OPTIMAL PLANNING OF TRANSMISSION FACILITIES

FOR

TELECOMMUNICATIONS NETWORKS

by

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We hereby recommend that the thesis entitled "Optimal Planning of Transmission Facilities for Telecommunications Networks" submitted by Mehlika Miraboğlu be accepted in partial fulfillment of the requirements for the Degree of Master of Science in Industrial Engineering, School of Engineering, Boğaziçi University.

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ABSTRACT

The aim in the transmission facilities planning problem for a telecommunication network is to decide on the routing of the trunk groups over the transmission network and types and capacities of facilities, so that, for a given set of trunk group requirements, the resulting installation cost is minimized and a certain degree of reliability is maintained.

In the present study, the optimization problem and its characteristics are presented. In the formulation of the problem, a new dimension, technology of the system, is introduced. This leads to a multi-fixed charge cost function with concave continuous portions. The concavity reflects the economies of scale of the investment costs. Also, alternative paths in routing the trunk groups are considered to assure a certain satisfactory degree of reliability.

Three binary mixed integer programming models are developed and applied to a test network to show their functionality and to set a comparison basis for future study.

Keywords : - Investment Planning
- Telecommunications Network

ÖZET

Aboneler arasındaki telefon bağlantılarını sağlayan telekomünikasyon şebekesi, santral ve iletişim şebekesi olarak ikiye ayrıştırılıp incelenmektedir. Santral şebekesi, birbirlerine devreler ile bağlanmış santrallardan oluşmaktadır. İletişim şebekesi ise santralları bağlayan iletişim sistemlerini içerir. Telekomünikasyon şebekesinin eniyilenmesi (optimizasyonu) iki etapta düşünülmektedir. Santral şebekesinin eniyilenmesi olan birinci kademenin çıktıları ikinci optimizasyon kademesi olan iletişim şebekesinin eniyilenmesi problemine girdi olarak kullanılmaktadır.

İletişim şebekelerinin eniyilenmesindeki hedef santrallar arasında doğan talebi iletecek eniyi (optimal) dağıtım yollarını saptamak ve seçilen iletişim sistemlerinin kapasitelerini belirlemektir. Bu hesaplamada, jonksiyon devre sayısı taleplerinin enküçük (minimum) maliyetle karşılanması ve abonelere yeterli bir servis ve güvenilirlik sağlamak amaçlanmaktadır.

Çalışmada, probleme yeni bir boyut getirilmiş ve sistemlerin teknolojilerinin seçimi de formülasyonda içerilmiştir. Bu boyut, problemin önemli bir özelliği olan ve yatırım

maliyetlerindeki ölçek ekonomisini temsil eden içbükey maliyet işlevinde sıçramalara yol açmaktadır. Literatürdeki sezgisel ve yaklaşık çözüm yordamları bu boyutu, genellikle ele almamaktadır.

Bu problemi çözmek amacıyla üç tane 0,1 karmaşık tamsayı programlama modeli geliştirilmiştir.

Bu modeller İrlanda telekomünikasyon şebekesinin bir bölümünü oluşturan bir test şebekesi için çalıştırılmış ve mukayeseli olarak değerlendirilmişlerdir.

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CHAPTER I

INTRODUCTION

A telecommunications network is the means of inter-connecting telephone customers on demand. It comprises telephone exchanges (switching nodes) inter-connected by transmission systems (links). A national network can be considered as a number of inter-linked discrete sub-networks and also being composed of a switched network and a transmission network.

The switched network comprises switching nodes inter-connected by groups of circuits (trunk groups) which carry the parcels of telephone traffic.

The transmission network consists of transmission systems inter-connecting switching nodes.

In real-life telecommunications networks the planning process consists of two major steps. The first step of the planning process termed as trunking analysis (Baybars, et.al., 1981), (Nivert, et.al.,1983) translates the originating traffic demand into trans- mission channels or trunks i.e. determines linkwise circuit requirements. This analysis employs a network hierarchy which permits blocked traffic on a direct route to be switched through other junctions, eventually reaching the intended destination (alternate routing). The output of the trunking analysis is a list of trunks between all point pairs.

The second step of the planning process considers the so-computed trunks as inputs to a facilities planning model by referring to them formally as circuits. The task at this stage may be termed the transmission facilities planning problem in telecommunications network: given point-pair circuit requirements, find a minimum cost facility installation plan by specifying the type of transmission systems and the links themselves on which the systems are to be installed as well as the number of circuits to be installed on each such link. Formally, this optimization problem is a fixed-charge multi-commodity flow synthesis problem .

The overall purpose of this thesis is to develop a transmission network optimization model. Three different models are developed and tested for comparison against each other.

In Chapter II, a general definition of the problem is presented. A brief review of the relevant literature on the transmission network optimization is given in Chapter III. The specific assumptions and the formulations of the developed models are described in Chapter IV. The general input requirements and the discussions on the numerical results are included in Chapter V, while the Appendices contain the related computer outputs and some mathematical calculations.

CHAPTER II

PROBLEM DEFINITION

A telecommunications network is a collection of junctions (or points) some or all which are joined by direct communication links. It can be pictorially represented by a graph whose "vertices" and "edges" correspond to the "points" and the "links" of the network, respectively. For instance the graph of Figure II.1 represents a real-life telecommunications network with 8 points and 15 direct links.

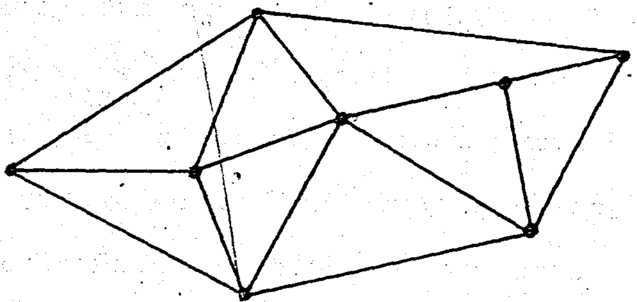

Figure II.1 A Telecommunications Network

A link is a collection of facilities known as transmission equipment which when taken together comprise various transmission systems. The main components of a transmission system are the circuits. One of the traditional transmission facilities is the cable consisting of a large number of wires. In real-life telephone networks there do not exist direct communication lines between all pairs of points. That is, in graph theoretic terms, telephone networks are, typically non-complete graphs (Baybars, et.al., 1981). However the graph is connected and therefore, it is possible to dial any point from any specific point. Traffic, in the form of voice telephone calls, originate at a junction A, such as a city, to be transmitted to another junction B, termed the destination. If between two points a direct communication link does not exist, then the call is transmitted through a sequence of links. For instance, in the network of Figure II.2 the traffic for the pair of points P_A and P_B can be also transmitted through the communication lines represented by links L_{AC} and L_{CB}. Depending on how dense (in terms of the number of links) the network is , there may be a few or many such sequences of links which could carry the traffic of a specific link. Such sequences will be referred to as alternate routes.

Furthermore, if the customer demand for some pair (P_A, P_B) exceeds the capacity of direct P_A to P_B communication link, excess demand can be switched to an alternate route.

The size of the problem grows exponentially with the number of "alternate routes" for each demand relation.

Routing implies that more than one set of circuits can be installed on a link to meet the requirements of several relations. However, the fixed cost of installing a system is in general so high and there is so much economies of scale involved in installing a larger system that, it often is less expensive to route them then otherwise.

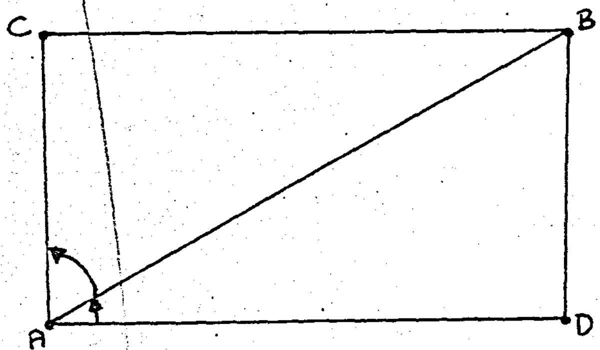

Figure II.2 Alternate Routes

The structure and dimensions of the transmission network are governed by the need to route circuits in the most cost effective manner and to give the customer a prescribed standard of service in terms of the proportion of successful calls during the busy hours of the day. Other factors that can degrade the quality of service perceived by the customer are caused by equipment failures and congestion due to unforeseen surges of traffic due to customer behaviour.

The main objective of the transmission network optimization is thus to route the trunk groups requirements in such a way that the overall system cost will be minimum.

Some related aspects and principles are presented below in order to give a better understanding of the transmission system technology.

II.1 Technological Aspects of Transmission Systems

Transmission systems exist to provide circuits for transmitting speech and other signals between the nodes of a telecommunications network. A circuit provides for the transmission of these signals in both directions. If the circuit uses a separate transmission path for each direction, then each of these unidirectonal paths is called a channel. In general, a complete channel consists of sending equipment at a terminal station, a transmission link, which may contain repeaters at intermediate stations, and receiving equipment at another terminal station (Flood, 1975).

Both transmission channels and the signals they convey may be classified in two broad classes: analogue and digital. An analogue signal is a continuous function of time; at any instant it may have any value between limits set by the maximum power that can be transmitted. Speech signals are an obvious example. A digital signal can only have discrete values. The most common digital signal is a binary signal, having only two values (e.g. 'mark' and 'space' or '1' and '0'). A telegraph signal is thus a digital signal. A television waveform is a mixture of analogue and digital signals, since it transmits both the picture contents and synchronising pulses.

To transmit an analogue signal without error the channel must be a linear system. Any departure from linearity causes nonlinear distortion of an analogue signal. Cable systems and radio-relay systems equipped with linear amplifiers are examples of analogue channels. A digital channel does not require to be linear, since its output provides a number of discrete conditions corresponding to the input signal. An example of a digital channel is a telegraph circuit, whose output signal is provided by the operation of a relay.

It does not follow that analogue signals must always be transmitted over analogue channels and digital signals over digital channels. Data communication and voice-frequency telegraphy over telephone lines are examples of transmitting digital signals over analogue channels. Analogue signals may be coded for transmission over digital channels by means of analogue-to-digital converters. An example is the transmission of speech by means of pulse-code modulation over lines equipped with regenerators.

If a link can provide adequate transmission over a band of frequencies which is wider than that of the signals to be sent, it can be used to provide a number of channels. At the sending terminal the signals of different channels are combined to form a composite signal of wider bandwidth. At the receiving terminal, the signals are separated and retransmitted over separate channels. This process is known as multiplexing. The separate channels that enter and leave the terminal stations are called baseband channels and the transmission link, which carries the multiplex signal, is called a broadband channel or a bearer channel.

CHAPTER III

LITERATURE SURVEY

Capacity expansion models have been extensively used for communication network applications (Luss, 1982).

Generally, in investment planning and capacity expansion problems, the major decisions are:

(i) investment and/or expansion capacities
(ii) time of investments and/or expansions
(iii) investment and/or expansion locations.

The first issue to be considered is the capacity of investment.

The second issue is the time of investments. In this study, the problem is solved for a target network which is aimed to be achieved after a certain time period and is considered static whereas in real-life it should be dynamic. But in the case of unsatisfied demand in other words a waiting list or backorder (as in inventory systems), the model can be visualized as a static one. So the decisions on the time of investments and/or expansions are not relevant in this type of formulation.

Decisions on investment and/or expansion location have vital importance i.e. "which link's capacity in the transmission network will be expanded" constitute the third issue. So, the question "which link's capacity" in communications networks replaces the "at which location" question of general investment location problems.

Furthermore, a new dimension which is generally not considered is introduced in this study. This is the technology or type of the transmission system (Luss, 1982). In investment planning models, the investment cost function being usually concave, exhibits economies of scale. Popular cost functions are

(i) the power cost function

$$f(x)=Kx^{a} \qquad (0<a<1) \qquad x\geqslant 0$$

(ii) the fixed charge cost function

$$f(x)=\begin{cases} 0 & \text{if } x=0 \\ A+Bx & \text{if } x>0 \end{cases}$$

(iii) or some combination of the two.

However in some applications such as telecommunication networks, the cost function is not continous. It is easily seen that when different technologies are considered, the cost function displayed is quasi-concave, i.e. it is concave in the range covered by any single technology (Figure III.1). The objective function might have jumps at the capacities of the systems.This point as emphasized in Luss's excellent paper [8], is not so simple to overcome. It causes additional difficulties in solution procedures.

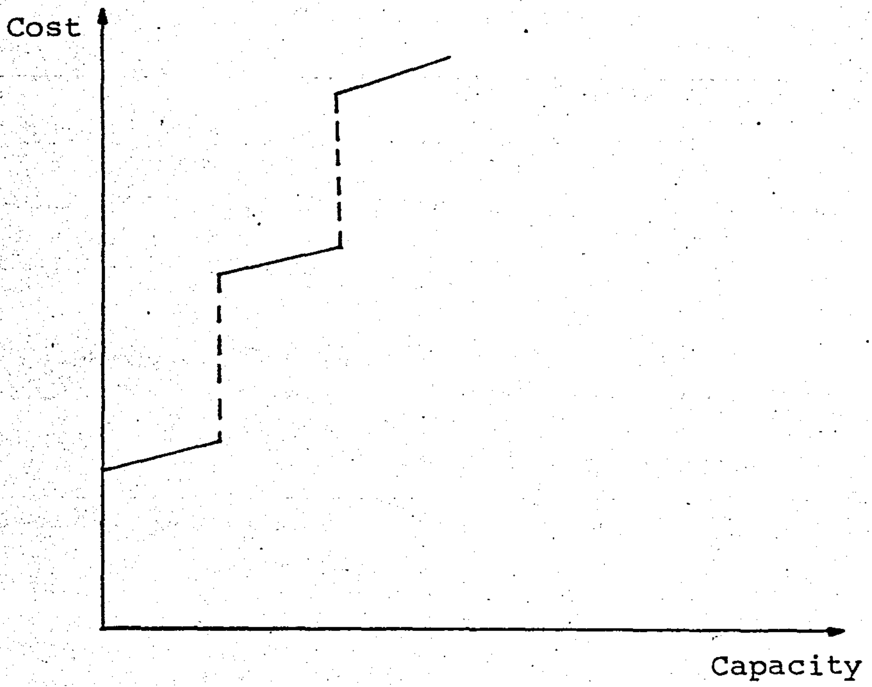

Figure III.1 Cost function

In another study, Ulusoy (1981) developed a heuristic algorithm which takes economies of scale into account. He considered that a unique technology will be applied. So he accepted an objective function such as:

$$f_{ij} = \begin{cases} 1.5\, x_{ij} & 0 \leqslant x_{ij} \leqslant 140 \\ 1.2x_{ij} + 42 & 140 < x_{ij} \leqslant 220 \\ 0.6x_{ij} + 174 & 220 < x_{ij} \end{cases}$$

Also,in Ulusoy's paper only one route is considered for each relation, so the reliability of the system is not included. Fixed charge is considered together with the impact of different fixed charge levels which carries the ingredients of a heuristic

Evranuz (1982), Nivert et.al. (1983) presents studies in which heuristic algorithms have been developed for large scale networks. In those studies, fixed charges are considered on the links and different technologies are represented by parallel edges.

In this thesis, the problem of transmission network optimization will be treated as a binary mixed integer problem. The first question that comes to the mind is "Why is an integer programming model not considered ?". It seems unreasonable to skip the possibility, for instance, to install two systems with thirty units of capacity instead of one system with ninety units of capacity if the required units are thirty-five.

As seen in Figure III.2 the characteristics of the selected cost functions are such that the costs incurred by adding a second capacity of thirty units and using five units of that system is much higher than selecting a system with ninety units. The breakdown point of the two systems is at point (30,875). Until and including thirty units, system I is cheaper. In the range between thirty-one and ninety units system II is the cheapest; from ninety-one system III is the less costly.
So to consider a second system unit of the same system type will not be an optimal solution. It will only increase the size of the problem as well as the computer time.

The cost functions (Baybars, et.al., 1981) which are multiple fixed charge in nature are graphically presented on Figure III.3 are shown below. These are the cost functions utilized in the example problem solved in Chapter V of the thesis.

$$f(x)=\begin{cases} 530000 + 3100x & 0 < x \leq 30 \\ 870000 + 1077x & 30 < x \leq 90 \end{cases}$$

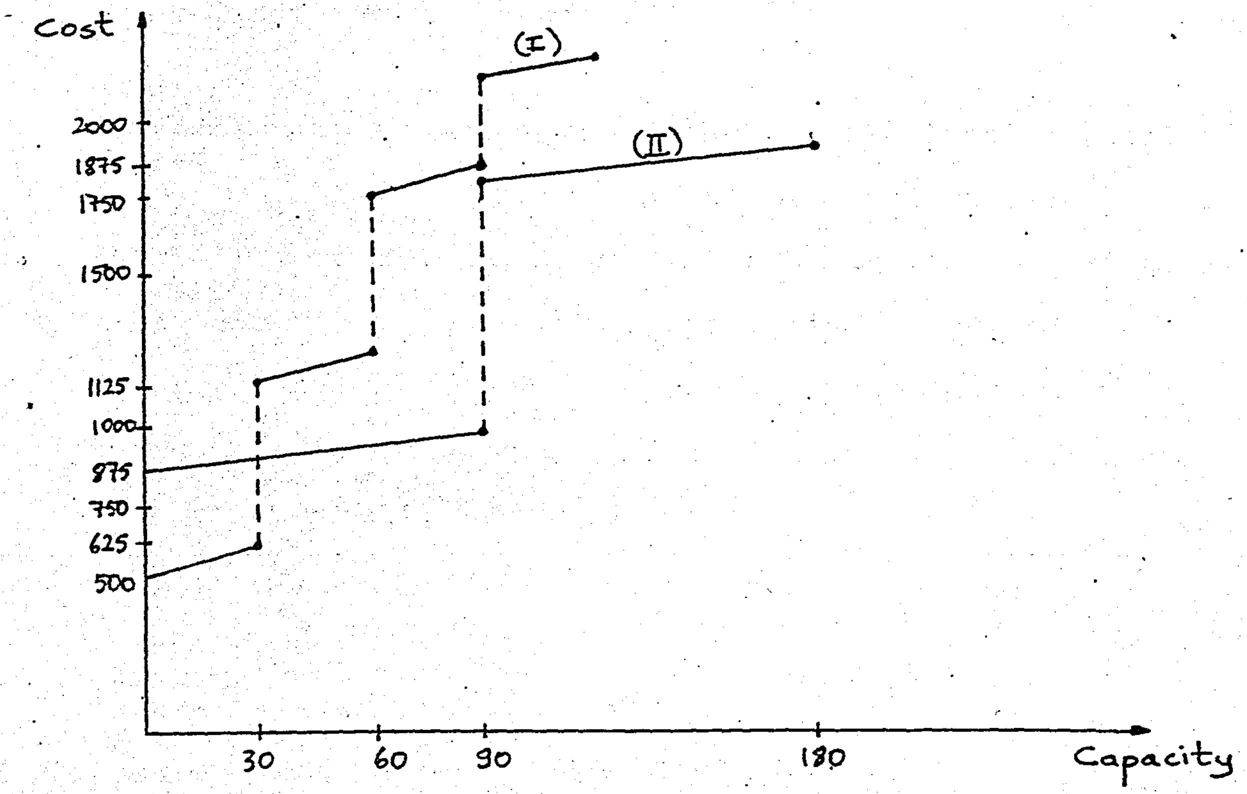

Figure III.2 Cost Functions

Opposite to the operational planning methods, the model developed by Claus et.al. (1981) allows a global cost-optimal network which is subject to a certain set of constraints. His paper describes a practical planning problem, taken from the work of the "Network Planning Department of the Telecommunication Administration of the Deutsche Bundespost". The actual planning problem is to utilize and to extend the capacity of an existing burried cable network in such a way that all (future) traffic requirements are met by minimal costs.

The mathematical structure of the entire planning problem is a mixed-integer program. The decision variables are first, discrete digital systems (PCM) set up on existing cable lines and second, new cable links required in the future. Furthermore, the formulation takes account of the circuit capacity of the system and path diversification required for reliability reasons.

FORMULATION:

(1) Cost function:

$$\text{min cost} = \ell_{ij}\Big(\sum_{i=1}^{n}\sum_{j=i+1}^{n+1} c_{ij} Z_{ij} + \sum_{i=1}^{n}\sum_{j=i+1}^{n} a_{ij} Y_{ij}$$

$$+ \sum_{i}\sum_{j} 100000\ p_{ij} + \sum_{i}\sum_{j} 100000\ V_{ij}\Big)$$

(2) Restriction of capacity: The flow on link (i,j) must be smaller than or equal to the existing circuit capacity plus a new circuit capacity achieved either by a new cable and/or by PCM systems (no PCM on new links).

$$F_{ij} \leq K_{ij} + 27\ p_{ij} + Z_{ij}$$

One PCM-system enables 27 additional circuits to be set up on an existing copper cable.

(3) Two-node connectivity: For reliability reasons, each source having more than a certain amount G of traffic must be connected via at least two node-disjoint paths i.e. the flow on this link must not exceed 65% of the total requirements.

$$X_{ij}^{k} \leq 0.65\ S^{k}$$

For two or more parallel existing links, it yields:

$$X_{ij}^{k} \leq \min[S^{k};L_{ij}^{k}] + 27\ p_{ij}$$

where

$$L_{ij}^{k} = \min[0.65S^{k};K_{ij}^{\ell}] + \min[0.65S^{k};K_{ij}^{2}]$$

K_{ij}^{1} = the circuit capacity of the first link

K_{ij}^{2} = the circuit capacity of the second link

(4) Flow conservation

$$\sum_{j=1}^{n+1} X_{kj}^{k} = S^{k} \qquad k = 1,\ldots,n$$

For possible transit nodes

$$\sum_{i\in k} X_{ij}^{k} - \sum_{i\in k} X_{ji}^{k} = 0 \qquad j \in T^{k},\ i \neq j,\ k=1,\ldots,n$$

Transit nodes T^{k} are only line-related nodes. The circuits for all relations k remain seperated along the entire path.The elements k are manually selected to avoid unreasonable links.

(5) Length Restriction: Cable paths w_{k} are not allowed to be longer than 25 km. due to the low-frequency transmission technique.

$$\ell_{i_jc} \leq 25 \qquad i_1, i_2, \ldots \in T^{k}$$

that means $\ell_{(wk)} \leq 25$

where n = number of commodities

I = integer 0,1,2...

S^k = number of circuits demanded

X_{ij}^k = flow of commodity k from node i to node j

K_{ij} = existing circuit capacity on link (i,j)

Z_{ij} = number of circuits to be newly installed on link (i,j)

Y_{ij} = 1 if a new link must be installed between (i,j)

= 0 otherwise

P_{ij} = number of PCM systems between (i,j)

V_{ij} = 1 if at least one PCM system is installed between (i,j)

= 0 otherwise

F_{ij} = total flow on link (i,j)

$= \sum_k x_{ij}^k + \sum_k x_{ji}^k$

C_{ij} = cost per circuit on link (i,j) per km.

a_{ij} = fixed charge component

T^k = set of possible transit nodes for commodity k

ℓ_{ij} = distance between nodes i and j

w_k = path between node k and center C

G =lower bound for traffic splitting

Claus used a CDC Cyber 175 for his calculations. Apex III was used as a mixed-integer program. The problem with seven nodes is calculated in 470 seconds in straightforward use, in 200 seconds with addition constraint saying that an existing link shall only be extended by PCM or new cable plants.

Baybars et.al. (1981) considered the transmission network optimization problem on switching network. However, most European countries consider additional transmission centers as well the transmission centers at switching centers (Evranuz, 1982), (Nivert, et.al.,1983) . This new network is called as transmission network and it contains all vertices of switching network. Indeed, the switching network is somehow a hypothetic network and it shows the topology of switching centers and trunk groups which connect them. In real life, the trunk groups are routed via transmission nodes. The switching network is shown in Figure III.3.a, and the corresponding transmission network is shown in Figure III.3.b.

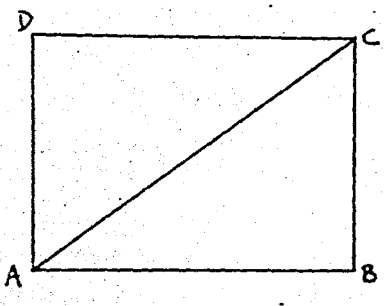

Figure III.3:a

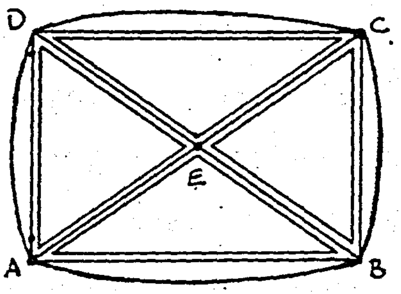

Figure III.3.b

The transmission network in Figure III.3.b contains the transmission node E plus four transmission nodes at every four switching center. As seen in the Figure III.3.b, part of the trunk groups are routed via E. The other part of the trunk groups between A and B has a direct link as a second path. This application increases the connectivity (or availability). On the other hand, the link between A and E carries some parts of trunk groups of AB, AD and AC. If that link is cut, those portions of trunk groups between the concerned relations are lost.

At switching centers, the traffic is switched. However, at transmission nodes there is no facility of switching but just multiplexing (COST PROJECT 201, 1980/1981).

Alternative routing is defined for switching networks. By multirouting (it is also called as diversification), a trunk group will be routed in more than one path. So, if one of the links on a path is cut, not all of the trunk groups for that relation will be lost. The formulation (Baybars, et.al., 1981) takes the alternative routing possibilities into account to increase the grade of service, but not the multi-routing in transmission. To increase the reliability,Baybars and Kortanek put a constraint to guarantee that not all transmission systems will be of the same type, but this is rather a weak protection measure. They also specify the links as high usage links and final choice links. In transmission networks, such a distinction on the transmission media is meaningless since a link may carry high usage and final choice trunk groups at the same time.

CHAPTER IV

MATHEMATICAL MODELS

As stated in chapter II, the problem is to find a minimum cost transmission network which will satisfy the circuit requirements calculated by a previous model called as trunking analysis (COST PROJECT 201, 1980/81). So, the output of the first phase of the telecommunication network optimization problem constitutes the input of the second phase.

The transmission network problem has all the characteristics of a mixed-integer problem. In this chapter three models will be developed and the differences between these models will be pointed out.

The properties related with the problem are as follows:

(i) The model is deterministic.

(ii) The flows are considered as undirected.

(iii) The problem is considered static.

(iv) The network topology (nodes and distances) is given;neither new links nor new switching equipment are to be installed.

(v) Each link cost associated with the installation of transmission systems is assumed to be a concave function of the link size (marginal cost decreases as size increases). This assumption is based upon the fact that the cost functions associated with installing transmission systems are concave, reflecting economies of scale.These functions may be decomposed -- approximately -- into a fixed charge and a variable part. The fixed charge part represents the initial investment cost of installing a transmission system on a link for the first time whereas the variable cost part represents the cost of installing the circuits of that system. It's furthermore assumed that both of these costs depend on the length of the individual links (Evranuz, 1981), that is the actual distance between the two points joined by that link (Figure IV.1).

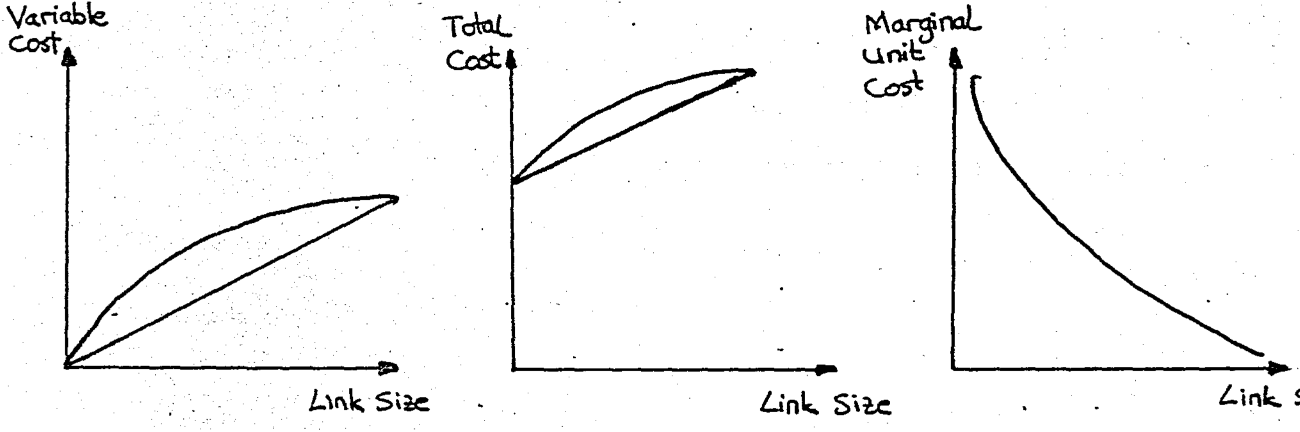

Figure IV.1 Cost Functions

(vi) There are alternate transmission systems such as cables, satellites, microwave radios.

(vii) In order to ensure system reliability, the flow between two demand points should not only be directed via a certain path; in case of failure of a link, the demand on that relation should have the capability to flow partly on another route. (This application is called as multi-routing).

(viii) Message flows have definite destinations and origins; each flow concerned with some demand relation is considered as a different item. Thus the restriction that the flows from certain sources must be sent to certain sinks, makes the problem a multicommodity problem (Baybars, 1981), (Hu, 1970).

IV.1 Model I

In addition to the above properties, one assumption will be included: there will be no prespecified routes for each flow requirement. So, a maximum graph which specifies the topology of the network will be considered (Hu,1970). This maximum graph is manually selected and is a subset of the complete graph. The purpose is to route the flows in such a way that the cost of installation will be minimum while satisfying all the generated demands between the nodes, regardless of the paths.

Formulation:

$$\text{Min } Z=\sum_{ijs} F_{ij}^{s}\ Y_{ij}^{s} + \sum_{ijs} c_{ij}^{s}\ u_{ij}^{s}$$

$$(1) \quad \sum_{i} v_{ij}^{pq} - \sum_{k} v_{jk}^{pq} = \begin{cases} -d_{pq} & j=p \\ 0 & j\neq p,q \quad \forall(p,q) \\ d_{pq} & j=q \end{cases}$$

$$(2) \quad \sum_{pq} v_{ij}^{pq} - \sum_{s} u_{ij}^{s} = 0 \qquad \forall(ij)$$

$$(3) \quad u_{ij}^{s} \leq p^{s}\, Y_{ij}^{s} \qquad \forall(ij),s$$

$$(4) \quad Y_{ij}^{s} = 0 \text{ or } 1 \qquad \forall(ij),s$$

$$v_{ij}^{pq} \geqslant 0 \qquad \forall(ij),(pq)$$

$$u_{ij}^{s} \geqslant 0 \qquad \forall(ij),(pq)$$

where (ij) denotes the arc A_{ij}

(pq) denotes the demand relation

p denotes the source node N_p

q denotes the sink node N_q

s denotes the system type

Inputs are;

d_{pq} = demand value from source N_p to the sink N_q

F_{ij}^{s} = fixed charge of installing a system s on the arc A_{ij}

c_{ij}^{s} = variable unit cost of installing one circuit of system s on the arc A_{ij}

P^{s} = capacity of system s

Decision Variables are;

Y_{ij}^{s} = 0,1 integer variable indicating whether communication systems will be installed on the arc A_{ij} or not.

v_{ij}^{pq} = arc flow on arc A_{ij} with source N_p and sink N_q.

u_{ij}^{s} = total arc flow on the arc A_{ij}, over system s.

The objective function is the sum of the fixed costs of installing transmission systems and variable costs of installing circuits on the links of the network.

Constraint (1) provides conservation of flows:

- the sum of flows diverging from a source N_p and related with a requirement should equal the demand
- the sum of flows merging to a sink N_q and related with a requirement should equal the demand

- the flows related with a certain requirement on intermediate nodes should not be lost.

Constraint (2) is the redundant equation specifying the relation between the decision variables on the links.

Constraint (3) limits the arc flow over system s on a link (ij).

Constraint (4) is the nonnegativity and integer constraints.

The size of the problem is too large to consider; since the problem assumes a maximum graph, each demand relation evaluates each link. The relationship between each two adjacent links are considered for each relation.

IV.2 Model II

Model II is a simplified version of the problem and assumes a network with predetermined routes for each demand relation. In order to avoid dependency only on one route, a reliability constraint is added to the model which guarantees that at least a specific percentage of the circuit demand will be routed on another route.

Formulation:*

$$\text{Min } Z = \sum_{ijs} F^{s}_{ij} \; Y^{s}_{ij} + \sum_{ijs} c^{s}_{ij} \; u^{s}_{ij}$$

$$(1) \quad \sum_{s} u^{s}_{ij} - \sum_{pq,n \in \mathcal{L}} v^{n}_{pq} = 0 \qquad \forall (ij)$$

$$(2) \quad \sum_{n} v^{n}_{pq} = d_{pq} \qquad \forall (pq)$$

$$(3) \quad v^{n}_{pq} \leq k^{n} \, d_{pq} \qquad \forall (pq), n$$

$$(4) \quad u^{s}_{ij} \leq p^{s} \, Y^{s}_{ij} \qquad \forall (ij), s$$

$$(5) \quad Y^{s}_{ij} = 0,1 \qquad \forall (ij), s$$

$$u^{s}_{ij} \geqslant 0 \qquad \forall (ij), s$$

$$v^{n}_{pq} \geqslant 0 \qquad \forall (pq), n$$

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* The author wishes to acknowledge Mr. Çetin Evranuz's help and guidance in the development of this model.

where n denotes the alternative path

k^n is reliability parameter associated with the alternative path n.

$I_{pq,n}$ is the set of links (ij) belonging to the n^{th} path of the relation pq.

Decision Variables are;

Y_{ij}^{s} = 0,1 integer variable indicating whether communication systems will be installed on the arc A_{ij} or not.

u_{ij}^{s} = total arc flow on the arc A_{ij}, over system s.

v_{pq}^{n} = arc flow over path n belonging to relation (pq)

Constraint (1) specifies that the sum of flows on arc A_{ij} over system s is the sum of flows of relation (pq) having an alternate route passing by (ij).

Constraint (2) indicates that the sum of flows of relation pq over all its alternative routes should satisfy its demand.

Constraint (3) is the reliability constraint providing that the flows be distributed among the alternative paths on a prespecified percentage, regardless of economies of scale.

Constraint (4) is the capacity constraint.

IV.3 Model III

A slight variation made to model II results in model III; namely, the variable cost is not considered.

Formulation:

$$\text{Min } Z = \sum_{ijs} F_{ij}^{s} \; Y_{ij}^{s}$$

$$(1) \sum_{n} v_{pq}^{n} \geqslant d_{pq} \qquad \forall (pq)$$

$$(2) \sum_{pq,n \in I} v_{pq}^{n} \leqslant \sum_{s} p^{s} \; Y_{ij}^{s} \qquad \forall (ij)$$

$$(3) \quad v_{pq}^{n} \leqslant k^{n} d_{pq} \qquad \forall (p,q), n$$

(4) $Y_{ij}^{s} = 0 \text{ or } 1$ $\quad \forall (i,j), s$

$v_{pq}^{n} \geqslant 0$ $\quad \forall (pq), n$

As it is seen on the formulation, the decision variable u_{ij}^{s} of model II is not considered here. In this case, the model does not give the information about the distribution of flows over different types of systems on a link. Once a system is installed, the variable costs being not considered in this model, the number of circuits used do not affect the incurred system cost. So, the existence of the variable u_{ij}^{s} is not necessary for this model.

Constraint (1) states that the sum of flows of the alternative paths concerning a certain demand relation should at least equal that demand. (Since additional flows don't charge any variable cost, there can be more than the demanded circuits in the capacity range installed).

Constraint (2) states that the sum of all flows on a link A_{ij} should at most equal the total capacity of the link.

The difference is that , in model II the total flows should equal the demand while in model III it can be equal or greater since an additional flow in the capacity range does not cause any additional cost once a system is installed.

CHAPTER V

NUMERICAL RESULTS

The mathematical models presented and developed in chapter IV are solved by means of the functional mathematical programming package called FMPS [11] on Univac 1106 Data Processing System which uses a branch and bound algorithm.

V.1 TEST NETWORK

The test network represented in Figure V.1 is a part of the Irish Telecommunication Network and it is commonly used by eleven European Countries in the context of COST (European CO-Operation in Scientific and Technical Research) PROJECT 201: "Methods for Planning and Optimisation of Telecommunication Networks". The switching nodes and the transmission nodes are denoted on the test network in Figure V.1 by squares (□) and circles (O), respectively. The switching nodes numbered as 1,2,3,4,5 and 6 are the originating and terminating nodes of trunk groups, whereas the transmission nodes denoted by 7,8 and 9 constitute the junctions (or multiplexing nodes over which the trunk groups are routed).

The sixteen links between nodes are denoted by A through P alphabetically.

V.2 INPUT DATA

All input data is taken from COST PROJECT 201 except for the system capacities and the related cost figures. These figures provide a better analysis and are taken from Baybars et.al. (1981), because of the limited number of alternative transmission systems in the original problem of Irish Telecommunication network.

V.2.1 Distance Matrix

The symmetric distance matrix given in Table V.1 indicates the length of each existing link between the source i and the sink j.

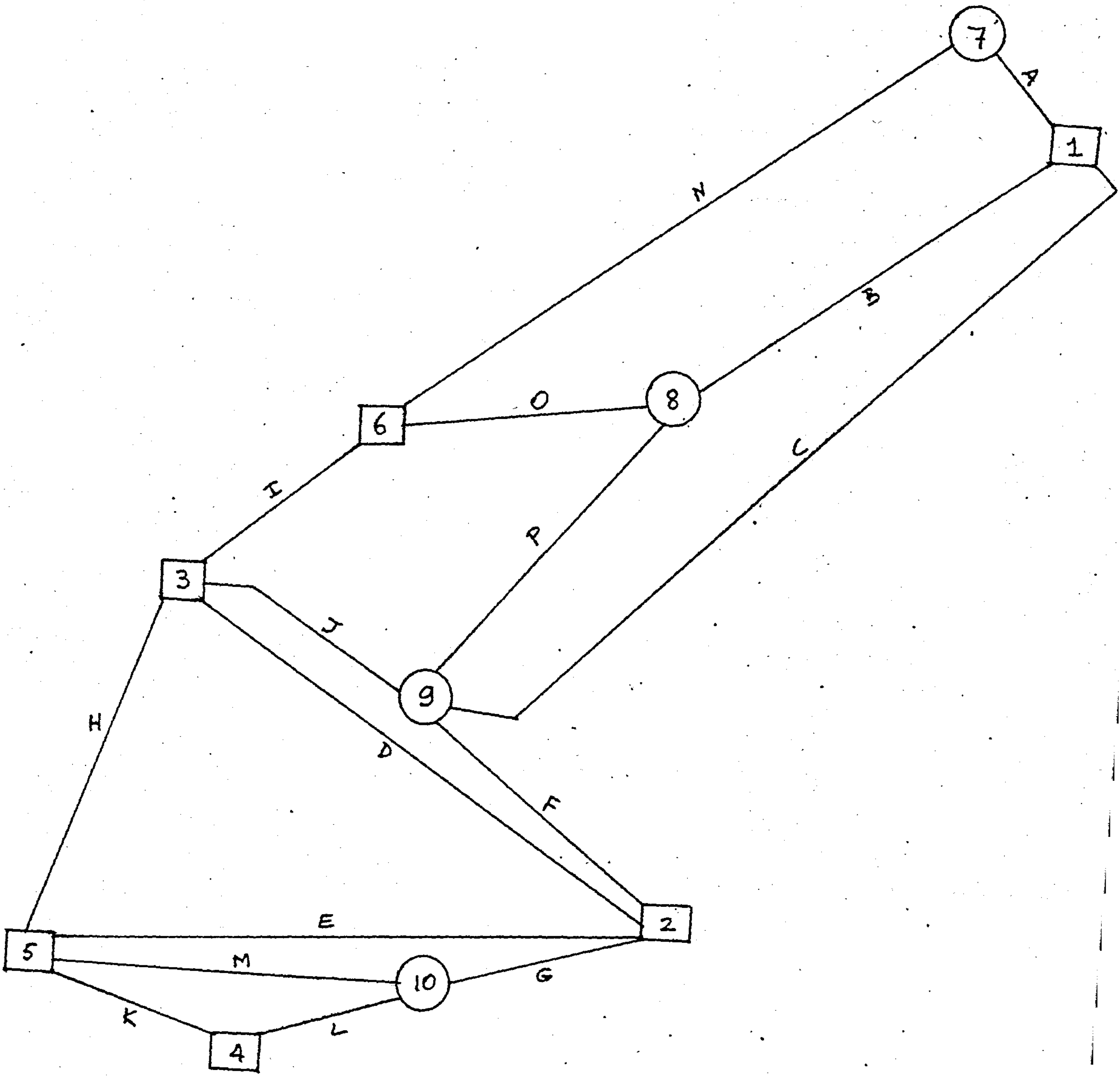

Figure V.1 Test Network

Table V.1. Distance Matrix (Kilometers)

j i	1	2	3	4	5	6	7	8	9	10
1							11	64	26	
2			75		144				20	14
3		75			40	15			46	
4					30					30
5		144	40	30						30
6		15					90	75		
7	11					90				
8	64					75			98	
9	26	20	46					98		
10		14		30	30					

V.2.2 System Costs and Capacities

System capacities and the related cost figures are tabulated in Table V.2. The costs are the Turkish Lira conversion of Baybars's cost figures, based on the exchange rate of 1 US$=200 TL.-.

The multiplication of the unit costs by the corresponding lengths of the links is detailed in Appendix A.

The operating costs of the transmission systems being negligible with respect to the variable costs (oral communication - Miss Arlanoğlu from Netaş) are not taken into account in this study.

Table V.2 Capacity and Cost Figures

	Capacity	Fixed Cost/km	Variable Cost/km
System I	30	10.600.000	62.000
System II	90	17.400.000	21.500

V.2.3 Demand Matrix

The demands between switching centers are shown in Table V.3. As stated in Chapter IV, the links being undirected, the demands on the same link but in opposite directions are to be additioned.

Table V.3. Demand Matrix

Terminating Node Originating Node	1	2	3	4	5	6
1		23	18	40	52	35
2	49		30	6	6	
3	30					16
4		30				
5		18				
6		14				

V.2.4 Alternative Paths

This input section is relevant only for the second and third type mathematical models, the first one being a maximum graph without specified routes. The demand originated between two nodes can be routed via different paths and thus allocating the demand onto various paths to increase the reliability; in case of failure of one link on a path, only a portion of trunk groups will be lost. In this test network, considering the three shortest paths, three alternative paths which are not necessarily disjoint are selected manually for each relation. These paths are tabulated in Table V.4.

Table V.4. Alternative Paths

Originating node	Terminating node	Path 1	Path 2	Path 3
1	2	1,9,2	1,8,9,3,2	1,7,6,3,5,
1	3	1,7,6,3	1,9,2,3	1,8,9,
1	4	1,9,2,10,4	1,7,6,3,5,4	1,8,9,3,2,5,
1	5	1,7,6,3,5	1,8,9,3,2,5	1,9,2,10,
1	6	1,7,6	1,8,6	1,9,3,
2	3	2,3	2,9,3	2,5,
2	4	2,10,4	2,5,4	2,3,5,
2	5	2,5	2,10,5	2,3,
3	6	3,6	3,9,8,6	3,9,1,7,

V.3 SOLUTIONS

The three models presented in chapter IV are attempted to be solved.

V.3.1 Model I

The size of this model with maximum graph applied to the test network is very large to handle. The program was unable to find any feasible solution by the end of 20 minutes of CPU time on the Univac 1106 system. Although the limits of the FMPS parameters such as FCUTOFF, IZTABZS, IENDNODE are progressivelly increased in several trials, no feasible solution is obtained. Since the computer usage time is so limited and the machine is rather slow, getting a solution for this model is given up.

V.3.2 Model II

As mentioned in Chapter IV, this model is completely different from the first model. For every trunk group relation three alternative paths are specified over which the trunk groups requirements will be routed. The model can select one, two or three paths according to the reliability constraints and cost optimization. The model also determines the amounts of trunk groups which will be routed over each path.

The model is run for three different reliability measures. The first run do not consider a reliability constraint. The parameters used in the second and third are 70% and 40%, respectively. These figures are selected in order to guarantee the distribution of the demand at least over two and three routes. The model should be tested with several different parameter values in order to make a better analysis. The reliability measure values employed here correspond to different level of service. The relationship governing this correspondence is rather complex and no attempt has been made to translate each reliability measure to its corresponding level of service value. The only hint we have here is that an increasing reliability measure implies an increasing level of service.

V.3.2.1 Solution 1

In the first run, no reliability constraint is considered. The model is free to choose the paths. The goal is to meet the demands in the most economical way. In the problem, there are 32 integer variables, 59 continuous variables

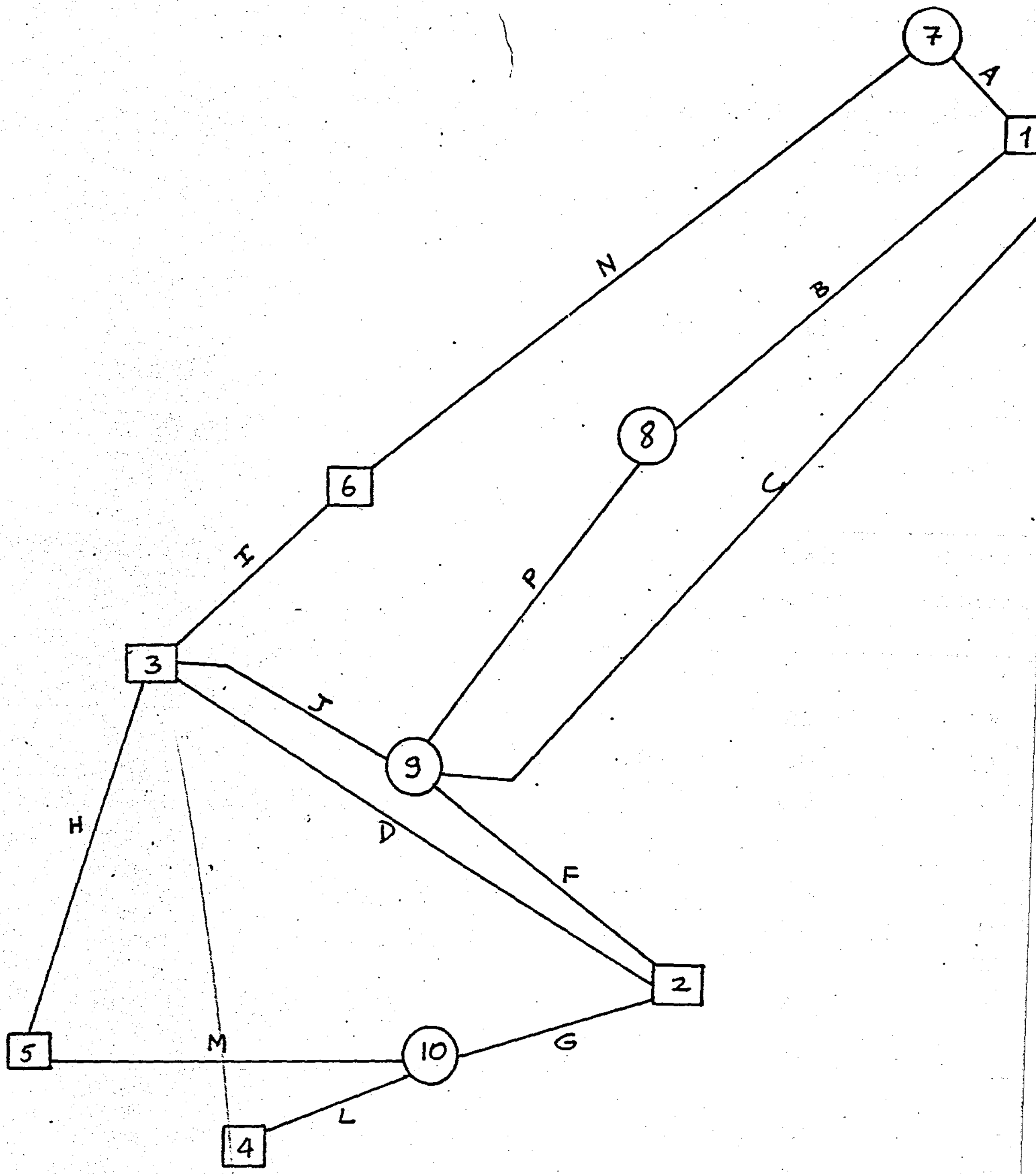

Figure V.2 Network Generated by Solution I and 57 constraints. Figure V.2 shows the network generated after 13.08 minutes of computer time. The optimal results are found in branch numbered 829, at iteration 2422.

Main results concerning system type choice are listed in Table V.5.

Table V.5. System Choice-Solution 1

Link	Source	Sink	System Type	Chosen Capacity	Used Capacity	Unused Capacity
A	1	7	II	90	83	7
B	1	8	II	90	80	10
C	1	9	II	90	90	0
D	2	3	II	90	90	0
E	2	5	-	-	-	-
F	2	9	II	90	90	0
G	2	10	II	90	90	0
H	3	5	II	90	46	44
I	3	6	II	90	90	0
J	3	9	II	90	86	4
K	4	5	-	-	-	-
L	4	10	II	90	64	26
M	5	10	I	30	26	4
N	6	7	II	90	83	7
O	6	8	-	-	-	-
P	8	9	II	90	80	10

As seen from Figure V.2 and Table V.5, 13 links out of 16 link are used. System II is chosen for all links except for the lin M. The second type transmission systems are mostly preferred i order to get the benefit of economies of scale. 5 relations ou of 9 are routed in two paths in this solution. The minimum tota cost minized equals 10.485.978.414 TL.-.The distribution of th trunk groups on the alternative paths is shown in Table V.6.

Table V.6. Distribution of Trunk Groups-Solution 1

Source	Sink	Demand	Path 1	Path 2	Path 3
1	2	72	40	32	-
1	3	48	-	-	48
1	4	40	40	-	-
1	5	52	45	-	7
1	6	35	35	-	-
2	3	60	57	3	-
2	4	24	24	-	-
2	5	20	-	19	1
3	6	48	45	-	3

V.3.2.2 Solution 2

The second run includes the reliability constraint which guarantees the use of at least two paths for every trunk group relations. This is achieved by specifying that the flow on each path will be at most 70% of the demand. The optimal results found at iteration 4829, branch 1101, at the end of 20.24 minutes of computer time are listed in Table V.7 and Table V.8 and the resulting network is shown in Figure V.3. The objective function value is equal to 12.109.445.402.-TL.. The number of constraints was increased from 57 of solution 1 to 84 at this solution.

This time, the number of unused edges dropped from 3 to 2. More importantly, two transmission systems are used together on the links C, F, and J. Only one link K has only system I with 30 capacity. In the solution, there is no relation for which all three paths are used. 8 relations use the first choice paths, 4 relations use the second choice paths and 6 relations use third choice paths.

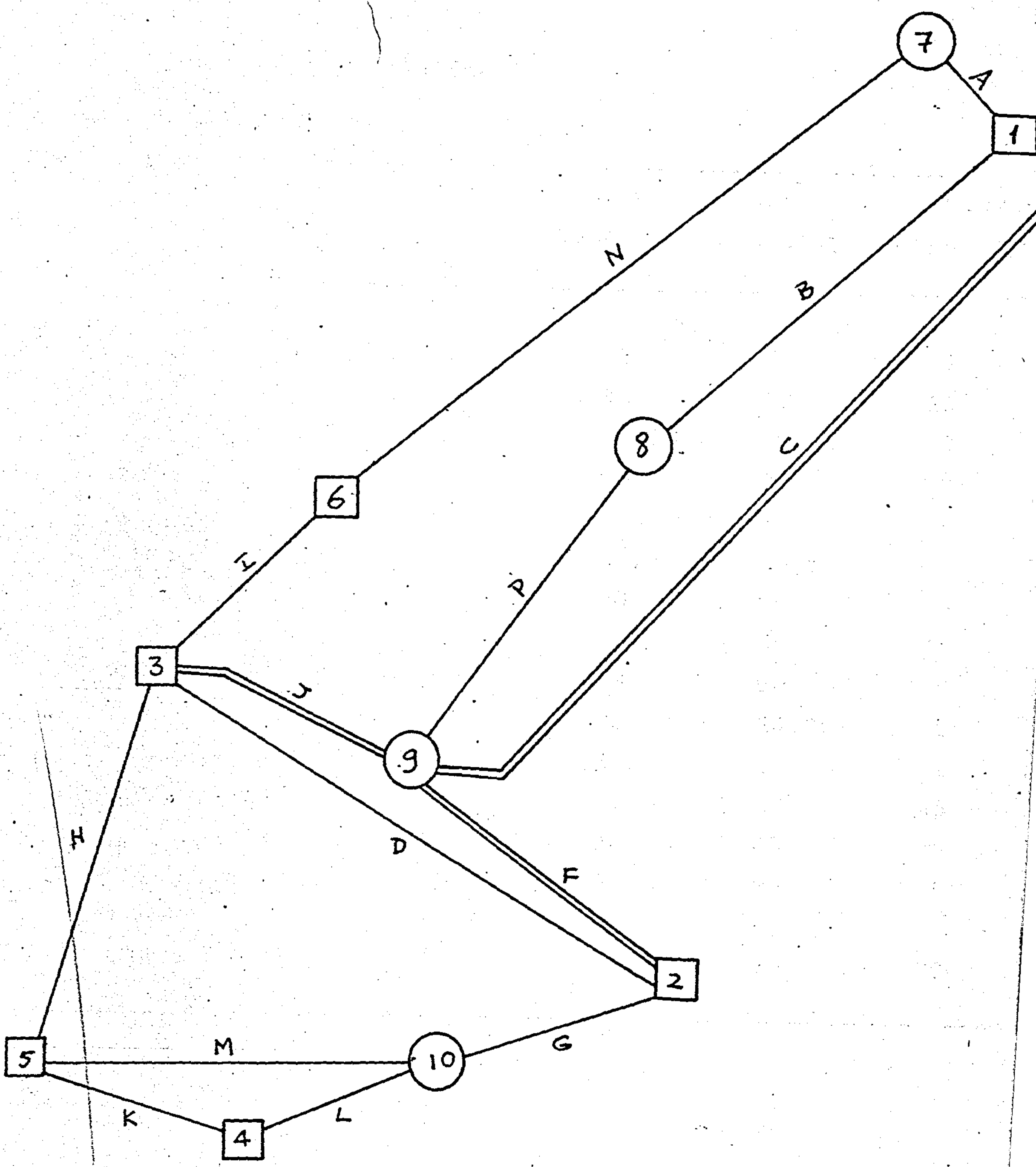

Figure V.3 Network Generated by Solution 2

Table V.7 System Choice-Solution 2

Link	Source	Sink	System Type	Chosen Capacity	Used Capacity	Unused Capacity
A	1	7	II	90	85	5
B	1	8	II	90	70	20
C	1	9	I,II	30,90	30,90	0
D	2	3	II	90	90	0
E	2	5	-	-	-	-
F	2	9	I,II	30,90	26,90	4
G	2	10	II	90	90	0
H	3	5	II	90	46	44
I	3	6	II	90	90	0
J	3	9	I,II	30,90	24,90	6
K	4	5	I	30	19	11
L	4	10	II	90	45	45
M	5	10	II	90	45	45
N	6	7	II	90	85	5
O	6	8	-	-	-	-
P	8	9	II	90	70	20

Table V.8 Distribution of Trunk Groups-Solution 2

Source	Sink	Demand	Path 1	Path 2	Path 3
1	2	72	36	36	-
1	3	48	14	-	34
1	4	40	28	12	-
1	5	52	20	-	32
1	6	35	25	-	10
2	3	60	40	20	-
2	4	24	17	-	7
2	5	20	-	13	7
3	6	48	34	-	14

V.3.2.3 Solution 3

The third run increases the measure of reliability by specifying that each path can carry at most 40% of the demand. So the usage of all the 3 paths are required but the distribution among these paths are to be determined. The size of the problem is the same as of the second solution. Total cost amounts to 15.652.868.633.-TL at the end of 8.57 minutes of computer time, at iteration 1982,

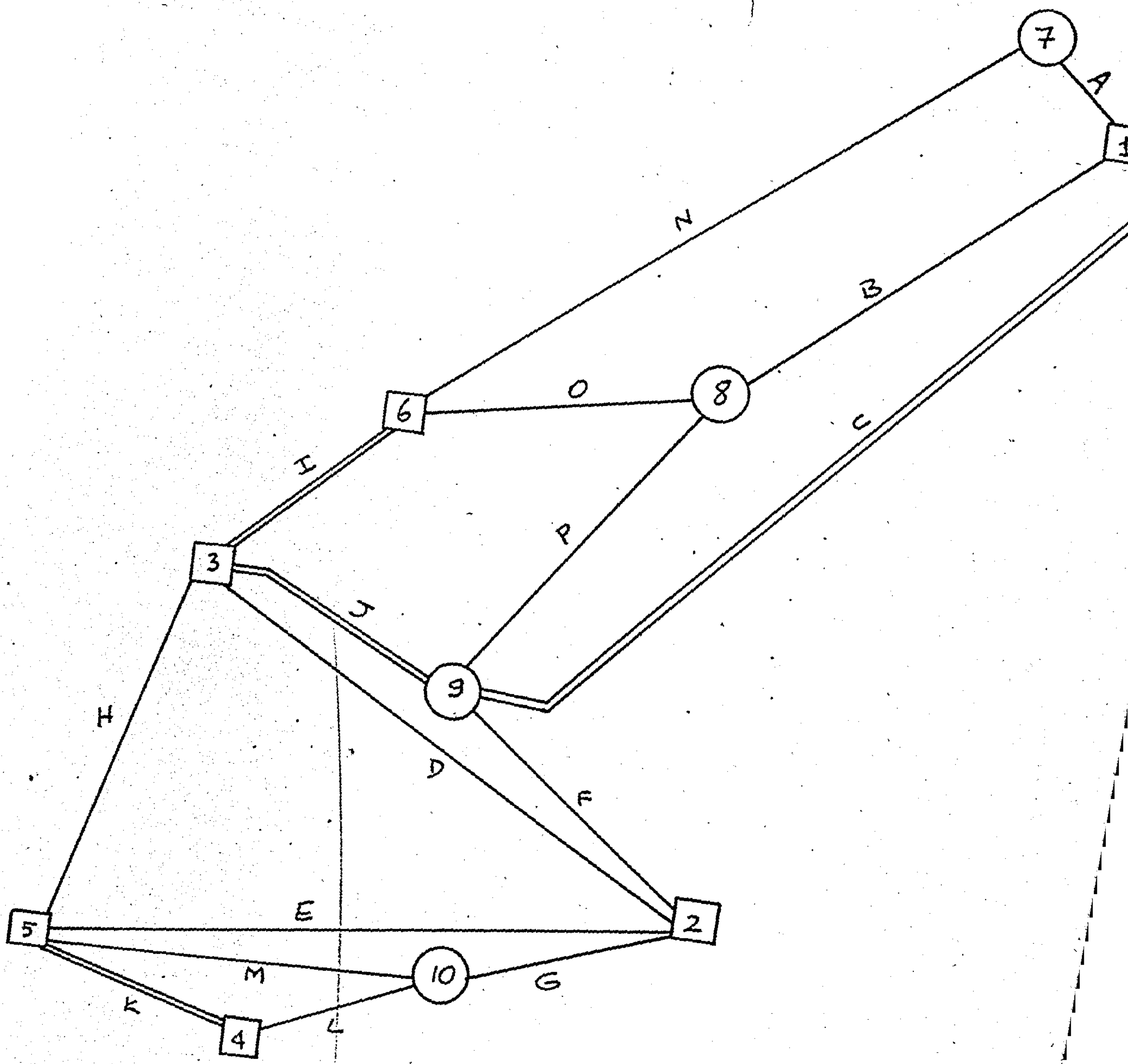

Figure V.3 Network Generated by Solution 3 and branch 499. The resulting network and the corresponding results are shown in Figure V.3 and Table V.9 and Table V.10.

Table V.9 System Choice-Solution 3

Link	Source	Sink	System Chosen Type	Capacity	Used Capacity	Unused Capacity
A	1	7	II	90	90	0
B	1	8	II	90	75.3	14.7
C	1	9	I,II	30,90	11.7,90	18.3
D	2	3	II	90	90	0
E	2	5	II	90	77.3	12.7
F	2	9	II	90	90	0
G	2	10	II	90	54	36
H	3	5	II	90	85.3	4.7
I	3	6	I,II	30,90	12.7,90	17.3
J	3	9	I,II	30,90	30,90	0
K	4	5	II	90	38	52
L	4	10	I	30	26	4
M	5	10	I	30	28	2
N	6	7	II	90	90	0
O	6	8	I	30	29.7	0.3
P	8	9	II	90	83.7	6.3

In the solution all links are used. Unexpectedly, the number of links which use only the first transmission system are increased from 1 to 3. The number of links using both systems are 3.

Table V.10 Distribution of Trunk Groups-Solution 3

Source	Sink	Demand	Path 1	Path 2	Path 3
1	2	72	29	27.7	15.3
1	3	48	17.3	11.7	19
1	4	40	16	16	8
1	5	52	21	10	21
1	6	35	10.3	10.7	14
2	3	60	23.7	12.3	24
2	4	24	10	10	4
2	5	20	8	7	5
3	6	48	19	19	10

V.3.3 Model III

It is a modified version of model II. The aim is to test the significance of variable portion of transmission systems costs. So the model is changed as stated in

Chapter IV. The size of the model is decreased. It consists now of 32 integer variables with 20 continuous variables. The model with 40% reliability measure is run and the new objective function value amounts to 14.123.400.000 TL. The network is presented in Figure V.4 and the results are listed in Tables V.11 and V.12.

Table V.11 System Choice-Model III

Link	Source	Sink	System Chosen Type	System Chosen Capacity	Used* Capacity	Unused Capacity
A	1	7	II	90	90	0
B	1	8	II	90	75.5	14.5
C	1	9	I,II	30,90	11.5,90	18.5
D	2	3	II	90	90	0
E	2	5	II	90	88	2
F	2	9	II	90	90	0
G	2	10	II	90	54	36
H	3	5	II	90	85	5
I	3	6	I,II	30,90	13,90	17
J	3	9	I,II	30,90	30,90	0
K	4	5	II	90	38	52
L	4	10	I	30	26	4
M	5	10	I	30	28	2
N	6	7	II	90	90	0
O	6	8	I	30	30	0
P	8	9	II	90	83.5	6.5

---

* The capacities can be determined either by the program or by summation of the link flows.

Table V.12 Distribution of Trunk Groups-Model III

Source	Sink	Demand	Path 1	Path 2	Path 3
1	2	72	29	21.5	21.5
1	3	48	17	12.5	18.5
1	4	40	16	10.5	13.5
1	5	52	21	11	20
1	6	35	10	11	14
2	3	60	23.5	12.5	24
2	4	24	10	10	4
2	5	20	8	8	4
3	6	48	19	19	10

V.4 OVERALL DISCUSSION

As expected, improved reliability costs more. As the number of paths increases, in other terms the upper limit of the flow on a path decreases, the total cost increases.With two-forced paths, the total cost is increased by 15% compared with the case of one-forced path. For the three-forced paths, the cost is increased

28% with respect to the second case and 48% with respect to the first case.

The variations in the system choice depending on the reliability measures are summarized in Table V.13 and Table.V.14.

Table V.13 System Choice Comparison of The Three Solutions

Link	1st run	2nd run	3rd run
A	II	II	II
B	II	II	II
C	II	I,II	I,II
D	II	II	II
E	-	-	II
F	II	I,II	II
G	II	II	II
H	II	II	II
I	II	II	I,II
J	II	I,II	I,II
K	-	I	II
L	II	II	II
M	I	II	I
N	II	II	II
O	-	-	II
P	II	II	II

Table V.14 Comparison of The Three Solutions

	1^{st} run(no)	2^{nd} run(%70)	3^{rd} run(%40)
Cost (TL)	10.485.978.414	12.109.445.402	15.652.868.633.-TL.
CPU time (min)	3.277	5.739	2.340
Iteration no.	2422	4829	1892
Branch no.	829	1.101	499

Another important point is that there is no change in the values of binary variables of the 40% reliability constraint with and without variable costs. That is all transmission systems selected are the same in both solutions but the number of circuits which are put in each system are different. The percentage of unused capacity decreases by 6.4% if variable costs are not considered. As expected, in the run with variable costs, the model looks for cheapest paths as much as possible, whereas in the run without variable costs the model tries to distribute evenly the circuits requirements among paths.

It is easily seen that if the variable costs are applied on the used capacities of the last run then the total variable costs amounts to 1.569.800.000 TL.- approximately

(Appendix B). The total costs increasing to 15.721.400.000 TL.- will exceed the total costs of the third run by 68.532.000 TL.-. The difference between the evenly routed flows in model III and the cheapest routed flows in model II being around 0.44% for the case discussed is negligible. So, the third model brings a small variable cost increment, but saves a lot of computer time with its size decreased.

The comparison of the three solutions of model II shows that the transmission node numbered as 7 is a redundant one. Since in all the three cases it only transmits the flows between link A and N and it does not have the multiplexing function , the result will not change if it is taken out. The elimination of one node and one link will also decrease the size of the model; two integer variables and two continuous variables may be taken out. Finally, consideration of one link instead of two will cause a decrease of total system cost.

CHAPTER VI

CONCLUSION AND SUGGESTIONS FOR FUTURE WORK

In this thesis, the problem of optimal planning for telecommunications networks are studied. For telecommunications networks, the planning process consists of two major stages. The first stage defined as switching network optimization problem, translates the originating traffic demand into transmission channels or trunks. The output of this stage is the list of trunks between all switching centers. The second stage of the planning process considers the trunk group requirements as inputs to a facilities planning model by which the routings of trunk groups over the transmission networks and the capacities of facilities are determined.

In an investment-expansion planning problem, the major issues are the capacities, times, and locations of the investment-expansion decisions. In this problem the capacities are defined as the number of trunk groups and the locations are considered as links on which facilities are installed. The problem is not formulated as time-phased, and is solved for a target network which reflects the demand at the end of the planning horizon. This is not a real drawback for the case of a country such as Turkey which faces a great deal of unsatisfied demand. The models developed can be easily modified to be time-phased, but the necessary computer time will be further increased.

In this study, another dimension, technology or type of transmission systems, which is generally not considered in other studies is introduced. The cost function reflecting economies of scale in investment costs are concave. Furthermore, the introduction of technologies of transmission system causes jumps at these concave cost functions and makes the problem harder to be solved. Most of the heuristic procedures (Evranuz, 1981), (Ulusoy, 1981), (Yaged, 1971) do not take the technologies of transmission systems on the same link into account.

The formulation also takes the reliability constraints into account. The trunk groups can be required to be routed in more than one path to guarantee that not all trunk groups are lost when an edge is cut. This feature is used before by Claus et.al. (1981), only for a special type network (a star-type network). The way of their formulation might cause some problems for a general network.

Three different models have been developed and numerically tested on a network which is a part of Irish Telecommunications Network. An important point to emphasize is that more reliability costs more. To ensure the reliability, additional transmission systems are needed and trunk groups are routed in longer paths. This may cause the increases in the ratios of capacities used on the systems.

Another point is that the variable costs can be neglected when compared to fixed costs; this will provide the same or very similar installation pattern and will take less computer time.

The problem can be defined as capacity expansions planning problem by taking the existing networks into account. The same model can also be used for this purpose. When this is the case, the cost of existing capacities should be taken as zero.

One way to test the reliability (or availability) is to evaluate the grade of service that will be practised in case of failures. In that case, the trade-off between the costs of reliability and increases in grade of service should be looked for. To increase the reliability in failure cases, the overprovision (provide more circuits than optimum value) and stand-by facilities might be considered.

To achieve more evenly distribution of the circuits and consequently better reliability, it can be suggested first to optimize the structure of the transmission network and then finding the optimum circuit routing on that network structure.

Finally, to handle bigger real life networks the development of more accurate and efficient heuristic algorithms should be developed.

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APPENDICES

APPENDIX A

COSTS OF TRANSMISSIONS
ON
DIFFERENT LINKS

Table A.1 Costs of System I (TL)

Link	Length (km)	Fixed Cost (106 TL/km)	Variable Cost (0.62 TL/km)
A	11	1,166	6.82
B	64	6,784	39.68
C	26	2,756	16.12
D	75	7,950	46.50
E	144	15,264	89.28
F	20	2,120	12.40
G	14	1,484	8.68
H	40	4,240	24.87
I	15	1,890	9.30
J	46	4,876	28.52
K	30	3,180	18.60
L	30	3,180	18.60
M	30	3,180	18.60
N	90	9,540	55.80
O	75	8,250	46.50
P	98	10,388	60.76

Table A.2 Costs of System II (TL)

Link	Length (km)	Fixed Cost (174 TL/km)	Variable Cost (2.15 TL/km)
A	11	1,914	2.36
B	64	11,136	13.76
C	26	4,524	5.59
D	75	13,050	16.12
E	144	25,056	30.96
F	20	3,480	4.30
G	14	2,436	3.01
H	40	6,960	8.60
I	15	2,610	3.22
J	46	8,004	9.89
K	30	5,220	6.45
L	30	5,220	6.45
M	30	5,220	6.45
N	90	15,660	19.35
O	75	13,050	16.12
P	98	17,052	21.07

APPENDIX B

COST COMPARISON OF MODEL II, SOLUTION 3 WITH MODEL III.

Table B.1 Variable Cost Calculations for Model III (TL)

Link	Amount (Circuits)	Unit Cost	Cost
A	90	2.36	212.85
B	75.5	13.76	1,038.88
C	11.5, 90	16.12, 5.59	188.07
D	90	16.12	503.10
E	88	30.96	1,451.25
F	90	4.30	2,724.48
G	54	3.01	387.00
H	85	8.60	162.54
I	13, 90	9.30, 3.22	731.00
J	30, 90	28.52, 9.89	117.80
K	38	6.45	290.25
L	26	18.6	855.60
M	28	18.60	890.10
N	90	19.35	245.10
O	30	46.50	483.60
P	83.5	21.07	520.80

TOTAL = 15,698.26 * 100,000 = 1,569,826,000 TL.-.

APPENDIX C

COMPUTER OUTPUTS

MODEL II SOLUTION 1

```
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TION 1 - ROWS                    PRIMAL-DUAL OUTPUT

BER  ..NAME..  AT  ..ACTIVITY..  SLACK ACTIVITY   .LOWER LIMIT  .UPPER LIMI
  1  COST      FR  104859.78414     -.10486+06           NONE           NON
  2  C1        EQ         .             .                .              .
  3  C2        EQ         .             .                .              .
  4  C3        EQ         .             .                .              .
  5  C4        EQ         .             .                .              .
  6  C5        EQ         .             .                .              .
  7  C6        EQ         .             .                .              .
  8  C7        EQ         .             .                .              .
  9  C8        EQ         .             .                .              .
 10  C9        EQ         .             .                .              .
 11  C10       EQ         .             .                .              .
 12  C11       EQ         .             .                .              .
 13  C12       EQ         .             .                .              .
 14  C13       EQ         .             .                .              .
 15  C14       EQ         .             .                .              .
 16  C15       EQ         .             .                .              .
 17  C16       EQ         .             .                .              .
 18  D1        EQ     72.00000          .            72.00000       72.0000
 19  D2        EQ     48.00000          .            48.00000       48.0000
 20  D3        EQ     40.00000          .            40.00000       40.0000
 21  D4        EQ     52.00000          .            52.00000       52.0000
 22  D5        EQ     35.00000          .            35.00000       35.0000
 23  D6        EQ     60.00000          .            60.00000       60.0000
 24  D7        EQ     24.00000          .            24.00000       24.0000
 25  D8        EQ     20.00000          .            20.00000       20.0000
 26  D9        EQ     48.00000          .            48.00000       48.0000
 27  R1        BS     40.00000      32.00000             NONE       72.0000
 28  R2        BS     32.00000      40.00000             NONE       72.0000
 29  R3        BS         .         72.00000             NONE       72.0000
 30  R4        BS         .         48.00000             NONE       48.0000
 31  R5        BS         .         48.00000             NONE       48.0000
 32  R6        UL     48.00000          .                NONE       48.0000
 33  R7        UL     40.00000          .                NONE       40.0000
 34  R8        BS         .         40.00000             NONE       40.0000
 35  R9        BS         .         40.00000             NONE       40.0000
 36  R10       BS     45.00000       7.00000             NONE       52.0000
 37  R11       BS         .         52.00000             NONE       52.0000
 38  R12       BS      7.00000      45.00000             NONE       52.0000
 39  R13       UL     35.00000          .                NONE       35.0000
 40  R14       BS         .         35.00000             NONE       35.0000
 41  R15       BS         .         35.00000             NONE       35.0000
 42  R16       BS     57.00000       3.00000             NONE       60.0000
 43  R17       BS      3.00000      57.00000             NONE       60.0000
 44  R18       BS         .         60.00000             NONE       60.0000
 45  R19       BS     24.00000          .                NONE       24.0000
 46  R20       BS         .         24.00000             NONE       24.0000
 47  R21       BS         .         24.00000             NONE       24.0000
```

```
ION 1 - ROWS                               PRIMAL-DUAL OUTPUT

ER  ..NAME..  AT  ..ACTIVITY..   SLACK ACTIVITY   .LOWER LIMIT  .UPPER LIMIT
48  R22       BS        .            24.00000           NONE       24.00000
49  R23       BS     19.00000         1.00000           NONE       24.00000
50  R24       BS      1.00000        19.00000           NONE       24.00000
51  R25       BS     45.00000         3.00000           NONE       48.00000
52  R26       BS        .            48.00000           NONE       48.00000
53  R27       BS      3.00000        45.00000           NONE       48.00000
54  P1        UL        .               .               NONE          .
55  P2        BS     -7.00000         7.00000           NONE          .
56  P4        UL        .               .               NONE          .
57  P5        BS    -10.00000        10.00000           NONE          .
58  P7        UL        .               .               NONE          .
59  P8        UL        .               .               NONE          .
60  P10       UL        .               .               NONE          .
61  P11       UL        .               .               NONE          .
62  P13       UL        .               .               NONE          .
63  P14       UL        .               .               NONE          .
64  P16       UL        .               .               NONE          .
65  P17       UL        .               .               NONE          .
66  P19       UL        .               .               NONE          .
67  P20       UL        .               .               NONE          .
68  P22       UL        .               .               NONE          .
69  P23       BS    -44.00000        44.00000           NONE          .
70  P25       UL        .               .               NONE          .
71  P26       UL        .               .               NONE          .
72  P28       BS        .               .               NONE          .
73  P29       BS     -4.00000         4.00000           NONE          .
74  P31       BS        .               .               NONE          .
75  P32       BS        .               .               NONE          .
76  P34       BS        .               .               NONE          .
77  P35       BS    -26.00000        26.00000           NONE          .
78  P37       BS     -4.00000         4.00000           NONE          .
79  P38       UL        .               .               NONE          .
80  P40       UL        .               .               NONE          .
81  P41       BS     -7.00000         7.00000           NONE          .
82  P43       UL        .               .               NONE          .
83  P44       UL        .               .               NONE          .
84  P46       UL        .               .               NONE          .
85  P47       BS    -10.00000        10.00000           NONE          .
```

```
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TION 2 - COLUMNS                         PRIMAL-DUAL OUTPUT

BER  ..NAME..  AT  ..ACTIVITY..  .INPUT COST.     .LOWER LIMIT .UPPER LIMIT
 86  YA1       IT       .          1166.00000           .           1.00000
 87  YA2       IT   1.00000        1914.00000           .           1.00000
 88  YB1       IT       .          6784.00000           .           1.00000
 89  YB2       IT   1.00000       11136.00000           .           1.00000
 90  YC1       IT       .          2756.00000           .           1.00000
 91  YC2       IT   1.00000        4524.00000           .           1.00000
 92  YD1       IT       .          7950.00000           .           1.00000
 93  YD2       IT   1.00000       13050.00000           .           1.00000
 94  YE1       IT       .         15264.00000           .           1.00000
 95  YE2       IT       .         25056.00000           .           1.00000
 96  YF1       IT       .          2120.00000           .           1.00000
 97  YF2       IT   1.00000        3480.00000           .           1.00000
 98  YG1       IT       .          1484.00000           .           1.00000
 99  YG2       IT   1.00000        2436.00000           .           1.00000
 00  YH1       IT       .          4240.00000           .           1.00000
 01  YH2       IT   1.00000        6960.00000           .           1.00000
 02  YI1       IT       .          1890.00000           .           1.00000
 03  YI2       IT   1.00000        2610.00000           .           1.00000
 04  YJ1       IT       .          4876.00000           .           1.00000
 05  YJ2       IT   1.00000        8004.00000           .           1.00000
 06  YK1       IT       .          3180.00000           .           1.00000
 07  YK2       IT       .          5220.00000           .           1.00000
 08  YL1       IT       .          3180.00000           .           1.00000
 09  YL2       IT   1.00000        5220.00000           .           1.00000
 10  YM1       IT   1.00000        3180.00000           .           1.00000
 11  YM2       IT       .          5220.00000           .           1.00000
 12  YN1       IT       .          9540.00000           .           1.00000
 13  YN2       IT   1.00000       15660.00000           .           1.00000
 14  YO1       IT       .          8250.00000           .           1.00000
 15  YO2       IT       .         13050.00000           .           1.00000
 16  YP1       IT       .         10388.00000           .           1.00000
 17  YP2       IT   1.00000       17052.00000           .           1.00000
 18  UA1       LL       .             6.82000           .              NONE
 19  UA2       BS  83.00000           2.36500           .              NONE
 20  UB1       LL       .            39.68000           .              NONE
 21  UB2       BS  80.00000          13.76000           .              NONE
 22  UC1       BS       .            16.12000           .              NONE
 23  UC2       BS  90.00000           5.59000           .              NONE
 24  UD1       BS       .            46.50000           .              NONE
 25  UD2       BS  90.00000          16.12500           .              NONE
 26  UE1       LL       .            89.28000           .              NONE
 27  UE2       BS       .            30.96000           .              NONE
 28  UF1       BS       .            12.40000           .              NONE
 29  UF2       BS  90.00000           4.30000           .              NONE
 30  UG1       LL       .             8.68000           .              NONE
 31  UG2       BS  90.00000           3.01000           .              NONE
 32  UH1       LL       .            24.87000           .              NONE
```

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CTION 2 - COLUMNS                    PRIMAL-DUAL OUTPUT

MBER   ..NAME..   AT   ..ACTIVITY..   .INPUT COST.   .LOWER LIMIT   .UPPER LIMIT
 133   UH2        BS       46.00000        8.60000          .               NONE
 134   UI1        BS        .              9.30000          .               NONE
 135   UI2        BS       90.00000        3.22500          .               NONE
 136   UJ1        LL        .             28.52000          .               NONE
 137   UJ2        BS       86.00000        9.89000          .               NONE
 138   UK1        LL        .             18.60000          .               NONE
 139   UK2        LL        .              6.45000          .               NONE
 140   UL1        LL        .             18.60000          .               NONE
 141   UL2        BS       64.00000        6.45000          .               NONE
 142   UM1        BS       26.00000       18.60000          .               NONE
 143   UM2        BS        .              6.45000          .               NONE
 144   UN1        LL        .             55.80000          .               NONE
 145   UN2        BS       83.00000       19.35000          .               NONE
 146   UO1        LL        .             46.50000          .               NONE
 147   UO2        BS        .             16.12500          .               NONE
 148   UP1        LL        .             60.76000          .               NONE
 149   UP2        BS       80.00000       21.07000          .               NONE
 150   V11        BS       40.00000        .                .               NONE
 151   V12        BS       32.00000        .                .               NONE
 152   V13        LL        .              .                .               NONE
 153   V21        LL        .              .                .               NONE
 154   V22        BS        .              .                .               NONE
 155   V23        BS       48.00000        .                .               NONE
 156   V31        BS       40.00000        .                .               NONE
 157   V32        BS        .              .                .               NONE
 158   V33        AL        .              .                .               NONE
 159   V41        BS       45.00000        .                .               NONE
 160   V42        BS        .              .                .               NONE
 161   V43        BS        7.00000        .                .               NONE
 162   V51        BS       35.00000        .                .               NONE
 163   V52        BS        .              .                .               NONE
 164   V53        LL        .              .                .               NONE
 165   V61        BS       57.00000        .                .               NONE
 166   V62        BS        3.00000        .                .               NONE
 167   V63        LL        .              .                .               NONE
 168   V71        BS       24.00000        .                .               NONE
 169   V72        LL        .              .                .               NONE
 170   V73        BS        .              .                .               NONE
 171   V81        LL        .              .                .               NONE
 172   V82        BS       19.00000        .                .               NONE
 173   V83        BS        1.00000        .                .               NONE
 174   V91        BS       45.00000        .                .               NONE
 175   V92        BS        .              .                .               NONE
 176   V93        BS        3.00000        .                .               NONE
```

MODEL II SOLUTION 2

```
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TION 1 - ROWS                       PRIMAL-DUAL OUTPUT

BER  ..NAME..  AT  ..ACTIVITY..  SLACK ACTIVITY   .LOWER LIMIT .UPPER LIMIT
  1  COST      FR  121094.45402      -.12109+06          NONE         NONE
  2  C1        EQ        .               .                 .            .
  3  C2        EQ        .               .                 .            .
  4  C3        EQ        .               .                 .            .
  5  C4        EQ        .               .                 .            .
  6  C5        EQ        .               .                 .            .
  7  C6        EQ        .               .                 .            .
  8  C7        EQ        .               .                 .            .
  9  C8        EQ        .               .                 .            .
 10  C9        EQ        .               .                 .            .
 11  C10       EQ        .               .                 .            .
 12  C11       EQ        .               .                 .            .
 13  C12       EQ        .               .                 .            .
 14  C13       EQ        .               .                 .            .
 15  C14       EQ        .               .                 .            .
 16  C15       EQ        .               .                 .            .
 17  C16       EQ        .               .                 .            .
 18  D1        EQ      72.00000          .             72.00000     72.00000
 19  D2        EQ      48.00000          .             48.00000     48.00000
 20  D3        EQ      40.00000          .             40.00000     40.00000
 21  D4        EQ      52.00000          .             52.00000     52.00000
 22  D5        EQ      35.00000          .             35.00000     35.00000
 23  D6        EQ      60.00000          .             60.00000     60.00000
 24  D7        EQ      24.00000          .             24.00000     24.00000
 25  D8        EQ      20.00000          .             20.00000     20.00000
 26  D9        EQ      48.00000          .             48.00000     48.00000
 27  R1        BS      36.00000      14.00000            NONE       50.00000
 28  R2        BS      36.00000      14.00000            NONE       50.00000
 29  R3        BS        .           50.00000            NONE       50.00000
 30  R4        BS      14.00000      20.00000            NONE       34.00000
 31  R5        BS        .           34.00000            NONE       34.00000
 32  R6        UL      34.00000          .               NONE       34.00000
 33  R7        UL      28.00000          .               NONE       28.00000
 34  R8        BS      12.00000      16.00000            NONE       28.00000
 35  R9        BS        .           28.00000            NONE       28.00000
 36  R10       BS      20.00000      16.00000            NONE       36.00000
 37  R11       BS        .           36.00000            NONE       36.00000
 38  R12       BS      32.00000       4.00000            NONE       36.00000
 39  R13       UL      25.00000          .               NONE       25.00000
 40  R14       BS        .           25.00000            NONE       25.00000
 41  R15       BS      10.00000      15.00000            NONE       25.00000
 42  R16       BS      40.00000       2.00000            NONE       42.00000
 43  R17       BS      20.00000      22.00000            NONE       42.00000
 44  R18       BS        .           42.00000            NONE       42.00000
 45  R19       UL      17.00000          .               NONE       17.00000
 46  R20       BS        .           17.00000            NONE       17.00000
 47  R21       BS       7.00000      10.00000            NONE       17.00000
```

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ION 1 - ROWS                                PRIMAL-DUAL OUTPUT

ER   ..NAME..   AT   ..ACTIVITY..   SLACK ACTIVITY   .LOWER LIMIT  .UPPER LIMIT
48   R22        BS           .           14.00000            NONE      14.00000
49   R23        BS      13.00000          1.00000            NONE      14.00000
50   R24        BS       7.00000          7.00000            NONE      14.00000
51   R25        UL      34.00000             .               NONE      34.00000
52   R26        BS           .           34.00000            NONE      34.00000
53   R27        BS      14.00000         20.00000            NONE      34.00000
54   P1         UL           .               .               NONE          .
55   P2         BS      -5.00000          5.00000            NONE          .
56   P4         UL           .               .               NONE          .
57   P5         BS     -20.00000         20.00000            NONE          .
58   P7         UL           .               .               NONE          .
59   P8         UL           .               .               NONE          .
60   P10        UL           .               .               NONE          .
61   P11        UL           .               .               NONE          .
62   P13        UL           .               .               NONE          .
63   P14        UL           .               .               NONE          .
64   P16        BS      -4.00000          4.00000            NONE          .
65   P17        UL           .               .               NONE          .
66   P19        UL           .               .               NONE          .
67   P20        UL           .               .               NONE          .
68   P22        UL           .               .               NONE          .
69   P23        BS     -44.00000         44.00000            NONE          .
70   P25        UL           .               .               NONE          .
71   P26        UL           .               .               NONE          .
72   P28        BS      -6.00000          6.00000            NONE          .
73   P29        UL           .               .               NONE          .
74   P31        BS     -11.00000         11.00000            NONE          .
75   P32        UL           .               .               NONE          .
76   P34        BS           .               .               NONE          .
77   P35        BS     -45.00000         45.00000            NONE          .
78   P37        UL           .               .               NONE          .
79   P38        BS     -45.00000         45.00000            NONE          .
80   P40        UL           .               .               NONE          .
81   P41        BS      -5.00000          5.00000            NONE          .
82   P43        UL           .               .               NONE          .
83   P44        UL           .               .               NONE          .
84   P46        UL           .               .               NONE          .
85   P47        BS     -20.00000         20.00000            NONE          .
```

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TION 2 - COLUMNS                     PRIMAL-DUAL OUTPUT

BER   ..NAME..   AT   ..ACTIVITY..   .INPUT COST.     .LOWER LIMIT .UPPER LIMIT.
 86   YA1        IT        .          1166.00000          .          1.00000
 87   YA2        IT    1.00000        1914.00000          .          1.00000
 88   YB1        IT        .          6784.00000          .          1.00000
 89   YB2        IT    1.00000       11136.00000          .          1.00000
 90   YC1        IT    1.00000        2756.00000          .          1.00000
 91   YC2        IT    1.00000        4524.00000          .          1.00000
 92   YD1        IT        .          7950.00000          .          1.00000
 93   YD2        IT    1.00000       13050.00000          .          1.00000
 94   YE1        IT        .         15264.00000          .          1.00000
 95   YE2        IT        .         25056.00000          .          1.00000
 96   YF1        IT    1.00000        2120.00000          .          1.00000
 97   YF2        IT    1.00000        3480.00000          .          1.00000
 98   YG1        IT        .          1484.00000          .          1.00000
 99   YG2        IT    1.00000        2436.00000          .          1.00000
100   YH1        IT        .          4240.00000          .          1.00000
101   YH2        IT    1.00000        6960.00000          .          1.00000
102   YI1        IT    1.00000        1890.00000          .          1.00000
103   YI2        IT    1.00000        2610.00000          .          1.00000
104   YJ1        IT    1.00000        4876.00000          .          1.00000
105   YJ2        IT    1.00000        8004.00000          .          1.00000
106   YK1        IT    1.00000        3180.00000          .          1.00000
107   YK2        IT        .          5220.00000          .          1.00000
108   YL1        IT        .          3180.00000          .          1.00000
109   YL2        IT    1.00000        5220.00000          .          1.00000
110   YM1        IT        .          3180.00000          .          1.00000
111   YM2        IT    1.00000        5220.00000          .          1.00000
112   YN1        IT        .          9540.00000          .          1.00000
113   YN2        IT    1.00000       15660.00000          .          1.00000
114   YO1        IT        .          8250.00000          .          1.00000
115   YO2        IT        .         13050.00000          .          1.00000
116   YP1        IT        .         10388.00000          .          1.00000
117   YP2        IT    1.00000       17052.00000          .          1.00000
118   UA1        LL        .             6.82000          .             NONE
119   UA2        BS   85.00000           2.36500          .             NONE
120   UB1        LL        .            39.68000          .             NONE
121   UB2        BS   70.00000          13.76000          .             NONE
122   UC1        BS   30.00000          16.12000          .             NONE
123   UC2        BS   90.00000           5.59000          .             NONE
124   UD1        LL        .            46.50000          .             NONE
125   UD2        BS   90.00000          16.12500          .             NONE
126   UE1        LL        .            89.28000          .             NONE
127   UE2        BS        .            30.96000          .             NONE
128   UF1        BS   26.00000          12.40000          .             NONE
129   UF2        BS   90.00000           4.30000          .             NONE
130   UG1        LL        .             8.68000          .             NONE
131   UG2        BS   90.00000           3.01000          .             NONE
132   UH1        LL        .            24.87000          .             NONE
```

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ECTION 2 - COLUMNS                    PRIMAL-DUAL OUTPUT

UMBER  ..NAME..  AT  ..ACTIVITY..  .INPUT COST.  .LOWER LIMIT  .UPPER LIMIT
  133  UH2       BS     46.00000       8.60000         .             NONE
  134  UI1       BS         .          9.30000         .             NONE
  135  UI2       BS     90.00000       3.22500         .             NONE
  136  UJ1       BS     24.00000      28.52000         .             NONE
  137  UJ2       BS     90.00000       9.89000         .             NONE
  138  UK1       BS     19.00000      18.60000         .             NONE
  139  UK2       BS         .          6.45000         .             NONE
  140  UL1       LL         .         18.60000         .             NONE
  141  UL2       BS     45.00000       6.45000         .             NONE
  142  UM1       LL         .         18.60000         .             NONE
  143  UM2       BS     45.00000       6.45000         .             NONE
  144  UN1       LL         .         55.80000         .             NONE
  145  UN2       BS     85.00000      19.35000         .             NONE
  146  UO1       BS         .         46.50000         .             NONE
  147  UO2       BS         .         16.12500         .             NONE
  148  UP1       LL         .         60.76000         .             NONE
  149  UP2       BS     70.00000      21.07000         .             NONE
  150  V11       BS     36.00000          .            .             NONE
  151  V12       BS     36.00000          .            .             NONE
  152  V13       LL         .              .            .             NONE
  153  V21       BS     14.00000          .            .             NONE
  154  V22       LL         .              .            .             NONE
  155  V23       BS     34.00000          .            .             NONE
  156  V31       BS     28.00000          .            .             NONE
  157  V32       BS     12.00000          .            .             NONE
  158  V33       LL         .              .            .             NONE
  159  V41       BS     20.00000          .            .             NONE
  160  V42       LL         .              .            .             NONE
  161  V43       BS     32.00000          .            .             NONE
  162  V51       BS     25.00000          .            .             NONE
  163  V52       BS         .              .            .             NONE
  164  V53       BS     10.00000          .            .             NONE
  165  V61       BS     40.00000          .            .             NONE
  166  V62       BS     20.00000          .            .             NONE
  167  V63       LL         .              .            .             NONE
  168  V71       BS     17.00000          .            .             NONE
  169  V72       BS         .              .            .             NONE
  170  V73       BS      7.00000          .            .             NONE
  171  V81       AL         .              .            .             NONE
  172  V82       BS     13.00000          .            .             NONE
  173  V83       BS      7.00000          .            .             NONE
  174  V91       BS     34.00000          .            .             NONE
  175  V92       LL         .              .            .             NONE
  176  V93       BS     14.00000          .            .             NONE
```

MODEL II SOLUTION 3

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ON 1 - ROWS                         PRIMAL-DUAL OUTPUT

R   ..NAME..   AT   ..ACTIVITY..   SLACK ACTIVITY   .LOWER LIMIT  .UPPER LIMIT
1   COST       FR   156528.68633     -.15652+06           NONE          NONE
2   C1         EQ          .               .                .             .
3   C2         EQ          .               .                .             .
4   C3         EQ          .               .                .             .
5   C4         EQ          .               .                .             .
6   C5         EQ          .               .                .             .
7   C6         EQ          .               .                .             .
8   C7         EQ          .               .                .             .
9   C8         EQ          .               .                .             .
0   C9         EQ          .               .                .             .
1   C10        EQ          .               .                .             .
2   C11        EQ          .               .                .             .
3   C12        EQ          .               .                .             .
4   C13        EQ          .               .                .             .
5   C14        EQ          .               .                .             .
6   C15        EQ          .               .                .             .
7   C16        EQ          .               .                .             .
8   D1         EQ      72.00000            .            72.00000      72.00000
9   D2         EQ      48.00000            .            48.00000      48.00000
20  D3         EQ      40.00000            .            40.00000      40.00000
1   D4         EQ      52.00000            .            52.00000      52.00000
22  D5         EQ      35.00000            .            35.00000      35.00000
23  D6         EQ      60.00000            .            60.00000      60.00000
24  D7         EQ      24.00000            .            24.00000      24.00000
5   D8         EQ      20.00000            .            20.00000      20.00000
26  D9         EQ      48.00000            .            48.00000      48.00000
27  R1         UL      29.00000            .                NONE      29.00000
28  R2         BS      27.66667         1.33333             NONE      29.00000
9   R3         BS      15.33333        13.66667             NONE      29.0000
30  R4         BS      17.33333         1.66667             NONE      19.0000
1   R5         BS      11.66667         7.33333             NONE      19.0000
32  R6         UL      19.00000            .                NONE      19.0000
3   R7         UL      16.00000            .                NONE      16.0000
4   R8         UL      16.00000            .                NONE      16.0000
5   R9         BS       8.00000         8.00000             NONE      16.0000
36  R10        UL      21.00000            .                NONE      21.0000
37  R11        BS      10.00000        11.00000             NONE      21.0000
38  R12        UL      21.00000            .                NONE      21.0000
39  R13        BS      10.33337         3.66667             NONE      14.0000
40  R14        BS      10.66667         3.33333             NONE      14.0000
41  R15        UL      14.00000            .                NONE      14.0000
42  R16        BS      23.66667          .33333             NONE      24.0000
43  R17        BS      12.33333        11.66667             NONE      24.0000
44  R18        UL      24.00000            .                NONE      24.0000
45  R19        UL      10.00000            .                NONE      10.0000
46  R20        UL      10.00000            .                NONE      10.0000
47  R21        BS       4.00000         6.00000             NONE      10.0000
```

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ON 1 - ROWS                        PRIMAL-DUAL OUTPUT

CR  ..NAME..   AT  ..ACTIVITY..  SLACK ACTIVITY  .LOWER LIMIT .UPPER LIMIT
8   R22        UL      8.00000          .              NONE       8.00000
9   R23        BS      7.00000         1.00000         NONE       8.00000
0   R24        BS      5.00000         5.00000         NONE       8.00000
1   R25        UL     19.00000          .              NONE      19.00000
2   R26        UL     19.00000          .              NONE      19.00000
3   R27        BS     10.00000         9.00000         NONE      19.00000
4   P1         BS          .            .              NONE           .
5   P2         BS          .            .              NONE           .
6   P4         UL          .            .              NONE           .
7   P5         BS    -14.66667        14.66667         NONE           .
8   P7         BS    -18.33333        18.33333         NONE           .
9   P8         UL          .            .              NONE           .
0   P10        BS          .            .              NONE           .
1   P11        UL          .            .              NONE           .
2   P13        UL          .            .              NONE           .
3   P14        BS    -12.66667        12.66667         NONE           .
4   P16        BS          .            .              NONE           .
5   P17        UL          .            .              NONE           .
6   P19        BS          .            .              NONE           .
7   P20        BS    -43.00000        43.00000         NONE           .
8   P22        UL          .            .              NONE           .
9   P23        BS     -4.66667         4.66667         NONE           .
0   P25        BS    -17.33333        17.33333         NONE           .
1   P26        UL          .            .              NONE           .
2   P28        UL          .            .              NONE           .
3   P29        UL          .            .              NONE           .
4   P31        UL          .            .              NONE           .
5   P32        BS    -52.00000        52.00000         NONE           .
6   P34        BS     -4.00000         4.00000         NONE           .
7   P35        UL          .            .              NONE           .
8   P37        UL          .            .              NONE           .
9   P38        UL          .            .              NONE           .
0   P40        UL          .            .              NONE           .
1   P41        UL          .            .              NONE           .
2   P43        BS      -.33333          .33333         NONE           .
3   P44        UL          .            .              NONE           .
4   P46        UL          .            .              NONE           .
5   P47        BS     -6.33333         6.33333         NONE           .
```

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ION 2 - COLUMNS                        PRIMAL-DUAL OUTPUT

ER  ..NAME..   AT   ..ACTIVITY..   .INPUT COST.    .LOWER LIMIT  .UPPER LIMI
36  YA1        IT      .            1166.00000          .          1.00000
37  YA2        IT     1.00000       1914.00000          .          1.00000
38  YB1        IT      .            6784.00000          .          1.00000
39  YB2        IT     1.00000      11136.00000          .          1.00000
90  YC1        IT     1.00000       2756.00000          .          1.00000
91  YC2        IT     1.00000       4524.00000          .          1.00000
92  YD1        IT      .            7950.00000          .          1.00000
93  YD2        IT     1.00000      13050.00000          .          1.00000
94  YE1        IT      .           15264.00000          .          1.00000
95  YE2        IT     1.00000      25056.00000          .          1.00000
96  YF1        IT      .            2120.00000          .          1.00000
97  YF2        IT     1.00000       3480.00000          .          1.00000
98  YG1        IT      .            1484.00000          .          1.00000
99  YG2        IT     1.00000       2436.00000          .          1.00000
00  YH1        IT      .            4240.00000          .          1.00000
01  YH2        IT     1.00000       6960.00000          .          1.00000
02  YI1        IT     1.00000       1890.00000          .          1.00000
03  YI2        IT     1.00000       2610.00000          .          1.00000
04  YJ1        IT     1.00000       4876.00000          .          1.00000
05  YJ2        IT     1.00000       8004.00000          .          1.00000
06  YK1        IT      .            3180.00000          .          1.00000
07  YK2        IT     1.00000       5220.00000          .          1.00000
08  YL1        IT     1.00000       3180.00000          .          1.00000
09  YL2        IT      .            5220.00000          .          1.00000
10  YM1        IT     1.00000       3180.00000          .          1.0000
11  YM2        IT      .            5220.00000          .          1.0000
12  YN1        IT      .            9540.00000          .          1.0000
13  YN2        IT     1.00000      15660.00000          .          1.0000
14  YO1        IT     1.00000       8250.00000          .          1.0000
15  YO2        IT      .           13050.00000          .          1.0000
16  YP1        IT      .           10388.00000          .          1.0000
17  YP2        IT     1.00000      17052.00000          .          1.0000
18  UA1        LL      .               6.82000          .             NON
19  UA2        BS    90.00000          2.36500          .             NON
20  UB1        LL      .              39.68000          .             NON
21  UB2        BS    75.33333         13.76000          .             NON
22  UC1        BS    11.66667         16.12000          .             NON
23  UC2        BS    90.00000          5.59000          .             NON
24  UD1        LL      .              46.50000          .             NON
25  UD2        BS    90.00000         16.12500          .             NON
26  UE1        LL      .              89.28000          .             NON
27  UE2        BS    75.33333         30.96000          .             NON
28  UF1        LL      .              12.40000          .             NON
29  UF2        BS    90.00000          4.30000          .             NON
30  UG1        LL      .               8.68000          .             NON
31  UG2        BS    54.00000          3.01000          .             NON
32  UH1        LL      .              24.87000          .             NON
```

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ION 2 - COLUMNS                          PRIMAL-DUAL OUTPUT

BER  ..NAME..  AT   ..ACTIVITY..  .INPUT COST.  .LOWER LIMIT  .UPPER LIMIT
33   UH2       BS       85.33333       8.60000        .            NONE
34   UI1       BS       12.66667       9.30000        .            NONE
35   UI2       BS       90.00000       3.22500        .            NONE
36   UJ1       BS       30.00000      28.52000        .            NONE
37   UJ2       BS       90.00000       9.89000        .            NONE
38   UK1       LL           .         18.60000        .            NONE
39   UK2       BS       38.00000       6.45000        .            NONE
40   UL1       BS       26.00000      18.60000        .            NONE
41   UL2       BS           .          6.45000        .            NONE
42   UM1       BS       28.00000      18.60000        .            NONE
43   UM2       BS           .          6.45000        .            NONE
44   UN1       LL           .         55.80000        .            NONE
45   UN2       BS       90.00000      19.35000        .            NONE
46   UO1       BS       29.66667      46.50000        .            NONE
47   UO2       BS           .         16.12500        .            NONE
48   UP1       LL           .         60.76000        .            NONE
49   UP2       BS       83.66667      21.07000        .            NONE
50   V11       BS       29.00000           .          .            NONE
51   V12       BS       27.66667           .          .            NONE
52   V13       BS       15.33333           .          .            NONE
53   V21       BS       17.33333           .          .            NONE
54   V22       BS       11.66667           .          .            NONE
55   V23       BS       19.00000           .          .            NONE
56   V31       BS       16.00000           .          .            NONE
57   V32       BS       16.00000           .          .            NONE
58   V33       BS        8.00000           .          .            NONE
59   V41       BS       21.00000           .          .            NONE
60   V42       BS       10.00000           .          .            NONE
61   V43       BS       21.00000           .          .            NONE
62   V51       BS       10.33333           .          .            NONE
63   V52       BS       10.66667           .          .            NONE
64   V53       BS       14.00000           .          .            NONE
65   V61       BS       23.66667           .          .            NONE
66   V62       BS       12.33333           .          .            NONE
67   V63       BS       24.00000           .          .            NONE
68   V71       BS       10.00000           .          .            NONE
69   V72       BS       10.00000           .          .            NONE
70   V73       BS        4.00000           .          .            NONE
71   V81       BS        8.00000           .          .            NONE
72   V82       BS        7.00000           .          .            NONE
73   V83       BS        5.00000           .          .            NONE
74   V91       BS       19.00000           .          .            NONE
75   V92       BS       19.00000           .          .            NONE
76   V93       BS       10.00000           .          .            NONE
```

MODEL III

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ION 1 - ROWS                      PRIMAL-DUAL OUTPUT

ER  ..NAME..    AT  ..ACTIVITY..   SLACK ACTIVITY   .LOWER LIMIT  .UPPER LIMIT
 1  COST        FR  141234.00000        -.14123+06          NONE           NONE
 2  C1          EQ         .                .               .              .
 3  C2          EQ         .                .               .              .
 4  C3          EQ         .                .               .              .
 5  C4          EQ         .                .               .              .
 6  C5          EQ         .                .               .              .
 7  C6          EQ         .                .               .              .
 8  C7          EQ         .                .               .              .
 9  C8          EQ         .                .               .              .
10  C9          EQ         .                .               .              .
11  C10         EQ         .                .               .              .
12  C11         EQ         .                .               .              .
13  C12         EQ         .                .               .              .
14  C13         EQ         .                .               .              .
15  C14         EQ         .                .               .              .
16  C15         EQ         .                .               .              .
17  C16         EQ         .                .               .              .
18  D1          EQ      72.00000       72.00000        72.00000            .
19  D2          EQ      48.00000       48.00000        48.00000            .
20  D3          EQ      40.00000       40.00000        40.00000            .
21  D4          EQ      52.00000       52.00000        52.00000            .
22  D5          EQ      35.00000       35.00000        35.00000            .
23  D6          EQ      60.00000       60.00000        60.00000            .
24  D7          EQ      24.00000       24.00000        24.00000            .
25  D8          EQ      20.00000       20.00000        20.00000            .
26  D9          EQ      48.00000       48.00000        48.00000            .
27  R1          AU      29.00000           .                NONE       29.00000
28  R2          BS      21.50000        7.50000             NONE       29.00000
29  R3          BS      21.50000        7.50000             NONE       29.00000
30  R4          BS      17.00000        2.00000             NONE       19.00000
31  R5          BS      12.50000        6.50000             NONE       19.00000
32  R6          BS      18.50000         .50000             NONE       19.00000
33  R7          AU      16.00000           .                NONE       16.00000
34  R8          BS      10.00000        5.50000             NONE       16.00000
35  R9          BS      13.50000        2.50000             NONE       16.00000
36  R10         AU      21.00000           .                NONE       21.00000
37  R11         BS      11.00000       10.00000             NONE       21.00000
38  R12         BS      20.00000        1.00000             NONE       21.00000
39  R13         BS      10.00000        4.00000             NONE       14.00000
40  R14         BS      11.00000        3.00000             NONE       14.00000
41  R15         AU      14.00000           .                NONE       14.00000
42  R16         BS      23.50000         .50000             NONE       24.00000
43  R17         BS      12.50000       11.50000             NONE       24.00000
44  R18         AU      24.00000           .                NONE       24.00000
45  R19         AU      10.00000           .                NONE       10.00000
46  R20         AU      10.00000           .                NONE       10.00000
47  R21         BS       4.00000        6.00000             NONE       10.00000
```

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ION 1 - ROWS                        PRIMAL-DUAL OUTPUT

ER   ..NAME..   AT   ..ACTIVITY..  SLACK ACTIVITY  ..LOWER LIMIT .UPPER LIMI
48   R22        AU       8.00000         .                NONE        8.0000
49   R23        AU       8.00000         .                NONE        8.0000
50   R24        BS       4.00000      4.00000             NONE        8.0000
51   R25        AU      19.00000         .                NONE       19.0000
52   R26        AU      19.00000         .                NONE       19.0000
53   R27        BS      10.00000      9.00000             NONE       19.0000
54   P1         BS          .            .                NONE          .
55   P2         BS          .            .                NONE          .
56   P4         UL          .            .                NONE          .
57   P5         BS     -14.50000     14.50000             NONE          .
58   P7         BS     -18.50000     18.50000             NONE          .
59   P8         AU          .            .                NONE          .
60   P10        AU          .            .                NONE          .
61   P11        AU          .            .                NONE          .
62   P13        UL          .            .                NONE          .
63   P14        AU          .            .                NONE          .
64   P16        AU          .            .                NONE          .
65   P17        AU          .            .                NONE          .
66   P19        BS          .            .                NONE          .
67   P20        BS     -44.00000     44.00000             NONE          .
68   P22        UL          .            .                NONE          .
69   P23        BS      -5.00000      5.00000             NONE          .
70   P25        BS     -17.00000     17.00000             NONE          .
71   P26        AU          .            .                NONE          .
72   P28        AU          .            .                NONE          .
73   P29        AU          .            .                NONE          .
74   P31        UL          .            .                NONE          .
75   P32        BS     -52.00000     52.00000             NONE          .
76   P34        BS      -4.00000      4.00000             NONE          .
77   P35        AU          .            .                NONE          .
78   P37        AU          .            .                NONE          .
79   P38        AU          .            .                NONE          .
80   P40        UL          .            .                NONE          .
81   P41        AU          .            .                NONE          .
82   P43        AU          .            .                NONE          .
83   P44        AU          .            .                NONE          .
84   P46        UL          .            .                NONE          .
85   P47        BS      -6.50000      6.50000             NONE          .
```

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ION 2 - COLUMNS                           PRIMAL-DUAL OUTPUT

ER  ..NAME..  AT  ..ACTIVITY..  .INPUT COST.    .LOWER LIMIT .UPPER LIMIT
36  YA1       IT        .          1166.00000         .          1.00000
37  YA2       IT     1.00000       1914.00000         .          1.00000
38  YB1       IT        .          6784.00000         .          1.00000
39  YB2       IT     1.00000      11136.00000         .          1.00000
90  YC1       IT     1.00000       2756.00000         .          1.00000
91  YC2       IT     1.00000       4524.00000         .          1.00000
92  YD1       IT        .          7950.00000         .          1.00000
93  YD2       IT     1.00000      13050.00000         .          1.00000
94  YE1       IT        .         15264.00000         .          1.00000
95  YE2       IT     1.00000      25056.00000         .          1.00000
96  YF1       IT        .          2120.00000         .          1.00000
97  YF2       IT     1.00000       3480.00000         .          1.00000
98  YG1       IT        .          1484.00000         .          1.00000
99  YG2       IT     1.00000       2436.00000         .          1.00000
00  YH1       IT        .          4240.00000         .          1.00000
01  YH2       IT     1.00000       6960.00000         .          1.00000
02  YI1       IT     1.00000       1890.00000         .          1.00000
03  YI2       IT     1.00000       2610.00000         .          1.00000
04  YJ1       IT     1.00000       4876.00000         .          1.00000
05  YJ2       IT     1.00000       8004.00000         .          1.00000
06  YK1       IT        .          3180.00000         .          1.00000
07  YK2       IT     1.00000       5220.00000         .          1.00000
08  YL1       IT     1.00000       3180.00000         .          1.00000
09  YL2       IT        .          5220.00000         .          1.00000
0   YM1       IT     1.00000       3180.00000         .          1.00000
1   YM2       IT        .          5220.00000         .          1.00000
2   YN1       IT        .          9540.00000         .          1.00000
3   YN2       IT     1.00000      15660.00000         .          1.00000
4   YO1       IT     1.00000       8250.00000         .          1.00000
5   YO2       IT        .         13050.00000         .          1.00000
6   YP1       IT        .         10388.00000         .          1.00000
7   YP2       IT     1.00000      17052.00000         .          1.00000
8   UA1       AL        .              .              .             NONE
9   UA2       BS    90.00000           .              .             NONE
0   UB1       LL        .              .              .             NONE
1   UB2       BS    75.50000           .              .             NONE
2   UC1       BS    11.50000           .              .             NONE
3   UC2       BS    90.00000           .              .             NONE
4   UD1       BS        .              .              .             NONE
5   UD2       BS    90.00000           .              .             NONE
6   UE1       LL        .              .              .             NONE
7   UE2       BS    88.00000           .              .             NONE
8   UF1       BS        .              .              .             NONE
9   UF2       BS    90.00000           .              .             NONE
0   UG1       AL        .              .              .             NONE
1   UG2       BS    54.00000           .              .             NONE
2   UH1       LL        .              .              .             NONE
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CTION 2 - COLUMNS                   PRIMAL-DUAL OUTPUT

MBER  ..NAME..  AT   ..ACTIVITY..  .INPUT COST.  .LOWER LIMIT  .UPPER LIMI
 133  UH2       BS     85.00000         .             .              NON
 134  UI1       BS     13.00000         .             .              NON
 135  UI2       BS     90.00000         .             .              NON
 136  UJ1       BS     30.00000         .             .              NON
 137  UJ2       BS     90.00000         .             .              NON
 138  UK1       LL         .            .             .              NON
 139  UK2       BS     38.00000         .             .              NON
 140  UL1       BS     26.00000         .             .              NON
 141  UL2       BS         .            .             .              NON
 142  UM1       BS     28.00000         .             .              NON
 143  UM2       BS         .            .             .              NON
 144  UN1       LL         .            .             .              NON
 145  UN2       BS     90.00000         .             .              NON
 146  UO1       BS     30.00000         .             .              NON
 147  UO2       BS         .            .             .              NON
 148  UP1       LL         .            .             .              NON
 149  UP2       BS     83.50000         .             .              NON
 150  V11       BS     29.00000         .             .              NON
 151  V12       BS     21.50000         .             .              NON
 152  V13       BS     21.50000         .             .              NON
 153  V21       BS     17.00000         .             .              NON
 154  V22       BS     12.50000         .             .              NON
 155  V23       BS     18.50000         .             .              NON
 156  V31       BS     16.00000         .             .              NON
 157  V32       BS     10.50000         .             .              NON
 158  V33       BS     13.50000         .             .              NON
 159  V41       BS     21.00000         .             .              NON
 160  V42       BS     11.00000         .             .              NON
 161  V43       BS     20.00000         .             .              NON
 162  V51       BS     10.00000         .             .              NON
 163  V52       BS     11.00000         .             .              NON
 164  V53       BS     14.00000         .             .              NON
 165  V61       BS     23.50000         .             .              NON
 166  V62       BS     12.50000         .             .              NON
 167  V63       BS     24.00000         .             .              NON
 168  V71       BS     10.00000         .             .              NON
 169  V72       BS     10.00000         .             .              NON
 170  V73       BS      4.00000         .             .              NON
 171  V81       BS      8.00000         .             .              NON
 172  V82       BS      8.00000         .             .              NON
 173  V83       BS      4.00000         .             .              NON
 174  V91       BS     19.00000         .             .              NON
 175  V92       BS     19.00000         .             .              NON
 176  V93       BS     10.00000         .             .              NON
```