RADIO COMMUNICATIONS INTERDICTION PROBLEM

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By

Türker TANERGÜÇLÜ

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We certify that we have read this dissertation and that in our opinion it is fully adequate, in scope and in quality, as a dissertation for the degree of Doctor of Philosophy.

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ABSTRACT

RADIO COMMUNICATIONS INTERDICTION PROBLEM

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Tactical communications have always played a pivotal role in maintaining effective command and control of troops operating in hostile, extremely fragile and dynamic battlefield environments. Radio communications, in particular, have served as the backbone of the tactical communications over the years and have proven to be very useful in meeting the information exchange needs of widely dispersed and highly mobile military units, especially in the rugged area.

Considering the complexity of today's modern warfare, and in particular the emerging threats from the latest electronic warfare technologies, the need for optimally designed radio communications networks is more critical than ever. Optimized communication network planning can minimize network vulnerabilities to modern threats and provide additional assurance of continued availability and reliability of tactical communications.

To do so, we present the Radio Communications Interdiction Problem (RCIP) to identify the optimal locations of transmitters on the battlefield that will lead to a robust radio communications network by anticipating the degrading effects of intentional radio jamming attacks used by an adversary during electronic warfare. We formulate RCIP as a binary bilevel (max–min) programming problem, present the equivalent single level formulation, and propose an exact solution method using a decomposition scheme. We enhance the performance of the algorithm by utilizing dominance relations, preprocessing, and initial starting heuristics.

To reflect a more realistic jamming representation, we introduce the probabilistic version of RCIP (P-RCIP) where a jamming probability is associated at each receiver site as a function of the prevalent jamming to signal ratios leading to an expected

coverage of receivers as an objective function. We approximate the nonlinearity in the jamming probability function using a piecewise linear convex function and solve this version by adapting the decomposition algorithm constructed for RCIP.

Our extensive computational results on realistic scenarios that reflect different phases of a military conflict show the efficacy of the proposed solution methods. We provide valuable tactical insights by analyzing optimal solutions on these scenarios under varying parameters.

Finally, we investigate the incorporation of limited artillery assets into communications planning by formulizing RCIP with Artillery (RCIP-A) as a trilevel optimization problem and propose a nested decomposition method as an exact solution methodology. Additionally, we present computational results and tactical insights obtained from the solution of RCIP-A on predefined scenarios.

Keywords: Radio communications, interdiction, electronic warfare, artillery fire support, bilevel and trilevel optimization, decomposition.

ÖZET

TELSİZ HABERLEŞME AĞINI SEKTEYE UĞRATMA PROBLEMİ

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Muhabere (askeri anlamda taktiksel haberleşme), oldukça dinamik ve hassas bir yapıya sahip olan muharebe sahasında harekât icra eden askeri birliklerin komuta ve kontrolünde her zaman oldukça önemli bir role sahip olmuştur. Telsiz haberleşmesi de, muhabere vasıtalarının özelinde, taktik haberleşmenin direnek noktası olarak muharebe sahasında çok uzak mesafelerde harekât icra eden, yüksek hareket kabiliyetine sahip askeri birliklerin haberleşme ihtiyaçlarını gidermekte oldukça başarılı bir vasıta olmuştur.

Günümüzde, son derece karmaşık hale gelen muharebe sahası ile beraber özellikle elektronik harp teknolojisi ile ortaya çıkan tehditler de düşünüldüğünde en iyi şekilde tasarlanmış olan bir telsiz haberleşme ağına olan ihtiyaç her zamankinden daha da fazladır. Böyle bir telsiz haberleşme ağı son dönemde ortaya çıkan bu tehditlere karşı hassasiyetleri azaltmakla beraber sürekli, kesintisiz ve güvenli bir muhabere imkânı da sunacaktır.

Bahse konu özelliklere sahip bir telsiz haberleşme ağını oluşturabilmek maksadıyla düşmanın elektronik harp imkân kabiliyetleri kapsamında kullanabilmesi muhtemel karıştırıcıların sebep olabileceği etkiyi de dâhil ederek vericilerimizin muharebe sahasındaki en uygun yerlerini bulabilen Radyo Haberleşme Ağını Sekteye Uğratma problemi tanımlanmıştır. Bu problem tam sayılı iki seviyeli bir matematiksel olarak formüle edilmiş, bu formülasyon tek seviyeli bir matematiksel modele dönüştürülmüş ve en iyi sonucu verecek bir çözüm yöntemi sunulmuştur. Çözüm yöntemi olarak sunulan algoritmanın performansını da üstünlük ilişkisi, önişlem ve daha iyi başlangıç çözümleri şeklinde sezgisel yöntemler ile geliştirilmiştir.

İletişim sinyallerinde yansıma, kırılma ve engellemeden dolayı oluşabilecek değişkenlik dolayısıyla alıcıların karıştırılma olasılığı ve müteakibinde oluşacak olan beklenen kaplama nedeniyle problemi daha gerçekçi olarak modelleyebilmek maksadıyla problemin olasılıklı versiyonunu formüle edilmiştir. Bu formulasyon doğrusal olmayan bir yapıda olduğundan amaç fonksiyonunu parçalı doğrusal bir fonksiyonla ifade edilmiş ve bir önceki model için önerilen çözüm yöntemi bu problem için uyarlanmıştır.

Kapsamlı hesaplamalar ile başlangıç durumu ile beraber harekâtın zamanla gelişerek oluşturabileceği düşünülen gerçekçi senaryolar için de taktiksel öngörüler elde edilmiştir. Aynı zamanda değişik parametreler altında ve değişik büyüklükteki problemler üzerinde önerilen çözüm metodunun performansı da değerlendirilmiş ve önerilen çözüm metodunun etkinliği ortaya konmuştur.

Son olarak, dost birlik imkan kabiliyetleri dahilinde olan topçu birliklerinin telsiz haberleşme ağının en iyileştirilmesine entegre edilebilmesi için sınırlı sayıda topçu ateşinin en etkin planlamasını ifade edebilecek üç seviyeli matematiksel modeli ortaya konmuştur ve modelin çözümü için iç içe geçmiş ayrıştırmaya dayalı bir çözüm yöntemi geliştirilmiştir. Çözüm yöntemi değişik senaryolar üzerinde test edeilerek taktiksel öngörüler elde edilmiştir.

Anahtar sözcükler: Telsiz haberleşme ağı, sekteye uğratma, elektronik karıştırma, topçu ateş desteği, iki/üç seviyeli matematiksel modelleme, dekompozisyon.

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Chapter 1

Introduction

Communication, in its simplest form, is the exchange of information and it is fundamental for conveying thoughts, ideas, feelings, needs, etc. Likewise, tactical communications, which is the communication among military units on the battlefield, enables the transfer of military orders, intelligence, reports, observations, and other useful information in order to provide the command and control of military operations at all levels.

Tactical communications, from the most primitive times of military conflict till the modern warfare of today, has always maintained its utmost importance and proved to be indispensable in this rapidly changing operational environment. Today's modern warfare strictly dictates commanders to gain and preserve tactical and operational initiative by applying basic principles of military operations which are all tightly dependent on the success of tactical communications. A secure, robust, reliable, and uninterrupted communications system provides commanders at all levels the means to incorporate necessary information required by the decision-makers and enables them to exercise authority and direct forces over large geographic areas and a wide range of conditions [1].

In correspondence with the rapidly changing environment of the battlefield, new requirements for tactical communications have emerged and means to meet those requirements have been developed and improved accordingly. As fronts become wider and deeper, with its wireless nature and practicality to meet maneuverability, radio has become the primary means to enable tactical communication among distant and highly mobilized military units. Besides, as being flexible, adaptable and modular communication devices, radios are easily used by a variety of military units (amphibious, mechanized, dismounted, etc.) in order to provide communication in diverse environments and operations.

Over the years, tactical communication techniques in general and radio communications, in particular, have evolved due to significant progress in the technology used on the battlefield and increased the command and control capability of military commanders. Tactical planners have strived hard to identify better communication architectures not only by improving the capabilities of available assets but also by building up new techniques and tactics for planning secure and continuous communications.

However, consistent with this progress, heavy dependence on the use of the electromagnetic spectrum has revealed potential vulnerabilities that may offset the advantages and capabilities offered. Consequently, Electronic Warfare (EW), which is defined as the use of the electronic spectrum to degrade or destroy an adversary's communication capability, has emerged as a potential threat. More specifically, within the context of EW, jamming has become a frequently used snd effective Electronic Attack (EA) instrument to prevent the transfer of information and ultimately disable the opponent's communication network. Both sides of the military conflict have investigated the optimal use of jammers to disrupt or prevent signal transmission of their adversary. As a result, security in tactical communication has become an important concern for planners and commanders.

As a consequence of fundamental changes in modern warfare requirements along with significant technological improvements, demands for tactical communications are currently greater than ever. Moreover, tactical communications are still challenged by distance, terrain, mobility, security, vulnerability, reliability, and other factors. Under these circumstances, tactical radio communications remain a key capability and a core asset to support all military units in the theater. Therefore, attaining reliable, secure and continuous radio communication obligates planners to provide a holistic approach that optimizes the communication network of friendly forces while taking into account the EW and particularly EA assets of the adversary.

In this context, the scope of this dissertation is to provide a game-theoretic approach for radio communications planners that aims to meet the current demands of modern warfare. To do so, we define Radio Communications Interdiction Problem (RCIP) under deterministic and probabilistic approach and also expand the proposed holistic planning approach by incorporating distinctive assets such as artillery fire planning into radio communications planning.

1.1 Motivation

Planning the radio communications network for a military unit involved in a military conflict is a fundamental issue for tactical communication planners. Signal corps is the sole military branch responsible for planning the radio communications network that should provide continuous, secure, and resilient communication service to widely disperse and highly mobilized military units operating at extended distances within the battlefield.

Basically, planning such a radio communications network simply depends on the analysis in terms of individual communication links between one radiation source (e.g. transmitter, jammer), a receiving device and everything that happens to the radiated signal as it propagates from the source to the receiver. On this link, communication takes place only if the resulting received power level is greater than a threshold value, which denotes the smallest signal power needed for proper reception [2]. Therefore, the vital decision for the planners is to identify the locations of transmitters since they regulate the power of electromagnetic transmission and signal level on each receiver in the communication network. Whitaker and Hurley [3] and Chapman et al. [4] emphasize that building an effective and efficient radio communication network that can maintain the minimum level of the desired signal on each receiver depends mainly on the locations of the transmitters and these sites must satisfy certain requirements such as high coverage area, high traffic capacity, and low infrastructure cost.

Additionally, an important aspect that should be considered in radio communication planning is the adversarial nature of the battlefield. Based on that, planners should incorporate the probable adverse effects of the opponents' EW assets and particularly must hedge their radio communications network against the adversarial effects of jammers that are powerful and prevalent means of present-day EA technology.

With its features pertinent to military context and its bilateral structure, radio communications planning apparently requires a game-theoretic approach to identify optimal radio communications network planning strategies, which enable the evaluation of mutual effects and identification of vulnerabilities [5]. However, compared to the broad studies that apply unilateral approaches either to optimize or to degrade the communications network, game-theoretic applications [5, 6, 7, 8] are limited. Thus, the radio communications network planning problem requires a bilateral approach in order to incorporate the adversarial effects of the opponent into friendly radio communications network planning and optimization. In this respect, this study applies the game-theoretic approach to radio communications network planning problem to mitigate the adverse effects of the opponent's radio jamming capability.

Another major interest in radio communications planning is to recognize the probabilistic nature of transmitting signals due to reflection, diffraction and scattering [9] that may be induced by the obstacles on the battlefield. This is basically called shadowing and described as the deviation of the power of the received electromagnetic signal from an average value [10]. Therefore, probabilistic propagation models that incorporate the shadowing effect should be used to predict the mean signal strength to be used in communication links. To do so, we incorporate shadowing effect into radio communications network planning in which the power of the received transmitter signal is random due to failing over the channel from the transmitter to the receiver.

Today's modern warfare dictates the commanders to exploit the combination of all their assets in order to create a sophisticated effect that can not be endured by the adversary. Taking this idea into account, it is quite clear that radio communications planning should not be considered as a separate problem isolated from other battlespace function domains that are maneuver control, fire support, air defense, combat service support, and intelligence. Among them, fire support, as the workhorse of modern armies, can be easıly used to provide suppression on the strategic enemy assets. Therefore, it is quite interesting how artillery assets as the main fire support units can be integrated into radio communications interdiction planning. Thus, integration of such different domains into the planning process and identification of the interaction of a variety of different assets belonging to different domains results in a practically interesting problem that needs to be investigated in the framework of modern warfare.

1.2 Contribution

In regards to the latest development in Electronic Warfare technology, we emphasize that any military radio communication planning should be constructed with a bilateral approach that considers not only the friendly forces' endeavor but also the destructive intention of the opponent. Therefore, we study the Radio Communication Interdiction Problem (RCIP) within the framework of a military context and apply a game-theoretic approach to be able to reflect a bilateral approach to identify optimal radio communications network planning strategies. Even though a number of studies that apply bilevel approach to wireless communication networks, our study is a distinctive example of a defender-attacker type of problem that optimizes military radio communication systems under jamming attacks on the battlefield. Where as, the existing studies deal with the optimization of the flow of information, we investigate whether the receivers are able to communicate or not and eventually this provides a broader approach.

We formulate RCIP as a bilevel programming problem and propose an exact solution method with enhancements. We evaluate the efficacy of our solution method by solving considerably large instances of the problem in reasonable times. Our study is the first that investigates the radio communications optimization on different military scenarios that reflects not only the initial but also the probable follow on phases of the warfare. Additionally, we derive valuable tactical insights to be considered in planning.

Next, we consider the stochastic nature of transmitting signals due to reflection, diffraction, and scattering that may be induced by the obstacles on the battlefield. In this regard, we study the probabilistic RCIP (P-RCIP) to provide a more realistic scheme. We introduce the probabilistic jamming to signal ratio in order to identify the probability that a receiver is able to communicate and formulate the problem as a bilevel programming problem.

An extended version of RCIP and P-RCIP, presented in Chapter 3, 4, and 5 of this dissertation is authored by Türker Tanergüçlü, Oya Karaşan, İbrahim Akgün, and Ezhan Karaşan and is published in Computers & Operations Research, 107:200-2017, 2019 [11].

Finally, we extend RCIP to incorporate artillery fire support into the radio communications planning and introduce RCIP with Artillery (RCIP-A). From a doctrinal perspective, RCIP-A reflects the idea of integration and coordination of cross-functional non-symmetric effects into warfare planning. To our knowledge, we are the first to consider the artillery fire support in radio communications planning. We formulate RCIP-A as a trilevel programming problem and propose a nested decomposition technique as an exact solution method. We test RCIP-A on a basic scenario and provide tactical insights on the use of artillery in communications planning.

1.3 Organisation of the Thesis

In Chapter 2, we provide a literature review on radio communications planning both from the optimization and the degradation perspective. Then, we present hierarchical mathematical optimization literature by putting stress on Stackelberg Games used for modeling and defending critical infrastructure.

In Chapter 3, we provide initial thoughts on the basic one-way communication link by describing how radio communication and jamming take place on a one-way communication link. In this framework, we define RCIP, provide its bilevel formulation, and propose an exact solution method. To improve the solution times, we propose three enhancements that utilize the dominance relations between possible location sites, preprocessing and initial starting heuristics. Additionally, we present two heuristic methods (i.e. Maximum Cover and Sequential Location heuristics) to solve RCIP.

In Chapter 4, we focus on the probable deviation of the power of the received electromagnetic signal and present P-RCIP, to provide a more realistic framework. We present the bilevel nonlinear formulation of P-RCIP and propose an exact solution method that approximates the nonlinearity in the formulation.

In Chapter 5, we present the computational results obtained both from RCIP and P-RCIP. We first investigate the performance of the decomposition method for the deterministic and probabilistic approaches in terms of the number of iterations, solution times, and objective function values on different problem instances with varying parameter settings that are defined on a brigade-level military unit with three battalions and test the efficacy of the proposed enhancements. In an attempt to provide tactical insights from the commander's perspective, we test the performance of the decomposition method on larger instances with four battalions by considering different scenarios that reflect not only the initial but also the probable subsequent phases of a military operation. Additionally, we evaluate heuristic methods for RCIP to assess the value of the exact solution method. Finally, we analyze how various parameters affect the performance of the solution method and decisions.

In Chapter 6, we extend RCIP by incorporating artillery fire support into the existing problem and define the new problem RCIP with Artillery (RCIP-A), which is a trilevel sequential game. We define a nested decomposition method that solves RCIP-A. Additionally, we conduct some experimental tests to identify the value of artillery fire and investigate the effects on location decisions of both sides.

Finally, we conclude with remarks and present possible improvement and future research directions in Chapter 7.

Chapter 2

Literature Review

2.1 Radio Communications Network

A radio is a device that enables communication utilizing various frequencies and waveforms on the electromagnetic spectrum. Tactical radios, in particular, are used by military units in all kinds of military operations to communicate valuable information, intelligence, and orders continuously, safely and in high quality. The versatile and adaptable design of today's tactical radios enables the radios to be used by a wide range of military units from individual soldiers to armored vehicles, fire support units, logistic centers, and headquarters. Resultingly, all these units constitute a radio communications network that must be configured and planned carefully by signal corps as the communication planners.

Because of its criticality and importance, the radio communications network is a common interest of friendly and enemy forces. While one side struggles hard to make this network effective, safe and secure by applying electronic protection measures, the other side, on the contrary, attempts to first identify the vulnerabilities of the resulting network and then expose and exploit them. Therefore, literature related to both purposes is organized accordingly in the following subsections.

2.1.1 Radio Communications Network Optimization

The design and configuration of the radio communication network for a particular military unit whose sub-units have several different units located on different geographical locations on the battlefield depends on multiple parameters such as the location of transmitters and receivers, transmitter power, operating frequency, receiver sensitivity, various antenna types, interference levels, etc.

Chapman et al. [12] and Hurley [13] consider the location of transmitters as a crucial activity that will form the basis in the radio communications network and state that the selected sites must satisfy certain requirements such as high area coverage and high traffic capacity while minimizing the infrastructure cost. Additionally, Nebro et al. [14] emphasize that transmitter location decisions affect the quality of the service and cost. Thus, the location decision of transmitters is vital since transmitters regulate the power of electromagnetic transmission and signal level on each receiver in the communication network and they must be located in a way that receivers must receive the desired signal with a power level that is greater than the receiver sensitivity threshold value.

We now discuss the literature on network optimization in terms of transmitter location from different perspectives. Objectives used in these research works mainly address the maximization of the total number of receivers that are able to communicate or able to receive the desired signal. Sometimes, it is also expressed in terms of the total demand served by the receivers [15] or the total number of accessible people [16]. Researchers also investigate the minimization of the number of receivers that will enable predefined coverage standards [14, 17]. Other types of objectives used in the transmitter location are listed in Table 2.1.

Table 2.1: Objective function classification of literature on transmitter location

Objectives	Ji et al. [18]	Mathar and Niessen[19]	Nebro [14]	Lakashimmar [20]	Ahmed et al. [21]	Lee and Murray [16]	Akella [15]	Eiselt and Marianov [22]	Alenhogena et al. [23]	Shillington and Ton [24]	Sherali [25]	Zimmerman [17]	Kouhbor [26]
Maximize coverage	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓			
Minimize path loss	✓										✓		✓
Minimize interference		✓										✓	
Minimize the required number of transmitters			✓									✓	
Minimize the cost				✓									
Minimize the energy consumption					✓								

Among different constraints identified in the mathematical formulation of radio communications network optimization problems, the limited number of transmitters to be located, the desired quality of coverage such as the number of receivers to be covered, signal power level above the threshold value and specific receivers to be covered are the ones that are considered widely.

Additionally, it is observed that problems in the literature are mainly formulated by using mixed-integer linear programming [14, 16, 19, 21, 22, 24] and non-linear programming [18, 25]. When it comes to solution techniques it is identified that researchers prefer to use heuristic techniques, especially Genetic Algorithms [14, 20, 21], Simulated Annealing [19], and various search methods [15, 18].

2.1.2 Radio Communications Network Jamming

In an adversarial environment, while military strategists and planners try to optimize wireless communication networks, it is highly expected that the adversary will intend to neutralize the opponent's communication network by malicious attacks. To do so, the adversary may use various different techniques and tactics within the framework of Electronic Attack, which is defined as the use of electromagnetic energy, directed energy, or anti-radiation weapons to attack personnel, facilities, or equipment with the intent of degrading, neutralizing, or destroying enemy combat capability [27]. Electronic attacks can be executed by various different means such as jamming, deception, directed energy, and anti-radiation missile. However, due to the exposed nature of wireless links, current wireless networks can be easily attacked by jamming technology [28].

Radio jamming is a commonly used Electronic Attack technique that aims to disable the opponent's communications network by deliberate radiation of electromagnetic energy. Although several techniques and strategies can be used in jamming, the basic technique adds an interfering jamming signal into the opponent's receiver that overrides any other communication signal at the receiver to deny the effective transfer of military information among tactical units [2].

Detailed information regarding the characteristics and descriptive features of different types of jammers and comparison among them, an overview of commonly used and new emerging jamming techniques and strategies can be found in [28, 29, 30, 31, 32, 33]. Additionally, a literature survey on jamming attacks on wireless networks and potential research areas for further investigation are presented in [34].

As wireless networks continue to emerge increasingly in various different application areas, jamming of these systems is attracting researchers to develop optimal attack and defense strategies accordingly. Among these, Commander et al. [35] present Wireless Network Jamming Problem, which is the first military application to identify the optimal location of a set of jammers and the minimum number of jamming devices needed to meet a certain threshold on the area that can be jammed. Commander et al. extended the problem for networks under complete uncertainty [36] and for robust networks [37]. Even there exists a growing interest in jamming wireless networks [34], military applications are still scarce.

2.1.3 Bilateral Research on Radio Communications Network

A wide variety of research has been carried out on effectively locating transmitters in communication network designs with different objectives. Alternatively, numerous optimization problems have been identified to increase the efficiency of radio jamming and hence disable the opponent's communication capability either by identifying the optimal locations of the jammers or optimal jamming strategies. However, these studies handle the problem unilaterally, either from the perspective of the communication network designer or the adversary that aims to disable the communication network.

2.2 Hierarchical Mathematical Optimization

To apply a bilateral approach to the radio communications network optimization that incorporates not only the defensive strategies of the communication network designer but also the attacking strategies of the adversary that aims to disable the communication network, it is crucial to apply decentralized optimization techniques, such as hierarchical mathematical programming.

Over the years there has been a considerable increase in interest for hierarchical mathematical programming models, which involve multiple decision-makers in different levels with different objective functions and mutually interacting with each other's optimal decisions by their own consecutive decisions in decentralized planning systems. Hierarchical Programming was first defined by [38, 39] as mathematical programming models in which the feasible region is implicitly determined by a series of optimization problems which must be solved in a predetermined sequence [40].

Bilevel and Trilevel optimization are major fields of interest in hierarchical mathematical programming. Therefore, we present the literature mainly on the bilevel but also on trilevel programming in the following subsections.

2.2.1 Bilevel Programming

Bilevel Programming Problem (BPP) is a Hierarchical Programming Problem with specifically two different levels, namely, the upper and lower level optimization problems, controlled by the leader and the follower, respectively.

The general formulation of a BPP, given by [41, 42] is

$$\min_{x \in X} \quad F(x, y) \tag{2.1}$$

$$\text{s.t.} \quad G(x, y) \leq 0 \tag{2.2}$$

$$\min_{y \in Y} \quad f(x, y) \tag{2.3}$$

$$\text{s.t.} \quad g(x, y) \leq 0 \tag{2.4}$$

$$\tag{2.5}$$

where $x \in X \subseteq \mathbb{R}^n$ are called upper level variables controlled by the leader and $y \in Y \subseteq \mathbb{R}^m$ are called lower level variables controlled by the follower. Similarly, the functions $F : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$ and $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$ are the upper level and lower level objective functions respectively, while the vector valued functions $G : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^p$ and $g : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^q$ are called the upper level and lower level constraints respectively.

2.2.1.1 Stackelberg Game

Bilevel optimization is first used in the field of game theory in 1934 as a Stackelberg game, which describes the sequential game between two non-cooperative players, the leader and follower [43]. Players have perfect information on both their own and their opponent's permissible strategies and consequent payoffs. First, the leader decides on his optimal strategy and then the follower reacts rationally after observing the leader's strategy. Therefore, if the leader wants to optimize his objective, then he needs to anticipate the optimal response of the follower. In this setting, the leader's optimization problem contains a nested optimization task that corresponds to the follower's optimization problem.

Stackelberg game applications can be encountered in many different areas such as transportation and traffic optimization [44, 45], economics [46, 47], toll pricing [48, 49, 50], facility location [51, 52, 53, 54, 55, 56], and supply chain management [57, 58, 59, 60].

Another important application area of the Stackelberg game, which has attracted significant interest especially after 2000 is defense and security [61]. A big majority of defense and security applications identify the vulnerabilities and plan defensive measures for critical infrastructures such as emergency services, energy, food, government, information and telecommunications, postal and shipping, public health, transportation, and water protection [62]. Generally, players in these bilevel programming applications are named as Attacker and Defender and depending on the sequence of play between the players, the problems are classified as Attacker-Defender, Defender-Attacker, and sometimes Defender-Attacker-Defender [63].

2.2.1.2 Attacker-Defender Models

Basically, an Attacker-Defender model is an optimization model of an infrastructure system whose objective function represents the system's value to society while it operates or the cost to society when the system loses functionality [63]. The Attacker has limited resources to interdict and therefore degrade the functionality of the underlying infrastructure and the objective of the Attacker is to determine how to use these limited assets in order to cause the maximum damage possible. Additionally, this model addresses the criticality, vulnerability, reconstitutability, and threat in a very different way than military planners [64].

The attacker-defender model is often called an "interdiction model" in the literature [65, 66]. Interdiction means to destroy, cut or damage by ground or aerial military assets to limit enemy effectiveness [67]. Though this is a military definition, interdiction is an important part of modern warfare and also there exist many different non-military applications of interdiction problems. Additionally, a big majority of interdiction problems are defined on networks with different structures and many researchers named these problems as network interdiction problems.

An eminent example of Attacker-Defender problem defined on a network is the Maximum Flow Network Interdiction Problem, in which Attacker aims to choose a limited number of arcs to interdict that minimizes the maximum flow from the source node to the sink node that can be routed via the remaining arcs [66, 68]. and attracted significant interest from the researchers. Complementing the early works

[69, 70, 71, 72, 73], Wood is the first to provide a mixed-integer linear programming model to solve the problem [66]. Other notable extensions of the same problem are multi-commodity flow interdiction [74, 75], bi-objective (i.e. minimizing total interdiction cost while minimizing maximum flow) [76], and uncertainty on arc capacities [77].

Shortest Path Network Interdiction Problem [78] is another remarkable example of Attacker-Defender problem defined on a network and aims to identify the arcs to be interdicted to maximize the length of the shortest path between the source and the sink. The basic idea used in this problem is encountered in project management to identify the optimal interventions to delay adversary's project as much as possible [79, 80].

Other examples of Attacker-Defender problems arise in electric power networks [64, 81], transportation networks [82, 83], homeland security [84], cyber security [85], and in various different facility location applications [86, 87].

2.2.1.3 Defender-Attacker Models

The solution of an Attacker-Defender model identifies the most critical components of a system that will be targeted and this may lead to some obvious heuristics for approximating the solution to identify a near-optimal defense plan, given a limited defense budget. However, an optimal defensive plan can only be devised by solving a Defender-Attacker problem, which basically differs from Attacker-Defender in terms of the objectives of the players and the order of play. In this problem, the Defender acts first by executing a predetermined defense plan and the attacker responds after observing the defense.

Some interesting examples of this model, which has a wide range of applications, are mentioned below.

- Pan et al. [88] identify the optimal locations to install detectors to minimize the evasion probability of nuclear material.
- Brown et al. [89] deal with the optimal pre-positioning of ballistic missile defense platforms to minimize the worst-case damage an attacker can achieve by launching tactical ballistic missiles.
- Brown et al. [90] propose a mathematical model for advantageously positioning port patrol vessels, and possibly shore-based radar too, to minimize the probability that an intelligent adversary in one or more speedboats will evade detection while mounting an attack.
- An et al. [91] investigate the best security schedules for the United States Coastal Guard to defend the port of Boston.
- Scaparra and Church [92] and Church et al. [93] investigate the need to determine q out of p facilities to fortify in order to provide the best protection to a subsequent optimal interdiction strike. Zhang et al. [94] handles the same problem under the assumption that attack resources are invisible to the defender.
- Watson et al. [95] deal with the optimal location of sensors to monitor drinking water networks to minimize the maximum expected impact of contaminated water and maximize the reaction time needed before contaminated water reaches to many users.

2.2.1.4 Solution of Bilevel Mathematical Models

Stackelberg games and other bilevel programming problems are generally difficult to solve with even the linear form being NP-Hard [96]. Detailed information for existing solution methods for BPPs can be found in surveys by Labbe [97], Colson et al. [41], Dempe [98] and in textbooks by Dempe [99] and Bard [100]. BPPs having integer variables only in the first stage or having a totally unimodular constraint matrix in the second stage problem are generally solved by taking the dual of the second stage problem and solving the resulting single level formulation [7, 66, 89, 90].

Nevertheless, solution methods for BPPs with integer and binary variables in the first and/or second stage are uncommon. Bard and Moore [101] and Moore and Bard [102] provide an implicit enumeration technique based on branch and bound to solve BPPs with integer variables on both stages and they are able to solve problems with 35 binary variables. However, this method has limited applicability to solve large-scale problems. Difficulties encountered while solving BPPs with integer decision variables enforce researchers to introduce solution methods that are tailored to the specific bilevel structure of their problems.

The general trend is to reformulate the bilevel model as a single level model and solve with appropriate methods typically involving decomposition [53, 80, 85, 93, 103, 104]. Some researchers enhance the decomposition method by adding super valid inequalities to the master problems [78, 83, 105]. Implicit enumeration methods that make use of some problem-specific observations are also common [87, 101, 102, 106, 107]. In addition to exact approaches, heuristic methods are also frequently used to find quick solutions to BPPs with integer decision variables [108, 109, 110].

2.2.2 Trilevel Programming

Tri-level programming is a hierarchical mathematical programming model, which interacts three hierarchical decision entities that are distributed throughout three levels, which is a subfamily of multilevel programming motivated by Stackelberg game theory [111]. Decision entities at the three hierarchical levels are respectively termed the top-level leader, the middle-level follower, and the bottom-level follower [112]. Similar to bilevel programming models, decision entities in trilevel programming models make their individual decisions in sequence from the top level to the middle level and then to the bottom level with the aim of optimizing their respective objectives [113].

Even trilevel programming is not prevalent in the literature, there is an increasing interest in some fields of application such as supply chain management [114], resource allocation [115, 116], and hierarchical production operations [117]. Another important area of application for trilevel programming models is the critical infrastructure protection and as in the bilevel programming, Defender-Attacker-Defender models are used to identify best defensive plans against an intelligent adversary, which will be discussed in the next section.

2.2.2.1 Defender-Attacker-Defender Models

Defender-Attacker-Defender model is a sequential game with three stages that are (i) Defender in the first stage decides on the defensive plan to protect critical components of the system by anticipating an intelligent adversary attack, (ii) Attacker in the second stage executes his optimal attack plan by attacking on the undefended or less defended components, and finally, (iii) Defender as an operator on the third stage observes the resulting system and minimize the damage caused on the residual system to optimize the functionality of the system, to minimize the operating costs, etc. Detailed theoretical information and proposed solution methods can be found in [88, 118].

The research on Defender-Attacker-Defender models is limited; however, there is an increasing interest in these models. Some application areas of this problem are listed below.

- Yao et al. [115], Wu and Conejo [119], and Alguacil et al. [120] provide applications to optimize electric power defense planning.
- Fard and Mostafa [121] propose tri-level location-allocation model for forward/reverse supply chain.
- Thomas [122] optimizes anti-submarine warfare mission planning.
- San Martin [123] considers the defense of the shortest path on a network and provides an application in homeland security.

2.3 Hierarchial Optimization on Radio Communications

Radio Communication Network literature generally contains studies that apply a unilateral approach either by the defender to find out optimal decisions in terms of location of transmitters, assignment of frequencies, setting power preferences or by the attacker to find out the location of jammers.

Shankar's study [5] is the first attempt to formulate and solve a bilevel optimization problem to assess the defense and attack strategies of wireless mesh networks bilaterally. In the first stage, the attacker intentionally locates a limited number of jammers to disrupt the network in the worst possible way. The defender in the second stage investigates the best strategy to optimize the flow of information after observing the location strategy of the attacker by solving the Simultaneous Routing and Resource Allocation (SRRA) problem of Xiao et al. [6]. Shankar solves moderately sized problem instances by enumerating all possible attacker strategies and devises several jammer location heuristics for larger instances. Different from Shankar's study, we design the transmitter locations and thus the communication network, consider the maximization of the number of receivers that can communicate rather than improving the flow of information in the given network and we incorporate the Jamming to Signal Ratio metric into our model rather than using the metric in the SRRA problem. Also, we manage to solve considerably larger instances to optimality within reasonable solution times.

Medal [7] also applies a game-theoretic approach to identify the locations of a set of jammers that will induce the largest degradation in a given wireless network and determines the most effective strategies such as channel hopping to mitigate these jamming attacks. This study is the first to optimize network throughput by modeling radio wave interference between transmitters. In our study, we ignore the radio interference effect since we assume that receivers belonging to different units communicate with transmitters by using different frequencies, which prevents the occurrence of interference. Additionally, we optimize and design the locations of the transmitters.

Nicholas and Alderson [124] are the first to apply the tri-level game theoretic optimization framework to design wireless mesh network topologies that are robust to jamming. In this problem, the network designer as the defender locates the access points in the first stage; after observing the locations of the access points an intelligent adversary as an attacker identifies the jammer locations in the second stage; and finally at the third stage designer as the operator optimizes the value of the network by using the SRRA and Coverage problem [8], in order to quantify the value of a particular wireless mesh network. This study is also the first to devise a solution algorithm that makes use of the Dividing Rectangles sampling algorithm [125] to design an electromagnetic interference robust wireless mesh networks. The authors extend Shankar's work [5] by considering a continuous space for jammer locations, rather than considering a set of predetermined potential jammer location sites. In contrast to this study, with our work, we intend to cover non-uniformly distributed receivers by depending on deterministic and probabilistic Jamming to Signal Ratio criteria rather than covering the maximum terrain.

With its features pertinent to military context only, our study is a distinctive example of a defender-attacker type of problem that optimizes military radio communication systems on the battlefield under jamming attacks. We incorporate Jamming to Signal Ratio into a bilevel formulation to identify the location of transmitters that will yield a jamming robust radio communication network. We assume that the transmitters are connected to each other via a backbone network, possibly having a mesh topology. Since directional antennas with very large gains are used between fixed transmitters, this backbone network is robust against jamming and thus the jamming effect in this backbone network is ignored in this paper. Different from the previous works, we do not deal with the flow of information but the coverage of the receivers since the flow of information is enabled whenever the receivers are covered as argued above.

Even though the aforementioned works consider locating facilities under deterministic conditions, Daskin [126] and Batta et al. [127] maximize the expected coverage by considering the probability that a facility may not be able to serve a demand point. Similarly, Patel et al. [128] determine locations of sensors over a time horizon to maximize the expected coverage of data by considering the probability of a link failure. In a similar fashion, to bring more realism to our problem, we consider the probability that a receiver is not able to communicate due to the deviation in the received signal power because of fading, which is generally caused by geographical obstacles on the battlefield. We define the Probabilistic Jamming to Signal Ratio which incorporates the randomness in the jamming to signal ratio and introduce and formulate the probabilistic version of RCIP, namely P-RCIP that maximizes the expected coverage of receivers. After approximating the jamming probability function as a piecewise linear convex function, we manage to adapt the decomposition approach for RCIP to solve P-RCIP efficiently.

Additionally, even though there exist trilevel programming problems for wireless mesh network optimization [124], to our knowledge we are the first to incorporate artillery fire support into radio communications network by formulating a defender-attacker-defender problem that mitigates jamming effects at the third level by limited artillery fire support.

Chapter 3

Radio Communications Interdiction Problem Under Deterministic Jamming

3.1 Radio Communications Interdiction Problem

Radio communications form the backbone of the tactical communications on the battlefield. We can assume radio communications as a large network with numerous node that needs to communicate with each other. Nodes in this network are composed of a wide variety of entities, such as individual soldiers acting on the frontline, observation posts on commanding heights, armored vehicles moving forward with high speed, command posts that manage the ongoing operations, artillery units at the rear field, higher headquarters, etc. The complex structure of the radio communications network is depicted in Figure 3.1.

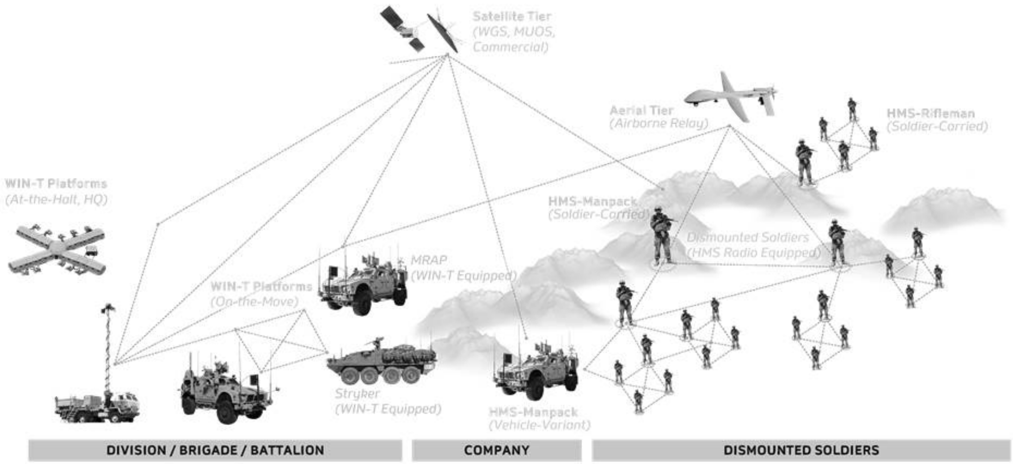


Figure 3.1: Tactical communications on the battlefield

To provide continuous, secure, and resilient communication among the nodes of this network, signal corps and tactical planners elaborate on communication planning and success in this planning heavily depends on the analysis in terms of individual communication links. However, due to the adversarial nature of the battlefield, while one side of the conflict tries to optimize his network, the other side considers degrading the opponent's network. Therefore, we call one side as the Defender (DF) who wants to optimize his communication network and the other side as the Attacker (AT) who wants to degrade the DF's communication network and simply Radio Communications Interdiction Problem (RCIP) is based on this military conflict between DF and AT.

Before presenting RCIP in detail, we present basic notions in radio communication technology in the following subsections and define the problem subsequently.

3.1.1 How Radio Communications Takes Place?

Any communication system can be analyzed in terms of individual communication links that include one radiation source (e.g. transmitter, jammer), a receiving device, and everything that happens to the radiated signal as it propagates from the transmitter to the receiver [2]. This one-way radio communication link is simply depicted in Figure-3.2. The signal is created at transmitter t as a source with a specified power level (P_t), which is expressed in watts. Before the signal leaves the transmitter, its power level is increased by the transmitter antenna gain (G_t), which is expressed in decibels (dB). As the signal propagates from the transmitter to the receiver, the power of the radiated signal attenuates with distance due to various factors. This power fall is commonly modeled by the path loss exponent rate (α), which is a function of the carrier frequency, environment, obstructions, and several other parameters. Aragon [129] states that the value of α ranges from 2 to 5 (where 2 is for propagation in free space and 5 is propagation for relatively rough and mountainous areas). When the signal arrives at the receiver r, the power of the residual signal is increased by the receiver's antenna gain (G_r), which is expressed in dB. Power level of the resulting signal (P_r) is defined as $P_t G_t \frac{1}{d_{tr}^{\alpha}} G_r$ where d_{tr} denotes the euclidean distance between transmitter t and receiver r.

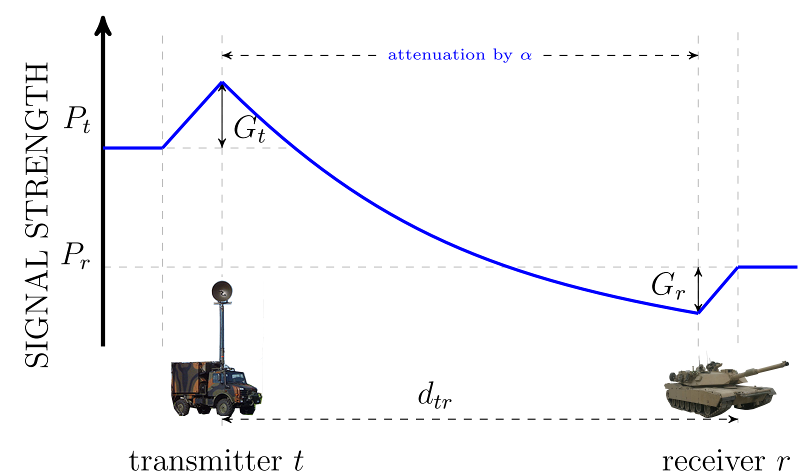


Figure 3.2: Visualization of one-way radio communication link

Finally, communication takes place on this link only if P_r is greater than the receiver sensitivity threshold value (γ), which denotes the smallest signal power needed for proper reception [2].

Let $\mathcal{R}$ represent the locations of receivers. All receivers are assumed to be identical with a receiver sensitivity threshold value γ, i.e., the minimum received power for a successful reception. DF is assumed to have a limited number (p) of transmitters each radiating a signal with a specific power level and a specific antenna gain. Signal corps determine the possible transmitter location sites by evaluating the geographical characteristics of the area of operation either by making a reconnaissance on the terrain or using a digital or printed map and considering the locations of all tactical units. We refer to this set of potential transmitter locations as $\mathcal{T}$. DF concludes the military decision-making process by selecting the locations of p transmitters from $\mathcal{T}$.

3.1.2 How Radio Communications Jamming Takes Place?

Since a jammer is also a radiation source, the communication link between a jammer and a receiver is the same as the link between a transmitter and a receiver. As depicted in Figure 3.3, the signal created by the jammer is transmitted with a power level of P_j , which is described in watts and increased by the antenna gain of the jammer G_j, which is expressed in decibels (dB). As the jamming signal propagates through the receiver, the power of the radiated signal attenuates with distance and the path loss exponent rate (β).

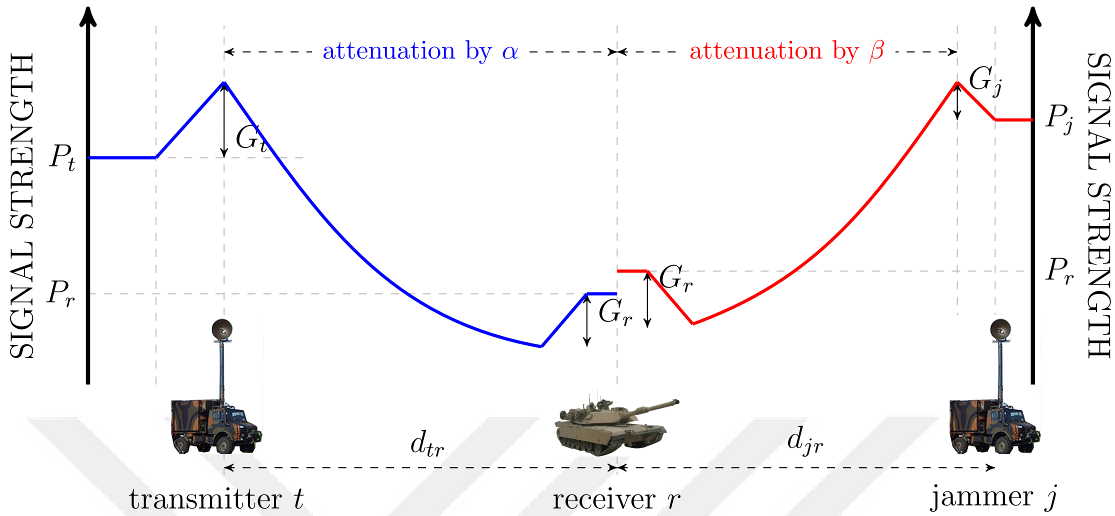


Figure 3.3: Jamming to Signal ratio visualization

Whether a receiver $r \in \mathcal{R}$ is jammed or not is determined by Jamming to Signal Ratio (JSR), which basically denotes the ratio of the received jamming signal power to the received communications signal power at the receiver. Considering the transmitter t, the jammer j in Figure 3.3, receiver r is defined as jammed if JSR at receiver r (JSR_r), given in equation 3.1, is greater than the jamming to signal ratio threshold value (ε).

$$JSR_r = \frac{P_j G_j \frac{1}{d_{jr}^{\beta}} G_r}{P_t G_t \frac{1}{d_{tr}^{\alpha}} G_r} \tag{3.1}$$

Based on the definition above, AT as the other side of the military conflict, aims to conduct an interdiction operation in order to interrupt or impede the flow of information and operational tempo [130]. For this purpose, AT has a limited number of (q) radio jammers with associated power levels and antenna gains. The objective is to locate q radio jammers so as to maximize the number of jammed receivers by conducting intentional jamming attacks. To achieve this objective, AT first, identifies possible jammer location sites $\mathcal{J}$, and later, after observing the locations of DF's tactical units and p transmitters, locates q radio jammers among these $\mathcal{J}$ sites. In this case, which contains multi-transmitter and multi-jammer, letting $\mathcal{T}_p$ be a subset of p transmitters from set $\mathcal{T}$ and $\mathcal{J}_q$ be a subset of q jammers from set $\mathcal{J}$ Scheleher [131] and Shankar [5] define the JSR_r, as the ratio of the sum of all individual undesired signal powers to the maximum of desired signal powers. More formally,

$$JSR_r = \frac{\sum_{j \in \mathcal{J}_q} P_j G_j G_r \frac{1}{d_{jr}^{\beta}}}{\max_{t \in \mathcal{T}_p} P_t G_t G_r \frac{1}{d_{tr}^{\alpha}}} \tag{3.2}$$

where d_{tr} (d_{jr}) is the Euclidean distance between the transmitter (jammer) and the receiver in kilometers and α (β) is the path loss exponent rate which defines the reduction in signal power attenuation of transmitter's (jammer's) electromagnetic wave as it propagates through space.

3.1.3 Problem Definition

Considering this basic information on radio communications, RCIP is based on a military conflict between two opposing forces, DF and AT. Both sides are composed of military units that are equipped and deployed on the battlefield according to their respective organizational structures and tactics. DF aims to establish a reliable tactical radio communications system among all tactical units. These tactical units are assumed to be the smallest maneuver units that have a military radio in their vehicles (e.g. tanks, armored personnel carriers, etc.) or the smallest combat support/combat service support units that have a military radio in their organizational structure.

RCIP is considered as a sequential game in which DF takes the first step and locates p transmitters to optimize his communication network. Thereafter, observing the locations of the transmitters, AT locates q radio jammers in order to degrade DF's communication network. The overall purpose of RCIP is to determine the optimal locations of DF's transmitters in order to maximize the total (expected) number of receivers that will be able to communicate even after AT's intentional jamming attacks are executed by optimally located radio jammers.

3.2 Mathematical Formulation of RCIP

We formulate RCIP as a Bilevel Programming Problem using the following notation.

Sets:

$\mathcal{T} = \{t_1, \ldots t_T\}$ potential location sites for transmitters

$\mathcal{J} = \{j_1, \ldots, j_J\}$ potential location sites for jammers

$\mathcal{R} = \{1, \ldots, R\}$ location sites of receivers on the battlefield

Parameters:

d_{kr} : distance between site $k \in \mathcal{T} \cup \mathcal{J}$ and $r \in \mathcal{R}$ (km)

α : path loss exponent for DF's transmitters

β : path loss exponent for AT's jammers

P_k : transmitting power of transmitter/jammer k located at $\mathcal{T} \cup \mathcal{J}$ (Watt)

G_k : antenna gain of transmitter/jammer/receiver k located at $\mathcal{T} \cup \mathcal{J} \cup \mathcal{R}$ (dB)

ε : threshold value for JSR (dB)

γ : receiver sensitivity (dBm)

p : maximum number of transmitters to be located

q : maximum number of jammers to be located

Decision Variables:

$$x_t = \begin{cases} 1 & \text{if a transmitter is located on transmitter site } t \in \mathcal{T}, \\ 0 & \text{otherwise,} \end{cases}$$

$$y_j = \begin{cases} 1 & \text{if a jammer is located on jammer site } j \in \mathcal{J}\ , \\ 0 & \text{otherwise,} \end{cases}$$

$$w_r = \begin{cases} 1 & \text{if the power of desired signal at receiver } r \in \mathcal{R} \text{ is greater than the receiver sensitivity } (\gamma), \\ 0 & \text{otherwise,} \end{cases}$$

$$z_r = \begin{cases} 1 & \text{if receiver } r \in R \text{ communicates,} \\ 0 & \text{otherwise.} \end{cases}$$

An important feature of RCIP is that # of available transmitters, # of available jammers , and # of receivers on the battlefield are common knowledge both for the DF and AT. Moreover, it is assumed that set of locations are also common knowledge.

Without loss of generality, we assume that all transmitters and jammers are identical among themselves and all receivers have omnidirectional antennas with the same antenna gain. Let $\lambda = (P_{j_1} G_{j_1})/(P_{t_1} G_{t_1})$ where j_1 is the first jammer location site and t_1 is the first transmitter location site.

Given the location plans $x \in \{0,1\}^T$ and $y \in \{0,1\}^J$, $JSR_r(x,y)$ is the jamming to signal ratio at receiver $r \in \mathcal{R}$, and is given as

$$JSR_r(x,y) = \lambda \, \frac{\sum\limits_{j \in \mathcal{J}} \frac{1}{d_{jr}^{\beta}} y_j}{\max\limits_{t \in \mathcal{T}} \frac{1}{d_{tr}^{\alpha}} x_t}. \tag{3.3}$$

For each $r \in \mathcal{R}$, let $\mathcal{T}(r) = \{t \in \mathcal{T} \mid P_t G_t G_r \frac{1}{d_{tr}^{\alpha}} \geq \gamma\}$ denote the potential transmitter locations that can communicate with receiver r.

A receiver $r \in \mathcal{R}$ is assumed to be jammed if $JSR_r(x,y) \geq \varepsilon$; see [108]. On the other hand, for a receiver to be deemed communicating, not only $JSR_r(x,y) < \varepsilon$ should hold but also there should exist a transmitter located within its communication range, i.e., $\exists t \in \mathcal{T}(r)$ such that $x_t = 1$.

The mathematical formulation of RCIP then becomes the following.

$$W^* = \max \quad \tau(x) \tag{3.4}$$

$$\text{s.t.} \quad \sum_{t \in \mathcal{T}} x_t \leq p, \tag{3.5}$$

$$x_t \in \{0,1\}, \qquad t \in \mathcal{T}. \tag{3.6}$$

where

$$\tau(x) = \min \sum_{r \in \mathcal{R}} z_r \tag{3.7}$$

$$\text{s.t.} \quad \sum_{t \in \mathcal{T}(r)} x_t \leq w_r \; p, \qquad r \in \mathcal{R}, \tag{3.8}$$

$$z_r + \frac{\lambda}{\varepsilon} \frac{\sum\limits_{j \in \mathcal{J}} \frac{1}{d_{jr}^{\beta}} y_j}{\max\limits_{t \in \mathcal{T}(r)} \frac{1}{d_{tr}^{\alpha}} x_t} \geq w_r, \qquad r \in \mathcal{R}, \tag{3.9}$$

$$\sum_{j \in \mathcal{J}} y_j \leq q, \tag{3.10}$$

$$y_j \in \{0, 1\}, \qquad j \in \mathcal{J}, \tag{3.11}$$

$$z_r, w_r \in \{0, 1\}, \qquad r \in \mathcal{R}. \tag{3.12}$$

The above bilevel formulation (3.4)-(3.12) is composed of the upper level DF's problem (3.4)-(3.6) and the lower level AT's problem (3.7)-(3.12). DF locates at most p transmitters (constraints (3.5) and (3.6)) so as to maximize the number of receivers that are able to communicate with these transmitters hedging against the best location decisions of AT. For a given set of transmitter locations, AT in turn solves model (3.7)-(3.12) and locates at most q jammers (constraints (3.10) and (3.11)) in order to minimize the number of communicating receivers of DF (objective (3.7)). Note that once the x values are fixed, constraints (3.9) become linear. For a given receiver $r \in \mathcal{R}$, if one of the locations in $\mathcal{T}(r)$ has a transmitter, constraints (3.8) will force $w_r = 1$. If $w_r = 1$ and $JSR_r(x, y) < \varepsilon$, then constraints (3.9) will force $z_r = 1$, i.e., if there is a close transmitter and the JSR is low, then receiver r will communicate. On the other hand, if $x_t = 0 \;\; \forall t \in \mathcal{T}(r)$, then w_r may take a value of 0 or 1 through constraints (3.8). However, through constraints (3.9) and the objective function (3.7), one can deduce that there exists an optimal solution with $w_r = 0$. In other words, without loss of generality, one may assume that $w_r = \lceil \frac{\sum_{t \in \mathcal{T}(r)} x_t}{p} \rceil$ and these auxiliary w variables simply indicate whether any transmitter in set $\mathcal{T}(r)$ is located or not.

3.2.1 Solving RCIP using decomposition

To solve RCIP, we present an equivalent single level formulation and propose an exact solution method that decomposes the single level formulation into a master problem and a subproblem. The master problem and the subproblem provide upper and lower bounds, respectively. We solve each problem sequentially until the lower and upper bounds coincide. A similar approach under a different context is used by Alekseeva et al. [53].

Let $\mathcal{Y} = \{y \in \{0,1\}^J \mid \sum_{j\in\mathcal{J}} y_j \leq q\}$ represent all possible AT strategies. For each receiver $r \in \mathcal{R}$, we introduce a new decision variable s_{ry}, which is defined as follows.

$$s_{ry} = \begin{cases} 1 & \text{if receiver } r \in \mathcal{R} \text{ is able to communicate when AT's strategy is } y \in \mathcal{Y}, \\ 0 & \text{otherwise.} \end{cases}$$

With the addition of an exponential number of such decision variables and an exponential number of constraints, we may reformulate $RCIP$ as the following linear mixed integer programming (MIP) problem, say $MP(\mathcal{Y})$, to stand for the master problem.

$$
\begin{aligned}
MP(\mathcal{Y}) \qquad\qquad \theta_{MP}(y) = \max \quad & \omega && && (3.13)\\
\text{s.t.} \quad & \omega \le \sum_{r\in\mathcal{R}} s_{ry}, && y \in \mathcal{Y}, && (3.14)\\
& s_{ry} \le \sum_{t\in T(r,y)} x_t, && r\in\mathcal{R},\ y\in\mathcal{Y}, && (3.15)\\
& \sum_{t\in\mathcal{T}} x_t \le p, && && (3.16)\\
& x_t \in \{0,1\}, && t\in\mathcal{T}, && (3.17)\\
& 0 \le s_{ry} \le 1, && r\in\mathcal{R},\ y\in\mathcal{Y}. && (3.18)
\end{aligned}
$$

In this model, ω is an auxiliary variable that will correspond to the number of communicating receivers when hedging against all possible AT strategies. Set $T(r,y)$ represents the transmitter location sites that will enable the communication of receiver $r \in \mathcal{R}$ when AT's strategy is y, i.e., $T(r,y) = \{t \in \mathcal{T}(r) \mid \lambda\ d_{tr}^{\alpha} / \sum_{j\in\mathcal{J}} d_{jr}^{\beta} y_j < \varepsilon\}$. Constraints (3.15) enforce one such transmitter to be located when s_{ry} variable takes the value of one. Through constraints (3.14), (3.18) and the objective function (3.13), the auxiliary variable ω will be equal to the minimum number of receivers that will be communicating when considering all possible AT strategies. Constraint (3.16) limits the number of transmitters to be located by p. Constraints (3.17) are domain restrictions for x_t variables. Note that constraints (3.18) relax the binary requirements of s_{ry} variables since once the transmitter location variables take integer values, the objective function and constraints (3.15) imply the integrality of these variables.

Set $\mathcal{Y}$ has $\binom{J}{q}$ elements and as such $MP(\mathcal{Y})$ is a huge model to solve directly. To this end, we propose a decomposition approach for its solution. At every iteration, we shall solve this master problem with only a subset of AT strategies, say with $Y \subseteq \mathcal{Y}$. Then, $MP(\mathcal{Y})$ restricted to only the strategies $y \in Y$, i.e. $MP(Y)$, constitutes the relaxed master problem. Its optimal solution will provide an upper bound (UB) for

$RCIP$. Let $\hat{x}$ be the optimal solution of the relaxed master problem $MP(Y)$. In order to generate new AT strategies to include in the relaxed master problem, we identify AT's optimal response to $\hat{x}$ by solving model (3.7)-(3.12) when $x = \hat{x}$ and the auxiliary w variables are eliminated as discussed. In other words, we solve the following equivalent subproblem $SP(\hat{x})$ where $\hat{R} = \{r \in \mathcal{R} : \sum_{t \in \mathcal{T}(r)} \hat{x}_t > 0\}$ is the set of all receivers having transmitters located within their communication ranges, i.e., set of all potential communicating receivers.

$$
\begin{aligned}
SP(\hat{x}) \qquad \theta_{SP}(\hat{x}) = \min \quad & \sum_{r \in \hat{R}} z_r && (3.19)\\
\text{s.t.} \quad & \lambda \frac{\sum_{j \in \mathcal{J}} \frac{1}{d_{jr}^{\beta}} y_j}{\max_{t \in \mathcal{T}(r)} \frac{1}{d_{tr}^{\alpha}} \hat{x}_t} \geq \varepsilon(1 - z_r), \quad & r \in \hat{R}, \quad & (3.20)\\
& \sum_{j \in \mathcal{J}} y_j \leq q, && (3.21)\\
& y_j \in \{0, 1\}, & j \in \mathcal{J}, \quad & (3.22)\\
& z_r \in \{0, 1\}, & r \in \hat{R}. \quad & (3.23)
\end{aligned}
$$

Let ŷ be the optimal solution to $SP(\hat{x})$. Obviously, $(\hat{x}, \hat{y})$ is a feasible solution of RCIP and $\theta_{SP}(\hat{x})$ is a lower bound (LB) to its optimal objective function value.

Until $LB = UB$, we solve the master and subproblems sequentially in this fashion, each time augmenting the set Y in the relaxed master problem with the optimal solution of the current subproblem. The proposed solution method is formalized with Algorithm 1.

Algorithm 1: Decomposition method to solve RCIP

Data: $\mathcal{T},\ \mathcal{R},\ \mathcal{J},\ \varepsilon,\ \gamma$

Result: $x^*,\ W^*$

begin

  $LB \longleftarrow 0,\ \ UB \longleftarrow R,\ \ Y \longleftarrow \emptyset$;

  Select an arbitrary $y \in \mathcal{Y}$ as an initial solution;

  $Y \longleftarrow Y \cup \{y\}$;

  while $LB < UB$ **do**

    Solve $MP(Y)$ for $\hat{x}$;

    if $\theta_{MP}(Y) < UB$ **then** $UB \longleftarrow \theta_{MP}(Y)$;

    if $LB = UB$ **then**

      $x^* \longleftarrow \hat{x}$, $W^* \longleftarrow UB$;

      break;

    Solve $SP(\hat{x})$ for $\hat{y}$;

    if $\theta_{SP}(\hat{x}) > LB$ **then** $LB = \theta_{SP}(\hat{x})$;

    if $LB = UB$ **then**

      $x^* \longleftarrow \hat{x}$, $W^* \longleftarrow LB$;

      break;

    $Y \longleftarrow Y \cup \hat{y}$;

  Print("x^* is the optimal strategy for DF that will enable W^* receivers to communicate")

3.2.2 Enhancements to the decomposition method

We propose three types of enhancements to our decomposition algorithm.

3.2.2.1 Initial solution

Our preliminary analyses have indicated that the overall computation time is sensitive to the choice of the initial solution y. In order to find an initial solution that will provide a tight upper bound and decrease the overall solution time, we propose a greedy logic for choosing the initial jammer sites. For each potential jammer site, we keep a count of the number of receivers whose closest jammer site is this particular site. We then order the jammer sites in nonincreasing order of their respective count values and simply choose the first q such sites in our initial solution y.

3.2.2.2 Preprocessing

For a fixed DF solution $\hat{x}$, among the receivers that have the potential to communicate, i.e., those defined by the set $\hat{R}$, some might not be jammable and others will be jammable regardless of AT's location decisions. Such receivers can be identified with the following proposition and the corresponding variables can simply be eliminated from the models.

Proposition. *Let $\hat{x}$ be a given DF solution and consider a particular receiver $r \in \hat{R}$. Assume without loss of generality that $d_{j_1 r} \leq d_{j_2 r} \leq \cdots \leq d_{j_{|J|} r}$. Then, the following statements are valid in any optimal solution to $SP(\hat{x})$.*

1. *If* $\left(\lambda \dfrac{\sum_{1 \leq i \leq q} \frac{1}{d_{j_i r}^{\beta}}}{\max_{t \in \mathcal{T}} \frac{1}{d_{tr}^{\alpha}} \hat{x}_t} \right) < \varepsilon$*, then $z_r = 1$ (i.e., receiver r is able to communicate).*
2. *If* $\left(\lambda \dfrac{\sum_{J-q+1 \leq i \leq J} \frac{1}{d_{j_i r}^{\beta}}}{\max_{t \in \mathcal{T}} \frac{1}{d_{tr}^{\alpha}} \hat{x}_t} \right) \geq \varepsilon$*, then $z_r = 0$ (i.e., receiver r is not able to communicate).*

Proof. The first statement establishes that if the cumulative power of even the closest q jammers to receiver r is not enough to jam for the specific transmitter locations $\hat{x}$, then receiver r will not be jammed in an optimal solution to $SP(\hat{x})$ and the corresponding decision variable can be fixed to 1 in this model. In contrast, the second statement considers the farthest q jammer locations to receiver r. If the jamming to signal ratio is at least the threshold value even when the jammers are located farthest away, then in an optimal solution to $SP(\hat{x})$ it will not be possible to achieve $z_r = 1$ and thus this variable can be fixed to zero in the model without loss of generality. For the specific DF solution $\hat{x}$ and any feasible AT solution y, i.e., $\sum_{j \in \mathcal{J}} y_j \leq q$, the above results simply follow from the following relationships:

$$\lambda \, \frac{\sum\limits_{1 \leq i \leq q} \frac{1}{d^{\beta}_{j_i r}}}{\max\limits_{t \in \mathcal{T}} \frac{1}{d^{\alpha}_{tr}} \hat{x}_t} \geq \lambda \, \frac{\sum_{j \in \mathcal{J}} \frac{1}{d^{\beta}_{jr}} y_j}{\max\limits_{t \in \mathcal{T}} \frac{1}{d^{\alpha}_{tr}} \hat{x}_t} = JSR_r(\hat{x}, y) \geq \lambda \, \frac{\sum\limits_{J-q+1 \leq i \leq J} \frac{1}{d^{\beta}_{j_i r}}}{\max\limits_{t \in \mathcal{T}} \frac{1}{d^{\alpha}_{tr}} \hat{x}_t}. \tag{3.24}$$

□

3.2.2.3 Dominance

Depending on the relative geographical dispersion of two distinct potential jammer location sites j' and j'', one may dominate the other one. More formally, if $d_{j'r} \leq d_{j''r}$ $\forall r \in \mathcal{R}$, then site j' dominates site j'' and site j'' cannot be selected unless site j' is selected. In other words, the constraints $y_{j'} \geq y_{j''}$ for each such pair $j', j'' \in \mathcal{J}$ can be incorporated into the subproblem without any loss of generality.

3.2.3 Heuristic Methods for RCIP

RCIP is a fairly large bilevel programming problem with binary variables both in the first and the second stage of the problem. Although we are able to solve large instances in reasonable times, in order to obtain quick solutions for the aforementioned instances and evaluate the exact solution method, we also propose two heuristic solution methods.

3.2.3.1 MaxCover Heuristic

In this method, we ignore the adversarial effect and the bilevel structure of RCIP and solve the maximum covering location problem [132] by the communication range covering criterion.

As in the bilevel formulation of RCIP, we let $\mathcal{T}(r) = \{t \in \mathcal{T} \mid P_t G_t G_r \frac{1}{d_{tr}^{\alpha}} \geq \gamma\}$ denote the potential transmitter locations that can communicate with receiver $r \in \mathcal{R}$ and use the following decision variables.

$$x_t = \begin{cases} 1 & \text{if a transmitter is located on transmitter site } t \in \mathcal{T}(r) \\ 0 & \text{otherwise} \end{cases}$$

$$z_r = \begin{cases} 1 & \text{if receiver } r \in R \text{ communicates} \\ 0 & \text{otherwise.} \end{cases}$$

The maximum covering location problem then becomes:

$$\text{Max} \quad \sum_{r \in R} z_r \tag{3.25}$$

$$\text{s.t.} \quad z_r \leq \sum_{t \in \mathcal{T}(r)} x_t \qquad r \in \mathcal{R} \tag{3.26}$$

$$\sum_{t \in \mathcal{T}} x_t \leq p \qquad r \in R \tag{3.27}$$

$$x_t \in \{0,1\} \qquad t \in \mathcal{T} \tag{3.28}$$

$$z_r \in \{0,1\} \qquad r \in \mathcal{R} \tag{3.29}$$

This mathematical model maximizes the total number of receivers (3.25) that are determined as covered (3.26) by locating at most p transmitters (3.27).

3.2.3.2 Sequential Location Heuristic

Inspecting the optimal transmitter locations as output by our exact solution method, we observe that each battalion has at least one transmitter located to cover the receivers within the battalion site and nearby. This observation is also in sync with the current practices that are used to locate transmitters in the field. The sequential Location heuristic solution method relies on these principles while locating transmitters. For each battalion, a transmitter with the highest cumulative signal power on the receivers of that battalion is chosen. If p is greater than the number of battalions, the remaining transmitters are sequentially located in nonincreasing order of their additional signal power considering all the receivers in the field. Algorithm 2 formalizes our method.

Algorithm 2: Sequential Location Heuristic

Result: Transmitter Location Decision
Let $\mathcal{B}$ be the number of battalions and $\mathcal{R}_b$ be the set of receivers of battalion b
for $t \in \mathcal{T}$ ***and*** $r \in \mathcal{R}$ **do**
  $SP_{tr} = P_t G_t G_r \frac{1}{d_{tr}^{\alpha}}$
end
$b \leftarrow 1$
while $b \leq p$ **do**
  if $b \leq \mathcal{B}$ **then**
    for $t \in \mathcal{T}$ *and* $r \in \mathcal{R}_b$ **do**
      $total\_SP_t \leftarrow total\_SP_t + SP_{tr}$
    end
    $\hat{t} = \underset{t \in \mathcal{T}: x_t = 0}{\arg\max}\{total\_SP_t\}$
    $x_{\hat{t}} \leftarrow 1$
    $b \leftarrow b + 1$
  end
  else
    for $r \in \mathcal{R}$ **do**
      $current\_SP_r \leftarrow \max_{t \in \mathcal{T}} SP_{tr} x_t$
    end
    for $t \in \mathcal{T}$ *and* $r \in \mathcal{R}$ **do**
      $additional\_SP_t \leftarrow additional\_SP_t + \max\{0, SP_{tr} - current\_SP_r\}$
    end
    $\hat{t} = \underset{t \in \mathcal{T}: x_t = 0}{\arg\max}\{additional\_SP_t\}$
    $x_{\hat{t}} \leftarrow 1$
    $b \leftarrow b + 1$
  end
end

3.3 Summary

Radio communications network, which constitutes the backbone of tactical communications on the battlefield and needs to optimized in terms of location decisions, frequency management, power usage, etc. In line with several works that highlight the importance of location decisions to improve the quality of the communications, we present RCIP to identify the optimal location of the limited number of transmitters by anticipating the degrading effects of jamming devices of an intelligent adversary.

We formulate RCIP as a bilevel programming problem in which the defender locates the limited number of transmitters to maximize the number of communicating receivers in the first stage and attacker locates the limited number of jammers after observing the location of transmitters and receivers. Doing so, we consolidate the adversary's jammers as possible electronic warfare assets and provide a bilateral approach to radio communications optimization.

To solve the problem, we present the single level formulation of the bilevel formulation and propose an exact solution method based on the decomposition method. In order to improve the solution times, we propose three enhancements that utilize the dominance relations between possible location sites, preprocessing and initial starting heuristics. Additionally, we propose two heuristic solution methods, a traditional one in the location literature and one that mimics the decision making process in practice.

RCIP provides a comprehensive framework for the bilateral evaluation of the problem. However, it does not consider the stochastic nature of transmitting signals due to reflection, diffraction, and scattering that may be induced by the obstacles on the battlefield. In this regard, to provide a more realistic scheme we study the probabilistic RCIP in the next chapter.

Chapter 4

Radio Communications Interdiction Problem Under Probabilistic Jamming

Electromagnetic wave propagation can be easily modeled by the one-way link equation as described in Section 3.1.1. However, once the signal is transmitted from the transmitter it can impinge upon a smooth surface of the earth and from buildings and walls, or it may be blocked by an object with large dimensions and sharp irregularities, or even it may be scattered because of objects such as street lights, signs, and leaves [9] until it reaches the receiver (Figure 4.1). This is basically called Shadowing and it implies a deviation of the power of the received electromagnetic signal from an average value [10]. Consequently, the deterministic one-way link equation described in Section 3.1.1 may not be appropriate to determine the power level of the received signal strength at the receiver. As seen in Figure 4.2, due to shadowing effects the power level of the received signal may be less than (for receiver A) or greater than (for receiver B) the mean power level obtained by the one-way link

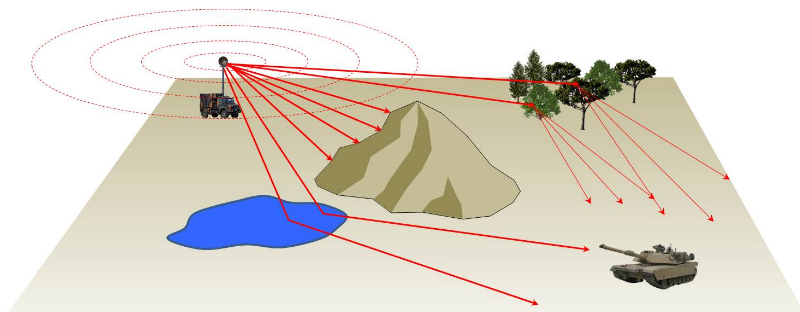


Figure 4.1: Shadowing in electromagnetic waves on the battlefield

equation and independent from the distance. Therefore, probabilistic propagation models that incorporate the shadowing effect are used to predict the mean signal strength. To do so, we incorporate shadowing effect into the jamming to signal ratio and formulate the probabilistic version of RCIP in which the power of the received transmitter signal is random due to failing over the channel from the transmitter to the receiver. To solve probabilistic RCIP, we adjust the solution method proposed in Chapter 3 and test the problem on different instances based on different scenarios.

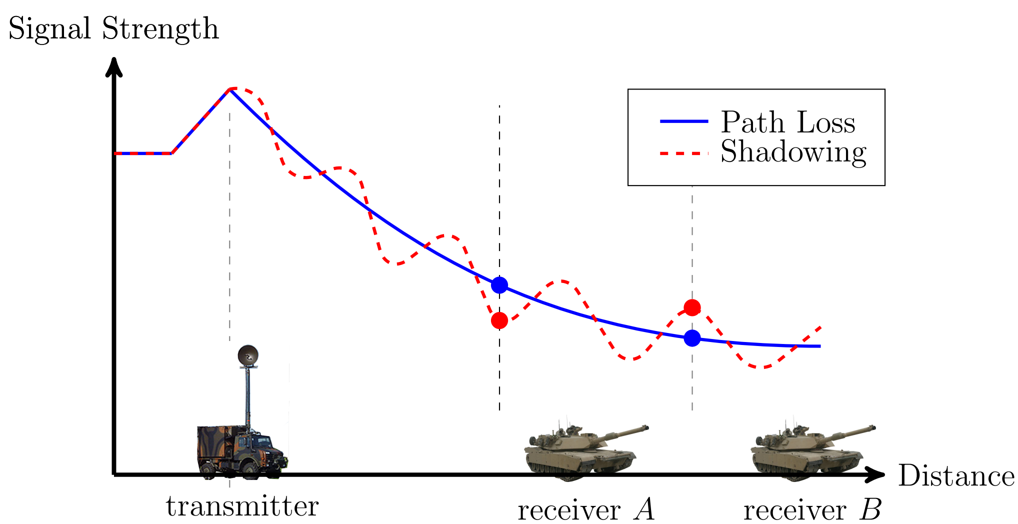


Figure 4.2: Power level of transmitted signal under path loss and shadowing effects

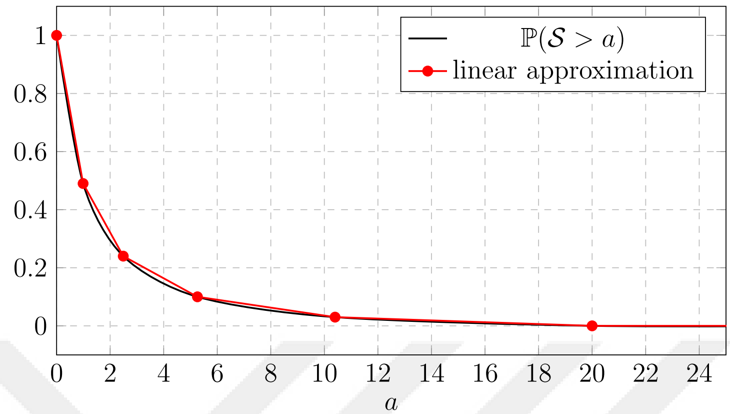
4.1 Mathematical Formulation of P-RCIP

Given the location plans $x \in \{0,1\}^T$ and $y \in \{0,1\}^J$, $PJSR_r(x,y)$ is the probabilistic jamming to signal ratio at receiver $r \in \mathcal{R}$, which is given as

$$PJSR_r(x,y) = \lambda \, \frac{\sum\limits_{j \in \mathcal{J}} \frac{1}{d_{jr}^{\beta}} y_j}{\max\limits_{t \in \mathcal{T}} \, \frac{1}{d_{tr}^{\alpha}} x_t} \frac{1}{\mathcal{S}} \tag{4.1}$$

where $\mathcal{S}$ is a random variable corresponding to the random fluctuations in the path loss over the channel from the transmitter to the receiver. Random variable $\mathcal{S}$ is defined to encompass random shadowing effects due to signal blockage by hills, trees, buildings etc. and it is also referred as log normal shadowing model [133]. Rappaport [134] states that independent form the distance, path loss at a particular location is distributed lognormally. Therefore, $\mathcal{S}$ is modelled as a lognormal distributed random variable, i.e., log $(\mathcal{S})$ has zero mean Gaussian distribution.

Let x_t for $t \in \mathcal{T}$ indicate transmitter locations, y_j for $j \in \mathcal{J}$ indicate jammer locations, and ε be jamming to signal ratio threshold value, respectively, as described in Section 3.2. The binary decision variables z_r for $r \in \mathcal{R}$ that indicate communicating receivers will be replaced with their probabilistic variants called pz_r. In this setting, pz_r corresponds to the probability that receiver r ,communicates i.e., $\mathbb{P}(PJSR_r(x,y) < \varepsilon)$. Letting $a = \lambda \frac{\sum_{j \in \mathcal{J}} \frac{1}{d_{jr}^{\beta}} y_j}{\max_{t \in \mathcal{T}} \frac{1}{d_{tr}^{\alpha}} x_t} \frac{1}{\varepsilon}$, we denote $\mathbb{P}(PJSR_r(x,y) < \epsilon)$ as $\mathbb{P}(\mathcal{S} > a)$ and the shape of this probability function is depicted in Figure 4.3. This nonlinear function can be approximated with a piecewise linear function and after a preliminary computational analysis, we chose to do this approximation using six segments as can be seen in the graph presented in Figure 4.3.



$$\mathbb{P}(S > a) = \begin{cases} -\frac{51}{99}a + 1 & \text{if } a \leq 0.99 \\ -\frac{1}{6}a + 0.655 & \text{if } 0.99 < a \leq 2.49 \\ -\frac{14}{277}a + 0.366 & \text{if } 2.49 < a \leq 5.26 \\ -\frac{7}{514}a + 0.172 & \text{if } 5.26 < a \leq 10.4 \\ -\frac{3}{960}a + 0.0625 & \text{if } 10.4 < a \leq 20 \\ 0 & \text{if } a > 20 \end{cases}$$

Figure 4.3: Cumulative distribution function and linear approximation of $\mathbb{P}(S > a)$

4.2 Solution Method for P-RCIP

A solution approach similar to that of Section 3.2.1 can be facilitated for this variation of RCIP. For each receiver $r \in \mathcal{R}$, we introduce a new decision variable ps_{ry}, which is defined as follows.

ps_{ry} = the probability that receiver $r \in R$ is able to communicate when AT's strategy is $y \in \mathcal{Y}$.

To this end, the probabilistic master problem becomes

$$
\begin{aligned}
MPp(\mathcal{Y}) \qquad \theta_{MP}(y) = \max \quad & p\omega && && (4.2)\\
\text{s.t.} \quad & p\omega \leq \sum_{r \in \mathcal{R}} ps_{ry}, && y \in \mathcal{Y}, && (4.3)\\
& ps_{ry} \leq \mathbb{P}(PJSR_r(\mathrm{x}, \mathrm{y}) < \varepsilon), && r \in \mathcal{R},\ y \in \mathcal{Y}, && (4.4)\\
& \sum_{t \in \mathcal{T}} x_t \leq p, && && (4.5)\\
& x_t \in \{0, 1\}, && t \in \mathcal{T} && (4.6)\\
& 0 \leq ps_{ry} \leq 1, && r \in \mathcal{R},\ y \in \mathcal{Y}. && (4.7)
\end{aligned}
$$

where $\mathcal{Y} = \{y \in \{0,1\}^J \mid \sum_{j \in \mathcal{J}} y_j \leq q\}$ and the auxiliary variable $p\omega$ keeps track of the expected number of receivers that are not jammed with respect to all possible AT solutions in $\mathcal{Y}$. Thus, objective of this formulation is to maximize the total expected coverage of the receivers.

Due to constraints (4.4), $MPp(\mathcal{Y})$ is a nonlinear MIP model. To linearize $MPp(\mathcal{Y})$, we introduce the parameter P_{try}, which denotes the probability that receiver $r \in \mathcal{R}$ can communicate when AT's strategy is $y \in \mathcal{Y}$ and a transmitter is located on possible transmitter location site $t \in \mathcal{T}$. The formal definition of P_{try} is

$$P_{try} = \mathbb{P}\Big(\lambda \, \frac{\sum_{j \in \mathcal{J}} \frac{1}{d_{jr}^{\beta}} y_j}{\frac{1}{d_{tr}^{\alpha}}} \frac{1}{\mathcal{S}} < \varepsilon\Big). \tag{4.8}$$

Additional variables to linearize $MPp(\mathcal{Y})$ are as follows.

$$\delta_r = \text{power level of the strongest transmitter signal received at receiver } r \in \mathcal{R} \tag{4.9}$$

$$u_{tr} = \begin{cases} 1 & \text{if transmitter } t \in \mathcal{T} \text{ transmits the strongest transmitter signal to receiver } r \in \mathcal{R}, \\ 0 & \text{otherwise.} \end{cases} \tag{4.10}$$

With these new parameters and variables, the MIP probabilistic master model is formalized as:

$$
\begin{aligned}
MPpl(\mathcal{Y}) \quad \theta_{MP}(y) = \max \quad & p\omega && && (4.11)\\
\text{s.t.} \quad & p\omega \leq \sum_{r\in\mathcal{R}} ps_{ry}, && y \in \mathcal{Y}, && (4.12)\\
& ps_{ry} \leq \sum_{t\in\mathcal{T}} P_{try} u_{tr}, && r \in \mathcal{R},\ y \in \mathcal{Y}, && (4.13)\\
& \sum_{t\in\mathcal{T}} u_{tr} = 1, && r \in \mathcal{R}, && (4.14)\\
& u_{tr} \leq x_t, && r \in \mathcal{R},\ t \in \mathcal{T}, && (4.15)\\
& \delta_r \geq \frac{1}{d_{tr}^{\alpha}} x_t, && r \in \mathcal{R},\ t \in \mathcal{T}, && (4.16)\\
& \delta_r \leq \frac{1}{d_{tr}^{\alpha}} x_t + M(1 - u_{tr}), && r \in \mathcal{R},\ t \in \mathcal{T}, && (4.17)\\
& \sum_{t\in\mathcal{T}} x_t \leq p, && && (4.18)\\
& x_t \in \{0, 1\}, && t \in \mathcal{T}, && (4.19)\\
& u_{tr} \in \{0, 1\}, && r \in \mathcal{R},\ t \in \mathcal{T}, && (4.20)\\
& 0 \leq ps_{ry} \leq 1 && r \in \mathcal{R},\ y \in \mathcal{Y}. && (4.21)
\end{aligned}
$$

By constraints (4.14) and domain restrictions (4.20), only one u_{tr} variable takes a value of 1 for each receiver and with constraints (4.15), (4.16), and (4.17) $u_{tr} = 1$ only for the transmitter that transmits the strongest transmitter signal to receiver r (M is a large enough number). Note that we no longer use set $\mathcal{T}(r)$ as we did in the deterministic formulation since any transmitter has a positive probability of transmitting to any receiver. By constraints (4.13), ps_{ry} will be bounded from above with the probability value corresponding to the strongest located transmitter signal and will be equal to this bound value at an optimal solution. The rest of the formulation is the same as that of $MPp(\mathcal{Y})$.

Let $\hat{x}$ be the optimal solution of the relaxed master problem $\mathbf{MP}_{pl}(Y)$ where the set of all AT strategies $\mathcal{Y}$ is replaced with a subset Y and define the constant

$$c_r(\hat{x}) = \frac{\lambda}{\epsilon \max\limits_{t \in \mathcal{T}} \frac{1}{d_{tr}^{\alpha}} \hat{x}_t} \text{ for } r \in \mathcal{R}.$$

The subproblem to be solved for this variant then becomes:

$$
\begin{aligned}
SP(\hat{x}) \qquad \theta_{SP}(\hat{x}) = \min \quad & \sum_{r\in\mathcal{R}} pz_r && && (4.22)\\
\text{s.t.} \quad & pz_r \geq -\frac{51}{99} c_r(\hat{x}) \sum_{j\in\mathcal{J}} \frac{1}{d_{jr}^{\beta}} y_j + 1, && r \in \mathcal{R}, && (4.23)\\
& pz_r \geq -\frac{1}{6} c_r(\hat{x}) \sum_{j\in\mathcal{J}} \frac{1}{d_{jr}^{\beta}} y_j + 0.655, && r \in \mathcal{R}, && (4.24)\\
& pz_r \geq -\frac{14}{277} c_r(\hat{x}) \sum_{j\in\mathcal{J}} \frac{1}{d_{jr}^{\beta}} y_j + 0.366, && r \in \mathcal{R}, && (4.25)\\
& pz_r \geq -\frac{7}{514} c_r(\hat{x}) \sum_{j\in\mathcal{J}} \frac{1}{d_{jr}^{\beta}} y_j + 0.172, && r \in \mathcal{R}, && (4.26)\\
& pz_r \geq -\frac{3}{960} c_r(\hat{x}) \sum_{j\in\mathcal{J}} \frac{1}{d_{jr}^{\beta}} y_j + 0.0625, && r \in \mathcal{R}, && (4.27)\\
& pz_r \geq 0 && r \in \mathcal{R} && (4.28)\\
& \sum_{j\in\mathcal{J}} y_j \leq q && && (4.29)\\
& y_j \in \{0,1\} && j \in \mathcal{J} && (4.30)
\end{aligned}
$$

Note that we would like pz_r take the probability value corresponding to the interval where $\mathbb{P}(PJSR_r(\hat{x}, y) < \varepsilon)$ falls, however, due to convexity, by taking the maximum of all these function values as in inequalities (4.23)-(4.28) we can guarantee that pz_r will take the correct value.

4.3 Summary

To realize the probable deviation in the power of received electromagnetic signals that may be caused by reflection, diffraction, and scattering due to obstacles on the battlefield, we present Probabilistic Jamming to Signal Ratio and define RCIP-P by incorporating this metric into the problem. We provide the bilevel mathematical formulation of the problem and propose a similar solution method, which presents the single level formulation and then decomposes the single level formulation into a mixed-integer linear master problem and a mixed-integer nonlinear subproblem. We approximate the nonlinearity in the subproblem caused by the probabilistic approach by using a piecewise linear function.

We present the results of the computational results of P-RCIP tested on different scenarios with varying parameters in Chapter 5. Results show that P-RCIP provides an alternative to RCIP with similar optimal locations obtained in shorter solution times.

Even the objective function is stated as maximizing the total excpected coverage, it can be restated as maximizing the # of receivers with probabilty higher than a threshold value. This can be a good direction for future works in P-RCIP.

Chapter 5

Computational Results

In this chapter, we present the computational results for RCIP and P-RCIP. We first investigate the performance of the decomposition method for the deterministic and probabilistic approaches in terms of the number of iterations, solution times, and objective function values. We conduct tests on different problem instances with varying parameter settings that are defined on a brigade-level DF unit with three battalions and test the efficacy of the proposed enhancements. In an attempt to provide tactical insights from the commander's perspective, we test the performance of the decomposition method on larger instances with four battalions by considering different scenarios that reflect not only the initial but also the probable subsequent phases of a military operation. Additionally, we present the results of two heuristic methods defined in Chapter 3, to assess the value of the exact solution method. Finally, we analyze how parameters like the Jamming to Signal Ratio threshold value (ε) and the path loss exponent rates (α, β) affect the performance of the solution method and the decisions.

All experiments are executed on a Lenovo Z580 computer with a 2.2 GHz Intel

Core i7-3632QM processor and 6 GB RAM by implementing the proposed solution method using Java and CPLEX 12.5.

5.1 Experimental setting

The number of receivers, R, largely depends on the number of battalions. Each battalion is supposed to have three companies and each company is composed of three platoons. Platoon, being the smallest combat unit, consists of four armored personnel carriers and/or tanks and each of them has a military radio mounted on its vehicle. Hence, a company with three platoons has 12 receivers. In addition to these maneuver units, for each company, we include one command and control vehicle and two combat support/combat service support vehicles with mounted military radios. In total, the number of receivers in a company sum up to 15 and a battalion with three companies, two command and control vehicles and three combat support/combat service support vehicles has 50 receivers. Finally, with 50 additional receivers regarding the combat support units such as artillery, air defense, corps of engineers and various combat service support units, the value of R is approximately 200 for a brigade with three battalions and 250 for a brigade with four battalions. Nevertheless, the number of receivers in a battalion may be incremented according to the type of operation to be conducted with military units having different capabilities and hence we let R vary from 200 to 245 and from 250 to 310 for the brigade with three battalions and four battalions, respectively, in our experiments. The number of the potential transmitter (T) and jammer location (J) sites are considered to range from 100 to 130 in proportion to the number of receivers.

For the test problems with three battalions, we assume that p ranges from 3 to 6 and for each p value q is assumed to range from 2 to $(p + 2)$. Similarly, for the test problems with four battalions, p ranges from 4 to 7 and q ranges from 2 to $(p + 2)$.

Unless otherwise stated, we use $\alpha = 2$ and $\beta = 2$, i.e., propagation is assumed to take place in free space, $\varepsilon = -3$ dB, i.e., the received signal power should be twice the received jammer power for proper reception, $\gamma = -10$ dBm, i.e., the received signal power should be at least $100\mu W$ for proper reception required by challenging tactical applications, and $\lambda = 1$, i.e., the transmitter power and antenna gain are the same as the jammer's transmitted power and antenna gain.

5.2 Experimental results

5.2.1 Experimental results for the brigade with three battalions

The generic scenario is depicted in Figure 5.1. The first and the second battalions are located along the frontline and the third battalion is located behind them. The border of transmitter location site surrounds the borders of the battalions and the jammer location site lies approximately 1 km away from the frontline with a depth of 2 km.

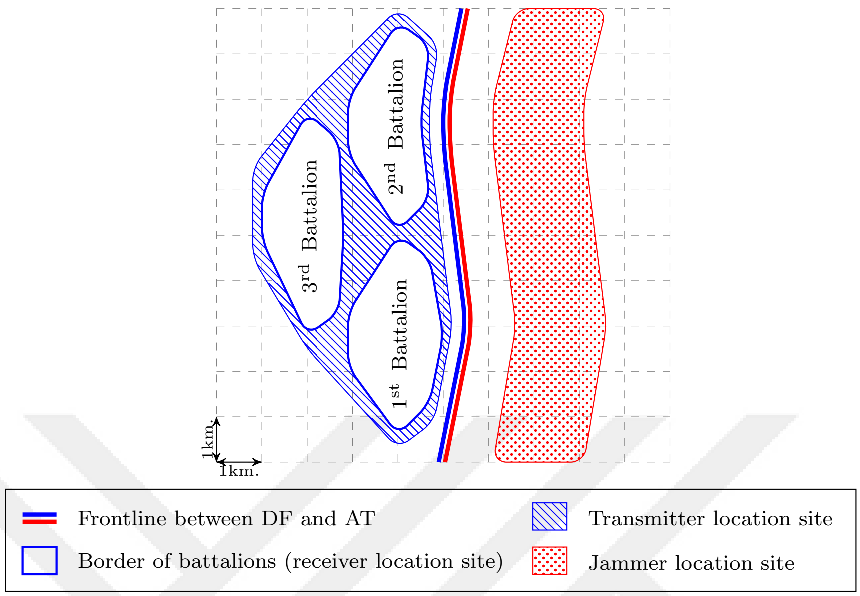


Figure 5.1: Sketch of the scenario for a brigade with 3 battalions

In an attempt to evaluate the proposed decomposition method, we solve both the deterministic and the probabilistic RCIP models with this scenario. For each parameter setting provided in Section 5.1, we generate 10 different problem instances by randomly determining the locations of receivers, possible transmitter and jammer location sites depending on the given width, depth and borders of the military unit's deployment on the battlefield. Each row in Table 5.1 displays the average number of iterations, solution times (in CPU seconds) and the objective function values of 10 randomly generated problem instances. The results for the deterministic and probabilistic approaches are depicted in separate multi-columns. The objective function value of the deterministic RCIP refers to the minimum number of receivers (out of R) that will be covered even under the smartest jamming attack, whereas that of the probabilistic RCIP expresses the expected coverage. We also present the average percentage coverages these objective values correspond to. The breakdown of solution times into master and subproblems as well as the average number of iterations during the decomposition method are also depicted under columns MP, SP and #

iterations in each approach, respectively.

Table 5.1: Solution statistics of deterministic and probabilistic RCIP for the brigade with 3 battalions

					Deterministic						Probabilistic					
						Solution Times (sec.)			Objective Value			Solution Times (sec.)			Objective Value	
R	T	J	p	q	#iterations	MP	SP	Total	# of receivers covered	Coverage percentage	#iterations	MP	SP	Total	# of receivers covered	Coverage percentage
200	100	100	3	2	6.1	1.4	10.5	11.9	143.2	71.6 %	1.7	28.0	0.5	28.5	133.3	66.7 %
				3	5.1	0.9	16.8	17.6	119.3	59.7 %	2	41.6	0.7	42.3	115.1	57.6 %
				4	6.2	1.2	10.1	11.3	106.3	53.2 %	2.3	47.5	0.8	48.3	102.7	51.4 %
				5	4.5	0.5	3.3	3.8	99.5	49.8 %	2	38.6	0.6	39.2	93.8	46.9 %
215	110	110	4	2	6.4	3.2	21.7	24.9	169.6	78.9 %	2.4	60.3	1.1	61.4	160.6	74.7 %
				3	7.3	3.6	78.9	82.5	152.6	70.9 %	2.3	60.4	0.9	61.3	142.4	66.2 %
				4	9.8	8.8	57.1	65.9	136.5	63.5 %	2.6	73.8	1.1	74.9	129.1	60.1 %
				5	9.3	7.3	33.9	41.2	127.5	59.3 %	2.4	67.8	0.8	68.6	119.2	55.4 %
				6	7.1	2.9	21.3	24.2	119.4	55.5 %	2.6	100.1	0.9	101.0	111.3	51.8 %
230	120	120	5	2	9.6	18.7	35.6	54.3	206.7	89.8 %	2.4	98.8	1.0	99.8	181.8	79.1 %
				3	8.9	12.9	143.9	156.8	194.8	84.7 %	2	68.0	1.0	69.0	164.2	71.4 %
				4	10.7	7.7	119.6	127.3	184.0	80.0 %	1.9	51.8	0.8	52.6	151.1	65.7 %
				5	11.1	13.6	65.5	79.1	155.1	67.4 %	2.5	85.9	1.2	87.0	140.4	61.1 %
				6	10.2	9.4	46.5	55.9	146.3	63.6 %	3	103.7	1.3	105.0	131.6	57.2 %
				7	8.2	4.3	44.3	48.6	137.2	59.7 %	3.2	122.4	1.4	123.9	124.5	54.1 %
245	130	130	6	2	8.5	13.5	32.6	46.1	234.1	95.5 %	3.1	208.9	1.7	210.6	203.8	83.2 %
				3	10.8	38.4	263.1	301.5	215.9	88.1 %	2.8	148.5	1.4	149.9	187.1	76.4 %
				4	9.2	14.2	285.2	299.4	199.9	81.6 %	1.7	58.5	0.9	59.4	174.3	71.1 %
				5	9.2	12.2	209.5	221.7	188.2	76.8 %	2.2	78.1	1.2	79.3	163.8	66.9 %
				6	8.8	6.6	136.1	142.7	176.8	72.2 %	2.6	98.0	1.3	99.3	155.1	63.3 %
				7	7.7	3.4	121.3	124.7	165.4	67.5 %	2.7	105.5	1.2	106.7	147.3	60.1 %
				8	8.7	5.2	82.1	87.3	154.9	63.2 %	2.5	121.2	1.6	122.8	140.6	57.4 %

It is readily observed that the coverage improves as the number of transmitters increases and worsens as the number of jammers increases. The results clearly show that both the deterministic and the probabilistic approaches are able to solve all the instances to optimality within reasonable solution times (under five minutes). As expected, solution times increase in both approaches as problem dimensions R, T, and J increase. On average, 88.3% of the total solution time is spent on solving the subproblems in the deterministic approach, while 97.9% of the total solution time is spent on solving the master problems in the probabilistic approach. This is an expected result as the master problem models for P-RCIP and the subproblem models for RCIP involve extra binary variables when compared with their counterpart variants and hence are computationally more challenging.

5.2.2 Effects of proposed enhancements

We apply each enhancement proposed in Section 3.2.2 both individually and collectively, solve RCIP with the same instances presented in Table 5.1 and observe the results in Table 5.2. The experiments reveal that starting with the initial AT solution provided by our heuristic reduces the average number of iterations by 14.9%. Through preprocessing, 59.4% of the z_r variables are fixed and the average solution time reduces by 49.6%. Finally, identifying dominance relations between possible jammer locations yields an average of 64.9% reduction in solution times. Table 5.2 also presents the results obtained by applying all enhancements simultaneously, which provides an average of 16.8% reduction in the number of iterations and 81.3% reduction in solution times. Additionally, the results show that after applying all the enhancements, the percentage of the total solution time spent to solve the sub problems reduces from 88.3% to 59.4%.

Table 5.2: Effects of proposed enhancements

					Heuristic Initial Solution				Preprocessing						Dominance Relation			All enhancements					
						Solution Times (seconds)			# preprocessed z_r variable			Solution Times (seconds)			Solution Times (seconds)			Iteration		Solution Times (seconds)			
R	T	J	p	q	#iter.	MP	SP	Total	$z_r = 0$	$z_r = 1$	Total	MP	SP	Total	MP	SP	Total	#iter.	imp.%	MP	SP	Total	imp.%
200	100	100	3	2	4.3	0.9	7.5	8.3	0.8	108.9	109.6	1.0	1.9	2.9	1.1	1.8	2.9	4.1	33	0.6	0.6	1.2	90
				3	4.1	0.5	12.3	12.9	5.1	93.1	98.2	0.9	11.3	12.2	0.9	4.9	5.7	4.2	18	0.6	3.1	3.7	79
				4	4.9	0.9	5.7	6.6	11.3	89.9	101.2	0.9	4.9	5.8	0.9	2.6	3.5	4.7	24	0.6	0.8	1.5	87
				5	3.3	0.3	1.5	1.8	22.0	84.8	106.9	0.5	2.1	2.7	0.4	1.3	1.6	3.4	24	0.2	0.3	0.5	87
215	110	110	4	2	5.3	2.3	18.8	21.1	0	148.7	148.7	3.2	5.9	9.1	4.4	4.1	8.5	5.5	14	2.6	0.9	3.4	86
				3	6.2	3.2	63.2	66.4	1.1	117.4	118.5	3.9	44.9	48.9	4.4	26.3	30.7	5.8	21	2.3	13.6	15.9	81
				4	9.2	8.1	46.8	54.9	6.4	106.2	112.6	9.3	26.1	35.4	9.3	15.2	24.4	9.6	2	8.8	9.8	18.5	72
				5	8.8	8.3	25.3	33.6	14.9	98.7	113.6	8.1	14.7	22.8	5.7	8.6	14.3	7.9	15	4.1	3.8	7.9	81
				6	5.7	1.6	14.9	16.5	28.9	96.8	125.7	3.3	11.9	15.2	2.8	6.2	9.0	5.7	20	1.3	1.9	3.3	87
230	120	120	5	2	7.5	7.1	29.6	36.6	7.8	164.7	172.5	13.4	4.9	18.3	7.5	4.1	11.7	7.2	25	5.7	1.0	6.7	88
				3	7.3	5.5	102.6	108.1	14.8	132.1	146.9	8.4	50.3	58.7	8.4	39.6	48.0	7.4	17	7.9	1.0	8.8	93
				4	10.4	13.6	129.6	143.2	1.9	122.2	124.1	13.8	59.8	73.7	17.3	44.1	61.4	7.7	28	6.2	21.7	27.9	82
				5	10.3	14.7	58.2	72.8	7.7	155.1	162.8	13.4	32.1	45.5	11.2	22.4	33.6	10	10	11.7	13.4	25.1	68
				6	8.5	5.9	42.2	48.1	18.2	108.5	126.7	6.2	20.2	26.4	6.5	14.6	21.1	8.7	15	6.9	9.2	16.1	71
				7	6.5	2.5	30.4	32.9	26.1	95.5	121.6	3.4	18.7	22.1	4.8	15.1	18.9	7	15	2.8	7.5	10.4	79
245	130	130	6	2	7.9	13.5	32.1	45.6	0	201.3	201.3	14.1	3.6	17.7	14.8	5.4	20.3	8.2	4	15.3	0.9	16.3	65
				3	10.1	30.1	223.5	253.6	0	163.6	163.6	38.9	117.5	156.4	43.1	82.5	125.6	10	7	29.7	40.4	70.1	77
				4	7.4	13.3	151.4	164.6	0.1	154.4	154.5	16.0	157.6	173.6	17.8	98.1	116.1	8.2	11	22.6	33.7	56.3	81
				5	7.3	5.6	108.6	114.1	3.1	142.1	145.2	9.8	78.5	88.3	8.8	43.8	52.7	7.2	22	4.9	19.7	24.7	89
				6	7.5	3.4	89.6	93.1	8.2	131.5	139.7	6.4	68.5	74.9	5.4	40.5	45.9	7.9	10	3.9	21.3	25.1	82
				7	6.7	2.4	82.4	84.8	12.9	121.1	134.0	3.9	63.2	67.1	2.7	36.5	39.3	6.1	21	1.6	17.1	18.6	85
				8	8.6	5.6	79.8	85.4	24.9	113.9	138.8	5.7	41.4	47.1	4.2	23.4	27.7	7.3	16	3.9	13.3	17.2	80

5.2.3 Experimental results for the brigade with four battalions

We solve RCIP and P-RCIP for a brigade level military unit with four battalions and test the performance of the proposed decomposition method on four different probable scenarios. While all enhancements are utilized for RCIP models, the preprocessing enhancement is not available for P-RCIP models.

5.2.3.1 Scenarios

We designe scenarios to reflect not only the initial but also the probable subsequent phases of a military operation. Sketches of all the scenarios are depicted in Figure 5.2. Scenario 1 (Figure 5.2a) reflects the initial phase of a military operation. We assume that three battalions are positioned along the frontline and the fourth battalion positioned behind serves as a reserve unit. In Scenario 2 (Figure 5.2b), we assume that the brigade improves its attacks from the north and thereupon the brigade commander deploys the reserve battalion to the north in order to support the improvement or exploit a possible breakthrough. A symmetric scenario can be visualized to represent a southern improvement. To investigate the effects of improvement from the middle of the frontline we provide Scenario 3 (Figure 5.2c). Finally, we investigate the effects of a withdrawal operation conducted by the brigade in Scenario 4 (Figure 5.2d). We assume that the battalions of DF, especially the second battalion, strive hard to prevent an AT breakthrough. Hence, the commander is keeping the reserved battalion very close to the second battalion in order to quickly exploit the situation in case of emergency.

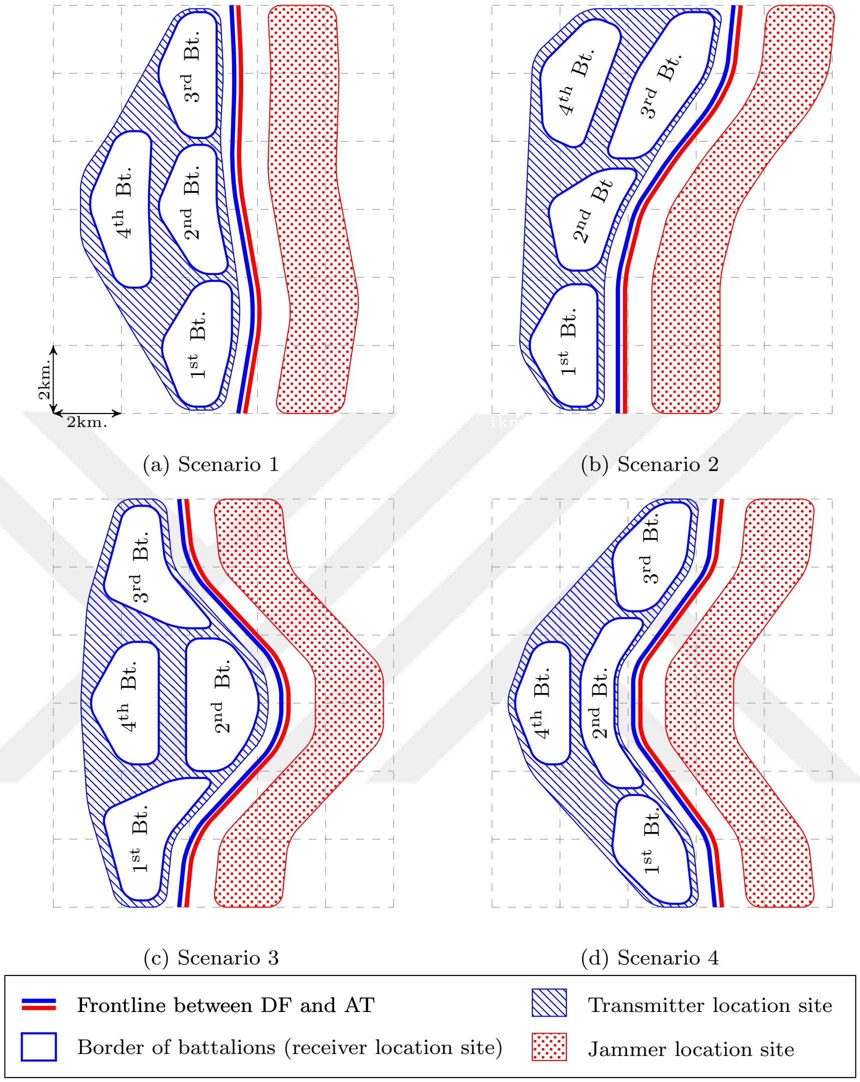


Figure 5.2: Scenario sketches

5.2.3.2 Numerical results

Table 5.3 presents the solution statistics of RCIP and P-RCIP based on the scenarios described above. Each row depicts the average results obtained by solving 10 randomly generated problem instances with the specified parameter choices.

Table 5.3: Solution times of RCIP and P-RCIP on different scenarios

					Solution Time (seconds)							
					Scenario 1		Scenario 2		Scenario 3		Scenario 4	
R	T	J	p	q	RCIP	P-RCIP	RCIP	P-RCIP	RCIP	P-RCIP	RCIP	P-RCIP
250	100	100	4	2	7.7	58.2	8.4	55.5	3.4	48.2	19.2	76.1
				3	**31.3**	70.7	**117.7**	54.8	**60.5**	56.4	**522.1**	77.1
				4	4.7	87.3	55.1	64.5	30.3	56.2	87.8	113.5
				5	4.2	**142.5**	12.7	80.6	14.9	**77.8**	27.6	**115.8**
				6	1.7	116.5	4.9	**97.1**	5.4	76.1	7.9	90.3
270	110	110	5	2	7.7	123.9	33.2	88.9	26.1	57.7	73.6	70.4
				3	**117.6**	95.5	**952.3**	81.3	**739.9**	89.8	**952.6**	101.2
				4	34.2	129.5	387.6	112.7	513.2	75.3	410.9	122.5
				5	17.6	217.7	110.7	124.9	250.4	126.3	166.5	125.3
				6	10.4	224.4	34.1	186.5	33.6	106.2	26.8	**148.5**
				7	4.1	**326.1**	11.7	**253.4**	21.2	**131.2**	16.6	123.6
290	120	120	6	2	24.2	207.6	146.3	134.1	27.3	88.4	171.5	83.8
				3	**893.6**	165.1	**2745.9**	150.1	1007.2	**161.5**	**4005.3**	172.4
				4	139.4	160.1	1049.8	133.7	**1254.4**	92,6	3729.6	142.9
				5	44.1	186.1	359.6	158.5	591.5	150.5	1405.9	156.7
				6	22.1	224.9	102.1	286.7	96.7	100.2	335.8	141.1
				7	20.6	365.9	45.5	372.8	49.2	147.8	147.6	216.8
				8	8.9	**477.5**	23.7	**390.5**	38.7	157.5	52.8	**216.9**
310	130	130	7	2	22.1	372.9	104.6	**248.8**	28.8	139.8	117.2	161.7
				3	**683.7**	220.8	2787.3	228.7	1463.3	195.8	3107.4	**330.5**
				4	266.5	215.2	**4244.6**	170.2	**4583.5**	170.6	**4875.4**	190.7
				5	89.6	225.4	1640.7	179.8	689.8	213.4	2466.8	130.8
				6	47.3	269.3	498.2	142.9	326.1	168.8	715.8	144.9
				7	26.7	418.1	278.6	155.6	158.8	143.2	258.8	179.1
				8	15.3	267.4	112.5	190.5	76.5	147.1	238.6	189.3
				9	12.3	**435.1**	94.8	215.6	66.4	**299.1**	116.3	202.6

For fixed R, T, J, and p values, solution times for RCIP increase rapidly as q increases in the beginning but decrease gradually afterwards. The main reason for this pattern is the number of preprocessed z_r variables as given in Figure 5.3, which directly affects the sizes of subproblems in RCIP. For small q values, DF is at a greater advantage and many receivers are identified as non-jammable. On the other hand, as q increases, AT gains power and the number of receivers that are surely jammed increases. Thus, for small and large values of q, a large number of z_r variables are fixed, reducing the subproblem sizes and thus resulting in smaller CPU times. In general, the overall solution time attains its maximum value (*printed bold in the table*) when the algorithm identifies the least number of preprocessed z_r

variables. We do not see this trend in the probabilistic case since the majority of the solution time is spent on tackling the master problem. When compared with the deterministic approach, we observe that 81.2% of the instances in which $q = 3$ or $q = 4$ are solved in shorter times by the probabilistic approach.

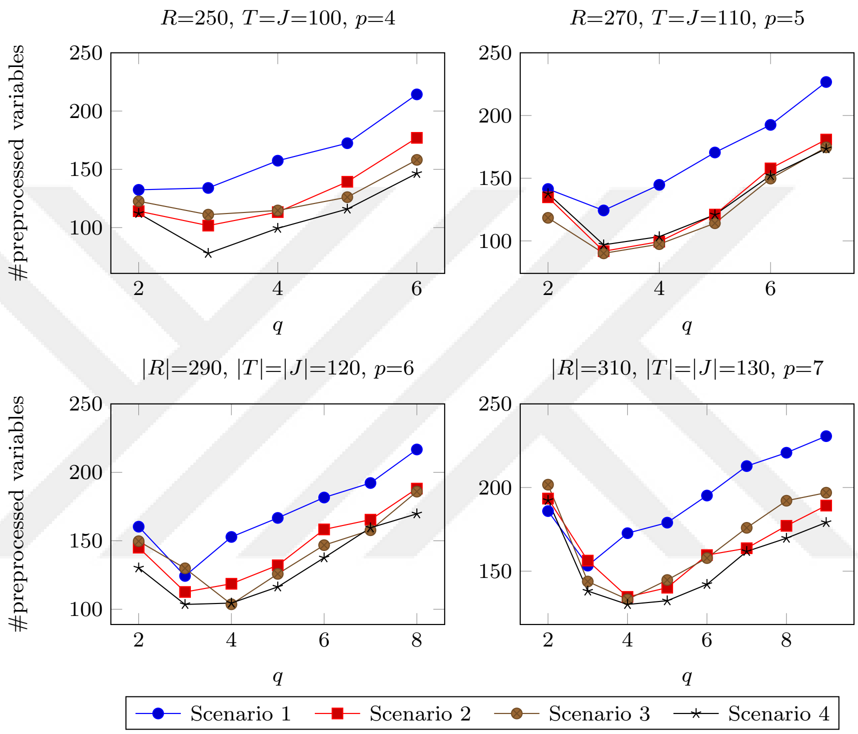


Figure 5.3: #preprocessed z_r variables against q in different scenarios for RCIP

The optimal solution values in different scenarios are also presented in Table 5.4. As expected, coverages in both the deterministic and the probabilistic models decrease as q increases. The marginal loss in the coverage due to the incremental change in the number of available jammers is high for small q values but gradually decreases as q increases. The reason for this gradual decrease stems from the fact that AT is restricted to locate all jammers in a particular area. Hence, as q increases AT wants to jam the communicating receivers that are far behind the frontline, but the restriction on the location area causes more overlap on the jammer coverage.

Also, we observe that the optimal solution value of the deterministic approach is greater than the optimal solution value of the probabilistic approach in 88% of the instances with $q \leq 3$ and less than the optimal solution value of the probabilistic approach in 90% of the instances with $q \geq 6$. This result stems from the fact that for small q values many of the receivers are counted as communicating ($z_r = 1$) in the deterministic case but only communicating with a high probability ($pz_r \approx 1$) in the probabilistic case. Similar reasoning explains the difference in large q values.

Table 5.4: Objective funnction values of RCIP and P-RCIP on different scenarios

					Objective Function Value							
					Scenario 1		Scenario 2		Scenario 3		Scenario 4	
R	T	J	p	q	RCIP	P-RCIP	RCIP	P-RCIP	RCIP	P-RCIP	RCIP	P-RCIP
250	100	100	4	2	172.6	156.1	165.1	154.5	167.3	151.9	166.9	151.6
				3	115.5	114.4	103.5	113.2	101.4	112.8	102.1	111.7
				5	102.6	103.1	91.6	101.1	86.6	101.2	86.9	99.1
				6	94.5	94.8	82.9	91.8	77.1	92.1	79.3	90.3
270	110	110	5	2	193.7	182.1	203.2	184.1	205.5	183.7	201.8	179.8
				3	159.2	155.5	160.3	157.2	155.8	155.7	159.5	154.9
				4	137.9	137.3	131.2	138.5	123.5	139.6	132.8	137.2
				5	125.0	124.7	114.8	124.8	113.6	125.9	116.9	123.8
				6	113.7	115.1	104.1	114.6	102.7	115.3	107.8	114.2
				7	107.1	107.5	98.6	106.6	95.7	107.1	101.1	106.7
290	120	120	6	2	231.4	206.7	227.6	206.1	235.1	210.8	233.6	209.8
				3	190.3	179.6	185.3	179.8	200.8	185.4	195.2	183.3
				4	162.7	161.1	159.0	161.8	162.9	164.9	164.0	163.2
				5	147.4	147.6	142.6	148.3	143.1	150.5	144.9	148.8
				6	135.7	137.2	131.6	137.8	132.0	139.4	130.1	137.6
				7	126.7	128.9	123.9	129.3	122.8	130.7	121.4	129.1
				8	120.3	122.1	117.7	122.3	115.3	123.5	113.2	121.6
310	130	130	7	2	260.0	230.5	264.9	236.5	265.9	235.6	265.9	231.9
				3	219.0	204.1	228.1	209.1	232.9	208.9	224.8	205.8
				4	195.4	185.1	195.4	189.1	196.6	187.7	191.6	186.6
				5	178.9	170.7	173.4	174.4	169.1	172.5	173.5	171.7
				6	165.6	159.5	159.8	162.7	155.1	160.4	157.3	159.7
				7	155.2	150.1	147.4	153.2	144.6	150.8	145.9	149.9
				8	146.6	142.6	138.3	145.1	135.0	142.4	136.5	141.8
				9	137.6	136.4	126.6	137.9	127.1	135.5	128.6	134.8

5.2.4 Performance of Heuristic Methods for RCIP

In this section we compare the solution statistics of the heuristic solution methods for RCIP, defined in Section 3.2.3

The optimality gaps ($100 * \frac{RCIP-Heuristic}{RCIP}$) of both heuristics for each problem instance in Table 5.3 and/or 5.4 are presented in Table 5.5. The optimal and heuristic objective values are also depicted in Figure 5.4. Inspecting these results, we observe that Sequential Location Heuristic clearly outperforms MaxCover Heuristic. The main reason behind this difference is the fact that the coverage criterion in MaxCover Heuristic, a simple yes or no value, ignores the level of signal power on receivers, which is utilized in Sequential Location Heuristic. Another apparent observation is that for both heuristics, the optimality gaps increase with increasing q values (with the adversary getting stronger) as well as with the dimensions of the instances.

Additionaly, solution times of both heuristic for the same problem instances are presented in Table 5.6. Results indicate that solution time of both heuristic methods are very close to each other and they are very small when compared with the solution times of exact solution methods presented in Table 5.3. Furthermore, solution times of both heuristics follow the same pattern with the solution times of RCIP under deterministic approach and they are almost negligible whjem compared with high solution times of RCIP.

In conclusion, even though the heuristic approaches are very efficient in terms of solution times, both of them fail to reflect the adversarial structure of the problem. For some parameter settings, the average gaps can be as large as 50% which clearly indicates the value of the bilevel solution approach for RCIP. Eventough the heuristics can be improved to obtain better solutions in terms of objective function value, they are intentionally kept in their original form as they have been defined in Section 3.2.3 in order to present the difference between the heuristics and the proposed exact solution and especially highlight the achievement.

Table 5.5: Optimality gaps of heuristic approaches in each scenario

					Gap values							
					Scenario 1		Scenario 2		Scenario 3		Scenario 4	
R	T	J	p	q	MaxCover	Seq. Loc.	MaxCover	Seq. Loc.	MaxCover	Seq. Loc.	MaxCover	Seq. Loc.
250	100	100	4	2	26.6	4.5	12.2	2.9	22.2	2.0	17.9	15.4
				3	29.0	10.1	19.2	7.7	28.6	5.5	23.4	18.3
				4	36.1	23.9	27.1	15.2	34.2	12.1	30.7	18.6
				5	40.0	30.9	35.5	25.0	39.7	21.2	38.8	21.6
				6	43.8	38.0	41.4	31.0	44.7	29.2	46.3	28.2
270	110	110	5	2	26.4	7.2	24.8	16.9	25.5	12.9	18.8	18.5
				3	34.5	16.1	34.9	21.5	26.5	16.3	24.0	22.4
				4	39.3	25.5	43.0	25.5	28.6	20.8	30.0	25.0
				5	46.1	34.9	49.7	32.3	38.6	29.9	35.6	28.1
				6	49.5	42.3	54.8	37.1	42.6	35.0	42.5	32.7
				7	53.2	46.5	58.7	44.0	47.9	39.6	48.6	37.8
290	120	120	6	2	30.3	22.2	26.4	23.0	28.4	23.5	21.4	20.0
				3	33.9	33.5	33.9	30.1	36.0	31.7	27.5	24.8
				4	37.8	38.6	40.1	33.8	38.1	32.5	31.8	27.9
				5	43.1	42.7	44.8	35.8	42.8	35.4	37.8	33.6
				6	47.5	46.4	49.0	38.6	47.2	40.1	43.0	38.7
				7	50.9	47.8	51.9	42.4	50.2	43.3	48.0	44.1
				8	55.8	49.8	54.5	45.9	52.1	46.6	51.1	46.9
310	130	130	7	2	28.5	14.5	28.0	17.2	24.0	16.5	26.6	22.0
				3	36.0	20.2	35.5	24.1	32.1	24.5	33.6	24.3
				4	42.4	27.2	38.6	27.4	34.8	29.1	38.0	26.0
				5	47.2	32.3	40.5	31.4	37.5	31.9	43.2	28.5
				6	50.4	36.8	44.2	35.9	39.9	37.6	46.6	30.5
				7	53.2	40.9	46.9	38.3	43.5	41.4	50.4	33.0
				8	55.5	44.0	50.2	40.9	45.3	44.7	53.6	35.2
				9	57.2	45.3	51.5	41.6	47.0	48.2	56.6	36.5

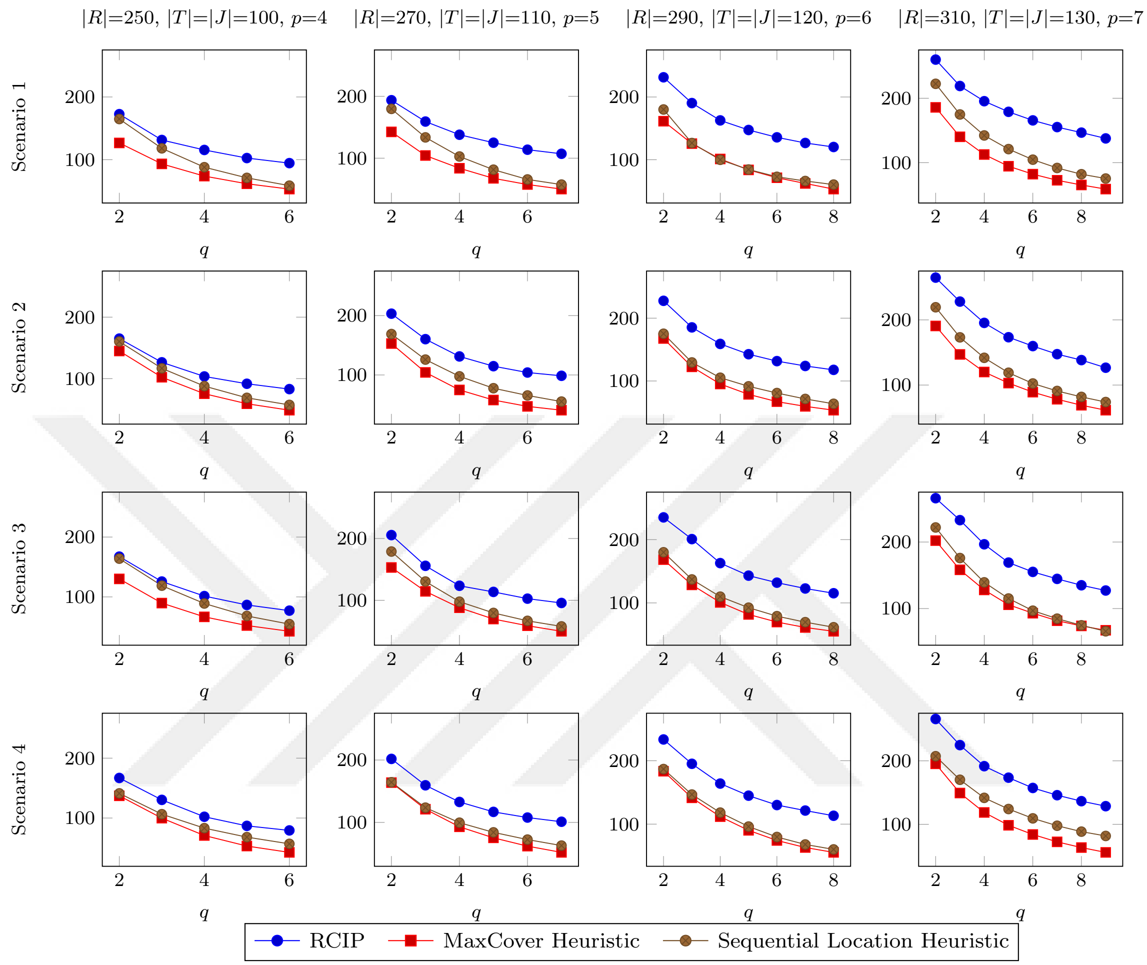


Figure 5.4: Comparison of exact and heuristic coverages for different problem instances in each scenario

Table 5.6: Solution times of heuristic approaches for each scenario in comparison with the exact solution method

					SOlution Times (seconds)											
					Scenario 1			Scenario 2			Scenario 3			Scenario 4		
R	T	J	p	q	MaxCover	Seq.Loc.	RCIP	MaxCover	Seq.Loc.	RCIP	MaxCover	Seq.Loc.	RCIP	MaxCover	Seq.Loc.	RCIP
250	100	100	4	2	2.8	1.7	7.7	2.2	2.4	8.4	1.2	1.3	3.4	28.0	31.2	19.2
				3	9.6	5.9	31.3	36.6	41.3	117.7	7.4	6.2	60.5	139.5	77.4	522.1
				4	3.2	1.6	4.7	16.7	18.8	55.1	4.3	4.5	30.3	61.4	66.4	87.8
				5	2.1	2.3	4.2	2.9	3.1	12.7	4.4	2.3	14.9	19.1	15.8	27.6
				6	1.8	1.1	1.7	2.6	1.7	4.9	1.5	1.0	5.4	3.7	1.9	7.9
270	110	110	5	2	13.7	8.1	7.7	15.4	16.3	33.2	1.3	0.8	26.1	32.9	22.9	73.6
				3	45.1	35.8	117.6	31.9	17.9	952.3	26.7	30.2	739.9	79.1	47.4	952.6
				4	4.1	3.4	34.2	14.1	15.0	387.6	11.1	7.5	513.2	31.9	20.3	410.9
				5	2.3	1.2	17.6	4.7	3.2	110.7	7.5	8.5	250.4	18.4	20.5	166.5
				6	2.2	2.2	10.4	3.4	2.1	34.1	3.6	1.8	33.6	7.1	3.8	26.8
				7	1.5	1.6	4.1	2.7	2.2	11.7	1.9	2.2	21.2	3.8	4.3	16.6
290	120	120	6	2	3.9	4.0	24.2	22.3	15.9	146.3	4.3	3.5	27.3	16.2	9.5	171.5
				3	23.5	16.3	893.6	24.8	17.9	2745.9	79.4	57.4	1007.2	327.9	339.2	4005.3
				4	19.0	9.9	139.4	20.6	13.6	1049.8	93.3	61.7	1254.4	97.1	49.8	3729.6
				5	11.2	11.6	44.1	15.6	11.9	359.6	26.5	22.0	591.5	71.9	43.5	1405.9
				6	5.6	5.8	22.1	5.8	3.2	102.1	9.4	5.6	96.7	20.7	17.4	335.8
				7	3.6	2.7	20.6	5.6	2.9	45.5	6.6	5.0	49.2	11.1	12.7	147.6
				8	2.4	2.6	8.9	4.8	3.0	23.7	7.0	7.4	38.7	9.7	5.5	52.8
310	130	130	7	2	28.8	20.2	22.1	45.7	29.4	104.6	4.5	2.9	28.8	58.2	47.8	117.2
				3	97.1	49.4	683.7	122.1	130.1	2787.3	223.1	182.7	1463.3	564.3	583.5	3107.4
				4	20.6	15.6	266.5	76.1	83.6	4244.6	166.8	96.1	4583.5	285.9	200.1	4875.4
				5	10.6	6.8	89.6	44.7	31.9	1640.7	47.5	53.3	689.8	131.4	68.9	2466.8
				6	7.5	4.4	47.3	8.5	9.3	498.2	8.9	5.5	326.1	71.6	75.4	715.8
				7	6.1	5.1	26.7	5.6	3.1	278.6	5.5	6.1	158.8	15.2	11.8	258.8
				8	3.1	3.1	15.3	4.1	2.6	112.5	2.8	1.6	76.5	7.4	5.8	238.6
				9	2.2	2.3	12.3	7.3	8.0	94.8	2.5	1.4	66.4	5.1	4.2	116.3

5.3 Tactical Insights

For each scenario introduced in Section 5.2, we generate a problem instance where $R = 250$ and $T = J = 100$. Based on this instance, we investigate optimal transmitter and jammer locations for RCIP and P-RCIP corresponding to different choices of p and q values for. Optimal solutions are shown in Figure 5.5 through Figure 5.8.

Regarding Scenario 1, which basically reflects the initial situation, it is observed that if p is equal to the number of battalions, then we have one transmitter located within the borderline of each battalion. With p value exceeding the number of battalions, the surplus transmitters are placed within the borderline starting from the locations that are closer to the frontline since they are exposed to more powerful jamming signals compared to the receivers far from the frontline.

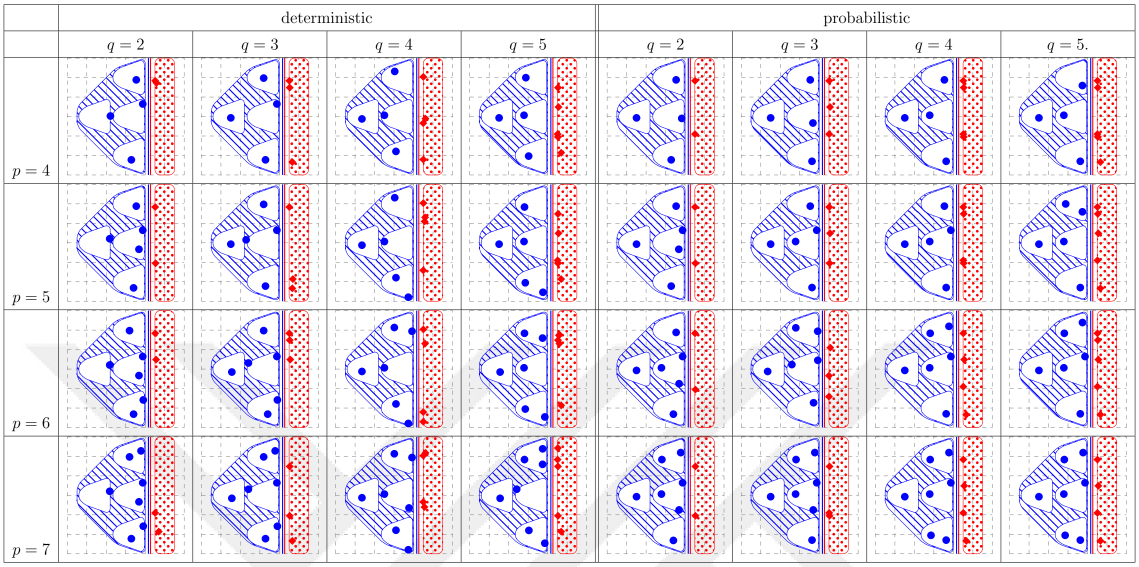


Figure 5.5: Optimal transmitter and jammer locations for different p and q values in Scenario 1

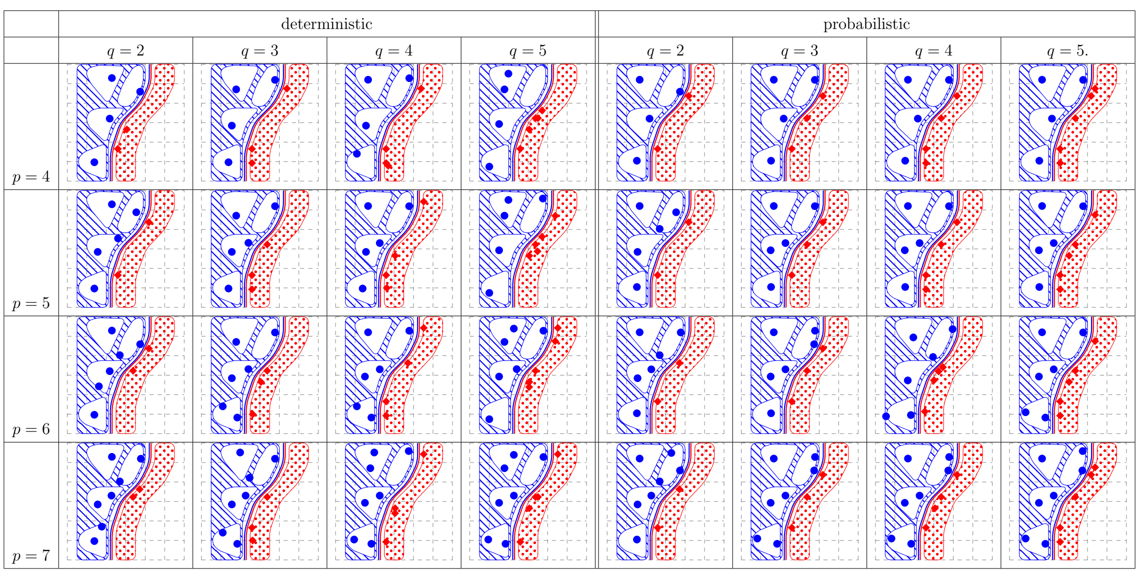


Figure 5.6: Optimal transmitter and jammer locations for different p and q values in Scenario 2

Scenario 2 (Figure 5.2b) assumes that the brigade improves its attacks from the north and thereupon the brigade commander deploys the reserve battalion to the north in order to support the improvement or exploit a possible breakthrough. When we investigate the deterministic and probabilistic solutions in Scenario 2, we observe that as q increases DF does not prefer to locate the surplus transmitters to the 3$^{\text{rd}}$ battalion that improved inwards the enemy lines but to the others. This implies that if a battalion accelerates its attacks and moves further forward than the others, it typically becomes more susceptible to jamming.

In Scenario 3, which investigates the effects of improvement from the middle of the frontline, we observe that optimal jammer locations are dispersed on the northern and southern parts of the possible jammer location site and as q increases, jammers are located collectively in order to increase their additive effect. To cope with the situation, the defender locates one transmitter to each battalion when $p = 4$ and generally locates more transmitters to the central region when $p \geq 5$.

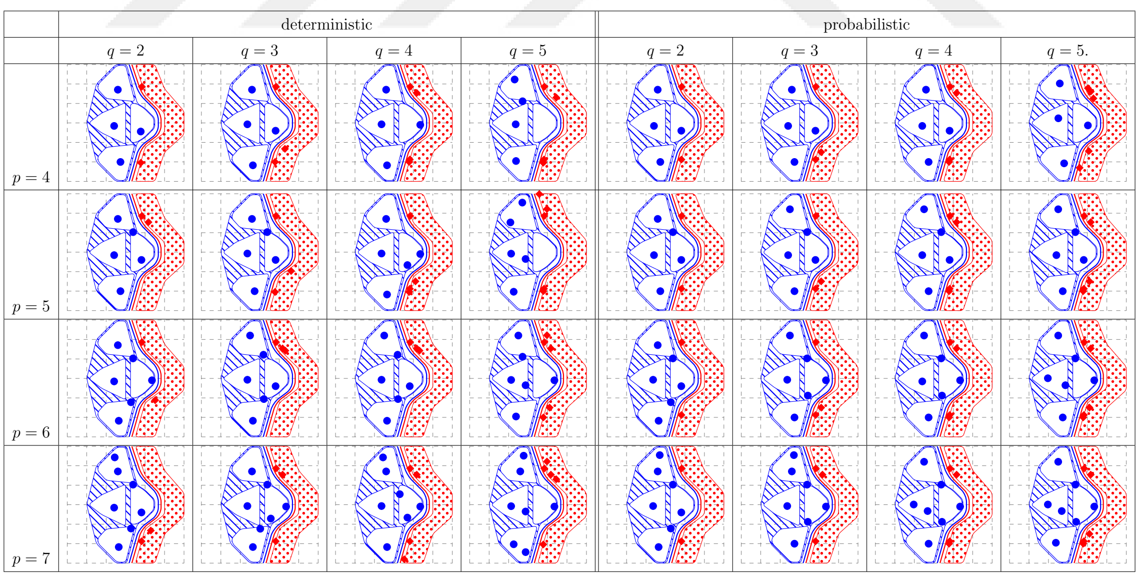


Figure 5.7: Optimal transmitter and jammer locations for different p and q values in Scenario 3

In Scenario 4 that reflects a withdrawal operation conducted by the brigade, we realize that optimal jammer locations are gathered in the center of the possible jammer location site since AT has the advantage of controlling the center of the tactical area in this scenario and uses this advantage to jam a larger portion of receivers. This makes the receivers in the center very susceptible to jamming. Therefore, defender locates more transmitters in the central region, especially when $p \geq 5$.

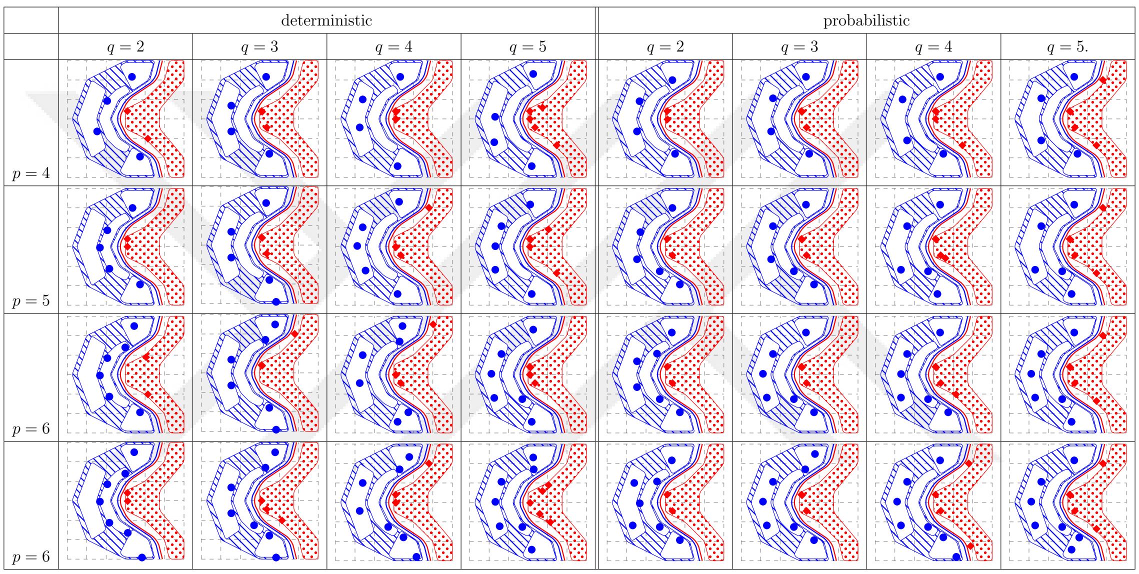


Figure 5.8: Optimal transmitter and jammer locations for different p and q values in Scenario 4

In conclusion, the results indicate that transmitter location decisions are getting complicated for scenarios 2, 3, and 4 that reflect the subsequent phases of a military operation. We suggest that rather than using the transmitters homogeneously, commanders must concentrate the effects of available transmitters in the decisive place by allocating minimum essential power to secondary places. To this end, RCIP can provide very useful courses of action in a very short time, especially for complex situations as in scenarios 2, 3, and 4.

One very fruitful observation common to all scenarios is the closeness of location decisions in RCIP and P-RCIP. Depending on the problem parameters, the approach more advantageous in solution time may be utilized to guide the commander.

5.4 Sensitivitiy analysis on parameters

5.4.1 Sensitivity analysis on the JSR threshold value (ε)

Table 5.7 presents the solution times and the optimal solution values of the deterministic approach when JSR threshold value (ε) varies between -3 dB and -7 dB for specific problem instances in each scenario. The results show that the algorithm attains the maximum solution time (highlighted in bold for each parameter setting) when $\varepsilon = -3$ dB except for two sets of 10 instances in Scenario 4 and decreases dramatically for each 1 dB decrement in ε. This decrease in solution times largely depends on the number of preprocessed z_r variables. As ε decreases, receivers become more susceptible to jamming and therefore the number of receivers that cannot be protected from jamming (i.e. $z_r = 0$) increases and the number of receivers that are not jammed (i.e. $z_r = 1$) decreases. Since the number of receivers that are close to the frontline is larger than the number of receivers located at the rear parts of the battlefield, the increment in the number of receivers for which $z_r = 0$ is more than the decrement in the number of receivers for which $z_r = 1$. Consequently, this enables the algorithm to preprocess more variables as ε decreases. Another consequence of this fact is that the optimal solution value uniformly decreases as ε decreases since receivers are more prone to jamming.

Table 5.7: Sensitivity analysis of JSR threshold value (ε)

Solution Times (seconds)

					Scenario 1					**Scenario 2**					**Scenario 3**					**Scenario 4**				
					ε in dB					ε in dB					ε in dB					ε in dB				
R	T	J	p	q	−3	−4	−5	−6	−7	−3	−4	−5	−6	−7	−3	−4	−5	−6	−7	−3	−4	−5	−6	−7
250	100	100	4	3	**140.0**	67.2	33.6	5.6	3.8	**261.8**	190.9	101.5	64.1	22.5	**136.9**	104.3	60.5	26.0	10.0	278.5	435.5	**740.5**	178.6	38.4
				4	**112.1**	10.8	4.4	4.8	3.3	**194.5**	100.1	48.4	10.9	7.9	**101.9**	67.0	30.3	13.8	5.5	**700.5**	566.8	135.8	27.8	10.4
				5	**8.5**	3.0	3.4	2.2	0.8	**111.6**	54.0	15.8	5.4	3.2	**70.9**	32.4	19.7	6.4	2.9	**502.4**	100.5	34.7	8.6	4.5
270	110	110	5	4	**556.1**	107.0	39.2	14.6	6.6	**1476.2**	1296.0	447.4	169.6	37.2	**1423.2**	1036.6	348.0	193.0	48.3	1865.1	**2850.8**	595.6	188.9	35.1
				5	**118.7**	26.6	19.0	7.3	3.7	**1867.5**	449.5	138.0	32.2	11.5	**1165.9**	608.5	261.8	47.0	17.6	**2139.6**	504.3	282.0	38.4	23.0
				6	**28.5**	12.2	16.2	3.2	1.4	**604.0**	212.0	40.0	12.1	11.0	**849.7**	299.1	44.1	21.6	13.1	**750.1**	80.8	36.0	18.2	11.2
290	120	120	6	5	**765.4**	245.7	47.1	30.5	12.7	**5776.7**	1326.1	452.1	123.3	43.6	**2818.7**	2527.5	715.3	116.8	101.3	**8408.7**	7501.0	3145.1	396.8	114.4
				6	**137.6**	60.2	24.9	10.0	5.4	**1194.6**	756.0	109.8	57.1	33.9	**2537.0**	583.2	131.8	73.3	45.1	**9011.7**	2292.3	335.6	158.6	58.9
				7	**85.9**	31.1	21.7	5.7	5.8	**431.8**	155.6	85.5	25.0	30.4	**1019.6**	188.7	67.6	53.5	33.1	**2583.5**	318.5	161.0	64.9	44.3
310	130	130	7	6	**414.6**	90.1	63.9	26.9	16.7	**2524.3**	1780.1	528.4	232.5	191.5	**5761.0**	2340.9	381.9	202.3	79.9	**2930.6**	2614.3	1096.1	138.3	39.9
				7	**132.9**	71.2	29.9	19.4	11.5	**1967.0**	792.6	309.6	221.8	41.8	**2892.7**	342.2	249.7	86.2	70.8	**15029.7**	3823.0	559.7	264.6	98.2
				8	**58.7**	35.5	17.6	8.3	5.3	**775.0**	325.3	163.7	106.7	35.0	**573.2**	259.1	106.5	69.5	34.7	**4989.5**	615.2	310.9	125.4	85.5

Coverages

					Scenario 1					**Scenario 2**					**Scenario 3**					**Scenario 4**				
					ε in dB					ε in dB					ε in dB					ε in dB				
R	T	J	p	q	−3	−4	−5	−6	−7	−3	−4	−5	−6	−7	−3	−4	−5	−6	−7	−3	−4	−5	−6	−7
250	100	100	4	3	175.3	152.6	131.4	118.7	105.7	168.4	147.2	126.5	108.3	93.9	168.6	148.4	125.7	106.1	89.6	171.7	148.2	129.1	106.5	90.6
				4	144.8	127.5	115.5	102.1	91.3	140.6	120.9	103.5	91.1	79.6	142.4	120.6	101.4	86.2	74.6	143.9	120.4	102.1	87.1	77.2
				5	128.1	115.9	102.6	91.8	81.4	120.9	103.5	91.6	80.3	72.6	121.7	101.8	86.6	75.1	68.2	120.5	100.8	86.9	77.7	69.6
270	110	110	5	4	175.8	153.3	137.9	124.6	111.3	182.3	158.3	131.2	114.1	101.7	186.1	159.9	132.9	113.3	100.1	175.4	150.5	132.8	116.1	105.3
				5	153.9	138.3	125.2	114.8	100.7	158.7	131.9	114.8	102.1	93.6	161.6	137.2	113.6	100.4	89.5	151.3	131.3	116.8	105.4	95.1
				6	141.1	127.6	113.7	103.3	93.1	136.2	118.5	104.1	95.8	86.8	137.7	116.9	102.7	92.4	81.7	134.1	119.9	107.8	97.9	86.2
290	120	120	6	5	183.9	163.1	147.4	133.1	120.2	180.7	158.3	142.6	129.1	117.2	192.4	163.5	143.1	126.2	111.8	185.2	162.6	144.9	144.1	114.3
				6	166.7	150.5	135.7	122.5	112.1	164.9	144.7	131.6	119.9	108.7	169.6	146.9	132.4	114.8	101.2	166.6	145.5	130.1	117.8	105.3
				7	155.8	140.7	126.7	116.7	104.4	151.4	135.9	123.9	112.7	100.7	153.4	136.4	122.8	105.9	93.1	153.2	135.6	121.4	108.9	96.6
310	130	130	7	6	199.1	181.9	165.6	149.9	133.5	198.8	178.5	159.8	141.9	126.5	202.8	173.7	155.1	138.8	122.5	200.8	176.5	157.3	144.6	124.1
				7	187.8	171.1	155.2	138.8	123.1	184.7	166.1	147.4	131.5	117.4	182.7	161.2	144.6	128.3	111.7	186.8	164.8	145.9	132.1	117.7
				8	178.5	162.2	146.6	129.4	116.1	172.6	155.5	138.3	122.8	109.8	169.6	151.3	134.9	118.9	103.1	172.3	152.5	136.5	122.4	108.6

5.4.2 Sensitivity analysis on the path loss exponent rates (α, β)

In order to discuss the effects of the path loss exponent rates, we conducted experiments for different values of α and β on a specific problem instance ($R = 290, T = J = 120, P = 6, q = 4$) in Scenario 1. We let α and β vary between 2 and 4 to be able to reflect the situations in which the propagation losses are low and high, respectively. Figure 5.9 depicts the solution times and the optimal solution values obtained from these experiments. The results indicate that solution times are larger for intermediate values of β ($\beta = 2.5$ and $\beta = 3$) but considerably lower for other values of β because the proposed solution algorithm can identify more z_r variables to preprocess when $\beta = 2$ (more jamming so rounding down to $z_r = 0$) or when $\beta = 4$ (less jamming so rounding up to $z_r = 1$).

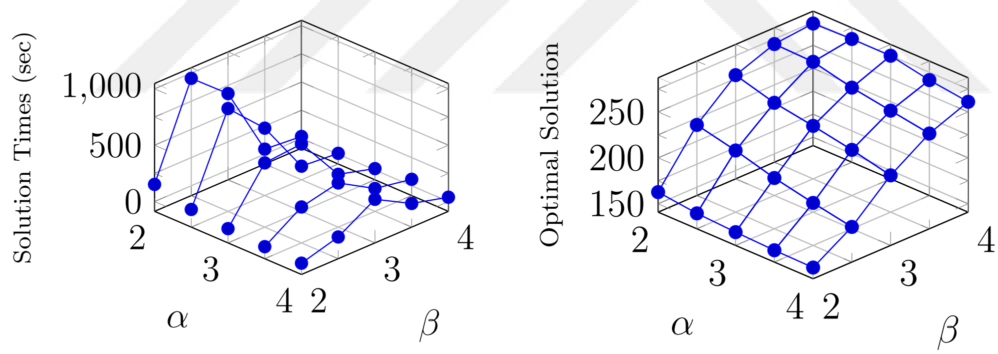


Figure 5.9: Sensitivity analysis for the path loss exponent

As expected, we obtain the highest coverage when $\alpha = 2, \beta = 4$ and the lowest coverage when $\alpha = 4, \beta = 2$. We also conclude that the optimal value is more sensitive to AT's path loss exponent β rather than DF's path loss exponent α because β becomes more decisive as we add jammer signals in calculating JSR when compared to single transmitter signal effect.

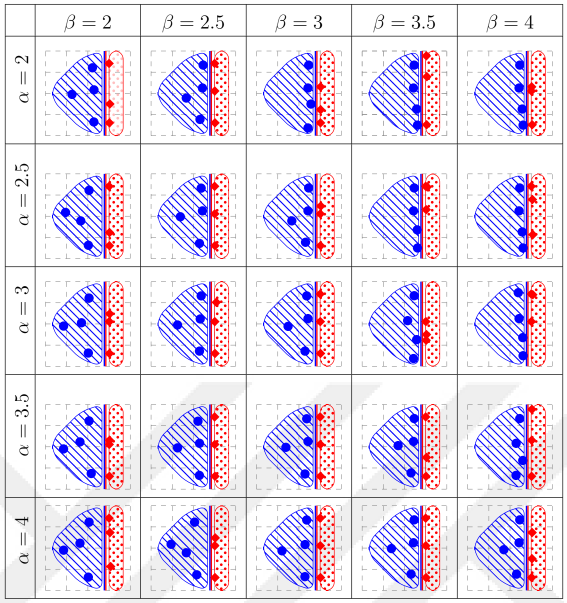


Figure 5.10: Optimal transmitter and jammer locations for different values of α and β when $R = 250$, $T = J = 100$, $p = 4$, and $q = 3$ in Scenario 1

In order to find out how path loss exponent rates α and β affect the location decisions of DF and AT, we solve an exemplary instance of Scenario 1 with parameters $R = 250$, $T = J = 100$, $p = 4$, and $q = 3$ and present the optimal locations of transmitters and jammers in Figure 5.10. The chosen locations indicate that DF locates transmitters very close to the frontline for high β values but prefers the interior of possible transmitter location area for low β values. The location decisions of DF are more sensitive to path loss exponent β and optimal transmitter locations differ only a little for different α values. For a fixed value of α, transmitter locations get closer to the frontline as β increases, i.e., jamming effect decreases. Moreover, we establish that optimal jammer locations of AT are independent of the path loss exponent rate and are always very close to the frontline. The average distance of transmitter locations to the frontline decreases gradually from 2.81 to 0.73 kilometers with a slope of -1.03 as we increase β from 2 to 4. In contrast, the average distance of transmitter locations to the frontline increases from 0.96 to 2.18 kilometers with a slope of 0.61 as we increase α from 2 to 4. When we compare the absolute values of both slopes we conclude that location decisions of transmitters are more sensitive to β than α.

5.5 Summary

The results of our intensive computational studies present the efficacy of the proposed solution methods for both the RCIP and P-RCIP. Thanks to the enhancements, significant improvement has been achieved in solution times and large instances of the problem are solved in reasonable times.

We showed that our treatment of formulating the problem with a bilevel formulation that incorporates the adversarial effect yields considerably better decisions when compared against two fast solution methods, a traditional one in the location literature and one that mimics the decision making process in practice.

We provided some useful tactical insights on transmitter and jammer location decisions by analyzing optimal solutions under varying p and q values in each scenario. The results showed that even though the optimal locations obtained in Scenario 1 are consistent with the expected layout, for other scenarios that reflect the subsequent phases of a military operation, solutions obtained by RCIP outperform the experiential results, highlighting the value of our treatment of RCIP, especially in complex military situations.

Additionally, sensitivity analysis on different problem parameters provided useful tactical insights to contribute to the radio communications network planning.

Chapter 6

RCIP with Artillery

In modern warfare, a variety of functions help the commander to build and sustain combat power. Commanders integrate and coordinate these functions to synchronize battle effects in time, space, and purpose. From a doctrinal perspective, it is essential to exploit the combination of all available combat functions to create a sophisticated effect that can not be endured by the adversary. Among these functions, fire support in general and artillery in particular, plays an eminent role in the application of fire, coordinated with the maneuver of forces to destroy, neutralize or suppress the enemy at the operational and tactical level.

Artillery, with its ability to project firepower on distant targets serially and accurately, is considered as the workhorse of modern armies. Artillery is mainly used to provide suppression on the strategic enemy assets, such as strategic reserve forces, central logistic units, principal supply roads, lines of communications, and other notable targets that can set the tone of the ongoing conflict. As an effective electronic warfare asset, jammers also be considered as an important type of target for artillery fire. Considering this fact, we extend RCIP to incorporate the artillery fire support.

Specifically, we assume Defender (DF) is capable of allocating a limited number of its planned artillery fire to destroy the jammers of the Attacker (AT) in order to decrease the effect of the AT's jammer capability and increase the reliability and quality of his communications system.

Accordingly, we extend RCIP by incorporating artillery fire support into the existing problem and define the new problem RCIP with Artillery (RCIP-A).

6.1 Problem Definition

RCIP-A is a trilevel sequential game. In the first stage of the problem, DF locates a limited number of (p) transmitters to maximize the number of receivers that are able to communicate. After observing the transmitter location decisiosn of DF, the AT locates the limited number of (q) jammers to minimize the number of communicating receivers. At the final stage, after observing the locations of p transmitters and AT's q jammers, DF uses his current artillery assets and utilizes a limited number of (s) artillery fire to destroy AT's located jammers to maximize the number of receivers that are able to communicate.

RCIP-A, similar to RCIP, is a type of Stackelberg game with three nested optimization problems. Therefore, We model this problem as a trilevel DF-AT-DF sequential game.

6.2 Mathematical formulation of RCIP-A

To ensure consistency, we continue to use the same nomenclature used to define the first and second stages of the problem. We define the following additional notation needed to formulate the third stage problem of RCIP-A.

Sets:

$\mathcal{L} = \{l_1, \ldots, l_L\}$ the set of possible targets identified during the artillery fire planning.

Parameters:

s the maximum number of artillery fire that can be executed against jammers.

η the length of the radius of impact when an artillery fire is executed on any possible target l.

d_{jl} the euclidean distance between possible transmitter location site $j \in \mathcal{J}$ and possible target $l \in \mathcal{L}$.

ϕ_{jl} the probability of jammer located on $j \in \mathcal{J}$ to survive from an artillery fire planned on target $l \in \mathcal{L}$ and it is equal to $\min\{1, \frac{d_{jl}}{\eta}\}$.

Decision Variable:

$$a_l = \begin{cases} 1 & \text{if artillery fire is planned on target } l \in \mathcal{L} \\ 0 & \text{otherwise} \end{cases}$$

To clarify the parameters and the variables, consider Figure 6.1. In this example, AT has 9 probable jammer location sites $(j_1, j_2, \ldots, j_9)$ and is to locate 2 jammers. Additionally, we assume that DF has 4 different targets to be used by DF's limited artillery assets and each fire planned on these targets has a radius of impact with length indicated by η in the figure.

Specifically for the jammer located at j_2, the probability of survivability from a fire planned at target l_1 is denoted by $\phi_{j_2 l_1}$ and equal to $\min\{1, \frac{d_{j_2 l_1}}{\eta}\}$. Apparently, function min returns $\frac{d_{j_2 l_1}}{\eta}$ because j_2 is within the radus of impact of l_1. However, if we assume that an artillery fire is planned on target l_3, the fact that $d_{j_2 l_3} > \eta$ (i.e. j_2 is located at the outside of the radius of impact of artillery fire executed on l_3), implies that $\phi_{j_2 l_3} = 1$.

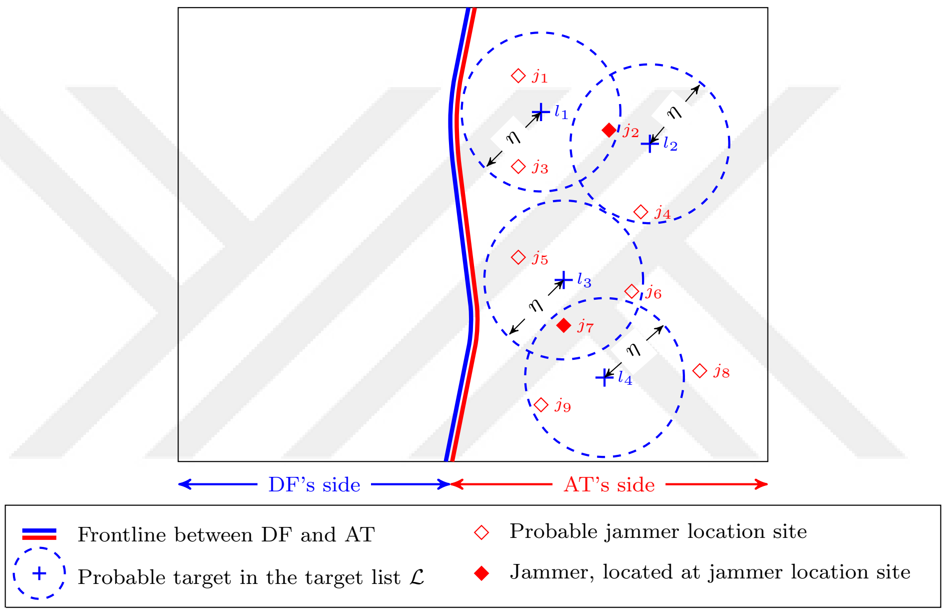


Figure 6.1: An example sketch for the notations used in RCIP-A

Using the notation above and given the transmitter location plan $\hat{\mathbf{x}}$ obtained from the first stage problem and jammer location plan $\hat{\mathbf{y}}$ obtained from the second stage problem, Jamming to Signal Ratio to be used in the third stage problem for each receiver $r \in \mathcal{R}$ is defined in equation (6.1).

$$JSR_r(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \mathrm{a}) = \lambda \frac{\sum\limits_{j \in \mathcal{J}} \frac{1}{d_{jr}^{\beta}} \hat{y}_j (\prod\limits_{l \in \mathcal{L}} \phi_{jl} a_l)}{\max\limits_{t \in \mathcal{T}} \frac{1}{d_{tr}^{\alpha}} \hat{x}_t} \tag{6.1}$$

The mathematical formulation of RCIP-A then becomes the following.

$$
\begin{aligned}
W^* = \max \quad & \tau(x) && && (6.2)\\
\text{s.t.} \quad & \sum_{t \in \mathcal{T}} x_t \leq p && && (6.3)\\
& x_t \in \{0,1\} && t \in \mathcal{T} && (6.4)\\
\text{where} \quad & \tau(x) = \min \ \mu(\hat{x}, y) && && (6.5)\\
& \quad \text{s.t.} \quad \sum_{y \in \mathcal{Y}} y_j \leq q && && (6.6)\\
& \quad y_j \in \{0,1\} && j \in \mathcal{J} && (6.7)\\
& \quad \text{where} \quad \mu(\hat{x}, \hat{y}) = \max \sum_{r \in \mathcal{R}} z_r && && (6.8)\\
& \quad \quad \text{s.t.} \quad z_r \leq \frac{JSR_r(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \mathrm{a})}{\varepsilon} && r \in \mathcal{R} && (6.9)\\
& \quad \quad \sum_{l \in \mathcal{L}} a_l \leq s && && (6.10)\\
& \quad \quad a_l \in \{0,1\} && l \in \mathcal{L} && (6.11)\\
& \quad \quad z_r \in \{0,1\} && r \in \mathcal{R} && (6.12)
\end{aligned}
$$

The above formulation (6.2)-(6.12) is composed of three different levels that are nested. The first level DF's problem locates at most p transmitters (6.3) by using binary transmitter location decision variables (6.4) so as to maximize the number of communicating receivers (6.2) by assuming the jamming effect and considering the supportive effect of the artillery fire planning. The second level AT's problem locates at most q jammers (6.6) by using binary jammer location decision variables (6.7) so as to minimize the number of communicating receivers. Having both the transmitter and jammer location decisions taken by the first and second level problems, DF in the third level formulation maximizes the number of communicating receivers (6.8) by planning at most s artillery fires. Each receiver is identified as communicating or jammed according to constraints (6.9) which evaluates the Jamming to Signal Ratio level at each receiver $r \in \mathcal{R}$. Finally (6.10) and (6.12) are sign constraints for binary decision variables.

6.3 Solving RCIP-A using decomposition

The proposed solution method is a nested decomposition method which is based on the idea used to solve deterministic RCIP. To solve the trilevel mathematical formulation of RCIP-A, we present an equivalent single level formulation and decompose it into a master problem and a bilevel subproblem.

To present the single level formulation, let $\mathcal{Y} = \{y \in \{0,1\}^J \mid \sum_{j\in\mathcal{J}} y_j \leq q\}$ represent all possible AT strategies and similarly, $\mathcal{A} = \{a \in \{0,1\}^L \mid \sum_{l\in\mathcal{L}} a_l \leq s\}$ represent all possible artillery planning strategies of DF. For each receiver $r \in \mathcal{R}$, we introduce a new decision variable $\bar{\bar{z}}_{rya}$, which is defined as follows

$$\bar{\bar{z}}_{rya} = \begin{cases} 1 & \text{if receiver } r \in \mathcal{R} \text{ is able to communicate when AT's strategy is } y \in \mathcal{Y} \\ & \text{and DF's artillery plan is } a \in \mathcal{A} \\ 0 & \text{otherwise.} \end{cases}$$

With the addition of an exponential number of such decision variables and an exponential number of constraints, we may reformulate RCIP-A as the following linear mixed-integer programming (MIP) problem, say $MP(\mathcal{Y} \times \mathcal{A})$, to stand for the master problem.

$$
\begin{aligned}
MP(\mathcal{Y}\times\mathcal{A}) \quad \theta_{MP}(y,a) = \max \quad & \omega && && (6.13)\\
\text{s.t.} \quad & \omega \le \sum_{r\in\mathcal{R}} \bar{\bar{z}}_{rya} && y\in\mathcal{Y},\ a\in\mathcal{A} && (6.14)\\
& \bar{\bar{z}}_{rya} \le \sum_{t\in T(r,y,a)} x_t && r\in\mathcal{R},\ y\in\mathcal{Y},\ a\in\mathcal{A} && (6.15)\\
& \sum_{t\in\mathcal{T}} x_t \le p && && (6.16)\\
& x_t \in \{0,1\} && t\in\mathcal{T} && (6.17)\\
& 0 \le \bar{\bar{z}}_{rya} \le 1 && r\in\mathcal{R},\ y\in\mathcal{Y},\ a\in\mathcal{A} && (6.18)
\end{aligned}
$$

In this model, ω is an auxiliary variable that will correspond to the number of communicating receivers when hedging against all possible AT strategies by considering all possible artillery fire planning of DF. Set $T(r,y,a)$ represents the transmitter location sites that will enable the communication of receiver $r\in\mathcal{R}$ when AT's strategy is $y\in\mathcal{Y}$ and $a\in\mathcal{A}$, i.e., $T(r,y,a) = \{t\in\mathcal{T}(r) \mid \lambda\ d_{tr}^{\alpha} \sum_{j\in\mathcal{J}} \frac{1}{d_{jr}^{\beta}} \hat{y}_j (\prod_{l\in\mathcal{L}} \phi_{jl} a_l) < \varepsilon\}$. Constraints (6.15) enforce one such transmitter to be located when s_{rya} variable takes the value of one. Through constraints (6.14) and the objective function (6.13), the auxiliary variable ω will be equal to the minimum number of receivers that will be communicating when considering all possible AT strategies and all possible artillery fire plannings. Constraint (6.16) limits the number of transmitters to be located by p. Constraints (6.17) are domain restrictions for x_t variables. Note that constraints (6.18) relax the binary requirements of s_{rya} variables since once the transmitter location variables take on integer values, the objective function and constraints (6.15) imply the integrality of these variables.

It is apparent that $MP(\mathcal{Y}\times\mathcal{A})$ is a huge model with set $\mathcal{Y}$ and set $\mathcal{A}$, which has $\binom{J}{q}\binom{L}{s}$ elements. Therefore, we shall solve the above-mentioned master problem with only a subset of AT's strategies, say $Y \subseteq \mathcal{Y}$ and a subset of DF artillery planning strategies, say $A \subseteq \mathcal{A}$, which is the relaxed master problem $MP(Y \times A)$ and its optimal solution gives an upper bound (UB) for RCIP-A.

Let $\hat{\mathbf{x}}$ be the optimal solution of the relaxed master problem $MP(Y \times A)$. In order to identify the optimal response of AT, who considers DF's artillery planning, we solve the model (6.5)-(6.12). However, the proposed model itself is a bilevel programming problem that can not be solved directly. Therefore, we propose a similar decomposition method to solve this problem. To do so, by making use of the previously defined set $\mathcal{A}$, we define a new decision variable for each receiver $r \in \mathcal{R}$, which is defined as follows.

$$\bar{z}_{ra}(\hat{\mathbf{x}}) = \begin{cases} 1 & \text{if receiver } r \in \mathcal{R} \text{ is able to communicate when DF's artillery plan is} \\ & a \in \mathcal{A}, \text{ given the transmitter location plan } \hat{\mathbf{x}} \\ 0 & \text{otherwise.} \end{cases}$$

With this new decision variable and exponential number of constraints, we define the model (6.5)-(6.12) by an equivalent single level formulation $MP_{\hat{\mathbf{x}}}(\mathcal{A})$ as follows.

$$MP_{\hat{\mathbf{x}}}(\mathcal{A}) \quad \rho_{MP_{\hat{\mathbf{x}}}}(y) = \min \quad \Delta \tag{6.19}$$

$$\text{s.t.} \quad \Delta \geq \sum_{r \in \mathcal{R}} \bar{z}_{ra}(\hat{\mathbf{x}}) \qquad a \in \mathcal{A} \tag{6.20}$$

$$\sum_{t \in \mathcal{T}(r)} \hat{x}_t \leq w_r \; p \qquad r \in \mathcal{R} \tag{6.21}$$

$$\bar{z}_{ra}(\hat{\mathbf{x}}) + \frac{JSR_r(\hat{\mathbf{x}}, y, \hat{\mathbf{a}})}{\varepsilon} \geq w_r \qquad r \in \mathcal{R},\; a \in \mathcal{A} \tag{6.22}$$

$$\sum_{j \in \mathcal{J}} y_j \leq q \tag{6.23}$$

$$y_j \in \{0, 1\} \qquad j \in \mathcal{J} \tag{6.24}$$

$$\bar{z}_{ra}(\hat{\mathbf{x}}) \in \{0, 1\} \qquad r \in \mathcal{R},\; a \in \mathcal{A} \tag{6.25}$$

Objective function (6.19) and the constraints (6.20) in $MP_{\hat{\mathbf{x}}}(\mathcal{A})$ minimizes the total number of communicating receivers for a given transmitter location plan ($\hat{\mathbf{x}}$) by considering all possible artillery planning of DF. By constraints (6.21), auxiliary variable w_r for each receiver $r \in \mathcal{R}$ takes the value of 1, only if there exists at least one transmitter located close enough to the receiver that can transmit a signal with a power greater than the receiver threshold value (γ). Given the transmitter location plan ($\hat{\mathbf{x}}$), constraints (6.22) imply that receiver $r \in \mathcal{R}$ is counted as able to communicate (i.e. $\bar{z}_{ra}(\hat{\mathbf{x}}) = 1$) if $w_r = 1$ and jamming to signal ratio value is less than ε. Otherwise, because of the objective function, receiver $r \in \mathcal{R}$ is counted as unable to communicate and either the receiver is jammed (i.e. $JSR_r(\hat{\mathbf{x}}, y, \hat{\mathbf{a}}) \geq \varepsilon$) or there does not exist any located transmitter that can transmit a signal with a power greater than the receiver threshold value (γ). Constraint (6.23) locates at most q jammers and finally constraints (6.24) and (6.25)) imply that associated variables are binary decision variables.

As in the previous master problem, at each iteration, we propose to solve this master problem by using a subset of DF's artillery planning strategies, say $A \subseteq \mathcal{A}$, which gives a lower bound for the model (6.5)-(6.12), the bilevel subproblem of the tri-level formulation.

Let $\hat{\mathbf{y}}$ be the optimal solution of the relaxed master problem $MP_{\hat{\mathbf{x}}}(A)$. Then, given the transmitter location plan $\hat{\mathbf{x}}$ and jammer location plan $\hat{\mathbf{y}}$, to identify the optimal artillery fire plan we solve (6.8)-(6.12). However, jamming to signal ratio given explicitly in equation (6.1) causes nonlinearity in constraint (6.9), which makes it difficult to solve. Therefore, in order to linearize the model, given the jammer location plan $\hat{\mathbf{y}}$, we define the set $\mathcal{A}(\hat{\mathbf{y}}) = \{a \in \{0,1\}^L \mid \sum_{l \in \mathcal{L}(\hat{\mathbf{y}})} a_l \leq s$ and $a_l = 0$ if $l \notin \mathcal{L}(\hat{\mathbf{y}})\}$, where $\mathcal{L}(\hat{\mathbf{y}}) = \{l \in \mathcal{L} \mid \phi_{jl} < 1$ and $\hat{y}_j = 1\}$. By definition, $\mathcal{A}(\hat{\mathbf{y}})$ is composed of all possible artillery fire planning strategies that are formed by the target lists $l \in \mathcal{L}(\hat{\mathbf{y}})$ and $\mathcal{L}(\hat{\mathbf{y}})$ contains the target lists that can be effective on located jammer sites (i.e. $\hat{y}_j = 1$).

Assuming that a set of K possible artillery fire plan has been identified for a given jammer location plan $\hat{\mathbf{y}}$ (i.e. $\mathcal{A}(\hat{\mathbf{y}}) = \{\mathbf{a}^1, \ldots, \mathbf{a}^K\}$), jamming to signal ratio at receiver r for artillery plan $a^k \in \mathcal{A}(\hat{\mathbf{y}})$ is easy to calculate. Thus, letting $f(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \mathbf{a}^k)$ to denote the total number of receivers that are able to communicate, an equivalent formulation of the model (6.5)-(6.12) can be defined as

$$\mu(\hat{x}, \hat{y}) = \max_{\mathbf{a}^1, \ldots, \mathbf{a}^K} f(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \mathbf{a}^k) \tag{6.26}$$

and it can be solved by enumerating all elements of the set $\mathcal{A}(\hat{\mathbf{y}}) \subseteq \mathcal{A}$, and without loss of generality the cardinality of this set can be reduced by assuming that $\sum_{l \in \mathcal{L}(\hat{\mathbf{y}})} a_l = s$ in the definition of $\mathcal{A}(\hat{\mathbf{y}})$.

The proposed solution method is formalized with Algorithm 3.

Algorithm 3: Decomposition method to solve RCIP-A

Data: $\mathcal{R},\ \mathcal{T},\ \mathcal{J}, \mathcal{L}\ \varepsilon,\ \gamma$
Result: $x^*,\ W^*$
begin
  $LB \longleftarrow 0,\ UB \longleftarrow R,\ Y \longleftarrow \emptyset\ A \longleftarrow \emptyset$;
  Select an arbitrary $y \in \mathcal{Y}$ and $a \in \mathcal{A}$ as an initial solution;
  $Y \longleftarrow Y \cup \{y\},\ A \longleftarrow A \cup \{a\}$;
  while $LB < UB$ **do**
    Solve $MP(Y \times A)$ for $\hat{x}$;
    if $\theta_{MP}(Y \times A) < UB$ **then** $UB \longleftarrow \theta_{MP}(Y \times A)$;
    $LB_{\hat{x}} \longleftarrow 0,\ UB_{\hat{x}} \longleftarrow R,\ A_{\hat{x}} \longleftarrow \emptyset$;
    Select an arbitrary $a \in \mathcal{A}$ as an initial solution;
    $A_{\hat{x}} \longleftarrow A_{\hat{x}} \cup \{a\}$;
    while $LB_{\hat{x}} < UB_{\hat{x}}$ **do**
      Solve $MP_{\hat{\mathbf{x}}}(A)$ for ŷ ;
      if $\rho_{MP_{\hat{x}}}(y) > LB_{\hat{x}}$ **then** $LB_{\hat{x}} = \rho_{MP_{\hat{\mathbf{x}}}}(y)$;
      Define $\mathcal{A}(\hat{\mathbf{y}})$;
      Solve (6.26) for $\hat{a}$ and identify $\mu^*(\hat{x}, \hat{y})$;
      if $\mu^*(\hat{x}, \hat{y}) < UB_{\hat{x}}$ **then** $UB_{\hat{x}} = \mu^*(\hat{x}, \hat{y})$;
      $A_{\hat{x}} \longleftarrow A_{\hat{x}} \cup \{\hat{a}\}$;
    if $LB_{\hat{x}} = UB_{\hat{x}}$ **then**
      $Y \longleftarrow Y \cup \{$ŷ$\}$;
      Define $\mathcal{A}(\hat{\mathbf{y}})$;
      Solve (6.26) for $\hat{a}$;
      $a^* \longleftarrow \hat{a}$;
      $A \longleftarrow A \cup \{a^*\}$;
    $Y \longleftarrow Y \cup$ ŷ;
  Print("x^* and a^* are the optimal transmitter location and artillery planning strategies, respectively for DF that will enable W^* receivers to communicate")

6.4 Computational study

In this section, we first present an illustrative example to be able to present the difference in terms of tactical thoughts and setup between RCIP and RCIP-A. We also present some tactical insights observed from different parametric solutions. Then, we present solution statistics of RCIP for different parameter values and conclude by a sensitivity analysis on the radius of impact of artillery fire.

6.4.1 An illustrative example for RCIP-A

An instance of the problem for a brigade with 4 battalions is presented in Figure 6.2. This illustrative example is based on Scenario 1, presented in Section 5.2, with 60 receivers, 30 probable transmitter location sites, 30 probable jammer location sites, 15 probable targets identified by DF on AT's side to use in the artillery fire planning. Additionally,we assume that DF has 5 transmitters ($p = 5$) and AT has 5 jammers ($q = 5$)to be located.

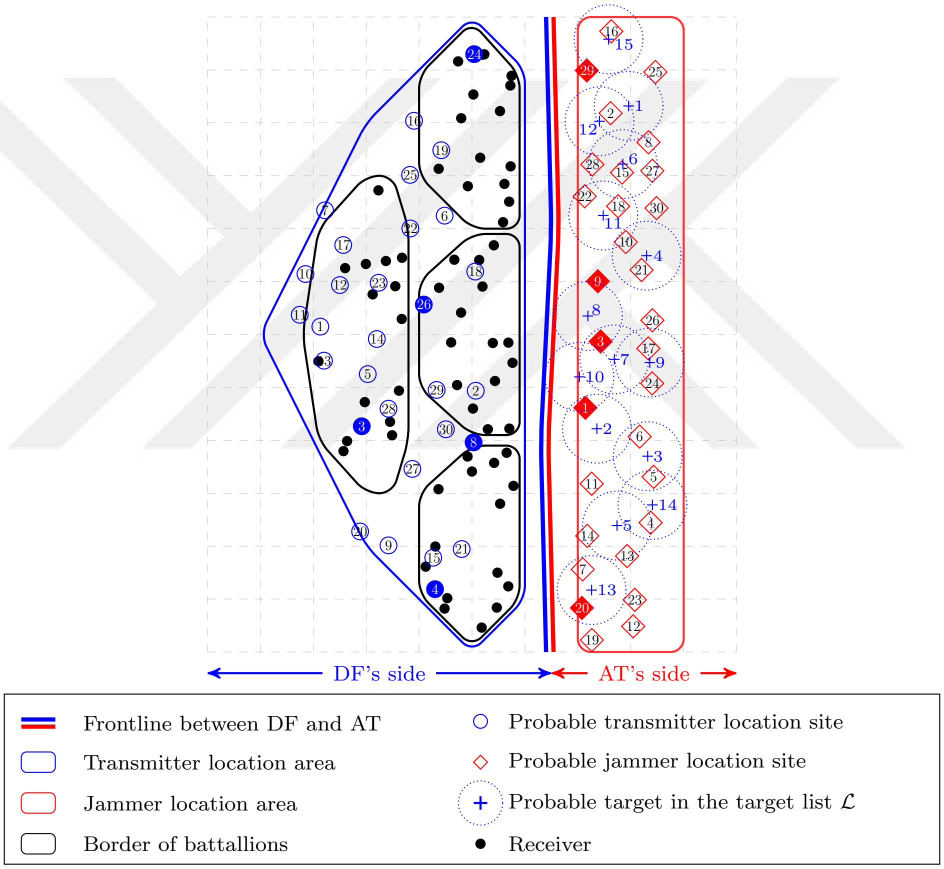


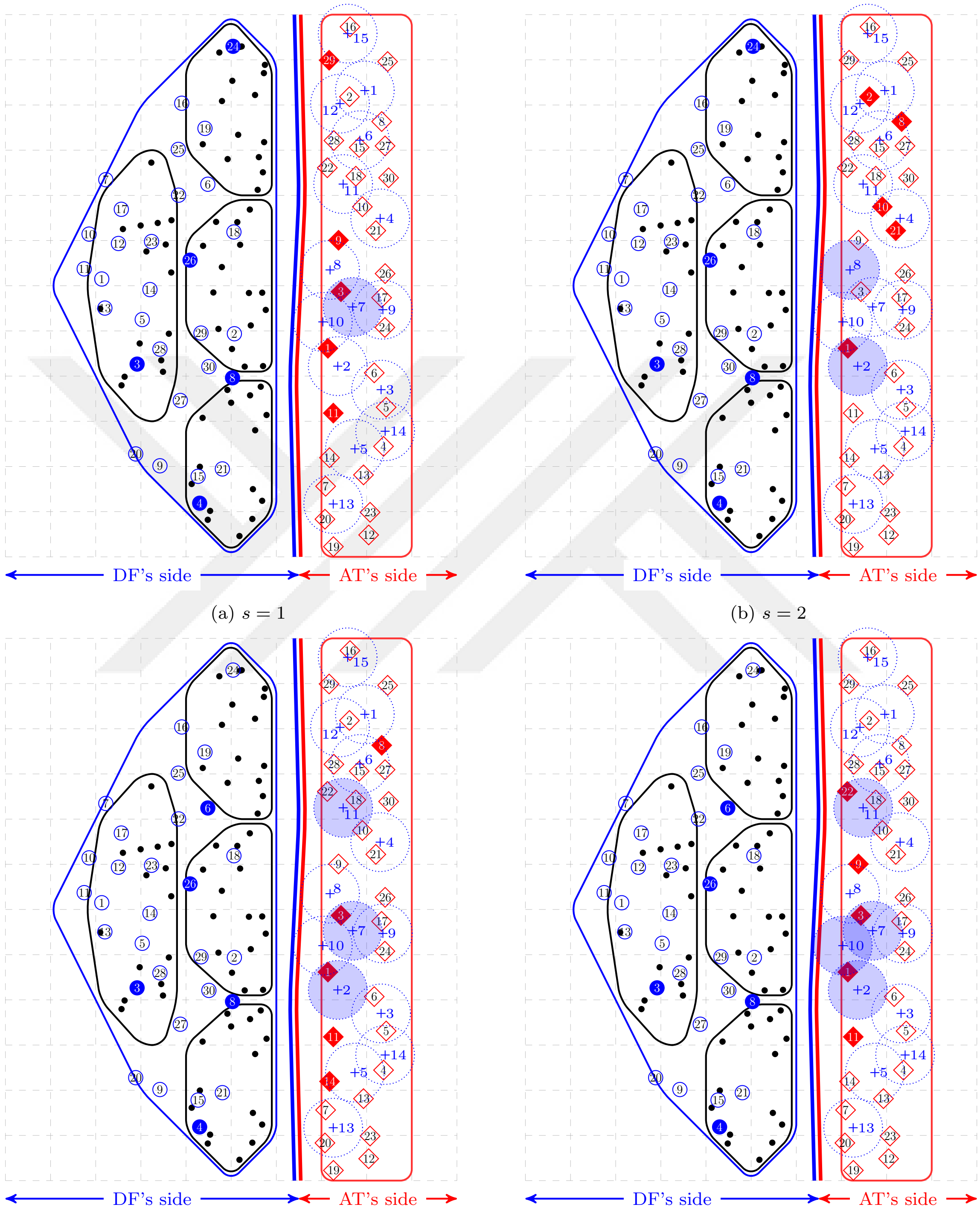
Figure 6.2: Sketch of the illustrative example

To compare the results between RCIP and RCIP-A, the problem is first solved by RCIP which does not evaluate the artillery planning of DF. In this case, DF locates transmitters at sites 3, 4, 8, 24, 26 and after observing these transmitter locations, AT locates jammers at sites 1, 3, 9, 20, 29. With this location plan, 26 receivers can communicate, while 34 is jammed.

Artillery planning is not involved in RCIP. Contrary to this fact, let's assume that DF will use his artillery fire capability after observing AT's jammer locations identified by RCIP. In this case, DF will fire at target 7 if $s = 1$ (i.e. DF has only one artillery fire support available) and this will imply the number of communication receivers to increase from 26 to 31. Similarly, number of jammed receivers will decrease from 34 to 29. However, different from RCIP, RCIP-A enables AT to anticipate DF's artillery fire capability and adjust his optimal jammer locations accordingly. To utilize this idea, we solve RCIP-A with the assumption that DF will use 1 artillery fire at the final stage and present the optimal solutions in Figure 6.3a. In this case, DF uses the same transmitter location plan and fires at the same target, but AT makes a slight change and relocates only 1 jammer from jammer location site 20 to 11. But this relocation implies 28 receivers to communicate, which is a more preferable course of action from AT's perspective.

The optimal solution values under different s values are also presented in Figure 6.3. When $s = 2$, we observe that AT locates jammers to the rear parts and tries to locate them closer to each other in order to benefit from the cumulative effect of the jammer signal. We also observe that DF does not change the location of the transmitters but fires targets 2 and 8, which are very close to the front line.

As artillery fire capability increases, we observe that DF does not change the transmitter locations but AT tries to gather jammers close to the center and front of his area and similarly DF uses his artillery fire capability in the same area. We conclude from the results that jammers in the center front of the AT'side are very powerful especially when they are gathered. Therefore, to prevent the cumulative effect of jammers, DF intensifies his artillery fire in the same place of the battlefield.



(a) $s = 1$ (b) $s = 2$ (c) $s = 3$ (d) $s = 4$

Figure 6.3: Optimal transmitter and jammer locations identified by RCIP-A for different s values

6.4.2 Numerical Results

Table 6.1 presents the solution statistics of RCIP-A based on the scenario described above. Each row depicts the average number of iterations, solution times (in CPU Seconds), and objective function values obtained by solving 5 randomly generated problem instances with the specified parameter choices for R, T, J, L, p, q, and s values. In the table, MP denotes the Master Problem that DF solves to identify the optimal transmitter locations and SP denotes the bilevel subproblem that AT solves to identify the optimal response to a given transmitter location. Therefore, both the number of iterations and solution times are provided in two different columns under MP and SP. Objective function value refers to the minimum number of receivers that will be able to communicate under the smartest jamming attack of AT and the artillery support of DF. We also present the average percentage coverages these objective values correspond to.

Table 6.1: Solution statistics of RCIP-A for the illustrative example

							# Iterations		Solution Times (sec.)			Objective Value	
R	T	J	L	p	q	s	MP	SP	MP	SP	Total	# receivers covered	Coverage percentage
60	20	20	20	4	3	1	3.4	4.6	2.4	11.5	13.9	35.4	58.9 %
						2	3.1	5.5	21.9	30.5	52.5	37.4	62.3 %
						3	2.6	5.9	15.6	16.1	31.7	38.2	63.7 %
						4	2.5	6.2	14.9	15.3	30.2	38.4	64.1 %
80	25	25	25	5	4	1	4.6	6.9	15.6	162.4	178.0	48.2	60.3 %
						2	3.5	7.6	45.8	745.8	791.6	51.2	64.0 %
						3	3.2	12.8	250.4	1931.4	2181.8	53.5	66.9 %
						4	3.1	8.4	26.4	110.1	136.4	55.1	68.9 %
						5	2.9	9.5	15.8	72.9	88.7	55.6	69.5 %

Results indicate that the average number of iterations for MP slightly decreases as s increases for fixed parameter values but the average number of iterations for SP for each MP increases and this causes a meaningful increase in the solution times. When we investigate the solution times, we observe that most of the solution time is spent on the bilevel subproblem to identify optimal jammer locations. Enhancements for RCIP are not applicable to RCIP-A. Therefore, we are not currently able to solve larger instances of the problem unless new enhancements or more prosperous solution methods or techniques are identified.

Table 6.2: Value of artillery fire in terms of objective function value

						Average objective function value					
							RCIP-A				
R	T	J	L	p	q	RCIP	$s=1$	$s=2$	$s=3$	$s=4$	$s=5$
60	20	20	20	4	1	35.1	41.3	43.2	44.2	44.6	44.8
					2	32.4	37.8	40.1	41.2	41.3	41.5
					3	30.6	35.4	37.4	38.2	38.4	38.5
					4	29.9	34.4	36.3	37.1	37.2	37.3
					5	39.4	33.8	35.7	36.5	36.7	36.7

Table 6.2 presents the average objective function values for the different number of available artillery fire and enables the comparison of RCIP and RCIP-A to evaluate the utility of artillery fire. Results indicate that if DF can make use of one artillery fire, this provides an average of 15% increase in the objective function value. Even the marginal utility is decreasing for each artillery fire available, the average increase in the objective function value is meaningful from the radio communications network planning perspective.

6.4.3 Sensitiviy anlaysis for the radius of impact (η)

Generally, it is accepted that the radius of impact for unprotected infantry personnel is 100 meters, but in addition to the lethal effect, artillery fire causes a material effect that destroys or damages the equipment. Besides, when we consider the electronic devices mounted on soft-skinned vehicles, it is acceptable to consider the radius of impact between 400 and 600 meters. Thus, Table 6.3 presents the solution times and the optimal solution values of RCIP-A when the radius of impact of artillery fire value (η) changes between 400 and 600 meters.

The results indicate that as the value of η is increases, optimal solution values increase almost linearly with a decreasing marginal utility. However, considering the high research and development and production cost of new ammunition with a higher radius of impact, results show that utilizing one or two more artillery fire provides almost the same or more benefit. For instance, when we consider the first line of the table optimal solution for 1 artillery fire is 30.2 when $\eta = 400\ m$ and 32.4 when $\eta = 450\ m$. But, using the same ammunition with $\eta = 400\ m$ and increasing s to 3 we obtain the same optimal solution.

Table 6.3: Sensitivity analysis for the radius of impact (η)

							$\eta = 400\ m.$		$\eta = 450m.$		$\eta = 500m.$		$\eta = 550m.$		$\eta = 600m.$	
R	T	J	L	p	q	s	Sol. Time	Obj. Value	Sol. Time	Obj. Value	Sol. Time	Obj. Value	Sol. Time	Obj. Value	Sol. Time	Obj. Value
60	20	20	20	4	3	1	2.7	30.2	4.4	32.4	6.1	34.8	9.2	35.5	6.1	36.5
						2	5.5	31.8	6.9	34.5	9.9	36.9	12.3	37.7	9.9	38.6
						3	3.6	32.4	3.4	35.1	5.2	37.5	5.5	38.6	5.2	39.4
						4	4.2	32.7	4.4	35.6	4.6	37.9	6.4	38.9	4.6	39.5
80	25	25	25	5	4	1	57.1	41.8	80.5	45.1	138	47.5	184	48.8	121.2	49.7
						2	79.5	45.5	175.3	48.1	750	50.1	954.5	51.8	654.8	52.3
						3	31.2	48.2	105.7	50.5	2010.4	52.3	3598.3	53.5	1265.4	54.4
						4	27	50.1	51.9	52.3	110.7	54.1	264.7	55.1	245.3	55.9
						5	28	50.9	24.5	53.5	70.2	55.3	164.3	55.9	69.4	56.5

Figure 6.4 depicts the change in the objective function depending on the radius of impact of the artillery fire. We observe objective function value increase with a decreasing marjinal utility per 50 meters increase in the radius of impact. We also observe the same pattern of increase in the objective function value for each increment in the number available artillery fire (s) to be used.

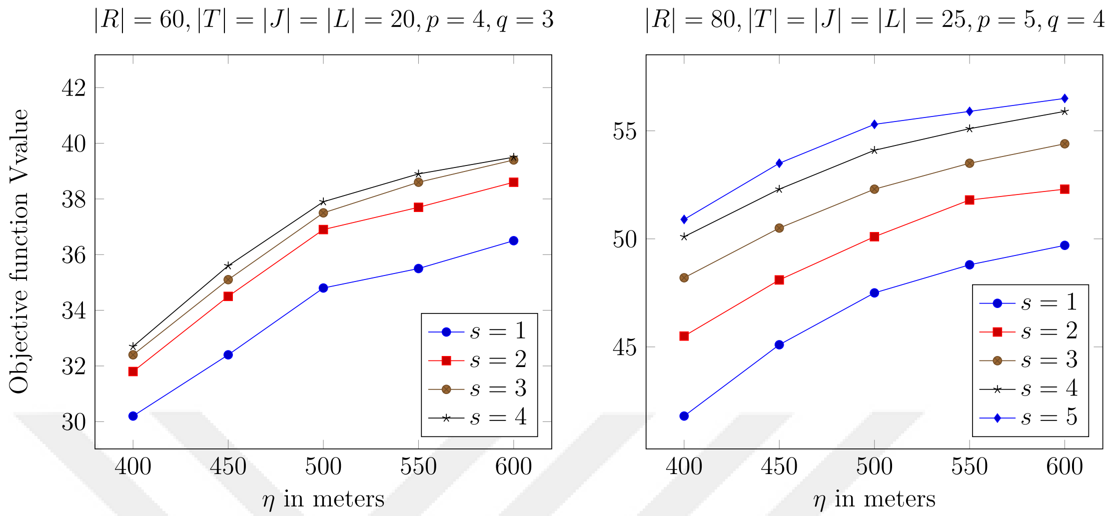


Figure 6.4: Objective function value for different radius of impact η values

From the computational perspective, we observe that solution times are increasing as η increases and obtain the highest solution times when $\eta = 550$. This derives from the fact that as η increases, the number of alternative artillery fires that cover a single jammer location increases and this complicates the solution of both the master and especially the bilevel subproblem.

6.5 Summary

In this chapter, we define RCIP-A that enables artillery fire capabilities of friendly forces to contribute to radio communications planning. This comprehensive approach helps to realize one of the essential principles and concepts of today's modern warfare, which is the comprehensive utilization of multiple, contested battlefield function domains simultaneously.

We formulate RCIP-A as a trilevel mathematical programming model that identifies optimal transmitter locations of the limited number of transmitters and optimal artillery fire planning to support radio communications planning considering an intelligent adversary that identifies optimal jammer location sites as his electronic warfare assets.

We propose a nested decomposition method to solve RCIP-A optimally. Additionally, solutions and insights obtained from an illustrative example and the results of limited computational studies executed by the proposed solution method are presented.

Admittedly, the work presented in this chapter needs improvement in terms of enhancement to the solution method to be able to solve larger instances of the problem defined by different scenarios. Doing so will enable additional computational studies to obtain more tactical insights. Another future line of development is to develop a more realistic survivability probability distribution function based on fragmentation directions for different types of artillery weapons and ammunition.

Chapter 7

Conclusion

7.1 Remarks

Radio communications, with its wireless nature and practicality to meet maneuverability, is generally well accepted as a primary means and backbone for tactical communications. It plays a vital role in the command and control of military units, and therefore it should be designed and planned to assure continuous, secure, and resilient communication service to widely dispersed and highly mobilized military units operating at extended distances within the battlefield.

Over the years, radio communications has evolved due to significant progress in technology. However, consistent with this progress, potential vulnerabilities that may offset the advantages and capabilities are identified. More specifically, jamming has became a frequently but effective electronic attack instrument to prevent the transfer of information and ultimately disable the opponent's communication network. Therefore, attaining reliable, secure and continuous radio communication obligates planners to provide a holistic approach that optimizes the communication network of friendly forces while incorporating the EW and particularly EA assets of the adversary.

To this end, we have presented and solved RCIP to integrate probable but rational adverse effects of an intelligent adversary's jamming attacks into the radio communications network planning of our friendly forces. To present a more realistic framework, we have incorporated the probabilistic jamming to signal ratio and introduced the probabilistic variant, P-RCIP, to include the possible deviation in the received signal power due to geographical obstacles on the battlefield

Adopting a game theoretic approach, RCIP and P-RCIP have been formulated as binary bilevel programming problems and solved by decomposition. In order to improve the solution times, we have proposed three enhancements that utilize the dominance relations between possible location sites, preprocessing, and initial starting heuristics. In anticipation of different probable subsequent phases of military operations, we have presented four different scenarios and investigated the computational efficacy of the proposed solution methods with different parameters based on these scenarios.

We have showed that our treatment of formulating the problem with a bilevel formulation that incorporates the adversarial effect yields considerably better decisions when compared against two fast solution methods, a traditional one in the location literature and one that mimics the decision making process in practice.

We have provided some useful tactical insights on transmitter and jammer location decisions by analyzing optimal solutions for different number of available transmitters and jammers in each scenario. The results have showed that even though the optimal locations obtained in Scenario 1 are consistent with the expected layout, for other scenarios that reflect the subsequent phases of a military operation, solutions obtained by RCIP outperform the experiential results, highlighting the value of our treatment of RCIP especially in complex military situations.

We also have presented sensitivity analyses for problem parameters to provide invaluable tactical insights in military communication network design.

Finally, we investigate the incorporation of limited artillery assets into communications planning by formalizing RCIP with Artillery (RCIP-A) as a trilevel optimization problem and propose a nested decomposition method as an exact solution methodology. We have observed useful tactical insights on how to utilize artillerry fire by solving RCIP-A on different scenarios. Additionally, sensitivity analysis on the radius of impact of artillery fire provided valuable information on artillery fire planning.

7.2 Future Research

Considering that armies are not willing to use a wide variety of transmitters and jammers, we assumed that all the transmitters and jammers are mutually identical in our study. However, as a future research direction, our treatment can be adapted not only to include non-identical transmitters and jammers having different technical and tactical capabilities but also to incorporate sophisticated jammers and transmitters that are far more proficient thanks to new emerging technologies. For instance, rather than using constant jamming, which is energy inefficient, easy to detect but also easy to launch and disruptive, deceptive, random or reactive jammers that can perform advanced jamming techniques may also be considered as a future research direction. The modeling framework will have to be enhanced to consolidate this type of jammers, which are harder to detect and more energy efficient. Finally, integrating transmitters that are capable of using state-of-the-art approaches to avoid jamming attacks such as channel and frequency hopping, jam mapping, spatial retreat, and hybrid techniques may certainly enrich the insights of such a research direction.

In addition to evaluating transmitters and jammers with different type and specifications, a fruitful research area can be to evaluate them not in a static manner we evaluated in our models but in a setting where these transmitters, jammers and even receivers are considered as mobile in the direction of the development of the operation. Reflecting the dynamism on the battlefield, not only the location but also the relocation decisions of transmitters and jammers may be included in the analyses. Additionally, rather than limiting location decisions to take place on a discrete set of all possible transmitter and jammer locations, RCIP can be extended to reflect any possible continuous location on the battlefield.

An important assumption in RCIP, P-RCIP, and RCIP-A was the sequential game between DF and AT, with full knowledge of the previous action of the opponent. To extend this research, this assumption can be relaxed by considering the simultaneous acts of both sides of the conflict to obtain a nash equilibrium. Furthermore, partial or no knowledge on the course of actions of opponent forces can be investigated to evaluate the value of correct information, named intelligence in the warfare framework, based on the observation of the rivalry's assets.

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