B A Y E S I A N A N A L Y S I S O F A P R O C E C T ' S N E T P R E S E N T V A L U E U N D E R R I S K

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B A Y E S I A N   A N A L Y S I S
O F   A   P R O J E C T'S   N E T   P R E S E N T   V A L U E
U N D E R   R I S K

C O N T E N T S

1.- INTRODUCTION

This paper considers the problem that arise because the net present value of a project cannot be deter - mined with the certainity in advance. Each decision taken during project evalution is a product of a set of assumptions conserning the future; about political and social developments, technological developments, changes in prices of inputs and outputs and so on. The problem that will be considered in the first part of the paper is how this uncertainity is to be taken into account to determine the project's net present value.

Ordinarily the decision-maker has only a forecast of the project's net present value. Based on the forcast he must decide whether to accept the project. If the project is accepted it may be possible to compare the cash flows that are actually achived with those that are forecasted. If the project is rejected, it will usually not be possible to compare the forecast with the results that would have occured if the project had been accepted. The decision maker would like to accept a project if its true net present value is greater than or equal to (0). At the time the decision must be made the true net present value is not known, but a forecast is available. The high - er the forecast value, the more likely the project is to be acceptable. Therefore the smallest forecast value that is acceptable should be determined.

The problem that will be considered in the second part of the paper is whether the minimal acceptable forecast value should be (0) or some other number inorder to maximize the expected net present value of the projects that are accepted.

In the third part of the paper the procedures dicussed in the previous parts, that is how uncertainity can be taken into account and how accept-reject decision can be made by considering the past exprience, will be applied on an agricultural machinery production project which is evaluated in TSKB. (X X)

In the last part of the paper a conclusion is assumed.

(X X) TSKB - TÜRKİYE SINAİ KALKINMA BANKASI

2.- ANALYSIS OF A PROJECT'S NET PRESENT VALUE UNDER UNCERTAINITY

Uncertainity which is inherent in project evaluation can be taken into account by determining the project's expected value and the dispersion of the possible outcomes around the expected value. To find the risk of the expected value, the possible range of each variable might be identified and a probability can be attached to each possible value of the variables within the range. Those Probabilities are generally subjective and reflect the state of belief of specialized analyst who was fully familiar with the specific variable. In most cases it is not necessary to analyze the variations of all variables since some variables have small effect on the net present value of the proposed project. In these circumstances Sensivity analyses may be used to determine the variables affecting the expected net present value in considerable amounts. In this part fist Sensivity analysis will be considered and after the crucial variables are determined how the risk of net present value can be calculated will be explained in section (2.2.-) .

For any project, net present value before taxes can be calculated with the following equation :

$$V_e = -\left[\sum_{t:o}^{n} \frac{I_t}{(1-k)^t} + \sum_{t:o}^{n} \frac{W_t}{(1-k)^t}\right] + \left[\sum_{t:o}^{n} \frac{R_t - E_t}{(1-k)^t}\right] \quad (1)$$

Where,

V_e Is the forecasted net present value of the project,

I_t Is the amount of the investment made in year (t),

W_t Is the amount of working Capital needed in year (t),

R_t Is the net cash inflows in year (t),

E_t Is the net cash outflows in year (t),

k Is the discount rate ,

n Is the life of the project.

2.1.- SENSIVITY ANALYSIS

Cash Flows of a proposed project are estimated under specific assumptions. Beause there is a large amount of uncertainity connectedwith these assumptions, actual values of cash flows may deviate from the expected values. Therefore it is important to investi - gate the impact of such deviations on the net present value of the project. Sensivity analyses is used in early stages of uncertainity analyses to identify the variables in the estimation of which special care should be taken. A simple method is to vary the magnitude of the variables by a certain percantage and than to deter - minethe change in the net present value, $\Delta V_e / V_e$ is determined due to percentage change in any variable, $\Delta X_t / X_t$. The elasticity of any variable in the net present value analyses can be defined as :

$$E = \frac{\Delta V_e / V_e}{\Delta X_t / X_t} \qquad (2)$$

To determine the most crucial variable in the calculation, elasticity of each variable must be found. The most crucial variable is the one with the highest elasticity.

Changes in the net present value can be found by taking the first derivative of equation (1) :

$$\frac{\partial V_e}{\partial X_i} = \frac{\partial \left[-\sum_{t:o}^{N} \frac{I_t}{(1+k)^t} - \sum_{t:o}^{n} \frac{W_t}{(1+k)^t} + \sum_{t:o}^{n} \frac{(R-E)_t}{(1+k)^t} \right]}{\partial X_i} \qquad (3)$$

Where X_i is any variable among I_t, W_t, R_t, E_t, Investment period, etc.

From equation (3) the following result is obtained :

$$\Delta V_e = \sum_{t:o}^{n} \frac{\Delta X_{i,t}}{(1+k)^t} \qquad (4)$$

Where $\Delta X_{i,t}$ is the change in the value of any variable X_i in the year (t).

Such sensivity analysis helps to provide a better under - standing of the critical variables but it does not guide the forecaster about the possible occurance of the varia - bles. It does not tell about which of the pessimistic and optimistic values have higher chance of occuring. In some situations sensivity analyses gives enough information to take a decision. That is a proposed project may be unpro - fitable under the best conditions of all variables or alternatively it may be profitable even in the worst cir - cumstances; however this will not often the case.

Sensivity analyses may be used in early stages of project evaluation. It is not necessary to analyze the variations of all possible variables. It will be sufficent to confine the analysis to the key variables affecting the project the most, either because they are large in value as para - meters or they are expected to vary considerably below or above the most likely magnitude. Therefore in this paper sensivity analysis used to determine the crucial variables which have great impact on the forecasting of the net present value of the project. Based on those variables risk of the project will be determined in the next section.

2.2.- R I S K A N A L Y S I S

The risk of investment projects are measured with certain ways under varying assumptions about cash flow behaviour. The main idea is to develop pertinent information about the expected value and the probability distribution of possible values. In general the probability distribution of cash flows for different periods are not necessarily the same. Both expected value of cash flows and dispersion of the probability distribution change over time. Therefore cash flow distribution for each period should be forecasted as a first step in the determination of the proposed project risk. The probability distribution of cash flows can be found by analysing variables affecting the cash flows. It is not necessary to determine pobability distribution of each variable entering into the cash flow calculation. Using sensivity analysis as dicussed in previous section, one can identify the variables for which probability distributions will be determined. Other variables can be taken into calculation at their most likely values.

In the present part the expected values and variances of the cash flows per period is calculated assuming the crucial variables which are identified by sensivity analysis have a normal probability distribution.

Expected value of net cash flow in period (t) is :

$$\bar{A}_t = \sum_{i:1}^{m} \bar{X}_{i,t} \qquad (5)$$

where,

$\bar{A}_t$ Is the expected value of the net cash flow in period (t),

$\overline{X}_{i,t}$ Is the expected value of the variable X_i in period (t),

M Is the number of variables in the calculation of net cash flow,

and variance of the cash flow period (t) can be found by equation (6) assuming that variables are independant in period (t).

$$\sigma_t^2 = \sum_{i:1}^{m} \sigma_{i,t}^2 \qquad (6)$$

Where,

σ_t^2 Is the variance of the cash flow in period (t),

$\sigma_{i,t}^2$ Is the variance of the variable X_i in period (t),

$\overline{X}_{i,t}$ and $\sigma_{i,t}^2$ which are used in equation (5) and (6) res - pectively, can be calculated as follows :

$$\overline{X}_{i,t} = \sum_{j:1}^{r} X_{i,t}^{J} \; P_{X_{i,t}^{J}} \qquad (7)$$

and $\sigma^2_{i,t} = \sum_{j:1}^{r} (X^J_{i,t} - \bar{X}_{i,t})^2 P_{X^J_{i,t}}$ (8)

Where , $X^J_{i,t}$ is a possible value of variable X_i in period t.

$P_{X^J_{i,t}}$ is the probability occurance of $X^J_{i,t}$

r is the number of possible values of variable X_i in period t.

Standart deviation of the preject's net present value, as a degree of risk can be obtained by considering the rela - tionships between cash flows from period to period. First the projects where cash flows are independant will be con - sidered.

2.2.1.- INDEPENDANT CASH FLOWS OVER TIME

With this type of cash flow the outcome in period (t) does not depend upon what happened in period (t-1). Stated differently there is no casuative relationships between cashflows from period to period.

Given the assumption of independant cash flows over time, the standart deviation of the project's net present value is :

$$\sigma_e = \sqrt{\sum_{t:o}^{n} \frac{\sigma_t^2}{(1+k)^{2t}}} \qquad (9)$$

The expected value and the standart deviation of the probability distribution of possible net present values give a considerable amount of information by which to evaluate the risk of the investment proposal.

2.2.2.- D E P E N D A N T   C A S H   F L O W S   O V E R   T I M E

For most investment proposals however the cash flow in one period depends upon the cash flows in the previous periods. If an invesment proposal turns bad in the early years the probability is high that cash flows in later years also will be lower than originally expected. In most investment situations it will be unrealistic to assume that the outcome in the early life of the proposed project does not affect the later outcomes.

The consequence of cash flows being correlated over time is that the variance of the probability disrtibution of the project's net present value is larger than it would be if independant cash flows is assumed.

The dispersion of the probability distribution will be greater as the degree of correlation increase. The expect - ed value of net present value however is the same regadless of degree of correlation over time.

In the remainder of this section variance of the net pre - sent value will be considered under two conditions. First in the case of perfect correlation and second with moderate correlation.

P e r f e c t   C o r r e l a t i o n

If the actual cash Flow for a period deviates from the expected cash flow of that period this implies that cash flows in all other periods will deviate in exactly the same relative manner. Therefore cash flows are said perfectly correlated over time. In other Words the cash flow in period (t) depends entirely upon what happened in previous periods. If the actual cash flow in period (t) is (k) standart deviation to the right of the expected value of the cash flow for that period; actual cash flows in all other periods will be (k) standart deviation to the right of their respective expected values. That is if,

$$A_1 = \bar{A}_1 - k\sigma_1 \tag{10}$$

then $$A_2 = \bar{A}_2 - k\sigma_2 \tag{11}$$

The formula for the standart deviation of a perfectly correlated stream of cash flows over time is :

$$\sigma_e = \sum_{t:o}^{n} \frac{\sigma t}{(1-k)^t} \tag{12}$$

The standart deviation for a perfectly correlated stream of cash flows is significantly greater than the deviation for the same stream under the assumption of independance. The probabilistic analysis of a project with a perfectly correlated stream of cash flows over time is the same as that illustrated previously for a project with an independant stream.

M o d e r a t e   C o r r e l a t i o n

When the cash flows of the project is neither appoxima - tely independant nor perfectly correlated over time, the standart deviation of the project's net present value can be found with a series of conditional probability distributions. The use of conditional probability distributions enables us to take into account the cor - relation of cash flows over time.

Un fortunately for complex situations the mathematical calculation of the standart deviation is infeasible. for these situations the probability distribution of possible net present values can be formed by using simulation techniques in which cash flows are calculated from randomly sellected values of variables.

It is seen that the assumption as to the degree of corre - lation of cash flows over time is an important one. The risk of a project will be considerably greater if the cash flows are highly correlated over time than if they are mutually independant, all other things being the same.

Although independancy of cash flows over time is often assumed for ease of calculation, this assumption greatly underestimetes project risk if in fact the cash flows are highly correlated over time. Thus it is important that a carefull consideration should be given to the degree of dependancy of cash flows over time. Otherwise the assesment of risk may well be distorted. Dealing with the problem, the use of conditional probabilities it is the most accurate way to consider the risk of the project's net present value.

3.- B A Y E S I A N A N A L Y S I S O F A C C E P T - R E J E C T D E C I S I O N

In the present part the acceptance criterion problem is analy sed . Edward M.Miller (I) has dealed with the question of whether the minimal acceptable forecast value should equal to (O). He has introduced a Bayesian statis - tical framework to analyze the problem. He interprets the forecast as a sample estimate of V which denotes the net pre - sent value of the proposed project, and assumes that it does not incorporate prior knowladge about the relative frequency of good and bad project proposals. When such prior knowledge is taken into account the smallest accept - able forecast may be different than (O).

In this part the Bayesian Statistical Framework suggested by Miller is used to analyze the acceptance criterion problem. In this context two forecasts of V are compared. These are the initial forecast which is discussed in the

I) See reference 1.

previous part and a revised forecast that incorporates both sample and prior information.

If accept-reject decisions are made using the initial forecast of V , then in general the correct minimal acceptable forecast of V will not be equal to (0). It may be greater or smaller than (0).And its exact determination will require the use of prior information.

An alternative procedure is to revise the initial forecast by taking prior information into account,and making a decision using the mean of the revised (posterior) distribution as the forecast of V. The revised forecast of V has a desirable property that project proposals should be accepted if the revised forecast is greater than / or equal to (0).

Considering these two decision procedures, one may wonder why initial forecast does not incorporate all relevant data. If it did ,the person preparing the forecast could make the decision. The more common practice is for the responsability of preparing forecasts and of making decisions about pro - jects to be assigned to different persons. This may reflect the belief that the desicion maker has some relevant data, that is not available to the forecaster. If this relevant data is embodied in the greater experince of the decision maker, it may be difficult to transmitit to the forecaster. These circumstances could favor the first decision procedure that is the accept-reject desicions are made using the ini - tial forecast of project's value.

3.1.- AVAILABLE KNOWLADGE OF THE DECISION-MAKER WITH BAYESIAN APPROACH

It is relatively common in project evaluation to prepare point estimates of the cash flows of the proposed project. Not much is known in any systematic way about the statis- tical characteristics of these cash flow estimates and their relation to subsequent realizations. Therefore the decision maker might calculate statistical distributions of the cash flow estimates and use them in the accept-re - ject decision.

The terminology used is summarized below :

V Is the true net present value of a proposed project. It is a random variable because its actual value is not known when the decision is made but it will be observed if the proposed project is accepted.

V_e Is the initial forecast which can be made available to the decision maker before a decision is made. The procedures used in the calculation of V_e were discused in the first part of this paper. V_e Takes into account all available project-specific information;but may not take into account all of the prior experience with similar projects. V_e Is the sort of forecast that might be prepared by a specialized analyst who was fully familiar with the specific project but who was not necessarily fully familiar with the previous experience with similar projects.

$\bar{V}_r$ Is the mean of the posterior (revised) distribution as the forecast of V.

The joint distribution of the random variables V and V_e is denoted by f(V, V_e). It is assumed, in principle, to be known to the desicion maker so that he can calculate a number of related distributions that are relevant for a decision making. Two of these related distributions are marginal distributions. One of these, the prior distribution of V is obtained from,

$$f (V) = \int f (V, V_e) \partial V_e$$

The value of f(V) represents the prior probability of the event such that a new project has a true net present value of V. The prior probability is assigned by management before examining the specific project, based on their previous experience with the similar projects. Hence the adjective " prior " is used to refer this fact. If there has been little relevant previous experience, the prior distribution will have a large variance.

The other marginal Distribution is :

$$f (V_e) = \int f (V,V_e) \partial V \qquad (13)$$

The value of $f(V_e)$ represents the marginal probability of observing a particular forecast V_e .

Other relevant distributions that can be calculated are two conditional distributions. To study forecasting accuracy, $f (V_e/ V)$ which is the conditional distribution of Ve given V is needed.

It is defined as :

$$f (V_e/V) = \frac{f (V,V_e)}{f (V_e)} \qquad (14)$$

The other conditional distribution is :

$$f\ (V/V_e) = \frac{f\ (V, V_e)}{f\ (Ve)} \tag{15}$$

$f(V/V_e)$ Is the revised or posterior distribution of V. It can be thought of as a revision of the prior distri - bution of V to reflect the additional information conta - ined in the forecast of V_e.

Using the revised distribution, $f\ (V/V_e)$ the decision maker can calculate another forecast that incorporates both prior distribution and the specific information contained in the forecast of V_e. The revised forecast is simply the expected value of V calculated from posterior distribution. That is :

$$\bar{V}_r = \int V\ f\ (V/V_e)\ \partial V \tag{16}$$

3.2.- A L T E R N A T İ V E   D E C I S I O N   P R O C E D U R E S   TO   A C C E P T   A   P R O J E C T

As discussed in section (3.1.-), before the accept-reject decision is made some statistical distributions of the cash flow estimates are available to the decision maker. These distributions are :

$f(V, V_e)$, The joint distribution of V and V_e, represents the state of knowladge of the decision maker before a fore - cast is made.

$f(V)$, The prior distribution, represents what the decision maker knows about the project before a forecast is made. If a decision had to be made before a forecast could be obtained, only the prior distribution would be relevant.

$f(V_e/V)$ Summarizes what the decision maker knows about the accuracy of the forecast in general.

$f(V/V_e)$, The posterior distribution represents all of the relevant information available to the decision maker after the results of the forecast have been obtained.

Therefore any rational decision process will take into account all of the available knowladge. The two decision procedures considered here do this in different ways. The rule of the first decision procedure is:

Accept the project when $V_e \geqslant C_e$.

And with the second decision procedure the rule is:

Accept the project when $\overline{V}_r \geqslant 0$.

These procedures are equivalant in the sense that they will make the same accept or reject decision.

3.2.1.- F I R S T D E C I S I O N P R O C E D U R E

ACCEPT THE PROJECT WHEN $V_e \geqslant C_e$

When this procedure is used the accept-reject decision is made, based on the initial forecast. The optimal value of C_e which is the smallest acceptable forecast of V is determined on the basis of the knowladge available to de - cision maker before the forecast results are known. In computation of C_e the objective is to maximize the expect - ed net present value of accepted projects. thus the value of C_e depends upon the prior distribution. A different prior distribution might sellect a different value of Ce .

H o w   t o   s e l l e c t   t h e   v a l u e   o f C_e

A project will be accepted if $V_e \geqslant C_e$ and rejected other - wise. The value of C_e used depends on the joint distribution of V and V_e. It would be the same for all projects that come from the same joint distribution.

If a project is accepted ($V_e \geqslant C_e$) it will contribute a net present value of V to the enterprise (V may be negative). If a project is rejected ($V_e \geqslant C_e$) its contribution is (0) regardless of V. The expected net present value that could be earned by accepting projects for which the cash flow forecast is exactly V_e is:

$$g(V_e) = \int V \quad f(V, V_e) \, \partial V \tag{17}$$

Where $g(V_e)$ is the expected net present value.

If $g(V_e)$ is negative we will want to reject the project. For any forecasting scheme $g(V_e)$ should be a monotonically increasing fuction of V_e. If so, to maximize the expected net present value, the fallowing decision rule can be used :

Accept if $V_e \geqslant C_e^*$ where $g(C_e^*) = 0$

Fallowing this rule the maximized expected net present value will be :

$$G(C_e^*) = \int_{(C_e^*)}^{\infty} g(V_e) \, \partial V_e \tag{18}$$

To summarize it is shown that if a decision maker wishes to maximize the expected net present value of the projects that he accepts and wishes to use the initial forecast of the preject's net present value as the decision rule he should accept projects for which $V_e \geqslant C_e^{*}$ where C_e^{*} is defined as the value of V_e that satisfies the fallowing equation :

$$\int V \quad f\,(V, V_e)\,\partial V = 0 \qquad (19)$$

2.2.- S E C O N D   D E C I S I O N   P R O C E D U R E

ACCEPT THE PROJECT WHEN $\overline{V}_r \geqslant 0$

$\overline{V}_r$ Is the mean of the posterior distribution and its calculation was shown in section (3.1.-) as :

$$\overline{V}_r = \int V \quad f\,(V/V_e)\,\partial\,V \qquad (16)$$

It reflects all of the knowladge available to the decision maker after the value of the forecast has been revealed. It is not surprising that the optimal value of the accep - tance is (0). Since the objective is to maximize the ex -

pected net present value of accepted projects,any project with a posotive value of $\overline{V}_r$ should be accepted.

Using this approach a different decision maker who had a different prior distribution might end up with a different value of $\overline{V}_r$,but not a different value of acceptance criterion .

3.3.- A SPECIAL CASE OF DECISION PROCEDURESUNDER NORMAL PRIOR DISTRIBUTION

To illustrate the accept-reject procedures, a special case which involves normal prior distribution and normal posterior distribution can be taken into account. It is true that a normal prior distribution may contradictthe decision-maker's belief that the certain values of the true value of V are impossible. Ordinarily, however, the assignment of some small probability to the impossible values of V will not make a material effect on the accept-reject decision.

Therefore as a special case normal prior distribution will be considered here because of two reasons:

First, The decision maker can best express his belief about the project's net present value by normal distribution without any mathematical skill.

Second, the use of normal prior distribution leads under certain conditions to an extremely simple posterior distribution. These conditions are (II) :

When

I. The prior distribution of V is normal with parameters $\overline{V}_0$ and σ_0 .

II. The distribution of forecast values f(V_e/V) is normal with parameters V_e and σ_e .

III. The value of σ_e is known.

The posterior distribution of V will also be normal with

(II). The revision of prior disributions are dicussed in SCHLAIFER (2, pp 441) parameters,

$$\bar{V}_r = \frac{\bar{V}_o \left[1/\sigma_o^2\right] + V_e \left[1/\sigma_e^2\right]}{1/\sigma_o^2 + 1/\sigma_e^2} \qquad (20)$$

$$\frac{1}{\sigma_r^2} = \frac{1}{\sigma_o^2} + \frac{1}{\sigma_e^2} \qquad (21)$$

substituting equation (21) into equation (20), the expeced value of the posterior distribution will be found as :

$$\bar{V}_r = \frac{\sigma_r^2}{\sigma_o^2} \bar{V}_o + \frac{\sigma_r^2}{\sigma_e^2} V_e \qquad (22)$$

and $$\sigma_r^2 = \frac{\sigma_o^2 \; \sigma_e^2}{\sigma_o^2 + \sigma_e^2} \qquad (23)$$

If the second decision procedure is used (that is accept the project if $\bar{V}_r \geq 0$), equation (22) can be used di - rectly.

If the first decision procedure will be used, we can find the smallest acceptable level of V_e from equation (22) by calculating the value of V_e for which $\bar{V}_r$ will be equal to (0) . This will be C_e^* .

Thus,

$$\sigma_o = \frac{G_r^2}{G_o^2}\,\bar{V}_o + \frac{G_r^2}{G_e^2}\,C_e^* \qquad (24)$$

And from equation (24) C_e^* will be found as :

$$C_e^* = -\frac{G_e^2}{G_o^2}\,\bar{V}_o \qquad (25)$$

4.- APPLICATION OF THE PROPOSED PROCEDUDES TO AN INVESMENT PROJECT

In the present part, the procedures discussed in part (2.-) and part (3.-) will be applied to a project which was evaluated in TSKB. In the project feasibility analysis of an agricultural machinery production was made by the specialized analysts of TSKB.

The real name of the project is not mentioned because of the secreacy reasons of the project.

4.1.- THE EXPECTED NET PRESENT VALUE OF THE PROJECT

TABLE I. Shows the forecested cash flows of the project. The forecasted values of the cash flows can be taken as the expected values.

T A B L E I.

	1977	1978	1979	1980	1981	1982	1983/89	1990
REVENUES	-	7.535	11.523	14.110	16.715	17.442	17.500	9.750
INVESTMENT	10.055	-	-	-	-	-	-	-
WORK CAPITAL	1.707	33	612	348	-	-	-	-
EXPENSES	- -	7.881	8.908	10.440	11.899	11.908	11.908	-
CASH FLOW	-11.762	- 379	-2.003	-3.322	-4.816	-5.534	-5.592	-9.750

And the expected net present value of the project is calculated using equation (1) and taking the discount rate as 20% which is given by the project's analyst.

$$V_e = -11.762 + \frac{-379}{(1+2)} + \frac{+2.003}{(1+2)^2} + \frac{+3.322}{(1+2)^3} + \frac{+4.816}{(1+2)^4} + \frac{+5.534}{(1+2)^5} -$$

$$+ P\ V\ I\ F_a \left[\frac{5.592}{(1+2)^5} \right] - \frac{9.750}{(1+2)^{13}} = 4.794$$

Here $P\ V\ I\ F_a$ is the present value factor of an annuity for seven years and k = 20% and the result of it is (3.6046.) (III)

4.2.- S E N S I V I T Y A N A L Y S I S O F T H E I N V E S T M E N T P R O J E C T

To determine the most crucial variables, the elasticities of revenue, expense, investment and investment period are calculated by using equation (4) the changes in the

(III) It is found from the present value tables (4 , pp 438) variables are calculated according to the analyst's belief :

E l a s t i c i t y   o f   r e v e n u e

Revenue might be decreased at most 10% , that is,

$$\frac{\Delta R_t}{R_t} = -0.10$$

	1978	1979	1980	1981	1982	1983/89	1990
ΔR_t	-754	-1.152	-1.411	-1.672	-1.774	-1.750	-975

Using equation (4), change in the net present value due to the revenue change is,

$$\Delta V_e = \frac{-754}{(1+2)} + \frac{-1.152}{(1+2)^2} + \frac{-1.411}{(1+2)^3} + \frac{-1.672}{(1+2)^4} + \frac{-1.744}{(1+2)^5} -$$

$$+ 36.046 \left[\frac{-1.750}{(1+2)^5} \right] + \frac{-9.75}{(1+2)^{13}} = -6.378$$

Elasticity of revenue is calculated using equation (2)

$$\varepsilon_R = \frac{-6.378 / 4.794}{-0.10} = 13.3$$

E l s t i c i t y   o f   e x p e n c e

Expense might be increased at most 30% , that is,

$$\frac{\Delta E_t}{Et} = 0.30$$

	1978	1979	1980	1981	1982	1983/89	1990
ΔE_t	2.364	2.672	3.132	3.570	3.572	3.572	-

Using equation (4) and (2) ,

$$\Delta V_e = \frac{2.346}{(1+2)} + \frac{2.672}{(1+2)^2} + \frac{3.132}{(1+2)^3} + \frac{3.570}{(1+2)^4} + \frac{3.572}{(1+2)^5} + 36.046\left[\frac{3.572}{(1+2)^5}\right]$$

$$\Delta V_e = -13.970$$

$$\varepsilon_E = \frac{-13.970 / 4.794}{0.30} = -9.71$$

E l a s t i c i t y   o f   I n v e s t m e n t

Investment amount might be increased by 20% , that is,

$$\frac{\Delta I_t}{I_t} = 0.20 \qquad \text{and,}$$

	1977
ΔI_t	-2.011

$$\Delta V_e = -2.011$$

$$\varepsilon_I = \frac{-2.011 / 4.794}{0.20} = -2.10$$

E l a s t i c i t y o f I n v e s t m e n t

P e r i o d

Investment period may increase 100% , that is 1 year.

changes in cash flow is,

	1977	1978	1979	1980	1981	1982	1983	1984/89	199
If Investment period is 2 years	-11.762	-	-379	+2.003	+3.322	+4.816	+5.534	+5.592	+9.7
Expected cashflows with 1 year	-11.762	-379	+2.003	+3.322	+4.816	+5.534	+5.592	+5.592	+9.7
Δ CASHFLOWS	-	+379	-2.382	-1.319	-1.494	-718	-58	-	-

Using equation (4) ,

$$\Delta V_e = \frac{379}{(1+2)} + \frac{-2.382}{(1+2)^2} + \frac{-1.319}{(1+2)^3} + \frac{-1.494}{(1+2)^4} + \frac{-718}{(1+2)^5} + \frac{-58}{(1+2)^6} = -3.130$$

$$\varepsilon_{IP} = \frac{-3.130 / 4.794}{1.0} = -0.65$$

Higher the elasticity,more crucial is the variable.

therefore the most crucial variables in this project are

Revenue , Expense and Investment amount.

4.3.- R I S K A N A L Y S I S O F T H E I N V E S T M E N T P R O J E C T

TABLE. $II_{1,2,3}$ Shows the possible values of the key variables which are identified using the sensivity analysis and their attached probabilities estimated by the specialized analyst. The expected values and the standart deviations are calculated using equations (7) and (8) respectively.

TABLE II$_1$. (REVENUES)

Probability	1978	1979	1980	1981	1982	1983/89	1990
.2	8.000	13.000	15.000	19.000	21.000	21.500	11.000
.5	7.670	11.846	13.200	17.430	17.484	17.100	10.900
.3	7.000	10.000	12.000	14.000	15.000	15.500	7.000
E (R)	7.535	11.523	14.110	16.715	17.442	17.500	9.750
σ_t^2	138.225	183.744	1.908.100	3.511.225	4.321.763	4.480.000	3.242.500
σ_t	371,8	1.088	1.381	1.874	2.078	2.116	1.800

TABLE II_2. (EXSPENSES)

Probabilitiy	1979	1979	1980	1981	1982	1983/89	1990
.3	9.000	10.000	12.000	13.500	14.000	14.200	-
.5	7.562	8.616	10.080	11.998	11.908	11.908	-
.2	7.000	8.000	9.000	10.000	10.500	10.600	-
E (CE)	7.881	8.908	10.440	11.899	11.908	11.908	-
σ_t^2	581.760	619.369	1.209.600	1.490.200	1.948.864	2.281.107	-
σ_t	763	787	1.099	1.220	1.396	1.510	-

TABLE II3. (INVESTMENTS)

Probability	1977
.35	11.000
.45	9.566
.20	9.500
E (I)	10.550
σ_t^2	481.474
σ_t	693

Assuming that key variables are independant in each period, equation (6) is used to calculate the variance of cash flows.

	1977	1978	1979	1980	1 981	1982	1983/89	1990
σ_t =	693	848	896	1765	2236	2504	2600	1800

Cash Flows over time are assumed to be perfectly correlated and the standart deviation of the expected net present value can be found using equation (12)

$$\sigma_e = 693 + \frac{848}{(1+2)} + \frac{896}{(1+2)^2} + \frac{1765}{(1+2)^3} + \frac{2236}{(1+2)^4} + \frac{2504}{(1+2)^5} + 36.046\left[\frac{2600}{(1+2)^5}\right] + \frac{1800}{(1+2}$$

$$\sigma_e = 9062$$

4.4.- ACCEPT-REJECT DECISION OF THE INVESTMENT PROJECT

Initial forecast of the distribution of the project's net present value is found as normal distribution with a mean of 4794 and a standart deviation of 9062 .

To select the decision criterion C_e^* ,the prior distribution of actual net present values should be determined. Analysts who are specialized in agricultural machinery projects believe that the prior distiribution in this group is normal such that 60% of the net present values have a negative value and the mean of the actual values is (- 1.000).

To find the parameters of normal prior distribution :

$$P (V \leq 0) = .60$$

$$P (V \geq 0) = P_N \left(Z \geq \frac{0 - (- 1.000)}{\sigma_o} \right) = .40$$

and from normal distribution tables (IV) it is found that

$$P_N (Z \geq .25) = .40$$

therefore

$$\frac{1.000}{\sigma_o} = .25 \quad \text{and} \quad \sigma_o = 4.000$$

Using equation(25) C_e^* is calculated as :

(IV) Schlaifer , (2 , pp 704 , Table 3)

$$C_e^* = - \frac{(9062)^2}{(4000)^2} (- 1000) = 5130$$

Thus the desicion rule is :

Accept the project if $V_e \geq 5130$

The initial forecast of the proposed project was 4794 which is less than the minimum acceptable level. We would reject the project if there were no intangibles associated with the project.

Now, the decision maker should decide whether the intangible considerations are sufficent to offset the difference between the acceptance criterion and the initial forecast of the project.

5.- CONCLUSION

The application of this analysis is quite straightforward if the management use an explicit decision process in deciding about proposed projects. Although the analysis in this paper is carried out in terms of net present value, a comparable analysis could be carried out for the internal rate of return, the payback period or other statistics that are frequently used in practice.

The theoretical sections of this paper proceed on the assumption that one can distinguish between the initial forecast (which excludes prior information),and the revised forecast. In practice it is not easy to know what information in included in the forecast.

In the decision Procedures developed in this paper to accept-reject a proposed project, the bias of forecasting was not considered. It was assumed that an unbiased initial forecast of V is made. This is almost never the case Initial forecast should be adjusted considering the bias. If there is a positive bias, decision criterion C_e^* must be increased.

The post-audit process can be very helpful to decision makers in understanding and controling the decision making process. If post-audits reveal that forecasted present values are typically greater than actually realized present values for a certain class of project's,this would support the conclusion that the forecasted net present values are biased or exclude some relevant information. If the post-audit process reveals that forecasted net present values of accepted projects are typically lower than realized net present values of them,the cause may be a systematic downward bias in the forecasting process, or a high proportion of good projects in the prior distribution.

An important area for further studies is to determine the nature of the prior distribution of projects for certian classes. Emprical studies of the prior distribution might be very helpfull to managers.

6.- A C K N O W L E D G E M E N T S

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