WEINBERG-SALAM MODEL AND $e^+e^- \rightarrow \mu^+\mu^-$ PROCESS

by

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To my lovely fiancée,

physicist and teacher,

ERANİ ELİZ KARAYEL

(Dec.18,1954 - Jan.28,1978)

C O N T E N T S

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ABSTRACT

In this work the Weinberg-Salam model of weak and electromagnetic interactions is reviewed. This model was proposed to cure the high energy behaviour of the classical weak phenomenology which was first put forward by Fermi as β-decay theory and later modified by Feynman and Gell-Mann. The behaviour of the theory at high energies was hoped to be modified by the introduction of a new particle, $W^{\pm}$. However, new difficulties were met especially in the reaction $\nu\bar{\nu} \rightarrow W^{+}W^{-}$. Weinberg's model was first proposed in 1967 and then presented with emphasis on the gauge invariance in 1968. The most important theoretical difficulty was surpassed by 't Hooft who proved that the theory was renormalizable. The successful gauge invariant formulation of the weak and electromagnetic interactions was verified in 1973, with the experimental observation of the neutral currents, predicted by the theory. Today, the model is considered to be true, waiting for the direct observation of the gauge bosons.

After the introduction of the model the reaction $e^{+}e^{-} \rightarrow \mu^{+}\mu^{-}$ is considered. The cross-section, the parity violation, the front-to-back ratio for the $e^{+}e^{-} \rightarrow \mu^{+}\mu^{-}$ decay has been calculated and compared with experimental data. Showing the already well known fact that, theory and experiment agree well at presently available energies.

I.1. CLASSICAL WEAK PHENOMENOLOGY AND ITS PROBLEMS

Weak interactions manifest themselves especially in the decays of elementary particles which are very slow. The lifetime of a weak decay ranges from 10^{-10} sec to 10^{3} sec, the range of the weak force is finite being about 10^{-16} cm and has coupling strength around $10^{-5}\, m_p^{-2}$ in natural units ($\hbar=c=1$). Weak interactions can be studied in three categories.

a. Purely leptonic processes, where only leptons appear in the initial and final states. Leptons don't show any internal structure and they behave as point-like particles. The known ones are $e^{\pm}, \mu^{\pm}, \tau^{\pm}$ (newly discovered), $\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau$ (where the last two are also newly discovered).

b. Semileptonic weak processes, where both leptons and hadrons (strongly interacting particles) are involved.

c. Purely hadronic weak processes among the hadrons, where leptons do not participate.

The weak interactions at low energies were described phenomenologically by the Hamiltonian density

$$\mathcal{H}(x) = \frac{G}{\sqrt{2}}\, J_\alpha^{+}(x)\, J_\alpha(x) + h.c. \tag{1.1.1}$$

where

$$J_\alpha(x) = L_\alpha(x) + j_\alpha(x) \tag{1.1.2}$$

Here $L_\alpha(x)$ involves the leptonic part while $j_\alpha(x)$ represents the hadronic current involved in semi-leptonic and non-leptonic interactions. In this work, only the purely leptonic part will be considered, so that the Hamiltonian density is

$$\mathcal{H}_\ell(x) = \frac{G}{\sqrt{2}}\, L_\alpha^{+}(x)\, L^{\alpha}(x) \tag{1.1.3}$$

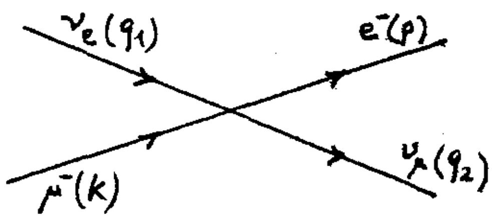
The charged currents L_α and L_α^+ having vector-axial vector form are given by,

$$L_\alpha(x) = \bar{\Psi}_e(x)\gamma_\alpha(1-\gamma_5)\Psi_{\nu_e}(x) + \bar{\Psi}_\mu(x)\gamma_\alpha(1-\gamma_5)\Psi_{\nu_\mu}(x) \tag{I.1.4a}$$

$$L_\alpha^+(x) = \bar{\Psi}_{\nu_e}(x)\gamma_\alpha(1-\gamma_5)\Psi_e(x) + \bar{\Psi}_{\nu_\mu}(x)\gamma_\alpha(1-\gamma_5)\Psi_\mu(x) \tag{I.1.4b}$$

(In this discussion Lagrangian and Hamiltonian densities can be used interchangeably because of the absence of derivative couplings, so that $\mathcal{L}(x) = -\mathcal{H}(x)$. In the following sections "density" will be omitted for brevity.)

Such a Hamiltonian describes a four-fermion point interaction in the lowest order of perturbation theory, explaining the experimental data at low energies. However, the high energy results are not compatible with such a four-fermion point interaction. Consider the interaction $e^-\nu_\mu \to \mu^-\nu_e$.



(I.1.5)

which is a point interaction with zero range. In the lowest order of perturbation theory the amplitude becomes,using Eqns.(I.1.3-4),

$$\mathcal{M} = \frac{G}{\sqrt{2}}\, \bar{u}_{\nu_\mu}\gamma_\alpha(1-\gamma_5)u_\mu\, \bar{u}_e\gamma^\alpha(1-\gamma_5)v_{\nu_e}\,. \tag{I.1.6}$$

In the high energy limit and in the centre of mass frame where lepton masses m_e and m_μ are negligible, one obtains,

$$|\mathcal{M}|^2 \sim G^2(q_1\cdot k)(q_2\cdot p)\,. \tag{I.1.7}$$

Since,

$$q_1\cdot k \sim q_2\cdot p \sim s\,, \tag{I.1.8}$$

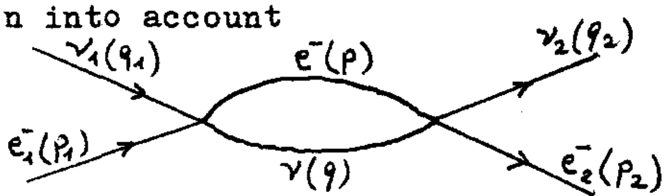
where s is the square of the center of mass energy,it is easy to deduce that,

$$\mathcal{M} \longrightarrow Gs \; . \qquad \text{(I.1.9)}$$

Thus, $\mathcal{M}$ grows quadratically with the c.m. energy and there exist an energy for which $\mathcal{M}$ can exceed unity. Indeed when,

$$\sqrt{s} = \frac{1}{\sqrt{G}} \sim 320 \text{ GeV} \qquad \text{(I.1.10)}$$

$\mathcal{M}$ is nearly unity. Above this energy it is apparent that $\mathcal{M}$ will violate the unitarity condition. Therefore Eq.(I.1.3) fails to describe weak interactions at such high energies. At first sight it is natural to think that the failure arises from the neglect of higher order effects. If the following diagram is taken into account



(I.1.11)

the amplitude is given by,

$$\mathcal{M}^{(2)} = \frac{G^2}{2} \frac{1}{(2\pi)^4} \int d^4q \left[\bar{u}_{e_2} \gamma_\alpha (1-\gamma_5) \frac{1}{\not{q}} \gamma_\beta (1-\gamma_5) u_{e_1} \right]$$

$$\times \left[\bar{u}_{\nu_2} \gamma^\alpha (1-\gamma_5) \frac{1}{\not{p} - m_\mu} \gamma^\beta (1-\gamma_5) u_{\nu_1} \right] \qquad \text{(I.I.12)}$$

with $\qquad p_1 + q_1 = p + q = p_2 + q_2 \; .$

However, since

$$\mathcal{M}^{(2)} \sim \frac{G^2}{2(2\pi)^4} \int \frac{d^4q}{pq} \sim G^2 q^2 \qquad \text{(I.1.13)}$$

the amplitude again diverges. In quantum electrodynamics divergent integrals also exists, and these divergencies may be removed at any order by charge and mass renormalization. In the present case if one follows an analogous procedure to cancel the divergence there will be higher order diagrams where the divergencies become more and more severe and each requires a new set of renormalization constants. Thus, if all possible diagrams are taken into account an infinite set of renormalization constants will be needed. It follows that Fermi theory of weak interactions is not a renormalizable theory.

I.2. INTERMEDIATE VECTOR BOSON HYPOTHESİS

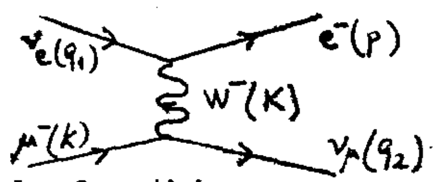
Another approach to cure the high enėrgy behaviour would be to change the assumption of a point interaction in lowest order, by a non-local interaction mediated by a vector boson. Then, there exist a vector field which couples to lepton fields through the following Hamiltonian,

$$\mathcal{H}(x) = f\left[L^{+}_{\alpha}(x)W^{\alpha}(x) + L^{\alpha}(x)W^{+}_{\alpha}(x)\right] \qquad \text{(I.2.1)}$$

where f is a coupling constant and W_α represents the intermediate vector boson field and L_α is the current given in Eq.(I.1.4). The vector boson W_α is charged ahd it can have the decay modes,

$$W^{\pm}_{\alpha} \rightarrow \mu^{\pm}\nu_{\mu}(\bar{\nu}_{\mu}) \;, \quad W^{\pm}_{\alpha} \rightarrow e^{\pm}\nu_{e}(\bar{\nu}_{e}) \qquad \text{(I.2.2)}$$

Consider the high energy behaviour of the crossed process described by Eq.(I.1.5) which now occurs via W-exchange.



(I.2.3)

The amplitude for this process, using Eq.(I.2.1) is,

$$\mathcal{M} = f^2\,\frac{g^{\alpha\beta} - K^{\alpha}K^{\beta}/m_W^2}{K^2 - m_W^2}\left[\bar{u}_e\gamma_\alpha(1-\gamma_5)u_{\nu_e}\right]\left[\bar{u}_{\nu_\mu}\gamma_\beta(1-\gamma_5)u_\mu\right] \qquad \text{(I.2.4)}$$

with $K = q_2 - k = p - q_1$.

This amplitude reduces to Eq.(I.1.6) if W is very heavy and,

$$\frac{G}{\sqrt{2}} = \frac{f^2}{m_W^2} \qquad \text{(I.2.5)}$$

In the high energy limit Eq.(I.2.4) behaves as,

$$\mathcal{M} \rightarrow f^2\,\frac{s}{K^2 - m_W^2} \qquad \text{(I.2.6)}$$

where,

$$K^2 \sim s \tag{I.2.7}$$

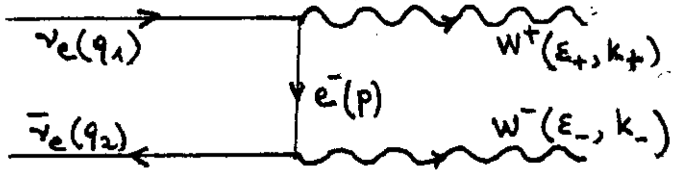
Therefore $\mathcal{M}$ approaches a constant value with increasing c.m. energy, an advantage arising from the introduction of the W-boson. It may seem that W-boson will modify the high energy behaviour and divergencies will be avoided. However, this is not so and the s-wave amplitude violates partial wave unitarity. Using Eq.(I.2.5) the amplitude in Eq.(I.2.4) can be written as,

$$f(\theta) = \left(2\frac{d\sigma}{d\Omega_{cm}}\right)^{1/2} = \frac{\sqrt{2}\,G\,p_{cm}}{\pi}\left[1+\frac{2p_{cm}^2}{m_W^2}(1-\cos\theta)\right]^{-1}$$
$$= \frac{1}{p_{cm}}\sum_{J=0}^{\infty}\left(J+\frac{1}{2}\right)P_J(\cos\theta)\,\mathcal{M}_J\,. \tag{I.2.8}$$

The s-wave amplitude is,

$$\mathcal{M}_0 = \frac{\sqrt{2}\,G\,p_{cm}^2}{\pi}\int_{-1}^{1} d(\cos\theta)\left[1+\frac{2p_{cm}^2}{m_W^2}(1-\cos\theta)\right]^{-1}$$
$$= \frac{G\,m_W^2}{\sqrt{2}\,\pi}\log\left(1+\frac{2m_\mu E_{\nu_e}}{m_W^2}\right), \tag{I.2.9}$$

showing that $\mathcal{M}_0$ is logarithmically divergent and partial waves eventually violate unitarity condition. Even in renormalizable theories and for small coupling constants, there is an energy above which the perturbation expansion, to any finite order, ceases to be meaningful. In the spesific example discussed above, $K^\alpha K^\beta/m_W^2$ term in Eq.(I.2.4) causes the theory to be unrenormalizable and violates unitarity. This term arises from the longitudinal polarization of the W-boson and it is a property common to all massive spin one particles. The introduction of W-boson also introduces new divergencies in other processes. As first explained by M. Gell-Mann, M.L. Goldberger, N. Kroll, F.E. Low(6,18) the most celebrated example is the reaction $\nu_e\bar{\nu}_e \rightarrow W^+W^-$.



(I.2.10)

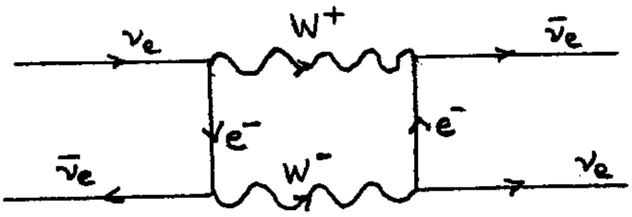
where ε_+ and ε_- are polarizations of W^+ and W^- respectively. At high energies,

$$\mathcal{M} \sim \frac{G}{\sqrt{2}} \bar{v}_{\bar{\nu}_e} \not{\varepsilon}_- (1-\gamma_5) \frac{1}{\not{p}-m_e} \not{\varepsilon}_+ (1-\gamma_5) u_{\nu_e} \qquad \text{(I.2.11)}$$

so,

$$|\mathcal{M}|^2 \sim s^2 \qquad \text{(I.2.12)}$$

and $\mathcal{M}$ grows quadratically with c.m. energy. Equivalently the second order contribution to the $\nu_e \bar{\nu}_e$ elastic scattering amplitude

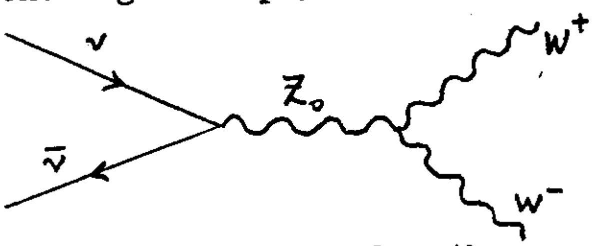


(I.2.13)

is also proportional to s^2. Since the $K^\alpha K^\beta / M_W^2$ piece of the W_α propagator give rise to more severe divergencies in higher orders it is necessary to find some means of eradicating the divergence order by order.

I.3. WEİNBERG-SALAM MODEL

One possibility, as proposed by S. Weinberg in 1967 is to introduce a new neutral particle in the s-channel, and adjust its couplings to cancel the undesirable growth of the electron exchange in Eq.(I.2.10)[4,5,6,7,8,20].



(I.3.1)

Since Z_o is a neutral particle, the new model predicts neutral currents which were first observed at CERN[5,7,8,9,22]. However it turns out that neutral currents are not sufficient to cancel all the divergencies or the leading high energy behaviour of all "bad" graphs in the theory. Recent developments in the last two decades made obvious that an acceptable theory which is renormalizable, is a gauge invariant theory.

The Physical appeal of gauge invariance stems from the old observation (Noether's theorem) that to every continous symmetry of the Lagrangian there corresponds a conservation law. In any field theory the classical action "I" constructed from the Lagrangian is,

$$I = \int_{-\infty}^{\infty} dt\, L(t) = \int d^4x\, \mathcal{L}\left(\psi(x), \partial_\mu \psi(x)\right) \tag{I.3.2}$$

The equation of motion is obtained by Hamilton's principle,

$$\delta \int_{t_1}^{t_2} dt\, L(t) = 0 \tag{I.3.3}$$

which gives the Euler-Lagrange equations if the variations of the endpoints are fixed.

$$\frac{\delta \mathcal{L}}{\delta \psi} - \partial^\mu \frac{\delta \mathcal{L}}{\delta(\partial_\mu \psi)} = 0 \tag{I.3.4}$$

Now, consider a transformation of the fields in the form,

$$\psi(x) \rightarrow e^{iq\alpha}\psi(x) \;, \quad \alpha \neq \alpha(x) \tag{I.3.5a}$$

$$\partial_\mu \psi(x) \rightarrow e^{iq\alpha}\partial_\mu \psi(x) \tag{I.3.5b}$$

which lead to the infinitesimal variations,

$$\delta\psi = iq(\delta\alpha)\psi \tag{I.3.6a}$$

$$\delta(\partial_\mu\psi) = iq(\delta\alpha)\partial_\mu\psi \tag{I.3.6b}$$

Such a space-time independent transformation is called a gauge transformation of the first kind or a global transformation. If the Lagrangian is invariant under such a global gauge transformation it should satisfy,

$$\delta\mathcal{L} = 0 \tag{I.3.7}$$

Explicitly,

$$\delta\mathcal{L} = \frac{\delta\mathcal{L}}{\delta\psi}\delta\psi + \frac{\delta\mathcal{L}}{\delta\psi^*}\delta\psi^* + \frac{\delta\mathcal{L}}{\delta(\partial_\mu\psi)}\delta(\partial^\mu\psi) + \frac{\delta\mathcal{L}}{\delta(\partial_\mu\psi^*)}\delta(\partial^\mu\psi^*) \quad (\text{I.3.8})$$

Using Eqns.(I.3.4-6a-6b),

$$\begin{aligned}\delta\mathcal{L} &= \partial^\mu \frac{\delta\mathcal{L}}{\delta(\partial_\mu\psi)} iq(\delta\alpha)\psi + \partial^\mu \frac{\delta\mathcal{L}}{\delta(\partial_\mu\psi^*)}(-iq)(\delta\alpha)\psi^* \\ &\quad + \frac{\delta\mathcal{L}}{\delta(\partial_\mu\psi)} iq(\delta\alpha)\partial^\mu\psi + \frac{\delta\mathcal{L}}{\delta(\partial_\mu\psi^*)}(-iq)(\delta\alpha)\partial^\mu\psi^* \\ &= i(\delta\alpha)\partial^\mu\left(\frac{\delta\mathcal{L}}{\delta(\partial_\mu\psi)} q\psi - \frac{\delta\mathcal{L}}{\delta(\partial_\mu\psi^*)} q\psi^*\right) \qquad (\text{I.3.9})\end{aligned}$$

Defining the current as,

$$j_\mu = \frac{\delta\mathcal{L}}{\delta(\partial_\mu\psi)} q\psi - \frac{\delta\mathcal{L}}{\delta(\partial_\mu\psi^*)} q\psi^* \qquad (\text{I.3.10})$$

it is easily seen that the invariance in Eq.(I.3.7) leads to a conserved quantity,

$$\partial^\mu j_\mu = 0 \qquad (\text{I.3.11})$$

However, the gauge transformation considered here is restricted, since the transformation is merely a phase transformation. In other words the gauge group is simply the rotation group U(1) or O(2) and the parameter " α " is space-time independent. Therefore, one can consider other groups and space-time dependent α which will lead to gauge transformation of second kind. A hint about the relevant gauge group can be obtained from the Lagrangian Eq.(I.2.1), (Henceforth only the electronic part of the Lagrangian will be written. To obtain the full Lagrangian same expression should be added with the substitution $e \rightarrow \mu$)

$$\begin{aligned}\mathcal{L} &= -\frac{g}{2\sqrt{2}}\left(\bar{\nu}_e \gamma_\alpha (1-\gamma_5) e W_+^\alpha + \bar{e}\gamma_\alpha(1-\gamma_5)\nu_e W_-^\alpha\right) \\ &= -\frac{g}{\sqrt{2}}\left\{(\bar{\nu}_e, \bar{e})(1+\gamma_5)\not{W}_+ (1-\gamma_5)\tau^+ \binom{\nu_e}{e}\right. \\ &\quad \left. + (\bar{\nu}_e, \bar{e})(1+\gamma_5)\not{W}_- (1-\gamma_5)\tau^- \binom{\nu_e}{e}\right\} \qquad (\text{I.3.12})\end{aligned}$$

where

$$\tau^{\pm} = \frac{1}{2}\left(\tau_1 \pm i\tau_2\right) \quad . \tag{I.3.13}$$

This form suggest that it is possible to define fields transforming as a left handed doublet under the "weak" SU(2) group,

$$L = \frac{1-\gamma_5}{2}\begin{pmatrix}\nu_e \\ e\end{pmatrix} \ , \tag{I.3.14a}$$

and,

$$\bar{L} = \left(\bar{\nu}_e, \bar{e}\right)\frac{1+\gamma_5}{2} \ . \tag{I.3.14b}$$

Describing the weak boson fields as,

$$W_{\pm}^{\alpha} = \frac{W_1^{\alpha} \mp i W_2^{\alpha}}{\sqrt{2}} \tag{I.3.15}$$

Eq.(I.3.12) takes the following form,

$$\mathcal{L} = -g\left(\bar{L}\,\frac{\tau_1}{2}.\not{W}_1 L + \bar{L}\,\frac{\tau_2}{2}.\not{W}_2 L\right) \ . \tag{I.3.16}$$

Obviously a " τ_3 " part is absent, and a neutral interaction with a neutral gauge boson Z_0 must be included. Since, a priori, nothing is known about the coupling of Z_0 to neutrinos and leptons, one should consider the most general renormalizable coupling. A vector boson coupling to the neutral lepton current with only left handed neutrinos is expressed by,

$$\mathcal{L}_Z = -\left\{g_1 \bar{e}\gamma_\alpha e + g_2 \bar{e}\gamma_\alpha\gamma_5 e + g_3 \bar{\nu}_e \gamma_\alpha (1-\gamma_5)\nu\right\} Z^{\alpha} \tag{I.3.17}$$

This Lagrangian may be written in terms of a singlet right handed electron field and the doublet L in Eq.(I.3.14). The right handed singlet is a new degree of freedom which automatically enters into the theory.

If the electromagnetic interactions of leptons are considered, the Lagrangian is given by,

$$\mathcal{L}_{em} = e_o \bar{e} \not{A} e \qquad (e_o\text{=electronic charge})$$

$$= e_o \bar{e}\left(\not{A}\frac{1+\gamma_5}{2} + \not{A}\frac{1-\gamma_5}{2}\right)e .$$

$$= e_o\left\{\bar{e}\not{A}\frac{1+\gamma_5}{2}e + (\bar{\nu}_e, \bar{e})\frac{1+\gamma_5}{2}\not{A}\begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}\frac{1-\gamma_5}{2}\begin{pmatrix}\nu_e\\ e\end{pmatrix}\right\}. \qquad (I.3.18)$$

Defining the right handed part of the electron field to be a singlet,

$$R = e_R = \frac{1+\gamma_5}{2} e , \qquad (I.3.19)$$

Eq.(I.3.18) takes the form,

$$\mathcal{L}_{em} = e_o\left\{\bar{R}\not{A}R + \frac{1}{2}\bar{L}\not{A}L - \bar{L}\not{A}\frac{\tau_3}{2}L\right\}. \qquad (I.3.20)$$

Writing the Lagrangian in Eq.(I.3.17) with R and L

$$\mathcal{L}_z = -\left\{f_1\bar{R}\gamma_\alpha R + \frac{f_2}{2}\bar{L}\gamma_\alpha L + f_3\bar{L}\gamma_\alpha\frac{\tau_3}{2}L\right\}Z^\alpha \qquad (I.3.21)$$

so that the τ_3 parts of the Lagrangians in Eqns.(I.3.20-21) are,

$$(\mathcal{L}_{em} + \mathcal{L}_z)_{\tau_3} = -\bar{L}\gamma_\alpha(e_o A^\alpha + f_3 Z^\alpha)\frac{\tau_3}{2}L . \qquad (I.3.22)$$

Adding the above expression to the Lagrangian in Eq.(I.3.16) the desired SU(2) gauge invariant interaction is obtained as,

$$\mathcal{L} + (\mathcal{L}_{em} + \mathcal{L}_z)_{\tau_3} = -\frac{1}{2}\bar{L}\left\{g\tau_1\not{W}_1 + g\tau_2\not{W}_2 + \sqrt{e_o^2+f_3^2}\,\tau_3\not{W}_3\right\}L \qquad (I.3.23)$$

where,

$$e_o A^\alpha + f_3 Z^\alpha = \sqrt{e_o^2 + f_3^2}\,W_3^\alpha . \qquad (I.3.24)$$

If,

$$g = \sqrt{e_o^2 + f_3^2} \qquad (I.3.25)$$

the above expression can be written with a single coupling constant,

$$\mathcal{L}+(\mathcal{L}_{em}+\mathcal{L}_{z})_{\tau_3}=-g\,\bar{L}\,\frac{\vec{\tau}}{2}\,L.\vec{W}, \qquad \text{(I.3.26)}$$

where,

$$\vec{W}=(W_1,W_2,W_3)\ . \qquad \text{(I.3.27)}$$

The remaining part of $\mathcal{L}_{em}+\mathcal{L}_{z}$ is,

$$e_0\bar{R}\not{A}R+e_0\frac{1}{2}\bar{L}\not{A}L-f_1\bar{R}\not{Z}R-\frac{1}{2}f_2\bar{L}\not{Z}L$$

$$=-\bar{R}(-e_0\not{A}+f_1\not{Z})R-\frac{1}{2}\bar{L}(-e_0\not{A}+f_2\not{Z})L\ . \qquad \text{(I.3.28)}$$

Defining the orthogonal combination,

$$Y_\alpha=\frac{-e_0 Z_\alpha+f_3A_\alpha}{\sqrt{e_0^2+f_3^2}} \qquad \text{(I.3.29)}$$

the total interaction Lagrangian takes the form,

$$\mathcal{L}_{int}=\mathcal{L}+\mathcal{L}_{em}+\mathcal{L}_{z}=-g\bar{L}\frac{\vec{\tau}}{2}.\vec{W}L+g'\left(\frac{1}{2}\bar{L}\not{Y}L+\bar{R}\not{Y}R\right). \qquad \text{(I.3.30)}$$

The coefficient of electromagnetic current contained in Y_α must be equal to electronic charge so that,

$$e_0=\frac{g'f_3}{\sqrt{e_0^2+f_3^2}} \qquad \text{(I.3.31)}$$

Using Eq.(I.3.25) and Eq.(I.3.31) one obtains,

$$e_0=\frac{g\,g'}{\sqrt{g^2+g'^2}}\ . \qquad \text{(I.3.32)}$$

The only parameter in the model θ_W the Weinberg angle is defined by,

$$\tan\theta_W=\frac{g'}{g} \qquad \text{(I.3.33)}$$

Thus ,one obtains the following relations,

$$e_0=g'\cos\theta_W=g\sin\theta_W\quad,\quad f_3=g\cos\theta_W \qquad \text{(I.3.34)}$$

Interms of Weinberg angle the fields W_3^α and Y^α are written as follows,

$$W_3^\alpha = \cos\theta_W Z^\alpha + \sin\theta_W A^\alpha \tag{I.3.35a}$$

$$Y_\alpha = -\sin\theta_W Z^\alpha + \cos\theta_W A^\alpha . \tag{I.3.35b}$$

Assuming that Gell-Mann Nishijima formula is also valid in electro-weak interactions one can define a weak "hypercharge" $\mathcal{Y}$ such that,

$$Q = T_3 + \mathcal{Y}/2 . \tag{I.3.36}$$

Then it follows that,

$$\mathcal{Y} R = (-2) R \tag{I.3.37a}$$

$$\mathcal{Y} L = (-1) L \tag{I.3.37b}$$

This makes meaningful the coefficient $1/2$ appearing in front of the second term in Eq.(I.3.30). The free Lagrangian without a mass term is,

$$\mathcal{L}_{free} = \bar{L} i \not{\partial} L + \bar{R} i \not{\partial} R \tag{I.3.38}$$

The interaction Lagrangian can be obtained from Eq.(I.3.38) by the following minimal substitutions,

$$i\partial_\mu (\text{on } R) \longrightarrow i\partial_\mu + g' Y_\mu \tag{I.3.39a}$$

$$i\partial_\mu (\text{on } L) \longrightarrow i\partial_\mu - g \frac{\vec{\tau}}{2} \cdot \vec{W}_\mu + g' \frac{1}{2} Y_\mu . \tag{I.3.39b}$$

As it can easily be seen Eq.(I.3.30) both contains electromagnetic and weak interactions. Eq.(I.3.30) is not simply the sum of independent weak and electromagnetic parts as is seen in the reaction $\nu\bar{\nu} \to W^+ W^-$ previously discussed. The quadratic leading order growth of the two weak amplitudes does not in this case compensate, but it cancells against the growth of electromagnetic term. As pointed out by weinberg "this cooperation of weak and electromagnetic currents in solving each others problems is one of the most satisfying feature of the theory[4,24]."

I.4. GAUGE THEORY OF WEAK AND ELECTROMAGNETIC INTERACTIONS

A gauge theory for strong interactions is first attempted by Yang and Mills in 1954 for the gauge group SU(2)(17). If L is subject the gauge transformation,

$$L' = e^{-ig\vec{\alpha}(x)\cdot\frac{\vec{\tau}}{2} + i\frac{g'}{2}\alpha(x)} L \,, \qquad \text{(I.4.1)}$$

where the infinitesimal form becomes,

$$L' = \left(1 - ig\vec{\alpha}(x)\cdot\frac{\vec{\tau}}{2} + i\,\frac{g'}{2}\alpha(x)\right)L \,, \qquad \text{(I.4.2)}$$

and the transformation property of the remaining fields are,

$$R' = \left(1 + i\,\frac{g'}{2}\alpha\right)R \,, \qquad \text{(I.4.3a)}$$

$$\vec{W}'_\mu = \vec{W}_\mu + \partial_\mu\vec{\alpha} + g\vec{\alpha}\times\vec{W}_\mu \,, \qquad \text{(I.4.3b)}$$

$$Y'_\mu = Y_\mu + \partial_\mu\alpha \qquad \text{(I.4.3c)}$$

then the Lagrangians in Eqns.(I.4.4a-4b) are SU(2)xU(1) gauge invariant,

$$\mathcal{L}_1 = \bar{L}\left(i\partial\!\!\!/ - g\frac{\vec{\tau}}{2}\cdot\vec{W\!\!\!/} + \frac{1}{2}g'Y\!\!\!/\right)L \,, \qquad \text{(I.4.4a)}$$

$$\mathcal{L}_2 = \bar{R}\left(i\partial\!\!\!/ + \frac{1}{2}g'Y\!\!\!/\right)R \,. \qquad \text{(I.4.4b)}$$

Defining the field tensor $\vec{F}_{\mu\nu}$ as,

$$\vec{F}_{\mu\nu} = \partial_\mu\vec{W}_\nu - \partial_\nu\vec{W}_\mu - g\vec{W}_\mu\times\vec{W}_\nu \,, \qquad \text{(I.4.5)}$$

the transformation of $\vec{F}_{\mu\nu}$ under Eq.(I.4.2) is seen to be,

$$\vec{F}'_{\mu\nu} = \vec{F}_{\mu\nu} + \vec{\alpha}\times\vec{F}_{\mu\nu} \,, \qquad \text{(I.4.6)}$$

leading to gauge invariant kinetic energy term for the W_α's,

$$\mathcal{L}_3 = -\frac{1}{4}\vec{F}_{\mu\nu}\cdot\vec{F}^{\mu\nu} \,. \qquad \text{(I.4.7)}$$

The Euler-Lagrange equation of motion for the non-interacting gauge bosons is,

$$\partial^{\mu}\frac{\delta\mathcal{L}_3}{\delta(\partial_{\mu}\vec{W}_{\nu})}=\frac{\delta\mathcal{L}_3}{\delta\vec{W}_{\nu}}\ . \qquad (I.4.8)$$

From Eq.(I.4.7) one obtains,

$$\frac{\delta\mathcal{L}_3}{\delta(\partial_{\mu}\vec{W}_{\nu})}=-\frac{1}{4}\vec{F}_{\mu\nu} \qquad (I.4.9a)$$

$$\frac{\delta\mathcal{L}_3}{\delta\vec{W}_{\nu}}=\frac{1}{4}g\vec{W}_{\mu}\times\vec{F}^{\mu\nu}\ . \qquad (I.4.9b)$$

Substitution of Eqns.(I.4.9a-9b) into Eq.(I.4.8) gives,

$$\partial^{\mu}\vec{F}_{\mu\nu}+g\vec{W}_{\mu}\times\vec{F}^{\mu\nu}=0\ . \qquad (I.4.10)$$

This is a non-linear equation which shows that non-abelian fields are carrying isospin and their source is their current.Also defining,

$$f_{\mu\nu}=\partial_{\mu}Y_{\nu}-\partial_{\nu}Y_{\mu}\ , \qquad (I.4.11)$$

it can easily seen that,

$$\mathcal{L}_4=-\frac{1}{4}f_{\mu\nu}f^{\mu\nu} \qquad (I.4.12)$$

is also gauge invariant.

The sum of the Lagrangians given in Eqns.(I.4.4a-4b-7-12) describing the interactions involving $\vec{W}_{\alpha}$, Y_{α} and the leptons becomes,

$$\mathcal{L}=-\frac{1}{4}\vec{F}_{\mu\nu}\cdot\vec{F}^{\mu\nu}-\frac{1}{4}f_{\mu\nu}f^{\mu\nu}+\bar{R}(i\not{\partial}+\frac{1}{2}g'Y_{\mu})R$$
$$+\bar{L}(i\not{\partial}-g\frac{\vec{\tau}}{2}\cdot\vec{W}+\frac{1}{2}g'Y_{\mu})L\ . \qquad (I.4.13)$$

This Lagrangian contains no mass terms either for the leptons or the gauge bosons. Therefore the gauge bosons described by this Lagrangian are massless contradicting nature, where only massless boson is the photon. Direct introduction of mass terms into the Lagrangian in Eq.(I.4.13) destroys gauge invariance, and the renormalizability of the theory.

I.5. HIGGS MECHANISM AND SPONTANEOUSLY BROKEN GAUGE THEORİES

Without destroying renormalizability and gauge invariance of the theory, the masses are generated through spontaneous symmetry breaking. To be able to understand Higgs Mechanism and spontaneous symmetry breaking(3,4,5,10,29,30,31,32) consider an abelian gauge theory containing only scalar and vector particles φ and A_μ . The Lagrangian is,

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \left[(\partial^\mu + ieA^\mu)\varphi^+\right]\left[(\partial_\mu - ieA_\mu)\varphi\right] - \mu^2\varphi^+\varphi - h(\varphi^+\varphi)^2 , \qquad \text{(I.5.1)}$$

being locally invariant under the transformations,

$$\varphi' = e^{-i\alpha(x)}\varphi \qquad \text{(I.5.2a)}$$

$$A'_\mu = A_\mu + \frac{1}{e}\partial_\mu\alpha(x) , \qquad \text{(I.5.2b)}$$

with φ expressed as,

$$\varphi = \frac{\varphi_1 + i\varphi_2}{\sqrt{2}} . \qquad \text{(I.5.3)}$$

Consider only the part of Lagrangian for φ .

$$\mathcal{L}_\varphi = \partial^\mu\varphi^+\partial_\mu\varphi - \mu^2\varphi^+\varphi - h(\varphi^+\varphi)^2 , \qquad \text{(I.5.4)}$$

The equation of motion now is,

$$(\Box + \mu^2)\varphi = -2h(\varphi^+\varphi)\varphi \qquad \text{(I.5.5)}$$

If φ is assumed to be constant i.e. the vacuum expectation value of the field φ , one has,

$$[\mu^2+2h(\varphi^+\varphi)]\varphi=0 \,, \tag{I.5.6}$$

and under the conditions $h>0$, $\mu^2>0$ the solution is,

$$\langle\varphi\rangle_0=0 \,. \tag{I.5.7}$$

But, under the conditions $h>0$, $\mu^2<0$ another solution is possible,

$$\langle\varphi^+\varphi\rangle_0=-\frac{\mu^2}{2h}=\frac{\lambda^2}{2} \,, \tag{I.5.8}$$

from which it follows that,

$$\langle\varphi\rangle_0=\frac{\lambda}{\sqrt{2}}\,e^{i\beta} \,. \tag{I.5.9}$$

The new field with zero vacuum expectation value is,

$$\varphi=\frac{\lambda+\Phi_1+i\Phi_2}{\sqrt{2}} \tag{I.5.10}$$

assuming that $\langle\Phi_1\rangle_0=\langle\Phi_2\rangle_0=0$. Using Eq.(I.5.10) and separating real and imaginary parts one obtains,

$$(\Box+\mu^2+3h\lambda^2)\Phi_1=-\mu^2\lambda-h\lambda^3+\cdots \tag{I.5.11a}$$

$$(\Box+\mu^2+h\lambda^2)\Phi_2=0+\cdots \tag{I.5.11b}$$

with $\mu^2+h\lambda^2=0$. Thus, the new fields are seen to have masses $m_1^2=-2\mu^2>0$ and $m_2^2=0$. One of the fields gaines mass and the other remaines massless. This situation arises from the fact that $\lambda\neq0$. The symmetry of the lowest energy state is not broken by the Lagrangian but by the vacuum itself. Such symmetry breakings are called "spontaneously broken symmetries" .

Taking into consideration the Lagrangian in Eq.(I.5.1) the equations of motion,

$$\partial^\mu \frac{\delta \mathcal{L}}{\delta(\partial^\mu A^\nu)} - \frac{\delta \mathcal{L}}{\delta A^\nu} = 0 \quad , \quad \partial^\mu \frac{\delta \mathcal{L}}{\delta(\partial^\mu \varphi^+)} - \frac{\delta \mathcal{L}}{\delta \varphi^+} = 0$$

lead to,

$$-\partial^\mu F_{\mu\nu} = e^2\lambda^2\left(A^\nu - \frac{1}{e\lambda}\partial^\nu \phi_2\right) + \cdots \qquad \text{(I.5.12a)}$$

$$(\Box + \mu^2 + 2h\varphi^+\varphi)\varphi = ie(\partial^\mu A_\mu)\varphi + 2ieA^\mu\partial_\mu\varphi + e^2A^\mu A_\mu\varphi. \qquad \text{(I.5.12b)}$$

Defining a new field,

$$B_\mu = A_\mu - \frac{1}{e\lambda}\partial_\mu \phi_2 \qquad \text{(I.5.13)}$$

one has,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = \partial_\mu B_\nu - \partial_\nu B_\mu = \widetilde{F}_{\mu\nu} \qquad \text{(I.5.14)}$$

so that,

$$-\partial^\mu \widetilde{F}_{\mu\nu} \approx e^2\lambda^2 B^\nu \qquad \text{(I.5.15)}$$

describing a massive vector field, the mass being $m = |e\lambda|$. The original massless vector field has acquired a longitudinal component $-e\lambda\partial_\mu\phi_2$ allowing it to have a non zero mass. In terms of new fields ϕ_1 and ϕ_2 Eq.(I.5.12b) becomes,

$$(\Box - 2\mu^2)\phi_1 + \Box i\phi_2 \approx i\Box\phi_2 + \cdots \qquad \text{(I.5.16)}$$

which is the Klein-Gordon equation for ϕ_1, and ϕ_2 is eaten up by vector field. The disappearance of ϕ_2 may be understood in another way. The defining equation of the new field B_μ Eq.(I.5.13), is akin to a gauge transformation with a gauge function $\alpha = -\phi_2/\lambda$. The Lagrangian in Eq.(I.5.1) is invariant if $\varphi \to \frac{\lambda+\phi_1}{\sqrt{2}} e^{i\phi_2/\lambda}$ simultaneously rotated. This form shows that shifted fields correspond to a gauge transformation if only linear terms are considered. Interms of $\frac{\lambda+\phi_1}{\sqrt{2}}$ and B_μ the

Lagrangian becomes,

$$\mathcal{L} = -\frac{1}{4}\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{1}{2}\{\partial_\mu\phi_1\partial^\mu\phi_1 + e^2 B_\mu B^\mu(\lambda+\phi_1)^2$$

$$-\mu^2(\lambda+\phi_1)^2 - \frac{h}{2}(\lambda+\phi_1)^4\}, \qquad \text{(I.5.17)}$$

showing explicitly the massive vector field B_μ and the massive Higgs scalar ϕ_1. The mechanism described above is called the Higgs mechanism.

The gauge boson masses can now be generated if the SU(2) symmetry of the Lagrangian in Eq.(I.4.13) is spontaneously broken. Consider the Higgs Lagrangian in Eq.(I.5.4) with $\mu^2<0$ and φ being SU(2) doublet scalar field,

$$\varphi = \begin{pmatrix}\varphi_+\\ \varphi_0\end{pmatrix} \qquad \text{(I.5.18)}$$

Such a choice is consistent because,

$$Y\varphi = +1\varphi \qquad \text{(I.5.19)}$$

and if the coupling of φ to the gauge fields is introduced through the covariant derivative

$$D_\mu(\varphi) = \partial_\mu + ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu - ig'\frac{1}{2}Y_\mu$$

the coupling $-g'/2$ to the hypercharge field is also consistent with φ having opposite hypercharge to the doublet L. Defining the new fields after the spontaneous breaking to be,

$$\begin{pmatrix}\varphi^+\\ \varphi_0\end{pmatrix} \longrightarrow \begin{pmatrix}0\\ \frac{\lambda+\phi_0}{\sqrt{2}}\end{pmatrix} \qquad \text{(I.5.20)}$$

the kinetic term becomes,

$$D^\mu\varphi^+ D_\mu\varphi = [(\partial^\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{W}^\mu + ig'\frac{1}{2}Y^\mu)\varphi^+]$$

$$\times[(\partial_\mu + ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu - ig'\frac{1}{2}Y_\mu)\varphi]$$

$$= \left|\frac{1}{\sqrt{2}}\{\partial_\mu\begin{pmatrix}0\\ \lambda+\phi_0\end{pmatrix} + ig\,\frac{W_{1\mu}-iW_{2\mu}}{2}\begin{pmatrix}\lambda+\phi_0\\ 0\end{pmatrix} - \frac{i}{2}\sqrt{g^2+g'^2}\,Z_\mu\begin{pmatrix}0\\ \lambda+\phi_0\end{pmatrix}\right|^2$$

$$= \frac{1}{2}\partial^\mu\phi_0\partial_\mu\phi_0 + \frac{g^2}{8}|W_{1\mu}-iW_{2\mu}|^2(\lambda+\phi_0)^2 + \frac{g^2+g'^2}{8}Z_\mu^2(\lambda+\phi_0)^2 \qquad \text{(I.5.21)}$$

so that the mass terms for $W^{\pm}$ and Z_0 are given by,

$$m_W^2 = \frac{1}{4} g^2 \lambda^2 \quad , \tag{I.5.22a}$$

$$m_Z^2 = \frac{1}{4}(g^2 + g'^2)\lambda^2 . \tag{I.5.22b}$$

However the electron and muon are still massless. A Yukawa-type of interaction between leptons and φ can be considered in order to give mass to the leptons. An SU(2)xU(1) gauge invariant yukawa interaction is,

$$\mathcal{L}_{int}(\varphi, \text{leptons}) = -f_e(\bar{R}\varphi^{+}L + \bar{L}\varphi R) . \tag{I.5.23}$$

Using Eq.(I.5.20), Eq.(I.5.23) takes the form,

$$\begin{aligned} \mathcal{L}_{int}(\varphi, \text{leptons}) &= -\frac{f_e}{\sqrt{2}}\Big\{\bar{e}\,\frac{1-\gamma_5}{2}(0, \lambda+\phi_0)\,\frac{1-\gamma_5}{2}\begin{pmatrix}\nu_e\\ e\end{pmatrix} \\ &\quad + (\bar{\nu}_e, \bar{e})\,\frac{1+\gamma_5}{2}\begin{pmatrix}0\\ \lambda+\phi_0\end{pmatrix}\frac{1+\gamma_5}{2}\,e\Big\} \\ &= -\frac{f_e}{\sqrt{2}}(\lambda+\phi_0)\,\bar{e}e \end{aligned} \tag{I.5.24}$$

leading to the electron mass,

$$m_e = \frac{1}{\sqrt{2}} f_e \lambda \tag{I.5.25}$$

while the neutrino stays massless.

The most general SU(2)xU(1) gauge invariant and renormalizable Lagrangian of weak and electromagnetic interactions is then the sum of the Lagrangians in Eq.(I.4.13), Eq.(I.5.4) and Eq.(I.5.23). Explicitly written as,

$$\mathcal{L} = -\frac{1}{4}\left(\partial_\mu \vec{W}_\nu - \partial_\nu \vec{W}_\mu + g\vec{W}_\mu \times W_\nu\right)^2 - \frac{1}{4}\left(\partial_\mu Y_\nu - \partial_\nu Y_\mu\right)^2$$

$$+ (\bar{\nu}_e, \bar{e})\left(i\not\partial - g\vec{\not W}\cdot\frac{\vec{\tau}}{2} + g'\frac{1}{2}\not Y\right)\begin{pmatrix}\nu_e \\ e\end{pmatrix} + \bar{e}\,\frac{1+\gamma_5}{2}\left(i\not\partial + g'\not Y\right)\frac{1-\gamma_5}{2}e$$

$$+ \left|\left(\partial_\mu + ig\vec{W}_\mu\cdot\frac{\vec{\tau}}{2} - ig'\frac{1}{2}Y_\mu\right)\varphi\right|^2 - \mu^2|\varphi|^2 - h\left(|\varphi|^2\right)^2$$

$$- f_e\left(\bar{e}\,\frac{1-\gamma_5}{2}(\varphi_+, \varphi_0)\,\frac{1-\gamma_5}{2}\begin{pmatrix}\nu_e \\ e\end{pmatrix} + (\bar{\nu}_e, \bar{e})\,\frac{1+\gamma_5}{2}\begin{pmatrix}\varphi_+ \\ \varphi_0\end{pmatrix}\frac{1+\gamma_5}{2}e\right).$$

(I.5.26)

The interaction part becomes,

$$\mathcal{L}_{Int.} = -\frac{1}{4}\left\{g^2\left(\vec{W}_\mu \times \vec{W}_\nu\right)\cdot\left(\vec{W}^\mu \times \vec{W}^\nu\right) - 2g\left(\vec{W}_\mu \times \vec{W}_\nu\right)\cdot\left(\partial^\mu \vec{W}^\nu - \partial^\nu \vec{W}^\mu\right)\right\}$$

$$+ e_0\left(\bar{e}\gamma^\mu e\right)A_\mu + e_0\tan\theta_W\, Z_\mu\left(-\bar{e}\gamma^\mu e - \operatorname{cosec}^2\theta_W\,\bar{L}\gamma^\mu\frac{\tau_3}{2}L\right)$$

$$-\frac{1}{2}g\left[\left(\bar{\nu}_e\gamma^\mu\frac{1-\gamma_5}{2}e\right)W_\mu^+ + h.c.\right] + \frac{1}{4}g^2|W_\mu|^2\left(\phi_0^2 + 2\lambda\phi_0\right)$$

$$+ \frac{g^2+g'^2}{8}Z_\mu^2\left(\phi_0^2 + 2\lambda\phi_0\right) - \frac{m_e}{\lambda}(\bar{e}e)\phi_0 - \frac{1}{4}h\left(\phi_0^4 + 4\lambda\phi_0^3\right)$$

(I.5.27)

The above Lagrangian describes a unified theory of weak and electromagnetic interactions, the so called Weinberg-Salam model[3,4,5,7,8,9,20,21].

Upon comparison of Eq.(I.5.22a) and Eq.(I.5.22b) it is seen that $m_Z > m_W$. The coupling of the leptonic current to the W-boson (Eq.(I.5.27)) must be equal to the coupling constant (Eq.(I.2.1)) f, therefore,

$$f = \frac{1}{2\sqrt{2}}\,g\;. \qquad \text{(I.5.28)}$$

Using Eq.(I.2.5) one has,

$$\frac{G}{\sqrt{2}} = \frac{g^2}{8m_W^2}\;. \qquad \text{(I.5.29)}$$

Substitution of m_W^2 from Eq.(I.5.22a) finally gives,

$$\lambda^2 = \frac{1}{\sqrt{2}\,G} \qquad (I.5.30)$$

Solving Eq.(I.5.29) for m_W^2 and using Eq.(I.3.34) one has,

$$m_W^2 = \frac{e_0^2\sqrt{2}}{8G\sin^2\theta_W} \geqslant \frac{\alpha\pi}{G\sqrt{2}} \qquad (I.5.31)$$

where $e_0^2 = 4\pi\alpha$, α being the fine structure constant. Using the experimental values, the lower limits for the masses become,

$$m_W \geqslant 37.3\ \mathrm{GeV}, \qquad (I.5.32)$$

$$m_Z^2 = \frac{e_0^2\sqrt{2}}{2G\sin^2 2\theta_W} \geqslant (74.6)^2\,\mathrm{GeV}^2. \qquad (I.5.33)$$

I.6. INCLUSION OF HADRONS IN THE WEINBERG-SALAM MODEL

In order to include hadrons into this scheme, the enlargement of the SU(2)xU(1) gauge group into at least SU(3) is required, if Cabibbo hypothesis and only three quarks are considered. According to Cabibbo's picture of weak interaction universality, the hadronic charged current is represented by the expression,

$$j_\alpha = \bar{u}\gamma_\alpha(1-\gamma_5)(d\cos\theta_c + s\sin\theta_c) \qquad (I.6.1)$$

where u, d, s are Gell-Mann quarks and θ_c is the Cabibbo angle. One can form the left handed quark doublet,

$$N_L = \frac{1-\gamma_5}{2}\begin{pmatrix} u \\ d_\theta \end{pmatrix} \qquad (I.6.2)$$

where,

$$d_\theta = d\cos\theta_c + s\sin\theta_c .$$

The SU(2) raising and lowering operators $\tau^{\pm}$, will then generate j_κ and j_κ^{+}. The remaining left handed component,

$$(s_\theta)_L = \frac{1-\gamma_5}{2}\left(-d\sin\theta_c + s\cos\theta_c\right) \tag{I.6.3a}$$

and the three right handed components,

$$u_R = \frac{1+\gamma_5}{2}u \quad , \quad (d_\theta)_R = \frac{1+\gamma_5}{2}\left(d\cos\theta_c + s\sin\theta_c\right)$$

$$(s_\theta)_R = \frac{1+\gamma_5}{2}\left(-d\sin\theta_c + s\cos\theta_c\right) \tag{I.6.3b}$$

may be assigned to the singlet represantations of the $SU(2)_L$. The couplings to the U(1) gauge vector meson Y_μ are adjusted as before, to get the correct charges. The coupling to the Z-boson is then completely determined. It has the form,

$$e_0\tan\theta_W Z^\mu\left[j_\mu^{em} - \mathrm{cosec}^2\theta_W \bar{N}_L\gamma_\mu\frac{\tau^3}{2}N_L\right]. \tag{I.6.4}$$

The last term of the above expression,

$$\bar{u}\gamma_\mu(1-\gamma_5)u - \bar{d}\gamma_\mu(1-\gamma_5)d\cos^2\theta_c - \bar{s}\gamma_\mu(1-\gamma_5)s\sin^2\theta_c$$

$$-\left(\bar{d}\gamma_\mu(1-\gamma_5)s + \bar{s}\gamma_\mu(1-\gamma_5)d\right)\cos\theta_c\sin\theta_c \tag{I.6.5}$$

is a strangeness changing neutral current and completely is unacceptable. It will lead to transitions of the form,

$$d\bar{s} \to \mu^+\mu^-$$

via Z-exchange. The $d\bar{s}$ has the same quantum numbers as K^0, thus the existence of the strangeness changing neutral current will give rise to the decay,

$$K^0 \to \mu^+\mu^- .$$

The amplitude of this decay is of the same order of magnitude as that for $K^+ \to \mu^+\nu_\mu$ (3). Thus the branching ratio $\frac{\Gamma(K_L \to \mu^+\mu^-)}{\Gamma(K_L)}$

becomes of the order of unity, whereas experimentally it is less than 10^{-5}. Further, the same Z-exchange mechanism will give rise to the transition,

$$d\bar{s} \longleftrightarrow s\bar{d}$$

or,

$$K^0 \longleftrightarrow \overline{K^0}$$

as a first order (in G) weak effect, which would lead to a mass difference of K_S^0 and K_L^0 of order G. Yet, this mass difference is entirely compatible with its being an order G^2 effect(3).

Glashow, Iliapoulos and Maiani (GIM Hypothesis)(5,23,24) proposed that the undesirable strangeness changing terms can be removed by introducing a fourth quark "c". Constructing a second doublet N_L' with the new quark c,

$$N_L' = \begin{pmatrix} c \\ s_\theta \end{pmatrix}_L \qquad \text{(I.6.6)}$$

results an additional contribution to the Z-interaction given by,

$$\overline{N_L'}\gamma_\mu \frac{\tau_3}{2} N_L' \sim \bar{c}\gamma_\mu(1-\gamma_5)c - \bar{d}\gamma_\mu(1-\gamma_5)d\sin^2\theta_c$$

$$-\bar{s}\gamma_\mu(1-\gamma_5)\cos^2\theta_c + \left(\bar{d}\gamma_\mu(1-\gamma_5)s + \bar{s}\gamma_\mu(1-\gamma_5)d\right)\cos\theta_c\sin\theta_c \qquad \text{(I.6.7)}$$

so the total neutral current becomes,

$$2\overline{N_L}\gamma_\mu\frac{\tau_3}{2}N_L + 2\overline{N_L'}\gamma_\mu\frac{\tau_3}{2}N_L' \sim \bar{u}\gamma_\mu(1-\gamma_5)u$$

$$+\bar{c}\gamma_\mu(1-\gamma_5)c - \bar{d}\gamma_\mu(1-\gamma_5)d - \bar{s}\gamma_\mu(1-\gamma_5)s \; . \qquad \text{(I.6.8)}$$

Thus, although the new quark c does not contribute to the strangeness changing neutral current, its existence makes possible to eliminate it.To the c quark a new quantum number "charm" which is conserved by strong interactions, is assigned.

The discovery of the family of $(c\bar{c})$ bound states known as psions[14,15] and the observation of charmed particles[16] which decay according to the $(c,s_\theta)_L$ pattern constitute a striking confirmation of the GIM hypothesis.

Thus, under the SU(2)xU(1) electro-weak gauge group the transformation properties of the quarks and the leptons would be as follows,

$$\text{doublets:} \quad \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L , \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L , \begin{pmatrix} u \\ d_\theta \end{pmatrix}_L , \begin{pmatrix} c \\ s_\theta \end{pmatrix}_L \qquad \text{(I.6.9a)}$$

$$\text{singlets:} \quad e_R , \mu_R ; u_R , d_R , c_R , s_R \quad . \qquad \text{(I.6.9b)}$$

The charged and neutral currents are[9] ,

$$j_\alpha = \bar{\nu}_e \gamma_\alpha \frac{1-\gamma_5}{2} e + \bar{\nu}_\mu \gamma_\alpha \frac{1-\gamma_5}{2} \mu + \bar{u} \gamma_\alpha \frac{1-\gamma_5}{2} d_\theta \cdot$$
$$+ \bar{c} \gamma_\alpha \frac{1-\gamma_5}{2} s_\theta \qquad \text{(I.6.10a)}$$

and,

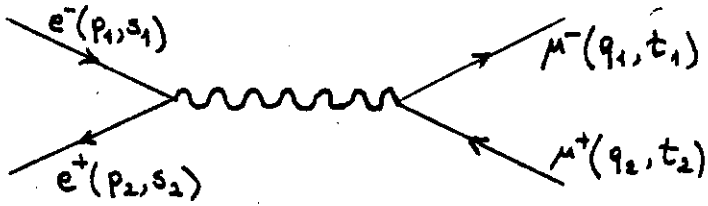
$$j_\alpha^0 = \sum_f \bar{f}_L \gamma_\alpha \frac{1-\gamma_5}{2} f_L \left[I_3(f_L) - Q(f_L) \sin^2\theta_W \right]$$
$$+ \sum_f \bar{f}_R \gamma_\alpha \frac{1+\gamma_5}{2} f_R \left[I_3(f_R) - Q(f_R) \sin^2\theta_W \right] \qquad \text{(I.6.10b)}$$

where $f = \nu_e , \nu_\mu , e , \mu , u , d , s , c .$

II. THE $e^+e^- \to \mu^+\mu^-$ PROCESS

II.1. KINEMATICS

The process $e^+e^- \to \mu^+\mu^-$ (studied in colliding beam machines) is possible only via neutral currents if lepton conservation is assumed. The lowest order diagram is,



(II.1.1)

The four-momentum and spin polarizations of corresponding particles are shown respectively in the paranthesis. In the lab. frame,

$$p_1 = \left(p_{10}^{l}, \vec{p}_1\right) \quad , \quad p_2 = (m_e, 0) \tag{II.1.2a}$$

and in the center of mass frame (cm),

$$p_1 = \left(p_{10}^{c}, \vec{p}\right) \quad , \quad p_2 = \left(p_{20}^{c}, -\vec{p}\right) . \tag{II.1.2b}$$

Since $p_1 \cdot p_2$ is a Lorentz-invariant,

$$p_{10}^{l}\, m_e = p_{10}^{c}\, p_{20}^{c} + |\vec{p}|^2 \tag{II.1.3}$$

from which, using Eqns.(II.1.2a-2b) it follows that,

$$\sqrt{m_e^2 + |\vec{p}_1|^2} = \sqrt{m_e^2 + |\vec{p}|^2}\sqrt{m_e^2 + |\vec{p}|^2} + |\vec{p}|^2 . \tag{II.1.4}$$

Therefore,

$$m_e^4 + |\vec{p}_1|^2 m_e^2 + |\vec{p}|^4 - 2|\vec{p}|^2\sqrt{m_e^2 + |\vec{p}_1|^2}\; m_e = m_e^4 + 2m_e^2|\vec{p}|^2 + |\vec{p}|^4 \tag{II.1.5}$$

$$|\vec{p}|^2 = \frac{|\vec{p}_1|^2 m_e^2}{2m_e^2 + 2p_{10}^{l} m_e} . \tag{II.1.6}$$

Since,

$$p_{10}^{c} = \sqrt{m_e^2 + |\vec{p}|^2} \qquad \text{(II.1.7)}$$

one obtains,

$$p_{10} = \frac{m_e^2 + p_{10}^{\ell} m_e}{\sqrt{2m_e^2 + 2p_{10}^{\ell} m_e}} \qquad \text{(II.1.8)}$$

while,

$$p_{10}^{c} = p_{20}^{c} \qquad \text{(II.1.9)}$$

The center of mass energy is,

$$s = (p_1 + p_2)^2 = (p_{10}^{\ell} + m_e\,,\ \vec{p}_1)^2$$

$$= 2m_e^2 + 2m_e p_{10}^{\ell} \qquad \text{(II.1.10)}$$

from which it follows that,

$$p_{10}^{\ell} = \frac{s - 2m_e^2}{2m_e} \ . \qquad \text{(II.1.11)}$$

Using Eq.(II.1.11) in Eq.(II.1.8),

$$p_{10}^{c} = \frac{m_e^2 + \frac{s - 2m_e^2}{2m_e} m_e}{\sqrt{s}}$$

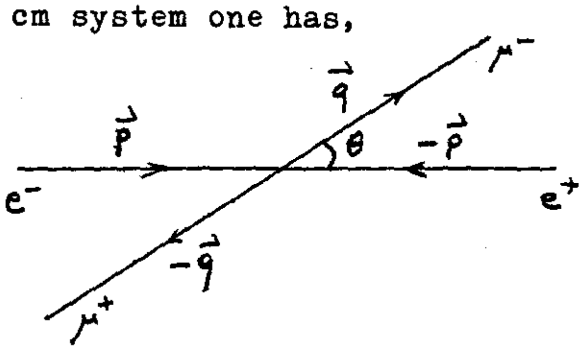
$$= \frac{\sqrt{s}}{2} = p_{20}^{c} \qquad \text{(II.1.12)}$$

The Eq.(II.1.6) takes the form,

$$|\vec{p}| = \frac{m_e\sqrt{(p_{10}^{\ell})^2 - m_e^2}}{\sqrt{s}}$$

$$= \frac{\sqrt{s^2 - 4sm_e^2}}{2\sqrt{s}} \ . \qquad \text{(II.1.14)}$$

In the cm system one has,



(II.1.15)

where θ is the scattering angle, so that the exchange momentum square is,

$$t = (p_1 - q_1)^2$$

$$= m_e^2 + m_\mu^2 + 2|\vec{p}||\vec{q}|\cos\theta - 2p_{10}^c q_{10}^c \ . \qquad \text{(II.1.16)}$$

Using Eqns.(II.1.12-14) one obtains,

$$t = m_e^2 + m_\mu^2 + \frac{\sqrt{s^2 - 4sm_e}\sqrt{s^2 - 4sm_\mu}}{2s}\cos\theta - \frac{s}{2} \ . \qquad \text{(II.1.17)}$$

In the high energy limit where m_e and m_μ are negligible these expressions reduces to,

$$|\vec{p}| = \frac{\sqrt{s}}{2} \qquad \text{(II.1.18a)}$$

and,

$$t = -\frac{s}{2}(1 - \cos\theta) \qquad \text{(II.1.18b)}$$

The third relativistic invariant is defined by,

$$u = (p_1 - q_2)^2 \qquad \text{(II.1.19)}$$

satisfying,

$$s + t + u = 2m_e^2 + 2m_\mu^2 \approx 0 \qquad \text{(II.1.20)}$$

which in the high energy limit becomes

$$u = -\frac{s}{2}(1 + \cos\theta) \ . \qquad \text{(II.1.21)}$$

Abbreviating $\cos\theta = z$, the Eqns.(II.1.18b-21) takes the form,

$$t = -\frac{s}{2}(1 - z) \ , \quad u = -\frac{s}{2}(1 + z). \qquad \text{(II.1.22)}$$

Using the energy-momentum conservation relation,

$$p_1 + p_2 = q_1 + q_2 \qquad \text{(II.1.23)}$$

the following relations are obtained:

$$s=(p_1+p_2)^2=2m_e^2+2p_1\cdot p_2\approx 2p_1\cdot p_2 \Rightarrow p_1\cdot p_2\approx q_1\cdot q_2\approx \frac{s}{2} \quad \text{(II.1.24a)}$$

$$t=(p_1-q_1)^2=m_e^2+m_\mu^2-2p_1\cdot q_1\approx -2p_1\cdot q_1$$

$$\therefore \quad p_1\cdot q_1\approx p_2\cdot q_2\approx \frac{s}{4}(1-z) \quad \text{(II.1.24b)}$$

$$u=(p_1-q_2)^2=m_e^2+m_\mu^2-2p_1\cdot q_2\approx -2p_1\cdot q_2$$

$$\therefore \quad p_1\cdot q_2\approx p_2\cdot q_1\approx \frac{s}{4}(1+z) \quad \text{(II.1.24c)}$$

The differential cross section is given by

$$\frac{d\sigma}{d\Omega}=\frac{1}{16\pi^2 s}\,\frac{1}{4}\sum_{\substack{s_1,s_2\\ t_1,t_2}}|M|^2 \quad \text{(II.1.25)}$$

where M is the invariant matrix element.

II.2. CALCULATION OF THE INVARIANT MATRIX ELEMENT

From the interaction Lagrangian the relevant terms for the $e^+e^-\to\mu^+\mu^-$ process, as seen in Eq.(I.5.27) become,

$$\frac{e_0}{\cos\theta_W\sin\theta_W}\Big[\sin\theta_W\left(\overline{\psi}_e(x)\,\gamma_\alpha\,\psi_e(x)+\overline{\psi}_\mu(x)\,\gamma_\alpha\,\psi_\mu(x)\right)$$

$$\times\left(\cos\theta_W A^\alpha(x)-\sin\theta_W Z^\alpha(x)\right)$$

$$+\frac{1}{2}\left(\overline{\psi}_e(x)\,\gamma_\alpha\,\frac{1-\gamma_5}{2}\,\psi_e(x)+\overline{\psi}_\mu(x)\,\gamma_\alpha\,\frac{1-\gamma_5}{2}\,\psi_\mu(x)\right)Z^\alpha(x)\Big]$$

Abbreviating as $\psi_e(x)=e(x)$ and $\psi_\mu(x)=\mu(x)$ the scattering matrix element takes the form in the lowest order,

$$\langle \mu^+\mu^- | S | e^+e^- \rangle = \frac{i^2}{2!}\left(\frac{e_0}{\cos\theta_W \sin\theta_W}\right)^2 \int dx\,dy \langle \mu^+\mu^- | T$$

$$\Big[\sin\theta_W\big(\bar{e}(x)\gamma_\alpha e(x) + \bar{\mu}(x)\gamma_\alpha \mu(x)\big)\big(\cos\theta_W A^\alpha(x) - \sin\theta_W Z^\alpha(x)\big)$$

$$+\frac{1}{2}\Big(\bar{e}(x)\gamma_\alpha \frac{1-\gamma_5}{2} e(x) + \bar{\mu}(x)\gamma_\alpha \frac{1-\gamma_5}{2}\mu(x)\Big) Z^\alpha(x)\Big]$$

$$\times\Big[\sin\theta_W\big(\bar{e}(y)\gamma_\beta e(y) + \bar{\mu}(y)\gamma_\beta \mu(y)\big)\big(\cos\theta_W A^\beta(y) - \sin\theta_W Z^\beta(y)\big)$$

$$+\frac{1}{2}\Big(\bar{e}(y)\gamma_\beta \frac{1-\gamma_5}{2} e(y) + \bar{\mu}(y)\gamma_\beta \frac{1-\gamma_5}{2}\mu(y)\Big) Z^\beta(y)\Big] | e^+e^- \rangle \qquad \text{(II.2.1)}$$

where T denotes the time ordering of field operators. Since α and β also distinguishes x and y, the cumbersome notation can be simplified by dropping x and y. So Eq.(II.2.1) takes the form,

$$\langle \mu^+\mu^- | S | e^+e^- \rangle = -\frac{e_0^2}{2\sin^2\theta_W \cos^2\theta_W} \int dx\,dy \langle \mu^+\mu^- | T$$

$$\Big[\sin^2\theta_W\big(\bar{e}\gamma_\alpha e + \bar{\mu}\gamma_\alpha \mu\big)\big(\bar{e}\gamma_\beta e + \bar{\mu}\gamma_\beta \mu\big)\big(\cos\theta_W A^\alpha - \sin\theta_W Z^\alpha\big)$$

$$\times\big(\cos\theta_W A^\beta - \sin\theta_W Z^\beta\big)$$

$$+\frac{1}{2}\sin\theta_W\big(\bar{e}\gamma_\alpha e + \bar{\mu}\gamma_\alpha \mu\big)\big(\cos\theta_W A^\alpha - \sin\theta_W Z^\alpha\big)\Big(\bar{e}\gamma_\beta \frac{1-\gamma_5}{2} e$$

$$+\bar{\mu}\gamma_\beta \frac{1-\gamma_5}{2}\mu\Big) Z^\beta$$

$$+\frac{1}{2}\sin\theta_W\Big(\bar{e}\gamma_\alpha \frac{1-\gamma_5}{2} e + \bar{\mu}\gamma_\alpha \frac{1-\gamma_5}{2}\mu\Big)\big(\bar{e}\gamma_\beta e + \bar{\mu}\gamma_\beta \mu\big)$$

$$\times\big(\cos\theta_W A^\beta - \sin\theta_W Z^\beta\big) Z^\alpha$$

$$+\frac{1}{4}\Big(\bar{e}\gamma_\alpha \frac{1-\gamma_5}{2} e + \bar{\mu}\gamma_\alpha \frac{1-\gamma_5}{2}\mu\Big)\Big(\bar{e}\gamma_\beta \frac{1-\gamma_5}{2} e + \bar{\mu}\gamma_\beta \frac{1-\gamma_5}{2}\mu\Big) Z^\alpha Z^\beta\Big] | e^+e^- \rangle$$

(II.2.2)

The only non-zero contributions are:

$$\langle \mu^+\mu^- | S | e^+e^- \rangle = -\frac{e_0^2}{2\sin^2\theta_W \cos^2\theta_W} \int dx\,dy \Big\{ \langle 0 | b_{\mu^+}(q_2,t_2)\, a_{\mu^-}(q_1,t_1)$$

$$\sin^2\theta_W \left(\bar e \gamma_\alpha e\, \bar\mu \gamma_\beta \mu + \bar\mu \gamma_\alpha \mu\, \bar e \gamma_\beta e\right) b^+_{e^+}(p_2,s_2)\, a^+_{e^-}(p_1,s_1) | 0 \rangle$$

$$\langle 0 | T(\cos^2\theta_W A^\alpha A^\beta + \sin^2\theta_W Z^\alpha Z^\beta) | 0 \rangle$$

$$- \langle 0 | b_{\mu^+}(q_2,t_2)\, a_{\mu^-}(q_1,t_1) \frac{1}{2} \sin^2\theta_W \Big(\bar e \gamma_\alpha e\, \bar\mu \gamma_\beta \frac{1-\gamma_5}{2} \mu$$

$$+ \bar\mu \gamma_\alpha \mu\, \bar e \gamma_\beta \frac{1-\gamma_5}{2} e \Big) b^+_{e^+}(p_2,s_2)\, a^+_{e^-}(p_1,s_1) | 0 \rangle \langle 0 | T(Z^\alpha Z^\beta) | 0 \rangle$$

$$- \langle 0 | b_{\mu^+}(q_2,t_2)\, a_{\mu^-}(q_1,t_1) \frac{1}{2} \sin^2\theta_W \Big(\bar e \gamma_\alpha \frac{1-\gamma_5}{2} e\, \bar\mu \gamma_\beta \mu$$

$$+ \bar\mu \gamma_\alpha \frac{1-\gamma_5}{2} \mu\, \bar e \gamma_\beta e \Big) b^+_{e^+}(p_2,s_2)\, a^+_{e^-}(p_1,s_1) | 0 \rangle \langle 0 | T(Z^\alpha Z^\beta) | 0 \rangle$$

$$+ \langle 0 | b_{\mu^+}(q_2,t_2)\, a_{\mu^-}(q_1,t_1) \frac{1}{4} \Big(\bar e \gamma_\alpha \frac{1-\gamma_5}{2} e\, \bar\mu \gamma_\beta \frac{1-\gamma_5}{2} \mu$$

$$+ \bar\mu \gamma_\alpha \frac{1-\gamma_5}{2} \mu\, \bar e \gamma_\beta \frac{1-\gamma_5}{2} e \Big) b^+_{e^+}(p_2,s_2)\, a^+_{e^-}(p_1,s_1) | 0 \rangle$$

$$\langle 0 | T(Z^\alpha Z^\beta) | 0 \rangle \Big\} . \qquad \text{(II.2.3)}$$

Z_o and photon propagators are given by,

$$\langle 0 | T\, Z^\alpha(x) Z^\beta(y) | 0 \rangle = -i(2\pi)^4 \int dk\, e^{-ik(x-y)} \frac{g^{\alpha\beta} - k^\alpha k^\beta / m_Z^2}{k^2 - m_Z^2 + i\varepsilon} \qquad \text{(II.2.4)}$$

$$\langle 0 | T\, A^\alpha(x) A^\beta(y) | 0 \rangle = -i(2\pi)^4 \int dk\, e^{-ik(x-y)} \frac{g^{\alpha\beta}}{k^2} \qquad \text{(II.2.5)}$$

where k is the transferred 4-momentum and,

$$k = p_1 + p_2 = q_1 + q_2 \qquad \text{(II.2.6)}$$

with

$$k^2 = (p_1+p_2)^2 = s \, . \tag{II.2.7}$$

In the high energy limit $k^\alpha k^\beta / m_Z^2$ is negligible. After expansion of field operators in terms of creation and annihilation operators as given in Eq.(A.7) and using anti-commutation relations of fermion fields as in Eq.(A.8) the Eq.(II.2.3) takes the form,

$$\begin{aligned}
\langle \mu^+\mu^- | S | e^+e^- \rangle = & \frac{e_0^2}{2\sin^2\theta_W \cos^2\theta_W} (-i)(2\pi)^4 \int dx\,dy\,dk \Big\{ \sin^2\theta_W \cos^2\theta_W \frac{g^{\alpha\beta}}{k^2} \\
& \cdot \Big(\bar{v}_{e^+}(p_2,s_2)\gamma_\alpha u_{e^-}(p_1,s_1)\, \bar{u}_{\mu^-}(q_1,t_1)\gamma_\beta v_{\mu^+}(q_2,t_2)\, e^{-i(p_1+p_2)x}\, e^{i(q_1+q_2)y} \\
& + \bar{u}_{\mu^-}(q_1,t_1)\gamma_\alpha v_{\mu^+}(q_2,t_2)\, \bar{v}_{e^+}(p_2,s_2)\gamma_\beta u_{e^-}(p_1,s_1)\, e^{i(q_1+q_2)x}\, e^{-i(p_1+p_2)y} \Big) e^{-ik(x-y)} \\
& + \Big[\sin^4\theta_W \Big(\bar{v}_{e^+}(p_2,s_2)\gamma_\alpha u_{e^-}(p_1,s_1)\, \bar{u}_{\mu^-}(q_1,t_1)\gamma_\beta v_{\mu^+}(q_2,t_2)\, e^{-i(p_1+p_2)x}\, e^{i(q_1+q_2)y} \\
& + \bar{u}_{\mu^-}(q_1,t_1)\gamma_\alpha v_{\mu^+}(q_2,t_2)\, \bar{v}_{e^+}(p_2,s_2)\gamma_\beta u_{e^-}(p_1,s_1)\, e^{i(q_1+q_2)x}\, e^{-i(p_1+p_2)y} \Big) \\
& - \frac{1}{2}\sin^2\theta_W \Big(\bar{v}_{e^+}(p_2,s_2)\gamma_\alpha u_{e^-}(p_1,s_1)\, \bar{u}_{\mu^-}(q_1,t_1)\gamma_\beta \frac{1-\gamma_5}{2} v_{\mu^+}(q_2,t_2)\, e^{-i(p_1+p_2)x} \\
& \qquad \cdot e^{i(q_1+q_2)y} \\
& + \bar{u}_{\mu^-}(q_1,t_1)\gamma_\alpha v_{\mu^+}(q_2,t_2)\, \bar{v}_{e^+}(p_2,s_2)\gamma_\beta \frac{1-\gamma_5}{2} u_{e^-}(p_1,s_1)\, e^{i(q_1+q_2)x}\, e^{-i(p_1+p_2)y} \Big) \\
& - \frac{1}{2}\sin^2\theta_W \Big(\bar{v}_{e^+}(p_2,s_2)\gamma_\alpha \frac{1-\gamma_5}{2} u_{e^-}(p_1,s_1)\, \bar{u}_{\mu^-}(q_1,t_1)\gamma_\beta v_{\mu^+}(q_2,t_2)\, e^{-i(p_1+p_2)x} \\
& \qquad \cdot e^{i(q_1+q_2)y} \\
& + \bar{u}_{\mu^-}(q_1,t_1)\gamma_\alpha \frac{1-\gamma_5}{2} v_{\mu^+}(q_2,t_2)\, \bar{v}_{e^+}(p_2,s_2)\gamma_\beta u_{e^-}(p_1,s_1)\, e^{i(q_1+q_2)x}\, e^{-i(p_1+p_2)y} \Big) \\
& + \frac{1}{4}\Big(\bar{v}_{e^+}(p_2,s_2)\gamma_\alpha \frac{1-\gamma_5}{2} u_{e^-}(p_1,s_1)\, \bar{u}_{\mu^-}(q_1,t_1)\gamma_\beta \frac{1-\gamma_5}{2} v_{\mu^+}(q_2,t_2)\, e^{-i(p_1+p_2)x}\, e^{i(q_1+q_2)y} \\
& + \bar{u}_{\mu^-}(q_1,t_1)\gamma_\alpha \frac{1-\gamma_5}{2} v_{\mu^+}(q_2,t_2)\, \bar{v}_{e^+}(p_2,s_2)\gamma_\beta \frac{1-\gamma_5}{2} u_{e^-}(p_1,s_1)\, e^{i(q_1+q_2)x}\, e^{-i(p_1+p_2)y} \Big) \Big] \\
& \cdot e^{-ik(x-y)} \frac{g^{\alpha\beta}}{k^2 - m_Z^2} \Big\}
\end{aligned} \tag{II.2.8}$$

After integration of Eq.(II.2.8) one obtains,

$$\langle \mu^+\mu^- | S | e^+e^- \rangle = -i(2\pi)^4 \delta(p_1+p_2-q_1-q_2) M \qquad \text{(II.2.9)}$$

where M is invariant matrix element and is equal to,

$$M = \frac{e_0^2}{s} \bar{v}_e \gamma_\alpha u_{e^-} \bar{u}_{\mu^-} \gamma^\alpha v_{\mu^+} + \frac{e_0^2 \tan^2\theta_W}{s-m_Z^2} \bar{v}_{e^+} \gamma_\alpha u_{e^-} \bar{u}_{\mu^-} \gamma^\alpha v_{\mu^+}$$

$$- \frac{e_0^2}{2\cos^2\theta_W (s-m_Z^2)} \left(\bar{v}_{e^+} \gamma_\alpha u_{e^-} \bar{u}_{\mu^-} \gamma^\alpha \frac{1-\gamma_5}{2} v_{\mu^+} + \bar{v}_{e^+} \gamma_\alpha \frac{1-\gamma_5}{2} u_{e^-} \bar{u}_{\mu^-} \gamma^\alpha v_{\mu^+} \right)$$

$$+ \frac{e_0^2}{4\sin^2\theta_W \cos^2\theta_W (s-m_Z^2)} \bar{v}_{e^+} \gamma_\alpha \frac{1-\gamma_5}{2} u_{e^-} \bar{u}_{\mu^-} \gamma^\alpha \frac{1-\gamma_5}{2} v_{\mu^+}$$

$$= \frac{e_0^2}{s} \bar{v}_{e^+} \gamma_\alpha u_{e^-} \bar{u}_{\mu^-} \gamma^\alpha v_{\mu^+} + \frac{e_0^2}{4\sin^2\theta_W \cos^2\theta_W (s-m_Z^2)} \bar{v}_{e^+} \gamma_\alpha \left(c_L \frac{1-\gamma_5}{2} + c_R \frac{1+\gamma_5}{2} \right)$$

$$\cdot u_{e^-} \bar{u}_{\mu^-} \gamma^\alpha \left(c_R \frac{1-\gamma_5}{2} + c_L \frac{1+\gamma_5}{2} \right) v_{\mu^+} \qquad \text{(II.2.10)}$$

where,

$$c_L = 2\sin^2\theta_W - 1 \quad , \quad c_R = 2\sin^2\theta_W \qquad \text{(II.2.11)}$$

The vector and axial-vector coupling constants are defined by,

$$g_V = \frac{c_L + c_R}{2} = 2\sin^2\theta_W - \frac{1}{2} \qquad \text{(II.2.12)}$$

$$g_A = \frac{c_L - c_R}{2} = -\frac{1}{2} \qquad \text{(II.2.13)}$$

The first term in Eq.(II.2.10) is purely electromagnetic, second term is purely weak amplitude. Therefore $|M|^2$ will contain interference of these two amplitudes.

$$|M|^2 = \frac{e_0^4}{s^2} \bar{v}_{e^+} \gamma_\alpha u_{e^-} \bar{u}_{\mu^-} \gamma^\alpha v_{\mu^+} \bar{u}_{e^-} \gamma_\beta v_{e^+} \bar{v}_{\mu^+} \gamma^\beta u_{\mu^-}$$

$$+ \frac{e_0^4}{4\sin^2\theta_W \cos^2\theta_W s(s-m_Z^2)} \bar{v}_{e^+} \gamma_\alpha u_{e^-} \bar{u}_{\mu^-} \gamma^\alpha v_{\mu^+} \bar{u}_{e^-} \gamma_\beta \left(c_L \frac{1-\gamma_5}{2} + c_R \frac{1+\gamma_5}{2} \right) v_{e^+}$$

$$\cdot \bar{v}_{\mu^+} \gamma^\beta \left(c_R \frac{1-\gamma_5}{2} + c_L \frac{1+\gamma_5}{2} \right) u_{\mu^-}$$

$$+ \frac{e_0^4}{4\sin^2\theta_W \cos^2\theta_W s(s-M_Z^2)} \bar{v}_{e^+} \gamma_\alpha \left(c_L \frac{1-\gamma_5}{2} + c_R \frac{1+\gamma_5}{2}\right) u_{e^-} \bar{u}_{\mu^-} \gamma^\alpha \left(c_R \frac{1-\gamma_5}{2} + c_L \frac{1+\gamma_5}{2}\right)$$

$$\cdot v_{\mu^+} \bar{u}_{e^-} \gamma_\beta v_{e^+} \bar{v}_{\mu^+} \gamma^\beta u_{\mu^-}$$

$$+ \frac{e_0^4}{16\sin^4\theta_W \cos^4\theta_W (s-M_Z^2)} \bar{v}_{e^+} \gamma_\alpha \left(c_L \frac{1-\gamma_5}{2} + c_R \frac{1+\gamma_5}{2}\right) u_{e^-} \bar{u}_{\mu^-} \gamma^\alpha \left(c_R \frac{1-\gamma_5}{2} + c_L \frac{1+\gamma_5}{2}\right)$$

$$\cdot v_{\mu^+} \bar{u}_{e^-} \gamma_\beta \left(c_L \frac{1-\gamma_5}{2} + c_R \frac{1+\gamma_5}{2}\right) v_{e^+} \bar{v}_{\mu^+} \gamma^\beta \left(c_R \frac{1-\gamma_5}{2} + c_L \frac{1+\gamma_5}{2}\right) u_{\mu^-} \quad \text{(II.2.14)}$$

To obtain the differential cross-section $|M|^2$ must be averaged over initial spins and summed over final spins. The spin projection operator is,

$$\lambda(p,s) = u(p,s)\bar{u}(p,s) = (\not p + m)\frac{1}{2}(1+\gamma_5 \not s) \qquad \text{(II.2.15)}$$

so that

$$\frac{1}{2}\left[\lambda(p,s) + \lambda(p,-s)\right] = \frac{1}{2}(\not p + m) \ . \qquad \text{(II.2.16)}$$

Using Eq.(I.5.33) one obtains,

$$\frac{1}{4}\sum_{\substack{s_1,s_2\\t_1,t_2}} |M|^2 = \frac{16\pi^2\alpha^2}{s^2} \operatorname{Tr} \gamma_\alpha \not p_1 \frac{1}{2} \gamma_\beta \not p_2 \frac{1}{2} \operatorname{Tr} \gamma^\alpha \not q_2 \gamma^\beta \not q_1$$

$$+ \frac{4\pi\alpha\, 2G M_Z^2}{\sqrt{2}\, s(s-M_Z^2)} \operatorname{Tr} \gamma_\alpha \not p_1 \frac{1}{2} \gamma_\beta \left(c_L \frac{1-\gamma_5}{2} + c_R \frac{1+\gamma_5}{2}\right) \not p_2$$

$$\cdot \frac{1}{2} \operatorname{Tr} \gamma^\alpha \not q_2 \gamma^\beta \left(c_R \frac{1-\gamma_5}{2} + c_L \frac{1+\gamma_5}{2}\right) \not q_1$$

$$+ \frac{4\pi\alpha\, 2G M_Z^2}{\sqrt{2}\, s(s-M_Z^2)} \operatorname{Tr} \gamma_\alpha \left(c_L \frac{1-\gamma_5}{2} + c_R \frac{1+\gamma_5}{2}\right) \not p_1 \frac{1}{2} \gamma_\beta \not p_2$$

$$\cdot \frac{1}{2} \operatorname{Tr} \gamma^\alpha \left(c_R \frac{1-\gamma_5}{2} + c_L \frac{1+\gamma_5}{2}\right) \not q_2 \gamma^\beta \not q_1$$

$$+ \frac{2G^2 M_Z^4}{(s-M_Z^2)^2} \operatorname{Tr} \gamma_\alpha \left(c_L \frac{1-\gamma_5}{2} + c_R \frac{1+\gamma_5}{2}\right) \not p_1 \frac{1}{2} \gamma_\beta \left(c_L \frac{1-\gamma_5}{2} + c_R \frac{1+\gamma_5}{2}\right) \not p_2$$

$$\cdot \frac{1}{2} \operatorname{Tr} \gamma^\alpha \left(c_R \frac{1-\gamma_5}{2} + c_L \frac{1+\gamma_5}{2}\right) \not q_2 \gamma^\beta \left(c_R \frac{1-\gamma_5}{2} + c_L \frac{1+\gamma_5}{2}\right) \not q_1 \quad \text{(II.2.17}$$

As it is easily seen that the contribution of the second and the third terms are equal. Using Eqns.(B.6-7a-7b) the above expression Eq.(II.2.17) becomes,

$$\text{First term} = \frac{16\pi^2\alpha^2}{4s^2} \operatorname{Tr} \gamma_\alpha \gamma_\sigma \gamma_\beta \gamma_\rho p_1^\sigma p_2^\rho \operatorname{Tr} \gamma^\alpha \gamma^\lambda \gamma^\beta \gamma^\mu q_{2\lambda} q_{1\mu}$$

$$= \frac{16\pi^2\alpha^2}{4s^2} 4\,\Theta_{\alpha\sigma\beta\rho} \cdot 4\,\Theta^{\alpha\lambda\beta\mu} p_1^\sigma p_2^\rho q_{2\lambda} q_{1\mu}$$

$$= \frac{128\pi^2\alpha^2}{s^2}\left(p_1\cdot q_2\, p_2\cdot q_1 + p_1\cdot q_1\, p_2\cdot q_2\right) \qquad \text{(II.2.18)}$$

$$\text{Second+Third term} = \frac{16\pi\alpha G m_Z^2}{4\sqrt{2}\, s(s-m_Z^2)} \operatorname{Tr} \gamma_\alpha \gamma_\sigma \gamma_\beta \gamma_\rho \left(c_L \frac{1+\gamma_5}{2} + c_R \frac{1-\gamma_5}{2}\right)$$

$$\cdot \operatorname{Tr} \gamma^\alpha \gamma^\lambda \gamma^\beta \gamma^\mu \left(c_L \frac{1-\gamma_5}{2} + c_R \frac{1+\gamma_5}{2}\right) p_1^\sigma p_2^\rho q_{2\lambda} q_{1\mu}$$

$$= \frac{64\pi\alpha G m_Z^2}{4\sqrt{2}\, s(s-m_Z^2)} \left(c_L \chi^+_{\alpha\sigma\beta\rho} + c_R \chi^-_{\alpha\sigma\beta\rho}\right)\left(c_L \chi_-^{\alpha\lambda\beta\mu} + c_R \chi_+^{\alpha\lambda\beta\mu}\right)$$

$$\cdot\, p_1^\sigma p_2^\rho q_{2\lambda} q_{1\mu}$$

$$= \frac{256\pi\alpha G m_Z^2}{4\sqrt{2}\, s(s-m_Z^2)} \Big[(c_L^2 + c_R^2)\, p_1\cdot q_2\, p_2\cdot q_1$$

$$+ 2 c_L c_R\, p_1\cdot q_1\, p_2\cdot q_2 \Big] \qquad \text{(II.2.19)}$$

Since,

$$c_L^2 + c_R^2 = \frac{1}{2}\left[(c_L+c_R)^2 + (c_L-c_R)^2\right]$$

and,

$$2 c_L c_R = \frac{1}{2}\left[(c_L+c_R)^2 - (c_L-c_R)^2\right]$$

Eq.(II.2.19) takes the following form,

$$\text{Second+Third term} = \frac{32\pi\alpha G m_Z^2}{\sqrt{2}\, s(s-m_Z^2)} \Big[(c_L+c_R)^2 \left(p_1\cdot q_2\, p_2\cdot q_1 + p_1\cdot q_1\, p_2\cdot q_2\right)$$

$$+ (c_L-c_R)\left(p_1\cdot q_2\, p_2\cdot q_1 - p_1\cdot q_1\, p_2\cdot q_2\right) \qquad \text{(II.2.20)}$$

Fourth term can be obtained from Eq.(II.2.20) by the substitutions $c_L \to c_L^2$ and $c_R \to c_R^2$, so

$$\text{Fourth term} = \frac{4G^2 m_Z^4}{(s-m_Z^2)^2}\Big[(c_L^2+c_R^2)^2(p_1\cdot q_2\, p_2\cdot q_1 + p_1\cdot q_1\, p_2\cdot q_2) + (c_L^2-c_R^2)(p_1\cdot q_2\, p_2\cdot q_1 - p_1\cdot q_1\, p_2\cdot q_2)\Big] \tag{II.2.21}$$

According to the Eq.(II.1.25), cross-sections may be easily calculated now in the high energy limit. Using Eqns.(II.1.24a -24b-24c) one obtains the following cross-sections:

The purely electromagnetic cross-section is,

$$\frac{d\sigma_{em}}{d\Omega} = \frac{1}{64\pi^2 s}\cdot\frac{128\pi^2\alpha^2}{s^2}\cdot\frac{s^2}{16}\left[(1+z)^2+(1-z)^2\right]$$
$$= \frac{\alpha^2(1+z^2)}{4s}\,. \tag{II.2.22}$$

Since,

$$\frac{d\sigma}{dz} = 2\pi\frac{d\sigma}{d\Omega} \tag{II.2.23}$$

then,

$$\frac{d\sigma_{em}}{dz} = \frac{\pi\alpha^2(1+z^2)}{2s} \tag{II.2.24}$$

thus,

$$\sigma_{em} = \int_{-1}^{1} dz\ \frac{d\sigma_{em}}{dz}$$
$$= \frac{4}{3}\ \frac{\pi\alpha^2}{s}\,. \tag{II.2.25}$$

Similarly, one has for the interference part,

$$\frac{d\sigma_{intf}}{d\Omega} = \frac{\alpha G m_Z^2}{16\pi\sqrt{2}\,(s-m_Z^2)}\left[(c_L+c_R)^2(1+z^2)+2(c_L-c_R)^2 z\right] \tag{II.2.26}$$

$$\frac{d\sigma_{intf}}{dz} = \frac{\alpha G m_Z^2}{8\sqrt{2}\,(s-m_Z^2)}\left[(c_L+c_R)^2(1+z^2)+2(c_L-c_R)^2 z\right] \tag{II.2.27}$$

$$\sigma_{intf} = \frac{\alpha G m_Z^2}{3\sqrt{2}\,(s-m_Z^2)}\,(c_L+c_R)^2\,. \tag{II.2.28}$$

For the purely weak part one obtains,

$$\frac{d\sigma_W}{d\Omega} = \frac{G^2 s\, m_Z^4}{128\pi^2 (s-m_Z^2)^2}\left[(c_L^2+c_R^2)^2(1+z^2)+2z(c_L^2-c_R^2)^2\right] \qquad \text{(II.2.29)}$$

$$\frac{d\sigma_W}{dz} = \frac{G^2 s}{64\pi\left(1-\frac{s}{m_Z^2}\right)^2}\left[(c_L^2+c_R^2]^2(1+z^2)+2(c_L^2-c_R^2)^2 z\right] \qquad \text{(II.2.30)}$$

$$\sigma_W = \frac{G^2 s}{24\pi\left(1-\frac{s}{m_Z^2}\right)^2}\left(c_L^2+c_R^2\right)^2 . \qquad \text{(II.2.31)}$$

The differential cross section is the sum of Eq.(II.2.22), Eq.(II.2.26) and Eq.(II.2.29),

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2(1+z^2)}{4s} + \frac{\alpha G m_Z{}^2}{16\pi\sqrt{2}(s-m_Z^2)}\left[(c_L+c_R)^2(1+z^2)+2(c_L-c_R)^2 z\right]$$

$$+\frac{G^2 s}{128\pi^2\left(1-\frac{s}{m_Z^2}\right)^2}\left[(c_L^2+c_R^2)^2(1+z^2)+(c_L^2-c_R^2)^2 2z\right] \qquad \text{(II.2.32)}$$

II.3. PARITY VIOLATION IN $e^+e^- \to \mu^+\mu^-$ AND THE EXPERIMENTAL RESULTS

Interference between photon and Z_0 exchange diagrams will in general give rise to parity violating effects in the angular distribution of μ-pairs, because the coupling of the photon is vectorial while Z_0 has an axial vector coupling. The parity violation can be measured by considering,

$$A = \frac{\frac{d\sigma(z)}{d\Omega} - \frac{d\sigma(-z)}{d\Omega}}{\frac{d\sigma(z)}{d\Omega} + \frac{d\sigma(-z)}{d\Omega}} . \qquad \text{(II.3.1)}$$

Which would be zero in the case of parity coservation. Using $z=\cos\theta$, one has from Weinberg-Salam model,

$$A = \frac{\dfrac{\alpha G m_Z^{2}}{4\pi\sqrt{2}(s-m_Z^2)} + \dfrac{G^2 s}{32\pi^2(1-s/m_Z^2)^2}(c_L^2-c_R^2)}{\dfrac{\alpha^2}{2s} + \dfrac{\alpha G m_Z^2(c_L+c_R)^2}{8\pi\sqrt{2}(s-m_Z^2)} + \dfrac{G^2 s(c_L^2+c_R^2)^2}{64\pi^2\left(1-\frac{s}{m_Z^2}\right)^2}} \frac{\cos\theta}{1+\cos^2\theta} \qquad \text{(II.3.2)}$$

The dominant terms are first terms in the numerator and denominator, so that

$$A \approx \frac{\alpha G m_Z^{2}}{4\pi\sqrt{2}(s-m_Z^2)} \cdot \frac{4s}{2\alpha^2} \cdot \frac{\cos\theta}{1+\cos^2\theta}$$

$$\approx \frac{G m_Z^{2} s}{2\pi\sqrt{2}\,\alpha(s-m_Z^2)} \cdot \frac{\cos\theta}{1+\cos^2\theta} \qquad \text{(II.3.3)}$$

which obviously differs from zero.

There is also a forward backward asymmetry $A_{\mu\mu}$ in the angular distribution of the μ-pairs. Forward scattering cross-section is,

$$F = \int_0^1 \frac{d\sigma(z)}{dz}\, dz$$

$$= \frac{2}{3}\frac{\pi\alpha^2}{s} + \frac{\alpha G m_Z^2}{8\sqrt{2}(s-m_Z^2)}\left[\frac{4}{3}(c_L+c_R)^2 + (c_L-c_R)^2\right]$$

$$+ \frac{G^2 s}{64\pi(1-s/m_Z^2)^2}\left[\frac{4}{3}(c_L^2+c_R^2)^2 + (c_L^2-c_R^2)^2\right] \qquad \text{(II.3.4)}$$

and backward scattering cross-section is,

$$B = \int_0^1 \frac{d\sigma(-z)}{dz}\, dz$$

$$= \frac{2}{3}\frac{\pi\alpha^2}{s} + \frac{\alpha G m_Z^2}{8\sqrt{2}(s-m_Z^2)}\left[\frac{4}{3}(c_L+c_R)^2 - (c_L-c_R)^2\right]$$

$$+ \frac{G^2 s}{64\pi(1-s/m_Z^2)^2}\left[\frac{4}{3}(c_L^2+c_R^2)^2 + (c_L^2-c_R^2)\right]. \qquad \text{(II.3.5)}$$

Therefore the forward-backward asymmetry is given by,

$$A_{\mu\mu} = \frac{F-B}{F+B}$$

$$\approx \frac{3}{4} \frac{G}{\sqrt{2}\pi\alpha} \frac{s m_Z^2}{s-m_Z^2} g_A^2 \tag{II.3.6}$$

where use has been made of Eq.(II.2.13). Eq.(II.3.6)can be used to test the Weinberg-Salam model,if the Weinberg angle θ_W is known, since m_Z depends on the value of θ_W , the free parameter of the model. θ_W is determined experimentally from lepton scattering. According to the Weinberg-Salam model, one has the cross-section for $\nu_\mu e$ and $\bar{\nu}_\mu e$ scattering as,

$$\sigma_L(\nu_\mu e \rightarrow \nu_\mu e) = \frac{1}{4} \sigma_0 \omega \left[1 - 4\sin^2\theta_W + \frac{16}{3} \sin^4\theta_W\right] \tag{II.3.7a}$$

$$\sigma_L(\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e) = \frac{1}{4} \sigma_0 \omega \left[\frac{1}{3} - \frac{4}{3} \sin^2\theta_W + \frac{16}{3} \sin^4\theta_W\right] \tag{II.3.7b}$$

where,

$$\sigma_0 = \frac{2G^2}{\pi} m_e^2 \quad , \quad \omega = \frac{E_\nu}{m_e} \; .$$

Three events have been identified using neutrino beams at CERN and at the Gargamelle bubble chamber as the process $\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e$. The cross-sections measured satisfy[22],

$$\sigma(\nu_\mu e \rightarrow \nu_\mu e) < 0.26 \times 10^{-41} \left(\frac{E_\nu}{1\,\mathrm{GeV}}\right) \mathrm{cm}^2 \tag{II.3.8a}$$

$$0.03 \times 10^{-41} \left(\frac{E_\nu}{1\,\mathrm{GeV}}\right) \mathrm{cm}^2 < \sigma(\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e) < 0.28 \times 10^{-41} \left(\frac{E_\nu}{1\,\mathrm{GeV}}\right) \mathrm{cm}^2 \tag{II.3.8b}$$

allowing the determination of θ_W , leading to the value[3,9,22]

$$0.1 < \sin^2\theta_W < 0.45 \; . \; (90\,\%\ \mathrm{c.l.})$$

Today, the world average value of Weinberg angle is,[13]

$$\sin^2\theta_W = 0.23 \pm 0.009 \; . \tag{II.3.9}$$

The masses of intermediate vector bosons $W^{\pm}$ and Z_o, using Eq.(I.5.31) and Eq.(I.5.33) are now determined to be,

$$m_W \approx 78\,\mathrm{GeV} \tag{II.3.10a}$$

$$m_Z \approx 89\,\mathrm{GeV} \tag{II.3.10b}$$

With the avaible energies today in the colliding beam machines the direct observation of $W^{\pm}$ and Z_o is not possible. In near future the machines will reach to threshold of $W^{\pm}$ and Z_o production and the model will be tested directly.

Also, in view of lage cross-sections obtained in deep inelastic scattering of neutrinos via the charged current, one can hope that similar results can be obtained from scattering by neutral currents. In the interactions,

$$\nu_l N \rightarrow \nu_l X \tag{II.3.11a}$$

$$\bar{\nu}_l N \rightarrow \bar{\nu}_l X \tag{II.3.11b}$$

where N represents the nucleon, l any lepton and X any hadronic state, according Weinberg-Salam model, the following branching ratios are obtained(25,26,27,28),

$$\sigma_Z(\nu N)/\sigma_W(\nu N) = \frac{20}{27}\sin^4\theta_W - \sin^2\theta_W + \frac{1}{2} \tag{II.3.12a}$$

$$\sigma_Z(\bar{\nu} N)/\sigma_W(\bar{\nu} N) = \frac{20}{9}\sin^4\theta_W - \sin^2\theta_W + \frac{1}{2} \tag{II.3.12b}$$

The neutral current cross-sections have been measured at CERN and NAL, and experimentally it is found that(22),

$$\sigma_Z(\nu N)/\sigma_W(\nu N) = 0.21 \pm 0.03 \tag{II.3.13a}$$

$$\sigma_Z(\bar{\nu} N)/\sigma_W(\bar{\nu} N) = 0.41 \pm 0.09 \tag{II.3.13b}$$

which gives for the Weinberg angle,

$$0.2 < \sin^2\theta_W < 0.4 \tag{II.3.14}$$

The forward backward asymmetry of μ-pair creation (Eq.(II.3.6)) for the cm energy s= 1000 Gev^2 is obtained to be,

$$A_{\mu\mu} = \frac{3}{4} \cdot \frac{1.026\times10^{-5}\times137}{\sqrt{2}\,\pi\times0.8804} \cdot \frac{1000\times89^2}{1000-89^2} \cdot \frac{1}{4}$$

$$= -0.077 \qquad \text{(II.3.15)}$$

if one uses $\sin^2\theta_W = 0.23$, m_z= 89 Gev and $g_A = -1/2$. The asymmetry data obtained by the various PETRA groups are[11],

Group :	JADE	MARK J	PLUTO	TASSO
$A_{\mu\mu}$(%) :	-8 ± 9	0 ± 9	7 ± 10	1 ± 12

The combined angular distribution is plotted in Fig.1 and it yields $\langle A_{\mu\mu}\rangle = -(0.9\pm4.9)\%$ which is very near the theoretically predicted value - 7.7 %.

The model also can be tested on the basis of experimental limits of the strenght of weak neutral currents in lepton pair production. The importance of a this kind of test arises from the fact that it is possible to determine weak couplings for the lepton sector in a model independent way and free from hadron uncertainties. The theoretical values of g_V and g_A defined by Eq.(II.2.12) and Eq.(II.2.13) using $\sin^2\theta_W = 0.23$ are,

$$g_V^2 = 0.0016 \text{ and } g_A^2 = 0.25 \qquad \text{(II.3.16)}$$

The JADE collaboration finds[12],

$$g_V^2 = 0.01\pm0.08 \quad \text{and} \quad g_A^2 = 0.18\pm0.16$$

Since these values are consistent with the theoretical values given in Eq.(II.3.16), one can use Eq.(II.2.32) to determine the value of $\sin^2\theta_W$ with given g_V and g_A. The result is,

$$0.04 < \sin^2\theta_W < 0.46 \qquad (95\%\ \text{c.l.})$$

The values for the vector and axial vector weak coupling constants thus found are in agreement with the predictions of Weinberg-Salam model and within the framework of this model experiments give a value of $\sin^2\theta_W$ which agrees with the current world average. The mass of the weak boson, with minimal assumptions appears to be greater than 51 Gev.

Various groups using the Weinberg-Salam model, used the data on $e^+e^- \to e^+e^-$, $e^+e^- \to \mu^+\mu^-$ to extract values on $\sin^2\theta_W$. The results are(11),

	Lower bound	Upper bound	$\sin^2\theta_W$
MARK J	0.07	0.42	0.24 0.11
JADE	-	0.55	0.25 0.18
PLUTO	-	0.57	0.23 0.17
TASSO	-	0.52	-

II.4. CONCLUDING REMARKS

The process $e^+e^- \to \mu^+\mu^-$ via photon and Z_o exchange is analysed in first order perturbation theory and the purely electromagnetic, the purely weak and the interference cross sections are obtained. Parity violation and front to back ratio in the reaction has been calculated and it is shown that the theoretical value agrees well with experiments.

Since it is not possible to determine the Weinberg angle, $\sin^2\theta_W$, within the model, the experimental values of θ_W are given and it is shown that the world average value of Weinberg angle, $\sin^2\theta_W \approx 0.23$ is consistent with the calculations and experimental data for the cross-section and front to back asymmetry in $e^+e^- \to \mu^+\mu^-$.

The Weinberg-Salam model now, being a well established theory describing electromagnetic and weak interactions, attemps have been made to try to unify strong interactions and even gravitation. Such models all have the $SU(2)_W xU(1)$ group as a subgroup i.e. contain the Weinberg-Salam model.

In the near future, it is expected that the new machines will enable the $W^{\pm}$ and Z_0 bosons to be observed directly.

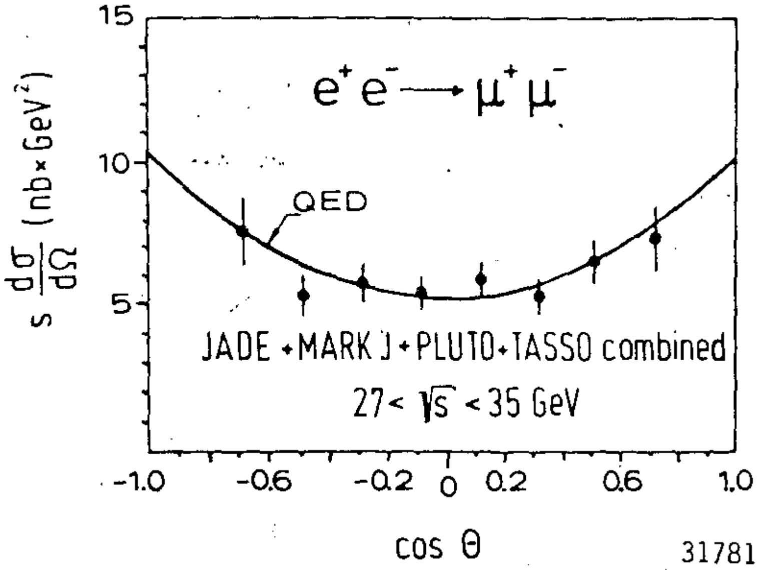


Fig.1

The combined angular distribution of $S\ d\sigma/d\Omega$ for $e^+e^- \to \mu^+\mu^-$ at c.m. energies between 27 and 35 GeV.

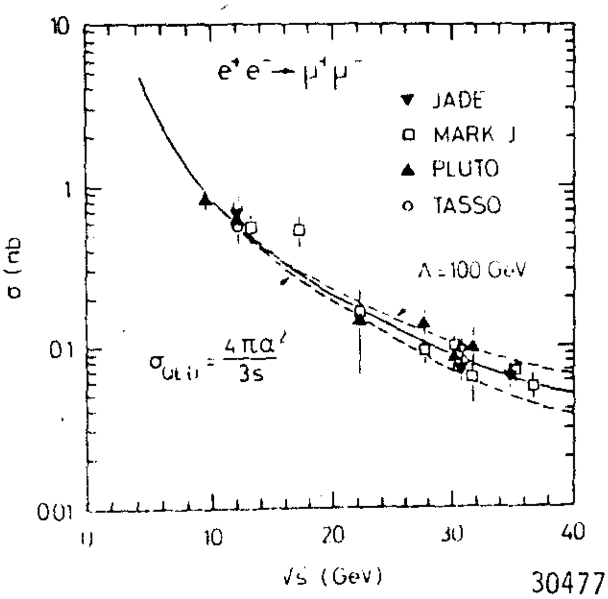


Fig.2

The total cross section for $e^+e^- \to \mu^+\mu^-$ measured by JADE, MARK J, PLUTO and TASSO plotted versus c.m. energy.

APPENDIX A

NOTATION(2,3)

A four vector is denoted by,

$$A = (A_0, \vec{A}) , \tag{A.1}$$

the scalar product of two four vectors is,

$$A.B = A_0 B_0 - \vec{A}.\vec{B} , \tag{A.2}$$

the employed metric being,

$$g_{00} = -g_{11} = -g_{22} = -g_{33} = 1, \; g_{\mu\nu} = 0 \; (\mu \neq \nu) \tag{A.3}$$

The Dirac γ-matrices satisfy,

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2 g^{\mu\nu} I_4 \; , \; \{\gamma_5, \gamma_\mu\} = 0 \tag{A.4}$$

and are represented as

$$\gamma_0 = \begin{pmatrix} I_2 & 0 \\ 0 & -I_2 \end{pmatrix} \; , \; \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix} \tag{A.5}$$

with

$$\gamma_5 = \frac{i}{4!} \epsilon_{\lambda\mu\nu\rho} \gamma^\lambda \gamma^\mu \gamma^\nu \gamma^\rho = i \gamma^0 \gamma^1 \gamma^2 \gamma^3 . \tag{A.6}$$

The expansion of field operators in terms of creation and annihilation operators are as follows(3):

If $\psi(x)$ is a fermion field,

$$\psi(x) = (2\pi)^{-3} \int d^3p \, (2p_0)^{-1} \sum_{\pm s} \left\{ a(p,s) u(p,s) e^{-ipx} + b^\dagger(p,s) v(p,s) e^{ipx} \right\} \tag{A.7}$$

where a(p,s) and b(p,s) are particle and anti-particle annihilation operators with the following anti-commutation relations

$$\begin{aligned} \{a(p,s), a^\dagger(p',s')\} &= \{b(p,s), b^\dagger(p',s')\} \\ &= (2\pi)^3 \, 2p_0 \, \delta(\vec{p} - \vec{p}') \, \delta_{ss'} \end{aligned} \tag{A.8}$$

If $\varphi(x)$ is a boson field,

$$\varphi(x) = (2\pi)^{-3} \int d^3\vec{K}\, (2K_0)^{-1} \sum_{\lambda=1}^{3} \epsilon_\alpha(K,\lambda) \left\{ a(K,\lambda) e^{-iKx} + b^+(K,\lambda) e^{iKx} \right\} \quad (A.9)$$

where $\epsilon_\alpha(K,\lambda)$ is the polarization vector, satisfying,

$$K \cdot \epsilon(K,\lambda) = 0 \,,\; \epsilon(K,\lambda) \cdot \epsilon(K,\lambda) = -\delta_{\lambda\mu} \quad \lambda,\mu = 1,2,3 \qquad (A.10)$$

and the commutation relations for the creation and annihilation operators being

$$\begin{aligned} [a(K,\lambda), a^+(L,\mu)] &= [b(L,\mu), b^+(K,\mu)] \\ &= (2\pi)^3\, 2K_0\, \delta(\vec{K}-\vec{L})\, \delta_{\lambda\mu} \,. \end{aligned} \qquad (A.11)$$

The operator $\frac{1-\gamma_5}{2}$ represents the left handed helicity projection operator whereas $\frac{1+\gamma_5}{2}$ projects the right handed helicity part of a spin 1/2 field. Feynman's slash notation is used throughout as, $\not{a} = \gamma^\mu a_\mu$

Dirac's particle and anti-particle spinors are normalized respectively as follows,

$$\bar{u}(p,\pm s)\, u(p,\pm s) = -\bar{v}(p,\pm s)\, v(p,\pm s) = 2m \,, \qquad (A.12)$$

and plane wave projection operators have the property,

$$u(p,s)\, \bar{u}(p,s) = (\not{p}+m)\, \frac{1}{2} (1+\gamma_5 \not{s}) \,, \qquad (A.13a)$$

$$v(p,s)\, \bar{v}(p,s) = (\not{p}-m)\, \frac{1}{2} (1+\gamma_5 \not{s}) \,. \qquad (A.13b)$$

APPENDIX B

SOME TRACE PROPERTIES OF γ-MATRICES

a. $Tr\gamma_\alpha\gamma_\beta = 4g_{\alpha\beta}$

Proof: Using Eq.(A.4) and cyclic property of the trace gives,

$$Tr\gamma_\alpha\gamma_\beta = -Tr\gamma_\beta\gamma_\alpha + 2g_{\alpha\beta}\,Tr\,I_4$$

$$2\,Tr\gamma_\alpha\gamma_\beta = 8g_{\alpha\beta}$$

$$Tr\gamma_\alpha\gamma_\beta = 4g_{\alpha\beta} \qquad (B.1)$$

b. $Tr\gamma_\alpha\gamma_\beta\gamma_\delta\gamma_\sigma = 4g_{\alpha\beta}g_{\delta\sigma} - 4g_{\alpha\delta}g_{\beta\sigma} + 4g_{\alpha\sigma}g_{\beta\delta}$

Proof: Using Eq.(A.4) successively one obtains,

$$Tr\gamma_\alpha\gamma_\beta\gamma_\delta\gamma_\sigma = -Tr\gamma_\beta\gamma_\delta\gamma_\sigma\gamma_\alpha + 2g_{\alpha\beta}Tr\gamma_\delta\gamma_\sigma - 2g_{\alpha\delta}Tr\gamma_\beta\gamma_\sigma + 2g_{\alpha\sigma}Tr\gamma_\beta\gamma_\delta$$

Cyclic property of trace and Eq.(B.1) yields,

$$Tr\gamma_\alpha\gamma_\beta\gamma_\delta\gamma_\sigma = 4g_{\alpha\beta}g_{\delta\sigma} - 4g_{\alpha\delta}g_{\beta\sigma} + 4g_{\alpha\sigma}g_{\beta\delta} \qquad (B.2)$$

c. $Tr\gamma_\alpha\gamma_\beta\gamma_\delta\gamma_\sigma\gamma_5 = -4i\,\epsilon_{\alpha\beta\delta\sigma}$

Proof: Case 1. $\alpha=\beta$, $\delta\neq\sigma$.

$$Tr\gamma_\alpha\gamma_\beta\gamma_\delta\gamma_\sigma\gamma_5 = i\,Tr\gamma_\alpha^2\gamma_\delta\gamma_\sigma\gamma_0\gamma_1\gamma_2\gamma_3$$

$$= i\,g_{\alpha\alpha}Tr\gamma_\delta\gamma_\sigma\gamma_0\gamma_1\gamma_2\gamma_3$$

(δ,σ) is a pair from the set (0,1,2,3). If (μ,ν) is the other pair, then as a result of Eq.(B.1),

$$Tr\gamma_\alpha\gamma_\beta\gamma_\delta\gamma_\sigma\gamma_5 \sim i\,g_{\alpha\alpha}g_{\mu\nu}$$

Since $\mu\neq\nu$, it follows that,

$$Tr\gamma_\alpha\gamma_\beta\gamma_\delta\gamma_\sigma\gamma_5 = 0 \qquad (B.3a)$$

Case 2. All indices are different.

$$Tr\gamma_\alpha\gamma_\beta\gamma_\delta\gamma_\sigma\gamma_5 = i\,Tr\gamma_\alpha\gamma_\beta\gamma_\delta\gamma_\sigma\gamma_0\gamma_1\gamma_2\gamma_3$$

$(\alpha,\beta,\delta,\sigma)$ is a permutation of (0,1,2,3). Therefore,

$$\mathrm{Tr}\,\gamma_\alpha\gamma_\beta\gamma_\delta\gamma_\sigma\gamma_5 = \begin{cases} -4i & \text{for positive permutations} \\ 4i & \text{for negative permutations} \end{cases} \qquad \text{(B.3b)}$$

Combining Eq.(B.3a) and Eq.(B.3b) one obtains,

$$\mathrm{Tr}\,\gamma_\alpha\gamma_\beta\gamma_\delta\gamma_\sigma\gamma_5 = -4i\,\epsilon_{\alpha\beta\delta\sigma} \qquad \text{(B.4)}$$

d. Summing and subtracting Eq.(B.4) to Eq.(B.2) the following relation emerges,

$$\frac{1}{4}\mathrm{Tr}\,\gamma_\alpha\gamma_\beta\gamma_\delta\gamma_\sigma(1\pm\gamma_5) = g_{\alpha\beta}g_{\delta\sigma} - g_{\alpha\delta}g_{\beta\sigma} + g_{\alpha\sigma}g_{\beta\delta} \mp i\epsilon_{\alpha\beta\delta\sigma} \qquad \text{(B.5)}$$

e. If one defines, $\Theta_{\alpha\sigma\beta\rho} = \frac{1}{4}\mathrm{Tr}\,\gamma_\alpha\gamma_\sigma\gamma_\beta\gamma_\rho$, then

$$\Theta_{\alpha\sigma\beta\rho}\Theta^{\alpha\lambda\beta\mu} = 2(\delta_\sigma^\lambda\delta_\rho^\mu + \delta_\rho^\lambda\delta_\sigma^\mu)$$

Proof:

$$\begin{aligned}\Theta_{\alpha\sigma\beta\rho}\Theta^{\alpha\lambda\beta\mu} &= (g_{\alpha\sigma}g_{\beta\rho} - g_{\alpha\beta}g_{\sigma\rho} + g_{\alpha\rho}g_{\sigma\beta})(g^{\alpha\lambda}g^{\beta\mu} - g^{\alpha\beta}g^{\lambda\mu} + g^{\alpha\mu}g^{\lambda\beta}) \\ &= \delta_\sigma^\lambda\delta_\rho^\mu - \delta_\sigma^\beta g_{\beta\rho}g^{\lambda\mu} + \delta_\sigma^\mu\delta_\rho^\lambda \\ &\quad - \delta_\beta^\lambda g_{\sigma\rho}g^{\beta\mu} + 4g_{\sigma\rho}g^{\lambda\mu} - \delta_\beta^\mu g_{\sigma\rho}g^{\lambda\beta} \\ &\quad + \delta_\rho^\lambda\delta_\sigma^\mu - \delta_\rho^\beta g_{\sigma\beta}g^{\lambda\mu} + \delta_\rho^\mu\delta_\sigma^\lambda\end{aligned}$$

since,

$$\delta_\sigma^\beta g_{\beta\rho}g^{\lambda\mu} = \delta_\beta^\lambda g_{\sigma\rho}g^{\beta\mu} = \delta_\beta^\mu g_{\sigma\rho}g^{\lambda\beta} = \delta_\rho^\beta g_{\sigma\beta}g^{\lambda\mu} = g_{\sigma\rho}g^{\lambda\mu}$$

it follows that,

$$\Theta_{\alpha\sigma\beta\rho}\Theta^{\alpha\lambda\beta\mu} = 2(\delta_\sigma^\lambda\delta_\rho^\mu + \delta_\rho^\lambda\delta_\sigma^\mu). \qquad \text{(B.6)}$$

f. If one defines, $\chi^\pm_{\alpha\sigma\beta\rho} = \frac{1}{4}\mathrm{Tr}\,\gamma_\alpha\gamma_\sigma\gamma_\beta\gamma_\rho(1\pm\gamma_5)$, then,

$$\chi^\pm_{\alpha\sigma\beta\rho}\chi_\pm^{\alpha\lambda\beta\mu} = 4\delta_\rho^\lambda\delta_\sigma^\mu \;,\; \chi^\pm_{\alpha\sigma\beta\rho}\chi_\mp^{\alpha\lambda\beta\mu} = 4\delta_\sigma^\lambda\delta_\rho^\mu$$

Proof:

$$\chi^{\pm}_{\alpha\sigma\beta\rho}\chi^{\alpha\lambda\beta\mu}_{\pm} = (\theta_{\alpha\sigma\beta\rho} \mp i\epsilon_{\alpha\sigma\beta\rho})(\theta^{\alpha\lambda\beta\mu} \mp i\epsilon^{\alpha\lambda\beta\mu})$$

$$= \theta_{\alpha\sigma\beta\rho}\theta^{\alpha\lambda\beta\mu} \mp i\epsilon^{\alpha\lambda\beta\mu}\theta_{\alpha\sigma\beta\rho} \mp i\epsilon_{\alpha\sigma\beta\rho}\theta^{\alpha\lambda\beta\mu} - \epsilon_{\alpha\sigma\beta\rho}\epsilon^{\alpha\lambda\beta\mu}$$

$\theta_{\alpha\sigma\beta\rho}$ is a symmetric tensor with respect to the indices α and β, therefore the contraction with the totally antisymmetric tensor $\epsilon^{\alpha\lambda\beta\mu}$ yields zero. Also using Eq.(B.6) one obtains,

$$\chi^{\pm}_{\alpha\sigma\beta\rho}\chi^{\alpha\lambda\beta\mu}_{\pm} = 2(\delta^{\lambda}_{\sigma}\delta^{\mu}_{\rho} + \delta^{\lambda}_{\rho}\delta^{\mu}_{\sigma}) - 2(\delta^{\lambda}_{\sigma}\delta^{\mu}_{\rho} - \delta^{\mu}_{\sigma}\delta^{\lambda}_{\rho})$$

$$= 4\delta^{\lambda}_{\rho}\delta^{\mu}_{\sigma} . \qquad \text{(B.7a)}$$

By similar reasoning,

$$\chi^{\pm}_{\alpha\sigma\beta\rho}\chi^{\alpha\lambda\beta\mu}_{\mp} = 4\delta^{\lambda}_{\sigma}\delta^{\mu}_{\rho} . \qquad \text{(B.7b)}$$

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