SHEAR STRENGTH OF REINFORCED CONCRETE NON-SLENDER MEMBERS SUBJECTED TO POINT LOADS

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ABSTRACT

SHEAR STRENGTH OF REINFORCED CONCRETE NON-SLENDER MEMBERS SUBJECTED TO POINT LOADS

For many years, shear strength of reinforced concrete members has been studied and researched. The main purpose of this thesis is development of simple equations to consider shear strength enhancement of non-slender members under point load.

Shear strength is enhanced when loads are applied within a distance 2-2.5d of support. European codes, namely Eurocode 2 and Model Code 2010 allow sectional shear strength enhancement instead of more refined models to account for the strut action in non-slender members. Eurocode2 and *fib* Model Code 2010 take account of shear enhancement by decrease the shear force by one factor. Recent research has shown that this approach gives very poor results. In this thesis shear enhancement methods in Model Code 2010 will be evaluated using ACI-DAfStb databases for point loaded members with no vertical design and with shear reinforcement. Results will be presented along with recommendations.

In the *fib* MC2010, when loads are performed near 2d distance of supports, the design shear force can be reduced by $a_v/2d$, where a_v is the clear shear span and d is the effective depth. The accuracy and conservativeness of the shear enhancement method in the *fib* MC2010 are evaluated. Based on the results obtained, recommendations are made to enhance the certainty of the *fib* MC2010 sectional shear enhancement method.

In the shear design of non-slender members, significant vertical compressive stresses occur by the loading and support conditions. Sectional models which ignore the beneficial effects of these vertical compressive stresses underestimate the shear strength of such members. In this thesis, a simple expression is derived to calculate the vertical compressive stress at mid-depth of point loaded non-slender beams and the influence of these compressive stresses is incorporated into the shear design procedures of *fib* Model Code 2010. It is concluded that sectional analysis with vertical compressive stresses yields better results than the sectional enhancement method in the *fib* MC2010.

ÖZET

TEKİL YÜKLER ALTINDA YÜKSEK KİRİŞLERİN KAYMA DAYANIMI

Uzun yıllar boyunca betonarme yüksek kirişlerin kesme dayanımı üzerinde çalışılmış ve araştırmalar yapılmıştır. Bu tezin asıl amacı, tekil yük altında yüksek kirişlerin kesme dayanımının tahmini için basit formüllerin geliştirilmesidir.

Mesnete 2-2.5d mesafe uzaklıkta uygulanan tekil yük, kemerlenme etkisi ile taşındığı için bu tip kirişlerin kayma dayanımı daha yüksektir. Eurocode2 ve *fib* Model Code 2010, tekil yükler altındaki yüksek kirişlerde kemerlenme etkisini hesaba katan daha ayrıntılı modeller yerine basit kayma dayanımı denklemlerinin kullanılmasına izin verir. Eurocode2 ve *fib* Model Code 2010, tasarım kesme kuvvetini bir faktör ile azaltarak hesaplanan kesme dayanımını arttırmayı amaçlamaktadır. Son araştırmalar bu yaklaşımın çok iyi sonuçlar vermediğini göstermiştir. Bu tez çalışmasında *fib* Model Code 2010'daki kesme dayanımı arttırma yöntemleri, kayma donatısı içeren ve içermeyen tekil yüklü kirişler için ACI-DAfStb veritabanları kullanılarak değerlendirilecektir.

fib Model Code'a göre, tekil yükler mesnete $2d$ mesafesi içinde uygulandığında, tasarım kesme kuvveti $a_v/2d$ ile azaltılabilir, burada a_v net açıklık ve d faydalı yüksekliktir. *fib* MC2010'da kayma dayanımı arttırma yönteminin doğruluğu ve güvenliği değerlendirilirmiştir. Elde edilen sonuçlara dayanarak, *fib* Model Code 2010 yüksek kirişler için kayma dayanımı hesabının doğruluğunu artırmak için önerilerde bulunulmuştur.

Yüksek kirişlerin kesme tasarımında, yük ve mesnet koşullarından dolayı belirgin düşey basınç gerilmeleri oluşur. Bu düşey basınç gerilmelerinin faydalı etkilerini içermeyen kesit analizi modelleri ile bu kirişlerin kayma dayanımı olduğundan daha az bulunur. Bu çalışmada, yüksek kirişlerde oluşan düşey basınç gerilmesinin hesabı için basit bir ifade geliştirilmiş ve bu basınç gerilmesinin etkisi, *fib* Model Code 2010'un kesme dayanım ilkelerinde gözönüne alınmıştır. Düşey basınç gerilmeli kesit analizinin, *fib* Model Code 2010'daki kesit kayma dayanımı arttırma yönteminden daha iyi sonuçlar verdiği görülmüştür.

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LIST OF SYMBOLS/ABBREVIATIONS

a	Shear span
b	Cross-section width
c	Depth of compression zone above flexural cracks
d	Effective depth
T	Tensile force at the bottom reinforcement
A_c	Concrete cross section area
A_s	Area of longitudinal reinforcement
A_{sl}	Tensile reinforcement area
A_{sw}	Cross-sectional area of the shear reinforcement
a_v	Clear shear span
b_w	Width of the member
c_s	Depth of compression on the cross crack
f_{avg}	Average compressive stress
f_c'	Nominal compressive strength of concrete
f_{cu}	Compressive cube strength of concrete
f_y	Yield strength of the longitudinal bar
f_{ye}	Effective yield strength of longitudinal reinforcement
f_{yv}	Characteristic strength of stirrups
f_{yw}	Yield strength of vertical shear reinforcement
f_z	Transverse compressive stress
k_ε	Strain effect
l_k	Bottom reinforcement elongation that causes from the critical crack
n_b	Number of stirrup legs
s_w	Shear reinforcement spacing
v_c	Design concrete shear stress
α	Angle indicating the shear reinforcement relative to the axis of the member
ρ_l	Ratio of longitudinal reinforcement

ρ_v	Ratio of shear reinforcement
ε_v	Strain of the stirrup in the middle of the shear span
θ	Angle between the concrete compression strut and the axis of the member
$\emptyset$	Stirrups diameter

BS8110	British standard
EC2	Eurocode2
LoA	Level of approximation
MC2010	Model Code 2010
NLFEM	Non-linear finite element models
SMCFT	Simplified modified compression field theory
ST	Strut-and-tie
STM	Strut-and-tie models

1. INTRODUCTION

1.1. GENERAL ASPECTS

Shear design of point loaded non-slender members occurs in the design of pile caps under column loads and thick mat foundations resting on piles. Reinforced concrete members may fail in shear, flexure or combination of both. Shear failure needs to be avoided, as it characteristically occurs suddenly with very little warning, as opposed to flexural failure. Shear failure mechanism is complicated and, in contrast to flexural failure, is still a subject of intensive research and argument. In the 1970 Federation international de la Precontrainte Congress, Professor Fritz Leonhardt [1] stated that the principal cause to the low quality of design conditions for torsion and shear was that the strengths were affected by approximate 20 variables. Many concrete structures built in the last few years in countries such as China, Japan and the USA do not comply the existing design requirements, especially in terms of ductility and shear capacity [2]. Most recent researches in developing design provisions for shear to prevent shear failures of current structures. On February 2011, a class separation bridge in Zhejiang Province failed abruptly due to shear and as a result of the collapse of the large overpass, 3 people are wounded (Figure 1.1.) [2]. Many reinforced concrete members were heavily damaged. Inadequate shear capacity and absence of ductility in principal slabs and members were defined as the reason of collapse. A similar collapse happened on Huairou Bridge, which served for 13 years. Failures like this are too many to mention.

Figure 1.1. A grade separation bridge collapsed in shear, China [2]

In the last 30 years, numerous experimental researches were conducted for establishing design equations that can estimate the shear strength of reinforced concrete short-span members.

1.2. STATEMENT OF THE PROBLEM

Reinforced concrete non-slender members are structural elements that have a relatively deep cross-section relative to the time during which a significant section of the load is changed summarily at the support by a direct strut. In general, non-slender members are used as transfer members in bridges, buildings or in the design of underground structures. In compliance with Eurocode2 (EC2) members, for a member to be a non-slender member, the span to depth ratio must be less than three. Moreover, ACI 318-14 categorizes the members like non-slender members that answer (i) clear span must not pass quadruple depth of the whole member; else (ii) shear span must not pass twice depth of the total member [3]. The strength of the non-slender members is often checked by shear instead of bending [4]. When practical load is straightly carried to the support with strut action, non-linear finite element models (NLFEM) or strut-and-tie models (STM) are the most reliable procedure for aims of design. NLFEM and STM models are usually cumbersome and time-consuming methods. Often all needed is a safe estimate of shear strength.

In addition, more reliable and elaborated empirical data are necessary to examine impact of basic variables like compressive strength of concrete, aggregate size, shear span-depth ratio and depth of member.

If the size of the reinforced concrete member increases, the shear strength is reduced, but many current shear codes do not consider this phenomena. The dimension impact at reinforced concrete slender members is well researched and concluded, but about reinforced concrete non-slender members, size effect still remains a topic of debate among researchers. Therefore, the size effect should be farther researched and evaluated to assess the safety of existing shear models.

1.3. OUTLINE OF THESIS

This thesis contains seven chapters.

Firstly, Chapter 1 introduces general features of the thesis. In general subject of the research is explained. Then, the objectives of this thesis are explained.

Chapter 2 discusses the shear strength of short-span members. Firstly, STM and NLFEM to predict the shear strength of non-slender members are mentioned. Then, as alternative methods, information was given about Zararis shear strength model for reinforced members [5], unified shear strength model [17] and two-parameter kinematic theory for shear members [19].

In the third chapter of the thesis, an introduction to shear enhancement approach for reinforced shear members in EC2 [6], BS8110 [20] and Model Code 2010 are given. Later, the MC2010 shear design equations that are used for non-slender members with and with no reinforcement are explained in detail.

In Chapter 4 firstly, *fib* Model Code 2010 shear strength provisions are used to calculate the shear strength of members in the database. The details of the database used for the tests are explained. For non-slender members (a/d<2.4) with shear reinforcement and with no shear reinforcement, shear strength predictions have been performed and the results are shown. In the last part of the section, a method for *fib* Model Code 2010 is recommended and tested.

In Chapter 5, the shear strength of point loaded non-slender members is described. In the first section, vertical compressive stresses sectional shear analysis was performed. In the last section, the shear strength experimental and predicted values are compared, for non-slender members with shear reinforcement and with no shear reinforcement.

In Chapter 6, a overall conclusion of the research in this thesis is mentioned. The results of the shear calculations were compared with each other. The comparison of experimental and predicted values of shear strength was mentioned.

Finally at Chapter 7, the study in this thesis is summarized and the conclusion is presented.

2. SHEAR STRENGTH OF SHORT-SPAN MEMBERS

2.1. NON-LINEAR FINITE ELEMENT METHODS AND STRUT-AND-TIE METHODS

The cross-sectional models based on the member theory assuming that the sections can remain plane even after the bending were created to analytically research the behaviour of reinforced concrete members. Non-slender members generally fail because of shear failure instead of bending failure, and can not be studied and designed by either bending analysis or beam theory. Regions are divided into two main sections: 'beam regions' are part of a member that suppositions for beam theory are correct; the second, 'disturbed regions' are regions where sudden changes in forces or geometries affect the support and loading details. The analysis method adapted from the sectional models is suitable for estimating the shear strength of beam regions. On the other hand, disturbed regions, must be analyzed and designed using models like STM or NLFEM which are capable of better estimate the force flow therein.

The results of the test performed by Kani in non-slender members with a height of 610 mm are given in the figure below for different shear span, a to effective depth, d ratios. The values indicated by points are the experimental results, also the two lines are estimated according to different methods. The dashed line shows the results of the analysis program Response2000 [23]. The straight line shows the result of the STM evolved by Collins and Mitchell [24].

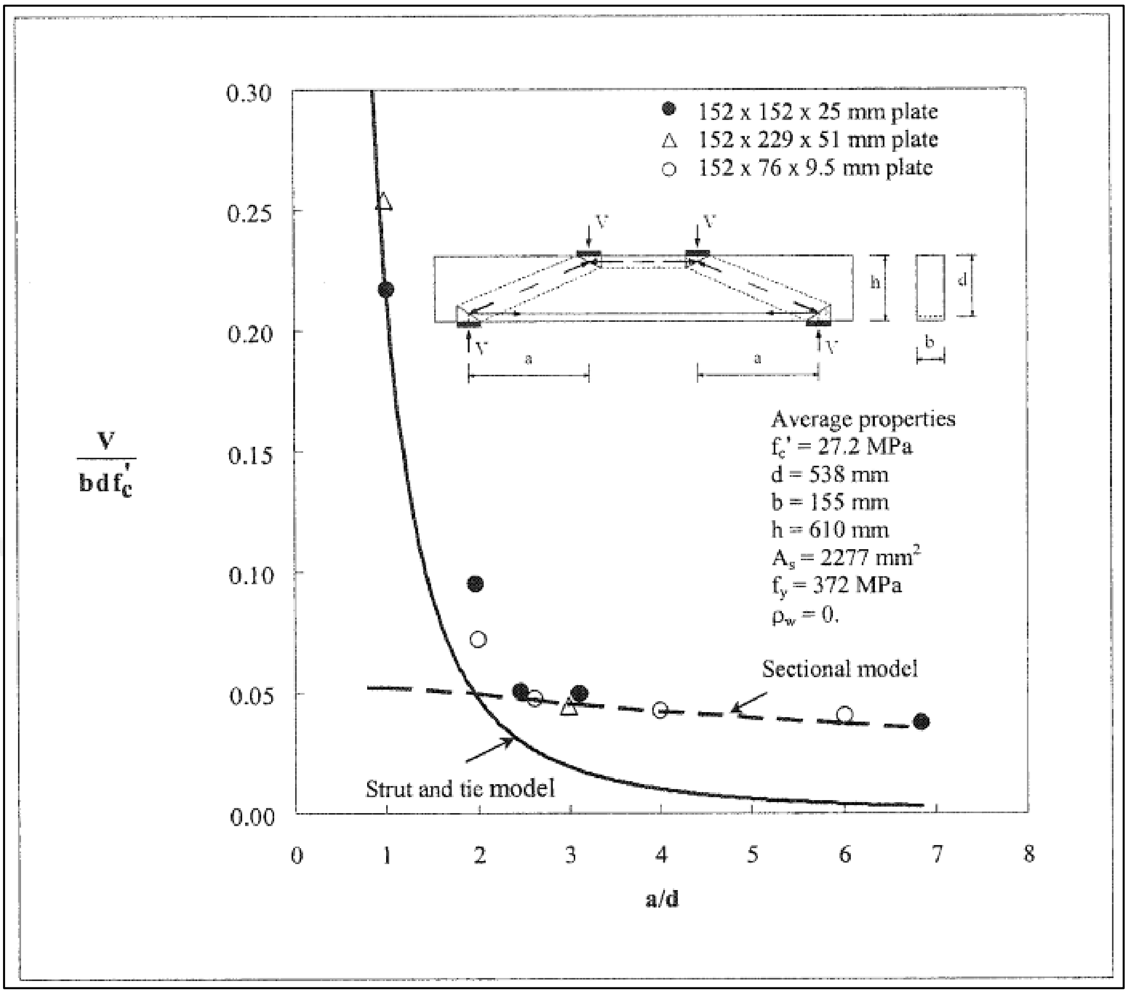

Figure 2.1. Estimation and experimental shear strength of reinforced concrete members tested by Kani[22]

As seen from Figure 2.1 for the members with a/d ratios of less than approximately 2 the shear strength estimates are quite conservative.

Many researchers have studied and contributed to the analysis of disturbed regions at various times, as a result of the STM and non-linear FEM have been developed to estimate the response of members.

Strut-and-tie models were first introduced by Ritter [26] and Mörsch [27]. Then, this analysis method was further developed and extended by Thürlimann et al. [28], Marti [29] and Schlaich and Schafer [30].

Nonlinear finite element analysis is a powerful method of estimating the response of disturbed regions. The use of finite element techniques to predict the response of reinforced concrete was first studied by Ngo and Scordelis [31]. An important aspect of finite element techniques is the constitutive relationship between streeses and strains in the cracked concrete.

2.2. ALTERNATIVE METHODS

NLFEA and ST models can be cumbersome. Below are examples of alternative shear analysis methods for short-span members. Design codes also have adapted fully empirical approaches named shear enhancement methods. These approaches will be discussed separately in detail.

2.2.1. Zararis Shear Strength Model For Reinforced Members

Zararis [5] suggested a theory to define the failure of shear compression in members of short span based on equilibrium importance in the crucial diagonal shear crack. Figure 2.2 demonstrates the forces in non-slender members with and with no shear reinforcement.

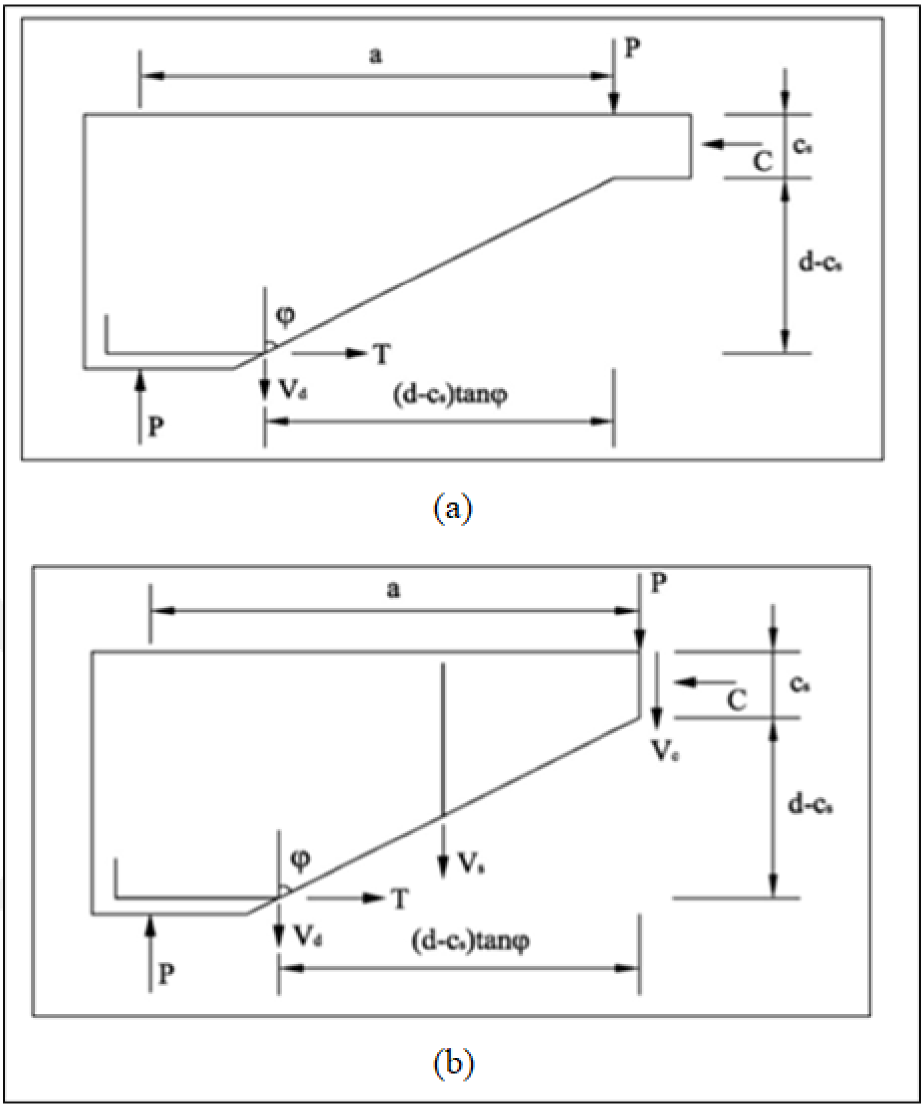

Figure 2.2. Forces acting on non-slender member at the time of failure: (a) without shear reinforcement; (b) with shear reinforcement[2]

In the Zararis model, it is supposed that the crack width is opened in a direction perpendicular to the direction of the crack and that the crack width is uniform throughout the cross crack. The single force affecting on the crack face is considered to be shear and flexural reinforcement [16]. Moreover, the dowel force in the web can be neglected as the web reinforcement is generally smaller than that of the main longitudinal reinforcement [5]. This model assumes different failure approaches for members with and with no shear reinforcement[2]. For members with no shear reinforcement, failure occurs as a result of separation of concrete on the compression zone over a horizontal crack extending from the end of the crucial diagonal crack to the bending region. A different failure approach is assumed in members with shear reinforcement where no horizontal cracks occur in beginning of the diagonal failure. For members with shear reinforcement, concrete cracking happens at the top of the crucial crack, because the moment of the force carried by stirrup is greater than the load point [2]. For this reason, the concrete region under the loading point is the weakest region of the member.

Equation (2.1) has been suggested to the calculation of the shear capacity of short span members with and with no shear reinforcement.

$$V_u = \frac{bd}{a/d}[\frac{c_s}{d}\left(1 - 0.5\frac{c_s}{d}\right)f_c' + 0.5\rho_v f_{ywd}\left(1 - \frac{c_s}{d}\right)^2\left(\frac{a}{d}\right)^2] \quad (2.1)$$

where;

c_s :is the depth of compression on a crucial on the cross crack,

f_c' :is the nominal compressive strength of concrete,

f_{ywd} :is the vertical web reinforcement yield strength,

$\rho = \frac{A_s}{bs}$.

The depth c_s is found by solving equations (2.2) and (2.3) as shown below [5]

$$\frac{c_s}{d} = \frac{1+0.27R(a/d)^2}{1+R(a/d)^2}\frac{c}{d} \quad (2.2)$$

$$\left(\frac{c}{d}\right)^2 = 600\frac{\rho_l}{f_c'}\frac{c}{d} - 600\frac{\rho_l}{f_c'} = 0 \quad (2.3)$$

where;

c :is the depth of compression zone above flexural cracks,

ρ_l :is longitudinal reinforcement ratio and

$R = 1 + \frac{\rho_v}{\rho_l}\left(\frac{a}{d}\right)^2$.

These equations describe the depth of the flexural compression region and the shear force at failure. Zararis has shown that these equations give good estimates of the shear resistance of non-slender members [2].

2.2.2. Unified Shear Strength Model

Kyoung-Kyu et al. [17] suggested a Unified Shear Strength model based on the default approaches of shear resistance. The model can be applied to reinforced concrete members with no and with shear reinforcement. The most important feature of this model is that the shear resistance of a member is primarily ascribed to the compression region and, the contribution of aggregate interlock or the contribution to the dowel action are ignored in shear strength calculations.

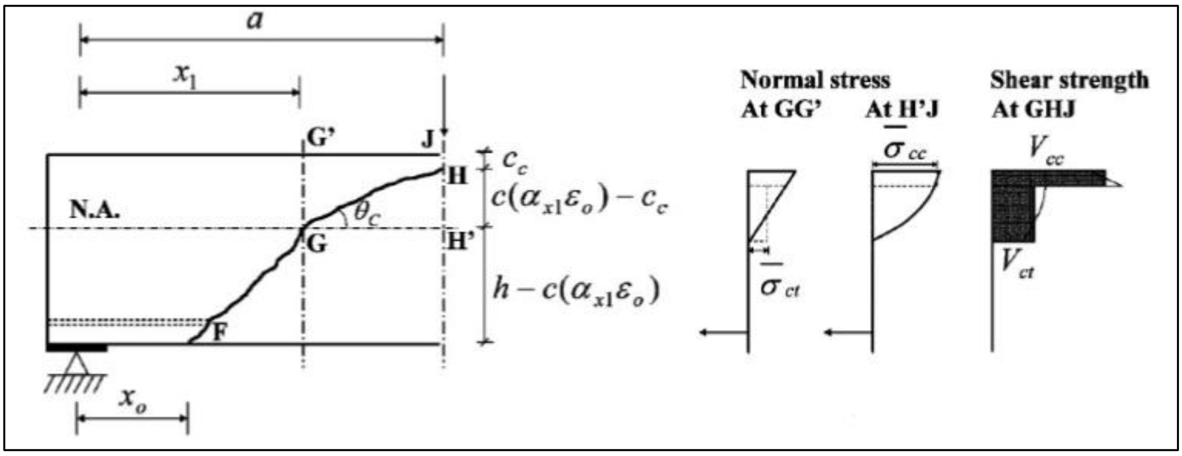

Figure 2.3. The geometry and shear stress in Unified Shear Strength model[17]

The shear resistance in the model is stated in the equation (2.4).

$$V_{Rd} = V_{cz} + V_s \tag{2.4}$$

where;

V_{cz} :is the shear contribution by compression region which is the sum of compression crushing force V_{cc} and tensile cracking force V_{ct},

V_s :is the contribution of stirrups, show equation (2.5) and (2.6).

$$V_{cz} = V_{ct} + V_{cc} \tag{2.5}$$

$$V_s = \rho_v f_{yw} b(d - 2c_c) \tag{2.6}$$

The strength of shear of the compression region V_{cz} is calculating taking the failure criteria for concrete beneath the sectional shear and axial load formulated in Rankine's equation (2.7).

$$V_{cz} = \sqrt{f_c'(f_c' - \sigma_{cc})}bc_c + \lambda_s\sqrt{f_t + (f_t + \sigma_{ct})}b.[c(\alpha_{x1}\varepsilon_0)] - c_c \tag{2.7}$$

c_c is the depth of the error surface of the compression crush, and $c(\alpha_{x1}\varepsilon_0)$ is the depth of the compression region in crucial cross-section, show equation (2.8) and (2.9). λ_s is the size effect factor that can be calculated using this equation (2.10) suggested by Zararis and Papadakis [18].

$$c_c = \left(1 - 0.43\left(\frac{a}{d}\right)\right).c(\alpha_{x1}\varepsilon_0) \tag{2.8}$$

$$c(\alpha_{x1}\varepsilon_0) = \frac{-\varepsilon_0 E_s d(\rho_{vh}+\rho_l)}{2(1-1/3\alpha_{x1})f_c'} + \frac{\sqrt{[\varepsilon_0 E_s d(\rho_{vh}+\rho_l)]^2 + 2(1-1/3\alpha_{x1})f_c'\varepsilon_0 E_s d^2(\rho_{vh}+2\rho_l)}}{2(1-1/3\alpha_{x1})f_c'} \tag{2.9}$$

$$\lambda_s = 1.2 - 200\left(\frac{a}{d}\right)d \geq 0.65 \tag{2.10}$$

Here, ε_0 is the compressive strain at which the concrete reaches its compressive capacity recommended to be 0.002 in EC2. E_s is Young's reinforcement module. ρ_{vh} is the longitudinal web reinforcement ratio, the ρ_l is ratio of tensile reinforcement. α_{x1} is a function of a/d expressed as $(1 - 0.44a/d)$.

Various assumptions were made in equation (2.7) to reduce the computational complexity. The stress σ_{cc} is assumed to be $0.8f_c'$, σ_{ct} is taken as $0.625\sqrt{f_c'}$ and f_t equals to $0.292\sqrt{f_c'}$. Then, with these assumptions, equation (2.7) is abbreviated as below:

$$V_{cz} = 0.52\lambda_s\sqrt{f_c'}b[c(\alpha_{x1}\varepsilon_0) - c_c] + 0.45f_c'bc_c \tag{2.11}$$

Consequently, the shear capacity can be calculated by dissolving equation (2.5), (2.6), (2.8), (2.9), (2.10) and (2.11).

2.2.3. Two-Parameter Kinematic Theory For Shear Members

This theory [19] is developed to estimating the strength of short-span and non-slender members**.** The widths of crack, maximum deviations and the full displacement area for the member can be determined by examining the balance of internal force flow and stress-strain relationships [19]. The fundamental assumption of this model is that the movement of the concrete block over the crucial crack is considered as a combination of the upper part of the crucial crack and a rotation of vertical translation Δc. In this way, the "two parameters" described in the theory are vertical translation Δc, and the average strain in the lower reinforcement $\varepsilon_{t.avg}$ because the strain is commeasurable to the block rotation of concrete [2].

The shear resistance is considered the total of the contributions of the compression region V_{cz}, aggregate interlock V_{ag}, dowel action V_d and transverse reinforcement V_s (if available).

$$V_{Rd} = V_{cz} + V_{ag} + V_d + V_s \tag{2.12}$$

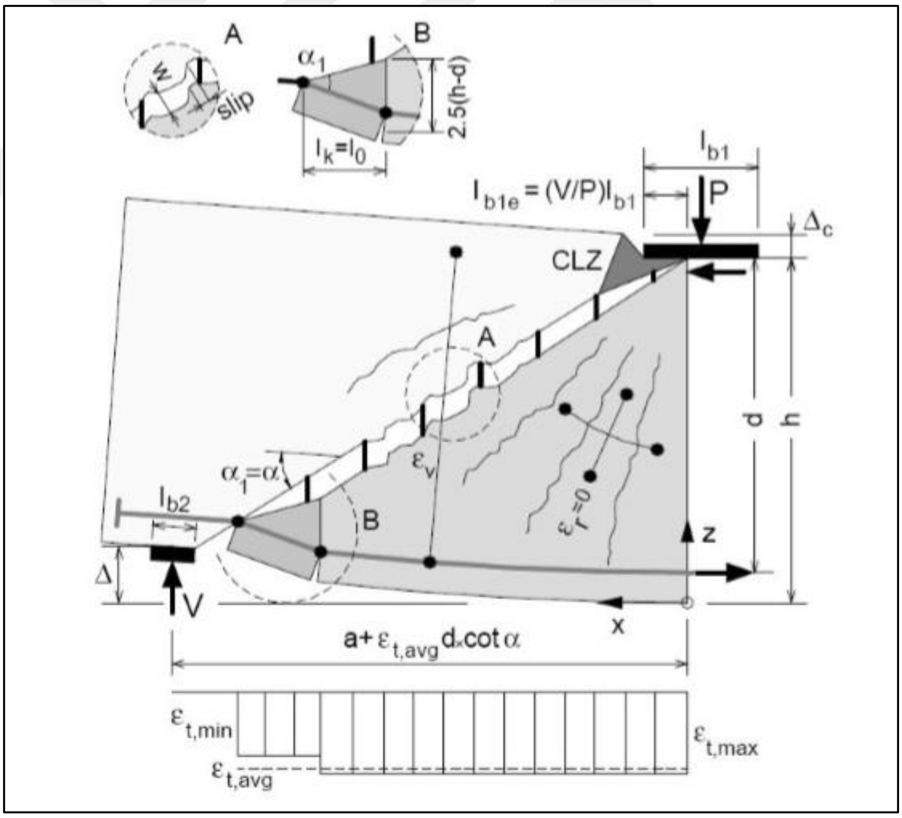

Figure 2.4. Details of model [19]

In the model, the shear strength of the crucial loading region is stated as follows:

$$V_{cz} = k f_{avg} b l_{b1e} \sin^2 \alpha \tag{2.13}$$

where;

k :is a default crack condition coefficient of 1.0 for short span and non-slender members,

f_{avg} :is the average compressive stress and stated as $1.43f_c'^{0.8}$,

l_{b1e} :is described as $(V/P)l_{b1}$, see Figure 2.4,

α :is the crucial diagonal shear crack angle that is defined by geometry [2].

The shear resistance caused by aggregate interlock:

$$V_{ag} = \frac{0.18\sqrt{f_c'}}{0.31+{}^{24w}\!/\!_{(a_{ge}+16)}}bd \tag{2.14}$$

where α_{ge} is the effective aggregate size equivalent to a_g while the concrete strength is lower than 60 MPa and equivalent to 0 if the concrete strength is above 60 MPa [2]. The shear reinforcement contribution is established by:

$$V_s = \rho_v b(d\cot\alpha_1 - l_0 - 1.5l_{b1e})f_{yw} \tag{2.15}$$

where

ρ_v :is the vertical stirrups ratio,

α_1 :is described as the maximum of θ and α, (look at the Figure 2.4.),

θ :is the cracks angle developing in a uniform stress area that can be calculated from the SMCFT [9]. Below the critical crack (Figure 2.4 Area B),

l_0 :is the length of the heavy crack region, that is equivalent to $1.5(h-d)\cot\alpha_1$,

f_{yw} :is the stress in the stirrup acquired from the equation (2.16).

$$f_{yw} = \varepsilon_v E_s \tag{2.16}$$

ε_v :is the is strain of the stirrup in the middle of the shear span given by

$$\varepsilon_v = 1.667\,{}^{\Delta c}\!/\!_{d} \tag{2.17}$$

contribution of the dowel action that is calculated as below:

$$V_d = {n_b f_{ye} \emptyset^3}/{3 l_k} \tag{2.18}$$

where;

$\emptyset$:is the stirrups diameter,

n_b :is the number of stirrup legs,

l_k :is the bottom reinforcement elongation that causes from the critical crack (Figure 2.4 Area B), see equation (2.19),

f_{ye} :is the effective yield strength of longitudinal reinforcement, see equation (2.20).

$$\begin{cases} l_k = l_0 + (\cot\alpha + \cot\alpha_1) \\ l_0 = 1.5(h-d)\cot\alpha_1 \end{cases} \tag{2.19}$$

$$f_{ye} = f_y\left[1 - \left({T}/{f_y A_s}\right)^2\right] \tag{2.20}$$

where;

T :is tensile stress at bottom reinforcement,

f_y :is yield strength of longitudinal bar,

A_s :is area longitudinal bar.

Two-parameter kinematic theory provides accurate prediction of the shear strength of the members as explained by Mihaylov et al. [19].

3. SECTIONAL SHEAR ENHANCEMENT METHODS

Many design codes allow sectional shear strength enhancement instead of more refined models to account for the strut action in members for which point loads are carried out at a distance of 2d to the near of the support.

In more recent times, many studies and research have been done on the subject of shear enhancement method. Examples are the research of Robert L. Vollum and Libin Fang's research and thesis [2].

3.1. SHEAR ENHANCEMENT APPROACH FOR REINFORCED CONCRETE MEMBERS IN EC2

3.1.1. Members With No Shear Reinforcement

In EC2 [6], shear resistance can be enhanced for members with loads applied from a support edge at a distance of $0.5d \leq x < 2d$. EC2 gives the equation for the with no shear reinforcement of reinforced concrete members as follows:

$$V_{Rd,c} = \left[C_{Rd,c}k(100\rho_l f_{ck})^{1/3}\left(\frac{2d}{x}\right) + 0.15\sigma_{cp}\right]b_w d \leq 0.5 b_w d v f_{cd} \tag{3.1}$$

$$\text{where } v = 0.6\left[1 - \frac{f_{ck}}{250}\right] \tag{3.2}$$

$k = 1 + \sqrt{\frac{200}{d}} \leq 2.0$

$\rho_l = \frac{A_{sl}}{b_w d} \leq 0.02$

A_{sl} : is the tensile reinforcement area, that expands $\geq (l_{bd} + d)$ beyond section take into account (show Figure 3.1).

b_w : is the width of the member

$\sigma_{cp} = N_{Ed}/A_c < 0.2 f_{cd}$ [MPa]

N_{Ed} : is the axial force in the cross-section because of loading [in N]. (if it is compression, N_{Ed} value can not be negative). The effect of loaded deformations on N_E can be neglected.

A_c : is the concrete cross section area

$V_{Rd,c}$: is [N]

Note that suggested value for $C_{Rd,c}$ is $0.18/\gamma_c$, that for v_{min} is given by equation (3.3) and that for k_1 is 0.15.

$$v_{min} = 0.035k^{3/2}{f_{ck}}^{1/2} \tag{3.3}$$

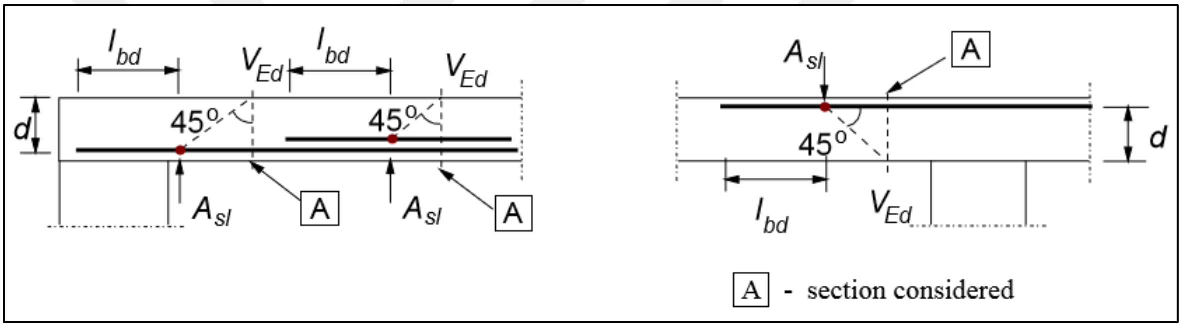

Figure 3.1. Definition of $\boldsymbol{A_{sl}}$ [6] (adapted from EC2)

This enhancement is valid only if the longitudinal reinforcement is fully secured to the support, the cross-sectional dimension being not decreased from the distance $x = 2d$ to the desired, and the enhanced resistance to the equation (3.1). If $x \geq 0.5d$, $x = 0.5d$ value can be used.

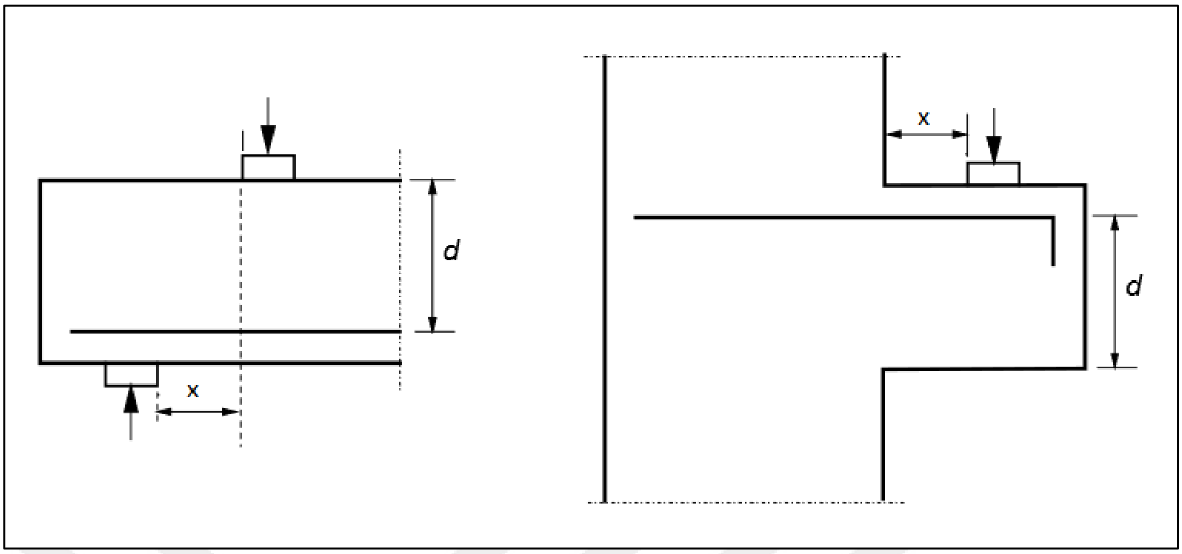

Figure 3.2. Loads beside supports [6] (adapted from EC2)

3.1.2. Members With Shear Reinforcement

In EC2 [6], shear resistance can be enhanced for members with loads applied from a support edge at a distance of $0.5d \leq x < 2d$. EC2 gives the equation for the with shear reinforcement of reinforced concrete members as follows:

$$V_{Rd} = V_{Rd,c} + A_{sw} f_{ywd} \sin \alpha \tag{3.4}$$

where $V_{Rd,c}$ is calculated in equation (3.1), $A_{sw}.f_{ywd}$ is strength of shear reinforcement among the shear crack of sloping and the areas of loaded (show Figure3.3). Just shear reinforcement in the center of 0.75 a_v can be considered.

If $x < 0.5d$, $x = 0.5d$ can be used.

The calculation of $V_{Rd,max}$ is as follows:

$$V_{Rd,max} = \alpha_c b_w z v f_{cd} (\cot \theta + \cot \alpha)/(1 + cot^2 \theta) \tag{3.5}$$

where;

α : is indicating the shear reinforcement angle relating to the member axis

θ : is the strut angle with respect to the axis of the member perpendicular to the shear force

b_w : is the min. width among compression and tension member

z : is the inner lever arm being the arm which corresponds to the maximum bending moment of the thought element in the fixed member depth. Ordinarily, z can be taken as $0.9d$ in the shear analysis of the reinforced concrete.

f_{ywd} : is the design yield strength of shear reinforcement

v : given in equation (3.2)

A_{sw} : is the cross-sectional area of the shear reinforcement.

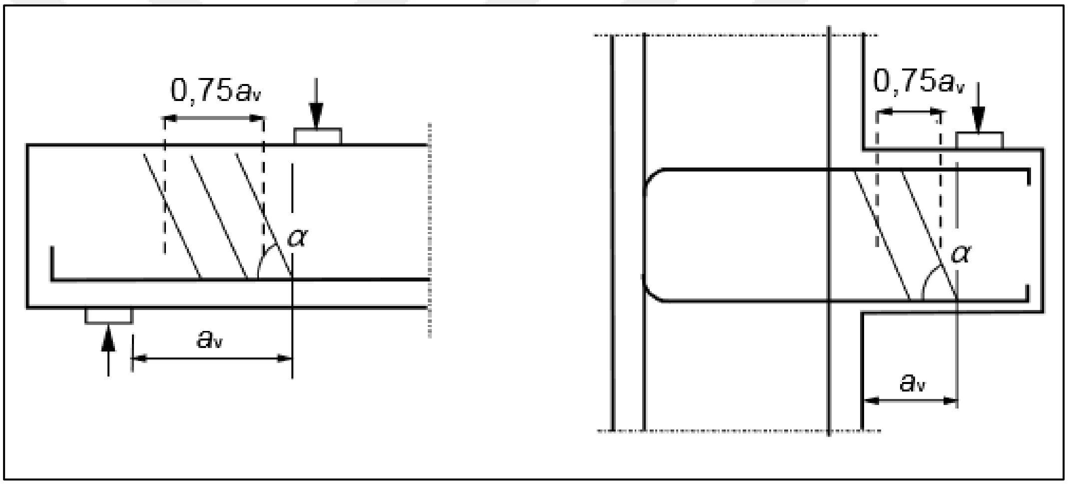

Figure 3.3. Shear reinforcement at non-slender member[6] (adapted from EC2)

3.2. SHEAR ENHANCEMENT APPROACH FOR REINFORCED CONCRETE MEMBERS IN BS8110

In BS8110 [20] the design shear strength of members with no shear reinforcement is calculated using the following equation.

$$V_{Rd,c} = 0.79 \left(\frac{100A_{sl}}{bd}\right)^{\frac{1}{3}} \left(\frac{400}{d}\right)^{\frac{1}{4}} \left(\frac{f_{cu}}{25}\right)^{\frac{1}{3}} bd \tag{3.6}$$

where

A_{sl} : is the tensile reinforcement area

f_{cu} : is the compressive cube strength of concrete

b : is the cross-section width

d : is effective depth

$100A_s/bd$ should not be larger than 3,

$(400/d)^{1/4}$ should not be less than 0.67 for members with no shear reinforcement; less than 1 for members with shear reinforcement.

Similar to EC2, the semi-empirical equation also takes in consideration the concrete strength, reinforcement ratio, size effect and dowel action. The BS8110, however, allows the concrete to be increased by multiplying the shear resistance ensured by the concrete by a factor of $\beta = \frac{2d}{a_v}$ to take into account the contribution of the arching effect for the short-span members or deep members. However, the BS8110 considers the shear force provided by shear reinforcement and concrete, differently from EuroCode2[6]. Consequentially, shear resistance (V_{Rd}) is the sum of, $V_{Rd,c}$ and $V_{Rd,s}$. Below is the BS8110 equation to calculate the shear reinforcement in parts close to the supports.

The increase in shear strength can be considered in the design of the sections close a support by enhancement shear stress of the design concrete v_c to $2dv_c/a_v$.

For shear reinforcement is necessary, the total area is as follows:

$$\sum A_{sw} = \left(v - \frac{2d}{a_v} v_c\right) \frac{a_v b_v}{0.87 f_{yv}} \tag{3.7}$$

v : is shear stress in a cross-section

v_c : is shear stress of the design concrete (show Figure 3.4.)

d : is the effective depth

a_v : is the length of that piece of a member passing through the shear failure plane

b_v : is the section width

f_{yv} : is characteristic strength of stirrups (this value should not be more than 500N/mm^2)

The reinforcement can be ensured in the middle of the three-quarters of a_v. Horizontal shear reinforcement can be more effective than vertical when a_v is smaller than d.

$\frac{100A_s}{b_v d}$	Effective depth mm							
	125	150	175	200	225	250	300	≥ 400
	N/mm^2	N/mm^2	N/mm^2	N/mm^2	N/mm^2	N/mm^2	N/mm^2	N/mm^2
≤ 0.15	0.45	0.43	0.41	0.40	0.39	0.38	0.36	0.34
0.25	0.53	0.51	0.49	0.47	0.46	0.45	0.43	0.40
0.50	0.67	0.64	0.62	0.60	0.58	0.56	0.54	0.50
0.75	0.77	0.73	0.71	0.68	0.66	0.65	0.62	0.57
1.00	0.84	0.81	0.78	0.75	0.73	0.71	0.68	0.63
1.50	0.97	0.92	0.89	0.86	0.83	0.81	0.78	0.72
2.00	1.06	1.02	0.98	0.95	0.92	0.89	0.86	0.80
≥ 3.00	1.22	1.16	1.12	1.08	1.05	1.02	0.98	0.91

NOTE 1 Allowance has been made in these figures for a γ_m of 1.25.
NOTE 2 The values in the table are derived from the expression:
$0.79\{100A_s/(b_v d)\}^{1/3} (400/d)^{1/4}/\gamma_m$
where
$\frac{100A_s}{b_v d}$ should not be taken as greater than 3;
$\frac{400}{d}$ should not be taken as less than 1.
For characteristic concrete strengths greater than 25 N/mm^2, the values in this table may be multiplied by $(f_{cu}/25)^{1/3}$. The value of f_{cu} should not be taken as greater than 40.

Figure 3.4. The design concrete shear stress values $\boldsymbol{v_c}$[20] (adapted from BS8110)

3.3. SHEAR ENHANCEMENT APPROACH FOR REINFORCED CONCRETE MEMBERS IN *FIB* MC2010

The MC2010[8] shear provisions for one-way shear strength of members are explained in detail by Sigrist et al. [7]. Herein, a brief summary of the equations is given.

3.3.1. *fib* MC2010 Shear Design Equations For Members With No Shear Reinforcement

The provisions of shear of the *fib* MC2010 [8] come from the Simplified Modified Compression Field Theory (SMCFT) for members with no shear reinforcement [9]. In this model the effect of longitudinal straining due to flexure and member size effect in shear, are taken into account when determining the shear strength of members. An approach called the "level of approximation" (LoA) approach [10] was adopted in the *fib* MC2010 where advanced models were conservatively simplified. Herein, Level II approximation is used for members with no shear reinforcement.

For reinforced concrete members with no shear reinforcement, Level II shear resistance, is given as:

$$V_{Rd,c} = k_v \frac{\sqrt{f_{ck}}}{\gamma_c} b_w z \ \ (f_{ck} \text{ in MPa}) \tag{3.8}$$

In Eq. (3.8), $\sqrt{f_{ck}}$ must not be taken greater than 8 MPa; effective shear depth z can be taken as $0.9d$ and k_v is defined as,

$$k_v(\text{II}) = \frac{0.4}{1+1500\varepsilon_x} \frac{1300}{1000+k_{dg}z} \ \ (z \text{ in mm}) \tag{3.9a}$$

$$k_{dg} = \frac{32}{16+d_g} \geq 0.75 \ \ (d_g \text{ in mm}) \tag{3.9b}$$

The term k_{dg} in Eq. (3.9b), accounts for the effect of different aggregate sizes, d_g, and should be taken as zero for concrete strengths greater than 70 MPa.

For non-prestressed members without axial loads, ε_x is the longitudinal strain that is represent the medium depth of the effective shear depth, is given in the follows:

$$\varepsilon_x = \frac{\frac{M_{Ed}}{z}+V_{Ed}}{2E_sA_s} \tag{3.10}$$

where M_{Ed} and V_{Ed} are respectively applied moment and shear at the critical section, E_s is the elastic modulus of longitudinal reinforcement; A_s is the area of longitudinal reinforcement. If the value of ε_x is less than 0, the result is always equal to 0 and ε_x cannot be greater than 0.003.

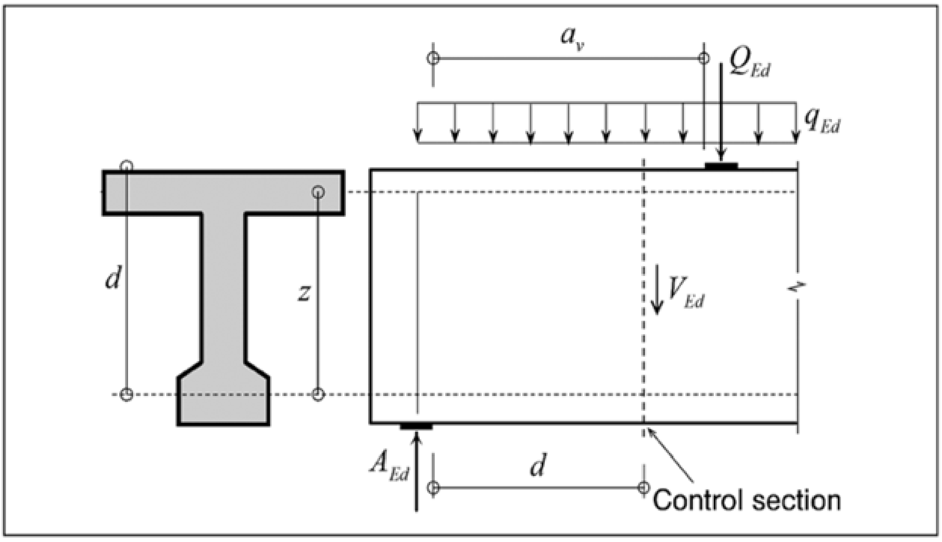

Figure 3.5. Definition of control section and distance $\boldsymbol{a_v}$ (adapted from the *fib* MC2010) [8]

In order to calculate the LoA II shear strength of a section with no shear reinforcement, an iteration process is necessary. If point loads are carried out at a distance of d to 2d to the near of the support, *fib* Model Code 2010 allows sectional shear enhancement unless further refined techniques are used to take into account the loads received on a direct support by strut action. *fib* MC2010[8] shear enhancement procedure is to decrease of the shear force, V_{Ed}, with using factor $a_v/2d$ where a_v is the distance from the face of the bearing plate, on which the point load is applied, to the face of the support as depicted in Fig. 3.5. adapted from the *fib* Model Code 2010. It must be noted that the *fib* Model Code does not state that the design moment associated with the design shear should also be reduced. Therefore, to account for shear enhancement, the strain in longitudinal stress in the medium depth of the effective shear depth, ε_x, is calculated below follows:

$$\varepsilon_x = \frac{\frac{M_{Ed}}{z} + \beta V_{Ed}}{2E_s A_s} \tag{3.11}$$

where β is equal to $a_v/2d$ and cannot be taken less than 0.5.

3.3.2. *fib* MC2010 Shear Design Equations For Members With Shear Reinforcement

The provisions of shear of *fib* Model Code 2010 [8] for members with shear reinforcement are come from the some special theory such as a general stress field approach ([11], [12]) related to SMCFT [9]. For members with shear reinforcement, *fib* MC2010 introduces three levels of accuracy. Herein, Level III approximation is used, where the shear resistance V_{Rd} is given as:

$$V_{Rd} = V_{Rd,c} + V_{Rd,s} \tag{3.12}$$

In the above equation $V_{Rd,c}$ is the shear strength provided by concrete and given by Eq. 3.8 and $V_{Rd,s}$ is the shear carried by stirrups,

$$V_{Rd,s} = \frac{A_{sw}}{s_w} z \frac{f_{yw}}{\gamma_s} \cot\theta \tag{3.13}$$

where A_{sw} is the cross-sectional area of shear reinforcement, s_w is the spacing of shear reinforcement, f_{yw} is the yield strength of shear reinforcement. The minimum value of ratio of stirrup reinforcement in accordance with the *fib* MC2010[8] is calculated as:

$$\rho_{w,min} = 0.08 \frac{\sqrt{f_{ck}}}{f_{yw}} \;\; (f_{ck} \text{ and } f_{yw} \text{ in MPa}) \tag{3.14}$$

The shear resistance of a section, V_{Rd}, is governed by crushing strength of struts, $V_{Rd,max}$, which is given as:

$$V_{Rd,max} = k_\varepsilon \eta_{fc} \frac{f_{ck}}{\gamma_c} b_w z \sin\theta \cos\theta \tag{3.15a}$$

$$\eta_{fc} = \left(\frac{30}{f_{ck}}\right)^{1/3} \leq 1 \;\; (f_{ck} \text{ in MPa}) \tag{3.15b}$$

$$k_\varepsilon = \frac{1}{1.2+55\varepsilon_1} \leq 0.65 \tag{3.15c}$$

$$\varepsilon_1 = \varepsilon_x + (\varepsilon_x + 0.002)cot^2\theta \tag{3.15d}$$

In Eq. 3.15a, k_ε accounts for the strain effect and η_{fc} accounts for brittleness of concrete with f_{ck}>30 MPa. $V_{Rd,c}$ is given by Eq. 3.8 and k_v value for members with stirrups considering LoA III is calculated as:

$$k_v\ (\mathrm{III}) = \frac{0.4}{1+1500\varepsilon_x}\left(1 - \frac{V_{Ed}}{V_{Rd,max}(\theta_{min})}\right) \geq 0 \quad (3.16a)$$

$$\theta_{min} = 20^{\circ} + 1000\varepsilon_x \quad (3.16b)$$

$V_{Rd,max}(\theta_{\min})$ corresponds to the crushing of struts at minimum inclination and it is calculated by substituting $\theta = \theta_{min}$ in Eq. (3.15a). Minimum angle of inclination, θ_{min}, is given in Eq. (3.16b).

In order to estimate the LoA III shear capacity of a section with shear reinforcement an iteration process, is necessary. To account for shear enhancement, the strain in longitudinal in the medium depth of the effective shear depth, ε_x, is calculated as given in equation (3.11), where the shear force of design, V_{Ed}, is decrease by the factor $\beta = a_v/2d$ and k_v(III) is calculated considering the reduction in the design shear force, V_{Ed}, as follows:

$$k_v(\mathrm{III}) = \frac{0.4}{1+1500\varepsilon_x}\left(1 - \frac{\beta V_{Ed}}{V_{Rd,max}(\theta_{min})}\right) \geq 0 \quad (3.17)$$

4. SHEAR STRENGTH PREDICTIONS USING THE *FIB* MODEL CODE 2010

Predictions of shear strength using the *fib* MC2010 shear enhancement method are carried out in this chapter.

4.1. ACI- DAFSTB DATABASE OF SHEAR TESTS FOR NON-SLENDER RC MEMBERS

Various check and filtration criterion have been practiced to reach databases that can be used to evaluate the correctness and conservatism of design provisions. Shear strength predictions of the tests in the two evaluation databases are calculated in accordance with the shear enhancement method in the *fib* MC2010[8]. Partial safety factors for concrete, γ_c, and steel reinforcement, γ_s, are taken as 1.0 when calculating strength predictions of the specimens in the databases. In this study, shear strength predictions are based on characteristic concrete strength, f_{ck}, which is given in the evaluation database. For both databases, failure loads that include the shear due to self-weight of the specimens and weight of loading equipment are considered. The design moment M_{Ed} is calculated at a section d away from the face of the support for beams with $a_v \leq 2d$ and at a section d away from the concentrated load if $a_v > 2d$. Note that if $a_v > 2d$ then no sectional shear enhancement is applied. For specimens with $a_v < d$, M_{Ed}/V_{Ed} is simply taken equal to the effective depth, d, of the section. When calculating the design bending moment, the lengths of support and loading plates are needed, therefore an average value of $0.2h$ is assumed as suggested in Reineck and Todisco [13], if plate sizes are not given.

4.2. SHEAR TESTS ON NON-SLENDER MEMBERS WITH NO SHEAR REINFORCEMENT

Details of ACI-DAfStb evaluation database of shear tests on non-slender members (a/d<2.4) without shear reinforcement are presented in a paper by Reineck and Todisco [13]. This database consists of 222 tests after applying several selection criteria as explained in Reineck and Todisco [13]. To calculate the shear resistance of members with no shear reinforcement using the MC2010 shear provisions, aggregate size, d_g, is needed. It is assumed that d_g is equal to 6 mm if relevant data is not provided in the evaluation database.

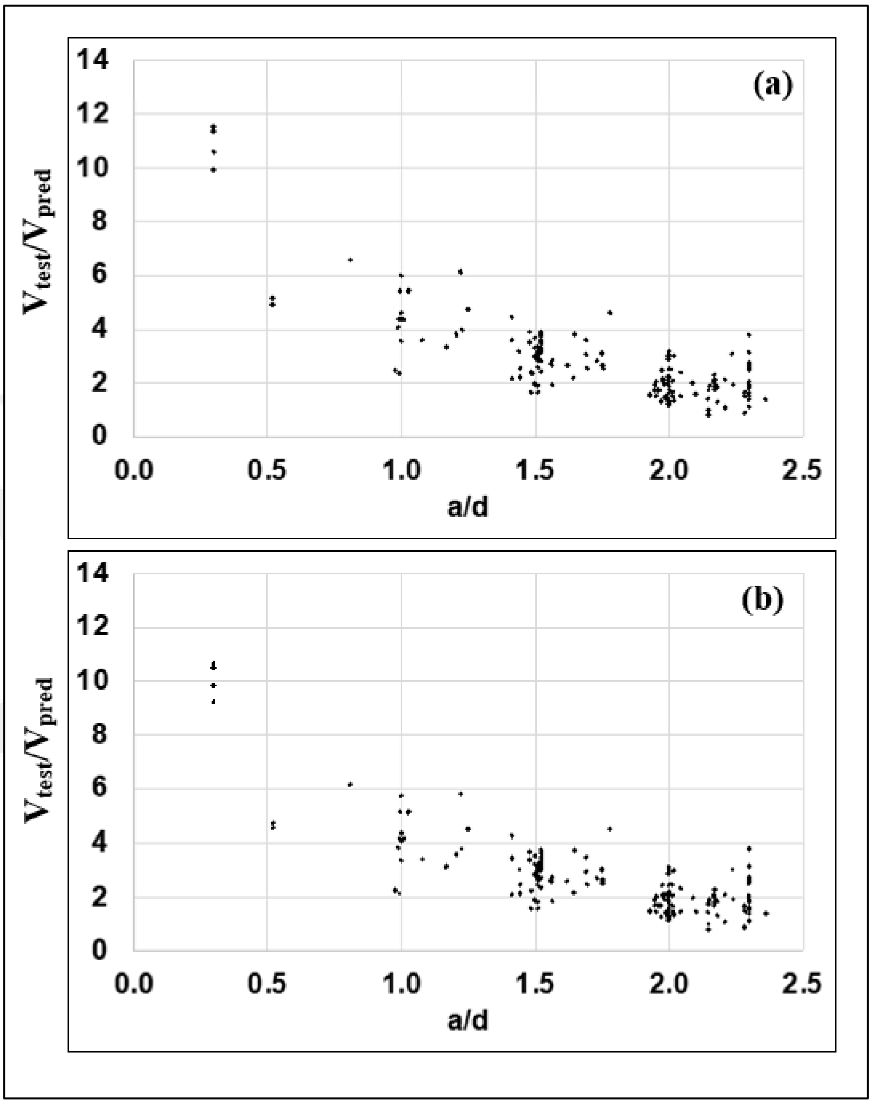

Figure 4.1. Shear capacity predictions of non-slender members without shear reinforcement using the *fib* MC2010 shear design provisions: (a) with no shear enhancement, (b) with shear enhancement

Fig. 4.1 compares the shear strength predictions by *fib* MC2010[8], with and with no considering shear enhancement for loads close to supports. The ratio of experimental shear strength to predicted shear strength for each test is plotted with respect to shear span-to-effective depth ratio, a/d. As can be seen from Figure 4.1, the sectional shear enhancement method explained in chapter 3.3.1. the *fib* Model Code gives very conservative predictions especially for very short specimens with no shear reinforcement. Applying the shear enhancement method given in the *fib* MC2010[8] does not change the quality of the shear strength predictions, substantially. The mean ratio of experimental-to-predicted shear strengths is 3.06 with a coefficient of variation of 57 percent even when the shear enhancement method in the *fib* MC2010 is considered for non-slender members with no shear reinforcement.

4.3. SHEAR TESTS ON NON-SLENDER MEMBERS WITH SHEAR REINFORCEMENT

ACI-DAfStb evaluation database of shear tests on non-slender members $(a/d < 2.4)$ with stirrups is presented in two recent papers by Todisco et al. [14], [15]. This evaluation database consists of 178 tests. In this paper, only 171 tests with rectangular sections are used for verification of shear enhancement models. Two of the tests do not satisfy the minimum shear reinforcement requirement given in Eq. 13 however, they are included in the evaluation database.

Figure 4.2 compares the shear strength predictions of non-slender members with shear reinforcement by the fib Model Code 2010, with and without applying the shear enhancement model. As can be seen from Figure 4.2, the sectional shear enhancement method explained in chapter 3.3.2. the fib Model Code gives very conservative predictions for most of the tests although the shear strength is predicted using the shear enhancement method in the fib MC2010. The mean ratio of experimental-to-predicted shear strengths of non-slender members with shear reinforcement is 1.59 with a coefficient of variation of 45 percent.

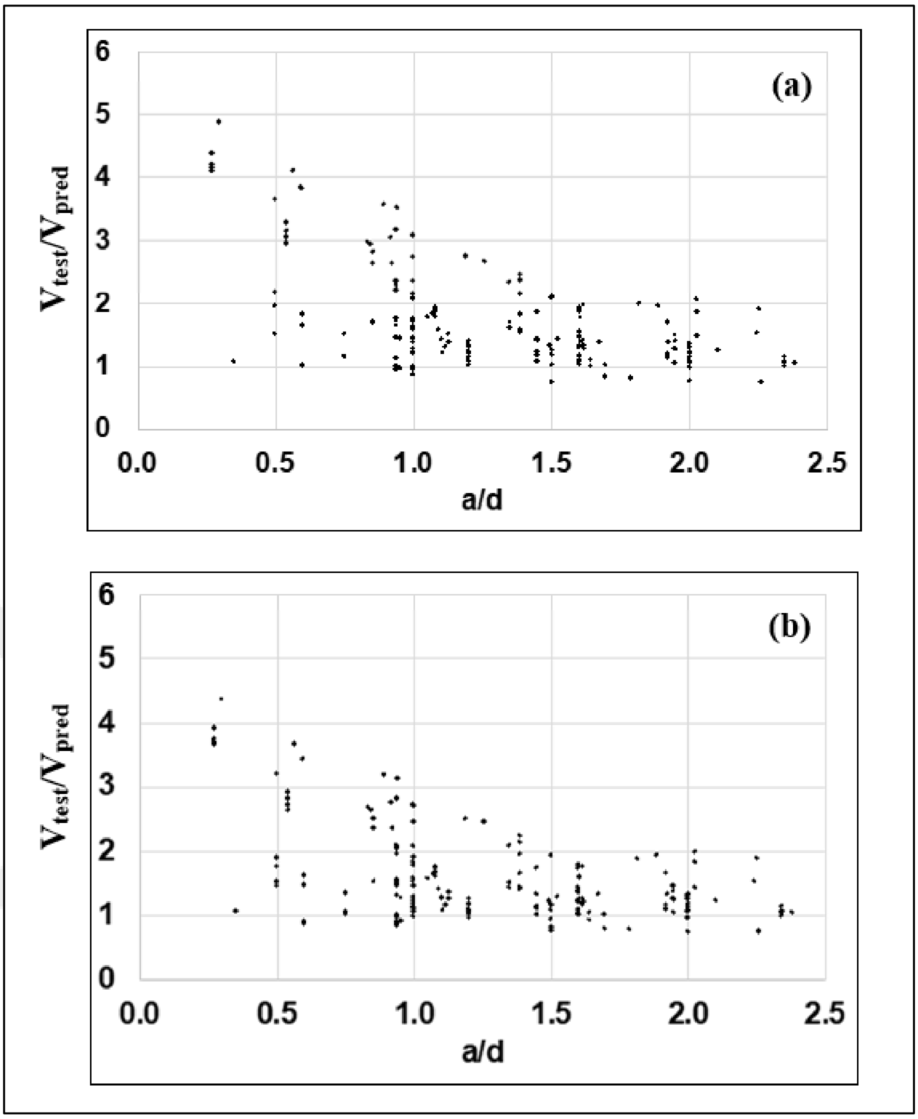

Figure 4.2. Shear strength predictions of non-slender members with shear reinforcement using the *fib* MC2010 shear design methods: (a) with no shear enhancement, (b) with shear enhancement

4.4. PROPOSED SECTIONAL SHEAR ENHANCEMENT METHOD FOR THE *FIB* MODEL CODE 2010

It is clear from Figures 4.1 and 4.2 that the shear enhancement approach adopted in the *fib* Model Code 2010 gives very conservative results particularly for beams with low a/d ratios. In this paper, a shear enhancement method, somewhat similar to the one in BS8110 [20], is proposed to increase the accuracy of the predictions. It must be recalled that the sectional shear enhancement method is merely an empirical approach to consider the increase in shear capacity of non-slender members, without applying more detailed models to study the flow of forces when loads are applied closer to supports.

For non-slender members with no shear reinforcement the enhanced shear capacity is simply be calculated with increasing the shear strength provided by concrete, $V_{Rd,c}$, by a factor of $(2d/a_v)$.

$$V_{enhanced} = \frac{2d}{a_v} V_{Rd,c} \tag{4.1}$$

If it happens non-slender members with shear reinforcement the enhanced capacity cannot exceed the strength given by crushing of struts at minimum inclination, $V_{Rd,max}(\theta_{min})$.It is proposed that the enhanced shear capacity for non-slender members with shear reinforcement can be calculated as follows:

$$V_{enhanced} = \frac{2d}{a_v} V_{Rd,c} + V_{Rd,s} \leq V_{Rd,max}(\theta_{min}) \tag{4.2}$$

As can be understood from Equation 4.2, only the shear capacity provided by concrete is increased if loads are applied within distance $2d$ of the supports of non-slender members with shear reinforcement.

Figure 4.3 compares the experimental shear strengths to predicted shear strengths in the two databases using the proposed shear enhancement method. It is clearly seen from Figure 4.3 that the proposed shear enhancement method yields better results.

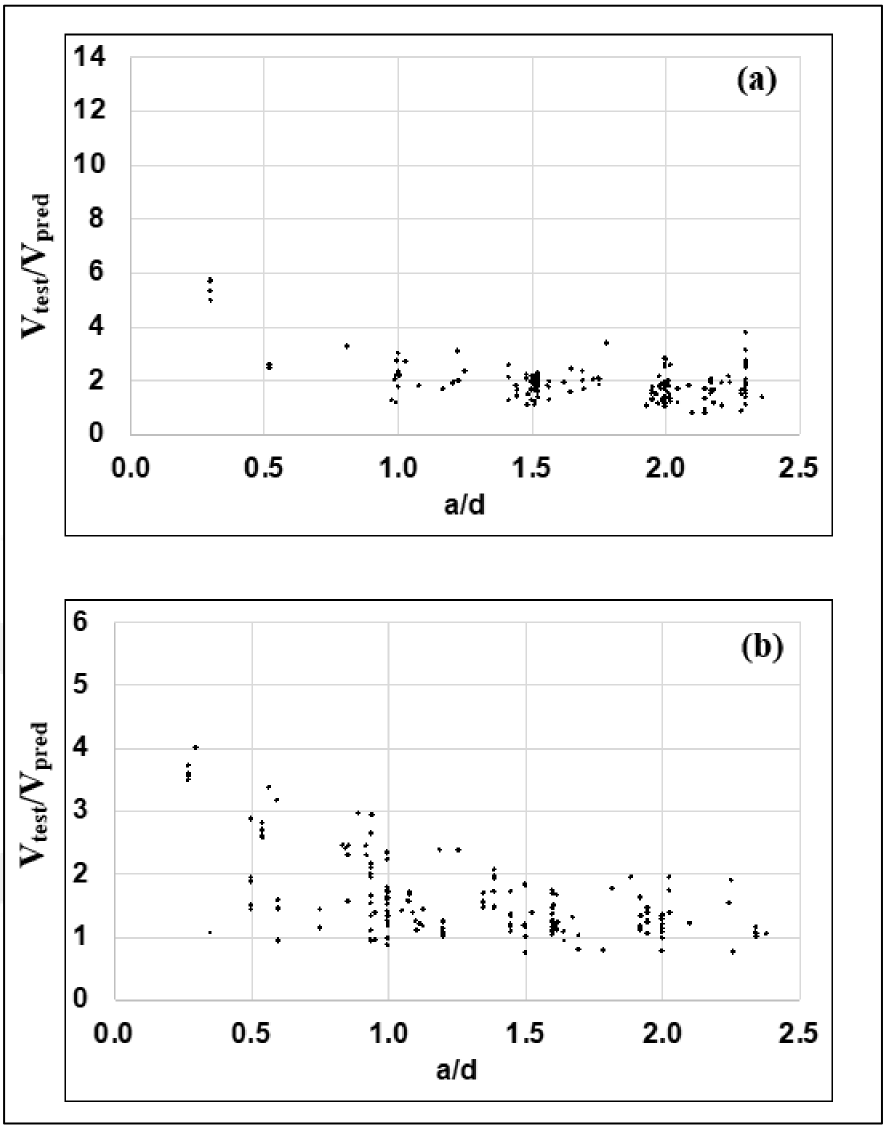

Figure 4.3. Shear strength predictions of non-slender members using the *fib* MC2010 shear design provisions with the proposed shear enhancement method: (a) with no shear reinforcement, (b) with shear reinforcement

Table 4.1 gives the overall statistical evaluation of the two enhancement methods. The statistical results show that the shear enhancement method proposed in this paper, gives better predictions of the experiments in the two databases.

Table 4.1. Statistical analysis of results

		***fib* MC2010**	***fib* MC2010 with shear enhancement**	***fib* MC2010 proposed shear enhancement**
Without stirrups (222 tests)	mean	3.20	3.06	2.03
	max	11.47	10.65	5.74
	min	0.79	0.79	0.77
	CoV(%)	59	57	41
With stirrups (171 tests)	mean	1.73	1.59	1.55
	max	4.88	4.36	3.99
	min	0.72	0.73	0.72
	CoV(%)	47	45	41

The mean ratio of experimental-to-predicted shear strengths is 2.03 with a CoV of 41 percent percent for non-slender members without shear reinforcement and 1.55 with a CoV of 41 percent for non-slender members with shear reinforcement, when the proposed shear enhancement method along with the *fib* MC2010[8] shear design provisions is used.

5. PREDICTING SHEAR STRENGTH OF POINT LOADED NON-SLENDER MEMBERS USING VERTICAL COMPRESSIVE STRESSES

In the shear design of non-slender members, significant vertical compressive stresses occur by the loading and support conditions. Sectional models which ignore the beneficial effects of these vertical compressive stresses underestimate the shear strength of such members. In this chapter, a simple expression is derived to calculate the vertical compressive stress at mid-depth of point loaded non-slender beams and the influence of these compressive stresses is incorporated into the shear design procedures of *fib* Model Code 2010. ACI-DAfStb evaluation databases of shear tests on point loaded simply supported reinforced concrete members with and without stirrups, are employed to evaluate the accuracy of this proposed sectional analysis method. It is concluded that sectional analysis with vertical compressive stresses yields better results than the sectional method in the *fib* Model Code 2010.

5.1. SECTIONAL SHEAR ANALYSIS WITH VERTICAL COMPRESSIVE STRESSES

In this section, a very simple expression is suggested to determine the vertical compressive stresses in point loaded short members. These vertical stresses are then incorporated into the shear design equations of *fib* MC2010[8]. The accuracy and conservativeness of this proposed sectional analysis method are assessed using ACI-DAfStb evaluation databases of shear tests on point loaded simply supported reinforced concrete members with and without shear reinforcement.

5.1.1. Simple Expression To Determine Vertical Compressive Stresses In Point Loaded Members

A simple expression to calculate the vertical compressive stress at mid-height of the control section is created with non-linear FEA results. Point loaded short members with different shear span, a to effective depth, d ratios are analyzed using VecTor2, a non-linear finite element program, which is based on Disturbed Stress Field Model [25]. As can be understood from equation (5.1), vertical compressive stresses at mid-height of control section is expressed in terms $V_{Ed}/(a.b_w)$, where V_{Ed} is the applied design shear, b_w is the width of the member cross-section and a is the shear span measured from the centre of loading plate to the centre of support.

$$f_z = \frac{V_{Ed}}{a \times b_w}\left(-0.6\frac{a}{d} + 1.4\right) \leq 0.7\frac{V_{Ed}}{a \times b_w} \tag{5.1}$$

Control section for shear design is taken at a distance d away from the face of support or loading plate, whichever produces a greater moment to shear ratio. When calculating vertical compressive stresses for members with a shear span-to-effective depth ratio, a/d of 1, the control section is assumed to be at the centre of clear span.

It is assumed that the vertical compressive stresses in the web of a non-slender member cannot be taken greater than $0.7V_{Ed}/(a \times b_w)$. It must be noted that this vertical compressive stress expression is good for members with shear span-to-effective depth values equal to or greater than 1.

5.1.2. Including Vertical Compressive Stresses In The *Fib* Model Code Shear Design Equations

Considering the vertical equilibrium of forces in the web of a member as shown in Figure 5.1 and the equations of *fib* Model Code 2010 [8], the beneficial effect of vertical compressive stresses can be included by adding a shear strength carried by the vertical compressive stresses as below,

$$V_{Rd} = V_{Rd,c} + V_{Rd,s} + V_{Rd,clamping} \geq V_{Ed} \tag{5.2}$$

It is clear that these vertical compressive stresses act in the same direction as the steel stresses provided by shear reinforcement and $V_{Rd,clamping}$, clamping can be expressed as:

$$V_{Rd,clamping} = f_z \cot\theta \, b_w z \tag{5.3}$$

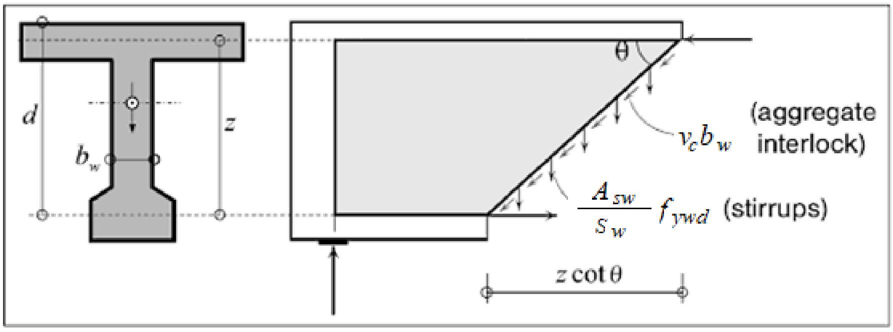

Figure 5.1. Forces in the web of a member [8]

As mention previous chapter, Simplified Modified Compression Field Theory (SMCFT) [9] are fundamental of the shear provisions of the fib Model Code 2010 [8] for members with no shear reinforcement. For members with no shear reinforcement there are two levels of accuracy in the *fib* Model Code 2010 [8]. Herein, approximation in Level II is used, where the shear resistance V_{Rd} is defined the combined with shear strength ensured by concrete and by vertical compressive stresses. The shear provisions of the *fib* Model Code 2010 [8] for members with shear reinforcement are based on a general stress field approach (Muttoni et al. [11], Sigrist [12]) combined with Simplified Modified Compression Field Theory [9]. For members with shear reinforcement, *fib* Model Code 2010 introduces three levels of accuracy. Herein, Level III approximation is used, where the shear resistance V_{Rd} is given as the total of the shear strength ensured by concrete, $V_{Rd,c}$, the shear carried by stirrups, $V_{Rd,s}$ and the shear strength provided by the vertical compressive stresses. The shear design equations of the *fib* Model Code 2010 (*fib* MC2010 [21]) for one-way shear strength of members are explained in detail by Sigrist et al. [7]. Herein, only proposed changes to include vertical compressive stresses in the shear strength calculations will be given.

The shear resistance of a section, V_{Rd}, is governed by the crushing of struts at minimum inclination, $V_{Rd,max}(\theta_{min})$ and it is calculated by substituting $\theta = \theta_{min}$ in equation (7.3-26) of the *fib* MC2010 [8]. In non-slender members a strut will form between the applide load and the support. For very nen-slender members the inclination of this strut will be very steep. The geometry of the member will dictate the inclination of compressive stress field in short members therefore the current expression of minimum stress field inclination will typically give very low values whereas the actual inclination is steeper. It is important to note that for steep crack inclinations i.e., high θ values, the contribution of web reinforcement to the shear strength of the member is less than that of for lower θ values.

Therefore, it is proposed that if vertical stresses are considered in the sectional analysis, the minimum angle of inclination is calculated using the expression below,

$$\theta_{min} = 40° + 10000\varepsilon_x \leq 50° \tag{5.4}$$

where ε_x is the strain in longitudinal in the medium depth of the effective shear depth and calculated as before.

5.2. COMPARISON OF EXPERIMENTAL AND PREDICTED VALUES OF SHEAR STRENGTH

Shear strength predictions of the tests in the two evaluation databases are calculated according to the proposed sectional analysis which includes the beneficial effects of vertical compressive stresses.

The explanations of partial safety factors for concrete, γ_c, steel reinforcement, γ_s, characteristic concrete strength, f_{ck}, and the design moment, M_{Ed} values to be used when calculating the strength predictions of the specimens in the databases are explained in detail in chapter 4 under title 4.1.ACI- DAfStb Database of shear tests for non-slender RC members.

5.2.1. Shear Strength Predictions Of Non-Slender Beams (A/D<2.4) With No Shear Reinforcement

As mentioned in Section 4.2. Shear tests on non-slender members without shear reinforcement, this database consists of 222 tests. But unlike that section, since the vertical compressive stress expression given in (5.1) is good for members with a/d ratios of greater than or equal to 1, the results of 37 tests with $a/d < 1$ are excluded.

Figure 5.2 compares the shear strength predictions of non-slender members with no shear reinforcement by *fib* MC2010[8], with and without vertical compressive stresses for members that are loaded close to supports. The ratio of experimental shear strength to estimated shear strength for each test is plotted with respect to ratio of shear span to effective depth, a/d.

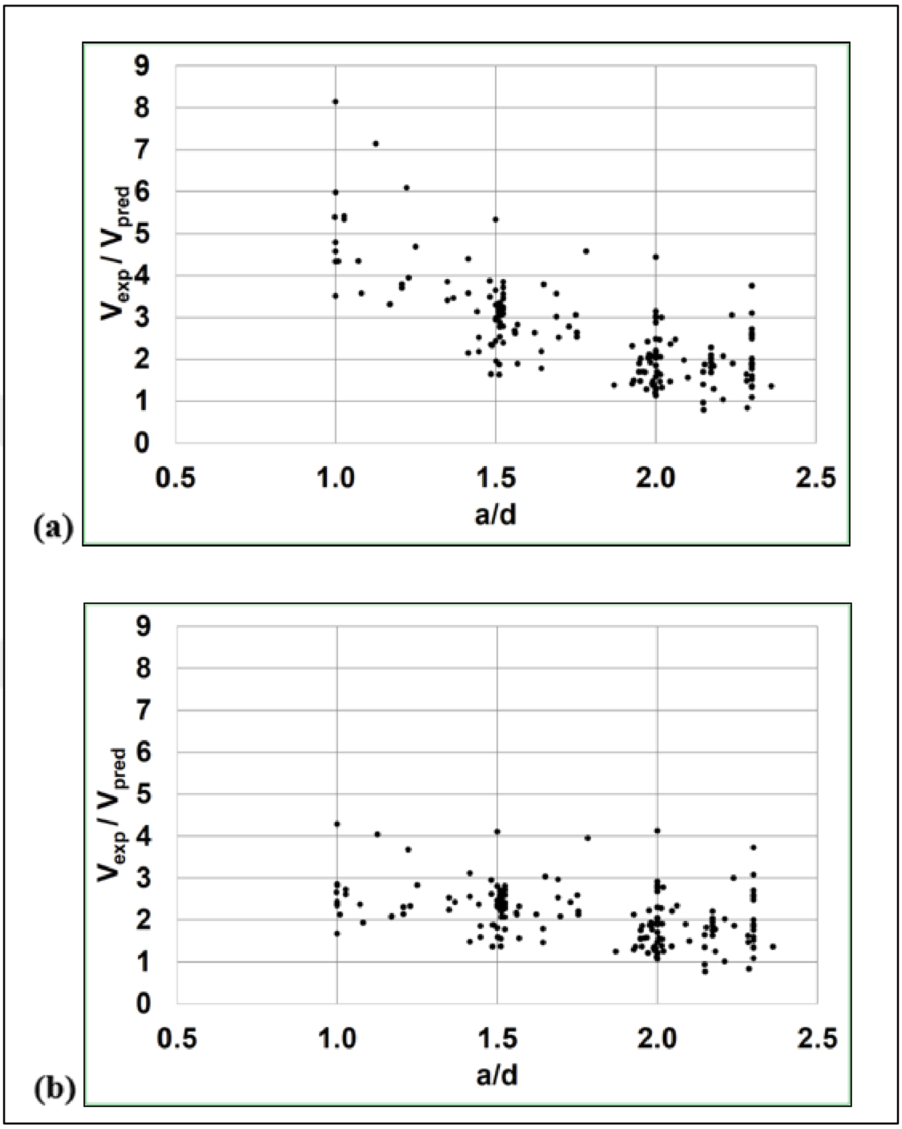

Figure 5.2. Shear strength predictions of non-slender members with no shear reinforcement by the *fib* Model Code 2010: (a) with no vertical compressive stresses, (b) with vertical compressive stresses

As it can be seen from Fig. 5.2, the sectional method given in the fib Model Code gives very conservative predictions especially for very short specimens. Applying the proposed sectional analysis method which includes the effect of vertical compressive stresses yields to better results for non-slender members without shear reinforcement with much less scatter.

The mean value of experimental-to-estimated shear capacity ratios is 2.13 with a coefficient of variation of 30 percent for the proposed method which includes the beneficial effects of vertical compressive stresses.

5.2.2. Shear Strength Predictions Of Non-Slender Beams (A/D<2.4) With Shear Reinforcement

As mentioned in section 4.3. Shear tests on non-slender members with shear reinforcement, this database consists of 171 tests. As explained before the results of 48 tests with $a/d < 1$ are excluded. Two of the tests from the remaining 123 tests, do not satisfy the minimum shear reinforcement requirement given in the *fib* Model Code 2010 [8], however they are involved in the evaluation database.

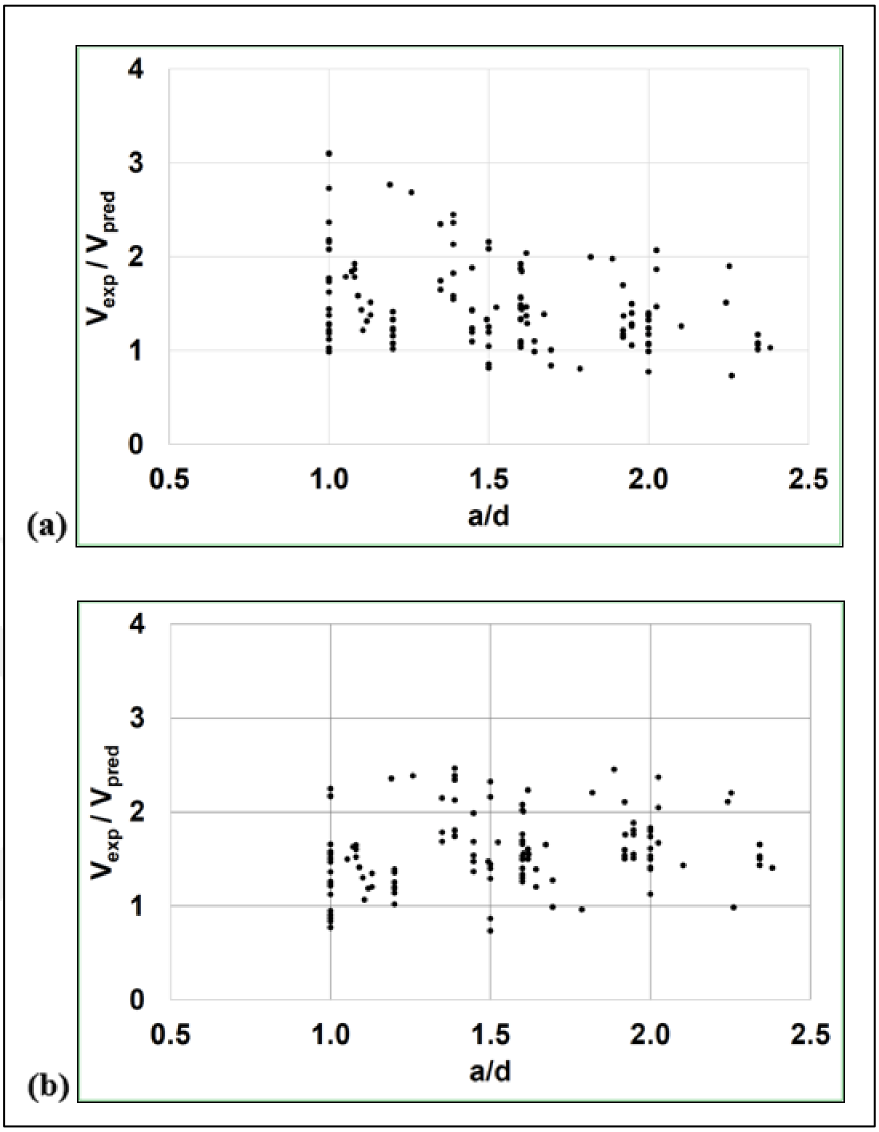

Figure 5.3. Shear strength predictions of non-slender members with shear reinforcement using by the *fib* Model Code 2010: (a) with no vertical compressive stresses, (b) with vertical compressive stresses

Figure 5.3 compares the shear strength predictions of non-slender members with shear reinforcement by MC2010, with and without including vertical compressive stresses for loads close to supports. As it can be seen from Figure 5.3, the sectional method given in the *fib* Model Code gives very conservative predictions for most of the tests. Applying the proposed sectional analysis method which includes the effect of vertical compressive stresses yields to better results for non-slender members with shear reinforcement. The mean value of experimental-to-estimated shear strength ratios is 1.58 with a coefficient of variation of 26 percent for the proposed method.

Table 5.1. Statistical analysis of results

		***fib* Model Code**	***fib* Model Code Sectional Analysis with vertical compressive stresses**
without stirrups (185 tests)	mean	2.72	2.13
	max	8.14	4.29
	min	0.79	0.77
	CoV(%)	43	30
with stirrups (123 tests)	mean	1.50	1.58
	max	3.10	2.47
	min	0.73	0.74
	CoV(%)	32	26

The statistical analyses of each of the methods used for shear strength calculations is summarized in Table 5.1. It is clear that sectional analysis with vertical compressive stresses yields to less scattered results for both members with and without shear reinforcement. It can be observed from Figures 5.2 and 5.3 that shear strength predictions, especially for very short members that have shear span to effective depth, a/d values of less than 1.5, are greatly improved when the shear strength provided by the vertical compressive stresses are included in the shear strength equations of *fib* MC2010 [8].

6. RESULTS AND DISCUSSION

The following results are obtained in this thesis.

NLFEM and ST methods are suitable methods for the design of short span members, that are the subject of this thesis. STM is coded in national design standards such as EC2, *fib* Model Code 2010. The most important challenge in the application of the ST method is because of the creation of a suitable STM and the recognition of node dimensions. Therefore, there are sometimes errors in applying this model.

Many shear resistance models are described in the literature. In this thesis, three shear resistance models were mentioned. These three models (the Zararis model [5], the Unified Shear Strength model of Kyoung-Kyu et al. [17], the Two-Parameter kinematic theory of Mihaylov et al. [19]) are attributed on the equilibrium and a default failure mechanism.

There are important differences in code design provisions for shear in non-slender members. Chapter 3.1. and Chapter 3.2. are described and explained. For example, the shear resistance for BS8110 consists of the sum of concrete and shear reinforcement, while the situation for EC2 is different. In the EC2, the shear resistance is completely countered by the shear reinforcement if any.

Initially, shear strength predictions of non-slender members are calculated by using the sectional enhancement method given in *fib* Model Code 2010. Later, it was suggested that for non-slender members with and with no shear reinforcement, the improved shear capacity can be calculated by enhance the shear strength provided by the concrete. After all, the results are calculated and shown in Table 4.1.

The effect of vertical compressive stresses on shear design equations of *fib* Model Code 2010 is shown in the Chapter 5. As a result, sectional shear strength predictions with the proposed method of these vertical compression stresses were compared for shear tests on non-slender reinforced concrete members with and without shear reinforcement. The results are presented in Figures 5.1 and 5.2 and statistical analyses of these results are given in Table 5.1.

7. CONCLUSION

The main purpose of this thesis is to develop simple equations to consider shear enhancement in members with a point load applied to the upper part of the member within a distance of 2d.

The *fib* Model Code 2010 allows the engineer to use sectional shear strength enhancement instead of using more refined models to account for the strut action in non-slender members. In order to assess the conservativeness and accuracy of the shear enhancement method in the *fib* Model Code 2010, shear strength predictions are evaluated using ACI-DAfStb databases for shear tests on non-slender reinforced concrete members, with and without shear reinforcement. It is shown that the shear enhancement method given in the MC2010 gives very conservative results especially for short members. In this thesis, a simple shear enhancement method is proposed to increase the accuracy of the shear strength predictions of non-slender members. It is shown that the proposed shear enhancement method, along with the *fib* Model Code 2010 shear provisions, gives better shear strength predictions for non-slender members with and with no shear reinforcement.

Although the shear enhancement method proposed in this thesis gives better predictions then the one in *fib* Model Code 2010, it is observed that the results are still not improved. It is believed that there is need for a more realistic method to predict shear strength of non-slender members.

Since it is practical to use sectional analysis methods in the design of non-slender members with disturbed regions where vertical compressive stresses may be of significant magnitude, instead of more complicated analyses based on non-linear finite element models or strut-and-tie models. The scope of this thesis the *fib* MC2010[8] shear design parameters are extended to include the effect of vertical compressive stresses. A simple expression for the vertical compressive stresses in point loaded non-slender members is developed based on non-linear finite element analyses. Sectional shear strength predictions with the proposed method which includes the beneficial effects of these vertical compressive stresses are compared to the tests results in two databases, namely, ACI-DAfStb databases for shear tests on non-slender reinforced concrete members, with and without shear reinforcement. As expected, the sectional analysis method given in the MC2010[8] gives very conservative results especially for short members. It is shown that the proposed sectional shear strength prediction method, gives better predictions for non-slender members with and with no shear reinforcement. It is concluded that a simple sectional model with minor modifications to include the effect of loads close to supports is a practical tool for engineers, to predict the shear strength of non-slender members.

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