BALANCING THE PRODUCTION OF BLOOD BAGS FROM DONATION THROUGH APPOINTMENT SCHEDULING

by

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ABSTRACT

BALANCING THE PRODUCTION OF BLOOD BAGS FROM DONATION THROUGH APPOINTMENT SCHEDULING

Blood is fundamental in several care treatments and surgeries, and plays a crucial role in the health care system. It is a limited resource as it can be produced only by donors and its shelf life is short. Blood Donation (BD) system aims at providing an adequate supply of blood units to transfusion centers and hospitals. Its main phases are blood collection, screening, storage, distribution and utilization. An effective collection of blood units from donors, through a suitable scheduling of donations, is fundamental for adequately feeding the entire BD system and optimizing blood usage. However, despite its relevance, to the best of our knowledge, donor scheduling is only marginally addressed in the literature. In this dissertation, we consider the Blood Donation Appointment Scheduling (BDAS) problem, which aims at balancing the production of the different blood types among days in order to provide a quite constant feeding of blood units to the BD system and avoid periodic overtimes. Deterministic models and corresponding stochastic versions are formulated. For the deterministic model a two-stage architecture is proposed and it consists of an offline Mixed Integer Linear Programming (MILP) model for preallocating time slots to blood types and an online prioritization policy to assign a preallocated slot when the donor calls to make the reservation. In order to establish the impact of the uncertainty in the model, we formulate both risk-neutral and risk-averse stochastic programming models and compare them with each other. Moreover, Conditional Value at Risk (CVaR) risk measure is analyzed in the stochastic models with either only risk terms or with mean-risk terms for the preallocation (offline) phase. We conduct an extensive computational study for the proposed deterministic and stochastic models with the use of real case of the Milan Department of the *Associazione Volontari Italiani Sangue* (AVIS), which is the main BD association in Italy.

ÖZET

BAĞIŞLARDAN ÜRETİLEN KAN TORBALARININ RANDEVU ÇİZELGELEME İLE DENGELENMESİ

Kan birçok tıbbi işlemde hayati önem taşıması sebebiyle, sağlık sisteminde önemli bir rol oynamaktadır. Kan insan vücudu dışında üretilemeyen ve raf ömrü kısa olan sınırlı bir kaynaktır. Kan bağışı (KB) sistemi, transfüzyon merkezlerine ve hastanelere yeterli sayıda kan torbası tedarik etmeyi hedeflemektedir. Kan bağış sisteminin ana fazları kan toplama, tarama, depolama, dağıtım ve kullanımdan oluşmaktadır. Kan bağış sistemini yeterli şekilde beslemek ve kan kullanımını eniyilemek için, bağışçılardan gelecek olan kanın etkin bir şekilde toplanması ve bağışçılara uygun bir şekilde randevu verilmesi gerekmektedir. Bildiğimiz kadarıyla mevcut çalışmalar dahilinde bağışçı randevu çizelgeleme konusu yeterince ele alınmamıştır. Bu tezde, kan bağış sistemine sabit akış sağlayacak, farklı kan türleri üretimini günler arası dengeleyecek ve periyodik mesaiden kaçınacak bir kan bağışçı randevu çizelgeleme (KBRÇ) modeli geliştirilmiştir. Deterministik ve ilgili stokastik modeller formüle edilmiştir. Deterministik modellerde yapı iki fazdan oluşmaktadır, ilk fazda çevrimdışı olarak tamsayılı doğrusal programlama modeli, kan tipleri için önatama yapmaktadır. İkinci fazda ise çevrimiçi olarak, önceliklendirme politikası ile rezervasyon yaptırmak için arayan bağışçılar, önataması yapılan zaman dilimlerine yerleştirilmektedir. Modeldeki belirsizliğin üstesinden gelebilmek için risk-nötr ve risk-yanlı stokastik programlama modelleri ele alınıp, karşılaştırmaları yapılmıştır. Buna ek olarak, sadece risk terimleri ve ortalama risk terimleri içeren stokastik modeller koşullu riske maruz değer (Conditional Value at Risk, CVaR) risk ölçütü ile çevrimdışı faz için analiz edilmiştir. Önerilen deterministik ve stokastik programlama modelleri ile İtalya'nın ana kan bağış kurumlarından biri olan Associazione Volontari Italiani Sangue (AVIS)'in Milan şubesinde gerçek bir vakaya kapsamlı analiz yapılmıştır.

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a_t^b	Number of already booked donors at day t with blood type b
B	Set of blood types
c_{tk}	Overall capacity of physicians (time) in period k of day t
d_b	Expected number of booked donors over T with blood type b
K	Set of time periods in a day
n_t^b	Expected number of non-booked donors at day t with blood type b
p_s	probability of each scenario s
p_{tk}	Dispersion amount in period k of day t
r	Standard time required for serving a donor
R_{tk}	Time amount for serving the already booked donors in period k of day t
S	Set of scenarios
T	Time horizon
v	Maximum of the variations $z_t^b \; \forall t \in T, b \in B$
w_{tk}^b	Number of preallocated slots for blood type b in period k of day t
x_t^b	Number of preallocated slots for blood type b in day t
y_t^b	Number of planned units for blood type b in day t
z_t^b	Absolute variation of y_t^b with respect its average value over T
α_k	Fraction of n_t^b in period k
δ_k	Weight of the dispersion amount in period k
η	Maximum variation weight (for the objective function)
μ	Maximum fraction of the total dispersion amount in a day with respect to the overall capacity in the same day
ϕ^s	The summation of the total absolute variation with respect to the average request for all blood types for each scenario
ψ^s	The summation of the total dispersion amount for each scenario
θ	A non-negative trade off coefficient

ε	Flexibility degree associated with d_b

BD	Blood donation
BDAS	Blood donation appointment scheduling
CVaR	Conditional value at risk
EOQ	Economic order quantity
EEV	Evaluation of expected value
EV	Expected value
FIFO	First-in first-out
HN	Here and now
LB	Lower bound
MBCP	Maximum blood collection problem
OF	Objective function
RD	Relative difference
Rh	Rhesus factor
RC	Robust counterpart
SP	Stochastic programming
TC	Transfusion center
USA	United States of America
UK	United Kingdom
VaR	Value at risk
VRP	Vehicle routing problem
WS	Wait and see

1. INTRODUCTION

Blood is a fundamental component for several care treatments, and it plays a crucial role in the health care system. For example, in 2012, the annual need for blood was about 10 million units in the USA, 2.1 million units in Italy, and 2 million units in Turkey [1]. Blood is also a limited resource because, at present, it cannot be produced in laboratory but only by humans. Thus, blood is usually collected from donors, i.e., unpaid individuals who donate their blood voluntarily. Furthermore, its short shelf life limits the period between donation and utilization, thus preventing long term storage.

Blood is provided through Blood Donation (BD) system, which is in charge of providing an adequate supply of blood units to transfusion centers and hospitals. Due to the short shelf life, BD system should meet the overall blood demand from hospitals and transfusion centres, but at the same time it should follow the temporal profile of the demand to avoid blood shortage and wasted units. The BD supply chain can be divided into four steps, as shown in Figure 1.1: *collection*, *transportation*, *storage* and *utilization* [2]. In short, blood is first collected: donors are registered and served by a physician to assess their eligibility for donation and, if eligible, they make the donation. Once the blood is gathered, tests are performed on each blood unit to prevent infectious diseases. Afterwards, blood units are transported and stored. Blood components are then distributed to hospitals and transfusion centers based on their inventory levels. Finally, blood is transferred to the end users (the patients) for transfusion.

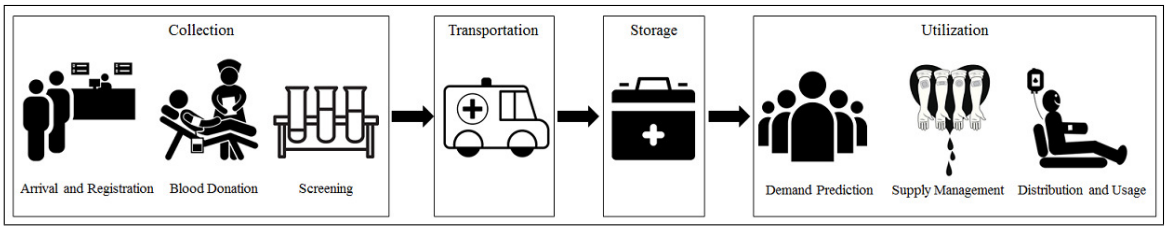

Figure 1.1. Steps of the BD supply chain.

In this dissertation, we focus on the blood collection step, which represents the first and the most critical step of the BD supply chain. Not only increasing the number of donations improves the throughput of the BD system, but also an effective management of donors' arrivals among the days may improve the performance of the system and provide a reliable supply of blood units considering the storage requests. In fact, the role of a blood collection center is to provide a reliable supply of blood units to the storage, in agreement with the storage request. There are two main storage policies, i.e., to store enough blood units to cope with any blood demand at any time, or to satisfy the demand from a hospital or a medical center while keeping the stored amount of blood units limited. When the second storage policy is considered, the goal might be to balance the production. Balancing the production, i.e., producing a constant number of blood units over the days, is the main goal in several cases when the customer is a large hospital with several elective patients.

Several blood collection centers are starting to implement a reservation system. In fact, reserving the donation appointment can reduce donors' waiting time and, thus, guarantee a better service to donors, which may help in increasing the number of donors and the frequency of donation. Moreover, by appropriately addressing donors to a suitable day, reservation may also balance the production of blood units among the days. In any case, centers also accept donors without reservation not to refuse any possible donation, because of the high need for blood units and to prevent donors from feeling that their donation is not important. Thus, generally speaking, both booked and non-booked donors are usually present in the collection centers, even though the effort is to increase the rate of booked ones. So far, appointments are manually assigned in the majority of collection centers where reservation is possible. Manual management may be able to reduce donors' waiting times and to take their preferences (donation day and hour) into account; however, it is a short term solution and may prevent effectively balancing blood production.

In this dissertation, we propose an appointment scheduling system for blood donation to balance the production of blood units of the different types (a combination of group and Rhesus factor) among days and to avoid periodic overtimes due to possible accumulation of donors within periods of the day, while taking into account both booked and non-booked donors. We first develop the deterministic models and then the corresponding stochastic

versions are formulated in order to analyze the effect of random arrivals of non-booked donors. Since this is a real life application, uncertainty of the system is inevitable and it must be handled carefully. For the deterministic case a two-stage architecture is proposed and it consists of two phases, i.e., an offline preallocation of time slots for donation and an online allocation of the preallocated slots, where a *time slot* is as an operational interval or service time interval suitable for a donor. The preallocation phase is responsible of reserving slots to the blood types, while the allocation phase is responsible of assigning a suitable preallocated slot to each donor when he/she calls for reservation. In other words, the preallocation phase prepares a number of spare slots for the different blood types, which are then used for the successive online booking phase. The architecture is based on a *Mixed Integer Linear Programming* (MILP) model for the preallocation phase and a prioritization policy for the allocation phase. In particular, stochastic versions of the deterministic models are proposed with risk-neutral and risk-averse methodologies for the preallocation (offline) phase.

Although the problem shares some features with other health care related appointment scheduling problems (see Section 2.3), balancing the production is not a common objective. Moreover, the characteristics of the BD system make the donation scheduling different from other appointment scheduling systems of different fields. Thus, to the best of our knowledge, this dissertation is the first attempt to deal with the Blood Donation Appointment Scheduling (BDAS) problem.

In this study, we particularly consider the case of the Milan Department of the *Associazione Volontari Italiani Sangue* (AVIS), denoted as AVIS Milan, which serves a large hospital with a quite constant demand for blood. AVIS was founded in 1927 and is the largest blood donors' association in Italy today, bringing together over one million of volunteer blood donors across the country. AVIS Milan covers the territory of Milan and is in charge of collecting blood for one of the main hospitals in Milan, i.e., the Niguarda hospital; in the last 4 years, it provided an average about 50 whole blood donations per day, with a total of about 18000 donations per year. AVIS is the largest network of BD collection centers in Italy, and AVIS Milan is one of the largest centers in the network. AVIS Milan is willing to implement a reservation system and currently accepts donors with (booked) and without (non-booked) reservation. Even the booked donors are the majority at the moment, AVIS

Milan still aims at increasing their rates. Its goal is to produce a constant amount of blood units for each blood type along with the days. In fact, Niguarda is a large hospital with a lot of elective surgeries and a quite constant amount of emergency requests. Thus, the request from Niguarda hospital is to feed the system with a constant (and possibly high) daily amount of units of the different blood types, even if unpredictable demand peaks may occur in specific periods and conditions. On the contrary, the lack of a constant feeding is the actual bottleneck of the entire system in practice, as explained by the AVIS Milan staff. Hence, the main goal of this thesis is to formulate and analyze alternative solutions for AVIS Milan blood collection center in order to balance the daily blood production while minimizing the dispersion amount. Dispersion amount term is used in stead of overtime because periodic accumulation of the donors has to be dispersed in the underutilized parts of the day. Moreover, it can be considered as a typical blood center, since it shares common features in terms of donors, activities and management with several other centers. Thus, the approach proposed in this dissertation can be considered as general and applicable to other blood collection centers [4].

1.1. CONTRIBUTIONS

There are several contributions of this work. The first one is to present a detailed research on BD supply chain literature and highlight the gaps in the literature. As a result, it is seen that the blood collection phase has not been adequately addressed so far. Particularly, an appointment scheduling for the collection phase that jointly considers nature of its service provider and its role in terms of production is still lacking in the literature. More details are presented in Chapter 2.

Following this point, it can be concluded that an effective management of a collection center must include the production of blood units in addition to its internal organization as a service provider. Principally, an effective management of blood collection is necessary to increase the throughput and keep the costs sustainable. However, a more general view should include an effective management of donors' arrivals throughout the days to optimize the daily production of blood units with respect to the storage requests. Neglecting this point may result in an imbalanced feeding of blood units to the rest of the BD supply chain, with

consequent blood shortage or wasted units. Although, throughput and costs are addressed in the literature, more structured strategies that also include the impact on the whole BD chain are still underrepresented. Thus, the second contribution is to propose an effective BD appointment scheduling system to fill the gap in the literature. We propose a deterministic model in which the aim is to balance the production of blood units of each type among the days, while also avoiding dispersion amounts associated with overtime and donor waiting times. Details are presented in Chapter 3.

Uncertainty is an unavoidable part of real life problems. The most important parameter that creates uncertainty in the BD system is the non-booked donors. While scheduling the appointments for the donors, random arrivals of the non-booked donors should also be considered. Therefore, third contribution is based on the development of risk-neutral and risk-averse stochastic programming models to provide an efficient appointment schedule, which balances the production and avoids overtime of the physician's while considering the uncertainty in the non-booked donor information (see Chapter 4).

These contributions have already been or will be published in journals and conference publications as follows:

- The presented literature review in Chapter 2 is published in *Health Care Systems Engineering for Scientists and Practitioners, Springer* conference proceeding with the title "Management of blood donation system: literature review and research perspectives" [4].
- Considering the highlighted gaps of the literature (Chapter 2), some research directions are presented in the journal publication of *Production Planning & Control* with the title "Unaddressed Problems and Research Perspectives in Scheduling Blood Collection from Donors" [5].
- The deterministic model for BDAS system (Chapter 3) is published in *European Journal of Operational Research* journal with the title "An Appointment Scheduling Framework to Balance the Production of Blood Bags from Donation", [6] and in *IMATI-CNR* technical report with the same name [7].
- Stochastic programming models that are presented in Chapter 4 will be considered for

journal publication.

1.2. OUTLINE

The structure of the thesis is presented in Figure 1.2 below:

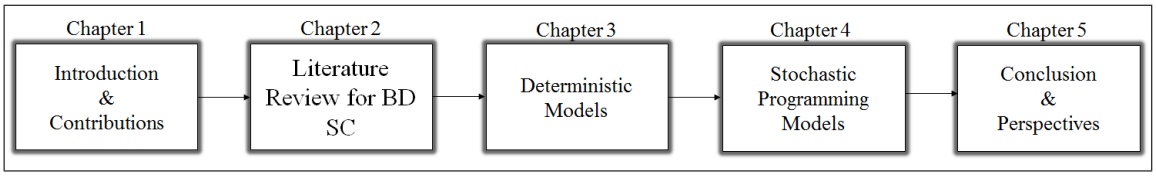

Figure 1.2. The structure of the dissertation.

In **Chapter 2** a detailed literature review on BD supply chain echelons is presented and open issues in the literature are discussed with a particular focus on the appointment scheduling systems. The details of the proposed architecture for the BDAS problem is explained in **Chapter 3**, including the deterministic MILP preallocation model and the prioritization policy. Moreover, sub-problems and valid inequalities are discussed and lower bounds are presented in order to increase the computational efficiency. Lastly, the computational tests that are performed for the offline and online procedure and the corresponding results are reported. In **Chapter 4**, the approaches for handling uncertainty are described. The stochastic programming models for donor appointment scheduling problem are presented and the related numerical results are discussed. Finally, the comparison of the proposed models and the driven conclusions together with the future perspectives are presented in **Chapter 5**.

2. LITERATURE REVIEW

In this chapter, we first review the literature dealing with the blood donation supply chain, focusing on the blood collection step and then we survey the literature on appointment scheduling systems. Blood collection is the first and most important step of the BD supply chain to maintain sustainability of the entire system. In BD system, to provide adequate supply of blood units to transfusion centers and hospitals with respect to the storage requests is significantly important. To enhance the stability of the BD system it is important to manage the first step of the chain which is the donor arrivals with an appointment scheduling model.

Many optimization problems are present in managing the BD supply chain, from donors' arrival and registration to final utilization of blood units. Most of them have been largely addressed in the literature, as underlined by recent surveys; e.g., Beliën and Forcé [8] reviewed the literature up to 2010, and Osorio *et al.* [9] presented a structured review on quantitative modeling for BD supply chain. Different problems related to the BD supply chain management have received attention in the literature: a high number of papers are related to storage and distribution, whereas other problems, such as donation scheduling, are not adequately addressed. In particular, even though blood collection step is one of the most important step in the chain at the operational level, the Blood Donation Appointment Scheduling (BDAS) problem has never been addressed so far to the best of our knowledge.

In order to highlight unexplored issues and to point out alternative perspectives and possible future research opportunities within the scope of this thesis, we review the literature related to the BD system management and classify the existing research based on the process phase. A literature analysis on BD supply chain management was conducted by Baş *et al.* [4] and in this study we grouped the studies of BD supply chain echelons as a percentage to point out the lack in the literature.

In the following part (Section 2.1), we first present the literature dealing with the blood donation supply chain echelons. In Section 2.2 future research directions are discussed

considering the lack in the literature for BD supply chain and the classification of donation scheduling within the appointment scheduling literature is presented in Section 2.3.

2.1. BLOOD DONATION SUPPLY CHAIN

The BD supply chain and the related decisions have been classified in the literature either based on decision levels or supply chain steps.

Sundaram and Santhanam [2] classified the system based on the main steps of a blood supply chain while Pierskalla [10] classified the decisions based on the strategic, tactical and operational decisions and Osorio *et al.* [9] grouped the decisions as strategic, tactical, operational decisions and high-level processes. The details of the papers are as follows.

Sundaram and Santhanam [2] divided the BD supply chain into four steps based on the main phases of a blood unit life: *collection*, *transportation*, *storage* and *utilization*. First, blood is collected: donors are checked and, if eligible, they make the donation. Then, blood undergoes a screening process to search for any infectious diseases, and it is possibly fractionated into components (erythrocytes, plasma and platelets). Afterwards, it is transported and stored. Its components are then distributed to hospitals according to their needs. Finally, blood is transferred to end users (the patients) for transfusion. A scheme of this chain is shown in Figure 1.1.

Pierskalla [10] classified the decisions based on the strategic and tactical operational decisions. They defined the strategic decisions such as locations of blood banks, the number of blood centers in a region and the coordination of supply and demand. Many tactical operation decisions involved in the paper as blood collection, controlling inventory level and allocating blood to hospitals.

Osorio *et al.* [9] classified the echelons of the blood supply chain as *collection* (including donors' arrival and registration, collection, and transportation of blood units to storage centers), *production* (including testing, primary fractionation which means separating blood to its components, secondary fractionation and storage of blood products), *inventory*

(including replenishment and management of hospital blood banks) and *distribution* (including picking and delivery to care units and patients).

With respect to the Figure 1.1, the details of each blood donation supply chain echelon and related studies are presented in detail as follows. For the sake of simplicity, the figure structure is used in the following sections.

2.1.1. Collection: Donors, Donation and Screening

BD process starts with the arrival of the donor at the blood center. Donors can be divided in *returning donors* who donate on an almost regular basis, and *walk-in donors*, who are entering the system for the first time. If the blood collection center has a reservation system, donors can be further classified as *booked* and *non-booked* donor. In any case, donations can be made after a defined rest period from the previous one, which is defined by law. For example in Italy, the rest period is defined as 90 days after the previous donation for men and 180 days for women and for Turkey it is 90 days for men and 120 days for women. As donors have a crucial importance in the system, their availability, frequency and motivation have been studied from both a statistical and a social perspective. After blood collection, some tests are performed on each blood unit and all the details with respect to the phases is defined in the next sections.

In the following parts, firstly donor arrival and registration step is defined and the related papers are detailed. Secondly blood collection and screening processes are defined.

2.1.1.1. Donor Arrival and Registration

When a donor enters in the system for the first time, they are requested to provide personal (e.g., name, address, age, job, gender) and medical/health (e.g., diagnosis, lab results, treatments) information, which are digitally or manually collected. Digital registration provides a good traceability of the transfusion cycle, from collection to blood distribution and transfusion. Furthermore, an effective system integrates with the information systems of hospitals, and interacts with the collection units and associations of donors. To be able

to evaluate the blood donors eligibility, final determination is made by a physicians. The registration also includes a visit from a physician (brief examination of temperature, pulse, blood pressure and hemoglobin measures), and it is followed by blood exams. Physicians discuss donor's health history as part of the donation process before donation. If the donor is eligible, blood collection centers check that they make the first donation within few days from the declaration of eligibility. A fixed number of days must pass between two donations which is defined in Section 2.1.1.

There are several papers that include social aspects of the donors. The main reasons for blood donation and their relative importance have been studied by Bani and Giussani [11]. Moreover, it is also documented that the organization of blood collection phase may have an impact on donors availability. They analyze several reasons for blood donation, such as "to help others" (56%), "influence of family/friends" (22%), "social/moral obligation" (11.2%) and evaluate their importance. Poor treatment, poor staff skills, and a bad experience are the main reasons of not returning to donate [12]. Giving the opportunity to check one's own health state also plays an important role for donation (6.9%), exclusively for male donors. Beside these facts, donation decision is fundamentally a personal choice (41.3%), also influenced by relatives (21.8%), friends (22.3%), or voluntary organizations (21.8%). Also prolonged queuing times are negatively correlated to BD satisfaction [13, 14]. Hence, a well-organized donation management has a strong impact on the availability of blood units, and also on donors motivation, thus possibly increasing/decreasing their availability.

Winston Churchill affirms that *we make a living by what we get, but we make a life by what we give* to expresses that voluntarism is a helping behaviour and it is one of the most essential parts of human altruism. Hoffman [15] defined altruism as a behaviour such as helping or sharing without the expectation of reciprocity or compensation for that action. It is a long-term act and organized activity for the benefits of others. In the recent years, psycho-sociological studies about voluntarism have incremented, and several theoretical models have also been developed [16, 17]. Omoto *et al.* [16] modelled a conceptual framework that explains psychological and behavioural features of voluntarism. In this study, a field study constructed to understand volunteer's motivations, social support and personal satisfaction. Clary and Snyder [18] designated the theory in order to be able to make a comment on

different types of motives participation. This theory contains that different people may have different point of views but in the end they might be volunteer to same activity with varied intentions at different times. Later on, they focus on six primary motives [19]: protective (to lessen destructive feelings), values (to express or act on important values), social (to strengthen social relationships), understanding (to learn about the world), career (to gain career-related experience) and enhancement (to enhance self-esteem). Furthermore, Okun and Schultz [20] added a new motivation that was not provided by Clary and Snyder [18]: establishing friendships. Later on, Ferguson *et al.* [21] asserted that blood donors exhibit compassion over donated subject rather than altruism, and declares compassion as a synthesis of self-interest and selflessness.

Several management problems arise, both at a planning level (e.g., blood collection center location or staff dimensioning) and at an operational level (e.g., appointment scheduling [22]). Michaels *et al.* [22] studied customer service and productivity issues for the American Red Cross blood mobiles and also proposed a simulation model based on a six-bed benchmark clinic; then different strategies of setting-up, staff distribution and work directions is used to improve the system. However, only few papers focus on optimization issues arising in the registration and donation phase, despite the strong impact of donors' arrivals on the overall system performance.

Some papers focus on estimating the supply of blood from donations, considering annual donor retention rates, donor recruitment rates, and mean numbers of donations per donor and per year [23]. Finally, the online applications and database systems for donors and bags management are also investigated in this stage [24, 25, 26].

In the following part, blood collection and screening processes are defined and the related papers are detailed.

2.1.1.2. Blood Collection and Screening

The blood collection process starts when the donor arrives at the blood collection center. Here, donors are registered and visited (by a physician, a nurse or a qualified personnel) to

assess their eligibility for donation; if eligible, donors make the donation [1]. Once the blood is drawn from an individual, tests are performed on it to search for any infectious diseases (screening process). The blood units that pass the tests are separated into components, if required, and sent for storage.

There are two main types of BD collection centers, i.e., centers in a fixed location and mobile centers that move in the territory. In mobile centers, generally a vehicle is used for BD which is equipped with everything necessary and these vehicles are located in public places such as universities and city squares. Moreover, centers are generally subject to regulatory control, designed to ensure the maximum quality and safety of blood products. They guarantee that blood units are produced according to standardized procedures to achieve consistency of each product [27].

In some countries, such as Italy, the vast majority of collection centers are fixed and there is a small number of complementary mobile centers in the main cities, which are devoted to specific collection activities for a limited time (e.g., in companies, schools and parishes). Instead, in other countries like in Turkey, a large number of centers are mobile (Only Turkish Red Crescent (TRC) performs mobile blood collection in Turkey) and change their location from time to time (e.g., from day to day or from week to week) and in Turkey there is also fixed location blood centers (e.g., Red Crescent, public hospitals, university hospitals, private hospitals and military hospitals).

Some BD centers have a reservation system, and donors may reserve a specific time slot in a future date to avoid long waiting times and queues. In this case, donors can be divided into *booked* donors (donors with reservation) and *non-booked* donors (walk-in donors without a reservation). In some centers, reservation is mandatory for all donors but, in general, booked and non-booked donors coexist.

Despite the importance of this phase, the literature on blood collection system planning is rare [28, 29, 30]. De Angelis *et al.* [31] proposed a stochastic methodology to analyse and certify the optimal configuration of servers by integrating simulation and optimization for a transfusion center in Rome. As mentioned, management problems that arise in the blood

collection phase can be classified based on the decision level [9, 10]: planning level (e.g., location of fixed and mobile blood collection centers, staff dimensioning) or operational level (e.g., appointment scheduling, screening policies, donation prediction). Problems at both levels have an impact on the entire BD chain, and some of them have been addressed in the literature.

There are several studies in donors and blood collection literature ([32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48]) and the details of the related papers are explained as follows. Pratt and Grindon [32] developed a simulation model to study work flow and queueing problems, and to compare different donor scheduling strategies. Brennan *et al.* [33] studied customer service and productivity issues through simulation for the Red Cross blood mobile centers. Michaels *et al.* [22] developed a simulation study to evaluate scheduling strategies for donors who arrive at the Red Cross blood mobile centers, and compared these strategies in terms of mean transit time to find out the most effective one. Bretthauer *et al.* [34] considered a capacity planning problem which involves the number of required donor beds and the size of the nursing and support staff. Flegel *et al.* [35] developed a logistic regression model to compute the donation probability within a given time frame, considering different regression coefficients for the different types of donors. Ferguson and Bibby [36] used a prospective design to predict the number of future blood donations. Blake *et al.* [37] proposed a desktop tool for planners to schedule staff and donors; they also developed a discrete event simulator to evaluate the proposed clinic schedules. More recently, Raven *et al.* [23] estimated the blood supply from donations using annual donor retention rates and mean numbers of donations per donor and year. Testik *et al.* [38] adopted clustering method, classification methods and regression trees to identify donor arrival patterns; then, they applied a queuing network model of the donation process to dimension the workforce. Boonyanusith and Jittamai [39] investigated donor behavior patterns and the factors that influence donation decision. Alfonso *et al.* [40] addressed the modelling and simulation of blood collection process for fixed blood collection sites; indeed, they proposed formal Petri net models to describe all relevant donor flows of various blood collection systems. Ritika [41] examined different classification algorithms to find out a fair classification technique for donation prediction. Van Dongen *et al.* [42] analyzed the factors that affect the intention to continue donating in new donors. Blake and Shimla

[43] presented a method that incorporates cost control and impact on donor experience, and calculated staffing requirements to minimize costs and optimize donor wait time metrics. Mobasher and Özener [44] coordinated appointment and pick-up times at blood donation sites to maximize platelet production while considering processing time requirements for platelet production. Alfonso *et al.* [46] presented a simulation-optimization approach based on mathematical programming representation of event dynamics for capacity planning and appointment scheduling in blood collection systems. Elalouf *et al.* [47] improved the structure of a three-echelon blood sample collection chain, which includes clinics, centrifuge centers, and a centralized testing laboratory. Osorio *et al.* [48] worked on a multi-objective stochastic optimization model for technology selection and donor assignment. Rabbani *et al.* [49] investigated two mathematical models for mobile blood collection system. While the first model is a multi-objective fuzzy mathematical programming that maximize the amount of blood collection and minimize the operational cost, the second model is a vehicle routing problem. Gunpinar and Centeno [50] proposed a vehicle routing problem which focused on the number of bloodmobiles to operate while minimizing the travelled distance.

After collection, screening phase starts with few tests performed against infectious diseases, e.g., HIV, Hepatitis B and C, and syphilis. They are repeated on each gathered blood sample, and are generally the same all around the world. Based on the screening results, the blood bag is either released for clinical and manufacturing use or discarded [28]. An effective, well- organized screening program and a good quality system are essential for provisioning safe blood units to patients and meeting the transfusion requirements.

2.1.2. Transportation and Storage of Blood Products

Once blood is collected from donors at regional or community blood centers, blood must be stored in storage centers or TC before it perishes. These locations serve as a depot until the blood is distributed to the demand points and sometimes deal with testing of the blood products.

If collection and storage or TC do not coincide, blood must be transported. Transportation is a rather simple task in this phase because all collected units are usually addressed to a

big TC or storage center from all BD centers in the related territory. Transportation must be carefully performed as the blood must be stored before perishing and requires particular transportation conditions. Inefficient and inadequate transportation may reduce the quality of end user care and increase costs. There is not much literature available about blood transportation between collection and storage centers; on the contrary, many papers on blood transportation focus on the distribution to hospitals. Ghandforoush and Sen [51] used a deterministic non-convex integer optimization model to schedule the shuttle transportation of whole blood products from the collection points to the regional processing centers.

The presence of blood collection vehicles is also considered in blood bag transportation. Ekici and Ozener [52] defined a variant of the Vehicle Routing Problem (VRP), with respect to the availability of collection vehicles and they aimed to maximize the amount of collected blood. There is no capacity limitation on the vehicles due to small size of the blood collection bags [53, 54]. Donated blood has to be delivered to the processing center within a certain amount of time. Certain standards are set for the collection and processing of each blood product. Whole blood cells and red blood cells are both kept refrigerated at 1.0 to 6.0 ^{o}C, with maximum permitted storage periods (shelf lives) of 35 and 42 days respectively.

More attention has been paid to the storage of blood products. During the past 20 years significant progresses have been made in the technology of blood component preparation and storage [55, 56]. Beliën and Forcé [8] included several works in their survey. Literature is mainly focused on inventory management problems [57, 58, 59, 60, 61, 62, 63], from both a deterministic [30, 57, 58, 59, 64] and a stochastic perspective [31, 60, 65, 66, 67, 68, 69, 70, 71, 72, 73]. First, some of the related stochastic inventory problems is detailed as follows. Dillon *et al.* [62] proposed a two-stage stochastic programming model to minimize operational costs while considering blood shortage, wastage and demand uncertainty. Sirelson and Brodheim [65] built a stochastic simulation model as a function of base stock levels to manage inventory level, outdated performance measures and shortage rates. Pereira [66] built a stochastic model for a hospital blood bank inventory system, in which the remaining shelf life of blood units and the number of days between consecutive shipments were analysed according to the daily transfusion mean and variation impact. Katsaliaki [67] used a stochastic simulation model for a cost-effective management of blood

in the UK: valuable recommendations are provided to the stakeholders for cost reductions and for increasing the level of services and safety of the processes. Pierskalla and Roach [74] grouped stock level into categories according to shelf age; to satisfy the current (deterministic) demand, a First-In First-Out (FIFO) optimal policy was then applied issuing the oldest unit. Kopach *et al.* [68] revisited a queuing model and, using level crossing techniques, determined an optimal policy to support the modeling of several trade-offs; the model was also combined with the current control techniques using simulation and the effectiveness of the model was verified with real data. Hemmelmayr *et al.* [75] evaluated the impact of switching from their present vendee (customer) managed inventory system to a vendor (supplier) managed inventory system by a stochastic integer programming-based approach.

Secondly, some of the deterministic inventory problems is detailed as follows. Some researchers extended inventory models like the Economic Order Quantity (EOQ) policy for including perishable products. For example, Giri and Chaudhuri [57] proposed an inventory model for a perishable product where the demand rate is a function of the on-hand inventory, and the holding cost is non-linear. Axsater [58] used an economic order quantity model with deterministic demand. Then, he found the optimal reorder point under uncertain demand with the use of the order quality. Kazemi *et al.* [63] used a mixed integer linear programming model for solving blood inventory-routing problem while applying older-first policy. Padmanabhan and Vrat [59] proposed a stock-dependent selling rate model where the backlogging function was assumed to be dependent on the amount of demand backlogged. Dye and Ouyang [61] extended their model by introducing a time-proportional backlogging rate.

The next echelon of the BD supply chain is blood distribution and utilization and it is defined in the following section.

2.1.3. Blood Distribution and Utilization

The last step of the BD chain includes distribution and utilization, which involves several management problems as detailed below. Distribution is highly important for efficient

blood usage and should meet the demand, which is often uncertain and requires accurate predictions. Distribution and utilization echelons includes demand prediction for blood products, supply management of blood products, distribution to users and usage which are detailed in the following sections.

2.1.3.1. Demand Prediction for Blood Products

In many countries, people still die because of inadequate supply of blood products [1]. Thus, the main issue in BD is improving blood availability to save lives and to meet the needs. Moreover, predicting the demand for blood products is crucial for BD process. However, meeting the demand may not be easy since also the number of donors is difficult to foresee. One of the most used strategy is holding each blood type in the inventory to meet the demand. Since there can be shift in donors or demand levels, different inventory management strategies can be applied as well.

Several works include an evaluation of the demand, even if papers that only focus on a stochastic prediction of the demand are not available. They can mainly classified based on the demand structure: deterministic [51, 74, 76, 77, 78, 79, 80, 81] or stochastic [66, 67, 68, 72, 76, 82, 83, 84, 85, 86, 87, 88]. Moreover, some of the works can also be classified with respect to the aggregation level: single hospital [66, 71, 72, 82, 86, 90, 91] or regional level [51, 68, 72, 82, 84, 92, 93, 94, 95, 96, 97]. Although both deterministic and stochastic demand prediction models have been widely traced in the literature, most of these works have been devoted to the stochastic ones since they better reflect the realistic situations. In the following, papers that are published in recent years (considering last 5 years) are explained in detail. For example, Kaspi and Perry [87] considered a system in which both arrival of blood products and demand are modelled via stochastic process as independent Poisson processes. Silva *et al.* [80, 81] developed a demand forecasting tool to make decisions about the weekly demand required by hospitals. Furthermore, they also improved the planning of the inventory balance process by forecasting the demand of blood components. Forecasting the monthly demand was also investigated in [68] by univariate time-series methods. Lau *et al.* [98] predicted the future blood demand of thalassemia major patients for the next 10 years for long-term management of blood supply. Osorio *et al.*

[88] proposed a multi-objective stochastic integer linear programming model to minimize the total cost while optimizing the total number of donors required by sample average approximation and the augmented epsilon-constraint algorithms. Zahiri and Pishvaee *et al.* [89] proposed a bi-objective mathematical programming model to minimize total cost considering maximum unsatisfied demand in blood donation supply chain.

In the following section, BD supply chain management process and the related papers are explained in detail.

2.1.3.2. Supply Management of Blood Products

Some papers deal with the decision making support in BD supply chain management, and on how to maintain or increase the supply of blood products [51, 72, 76, 84, 86, 96, 99, 100, 101]. Recent studies in the literature are summarized as follows. Sahin *et al.* [76] established several deterministic mathematical models to solve the location problem of blood services. Hemmelmayr *et al.* [84] used an integer programming model to generate low costs and robust delivery routes for the supply of blood products to hospitals by a blood bank, and showed the impact of the uncertain demand on the resulting routes. Van Dijk *et al.* [72] combined stochastic dynamic programming and computer simulation for the inventory management problem; the first approach is used to obtain optimal solutions whereas the latter to investigate various what-if questions as extending the shelf life, changes in the cost ratio, delivering policy, errors in the demand estimates, and uncertain supply by donors. Ramezanian and Behboodi [99] proposed a mixed integer linear programming model and the aim is to increase utility and motivation of the donors to donate blood. First they propose a deterministic location-allocation model and than proposed a robust optimization approach that takes into account the stochastic nature of demand and cost parameters with scenario-based solution methods. Salehi *et al.* [100] proposed a robust two-stage multi-period stochastic model for the blood supply network design considering possible natural disasters. Fazli-Khalaf *et al.* [101] proposed a multi-objective mathematical model for emergency blood supply chain network design. The aim of the model is to minimize total supply chain costs and transportation time between facilities while maximizing total testing reliability of the donated blood in the laboratories. Arvan *et al.* [102] designed a bi-objective, multi-

product supply chain network for blood supply and the aim is to minimize the cost of the supply chain while minimizing the time of the blood products before it is consumed. Fahimnia *et al.* [103] presented a stochastic bi-objective supply chain design model with the aim of cost minimization and delivery time minimization in a case of disasters. Finally, Elalouf *et al.* [104] formulated a mixed-integer programming to obtain the optimal number of vehicles to be deployed and the scheduling of the pickup process.

In the following part, the last echelon of the BD supply chain is defined and the related papers in the literature is summarized.

2.1.3.3. Distribution to Users and Usage

Distribution starts with the delivering of components to hospitals, where they are transfused into patients. TCs are usually responsible for the provisioning of blood products to hospitals, and the delivered quantities are limited by the shelf-life of blood products as well as by the holding capacity. In the BD distribution literature, models for determining the number of distribution centers and their locations are studied. Some of the papers ignore demand uncertainty [77, 79, 105, 106] while some of them not [107].

Two types of blood distribution systems were outlined by Hirsch and Cazal [77]: the reactive type, where the inventory level of the hospital is managed with respect to demand, and the predictive type, where the demand is fixed on schedule. Prastacos [79] focused on a deterministic mathematical programming model, whose target is to streamline the distribution of the regional blood resources while viewing plan commitments. It is characterized by a centralized management of blood, rather than management by individual hospitals, pre-scheduled deliveries, and a distribution system according to which blood is rotated among the hospitals.

Generally, redistributing the blood among hospitals is equally important for preventing out dating. In the event that there is a pressing need of a particular blood type in a clinic, they may use the blood with the closest decay date from an alternative facility to avoid spoilage of blood units. Kendall and Lee [105] focused on this redistribution problem with

goal programming: their model has distinctive goals, e.g., anticipation blood shortages and overages in hospitals, minimization of the quantity of old units, and minimization of the working expenses.

Recently, Le *et al.* [106] combined inventory and routing management into one model: they proposed a column generation-based heuristic to solve the problem, and showed significant savings when using their model. Shen *et al.* [107] presented a joint location-inventory model for blood distribution system, with non-linear working-inventory costs and non-linear safety stock inventory costs.

As out literature review indicates, there is a considerable body of work related to BD, however, not all phases of the BD supply chain have been adequately investigated in the literature so in Section 2.2, we have highlighted relevant lacks and proposed some research directions in this field.

2.2. OPEN ISSUES IN LITERATURE

The BD supply chain should meet the overall blood demand from hospitals and transfusion centers, and at the same time the blood supply should take into account the temporal profile of the overall demand. In fact, as mentioned, blood is a limited resource and its short shelf life limits the period between donation and utilization; thus, imbalanced feeding of blood units could trigger alternate periods of blood shortage and wastage. Blood collection, as the first stage of the BD supply chain (see Figure 1.1), has a great impact on the entire system in terms of blood units flow. Moreover, it is responsible for the perceived quality of service from the donors viewpoint. Unfortunately, this phase is not adequately addressed in the literature. Table 2.1 summarizes a literature analysis on BD supply chain management conducted by Baş *et al.* [5]. Percentage of the existing work for each phase are calculated based on the 229 papers on blood management (papers on social and physiological aspect neglected) that are available on Scopus and on the other main scientific databases. This research is based on Baş *et al.* [5] which is updated in February 2018. It shows that only the 3% of the research deals with the first step of a blood unit life, i.e. with donors' arrival and scheduling.

In particular, as in other domains, an effective application system is also needed in the BD domain, in order to combine the registration system with donors' and physicians' preferences and their points of view. Such an application system (e.g., an online system) could be a solution to join donors and physicians at the same platform and to encourage volunteer donation.

Table 2.1. Percentage of the existing works for each phase on blood management that are available in the literature and this research is based on Baş *et al.* [5].

Phase	**Process**	**Percentage**
Collection	arrival and registration	3%
	donation	11%
	screening	3%
Transportation		0%
Storage		30%
Utilization	demand prediction	18%
	supply management	19%
	distribution	16%

Another important issue of the BD system is the management of donors' appointments and visits, as it has a significant impact on the effectiveness of the entire BD chain and on donors' motivation. Also this point has been slightly studied in the literature (Fig. 2.1) and it can be improved. Definitely, increasing the number of donations improves the performance of the system, but also an effective management of donor arrivals among the days may optimize the daily production of units with respect to the demand in order to have better management of BD system. For instance, booked donors' appointments could be scheduled in advance, but not all donors are willing to accept prescheduled appointments, or they often require appointments at the beginning or at the ending of the day rather than at noon. Thus, an important lack in the literature is the development of mathematical models and techniques for providing an efficient appointment scheduling.

In the following section, first the related appointment scheduling systems in the literature is discussed and then an appointment scheduling for blood donation system is proposed.

2.3. APPOINTMENT SCHEDULING SYSTEMS IN THE LITERATURE

Scheduling is the decision making process to determine when, where and with which resources to produce a set of products, or to provide a set of services. Generally speaking, scheduling an operation includes deciding when and how to process and deliver the product/service, in order to achieve the maximum benefit according to an objective [108, 109, 110, 111].

Scheduling problems are widely studied in the literature and have been classified according to several criteria (e.g., number and sequence of machines, processing times, job arrival rates and objective function) for both manufacturing and service (including health care) systems [112]. Usually, scheduling problems deal with limited resources and some other requirements (e.g., compatibility, prioritization) over a specific time period. Typical objectives are: minimizing time and cost, maximizing the total amount of work completed, reducing inventory, increasing efficiency, etc.

Effective schedules are widely studied in manufacturing [113, 114, 115]. However, scheduling in service systems is different than in manufacturing, mainly because the system capacity in manufacturing may exploit inventories. On the other hand, in service systems, a service is provided together with its utilization; thus, service capacity cannot be stored and it is lost if unused [116, 117]. In service systems, customers want to spend the minimum waiting time and receive good quality service, whereas service providers want to perform the schedule at the minimum cost. In particular, service systems try to satisfy the demand through appointments. Thus, appointment scheduling represents the interface between demand and service provider, and balances between the needs of the two stakeholders.

In health care services, the appointment scheduling process allocates service times to individual patients [118] while pursuing two main goals. The first one is to provide a good quality of service to customers, e.g., by assigning them a short service time window during

which they are guaranteed to receive the service. The second goal is to protect the system from daily fluctuations of demand, which may result in low utilization levels in some days and overloads in others [119].

Researchers focused on several objectives while designing appointment scheduling frameworks in health care systems [120]; reducing waiting times, improving the continuity of care, optimizing clinical resource utilization, improving patient satisfaction and reducing costs are widely applied objectives. Examples of applications are medical patient scheduling, scheduling systems for ambulatory care services, home care scheduling, physician coverage scheduling, appointment scheduling in outpatient clinics, and operating room scheduling. For instance, Cayirli *et al.* [121] investigated scheduling systems for ambulatory care services; they highlighted the great impact of patient sequencing on ambulatory performance and showed that panel characteristics (such as walk-in patients, no-shows, punctuality and numerousness) strongly influence the effectiveness of appointment systems. Alvarado and Ntaimo [122] developed three (chemotherapy patients, chairs, and nurses) different mean-risk stochastic integer programming (SIP) models, to schedule individual chemotherapy patient appointments and resources. Appointment scheduling in outpatient clinics is also widely studied. The negative effects of no-shows are studied in terms of provider underutilization and delayed patient access [123, 124, 125, 126, 127, 128, 129, 130]. In such case, most of the applied solutions use overbooking in order to increase the utilization; the goal is to maximize the number of visited patients while minimizing waiting times, physician idle times and overtimes [131, 132]. Finally, Muthuraman and Lawley [125] showed that the near-term schedule tends to be fully utilized for outpatient clinics when the health care service provider works close to capacity.

In practice, appointment scheduling decisions in BD systems are made by hand (physicians and/or support staff spend a lot of time to assign donors to slots) or supported by tools. Since the papers in the literature generally deals with the social and physiological aspects, quantitative models are still missing in the donation scheduling literature. However, to the best of our knowledge, the available systems do not include an analysis of the daily blood production with respect to storage requests while allocating time slots to donors. Moreover, integrating the production point of view makes the scheduling problem more complex and

almost impossible to consider any manual scheduling made by the physicians or the support staff. Hence, the main goal and benefit of new effective scheduling systems will be to combine these contrasting needs: to improve the operational level while optimizing the produced blood units based on storage request, its pattern and storage capacity (free and occupied).

Unfortunately, several issues complicate developing such an effective system and prevent from using solutions imported from other health care services. One of these issues is the uncertainty related to donor arrivals. While scheduling the appointment for a donor, unexpected arrivals of additional non-booked donors and the other future reservations should be also considered. Hence, the system should predict these arrival rates while assigning time slots to donations. Another issue is the uncertainty of blood demand, strongly affecting storage levels, which can be only partially predicted due to daily and weekly fluctuations. The fluctuations depend on the institution structure; for example, the demand is highly variable for blood collections centers that serve emergency departments.

To summarize, appointment scheduling in BD systems is different from all of the presented cases, since blood collection includes both the features of a service system (time windows, operators' capacity, possibility of no show-up) and those of a production system (supply of blood units to the rest of the BD supply chain). For this reason, the blood donation scheduling problem cannot be strictly classified within the existing literature, and this may also explain the lack in the literature on BD appointment scheduling systems that include both system and production points of view.

From the service provider point of view, there are some priorities to consider. The first priority is to reduce waiting times for booked donor, to improve the quality of service perceived by them. The second priority is to satisfy donors' preferences while making a reservation, in terms of desired donation day and time, and possibly place where to make the donation. In fact, donors are more willing to donate if they are satisfied with the service [133], e.g., if the BD collection center is in a suitable location, the donation day and time fit with donors' needs, and/or the waiting time is limited. Thus, optimizing the donation phase can increase the donation frequency, but it must involve several planning levels and

decisions to coordinate.

As mentioned, an important issue in optimizing the blood collection phase is the uncertainty associated with some inputs of the system. One of the most uncertain parameters is the number of non-booked donors, whose uncertainty must be considered when scheduling the appointments. More in general, donor arrival rates may highly vary according to the time of the day, the day of the week, etc. Thus, predicting the numbers of booked and non-booked donors and their temporal patterns is necessary to evaluate the number of needed appointments and to get more effective appointment solutions. Moreover, predicting the number of donors is also useful for managing and dimensioning the system. Thus, an appointment scheduling system that includes these priorities and a fair prediction of the arrivals would certainly improve the BD collection phase.

In Section 3, the proposed architecture for the blood donation scheduling problem is presented which include both system and production points of view. The architecture is based on a deterministic Mixed Integer Linear Programming (MILP) model for the preallocation phase and a prioritization policy for the allocation phase.

3. DETERMINISTIC MODEL

3.1. INTRODUCTION

In this dissertation, we propose a new architecture for AVIS Milan in the BDAS problem. In the following section, primarily the current architecture of the AVIS Milan scheduling system is explained and then the proposed architecture is defined.

The current architecture of the AVIS Milan scheduling system is shown in Figure 3.1, which also shares many features with several blood collection centers. Some donors call to book the donation day and time slot beforehand, and slots are assigned (booked) until a maximum percentage of the daily capacity is reached, regardless of blood type. The daily capacity is expressed in terms of the total physician working time without incurring overtime. While AVIS Milan has a large donation room where a seat is almost always available when a donor arrives, the physician's service before donation is the bottleneck of the system that generates the queue; thus, we consider the physician working time as the scarce resource and the time slot refers to the time spent for serving a donor.

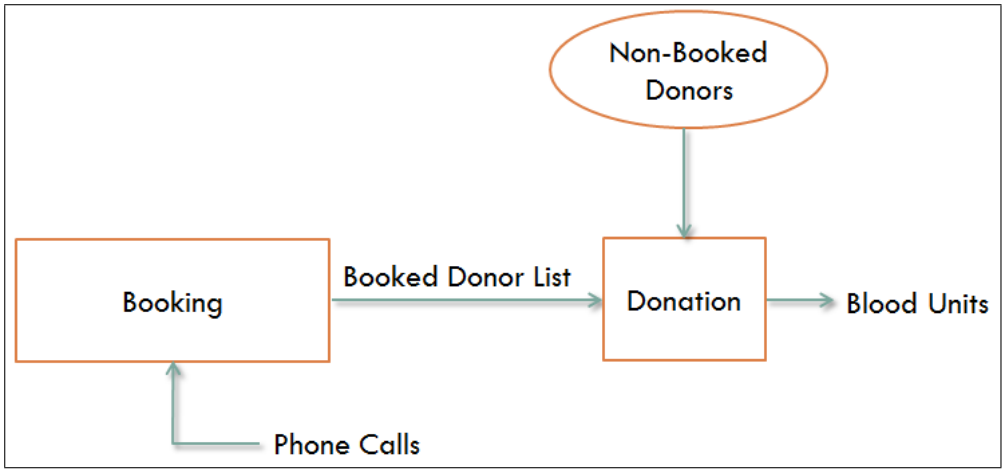

Figure 3.1. Current architecture of AVIS Milan.

Another problem is related with the allocation of time slots to the booked donors without

considering the arrival of non-booked donors. This generally creates unavoidable overtime for the physicians at the end of the day. Furthermore, as it is discussed in 1, the actual problem of the entire system is an imbalanced production. In order to reduce the outdated blood units and blood shortages, it is important to balance the daily blood production. The daily donations are given by the amount of booked and non-booked donors who show up at the blood collection center. Historical data from AVIS Milan show that the number of produced units is not constant among days.

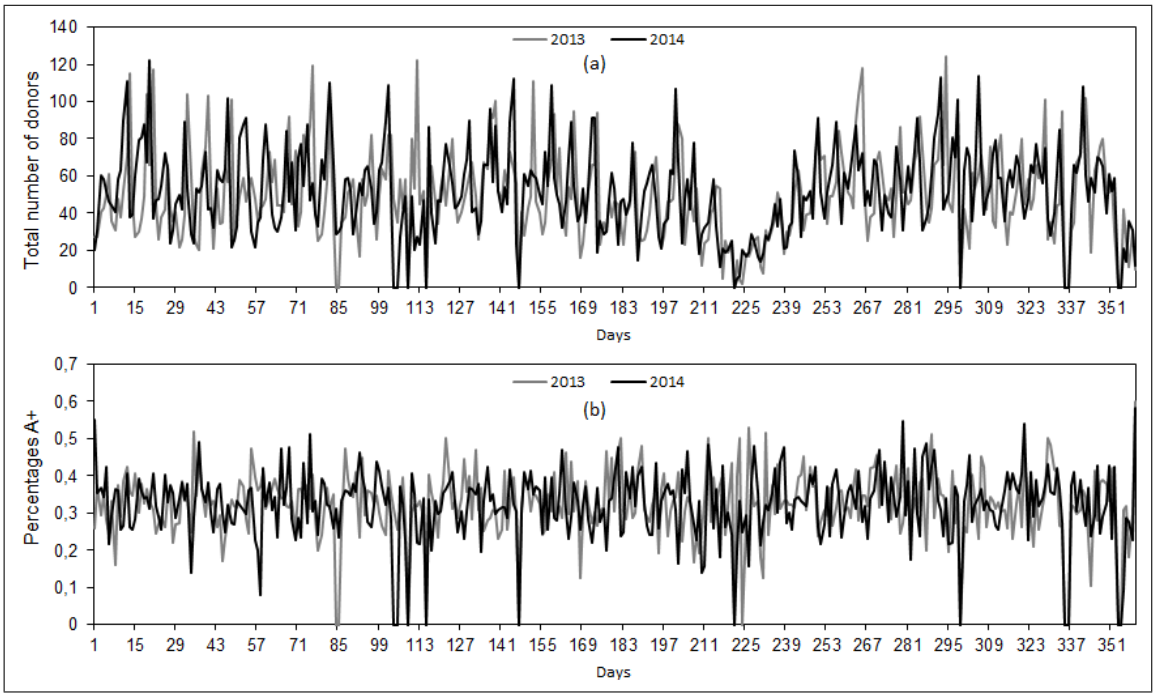

Figure 3.2. Daily number of whole blood donations in 2013 and 2014 according to the historical information of AVIS Milan: total number of donations (a) and percentage of type A Rh+ (b).

Figure 3.2 (a) reports the daily number of whole blood units produced per day, and Figure 3.2 (b) gives the relative percentage of units with type A Rh+ (data refer to 2013 and 2014, i.e., two years in which production balancing was not considered). It is observable that the number of blood units is not evenly balanced among the days, despite the goal of flattening the production both in terms of total number of units per day and for the different blood types. In particular, AVIS Milan would like to avoid high frequency oscillations,

while low frequency oscillations do not depend on scheduling and cannot be avoided. For example, the decreased production around days 220-240 in Figure 3.2 (a) corresponds to August when people are usually on holiday and do not donate. In some days no donation occurs because of the holidays. In current practice, the appointment scheduling decisions are manually made or supported by short-sighted tools. Even though these tools are able to reduce donor waiting times and physician overtime, and/or optimize other operational issues, they do not include any analysis of the daily blood production with respect to the storage while allocating time slots to donors. Hence, a comprehensive scheduling system must accommodate these contrasting aspects: improving operational level while providing a reliable supply of blood units in agreement with the storage requests.

To overcome these drawbacks, this dissertation proposes an architecture for planning the donations that consists of two phases, i.e., an offline preallocation of time slots for donation based on the blood type, and an online allocation. The output of the preallocation acts as an input for the allocation, in which the daily layout of prereserved slots is filled while the donors call for booking. Infact, when a donor calls for reservation, the allocation phase assigns a preallocated slot to him/her, among those slots prepared in the preallocation phase. The first reason of such decomposition into two phases is to balance the daily production of all blood types while assigning the slots to different blood types. The second reason is to assign these slots to the donors with their specific blood types.

The list of preallocated slots is refreshed (regenerated) after a certain number of reservations are received and/or at a fixed frequency (e.g., each day). The number of preallocated slots which have been converted into reserved slots is fed back to the preallocation phase (the assigned slots are no longer available and have to be considered as occupied) and the process is repeated. As a result, the plan for each day is given by the list of booked donors for that day, together with the number of empty slots that are left free for the non-booked donors who may arise to donate.

Besides the goal of production balancing, the daily layout of prereserved slots should meet some other requirements: the total number of slots should be around the expected number of donors, the slots should respect the proportions of the blood types, and an

appropriate number of spare slots should be preserved for non-booked donors. To meet these requirements, the future number of donors (both booked and non-booked) is required and should be predicted, e.g., based on the available historical data.

The proposed architecture is summarized in Figure 3.3. The preallocation phase receives the expected number of booked and non-booked donors, together with the number of occupied (already booked) slots, and provides the preallocated slots. Then, the allocation phase uses these preallocated slots to respond to the phone calls for reservation, and updates the list of occupied slots.

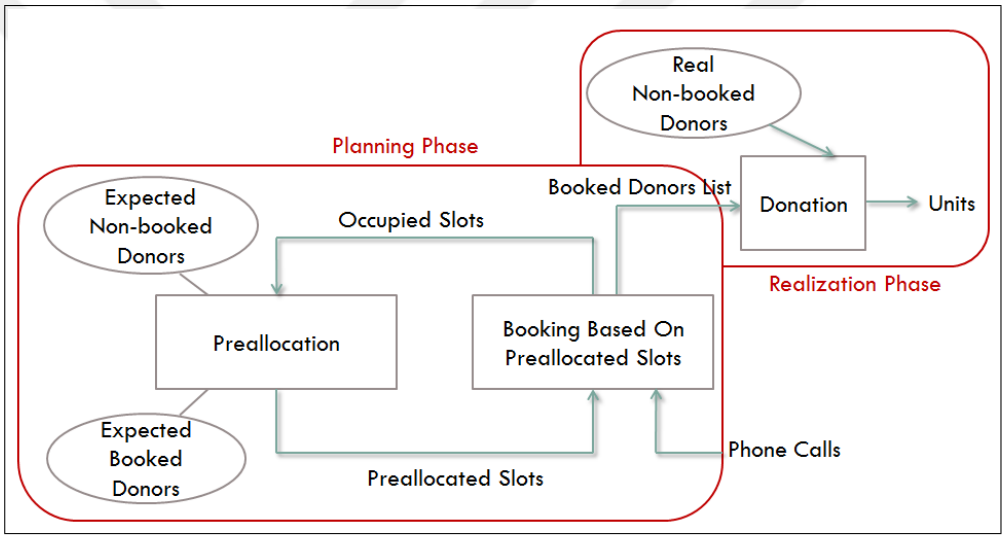

Figure 3.3. Proposed architecture for AVIS Milan.

As mentioned in the Introduction, the preallocation phase is based on an *MILP* model whereas the allocation phase assigns a prereserved slot to each donor with a prioritization policy. The developed model is presented by Baş *et al.* [6, 7]. They are presented in detail in the following subsections. The proposed deterministic model is described in Section 3.2.Iin Section 3.3 some valid inequalities are added to the model in order to reduce computational times, the prioritization policy is defined in Section 3.4, the overall framework for the deterministic model is defined in Section 3.5.3 and the numerical results for the deterministic model is presented in Section 3.5.

3.2. PROPOSED MODEL

The preallocation of the slots is optimized through an *MILP* model, whose aim is to preallocate a balanced number of slots for each blood type close to the expected number of booked donors in the considered time horizon. While doing so, some spare time slots are left empty for non-booked donors, physicians' peak loads are dispersed within each day by means of periodic physician capacities, and the total system capacity is restricted by considering maximum daily physician capacities.

A set of days T represents the considered time horizon, and all days $t \in T$ are divided into a set K of periods. Moreover, the set of blood types is denoted as B. We consider for each day t and each blood type b a number of slots x_t^b to preallocate (non-negative integer decision variable) and a number of already allocated slots a_t^b coming from previous reservations.

We assume an expected number d_b of booked donors for blood type b over T. Ideally, for each blood type, the summation over T of the already booked slots and the slots to preallocate should be equal to d_b, i.e., $\sum_{t \in T} \left(x_t^b + a_t^b\right) = d_b$. However, as mentioned, we do not know the exact number of booked donors in advance. Thus, we include a flexibility degree in complying with the summation by imposing that $\sum_{t \in T} \left(x_t^b + a_t^b\right)$ can lie in the interval from $(1 - \varepsilon)\, d_b$ to $(1 + \varepsilon)\, d_b$ for each blood type b. Parameter ε ($0 \leq \varepsilon \leq 1$) is an index of the associated flexibility: small ε values close to 0 refer to low flexibility, whereas higher values can be assumed in case of highly unknown donor arrivals. Forcing the system to allocate a given number of slots (actually a number in a range) is necessary in the presence of an objective function that aims at balancing the production of blood units among days and at avoiding periodic accumulation of donors. In fact, a perfect balancing with no overtime can be obtained with a null production. Furthermore, preallocating a number of slots higher than the necessary amount will lead to several empty slots because of fewer calls for reservation; thus, even though the preallocated slots are balanced, the actually occupied slots could be imbalanced. Hence, an appropriate selection of ε value is crucial, and too high values are not of interest for a practical application, meaning no information about the d_b parameters. In particular, we remark that high values of ε close to 1 may nullify the number of preallocated slots or generate an unnecessary higher number of slots.

As indicated above, a certain amount of slots should be left empty for non-booked donors, which is represented by n_t^b for blood type b and day t. Since non-booked donors may arrive in any period k of the day ($k \in K$), the fraction of n_t^b for period k is denoted with α_k (same or different fractions $\forall t \in T$ can be assumed).

The standard time r required for the service of a donor (considered while allocating new slots x_t^b) is assumed to be constant and equal for all donors. In addition, for the already booked slots a_t^b, a specific service duration can be set for each donor; we denote by R_{tk} the total time for the already allocated donors in period k of day t. Note that, on each day t, the number of already allocated slots a_t^b are grouped by blood type b, while the associated times R_{tk} are grouped by period k.

The overall capacity of the physicians in period k of day t is denoted by c_{tk}, and the service time required at day t and period k above the capacity c_{tk} is denoted by p_{tk}. We refer to p_{tk} as a *dispersion amount* rather than overtime because overtime is generally considered as the time beyond the overall daily capacity, while we consider periodic overtime due to possible accumulation of donors within periods of the day (e.g., in the morning). Hence, rather than penalizing the overall overtime, it may be useful to penalize the periodic accumulations of donors (i.e., the overtime in each period of the day) in order to disperse them towards the underutilized parts of the day. This also compensates for the higher arrival of non-booked donors in certain periods by allocating the booked donors in the others. Let us consider two examples that motivate the implementation of the periodic dispersion. In the first example, assume that no donors arrive at period $k = 1$ and that the overall service time in the other periods $k = 2, \ldots, K$ exceeds the corresponding periodic capacities (i.e., $\sum_{k=2,\ldots,K} c_{tk}$), while the overall system capacity ($\sum_{k=1,\ldots,K} c_{tk}$) is not exceeded. We might not have daily overtime according to the classical definition, even though we observe periodic ones. Thus, focusing on the daily overtime is not accurate since additional service time is actually required to serve donors arriving from $k = 2$. In the second example, assume that the service time required in the first period $k = 1$ is higher than the corresponding capacity c_{t1} and that the service times required in the other periods are not over their periodic capacities. Even though this situation does not result in daily overtime, it is not desirable since waiting times in the first period might be high. Hence, a dispersion amount for each period helps to

both balance service times among periods and reduce waiting times.

Table 3.1. Sets, parameters and decision variables for the preallocation model.

Sets	
B	set of blood types
T	time horizon
K	set of time periods in a day (same $\forall t \in T$)
Parameters	
d_b	expected number of booked donors over T with blood type b
ε	flexibility degree associated with d_b (same $\forall b \in B$)
a_t^b	number of already booked donors at day t with blood type b
n_t^b	expected number of non-booked donors at day t with blood type b
α_k	fraction of n_t^b in period k (same $\forall t \in T$)
c_{tk}	overall capacity of physicians (time) in period k of day t
r	standard time required for serving a donor
R_{tk}	time amount for serving the already booked donors in period k of day t
μ	maximum fraction of the total dispersion amount in a day with respect to the overall capacity in the same day
η	maximum variation weight (for the objective function)
δ_k	weight of the dispersion amount in period k (same $\forall t \in T$, for the objective function)
Decision variables	
x_t^b	number of preallocated slots for blood type b in day t
w_{tk}^b	number of preallocated slots for blood type b in period k of day t
y_t^b	number of planned units for blood type b in day t
z_t^b	absolute variation of y_t^b with respect its average value over T
v	maximum of the variations z_t^b $\forall t \in T, b \in B$
p_{tk}	dispersion amount in period k of day t

Variables are subject to the following constraints:

$$y_t^b = x_t^b + n_t^b + a_t^b \qquad \forall t \in T, b \in B \tag{3.1}$$

$$\sum_{\tau \in T} y_\tau^b - y_t^b |T| \leq z_t^b |T| \qquad \forall t \in T, b \in B \tag{3.2}$$

$$y_t^b |T| - \sum_{\tau \in\ T} y_\tau^b \leq z_t^b |T| \qquad \forall t \in T, b \in B \tag{3.3}$$

$$v \geq z_t^b \qquad \forall t \in T, b \in B \tag{3.4}$$

$$(1 - \varepsilon)\, d_b \leq \sum_{t \in T} \left(x_t^b + a_t^b\right) \qquad \forall b \in B \tag{3.5}$$

$$\sum_{t \in T} \left(x_t^b + a_t^b\right) \leq (1 + \varepsilon)\, d_b \qquad \forall b \in B \tag{3.6}$$

$$x_t^b = \sum_{k \in K} w_{tk}^b \qquad \forall t \in T, b \in B \tag{3.7}$$

$$r \sum_{b \in B} \left(w_{tk}^b + \alpha_k n_t^b\right) + R_{tk} \leq c_{tk} + p_{tk} \qquad \forall k \in K, t \in T \tag{3.8}$$

$$\sum_{k \in K} p_{tk} \leq \mu \sum_{k \in K} c_{tk} \qquad \forall t \in T \tag{3.9}$$

$$x_t^b \in \mathbb{N} \qquad \forall t \in T, b \in B \tag{3.10}$$

$$y_t^b \in \mathbb{N} \qquad \forall t \in T, b \in B \tag{3.11}$$

$$p_{tk} \geq 0 \qquad \forall k \in K, t \in T \tag{3.12}$$

$$z_t^b \geq 0 \qquad \forall t \in T, b \in B \tag{3.13}$$

$$w^b_{tk} \in \mathbb{N} \qquad \forall k \in K, t \in T, b \in B \tag{3.14}$$

$$v \geq 0 \tag{3.15}$$

Some additional decision variables are finally included to model the preallocation problem. The number of preallocated slots for blood type b in day t and period k is represented by a non-negative integer variable w^b_{tk}, whose sum over $k \in K$ provides x^b_t. The overall number of planned donations for blood type b at day t is y^b_t, which is given by $x^b_t + a^b_t + n^b_t$. The absolute variation of y^b_t with respect to its average value over the days t is denoted as z^b_t; thus, the summation and the maximum of z^b_t over t are linear terms to represent the variance of y^b_t. Sets, parameters and decision variables are summarized in Table 3.1.

Constraints (3.1) compute the number of blood units y^b_t for each day t and blood type b. Constraints (3.2) and (3.3) calculate the absolute variation z^b_t between y^b_t and its average value over T, and constraints (3.4) compute the maximum of such absolute variations. Constraints (3.5) and (3.6) force the total number of slots of type b to be around d_b, with tolerance ε; obviously, the number of slots is an integer number, so that the effect of these constraints is to bound $\sum_{t \in T} \left(x^b_t + a^b_t\right)$ between $\lceil (1-\varepsilon) d_b \rceil$ and $\lfloor (1+\varepsilon) d_b \rfloor$. Constraints (3.7) calculate, for each blood type b, the total number of preallocated slots x^b_t in day t based on the w^b_{tk} amounts. Constraints (3.8) calculate the dispersion amount p_{tk} based on service times and physicians' capacities. Constraints (3.9) limit the total dispersion amount in a day to be at most a given fraction of the overall capacity in the same day, where μ is such fraction.

In this formulation, we assume that all arriving donors make a donation, that all booked donors show up at the right period and day, and we do not consider different types of donations other than the whole blood donation (e.g., apheresis).

The primary objective of the model is to balance the production of each blood type b among the days, which corresponds to obtaining low z^b_t values. Moreover, the secondary goal is to minimize the dispersion amounts p_{tk}, where the amount of each period $k \in K$ is weighted

through a specific parameter δ_k. Hence, the following objective function is considered, which is composed by three terms:

$$\min \left\{ \sum_{b \in B} \sum_{t \in T} z_t^b + \eta v |T||B| + \sum_{t \in T} \sum_{k \in K} \delta_k p_{tk} \right\} \tag{3.16}$$

The first two terms (named OF1 and OF2, respectively) balance the production among days by reducing the absolute variations z_t^b; OF1 minimizes the total absolute variation with respect to the average production, while OF2 minimizes the maximum absolute variation among all days and all blood types. The third term (named OF3) minimizes the total weighted dispersion amount. The objective function may contain all three terms, as reported in (3.16), or alternatively it may include only one or two of them. If OF2 is neglected, constraints (3.4) can be removed from the model, while constraints (3.8) and (3.9) can be removed if OF3 is not considered.

Let us focus on the first two terms OF1 and OF2, which both aim at balancing the production. η is a positive parameter that represents the relative weight of the maximum absolute variation with respect to the total one: a low value of η favors the total variation, whereas higher values favor controlling the maximum variation. Decision variable v is multiplied by $|T|$ and $|B|$ to obtain, with $\eta = 1$, the same order of magnitude for the two terms. It is common in optimization problems that both the summation and the maximum of a set of decision variables are optimized. But, in our case, these two terms may lead to allocate a different number of slots x_t^b, since y_t^b is given by $x_t^b + n_t^b + a_t^b$ and the summation $\sum_{t \in T} x_t^b + a_t^b$ is not constrained to a value but to a range, due to (3.5) and (3.6). On the contrary, in several other problems, the overall amount is generally fixed and just differently allocated. Further details will be provided in Section 3.3.

We finally underline that our framework assigns a day t and a period k to each donor, and we can tune the granularity of the assignments based on the number K of periods in a day. For example, with $K = 3$, the donor is assigned to a period that is obviously longer than the actual duration of the slots; thus, for a practical application, the appointment can be further refined considering a real scheduling within such period. Alternatively, with higher values

of K, the length of the periods could be comparable with that of slots and the assigned period could also refer to the scheduled time.

3.3. SUBPROBLEMS AND VALID INEQUALITIES

In this section some subproblems are analyzed for some particular cases of the preallocation problem, to show the different behaviors of OF1 and OF2, and to derive valid inequalities that could speed up the computational times (See Section 3.3.1). In case of unavoidable imbalance, it can be time consuming to close the gap between the integer solution and the continuous relaxation in commercial solvers (e.g., CPLEX solver). In mixed integer programming, some variables are required to be integer and to be able to solve the mixed integer programming problems by the branch-and-bound method, the approach branches only the variables that are required to be integer. Branch-and-bound procedure solves the linear relaxation of the problem and stops if the solution is integer. If not, the branch-and-bound procedure continues, systematically generating sub problems to analyze and discarding those that do not improve the objective lower bound.

Subproblem 1: Let us first consider the case of one blood type b^* alone ($|B| = 1$), no preallocated slots ($a_t^{b^*} = 0$), a constant number of non-booked donors ($n_t^{b^*} = \bar{n}^{b^*}, \forall t$), and infinite capacities ($c_{tk} \to \infty, \forall t, k$).

Definition 1: Given a time horizon T and two values M_{min} and M_{max}, the Subproblem 1 consists of finding an integer value $N \in [M_{min}, M_{max}]$ and an allocation of slots to days $x_t^{b^*}$ ($t \in [1, \ldots, |T|]$) such that $\sum_{t \in T} x_t^{b^*} = N$ and $\sum_{t \in T} \left| x_t^{b^*} - \frac{N}{T} \right|$ is minimized. The range for $M = \sum_{t \in T} x_t^{b^*} + a_t^{b^*} = \sum_{t \in T} x_t^{b^*}$ is constrained between $M_{min} = \lceil (1 - \varepsilon) d_{b^*} \rceil$ and $M_{max} = \lfloor (1 + \varepsilon) d_{b^*} \rfloor$ because of constraints (3.5) and (3.6).

If there exists a multiple of $|T|$ in $[M_{min}, M_{max}]$, then a perfect balancing with $z_t^b = 0, \forall t$ is possible. Otherwise, the best possible balancing is given by allocating blocks of $|T|$ time slots (one slot for each day $t \in T$) until the remaining number of slots to allocate is lower than $|T|$. This remaining number N (with $0 < N < |T|$) is responsible of an unavoidable

imbalance because, at the optimum, N slots are allocated in N days (one for each day), while no slots are allocated in the other $|T| - N$ days. The remaining number N must be decided within the range $[N_{min}, N_{max}]$, where N_{min} and N_{max} are the remaining parts of M_{min} and M_{max}, respectively. Consequently, $z_t^{b^*} = 1 - \frac{N}{|T|}$ in the N days in which a remaining slot is allocated, while $z_t^{b^*} = \frac{N}{|T|}$ in the $|T| - N$ days in which no remaining slots are allocated. In the best case, the objective function (OF1) is:

$$\sum_{t \in T} z_t^{b^*} = N(1 - \frac{N}{|T|}) + (|T| - N)\frac{N}{|T|} = 2(N - \frac{N^2}{|T|}) \tag{3.17}$$

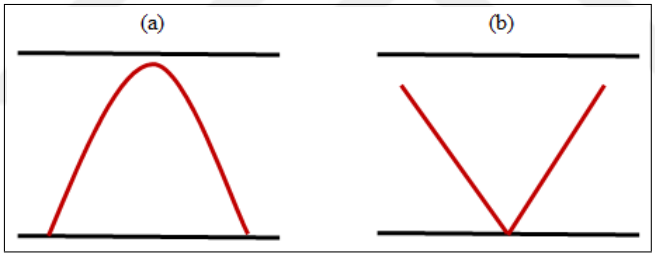

Figure 3.4. Demonstration of a concave parabola in (a) and a v-shaped function in (b).

The expression in (3.17) is a concave parabola (see Figure 3.4 (a)) with maximum in $N = \frac{|T|}{2}$ and null value in $N = 0$ and $N = |T|$.

If we consider the second objective function term (OF2):

$$v = \max\left\{z_t^{b^*}, t \in T\right\} = \begin{cases} 0 & N = \{0; |T|\} \\ \max\left\{1 - \frac{N}{|T|}; \frac{N}{|T|}\right\} & N \in [1, |T| - 1] \end{cases} \tag{3.18}$$

The expression in (3.18) assumes a null value for $N = 0$ and $N = |T|$ while for $N \in [1, |T| - 1]$ it is a V-shaped function (see Figure 3.4 (b)) with minimum value 0.5 in $N = \frac{|T|}{2}$.

The flexibility ε is responsible of the different behaviors between OF1 and OF2 in terms of allocated x_t^b. By constraining the domain of N to $[N_{min}, N_{max}]$, the minimum of (3.17) is in the farthest point from the maximum of the parabola, i.e., in N_{min} if $N_{min} < |T| - N_{max}$, or in N_{max} if $N_{min} > |T| - N_{max}$. As a consequence, OF1 prefers to allocate a number of M slots as close as possible to a multiple of $|T|$. On the contrary, if a perfect balance is not possible, the minimum of (3.18) is obtained by allocating a number of slots M as close as possible to the intermediate value between two consecutive multiples of $|T|$.

Intermediate behaviors can be obtained when both OF1 and OF2 are present, which can be adjusted by varying the relative weight η.

Let us consider the given numerical example with $T = 7$, $\varepsilon = 0.25$ and $d_b = 4$ in Figure 3.5.

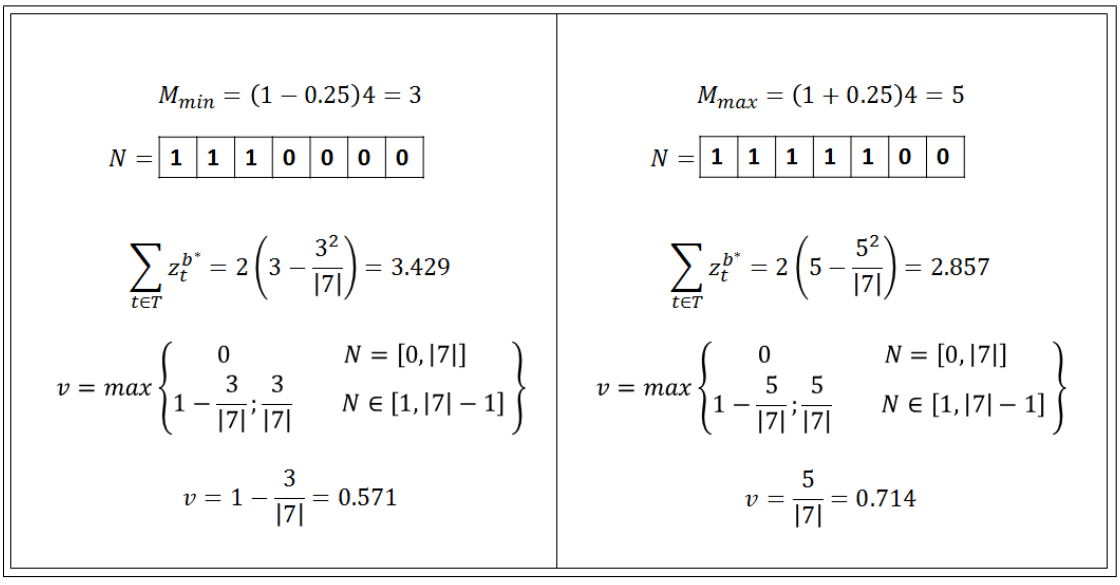

Figure 3.5. Numerical example for the valid inequalities.

The domain of N is constrained to $[N_{min} = 3, N_{max} = 5]$. Best possible balance is given

by allocating a first slot for each day until the remaining number N of slots is lower than T. When we consider $M_{min} = 3$, three slots are allocated one by one for each day and for the remaining four days, no slot is assigned to the day. So, the variate part with respect to the mean value of $y_t^{b^*}$ is given by $\frac{N}{|T|}$. Consequently, $z_t^b = 1 - \dfrac{3}{|7|}$ in the 3 days in which a remaining slot is allocated (actual value minus the average value), while $z_t^b = \dfrac{3}{|7|}$ in the $|7| - 3$ days in which no remaining slots are allocated. The expression in (3.17) is calculates as follows: $3(1 - \dfrac{3}{|7|}) + (|7| - 3)\dfrac{3}{|7|} = 2(3 - \dfrac{3^2}{|7|}) = 3.429$. Furthermore, since the remaining part $N = 3$, the expression in (3.18) is calculated as: $\max\left\{1 - \frac{3}{|7|}; \frac{3}{|7|}\right\}$. The minimum of (3.17) is in the farthest point from the maximum of the parabola (in $N_{max} = 5$ if $N_{min} = 3 > |T| = 7 - N_{max} = 5$). In contrast, the minimum of (3.18) is obtained by allocating a number of slots M as close as possible to the intermediate value (see Figure 3.4 (b)).

Subproblem 2: Let us consider again one blood type b^* alone ($|B| = 1$), a constant number of non-booked donors ($n_t^{b^*} = \bar{n}^{b^*}, \forall t$), and infinite capacities ($c_{tk} \to \infty, \forall t, k$). But, now, let us consider some preallocated slots $a_t^{b^*} \geq 0$. Two cases may occur:

- *Subproblem 2a:* if $a_t^{b^*} \leq \xi_t^{b^*}$, where $\xi_t^{b^*}$ denotes the optimal value of $x_t^{b^*}$ in the corresponding *Subproblem 1* where $a_t^{b^*} = 0, \forall t$ the same considerations derived for *Subproblem 1* still hold, and (3.17) and (3.18) are valid. Indeed, slots either belong to $a_t^{b^*}$ or $x_t^{b^*}$, but the constraint on the summation $\sum_{t \in T} x_t^{b^*} + a_t^{b^*}$ acts in the same way and the same values of OF1 and OF2 are reached.
- *Subproblem 2b:* if $a_t^{b^*} \leq \xi_t^{b^*}$, it is not possible to reach the same balancing of *Subproblem 2a*, and higher values of OF1 and OF2 are obtained. Even though we allocate the slots $x_t^{b^*}$ in a balanced way with $x_t^{b^*} = 0$, the higher value of $y_t^{b^*}$ with respect to the mean daily production remains, thus giving an imbalanced solution. In this subproblem, an analytical expression cannot be deriven as for *Subproblem 1*, but the best possible balancing can be derived with an algorithm (which is out the scope of this dissertation).

Subproblem 3: Let consider again infinite capacities ($c_{tk} \to \infty$, $\forall t, k$) but more than one blood type, i.e., $|B| > 1$. Due to the presence of unlimited capacity without overtime, the problem can be decomposed by balancing the blood types individually. Hence, for each blood type $b \in B$, a *Subproblem 1* or *Subproblem 2* can be considered.

Other problems: To move towards the complete problem, the assumption of infinite capacities (while also including OF3 in the objective function) is removed or a variable amount of non-booked donors n_t^b among days t are considered. In the most general case, both these aspects can be included.

By removing the assumption of infinite capacity, the slots of the different blood types cannot be preallocated individually, and we cannot decompose the problem anymore. Due to the competing blood types and the resulting dispersion amount costs, the best balancing previously obtained with *Subproblem 1* or *Subproblem 2a* could not be achieved. Indeed, while improving the balancing, the additional dispersion amount cost in OF3 could be more expensive than the corresponding reduction of OF1 and/or OF2, and the system would prefer more imbalanced solutions.

As for variable n_t^b values, an expression for the best possible balance can be derived while considering blood types individually, but a close analytical formula does not exist and an algorithm is required, as for the $a_t^{b^*} > \xi_t^{b^*}$ case of *Subproblem 2*.

In the following part, lower bound formulation for improving computational efficiency is derived.

3.3.1. Lower Bounds for Improving Computational Efficiency

The described problems that are explained in Section 3.3, can be used to derive some lower bounds on the value of OF1 and OF2. Valid inequalities can be added to reduce computational times, i.e., additional cuts that reduce the admissible region of only the continuous relaxation by bounding the values of OF1 and OF2.

In particular, in case of constant n_t^b, we bound OF1 and OF2 with the best possible balancing obtained for *Subproblem 1* and *Subproblem 2a*, respectively, in a closed analytical form which correspond to the minimum of (3.17) and (3.18).

The lowest value of OF1 for a given blood type b^* is given by (see *Subproblem 1*):

$$LB_{OF1} = 2 \min \left\{ \left(N_{min} - \frac{N_{min}^2}{|T|} \right) ; \left(N_{max} - \frac{N_{max}^2}{|T|} \right) \right\}$$

Its summation over the blood types, assuming a null value for those types where a perfect balancing is possible, gives the lower bound LB_{OF1}. Hence, the following lower bound constraint LB1 can be added to the model:

$$\sum_{b \in B} \sum_{t \in T} z_t^b \geq LB_{OF1} \tag{3.19}$$

We remark that LB_{OF1} is computed from the available data before the model is run and, thus, it is another model parameter.

The lowest value of OF2 for a given blood type b^* is given by (see (3.18)):

$$LB_{OF2} = max \begin{cases} 0 & N_{min} = 0 \text{ or } N_{max} = |T| \\ 1 - \frac{N_{max}}{|T|} & N_{min} > 0 \text{ and } N_{max} \leq \frac{|T|}{2} \\ \frac{N_{min}}{|T|} & N_{min} \geq \frac{|T|}{2} \text{ and } N_{max} < |T| \\ \frac{1}{2} & 0 < N_{min} < \frac{|T|}{2} \text{ and } \frac{|T|}{2} < N_{max} < |T| \end{cases}$$

Then, the highest of the values among the blood types b gives the lower bound LB_{OF2}. Hence, the following lower bound constraint LB2 can be added to the model:

$$v \geq LB_{OF2} \tag{3.20}$$

We remark that also LB_{OF2} is computed from the available data and it is another model parameter.

We underline that, due to the opposite behaviors of OF1 and OF2, the lower bounds LB_{OF1} and LB_{OF2} cannot be reached at the same time (when greater than 0). Though, the lower bound of their summation is for sure higher than $LB_{OF1} + \eta|T||B|LB_{OF2}$. Another constraint could be introduced to bound such summation; however, no closed formula are possible in this case.

We should compute $\min \sum_{b \in B} \sum_{t \in T} z_t^b + \eta v|T||B|$ for each possible combination of the values N of the different blood types, between their respective N_{min} and N_{max}; the minimum of the computed values is the lower bound for the summation.

In case of variable n_t^b, the absolute variations z_t^b also depend on the temporal patterns of n_t^b and the lower bounds cannot be computed by exploiting (3.17) and (3.18). They can be again computed for individual blood types with simple algorithms that search for the most balanced pattern $y_t^{b^*}, t \in T$ among the possible ones; however, in this thesis we only focus on lower bounds that can be analytically expressed.

3.4. PRIORITIZATION POLICY FOR THE ONLINE ALLOCATION OF SLOTS

The goal of the prioritization policy is to decide the best preallocated slot to propose when a donor calls to make a reservation. However, proposing only one day to the donor is not enough because the donor may have other constraints and could not accept the proposal. Thus, it is preferable to propose a list of possible days t and periods k, and let the donor choose among them. This might increase the donation frequency and the perceived usefulness of the donation from the donor. Hence, the goal of this second phase is to assign a score to each slot of the donor's blood type, such that the slots can be proposed one by one to the donor in a decreasing order of score until a slot is accepted. This is a good compromise between donor's needs (propose several alternatives) and production needs (propose the best alternative).

Basically, there are two points behind the prioritization of the slots and the assignment of the score: to fill the first available day and to keep the flexibility of the reservation system. The first point requires assigning the donor in the first available day according to his/her

blood type. In fact, keeping the first available slots empty may negatively affect the system if no further donors of the same blood type will ask to reserve a donation slot, because such slots will remain empty. The second point requires not to fill all of the preassigned slots of a day; otherwise, the range of choice for the next calling donor is reduced. Hence, flexibility means to assign donors in the day with the highest number of preallocated slots still available. Both points are taken into account while assigning scores, each one weighted by a value. The score S_{tkb} of slots w^b_{tk} is computed ($\forall t, k, b$) by the following linear formula:

$$S_{tkb} = \lambda_f w^b_{tk} - \lambda_d t \tag{3.21}$$

where t represents, according to the MILP model, the day in the time horizon, starting from the current one in which reservations are arriving ($t = 1$).

The first term generates higher scores for higher values of w^b_{tk}, i.e., when the flexibility remains higher if the donor of blood type b is allocated to t and k; the second term, due to the minus sign, generates higher scores when the donor is allocated to as low as possible values of t (i.e., to a closer day). λ_f is the weight of the flexibility term, while λ_d is the weight of the early allocation term.

Preallocated time slots are thus sorted and proposed one by one in a decreasing order of score. If the donor accepts the first proposed slot, this maximizes the goals of the system. In any case, we remark that every request for reservation is accepted: if no slots are available in the donor's suitable days, an additional slot is forced in addition to the preallocated ones.

Each time a reservation is made, the corresponding value x^b_t is reduced by 1 in view of the next calls, in order to respect the capacity. Moreover, before rerunning the preallocation model, all a^b_t values are updated with the new reservations.

Alternative scoring schemes have been also considered. However, the one we propose includes the two priorities highlighted by the staff of AVIS Milan (i.e., filling the first available day and keeping the flexibility of the reservation system) which are also common to several other blood collection centers.

When a donor calls to make a reservation, it might happen that either no more slots are available for his/her blood type, or slots are available only in days that are not accepted by the donor. In these cases, the donation slot is chosen considering the slots still available from the other blood types. In addition, in case no existing slots are accepted by the donor, a new slot is added to the plan. In this way, no donor is rejected. This choice may create an imbalance in the plan; however, due to the cyclic approach (Figure 3.3), this local imbalance is quickly reabsorbed. Anyway, we would like to mention that this situation is extremely rare in the case of well dimensioned d_b. If such an event happens, this means that the BD collection center has to revise the values of parameters d_b.

3.5. NUMERICAL RESULTS

In this section, we first present the results of the numerical experiments that analyze the behavior of the preallocation model, considering the impact of the modeling assumptions and the related parameters (Section 3.5.1). Moreover, further experiments to test computational aspects of the preallocation model and additional figures related to the overall framework are presented in the Section 3.5.2. Then, we evaluate the performance of the overall framework in Section 3.5.3 (preallocation model and prioritization policy) over a period of time with realistic instances derived from the AVIS Milan case (Section 3.5.3.1) and randomly generated instances (Section 3.5.3.2).

The preallocation model is implemented in IBM ILOG OPL and solved via CPLEX 12. The overall framework is implemented in Microsoft Visual Basic, and the developed solution integrates the data and the prioritization policy with the input and the output of the OPL model. All experiments are run on a Windows OS installed on a server with CPU Intel® Core™ i3, 2.40 GHz, and 12 GB of dedicated RAM.

3.5.1. Modeling Assumptions and Parameters

We test our modeling assumptions (i.e., the impact of d_b flexibility through ε, dispersion amount weights δ_k, and maximum fraction μ) and the behavior of the model in response to different parameter values. Tests are conducted with two classes of instances, namely, class

A and B; a time limit of 5400 seconds and a memory limit of 3 GB have been imposed in all experiments.

Instances of class A (to test ε and p_{tk}) are divided in two groups, denoted by A.1 and A.2, respectively, where the differences between the groups refer to the number of non-booked donors (n_t^b) for each day and blood type. Group A.1 includes the *balanced* instances, in which each n_t^b is randomly generated close to a nominal value, and the summation $\sum_t n_t^b$ over the days is the same for each blood type b. Group A.2 includes the *imbalanced* instances, in which the summation $\sum_t n_t^b$ is again the same for each blood type b as in Group A.1. But, in this group, an imbalance among the days is included by considering higher values in the first days of the planning horizon and lower values in the last days. The goal is to replicate practical cases, where there can be more non-booked donors than usual in some days, especially after holiday periods.

Table 3.2. Summary of the instances in Group A.

Group	**Non-booked level**	$\boldsymbol{d_b}, \forall \boldsymbol{b}$	$\sum_t \boldsymbol{n_t^b}, \forall \boldsymbol{b}$
A.1 & A.2	Null (N)	51	0
	Medium (M)	34	17
	High (H)	17	34

In both groups we further consider three levels for the fraction of non-booked donors with respect to the total number of donors: Null (N), Medium (M), and High (H). The list of the instances is reported in Table 3.2. Note that in all cases, for the sake of simplicity, booked donors are not considered ($a_t^b = 0, \forall t, b$). All instances are generated by considering 8 blood types ($|B| = 8$), 7 days of time horizon ($|T| = 7$) with 3 periods ($|K| = 3$), and capacities c_{tk} equal to 240, 300 and 180 minutes for $k = 1, 2, 3$, respectively, in all days t. Service durations are assumed to be 15 minutes ($r = 15$) and α_k fractions are considered equal to 0.5, 0.3 and 0.2 for $k = 1, 2, 3$, respectively. Equal fractions are also tested, moreover these parameter values are chosen to fit the tested case and to show the impact and accumulation

of non-booked donors.

Several experiments are conducted by varying ε and δ_k values, to test d_b flexibility and the weights of the dispersion amount with respect to the production balancing (OF1 and OF2). For each instance group and level of non-booked donors, 20 different combinations of ε and δ_k values are tested. In all these cases, we have considered the entire objective function (OF1 + OF2 + OF3) while setting $\eta = 1$. Moreover, as we want to exclude constraints (3.9) in the analysis, we assume a high μ value equal to the 100%, which is never reached in the considered instances.

Results are reported in Tables 3.3 and 3.4. It can be observed that, for higher $\boldsymbol{\delta} = [\delta_1\ \delta_2\ \delta_3]$ values (i.e., $\boldsymbol{\delta} = [8\ 6\ 3]$ and $\boldsymbol{\delta} = [0.8\ 0.6\ 0.3]$), the dispersion amount term OF3 is privileged, with consequent higher OF1 and OF2 values (meaning an imbalanced system) for lower ε values. For higher ε values, as expected, the system remains balanced also with high δ_k values, because of the flexibility given by the larger range around d_b.

Furthermore, lower δ_k values (i.e., $\boldsymbol{\delta} = [0.08\ 0.06\ 0.03]$ and $\boldsymbol{\delta} = [0.008\ 0.006\ 0.003]$) result in completely balanced solutions as long as $\varepsilon > 0$, which also show decreasing OF3 values while increasing ε. Only for Level H of Group A.2, the high imbalanced arrival of non-booked donors always prevents from a perfect balancing (OF1$\neq$0 and OF2$\neq$0, $\forall\varepsilon$) and determines increased OF3 values with ε, because the system tries to compensate the imbalance by adding slots. When $\varepsilon = 0.25$ case is considered for Level H of Group A.2, OF1 and OF2 values are decreased while δ_k values are decreased. But when $\varepsilon = 0.5$ case is considered, for lower values of δ_k values (i.e., $\boldsymbol{\delta} = [0.08\ 0.06\ 0.03]$ and $\boldsymbol{\delta} = [0.008\ 0.006\ 0.003]$), OF1 and OF2 term values are increased and the system allocated more slots in order to compensate the imbalancing. But the total objective function (summation of the three terms) value is reduced, although the terms (OF1 and OF2) in the objective function are increased.

CPU times for Group A.1.1 and Group A.1.2 instances are presented in Appendix A (CPU times of the Tables 3.3 and 3.4). In the tables, the first two columns show non-booked donor levels with five different ε values respectively.

Table 3.3. Impact of ε and δ_k on the objective function terms for Group A.1. * and $^\bullet$ indicate that the run is terminated because the memory limit or the time limit has been reached, respectively; the maximum optimality gap over these cases is 2.69%.

Non-booked		$\boldsymbol{\delta = [8\ 6\ 3]}$			$\boldsymbol{\delta = [0.8\ 0.6\ 0.3]}$			$\boldsymbol{\delta = [0.08\ 0.06\ 0.03]}$			$\boldsymbol{\delta = [0.008\ 0.006\ 0.003]}$		
level	ε	**OF1**	**OF2**	**OF3**	**OF1**	**OF2**	**OF3**	**OF1**	**OF2**	**OF3**	**OF1**	**OF2**	**OF3**
Null	0.00	22.86	40.00	3240.00	22.86	40.00	324.00	22.86	40.00	32.40	22.86*	40.00*	3.24*
(N)	0.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Medium	0.00	22.86	40.00	3249.00	22.86	40.00	324.90	22.86	40.00	32.49	22.86	40.00	3.25
(M)	0.25	13.71	48.00	369.00	13.71	48.00	36.90	0.00	0.00	25.29	0.00	0.00	2.53
	0.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
High	0.00	22.86	40.00	5058.00	22.86	40.00	505.80	22.86$^\bullet$	40.00$^\bullet$	50.58$^\bullet$	22.86*	40.00*	5.06*
(H)	0.25	22.86	40.00	3618.00	22.86*	40.00*	361.80*	0.00	0.00	43.38	0.00	0.00	4.34
	0.50	27.43*	32.00*	2898.00*	27.43*	32.00*	289.80*	0.00	0.00	43.38	0.00	0.00	4.34
	0.75	0.00	0.00	2898.00	0.00	0.00	289.80	0.00	0.00	28.98	0.00	0.00	2.90
	1.00	0.00	0.00	2898.00	0.00	0.00	289.80	0.00	0.00	28.98	0.00	0.00	2.90

Table 3.4. Impact of ε and δ_k on the objective function terms for Group A.2. * and $^\bullet$ indicate that the run is terminated because the memory limit or the time limit has been reached, respectively; the maximum optimality gap over these cases is 13.63%.

Non-booked		$\boldsymbol{\delta = [8\ 6\ 3]}$			$\boldsymbol{\delta = [0.8\ 0.6\ 0.3]}$			$\boldsymbol{\delta = [0.08\ 0.06\ 0.03]}$			$\boldsymbol{\delta = [0.008\ 0.006\ 0.003]}$		
level	ε	**OF1**	**OF2**	**OF3**	**OF1**	**OF2**	**OF3**	**OF1**	**OF2**	**OF3**	**OF1**	**OF2**	**OF3**
Null	0.00	22.86	40.00	3240.00	22.86	40.00	324.00	22.86	40.00	32.40	22.86*	40.00*	3.24*
(N)	0.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Medium	0.00	22.86	40.00	3666.00	22.86	40.00	366.60	22.86	40.00	36.66	22.86	40.00	3.67
(M)	0.25	13.71	48.00	786.00	13.71	48.00	78.60	0.00	0.00	29.46	0.00	0.00	2.94
	0.50	0.00	0.00	480.00	0.00	0.00	48.00	0.00	0.00	4.80	0.00	0.00	0.48
	0.75	0.00	0.00	480.00	0.00	0.00	48.00	0.00	0.00	4.80	0.00	0.00	0.48
	1.00	0.00	0.00	480.00	0.00	0.00	48.00	0.00	0.00	4.80	0.00	0.00	0.48
High	0.00	54.86	152.00	6402.00	54.86	152.00	640.20	54.86	152.00	64.02	54.86	152.00	6.40
(H)	0.25	69.14*	184.00*	5304.00*	68.57	176.00	533.10	41.14$^\bullet$	120.00$^\bullet$	78.42$^\bullet$	41.14*	120.00*	7.84*
	0.50	69.14*	184.00*	5304.00*	68.57	176.00	533.10	27.43	96.00	89.22	34.29	88.00	9.28
	0.75	69.14*	184.00*	5304.00*	68.57	176.00	533.10	16.00	56.00	107.22	16.00	56.00	10.72
	1.00	69.14*	184.00*	5304.00*	68.57	176.00	533.10	13.71	48.00	110.82	27.43*	32.00*	11.80*

The remaining columns show the corresponding CPU times with four different δ_k values. While non-booked donor level is increased, CPU times are increased in parallel.

Table 3.5. Impact of η on the objective function terms for Group A.1; * indicates that the run is terminated because the memory limit has been reached.

Non-booked		$\eta = 0.1$			$\eta = 1$			$\eta = 10$		
level	ε	**OF1**	**OF2**	**OF3**	**OF1**	**OF2**	**OF3**	**OF1**	**OF2**	**OF3**
Medium	0.00	22.86	4.00	32.49	22.86	40.00	32.49	22.86	400.00	32.49
	0.25	13.71	4.80	3.69	0.00	0.00	25.29	0.00	0.00	25.29
	0.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
High	0.00	22.86	4.00	50.58	22.86	40.00	50.58	22.86*	400.00*	50.58*
	0.25	0.00	0.00	43.38	0.00	0.00	43.38	0.00	0.00	43.38
	0.50	0.00	0.00	43.38	0.00	0.00	43.38	0.00	0.00	43.38
	0.75	0.00	0.00	28.98	0.00	0.00	28.98	0.00	0.00	28.98
	1.00	0.00	0.00	28.98	0.00	0.00	28.98	0.00	0.00	28.98

Table 3.6. Impact of η on the objective function terms for Group A.2; * indicates that the run is terminated because the memory limit has been reached.

Non-booked		$\eta = 0.1$			$\eta = 1$			$\eta = 10$		
level	ε	**OF1**	**OF2**	**OF3**	**OF1**	**OF2**	**OF3**	**OF1**	**OF2**	**OF3**
Medium	0.00	22.86	4.00	36.66	22.86	40.00	36.66	22.86	400.00	36.66
	0,25	13.71*	4.80*	7.86*	0.00	0.00	29.46	0.00	0.00	29.46
	0,50	0.00	0.00	4.80	0.00	0.00	4.80	0.00	0.00	4.80
	0,75	0.00	0.00	4.80	0.00	0.00	4.80	0.00	0.00	4.80
	1,00	0.00	0.00	4.80	0.00	0.00	4.80	0.00	0.00	4.80
High	0.00	54.86	15.20	64.02	54.86	152.00	64.02	54.86	1520.00	64.02
	0.25	45.71	13.60	71.22	41.14	120.00	78.42	41.14	1200.00	78.42
	0.50	32.00	11.20	82.02	27.43	96.00	89.22	34.29	880.00	92.82
	0.75	32.00	11.20	82.02	16.00	56.00	107.22	16.00	560.00	107.22
	1.00	32.00	11.20	82.02	13.71	48.00	110.82	27.43*	320.00*	118.02*

The realization of memory limit started at this point. The gap between the integer solution and the continuous relaxation could not close from a computational point of view in some of the instances. The impact of the two terms related to production balancing in the objective function (OF1 and OF2) is analyzed by varying the relative weight η.

Indeed, three different values of η (0.1, 1, and 10) are tested, considering the same values of ε used for the previous analysis, fixing $\delta_k = \{0.08; 0.06; 0.03\}$, and including all of the three terms in the objective function. Results in terms of the values obtained for the terms of the objective function are reported in Tables 3.5 and 3.6 for the medium and high levels of non-booking donors, respectively.

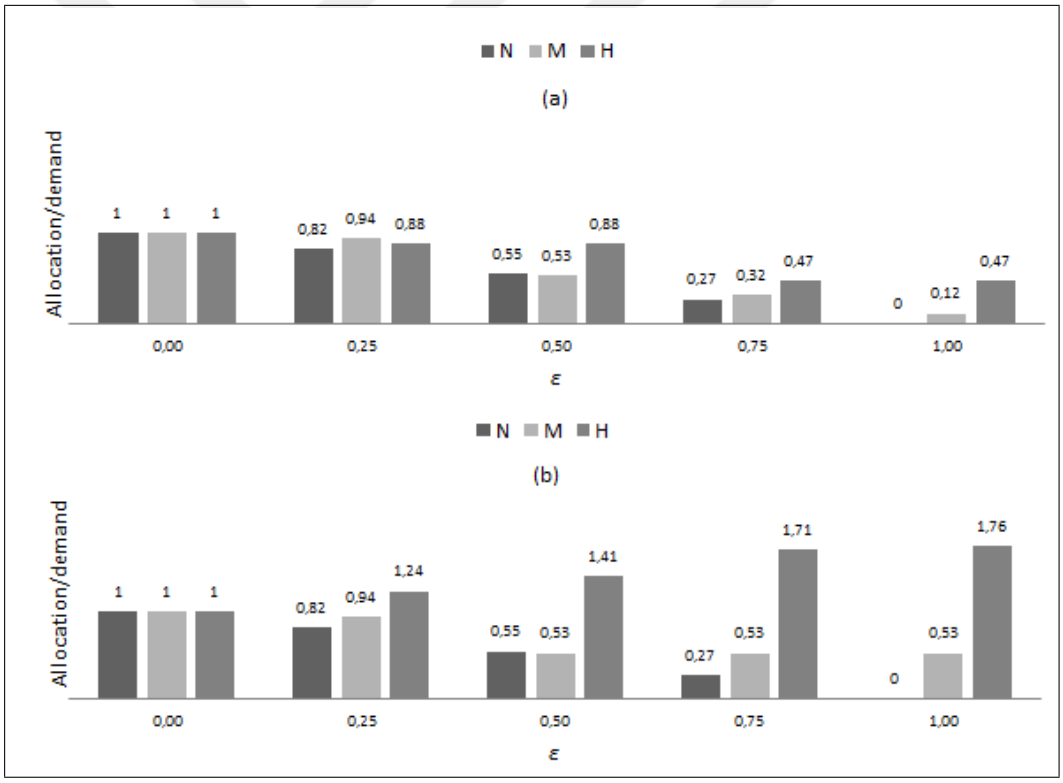

Figure 3.6. Allocated slots ($\sum_t \sum_b x_t^b$) over demand ($\sum_b d_b$) for 5 different ε values and 3 non-booked donor levels: null (N), medium (M) and high (H). Subfigure (a) refers to Group A.1 and subfigure (b) to Group A.2.

Figure 3.6 shows the ratio between the number of allocated slots ($\sum_t \sum_b x_t^b$) and the total

number of expected donors ($\sum_b d_b$) as a function of ε for both groups and all levels of non-booked donors (for $\boldsymbol{\delta} = [0.08\ 0.06\ 0.03]$). It can be seen that, as expected, the number of allocated slots decreases while ε increases (obtaining a null production with $\varepsilon = 0$), except in level H of Group A.2.

Thus, better balancing and lower dispersion amounts for higher ε values are observed due to the reduced number of assigned slots. On the contrary, as for level H of Group A.2, the model allocates more slots to partially compensate the imbalance given by the high and imbalanced amount of non-booked donors. Anyway, as mentioned in Section 3.2 too high values of ε are not of interest for a practical application, meaning no information about the d_b parameters.

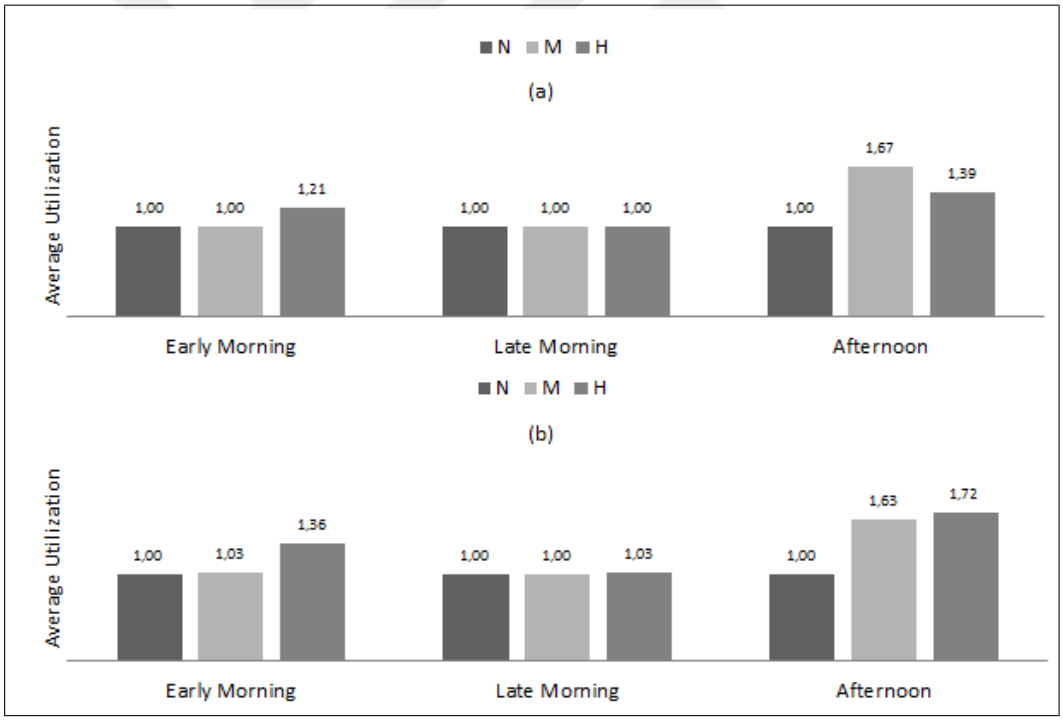

Figure 3.7. Average utilization for 3 different periods, namely *early morning* ($k = 1$), *late morning* ($k = 2$), and *afternoon* ($k = 3$). Subfigure (a) refers to Group A.1 and subfigure (b) to Group A.2, both including the 3 levels of non-booked donors.

Other analyses are conducted to investigate how the dispersion amount is divided among

periods $k \in K$ (Figure 3.7), how booked and non-booked donors are scheduled in a day (Figure 3.8), and how many units per day are produced while trying to balance the production (Figure 3.9). All figures refer to the case with all terms in the objective function (OF1, OF2 and OF3), and with parameters $\boldsymbol{\delta} = [0.08\ 0.06\ 0.03]$, $\eta = 1$ and $\varepsilon = 0.25$.

Figure 3.7 shows the average utilization among days t for each period k, where utilization in t and k is given by $\left(r \sum_{b \in B} \left(w_{tk}^{b} + \alpha_k n_t^b\right) + R_{tk}\right) / c_{tk}$, and for the 3 levels of non-booked donors (N, M and H). In general, results show the possibility of shifting the donor accumulation to the period k with the lowest weight δ_k. However, for level H, over-utilization is also present in the most weighted period of the day (i.e., $k = 1$ in our case) because of the high and imbalanced number of non-booked donors, which are not controllable.

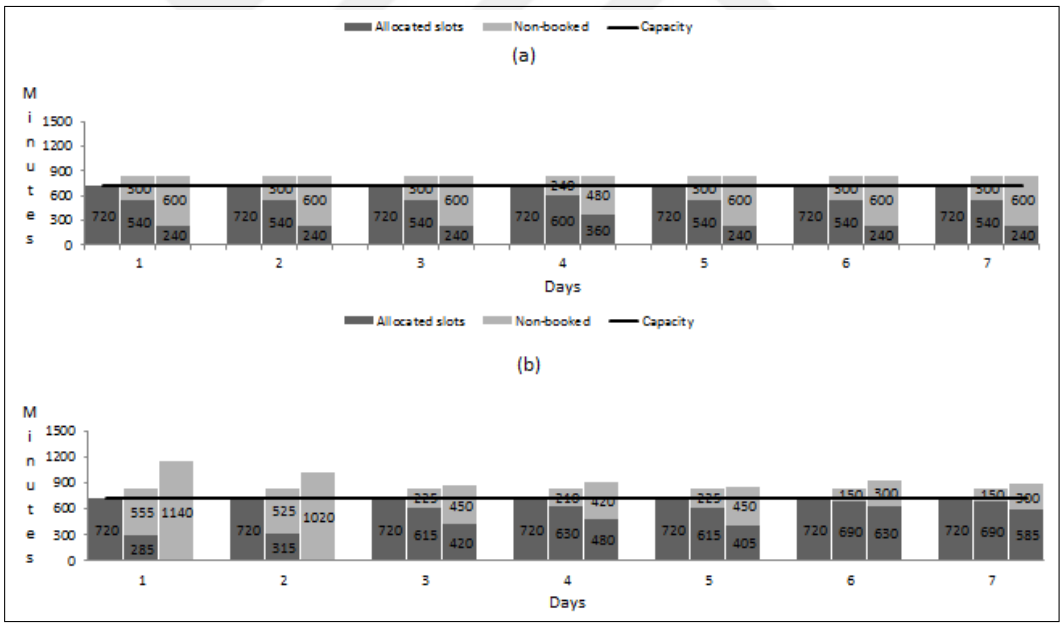

Figure 3.8. Daily workload for the 3 levels of non-booked donor: N (first column in each day), M (second column in each day), and H (third column in each day). Subfigure (a) refers to Group A.1 and subfigure (b) to Group A.2.

Figure 3.8 shows the daily workload (compared with the capacity) for each day t. It can be seen that the model equally divides the total workload among days, as production balancing

is the primary objective. Moreover, equal proportions of booked and non-booked donors are found in all days for Group A.1, while the proportions vary from day to day in Group A.2. This means that, in the presence of balanced non-booked donors, the system equally allocates slots in the days to keep the situation balanced, while slots are preallocated to compensate the imbalanced input in the presence of imbalanced non-booked donors.

Lastly, Figure 3.9 shows the minimum, average and maximum daily production of blood units among days t for a given blood type b (values are the same for all blood types, as the same d_b values are used $\forall b$). The model perfectly balances the daily production; the only difference is again due to the imbalanced number of non-booked donors that affects the production in the H case of Group A.2.

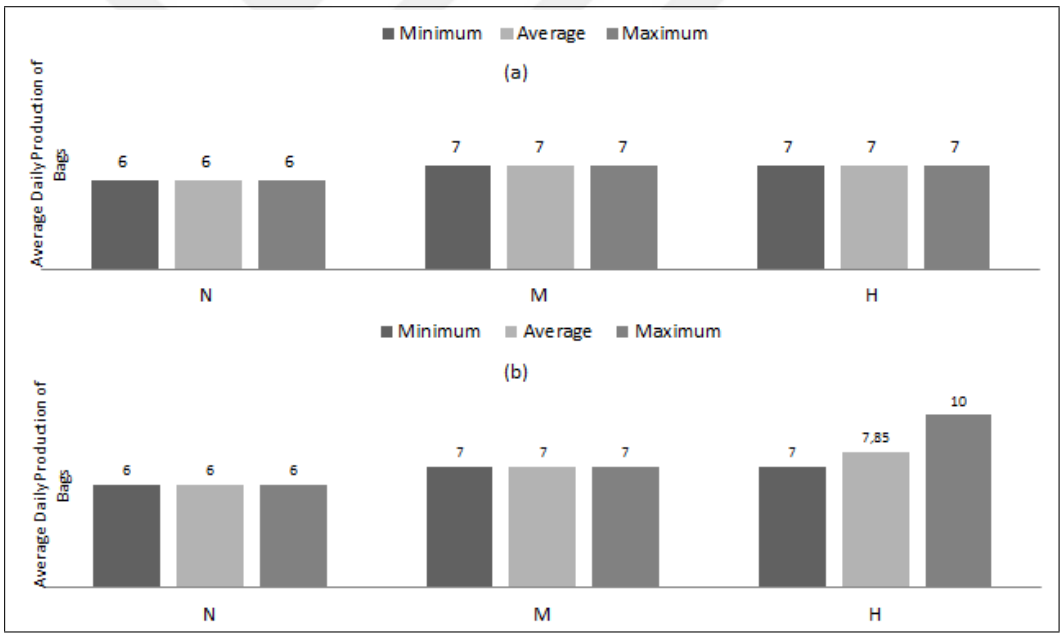

Figure 3.9. Minimum, average and maximum daily production for a blood type (same values for all types) for the 3 levels of non-booked donors: null (N), medium (M) and high (H). Subfigure (a) refers to Group A.1 and subfigure (b) to Group A.2.

It can be seen from the analyses that the amount of non-booked donors, in the presence of imbalanced arrivals, has a great impact on the system, both in terms of utilization dispersion and balancing (see in particular level H of Group A.2). However, with an appropriate set

of parameters, the model is able to find a good trade-off between production balancing and accumulation reduction also in this case. Thus, the decision maker can choose the preferred set of parameters based on his/her priorities and the features of the blood collection center.

Another parameter with a high impact is ε, which models the flexibility degree associated with d_b. As shown, high values of ε may deteriorate the quality of the solution and, in particular, reduce the amount of produced units. Thus, the decision maker should accurately set this value not to constrain the solution on a number of donors different from the actual one, but also not to reduce the production without a real motivation coming from the data.

Instances of class B (to test μ and its relationship with c_{tk} and δ_k) are divided in two groups, namely, B.1 and B.2. Instances of Group B.1 are derived from the *balanced* Group A.1, with non-booked level M and $\varepsilon = 0.25$, while instances of Group B.2 from the *imbalanced* Group A.2, again with non-booked level M and $\varepsilon = 0.25$. The alternative values for the parameters in both B.1 and B.2 are reported below:

- μ: equal to either 0, 0.5, or 1;
- $\mathbf{c_t} = [c_{t1}\ c_{t2}\ c_{t3}]$: equal to either $[240\ 300\ 180]\ \forall t$ (with $\sum_k c_{tk} = 720$), or $[270\ 330\ 210]\ \forall t$ (with $\sum_k c_{tk} = 810$);
- $\boldsymbol{\delta} = [\delta_1\ \delta_2\ \delta_3]$: equal to either $[0.095\ 0.05\ 0.005]$, $[0.08\ 0.06\ 0.03]$, or $[0.055\ 0.05\ 0.045]$.

Instances of class B outcomes are reported in Table 3.7. All tests have been solved to optimality and the optimal solutions always show $OF1 = OF2 = 0$ in all cases. Results show that the preallocation model successfully takes into account the dispersion of donors, i.e., it first allocates slots to fill all periodic capacities c_{tk} and then allocates slots in the periods with lower δ_k values. Moreover, as expected, infeasibilities can be removed by increasing the parameter μ. Finally, we can observe that, in the *balanced* Group B.1, the slots are assigned in such a way to first saturate the capacity of the period with the lowest δ_k ($k = 3$ in our case) and then the capacity of the period with the second last δ_k ($k = 2$ in our case). On the contrary, in the *imbalanced* Group B.2, slots above the capacity are exploited in $k = 1$ and $k = 2$ at the same time.

Table 3.7. Impact of μ, δ_k and c_{tk} on OF3 for Group B.1 (a) and B.2 (b); *inf.* indicates infeasibility.

$\sum_k c_{tk}$		$\boldsymbol{\delta} = [\mathbf{0.095\ 0.05\ 0.005}]$				$\boldsymbol{\delta} = [\mathbf{0.08\ 0.06\ 0.03}]$				$\boldsymbol{\delta} = [\mathbf{0.055\ 0.05\ 0.045}]$			
$\forall t$	μ	$\sum_t p_{t1}$	$\sum_t p_{t2}$	$\sum_t p_{t3}$	**OF3**	$\sum_t p_{t1}$	$\sum_t p_{t2}$	$\sum_t p_{t3}$	**OF3**	$\sum_t p_{t1}$	$\sum_t p_{t2}$	$\sum_t p_{t3}$	**OF3**
720	0.00	inf.	inf.	inf.	inf.	inf.	inf.	inf.	inf.	inf.	inf.	inf.	inf.
	0.50	0.00	0.00	843.00	4.22	0.00	0.00	843.00	25.29	0.00	12.00	828.00	37.86
	1.00	0.00	0.00	843.00	4.22	0.00	0.00	843.00	25.29	0.00	12.00	828.00	37.86
810	0.00	0.00	0.00	0.00	1.07	0.00	0.00	0.00	6.39	0.00	0.00	0.00	9.51
	0.50	0.00	0.00	213.00	1.07	0.00	0.00	213.00	6.39	0.00	12.00	198.00	9.51
	1.00	0.00	0.00	213.00	1.07	0.00	0.00	213.00	6.39	0.00	12.00	198.00	9.51

(a)

$\sum_k c_{tk}$		$\boldsymbol{\delta} = [\mathbf{0.095\ 0.05\ 0.005}]$				$\boldsymbol{\delta} = [\mathbf{0.08\ 0.06\ 0.03}]$				$\boldsymbol{\delta} = [\mathbf{0.055\ 0.05\ 0.045}]$			
$\forall t$	μ	$\sum_t p_{t1}$	$\sum_t p_{t2}$	$\sum_t p_{t3}$	**OF3**	$\sum_t p_{t1}$	$\sum_t p_{t2}$	$\sum_t p_{t3}$	**OF3**	$\sum_t p_{t1}$	$\sum_t p_{t2}$	$\sum_t p_{t3}$	**OF3**
720	0.00	inf.	inf.	inf.	inf.	inf.	inf.	inf.	inf.	inf.	inf.	inf.	inf.
	0.50	60.00	0.00	843.00	9.22	60.00	12.00	798.00	29.46	75.00	27.00	738.00	38.69
	1.00	60.00	1.50	828.00	9.22	60.00	12.00	798.00	29.46	75.00	27.00	738.00	38.69
810	0.00	inf.	inf.	inf.	inf.	inf.	inf.	inf.	inf.	inf.	inf.	inf.	inf.
	0.50	7.50	1.50	258.00	2.08	7.50	27.00	198.00	8.16	30.00	27.00	153.00	9.89
	1.00	7.50	0.00	273.00	2.08	7.50	27.00	198.00	8.16	30.00	27.00	153.00	9.89

(b)

3.5.2. Run Times and Lower Bounds

As it is discussed in Section 3.3, some valid inequalities are derived and in Subsection 3.3.1, lower bound equations are formulated to speed up the computational times. In this section, different model formulations (i.e., with alternative objective functions) are solved both neglecting and including LB1 and LB2, to analyze their impact on the computational times.

Eight test instances are considered, to give a wide range of situations, which have been generated as follows. The time horizon T is set either equal to 14 or 28 days, and each day is divided in $|K| = 3$ parts; the set B includes 8 blood types; d_b vector for the 8 blood types is assumed to be $[140\ 28\ 56\ 5\ 14\ 4\ 140\ 42]$ for $T = 14$ and $[280\ 56\ 112\ 10\ 28\ 9\ 280\ 84]$ for $T = 28$; two flexibility values ε are chosen, i.e., 0.1 and 0.25; dispersion amount weights δ_k are selected to be 0.08, 0.06 and 0.03 for $k = 1, 2, 3$, respectively; α_k fractions are taken equal to 0.4, 0.3 and 0.3 for $k = 1, 2, 3$; respectively; the number of non-booked donors n_t^b is assumed constant over the days and the values for the different blood types are set equal to $[2\ 0\ 1\ 0\ 0\ 0\ 2\ 1]$; the capacity c_{tk} is assumed equal to 450 minutes for each day t and period k; service duration r is assumed to be 20 minutes and such value is also used to compute R_{tk} when required. Finally, the maximum variation weight η in the objective function and μ in constraints (3.9) are set equal to 1% and to the 5%, respectively.

Different values are considered for a_t^b, to evaluate both the case without ($a_t^b = 0,\ \forall t, b$) and with previously allocated slots. In the latter case, each allocated slot a_t^b is randomly generated within the 40%-80% range of the corresponding optimal values ξ_t^b of x_t^b obtained without previously booked donors ($a_t^b = 0$). Moreover, the corresponding R_{tk} values are generated by randomly splitting a_t^b over the $|K|$ periods within the day t. Names and characteristics of the instances are summarized in Table 3.8.

A total of 8 configurations of objective functions and LBs have been analyzed for each instance, thus obtaining 64 combinations for the pair instance-configuration. A 5400 seconds time limit and a 3 GB memory limit have been set in all cases.

Results are reported in Tables 3.9–3.11 in terms of CPU time (second), objective function (OF) value, and lower bound value of the objective function (OFlow), i.e., best bound given by CPLEX.

Table 3.8. Instances for the analysis of computational times and LBs.

Instance	**T**	ε	a_t^b
I.1	14	0.1	0
I.2	14	0.1	40-80%
I.3	14	0.25	0
I.4	14	0.25	40-80%
I.5	28	0.1	0
I.6	28	0.1	40-80%
I.7	28	0.25	0
I.8	28	0.25	40-80%

Tables also compare the case with and without LB by showing the time reduction and the per cent OFlow increase when LB is included.

Table 3.9. Results for the cases OF1+OF3; * indicates that the run is terminated because the memory limit has been reached.

	OF1+OF3			**OF1+OF3 with LB1**			**Time**	**OFlow**
Inst.	**CPU time**	**OF**	**OFlow**	**CPU time**	**OF**	**OFlow**	**reduction**	**improvement**
I.1	0.27	12.14	12.14	0.24	12.14	12.14	0.03	0.00%
I.2	0.18	12.14	12.14	0.17	12.14	12.14	0.02	0.00%
I.3	16.67	10.43	10.43	0.14	10.43	10.43	16.54	0.00%
I.4	0.21	12.14	12.14	0.20	12.14	12.14	0.01	0.00%
I.5	17.10	24.43	24.43	0.40	24.43	24.43	16.69	0.00%
I.6	0.30	24.43	24.43	0.24	24.43	24.43	0.07	0.00%
I.7	3352.40*	21.93*	20.12*	0.97	21.93	21.93	3351.44	9.01%
I.8	0.49	24.43	24.43	2.04	24.43	24.43	-1.55	0.00%

We first observe very low computational times, which increase with the number of days in T, with parameter ε, and in the absence of preallocated slots ($a_t^b = 0 \; \forall t, b$). In particular, the absence of preallocated slots introduces symmetries among days, which result in longer computational times.

Table 3.10. Results for the cases OF2+OF3.

	OF2+OF3			OF2+OF3 with LB2			Time	OFlow
Inst.	**CPU time**	**OF**	**OFlow**	**CPU time**	**OF**	**OFlow**	**reduction**	**improvement**
I.1	0.47	80.00	80.00	0.46	80.00	80.00	-0.01	0.00%
I.2	0.23	80.00	80.00	0.18	80.00	80.00	0.05	0.00%
I.3	1.95	72.00	72.00	0.26	72.00	72.00	1.69	0.00%
I.4	0.19	72.00	72.00	0.34	72.00	72.00	-0.16	0.00%
I.5	0.67	152.00	152.00	1.70	152.00	152.00	-1.03	0.00%
I.6	0.27	152.00	152.00	0.28	152.00	152.00	-0.01	0.00%
I.7	15.74	136.00	136.00	0.37	136.00	136.00	15.37	0.00%
I.8	0.24	136.00	136.00	0.22	136.00	136.00	0.01	0.00%

In most of the cases, the problem is solved in few seconds or even in less than 1 second. This guarantees the applicability of the model and allows refreshing the preallocation of the slots with a high frequency. Even though we suggest a daily refresh, higher frequencies are possible, which may be useful when a high number of reservation calls arrive. Longer computational times and memory or time limit stops are observed in instances I.7 and I.8, which are the most demanding instances in terms of higher variability ε and longer horizon T.

Comparing the results with and without LB, we can see the benefit of including LBs in terms of CPU time reduction. As for OF1+OF3, the respective LB guarantees to get the optimum also in instance I.7 for which a memory limit is reached without LB.

Moreover, both in OF1+OF3 and OF2+OF3, the presence of LB makes almost all CPU times lower than one second. Concerning OF1+OF2+OF3, LBs generally improve the CPU

times but none of the alternatives (LB1, LB2 or LB1+LB2) seems to be the best one, e.g.,

Table 3.11. Results for the cases OF1+OF2+OF3 without LBs and with LB1 (a), with LB2 (b), and with both LB1 and LB2 (c); * indicates that the run is terminated because the memory limit has been reached; TL indicates the time limit has been reached.

	OF1+OF2+OF3			OF1+OF2+OF3 with LB1			Time	OFlow
Inst.	**CPU time**	**OF**	**OFlow**	**CPU time**	**OF**	**OFlow**	**reduction**	**improvement**
I.1	0.37	92.14	92.14	0.27	92.14	92.14	0.10	0.00%
I.2	0.16	92.14	92.14	0.15	92.14	92.14	0.01	0.00%
I.3	501.95	84.86	84.86	60.21	84.86	84.86	441.74	0.00%
I.4	0.29	84.86	84.86	0.28	84.86	84.86	0.01	0.00%
I.5	3095.13*	176.43*	172.75*	0.51	176.43	176.43	3094.62	2.13%
I.6	0.34	176.43	176.43	0.20	176.43	176.43	0.14	0.00%
I.7	4653.57*	162.71*	147.99*	TL	162.71	158.36	NA	7.01%
I.8	45.08	162.71	162.71	164.00	162.71	162.71	-118.92	0.00%

(a)

	OF1+OF2+OF3 with LB2			Time	OFlow
Inst.	**CPU time**	**OF**	**OFlow**	**reduction**	**improvement**
I.1	0.16	92.14	92.14	0.21	0.00%
I.2	0.17	92.14	92.14	-0.01	0.00%
I.3	19.20	84.86	84.86	482.76	0.00%
I.4	0.28	84.86	84.86	0.01	0.00%
I.5	183.05	176.43	176.43	2912.08	2.13%
I.6	0.33	176.43	176.43	0.00	0.00%
I.7	TL	162.71	160.72	NA	8.60%
I.8	111.98	162.71	162.71	-66.90	0.00%

(b)

	OF1+OF2+OF3 with LB1+LB2			Time	OFlow
Inst.	**CPU time**	**OF**	**OFlow**	**reduction**	**improvement**
I.1	0.25	92.14	92.14	0.12	0.00%
I.2	0.16	92.14	92.14	0.00	0.00%
I.3	49.78	84.86	84.86	452.17	0.00%
I.4	0.28	84.86	84.86	0.01	0.00%
I.5	0.29	176.43	176.43	3094.84	2.13%
I.6	0.20	176.43	176.43	0.13	0.00%
I.7	TL	162.71	157.93	NA	6.72%
I.8	139.51	162.71	162.70	-94.43	0.00%

(c)

LB1 is the best alternative for I.5 while LB2 is the best alternative for I.3.

Also, LB1+LB2 does not seem to improve the performance with respect to the single LB cases. Negative time reductions are sometimes observed. Most of them are lower than 1 second, and due to the randomness of the Windows OS. As for the higher ones, we remark that we did not fix any branch and bound strategy in CPLEX; thus, the chosen strategy to find a solution can be different with or without LBs, and this originates the negative reductions. Anyway, the worsening in terms of negative reduction with LBs is limited in front of the improvement observed in other instances (I.7).

We remind that we respect the assumptions of Section 3.3.1 in the 8 instances, because we are considering constant n_t^b values over t for each blood type b and because, when we include already booked donors, we are assuming that $a_t^b > 0$ values are always lower than the ξ_t^b values of the corresponding instance with non-booked donors ($a_t^b = 0, \forall t, b$). For this reason, the value of the objective function does not change within each pair of corresponding instances with non-booked and previously booked donors (e.g., I.1 and I.2).

3.5.3. Overall Framework

The numerical results for the overall framework is discussed for two cases. In Section 3.5.3.1, the numerical results for AVIS Milan case are presented while in Section 3.5.3.2 the results for randomly generated instances are represented.

In this section, the structure of overall framework for the deterministic model is described. Experiments are conducted over a rolling horizon; the preallocation model is run, at each rolling day, considering the previously assigned slots (a_t^b), and then the newly arriving calls for reservation are assigned to one of the preallocated slots x_t^b. The corresponding value x_t^b is reduced by 1 after each reservation is made; moreover, a_t^b values are updated at the end of the day with the new reservations, and the day t is shifted to $t + 1$. On the first day of the horizon, we start from an initial condition without booked donors ($a_t^b = 0, \forall t, b$). Then, the two phases are repeated, and so forth.

3.5.3.1. AVIS Milan Case

We test the effectiveness of the overall framework on a realistic instance derived from AVIS Milan case. The considered rolling period consists of 200 days, and the preallocation model is run at each rolling day with a planning period of $|T| = 28$ horizon days. At the first rolling day, we start from a null booked donors ($a_t^b = 0, \forall t, b$) condition.

The number of donors at each rolling day and their blood types are directly taken from the historical data of AVIS Milan, considering the whole blood donations over 200 days, from April 6 to October 22, 2014. In the dataset, the daily list of donations with the associated donor ID (from which all other information can be extracted) are available. Over these days, about 51 whole blood donations were made on average per day with a total of 10124 donations. The percentages of blood groups and Rhesus factor were as follows: 33.67% for A Rh+, 5.49% for A Rh-, 10.25% for B Rh+, 1.71% for B Rh-, 3.68% for AB Rh+, 0.56% for AB Rh-, 37.60% for 0 Rh+, and 7.02% for 0 Rh-. The historical data show that the number of produced units over these 200 days is highly variable among the days, as shown in Figure 3.2.

To create the instance for the test, we have simulated the subsets of booked and non-booked donors, as the possibility of reserving a donation in AVIS Milan is quite new and no significant historical information are available. Thus, to generate the portion of booked donors, existing donors in the historical data are randomly assigned to booked or non-booked class. From a discussion with the managers of AVIS Milan, they declared that a good percentage of booked donors should be at least the 80%. Thus, each donor is independently considered to be booked with probability 0.8, and non-booked with probability 0.2.

For the non-booked donors, we assume that they arrive in the same day as in the historical data. For each booked donor, we use the previous donation date and we compute the first available donation day (90 days after the previous donation for men and 180 days for women); then, date of the reservation call is generated by adding a random number of days, uniformly distributed between 0 and 30, to the first available day.

As for the preallocation model, we consider an appropriate parametric setting from the analysis made in Section 3.5.1; moreover, we analyze the impact of the coefficients λ_d and λ_f for the prioritization policy of the allocation phase.

Indeed, the preallocation model has been solved considering either the configuration OF1+OF3 (including LB1 in the formulation) and the configuration OF2+OF3 (including LB2 in the formulation), to evaluate the two opposite cases in terms of balancing. Two levels for the flexibility parameter ε are also considered, i.e., either $\varepsilon = 0$ or $\varepsilon = 0.25$ (the latter to model the observed fluctuations). Moreover, the following parameters have been considered: each day divided in $|K| = 3$ parts; set B made of 8 blood types; $\boldsymbol{\delta} = [0.08\ 0.06\ 0.03]$ and $\mu = 0.05$; fractions α_k equal to 0.4, 0.3 and 0.3 for $k = 1, 2, 3$, respectively; capacity c_{tk} equal to 450 minutes $\forall t, k$; all service durations equal to 20 minutes (for both r and R_{tk}). As for a_t^b and R_{tk}, they are daily updated by the rolling approach, starting from no preassignments at the first day. Differently from Section 3.5.2 where the time associated with each a_t^b is randomly split among the corresponding R_{tk}, here we exactly track the assigned period k and each preallocated slot directly determines both a_t^b and R_{tk}. Finally, a longer time horizon T equal to 28 days is taken. The remaining parameters are chosen to fit the tested case: the vector of d_b values for the 8 blood types with $|T| = 28$ is assumed as $[503\ 76\ 151\ 22\ 50\ 8\ 602\ 98]$; the number of non-booked donors n_t^b is assumed to be constant over the days (no trend is observed but just noise) and the vector for the different blood types is set equal to $[3\ 1\ 1\ 0\ 0\ 0\ 4\ 1]$. To simulate the real functioning of the BD collection center, the adopted values of parameters d_b and n_t^b have been set to replicate the historical data of the same period (April – October) in previous years (up to 2013). The blood type index b in d_b and n_t^b is intended as follows: $b = 1$ for A Rh+, $b = 2$ for A Rh-, $b = 3$ for B Rh+, $b = 4$ for B Rh-, $b = 5$ for AB Rh+, $b = 6$ for AB Rh-, $b = 7$ for 0 Rh+, $b = 8$ for 0 Rh-.

Three different configurations for the prioritization policy are considered. Either we include only the system flexibility (with $\lambda_d = 0$ and $\lambda_f = 1$) or the first available slot policy (with $\lambda_d = 1$ and $\lambda_f = 0$), or we consider both of them together with equal weights ($\lambda_d = 0.5$ and $\lambda_f = 0.5$). The scores S_{tkb} are recomputed after each reservation call is accepted.

A time limit of 1800 seconds and a memory limit of 3 GB have imposed set in each single run of the preallocation model. The lower time limit with respect to the previous analyses has been chosen because of the overall framework, to fasten the process. Outcomes show that the preallocation model always provides the optimal solution when $\varepsilon = 0.25$, while with $\varepsilon = 0$ the model does not always find the optimal solution in 3 experiments, for a total of 20 out of 600 overall runs. Anyway, even in these cases, the maximum observed optimality gap is 1.18%; thus, we can assess that the considered time limit is enough to derive conclusions.

Results are reported in Figures 3.10–3.15 for the six cases with $\varepsilon = 0.25$, while in the Appendix B for the six cases with $\varepsilon = 0$. In all figures, subfigures (a) report, for the 200 rolling days, the number of donations (total number, booked and non-booked) and the $\sum_b x_t^b + a_t^b$ values for the first day of the respective planning horizon (with $t = 1$). Subfigures (b) report the comparison between the total number of donations in the test case and in the historical data. Moreover, the waiting times between the reservation call and the donation are reported in Table 3.12.

Figures show that the approach is able to balance the production of blood units among days. The part related to the booked donations, which can be optimized, is highly balanced in all of the tests. On the contrary, the part related to non-booked donations obviously fluctuates as in the historical data. Globally, comparing the outcomes with the historical data, daily fluctuations are reduced even despite the remaining 20% of uncontrolled non-booked donor arrivals. We remark that the 80% of booked donors was considered because this represents the first goal of AVIS Milan while introducing the reservation system. However, our results show that, despite the good behavior of the approach, a remaining detriment of the balancing is present due to the 20% of non-booked donors. Thus, our suggestion is to implement all promotion policies to bring the highest number of donors to reserve the donation in advance. We also remark that some zeros are present in the historical data, related to holiday days (e.g., Christmas, Easter) that are not considered in our experiments. Anyway, also neglecting these days, we can confirm the reduced fluctuations in our results.

Results presented above refer to all blood types together. However, a similar balancing is obtained while considering each blood type singularly. For instance, we report in Figure

3.16 the number of booked donations and the total number of donations, divided by blood type, for the case OF1+OF3 with $\lambda_d = 1$ and $\lambda_f = 0$.

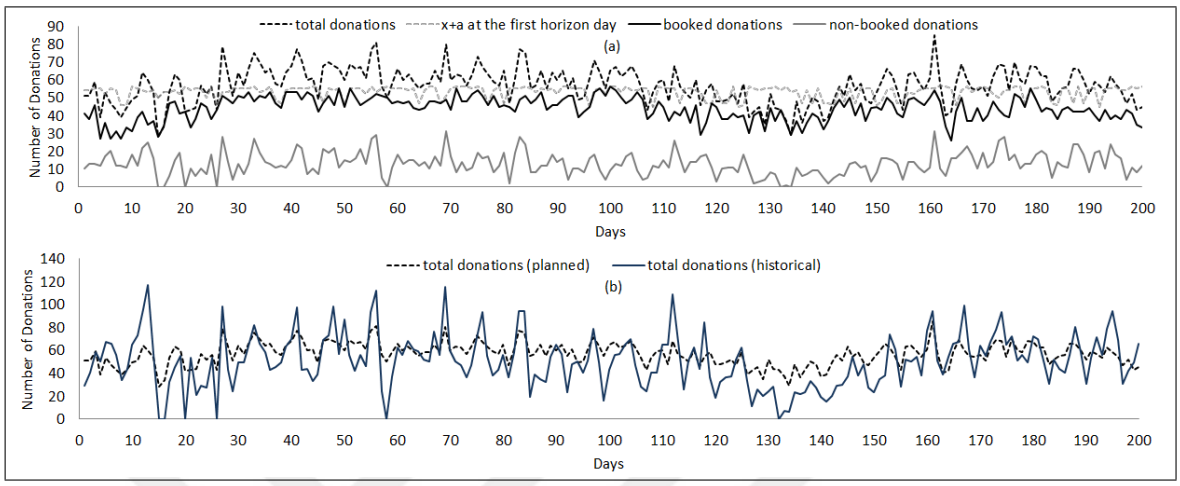

Figure 3.10. Number of donations per day for objective function OF1+OF3, $\varepsilon = 0.25$, and $\lambda_d = 1$ and $\lambda_f = 0$: (a) total number of donations, booked donations, non-booked donations, and $\sum_b x_1^b + a_1^b$; (b) comparison between the total number of donations in the test case and in the observed historical data.

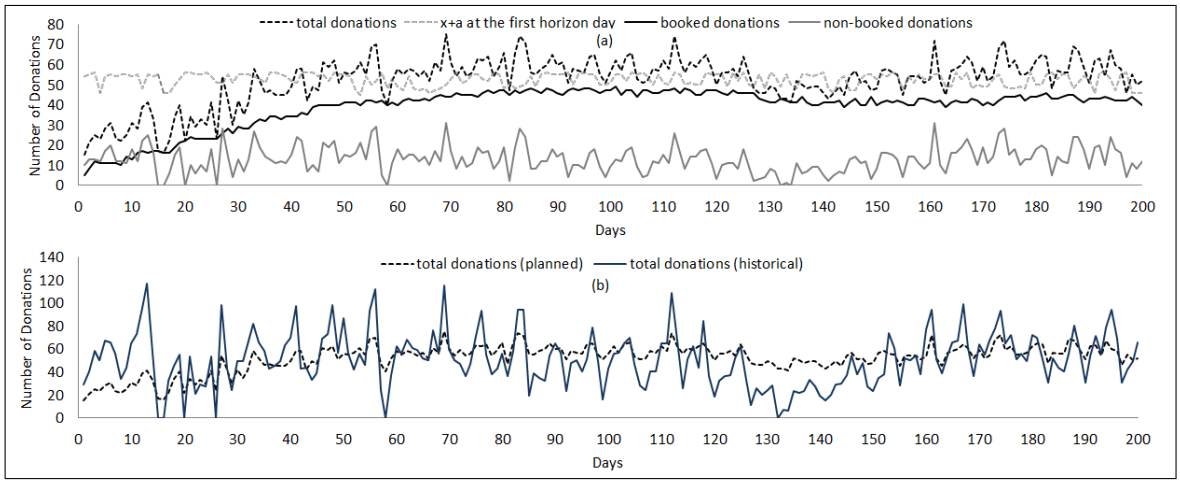

Figure 3.11. Number of donations per day for objective function OF1+OF3, $\varepsilon = 0.25$, and $\lambda_d = 0$ and $\lambda_f = 1$: (a) total number of donations, booked donations, non-booked donations, and $\sum_b x_1^b + a_1^b$. Reported data are as in Figure 3.10.

Even though the variability among days is slightly higher than in the total amount of donations, the balancing is mainly guaranteed.

With the first available slot policy, the preallocation model is able to serve most of the donors within the first week, while with the system flexibility policy they are shifted towards the end of the planning period (as shown in Table 3.12). Moreover, it can be observed that decreasing ε slightly decreases the waiting times of donors. Thus, keeping the flexibility of the system without prioritizing the first available slot is not very effective, with significantly longer waiting times between reservation call and donation. This has a negative impact on the amount of donations, as longer waiting times reduce the donation frequency. Moreover, without weighting the first available slot, the closest slots might remain empty, thus reducing the daily throughput of the system.

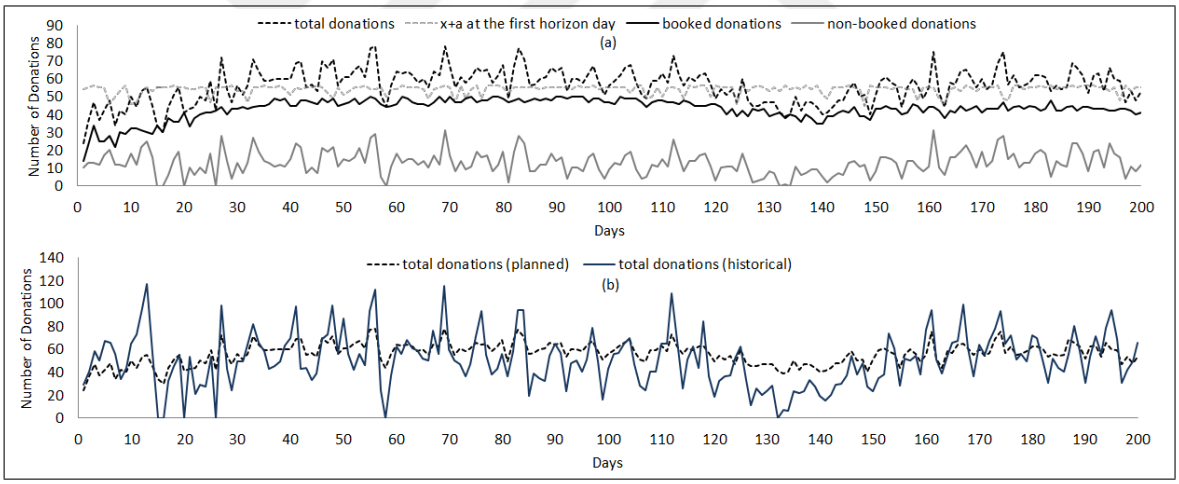

Figure 3.12. Number of donations per day for objective function OF1+OF3, $\varepsilon = 0.25$, and $\lambda_d = 0.5$ and $\lambda_f = 0.5$: (a) total number of donations, booked donations, non-booked donations, and $\sum_b x_1^b + a_1^b$. Reported data are as in Figure 3.10.

It is worth remarking that, in our tests, we assume that donors always accept the first suggested slot (with the highest score S_{tkb}) without evaluating donors' preferences, who might also ask to donate in a day without any empty preallocated slots. This evaluation requires data that are not included in the AVIS Milan database.

The two weights λ_d and λ_f also affect the ramp-up period. The number of booked donations are not stabilized until about the 40th day for the cases with $\lambda_d = 0$ and $\lambda_f = 1$.

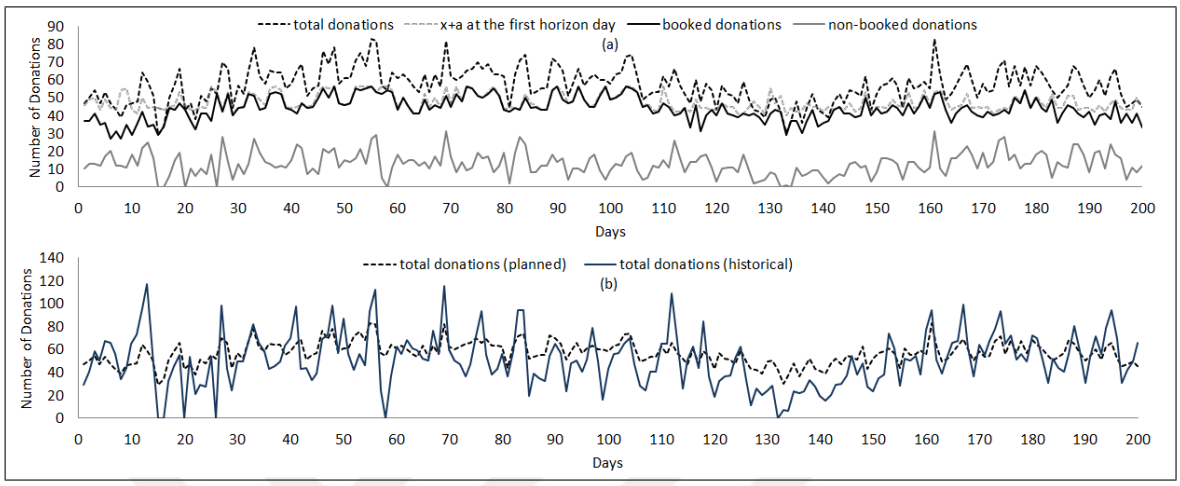

Figure 3.13. Number of donations per day for objective function OF2+OF3, $\varepsilon = 0.25$, and $\lambda_d = 1$ and $\lambda_f = 0$: (a) total number of donations, booked donations, non-booked donations, and $\sum_b x_1^b + a_1^b$. Reported data are as in Figure 3.10.

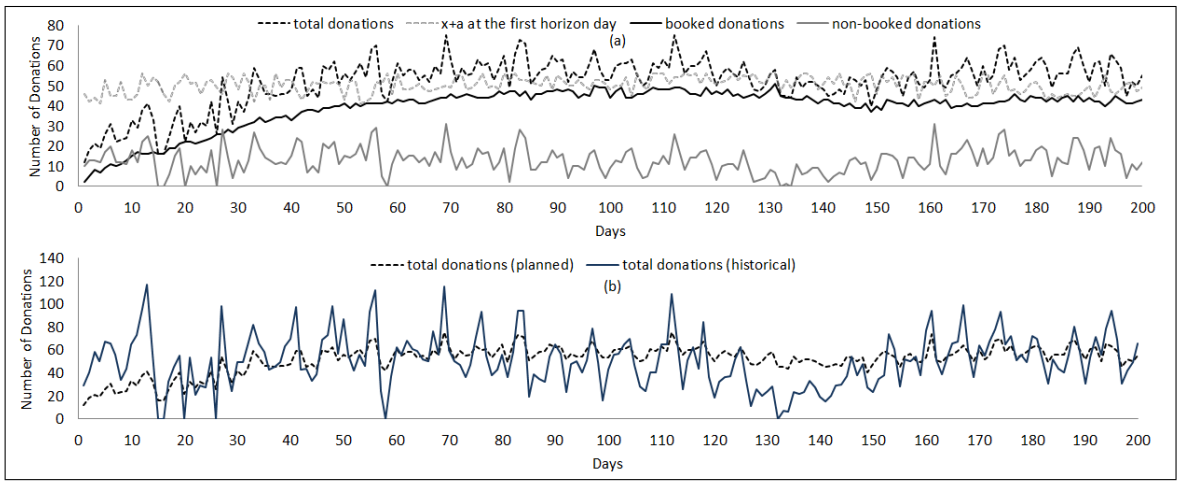

Figure 3.14. Number of donations per day for objective function OF2+OF3, $\varepsilon = 0.25$, and $\lambda_d = 0$ and $\lambda_f = 1$: (a) total number of donations, booked donations, non-booked donations, and $\sum_b x_1^b + a_1^b$. Reported data are as in Figure 3.10.

As mentioned, flexibility spreads the donation days over the time horizon, thus letting some slots empty, while on the contrary assigning slots based only on the first available day fills

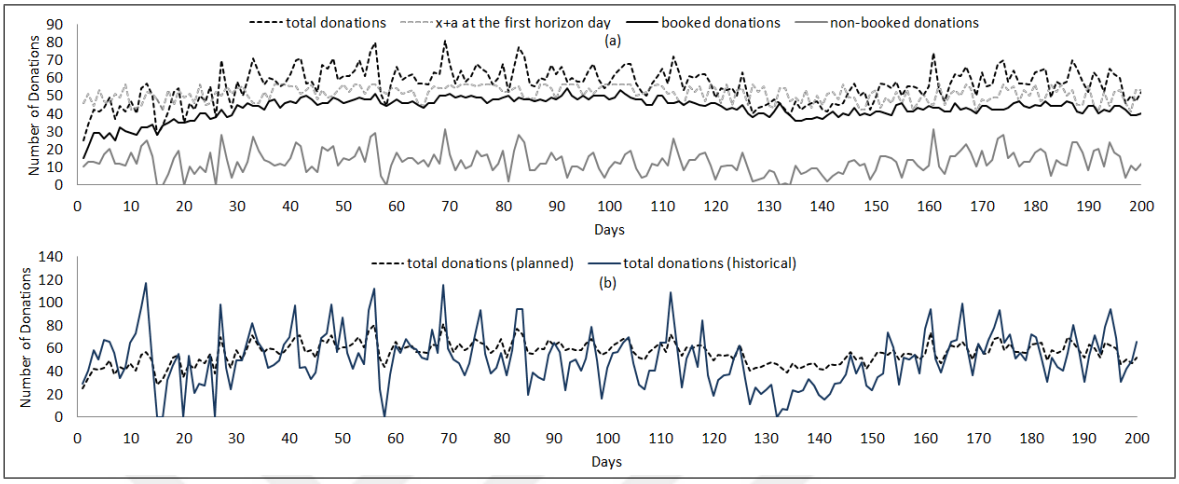

Figure 3.15. Number of donations per day for objective function OF2+OF3, $\varepsilon = 0.25$, and $\lambda_d = 0.5$ and $\lambda_f = 0.5$: (a) total number of donations, booked donations, non-booked donations, and $\sum_b x_1^b + a_1^b$. Reported data are as in Figure 3.10.

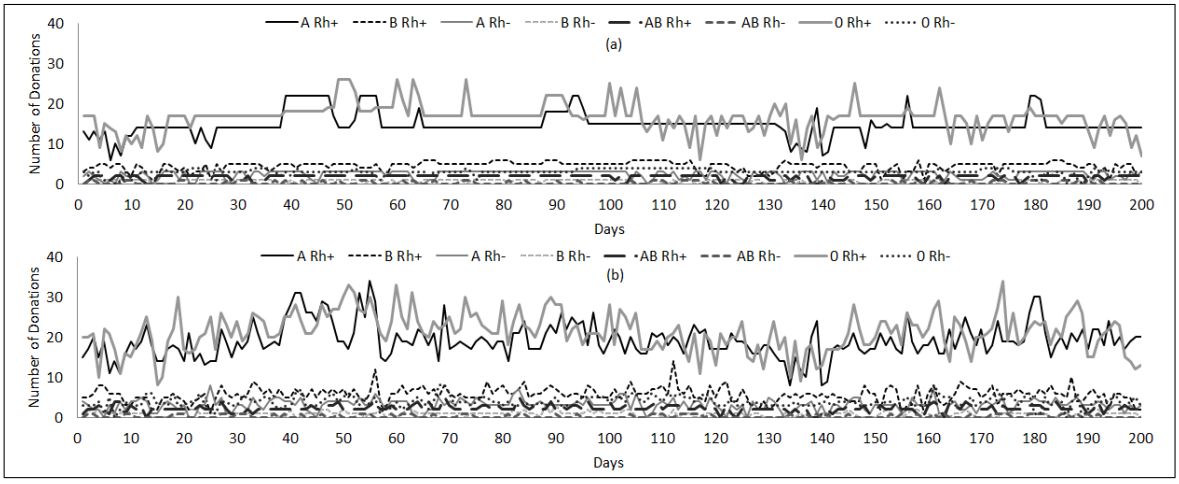

Figure 3.16. Number of donations per day, divided by blood type, for objective function OF1+OF3 with $\lambda_d = 1$ and $\lambda_f = 0$: (a) booked donations and (b) total number of donations. Labels of blood types are reported in increasing order of the associated index b.

the slots from early beginning, thus avoiding empty slots in the first days when the system starts with $a_t^b = 0$.

Table 3.12. Waiting time in days between reservation call and donation for booked donors: average and distribution in the last 160 days (excluding the initial ramp-up period of 40 days).

Case	Average	Waiting days distribution			
		0-7	8-14	15-21	$\geq$22
OF1+OF3, $\varepsilon = 0.25$, $\lambda_d = 1$ and $\lambda_f = 0$	0.48	98.94%	0.99%	0.05%	0.02%
OF1+OF3, $\varepsilon = 0.25$, $\lambda_d = 0$ and $\lambda_f = 1$	22.49	10.15%	6.42%	6.92%	76.50%
OF2+OF3, $\varepsilon = 0.25$, $\lambda_d = 1$ and $\lambda_f = 0$	1.02	98.31%	1.61%	0.08%	0.00%
OF2+OF3, $\varepsilon = 0.25$, $\lambda_d = 0$ and $\lambda_f = 1$	22.60	8.63%	6.00%	9.42%	75.94%
OF1+OF3, $\varepsilon = 0.25$, $\lambda_d = 0.5$ and $\lambda_f = 0.5$	4.93	72.36%	26.73%	0.91%	0.00%
OF2+OF3, $\varepsilon = 0.25$, $\lambda_d = 0.5$ and $\lambda_f = 0.5$	5.27	70.17%	27.85%	1.98%	0.00%
OF1+OF3, $\varepsilon = 0$, $\lambda_d = 1$ and $\lambda_f = 0$	0.30	99.83%	0.17%	0.00%	0.00%
OF1+OF3, $\varepsilon = 0$, $\lambda_d = 0$ and $\lambda_f = 1$	22.27	10.25%	6.42%	8.04%	75.28%
OF2+OF3, $\varepsilon = 0$, $\lambda_d = 1$ and $\lambda_f = 0$	0.33	99.78%	0.22%	0.00%	0.00%
OF2+OF3, $\varepsilon = 0$, $\lambda_d = 0$ and $\lambda_f = 1$	22.42	9.65%	5.89%	8.84%	75.61%
OF1+OF3, $\varepsilon = 0$, $\lambda_d = 0.5$ and $\lambda_f = 0.5$	4.52	75.51%	24.08%	0.41%	0.00%
OF2+OF3, $\varepsilon = 0$, $\lambda_d = 0.5$ and $\lambda_f = 0.5$	4.46	76.81%	22.84%	0.35%	0.00%

In all cases, after the ramp-up period, $\sum_b x_1^b + a_1^b$ at the first day of the planning horizon is really close to the number of booked donations (equal or slightly higher). This indicates both that the d_b parameters have been appropriately set and that, once a fair prediction of d_b is considered, our system does not leave many empty preallocated slots. A slightly higher number of empty slots is present with OF1+OF3, but this amount is anyway limited.

3.5.3.2. Randomly Generated Instances

We also test the effectiveness of the overall framework on randomly generated instances. We consider a subset of the configurations tested in Section 3.5.3.1, to show the effectiveness of

the overall framework on other cases through the random generation of scenarios. Indeed, both OF1+OF3 and OF2+OF3 alternatives are considered, with only $\varepsilon = 0.25$ and one configuration for the prioritization policy (equal weights $\lambda_d = 0.5$ and $\lambda_f = 0.5$). All other parameters (e.g., $|T|$, $|K|$, δ_k) are set at the same value than in the previous section.

Table 3.13. Mean daily number of booked and non-booked donors (for the Poisson distribution) and d_b vector for the 8 blood types in Groups C.1 and C.2.

Instance		A Rh+	A Rh-	B Rh+	B Rh-	AB Rh+	AB Rh-	0 Rh+	0 Rh-	Total
C.1	Mean daily booking	15	3	1	1	3	1	18	3	45
C.1	Mean daily non-booking	4	1	0	0	1	0	5	1	12
C.1.1	d_b	418	70	43	20	82	29	461	75	1198
C.1.2	d_b	451	90	28	32	83	20	492	77	1273
C.1.3	d_b	433	74	28	40	85	24	511	98	1293
C.1.4	d_b	413	82	24	33	69	28	526	85	1260
C.2	Mean daily booking	13	3	3	1	1	1	15	3	40
C.2	Mean daily non-booking	3	1	1	0	0	0	4	1	10
C.2.1	d_b	340	75	83	29	28	23	429	73	1080
C.2.2	d_b	365	83	75	33	22	29	464	83	1154
C.2.3	d_b	403	73	102	21	28	29	445	90	1191
C.2.4	d_b	366	85	97	33	24	17	469	97	1188

The data generation mechanism is as follows. The number of booking donors who call at each day and the number of non-booking donors who arrive at each day are randomly generated for each blood type according to a Poisson distribution. Different mean values are considered for each blood type and booking/non-booking alternative, while the mean values of the Poisson distributions are the same for each day. Alternative values are considered to generate different layouts. In all cases, the common proportion among the blood types all around the world is respected, being the mean amount of donors belonging to groups A Rh+ and 0 Rh+ about the 70-80% of the total amount. Moreover, we assume that booked donors are about the 80% of the total, while non-booked ones the remaining 20%. Given a realization of booked and non-booked donors, d_b values are generated with another random

process. Indeed, for each blood type b, the sum of the generated booked donors over the rolling days (200 days) is scaled by the ratio between the time horizon T (28 days) and the rolling days.

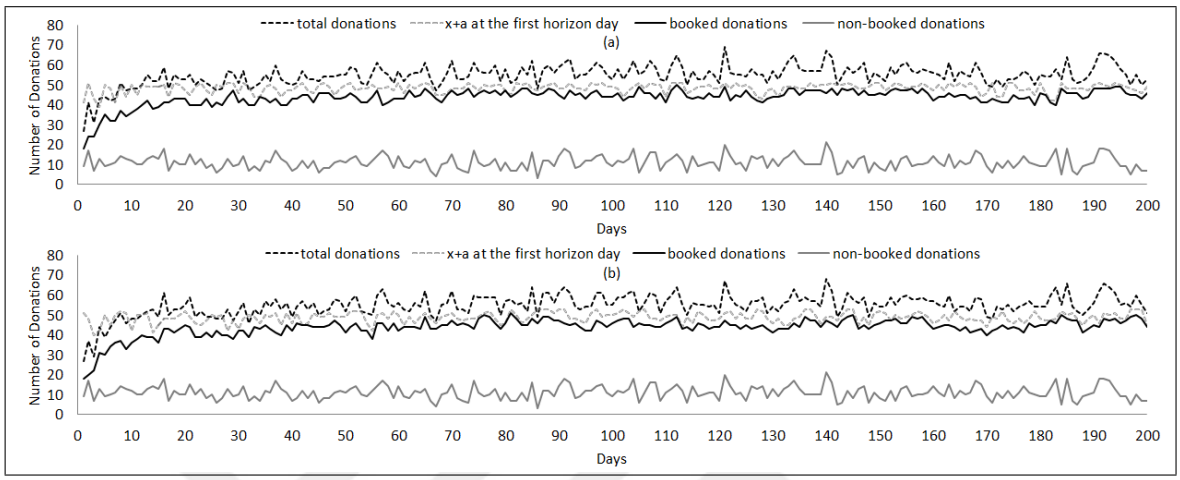

Figure 3.17. Number of donations per day in Group C.1.1 for objective function OF1+OF3 (a) and OF2+OF3 (b), with $\varepsilon = 0.25$, $\lambda_d = 0.5$ and $\lambda_f = 0.5$: total number of donations, booked donations, non-booked donations, and $\sum_b x_1^b + a_1^b$.

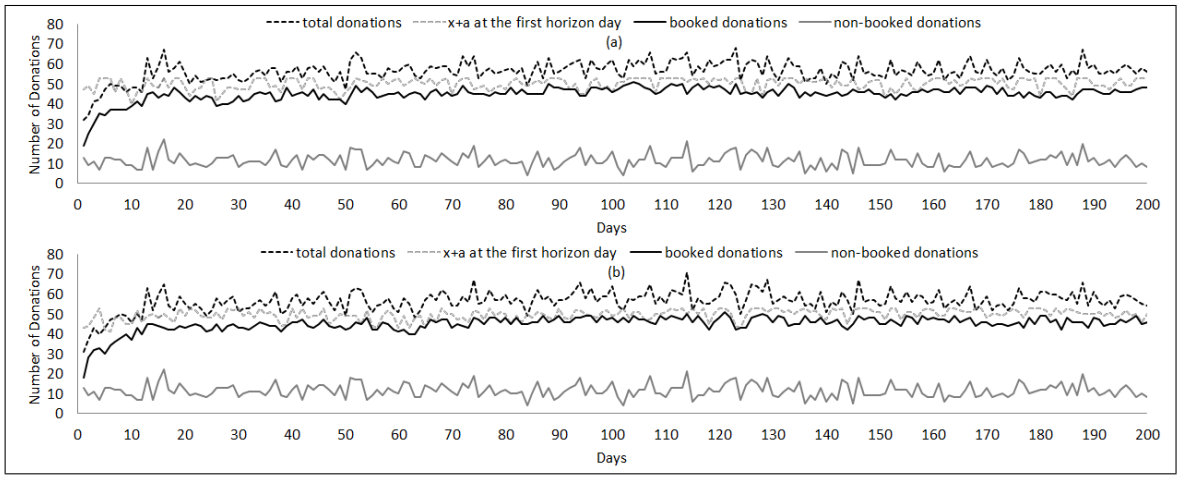

Figure 3.18. Number of donations per day in Group C.1.2 for objective function OF1+OF3 (a) and OF2+OF3 (b), with $\varepsilon = 0.25$, $\lambda_d = 0.5$ and $\lambda_f = 0.5$. Reported data are as in Figure 3.17.

The scaled value is assumed as the mean value of another Poisson distribution, and the value of d_b is drawn from this distribution.

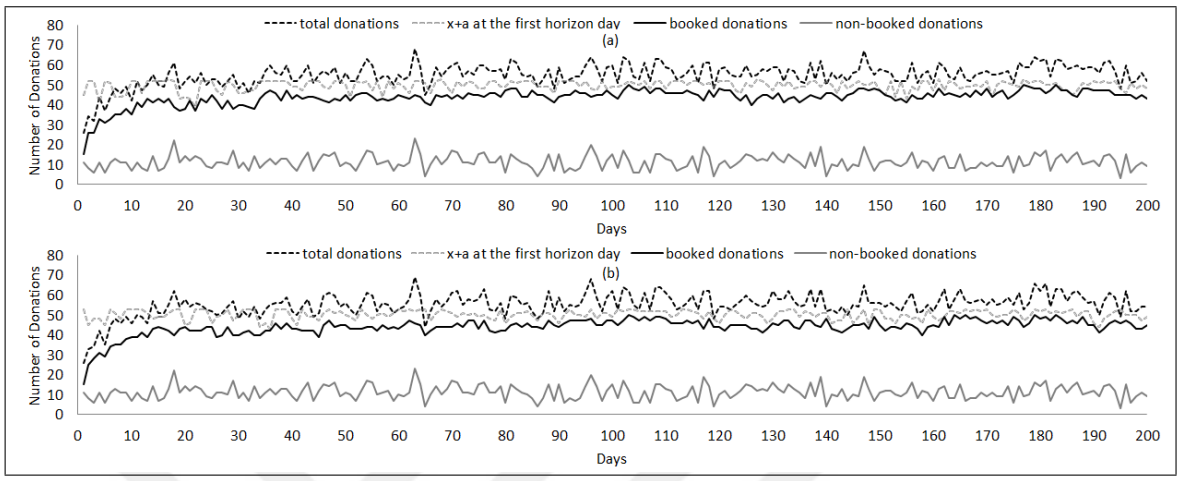

Figure 3.19. Number of donations per day in Group C.1.3 for objective function OF1+OF3 (a) and OF2+OF3 (b), with $\varepsilon = 0.25$, $\lambda_d = 0.5$ and $\lambda_f = 0.5$. Reported data are as in Figure 3.17.

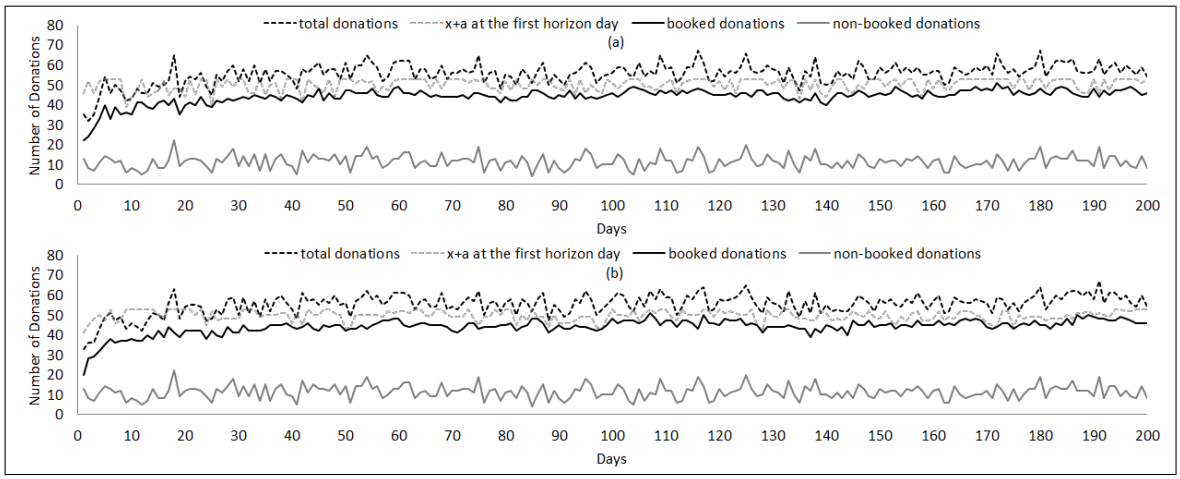

Figure 3.20. Number of donations per day in Group C.1.4 for objective function OF1+OF3 (a) and OF2+OF3 (b), with $\varepsilon = 0.25$, $\lambda_d = 0.5$ and $\lambda_f = 0.5$. Reported data are as in Figure 3.17.

Two groups of instances are defined with this mechanism (as shown in Table 3.13) and 4 random generations are extracted, each one is characterized by specific d_b values.

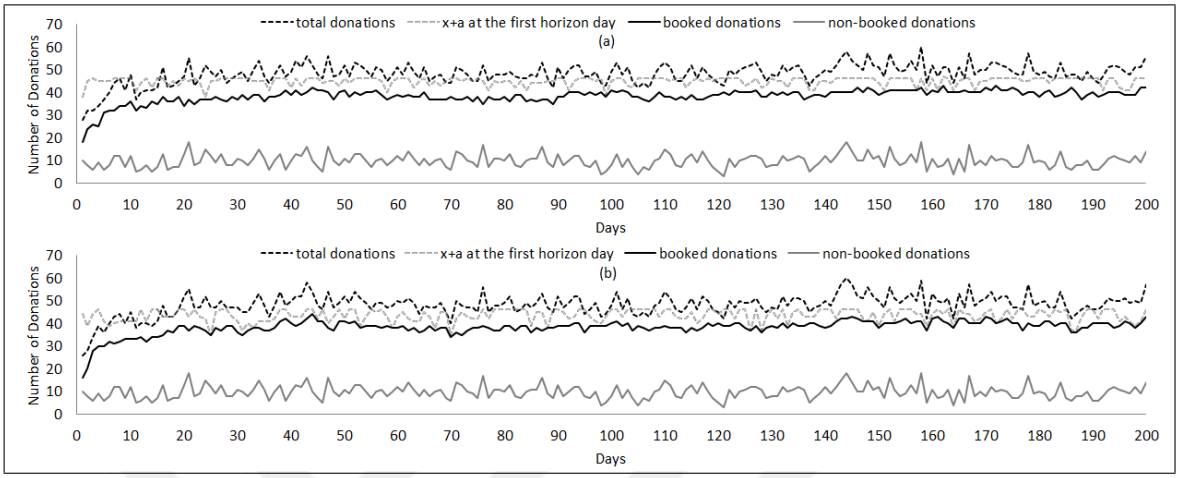

Figure 3.21. Number of donations per day in Group C.2.1 for objective function OF1+OF3 (a) and OF2+OF3 (b), with $\varepsilon = 0.25$, $\lambda_d = 0.5$ and $\lambda_f = 0.5$. Reported data are as in Figure 3.17.

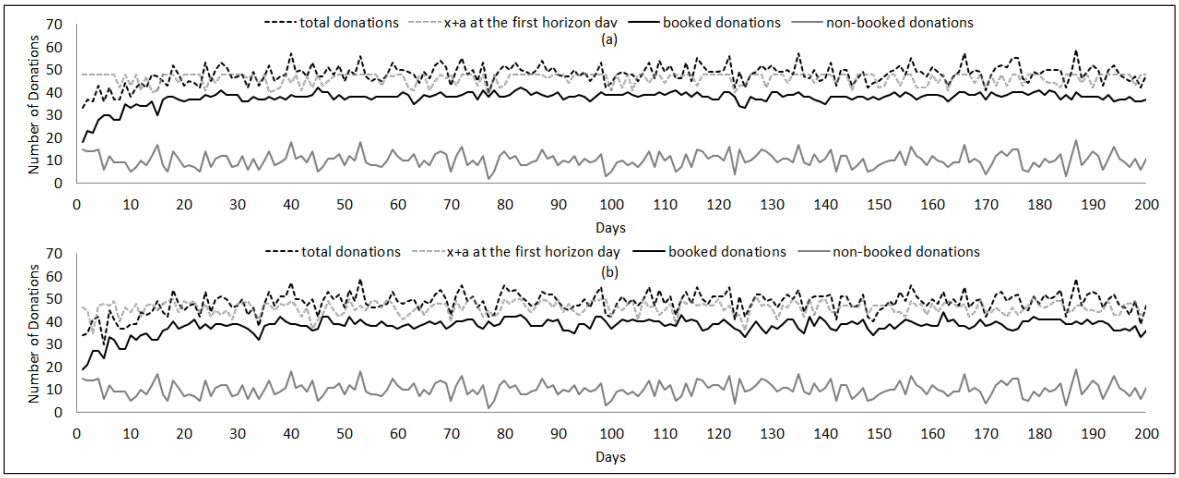

Figure 3.22. Number of donations per day in Group C.2.2 for objective function OF1+OF3 (a) and OF2+OF3 (b), with $\varepsilon = 0.25$, $\lambda_d = 0.5$ and $\lambda_f = 0.5$. Reported data are as in Figure 3.17.

The number of donations per day for all of the 8 instances are reported in the Figures 3.17 to 3.24.

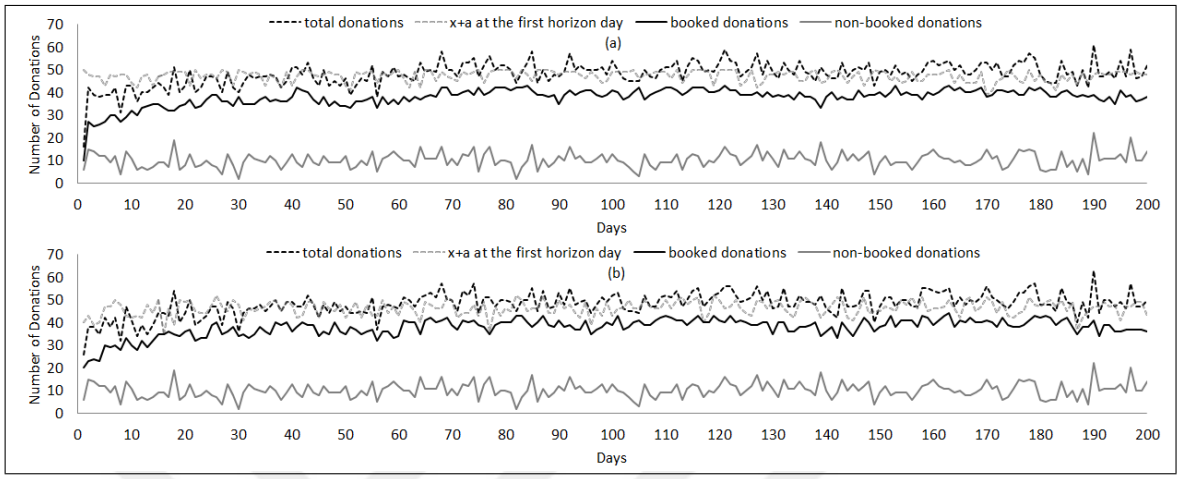

Figure 3.23. Number of donations per day in Group C.2.3 for objective function OF1+OF3 (a) and OF2+OF3 (b), with $\varepsilon = 0.25$, $\lambda_d = 0.5$ and $\lambda_f = 0.5$. Reported data are as in Figure 3.17.

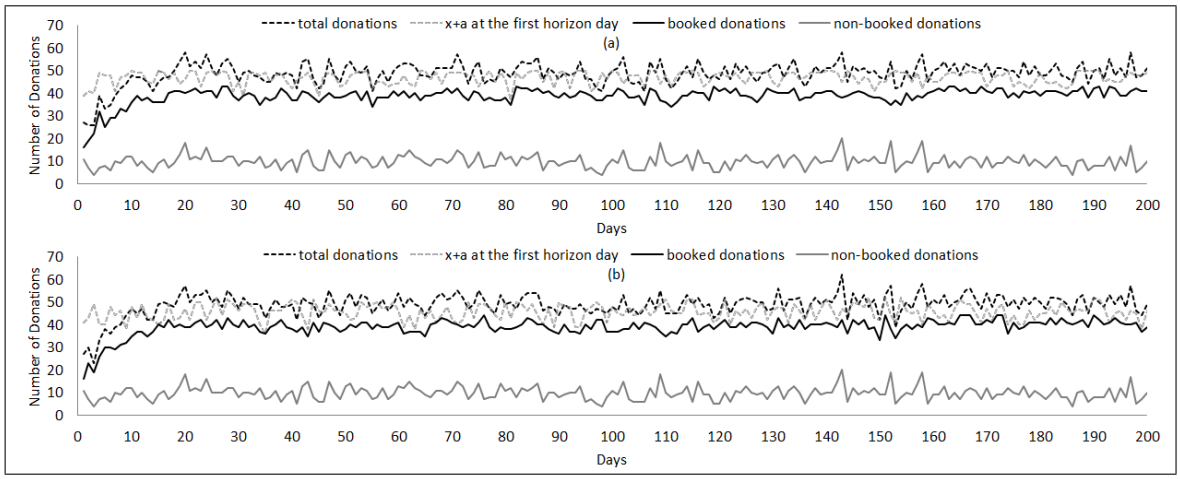

Figure 3.24. Number of donations per day in Group C.2.4 for objective function OF1+OF3 (a) and OF2+OF3 (b), with $\varepsilon = 0.25$, $\lambda_d = 0.5$ and $\lambda_f = 0.5$. Reported data are as in Figure 3.17.

These figures show that also in this case the proposed approach is able to balance blood production over the 200 days.

Table 3.14. Average, minimum and maximum values of booked donors (overall over the blood types) in the last 160 days (excluding the initial ramp-up period of 40 days) for Groups C.1 and C.2.

Instance	Objective function	Average	Minimum	Maximum
C.1.1	OF1+OF3	45.03	40	50
C.1.1	OF2+OF3	45.20	38	51
C.1.2	OF1+OF3	45.96	40	51
C.1.2	OF2+OF3	46.09	40	51
C.1.3	OF1+OF3	45.05	40	50
C.1.3	OF2+OF3	45.24	39	50
C.1.4	OF1+OF3	45.44	40	51
C.1.4	OF2+OF3	45.23	39	51
C.2.1	OF1+OF3	39.23	35	43
C.2.1	OF2+OF3	39.16	34	44
C.2.2	OF1+OF3	38.48	33	42
C.2.2	OF2+OF3	38.91	33	44
C.2.3	OF1+OF3	39.11	33	43
C.2.3	OF2+OF3	39.04	32	44
C.2.4	OF1+OF3	39.53	34	43
C.2.4	OF2+OF3	39.64	33	44

Indeed, compared to the outcomes of the AVIS Milan case (previous section), we even observe slightly lower deviations for the total number of donations. In both cases, production imbalance remains due to the uncontrolled arrivals of non-booked donors.

To briefly compare the 8 instances, we show in Table 3.14 the number of booked donors for each instance in terms of average, minimum and maximum values. Results shows that in all cases the approach is able to allocate slots according to the mean number of booked donors, which is 45 or for Group C.1 and 40 for Group C.2, respectively (Table 3.13). Moreover, the small minimum to maximum ranges confirm once again the effectiveness of the proposed

approach on production balancing.

As for the waiting times, we observe slightly higher values with respect to the AVIS Milan case.

Table 3.15. Waiting time in days between reservation call and donation for booked donors: average and distribution in the last 160 days (excluding the initial ramp-up period of 40 days).

	Objective		**Waiting days distribution**			
Instance	**terms**	**Average**	**0-7**	**8-14**	**15-21**	**$\geq$22**
C.1.1	OF1+OF3	6.99	54.92%	39.75%	5.01%	0.32%
C.1.1	OF2+OF3	6.90	55.84%	39.53%	3.49%	1.14%
C.1.2	OF1+OF3	6.40	60.43%	36.04%	3.53%	0.00%
C.1.2	OF2+OF3	6.34	59.09%	38.35%	2.53%	0.04%
C.1.3	OF1+OF3	6.96	57.52%	33.07%	8.68%	0.73%
C.1.3	OF2+OF3	6.24	61.13%	35.18%	3.68%	0.00%
C.1.4	OF1+OF3	6.48	61.02%	34.58%	4.28%	0.12%
C.1.4	OF2+OF3	6.97	56.75%	37.01%	5.50%	0.73%
C.2.1	OF1+OF3	6.53	57.24%	39.53%	3.23%	0.00%
C.2.1	OF2+OF3	6.63	55.62%	41.47%	2.91%	0.00%
C.2.2	OF1+OF3	6.60	61.81%	30.62%	6.82%	0.75%
C.2.2	OF2+OF3	5.45	69.34%	27.53%	2.11%	1.02%
C.2.3	OF1+OF3	5.44	68.07%	28.79%	2.37%	0.77%
C.2.3	OF2+OF3	5.48	69.63%	27.99%	1.62%	0.76%
C.2.4	OF1+OF3	5.24	69.04%	29.13%	1.83%	0.00%
C.2.4	OF2+OF3	4.97	72.53%	27.06%	0.40%	0.02%

As shown in Table 3.15 even though the system is able to serve most of the donors within 14 days as in the AVIS Milan case, here we observe that the ratio of donors served in the first week decreases and donors are shifted to the second week.

3.6. CONCLUSION

In this chapter, we define and formalize the first BDAS problem to the best of our knowledge, and we propose an appointment scheduling framework to solve it.

Our framework for planning the assignments consists of two phases: an MILP model to preallocate time slots of the different blood types, and a prioritization policy to assign the preallocated slots. The goal is to balance the production of blood units of each type among the days, while also avoiding dispersion amounts associated with dispersion amount and donor waiting times. The main points of our framework are, besides the decomposition in two phases, the presence of both booked and non-booked donors and the degree of freedom for the number of slots to preallocate (due to the flexibility associated with d_b). The latter point makes our preallocation model different from the allocation and scheduling models available in the literature, since here the amount of entities to allocate is another decision variable, whereas it is fixed in several other cases.

The proposed approach has been successfully applied to the real case of a large blood collection center operating in Italy, the AVIS Milan, and the results confirm the capability of the approach to balance the production of each blood type among days.

To improve the quality of the solution, we investigate the possibility of creating a stochastic version of the preallocation model. At present, d_b variation is modeled through the flexibility parameter ε, but the model is deterministic. On the contrary, a stochastic version would include uncertain parameters, at least for n_t^b. In the proposed deterministic model, the number of non-booked donors n_t^b is assumed to be constant over the days. In deterministic models the model output is determined by the given parameter values. On the contrary, in stochastic models, there is unavoidable randomness and the model has the capacity to handle the uncertainty. In real life problems, all of the necessary information is not known in advance at the beginning of planning horizon. As it is discussed, the amount of non-booked donors has a great impact on the entire system. Therefore, it is important to consider the uncertainty created by the non-booked donors.

In the following chapter, the stochastic programming approaches for handling the uncertainty in optimization problems are defined. First, risk-neutral and risk-averse stochastic programming models are defined and then the proposed BDAS models are formulated.

Different model formulations with alternative objective functions are solved both neglecting and including LB1 and LB2, to analyze their impact on the computational times. OF1 minimizes the total absolute variation with respect to the average production, while OF2 minimizes the maximum absolute variation among all days and all blood types. The benefits of including LBs in terms of CPU time reduction is observed while comparing the results with and without LBs. Furthermore, both in OF1+OF3 and OF2+OF3, almost all CPU times are lowered with the presence of LB. But when OF1+OF2+OF3 is considered, none of the LB alternatives seems to be the best one. When both terms are taken into consideration, there is no benefit in terms of CPU time. As it is discussed, it is common in multi-objective optimization problems that both the summation and the maximum of a set of decision variables are optimized in order to prevent multi-optimal solutions. But, in our case, these two terms may lead to allocate a different number of slots and different lower bound calculations are driven for OF1 and OF2 as it is discussed in Section 3.3. OF1 prefers the farthest point from the maximum of the parabola which means OF1 prefers to allocate a number of slots as close as possible to multiple of time horizon. But OF2 prefers to allocate a number of slots as close as possible to the intermediate value of the time horizon. In the stochastic programming models the OF2 term is not included for the sake of simplicity.

4. STOCHASTIC PROGRAMMING MODELS

4.1. INTRODUCTION

In this chapter, we define the approaches for handling uncertainty. The optimization problems based on real case applications are subjected to uncertainty and the decision maker generally makes a decision without knowing the effect in the future. According to the context of a problem, several parameters can be considered uncertain and treated as random variables. The current decisions that are given against these uncertain parameters may lead to undesirable consequences in the future. Therefore it is important to identify the impact of random variables on a future decision in a given problem.

In this dissertation, we address BDAS model where the uncertainty is realized through non-booked donor information. In this study, the demand variation is considered through the flexibility parameter ε. However, the random arrival of non-booked donors has not been considered yet. Unpredictable non-booked donor arrivals introduce a variability in the entire schedule and thus variations in the non-booked donor arrivals must be considered in the proposed framework.

Over the past decades the majority of the research on data uncertainty has focused on different techniques (such as recourse-based stochastic programming, robust programming, chance-constraint programming, fuzzy programming, and stochastic dynamic programming) but in this study, we consider two families of techniques, robust optimization and stochastic programming.

Robust optimization is used to process optimization problems in which the data is uncertain and the only acquired information is that the data belongs to some uncertainty set [134]. Robust optimization provides probabilistic guarantee on the feasibility of solutions obtained from an uncertainty set for any realization of uncertain coefficients [135]. Robust optimization tries to find a feasible solution against all data variations.

The history of robust optimization starts with the use of worst case analysis as a tool for the uncertainty by the establishment of modern decision theory in the 1950s. The aim of robust optimization is to produce solutions that remain feasible with respect to the parameters that take values in the uncertainty region. It takes into account a solution that is acceptable under most realizations of the uncertain inputs. However, there are several assumptions made on the probability distribution of the uncertain parameters, and the region defining all of their possible realizations is supposed to be completely known. If the optimization problem minimizes the robust value of the objective among all robust feasible solutions, it is called the *robust counterpart* of the uncertain linear optimization problem. It is a conservative, worst-case oriented methodology. Researchers evolve robust counterparts of the nominal problem to eliminate over-conservatism through estimation processes for some of the uncertain parameters.

The main contributors to robust optimization approaches who deal with the uncertainty are, Ben-Tal *et al.*, Charnes & Cooper, Soyster, Mulvey *et al.* and Bertsimas & Sim [136, 137, 138, 139, 140, 141]. Soyster [138] proposed a model which is very conservative in the sense that the approach protects against the worst-case scenario so that each parameter can deviate. The solution of the model has an objective function value much worse than the solution of the nominal linear optimization problem. Mulvey *et al.* [142] proposed a model to avoid an over-conservative worst-case solution that combines problem data which is described as scenario-based with goal programming. In order to handle over-conservatism, they introduced a penalty function but unfortunately scenario generation increases the computational effort to solve the problem. Ben-Tal *et al.* [139, 143] guarantee the feasibility of the solution only if, for each row, the data lie within an ellipsoidal set. By the help of this assumption parameters arising in the same row are restricted from taking their worst values. Ben-Tal *et al.*'s [139] model is less conservative. This is like changing a random quantity with its expectations minus a constant (two or three) times its standard deviation. The assumption is that, the random data will not take the worst possible values simultaneously. The biggest problem for Ben-Tal *et al.* [139] is finding the appropriate uncertainty set. In that case, they choose a solution that minimizes the objective function value. Since Ben-Tal *et al.*'s [139] approach is a conic quadratic problem which is nonlinear and not suitable for large sized optimization problems, they proposed an uncertain linear

optimization problem that can be reformulated with the robust counterpart and it has the same objective.

Considering these approaches chronologically, in robust optimization there is significant improvement starting from 1973 by Ben-Tal *et al.* and Soyster [134, 138, 139, 143]. Over conservatism issue is addressed in the cited papers. Furthermore, these papers proposed uncertain linear problems with ellipsoidal uncertainties that are less conservative models concerned with conic quadratic form problems of the robust counterparts of the nominal problems [134, 138, 139, 143].

Bertsimas and Sim [141] proposed a new method to reduce the over-conservatism in the method of Soyster [138]. Instead of allowing all of the coefficients to deviate, they limit the number of coefficient deviations. The main idea behind the approach is that, given a row of the constraint matrix, the coefficients are not likely to take worst values simultaneously. Considering this, they introduce a parameter for each row, which can be set in a continuous range and rather than protecting against the case where each parameter can deviate, as in the original model of Soyster [138], they allow at most decided coefficients to deviate. Since this approach is a worst case analysis, we search for the coefficient that results in the maximum variation. The index of maximum variation is seeked for a preselected set of rows. Furthermore, it is assumed that only these parameters will be affected by the variation. For a detailed comparison, see Li and Lerapetritou [144] who explains the difference between robust optimization formulations.

In our BDAS problem, uncertainty is based on non-booked donors. As discussed before, in robust optimization the objective function is determined under the worst-case condition with a protection function which is obtained through the duality. Duality is used in the linearization of the protection function which has to be defined over the constraints. According to Bertsimas and Sim [141], we have to select the parameters for each constraint independently. Since the main objective in the BDAS model is to minimize the total variation, the protection function should be written for the first objective function term, total variation, z_t^b. So a subset of the parameters in the constraints (3.2) and (3.3) should be selected such that they reach the maximum value. The aforementioned variable z_t^b computes

the variation between the average and the daily production by the index t in the constraints (3.2) and (3.3). However, obtaining a protection function in robust optimization requires independency among the constraints. In order to calculate the total variation, the overall schedule has to be done for the given time horizon. So that, the average production values of the entire schedule for the blood types can be found. In this case, independency assumption is violated. So our model is not suitable for the robust optimization approach.

Stochastic programming involves data that is not certainly known. These uncertain parameters can follow a probability distribution that is known or can be estimated. Furthermore, uncertain parameters are modeled as random variables. The aim of the stochastic programming is to minimize the expected value of the objective. So, the key idea is to solve the optimization model considering all possible realizations that are generated from a probability distribution of uncertain variables. Therefore, it incorporates all possible future realizations into the model through the generated scenarios.

The traditional stochastic programming approaches are commonly based on expected value calculation in their objective functions. Usually in stochastic programming, risk preferences can be *risk-seeking*, *risk-neutral* and *risk-averse*. *Risk-seeking* method is attracted to risk, therefore, greater risk and lower expected returns are preferable. In the *risk-neutral* case, the decision maker focuses only on the expected total objectives. The standard deviation of the outcomes are high in risk-neutral models and any decision policy will result in high recourse objectives. The objective function of the risk-neutral stochastic programming model is calculated based on the expected value of each realization. On the other hand, in *risk-averse* stochastic programming, it is important to consider the effect of unpredictable variability. More precisely, it involves maximizing the profit or minimizing the loss under the worst conditions. With risk-averse models, the solutions provide more protection, although there might be an increase in the expected objective values [145]. Risk-averse models can provide solutions which may perform better under different realizations of uncertain parameters.

In the following section risk-neutral (see Section 4.2) and risk-averse (see Section 4.3) stochastic programming approaches are presented. The advantage of stochastic programming is to produce solutions that are usually not as over-conservative as their robust

counterpart but they are still protective against the most likely realizations.

4.2. RISK-NEUTRAL STOCHASTIC PROGRAMMING MODELS

In this section, a special focus is given to risk-neutral stochastic programming approaches to model stochastic optimization problems. There are several solution techniques presented in the literature but the most applied ones are Expected Value (EV), Evaluation of the Expected Value (EEV), Here and Now (HN) and Wait and See (WS) approaches [146].

Expected Value approach: EV method optimizes the model by replacing the random parameters with their respective average values [147]. Simply, the deterministic version of the considered model is solved while replacing the stochastic parameters with their average values and the objective function value is defined as OF_{EV}.

Evaluation of the Expected Value approach: EEV stands for expected result of the EV approach, such that OF^*_{EV} is the solution for a scenario s in the corresponding deterministic model where the solution for the deterministic model has been fixed to the obtained solution from the EV model. Let's consider S different scenarios s with probability p_s to occur. Once the decision of x^* is obtained, objective function OF^*_s for each scenario s is post calculated with x^*. The decision of x^* is evaluated for all possible realization. There can be feasible solutions to a scenario that is not feasible for another scenario. So, the overall value of the objective function value is computed by the weighted average over all scenarios' objective function values (expected total objective):

$$OF_{EEV} = \sum_{s=1}^{S} OF^*_s p_s \tag{4.1}$$

Wait and See approach: In WS approach, before implementing the optimal decisions, it is assumed that the decision maker can wait until the resolution of uncertainty [146]. In the

literature, it is referred as scenario analysis which actually delays all the decisions before all uncertainties have been solved [148]. The deterministic model is solved for each scenario s and s decision variables are obtained independently. Objective function OF_s in scenario s is directly computed by solving the deterministic model. The objective function values that are obtained from the deterministic model is multiplied with the probabilities of the scenarios to post calculate the overall objective function value of the WS approach. Therefore, the overall value of the objective function is post calculated with:

$$OF_{WS} = \sum_{s=1}^{S} OF_s p_s, \tag{4.2}$$

Here and Now approach (Two-stage Problem): The third approach is HN in which the optimization model is solved once by expanding original LP problem with the set of constraints corresponding to all generated scenarios. In other words, the model is solved by considering all the possible scenarios together. In this approach the obtained solution takes precaution against all the possible events that may happen in the future [146]. Consequently, the solution is feasible against all the realizations of scenarios. It is basically a two-stage optimization technique. In two-stage stochastic programming models, the first stage decisions must be taken before uncertainty is taken into account and the second stage decisions are provided considering the set of scenarios. Obviously there is one unique first stage decision while in the second stage there can be a one unique decision or decisions as many as the number of scenarios that is one for each scenario. So the problem is feasible for each scenario and the decision vector that is obtained from the first stage is feasible over all scenarios.

To summarize, there are three commonly used modeling and solution approaches for stochastic LP problems: 1) *Expected value* which is solved with a single model (the deterministic model). 2) *Wait and see* approach in which the deterministic model is solved for each scenario independently. Post calculation is made to obtain the expected

total objective function. The objective functions that are obtained from the scenarios are multiplied by the scenario probabilities. 3) *Here and now* approach which optimizes one expanded model with s constraints. HN approach produce solutions that are usually not as over-conservative as their robust counterpart but they are still protective against the most likely realizations.

The relationship between the approaches and the objective function values for the minimization problems are satisfied [149, 150] with the following inequality (see Figure 4.1):

$$OF_{WS} \leq OF_{HN} \leq OF_{EEV} \tag{4.3}$$

In the literature, two measures are defined for stochastic solutions that can be used to judge how well the obtained solution is. *Expected Value of Perfect Information* (EVPI) is a distance metric that measures the willingness of decision maker to obtain perfect information about the future [149]. The EVPI is defined as the difference between WS and the HN solution [150] (see Figure 4.1).

$$EVPI = OF_{HN} - OF_{WS} \tag{4.4}$$

While large EVPI shows that the incomplete information about the future may be costly, small EVPI may indicate that better forecast will not lead to a considerable amount of improvement. As it can be seen from the Figure 4.1, the desired objective value to be reached is WS, so it is important to obtain significant amount of difference.

The second measure is the *Value of Stochastic Solution* (VSS) which allows to obtain more

precisely the goodness of the EV solution comparing with the HN solution. It measures the expected gain from solving a stochastic model rather than its deterministic counterpart. If the difference between the EEV and HN solution is not large, we may not benefit from stochastic modeling. So it means no further information is incorporated in the deterministic model by applying stochastic approaches.

$$VSS = OF_{EEV} - OF_{HN} \tag{4.5}$$

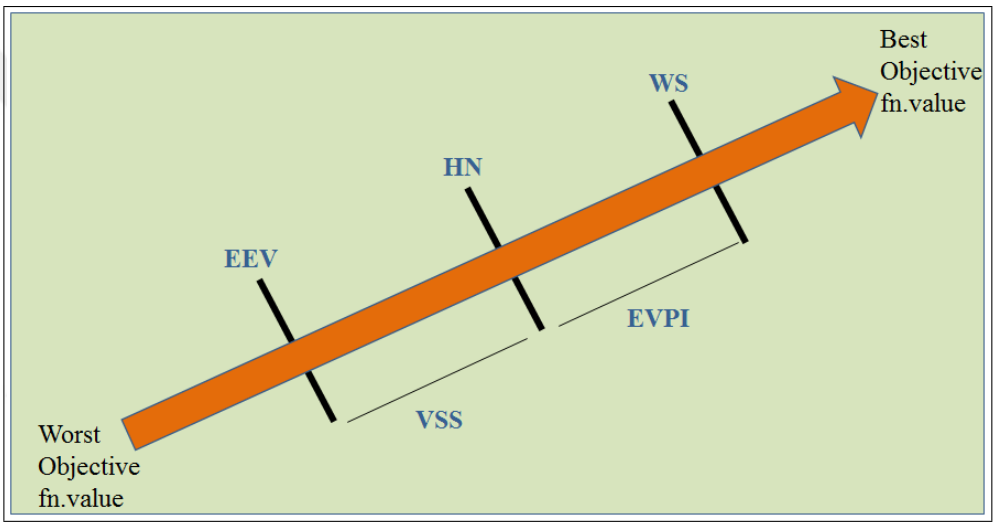

Figure 4.1. Ordered relationship between the approaches and measures for the stochastic solutions.

In the following, the proposed risk-neutral stochastic programming approaches are formulated for the proposed BDAS model.

4.2.1. Proposed Models

BD centers have a crucial role in feeding the needs of hospitals. The BDAS problem is stochastic in nature and it is important to model the scheduling process while considering the unpredictable arrival of non-booked donors. In this work, we consider the source of uncertainty as the number of non-booked donors. Since the booked donors call for

reservation, we know the number of booked donors in advance. In contrast to booked donors, we do not know the exact number of non-booked donors. The BDAS model described in Section 3.2 includes only the average values for the non-booked donors. The information about non-booked donor variability is not included in our model, so the uncertainty is not considered. In summary, the BDAS model is optimized only for a specific scenario in which the average values for non-booked donors are used. In order to obtain preallocated slots that account for stochastic nature of non-booked donors, we have to consider more than one possible outcome for the future.

To be able to analyze the effect of stochastic parameter (non-booked donors) on decision variables, we analyze either OF1 or OF1+OF3 objective terms for the sake of simplicity. In the following stochastic programming approaches, S indexes the set of scenarios where p_s is the probability of s scenario $s \in S$.

EV approach for OF1+OF3: Here, the deterministic BDAS model is used as it is explained in Section 3.2 which includes only one scenario (i.e., the average values of the non-booked donors). The considered scenario is given by the expected values. Since we consider the first and third objective function terms, OF1+OF3, we remove Constraints (3.4) and (3.15).

Evaluation of the Expected Value for OF1+OF3: EEV is the way to evaluate the obtained solution of the EV approach. That means fixing the solution of the EV problem, ($x_t^{b^*}$) for constraint (3.1) and evaluating the result against each scenario. Normally, it is important to check the feasibility of the obtained solution for each scenario since a feasible solution to a given scenario might not be feasible for another one. Since there is not a binding capacity constraint in the model, here we do not have to check the feasibility for each scenario.

First, in Equation (4.6), the summation of the total absolute variation with respect to the average request for all blood types for scenarios, ϕ^s is calculated as follows:

$$\phi^s = \sum_{b \in B} \sum_{t \in T} z_t^{bs} \qquad \forall s \in S \tag{4.6}$$

In equation (4.7), the summation of the total dispersion amount for each scenario, ψ^s is calculated as follows:

$$\psi^s = \sum_{t \in T} \sum_{k \in K} \delta_k p^s_{tk} \qquad \forall s \in S \tag{4.7}$$

Since $x^{b^*}_t$ is the EV solution, this solution is fixed and then the overall expected value of the objective function is post calculated by multiplying the objective function terms of each scenario with the scenario probabilities.

$$(\sum_{s \in S} \phi^s + \sum_{s \in S} \psi^s) p_s \tag{4.8}$$

EV approach for OF1: The deterministic BDAS model is used in EV approach which is explained in Section 3.2. Since the first objective function term is considered, OF1, the Constraints (3.4), (3.7), (3.8), (3.12), (3.14) and (3.15) are removed.

Evaluation of the Expected Value for OF1: EEV is the way to evaluate the obtained solution of the EV approach as it is explained. The evaluation of the EEV for OF1 is as follows: First, the Equation (4.6) is used to calculate the summation of the total absolute variation with respect to the average request for all blood types for each scenario, ϕ^s is calculated. The expected total objective function is post calculated by multiplying objective function values obtained from the EV approach with their probabilities, p_s against each scenario. Fixing the obtained solution from EV $x^{b^*}_t$, the total expected absolute variation is calculated as follows.

$$\sum_{s \in S} \phi^s p_s \tag{4.9}$$

WS approach for OF1+OF3: As mentioned before, in this case for each scenario s, the deterministic BDAS model is solved independently. Since the first and third objective function terms are considered, OF1+OF3, the Constraints (3.4) and (3.15) are removed. So

all the decision variables are decided for each scenario independently. The expected total objective is post calculated by multiplying objective function values of the scenarios with their probabilities, p_s. Equations (4.6), (4.7), (4.8) are used to calculate the overall objective function value.

WS approach for OF1: For each scenario s, the deterministic BDAS model is solved independently. Since the first objective function term is considered, OF1, the Constraints (3.4), (3.7), (3.8), (3.12), (3.14) and (3.15) are removed. So all the decision variables are obtained for each scenario with the perfect information. The overall objective function is post calculated by multiplying objective function values that are obtained from the scenarios with their probabilities, p_s. Equations (4.6), (4.9) are used to calculate the overall objective function value.

HN approach for OF1+OF3: In the HN approach, BDAS model is used to plan preallocated slots considering s scenario. The decision variables in this model can be divided into two stages. In the first stage, the decision variables are the number of preallocated slots. Once we observe the number of preallocated slots, we calculate the total number of produced blood units and then the total variation and total dispersion amount (see Constraints (4.11)-(4.14)). Hence, the decision variables in the second stage are the total number of produced blood units for each scenario s. Note that for different set of scenarios, there will be a different second stage decision and thus a different second stage objective value can be obtained.

The difference between the deterministic BDAS model and HN model is that the number of preallocated slots, x_t^b is decided by taking into account all of the considered scenarios, which means finding a single solution that satisfies all of the scenarios. The model is:

$$\min \left\{ (\sum_{s \in S} \phi^s + \sum_{s \in S} \psi^s) p_s \right\} \tag{4.10}$$

$$y_t^{bs} = x_t^b + n_t^{bs} + a_t^b \qquad \forall t \in T, b \in B, s \in S \qquad (4.11)$$

$$\sum_{\tau \in T} y_\tau^{bs} - y_t^{bs}|T| \leq z_t^{bs}|T| \qquad \forall t \in T, b \in B, s \in S \qquad (4.12)$$

$$y_t^{bs}|T| - \sum_{\tau \in\ T} y_\tau^{bs} \leq z_t^{bs}|T| \qquad \forall t \in T, b \in B, s \in S \qquad (4.13)$$

$$r \sum_{b \in B} \left(w_{tk}^b + \alpha_k n_t^{bs}\right) + R_{tk} \leq c_{tk} + p_{tk}^s \qquad \forall k \in K, t \in T, s \in S \qquad (4.14)$$

$$\phi^s = \sum_{b \in B} \sum_{t \in T} z_t^{bs} \qquad \forall s \in S \qquad (4.15)$$

$$\psi^s = \sum_{t \in T} \sum_{k \in K} \delta_k p_{tk}^s \qquad \forall s \in S \qquad (4.16)$$

Constraints (3.5) – (3.7) (4.17)

Constraints (3.10), (3.14) (4.18)

$$\phi^s \geq 0 \qquad \forall s \in S \qquad (4.19)$$

$$\psi^s \geq 0 \qquad \forall s \in S \qquad (4.20)$$

$$p_{tk}^s \geq 0 \qquad \forall s \in S \qquad (4.21)$$

$$y_t^{bs} \in \mathbb{N} \qquad \forall t \in T, b \in B, s \in S \qquad (4.22)$$

$$z_t^{bs} \geq 0 \qquad \forall t \in T, b \in B, s \in S \qquad (4.23)$$

The objective function is obtained by multiplying the summation of the total variation

and total dispersion amounts with the probability of the scenarios for each scenario independently.

HN approach for OF1: In the HN approach, BDAS model is used with s scenarios all together to plan preallocated slots. The values in the scenarios are replaced with the stochastic variables to obtain a solution that is in the feasible region considering all the scenarios. First stage decision variables are determined and considering the realization of the scenarios second stage decision variables are specified in order to minimize the objective function value (total variation and total dispersion amount). The modified BDAS model is:

$$\min \left\{ \sum_{s \in S} \phi^s p_s \right\} \tag{4.24}$$

Constraints (3.5) (4.25)

Constraints (3.6) (4.26)

Constraints (4.11) $-$ (4.13) (4.27)

Constraints (4.15) (4.28)

and the non-negativity constrainst are included.

To sum up, the literature on risk-neutral stochastic programming are described in this section. In the EV approach, the deterministic model is solved using the average value of the uncertain parameters. In the WS approach the deterministic model is solved after an observation is made on the random elements. That means, the deterministic model is solved independently for each scenario and with the obtained objective function values of each scenario is multiplied with the probabilities of the scenarios. In many real life problems,

some of the parameters are uncertain at the moment, and the decisions are made in the presence of this uncertainty as in the HN approach. The only assumption that can be made is that the random variable is sampled from known probability distribution.

In the following section, the numerical results for the proposed risk-neutral stochastic programming models are presented.

4.2.2. Numerical Results

In this section, the results of the stochastic programming models are presented. The risk-neutral stochastic programming models are implemented in GAMS 24.4 and solved via CPLEX. All experiments are run on a server on Windows OS with CPU Intel® Core™ i3, 2.40 GHz, and 3 GB of dedicated RAM.

According to the historical data of AVIS, looking at the donor frequencies, mean values of the non-booked donor are calculated. We assume that non-booked donors arrive independently to the BD center, and follow a given probability distribution. First, let us define the data generation mechanism for the scenarios. We use two different distributions for the non-booked donor arrivals, namely Poisson and Normal distributions.

Poisson distribution: A Poisson process with a constant rate of λ is used to approximate the arrival process of donors without reservation. The number of non-booked donors who arrive at each day are randomly generated for each blood type according to a Poisson distribution. Different mean values are generated with Monte Carlo (MC) technique for each blood type and non-booked alternative. MC technique is generally named as probability simulation which is used to model the probability of different outcomes while the random variables are not easily predicted. It is important to measure the impact of risk and uncertainty in the model. For each scenario, these non-booked donor numbers are automatically assigned for each day and for each blood type. Therefore, for each day a λ value is generated with the MC technique.

Normal distribution: It is assumed that the defined mean values and the standard deviation

values of the truncated Normal distributions are used across days. We consider both $N(\mu, \sigma)$ and $N(\mu, 3\sigma)$. First of all, uniform random numbers are generated for the number of non-booked donors who arrive at each day according to a Normal distribution with the mean values, μ and standard deviation, σ for each blood type separately with MC technique. For each scenario, the generated non-booked donor numbers are automatically assigned for each day, and blood type.

The proposed models are solved considering the first term of the objective function, OF1 and the first and the third objective function terms, OF1+OF3 for five random replications. MC technique is used to create different replications with constant mean values. We consider different number of scenarios as $|S| = [10, 50, 100, 250, 1000]$. In these scenarios, maximum number of scenarios are generated and these generated scenarios are decomposed as $10, 50, 100, 250$. The experiments are repeated for various values of scenarios to evaluate the impact of the scenarios that are used in the approaches.

Appropriate parameter settings are obtained from the deterministic model computational analysis. Two levels for the flexibility parameter, ε, are also considered, i.e., either $\varepsilon = 0$ or $\varepsilon = 0.25$. All instances are generated by considering 8 blood types ($|B| = 8$), 7 days of time horizon ($|T| = 7$) with 3 periods ($|K| = 3$), and capacities c_{tk} equal to 240, 240 and 240 minutes for $k = 1, 2, 3$, respectively, in all days t. Service duration is assumed to be 20 minutes ($r = 20$) and α_k fractions are considered equal to 0.5, 0.3 and 0.2 for $k = 1, 2, 3$, respectively.

The remaining parameters are chosen to fit the tested case: the vector of d_b values for the 8 blood types with $|T| = 7$ is assumed as $[80\ 20\ 24\ 5\ 8\ 2\ 73\ 14]$; the number of non-booked donors n_t^b mean values are assumed to be constant over the days and the vector for the different blood types is set equal to $[3\ 1\ 1\ 0\ 0\ 0\ 4\ 1]$ for Poisson distribution for each day taking into account the historical data. For the Normal distribution, $N(\mu, \sigma)$, mean values are assumed to be $[3.72\ 0.56\ 1.14\ 0.06\ 0.35\ 0.01\ 4.20\ 0.75]$ and the standard deviation values are assumed to be $[1.73\ 0.55\ 0.77\ 0.24\ 0.50\ 0.08\ 1.83\ 0.56]$. For Normal distribution, $N(\mu, 3\sigma)$, the standard deviations are multiplied by three. Since the result of Normal distribution are similar to the ones of the Poisson distribution results, so here, we

present the results of the Poisson distribution. The result tables for Normal distribution is presented in Appendix C.

Table 4.1. Results for stochastic programming approaches for Poisson distribution with half-width %95 confidence interval for OF1: (a) is with $\varepsilon = 0.25$, $t = 7$, (b) is with $\varepsilon = 0$, $t = 7$.

Scenario Number	OF_{EEV}	OF_{WS}	OF_{HN}
10	31.62±(0.95)	6.31±(0.06)	30.06±(1.42)
50	31.25±(0.74)	6.37±(0.00)	31.11±(0.78)
100	31.19±(0.70)	6.44±(0.04)	31.19±(0.70)
250	31.19±(0.22)	6.46±(0.00)	31.09±(0.22)
1000	31.11±(0.12)	6.45±(0.01)	31.11±(0.12)

(a)

Scenario Number	OF_{EEV}	OF_{WS}	OF_{HN}
10	35.89±(1.25)	20.48±(0.72)	33.35±(1.48)
50	35.70±(0.73)	20.84±(0.31)	34.53±(0.89)
100	35.61±(0.68)	20.70±(0.14)	34.77±(0.69)
250	35.51±(0.24)	20.59±(0.05)	34.86±(0.33)
1000	35.48±(0.11)	20.67±(0.05)	35.18±(0.12)

(b)

Results for the first objective function term (OF1): Table 4.1 show the half-width $\%95$ confidence interval for OF1 objective term among five repetitions. Two levels for the flexibility parameter are considered; Table 4.1 (a) is for $\varepsilon = 0.25$ and Table 4.1 (b) is for $\varepsilon = 0$. We can observe that increasing the number of scenarios proportionally decreases the confidence interval range. Detailed results for five different random replications and the CPU times are reported in Appendix D.

Since we use daily λ values for the non-booked donor arrivals, in the EV approach the results are the same for all the replications. For $\varepsilon = 0.25$ and $\varepsilon = 0$ cases, the objective function

value, OF_{EV}, is equal to 4.57 and 19.42 respectively. The EVPI represents the willingness to obtain perfect information for the non-booked donor arrivals in the proposed BDAS model. According to the table results, maximum willingness is obtained 24.51 unit on the average over the scenarios for $\varepsilon = 0.25$ and 13.88 unit for $\varepsilon = 0$. In contrast, from Table 4.1 (a), it can be observed that with $\varepsilon = 0.25$, the value of stochastic solution ($VSS = OF_{EEV} - OF_{HN}$) is too low and by increasing the number of scenarios, the objective function values for OF_{EEV} and OF_{HN} converge to the same value. If the deterministic solution is far from the stochastic solution for any given time, that would indicate the value of the stochastic solution. Table 4.1 (b) is analyzed with $\varepsilon = 0$ and without the flexibility term, ε. The average VSS values are increased from 0.36 to 1.10 compared to $\varepsilon = 0.25$ for the scenarios. The reason behind that is, we use the same mean values for each day and low values of VSS indicates that, ε involves another stochasticity to the problem.

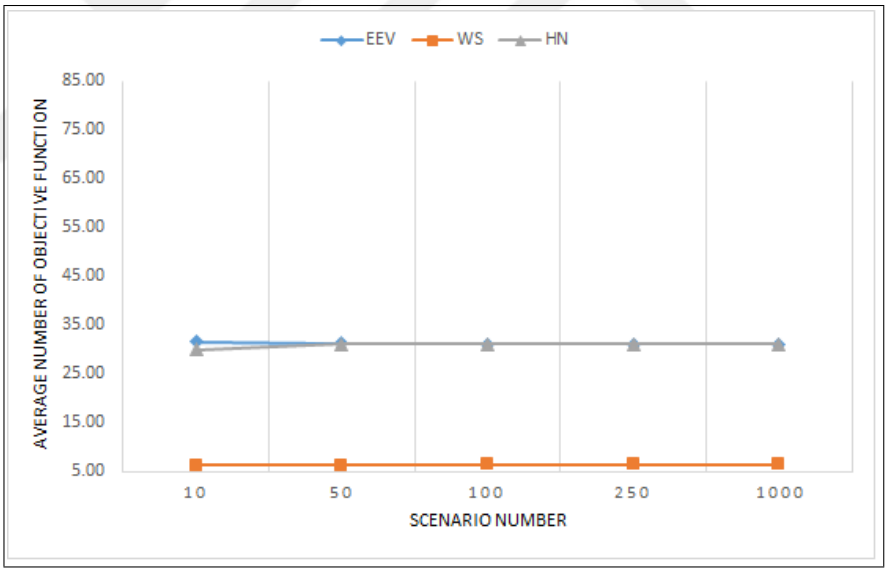

Figure 4.2. Comparison of the average objective function values of the replications with $\varepsilon = 0.25$, $t = 7$ for EEV, WS and HN approaches for OF1.

Figures 4.2 and 4.3 demonstrate the average objective function values of the proposed stochastic programming models. These figures are the summary of the half-width $\%95$ confidence interval tables for OF1. A large difference between OF_{EEV} and OF_{HN} shows that we are able to resolve the uncertainty or we can improve the solution. As it is discussed,

while the number of scenarios is increased, OF_{EEV} and OF_{HN} results converges to the same value. It can also seen from the figures that without the flexibility term, the solution is improved in terms of VSS. Therefore, we are far from the perfect information and it means the solution is almost the same with EV (deterministic model) approach. Since the expected gain is not too high while solving the stochastic model rather than the deterministic counterpart, the demand flexibility term, ε, is removed from our model in the stochastic programming analysis. We want to analyze a single randomness in the proposed stochastic programming models.

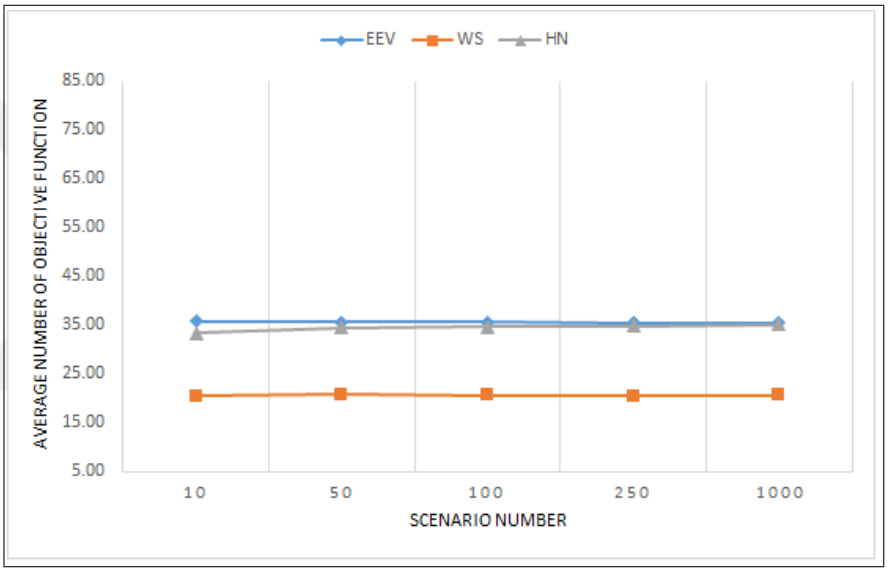

Figure 4.3. Comparison of the average objective function values of the replications with $\varepsilon = 0$, $t = 7$ for EEV, WS and HN approaches for OF1.

We know that EEV is the average values that are given by EV approach. So all the feasible solutions of the average value approach is already considered in the HN approach. Therefore, the optimal solution from HN approach must be better. According to the ordered relationship between the approaches, our objective function values always follow the order, ($OF_{WS} \leq OF_{HN} \leq OF_{EEV}$) as expected.

Furthermore, additional experiments are conducted to see the effect of longer time horizon ($t = 14$) . The half-width $\%95$ confidence interval tables are reported in Table 4.2.

For the detailed results for each replication see Appendix D. For $\varepsilon = 0.25$ and $\varepsilon = 0$ cases, the objective function value OF_{EV} is equal to 16.71 and 49.28 respectively. Setting the time horizon to 14 days did not change the previous observations.

Results for the first and the third objective function terms (OF1+OF3): Since the effect of ε is indicated in the analysis with OF1, the flexibility degree associated with d_b is not included for OF1+OF3 cases.

Table 4.2. Results for stochastic programming approaches for Poisson distribution with half-width %95 confidence interval for OF1: (a) is with $\varepsilon = 0.25$, $t = 14$, (b) is with $\varepsilon = 0$, $t = 14$.

Scenario Number	$\boldsymbol{OF_{EEV}}$	$\boldsymbol{OF_{WS}}$	$\boldsymbol{OF_{HN}}$
10	61.77±(4.21)	9.95±(0.86)	55.31±(3.90)
50	62.08±(1.20)	10.55±(0.18)	59.83±(1.24)
100	62.33±(1.09)	10.65±(0.24)	60.32±(1.18)
250	62.41±(0.47)	10.67±(0.12)	60.75±(0.43)

(a)

Scenario Number	$\boldsymbol{OF_{EEV}}$	$\boldsymbol{OF_{WS}}$	$\boldsymbol{OF_{HN}}$
10	76.87±(3.63)	48.14±(1.84)	70.71±(3.89)
50	77.01±(1.75)	48.21±(1.11)	74.54±(1.45)
100	76.87±(1.49)	48.03±(0.54)	74.92±(1.06)
250	77.21±(0.93)	48.18±(0.55)	75.77±(0.59)

(b)

The results are given in Table 4.3 with the stochastic programming approaches for the objective function terms, OF1+OF3. When we compare the objective terms, OF1 and OF1+OF3, the VSS measure improvement for OF1+OF3 is increased from $\%3.18$ to $\%5.65$ considering the combination of two objective function terms, OF1+OF3. High values of $VSS = OF_{EEV}$-OF_{HN} is obtained with OF1+OF3 in each replication.

The measurably small differences show that the optimal solutions are not sensitive to the value of random variables. In that case, finding the optimal solution with deterministic model yields the same result with the stochastic model. Hence if the prediction of non-booked donor arrivals is good enough, the HN approach does not improve the solution a lot. To this end, modeling the randomness based on the risk-neutral approach does not play a significant role on the obtained solutions for BDAS model.

Table 4.3. Results for stochastic programming approaches for Poisson distribution with half-width %95 confidence interval for OF1+OF3 with $\varepsilon = 0$, $t = 7$.

Scenario Number	OF_{EEV}	OF_{WS}	OF_{HN}
10	82.19±(2.87)	60.51±(2.62)	75.93±(2.33)
50	81.52±(1.19)	60.71±(0.68)	77.24±(1.19)
100	81.57±(0.88)	60.65±(0.34)	77.53±(0.95)
250	81.36±(0.76)	60.43±(0.41)	77.56±(0.45)
1000	81.30±(0.40)	60.44±(0.15)	77.86±(0.19)

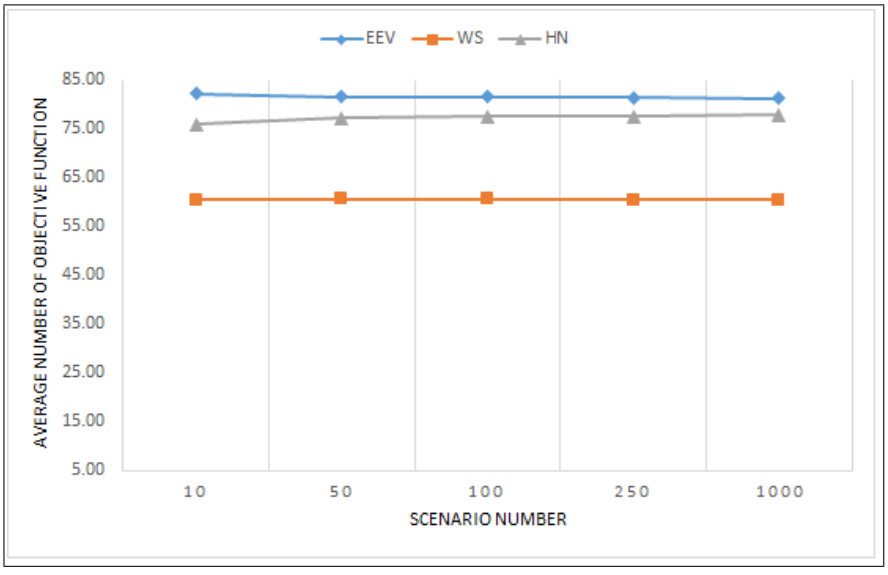

Figure 4.4. Comparison of the average objective function values of the replications with $\varepsilon = 0$, $t = 7$ for EEV, WS and HN approaches for OF1+OF3.

As it can be also seen from the Figure 4.4, the value of stochastic solution is increased with the combination of two objective term. Since increasing the planning period does not change the observations, longer planning period analysis is not considered for OF1+OF3.

Since risk-neutral stochastic programming approaches consider the expected values in their objectives, risk is not taken into account in these approaches. According to the results of risk-neutral stochastic programming approaches, the solution is far from that of the perfect information, so it might be better to consider an effective approach that is also aware of the risk since risk-neutral models cannot capture the risk as explained. To this end, in the following section we propose a risk-averse stochastic programming approach that also includes the risk terms.

4.3. RISK-AVERSE STOCHASTIC PROGRAMMING MODELS WITH ONLY RISK TERMS

The formulation in classical stochastic programing is based on the expected value of an objective function and such a formulation is assumed as a risk-neutral. So the decision makers do not account for the large losses in some of the realizations. Therefore, in risk-neutral stochastic programming models, expectations can be found analytically but these formulations cannot capture the risk since random variables are replaced with their mean values. When all certain realizations are considered, decisions based on the expected value may perform poorly (can be seen from Section 4.2). Therefore, it is important to consider the effect of variability which is actually defined as the *risk* concept. More specifically, risk-averse stochastic programming models imply that the probability for each possible random outcome is known in advance. So according to the random outcomes we have different risk preferences. The aim is lowering the effects of undesirable results for the realizations. Risk-averse models measure risk with different approaches such as value-at-risk, conditional value-at-risk, mean-risk analysis, expected utility theory, stochastic dominance and chance constraints [151].

In this dissertation, we consider two types of models that incorporate risk measures; in Section, 4.3.1 we only consider the risk terms whereas in Section 4.3.2, we consider the

mean-risk approach. We first introduce the definitions of VaR and CVaR. VaR is used in the CVaR interpretation, therefore, first VaR definition is presented as follows.

Value-at-risk (VaR) is widely used as a risk measure which calculates the maximum loss with the confidence level α [152]. VaR does not take into account the size of losses that may occur when the VaR is exceeded. VaR is a relatively simple risk management concept and has a clear interpretation. VaR focuses on the part of the distribution specified by the confidence level and it actually does not consider the properties of the distribution beyond the confidence level. It is exactly obtained by taking the percentile of the loss distribution with different α confidence levels. VaR can be formally defined as follows.

Definition 1 *Let $F_Z(.)$ represent the cumulative distribution function of a random variable Z. The α-quantile*

$$inf\left\{\eta : F_Z(\eta) \geq \alpha\right\} \tag{4.29}$$

is called the Value-at-Risk (VaR) at the confidence level α and denoted by $VaR_\alpha(Z)$, $\alpha \in (0, 1]$.

Conditional Value-at-risk (CVaR) is proposed by Rockafellar and Uryasev [152] due to the limitations of VaR approach. CVaR measure takes into account both the α-quantile (it takes into account the impacts of all extreme values in the tail of distribution) and the conditional expectation of the least favorable outcomes. It means the conditional mean value of the worst $1 - \alpha * 100\%$ losses. It is also known as Mean Excess Loss, Mean Shortfall, or Tail VaR. We consider CVaR as a risk measure which includes the effects of the stochastic nature of the system.

Definition 2 *The Conditional Value-at-Risk (CVaR) of a random variable Z at the confidence level α is given by*

$$CVaR_{\alpha}(Z) = inf_{\eta \in \mathbb{R}} \left\{ \eta + \frac{1}{1-\alpha} E(max[Z-\eta], 0) \right\} \tag{4.30}$$

$CVaR_{\alpha}(Z)$ is the expectation value that exceeds the VaR value at the confidence level α. VaR provides an upper bound that is exceeded with a small probability of $1-\alpha$.

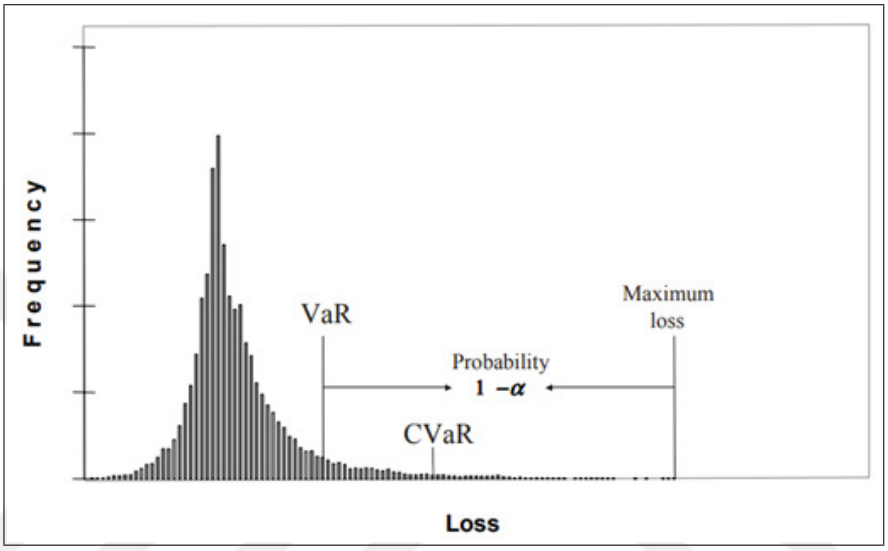

Figure 4.5. The relation between Var and CVaR [151].

The definitions guarantee that VaR cannot be greater than CVaR value. While VaR calculates a quantile of a loss distribution, CVaR calculates the weighted sum of the least favorable outcomes that are exceeding VaR. In comparison to VaR, CVaR provides additional information on the magnitude of the excess loss. CVaR is designed to measure the risk of extreme losses, so CVaR is an extension of VaR that gives the total amount of loss.

The relation between VaR and CVaR can be seen from the Figure 4.5 explicitly [151].

According to the preferences, different CVaR objectives with several α-levels can be tested and that can reshape the loss distribution taking into account the α change. These preferences are decided with different percentile terms. Actually, α represents risk-aversion

degree of a decision maker. To express decision makers' risk preferences, we can specify various confidence levels that allows higher flexibilities.

In order to model the linear optimization model, Rockafellar and Uryasev [152] also provided a transformation with the linearization of the CVaR equation. Since the maximum operator in Equation (4.30) is not linear, we present the linearization of the CVaR equation which is provided by Rockafellar and Uryasev [152].

Linearization of the CVaR approach: Let Z be a random variable with realizations $z_1, \ldots, z_s$ and the probabilities $s = 1, 2, \ldots, S$ respectively. The probability will not exceed a threshold η is then given by the VaR equation (4.29) as in $\{\eta : F_Z(\eta) \geq \alpha\}$. As a function of η, $F_Z(\eta)$ is the cumulative distribution function for the loss associated with z. By using dummy variables h_s, $s = 1, 2, \ldots, S$, the function $F_Z(\eta)$ can be replaced by the linear function $\eta + \dfrac{1}{1-\alpha}\sum_{s\in S} h_s p_s$ where p_s are the probabilities of the scenarios. For a given confidence level $\alpha \in [0, 1)$, the linear optimization problem that approximates, $CVaR_\alpha(Z)$ is equal to:

$$\min \eta + \frac{1}{1-\alpha}\sum_{s\in S} h_s p_s \tag{4.31}$$

$$h^s \geq Z^s - \eta \qquad \forall s \in S \tag{4.32}$$

$$h^s \geq 0 \qquad \forall s \in S \tag{4.33}$$

where $h^s = max(Z_s - \eta, 0)$. Therefore, CVaR risk measure can be written as linear functions for the optimization problems [151].

As we mentioned, we consider two types of models that incorporate risk measure. In the CVaR part, we only consider the risk terms in the models, whereas in the following definition

we consider both the expectation and the risk measure (which is called mean-risk approach).

Mean-risk approach firstly proposed by Markowitz [153] for a portfolio optimization problem. For the mean-risk approach, the expected outcome of interest is explained as expected value and the value of a risk measure. Thus, the trade-off between the mean and the risk is considered in mean-risk approach. Briefly, the aim is to minimize the weighted combination of these. The general formulation of the mean-risk function with CVaR is as follows:

$$E[Z] + \theta CVaR_{\alpha}[Z] \tag{4.34}$$

where Z is the random outcome and the parameter θ is a non-negative trade off coefficient. The overall objective is calculated by the summation of the expected outcome and a weight multiplied by the CVaR value. $E[Z]$ is denoted as $E(.)$ and $CVaR_{\alpha}[Z]$ is denoted as $CVaR(.)$ for the rest of the dissertation.

When the health care literature is considered, Noyan [154] presented alternate risk measures for emergency medical service system design and Chan *et al.* [155] presented a robust-CVaR optimization approach with application to breast cancer therapy. To the best of our knowledge, this study is the first attempt that uses a CVaR approach in a BD system. Since controlling the random arrival of non-booked donors is not easy in a BDAS system, it causes an imbalanced production for the entire system. Since CVaR is a policy that embeds risk, the possible large realizations of non-booked donor numbers for BDAS model may be considered. Risk may effect the entire BD system and to be able to measure risk, generally the loss resulting from a donor appointment schedule is measured. In the considered BD system, the result is an imbalanced production. However, instead of minimizing total variation and total weighted dispersion amount, we can minimize the expected total variation and expected total weighted dispersion amount considering the risk measure CVaR. In the following section, the proposed risk-averse stochastic programming models are presented.

4.3.1. Proposed Models

In this study, we develop two-stage risk-averse stochastic programming models for the considered BDAS problem. The models try to optimize the appointment schedule while minimizing total variation and/or total weighted dispersion amount, under random non-booked donor arrivals.

Minimizing OF1+OF3, by Using CVaR ($CVaR^{RA-HN}$): In this model, we try to minimize $CVaR_\alpha \left\{\sum_{b\in B}\sum_{t\in T} z_t^{bs} + \sum_{t\in T}\sum_{k\in K}\delta_k p_{tk}^s\right\}$ and the corresponding model is as follows:

$$\min\left\{\eta + \frac{1}{1-\alpha}\sum_{s\in S} p^s h^s\right\} \tag{4.35}$$

$$h^s \geq \phi^s + \psi^s - \eta \qquad \forall s \in S \tag{4.36}$$

$$\text{Constraints (3.5)} - \text{(3.7)} \tag{4.37}$$

$$\text{Constraints (4.11)} - \text{(4.23)} \tag{4.38}$$

$$h^s \geq 0 \qquad \forall s \in S \tag{4.39}$$

$$\eta \geq 0 \tag{4.40}$$

where h^s variables are used to compute the $CVaR_\alpha$ value on the total variation and total weighted dispersion amount under different scenarios. Constraints (4.36) and (4.40) are introduced to linearize the maximum operator in the equation (4.30) under each scenario. Due to the minimization objective, if the difference in constraint (4.36) is negative, then h^s gets value 0. Furthermore, the non-negativity constraints are included.

Minimizing OF1, by Using CVaR ($CVaR^{RA-HN}$): In this model, we focus on the random variable, total variation and the model's aim is to minimize $CVaR_\alpha \left\{\sum_{b\in B}\sum_{t\in T} z_t^{bs}\right\}$.

$$\min\left\{\eta + \frac{1}{1-\alpha}\sum_{s\in S} p^s h^s\right\} \tag{4.41}$$

$$h^s \geq \phi^s - \eta \qquad \forall s \in S \tag{4.42}$$

Constraints (3.5) (4.43)

Constraints (3.6) (4.44)

Constraints (4.11) – (4.13) (4.45)

Constraints (4.15) (4.46)

where h^s variables are used to compute the $CVaR_\alpha$ value on the total variation under different scenarios. Constraints (4.42) and (4.40) are introduced to linearize the maximum operator in the equation (4.30) under each scenario. Due to the minimization objective, if the difference in constraint (4.42) is negative, then h^s gets value 0. Furthermore, the non-negativity constraints are included.

4.3.2. Numerical Results

In this section, we provide the numerical results for the risk-averse models that are focused on analyzing how the decisions change by incorporating the risk terms and the effects of the risk parameters.

We also provide the relative differences of the risk-averse and risk-neutral models based on the considered random outcomes. More specifically, for the $CVaR(.)$ values we define the relative difference (RD) as follows:

$$RD = \frac{CVaR_{\alpha}^{RA} - CVaR_{\alpha}^{RN}}{CVaR_{\alpha}^{RN}} \tag{4.47}$$

where $CVaR_{\alpha}^{RA}$ and $CVaR_{\alpha}^{RN}$ correspond to the risk-averse and the risk-neutral (HN and EEV approach) models, respectively. $CVaR_{\alpha}^{RA}$ is the model outcome while $CVaR_{\alpha}^{RN}$ is the calculation based on the risk-neutral model outcome. The relative differences of other indicators (i.e. $E(.)$) are also calculated similarly.

Several problem instances are considered with different sizes to test risk-averse BDAS models. The generated instances are described in Section 4.2.2. So we either test for the first objective function term (OF1) or we test for the first and the third objective function terms (OF1+OF3) with or without flexibility degree, $\varepsilon = 0.25$ or $\varepsilon = 0$. Previously, analyzed risk-neutral (HN and EEV apparoaches) solutions (see Section 4.2.2) are compared with the risk-averse model solutions in the following part. When we calculate $CVaR(.)$, based on the solutions that are obtained in HN and EEV approaches (without directly optimizing), they are named as $CVaR^{RN-HN}$ and $CVaR^{RN-EEV}$, respectively.

For OF1 and OF1+OF3, 2 different flexibility degree $\varepsilon = 0$ or $\varepsilon = 0.25$ are considered. We consider five different replications and each combination of $\alpha = [0.8, 0.9, 0.99]$, and five alternative scenario number $|S| = [10, 50, 100, 250, 1000]$. All the scenario probabilities are equally likely or sampled from the Uniform distribution on the interval $[0.2, 0.6]$ and then normalized. We also show the relative differences of $E(.)$ and $CVaR(.)$ values with respect to their risk-neutral models. The computational study with the proposed models are conducted by using GAMS 24.4 modeling language. Note that all the numerical experiments are performed with CPU Intel® Core™ i3, 2.40 GHz, and 3 GB of dedicated RAM.

Results for the first objective function term ($OF1-CVaR^{RA}$): In Table 4.4, we calculate the relative difference (RD) values of the replications for each number of scenarios with

$\varepsilon = 0$. At the last column of the table, the presented results are confidence intervals for the average values over replications.

Table 4.4. Relative difference values of each replication and confidence intervals of the relative differences for different number of scenarios ($OF1 - CVaR^{RA-HN}$, $\varepsilon = 0, t = 7$), * indicates the case is solved with an optimality gap$< 1.3\%$.

α=0.8 with equal probability						
Number of Scenarios	**RD - Rep. 1**	**RD - Rep. 2**	**RD - Rep. 3**	**RD - Rep. 4**	**RD - Rep. 5**	**Confidence Interval**
10	-10.24%	-4.40%	-4.44%	-11.53%	-5.51%	-7.22 $\pm$ 4.22
50	-3.09%	-1.92%	-4.52%	-3.37%	-3.07%	-3.19 $\pm$ 1.15
100	-1.90%	-1.84%	-2.15%	-1.96%	-3.52%	-2.28 $\pm$ 0.87
250	-2.14%	-1.48%	-1.59%	-0.87%	-1.41%	-1.50 $\pm$ 0.57
1000	-0.56%*	-0.28%*	-0.75%*	-0.44%*	-0.44%*	-0.49 $\pm$ 0.22
α=0.9 with equal probability						
Number of Scenarios	**RD - Rep. 1**	**RD - Rep. 2**	**RD - Rep. 3**	**RD - Rep. 4**	**RD - Rep. 5**	**Confidence Interval**
10	-10.81%	-4.76%	-4.44%	-11.49%	-5.47%	-7.39 $\pm$ 4.29
50	-5.16%	-2.60%	-5.91%	-6.41%	-4.94%	-5.01 $\pm$ 1.82
100	-3.33%	-3.49%	-3.43%	-3.76%	-6.10%	-4.02 $\pm$ 1.46
250	-3.23%	-3.30%	-2.59%	-1.32%	-2.31%	-2.55 $\pm$ 1.00
1000	-0.69%*	-0.40%*	-1.02%*	-0.77%*	-1.02%*	-0.78 $\pm$ 0.32
α=0.99 with equal probability						
Number of Scenarios	**RD - Rep. 1**	**RD - Rep. 2**	**RD - Rep. 3**	**RD - Rep. 4**	**RD - Rep. 5**	**Confidence Interval**
10	-12.25%	-5.97%	-4.44%	-11.97%	-6.65%	-8.26 $\pm$ 4.48
50	-13.18%	-7.78%	-10.25%	-12.38%	-7.84%	-10.29 $\pm$ 3.10
100	-7.50%	-11.88%	-8.18%	-14.12%	-13.75%	-11.08 $\pm$ 3.84
250	-8.22%	-5.82%	-6.18%	-3.61%	-3.60%	-5.49 $\pm$ 2.42
1000	-4.09%*	-3.63%*	-2.27%*	-4.01%*	-3.73%*	-3.55 $\pm$ 0.92

For the detailed risk-averse and risk-neutral comparison tables for each replication with $\varepsilon = 0$ please see Appendix E (Tables E.1-E.7).

As seen from Table 4.4, improvements of the $CVaR(.)$ values are higher with respect to the risk-neutral cases. We are successful to obtain significant amount of decreases in the objective function values with CVaR approach. A negative percentage value represents a lower $CVaR(.)$ value in the objective function. Increasing α, results an increase in the RD values.

Relative differences which show the level of improvement in $CVaR(.)$ values is generally data dependent and we are able to achieve on the average up to $\%11.08$ reduction amounts in the $CVaR(.)$ values with respect to risk-neutral cases for 5 different replication with $\varepsilon = 0$ and for $\alpha = 0.99$. When the number of scenarios are increased, there is generally a reduction in the CVaR improvements due to the correlation between days or because of the constant λ value that is used to generate scenarios for different days.

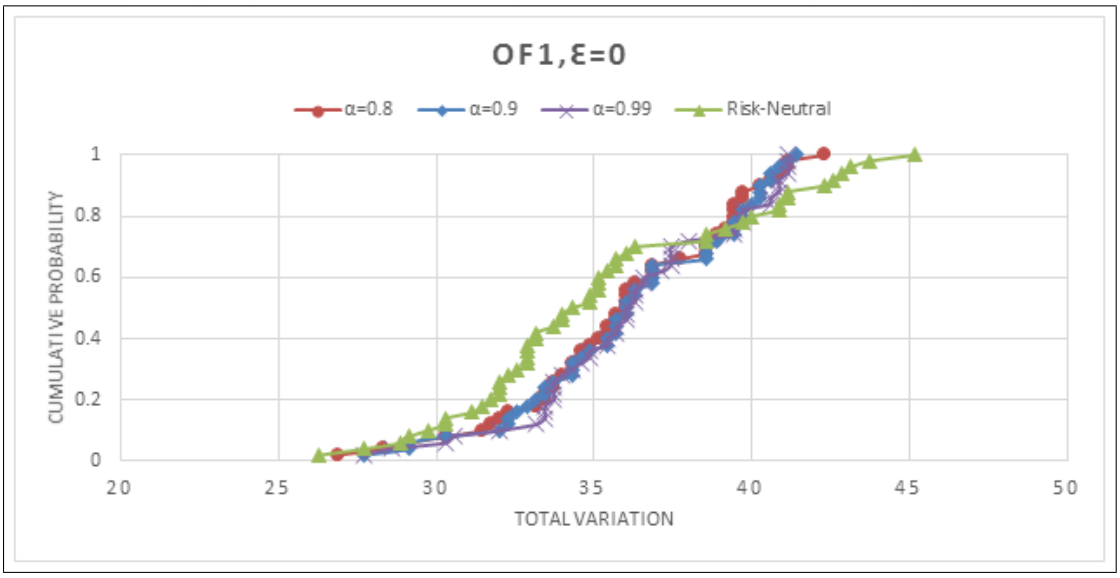

Figure 4.6. Cumulative distribution functions of the total variation for different values of α with $|S| = 50, \varepsilon = 0$.

In addition, we provided the cumulative distribution function (cdf) plots associated with

risk-neutral and risk-averse models which are presented in the Figure 4.6 with $\alpha = [0.8, 0.9, 0.99]$. Risk-averse approach necessarily shapes the cdf according to the decision makers' risk preferences. An increase in the α values results with a reduction in the critical scenarios and results an increase in the importance of the extreme realizations. Because, the critical values that are exceeding VaR value are decreased while the α value is increased. Therefore, it principally results with a shift to the left (right tail of the cdf).

The critical scenarios can be calculated easily by $(1 - \alpha)|S|$ that are exceeding VaR value. Figure 4.6 shows the cdf plot for a replication and for the further figures please see Appendix F (Figures F.1-F.5). When the cumulative distribution function is considered, there is a trade-off between $E(.)$ and $CVaR(.)$ improvement. The expectation is increased and as a result, cdf is shifted to the left with respect to the right tail of the cdf.

In contrast to $CVaR(.)$ improvements, there is an increase in the $E(.)$ values. The reason behind this trade-off is risk-neutral models are based on the expected value calculation in their objective functions while considering all the possible realizations. Therefore, an equal importance is given for each scenario. But in risk-averse models, it is important to consider the effect of randomness. The importance is given to the values that are exceeding VaR value and try to minimize these values in the model. The trade-off is obtaining an improved solution in terms of CVaR while obtaining a higher $E(.)$ values.

We also compare all the results against HN and EEV approaches to see the effect of risk factor on the corresponding approaches. We know from the risk-neutral models that EEV is the expected result of EV approach. A post $CVaR(.)$ calculation is done for the EEV solution. From the analysis made for risk-neutral models, once the number of scenarios is increased, OF_{EEV} and OF_{HN} results tends to converge to the similar value.

However, as it can be seen from the Tables 4.4 and 4.5, with risk-averse models we are able to improve the solution of $CVaR^{RA-HN}$ and $CVaR^{RN-EEV}$ approaches. More specifically, we are able to achieve on the average up to $\%14.88$ reduction amounts in the $CVaR(.)$ values with respect to risk-neutral cases (with $\varepsilon = 0$). For instance, when we consider $CVaR^{RN-EEV}$ calculations with $\alpha = 0.8, 0.9, 0.99$, we get on the average $\%13.43$, $\%14.88$

and $\%14.88$ improvement for 10 scenarios respectively.

Table 4.5. Relative difference values of each replication and confidence intervals of the relative differences for different number of scenarios ($OF1 - CVaR^{RN-EEV}$, $\varepsilon = 0$, $t = 7$).

α=0.8 with equal probability						
Number of Scenarios	**RD - Rep. 1**	**RD - Rep. 2**	**RD - Rep. 3**	**RD - Rep. 4**	**RD - Rep. 5**	**Confidence Interval**
10	-8.04%	-14.34%	-12.54%	-14.98%	-17.24%	-13.43 $\pm$ 4.28
50	-4.57%	-4.45%	-4.84%	-9.34%	-3.57%	-5.35 $\pm$ 2.83
100	-2.88%	-3.52%	-3.69%	-6.40%	-3.97%	-4.09 $\pm$ 1.68
250	-2.94%	-2.54%	-2.04%	-4.99%	-3.78%	-3.26 $\pm$ 1.44
1000	-1.68%	-0.51%	-0.90%	-1.44%	-1.46%	-1.20 $\pm$ 0.59
α=0.9 with equal probability						
Number of Scenarios	**RD - Rep. 1**	**RD - Rep. 2**	**RD - Rep. 3**	**RD - Rep. 4**	**RD - Rep. 5**	**Confidence Interval**
10	-8.33%	-18.37%	-16.23%	-14.94%	-16.55%	-14.88 $\pm$ 4.79
50	-5.93%	-5.59%	-6.77%	-10.91%	-5.34%	-6.91 $\pm$ 2.85
100	-3.97%	-5.21%	-5.09%	-7.01%	-5.12%	-5.28 $\pm$ 1.36
250	-4.32%	-4.54%	-2.67%	-5.67%	-4.56%	-4.35 $\pm$ 1.34
1000	-2.01%	-0.71%	-1.24%	-1.80%	-2.05%	-1.56 $\pm$ 0.72
α=0.99 with equal probability						
Number of Scenarios	**RD - Rep. 1**	**RD - Rep. 2**	**RD - Rep. 3**	**RD - Rep. 4**	**RD - Rep. 5**	**Confidence Interval**
10	-8.33%	-18.37%	-16.23%	-14.94%	-16.55%	-14.88 $\pm$ 4.79
50	-9.62%	-8.11%	-8.28%	-12.10%	-6.90%	-9.00 $\pm$ 2.46
100	-9.76%	-11.88%	-7.01%	-7.59%	-13.21%	-9.89 $\pm$ 3.32
250	-12.39%	-10.56%	-3.47%	-5.68%	-7.53%	-7.93 $\pm$ 4.47
1000	-5.42%	-2.00%	-2.76%	-1.71%	-2.88%	-2.95 $\pm$ 1.82

When we consider $CVaR^{RA-HN}$ results with $\alpha = 0.8, 0.9, 0.99$, we get on the average $\%7.22$, $\%7.39$ and $\%8.26$ improvement for 10 scenarios respectively (see Table 4.4 and Table 4.5). When we compare the results for $CVaR^{RN-EEV}$ calculations with $CVaR^{RA-HN}$ results, the $CVaR^{RN-EEV}$ average improvements are lower than the $CVaR^{RA-HN}$ results. So in contrast to risk-neutral models, modeling the randomness based on the risk averse models plays a significant role for BDAS system.

Furthermore, we also analyze the first objective function term with $\varepsilon = 0.25$, but as it is discussed ε involves another stochasticity to the problem.

For the results with $\varepsilon = 0.25$ please see Appendix E (Tables E.8-E.14) for the tables and Appendix F (Figures F.6-F.8) for the figures for the total variation (OF1).

As it is explained, we test with equal scenario probabilities and with different scenario probabilities that are drawn from a Uniform distribution. Since the $CVaR(.)$ relative differences with equal scenario probability cases are similar to the different scenario probability cases, the results are demonstrated in the Appendix E.

Results for the first and third objective function terms ($OF1OF3 - CVaR^{RA}$): We also analyze the objective function terms OF1+OF3 with $\varepsilon = 0$. The results are represented in Table 4.6 for different replications and the cdf plot is shown in Figure 4.7.

We are able to observe same comments as in OF1 with $\varepsilon = 0$. We are able to achieve on the average up to $\%9.26$ reduction amounts in the $CVaR(.)$ values with respect to risk-neutral cases. For instance when we consider 10 scenario case, we get on the average up to $\%4.6$, $\%5.58$ and $\%7.28$ improvement for $\alpha = [0.8, 0.9, 0.99]$, respectively.

When $CVaR^{RN-EEV}$ calculations with $\alpha = [0.8, 0.9, 0.99]$ are considered, we get on the average $\%13.82$, $\%15.58$ and $\%15.58$ improvement for 10 scenarios respectively. When $CVaR^{RA-HN}$ results are considered with $\alpha = [0.8, 0.9, 0.99]$, we get on the average $\%4.60$, $\%5.58$ and $\%7.28$ improvement for 10 scenarios respectively (see Table 4.6 and Table 4.7).

As a result of the conducted computational study we observe that the proposed risk-averse

Table 4.6. Relative difference values of each replication and confidence intervals of the relative differences for different number of scenarios ($OF1OF3 - CVaR^{RA-HN}$, $\varepsilon = 0$, $t = 7$), * indicates the case is solved with an optimality gap$< 0.3\%$.

α=0.8 with equal probability						
Number of Scenarios	**RD - Rep. 1**	**RD - Rep. 2**	**RD - Rep. 3**	**RD - Rep. 4**	**RD - Rep. 5**	**Confidence Interval**
10	-3.31%*	-4.87%*	-4.58%*	-5.82%*	-4.40%*	-4.6 $\pm$ 1.12
50	-2.45%*	-1.15%	-2.93%	-3.42%	-2.70%	-2.53 $\pm$ 1.05
100	-1.12%	-1.70%	-1.68%	-1.96%*	-1.94%	-1.68 $\pm$ 0.42
250	-1.56%	-2.06%	-1.86%	-1.66%	-1.45%*	-1.72 $\pm$ 0.30
1000	-0.88%*	-1.00%*	-1.13%*	-1.13%*	-1.36%	-1.10 $\pm$ 0.22
α=0.9 with equal probability						
Number of Scenarios	**RD - Rep. 1**	**RD - Rep. 2**	**RD - Rep. 3**	**RD - Rep. 4**	**RD - Rep. 5**	**Confidence Interval**
10	-3.57%*	-5.62%*	-6.03%*	-6.52%*	-6.16%*	-5.58 $\pm$ 1.45
50	-3.56%*	-3.16%	-3.47%*	-6.15%	-4.77%*	-4.22 $\pm$ 1.54
100	-1.71%*	-3.29%	-2.44%	-3.70%	-2.78%*	-2.79 $\pm$ 0.95
250	-2.16%	-3.02%	-2.48%	-2.77%	-1.68%*	-2.42 $\pm$ 0.65
1000	-1.35%*	-1.56%*	-1.44%*	-1.56%*	-1.89%*	-1.56 $\pm$ 0.25
α=0.99 with equal probability						
Number of Scenarios	**RD - Rep. 1**	**RD - Rep. 2**	**RD - Rep. 3**	**RD - Rep. 4**	**RD - Rep. 5**	**Confidence Interval**
10	-4.05%*	-7.05%*	-8.39%*	-7.79%*	-9.12%*	-7.28 $\pm$ 2.43
50	-7.52%	-10.47%*	-7.21%*	-13.94%*	-7.31%*	-9.29 $\pm$ 3.64
100	-3.24%*	-6.26%*	-3.07%*	-6.93%*	-9.21%*	-5.74 $\pm$ 3.23
250	-6.55%*	-7.71%*	-8.59%*	-6.87%*	-4.88%*	-6.92 $\pm$ 1.72
1000	-3.45%*	-2.43%*	-2.35%*	-4.37%*	-4.70%*	-3.46 $\pm$ 1.35

models are successful to achieve a better OF1+OF3 values and the previous observations did not change.

Table 4.7. Relative difference values of each replication and confidence intervals of the relative differences for different number of scenarios ($OF1OF3 - CVaR^{RN-EEV}$, $\varepsilon = 0$, $t = 7$).

α=0.8 with equal probability						
Number of Scenarios	**RD - Rep. 1**	**RD - Rep. 2**	**RD - Rep. 3**	**RD - Rep. 4**	**RD - Rep. 5**	**Confidence Interval**
10	-7.28%	-13.88%	-13.46%	-15.45%	-19.01%	-13.82 $\pm$ 5.28
50	-9.72%	-9.24%	-9.18%	-9.78%	-10.17%	-9.62 $\pm$ 0.51
100	-8.39%	-8.56%	-8.23%	-8.40%	-9.32%	-8.58 $\pm$ 0.53
250	-8.02%	-7.85%	-7.96%	-7.47%	-9.05%	-8.07 $\pm$ 0.73
1000	-7.63%	-7.15%	-6.98%	-7.22%	-7.75%	-7.34 $\pm$ 0.41
α=0.9 with equal probability						
Number of Scenarios	**RD - Rep. 1**	**RD - Rep. 2**	**RD - Rep. 3**	**RD - Rep. 4**	**RD - Rep. 5**	**Confidence Interval**
10	-9.56%	-14.37%	-14.35%	-19.75%	-19.87%	-15.58 $\pm$ 5.38
50	-11.65%	-10.65%	-9.55%	-12.53%	-12.98%	-11.47 $\pm$ 1.73
100	-5.96%	-6.75%	-6.05%	-4.24%	-5.36%	-5.67 $\pm$ 1.17
250	-8.90%	-8.55%	-8.47%	-8.94%	-9.88%	-8.95 $\pm$ 0.70
1000	-7.92%	-8.05%	-7.50%	-7.96%	-8.56%	-8.00 $\pm$ 0.47
α=0.99 with equal probability						
Number of Scenarios	**RD - Rep. 1**	**RD - Rep. 2**	**RD - Rep. 3**	**RD - Rep. 4**	**RD - Rep. 5**	**Confidence Interval**
10	-9.56%	-14.37%	-14.35%	-19.75%	-19.87%	-15.58 $\pm$ 5.38
50	-12.72%	-14.33%	-10.38%	-17.01%	-16.27%	-14.14 $\pm$ 3.34
100	-11.18%	-13.45%	-9.81%	-15.94%	-13.08%	-12.69 $\pm$ 2.90
250	-13.78%	-10.81%	-14.31%	-13.56%	-12.30%	-12.95 $\pm$ 1.75
1000	-9.60%	-11.09%	-9.33%	-11.03%	-11.84%	-10.58 $\pm$ 1.33

For the detailed results of each replication, please see Appendix E for the tables (Tables E.15-E.19) and see Appendix F for the figures (Figures F.9-F.11).

Furthermore, we provided the cumulative distribution function (cdf) plots associated with risk-neutral and risk-averse models which are presented in the Figure 4.7 with $\alpha = [0.8, 0.9, 0.99]$. When the cumulative distribution function is considered, as it is discussed, there is a trade-off between $E(.)$ and $CVaR(.)$ improvement. The $E(.)$ is increased and as a result, cdf is shifted to the left with respect to the right tail of the cdf.

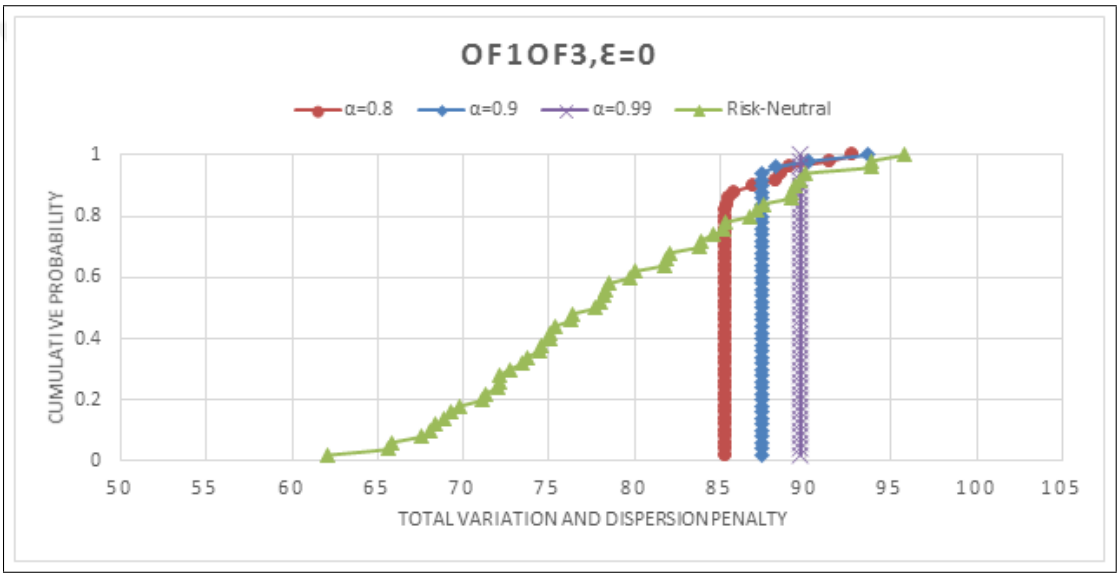

Figure 4.7. Cumulative distribution functions of the total variation and total weighted dispersion amounts for different values of α with $|S| = 50$, $\varepsilon = 0$.

In CVaR objective, the values that are not exceeding the VaR value is not considered. The only thing that changes the $CVaR(.)$ objective value is the values that passes the VaR value. Since the two objective function terms, OF1 and OF3 are conflicting, the effect of $CVaR(.)$ on the objective function terms converge to the same value once we analyze Figure 4.7.

Thus, to better analyze this situation, we also examine the models with mean-risk terms in the following section. Two aspects are considered in mean-risk approach: $E(.)$ and the risk.

Due to the uncertainty in the model, we usually consider its $E(.)$ and risk is represented in terms of variability. Therefore, the trade-off of these two factors against each other has to be measured.

4.4. RISK-AVERSE STOCHASTIC PROGRAMMING MODELS WITH MEAN-RISK TERMS

As it is discussed in Section 4.3, the objective of the mean-risk approach is to minimize the weighted combination of the mean ($E(.)$) and the risk measure. The general representation of the mean-risk function with $CVaR_\alpha$ is as follows:

$$E[Z] + \theta CVaR_\alpha[Z] \tag{4.48}$$

where θ is the trade-off coefficient. It is also called as a risk coefficient, which is specified by decision makers according to their risk preferences.

4.4.1. Proposed Models

By using the previously stated two risk-averse models (for OF1, OF1+OF3) that are explained in Section 4.3, the mean-risk versions are developed as follows:

- Minimizing OF1

$$E[\sum_{s\in S}\phi^s] + \theta CVaR_\alpha[\sum_{s\in S}\phi^s] = \sum_{s\in S}\phi^s pp^s + \theta(\eta + \frac{1}{1-\alpha}\sum_{s\in S}p^s h^s) \tag{4.49}$$

  where $h^s, s \in S$, variables satisfy the constraints (4.42) and (4.39). Exactly the same model is used which is named as "$CVaR^{RA-HN}$" (see Section 4.3.1, Constraints (4.42) to (4.46)).

- Minimizing OF1+OF3

$$E[\sum_{s\in S}\phi^s + \sum_{s\in S}\psi^s] + \theta CVaR_\alpha[\sum_{s\in S}\phi^s + \sum_{s\in S}\psi^s] =$$

$$\sum_{s\in S}(\phi^s + \psi^s)p^s + \theta(\eta + \frac{1}{1-\alpha}\sum_{s\in S}p^s h^s) \tag{4.50}$$

where $h^s, s \in S$, variables satisfy the constraints (4.36) and (4.39). Exactly the same model is used which is named as "$CVaR^{RA-HN}$" (see Section 4.3.1, Constraints (4.36) to (4.39)).

Then, the corresponding models can be obtained by changing the objective functions of the problems that are presented in Section 4.3.1. The effects of risk parameters and comparative results of the proposed models are presented in the following subsection.

4.4.2. Numerical Results

In this section, we present results for the models of OF1 and OF1+OF3 with mean-risk terms. Here, we discuss the effect of risk parameters on the solutions and we report the results for different values of risk parameters: two different $\alpha = [0.8, 0.9]$ values and four different $\theta = [0, 0.1, 1, 10]$ values are used for the analysis.

Results for the first objective function term (OF1): Tables 4.8 and 4.9 show how these risk parameters effect the obtained solutions for different values of risk parameters α and θ. These are the results of first replication with $\varepsilon = 0$. For the other analysis please see Appendix G for the mean-risk term tables.

The Tables that are presented in Appendix G is as follows: G.1-G.8 are for OF1 with $\varepsilon = 0$ and Tables G.9-G.18 are for OF1 with $\varepsilon = 0.25$.

As presented in the tables, increasing the risk parameter, θ, results an increase in the relative

importance of the risk term, which means more explicitly if we increase θ, we may obtain higher CVaR improvements.

Table 4.8. Replication 1 results for the mean-risk model for each number of scenarios (OF1, $\varepsilon = 0$, $\alpha = 0.8$).

α=0.8 and Number of Scenarios=10				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	41.86	34.69	**CVaR$_\alpha$**	**Exp.**
0.1	39.86	34.80	-4.78%	0.33%
1	37.71	35.49	-9.90%	2.31%
10	37.57	35.80	-10.24%	3.21%
α=0.8 and Number of Scenarios=50				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	40.66	35.26	**CVaR$_\alpha$**	**Exp.**
0.1	40.06	35.26	-1.48%	0.00%
1	39.46	35.38	-2.95%	0.36%
10	39.40	35.53	-3.09%	0.78%
α=0.8 and Number of Scenarios=100				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	41.26	35.47	**CVaR$_\alpha$**	**Exp.**
0.1	41.11	35.48	-0.35%	0.02%
1	40.71	35.71	-1.32%	0.68%
10	40.47	36.03	-1.90%	1.57%
α=0.8 and Number of Scenarios=250				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	41.31	35.21	**CVaR$_\alpha$**	**Exp.**
0.1	41.31	35.21	0.00%	0.00%
1	40.42	35.47	-2.14%	0.75%
10	40.42	35.47	-2.14%	0.75%

Because, giving a higher importance to $CVaR(.)$ term decreases the importance of the $E(.)$ term in Equation (4.48). So with larger θ values, we obtain more risk-averse policies.

Table 4.9. Replication 1 results for the mean-risk model for each number of scenarios (OF1, $\varepsilon = 0$, $\alpha = 0.9$).

α=0.9 and Number of Scenarios=10				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	42.29	34.69	**CVaR$_\alpha$**	**Exp.**
0.1	40.29	34.80	-4.73%	0.33%
1	37.71	35.49	-10.81%	2.31%
10	37.71	35.49	-10.81%	2.31%
α=0.9 and Number of Scenarios=50				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	42.06	35.26	**CVaR$_\alpha$**	**Exp.**
0.1	40.40	35.31	-3.94%	0.16%
1	39.89	35.37	-5.16%	0.32%
10	39.89	35.37	-5.16%	0.32%
α=0.9 and Number of Scenarios=100				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	42.86	35.47	**CVaR$_\alpha$**	**Exp.**
0.1	42.57	35.48	-0.67%	0.02%
1	41.46	35.89	-3.27%	1.19%
10	41.43	36.02	-3.33%	1.55%
α=0.9 and Number of Scenarios=250				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	43.13	35.21	**CVaR$_\alpha$**	**Exp.**
0.1	42.78	35.24	-0.82%	0.07%
1	41.74	35.50	-3.23%	0.82%
10	41.74	35.50	-3.23%	0.82%

In $E(.)$, equal importance is given to each scenario but in $CVaR(.)$, the importance is given to the values that are exceeding VaR value. For instance, the CVaR improvement is reached up to $\%10.24$ with $\theta = 10$ (see Table 4.8).

Table 4.10. Replication 1 results for the mean-risk model (OF1+OF3, $\varepsilon = 0$, $\alpha = 0.8$), * indicates the case is solved with an optimality gap $< 0.2\%$.

α=0.8 and Number of Scenarios=10				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	82.43	75.89	**CVaR$_\alpha$**	**Exp.**
0.1	81.67	75.91	-0.92%	0.02%
1	80.04*	76.36	-2.90%	0.61%
10	79.69*	77.43	-3.31%	2.03%
α=0.8 and Number of Scenarios=50				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	87.59	78.02	**CVaR$_\alpha$**	**Exp.**
0.1	86.97	78.08	-0.71%	0.07%
1	85.67	78.49	-2.20%	0.59%
10	85.45*	78.81	-2.45%	1.01%
α=0.8 and Number of Scenarios=100				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	87.63	78.48	**CVaR$_\alpha$**	**Exp.**
0.1	87.47	78.48	-0.18%	0.01%
1	86.71	78.86	-1.05%	0.49%
10	86.65*	78.93	-1.12%	0.58%
α=0.8 and Number of Scenarios=250				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	88.62	77.98	**CVaR$_\alpha$**	**Exp.**
0.1	88.62	77.98	0.00%	0.00%
1	87.26	78.42	-1.53%	0.55%
10	87.23	78.44	-1.56%	0.59%

As the θ is increased, we reach the solutions that are presented in Section 4.3.2.

Table 4.11. Replication 1 results for the mean-risk model (OF1+OF3, $\varepsilon = 0$, $\alpha = 0.9$), * indicates the case is solved with an optimality gap $< 0.2\%$.

α=0.9 and Number of Scenarios=10				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	82.69	75.89	**CVaR$_\alpha$**	**Exp.**
0.1	81.69	75.93	-1.21%	0.05%
1	80.05*	76.36	-3.19%	0.61%
10	79.74*	77.27	-3.57%	1.81%
α=0.9 and Number of Scenarios=50				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	89.84	78.02	**CVaR$_\alpha$**	**Exp.**
0.1	88.87	78.10	-1.07%	0.10%
1	86.64	78.81	-3.56%	1.01%
10	86.64	78.81	-3.56%	1.01%
α=0.9 and Number of Scenarios=100				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	87.04	78.48	**CVaR$_\alpha$**	**Exp.**
0.1	89.18	78.50	-0.54%	0.03%
1	88.37	78.73	-1.45%	0.33%
10	88.13	79.14	-1.71%	0.85%
α=0.9 and Number of Scenarios=250				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	91.33	77.98	**CVaR$_\alpha$**	**Exp.**
0.1	91.33	77.98	0.00%	0.00%
1	89.37	78.46	-2.15%	0.61%
10	89.37	78.46	-2.15%	0.61%

In addition to that, increase in the α values results with a reduction in the critical scenarios and results an increase in the importance of the extreme realizations. Because, the critical values that are exceeding VaR value are decreased while the α value is increased (see Table 4.9).

Results for the first objective function term (OF1+OF3): Tables 4.10 and 4.11 show how these risk parameters effect the obtained solutions for different values of risk parameters α and θ.

For the other analysis please see Appendix G, Tables G.19-G.26 G.19-G.26 are for OF1+OF3 with $\varepsilon = 0$. The observations for the θ parameter is similar to the ones that are presented in OF1.

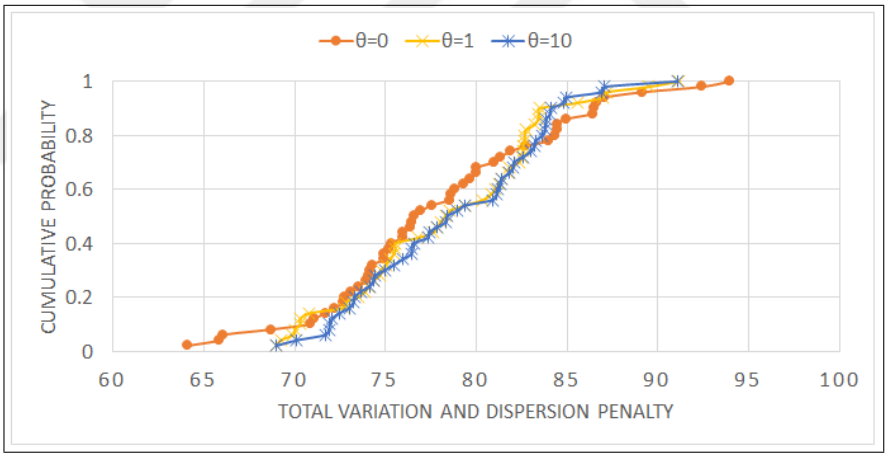

Figure 4.8. Cumulative distribution functions of the OF1+OF3 with different θ values ($|S| = 50$, $\alpha = 0.8$, $\varepsilon = 0$).

Figures 4.8 and 4.9 show the cumulative distribution functions with mean-risk terms for different θ values. In CVaR objective, the values that are not exceeding the VaR value is not considered as it is explained. The only contribution for the objective is the ones that are exceeding the VaR value. As it is discussed in Section 4.3.2, the two objective function terms, OF1 and OF3 are conflicting and objective function terms converged to the same value. After two new terms have been added to the objective ($CVaR(.)$ and $E(.)$),

we cannot observe similar figures as in Section 4.3.2. When the figures are taken into consideration, it is observed that the model is able to balance the production and minimize the dispersion amounts at the same time with mean-risk approach. As a result, the added $E(.)$ term provided a contribution in mean-risk analysis for OF1+OF3.

Furthermore, increasing θ increases the relative importance of the risk term, thus it is obvious to observe a left shift in the right tail of the cdfs. In addition, increasing α, results a shift to the left as it can be seen from the Figure 4.9 (the graphs are overlapped for both values of $\theta = 0$ and $\theta = 1$). All of the values after VaR value remain on the left when it is compared to the risk-neutral solution.

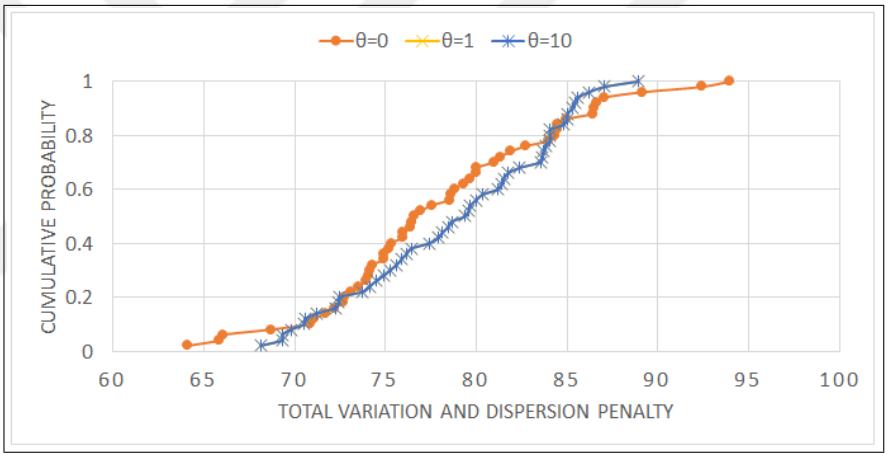

Figure 4.9. Cumulative distribution functions of the OF1+OF3 with different θ values ($|S| = 50$, $\alpha = 0.9$, $\varepsilon = 0$).

To summarize, according to the risk-averse result with only risk term and with mean-risk terms together, CVaR help us to obtain improvements in the objective functions with respect to the risk-neutral cases. We also obtain significant amount of reduction amounts in the risk-averse cases and it shows the improvement in the production balancing and minimization of total weighted dispersion amounts.

4.5. CONCLUSION

In this chapter, we applied the BDAS model in which there are uncertainties associated with the non-booked donor arrivals and considered two different stoctastic programming approaches, namely risk-neutral and risk-averse approaches. Different possible scenarios of non-booked donor arrivals are generated using Monte Carlo sampling technique. We sampled number of non-booked donor arrivals from Poisson and Normal distributions that are parameterized by the historical data of AVIS.

Since the risk-neutral objectives are based on the expected value of the realizations, significant improvements have not been achieved. Thus, risk-averse stochastic programming approaches are formulated in order to analyze the effect of risk. Moreover, risk measures are introduced to be able to incorporate the risk behaviour of decision makers. In Section 4.3, only the risk terms are considered in the models, whereas in Section 4.4 both the expectation and the risk measure in mean-risk approach of the two different objective functions are considered, OF1 and OF1+OF3. The risk terms and mean-risk terms are investigated on the total variation and total weighted dispersion amounts. These are also compared with the risk-neutral models in order to analyze the impact of incorporating the risk measures.

Finally, a comprehensive computational study is conducted to analyze the effects of different policies and present comparative results for the proposed alternative models. Risk incorporated production balancing models ($CVaR^{RA-HN}$) provided significant reduction in the OF1 and OF1+OF3 over the risk-neutral models. The risk-averse models with mean-risk terms demonstrated a more conservative behavior which means better objective function values. In mean-risk approach it can be achieved by increasing the weight of CVaR part (θ) and decreasing the value of α in CVaR.

In conclusion, we compared the risk-neutral stochastic programming models with the risk-averse stochastic programming models and showed that the model with risk aversion take precaution against the uncertainty that is created by the random non-booked donor arrivals. Since risk-neutral models include only the expectation in their objectives, risk-averse models are considered in order to show the effects of the variability of random outcomes. To the

best of our knowledge, in the literature two-stage stochastic optimization with CVaR risk measure is formulated and analyzed for the first time.

5. CONCLUSIONS AND PERSPECTIVES

In this thesis, we first define and formalize a framework for the BDAS problem whose goal is to balance the production of blood units of each type across the days, while also avoiding dispersion amounts associated with overtime. The framework for the BDAS problem is both a real life problem and an advancement with respect to the state of the art. Our framework consists of two phases: an MILP model to preallocate time slots of the different blood types, and a prioritization policy to assign the preallocated slots. Since the amount of entities to allocate is fixed and known in several cases, for our framework it is another decision variable due to the flexibility associated with d_b. Therefore, our preallocation model is different from the allocation and scheduling models usually available in the literature. Two phases (offline and online) are analyzed for the deterministic model and the proposed approach is successfully applied for the historical data of AVIS. Considering the goal of balanced production, the results confirm the capability of the approach.

In real life problems we cannot ignore the uncertainties in the systems. In this dissertation, risk-neutral stochastic programming approaches are formulated in order to schedule the appointments of the donors while considering the non-booked donor uncertainty. Moreover, to improve the quality of the deterministic solution, initially risk-neutral stochastic programming approaches are implemented. In this stochastic context, the uncertain non-booked donor arrivals are represented by random variables and different realizations are generated by sampling from a set of scenarios from historical data of AVIS. Traditional two-stage stochastic programming for analysis of the risk-neutral solutions is implemented where only the expectation of random variables is considered in the objective functions. When all certain realizations are considered, decisions based on the expected value performed poorly and no significant improvement is observed. In order to analyze and improve the solution of risk-neutral stochastic programming approaches, it is important to consider the risk caused by the uncertainties in the system.

Therefore, risk-averse two-stage stochastic programming models are formulated. In order to incorporate risk measures into the objective functions, the two models are formulated. The

first one includes only the risk measures while the second one considers the expectations together with the risk measures. In our formulation, we have considered the risk aversion in the objective function using the CVaR risk measure with non-booked donor arrivals as the random parameter and compared it against the risk-neutral formulations. Numerical examples are designed and solved both for risk-neutral and risk-averse models. For the models incorporating the risk, we succeeded to obtain significant amount of improvement with respect to the risk-neutral models.

Future work can be listed as follows:

- The model can be extended, such as donations different other than whole blood can be included to the model.
- No-show donations can be considered.
- Different scenario generation techniques can be developed to analyze the performance of the proposed models.
- Furthermore, stochastic programming models may be applied for the overall framework (using a rolling horizon mechanism).
- The first step of the BD supply chain can be integrated with the other steps of the chain, e.g., getting the demand information from the next steps and optimizing the next level driving the production of blood units (a feedback mechanism).
- Finally, since the first objective function term is not common in the literature, it might be useful to model the variation minimization in other real life applications, e.g., other systems which produces perishable products.

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APPENDIX A: NUMERICAL RESULTS FOR MODELING ASSUMPTIONS AND PARAMETERS

Table A.1. CPU times (in seconds) for Group A.1.1, objective function OF1+OF2+OF3.

Non-booked level	ε	$\boldsymbol{\delta} = [8\ 6\ 3]$	$\boldsymbol{\delta} = [0.8\ 0.6\ 0.3]$	$\boldsymbol{\delta} = [0.08\ 0.06\ 0.03]$	$\boldsymbol{\delta} = [0.008\ 0.006\ 0.003]$
Null	0.00	0.40	0.45	1.96	1.96
	0.25	0.15	0.15	0.16	0.16
	0.50	0.17	0.16	0.17	0.17
	0.75	0.16	0.15	0.16	0.16
	1.00	0.11	0.12	0.11	0.11
Medium	0.00	0.52	0.44	0.50	81.29
	0.25	59.30	23.43	0.36	0.16
	0.50	0.15	0.14	0.15	0.15
	0.75	0.16	0.15	0.16	0.17
	1.00	0.12	0.12	0.12	0.14
High	0.00	0.73	0.61	5399.82	2081.73
	0.25	8.08	2063.60	0.17	0.17
	0.50	2873.94	2414.34	0.58	0.17
	0.75	0.16	0.14	0.15	0.14
	1.00	0.15	0.15	0.15	0.14

Table A.2. CPU times (in seconds) for Group A.1.2, objective function OF1+OF2+OF3.

Non-booked level	ε	$\boldsymbol{\delta} = [8\ 6\ 3]$	$\boldsymbol{\delta} = [0.8\ 0.6\ 0.3]$	$\boldsymbol{\delta} = [0.08\ 0.06\ 0.03]$	$\boldsymbol{\delta} = [0.008\ 0.006\ 0.003]$
Null	0.00	0.41	0.50	2.07	2122.30
	0.25	0.15	0.16	0.15	0.17
	0.50	0.15	0.15	0.36	0.16
	0.75	0.16	0.16	0.16	0.16
	1.00	0.12	0.12	0.12	0.13
Medium	0.00	1.49	2.73	9.05	1225.58
	0.25	3.27	22.38	0.66	0.17
	0.50	0.16	0.14	0.14	0.14
	0.75	0.17	0.17	0.19	0.17
	1.00	0.17	0.16	0.16	0.16
High	0.00	0.28	0.29	0.28	0.18
	0.25	2663.27	90.93	5399.69	2420.77
	0.50	2955.22	75.27	1.19	56.96
	0.75	2753.70	2242.97	0.22	0.22
	1.00	2632.92	1102.01	4314.94	1776.50

APPENDIX B: NUMERICAL RESULTS FOR OVERALL FRAMEWORK (AVIS MILAN CASE)

Additional figures for the analysis of the overall framework for the AVIS Milan case are reported here. In particular, those with $\varepsilon = 0.25$ are reported in the dissertation, while we show here those with $\varepsilon = 0$.

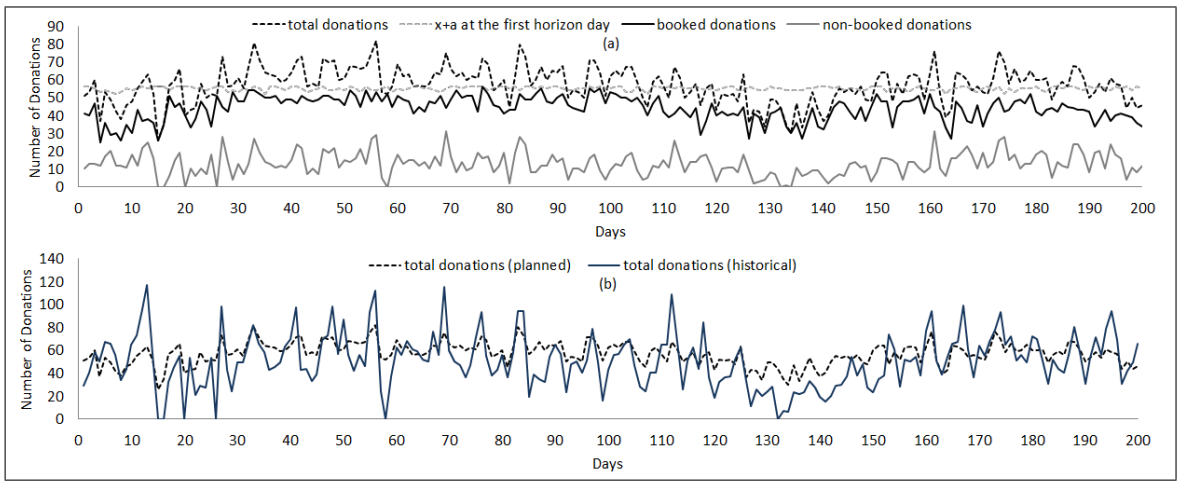

Figure B.1. Number of donations per day for objective function OF1+OF3, $\varepsilon = 0$, and $\lambda_d = 1$ and $\lambda_f = 0$: (a) total number of donations, booked donations, non-booked donations, and $\sum_b x_1^b + a_1^b$; (b) comparison between the total number of donations in the test case and in the observed historical data.

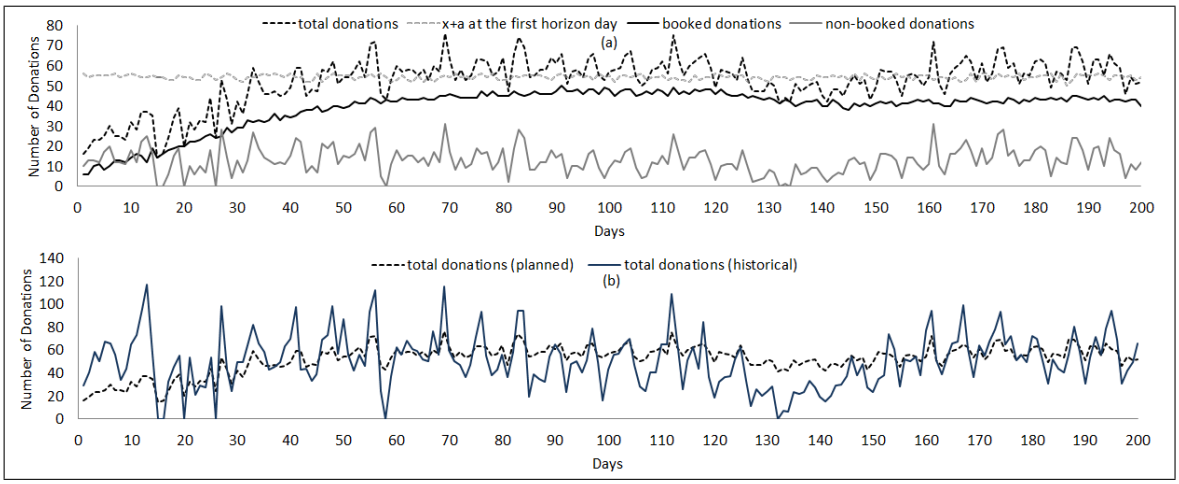

Figure B.2. Number of donations per day for objective function OF1+OF3, $\varepsilon = 0$, and $\lambda_d = 0$ and $\lambda_f = 1$. Reported data are as in Figure B.1.

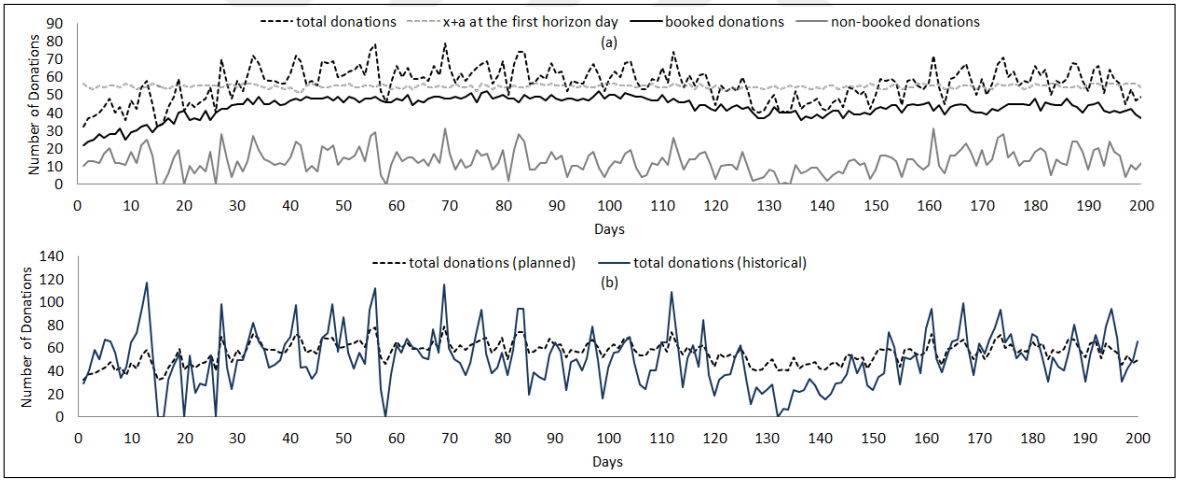

Figure B.3. Number of donations per day for objective function OF1+OF3, $\varepsilon = 0$, and $\lambda_d = 0.5$ and $\lambda_f = 0.5$. Reported data are as in Figure B.1.

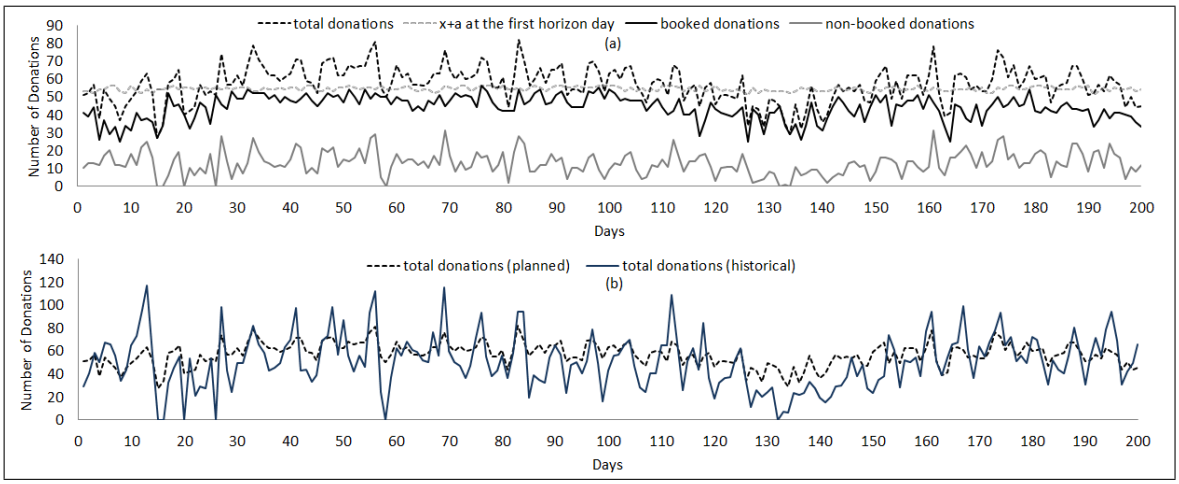

Figure B.4. Number of donations per day for objective function OF2+OF3, $\varepsilon = 0$, and $\lambda_d = 1$ and $\lambda_f = 0$. Reported data are as in Figure B.1.

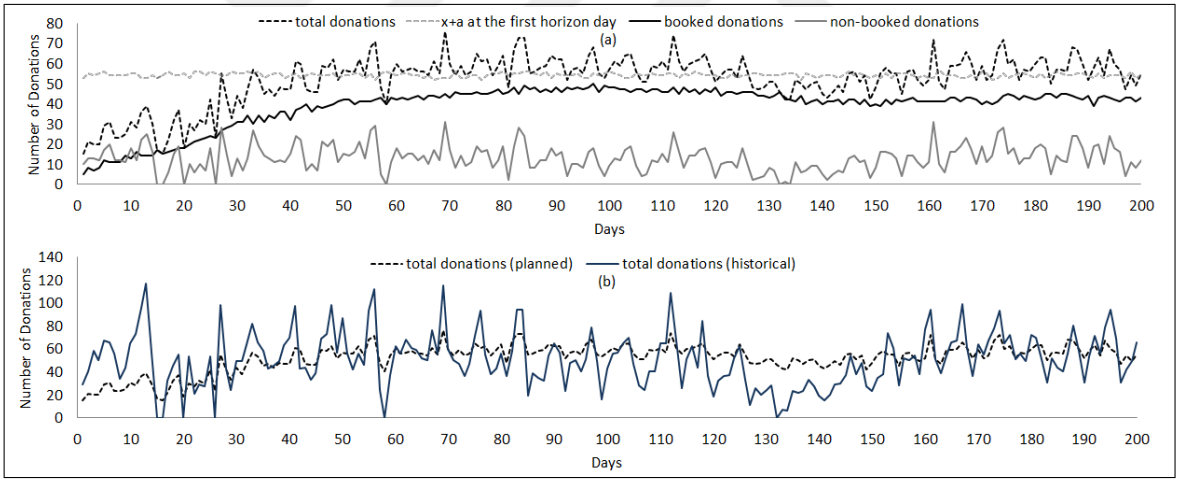

Figure B.5. Number of donations per day for objective function OF2+OF3, $\varepsilon = 0$, and $\lambda_d = 0$ and $\lambda_f = 1$. Reported data are as in Figure B.1.

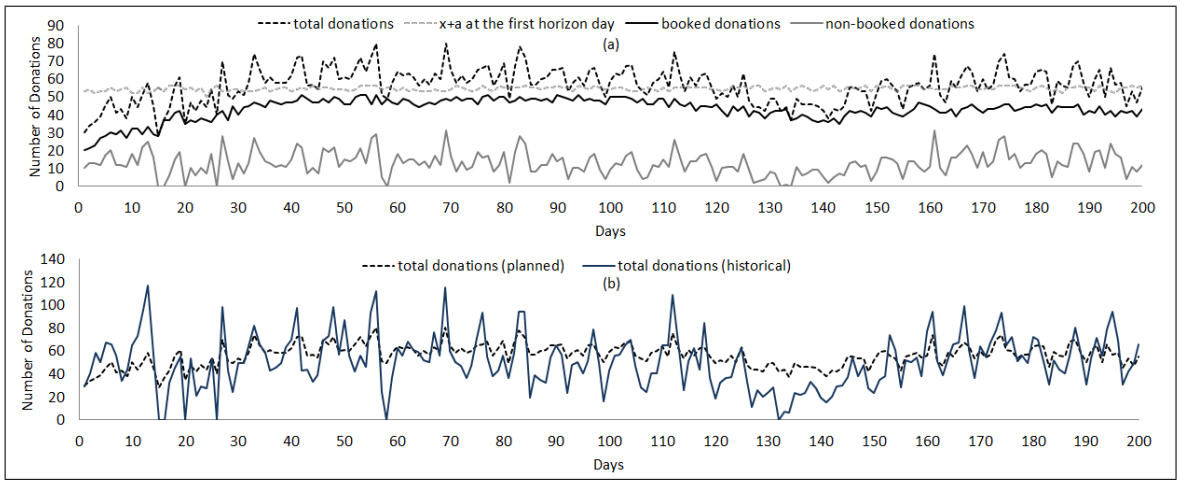

Figure B.6. Number of donations per day for objective function OF2+OF3, $\varepsilon = 0$, and $\lambda_d = 0.5$ and $\lambda_f = 0.5$. Reported data are as in Figure B.1.

APPENDIX C: NORMAL DISTRIBUTION RESULTS FOR RISK-NEUTRAL STOCHASTIC PROGRAMMING MODELS

Table C.1. Results for stochastic programming approaches for $N(\mu, \sigma)$ with OF1, $t = 7$ and $\varepsilon = 0.25$.

Scenario	Replication 1			Replication 2			Replication 3		
Number	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}
50	35.96	5.54	35.56	36.29	5.50	35.98	36.23	5.41	35.97
100	36.46	5.53	36.18	36.59	5.58	36.50	36.62	5.58	36.54
250	36.39	5.52	36.34	36.39	5.53	36.36	36.35	5.50	36.32
1000	36.15	5.51	36.15	36.18	5.53	36.18	36.44	5.49	36.43

Table C.2. Results for stochastic programming approaches for $N(\mu, 3\sigma)$ with OF1, $t = 7$ and $\varepsilon = 0.25$.

Scenario	Replication 1			Replication 2			Replication 3		
Number	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}
50	76.08	12.02	75.28	78.02	11.97	77.44	77.08	11.96	75.99
100	75.70	11.62	75.52	77.34	11.92	76.90	77.02	11.62	76.40
250	75.85	11.90	75.77	76.44	11.41	76.36	76.18	11.52	75.89
1000	75.63	11.71	75.60	76.18	11.67	76.17	76.54	11.65	76.52

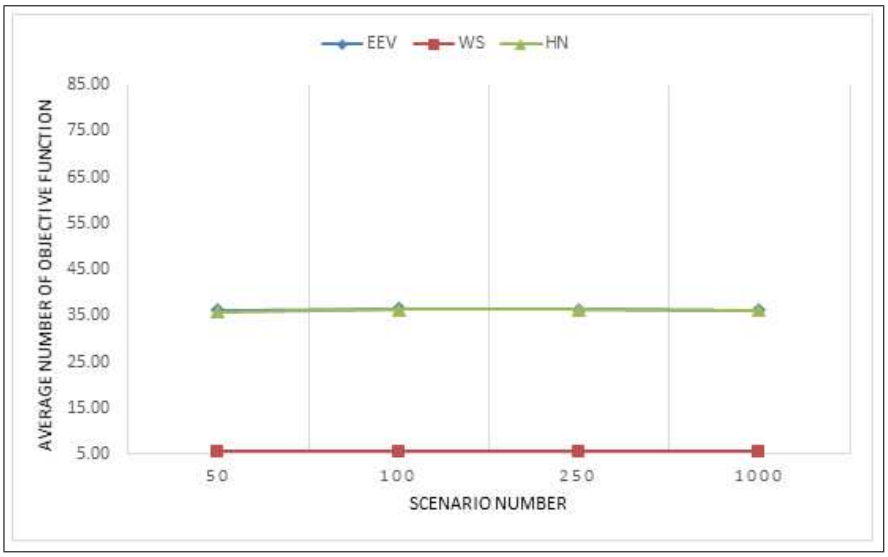

Figure C.1. Comparison of the average objective function values of the replications with $\varepsilon = 0.25$, $t = 7$ for EEV, WS and HN approaches for OF1, $N(\mu, \sigma)$.

Table C.3. Results for stochastic programming approaches for $N(\mu, \sigma)$ with OF1, $t = 7$ and $\varepsilon = 0$.

Scenario	Replication 1			Replication 2			Replication 3		
Number	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}
50	40.42	20.49	38.13	40.34	20.23	39.04	40.40	20.96	38.94
100	40.53	20.75	39.13	40.32	20.46	39.86	40.75	20.92	39.92
250	40.49	20.91	39.77	40.38	20.84	39.93	40.43	20.81	39.88
1000	40.10	20.75	39.93	40.34	20.76	39.96	40.42	20.74	40.24

Table C.4. Results for stochastic programming approaches for $N(\mu, 3\sigma)$ with OF1, $t = 7$ and $\varepsilon = 0$.

Scenario	Replication 1			Replication 2			Replication 3		
Number	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}
50	77.95	26.29	76.21	79.00	26.04	77.85	78.67	26.40	76.72
100	77.27	25.60	76.40	78.83	25.83	77.57	78.29	25.79	77.27
250	77.57	25.81	76.86	77.70	25.28	77.21	77.59	25.49	77.03
1000	77.35	25.85	76.96	77.69	25.74	77.44	78.01	25.76	77.86

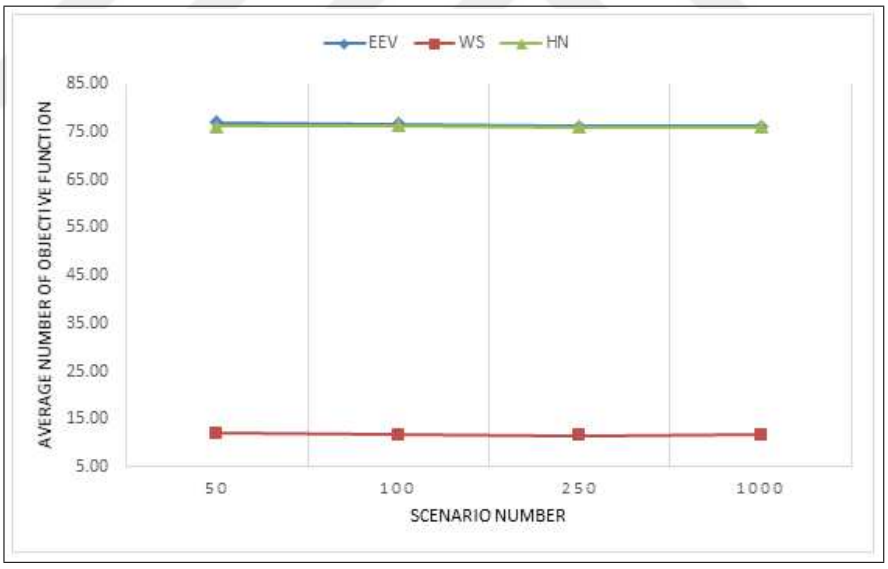

Figure C.2. Comparison of the average objective function values of the replications with $\varepsilon = 0.25$, $t = 7$ for EEV, WS and HN approaches for OF1, $N(\mu, 3\sigma)$.

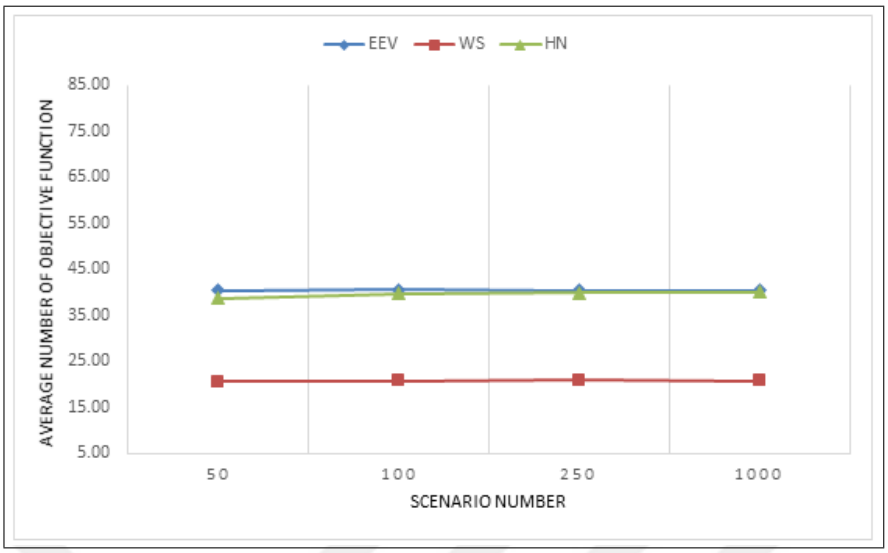

Figure C.3. Comparison of the average objective function values of the replications with $\varepsilon = 0, t = 7$ for EEV, WS and HN approaches for OF1, $N(\mu, \sigma)$.

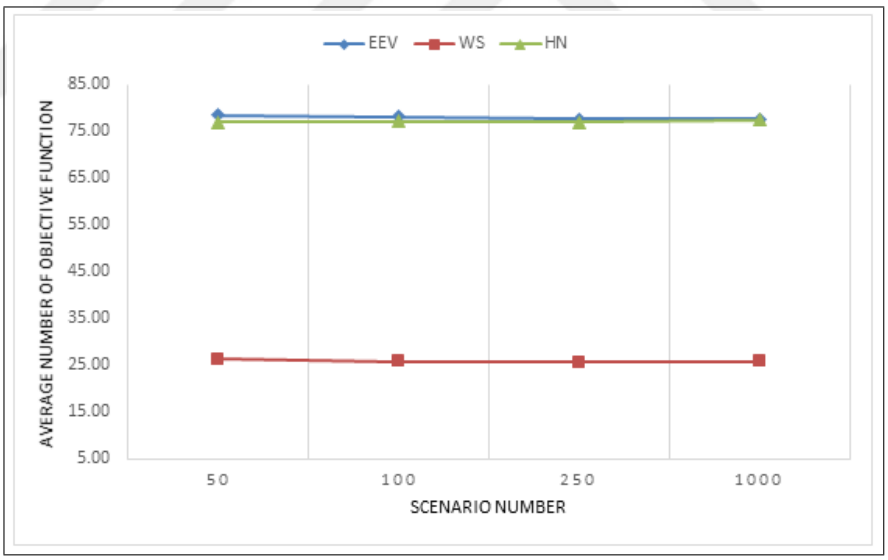

Figure C.4. Comparison of the average objective function values of the replications with $\varepsilon = 0, t = 7$ for EEV, WS and HN approaches for OF1, $N(\mu, 3\sigma)$.

Table C.5. Results for stochastic programming approaches for $N(\mu, \sigma)$ with OF1+OF3, $t = 7$ and $\varepsilon = 0$.

Scenario	Replication 1			Replication 2			Replication 3		
Number	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}
50	80.48	55.93	77.77	80.25	54.97	78.21	81.35	56.14	78.56
100	80.75	56.07	78.60	81.16	55.43	79.22	80.85	55.53	78.85
250	81.32	56.34	79.40	81.21	56.28	79.53	80.79	55.87	79.09
1000	81.01	55.99	79.47	80.96	55.95	79.47	81.09	55.88	79.54

Table C.6. Results for stochastic programming approaches for $N(\mu, 3\sigma)$ with OF1+OF3, $t = 7$ and $\varepsilon = 0$.

Scenario	Replication 1			Replication 2			Replication 3		
Number	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}
50	179.40	110.43	165.64	181.38	109.97	166.82	182.47	11.82	167.66
100	177.02	108.11	164.22	179.81	109.19	166.06	180.39	109.58	166.76
250	177.53	108.56	164.98	178.67	108.39	165.62	176.81	107.31	164.32
1000	176.41	107.78	164.14	177.94	108.36	165.47	178.40	108.53	166.09

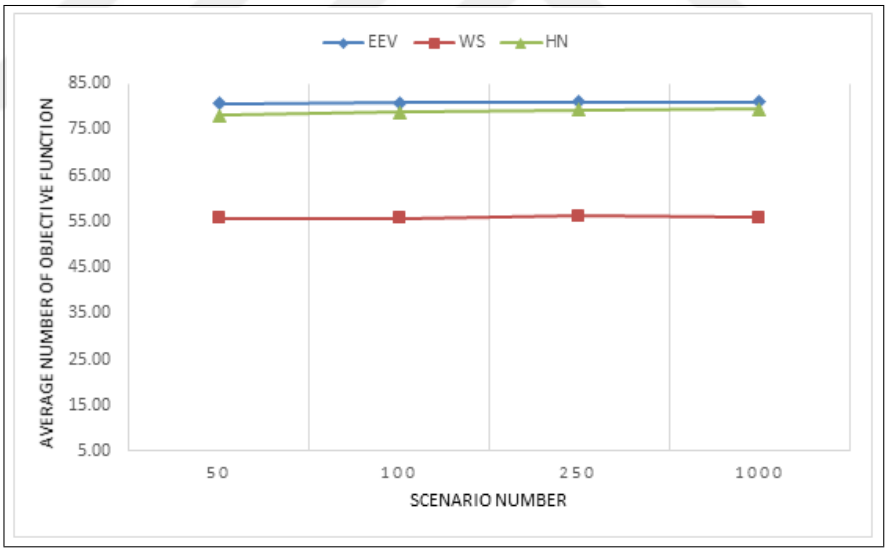

Figure C.5. Comparison of the average objective function values of the replications with $\varepsilon = 0$, $t = 7$ for EEV, WS and HN approaches for OF1, $N(\mu, 3\sigma)$.

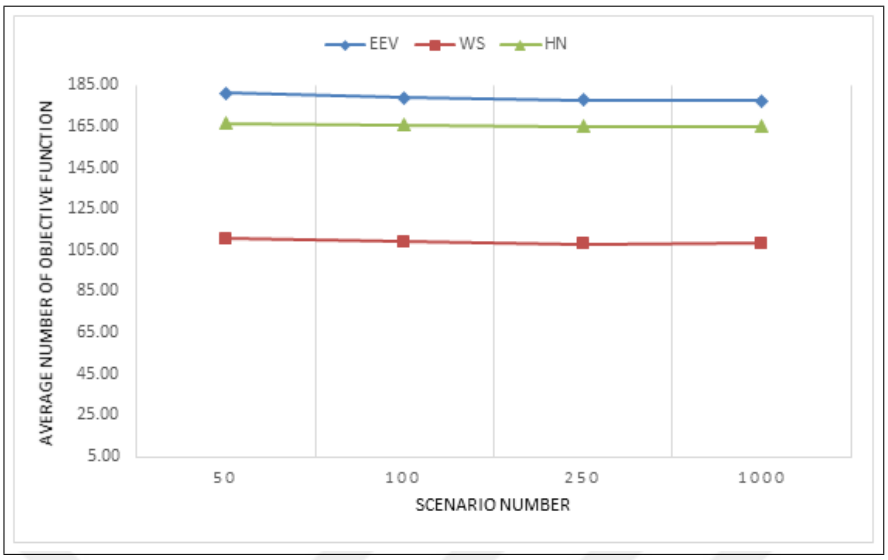

Figure C.6. Comparison of the average objective function values of the replications with $\varepsilon = 0$, $t = 7$ for EEV, WS and HN approaches for OF1, $N(\mu, 3\sigma)$.

APPENDIX D: NUMERICAL RESULTS FOR RN STOCHASTIC PROGRAMMING MODELS WITH OF1 AND OF1+OF3

Table D.1. Results for stochastic programming approaches with OF1, $\varepsilon = 0.25$, $t = 7$.

Scenario	Replication 1			Replication 2			Replication 3			Replication 4			Replication 5		
Number	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}
10	32.94	6.28	31.14	31.20	6.29	29.54	31.57	6.28	30.54	31.31	6.29	30.77	31.06	6.40	28.31
50	31.93	6.37	31.85	31.32	6.37	31.13	31.72	6.37	31.58	30.57	6.37	30.32	30.72	6.37	30.68
100	32.02	6.46	32.02	31.08	6.46	31.08	31.47	6.46	31.47	30.67	6.46	30.67	30.7	6.38	30.7
250	31.33	6.46	31.33	31.15	6.46	31.15	31.13	6.46	31.13	30.85	6.46	30.85	30.99	6.46	30.99
1000	31.19	6.46	31.19	31.13	6.45	31.13	31.18	6.45	31.18	30.95	6.45	30.95	31.08	6.46	31.08

Table D.2. Results for stochastic programming approaches with OF1, $\varepsilon = 0$, $t = 7$.

Scenario	Replication 1			Replication 2			Replication 3			Replication 4			Replication 5		
Number	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}
10	37.51	20.17	34.69	35.63	19.71	32.31	35.51	20.57	33.89	36.00	21.26	33.97	34.80	20.69	31.89
50	36.29	20.78	35.25	35.79	20.94	34.41	36.18	21.23	35.30	35.40	20.67	33.76	34.85	20.59	33.94
100	36.43	20.56	35.47	35.31	20.62	34.49	35.73	20.81	35.25	35.61	20.80	34.46	34.97	20.70	34.18
250	35.77	20.64	35.21	35.45	20.56	34.81	35.31	20.62	35.07	35.64	20.56	34.62	35.36	20.56	34.61
1000	35.63	20.64	35.25	35.43	20.65	35.19	35.45	20.67	35.29	35.41	20.73	35.06	35.48	20.65	35.11

Table D.3. CPU times of stochastic programming approaches with OF1, $\varepsilon = 0.25$, $t = 7$.

Scenario	Replication 1			Replication 2			Replication 3			Replication 4			Replication 5		
Number	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}
10	0.02	0.23	0.19	0.00	0.34	0.20	0.00	0.30	0.30	0.01	0.23	0.17	0.00	0.27	0.17
50	0.03	1.17	2.02	0.03	1.36	2.33	0.03	1.17	3.62	0.01	1.22	2.05	0.03	1.25	2.13
100	0.06	3.37	9.03	0.08	3.98	24.70	0.06	3.33	6.96	0.06	3.60	7.16	0.06	3.33	13.71
250	0.17	20.98	33.21	0.16	22.00	41.81	0.12	19.17	39.68	0.15	19.23	40.34	0.17	18.97	38.17
1000	0.57	79.06	615.72	0.66	89.88	570.28	0.59	94.74	542.47	0.63	81.31	577.05	0.61	70.87	623.37

Table D.4. CPU times of stochastic programming approaches with OF1, $\varepsilon = 0$, $t = 7$.

Scenario	Replication 1			Replication 2			Replication 3			Replication 4			Replication 5		
Number	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}
10	0.00	0.39	0.20	0.00	0.47	0.19	0.02	0.55	0.13	0.01	0.53	0.14	0.02	0.53	0.16
50	0.03	1.13	0.46	0.03	1.08	0.44	0.00	1.10	0.53	0.03	1.16	0.43	0.03	1.69	0.58
100	0.06	2.41	0.66	0.06	2.26	0.71	0.05	2.29	0.69	0.06	2.24	0.61	0.03	2.32	0.66
250	0.16	8.08	1.48	0.16	8.27	1.62	0.14	8.01	1.67	0.17	8.06	1.59	0.17	7.68	1.70
1000	0.68	56.11	8.74	0.60	54.56	8.02	0.60	54.93	7.58	0.61	54.23	7.54	0.64	50.76	8.07

Table D.5. Results for stochastic programming approaches with OF1, $\varepsilon = 0.25$.

Scenario	Replication 1			Replication 2			Replication 3			Replication 4			Replication 5		
Number	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}
10	59.51	10.79	53.56	66.77	9.77	59.37	63.84	8.97	58.00	59.41	9.83	52.64	59.30	10.39	52.96
50	60.74	10.54	58.41	63.26	10.62	61.07	62.60	10.66	60.39	62.22	10.63	59.79	61.59	10.31	59.49
100	61.81	10.76	59.75	63.20	10.71	61.25	63.22	10.81	61.36	62.20	10.63	60.02	61.21	10.33	59.21
250	62.27	10.56	60.57	62.66	10.64	61.05	62.91	10.72	61.17	62.25	10.61	60.61	61.95	10.81	60.34

Table D.6. Results for stochastic programming approaches with OF1, $\varepsilon = 0$, $t = 14$.

Scenario	Replication 1			Replication 2			Replication 3			Replication 4			Replication 5		
Number	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}
10	73.86	47.03	67.87	80.56	50.43	74.54	78.33	48.29	72.86	73.93	46.64	67.27	77.67	48.30	71.01
50	75.49	46.67	72.70	78.98	48.69	75.54	77.77	48.54	75.35	75.92	48.21	74.09	76.87	48.92	75.03
100	76.58	47.53	74.13	78.65	48.24	75.56	77.42	48.54	76.08	75.60	47.62	74.34	76.09	48.23	74.48
250	77.34	47.85	75.27	77.21	48.06	75.96	78.36	48.67	76.49	76.34	47.70	75.51	76.83	48.61	75.63

Table D.7. CPU times of stochastic programming approaches with OF1, $\varepsilon = 0.25$, $t = 14$.

Scenario	Replication 1			Replication 2			Replication 3			Replication 4			Replication 5		
Number	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}
10	0.00	1.33	0.91	0.01	1.25	3.04	0.02	1.22	0.92	0.02	1.09	2.87	0.01	1.19	1.05
50	0.06	10.34	55.63	0.06	7.26	90.35	0.04	9.51	73.87	0.06	9.48	93.90	0.05	5.37	77.69
100	0.12	36.56	211.47	0.14	42.76	1000.09	0.10	40.64	395.76	0.11	47.83	899.20	0.10	45.83	216.31
250	0.28	515.98	1000.19	0.33	116.56	7811.87	0.28	117.20	13916.16	0.30	502.17	9013.45	0.29	98.00	3712.22

Table D.8. CPU times of stochastic programming approaches with OF1, $\varepsilon = 0$, $t = 14$.

Scenario	Replication 1			Replication 2			Replication 3			Replication 4			Replication 5		
Number	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}
10	0.02	1.28	0.29	0.00	0.76	0.20	0.02	0.63	0.33	0.02	0.77	0.28	0.01	1.83	0.41
50	0.06	2.19	0.81	0.06	2.46	0.91	0.04	2.20	0.82	0.06	2.62	0.75	0.04	2.15	0.82
100	0.11	4.99	1.70	0.14	4.85	1.70	0.13	4.80	1.64	0.11	5.29	1.64	0.11	5.00	1.66
250	0.33	15.52	3.70	0.31	16.14	3.48	0.33	13.78	3.17	0.30	14.90	3.99	0.31	15.60	3.35

Table D.9. Results for stochastic programming approaches with OF1+OF3, $\varepsilon = 0$, $t = 7$.

Scenario	Replication 1			Replication 2			Replication 3			Replication 4			Replication 5		
Number	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}
10	79.35	58.28	75.89	80.60	58.72	74.13	84.45	61.24	77.10	84.56	63.51	78.42	81.98	60.78	74.13
50	81.66	60.44	78.02	81.54	60.42	76.88	82.83	61.58	78.45	80.13	60.21	76.16	81.43	60.92	76.67
100	82.39	60.59	78.47	81.10	60.27	77.03	82.19	60.96	78.26	80.72	60.56	77.05	81.47	60.86	76.86
250	81.62	60.51	77.98	81.48	60.33	77.54	81.19	60.29	77.66	80.43	60.07	76.99	82.09	60.95	77.62
1000	81.44	60.29	77.91	81.66	60.57	78.00	81.26	60.40	77.95	80.78	60.36	77.61	81.34	60.56	77.84

Table D.10. CPU times for stochastic programming approaches with OF1+OF3, $\varepsilon = 0$, $t = 7$.

Scenario	Replication 1			Replication 2			Replication 3			Replication 4			Replication 5		
Number	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}	OF_{EEV}	OF_{WS}	OF_{HN}
10	0.00	0.58	0.20	0.00	0.47	0.13	0.02	0.64	0.28	0.01	0.77	0.24	0.00	0.69	0.16
50	0.03	1.84	0.59	0.04	1.77	0.60	0.02	2.20	0.64	0.02	2.43	0.73	0.03	2.28	0.61
100	0.06	3.60	0.94	0.06	3.83	1.00	0.08	4.65	1.20	0.08	4.79	1.21	0.10	4.92	1.22
250	0.12	11.46	4.02	0.14	11.42	3.99	0.16	15.59	4.77	0.18	15.24	5.10	0.20	15.49	5.18
1000	0.51	74.20	49.68	0.58	77.34	49.45	0.75	93.69	66.11	0.71	100.16	66.05	0.78	97.63	71.36

APPENDIX E: NUMERICAL RESULTS (TABLES) FOR RISK-AVERSE MODELS WITH ONLY RISK TERM

The presented results are obtained according to the risk-averse models with only risk term both with equal probabilities and different scenario probabilities.

Table E.1. Comparative results of RA solutions against RN (HN) with OF1, $t = 7$ and $\varepsilon = 0$ for Replication 1, * indicates the case is solved with an optimality gap$< 0.7\%$.

α=0.8 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	37.57	36.69	41.86	34.69	-10.24%	5.77%
50	39.40	35.53	40.66	35.26	-3.09%	0.78%
100	40.47	36.03	41.26	35.47	-1.90%	1.57%
250	40.42	35.55	41.31	35.21	-2.14%	0.95%
1000	40.50*	35.33	40.73	35.26	-0.56%	0.21%
α=0.9 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	37.71	36.77	42.29	34.69	-10.81%	6.01%
50	39.89	35.37	42.06	35.26	-5.16%	0.32%
100	41.43	36.13	42.86	35.47	-3.33%	1.85%
250	41.74	35.52	43.13	35.21	-3.23%	0.88%
1000	41.85*	35.28	42.15	35.26	-0.69%	0.07%
α=0.99 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	37.71	36.77	42.98	34.69	-12.25%	6.01%
50	40.43*	35.81	46.40	35.26	-12.88%	1.56%
100	42.29	36.11	45.71	35.47	-7.50%	1.80%
250	43.66	36.03	47.57	35.21	-8.22%	2.32%
1000	44.91*	35.54	46.83	35.26	-4.09%	0.82%

Table E.2. Comparative results of RA solutions against RN (HN) with OF1, $t = 7$ and $\varepsilon = 0$ for Replication 2, * indicates the case is solved with an optimality gap $< 1.3\%$.

α=0.8 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	34.14	33.66	35.71	32.31	-4.40%	4.16%
50	38.00	34.65	38.74	34.41	-1.92%	0.68%
100	38.79	34.74	39.51	34.49	-1.84%	0.72%
250	39.97	34.93	40.57	34.81	-1.48%	0.33%
1000	40.68*	35.31	40.79	35.19	-0.28%	0.35%
α=0.9 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	34.29	33.66	36.00	32.31	-4.76%	4.16%
50	38.57	34.89	39.60	34.41	-2.60%	1.39%
100	39.51	34.87	40.94	34.49	-3.49%	1.09%
250	40.89	35.02	42.29	34.81	-3.30%	0.59%
1000	42.04*	35.35	42.21	35.19	-0.40%	0.45%
α=0.99 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	34.52	33.66	36.46	32.31	-5.34%	4.16%
50	38.86	34.89	42.13	34.41	-7.78%	1.39%
100	40.29	34.97	45.71	34.49	-11.88%	1.39%
250	42.57	35.24	45.20	34.81	-5.82%	1.24%
1000	44.74*	35.54	46.43	35.19	-3.63%	0.99%

Table E.3. Comparative results of RA solutions against RN (HN) with OF1, $t = 7$ and $\varepsilon = 0$ for Replication 1.

α=0.8 with different probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	37.39	36.53	40.60	34.99	-7.90%	4.38%
50	38.98	35.85	40.57	34.37	-3.92%	4.30%
100	39.62	35.74	40.14	35.35	-1.30%	1.12%
250	39.66	34.59	40.59	34.11	-2.31%	1.41%
α=0.9 with different probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	37.59	36.79	42.05	34.99	-10.62%	5.14%
50	39.91	35.85	42.58	34.37	-6.28%	4.30%
100	40.28	36.16	41.29	35.35	-2.44%	2.31%
250	40.70	34.92	42.05	34.11	-3.21%	2.37%
α=0.99 with different probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	37.71	36.89	41.43	34.99	-8.97%	5.43%
50	40.29	35.68	45.14	34.37	-10.76%	3.79%
100	44.48	36.03	47.71	35.35	-6.77%	1.94%
250	44.07	35.37	45.52	34.11	-3.18%	3.70%

Table E.4. Comparative results of RA solutions against RN (HN) with OF1, $t = 7$ and $\varepsilon = 0$ for Replication 2.

α=0.8 with different probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	34.69	32.20	35.57	32.10	-2.49%	0.31%
50	37.94	35.46	38.79	34.74	-2.21%	2.07%
100	38.35	34.46	39.40	33.71	-2.68%	2.20%
250	39.75	35.13	40.22	34.77	-1.16%	1.01%
α=0.9 with different probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	35.03	32.64	36.91	32.10	-5.09%	1.69%
50	38.73	35.20	39.68	34.74	-2.40%	1.33%
100	39.11	34.56	40.29	33.71	-2.93%	2.52%
250	40.90	35.08	42.07	34.77	-2.78%	0.88%
α=0.99 with different probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	34.29	33.67	37.71	32.10	-9.09%	4.90%
50	38.86	35.58	42.86	34.74	-9.33%	2.43%
100	40.19	34.93	41.94	33.71	-4.18%	3.60%
250	42.58	35.94	47.63	34.77	-10.59%	3.36%

Table E.5. Comparative results of RA solutions against RN (HN) with OF1, $t = 7$ and $\varepsilon = 0$ for Replication 3, * indicates the case is solved with an optimality gap $< 1.3\%$.

α=0.8 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	36.86	35.51	38.57	33.89	-4.44%	4.81%
50	40.46	35.99	42.37	35.30	-4.52%	1.94%
100	40.30	35.79	41.19	35.25	-2.15%	1.54%
250	40.61	35.40	41.26	35.08	-1.59%	0.91%
1000	40.58*	35.45	40.89	35.30	-0.75%	0.43%
α=0.9 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	36.86	35.51	38.57	33.89	-4.44%	4.81%
50	40.91	36.16	43.49	35.30	-5.91%	2.43%
100	41.06	35.95	42.51	35.25	-3.43%	1.97%
250	41.63	35.51	42.74	35.08	-2.59%	1.24%
1000	41.78*	35.55	42.21	35.30	-1.02%	0.72%
α=0.99 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	36.86	35.51	38.57	33.89	-4.44%	4.81%
50	41.14	36.27	45.84	35.30	-10.25%	2.74%
100	41.71	35.75	45.43	35.25	-8.18%	1.42%
250	42.91	35.51	45.74	35.08	-6.18%	1.25%
1000	44.29*	35.41	45.31	35.30	-2.27%	0.31%

Table E.6. Comparative results of RA solutions against RN (HN) with OF1, $t = 7$ and $\varepsilon = 0$ for Replication 4, * indicates the case is solved with an optimality gap $< 1.2\%$.

α=0.8 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	37.29	35.74	42.14	33.97	-11.53%	5.21%
50	38.54	34.78	39.89	33.77	-3.37%	3.00%
100	39.91	34.80	40.71	34.47	-1.96%	0.95%
250	39.86	34.72	40.21	34.63	-0.87%	0.28%
1000	40.37*	35.15	40.55	35.07	-0.44%	0.24%
α=0.9 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	37.43	35.29	42.29	33.97	-11.49%	3.87%
50	39.20	35.14	41.89	33.77	-6.41%	4.08%
100	40.91	35.00	42.51	34.47	-3.76%	1.53%
250	41.10	34.69	41.65	34.63	-1.32%	0.19%
1000	41.73*	35.22	42.06	35.07	-0.77%	0.44%
α=0.99 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	37.43	35.29	42.52	33.97	-11.97%	3.87%
50	39.43	35.69	45.00	33.77	-12.38%	5.69%
100	41.71	35.12	48.57	34.47	-14.12%	1.88%
250	42.71	35.27	44.29	34.63	-3.55%	1.86%
1000	44.43*	35.36	46.29	35.07	-4.01%	0.84%

Table E.7. Comparative results of RA solutions against RN (HN) with OF1, $t = 7$ and $\varepsilon = 0$ for Replication 5, * indicates the case is solved with an optimality gap $< 1.2\%$.

α=0.8 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	34.29	32.66	36.29	31.89	-5.51%	2.42%
50	37.83	34.58	39.03	33.94	-3.07%	1.89%
100	38.39	34.61	39.79	34.19	-3.52%	1.23%
250	39.60	34.95	40.17	34.61	-1.41%	0.96%
1000	40.56*	35.24	40.74	35.12	-0.44%	0.35%
α=0.9 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	34.57	32.66	36.57	31.89	-5.47%	2.42%
50	38.46	35.16	40.46	33.94	-4.94%	3.59%
100	39.14	34.97	41.69	34.19	-6.10%	2.28%
250	40.62	35.07	41.58	34.61	-2.31%	1.33%
1000	41.83*	35.27	42.26	35.12	-1.02%	0.44%
α=0.99 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	35.03	32.66	37.03	31.89	-5.40%	2.42%
50	38.57	35.16	41.85	33.94	-7.84%	3.59%
100	39.43	34.76	45.71	34.19	-13.75%	1.68%
250	42.11	35.08	43.69	34.61	-3.60%	1.35%
1000	44.29*	35.29	46.00	35.12	-3.73%	0.47%

Table E.8. Comparative results of RA solutions against RN (HN) with OF1, $t = 7$ and $\varepsilon = 0.25$ for Replication 1, * indicates the case is solved with an optimality gap $< 3.2\%$.

α=0.8 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	33.29	31.66	36.14	31.14	-7.91%	1.65%
50	36.00	32.38	36.46	31.86	-1.25%	1.63%
100	36.86	32.03	36.87	32.02	-0.04%	0.02%
250	36.92	31.33	36.92	31.33	0.00%	0.00%
1000	36.43*	31.19	36.43	31.19	0.00%	0.00%
α=0.9 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	33.43	31.89	36.86	31.14	-9.30%	2.39%
50	36.69	32.38	37.66	31.86	-2.58%	1.65%
100	38.03	32.69	38.46	32.02	-1.11%	2.10%
250	38.62	31.69	38.85	31.33	-0.59%	1.14%
1000	37.91*	31.19	37.91	31.19	0.00%	0.00%
α=0.99 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	33.43	32.00	36.14	31.14	-7.51%	2.75%
50	37.28*	33.03	44.81	31.86	-16.79%	3.68%
100	39.14*	33.21	41.71	32.02	-6.16%	3.72%
250	40.63*	32.33	43.17	31.33	-5.89%	3.18%
1000	41.97*	31.73	41.94	31.19	0.07%	1.72%

Table E.9. Comparative results of RA solutions against RN (HN) with OF1, $t = 7$ and $\varepsilon = 0.25$ for Replication 2, * indicates the case is solved with an optimality gap $< 3.3\%$.

α=0.8 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	31.57	30.54	33.00	29.54	-4.33%	3.38%
50	34.91	31.81	35.46	31.13	-1.53%	2.17%
100	35.49	31.38	35.63	31.08	-0.40%	0.97%
250	36.60	31.15	36.60	31.15	0.00%	0.00%
1000	36.60	31.14	36.60	31.14	0.00%	0.00%
α=0.9 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	31.71*	30.54	33.14	29.54	-4.31%	3.38%
50	35.26	32.09	36.00	31.13	-2.06%	3.08%
100	36.37	31.44	36.83	31.08	-1.24%	1.15%
250	37.91	31.50	38.09	31.15	-0.48%	1.12%
1000	37.88	31.14	37.88	31.14	0.00%	0.00%
α=0.99 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	31.71	30.91	33.37	29.54	-4.97%	4.64%
50	35.85*	32.36	37.70	31.13	-4.90%	3.95%
100	37.14*	31.84	38.86	31.08	-4.41%	2.44%
250	39.54*	32.27	41.74	31.15	-5.27%	3.58%
1000	41.20*	32.01	40.86	31.14	0.84%	2.79%

Table E.10. Comparative results of RA solutions against RN (HN) with OF1, $t = 7$ and $\varepsilon = 0.25$ for Replication 1, * indicates an optimality gap $< 2.8\%$.

α=0.8 with different probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	33.15	31.98	37.90	30.53	-12.54%	4.73%
50	35.57	32.15	37.14	31.12	-4.24%	3.33%
100	36.26*	32.46	36.68	32.18	-1.13%	0.87%
250	36.15	30.93	36.44	30.71	-0.79%	0.72%
α=0.9 with different probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	33.17	32.10	38.32	30.53	-13.42%	5.14%
50	36.40*	32.28	39.30	31.12	-7.38%	3.75%
100	36.95*	33.03	37.95	32.18	-2.63%	2.66%
250	37.68*	31.61	38.20	30.71	-1.36%	2.91%
α=0.99 with different probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	33.43	32.15	38.57	30.53	-13.33%	5.30%
50	38.57*	32.36	44.29	31.12	-12.90%	4.01%
100	40.29*	33.30	41.68	32.18	-3.33%	3.51%
250	41.27*	32.59	41.98	30.71	-1.68%	6.14%

Table E.11. Comparative results of RA solutions against RN (HN) with OF1, $t = 7$ and $\varepsilon = 0.25$ for Replication 2, * indicates an optimality gap $< 3.4\%$.

α=0.8 with different probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	30.20	29.13	33.38	28.29	-9.53%	2.96%
50	34.55	31.78	35.00	31.23	-1.30%	1.74%
100	35.07*	31.44	36.17	30.79	-3.06%	2.10%
250	36.55	31.60	36.72	31.43	-0.48%	0.55%
α=0.9 with different probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	30.68	29.13	34.19	28.29	-10.26%	2.96%
50	35.03*	32.21	36.05	31.23	-2.83%	3.13%
100	35.83*	31.99	37.45	30.79	-4.33%	3.90%
250	37.67*	32.11	38.45	31.43	-2.04%	2.19%
α=0.99 with different probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	31.71	30.86	34.57	28.29	-8.26%	9.06%
50	36.00*	32.12	38.29	31.23	-5.97%	2.84%
100	36.89*	32.00	40.11	30.79	-8.04%	3.91%
250	39.77*	31.83	41.44	31.43	-4.04%	1.30%

Table E.12. Comparative results of RA solutions against RN (HN) with OF1, $t = 7$ and $\varepsilon = 0.25$ for Replication 3, * indicates an optimality gap $< 3.6\%$.

α=0.8 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	34.29*	32.66	36.29	31.89	-5.51%	2.42%
50	36.80	32.39	37.74	31.59	-2.50%	2.53%
100	36.70	31.53	36.80	31.47	-0.27%	0.18%
250	36.59	31.14	36.59	31.14	0.00%	0.00%
1000	36.29*	31.19	36.29	31.19	0.00%	0.00%
α=0.9 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	33.14*	31.57	36.57	31.89	-9.38%	-0.99%
50	37.49*	32.45	39.31	31.59	-4.65%	2.71%
100	37.54*	32.15	37.83	31.47	-0.76%	2.15%
250	37.95*	31.14	37.95	31.14	0.00%	0.00%
1000	37.51*	31.19	37.51	31.19	0.00%	0.00%
α=0.99 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	33.37*	31.31	37.03	31.89	-9.88%	-1.79%
50	37.71*	32.45	40.43	31.59	-6.71%	2.71%
100	38.29*	31.85	39.14	31.47	-2.19%	1.21%
250	39.66*	31.70	39.86	31.14	-0.50%	1.79%
1000	40.34*	31.19	40.34	31.19	0.00%	0.00%

Table E.13. Comparative results of RA solutions against RN (HN) with OF1, $t = 7$ and $\varepsilon = 0.25$ for Replication 4, * indicates an optimality gap $< 3.7\%$.

α=0.8 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	32.86	32.14	38.29	30.77	-14.18%	4.46%
50	35.69*	31.86	37.17	30.32	-4.00%	5.09%
100	36.74*	32.08	37.46	30.67	-1.91%	4.58%
250	36.73*	31.31	36.83	30.86	-0.26%	1.46%
1000	36.18*	30.96	36.18	30.96	0.00%	0.00%
α=0.9 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	32.86*	32.20	39.14	30.77	-16.06%	4.64%
50	36.17*	32.49	39.60	30.32	-8.66%	7.14%
100	37.69*	31.91	38.74	30.67	-2.73%	4.04%
250	38.02*	31.37	38.27	30.86	-0.66%	1.67%
1000	37.68*	30.96	37.68	30.96	0.00%	0.00%
α=0.99 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	32.86*	32.14	39.14	30.77	-16.06%	4.46%
50	36.29*	32.62	44.00	30.32	-17.53%	7.60%
100	38.57*	32.19	41.71	30.67	-7.53%	4.97%
250	39.71*	32.52	40.63	30.86	-2.25%	5.41%
1000	40.51*	31.32	40.51	30.96	0.00%	1.18%

Table E.14. Comparative results of RA solutions against RN (HN) with OF1, $t = 7$ and $\varepsilon = 0.25$ for Replication 5, * indicates an optimality gap $< 2.3\%$.

α=0.8 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	31.14	29.97	34.57	28.31	-9.92%	5.85%
50	34.51	31.38	35.60	30.69	-3.05%	2.27%
100	34.96	31.27	35.46	30.70	-1.41%	1.87%
250	36.11*	31.14	36.12	31.00	-0.02%	0.46%
1000	36.28*	31.08	36.28	31.08	0.00%	0.00%
α=0.9 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	31.14	29.97	35.43	28.31	-12.10%	5.85%
50	34.91	31.23	36.29	30.69	-3.78%	1.77%
100	35.63	31.31	36.43	30.70	-2.20%	2.00%
250	37.22*	31.15	37.51	31.00	-0.76%	0.49%
1000	37.71*	31.08	37.71	31.08	0.00%	0.00%
α=0.99 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	31.14	29.97	35.43	28.31	-12.10%	5.85%
50	35.43*	31.74	37.43	30.69	-5.34%	3.45%
100	36.57*	31.70	39.14	30.70	-6.57%	3.25%
250	38.51*	31.85	40.86	31.00	-5.73%	2.75%
1000	40.89*	31.86	40.80	31.08	0.21%	2.50%

Table E.15. Comparative results of RA solutions against RN (HN) with OF1+OF3, $t = 7$ and $\varepsilon = 0$ for Replication 1, * an optimality gap $< 0.3\%$.

α=0.8 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	79.69*	79.66	82.43	75.89	-3.31%	4.96%
50	85.45*	84.12	87.59	78.02	-2.45%	7.82%
100	86.65	84.47	87.63	78.48	-1.12%	7.64%
250	87.23	84.26	88.62	77.98	-1.56%	8.05%
1000	87.43*	84.02	88.20	77.92	-0.88%	7.83%
α=0.9 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	79.74*	79.74	82.69	75.89	-3.57%	5.06%
50	86.64*	85.55	89.84	78.02	-3.56%	9.65%
100	88.13*	87.09	87.04	78.48	1.26%	10.98%
250	89.36	87.27	91.33	77.98	-2.16%	11.90%
1000	90.07*	87.04	91.31	77.92	-1.35%	11.71%
α=0.99 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	79.74*	79.74	83.11	75.89	-4.05%	5.06%
50	87.62	87.62	94.75	78.02	-7.52%	12.31%
100	89.17*	89.17	92.15	78.48	-3.24%	13.62%
250	92.35*	92.31	98.83	77.98	-6.55%	18.37%
1000	96.70*	95.45	100.16	77.92	-3.45%	22.50%

Table E.16. Comparative results of RA solutions against RN (HN) with OF1+OF3, $t = 7$ and $\varepsilon = 0$ for Replication 3, * an optimality gap $< 0.2\%$.

α=0.8 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	81.66*	81.61	85.58	77.10	-4.58%	5.85%
50	87.92	85.85	90.57	78.46	-2.93%	9.42%
100	87.54	85.16	89.03	78.27	-1.68%	8.80%
250	87.34	84.88	88.99	77.67	-1.86%	9.28%
1000	87.50*	84.37	88.50	77.96	-1.13%	8.23%
α=0.9 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	81.73*	81.73	86.97	77.10	-6.03%	6.00%
50	89.43*	87.66	92.64	78.46	-3.47%	11.73%
100	89.13	87.30	88.58	78.27	0.61%	11.54%
250	89.43	87.22	91.70	77.67	-2.48%	12.30%
1000	90.04*	87.11	91.35	77.96	-1.44%	11.75%
α=0.99 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	81.73*	81.73	89.21	77.10	-8.39%	6.00%
50	89.72*	89.72	96.69	78.46	-7.21%	14.35%
100	90.29*	90.29	93.15	78.27	-3.07%	15.36%
250	92.05*	91.99	100.70	77.67	-8.59%	18.44%
1000	96.28*	94.48	98.59	77.96	-2.35%	21.19%

Table E.17. Comparative results of RA solutions against RN (HN) with OF1+OF3, $t = 7$ and $\varepsilon = 0$ for Replication 2, * indicates an optimality gap $< 0.3\%$.

α=0.8 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	79.79*	79.23	83.88	74.13	-4.87%	6.88%
50	84.64	83.53	85.62	76.88	-1.15%	8.65%
100	84.84	83.87	86.31	77.04	-1.70%	8.87%
250	86.78	84.30	88.61	77.55	-2.06%	8.70%
1000	87.74*	84.71	88.63	78.00	-1.00%	8.60%
α=0.9 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	79.92*	79.92	84.68	74.13	-5.62%	7.81%
50	85.12	84.67	87.90	76.88	-3.16%	10.13%
100	85.57	85.06	86.07	77.04	-0.58%	10.41%
250	88.73	86.81	91.48	77.55	-3.02%	11.94%
1000	89.96*	87.40	91.98	78.00	-1.56%	12.05%
α=0.99 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	79.92*	79.92	85.98	74.13	-7.05%	7.81%
50	85.40*	85.40	95.39	76.88	-10.47%	11.08%
100	86.27*	86.27	92.03	77.04	-6.26%	11.99%
250	91.37*	91.29	99.00	77.55	-7.71%	17.72%
1000	94.73*	93.71	97.09	78.00	-2.43%	20.13%

Table E.18. Comparative results of RA solutions against RN (HN) with OF1+OF3, $t = 7$ and $\varepsilon = 0$ for Replication 4, * indicates an optimality gap $< 0.2\%$.

α=0.8 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	85.47*	85.40	90.75	78.42	-5.82%	8.91%
50	86.10	83.09	89.14	76.17	-3.42%	9.09%
100	88.56*	84.69	90.33	77.05	-1.96%	9.91%
250	87.90	84.46	89.38	77.00	-1.66%	9.69%
1000	87.51*	84.24	88.51	77.61	-1.13%	8.55%
α=0.9 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	85.55*	85.55	91.52	78.42	-6.52%	9.09%
50	87.89	87.04	93.65	76.17	-6.15%	14.28%
100	91.20	89.00	94.70	77.05	-3.70%	15.51%
250	90.72	87.75	93.30	77.00	-2.77%	13.97%
1000	90.15*	86.84	91.58	77.61	-1.56%	11.89%
α=0.99 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	85.55*	85.55	91.52	78.42	-6.52%	9.09%
50	88.47*	88.47	100.46	76.17	-11.93%	16.15%
100	92.30*	92.30	99.17	77.05	-6.93%	19.78%
250	93.27*	93.19	100.03	77.00	-6.75%	21.03%
1000	95.36	94.45	99.73	77.61	-4.37%	21.70%

Table E.19. Comparative results of RA solutions against RN (HN) with OF1+OF3, $t = 7$ and $\varepsilon = 0$ for Replication 5, * indicates an optimality gap $< 0.2\%$.

α=0.8 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	81.92*	81.81	85.69	74.14	-4.40%	10.35%
50	84.76	83.21	87.11	76.67	-2.70%	8.52%
100	85.19	83.12	86.88	76.87	-1.94%	8.14%
250	86.91*	83.71	88.19	77.63	-1.45%	7.83%
1000	87.78	84.29	88.99	77.85	-1.36%	8.28%
α=0.9 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	82.06*	82.06	87.45	74.14	-6.16%	10.69%
50	85.46*	85.18	89.74	76.67	-4.77%	11.10%
100	86.96*	85.40	89.45	76.87	-2.78%	11.10%
250	89.25*	87.02	90.77	77.63	-1.68%	12.10%
1000	90.44*	87.33	92.18	77.85	-1.89%	12.19%
α=0.99 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	82.06*	82.06	87.45	74.14	-6.16%	10.69%
50	85.75*	85.75	92.03	76.67	-6.82%	11.85%
100	89.02*	89.02	98.05	76.87	-9.21%	15.81%
250	92.60*	92.26	97.29	77.63	-4.83%	18.85%
1000	95.75*	94.44	100.48	77.85	-4.70%	21.32%

Table E.20. Comparative results of RA solutions against RN (EEV) with OF1, $t = 7$ and $\varepsilon = 0$ for Replication 1.

α=0.8 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	37.57	36.69	40.86	37.51	-8.04%	-2.21%
50	39.40	35.53	41.29	36.30	-4.57%	-2.11%
100	40.47	36.03	41.67	36.43	-2.88%	-1.11%
250	40.42	35.55	41.65	35.77	-2.94%	-0.64%
1000	40.50	35.33	41.19	35.64	-1.68%	-0.86%
α=0.9 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	37.71	36.77	41.14	37.51	-8.33%	-1.98%
50	39.89	35.37	42.40	36.30	-5.93%	-2.55%
100	41.43	36.13	43.14	36.43	-3.97%	-0.84%
250	41.74	35.52	43.62	35.77	-4.32%	-0.71%
1000	41.85	35.28	42.71	35.64	-2.01%	-1.01%
α=0.99 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	37.71	36.77	41.14	37.51	-8.33%	-1.98%
50	40.29	35.81	44.57	36.30	-9.62%	-1.35%
100	42.29	36.11	46.86	36.43	-9.76%	-0.89%
250	43.66	36.03	49.83	35.77	-12.39%	0.72%
1000	44.91	35.54	47.49	35.64	-5.42%	-0.26%

Table E.21. Comparative results of RA solutions against RN (EEV) with OF1, $t = 7$ and $\varepsilon = 0$ for Replication 2.

α=0.8 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	34.14	33.66	39.86	35.63	-14.34%	-5.53%
50	38.00	34.65	39.77	35.79	-4.45%	-3.21%
100	38.79	34.74	40.20	35.31	-3.52%	-1.61%
250	39.97	34.93	41.01	35.45	-2.54%	-1.49%
1000	40.68	35.31	40.89	35.44	-0.51%	-0.34%
α=0.9 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	34.29	33.66	42.00	35.63	-18.37%	-5.53%
50	38.57	34.89	40.86	35.79	-5.59%	-2.52%
100	39.51	34.87	41.69	35.31	-5.21%	-1.25%
250	40.89	35.02	42.83	35.45	-4.54%	-1.24%
1000	42.04	35.35	42.34	35.44	-0.71%	-0.24%
α=0.99 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	34.29	33.66	42.00	35.63	-18.37%	-5.53%
50	38.86	34.89	42.29	35.79	-8.11%	-2.52%
100	40.29	34.97	45.71	35.31	-11.88%	-0.95%
250	42.57	35.24	47.60	35.45	-10.56%	-0.60%
1000	44.74	35.54	45.66	35.44	-2.00%	0.29%

Table E.22. Comparative results of RA solutions against RN (EEV) with OF1, $t = 7$ and $\varepsilon = 0$ for Replication 3.

α=0.8 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	36.86	35.51	42.14	35.51	-12.54%	0.00%
50	40.46	35.99	42.51	36.19	-4.84%	-0.55%
100	40.30	35.79	41.84	35.73	-3.69%	0.18%
250	40.61	35.40	41.45	35.32	-2.04%	0.22%
1000	40.58	35.45	40.95	35.45	-0.90%	-0.01%
α=0.9 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	36.86	35.51	44.00	35.51	-16.23%	0.00%
50	40.91	36.16	43.89	36.19	-6.77%	-0.08%
100	41.06	35.95	43.26	35.73	-5.09%	0.60%
250	41.63	35.51	42.78	35.32	-2.67%	0.54%
1000	41.78	35.55	42.31	35.45	-1.24%	0.28%
α=0.99 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	36.86	35.51	44.00	35.51	-16.23%	0.00%
50	41.14	36.27	44.86	36.19	-8.28%	0.22%
100	41.71	35.75	44.86	35.73	-7.01%	0.06%
250	42.91	35.51	44.46	35.32	-3.47%	0.55%
1000	44.29	35.41	45.54	35.45	-2.76%	-0.14%

Table E.23. Comparative results of RA solutions against RN (EEV) with OF1, $t = 7$ and $\varepsilon = 0$ for Replication 4.

α=0.8 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	37.29	35.74	43.86	36.00	-14.98%	-0.71%
50	38.54	34.78	42.51	35.41	-9.34%	-1.78%
100	39.91	34.80	42.64	35.62	-6.40%	-2.30%
250	39.86	34.72	41.95	35.64	-4.99%	-2.57%
1000	40.37	35.15	40.96	35.41	-1.44%	-0.73%
α=0.9 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	37.43	35.29	44.00	36.00	-14.94%	-1.98%
50	39.20	35.14	44.00	35.41	-10.91%	-0.74%
100	40.91	35.00	44.00	35.62	-7.01%	-1.74%
250	41.10	34.69	43.57	35.64	-5.67%	-2.66%
1000	41.73	35.22	42.50	35.41	-1.80%	-0.53%
α=0.99 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	37.43	35.29	44.00	36.00	-14.94%	-1.98%
50	39.43	35.69	44.86	35.41	-12.10%	0.79%
100	41.71	35.12	45.14	35.62	-7.59%	-1.40%
250	42.69	35.27	45.26	35.64	-5.68%	-1.04%
1000	44.43	35.36	45.20	35.41	-1.71%	-0.13%

Table E.24. Comparative results of RA solutions against RN (EEV) with OF1, $t = 7$ and $\varepsilon = 0$ for Replication 5.

α=0.8 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	34.29	32.66	41.43	34.80	-17.24%	-6.16%
50	37.83	34.58	39.23	34.86	-3.57%	-0.79%
100	38.39	34.61	39.97	34.97	-3.97%	-1.05%
250	39.60	34.95	41.15	35.36	-3.78%	-1.18%
1000	40.56	35.24	41.16	35.48	-1.46%	-0.69%
α=0.9 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	34.57	32.66	41.43	34.80	-16.55%	-6.16%
50	38.46	35.16	40.63	34.86	-5.34%	0.87%
100	39.14	34.97	41.26	34.97	-5.12%	-0.02%
250	40.62	35.07	42.56	35.36	-4.56%	-0.82%
1000	41.83	35.27	42.71	35.48	-2.05%	-0.60%
α=0.99 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	34.57	32.66	41.43	34.80	-16.55%	-6.16%
50	38.57	35.16	41.43	34.86	-6.90%	0.87%
100	39.43	34.76	45.43	34.97	-13.21%	-0.61%
250	42.11	35.08	45.54	35.36	-7.53%	-0.80%
1000	44.29	35.29	45.60	35.48	-2.88%	-0.56%

Table E.25. Comparative results of RA solutions against RN (EEV) with OF1, $t = 7$ and $\varepsilon = 0.25$ for Replication 1.

α=0.8 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	33.29	31.66	36.57	32.94	-8.98%	-3.90%
50	36.00	32.38	36.37	31.94	-1.02%	1.38%
100	36.86	32.03	36.87	32.02	-0.04%	0.02%
250	36.92	31.33	36.92	31.33	0.00%	0.00%
1000	36.43	31.19	36.43	31.19	0.00%	0.00%
α=0.9 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	33.43	31.89	37.14	32.94	-10.00%	-3.21%
50	36.69	32.38	37.60	31.94	-2.43%	1.40%
100	38.03	32.69	38.46	32.02	-1.11%	2.10%
250	38.62	31.69	38.85	31.33	-0.59%	1.14%
1000	37.91	31.19	37.91	31.19	0.00%	0.00%
α=0.99 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	33.43	32.00	37.14	32.94	-10.00%	-2.86%
50	37.14	33.03	41.71	31.94	-10.96%	3.42%
100	39.14	33.21	41.71	32.02	-6.16%	3.72%
250	40.63	32.33	43.14	31.33	-5.83%	3.18%
1000	41.97	31.73	41.94	31.19	0.07%	1.72%

Table E.26. Comparative results of RA solutions against RN (EEV) with OF1, $t = 7$ and $\varepsilon = 0.25$ for Replication 2.

α=0.8 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	31.57	30.54	36.29	31.20	-12.99%	-2.11%
50	34.91	31.81	35.29	31.32	-1.05%	1.55%
100	35.49	31.38	35.63	31.08	-0.40%	0.97%
250	36.60	31.15	36.60	31.15	0.00%	0.00%
1000	36.60	31.14	36.60	31.14	0.00%	0.00%
α=0.9 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	31.71	30.54	36.86	31.20	-13.95%	-2.11%
50	35.26	32.09	36.00	31.32	-2.06%	2.46%
100	36.37	31.44	36.83	31.08	-1.24%	1.15%
250	37.91	31.50	38.09	31.15	-0.48%	1.12%
1000	37.88	31.14	37.88	31.14	0.00%	0.00%
α=0.99 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	31.71	30.91	36.86	31.20	-13.95%	-0.92%
50	35.71	32.36	36.86	31.32	-3.10%	3.32%
100	37.14	31.84	38.86	31.08	-4.41%	2.44%
250	39.54	32.27	41.66	31.15	-5.08%	3.58%
1000	41.20	32.01	40.86	31.14	0.84%	2.79%

Table E.27. Comparative results of RA solutions against RN (EEV) with OF1, $t = 7$ and $\varepsilon = 0.25$ for Replication 3.

α=0.8 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	33.00	31.31	36.57	31.57	-9.77%	-0.81%
50	36.80	32.39	37.23	31.72	-1.15%	2.11%
100	36.70	31.53	36.80	31.47	-0.27%	0.18%
250	36.59	31.14	36.59	31.14	0.00%	0.00%
1000	36.29	31.19	36.29	31.19	0.00%	0.00%
α=0.9 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	33.14	31.57	38.86	31.57	-14.71%	0.00%
50	37.49	32.45	38.29	31.72	-2.09%	2.29%
100	37.54	32.15	37.83	31.47	-0.76%	2.15%
250	37.95	31.14	37.95	31.14	0.00%	0.00%
1000	37.51	31.19	37.51	31.19	0.00%	0.00%
α=0.99 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	33.14	31.31	38.86	31.57	-14.71%	-0.81%
50	37.71	32.45	39.14	31.72	-3.65%	2.29%
100	38.29	31.85	39.14	31.47	-2.19%	1.21%
250	39.66	31.70	39.83	31.14	-0.43%	1.79%
1000	40.34	31.19	40.34	31.19	0.00%	0.00%

Table E.28. Comparative results of RA solutions against RN (EEV) with OF1, $t = 7$ and $\varepsilon = 0.25$ for Replication 4.

α=0.8 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	32.86	32.14	38.29	31.31	-14.18%	2.65%
50	35.69	31.86	37.09	30.57	-3.78%	4.22%
100	36.74	32.08	37.46	30.67	-1.91%	4.58%
250	36.73	31.31	36.83	30.86	-0.26%	1.46%
1000	36.18	30.96	36.18	30.96	0.00%	0.00%
α=0.9 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	32.86	32.20	39.71	31.31	-17.27%	2.83%
50	36.17	32.49	39.09	30.57	-7.46%	6.26%
100	37.69	31.91	38.74	30.67	-2.73%	4.04%
250	38.02	31.37	38.27	30.86	-0.66%	1.67%
1000	37.68	30.96	37.68	30.96	0.00%	0.00%
α=0.99 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	32.86	32.14	39.71	31.31	-17.27%	2.65%
50	36.29	32.62	41.71	30.57	-13.01%	6.71%
100	38.57	32.19	41.71	30.67	-7.53%	4.97%
250	39.71	32.52	40.63	30.86	-2.25%	5.41%
1000	40.51	31.32	40.51	30.96	0.00%	1.18%

Table E.29. Comparative results of RA solutions against RN (EEV) with OF1, $t = 7$ and $\varepsilon = 0.25$ for Replication 5.

α=0.8 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	31.14	29.97	36.71	31.06	-15.18%	-3.50%
50	34.51	31.38	35.29	30.73	-2.19%	2.14%
100	34.96	31.27	35.46	30.70	-1.41%	1.87%
250	36.11	31.14	36.12	31.00	-0.02%	0.46%
1000	36.28	31.08	36.28	31.08	0.00%	0.00%
α=0.9 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	31.14	29.97	36.86	31.06	-15.50%	-3.50%
50	34.91	31.23	35.94	30.73	-2.86%	1.64%
100	35.63	31.31	36.43	30.70	-2.20%	2.00%
250	37.22	31.15	37.51	31.00	-0.76%	0.49%
1000	37.71	31.08	37.71	31.08	0.00%	0.00%
α=0.99 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	31.14	29.97	36.86	31.06	-15.50%	-3.50%
50	35.43	31.74	36.86	30.73	-3.88%	3.31%
100	36.57	31.70	39.14	30.70	-6.57%	3.25%
250	38.51	31.85	40.86	31.00	-5.73%	2.75%
1000	40.89	31.86	40.80	31.08	0.21%	2.50%

Table E.30. Comparative results of RA solutions against RN (EEV) with OF1 and OF3, $t = 7$ and $\varepsilon = 0$ for Replication 1.

α=0.8 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	79.69	79.66	85.95	79.35	-7.28%	0.39%
50	85.45	84.12	94.65	81.67	-9.72%	3.00%
100	86.65	84.47	94.59	82.40	-8.39%	2.51%
250	87.23	84.26	94.84	81.62	-8.02%	3.24%
1000	87.43	84.02	94.65	81.45	-7.63%	3.15%
α=0.9 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	79.74	79.74	88.17	79.35	-9.56%	0.49%
50	86.64	85.55	98.07	81.67	-11.65%	4.75%
100	88.13	87.09	93.71	82.40	-5.96%	5.70%
250	89.36	87.27	98.08	81.62	-8.90%	6.91%
1000	90.07	87.04	97.82	81.45	-7.92%	6.86%
α=0.99 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	79.74	79.74	88.17	79.35	-9.56%	0.49%
50	87.62	87.62	100.39	81.67	-12.72%	7.29%
100	89.17	89.17	100.39	82.40	-11.18%	8.22%
250	92.35	92.31	107.11	81.62	-13.78%	13.09%
1000	96.70	95.45	106.97	81.45	-9.60%	17.19%

Table E.31. Comparative results of RA solutions against RN (EEV) with OF1 and OF3, $t = 7$ and $\varepsilon = 0$ for Replication 2.

α=0.8 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	79.79	79.23	92.66	80.69	-13.88%	-1.81%
50	84.64	83.53	93.25	81.55	-9.24%	2.43%
100	84.84	83.87	92.78	81.11	-8.56%	3.41%
250	86.78	84.30	94.18	81.49	-7.85%	3.45%
1000	87.74	84.71	94.50	81.67	-7.15%	3.72%
α=0.9 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	79.92	79.92	93.34	80.69	-14.37%	-0.95%
50	85.12	84.67	95.27	81.55	-10.65%	3.83%
100	85.57	85.06	91.76	81.11	-6.75%	4.87%
250	88.73	86.81	97.02	81.49	-8.55%	6.53%
1000	89.96	87.40	97.83	81.67	-8.05%	7.02%
α=0.99 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	79.92	79.92	93.34	80.69	-14.37%	-0.95%
50	85.40	85.40	99.68	81.55	-14.33%	4.72%
100	86.27	86.27	99.68	81.11	-13.45%	6.37%
250	91.37	91.29	102.44	81.49	-10.81%	12.02%
1000	94.73	93.71	106.54	81.67	-11.09%	14.74%

Table E.32. Comparative results of RA solutions against RN (EEV) with OF1 and OF3, $t = 7$ and $\varepsilon = 0$ for Replication 3.

α=0.8 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	81.66	81.61	94.36	84.45	-13.46%	-3.36%
50	87.92	85.85	96.81	82.84	-9.18%	3.63%
100	87.54	85.16	95.39	82.20	-8.23%	3.61%
250	87.34	84.88	94.90	81.20	-7.96%	4.53%
1000	87.50	84.37	94.06	81.27	-6.98%	3.82%
α=0.9 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	81.73	81.73	95.42	84.45	-14.35%	-3.22%
50	89.43	87.66	98.87	82.84	-9.55%	5.83%
100	89.13	87.30	94.86	82.20	-6.05%	6.21%
250	89.43	87.22	97.70	81.20	-8.47%	7.42%
1000	90.04	87.11	97.34	81.27	-7.50%	7.19%
α=0.99 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	81.73	81.73	95.42	84.45	-14.35%	-3.22%
50	89.72	89.72	100.11	82.84	-10.38%	8.31%
100	90.29	90.29	100.11	82.20	-9.81%	9.85%
250	92.05	91.99	107.43	81.20	-14.31%	13.29%
1000	96.28	94.48	106.19	81.27	-9.33%	16.25%

Table E.33. Comparative results of RA solutions against RN (EEV) with OF1 and OF3, $t = 7$ and $\varepsilon = 0$ for Replication 4.

α=0.8 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	85.47	85.40	101.08	84.56	-15.45%	1.00%
50	86.10	83.09	95.43	80.14	-9.78%	3.69%
100	88.56	84.69	96.67	80.73	-8.40%	4.91%
250	87.90	84.46	95.00	80.44	-7.47%	5.00%
1000	87.51	84.24	94.31	80.78	-7.22%	4.29%
α=0.9 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	85.55	85.55	106.60	84.56	-19.75%	1.17%
50	87.89	87.04	100.48	80.14	-12.53%	8.62%
100	91.20	89.00	95.23	80.73	-4.24%	10.25%
250	90.72	87.75	99.63	80.44	-8.94%	9.09%
1000	90.15	86.84	97.94	80.78	-7.96%	7.50%
α=0.99 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	85.55	85.55	106.60	84.56	-19.75%	1.17%
50	88.47	88.47	106.60	80.14	-17.01%	10.40%
100	92.30	92.30	109.79	80.73	-15.94%	14.33%
250	93.27	93.19	107.91	80.44	-13.56%	15.85%
1000	95.36	94.45	107.18	80.78	-11.03%	16.93%

Table E.34. Comparative results of RA solutions against RN (EEV) with OF1 and OF3, $t = 7$ and $\varepsilon = 0$ for Replication 5.

α=0.8 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	81.92	81.81	101.15	81.98	-19.01%	-0.20%
50	84.76	83.21	94.36	81.43	-10.17%	2.18%
100	85.19	83.12	93.94	81.48	-9.32%	2.02%
250	86.91	83.71	95.56	82.10	-9.05%	1.96%
1000	87.78	84.29	95.15	81.34	-7.75%	3.62%
α=0.9 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	82.06	82.06	102.42	81.98	-19.87%	0.11%
50	85.46	85.18	98.20	81.43	-12.98%	4.60%
100	86.96	85.40	91.89	81.48	-5.36%	4.82%
250	89.25	87.02	99.03	82.10	-9.88%	5.99%
1000	90.44	87.33	98.90	81.34	-8.56%	7.36%
α=0.99 with equal probability						
Number of	**Risk-averse Model**		**Risk-neutral Model**		**Relative Difference**	
Scenarios	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**	$CVaR_\alpha$	**Exp.**
10	82.06	82.06	102.42	81.98	-19.87%	0.11%
50	85.75	85.75	102.42	81.43	-16.27%	5.30%
100	89.02	89.02	102.42	81.48	-13.08%	9.25%
250	92.60	92.26	105.59	82.10	-12.30%	12.38%
1000	95.75	94.44	108.61	81.34	-11.84%	16.10%

APPENDIX F: NUMERICAL RESULTS (FIGURES) FOR RISK-AVERSE MODELS WITH ONLY RISK TERM

The figures presented in Appendix E are obtained according to the risk-averse models with only risk term both with equal probabilities.

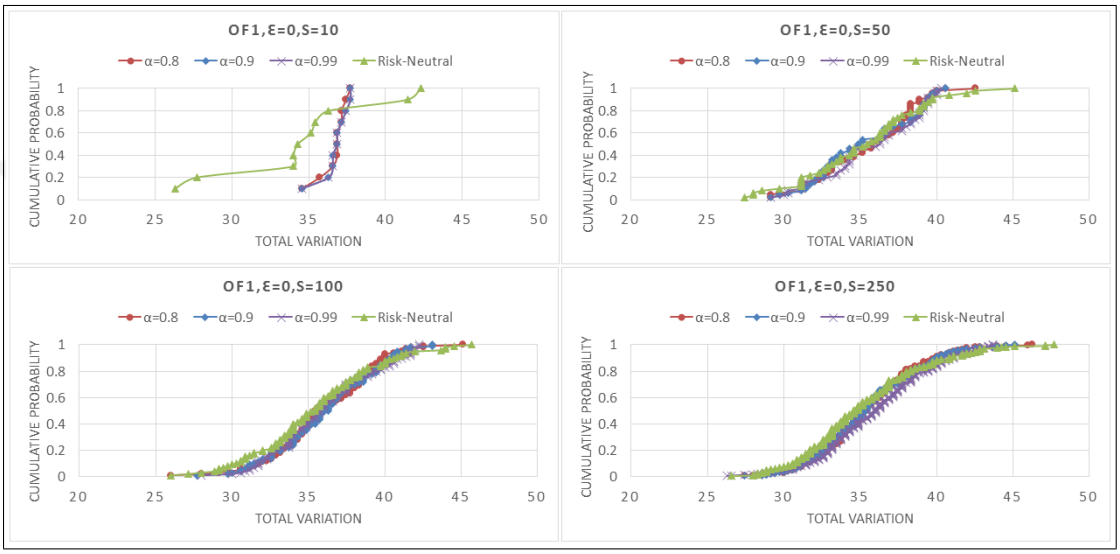

Figure F.1. Cumulative distribution functions of the total variation for different values of α for Replication 1 with $\varepsilon = 0$.

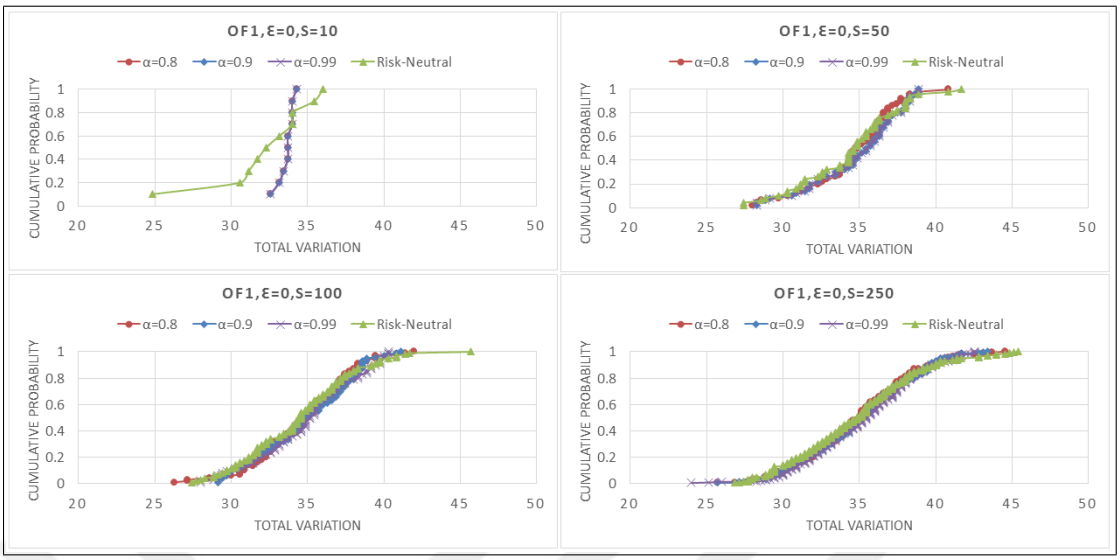

Figure F.2. Cumulative distribution functions of the total variation for different values of α for Replication 2 with $\varepsilon = 0$.

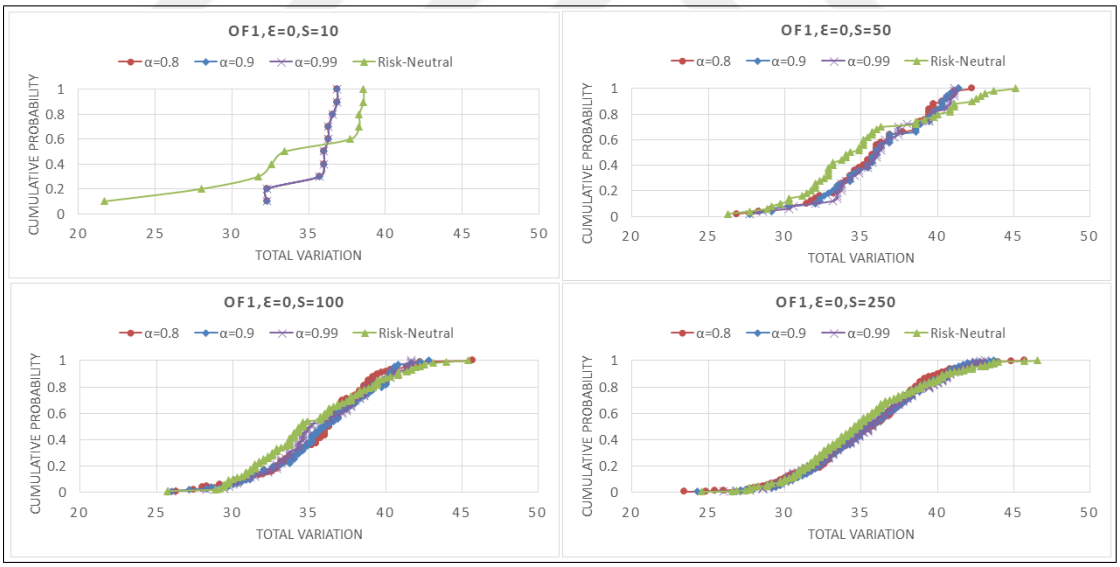

Figure F.3. Cumulative distribution functions of the total variation for different values of α for Replication 3 with $\varepsilon = 0$.

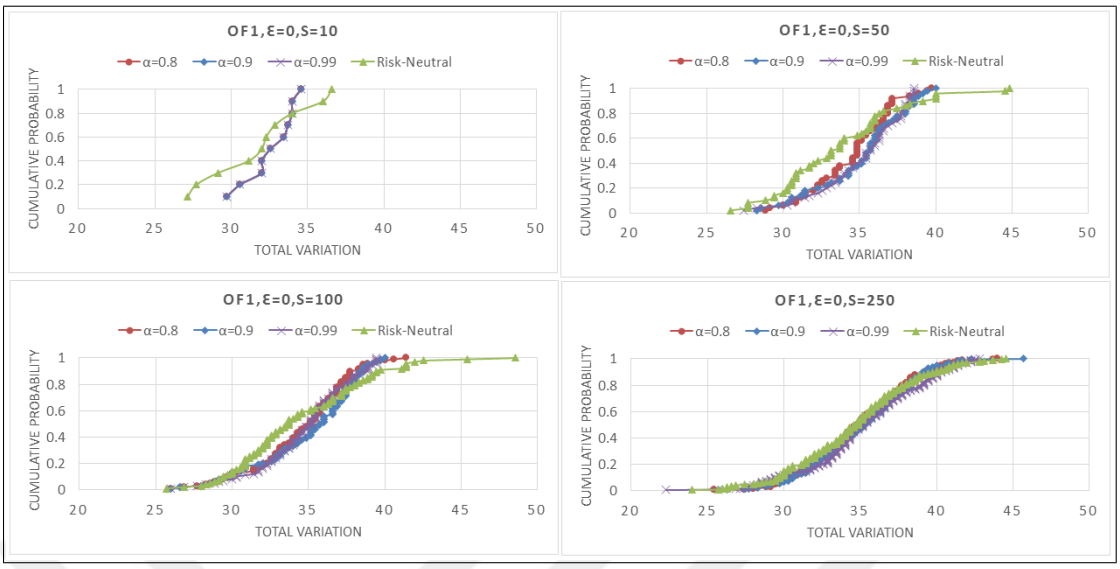

Figure F.4. Cumulative distribution functions of the total variation for different values of α for Replication 4 with $\varepsilon = 0$.

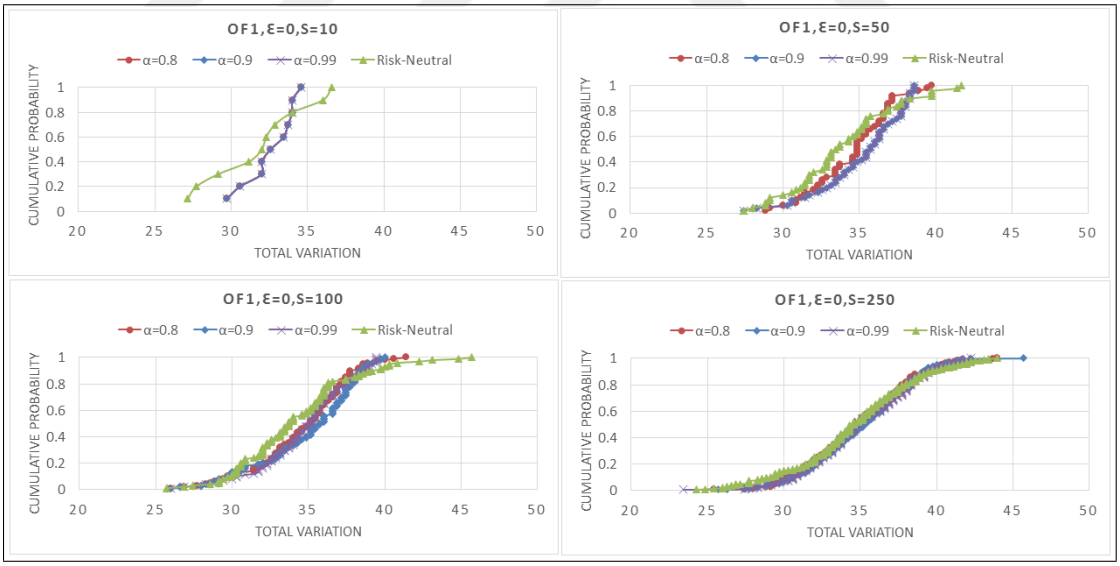

Figure F.5. Cumulative distribution functions of the total variation for different values of α for Replication 5 with $\varepsilon = 0$.

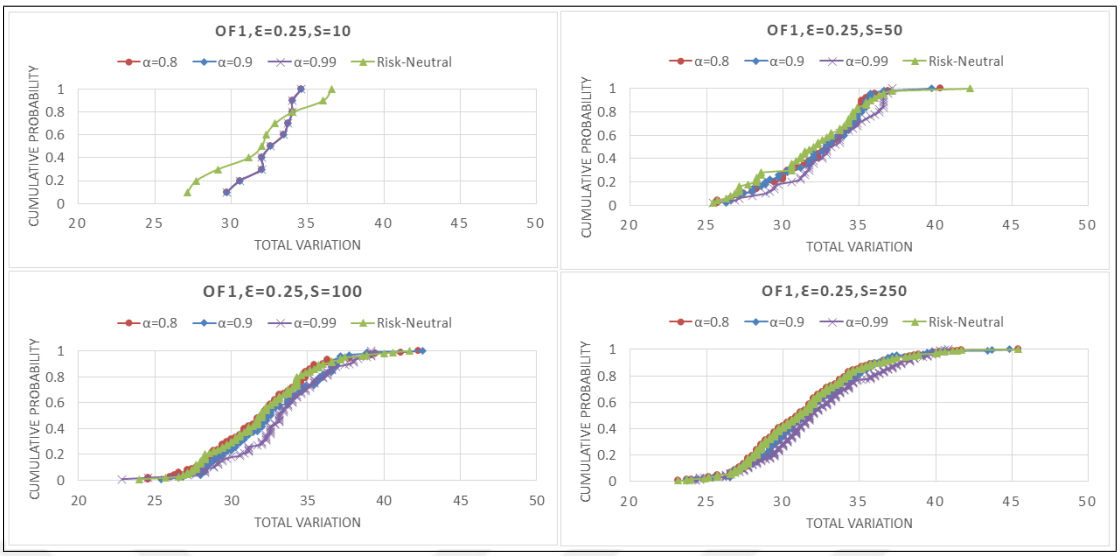

Figure F.6. Cumulative distribution functions of the total variation for different values of α for Replication 1 with $\varepsilon = 0.25$.

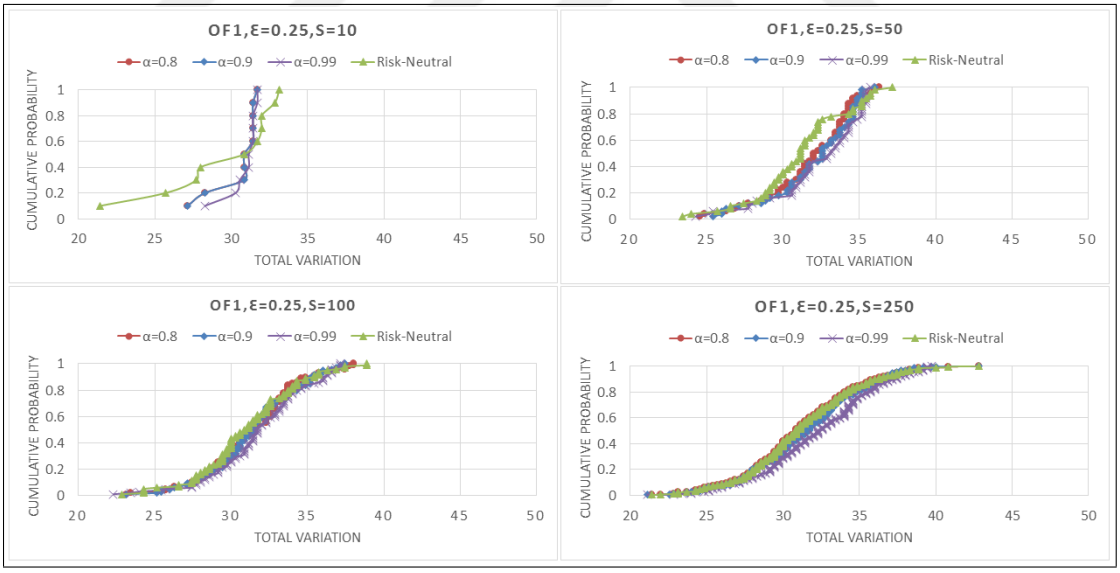

Figure F.7. Cumulative distribution functions of the total variation for different values of α for Replication 2 with $\varepsilon = 0.25$.

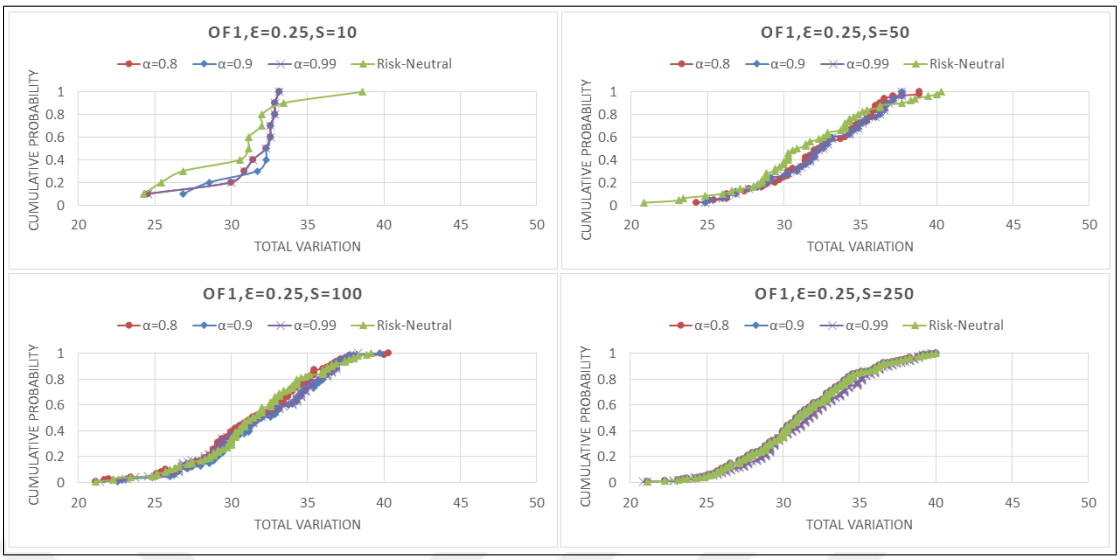

Figure F.8. Cumulative distribution functions of the total variation for different values of α for Replication 3 with $\varepsilon = 0.25$.

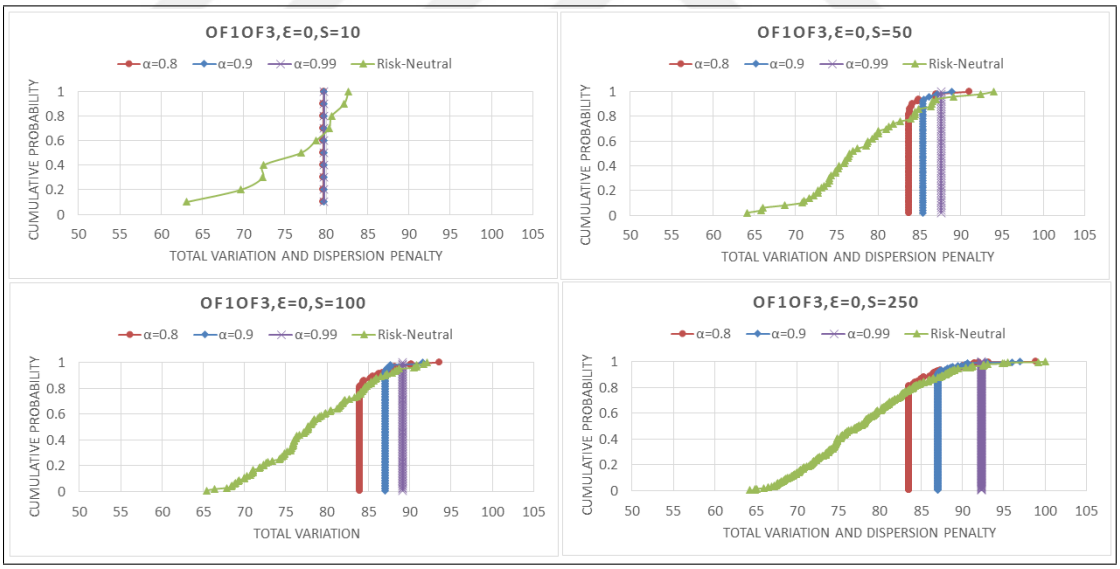

Figure F.9. Cumulative distribution functions of the total variation and total weighted dispersion amounts for different values of α for Replication 1 with $\varepsilon = 0$.

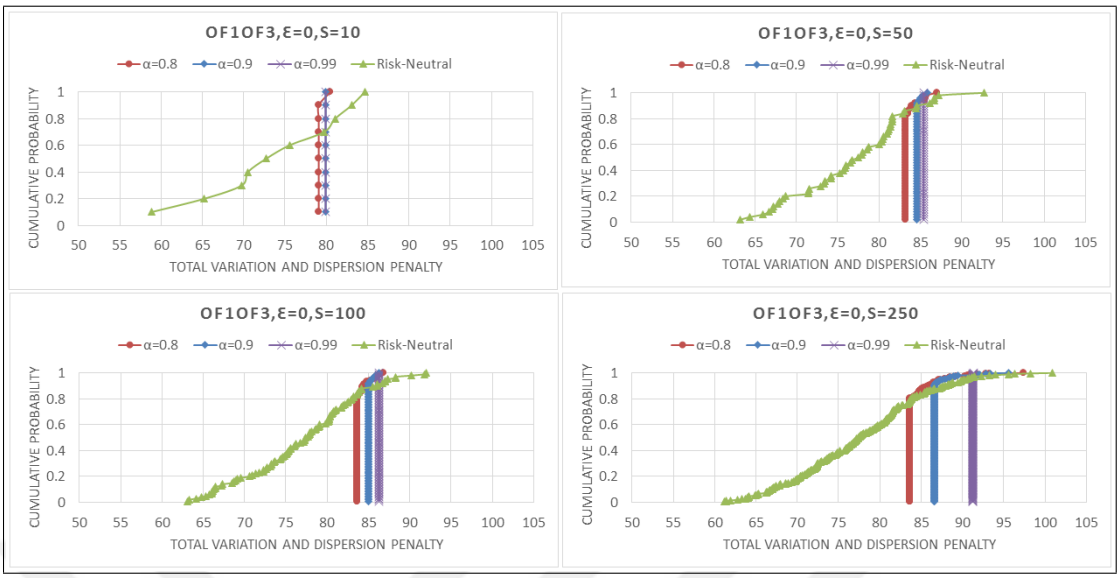

Figure F.10. Cumulative distribution functions of the total variation and total weighted dispersion amounts for different values of α for Replication 2 with $\varepsilon = 0$.

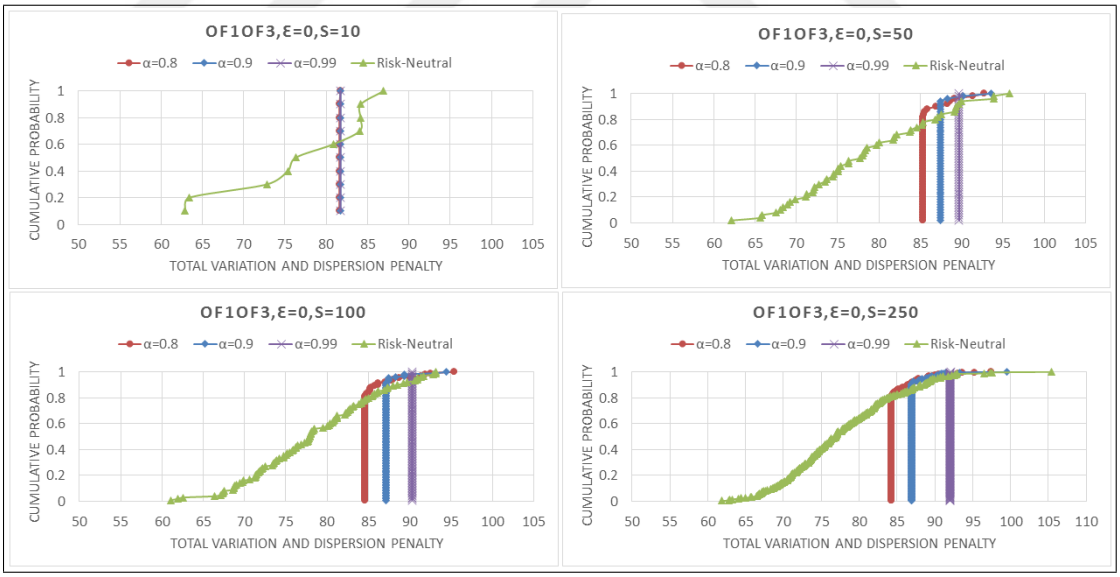

Figure F.11. Cumulative distribution functions of the total variation and total weighted dispersion amounts for different values of α for Replication 3 with $\varepsilon = 0$.

APPENDIX G: NUMERICAL RESULTS FOR RISK-AVERSE MODELS WITH MEAN-RISK TERM

Table G.1. Replication 2 results for the mean-risk model (OF1, $\varepsilon = 0$, $\alpha = 0.8$).

α=0.8 and Number of Scenarios=10				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	35.71	32.31	**CVaR$_\alpha$**	**Exp.**
0.1	35.71	32.31	0.00%	0.00%
1	35.00	32.46	-2.00%	0.44%
10	34.14	33.66	-4.40%	4.16%
α=0.8 and Number of Scenarios=50				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	38.74	34.41	**CVaR$_\alpha$**	**Exp.**
0.1	38.74	34.41	0.00%	0.00%
1	38.00	34.65	-1.92%	0.68%
10	38.00	34.65	-1.92%	0.68%
α=0.8 and Number of Scenarios=100				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	39.51	34.49	**CVaR$_\alpha$**	**Exp.**
0.1	39.51	34.49	0.00%	0.00%
1	38.79	34.74	-1.84%	0.72%
10	38.79	34.74	-1.84%	0.72%
α=0.8 and Number of Scenarios=250				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	40.57	34.81	**CVaR$_\alpha$**	**Exp.**
0.1	40.14	34.83	-1.04%	0.04%
1	40.03	34.84	-1.32%	0.08%
10	39.97	34.93	-1.48%	0.33%

Table G.2. Replication 2 results for the mean-risk model (OF1, $\varepsilon = 0$, $\alpha = 0.9$).

α=0.9 and Number of Scenarios=10				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	36.00	32.31	**CVaR$_\alpha$**	**Exp.**
0.1	36.00	32.31	0.00%	0.00%
1	34.57	33.03	-3.97%	2.21%
10	34.29	33.66	-4.76%	4.16%
α=0.9 and Number of Scenarios=50				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	39.60	34.41	**CVaR$_\alpha$**	**Exp.**
0.1	39.60	34.41	0.00%	0.00%
1	38.69	34.70	-2.31%	0.85%
10	38.57	34.89	-2.60%	1.39%
α=0.9 and Number of Scenarios=100				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	40.94	34.49	**CVaR$_\alpha$**	**Exp.**
0.1	40.74	34.50	-0.49%	0.02%
1	39.66	34.70	-3.14%	0.60%
10	39.51	34.87	-3.49%	1.09%
α=0.9 and Number of Scenarios=250				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	42.29	34.81	**CVaR$_\alpha$**	**Exp.**
0.1	41.41	34.83	-2.08%	0.04%
1	40.89	35.02	-3.30%	0.59%
10	40.89	35.02	-3.30%	0.59%

Table G.3. Replication 3 results for the mean-risk model (OF1, $\varepsilon = 0$, $\alpha = 0.8$).

α=0.8 and Number of Scenarios=10				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	38.57	33.89	**CVaR$_\alpha$**	**Exp.**
0.1	38.57	33.89	0.00%	0.00%
1	37.00	34.43	-4.07%	1.60%
10	36.86	34.60	-4.44%	2.11%
α=0.8 and Number of Scenarios=50				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	42.37	35.30	**CVaR$_\alpha$**	**Exp.**
0.1	41.74	35.35	-1.48%	0.13%
1	40.46	35.99	-4.52%	1.94%
10	40.46	35.99	-4.52%	1.94%
α=0.8 and Number of Scenarios=100				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	41.19	35.25	**CVaR$_\alpha$**	**Exp.**
0.1	40.89	35.27	-0.73%	0.06%
1	40.56	35.42	-1.53%	0.49%
10	40.30	35.79	-2.15%	1.54%
α=0.8 and Number of Scenarios=250				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	41.26	35.08	**CVaR$_\alpha$**	**Exp.**
0.1	41.18	35.08	-0.21%	0.02%
1	40.65	35.23	-1.48%	0.43%
10	40.61	35.40	-1.59%	0.91%

Table G.4. Replication 3 results for the mean-risk model (OF1, $\varepsilon = 0$, $\alpha = 0.9$).

α=0.9 and Number of Scenarios=10				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	38.57	33.89	**CVaR$_\alpha$**	**Exp.**
0.1	38.57	33.89	0.00%	0.00%
1	37.14	34.43	-3.70%	1.60%
10	36.86	34.94	-4.44%	3.12%
α=0.9 and Number of Scenarios=50				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	43.49	35.30	**CVaR$_\alpha$**	**Exp.**
0.1	42.34	35.38	-2.63%	0.23%
1	41.03	35.90	-5.65%	1.68%
10	40.91	36.16	-5.91%	2.43%
α=0.9 and Number of Scenarios=100				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	42.51	35.25	**CVaR$_\alpha$**	**Exp.**
0.1	42.51	35.25	0.00%	0.00%
1	41.09	35.65	-3.36%	1.12%
10	41.09	35.65	-3.36%	1.12%
α=0.9 and Number of Scenarios=250				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	42.74	35.08	**CVaR$_\alpha$**	**Exp.**
0.1	41.99	35.11	-1.76%	0.09%
1	41.76	35.22	-2.30%	0.41%
10	41.63	35.51	-2.59%	1.24%

Table G.5. Replication 4 results for the mean-risk model (OF1, $\varepsilon = 0$, $\alpha = 0.8$).

α=0.8 and Number of Scenarios=10				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	42.14	33.97	**CVaR$_\alpha$**	**Exp.**
0.1	39.57	34.00	-6.10%	0.08%
1	37.71	34.57	-10.51%	1.77%
10	37.29	35.74	-11.53%	5.21%
α=0.8 and Number of Scenarios=50				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	39.89	33.77	**CVaR$_\alpha$**	**Exp.**
0.1	39.89	33.77	0.00%	0.00%
1	38.60	34.41	-3.22%	1.90%
10	38.54	34.78	-3.37%	3.00%
α=0.8 and Number of Scenarios=100				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	40.71	34.47	**CVaR$_\alpha$**	**Exp.**
0.1	40.63	34.47	-0.21%	0.01%
1	39.91	34.80	-1.96%	0.95%
10	39.91	34.80	-1.96%	0.95%
α=0.8 and Number of Scenarios=250				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	40.21	34.63	**CVaR$_\alpha$**	**Exp.**
0.1	39.99	34.64	-0.53%	0.03%
1	39.87	34.70	-0.84%	0.21%
10	39.86	34.72	-0.87%	0.28%

Table G.6. Replication 4 results for the mean-risk model (OF1, $\varepsilon = 0$, $\alpha = 0.9$).

α=0.9 and Number of Scenarios=10				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	42.29	33.97	**CVaR$_\alpha$**	**Exp.**
0.1	39.43	34.06	-6.76%	0.25%
1	37.43	35.09	-11.49%	3.28%
10	37.43	35.09	-11.49%	3.28%
α=0.9 and Number of Scenarios=50				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	41.89	33.77	**CVaR$_\alpha$**	**Exp.**
0.1	41.31	33.82	-1.36%	0.15%
1	39.54	34.41	-5.59%	1.90%
10	39.20	35.04	-6.41%	3.77%
α=0.9 and Number of Scenarios=100				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	42.51	34.47	**CVaR$_\alpha$**	**Exp.**
0.1	42.34	34.47	-0.40%	0.01%
1	41.09	34.78	-3.36%	0.91%
10	40.91	35.00	-3.76%	1.53%
α=0.9 and Number of Scenarios=250				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	41.65	34.63	**CVaR$_\alpha$**	**Exp.**
0.1	41.15	34.66	-1.18%	0.10%
1	41.10	34.69	-1.32%	0.19%
10	41.10	34.69	-1.32%	0.19%

Table G.7. Replication 5 results for the mean-risk model (OF1, $\varepsilon = 0$, $\alpha = 0.8$).

α=0.8 and Number of Scenarios=10				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	36.29	31.90	**CVaR$_\alpha$**	**Exp.**
0.1	34.86	31.97	-3.94%	0.27%
1	34.86	31.97	-3.94%	0.27%
10	34.29	32.66	-5.51%	2.42%
α=0.8 and Number of Scenarios=50				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	39.03	33.94	**CVaR$_\alpha$**	**Exp.**
0.1	39.03	33.94	0.00%	0.00%
1	37.83	34.58	-3.07%	1.89%
10	37.83	34.58	-3.07%	1.89%
α=0.8 and Number of Scenarios=100				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	39.79	34.19	**CVaR$_\alpha$**	**Exp.**
0.1	39.19	34.21	-1.51%	0.08%
1	38.53	34.31	-3.16%	0.36%
10	38.39	34.61	-3.52%	1.23%
α=0.8 and Number of Scenarios=250				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	40.17	34.61	**CVaR$_\alpha$**	**Exp.**
0.1	39.97	34.62	-0.48%	0.03%
1	39.73	34.73	-1.08%	0.33%
10	39.60	34.95	-1.41%	0.96%

Table G.8. Replication 5 results for the mean-risk model (OF1, $\varepsilon = 0$, $\alpha = 0.9$).

α=0.9 and Number of Scenarios=10				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	36.57	31.89	**CVaR$_\alpha$**	**Exp.**
0.1	35.14	31.97	-3.91%	0.27%
1	34.86	32.14	-4.69%	0.81%
10	34.57	32.66	-5.47%	2.42%
α=0.9 and Number of Scenarios=50				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	40.46	33.94	**CVaR$_\alpha$**	**Exp.**
0.1	39.83	33.95	-1.55%	0.02%
1	38.51	34.66	-4.80%	2.12%
10	38.46	35.16	-4.94%	3.59%
α=0.9 and Number of Scenarios=100				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	41.69	34.19	**CVaR$_\alpha$**	**Exp.**
0.1	40.31	34.26	-3.29%	0.23%
1	39.31	34.46	-5.69%	0.80%
10	39.14	34.97	-6.10%	2.28%
α=0.9 and Number of Scenarios=250				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	41.58	34.61	**CVaR$_\alpha$**	**Exp.**
0.1	41.39	34.62	-0.44%	0.03%
1	40.74	34.88	-2.01%	0.79%
10	40.62	35.07	-2.31%	1.33%

Table G.9. Replication 1 results for the mean-risk model (OF1, $\varepsilon = 0.25$, $\alpha = 0.8$), * indicates the case is solved with an optimality gap $< 0.3\%$.

α=0.8 and Number of Scenarios=10				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	36.14	31.14	**CVaR$_\alpha$**	**Exp.**
0.1	35.14	31.14	-2.77%	0.00%
1	33.29	31.66	-7.91%	1.65%
10	33.29	31.66	-7.91%	1.65%
α=0.8 and Number of Scenarios=50				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	36.46	31.86	**CVaR$_\alpha$**	**Exp.**
0.1	36.46	31.86	0.00%	0.00%
1	36.03	32.02	-1.18%	0.50%
10	36.03	32.02	-1.18%	0.50%
α=0.8 and Number of Scenarios=100				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	36.87	32.02	**CVaR$_\alpha$**	**Exp.**
0.1	36.87	32.02	0.00%	0.00%
1	36.86	32.03	-0.04%	0.02%
10	36.86	32.03	-0.04%	0.02%
α=0.8 and Number of Scenarios=250				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	36.92	31.33	**CVaR$_\alpha$**	**Exp.**
0.1	36.92	31.33	0.00%	0.00%
1	36.92	31.33	0.00%	0.00%
10	36.92*	31.33	0.00%	0.00%

Table G.10. Replication 1 results for the mean-risk model (OF1, $\varepsilon = 0.25$, $\alpha = 0.9$), * indicates the case is solved with an optimality gap $< 0.6\%$.

α=0.9 and Number of Scenarios=10				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	36.86	31.14	**CVaR$_\alpha$**	**Exp.**
0.1	35.43	31.14	-3.88%	0.00%
1	33.43	31.66	-9.30%	1.65%
10	33.43	31.66	-9.30%	1.65%
α=0.9 and Number of Scenarios=50				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	37.66	31.86	**CVaR$_\alpha$**	**Exp.**
0.1	37.66	31.86	0.00%	0.00%
1	36.97	32.02	-1.82%	0.50%
10	36.69	32.38	-2.58%	1.65%
α=0.9 and Number of Scenarios=100				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	38.46	32.02	**CVaR$_\alpha$**	**Exp.**
0.1	38.46	32.02	0.00%	0.00%
1	38.23	32.20	-0.59%	0.54%
10	38.03*	32.69	-1.11%	2.10%
α=0.9 and Number of Scenarios=250				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	38.85	31.33	**CVaR$_\alpha$**	**Exp.**
0.1	38.85	31.33	0.00%	0.00%
1	38.70*	31.41	-0.38%	0.23%
10	38.62*	31.69	-0.59%	1.14%

Table G.11. Replication 2 results for the mean-risk model (OF1, $\varepsilon = 0.25$, $\alpha = 0.8$), * indicates an optimality gap $< 0.5\%$.

α=0.8 and Number of Scenarios=10				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	33.00	29.54	**CVaR$_\alpha$**	**Exp.**
0.1	33.00	29.54	0.00%	0.00%
1	31.86	30.03	-3.46%	1.64%
10	31.57	30.54	-4.33%	3.38%
α=0.8 and Number of Scenarios=50				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	35.46	31.13	**CVaR$_\alpha$**	**Exp.**
0.1	35.46	31.13	0.00%	0.00%
1	34.94	31.32	-1.45%	0.61%
10	34.91*	31.43	-1.53%	0.95%
α=0.8 and Number of Scenarios=100				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	35.63	31.08	**CVaR$_\alpha$**	**Exp.**
0.1	35.63	31.08	0.00%	0.00%
1	35.63	31.08	0.00%	0.00%
10	35.49	31.38	-0.40%	0.97%
α=0.8 and Number of Scenarios=250				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	36.60	31.15	**CVaR$_\alpha$**	**Exp.**
0.1	36.60	31.15	0.00%	0.00%
1	36.60	31.15	0.00%	0.00%
10	36.60*	31.15	0.00%	0.00%

Table G.12. Replication 2 results for the mean-risk model (OF1, $\varepsilon = 0.25$, $\alpha = 0.9$), * indicates an optimality gap $< 1.0\%$.

α=0.9 and Number of Scenarios=10				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	33.14	29.54	**CVaR$_\alpha$**	**Exp.**
0.1	33.14	29.54	0.00%	0.00%
1	32.00	30.03	-3.45%	1.64%
10	31.71	30.31	-4.31%	2.61%
α=0.9 and Number of Scenarios=50				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	36.00	31.13	**CVaR$_\alpha$**	**Exp.**
0.1	36.00	31.13	0.00%	0.00%
1	35.31	31.40	-1.90%	0.86%
10	35.31*	31.40	-1.90%	0.86%
α=0.9 and Number of Scenarios=100				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	36.83	31.08	**CVaR$_\alpha$**	**Exp.**
0.1	36.83	31.08	0.00%	0.00%
1	36.37	31.44	-1.24%	1.15%
10	36.37*	31.44	-1.24%	1.15%
α=0.9 and Number of Scenarios=250				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	38.09	31.15	**CVaR$_\alpha$**	**Exp.**
0.1	38.09	31.15	0.00%	0.00%
1	38.09	31.15	0.00%	0.00%
10	37.91*	31.50	-0.48%	1.12%

Table G.13. Replication 3 results for the mean-risk model (OF1, $\varepsilon = 0.25$, $\alpha = 0.8$), * indicates an optimality gap $< 0.1\%$.

α=0.8 and Number of Scenarios=10				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	36.29	31.89	**CVaR$_\alpha$**	**Exp.**
0.1	36.00	30.54	-0.79%	-4.21%
1	33.00	31.11	-9.06%	-2.42%
10	33.00	31.11	-9.06%	-2.42%
α=0.8 and Number of Scenarios=50				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	37.74	31.59	**CVaR$_\alpha$**	**Exp.**
0.1	37.66	31.59	-0.23%	0.00%
1	37.29	31.64	-1.21%	0.16%
10	36.80	32.39	-2.50%	2.53%
α=0.8 and Number of Scenarios=100				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	36.80	31.47	**CVaR$_\alpha$**	**Exp.**
0.1	36.80	31.47	0.00%	0.00%
1	36.70	31.53	-0.27%	0.18%
10	36.70	31.53	-0.27%	0.18%
α=0.8 and Number of Scenarios=250				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	36.59	31.14	**CVaR$_\alpha$**	**Exp.**
0.1	36.59	31.14	0.00%	0.00%
1	36.59	31.14	0.00%	0.00%
10	36.59*	31.14	0.00%	0.00%

Table G.14. Replication 3 results for the mean-risk model (OF1, $\varepsilon = 0.25$, $\alpha = 0.9$), * indicates an optimality gap $< 0.5\%$.

α=0.9 and Number of Scenarios=10				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	36.57	31.89	**CVaR$_\alpha$**	**Exp.**
0.1	36.00	30.71	-1.56%	-3.67%
1	33.14	31.11	-9.38%	-2.42%
10	33.14*	31.11	-9.38%	-2.42%
α=0.9 and Number of Scenarios=50				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	39.31	31.59	**CVaR$_\alpha$**	**Exp.**
0.1	38.97	31.59	-0.87%	0.00%
1	37.71	32.05	-4.07%	1.47%
10	37.49	32.45	-4.657%	2.717%
α=0.9 and Number of Scenarios=100				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	37.83	31.47	**CVaR$_\alpha$**	**Exp.**
0.1	37.83	31.47	0.00%	0.00%
1	37.83	31.47	0.00%	0.00%
10	37.58*	31.85	-0.68%	1.21%
α=0.9 and Number of Scenarios=250				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	37.95	31.14	**CVaR$_\alpha$**	**Exp.**
0.1	37.95	31.14	0.00%	0.00%
1	37.95	31.14	0.00%	0.00%
10	37.95*	31.14	0.00%	0.00%

Table G.15. Replication 4 results for the mean-risk model (OF1, $\varepsilon = 0.25$, $\alpha = 0.8$), * indicates an optimality gap $< 0.6\%$.

α=0.8 and Number of Scenarios=10				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	38.29	30.77	**CVaR$_\alpha$**	**Exp.**
0.1	36.00	30.91	-5.97%	0.46%
1	32.86	31.69	-14.18%	2.97%
10	32.86	31.69	-14.18%	2.97%
α=0.8 and Number of Scenarios=50				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	37.17	30.32	**CVaR$_\alpha$**	**Exp.**
0.1	36.49	30.37	-1.84%	0.17%
1	36.23	30.51	-2.54%	0.62%
10	35.69*	31.86	-4.00%	5.09%
α=0.8 and Number of Scenarios=100				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	37.46	30.67	**CVaR$_\alpha$**	**Exp.**
0.1	37.23	30.68	-0.61%	0.02%
1	36.87	30.90	-1.56%	0.75%
10	36.76	31.22	-1.87%	1.78%
α=0.8 and Number of Scenarios=250				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	36.83	30.86	**CVaR$_\alpha$**	**Exp.**
0.1	36.83	30.86	0.00%	0.00%
1	36.83*	30.86	0.00%	0.00%
10	36.73*	31.31	-0.26%	1.46%

Table G.16. Replication 4 results for the mean-risk model (OF1, $\varepsilon = 0.25$, $\alpha = 0.9$), * indicates an optimality gap $< 1.0\%$.

α=0.9 and Number of Scenarios=10				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	39.14	30.77	**CVaR$_\alpha$**	**Exp.**
0.1	36.00	30.91	-8.03%	0.46%
1	32.86	31.69	-16.06%	2.97%
10	32.86	31.69	-16.06%	2.97%
α=0.9 and Number of Scenarios=50				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	39.60	30.32	**CVaR$_\alpha$**	**Exp.**
0.1	38.91	30.37	-1.73%	0.17%
1	37.03*	30.95	-6.49%	2.09%
10	36.17*	31.96	-8.66%	5.41%
α=0.9 and Number of Scenarios=100				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	38.74	30.67	**CVaR$_\alpha$**	**Exp.**
0.1	38.74	30.67	0.00%	0.00%
1	38.11*	31.09	-1.62%	1.35%
10	37.69*	31.91	-2.73%	4.04%
α=0.9 and Number of Scenarios=250				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	38.27	30.86	**CVaR$_\alpha$**	**Exp.**
0.1	38.27	30.86	0.00%	0.00%
1	38.11*	31.01	-0.42%	0.51%
10	38.02*	31.37	-0.66%	1.67%

Table G.17. Replication 5 results for the mean-risk model (OF1, $\varepsilon = 0.25$, $\alpha = 0.8$), * indicates an optimality gap $< 0.5\%$.

α=0.8 and Number of Scenarios=10				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	34.57	28.31	**CVaR$_\alpha$**	**Exp.**
0.1	34.57	28.31	0.00%	0.00%
1	31.29	29.00	-9.50%	2.42%
10	31.14	29.97	-9.92%	5.85%
α=0.8 and Number of Scenarios=50				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	35.60	30.69	**CVaR$_\alpha$**	**Exp.**
0.1	34.66	30.70	-2.65%	0.06%
1	34.66	30.70	-2.65%	0.06%
10	34.51	31.38	-3.05%	2.27%
α=0.8 and Number of Scenarios=100				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	35.46	30.70	**CVaR$_\alpha$**	**Exp.**
0.1	35.46	30.70	0.00%	0.00%
1	35.01	30.98	-1.25%	0.92%
10	34.96	31.27	-1.41%	1.87%
α=0.8 and Number of Scenarios=250				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	36.12	31.00	**CVaR$_\alpha$**	**Exp.**
0.1	36.12*	31.00	0.00%	0.00%
1	36.12	31.00	0.00%	0.00%
10	36.12*	31.00	0.00%	0.00%

Table G.18. Replication 5 results for the mean-risk model (OF1, $\varepsilon = 0.25$, $\alpha = 0.9$), * indicates an optimality gap $< 0.9\%$.

α=0.9 and Number of Scenarios=10				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	35.43	28.31	**CVaR$_\alpha$**	**Exp.**
0.1	33.43	28.51	-5.65%	0.71%
1	31.43	28.91	-11.29%	2.12%
10	31.14	29.97	-12.10%	5.85%
α=0.9 and Number of Scenarios=50				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	36.29	30.69	**CVaR$_\alpha$**	**Exp.**
0.1	35.49	30.70	-2.20%	0.06%
1	35.09	31.03	-3.31%	1.12%
10	34.91	31.23	-3.78%	1.77%
α=0.9 and Number of Scenarios=100				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	36.43	30.70	**CVaR$_\alpha$**	**Exp.**
0.1	36.43	30.70	0.00%	0.00%
1	35.94	30.98	-1.33%	0.92%
10	35.63	31.31	-2.20%	2.00%
α=0.9 and Number of Scenarios=250				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	37.51	31.00	**CVaR$_\alpha$**	**Exp.**
0.1	37.51	31.00	0.00%	0.00%
1	37.22*	31.14	-0.76%	0.46%
10	37.15*	31.38	-0.94%	1.22%

Table G.19. Replication 2 results for the mean-risk model (OF1+OF3, $\varepsilon = 0$, $\alpha = 0.8$), * indicates an optimality gap $< 0.01\%$.

α=0.8 and Number of Scenarios=10				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	83.88	74.13	**CVaR$_\alpha$**	**Exp.**
0.1	83.02	74.13	-1.02%	0.00%
1	80.51	74.87	-4.02%	0.99%
10	79.85	75.75	-4.80%	2.19%
α=0.8 and Number of Scenarios=50				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	85.62	76.88	**CVaR$_\alpha$**	**Exp.**
0.1	85.62	76.88	0.00%	0.00%
1	84.96	77.22	-0.78%	0.44%
10	84.65	78.05	-1.14%	1.51%
α=0.8 and Number of Scenarios=100				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	86.31	77.04	**CVaR$_\alpha$**	**Exp.**
0.1	86.11	77.04	-0.23%	0.01%
1	84.97*	77.57	-1.55%	0.70%
10	84.84	77.87	-1.70%	1.08%
α=0.8 and Number of Scenarios=250				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	88.61	77.55	**CVaR$_\alpha$**	**Exp.**
0.1	87.77	77.57	-0.94%	0.03%
1	86.99	77.99	-1.83%	0.56%
10	86.78	78.20	-2.06%	0.83%

Table G.20. Replication 2 results for the mean-risk model (OF1+OF3, $\varepsilon = 0$, $\alpha = 0.9$), * indicates an optimality gap $< 0.2\%$.

α=0.9 and Number of Scenarios=10				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	84.68	74.13	**CVaR$_\alpha$**	**Exp.**
0.1	83.07	74.27	-1.90%	0.19%
1	80.53	74.87	-4.90%	0.99%
10	79.92*	76.46	-5.62%	3.14%
α=0.9 and Number of Scenarios=50				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	87.90	76.88	**CVaR$_\alpha$**	**Exp.**
0.1	87.35	76.92	-0.63%	0.04%
1	85.47	77.60	-2.77%	0.93%
10	85.12	78.52	-3.16%	2.13%
α=0.9 and Number of Scenarios=100				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	86.07	77.04	**CVaR$_\alpha$**	**Exp.**
0.1	87.54*	77.08	1.71%	0.06%
1	85.77*	77.57	-0.35%	0.70%
10	85.57	77.84	-0.58%	1.04%
α=0.9 and Number of Scenarios=250				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	91.48	77.55	**CVaR$_\alpha$**	**Exp.**
0.1	90.03	77.58	-1.59%	0.04%
1	88.73	78.15	-3.02%	0.77%
10	88.73*	78.15	-3.02%	0.77%

Table G.21. Replication 3 results for the mean-risk model (OF1+OF3, $\varepsilon = 0$, $\alpha = 0.8$), * indicates an optimality gap $< 0.1\%$.

α=0.8 and Number of Scenarios=10				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	85.58	77.10	**CVaR$_\alpha$**	**Exp.**
0.1	85.22	77.13	-0.42%	0.04%
1	81.93	78.28	-4.27%	1.53%
10	81.66*	79.58	-4.58%	3.22%
α=0.8 and Number of Scenarios=50				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	90.57	78.46	**CVaR$_\alpha$**	**Exp.**
0.1	89.38	78.51	-1.32%	0.07%
1	88.22	78.99	-2.60%	0.67%
10	87.92	79.68	-2.93%	1.56%
α=0.8 and Number of Scenarios=100				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	89.03	78.27	**CVaR$_\alpha$**	**Exp.**
0.1	88.34	78.31	-0.78%	0.05%
1	87.62	78.67	-1.59%	0.51%
10	87.54	78.95	-1.68%	0.87%
α=0.8 and Number of Scenarios=250				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	88.99	77.67	**CVaR$_\alpha$**	**Exp.**
0.1	88.46	77.68	-0.60%	0.01%
1	87.64*	77.98	-1.52%	0.40%
10	87.34	78.40	-1.86%	0.94%

Table G.22. Replication 3 results for the mean-risk model (OF1+OF3, $\varepsilon = 0$, $\alpha = 0.9$), * indicates an optimality gap $< 0.2\%$.

α=0.9 and Number of Scenarios=10				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	86.97	77.10	**CVaR$_\alpha$**	**Exp.**
0.1	85.97	77.14	-1.15%	0.05%
1	82.13*	78.16	-5.57%	1.38%
10	81.73*	79.50	-6.03%	3.11%
α=0.9 and Number of Scenarios=50				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	92.64	78.46	**CVaR$_\alpha$**	**Exp.**
0.1	91.55	78.51	-1.18%	0.07%
1	89.43*	79.61	-3.47%	1.47%
10	89.43*	79.61	-3.47%	1.47%
α=0.9 and Number of Scenarios=100				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	88.58	78.27	**CVaR$_\alpha$**	**Exp.**
0.1	90.43*	78.32	2.09%	0.07%
1	89.28	78.70	0.79%	0.557%
10	89.13*	79.21	0.61%	1.20%
α=0.9 and Number of Scenarios=250				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	91.70	77.67	**CVaR$_\alpha$**	**Exp.**
0.1	90.83	77.70	-0.95%	0.04%
1	89.64*	78.15	-2.24%	0.62%
10	89.43	78.43	-2.48%	0.99%

Table G.23. Replication 4 results for the mean-risk model (OF1+OF3, $\varepsilon = 0$, $\alpha = 0.8$), * indicates an optimality gap $< 0.02\%$.

α=0.8 and Number of Scenarios=10				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	90.75	78.42	**CVaR$_\alpha$**	**Exp.**
0.1	89.32	78.51	-1.57%	0.11%
1	81.93*	78.28	-9.72%	-0.18%
10	81.66*	79.58	-10.01%	1.48%
α=0.8 and Number of Scenarios=50				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	89.14	76.17	**CVaR$_\alpha$**	**Exp.**
0.1	87.95	76.22	-1.34%	0.07%
1	88.22*	78.99	-1.04%	3.70%
10	87.92	79.68	-1.38%	4.61%
α=0.8 and Number of Scenarios=100				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	90.33	77.05	**CVaR$_\alpha$**	**Exp.**
0.1	89.65	77.08	-0.75%	0.03%
1	87.62	78.67	-3.00%	2.09%
10	87.54	78.95	-3.09%	2.45%
α=0.8 and Number of Scenarios=250				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	89.38	77.00	**CVaR$_\alpha$**	**Exp.**
0.1	89.27	77.00	-0.12%	0.00%
1	87.64	77.98	-1.95%	1.28%
10	87.34	78.40	-2.28%	1.82%

Table G.24. Replication 4 results for the mean-risk model (OF1+OF3, $\varepsilon = 0$, $\alpha = 0.9$), * indicates an optimality gap $< 0.1\%$.

α=0.9 and Number of Scenarios=10				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	91.52	78.42	**CVaR$_\alpha$**	**Exp.**
0.1	90.09	78.45	-1.56%	0.04%
1	82.13*	78.16	-10.26%	-0.33%
10	81.73*	79.50	-10.70%	1.37%
α=0.9 and Number of Scenarios=50				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	93.65	76.17	**CVaR$_\alpha$**	**Exp.**
0.1	91.84	76.25	-1.93%	0.11%
1	89.43*	79.61	-4.51%	4.52%
10	89.43	79.61	-4.51%	4.52%
α=0.9 and Number of Scenarios=100				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	94.70	77.05	**CVaR$_\alpha$**	**Exp.**
0.1	93.63	77.08	-1.13%	0.03%
1	89.28*	78.70	-5.73%	2.14%
10	89.13	79.21	-5.89%	2.79%
α=0.9 and Number of Scenarios=250				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	93.30	77.00	**CVaR$_\alpha$**	**Exp.**
0.1	92.93	77.00	-0.40%	0.00%
1	89.64*	78.15	-3.92%	1.49%
10	89.43	78.43	-4.15%	1.87%

Table G.25. Replication 5 results for the mean-risk model (OF1+OF3, $\varepsilon = 0$, $\alpha = 0.8$), * indicates an optimality gap $< 0.01\%$.

α=0.8 and Number of Scenarios=10				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	85.69	74.14	**CVaR$_\alpha$**	**Exp.**
0.1	85.15	74.17	-0.63%	0.05%
1	82.48	74.88	-3.75%	1.00%
10	82.04	76.08	-4.27%	2.61%
α=0.8 and Number of Scenarios=50				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	87.11	76.67	**CVaR$_\alpha$**	**Exp.**
0.1	87.00	76.68	-0.12%	0.01%
1	85.44	77.24	-1.92%	0.73%
10	84.79*	78.42	-2.66%	2.28%
α=0.8 and Number of Scenarios=100				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	86.88	76.87	**CVaR$_\alpha$**	**Exp.**
0.1	86.63	76.89	-0.28%	0.03%
1	85.19	77.41	-1.94%	0.70%
10	85.19	77.41	-1.94%	0.70%
α=0.8 and Number of Scenarios=250				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	88.19	77.63	**CVaR$_\alpha$**	**Exp.**
0.1	88.03	77.63	-0.18%	0.00%
1	87.01	77.99	-1.33%	0.46%
10	86.91	78.12	-1.45%	0.63%

Table G.26. Replication 5 results for the mean-risk model (OF1+OF3, $\varepsilon = 0$, $\alpha = 0.9$), * indicates an optimality gap $< 0.08\%$.

α=0.9 and Number of Scenarios=10				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	87.45	74.14	**CVaR$_\alpha$**	**Exp.**
0.1	85.42	74.23	-2.32%	0.13%
1	82.48*	75.15	-5.68%	1.37%
10	82.09*	76.32	-6.13%	2.94%
α=0.9 and Number of Scenarios=50				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	89.74	76.67	**CVaR$_\alpha$**	**Exp.**
0.1	87.98	76.76	-1.96%	0.12%
1	85.65*	77.86	-4.55%	1.55%
10	85.46*	78.17	-4.77%	1.95%
α=0.9 and Number of Scenarios=100				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	89.45	76.87	**CVaR$_\alpha$**	**Exp.**
0.1	88.51	76.92	-1.06%	0.07%
1	87.11*	77.42	-2.62%	0.71%
10	87.00*	77.61	-2.74%	0.96%
α=0.9 and Number of Scenarios=250				
Trade-off Par. θ	**CVaR$_\alpha$**	**Exp.**	**Relative Difference**	
0	90.77	77.63	**CVaR$_\alpha$**	**Exp.**
0.1	90.15	77.65	-0.69%	0.03%
1	89.26*	78.04	-1.67%	0.53%
10	89.26*	78.04	-1.67%	0.53%