TEMPERATURE DISTRIBUTION FOR ROTATING SOLID CYLINDERS HEATED BY RADIATION

by

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Submitted to the Faculty of the Graduate School
of Engineering in Partial Fulfillment
of the Requirements of

MASTER OF SCIENCE

in

MECHANICAL ENGINEERING

Bogazici University

1981

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ACKNOWLEDGEMENT

The author wishes to express her sincere thanks to her thesis supervisor, Prof. Dr. Ahmet Rasim Büyüktür who guided this study and to Dr. Emre Aksan who made this final text possible.

Thanks are due to Prof. Dr. Akın Tezel for his valuable suggestions.

A note of appreciation is due to Doç. Dr. Aksel Öztürk for his patience, ever unending help and recommendations throughout the study.

Thanks also go to all the authors friends in the Environmental Engineering Group for their encouragement and understanding.

A word of appreciation to Gülsen Karşit for her expert typing of the manuscript.

ABSTRACT

The temperature distribution in rotating circular cylinders heated by radiation has been studied in this thesis. Analytical solutions have been obtained for solid cylinders rotating with constant angular velocity, with incoming radiation taken perpendicular to the axis of rotation and incident on one side. Thermal equilibrium has been achieved by means of reradiation to the surrounding opposite half-space. The treatment employed makes use of the method of separation of variables with linearized boundary conditions.

Results of the study agree very well with works cited where similar problems have been treated employing different mathematical techniques.

ÖZET

Bu çalışmada, radyasyona tabi, dönen silindirlerdeki sıcaklık dağılımı incelenmiştir. Analitik sonuçlar, kendi ekseni etrafında sabit bir açısal hızla, ışıma yönüne dik bir eksen etrafında dönen katı silindirler için elde edilmiştir. Diferansiyel denklemin lineer hale getirilmiş sınır şartlarına göre çözümü, değişkenlere ayırma metodu ile bulunmuştur.

Sonuçlar benzer problemlerin sonuçları ile karşılaştırılmış ve büyük ölçüde uygunluk gösterdiği saptanmıştır.

CONTENTS

LIST OF FIGURES

NOMENCLATURE

a	Thermal Diffusivity (ft^2h^{-1})
a_n, b_n	Real series coefficients
B_n	Fourier series coefficients for radiation input functions
k	Thermal conductivity (Btu $ft^{-1}h^{-1}R^{-1}$)
q_o''	Density of Radiation (Btu $ft^{-2}h^{-1}$)
r	Radial coordinate (ft)
R	Radius of cylinder, (ft)
t, t'	Time, (h)
T	Solid cylinder temperature (^oR)
T_o	Constant temperature in equation (4.11)(^oR)

GREEK SYMBOLS

α	Absorptivity of surface exposed to radiation
β	$\alpha q_o''R/(T_o k)$
γ	$4\sigma\varepsilon T_o^3 R/k$
ε	Emissivity
θ	Angular coordinate fixed in space, (rad)
λ	$\omega R^2/a$
ψ^*	Temperature variable in equation (4.11)(^oR)

ψ	Temperature variable in equation (4.13) (^{o}R)
σ	Stefan-Boltzmann constant (0.171×10^{-8} Btu ft^{-2} $h^{-1}R^{-1}$)
ϕ	Angular coordinate fixed in the cylinder
ω	Speed of angular rotation (rad h^{-1})

CHAPTER 1

INTRODUCTION

This thesis is concerned with the temperature distribution in a solid rotating cylinder exposed to incoming parallel radiation.

Literature, in recent years, contains studies of heat conduction in various types of bodies, with radiant energy interchange prescribed on the surface. The current emphasis on satellites and space vehicles has created an interest in the temperature distribution in rotating bodies exposed to solar radiation. Studies can be grouped in several categories such as the analysis of thin-walled bodies, thick-walled bodies and solid bodies.

In this work, a solid cylinder has been considered rotating with constant angular velocity "ω". This state implies that a balance has been attained between the total heat absorbed by the cylinder and the total heat re-radiated into space. The resulting temperature distribution is typical of a quasi-steady state, since each point of the cylinder fixed with respect to the incoming radiation would have a time-independent temperature. In other words the temperature distribution would vary periodically with time.

Although the differential equation leading to the temperature distribution is linear, the true radiation boundary condition turns out to be non-linear. Nonetheless, analytical solutions of the differential equation, have been obtained here for linearized boundary conditions. Expressing the governing equation in a rotating coordinate system and using separation of variables, the case has been reduced to an eigenvalue problem. Temperature distributions thus obtained are in very good agreement with results | 7 | published in similar works.

CHAPTER 2

LITERATURE SURVEY

The current emphasis on satellites and space vehicles has created an interest in the temperature distribution of various types of rotating bodies, especially in the past fifteen years, several analysis related to this subject have appeared in the literature. In general these papers consider the body to be placed in a vacuum and receiving radiant energy from a distant source, reradiating energy to a heat sink at absolute zero.

The problem of the solar heating of a rotating cylindrical shell has been considered first by Raynor and Charnes in |11|. Nichols |6| and Roberts |12| have individually obtained the approximate formulas for thin-walled solid cylinders, but neither considered the possibility of rotation.

In a closely related paper, Olmstead and Raynor |7|

have undertaken the problem of the rotating solid cylinder. They have obtained analytical solutions by the method of Green's functions. However, in this study, the solution has been examined only for the limiting cases of either very slow or very fast rotational speeds, indicating a need for further analysis of the same problem.

Ölçer |8| considered the problem of the solar heating of rotating space vehicles, and obtained expressions to unsteady temperature distributions by making use of eigen vector techniques.

In a similar analysis made by Raynor and Petrof, |10| for solar heating of rotating thick-walled cylinders, analytical solutions were found as series expressions in terms of orthogonal functions for the hollow cylinder with the adiabatic hole. The resulting temperature distributions were examined for various rotational speeds at the surface of the cylinder.

Arpacı |2| and Carslaw and Jaeger |3| explain in detail the formulation and solution methods of various types of problems in conduction heat transfer. Schneider |13| gives the transformation formulas for the problems which have moving heat sources.

CHAPTER 3

HEAT CONDUCTION EQUATION FOR ROTATING BODIES

3.1. GENERAL

The formulation of conduction phenomena can be defined so as to obtain the mathematical expression in light of the physics of the problem under consideration and to specify the initial and/or boundary conditions pertinent to the governing equation. The governing equation of a conduction problem can be obtained either by the mathematical interpretation of general formulation or by following, from the start, an individual formulation suitable to the problem. The latter method is especially appropriate for practical application of the study of heat conduction. Detailed exploration of methods of formulation can be found in |2| and |3|.

Heat conduction equation for the cylindrical coordinate system which can be obtained by either one of the above procedures is given as:

$$\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial T}{\partial r}) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} + \frac{u'''}{k} = \frac{1}{a}\frac{\partial T}{\partial t} \tag{3.1}$$

where

$$T \equiv T(r,\phi,z,t)$$

and

$$a = \frac{k}{\rho c_p}$$

Equation (3.1) is applicable for homogeneous isotropic solids and for frictionless incompressible fluids. For $u''' \equiv 0$, Equation (3.1) becomes:

$$\nabla^2 T = \frac{1}{a}\frac{\partial T}{\partial t} \tag{3.2}$$

3.2. TRANSFORMATION FORMULA FOR MOVING HEAT SOURCES

The approximate theory of moving heat sources has been considered by Sprarapen and Claussen and the exact analytical theory was developed by Rosenthal. Derivation of the transformation formula for moving heat sources is discussed by Schneider |13| for rectangular coordinates. The transformation formula from stationary to moving systems can be obtained for polar coordinates by applying the same procedure as in |13|.

In the stationary system, $T(r,\theta,z)$ the temperature must satisfy

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{a}\frac{\partial T}{\partial t} \tag{3.3}$$

If we define two new variables

$$\phi = \theta - \omega t \; ; \qquad t' = t$$

then

$$\frac{\partial \phi}{\partial \theta} = 1; \qquad \frac{\partial \phi}{\partial t} = -\omega \qquad \frac{\partial t'}{\partial \theta} = 0 \; ; \qquad \frac{\partial t'}{\partial t} = 1$$

$$\frac{\partial T}{\partial \theta} = \frac{\partial T}{\partial \phi}\frac{\partial \phi}{\partial \theta} + \frac{\partial T}{\partial t'}\frac{\partial t'}{\partial \theta} = \frac{\partial T}{\partial \phi} \; ; \qquad \frac{\partial^2 T}{\partial \theta^2} = \frac{\partial^2 T}{\partial \phi^2} \tag{3.4}$$

and

$$\frac{\partial T}{\partial t} = \frac{\partial T}{\partial \phi} \cdot \frac{\partial \phi}{\partial t} + \frac{\partial T}{\partial t'}\frac{\partial t'}{\partial t} = -\omega \frac{\partial T}{\partial \phi} + \frac{\partial T}{\partial t'} \tag{3.5}$$

or

$$\frac{\partial T}{\partial t} = -\omega \frac{\partial T}{\partial \phi} + \frac{\partial T}{\partial t'} \tag{3.6}$$

Substituting Equations (3.4) and (3.6) into Equation (3.3), we get,

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{a}\left(-\omega \frac{\partial T}{\partial \phi} + \frac{\partial T}{\partial t'}\right) \qquad (3.7)$$

An observer of the ϕ direction would notice a change in temperature of his surroundings, but he would notice no such change in temperature if he were stationed at a point on the moving θ axis. This condition of apparent steady state temperature has come to be known as the quasi-steady state and it is represented mathematically by $\frac{\partial T}{\partial t} = 0$ in the moving coordinate system.

Since this puts us in the moving coordinate system $\frac{\partial T}{\partial t'} = 0$ and

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\omega}{a}\frac{\partial T}{\partial \phi} = 0 \qquad (3.8)$$

Equation (3.8) is applicable as heat conduction equation for rotating bodies.

3.3 BOUNDARY CONDITIONS

The formulation of any specified problem can be realized either from the equation of conduction given by Equation (3.1) or from the mathematical interpetations of it in the light of the physics of the problem. These equations involve a partial differential equation in terms of the unknown temperature and their solution involves a number of integration constants. Therefore, an equal number of appropriate conditions in space and time is necessary in order to determine these constants. These conditions are initial and boundary conditions.

The boundary conditions specify the temperature on the heat flow situation at the boundaries of the region. The most frequently encountered boundary conditions are prescribed temperature, prescribed heat flux and heat transfer to the ambient by convection. According to the physics of the problem, combinations of the above can also appear as the boundary conditions of the problem |2,8|.

The boundary conditions for heat transfer problems involving the fourth-power radiation law, free convection and so on, are called nonlinear boundary conditions because they involve a power of temperature. Since this study is about the radiation heating of a solid cylinder, it will be useful to give some information about this type of boundary conditions.

Consider two isothermal surfaces A_1 and A_2 having the absolute temperatures T_1, T_2. It has been shown experimentally by Stefan and later proved thermodynamically by Boltzmann that the radiant heat flux q_{12} between the surfaces A_1 and A_2 can be expressed by the following equation as

$$q_{12} = \sigma \overline{F}_{12}(T_1^4 - T_2^4) \tag{3.9}$$

where σ is the Stefan-Boltzmann constant and $\overline{F}_{12}$ is the overall interchange factor

For an enclosure composed of two concentric, very long cylinders, $\overline{F}_{12}$ becomes |9|,

$$\overline{F}_{12} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)} \tag{3.10}$$

where A_1 and ε_1 are the area and emissivity of inner cylinder, respectively.

As $A_1/A_2 \rightarrow 0$, Equation (3.10) becomes,

$$\overline{F}_{12} = \varepsilon_1 \tag{3.11}$$

Equation (3.11) is applied to calculate the radiation energy loss from an object to its surroundings. When this object is in vacuum, the radiation energy loss would be

$$q = \sigma\varepsilon_1 T_1^4 = \sigma\varepsilon T^4 \tag{3.12}$$

To express the heat flux from the surfaces of a solid cylinder exposed to radiation by conduction and radiation, the required boundary condition can be written in the form,

$$\mp k\left(\frac{\partial T}{\partial n}\right)_\sigma = \sigma\varepsilon T^4 - q_n'' \tag{3.13}$$

where q_n'' is the radiation term in the direction of surface normal and plus or minus signs of the conduction term correspond to the direction of inward and outward normals, respectively.

CHAPTER 4

4.1. FORMULATION

4.1.1. Definition of the Problem

In this study, a solid cylinder is considered to be rotating with constant angular velocity about its geometric axis, the axis being perpendicular to the direction of incoming radiation. Thermal radiation is heating the body and thermal equilibrium is attained by means of reradiation to the surrounding space.

There is no conduction and convection effects with the surroundings at the surface of the body based on the assumption that the importance of radiation relative to convection is larger for low rates of convection. Internal reradiation is not considered on the basis of the conclusion reached in |9| that the body rotation influences the temperatures to a much greater degree than internal radiation. Also longitudinal conduction is neglected according to the analysis of an infinitely long cylinder. Therefore, the heat flow is radial and circumferential and the resulting temperature distribution is two dimensional. The body is considered to be diffusely emitting and diffusely reflecting. The thermal conductivity, k is the same in all direction, i.e., isotropic material.

In the light of this definition of the present problem the temperature distribution of the cylinder can be determined from the solution of the differential equation of temperature subject to the necessary boundary conditions.

4.1.2. <u>Differential Equation for Rotating Cylinder</u>

The cylindrical coordinate system (r,ϕ,z) fixed in the body is shown in Figure 1 with θ measured in a sense opposite to the rotation, ω. Taking diffusivity "a" as constant, the governing partial differential equation can be written in cylindrical coordinate system as,

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} = \frac{1}{a}\frac{\partial T}{\partial t} \qquad (4.1)$$

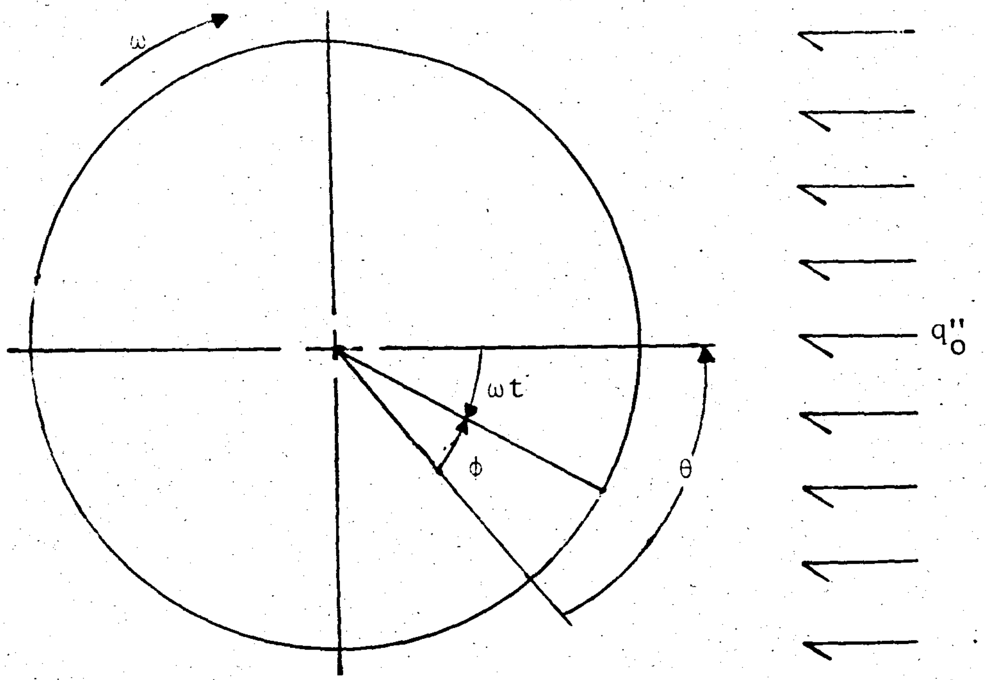

FIGURE 1. Schmatic Representation of the Cylindrical Co-ordinate System (r,ϕ,z) Fixed in the Body with θ Measured in a Sense Opposite to the Rotation ω.

A transformation that has been useful in solving problems with moving heat sources eliminates the explicit dependence of the temperature on time. As it is mentioned in the previous chapter, for polar coordinates this transformation is

$$\theta = \phi - \omega t \qquad t' = t \tag{4.2}$$

Fixed in a stationary frame of reference, the temperature becomes the quasi-steady temperature

$$T(r,\theta) = T(r,\phi,t) \tag{4.3}$$

characterized by

$$\frac{\partial T}{\partial t'} = 0 \tag{4.4}$$

Using Equations (4.2) - (4.4), Equation (4.1) becomes,

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\omega}{a}\frac{\partial T}{\partial \theta} = 0 \tag{4.5}$$

4.1.3. Boundary Conditions

The boundary conditions for the problem involve prescribed surface values for the heat flux. The external surface of the body is heated by the absorption of radiant energy from the distant source and suffers an energy loss by reradiation to space. The gradient of temperature is determined by the net rate of heat conduction per unit area with the local absorbed radiation by Lambert's cosine law and the reradiation governed by the Stefan-Boltzmann law. From Equation (3.13) we get,

$$k\frac{\partial T}{\partial r} + \sigma\varepsilon T^4 = \alpha q_o'' \, Cos^+(\phi - \omega t) \quad \text{on } r=R \tag{4.6}$$

where

α - the average absorptivity of the cylinder surface,

q_o'' - the radiant energy flux in a plane normal to the direction of incoming radiation

σ - Stefan-Boltzmann constant

ε - the average emissivity of the cylinder surface.

The Cosine function is defined

$$\mathrm{Cos}^{+}(\phi-\omega t) = \begin{cases} \mathrm{Cos}(\phi-\omega t) & -\frac{\Pi}{2} \leq (\phi-\omega t) \leq \frac{\Pi}{2} \\ 0 & \frac{\Pi}{2} \leq (\phi-\omega t) \leq \frac{3\Pi}{2} \end{cases} \qquad (4.7)$$

Boundary condition at the center of the cylinder is

$$T = \text{Finite} \qquad \text{at } r=0 \qquad (4.8)$$

When the transformation Equations (4.2) and (4.3) are introduced into the boundary conditions Equations (4.6) and (4.7) become

$$k\,\frac{\partial T}{\partial r} + \sigma\varepsilon T^4 = \alpha q_o''\mathrm{Cos}^{+}\theta \qquad \text{on } r=R \qquad (4.9)$$

$$\mathrm{Cos}^{+}\theta = \begin{cases} \mathrm{Cos}\theta & -\frac{\Pi}{2} \leq \theta \leq \frac{\Pi}{2} \\ 0 & \frac{\Pi}{2} \leq \theta \leq \frac{3\Pi}{2} \end{cases}$$

and

$$T = \text{Finite} \qquad \text{at } r=0 \tag{4.10}$$

In order to linearize the reradiation heat flux term in Equation (4.6), the following substitution is made,

$$\psi^* \equiv \frac{T-T_o}{T_o} \qquad \text{or} \qquad T = T_o(1 + \psi^*) \tag{4.11}$$

It is assumed that ψ^* is small compared to unity, and using Maclaurin's expansion, we get

$$T^4 = T_o^{\ 4}(1 + \psi^*)^4 \cong T_o^{\ 4}(1 + 4\psi^* + 6\psi^{*2} + \ldots)$$

or

$$T^4 \cong T_o^{\ 4}\ (1 + 4\psi^*) \tag{4.12}$$

$$T^4 \cong 4T_o^{\ 4}\ (1/4 + \psi^*) = 4T_o^{\ 4}\psi \tag{4.13}$$

where

$$\psi = \frac{1}{4} + \psi^*$$

Substituting the value of ψ into Equation (4.11), yields

$$T = T_o\ (\tfrac{3}{4} + \psi) \tag{4.14}$$

Then Equation (4.9) becomes

$$\frac{\partial\psi}{\partial r} = \frac{\alpha q_o''}{T_o k} \mathrm{Cos}^{+}\theta - \frac{4\sigma\varepsilon T_o^{\,3}}{k}\psi \tag{4.15}$$

Introducing Equation (4.14) into Equation (4.5), the differential equation becomes,

$$\frac{\partial^2\psi}{\partial r^2} + \frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\theta^2} + \frac{\omega}{a}\frac{\partial\psi}{\partial\theta} = 0$$

Finally, the complete formulation of the problem in ψ becomes

$$\frac{\partial^2\psi}{\partial r^2} + \frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\theta^2} + \frac{\omega}{a}\frac{\partial\psi}{\partial\theta} = 0 \tag{4.16}$$

and subject to the boundary conditions,

$$\frac{\partial\psi}{\partial r} = \frac{\alpha q_o''}{T_o k} \mathrm{Cos}^{+}\theta - \frac{4\sigma\varepsilon T_o^{\,3}}{k}\psi \qquad \text{on } r=R \tag{4.17}$$

and

$$\psi = \text{Finite} \qquad \text{on } r=0 \tag{4.18}$$

In addition, the following non-dimensional quantities are introduced

$$\rho \equiv \frac{r}{R} \tag{4.19}$$

$$\beta \equiv \frac{\alpha q_o'' R}{T_o k} \tag{4.20}$$

$$\gamma \equiv \frac{4\sigma\varepsilon T_o^{3} R}{k} \tag{4.21}$$

$$\lambda \equiv \frac{\omega R^2}{a} \tag{4.22}$$

Substituting Equations (4.19) to (4.22) into Equations (4.16) to (4.18), the formulation of the problem in non-dimensional quantities becomes,

$$\frac{\partial^2\psi}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial\psi}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2\psi}{\partial\theta^2} + \lambda\frac{\partial\psi}{\partial\theta} = 0 \tag{4.23}$$

$$\left.\frac{\partial\psi}{\partial\rho} + \gamma\psi\right|_{\rho=1} = \beta \mathrm{Cos}^{+}\theta \tag{4.24}$$

$$\psi\Big|_{\rho=0} = \text{Finite} \tag{4.25}$$

4.2. SOLUTION OF THE PROBLEM FOR THE TEMPERATURES IN THE SOLID CYLINDER

4.2.1. Separation of Variables

After the formulation in non-dimensional quantities is completed, a solution is sought by the method of separation of variables.

Assume the existence of a product solution

$$\psi(\rho,\theta) = R(\rho)\ \Theta\ (\theta) \tag{4.26}$$

Introducing Equation (4.26) into Equation (4.23), we obtain

$$(R'' + \frac{1}{\rho} R')\ \Theta + \frac{1}{\rho^2} R\ \Theta'' + \lambda R\ \Theta' = 0 \tag{4.27}$$

Rearranging Equation (4.27), yields

$$\rho^2 \frac{R''}{R} + \rho \frac{R'}{R} = - \frac{\Theta''}{\Theta} - \lambda\rho^2 \frac{\Theta'}{\Theta} \tag{4.28}$$

The left-hand side of this equation is independent of θ while the right-hand side is a function of ρ and θ. Since both sides of Equation (4.28) are not independent of either variable they cannot be equal to a constant. Because of the equality of both sides to each other, the right-hand side of this equation must yield to a funcion of ρ only. Accordingly, the functions $\frac{\Theta''}{\Theta}$ and $\frac{\Theta'}{\Theta}$ are necessarily constants. On the basis of this conclusion, assume

$$\frac{\Theta''}{\Theta} = -\nu^2 \tag{4.29}$$

and

$$\frac{\Theta'}{\Theta} = \kappa \tag{4.30}$$

From differentiation of Equation (4.30) with respect to θ, we obtain

$$\Theta'' = \kappa\,\Theta' \tag{4.31}$$

Substituting Equation (4.30) into Equation (4.31) and comparing with Equation (4.29), a relation between κ and ν^2 can be obtained

$$\kappa^2 = -\nu^2 \tag{4.32}$$

or equally

$$\kappa = \mp i\nu \tag{4.33}$$

Consider the possible two cases:

CASE 1

Let,

$$\kappa = \frac{\Theta'}{\Theta} = -i\nu \tag{4.34}$$

A solution satisfying this value of κ is,

$$\Theta = A e^{-i\nu\theta} \tag{4.35}$$

where A being a complex number.

The above separation constants so introduced, the differential equation in ρ direction becomes,

$$R'' + \frac{1}{\rho} R' - (i\nu\lambda + \frac{\nu^2}{\rho^2}) R = 0 \tag{4.36}$$

and the complex solution of this equation can be written in the forms

$$R(\rho) = BJ_\nu(i\sqrt{i\lambda\nu}\,\rho) + CY_\nu(i\sqrt{i\lambda\nu}\,\rho) \tag{4.37}$$

where B and C may take complex values.

For a given value of ν, the product solution is,

$$T_{\nu}^{-}(\rho,\theta) = e^{-i\nu\theta}\left|B^{-}J_{\nu}(i\sqrt{i\lambda\nu}\ \rho) + C^{-}Y_{\nu}(i\sqrt{i\lambda\nu}\ \rho)\right. \quad (4.38)$$

CASE 2

Consider the second case, i.e.,

$$\kappa = \frac{\dot{\Theta}}{\Theta} = i\nu \quad (4.39)$$

A solution satisfying Equation (4.39) and Equation (4.29) is of the form

$$\Theta = A\ e^{i\nu\theta} \quad (4.40)$$

where A being a complex number.
Then Equation (4.36) becomes

$$\rho^2 \ddot{R} + \rho\dot{R} + \left|i\lambda\nu\rho^2 - \nu^2\right| R = 0 \quad (4.41)$$

and the complex solution to the differential Equation (4.41) can be written in the form

$$R(\rho) = BJ_{\nu}(\sqrt{i\lambda\nu}\ \rho) + CY_{\nu}(\sqrt{i\lambda\nu}\ \rho) \quad (4.42)$$

where B and C may take complex values.
For a given value of v the product solution is

$$T_v^{+}(\rho,\theta) = e^{iv\theta}|B^{+}J_v(\sqrt{i\lambda v}\ \rho) + C^{+}Y_v(\sqrt{i\lambda v}\ \rho)| \quad (4.43)$$

The linear combination of these solutions is the solution of this problem.

$$\psi(\rho,\theta) = \sum_{v=-\infty}^{\infty} T_v^{+}(\rho,\theta) + T_v^{-}(\rho,\theta) \quad (4.44)$$

4.2.2. Solution for the Solid Cylinder

Since periodic behavior is required in θ-direction, v values must be integers, i.e., v=n. Also requirement of finite temperature for ρ=0 leads to the conclusion that the complex constants C^{-} and C^{+} in Equations (4.38) and (4.43) should be set equal to zero. Therefore, the general solution to the problem can be written in the following complex form,

$$\psi(\rho,\theta) = \sum_{n=-\infty}^{\infty} e^{-in\theta}|B_n^{-}J_n(i\sqrt{i\lambda n}\rho)| + e^{in\theta}|B_n^{+}J_n(\sqrt{i\lambda n}\rho)| \quad (4.45)$$

or

$$\psi(\rho,\theta) = \sum_{n=-\infty}^{\infty} e^{-in\theta}|B_n^- J_n(i\sqrt{i\lambda n}\rho)| + e^{in\theta}|B_n^+ J_n(\sqrt{i\lambda n}\rho)| \quad (4.4\ )$$

Now, in this expression if we set -n instead of n in the second term, which also changes the argument $(i\lambda n)^{\frac{1}{2}}$ to $i(i\lambda n)^{\frac{1}{2}}$, we obtain

$$\psi(\rho,\theta) = \sum_{n=-\infty}^{\infty} B_n^- e^{-in\theta} J_n(i\sqrt{i\lambda n}\ \rho)$$

$$+ \sum_{n=-\infty}^{\infty} B_n^+ e^{-in\theta} J_{-n}(i\sqrt{i\lambda n}\ \rho) \quad (4.47)$$

Using the relation

$$J_{-n}(x) = (-1)^n J_n(x) \quad \text{for n integer}$$

in the Equation (4.47), the general solution for the solid cylinder becomes,

$$\psi(\rho,\theta) = \sum_{n=-\infty}^{\infty} B_n^* e^{-in\theta} J_n(i\sqrt{i\lambda n}\ \rho) \quad (4.48)$$

where $B_n^* = (B_n^- + (-1)^n B_n^+)$

or

$$\psi(\rho,\theta) = B_o^* + \sum_{\substack{n=-\infty \\ \neq 0}}^{\infty} B_n^* J_n(i\sqrt{i\lambda n}\ \rho) e^{-in\theta} \quad (4.49)$$

4.2.3. Determination of the Constant

By means of the outside boundary condition the remaining constant of integration is determined. Along the boundary we have

$$\frac{\partial\psi}{\partial\rho} + \gamma\psi \Big|_{\rho=1} = \beta \mathrm{Cos}^{+}\theta \tag{4.50}$$

$$\mathrm{Cos}^{+}\theta = \begin{cases} \mathrm{Cos}\theta & -\frac{\Pi}{2} \leq \theta \leq \frac{\Pi}{2} \\ 0 & \frac{\Pi}{2} \leq \theta \leq \frac{3\Pi}{2} \end{cases}$$

Accordingly

$$\frac{\partial\psi}{\partial\rho} = \sum_{\substack{n=-\infty \\ \neq 0}}^{\infty} B_n^{*}(n\lambda)^{\frac{1}{2}} i^{3/2} J_n'(n\lambda)^{\frac{1}{2}} i^{3/2}\rho) e^{-in\theta} \tag{4.51}$$

Substituting Equations (4.51) and (4.49) into Equation (4.50) yields,

$$\gamma B_o^{*} + \sum_{\substack{n=-\infty \\ \neq 0}}^{\infty} B_n^{*} \Big| (n\lambda)^{\frac{1}{2}} i^{3/2} J_n'((n\lambda)^{\frac{1}{2}} i^{3/2}) + \gamma J_n((n\lambda)^{\frac{1}{2}} i^{3/2}) \Big| e^{-in\theta} = \beta \mathrm{Cos}^{+}\theta \tag{4.52}$$

$$\gamma B_o^{*} + \sum_{\substack{n=-\infty \\ \neq 0}}^{\infty} B_n e^{-in\theta} = \beta \mathrm{Cos}^{+}\theta \tag{4.53}$$

where

$$B_n = B_n^* \left| (n\lambda)^{\frac{1}{2}} i^{3/2} J_n'((n\lambda)^{\frac{1}{2}} i^{3/2}) + \gamma J_n((n\lambda)^{\frac{1}{2}} i^{3/2}) \right| \quad (4.53a)$$

Introducing $e^{-in\theta} = \text{Cos}n\theta - i\text{Sin}n\theta$ into Equation (4.53) yields,

$$\gamma B_o^* + \sum_{n=1}^{\infty} \left| (B_n + B_{-n})\text{Cos}n\theta - i(B_n - B_{-n})\text{Sin}n\theta \right| = \beta\text{Cos}^+\theta \quad (4.54)$$

or

$$\gamma B_o^* + \sum_{n=1}^{\infty} \left| a_n\text{Cos}n\theta + b_n\text{Sin}n\theta \right| = \beta\text{Cos}^+\theta \quad (4.55)$$

where

$$a_n = B_n + B_{-n}$$

$$b_n = -i\left| B_n - B_{-n} \right|$$

or

$$B_n = \frac{1}{2}(a_n + ib_n)$$

$$B_{-n} = \frac{1}{2}(a_n - ib_n) \quad (4.56)$$

Equation (4.55) is the complete Fourier series representation of $\beta\text{Cos}^+\theta$ with complex coefficients.

$$\gamma B_o^* = \frac{\beta}{\Pi} \quad (4.57)$$

$$a_n = \begin{cases} \frac{2\beta}{\Pi} \frac{(-1)^{n/2+1}}{n^2-1} & n \text{ even} \\ 0 & n \text{ odd} \\ \frac{\beta}{2} & |n| = 1 \end{cases}$$

$b_n = 0$ for all n.

Integrals are given in detail in Appendix A. Therefore, B_n and B_{-n} becomes,

$$B_n = \frac{a_n}{2}$$
$$B_{-n} = \frac{a_n}{2} \tag{4.58}$$

or

$$B_n = B_{-n} = \begin{cases} \frac{\beta}{2} \frac{(-1)^{\frac{n+2}{2}}}{n^2-1} & n \text{ even} \\ 0 & n \text{ odd} \\ \frac{\beta}{4} & |n| = 1 \end{cases} \tag{4.59}$$

Substituting the value of B_n into Equation (4.53a), Equation (4.49) becomes,

$$\psi(\rho,\theta) = \frac{\beta}{\Pi\gamma} + \sum_{\substack{n=-\infty \\ \neq 0}}^{\infty} \frac{B_n J_n((n\lambda)^{\frac{1}{2}} i^{3/2} \rho) \quad e^{-in\theta}}{(n\lambda)^{\frac{1}{2}} i^{3/2} J_n'((n\lambda)^{\frac{1}{2}} i^{3/2}) + \gamma J_n((n\lambda)^{\frac{1}{2}} i^{3/2})} \tag{4.60}$$

Using this result in temperature expression, Equation (4.60) can be represented in a new form as follows:

$$\psi(\rho,\theta) = \frac{\beta}{H\gamma} + \sum_{n=1}^{\infty} B_n \phi_n(k_n;\rho) e^{-in\theta} + \sum_{n=1}^{\infty} B_{-n} \phi_{-n}(k_{-n};\rho) e^{in\theta} \tag{4.61}$$

where ϕ_n is defined as

$$\phi_n = \frac{J_n(k_n\rho)}{k_n\ J_n'(k_n) + \gamma J_n(k_n)} \tag{4.62}$$

and

$$k_n = i\sqrt{i\lambda n}$$

Noting that

$$\begin{aligned} B_n &= B_{-n} \\ \overline{k_n} &= \overline{i\sqrt{i\lambda n}} = \sqrt{i\lambda n} \\ k_{-n} &= i\sqrt{i\lambda(-n)} = -\sqrt{i\lambda n} = -\overline{k_n} \end{aligned} \tag{4.63}$$

and also

$$\begin{aligned} J_n(-\overline{k}_n\rho) &= (-1)^n J_n(\overline{k}_n\rho) \\ J_n'(-\overline{k}_n\rho) &= (-1)^n J_n'(\overline{k}_n\rho) \\ J_{-n}(\overline{k}_n\rho) &= (-1)^n J_n(\overline{k}_n\rho) \end{aligned} \tag{4.64}$$

It can be shown that

$$\phi_{-n}(k_{-n};\rho) = \frac{J_n(\bar{k}_n\rho)}{\bar{k}_n J_n'(\bar{k}_n) + \gamma J_n(\bar{k}_n)} = \overline{\phi_n(k_n;\rho)} \qquad (4.65)$$

The complex exponential form of the series is then,

$$\psi(\rho,\theta) = \frac{\beta}{\Pi\gamma} + \sum_{n=1}^{\infty} B_n \left| \phi_n e^{-in\theta} + \overline{\phi_n} e^{-in\theta} \right| \qquad (4.66)$$

If ϕ_n and $\overline{\phi_n}$ are defined as follows:

$$\phi_n = \frac{a_n + ib_n}{2} \qquad \text{and} \qquad \overline{\phi_n} = \frac{a_n - ib_n}{2}$$

Then Equation (4.66) becomes

$$\psi(\rho,\theta) = \frac{\beta}{\Pi\gamma} + \sum_{n=1}^{\infty} B_n \left| a_n \mathrm{Cos} n\theta + b_n \mathrm{Sin} n\theta \right| \qquad (4.67)$$

where

$$a_n = \left| \phi_n + \overline{\phi_n} \right| = 2\mathrm{Re}\{\phi_n\}$$

$$b_n = -i \left| \overline{\phi_n} - \phi_n \right| = -2\mathrm{Im}\{\phi_n\}$$

$$B_n = \begin{cases} \dfrac{\beta}{\Pi} \dfrac{(-1)^{\frac{n+2}{2}}}{n^2-1} & n \text{ even} \\ 0 & n \text{ odd} \\ \dfrac{\beta}{4} & |n| = 1 \end{cases}$$

Besides that, the relationship between the complex function $J_n((n\lambda)^{\frac{1}{2}}\rho)$ and the real functions, $ber_n((n\lambda)^{\frac{1}{2}}\rho)$ and $bei_n((n\lambda)^{\frac{1}{2}}\rho)$ is

$$J_n((n\lambda)^{\frac{1}{2}}i^{3/2}\rho) = ber_n((n\lambda)^{\frac{1}{2}}\rho) + ibei_n((n\lambda)^{\frac{1}{2}}\rho) \quad (4.68)$$

from which follows

$$(n\lambda)^{\frac{1}{2}}i^{3/2}J_n'((n\lambda)^{\frac{1}{2}}i^{3/2}\rho) = (n\lambda)^{\frac{1}{2}}|ber_n'((n\lambda)^{\frac{1}{2}}\rho) + ibei_n'((n\lambda)^{\frac{1}{2}}\rho)| \quad (4.69)$$

where the prime indicates differentiation with respect to argument.

Substituting Equations (4.68) and (4.69) first into Equation (4.62) and then into Equation (4.67), one finds,

$$\psi(\rho,\theta) = \frac{\beta}{\Pi\gamma} + 2\sum_{n=1}^{\infty} B_n(a_n(\rho)\text{Cos}n\theta - b_n(\rho)\text{Sin}n\theta) \quad (4.70)$$

where

$$a_n(\rho) = \frac{1}{\Omega}\,|ber_n(n\lambda)^{\frac{1}{2}}\rho\,|(n\lambda)^{\frac{1}{2}}ber_n'(n\lambda)^{\frac{1}{2}} + \gamma ber_n(n\lambda)^{\frac{1}{2}}| + bei_n(n\lambda)^{\frac{1}{2}}\rho\,|(n\lambda)^{\frac{1}{2}}bei'_n(n\lambda)^{\frac{1}{2}} + \gamma bei_n(n\lambda)^{\frac{1}{2}}|| \quad (4.71)$$

$$b_n(\rho) = -\frac{1}{\Omega}\left| ber_n(n\lambda)^{\frac{1}{2}}\rho \left| (n\lambda)^{\frac{1}{2}} bei_n'(n\lambda)^{\frac{1}{2}} + \gamma bei_n(n\lambda)^{\frac{1}{2}} \right| - bei_n(n\lambda)^{\frac{1}{2}}\rho \left| (n\lambda)^{\frac{1}{2}} bei_n'(n\lambda)^{\frac{1}{2}} + \gamma ber_n(n\lambda)^{\frac{1}{2}} \right| \right| \qquad (4.72)$$

and

$$\Omega = \left| (n\lambda)^{\frac{1}{2}} ber_n'(n\lambda)^{\frac{1}{2}} + \gamma ber_n(n\lambda)^{\frac{1}{2}} \right|^2 + \left| (n\lambda)^{\frac{1}{2}} bei_n'(n\lambda)^{\frac{1}{2}} + \gamma bei_n(n\lambda)^{\frac{1}{2}} \right|^2 \qquad (4.73)$$

Then the actual temperature distribution can be obtained by inserting Equation (4.69) into Equation (4.14)

$$T = T_o\left(\frac{3}{4} + \frac{\beta}{\Pi\gamma} + 2\sum_{n=1}^{\infty} (a_n(\rho)\,\mathrm{Cos}\,n\theta - b_n(\rho)\,\mathrm{Sin}\,n\theta)\right) \qquad (4.74)$$

The reference temperature T_o has not yet been specified. A logical choice for T_o is the value of temperature at the centerline. Since the temperature distribution given by Equation (4.74) is based on the linearization which assumes that variation about T_o is small, T_o value can be found easily from Equation (4.74) as

$$T_o = T(0,\theta) = T_o\left\{ \frac{3}{4} + \frac{\beta}{\Pi\gamma} \right\} \qquad (4.75)$$

Equation (4.75) gives us,

$$\frac{1}{4} = \frac{\beta}{\Pi\gamma} \tag{4.76}$$

Substituting for β and γ from Equations (4.20) and (4.21), respectively gives the result

$$T_o = \left(\frac{\alpha q_o''}{\sigma\varepsilon\Pi}\right)^{\frac{1}{4}} \tag{4.77}$$

With the use of Equation (4.77), the expression for γ and β become

$$\gamma = \frac{R}{k}\left(\frac{256}{\Pi^3}\sigma\varepsilon\alpha^3\ q_o''^3\right)^{\frac{1}{4}} \tag{4.78}$$

$$\beta = \frac{\Pi}{4}\gamma \tag{4.79}$$

Then the expression for the temperature distribution with the use of Equation (4.76) and (4.77) become

$$T(\rho,\theta) = T_o\{1 + \gamma\frac{\Pi}{8}\ |a_1(\rho)\text{Cos}\theta - b_1(\rho)\text{Sin}\theta +$$

$$\frac{\gamma}{2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{4n^2-1}\ |a_{2n}(\rho)\text{Cos}2n\theta - b_{2n}(\rho)\text{Sin}2n\theta|\} \tag{4.80}$$

where the quantities T_o, γ, $a_n(\rho)$ and $b_n(\rho)$ are given by Equations (4.77), (4.78), (4.71) and (4.72) respectively.

CHAPTER 5

5.1. EVALUATION OF THE SOLUTION

The initial step in the evaluation of the solution consists in replacing the Bessel function derivatives with different orders of the same function according to the equalities |1-5|.

$$2z'_\nu(x) = z_{\nu-1}(x) - z_{\nu+1}(x)$$

$$xz'_\nu(x) = \nu z\ (x) - xz_{\nu+1}(x) \tag{5.1}$$

Although the real and imaginary parts of J_n can be resolved into Kelvin functions, the tabulated values of these functions for order other than n=0 and n=1 are not available for argument values x>10. Tables for Kelvin functions only for orders zero and one are given by

Abramowitz |1| and these functions for orders $0 \leq n \leq 5$ are given by Mclachlan |5| for argument values limited to $x \leq 10$. Therefore, instead of using tabulated values of these functions, examination of behavior of these functions according to the argument is preferred.

In evaluating the general solution, it is difficult to justify a limitation of the argument range, because the arguments are functions not only of index n, but also thermal diffusivity, radius and speed of angular rotation. Therefore, it is necessary to the calculation of a single value of temperature to determine the Bessel functions of many orders and for a like number of argument values.

The first step in evaluation of the solution is the consideration of temperatures in the solid cylinder for the case of no rotation which are determined by considering the behavior of the series coefficients as ω, or equivalently λ, as it approaches zero. It is appropriate to consider the small argument approximation for the Bessel function |1|,

$$J_n(x) \cong \frac{x^n}{(2^n\, n!)} \quad (x \to 0) \qquad (5.2)$$

The derivative in the expression for the coefficients is replaced by the identity in Equation (5.1). Substituting Equations (5.1) and (5.2) into Equation (4.53a) yields,

$$B_n^*(\rho) = \frac{B_n \; \rho^n}{n + \gamma} \tag{5.3}$$

It is noted that in Equation (5.3), the complex series coefficient is a real quantity and it follows that the temperature is symmetrical in the body about $\theta=0$. Then the temperature distribution for $\omega=0$ or equivalently $\lambda=0$ becomes,

$$T(\rho,\theta) = T_o \{1 + \gamma | \frac{\Pi}{8} \frac{\rho}{1+\gamma} \operatorname{Cos}\theta +$$

$$\frac{1}{2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{4n^2-1} \frac{\rho^{2n}}{2n + \gamma} \operatorname{Cos}2n\theta \} \tag{5.4}$$

For small values of the argument with respect to n, power series representation is useful in calculating $J_n(x)$. Then considering the series representations of $J_n(x)$ and $J_n'(x)$ as follows,

$$J_n(x) = \frac{x^n}{2^n n!} \left| 1 - \frac{x^2}{4(n+1} \right|$$

$$J'_n(x) = \frac{x^{n-1}}{2^n n!} \left| n - \frac{(n+2)x^2}{4(n+1)} \right| \qquad (5.5)$$

we obtain

$$\frac{T(\rho,\theta)}{T_o} = 1 + \gamma\{\frac{\Pi}{2} |C_1 \text{Cos}\theta - D_1 \text{Sin}\theta|$$

$$\frac{1}{2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{4n^2-1} |C_{2n} \text{Cos} 2n\theta - D_{2n} \text{Sin} 2n\theta|\} \qquad (5.6)$$

where

$$C_n = \frac{|(16(n+1)^2(n+\gamma) + n^2\lambda^2\rho^2(n+\gamma+2))|\rho^n}{|16(n+1)^2(n+\gamma) + (n\lambda)^2(n+\gamma+2)^2|}$$

$$D_n = \frac{4n\lambda(n+1) \quad |(n+\gamma+2) - \rho^2(n+\gamma)|\rho^n}{|16(n+1)^2(n+\gamma) + (n\lambda)^2(n+\gamma+2)^2|} \qquad (5.7)$$

When $|x|$ is large with respect to n, asymptotic expansions of $J_n(x)$ is useful |1|. In this case, coefficients C_n and D_n for Equation (5.6) becomes

$$C_n = \frac{e^{(\frac{n\lambda}{2})^{\frac{1}{2}}(\rho-1)}\{|\text{Cos}(\frac{n}{2})^{\frac{1}{2}}(\rho-1)||\gamma+(\frac{n\lambda}{2})^{\frac{1}{2}}|+(\frac{n\lambda}{2})^{\frac{1}{2}}\text{Sin}(\frac{n\lambda}{2})^{\frac{1}{2}}(\rho-1)\}}{\sqrt{\rho} \qquad |\frac{n\lambda}{2} + (\gamma + (\frac{n\lambda}{2})^{\frac{1}{2}})^2|} \qquad (5.8)$$

$$D_n = \frac{e^{(\frac{n\lambda}{2})^{\frac{1}{2}}(\rho-1)}\{(\frac{n\lambda}{2})^{\frac{1}{2}}\mathrm{Cos}(\frac{n\lambda}{2})(\rho-1) - (\gamma+\frac{n\lambda}{2})^{\frac{1}{2}}\mathrm{Sin}(\frac{n\lambda}{2})^{\frac{1}{2}}(\rho-1)\}}{\sqrt{\rho}\quad |\frac{n\lambda}{2} + (\gamma + (\frac{n\lambda}{2})^{\frac{1}{2}})^2|}$$

For the cases in which both n and $|x|$ are large the Bessel functions can be derived from recurrence technique as explained in |1|. For these large arguments a procedure based on asymptotic series is adopted and the final temperature distribution again becomes the same as Equation (5.8).

For values of λ between $0.1 \leq \lambda \leq 10$ these functions are not applicable. The temperature distributions for these values of λ are directly programmed for digital computer and the values of temperature at all ranges of θ and ρ values are obtained and plotted. This computer program with the ones which compute the above functions are given in Appendix B.

5.2. NUMERICAL PROCEDURE

In the previous section, the solution for the temperature distribution in a solid cylinder is obtained and the forms of the solution for the different cases are improved. Because physical insight into the phenomena is not well served by the equations of the solution, a series of numerical results have been obtained.

In order to obtain the temperature values for the cases of no rotation ($\lambda=0$), for very slow rotation ($\lambda<<1$) and for a high rotational speeds ($\lambda>>1$), temperature distributions were computed from Equations (5.4), (5.6,7) and (5.8), respectively. Also for the values of λ which are neither in the range of asymptotic expansion nor series expansion, direct computation of Equation (4.70) is preferred by using a subprogram BESCJ. BESCJ can calculate $J_n(z)$ for maximum value of n=100 and for each complex z (except Re(z)=0 and Im(z)=-1) with desired accuracy. A detailed inspection of the resulting temperature values in these programs show us that, asymptotic expansion of the Bessel functions is applicable only for $\lambda>10$ or equally $\omega>0.04$ rad/hr (for $a=0.004$ ft^2/hr). Also series expansion approximation is correct only for $\lambda<0.1$ (i.e., $\omega=0.0004$ rad/hr for $a=0.004$ ft^2/hr). For the values of $0.1\leq\lambda\leq10$ temperature distribution must be calculated by using BESCJ.

In each of these computer programs, infinite series are calculated with 6 decimal accuracy and it is observed that after the terms for n=10, series are converged with the desired accuracy. Therefore, the results of the infinite series are truncated after the terms for n=10.

To compute temperature values, physical parameters were chosen for an aluminum alloy body analogous to that used by Olmstead and Raynor. These values are: radius R=1ft, solar constant q''_o=442 Btu/ft^2-hr, Stefan-Boltzmann constant σ=0.1717x10^{-8} Btu/ft^2-hr^{o}R^4, absorptivity α=1, emissivity ε=1, thermal diffusivity a=3ft^2/hr, thermal conductivity k=100 Btu/ft hr^{o}R. It follows that:

$$T_o = 535.03^oR \qquad \gamma = 0.010478 \qquad \lambda = \omega/3$$

where ω is given in radian per hour. As a second choice, k is taken equal to 0.01 Btu/ft hr^{o}R and a=0.004 ft^2/hr. From this choice of values follows:

$$T_o = 535.03^oR \qquad \gamma = 10.4758 \qquad \lambda = \omega/0.004$$

Using these physical parameters, temperature values were obtained for the full range of λ values.

5.3. GRAPHICAL RESULTS

In this section, the resulting temperature distributions are examined to ascertain the significance of the rotation radial temperature gradients, thermal diffusivity, etc. Since the dimensionless parameters λ and γ depend on these, the temperature distribution of solid cylinder are obtained for the full range of λ and γ values.

Variation of surface temperature around the circumference for all values of λ is shown in Figure 2. Since λ depends on both rotational speed and thermal diffusivity of the material, these temperature distributions become helpful for the variation of one variable while the other remains constant. For a stationary cylinder, the temperatures are symmetric about the point on the cylinder nearest to the radiant source. As easily noticed in Equation(5.3), temperature variation is not sinusoidal as it is for the thin-walled solutions |6|. As the cylinder begins to rotate, rotation destroys symmetry and even for slow rotation $0 \leq \lambda \leq 20$, there is a noticeable shift in the positions of maximum and minimum temperature into the rotational direction. This shifting effect with increasing speed values or decreasing thermal diffusivity is observed to reach a maximum condition for an arc of about 30° on the bright side in the direction of rotation but the minimum temperature shifts an angle of about 90°. Rotation decreases both maximum and minimum values. As λ reaches higher higher values (i.e., rotational speeds approaching infinity or thermal diffusivity diminishes) variation in temperature values becomes smaller and smaller and the temperature of the overall body is found to approach T_o as required by thermodynamic equilibrium.

In Figure 3, the ratio of two temperature values at the surface of the cylinder, which are obtained from asymptotic series and from the direct computation of the Bessel functions with the aid of BESCJ is given. For the values of $1. \leq \lambda \leq 10$, Figure 3 shows that, as λ approaches 10, this ratio becomes nearly 1, i.e., asymptotic approach gives nearly correct temperature values, and for $\lambda=10$ these two values coincide.

Figure 4 gives results for the similar case of Figure 1, but for an internal circumferential surface within the cylinder at $\rho=0.8$. For corresponding values of λ values, there is a greater shifting effect at the extreme temperatures. It is easily concluded that the tendency for the λ to drop the temperature gradient is more effective in the interior than on the surface.

Radial variation of temperature and dependence of this variation on λ, are shown in Figure 5 and Figure 6, for two angular positions, $\theta=0$ and $\theta=\frac{3\Pi}{2}$, respectively. These results show that the radial gradient increases as λ

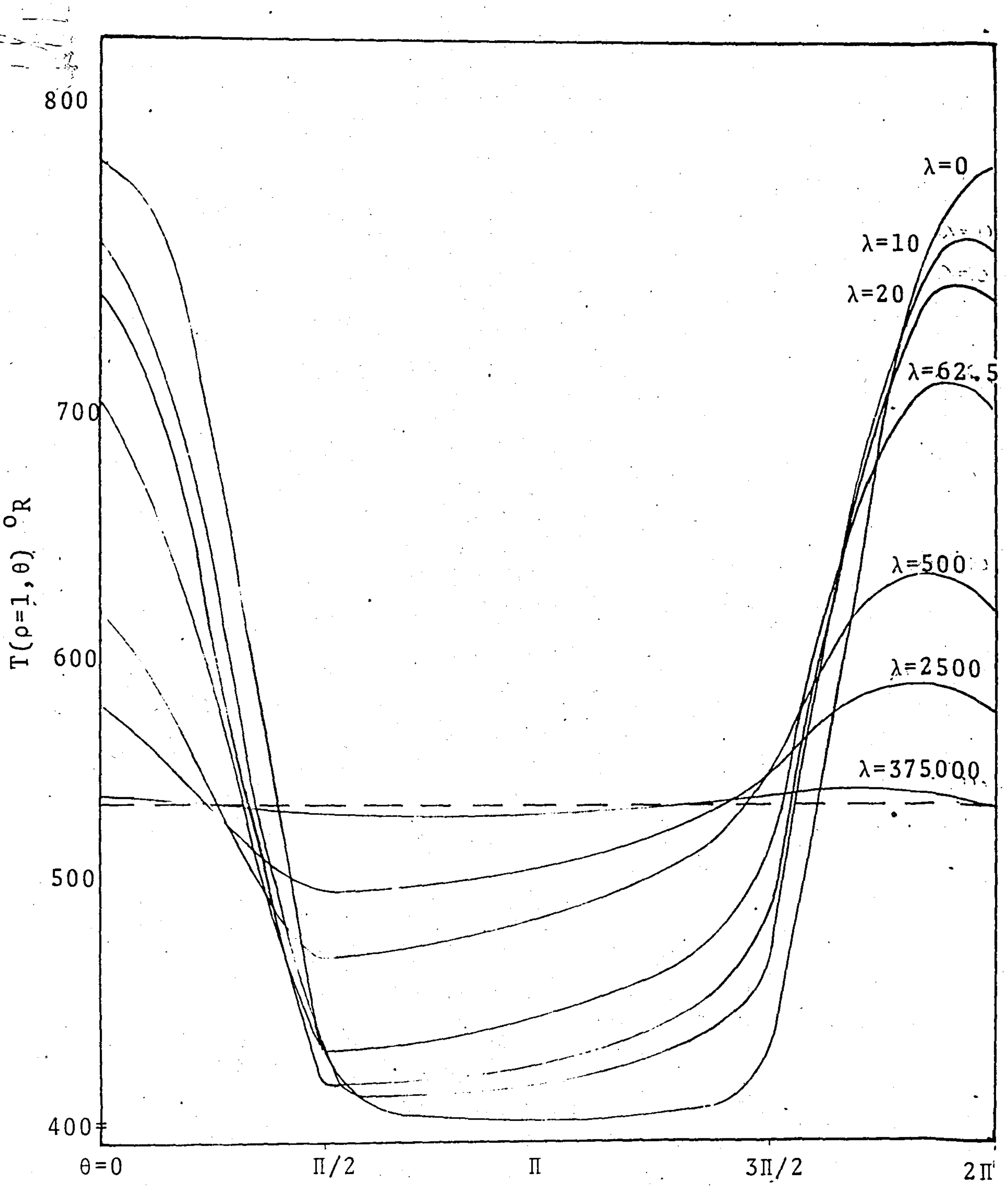

FIGURE 2. Outer Surface Temperature Variation for Different Rotational Speeds for the Solid Cylinder (R=1ft, k=0.1 Btu $ft^{-1}hr^{-1}R^{-1}$, a=0.004 ft^2hr^{-1}, q"=442 Btu $ft^{-2}hr^{-1}$)

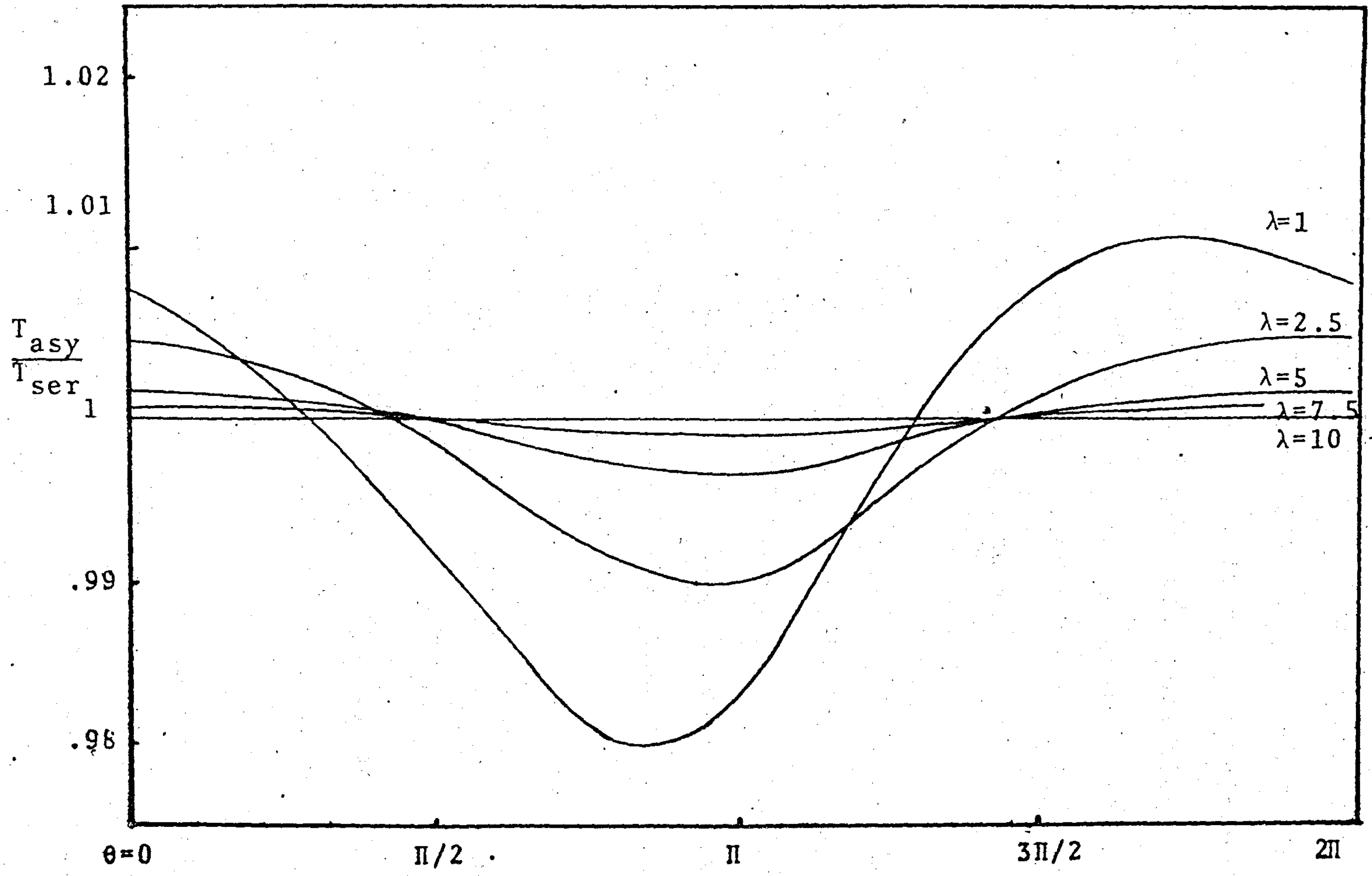

FIGURE 3. Variation of Ratio of Two Temperature Values ($T_{asymptotic}/T_{series}$) Around the Circumference for λ Values Between 1. and 10.

increases while the circumferential gradients always diminish with increasing λ values. Since λ is directly proportional with rotational speeds, the above conclusion is also valid for the increasing ω values. This result is in agreement with results of |11| and |6|, circumferential gradients always diminish with increasing λ and also ω. However, Figure 5 and Figure 6, indicate that larger gradients can be encountered with rotation that would occur for a stationary cylinder.

For the solid cylinder, in Figure 7 and Figure 8 radial temperature distribution for those radial lines, θ=constant for two rotational speeds also show that temperature approaches to T_0 as ω increases. The effect of rotation cause the shift in maximum temperature into the rotational direction.

Effect of the variation of γ values of temperature distribution is given in Figure 9. It is noted that gamma can be considered as the inverse of Planck number which is the ratio of rate of heat conduction in the body to the radiation emitted by the body. At very low γ values, temperature nearly stays constant and equal to corresponding T_0 value. As gamma increases, or equivalently Planck number decreases, temperature distribution is not uniform

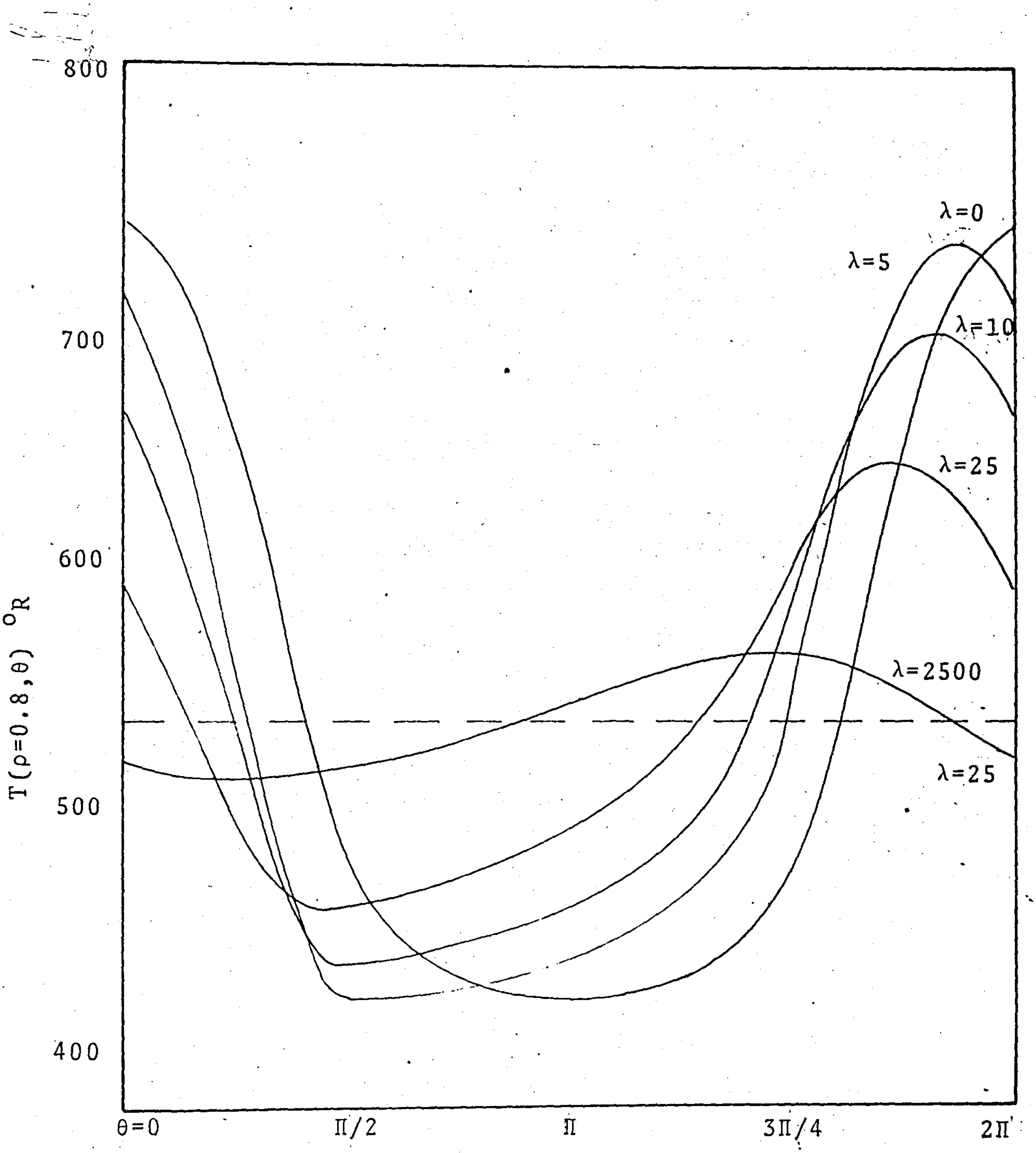

FIGURE 4, Solid Cylinder Temperatures on the $\rho=0.8$ as a Function of Angle ϕ for Various λ Values.

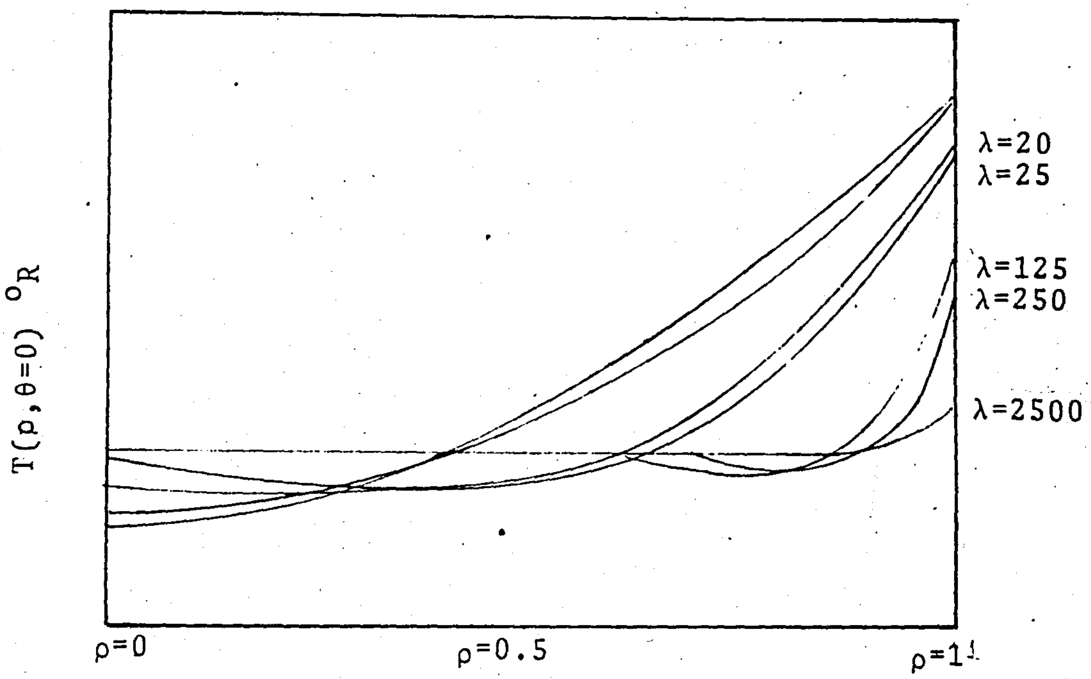

FIGURE 5. Temperature Distribution as a Function of Radius for Various λ Values (Same Physical Parameters on Figure 4)

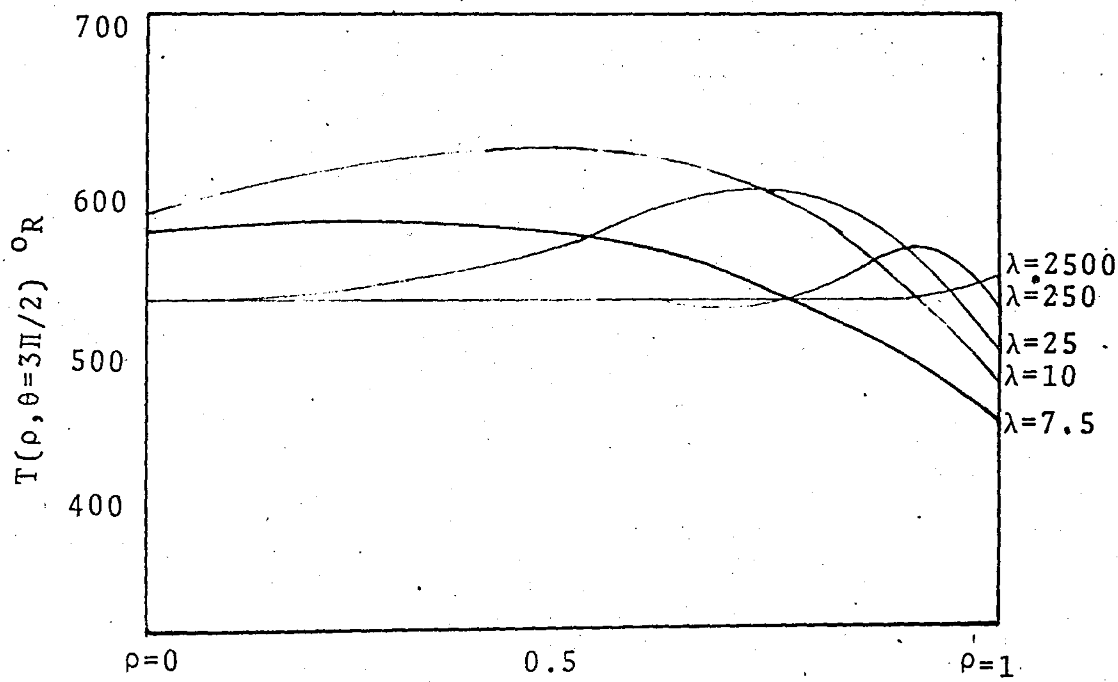

FIGURE 6. Temperature Distribution as a Function of Radius at θ=3Π/2.

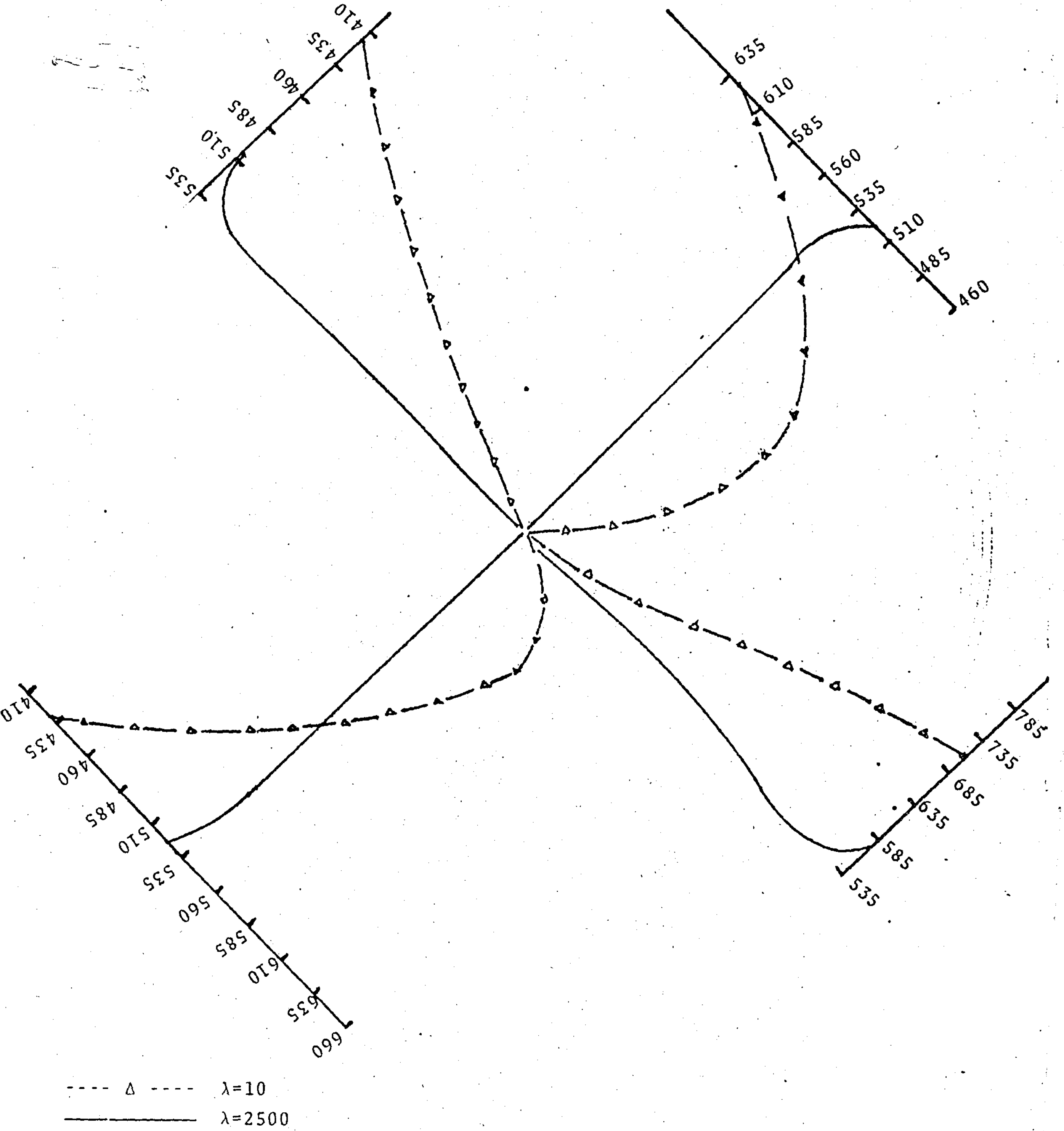

FIGURE 7. Radial Temperature Distribution at $\theta=\Pi/4$ and for $\lambda=10$ and $\lambda=2500$ (Same parameters as Figure 2).

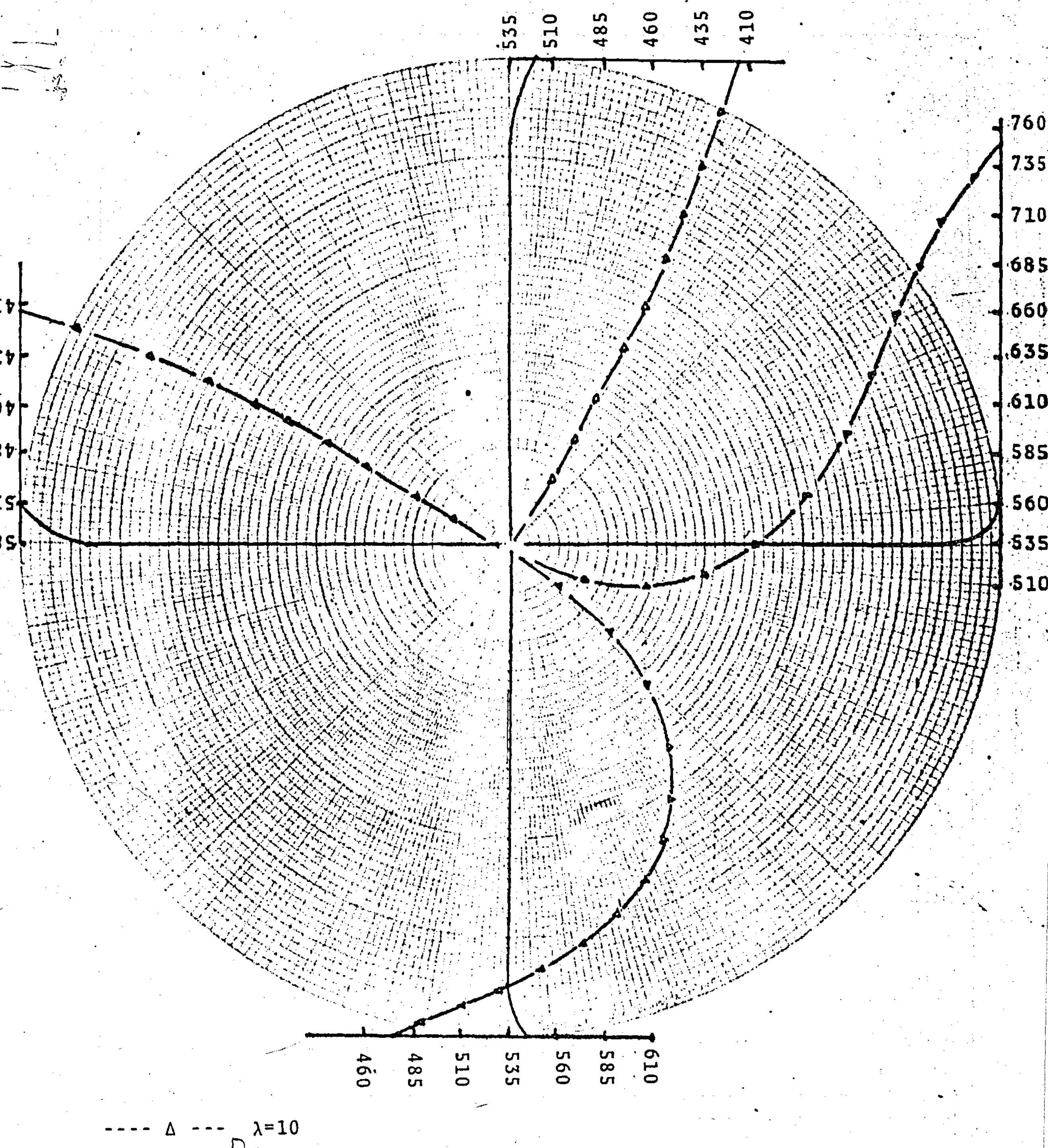

FIGURE 8. Radial Temperature Distribution for θ=Constant for λ=10 and λ=2500.

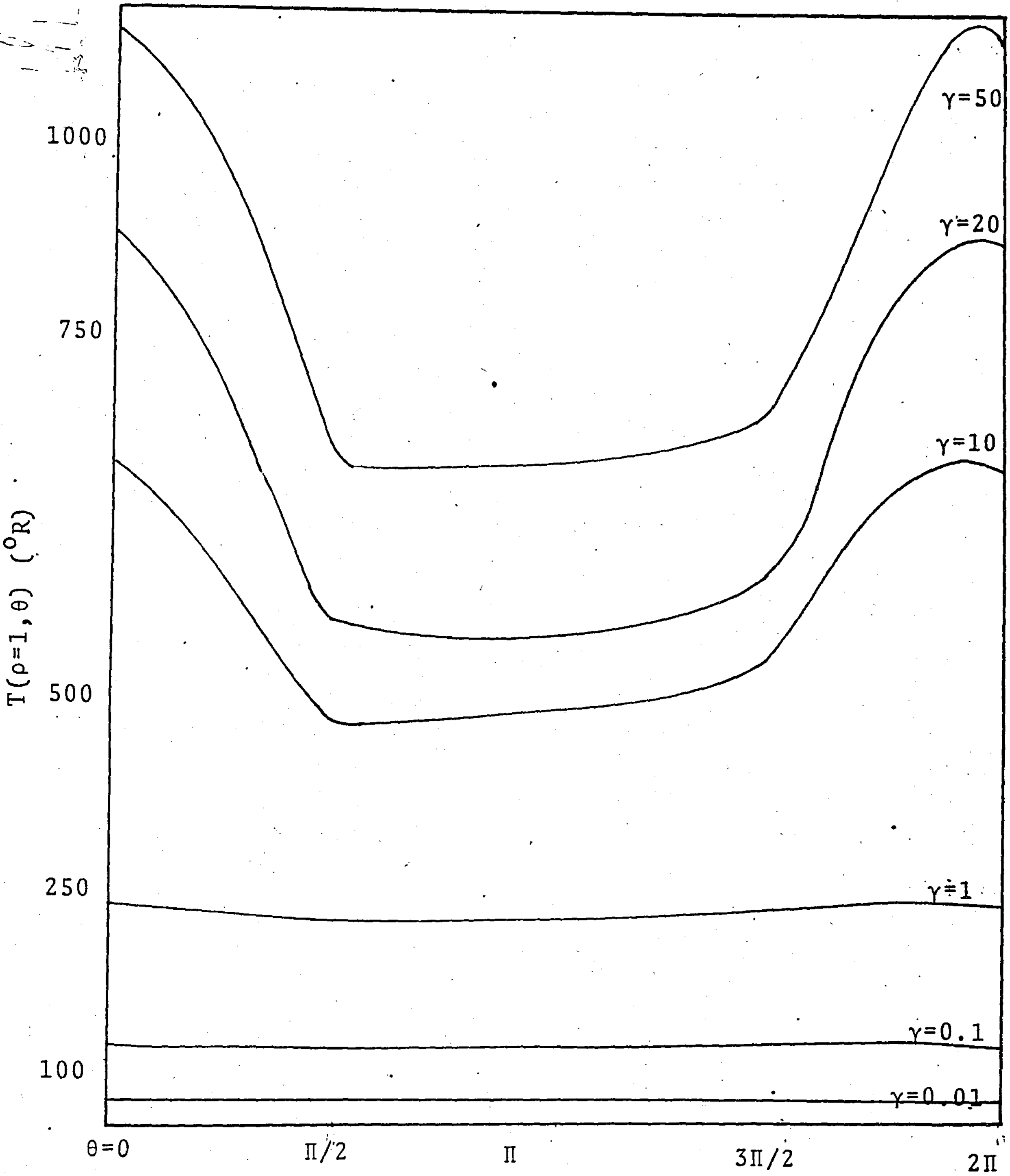

FIGURE 9. Surface Temperature Distribution for Full Range of Gamma Values at ω=0.25 rad/hr.

anymore and show the expected variation from T_o.

Also temperature distributions are shown in Figure 10 for various values of γ at $\rho=0.8$. At low gamma values such as $\gamma=0.1$ temperature is just equal to the corresponding T_o value or very small variation from T_o is observed as in the case of $\gamma=1$. After this value of gamma, there is a noticable variation from T_o values as gamma increases.

5.4. COMPARISON WITH THE OTHER STUDIES

Isothermals at different rotational speeds which are compared with the results obtained in |7| are shown in Figure 11, Figure 12 for stationary cylinder ($\lambda=0$) and for $\lambda=0.25$, respectively. The thermal property values used in the calculations are typical of a high termal conductivity material. The radiation intensity corresponds to the ambient thermal energy density for solar radiation near the earth. The cylinder surface is assumed to be a black body so that $\alpha=\varepsilon=1$. For a corresponding gray body the temperature gradients would be smaller. Using these parameters in Equation (4.77) provides $T_o=535.03^oR$. Using these values of T_o and α isothermals are drawn for the present case. As easily noticed these curves exactly coincide with the ones obtained in |7| by the method of

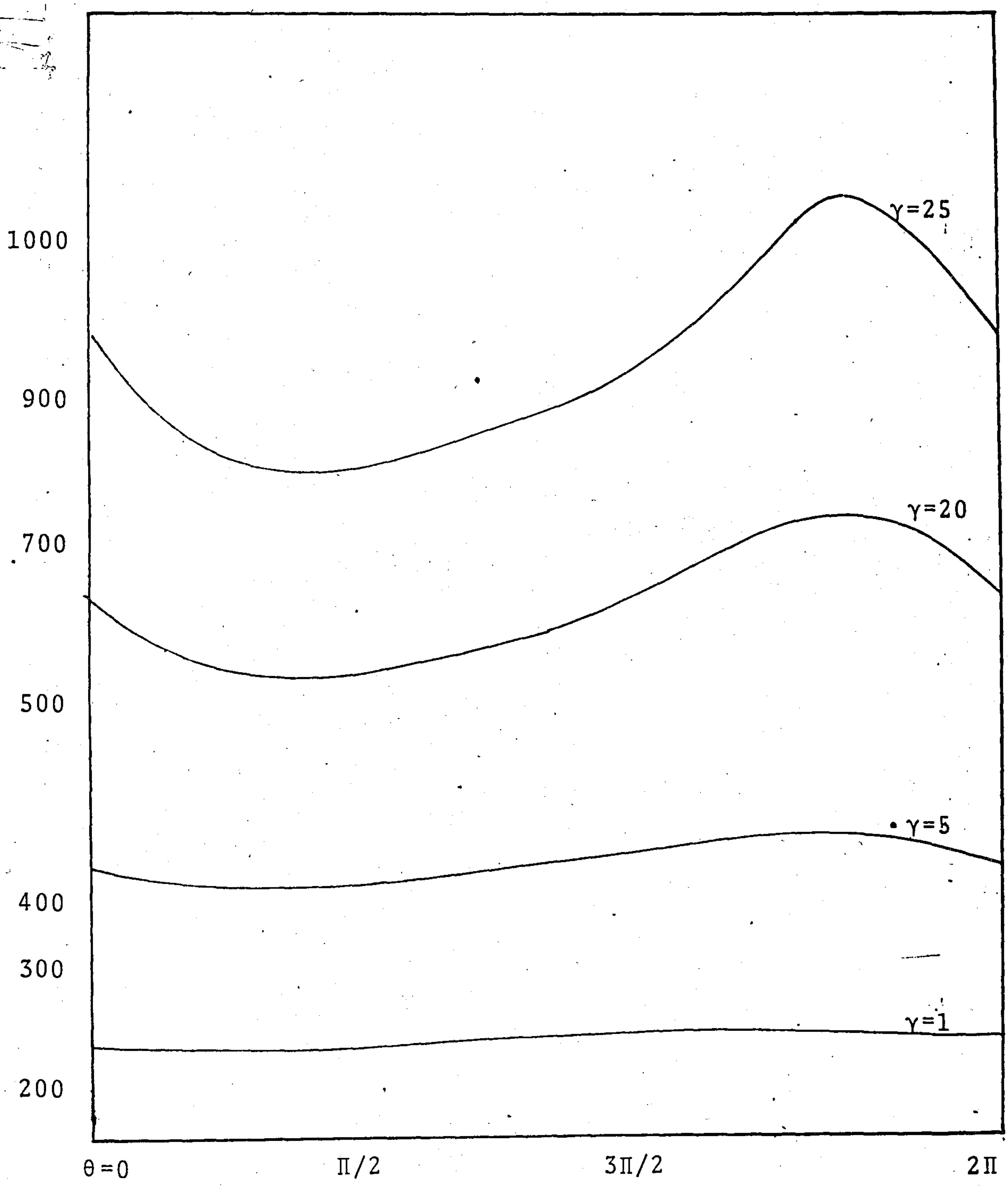

FIGURE 10. Temperature Distribution Versus θ for Different Values of Gamma at ρ=0.8, ω=0.25 rad/hr.

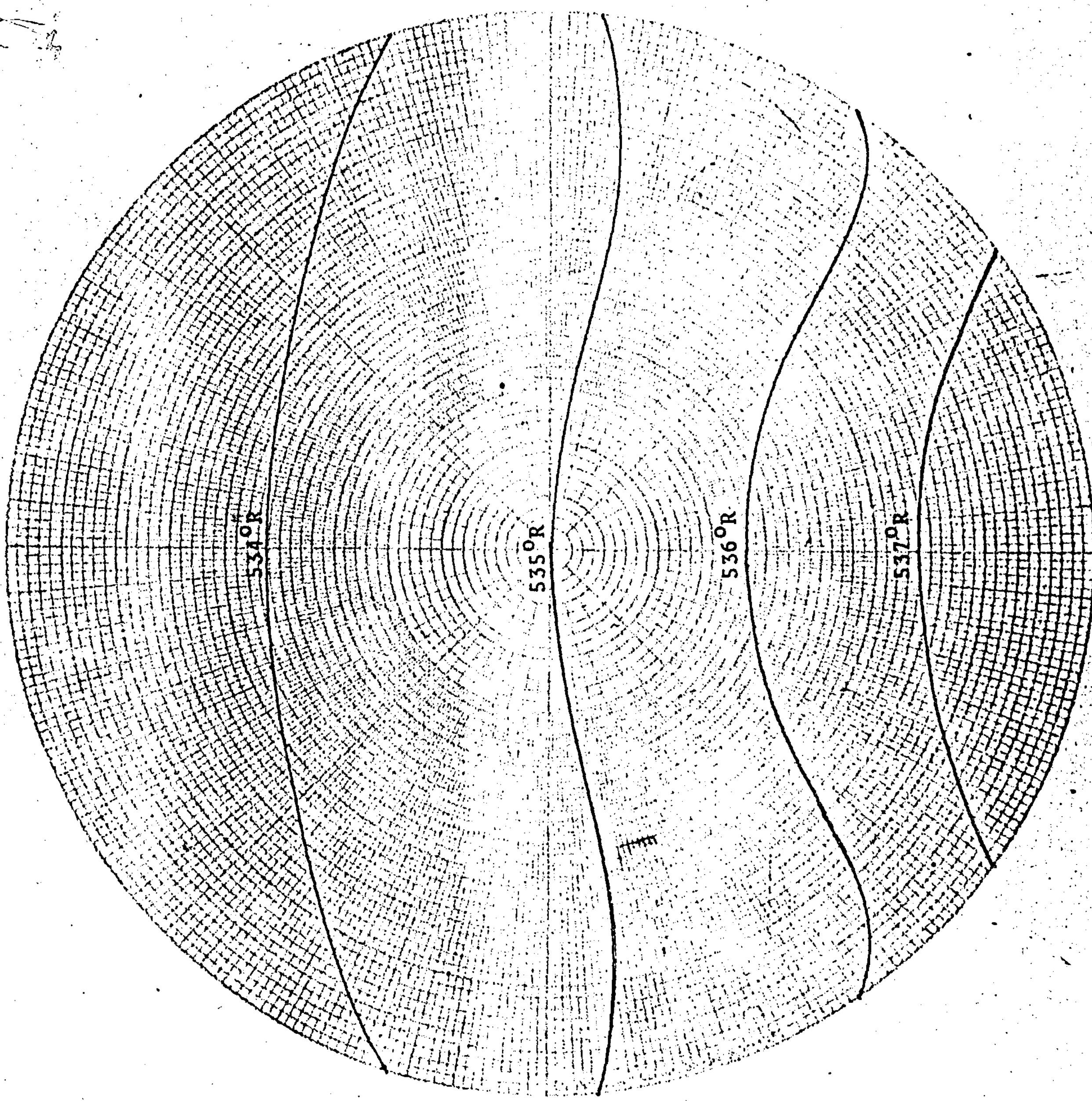

FIGURE 11. Isothermals for the Stationary Solid Cylinder; ω=0 rad/hr. (a=3ft^2/hr, q_o''=442 Btu/ft^{2}hr, $\alpha=\epsilon=1$).

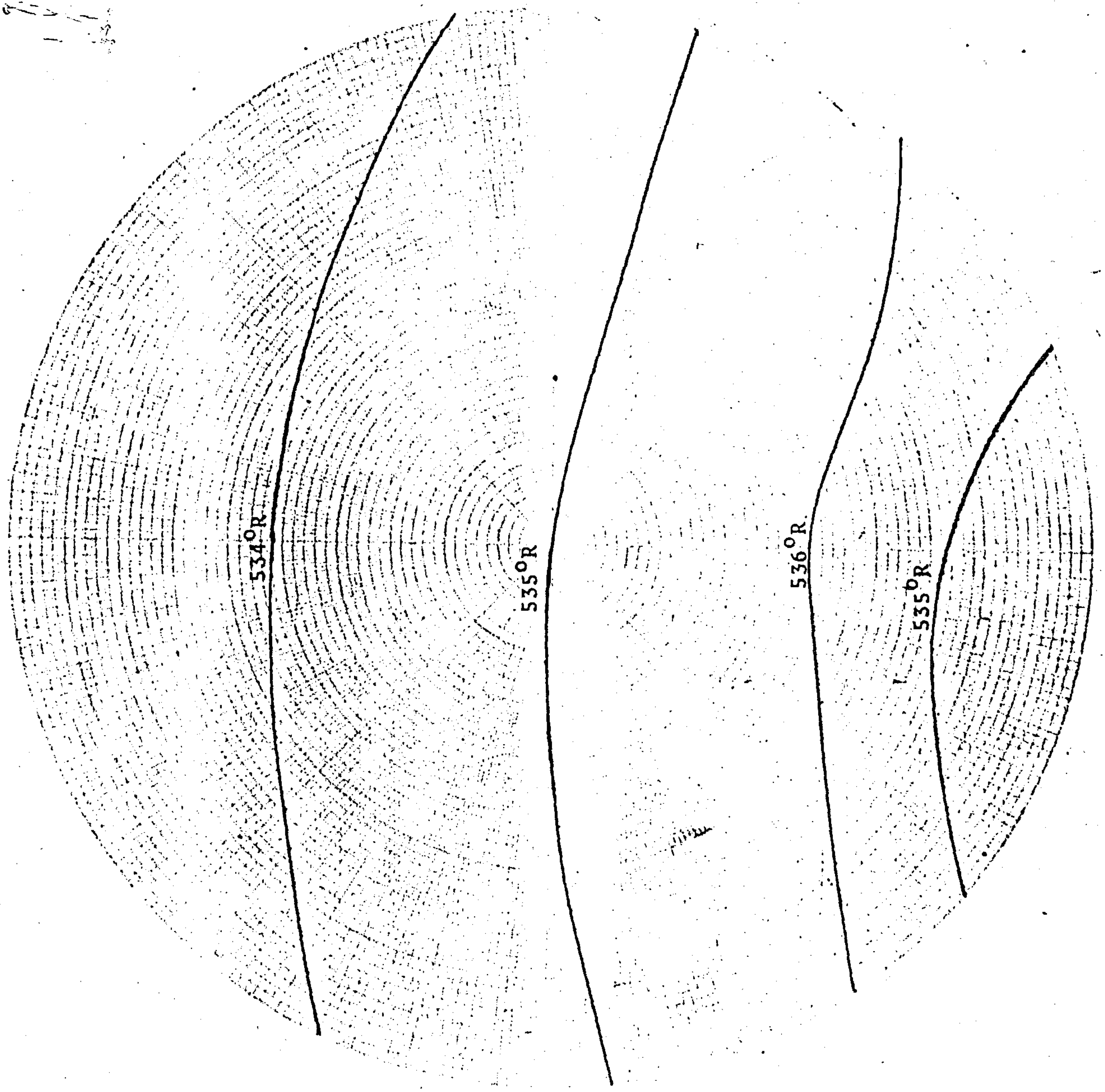

FIGURE 12. Isothermals for Slow Rotation, $\lambda=0.25$ ($a=3ft^2/hr$, $q''_o=442$ Btu/ft^2hr, $\alpha=\varepsilon=1$)

Green's functions.

5.5. DISCUSSION OF THE RESULTS

An approximate analytical solution for the radiation heating of the solid cylinder has been obtained in previous sections followed by a series of numerical results. The numerical results presented in the graphs show a symmetric temperature distribution in θ, for no rotation (i.e., $\lambda \equiv 0$) with the maximum temperature occurring at $\theta \equiv 0$. When rotation is present the temperature distribution in no more symmetric with respect to θ, and the maximum temperature occurs for values of θ in between 0^o to -30^o, depending on the magnitude of λ. The shift occurs in the direction of rotation. Similarly, a shift is observed for the location of the minimum temperature with respect to the "no rotation" case with values occurring in the range $0 \leq \theta \leq \Pi/2$.

For very high rotational speeds, the temperature of the cylinder approaches T_o, more so in the inside as on the outside.

Results presented were based on a solution obtained by linearizing the radiation boundary condition. Therefore, the temperature distributions obtained are approximate more so at points on the surface where the temperature departs the most from the uniform equilibrium value, T_o.

Based on Petrof and Raynor's |10| studies, the results obtained by a linear approximation lead to higher temperatures than those that would actually occur.

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APPENDIX A

DETERMINATION OF FOURIER COEFFICIENTS

Fourier series expansion of any function f(x) on the interval $(-\frac{\Pi}{2}, \frac{3\Pi}{2})$ is

$$f(x) = \frac{a_o}{2} + \sum_{k=1}^{\infty} a_k Cos_k x + b_k Sinkx \qquad (1)$$

where

$$a_k = \frac{1}{\Pi} \int_{-\Pi/2}^{3\Pi/2} f(x) Coskx \, dx$$

$$b_k = \frac{1}{\Pi} \int_{-\Pi/2}^{3\Pi/2} f(x) Sinkx \, dx$$

$$a_o = \frac{2}{\Pi} \int_{-\Pi/2}^{3\Pi/2} f(x) \, dx$$

Then, the Fourier series expansion of $Cos^+\theta$ becomes;

$$\beta Cos^+\theta = \frac{a_o}{2} + \sum_{n=1}^{\infty} a_n Cosnx + b_n Sinnx \qquad (2)$$

where

$$a_o = \frac{1}{2\Pi} \int_{-\Pi/2}^{3\Pi/2} \beta Cos^{+}\theta \; d\theta \qquad (3)$$

$$a_n = \frac{1}{\Pi} \int_{-\Pi/2}^{3\Pi/2} \beta Cos^{+}\theta Cosn\theta \; d\theta \qquad (4)$$

$$b_n = \frac{1}{\Pi} \int_{-\Pi/2}^{3\Pi/2} \beta Cos^{+}\theta Sinn\theta \; d\theta \qquad (5)$$

$$Cos^{+}\theta = \begin{cases} Cos\theta & -\frac{\Pi}{2} \leq \theta \leq \frac{\Pi}{2} \\ 0 & \frac{\Pi}{2} \leq \theta \leq \frac{3\Pi}{2} \end{cases}$$

Then a_o, a_n and b_n becomes,

$$a_o = \frac{1}{2\Pi} \int_{-\Pi/2}^{3\Pi/2} \beta Cos^{+}\theta d\theta = \frac{\beta}{2n} \int_{-\Pi/2}^{\Pi/2} Cos\theta d\theta$$

$$a_o = \frac{\beta}{2\Pi} \left| Sin\theta \right|_{-\Pi/2}^{\Pi/2} = \frac{\beta}{\Pi}$$

and

$$a_n = \frac{1}{\Pi} \int_{-\Pi/2}^{3\Pi/2} \beta Cos^{+}\theta Cosn\theta d\theta = \frac{1}{\ } \int_{-\Pi/2}^{\Pi/2} \beta Cos\theta Cosn\theta d\theta$$

$$= \frac{\beta}{\Pi} \left| \frac{Cosn\theta Sin\theta \Big|_{-\Pi/2}^{\Pi/2} - nSin\theta Cos\theta \Big|_{-\Pi/2}^{\Pi/2}}{1 - n^2} \right|$$

$$a_n = \begin{cases} \frac{2\beta}{\Pi} \frac{(-1)^{n/2+1}}{n^2-1} & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$$

if $|n| = 1$

$$a_1 = \frac{\beta}{\Pi} \int_{-\Pi/2}^{\Pi/2} \text{Cos}^2\theta d\theta = \frac{\beta}{\Pi} \left|\frac{1}{2}(\theta + \text{Cos}\theta\text{Sin}\theta)\right|_{-\Pi/2}^{\Pi/2}$$

$$a_1 = \frac{\beta}{2\Pi} \times \Pi = \frac{\beta}{2}$$

Also, from Equation (5), b_n becomes;

$$b_n = \frac{1}{\Pi} \int_{-\Pi/2}^{3\Pi/2} \beta\text{Cos}^{+}\theta\text{Sinn}\theta d\theta = \frac{\beta}{\Pi} \int_{-\Pi/2}^{\Pi/2} \text{Cos}\theta\text{Sinn}\theta d\theta$$

$$b_n = \frac{\beta}{\Pi} \left| \frac{\text{Sinn}\theta\text{Sin}\theta \Big|_{-\Pi/2}^{\Pi/2} + n\ \text{Cos}\theta\text{Cosn}\theta \Big|_{-\Pi/2}^{\Pi/2}}{1 - n^2} \right|$$

$$b_n = \frac{\beta}{\Pi} \frac{1}{1-n^2} \left|\text{Sinn}\ \frac{\Pi}{2} - \text{Sin}(-\frac{n\Pi}{2})\ \text{Sin}\ (-\frac{\Pi}{2})\right|$$

$$b_n = \frac{\beta}{\Pi(1-n^2)} \left|\text{Sinn}\ \frac{\Pi}{2} - \text{Sin}\ \frac{n\Pi}{2}\right| = 0 \quad \text{for all n.}$$

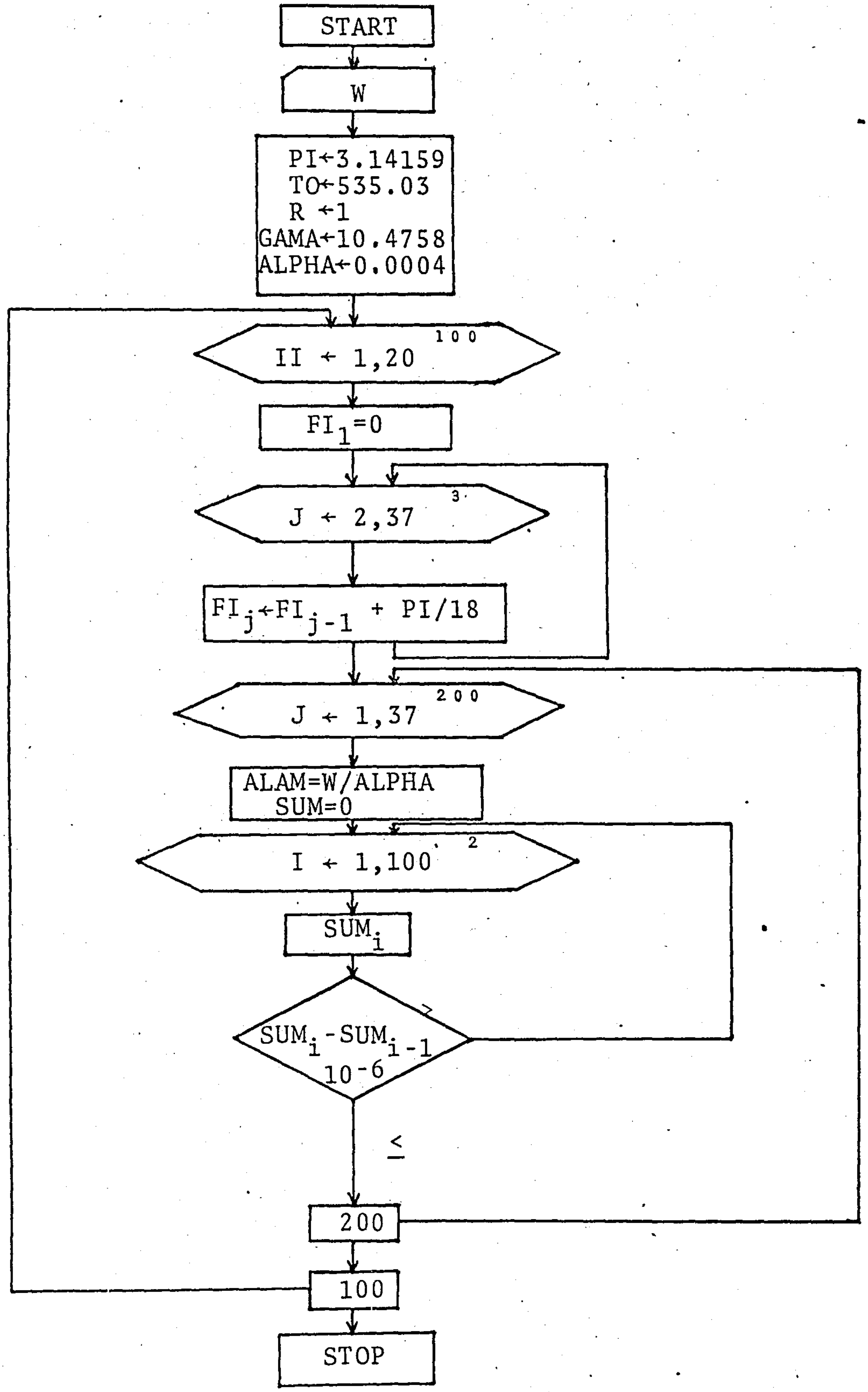
APPENDIX B

1. Calculation of temperature distribution by using asymptotic expansion of Bessel functions. Applicable only for $\lambda > 10$.

2. Direct computation of temperature values by using BESCJ (for $1 \leq \lambda \leq 10$).

3. Temperature distribution for stationary solid cylinders.

Each computer program reads ω, angular velocity according to the format F6.0 and computes the temperature values, TEMP(J) for the circumference of the cylinder at each values of angle FI(J). Also ϕ values and the corresponding temperature values are printed.


START
W
PI←3.14159
TO←535.03
R ←1
GAMA←10.4758
ALPHA←0.0004
II ← 1,20
100
FI1=0
J ← 2,37
3
FIj←FIj-1 + PI/18
J ← 1,37
200
ALAM=W/ALPHA
SUM=0
I ← 1,100
2
SUMi
SUMi-SUMi-1
10-6
>
≤
200
100
STOP

Angular velocity, ω, must be read according to the format F6.0. Angular velocity values can be given in two cards in the following manner.

```
0.001 0.004 0.005 0.008 0.01  0.04  0.05  0.08  0.1   0.5   0.8   1.    2.5
```

```
 5.    7.5   10.   15.   25.   50.   100.
```

```
NNN         NN      UU         UU     RRRRRRRRRRR      DDDDDDDDDD
NNNN        NN      UU         UU     RR       RR      DD       DD
NNNNN       NN      UU         UU     RR       RR      DD       DD
NN NNN      NN      UU         UU     RR       RR      DD       DD
NN  NNN     NN      UU         UU     RRRRRRRRRRR      DD       DD
NN   NNN    NN      UU         UU     RRRRRRRRRR       DD       DD
NN    NNN NN        UU         UU     RR    RR         DD       DD
NN     NNNNN        UU         UU     RR     RR        DD       DD
NN      NNNN        UUU       UUU     RR      RR       DD       DD
NN       NNN         UUUUUUUUUU       RR       RR      DDDDDDDDDDD
NN        NN          UUUUUUUUU       RR       RR      DDDDDDDDDD
UNIVAC 1106 -- BOGAZICI UNIVERSITESI KOMPUTER MERKEZI --ISTANBUL VER

.D * NURDIO            USER ID *                    PART NUMBER * 00       INPUT

:LE NAME * PR@U00NURDIO                 CREATED AT:   16;09;33 JAN 26,1981

590123456789012345678901234567890123456789012345678901234567890123456789012345678901
NURDIL,111-11-202,PROJECT          .NURDIL.
  SYS$*READT.
NING          040000000200
SYS$*READT.CLEAR
   .MAIN
-1 01/26/81-16:09(,0)
          DIMENSION FI(37),FF(37),TEMP(37),ARRAY(1,20),SUM(101)
          READ(5,10) (ARRAY(1,J),J=1,20)
       10 FORMAT(10F6.0)
          PI=3.141592654
          ALPHA=0.004
          TO=535.03
          R=1.
          DO 100 II=1,20
          W=ARRAY(1,II)
          WRITE(6,1) W
        1 FORMAT(10X,'W=',F10.4)
          FI(1)=0.
          DO 3 J=2,37
        3 FI(J)=FI(J-1)+PI/18.
          DO 200 J=1,37
          ALAM=W/ALPHA
          DO 2 I=2,100
          SUM(1)=0.
          E=ALAM/2.
          FF(J)=2.*FLOAT(I)*FI(J)
          ED=(E*FLOAT(I)*2.)**0.5
          EE=ED*(R-1.)
          EF=GAMA+ED
          PAY=(R**0.5)*((FLOAT(I)*E*2.)+((EF)**2))
          C2=(COS(EE)*EF+ED*SIN(EE))/PAY
          D2=(ED*COS(EE)-EF*SIN(EE))/PAY
          CC=(C2*COS(FF(J))-D2*SIN(FF(J)))
          SUM(I)=SUM(I-1)+(((-1.)**(I+1))/(4.*FLOAT(I)**2-1.))*CC
          IF((SUM(I)-SUM(I-1)).LE.0.000001) GO TO 15
        2 CONTINUE
       15 EA=E**0.5
          EB=EA*(R-1.)
          EC=GAMA+EA
          PA=(R**0.5)*(E+EC**2)
          C1=(COS(EB)*EC+EA*SIN(EB))/PA
          D1=(COS(EB)*EA-EC*SIN(EB))/PA
          EXP1=PI*(C1*COS(FI(J))-D1*SIN(FI(J)))/8.
```

```
          SUM(J)=SUMI
          TEMP(J)=TO+TO*GAMA*(EXP1+SUMI/2.)
          WRITE(6,111) FI(J),TEMP(J)
      111 FORMAT(5X,2(F12.8,5X))
      200 CONTINUE
      100 CONTINUE
          STOP
          END

220 IBANK 353 DBANK

72R1UT 01/26/81 16:10:05

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            040000 043443       1828 DBANK WORDS DECIMAL
 ADDRESS    032574



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                    $(1)      001200 002114
                    $(1)      002115 002131      $(2)      040000 040002
                    $(3)      002132 002146
                    $(5)      002147 002147
                                                 $(2)      040003 040314
                                                 $(2)      040315 040315
                                                 $(0)      040316 040322
                                                 $(034)    040323 040364
                                                 $(2)      040365 040432
                    $(1)      002150 005151
YS73R1Q8            $(1)      005152 005346
                    $(1)      005347 007524
                    $(1)      007525 012315
                    $(1)      012316 012644
                    $(1)      012645 013002
                    $(1)      013003 015125
                    $(1)      015126 016562
                    $(1)      016563 017540
                    $(1)      017541 024106
                    $(1)      024107 026425
                    $(1)      026426 026457      $(0)      040433 040435
COMMONBLOCK)                                               040436 040436
                                                 $(2)      040437 042160
                                                 $(034)    MOERO$
                                                 $(036)    PMD$COM
                                                 $(2)      042161 042432
73R1
FORFTN                                           $(2)      042433 042440

                                                 $(2)      042441 042474
MATH                $(1)      026460 026631      $(2)      042475 042527
                    $(037)    INFO-010-LC        $(034)    MOERO$
TH                  $(1)      026632 026716      $(2)      042530 042535
                    $(037)    INFO-010-LC        $(034)    MOERO$
OMMONBLOCK)                                                042536 042541
TH                  $(1)      026717 027123      $(2)      042542 042616
                    $(037)    INFO-010-LC        $(034)    MOERO$
```

```
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  NNNNN        NN      UU          UU     RR         RR      DD         DD
  NN NNN       NN      UU          UU     RR         RR      DD         DD
  NN  NNN      NN      UU          UU     RRRRRRRRRRR        DD         DD
  NN   NNN     NN      UU          UU     RRRRRRRRRR         DD         DD
  NN    NNN NN         UU          UU     RR     RR          DD         DD
  NN     NNNNN         UU          UU     RR      RR         DD         DD
  NN      NNNN         UUU        UUU     RR       RR        DD         DD
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  NN        NN           UUUUUUUU         RR         RR      DDDDDDDDDD
 UNIVAC 1106 -- BOGAZICI UNIVERSITESI KOMPUTER MERKEZI --ISTANBUL V

NID * NURDIL          USER ID *                  PART NUMBER * 00        INPU

FILE NAME * PR@OOONURDIL                   CREATED AT:    15:57:49 JAN 26,1981

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 SYS$*READT.CLEAR
   BOGAZICI*CERN
RNING          040000100200
S     .MAIN
1-1 01/26/81-15:58(,0)
               INTEGER D
               COMPLEX V,VO,V1,SER,SER1,U(101),Z,Z1
               DIMENSION FI(37),FF(37),TEMP(37),ARRAY(1,20),SUM(101)
               READ(5,10) (ARRAY(1,J),J=1,20)
            10 FORMAT(10F6.0)
               PI=3.141592654
               R=1.
               GAMA=10.47587975
               T0=535.03
               DO 100 II=1,20
               W=ARRAY(1,II)
               WRITE(6,1) W
             1 FORMAT(10X,'W=',F12.8)
               FI(1)=0.
               DO 3 J=2,37
             3 FI(J)=FI(J-1)+PI/18.
               DO 200 J=1,37
               DLAMDA=W/ALPHA
               A=0.
               D=6
               DO 2 I=2,50
               SUM(1)=0.
               V=CMPLX(-SQRT(DLAMDA*FLOAT(I)),SQRT(DLAMDA*FLOAT(I)))
               NMAX=2*I
               CALL BESCJ(V,A,NMAX,D,U)
               Z=(-2.*FLOAT(I)+GAMA)*U(NMAX)+V*U(NMAX-1)
               V1=R*V
               CALL BESCJ(V1,A,NMAX,D,U)
               SER=U(NMAX)/Z
               X1=REAL(SER)
               Y1=AIMAG(SER)
               FF(J)=2.*FLOAT(I)*FI(J)
               CC=X1*COS(FF(J))-Y1*SIN(FF(J))
               SUM(I)=SUM(I-1)+(((-1.)**(I+1))/(4.*FLOAT(I)**2-1.))*CC
               IF((SUM(I)-SUM(I-1)).LE.0.000001) GO TO 15
```

```
      2  CONTINUE
     15  NMAX=2
         V=CMPLX(-SQRT(DLAMDA/2.),SQRT(DLAMDA/2.))
         CALL BESCJ(V,A,NMAX,D,U)
         Z1=(1.+GAMA)*U(1)-V*U(2)
         V0=R*V
         CALL BESCJ(V0,A,1,6,U)
         SER1=U(1)/Z1
         X0=REAL(SER1)
         Y0=AIMAG(SER1)
         EXP1=PI/8.*(X0*COS(FI(J))-Y0*SIN(FI(J)))
         SUM(J)=SUMI
         TEMP(J)=T0+T0*GAMA*(EXP1+SUMI/2.)
         WRITE(6,11) FI(J),TEMP(J)
     11  FORMAT(5X,2(F12.8,5X))
    200  CONTINUE
    100  CONTINUE
         STOP
         END

| 287 IBANK 577 DBANK
 BOGAZICI*CERN.
 27R2  RL72R1 01/26/81 15:59:41
:P.

: 72R1U1 01/26/81 16:00:55
      LIB BOGAZICI*CERN


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             040000 045276       2751 DBANK WORDS DECIMAL
NG ADDRESS   034613



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/$                    $(1)     002115 002131      $(2)     040000 040002
                      $(3)     002132 002146
                      $(5)     002147 002147
                                                  $(2)     040003 040314
5                                                 $(2)     040315 040315
                                                  $(0)     040316 040322
                                                  $(034)   040323 040364
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5                                                 $(2)     040455 040522
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/SYS73R1Q8            $(1)     005516 005712
                      $(1)     005713 010070
                      $(1)     010071 012661
                      $(1)     012662 013210
                      $(1)     013211 013346
                      $(1)     013347 015471
                      $(1)     015472 017126
```

```
1234567890123456789012345678901234567890123456789012345678901234567890123456

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                NN NNN      NN         UU           UU       RR          RR
                NN  NNN     NN         UU           UU       RRRRRRRRRRR
                NN   NNN    NN         UU           UU       RRRRRRRRRR
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                NN     NNNNN           UU           UU       RR     RR
                NN      NNNN           UUU         UUU       RR      RR
                NN       NNN             UUUUUUUUUUU         RR       RR
                NN        NN              UUUUUUUUU          RR        RR
 *  *  *  UNIVAC 1106 -- BOGAZICI  UNIVERSITESI  KOMPUTER  MERKEZI

    RUNID * NURDIL            USER ID *                        PART NUMBER *

      FILE NAME * PRA000NURDIL                      CREATED AT:     12:13:23

1234567890123456789012345678901234567890123456789012345678901234567890123456
ΔRUN,E NURDIL,111-11-202,PROJECT          .NURDIL.
ΔFTN,IS  .MAIN
FTN 7R1-1 01/28/81-12:13(,0)
              1.         DIMENSION FI(37),TEMP(37),FF(37)          Δ
              2.         PI=3.141592654
              3.         R=0.5    Δ                                59 OT OG )1VA
              4.         FI(1)=0.
              5.         DO 3 J=2,37            Δ
   1          6.       3 FI(J)=FI(J-1)+PI/18.        Δ
              7.         WRITE(6,12)           Δ
              8.      12 FORMAT(5X,,W=0.,)       Δ
              9.         DO 200 J=1,37          Δ
   1         10.         SUM=0.
   1         11.         DO 8 I=1,50
   2         12.         FF(J)=2*I*FI(J)                                  Δ
   2         13.         BI=((-1.)**(I+1))/(4.*FLOAT(I)**2-1.)           Δ
   2         14.         CI=GAMA+2.*FLOAT(I)
   2         15.       8 SUM=SUM+BI*((R**2)**I)*COS(FF(J))/CI
   1         16.         DE=(PI*R*COS(FI(J)))/((1.+GAMA)*8.)
   1         17.         EXP1=GAMA*(DE+SUM/2.)
   1         18.         TEMP(J)=TO+TO*EXP1
   1         19.         WRITE(6,111) FI(J),TEMP(J)
   1         20.     111 FORMAT(5X,2(F12.8,5X))
   1         21.     200 CONTINUE
             22.         STOP
             23.         END

END FTN 102 IBANK 174 DBANK
ΔXQT
MAP28R2 72R1U1 01/28/81 12:13:39

ADDRESS LIMITS     001000 032534     13149 IBANK WORDS DECIMAL
                   040000 043103      1604 DBANK WORDS DECIMAL
STARTING ADDRESS   032367



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