PLANNING OF ENERGY STORAGE SYSTEMS IN POWER NETWORKS: IMPROVING RELIABILITY OF TWO-STAGE ROBUST OPTIMIZATION ALGORITHMS

by

Görkem Vardar

Submitted to Graduate School of Natural and Applied Sciences
in Partial Fulfillment of the Requirements
for the Degree of Master of Science in
Industrial and Systems Engineering

Yeditepe University
2020

PLANNING OF ENERGY STORAGE SYSTEMS IN POWER NETWORKS: IMPROVING RELIABILITY OF TWO-STAGE ROBUST OPTIMIZATION ALGORITHMS

APPROVED BY:

Assist. Prof. Dr. Uğur Yıldıran ………………………………
(Thesis Supervisor)
(Yeditepe University)

Prof. Dr. Linet Özdamar ………………………………
(Yeditepe University)

Assoc. Prof. Dr. Engin Maşazade ………………………………
(Marmara University)

DATE OF APPROVAL: ..../..../2020

ACKNOWLEDGEMENTS

It is with immense gratitude that I acknowledge the support and help of my advisor Uğur Yıldıran for his endless support, contributions, and patience. His knowledge and experience were my greatest guide during my thesis. Pursuing my thesis under his supervision has been an experience that broadens the mind and presents an unlimited source of learning.

I thank my friend Nazlı Kavas and also Yeditepe University Family.

Finally, I would like to thank my family for their endless love and support, which makes everything more beautiful.

ABSTRACT

PLANNING OF ENERGY STORAGE SYSTEMS IN POWER NETWORKS: IMPROVING RELIABILITY OF TWO-STAGE ROBUST OPTIMIZATION ALGORITHMS

The usage of renewable energy sources such as wind and solar in power networks has increased in recent years, introducing uncertainty and variability into the system. Energy storage plays a critical role in mitigating these problems. Thus, placement, sizing, and operation of energy storage units constitute important problems and there are many related studies. In this thesis, the problem of placement of energy storage systems in power networks is studied by modeling it as a two-stage stochastic robust optimization. A hybrid method is used, combining deterministic equivalent for the stochastic part and column and constraint generation for the robust part, to solve the problem. Two new algorithms are proposed for the solution of the second-stage max-min optimization. The proposed methods are applied to the 6-bus, IEEE 14-bus, and IEEE 30-bus systems, and numerical analyses are performed to compare the proposed methods with other methods from the literature.

ÖZET

ENERJİ AĞLARINDA ENERJİ DEPOLAMA SİSTEMLERİNİN PLANLAMASI: İKİ-AŞAMALI GÜRBÜZ OPTİMİZASYON ALGORİTMALARININ GÜVENİLİRLİĞİNİN ARTIRILMASI

Enerji şebekelerinde rüzgar ve güneş gibi yenilenebilir enerji kaynaklarının kullanımı son yıllarda artmış ve sisteme belirsizlik ve değişkenlik getirmiştir. Enerji depolama, bu sorunların azaltılmasında kritik bir rol oynar. Bu nedenle, enerji depolama birimlerinin yerleştirilmesi, boyutlandırılması ve işletilmesi önemli problemler oluşturmaktadır ve bunlarla ilgili birçok çalışma bulunmaktadır. Bu tezde, enerji depolama sistemlerinin güç şebekelerine yerleştirilmesi problemi, iki-aşamalı stokastik gürbüz optimizasyon olarak modellenerek incelenmiştir. Problemi çözmek için, stokastik kısım için deterministik eşdeğer ile gürbüz kısım için sütun ve kısıt üretim metodunu birleştiren hibrit bir yöntem kullanılmıştır. İkinci-aşama maksimum-minimum optimizasyonunun çözümü için iki yeni algoritma önerilmiştir. Önerilen yöntemler 6-baralı, IEEE 14-baralı ve IEEE 30-baralı sistemlerine uygulanmıştır ve önerilen yöntemleri literatürdeki diğer yöntemlerle karşılaştırmak için sayısal analizler yapılmıştır.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF SYMBOLS/ABBREVIATIONS

$\mathcal{B}$	Index set of all buses
b, j	Indexes of buses
B_{jb}:	Susceptance of the line between bus j and bus b
CG_i	Generation cost of thermal generator i
d_{bt}^{n}	Distributed nominal demand at bus b in time t
$\overline{E_b}$	Maximum energy capacity of storage unit can be built at bus b
$\underline{E_b}$	Minimum energy capacity of storage unit can be built at bus b
E_{b0}	Initial energy stored in storage unit can be built at bus b
E_{b0s}	Initial energy stored in storage unit can be built at bus b for scenario s
E_{bt}''	Energy stored in the storage unit at bus b in time t
E_{bts}'	Energy stored in the storage unit at bus b in time t and for scenario s
g_{i0}	Initial electricity generation of thermal generator i
g_{i0s}	Initial electricity generation of thermal generator i for scenario s
g_{it}''	Power generation by thermal generator i in time t
g_{its}'	Power generation by thermal generator in time t and for scenario s
i	Index of generators
k	Contribution weight of the stochastic and robust parts in the objective function
L_{jb}	Flow limit of the line between bus j and bus b
$\overline{PG_i}$	Maximum power generation of thermal generator i
$\underline{PG_i}$	Minimum power generation of thermal generator i
$\overline{p_b}$	Maximum charging capacity of storage unit can be built at bus b
$\underline{p_b}$	Minimum charging capacity of storage unit can be built at bus b
p_{bt}''	Power charged into the storage unit at bus b in time t
p_{bts}'	Power charged into the storage unit at bus b in time t and for scenario s
$\overline{q_b}$	Maximum discharging capacity of storage unit can be built at bus b
$\underline{q_b}$	Minimum discharging capacity of storage unit can be built at bus b

q''_{bt}	Power discharged from the storage unit at bus b in time t
q'_{bts}	Power discharged from the storage unit at bus b in time t and for scenario s
RR_i	Ramp-rate limit of thermal generator i
$\mathcal{S}$	Index set of all scenarios
s	Index of scenarios
SC_b	Storage investment cost at bus b
T	Time horizon
t	Index of time
$\mathcal{W}_b$	Index set of wind farms at bus b
w	Index of wind farms
w^n_{wt}	Nominal wind power of wind farm w in time t
w_{wts}	Wind power of wind farm w in time t and for scenario s
α_b	Binary variable equal to one if storage is opened at bus b and equal to zero otherwise
$\overline{\Delta w}_{wt}$	Upper-bound on wind deviation of wind farm w in time t
$\underline{\Delta w}_{wt}$	Lower-bound on wind deviation of wind farm w in time t
Δw_{wt}	Wind power deviation of wind farm w in time t
θ''_{bt}	Phase angle of bus b in time t
θ'_{bts}	Phase angle of bus b in time t and for scenario s
Λ	Index set of all thermal generators
$\mathit{\Lambda}_b$	Index set of thermal generators connected to bus b
Ω_b	Index set of buses linked to bus b
BD	Bender’s decomposition
BLP	Bilinear programming
CCG	Column and constraint generation
DE	Deterministic equivalent
EP	Extreme point
ESS	Energy storage system
KKT	Karush-Kuhn-Tucker

LB	Lower-bound
LP	Linear programming
MC	Mountain climbing
MILP	Mixed-integer linear programming
OA	Outer approximation
RO	Robust optimization
SAA	Sample average approximation
SP	Stochastic programming
SRO	Stochastic robust optimization
UB	Upper-bound

1. INTRODUCTION

The usage of renewable energy sources such as wind and solar in power networks has increased in recent years, introducing uncertainty and variability into the system. Energy storage systems (ESSs) play a crucial role in reducing these problems in power networks. Thus, placement, sizing, and operation of ESSs in power networks constitute important problems and there are many related studies in the literature.

Mainly two approaches can be used for planning of storage systems under uncertainty, namely stochastic programming (SP) and robust optimization (RO). In the former an expected cost is optimized while in the latter cost is optimized for wort-case scenarios.

Because the computation of expectation exactly is difficult in general, SP based approaches make use of sampling-based approximations. As a result of sampling, worst-case uncertainties that can lead to infeasible operation can be missed, hurting the safety of the system. Moreover, the SP requires probabilistic information about uncertainties, which can be difficult to obtain in general. The RO, on the other hand, does not require any probabilistic information. It is enough to know sets from which uncertainties take values. In addition, solution algorithms do not rely on sampling. The uncertainties are regarded as continuous variables whose all possible values are taken into account in the associated optimization algorithms. As a result, worst-case uncertainties are not missed, and hence, the solutions found can ensure the safety of the system.

Due to the aforementioned advantages, the RO has been used extensively for the operation and planning of power systems in the literature. Jabr et al. [1] studied two-stage robust investment planning of ESSs on transmission networks with renewable energy resources. Liu et al. [2] built a two-stage RO model for coordinated planning of generation expansion and ESSs. Dehghan and Amjady [3] proposed a two-stage RO model for transmission expansion and ESS planning problems in wind farm-integrated power network. Furthermore, Ye et al. [4] solved a two-stage robust security-constraninted unit commitment problem. Baringos [5] proposed a RO approach for the generation and transmission expansion planning. Jabr [6] presented a two-stage RO method for transmission network expansion planning problem under uncertain renewable generation and demand. Wei et al. [7] studied robust energy and reserve dispatch planning under uncertain renewable generation.

Bertsimas et al. [8] built a two-stage adaptive RO model for the security constrainted unit commitment problem in the presence of load uncertainty.

Despite the advantages of the RO described above, it has a serious shortcoming. Because the cost is optimized for worst-case scenarios, solutions found can be highly conservative, making this approach unattractive economically. There have been some efforts in the literature to alleviate this conservativeness. The most widely used solution is to combine the RO with SP to obtain a hybrid approach, called Stochastic Robust Optimization (SRO). Zhao and Guan [9] proposed a two-stage SRO approach for the unit commitment problem. Liu et al. [10] presented a stochastic and robust optimization model for transmission expansion planning.

Deterministic Equivalent (DE) is a method that can be used to solve both SP and RO problems. The method relies on defining uncertainty with a finite number of scenarios and these scenarios are considered while computing the optimal decisions. Unfortunately, the method is intractable for a large number of scenarios. This is especially problematic for the RO which requires enumeration of all extreme scenarios whose size increases exponentially with the dimension of uncertainties.

Bender's Decomposition (BD) is another method that can be used for the solution of SP and RO problems. The problem of interest is assumed to occur in two or more stages and the method decomposes the problem into master and slave problems. The master problem computes the first-stage decision while the slave problem sends information (cuts) to the master problem based on first-stage decisions. It is well known that this method converges slowly since it is a cutting plane based approach. Jabr [6] used the BD scheme to solve the problem he proposes in his work. Wei et al. [7] established the BD method to solve their optimization problem. Also, Bertsimas et al. [8] developed a solution methodology based on a BD-type algorithm.

Recently, Zeng and Zhao [11] introduced a novel algorithm called the column and constraint generation (CCG) method for the solution of RO problems. It works quite fat alleviating the solve convergence issues encountered in BD. Due to this property, it has been used widely for the operation and planning of power systems. Jabr et al. [1] developed a computational engine called ROSION which is an implementation of the CCG method. Liu et al. [2] decomposed and solved their proposed min-max-min type problem CCG scheme. Also, Dehghan and Amjady [3] introduced a CCG type primal cutting plane decomposition algorithm to obtain an optimal solution. Finally, Ye et al. [4] and Baringos [5] used modified versions of the CCG method to solve their two-stage RO models.

In [9], where the SRO approach was introduced, they combined DE with CCG to solve the two-stage SRO problem. DE represented the stochastic part and CCG was employed for the robust part. It merges a scenario-based SP procedure with a two-stage RO solution scheme. Using DE for the stochastic part is realistic because good approximations for the SP can usually be obtained by using a reasonable number of scenarios and employing CCG for the robust part is practical because the method terminates after adding a reasonable number of worst-case scenarios. This method was used by Zhao and Guan [9] and Liu et al. [10].

BD and CCG methods that can be used for the solution of the RO problem decompose it into master and slave problems as mentioned above. The slave problem obtained has the same form for both approaches. It is a max-min type problem, which is difficult to solve in general. Some methods have been proposed for its solution in the literature, which can be categorized into three groups.

The first set of methods are dedicated algorithms that making use of tailored versions of optimization strategies. However, in practice, for instance, in the power system literature, they are not preferred because they require a lot of coding and the algorithm obtained may not work reliably due to numerical issues that can arise from technical details.

The second set of methods converts the max-min problem into an equivalent mixed-integer linear programming (MILP) problem by algebraic manipulations [3,5,6]. Then the equivalent formulation can be solved reliably without too much coding effort using off-the-shelf solvers. One approach is the extreme point (EP) method [4,7] which solves feasibility type problems and works reliably. Another approach is Fortuny-Amat Formulation where Karush-Kuhn-Tucker (KKT) conditions are used [2,10–13]. It can be used to solve optimization type problems. But the method is based on Big-M formulation which causes numerical issues and the choice of M is problematic [12,13].

The third set of methods is approximations. They are preferred because max-min is a hard problem. But they produce suboptimal solutions, which can lead to safety issues because the robustness cannot be assured due to suboptimality. One approach is the Mountain Climbing (MC) method [4,9] where two linear problems are solved iteratively to obtain a solution and the other approach is the Outer Approximation (OA) method [8] where linear approximations of the objective function are iteratively added to the problem to solve it.

Our aim, in this thesis, is to solve a special type of problem for the planning of energy storage systems in power networks, namely, determining the locations of storage units so that the system will work reliably and installation and operation costs will be optimized. To ensure the safety of the system under all possible values of uncertainties without causing a high degree of conservativeness, a stochastic and robust optimization formulation will be employed. A hybrid method combining DE for the stochastic part and CCG for the robust part will be utilized as the solution approach. As mentioned above, solving the inner max-min problem reliably and efficiently is an important issue. Two new algorithms are proposed for this problem, one being an exact method while the other an approximation scheme. The advantages of these methods are demonstrated by comparing them with the other alternatives available in the literature and summarized above. The proposed methods are not only applicable to storage locationing problem but also can be used for two-stage robust optimization in general. Therefore, they can be considered as contributions to RO literature.

The rest of this thesis is organized as follows: In Section 2 the details of two-stage stochastic and robust optimization problems and their deterministic equivalents are given. Also, the modified CCG method is introduced in this section. Next, in Section 3 the solution methods for the second-stage max-min problem are discussed; as some exact and approximation methods and our proposed algorithms are explained. Then, in Section 4 the details of the energy storage planning problem are given and the mathematical model is formulated.

2. TWO-STAGE STOCHASTIC AND ROBUST OPTIMIZATION

In the optimization theory, there are problems that include uncertainties. These problems can be classified as stochastic or robust optimization problems depending on how the uncertainty is characterized. In SP, the probability distribution of uncertainties is known and the solution is found optimizing an expected cost. On the other hand, in RO, sets to which uncertainties belong are known and optimization is performed for worst-case scenarios. Several real-world problems, such as problems in the supply chain, finance, transportation, and energy, can be formulated as SP, RO, or a combination of them.

In the following sections, two-stage stochastic and robust optimization problems are introduced and the solution approaches used frequently for them are explained which are also employed in this thesis.

2.1. TWO-STAGE STOCHASTIC OPTIMIZATION AND ITS DETERMINISTIC EQUIVALENT

The two-stage stochastic optimization problems have two stages. The decisions made in the first stage are called "here and now" and the decisions made in the second stage are called "wait and see". In general, a two-stage SP problem can be expressed as follows:

$$\min_{\mathbf{y}} \boldsymbol{c}^T \boldsymbol{y} + \mathbb{E}[\min_{\mathbf{x}} \boldsymbol{b}^T \boldsymbol{x}(\boldsymbol{\xi})] \tag{2.1}$$

subject to:

$$\boldsymbol{A}\boldsymbol{y} \leq \boldsymbol{d} \tag{2.2}$$

$$\boldsymbol{E}\boldsymbol{y} + \boldsymbol{G}\boldsymbol{x}(\boldsymbol{\xi}) \leq \boldsymbol{h} + \boldsymbol{M}\boldsymbol{\xi} \tag{2.3}$$

$$\boldsymbol{x} \in \mathbb{R}^m, \qquad \boldsymbol{y} \subset \mathbb{R}^n, \qquad \boldsymbol{\xi} \in \Xi \tag{2.4}$$

where $\Xi \subset \mathbb{R}^{\mathrm{k}}$. In the above formulation, $\boldsymbol{y}$ represents the vector of first-stage decision variables which are made before the observation of the vector of random data $\boldsymbol{\xi}$ and $\boldsymbol{x}$ represents the vector of second-stage variables decided after $\boldsymbol{\xi}$ is revealed. The objective function (2.1) minimizes the sum of first stage cost and expected cost of the second stage decisions. The constraints are given in (2.2)-(2.4).

In general, computing expectation for continuous random variables is intractable. Hence, sampling-based approximations are usually employed. In this approximation, uncertain data $\boldsymbol{\xi}$ is represented by a set of finite number of samples taken from the domain of its distribution, which can be expressed as $\mathcal{S} = \{\boldsymbol{\xi}^1, \boldsymbol{\xi}^2, \dots, \boldsymbol{\xi}^N\}$. The N samples $\boldsymbol{\xi}^1, \boldsymbol{\xi}^2, \dots, \boldsymbol{\xi}^N$ are called the scenarios and they can be obtained by scenario generation techniques, like Monte Carlo sampling. If the respective probabilities are $p_1, p_2, \dots, p_N$, then the expectation in the objective function (2.1) can be approximated as:

$$\mathbb{E}\left[\min_{\mathbf{x}} \boldsymbol{b}^T \boldsymbol{x}(\boldsymbol{\xi})\right] \approx \min_{\mathbf{x}} \sum_{s=1}^{N} p_s \boldsymbol{b}^T \boldsymbol{x}(\boldsymbol{\xi}^s) \tag{2.5}$$

where s is the scenario index. In many practical applications, scenarios are generated by drawing N independent samples from the distribution of $\boldsymbol{\xi}$ and assigning them equal probabilities, that is, $p_s = \frac{1}{N}$, $s = 1{:}\,N$. In this case, the Equation (2.5) can be expressed as:

$$\sum_{s=1}^{N} p_s \boldsymbol{b}^T \boldsymbol{x}(\boldsymbol{\xi}^s) = \frac{1}{N} \sum_{s=1}^{N} \boldsymbol{b}^T \boldsymbol{x}(\boldsymbol{\xi}^s) \tag{2.6}$$

This is called the sample average approximation (SAA).

The simplest approach that can be used to solve the SP (2.1)-(2.4), is to construct the so-called deterministic equivalent, which is given below for the SAA objective (2.6).

$$\min_{\mathbf{y}} \boldsymbol{c}^T \boldsymbol{y} + \frac{1}{N} \sum_{s=1}^{N} \boldsymbol{b}^T \boldsymbol{x}(\boldsymbol{\xi}^s) \tag{2.7}$$

subject to:

$$\boldsymbol{A}\boldsymbol{y} \leq \boldsymbol{d} \tag{2.8}$$

$$\boldsymbol{E}\boldsymbol{y} + \boldsymbol{G}\boldsymbol{x}(\boldsymbol{\xi}^s) \leq \boldsymbol{h} + \boldsymbol{M}\boldsymbol{\xi}^s, \qquad s = 1{:}\,N \tag{2.9}$$

$$\boldsymbol{x} \in \mathbb{R}^m, \qquad \boldsymbol{y} \subset \mathbb{R}^n \tag{2.10}$$

Given that some elements of the first stage decision vector, $\boldsymbol{y}$, can be an integer, this is a MILP in general and can be solved using off-the-shelf solvers reliably. But this problem becomes intractable when N is large. In this case, one may use Bender's Decomposition approach to split the SP into master and slave problems. In this way, an optimal solution can be obtained by optimizing problems of smaller sizes iteratively. However, since it is a cutting plane based method, generally, Bender's Decomposition converges slowly to an optimal solution.

2.2. TWO-STAGE ROBUST OPTIMIZATION AND ITS DETERMINISTIC EQUIVALENT

A two-stage RO has a structure similar to a two-stage SP. There are "here and now" decisions to be made before the realization of uncertainties and "wait and see" decisions to be given after uncertainties are revealed. However, different from the SP, the second stage objective is not to minimize an expected cost but a worst-case cost that can be achieved under all possible realizations of uncertainties. This problem can be formulated as follows:

$$\min_{\boldsymbol{y}} \boldsymbol{c}^T \boldsymbol{y} + \max_{\boldsymbol{\zeta} \in \Xi} \min_{\boldsymbol{x}} \boldsymbol{b}^T \boldsymbol{x}(\boldsymbol{\zeta}) \tag{2.11}$$

subject to:

$$\boldsymbol{Ay} \leq \boldsymbol{d} \tag{2.12}$$

$$\boldsymbol{Ey} + \boldsymbol{Gx}(\boldsymbol{\zeta}) \leq \boldsymbol{h} + \boldsymbol{M\zeta} \tag{2.13}$$

$$\boldsymbol{x} \in \mathbb{R}^m, \qquad \boldsymbol{y} \subset \mathbb{R}^n \tag{2.14}$$

In the above formulation, $\boldsymbol{y}$ is the vector of first stage decisions whose elements can be continuous, integer, or binary variables, $\boldsymbol{x}$ is the vector of second-stage decisions and $\boldsymbol{\zeta}$ is the vector of uncertainties. The objective of (2.11) is the summation of the first stage decision cost and the worst-case cost of the second stage. The uncertainty vector $\boldsymbol{\zeta}$ can take values from the set Ξ. This uncertainty set can be characterized in different ways using box, polytopic, or other convex constraints.

There are two main approaches to solve two-stage RO problems, Bender's Decomposition and CCG. Since BD works slow in general we will use the CCG method, proposed by Zeng and Zhao [11], in this thesis similar to several works in the literature. These algorithms can terminate by finding an optimal solution when the uncertainty sets are box or polytopic. In this thesis, we assume that they are characterized by box constraints.

CCG algorithm can be explained by firstly understanding the DE formulation of the RO problem (2.11)-(2.14). In the second stage of the RO problem, uncertainty can be considered as an adversary trying to maximize the minimum cost that can be achieved by the “wait and see” decisions $\boldsymbol{x}$. Although the vector $\boldsymbol{\zeta}$ can take values from a continuous domain, for polytopic sets it can be easily shown that worst-case solutions lie at the corners of the polytope. Hence the maximum can be taken over a finite set of corner points $\boldsymbol{\zeta}^1, .., \boldsymbol{\zeta}^R$. This leads to the following DE which is a single-stage optimization problem:

$$\min_{\mathbf{y}} \boldsymbol{c}^T \boldsymbol{y} + \eta \tag{2.15}$$

subject to:

$$\boldsymbol{A}\boldsymbol{y} \leq \boldsymbol{d} \tag{2.16}$$

$$\eta \geq \boldsymbol{b}^T \boldsymbol{x}(\boldsymbol{\zeta}^r), \qquad r = 1{:}\,R \tag{2.17}$$

$$\boldsymbol{E}\boldsymbol{y} + \boldsymbol{G}\boldsymbol{x}(\boldsymbol{\zeta}^r) \leq \boldsymbol{h} + \boldsymbol{M}\boldsymbol{\zeta}^r, \qquad r = 1{:}\,R \tag{2.18}$$

$$\eta \in \mathbb{R}, \qquad \boldsymbol{x} \in \mathbb{R}^m, \qquad \boldsymbol{y} \subset \mathbb{R}^n \tag{2.19}$$

Although it has a simple form, the above given DE is difficult to solve in general because R increases exponentially with the dimension of $\boldsymbol{\zeta}$.

The CCG algorithm is based on the following key observation about the DE problem. Instead of considering all corners, if a subset of them is used in (2.16)-(2.19), a relaxed problem can be obtained giving a lower bound on the DE. By adding new uncertainties to this relaxation incrementally, one can obtain improving approximations. If these approximations converge to the optimal solution after adding a reasonable number of corners, the optimal solution of the RO problem can be obtained tractably.

CCG algorithm is given in Algorithm 2.1. It solves the following master and slave problems iteratively. Here the master problem is the relaxation of the DE mentioned above. The slave problem finds a new corner to be added to the uncertainty set in each iteration of the algorithm.

Master Problem:

$$\min_{\mathbf{y}} \boldsymbol{c}^T \boldsymbol{y} + \eta \tag{2.20}$$

subject to:

$$\boldsymbol{A}\boldsymbol{y} \leq \boldsymbol{d} \tag{2.21}$$

$$\eta \geq \boldsymbol{b}^T \boldsymbol{x}(\boldsymbol{\zeta}^r), \qquad \forall r \in \Omega \tag{2.22}$$

$$\boldsymbol{E}\boldsymbol{y} + \boldsymbol{G}\boldsymbol{x}(\boldsymbol{\zeta}^r) \leq \boldsymbol{h} + \boldsymbol{M}\boldsymbol{\zeta}^r, \qquad \forall r \in \Omega \tag{2.23}$$

$$\eta \in \mathbb{R}, \qquad \boldsymbol{x} \in \mathbb{R}^m, \qquad \boldsymbol{y} \subset \mathbb{R}^n \tag{2.24}$$

Slave Problem:

$$\phi(\boldsymbol{y}) = \max_{\zeta \in \Xi} \min_{x} \boldsymbol{b}^T \boldsymbol{x}(\boldsymbol{\zeta}) \tag{2.25}$$

subject to:

$$\boldsymbol{E}\boldsymbol{y} + \boldsymbol{G}\boldsymbol{x}(\boldsymbol{\zeta}) \leq \boldsymbol{h} + \boldsymbol{M}\boldsymbol{\zeta} \tag{2.26}$$

$$\boldsymbol{x} \in \mathbb{R}^{\mathrm{m}} \tag{2.27}$$

where $\Xi = \{\boldsymbol{\zeta} \subset \mathbb{R}^u : \underline{\boldsymbol{\zeta}} \leq \boldsymbol{\zeta} \leq \overline{\boldsymbol{\zeta}}\}$.

In the CCG method given in Algorithm 2.1, the uncertain scenarios are added to the master problem gradually in Step 4a and 4b, unlike DE which involves all extreme point scenarios. The optimal solution of the slave problem, which is solved in Step 3, gives the scenario to be added to the set Ω of the master problem. The procedure expands the set Ω which is a subset of Ξ, at the end of 4a and 4b. Since the constraints (2.22) are defined on the set Ω, a subset of all possible uncertain scenarios, the master problem gives a lower-bound (LB) to the original two-stage RO problem which is calculated on Step 2, and adding more scenarios can give stronger lower bounds. The objective value of the master problem is added to the objective value of the slave problem and a new value is calculated in Step 3. This gives an upper-bound (UB) to the original two-stage RO problem because the summation of the cost of a first-stage feasible solution with the cost of a second-stage optimal solution yields an upper-bound to a minimization problem. The master problem is a MILP and can be solved by off-the-shelf solvers in Step 2. On the other hand, it is assumed that there is an oracle to solve the given subproblem in Step 3. Algorithms that can be used to solve the subproblem will be introduced in the next section.

Algorithm 2.1. Column and constraint generation method

1. Set $LB = -\infty$, $UB = +\infty$, $k = 0$, $\Omega = \emptyset$.
2. Solve the master problem (2.20)-(2.24) and get $\boldsymbol{y}_{k+1}^*$, η_{k+1}^*, $\boldsymbol{x}(\boldsymbol{\zeta}^1)^*$, …, $\boldsymbol{x}(\boldsymbol{\zeta}^k)^*$ and update $LB = \boldsymbol{c}^T\boldsymbol{y}_{k+1}^* + \eta_{k+1}^*$.
3. Solve the slave problem $\phi(\boldsymbol{y}_{k+1}^*)$ and update $UB = \min\{UB,\ \boldsymbol{c}^T\boldsymbol{y}_{k+1}^* + \phi(\boldsymbol{y}_{k+1}^*)\}$.
4. If $UB - LB \leq \varepsilon$, return $\boldsymbol{y}_{k+1}^*$ and terminate. Otherwise, do
   a. if $\phi(\boldsymbol{y}_{k+1}^*) < +\infty$, create variables $\boldsymbol{x}(\boldsymbol{\zeta}^{k+1})$ and add the following constraints
      $\eta \geq \boldsymbol{b}^T\boldsymbol{x}(\boldsymbol{\zeta}^{k+1})$
      $\boldsymbol{Ey} + \boldsymbol{Gx}(\boldsymbol{\zeta}^{k+1}) \leq \boldsymbol{h} + \boldsymbol{M}\boldsymbol{\zeta}^{k+1^*}$
      to the master problem where $\boldsymbol{\zeta}^{k+1^*}$ is the optimal solution on $\phi(\boldsymbol{y}_{k+1}^*)$. Update $k = k + 1$, $\Omega = \Omega \cup \{\boldsymbol{\zeta}^{k+1^*}\}$ and go to Step 2.
   b. if $\phi(\boldsymbol{y}_{k+1}^*) = +\infty$, create variables $\boldsymbol{x}(\boldsymbol{\zeta}^{k+1})$ and add the following constraints
      $\boldsymbol{Ey} + \boldsymbol{Gx}(\boldsymbol{\zeta}^{k+1}) \leq \boldsymbol{h} + \boldsymbol{M}\boldsymbol{\zeta}^{k+1^*}$
      to the master problem where $\boldsymbol{\zeta}^{k+1^*}$ is the scenario for which $\phi(\boldsymbol{y}_{k+1}^*) = +\infty$. Update $k = k + 1$, $\Omega = \Omega \cup \{\boldsymbol{\zeta}^{k+1^*}\}$ and go to Step 2.

In the algorithm given above, the constraints formulated in Step 4a present as optimality cuts, and constraints formulated in Step 4b present as feasibility cuts. This algorithm is guaranteed to converge to an optimal solution of (2.11)-(2.14) after finitely many steps. ε is the tolerance used for terminating the algorithm.

2.3. TWO-STAGE STOCHASTIC ROBUST OPTIMIZATION AND ITS DETERMINISTIC EQUIVALENT

SP and RO approach described in previous sections have their own advantages and disadvantages. The former is usually preferred in many applications because optimization of expected cost constitutes a realistic objective. However, since one needs to approximate the expectation by taking samples, all possible values of uncertainties cannot be taken into account. Thus, the solutions found may lead to infeasibilities if such uncertainties occur. RO, on the other hand, ensures feasibility for all possible realization of uncertainties but the worst-cost objective is conservative so that the solutions found lead to high costs on average.

In order to overcome the problems mentioned and take the advantage of the desired features of SP and RO, a new method named Stochastic Robust Optimization was proposed in [9]. The problem solved in SRO can be formulated as follows:

$$\min_{\mathbf{y}} \boldsymbol{c}^T \boldsymbol{y} + \alpha\ \mathbb{E}[\min_{\mathbf{x}} \boldsymbol{b}^T \boldsymbol{x}(\boldsymbol{\xi})] + (1-\alpha) \max_{\boldsymbol{\zeta}\in\Xi} \min_{\mathbf{x}} \boldsymbol{b}^T \boldsymbol{x}(\boldsymbol{\zeta}) \tag{2.28}$$

subject to:

$$\boldsymbol{A}\boldsymbol{y} \leq \boldsymbol{d} \tag{2.29}$$

Stochastic Constraints:

$$\boldsymbol{E}\boldsymbol{y} + \boldsymbol{G}\boldsymbol{x}(\boldsymbol{\xi}) \leq \boldsymbol{h} + \boldsymbol{M}\boldsymbol{\xi} \tag{2.30}$$

$$\boldsymbol{\xi} \in \Xi \tag{2.31}$$

Robust Constraints:

$$\boldsymbol{E}\boldsymbol{y} + \boldsymbol{G}\boldsymbol{x}(\boldsymbol{\zeta}) \leq \boldsymbol{h} + \boldsymbol{M}\boldsymbol{\zeta} \tag{2.32}$$

$$\boldsymbol{x} \in \mathbb{R}^m, \qquad \boldsymbol{y} \subset \mathbb{R}^n \tag{2.33}$$

In the above, the objective function (2.28) is composed of three parts. The first part minimizes the first-stage decision cost, the second part is the expectation of the second-stage stochastic decision cost and the third part minimizes the second-stage cost under worst-case uncertainty. The weight factors, α for the stochastic optimization objective and $(1-\alpha)$ for the robust optimization objective, are used in order to determine the contribution of stochastic and robust parts to the overall objective function value. $\boldsymbol{x}$ and $\boldsymbol{y}$ represent second and first stage decisions, respectively. The DE of the above two-stage stochastic robust problem can be given as below:

$$\min \boldsymbol{c}^T \boldsymbol{y} + \alpha \frac{1}{N} \sum_{s=1}^{N} \boldsymbol{b}^T \boldsymbol{x}(\boldsymbol{\xi}^s) + (1-\alpha)\eta \tag{2.34}$$

subject to:

$$\boldsymbol{Ay} \leq \boldsymbol{d} \tag{2.35}$$

Stochastic Constraints:

$$\boldsymbol{Ey} + \boldsymbol{Gx}(\boldsymbol{\xi}^s) \leq \boldsymbol{h} + \boldsymbol{M\xi}^s, \qquad s = 1{:}N \tag{2.36}$$

Robust Constraints:

$$\eta \geq \boldsymbol{b}^T \boldsymbol{x}(\boldsymbol{\zeta}^r), \qquad r = 1{:}R \tag{2.37}$$

$$\boldsymbol{Ey} + \boldsymbol{Gx}(\boldsymbol{\zeta}^r) \leq \boldsymbol{h} + \boldsymbol{M\zeta}^r, \qquad r = 1{:}R \tag{2.38}$$

$$\eta \in \mathbb{R}, \qquad \boldsymbol{x} \in \mathbb{R}^m, \qquad \boldsymbol{y} \subset \mathbb{R}^n \tag{2.39}$$

This problem can be solved by using a combination of DE and CCG. To be more specific, in practical applications, the expected objective is usually approximated well by using a relatively small number of scenarios. Hence, the SP part can be represented by a DE. For the RO part, on the other hand, one may employ the CCG algorithm because it is known to terminate after adding a reasonable number of worst-case scenarios in practice. This idea leads to Algorithm 2.2 which solves the following master and slave problems repeatedly to obtain an optimal solution of the SRO problem given in (2.34)-(2.39).

Master Problem:

$$\min_{\mathbf{y}} \boldsymbol{c}^T \boldsymbol{y} + \alpha \, \frac{1}{N} \sum_{s=1}^{N} \eta_s + (1 - \alpha)\, \eta \tag{2.40}$$

subject to:

$$\boldsymbol{Ay} \leq \boldsymbol{d} \tag{2.41}$$

Stochastic Constraints:

$$\eta_s \geq \boldsymbol{b}^T \boldsymbol{x}(\boldsymbol{\xi}^s), \qquad s = 1{:}N \tag{2.42}$$

$$\boldsymbol{Ey} + \boldsymbol{Gx}(\boldsymbol{\xi}^s) \leq \boldsymbol{h} + \boldsymbol{M\xi}^s, \qquad s = 1{:}N \tag{2.43}$$

$$\eta_s \in \mathbb{R}, \qquad s = 1{:}N \tag{2.44}$$

Robust Constraints:

$$\eta \geq \boldsymbol{b}^T \boldsymbol{x}(\boldsymbol{\zeta}^r), \qquad \forall r \in \Omega \tag{2.45}$$

$$\boldsymbol{Ey} + \boldsymbol{Gx}(\boldsymbol{\zeta}^r) \leq \boldsymbol{h} + \boldsymbol{M\zeta}^r, \qquad \forall r \in \Omega \tag{2.46}$$

$$\eta \in \mathbb{R}, \qquad \boldsymbol{x} \in \mathbb{R}^m, \qquad \boldsymbol{y} \subset \mathbb{R}^n \tag{2.47}$$

Slave problem:

$$\phi(\boldsymbol{y}) = \max_{\zeta \in \Xi} \min_{\boldsymbol{x}} \boldsymbol{b}^T \boldsymbol{x}(\boldsymbol{\zeta}) \tag{2.48}$$

subject to:

$$\boldsymbol{Ey} + \boldsymbol{Gx}(\boldsymbol{\zeta}) \leq \boldsymbol{h} + \boldsymbol{M\zeta} \tag{2.49}$$

$$\boldsymbol{x} \in \mathbb{R}^{\mathrm{m}} \tag{2.50}$$

where $\Xi = \left\{ \boldsymbol{\zeta} \in \mathbb{R}^u : \underline{\boldsymbol{\zeta}} \leq \boldsymbol{\zeta} \leq \overline{\boldsymbol{\zeta}} \right\}$ and N is the number of generated stochastic scenarios.

Algorithm 2.2. Modified CCG method

1. Set $LB = -\infty$, $UB = +\infty$, $k = 0$, set $\Omega = \{\boldsymbol{\xi}^s, s = 1{:}\,N\}$.
2. Solve the master problem and get $\boldsymbol{y}_{k+1}^*$, η_{k+1}^*, $\eta_s^*, s = 1{:}\,N$, $\boldsymbol{x}(\boldsymbol{\xi}^1)^*$, …, $\boldsymbol{x}(\boldsymbol{\xi}^N)^*$, $\boldsymbol{x}(\boldsymbol{\zeta}^1)^*$, …, $\boldsymbol{x}(\boldsymbol{\zeta}^k)^*$ and update $LB = \boldsymbol{c}^T \boldsymbol{y}_{k+1}^* + \alpha \frac{1}{N} \sum_{s=1}^N \eta_s^* + (1-\alpha)\, \eta_{k+1}^*$.
3. If $UB - LB \leq \varepsilon$, return $\boldsymbol{y}_{k+1}^*$ and terminate. Otherwise,
4. Check the feasibility of the slave problem under $\boldsymbol{y}_{k+1}^*$.
   a. If the slave problem is infeasible, create variables $\boldsymbol{x}(\boldsymbol{\zeta}^{k+1})$ and add the following constraints
      $\boldsymbol{Ey} + \boldsymbol{Gx}(\boldsymbol{\zeta}^{k+1}) \leq \boldsymbol{h} + \boldsymbol{M\zeta}^{k+1*}$
      to the master problem where $\boldsymbol{\zeta}^{k+1*}$ is the scenario that makes the subproblem infeasible. Update, $k = k + 1$, $\Omega = \Omega \cup \{\boldsymbol{\zeta}^{k+1*}\}$ and go to Step 2.
   b. Otherwise, solve the subproblem $\phi(\boldsymbol{y}_{k+1}^*)$, create variables $\boldsymbol{x}(\boldsymbol{\zeta}^{k+1})$ and add the following constraints
      $\eta \geq \boldsymbol{b}^T \boldsymbol{x}(\boldsymbol{\zeta}^{k+1})$
      $\boldsymbol{Ey} + \boldsymbol{Gx}(\boldsymbol{\zeta}^{k+1}) \leq \boldsymbol{h} + \boldsymbol{M\zeta}^{k+1*}$
      to the master problem where $\boldsymbol{\zeta}^{k+1*}$ is the scenario solving the slave problem. Update $UB = \min\{UB, \boldsymbol{c}^T \boldsymbol{y}_{k+1}^* + \alpha \frac{1}{N} \sum_{s=1}^N \eta_s^* + (1 - \alpha)\, \phi(\boldsymbol{y}_{k+1}^*)\}$, $k = k + 1$, $\Omega = \Omega \cup \{\boldsymbol{\zeta}^{k+1*}\}$ and go to Step 2.

As can be seen, Algorithm 2.2. is very similar to the CCG algorithm given in Algorithm 2.1. The main difference is that scenarios used for the DE of the stochastic part are added at the very beginning. Besides, the lower and upper bounds computed combines stochastic objective with the bounds of the RO part.

3. SOLUTION METHODS FOR SECOND-STAGE MAX-MIN PROBLEM AND PROPOSED METHODS

CCG algorithms introduced in Algorithm 2.1. and Algorithm 2.2 relies on the solution of the slave problem (2.48)-(2.50). This is a max-min type problem, which is NP-hard in general, and hence, difficult to solve. Several methods have been proposed in the literature to solve this type of problem, which can be categorized into three different classes.

The first class contains dedicated algorithms making use of tailored versions of optimization strategies such as branch-and-bound, vertex enumeration, etc. Although, there is a large body of literature on these methods, in practice, particularly in power systems literature, they are seldom used. This is because they require considerable coding effort and making them numerically reliable is a difficult task.

The second class tries to convert the max-min problem into an equivalent MILP via some transformations. This allows for solving the problem by making use of reliable off-the-shelf software packages. Thus, coding effort and numerical problems are easily avoided. This approach is mostly preferred in studies applying RO techniques to specific areas such as power systems.

The last class involves approximation methods. Because the max-min problem is NP-hard in general, it can be difficult to solve (2.48)-(2.50) to optimality for large instances. In this case, one may need to resort to some cheaply computable approximations.

In the first two sections given below, solution techniques employed in the literature for the last two classes summarized above will be described. Then, two new techniques proposed in this thesis will be introduced. The first class of methods is omitted since they are rarely used in the power systems literature.

3.1. EXACT SOLUTION METHODS

3.1.1. Extreme Point Method

The EP method is proposed for robust optimization problems in which only robust feasibility is considered. In the case of the two-stage robust optimization, one can check if the second-stage problem is robustly feasible under the first-stage decision and find the uncertain scenario that makes the system infeasible.

To check feasibility, two positive slack variables, $\boldsymbol{s}^+$ and $\boldsymbol{s}^-$, are added to the slave problem and it is reformulated as follows:

$$\psi(\boldsymbol{y}) = \max_{\boldsymbol{\zeta}\in\Xi} \min_{\boldsymbol{x},\boldsymbol{s}^+,\boldsymbol{s}^-} \boldsymbol{s}^+ + \boldsymbol{s}^- \tag{3.1}$$

subject to:

$$\boldsymbol{Ey} + \boldsymbol{Gx}(\boldsymbol{\zeta}) \leq \boldsymbol{h} + \boldsymbol{M\zeta} + \boldsymbol{s}^- - \boldsymbol{s}^+ \tag{3.2}$$

$$\boldsymbol{s}^+, \boldsymbol{s}^- \geq 0, \qquad \boldsymbol{x} \in \mathbb{R}^{\mathrm{m}} \tag{3.3}$$

where $\Xi = \left\{ \boldsymbol{\zeta} \in \mathbb{R}^u : \underline{\boldsymbol{\zeta}} \leq \boldsymbol{\zeta} \leq \overline{\boldsymbol{\zeta}} \right\}$. In order to convert the problem (3.1)-(3.3) into single-level optimization, the dual of the inner minimization problem is taken, which yields to:

$$\psi(\boldsymbol{y}) = \max_{\boldsymbol{\zeta},\boldsymbol{\beta}} \boldsymbol{\beta}^T(\boldsymbol{Ey} - \boldsymbol{h}) - \boldsymbol{\beta}^T\boldsymbol{M\zeta} \tag{3.4}$$

subject to:

$$\boldsymbol{G}^T\boldsymbol{\beta} = 0 \tag{3.5}$$

$$0 \leq \boldsymbol{\beta} \leq 1 \tag{3.6}$$

$$\underline{\boldsymbol{\zeta}} \leq \boldsymbol{\zeta} \leq \overline{\boldsymbol{\zeta}} \tag{3.7}$$

where $\boldsymbol{\beta}$ represents the vector of dual variables. The problem above is a bilinear optimization problem because there is a product of terms in the objective function (3.4). It can be transformed into a MILP applying the reformulation-linearization technique introduced in [14] and described below.

Because uncertainties maximizing the slave problem (3.1)-(3.3) are extreme points of Ξ, they can be expressed by introducing binary variables z_j^+ and z_j^- as follows:

$$\zeta_j = \underline{\zeta_j}\, z_j^- + \overline{\zeta_j}\, z_j^+, \qquad j = 1{:}\, b \tag{3.8}$$

$$z_j^+ + z_j^- \le 1, \qquad j = 1{:}\, b \tag{3.9}$$

$$z_j^+, z_j^- \in \{0,1\}, \qquad j = 1{:}\, b \tag{3.10}$$

In the above formulation, constraints (3.8) implies that uncertainties occur at the extreme points. If $z_j^+ = 1$, then $\zeta_j = \overline{\zeta_j}$, which is the upper-bound of uncertainty. If $z_j^- = 1$, then $\zeta_j = \underline{\zeta_j}$, which is the lower-bound of uncertainty.

Defining auxiliary continuous variables $v_{ij}^+ = \beta_i z_j^+$ and $v_{ij}^- = \beta_i z_j^-$, where i is an index for dual variables, and using the equations (3.8)-(3.10), (3.4)-(3.7) can be equivalently written as follows:

$$\psi(\boldsymbol{y}) = \max_{\boldsymbol{\beta},\boldsymbol{z}^+,\boldsymbol{z}^-,\boldsymbol{v}^+,\boldsymbol{v}^-} \boldsymbol{\beta}^T(\boldsymbol{E}\boldsymbol{y} - \boldsymbol{h}) - \sum_i \sum_j m_{ij}(\underline{\zeta_j}\, v_{ij}^- + \overline{\zeta_j}\, v_{ij}^+) \tag{3.11}$$

subject to:

$$\boldsymbol{G}^T\boldsymbol{\beta} = 0 \tag{3.12}$$

$$v_{ij}^+ = \beta_i z_j^+, \qquad i = 1{:}\, a, j = 1{:}\, b \tag{3.13}$$

$$v_{ij}^- = \beta_i z_j^-, \qquad i = 1{:}\, a, j = 1{:}\, b \tag{3.14}$$

$$z_j^+ + z_j^- \le 1, \qquad j = 1{:}\, b \tag{3.15}$$

$$0 \le \boldsymbol{\beta} \le 1 \tag{3.16}$$

$$z_j^+, z_j^- \in \{0,1\}, \qquad j = 1{:}\, b \tag{3.17}$$

where m_{ij} represents the elements of the matrix $\boldsymbol{M}$. The problem above is still bilinear due to Constraints (3.13) and (3.14). They can be linearized, by adding new valid constraints that can be derived from McCormick envelopes [15]. The new problem takes the following form:

$$\psi(\boldsymbol{y}) = \max_{\boldsymbol{\beta},\boldsymbol{z}^+,\boldsymbol{z}^-,\boldsymbol{v}^+,\boldsymbol{v}^-} (\boldsymbol{E}\boldsymbol{y} - \boldsymbol{h})^T\boldsymbol{\beta} - \sum_i \sum_j m_{ij}(\underline{\zeta_j}\, v_{ij}^- + \overline{\zeta_j}\, v_{ij}^+) \tag{3.18}$$

subject to:

$$\boldsymbol{G}^T\boldsymbol{\beta} = 0 \tag{3.19}$$

$$z_j^+ + z_j^- \leq 1, \qquad j = 1{:}\,b \tag{3.20}$$

$$0 \leq \boldsymbol{\beta} \leq 1 \tag{3.21}$$

$$z_j^+, z_j^- \in \{0,1\}, \qquad j = 1{:}\,b \tag{3.22}$$

$$v_{ij}^+ \geq 0, \qquad i = 1{:}\,a, j = 1{:}\,b \tag{3.23}$$

$$v_{ij}^+ \geq z_j^+ + \beta_i - 1, \qquad i = 1{:}\,a, j = 1{:}\,b \tag{3.24}$$

$$v_{ij}^+ \leq z_j^+, \qquad i = 1{:}\,a, j = 1{:}\,b \tag{3.25}$$

$$v_{ij}^+ \leq \beta_i, \qquad i = 1{:}\,a, j = 1{:}\,b \tag{3.26}$$

$$v_{ij}^- \geq 0, \qquad i = 1{:}\,a, j = 1{:}\,b \tag{3.27}$$

$$v_{ij}^- \geq z_j^- + \beta_i - 1, \qquad i = 1{:}\,a, j = 1{:}\,b \tag{3.28}$$

$$v_{ij}^- \leq z_j^-, \qquad i = 1{:}\,a, j = 1{:}\,b \tag{3.29}$$

$$v_{ij}^- \leq \beta_i, \qquad i = 1{:}\,a, j = 1{:}\,b \tag{3.30}$$

It is easy to verify that, in above, constraints (3.23)-(3.26) ensure that v_{ij}^+ is equal to β_i if $z_j^+ = 1$ and it is zero otherwise. The same also applies to v_{ij}^-, z_j^- and β_i.

The problem (3.18)-(3.30) is a MILP and can be solved reliably using commercially available software packages. An important feature of this problem, owing to the bounds on $\boldsymbol{\beta}$ imposed by (3.21), the use of the big-M approach is avoided, which leads to numerical problems and reliability issues as described in the sequel.

3.1.2. Fortuny-Amat Formulation

Consider the max-min optimization slave problem introduced in Section 2, which is given below for convenience.

$$\phi(\boldsymbol{y}) = \max_{\boldsymbol{\zeta}\in\Xi} \min_{\boldsymbol{x}} \boldsymbol{b}^T\boldsymbol{x}(\boldsymbol{\zeta}) \tag{3.31}$$

subject to:

$$\boldsymbol{E}\boldsymbol{y} + \boldsymbol{G}\boldsymbol{x}(\boldsymbol{\zeta}) \leq \boldsymbol{h} + \mathbf{M}\boldsymbol{\zeta} \tag{3.32}$$

$$\boldsymbol{x} \in \mathbb{R}^{\mathrm{m}} \tag{3.33}$$

where $\Xi = \left\{ \boldsymbol{\zeta} \in \mathbb{R}^u : \underline{\boldsymbol{\zeta}} \leq \boldsymbol{\zeta} \leq \overline{\boldsymbol{\zeta}} \right\}$. A popular method to convert it into a MILP is the Fortuny-Amat method introduced in [16]. In this method, the inner minimization is replaced with KKT conditions, which are added as constraints to the outer maximization. This leads to the following single-level problem:

$$\phi(\boldsymbol{y}) = \max_{\boldsymbol{\zeta} \in \Xi, \boldsymbol{x}, \boldsymbol{\beta}} \boldsymbol{b}^T \boldsymbol{x}(\boldsymbol{\zeta}) \tag{3.34}$$

subject to:

$$\boldsymbol{E}\boldsymbol{y} + \boldsymbol{G}\boldsymbol{x}(\boldsymbol{\zeta}) \leq \boldsymbol{h} + \boldsymbol{M}\boldsymbol{\zeta} \tag{3.35}$$

$$\boldsymbol{G}^T \boldsymbol{\beta} = -\boldsymbol{b} \tag{3.36}$$

$$(\boldsymbol{E}\boldsymbol{y} + \boldsymbol{G}\boldsymbol{x}(\boldsymbol{\zeta}) - \boldsymbol{h} - \boldsymbol{M}\boldsymbol{\zeta})_i \beta_i = 0, \qquad i = 1{:}\, a \tag{3.37}$$

$$\boldsymbol{\beta} \geq 0 \tag{3.38}$$

$$\boldsymbol{x} \in \mathbb{R}^m \tag{3.39}$$

where $\Xi = \left\{ \boldsymbol{\zeta} \in \mathbb{R}^u : \underline{\boldsymbol{\zeta}} \leq \boldsymbol{\zeta} \leq \overline{\boldsymbol{\zeta}} \right\}$ and $\boldsymbol{\beta}$ represents the vector of dual variables of the problem (3.31)-(3.33). Constraints (3.37) are complementary slackness conditions, where i is the index of the dual variables and their corresponding constraints. To obtain a MILP, these constraints can be replaced with the following using the big-M technique by introducing binary variables v_i:

$$(-\boldsymbol{E}\boldsymbol{y} - \boldsymbol{G}\boldsymbol{x}(\boldsymbol{\zeta}) + \boldsymbol{h} + \boldsymbol{M}\boldsymbol{\zeta})_i \leq (1 - v_i)\, M_i^P, \qquad i = 1{:}\, a \tag{3.40}$$

$$\beta_i \leq v_i\, M_i^D, \qquad i = 1{:}\, a \tag{3.41}$$

$$v_i \in \{0,1\}, \qquad i = 1{:}\, a \tag{3.42}$$

where M_i^P, M_i^D are large enough constants. The new problem, which is a MILP, becomes:

$$\phi(\boldsymbol{y}) = \max_{\boldsymbol{\zeta}\in\Xi,\boldsymbol{x},\boldsymbol{\beta}} \boldsymbol{b}^T\boldsymbol{x}(\boldsymbol{\zeta}) \tag{3.43}$$

subject to:

$$\boldsymbol{Ey} + \boldsymbol{Gx}(\boldsymbol{\zeta}) \leq \boldsymbol{h} + \boldsymbol{M\zeta} \tag{3.44}$$

$$\boldsymbol{G}^T\boldsymbol{\beta} = -\boldsymbol{b} \tag{3.45}$$

$$(-\boldsymbol{Ey} - \boldsymbol{Gx}(\boldsymbol{\zeta}) + \boldsymbol{h} + \boldsymbol{M\zeta})_i \leq (1 - v_i)\, M_i^P, \qquad i = 1{:}\,a \tag{3.46}$$

$$\beta_i \leq v_i\, M_i^D, \qquad i = 1{:}\,a \tag{3.47}$$

$$v_i \in \{0,1\}, \qquad i = 1{:}\,a \tag{3.48}$$

$$\boldsymbol{\beta} \geq 0 \tag{3.49}$$

$$\boldsymbol{x} \in \mathbb{R}^m \tag{3.50}$$

where $\Xi = \left\{ \boldsymbol{\zeta} \in \mathbb{R}^u : \underline{\boldsymbol{\zeta}} \leq \boldsymbol{\zeta} \leq \overline{\boldsymbol{\zeta}} \right\}$. In the problem above, M_i^P and M_i^D are the upper bounds of the primal and dual variables, respectively. Selecting an appropriate value to M_i^P is usually easy because bounds on the primal variables can be obtained from physical limitations. However, it is a difficult task to find appropriate value to M_i^D since the upper limits of dual variables do not have any physical interpretation. This complication was discussed in [12] in the context of more general bilevel optimization problems. The main issue is that if one chooses values for M_i^D which are not big enough, then the optimal solution will be incorrect because (3.47) will be binding constraints which is not a part of the actual problem. In order to avoid this, if one picks too big values for M_i^D, then numerical problems may easily arise making the MILP solver to fail to find an optimal solution. This constitutes a fundamental difficulty in the solution of max-min problems using the Fortuny-Amat approach.

Another point that should be taken into account with the method described above is the validity of the KKT conditions (3.36)-(3.37). They are applicable if the max-min problem (3.31)-(3.33) has a feasible solution. Thus, in practice, before using this method, one may perform a feasibility check, which can be done using the EP approach introduced in the previous section.

3.2. APPROXIMATION METHODS

3.2.1. Mountain Climbing Method

Mountain climbing is a greedy approach to find a solution to the inner max-min problem. The method relies on iteratively solving two linear programs [9]. It is an efficient method but only guarantees local optimality. To apply the method, first, the inner max-min problem is converted into its single-level equivalent by taking the dual of the lower-level minimization problem. So, the problem (3.31)-(3.33) can be reformulated into its single-level maximization problem as the following:

$$\phi(\boldsymbol{y}) = \max_{\boldsymbol{\zeta}\in\Xi,\boldsymbol{\beta}} (\boldsymbol{Ey} - \boldsymbol{h})^T\boldsymbol{\beta} - (\boldsymbol{M\zeta})^T\boldsymbol{\beta} \tag{3.51}$$

subject to:

$$\boldsymbol{G}^T\boldsymbol{\beta} = -\boldsymbol{b} \tag{3.52}$$

$$\boldsymbol{\beta} \geq 0 \tag{3.53}$$

where $\Xi = \left\{ \boldsymbol{\zeta} \in \mathbb{R}^u : \underline{\boldsymbol{\zeta}} \leq \boldsymbol{\zeta} \leq \overline{\boldsymbol{\zeta}} \right\}$ and $\boldsymbol{\beta}$ represents the vector of dual variables of the lower-level minimization problem. Then, two linear programs obtained from the bilinear problem given above can be expressed as follows:

First Problem:

$$\phi^1(\boldsymbol{y},\boldsymbol{\zeta}) = \max_{\boldsymbol{\beta}} (\boldsymbol{Ey} - \boldsymbol{h} - \boldsymbol{M\zeta})^T\boldsymbol{\beta} \tag{3.54}$$

subject to:

$$\boldsymbol{G}^T\boldsymbol{\beta} = -\boldsymbol{b} \tag{3.55}$$

$$\boldsymbol{\beta} \geq 0 \tag{3.56}$$

Second Problem:

$$\phi^2(\boldsymbol{y},\boldsymbol{\beta}) = \max_{\boldsymbol{\zeta}\in\Xi} -\boldsymbol{\beta}^T\boldsymbol{M\zeta} + (\boldsymbol{Ey} - \boldsymbol{h})^T\boldsymbol{\beta} \tag{3.57}$$

The first LP starts with an initial guess $\boldsymbol{\zeta}$ and finds a dual variable vector $\boldsymbol{\beta}$ and gives it to the second LP. The second LP finds an uncertainty vector $\boldsymbol{\zeta}$ for the given $\boldsymbol{\beta}$ and produces a decision vector $\boldsymbol{\zeta}$ to the first LP. The procedure goes on until the method terminates. The pseudo-code of this algorithm is given in Algorithm 3.1.

Algorithm 3.1. Mountain Climbing method

1. Pick an extreme point $\boldsymbol{\zeta}$ in Ξ.
2. Solve the problem (3.54)-(3.56). If the problem (3.54)-(3.56) is infeasible, let $\phi^1(\boldsymbol{y},\boldsymbol{\zeta}) = -\infty$. Otherwise, keep the objective value $\phi^1(\boldsymbol{y},\boldsymbol{\zeta})$ and the optimal solution $\boldsymbol{\beta}$.
3. If $\phi^1(\boldsymbol{y},\boldsymbol{\zeta}) = -\infty$, let $\phi(\boldsymbol{y}) = +\infty$, return $\boldsymbol{\zeta}$ and terminate. Otherwise, Solve the problem (3.57) and keep the objective value $\phi^2(\boldsymbol{y},\boldsymbol{\beta})$ and optimal solution $\boldsymbol{\zeta}^*$.
4. If $\phi^2(\boldsymbol{y},\boldsymbol{\beta}) - \phi^1(\boldsymbol{y},\boldsymbol{\zeta}) > \varepsilon$, let $\boldsymbol{\zeta} = \boldsymbol{\zeta}^*$ and go to Step 2. Otherwise, return $\boldsymbol{\zeta}$, $\phi'(\boldsymbol{y}) = (\phi^1(\boldsymbol{y},\boldsymbol{\zeta}) + \phi^2(\boldsymbol{y},\boldsymbol{\beta}))/2$ and terminate.

The algorithm described above finds a value for the objective function $\phi(\boldsymbol{y})$ and the worst-case uncertainty $\boldsymbol{\zeta}$. Since it is a local optimization method, it does not guarantee to find a globally optimal solution. Hence, the solution found gives a lower bound on the optimal solution, in general. Note that this method also performs a feasibility check. But since this test is also based on local optimization, the problem can be incorrectly reported as feasible for infeasible instances. At the first step of the algorithm, one has to start with an initial guess of uncertainty. When there is no other information, one can choose a special extreme point such as all coordinates taking maximum or minimum values or a randomly generated extreme point. These strategies will be tested in Sections 5.1.2, 5.2.2, and 5.3.2.

3.2.2. Outer Approximation Method

The outer approximation is another method that can find a locally optimal solution to solve the inner max-min problem. In order to apply the method, first, the dual of the lower-level minimization problem is taken and it can be given as the following:

$$\tau(\boldsymbol{y},\boldsymbol{\zeta}) = \max_{\boldsymbol{\beta}}(\boldsymbol{E}\boldsymbol{y} - \boldsymbol{h})^T\boldsymbol{\beta} - (\boldsymbol{M}\boldsymbol{\zeta})^T\boldsymbol{\beta} \tag{3.58}$$

subject to:

$$\boldsymbol{G}^T\boldsymbol{\beta} = -\boldsymbol{b} \tag{3.59}$$

$$\boldsymbol{\beta} \geq 0 \tag{3.60}$$

where $\boldsymbol{\beta}$ represents the vector of dual variables. Now the inner max-min problem (3.31-3.33) is equivalent to a BLP as follows:

$$\phi(\boldsymbol{y}) = \max_{\boldsymbol{\zeta} \in \Xi, \boldsymbol{\beta}} (\boldsymbol{E}\boldsymbol{y} - \boldsymbol{h})^T \boldsymbol{\beta} - (\boldsymbol{M}\boldsymbol{\zeta})^T \boldsymbol{\beta} \tag{3.61}$$

subject to:

$$\boldsymbol{G}^T \boldsymbol{\beta} = -\boldsymbol{b} \tag{3.62}$$

$$\boldsymbol{\beta} \geq 0 \tag{3.63}$$

where $\Xi = \left\{ \boldsymbol{\zeta} \in \mathbb{R}^u : \underline{\boldsymbol{\zeta}} \leq \boldsymbol{\zeta} \leq \overline{\boldsymbol{\zeta}} \right\}$. The method is used to solve the above BLP, where the linear approximations of the bilinear term in the objective function are added to the OA formulation and the bilinear term is linearized around intermediate solution points [8]. A pseudo-code of the OA method is given below:

Algorithm 3.2. Outer Approximation method

1. Pick an extreme point $\boldsymbol{\zeta}_1$ in Z. Set $LB = -\infty$, $UB = +\infty$, $j = 1$.
2. Solve the problem $\tau(\boldsymbol{y}, \boldsymbol{\zeta}_j)$ defined by (3.58)-(3.60). Let $\boldsymbol{\beta}_j$ be the optimal solution. Set $LB = \tau(\boldsymbol{y}, \boldsymbol{\zeta}_j)$. Define $L_j(\boldsymbol{M\zeta}, \boldsymbol{\beta})$, the linearization of the bilinear term $(\boldsymbol{M\zeta})^T\boldsymbol{\beta}$ at $(\boldsymbol{M\zeta}_j, \boldsymbol{\beta}_j)$, as follows:
   $L_j(\boldsymbol{M\zeta}, \boldsymbol{\beta}) = (\boldsymbol{M\zeta}_j)^T\boldsymbol{\beta}_j + \left(\boldsymbol{\beta} - \boldsymbol{\beta}_j\right)^T(\boldsymbol{M\zeta}_j) + \left(\boldsymbol{M\zeta} - \boldsymbol{M\zeta}_j\right)^T\boldsymbol{\beta}_j$.
3. Check if $UB - LB < \varepsilon$, then terminate and return the current solution. Otherwise, set $j = j + 1$ and go to Step 4.
4. Solve the linearized version of the problem $\phi(\boldsymbol{y})$, defined as follows:

$$U\left(\boldsymbol{y}, \boldsymbol{\zeta}_j, \boldsymbol{\beta}_j\right) = \max_{\zeta \in \Xi, \boldsymbol{\beta}, \lambda} (\boldsymbol{Ey} - \boldsymbol{h})^T\boldsymbol{\beta} - (\boldsymbol{M\zeta})^T\boldsymbol{\beta} + \lambda \tag{3.64}$$

subject to:

$$\lambda \leq L_i(\boldsymbol{M\zeta}, \boldsymbol{\beta}), \quad \forall i = 1, \dots, j \tag{3.65}$$

$$\boldsymbol{G}^T\boldsymbol{\beta} = -\boldsymbol{b} \tag{3.66}$$

$$\boldsymbol{\beta} \geq 0 \tag{3.67}$$

$$\Xi = \left\{ \boldsymbol{\zeta} \in \mathbb{R}^u : \underline{\boldsymbol{\zeta}} \leq \boldsymbol{\zeta} \leq \overline{\boldsymbol{\zeta}} \right\} \tag{3.68}$$

5. Denote $(\boldsymbol{\zeta}_{j+1}, \boldsymbol{\beta}_{j+1}, \lambda_{j+1})$ as the optimal solution. Set the $UB = U\left(\boldsymbol{y}, \boldsymbol{\zeta}_j, \boldsymbol{\beta}_j\right)$.

The OA method guarantees the local optimum since the problem (3.61)-(3.63) is nonconcave [8].

3.3. METHODS PROPOSED IN THIS THESIS

Two new methods are proposed in this thesis for the solution of the max-min slave problem. The first one combines EP and MC algorithms to obtain a new local optimization method that can give better approximations and guarantee robust feasibility. The second one is an interval partitioning based method that can find a globally optimal solution to the max-min optimization problem by employing off-the-shelf solvers and avoiding the use of the big-M technique.

3.3.1. A Hybrid Optimization Method

EP method introduced in Section 3.1.1 can be used to check the feasibility of the max-min problem reliably because it does not make use of the big-M technique owing to the bounds on the dual variables. But it does not solve the optimality problem (2.48)-(2.50). On the other hand, the MC method can find a locally optimal solution to the max-min problem but the global optimality will depend on the initial guess which is chosen at Step 1 of Algorithm 3.1. As described in Section 3.2.1, when there is no other information, the initial guess can be taken as a randomly chosen extreme point or a special extreme point such as all maximum or all minimum.

The proposed hybrid optimization algorithm combines EP and MC methods. It first applies the EP to check feasibility. If the problem is feasible, its solution is used to obtain an initial guess for the MC. In this way, the MC algorithm can start from a better guess than the ones mentioned above. This proposed hybrid method is given in Algorithm 3.3.

Algorithm 3.3. Hybrid optimization method

1. Check the feasibility of the problem (2.48)-(2.50) with the EP method.
2. If the problem is infeasible, return $\boldsymbol{\zeta}^*$ and terminate.
3. Otherwise, keep $\boldsymbol{\zeta}^*$. Solve the problem (2.48)-(2.50) with MC method using initial guess as $\boldsymbol{\zeta}^*$.
4. Return $\phi'(\boldsymbol{y})$, $\boldsymbol{\zeta}$, and terminate.

3.3.2. An Interval Partitioning Method

The second proposed method can find the global optimal solution of the max-min problem. It uses the EP method to find iteratively improving intervals that contain the optimal objective value. The lengths of these intervals are reduced by half-width at each iteration. Hence they converge to the optimal solution quickly after a reasonable number of steps.

One needs an initial interval for this method. For the upper bound of the interval, the inner-minimization of the original max-min problem is turned into maximization. This new problem is given in (3.69)-(3.72) and since the objective is replaced with a greater objective, the solution of this problem gives an upper-bound. As for the lower bound, the outer-maximization of the original max-min problem is turned into minimization. This problem is given in (3.73)-(3.76) and since the objective is replaced with a smaller objective, the solution of this problem gives a lower-bound.

$$\overline{\phi}(\boldsymbol{y}) = \max_{\zeta,x} \boldsymbol{b}^T \boldsymbol{x}(\boldsymbol{\zeta}) \tag{3.69}$$

subject to:

$$\boldsymbol{E}\boldsymbol{y} + \boldsymbol{G}\boldsymbol{x}(\boldsymbol{\zeta}) \leq \boldsymbol{h} + \boldsymbol{M}\boldsymbol{\zeta} \tag{3.70}$$

$$\boldsymbol{x} \in \mathbb{R}^m \tag{3.71}$$

$$\underline{\boldsymbol{\zeta}} \leq \boldsymbol{\zeta} \leq \overline{\boldsymbol{\zeta}} \tag{3.72}$$

$$\underline{\phi}(\boldsymbol{y}) = \min_{\zeta,x} \boldsymbol{b}^T \boldsymbol{x}(\boldsymbol{\zeta}) \tag{3.73}$$

subject to:

$$\boldsymbol{E}\boldsymbol{y} + \boldsymbol{G}\boldsymbol{x}(\boldsymbol{\zeta}) \leq \boldsymbol{h} + \boldsymbol{M}\boldsymbol{\zeta} \tag{3.74}$$

$$\boldsymbol{x} \in \mathbb{R}^m \tag{3.75}$$

$$\underline{\boldsymbol{\zeta}} \leq \boldsymbol{\zeta} \leq \overline{\boldsymbol{\zeta}} \tag{3.76}$$

It is hard to solve the inner max-min optimization problem exactly. Thus, we iteratively convert this max-min problem into feasibility problems. Feasibility problems are obtained as follows. The objective function of the max-min problem is put as a constraint to the problem by setting its right-hand side as e. If the problem is feasible this means that e is greater than the optimal objective value. Otherwise, e is less than the optimal objective value. The feasibility problem defined is as follows:

$$\omega(\boldsymbol{y}) = \max_{\zeta \in \Xi} \min_{x,s^+,s^-} \boldsymbol{s}^+ + \boldsymbol{s}^- \tag{3.77}$$

subject to:

$$\boldsymbol{E}\boldsymbol{y} + \boldsymbol{G}\boldsymbol{x}(\boldsymbol{\zeta}) \leq \boldsymbol{h} + \boldsymbol{M}\boldsymbol{\zeta} + \boldsymbol{s}^- - \boldsymbol{s}^+ \tag{3.78}$$

$$\boldsymbol{b}^T \boldsymbol{x}(\boldsymbol{\zeta}) \leq e \tag{3.79}$$

$$\boldsymbol{s}^{+}, \boldsymbol{s}^{-} \geq 0, \qquad \boldsymbol{x} \in \mathbb{R}^{\mathrm{m}} \tag{3.80}$$

Algorithm 3.4. Interval partitioning method

1. Set $UB = \overline{\phi}(\boldsymbol{y})$ and $LB = \underline{\phi}(\boldsymbol{y})$.
2. If $UB - LB < \varepsilon$, return $\boldsymbol{\zeta}^*$ and terminate. Otherwise, go to Step 3.
3. Set $e = \frac{UB+LB}{2}$ and solve the problem (3.77)-(3.80).
4. If $\omega(\boldsymbol{y}) > \varepsilon$, set $LB = e$. Otherwise, set $UB = e$ and go to Step 2.

In the algorithm above, in Step 1 UB and LB are obtained by solving problems (3.69)-(3.72) and (3.73)-(3.76), respectively. In Step 2, the difference between UB and LB is calculated and if it is under a certain threshold value, ε, the algorithm terminates and the optimal solution is returned. If it is not, the algorithm moves to Step 3. In Step 3, first, e is calculated as the midpoint of the interval the feasibility problem given in (3.77)-(3.80), is solved with the EP method. If $\omega(\boldsymbol{y}) > \varepsilon$, then the original objective value of the max-min problem is greater than e, thus LB is updated as e. Otherwise if $\omega(\boldsymbol{y}) = 0$, then the optimal objective value of the max-min problem is less than e, therefore UB is updated and the algorithm goes to Step 2.

4. ENERGY STORAGE PLANNING PROBLEM AND ITS FORMULATION

In this thesis, we study the sitting of energy storage devices in power networks. This problem is modeled as a two-stage stochastic robust optimization problem and solved using the method described in Section 2.3 that combines DE for the stochastic part with CCG for the robust part. For the second stage, the proposed exact and hybrid optimization methods are employed and compared with the other alternative described in Section 3.1.3. In below, first, a mathematical description of the problem is given and then, the explicit and abstract formulations of the two-stage stochastic robust optimization model are presented.

4.1. PROBLEM DESCRIPTION

The aim of the problem is to decide the locations of the energy storage units to be built in a power network such that installation and operation costs are minimized.

Such a system is composed of the following components. There are buses to which several components of power networks such as conventional generators, energy storage devices, and loads are connected. The buses are linked to each other with lines that transfer limited amounts of power.

Conventional thermal generators use non-renewable energy sources like fossil fuels; coal, natural gas, or nuclear power. Thermal generators have limited production capacity. Besides, they have hourly ramping-limits meaning that there is a bound on the change in the power produced between consecutive hours.

Renewable energy generators use renewable energy sources like solar energy, wind, falling water, geothermal, biomass, and tidal energy. Wind generators are used commonly to produce power and in this thesis, we also use wind generators as renewable sources.

Energy storage systems are the units that can store electric energy in different forms. They can be used to reduce line congestions, mitigate voltage deviations, regulate the system frequency, perform load shifting, shave energy peaks, and facilitate the integration of renewable energy sources [17]. There are different kinds of storage devices based on the technology used. Some examples are superconducting magnetic energy storage, flywheel energy storage, battery energy storage, compressed air energy storage, and pumped hydro energy storage devices [18]. Mathematically, they can be modeled as energy inventories having physical limits on the amount of energy stored and charge/discharge power.

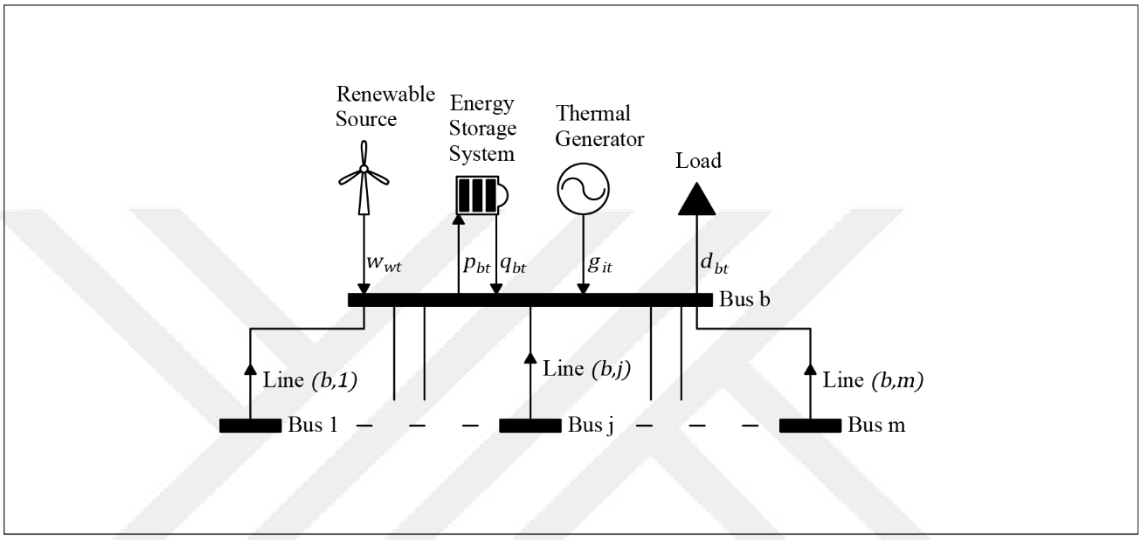

Figure 4.1. M-bus power network

Figure 4.1 is a representation of m-bus power network. The components that are explained above and lines from bus b to other buses are depicted in the figure. In general, there can be connections from any bus to any other. In order to develop a mathematical model of the storage sitting problem, one needs to have constraints associated with the components of the power network summarized above.

- *Bus Energy Balance Constraints:*

$$\sum_{i\in\Lambda_b} g_{it} + q_{bt} - p_{bt} + \sum_{j\in\Omega_b} B_{bj}(\theta_{jt} - \theta_{bt}) = d_{bt} - \sum_{w\in\mathcal{W}_b} w_{wt}\,, \quad t = 1{:}T \tag{4.1}$$

Constraints (4.1) maintains energy balance on a bus. It ensures that the total power generation of thermal generators at bus b, plus energy discharged from storage unit minus energy charged into storage plus power coming from other buses linked to bus b is equal to power demand minus total wind generation of wind farms at that bus.

- *Storage Energy Balance Constraints:*

$$E_{bt} = E_{b(t-1)} + p_{bt} - q_{bt}, \qquad t = 1{:}T \tag{4.2}$$

Constraints representing the dynamics of energy storage systems are given in equation (4.2). As can be seen, in this equation energy level in storage in time t is given by energy stored in the device in time $(t-1)$ plus charge rate minus discharge rate.

- *Line Flow Capacity Constraints:*

$$B_{bj}\left(\theta_{bt} - \theta_{jt}\right) \le L_{bj}, \qquad \forall j \in \Omega_b, t = 1{:}T \tag{4.3}$$

The amount of power that a line can transfer is limited. This is given in constraints (4.3). Power flowing on the line is equal to susceptance of a line times phase difference between buses that are linked by that line.

- *Generator Capacity Constraints*

$$\underline{PG_i} \le g_{it} \le \overline{PG_i}, \qquad \forall i \in \Lambda, t = 1{:}T \tag{4.4}$$

Power generation by a thermal generator is limited, lying between and upper and lower limit.

- *Ramping Capacity Constraints:*

$$-RR_i \le g_{it} - g_{i(t-1)} \le RR_i, \qquad \forall i \in \Lambda, t = 1{:}T \tag{4.5}$$

These constraints ensure that change in the power produced by a thermal unit in consecutive time instances is bounded below and above.

- *Storage Energy Capacity Constraints*

$$\underline{E_b} \le E_{bt} \le \overline{E_b}, \qquad t = 1{:}T \tag{4.6}$$

The amount of energy that a storage unit can store is limited. constraints (4.6) ensure that energy stored in the storage at bus b is smaller than maximum energy capacity and greater than minimum energy capacity of the unit

- *Storage Charging Capacity Constraints*

$$\underline{p_b} \leq p_{bt} \leq \overline{p}_b, \qquad t = 1{:}T \tag{4.7}$$

Constraints (4.7) ensure that energy charged to the ESS at bus b is limited.

- *Storage Discharging Capacity Constraints*

$$\underline{q_b} \leq q_{bt} \leq \overline{q_b}, \qquad t = 1{:}T \tag{4.8}$$

Constraints (4.8) guarantee that energy discharged from the ESS at bus b is limited.

4.2. FORMULATION AS A TWO-STAGE STOCHASTIC ROBUST OPTIMIZATION PROBLEM

In this study, we build a two-stage stochastic robust programming model for the sitting of ESSs on power networks. The power network is composed of buses connected with transmission lines. Different components can be connected to each bus including, conventional thermal generators, energy storage units, wind generators as renewable energy sources, and loads. The mathematical model of the problem solved and its nomenclature are given below:

Sets:

$\mathcal{B}$: index set of all buses
Ω_b: index set of buses linked to bus b
Λ: index set of all thermal generators
Λ_b: index set of thermal generators connected to bus b
$\mathcal{W}_b$: index set of wind farms at bus b
$\mathcal{S}$: index set of all scenarios

Indexes:

b, j: indexes of buses

i: index of generators

t: index of time

w: index of wind farms

s: index of scenarios

Parameters:

T: time horizon

SC_b: storage investment cost at bus b

CG_i: generation cost of thermal generator i

B_{jb}: susceptance of the line between bus j and bus b

w_{wts}: wind power of wind farm w in time t and for scenario s

d_{bt}^n: distributed nominal demand at bus b in time t

w_{wt}^n: nominal wind power of wind farm w in time t

$\overline{\Delta w}_{wt}$: upper-bound on wind deviation of wind farm w in time t

$\underline{\Delta w}_{wt}$: lower-bound on wind deviation of wind farm w in time t

L_{jb}: flow limit of the line between bus j and bus b

$\overline{PG_i}$: maximum power generation of thermal generator i

$\underline{PG_i}$: minimum power generation of thermal generator i

RR_i: ramp-rate limit of thermal generator i

$\overline{E_b}$: maximum energy capacity of storage unit can be built at bus b

$\underline{E_b}$: minimum energy capacity of storage unit can be built at bus b

$\overline{p_b}$: maximum charging capacity of storage unit can be built at bus b

$\underline{p_b}$: minimum charging capacity of storage unit can be built at bus b

$\overline{q_b}$: maximum discharging capacity of storage unit can be built at bus b

$\underline{q_b}$: minimum discharging capacity of storage unit can be built at bus b

E_{b0s}: initial energy stored in storage unit can be built at bus b for scenario s

g_{i0s}: initial electricity generation of thermal generator i for scenario s

E_{b0}: initial energy stored in storage unit can be built at bus b

g_{i0}: initial electricity generation of thermal generator i

k: contribution weight of the stochastic and robust parts in the objective function

Decision Variables:

α_b: binary variable equal to one if storage is opened at bus b and equal to zero otherwise

g'_{its}: power generation by thermal generator in time t and for scenario s

θ'_{bts}: phase angle of bus b in time t and for scenario s

E'_{bts}: energy stored in the storage unit at bus b in time t and for scenario s

p'_{bts}: power charged into the storage unit at bus b in time t and for scenario s

q'_{bts}: power discharged from the storage unit at bus b in time t and for scenario s

g''_{it}: power generation by thermal generator i in time t

θ''_{bt}: phase angle of bus b in time t

E''_{bt}: energy stored in the storage unit at bus b in time t

p''_{bt}: power charged into the storage unit at bus b in time t

q''_{bt}: power discharged from the storage unit at bus b in time t

Δw_{wt}: wind power deviation of wind farm w in time t

$$\min_{\boldsymbol{\alpha}} \sum_{b\in\mathcal{B}} \alpha_b SC_b + k\frac{1}{N}\min_{\mathbf{g}'_s} \sum_{t=1}^{T}\sum_{i\in\Lambda}\sum_{s\in\mathcal{S}} g'_{its}CG_i + (1-k) \max_{\underline{\Delta w}\leq\Delta\mathbf{w}\leq\overline{\Delta w}} \min_{\mathbf{g}''} \sum_{t=1}^{T}\sum_{i\in\Lambda} g''_{it}CG_i \tag{4.9}$$

subject to:

Stochastic Constraints:

$$\sum_{i\in\Lambda_b} g'_{its} + q'_{bts} - p'_{bts} + \sum_{j\in\Omega_b} B_{jb}\left(\theta'_{jts} - \theta'_{bts}\right) = d^n_{bt} - \sum_{w\in\mathcal{W}_b} w_{wts}\,, \quad \forall b\in\mathcal{B}, \forall s\in\mathcal{S}, t=1{:}T \tag{4.10}$$

$$E'_{bts} = E'_{b(t-1)s} + p'_{bts} - q'_{bts}\,, \qquad \forall b\in\mathcal{B}, \forall s\in\mathcal{S}, t=1{:}T \tag{4.11}$$

$$B_{jb}\left(\theta'_{bts} - \theta'_{jts}\right) \leq L_{jb}, \qquad \forall j\in\Omega_b, \forall b\in\mathcal{B}, \forall s\in\mathcal{S}, t=1{:}T \tag{4.12}$$

$$\underline{PG_i} \leq g'_{its} \leq \overline{PG_i}, \qquad \forall i\in\Lambda, \forall s\in\mathcal{S}, t=1{:}T \tag{4.13}$$

$$-RR_i \leq g'_{its} - g'_{i(t-1)s} \leq RR_i, \qquad \forall i\in\Lambda, \forall s\in\mathcal{S}, t=1{:}T \tag{4.14}$$

$$\underline{E_b}\alpha_b \leq E'_{bts} \leq \overline{E_b}\alpha_b, \qquad \forall b\in\mathcal{B}, \forall s\in\mathcal{S}, t=1{:}T \tag{4.15}$$

$$\underline{p_b} \leq p'_{bts} \leq \overline{p_b}, \qquad \forall b\in\mathcal{B}, \forall s\in\mathcal{S}, t=1{:}T \tag{4.16}$$

$$\underline{q_b} \leq q'_{bts} \leq \overline{q_b}, \qquad \forall b\in\mathcal{B}, \forall s\in\mathcal{S}, t=1{:}T \tag{4.17}$$

Robust Constraints:

$$\sum_{i\in\Lambda_b} g''_{it} + q''_{bt} - p''_{bt} + \sum\nolimits_{j\in\Omega_b} B_{jb}\left(\theta''_{jt} - \theta''_{bt}\right) = d^n_{bt} - \sum_{w\in\mathcal{W}_b} w^n_{wt} + \Delta w_{wt}, \quad \forall b \in \mathcal{B}, t = 1{:}T \tag{4.18}$$

$$E''_{bt} = E''_{b(t-1)} + p''_{bt} - q''_{bt}, \quad \forall b \in \mathcal{B}, t = 1{:}T \tag{4.19}$$

$$B_{jb}\left(\theta''_{bt} - \theta''_{jt}\right) \leq L_{jb}, \quad \forall j \in \Omega_b, \forall b \in \mathcal{B}, t = 1{:}T \tag{4.20}$$

$$\underline{PG_i} \leq g''_{it} \leq \overline{PG_i}, \quad \forall i \in \Lambda, t = 1{:}T \tag{4.21}$$

$$-RR_i \leq g''_{it} - g''_{i(t-1)} \leq RR_i, \quad \forall i \in \Lambda, t = 1{:}T \tag{4.22}$$

$$\underline{E_b}\alpha_b \leq E''_{bt} \leq \overline{E_b}\alpha_b, \quad \forall b \in \mathcal{B}, t = 1{:}T \tag{4.23}$$

$$\underline{p_b} \leq p''_{bt} \leq \overline{p_b}, \quad \forall b \in \mathcal{B}, t = 1{:}T \tag{4.24}$$

$$\underline{q_b} \leq q''_{bt} \leq \overline{q_b}, \quad \forall b \in \mathcal{B}, t = 1{:}T \tag{4.25}$$

$$\alpha_b \in \{0,1\}, \forall b \in \mathcal{B} \tag{4.26}$$

where $N = |\mathcal{S}|$. The objective function (4.19) is composed of three parts. The first part is the minimization of the total investment cost of storage devices based on first-stage decisions α_b. The second part is the minimization of the second stage expected economic dispatch cost for the stochastic optimization part. The third part is the minimization of the second stage worst-case economic dispatch cost for the robust part. The constraints are given for the stochastic and robust parts separately as described below:

Constraints (4.10)-(4.17) are for the stochastic optimization part and they are defined for all scenarios. Constraints (4.10) are the bus energy balance constraints. Constraints (4.11) are storage energy balance constraints. The line flow capacity constraints are given in (4.12). Constraints (4.13) are the generation capacity constraints of thermal generators. The generation ramping capacities are given in the constraints (4.14). Constraints (4.15) are storage energy capacity constraints. As can be seen α_b determines if a storage system is installed or not. If α_b is zero, the energy level of the storage will be enforced to be zero implying that no storage device will be installed to bus b. If α_b is one there will be a single storage device installed on the bus. The charging and discharging capacities of the storage units are given in the constraints (4.16) and (4.17), respectively.

Constraints (4.18)-(4.25) are for the robust optimization part. Constraints (4.18) are energy balance equations of the buses. Constraints (4.19) are storage energy balance constraints. Line power flow capacities are enforced by (4.20). Constraints (4.21) are for generation capacities of the thermal power plants. Ramp-rate limits are given in (4.22). Constraints (4.23) are storage energy capacity constraints. Constraints (4.24) and (4.25) ensure the charging/discharging capacities of ESSs are not exceeded. Constraints (4.26) ensure that the locationing decisions of the storage systems α_b are binary.

The abstract formulation of the model given above can be expressed as follows:

$$\min_{\mathbf{y}} \boldsymbol{c}^T \boldsymbol{y} + \alpha \frac{1}{N} \sum_{s=1}^{N} \boldsymbol{b}^T \boldsymbol{x}(\boldsymbol{\xi}^s) + (1-\alpha) \max_{\underline{\zeta} \leq \zeta \leq \bar{\zeta}} \min_{\mathbf{x}} \boldsymbol{b}^T \boldsymbol{x}(\boldsymbol{\zeta}) \tag{4.27}$$

subject to:

Stochastic Constraints:

$$\boldsymbol{E}\boldsymbol{y} + \boldsymbol{G}\boldsymbol{x}(\boldsymbol{\xi}^s) \leq \boldsymbol{h} + \boldsymbol{M}\boldsymbol{\xi}^s, \qquad s = 1{:}N \tag{4.28}$$

Robust Constraints:

$$\boldsymbol{E}\boldsymbol{y} + \boldsymbol{G}\boldsymbol{x}(\boldsymbol{\zeta}) \leq \boldsymbol{h} + \boldsymbol{M}\boldsymbol{\zeta} \tag{4.29}$$

$$\boldsymbol{x} \in \mathbb{R}^m, \qquad \boldsymbol{y} \subset \mathbb{R}^n \tag{4.30}$$

In the above formulation, $\mathbf{y}$ represents the vector of binary decision variables $\boldsymbol{\alpha}$, $\boldsymbol{x}(\boldsymbol{\xi})$ represents the vector of continuous decision variables $\boldsymbol{g}'_s$ of the stochastic part, $\boldsymbol{x}(\boldsymbol{\zeta})$ represents the vector of continuous decision variables $\boldsymbol{g}''$ of the robust part and $\boldsymbol{\zeta}$ represents the vector of uncertain decision variables $\boldsymbol{\Delta w}$. Equation (4.27) corresponds to the objective function (4.9). For the stochastic part, constraints (4.28) correspond to constraints (4.10)-(4.17). For the robust part, constraints (4.29) correspond to constraints (4.18)-(4.25).

5. NUMERICAL EXPERIMENTS

In this section, the proposed methods and alternatives are tested on three different bus network systems. The systems considered are; 6-bus system [19], IEEE 14-bus system, and IEEE 30-bus system [20]. The details of these systems and the results of numerical analysis are presented in the next sections.

For these power network systems, the SRO problem formulated in Section 4.2. is solved using the CCG algorithm introduced in Section 2.3. The inner max-min problem is solved using the two methods proposed in this thesis and the MC approach described in Section 3.2.1. The Fortuny-Amat and OA methods are not considered. The former is not considered because it leads to numerical issues and initial experiments carried out show that this method takes a too long time to finish for the problems considered. The OA is not also considered because it is a local optimization method as the MC, and hence, it is expected to produce similar results.

For each bus system, two sets of experiments are carried out. In the first one, the interval partitioning based exact solution method is used and the positioning of storage devices are analyzed under different conditions obtained by changing problem parameters. In the second one, a larger set of parameters are considered and max-min optimization methods considered are compared in terms of optimality and computation times. In the experiments, the MC method is run repeatedly for seven different initial guesses. Five of them are generated by choosing extreme points of uncertainties randomly while in two of them maximum and minimum values are taken as initial guesses.

All numerical analyses are carried out using CPLEX 12.8.0 on a computer having an i7-8550U CPU @ 1.80 GHz processor and a 16.0 GB RAM. The optimality gap tolerance ε used in all algorithms is set to 10^{-3}.

The demand profiles used in the numeric analyses are common for all test systems. There are two types; 24-hour profile and 12-hour profile. These profiles are generated by making use of data provided in Jabr et al. [21]. 24-hour and the 12-hour profiles are given in Figure 5.1 and Figure 5.2, respectively.

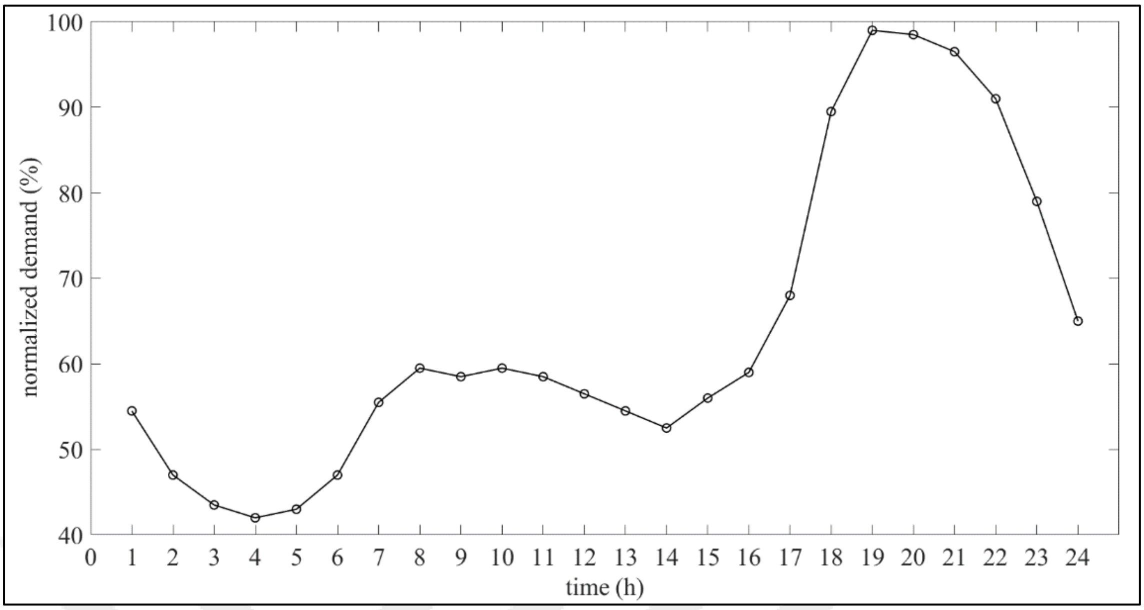

Figure 5.1. 24-hour demand profile

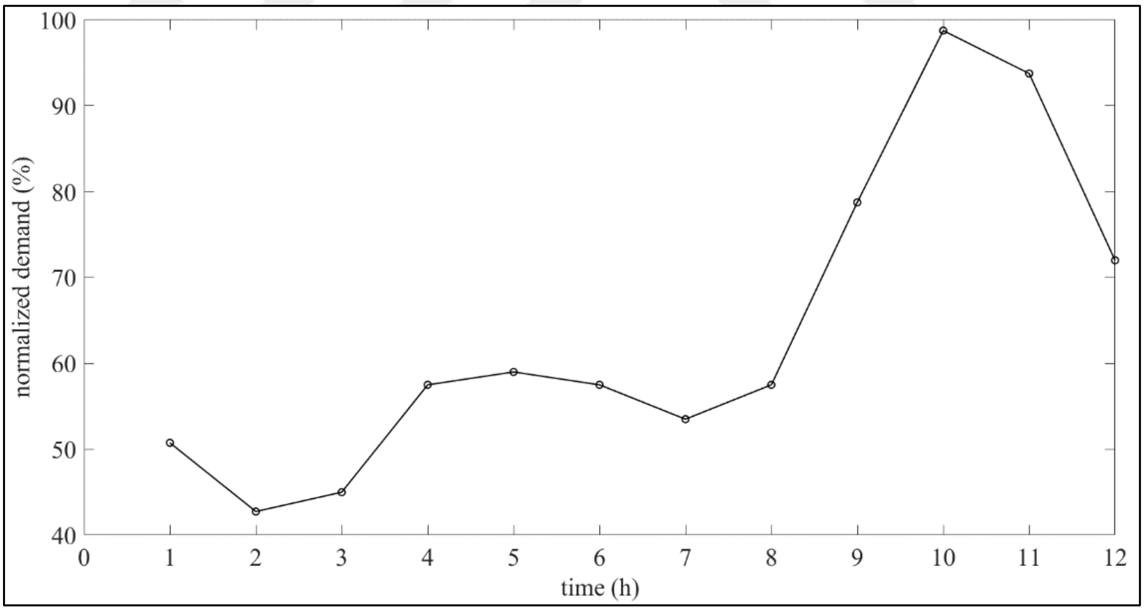

Figure 5.2. 12-hour demand profile

Values of the 12-hour profile are obtained by taking averages of values in the 24-hour profile in consecutive time instances. As can be seen from the figures, values are between zero and

100. For every system, these percent values are multiplied with a maximum demand value to obtain nominal demands.

The normalized values depicted in Figure 5.3 and Figure 5.4, which are taken from [22], are used for obtaining nominal wind generations. Similar to demand data, percent values are multiplied with a maximum wind power value to generate nominal wind values.

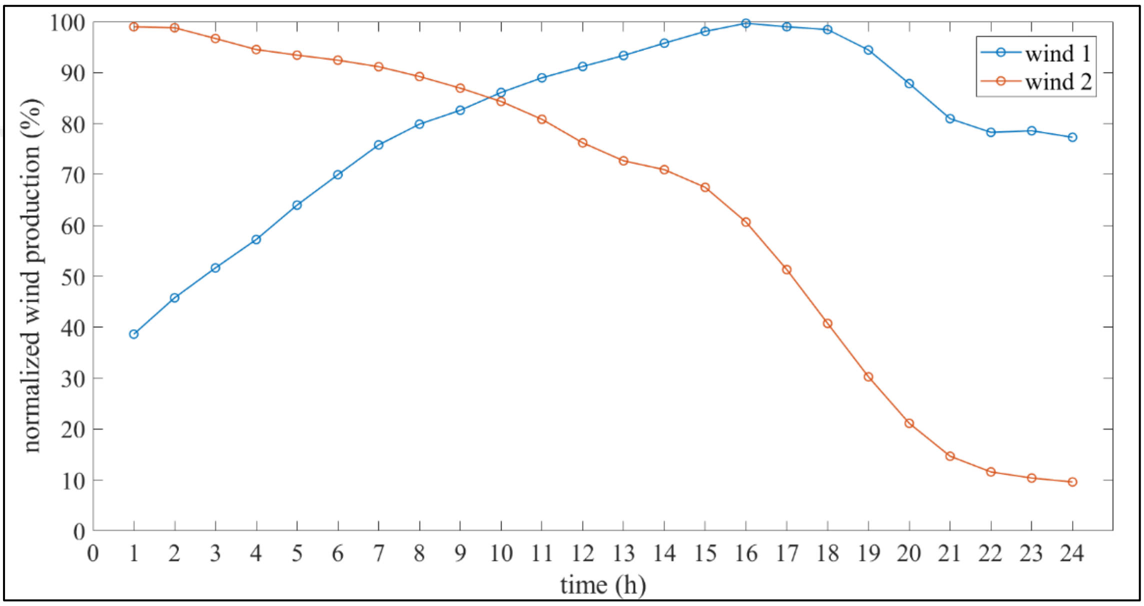

Figure 5.3. 24-hour wind profiles

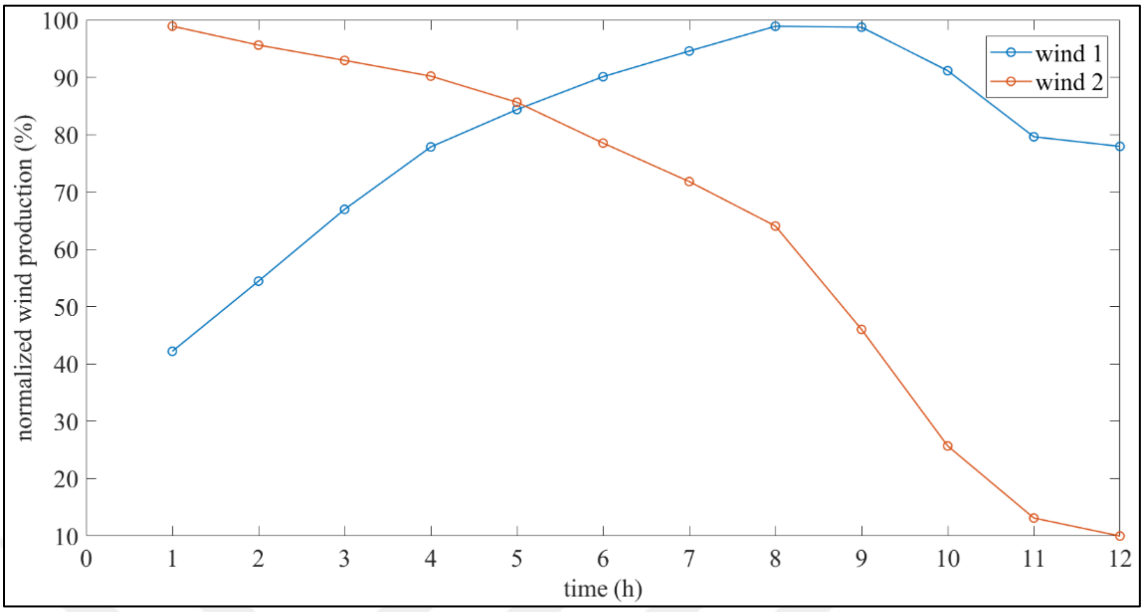

Figure 5.4. 12-hour wind profiles

Stochastic wind scenarios are produced by adding Gaussian noise to the nominal wind generation at each hour. The standard deviation of the noise is chosen as the one-third of the uncertainty interval length used for the robust part. This will ensure that stochastic scenario values will fall into the RO intervals with almost probability one.

5.1. 6-BUS SYSTEM

This test system includes six buses. There are three generators, 11 lines, and three demand points. The generators are located at buses 1, 2, and 3 and the demands are at buses 4,5, and 6. Parameters of generators and lines are given in Table 5.1 and Table 5.2, respectively.

Table 5.1. Parameters of generators for the 6-bus system

Generator	$\overline{PG_i}$ **(MW)**	CG_i **($/MW)**
1	200	11.67
2	150	10.33
3	180	10.83

The minimum generation capacity is set to zero for all generators. Ramp-rate limits are calculated by multiplying maximum generation capacity with a scaling factor. The details will be given in the next section.

Table 5.2. Parameters of lines for the 6-bus system

Line	**From**	**To**	L_{jb} **(MW)**	B_{jb}
1	1	2	40	0.04
2	1	4	60	0.04
3	1	5	40	0.06
4	2	3	40	0.06
5	2	4	60	0.02
6	2	5	30	0.04
7	2	6	90	0.05
8	3	5	70	0.05
9	3	6	80	0.02
10	4	5	20	0.08
11	5	6	40	0.06

Tests on the 6-bus system are carried out for 24 hours using the normalized demand profile depicted in Figure 5.1. The system is tested for different maximum demand values. Total nominal demand values of different test cases are obtained by multiplying the normalized profile with different maximum values. Total nominal demands are distributed to buses proportional to the demands provided in the data file of the bus system. We use this approach in all test systems and numerical analysis.

The original 6-bus system is modified by adding two find farms of 40 MW capacity to buses 1 and 3. The modified system is depicted in Figure 5.5.

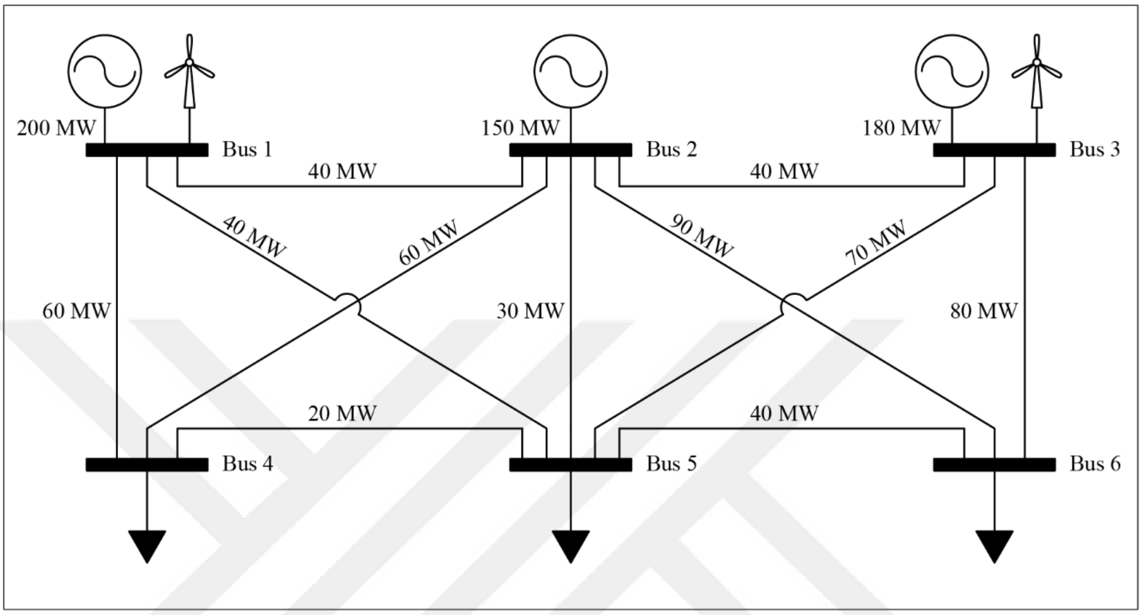

Figure 5.5. Modified 6-bus system

Similar to demand, nominal wind values are obtained by scaling normalized profiles shown in Figure 5.3 with maximum generation capacities of wind farms. Four different sets of stochastic wind scenarios obtained by adding Gaussian noise are given in Figure 5.6, Figure 5.7, Figure 5.8, and Figure 5.9. They correspond to 20 and 40 percent deviations respectively.

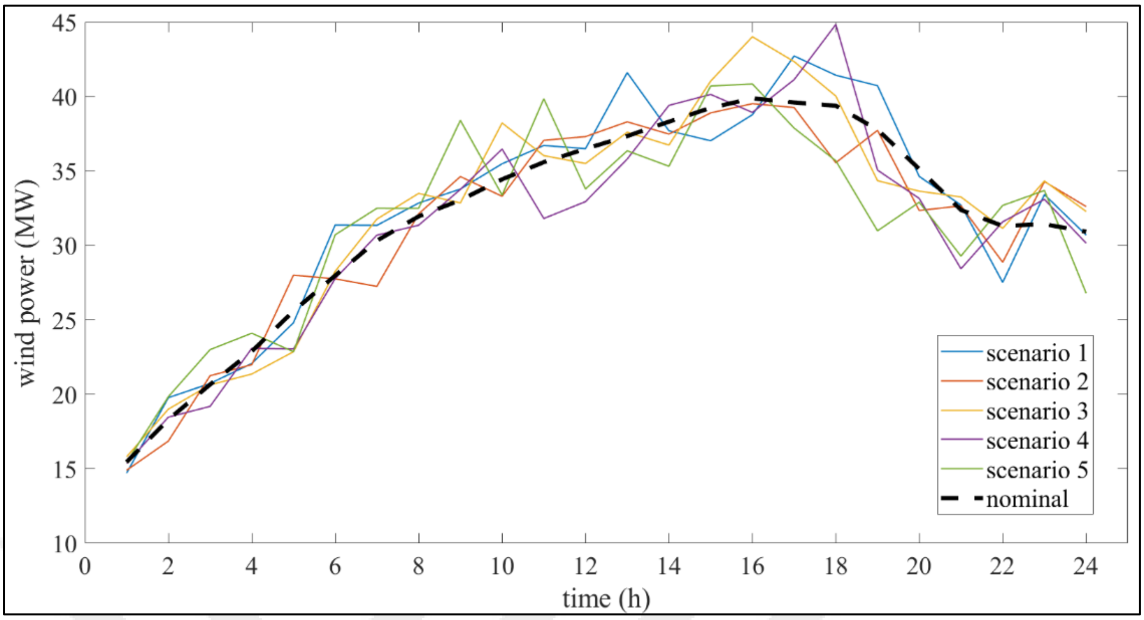

Figure 5.6. Wind scenarios of first wind farm for 20 percent wind deviation

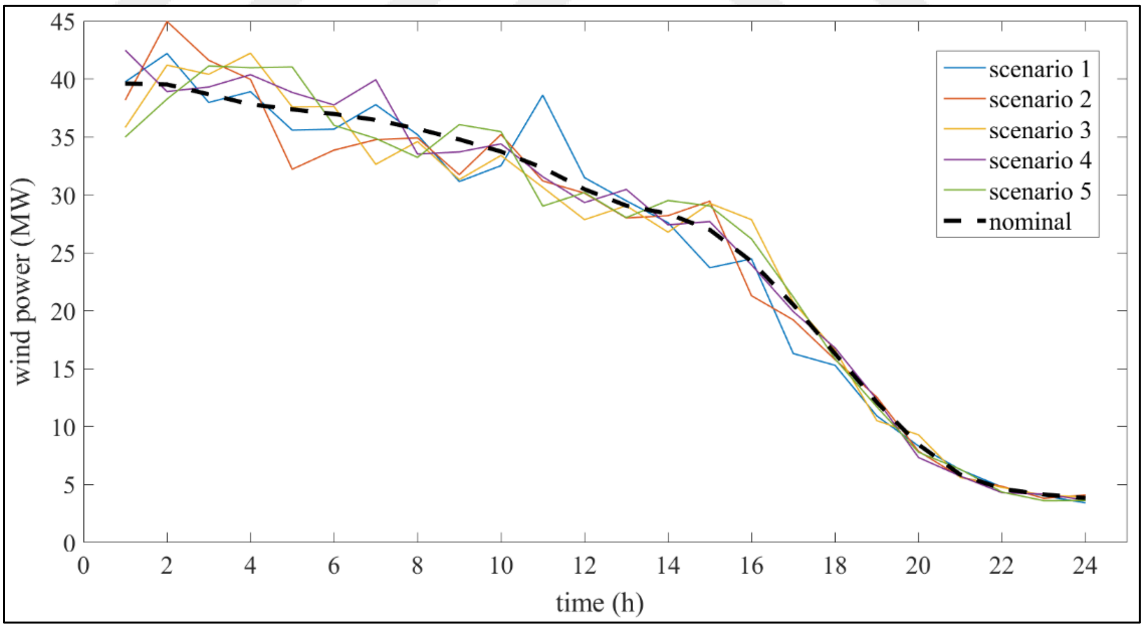

Figure 5.7. Wind scenarios of second wind farm for 20 percent wind deviation

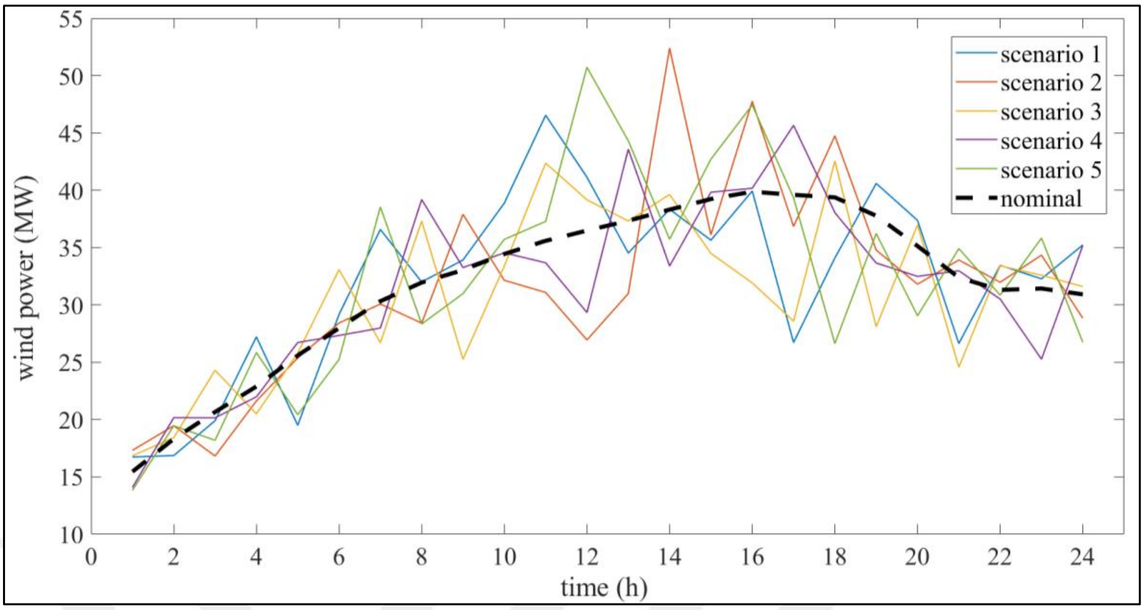

Figure 5.8. Wind scenarios of first wind farm for 40 percent wind deviation

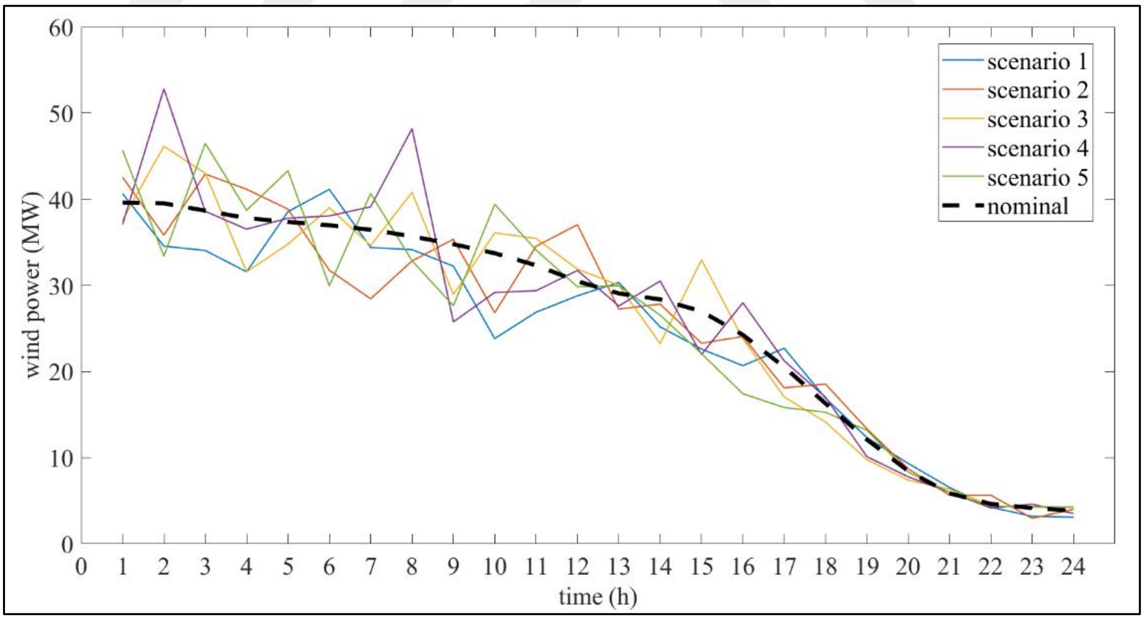

Figure 5.9. Wind scenarios of second wind farm for 40 percent wind deviation

The candidate ESSs are assumed to have 30 MWh maximum energy storage capacity and 6 MW rated power for charge and discharge. ESS installation cost is taken as $160 for all buses and it is assumed that ESS units can be placed on every bus.

5.1.1. Analysis of ESS Planning

In this section, we analyze the placement of ESSs under different conditions. We use the original line capacities of the 6-bus system. We create eight different cases for different values of maximum demand power, wind power deviation and ramp-rate limit scaling factor and solve our planning problem for these cases. Cases considered and results are given in Table 5.3.

In this table, low and high maximum loads are taken as 150 MW and 250 MW, respectively. Low and high wind power deviations are 20 and 40 percent, respectively. Lastly, low and high ramp rate limits are considered correspond to 25 percent and 50 percent of generation capacities of thermal units.

Table 5.3. ESS planning for the 6-bus system

Maximum Load Level	**Wind Power Deviation Level**	**Ramp-Rate Limit Factor Level**	**Investment Cost ($)**	**Total Stochastic Robust Operation Cost ($)**	**Location**	**No. Of Iterations**
low	low	low	320	10911.86	1, 6	3
low	low	high	320	10911.86	1, 5	3
low	high	low	960	12400.19	1, 2, 3, 4, 5, 6	3
low	high	high	960	12400.19	1, 2, 3, 4, 5, 6	3
high	low	low	160	27171.31	4	2
high	low	high	160	27163.12	4	2
high	high	low	160	28694.83	4	2
high	high	high	160	28690.39	4	2

Table 5.3 summarizes the planning results including the investment cost, the total operation cost of thermal generators of stochastic and robust parts, storage locations, and the number

of iterations of the CCG algorithm. The results show that for low demand low deviation cases the investment cost is $320 and the total operational cost is $1092 with two ESSs placement. When the wind deviation is high, the investment cost is $960 and the operational cost is $12400. ESSs are installed at every bus this time. The need for ESS placement in these cases is energy peak. Since the generated power is higher than the demand, especially when wind deviation is high, storage devices are installed near generating units to store excess energy and keep the system safe. For high demand cases, the situation is different since, for all high demand cases, there is one ESS installed at bus 4 with a $160 investment cost. Total operational costs are changing with wind deviations and ramping limits. ESSs are used to overcome the congestion problem in these cases. To be more specific, because capacities of lines connected to bus 4 are lower than the other buses, congestion may occur while satisfying the demand at this bus. By installing a storage device, the system can avoid congestion by storing energy when demand is low and using it when the demand is high at this bus.

5.1.2. Comparison of Solution Methods for Max-Min Problem

In order to compare performance of the algorithms used for the max-min optimization, additional numerical experiments are performed for 36 test cases including the ones used in the previous section. Details of the results obtained are provided in Appendix A. Since the exact method proposed in this thesis give the global optimal solution, it is used to measure and compare optimality gaps of local optimization based methods. Also, computation times of all methods considered are compared.

Test cases are created by considering three different levels for the total maximum demand as low (150 MW), medium (200 MW), and high (250 MW). For ramping limits and wind deviations, the same values used in the previous experiment are employed as high and low levels. In addition to cases in which original line flow limits are employed, new cases are created by scaling flow limits to smaller values. All parameters of the cases investigated can be found in Appendix A.

Optimality gaps of seven MC strategies and our hybrid method for 36 cases are given in Figure 5.10.

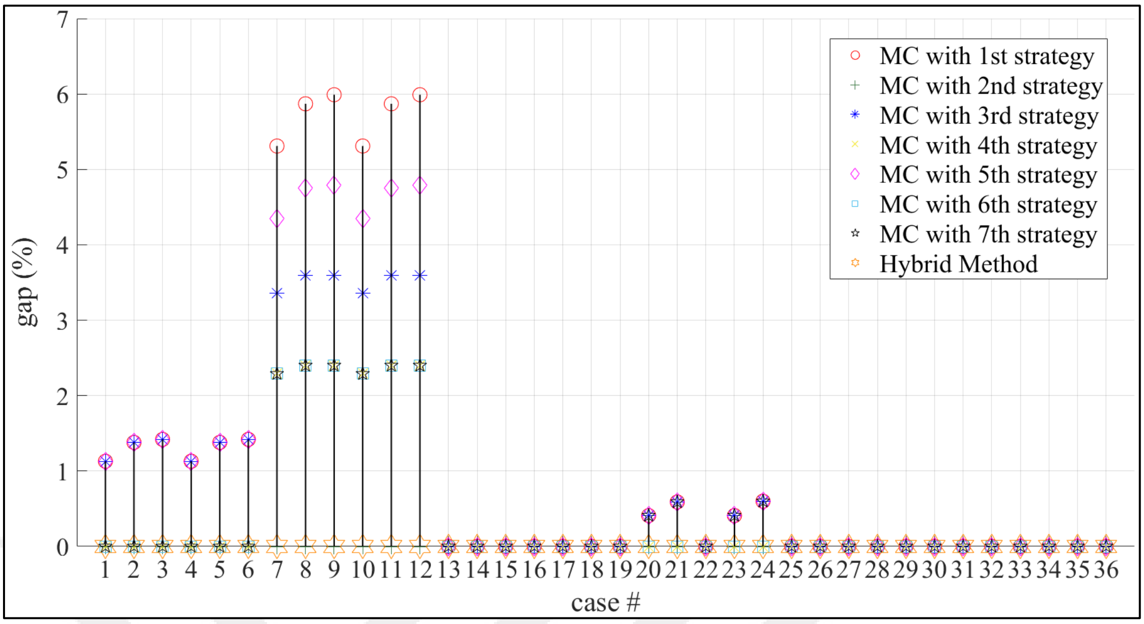

Figure 5.10. Optimality gaps for the 6-bus system

The maximum gap value is about six percent and it is observed in cases 9 and 12. For all cases, there is at least one MC strategy that can find the global optimal solution means that the optimality gap is equal to zero. Especially for cases 13-19 and 25-26, optimality gap values are zero for all MC strategies. Furthermore, all of the gap values are equal to zero when the hybrid method is used meaning it works well for the 6-bus system.

In Figure 5.11 computation times are shown. Here, total time is the computation time of the CCG algorithm to solve a given case.

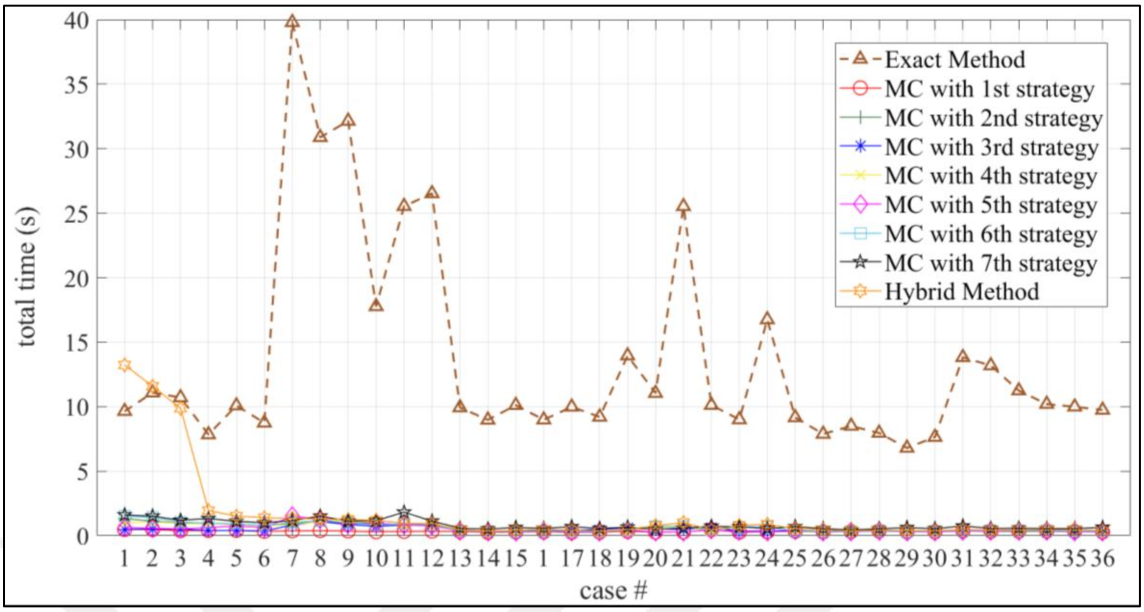

Figure 5.11. Computation times for the 6-bus system

The proposed exact method leads to the highest computation time as can be expected. But the solutions could be obtained in a reasonable time. The MC is the fastest method and it is closely followed by the proposed hybrid method. To be more specific, these two approaches yield almost the same computation times except the first three cases for which the hybrid method performed considerably slower. But one should keep in mind that the hybrid method ensures robust feasibility while the MC approach does not have such a guarantee.

5.2. IEEE 14-BUS SYSTEM

There are 14 buses in this test system. It includes two generators, 20 lines, and 11 demand points. The generators are at buses 1, 2 and the demands are at buses 2-6 and 9-14. Parameters of generators and lines are given in Table 5.4 and Table 5.5, respectively.

Table 5.4. Parameters of generators for the 14-bus system

Generator	$\overline{PG_i}$ (MW)	CG_i ($/MW)
1	340	7.92
2	59	23.27

The minimum generation capacity is set to zero for all generators. Ramp-rate limits are calculated by multiplying maximum generation capacity with a scaling factor.

Table 5.5. Parameters of lines for the 14-bus system

Line	From	To	L_{jb} (MW)	B_{jb}
1	1	2	472	0.059
2	1	5	128	0.223
3	2	3	145	0.198
4	2	4	158	0.176
5	2	5	161	0.174
6	3	4	160	0.171
7	4	5	664	0.042
8	4	7	141	0.209
9	4	9	53	0.556
10	5	6	117	0.252
11	6	11	134	0.199
12	6	12	104	0.256
13	6	13	201	0.13
14	7	8	167	0.176
15	7	9	267	0.11
16	9	10	325	0.085
17	9	14	99	0.27
18	10	11	141	0.192
19	12	13	99	0.2
20	13	14	76	0.348

Tests on the 14-bus system are carried out for 12 hours using the normalized demand profile depicted in Figure 5.2. The system is tested for different maximum demand values.

The original 14-bus bus system is modified by adding two find farms of 40 MW capacity to buses 2 and 3. The modified system is illustrated in Figure 5.12.

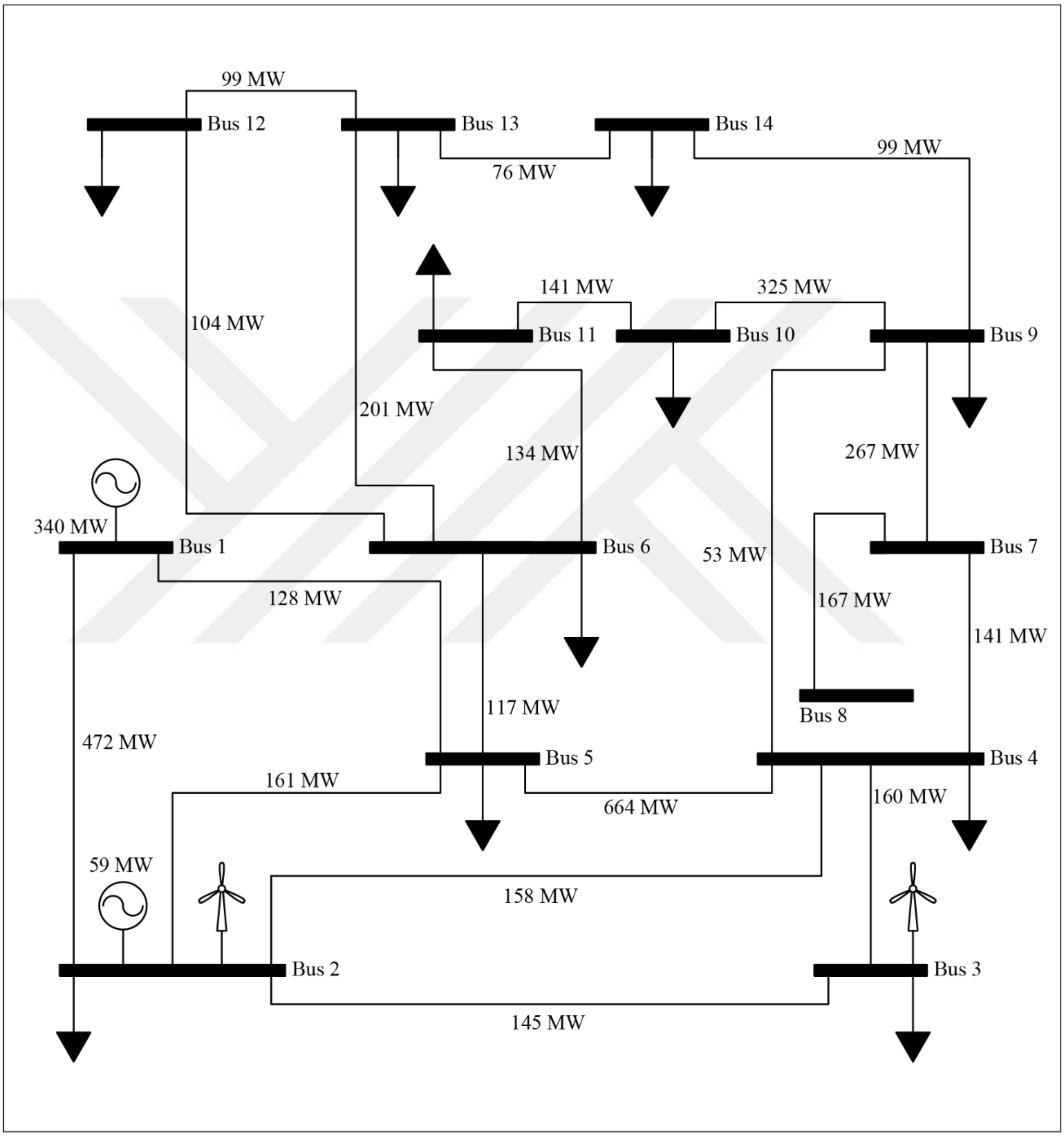

Figure 5.12. Modified 14-bus system

Nominal wind values are obtained by scaling normalized profiles shown in Figure 5.4 with maximum generation capacities of wind farms. Stochastic wind scenarios used are given in the figures below. Wind deviations are taken as 25 and 50 percent.

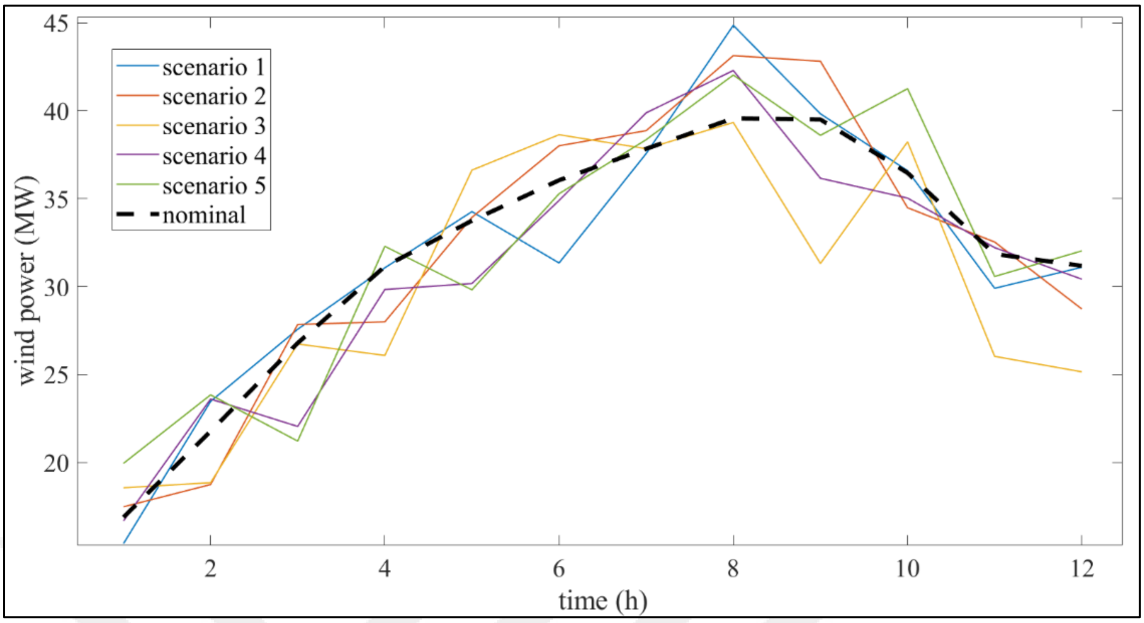

Figure 5.13. Wind scenarios of first wind farm for 25 percent wind deviation

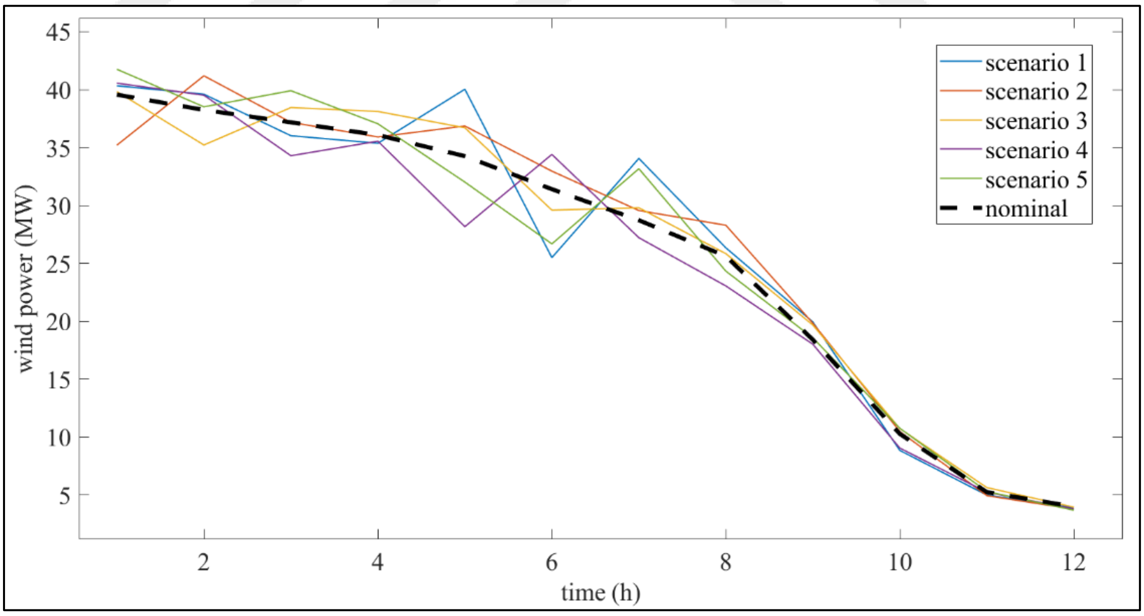

Figure 5.14. Wind scenarios of second wind farm for 25 percent wind deviation

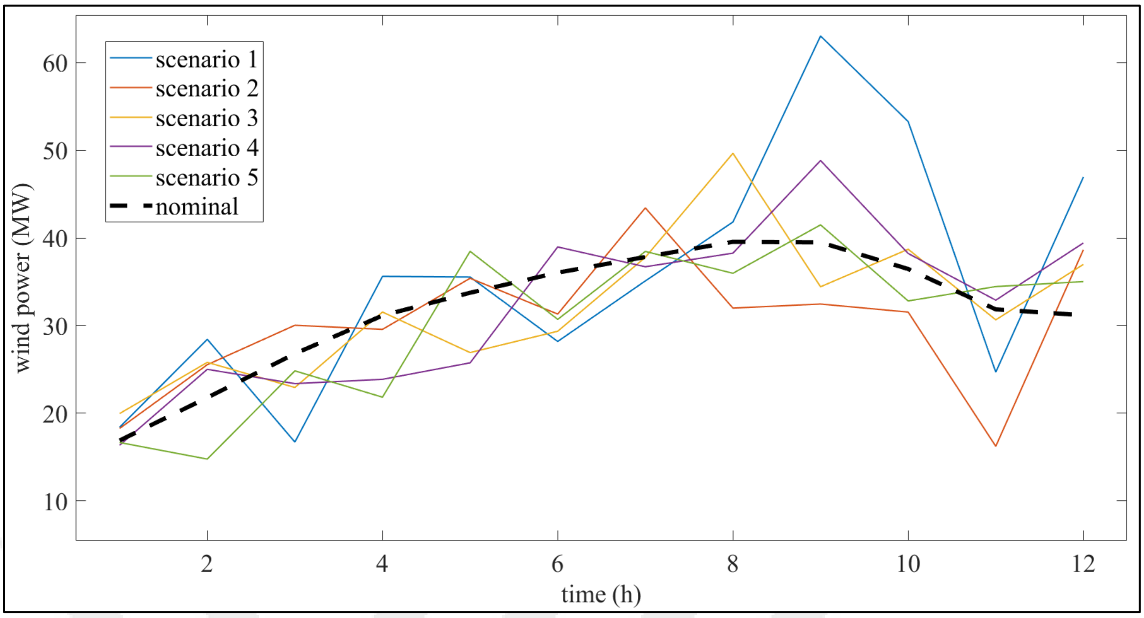

Figure 5.15. Wind scenarios of first wind farm for 50 percent wind deviation

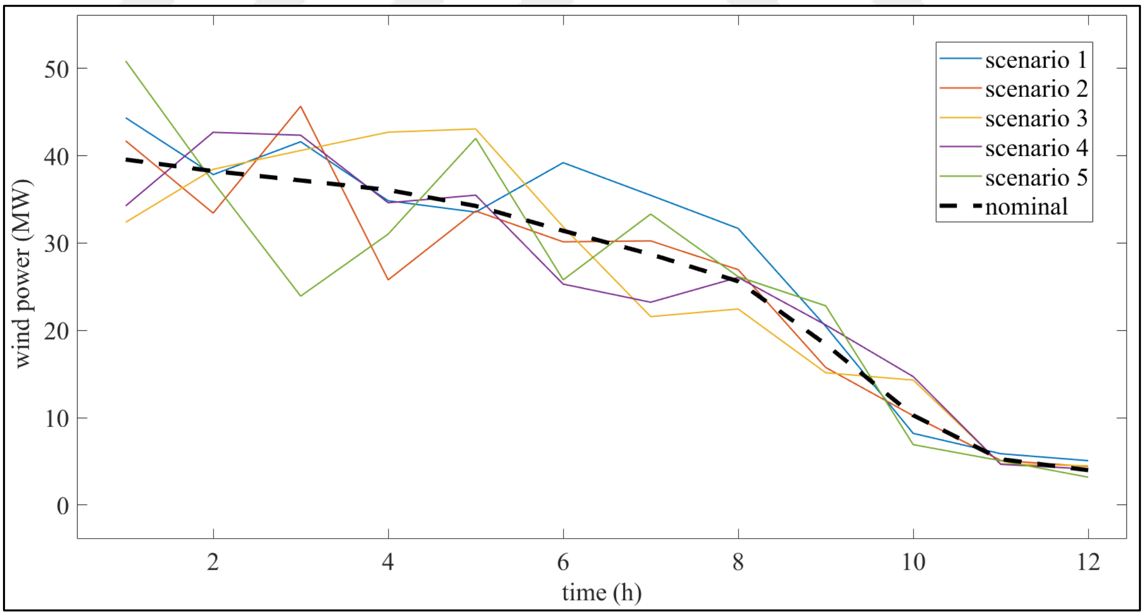

Figure 5.16. Wind scenarios of second wind farm for 50 percent wind deviation

The candidate ESSs are assumed to have 60 MWh maximum energy storage capacity and 12 MW rated power for charge and discharge. ESS installation cost is taken as $160 for all buses and it is assumed that ESS units can be placed to every bus.

5.2.1. Analysis of ESS Planning

Similar to the 6-bus system analysis, we use the original line capacities of the 14-bus system. We create eight different cases for different values of maximum demand power, wind power deviation and ramp-rate limit scaling factor and solve our planning problem for these cases. Cases considered and results are given in Table 5.6.

In this table, low and high maximum total loads are taken as 100MW and 300 MW, respectively. Low and high wind power deviations are 25 and 50 percent, respectively. Lastly, low and high ramp rate limits correspond to 25 percent and 50 percent of generation capacities of thermal units.

Table 5.6. ESS planning for the IEEE 14-bus system

Maximum Load Level	**Wind Power Deviation Level**	**Ramp-Rate Limit Factor Level**	**Investment Cost ($)**	**Total Stochastic Robust Operation Cost ($)**	**Location**	**No. Of Iterations**
low	low	low	640	1290.21	2, 3, 10, 14	3
low	low	high	640	1290.21	2, 4, 5, 12	3
low	high	low	960	1931.77	1, 2, 4, 9, 12, 14	3
low	high	high	960	1931.77	2, 3, 4, 5, 6, 11	3
high	low	low	960	14854.23	5, 6, 10, 11, 12, 13	2
high	low	high	960	13948.67	5, 6, 10, 11, 12, 13	2
high	high	low	1120	15482.77	5, 6, 10, 11, 12, 13, 14	2
high	high	high	1120	14577.21	5, 6, 10, 11, 12, 13, 14	2

Table 5.6 shows the planning results including the investment cost, the total operation cost of thermal generators, locations of ESSs, and the number of iterations of the CCG algorithm. The results show that for low demand low deviation cases the investment cost is $640 and the total operational cost is $1290 with four ESSs placement. When the wind deviation is high, the investment cost is $960 and the operational cost is $1932, the ESSs are installed at six buses. Furthermore, in these cases, storage units are installed near wind farms and thermal generators to store excess power production to maintain system balance. For cases for which maximum demand is high, there are more ESSs installed in the system. For the last two cases, there are seven ESSs installed with an investment cost of $1120. For high demand cases are installed to buses having relatively high demands. By this way, possible line congestions are avoided by storing energy when the demand is low and using it in peak demand periods.

5.2.2. Comparison of Solution Methods for Max-Min Problem

In this section, again experiments are performed for 36 test cases including the ones used in the previous section to compare the performance of the algorithms used for the max-min optimization. Details of the results obtained are provided in Appendix A.

Test cases are created by considering three different levels for the total maximum demand as low (100 MW), medium (200 MW), and high (300 MW). For ramping limits and wind deviations, the same values used in the previous experiment are employed as high and low levels. In addition to cases in which original line flow limits are employed, new cases are created by scaling flow limits to smaller values. All parameters of the cases investigated can be found in Appendix A. Optimality gaps are given in Figure 5.17.

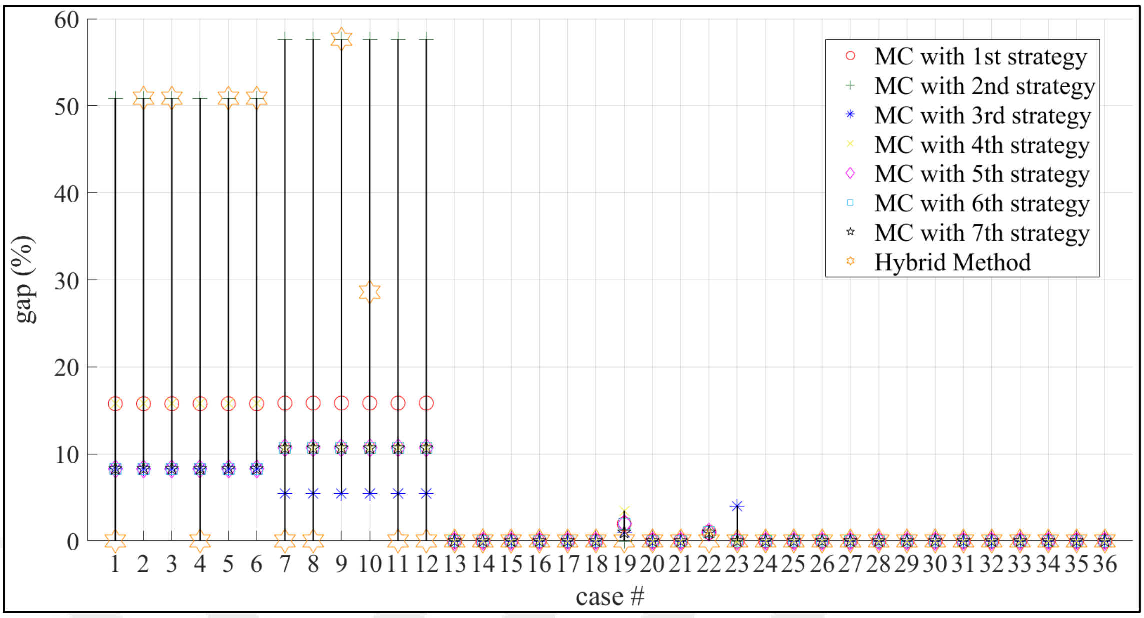

Figure 5.17. Optimality gaps for the 14-bus system

As can be seen for the first 12 cases, none of the MC strategies could find a global optimal solution and their gaps range between 8 and 58 percent. On the other hand, the proposed hybrid method led to zero optimality gap for cases 1,4,7,8,11,12 while having a high gap for cases 2,3,5,6,9. Given that this method ensures robust feasibility, it can be said that the hybrid method performed better than the MC in terms of optimality and feasibility.

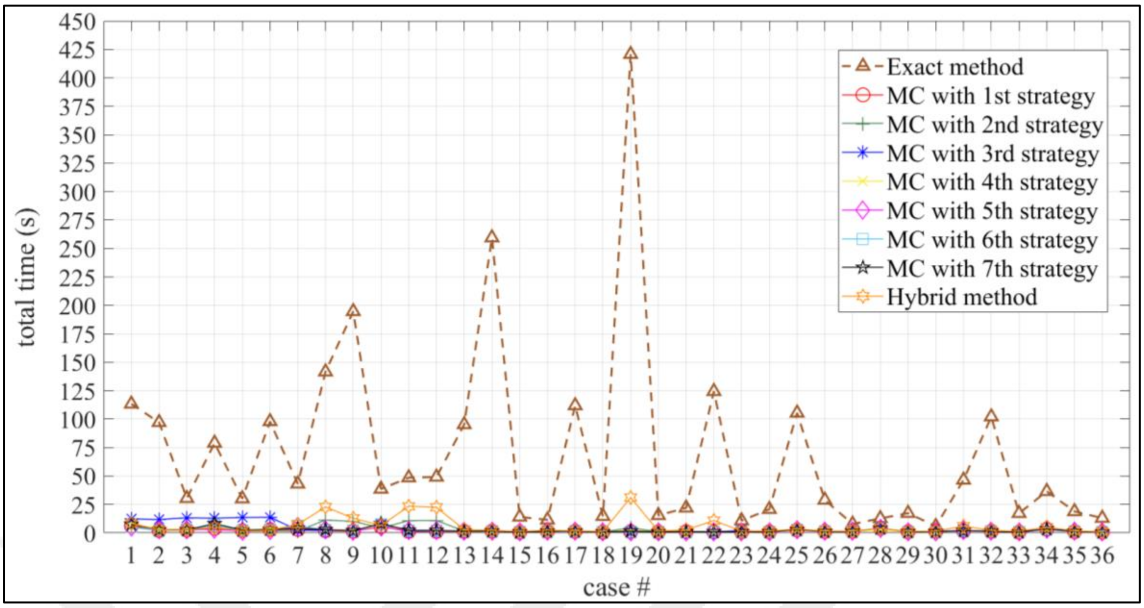

Figure 5.18. Computation times for the 14-bus system

Computation times are shown in Figure 5.18. As for the 14-bus system, the interval partitioning method leads to the highest computation times while the hybrid method follows the MC strategy closely. Although the interval partitioning method is computationally expensive it could find optimal solutions in a reasonable time for all case cases considered.

5.3. IEEE 30-BUS SYSTEM

This test system includes 30 buses. There are two generators, 41 lines, and 21 demand points. The generators are located at buses 1 and 2. Parameters of generators and lines are given in Table 5.7 and Table 5.8, respectively.

Table 5.7. Parameters of generators for the 30-bus system

Generator	$\overline{PG_i}$ **(MW)**	CG_i **($/MW)**
1	271	18.42
2	92	52.18

The minimum generation capacity is set to zero for all generators. Ramp-rate limits are calculated by multiplying maximum generation capacity with a scaling factor.

Table 5.8. Parameters of lines for the 30-bus system

Line	From	To	L_{jb} (MW)	B_{jb}
1	1	2	138	0.058
2	1	3	152	0.165
3	2	4	139	0.174
4	3	4	135	0.038
5	2	5	144	0.198
6	2	6	139	0.176
7	4	6	148	0.041
8	5	7	127	0.116
9	6	7	140	0.082
10	6	8	148	0.042
11	6	9	142	0.208
12	6	10	53	0.556
13	9	11	142	0.208
14	9	10	267	0.11
15	4	12	115	0.256
16	12	13	210	0.14
17	12	14	29	0.256
18	12	15	29	0.13
19	12	16	30	0.199
20	14	15	20	0.2
21	16	17	38	0.192
22	15	18	29	0.219
23	18	19	29	0.129
24	19	20	29	0.068
25	10	20	30	0.209
26	10	17	33	0.085
27	10	21	30	0.075
28	10	22	29	0.15
29	21	22	29	0.024
30	15	23	29	0.202
31	22	24	26	0.179
32	23	24	29	0.27
33	24	25	27	0.329
34	25	26	25	0.38
35	25	27	28	0.209
36	28	27	75	0.396
37	27	29	28	0.415
38	27	30	28	0.603
39	29	30	28	0.453
40	8	28	140	0.2
41	6	28	149	0.06

Tests on the 30-bus system are carried out for 12 hours using the normalized demand profile depicted in Figure 5.2. The system is tested for different maximum demand values.

The original 30-bus system is modified by adding two find farms of 40 MW capacity to buses 5 and 11. The modified system is depicted in Figure 5.19.

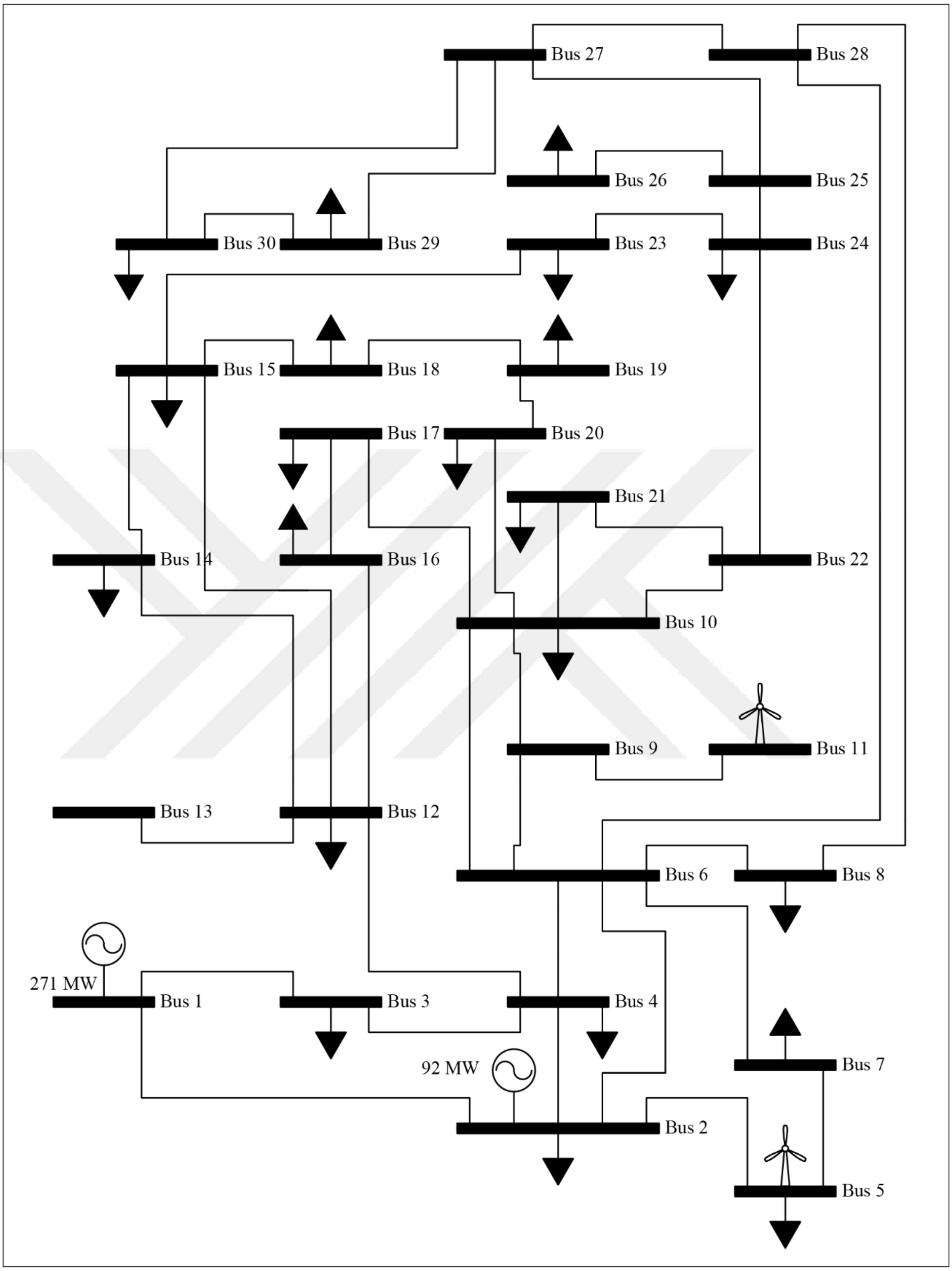

Figure 5.19. Modified 30-bus system

Nominal wind values are obtained by scaling normalized profiles shown in Figure 5.4 with maximum generation capacities of wind farms. Stochastic wind scenarios used are given in Figure 5.20-Figure 5.23. Wind deviations are taken as 20 percent and 30 percent.

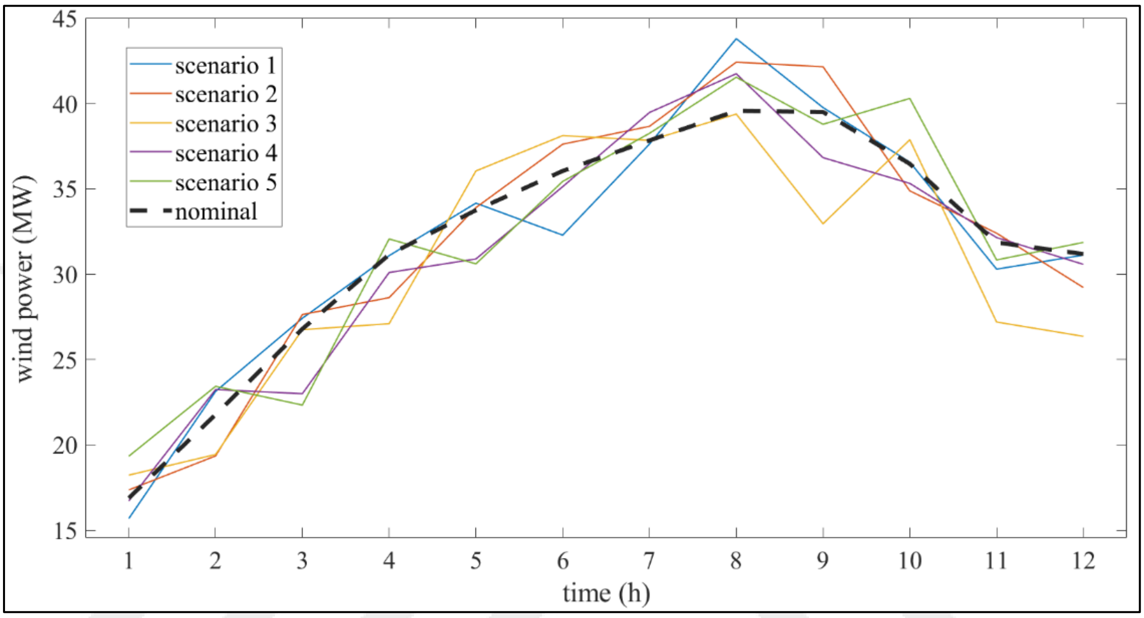

Figure 5.20. Wind scenarios of first wind farm for 20 percent wind deviation

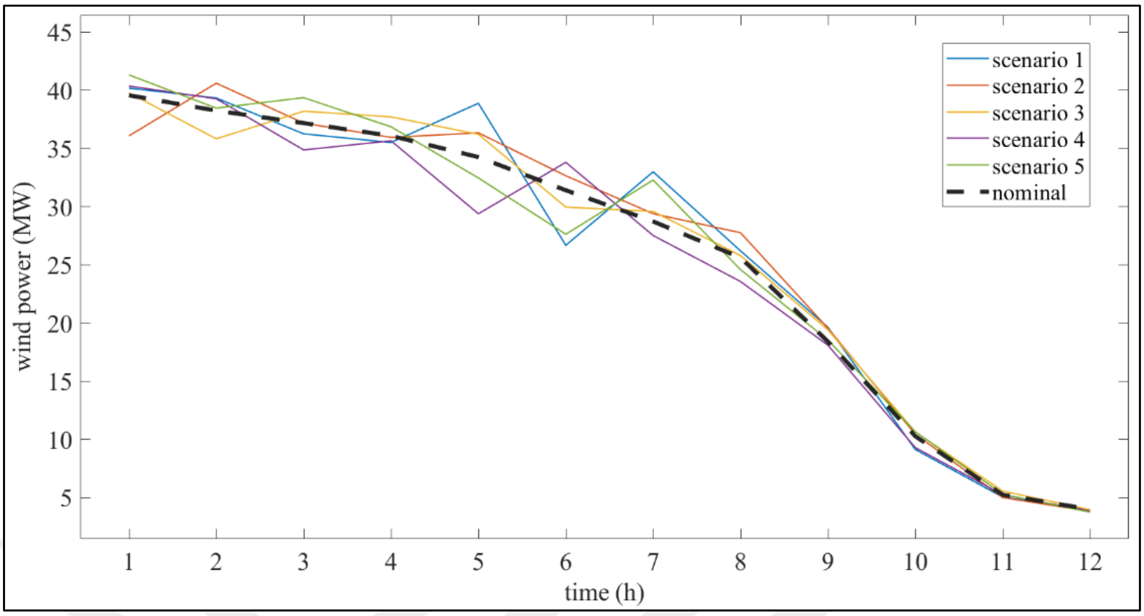

Figure 5.21. Wind scenarios of second wind farm for 20 percent wind deviation

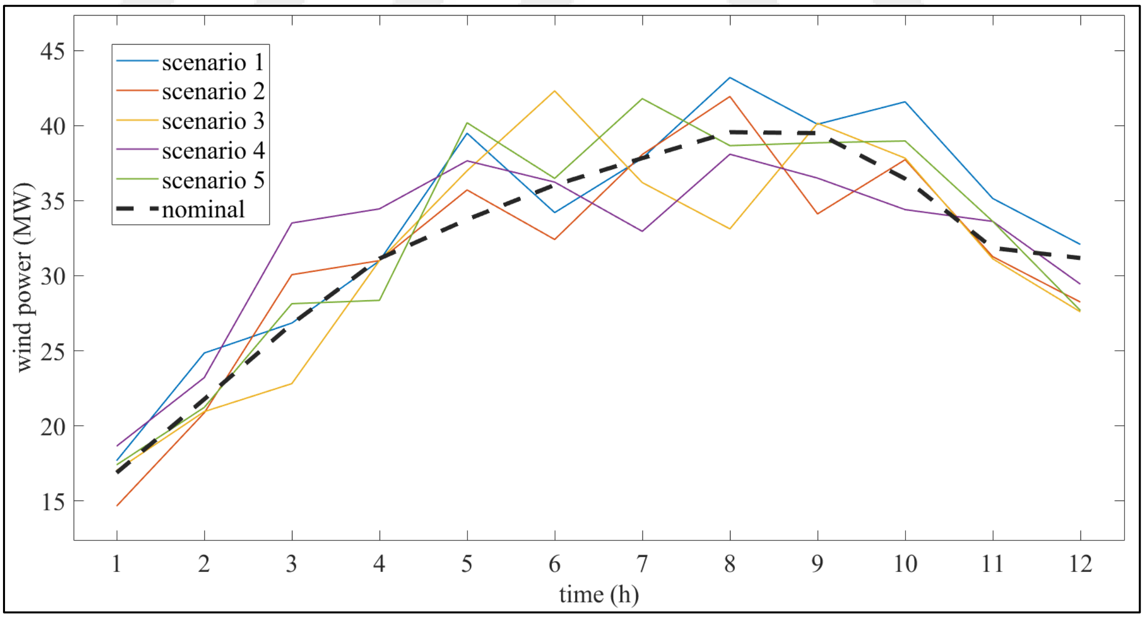

Figure 5.22. Wind scenarios of first wind farm for 30 percent wind deviation

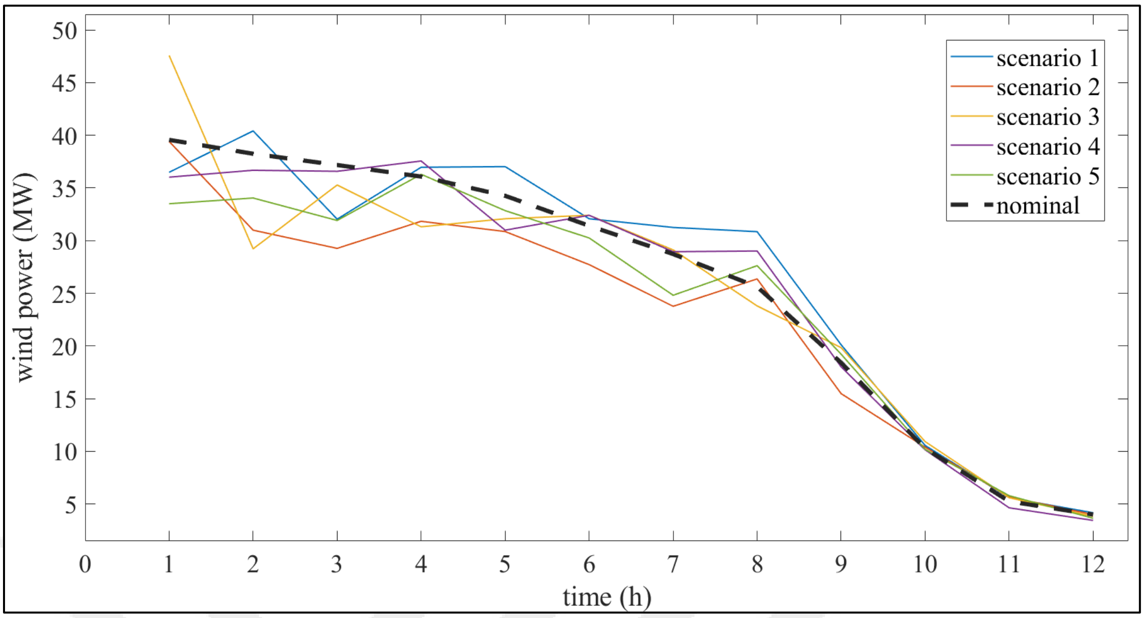

Figure 5.23. Wind scenarios of second wind farm for 30 percent wind deviation

ESSs to be installed are assumed to have 90 MWh maximum energy storage capacity and 18 MW rated power for charge and discharge. ESS installation cost is taken as $160 for all buses and it is assumed that ESS units can be placed to every bus.

5.3.1. Analysis of ESS Planning

The placement of ESSs under different conditions is analyzed in this section. Original line capacities of the 30-bus system are used. We create eight different cases for different values of maximum demand power, wind power deviation and ramp-rate limit scaling factor and solve our planning problem for these cases. Cases considered and results are given in Table 5.9.

In this table, low and high total maximum loads are taken as 100 MW and 300 MW, respectively. Low and high wind power deviations are 20 and 30 percent, respectively. Lastly, low and high ramp rate limits correspond to 25 percent and 50 percent of generation capacities of thermal units.

Table 5.9. ESS planning for the IEEE 30-bus system

Maximum Load Level	Wind Power Deviation Level	Ramp-Rate Limit Factor Level	Investment Cost ($)	Total Stochastic Robust Operation Cost ($)	Location	No. Of Iterations
low	low	low	480	2676.79	1, 2, 30	3
low	low	high	480	2676.79	2, 7, 24	3
low	high	low	480	3353.12	5, 17, 24	3
low	high	high	480	3353.12	2, 6, 9	3
high	low	low	320	31050.52	2, 5	2
high	low	high	320	31050.52	2, 5	2
high	high	low	480	31602.54	2, 5, 7	2
high	high	high	480	31602.54	2, 5, 7	2

Planning results are given in Table 5.9. The results show that for all low demand cases the investment cost is $480 with four ESSs placement. As wind-power deviation increases total operational cost increases from $2678 to $3353. For high demand cases, the number of ESSs installed in the system changes for different levels of wind power deviation. As the deviation increases, the number of ESSs increases. For low deviation, the investment cost is $320 and the total operational cost is $31050 but when the deviation becomes high investment cost is $480 and the total operational cost is $31602. ESSs are placed near wind farms and thermal generators to shave energy peaks.

5.3.2. Comparison of Solution Methods for Max-Min Problem

In this section, additional numerical experiments are performed for 36 test cases including the ones used in the previous section to compare the performance of the algorithms used for the max-min optimization. Optimality gaps of local optimization based methods and computation times of all methods considered are presented. Details of the result obtained are provided in Appendix A.

Test cases are created by considering three different levels for the total maximum demand as low (100 MW), medium (200 MW), and high (300 MW). For ramping limits and wind deviations, the same values used in the previous experiment are employed as high and low levels. In addition to cases in which original line flow limits are employed, new cases are created by scaling flow limits to smaller values. All parameters of the cases investigated can be found in Appendix A. Optimality gaps are given in Figure 5.24.

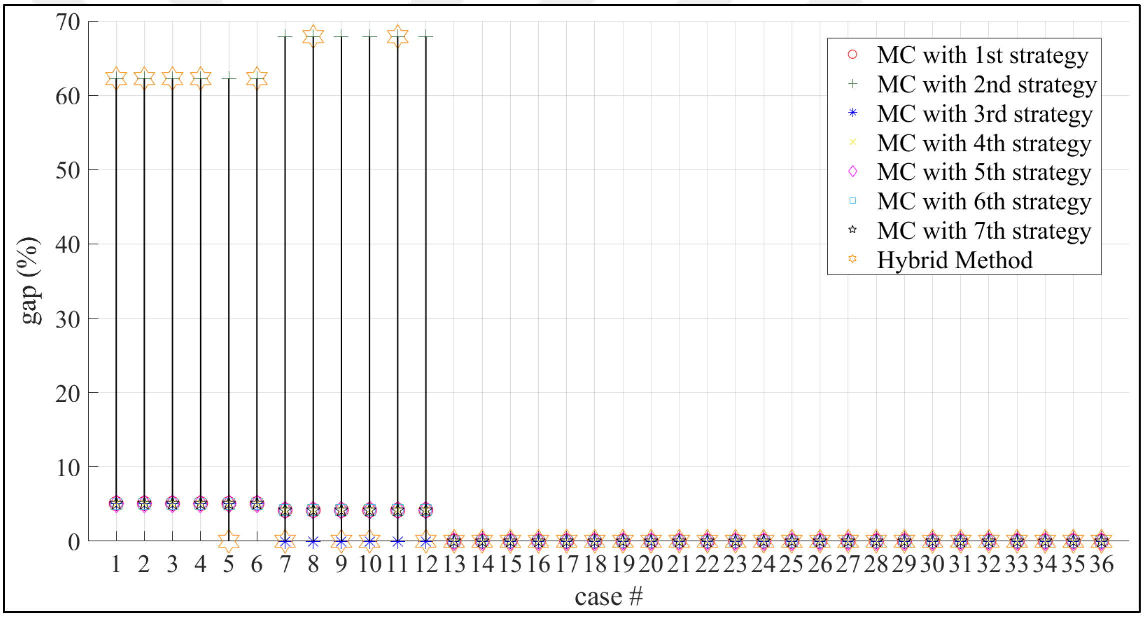

Figure 5.24. Optimality gaps for the 30-bus system

As can be seen, the MC method could not find a global optimal solution for the first six cases for any initialization strategy and the gaps obtained range between 4 and 68 percent. Among the first six cases, the hybrid method could found the global optimal solution for case five while leading to a high optimality gap for the others. For cases 13-36 non of the methods had an optimality gap. Lastly, for cases 7-12 there are some initialization strategies for the MC leading to zero optimality gap, and for four cases the hybrid method yields the global optimal solution.

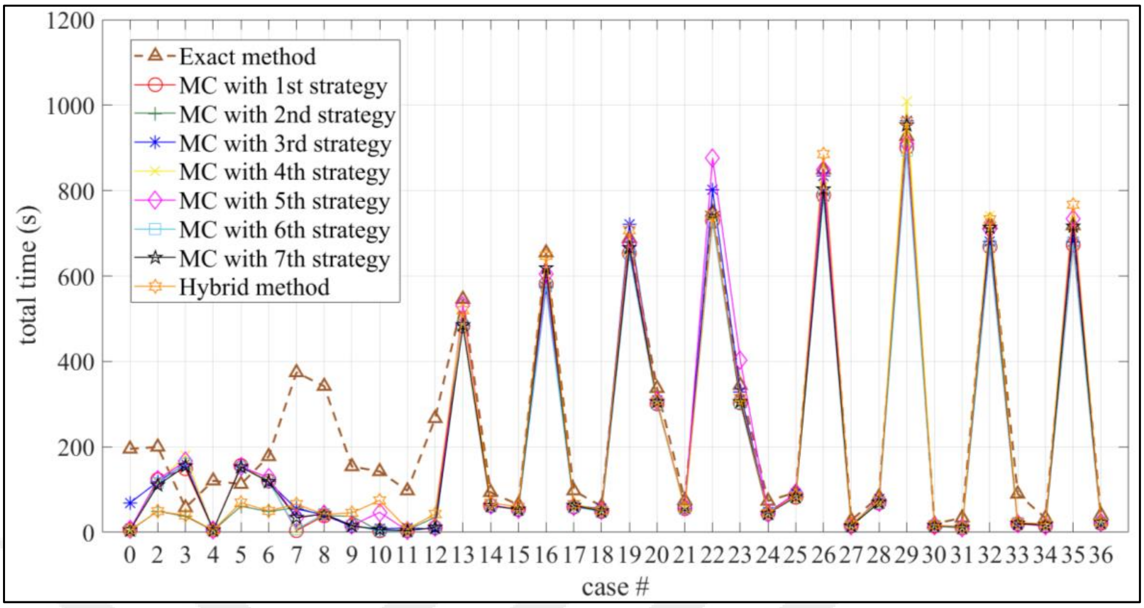

Figure 5.25. Computation times for the 30-bus system

Computational times are given in Figure 5.25. Different than other bus systems, the MC method did not exhibit superior performance, especially for cases 13-36. The reason behind this is that unlike the previous cases the first stage problem turns out to be much more computationally intensive dominating the solution time of the second stage max-min optimization. Although the interval partitioning method worked slower than the other methods for the first 12 cases, the solution times are comparable.

6. CONCLUSION AND FUTURE WORK

In this thesis, the problem of finding optimal locations of storage systems in power networks is studied. A stochastic robust optimization approach is employed to model and solve the problem. As in two-stage robust optimization, the underlying CCG algorithm relies on the solution of a max-min type problem, which is difficult in general. Two new algorithms, one can find a locally optimal solution while the other is guaranteed to produce a globally optimal solution using off-the-shelf solvers, are proposed. The proposed methods and some alternatives available in the literature are implemented and tested on IEEE benchmark bus systems.

The results show that the algorithms developed works successfully. To be more specific, the exact method could find a global optimal solution for the test cases in acceptable time limits. Besides, it worked reliably without any numerical issue because it avoids the use of big-M values as opposed to the well known Fortuny-Amat approach. On the other hand, the hybrid local optimization algorithm proposed does not guarantee global optimality but ensures robust feasibility. Moreover, it could find solutions with relatively low optimality gaps in several test cases. Thus, this method provides a good trade-off between optimality and computational efficiency.

Although proposed max-min optimization algorithms are employed for energy storage planning, they are applicable to two-stage robust optimization in general and can be used to solve different problems in power networks including unit commitment, transmission expansion planning, generation expansion planning, and reserve scheduling.

REFERENCES

1. Jabr RA, Džafić I, Pal BC. Robust optimization of storage investment on transmission networks. *IEEE Transactions on Power Systems*. 2015;30(1):531–9.

2. Liu D, Shang C, Cheng H. A two-stage robust optimization for coordinated planning of generation and energy storage systems. *2017 IEEE Conference on Energy Internet and Energy System Integration (EI2) on*; 2017: IEEE.

3. Dehghan S, Amjady N. Robust transmission and energy storage expansion planning in wind farm-integrated power systems considering transmission switching. *IEEE Transactions on Sustainable Energy*. 2016;7(2):765–74.

4. Ye H, Wang J, Li Z. MIP reformulation for max-min problems in two-stage robust SCUC. *IEEE Transactions on Power SystemIEEE Transactions on Power Systems*. 2017;32(2):1237–47.

5. Baringo L, Baringo A. A stochastic adaptive robust optimization approach for the generation and transmission expansion planning. *IEEE Transactions on Power Systems*. 2018;33(1):792–802.

6. Jabr RA. Robust transmission network expansion planning with uncertain renewable generation and loads. *IEEE Transactions on Power Systems*. 2013;28(4):4558–67.

7. Wei W, Liu F, Mei S, Hou Y. Robust energy and reserve dispatch under variable renewable generation. *IEEE Transactions on Smart Grid*. 2015;6(1):369–80.

8. Bertsimas D, Litvinov E, Sun XA, Zhao J, Zheng T. Adaptive robust optimization for the security-constrained unit commitment problem. *IEEE Transactions on Power Systems*. 2013 Feb;28(1):52–63.

9. Zhao C, Guan Y. Unified stochastic, and robust unit commitment. *IEEE Transactions on Power Systems*. 2013;28(3):3353–61.

10. Liu D, Cheng H, Fang S. Coupled stochastic and robust transmission expansion planning. *2017 IEEE Power & Energy Society General Meeting on*; 2017: IEEE.

11. Zeng B, Zhao L. Solving two-stage robust optimization problems using a column-and-constraint generation method. *Oper Res Lett*. 2013 Sep;41(5):457–61.

12. Pineda S, Morales JM. Solving linear bilevel problems using big-ms: not all that glitters is gold. *IEEE Transactions on Power Systems*. 2019;34(3):2469–71.

13. Pineda S, Bylling H, Morales JM. Efficiently solving linear bilevel programming problems using off-the-shelf optimization software. *Optimization and Engineering*. 2018 Mar 4;19(1):187–211.

14. Sherali HD, Alameddine A. A new reformulation-linearization technique for bilinear programming problems. *Journal of Global Optimization*. 1992;2(4):379–410.

15. McCormick GP. Computability of global solutions to factorable nonconvex programs: Part I-Convex underestimating problems. *Mathematical Programming*. 1976;10(1):147–75.

16. Fortuny-Amat J, McCarl B. A representation and economic interpretation of a two-level programming problem. *Journal of Operational Research Society*. 1981;32(9):783–92.

17. Das CK, Bass O, Kothapalli G, Mahmoud TS, Habibi D. Overview of energy storage systems in distribution networks: Placement, sizing, operation, and power quality. *Renewable Sustainable Energy Review*. 2018;91(March 2018):1205–30.

18. Wong LA, Ramachandaramurthy VK, Taylor P, Ekanayake JB, Walker SL, Padmanaban S. Review on the optimal placement, sizing, and control of an energy storage system in the distribution network. *Journal of Energy Storage*. 2019;21(December 2018):489–504.

19. Zimmerman RD, Murillo-sánchez CE, Thomas RJ. MATPOWER : Steady-state operations, systems research, and education. *IEEE Transactions on Power Systems*. 2011;26(1):12–9.

20. Babaeinejadsarookolaee S, Birchfield A, Christie RD, Coffrin C, DeMarco C, Diao R, et al. The power grid library for benchmarking AC optimal power flow algorithms. *arXiv*. 2019;1(1):1–17.

21. Jabr RA, Karaki S, Korbane JA. Robust multi-period OPF with storage and renewables. *IEEE Transactions on Power Systems*. 2015;30(5):2790–9.

22. Pena I, Brancucci C, Hodge BM. An extended IEEE 118-bus test system with high renewable penetration. *IEEE Transactions on Power Systems*. 2017;33(1):281–9.

APPENDIX A: DETAILED RESULTS OF THE ANALYSIS

Table A.1. Results of 6-bus test system for the exact solution method

Case #	Maximum Demand (MW)	Wind Power Deviation	Ramp-Rate Limit Factor	Flow Limit Factor	Total Cost ($)	Location	No. Of Iterations	Total Time (s)
1	150	20	0.25	0.61	11308.59	4, 5	3	9.64
2	150	20	0.25	0.805	11231.86	4, 6	3	11.08
3	150	20	0.25	1	11231.86	1, 6	3	10.71
4	150	20	0.5	0.61	11308.59	4, 5	3	7.84
5	150	20	0.5	0.805	11231.86	4, 5	3	10.08
6	150	20	0.5	1	11231.86	1, 5	3	8.75
7	150	40	0.25	0.61	13409.18	1, 2, 3, 4, 5, 6	3	39.8
8	150	40	0.25	0.805	13360.19	1, 2, 3, 4, 5, 6	3	30.88
9	150	40	0.25	1	13360.19	1, 2, 3, 4, 5, 6	3	32.15
10	150	40	0.5	0.61	13406.55	1, 2, 3, 4, 5, 6	3	17.77
11	150	40	0.5	0.805	13360.19	1, 2, 3, 4, 5, 6	3	25.54
12	150	40	0.5	1	13360.19	1, 2, 3, 4, 5, 6	3	26.53
13	200	20	0.25	0.79	19262.44	4	2	9.93
14	200	20	0.25	0.895	19056.76	-	2	8.98
15	200	20	0.25	1	18946	-	2	10.11
16	200	20	0.5	0.79	19262.03	4	2	8.99
17	200	20	0.5	0.895	19056.76	-	2	9.98
18	200	20	0.5	1	18944.72	-	2	9.2
19	200	40	0.25	0.79	20786.16	4	2	13.95
20	200	40	0.25	0.895	20659.78	4	3	11.05
21	200	40	0.25	1	20581.27	4	3	25.53
22	200	40	0.5	0.79	20785.89	4	2	10.12
23	200	40	0.5	0.895	20659.65	4	3	9
24	200	40	0.5	1	20580.85	4	3	16.75
25	250	20	0.25	1	27331.31	4	2	9.16
26	250	20	0.25	1.08	27147.23	-	2	7.88
27	250	20	0.25	1.34	26887.75	-	2	8.5
28	250	20	0.5	1	27323.12	4	2	7.96
29	250	20	0.5	1.08	27145.47	-	2	6.8
30	250	20	0.5	1.34	26887.09	-	2	7.64
31	250	40	0.25	1	28854.83	4	2	13.82
32	250	40	0.25	1.08	28669.18	-	2	13.19
33	250	40	0.25	1.34	28395.55	-	2	11.25
34	250	40	0.5	1	28850.39	4	2	10.2
35	250	40	0.5	1.08	28664.85	-	2	9.99
36	250	40	0.5	1.34	28394.74	-	2	9.74

Table A.2. Results of 6-bus test system for MC method with the first strategy

Case #	Total Cost ($)	Location	No. Of Iterations	Total Time (s)	Gap (%)
1	11180.74	4	2	0.48	1.13
2	11076.88	5	2	0.48	1.38
3	11071.87	6	2	0.39	1.42
4	11180.74	4	2	0.4	1.13
5	11076.88	5	2	0.4	1.38
6	11071.87	6	2	0.34	1.42
7	12697.54	4	2	0.38	5.31
8	12576.32	5	2	0.39	5.87
9	12560.19	4	2	0.36	5.99
10	12694.9	4	2	0.3	5.31
11	12576.32	5	2	0.33	5.87
12	12560.19	4	2	0.35	5.99
13	19262.44	4	2	0.33	0
14	19056.77	-	2	0.26	0
15	18946.01	-	2	0.28	0
16	19262.04	4	2	0.3	0
17	19056.77	-	2	0.26	0
18	18944.73	-	2	0.27	0
19	20786.17	4	2	0.34	0
20	20574.99	-	2	0.28	0.41
21	20458.95	-	2	0.29	0.59
22	20785.9	4	2	0.45	0
23	20574.99	-	2	0.27	0.41
24	20458.34	-	2	0.31	0.6
25	27331.32	4	2	0.35	0
26	27147.23	-	2	0.29	0
27	26887.75	-	2	0.32	0
28	27323.12	4	2	0.36	0
29	27145.48	-	2	0.3	0
30	26887.09	-	2	0.29	0
31	28854.83	4	2	0.36	0
32	28669.18	-	2	0.34	0
33	28395.56	-	2	0.34	0
34	28850.4	4	2	0.35	0
35	28664.85	-	2	0.34	0
36	28394.74	-	2	0.29	0

Table A.3. Results of 6-bus test system for MC method with the second strategy

Case #	Total Cost ($)	Location	No. Of Iterations	Total Time (s)	Gap (%)
1	11308.60	4, 5	3	1.31	0.00
2	11231.87	4, 6	3	1.13	0.00
3	11231.87	1, 6	3	0.97	0.00
4	11308.60	4, 5	3	0.94	0.00
5	11231.87	4, 5	3	0.87	0.00
6	11231.87	1, 5	3	0.78	0.00
7	13409.19	1, 2, 3, 4, 5, 6	3	0.97	0.00
8	13360.19	1, 2, 3, 4, 5, 6	3	1.04	0.00
9	13360.19	1, 2, 3, 4, 5, 6	3	1.01	0.00
10	13406.55	1, 2, 3, 4, 5, 6	3	0.98	0.00
11	13360.19	1, 2, 3, 4, 5, 6	3	0.82	0.00
12	13360.19	1, 2, 3, 4, 5, 6	3	0.83	0.00
13	19262.44	4	2	0.40	0.00
14	19056.77	-	2	0.34	0.00
15	18946.01	-	2	0.31	0.00
16	19262.04	4	2	0.46	0.00
17	19056.77	-	2	0.30	0.00
18	18944.73	-	2	0.28	0.00
19	20786.17	4	2	0.43	0.00
20	20659.79	4	3	0.68	0.00
21	20581.27	4	3	0.67	0.00
22	20785.90	4	2	0.37	0.00
23	20659.66	4	3	0.68	0.00
24	20580.86	4	3	0.67	0.00
25	27331.32	4	2	0.38	0.00
26	27147.23	-	2	0.31	0.00
27	26887.75	-	2	0.33	0.00
28	27323.12	4	2	0.35	0.00
29	27145.48	-	2	0.32	0.00
30	26887.09	-	2	0.32	0.00
31	28854.83	4	2	0.38	0.00
32	28669.18	-	2	0.33	0.00
33	28395.56	-	2	0.32	0.00
34	28850.40	4	2	0.34	0.00
35	28664.85	-	2	0.30	0.00
36	28394.74	-	2	0.32	0.00

Table A.4. Results of 6-bus test system for MC method with the third strategy

Case #	**Total Cost ($)**	**Location**	**No. Of Iterations**	**Total Time (s)**	**Gap (%)**
1	11180.74	4	2	0.48	1.13
2	11076.88	5	2	0.50	1.38
3	11071.87	6	2	0.50	1.42
4	11180.74	4	2	0.40	1.13
5	11076.88	5	2	0.41	1.38
6	11071.87	6	2	0.36	1.42
7	12958.50	1, 4, 5	3	0.82	3.36
8	12880.49	1, 4, 5	3	1.10	3.59
9	12880.19	1, 5, 6	3	0.86	3.59
10	12955.87	1, 4, 5	3	0.74	3.36
11	12880.49	1, 4, 5	3	0.82	3.59
12	12880.19	4, 5, 6	3	0.85	3.59
13	19262.44	4	2	0.40	0.00
14	19056.77	-	2	0.31	0.00
15	18946.01	-	2	0.33	0.00
16	19262.04	4	2	0.42	0.00
17	19056.77	-	2	0.39	0.00
18	18944.73	-	2	0.48	0.00
19	20786.17	4	2	0.63	0.00
20	20574.99	-	2	0.53	0.41
21	20458.95	-	2	0.49	0.59
22	20785.90	4	2	0.61	0.00
23	20574.99	-	2	0.36	0.41
24	20458.34	-	2	0.35	0.60
25	27331.32	4	2	0.43	0.00
26	27147.23	-	2	0.33	0.00
27	26887.75	-	2	0.37	0.00
28	27323.12	4	2	0.39	0.00
29	27145.48	-	2	0.31	0.00
30	26887.09	-	2	0.30	0.00
31	28854.83	4	2	0.37	0.00
32	28669.18	-	2	0.33	0.00
33	28395.56	-	2	0.38	0.00
34	28850.40	4	2	0.35	0.00
35	28664.85	-	2	0.31	0.00
36	28394.74	-	2	0.29	0.00

Table A.5. Results of 6-bus test system for MC method with the fourth strategy

Case #	Total Cost ($)	Location	No. Of Iterations	Total Time (s)	Gap (%)
1	11308.60	4, 5	3	1.00	0.00
2	11231.87	4, 5	3	1.07	0.00
3	11231.87	1, 6	3	0.93	0.00
4	11308.60	4, 5	3	0.92	0.00
5	11231.87	4, 5	3	0.90	0.00
6	11231.87	1, 6	3	0.73	0.00
7	13102.34	1, 4, 5, 6	3	0.89	2.29
8	13040.19	1, 4, 5, 6	3	1.06	2.40
9	13040.19	3, 4, 5, 6	3	1.26	2.40
10	13099.70	1, 4, 5, 6	3	0.97	2.29
11	13040.19	1, 4, 5, 6	3	0.81	2.40
12	13040.19	1, 4, 5, 6	3	0.95	2.40
13	19262.44	4	2	0.38	0.00
14	19056.77	-	2	0.28	0.00
15	18946.01	-	2	0.33	0.00
16	19262.04	4	2	0.34	0.00
17	19056.77	-	2	0.30	0.00
18	18944.73	-	2	0.30	0.00
19	20786.17	4	2	0.38	0.00
20	20659.79	4	3	0.66	0.00
21	20581.27	4	3	0.69	0.00
22	20785.90	4	2	0.39	0.00
23	20659.66	4	3	0.66	0.00
24	20580.86	4	3	0.65	0.00
25	27331.32	4	2	0.41	0.00
26	27147.23	-	2	0.33	0.00
27	26887.75	-	2	0.34	0.00
28	27323.12	4	2	0.36	0.00
29	27145.48	-	2	0.31	0.00
30	26887.09	-	2	0.30	0.00
31	28854.83	4	2	0.38	0.00
32	28669.18	-	2	0.34	0.00
33	28395.56	-	2	0.35	0.00
34	28850.40	4	2	0.38	0.00
35	28664.85	-	2	0.35	0.00
36	28394.74	-	2	0.33	0.00

Table A.6. Results of 6-bus test system for MC method with the fifth strategy

Case #	Total Cost ($)	Location	No. Of Iterations	Total Time (s)	Gap (%)
1	11180.74	4	2	0.65	1.13
2	11076.88	5	2	0.57	1.38
3	11071.87	6	2	0.52	1.42
4	11180.74	4	2	0.63	1.13
5	11076.88	5	2	0.75	1.38
6	11071.87	6	2	0.62	1.42
7	12825.52	4, 5	3	1.51	4.35
8	12726.20	4, 5	3	1.27	4.75
9	12720.19	1, 5	3	0.91	4.79
10	12822.89	4, 5	3	0.98	4.35
11	12726.20	4, 5	3	0.83	4.75
12	12720.19	4, 5	3	0.77	4.79
13	19262.44	4	2	0.37	0.00
14	19056.77	-	2	0.30	0.00
15	18946.01	-	2	0.30	0.00
16	19262.04	4	2	0.37	0.00
17	19056.77	-	2	0.28	0.00
18	18944.73	-	2	0.27	0.00
19	20786.17	4	2	0.41	0.00
20	20574.99	-	2	0.30	0.41
21	20458.95	-	2	0.29	0.59
22	20785.90	4	2	0.43	0.00
23	20574.99	-	2	0.34	0.41
24	20458.34	-	2	0.29	0.60
25	27331.32	4	2	0.39	0.00
26	27147.23	-	2	0.32	0.00
27	26887.75	-	2	0.31	0.00
28	27323.12	4	2	0.36	0.00
29	27145.48	-	2	0.31	0.00
30	26887.09	-	2	0.29	0.00
31	28854.83	4	2	0.37	0.00
32	28669.18	-	2	0.33	0.00
33	28395.56	-	2	0.36	0.00
34	28850.40	4	2	0.37	0.00
35	28664.85	-	2	0.30	0.00
36	28394.74	-	2	0.29	0.00

Table A.7. Results of 6-bus test system for MC method with the sixth strategy

Case #	Total Cost ($)	Location	No. Of Iterations	Total Time (s)	Gap (%)
1	11308.60	4, 5	3	1.51	0.00
2	11231.87	4, 5	3	1.40	0.00
3	11231.87	1, 5	3	1.06	0.00
4	11308.60	4, 5	3	0.97	0.00
5	11231.87	4, 5	3	0.94	0.00
6	11231.87	2, 5	3	0.78	0.00
7	13102.34	1, 4, 5, 6	3	1.02	2.29
8	13040.19	1, 4, 5, 6	3	1.16	2.40
9	13040.19	2, 4, 5, 6	3	1.02	2.40
10	13099.70	1, 4, 5, 6	3	0.83	2.29
11	13040.19	1, 4, 5, 6	3	0.92	2.40
12	13040.19	1, 4, 5, 6	3	0.84	2.40
13	19262.44	4	2	0.38	0.00
14	19056.77	-	2	0.33	0.00
15	18946.01	-	2	0.32	0.00
16	19262.04	4	2	0.37	0.00
17	19056.77	-	2	0.33	0.00
18	18944.73	-	2	0.30	0.00
19	20786.17	4	2	0.43	0.00
20	20659.79	4	3	0.72	0.00
21	20581.27	4	3	0.74	0.00
22	20785.90	4	2	0.43	0.00
23	20659.66	4	3	0.67	0.00
24	20580.86	4	3	0.68	0.00
25	27331.32	4	2	0.36	0.00
26	27147.23	-	2	0.34	0.00
27	26887.75	-	2	0.33	0.00
28	27323.12	4	2	0.38	0.00
29	27145.48	-	2	0.32	0.00
30	26887.09	-	2	0.34	0.00
31	28854.83	4	2	0.41	0.00
32	28669.18	-	2	0.38	0.00
33	28395.56	-	2	0.39	0.00
34	28850.40	4	2	0.38	0.00
35	28664.85	-	2	0.35	0.00
36	28394.74	-	2	0.33	0.00

Table A.8. Results of 6-bus test system for MC method with the seventh strategy

Case #	Total Cost ($)	Location	No. Of Iterations	Total Time (s)	Gap (%)
1	11308.60	4, 5	3	1.61	0.00
2	11231.87	4, 5	3	1.51	0.00
3	11231.87	1, 5	3	1.19	0.00
4	11308.60	4, 5	3	1.33	0.00
5	11231.87	4, 5	3	1.11	0.00
6	11231.87	1, 5	3	1.00	0.00
7	13102.34	1, 4, 5, 6	3	1.17	2.29
8	13040.19	1, 4, 5, 6	3	1.51	2.40
9	13040.19	1, 4, 5, 6	3	1.14	2.40
10	13099.70	1, 4, 5, 6	3	1.17	2.29
11	13040.19	1, 4, 5, 6	3	1.82	2.40
12	13040.19	1, 4, 5, 6	3	1.13	2.40
13	19262.44	4	2	0.57	0.00
14	19056.77	-	2	0.51	0.00
15	18946.01	-	2	0.64	0.00
16	19262.04	4	2	0.57	0.00
17	19056.77	-	2	0.69	0.00
18	18944.73	-	2	0.56	0.00
19	20786.17	4	2	0.65	0.00
20	20574.99	-	2	0.49	0.41
21	20458.95	-	2	0.59	0.59
22	20785.90	4	2	0.73	0.00
23	20574.99	-	2	0.69	0.41
24	20458.34	-	2	0.55	0.60
25	27331.32	4	2	0.70	0.00
26	27147.23	-	2	0.55	0.00
27	26887.75	-	2	0.43	0.00
28	27323.12	4	2	0.53	0.00
29	27145.48	-	2	0.63	0.00
30	26887.09	-	2	0.54	0.00
31	28854.83	4	2	0.75	0.00
32	28669.18	-	2	0.56	0.00
33	28395.56	-	2	0.58	0.00
34	28850.40	4	2	0.55	0.00
35	28664.85	-	2	0.55	0.00
36	28394.74	-	2	0.65	0.00

Table A.9. Results of 6-bus test system for the hybrid method

Case #	**Total Cost ($)**	**Location**	**No. Of Iterations**	**Total Time (s)**	**Gap (%)**
1	11308.60	4, 5	3	13.23	0.00
2	11231.87	4, 6	3	11.58	0.00
3	11231.87	1, 6	3	9.87	0.00
4	11308.60	4, 5	3	1.95	0.00
5	11231.87	4, 5	3	1.51	0.00
6	11231.87	1, 5	3	1.39	0.00
7	13409.19	1, 2, 3, 4, 5, 6	3	1.34	0.00
8	13360.19	1, 2, 3, 4, 5, 6	3	1.30	0.00
9	13360.19	1, 2, 3, 4, 5, 6	3	1.25	0.00
10	13406.55	1, 2, 3, 4, 5, 6	3	1.19	0.00
11	13360.19	1, 2, 3, 4, 5, 6	3	0.96	0.00
12	13360.19	1, 2, 3, 4, 5, 6	3	0.94	0.00
13	19262.44	4	2	0.46	0.00
14	19056.77	-	2	0.35	0.00
15	18946.01	-	2	0.41	0.00
16	19262.04	4	2	0.48	0.00
17	19056.77	-	2	0.42	0.00
18	18944.73	-	2	0.35	0.00
19	20786.17	4	2	0.48	0.00
20	20659.79	4	3	0.78	0.00
21	20581.27	4	3	1.01	0.00
22	20785.90	4	2	0.45	0.00
23	20659.66	4	3	0.79	0.00
24	20580.86	4	3	0.89	0.00
25	27331.32	4	2	0.53	0.00
26	27147.23	-	2	0.47	0.00
27	26887.75	-	2	0.38	0.00
28	27323.12	4	2	0.44	0.00
29	27145.48	-	2	0.38	0.00
30	26887.09	-	2	0.38	0.00
31	28854.83	4	2	0.47	0.00
32	28669.18	-	2	0.43	0.00
33	28395.56	-	2	0.44	0.00
34	28850.40	4	2	0.46	0.00
35	28664.85	-	2	0.46	0.00
36	28394.74	-	2	0.38	0.00

Table A.10. Results of 14-bus test system for the exact solution method

Case #	Maximum Demand (MW)	Wind Power Deviation	Ramp-Rate Limit Factor	Flow Limit Factor	Total Cost ($)	Location	No. Of Iterations	Total Time (s)
1	100	25	0.25	0.24	1930.21	1, 2, 4, 12	3	113.36
2	100	25	0.25	0.62	1930.21	2, 3, 10, 14	3	97.16
3	100	25	0.25	1	1930.21	2, 3, 10, 14	3	30.39
4	100	25	0.5	0.24	1930.21	2, 4, 8, 10	3	79.01
5	100	25	0.5	0.62	1930.21	2, 4, 5, 12	3	30.14
6	100	25	0.5	1	1930.21	2, 4, 5, 12	3	97.92
7	100	50	0.25	0.24	2891.77	1, 2, 3, 4, 7, 8	3	43.11
8	100	50	0.25	0.62	2891.77	2, 4, 5, 9, 12, 14	3	141.62
9	100	50	0.25	1	2891.77	1, 2, 4, 9, 12, 14	3	194.70
10	100	50	0.5	0.24	2891.77	2, 3, 4, 5, 7, 8	3	38.61
11	100	50	0.5	0.62	2891.77	2, 3, 4, 5, 6, 11	3	48.54
12	100	50	0.5	1	2891.77	2, 3, 4, 5, 6, 11	3	49.17
13	200	25	0.25	0.29	14012.18	3, 4, 7, 8, 9, 14	3	95.29
14	200	25	0.25	0.49	9007.59	5, 6, 10, 11, 12, 13	2	259.49
15	200	25	0.25	0.69	8544.02	5, 6, 12	2	14.03
16	200	25	0.5	0.29	14012.19	3, 4, 7, 8, 9, 14	3	11.47
17	200	25	0.5	0.49	8551.60	5, 6, 10, 11, 12, 13	2	111.97
18	200	25	0.5	0.69	8088.03	5, 6, 12	2	14.62
19	200	50	0.25	0.29	16235.82	2, 3, 4, 5, 6, 7, 8, 9	3	420.72
20	200	50	0.25	0.49	10511.76	5, 6, 11, 12, 13	2	15.42
21	200	50	0.25	0.69	9281.90	5, 6, 11, 12	2	22.00
22	200	50	0.5	0.29	16075.81	3, 4, 5, 6, 7, 8, 14	3	124.47
23	200	50	0.5	0.49	10294.37	5, 6, 11, 12, 13	2	10.75
24	200	50	0.5	0.69	8850.34	5, 6, 11, 12	2	20.80
25	300	25	0.25	0.62	21430.33	5, 6, 9, 10, 11, 12, 13, 14	3	105.45
26	300	25	0.25	1	15814.23	5, 6, 10, 11, 12, 13	2	28.88
27	300	25	0.25	1.26	15267.75	5, 6	2	7.81
28	300	25	0.5	0.62	21430.33	5, 6, 9, 10, 11, 12, 13, 14	3	12.72
29	300	25	0.5	1	14908.67	5, 6, 10, 11, 12, 13	2	17.34
30	300	25	0.5	1.26	14362.19	5, 6	2	5.80
31	300	50	0.25	0.62	23493.96	5, 6, 8, 9, 10, 11, 12, 13,14	3	46.50
32	300	50	0.25	1	16602.77	5, 6, 10, 11, 12, 13,14	2	102.12
33	300	50	0.25	1.26	16001.50	5, 6, 12	2	17.20
34	300	50	0.5	0.62	23493.96	5, 6, 7, 9, 10, 11, 12, 13, 14	3	36.56
35	300	50	0.5	1	15697.21	5, 6, 10, 11, 12, 13, 14	2	18.85
36	300	50	0.5	1.26	15095.95	5, 6, 12	2	12.99

Table A.11. Results of 14-bus test system for MC method with the first strategy

Case #	Total Cost ($)	Location	No. Of Iterations	Total Time (s)	Gap (%)
1	1625.89	4, 5	2	6.20	15.77
2	1625.89	2, 3	2	2.07	15.77
3	1625.89	2, 3	2	2.46	15.77
4	1625.89	4, 5	2	4.03	15.77
5	1625.89	2, 3	2	1.95	15.77
6	1625.89	2, 3	2	2.93	15.77
7	2433.65	4, 5, 6	2	3.10	15.84
8	2433.65	2, 3, 4	2	1.96	15.84
9	2433.65	2, 3, 14	2	1.40	15.84
10	2433.65	4, 5, 6	2	5.51	15.84
11	2433.65	2, 4, 6	2	1.27	15.84
12	2433.65	2, 4, 6	2	1.27	15.84
13	14012.19	3, 4, 7, 8, 9, 14	3	1.29	0.00
14	9007.60	5, 6, 10, 11, 12, 13	2	1.46	0.00
15	8544.03	5, 6, 12	2	0.65	0.00
16	14012.19	3, 4, 7, 8, 9, 14	3	1.30	0.00
17	8551.60	5, 6, 10, 11, 12, 13	2	1.37	0.00
18	8088.03	5, 6, 12	2	0.67	0.00
19	15915.82	3, 4, 7, 8, 9, 14	2	1.09	1.97
20	10511.77	5, 6, 11, 12, 13	2	0.81	0.00
21	9281.91	5, 6, 11, 12	2	1.06	0.00
22	15915.82	3, 4, 7, 8, 9, 14	2	0.92	1.00
23	10294.38	5, 6, 11, 12, 13	2	0.95	0.00
24	8850.34	5, 6, 11, 12	2	0.89	0.00
25	21430.34	5, 6, 9, 10, 11, 12, 13, 14	3	2.58	0.00
26	15814.24	5, 6, 10, 11, 12, 13	2	1.16	0.00
27	15267.76	5, 6	2	1.27	0.00
28	21430.34	5, 6, 9, 10, 11, 12, 13, 14	3	3.99	0.00
29	14908.68	5, 6, 10, 11, 12, 13	2	0.96	0.00
30	14362.19	5, 6	2	1.22	0.00
31	23493.97	5, 6, 8, 9, 10, 11, 12, 13, 14	3	1.86	0.00
32	16602.78	5, 6, 10, 11, 12, 13, 14	2	1.38	0.00
33	16001.52	5, 6, 12	2	0.64	0.00
34	23493.97	5, 6, 7, 9, 10, 11, 12, 13, 14	3	3.27	0.00
35	15697.22	5, 6, 10, 11, 12, 13, 14	2	1.11	0.00
36	15095.95	5, 6, 12	2	0.59	0.00

Table A.12. Results of 14-bus test system for MC method with the second strategy

Case #	Total Cost ($)	Location	No. Of Iterations	Total Time (s)	Gap (%)
1	948.28	1, 2, 4, 6	2	5.02	50.87
2	948.28	2, 5, 6, 7	2	2.24	50.87
3	948.28	1, 2, 3, 7	2	2.48	50.87
4	948.28	2, 3, 4, 12	2	3.36	50.87
5	948.28	2, 3, 5, 7	2	1.50	50.87
6	948.28	2, 5, 9, 12	2	1.94	50.87
7	1224.92	1, 2, 3, 4, 7, 8	2	1.76	57.64
8	1224.92	1, 2, 5, 6, 12, 13	2	11.24	57.64
9	1224.92	1, 2, 5, 6, 12, 13	2	10.09	57.64
10	1224.92	1, 2, 3, 4, 7, 8	2	4.35	57.64
11	1224.92	1, 2, 3, 4, 5, 7	2	10.75	57.64
12	1224.92	1, 2, 3, 4, 5, 7	2	10.86	57.64
13	14012.19	3, 4, 7, 8, 9, 14	3	1.57	0.00
14	9007.60	5, 6, 10, 11, 12, 13	2	1.63	0.00
15	8544.03	5, 6, 12	2	0.75	0.00
16	14012.19	3, 4, 7, 8, 9, 14	3	1.40	0.00
17	8551.60	5, 6, 10, 11, 12, 13	2	1.54	0.00
18	8088.03	5, 6, 12	2	0.76	0.00
19	16235.82	2, 3, 4, 5, 6, 7, 8, 9	3	4.58	0.00
20	10511.77	5, 6, 11, 12, 13	2	0.89	0.00
21	9281.91	5, 6, 11, 12	2	1.17	0.00
22	15915.82	3, 4, 7, 8, 9, 14	2	1.05	1.00
23	10294.38	5, 6, 11, 12, 13	2	1.03	0.00
24	8850.34	5, 6, 11, 12	2	1.03	0.00
25	21430.34	5, 6, 9, 10, 11, 12, 13, 14	3	2.91	0.00
26	15814.24	5, 6, 10, 11, 12, 13	2	1.28	0.00
27	15267.76	5, 6	2	1.31	0.00
28	21430.34	5, 6, 9, 10, 11, 12, 13, 14	3	4.21	0.00
29	14908.68	5, 6, 10, 11, 12, 13	2	1.08	0.00
30	14362.19	5, 6	2	1.22	0.00
31	23493.97	5, 6, 8, 9, 10, 11, 12, 13, 14	3	1.88	0.00
32	16602.78	5, 6, 10, 11, 12, 13, 14	2	1.49	0.00
33	16001.52	5, 6, 12	2	0.74	0.00
34	23493.97	5, 6, 7, 9, 10, 11, 12, 13, 14	3	3.55	0.00
35	15697.22	5, 6, 10, 11, 12, 13, 14	2	1.29	0.00
36	15095.95	5, 6, 12	2	0.69	0.00

Table A.13. Results of 14-bus test system for MC method with the third strategy

Case #	Total Cost ($)	Location	No. Of Iterations	Total Time (s)	Gap (%)
1	1770.22	3, 5, 7	3	12.30	8.29
2	1770.22	4, 8, 12	3	11.75	8.29
3	1770.22	4, 8, 12	3	13.21	8.29
4	1770.22	2, 4, 11	3	12.87	8.29
5	1770.22	1, 2, 11	3	13.55	8.29
6	1770.22	1, 2, 11	3	13.74	8.29
7	2733.65	1, 2, 3, 4, 11	3	2.32	5.47
8	2733.65	1, 2, 3, 8, 10	3	2.15	5.47
9	2733.65	2, 3, 4, 10, 12	3	2.08	5.47
10	2733.65	1, 2, 3, 4, 7	3	7.68	5.47
11	2733.65	1, 2, 3, 4, 11	3	1.61	5.47
12	2733.65	1, 2, 3, 4, 11	3	1.86	5.47
13	14012.19	3, 4, 7, 8, 9, 14	3	1.54	0.00
14	9007.60	5, 6, 10, 11, 12, 13	2	1.65	0.00
15	8544.03	5, 6, 12	2	0.80	0.00
16	14012.19	3, 4, 7, 8, 9, 14	3	1.41	0.00
17	8551.60	5, 6, 10, 11, 12, 13	2	1.59	0.00
18	8088.03	5, 6, 12	2	0.75	0.00
19	16075.82	3, 4, 7, 8, 9, 10, 14	3	1.69	0.99
20	10511.77	5, 6, 11, 12, 13	2	0.91	0.00
21	9281.91	5, 6, 11, 12	2	1.21	0.00
22	15915.82	3, 4, 7, 8, 9, 14	2	1.02	1.00
23	9879.93	5, 6, 11, 12, 13	3	2.02	4.03
24	8850.34	5, 6, 11, 12	2	1.04	0.00
25	21430.34	5, 6, 9, 10, 11, 12, 13, 14	3	2.94	0.00
26	15814.24	5, 6, 10, 11, 12, 13	2	1.23	0.00
27	15267.76	5, 6	2	1.36	0.00
28	21430.34	5, 6, 9, 10, 11, 12, 13, 14	3	4.24	0.00
29	14908.68	5, 6, 10, 11, 12, 13	2	1.05	0.00
30	14362.19	5, 6	2	1.25	0.00
31	23493.97	5, 6, 8, 9, 10, 11, 12, 13, 14	3	2.09	0.00
32	16602.78	5, 6, 10, 11, 12, 13, 14	2	1.52	0.00
33	16001.52	5, 6, 12	2	0.75	0.00
34	23493.97	5, 6, 7, 9, 10, 11, 12, 13, 14	3	2.89	0.00
35	15697.22	5, 6, 10, 11, 12, 13, 14	2	1.30	0.00
36	15095.95	5, 6, 12	2	0.70	0.00

Table A.14. Results of 14-bus test system for MC method with the fourth strategy

Case #	Total Cost ($)	Location	No. Of Iterations	Total Time (s)	Gap (%)
1	1625.89	4, 5	2	8.50	15.77
2	1625.89	2, 3	2	3.23	15.77
3	1625.89	2, 3	2	3.22	15.77
4	1625.89	4, 5	2	5.19	15.77
5	1625.89	2, 3	2	2.12	15.77
6	1625.89	2, 3	2	3.65	15.77
7	2583.16	1, 2, 3, 6	3	4.93	10.67
8	2583.16	2, 3, 9, 10	3	2.89	10.67
9	2583.16	2, 3, 9, 10	3	2.19	10.67
10	2583.16	3, 4, 5, 12	3	6.13	10.67
11	2583.16	1, 2, 3, 7	3	1.91	10.67
12	2583.16	4, 5, 8, 13	3	2.24	10.67
13	14012.19	3, 4, 7, 8, 9, 14	3	1.64	0.00
14	9007.60	5, 6, 10, 11, 12, 13	2	1.60	0.00
15	8544.03	5, 6, 12	2	0.75	0.00
16	14012.19	3, 4, 7, 8, 9, 14	3	1.42	0.00
17	8551.60	5, 6, 10, 11, 12, 13	2	1.80	0.00
18	8088.03	5, 6, 12	2	0.80	0.00
19	15672.64	3, 4, 7, 8, 9, 14	2	2.25	3.47
20	10511.77	5, 6, 11, 12, 13	2	0.95	0.00
21	9281.91	5, 6, 11, 12	2	1.13	0.00
22	15915.82	3, 4, 7, 8, 9, 14	2	1.07	1.00
23	10294.38	5, 6, 11, 12, 13	2	1.16	0.00
24	8850.34	5, 6, 11, 12	2	1.15	0.00
25	21430.34	5, 6, 9, 10, 11, 12, 13, 14	3	2.83	0.00
26	15814.24	5, 6, 10, 11, 12, 13	2	1.25	0.00
27	15267.76	5, 6	2	1.40	0.00
28	21430.34	5, 6, 9, 10, 11, 12, 13, 14	3	2.73	0.00
29	14908.68	5, 6, 10, 11, 12, 13	2	1.10	0.00
30	14362.19	5, 6	2	1.25	0.00
31	23493.97	5, 6, 7, 9, 10, 11, 12, 13, 14	3	1.69	0.00
32	16602.78	5, 6, 10, 11, 12, 13, 14	2	1.83	0.00
33	16001.52	5, 6, 12	2	1.04	0.00
34	23493.97	5, 6, 7, 9, 10, 11, 12, 13, 14	3	2.90	0.00
35	15697.22	5, 6, 10, 11, 12, 13, 14	2	1.75	0.00
36	15095.95	5, 6, 12	2	1.02	0.00

Table A.15. Results of 14-bus test system for MC method with the fifth strategy

Case #	Total Cost ($)	Location	No. Of Iterations	Total Time (s)	Gap (%)
1	1770.22	1, 4, 5	3	5.32	8.29
2	1770.22	2, 4, 8	3	2.64	8.29
3	1770.22	2, 4, 8	3	3.03	8.29
4	1770.22	2, 4, 10	3	3.03	8.29
5	1770.22	2, 4, 12	3	1.65	8.29
6	1770.22	2, 4, 12	3	2.26	8.29
7	2583.16	2, 3, 4, 6	3	5.98	10.67
8	2583.16	2, 3, 8, 10	3	2.65	10.67
9	2583.16	2, 3, 10, 14	3	1.77	10.67
10	2583.16	1, 2, 4, 7	3	4.89	10.67
11	2583.16	2, 4, 5, 13	3	2.11	10.67
12	2583.16	1, 2, 3, 11	3	2.27	10.67
13	14012.19	3, 4, 7, 8, 9, 14	3	1.39	0.00
14	9007.60	5, 6, 10, 11, 12, 13	2	1.48	0.00
15	8544.03	5, 6, 12	2	0.70	0.00
16	14012.19	3, 4, 7, 8, 9, 14	3	1.33	0.00
17	8551.60	5, 6, 10, 11, 12, 13	2	1.43	0.00
18	8088.03	5, 6, 12	2	0.68	0.00
19	15915.82	3, 4, 7, 8, 9, 14	2	1.16	1.97
20	10511.77	5, 6, 11, 12, 13	2	0.90	0.00
21	9281.91	5, 6, 11, 12	2	1.12	0.00
22	15915.82	3, 4, 7, 8, 9, 14	2	1.02	1.00
23	10294.38	5, 6, 11, 12, 13	2	1.03	0.00
24	8850.34	5, 6, 11, 12	2	1.02	0.00
25	21430.34	5, 6, 9, 10, 11, 12, 13, 14	3	2.78	0.00
26	15814.24	5, 6, 10, 11, 12, 13	2	1.25	0.00
27	15267.76	5, 6	2	1.31	0.00
28	21430.34	5, 6, 9, 10, 11, 12, 13, 14	3	4.14	0.00
29	14908.68	5, 6, 10, 11, 12, 13	2	1.04	0.00
30	14362.19	5, 6	2	1.18	0.00
31	23493.97	5, 6, 8, 9, 10, 11, 12, 13, 14	3	1.98	0.00
32	16602.78	5, 6, 10, 11, 12, 13, 14	2	1.59	0.00
33	16001.52	5, 6, 12	2	0.78	0.00
34	23493.97	5, 6, 7, 9, 10, 11, 12, 13, 14	3	3.51	0.00
35	15697.22	5, 6, 10, 11, 12, 13, 14	2	1.28	0.00
36	15095.95	5, 6, 12	2	0.64	0.00

Table A.16. Results of 14-bus test system for MC method with the sixth strategy

Case #	Total Cost ($)	Location	No. Of Iterations	Total Time (s)	Gap (%)
1	1770.22	1, 4, 5	3	5.35	8.29
2	1770.22	2, 4, 11	3	2.29	8.29
3	1770.22	2, 4, 11	3	2.59	8.29
4	1770.22	4, 5, 7	3	6.93	8.29
5	1770.22	2, 4, 12	3	1.98	8.29
6	1770.22	2, 4, 12	3	2.52	8.29
7	2583.16	2, 3, 4, 6	3	6.28	10.67
8	2583.16	1, 2, 3, 8	3	2.56	10.67
9	2583.16	1, 2, 3, 14	3	1.82	10.67
10	2583.16	2, 3, 4, 12	3	7.79	10.67
11	2583.16	1, 2, 3, 11	3	2.26	10.67
12	2583.16	1, 2, 3, 11	3	1.84	10.67
13	14012.19	3, 4, 7, 8, 9, 14	3	1.42	0.00
14	9007.60	5, 6, 10, 11, 12, 13	2	1.47	0.00
15	8544.03	5, 6, 12	2	0.72	0.00
16	14012.19	3, 4, 7, 8, 9, 14	3	1.29	0.00
17	8551.60	5, 6, 10, 11, 12, 13	2	1.46	0.00
18	8088.03	5, 6, 12	2	0.71	0.00
19	15915.82	3, 4, 7, 8, 9, 14	2	1.18	1.97
20	10511.77	5, 6, 11, 12, 13	2	0.85	0.00
21	9281.91	5, 6, 11, 12	2	1.11	0.00
22	15915.82	3, 4, 7, 8, 9, 14	2	1.04	1.00
23	10294.38	5, 6, 11, 12, 13	2	1.04	0.00
24	8850.34	5, 6, 11, 12	2	1.00	0.00
25	21430.34	5, 6, 9, 10, 11, 12, 13, 14	3	2.73	0.00
26	15814.24	5, 6, 10, 11, 12, 13	2	1.23	0.00
27	15267.76	5, 6	2	1.28	0.00
28	21430.34	5, 6, 9, 10, 11, 12, 13, 14	3	4.07	0.00
29	14908.68	5, 6, 10, 11, 12, 13	2	1.03	0.00
30	14362.19	5, 6	2	1.15	0.00
31	23493.97	5, 6, 8, 9, 10, 11, 12, 13, 14	3	1.83	0.00
32	16602.78	5, 6, 10, 11, 12, 13, 14	2	1.45	0.00
33	16001.52	5, 6, 12	2	0.74	0.00
34	23493.97	5, 6, 7, 9, 10, 11, 12, 13, 14	3	3.37	0.00
35	15697.22	5, 6, 10, 11, 12, 13, 14	2	1.21	0.00
36	15095.95	5, 6, 12	2	0.66	0.00

Table A.17. Results of 14-bus test system for MC method with the seventh strategy

Case #	Total Cost ($)	Location	No. Of Iterations	Total Time (s)	Gap (%)
1	1770.22	4, 5, 6	3	7.57	8.29
2	1770.22	1, 4, 10	3	2.59	8.29
3	1770.22	1, 4, 10	3	2.69	8.29
4	1770.22	4, 5, 11	3	8.09	8.29
5	1770.22	1, 2, 11	3	2.51	8.29
6	1770.22	2, 4, 6	3	2.99	8.29
7	2583.16	2, 3, 4, 6	3	4.06	10.67
8	2583.16	2, 3, 10, 14	3	2.91	10.67
9	2583.16	2, 3, 8, 10	3	1.88	10.67
10	2583.16	2, 3, 4, 12	3	8.13	10.67
11	2583.16	1, 2, 3, 11	3	2.23	10.67
12	2583.16	1, 2, 3, 11	3	2.14	10.67
13	14012.19	3, 4, 7, 8, 9, 14	3	1.45	0.00
14	9007.60	5, 6, 10, 11, 12, 13	2	1.62	0.00
15	8544.03	5, 6, 12	2	0.75	0.00
16	14012.19	3, 4, 7, 8, 9, 14	3	1.45	0.00
17	8551.60	5, 6, 10, 11, 12, 13	2	1.51	0.00
18	8088.03	5, 6, 12	2	0.77	0.00
19	16075.82	3, 4, 7, 8, 9, 10, 14	3	1.55	0.99
20	10511.77	5, 6, 11, 12, 13	2	0.88	0.00
21	9281.91	5, 6, 11, 12	2	1.12	0.00
22	15915.82	3, 4, 7, 8, 9, 14	2	0.95	1.00
23	10294.38	5, 6, 11, 12, 13	2	1.05	0.00
24	8850.34	5, 6, 11, 12	2	0.99	0.00
25	21430.34	5, 6, 9, 10, 11, 12, 13, 14	3	2.76	0.00
26	15814.24	5, 6, 10, 11, 12, 13	2	1.20	0.00
27	15267.76	5, 6	2	1.28	0.00
28	21430.34	5, 6, 9, 10, 11, 12, 13, 14	3	4.06	0.00
29	14908.68	5, 6, 10, 11, 12, 13	2	1.02	0.00
30	14362.19	5, 6	2	1.16	0.00
31	23493.97	5, 6, 8, 9, 10, 11, 12, 13, 14	3	1.82	0.00
32	16602.78	5, 6, 10, 11, 12, 13, 14	2	1.50	0.00
33	16001.52	5, 6, 12	2	0.71	0.00
34	23493.97	5, 6, 7, 9, 10, 11, 12, 13, 14	3	3.43	0.00
35	15697.22	5, 6, 10, 11, 12, 13, 14	2	1.29	0.00
36	15095.95	5, 6, 12	2	0.67	0.00

Table A.18. Results of 14-bus test system for the hybrid method

Case #	Total Cost ($)	Location	No. Of Iterations	Total Time (s)	Gap (%)
1	1930.22	1, 2, 4, 12	3	8.23	0.00
2	948.28	2, 5, 6, 7	2	2.65	50.87
3	948.28	1, 2, 3, 7	2	2.74	50.87
4	1930.22	2, 4, 8, 10	3	4.69	0.00
5	948.28	2, 3, 5, 7	2	1.81	50.87
6	948.28	2, 5, 9, 12	2	2.37	50.87
7	2891.78	1, 2, 3, 4, 7, 8	3	7.46	0.00
8	2891.78	2, 4, 5, 9, 12, 14	3	22.82	0.00
9	1224.92	1, 2, 5, 6, 12, 13	2	12.84	57.64
10	2065.04	2, 3, 4, 5, 7, 8	3	6.52	28.59
11	2891.78	2, 3, 4, 5, 6, 11	3	23.41	0.00
12	2891.78	2, 3, 4, 5, 6, 11	3	22.49	0.00
13	14012.19	3, 4, 7, 8, 9, 14	3	2.92	0.00
14	9007.60	5, 6, 10, 11, 12, 13	2	2.16	0.00
15	8544.03	5, 6, 12	2	1.30	0.00
16	14012.19	3, 4, 7, 8, 9, 14	3	2.26	0.00
17	8551.60	5, 6, 10, 11, 12, 13	2	1.90	0.00
18	8088.03	5, 6, 12	2	1.17	0.00
19	16235.82	2, 3, 4, 5, 6, 7, 8, 9	3	31.14	0.00
20	10511.77	5, 6, 11, 12, 13	2	1.51	0.00
21	9281.91	5, 6, 11, 12	2	2.62	0.00
22	16075.82	3, 4, 5, 6, 7, 8, 14	3	10.84	0.00
23	10294.38	5, 6, 11, 12, 13	2	1.52	0.00
24	8850.34	5, 6, 11, 12	2	1.60	0.00
25	21430.34	5, 6, 9, 10, 11, 12, 13, 14	3	3.38	0.00
26	15814.24	5, 6, 10, 11, 12, 13	2	1.63	0.00
27	15267.76	5, 6	2	1.62	0.00
28	21430.34	5, 6, 9, 10, 11, 12, 13, 14	3	3.95	0.00
29	14908.68	5, 6, 10, 11, 12, 13	2	1.37	0.00
30	14362.19	5, 6	2	1.43	0.00
31	23493.97	5, 6, 8, 9, 10, 11, 12, 13, 14	3	5.97	0.00
32	16602.78	5, 6, 10, 11, 12, 13, 14	2	2.03	0.00
33	16001.52	5, 6, 12	2	1.39	0.00
34	23493.97	5, 6, 7, 9, 10, 11, 12, 13, 14	3	5.01	0.00
35	15697.22	5, 6, 10, 11, 12, 13, 14	2	1.65	0.00
36	15095.95	5, 6, 12	2	1.02	0.00

Table A.19. Results of 30-bus test system for the exact solution method

Case #	Maximum Demand (MW)	Wind Power Deviation	Ramp-Rate Limit Factor	Flow Limit Factor	Total Cost ($)	Location	No. Of Iterations	Total Time (s)
1	100	20	0.25	0.29	3156.79	2, 11, 30	3	195.27
2	100	20	0.25	0.645	3156.79	1, 2, 10	3	200.00
3	100	20	0.25	1	3156.79	1, 2, 30	3	57.89
4	100	20	0.50	0.29	3156.79	1, 11, 26	3	120.88
5	100	20	0.50	0.645	3156.79	1, 2, 24	3	112.71
6	100	20	0.50	1	3156.79	2, 7, 24	3	177.84
7	100	30	0.25	0.29	3833.12	2, 11, 30	3	374.31
8	100	30	0.25	0.645	3833.12	1, 4, 30	3	342.19
9	100	30	0.25	1	3833.12	5, 17, 24	3	154.07
10	100	30	0.50	0.29	3833.12	2, 11, 27	3	142.65
11	100	30	0.50	0.645	3833.12	2, 19, 30	3	97.66
12	100	30	0.50	1	3833.12	2, 6, 9	3	266.76
13	200	20	0.25	0.41	17441.49	2, 5, 7, 10	2	545.85
14	200	20	0.25	0.48	17302.98	2, 5, 9	2	93.75
15	200	20	0.25	0.55	17231.20	2, 5	2	64.70
16	200	20	0.50	0.41	17441.49	2, 5, 7, 10	2	654.54
17	200	20	0.50	0.48	17302.98	2, 5, 9	2	97.55
18	200	20	0.50	0.55	17231.20	2, 5	2	61.17
19	200	30	0.25	0.41	18161.50	2, 5, 7, 10	2	688.25
20	200	30	0.25	0.48	18105.73	2, 5, 11	2	337.31
21	200	30	0.25	0.55	17957.83	2, 5, 7	2	75.11
22	200	30	0.50	0.41	18161.50	2, 5, 7, 10	2	749.64
23	200	30	0.50	0.48	18105.73	2, 5, 11	2	344.29
24	200	30	0.50	0.55	17957.83	2, 5, 7	2	73.49
25	300	20	0.25	0.64	33542.06	2, 5, 6, 7, 10, 11	2	93.64
26	300	20	0.25	0.82	31726.20	2, 5, 6, 7, 21	2	848.87
27	300	20	0.25	1	31370.52	2, 5	2	28.26
28	300	20	0.50	0.64	33542.06	2, 5, 6, 7, 10, 11	2	83.22
29	300	20	0.50	0.82	31726.20	2, 5, 6, 7, 11	2	927.15
30	300	20	0.50	1	31370.52	2, 5	2	19.66
31	300	30	0.25	0.64	35337.22	2, 5, 7, 9, 10, 11	2	34.18
32	300	30	0.25	0.82	32402.54	2, 5, 6, 7, 9	2	720.67
33	300	30	0.25	1	32082.54	2, 5, 7	2	90.00
34	300	30	0.50	0.64	35337.07	2, 5, 6, 7, 10, 11	2	33.35
35	300	30	0.50	0.82	32402.54	2, 5, 6, 7, 11	2	718.07
36	300	30	0.50	1	32082.54	2, 5, 7	2	41.32

Table A.20. Results of 30-bus test system for MC method with the first strategy

Case #	Total Cost ($)	Location	No. Of Iterations	Total Time (s)	Gap (%)
1	2996.80	11, 27	2	6.01	5.07
2	2996.80	3, 19	2	123.38	5.07
3	2996.80	6, 10	2	149.19	5.07
4	2996.80	2, 11	2	5.38	5.07
5	2996.80	3, 21	2	156.22	5.07
6	2996.80	2, 9	2	120.22	5.07
7	3673.13	11, 29	2	4.12	4.17
8	3673.13	1, 12	2	38.95	4.17
9	3673.13	7, 10	2	15.74	4.17
10	3673.13	11, 29	2	3.69	4.17
11	3673.13	2, 11	2	3.93	4.17
12	3673.13	5, 18	2	11.07	4.17
13	17441.50	2, 5, 7, 10	2	482.60	0.00
14	17302.99	2, 5, 9	2	62.51	0.00
15	17231.21	2, 5	2	53.59	0.00
16	17441.50	2, 5, 7, 10	2	581.51	0.00
17	17302.99	2, 5, 9	2	59.25	0.00
18	17231.21	2, 5	2	49.20	0.00
19	18161.52	2, 5, 7, 10	2	653.16	0.00
20	18105.74	2, 5, 11	2	300.79	0.00
21	17957.84	2, 5, 7	2	55.77	0.00
22	18161.52	2, 5, 7, 10	2	728.81	0.00
23	18105.74	2, 5, 11	2	303.26	0.00
24	17957.84	2, 5, 7	2	43.57	0.00
25	33542.07	2, 5, 6, 7, 10, 11	2	82.25	0.00
26	31726.21	2, 5, 6, 7, 21	2	788.98	0.00
27	31370.53	2, 5	2	14.86	0.00
28	33542.07	2, 5, 6, 7, 10, 11	2	68.85	0.00
29	31726.21	2, 5, 6, 7, 11	2	900.54	0.00
30	31370.53	2, 5	2	14.51	0.00
31	35337.09	2, 5, 6, 7, 9, 10	2	10.54	0.00
32	32402.55	2, 5, 6, 7, 9	2	668.08	0.00
33	32082.55	2, 5, 7	2	20.15	0.00
34	35337.09	2, 5, 6, 7, 10, 11	2	14.86	0.00
35	32402.55	2, 5, 6, 7, 11	2	672.83	0.00
36	32082.55	2, 5, 7	2	21.36	0.00

Table A.21. Results of 30-bus test system for MC method with the second strategy

Case #	Total Cost ($)	Location	No. Of Iterations	Total Time (s)	Gap (%)
1	1191.72	2, 11, 27	2	4.23	62.25
2	1191.72	2, 26, 30	2	49.61	62.25
3	1191.72	1, 2, 17	2	37.29	62.25
4	1191.72	6, 11, 27	2	4.30	62.25
5	1191.72	2, 4, 11	2	61.29	62.25
6	1191.72	1, 2, 9	2	48.82	62.25
7	1230.90	11, 17, 27	2	58.24	67.89
8	1230.90	1, 2, 29	2	39.16	67.89
9	1230.90	2, 11, 14	2	37.59	67.89
10	1230.90	2, 9, 11	2	3.41	67.89
11	1230.90	2, 4, 11	2	5.74	67.89
12	1230.90	2, 9, 24	2	35.27	67.89
13	17441.50	2, 5, 7, 10	2	483.60	0.00
14	17302.99	2, 5, 9	2	62.74	0.00
15	17231.21	2, 5	2	53.72	0.00
16	17441.50	2, 5, 7, 10	2	572.74	0.00
17	17302.99	2, 5, 9	2	60.19	0.00
18	17231.21	2, 5	2	48.80	0.00
19	18161.52	2, 5, 7, 10	2	656.97	0.00
20	18105.74	2, 5, 11	2	302.29	0.00
21	17957.84	2, 5, 7	2	55.40	0.00
22	18161.52	2, 5, 7, 10	2	735.81	0.00
23	18105.74	2, 5, 11	2	304.38	0.00
24	17957.84	2, 5, 7	2	43.99	0.00
25	33542.07	2, 5, 6, 7, 10, 11	2	82.60	0.00
26	31726.21	2, 5, 6, 7, 21	2	793.08	0.00
27	31370.53	2, 5	2	15.26	0.00
28	33542.07	2, 5, 6, 7, 10, 11	2	69.26	0.00
29	31726.21	2, 5, 6, 7, 11	2	902.83	0.00
30	31370.53	2, 5	2	14.74	0.00
31	35337.09	2, 5, 6, 7, 9, 10	2	11.25	0.00
32	32402.55	2, 5, 6, 7, 9	2	674.05	0.00
33	32082.55	2, 5, 7	2	20.04	0.00
34	35337.09	2, 5, 6, 7, 10, 11	2	14.79	0.00
35	32402.55	2, 5, 6, 7, 11	2	678.61	0.00
36	32082.55	2, 5, 7	2	21.70	0.00

Table A.22. Results of 30-bus test system for MC method with the third strategy

Case #	Total Cost ($)	Location	No. Of Iterations	Total Time (s)	Gap (%)
1	2996.80	2, 11	3	68.86	5.07
2	2996.80	3, 19	2	118.60	5.07
3	2996.80	6, 10	2	160.44	5.07
4	2996.80	2, 11	2	5.35	5.07
5	2996.80	3, 21	2	154.53	5.07
6	2996.80	2, 9	2	118.89	5.07
7	3833.13	2, 11, 27	3	54.69	0.00
8	3833.13	4, 9, 22	3	41.54	0.00
9	3833.13	2, 14, 29	3	13.37	0.00
10	3833.13	2, 11, 21	3	8.54	0.00
11	3833.13	1, 2, 9	3	10.17	0.00
12	3833.13	2, 14, 22	3	8.08	0.00
13	17441.50	2, 5, 7, 10	2	485.29	0.00
14	17302.99	2, 5, 9	2	62.92	0.00
15	17231.21	2, 5	2	53.78	0.00
16	17441.50	2, 5, 7, 10	2	574.14	0.00
17	17302.99	2, 5, 9	2	63.41	0.00
18	17231.21	2, 5	2	55.20	0.00
19	18161.52	2, 5, 7, 10	2	721.58	0.00
20	18105.74	2, 5, 11	2	308.86	0.00
21	17957.84	2, 5, 7	2	55.32	0.00
22	18161.52	2, 5, 7, 10	2	801.18	0.00
23	18105.74	2, 5, 11	2	329.12	0.00
24	17957.84	2, 5, 7	2	46.91	0.00
25	33542.07	2, 5, 6, 7, 10, 11	2	90.66	0.00
26	31726.21	2, 5, 6, 7, 21	2	834.15	0.00
27	31370.53	2, 5	2	17.12	0.00
28	33542.07	2, 5, 6, 7, 10, 11	2	77.61	0.00
29	31726.21	2, 5, 6, 7, 11	2	963.57	0.00
30	31370.53	2, 5	2	16.77	0.00
31	35337.09	2, 5, 6, 7, 9, 10	2	13.19	0.00
32	32402.55	2, 5, 6, 7, 9	2	680.93	0.00
33	32082.55	2, 5, 7	2	21.92	0.00
34	35337.09	2, 5, 6, 7, 10, 11	2	15.67	0.00
35	32402.55	2, 5, 6, 7, 11	2	680.13	0.00
36	32082.55	2, 5, 7	2	21.65	0.00

Table A.23. Results of 30-bus test system for MC method with the fourth strategy

Case #	Total Cost ($)	Location	No. Of Iterations	Total Time (s)	Gap (%)
1	2996.80	11, 27	2	5.24	5.07
2	2996.80	3, 19	2	117.55	5.07
3	2996.80	6, 10	2	180.20	5.07
4	2996.80	2, 11	2	5.43	5.07
5	2996.80	3, 21	2	154.74	5.07
6	2996.80	2, 9	2	120.38	5.07
7	3673.13	2, 11	3	6.98	4.17
8	3673.13	1, 12	2	43.70	4.17
9	3673.13	7, 10	2	16.07	4.17
10	3673.13	11, 27	3	5.83	4.17
11	3673.13	2, 11	2	3.80	4.17
12	3673.13	5, 18	2	11.44	4.17
13	17441.50	2, 5, 7, 10	2	487.63	0.00
14	17302.99	2, 5, 9	2	62.91	0.00
15	17231.21	2, 5	2	54.21	0.00
16	17441.50	2, 5, 7, 10	2	579.02	0.00
17	17302.99	2, 5, 9	2	60.16	0.00
18	17231.21	2, 5	2	49.36	0.00
19	18161.52	2, 5, 7, 10	2	657.54	0.00
20	18105.74	2, 5, 11	2	300.66	0.00
21	17957.84	2, 5, 7	2	55.21	0.00
22	18161.52	2, 5, 7, 10	2	736.21	0.00
23	18105.74	2, 5, 11	2	303.87	0.00
24	17957.84	2, 5, 7	2	44.10	0.00
25	33542.07	2, 5, 6, 7, 10, 11	2	82.86	0.00
26	31726.21	2, 5, 6, 7, 21	2	809.73	0.00
27	31370.53	2, 5	2	14.92	0.00
28	33542.07	2, 5, 6, 7, 10, 11	2	68.61	0.00
29	31726.21	2, 5, 6, 7, 11	2	1008.84	0.00
30	31370.53	2, 5	2	14.88	0.00
31	35337.09	2, 5, 6, 7, 9, 10	2	10.79	0.00
32	32402.55	2, 5, 6, 7, 9	2	740.25	0.00
33	32082.55	2, 5, 7	2	24.36	0.00
34	35337.09	2, 5, 6, 7, 10, 11	2	15.29	0.00
35	32402.55	2, 5, 6, 7, 11	2	729.48	0.00
36	32082.55	2, 5, 7	2	23.50	0.00

Table A.24. Results of 30-bus test system for MC method with the fifth strategy

Case #	Total Cost ($)	Location	No. Of Iterations	Total Time (s)	Gap (%)
1	2996.80	6, 11	3	6.95	5.07
2	2996.80	3, 19	2	123.65	5.07
3	2996.80	6, 10	2	166.44	5.07
4	2996.80	2, 11	2	5.43	5.07
5	2996.80	3, 21	2	154.23	5.07
6	2996.80	2, 9	2	127.84	5.07
7	3673.13	5, 11	3	33.66	4.17
8	3673.13	1, 12	2	43.14	4.17
9	3673.13	7, 10	2	16.42	4.17
10	3673.13	1, 11	3	46.72	4.17
11	3673.13	2, 11	2	3.91	4.17
12	3673.13	5, 18	2	11.53	4.17
13	17441.50	2, 5, 7, 10	2	530.85	0.00
14	17302.99	2, 5, 9	2	64.54	0.00
15	17231.21	2, 5	2	53.35	0.00
16	17441.50	2, 5, 7, 10	2	605.64	0.00
17	17302.99	2, 5, 9	2	61.00	0.00
18	17231.21	2, 5	2	50.62	0.00
19	18161.52	2, 5, 7, 10	2	675.28	0.00
20	18105.74	2, 5, 11	2	308.39	0.00
21	17957.84	2, 5, 7	2	59.00	0.00
22	18161.52	2, 5, 7, 10	2	876.32	0.00
23	18105.74	2, 5, 11	2	403.57	0.00
24	17957.84	2, 5, 7	2	44.52	0.00
25	33542.07	2, 5, 6, 7, 10, 11	2	92.42	0.00
26	31726.21	2, 5, 6, 7, 21	2	847.94	0.00
27	31370.53	2, 5	2	15.31	0.00
28	33542.07	2, 5, 6, 7, 10, 11	2	69.56	0.00
29	31726.21	2, 5, 6, 7, 11	2	909.73	0.00
30	31370.53	2, 5	2	14.66	0.00
31	35337.09	2, 5, 6, 7, 9, 10	2	11.05	0.00
32	32402.55	2, 5, 6, 7, 9	2	710.36	0.00
33	32082.55	2, 5, 7	2	20.44	0.00
34	35337.09	2, 5, 6, 7, 10, 11	2	15.80	0.00
35	32402.55	2, 5, 6, 7, 11	2	733.32	0.00
36	32082.55	2, 5, 7	2	23.64	0.00

Table A.25. Results of 30-bus test system for MC method with the sixth strategy

Case #	Total Cost ($)	Location	No. Of Iterations	Total Time (s)	Gap (%)
1	2996.80	11, 27	2	4.82	5.07
2	2996.80	3, 19	2	114.97	5.07
3	2996.80	6, 10	2	161.40	5.07
4	2996.80	2, 11	2	5.50	5.07
5	2996.80	3, 21	2	153.39	5.07
6	2996.80	2, 9	2	119.00	5.07
7	3673.13	2, 11	3	6.74	4.17
8	3673.13	1, 12	2	43.23	4.17
9	3673.13	7, 10	2	15.97	4.17
10	3673.13	11, 29	2	3.63	4.17
11	3673.13	2, 11	2	3.92	4.17
12	3673.13	5, 18	2	11.22	4.17
13	17441.50	2, 5, 7, 10	2	483.94	0.00
14	17302.99	2, 5, 9	2	62.79	0.00
15	17231.21	2, 5	2	53.60	0.00
16	17441.50	2, 5, 7, 10	2	579.53	0.00
17	17302.99	2, 5, 9	2	59.74	0.00
18	17231.21	2, 5	2	49.01	0.00
19	18161.52	2, 5, 7, 10	2	653.31	0.00
20	18105.74	2, 5, 11	2	301.85	0.00
21	17957.84	2, 5, 7	2	55.64	0.00
22	18161.52	2, 5, 7, 10	2	729.98	0.00
23	18105.74	2, 5, 11	2	302.60	0.00
24	17957.84	2, 5, 7	2	43.51	0.00
25	33542.07	2, 5, 6, 7, 10, 11	2	82.59	0.00
26	31726.21	2, 5, 6, 7, 21	2	787.99	0.00
27	31370.53	2, 5	2	14.92	0.00
28	33542.07	2, 5, 6, 7, 10, 11	2	69.13	0.00
29	31726.21	2, 5, 6, 7, 11	2	894.00	0.00
30	31370.53	2, 5	2	14.63	0.00
31	35337.09	2, 5, 6, 7, 9, 10	2	10.69	0.00
32	32402.55	2, 5, 6, 7, 9	2	671.86	0.00
33	32082.55	2, 5, 7	2	22.21	0.00
34	35337.09	2, 5, 6, 7, 10, 11	2	16.78	0.00
35	32402.55	2, 5, 6, 7, 11	2	677.97	0.00
36	32082.55	2, 5, 7	2	21.64	0.00

Table A.26. Results of 30-bus test system for MC method with the seventh strategy

Case #	Total Cost ($)	Location	No. Of Iterations	Total Time (s)	Gap (%)
1	2996.80	11, 27	2	5.58	5.07
2	2996.80	3, 19	2	110.48	5.07
3	2996.80	6, 10	2	154.89	5.07
4	2996.80	2, 11	2	5.21	5.07
5	2996.80	3, 21	2	153.03	5.07
6	2996.80	2, 9	2	118.35	5.07
7	3673.13	2, 11	3	34.88	4.17
8	3673.13	1, 12	2	43.03	4.17
9	3673.13	7, 10	2	15.85	4.17
10	3673.13	4, 11	3	6.34	4.17
11	3673.13	2, 11	2	3.86	4.17
12	3673.13	5, 18	2	11.40	4.17
13	17441.50	2, 5, 7, 10	2	485.34	0.00
14	17302.99	2, 5, 9	2	62.31	0.00
15	17231.21	2, 5	2	53.89	0.00
16	17441.50	2, 5, 7, 10	2	618.69	0.00
17	17302.99	2, 5, 9	2	62.68	0.00
18	17231.21	2, 5	2	50.13	0.00
19	18161.52	2, 5, 7, 10	2	666.14	0.00
20	18105.74	2, 5, 11	2	306.10	0.00
21	17957.84	2, 5, 7	2	59.22	0.00
22	18161.52	2, 5, 7, 10	2	745.97	0.00
23	18105.74	2, 5, 11	2	305.85	0.00
24	17957.84	2, 5, 7	2	44.22	0.00
25	33542.07	2, 5, 6, 7, 10, 11	2	83.68	0.00
26	31726.21	2, 5, 6, 7, 21	2	803.05	0.00
27	31370.53	2, 5	2	15.04	0.00
28	33542.07	2, 5, 6, 7, 10, 11	2	69.81	0.00
29	31726.21	2, 5, 6, 7, 11	2	953.29	0.00
30	31370.53	2, 5	2	15.82	0.00
31	35337.09	2, 5, 6, 7, 9, 10	2	11.09	0.00
32	32402.55	2, 5, 6, 7, 9	2	713.57	0.00
33	32082.55	2, 5, 7	2	20.92	0.00
34	35337.09	2, 5, 6, 7, 10, 11	2	17.73	0.00
35	32402.55	2, 5, 6, 7, 11	2	716.41	0.00
36	32082.55	2, 5, 7	2	23.00	0.00

Table A.27. Results of 30-bus test system for the hybrid method

Case #	Total Cost ($)	Location	No. Of Iterations	Total Time (s)	Gap (%)
1	1191.72	2, 11, 27	2	5.49	62.25
2	1191.72	2, 26, 30	2	49.70	62.25
3	1191.72	1, 2, 17	2	39.75	62.25
4	1191.72	6, 11, 27	2	5.05	62.25
5	3156.80	1, 2, 24	3	69.64	0.00
6	1191.72	1, 2, 9	2	50.50	62.25
7	3833.13	2, 11, 30	3	65.96	0.00
8	1230.90	1, 2, 29	2	42.04	67.89
9	3833.13	5, 17, 24	3	45.90	0.00
10	3833.13	2, 11, 27	3	74.86	0.00
11	1230.90	2, 4, 11	2	8.03	67.89
12	3833.13	2, 6, 9	3	42.79	0.00
13	17441.50	2, 5, 7, 10	2	520.13	0.00
14	17302.99	2, 5, 9	2	69.21	0.00
15	17231.21	2, 5	2	59.80	0.00
16	17441.50	2, 5, 7, 10	2	647.33	0.00
17	17302.99	2, 5, 9	2	66.21	0.00
18	17231.21	2, 5	2	51.62	0.00
19	18161.52	2, 5, 7, 10	2	707.16	0.00
20	18105.74	2, 5, 11	2	307.88	0.00
21	17957.84	2, 5, 7	2	56.75	0.00
22	18161.52	2, 5, 7, 10	2	742.86	0.00
23	18105.74	2, 5, 11	2	314.94	0.00
24	17957.84	2, 5, 7	2	45.48	0.00
25	33542.07	2, 5, 6, 7, 10, 11	2	85.61	0.00
26	31726.21	2, 5, 6, 7, 21	2	885.24	0.00
27	31370.53	2, 5	2	18.23	0.00
28	33542.07	2, 5, 6, 7, 10, 11	2	75.35	0.00
29	31726.21	2, 5, 6, 7, 11	2	962.46	0.00
30	31370.53	2, 5	2	17.42	0.00
31	35337.09	2, 5, 6, 7, 9, 10	2	12.21	0.00
32	32402.55	2, 5, 6, 7, 9	2	732.68	0.00
33	32082.55	2, 5, 7	2	23.56	0.00
34	35337.09	2, 5, 6, 7, 10, 11	2	19.40	0.00
35	32402.55	2, 5, 6, 7, 11	2	766.67	0.00
36	32082.55	2, 5, 7	2	22.78	0.00