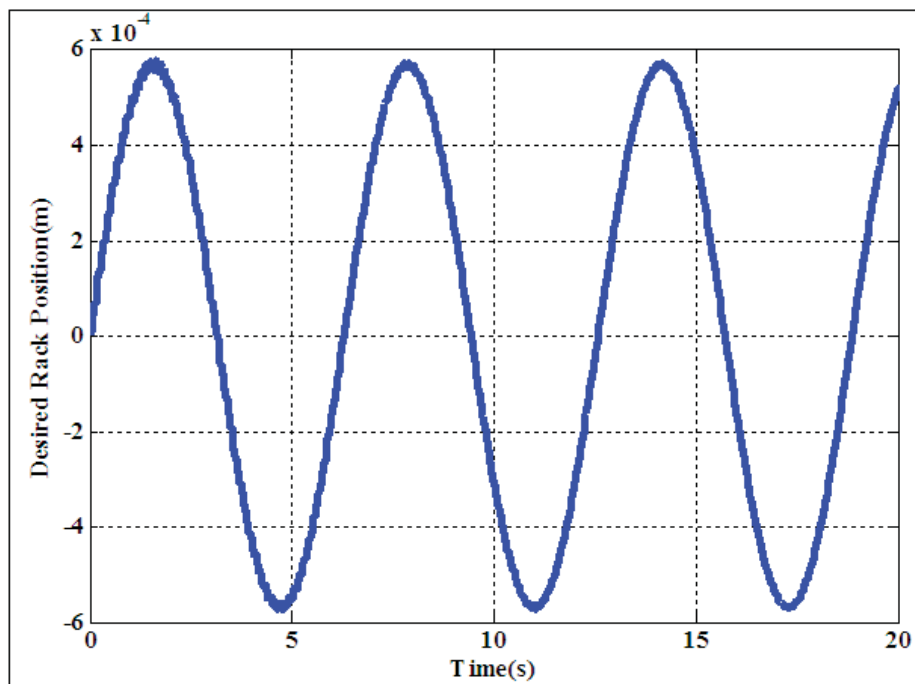
Figure 4.4. Desired rack position when $b_{sw} = 500$ and $J_C = 0.16$

It could be noticed that when b_{sw} was selected as 500, we could not reach the desired position 0.0248m. Thus, J_C was varied to obtain the desired rack position close to 0.0248m shown in Table 4.3. J_C is steering column inertia in a hydraulic steering system, which is an important parameter in the driver interaction unit written in Equation 3.3.

Table 4.3. The values of J_C when $b_{sw} = 500$

Test	$J_C (kgm^2)$
14	0.16
15	0.01
16	0.001
17	0.0001
18	0.0002
19	0.0003
20	0.00025

If $J_C = 0.01$, then the desired rack position had reached to 0.00058m shown in Figure 4.5, which was not actually close to 0.0248 m.

Figure 4.5. Desired rack position when $J_C = 0.01$ and $b_{sw} = 500$

Thus, we decreased J_C and selected 0.00025, the desired steer wheel angular position was obtained approximately 3.26 rad and the desired rack position reached to 0.023 m, which was close to 0.0248m with 4% error shown in the Figure 4.6. It was seen that J_C affects the ability of steer wheel rotation.

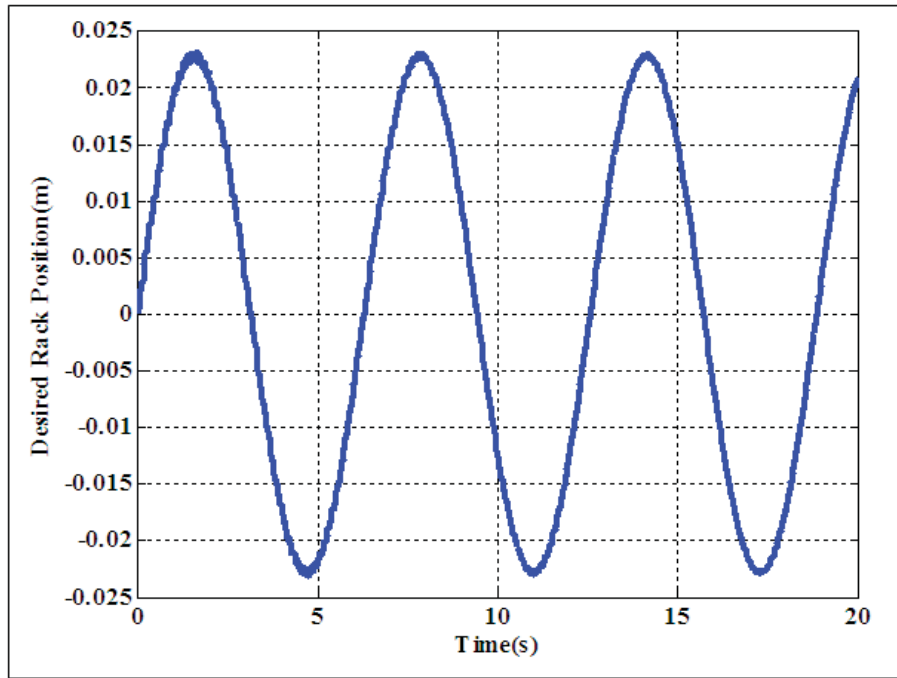


Figure 4.6. Desired rack position when $J_C = 0.00025$ and $b_{SW} = 500$

As a result in this thesis, if $J_C = 0.00025$ and $b_{SW} = 500$ were selected for the parameters of the driver interaction unit, then the desired rack position value was obtained as close to 0.0248m. The other parameters of a_{SW} , c_{SW} , and f_{SW} were selected 2, 40, and 1, respectively [1].

In the second experiment, the best parameters of the vehicle directional control unit (a_R , b_R , and f_R) were found to track the desired rack position 0.0248m that was obtained above. Proportional-Derivative (PD) control of the vehicle directional control unit was used to track the desired rack position (details were given in Section 2.3.2). Initially, PD controllers gains K_V and K_P were kept at 10 and 1500, respectively [1]. However, if K_V

and K_p were selected as 10 and 1500 respectively, then the actual rack position exploded. Thus, the gains were decreased to $K_v = 10$ and $K_p = 150$. These values were found by trial and error.

The rack viscous damping a_R is B_R / M_R , which depends on rack viscous friction including the driver interaction motor and gear box viscous frictions and rack mass including the driver interaction motor and gear box inertia. Initially, a_R was selected as 7.5, when $b_R = 1$, $f_R = 2$, $K_v = 10$ and $K_p = 150$. The error between the desired and actual rack positions was calculated with different a_R values written in Table 4.4. The maximum, minimum, mean, and standard deviation of the error were found written in Table 4.5.

Table 4.4. The values of a_R when $b_R = 1$, $f_R = 2$, $K_v = 10$, and $K_p = 150$

Test	a_R
1	7.5
2	5
3	10
4	15
5	20
6	30
7	50
8	100
9	200
10	500
11	1000
12	2000

When $a_R = 7.5$ was selected shown in Test 1 at Table 4.4, the actual rack position increased to 0.055m shown in Figure 4.7. The maximum error between the actual rack position and the desired rack position was 0.0330m shown in Test 1 in Table 4.5.

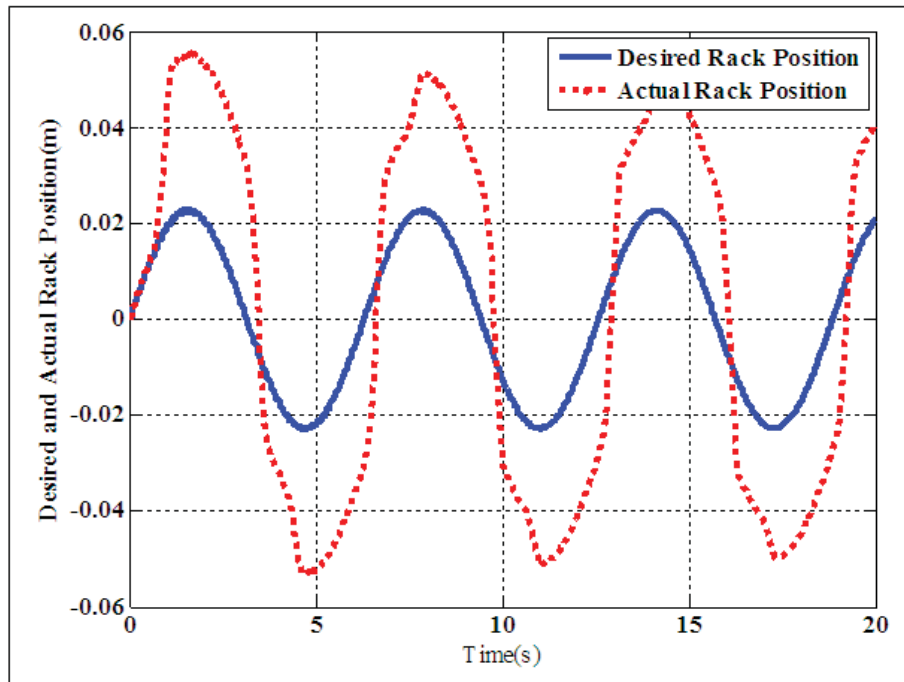


Figure 4.7. Desired and actual rack position when $a_R = 7.5$, $b_R = 1$, $f_R = 2$, $K_V = 10$, and $K_p = 150$

If a_R decreased, the actual rack position was obtained as 0.065m. Thus, error between the actual rack position and the desired rack position was increased which was not desirable shown in Test 2 at Table 4.5. When the value of a_R was increased, then the error between the actual rack position and desired position started to decrease written in Table 4.5. It was noticed that when the values of a_R was increased to 1000 shown in Test 11 at Table 3.4, the actual rack position was tracked in 0.0022m error shown in Test 11 at Table 4.5 in Figure 4.8.

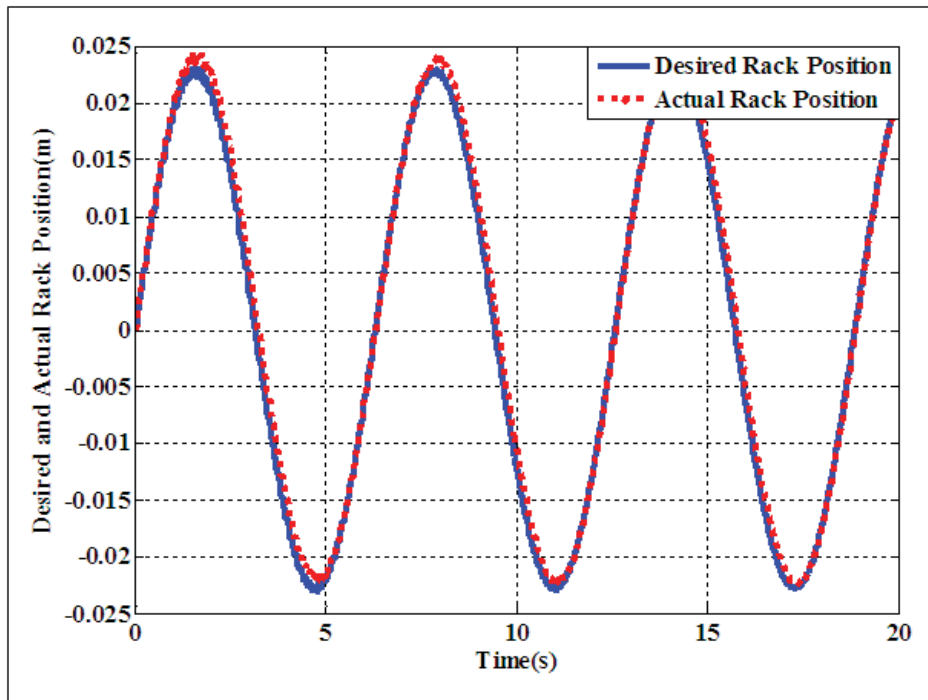


Figure 4.8. Desired and actual rack position when $a_R = 1000$, $b_R = 1$, $f_R = 2$, $K_V = 10$, and $K_P = 150$

It was observed in these tests that if a_R was increased, then the vehicle moved close to the desired value. Thereby, a_R affects the ability of the movement of rack. If a_R was greater than 1000, then the error increased shown in Test 11 at Table 4.5. However, a_R can not be 1000 in real applications. Thus, a_R was kept at 7.5 like in the paper [1].

Table 4.5. The error analysis of a_R when $b_R = 1$, $f_R = 2$, $K_V = 10$, and $K_p = 150$

Test	Maximum of Error	Minimum of Error	Mean of Error	Standard Deviation
1	0.0330	0	0.0234	0.0084
2	0.0446	0	0.0296	0.0103
3	0.0288	0	0.0218	0.0088
4	0.0247	0	0.0192	0.0076
5	0.0240	0	0.0184	0.0075
6	0.0219	0	0.0164	0.0070
7	0.0180	0	0.0125	0.0055
8	0.0124	0	0.0072	0.0033
9	0.0077	0	0.0038	0.0020
10	0.0038	0	0.0016	0.0011
11	0.0022	0	9.0701×10^{-4}	7.0262×10^{-4}
12	0.0027	0	0.0011	8.6977×10^{-4}

Later, the parameter b_R of vehicle directional control unit was changed. The rack input gain b_R is $i_G k_A / M_R$, which depends on driver interaction motor gear box ratio (i_G), torque constant of driver interaction motor (k_A), and rack mass including the driver interaction motor and gear box inertia (M_R). b_R was changed when a_R was selected as 7.5 shown in Table 4.6. The error between the desired and actual rack position were calculated with different b_R values in Table 4.6. The maximum, minimum, mean, and standard deviation of the error was found shown in Table 4.7.

Table 4.6. The values of b_R when $a_R = 7.5$,
 $f_R = 2$, $K_V = 10$, and $K_P = 150$

Test	$b_R(N / A)$
14	2
15	1

If b_R was selected as 2 written in Test 14 at Table 4.6, then the actual rack position exploded and the error between the actual and desired rack position was 2.8583×10^{147} m at Test 14 in Table 4.7. It was also observed that the error between desired and actual position continuously increased shown in Table 4.7, which was not desirable. If b_R was increased, the actual rack position moved away from the desired rack position. If b_R was decreased to 1, then the maximum error between the actual and desired rack position was 0.0330m shown in Test 15 at Table 4.7. The actual and desired rack positions were shown in Figure 4.7. However, b_R can not be less than 1 in real applications. Thus, b_R was kept at 1. Also, b_R affects the ability of the movement of rack.

Table 4.7. The error analysis of b_R when $a_R = 7.5$, $f_R = 2$, $K_V = 10$, and $K_P = 150$

Test	Maximum of Error	Minimum of Error	Mean of Error	Standard Deviation
14	2.8583×10^{147}	0	8.2275×10^{144}	1.0860×10^{146}
15	0.0330	0	0.0234	0.0084

The last parameter f_R is rack static friction. The error between the desired and actual rack position were calculated with different f_R values shown in Table 4.8. The maximum, minimum, mean, and standard deviation of the error were found shown in Table 4.9. Initially, f_R was selected as 2 written in Table 4.8 [1] and the actual and desired rack position were obtained same as in Figure 4.8.

Table 4.8. The values of f_R when $b_R = 1$,
 $a_R = 7.5$, $K_V = 10$, and $K_P = 150$

Test	f_R
16	2
17	1
18	3
19	4
20	5

If f_R was increased to 5, the maximum error between the actual and desired rack position was 0.0043m. This value of the maximum error was higher than the value of the maximum error at $f_R = 4$ written in Test 19 at Table 4.9), which was 0.0039m. The actual and desired rack positions were shown in Figure 4.9. The desired position was tracked with 15% error when $f_R = 4$. It was seen that when f_R was selected as 4, the error was the smallest one shown in Table 4.9, thus f_R was kept at 4. Also, f_R affects the ability of the movement of rack.

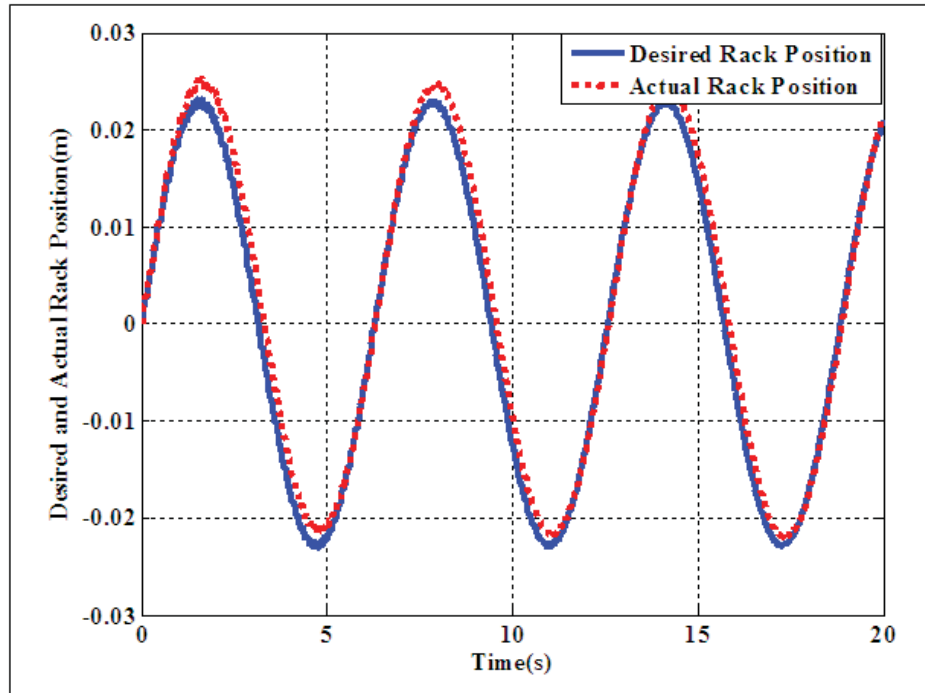


Figure 4.9. Desired and actual rack position when $f_R = 4$, $b_R = 1$, $a_R = 7.5$, $K_V = 10$, and $K_p = 150$

Table 4.9. The error analysis of f_R when $b_R = 1$, $a_R = 7.5$, $K_V = 10$, and $K_p = 150$

Nr.	Maximum of Error	Minimum of Error	Mean of Error	Standard Deviation
16	0.0330	0	0.0234	0.0084
17	0.0455	0	0.0319	0.0171
18	0.0178	0	0.0142	0.0048
19	0.0039	0	0.0016	0.0012
20	0.0043	0	0.0018	0.0013

When the parameters of driver interaction unit and vehicle directional control unit were determined, it was noticed that the PD controller gains could be adjusted to track the desired rack position for the selected parameters. The error between the desired and actual rack position were calculated with different PD control gains written in Table 4.10. The

maximum, minimum, mean, and standard deviation of the error were found written in Table 4.11.

Table 4.10. The values of K_p and K_v when $b_R = 1$, $a_R = 7.5$, and $f_R = 4$

Test	K_p	K_v
21	150	10
22	100	10
23	50	10
24	50	1
25	10	1
26	1	1

If K_p and K_v were selected as 100 and 10 respectively, the maximum error was 0.0030m shown at Figure 4.10. The actual and desired rack positions were shown in Figure 4.10.

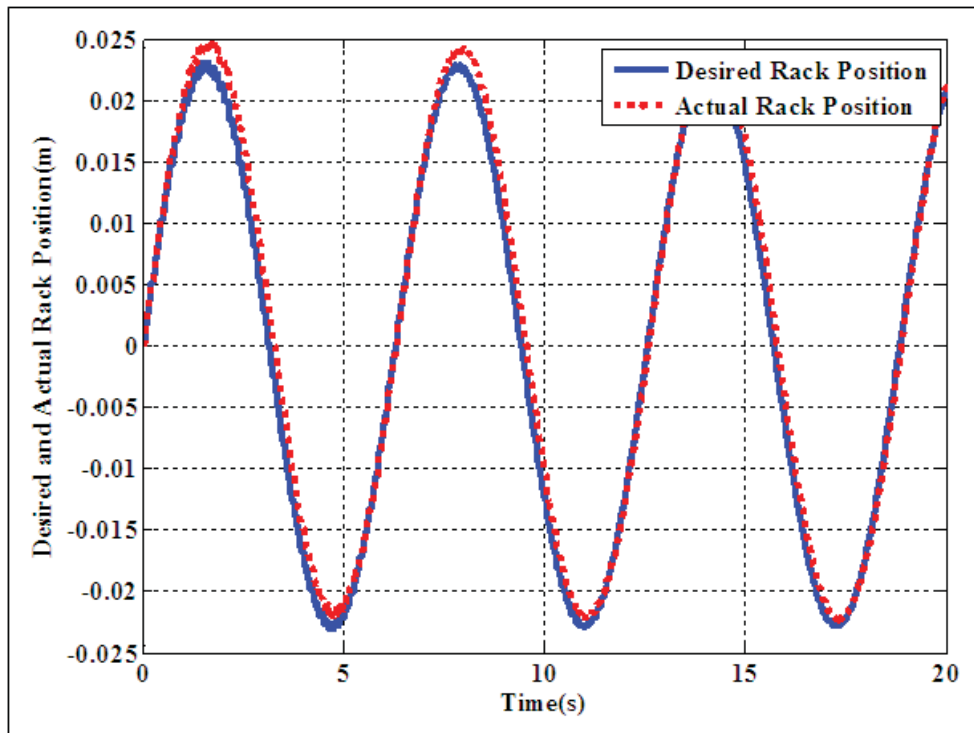


Figure 4.10. Desired and actual rack position when $b_R = 1$, $a_R = 7.5$, $f_R = 4$, $K_p = 100$,
and $K_v = 10$

It was observed that if K_p and K_v were selected as 1, the error was decreased to 2.1284×10^{-4} m shown in Test 26 at Table 4.10. The actual and desired rack positions were shown in Figure 4.11.

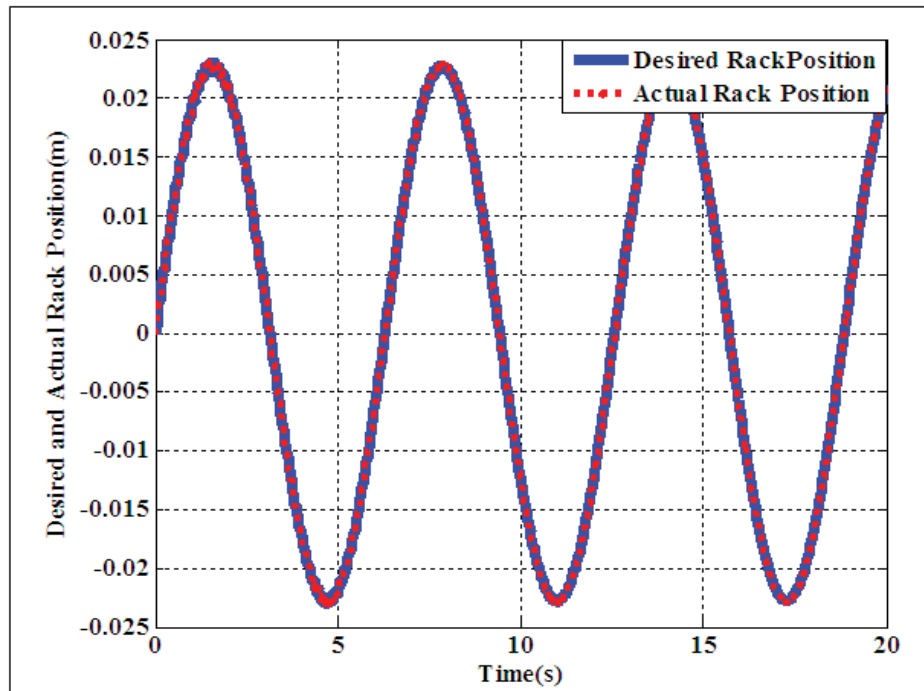


Figure 4.11. Desired and actual rack position when $b_R = 1$, $a_R = 7.5$, $f_R = 4$, $K_p = 1$, and $K_v = 1$

Table 4.11. The error analysis of PD controller gains K_p and K_v when $b_R = 1$, $a_R = 7.5$, and $f_R = 4$

Test	Maximum of Error	Minimum of Error	Mean of Error	Standard Deviation
21	0.0039	0	0.0016	0.0012
22	0.0030	0	0.0012	9.0038×10^{-4}
23	0.0015	0	5.0726×10^{-4}	3.7533×10^{-4}
24	0.0015	0	6.3325×10^{-4}	4.2302×10^{-4}
25	5.9162×10^{-4}	0	2.7978×10^{-4}	1.4980×10^{-4}
26	2.1284×10^{-4}	0	9.0852×10^{-4}	5.2820×10^{-5}

5. DISCUSSION AND CONCLUSION

Nowadays, the electrical systems such as steer-by-wire, which is a term referring to the replacement of mechanical or hydraulic systems by electronic ones, are being developed in automotive industry. Steer-by-wire is a clear trend of automotive development due to the advantages of the electronic components for enhancing safety, functionality and reducing cost.

In this thesis, an impedance control for the driver interaction unit of a steer-by-wire system has been used to obtain desired rack position and a PD controller has been used for the vehicle directional control unit of the same steer-by-wire system to track the desired rack position in an accurate manner. The parameters of the driver interaction and vehicle directional control units have been selected to follow the desired rack position in an accurate manner.

It was noticed that the parameters of driver interaction unit a_{sw} , c_{sw} , and f_{sw} did not affect the rack position. These parameters only influenced the current of driver interaction unit motor. In this thesis, we did not use real time system, thus we did not need to track the current. Physically, b_{sw} affects the vibration at the steer wheel and the ability of rotation of steer wheel. When the steer wheel input gain b_{sw} decreased, the oscillation at the desired rack position was noticed. If the value of b_{sw} was increased, the rack position was not close to the desired value. Therefore, steering column inertia in a hydraulic steering system J_c was varied to obtain the desired rack position when b_{sw} was kept at 500. J_c was chosen as 0.00025. J_c affects the ability of steer wheel rotation. If b_{sw} and J_c were selected as 500 and 0.00025 respectively, the desired rack position was 0.023m, which was close to 0.0248m with 4% error. After determining of the parameters of driver interaction unit, the parameters of vehicle directional control unit were analyzed. Physically, a_r , b_r , and f_r affect the ability of the movement of rack. It was observed that if a_r was increased, the vehicle moved close to the desired value. When a_r was selected as 1000, the actual rack position was close to the desired rack position. However, rack

viscous damping a_r cannot have a high value like 1000 in real time applications. Thus, a_r was kept at 7.5 like in the paper [1]. If the rack input gain b_r was increased to 2, the actual rack position moved away from the desired rack position. However, b_r can not be less than 1 in real applications. Thus, b_r was kept at 1. f_r was analyzed between 1 and 5. When f_r was selected as 4, the actual rack position was close to the desired rack position with 15% error. When the parameters of driver interaction unit and vehicle directional control unit were determined, it was noticed that the PD controller gains could be adjusted to track the desired rack position for the selected parameters. It was noticed that if proportional gain K_p and derivative gain K_v were selected as 1, the error was decreased to $2.1284 \cdot 10^{-4}$ m. As a result, the best parameters of driver interaction unit and vehicle directional control unit were selected based on the least error.

In this thesis, it is shown that the desired rack position can be followed in an accurate manner when the driver applies a torque. As a future work, the tracking performance of the steer-by-wire system is required to be evaluated when a disturbance such as the effect of wind, the effect of slippery road is applied to the system. Also, the velocity of the vehicle can be considered to track the performance.

APPENDIX A: MATLAB/SIMULINK BLOCKS

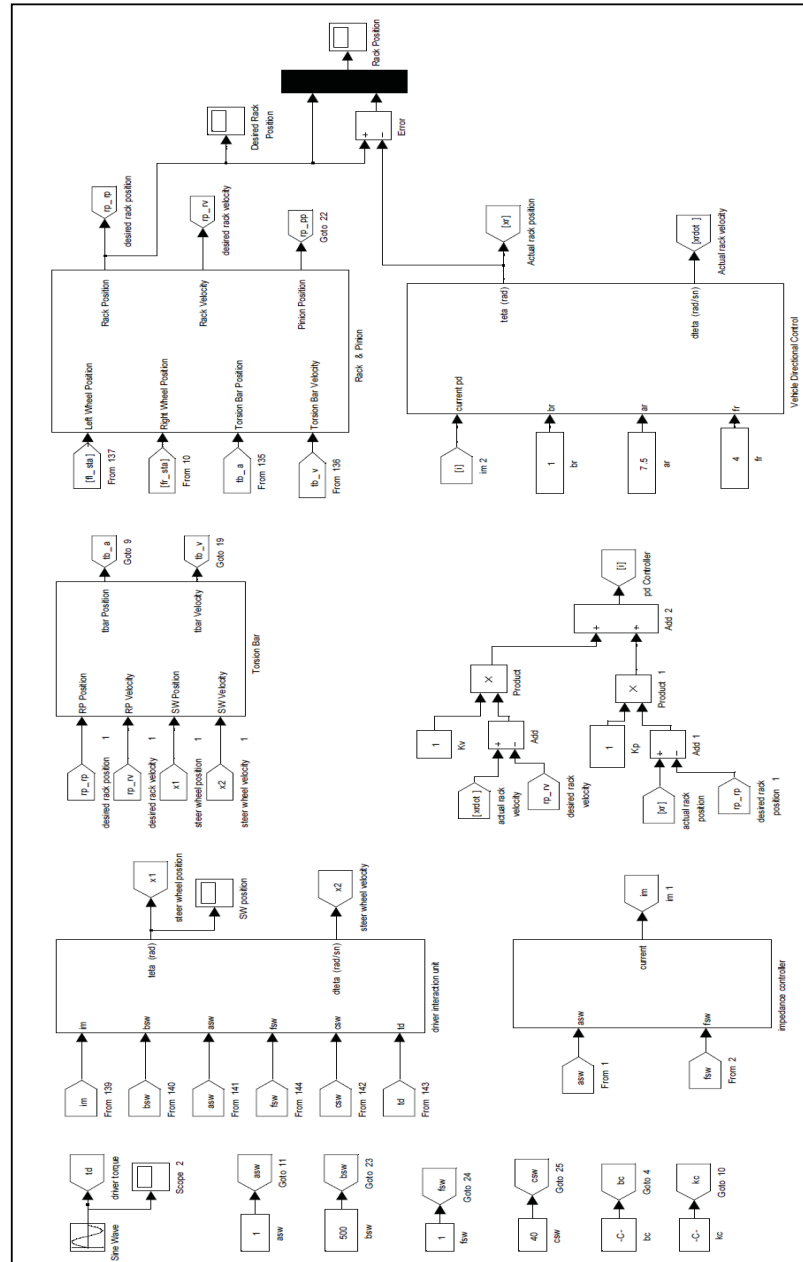


Figure A.1. Blocks of system architecture

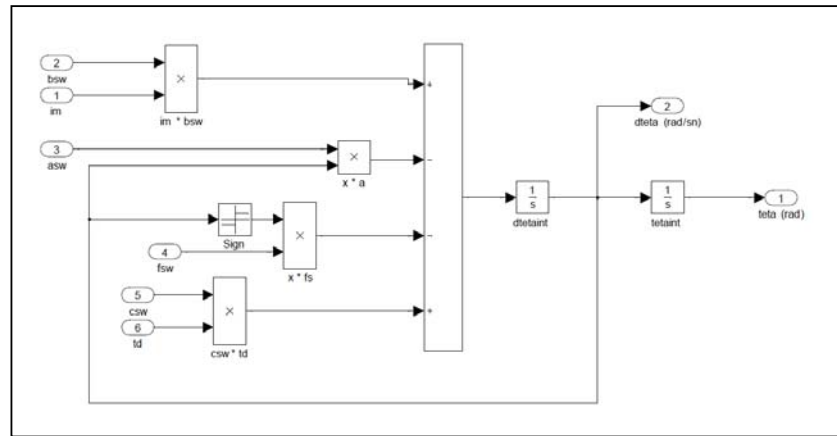


Figure A.2. Driver interaction unit block

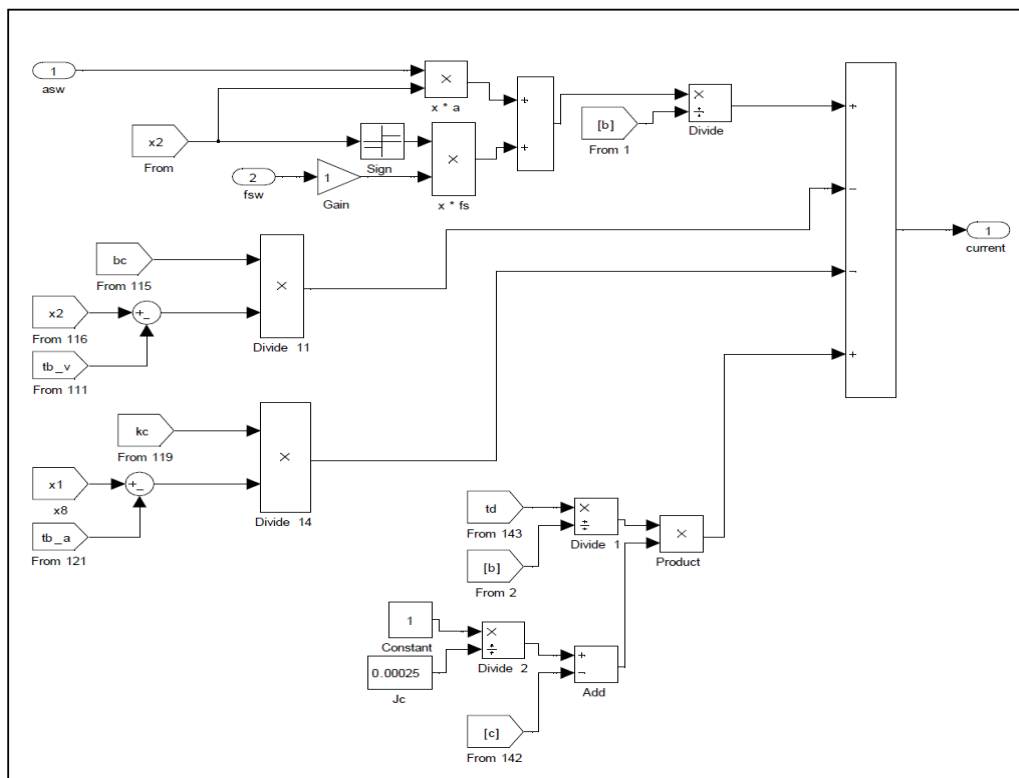


Figure A.3. Impedance controller block

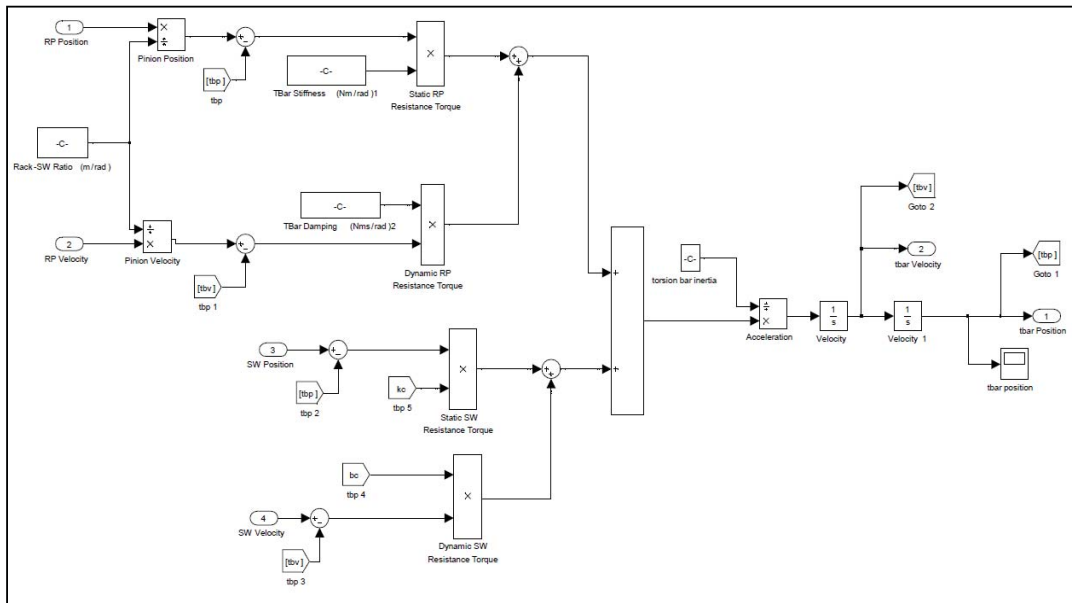


Figure A.4. Torsion bar block

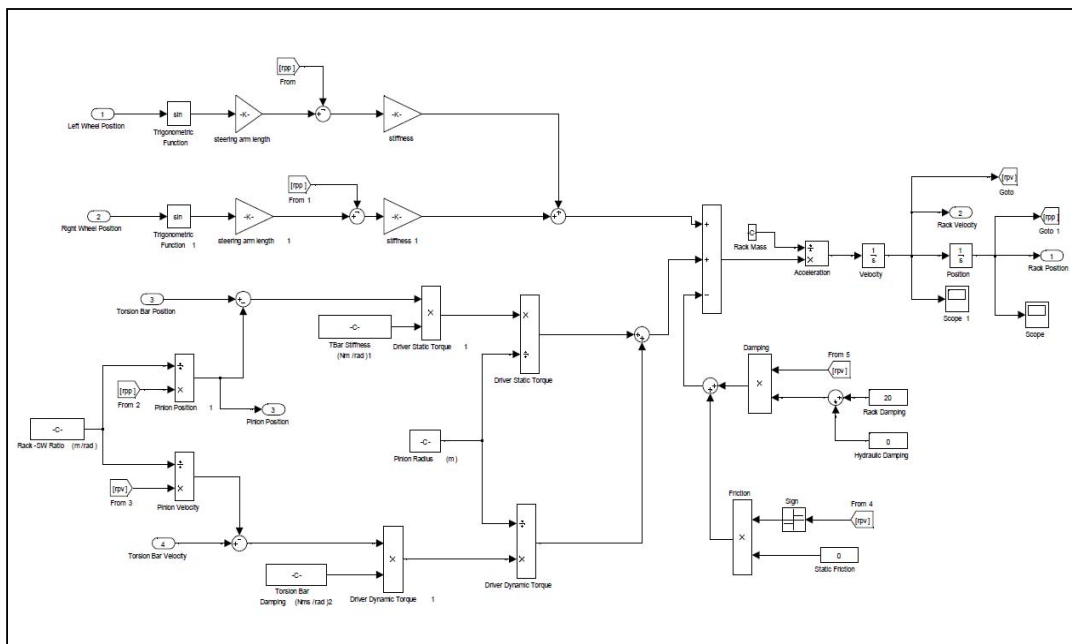


Figure A.5. Rack and pinion block

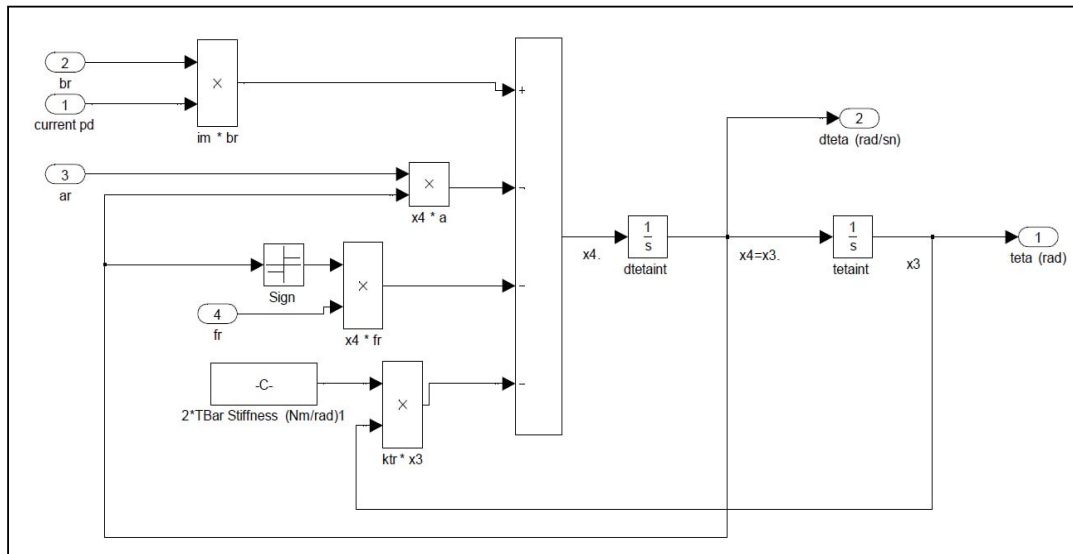


Figure A.6. Vehicle directional control unit block

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